AN EXPLORATION OF PRESERVICE TEACHERS' USE OF EDUCATIONAL TECHNOLOGIES AS VISUALIZATION TOOLS WHEN TEACHING MATHEMATICS

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Abstract

This interpretive qualitative study explores the use of educational technologies by preservice teachers as visualization tools during mathematics teaching at secondary schools. Sfard's commognitive framework and Koehler and Mishra's technological pedagogical content knowledge theoretical frameworks undergird the study. Data were collected from ten preservice mathematics teachers at a university in the province of KwaZulu-Natal, South Africa. Performance tests, semi-structured interviews, focus group discussions, and observations were employed to collect data, which was analyzed using reflexive thematic analysis. The study found that preservice teachers employed two primary visualization strategies when they engaged in mathematical problem-solving: symbolic mental visualization, which they combined with their understanding of word usage, endorsed narratives and routines to arrive at a solution; and graphic visual mediators, such as diagrams, which they sketched to contextualize the problem statement and verify they solutions and use of mathematical word usage, routines, and endorsed narratives. Participants were found to be unable to solve a mathematics problem if they had not visualized it effectively; using a graphic visual mediator to understand the problem statement did not, however, guarantee success when solving a problem. A relationship was found between the visualization techniques that the participants used in their own attempts to solve mathematical problems and the visualization techniques they used in their lesson planning and teaching of mathematics in the same content area. Participants used innovative strategies to mediate learning, including educational technologies that facilitated visual mediators to enhance learners' engagement with concepts. Synergies were found between the elements of the commognition and TPACK frameworks as these were used in tandem to analyze data. A model was developed (C+TPACK) that integrates the key elements of these frameworks. Further research is recommended to establish the viability, credibility and generalizability of the model.

Keywords: Commognition; C+TPACK; Educational technology; Mathematical problem-solving; Preservice mathematics teachers; TPACK; Visualization.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

C+TPACK	Commognitive Technological Pedagogical and Content Knowledge
CK	Content Knowledge
DBE	Department of Basic Education
DGS	Dynamic Geometry Software
DHET	Department of Higher Education and Training
DoE	Department of Education
FET	Further Education and Training
HEI	Higher Education Institution
HSSREC	Humanities and Social Sciences Research Ethics Committee
ICT	Information and Communication Technology
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
RTA	Reflexive Thematic Analysis
SITES	Second Information Technology in Education Study
TCK	Technological Content Knowledge
TK	Technological Knowledge
TPCK/TPACK	Technological Pedagogical Content Knowledge
TPK	Technological Pedagogical Knowledge

Statement of Original Authorship

The research presented in this thesis has never before been submitted to fulfil requirements for a degree at this university or any other. The thesis does not contain any information that has already been published or written by someone else, to the best of my knowledge and belief, unless appropriate citation is made.

Signature: [redacted]

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Chapter 1: Introduction

1.1 INTRODUCTION

Mathematics is an abstract scientific language that is learned by individuals through physical engagement with its concepts, social interaction with knowledgeable others, and through logical reasoning that is characterized by the ability to engage in worthwhile problem-solving activities. Thus, the value of educational technologies in mathematics teaching and learning is paramount as it affords both teachers and learners the opportunity to physically engage with its concepts and socialize in an enhanced manner. This implies that those who facilitate the learning of this subject ought to be well equipped, creative, and dynamic in their approach when they employ educational technologies to harness the acquisition of this knowledge by those learning it.

The utilization of different educational technologies for the facilitation of mathematics learning at different levels has been studied over the past decades to establish their effectiveness during instruction. Several studies have focused on the relationship between the visualization of mathematical concepts and the use of information and communication technology (ICT). However, there is a lack of research to determine which educational technologies are employed by preservice teachers as visualization tools during mathematics teaching, especially within the South African context. This study aims to explore the use of such technologies by preservice teachers at secondary schools to facilitate the learning of mathematics, with particular focus on enhancing learners' abilities to visualize mathematical concepts.

This chapter outlines the background, context, and purpose of the study. Furthermore, the significance and scope of this research are discussed and definitions of the terms used in the study are provided. The delimitations of the study are presented. Finally, the chapter concludes with an outline of the thesis.

1.2 BACKGROUND AND CONTEXT OF THE STUDY

As a secondary school mathematics teacher in a township neighborhood, I witnessed low performance in mathematics and a shortage of mathematics teachers. These circumstances motivated me to be proactive, innovative, and creative within the teaching and learning environment. The township school to which I was assigned as a novice mathematics teacher had two classrooms that were fully equipped with smartboard technologies, soundbars, projectors and laptops. Tablets with educational software were provided to learners. A private company had donated these advanced educational technologies intending to improve learner achievement in science and mathematics in the schools in the Umlazi District. However, there were not enough devices to accommodate all of the learners. This was an external contextual challenge that is common at public township schools.

Even though I had little to no training in using these technologies, I started to incorporate them into my mathematics teaching, albeit ineffectively. While a qualified information technology (IT) expert was later invited to provide instruction on how to use the technologies that had been donated, the training focused on how to use the hardware and software components and did not cover how to incorporate technological resources into the teaching of specific mathematics lessons, since the trainer possessed minimal knowledge of mathematical content and did not have the requisite knowledge for teaching.

Nonetheless, as I began to use these technologies more frequently, I became more adept at incorporating them into my mathematics instruction. I discovered that the use of these technologies supported different types of learners in the classroom during instruction. For instance, auditory, kinaesthetic, and visual learners' needs were equally supported by the technologies that I used in my lessons.

When I was hired by a higher education institution (HEI) as a teacher educator, I continued to use more advanced educational technologies in my teaching and I encouraged the preservice mathematics teachers that I lectured to do the same. Through these experiences, I witnessed the benefits that various educational technologies could bring to the teaching and learning of mathematics at both school and university levels.

Thus, as I began to engage with the South African literature on the use of educational technologies during mathematics instruction, I could relate to the findings. For instance, Saal, Graham, and Van Ryneveld (2020) established that mathematics teachers have utilized various educational technologies as a tool to develop and reinforce learners' higher order thinking skills. They found that some teachers adopted technologies (such as YouTube videos) as methods of teaching to enhance learning – for instance, when they wanted to expose learners to different ways to understand certain mathematics concepts that they found difficult to comprehend. Other affordances included enhanced communication and sharing of resources (via Google Classroom) with learners and teachers from different schools (Saal et al., 2020). Online assessment tools were also identified by mathematics teachers as being beneficial; these teachers were of the opinion that frequent assessment with immediate feedback is essential for learners to demonstrate improvement in their mathematics performance. Other research studies (Joshi, 2017; Netsianda & Ramaila, 2021) demonstrated that mathematics teachers employed particular digital educational technologies because they had observed that these motivated learning and promoted mathematical understanding. Other studies (Alata, 2017; Das, 2019; Eleftheriadi, Lavidas, & Komis, 2021) revealed that the affordances of educational technologies, as reported by mathematics teachers, included self-directed learning, promoting a creative environment for learning, and promoting virtual collaborative learning. These studies demonstrated that in-service mathematics teachers intentionally sought to enhance their teaching by implementing the various educational technologies they had access to at their respective schools.

In my quest to implement educational technologies in the teaching of mathematics, I encountered several challenges that hindered consistent and effective integration. One challenge was my lack of training when I first attempted to use some advanced educational technologies. However, with experience and practice, I was able to overcome this challenge. Another contextual challenge at the national level was the unexpected electric power cuts during school hours which rendered the use of digital educational technologies impossible.

When I engaged with mathematics teachers from other schools, I established that these challenges were not unique to my experience. The local literature verified this, too. Recent local studies (Aruleba & Jere, 2022; Munje & Jita, 2020) reported that South African teachers encounter numerous challenges in their attempt to integrate various educational technologies, including software licensing issues, maintenance plans, and other technical obstacles (Masango, Van Ryneveld, & Graham, 2022). Nevertheless, these constraints have not prevented in-service teachers from attempting to be innovative and creative in their teaching of mathematics by implementing educational technologies to the extent they are able (Mwapwele, Marais, Dlamini, & Van Biljon, 2019).

Most of the teacher training institutions in South Africa provide some exposure to preservice teachers to the range educational technologies for teaching and learning (Umugiraneza, Bansilal, & North, 2018). Anecdotal evidence demonstrates that their curricula are designed to equip preservice teachers with technological skills despite the socio-economic inequalities that perpetuate the digital divide in South African schools (Faloye & Ajayi, 2021). Their aim is to prepare preservice teachers for any school environment they might encounter. Also, with the increase in demand for the integration of technology in teaching and learning, private stakeholders have assisted the government by donating advanced educational technologies to selected schools (Isaacs 2007; Mukhari, 2016). Private stakeholders donate these advanced technologies to be utilized for the teaching and learning of mathematics and science subjects, predominantly, as these subjects are given high priority by the departments of basic and higher education in South Africa (Mapaire, 2016; Letsoalo, Masha, & Maoto, 2019). Several studies (Draper, 2010; Dzansi & Amedzo, 2014; Burns et al., 2019) have established that numerous township and rural schools in South Africa have been sponsored to receive various educational technologies to facilitate the teaching and learning of these prioritized school subjects at different grades. This suggests that, despite the digital divide that exists within the basic education sector in South Africa, teachers need to be trained to be able to integrate various educational technologies into teaching and learning, should they be given the opportunity.

Preservice teachers thus may find themselves placed at schools with varying levels of access to teaching technologies, depending on the backgrounds of the schools and whether they have been the beneficiaries of initiatives to equip schools with technologies. Despite this, there has been little empirical research in the South African context to investigate preservice teachers' teaching experiences with educational technologies. Moreover, empirical research on the utilization of educational technologies by preservice teachers for a predetermined purpose within the area of mathematics teaching in South Africa is minimal.

1.3 PROBLEM STATEMENT

Several scholars (Alhumaid, 2019; Brasiel et al., 2016; Costley, 2014; Mata, Monteiro, & Peixoto, 2012) have argued that the use of educational technologies in mathematics education may impact teaching and learning negatively or positively, depending on how they are utilized. One of the prominent benefits that educational technologies can offer in mathematics education is that they can enhance the visualization of mathematical concepts and problems (Kissane, McConney, & Ho, 2015). In mathematics education, educational technologies have been used as an important tool to facilitate an understanding of mathematical concepts by both teachers and learners. Studies exploring the impact of educational technologies have found a relationship between the use of educational technologies in mathematics and enhanced visualization of mathematics concepts (Bhagat & Chang, 2015; Kissane, McConney, & Ho, 2015).

Mathematics education has long used visualization to enhance teaching and learning, even before the development of modern educational technologies. According to Kadunz and Yerushalmy (2015), since the early 1980s, mathematics teacher educators and researchers alike have been interested in the practical difficulties and successes associated with teaching and learning visualization techniques for mathematics problem-solving. For instance, Budram's (2020) study, conducted in South Africa, demonstrated that preservice teachers struggled to create effective visual representations during problem-solving. This study argued that it is crucial to emphasize the organization of visual representation in teacher education. A Turkish study by Özsoy (2018) discovered that some preservice teachers had a limited understanding of how to use mathematical and visualization techniques to solve problems. Another Turkish study, by Horzum and Ünlü (2017), revealed that preservice mathematics teachers believed that the use of GeoGebra could assist in improving their visualization abilities and that of their learners when engaging with mathematical problems. In this study, participants were trained to use this single educational programme but were not observed implementing it in their teaching.

Studies conducted in developing countries in other parts of Africa have explored the general usage of educational technologies and investigated the level of skills possessed by preservice mathematics teachers regarding the integration of digital technologies (Kafyulilo, Fisser, Pieters, & Voogt, 2015; Gonscherowski & Rott, 2022; Njiku, Mutarutinya, & Maniraho, 2022). Some studies have focused on preservice teachers' self-efficacy in technology integration and the factors that hinder them from being able to integrate these technologies into their teaching (Batane & Ngwako, 2017; Olugbara & Letseka, 2020).

The literature thus demonstrates that understanding educational technologies as visualization tools in mathematics teaching is crucial for preservice teachers. However, negligible conclusions have been drawn from the available research about preservice teachers' understanding of the use of educational technologies as visualization tools in mathematics instruction. Also, minimal research exists regarding how visualization strategies employed by preservice teachers during mathematical problem solving influence their decisions to employ educational technologies as a means of enhancing visualization for learners. Thus, the existing literature does not provide an adequate understanding of preservice mathematics teachers' utilization of different educational technologies, especially in the current context of rapid and technological advancement. This gap in the literature is particularly profound with regard to the use of educational technologies by preservice mathematics teachers in developing countries, such as those in Africa.

1.4 PURPOSE OF THE STUDY

The purpose of this study was to explore preservice teachers' utilization of educational technologies as visualization tools when teaching mathematics. To engage preservice mathematics teachers in a discourse about their use of educational technologies as visualization tools, the researcher needed first to understand the visualization strategies that they used to solve mathematical problems themselves. Thereafter, the researcher needed to determine whether these visualization strategies that preservice mathematics teachers use in their own problem-solving influence their use of educational technologies to aid their learners' visualization during teaching and learning. Consequently, this was done to achieve the overarching aim of the study which was to explore and understand how preservice mathematics teachers' visualization strategies influence their utilization of educational technologies as visualization tools during practice teaching.

To achieve the purpose and aim of this study, the following objectives were defined:

1. Identify the visualization strategies utilized by preservice mathematics teachers to solve mathematics problems.
2. Establish the educational technologies that mathematics preservice teachers intend to utilize in their teaching.
3. Explore why mathematics preservice teachers believe that educational technologies can influence their teaching and concept development.
4. Establish how mathematics preservice teachers demonstrate the implementation of these educational technologies in their teaching during teaching practice.

1.5 RESEARCH QUESTIONS

Cohen, Manion, and Morrison (2018) remark that research questions for all types of research emanate from the purposes, aims, and objectives of the research. In qualitative research, however, research questions are typically more exploratory and open than in quantitative research (Cohen et al., 2018). To facilitate a comprehensive investigation into the phenomenon under study in this qualitative research, the general guiding questions "what", "why", and "how" (Cohen et al., 2018) were used to frame the specific research questions. To achieve the purpose, aim and objectives of this study, the following research questions were used to guide the study.

Main research question

Why and how do preservice mathematics teachers' visualization strategies influence their use of educational technologies as visualization tools during teaching?

Supporting research questions

1. What visualization strategies are used by preservice mathematics teachers to solve mathematics problems?
2. What educational technologies do mathematics preservice teachers intend to use in their teaching?
3. Why do mathematics preservice teachers believe that their chosen educational technologies can influence their teaching and concept development?
4. How do mathematics preservice teachers demonstrate the implementation of these educational technologies in their teaching during teaching practice?

Two theoretical frameworks – Sfard's (2008) commognition framework and Mishra's and Koehler's (2006) technological pedagogical content knowledge framework (TPCK/TPACK) – were employed to enable a deep understanding of the phenomenon under study.

1.6 SIGNIFICANCE OF THE STUDY

In recent years, both basic and higher education sectors in South Africa have recognised the value of using more advanced educational technologies to enhance teaching and learning. Several White Papers, policy documents, and technology integration frameworks have been published that promote the provision of educational technologies to both sectors (Wilson-Strydom, Thomson, & Hodgkinson-Williams, 2005; Isaacs, 2007; Mdlongwa, 2012; Department of Telecommunications and Postal Services, 2016) . While numerous challenges to the integration of technology by teachers that have been reported, several studies (Meyer & Gent, 2016; Jita, 2016; Jita, 2018) have demonstrated that training in the integration of educational technologies into teaching and learning does occur through both in-service and preservice training, although the extent and depth of the training varies from school to school and institution to institution.

In this context, this empirical investigation into the knowledge, skills, and abilities of senior preservice teachers regarding the usage and integration of various educational technologies as visualization tools in the teaching of mathematics at secondary schools makes a valuable contribute to the literature. It establishes whether the visualization strategies used by preservice teachers during their own mathematical problem solving influences their choice and use of technologies for teaching, and how these tools impact the mathematical concept development of their learners.

This study also addresses a gap in the literature by revealing the experiences of preservice teachers with regard to their implementation of educational technologies in the context of the South African basic education environment. The insights this study offers into the challenges and successes experienced by preservice teachers in their implementation of the knowledge and skills they acquired during their training represent a valuable contribution to institutions that train mathematics teachers, particularly in developing countries, providing the basis for them to evaluate their curriculum and improve their training approach.

The results of this empirical study are expected to inform the basic education sector about preservice teachers' knowledge of how to integrate educational technologies into the teaching of mathematics and their expectations of the school environment that would permit them to improve their teaching through visualization tools. This knowledge will assist in making both basic and higher education sectors aware of the systematic challenges that might exist regarding the availability of educational technologies, particularly given the digital divide between the two sectors.

This study also is expected to broaden the knowledge and understanding of those teacher educators (my peers) who intentionally model the integration of a variety of educational technologies to preservice mathematics teachers while actively equipping them with the necessary skills to employ these technologies.

On a more personal level, this study has developed my professional understanding and will also impact my future practice. From this study, I realized the importance of ensuring that preservice teachers are able to think in imaginative and dynamic ways in their approach to mathematics teaching using various educational technologies to maximize the knowledge acquisition of their learners in any teaching and learning context. Equipping and preparing preservice mathematics teachers to be more resilient and innovative in their approach to enhanced teaching in the South African context where they will find themselves placed in schools that range from the extreme of poorly resourced schools to well-resourced schools, is something that I am going to be conscious about in my practice.

1.7 STUDY DELIMITATION

This study was conducted at a university in the province of KwaZulu-Natal (KZN), South Africa. The university has six colleges with nineteen schools offering qualifications from undergraduate to post-graduate levels. As the focus of this study was on preservice mathematics teachers, the study was confined to the discipline of mathematics education in the School of Education in the College of Humanities. The third-year preservice mathematics teachers who constituted the population of this study had supervised teaching experience and had completed more than two mathematics pedagogical content knowledge modules.

1.8 DEFINITIONS OF TERMS

This section defines the conceptual terms used throughout this thesis. Operational terms are discussed in the methodology, theoretical framework, and literature review.

Preservice teachers: (also known as student teachers) refers to students enrolled in a higher education institution who are pursuing a teaching certificate in order to work as professional school teachers.

Teaching practicum: is a school-based experience that is regarded as a crucial part of the teacher education program since it offers student teachers supervised experience and assists them to comprehend the general expectations of a fully-qualified professional teacher (Tuli & File, 2009).

Public school: refers to a school that was founded by the government and whose operations are governed by the school management team on behalf of the government.

Private school: is an independent non-governmental school whose establishment and administration are governed by a separate board of directors.

Teacher educators: In this study, teacher educators are practitioners who offer instruction, support, and guidance to preservice teachers within higher education institutions (Koster, Brekelmans, Korthagen, & Wubbels, 2005) thereby making a significant contribution to the transformation of preservice teachers into competent, fully-fledged professionals.

In-service teachers: refers to certified professional teachers who are classroom practitioners within a school.

Training: refers to a process of learning the skills necessary to perform a specific task or activity. In this study, the training that is referred to is that of teachers for teaching purposes.

Basic education: refers to primary and secondary school education.

Higher education: refers to post-secondary school education.

Module: one of the discrete components of a college- or university-taught course. Each module is assessed separately from other modules and covers only one subject. A student must obtain passes in a minimum number of modules in order to receive a qualification for a particular course.

Technology integration: The integration of technology has been interpreted and defined differently by different individuals depending on the context. From the perspective of teaching and learning, this study defines technology integration as the usage of any form of technology – either digital or non-digital – including computers, projectors, smartboards, tablets, speakers, manipulatives, charts, whiteboards, chalkboards, chalk, and other similar tools incorporated for the facilitation of the learning process.

1.9 OUTLINE OF THE THESIS

This study has six chapters. The following provides a brief synopsis of each chapter.

Chapter One: Introduction

The study's context has been described in Chapter One. The statement of the problem and the importance of the study have been discussed. The objectives and core research questions were also identified. Finally, the study's delimitations were discussed.

Chapter Two: Literature Review

Literature was reviewed on the following topics relevant to the study: the purpose and impact of educational technologies in mathematics teaching and learning; visualization and mathematical thinking in mathematics; preservice mathematics teachers' knowledge and skills of the utilization of educational technologies; and the status of the availability and use of educational technologies in South African public schools.

Chapter Three: Theoretical framework

This chapter presents the rationale for employing the two theoretical frameworks selected to deepen the analysis of data in this study – Sfard's (2008) commognition framework and Mishra's and Koehler's (2006) technological pedagogical content knowledge (TPACK) framework – and presents the key elements of these frameworks.

Chapter Four: Research design

The research paradigm, approach, methods, design, data generating techniques, and data analysis techniques pertinent to this study are described in this chapter.

Chapter Five: Data presentation and analysis

This chapter presents and analyzes the findings of the study in accordance with relevant themes that address the primary research questions of the study.

Chapter Six: Discussion of results

This chapter discusses and interprets the findings presented in Chapter Five. It also presents and elucidates the proposed model that was developed organically during the course of employing the commognitive and TPACK theoretical frameworks.

Chapter Seven: Concluding remarks

This chapter summarises the major findings of the study in relation to the objectives and main research questions. The chapter further discusses the study's contribution, the implications of the study's findings, considers the study's limitations, and makes recommendations for future research.

Chapter 2: Literature Review

2.1 INTRODUCTION

The previous chapter delineated the background, aim and purpose of the study and outlined the structure of the thesis. This chapter commences with a historical background of the uses of educational technologies in mathematics education. Thereafter, the chapter reviews the relevant and recent literature on the purpose and impact of using educational technologies in the teaching and learning of mathematics. Studies exploring the expertise of preservice mathematics teachers with regard to the use of educational technologies are reviewed and the availability of educational technologies in South African schools and higher education institutions is discussed. The last section summarises the reviewed literature and identifies the gap that this study addresses.

2.2 THE HISTORICAL USE OF TECHNOLOGIES IN MATHEMATICS EDUCATION

"[M]an has been using technology in mathematics for thousands of years, starting with own fingers and stones for counters." (Alacaci & McDonald, 2012, p. 21).

Over the past decades, the rapid development of technology has begun to influence education across the globe increasingly (Oke & Fernandes, 2020). Raja and Nagasubramani (2018) assert that technology is now prevalent in every facet of the education sector.

In the context of mathematics education, Roberts (2014) notes that educational technologies include not only modern digital technologies but every tool that is utilized to aid the teaching and learning of a subject. From this perspective, the educational technologies used in mathematics education may include mathematical instruments, the geoboard, abacus, rod, counting board, graph paper, handheld calculator, graphing calculator, and mathematical computer software (such as GeoGebra, Geometer's Sketchpad, Microsoft Mathematics, and Math Editor), among others (Freiman & Volkov, 2018). For this study, educational technologies are understood using this broad definition.

Roberts (2014) notes that the educational technologies used in mathematics can be divided into non-specialized technologies (general-purpose teaching and learning tools such as chalkboards, textbooks, paper, and pencils) and specialized technologies (technologies specifically used for the teaching and learning of mathematics). The use of specialized educational technologies in the field of mathematics dates back to the 12th century, with evidence of their use by Asians, Americans, Russians, Greeks, and Europeans (Roberts, 2014). According to Roberts (2014), the use of more specialized educational technologies, included calculating tools, tools for drawing, and tools for physical manipulation, was found across countries like Greece, China, Egypt, Russia, and the United States before the 20^{th} century.

While today specialized educational technologies are used across the world for the teaching and learning of mathematics, with the advent of digital devices the world has witnessed a transformation in terms of the educational technologies available to facilitate and enhance the teaching and learning of mathematics. This has resulted in a global debate about the value of these technologies and how they impact mathematics education. For instance, in the early years of the electronic calculator's introduction into mathematics, Rogers (1976) contended that "the use of electronic calculator is an example of modern mathematics being transmitted to the general culture, but this is only a technological improvement which increases the power of the individual in mathematics already transmitted" (p. 8). He further argued that such technologies may not necessarily improve the understanding or appreciation of mathematical concepts.

In 2000, Williams, Coles, Wilson, Richardson, and Tuson (2000) reported negative attitudes on the part of mathematics and science teachers towards the use of non-specialized digital technologies. Drijvers (2003) however, found that the use of handheld computer algebra devices played a significant role in developing students' conceptual understanding of algebra, although this depended on the educational setting. The debate over the impact and value of educational technologies in mathematics education is ongoing.

2.3 THE PURPOSE AND IMPACT OF INTEGRATING EDUCATIONAL TECHNOLOGIES IN MATHEMATICS TEACHING

Educational technologies are used in the teaching of mathematics for several reasons. For instance, SRI International (2007) states that "many societies […] introduce arithmetic with an abacus, for two reasons. First, the abacus supports computation. Second, the abacus presents a tangible image of mathematics, which helps students understand difficult concepts" (p. 1). Thus, in as much as arithmetic can be done with non-specialized educational technologies (pen, paper, chalkboard and chalk), with the benefit of a specialized educational technology (the abacus) arithmetic computations can be better taught and learned. Also, the use of digital calculators for enhanced, faster arithmetic computation increases as the level of arithmetic problems gets more complex (Miles, 2008).

Many of the educational technologies that have been developed to aid mathematics teaching and learning are designed to aid learners' visualization of mathematical concepts that are not easy to comprehend (Presmeg, 2006; Kadunz & Yerushalmy, 2015; Azizah, Kusmayadi, & Fitriana, 2021). This particular purpose and use of educational technologies is a key focus of this study.

Cha, Park, and Seo (2020) argue that teachers use specific educational technologies to enhance the teaching of particular concepts that are difficult for students to grasp. For example, a South African study found that students grasped the concepts of Euclidean geometry, which is generally considered difficult, more easily when they used the interactive mathematics app GeoGebra to help them visualize concepts (Hlalele, 2020). Mthethwa, Bayaga, Bossé, and Williams (2020) also found that students used GeoGebra as a learning tool to be able to solve geometry problems and provide correct justifications for their statements. In response to the challenges learners experienced, lessons were strategically designed for the GeoGebra application to be integrated into teaching and learning with the aim of facilitating learners' understanding of geometry. Bayaga, Mthethwa, Bossé, and Williams (2019) found that 90% of the students in their study demonstrated improved visualization abilities after completing geometry lessons through the GeoGebra app. They also found that when GeoGebra was used to improve the teaching of geometry, students' mental schemata were maximized.

Bagni (1998) established that the study of functions is strongly grounded in visual techniques because of its close association with graphical representation and their interpretation. In this regard, Alexander (1994) claims that the visualization aspects of educational technologies like graphing calculators allow students to fit functions to pictures and relate them to real-world situations, thereby enhancing their modelling and visualization skills. The use of educational technology in the teaching and learning of functions and graphs enables students to formulate concepts on their own while benefiting from visualization in the process (Tall, 1991). Several studies (Doorman, Drijvers, Gravemeijer, Boon & Reed, 2012; Bedada & Machaba, 2022) have established these educational technologies can aid students to develop deeper insight into the relationship between functions and their graphs. Elliott (1998) cautions, however, that excessive use of these technologies can pose a risk as students may become dependent on graphical representations and overloaded with images while ignoring other alternatives for representing functions. For this reason, OECD (2016) suggests that such technologies only be utilized to support the traditional methods of teaching and learning. This raises again the possibility that educational technologies can have both positive and negative impacts on teaching and learning mathematics, irrespective of the purpose. These findings highlight the significance of educational technologies in the teaching of mathematics to complement the traditional symbolic and analytical approaches to the learning of functions.

In another study, it was established that the use of educational technologies was adopted to make the teaching and learning of fractions easier for both teachers and learners (Nicolete, Bilessimo, Cristiano, Simão, & da Silva, 2017). The study further revealed that the type of educational technology that was used during the study enabled both teachers and students to visualize different components of fractions, thus forming concrete base knowledge of fractions. Without the specialized educational technology that was used during the study, the results would potentially demonstrate that students lacked conceptual understanding, demonstrated by poor computation of problems involving fractions. Several studies globally (Gabriel et al., 2013; Aksoy & Yazlik, 2017; Coetzee & Mammen, 2017; Kusuma & Retnawati, 2019) have indicated that students experience difficulties when dealing with fraction computations without the aid of educational technology like a handheld calculator. It is thus, apparent that the purpose of technology integration in mathematics teaching and learning is to enhance how the content is delivered and received by those engaging with it. Several other studies (Davies & West, 2014; Kimmons, 2018; Umugiraneza, Bansilal, & North, 2018; Viberg, Grönlund, & Andersson, 2020) revealed that the fundamental purpose of technology utilization in mathematics education was to offer more opportunities for improved mathematics teaching approaches and to enhance understanding of content.

Baralis, Malafekas, Rappos, and Vlamos (2000) note that educational mathematics-based software can be beneficial when some students do not have a high level of understanding of the language used as the medium of instruction, because they "constitute an extremely fruitful educational object in multilingual classes since it helps the students to understand geometrical theorems by visualization" (p. 105). Furthermore, they contend that the use of certain educational technologies (like computer software) to assist in the teaching of geometry aids to eradicate unequal access to learning among students due to language. These findings advance one of the prominent reasons why educational technologies may be integrated into mathematics teaching and learning. Hence, it can be justified that the use of any form of educational technology is adopted with the same notion and purpose of enhancing the learning and teaching of mathematics.

2.3.1 Visualization in mathematics

The term ‘visualization’ has been utilized in different ways in the literature over the past decades (Strecker, 2012); its use in this study thus needs to be clarified.

According to Presmeg (1997), in the context of mathematics, the concept of visualization involves processes of creating and transforming visual mental images as well as the associated writings of spatial nature. In addition, Zimmermann and Cunningham (1991) define visualization as

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings (p. 217).

For this review, spatial thinking includes visual processing and interpreting figural information, as described by Bishop (1980). This concept relates harmoniously with the commognitive theoretical framework underpinning this study that will be discussed in the subsequent chapter. This study accepts Espinosa’s (1997) perspective that visualization of mathematical concepts is not the same as seeing with the naked eye, but refers to the ability to create rich mental images that can be manipulated by the brain by rehearsing different illustrations of the concept, which could then be represented visibly on paper or screen. Arcavi (2003) attests to the importance of mental images (otherwise known as visual images) in human cognitive activities. Presmeg (1985) locates visual imagery in the brain, where they form a mental scheme representing spatial or visual information. These views are consistent with the context and theories undergirding this study, which are discussed in subsequent chapters.

Kissane, McConney and Ho (2015) emphasize that visualization is one of the prominent benefits that is achieved consciously – and, occasionally, unconsciously – by teachers. As such, it can be justified that most mathematicians adopt educational technologies to facilitate a better understanding of mathematical problems and, most importantly, to visualize mathematical concepts. From the studies that have been conducted focusing on the impact and purpose of educational technologies, there has been a noticeable relationship between the use of educational technologies in mathematics and visualization (Bhagat & Chang, 2015; Kissane, McConney, & Ho, 2015).

Irawan, Mukhlash, Adzkiya, and Sanusi (2019) explain that visualization is important in the learning of mathematics as it provides the mental image of a concept which is necessary for high-level learning of the subject. Nevertheless, Presmeg (1986) cautions that it is crucial to recognize that concrete visual imagery can be a source of difficulty for students when dealing with certain concepts in mathematics. Thus, Haciomeroglu, Aspinwall, and Presmeg (2010) advocate for dynamic visual imagery as it can be more effective – for example, in calculus when dealing with slope changes of tangent lines or transforming the graphs of the derivative.

It is crucial to note that individuals use visual methods by choice (Presmeg, 2006) as there are other methods to comprehend mathematical concepts. Those who prefer to use concrete and dynamic visual images (mental constructs representing spatial information) are referred to as 'visualizers' by Presmeg (2006); while 'non-visualizers' are those who opt for other methods of representing mathematics. It is also possible to use a mixture of methods depending on individuals' preferences and the nature of mathematical concepts being studied. As such, having multiple ways of representing concepts in mathematics is beneficial (Berry & Nyman, 2003).

The literature suggests that visualization is not a new concept in the teaching and learning of mathematics. Kadunz and Yerushalmy (2015) assert that since the early 1980s mathematics educators and researchers have been interested in the practical challenges of teaching visualization. Thus, scholars like Cunningham and others have published numerous manuscripts outlining strategies for using visualization in mathematics teaching. The potential for using technology to encourage and promote skills of visualization has since been demonstrated by many researchers in the discipline of mathematics (Smart, 1995; Souza & Borba, 1995). In addition, the benefits of the utilization of visualization technology in natural and mathematical science have been confirmed by the analysis of the methodological and scientific sources (Semenikhina & Drushlyak, 2015; Udovychenko, Shamonya, & Yurchenko, 2015).

With the advent of modern digital educational technologies which afford the grasp of an idea at a single glimpse, visualization has become increasingly significant in mathematics education (Budaloo, 2015). Shatri and Buza (2017) attest that educational technologies offer new approaches to teaching and learning, one of them being a visualized approach where students can formulate concepts on their own. Moreover, Irawan et al. (2019) argue that students are motivated to learn mathematics when visualization technologies are incorporated during instruction.

It is therefore imperative that teachers teach in a manner that allows students to develop their ability to visualize to advance their skills for dealing with mathematical concepts and inculcating conceptual understanding. This means educators need to be adroit in using modern educational technologies to engage students' abilities to visualize mathematical concepts (Johari, Azli, & Idrus, 2018). Semenikhina and Yurchenko (2016) assert that "given the exponential increase in information content, there is a need to look into the levels and components of the professional readiness of teachers to use computer visualization tools" (p. 174). Consequently, preservice teachers' knowledge of educational technologies as visualization tools for mathematics teaching is essential.

Shatri and Buza (2017, p. 71) also raise the importance of visual learning in the teaching of 21st-century skills, including the development of critical thinking. They argue that critical thinking is vital, and any method utilized to encourage and develop it is rendered useful. The following section discusses the development of mathematical thinking, as this emphasizes the concept of critical thinking that is manifested in visualization.

2.3.2 The development of mathematical thinking

According to Samo and Kartasasmita (2017), mathematics can be regarded as a science that emphasizes the development of particular thinking abilities. Mathematical thinking is considered a complex activity that has received increasing scholarly attention globally over the past decades (Stacey, 2007). Uyangör (2019) defines mathematical thinking as the use of mathematical concepts, techniques, and methods, directly or indirectly, in the process of solving problems. Alkan and Bukova-Güze (2005) assert that the process of mathematical thinking is distinctly different from other ways of thinking. Its characteristics involve gaining new knowledge through specializing, generalizing, conjecturing, testing and reasoning, as well as proving (Yildiz, 2016,). It can be contended that engaging deeply with mathematical thinking means requires the thinker to be aware of their thinking while solving the problems, thus thinking about their thinking in the process.

The idea of mathematical thinking encountered during the process of mathematics instruction, according to Schoenfeld (1992), means to look at the world from a mathematical viewpoint. Stacey (2007) argues that the ability to utilize mathematical thinking during problem-solving is one of the most important goals of mathematics teaching, but is also one of the most elusive goals. This means that both the teacher training institution and school curricula must provide an explicit directive in terms of how mathematical problem-solving can be successfully incorporated into the classroom. At teacher training institutions, mathematics education courses engage preservice teachers in mathematical thinking through authentic problem solving and modelling (Jacobs & Durandt, 2017). Villa-Ochoa, Sánchez-Cardona, and Rendón-Mesa (2021) found that mathematics preservice teachers demonstrated improvement in their teaching of modelling to enhance learners' mathematical thinking skills.

Mathematics problems that stimulate mathematical thinking are generally not part of the core curriculum for most schools around the world, however, and tend to be more commonly used in the context of mathematics competitions (Wijers & de Haan, 2020). For instance, the South African Curriculum Assessment Policy Statement (CAPS) specifies for only 15% of high school mathematics exams to be comprised of problem-solving questions (Department of Basic Education, 2011). However, the curriculum is not clear about how mathematical problem-solving ought to be incorporated during teaching and learning. This places a burden on mathematics teachers to establish how to expose their students to problem-solving questions that will stimulate mathematical thinking.

Schoenfeld (1989) indicates that most school-based textbook activities assigned to learners as homework and classwork 'problems' are not actually problems, according to the definition of problem-solving put forward by Polya (1980). Schoenfeld (1989) states that most textbook practice activities can be solved through the direct application of the method shown in the chapter. Chirinda and Barmby (2018) postulate that the exercises that actually qualify as problems in the prescribed textbooks in South Africa are limited to "a few non-routine problems at the end of each chapter" (p. 7) that teachers often avoid. An analysis by Jäder, Lithner, and Sidenvall (2020) of mathematical problem-solving in textbooks from twelve countries demonstrated that there is a dearth of problem-solving tasks in secondary school textbooks compared to procedural tasks. This places the burden on mathematics teachers to design learning materials that incorporate real problem-solving questions that encourage mathematical thinking development.

Van Zoest et al. (2015) note that the continuing challenge in mathematics education is establishing how to support teachers' effective utilization of their students' mathematical thinking in the classroom. Wijers and de Haan (2020) recommend that "to really implement mathematical thinking for all students and help them develop the appropriate skills this should be part of the regular curriculum, which means that suitable assignments and problems are needed that fit within regular 50-min mathematics lessons" (p. 28). The National Council of Teachers of Mathematics (2000, 2014, 2020) in the United States of America regularly reiterates the need for the type of teaching that emphasizes students' development of mathematical thinking. The Council suggests that, to promote mathematical reasoning and mathematical thinking, students ought to concentrate on exploring problem-solving tasks, state and test conjectures, and create arguments to validate their conjectures. Stein, Grover, and Henningsen (1996) also recommend that, for students to think mathematically, they must be exposed to problem-solving tasks with high cognitive demand. Rigelman (2007, p. 314) asserts that if teachers carefully choose tasks that necessitate students to engage in problem-solving and mathematical thinking, students are likely to gain significant mathematical understanding while attaining higher levels of achievement for the subject.

Van Zoest et al. (2015) assert that teachers must be able to respond to students' mathematical thinking on the spot because some students engage with mathematical activities that are beyond the curriculum. They argue that if students pose questions that indicate that they are grappling with mathematically important ideas, such opportunities should be embraced to create rich teachable moments in the classroom. This demonstrates that there are numerous expectations of teachers regarding the development of students' mathematical thinking. Thus, teachers' mathematical thinking abilities are of vital importance because they cannot develop mathematical thinking in their students if they lack it themselves. Chapman (2015) claims that teachers ought to possess broader knowledge of mathematical problem-solving for their benefit as problem-solvers to be able to assist students to become improved problem-solvers.

Pehkonen, Naveri, and Laine (2013) caution that it is not easy to bring about change in teachers' approaches to teaching mathematics to encourage mathematical thinking; they can be presented with suggestions and ideas for how to teach in a way that promotes mathematical thinking. Pehkonen et al. (2013) note that, in Finland, mathematics is learned for the purpose of understanding the structures of mathematics as well as for the development of mathematical thinking, and not for merely mastering procedural calculations. It can be concluded that teachers ought to be willing and interested to implement such teaching for it to be successful or they should be located in schools where the curriculum advocates for such teaching and monitors it, otherwise teaching for mathematical thinking cannot be implemented.

Sherman (2014) posits that teachers' effective use of specialized educational technologies – such as dynamic geometry software (DGS) and graphing calculators – has the potential to deepen students' mathematical subject knowledge while supporting their mathematical thinking and discourse. Santos-Trigo, Reyes-Martínez, and Aguilar-Magallón (2015) found that the use of digital educational technologies in mathematics teaching extends mathematical thinking when dealing with problem-solving. Dubinsky and Tall (2002) also claim that computers can be utilized as a tool to complement the development of advanced mathematical thinking in many ways. Several scholars (Pea, 1987; Symons & Pierce, 2019; Yao & Manouchehri, 2019; Jacinto & Carreira, 2021) have also established that, when utilized appropriately, digital technologies (which they refer to as 'cognitive technologies/computing') prompt mathematical thinking development as a means of problem-solving. Although Santos-Trigo et al. (2015) caution that the use of technology for the development of mathematical thinking has some limitations, it appears that the correct use of educational technologies can assist in the development of mathematical thinking among students. For example, Joshi (2017) points out that, to a certain extent, the use of educational technology for the development of mathematical thinking presents an opportunity for better visualization of mathematical problems. Seeley (2017) advocates for mathematical discourse as another way of developing advanced mathematical thinking. The National Council of Teachers of Mathematics (1991) recommends that for students to develop mathematical thinking, teachers should give priority to listening while students engage in reasoning, modelling, talking, and explaining. Essentially, teachers are expected to carefully facilitate mathematical discourse among students to promote learning.

2.3.3 The development of mathematical thinking through discourse

The Oxford Advanced Dictionary (2010) defines the term 'discourse' as the utilization of language in writing or speech in order to produce meaning about something through the exchange of ideas. Sfard (2001, p. 28) defines 'discourse' as any explicit act of communication, be it synchronous or diachronous, with oneself or others, with the aid of symbolic systems or verbal communication. Seeley (2017) asserts that, in recent times, the term 'discourse' has come to take on a broader range of meanings, including debating, arguing, expressing, representing, interpreting, reflecting, and even thinking. Thus, this rich meaning illuminates the idea that discourse presents students with the platform to express their thoughts, ideas, and reasoning as they make conjectures and arguments that can be justified with evidence or proof. Essentially, Sfard (2007) outlines that "the different types of communication that bring some people together while excluding some others are called discourses" (p. 573). She adds that discourse is regarded as mathematical if it includes mathematical words, statements, or phrases.

Mathematical discourses differ across dissimilar communities; for instance, among statisticians, mathematician scholars, primary and secondary school teachers, or reform-oriented classrooms (Moschkovich, 2007). Mathematical discourses at any level may involve several genres, such as unique visual mediators (numerals, graphs, algebraic symbols), mathematically endorsed narratives (computational rules, theorems and axioms), algebraic modelling, algebraic proofs, geometric proofs, and conference presentations, among others (Moschkovich, 2007 & Sfard, 2012). This study, however, focuses on the mathematical discourses that influence students' development of mathematical thinking in the school environment.

McCarthy, Sithole, McCarthy, Cho, and Gyan (2016) found that guiding both preservice and in-service mathematics teachers to be able to analyze the types of questions they posed and responses they received from students through mathematical discourse allowed them to identify effective and ineffective question tactics. This implies that reflection by teachers after engaging in mathematical discourse in the classroom is critical, since it allows them to analyze the discourse further and ascertain if there was any development in students' mathematical thinking.

In facilitating mathematical discourse among students, Kersaint (2015) points to the importance of establishing learning spaces and strategies that support all types of productive mathematics conversations with students. In particular, she advocates that, initially, students need to be allowed to work independently on problem-solving questions to gather their thoughts by engaging in a cognitive mathematics conversation with themselves before they converse verbally with their peers or a knowledgeable other. From this, it can be inferred that for productive mathematical discourse to occur, a guided, conducive, learner-centred learning atmosphere must be promoted. This implies that teachers act as mediators of learning to provide a platform for students to engage in mathematical discourses designed to develop students' mathematical thinking abilities. Mediation of the process of learning has been proven by many scholars (Vygotsky, 1962; Vygotsky, 1978; Kozulin, 1998; Guerrero-Nieto 2007) to play an important role in cognitive development. Guerrero-Nieto (2007) asserts that mediation is a way in which people establish a connection between the world and their mental representations.

Since any form of discourse (verbal or non-verbal) involves the utilization of some language, it is maintained that mathematical discourses may not be a simple exercise in classroom environments where multiple home languages are represented, as is common in South Africa. For instance, Pourdavood, Carignan, King, Webb, and Glover (2005) conducted a study to explore teaching methods and mathematical discourse at a school where most students' home language was not English, which was the medium of instruction at school. Their study revealed that students whose home language was not English experienced challenges articulating their reasoning and thinking in English. They concluded that restricting students to the use of English during verbal discourse in the classroom limits the students' ability to demonstrate their mathematical thinking. Several other South African studies have had similar findings (Webb, Nemer, & Ing, 2006; Setati, Chitera, & Essien, 2009; Phakeng, 2018; Robertson & Graven, 2020).

In Sepeng's (2013) study, participants were permitted to converse in their vernacular languages during group peer discourses. He found that the level of whole-classroom discourse with the teacher was low since the teacher predominantly utilized English, but during small group discussions the level of discourse was higher. It can thus be inferred that the role of language during classroom mathematical discourse is vital. Literature from other several countries reveal similar results (Moschkovich, 2015; Planas, 2018; Planas, Farrugia, Ingram, & Schütte, 2019; Ingram, Chesnais, Erath, Rønning, & Schüler-Meyer, 2020).

As the literature reviewed in the previous subsection indicates that the use of educational technologies in mathematics teaching has the potential to extend students' mathematical thinking abilities, it is crucial to review the existing literature regarding the preparedness of preservice teachers to use educational technologies in teaching and learning.

2.4 IMPACT OF EDUCATIONAL TECHNOLOGIES IN MATHEMATICS

The use of technologies has become pervasive across all levels of mathematics teaching and learning in recent years. IvyPanda (2019) observes that since mathematics has had significant interactions with technology, this has led to numerous impacts on the teaching and learning of the subject. Research has demonstrated that the use of educational technologies in mathematics teaching and learning may have both positive and negative impacts depending on how they are utilized (Brasiel, et al., 2016). The impact of educational technologies used in mathematics can be viewed from both teacher and learner perspectives, as it should benefit both teaching and learning.

2.4.1 Impact of technologies on teaching in mathematics education

The use of any type of educational technology for the teaching of mathematics may be successful or unsuccessful in achieving the educational goals depending on a range of factors, such as the type of technology adopted, the intensity of the utilization of the technology, and the teacher's attitude and beliefs (Brasiel, et al., 2016). Many researchers have reported positive outcomes. Young, Gorumek, and Hamilton (2018) assert that educational technologies are pedagogical tools that have the potential to enhance the delivery of mathematics content with clarity and precision. Nepo (2017) goes so far as to say that the incorporation of educational technologies during instruction is indispensable to the success of learners in mathematics. Stols et al. (2015), in their study on South African mathematics teachers' perceptions and needs regarding the use of technology for teaching, established that "teachers were in agreement that using technology in the classroom could improve both teaching and learning" (p. 7).

Ghavifekr and Rosdy (2015) caution that the positive impact of educational technologies can depend on certain conditions, however. For instance, Gómez-García, Hossein-Mohand, Trujillo-Torres, and Hossein-Mohand (2020) emphasize that teachers must have adequate training to be able to effectively integrate specific educational technology into mathematics teaching. In addition, teachers' positive attitude towards the use of educational technologies in mathematics teaching contributes to the technologies having a constructive impact (Griffith, Hagan, Heymann, Heflin, & Bagner, 2020).

The training, attitudes, and beliefs of teachers can also be factors in educational technologies having a negative impact on the teaching and learning of mathematics. When teachers hold a negative attitude towards the use of educational technologies in mathematics teaching, their utilization of these technologies is likely to have poorer outcomes (Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012). Johnson, Jacovina, Russell, and Soto (2016) assert that the way teachers implement technology is ultimately influenced by their attitudes and beliefs towards both educational technology and pedagogy as a whole. (p. 23). A study by Mthethwa et al. (2020) established that the usage of GeoGebra software improved students' learning even in high-poverty, rural areas with limited access because teachers' beliefs about teaching and learning with advanced digital technologies were positive.

Ertmer (1999) notes that another barrier to educational technologies yielding a positive impact is the belief on the part of teachers that they do not have the required experience to utilize such technologies. Teachers holding this view may resort to traditional ways of teaching using general-purpose educational technologies (Roberts, 2014) such as chalkboard, chalk, pencil, and paper; in this case, cutting-edge educational technologies that are available in the education sector have no impact on teaching and learning as they are excluded from teaching and learning.

A factor influencing teachers' confidence to use technology for teaching mathematics is that the digital literacy of their students may be higher than their own, causing them embarrassment. Learners born in the digital era have been called "digital natives" (Prensky, 2001) due to the ease with which they integrate technologies into their lives. Johnson et al. (2016) argues that teachers lacking experience with more recent educational technologies are more likely to feel less in control of the classroom when they are aware that their learners are more competent than they are in the use of technology. The reality that learners may have a higher level of digital competence than their teachers must be acknowledged.

2.4.2 Impact of technologies on learning in mathematics education

It is essential to note that there is a strong relationship between the impact of the technologies used in mathematics teaching and the impact of the technologies used in mathematics learning. The manner in which technologies are utilized by mathematics teachers has an impact on student learning.

In a study conducted in Malaysia, Ghavifekr and Rosdy (2015) established that teachers' professional development programmes that are based on teaching and learning with technology played a significant role in enhancing the quality of learning for learners. Similarly, Higgins, Huscroft-D'Angelo, and Crawford (2019) note that teachers can create learning environments where educational technologies are used in tandem with traditional teaching strategies to enhance learning. They further assert that the effective utilization of educational technologies accelerates the rate of students' learning. This suggests that the use of educational technologies can positively impact student learning.

In a South African study, Oke and Fernandes (2020) found that technology enhanced students' motivation to learn because it provided students with prompt and reliable feedback on assessments. Dahal (2016) notes that when feedback on mathematics tasks is delayed, students are more likely to lose interest in mathematics learning. Similarly, Fyfe (2016) notes that rapid computer-based feedback in an authentic learning environment has been found to be beneficial to both low-knowledge and high-knowledge students when engaging in mathematics problem solving. However, Trenholm, Alcock, and Robinson (2015), in their investigation of feedback practices in fully asynchronous online (FAO) mathematics courses, found that feedback given by mathematics instructors during FAO "has limited potential for advancing student learning" (p. 1215). Kalogeropoulos, Roche, Vats, and Russo (2021) found that mathematics teachers were concerned by the lack of real-time synchronous feedback that results in virtual learning if the available technologies did not allow for this. Thus, the findings regarding the use of digital educational technologies and feedback vary.

The 2018 International Computer and Information Literacy Study (ICILS) revealed that 91% of participating teachers indicated that educational technologies assist students to develop high levels of interest in learning (Fraillon, Ainley, Schulz, Friedman, & Duckworth, 2019). Furthermore, a meta-analysis study conducted by Hillmayr, Ziernwald, Reinhold, Hofer, and Reiss (2020) found that, overall, the use of educational technologies has a medium significant impact on student learning outcomes as well as a small significant positive impact on the attitudes of students.

While these findings suggest that educational technologies have the potential to impact the learning of mathematics positively, not all of the literature supports a conclusion that educational technologies necessarily contribute to improved learning (National Academies of Sciences, 2018). Higgins et al. (2019) posit that technology can only be truly impactful to learning when several facets of the technological learning atmosphere are considered to optimize the potential of individual student learning. For instance, Fraillon et al. (2019) outline that when learners are not properly guided, the use of digital educational technology impedes concept development by students and distracts them from learning. Additionally, large-scale international studies, like PISA, have revealed that a considerable number of students worldwide continue to struggle to learn mathematics regardless of the learning innovations that exist within the education sector (OECD, 2019). As such, Hillmayr et al. (2020) note that the impact of utilizing digital educational technologies remains unclear as the findings of different studies differ.

2.5 PRESERVICE MATHEMATICS TEACHERS' KNOWLEDGE OF THE USE OF EDUCATIONAL TECHNOLOGIES

The review of the literature thus far has emphasized the pertinence of educational technologies in the schooling environment. Specifically, the review has focused on the central role played by teachers in teaching mathematics in an enhanced manner using educational technologies. Drijvers et al. (2016) argue that the latest advances in educational technologies, combined with access to these technologies in mathematics education, offer teachers and researchers a new standpoint for understanding how students learn. However, the knowledge of preservice mathematics teachers regarding the use of these educational technologies has not yet been discussed.

2.5.1 Studies in the United States, Europe and Asia

Dieker et al. (2014) mention that in the education system in the United States (US), preservice mathematics teachers are afforded enough time to gain technological and pedagogical skills to work within the online classroom environment. This suggests that the use of an online platform for teaching and learning would promote the complete integration of educational technologies, even though the intrinsic rewards of the online teaching environment differ from the face-to-face environment. However, Dieker et al. (2014, p. 38) claim that online teaching and learning has brought about a major shift in basic education in the US, which has resulted in educational leaders, district leaders, teacher educators, and teachers working towards purposefully shaping the skills of preservice and in-service teachers to impact the learning environment that effectively utilizes blended and online practices. Kent and Giles (2017), in their study on preservice teachers' technology self-efficacy in the US, established that 91% of the participants integrated technology into their teaching with confidence, while 90% indicated that they could incorporate technology for all the subjects in the curriculum. Admiraala et al. (2017) found that teacher educators modelling the use of technology in the classroom had an impact on the development of preservice teachers' skills and knowledge of technology integration. The findings of Graziano, Foulger, Schmidt-Crawford, and Slykhuis (2017) supported this. They recommended that all preservice teachers be trained thoroughly to integrate different types of educational technologies from their first day in the classroom.

In Thailand, Adulyasas' (2018) study on fostering preservice mathematics teachers' technological pedagogical content knowledge (TPACK) through a learning community revealed interesting results: participating preservice mathematics teachers demonstrated a high level of TPACK competence, suggesting that they had effectively integrated educational technologies into their teaching during their teaching practice. Adulyasas (2018) notes the curriculum at the institution where the participants were enrolled included a mandatory fourth year course titled 'Technology Innovation for Mathematics Teachers'. This course introduced the preservice mathematics teachers to several educational technologies and mathematical software packages, acquainting them with the benefits of technology in mathematics teaching. Students were given a set of examples of how to design lessons using technology, then asked to create their own.

In Turkey, Saylan, Onal and Onal (2018) had similar findings in their study on the use of technology in education by mathematics and science preservice teachers. Developing the knowledge and skills of preservice teachers to use educational technologies during instruction is thus given priority in some contexts. The results of such developmental initiatives within the Turkish context were confirmed by Sahal and Ozdemir's (2020) study that demonstrated that 83.86% of primary school preservice mathematic were eager to utilize educational technologies in their professional teaching upon employment. The remaining percentage reported, however, that they would not utilize advanced digital technologies since they believed that concrete materials were more effective and reliable, especially in rural school physical conditions characterized by limited access.

A study conducted at six universities in Vietnam revealed that in spite of the availability of educational technologies, technology utilization in preservice teacher preparation was still in its infancy (Tran, Phan, Van Le, & Nguyen, 2020). Additionally, it established that due to the wide disparity in ICT skill levels among mathematics lecturers, ICT integration in classroom activities was not particularly effective. Also, due to lecturers' restricted access to advanced educational technologies, ICT integration was not practical in the contemporary environment of Vietnamese pedagogical universities (Tran et al., 2020). This has a direct negative impact on preservice mathematics teachers' knowledge regarding the utilization of educational technologies.

Marcelo and Yot (2015) note, in their review of pedagogies using educational technology in Spain, that it is established that the process of implementing technology in the training of teachers ought to be on equal ground with any form of educational innovation. They further stress the importance of teacher educators acting as role models for preservice teachers, as teachers in training appear to adapt to the use of technology more easily if they see teacher educators model its use. Thus, it can be advanced that preservice teachers' knowledge of the use of educational technology is most likely to develop if they witness the effective use of such teaching resources during their instruction. Another Spanish study recommended that more emphasis be given to preservice mathematics teachers' technological knowledge as the study established that these teachers possessed greater knowledge of content and pedagogy than of educational technologies (Marbán & Sintema, 2020).

The European Commission (2019) has also indicated that all newly appointed teachers in European schools must possess some technology integration skills pertinent to the subject(s) they will be appointed to teach. The Commission further outlines that many teacher training institutions in European countries train preservice teachers to be able to tactically integrate digital educational technologies with confidence. Given the advanced development of most European countries, it can thus be expected that new teachers begin their careers proficient in the integration of modern digital educational technology into teaching and learning.

Additionally, a German study by Pozas and Letzel (2021) argues that, in as much as preservice mathematics teachers are trained to integrate technology into their teaching, their training ought to also emphasize preservice teachers' self-efficacy and attitudes. These authors concluded that, given the crisis in education that occurred in 2020/2021 due to the Covid-19 pandemic, preservice teachers require comprehensive ICT training based on real-world experiences to prepare them for their future roles as teachers who can successfully integrate ICT into their everyday teaching practices.

In Greece, Tzifopoulos (2020) found that preservice teachers seemed to be more proficient with non-academic digital technologies and encountered challenges integrating educational digital technologies into their teaching practice effectively. A study that included nine European countries assessing the use of educational technologies by preservice mathematics teachers revealed that schools had access to a wide range of educational technologies; however, these were typically used by students for passive learning rather than to engage in active learning in the classroom (Nantschev et al., 2020); the full potential of these technologies to enhance teaching and learning was thus not realized by novice and preservice teachers.

2.5.2 Studies in other African countries

In Tanzania, Kafyulilo, Fisser, Pieters, and Voogt (2015) conducted a study that employed Technological Pedagogical Content Knowledge (TPACK) as a framework to describe the knowledge and skills that Tanzanian preservice teachers ought to develop to effectively use technology in mathematics teaching. They discovered that preservice teachers improved in the knowledge domains of their TPACK competence during the intervention. In addition, their findings indicate that the professional training program was successful in enhancing the technology-related domain of preservice teachers' TPACK (Kafyulilo et al., 2015).

These results pose a logical question, however, as to why preservice teachers would be offered an additional professional development programme while they are still training as teachers, and why this component was not integrated into their core undergraduate coursework. Normally, professional development programmes are offered to in-service teachers who have completed their teacher training course (Kim, 2018). This might imply that at teacher training institutions in some developing countries, preservice mathematics teachers are not adequately equipped with the skills and knowledge necessary to integrate educational technologies. Nevertheless, Barakabitze et al. (2019) assert that continuous and consistent preservice and in-service training is viewed as an acceptable approach to support teachers in their use of educational technologies for science and mathematics teaching in Africa.

In an in-depth study in Ghana investigating the knowledge and skills of preservice mathematics teachers to design and enact spreadsheet-supported activity-based learning lessons found that participants were able to progress and demonstrate their knowledge and skills sufficiently during the design and enactment stages of activity-based mathematical lessons supported with technology (Agyei & Voogt, 2014). The study revealed that the utilization of spreadsheets by these preservice mathematics teachers as a teaching tool encouraged "student in-depth mathematical concept formation and an activity-based learning approach to make lessons less teacher-centred and more interactive" (Agyei & Voogt, 2014, p. 1). As in the Tanzanian study, teachers were trained through a professional development programme rather than as a component of the main curriculum. Another Ghanaian study on the use of educational technology by preservice mathematics teachers revealed that university tutors struggled to use technology during their tutorial sessions (Sedega, Mishiwo, Awuitor, & Nyamadi, 2018). The study concluded that tutors (preservice teachers) require special educational technology training courses to effectively utilize educational technologies during the teaching and learning of mathematics. Thus, this study also infers that there is a lack of technology integration skills among some African countries' preservice mathematics teachers.

In Botswana, Batane and Ngwako (2017) established that most of the preservice teachers in their study did not utilize the available educational technologies at their disposal during their lesson delivery, despite having reported high capability levels in the utilization of educational technology. This highlighted the need to develop a comprehensive and systematic strategy concerning technology implementation to guarantee a smooth transition for final year teachers-in-training as they move into the school environment where the use of technology is expected. This implies that preservice teachers may possess the skills and knowledge necessary to use educational technologies but lack the courage to implement their skills within an actual classroom environment. Batane and Ngwako (2017) maintain that providing pre-service teachers with technology skills and knowledge is crucial in any teacher-training program to empower them to address the educational requirements of the current century.

2.5.3 Studies in South Africa

In a South African study conducted by Tatira (2020), it was established that the participants (preservice teachers) demonstrated a lack of pedagogical content knowledge for utilizing any type of additional educational technology except for the general-purpose type. Ramatlapana (2017), in a study conducted in Johannesburg, also found that while preservice mathematics teachers' content knowledge and pedagogical content knowledge for geometry were above average, their knowledge of technological content knowledge was below average, which made it challenging for them to use GeoGebra to teach geometry. Neither of these studies indicate the reason for the preservice teachers' lack of knowledge and skills to use technology in their teaching. However, Chigona and Chigona (2013) found that preservice teachers in South Africa demonstrated a lack of knowledge and skills for the use of educational technology because there was a lack of institutional support during their teacher training course.

In a study with mathematics and science preservice teachers placed at 103 South African public schools, Jita (2018) found an uneven distribution of educational technologies across schools that corresponded to socio-economic status. As result, not all preservice teachers were able to utilize educational technologies during their teaching practice owing to limited access to these facilities. In this case, the prospective teachers' TPACK level was satisfactory; the only challenge was the poor infrastructure at schools where some preservice teachers were placed for teaching practice. Bansilal (2015) found that student teachers utilized educational technology more in their learning than in their teaching because of the lack of resources at many South African schools. Thus, it appears that in the South African context multiple challenges hinder preservice teachers' use of educational technology in their teaching practice. It is thus difficult to measure the practical skills of preservice teachers to integrate educational technologies as there are inadequate resources for them to use these skills.

The local and international literature established that most teacher training institutions invest in equipping mathematics preservice teachers with the skills and knowledge to integrate technology into their teaching. While extensive research has been conducted in this area investigating and exploring several concomitant aspects, the number of studies investigating the equipping of preservice mathematics teachers with technology integration skills and knowledge specifically for visualization purposes is negligible. Thus, this study seeks to fill the gap in the existing literature by exploring preservice mathematics teachers' use of educational technologies for visualization.

Next, it is essential to contextualize the availability of educational technologies in the South African educational landscape.

2.6 AVAILABILITY OF EDUCATIONAL TECHNOLOGIES AT SOUTH AFRICAN SCHOOLS AND HIGHER EDUCATION INSTITUTIONS (HEIS)

When student teachers register at an institution of higher education, they are required to have completed their basic education. As such, it is essential to initially understand the general background of students' exposure to educational technologies in their own schooling by interrogating the status of the availability of these technologies at schools.

2.6.1 Availability of educational technologies at South African schools

With the transition to democracy in South Africa in 1994, the basic education curriculum underwent extensive changes to meet the needs of the country (du Plessis, 2013). In 1995, the first Information and Communication Technology (ICT) policy for education was implemented with the formation of the Technology Enhanced Learning Initiatives (TELI), a strategic committee that led several projects which aimed to develop a network of 'technology-enhanced learning' to directly benefit learners (Isaacs, 2007). This policy's goal was that every learner at school should be ICT proficient by 2013; this goal was to be realized by converting schools to 'e-schools' (Isaacs, 2007). In 2004, the Department of Education (DoE) adopted the e-Education White Paper to reinforce the use of modern educational technologies in schools to enhance the quality of teaching and learning in the country (Department of Education (DoE), 2004). It was accepted that private schools and some other well-resourced schools in the country had already started to use digital educational technologies for teaching and learning (Mdlongwa, 2012). Nonetheless, the White Paper outlined that public and private sectors must work collaboratively to ensure all schools had access to ICT infrastructure for e-learning to be a success. The White Paper further stipulated that students ought to have regular access to reliable infrastructure for digital educational technologies to be impactful and effective (DoE, 2004). However, Vandeyar (2013) found that the policies introduced in the e-Education White Paper existed only on paper, as the goals of these policies were not even close to being actualized within the South African schools' context by 2013.

A 2012 report on the status of the implementation of the ICT policy at South African schools indicated that the major challenge to the implementation of ICT in schools across the country was the insufficient provision of funds by the government for computers and adequate infrastructure to support ICT (Mdlongwa, 2012). The report indicated that the government prioritized the provision of basic services to communities over the provision of ICT to schools. As a result, progress toward the implementation of ICT in 2012 was irregular and slow, with only three out of nine provinces making significant progress (ibid). Van Wyk (2012) also noted that since the implementation of the e-Education White Paper in 2004, slow and inadequate progress was made regarding ICT infrastructure provision to schools. These 2012 reports demonstrate that South African schools were not adequately equipped with digital educational technologies.

Five years later, a 2017 survey on the integration of ICT in South African schools found the key challenges to be a lack of infrastructure and the prohibitive costs of ICT, among other challenges to digital educational technology integration (Padayachee, 2017). Other studies conducted in different parts of South Africa in 2017 and 2018 indicated that, because of the high costs of educational technologies coupled with the lack of funding from the various stakeholders expected to support public schools, the availability of these technologies continued to be limited (Ojo & Adu, 2018; Umugiraneza et. al, 2018; Chisango & Lesame, 2017). A 2019 study conducted by Hannaway also revealed that the barriers to integrating technology-based teaching and learning include the high cost of equipment and the lack of financial support from the government to purchase and implement the modern technologies required by schools. There was thus little change in the findings reported from 2012 to 2019 regarding the availability of modern educational technologies (or ICTs) in schools.

Hannaway (2019) also found that expensive educational technologies in South African schools are vulnerable to theft, due to pervasively high levels of crime, which presents another limitation to the availability of modern educational technologies in South African schools. Mwapwele, Marais, Dlamini, and Van Biljon (2019) observe that several interventions to provide ICT have failed when the schools have struggled to protect and sustain the resources they have been given. Munje and Jita (2020) recommend that schools form partnerships with the communities where they are located to ensure the safety of ICT tools.

Notwithstanding this issue, Durodolu and Mojapelo (2020) report that there has been a noticeable increase in the quantity of South African schools with instructional media and computers for teaching and learning, but there is still a significantly wide gap between provinces regarding the availability of these facilities in schools. Munje and Jita (2020) also observe that the distribution of digital educational technologies in South African schools is uneven since the schools are located in areas with different standards of living. For instance, in socioeconomically disadvantaged communities, including most townships and rural areas, schools are usually under-resourced and lack basic necessities. Durodolu and Mojapelo (2020) assert that since

> South Africa is one of the countries struggling with problems related to the digital divide…learners from educationally disadvantaged environments continue to struggle with the use of modern technology, especially with instructional media and web facilities that have been incorporated into the school system (p. 64).

This means that not all South African schools are able to comply with the White Paper on e-Education (DoE, 2004) due to multiple intersecting challenges. For instance, Mwapwele et al. (2019) found that in many rural South African schools there are policies in place that prohibit students from using digital devices on the school premises, except for non-programmable scientific calculators. While schools may have valid reasons for implementing such policies, it can be argued that such policies have the potential to limit learners from being exposed to other avenues of learning. The number of well-resourced schools (Durodolu & Mojapelo, 2020; Munje & Jita, 2020) where both learners and teachers benefit from modern digital educational technologies are far fewer.

The impact of the Covid-19 epidemic on education across the country further exposed the digital divide that exists within South African schools (Chisango & Marongwe, 2021; Mhlanga, Denhere, & Moloi, 2022). The imposition of a state of emergency by the South African government that necessitated that educational institutions switch to a remote (online) mode of teaching and learning demonstrated that most public schools did not have the digital educational technologies required to this mode of instruction (Zubane, Khoza, & Mlambo, 2022). Several studies (Shepherd & Mohohlwane, 2021; Soudien, Reddy, & Harvey, 2022; Zubane et al., 2022) revealed that most public school learners came from disadvantaged backgrounds and could not afford the digital technologies required for them to participate in online learning from their homes. The Department of Basic Education (DBE) thus had to devise other strategies to ensure that teaching and learning continued at public schools across the country. Duby et al. (2022) reported that most private schools were able to progress with teaching and learning remotely after the online mode of instruction was configured. They further indicated that "learners attending private schools, or well-resourced public schools, were able to migrate relatively smoothly to remote learning, due to their ability to access suitable devices and internet connectivity, as well as benefiting from support from teachers and parents" (2022, p. 10).

As a result of this situation in basis education, many students enter tertiary institutions of higher education without prior knowledge of modern educational technologies (de Jager & Nassimbeni, cited in Durodolu & Mojapelo, 2020). Consequently, Makgati and Awolusi (2019) argue, that owing to the lack of technology infrastructure in schools, most students coming from poorly resourced schools face serious challenges during their first year of study at the tertiary institution because they are expected to swiftly adjust to new ways of learning. Thus, exploring the availability of educational technologies at HEIs is essential.

2.6.2 Availability of educational technologies in South African HEIs

Ng'ambi, Brown, Bozalek, Gachago, and Wood (2016) outline the four phases that the HEIs have experienced from 1996 to 2016 concerning the utilization of digital educational technologies in teaching and learning. The first phase (1996 to 2000) was characterized by a deepening 'digital divide' in education – both internationally (with the divide deepening between countries in the global South and North) and nationally (between HEIs who had adequate funding to invest in educational technologies and those and that did not). HEIs operated independently (Czerniewicz, 2004) since not much funding was received from the government to support the changing environment.

In the second phase (2001 to 2005) technology was seen as a resource for achieving 'radical democratization' (Pejout, 2004). Several institutional and national policies regarding the utilization of ICT were published during this phase. For instance, the White Paper on e-Education (DoE, 2004) stipulated that the national and provincial departments of education were expected to collaborate with institutions of higher education to design and deliver preservice and in-service training curricula for teachers. The policy further stipulated that the DoE "will ensure the inclusion of ICT integration competencies for teachers, administrators and managers in accredited pre-service teacher training programmes delivered by higher education institution" (DoE, 2004, p. 28). However, the policy does not specifically mention anything about a budget for the provision of these technological facilities to higher education institutions. Thus, Czerniewicz, Ravjee, and Mlitwa (2006) point out that many South African higher education institutions have used their own funds to purchase educational technologies. During this phase, the utilization of advanced educational technologies was minimal at several HEIs due to infrastructural constraints, a lack of digital instructional materials, and other challenges (Butcher, 2003). At this stage, there was no clear directive and proactive support from the DoE regarding the use of ICT (Cross & Adam, 2007). Viable ICT strategies were developed by only a small number of HEIs (such as the University of the Western Cape, the University of Pretoria, the University of Cape Town, Stellenbosch University, and the Tshwane University of Technology) during this phase (Czerniewicz & Brown, 2005).

During the third phase (2006 to 2010), higher education institutions began to expand their digital technology infrastructure, with portable devices and mobile communications occupying the space in the higher education environment (Ng'ambi et al., 2016). Despite an increasing understanding of the possibilities for pedagogical innovations through ICTs during this phase, South African HEIs were still unable to employ ICTs for the advancement and innovation of teaching and learning (ibid). The utilization of these technologies was still restricted to merely supporting traditional methods of instruction (Schmidt, 2008). During this phase, inequality in terms of students' access to digital technologies for learning became more pronounced as the majority of students at HEIs were from disadvantaged socio-economic backgrounds (Czerniewicz & Brown, 2009).

The fourth phase (2011 to 2016) was characterized by more advanced digital technology infrastructure together with high individual control through cloud-based resources (Ng'ambi et al., 2016). The utilization of Learning Management Systems (LMS) during this phase expanded across HEIs in the country although a high degree of inequality in the distribution of technologies among the institutions and within the departments or faculties was noted (Gachago, Ivala, Backhouse, Bosman, & Bozalek, 2013). Nevertheless, ICT use in post-school education was also emphasized in the Green Paper for Post-School Education and Training, as was the need to ensure equitable access through supporting infrastructure development (Department of Higher Education and Training (DHET), 2012). However, Makura (2014) found that some higher education institutions in South Africa appeared not to have invested adequately in ICT for teaching-related purposes. This reveals that no support had been received from DHET in line with the 2012 Green Paper. There was still a lack of comprehensive national policy on the use of ICTs in higher education, as of 2016 (Ng'ambi et al., 2016).

Following these four phases of technology development within the HEIs, in 2017, the National Student Financial Aid Scheme (NSFAS) made the first rollout of laptops for first-year students at one of the South African universities (ICS, 2017). Roll-out to other universities followed for all NSFAS-funded students, mitigating, to some extent, the unequal access to technology. However, provision was not made equally to all public HEIs, as Technical and Vocational Education and Training (TVET) colleges only received their first rollout of laptops for NSFAS-funded students in April 2021 (Siebritz, 2022) following the 2020 state of emergency. Thus, a state of unequal access was still evident within the HEIs even during the Covid-19 pandemic when online instruction was essential.

These challenges persisted despite the DHET having committed to “an expansion of open and distance education and the establishment of more ‘satellite’ premises where universities or colleges provide classes at places and times convenient to students (including in rural areas)” (Letseka, Letseka, & Pitsoe 2018, p. 121). This bold commitment by the DHET presumes that most South African universities possess modern educational technologies as it indicates confidence in its institutions to deliver a technology-dependent form of education. However, these directives on the part of DHET have not been followed through with tangible support, as evidenced by the disparity among HEIs when they were forced to implement the online mode of instruction during the 2020/2021 academic years.

Fomunyam (2019) attests that HEIs in South Africa are rated high for their modern educational technology capacity when compared to other African tertiary institutions. Moyo (2019) also highlights that the availability of educational technologies in South African HEIs has been acknowledged although in some institutions it is not effectively integrated. Several reports and studies (Mpungose, 2020; Louton & Hugo; 2021, Ndlovu, Ndebele, & Mbambo, 2022), however, indicate that when all institutions of learning were forced to migrate abruptly to online learning when South Africa instituted a state of emergency in 2020 response to the Covid-19 epidemic, many HEIs (especially TVET colleges) lacked the capacity to engage effectively with online learning due to the lack of infrastructure and training. Similarly, du Plessis et al (2022) revealed that while the HEIs were vulnerable and disordered at the beginning of Covid-19, various obstacles emerged regarding the disparities within and among universities, as some institutions were prepared to shift to online teaching and proceed with the academic semester, while others encountered significant limitations.

Mhlanga and Moloi (2020) conducted a study that sought to evaluate the influence of the national pandemic (Covid-19) in motivating digital technology transformation in the South African education sector. They found that 'Fourth Industrial Revolution (4IR)' technologies were being introduced from basic to tertiary education with teaching and learning activities migrating to online learning. These technologies included artificial intelligence; new computing technologies; neurotechnology; distributed ledger technology; geoengineering technologies; and space technologies. Mhlanga and Moloi (2020) report that because of the duration of lockdown in South Africa caused by the Covid-19 pandemic, schools and tertiary institutions that did not have basic digital teaching and learning technologies on hand were forced to shut down until the lockdown regulations were eased.

The literature before the 2020 state of emergency regarding the availability of educational technology in HEIs may have indicated that South Africa was well positioned to engage with online learning, however, the demands of the 2020/2021 academic years proved otherwise. The majority of tertiary institutions struggled to migrate smoothly to 100% remote (online) teaching and learning in a space of two to three months, while some managed to adjust. The recent literature demonstrates that inequalities persist in terms of the availability of educational technologies at South African HEIs. This also implies that instructors at HEIs lacked adequate knowledge and skill to effectively optimize the utilization of digital technologies for teaching and learning and required extensive skills development to ensure that the HEIs continued to implement the 2020/2021 academic programme.

Hedding, Greve, Breetzke, Nel, and Jansen van Vuuren (2020) point out that the 2015 #FeesMustFall protests may have prepared some tertiary institutions for the teaching and learning transformation, but the complete shutdown of almost all sectors of society brought about unprecedented challenges. They further state that the South African academics had to upskill and quickly familiarize themselves with remote teaching and learning platforms that entail the effective use of digital educational technologies.

While higher education institutions in the country are thus not fully equipped to make optimal use of educational technologies, several recent studies (Radović, Marić, & Passey, 2019; Rybak, 2021; Haleem, Javaid, Qadri, & Suman, 2022) have shown that many possibilities are offered by the increasing use of educational technologies within the education sector when implemented effectively.

2.7 SUMMARY AND IMPLICATIONS

The chapter has reviewed several inter-related topics in the literature regarding the utilization of educational technologies in mathematics teaching and learning. As the use of educational technologies in mathematics can be traced back to the 12th century, the literature has demonstrated that the purpose for its utilization from then to the 21st century has been to help students understand and work with mathematical concepts effectively. From ancient technologies like the abacus, to modern technologies like the interactive learning pad, educational technologies have made challenging mathematical concepts more accessible. The literature thus demonstrates that the purpose of using any form of specialized technology in mathematics has been to enhance the teaching and learning of mathematical concepts.

Studies across the world have found, however, that the benefit of using technologies to aid the teaching and learning of mathematics is highly dependent on how they are utilized. This implies that both teachers and learners need to be knowledgeable about the technology they intend to incorporate in teaching and learning while understanding the reasons for its use.

Several scholars (Czerniewicz et al., 2006; Raja & Nagasubramani, 2018; Young et al., 2018; Cha et al., 2020; Gómez-García et al., 2020) have reported on the different educational technologies used in mathematics education and the various reasons they are used. The literature suggests that the use of educational technologies for visualization is of vital importance. This highlights the importance of mathematics teachers possessing TPACK competence to be able to integrate educational technologies during instruction. Furthermore, the literature suggests that the utilization of educational technologies is a vital tool for the development of mathematical thinking for students in the classroom.

The research identifies several challenges that arise in the use of technologies at schools. In developing countries, including South Africa, the resource-constrained environment, coupled with a lack of skills and knowledge on the part of mathematics teachers, presents obstacles to the integration of technology. This appeared to be less of a problem in higher education, despite the lack of policy from the Department of Higher Education regarding the use of technology for teaching and learning. Some studies have found that preservice mathematics teachers lacked adequate expertise to use educational technologies during their teaching practicums, however, indicating that educational technologies are not always introduced effectively in teacher training programmes at higher education institutions in South Africa. On the other hand, the literature revealed that, in some cases, preservice mathematics teachers are disadvantaged by the poor infrastructure at the schools where they are placed for their teaching practicum and thus do not have the opportunity to demonstrate their ability to integrate technology into their teaching.

While numerous scholars (Agyei & Voogt, 2014; Kafyulilo et al., 2015; Adulyasas, 2018; Tatira, 2020) have investigated the TPACK competence of preservice teachers and their integration of technology to promote a learner-centred classroom environment, the research on the knowledge of preservice mathematics teachers has focussed mainly on in-service teachers. This study addresses the gap in the literature by explores preservice mathematics teachers' use of educational technologies as visualization tools in the teaching and learning of mathematics.

2.8 CONCLUSION

This chapter has presented a brief historical background of the utilization of educational technologies in mathematics education. The purpose and impact of educational technologies during mathematics instruction were established from the existing literature. Moreover, the utilization of educational technologies for mathematical thinking and visualization was discussed. Thereafter, the chapter outlined preservice mathematics teachers' knowledge regarding the integration of educational technologies in mathematics teaching. The status of the availability of educational technologies within basic and higher education was also presented. A review of the relevant literature found, however, that the focus on preservice mathematics teachers' utilization of educational technologies as visualization tools during instruction is minimal. The next chapter discusses the theoretical framework that underpins this study.

Chapter 3: Theoretical Framework

3.1 INTRODUCTION

This chapter deliberates on the two theoretical lenses used in this study: namely, the theory of commognition and Technological Pedagogical and Content Knowledge (TPACK). The definitions, tenets, benefits, and shortcomings of these frameworks used in mathematics education are discussed and their relevance and application to the South African context is explored. The chapter investigates the synergy between these theories and their potential, when used in tandem, to facilitate the aims of this study.

3.2 COMMOGNITION

In an attempt to strengthen some of the traditional human developments, the concept of commognition emerged within the mathematics education context (Sfard, Commognition, 2020). The commognitive framework was developed by Anna Sfard, who put forward the framework in 2007 in her research article, *When the Rules of Discourse Change, but Nobody Tells You: Making Sense of Mathematics Learning from a Commognitive Standpoint* (Sfard, 2007) and presented the framework in depth in her 2008 work, *Thinking as communicating: Human development, the growth of discourses, and mathematizing* (Sfard, 2008).

Sfard was interested in the type of discourses that students engage in with themselves during mathematical problem-solving. She established that thinking is essentially another type of communication. This approach to communication in the exploration of human cognition rests on the premise that thinking can be understood as a form of intrapersonal communication, – specifically, as one's dialogue with oneself. Commognition is an interpretive framework that is grounded in the assumption that thinking is a form of communication and that mathematics learning is essentially a form of extending and modifying one's internal discourse (Sfard, 2007). Commognition is thus, essentially, a coherent theory for thinking about *thinking within oneself*. Sfard coined the term 'commognitive', combining the terms 'cognitive' and 'communicational' (Sfard, 2007). This combination is put into perspective by her definition of commognition, which emphasizes that "interpersonal communication and individual thinking are two facets of the same phenomenon" (Sfard 2008, p. xvii).

Ioannou (2016) maintains that the commognitive framework comprises various elements, such as metaphor, thinking, communication, and commognition, that stem from the correlation between interpersonal communication and cognitive operations. These constructs form the fundamental basis for comprehending mathematics learning by students. According to Sfard (2008), interpersonal and intrapersonal (cognitive) communication processes are dissimilar manifestations of the same phenomenon.

The commognitive framework thus brings together a broad range of concepts. Sfard (2007) holds that the framework is not specific to one discipline and can be applied to any discourse that occurs during learning, as all discourses follow similar rules. Presmeg (2016), too, argues that the framework is sufficiently broad to be appropriately utilized as a theoretical lens in various settings. She further asserts that this theoretical view does not only address issues in mathematical teaching and learning but, as it engaged with what it means to be human beings capable of thinking, has relevance to human development on a larger scale.

3.2.1 Key tenets of commognition

Commognition relies on four basic original tenets that guide its application to specific research. According to Sfard (2007), these tenets are summarized as "thinking as individualization of communication; mathematics as a form of discourse; learning mathematics as changing a discourse, and commognitive conflict as a source of mathematical learning" (pp. 569-573). The following sections elaborate on each of these.

3.2.1.1 Thinking as individualization of communication

Sfard (2008) contends that human thinking is an individualized form of communication with oneself which is similar to the interpersonal process of communication that is naturally acknowledged. Geurts (2018) notes that people do not communicate with other beings only, however: overt and/or covert self-talk is also common. The commognitive perspective assumes that the process of thinking is fundamentally dialogical, as it involves interactions within ourselves such as notifying ourselves, arguing, questioning, and responding to our thoughts. This perspective is transferable to the process of engaging with mathematical problems, where an individual can think about their own thinking during the process of solving a problem. Sfard (2015) asserts that the commognitive framework is based on the notion that thought and its verbal or nonverbal expression are indivisible.

3.2.1.2 Mathematics as a form of discourse

Sfard (2007) holds that certain rules must be followed during the process of individualized intrapersonal (thinking) and interpersonal communication. Individuals would not consciously follow these rules, since they are neither required nor are natural in any way. She adds that it is likely, for this contingent reason, that Wittgenstein (1953) suggested that communication is similar to a game where some individuals may participate in some games and not others – games that are played with varied tools according to a set of numerous rules. Individuals may be able to engage in some forms of communicational activity but be unable to engage in others, just like in games. From the commognitive perspective, discourse relates to the various forms of communication that unite some individuals while alienating others. Individuals may engage in different or overlapping discourses depending on their interests and their ability to follow certain rules. As such, mathematical discourse would be classified as a 'discourse community' where the rules of engagement within this community comprise mathematics rules. The first two objectives of the current study engaged preservice teachers in individualized and interpersonal mathematical discourses that were expected to assist in comprehending their choices of instructional methods.

According to Sfard (2007), visual mediators are ways through which participants in mathematical discourses recognize the objects of their conversation and coordinate their communication – such as the use of drawings, diagrams, graphs, and formulae. These mediators are utilized during discourse between two or more people and as well as by an individual engaged in intrapersonal discourse. Thus, it can be deduced that the individuals engaging in intrapersonal discourse utilize their available visualization techniques.

3.2.1.3 Learning mathematics as changing a discourse

Sfard (2007) establishes that since the mathematical discourse is a form of modifying students' existing everyday discourses, mathematics learning may be viewed as transforming colloquially learned discourses, rather than constructing new ones. Thus, mathematics learning is essentially associated with learning to participate in different classroom mathematical discourses (Esmonde, 2009). For instance, once students have acquired knowledge related to algebra or trigonometric ideas, they can extend and change their skills of discourse to be able to implement this type of communication during mathematical problem-solving. Sfard (2007) claims that the students' discursive development can be observed and monitored by recognizing that transformations that take place in four discursive characteristics: “the use of words characteristic of the discourse, the use of mediators, endorsed narratives, and routines” (Sfard, 2007, p. 573). Therefore, by observing the development of each of the discursive characteristics among individuals in the ways they communicate, it can be concluded that mathematics learning is a way of changing a discourse. This development may not be apparent during normal teaching and learning, where the teachers’ focus is not on investigating the development of these discursive characteristics. As such, Nathan and Knuth (2003) contend that some school teachers believe that, during mathematics instruction, they should emphasize student mastery of procedures and symbols with the result that they neglect the discursive processes in which students engage.

3.2.1.4 Commognitive conflict as a source of mathematical learning

Sfard (2007) theorizes that learning can take on two forms. Firstly, there is object-level learning that assumes that learning is merely an expansion of pre-existing discourse through the extension of vocabulary and the construction of new routines together with endorsed narratives. Secondly, there is meta-level learning that involves alterations in meta-rules of discourse, which means that, over time, familiar tasks will begin to be done in a new way.

Meta-level learning is only possible via direct encounters with unfamiliar discourses that are governed by the rules of meta-level learning that are different from the rules governing the current discourses in which learners are engaging (Sfard, 2007). Can, Barnett, and Clark (2018) posit that these encounters usually entail a commognitive conflict, when the discussants unwittingly operate under meta-level rules that are entirely different. Sfard (2007) cautions, however, that commognitive conflict should not be confused with the idea of cognitive conflict. Commognitive conflict arises when discourses do not share any criteria for determining whether a specified narrative ought to be endorsed or not, and are thus incommensurable because they cannot be rendered mutually exclusive as they sound, or seem, contradictory. Cognitive conflict, in contrast, arises when discourses are incompatible, but can be resolved with decisive empirical evidence, in which one of the conflicting claims is refuted and the other proven true. Sfard (2007) notes that commognitive conflict is commonly encountered and is often evident during mathematics or science invention discourses.

Given the focused nature of this inquiry, the commognitive conflict tenet of the commognitive framework was not evident in the data. However, the other three tenets were evidenced to varying degrees as the study explored some mathematical discourses manifested by preservice mathematics teachers’ communicative practices during problem-solving.

3.2.2 Key commognitive constructs

Sfard's (2008) commognitive framework identifies four distinguishing characteristics of mathematical discourses: word uses; endorsed narratives; routines; and visual mediators.

Mbhiza (2021) explains that word utilization encompasses the words that are commonly utilized by teachers and learners when engaging in specific discourses in the classroom during teaching and learning. Endorsed narratives are concerned with sequential utterance patterns referring to mathematical objects, and relations that are performed on those objects which can be endorsed or denied within the community of mathematics (Sfard, 2008). Examples of such narratives may include theorems, certain equations, and mathematical definitions. Routines, in the context of commognitive theory, refer to the repetitive actions that characterize the discourse – such as the drawing of graphs and execution of mathematical calculations. Lastly, visual mediators denote the objects of mathematics; these include diagrams, graphs, mathematical symbols and other objects that are utilized for communication in a mathematical context. In this study, participants engaged with these four constructs that characterize the commognitive framework during the first phase of data collection.

3.2.3 Usefulness of the commognitive framework

The framework provides a theoretical and analytical lens to better comprehend students' engagement in mathematical thinking and reasoning as discourse (Kim et al., 2017). The advantages and capabilities of this framework are established in Sfard's (2007) original work on the topic and demonstrated in the empirical studies that have used the framework.

The framework has been used by numerous scholars (Presmeg, 2016; Ioannou, 2016; Ngin, 2018; Zayyadi, Nusantara, Subanji, Hidayanto, & Sulandra, 2019; Mpofu & Mudaly, 2020; Mbhiza, 2021) to study students' thinking when they are engaging in solving mathematical problems. Daher (2020) established that the commognitive framework assisted in exploring the process of the trigonometric function 'sine' to a group of Grade 10 learners. He asserts that the framework was found invaluable in studying students' routines and the process of mathematization when broadening their understanding of trigonometric ideas. His findings concurred with the results of a South African study by Ndlovu and Mwakapenda (2019) on the concepts that are linked with number pattern activities. Moreover, in a study involving commognitive analysis of teachers' mathematical discourses on derivatives, Tasara (2018) deduced that one of the main advantages of Sfard's framework is that it enables the examination, on a micro-level, of how teachers teach mathematics and how students develop mathematical discourses. In addition, Cobb (2009) maintains that the commognitive framework can address three levels of mathematical discourse: the macro-level, which concerns historically established mathematical discourse; the meso-level, which involves discourse practices jointly established by teachers and students in a classroom; and the micro-level, which relates to individual students' evolving mathematical discourses. The framework has also been applied to the South African context in several other empirical studies (Mudaly & Mpofu, 2019; Roberts & Le Roux, 2019; Mpofu & Mudaly, 2020; Mbhiza, 2021).

In this study, participants engaged in mathematical discourses relating to the visualization of geometric and trigonometric ideas during problem-solving. Using commognitive discourse analysis, participants' responses were analyzed to determine whether their visualization strategies influenced their style of teaching.

3.2.4 Critique of the commognitive framework

Despite its demonstrated usefulness, the commognitive framework has received criticism from some scholars. Fredriksen (2020) points out that Sfard's notion of 'participationist' neglects several means of communication – such as body language and physical gestures – by focusing on specific mathematics discourses. Sfard (2007) states that "participationists conceptualize developmental transformations as changes in what and how people are doing and claim that patterned collective activities are developmentally prior to those of the individual" (p. 568). Thoma (2018) points out that as the relationships between the discourses are complex, analysis of these in the data is a detailed and time-consuming process. However, beyond this negligible or generic criticism of the commognitive framework, there has been no intensive and extensive engagement with it. As such, Presmeg (2016) observes that the commognitive theoretical framework has significant untapped potential to be beneficial in research in mathematics education across all levels.

This study used the commognitive framework to gain a more holistic understanding of the phenomenon under inquiry. Exploring the intrapersonal communication processes of mathematics preservice teachers (the participants in the study) when dealing with mathematics problems enabled the researcher to gain a deeper understanding of how their individualized self-communication informed their use of educational technologies when teaching.

Thus, the use of the pedagogical framework was required to provide an important lens for investigating the aspect of teaching using educational technologies in this study. This framework is discussed next.

3.3 TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE

In the early 2000s, in the context of the rapid development of new educational technologies, there was increasing discussion amongst scholars about the need for a grounding theory in the field of educational technology. To address this gap, scholars Punya Mishra and Matthew Koehler at Michigan State University in the United States began working on a conceptual framework for educational technology. After five years of work, they published their framework in 2006.

The framework used as its basis Shulman's (1986, 1987) work on the different types of knowledge – using the concept of Pedagogical Content Knowledge (PCK) – which had already informed research in this area for approximately two decades. Mishra and Koehler realized that, in addition to Shulman's types of knowledge, there was an additional type of knowledge that was required for the utilization of technology during instruction. Building on Shulman's (1986, 1987) work, Koehler and Mishra (2006) formulated a framework which they termed 'Technological Pedagogical Content Knowledge' (TPACK).

The model adds technical knowledge to Shulman's pedagogical and content knowledge; an additional three types of knowledge arise at the intersection points of these three types of knowledge: technical pedagogic knowledge (TPK), technical content knowledge (TCK) and pedagogical content knowledge (PCK). Figure 3.3.1 depicts the intersection of the three main knowledge domains (content, pedagogical, and technological knowledge) which form the basis of three pairs of additional knowledge types, together constituting the TPACK framework.

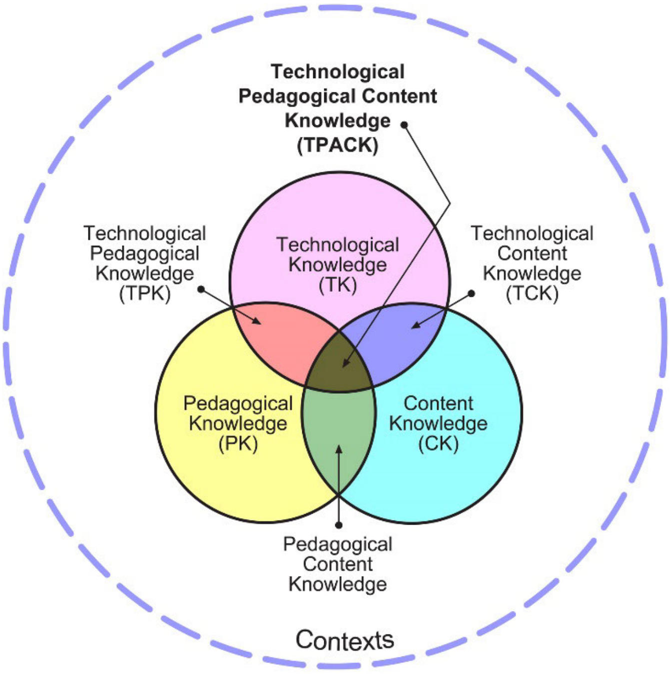


Figure 3.3.1: TPACK (adapted from Mishra and Koehler, 2006)

3.3.1 Key elements of the TPACK framework

The following types, and combinations, of knowledge, comprise the TPACK framework.

3.3.1.1 Content knowledge (CK)

The first basic type of knowledge that exists within the TPACK framework is content knowledge (CK). CK is defined by Shulman (1986) as the quantity and organization of knowledge that is within a teacher's mind. Ball, Thames and Phelps (2008) assert that this type of knowledge is the subject matter knowledge that a teacher needs to possess. It goes beyond concepts and facts to include why a particular concept is the way it is and why certain topics are central to a subject, while others are perhaps peripheral. With this knowledge, teachers should be able to tell learners why mathematical formulae work, and where they come from.

3.3.1.2 Pedagogical knowledge (PK)

While it is essential that teachers possess content knowledge, Shulman (1986) argues that one cannot successfully engage in a process of teaching without pedagogical knowledge (PK). Auerbach and Andrews (2018) posit that this type of knowledge is concerned with teaching and learning that is not specific to a particular topic. It includes classroom management, knowledge of the theories of learning, and the ability to motivate students, among other things. Auerbach and Andrews (2018) indicate that PK is significant for teachers and teacher educators as it assists in the development of other knowledge domains.

3.3.1.3 Pedagogical Content Knowledge (PCK)

Shulman (1987) notes that pedagogical knowledge can be specific to certain kinds of content knowledge; pedagogical knowledge and content knowledge in this case combine as pedagogical content knowledge (PCK). Gess-Newsome (2015) describes PCK as the teaching and learning knowledge specific to a subject or topic within a specific area of the curriculum. For instance, one may speak of the geometry or trigonometry knowledge of teaching that preservice teachers ought to develop in addition to their content knowledge of geometry or trigonometry. Ball et al. (2008) argues that PCK has been identified as the most fundamental content-related domain of all types of knowledge. Shulman (1986) notes that PCK comprises "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9). This implies that PCK incorporates the learner knowledge, together with curriculum knowledge, that teachers ought to attain during the interactions with their students in the process of teaching and learning. PCK thus brings together major components of knowledge that are vital for teachers to possess for basic instruction to be successful.

3.3.1.4 Technical knowledge (TK)

Despite the importance of content and pedagogical knowledge, Mishra and Koehler (2006) argue that these are not sufficient to make a successful teacher in an evolving world with the developments in the technology sector that influence the entire education system. In their model, they add a third basic type of knowledge: technological knowledge (TK). TK is concerned about the knowledge of both traditional and modern, novel and advanced technologies together with the skills required to use these technologies (Edwards & Cheok, 2018). This may include knowledge about the general and specialized educational technologies mentioned in the previous chapter.

3.3.1.5 Technological content knowledge (TCK)

Technological content knowledge (TCK) captures the reciprocal relation between technology and content. Koehler and Mishra (2009) posit that there is a deep and ancient relationship between technology and content knowledge. They maintain that this type of knowledge is the comprehension of the way content and technology constrain and influence one another. From this perspective, it follows that teachers need to carefully consider which educational technologies could promote more diverse and innovative representations of conceptual content and which might constrain representations of content (Koehler & Mishra, 2009). Essentially, this type of knowledge is vital to teachers' critical decision-making about which content knowledge can best be taught using which specific forms of educational technology. This is because not all educational technologies can be utilized to teach every subject or topic within the subject. For instance, Geometers' Sketchpad has been found to be a brilliant educational technology for the teaching and learning of geometry and functions (Eu, 2013; Kanandjebo & Ngololo, 2019), however, it may not necessarily be best suited for the teaching and learning of probability and statistics. This illustrates the importance of technological content knowledge. This type of knowledge may not be sufficient to make informed pedagogical decisions about the use of technology for specific content areas, however.

3.3.1.6 Technological pedagogical knowledge (TPK)

Koehler and Mishra (2009) state that technological pedagogical knowledge (TPK) focuses on the knowledge of how technology can be used pedagogically to transform the way learning and teaching are mediated. They further observe that this relies upon a comprehension of the TCK and disciplinary contexts within which teachers are functioning. Edwards and Cheok (2018) note that TPK encompasses the capabilities of the range of technologies used in education and how these might impact the teaching and learning process. Koehler and Mishra (2009) caution that many software programs and internet-based technologies are not necessarily designed for purposes of education, however (for example, Microsoft Office and various blog and podcast applications were designed for business, communication, and entertainment purposes). Teachers thus need to think laterally and creatively to be able to use these technologies effectively in learning and teaching.

3.3.1.7 Combining the types of knowledge into TPACK

The basic types of knowledge – content knowledge (CK), pedagogical knowledge (PK) and technical knowledge (TK), together with the knowledge blends (PCK, TCK and TPK) constitute the TPACK theoretical framework that undergirds the second aspect of this study: the exploration of preservice teachers' use of technology to promote visualization during their practice teaching. All of the knowledge elements are co-dependent constructs that together form TPCK (which later came to be called 'TPACK'), which encompasses the knowledge that all teachers at all levels should possess. As such, Koehler and Mishra (2006) define TPCK as an "emergent form of knowledge that goes beyond all three components (content, pedagogy, and technology). This knowledge is different from knowledge of a disciplinary or technology expert and also from the general pedagogical knowledge shared by teachers across disciplines" (p.

1029). They add that TPCK forms a fundamental basis for teaching using technology since it requires a comprehension of the depiction of content and concepts utilizing technologies. They stress that TPCK requires an understanding of pedagogical strategies that productively utilize technologies to teach content, as well as the knowledge of how technologies can assist to make concepts that are difficult or problematic for students easy to learn. They contend that TPCK represents a specialized knowledge class that is specific to teachers' utilization of technology in their work. This type of knowledge is typically not possessed by technology experts who are proficient in their subject matter or by educators who lack knowledge about the subject or technology (Koehler & Mishra, 2006). Essentially, TPCK is a framework for teachers' knowledge of the integration of technology into teaching and learning. Over time, 'TPCK' has been recast as 'TPACK'.

Adnan and Tondeur (2018) affirm that TPACK involves fundamental comprehension of the integration of technological skills and knowledge, as well as knowledge of learners, subject matter content, and pedagogies that are essential for teachers to be capable of teaching with technology effectively in the classroom. When teachers engage in the process of teaching and learning, it is imperative to be technologically knowledgeable for the use of technology to achieve its intended purposes. Hence, Koehler and Mishra (2008) add that teachers' TPACK development is crucial for effective and successful teaching with technology.

3.3.1.8 Teaching as a complex cognitive skill

Mishra and Koehler (2006) assert that the basis of TPACK as a framework for the knowledge of teachers is grounded in the understanding that the process of teaching is a cognitive skill that is complex in nature and occurs in a dynamic, ill-structured environment. It can be deduced that teachers' cognitive skills are vital when they are to engage in the process of instruction for it to be successful. Although the contexts may differ, the cognitive concepts used in TPACK may be compared to the cognitive aspect of the commognitive ('communication-cognitive') theory discussed earlier in the chapter. It can be asserted that the cognitive skill of the individuals is key to both of these theories, whether it is used in the service of mathematical problem solving or in the service of using technology effectively to enhance teaching and learning. In addition, Koehler, et al. (2011) accentuate that the cognitive skills that teachers need to possess draw on and synthesize different domains of knowledge.

3.3.2 Application of TPACK

In a 2008 TPACK handbook titled *Handbook of Technological Pedagogical Content Knowledge (TPACK) for Educators* as edited by the AACTE Committee on innovation and technology, different authors contributed chapters outlining how the TPACK framework by Mishra and Koehler could be integrated into different subjects such as languages, social studies, arts education, physical education, literacy education, science, technology, and mathematics education in different contexts (American Association of Colleges for Teacher Education, 2008). This study focuses on the integration of TPACK into mathematics education. Grandgenett (2008), who contributed a chapter on this topic in the TPACK handbook, indicates that when and how technology should be utilized within the field of mathematics has never been a question that is taken frivolously nor an easy question. He asserts that if the use of technology is neglected in the subject, teachers miss out on vital opportunities to enhance their students' understanding of the content. On the other hand, if teachers use technology inappropriately (for example, allowing students to rely heavily on calculators or similar technologies for fundamental arithmetic and graphs), they run the risk of perpetuating students' misconceptions and undermining their ability to think independently. Therefore, it is imperative for mathematics teachers to craft pedagogical approaches that engage educational technologies in a way that promotes learning and does not undermine it.

Durdu and Dag (2017) argue that preservice teachers should be exposed to more courses that require them to develop technology-based instructional materials. Birel and Çakıroğlu (2018) concur, and further posit that mathematics preservice teachers' TPACK develops through microteaching lessons. Similarly, Stapf and Martin (2019) advise that the provision of additional support and opportunities for mathematics preservice teachers to practice and further advance their comprehension of TPACK builds their confidence to engage with technology in teaching and learning. Modelling how to use technology by in-service teachers and teacher educators is thus critical to the general professional development of preservice teachers' TPACK (Wang, Schmidt-Crawford & Jin, 2018). A range of possibilities exists for how to develop preservice teachers' TPACK expertise and ability to integrate technology effectively into teaching and learning.

3.3.3 Limitations of the TPACK framework

Despite the possibilities offered by TPACK, some research studies have identified limitations and disadvantages to the TPACK framework. Handal, Campbell, Cavanagh, Petocz, and Kelly (2013) point out that as the framework encompasses education broadly, it does not provide a means to assess its effectiveness in different disciplinary settings. In a similar vein, Lai and Lin (2015) note that TPACK is generic and does not specifically address the teaching of mathematics. They also note that if teachers lack the autonomy and creativity necessary to integrate technology into their lessons, particularly given the rapid evolution of these technologies, the TPACK framework may not be appropriate. Ozudogru and Ozudogru's (2019) found that teachers needed to be comfortable with their competencies before they could confidently integrate technology into their mathematics teaching. Thus, teacher educators need to be strategic in their approach during preservice teacher training by ensuring that knowledge facilitation is sequential and only interlaced where appropriate. Bonafini and Lee (2021) also postulate that if preservice mathematics teachers are learning how to teach mathematics and simultaneously learning how to integrate technology, they might encounter challenges and the TPACK framework may be less effective.

3.3.4 Usefulness of the TPACK framework

Niess, et al. (2009) advocate that the TPACK framework, as a model, may be useful for researchers, professional development consultants, teachers, teacher educators, and school administrators, depending on their varying needs. They further claim that the framework is useful for assessing mathematics teachers' level of integrating instructional technologies during teaching and planning for their personal professional development. Similarly, Young (2016) asserts that TPACK is an advantageous framework as it offers theoretical, empirical, and practical considerations for the use of technologies in the mathematics classroom. In addition, studies by da Silva Bueno, Lieban and Ballejo (2021) and Hill and Uribe-Florez, (2020) found that the TPACK framework was beneficial when attempting to understand the integration of educational technologies by both preservice and in-service mathematics teachers. As such, Schmidt, et al. (2009) developed a TPACK scale to obtain a more specific and reliable level of teachers' TPACK. Thus, although TPACK may have its shortcomings, the majority of the literature supports the view that the framework can be helpful in a range of contexts, including mathematics education.

3.3.4.1 The use of the TPACK framework by researchers in South Africa

In the South African context, the TPACK framework has been utilized by several researchers (Hannaway, 2016; Padayachee, 2017; Mokotjo & Mokhele-Makgalwa, 2021) in mathematics education and other disciplines. Leendertz, Blignaut, Nieuwoudt, Els and Ellis (2013) conducted a study on the dataset for mathematics teachers from the Second Information Technology in Education Study (SITES) in South Africa. Their results were presented according to three categories which associated the variables of the SITES 2006 teachers' questionnaire with the main elements of the TPACK.

A study conducted in Gauteng explored the barriers that mathematics teachers experienced to integrating technology into their teaching and their levels of TPACK as a way of informing their needs for professional development (de Freitas & Spangenberg, 2019). The study revealed that consistent professional development dealing with specific technology integration barriers had a significant impact on teachers' TPACK that was likely to result in improved teaching of mathematics. In addition, Khoza and Biyela (2019) conducted a study on the decolonization of preservice mathematics teachers' knowledge of pedagogy, content, and technology at one of South Africa's universities. They found the TPACK framework useful when utilized as the learning framework to generate curriculum concepts to support preservice mathematics teachers' knowledge of content, technology, and pedagogy.

Similarly, Ramnarain, Pieters and Wu (2021) assessed the TPACK of preservice science teachers at a South African university. Their findings showed that most preservice teachers were on TPACK proficiency level 3 regarding their practical knowledge. They advise that their findings could be utilized by teacher training institutions to review their programmes for preparedness of science preservice teachers to harness the affordances of educational technologies in teaching. Furthermore, a study by Shilenge and Ramaila (2020) utilized TPACK in township schools in South Africa. Their findings revealed that teachers lacked the ability to identify and integrate appropriate educational technologies into teaching and learning. They further established that teachers' ability to meaningfully integrate TPACK was hampered by a general lack of access to essential educational technologies. Thus, it is evident that the TPACK framework has been used extensively in different South African contexts.

3.3.4.2 The usefulness of TPACK for preservice teachers

The literature suggests that the TPACK framework is useful for preservice teachers and not necessarily in-service teachers only. As such, this framework has been identified as being relevant to support the last three objectives of this inquiry as it relates to the envisioned utilization of educational technologies by preservice mathematics teachers as visualization tools in their teaching.

3.4 CONCLUSION

This chapter has explored the commognition and TPACK theoretical frameworks and established their compatibility for the purpose of this study. For this study, it was realized that when preservice mathematics teachers are to be engaged in the discourse about the use of educational technologies as visualization tools, the researcher ought to first understand their visualization strategies when they engage with mathematical problems themselves. Therefore, the researcher will further establish whether preservice teachers' visualization strategies influence their adoption of educational technologies in mathematics teaching. Hence, the commognition theoretical framework was found to be best suited to study this aspect of the phenomenon as it was established that this discursive framework is better suited to explore individuals' thinking especially when they engaged in mathematical problem solving. The data assisted in establishing the relationship between the preservice teachers' mathematical problem-solving skills and their mathematics teaching skills using technology. The TPACK theoretical framework was of benefit for exploring the aspect of teaching using educational technologies.

The methodological approach used in this study is presented in the next chapter.

Chapter 4: Research Methodology

4.1 INTRODUCTION

This chapter presents the basis on which this research study was designed, organized, planned, and executed (Saunders, Lewis, & Thornhill, 2007; Robson, 2011). It presents the methodological epistemologies used to explore preservice teachers' uses of educational technologies as visualization tools when teaching mathematics. This exploration was conducted by responding to the main research questions as articulated in the first chapter and presented in Table 4.5.1 of this chapter. This chapter delineates the research paradigm, approach and style adopted to answer these main questions. The research site and sample are identified and the methods used to collect and analyze data are discussed. Ethical issues and the approach taken to ensure the trustworthiness of the study are presented.

4.2 RESEARCH PARADIGM AND APPROACH

4.2.1 Research Paradigm

In educational research, the term 'paradigm' refers to a researcher's philosophical view of thinking, or their worldview (Kivunja & Kuyini, 2017). These worldviews ground the approaches researchers take to determining the types of questions to ask for their research and how data will be gathered and interpreted systematically, and identifying the variables that will be observed and investigated (Bertram & Christiansen, 2014). Guba (1990) demonstrates that the term 'paradigm' constitutes ontology, epistemology, and methodology. Guba and Lincoln (1994) postulate that the researcher must explicitly elaborate on how these pillars are fused within a study before it is conducted.

The principal aim of this study was to explore a phenomenon for the purpose of existential understanding. An interpretive paradigm is concerned with interpreting thick existential understanding of a phenomenon being studied from its naturalistic setting (Lukenchuk, 2013). The interpretivist paradigm was used to answer the main research questions in this study.

An interpretive worldview requires an exploratory approach to investigate participants' interpretations of their situations and to understand their behaviours, interactions, and attitudes (Cohen, Manion, & Morrison, 2018). Within this paradigm, researchers interact with individuals in their natural setting and attempt to comprehend their interpretations of the world they work and live in. Interpretivists essentially are interested in understanding the multiple and varied subjective meanings that individuals make of their lived experiences (Creswell, 2009).

This aspect of studying participants' lived experiences in their everyday context directly relates to the ontological pillar that exists within interpretivist research studies (Guba & Lincoln, 1989). The ontological assumptions for this study lie within a social world of meanings and a physical world. In this study, the researcher's assumption was that the world that was being explored was populated by social beings with their own interpretations, thoughts, and meanings (Ahmed, 2008). As such, the use of multiple research methods was essential to capture the preservice teachers' inner thoughts, opinions, and experiences regarding their utilization of educational technologies as visualization tools when teaching mathematics. Further, Ahmed (2008) argues that the physical world that social beings exist in can be understood as a realistic ontology in educational research. Hence, the researcher acknowledged that the participants exist in a real physical world where their actions may sometimes be shaped by the contextual factors within their social and geographical environments. For instance, during the teaching practicum, participants were immersed in different real physical school environments and the researcher sought to understand these.

This study held subjectivism as the epistemological stance since the knowledge acquired in this study was grounded in the preservice teachers' lived experiences and bound to the natural contexts in which they found themselves (Denzin & Lincoln, 2005). Additionally, the phenomenon under study was understood and explained through the thick descriptions gathered from participants' perspectives. Crotty (2003) avows that epistemological assumptions are concerned with "a way of understanding and explaining how we know what we know" (p. 3). Consequently, throughout this study, the researcher deliberately explored the complexity of views and meanings that were in accordance with participants' contexts, situations, and perspectives (Creswell & Creswell, 2018). Thus, this research study was situated within the interpretivist philosophical perspective. The methodology of this interpretive study was focused on comprehending the utilization of educational technologies as visualization tools by preservice mathematics teachers.

4.2.2 Research Approach

In addition to researchers' philosophical views that shape their research paradigm, researchers must choose an appropriate research approach to guide their investigation (Creswell & Poth, 2018). Researchers may choose to adopt a qualitative, quantitative, or mixed-method research approach (Creswell, 2009).

A qualitative research approach, which is aligned with the interpretive paradigm, was found to be most appropriate for this study because of the textual data that was collected to respond to the central research questions. Qualitative research is a form of social science research that gathers and interprets textual data to arrive at meaning (Punch, 2013). Qualitative data can provide an understanding of the social phenomena under study using targeted populations of interest or places. Silverman (2013) notes that topics for this type of research often emerge by realizing that "apparently 'obvious' features of the social world depend upon intricate social organization" (p. 7). Hence, individuals make meaning of the world they live in by interacting within their social organizations. Strauss and Corbin (2008) explain that, as qualitative research is inductive, qualitative researchers usually explore insights and meanings from the given context of their targeted population. Creswell (2009) notes that the qualitative approach is thus a productive model that materializes in a natural environment where a researcher can develop a level of understanding from a high level of engagement with the participants' actual experiences.

In this study, the researcher engaged the participants in their everyday setting on the university premises to explore their visualization abilities when engaging with mathematics problems. The participants were engaged further to explore how they utilized educational technologies in a physical classroom environment to enhance learners' abilities to visualize when solving mathematics problems.

Dudwick, Kuehnast, Jones, and Woolcock (2006) note that qualitative research can use a wide range of data collection methods and analysis techniques; purposive sampling and open-ended and semi-structured interviews are common choices. In this study, a suite of qualitative data collection methods was used to investigate the phenomenon. These included performance tests, semi-structured interviews, focus groups and direct observation. These are described in detail in Section 4.5.

4.3 RESEARCH STYLE

In Mann (2006) defines a research style as a "particular perspective toward conducting educational research, determined by the psychological or sociological context (not by personal preference)" (p. 3). The choice of any research style utilized in any form of research is influenced by a variety of factors which include the research paradigm and approach, the research questions, the objectives of the study, and the methods of data collection (Bertram & Christiansen, 2014).

Cohen, Manion, and Morrison (2005) confirm that case study as a research style is particularly effective for research in education and the other social sciences. The case study research style has been defined in so many different ways over the years that Yazan (2015) terms it a ‘contested terrain’. Stake (1995) suggests that a case study is a study of one case’s complexity and particularity, an attempt to understand its activity within significant circumstances. This study uses Creswell’s (2013) definition of case study research as a qualitative approach in which the investigator explores a real-life, contemporary, bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audio-visual material, and documents and reports), and reports a case description and case themes.

Yin (2009) claims that in case study research “the boundary line between the phenomenon and its context is blurred as a case study is a study of a case within a real-life, contemporary context or setting…” (p. 18). Thus, the fundamental interpretive element of the phenomenon studied within a real-world context lingers consistently within the qualitative research styles. Different researchers (Merriam, 1998; Sturman, 1999; Yin, 2009) have identified various types of case studies that can be utilized depending on the researcher’s type of study and their purpose. Stake (1995, 2005) identifies three types of case studies: namely, instrumental, intrinsic, and multiple/collective case studies. Hamilton and Corbett-Whittier (2013) extended Stake’s three types of case studies to include longitudinal, reflexive, cumulative, collaborative, and collective case studies. Some of these case study types are closely related, as such a brief account of these is presented in Table 4.3.1.

Table 4.3.1: Types of case studies

Case study type	**Description**
Instrumental	Used to shed light on a problem or to make a generalization (Stake, 1995).
Intrinsic	Places strong emphasis on the case itself. Examples: cases involving individuals, certain groups, professions, departments, and organizations (Stake, 2005).
Longitudinal	Used to track changes over an extended period of time, understand the dynamics of changing circumstances, and get a sense of the history of an occurrence or occurrences (Cohen et al., 2018)

Reflexive	A researcher (as the case) explores their own particular experiences with the phenomenon under study (Cohen et al., 2018).
Cumulative	Studies of cumulative cases combine data gathered over time from several sources about a particular phenomenon (Hamilton & Corbett-Whittier, 2013).
Collaborative	Studies conducted in collaboration with other researchers across and within institutions to collect and report on different several contexts (Hamilton & Corbett-Whittier, 2013).

The unit of analysis in the case study might be multiple cases (a multi-site study) or a single case (a within-site study). A multiple (or collective) case study research style involves a group of individual cases that are conducted to obtain a more general, or fuller, picture of a topic being studied (Cohen et al., 2018). Stake (2006) suggests that with multiple case studies, cases need to exhibit some similarities; for example, the cases could all be teachers, or clinics, or staff development sessions, or security stations at an airport. The phenomenon would be studied across all the different cases. Yin (2014) shares a classic example of a multiple case study in which the research focus was to explore the implementation of innovations at schools (such as the use of new educational technology or new curricula); the study as a whole included several schools, with each school being the subject of an individual case study. Herriott and Firestone (1983) argue that the multiple case study research style is considered to produce robust data that is frequently compelling.

Yin (1994) argues that a multiple case study should follow a 'replication' design – similar to the way multiple experiments are designed for a quantitative study. Yin (2014) explains that each case should be selected on the basic that either similar findings (a literal or direct replication) are predicted, or that contrasting findings are predicted intentionally (a theoretical replication). Figure 4.3.1 illustrates the multiple case study replication approach, where each case forms part of the whole study, and convergent evidence is reached. The conclusions drawn from each case are then utilized as the information requiring replication by other separate cases within the study. In the final stage, the focus of the summary report comes from the individual cases or the multiple-case findings. Additionally, Yin (2014) concludes that across all cases, the report ought to indicate the degree of replication logic.

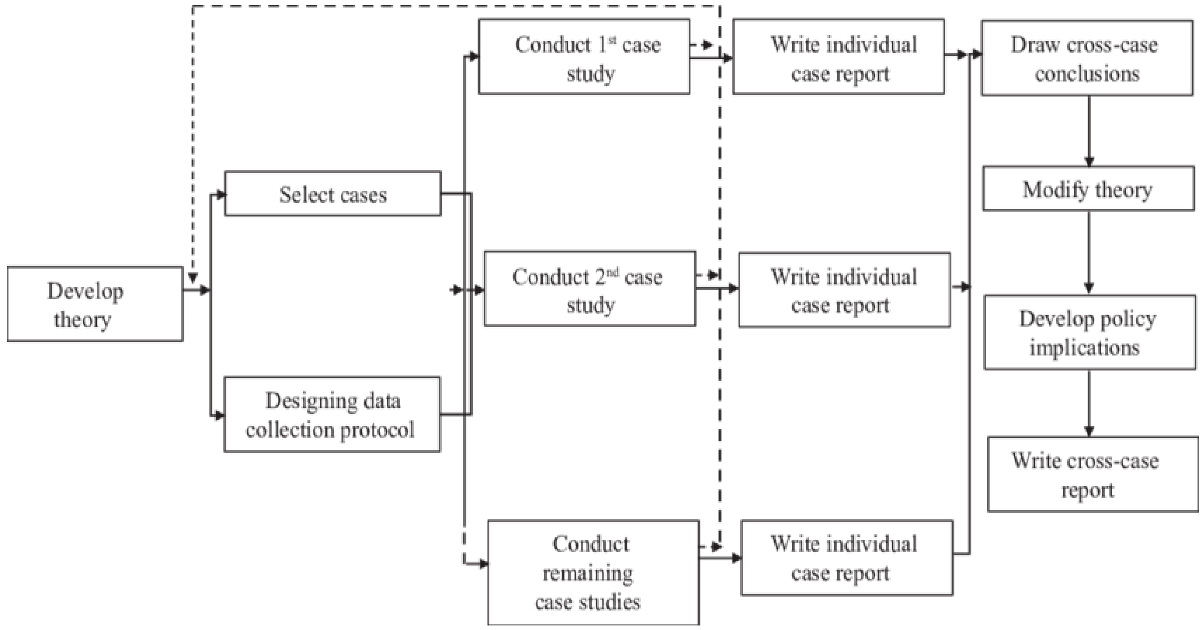


Figure 4.3.1: Multiple case study replication logic (Yin, 2014, p. 60)

As such, Yin (2014) argues that having more than two cases to conclude from blunts the skepticism and criticism of other scholars that may arise if empirical data is collected from only a single case study.

To explore the phenomenon under study effectively, the researcher chose to treat each participant as an individual case to be systematically studied. The choice of a multiple case study research style was thus employed to achieve the study's research objectives and to align it with the data collection methods. In this study, in-depth data were collected from multiple cases over time from the participants in their real-life contexts. The study included ten case studies, with each participant representing a single case. The individual case studies exhibited some similarities and differences. Hence, the researcher adopted a multiple case study research style because of the contextual factors that differed between cases. For instance, while the ten participants attended the same mathematics lectures at the university level, they were placed at different schools for their teaching practice experiences. Because these contexts were so different, it was not possible to treat them as a single case.

Furthermore, this study sought to achieve analytical conclusions through direct replication to produce robust and stronger findings on the use of educational technologies by preservice teachers as visualization tools in mathematics teaching. Each participant was studied as an individual case to make the final findings more compelling as all cases were expected to complement each other, thus strengthening the cross-case conclusions and the proposed model report.

4.4 SAMPLE POPULATION AND LOCATION

The target population for this study was preservice mathematics teachers from the province of KwaZulu-Natal in South Africa. Convenience sampling may be chosen for a case study or multiple case studies (Cohen et al., 2018). The researcher selected a Higher Education Institution (HEI) in the KwaZulu-Natal province on the basis of the convenience that its geographical proximity to the researcher afforded. Stake (2006) argues that for an interpretive qualitative inquiry, researchers ought to be immerse in their participants' context to gather rich data. The researcher was familiar with the culture of teaching and learning at this institution, having been a student tutor and a contract lecture at the institution for over five years, which influenced the selection choice.

Purposive sampling was employed to select participants for this interpretive qualitative inquiry. Yin (2016) explains that a purposive sample is a deliberate selection of participants or data sources for a study, based on their potential to provide valuable and relevant information for the study's research questions. Bertram and Christiansen (2014) and Patton (2015) add that purposive sampling involves choosing cases of interest that satisfy a predetermined criterion.

Mathematics preservice teachers who were registered with the School of Education from the discipline of mathematics and computer science education were selected for the study. The School of Education, located within the College of Humanities at the institution, is a well-resourced teaching and learning facility that has been producing professional teachers for more than two decades.

The researcher selected the third-year students who specialized in mathematics at the Further Education and Training (FET) level to participate in the study. The reason for this selection is that they were deemed likely to possess the requisite understanding of the mathematics problems in a performance test and to be able to participate meaningfully in the interview sessions. Third-year students were chosen also on the basis that they had had some experience teaching mathematics during their practical placements at schools at this point in their programme. They had thus been exposed to different types of teaching strategies by their instructors and mentors at the schools to which they had been assigned for teaching practice.

The researcher identified a module that the students had taken during their first year of studies that incorporate the integration of educational technology and visualization in the teaching of geometry and trigonometry. The ten students who had achieved the highest marks for this module were recruited for the study. Qualitative research in the social sciences aims to comprehensively examine a phenomenon in detail (Hong & Cross Francis, 2020) rather than simply representing the study population. Subedi (2021) maintain that the number of participants involved in qualitative research methods is determined by the specific methodology used. Thus, approaches such as phenomenology, narrative inquiry, and case study are designed for small sample sizes, typically ranging from a single case up to 20 (Subedi, 2021). Therefore, the ten participants who formed ten cases for this study were selected to comprehensively explore the phenomenon, rather than simply generalizing the results.

4.5 DATA COLLECTION METHODS

Bertram and Christiansen (2014) note that researchers' choice of collection methods is informed by the type of research questions to be answered. The research questions in this study focused on preservice mathematics teachers' utilization of educational technologies as visualization tools when teaching. Only textual data was collected as this facilitated the collection of in-depth and rich data through a systematic process to enable the researcher to gain deep insight into the main research questions. Multiple qualitative data collection methods were used, including performance tests, interviews, focus group discussion and observation. Table 4.5.1. depicts the data collection methods used to respond to the research questions in this study.

Table 4.5.1 Research questions and corresponding data collection methods

	Research Question 1	Research Question 2	Research Question 3	Research Question 4
	What visualization strategies are used by preservice mathematics teachers to solve mathematics problems?	What educational technologies do mathematics preservice teachers intend to use in their teaching?	Why do mathematics preservice teachers believe that their chosen educational technologies can influence their teaching and concept development?	How do mathematics preservice teachers demonstrate the implementation of these educational technologies in their teaching during teaching practice?
Data collection methods	Phase 1: Performance test			
	Phase 2: Semi-Structured interview			
		Phase 3: Focus group discussion	Phase 3: Focus group discussion	
				Phase 4: Observation

The following is a detailed description of each phase involved in the data collection process corresponding to Table 4.5.1. It provides an account of the processes that were involved during the chronological execution of each of the phases.

4.5.1 Phase 1: Performance test

The first phase of the data collection used a performance test to gather data. For this study, a performance test is defined according to Sweet (1993) to mean a method of testing that requires participants to complete a task instead of selecting a choice from a list of readily available options. For instance, participants may be requested to generate scientific hypotheses, have a dialogue in a foreign language, explain historical events, or solve mathematics problems. Experienced assessors in the field are then expected to assess the quality of students' or participants' responses based on specific criteria. Sweet (1993) adds that performance tests may be a valid indicator of the knowledge and abilities of those tested.

In this study, participants were asked to respond to a two-part mathematics performance test (see Appendix C). Part A consisted of two-word problems from analytical geometry and trigonometry of 3-dimensional (3D) shapes. Part B consisted of the same questions but with accompanying diagrams for each question. Participants first completed and submitted their responses to part A of the test before sighting part B. Participants were allocated approximately 30 minutes to respond to Part A of the performance test. Thereafter, they were given 20 minutes to respond to Part B, which was expected to be easier to complete after attempting Part A.

The researcher met with five participants at the university to conduct the short test. The other five participants were sent the questions separately via email to complete the test at their homes, as it was not feasible for the researcher to meet them due to Covid-19 restrictions. These participants had remote access to the researcher during the duration of the test. This phase aimed to assess participants' visualization abilities when solving mathematical problems and their answers assisted in responding to the first research question. After each participant completed the performance test (Phase 1 of the data collection process) and their responses were reviewed by the researcher – and while their test responses were still fresh in their minds – they moved on to Phase 2 of the data collection process: the semi-structured interview. As the interview questions were based on their responses on the performance test, it was important to proceed to the interviews immediately, while the thinking processes they had used during the performance test were still fresh in their minds.

4.5.2 Phase 2: Interviews

For this qualitative inquiry, interviews were selected as a useful data gathering tool as they afforded the researcher the necessary flexibility to conduct an in-depth exploration of issues (Kvale, 1996). The researcher was able to gain insight into why and how participants framed their thoughts the way they did through the interpersonal social encounter that an interview creates (Hochschild, 2009; Cohen et al., 2018). Kvale (1996) posits that qualitative research interviews are significantly different from quantitative research interviews, in that they attempt to understand the world from the participant's perspective.

Semi-structured individual interviews were employed in this study as they enable both closed-ended and open-ended questions to be posed to the participant in a conversational mode, allowing the researcher to probe for a deeper understanding (Adams, 2015). With semi-structured interviews, the researcher develops an interview guide consisting of a list of prepared questions that are asked in sequence (Cohen & Crabtree, 2006). In this study, the semi-structured interview questions (see Appendix D) were developed after reviewing participants' responses to a performance test, with the aim of understanding their thought processes and visualization abilities after engaging in a mathematical problem-solving exercise. DeJonckheere and Vaughn (2019) state that semi-structured interviews provide comparable and reliable qualitative data. They stress that the researcher should audio-record the interview session to review later rather than taking notes during the interview, which interrupts the flow of the interview.

The participants were given absolute freedom to express their thoughts about the performance test they had just completed and to expand on the visualization techniques they used during problem-solving. The duration of the interviews with each participant ranged from 45 to 60 minutes. The first three interviews were conducted face-to-face with each participant and were recorded using a smartphone. However, due to the increase in Covid-19 cases within the university, the remaining interviews were conducted online via the Zoom platform. These sessions were recorded via the platform. Participants' responses during the interviews yielded comprehensive data relevant to the first research question.

4.5.3 Phase 3: Focus group discussions

Jamieson and Williams (2003) maintain that the philosophical perspectives that underpin the focus group as a data collection method are grounded on the premise that ideas and attitudes are developed through interaction with others and not in isolation. Focus group discussions for this study were conducted to generate data for the second and third research questions. The researcher was of the view that interaction between the participants was necessary to gather data for these two main research questions. Denscombe (2007) avers that a focus group may consist of a small number of individuals assembled by an interviewer to explore their perceptions, ideas, attitudes, and feelings regarding a predetermined topic of interest. The focused discussions were guided by ten questions (see Appendix E) that related to the second and third research questions of the study.

Smithson (2008) advise that, ideally, a focus group should consist of five to twelve comparatively homogenous participants. In this study, the same sample of ten participants who had participated in the first two phases was grouped into two groups of five individuals in each group. The participants were randomly grouped with no specific criteria. Harvey-Jordan and Long (2002) caution that focus groups must not be too large to prevent interactions from becoming unmanageable, and also must not be too small to permit one participant to dominate the discussion. The two focus groups for this study were manageable with no one participant dominating the others. Each group discussion was allocated about an hour for the discussion, which was chaired by the researcher via Zoom. This completed the third phase of data collection for the study, following the semi-structured interviews.

4.5.4 Phase 4: Observation

This study sought to investigate the participants' environments, teaching strategies, and behaviours during their teaching practicums to obtain data that would answer the fourth research question (Simpson & Tuson, 2003; Marshall & Rossman, 2016). Thus, participant observation was essential to gather this form of data. Cohen et al. (2018) contend that the distinctive characteristic of observation as a data collection method is that it provides an investigator with an opportunity to collect "first-hand, live data *in situ* from naturally occurring social situations" (p. 542). Morrison (1993) adds that observations allow researchers to collect data on

> the *physical setting* (e.g., the physical environment and its organization); the *human setting* (e.g., the organization of people, the characteristics and make-up of the groups or individuals being observed, for instance, gender, class); the *interactional setting* (e.g., interactions that are taking place, formal, informal, planned, unplanned, verbal, non-verbal); the *programme setting* (e.g., resources and their organization, pedagogic styles, curricula, and their organization) (p. 80).

For this study, observations were utilized as the final and last method of data collection. The purpose was to physically gather first-hand, natural evidence of participants using educational technologies in a classroom experience during mathematics teaching. After meticulous planning for all ten observations at different locations, the third wave of the Covid-19 pandemic in South Africa necessitated that an alternative plan be made. At that time, the country was under an adjusted Level 3 lockdown. For this level, learners from public schools with large numbers had to attend school on alternate days. As the size of schools varied, this was determined on a case-by-case basis. However, most of the schools where the participants were placed for their teaching practice were required to implement this directive. As a result, most school principals had stringent restrictions preventing access to outsiders. Furthermore, six of the ten participants reported that their schools would sometimes close for two to three days at a time if a positive Covid-19 case was reported within the school. The entire school was closed for disinfection after identifying the infected teacher(s) or learner(s), who were sent home to quarantine for ten days. These cases negatively impacted the observation sessions since the researcher had to visit all of the participants' schools.

Table 4.5.4.1 depict the profiles of the ten schools where the preservice teachers were placed during their teaching practicums

Table 4.5.4.1: Profiles of schools visited by the participants

School (identified by participant pseudonym)	**Quintile**	**Location**	**School enrolment**	**Teacher/ learner ratio**	**Maths performance (Grade 12)**	**Learning facilities**
Zot	2	Kranskop	1050	1:45	27.0%	Chalkboard Textbook
Sfi	2	Kwamaphulo	800	1:35	28.4%	Chalkboard Textbook
Lunge	3	Umlazi	1100	1:50	64.7%	Chalkboard Smartboard Tablet Textbook
Qhaw	3	Inanda	950	1:40	73.1%	Chalkboard Smartboard Projector Tablet Textbook
Lethu	2	Nquthu	1009	1:43	53.3%	Chalkboard Textbook
Zakes	2	Nquthu	996	1:42	42.1%	Chalkboard Projector, Textbook
Sandi	3	Bergville	1310	1:60	57.4%	Chalkboard Textbook
Shan	4	Pinetown	1253	1:58	65.9%	Chalkboard Projector, Textbook
Phume	3	Ndengezi	1350	1:65	31.4%	Chalkboard Textbook

Amah	4	Pinetown	861	1:38	25.8%	Chalkboard Textbook

Since the researcher had already obtained permission from the provincial Department of Basic Education (DoE) but could not visit the schools due to Covid-19 restrictions, alternatives had to be considered to obtain the observational data. Hence, with assistance from the participants, the researcher approached the participants' school mentors and asked them to observe and video-record the preservice teachers while they taught the lesson that they would have taught during the researcher's visit. The ten school mentors were briefed (telephonically) about the study and instructed on what to observe during the preservice teachers' lesson while they also recorded. After they verbally consented to assist with this phase of data collection, the observation schedule was provided to the school mentors to complete. The schedule consisted of an open-ended observation questionnaire. This type of observation is what Denscombe (2014) and Cohen et al. (2018) term a 'semi-structured observation' – where the observer uses a predetermined agenda to inform their observation. All participants were observed by their respective mathematics mentor teachers who were physically present in the classroom. Mentor teachers completed the open-ended observation questionnaire during one lesson with the participants. Mentors were to observe preservice teachers (participants) teaching any mathematics lesson in which they were to implement the use of any educational technologies as visualization tools. The observations lasted for 55 minutes, on average, for all ten participants. The observation schedules and video recordings were then submitted electronically by the mentors once the observation had been completed.

4.6 ANALYSIS

Analysis of data, in this study, involved making sense of all data that had been collected and organizing it so it could be used to respond to the key research questions. Ngulube (2015) explains that qualitative data analysis is concerned with the transformation of raw data by searching, recognizing, mapping, exploring, evaluating, and describing trends, patterns, categories, and themes found in the raw data, thus interpreting the data and arriving at fundamental meanings. Cohen et al. (2018) attest that qualitative data analysis focuses on rich, in-depth, context-specific, subjective data and participants' meanings in a situation where a researcher is a principal research instrument. This study focused on classifying, interpreting, and transforming raw data collected from performance tests, interviews, focus groups and observations. The fundamental meanings and interpretations of this study's findings were drawn from the patterns, themes, and trends that emerged from the raw empirical data.

The qualitative data analysis for this study aimed at describing the phenomenon by comparing multiple case studies and proposing a model from the empirical data that was collected and analyzed. Cohen et al. (2018) state that the methods used to analyze and present qualitative data should be selected on the basis of fitness for purpose*, as* there is no one correct method for this process.

For this study, thematic analysis was employed as an analytical instrument to analyze the collected data systematically. This approach was adopted because of the thick descriptive data collected, which needed extensive exploration.

Braun & Clarke (2006) describe thematic analysis as a technique for analysing qualitative data that involves searching through a dataset to find, analyze, and document recurring patterns. They argue that the distinctive quality of thematic analysis is its adaptability for use across a broader spectrum of epistemological and theoretical frameworks and for application to a variety of study designs and questions – as well as sample sizes. Its status as a method – instead of a methodology – is advantageous: in contrast to many qualitative methodologies, it is not bound by a certain theoretical or epistemological stance and thus has greater flexibility (Clarke & Braun, 2013; Maguire & Delahunt, 2017). In addition to providing a technique for describing data, its processes of theme construction involve substantial interpretation of data (Braun & Clarke, 2006). The dataset is thoroughly organized, described, and interpreted in rich detail to cover all aspects of the phenomenon being studied (Braun & Clarke, 2006). According to Cohen et al. (2018), this is a process of moving from data, to explanation, to new knowledge.

Since this study sought to study the phenomenon through interpretation, thematic analysis was found to be the most suitable approach to analysis. In this study, thematic analysis was utilized as a constructionist method to report on participants' experiences, realities, and meanings concerning the phenomenon. Marks and Yardley (2004) note that this analytical tool offers an opportunity to comprehend the phenomenon comprehensively. In this study, it allowed the researcher to determine the associations between concepts that were being studied and to compare them with the replicated data from multiple case studies (Alhojailan, 2012).

Several scholars (Alhojailan, 2012; Javadi & Zarea, 2016) have indicated that there are numerous approaches to thematic analysis, which has raised questions about its nature and how it differs from qualitative content analysis (Vaismoradi, Turunen, & Bondas, 2013; Maguire &

Delahunt, 2017). As a result, Braun and Clarke (2019) introduced reflexive thematic analysis (RTA) to mitigate misconceptions around the numerous approaches to traditional thematic analysis and to clarify the nature of traditional thematic analysis. Braun and Clarke (2016) posit that RTA is an interpretive qualitative approach to data analysis that offers a simple and conceptually flexible way to identify and analyze patterns or themes in a dataset. It is regarded as the researcher's reflection on the analysis of data undertaken at the junction of the dataset; the theoretical presuppositions of the study and the researchers' analytical skills (Byrne, 2022). Braun and Clarke (2019) emphasize that it is concerned with "the researcher's reflective and thoughtful engagement with their data and their reflexive and thoughtful engagement with the analytic process" (p. 594).

The six phases of thematic analysis are retained from Braun and Clarke's (2006) traditional thematic analysis in their (2019) contemporary reflexive thematic analysis. This study employed the six-phase reflexive approach as it offers a flexible theoretical interpretive approach to the analysis of qualitative data (Byrne, 2022; Maguire & Delahunt, 2017). The decision to situate this study within this approach was grounded on this study's research aims and objectives. Additionally, this approach was beneficial since this study sought to comprehend the thoughts, behaviours, and experiences of the participants across the dataset in relation to the primary research questions. Furthermore, Kiger and Varpio (2020) established that this method is intended to assist researchers in conducting a systematic search for common or shared interpretations. However, they caution that it is not ideal for analyzing distinct interpretations or experiences from a single individual or data item. The collected data needed to be systematically grouped and organized according to categories from the ten cases to arrive at the core categories that integrate the ten responses to each research question. Data collected from all four phases of the collection were incorporated during this rigorous and systematic process.

The data collected from the ten participants were coded according to the main research questions. The coding process commenced after the researcher had familiarized himself with the data. Thereafter, all codes were collated into potential themes that were tailored to the research questions.

The themes or patterned responses (Braun & Clarke, 2006) derived from the dataset for all ten cases were constructed, presented, interpreted, and analyzed according to the primary research questions. This approach to data organization preserves the coherence of the information while bringing all the pertinent information together for the phenomenon being studied (Cohen et al., 2018). This approach to generating themes allowed the researcher to systematically collate all the pertinent data from the four data collection instruments to provide a collective response to the principal research question. According to Braun and Clarke (2019), the approach adopted by this study is termed deductive (or theoretical) thematic analysis and is adopted by researchers who produce codes and themes relative to a predetermined framework or specific research questions. Deductively analyzed and coded data may enable a thorough analysis of a specific aspect of the dataset through the lens of a specific theoretical framework (Braun & Clarke, 2021). Additionally, the first theoretical framework for this study was used to develop most of the subthemes within the first candidate theme. According to Byrne (2022), both deductive and inductive approaches are possible within reflexive thematic analysis. However, Braun and Clarke (2013, 2019, 2021) have consistently declared that rarely do coding and analysis fit neatly into one of these categories; instead, they frequently combine the two. Thus, in as much as the deductive analysis was a predominant approach for this study, inductive analysis was employed to produce other relevant and meaningful subthemes that were reflective of the data's content.

Braun & Clarke (2006) note that themes can be generated at semantic or latent levels. At the semantic level, themes address the more explicit or obvious meanings of the data. At the latent level, themes engage deeper interpretations of data, focusing on underlying assumptions, meanings, ideas, and ideologies.

In this study, the researcher's analytic interest informed the development of latent level, rather than semantic level, themes. These initial themes were then reviewed to establish whether they were functional and relevant for their intended purpose. Thereafter, the subthemes emanated from the 'candidate' (primary) themes. After an ongoing analysis, the themes and their subthemes were refined. Nevertheless, since this study adopted the reflexive thematic analysis style to data report, data were contextualized and synthesized as they were being presented. A diagram of the themes and subthemes is depicted and discussed in Chapter 5.

4.7 ENSURING THE TRUSTWORTHINESS OF THE RESEARCH

Trustworthiness is concerned with the criteria used to establish the quality of qualitative research (Korstjens & Moser, 2018). Although validity and reliability are post-positivism constructs for ensuring the credibility of quantitative data collection instruments, numerous trustworthiness criteria for qualitative data exist (Lincoln & Guba, 1985). The seminal criteria that were considered for this qualitative study are credibility, dependability, confirmability, and transferability. These criteria relate to how qualitative researchers can convince themselves and their audience that their research results can be trusted and are worthy of consideration (Lincoln & Guba, 1985; Korstjens & Moser, 2018). This study included member checking as a method to strengthen trustworthiness. The subsequent subsections delineate how this study conducted a reflexive thematic analysis that satisfied the criteria for trustworthiness.

4.7.1 Credibility

Tobin and Begley (2004) state that the criteria for credibility address the 'fit' between the views of participants and the researcher's representation of those views. As such, Lincoln and Guba (1985) recommend prolonged engagement, data collection triangulation, persistent observation, and research triangulation as techniques to address credibility.

In this study, the researcher worked as a lecturer teaching mathematics education modules for five semesters at the same institution where participants were being trained as preservice mathematics teachers. Having spent time in the participants' environment, the researcher had an understanding of the written, taught, and enacted curriculum that they experienced. This knowledge assisted in planning the study and influenced the development of the data collection instruments. The prolonged engagement in the participants' environment allowed the researcher to adequately plan for data collection triangulation by utilizing multiple data collection methods (as outlined in Section 4.5 of this chapter). Webb, Campbell, Schwartz, and Sechrest (1966) and Patton (1999) convey that triangulation is the utilization of multiple data collection instruments in qualitative research and is worth doing to develop a believable, as well as a comprehensive, understanding of phenomena.

To further ensure credibility, the researcher recruited two disinterested, knowledgeable peers, who were senior researchers, for peer debriefing sessions (Lincoln & Guba, 1985). The first peer debriefing session was held during the proposal stage of this study with one peer that possessed expert knowledge of research conducted in the field of mathematics education. Thereafter, during and after the course of the study, the second debriefer offered advice about methodological issues, ethical matters, fieldwork challenges, analysis strategies, and model development. The debriefing sessions were found to be particularly useful as they assisted the researcher to remain sober-minded and clear of feelings and emotions that could have clouded his decision-making. Figg, Wenrick, Youker, Heilman, and Schneider (2010) contend that critical friends as can be referred to as peer debriefers if they assist in ensuring that the research is successful by offering an unbiased external perspective, thereby certifying that the research study is rigorously trustworthy and credible.

4.7.2 Dependability

Tobin and Begley (2004) declare that dependability is achieved by researchers when they can ensure that the process of their research is logical, documented clearly, and traceable. With assistance from peer debriefers, the process used in this study was confirmed to be logical and clearly documented, with the thesis structured with adequate subsections and headings to facilitate understanding.

In addition, Lincoln and Guba (1985) postulate that, to further achieve dependability, the process of a research study must be externally audited. In addition to peer debriefing, an audit was conducted of the researcher's decisions and choices about methodological and theoretical issues. As such, this study contends that if another researcher were to conduct a similar study using the current study's data in a similar context, they would arrive at uncontradictory and comparable conclusions. This is the dependability criterion, as articulated by Koch (1994) and Shenton (2004). Thus, all records of raw data and researchers' reflexive journals will be kept safe for five years should the verification of results be required. The keeping of all records related to this study is expected to further assist with cross-referencing of data and ease the reporting of the process of research as Nowell, Norris, White, and Moules (2017) advocate.

4.7.3 Transferability

Transferability is concerned with the generalizability of a qualitative research study (Nowell et al., 2017). Tobin and Begley (2004) add that, with qualitative research studies, transferability is only possible on a case-to-case basis. Hence, this study provided thick descriptions regarding the contextual factors relating to the research site, methods, analysis, and participants' backgrounds to allow for the transferability of the findings to different sites (Lincoln & Guba, 1985). As such, other researchers and readers of this study's findings may transfer its results to other cases provided they exhibit features similar to those of this study. Korstjens and Moser (2018) confirm that qualitative researchers facilitate the judgment of transferability "by a potential user through thick description" (p. 121).

4.7.4 Confirmability

Confirmability refers to the extent to which the results of the empirical research study could be verified by other researchers (Korstjens & Moser, 2018). They further proclaim that this trustworthy criterion is concerned with the confirmation of collected data against the findings' interpretations by the researcher. The results of this study are free from the fabrication and creative imagination of the researcher, they are the true interpretations of the collected data and can be verified. Additionally, the discussions, interpretations, and conclusions for this inquiry were reached solely from the data collected from the ten recruited participants, as earlier stated. Guba and Lincoln (1989) claim that confirmability can only be established once credibility, dependability, and transferability have been achieved. Throughout this study, justifications for methodological, theoretical, and analytical choices were made as Koch (1994) suggests that this exercise is essential for confirmability, so that the reader can assess why and how decisions were taken.

4.7.5 Member checks

Member checking is another method of enhancing rigor in a qualitative research study (Birt, Scott, Cavers, Campbell, & Walter, 2016). For this study, participants took part in verifying the results for credibility. Harper and Cole (2012) attest that member checking is a quality control process whereby a researcher attempts to enhance the validity, accuracy, and credibility of the recorded research interviews. For the semi-structured interviews, after interviewing all the participants, the researcher summarized information for each participant and questioned them to determine the accuracy of their recorded responses (Lincoln & Guba, 1985; Creswell, 2007). The transcribed verbatim data was shared with each participant individually for the verification of their responses. Birt et al. (2016) confirm the essentiality of returning transcribed verbatim data for member checks in qualitative research studies to allow participants to reconstruct their narratives if they feel they were misinterpreted. Thus, all ten participants individually confirmed the interview transcripts reflected in the next chapter.

Since this study utilized focus group discussions to generate additional data, a member checking exercise was implemented for the focus groups as well (Nyumba, Wilson, Derrick, & Mukherjee, 2018). Two participants from each of the originally selected focus group members were used to member check the transcripts. This is an essential process to ensure credibility and rigor prior to presenting the findings.

4.7.6 Ensuring trustworthiness in every phase of reflexive thematic analysis

Trustworthiness was ensured at every phase of the six reflexive thematic analysis phases that were followed to analyze and present the data. This section describes how the criteria for trustworthiness – credibility, dependability, transferability and confirmability – were upheld in each phase. While this account is presented in a linear format, it was impossible for the actual process of analysis to proceed in a linear fashion; rather, it was recursive across the six phases.

Phase 1: Familiarization with data. Familiarization with data for this study occurred in stages since the researcher collected data in sequential phases. The first stage of data familiarization commenced immediately after the participants had completed the performance test and before the semi-structured interviews. Prolonged engagement with the first dataset was crucial since the researcher needed to be familiar with this initial data to be able to predetermine some of the probing questions for the individual interviews. Strengthened triangulation with different data-gathering instruments was achieved in this manner. Similarly, after the interviews, the researcher actively listened to each individual interview recording and documented some theoretical and reflective thoughts about this second set of data. The researcher did not transcribe the recordings at this stage. This process was key to the preparation for the focus group discussions as all the data collection modes worked together to produce a coherent dataset. The same active listening procedure was performed with the recordings of the two focus group discussions. Thereafter, while the observation sessions were being conducted, the researcher began to transcribe the individual interviews, and subsequently, the focus group discussion recordings. The researcher then carefully read each individual transcript of the semi-structured interviews and group discussions and noted initial observations of some trends in these data. The same was done with the observation questionnaires that were received from the participants' mentors. Finally, all records of raw data collected, including the field notes, were stored in organized archives to prepare for the next phase of analysis and for the audit trial.

Phase 2: Initial code generation. Using the participants' scripts from the performance test, the researcher manually coded (by hand) each response. Once all the scripts were coded, the individual interview transcripts were coded using the Microsoft Word 'comment' feature. This feature allowed the researcher to highlight the piece of text that corresponded to each respective code. The same feature was utilized to code the verbatim textual data from the focus discussions and the open-ended questionnaire responses from the participants' mentors. To ensure credibility in the analysis in this phase, the researcher sought counsel from the supervisor to evaluate the initial codes that had been established. The supervisor evaluated the initial codes from three performance tests, three individual interview transcripts, one focus discussion transcript, and three observation responses. This was a recursive non-linear systematic process since the peer debriefers were equally involved in this code generation phase. During and after the debriefing session meetings, reflexive notes were documented by the researcher to ensure the trustworthiness of this study.

Phase 3: Discovering themes. The candidate themes were derived deductively using the primary research questions for this study. All datasets were utilized equally to interpret and deeply analyze the data that related to the primary research questions. Thematic maps were drafted to establish connections between themes and make sense of the subthemes that were emanating from the candidate (primary) themes. The researcher kept detailed notes during this process of initial theme development.

Phase 4: Reviewing themes. The detailed notes taken during the previous phase (which included themes and subthemes) and records of raw data stored during the first phase were reviewed thoroughly by the researcher. Input was obtained from the peer debriefers, as well, during this phase to establish whether or not the candidate themes and their respective subthemes were coherent. From this exercise, it was established that there was not enough data to support two of the themes (from the third phase) as independent themes. During this phase, the researcher constantly tested the generated themes against the raw data for referential adequacy.

Phase 5: Defining and naming themes. During this phase, the researcher formally presented a detailed analysis of every theme and their respective subthemes in relation to both the primary research questions and the dataset (Braun & Clarke, 2006). Some of the themes and subthemes' names were refined to make them more succinct and ensure that they depicted the contents of each theme. One meeting with the peer debriefers assisted in ensuring that the refinement process was effective and that themes fitted into a lucid overall picture of the phenomenon being studied. Additionally, the researcher's supervisor contributed to ensuring that the findings were credible and were a true reflection of the data by reading the data from each theme and subtheme in relation to raw data (performance test responses and verbatim transcripts).

Phase 6: Producing the report. The researcher's task during this phase was mainly to establish cohesiveness, conciseness, and logical reporting of data within the themes and their respective subthemes. This is because the write-up had already commenced in the fifth phase, although it was not yet completely coherent. Braun and Clarke (2013) point out that qualitative research write-up is intertwined with the process of data analysis. An important aspect of this phase was to further finalize member checking for both individual interviews and focus group discussion transcripts to ensure the criteria for trustworthiness of the analysis were met. In this phase, the findings were interpreted and analyzed dialogue with the literature that had been reviewed (Chapter 2).

4.8 ETHICAL ISSUES

Research ethics are focused, essentially, on what is morally and legally acceptable and permissible (Bickman & Rog, 2009; Fouka & Mantzorou, 2011). Brooks et al. (2014) advocate for educational researcher to consider certain ethical principles before, during, and after the study has been conducted. This study was guided by the ethical principles of respect, dignity and ensuring no harm identified by Howe and Moses (1999) as well as the principles of access, participant autonomy, anonymity, confidentiality, non-maleficence and beneficence.

4.8.1 Permission to conduct research

As the target group in this study comprised students registered at the selected institution, access had to be obtained initially from the gatekeeper at the university (the registrar) prior to engaging the participants. Singh and Wassenaar (2016) describe a gatekeeper as an individual who is in control of allowing access to a privately-owned organization or institution, such as an administrator, school principal, or managing director. After access was granted by the university, further permission was required from the Department of Basic Education (DBE) to be able to observe preservice teachers during their teaching practice at schools. The DoE head of department for the province of KwaZulu-Natal granted the researcher access to schools (Reference number: 2/4/8/7094) for the period 20 April 2021 to 10 October 2023 (see Appendix G). Upon being granted access by both gatekeepers, ethical clearance was applied for and subsequently approved on 5 May 2021 (see Appendix A) by the Humanities and Social Sciences Research Ethics Committee (HSSREC) of the University of KwaZulu-Natal (Ethics Protocol Reference Number: HSSREC/00002656/2021). The HSSREC board ensures that every study conducted by either staff or students at the university is conducted in a way that safeguards the rights, safety, and dignity of the participants.

4.8.2 Participant autonomy

Participant autonomy relates to the informed consent principle which supports the participant’s right to freedom and self-determination (Lincoln & Guba, 1985). In this study, immediately after obtaining the ethical clearance letter the process of recruiting the participants commenced. All potential participants were emailed an invitation letter that included a brief description of the study. The informed consent letter, with information about the study, was also attached. The letter included a declaration page for participants to indicate their willingness to participate and, should they consent, their signature was required. With their signatures, participants legally bound themselves to take part in the study (Howe & Moses, 1999).

However, the right to withdraw from the study was reaffirmed (Nijhawan, et al., 2013). All of the prospective participants who were approached indicated their willingness to participate.

4.8.3 Participants' anonymity and confidentiality

Bos (2020) declares that participants' anonymity and confidentiality are important ethical principles. In this study, participants' anonymity was ensured as they were assigned unidentifiable pseudonyms. Additionally, the informed consent form stipulated that their personal information would not be disclosed by the researcher, supervisor of the study or by their school mentors who observed their lessons (see Appendix B). Confidentiality assurance was emphasized to the participants in writing and verbally before, during and after all the data collection phases. This was done because participants indicated some concern that the researcher might share information from the study with their lecturers, as the research was a mathematics education lecturer at the same institution.

4.8.4 Non-maleficence and beneficence

The principle of non-maleficence (do no harm) and beneficence (beneficial and positive research outcome) stipulate that ethical research must ensure that the "benefits outweigh the potential for harm" (Murphy & Dingwall, 2001, p. 340). Research should not inflict physical pain on participants nor should it damage participants psychologically, personally, emotionally, or professionally (Oliver, 2003). This study entailed no foreseeable risks of harm that could have been inflicted on participants. They were to be engaged in their natural setting and neither the interviews nor the focus group discussions contained questions that had the potential to cause them harm in any form.

4.9 CONCLUSION

This chapter has discussed the methodological approach employed in this study. The researcher's choice of research paradigm, approach, design, sample of the population, methods of sampling, data collection methods, and data analysis were justified and discussed to provide a comprehensive understanding of how the study was conducted. The chapter presented the measures that were taken to ensure trustworthiness and rigor in the study and addressed the process taken to ensure that the study was ethically sound and approved by the relevant institutions. The next chapter presents the reflexive thematic analysis of the data collected from multiple cases.

Chapter 5: Data Presentation and Analysis

5.1 INTRODUCTION

This chapter presents the data in response to the following main research questions:

1. What visualization strategies are used by preservice mathematics teachers to solve mathematics problems?
2. What educational technologies do mathematics preservice teachers intend to use in their teaching?
3. Why do mathematics preservice teachers believe that their chosen educational technologies can influence their teaching and concept development? and
4. How do mathematics preservice teachers demonstrate the implementation of these educational technologies in their teaching during teaching practice?

The chapter commences with a brief description of the participants. Thereafter, the findings from the performance test and semi-structured interviews are presented and analyzed concurrently according to the main tenets of the commognitive framework. Subsequently, the themes and sub-themes that emerged from the focus group discussions and observational findings are presented and analyzed in relation to the TPACK framework.

5.2 PROFILE OF PARTICIPANTS

Key characteristics of the ten third-year preservice mathematics teachers are presented in Table 5.2.1. The participants were each considered as an individual case as they exhibited some differences, as well as similarities. The last column of the table indicates the focus group to which the participant was assigned.

Table 5.2.1: Profile of participants

Name	Age	Gender	Secondary school attended	School Quintile	Year of study	Mathematics teaching experience	Focus group
Shan	21	M	Daleview Secondary School	4	3	1.5yrs	FG2
Zakhe	26	M	Zindlalele Secondary School	4	3	3yrs	FG1
Zot	22	M	Khombindlela High School	2	3	1yr	FG2
Lunge	25	M	Zwelibanzi High School	3	3	3.5yrs	FG1
Sandi	22	M	Bambazi High School	3	3	1yr	FG2
Phume	22	F	Buhlebemfundo Secondary School	2	3	4 months	FG2
Lethu	22	F	Maceba High School	2	3	0.5yr	FG1
Amah	22	F	Merebank Secondary School	4	3	1yr	FG2
Qhaw	23	M	Nqabakazulu Secondary School	3	3	1.5yr	FG1
Sfi	22	M	Mehlomlungu Secondary School	2	3	1.5yrs	FG1

As depicted in Table 5.2.1, all participants were in the third year of study. They were and enrolled in the same faculty at the same institution. However, they differed in terms of gender, age, secondary schools attended, and their previous teaching experience of mathematics. Their profiles revealed that male participants possessed more previous teaching experience in mathematics than females. Although the ratio of male to female participants in this study was 7:3, on average the ratio of male to female participants' experience with practice teaching was 88% :12% in the school calendar days respectively.

The secondary school that each participant had attended had significant contextual implications for this study, as anecdotal evidence suggests that preservice teachers are most likely to adopt teaching techniques similar to those they were exposed to during their experience as learners. Additionally, those participants who possessed more years of teaching experience indicated their exposure occurred when they volunteered at their former schools during semester breaks and on weekends. In addition, they received more exposure when the universities and schools suspended face-to-face teaching due to the Covid-19 restrictions imposed during the national lockdown. For instance, Lunge, Zakhe, Sfi, Qhaw, and Shan indicated that they had assisted Grade 10, 11, and 12 learners at their former secondary schools with mathematics when learners took turns to come to school to maintain social distancing. These learners experienced significant challenges with mathematics during the first wave of the pandemic in South Africa as the learners and their schools did not have access to the technological tools required for remote learning.

5.3 IDENTIFICATION OF THEMES AND SUB-THEMES IN THE DATA

Figure 5.3.1. provides a thematic map of the three primary themes that emerged from the data in relation to the three main research questions posed in this study, along with their respective sub-themes. Five sub-themes were identified within the first primary theme which, together, responded substantially to the first two research questions. The first four sub-themes were developed deductively using the tenets of the commognitive framework, while the fifth sub-theme emerged inductively from the data. Five themes were developed inductively within the second primary theme. Similarly, within the last primary theme, five sub-themes emerged inductively. Together, these themes and sub-themes provided a comprehensive set of findings that addressed the main research questions.

These three primary themes, along with their sub-themes, are presented in the sections that follow.

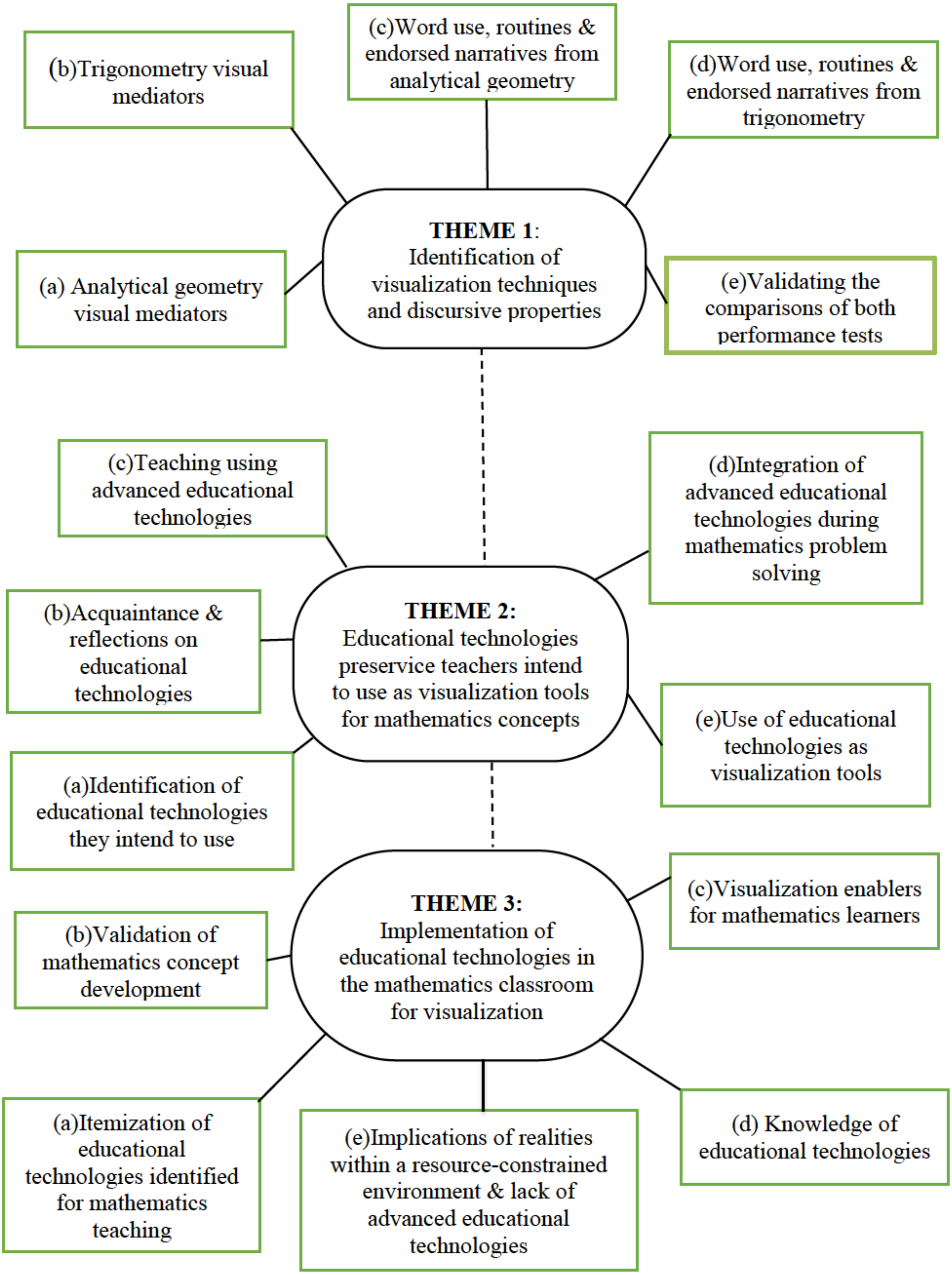
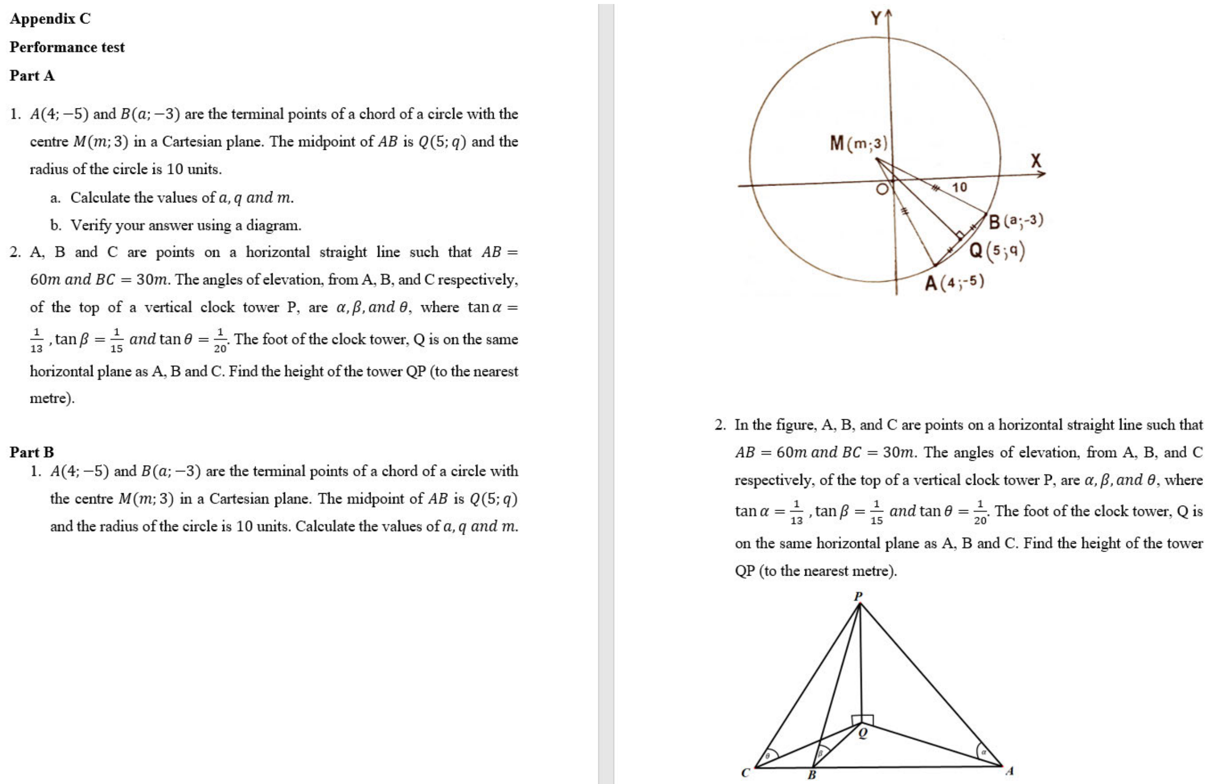


Figure 5.3.1: Thematic map depicting three themes and sub-themes.

5.4 THEME 1: IDENTIFICATION OF VISUALIZATION TECHNIQUES AND DISCURSIVE PROPERTIES

The initial phase of this study investigated the visualization strategies and techniques used by participants when engaging in mathematics problem-solving tasks. Using reflexive thematic analysis, the data collected from performance tests and semi-structured individual interviews were coded deductively and thematized concurrently. These data were interpreted using the commognitive theoretical lenses while framed by the first research question of the inquiry.

The study found that preservice participants' mathematical self-discourse was characterized by four of the four discursive constructs identified in Sfard's (2007, 2008) theory of commognition (discussed in Chapter 3): visual mediators, endorsed narrative, routines and word use. Four themes emanated from participants' responses to the analytical geometry and trigonometry questions and their interview narratives that aligned with the four discursive constructs of the commognitive framework. For ease of reference, performance test questions are depicted in Figure 5.4.1.



Appendix C

Performance test

Part A

1. $A(4;-5)$ and $B(a;-3)$ are the terminal points of a chord of a circle with the centre $M(m;3)$ in a Cartesian plane. The midpoint of AB is $Q(5;q)$ and the radius of the circle is 10 units.
   a. Calculate the values of a, q and m.
   b. Verify your answer using a diagram.
2. A, B and C are points on a horizontal straight line such that $AB = 60m$ and $BC = 30m$. The angles of elevation, from A, B, and C respectively, of the top of a vertical clock tower P, are $\alpha, \beta, and\ \theta$, where $\tan\alpha = \frac{1}{13}$, $\tan\beta = \frac{1}{15}$ and $\tan\theta = \frac{1}{20}$. The foot of the clock tower, Q is on the same horizontal plane as A, B and C. Find the height of the tower QP (to the nearest metre).

Part B

1. $A(4;-5)$ and $B(a;-3)$ are the terminal points of a chord of a circle with the centre $M(m;3)$ in a Cartesian plane. The midpoint of AB is $Q(5;q)$ and the radius of the circle is 10 units. Calculate the values of a, q and m.

2. In the figure, A, B, and C are points on a horizontal straight line such that $AB = 60m$ and $BC = 30m$. The angles of elevation, from A, B, and C respectively, of the top of a vertical clock tower P, are $\alpha, \beta, and\ \theta$, where $\tan\alpha = \frac{1}{13}$, $\tan\beta = \frac{1}{15}$ and $\tan\theta = \frac{1}{20}$. The foot of the clock tower, Q is on the same horizontal plane as A, B and C. Find the height of the tower QP (to the nearest metre).

Figure 5.4.1: Phase1: Performance test questions

5.4.1 Analytical Geometry visual mediators

Iconic (graphic, geometric) and symbolic (formulas and symbols) visual mediators were identified in the participants' responses. Figure 5.4.1.1 depicts Lunge's response to Question 1 of Part A of the test.

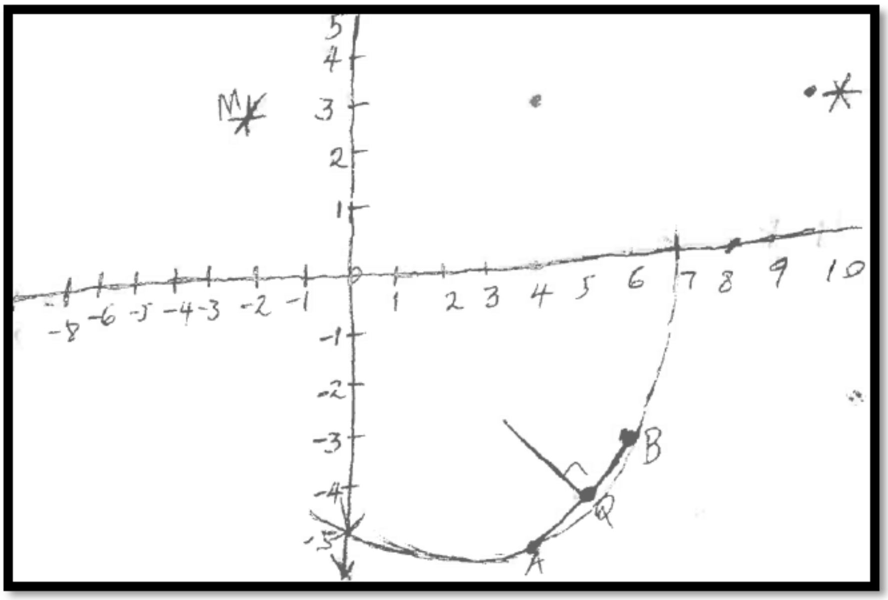


Figure 5.4.1.1: Lunge's visual mediator for Question 1 of Part A

Lunge's visual mediator, shown in Figure 5.4.1.1, depicts his visual interpretation of the problem. Although his sketch was not accurate, the labels on the diagram correspond with the problem statement and demonstrate his understanding of the problem. During the semi-structured interview, Lunge explained that when he encounters a mathematics word problem, he commences by visualizing the problem statement using concrete visual images on paper:

> *I'd say, scribbling around on the question paper and trying to sketch something visually is the way to get away with these types of questions; just reading the statement only is not enough to see what you need to do* (Lunge).

This process of scribbling and making sketches assisted Lunge to visualize and solve this problem. According to Supardi, Zayyadi, Lanya, Hasanah, and Hidayati (2021), it is common for students to utilize graphic visual mediators when dealing with word problems in mathematics. Other participants, including Shan, Qhaw, Sfi, Lethu, and Zot successfully utilized a similar visualization strategy for the first problem in Part A. Shan and Sfi explained as follows:

I think I'm more of a visual learner so I always try to draw it out if I can…so when I saw the coordinates the first thing I thought was ehmm; if they give us the coordinates A(4; -5), so I thought if I can just plot it on the Cartesian plane so I can see how the circle could possibly look like or where the chord was… (Shan).

The strategies to use to solve these types of problems is to sketch some diagrams that depict what you are internalizing as you read to see what you want to solve and whether the answer makes sense on your drawing…(Sfi).

These narratives suggest that these participants preferred a 'visual' sketch in order to comprehend mathematics word problems. For Shan, it was vital to see how the circle looked on the Cartesian plane given the centre and other coordinates that defined the circle in question.

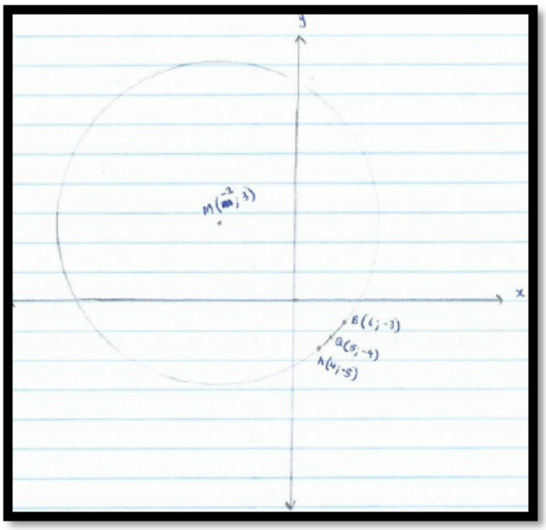


Figure 5.4.1.2: Shan's visual mediator for Question 1 of Part A

Figure 5.4.1.2 represents the visual mediator Shan utilized to see how the circle was located on the Cartesian plane inclusive of all the other pertinent details. Likewise, for Sfi, viewing what ought to be solved in the form of sketches is significant during the process of internalization and making sense of the solution. Arcavi (2003) proclaims that "seeing" is a significant element of the visualization of mathematical problems. Sfi's visual representation of the problem statement developed as he was solving the problem and determining the values of unknowns. When expanding on Figure 5.4.1.3, he elaborated that the final sketch he drew was the one appearing on the Cartesian plane, while the other three circles floating around the axis were merely initial sketches of what he was internalizing as he was reading the statement.

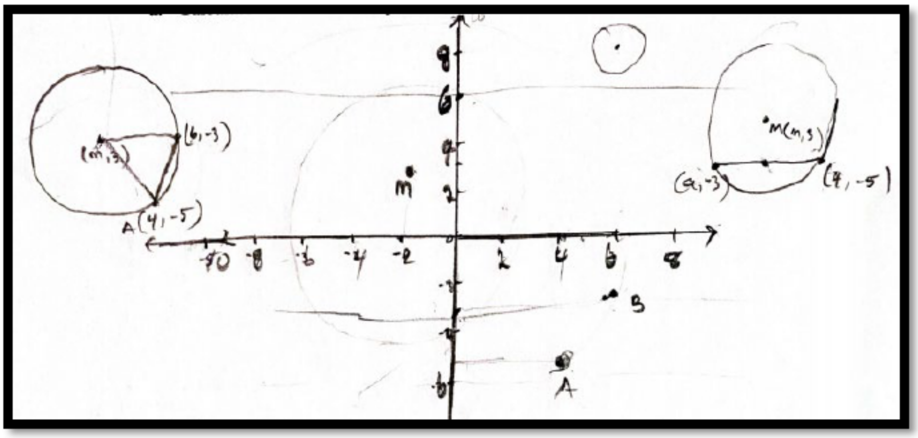


Figure 5.4.1.3: Sfi's visual mediator for Question 1 of Part A

Moreover, his explanation depicts the visualization processes as described by Mudaly and Rampersad (2010) in their model of processes involved in visualization. They established that internalization and externalization of knowledge occur when the meaning of mental images is formed through "reflection, interaction with the new stimuli and other given data" (Mudaly & Reddy, 2016, p. 181). Similar to Mudaly and Rampersad's (2010) description of the visualization processes, Sfi utilized the internalized new knowledge to influence what he added to his visual mediator as it gradually developed to the final image. Additionally, Qhaw in his narrative pointed out that depicting the information from a problem statement with the aid of sketches contributed to making sense of a problem. He maintained that

> *It is to first note every information they are giving you from the statement of the problem, that is strategy number 1…then sketch the given information diagrammatically and try to add more information into your sketch as you read the statement until it makes sense…* (Qhaw).

Qhaw and Phume's approach was similar to Sfi's as they all progressively developed their iconic visual mediators as they acquired additional information from the problem statement and the steps of their solution as they calculate the values of other unknown variables.

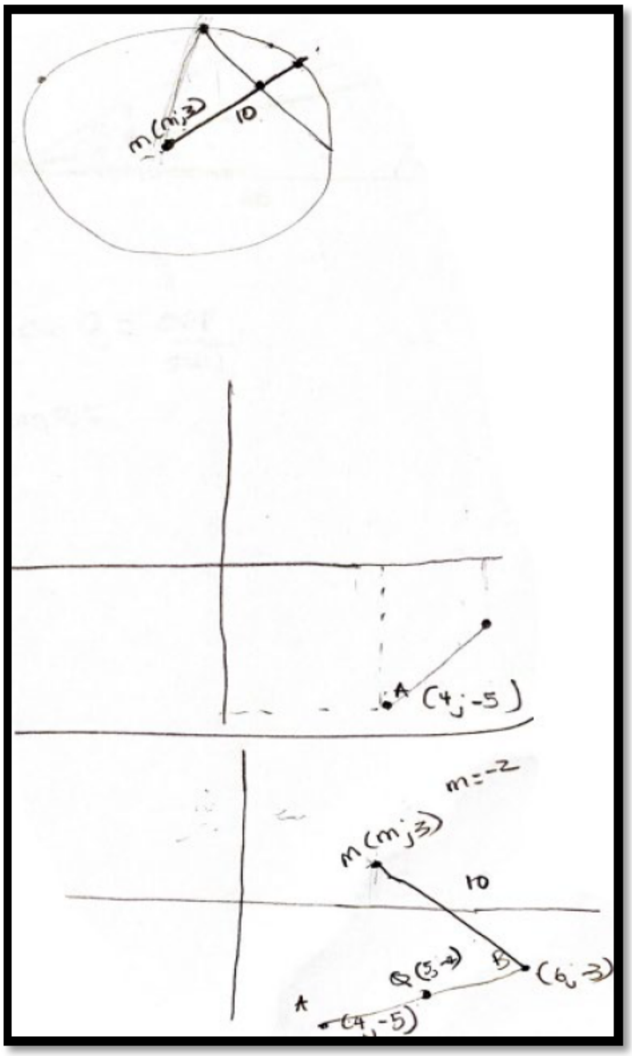


Figure 5.4.1.4: Qhaw's visual mediator for Question 1 of Part A

Figure 5.4.1.4 illustrates the visual mediators that Qhaw used to visualize the problem. His narrative also concurred with Mudaly and Rampersad's (2010) concept of visualization processes that are rooted within internalization and externalization activity. Additionally, Qhaw's writing down of information diagrammatically and adding information onto the diagram to make sense of the question correlates with Diezmann's (2000) and Mudaly and Reddy's (2016) interpretation of how students comprehend questions. Diezmann (2000) explains that writing information down in a visual mediator (diagram) is a process of translation that involves decoding linguistic information and encoding visual information. These interpretations of the visualization of mathematics problems demonstrate that the process of individualization, a fundamental aspect of the commognitive framework, occurred.

Similarly, Phume's visual mediators, shown in Figure 5.4.1.5, demonstrate how the visualization process progressed, first as she initially internalized the problem statement, and then after she solved the problem.

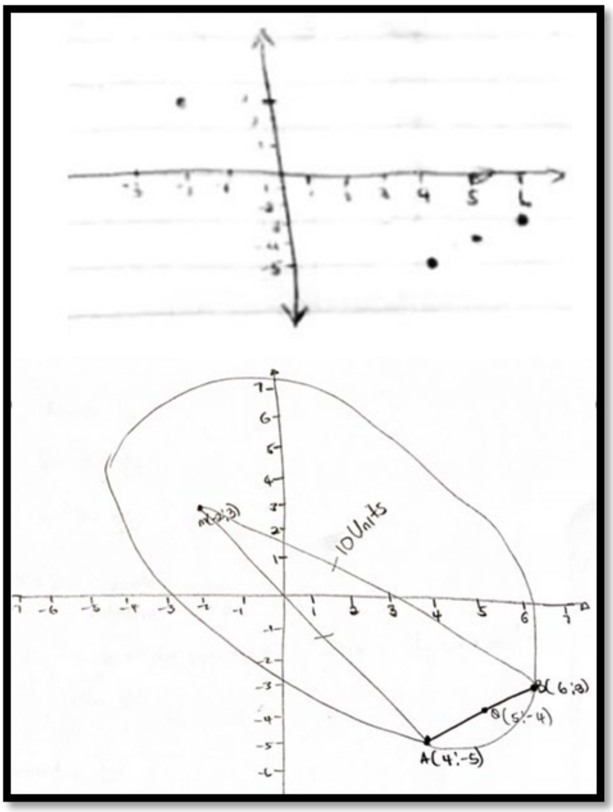


Figure 5.4.1.5: Phume’s visual mediator for Question 1 of Part A

It is apparent that these visual mediators were not accurate but were merely individual representations of the interpretation and mental images of these problems in their heads.

Figure 5.4.1.6 represents a visual mediator Amah sketched to verify her solution for Question 1a. From her diagram, she realized that her solution was incorrect, which she also mentioned in her narrative. However, if there had not been a follow-up question requiring her to verify her solution using a diagram, she would have assumed that her calculations were correct. The sketch of her diagram using the calculated unknowns revealed that she was incorrect, since line segment MQ was supposed to be perpendicular to the chord AB, which was not represented accurately in the diagram indicated. She stated that

> *…when I had to verify my answer using a diagram my values for a, m, and q were incorrect because they didn’t come out perfectly as a circle with other details corresponding like a chord and perpendicular bisector* (Amah).

Her narrative demonstrates that blind manipulation of mathematical symbols and routines may not always yield accurate results. In this regard, Vale and Barbosa (2018) assert that the use of visual methods during mathematical problem-solving is vital, as without them the chances are greater that students will attempt to solve problems without adequate understanding.

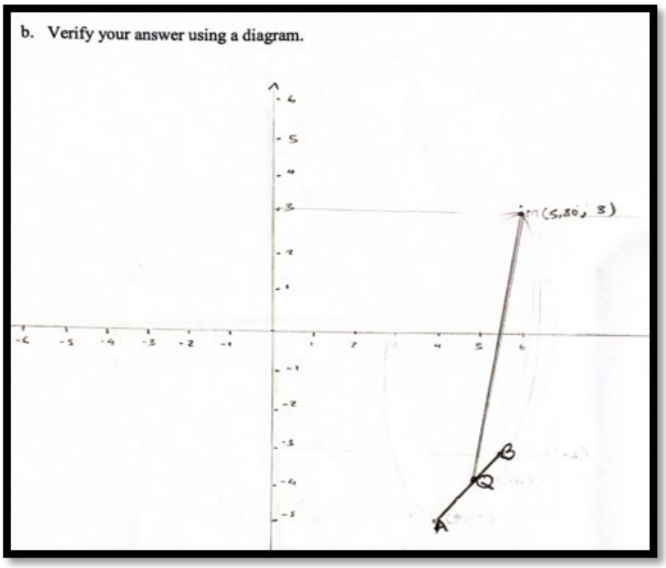


Figure 5.4.1.6: Amah's visual mediator for Question 1 of Part A

All the participants' narratives revealed that the visualization strategies they employed assisted them in verifying their solutions. In addition to the example of Amah being able to identify that there was a problem with her calculations after sketching her solution, other participants were able to verify that their solutions were correct using visual mediators. These findings concur with Pantziara, Gagatsis, and Elia (2009), who report that the "mathematics education community has long recognized the importance of diagrams in the solution of mathematical problems" (p. 39).

Lethu and Sandi, however, shared a slightly different approach in terms of their visualization strategies:

> *I create mental images without even drawing them down on paper, I visualize them in my head, especially when it is an easy problem like number 1…*(Lethu).

> *For analytical geometry, every time – before I even begin solving the problem – in my mind I sketch the Cartesian plane. Then I try to plot the coordinates as they appear when I read the problem statement* (Sandi).

These results demonstrate that, depending on the level of difficulty of a mathematics problem, it is possible to mentally visualize a problem without any concrete visual mediators on paper. Mudaly's (2021) empirical study revealed that students can organize the data in their heads, devise strategies, and work towards the solution to a mathematics problem with diagrams existing only in their heads. These visualization strategies can only be possible when those engaging in such mathematics problems are consciously 'thinking about their own thinking' during the process of solving the problem. This is what Sfard (2007) terms 'intrapersonal communication'. Both of these preservice participants only utilized graphic visual mediators after solving the problem as a way of verifying their solutions. Figure 5.4.1.7 shows Lethu's verification diagram on the left and Sandi's on the right. Both of these graphic visual mediators (not drawn to scale) depicted correct solutions to the first problem after the rough sketches they drew to interpret the problem.

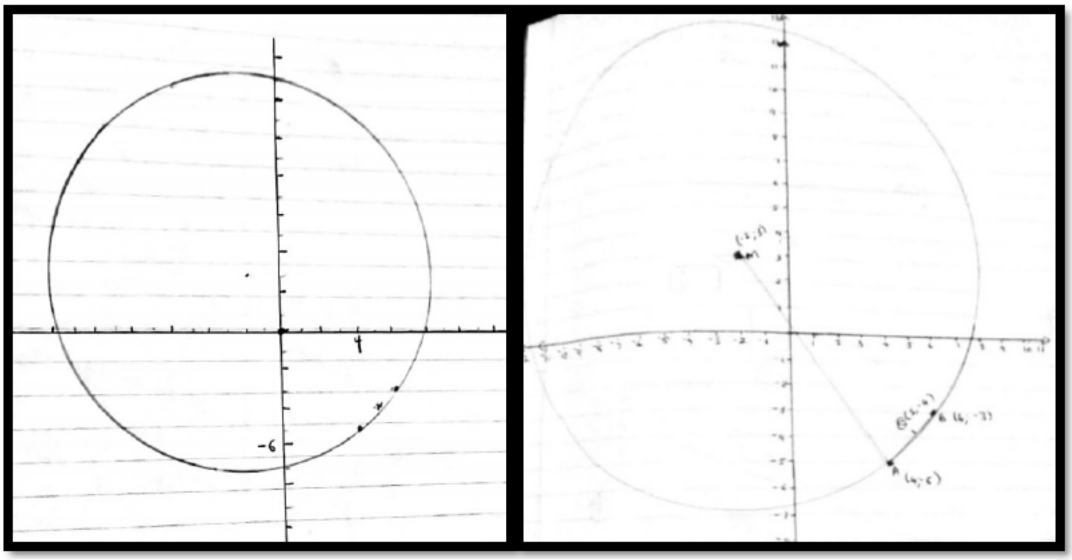


Figure 5.4.1.7: Lethu's and Sandi's visual mediators for Question 1 of Part A

Zot had a similar response, as he also relied on his mental visual images while he was solving the first problem and only made a visual sketch on paper for verification purposes. He maintained:

> *I was able to solve number 1 without sketching a diagram on the paper, it was easy to visualize it…*(Zot).

Figure 5.4.1.8 depicts Zot's graphic visual mediator illustrating the radius perpendicular to a chord on circle centre M as a way to verify his solution to the first question.

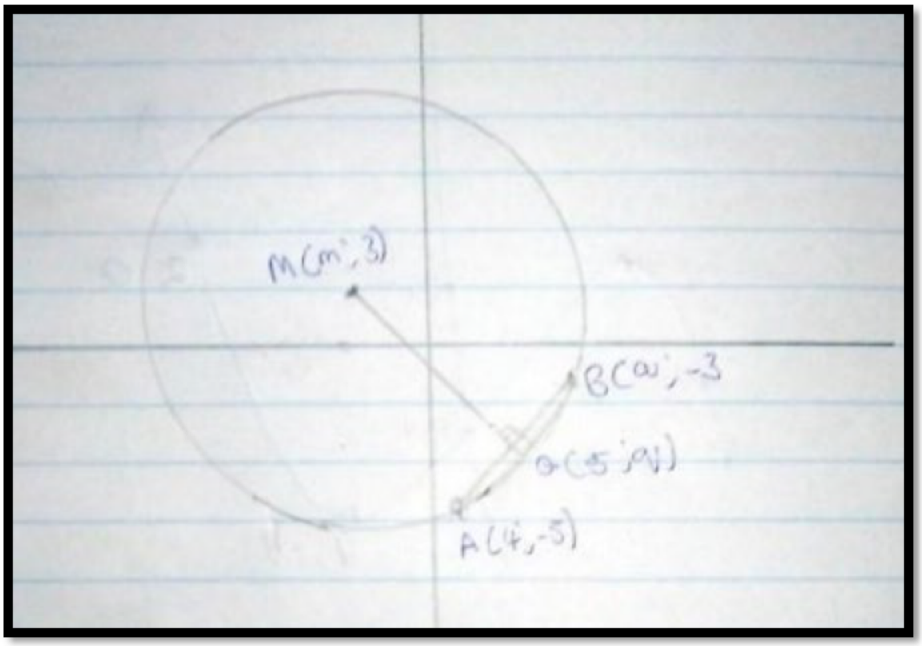


Figure 5.4.1.8: Zot's visual mediator for Question 1 of Part A

All of the preservice participants were able to verify their responses to the first question in Part A through the use of visual mediators such as the graphic representation of a circle, line segments indicating the chord and its perpendicular bisector. Other symbolic visual mediators, including the equation of a circle as well as the midpoint, distance, and slope formulae, were also in their responses. These symbolic visual mediators were crucial to solving the first question; they are presented in the next subsection on mathematical word use and routines.

5.4.2 Trigonometry visual mediators

For the second question in Part A of the test, preservice participants responded to a problem involving 3D trigonometry presented without a diagram (See Figure 5.4.1). Although this problem was more difficult than the first question, it required similar skills of visualization. Preservice participants experienced difficulty with this problem and only two of the ten arrived at a correct answer. Even though the focus of the exercise was not on obtaining correct answers, their approaches demonstrated a limitation in their ability to visualize higher-order mathematics problems. Eight participants indicated that they were unaware that the problem statement was referring to a 3D problem and produced visualization sketches in 2D format. During the individual interview sessions, they explained that they had not been able to visualize this problem and, thus, had been unable to solve it. Sandi described his attempt to solve the problem:

> *For the second one, yes, I tried to sketch my diagram to have a visual image and I tried to calculate. But then I ended up having more unknowns – like two or more in one*

equation – which was then confusing. And also, with my angles, I could not tell as to how I was going to manipulate them accordingly to help me get something useful and not confusing. So, I can also say that it was hard for me to have a clear diagram for number2; that's why I ended up just putting ABC that way on the straight line (Sandi).

Sandi's narrative confirmed that he was unable to visualize the second problem, as is evident in Figure 5.4.2.1.

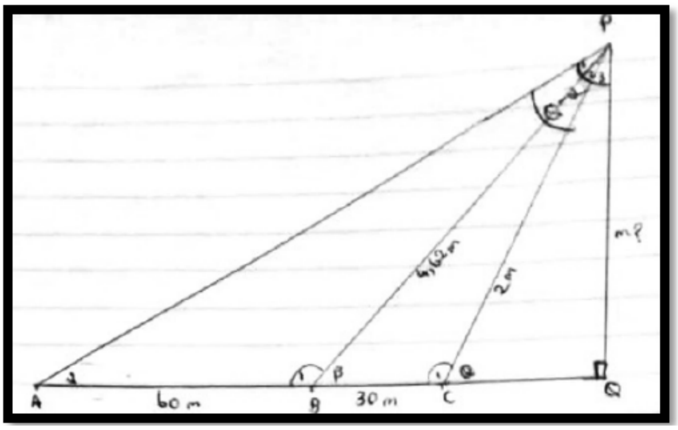


Figure 5.4.2.1: Sandi's visual mediator for Question 2 of Part A.

Failure to visualize the problem correctly resulted in failure to solve the entire problem. Osman, et al. (2018) emphasize this issue. In their study, they established that the ability to visualize mathematical problems is vital for success in problem-solving. They argue that visualization acts as an essential element in mathematics problem-solving as it reveals students' level of understanding and imagination. The participants in this study demonstrated a superficial level of understanding and imagination in their attempt to answer the second question in Part A. For instance, A, B, C, and Q on the straight line indicates that this participant (Sandi) misinterpreted the statement of the question, which indicated that A, B, C, and Q are on the same horizontal plane. This was a common mistake among the other participants, including Sfi, whose geometric visual mediator is depicted in Figure 5.4.2.2. Both of these participants could identify the height of the tower PQ as well as the angles of elevation from each point A, B, and C visually, however, they could not visualize the footing of the tower at Q. Sfi explained his thought process as follows:

For the second question of part A, I honestly thought they were talking about the 2D problem. That's the only thing I could think of until I saw part B – then I realized I missed the point of the question there and my sketches were not correct (Sfi).

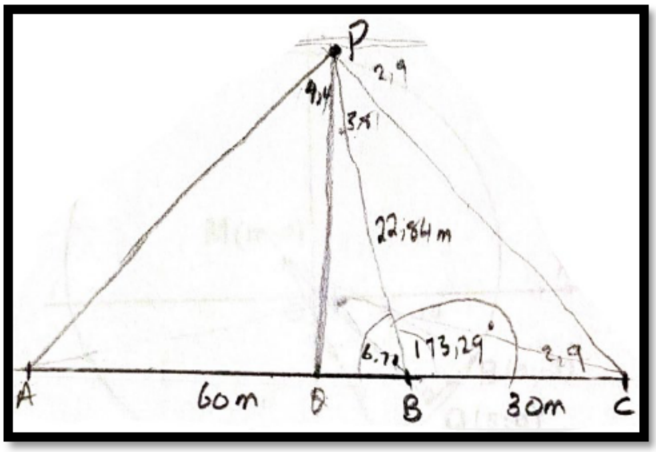


Figure 5.4.2.2: Sfi's visual mediator for Question 2 of Part A

His narrative indicates that he could not locate the correct footing of the tower PQ because he assumed he was dealing with a 2D problem, which demonstrated limited visual-spatial thinking. Hawes and Ansari (2020) postulate that there is increased consensus that visual-spatial thinking has a significant impact on how individuals deal with mathematical problems. This participant (Sfi) only realized once he received the second set of questions (Part B) that his solution for the second question of Part A was incorrect. A similar challenge was identified in Lunge's attempt to solve the same question as he also failed to locate the foot of the tower that the problem statement referred to. Figure 5.4.2.3 represents the geometric visual mediators Lunge sketched to visualize the problem.

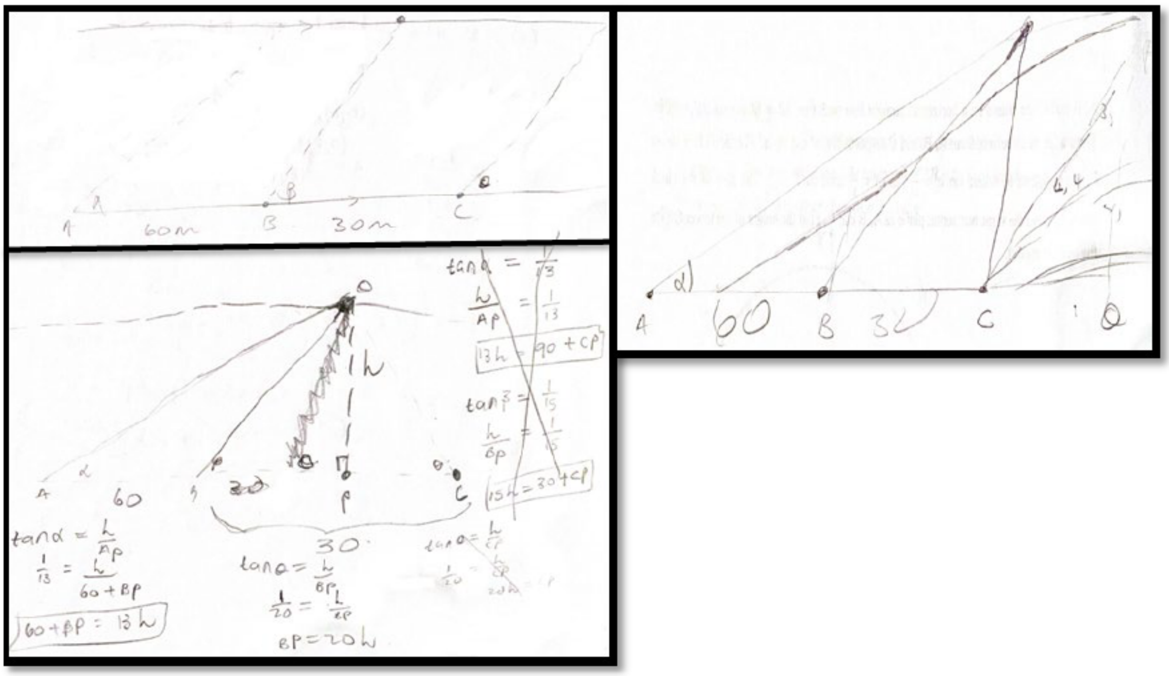


Figure 5.4.2.3: Lunge's visual mediator for Question 2 of Part A.

In each of the three draft sketches it is apparent that he did not visualize this problem as he plotted Q at a position that did not allow a 3D diagram to be drawn. During the interview session, he explained:

> *Number 2 was very tricky, because if you are trying to visualize the diagrams and once you have the wrong idea in your head then you will be out in terms of getting the correct answer. Wrong interpretation will result in an incorrect visual image and wrong solution based on the misinterpretation of the statement* (Lunge).

His narrative confirmed his inaccurate visual mediator of the problem, which was a result of his misinterpretation of the problem statement, to which he alluded. Lunge acknowledged that if someone is failing to visualize a mathematics problem they will be less likely to arrive at a reasonable or correct solution to the problem. This reveals his cognizance of the importance of the ability to visualize during mathematics problem solving – which may positively impact on his teaching as a mathematics teacher. He explained his process while trying to solve the problem further:

> *…when I first read this question I realized that I had to do some drawing since I saw a statement talking about the angle of elevation…I thought that was all happening in 2D, which is why everything was just incorrect… when they said you have ABCQ on the same plane I wrongly assumed that they were saying ABCQ is on the same straight line – as you can see on my first sketch* (Lunge).

His sketches confirm that he assumed that the question was referring to a 2D problem as the points A, B, C, and Q can be observed lying on the horizontal straight line instead of on a horizontal plane with Q on the other side of straight-line ABC. His narrative also demonstrates that when students are challenged to visualize and understand a mathematical word problem, they are prone to making assumptions just so they can apply their procedural knowledge. A similar misinterpretation was noted in Phume's visual mediator, shown in Figure 5.4.2.4. She explained

> *For number 2, I couldn't solve it fully, and I don't feel comfortable explaining it because I feel like I'm dismally wrong. I was just guessing, for this one, from the ratios that were given of tan. I was just guessing, because I didn't want to leave a blank space; I couldn't really solve it. I only reasoned about the ratios…(Phume).*

Her narrative, together with her sketch, shown in Figure 5.4.2.4, indicate that she did not understand the problem statement and was unable to visualize the problem. This further infers that the inability to understand the mathematics word problem resulted in failure to visualize it and thus failure to solve it.

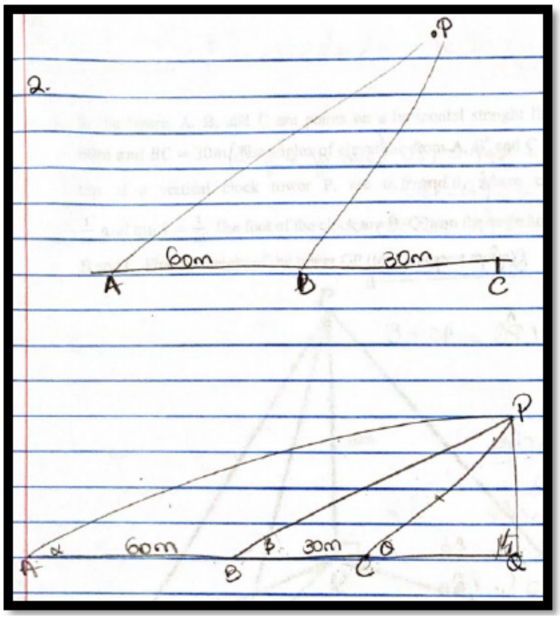


Figure 5.4.2.4: Phume's visual mediator for Question 2 of Part A

Amah's narrative also demonstrated an inability to create visual mediators for a mathematics word problem that required some form of visualization, which resulted in an incorrect solution and demonstrated a lack of deep understanding. All her visual mediators, as indicated in Figure 5.4.2.5, revealed that she was unable to accurately visualize the question according to the problem statement.

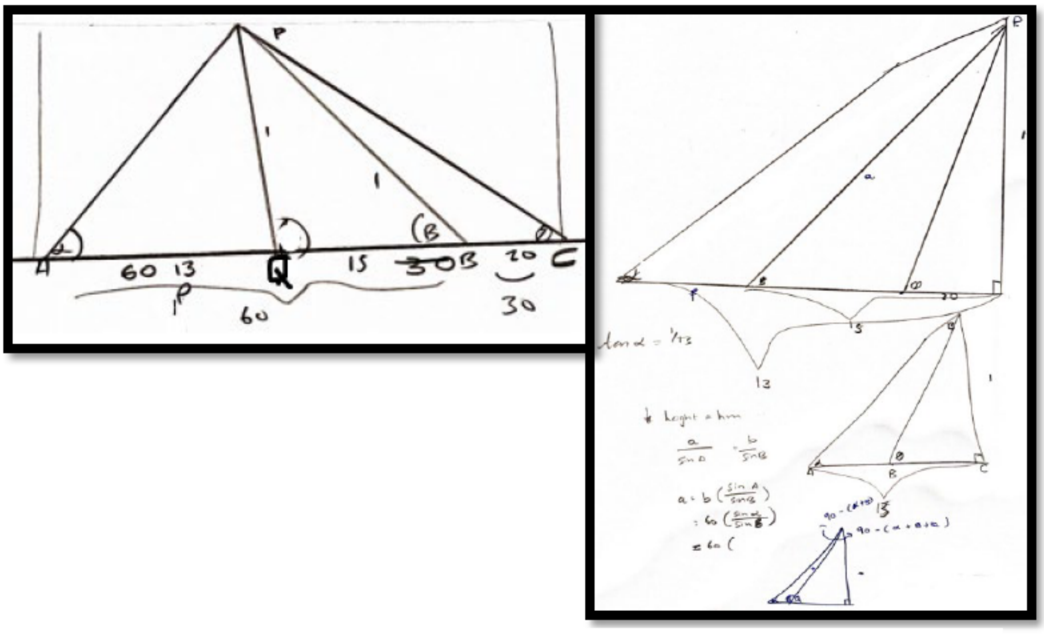


Figure 5.4.2.5: Amah's visual mediator for Question 2 of Part A

In addition, Amah found that the second problem of Part A was also challenging as she found it difficult to visualize and to fully comprehend.

> *For number 2, I had two diagrams there. The first one is incomplete. I realized that I was misinterpreting some details from the problem statement, then I left it like that. Then I reread the info to come up with the second one which I was still not sure of its accuracy. To be honest, I couldn't solve number 2; it was difficult for me to understand. I tried to attempt it but I ended up scratching it out because I could tell that I was not doing it correctly. I didn't think that the number 2 problem of Part A was in 3D at all, so that's why I was lost* (Amah).

This aspect is captured by Moleko (2021) who claimed that reading without full comprehension makes it challenging to reason for most mathematical word problems and even more difficult to create useful mental pictures or geometric visual mediators of the problems to be solved. Thus, for Amah, a lack of reading for full comprehension coupled with a limited ability to visualize resulted in her experiencing difficulty in dealing with the second question in Part A of the performance test.

A similar challenge was identified in Zot's response to the same question as depicted in Figure 5.4.2.6, illustrating his attempt to visualize this problem. His geometric visual mediator also demonstrated that he had misinterpreted the problem statement by assuming that the question was in 2D, as was the case with the other seven participants. He was unable to depict the unknown height of the tower PQ as required to answer the question. Zot explained his thinking process about the question as follows:

> *For number 2, again, you needed to understand the terminology – like what is the angle of elevation, and not confuse it with the angle of depression. Then, also, it was important to understand the concept of trig ratios and make a visual sketch* (Zot).

From his rationalization, Zot appeared to understand what was required for the second question in Part A, however, his geometric visual mediator and solution indicated otherwise. Perhaps if he had fully comprehended the terminology that was utilized in the problem statement, his visual mediator would have been more accurate.

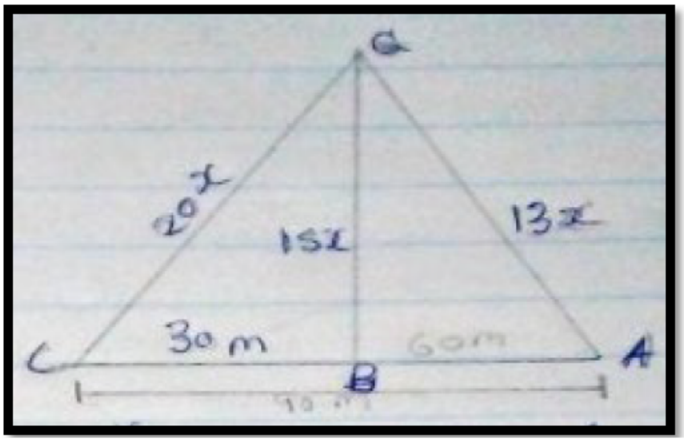


Figure 5.4.2.6: Zot’s visual mediator for Question 2 of Part A

In Zakhe’s case, he was able to establish that the problem statement was referring to a 3D problem. However, his geometric visual mediator, as illustrated in Figure 5.4.2.7, did not depict the problem statement with accuracy.

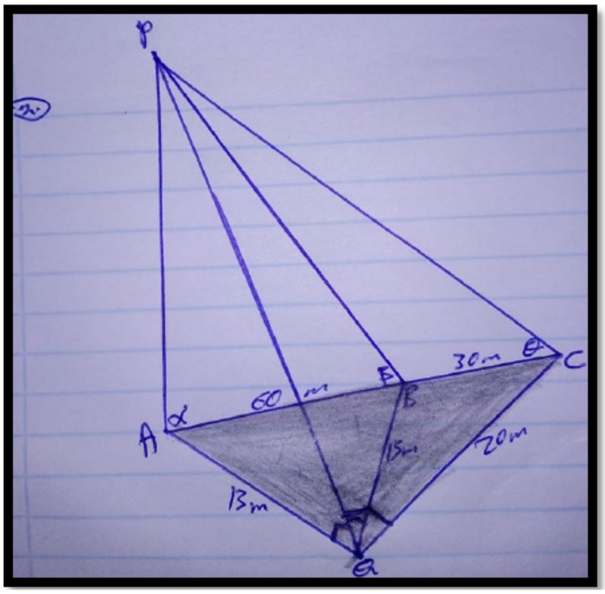


Figure 5.4.2.7: Zakhe’s visual mediator to question 2 of Part A

He explained during the interview session that he found Question 2 to be more challenging than Question 1. He explained his experience as follows:

> *...number 2 was very challenging because of the diagram that I had to sketch before I could even answer the question...So what I first did was to draw my horizontal straight line with ABC. Then I tried to label the lengths that were given:* $AB = 60$ *and* $BC = 30$. *Then I isolated that information about the ratios of* $\tan$. *Afterward, I saw that they were also giving us that Q was on the same horizontal plane as ABC. Then I plotted that Q a bit below ABC. Then I joined AQ, QC, and QB...Then when they asked us to*

calculate the height of the tower QP, it was then clear that I had to join Q and P to make a height or a pole (Zakhe).

Essentially, Zakhe indicated that he was thinking visually since he immediately thought of doing a visual sketch before executing any form of calculation. He was the only participant who offered in-depth detail about his spatial-visual thinking that resulted in the diagram he submitted (Figure 5.4.2.7). From his account of the steps he took to develop the geometric visual mediator, the source of the error became clear. He positioned Q (the foot of the tower) at a location that made PQ to be not perpendicular with the horizontal plane (the ground), since the tower was supposed to be vertical. Thus, labeling vertex Q as 90º was incorrect with P located on the other side of the straight line ABC (see Figure 5.4.2.7). However, at that stage he realised that there were trigonometric ratios, which meant that his diagram had to contain right-angled triangle faces.

Lethu's set of sketches revealed that she began to use spatial-visual thinking while she was attempting to answer the question. She realized that the 2D representation was incorrect and that she was dealing with a 3D problem. Figure 5.4.2.8 depicts her set of geometric visual mediators for the second question in Part A.

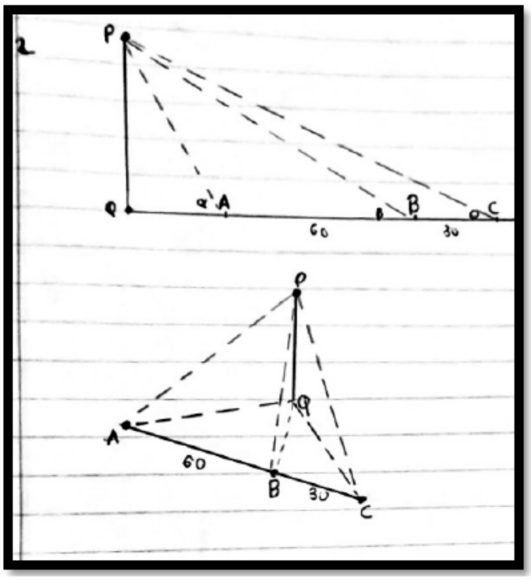


Figure 5.4.2.8: Lethu's visual mediator for Question 2 of Part A.

Lethu's visual mediators, as was the case with those of other participants (Lunge, Sfi, Amah, Zot & Phume), showed that she had initially misinterpreted the concept of A, B, C, and Q being on the horizontal plane as being on a straight line; however, on her next sketch she was more accurate. She requested extra time to submit Part A as she was contemplating her solution for the second question. During the interview session, she explained:

> *...when I first attempted that question, I thought points ABCQ were on the straight line. But, instead, they were on the same horizontal plane. So then it was challenging to identify where the tower was going to be located. I was not sure whether it was going to be on A, B, or C. So I can say that part gave me a hard time. Then later I realized that the statement did not say Q was on the straight line. So that perhaps meant the diagram could look different* (Lethu).

Her narrative validates her thought process as she worked out that the question was referring to a 3D problem. Nevertheless, even with an accurate sketch demonstrating that she understood the problem visually, she failed to obtain the height of the tower PQ in meters.

Qhaw was one of the few participants who managed to visualize from his first attempt that the second problem of Part A was in 3D. Nevertheless, like Lethu, he failed to determine the height of the tower PQ. Figure 5.4.2.9 illustrates his visual mediator for this question and the only two steps he wrote below the diagram.

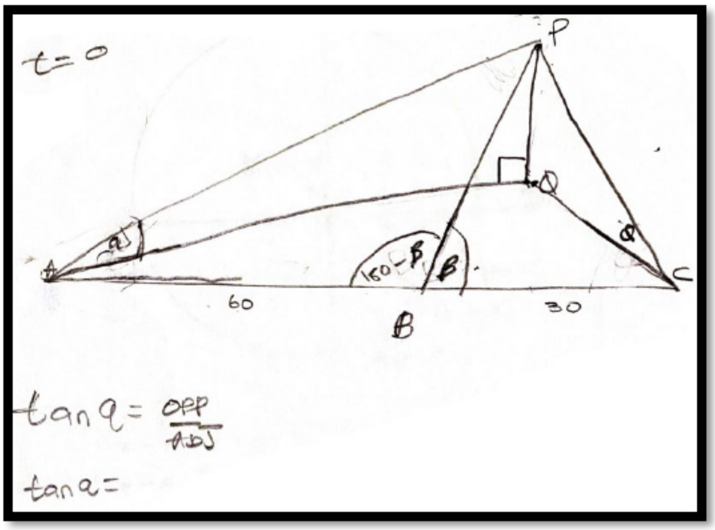


Figure 5.4.2.9: Qhaw's visual mediator for Question 2 of Part A.

On his geometric visual mediator, Qhaw was unable to visualize the line segment from B to the foot of the tower Q, thus forming an inaccurate angle of elevation β, as depicted in Figure 5.4.2.9. Another challenge for him was to strategically manipulate the given information, together with his visual sketch, to find the height of the tower PQ.

Two assumptions can be drawn from these participants' inability to solve this question. Firstly, they lacked the content knowledge for basic trigonometry. The question's level of difficulty would be considered advanced at the secondary school level in the South African curriculum, but of average difficulty for a third-year university mathematics course. Secondly, the students had not prepared in any way to take the test. Even so, their performance was alarming given that these participants were in their third year of study as mathematics education specialist students and their trigonometric content knowledge should have advanced far beyond the secondary school content covered in the test.

Qhaw identified the language used in the question as increasing the level of difficult of the question for him:

> *Part A questions were challenging because you have to visualize the diagram on your own using the information you are given and sometimes it is not clear enough – or, should I say, the type of English they use is a little bit tricky. You have to really understand the entire sentence to know what they are saying. For example, about the angle of elevation or depression…it was not easy to put everything together and solve the second problem* (Qhaw).

From his response, it can be inferred that the language used in the mathematics problem had an impact on his comprehension of the question. Several studies (Chitera, Kasoka, & Thomo, 2016; Robertson & Graven, 2020; Robertson & Graven, 2020a) have established that mathematics students who are second language English speakers experience difficulties when engaging with mathematics word problems where language proficiency is vital. The language barrier was not expected to be a problem for these participants since they were third year students, however Qhaw indicated that his difficult solving the second problem was related to language, in part. Although Moleko (2021) contends that proficiency in English, reading skills, and good comprehension of mathematical terminology are essential for success in visualizing mathematics word problems, it is apparent that English proficiency continues to be problematic for mathematics students. This is the case even while English is the medium of instruction at the institution where the participants were sampled.

Shan was the only participant who correctly visualized this problem on his first attempt. Figure 5.4.2.10 represents the geometric visual mediator that emanated from Shan's spatial-visual thinking as he engaged with this problem. Shan's sketch was accurate and correct; however, he made one assumption that was not stated in the problem statement, that is $C\hat{Q}A = 90^o$, for the problem to be solvable, according to his interpretation.

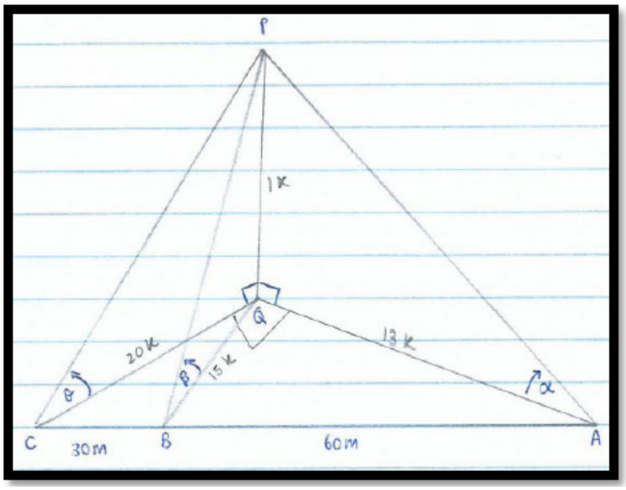


Figure 5.4.2.10: Shan's visual mediator for Question 2 of Part A.

5.4.3 Word use, routines, and endorsed narratives from the analytical geometry question

Several instances of mathematical word use, endorsed narratives, and routines were identified from participants' solutions and the interview responses, in which they were asked to describe their internal dialogues after completing the performance test. The use of mathematical words were noted in the interview responses and symbolic visual mediators were observed in their solutions. Figure 5.4.3.1 depicts Phume's solution, where symbolic visual mediators such as the equation of a circle, midpoint, and distance formula, and several arithmetic operators were evident.

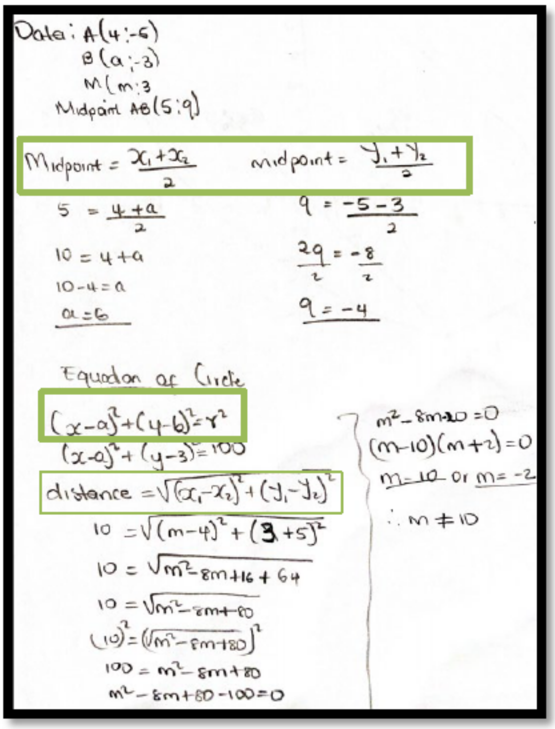


Figure 5.4.3.1: Symbolic visual mediators from Phume's solution.

Phume described how she approached solving the problem:

> *After reading the statement word by word and sentence by sentence, I was picking up those important things to help me solve the problem…So I realized that using a midpoint formula right away was a better option, looking at the information that was given. So I substituted everything correctly then solved for both unknowns simultaneously. Midpoint was the first and most important keyword that was giving a direction. So, for the center, I first needed to know the equation of the circle, but since the radius was already given, I substituted everything there then I solved for m and according to my diagram* [Figure 5.4.3.1]*, it was clear that* $m = -2$ *and not 10* (Phume).

Her knowledge of the mathematical concepts represented by the terms 'midpoint', 'centre', and 'radius' was found to be integral to her solution as they allowed her to identify the relevant formulae. Furthermore, her explanation revealed that carefully reading the problem statement assisted her to identify vital pieces of information that were key to solving the problem – the key symbolic visual mediators highlighted on her solution (Figure 5.4.3.1) which provided direction, as indicated in her explanation. This aligns with the findings of Moleko's (2021) study, which demonstrated that in an attempt to teach learners problem solving and visualization skills, mathematics teachers adopted a strategy of highlighting keywords as they read a problem statement to help them visualize it.

Similar symbolic visual mediators and mathematical word use were identified in the responses of the other nine participants; as their responses were similar to Phume's, they are not presented here. Amah's use of the mathematical word 'distance' differed slightly from that of the other participants, however, as she avoided the use of any formulae in her first solution. Figure 5.4.3.2 depicts Amah's response to the first question, where she used her conceptual understanding of a midpoint and applied an endorsed mathematical narrative.

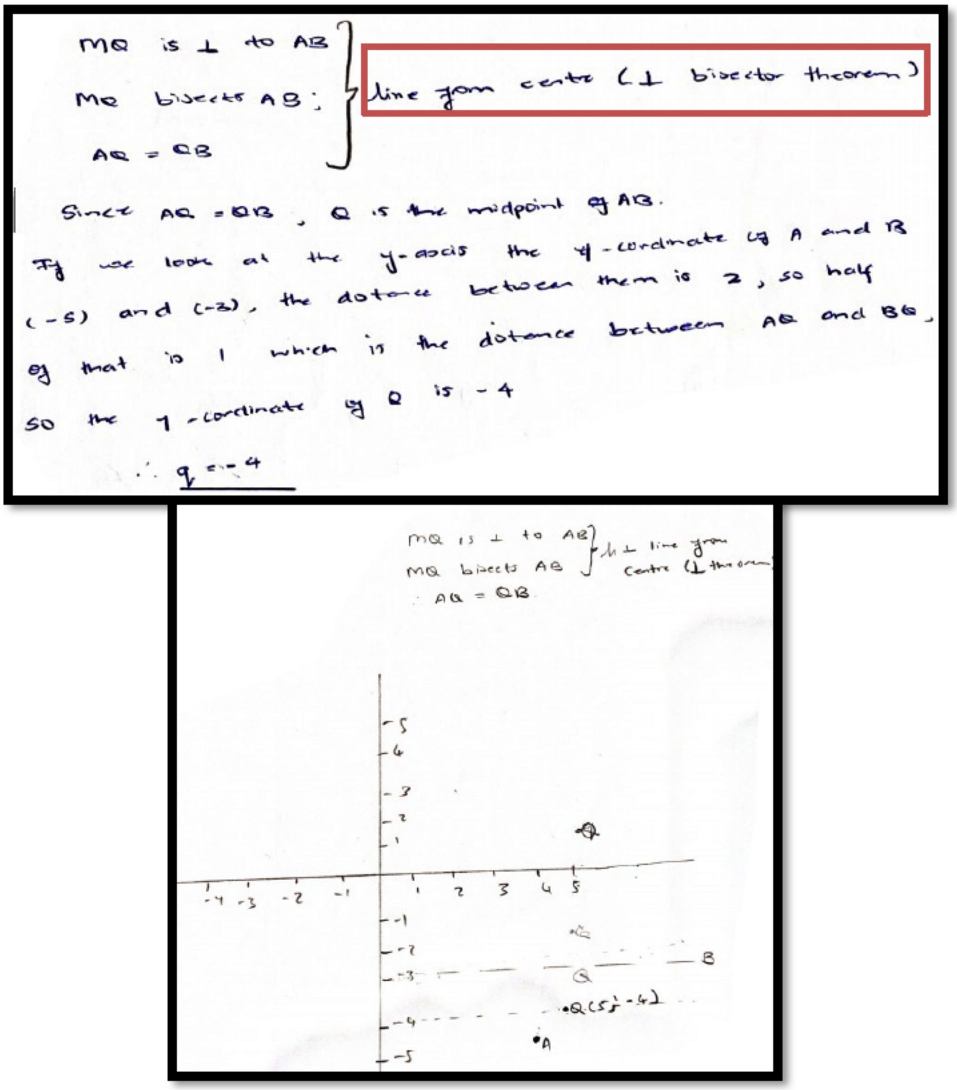


Figure 5.4.3.2: Amah's solution to number 1 of Part A. ▭ Endorsed narrative

By using a concept of an endorsed narrative 'a line drawn from the centre of a circle to a midpoint of a chord is perpendicular to the chord', as highlighted in Figure 5.4.3.2, she could visualize using the y-axis of a Cartesian plane, such that if one end of a line segment AB (a chord) was −5 and the other was −3 then the ordinate of the center of that line segment should be −4. She explained her thinking process as follows:

> *In calculating Q, I used my diagram, which assisted me to recognize a better and quicker method of getting the unknown. I used the fact that if they say Q is a midpoint of AB, then the y coordinate of Q could be calculated by saying: if the distance between A and B is 2, then it means the distance from A to Q is 1 and from B to Q is also 1 as well, given the y coordinate of B being −3 and y coordinate of B being* 5. *Then I saw that the y coordinate of Q had to be −4 since the distance was* 1. *So, I didn't use any*

formula for this one, I visualized the coordinates and distances on the Cartesian plane… (Amah).

From her explanation, it is apparent that Amah used her visualization skills to find the value of the variable as she did not use any specific formula as the other participants had. She identified her utilization of a graphic visual mediator (diagram) as an efficient method for obtaining the value that was required. The findings from Mudaly's (2021) study also indicated that it is helpful to encourage students to exercise their mental imagery abilities during problem-solving. Nonetheless, Amah's answer was similar to those of her peers who had used algebraic techniques. The use of another mathematical word, 'distance' – which is also used in common language – was identified in her explanation of the solution. Amah used a conceptual understanding of the word 'distance' within the context of coordinate geometry where she mentally visualized the coordinate system and counted the number of units according to the information that was provided.

In addition, a substantiation routine, 'solution verification' – which is defined by Gavilán-Izquierdo and Gallego-Sánchez (2021) as a routine for checking results through the use of geometric definitions – was observed in participants' responses. Qhaw explained his use of solution verification;

> *…so I used the distance formula and I substituted the length of the radius and the other unknowns, then I solved it where I got* $m = 14$ *and* $m = -2$. *Then I went to my diagram to ask myself that, if* m *was equal to* 14, *is it corresponding with the given information which stated that MQ is a perpendicular bisector of AB, since M was given to be a centre? Then I realized that* 14 *is too far for it to make sense and to correspond with the given info; then the option for* m *is* -2 (Qhaw).

His rationale for how he arrived at one final value of the abscissa of the centre was guided by his conceptual understanding of geometry – that if a line segment is to be a perpendicular bisector to another given line segment on a Cartesian plane, then it has to slant in a particular direction that satisfies the definition of a perpendicular bisector. In this study, the problem statement for the question provided an ordinate of the centre; thus, they had to verify their result using analytical geometric reasoning if they adopted a method that yielded two values for the unknown. A similar substantiation routine, with a slightly more detailed rationale, was observed in Lunge's justification:

I substituted everything into the formula of a circle, then I arrived at $m = 10$ *or* $m = -2$. *Then I had to choose one answer. So, looking at my sketch here* [Figure 5.4.2.1], *I tried to plot the coordinates according to these values, where I realized the value that was making sense is* -2. *Simple, because if I have a chord, these have to be radius perpendicular to the chord and it has to cut it at the midpoint. Then I saw that it has to be on this side* [pointing at his diagram] *and it can't be perpendicular coming from* 10; *then I chose* $m = -2$ (Lunge).

Lunge's justification demonstrates that much of his reasoning about the solution for the first question was based on visualizing the problem. This assisted him to verify his solution by recalling specific axioms (endorsed narrative) through the process of intrapersonal communication based on his visual images. Rif'at (2018) contends that an ability to visualize is more than just a type of thinking, or a tool, or an approach: it is also the interconnected sequence "of reasoning to achieve the formal analytic abilities" (p. 75). As such, it can be inferred that mathematical visualization during the problem-solving process is interlinked with commognition.

In addition to the use of solution verification as a substantiation routine, the use of the endorsed narrative 'radius perpendicular to a chord cut it at the midpoint' was noted in Lunge's justification. This further substantiation routine was classifiable within the 'verification of sufficient conditions' also identified in Gavilán-Izquierdo and Gallego-Sánchez's (2021) case study. Lunge's determination of the value of m was further guided by the condition of one of the theorems he had learned at the secondary school level. According to his explanation, his solution had to satisfy the condition of the theorem he was applying to the problem statement. Thus, it can be assumed that, in his practice as a teacher, when he uses these types of questions he will be able to teach his learners to integrate different mathematics topics, as this is a vital skill that learners must acquire.

In Zot's response, the endorsed narrative 'radius perpendicular to chord' and a substantiation routine of 'algorithms' prompted by the recall of the endorsed narrative that 'a product of the gradients of two perpendicular line segments is -1' were noted. Viirman (2015) contend that substantiation routines employ algorithms to support the justification of claims. Zot indicated that:

...to get the value of 'm' I used the fact that the product of perpendicular lines is -1, so the gradient of QM times the gradient of AB must give -1. So since I had already

calculated the other unknowns for the coordinates, then after substitution there was only one unknown on my equation, which was very easy to solve to get $m = -2$ (Zot).

Figure 5.4.3.3 depicts Zot's calculation of the abscissa of the centre of the circle. The recall routine that was inferred from his response assisted him to relate geometrical and algebraic concepts to answer the question. Gavilán-Izquierdo & Gallego-Sánchez (2021) affirms that recall routines are vital from a didactic perspective since the nature of mathematics is interconnected and cumulative.

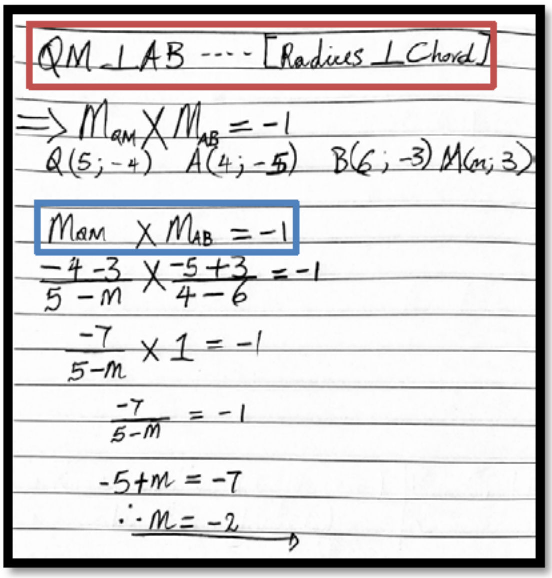


Figure 5.4.3.3: Zot's solution to number 1 of Part A
Endorsed narrative and Routine.

Furthermore, self-discourse – one of the tenets of commognition – was observed in all participants' explanations, providing evidence that they communicated with themselves during the completion of the assigned individual activity. They reported their internal dialogues as follows:

I saw that the question asked us to calculate the value. So I was thinking: if Q is the midpoint – then I thought about what formulas could we use when we calculate the midpoint. So then I thought about the midpoint formula, so I said to myself: I can use that to calculate the unknown values for A and Q. Then for the next one, the first thing I thought was the distance formula, I said…because we're given the radius of 10 units, I decided we can use the coordinates of A and the midpoint to find the unknown 'm'. Afterward, I plotted everything on the Cartesian plane to confirm my answers (Shan).

Well, it's to ask yourself questions like: 'what are the given angles and lengths?' and 'how can I use that information to answer the questions that are being asked?' Then if that is not enough I ask myself more questions and try to identify key information and make some drawings to see what it would look like (Qhaw).

For me, after getting the first part about figuring out the diagram, then I enjoy the rest of the problem…So, for example in number1, I asked myself questions like: 'where will the center of this circle be located on the Cartesian – like which quadrant' Then I marked the possible location, then did the actual calculation, then went back to the diagram and asked myself again if my solution made sense…Then I did the same back and forth – talking to myself –by checking if everything I was calculating made sense (Lethu).

These narratives reveal the type of self-discursive activities that the participants engaged in during the process of mathematical problem solving. It is apparent from their narratives that during these self-discursive activities they used the information given in the problem statement and identified which information was key to solving the problem. Furthermore, this was an iterative exercise for the participants, as they indicated that they went back and forth verifying their methods of calculation. Ripardo (2017) advocates that solving mathematics problems means being capable of engaging in a process of individualized mathematical discourses that "involves passing from one unobjectified discourse, that is, a personal discourse, in which the interlocutor talks about actions taken onto the object, to an objectified, impersonal discourse, in which the interlocutor talks about the relations with the object" (p. 905). The participants' narratives also suggest that the utilization of symbolic visual mediators was central to their self-discursive exercises.

5.4.4 Word use, endorsed narratives and routines from the trigonometry question

More mathematical word use, endorsed narratives, and different types of routines were identified in the participants' responses to the trigonometry section that they completed. Some of the mathematical 'word uses' were interrelated in participants' comments as they justified their visual mediators (discussed in Section 5.4.2). For instance, the use of 'horizontal plane' appeared frequently in their explanations of their visual mediators since it played a central role in their visualization of the second problem. This term is used in various disciplines, including mathematics, and has meaning in common language. In a mathematical context, it refers to a horizontal surface parallel to the horizon and can have vertical surface(s) perpendicular to it. It features in many mathematical content areas, such as spherical geometry, 3D dimensional coordinate geometry, and trigonometry.

In the second problem, ‘horizontal plane’ was used within the context of 3D trigonometry problems and it was key that participants understood its use in this context to be able to solve the problem. The failure of eight of the participants to respond correctly to this problem may be attributed to their lack of a comprehensive understanding of what was meant by this mathematical ‘word use’. According to Ho, Hong, Tay, Leong, and Ming (2019), individuals engaging in a mathematical problem-solving exercise are expected to possess a knowledge of the meanings of standard mathematical terms used (‘word use’). Also, Sfard (2008) proclaims that ‘word use’ is holistically an “important matter because…it is responsible for what the user is able to say about (and thus to see in) the world” (p. 133).

Additionally, the mathematical word ‘elevation’ was another crucial term that the participants were expected to comprehend completely and be able to use together with ‘horizontal plane’ to arrive at an accurate geometric visual mediator so as to solve the second question. The participants demonstrated a thorough understanding of how to use the word ‘elevation’ in their visual mediators and interview responses. Shan commented as follows:

> *… for Question 2, I tried to visualize that Q was on the same horizontal plane as the angle of elevation. So I tried – I tried my best – to visualize what the diagram would possibly look like…*(Shan).

The two key mathematical words, ‘horizontal plane’ and ‘elevation’ appear chronologically to contribute to the meaning of the bigger picture of what is required to be visualized. Participants needed to comprehend these two terms to be able to visualize the entire problem statement to determine that it referred to a 3D structure. It is evident from Shan’s response that his visualization of the problem was guided by his understanding of the usage of these two words. His first description proved that he could visualize what was stated in the problem statement as it appeared in his visual mediator (Figure 5.4.2.10). Zakhe also demonstrated an understanding of these terms as his visual mediator sketch (Figure 5.4.2.7) depicted both concepts, even though he made an error in his attempt to locate the foot of the tower (as explained in the previous subsection). Zakhe explained:

> *…then I labelled that vertex Q at the bottom with 90º, to be able to use the trig ratios. And I also placed the angles of elevation accordingly, as well* (Zakhe).

Lethu’s account was similar to that of the other participants. Lethu commented:

> *At first, I thought I was dealing with a 2D kind of problem, but then as I read the statement over and over again in relation to my first sketch I started to see that*

something was not right: Q is not on the same straight line with the other three points. After moving it away from the straight line, I could then form a horizontal plane which was the part I didn't pick up when I started…then the elevation angles were easy to form after that.

Other participants who failed to visualize the second problem (judging from their visual mediators) experienced challenges representing the mathematical concepts (word uses) 'angle of elevation' and 'horizontal plane'. The challenge was primarily with visualizing the 3D horizontal plane, then the vertical tower mounted on a specific position that could have consequently formed the angles of elevation, thereby forming one visual image. Phume and Zot reflected on this as follows:

> *…mine was not correct for Part A of number 2, because I thought those points – A, B, C, Q – were collinear – like on the straight line – and I thought I was dealing with a 2D problem – hence my sketches* [Figure 5.4.2.4]. *It didn't click to me that it was a 3D shape they were talking about, but then when I got stuck with my calculations I realized that there was something wrong with my diagram, which made me fail to get the solution because I visualized it differently. Those concepts of angles of elevation and horizontal plane were a bit tricky for me* (Phume).
>
> *It's just for Part A of number 2: at first I was not sure if I was dealing with a 2D or 3D. Then I realized later on that it was 3D because of the keyword 'horizontal plane' and 'three angles of elevation' and 'vertical tower'. But there on my solution* [pointing at Figure 5.4.2.6], *I have a 2D diagram, which is wrong… the terminology they were using needed to be sketched somewhere to make sense of it, otherwise it was going to be very hard to even visualize what needed to be calculated* (Zot).

The interview responses of these two participants demonstrated they were acquainted with the mathematical words that were important to comprehend for the second problem in Part A. They could tell that the terms were crucial for a successful visualization of the problem and consequently solving it. However, they were unable to visualize how these should look as iconic visual mediators useful for solving a problem. This demonstrated a lack of deep understanding of the mathematical terminology. This finding concurred with (Vula & Kurshumlia, 2015; Moleko, 2021) findings which demonstrated that students encounter difficulties in solving word problems due to their inability to conceptualize the problems and a lack of understanding and familiarity with the crucial mathematical terminology. It can be deduced that problem-solving requires a great deal of deep understanding of mathematical concepts to enable visualization within one's mental processes.

The usage of the mathematical words 'opposite and adjacent' were also noted in Shan's account of his self-discourse:

> *...then I took my pencil and tried to draw it out first and then label all the figures, as they gave us* $tan\,\alpha = \frac{1}{13}$*. So, I saw that was opposite and adjacent sides, so I put* $1k$ *for the opposite side, and then I put* $13k$ *for the adjacent. So, I tried to just label everything in pencil and then I tackled the question* (Shan).

The words 'opposite and adjacent' were used to demonstrate an understanding of a trigonometric definition of tangent that was also crucial in solving the second question. Shan's comment revealed that it was essential for him to visualize these terms by sketching on a piece of paper. Rellensmann, Schukajlow, and Leopold's (2017) empirical study established that students construct drawings during mathematics problem modelling to increase their chances of successfully solving the problem. Shan introduced the variable 'k' after realizing that $\frac{1}{13}$ was merely ratio of the height of the tower to the adjacent side forming the angle of elevation alpha. He added:

> *We were given that* $tan\alpha = \frac{1}{13}$*; so I thought that* $\frac{1}{13}$ *could be a simplification...It could be* $\frac{2}{26}$*, so we don't know the exact lengths...so I decided that to make it simple, I will make* $PQ = k$ *and then I will express the other sides to also have* k*, so* $tan\alpha = \frac{1k}{13k}$*, also with* $tan\beta$ *I said, since* $PQ = k$ *it will be* $tan\beta = \frac{k}{15k}$ *and the other one would be* $tan\theta = \frac{k}{20k}$ *...and then I wrote it with pencil on the diagram* [Figure 5.3.3.10]. *So I wrote* $QC = 20k$*,* $QA = 13k$*, and* $QB = 15k$*...*(Shan).

Similarly, in Zot's description of his self-discourse, other mathematical word uses were identified as he arrived at the final answer and explained his solution. Although he misinterpreted the problem due to being unable to accurately visualize the statement of the question as depicted in Figure 5.4.4.1, he revealed:

> *I started off with a drawing – as I've said – and I realized that we were given ratios – like that* $tan\alpha = \frac{1}{13}$*. Then I said, 'let me assign an arbitrary variable x for those ratios to have something like* $tan\alpha = \frac{1x}{13x}$*'. Then I labelled that info into my diagram using the fact that*

tan is $\frac{opposite}{adjacent}$ *so* $PQ = 1x$ *and the adjacent side* $= 13x$*. Then I did the same for other ratios, as you can see* [pointing at Figure 5.4.4.1]. *So I was putting the variable x to show that it is a ratio and not the length like that 30m and 60m…Then to solve for x, which I treated as a vertical height of the tower, I used triangle CQA – the big one– to apply the cosine rule. So I took the length of CB first and substituted, as you can see, to get my first equation* [pointing at Figure 5.3.4.1]. *Also, I used triangle BQA – that one on the right – and substituted into a cosine rule as shown. Then I labeled that as equation (2). So then I solved them simultaneously, since there is something common which is* $cosA$*. Then I*

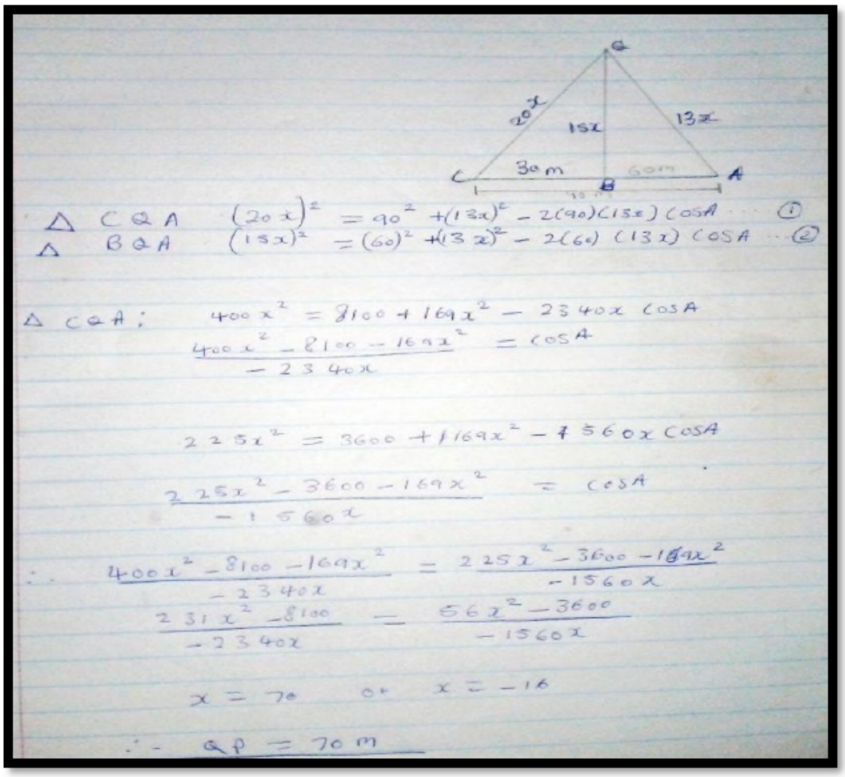


Figure 5.4.4.1: Zot's solution for Question 2 of Part A.

equated them to solve for x and I got it to be 70 or -16*. But then a height cannot be negative, so* $PQ = 70m$ *was the answer. I only skipped one step of factorizing the quadratic equation, which produced these two values here. Although my diagram was not drawn correctly, as I oversimplified it, but I managed to get an answer* (Zot).

The same word uses found in Shan's response were found again in Zot's account, and in a similar context; thus, the word uses in Zot's account are not analyzed here. However, Zot included the use of the term 'arbitrary variable' in his explication to specify that he assigned a symbol that was going to contain the value of the unknown (height of the tower that was to be calculated). The introduction of this variable by Zot and Shan was a good strategy and its use was correct, as it remained unaffected by the changes in the equations he manipulated, which fitted the mathematical definition of 'arbitrary variable'. Prediger and Krägeloh (2015) assert that "x-arbitrary means any number, but you do not know which one" (p. 89). However, since his sketch was already incorrect, his subsequent steps to solve the problem were unsuccessful. Further, the use of everyday language words, which have specific mathematical meanings like 'vertical height', was noted in his self-discourse. 'Vertical height', in mathematical terms, has a connotation that a line segment is perpendicular to the other; however, when used in common language the perpendicular meaning is not implied. Moreover, in his comment, a recall routine is suggested about one of the rules to solve triangles, which is a 'cosine rule'. This is one of the key rules introduced to learners in secondary school, according to the South African curriculum, and needed to be remembered for the second problem that the participants experienced difficulties with. Sfard (2007) declares that recall type routines are imperative for the discursive fluency of individuals engaging in any mathematical activities.

There were no specific endorsed narratives that were identified during the discursive activity based on the second question. Several mathematical 'word uses' and 'routines' were identified during the discursive activity, which were similar to those presented in the analysis above and fulfilled the same function. In addition, the participants were requested to compare tasks A and B and indicate which they found easier to comprehend. Their responses are discussed next.

5.4.5 Validating the comparisons of both parts of the performance tests

To reiterate, Part A of the performance test consisted of two sets of mathematics word problems where the participants had to create their graphical and geometric visual mediators from inception while solving both problems. In Part B of the test, participants were provided these visual mediators (diagrams) alongside the same two questions they had responded to in Part A (see appendix C). During the interview sessions, all participants were requested to compare the level of difficulty of Parts A and B to establish if the accompanying visual mediators assisted them to visualize these problems better. All of the participants indicated that Part B, which had the accompanying visual mediators, was easier. Shan and Qhaw commented:

> *I think task B was fair because the diagram was there so it was much easier to put it all together…for performance test A, I think it took me very long to try to think how to get the diagrams to look like that, especially for the trigonometry question, to visualize where that Q was and B was located, so it made it easier when there was an accompanying diagram below the statement of the question* (Shan).

The one that was easy and enjoyable between the two was the one with diagrams, and that will be Part B…I am saying that because if you visualize on your own and try to come up with the diagram from the question trying to understand that English, you could get it wrong but with the ones where diagrams are given all the information they give you is always accurate…so visualizing the problem and trying to come up with that correct diagram is challenging (Qhaw).

From their explanation, it is apparent that they realized Part B of the test was easy to comprehend because the accompanying visual mediators accurately depicted the visual images that the problem statements were referring to. Further, the issue of the language challenge Qhaw alluded to was mitigated by the presence of visual mediators alongside the statement of the problem. Supardi, et al. (2021) assert that "visual mediators will make it easier for students to solve problems with the help of illustrating problems in the form of images so that will minimize the occurrence of errors" (p. 959). With the assistance of the accurate visual mediators that were provided in Part B, Zot, Qhaw, and Amah managed to arrive at the different solutions to the second problem – which they found to be more challenging – than they had for the same problem in Part A. Zot explained

As such, for Number 2 of Parts A and B my answers are not the same. I think the correct answer is from part B because there I used the correct diagram, but for Part A my diagram was not correct, then my answer is not correct there…so the correct answer is supposed to be 4.3m, and not 70m (Zot).

Figure 5.4.5.1 depicts Zot's responses to the second problem in parts A and B.

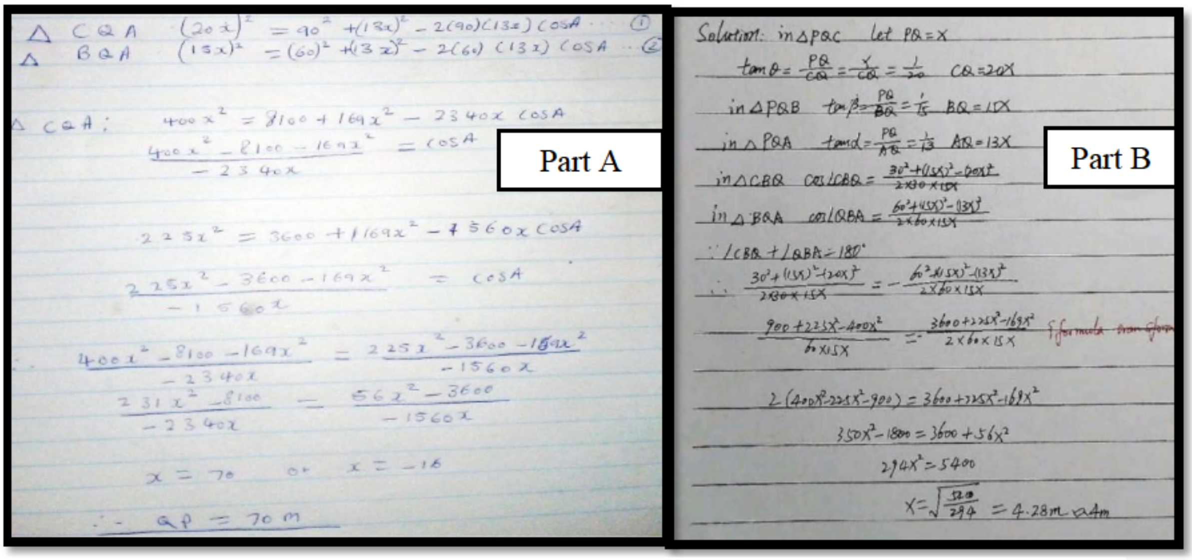


Figure 5.4.5.1: Zot's solutions to Question 2 in Part A (left) and Part B (right).

It is apparent that the solutions are different from each other, with problem-solving for the question in Part B showing significant improvement over that for the same question in Part A, arriving at the correct answer. The same was evident in Amah's solution, as illustrated in Figure 5.4.5.2.

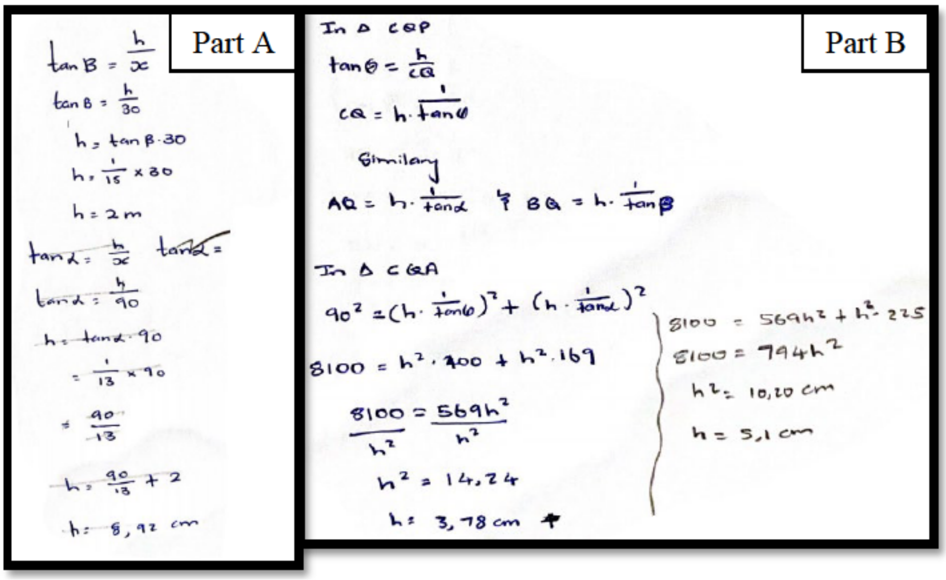


Figure 5.4.5.2: Amah's solutions to Question 2 in Part A (left) and Part B (right).

Although her solution for the second question in Part B was still incorrect, Amah explained that

> *Part B was not difficult to understand because it came with diagrams. I was even more convinced of my answers that were on the right track, since in Part A I doubted all my answers. If you are given a statement and you can also see the correct diagram next to it, then you can sort of verify your answers using a diagram to see if they're making sense. Unlike if you're given a statement only, then you have to visualize on your own as to what the question is talking about. So the statement together with the diagram make things easy for one to understand better* (Amah).

Despite the benefit of the supplementary geometric visual mediator, eight of the participants still experienced difficulty arriving at the correct solution to the second problem in Part B. Sfi commented on his experience:

> *Part B was much better for me because, as I've indicated, the key to having a correct picture/diagram to work with, make it much easier to visualize things even further. So, there are more clues to help with understanding the entire question. Even though I am still not getting to the final answer for the trig problem, at least now I have more clue as to what could work, though I'm not sure for now* (Sfi).

Sfi's description of the challenge he experienced solving the second problem is confirmed by Figure 5.4.5.3 which depicts his first and second attempts. It is evident that in both attempts he could not arrive at the correct solution to the problem, even in Part B where the correct visual mediator was provided.

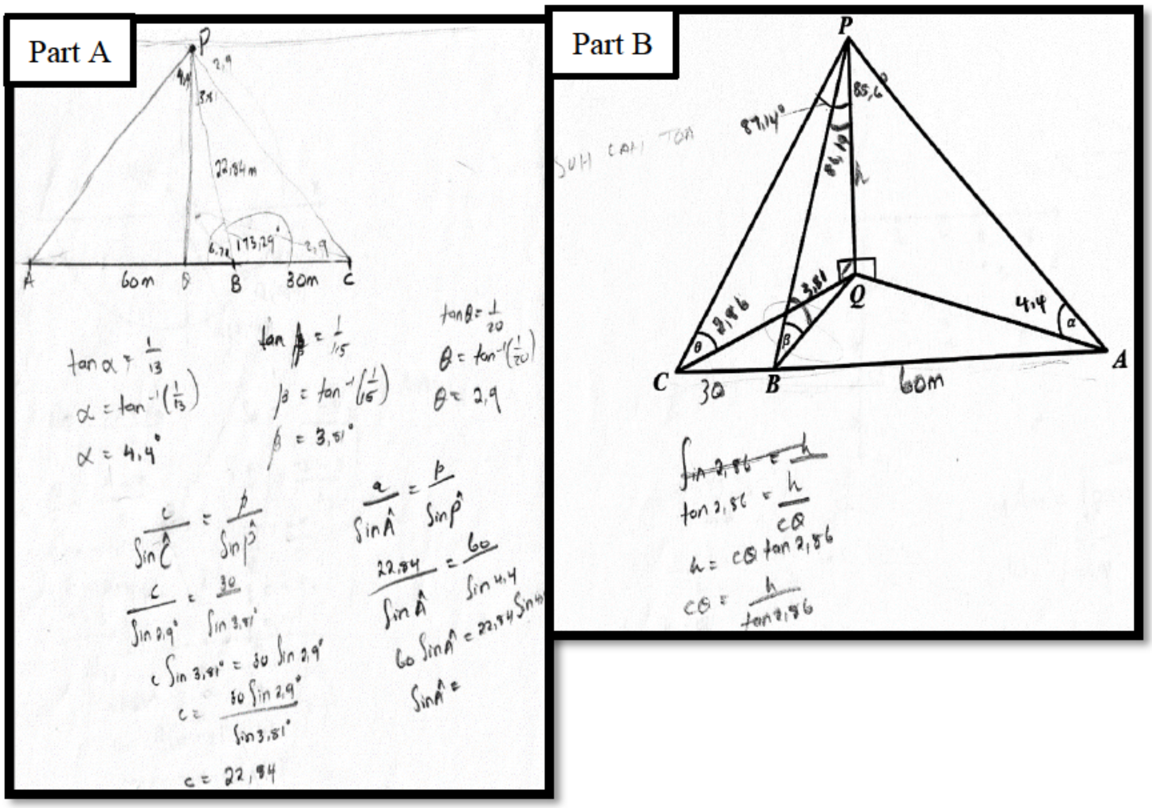


Figure 5.4.5.3: Sfi's solutions to Question 2 in Part A (left) and Part B (right).

Lunge described his experience as follows:

> *I could understand better what the question was talking about for Part B. Then I had to label and try to come up with a solution by using all the given information. But I still got stuck. If you look at my attempt, I'm still not sure what is it that I am doing wrong, because I have everything visually. Number 2 is very tricky* (Lunge).

In Lunge's case, it is evident that he arrived at the final step where he obtained the length of the height of the pole, although in both cases his solutions were incorrect. Similar to Shan's case, in Part B Lunge made the wrong assumption that the horizontal plane AQC was a right-angled triangle and, on this basis, applied the Pythagorean theorem. His solutions to both questions are depicted in Figure 5.4.5.4.

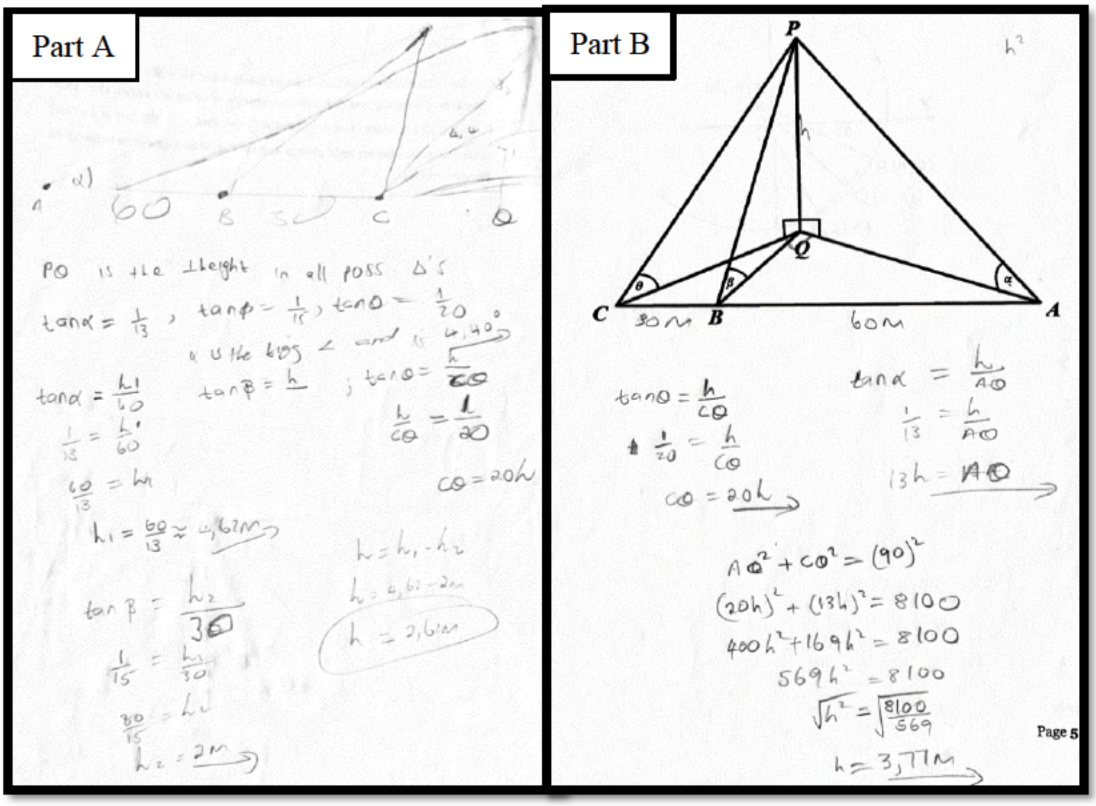


Figure 5.4.5.4: Lunge's solutions to Question 2 in Part A (left) and Part B (right).

Phume's experience was similar to Sfi's, as neither could get to the final step in their attempts to solve the problems. She indicated that:

> *Number1 of both tasks was easy for me. I just had challenges with both Number 2s from Parts A and B. I still don't know how to do it* (Phume).

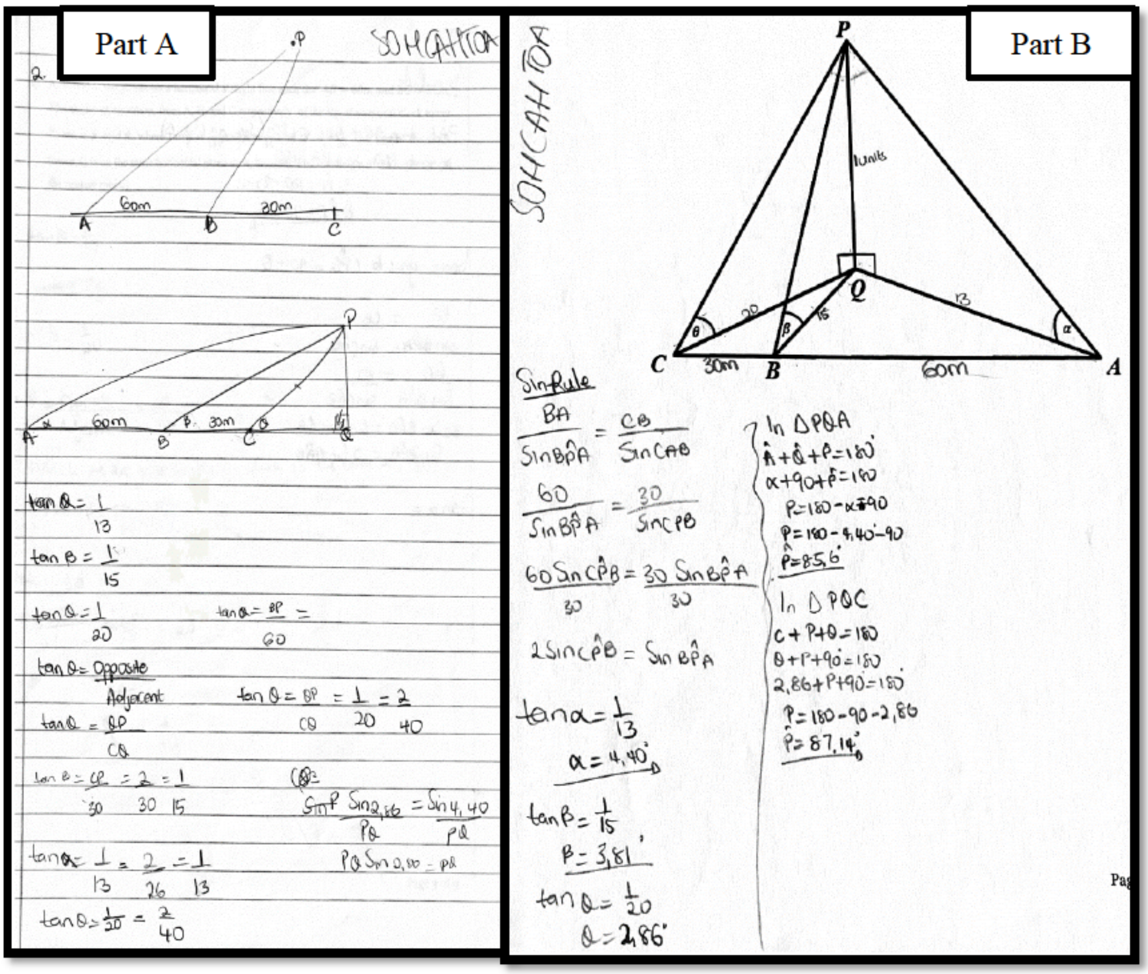


Figure 5.4.5.5: Phume's solutions to Question 2 in Part A (left) and Part B (right).

Lethu's experience was similar to that of Phume and Sfi. She noted:

> *Part B eliminated all those challenges since everything was already drawn. For Part A, I couldn't come up with a correct diagram at first until very late…and still after that, I couldn't work it out and I decided to skip it. I think what challenged me the most is that I haven't worked with these types of problems for a long time* (Lethu).

These comments and illustrations demonstrate that the challenges participants experienced were not limited to their visualization abilities. For instance, Lethu indicated that she had not engaged recently with the type of problem she encountered on the performance test; because she was no longer familiar with this kind of problem she was unable to solve it even when the visual mediator was provided in Part B for the second question. These findings coincide with Supardi, et al.'s (2021) empirical study, where they found that "some students use visual mediators but are still wrong" (p. 959). Rellensmann, et al. (2017) also found that the use of accurate and correct geometric visual mediators by students does not automatically result in correct solutions. It can be contended in these instances that students' inabilities to solve a problem with the visual mediators provided is due to not knowing the correct strategy to solve the problem they encounter. Nonetheless, participants obtaining correct or incorrect answers to the performance test problems was not a unit of analysis for this study; thus, their errors and challenges were not analyzed further.

The participants were asked to share how they could mediate learning using educational technologies to assist secondary school learners with these types of mathematical problems during their teaching practicums. Thus, the next section presents and analyzes the focus group discussions data that established the type of educational technologies they intended to utilize during teaching practicum as visualization tools.

5.5 THEME 2: THE EDUCATIONAL TECHNOLOGIES PRESERVICE TEACHERS INTEND TO USE AS VISUALIZATION TOOLS FOR CONCEPT DEVELOPMENT

This section addresses the second and third research questions, as shown in Table 4.5.1. in the previous chapter. Data was gathered through focus group discussions to answer these two research questions. The findings are presented according to the sub-themes that emerged.

5.5.1 Preservice teachers' identification of educational technologies for use in teaching practice

Two distinctive components emerged from the data regarding what participants classified as educational technologies: digital technologies, on the one hand, and other non-digital resources that enhance learning and teaching, on the other.

Shan and Phume identified digital technologies that could be used to enhance teaching and learning as follows:

> *For me, educational technologies are things like projectors, computers, tablets, cell phones, and software that runs in them used for teaching and learning…* (Shan, FG.1).
>
> *For me I think…other than using textbooks, chalkboards, chalks and so on…using things like advanced technologies that can make improvements in our teaching – like keeping up with the current revolution, called the 4th industrial revolution. Just like how we are also learning online with these different technological resources – even learning using WhatsApp sometimes…So I would say cell phones, laptops, tablets, and different software are the current resources that I understand as educational technologies… So, to me,*

educational technologies basically mean only digital tools we are introduced to for learning and teaching… (Phume, FG.2).

Phume acknowledged the use of traditional non-digital technologies but also spoke to the prevalence of digital educational technologies and their purpose. These narratives suggest that these participants defined educational technologies as those resources that are expected to improve and advance teaching, which had been necessitated particularly during the Covid-19 pandemic. Phume expressed her understanding of how educational technologies have evolved, with increasing reliance on digital technologies:

…for example, I think mathematics instruments were once a technological resource until we started to use things like GeoGebra, which allows us to be able to measure angles, lengths, and draw using laptops and cell phones after downloading the software. Digital technologies are replacing all those old technological resources we used to learn with (Phume, FG.2).

Thus, her narrative depicts how her understanding of what constitutes educational technologies has evolved progressively. This aligns with Dron’s (2021) assertion that “technologies evolve and the technological ecosystem constantly expands and diversifies” (p. 5). Viberg, Grönlund, and Andersson (2020) also argue that, because of this transformation over time, teachers tend to consider only digital technologies when they think of using educational technologies, because of the popularity of these new technologies, whereas other non-digital technologies may also be strategically useful.

In the South African context, however, teachers have been found to choose both non-digital and digital technologies to complement each other during instruction (Ndlovu & Moll, 2016). In a focus group, Sandi noted that in his context, the use of educational technologies should not be restricted to contemporary and advanced digital resources; Zakhe, Qhaw, and Lethu concurred with Sandi’s view:

…in a mathematics class, the teaching and learning technologies could be maybe a calculator, mathematics instruments [maths set]; *and maybe it will depend on the school, whether they teach using chalkboards or using the transparencies or other modern technologies* (Sandi, FG.1).

I also agree with Sandi: even those big rulers, protractors, setsquares that mathematics teachers use on the chalkboard could be included as types of educational technologies, because they assist during the lesson for learners to see how to measure angles – for *example, if there are no computers or projectors to use software to project for a bigger class* (Lethu, FG.1).

My peers are correct – I also think that those form part of technologies, because before all these new digital technologies existed the mathematicians that existed before the digital age considered their mathematical tools as technologies maybe…so yeah (Qhaw, FG.1).

Yes, I also agree with them sir…it depends on the teacher as to what they feel comfortable using or what they have available to teach a particular topic; whatever they choose, I think it is a technology (Zakhe, FG.1).

Their narratives affirm the importance of non-digital technologies and the preferences of different mathematics teachers given the resources available at individual schools. Additionally, from their narratives, it is also apparent that availability is another factor that shapes what is considered educational technologies in the space of teaching and learning. Furthermore, they included digital technologies like calculators and transparencies as viable and useful technologies, even though these are not so modern. These broad interpretations of what educational technologies constitute, as per the members of the first focus group, relate to the definition of educational technology used in this study, adapted from Roberts (2014).

Sfi alluded to the possibility of an even broader and more flexible range of aids constituting educational technologies:

I think it is all those things – like objects – that teachers bring with them to class to enhance their teaching maybe, like in addition to the normal chalkboard…it could be anything depending on the teacher and the topic they are teaching (Sfi, FG.2).

Sfi did not suggest what 'things' or 'objects' could constitute, however, the researcher understood that he was referring to various artefacts (for example, a coin, deck of cards, die, marbles) or tools that teachers could integrate into their lessons after establishing the pedagogical relationship between the object and what they intended to teach. These artefacts thus become instruments of value when integrated effectively and enhance the quality of teaching and learning. Lagrange, Artigue, Laborde, and Trouche (2003) refer to this process of identifying artefacts to be integrated during instruction as "instrumental genesis". It can be contended that Sfi was referring to this concept in his narrative. Nonetheless, Trujillo-Torres, Hossein-Mohand, Gómez-García, Hossein-Mohand, and Cáceres-Reche (2020) aver that instrumental genesis is a complex process that requires extensive planning and is thus not common for mathematics teaching at the secondary school and higher education level.

Zot and Lunge commented on the usefulness of educational technologies to learners:

Educational technologies, for me, are things that make learning easy for learners to understand a concept…So ja, even OHP [overhead projector] *comes to mind as I am thinking about others…I think it is easier for learners to visualize something that is in front of them through the use of educational technologies, than to try and imagine it in their minds without any aid…Like how we had to imagine those diagrams when we were doing the performance test: with some technology it could've been easier. Even the flashcards, interactive charts, and math instruments form part of the educational technologies that can be used for teaching and learning* (Zot, FG.2).

Oh yes! I also agree with Zot. Sorry to step in like that, Sir, but for those types of problems we did in the test: as a teacher, one could use these technologies Zot is mentioning to lessen their level of difficulty. Especially for 3D questions: to have the diagram moving and showing all dimensions, you know…this can really help, and I would try to use some during my own teaching of 3D trig… (Lunge, FG.2).

From Zot's perspective, anything that can facilitate learner understanding such as flashcards and interactive charts, can be considered to be an educational technology. Zot and Lunge viewed these as helpful specifically to assist learners with learning mathematical concepts that require visualization. Integrating technology into the teaching of trigonometry has been advocated in other empirical studies (Ibrahim & llyas, 2016; Mosese & Ogbonnaya, 2021; Bedada & Machaba, 2022). Also, Prabowo, Anggoro, Adiyanto, and Rahmawati's (2018) study revealed that the implementation of interactive multimedia-based teaching to deliver trigonometric concepts was beneficial to both teachers and learners.

In the second focus group, the conceptualization of educational technologies not necessarily being digital emerged as well. Amah, too, raised the usefulness of interactive charts:

…I also think everything that can help them have a better understanding of mathematics content may be regarded as educational technologies when teaching…for example, when you are using interactive charts, learners would have that visual image – then they can understand better the content you are delivering as a teacher (Amah, FG.2).

This affirmed the view expressed in the first focus group that anything that is used during teaching to aid learners' understanding of mathematical concepts can be classified as educational technology. Serdyukov's (2017) empirical study also established that teachers' understanding of educational technologies may include any resource capable of enhancing teaching and learning.

Amah's response also demonstrates that she perceived educational technology as a tool that could be utilized by teachers to afford learners an opportunity to visualize mathematics concepts. This perception concurs with Shara's (2020) assertion that "students can compute and visualize mathematical relationships using technology more quickly, and more of their mental resources are freed to ask new questions, interpret mathematical information and solve more difficult mathematical problems" (p. 5).

Focus group participants also reflected on the educational technologies they had encountered as third-year preservice teachers during their learning and practice teaching experience.

5.5.2 Acquaintance with and reflections on educational technologies

Participants' narratives focused on their acquaintance with educational technologies throughout their learning career. They demonstrated immediately the disjuncture between their basic and higher education experiences. Their reflective accounts attested to the diversity of their basic education experiences, especially in the poorly resourced schools they had attended as learners. Shan described his experience:

> *…at university, my lecturers used to use data projectors to display power-point presentations, sketchpads, view pictures, and play videos. Unfortunately, my school did not have the resources to incorporate digitally advanced educational technologies during teaching and learning* (Shan, FG.1).

Shan's limited exposure to educational technologies during his basic education was due to the fact that the school he had attended as a learner was under-resourced; however, he was exposed to more advanced technologies in higher education. For Shan, the use of GeoGebra and Geometer's Sketchpad by his lecturers in first and second year of university helped his learning as they aided his visualization of the concepts of geometry:

> *I was introduced to geometry software, such as Sketchpad and GeoGebra – both of which helped me better visualize geometry and thus made learning easier, compared to when I was still in high school* (Shan, FG.1).

Shan's account revealed how his learning experience developed due to the use of more advanced technologies at the university. Lethu, Qhaw, and Sandi described encountering an overhead projector using transparencies at university for the first time:

> *…there's this machine that I've forgotten its name…– the one that can display a transparent sheet – I was exposed to that here at the university during micro-mini* (Lethu, FG.1).
>
> *Oh yes! The transparencies, I only saw those for the first time here during my first year* (Qhaw, FG.1).
>
> *Those were also new to me as well* (Sandi, FG.1).

As overhead projectors have been used as an educational technology resource in teaching and learning for many decades, the fact that these three participants had not seen one before highlighted the fact that the schools they had attended were poorly resourced. They indicated that they were able to use these technologies themselves, however, once they had been trained. The "micro-mini" mentioned by Lethu was a first-year module called Teaching Practice 120: Classroom Technology, that was designed to introduce preservice teachers to the teaching experience at the university before their first practicum at a school. This module taught preservice teachers how to implement a range of educational technologies, from non-digital to digitally advanced technologies. Lei (2009) claims that exposure to, and systematic training in, the use of technology is essential to assist preservice teachers in developing their knowledge of advanced classroom technologies while helping them to build connections between teaching and technology.

Zakhe was the only participant who indicated that he had been exposed to a projector at school:

> *For me, it's the projector that we were using in 2018 when I was doing Grade 12. Our math teachers would connect the projector to their personal laptops and teach with it…but the projectors were only for Grade 12 classrooms since they were not enough for all classes* (Zakhe, FG.1).

In the first focus group, participants expressed similar, positive views about being taught using more advanced teaching and learning technologies at university, given their similar experiences at school. Qhaw, Lethu and Sandi commented as follows:

…the difference is very significant; the experience was very good compared to being taught a lesson with chalk and the board …For example, in a lesson that talks about a shift – say, maybe, the graph is shifted, or maybe a shape is being translated – if technology like GeoGebra is used, it is easy to grasp the explained concept faster because you can also see the shift and movement happening…so the experience was very great (Qhaw, FG.1).

I agree with Qhaw as well, the experience of learning geometry with these technologies made learning very enjoyable, because you don't need to imagine how a diagram for a theorem will look when an angle is moved across the circumference, you can actually see it as it is being dragged using Sketchpad or GeoGebra app…that gives you that 'wow' factor as you visualize it (Lethu, FG.1).

The experience is really good, Sir; I also agree with them. It confirms that something will always be true after learning its theory or theorem – as others have put it. You get that good sense of conviction about mathematics facts on theorems (Sandi, FG.1).

Their narratives further demonstrate how beneficial these technologies are for learning mathematics concepts that require substantial visualization abilities.

In the second focus group, participants' experiences were similar. Lunge and Sfi also described being struck by their first exposure to an overhead projector at university. Lungi, Sfi and Zot assisted each other to describe this technology as follows:

So, since at school it was only chalkboard, at the university during our first year we were exposed to… what do you call this thing with a light bulb? [checking with other participants] (Lunge, FG.2).

Oh! Do you mean the transparencies? (Sfi, FG.2)

Yes, that! It's called transparencies. I saw that for the first time in 2019… (Lunge, FG.2).

Yes, but it is also called an overhead projector: "OHP" … (Zot, FG.2).

This further affirms that the participants were fairly homogenous in terms of their lack of exposure to advanced educational technologies at the poorly-resourced schools they had attended. While the continued shortage of resources at the schools within the province reported by participants when they returned to these schools for their teaching practicum was not the focus of this study, it was nevertheless alarming. This situation has also been noted in other recent research studies in KwaZulu-Natal (Gumbi, 2020; Kafu-Quva, 2021) and also in other provinces in South Africa (Philbert, John, & Cosmas, 2014; Adukaite & Cantoni, 2016; Mlambo, Rambe, & Schlebusch, 2020; Soudien, Reddy, & Harvey, 2021). This implies that the systemic hallenges identified by participants in their reflective narratives constitute a national crisis in the basic education landscape and are persisting from one cohort of learners (as documented in the participants' accounts of their earlier experience as learners) to the next (as documented in the participants' accounts of their current experiences as preservice teachers).

Furthermore, several studies (Van Dyk & White, 2019; Buys, Du Plessis, & Mestry, 2020) have demonstrated that many of the schools that receive more government funding (those that fall within Quintiles 1 to 3) allocate these funds to other infrastructure needs, such as furniture and maintenance. Thus, the development of the school's teaching and learning resources and the maintenance of the few modern technologies they may possess remain neglected. Theft has been found to be a major challenge at these poorly resourced schools, which impairs the development of the school's technological capacity, as equipment is frequently stolen after being installed (Msiza, Malatji, & Mphahlele, 2020; Chisango & Marongwe, 2020). Thus, if these schools invest in advanced educational technologies they also need to invest in upgrading their security measures, which incurs additional costs. As a result, this systemic challenge of resource shortages identified by the participants is sustained. In addition, since the implementation of advanced educational technologies in mathematics is associated with improved results (Saal, Van Ryneveld, & Graham, 2019; Namome & Moodley, 2021), it could be argued that lack of access to these technologies at South African schools may contribute to the poor mathematics scores of learners across the country.

Like Lunge and Sfi, Amah and Phume indicated that they were first exposed to teaching technologies other than the chalkboard at the university level. Phume described her experience:

> *Based on the school I come from, I was exposed to using textbooks and chalkboards. They used to divide us in my school – like if you are doing geography, you would be different from someone in a computer application technology class because they would learn with and using technologies in their computer labs. And for us doing geography, we were not exposed at all: we were only dealing with the textbook and chalkboard…* (Phume, FG.2)

Phume's response raises another distinction, with exposure to advanced technologies being associated with subject choice. Her access was limited purely because of her subject choices. However, there has been minimal empirical research on this type of division of resources within a school. Mwapwele, Marais, Dlamini, and Van Biljon (2019) established that it is a common practice within under-resourced schools to reserve computer laboratories for the teaching and learning of computer-related subjects only. They further note that the possibilities for resource-sharing are limited, thus room for integration of these advanced technologies into other subjects is minimal.

In the second focus group, it was only Zot who indicated that he had been exposed to some advanced educational technologies at the school level:

> *In my experience, most of the time it was a projector that I was exposed to, and OHP* [overhead projector] *was used at my high school ...so they would connect to a laptop especially when they were teaching geometry to save time when drawing the diagrams, instead of drawing them on the chalkboard* (Zot, FG.2).

It is apparent from Zot's response that his teachers were integrating technology into mathematics teaching to save the time that would normally be taken if mathematics teachers sketched geometric visual mediators on the board during the lesson.

The second focus group shared similar sentiments about their experiences of learning through advanced technologies at the university. However, Phume indicated a preference for using traditional technologies that provided learners with tactile learning experiences:

> *I think for me, currently, I still like the old method of learning – like using those math instruments – so that at least learners can know how to hold them and measure angles using a protractor. With GeoGebra, it's too easy because you just draw a shape, then click a few buttons, then it tells you the size of an angle – without getting to feel it and do it practically. like with instruments…but I can teach learners both ways* (Phume, FG.2).

Sunzuma & Maharaj (2019) support the continued use of traditional mathematical instruments as they provide a physical learning experience that is associated with the long-term retention of knowledge in geometry.

5.5.3 Teaching using advanced educational technologies

Since all the preservice participants were in their third year of study during data collection, it was assumed that they possessed some experience of delivering some mathematics lessons in a school environment or during peer-teaching at the university. Hence, during the focus group discussion one question was to determine if they had an opportunity to conduct a lesson using some of the advanced educational technologies they have been exposed to during their learning at university. Since in addition to the first year "classroom technology" module they receive additional exposure to more advanced technologies to integrate during mathematics teaching. The other first year module focuses specifically on the teaching of geometry and trigonometry using mathematical software and other related technologies such as smartboards. Nevertheless, their experiences varied since three (Shan, Sandi & Sfi) from the first focus group indicated that they did not have exposure to advanced technologies at the school level as validated by Shan and Sandi

> *No, unfortunately in the schools I have been to for my first teaching practice they did not have the advanced technologies in their Maths classroom like projectors…I only used these technologies here at the university during the classroom technology module* (Shan, FG.1).

> *Me neither, I have never used technology that much like the ones we are taught with by our lecturers, I only created a video but I was presenting a lesson while recording myself and not necessarily using any specialized software or app. It was for the method module where they wanted to access our teaching skills. But I never used any advanced technology for my lessons because it was not compulsory, it was up to you* (Sandi, FG.1).

From these two narratives, it seems Shan could not adopt any advanced technologies in school because of a resource constraining environment whereas with Sandi it was a conscious decision not to implement any technologies at his disposal within the university. As indicated earlier in the study, the university where the participants are registered is well resourced with advanced educational technologies, and the teacher educators are experienced with the utilization of these technologies. However, it is individual preservice teachers' discretion to implement these technologies into their own teaching after receiving training. For instance, Sandi asserts that it is an individual decision to adopt advanced technologies in some of the mathematics method module assessment tasks. The mathematics method module he is referring to is designed to improve and assess preservice teachers' PCK while at a later stage another method module incorporates the entire TPACK. Nonetheless, there is no set percentage of how much advanced educational technologies can be implemented by preservice teachers in their lessons during teaching practicum, although teacher educators highly recommend its implementation. Additionally, these narratives demonstrated the sustained lack of resources at the basic education level, which is a systemic challenge that has persisted since they were pupils. It also demonstrates how the cycle of being a pupil, student, and preservice teacher allows them this reflective account of noticing how educational technologies in mathematics teaching has not changed substantially at the basic education level.

In addition to their reflective accounts, the recent findings from (Du Plessis & Mestry, 2019; Mwapwele, Marais, Dlamini, & Van Biljon, 2019; University of Chicago Law School - Global Human Rights Clinic, 2020) has established that several South African schools experience a shortage of advanced educational technologies for teaching and learning. The Covid-19 pandemic that necessitated the use of more advanced educational technologies to continue with the teaching and learning process amid the lockdown, confirmed this systemic challenge participants allude to. For instance, findings from several studies (Mukute, Francis, Burt, & De Souza, 2020; Chirinda, Ndlovu, & Spangenberg, 2021; Makgahlela, Mothiba, Mokwena, & Mphekgwana, 2021) conducted during and after the Covid-19 state of the emergency have demonstrated that many schools within the country could not continue with the teaching and learning process due to a shortage of advanced educational technologies.

Nevertheless, other participants incorporated some of the advanced technologies, which they had exposure to while at the university. Qhaw and Lethu clarify that

> *I have used an overhead projector and a PowerPoint in a classroom technology module here at varsity during my first year. I was using it to teach a grade 10 Trigonometry lesson, it was very interesting* (Qhaw, FG.1).

> *I can say I have used mathematics software this year, there was this lesson I wanted to prepare…a grade 9 lesson, to teach them about transformation and translations; things like that. I was using this software from my laptop projected for learners…I enjoyed using it…in fact, I think the software made the teaching of that topic even easier compared to when I would have prepared to teach it using a chalkboard* (Lethu, FG.1).

It is apparent from Qhaw's narrative that the mathematic method and classroom technology module (Teaching Practice 120) equipped him with TPACK competencies as he was able to confidently integrate advanced technologies during a mathematics content lesson. Santos and Castro (2021) proclaim that preservice teachers' training institutions ought to design instructional courses that promote the implementation of advanced technologies to remedy the lack of TPACK competence by in-service teachers. These findings further correlate with Tinh's et al., (2021) assertion highlighting that "pre-service teachers heavily depend on their teacher-training programmes to be confident and ready in their future teaching task" (p. 9). Furthermore, Lethu reaffirmed how the utilization of educational software enhanced teaching as compared to the choice of less advanced technology. Several scholars (Bulent, 2012; Denton, 2017; Setyawan, Kristanto, & Ishartono, 2018; Segal, Oxman, & Stupel, 2021) have demonstrated that using dynamic geometry software for transformation geometry improves the learning experience for learners. Lethu further elaborated that

> *…the software was showing the tracing of all the movements of the objects visually on the screen, but with the chalkboard, it was going to take time and sometimes end up confusing. With the software, I could do a lot of transformations in a short space of time* (Lethu, FG.1).

The inherent benefits for Lethu were undoubted, especially in comparison to primitive technologies. In addition, her narrative demonstrates TPACK competence and confidence as she successfully planned and executed a lesson with technology for visualization purposes.

All the other participants in the second focus group had an opportunity to implement some advanced educational technologies into their teaching of mathematics during their first and second years of study. Thus, it can be contended that they had an opportunity to improve their TPACK proficiencies. Lunge and Sfi specified that

> *We have used the ones we just discussed on the first and second questions, we were first taught how to use them and then we used them in our first teaching practice* (Sfi and Lunge agreeing with each other).

Lunge and Amah further shared their experience of teaching using the technologies

> *The one for transparencies for me was very useful in my teaching during first year TP compared to what I was used to during my own learning of mathematics at high school…it was particularly nice, especially in a topic where you have to first show the first step before you go to the second… whereas with chalkboard it is different, I can't explain but it's not the same…but I can try to use the same logic from transparencies with chalkboard because I will be going to a township school without these technologies* (Lunge, FG.2).
>
> *Oh yes! I think I also like that kind of transfer of logic when going to teach in the schools with none of these technologies because I will be going to a township school as well and there is only a chalkboard for teaching* (Amah, FG.2).

Lunge's narrative indicates his adaptability to different contexts. He is aware that in under-resourced schools he might have to revert to the chalkboard. Other participants in Lunge's group like Amah offered similar interpretations. Their adaptability and innovation are evident as they navigate the resource constraining teaching environment. Their concept of transfer of logic demonstrates that they learned beyond just being able to use these advanced technologies. However, they also identified a valuable teaching strategy to use with less advanced technologies. The rest of the group concurred with Lunge's narrative regarding the use of transparencies, Amah and Sfi mentioned that

> *Yeah…in fact, I was also seeing the transparencies for the first time here at varsity, they are interesting to work with they give you that sense of explaining something better with confidence where you have everything according to steps…* (Sfi, FG.2).
>
> *…Oh yes! for me as well, I've also used transparencies when I taught trigonometry in grade 10…in most cases that helped a lot because learners were able to understand better what was happening, they were able to build that mental image* (Amah, FG.2).

Clearly, from the narratives advanced educational technologies are preferred. Amah's response emphasizes comprehension of the content while creating those mental images which Presmeg (2020) identifies as being crucial in the learning of mathematics. While the former narratives allude to the self-confidence they develop when explaining a concept through the use of advanced technology. Their reflective accounts demonstrate that indeed their teacher training institution has equipped them with technological knowledge that is central to pedagogical content for mathematics teaching. Panthi and Belbase (2017) maintained that when teachers value technology for their teaching of mathematical concepts and have acquired adequate skills to utilize it, they are more likely to demonstrate confidence in implementing it in classroom practice. Moreover, Phume added on her adapting

> *Yes, in my first year I used an interactive chart, whiteboard, transparencies, projector, and a laptop because we were told that for introducing a lesson, it has to be interesting; so I had to download a video and play it in class just before I start a lesson to grab learner's attention, I then used the transparencies to teach graphs …then I went for TP, classes were overcrowded and the school did not have the resources to teach using technology… so I could only use interactive chart and chalkboard because of that situation, but I could still apply the same logic of transparencies…* (Phume, FG.2).
>
> *Yeah, same with me as well, at the university, we would just use technology anytime when presenting lessons but when you go to school things change…you can think of using like a laptop for a lesson with a nice power-point presentation or video explaining something but end up not playing it in class* (Zot, FG.2).

I also agree, Zot…especially if the class is too big and you can't use a laptop screen (Amah, FG.2).

Phume's comment indicates how their TPACK is developed from the first year of study as they commence with their teacher training courses. The TPACK theoretical framework proponents advocate the integration of technology. Such integration is emphasized for teaching and learning using technology. The basis of TPACK suggests that technology is not a separate element to be added to a teaching and learning process, however, it should be a cohesive unit with instruction. This transition would only be possible if preservice teachers were to receive consistent exposure to the use of educational technologies during their teaching experience.

Furthermore, their narratives reveal the limitations presented by the sustained under-resourced environments within the basic education sector, which the participants repeatedly alluded to during both focus group discussions.

5.5.4 Integration of advanced educational technologies during mathematical problem-solving

All of the preservice participants had been exposed to the integrative element of advanced educational technology during their TP120 module and mathematics method geometry modules. During the focus group discussions, participants were asked about their ability to integrate technology into their teaching of mathematical problem-solving using visualization. Their responses demonstrated that they held positive attitudes towards implementing advanced technologies, but their strategies for implementation varied:

Yes, it would help to visualize the figures. These software – like GeoGebra and Sketchpad – also measure the length of sides and angles, so it is great for confirming whether the answer is correct or not (Shan, FG.1).

I also think they could really help to understand those kinds of word problems to visualize them better – you know, like the ones we were doing (Lethu, FG.1).

That's true, guys. I also believe those advanced technologies can be very helpful because they can also move the objects and you can see all the faces and angles of the 3D shape, and that can make it easy to see a key strategy to solve the problem with (Qhaw, FG.1).

The participants described how they could integrate technology to understand problems similar to the one they had attempted during the first phase of data collection (performance test). Shan's response reveals that he believed that the use of these technologies could assist with visualization; this verified his intention to integrate advanced technology and supported his later assertion that he had used Sketchpad in his practice teaching. This aligns with numerous scholarly findings (Dogan, Dogan, & Celik, 2021; Luik & Taimalu, 2021; Watson & Rockinson-Szapkiw, 2021) that demonstrate that the intention to use technology by preservice teachers suggests their positive attitudes and beliefs towards technology as well as perceived usefulness. Next, Shan outlined the specific capabilities of these advanced educational technologies and that they could be utilized to verify the solution to a problem. According to Polya (1973), solution verification is a crucial step in the mathematics problem-solving cycle since it assists to make sense of the entire problem after solving it and to predict strategies to use to solve other problems in the future. Thus, integrating these advanced technologies allows for visualization and confirmation of results. This correlates with Moreno and Llinares (2018), who argue for "organizing a lesson plan taking into account mathematical activity and taking advantage of technological resources in problem solving" (p. 127) as one of three ways mathematics teachers can integrate educational technologies.

Qhaw also emphasized how integrating technology strengthens comprehension of the problem, thereby enhancing learners' visualization abilities. Similarly, other research studies (Greefrath, Hertleif, & Siller, 2018; Hillmayr et al., 2020; Liburd & Jen, 2021) have found that the utilization of technology can sustain strategies and skills relevant to mathematics content areas such as visualization of abstract relationships or problem-solving. Another important contribution was made by Qhaw during the first focus group discussions. He indicated that using advanced technologies in mathematics teaching offers dynamic capabilities to better visualize 3D geometric visual mediators, thereby increasing opportunities for successful problem-solving. This aligns with Mavani, Mavani, and Schäfer's (2018) empirical findings that dynamic geometry software (DGS) can play a significant role for teachers when they develop the visualization skills for learners while developing their conceptual comprehension of mathematics concepts.

Zakhe shared a creative idea for creating a resource that could serve the same aims as a digital technology. Sandi and Qhaw affirmed his idea that this kind of a resources would assist learners in understanding problems in mathematics:

> *I was thinking about the kind of things, or resources, I could create by myself for learners, something like maybe… like, for that task A number 2, I could make something that could depict that whole scenario using cardboard maybe, for learners to see visually. Something that is created – like a resource – to use not software or digital*

technology, but something that learners can also move around and see all of its sides and corners…like do something they can see and touch, maybe if I am failing to use technology or if I think that I won't be able to use it properly (Zakhe, FG.1).

Oh yes! that is a great idea, to have learners look at those created 3D objects to make sense of the problem in writing as well as visually (Sandi, FG.1).

Yes! That can also help them visualize these types of problems even during exams, after they have seen the visual objects (Qhaw, FG.1).

Zakhe shared a creative idea that could also achieve a similar goal as with digital technology that supports DGS to a certain extent. However, his idea was most appropriate for under-resourced schools. He elucidated the value for the teaching and learning of 3D geometry as was also suggested by Fonseca and Roberts (2017) in their empirical study. Other participants also concurred with Zakhe by further sharing the extent to which these types of resources would assist learners a long term with the understanding of word problems in mathematics.

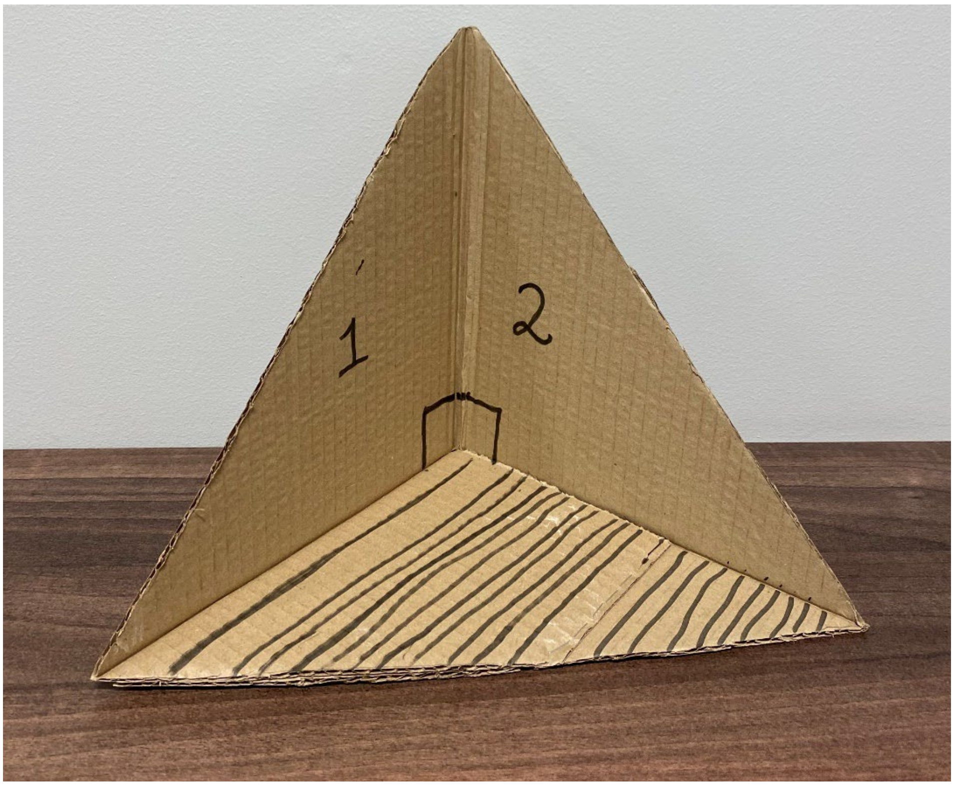


Figure 5.5.4.1: Teaching resource made by Zakhe.

Figure 5.5.4.1 depicts one of these resources that was created by Zakhe a few weeks after the focus group discussions. He shared this resource with his peers. Figure 5.5.4.2 adds labels for common terms in trigonometry to Zakhe's learning aid. This form of a mathematics resource can be classified as a manipulative that learners can utilize to develop their visual imagery for 3D geometric shapes.

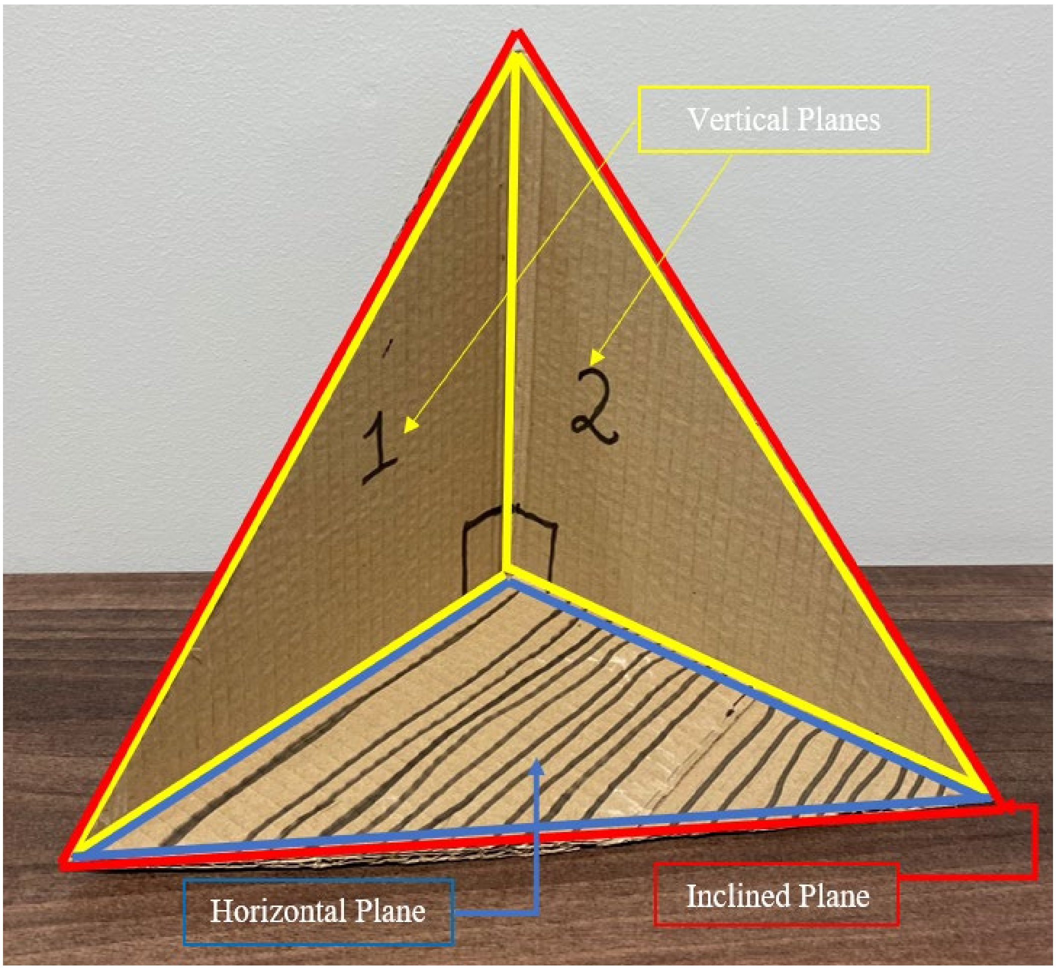


Figure 5.5.4.2: Adaption of teaching resource created by Zakhe.

In the second focus group, Sfi suggested another innovation for teaching 3D trigonometry concepts in an under-resourced environment without advanced technologies:

> *The first one is the one I just explained* [the mountain or the highest tree]*, where you will go out as a teacher and point at the mountain pick or the top of the tree and try to explain the concepts of trigonometry – like angles of elevation and depression – when you are standing on a hill or climbed right at the top of the tree…* (Sfi, FG.2).

His narrative is also framed within an under-resourced environment, hence, the innovation in the absence of technology. Sukendra (2020) contends that it is common for mathematics educators to use real-world examples and scenarios to explain 2D and 3D trigonometric concepts during instruction. Although Sfi's innovation is not a technology, such an alternative is not peculiar in mathematics teaching as anecdotal evidence also suggests. Lunge and Zot shared an idea similar to Zakhe's of using physical aids to teach 3D concepts:

> *You can also buy some stuff like those 3D objects and bring them to class for learners to visualize those concepts – like perpendicular heights, different planes* [faces]*, and angle of elevation…which brings me to my question: where do you get those things?* (Lunge, FG.2).

> *You can actually make these things using materials like cardboard and old containers for beverages if you don't find them at the stationery shops* (Zot, FG.2).

In the absence of advanced technologies, participants considered other available resources, such as non-digital manipulatives. These narratives appear to reveal that not being exposed to advanced modern digital technologies influences the adoption of certain teaching strategies and a kind of selection for resources to enhance teaching, although this may be the case as well for teachers who come from a digital background when they find themselves in a resource-constrained environment. Nevertheless, their TPACK competencies were used to integrate other mathematical resources that they perceived to be useful and effective in improving the teaching process.

Zot and Amah spoke about using digital technology to help themselves, and their learners, with visualization for problem-solving:

> *Yes, technology could have helped because I could've drawn the diagrams using technology before answering so that I can visualize; and after visualizing, it would've made me understand the problem better and then attempt the question* (Amah, FG.2).

> *I would also consider that as well in my own teaching – like using GeoGebra or Sketchpad before and during the lessons – so that learners can also see how to use it* (Zot, FG.2).

Phume and Shan, on the other hand, favoured teaching students to analyze problems abstractly before using technology to enhance visualization to help them solve a problem:

> *What I can say regarding that task is that it is important first for the learners to learn the skill of how to analyze those types of problems for understanding, and then any technologies can then be used later if learners are still struggling. What is important is for the teacher to teach learners how to tackle such problems* (Phume, FG.2).

> *Yes! I also agree with Phume. Sometimes it is useful to learn the skill of solving the problem analytically and logically before adopting more – any form of technology…But then it depends on the type of problem or concept one is dealing with* (Shan, FG.2).

The ability of a teacher to equip learners with analytical skills was identified as imperative by these participants. They maintained that technology integration can follow at a later stage should learners demonstrate any difficulties. In a recent study, Netsianda and Ramaila (2021) found that mathematics teachers only implemented advanced educational technologies later in their teaching to supplement traditional teaching. Phume explained that her view was informed by her lived experiences both as a learner and a preservice teacher in a resource-constrained environment:

> *So, as a teacher, for now I believe in teaching with less advanced technologies because of my background. I have taught in schools where learners do not even have cell phones to download GeoGebra. So, I would not like to teach with advanced technologies where there are so many challenges in our schools with resources* (Phume, FG.2).

Her narrative reveals a sense of acceptance of the systemic challenges that prevail at public schools in South Africa.

5.5.5 Use of educational technologies as visualization tools

Participants were asked about how their use of advanced educational technologies would influence learners' abilities to visualize. Their responses were optimistic. For instance, Sandi responded:

> *Yes, I agree – to an extent that I am going to try, myself – in this school I am going to visit for my teaching practice – to use these technologies for visualization…and if I need to hire a projector, I will do that…* (Sandi, FG.1).

It is apparent that this participant was committed to implementing advanced technologies in his lessons to enhance learners' abilities to visualize. This further implies that the participant was confident about his TPACK for the teaching of mathematics because of his demonstrated willingness. Ndlovu, Ramdhany, Spangenberg, and Govender (2020) also found that, when preservice teachers demonstrated confidence and the intention to integrate technology in the classroom, it correlated with competence in terms of their skills and knowledge. Lethu prioritized learners' perspectives and the visualization enablers that they could be offered:

> *I can say that it depends on which grade you are teaching and what topic are you teaching…for example, if you are going to teach 3D shapes in Grade 8 or 9, it would be a challenge if you bring the laptop and try to show them a cube or triangular prisms maybe. Like, trying to show them a picture in a computer or a piece of paper would be harder, but it would be easy for them to visualize if they can see an actual cube – like*

bring a die or try to create those triangular prisms for them to touch and visualize (Lethu, FG.1).

Yildiz and Gokcek (2017) stress the value of determining learners' prior knowledge before preparing a lesson to inform the level of teaching. Furthermore, several empirical studies (Golafshani, 2013; Larbi & Mavis, 2016; Hidayah, Dwijanto, & Istiandaru, 2018; Peltier, et al., 2020) have established that teaching mathematics with manipulatives has been associated with an improved level of comprehension among learners across grades. Liggett (2017) also affirms that manipulatives are a valuable resource that offer concrete ways for learners to understand abstract mathematical ideas. Thus, it is not only digital technologies that are capable of equipping learners with visualization skills for mathematics concept development. Qhaw and Phume shared this view:

> *For me as well, to make learners visualize I also believe in making 3D objects from cardboards and giving learners to have an opportunity to see and touch all the different views. And give them keywords of how they will then actually visualize these things when they are drawn or written in words during the exam sessions. Because the problem statements explain everything in words that you need to visualize as you read* (Phume, FG.2).

> *I also agree it helps to physically show learners what you are trying to explain to them. Then, when they are expected to visualize it – maybe in a different format later, in their minds when it is discussed, or coming up in the activity or test as a diagram or statement – they don't struggle that much, because of how it was introduced* (Qhaw, FG.2).

Phume indicated she supported equipping her learners with visualization skills using 3D geometric objects to enhance mathematics teaching and learning, as highlighted in the previous section. Zot contended that the use of educational technologies during mathematics teaching could influence learners' abilities to visualize a range of mathematics topics:

> *Yes, it does help a lot, in many concepts…it could be algebra, geometry, even the number patterns: you can show them different colourful patterns that are interesting – to visualize the next pattern, and so on…* (Zot, FG.2).

Amah agreed with these views and added her perspective that technologies could broaden access to concepts to different learners:

> *Yeah, and I can also say that educational technologies cater to more types of learners in class during the lesson to be able to understand and participate at the same time, so*

yes, I can say that it does make a difference and it is important for visualization and concept development…I agree with others as well (Amah, FG.2).

Yilmaz and Argun (2018), in their study, established that educational technology can be integrated for the visualization of any mathematics topic as long as proper preparation is made prior to the lesson. Additionally, Amah recognised that learners can have different learning styles and perceived technology as a resource that could assist visual learners during mathematics concept development. Alabdulaziz (2021) argues that the use of educational technologies during instruction is particularly beneficial for learners since "they allow visual, auditory and kinaesthetic processes to be integrated at the same time" (p. 7612).

The next section presents the findings and analysis drawn from data collected during observations of participants while teaching in the classroom.

5.6 THEME 3: IMPLEMENTATION OF EDUCATIONAL TECHNOLOGIES DURING TEACHING PRACTICE

Data obtained using observation during the last phase of the study is presented in themes and sub-themes in this section. Due to the Covid-19 pandemic, physical access to schools in South Africa was restricted, and a limited number of learners and staff were allowed to enter the school each day. As the researcher was not granted access due to these restrictions, the data was collected by the preservice teachers' school mentors using the observation questionnaire presented in the previous chapter (Appendix F). Both school mentors and the preservice teachers completed specified sections of the questionnaire. Their responses are presented and analyzed in this section.

5.6.1 Itemization of educational technologies identified for mathematics teaching

The school mentors from all ten schools reported the educational technologies (laptop, overhead projector, interactive chart, whiteboard, and chalkboard) available to be used by preservice teachers at their schools. These were used differently by the various participants in their teaching, depending on their availability per school. For instance, the mentors observing Shan, Amah, Lethu, Qhaw, Lunge, and Sfi indicated that, despite the challenges encountered (which are presented in the subsequent subsections), these participants included video clips in their lessons to enhance teaching and learning. Zakhe and Zot were reported to have utilized laptops to teach using different mathematics applications – GeoGebra and Sketchpad, respectively. Only Zakhe and Zot were reported to have used an overhead projector. The use of interactive charts was witnessed in the lessons taught by Lethu, Sandi, and Phume; however, Shan was the only participant who utilized the whiteboard. The chalkboard – one of the oldest forms of educational technology – was used by nine participants. Thus, preservice teachers implemented a variety of educational technologies in their teaching of mathematics.

5.6.2 Validation for mathematics concept development

Both mentors and participants, at different levels, noted the validation of concept development provided by educational technologies. For instance, Shan, who did not have a projector at his school, indicated that

> *[e]ducational technology helps learners visualize the content being taught. For example, I could nicely show the learners that transversal cutting parallel lines, form corresponding and alternate angles that are respectively equal to each other…I could do this by measuring the angles and proving that they are equal. Likewise, I could show that the transversal cutting parallel lines form co-interior angles that are supplementary. In fact, this is something I had always wanted to do in a Grade 10 class before introducing the section about quadrilaterals, you know; parallelogram properties and others...But the school did not have the projector for me to do this better, I had to rely on my small laptop screen and make learners sit in groups to see...*(Shan).

This narrative demonstrates the confidence and justification that this preservice teacher had concerning his implementation of educational technologies, regardless of the constraining circumstances. This confidence and ease aligns with Bonafini and Lee's (2021) recommendation that teacher educators at teacher training institutions ought to develop within their students as they develop their TPACK. It can further be inferred that the participant was confident about utilizing educational technologies in the teaching of geometry as he seemed to understand how its implementation assisted learners during instruction.

In Zakhe's case, his mentor reported observing him successfully implement the use of GeoGebra in a geometry lesson:

> *I believe that his chosen educational technology influenced his teaching and concept development, because he was able to convey the lesson expectations to the learners using the GeoGebra software that he brought with his computer to class. The use of his technology did not get in the way of a successful lesson he presented* (Mentor: Zakhe).

When the use of GeoGebra software is integrated successfully into a geometry lesson, learners are more likely to demonstrate a strong foundation of geometric concepts (Mukamba & Makamure, 2020). This foundation is commonly associated with long-term retention of conceptual understanding, as Adelabu, Makgato, and Ramaligela's (2019) empirical study also confirmed. Similarly, Lethu's mentor reported that the implementation of educational technologies in mathematics teaching assisted with scaffolding in different ways:

> *Using visual resources during scaffolding in class: number1, it draws learners' attention since they all want to feel as a part of the activity; and number2, this isn't the same as narrating content to learners: the use of these well-prepared videos develops a good imagination for learners…* (Mentor: Lethu).

The use of educational technologies during scaffolding in mathematics teaching has been advocated by numerous scholars (Bakker, Smit, & Wegerif, 2015; Abdu, Schwarz, & Mavrikis, 2015; Belland, Walker, Olsen, & Leary, 2015; Sun, Ruokamo, Siklander, Li, & Devlin, 2020) on the basis of their empirical findings. Reinhold, Hoch, Werner, Richter-Gebert, and Reiss (2020) confirm that educational technologies can be characterized as scaffolds that play an integral role in supporting learners' conceptual development. Lethu's mentor identified this role of educational technologies during the observation session. The mentors who observed Phume, Lunge, and Qhaw also validated that the adoption of educational technologies by participants held learners' interest throughout the lesson and helped to extend their concentration span. Lunge's mentor further reported that

> *[l]earners were so focused on the lesson and happy to experience new teaching methods – they have never experienced a video lesson before. They concentrated better than they do if it is a chalkboard lesson…the 'normal way' we employ as older teachers* (Mentor: Lunge).

The different visual modalities became a catalyst for creating a conducive mathematics learning environment. The school mentors' narratives indicated that this form of positive transformation was noted to some extent in all ten cases. The mentors observing Phume, Lunge, and Qhaw concurred with findings from other research studies (Sacristán, 2017; Lalian, 2019; Hillmayr, Ziernwald, Reinhold, Hofer, & Reiss, 2020) that when learners are taught using both auditory and visual inputs, they can organize new information through both channels, enhancing these two cognitive structures (auditory and visual channel). Hence, learners' ability to concentrate is strengthened since cognitive overload is prevented with the use of video graphics, sound, or visual images in combination with spoken and written texts. This validation of educational technologies for mathematics concept development was confirmed by the mentors and is supported by other empirical studies. Saylan, Onal, and Onal (2018), in a study of the views of preservice mathematics teachers regarding the use of technology, established that the use of videos and presentations made the subject enjoyable, interesting, visualizable, and easy to learn while enabling permanent learning.

In addition, Sandi's mentor mentioned that the use of interactive charts assisted with continuous assessment during the lesson; thus, it kept learners focused and attentive. His mentor reported that

> *An interactive chart was used to check if learners follow the concepts of a lesson by using continuous assessment* (Mentor: Sandi).

Essentially, the use of interactive charts in Sandi's classroom was implemented as a way of consolidating the mathematics content that was taught while learning occurred. This was an ideal approach, as other studies (Mayer, 2014; Hillmayr et al., 2020) have also confirmed that learners need to be actively engaged with mathematics learning content through the use of interactive learning tools to comprehend new knowledge. Hillmayr et al. (2020) point out that interactive tools support learning by providing learners with opportunities to engage in constructive dialogues or manipulations. Thus, the use of interactive charts in Sandi's lesson provided similar opportunities for learners, especially as the interactive chart was used during the assessment phase.

5.6.3 Visualization enablers for mathematics learners

Six of the school mentors alluded to the common educational technologies that were being implemented that enabled learners to visualize mathematics concepts. For instance, Zakhe's mentor stated:

> *I do believe that the use of educational technologies by the student teacher influenced learners' abilities to visualize mathematics, because he was able to deliver concepts to learners visually with GeoGebra software. His entire lesson using this tool was well executed, from my opinion* (Mentor: Zakhe).

Although the scope of this study did not include the aspect of actually examining the learners' ability to visualize after the implementation of these technologies for teaching and learning, the observational reports suggested that their use was effective. When used effectively, GeoGebra has the potential to improve the quality of learning by allowing learners to visualize, explore and construct mathematical concepts (Vágová & Kmetová, 2019; Tamam & Dasari, 2021). From Zakhe's mentor report, no micro-challenges were observed regarding Zakhe's implementation of educational technologies. Thus, it can be inferred that the positive impacts of using GeoGebra in a mathematics lesson were actualized in his teaching experience. Lethu's mentor described learners' responses during the lesson:

> *I saw this question while she was assessing them during the lesson; there was this Question: 'Shift the graph of $f(x) = \sin x$, 4 units up and label it g(x). Now if you are done shifting the graph of f(x) 4 units down then label if h(x)'. I saw a couple of learners before even doing the question on paper; they were busy demonstrating using hand gestures, trying to communicate their answer without words. They kept on drawing their graphs on air immediately after their teacher had asked them to think about what will happen to the original graph of f(x). The video clip that was used by the student teacher illustrated these shifts very well for learners* (Mentor: Lethu).

This indicates that the implementation of educational technologies enhanced learners' abilities to visualize in two ways. Firstly, learners were witnessed self-communicating – using hand gestures in the process of responding to a question – which could be classified as some form of commognition (individualization) since they initially attempted to make sense of their gestures prior to explaining to each other or their teacher. Secondly, drawing their graphs in the air indicated that they were visualizing the graphs in their minds, which is a form of visualization. Studies by Ng (2018) and Daher (2020) demonstrated that the use of digital technologies during the learning of functions is likely to promote new modes of communication and interaction amongst learners to build their comprehension of functions and their graphs as they advance.

Sandi's mentor reported that the use of educational technologies in his lesson helped to simplify mathematical concepts and problems as it enabled both visual and auditory learning:

> *Using educational technology help by simplifying mathematics problems; since learners see mathematics problems and examples then they can learn easily from visualizing than listening to the teachers' voice only* (Mentor: Sandi).

Sandi's mentor emphasized that learners learn mathematics differently. For some, learning by listening may not be sufficient; thus, using educational technology to enhance the visual experience for learners who rely heavily on visual representation is also necessary. Bosman and Schulze (2018), in their study, recommended that mathematics teachers create a positive learning atmosphere in the classroom by using a variety of teaching techniques that accommodate learners with different learning styles. In addition, Sandi's mentor noted that the use of educational technology also simplified mathematics problems for learners, as Liburd and

Jen (2021) also found. Furthermore, the facilitative capacity of educational technologies was established from mentors' reports. Amah's mentor, for example, mentioned that

> *[h]er lesson was highly visual. Learners were able to visualize the content delivered to them. They were also able to recall answers when asked* (Mentor: Amah).

Similarly, in Lunge's trigonometry lesson, his mentor observed that the use of video assisted learners with visualizing the 3D shapes from all views:

> *The video he played for learners from his laptop showed the 3D shapes in all angles and sides/planes. Whereas on the chalkboard the diagrams would look more like 2D shapes with intersecting lines. But it was easy for him to then quickly tell learners to think about the 3D shapes he showed from his laptop when looking at his 3D drawings on the chalkboard. Learners could simple relate and visualize* (Mentor: Lunge).

If the chalkboard had been used, more verbal explanation would have been needed by the teacher to enable learners to project the 2D representation into three dimensions in their minds. Lunge's mentor described this as follows

Not all of the preservice teachers were able to use technology to enhance learners' abilities to visualize, however. Some challenges were mentioned by both participants and their mentors. Sfi's mentor mentioned that other alternatives could enable learners' abilities to visualize where more advanced technologies were not available:

> *Our school is very poorly resourced to assist student teachers who want to teach using modern technologies of today for things like visualization of mathematical content. With us old teachers, it's easy because we just use our experience to verbally ask learners to create mental images or put a sketch on a piece paper when doing maths that requires some visualization* (Mentor: Sfi).

This mentor demonstrated that they were aware that learners' abilities to visualize needed to be cultivated and described other ways of achieving this in an under-resourced school environment, in the absence of more advanced technologies. Murphy (2019) found that gestures and verbal-linguistic codes played a role in the creation of visual mental imagery during mathematics learning and problem-solving. Similarly, Clements (1981) reports that several visualization studies associate learning with a constructive process of generating abstract mental imagery representations that align with logical verbal communication and thought. Sfi's mentor's narrative reveals a recognition of this process, although in a simpler form. Additionally, Sfi's mentor acknowledged the role of modern educational technologies as visualization enablers in mathematics learning, although as ‘old teachers’ they were adept at fostering learners’ visualization without these technologies.

5.6.4 Preservice teachers’ knowledge of educational technologies

The final two questions on the observation schedule were directed to the preservice teachers. Their responses regarding their knowledge of the use of educational technologies revealed that they had received training on the use and integration of different educational technologies. Their responses further unveiled their TPACK status pertaining to the teaching of mathematics at secondary school. Shan explained:

> *I believe I have the necessary knowledge to use maths software such as Sketchpad and GeoGebra. I have used a little bit of both – depending on what I want to achieve in a lesson – and I have seen my mathematics education lecturers efficiently use both all the time for different mathematics topics such as geometry, trigonometry and functions. I believe that a teacher has to be prepared for any lesson, and I believe that if I were to use these educational technologies every day, I would have to be fully prepared all the time to use them effectively* (Shan).

Shan expressed confidence that he could select and use supplementary technologies regularly in his lessons if they were available. This indicates that his TPACK for teaching mathematics had developed to the extent that it achieves the aims of TPACK framework laid out by Gillow-Wiles and Niess (2014). In a similar vein, Kartal and Çınar (2022) state that the TPACK framework should assist preservice mathematics teachers to strategically cogitate “about when, where, and how to use mathematics-specific technologies” (p. 2). Additionally, from Shan’s response, it was evident that his mathematics education instructors had provided a positive environment for the development of his TPACK by modelling the efficient utilization of educational technologies for the teaching of mathematics content he would also probably teach in the near future. This particular result concurs with the findings discussed in the literature review chapter pertaining to education professors or teacher educators modelling the integration of educational technologies in the facilitation of mathematical content. Furthermore, Shan’s narrative revealed that in the development of his knowledge about the teaching of mathematical content using technology, he had also formed some positive beliefs about teaching in this manner. Zakhe, too, mentioned that

…when I was doing my first year in university, we were taught how to use these kinds of technologies to teach maths, so it was easy for me to use them effectively in my teaching since our lecturers were also using them for all lectures (Zakhe).

Zakhe was referring to the use of the mathematics software GeoGebra and its supplementary technologies. It can be contended that being taught how to utilize these educational technologies by instructors who utilize the same tools for their own practice contributed to these participants' development of TPACK confidence. Amah indicated that, in addition to her mathematics lecturers, her tutors also taught her how to implement different educational technologies:

…I've been taught by my teachers (lecturers) and tutors how to use some of these technologies for teaching maths (Amah).

As mathematics tutors are tasked with facilitate the learning of mathematics content (CK) in a module, and not necessarily with addressing the pedagogical aspect — be it PCK or TPK – this was exceptional. However, this further accounted for participants' level of TPACK confidence and how they were able to cogitate about ways of integrating technology to deal with mathematical problem-solving questions, like those they encountered in the first phase of the study.

While reporting her knowledge of the use of educational technologies, Lethu raised another issue encountered by all of the participants:

I have used it before although I haven't used it together with a projector… (Lethu).

Lethu made this comment in the context of a focus group discussion after the focus group, where she recounted how she had successfully utilized different educational technologies in her practical lessons at the university but had been confronted by some limitations during her teaching practicum at a school. Sfi also added that

We were taught how to use these technologies in our first and second years at varsity (Sfi).

It can be concluded from the participants' narratives that there is some form of knowledge about the use of technologies (TK) that preservice teachers were equipped with in preparation for the teaching practicum. Although all the participants experienced various challenges when attempting to implement educational technologies in their mathematics lessons, they were not hesitant to use the available technologies. These results are consistent with the research conducted by Akaadom (2020), which revealed that despite the difficulties that pre-service teachers encounter in accessing technology, they make every effort to integrate technology into their teaching wherever feasible. Thus, it can be concluded that participants had been trained to implement technologies into mathematics teaching and had some degree of confidence to implement this knowledge during their practicums. However, one participant, who taught statistics in his lesson, indicated that he did not have adequate knowledge of a technology to use it competently:

> *I do not have enough knowledge to use it to its full capacity: I only know its basics. Since I taught myself, it will take some time for me to know how to use it – like, how I can use Sketchpad, for example* (Zot).

Here the participant was referring to the use of a statistical package that could be utilized during the teaching of statistics at school. Self-learning for students at the university level is a key component that is promoted by the curriculum. For instance, preservice mathematics teachers are encouraged to self-learn about the use of some educational technologies for teaching, while they are taught how to use some (such as the Sketchpad and GeoGebra software) during lectures. The rationale for prioritizing the use of these two mathematical software packages is that they are not limited to one topic (like SPPS, which is used for statistics only) but can be integrated into a variety of mathematical topics. Moreover, as that there is a variety of educational software available for the teaching and learning of mathematics, not all can be taught to the same extent by the university. In some instances, preservice teachers are only introduced to mathematics software by their lecturers. Thereafter they are instructed and encouraged to learn independently more about the technicalities of the software.

The responses given by participants in the focus groups concurred with their responses on the observation questionnaire with regard to their competence to implement educational technologies in mathematics teaching.

5.6.5 Implications of teaching in a resource-constrained environment

The data revealed that participants encountered micro and macro challenges with integrating educational technologies into their teaching. It was evident that results demonstrated commonalities across all ten cases. For instance, eight out of ten participants (Shan, Lethu, Amah, Phume, Qhaw, Sfi, Phume, and Lunge) who utilized a laptop did not have access to a projector to display to the entire class. Instead, they grouped the students to watch the video on the one laptop screen but had no soundbar to enable adequate volume. Various empirical studies in South Africa (Van Dyk & White, 2019; Nakidien, Singh, & Sayed, 2021) have indicated that schools classified in Quintiles 1-3 are characterized by having limited access to resources such as advanced educational technologies for teaching and learning. In addition, while Shan was placed at a Quintile 5 school for his teaching practicum, his mentor indicated that Shan had hoped to use Sketchpad software in his lesson but had not had access to a projector. Shan described how he would have approached the lesson if he had had access to a projector, and how he approached it in the absence of this educational technology:

> *If there was a projector in the class, I would have constructed parallel and perpendicular lines using the Sketchpad software for the entire class to see at once. I only showed a few learners just before the end of the lesson in smaller groups. I did this for like only one quarter of the class* (Shan).

This highlights the resource-constrained environment in which preservice teachers are expected to optimize learning. Amah's mentor reported that Amah had encountered the same constraint that Shan had:

> *Yes, she did experience challenges, unfortunately. Her laptop screen was too small so the majority of the learners weren't able to see clearly since our school does not have projectors, so she had to group them, and that was time-consuming. The lesson was delayed. That's why we, as old teachers, just use chalkboards all the time* (Mentor: Amah).

This demonstrates two major challenges. First, a shortage of supplementary technologies (like a data projector) to execute the lesson is common. Second, while the preservice teacher was motivated to use technology in the lesson, technical obstacles delayed teaching and learning, with the lesson time limited to 60 minutes. As such, some scholars (Pierce & Ball, 2009; Radović, Marić, & Passey, 2018; Thurm & Barzel, 2020) have argued that the implementation of technologies in mathematics teaching can have a detrimental effect because they are time-consuming, especially in resource-constrained schools. Amah's mentor explained that experienced teachers only use chalkboards due to the lack of resources necessary for the successful execution of the lesson without delays.

Lethu's mentor reported a similar experience, as the school did not have projectors and the preservice teacher had to use the laptop screen for the entire class, grouping learners in groups of four:

> *While she was circulating the laptop, a few learners were making some noise, but I managed that for her* (Mentor: Lethu).

This further revealed that, while dealing with the shortage of technologies and trying to improvise, a lesson can be disrupted by learners when they are not attended to, especially in a bigger classroom.

Similarly, Sfi's mentor attested to the limited resources, with access to only one laptop:

> *…he had to then create groups so that each and every learner will see what they were talking about, rather. If he was using a projector, it would've been easy for learners to observe what he was talking about. Moving around to each group took a lot of time because they were observing pictures and videos in small groups, and he was using one laptop* (Mentor: Sfi).

The poor use of time that resulted as the preservice teachers attempted to innovate to deal with the limited resources was mentioned repeatedly by different mentors as a factor that negatively affected the teaching and learning process. Hence, it can be concluded that the senior teachers in the system tend to avoid the use of advanced educational technologies as it proved too time-consuming. Sfi's mentor attested to this:

> *…it was easy for chalk and chalkboard because learners are used to learning with those resources. The challenge started when he was using a laptop because pictures and videos that are shown were small…* (Mentor: Sfi).

This mentor again alluded to the justification for the continued use of the chalkboard as more effective than trying to adapt the use of advanced technologies to a resource-constrained environment. This suggests that the higher education curriculum is not cognizant of the limited resources provided to basic education institutions, while universities have adequate access to educational technologies to train prospective teachers to implement these technologies. This results in teachers not being able to implement the knowledge and skills they have developed at university in their teaching at schools. Furthermore, from the comments by mentors' about resorting to using chalkboards, it is apparent that the preservice teachers were not sufficiently utilizing their knowledge of advanced educational technology acquired from their training.

The micro-challenges faced by under-resourced schools in the province of KwaZulu-Natal for schools falling in Quintiles 1-3 have persisted even though these schools receive financial support. Van Dyk and White (2019) posit that "although Quintile 5 schools receive less financial support from the state than Quintile 1 schools, the latter are still worse off in terms of school resources and school composition" (p. S3). The implication of these findings suggests that comprehending the challenges that exist in these resource-constrained schools requires focused attention to ensure adequate financial support to optimize learning.

In addition to these micro challenges, several macro challenges were also identified in the mentors' narratives. A pertinent comment in the report from Shan's mentor revealed restricted access to educational technologies resulting from the Covid-19 pandemic:

> *...I told him that before Covid-19 we used to have portable projectors that we would carry with us to class for some lessons and bring them back to our head of department offices. This system worked fine, but for health reasons this system had to halted. We will resume with it after the Covid-19 restrictions* (Mentor: Shan).

As the Covid-19 virus was easily transmitted through dermal contact, the sharing of portable projectors was discontinued for health reasons; this negatively affected the implementation of technologies during teaching. The World Health Organization (WHO) indicated that "individuals can also be infected from and [sic] touching surfaces contaminated with the virus and touching their face (e.g., eyes, nose, mouth). The COVID-19 virus may survive on surfaces for several hours" (2020, p. 2). This macro challenge prevented Shan from accessing the supplementary technologies he required for his teaching due to the school's health and safety policies. For Zakhe and Zot, lessons were negatively affected by the load-shedding (regular electrical power outages) that has been implemented increasingly in South Africa. Zakhe's mentor reported:

> *Yes, he experienced a challenge: there was load shedding during the lesson and that meant that he could not use a projector to project the information to learners. So he had to use his laptop. And learners made noise, and it was disrupted by that* (Mentor: Zakhe).

Zot's mentor reported that similar difficulties were experienced by Zot during his lesson:

> *Yes, he experienced some challenges using technology during his lesson. Everything was going well during his lesson using Sketchpad to teach a geometry lesson, but everything went off half an hour into his lesson when load-shedding arrived in our area. He then used a chalkboard to continue with his lesson because his laptop could not work without electricity as well* (Mentor: Zot).

Interruptions in electrical supply impacted the use of modern digital technologies by Zakhe and Zot. Coughlan (2015) asserts that the power cuts implemented in South Africa for load-shedding negatively impact teaching and learning at schools that do not have generators for backup. She reports that "the effect of load-shedding on learning, and particularly on South Africa's ambitious plans to drive education towards the digital era, is far harder to quantify in the short term, but equally devastating in the medium to long term" (2015, p. 1). As such, the only option for teachers during such times is to revert to less advanced technologies that do not require external power.

The electric power outages in South Africa are a persistent national challenge that affects schools, hospitals, and different sectors of the economy alike. Since most schools do not have the resources to invest in backup forms of energy to support digital technologies, teachers are unable to rely on digital technologies.

5.7 CONCLUSION

This chapter has presented, analyzed, and discussed the findings drawn from the empirical data that were collected from the ten participants and their school mentors. The first data set, taken from the performance tests and semi-structured interviews, was thematically categorized according to the four tenets of the commognitive theoretical framework. Thereafter, several themes from the second data set generated by the two focus group discussions and observations shaped the findings and discussions that are underpinned by the TPACK theoretical framework.

The next chapter discuss the main findings in relation to the research questions and further delineate this proposed model.

Chapter 6: Discussion of results

6.1 INTRODUCTION

This study aimed to explore preservice teachers' utilization of educational technologies as visualization tools during mathematics instruction at schools. Based on the qualitative analysis of systematically collected data, the study revealed a significant relationship between preservice teachers' visualization strategies when engaging with mathematics problems themselves and when engaged in teaching in the classroom. This chapter discusses the findings presented in the previous chapter. It also presents and elucidates a model that was developed organically during the course of employing the commognitive and TPACK theoretical frameworks together in an integrated way.

While the use of the performance test as the first data generation tool not only revealed the visualization strategies adopted by participants when engaging with mathematics problems, the use of semi-structured individual interviews immediately after the performance test facilitated reflection by participants on the visualization strategies they had employed to solve geometrical and trigonometrical problems in the test. The use of individual interviews also generated qualitative data that yielded deep insights into the phenomenon under study, particularly with regard to problem-solving strategies used by participants and their application of these to their teaching.

6.2 C+TPACK: A MODEL RELATING THE COMMOGNITIVE FRAMEWORK TO THE TPACK FRAMEWORK

The analysis of data in the previous chapter revealed that both traditional and advanced educational technologies can be used effectively to aid learners' visualization as part of their conceptual development in mathematics. In addition, it was established that all of the participants had received training to utilize various forms of technology in their mathematics teaching and had been given opportunities to practice and develop their TPACK competence.

The ideas put forward by participants about how different forms of educational technologies could be integrated into mathematics learning and teaching were influenced by their engagement with the first phase of the study. This first phase revealed in detail how participants' commognitive methods related to visualization. It is proposed that the tenets of commognition that formed the themes and sub-themes for the analysis activated the participants' TPACK as they had first engaged in a self-discursive exercise before being asked to think actively about different teaching approaches that involved the use of educational technologies. The nature of the relationship between the tenets of the commognition framework and the elements of the TPACK framework, as underpinned this study, resulted in development of a new model bringing these two frameworks together. Figure 6.2 depicts the proposed commognitive technological pedagogical content knowledge (C+TPACK) model that demonstrates the relationship between the commognitive tenets and TPACK elements based on the analysis of data presented in the previous chapter.

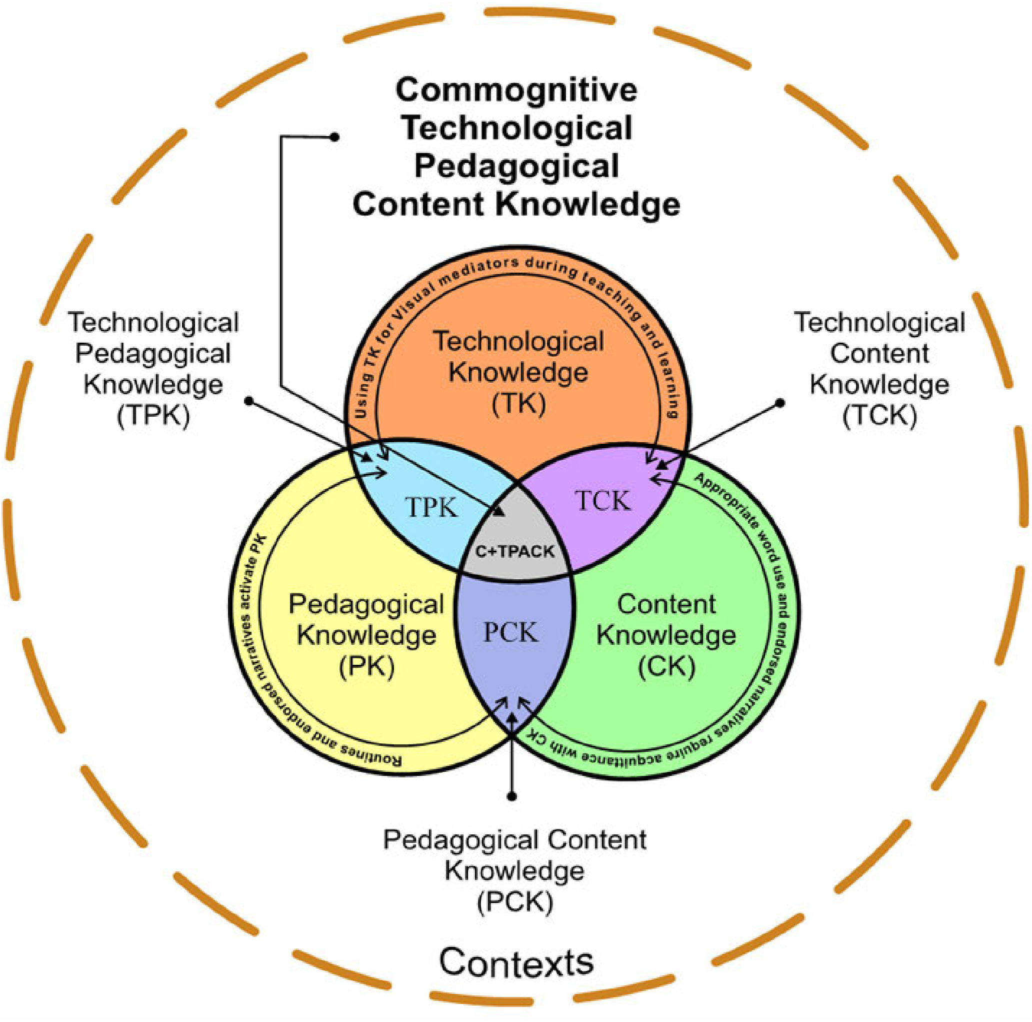


Figure 6.2: C+TPACK model illustrating the relationship between the tenets of the commognitive model and elements of the TPACK model

The findings revealed that when preservice mathematics teachers engaged with mathematics problem-solving questions themselves, they were prone to employ visualization skills that required them to utilize several visual mediators. The results further established that they preferred to utilize their knowledge of technology to mediate learning that supports and enhances iconic visual mediators (diagrams or sketches). Thus, as shown in Figure 6.2, the participants used their knowledge of technology and content (TCK) to employ iconic visual mediators to enhance learning in the classroom. The model demonstrates that their use of educational technologies based on their TK and TCK was informed by the self-discursive activities they engaged in that involved the use of commognitive tenets.

The data revealed that when participants attempted to solve mathematical problems during a self-discursive activity, they used several specific mathematical words and endorsed narratives. The self-discursive activity engaged their mathematical content knowledge; those participants with weaker content knowledge experienced difficulty solving the problems. Hence, a strong association between two tenets of the commognitive framework (‘word use’ and ‘endorsed narratives’) and the TPACK element ‘content knowledge’ (CK) was established. As indicated in Figure 6.2, appropriate use of content-specific words and endorsed narratives requires content knowledge (CK). In addition, as shown with arrows in Figure 6.2, these two tenets intersect with technological content knowledge (TCK), as some of the endorsed narratives and word uses can be depicted using visual mediators that may require technological knowledge (TK). Equally, as the knowledge of mathematics for teachers (preservice or in-service), should be appropriately organized for teaching purposes, suitable and accurate word use and endorsed narratives are particularly crucial within the realm of pedagogical content knowledge (PCK). Word usage and endorsed narratives need to be employed appropriately – not only for solving mathematical problems but, more specifically, for teaching the subject. The model thus harnesses the synergies between the core components of both frameworks.

From the findings, the routines and endorsed narratives that were noted in the participants’ self-discursive activities activated their PK as they went on to made inferences about how they would mediate the learning of concepts similar to those they had encountered during the performance test. These commognitive tenets had the same implications for participants’ TPK and PCK, since these TPACK components are interrelated. This interactive, mutual relationship is shown in Figure 6.2. Thus, all four commognitive tenets identified in participants’ responses are embedded within all of the elements of TPACK.

By integrating the commognitive and TPACK frameworks, this model provides a novel perspective on preservice teachers’ mathematics problem-solving discourses in relation to their teaching of problem-solving using educational technologies. The model thus offers a new analytical tool to researchers in this area.

Moreover, since this proposed model pertains to the knowledge of teaching using technology by preservice mathematics teachers in relation to their discourses during mathematics problem solving, it may also be utilized by mathematics teacher educators to inform their curriculum. Teacher educators may design their teaching and assessment for preservice mathematics teachers to involve the use of problem-solving in which they are actively encouraged to use their TPACK to integrate educational technologies to enhance the teaching of mathematical problem-solving. Capacitating teachers to actively combine the key elements of both commognition and TPACK consciously in their teaching could substantially boost their expertise to build learners' problem-solving skills.

6.3 PRESERVICE TEACHERS' USE OF VISUAL MEDIATORS WHEN ENGAGING WITH MATHEMATICS TASKS AND WHEN TEACHING

Participants were found to use various visual mediators to solve problems on the mathematics performance test completed for the study. Participants indicated that this use of visualization aided them significantly in comprehending and solving geometric and trigonometric problems and verifying their solutions. This finding is particularly pertinent to the South African mathematics education context where strategies are needed to improve learners' mathematics competence. This finding also highlights that the mathematics preservice teachers who participated in the study understood the value of novel mathematical visualization. As such, they attempted to employ non-traditional novel teaching methods that were well-matched to the contexts in which they taught. Gafoor and Kurukkan (2015) maintain that visual cognition can be an effective alternative resource in mathematics learning compared to rote learning.

These findings correspond with those in a study by Zayyadi, et al. (2019), where students tended to utilize visual mediators and mathematical word use to comprehend a mathematical word problem. Similar to their findings, this study found that participants used visual mediators in the form of graphic and geometric sketches to explain the different strategies they had employed to solve the problems. Their explanations revealed the visualization strategies they had employed during the problem-solving exercise.

Guzman (2002) classifies visualization strategies into isomorphic, homeomorphic, analogical, and diagrammatic visualization. Isomorphic visualization pertains to a set of guidelines that may be developed theoretically to translate the components of our visual representation and the mathematical relationships between the items they represent (Guzman, 2002). Guzman further states that elements within a homeomorphic visualization have certain interrelationships that sufficiently mimic the relationships between abstract objects to provide support and lead individuals' imaginations as they engage in mathematical proofs and conjecturing. In analogical visualization, mathematical objects are mentally replaced with other objects that individuals are manipulating whose relationships to one another are analogous and whose behaviour is more understood, or possibly simpler to handle from previous experience. Diagrammatic visualization refers to representing mathematical mental objects diagrammatically to externalize thinking processes, as was seen in this study. Diagrammatic visualization includes common types of visual imagery which in turn, can be classified as pattern, kinesthetic, dynamic, memory imagery of formulae, and concrete imageries, which Presmeg (1986) argues are vital to mathematical understanding.

In this study, one participant's utilization of interactive charts during instruction provided an example of kinesthetic visual imagery as the learning was predominantly hands-on and visually oriented. Furthermore, the usage of software packages (GeoGebra and Sketchpad) by a few participants could be classified as dynamic visual learning and teaching of geometry as both were respectively implemented to demonstrate the visual aspect of the Euclidean theorems that were being taught. Diagrammatical concrete imageries were commonly observed in all of the participants' lessons, irrespective of the topic they taught and/or the form of educational technologies they employed. These were utilized by both the participants and their learners during instruction. Moreover, memory imagery of formulae was also commonly utilized by the participants as a teaching strategy to assist their learners with memory recall for special formulae that were being adopted for the respective content areas they taught.

Irrespective of the type of visualization employed by teachers, as discussed in Chapter 2 numerous studies (Berry & Nyman, 2003; Yilmaz & Argun, 2018; Irawan et al., 2019) have demonstrated the pertinent role of visualization in the teaching of mathematics at any level. The findings in this study were particularly significant as all of the ten participants demonstrated an eagerness to implement teaching approaches that supported visual mediators.

This aligns with Lavy's (2006) argument for visualization on the basis that it allows for a variety of thinking processes, as opposed to traditional teaching methods that emphasize symbolism and formalism. Symbolism and formalism in the teaching of mathematics has been criticized by several scholars (Meletiou-Mavrotheris & Stylianou, 2003; Abramovich, Grinshpan, & Milligan, 2019) for its lack of real-world application which often leads to learners being unstimulated and unmotivated to learn.

The mathematics education curriculum at the institution at which the participants in this study were training encouraged transformative teaching that focuses on applying theory in practice and relating mathematics to everyday realities. The assessments for the three mathematics methods modules were designed to ensure that the students were regularly required to apply the mathematics content they were learning to real-world problems.

Additionally, in these three modules, they were taught and actively encouraged to also teach transformatively. For example, a participant who was teaching mathematics of finance to a Grade 10 class demonstrated this type of teaching by intentionally using examples that were relevant to learners' contexts and adapting the prescribed textbooks' examples and exercises to develop a worksheet that incorporated problems that learners could identify with, instead of merely using the exact examples and exercises given in the textbooks. Several studies (Alex & Roberts, 2019; Mabena, Mokgosi, & Ramapela, 2021) have recommended that new mathematical knowledge be presented in relevant real-world contexts and that suitable classroom context approaches be used for solving mathematical problems.

Although most of the participants in this study found the trigonometry question on the test challenging, they were certain that they needed to utilize visual mediators to depict the problem statement. All of the participants used geometric sketches as their first step and reaffirmed the role of a visual mediator in successfully solving the second problem. Strohmaier et al. (2022) found that the use of visual representations during problem-solving may appear as merely incidental, however, being able to accurately distinguish between useful and irrelevant information influences the probability that the person will solve the problem successfully. Thus, these findings reaffirm the pivotal significance of visual mediators for mathematics problem-solving.

Another pertinent finding was that the use of mental images served as another effective visualization strategy. This concurred with findings from Mudaly's (2021) study exploring students' construction of mental images when engaging with mathematics problem-solving questions. The current study found that participants first used mental images as a visualization strategy when they encountered a mathematical word problem for the first time; then, depending on the complexity of the problem, created concrete pictorial or diagrammatical images as visual mediators. The use of both kinds of visualization strategies was thus identified in the study. These visualization strategies were used by participants during their teaching practicum to facilitate learning of the content they were teaching. This was observed to be a common practice but occurred most frequently when learners were experiencing challenges understanding the language (English) used in the statement of the question. By switching to a language that was more familiar to learners and diagrammatizing the questions using visual mediators, improvement was observed in terms of learners' responses. These visualization strategies resonate with South Africa's mathematics teaching framework (DBE, 2018) which advocates for teachers to facilitate the visualization of mathematical objects by learners to aid their conceptual understanding. This framework speaks to international best practice while foregrounding South Africa's unique challenges and needs associated with the teaching and learning of mathematics, with an aim to improve the competence of learners in mathematics.

As the preservice teachers that participated in this study were asked to complete a test individually, it can be inferred that they were engaging in a process of individualization as they solved problems on the test through visualization. Zayyadi et al. (2019) argue that individualization is a fundamental commognitive process that occurs in the solving of mathematical problems. Individualization represents a form of internal self-communication, and constitutes one component of the main principles of the commognitive framework. Sfard (2008), too, affirms that interpersonal communication and the individualization process (intrapersonal) are reflexively interconnected. In this study, a relationship was observed between the intrapersonal communication (individualization) that the preservice teachers used to solve mathematical problems and their approaches to teaching mathematics, as depicted in the C+TPACK model discussed in the previous section.

When the participants reflected on the problem-solving strategies they had used in the test and were immediately asked to think about how they taught problem-solving for the same kinds of mathematical problems to their learners, they recognized that the manner in which they engaged with mathematical problems informed their pedagogical practices. Participants indicated that the curriculum at the institution at which they were training encouraged preservice teachers to be mindful of the socioeconomic and socio-social factors that affect the enacted curriculum in South Africa. This suggests that preservice teachers' approaches to mathematics teaching are influenced by their personal strategies for visualization while considering the type of teaching technologies they would have access to for facilitating learning. On these grounds, it can be argued that, as preservice teachers are exposed to mathematics problem solving activities, if their problem-solving visualization strategies continue to improve with practice, this will ultimately influence and impact on their teaching. For instance, a qualified teacher who has had significant exposure to mathematical thinking would be more likely to teach in ways that would develop the mathematical thinking skills of learners, rather than only teaching instrumentally. This was evident in this study's findings, as demonstrated in the previous chapter.

The participants' teaching plans included creating a platform for learners to learn through the use of visual mediators facilitated by advanced educational technologies. This implied that they valued their visualization strategies using iconic visual mediators as these enabled them to create a more conducive learning environment when engaging with mathematical problems. This demonstrates that the way they engaged with mathematics problems influenced their selection of pedagogical approaches. The data obtained during classroom observations verified this, as most of them tried to integrate both traditional, non-digital and digitally advanced educational technologies that supported learning with iconic visual mediators. These findings have direct relevance to the state of mathematics education in South Africa, which was discussed in Chapter 2, as the use of any form of educational technology for the purpose of enhancing teaching and learning addresses the challenges to developing learners' mathematical competence which persist in South Africa.

Participants were observed using non-digital educational technologies (such as interactive charts, mathematics instruments, and flashcards) to work effectively with the limited resources available at the schools at which they completed their teaching practicums. Few participants had access to advanced digital technologies such as laptops, projectors, smartboards, and other specialized mathematical software like GeoGebra and Sketchpad at the schools. This was consistent with the literature reviewed in Chapter 2, which demonstrated that the vast majority of public schools in South Africa are still under-resourced, with only a small percentage of public schools (such as ex-model C schools) being well-resourced (Kanyopa & Hlalele, 2021). The findings of this study thus confirm the enduring challenges experienced in resource-constrained school environments across the country.

All of the educational technologies that were employed by participants during practice teaching at schools supported the visualization aspect of mathematics learning for learners. This correlated with Tiwari, Obradovic, Rathour, Mishra, and Mishra's (2021) argument that visualization and educational technologies represent essential components in mathematics education that should be applied across all teaching domains because of their value in facilitating improved understanding of mathematical concepts by learners.

Thus, the interconnectedness between visual mediators and TK in the teaching context is affirmed by the findings of this study. This relationship is not restricted to only these two features of the commognitive and TPACK frameworks, however, since the teaching of mathematics is not restricted to technological knowledge and the mediation of learning through the use of visual mediators: it also requires teachers to use their knowledge of technology in conjunction with their knowledge of the subject matter (CK) and the knowledge they possess about teaching approaches (PK). In cognizance of this, the training institution had developed a sequence of modules designed to equip preservice mathematics teachers with the necessary skills necessary to apply all these different types of knowledge (CK, PK, TK, and PCK) in practice. This brings together the concept of visual mediators (from the commognitive framework) and the three core elements of TPACK (TK, CK, and PK), as depicted in the C+TPACK model that has been developed in this study. In their own problem-solving processes, the participants engaged in individualized discursive activities where they chose to create visual mediators and then reflected on the importance of these in learning and teaching. In their reflections, they further considered how they could apply their knowledge of technology, in conjunction with their pedagogical knowledge, to integrate a rich use of visual mediators into their teaching for the benefit of their learners.

This study also found that other tenets of the commognitive framework (word use and endorsed narratives) became apparent during participants' engagement with mathematics problems, and then informed their planning of lessons. During the practice teaching experience, they were found to use teaching aids that involved the utilization of resources that were purposefully selected to enable the demonstration of certain mathematically-endorsed narratives and word use. This was influenced by the manner in which they engaged with mathematical problems themselves, as they realized from their own problem-solving experiences that they used the same approaches that they had found useful for their own understanding of certain mathematical concepts in their teaching.

6.4 MATHEMATICAL WORD-USE AND ENDORSED NARRATIVES EMBEDDED WITHIN CK AND PCK

As discussed in the previous chapter, several instances of word-use and endorsed narratives were identified in the participants' responses to the performance test and individual semi-structured interviews, all of which had implications for their mathematical content and pedagogical content knowledge.

Participants also indicated that they believed that the integration of educational technologies into teaching and learning would promote learners' concept development in mathematics. Schoenfeld (1998) argues that teachers' decisions while teaching mathematics are linked to their beliefs, which are influenced by their knowledge and goals. Daher, Baya'a, and

Anabousy (2018), and also Schoen and LaVenia (2019), established that mathematics teachers' beliefs impact their instructional practices as well as their interaction with their learners. Thus, the beliefs of participants in this study regarding the role that educational technologies could play in the learning of mathematics concepts influenced their pedagogical practices to facilitate concept development during instruction.

This also demonstrates that the pedagogical training they had received from their institution for mathematics teaching at the secondary school level was aligned with modern teaching practices and that they had developed their confidence to implement these. It is contended that preservice teachers' pedagogical practices are affected by their skills, beliefs, and the environmental context in which teaching and learning occur. A recent study by Oke and Fernandes (2020) revealed several modern, interactive, exploratory, and learner-centred pedagogical practices that have become more popular, particularly with the advent of the 4IR. Thus, the traditional pedagogical practices that are teacher-centred and exercise-based are gradually falling out of use in the current mathematical teaching landscape. Despite the challenges that teachers in South Africa face to access the technologies of the 4IR, the teacher training institution where the participants were enrolled was found to be actively training preservice teachers in the use of modern and novel pedagogical practices. A shift in this regard was found to have occurred after the institution redesigned its bachelor of education curriculum prior to 2019. The participants in this study were part of the first cohort to use the new curriculum, which was designed in line with the 4IR and 21st century goals.

Furthermore, the analysis revealed that the mathematical word-use and endorsed narratives preservice teachers used during mathematics problem-solving also manifest within the content and pedagogical content knowledge domains. This relationship is particularly significant in that if they demonstrated weaknesses in the use of words and endorsed narratives while solving a problem, this would indicate weakness in their CK. This is consonant with findings of a large-scale literature review conducted by Knowledge Management and Dissemination (2010) that indicated that teachers with in-depth CK are more prone to respond to learners' mathematical ideas accurately while making negligible language or mathematical errors during teaching. This study demonstrated that participants' CK was not excellent, particularly in terms of the concepts of trigonometry, as indicated in the previous chapter. As such, some of them were challenged by their learners during the activities sessions when learners were experiencing difficulties with the content. In essence, their main challenge was explaining the basis of certain concepts to learners. Their inability to adequately justify trigonometrical concepts using appropriate mathematical language in conjunction with the common language familiar to learners posed a challenge. This, in turn, had implications for the learners, who had to apply trigonometric rules and routines without adequate conceptual and relational understanding.

Relational understanding of mathematical concepts is particularly vital as it ensures learners' awareness of suitable procedures and routines during problem-solving, thereby demonstrating logical reasoning (Utomo, 2020). Thus, the form of language used by teachers during instruction should be directed toward ensuring relational and conceptual understanding. Although it may be construed to be a minor issue, it can impact significantly on learners' performance in mathematics at the provincial and national levels since instrumental understanding is associated with short-term memory retention (Skemp, 1976).

Moreover, the issue of language in the province of KwaZulu-Natal, in particular, is of concern since the majority of the population in this province speaks English as an additional language and not as their home language. As such, nine of the ten participants had to confront not only the issue of mathematical language but also had to translate to learners' vernacular (isiZulu) to aid the comprehension of mathematical concepts. This was even a greater challenge for one of the participants, who spoke English only but had multilingual learners in his class. In relation to this, Setati (1998) proposes that code-switching as a communicative practice for mathematics teaching in multilingual classrooms may help to scaffold learning to enable learners' performance to improve, although it also runs the risk of introducing ambiguities during mathematics learning (Maluleke, 2019). This could be due to the lack of evidence that preservice teachers in South Africa are trained to code-switch during mathematics teaching (Cekiso, Meyiwa, & Mashige, 2019). Several studies (Sert, 2005; Goosen & Sikhondze, 2010; Bilgin, 2016) have revealed that code-switching is a spontaneous act that teachers utilize as a strategy to enhance their teaching.

In this study, the curriculum at the institution where the participants were enrolled had not actively trained them to use code-switching in their mathematics teaching; however, participants' strategic use of iconic visual mediators may have mitigated the language barrier in their classrooms to some extent. However, Knowledge Management and Dissemination (2010) note that a lack of CK would appear to limit teachers' teaching as they are challenged by mathematically specialized terminology. Thus, these can be traced to teachers' recall routine and comprehension of the mathematical endorsed narratives and word use that they demonstrate during mathematical problem-solving which is reinforced through the core mathematics content modules that are compulsory for preservice mathematics teachers.

This would have inferences for PCK, should preservice mathematics teachers exhibit weaknesses in terms of their ability to use a range of endorsed mathematical narratives that relate to the various content areas that may be invoked in a mathematical problem. The comprehension of endorsed mathematical narratives is vital for success during a mathematics problem-solving exercise. These narratives were found to be key when participants were applying various routines to obtain solutions. They were also significant as they revealed participants' level of CK which, in turn, had implications for their PCK. Both the local and international research reviewed in this study emphasizes that if preservice mathematics teachers lack subject matter knowledge (CK) this leads to a further lack of knowledge about teaching the subject (PCK). This can be attributed to poor comprehension of the significant endorsed narratives and word uses they encounter during engagement with mathematical problems. It is logical to infer that poor comprehension of significant endorsed narratives and word uses results in challenges in applying mathematical routines, as understood from Sfard's (2008) perspective. In turn, all of these aspects have direct implications for the forms of knowledge (CK and PCK) that are pivotal for preservice teachers' teaching of the subject.

Nonetheless, during the discursive activities in which the participants engaged in this study, it was found that their discourses were rich and explorative, which implies that they had met the curriculum requirements. Roberts and Le Roux (2019) found that an explorative discourse about the mathematical concepts in the syllabi with secondary school learners demonstrates their achievement of the curriculum requirements. They found that learners were inducted into this form of discourse primarily by their mathematics teachers, with additional support from peers and textbooks. In this study, preservice mathematics teachers were found to be capable of discursively mediating learners' routines as they engaged with mathematical tasks during instruction. This indicated that participants' PCK competence was of an acceptable standard for the content areas they taught. This was confirmed through the observation of their teaching in the classroom during their teaching practice.

6.5 PRESERVICE TEACHERS' IMPLEMENTATION OF EDUCATIONAL TECHNOLOGIES AS VISUALIZATION TOOLS DURING TEACHING PRACTICE

The methodological approaches used in this study provided the researcher with lived experiences and evidence of how the participants implemented different educational technologies as visualization tools during mathematics practice teaching at schools. Since this study adopted a naturalistic inquiry to research, participants were not conditioned or compelled by the researcher to visit selected schools equipped especially with educational technologies for mathematics teaching. As such, participants' motivation to implement technologies was observed in the presence of the various contextual challenges that they encountered. As shown in the previous chapter, these contextual factors did not prevent the participants from implementing their TPACK skills as they improvised to mitigate the different resource challenges they encountered. The degree of flexibility they demonstrated in implementing different kinds of educational technologies (interactive charts, mathematical instruments, whiteboards, laptops, projectors, Sketchpad, and GeoGebra) indicated that their training had equipped them to implement these technologies during instruction.

The use of such educational technologies in the 21st century classroom environment is advantageous for numerous reasons. For instance, the use of interactive charts by some participants was observed to be beneficial to both learners and preservice teachers in that it enhanced the lesson by accommodating different learning styles – such as learning through touch (while interacting with the chart), hearing, and visualizing. The interactive charts allowed learners to actively interact and engage with the learning material. Similarly, the use of mathematical instruments by one participant enhanced the learning process by affording learners an opportunity to engage in hands-on learning in which they were involved in constructing knowledge for themselves. Kinesthetic, auditory, and visual learners were supported during the teaching of geometry by the use of a whiteboard: the participant explained and demonstrated on the whiteboard how to construct geometric shapes and measure their angles. In addition, the collaborative use of laptops, projectors, Sketchpad and GeoGebra partially enhanced some of the participants' lessons although interruptions were observed during some of the lessons where these educational technologies were employed. During the use of these particular digital educational technologies, learners responded positively to the learning of geometry. Nevertheless, the utilization of laptops without a projector and a soundbar was found to be counter-productive as it proved time-consuming and derailed learning. However, the advantages of the various educational technologies that were employed by the participants as visualization tools in their respective classroom experiences outweighed the shortcomings that were observed.

These results align with the findings of several studies (Das, 2019; Gómez-García et al., 2020) which emphasize that if teachers are effectively trained to integrate educational technologies in mathematics teaching, then they should possess a good command of these and be able to integrate them during instruction. This study's findings concur with those of Dogan, Dogan, and Celik (2021) that newly trained teachers with the skills to integrate technology are inclined to find ways to integrate it during their practice. It can further be contended that the technology integration training they receive during their training builds confidence and capacity to improvise when they find themselves in resource-constrained environments. Thus, such training should be sustained in the curriculum of teacher training institutions, particularly in the South African and similar contexts where they are preparing teachers to enter school environments where the resources vary dramatically teachers thus need versatility.

The study revealed that several challenges restricted participants from implementing advanced educational technologies in their lessons; as a consequence, they resorted to less advanced technologies, like interactive charts and hand-held mathematical instruments. Their motivation was evident, however, as despite the systemic challenges and limited resources that characterize South African public schools at present, to a certain extent all of the participants strived to implement educational technologies in the delivery of their lessons. As indicated in the literature review chapter, the scarcity of educational technologies in South Africa is a key constraint. Several contextual factors (poverty, deteriorated facilities, overcrowded classrooms, corruption, high crime, and drug abuse) revealed by the recent studies (Mathevula & Uwizeyimana, 2014; Padayachee, 2017; Munje & Jita, 2020; Parker, Morris, & Hofmeyr, 2020; Aruleba & Jere, 2022) contribute to the shortage of modern technologies at school that teachers may be trained to use at their teacher training institution. While this suggests a degree of misalignment in terms of how the preservice mathematics teachers are trained to implement advanced technologies in their teaching and the realities in most public South African schools, HEIs continue to train these teachers to implement various forms of educational technologies as some teachers find themselves in well-resourced schools where they will be expected to implement and utilize these technologies.

6.6 CONCLUSION

This chapter has discussed and interpreted the study's findings as per the objectives of the research, with reference to the reviewed literature and the theoretical frameworks that underpinned the study. The next chapter concludes the thesis by relating the key findings to the research questions. The implications, contributions, limitations and recommendations for future research are also delineated.

Chapter 7: Concluding remarks

7.1 INTRODUCTION

This chapter presents the conclusions of this study. The chapter commences with a summary of the major findings in relation to the objectives and main research questions. For easy reference, the objectives of this study were to: (1) identify the visualization strategies utilized by preservice mathematics teachers to solve mathematical problems; (2) establish the educational technologies that mathematics preservice teachers intended to utilize in their teaching; (3) explore why mathematics preservice teachers believed that educational technologies could influence their teaching and concept development; and (4) establish how mathematics preservice teachers demonstrated the implementation of these educational technologies in their teaching during teaching practice. This chapter further discusses the contribution and implications of the study. It also reviews the study's limitations and suggests recommendations for future research.

7.2 SUMMARY OF THE OVERALL RESULTS IN RELATION TO THE STUDY'S OBJECTIVES

The aim of this study was to conduct an exploration of preservice teachers' usage of educational technologies as visualization tools during mathematics instruction in secondary schools. In order to engage preservice mathematics teachers in a discussion about the utilization of educational technologies as visualization tools, it was essential to first determine the visualization techniques they used for mathematical problem-solving. The results indicated that the first visualization strategy they employed during mathematical problem-solving was to create a mental image to represent the problem statement. It was established that when a word problem was relatively easy, mental imagery (seeing a visual mediator only in one's mind) was sufficient for solving a problem, where they would merely employ visual mediators and use their understanding of word usage, endorsed narratives and routines to arrive at the solution.

In addition, the results demonstrated that the preservice mathematics teachers predominantly made use of iconic visual mediators as visualization strategies when engaged with more complex problem-solving questions. They adopted this strategy for two reasons. Firstly, they utilized these visual mediators for to verify their solution after solving a problem. The findings established that the use of this strategy was particularly significant at the verification stage of problem-solving as could be used to confirm their application of routines, endorsed narratives, and mathematical word usage in the problem.

Secondly, they utilized iconic visual mediators to conceptualize the problem statement prior to attempting to solve the problem. This strategy was used by all ten participants, where it was established that a sound comprehension of mathematical word uses and endorsed narratives was fundamental to attempting to visualize a mathematical word problem using iconic visual mediators. Further, the findings revealed that failure to visualize a problem using iconic visual mediators automatically resulted in the inability to solve a problem. However, while some were able to construct accurate iconic visual mediators to represent the problem statement, this did not guarantee that they would be able to solve it correctly. This was evident particularly with the low success rate for solving the 3D trigonometric problem.

Having examined the participants' visualization strategies, the study established that there was a relationship between the visualization strategies that the participants used personally for problem-solving and their teaching and planning of the same topic. It was observed that they found innovative ways to mediate learning that included the presentation of iconic visual mediators. However, they were found to encounter several environmental challenges in their attempts. These included poor infrastructure at the schools, unexpected electric power cuts, and large classes. It was evident from the results that they explored different ways to improvise and mitigate the challenges that they encountered during their practice teaching experience.

7.3 CONTRIBUTION OF THE STUDY

Chapter Two demonstrated that visualization is a crucial skill that learners should develop in order to be successful with problem-solving in mathematics. In addition, the literature confirmed an upward trajectory in the adoption of educational technologies to enhance the visualization of mathematical objects and concepts by learners. In line with this, the utilization of such technologies by in-service mathematics teachers has become the topic of more research. Few studies have investigated the knowledge of preservice teachers regarding the use of educational technologies specifically as visualization tools in mathematics education, however. The contribution of this study, conducted in the South African context, is particularly significant as there is an urgent need to improve South African learners' competence to solve mathematical problems as they have performed poorly in the past. Thus, this study is significant as it fills the gap in the literature about preservice teachers' knowledge of educational technologies implemented in mathematics teaching as visualization tools in the South African context.

7.3.1 Understanding of the role of the influence of visualization strategies used in mathematical problem-solving on preservice teachers' pedagogical practices

As detailed in the previous chapter, in this study, participants' use of visualization skills while solving mathematical problems were identified and scrutinized in relation to their teaching of the subject at the secondary school level. A direct relationship was observed between their own visualization strategies and their pedagogical approaches to mathematics teaching. On this basis, this study asserts that preservice teachers' mathematical thinking processes are the influencing factor in how they plan and execute their mathematics lessons. Brown, Mc Auliffe, and Lampen (2019) argue that effective mathematics instructions relies not only on teachers having strong content knowledge, but also on their ability to think mathematically. This study validates the significance of teachers' mathematical thinking through visualization for improved mathematical teaching.

Jojo (2019) states that "the teaching of mathematics in South African schools has been pronounced to be among the worst in the world" (p.1). She notes that many learners are denied access to higher education and to contemporary, knowledge-intensive jobs due to the widespread failure of mathematics education at public schools to develop learners' mathematical competence to the required level. This study asserts that iconic visual mediators are crucial for mathematical problems to be solved successfully by teachers and learners with understanding. Preservice teachers' visualization strategies formed the basis for their planning and execution of lessons in ways that could support digital and non-digital forms of visualization for mathematics learners. This study thus contributes to the body of knowledge about mathematics education in the South African context and contributes key insights into the role that visualization can play in improving the effectiveness of mathematics instruction so as to improve learners' performance.

In addition, this study has contributed to the body of work engaging the theory of commognition by demonstrating that the individualized discourses that are essential for mathematics problem-solving are valuable to mathematics teachers whose knowledge is structured for teaching purposes because they later inform their instruction. Equally, it also contributed to the body of work engaging the TPACK framework by showing that the tenets of commognition that are operative during intrapersonal or interpersonal communication when preservice teachers engage in problem-solving evoke their mathematical knowledge of teaching with technology.

7.3.2 Identification of factors affecting preservice teachers' implementation of educational technologies during mathematics teaching in South African schools

In the South African context, where there is an urgent need to address the challenge of ineffective mathematics teaching, this study demonstrated that the preservice mathematics teachers who participated in the study were being equipped by their teacher training institution with the knowledge and skills to implement a variety of educational technologies to enhance teaching and learning. All of the ten participants demonstrated confidence in their TPACK abilities. However, several factors were identified that influenced their ability to integrate educational technologies into their teaching effectively.

From the results, it was established that preservice teacher' attitudes and beliefs were a determining factor in whether or not they would implement advanced educational technologies into their teaching. Only a few of the preservice teachers preferred to use traditional teaching methods (chalkboard and chalk), with no use of digital educational technologies – despite having been trained in these more advanced technologies. These participants affirmed that advanced educational technologies were used as a last resort when they established that learners were struggling to comprehend a concept after several attempts with traditional teaching. These results concur with findings from other studies (Marbán & Mulenga, 2019; Arthur, 2022). Thus, it is important for teacher educators to consider the attitudes and beliefs of preservice mathematics teachers toward the implementation of advanced educational technologies during their training to integrate technology into their teaching. Teacher educators can then actively develop their students' understanding of the value of these technologies for promoting learners' visualization abilities, build their students' competence and confidence in using these technologies, and promote their students' implementation of these technologies in their teaching.

A resource-constrained environment has been identified as a systemic challenge at South African schools. The lack of infrastructure and resources to support the use of technology in teaching and learning at the schools where the preservice teachers were placed for practical school experience was found to have restricted them from implementing several of the technologies they had been trained to use – such as Sketchpad and GeoGebra. However, their attitudes and beliefs with regard to the value of technology and their competence for using it impacted their ability to innovate under these conditions. Those who were positive and motivated about the integration of technology into their teaching found ways to mitigate the systemic challenge of the lack of infrastructure and resources by employing the principles of teaching with advanced technologies and adapting them to their situation. For instance, they found ways to employ the principles of teaching with transparencies and an overhead projector in a classroom environment without these resources. In addition, in the absence of access to advanced technologies that could effectively represent 3D objects, they mitigated this challenge by creating and using manipulatives to effectively support teaching and learning.

7.4 STUDY IMPLICATIONS

This study found a correlation between the participating preservice teachers' own strategies for solving mathematical problems and the pedagogical approaches they used to facilitate learning in the classroom. Thus, comprehending their problem-solving visualization strategies from a commognitive perspective was identified as being vital since these strategies can be expected to influence their teaching approaches. The implication of these findings for teacher educators and assessors at teacher training institutions is that the assessment of preservice teachers should not be confined to simple marking ('correct'/'incorrect') or assessing their own ability to solve mathematics problems. It should take a holistic approach to understand the thinking processes preservice teachers are using to solve problems and explore how and why they are using particular techniques for problem-solving, as these processes and choices will shape the way they teach learners in the classroom. Assessment of preservice mathematics teachers by teacher training institutions should thus focus on facilitating reflection by preservice teachers on their own mathematical problem-solving techniques in relation to their teaching techniques.

This study affirmed that, for preservice teachers, visual mediators are key to solving geometric and trigonometric word problems. Their approach to teaching mathematical concepts was consistent with their beliefs. In addition, the study validated the use of educational technologies by preservice mathematics teachers as visualization tools. Thus, mathematics instructors should be intentional about their use of educational technologies as tools to enhance visualization, especially during the teaching and learning of complex problem-solving content areas.

7.5 LIMITATIONS OF THE STUDY

While the findings of the study have answered the research question that was identified in Chapter 1 and contribute significantly to the literature, is necessary to acknowledge the shortcomings of the study.

Since this study employed convenience non-probability sampling to recruit participants, participants were selected on the basis of performing well in a module designed to equip them with the skills to integrate technology into their teaching of geometry and trigonometry. Hence, this sampling technique may have decreased the generalizability of this study's findings. Thus, it cannot be guaranteed that a similar study with preservice mathematics teachers who had not received the same training, or performed as well, would yield similar results.

Another limitation of this study is the size of the sample. Ten preservice mathematics teachers were selected from one higher education institutions. The results thus cannot be considered to represent all preservice mathematics teachers or all higher education institutions in the country.

In additions, regulations put in place by the government and educational institutions in response to the Covid-19 epidemic restricted the researcher from engaging with the participants face-to-face to determine non-verbal cues. DeJonckheere and Vaughn (2019) explain that active listening to an interviewee includes observing non-verbal social cues such as intonation, body language, and gestures which provide additional data in the interview process. However, the role placed by participants' mentor teachers to assist with gathering data mitigated this limitation to a certain extent. In addition, the study was designed to include two lesson observations during the final phase of data collection, but due to the Covid-19 restrictions, only one observation was possible.

7.6 RECOMMENDATIONS FOR FUTURE RESEARCH

It is recommended that future studies on this topic select participants from at least two different teacher training institutions using a probability sampling technique. Additionally, a larger sample size is recommended to increase the generalizability of results.

Further research is recommended to investigate preservice teachers' abilities to visualize and solve 3D trigonometric problems in relation to their secondary school learners' performance in the same content area. In this study, several participants performed poorly on the 3D trigonometric problem-solving question during the first phase of data collection; however, their errors and inability to visualize and successfully solve the problem were not closely interrogated as these aspects did not fall within the ambit of this study's research objectives.

It is also recommended that researchers conduct similar studies utilizing both commognition and TPACK as theoretical lenses to test and strengthen – and possibly refine or extend – the C+TPACK model put forward in this study. This model was developed organically in the course of using the two theoretical lenses in an integrated way in this study. As it was not an objective of this study to develop the model, it would be of value for future research to interrogate the viability of the model by applying it to a different sample in the context of another study.

It is recommended that mathematics teacher educators continue to develop preservice teachers' TPACK skills explicitly through the curriculum. A key aspect of this is teaching them to recognize the visualization strategies that they use. This, in turn, can be expected to impact learners' abilities to visualize mathematical concepts. Developing learners' visualization strategies is key to their success in mathematical problem-solving as it advances their mathematical and critical thinking abilities, which is an ultimate goal of mathematics teaching, according to the South African curriculum.

7.7 CONCLUSION

This chapter has concluded the exploration of preservice teachers' utilization of educational technologies as visualization tools when teaching mathematics. The main findings revealed that the different visualization strategies utilized by preservice mathematics teachers during their own mathematical problem-solving influenced their teaching strategies. This reveals a clear link between the problem-solving visualization strategies used by the preservice teacher themselves and their implementation of TPACK skills during mathematics teaching. The preservice teachers were found to attempt to implement a variety of educational technologies to enhance learners' opportunities to visualize mathematical concepts, despite the contextual challenges in South African schools that restrict access to advanced educational technologies.

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Appendices

Appendix A: Ethical clearance letter (UKZN)

Appendix B: Informed consent letter

Appendix C: Performance test

Appendix D: Semi-structured interview schedule

Appendix E: Focus group discussion questions

Appendix F: Observation schedule

Appendix G: Permission to conduct research in the KZN DoE institutions (KZN DoE)

Appendix H: Turnitin Report

Appendix I: Letter from the language editor

APPENDIX A: ETHICAL CLEARANCE LETTER (UKZN)

UNIVERSITY OF KWAZULU-NATAL™
INYUVESI YAKWAZULU-NATALI

05 May 2021

Mr Mzwandile Wiseman Zulu (212507659)
School Of Education
Edgewood Campus

Dear Mr Zulu,

Protocol reference number: HSSREC/00002656/2021
Project title: An exploration of preservice teachers' uses of educational technologies as visualization tools when teaching mathematics.
Degree: PhD

Approval Notification – Expedited Application

This letter serves to notify you that your application received on 16 March 2021 in connection with the above, was reviewed by the Humanities and Social Sciences Research Ethics Committee (HSSREC) and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

This approval is valid until 05 May 2022.
To ensure uninterrupted approval of this study beyond the approval expiry date, a progress report must be submitted to the Research Office on the appropriate form 2 - 3 months before the expiry date. A close-out report to be submitted when study is finished.

All research conducted during the COVID-19 period must adhere to the national and UKZN guidelines.

HSSREC is registered with the South African National Research Ethics Council (REC-040414-040).

Yours sincerely,

[redacted]

Professor Dipane Hlalele (Chair)

/dd

Humanities and Social Sciences Research Ethics Committee
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Telephone: +27 (0)31 260 8350/4557/3587 **Email:** hssrec@ukzn.ac.za **Website:** http://research.ukzn.ac.za/Research-Ethics

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

INSPIRING GREATNESS

APPENDIX B: INFORMED CONSENT LETTER

Informed Consent Letter to the Student

Dear student

Ms/Mrs/Mr/…………………………………………...

Name of university ………………………………..…..

Re: Permission to conduct a research study with you

I am writing to request your permission to conduct a research study with you. This research study is entitled:

An exploration of preservice teachers' use of educational technologies as visualizationtools when teaching mathematics

My name is Mzwandile Zulu and I am currently studying towards a Doctorate Degree at the University of KwaZulu-Natal (UKZN). As part of the requirements of this degree, I am required to complete a research thesis. This study focuses on exploring preservice teachers' uses of educational technologies as visualization tools when teaching mathematics.

I require 15 third-year preservice mathematics teachers of any gender or age, who are specializing in the Further Education and Training (FET) phase to participate in this research. I would be very grateful if you would agree to be one of the participants of this study.

If you agree to this, you will be invited to respond to a short easy mathematics performance task, be interviewed individually as well as a group, and finally be observed teaching any mathematics topic using any form of educational technology.

All discussions, interviews, and dialogues with participants will be audio recorded using a dictaphone, and thereafter transcribed verbatim to produce transcriptions. This research information (data) is required for the analysis of data and completion of the actual write-up of the thesis. Collecting research information for this study will take approximately 90 minutes. The process of data collection, that is: response to the performance tasks and interviews will take place on the university premises, with your permission. However, this is subject to the Covid19 regulations. To adhere to these regulations, the performance tasks will be completed online using Google forms and the interviews will be conducted via Zoom or Google teams. There will be compensation for Internet bundle costs incurred. The last process of data collection will take place at a school where you will be placed for teaching practice. During this process, you will be asked for 3 video recordings of yourself teaching any mathematics topic using some form of educational technology as a visualization tool. Arrangements of how this process will unfold will be discussed with each participant in due course.

Times and dates will be discussed and arranged with you at a later stage. I will try to ensure that this takes place during your free time, in an attempt to avoid any disruptions during lessons.

Please note:

* Times and dates of this data generation process will be at your sole discretion. I have merely presented you with an outline of what I intend to do, however, you are free to make any changes and suggestions, if necessary.
* Participation is completely voluntary and participants have the right to withdraw from this study at any time. They will not be penalized if they choose to do so.
* Confidentiality and anonymity will be maintained at all times. Participants' identities will not be revealed at any time, as pseudonyms (different names) will be used to protect everyone's right to privacy.
* Any information provided by the participants will not be used against them and will be used for purposes of this research only.
* Participation in this study will not result in any cost to the participants.
*No participants will receive financial remuneration. However, costs incurred by participants as a result of their involvement in this project will be covered (this includes the cost of data bundles for the communication via Zoom or Teams).
* This study does not intend to harm the participants in any way.
* All participants will be handed letters of consent which they will have to carefully read and sign before I begin data collection.

I may be contacted at:

wandilezulu2@gmail.com/ 212507659@stu.ukzn.ac.za

Tel: 078 383 6823/ 067 101 0862

My supervisor's contact details are:

mudalyv@ukzn.ac.za

Tel: 031 260 3682

You may also contact the Research Office through:

HSSREC Research Office,

E-mail: HSSREC@ukzn.ac.za

If you would like any further information or if you are unclear about anything, please feel free to contact me at any time**.** Your co-operation and consent will be greatly appreciated.

If you grant permission to conduct this research with you, please complete the form below and return it to me.

Warm regards,

Mzwandile.

DECLARATION

I ……………………………………………………………………….(full name/s) of ………………………………………….….. (name of university) hereby confirm that I understand the contents of this document and the nature of this research project, and I agree toparticipate in this research project willingly.

Additional consent

I understand that interviews will be audio-recorded and I grant permission for this.(*Please Tick*)

Yes	No

I understand that I am free to withdraw from the research project at any time.(*Please Tick*)

Yes	No

Participant signature **DATE**

……………………………………….. **……………………**…

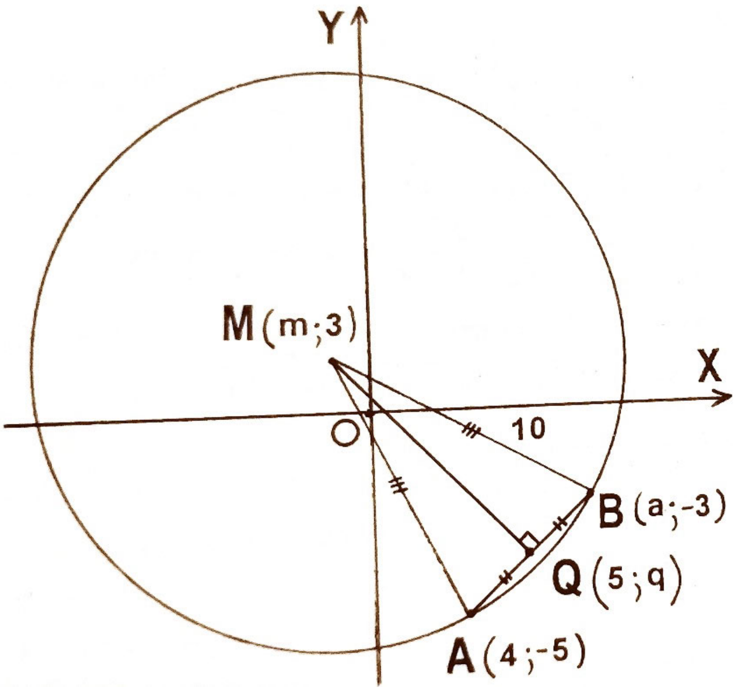
APPENDIX C: PERFORMANCE TEST

Part A

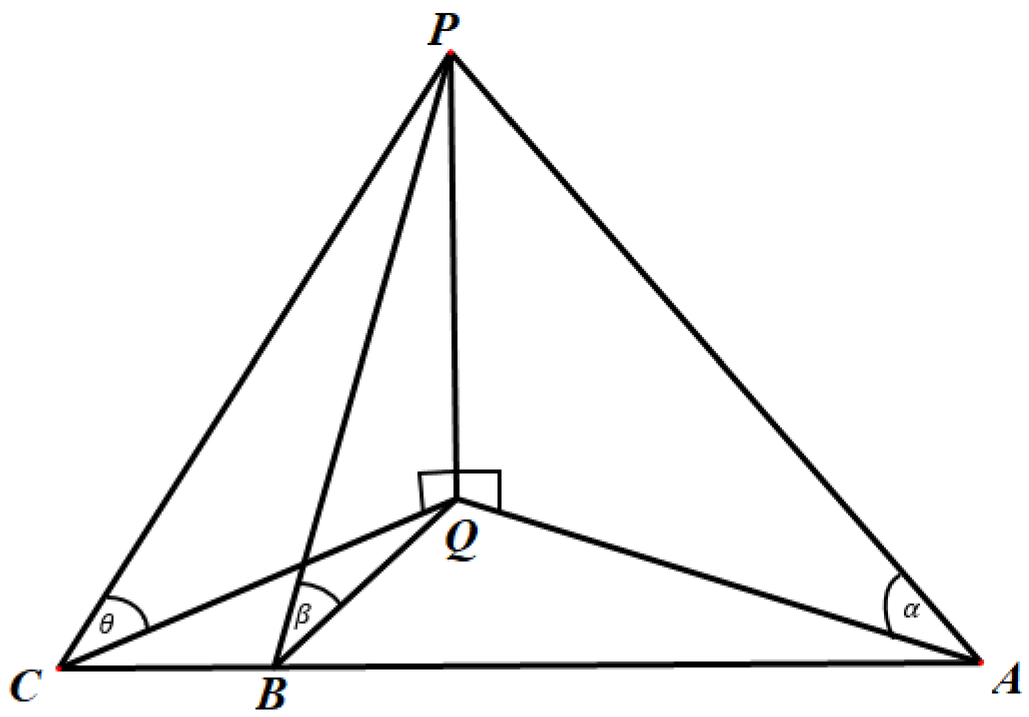
1. $A(4;-5)$ and $B(a;-3)$ are the terminal points of a chord of a circle with the centre $M(m;3)$ in a Cartesian plane. The midpoint of AB is $Q(5;q)$ and the radius of the circle is 10 units.
    a. Calculate the values of $a, q\ and\ m$.
    b. Verify your answer using a diagram.
2. A, B and C are points on a horizontal straight line such that $AB = 60m\ and\ BC = 30m$. The angles of elevation, from A, B, and C respectively, of the top of a vertical clock tower P, are $\alpha, \beta, and\ \theta$, where $\tan\alpha = \frac{1}{13}$, $\tan\beta = \frac{1}{15}\ and \tan\theta = \frac{1}{20}$. The foot of the clock tower, Q is on the same horizontal plane as A, B and C. Find the height of the tower QP (to the nearest metre).

Part B

1. $A(4;-5)$ and $B(a;-3)$ are the terminal points of a chord of a circle with the centre $M(m;3)$ in a Cartesian plane. The midpoint of AB is $Q(5;q)$ and the radius of the circle is 10 units. Calculate the values of a, q *and* m.



2. In the figure, A, B, and C are points on a horizontal straight line such that $AB = 60m$ *and* $BC = 30m$. The angles of elevation, from A, B, and C respectively, of the top of a vertical clock tower P, are $\alpha, \beta,$ *and* θ, where $\tan\alpha = \frac{1}{13}$, $\tan\beta = \frac{1}{15}$ *and* $\tan\theta = \frac{1}{20}$. The foot of the clock tower, Q is on the same horizontal plane as A, B and C. Find the height of the tower QP (to the nearest metre).


P
Q
θ
β
α
C
B
A

APPENDIX D: SEMI-STRUCTURED INTERVIEW SCHEDULE

i. How did you feel about the type of questions that were asked in Part A of the performance test? Did you find them challenging, fair, or easy? Explain.

ii. When you come across questions such as those in Part A, what strategies do you use to unpack them to be able to solve them with understanding?

iii. For the first question in Part A, what was the first thing you did after reading the problem statement?

iv. For the second question in Part A of the performance test, what was the first thing you did after reading the problem statement?

~~v.~~ What mental conversations do you engage in with yourself when solving such mathematics problems (Part A)?

vi. Now that you have solved the problems, explain what you did in each step to arrive at your answers for Part A?

vii. Which part of your solution process would you consider to be the most important one when solving both problems for Part A?

viii. Which performance task did you find easy to understand and solve between Part A and Part B? Why?

ix. Did the accompanying diagrams for Part B of the performance test make it easy for you to visualize the problem statements for both questions?

x. Did you engage in the same mental conversations with yourself when you were solving Part B of the performance test as when you were solving Part A?

APPENDIX E: FOCUS GROUP DISCUSSION QUESTIONS

i. What is your understanding of educational technologies in teaching and learning?

ii. What educational technologies were you exposed to during your learning career?

iii. Have you experienced the learning of mathematics through the use of any educational technologies?

iv. What were those educational technologies? And how was the experience of learning using those educational technologies?

v. As a preservice mathematics teacher, have you used any of the educational technologies in your teaching? If yes, did you find the use of those educational technologies useful for your teaching of the subject? Explain.

vi. Do you think the use of educational technologies could be of any assistance in understanding mathematics problems like the one you did on Part A of the performance test? Why?

vii. What are some of the educational technologies would you use for your teaching of mathematics at school? Why these?

viii. How do you intend to use these educational technologies?

ix. Do you believe your chosen educational technologies would influence your teaching and concept development of mathematics?

x. Do you believe your use of these educational technologies would influence learners' abilities to visualize mathematics concepts?

APPENDIX F: OBSERVATION SCHEDULE

Name of a preservice teacher:		**Role:**	**Observed by:**
Date of observation:		**Activity observed:**	**Location:**
Purpose of observation: Mathematics preservice teachers' usage of educational technologies in the classroom.			
Educational technologies used during the lesson:			
Time:	**Observation:**		

APPENDIX G: PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE HEAD OF DEPARTMENT

Private Bag X9137, PIETERMARITZBURG, 3200
Anton Lembede Building, 247 Burger Street, Pietermaritzburg, 3201
Tel: 033 392 1063
Email: Phindile.duma@kzndoe.gov.za

Enquiries: Phindile Duma **Ref.**:2/4/8/1726

Mr MW Zulu
C2020
Qaphela Lane
KWADABEKA
3610

Dear Mr Zulu

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"AN EXPLORATION OF PRESERVICE TEACHERS' USES OF EDUCATIONAL TECHNOLOGIES AS VISUALIZATION TOOLS WHEN TEACHING MATHEMATICS"**, in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 26 April 2021 to 31 August 2023.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

[redacted]

Dr. EV Nzama
Head of Department: Education
Date: 28 April 2021

GROWING KWAZULU-NATAL TOGETHER

APPENDIX H: TURNITIN REPORT

AN EXPLORATION OF PRESERVICE TEACHERS' USES OF EDUCATIONAL TECHNOLOGIES AS VISUALIZATION TOOLS WHEN TEACHING MATHEMATICS

ORIGINALITY REPORT

9%	8%	4%	3%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

MATCH ALL SOURCES (ONLY SELECTED SOURCE PRINTED)

2%

★ hdl.handle.net

Internet Source

Exclude quotes	On	Exclude matches	< 7 words
Exclude bibliography	On		

APPENDIX I: LETTER FROM THE LANGUAGE EDITOR

CERTIFICATE OF PROFESSIONAL EDITING

I, Barbara L. Louton, do hereby declare that I am a professional editor with a Bachelor of Arts in Professional Writing and sixteen years of experience as an editor, researcher and writer.

I declare that I was contracted by Mzwandile Wiseman Zulu (Student number: 212507659), a doctoral candidate under the supervision of Professor Vimolan Mudaly in the School of Education at the University of KwaZulu-Natal, to complete a professional edit of his dissertation: 'An Exploration of Preservice Teachers' Uses of Educational Technologies as Visualization Tools When Teaching Mathematics'.

I declare that I have completed a two-stage professional edit of the document, addressing structural and logical issues, the clarity and flow of language, and correcting grammatical and spelling errors. Changes were tracked and comments were used to engage with outstanding issues.

Disclaimer:

Responsibility for the originality and accuracy of the material presented in the edited document lies with the client. I have not verified the originality or accuracy of statements, quotations or citations and references presented in the thesis. Where I have detected inaccuracies I have rectified them or reported them to the client. In addition, the client was free to make further changes to the edited material after the edit was complete.

I can be contacted at:

Cell: 073 766 1139

Email: bellway@gmail.com

Barbara L Louton	[redacted]	2 December 2022
Name	Signed	Date