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Estimating the size of the underground economy in South Africa using the Multiple Indicators Multiple Cause Model (MIMIC) and the Currency Demand Approach (CDA)

A thesis submitted in fulfillment of the requirement for the Masters Degree in Statistics

by

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Abstract

The underground economy is a major challenge across the world affecting both developed and developing economies. South Africa is no exception to this phenomenon and has lost billions of rands due to the underground economy. Tax revenue loss due to illicit trade was estimated to be approximately R36.5 billion in 2019, with illicit cigarettes and tobacco and undervalued clothing and textiles perceived to be the main contributors to this economy. The objective of this research is to estimate the size of the underground economy in South Africa using the Currency Demand Approach (CDA) and the Multiple Indicators Multiple Cause (MIMIC) models. To accomplish this, secondary economic data was obtained from Statistics South Africa (STATSSA), World Bank, South African Reserve Bank (SARB) and the International Monetary Fund (IMF) for the period 2000 to 2020. The results from the MIMIC model showed that the underground economy in South Africa was growing with estimates ranging from 25.4% to 32.3% of GDP for 2003 to 2020.The model further indicated that mining employment rate, tax burden and government expenditure are the causes of the underground economy and Nominal Gross Domestic Product (NGDP) and labour force participation rate are the indicators of the underground economy. Similarly, the CDA model showed a steadily increasing underground economy estimated at 28.8% of GDP on average for 2003 to 2020. Furthermore, the CDA model showed that NGDP, tax burden, interest rate, unemployment rate, self-employment rate and social benefits granted by the government are determinants of the underground economy. This study makes a significant contribution to the body of knowledge in this research area and will provide much needed insights on the relative magnitude of the underground economy, drivers of the underground economy and the extent of tax evasion in South Africa, ultimately contributing towards an improved tax base and compliance. It will further serve as a basis for future research in this topic by academia, private sector, government, multilateral bodies and all other interest groups.

Keywords: Underground economy, regression model, confirmatory factor analysis, vector error correction model, structural equation modelling

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Definitions

Term	Description
Dependent Variable	A variable which is influenced by other variables in a model.
Endogenous Constructs / Variables	Variables that originate internally in the model.
Exogenous Constructs / Variables	Variables that originate outside the model and are not dependent on any other variables in the model.
Factor	A latent variable which can either be exogenous or endogenous and is not defined by its indicators.
Independent Variable	A variable whose variation does not depend on other variables in the model.
Indicator	An observed variable which can either be exogenous or endogenous in a measurement model.
Latent Variable	A variable which is not directly measured or observed.
Manifest Variable	A variable which is measured and observed directly. It can also be referred to as an indicator variable.
Observed Variable	A variable that exists in the data.
Parsimony	Refers to a minimum number of parameters used to achieve a given goodness of fit.
Structural Equation Modelling	A multivariate technique used to investigate causal relationships in a model.

Abbreviations and Acronyms

Abbreviation	Description
ADF	Asymptotic Distribution Free
AD-F	Augmented Dicky-Fuller
AGFI	Adjusted Goodness of Fit Index
AIC	Akaike Information Criterion
ARDL	Auto Regressive Distributed Lag
ARIMA	Autoregressive Integrated Moving Average
BIC	Bayesian Information Criterion
CDA	Currency Demand Approach
CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
CPI	Consumer Price Index
CSA	Covariance Structure Analysis
DTI	Department of Trade and Industry
DYMIMIC	Dynamic Multiple Indicators Multiple Cause Model
EBRD	European Bank for Reconstruction and Development
ECB	European Central Bank
EDA	Exploratory Data Analysis
EFA	Exploratory Factor Analysis
FPE	Final Prediction Error
GDP	Gross Domestic Product
GFI	Goodness of Fit Index

Abbreviation	Description
GLS	Generalised Least Squares
GNP	Gross National Product
HQC	Hannan-Quinn Criterion
IDC	Industrial Development Corporation
ILO	International Labour Organisation
IMF	International Monetary Fund
LR	Likelihood Ratio
MCSA	Mean and Covariance Structure Analysis
MIMIC	Multiple Indicators Multiple Cause Model
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimation
OECD	Organisation for Economic Co-operation and Development
OLS	Ordinary Least Squares
PCBS	Palestinian Central Bureau of Statistics
PMA	Palestine Monetary Authority
RGDP	Real Gross Domestic Product
RMSEA	Root Mean Square Error of Approximation
SARB	South African Reserve Bank
SBC	Schwarz Bayesian Criterion
SEM	Structural Equation Modelling
SMME	Small, Medium and Micro Enterprises
SPRTB	Start-up Procedures to Register a Business
SRMR	Standardised Root Mean Square Residual

Abbreviation	Description
STATSSA	Statistics South Africa
TLI	Tucker Lewis Index
UN	United Nations
UNCTAD	United Nations Conference on Trade and Development
VAR	Vector Autoregression
VECM	Vector Error Correction Model

Chapter 1: Introduction

1.1 Background of study

Hassan and Schneider (2016a:311) defines the underground economy as “every unrecorded economic activity which would contribute to the officially calculated Gross Domestic Product (GDP) if it was registered.” Underground economic activities are a significant issue for most countries, although to varying degrees. In fact, these activities are universally widespread and are present in developing as well as in advanced economies, but are more endemic in developing countries. The estimates of the size of the underground economy by the Organisation for Economic Co-operation and Development (OECD) (2017) vary from under 1% of GDP in some countries to over 20% in others. The underground economic activities occur as a result of individuals and companies avoiding paying taxes and social security contributions as well as non-compliance with labour regulations such as the minimum wage.

South Africa is no exception to this phenomenon. Similar to most developing countries, the South African economy is not spared from the consequences of the underground economy. The country is losing approximately R250m daily in tax revenue due to illicit trade. Tax revenue loss due to illicit trade was estimated to be R36.5 billion in 2019. The illicit trade segment comprises by value of counterfeits (38%), smuggling (39.5%) and artisanal products (17.7%) (Liedtke, 2020).

Cigarettes and tobacco make up a significant proportion of the revenue loss. Luckhoff (2021) states that tax revenue is dropping at an alarming rate as three out of four retailers are selling illicit cigarettes. Illicit cigarettes were already a concern in 2018, but the outbreak of COVID-19 in 2020 with the resultant lock down and ban on the sale of tobacco and alcohol products further exacerbated the problem with billions of rands lost in tax revenue (News24Wire, 2021 and Neves, 2021). The South African Revenue Services (SARS) estimated the loss in revenue due to illicit cigarettes to be around R6 billion for the 2015/16 financial year (Chelin and Nyoni, 2020). In agreement with Liedtke (2020), Chelin and Nyoni (2020) further assert that the illegal cigarette trade is part of a wider underground economy involving clothing and textiles, fake goods, music and movies. The illegal alcohol market is valued at R12.9 billion and is 14.5% in volume.

Furthermore, under declaration of customs value in clothing and textiles has grown from R5.2 billion in 2014 to R8.5 billion in 2018. As a result, thousands of jobs have been lost in this industry (Liedtke, 2020).

South Africa has been experiencing a slowdown in economic growth in recent years with many individuals finding themselves unemployed. In fact, the latest quarterly figures from Statistics South Africa (STATSSA) estimates unemployment to be 34.4% in 2021Q1, which is the highest in the world, with youth unemployment at an alarming 74.7%. In an effort to be frugal, businesses and individuals will most likely gravitate towards cheaper illegal items and engage in tax avoidance and evasion schemes.

1.2 Problem Statement

The presence of the underground economy can have a devastating effect on the formal economy of a country, contributes to crime, job losses, reputational damage to legitimate brands and disregards laws and regulations of a country. The underperformance in tax revenue brought about by the underground economy erodes the tax base and ultimately results in the South African government not being able to deliver essential services and goods to its people. The underground economy is not normally considered when making government policy choices. This omission may lead to inefficient policies because the underground economy effectively reallocates income and wealth in ways that are inconsistent with the redistribution intentions of the tax system (Tedds, 2005).

There is a literature gap in the studies devoted to studying the South African underground economy in comparison to other countries, especially the estimation of its size and development.

1.3 Aim and Objectives

This study aims to estimate the size of the underground economic activities in South Africa by using the Multiple Indicators Multiple Cause Model (MIMIC) and the Currency Demand Approach (CDA). This includes an estimation of the magnitude of the underground economy as a percentage of GDP as well as a determination of the factors that contribute to the growth of such activities.

Therefore, the main objectives of the study are to:

1) estimate the extent of the underground economy in South Africa.
2) determine the drivers of the underground economy in South Africa.
3) determine the potential revenue loss to the South African government as a result of the underground economy.
4) determine the impact of the underground economy on each economic sector.

1.4 Significance of the study

This study makes a significant contribution to the body of knowledge in this research area and will provide much needed insights on the relative magnitude of the underground economy, drivers of the underground economy and the extent of tax evasion in South Africa, ultimately contributing towards an improved tax base and tax compliance. It will further serve as a basis for estimating the magnitude of the underground economy of South Africa for future related studies by academia, private sector, government, multilateral bodies and all other interest groups.

1.5 Chapter division

The research report is presented in the following structure:

Chapter 1	**Introduction** Provides an introduction to the context of this study. It includes the background, problem statement, objectives of the study, research questions, significance of the study and concludes with the layout of the research report.
Chapter 2	**Literature review** Explores the different definitions and explanations of the underground economy, discusses causes and indicators of the underground economy, examines the methods used to estimate the underground economy and finally provides empirical studies conducted using the Currency Demand Approach (CDA), Multiple Indicators Multiple Cause Model (MIMIC) and the joint approach.

Chapter 3	**Exploratory Data Analysis** Presents data analysis on the variables used for the CDA and MIMIC models, which includes descriptive statistics, distributional properties of the variables, correlation analysis and an investigation of the linear relationship amongst variables.
Chapter 4	**Research Methodology** Presents a comprehensive explanation of the research methodology used in the study, with specific emphasis on Structural Equation Modelling (SEM), multiple linear regression, Confirmatory Factor Analysis (CFA) and Vector Error Correction Model (VECM). The joint model as well as the benchmarking procedure used in the MIMIC model are discussed at the end of the chapter.
Chapter 5	**Application and Results** Presents the application and modelling results from the CDA and MIMIC models and shows the estimates of the size of the underground economy as % of GDP. Finally sectoral estimates of the underground economy are computed using proportional contributions to total GDP.
Chapter 6	**Discussion and Conclusion** Discusses the findings, limitations, conclusions and recommendations for future research.

1.6 Summary

This introduction provided the background to the study. A background was given on the underground economic activities, with specific emphasis on South Africa. Key commodities that contributes to the growth of the underground economy were highlighted. The problem statement highlighting the impact of such activities on the formal economy was presented and the objectives and research questions to be addressed by the study were stated. The importance of the study, particularly for the tax authority and the South African government, was emphasised. Finally, a brief outline of the structure of the study was given.

Chapter 2: Literature Review

2.1 Introduction

The aim of this literature review is to define and gain understanding of the underground economy, its causes and indicators and to research different methods used to estimate the underground economy. This chapter also reviews previous articles on estimating the size of the underground economy.

2.2 Defining the underground economy

Empirical research on the underground economy reveals little to no agreement on the definition and scope of the underground economy. This lack of consensus has resulted in different naming conventions of the same phenomenon namely, shadow, illicit, hidden, unofficial, subterraneous, grey, black, clandestine, unrecorded, parallel, non-observed, illegal, irregular, cash or informal economy (Hassan and Schneider, 2016a and Dell'Anno, 2003). In most occasions, the definition adopted depend on the approach chosen by the author to estimate the underground economy.

Hassan and Schneider (2016a:311) defines the underground economy as "every unregistered economic activity which would contribute to the officially calculated Gross Domestic Product (GDP) if it was registered." Similarly, the OECD (2017) defines the underground economy as economic activities, legal or illegal, which by law should be reported to the tax authorities, but are not reported and therefore remain untaxed. In the International Monetary Fund (IMF) working paper, Medina and Schneider (2018) refer to the underground economy as all economic activities that are concealed from officials for regulatory, institutional and monetary reasons. Regulatory reasons refers to government bureaucracy and regulatory burdens, monetary reasons refers to tax avoidance and social security contributions and institutional reasons refers to quality of political institutions, corruption and the rule of law.

The underground economy may refer either to legal activities carried out without the required licenses and payment of taxes or illegal activities. Examples of legal activities in the underground economy include self-employment income which is unreported to the tax authorities. Illegal activities include smuggling, trading in stolen goods, drug dealing, fraud and illegal gambling (Hall, 2019).

According to Thomas (1992) the underground economy has four parts which are not accounted for in the national accounts: household consumer goods, unofficial (retail producers not required to keep accounting books), irregular (illegal production of legal goods) and illicit (production of illicit goods). Table 2.2 below shows the type of legal and illegal activities under the underground economy.

Table 2.2: Underground economic activities. Source: Lippert and Walker (1997).

Type of Activity	Monetary Transactions		Non-Monetary Transactions	
Illegal Activities	Trade with stolen goods,drug dealing and manufacturing, prostitution, gambling, smuggling, fraud etc.		Barter of drugs, stolen goods,smuggling etc. Produce or growing drugs for own use.Theft for own use.	
	Tax Evasion	**Tax Avoidance**	**Tax Evasion**	**Tax Avoidance**
Legal Activities	Unreported income from self-employment, wages, salaries and assets from unreported work related to legal services and goods.	Employee discounts, fringe benefits.	Barter of legal services and goods.	All do-it-yourself work and neighbour help.

This study adopts the definition given by Hassan and Schneider (2016a) and will only focus on the unreported legal activities that would if otherwise reported contribute to national GDP. Therefore, this study does not focus on any illegal activities and legal activities such as do-it-yourself activities.

2.3 The causes and indicators of the underground economy

The magnitude of the underground economy is influenced by various factors specific to the economic dynamics of each country. Literature reveals causes and indicators of underground economies shown in Table 2.3 below.

Table 2.3: Main determinants of the underground economy.

CAUSES	
Variables	**Theoretical reasoning for consideration**
1. Tax burden	Taxes and social security contributions generally place a burden on taxpayers. The higher the difference between the total labour cost in the official economy and after-tax earnings from work, the greater the incentive to reduce the tax wedge and work in the underground economy. This tax wedge depends on social security payments and the overall tax burden (Hassan and Schneider, 2016b and Schneider and Williams, 2013). A growth in the tax burden will cause an increase in the underground economy (Klaric, 2010). Only three million South Africans carry the tax burden out of the total population of 59.5 million (IOL, 2020).
2. Social benefits paid by the government	Social benefits can have an ambiguous effect on the underground economy. Social benefits can encourage individuals not to be formally employed but to enjoy extra revenue from the underground economy, by dedicating all their available working time to the underground economy. However, Dell'Anno et al. (2007) argues that social benefits can reduce the underground economic activities. Social benefits increase the cost of engaging in the underground economic activities, because informal workers do not have access to unemployment allowances and financial aid.
3. Tax morale	The quality of public institutions and the perceived lack of corruption has a positive impact on the formal economy as it affects tax morale. Taxpayers are more likely to pay their due taxes if they get valuable public services. Individuals with a high tax morale and strong social norms will most probably not engage in the underground economy. The relationship between the spread of corruption and tax morale is

CAUSES	
Variables	**Theoretical reasoning for consideration**
	negative (Jahnke and Weisser, 2019), thus with the perceived high level of corruption in South Africa, one can expect the tax morale to be low which will have a negative effect on the formal economy.
4. Tax complexity	A simplified tax return and easier tax regulations will have a positive effect on tax compliance as taxpayers will spend less time in ensuring the correctness and completeness of their returns (Slemrod, 1989). The more complicated and sophisticated the tax return is, the more likely is the non-filing of tax returns.
5. Corruption or quality of public institutions	A government with highly corrupt officials have a tendency to increase underground economic activities while good rule of law increases tax compliance resulting in less underground economic activities. An increase in the underground economy reduces government revenues which leads to reduced public service delivery. Eventually, the government will be forced to increase tax rates in order to increase state revenue, which will further encourage participation in the underground economy (Schneider and Williams, 2013). According to BusinessTech (2020), the high perception of corruption in South Africa increases tax evasion and the withholding of tax payments due.
6. Criminality level	Criminality can have an ambiguous effect on the underground economy. There is a positive correlation between the levels of criminality and the size of the hidden economy. Inversely, there can be a negative correlation as criminality weakens incentives to be in the shadow (Bilonizhko, 2006). According to the OECD (2017), most administrators are experiencing a surge in labour market crime, cross-border frauds and sophisticated use of technology in the underground economy.
7. Trade Openness	Trade openness will cause the underground economy to increase in the short run as domestic firms move underground in order to decrease costs to effectively deal with foreign competition. However, trade openness contributes to the transformation of institutions in the long run and ultimately more economic freedom that raises the opportunity cost of producing underground (Goel et al., 2019).
8. Business Regulations	Business regulations reduces the freedom of choice for individuals in the formal economy e.g. barriers to entry. They lead to a significant growth in labour costs in the official economy and thus encourage

CAUSES	
Variables	**Theoretical reasoning for consideration**
	working in the underground economy. Highly regulated economies may limit choices available to individuals and might encourage underground economic activities (Oviedo et al., 2009, McMillan, 2006 and Dabla-Norris et al., 2008). Regulations introduced in South Africa to address social cohesion and equality will most probably increase underground economic activities (Frontier Economics, 2012).
9. Self-employment	A growth in self-employment increases the number of opportunities to hide revenue from the tax administrators, thus increasing underground economic activities (Dell'Anno et al., 2007). Self-employed workforce have greater probability of tax evasion than large entities as there are generally fewer auditing controls in their businesses and they work very closely with their clients (Gauci and Rapa, 2020).The absence of reliable, comprehensive and consistent data on Small, Medium and Micro Enterprises (SMME) in South Africa has led to differing estimates on their number, employment figures, the sectors they dominate and their contribution to the country's GDP. Finscope's 2010 study estimate the number of SMME's to be 5.9 million while the Bureau of Economic Research 2016 study's estimate is around 2.2 million (Small Business Institute, 2018). With the existence of unregistered and unknown SMME's, the likelihood of tax evasion will be higher.
10. Unemployment	Unemployment can encourage individuals to seek employment work in the informal economy. An increase in the unemployment rate will most likely result in an increase in the underground economic activities (Gauci and Rapa, 2020). South Africa has long suffered from extreme levels of unemployment, even before the COVID-19 pandemic. In the second quarter of 2021, the unemployment rate reached a record high of 34.4%, the highest in the world.
11. Government Employment	The higher the size of the public sector, the more decision power bureaucrats have, resulting in a high likelihood of corruption. Dishonesty of civil servants and briberies have a positive impact on the shadow economy (Schneider and Enste, 2002). In South Africa, the public wage bill is regarded to be extremely high and the national treasury has imposed a salary freeze over the next three years which will result in R300bn worth of savings from the payment of public service salaries (Fin24, 2021).

CAUSES	
Variables	**Theoretical reasoning for consideration**
12. Degree of economic freedom (Recurrent government expenditure used as a proxy)	Government expenditure includes employee compensation, social benefits, spending on goods and services consumed by the government during the production process and any other recurrent expenses. A rise in government expenditure will give rise to the underground economy (Gauci and Rapa, 2020). The South African government spending in 2018/19 was R1.79 trillion, 4.7% higher than the 2017/18 value of R1.71 trillion, which was 7.3% higher than in 2016/17 (STATSSA, 2019).
13. Subsidies	Subsidies have a contradicting effect on the underground economy. Only qualifying businesses have access to government subsidies. The criteria for subsidy allocation could discriminate between entities and this could encourage entities towards irregular activities. On the other hand, entities find no incentives to engage in the underground economy as increased subsidies raises the opportunity costs to be informal, thus discouraging underground economic activities (Dell'Anno, 2007). The Land Bank, the Department of Trade and Industry (DTI) and the Industrial Development Corporation (IDC) offer funding and grants to qualifying businesses from various sectors with the aim of stimulating economic growth and development.
14. Electronic payments value to GDP	Card payments leave electronic trace, making it difficult to conceal transactions from the authorities. Therefore a growth in card payments will most likely result in a drop in the underground economic activities (Vorobyev, 2015).
15. Card payments per capita	An increase in these two variables may result in either a decrease in the underground economy (the likelihood of not reporting transactions is reduced) or no impact on the underground economy as electronic payments are just replacing registered cash transactions.
16. Number of point of sale terminals relative to card payments	
17. Inflation rate / Consumer Price Index	Inflation encourages tax evasion and participation in the underground economy (Goel and Nelson, 2016).The average annual inflation rate for December 2020 was 3.3%, which is the lowest average rate since 2004 (1.4%) and the second lowest since 1969 (3%) (BusinessTech, 2021).

CAUSES	
Variables	**Theoretical reasoning for consideration**
18. Real interest rate on deposits	Increasing interest rates may encourage individuals to deposit money since they can achieve a higher return on it while low interest rates might disincentivise individuals to deposit money. The higher the deposit interest rate, the less likely individuals will participate in underground economic activities.
19. Agriculture GDP	It is easier to hide activities in the agricultural sector than in an industrial sector (Bilonizhko, 2006). Ercolani (2000) further states that in developing countries where taxation is problematic, the hidden economy is mostly made up of the agricultural sector, which is often not directly taxed as the costs of levying taxes are higher than the potential revenues.
20. Mining GDP	Mining has been a very significant component of the South African economy. It led to the establishment of Johannesburg, Welkom and Kimberley, just to name a few. A United Nations Conference on Trade and Development (UNCTAD) (2016) study alleges that South African miners of platinum, iron ore, silver and gold had fraudulently and intentionally engaged in under invoicing to evade taxes. In addition, the development and advancement of illegal mining activities in South Africa has increased the magnitude of the underground economy as most of the output from the mines is sold in the black market.
21. Immigration	Foreign workers are more likely to engage in informal economic activities. Immigrants usually face bureaucratic problems of registration and finding work, so they are highly likely to engage in informal activities (Maurin et al., 2003). Since the dawn of democracy, there have been an influx of foreigners into South Africa seeking greener pastures. According to STATSSA, the number of foreigners living in South Africa in 2020 was around 3.9 million while the United Nations (UN) population division estimates this number to be around 4.2 million in 2019. 47% of migrants in South Africa are employed in hazardous and unregulated environments. Approximately 39% of these migrants are employed in the shadow economy (ISS, 2020).

INDICATORS	
Variables	**Theoretical reasoning for consideration**
22. The official economy / GDP	On the one hand, a rise in the underground economy leads to a reduction in the official economy because productive resources and factors are being used by the underground economy. Inversely, poor people are able to produce and sell cheap products in the underground economy as a way of generating income. The increased demand in the underground economy will have a positive spillover effect on the formal economy. Schneider (2005) contends that in developing countries, the relationship is negative and it is positive in the developed and transition countries.
23. Currency in circulation outside the banks	Underground transactions are normally conducted in cash rather than with cheques, credit cards or electronic transfers. Thus, the larger the currency in circulation, the larger the underground economy. South Africa has a large informal sector dependent on cash as a means of exchange. The township economy which includes spaza shops, salons and the taxi industry mostly uses cash for trading and these transactions are usually unrecorded and remain unnoticed by the tax authorities. Cash is a preferred means of payment for a large sector of the population.
24. Labour force participation	On the one hand, resources are absorbed by the underground economy from the official economy showing that a growth in the underground economy will lead to lower labour force participation rate (Gauci and Rapa, 2020). On the other hand, individuals have a tendency to participate in both the economies simultaneously. Labour force might participate in the underground economic activities during weekends, holidays or after working hours (Dell’Anno et al., 2007).

2.4 Methods used to estimate the size of the underground economy

It is challenging to measure the underground economy because of its latent nature, as a result, there is no agreement on the most appropriate methodology to use in order to quantify the phenomenon (Orsi et al., 2014 and Sharapenko, 2009). Schneider and Buehn (2016) and Schneider (2005) provide descriptions of various methodologies and approaches used in estimating the underground economy. There are three different approaches as shown in Figure 2.4 below. These are also briefly discussed in this section.

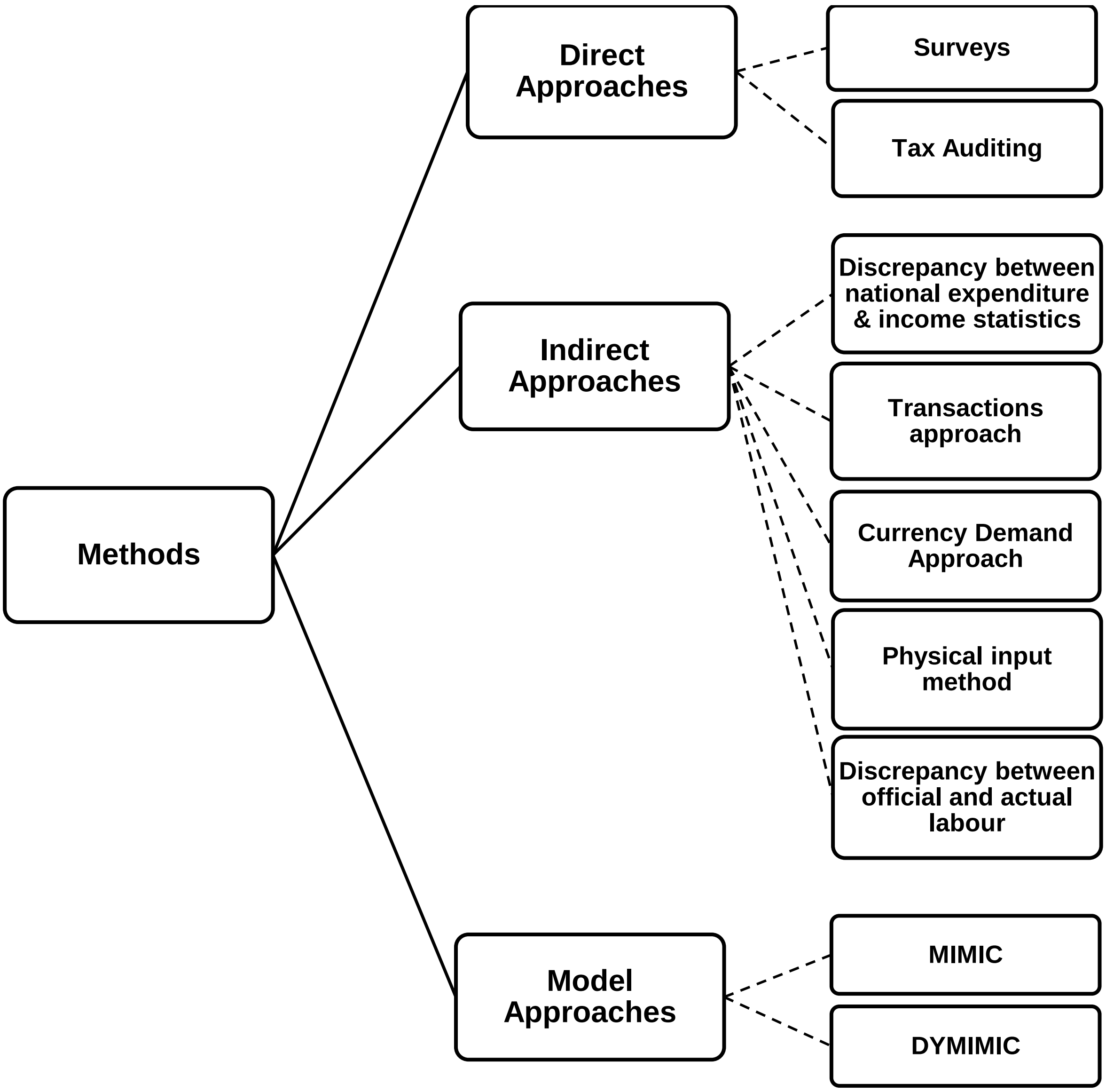


Figure 2.4: Approaches used in estimating the underground economy.

2.4.1 Direct Approaches

Schneider and Enste (2013) identifies two direct approaches in modelling the underground economy, namely surveys and tax audits. These are micro approaches that use either surveys and samples based on voluntary responses or tax auditing. The key drawback of surveys are the errors that occur naturally in all surveys, which are that the accuracy of survey results are highly dependent on the willingness of the respondent to cooperate. Most respondents would not ordinarily disclose their past fraudulent behavior and it is not easy to determine the quantity of undeclared work from a survey as responses are sometimes not reliable. This makes it challenging to compute a true monetary estimate of the magnitude of undeclared work.

Since tax audits quantify undeclared taxable income, they may also be used to calculate the magnitude of the underground economy. The main disadvantages of this method is that tax audits can sometimes offer a biased sample of the population as the selection of taxpayers for audits is based on the risk of tax fraud. Quantifying the underground economy from a biased sample may be inaccurate. In addition, tax audits show only that part of the underground economy income that the tax administrators manage to expose, and this is most probably only a small portion of the concealed income.

Although the direct approaches provide comprehensive information about the composition and structure of the underground economy, their main disadvantage is that not all underground economic activities can be revealed by these methods and they result in only point estimates of the size of the underground economy and may be seen as lower-bound estimates compared to other approaches (Schneider and Buehn, 2016). These methods are only suitable for short term estimation of the underground economy.

2.4.2 Indirect Approaches

As defined by Schneider and Enste (2013), indirect approaches are macro approaches which utilises various economic indicators that have information about the growth and advancement of the underground economy over time. The five indirect approaches used to measure the magnitude of the underground economy will be described below.

2.4.2.1 The discrepancy between national expenditure and income statistics

This approach is based on the differences between expenditure and income statistics. In national accounting, the income and expenditure measure of Gross National Product (GNP) should be equal. Gross National Product measures the value of goods and services produced by the citizens of a country both domestically and internationally. The variance between the expenditure and the income measures can be used as an indicator of the size of the underground economy (Schneider and Buehn, 2016). The initial discrepancy and not the published discrepancy should be used. If there were no errors in disclosing expenditure, then this approach would give good estimates of the underground economy. Unfortunately, this discrepancy does not only reflect underground economic activities but also all errors and omissions in the national accounts statistics. Therefore estimates derived using this approach may not be reliable (Schneider and Enste, 2002).

2.4.2.2 The discrepancy between official and actual labour force

A decline in participation of the labour force in the official economy can indicate an increase in the underground economy. Estimates derived this way are however seen as weak indicators of the magnitude and development of the underground economy as individuals can concurrently participate in both formal and informal economies (Schneider, 2005).

2.4.2.3 The transactions approach

The premise of this approach is on the assumption that there is a constant relationship between the volume of transactions and official GNP over time, as summarised by the famous Fisher's quantity equation,

$$MV = pT \tag{2.1}$$

where M = money, V = velocity, p = prices, and T = total transactions. To get consistent underground economy estimates, accurate figures of the total volume of transactions should be available which can be challenging for cash transactions, which depend, among other factors, on the durability of bank notes. Other weaknesses of this approach are the assumptions:

- that the gap between the total volume of transactions and officially measured GNP might be due to the underground economy, which is not always the case (Schneider, 2005).
- of a base year with no underground economy (Schneider and Enste, 2000).
- that the proportion of transactions to official GNP does not change over time (Schneider and Enste, 2000).

2.4.2.4 The physical input method (electricity consumption)

Two methods are identified in this approach, the Kaufmann - Kaliberda Method and the Lackó method. The single best physical indicator of overall economic activity (official and unofficial), as mentioned by Kaufmann and Kaliberda (1996), is electricity consumption. The increase in electricity consumption is an indicator of the growth in the overall GDP. Kaufmann and Kaliberda derived an estimate of unofficial GDP by deducting the estimates of official GDP from this overall measure. This approach however, has its shortcomings as cited by Schneider (2005), namely:

- some underground economic activities do not require a substantial amount of electricity as other energy sources can be used. Therefore, only a portion of the underground economy will be shown.
- electricity production and usage has become more efficient over the years.
- there have been substantial variations in the elasticity of electricity/GDP over time and among countries.

Lackó (2000) assumes that household consumption contributes a certain percentage to the underground economy. Consumption of electricity by households consists of activities such as do it yourself activities and household production and services. Lackó also assumes that, in countries where the part of the underground economy linked to household consumption is high, the rest of the underground economy will be high. The major drawbacks of this method are:

- some underground economic activities do not require a substantial amount of electricity as other energy sources can be used. Therefore, only a portion of the underground economy will be shown.
- underground economic activities can take part in other sectors other than the household sector.
- the proportion of social welfare expenditure cannot be utilised as a reliable predictor of the underground economy, especially in developing and transition countries.

2.4.2.5 The Currency Demand Approach (CDA)

This widely used approach was initially proposed by Cagan (1958), and further improved by Tanzi (1980, 1983). The CDA assumes that hidden or underground transactions are normally conducted in cash so as to leave no obvious traces for the tax authorities. In fact, Isachsen and Strøm (1985) found that approximately 80 percent of underground economic transactions are conducted in cash. Increased activities in the underground economy will therefore give rise to increased demand for currency. This approach assumes that the major reason people participate in the underground economy is to avoid tax. It is considered the most important determinant of the underground economy. Therefore, if the tax base is given, increasing (or decreasing) tax rates may encourage more (or less) underground economy activities which in turn may increase (or decrease) the demand for currency (Pickhardt and Pons, 2006).

Although this approach is extensively used around the world, it has the following disadvantages:

- In most cases, the time series model does not consist of all causes of the underground economy. Important factors such as tax morality, tax complexity, regulation impact and development of electronic payment system are most often excluded due to the unavailability of reliable data (Dybka et al., 2018).
- Some transactions in the underground economy are not conducted in cash, therefore the CDA may underestimate the magnitude of the underground economy.
- The growth in currency demand deposits is not necessarily due to increasing underground economic activities but may be due to a slowdown in demand deposits, as discovered by Garcia (1978) and Park (1979) in the United States.

- The velocity of currency is not the same for both formal and underground economies, as assumed by this approach. There is still a great uncertainty about the velocity of currency in the official economy and it's even more difficult to estimate in the underground economy (Hill and Kabir, 1996 and Klovland, 1984). A correction method developed by Ahumada et al. (2007) is proposed.
- The CDA often estimates the underground economy using a scenario of an economy with zero tax which does not exist. Dybka et al. (2018) suggests calibration of the lowest possible level of the underground economy by using the lowest recorded tax and social security contributions inflows.

2.4.3 Model Approaches

2.4.3.1 The Multiple Indicators Multiple Cause (MIMIC) model and the Dynamic Multiple Indicator Multiple Cause (DYMIMIC) models

Frey and Weck-Hannemann (1984) and Giles (1999) proposed a factor model known as the MIMIC model in which the underground economy is linked to a set of indicators and causal variables through a system of linear equations. Estimating the underground economy involves predicting the unobserved or latent variable, through standard regression techniques. Therefore, the MIMIC model can be seen as a purely statistical model, in which no assumptions are made a priori about the relationship between the variables and the underlying economic structure. The relationship between the latent, causal and indicator variables is demonstrated by Figure 2.5 below.

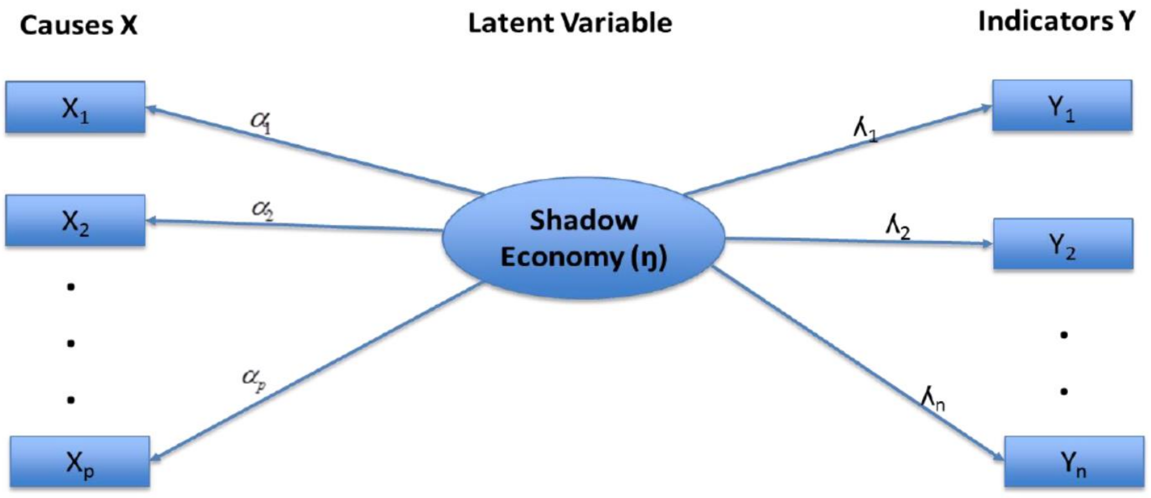


Figure 2.5: General structure of the MIMIC model.

The MIMIC model is a special type of structural equation modelling based on the statistical theory of unseen variables (Hassan and Schneider, 2016a). It is a theory-based approach which confirms the influence of a number of exogenous causal variables on the latent variable (underground economy), and the effect of the underground economy on observed macroeconomic indicator variables (Farzanegan, 2009). The changes in the size of the underground economy are reflected in the indicators and the causal variables are the most significant factors determining unrecorded economic activities (Vorobyev, 2015). The MIMIC model can therefore be considered a confirmatory rather than an explanatory method (Feld and Schneider, 2010 and Schneider et al., 2010).

The MIMIC model is considered superior to other statistical methods and offers the following advantages for estimating underground economic activities:

- According to Giles and Tedds (2002) and Hassan and Schneider (2016), the MIMIC model is more comprehensive than most other models as it takes into consideration multiple indicators and causal variables simultaneously.
- The MIMIC model allows one to vary the choice of causal and indicator variables depending on the dynamics of the underground economy being studied and the availability of data. Schneider and Enste (2000) agrees and states that because of the model's flexibility in its application, it has advanced studies in the development of the shadow economy.

- The MIMIC approach employs Maximum Likelihood Estimation (MLE) procedures, which are generally well accepted and are optimal if the sample size is large enough.

Some of the disadvantages of the MIMIC approach is that it is too dependent on the causal and indicators variables chosen and the benchmarking procedure utilised is still subject to academic debate (Pickhardt and Pons, 2006). This model also requires a large amount of data which is not often available in transition countries (Sharapenko, 2009). Gurkov (2015) defines a transition country as a country whose economy is moving from being centrally planned, that is, one controlled by the government, to a free market economy. Countries such as China, Mongolia, Vietnam, Russia, Poland, Croatia and Serbia are considered to be in a transition economy.

According to Hassan and Schneider (2016a) and (Thakkar, 2020), the main assumptions that need to be met when estimating the underground economy using the MIMIC/SEM model are:

- multivariate normal distribution: the Maximum Likelihood (ML) technique is used and therefore the variables must be normally distributed.
- linearity: endogenous and exogenous variables must have linear relationships.
- outliers affect the significance of the model and therefore the data must be free of outliers.
- sequence: the cause should occur before the event. Endogenous and exogenous variables should have an effect and cause relationship.
- non-spurious relationship: the observed covariance should be correct.
- the distribution of the residuals must be homoscedastic.
- large sample sizes: the sample size should be greater than 50 or more than 200 observations.
- the time series must be stationary.
- the variables (causes and indicators) must be co-integrated.

The time series may be transformed to stationarity by employing a differencing operator in order to avoid spurious regressions. In this form, the MIMIC model is referred to as the DYMIMIC model (Buehn and Schneider, 2008).

2.5 Overview of research conducted in estimating the size of the underground economy

This section provides a review of the existing literature on estimating the size of the underground economy using the CDA model, the MIMIC model, and a joint approach of the two models.

2.5.1 Empirical studies conducted using the CDA model

Bouriche and Bennihi (2020) estimated the underground economy in Algeria from 1980 to 2019 using the CDA model. Annual data was obtained from Official National Statistics and the World Bank and a regression model was built using the following variables: currency, money supply, inflation rate, nominal deposit interest rate, total tax outside oil tax revenues and real non-oil GDP. The real non-oil GDP was chosen instead of GDP to highlight the development of the official economy by taking out the repeated oil shocks which may lead to misleading results. The study revealed that the informal economy in Algeria is on average 21% of the official GDP and that the tax burden is a key predictor of the informal economy.

Awad and Alazzeh (2020) estimated the Palestinian underground economy by using the CDA model. The data for the period 2008-2017 was collected from the Palestine Monetary Authority (PMA) and the Palestinian Central Bureau of Statistics (PCBS) on the following variables: cash outside the bank system, supply of money, taxes, GDP, wages and government salaries, saving deposits, rate of workers in their private business to the real ratio of workers in Palestine and GDP per capita. The study revealed that the underground economy of Palestine has been steadily declining from 28.6% in 2010 to 13.80% in 2016. The underground economy slightly increased to 18.37% in 2017.

Manzoor et al. (2018) estimated Pakistan's underground economy using a modified CDA model. An Auto Regressive Distributed Lag (ARDL) bound test was applied to estimate dynamic monetary model and estimated the size of the underground economy in Pakistan. The ARDL model can use variables integrated of order I(0) and I(1) and produces long run relationships between the dependent and the independent variables. The model investigated the relationship between currency ratio and tax burden by controlling for the effect of financial development, education, interest rate and the strength of the government regime. Data for the period 1972-2015 was obtained from Pakistan's Handbook of Statistics, International Financial Statistics, World Development Indicators and Polity IV on the following variables: currency in circulation outside the banks, money supply, NGDP, tax burden, interest rate, domestic credit to private sector percentage (proxy for financial development), regime durability as well as total enrolment in universities and colleges (proxy for education). The study revealed a positive and significant long-run equilibrium relationship between the tax burden and currency ratio. Financial development, regime durability, education and interest rate have indirect relationship with currency demand. The size of the underground economy of Pakistan was estimated to be 49.38% of GDP in 1998 and it dropped to 27.16% of GDP in 2015.

Nchor and Konderla (2016) investigated the underground economy of the Czech Republic using the CDA model. The following predictor variables were used: currency in circulation, number of automatic teller machines, the deposit interest rate, GDP deflator, tax rates, velocity of currency, NGDP and nominal money supply. Annual data on the variables was obtained from the World Bank and the International Financial Statistics for the period 1993-2013. Extrapolation method was used to derive the data for 1991 and 1992 to ensure that the analysis period covered the period 1991-2013. The analysis revealed that the underground economy of the Czech Republic was 20.9% of GDP.

Raut et al. (2014) conducted a study to estimate the underground economy in Nepal by applying the CDA model. The data for the period 1985-2011 was obtained from the Quarterly Economic Bulletin of Nepal Rastra Bank (the Central Bank of Nepal), Economic Survey of Ministry of Finance, the government of Nepal and the World Bank. The regression model was built using the following variables: the proportion of cash to deposit accounts, tax revenue per GNP, proportion of private consumption in national income, interest paid on savings deposits, income per capita and inflation rate. The study revealed a visible growth of the underground economy between 2011 and 2012 with an unexpected rise of 31.7%. The growth rate for earlier years was much lower at 19.21% and 16.68% for the period 1991-2000 and 2001-2010 respectively.

2.5.2 Empirical studies conducted using the MIMIC model

Nchor (2020) conducted research to estimate the shadow economies and tax evasion in Czech Republic, Poland and Hungary by using the MIMIC model. The time series was sourced from the World Bank for the period 1990-2019 on the following variables: currency, labour force participation rate, corruption, unemployment rate, GDP per capita, tax burden, social security contributions, business regulations, business registration procedures and self-employment rate. The study showed that the sizes of the underground economies of the Czech Republic, Hungary and Poland were 10.44%, 11.18% and 20.47% respectively. The study also revealed that the key drivers of the underground economic activities in the Czech Republic are GDP per capita, unemployment rate and self-employment rate. In Hungary, the underground economic activities are caused by GDP per capita, taxes on goods and services, corporate taxes (taxes on income and profits) and unemployment. The drivers of the shadow economy in Poland are corporate taxes, GDP per capita, start-up procedures to register a business (SPRTB), taxes on goods and services and the unemployment rate.

Dell'Anno et al. (2018) applied the MIMIC model in estimating the shadow economy of Tanzania from 2003 to 2015 and investigated the relationship between the shadow economy and its potential causes and indicators. Annual historical data was collected for the period 1994 to 2015 on the following variables: tax burden, Real Gross Domestic Product (RGDP) per capita index, currency ratio, government expenditure, inflation rate, unemployment rate, exports of goods and services as a ratio of GDP, regulatory quality, dependent people as a percentage of the working age population, a percentage of the rural population with access to improved drinking water, value added in the services sector as a percentage of GDP, agriculture, rule of law, effectiveness of the government, corruption, political stability and the cost of starting a business as a percentage of income per capita. All these variables were sourced from the World Development Indicators, Worldwide Governance Indicators and the Bank of Tanzania. It was found that the underground economy ranges from 52% to 61% of the official GDP. The main drivers of the shadow economy are inflation, unemployment and government spending.

Asllani (2018) conducted research to estimate and analyse the underground economy of ten countries in the Balkan Peninsula region by applying the MIMIC model. Annual panel data from 1996 to 2014 was collected on the following variables: tax burden, regulatory burden, government expenditure and effectiveness as proxies of tax morality, inflation rate, strength of political regime, absence of violence / terrorism, rule of law, government integrity index, corruption index, agriculture value added as percentage of GDP, urban population as a proportion of total population, unemployment rate, currency in circulation, labour force participation ratio, GDP growth rate , GDP per capita and electricity consumption. The data was collected from numerous sources such as the World Bank, Central Banks from the Balkan countries, the Heritage Foundation, IMF, European Bank for Reconstruction and Development (EBRD), European Central Bank (ECB) and Polity. The study revealed that the size of the underground economy in most of these countries is decreasing. The annual average size of the underground economy in these ten countries was 31% in 1996 and dropped to around 26% in 2014. The study also revealed that the main drivers of the underground economy in these countries are the regulation burden, corruption level, agriculture sector, urbanisation, macroeconomic developments and the size of the government.

Nchor and Adamec (2015) investigated the magnitude and direction of the underground economies in Kenya, Namibia, Ghana and Nigeria by applying the MIMIC model. The data for the period 1990 to 2012 was obtained from the World Bank and the International Financial Statistics on the following variables: tax rates, the size of the government, business regulation, GDP per capita, deposit interest rate, unemployment rate, quality of public services, labour force participation rate, currency in circulation and GDP growth per capita. The research revealed that the underground economies in Kenya, Namibia, Ghana and Nigeria were 33.7%, 29.1%, 36% and 47%, respectively. The main predictors of the underground economy in Kenya are GDP per capita, the size of government, unemployment rate and the deposit interest rate. In Namibia, unemployment rate, the size of the government and GDP per capita are the main predictors of the underground economy. Ghana’s informal economy is caused by unemployment rate, the size of the government and total taxes. The shadow economy in Nigeria is caused by unemployment rate, the size of government, quality of public services, business regulation and total taxes.

Vorobyev (2015) determined the underground economy in connection with the quality of life in Russia and Ukraine by using factor analysis and the MIMIC model. Factor analysis was conducted on the relationship of quality and standard of living in Russia for the period 2002-2013 and in the Ukraine for the period 2004-2013. The analysis revealed that demographic and criminogenic factors are the most common. Formal employment and monetary income have a weak correlation in Russia while in Ukraine they are highly correlated. The MIMIC model was developed using factor analysis results. The research showed that the underground economy in Russia vary between 48% and 62% for the period 2000 to 2013.

Schneider and Savasan (2007) conducted research to estimate the shadow economy of Turkey using DYMIMIC approach for the period 1999-2005. Annual data which comprised of the following variables: taxation, public employment as percentage of total employment, unemployment rate, GDP per capita, currency ratio and employment rate, was obtained from the World Bank International Comparison Program, OECD, World Bank National Account and World Bank Governance Indicators. The results showed that the size of the Turkish underground economy was 31.1% in 1999 and increased to 35.1% in 2005. The analysis revealed that the key drivers of the shadow economy in Turkey are taxation, unemployment and GDP per capita.

2.5.3 Empirical studies conducted using the joint approach

Gauci and Rapa (2020) analyzed the shadow economy in Malta using the data from 2010-2019. Both the CDA and MIMIC models were used for the study. Although the results of the models differ on the historical trend of the Maltese shadow economy, they do however reveal that the underground economy in Malta has been stable over the sample period, averaging around 21% of GDP between 2010 and 2019. The main predictors of the underground economy in Malta are self-employment rate, tax burden, recurrent government expenditure and the unemployment rate. Self-employment rate had the largest effect on the shadow economy.

Dybka et al. (2018) reviewed the dominant approaches in estimating the underground economy namely, the CDA and MIMIC models, to address the misspecification issues in CDA equations and the vague transformation of the latent variable obtained using the MIMIC model. A novel hybrid procedure referred to as reverse standardisation was proposed wherein the MIMIC model is supplied with the panel-structured information on the latent variable’s mean and variance, treating this information as given in the restricted full information ML function. With this approach, some controversial steps such as choosing an externally estimated reference point for benchmarking or adopting other ad hoc identifying assumptions are avoided. The underground economy was estimated for 43 countries using quarterly data from 2005 to 2015 obtained from various sources including the IMF, Eurostat, European Central Bank and the World Bank. The variables used for the CDA model were taxes, social security contributions in GDP, time to prepare and pay taxes for business, the rule of law index, unemployment rate, number of payment cards per capita, ratio of the number of point of sale terminals to the number of payment cards, RGDP per capita, inflation rate, deposit interest rate, the share of domestic credit to private sector in GDP and agriculture share in employment. The causal variables used for the MIMIC model were unemployment rate, taxes and social security contributions, rule of law, tax time and electronic payments value to GDP. Electricity share and currency were the only indicators specified. Shadow economy country estimates ranged from 2.8% to 29.9% of GDP.

Hassan and Schneider (2016a) conducted research to estimate the size and trend of the underground economy in Egypt by using the CDA and MIMIC models. Annual data covering the period 1976 to 2013 was sourced from the International Labour Organisation (ILO), the National Bank of Egypt, World Bank and Central Bank of Egypt. The MIMIC model was constructed using the following variables: agricultural sector, tax burden, unemployment rate, institutional quality of democratic institutions, self-employment rate, RGDP, currency in circulation and total employment. The following variables were used in the CDA model: currency in circulation, RGDP, tax burden, deposit interest rate, business regulation as a proxy for public employment and self-employment. The study revealed a decrease in the size of the underground economy with the MIMIC model generally recording higher values than the CDA for most of the period under study. The main predictors of the underground economy in Egypt are self-employment, tax burden, agricultural sector, regulation and unemployment rate.

Sharapenko (2009) investigated the size of the Russian shadow economy by using the MIMIC and the CDA models. Annual data from 1995 to 2008 was collected from Rosstat, Russia High School of Economics and the Central Bank of Russia. The variables used for the CDA model were cash outside banks, GDP, Consumer Price Index (CPI), tax burden and the nominal interest rate. GDP and currency were used as indicators in the MIMIC model while unemployment rate, total tax contributions, number of crimes, real government expenditure, nominal oil barrel prices, a proportion of the population with income lower than the subsistence level, a proportion of personal income constituted of social transfers, corruption perception index and the rule of law indicator were used as causal variables. The study revealed an upward trending trajectory of the shadow economy with an estimate of 22.2% in 1995 and 70% in 2008. It was also found that subsistence level and rule of law causes a decrease in the shadow economy of Russia and that the underground economy and corruption are correlated in transition countries.

Pickhardt and Pons (2006) applied the CDA and MIMIC models to improve estimations of the shadow economy in Germany. Annual data spanning from 1980 to 2001 was collected from Statistisches Jahrbuch, Bundesamt für Statistik and OECD Statistical Compendium 1/2002. GDP, price index, total tax and nominal interest rate were used in the CDA model while the MIMIC model used GDP growth rate, male labour force participation rate and cash as indicators and inflation rate, unemployment rate, the number of individuals fully employees and taxes as causal variables. The models revealed that the shadow economy in Germany increased for a period until 1999 and then fell back to its 1996 level in 2001. The MIMIC model shows slightly higher figures than the CDA, whereas the figures of the joint model fall in between, except in 1991, but are usually closer to the figures of the CDA model. According to the CDA, MIMIC and joint models, the size of the underground economy in Germany for 2001 was 15.18%, 15.84% and 15.27% respectively.

2.6 Summary

This chapter introduced the concept of the underground economy by defining the construct and by identifying the causes and indicators of the underground economy. Different approaches to estimating the underground economy were examined and finally literature was reviewed on empirical studies conducted to estimate the underground economy using the CDA, MIMIC and a joint approach.

Chapter 3: Exploratory Data Analysis

3.1 Introduction

Exploratory Data Analysis (EDA), also known as Visual Analytics or Descriptive Statistics, is a first and critical step in analysing data with the aim of determining relationships among the predictor variables, determining the direction and the significance of relationships between predictor and outcome variables, checking assumptions, detecting mistakes and preliminary selection of appropriate models (Seltman, 2018). Data analysis in this study was carried out using STATA 15.0 software.

This chapter explores the distributional properties of the variables used for the CDA and MIMIC models by utilising descriptive statistics, histograms, scatterplots, normal probability plots and correlation analysis.

3.2 Data Description

Secondary economic data was obtained from STATSSA, World Bank, South African Reserve Bank (SARB) and IMF. A detailed description of the variables is provided in Appendix A.3. Critical employment data was not available from STATSSA for periods earlier than 2000 as previous estimates were derived using the October Household Survey and these estimates have not been adjusted to the current methodology. Therefore, quarterly data on the variables of interest was only available for the period 2000 to 2020. This translates into 84 observations per variable. According to Bentler and Chou (1987), a sample size of five or ten observations per estimated parameter is the minimum norm. Similarly, Bollen (1989a) indicates that a sample size for SEM analysis should encompass at least ten cases per variable.

Table 3.1 displays the variables that were used for the CDA and MIMIC models and their units of measurement. All the variables are continuous and measured in percentages and R million.

Table 3.1: Variable description.

Variable	Symbol used	Unit	Type
Currency in circulation outside the banks	CU	R million	Continuous
Currency in circulation outside the banks per NGDP	CU_NGDP	Percentage	Continuous
Currency growth rate	CU_GR	Percentage	Continuous
Nominal Gross Domestic Product	NGDP	R million	Continuous
NGDP growth rate	NGDP_GR	Percentage	Continuous
NGDP per capita	NGDP_CAP	Percentage	Continuous
Labour Force Participation rate	LFPR	Percentage	Continuous
Total Employment	TEMPLOY	Millions	Continuous
Employment rate	EMPLOY_R	Percentage	Continuous
Employment per capita	EMPLOY_CAP	Percentage	Continuous
Self-employment rate	SELFEMPLOY_R	Percentage	Continuous
Unemployment rate	UNEMPLOY_R	Percentage	Continuous
Government Employment rate	GOVTEMPLOY_R	Percentage	Continuous
Agriculture, Forestry & Fishing NGDP	AGRIC_NGDP	R million	Continuous
Agricultural Employment rate	AGRICEMPLOY_R	Percentage	Continuous
Mining Employment rate	MINEMPLOY_R	Percentage	Continuous
Mining NGDP	MIN_NGDP	R million	Continuous
Tax Burden	TAX_BUR	Percentage	Continuous
Tax Revenue	TAX_REV	R million	Continuous
Deposit interest rate	INT_R	Percentage	Continuous
Consumer Price Index	CPI	Percentage	Continuous
Social benefits paid by the government as a proportion of total government expenditure	SOCBEN_GOVTEXP	Percentage	Continuous
Government expenditure as % of GDP	GOVTEXP_NGDP	Percentage	Continuous

3.3 Results of Exploratory Data Analysis

In this section, the data will be explored to determine the distributional properties of the variables and investigate relationships among variables by using descriptive statistics, histograms, normal probability plots, scatterplots, Shapiro-Wilk W test for normality and correlation analysis.

3.3.1 Descriptive Statistics

Table 3.2 shows the summary statistics of the variables used in the CDA and MIMIC models. The variables were chosen based on previous research conducted by various authors, availability of data and the economic landscape of South Africa.

Nine variables, that is, CU_NGDP, NGDP_CAP, LFPR, TEMPLOY, EMPLOY_R, EMPLOY_CAP and UNEMPLOY_R had means very close to the median (< 1% difference) indicating a symmetric distribution. The rest of the variables were asymmetric with the highest difference on the two measures evident in AGRIC_NGDP. In addition, the variables did not have zero skewness, although NGDP_CAP, CU_GR and TEMPLOY displayed skewness very close to zero at values of 0.03, 0.06, -0.07 respectively. Skewness measures how symmetrical the data is around the mean. In a perfect normal distribution, the tails on either side of the mean are exact mirror images of one another. Similarly, kurtosis was not equal to zero for each variable indicating absence of a perfect normal distribution. Kurtosis detects the presence of outliers through the observed tails of the curve. A perfect normal distribution has zero kurtosis.

There was less dispersion in the MINEMPLOY_R as the values were centered around a mean of 2.11 with standard deviations of 0.34 and a range of 1.35. NGDP, CU and TAX_REV had the highest variance and this may skew the mean to higher values. The overdispersion was also reflected in the high range values for the three variables. Overdispersion occurs when there is more variability in the data than would have been expected based on a given statistical model.

Table 3.2: Summary statistics of the variables.

Variables	N	Mean	Median	St. Dev.	Skewness	Kurtosis	Min	Max	Range
CU	84	259 208.51	223 958.50	**130 472.54**	0.36	1.85	82 879.00	526 069.00	**443 190.00**
CU_NGDP	84	35.53	35.38	2.00	1.55	9.53	31.68	45.54	13.86
CU_GR	84	2.25	2.15	4.35	**0.06**	2.39	-6.52	11.63	18.15
NGDP	84	724 236.71	693 527.60	**345 331.48**	0.19	1.72	219 943.10	1 352 650.80	**1 132 707.70**
NGDP_CAP	84	13 622.27	13 614.20	5 500.28	**0.03**	1.67	4 943.00	22 687.87	17 744.87
NGDP_GR	84	2.25	2.53	3.52	-0.76	14.64	-16.20	17.93	34.13
LFPR	84	57.62	57.90	1.94	-1.80	10.88	47.26	60.79	13.53
TAX_REV	84	173 852.32	159 757.00	**89 321.62**	0.34	1.95	43 360.00	363 729.00	**320 369.00**
TAX_BUR	84	23.63	23.19	2.27	0.38	2.53	18.71	28.96	10.25
TEMPLOY	84	14.13	14.22	1.52	**-0.07**	1.81	11.66	16.53	4.87
EMPLOY_R	84	74.72	74.96	2.22	-0.64	3.41	67.50	78.97	11.47
EMPLOY_CAP	84	27.45	27.61	1.18	-0.68	3.08	23.94	29.67	5.73
SELFEMPLOY_R	84	7.55	7.09	1.30	1.72	6.04	5.96	12.55	6.59
UNEMPLOY_R	84	25.28	25.04	2.28	0.65	3.41	21.03	32.50	11.47
GOVTEMPLOY_R	84	13.59	13.78	1.23	-0.30	2.19	10.70	16.09	5.39
AGRICEMPLOY_R	84	4.45	4.06	1.12	1.73	5.67	3.31	8.47	5.16

Variables	N	Mean	Median	St. Dev.	Skewness	Kurtosis	Min	Max	Range
AGRIC_NGDP	84	17 061.26	14 375.35	9 642.05	0.93	2.96	4 632.50	43 731.00	39 098.50
INT_R	84	7.67	7.15	2.14	0.75	2.71	3.92	12.79	8.87
MINEMPLOY_R	84	2.11	2.00	**0.34**	1.03	2.94	1.61	2.96	**1.35**
MIN_NGDP	84	54 628.15	57 999.55	26 821.99	0.11	1.85	14 522.80	111 519.30	96 996.50
CPI	84	73.38	70.82	24.11	0.29	1.78	38.60	117.17	78.57
SOCBEN_GOVTEXP	84	11.33	13.63	5.83	-0.51	2.02	1.89	24.58	22.69
GOVTEXP_NGDP	84	27.55	27.19	3.72	0.54	3.85	19.91	40.39	20.48

3.3.2 Distributional properties of the variables

Figure 3.1 displays the histograms of the explanatory and the outcome variables of the CDA and MIMIC models. The histogram assists in providing an understanding of the distribution of the data, mean, median, mode, dispersion, skewness and whether there is a presence of outliers.

As can be seen from Figure 3.1, some variables such as CU_GR, NGDP_GR, GOVTEXP_NGDP and GOVTEMPLOY_R display a shape close to the characteristic bell shape of a normal distribution. However, NGDP_GR show two outlier values of -16.20% and 17.93% due to lower than expected NGDP 2020Q2 value of R1 073 725 million. This low figure was due to the nationwide lockdown imposed by the South African government due to the COVID-19 pandemic. The lockdown negatively impacted every economic sector except for agriculture which grew by 15.1%. Most of the other variables namely, CU, NGDP, SELFEMPLOY_R, UNEMPLOY_R, TAX_BUR, INT_R, CPI, AGRICEMPLOY_R, MINEMPLOY_R, CU_NGDP, TAX_REV, and AGRIC_NGDP have a right skewed distribution. The impact of the nationwide lockdown is also evident in CU_NGDP which shows two outlier values of 45.54% and 41.39% due to lower than expected NGDP value in the second quarter of 2020.

The variables EMPLOY_R, LFPR and EMPLOY_CAP have a left skewed distribution. LFPR, which measures the formal employment participation, was also negatively affected by the lockdown. A comparatively lower value of 47.26% was observed in 2020Q2, identifiable as an outlier in Figure 3.1. A significant number of South Africans lost their jobs during this period as a direct consequence of the lockdown. The distribution of TEMPLOY, MIN_NGDP and NGDP_CAP appears to be multimodal while SOCBEN_GOVTEXP seems to be bimodal. The normal probability plot in Figure 3.2 and the Shapiro-Wilk W test in Table 3.3 further confirm that EMPLOY_R, UNEMPLOY_R, TAX_BUR, GOVTEXP_NGDP and GOVTEMPLOY_R are normally distributed. The rest of the variables have p-values less than 0.05. Furthermore the normal probability plots show that LFPR and CU_NGDP are normally distributed as the data points are close to the diagonal line. If the data points deviate from the line in an obvious non-linear manner, the data is not normally distributed. For the rest of the variables, the normal probability plots do not indicate a normal distribution.

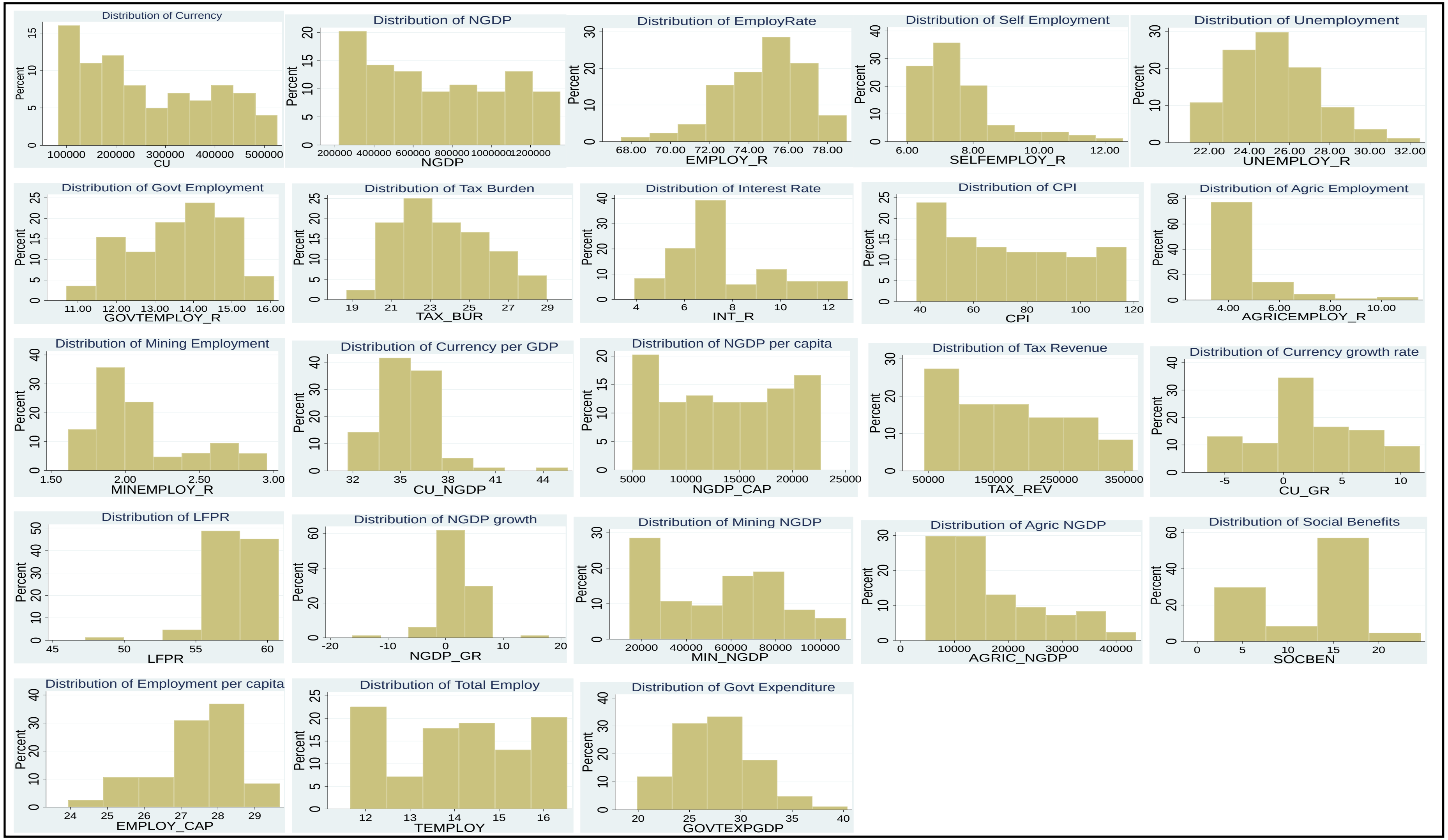


Figure 3.1: Histogram of the variables.

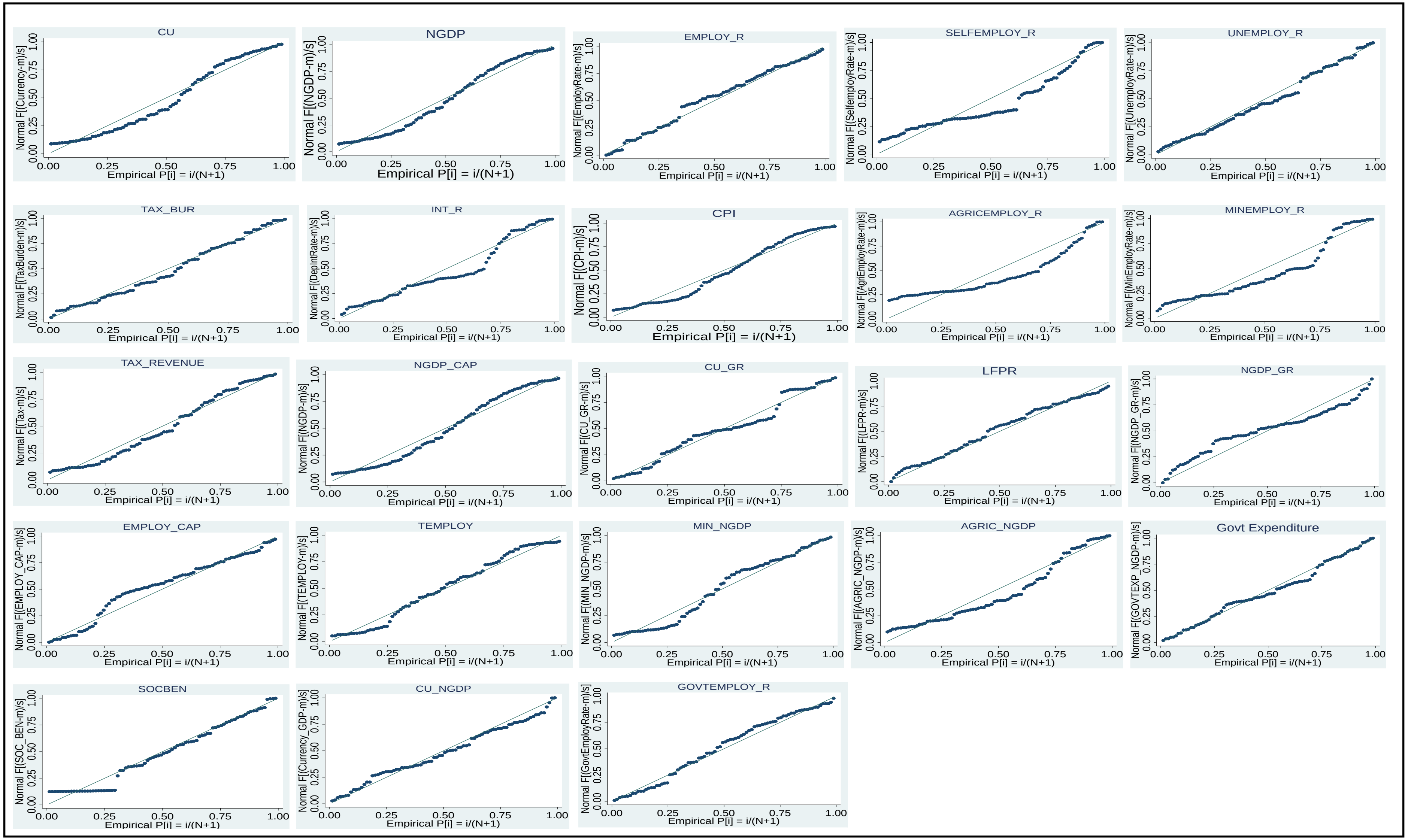


Figure 3.2: Normal Probability Plots of the variables.

Table 3.3: Shapiro-Wilk W tests for normality.

Variable	Number of Observations	W	V	z	Prob > z
CU	84	0.930	5.009	3.540	0.000
NGDP	84	0.934	4.718	3.409	0.000
EMPLOY_R	84	0.971	2.044	1.570	0.058
SELFEMPLOY_R	84	0.825	12.507	5.550	0.000
UNEMPLOY_R	84	0.971	2.086	1.615	0.053
GOVTEMPLOY_R	84	0.972	1.970	1.489	0.068
TAX_BUR	84	0.975	1.779	1.265	0.103
INT_R	84	0.925	5.379	3.697	0.000
CPI	84	0.930	4.991	3.532	0.000
AGRICEMPLOY_R	84	0.703	21.189	6.709	0.000
MINEMPLOY_R	84	0.867	9.469	4.939	0.000
CU_NGDP	84	0.892	7.701	4.485	0.000
NGDP_CAP	84	0.936	4.570	3.339	0.000
TAX_REV	84	0.943	4.062	3.079	0.001
CU_GR	84	0.967	2.348	1.875	0.030
LFPR	84	0.874	9.018	4.832	0.000
NGDP_GR	84	0.808	13.689	5.749	0.000
MIN_NGDP	84	0.939	4.332	3.221	0.001
AGRIC_NGDP	84	0.902	6.986	4.271	0.000
SOCBEN_GOVTEXP	84	0.821	12.797	5.601	0.000
EMPLOY_CAP	84	0.953	3.338	2.648	0.004
TEMPLOY	84	0.938	4.422	3.266	0.001
GOVTEXP_NGDP	84	0.977	1.678	1.137	0.128

3.3.3 Correlation between variables

Correlation analysis using Pearson's product-moment correlation was conducted to determine whether the variables were significantly correlated and to assess the strength of the relationships between those variables. A Pearson correlation coefficient for a sample is symbolised by *r* and measures the strength of the linear relationship between two variables. The correlation coefficient ranges from -1 to 1, where -1 indicates a perfectly negative linear relationship and +1 indicates a perfectly positive linear relationship. A value of 0 indicates that there is no linear relationship between the two variables (Blumberg et al., 2014).

Guilford (1956) offers the following rules in interpreting Pearson's correlation coefficient:

Table 3.4: Guilford's informal interpretations of the magnitude of r.

Value of r (+ or -)	Informal interpretation
< 0.2	Slight; almost no relationship
0.2 - 0.4	Low correlation; definite but small relationship
0.4 – 0.7	Moderate correlation; substantial relationship
0.7 – 0.9	High correlation; strong relationship
0.9 – 1.0	Very high correlation; very dependable relationship

Multicollinearity arises when explanatory variables in a regression model are correlated. If the variables are highly correlated, it can cause problems with model fitness and interpretation of the results. The coefficients become very sensitive to small changes in the model and their accuracy is reduced which weakens the statistical power of the regression model. In other words, the correct independent variables might not be chosen because of non-significant p-values.

The section that follows will show the correlation analysis for the variables chosen for the CDA and MIMIC models.

3.3.3.1 CDA model

The CDA model is a regression model where currency in circulation outside the banks is regressed against specified explanatory variables.

Appendix A.1 shows the correlation matrix for the variables used in the CDA model. All the r values greater than 0.7 are highlighted in red signifying high to very high correlation. These values are few compared to the rest of the values and indicate that there is no significant problem of multicollinearity. TAX_REV is very highly correlated with NGDP, CPI and NGDP_CAP and highly correlated with GOVTEMPLOY_R. Therefore, TAX_REV was excluded from the final model as it is also represented in the numerator of the TAX_BUR.

The variable CPI is very highly correlated with NGDP_CAP, TAX_REVENUE and NGDP, so NGDP_CAP, TAX_REVENUE and CPI were excluded from the model. A perfect negative correlation exists between EMPLOY_R and UNEMPLOY_R. As unemployment has become endemic in South Africa with a record high of

34.4% reached in the second quarter of 2021, it has become imperative to include it in the CDA model to assess its impact on the underground economy and leave out employment rate.

Thus, NGDP, GOVTEMPLOY_R, AGRICEMPLOY_R, SOCBEN_GOVTEXP, SELFEMPLOY_R, UNEMPLOY_R, TAX_BUR, INT_R and MINEMPLOY_R were considered for inclusion as explanatory variables in the final CDA model.

3.3.3.2 The MIMIC model

A MIMIC model is a structural equation model which confirms the influence of exogenous causal variables on the latent variable (underground economy) and the effect of the underground economy on observed macroeconomic indicator variables. The MIMIC model comprises of two models namely, the structural equation model and the measurement model. The structural equation model regresses the underground economy against identified causal variables and the measurement model assesses the effect of the underground economy on the specified indicator variables.

Numerous research on the underground economy have identified a combination of three indicator variables to use in the measurement model, viz.: CU (variations of this variable such as CU_NGDP, CU_GR are also found in literature), NGDP (RGDP is also used) and LFPR. However, different causal variables are used depending on the economic dynamics of the country under study.

Appendix A.2 displays the correlation matrix for the causal variables used in the MIMIC model. All the r values greater than 0.7 are highlighted in red signifying high to very high correlation. These values are few compared to the rest of the values and indicate that there is no significant problem of multicollinearity. The variable MIN_NGDP is very highly correlated with CPI, TEMPLOY and TAX_REV. It is highly correlated with SOCBEN_GOVTEXP, SELFEMPLOY_R, GOVTEMPLOY_R and GOVTEXP_NGDP. Therefore, MIN_NGDP was excluded from the final model. The variable TAX_REV has very high correlation with MIN_NGDP, CPI and TEMPLOY and has high correlation with SOCBEN_GOVTEXP and GOVTEMPLOY_R. Therefore, TAX_REV was excluded from the final model as it is also represented in the numerator of the TAX_BUR.

Although SOCBEN_GOVTEXP has a high correlation with CPI, TEMPLOY, MIN_NGDP, MINEMPLOY_R and TAX_REV, it was still considered for the MIMIC models, but not in combination with the variables it's correlated with. Similarly, GOVTEXP_NGDP was also considered in the final model despite high correlation with CPI.

Thus, the causal variables that were considered in the MIMIC models were TAX_BUR, MINEMPLOY_R, AGRIC_NGDP, AGRICEMPLOY_R, SOCBEN_GOVTEXP, UNEMPLOY_R, SELFEMPLOY_R, GOVTEMPLOY_R, CPI, EMPLOY_CAP, EMPLOY_R, TEMPLOY and GOVTEXP_NGDP.

3.3.4 Investigating linear relationships among variables

First order (linear) relationships among variables were explored through the utilisation of bivariate scatterplots to determine whether the assumption of linearity for the MIMIC model holds. Depending on the nature of the relationship discovered, appropriate transformation will be applied e.g. if the relationship between two variables is quadratic, power transformation will be applied. Appendix A.4 shows the scatterplots for the variables used in the MIMIC model. Some form of linear relationships is evident between the following variables: CU vs CPI, CU vs TEMPLOY and CPI vs TEMPLOY.

3.4 Summary

In this chapter the variables used for the CDA and MIMIC models were explored to determine their distributional properties and to check if there were any outliers. The histogram, normal probability plots and the Shapiro-Wilk W test confirmed that GOVTEMPLOY_R and GOVTEXP_NGDP are normally distributed while inconsistent results were obtained for the other variables from the three tests. Based on the correlation results, the explanatory variables that were considered for inclusion in the CDA model are NGDP, GOVTEMPLOY_R, AGRICEMPLOY_R, SOCBEN_GOVTEXP, SELFEMPLOY_R, UNEMPLOY_R, TAX_BUR, INT_R and MINEMPLOY_R and the causal variables that were considered for inclusion in the MIMIC model are TAX_BUR, MINEMPLOY_R, AGRIC_NGDP, AGRICEMPLOY_R, SOCBEN_GOVTEXP, UNEMPLOY_R, SELFEMPLOY_R, GOVTEMPLOY_R, CPI, EMPLOY_CAP, EMPLOY_R, TEMPLOY and GOVTEXP_NGDP. CU, NGDP and LFPR were considered as indicator variables.

Chapter 4: Research Methodology

4.1 Introduction

Structural Equation Modeling (SEM), which is the basis of the MIMIC model, will be presented in this chapter. The main objective of SEM analysis is to determine the extent to which the sample data supports the specified theoretical model. The MIMIC and CDA models are described extensively, including the VECM which is used to compute the currency demand in the CDA model. Finally, the benchmarking procedure used to convert the ordinal estimates computed from the MIMIC model into more interpretable estimates of the underground economy is discussed.

4.2 Structural Equation Modeling

Structural Equation Modeling is a powerful, comprehensive multivariate statistical tool which is used to analyse the structural relationship between observed and latent variables/constructs. Latent variables are not observable and cannot be measured directly but are rather deduced from observed, measured or indicator variables. Structural Equation Modeling is comprehensive as it combines inputs from factor analysis and multiple regression analysis.

Structural Equation Modeling has grown extensively in popularity and is being applied in various fields such as humanities, social science, marketing, engineering and behavioural sciences. This is mainly due to its ability to concurrently estimate multiple equations by considering the relationships between exogenous variables and measured indicator variables. This is performed by conducting Confirmatory Factor Analysis (CFA) and multiple regression analysis in one step only.

The following relationships are possible in SEM:

- observed to observed variables (γ, e.g. regression)
- latent to observed variables(λ, e.g.confirmatory factor analysis)
- latent to latent variables(γ,β, e.g. structural regression)

The objectives of SEM analysis are:

- to understand the correlation or covariance pattern among variables. Both covariance and correlation measure the linear relationship between two variables, with the latter measuring the strength of the linear relationship. The covariance matrix is the most important component of a structural equation model and serves as a dataset of a sample. The proposed model should produce a population variance-covariance matrix ($\sum$) which should be consistent with the sample variance-covariance matrix (***S***).
- it explains as much of the variance among variables as possible with the specified model.

Schumacker and Lomax (2010) emphasise that the main goal of SEM analysis is to determine the extent to which the theoretical model is supported by sample data.

SEM encompasses two models namely, the structural model and the measurement model. The measurement model is very similar to factor analysis, however the implementation of these two approaches is quite different. Factor analysis is considered exploratory in that it assesses the structure among variables and defines the factors in terms of the variables. In contrast, SEM requires a prior definition of association of variables with each construct. Consequently, loadings are estimated for the variables associated with constructs. No specification is required in factor analysis whereas SEM demands complete specification in the measurement model. The structural model, on the other hand, is based on multiple regression analysis whereby components of the measurement model are tied together and relations are made to one or more dependent variables.

Measurement error is explicitly taken into account when SEM techniques are employed. Analysis can include the latent variable, observed variables as well as measurement error terms in certain structural equation models. Interaction terms can also be included in a structural equation model so that the main effects and interaction effects can be tested (Thakkar, 2020).

4.2.1 Confirmatory Factor Analysis

The method for measuring latent variables is known as Confirmatory Factor Analysis (CFA). The method estimates latent variables on the basis of causal or correlated variation of the dataset and decreases the dimensions of data. It can also standardise the scale of multiple indicators and justify correlations present in the dataset. The latent variable is not in the data set but it is a derived common factor of other variables and could indicate a model's cause or effect (Byrne, 2013). In other words, the observable variables may appear as effects (that is, indicators) of the latent variable (Goldberger and Jöreskog, 1975).

In addition to CFA, there is Exploratory Factor Analysis (EFA), which is another form of factor analysis. Exploratory Factor Analysis is used to investigate the underlying latent variables while CFA is used when the indicators for each latent variable are well specified according to either related theories or prior knowledge.

Figure 4.1 below shows an example of a general CFA model with two latent factors (constructs) and five observed indicator variables.

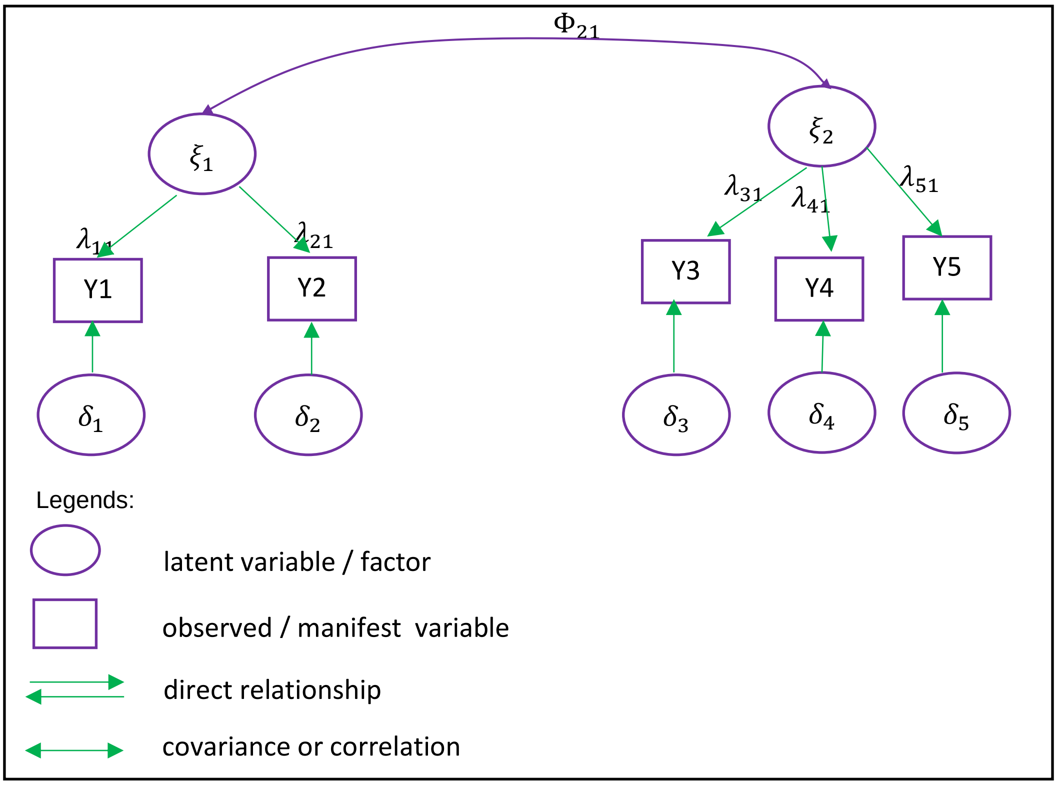


Figure 4.1: General CFA model. Source: Thakkar (2020).

Common notations used in the measurement model are the following:

- Y_1, Y_2, Y_3, Y_4, Y_5 are the observed indicator variables
- λ_y are factor loadings

- δ_y are the residual or measurement errors
- ξ are latent variables
- Φ is the correlation between the constructs (represented by latent variables)

Equation (4.1) - (4.5) shows the influences of the exogenous latent variables (ξ) on their indicators (*Y*).

$$Y_1 = \lambda_{11}\xi_1 + \delta_1 \quad (4.1)$$

$$Y_2 = \lambda_{11}\xi_1 + \delta_2 \quad (4.2)$$

$$Y_3 = \lambda_{11}\xi_2 + \delta_3 \quad (4.3)$$

$$Y_4 = \lambda_{42}\xi_2 + \delta_4 \quad (4.4)$$

$$Y_5 = \lambda_{52}\xi_2 + \delta_5 \quad (4.5)$$

Equation (4.6) relates the observed indicator variables to the unobserved latent variables through a factor analytic model and constitutes the measurement part of the model.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ 0 & \lambda_{32} \\ 0 & \lambda_{42} \\ 0 & \lambda_{52} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{bmatrix} \quad (4.6)$$

Equation (4.7) is the final measurement equation.

$$\boldsymbol{Y} = \boldsymbol{\Lambda\xi} + \boldsymbol{\delta} \quad (4.7)$$

4.2.2 Multiple Linear Regression

Multiple regression analysis is used to specify the causal relationships between exogenous and endogenous variables which can be observed or unobserved. The observable variables may appear as causes of the latent variable (Goldberger and Jöreskog, 1975). This is the structural model part of the SEM analysis linking the hypothesized constructs of the model to specify a set of interdependent relationships through the use of simultaneous multiple regression equations. The purpose of the model is to define the structural relationships among the constructs.

The multiple regression equation for a group of exogenous variables against a single endogenous outcome variable is represented by (4.8).

$$y_1 = \alpha_1 + \mathbf{x}\boldsymbol{\gamma} + \zeta_1 \qquad (4.8)$$

where y_1 is the single endogenous variable, α_1 is the intercept for y_1, $\mathbf{x}$ is a vector (1 x *p*) of exogenous variables, $\boldsymbol{\gamma}$ is a vector (*p* x 1) of regression coefficients where *p* is the total number of exogenous variables, ζ_1 is the residual of y_1 , ϕ is the variance-covariance of the exogenous variable and ψ is the residual variance or covariance of the endogenous variable.

SEM parameter estimation was initially conducted using Ordinary Least Squares (OLS) regression. Subsequently, an alternative, more efficient and unbiased estimator under the assumption of multivariate normality was applied, that is, Maximum Likelihood Estimation (MLE) (Thakkar, 2020). Maximum Likelihood Estimation measures the closeness of the population variance-covariance matrix represented by $\boldsymbol{\Sigma}$ to the sample variance-covariance matrix which is represented by ***S*** and finds parameter values that produce $\boldsymbol{\Sigma}$ that is as close as possible to ***S***. The main aim is to minimise the distance between $\boldsymbol{\Sigma}$ and ***S*** in order to reach perfect model fitness (Schumacker and Lomax, 2010). Maximum Likelihood Estimation is frequently used as it provides the estimates of models at the same time.

4.2.3 Parameter estimation methods

There are four main parameter estimation methods in SEM namely, OLS, Generalised Least Squares (GLS), ML method and Asymptotic Distribution Free (ADF) method. The GLS and the ML method are based on the assumption of multivariate normality of the variables to be analysed. Schumacker and Lomax (2010) states that if the observed variables are normally distributed, the ML estimates are unbiased, consistent, efficient, scale invariant, scale free and normally distributed. The OLS method does not conform to any distribution and the ADF method (a minimum χ^2 method) provides accurate statistics regardless of the distribution of variables. The ML method is preferred for SEM as it is not scale dependent and offers consistent and asymptomatically efficient results in large samples (Hayashi et al., 2008). Hence parameter estimation and model evaluation by ML will be discussed in detail henceforth.

Consider a random sample $X_1, X_2, X_3, \ldots, X_n$ from a distribution with a parameter θ. Given the observed variables $X_1 = x_1, X_2 = x_2, X_3 = x_3, \ldots, X_n = x_n$, a ML estimate of θ, shown by $\hat{\theta}_{ML}$ is a value of θ that maximises the likelihood function:

$$L(x_1, x_2, \ldots, x_n; \theta) \tag{4.9}$$

A MLE of the parameter θ, is $\hat{\theta}_{ML}$ which in itself is a random variable $\hat{\theta}_{ML}(X_1, X_2, \ldots, X_n)$ estimated from a random sample $\{X_1 = x_1, X_2 = x_2, X_3 = x_3, \ldots, X_n = x_n\}$.

The MLE $\hat{\theta}_{ML}$ can be obtained by minimising the fit function or discrepancy function. All discrepancy functions are special cases of a general family of functions meant for the examination of covariance structures (Browne, 1974). The discrepancy function for ML method (F_{ML}) is shown by (4.10) below.

$$F_{ML} = log|\boldsymbol{S}| - log|\boldsymbol{\Sigma}| + tr(\boldsymbol{S}\boldsymbol{\Sigma}^{-1}) - k \tag{4.10}$$

where *log* is the natural logarithm function, k is the number of variables in the correlation (or covariance) matrix, *tr* is the trace, $\boldsymbol{S}$ is the observed covariance matrix, $\boldsymbol{\Sigma}$ is the model implied covariance matrix and $\boldsymbol{\Sigma}^{-1}$ is the inverse of the matrix $\boldsymbol{\Sigma}$ (Gana and Broc, 2019 and Jöreskog and Sörbom, 1982).

4.2.3.1 Maximum likelihood estimation with normal data

Under the null hypothesis of correct model structure:

a) The test statistic

$$T_{ML} = (n-1)F_{ML} \tag{4.11}$$

converges to a χ^2 distribution with $df = \frac{p(p+1)}{2} + p - q_1$ for mean and covariance structure analysis (MCSA) and with $df = p(p+1)/2 - q_2$ for covariance structure analysis (CSA), where n is the sample size, p is the number of observed variables and q_1 and q_2 are the number of parameters to be estimated.

b) *Asymptotic normality*: When the data is from a multivariate normal distribution with true population mean vector ($\boldsymbol{\mu}$) and true population covariance matrix ($\boldsymbol{\Sigma}$), the estimators are consistent and asymptotic normal, that is,

$$\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \longrightarrow N(\mathbf{0}, \boldsymbol{\Omega}_{ML}) \tag{4.12}$$

where the covariance matrix is $\boldsymbol{\Omega}_{ML} = (\dot{\boldsymbol{\beta}}'\boldsymbol{W}^*\dot{\boldsymbol{\beta}})^{-1}$ with the weight matrix $\boldsymbol{W}^* = \begin{pmatrix} \boldsymbol{\Sigma}^{-1} & 0 \\ 0 & \boldsymbol{W} \end{pmatrix}$ with $\boldsymbol{W} = (1/2)\boldsymbol{D}'_p(\boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{\Sigma}^{-1})\boldsymbol{D}_p$ where the duplication matrix $\boldsymbol{D_p}$ is defined such that $vec(\boldsymbol{\Sigma}) = \boldsymbol{D}_p vech(\boldsymbol{\Sigma})$, $\dot{\boldsymbol{\beta}}$ is the first derivative of the coefficient matrix $\boldsymbol{\beta}$ and $\otimes$ is the Kronecker product (Hayashi et al., 2008).

4.2.3.2 Maximum likelihood estimation with non-normal data

If the data is not from a multivariate normal distribution, Hayashi et al. (2008) has shown that:

a) the estimates of parameters are consistent as long as $\boldsymbol{\beta}(\boldsymbol{\theta})$ is accurately specified and identified.
b) asymptotic normality still holds with a modified covariance matrix of the estimator given as

$$\sqrt{n}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \longrightarrow N(\mathbf{0}, \boldsymbol{\Omega}_{SW}) \tag{4.13}$$

where $\boldsymbol{\Omega}_{SW}$ represents the sandwich-type covariance matrix given by

$$\boldsymbol{\Omega}_{SW} = (\dot{\boldsymbol{\beta}}'\boldsymbol{W}^*\dot{\boldsymbol{\beta}})^{-1}(\dot{\boldsymbol{\beta}}'\boldsymbol{W}^*\boldsymbol{V}\boldsymbol{W}^*\dot{\boldsymbol{\beta}})(\dot{\boldsymbol{\beta}}'\boldsymbol{W}^*\dot{\boldsymbol{\beta}})^{-1} \tag{4.14}$$

$\boldsymbol{V} = \begin{pmatrix} \boldsymbol{V}_1 & \boldsymbol{V}_{12} \\ \boldsymbol{V}_{21} & \boldsymbol{V}_2 \end{pmatrix}$, $\boldsymbol{V}_1 = \boldsymbol{\Sigma}$, the elements of $\boldsymbol{V}_{12}$ are

$$E\{(X_i - \mu_i)(X_j - \mu_j)(X_k - \mu_k)\} \tag{4.15}$$

$\boldsymbol{V}_{21} = \boldsymbol{V}'_{12}$, the elements of $\boldsymbol{V}_2$ are

$$E\{(X_i - \mu_i)(X_j - \mu_j)(X_k - \mu_k)(X_l - \mu_l\} - \sigma_{ij}\sigma_{kl} \qquad (4.16)$$

and **W*** is as defined above (Bentler, 1983). When the data is from a normal distribution, $\boldsymbol{W}^* = \boldsymbol{V}^{-1}$ in (4.14) and $\boldsymbol{\Omega}_{SW}$ is reduced to $\boldsymbol{\Omega}_{ML}$ in (4.12).

The sandwich-type covariance matrix, also known as the robust covariance matrix estimator, provides consistent estimates of the covariance matrix even when a statistical model does not hold, or is not known (Carroll et al., 1998).

c) *Satorra–Bentler rescaled statistic*:

i. When models are accurately specified in CSA, T_{ML} can be estimated by a weighted sum of independent χ^2 distributions each with 1 degree of freedom, that is,

$$T_{ML} \longrightarrow \sum_{i=1}^{df} \kappa_i \chi^2_{i(1)} \;\; as\; n \longrightarrow \infty \qquad (4.17)$$

where $\kappa_i{}'s$ are the nonzero eigenvalues of $\boldsymbol{UV}_2$, with

$$\boldsymbol{U} = \boldsymbol{W} - \boldsymbol{W}\dot{\boldsymbol{\sigma}}(\dot{\boldsymbol{\sigma}}'\boldsymbol{W}\dot{\boldsymbol{\sigma}})^{-1}\dot{\boldsymbol{\sigma}}'\boldsymbol{W} \qquad (4.18)$$

When the data is normal, the weights $\kappa_i{}'s$ are all 1 and T_{ML} approaches a χ^2 distribution with $df = \frac{p(p+1)}{2} - q_2$. Satorra and Bentler (2001) observed the relation $tr(UV_2) = \sum_{i=1}^{df} \kappa_i$ and proposed

$$T_{RML} = T_{ML}/\hat{\kappa} \qquad \text{with} \quad \hat{\kappa} = tr(\widehat{\boldsymbol{U}}\widehat{\boldsymbol{V}}_2)/df \qquad (4.19)$$

which is known as the Satorra-Bentler rescaled statistic. The Satorra-Bentler rescaled statistic only rectifies the scaling so that the expected ML test statistic corresponds to the model degrees of freedom, that is, $E(T_{ML}) = \;\; df$ (Bentler and Yuan, 1999).

ii. When models are accurately specified in MCSA, T_{ML} can be approximated by the weighted sum of independent χ^2 distributions with 1 degree of freedom, with the weights being the nonzero eigenvalues of $\boldsymbol{U}^*\boldsymbol{V}$ where

$$\boldsymbol{U}^* = \boldsymbol{W}^* - \boldsymbol{W}^*\dot{\boldsymbol{\beta}}(\dot{\boldsymbol{\beta}}'\boldsymbol{W}^*\dot{\boldsymbol{\beta}})^{-1}\dot{\boldsymbol{\beta}}'\boldsymbol{W}^* \quad (4.20)$$

Therefore, the Satorra–Bentler rescaled statistic for MCSA is a simple extension of CSA, that is, $T_{ML} = T_{ML}/\hat{\kappa}^*$ with $\hat{\kappa}^* = \frac{tr(\widehat{\boldsymbol{U}}^*\widehat{\boldsymbol{V}})}{df}\, with\, df = \frac{p(p+1)}{2} + p - q_1$.

d) *Corrected ADF and F-statistics*: For CSA, Browne (1984) proposed a residual-based ADF statistic which asymptotically follows a χ^2 distribution regardless of the distribution of the data. However, this statistic requires a huge sample size to follow a χ^2 distribution. For MCSA, a corrected ADF statistic and an F-statistic were developed by Yuan and Bentler (1997) and Yuan and Bentler (1999), respectively.

e) *Finite mixtures*: Finite mixtures may be suitable when the data is not normal. Finite mixture SEMs have become very popular but can be challenging to use as they can falsely discover typologies when none exist (Hayashi et al., 2008).

4.2.4 Model diagnostics

There are various goodness of fit criteria which can be used when dealing with SEM models, depending on the nature of the model and properties of the criteria. The few criteria that are used in the MIMIC model will be covered below. These include the Root Mean Square Error of Approximation (RMSEA), the Standardised Root Mean Square Residual (SRMR), the Root Mean Square Residual (RMR), the Goodness of Fit Index (GFI), the Adjusted Goodness of Fit Index (AGFI), the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Comparative Fit Index (CFI) and the Tucker Lewis Index (TLI). Table 4.2 summarises the goodness of fit indices used in MIMIC models.

Table 4.2: Goodness of fit indices used in MIMIC models. Source: Gana and Broc (2019).

Fit Type	Index	Interpretation for guidance
Absolute	χ^2	Low χ^2 relative to degrees of freedom with an insignificant p value ($p > 0.05$)
	RMR	Good models have a small RMR. The average squared differences between the residuals of the sample covariances and the residuals of the estimated covariances. Unstandardised.
	SRMR	≤ 0.08 = good fit. Standardised version of the RMR.
	GFI	> 0.90. Higher values indicates better model fit. This statistic should be used with caution.
	AGFI	> 0.90. Adjusts the GFI based on the number of parameters in the model.
Parsimonious	RMSEA	≤ 0.05 = very good fit
		≤ 0.06 and ≤ 0.08 = good fit
	AIC	Comparative index: the lower the value of this index, the better the fit.
	BIC	Comparative index: the lower the value of this index, the better the fit.
Incremental	CFI	≥ 0.90 and ≤ 0.94 = good fit
		≥ 0.95 = very good fit
	TLI	≥ 0.90 and ≤ 0.94 = good fit
		≥ 0.95 = very good fit

4.2.4.1 Standardised Root Mean Square Residual (SRMR)

The SRMR is an absolute fit index based on the comparison between the sample covariance matrix **S** and the model-implied covariance matrix $\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})$ (Gana and Broc, 2019). SRMR is the square root of the sum of squares of the residuals in a correlation matrix. It is represented by:

$$SRMR = \sqrt{\frac{2}{p(p+1)} \sum_{i \le j} \frac{\{(S_{ij} - \sigma_{ij}(\widehat{\boldsymbol{\theta}})\}^2}{S_{ii}S_{jj}}} \quad (4.21)$$

where $\sigma_{ij}(\widehat{\boldsymbol{\theta}})$ is the (*i*, *j*) element of $\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})$, S_{ij} is the (*i*, *j*) element of the sample covariance matrix (**S**) and when *i* = *j*, S_{ii} and S_{jj} are variances and p is the number of observed variables. The value ranges from 0 to 1. When the value of SRMR is small and close to zero, the fit is good (Hayashi et al., 2008).

4.2.4.2 Root Mean Square Error of Approximation (RMSEA)

The RMSEA with confidence interval of 90% is the most recommended index among the parsimonious fit indices. Steiger (1990) defines the RMSEA as a measure of approximate fit in the population and is therefore concerned with the discrepancy due to approximation. It is given as

$$RMSEA = \sqrt{\frac{\chi^2 - df}{df(N-1)}} \quad (4.22)$$

where χ^2 is the χ^2 value of the specified and estimated theoretical model, *df* is the degrees of freedom of the specified and estimated theoretical model and *N* is the sample size (Gana and Broc, 2019).

4.2.4.3 Akaike Information Criterion (AIC)

This parsimonious fit index assists in discerning the best fitting alternative non-nested model and is given as

$$AIC = -2logL + 2p \quad (4.23)$$

where $logL$ is the maximised value of the log likelihood function for the estimated model and *p* is the number of parameters to be estimated in the model.

4.2.4.4 Bayesian Information Criterion (BIC)

The BIC also assists in discerning the best fitting alternative non-nested model and is given by

$$BIC = -2logL + 2p * \log(N) \quad (4.24)$$

where $logL$ is the maximised value of the log likelihood function for the estimated model, p is the number of parameters to be estimated in the model and $\log(N)$ is the logarithm of the sample size.

AIC and BIC values should be the lowest, or even negative for the best model. The BIC penalises the lack of parsimony more severely than the AIC (Gana and Broc, 2019).

4.2.4.5 Comparative Fit Index (CFI)

The CFI is one of the two incremental fit indices which will be discussed in this section.

$$CFI = \frac{1-\max[(\chi_t^2 - df_t), 0]}{\max[(\chi_t^2 - df_t), (\chi_n^2 - df_n), 0]} \quad (4.25)$$

where χ_t^2 is the χ^2 value of the specified and estimated theoretical model, df_t is the degrees of freedom of the specified and estimated theoretical model, χ_n^2 is the χ^2 value of the baseline model and df_n is the degrees of freedom of the baseline model (Gana and Broc, 2019).

4.2.4.6 Tucker Lewis Index (TLI)

This is the other incremental fit index represented by:

$$TLI = \frac{\left(\frac{\chi_n^2}{df_n}\right) - \left(\frac{\chi_t^2}{df_t}\right)}{\left(\frac{\chi_n^2}{df_n}\right) - 1} \quad (4.26)$$

where χ_t^2 , χ_n^2 , df_n and df_t are as specified in the CFI above.

The TLI can be used to compare different alternative models or to compare a proposed model against a null model. The advantage of TLI is that it penalises the model's lack of parsimony. The less a model is parsimonious, the more likely it is to fit the data.

4.2.5 Mathematical specification of SEM

A SEM is a hypothesis of a specific pattern of relations among a set of observed and latent variables. To demonstrate the fundamental components of a SEM, Figure 4.2 is used. The three equations (4.27-4.29) shown below are central to SEM. Equation (4.27) represents the directional influences of the exogenous latent variables (ξ) on their indicators (*x*). Equation (4.28) represents the directional influences of the endogenous latent variables (η) on their indicators (*y*). Equations (4.27) and (4.28) link the observed indicator variables to unobserved latent variables through a factor analytic model and constitute the measurement portion of the model. Equation (4.29) represents the endogenous latent variables (η) as linear functions of other exogenous latent variables (ξ) and endogenous latent variables plus residual terms (ζ). Therefore, equation (4.29) specifies relationships between latent variables through a structural equation model and constitutes the structural portion of the model.

Figure 4.2 is a schematic representation of the general SEM model.

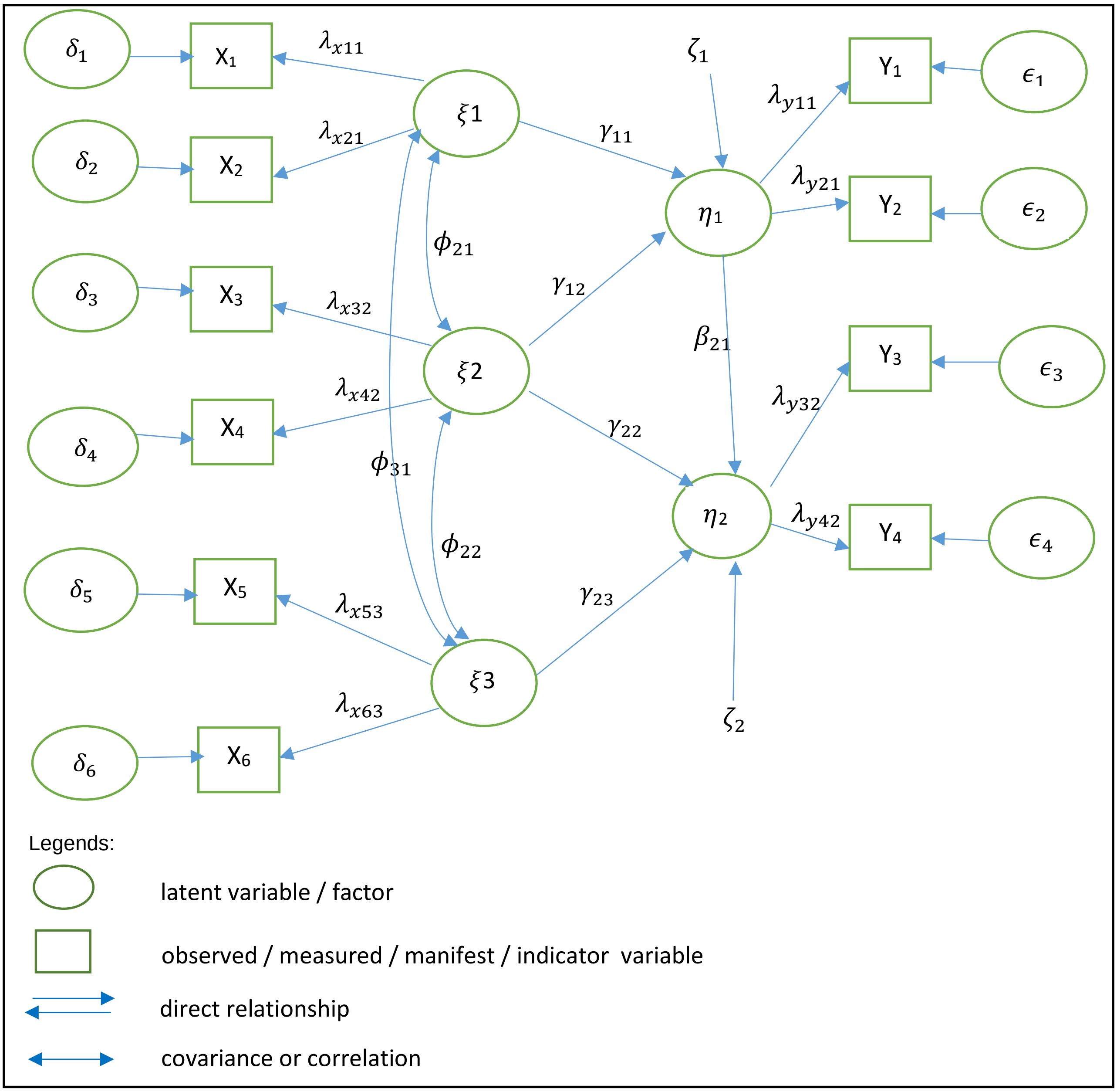


Figure 4.2: General SEM model. Source: Thakkar (2020).

where

$$\boldsymbol{\eta} = \boldsymbol{\eta}' = [\eta_1 \quad \eta_2] = endogenous\ variable(dependent\ variable)$$

$$\boldsymbol{\zeta} = \boldsymbol{\zeta}' = [\zeta_1 \quad \zeta_2] = error\ term\ related\ to\ \eta$$

$$\boldsymbol{\xi} = \boldsymbol{\xi}' = [\xi_1 \quad \xi_2 \quad \xi_3] = exogenous\ variable(independent\ variable)$$

$$\boldsymbol{Y} = \boldsymbol{Y}' = [Y_1 \quad Y_2 \quad Y_3 \quad Y_4] = dependent\ indicator\ variable$$

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}' = [\epsilon_1 \quad \epsilon_2 \quad \epsilon_3 \quad \epsilon_4] = error\ term\ related\ to\ Y$$

$$\boldsymbol{X} = \boldsymbol{X}' = [X_1 \quad X_2 \quad X_3 \quad X_4] = independent\ indicator\ variable$$

$$\boldsymbol{\delta} = \boldsymbol{\delta}' = [\delta_1 \quad \delta_2 \quad \delta_3 \quad \delta_4] = error\ term\ related\ to\ X$$

It can be written in matrix format as follows:

$$\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} = \begin{bmatrix}0 & 0\\ \beta_{21} & 0\end{bmatrix}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} + \begin{bmatrix}\gamma_{11} & \gamma_{12} & 0\\ 0 & \gamma_{22} & \gamma_{23}\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} + \begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix}$$

Thus the structural equations are as follows:

$$\eta_1 = \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \zeta_1$$

$$\eta_2 = \gamma_{22}\xi_2 + \gamma_{23}\xi_3 + \zeta_2$$

$$\boldsymbol{\eta} = \boldsymbol{\Gamma\xi} + \boldsymbol{B\eta} + \boldsymbol{\zeta}$$

and the measurement equations are given as

$$\boldsymbol{X} = \boldsymbol{\Lambda_x\xi} + \boldsymbol{\delta}$$

$$\boldsymbol{y} = \boldsymbol{\Lambda_y\eta} + \boldsymbol{\varepsilon}$$

Hence, to conclude

$$\boldsymbol{x} = \boldsymbol{\Lambda_x\xi} + \boldsymbol{\delta} \qquad (4.27)$$

$$\boldsymbol{y} = \boldsymbol{\Lambda_y\eta} + \boldsymbol{\varepsilon} \qquad (4.28)$$

$$\boldsymbol{\eta} = \boldsymbol{\Gamma\xi} + \boldsymbol{B\eta} + \boldsymbol{\zeta} \qquad (4.29)$$

where δ are the error terms related to $\boldsymbol{x}$, $\boldsymbol{\Lambda_x}$ is the effect of exogenous latent variables on their observed variables matrix, $\boldsymbol{y}$ measures the endogenous indicator variables, $\boldsymbol{\Lambda_y}$ measures the effect of endogenous latent variables on their observed variables matrix, $\boldsymbol{\varepsilon}$ is the error of measurement in endogenous indicator variables, $\boldsymbol{\xi}$ are the latent exogenous constructs, $\boldsymbol{\eta}$ are the latent endogenous constructs, $\boldsymbol{\Gamma}$ is the effect of exogenous constructs on endogenous constructs (matrix), ***B*** is the effect of endogenous constructs on each of the other endogenous constructs (matrix) and $\boldsymbol{\zeta}$ is the errors in equations or residuals.

The covariance matrices of SEM are as follows:

1. $\mathbf{\Phi} = \mathrm{E}(\xi\xi')$ is a covariance matrix for the exogenous latent variables.
2. $\boldsymbol{\theta}_{\boldsymbol{\delta}} = \mathrm{E}(\delta\delta')$ is a covariance matrix for the measurement errors in the exogenous observed variables.
3. $\boldsymbol{\theta}_{\boldsymbol{\varepsilon}} = \mathrm{E}(\varepsilon\varepsilon')$ is a covariance matrix for the measurement errors in the endogenous observed variables.
4. $\boldsymbol{\psi} = \mathrm{E}(\zeta\zeta')$ is a covariance matrix for the errors in equation for the endogenous latent variables.

The population covariance matrix for the observed variables is a function of eight parameter matrices namely, $\mathbf{\Lambda}_{\boldsymbol{x}}$, $\mathbf{\Lambda}_{\boldsymbol{y}}$, $\mathbf{\Gamma}$, $\boldsymbol{B}$,$\mathbf{\Phi}$, $\boldsymbol{\theta}_{\boldsymbol{\delta}}$, $\boldsymbol{\theta}_{\boldsymbol{\varepsilon}}$ and $\boldsymbol{\psi}$. Once the model has been hypothesized, including fixed and free parameters of the eight parameter matrices and a sample covariance matrix for the observed variables, it is possible to determine estimates of the free parameters of the model.

The model is fitted to the data by obtaining ML estimates of parameters and the likelihood ratio χ^2 test of the null hypothesis that the model holds in the population.

4.2.6 Procedural steps in SEM

Structural Equation Modelling involves five key steps and these will be listed in Table 4.3 below:

Table 4.3: SEM Procedural Steps.

Step	Description
Model Specification	• Develop a theoretical model defining variables and their relationships using existing theory and research. • Theoretical model should be confirmed using variance-covariance data. Every relationship and parameter in the model should be determined. • A misspecified model might result in biased parameter estimates which are systematically different from the true model i.e. specification error. • The measurement model should be specified with all the latent constructs. • The structural model is specified when the latent construct in the measurement model is adequately defined by the observed variables.
Model Identification	• Determine a unique solution for the model. A unique estimate for each parameter must be established. • A parameter can either be free, fixed or constrained. A free parameter must be estimated and is unknown. A fixed parameter is takes only specific values, that is, 0 or 1. A constrained parameter is not known but is limited to equal one or more other parameters. • Once the model and the parameters are specified, the parameters are combined to form a model implied variance-covariance matrix, $\sum$.
Model Estimation	• Estimates the theoretical model parameters so that the theoretical parameter values give a covariance matrix close to observed covariance matrix ***S***. • A fitting function is used to minimise the difference between the observed covariance matrix ***S*** and the estimated theoretical covariance matrix $\sum$. Hence it improves the primary estimates of the parameters with iterative calculation cycle. • The final estimates give the best fit parameter to the observed covariance matrix ***S***.
Model Testing	• Determines how well the data fits the model i.e. is the theoretical model supported by the sample data? This involves fitting the entire model and fitting the individual parameters. • There are several model evaluation indices based on a comparison of the model implied variance-covariance matrix $\sum$ to the sample covariance matrix *S* • If $\sum$ and ***S*** are similar, then the data fit the theoretical model. • All free parameters should be in the expected direction, be statistically different from zero and be meaningfully interpreted.
Model Modification	• If the fit is not as strong as intended, the model must be modified and the new modified model must be subsequently evaluated. • A number of procedures can be used to detect specification errors to ensure that properly specified alternative models may be evaluated during the respecification process. • This process involves eliminating non-significant parameters from the model and examining the model's standardised residual matrix.

4.3 The MIMIC Model

The MIMIC model is a special type of SEM model whereby latent variables are predicted by observed variables. The MIMIC model for the underground economy based on the potential indicator and causal variables is illustrated by Figure 4.3 below. This representation is based on the initial variables identified in Chapter 3.

The latent variable, the underground economy, is defined by CU, NGDP and LFPR with separate measurement error terms, ε_i , for each. This is the measurement part of the MIMIC model that defines the latent variable with respect to indicator variables.

The underground economy is predicted by the observed variables, TAX_BUR, MINEMPLOY_R, AGRIC_NGDP, GOVTEXP_NGDP, UNEMPLOY_R, SELFEMPLOY_R, GOVTEMPLOY_R, CPI and EMPLOY_CAP. This is the structural part of the MIMIC model that uses observed causal variables to predict a latent variable. The prediction error for the latent variable, ζ_1, is also shown in the diagram.

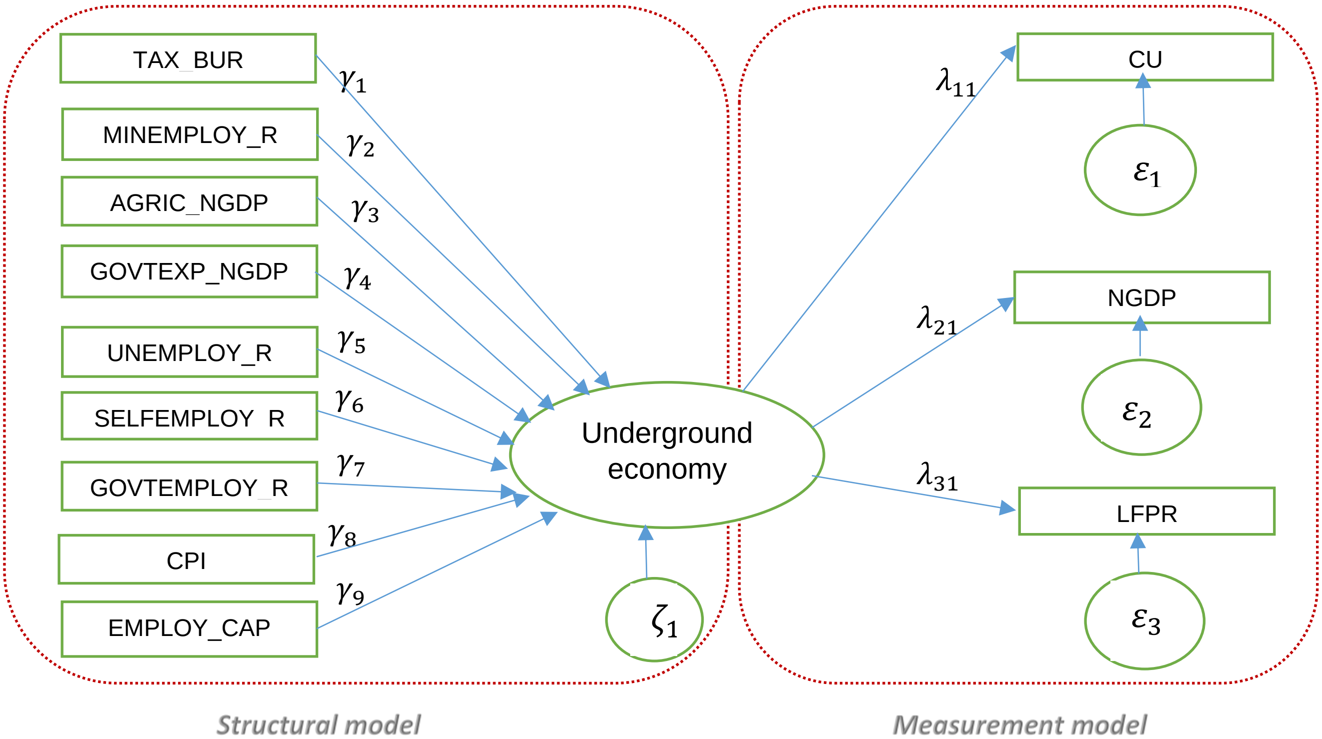


Figure 4.3: A MIMIC model of the underground economy.

The MIMIC model only produces an index of the trend of the size of the underground economy, therefore an extra step is necessary to calibrate this index in order to calculate the size of the underground economy as percentage of GDP. This is what is referred to as the benchmarking procedure which requires an exogenous estimate of the size of the underground economy at a certain point, preferably by another estimation method. In this case, the CDA model was utilised.

4.3.1 Measurement Model

Equation (4.30) links the latent variable to the vector $\boldsymbol{y}' = [y_1, \dots, y_p]$ encompassing the endogenous consequences (indicators) of the underground economy:

$$\boldsymbol{y} = \boldsymbol{\lambda\eta} + \boldsymbol{\varepsilon} \tag{4.30}$$

where $\boldsymbol{\lambda}$ is a (px1) vector of unknown coefficients, $\boldsymbol{\varepsilon}$ is a (px1) random error with a normal distribution and mutually independent. The latent variable, $\boldsymbol{\eta}$, determines linearly a set of observable endogenous indicators $y_1, y_2, \dots, y_p$ subject to disturbances $\boldsymbol{\varepsilon}$. All variables are taken to have expectation zero. If the estimated relationship is significant at the 5% significance level, it confirms the reliability of the variables used as indicators of the underground economy.

It is always recommended to first test the fit and construct validity of the proposed measurement model and then as a second step, the structural model should be tested.

4.3.2 Structural Model

This model defines the latent variable as a function of the vector $\boldsymbol{x}' = [x_1, \dots, x_q]$ encompassing the exogenous causes of the underground economy:

$$\boldsymbol{\eta} = \boldsymbol{\gamma}'\boldsymbol{x} + \zeta \tag{4.31}$$

where $\boldsymbol{\gamma}$ represents a (qx1) vector of unknown coefficients, $\boldsymbol{\eta}$ is the latent variable (underground economy) and ζ is a scalar random error with variance ψ and mutually independent. The latent variable $\boldsymbol{\eta}$ is linearly determined by a set of observable exogenous causes x_1, $x_2, \dots, x_q$ subject to disturbances ζ. All variables are taken to have expectation zero. Statistically significant coefficients imply that the variables are the predictors of the underground economy.

4.4 Currency Demand Approach

A CDA is based on the assumption that most of the transactions in the underground economy are undertaken using cash. The amount of cash used for informal transactions can predict the size of the underground economy.

Cagan (1958) derived a currency demand function as

$$C_0 = A(1+\theta)^{\alpha} Y_0^{\beta} \exp(-\gamma i) \tag{4.32}$$

where C_0 is observed cash, θ represents tax revenue to GDP, that is, total tax burden, Y_0 is the official GDP which approximates the level of transactions in the economy, i is the deposit interest rate representing the opportunity cost of holding cash and A, α, β, γ are the positive parameters. The parameter θ is the main variable behind all currency models because an increase in θ is expected to have a positive impact on currency demand. Therefore individuals hold onto their cash in order to use it in the informal economy. It is a variable that encourages individuals to participate in the underground economy. This variable has previously been calculated using government consumption normalised by GDP, tax rates or tax burden.

The regression equation proposed by Tanzi (1983) to model the currency demand is

$$ln\left(\frac{C}{M_2}\right)_t = \beta_0 + \beta_1 \ln(1+TW)_t + \beta_2 \ln\left(\frac{WS}{Y}\right)_t + \beta_3 \ln R_t + \beta_4 \ln\left(\frac{Y}{N}\right)_t + \varepsilon_t \tag{4.33}$$

where C is the currency in circulation outside the banks, M_2 is cash and deposit accounts, TW is a weighted average tax rate, WS/Y is a percentage of wages and salaries to GDP, R is the interest paid on savings deposits and Y/N is per capita income. It is expected that $\beta_1 > 0$, $\beta_2 > 0$ while $\beta_3 < 0$, $\beta_4 < 0$. This model assumes that the tax burden is one of the main causes of the underground economy and can be observed through the excessive demand and use of cash.

In order to capture the long run relationships of the possible predictor variables on currency demand, the CDA model can be represented by (4.34). The model is based on the initial variables identified in Chapter 3. All the variables are in natural logarithms.

$$C_t = \beta_0 + \beta_1 Y_t + \beta_2 TAX_t + \beta_3 REG_t + \beta_4 SELF_t + \beta_5 INT_t + \beta_6 UNEMP_t$$
$$+\beta_7 AGRICEMP_t + \beta_8 MINEMP_t + \varepsilon_t \quad (4.34)$$

where C_t represents the currency in circulation outside the banks, Y_t represents NGDP, TAX_t represents total tax revenues normalised by NGDP, REG_t represents business regulation (government employment rate is used as a proxy), $SELF_t$ represents self-employment rate, INT_t represents the deposit interest rate, $UNEMP_t$ represents unemployment rate, $AGRICEMP_t$ represents agricultural employment rate, $MINEMP_t$ represents mining employment rate and ε_t represents the error term. It is expected that tax burden, government employment rate, self-employment rate, unemployment rate, agricultural employment rate and mining employment rate (β_2, β_3, β_4, β_6, β_7, $\beta_8 > 0$) will have a positive effect on currency while NGDP and interest rates will have a negative effect on currency (β_1,$\beta_5 < 0$). The increase in interest rates is expected to encourage individuals to avoid holding excessive amounts of cash. In the case of NGDP, Schneider (2005) contends that as informal economy increases, resources are taken from the formal economy and the relationship will thus be negative for developing countries but positive for the developed and transition countries.

The variable $\hat{C}$ can be estimated from equation (4.34). Setting the tax variable to a minimum and leaving all the other variables, yields $\tilde{C}$. This is done to analyse currency when the tax burden is not a factor. Extra Currency (EC) in the economy is shown by (4.35) below. This is the illegal money used to transact in the underground economy.

$$EC = \hat{C} - \tilde{C} \quad (4.35)$$

As suggested by Tanzi (1983), equal velocity is assumed for the formal and underground economies and is shown by

$$V = \frac{Y}{C} \quad (4.36)$$

The velocities of both the economies are equal only when the coefficient of Y is equal to one (β_1=1). The size of the underground economy is estimated by multiplying EC with the velocity of money in circulation.

$$Y_{informal} = EC * V \tag{4.37}$$

Inferences about the size of the underground economy as a percentage of GDP can be made using equation (4.38). If $\beta_1 \neq 1$, a correction method needs to be applied to the results to obtain correct estimates of the South African underground economy.

The following correction method by Ahumada et al. (2007) was applied:

$$\frac{Y_{informal}}{Y_{formal}} = \left[\frac{C_{informal}}{C_{formal}}\right]^{\frac{1}{\beta}} = \left[\frac{\hat{Y}_{informal}}{\hat{Y}_{formal}}\right]^{\frac{1}{\beta}} \tag{4.38}$$

where Y represents GDP, C represents currency and β represents the income elasticity.

4.4.1 The Vector Error Correction Model (VECM)

Various authors such as Faal (2003) and Hassan and Schneider (2016) have used the VEC currency demand model to estimate the size of the underground economy. A VECM is a system with a vector of two or more variables. All the variables in a VECM are endogenous and there are no exogenous variables. The model is only constructed if the variables are cointegrated, that is, if there is a long run relationship between the variables. It confines the long-run behaviour of endogenous variables to converge to their cointegrating relationships.

A VECM is a special case of the VAR model for variables that are stationary and recognises any cointegrating relationships amongst variables. A VAR model with 3 variables is represented by (4.39)-(4.41), where the dependent variable is a function of its own lag and the lag values of other variables in the model.

$$lnCU_t = \alpha + \sum_{i=1}^{k} \beta_i lnCU_{t-i} + \sum_{j=1}^{k} \phi_j lnNGDP_{t-j} + \sum_{m=1}^{k} \varphi_m lnINTR_{t-m} + \varepsilon_{1t} \tag{4.39}$$

$$lnNGDP_t = c + \sum_{i=1}^{k} \beta_i lnCU_{t-i} + \sum_{j=1}^{k} \phi_j lnNGDP_{t-j} + \sum_{m=1}^{k} \varphi_m lnINTR_{t-m} + \varepsilon_{2t} \tag{4.40}$$

$$lnINTR_t = d + \sum_{i=1}^{k} \beta_i lnCU_{t-i} + \sum_{j=1}^{k} \phi_j lnNGDP_{t-j} + \sum_{m=1}^{k} \varphi_m lnINTR_{t-m} + \varepsilon_{3t} \quad (4.41)$$

The VECM model, which is a differenced VAR model, with cointegrated variables will thus be represented by (4.42)-(4.44) below.

$$\Delta lnCU_t = \alpha + \sum_{i=1}^{k-1} \beta_i \Delta lnCU_{t-i} + \sum_{j=1}^{k-1} \phi_j \Delta lnNGDP_{t-j} + \sum_{m=1}^{k-1} \varphi_m \Delta lnINTR_{t-m} + \lambda_1 ECT_{t-1} + \varepsilon_{1t} \quad (4.42)$$

$$\Delta lnNGDP_t = c + \sum_{i=1}^{k-1} \beta_i \Delta lnCU_{t-i} + \sum_{j=1}^{k-1} \phi_j \Delta lnNGDP_{t-j} + \sum_{m=1}^{k-1} \varphi_m \Delta lnINTR_{t-m} + \lambda_2 ECT_{t-1} + \varepsilon_{2t} \quad (4.43)$$

$$\Delta lnINTR_t = d + \sum_{i=1}^{k-1} \beta_i \Delta lnCU_{t-i} + \sum_{j=1}^{k-1} \phi_j \Delta lnNGDP_{t-j} + \sum_{m=1}^{k-1} \varphi_m \Delta lnINTR_{t-m} + \lambda_3 ECT_{t-1} + \varepsilon_{3t} \quad (4.44)$$

where β_i, ϕ_j, φ_m represents the short-run dynamic coefficients of the model's adjustment long-run equilibrium, λ_i is the speed of adjustment parameter with a negative sign, ECT_{t-1} is the error correction term and ε_{it} are the residuals or stochastic error terms. The error correction term is the lagged value of the residuals obtained from the cointegrating regression of the outcome variable on the predictor variables. It explains previous deviations from the long run equilibrium (Hill et al., 2018 and Estimating Vector Error Correction Models, 2018).

4.4.2 The Augmented Dickey-Fuller Test

The Augmented Dickey-Fuller (AD-F) test is a commonly used unit root test for time series data. Fitting an autoregressive (AR) model, this test examines the null hypothesis of an autoregressive integrated moving average (ARIMA (*p*, 1, 0)) process against the alternative stationary ARIMA (*p*+1, 0, 0) (Cheung and Lai, 1995).

AD-F test investigates whether there is any unit root in the data by using OLS estimator $\hat{\rho}_n$ of ρ obtained by fitting the regression equation (4.45) to the observed data.

$$Y_t = \rho Y_{t-1} + \sum_{j=1}^{p} a_j \Delta Y_{t-j} + e_t \quad (4.45)$$

Where Y_t is the variable of interest at time t, Δ is the difference operator, ρ is a coefficient that defines the unit root, while the order p is allowed to depend on n, and

e_t is white noise. Under the null hypothesis $H_0: \rho = 1$, Y_t is obtained by integrating a linear, infinite order AR (∞), that is,

$$Y_t = Y_{t-1} + U_t \qquad t = 1, 2, \ldots, \tag{4.46}$$

where $Y_0 = 0$ and

$$U_t = \sum_{j=1}^{\infty} a_j U_{t-j} + e_t \tag{4.47}$$

e_t represents independent, identically distributed normal random variables with mean zero and variance $0 < \sigma_e^2 < \infty$. To test H_0, Dickey and Fuller (1979) propose the studentized statistic:

$$t_n = \frac{\hat{\rho}_n - 1}{\widehat{Std}(\hat{\rho}_n)} \tag{4.48}$$

where $\widehat{Std}(\hat{\rho}_n)$ denotes an estimator of the standard deviation of the OLS estimator $\hat{\rho}_n$. The hypothesis that $\rho = 1$ is useful in many applications as it confirms that transformation by differencing is appropriate.

The time series Y_t converges to stationarity if $|\rho| < 1$. If $|\rho| = 1$, Y_t is not stationary. The time series with $\rho = 1$ is sometimes called a random walk. If $|\rho| > 1$, Y_t is also not stationary and the variance of the time series increases exponentially as t increases.

4.5 The joint model

The joint model consists of deriving the estimate using the CDA model and using that estimate in the MIMIC model benchmarking procedure as a value in the base year.

Substituting (4.31) into (4.30) yields,

$$y = \prod \boldsymbol{x} + \boldsymbol{z} \tag{4.49}$$

where the reduced form coefficient matrix is:

$$\prod = \boldsymbol{\lambda}\boldsymbol{\gamma}' \tag{4.50}$$

and the reduced form disturbance vector,

$$\boldsymbol{z} = \lambda\zeta + \varepsilon \tag{4.51}$$

has covariance matrix

$$\boldsymbol{\Omega} = E(zz') = \sigma^2\lambda\lambda' + \Theta^2 \tag{4.52}$$

The joint model, therefore, consists of (4.34) and (4.49) where equation (4.34) is used to estimate an exogenous value of the underground economy at a certain point and the estimate is used as an input into the MIMIC model procedure.

In the MIMIC model estimation, an indicator variable needs to be fixed in the measurement model (4.30) to have a reference variable to set a measurement scale for the underground economy because it is, by nature, unobserved (Bollen, 1989b). Therefore, λ needs to be constrained to some pre-assigned value.

4.6 The benchmarking procedure

The MIMIC model only yields ordinal estimates of the size of the underground economy. Therefore, an extra final step is necessary to calculate the size of the underground economy as a percentage of GDP. This last step is called a benchmarking step or procedure and is sometimes referred to as calibration methods. There are several benchmarking procedures used by different researchers but there's no consensus yet on the best and most reliable procedure (Medina and Schneider, 2018). The four benchmarking procedures found in literature will be discussed below:

1. Dell'Anno and Schneider (2003) solved a system of *t-1* linear equations:

$$\hat{\eta}_t = \eta^* + \hat{\eta}_{t-1} + \hat{\gamma}_q' \Delta x_{qt} \tag{4.53}$$

   by choosing η^* to satisfy the following condition: $\frac{\hat{\eta}_{1978}}{GDP_{1978}} = 0.197$ which is the estimate of the underground economy in the base year (1978) using an auxiliary method. The last step is to calculate the other *t-1* ratios ($\frac{\hat{\eta}_t}{GDP_t}$).

2. The benchmarking strategy applied by Bajada and Schneider (2004) is as follows:

$$g_t^{SE} = g_{1978}^* + \hat{\gamma}_q' \Delta x_t \tag{4.54}$$

where g_t^* satisfies the condition $g_{1978}^{SE} = g_{1978}^*$, g_t^{SE} shows the growth rate of the underground economy and g_{1978}^* represents the growth rate of the underground economy estimated in the base year (1978) by another method.

3. Dell'Anno and Schneider (2006) proposed a different benchmarking procedure which requires an alternative indicator, (GDP_t/GDP_{1978}), to be used as a reference variable (Y_1) instead of GDP growth rate. According to the identification rule ($\lambda_1 = -1$), the index of the underground economy as a percentage of GDP in 1978 is related to the chain index of real GDP as follows:

$$Measurement\ Equation: \frac{GDP_t - GDP_{t-1}}{GDP_{1978}} = -\frac{\hat{\eta}_t - \hat{\eta}_{t-1}}{GDP_{1978}} \quad (4.55)$$

The estimates of the structural model are used to obtain an ordinal time series index for the latent variable:

$$Structural\ Equation: \frac{\hat{\eta}_t}{GDP_{1978}} = \gamma' x_{qt} \quad (4.56)$$

The index is scaled to take values less than or equal to 19.7% in 1978 and is further changed to the underground economy as a percentage of GDP. These calculations are shown in the benchmarking equation below:

$$\frac{\hat{\eta}_t}{GDP_{1978}} \left[\frac{\eta_{1978}^*}{GDP_{1978}} \frac{GDP_{1978}}{\hat{\eta}_{1978}} \right] \frac{GDP_{1978}}{GDP_t} = \frac{\eta_t}{GDP_t} \quad (4.57)$$

where: $\frac{\hat{\eta}_t}{GDP_{1978}}$ is the index calculated by (4.56), $\frac{\eta_{1978}^*}{GDP_{1978}} = 19.7\%$ represents exogenous estimate of the underground economy, $\frac{\hat{\eta}_{1978}}{GDP_{1978}}$ represents the value of the index estimated by (4.53) in 1978, $\frac{GDP_{1978}}{GDP_t}$ converts the index of the underground economy with respect to GDP in the base year and current year and $\frac{\eta_t}{GDP_t}$ is the estimate of the underground economy as % of GDP.

4. Giles and Tedds (2002) applied the following benchmarking strategy:

$$\frac{\eta_t}{GDP_t} = 0.197 * \left(\frac{\hat{\eta}_t}{\hat{\eta}_{1978}} \right) \quad (4.58)$$

where $\frac{\hat{\eta}_{1978}}{GDP_{1978}} = 0.197$ is the underground economy estimated by another method. This is the benchmarking procedure which will be adopted for this study. This procedure is the latest and most widely used by various authors including Buehn and Schneider (2012), Hassan and Schneider (2016a) and Schneider et al. (2010). A previous estimate of the underground economy is required to be calculated. The theoretical formula of this benchmarking procedure is as follows:

$$\hat{\eta}_t = \frac{\tilde{\eta}_t}{\eta_{base\_year}} \eta * base\_year \quad (4.59)$$

where: $\tilde{\eta}_t$ represents the value of the MIMIC index at time *t* according to the structural equation (4.30), η_{base_year} represents the value of the MIMIC index in the base year and $\eta * base\_year$ is the exogenous estimate of the size of the underground economy based on a base year and usually this is either taken as a secondary value from existing literature, or it can be computed using the CDA model, and then a base year value used as the benchmark for calculations.

4.7 Rationale for using both the CDA and the MIMIC models to estimate the underground economy

As a stand-alone estimation tool, the MIMIC model is of very limited use, as a result a considerable number of authors such as Hassan and Schneider (2016a), Pickard and Pons (2006) and Gauci and Rapa (2020) recommends a joint application of CDA and MIMIC. This improves the efficiency of the estimation due to the inclusion of correlation between the CDA residuals and the MIMIC reduced-form residual vector (Dybka et al., 2018). Furthermore, the procedure addresses the calibration problem which is usually encountered with a MIMIC model, that is, converting the resulting index into a series of levels (Pickhardt and Pons, 2006).

4.8 Summary

The research methodology used to derive the estimates of the underground economy was discussed. Structural Equation Modelling, including the preferred parameter estimation method, MLE, were discussed in detail in order to provide a thorough background for the MIMIC model. Typical SEM fit indices used to select the best model, with specific reference to the MIMIC model were covered in detail. Finally, the CDA model, including the VECM, and the benchmarking procedure used in the MIMIC model were discussed at length.

Chapter 5: Application and Results

5.1 Introduction

In this chapter, the data explored in Chapter 3 are applied to the models described in the previous chapter. The time series data points for the period 2000 to 2020 (84 quarterly observations) were used in constructing the CDA and MIMIC model. A description of the variables and their sources is provided in Chapter 3 and Appendix A.3. The MIMIC model was derived using RStudio 4.1.0 as it is more comprehensive and encompasses numerous data manipulation capabilities. The CDA model does not require sophisticated data manipulation skills, therefore STATA 15.0 was deemed appropriate to use.

5.2 The CDA model

The CDA models were constructed using several combinations of the explanatory variables that were identified in Chapter 3. The study sought to search for the most parsimonious model by systematically dropping non-significant variables. Furthermore, the models were tested for serial correlation, normal distribution of residuals, stationarity, cointegration and hypothesized signs of the variables before finally settling for the model which provided the best results. When modelling supervised data, the variables need to be stationary and cointegrated to avoid estimating spurious regressions.

Thus, the results presented below are summaries of the final CDA model estimated for the period 2003 to 2020.

5.2.1 Unit root test

As observed from Chapter 3, the distribution of CDA variables showed skewness and did not follow the characteristic bell shape of a normal distribution. This necessitated a natural log transformation to be applied to the variables to render the data as normal as possible so that statistical analysis from this data becomes more valid. The variables were first transformed to approximately conform to normality and then tested for stationarity and cointegration. The transformed variables in Figure 5.1 show some form of trend indicating non-stationarity over time. CU and NGDP are trending upwards while SELFEMPLOY_R and INT_R are trending downwards. There is a distinct time series break in SOCBEN_GOVTEXP which was as a result of a drastic increase in social benefits paid by government in 2006 whereby social benefits increased by over 800%. The social benefits were also increased in 2020 to mitigate the COVID-19 impact. TAX_BUR appears to be a bit more stationary around the mean with a slight upward trend which is also evident in UNEMPLOY_R. Seasonality is also evident in TAX_BUR and CU.

The AD-F tested for the existence of unit roots on the transformed variables. The presence of unit roots indicates that the data is non-stationary. Before running an AD-F test, the data should be inspected to determine the appropriate regression model to apply e.g. a nonzero mean indicates that the regression will have a constant term. The model is chosen based on theory and visual examination of the data. There are three basic regression models:

- No intercept, no trend: $\Delta y_t = \rho y_{t-1} + \varepsilon_t$
- intercept, no trend: $\Delta y_t = \alpha + \rho y_{t-1} + \varepsilon_t$
- intercept and trend: $\Delta y_t = \alpha + \rho y_{t-1} + \delta_t + \varepsilon_t$

where y_t is the variable of interest, y_{t-1} is the lagged dependent variable, α is the intercept, ρ is the AR-coefficient, δ_t is the time trend and ε_t is the error term.

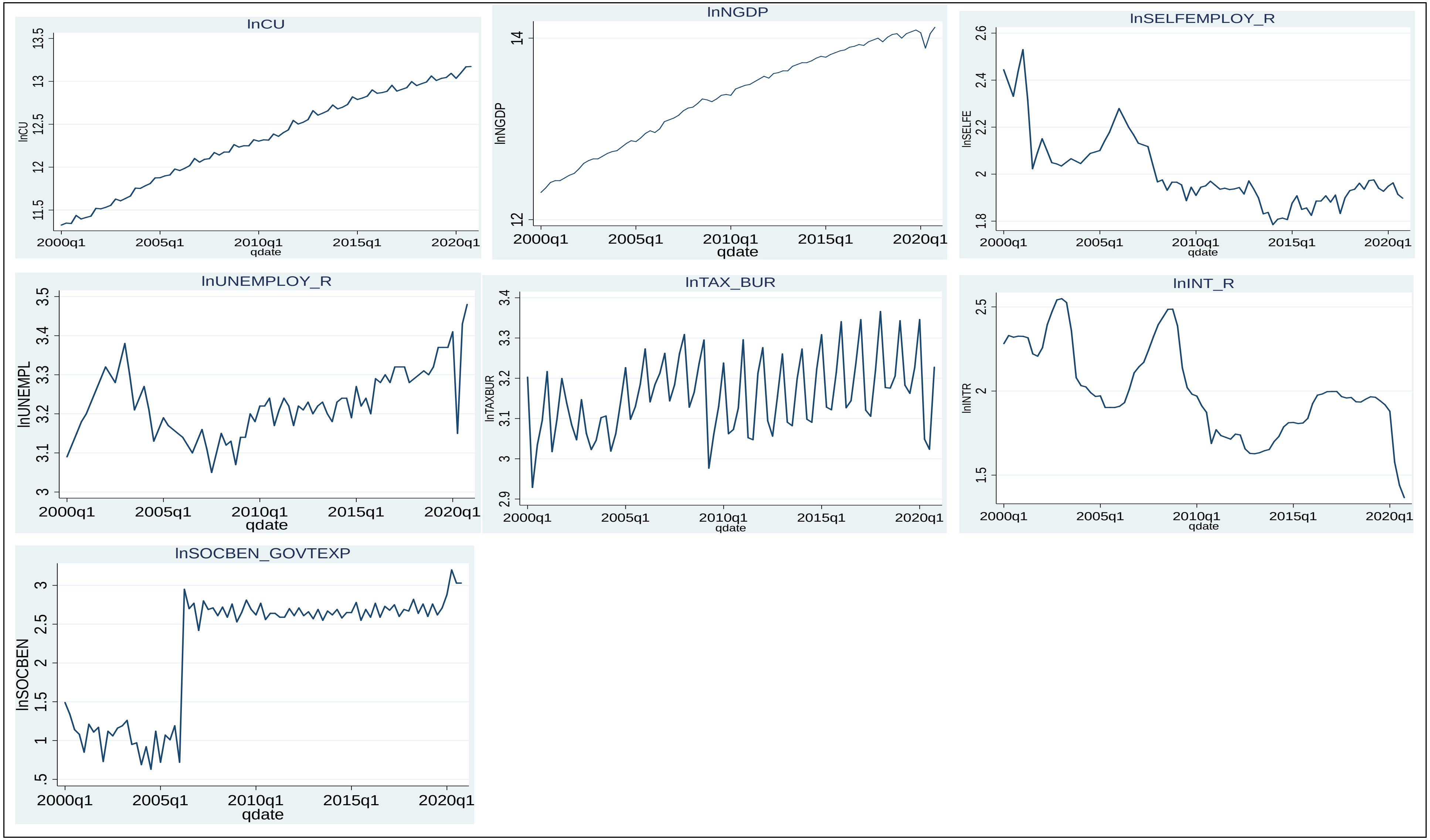


Figure 5.1: Time series of transformed CDA variables.

Table 5.1 shows the test statistic and the respective critical values of the AD-F unit root test for the levels and first differences of the variables. The variables exhibit unit roots at level (0) and become stationary after first differencing since the calculated t-statistics from the AD-F equations are larger in absolute terms than the critical values at 5% level of significance. Detailed output from STATA 15.0 is shown in Appendix B.1.

Table 5.1: AD-F unit root test at level and first difference.

Unit root at:	Statistic	CU	NGDP	SOCBEN_GOVTEXP	UNEMPLOY_R	TAX_BUR	INT_R	SELFEMPLOY_R
I (0)	Test Statistic	-1.436	1.619	-2.053	-1.309	0.138	-2.851	-3.295
	5% critical value	-3.472	-3.471	-3.469	-3.471	-1.950	-3..469	-3.468
I (1)	Test Statistic	-3.730	-3.997	-6.863	-3.985	-3.888	-4.080	-8.736
	5% critical value	-3.473	-3.472	-3.470	-3.472	-1.950	-3.470	-3.469

5.2.2 Lag length

The variables are stationary after first differencing, therefore Johansen cointegration test can now be performed on the variables. A cointegration test determines whether a long-run relationship exist among the variables. If the long-run relationship exists, then the series can be linearly combined and shocks in the short run which may affect movement in the individual series would converge with time in the long run. In this case, a VAR and VECM would be appropriate to use in estimating the model. However, if the series is not cointegrated, then the model can only be estimated using the VAR (short-run) model and not the VECM.

The optimal lag length of the series must first be determined before running the Johansen cointegration test. It is essential to determine the optimal lag length as too few lags might result in specification errors and too many lags might yield multicollinearity, insignificant coefficients and loss of degrees of freedom. The optimal length is determined by estimating a VAR model using the original or log transformed data. A VAR model must be specified in levels and not in differences, otherwise the model will be mis-specified. The selection order criteria in Table 5.2 below shows lag lengths of 2 from Schwarz Bayesian Criterion (SBC) and lag length of 8 from Hannan-Quinn Criterion (HQC), Likelihood Ratio (LR), Final Prediction Error (FPE) and AIC.

Table 5.2: Selection order criteria.

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	315.032				7.1E-13	-8.106	-8.020	-7.891
1	810.448	990.83	49	0.000	5.7E-18	-19.854	-19.168	-18.137
2	949.195	277.49	49	0.000	5.5E-19	-22.216	-20.929	-18.996*
3	1028.46	158.54	49	0.000	2.7E-19	-23.012	-21.125	-18.289
4	1094.34	131.76	49	0.000	2.0E-19	-23.456	-20.968	-17.231
5	1151.01	113.34	49	0.000	2.2E-19	-23.658	-20.570	-15.930
6	1252.87	203.72	49	0.000	9.0E-20	-25.049	-21.360	-15.818
7	1332.94	160.13	49	0.000	8.6E-20	-25.867	-21.577	-15.133
8	1435.37	204.86*	49	0.000	7.7E-20*	-27.273*	-22.383*	-15.037

Endogenous: ln CU, lnNGDP, lnUNEMPL, lnTB, lnSELFEMPL, lnSOCBEN_GOVTEX
Exogenous: _cons

Choosing the lag length of 2 resulted in the coefficients of the explanatory variables assuming theoretically incorrect signs. For instance, unemployment rate and tax burden had negative signs. Numerous literature showed that a growth in unemployment, will result in an increase in underground economic activities. Therefore, the expected sign for unemployment rate is positive. Similarly, taxes generally place a higher burden on taxpayers. The higher the tax burden, the higher the underground economic activities. South Africa has a particularly skewed tax population distribution as only 3 million individuals are carrying the tax burden of almost 60 million citizens. The expected sign for the tax burden is positive, but the model with 2 lags allocated a negative sign. The CDA model's premise is on estimating the extra currency when there is tax burden and when the tax burden is minimal. Therefore, a negative tax burden defies the model foundation and the literature reviewed. Hence the lag length of 3 which proved to be more optimal was chosen for the final model.

5.2.3 Johansen cointegration test

Similar to the procedure for determining the optimal lag length, the cointegration test should be performed on original or transformed variables and not on the first differences. Log transformed variables can also be utilised.

Table 5.3 shows the results of the Johansen cointegration test. The test indicates that there are at least 2 cointegrating equations.

Table 5.3: Johansen cointegration test.

Sample: 2000 - 2020					Lags = 3
maximum rank	parameters	LL	eigenvalue	trace statistic	5% critical value
0	105	948.244	.	183.562	124.24
1	118	985.936	0.606	108.178	94.15
2	129	1005.899	0.389	68.253*	68.52
3	138	1022.186	0.331	35.679	47.21
4	145	1030.446	0.184	19.159	29.68
5	150	1036.440	0.138	7.171	15.41
6	153	1039.873	0.081	0.305	3.76
7	154	1040.025	0.004		

5.2.4 VEC model results

Given the absence of unit root, the presence of a long run association between the variables and the cointegration of the variables, currency demand can now be estimated using the VECM. This type of model is considered superior to other estimation methods as it caters for long and short run effects. The model was estimated with one cointegrating equation and the detailed results are shown in Appendix B.2. The general equation of the dependent variable (Currency) can be written as:

$$\Delta lnCU_t = 0.0176 + \sum_{i=1}^{2} \theta_i \Delta lnCU_{t-i} + \sum_{i=1}^{2} \gamma_i \Delta lnNGDP_{t-i} + \sum_{i=1}^{2} \delta_i \Delta lnINT_{t-i} + \sum_{i=1}^{2} \vartheta_i \Delta lnUNEMPL_{t-i} + \sum_{i=1}^{2} \alpha_i \Delta lnTB_{t-i} + \sum_{i=1}^{2} \omega_i \Delta lnSELFEMPL_{t-i} + \sum_{i=1}^{2} \varphi_i \Delta lnSOCBEN\_GOVTEXP_{t-i} - 0.0289ECT_{t-1} + \varepsilon_t \quad (5.1)$$

where coefficients are as follows:

Lag$(t-i)$	θ_i	γ_i	δ_i	ϑ_i	α_i	ω_i	φ_i
1	-0.6091	-0.1330	-0.0562	-0.1188	-0.2678	0.0598	-0.0036
2	-0.2000	0.2876	0.0066	-0.2274	-0.2963	-0.0241	0.0179

$i = 1,2.$

The speed of adjustment term (-0.0289) is statistically significant at 1% level suggesting that previous year's errors (or deviation from long run equilibrium) are corrected for within the current year at a convergence speed of 2.9%. In other words, there's long causal effect in the $lnCU$ equation at 1% level. For short run causality, with the exception of INT_R and SELFEMPLOY_R, all the variables are significant at 5% level on either lag 1 or 2. The overall explanatory power of the model is strong, with an R-squared of 0.87 which indicates that these variables considerably explain the variations of currency demand in South Africa.

The long run equation is shown by the Johansen normalisation restriction in Table 5.4.

Table 5.4: The long run equation.

beta	Coefficient	Standard Error	z	P(z)	[95% confidence interval]	
_ce1						
lnCU	1	.	.	.	.	.
lnNGDP	1.094	0.263	4.150	0.000	0.578	1.610
lnINT	1.110	0.180	6.180	0.000	0.758	1.462
lnUNEMPL	-2.139	0.590	-3.620	0.000	-3.296	-0.982
lnTB	-7.798	1.081	-7.210	0.000	-9.918	-5.679
lnSELFEMPL	1.654	0.360	4.590	0.000	0.948	2.360
lnSOCBEN_GOVTEXP	-0.282	0.093	-3.020	0.003	-0.465	-0.099
_cons	-1.020	.	.	.	.	.

The long run model or cointegrating equation can be written as follows:

$$ECT_{t-1} = 1.000lnCU_{t-1} + 1.094lnNGDP_{t-1} + 1.110lnINT_{t-1} - 2.139lnUNEMPL_{t-1} - 7.798lnTB_{t-1} - 0.282lnSOCBEN\_GOVTEXP_{t-1} + 1.654lnSELFEMPL_{t-1} - 1.020 \quad (5.2)$$

The signs of the coefficients must be reversed for interpretation of the results. The restriction is placed on CU which is shown as the dependent or target variable. UNEMPLOY_R, TAX_BUR and SOCBEN_GOVTEXP have a positive effect on CU while NGDP, INT_R and SELFEMPLOY_R have a negative effect on CU. All the coefficients are statistically significant at 1% level.

Table 5.5 shows the hypothesized signs of the explanatory variables according to literature and the results from the final model chosen. The coefficient of deposit interest rate is negative and significant reflecting the opportunity cost of holding money. The increase in deposit interest rate serves as an incentive for individuals not to indulge in underground economic activities but rather to save money and earn interest on it. Similarly, increased formal economy activities will result in decreased usage of currency denoting less activities in the underground economy. In consistent with the literature reviewed, an increase in unemployment and tax burden will raise the currency demand thereby fueling underground economic activities. A growth in social benefits, in this case, will cause a surge in the underground economic activities identified through the increased usage of currency.

Table 5.5: Hypothesized signs of the explanatory variables.

	Variable	Hypothesized sign	Model Result
1.	NGDP	Negative (-)	Confirmed
2.	Deposit Interest Rate	Negative (-)	Confirmed
3.	Unemployment Rate	Positive (+)	Confirmed
4.	Tax Burden	Positive (+)	Confirmed
5.	Social benefits	Ambiguous effect (+) or (-)	Positive
6.	Self-Employment Rate	Positive (+)	Not Confirmed

5.2.5 Model diagnostics

A Lagrange multiplier test is used to test for serial correlation in the residuals of VAR / VEC models. Table 5.6 shows that there is no autocorrelation at lags 2 and 4.

Table 5.6: The Lagrange-multiplier test.

lag	χ^2	df	Prob > χ^2
1	71.667	49	0.019
2	62.038	49	0.100
3	70.941	49	0.022
4	36.783	49	0.901

H_0: no autocorrelation at lag order.

The Jarque-Bera test in Table 5.7 examines whether the residuals are normally distributed. As VECM is a system of equations with each variable having a function of its own, all seven models were tested to determine whether the residuals are normally distributed. The residuals are normally distributed for the models: *D_lnCU*, *D_lnUNEMPL* and *D_lnTB*. Of importance is that the target model, *D_lnCU* has normally distributed residuals.

Table 5.7: The Jarque-Bera test.

Equation	χ^2	df	Prob > χ^2
D_lnCU	1.285	2	0.526
D_lnNGDP	2015.875	2	0.000
D_lnINT	31.455	2	0.000
D_ lnUNEMPL	1.963	2	0.375
D_lnTB	0.628	2	0.731
D_lnSELFEMPL	57.444	2	0.000
D_lnSOCBEN_GOVTEXP	3560.964	2	0.000
ALL	5669.613	14	0.000

The model stability test is shown in Table 5.8 below. If the process is stable, the moduli of the remaining eigenvalues are less than one. As can be observed from Table 5.8, the moduli of the remaining 15 eigenvalues are less than one indicating a relatively stable process.

Table 5.8: The model stability test.

Eigenvalue stability condition

Eigenvalue		Modulus
1		1
1		1
1		1
1		1
1		1
1		1
-0.003	+ 0.971i	0.971
-0.003	- 0.971i	0.971
-0.942		0.942
-0.236	+ 0.780i	0.815
-0.236	- 0.780i	0.815
0.773		0.773
0.681		0.681
0.076	+ 0.627i	0.631
0.076	- 0.627i	0.631
-0.578		0.578
-0.736	+ 0.551i	0.556
-0.074	- 0.551i	0.556
-0.381		0.381
0.126	+ 0.302i	0.327
0.126	- 0.302i	0.327

The VECM specification imposes 6 unit moduli

5.2.6 Estimation of the size of the underground economy using the CDA model

The size of the underground economy in South Africa can be estimated using the VECM currency demand model shown in equation (5.1). Firstly, currency in circulation outside of banks is estimated with the tax burden and thereafter with minimal tax burden to minimise the incentive for individuals to participate in the underground economy. With the tax burden minimised, currency in circulation outside of banks would be lower because there is less demand for cash payments of goods and services to evade tax. The difference between the two estimates yields Extra Currency (EC), which is the illegal money used in the underground economy. The size of the underground economy is the product of EC and velocity of currency. The coefficient of the NGDP is 1.094 ≠ 1, therefore Ahumada et al. (2007)'s correction method was applied to the results to obtain correct estimates of the South African underground economy. Figure 5.2 shows the size of the underground economy as percentage of GDP.

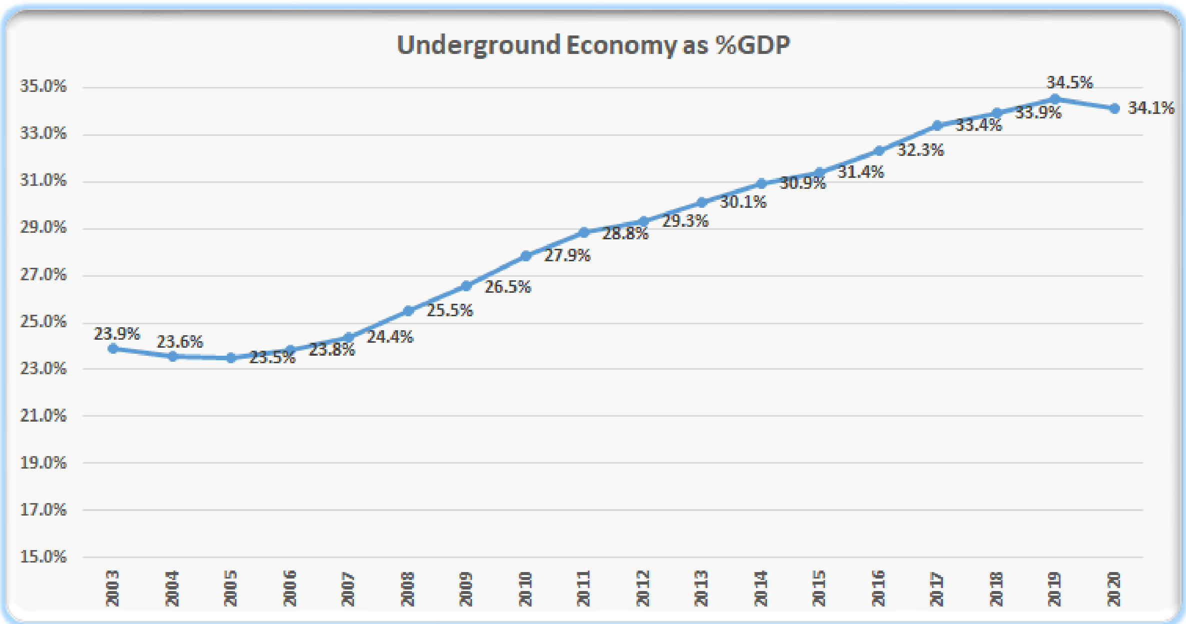


Figure 5.2: South Africa: Underground Economy as % of GDP (CDA).

The underground economy steadily increased from 23.9% in 2003 to 34.5% in 2019 and slightly declined to 34.1% in 2020. The 2010 estimate of 27.9% was used as input to the MIMIC model. The year 2010 was selected as a base year in consistent with

STATSSA which normalises GDP with constant 2010 prices. On average, the underground economy in South Africa was 28.8% of GDP for the period 2003 to 2020.

5.3 MIMIC model

As mentioned in Chapter 4, the MIMIC model consist of the measurement and the structural models. The indicator variables that were used in the measurement model are NGDP and LFPR. Several combinations of the causal variables that were identified in Chapter 3 were used to construct different specifications of the MIMIC model and only three models are presented in this section. Different specifications were computed in order to capture the magnitude and the effect of different causal variables on the size of the underground economy. The identified causal variables are TAX_BUR, MINEMPLOY_R, AGRIC_NGDP, AGRICEMPLOY_R, SOCBEN_GOVTEXP, UNEMPLOY_R, SELFEMPLOY_R, GOVTEMPLOY_R, CPI, EMPLOY_CAP, EMPLOY_R, TEMPLOY and GOVTEXP_NGDP.

5.3.1 Fitting the MIMIC model

The underground economy is by nature unobserved and so it is important to fix an indicator variable in the measurement model in order to have a reference variable to set a measurement scale for the underground economy (Bollen, 1989b). The underground economy is thus interpreted as % of GDP. The importance of fixing this indicator variable is to have a unit of measurement to estimate the other variables as a function of this scale variable and to make the estimated coefficients more comparable (Dell'Anno et al., 2007). Schumacker and Lomax (2010) concurs and states that this is one of the three methods used for avoiding identification problems. The authors suggest that one indicator variable must have a factor loading fixed to 1, or the variance of each latent variable be fixed to 1. In addition, various researchers recommend normalising NGDP to -1 or +1 based on the theoretical assumptions of the observed variables in the SEM model. After fixing the reference indicator, the signs of the observed causal variables should be consistent with empirical evidence found in the literature of the underground economy.

It is for this reason that NGDP was thus fixed to -1 in all the MIMIC model specifications. This is also consistent with the CDA model results where NGDP was found to have a negative effect on the underground economy. Productive resources and factors are absorbed by the underground economy creating a depressing effect on the development of the official economy.

Before estimating the models, all the variables were transformed and differenced to ensure normality and stationarity. The MLE might produce biased standard errors and an improper χ^2 test for overall model fitness when the variables are not normally distributed. Table 5.9 shows the three MIMIC specifications.

Table 5.9: MIMIC estimation of the size of the underground economy from 2000 to 2020, quarterly data.

Variables/specifications	**MIMIC1 (4-1-2)**	**MIMIC2 (3-1-2)**	**MIMIC3 (4-1-2)**
Causes			
1. Tax burden	0.514 (6.111)*	0.334 (4.075)*	0.342 (4.187)*
2. Agriculture NGDP	0.000 (4.023)*		
3. Mining employment rate	0.146 (2.331)**	0.189 (2.550)**	0.171 (2.480)**
4. Consumer Price Index			0.124 (1.823)***
5. Government Expenditure	0.283 (3.775)*	0.275 (3.491)*	0.232 (3.135)*
Indicators			
1. NGDP	-1	-1	-1
2. Labour Force Participation Rate	0.460 (2.279)**	0.606 (2.391)**	0.747 (2.674)*

Statistical tests			
Chi-square (χ^2)	5.244	0.074	4.102
p-value	0.263	0.964	0.251
RMSEA	0.061	0.000	0.067
p-value	0.364	0.970	0.335
SRMR	0.051	0.008	0.047
GFI	0.977	0.999	0.964
AGFI	0.880	0.995	0.746
CFI	0.980	1.000	0.974
TLI	0.924	1.190	0.922
AIC	1132.814	440.174	433.198
BIC	1171.515	457.106	452.452
Sample-size adjusted BIC	1121.047	435.026	427.220

Notes: Absolute z-statistics are reported in parenthesis. *, **, *** denote significance at 1, 5 and 10% significance levels.

5.3.2 MIMIC Specification 1 (4-1-2)

This model consist of four causal variables and two indicator variables. All the variables are significant and have the expected signs as identified from literature. Tax burden has the highest effect on the underground economy followed by agricultural GDP. According to the value of the χ^2(4, N=83) = 5.244, p=0.263, the hypothetical model is able to adequately reproduce the observed variance-covariances matrix. All the indices indicate a good fit.

5.3.3 MIMIC Specification 2 (3-1-2)

This model consist of three causal variables and two indicator variables. All the variables are significant and have the expected signs as identified from literature.

Similar to MIMIC Specification 1, tax burden has the highest effect on the underground economy followed by government expenditure.

According to the value of the χ^2(2, N=83) = 0.074, p=0.964, the hypothetical model is able to adequately reproduce the observed variance-covariances matrix. All the indices indicate a good fit.

5.3.4 MIMIC Specification 3 (4-1-2)

This model consist of four causal variables and two indicator variables. All the variables are significant and have the expected signs as identified from literature. Similar to MIMIC Specifications 1 and 2, tax burden has the highest effect on the underground economy followed by government expenditure. According to the value of the χ^2(3, N=82) = 4.102, p=0.251, the hypothetical model is able to adequately reproduce the observed variance-covariances matrix. All the indices indicate a good fit.

5.3.5 Final Model

Most fit statistics require models to be nested in order for the tests to be valid. Nested models contain at least all of the same observed variables contained in less complicated model. For nested models, one can only free or fix parameter(s), not do both. The underlying data matrix must be exactly the same between models (Ruginski, 2019). As the three models were computed using different set of observed variables and both free and fixed parameters were used, ANOVA could not be used to compare the models. Instead, the model was chosen on the basis of fit indices and validity of the parameter signs. Hence, MIMIC2 (3-1-2) was used to estimate the underground economy in South Africa.

The model is represented by the path diagram in Figure 5.3 below. The estimates of the causal variables, tax burden (dTA), mining employment rate (dMI) and government expenditure (dGO) are shown by the arrows pointing to the latent variable, underground economy (n) and the indicator estimates are shown by arrows pointing to labour force participation rate (dLF) and NGDP (dNG). As earlier mentioned, dNG is a reference indicator and its value is fixed to a value of -1. The estimated variance of dNG is 0.72 and the estimated variance of dLF is 0.89. The estimated covariance between the two variables is 0.23. Similarly, the variance of the three causal variables and the covariance estimates are shown on the path diagram. All the variables with the exception of the reference indicator dNG, have a positive effect on the underground economy implying that an increase in these variables would increase underground economic activities.

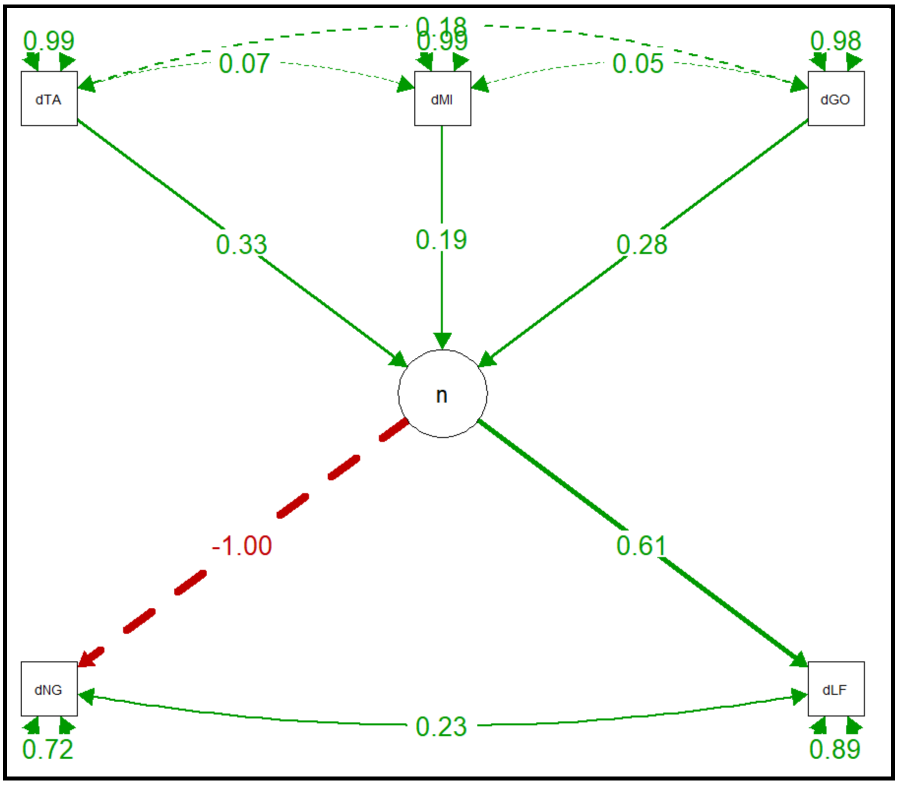


Figure 5.3: Path diagram of MIMIC2 (3-1-2).

5.3.6 Estimation of the size of the underground economy using the MIMIC model

The MIMIC model only produces an index of the trend of the size of the underground economy, that is, it only shows the variations in the ratio of the size of the underground economy yearly. Therefore, a benchmarking procedure used by Giles and Tedds (2002) which was described in Section 4.6 was used to convert the indices into meaningful estimates of the underground economy, that is, expressed as % of GDP. As mentioned in section 5.2.4, a base estimate of 27.9% for 2010 derived from the CDA model, was used as an input into the benchmarking procedure. Figure 5.4 shows the estimates of the underground economy as % of GDP.

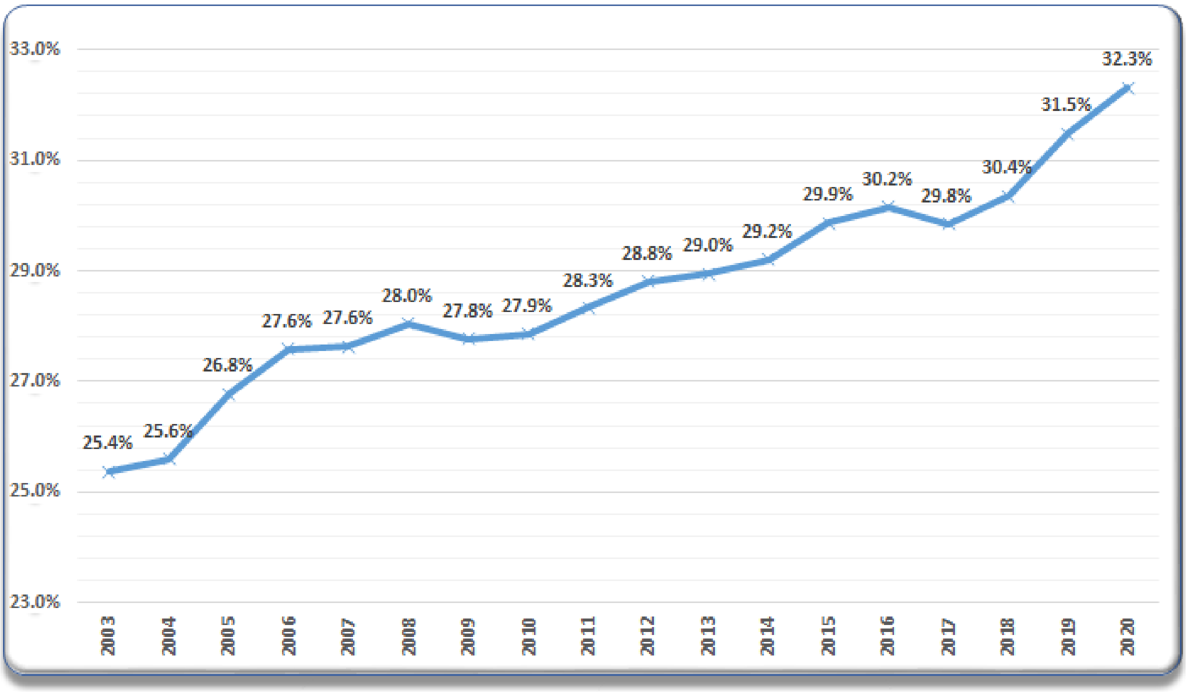


Figure 5.4: South Africa: Underground Economy as % of GDP (MIMIC).

The estimates of the underground economy range between 25.4% and 32.3% of GDP over the estimation period. The underground economy in South Africa is estimated to be on average 28.7% of GDP over the past 18 years. These results are in agreement with the results obtained from the CDA model.

According to the results, in 2020 South Africa has lost 32.3% of NGDP to the underground economy. Based on the total NGDP value for 2020 of R 4 973 974 million, the size of the underground economy translates into R1 607 107 million.

The 2020 tax to GDP ratio is 23.96% which means that the potential tax revenue loss based on R1 607 107 million is R 385 063 million.

5.3.6.1 Impact on the economic sectors

The impact of the underground economy on each sector was calculated based on their contribution to the 2020 total NGDP e.g. Finance, general government services and wholesale & retail trade contribute 18%, 17% and 13% respectively to NGDP. However, this is a very simplistic approach and necessitates more detailed future analysis on the contribution of each sector to the underground economy, considering significant causal variables per sector. Table 5.10 shows the estimated magnitude of the underground economy per sector.

Table 5.10: Estimates of the size of the underground economy per sector. Source: STATSSA and Author's computation.

Economic Sectors	2020 NGDP (R'm)	Estimate of the underground economy (R'm)
Agriculture, Forestry & Fishing	119 588	38 639
Construction	140 171	45 290
Electricity, Gas & Water	167 196	54 022
Finance, Real Estate & Business Services	879 512	284 173
General Government Services	859 229	277 620
Manufacturing	573 369	185 257
Mining & Quarrying	372 936	120 497
Personal Services	266 051	85 962
Taxes Less Subsidies On Products	544 264	175 853
Transport, Storage & Communication	396 436	128 090
Wholesale & Retail Trade, Hotels & Restaurants	655 222	211 704
Grand Total	**4 973 974**	**R 1 607 107**

5.4 Summary

In this chapter, the size of the underground economy in South Africa was estimated using the CDA and MIMIC models. According to the CDA model, the determinants of the underground economy in South Africa are the NGDP, interest rate, tax burden, unemployment rate, self-employment rate and social benefits granted by the government. The average size of the underground economy from 2003 to 2020 is 28.8% of NGDP, with the lowest value of 23.9% in 2003 and the highest value of 34.5% in 2019. Using the 2010 benchmark value of 27.9% from the CDA model, the MIMIC model confirms tax burden, mining employment rate and government expenditure as the causes of the underground economy. The average size of the underground economy from 2003 to 2020 is 28.7%, with the lowest value of 25.4% in 2003 and the highest value of 32.3% in 2020. The underground economy is estimated to be R1 607 107 million in monetary terms.

Chapter 6: Discussion and Conclusion

6.1 Introduction

Empirical results as presented in the Chapter 5, are discussed and interpreted in this chapter. This chapter will also ascertain whether the findings and the results addresses the objectives of the research as set out in Chapter 1.

The chapter starts with a discussion on the findings of the study, followed by limitations, recommendations and finally a conclusion is provided at the end of the chapter.

6.2 Discussion

Gross Domestic Product is closely related to employment. During the 2008/09 global financial crisis, over one million jobs were lost as South Africa entered into a recession. The recovery from 2010 to 2013 saw approximately 650 000 jobs gained (STATSSA, 2013). Prior to the global financial crisis, the economic growth rate was much higher (averaging over 5% per year). In recent times, South Africa's economy has been stagnant recording annual GDP growth rates of 0.79% and 0.15% in 2018 and 2019 respectively. The COVID-19 pandemic further exacerbated the poor performance resulting in a contraction of 6.96% in 2020 (World Bank, 2021). This type of environment serves as a fertile ground for underground economic activities.

In this study, CDA and MIMIC models were used to compute estimates of the underground economy in South Africa. In addition to providing its own estimates of the size of the underground economy, the CDA model provided an exogenous estimate for input into the MIMIC model in order to calculate the final estimates. According to the CDA model, unemployment, tax burden and social benefits granted by the government have a positive impact on the underground economy. The growth in the unemployment rate from 2007 to 2020 correlates with the growth in the underground economic activities. The unemployment rate grew by 6.90% in this period compared to the growth of 9.76% in the underground economy as estimated by the CDA model. With the current rising unemployment rates and the government attempting to mitigate the rise in poverty by increasing social benefits, one can expect underground economic activities to increase substantially. In addition, South Africans are over burdened with tax obligations. According to the IMF (2021), South Africa ranks among the ten most taxed countries in the world, making the country an ideal breeding ground for underground economic activities.

Conversely, NGDP, deposit interest rate and self-employment have a negative effect on the underground economy. A rise in these factors will result in a decrease in the underground economic activities and vice versa. The low deposit interest rates from 2011 to 2019 correlates with the increasing underground economic activities as shown by the CDA model. The model estimates the underground economy to be on average 28.8% of GDP for the period 2003 to 2020, with the lowest value of 23.5% in 2005 and the highest value of 34.5% in 2019.

After a lengthy search procedure, MIMIC2 (3-1-2) was chosen as the best model to predict the underground economy of South Africa. According to the model, tax burden, government expenditure and mining employment rate cause underground economic activities. Furthermore, the model indicated that individuals tend to participate in both the official and unofficial economies simultaneously shown by the positive labour force participation rate coefficient of 0.606 which is significant at 5% level. Similar to the CDA model, the MIMIC model estimates the underground economy to be on average 28.7% of GDP for the estimation period, with the lowest value of 25.4% in 2003 and the highest value of 32.3% in 2020.

During the periods 2009 to 2015, South Africa experienced increasing tax rates where the tax burden rose from 24.40% in 2009 to 27.34% in 2015. This correlates with the results from both the CDA and the MIMIC models. The underground economy grew from 27.8% to 29.9%, representing a growth rate of 2.1%, according to the MIMIC model, while the CDA model records a marginally higher growth rate of 4.4%.

Overall, the two models predict an increasing underground economy over the period of study.

6.3 Limitations

The notable limitation to this study was the unavailability of data for key variables. Employment data such as unemployment rate, labour force participation rate and self-employment rate was not available from STATSSA for periods earlier than 2000 as previous estimates were derived using the October Household Survey and these estimates have not been adjusted to the current methodology. Therefore, quarterly data on the variables of interest was only available for the period 2000 to 2020. This limitation affected the ability of the MIMIC models to produce reasonable and stable parameter estimates which resulted in a lengthy search procedure to find an appropriate model. Schumacker and Lomax (2010) recommends a sample size of 100 or more to produce reasonable and stable parameter estimates.

Furthermore, limited to no data was available for key variables such as the corruption index, tax morale, tax complexity, electronic payments value per GDP, economic freedom index, number of payment cards per capita, the ratio of point of sale terminals to the number of payments and immigration. Hence, their exclusion from the analysis.

6.4 Recommendations for future research

There is still a lot of uncertainty regarding the benchmarking procedure which is the final important stage in the estimation of the MIMIC model and from which the estimates of the underground economy are derived. The estimates of the underground economy are too sensitive to the benchmarking procedure utilised which makes the attained estimates very biased and as a result questions the reliability of the benchmarking tool. Future research needs to focus on the most reliable and appropriate benchmarking tool to use during the MIMIC estimation procedure.

Model specification, identification, estimation and testing requires a lengthy specification search procedure. A delicate balance has to be achieved with respect to model fitness, fit indices and suitable signs on the variable coefficients. Unfortunately a well-fitting model does not guarantee practical and reasonable parameter estimates or a correctly specified model. Negative variance estimates, known as Heywood case, often occur and these affect the predicted signs of the ordinal estimates. There is no single existing procedure sufficient to properly specify an appropriate model. There are a few guidelines provided in literature such as evaluating the significance and ranges of each parameter and examining the residual matrix. However all these are not definitive.

Although an attempt was made at calculating the size of the underground economy per sector, unfortunately these results are only an indication of the potential size of the underground economy per sector as the calculations are based on the contribution of each sector to NGDP. A high contribution does not necessarily mean high underground economic activities and vice versa. Several authors assert that the underground economy is likely higher in the agricultural sector as the sector is normally not fully controlled and tax authorities do not have full sight of all the transactions conducted in this sector, which would render the estimated 2% contribution inadequate. In South Africa's case, there's a considerable amount of illegal mining activities which might imply that the underground economy in the mining sector can be more than the estimated 7% contribution. Future work should focus on a comprehensive sectoral analysis considering all the causal and indicator variables of significance specific to that sector.

6.5 Conclusion

The objectives of the study as set out in Chapter 1 were achieved. The size of the underground economy in South Africa is on average 28.7% and is positively influenced by the growing unemployment rate, social benefits granted by the government, the rising tax burden, government expenditure and mining employment rate. Unless the South African government reprioritise their plans to tackle the poor economic growth and create a conducive environment for SMMEs and entrepreneurs, the underground economy will continue to rise at the detriment of the country's development.

Appendices

Appendix A.1: Correlation Matrix for the CDA model.

Variable	NGDP	EMPLOY_R	SELFEMPLOY_R	UNEMPLOY_R	GOVTEMPLOY_R	TAX_BUR	INT_R	CPI	AGRICEMPLOY_R	MINEMPLOY_R	NGDPCAP	TAX_REV	SOCBEN_GOVT EXP
NGDP	1.000												
EMPLOY_R	-0.534	1.000											
SELFEMPLOY_R	-0.697	0.371	1.000										
UNEMPLOY_R	0.533	-1.000	-0.368	1.000									
GOVTEMPLOY_R	0.765	-0.211	-0.840	0.209	1.000								
TAX_BUR	0.349	-0.087	-0.177	0.087	0.160	1.000							
INT_R	-0.614	0.186	0.448	-0.185	-0.696	-0.123	1.000						
CPI	0.995	-0.556	-0.689	0.555	0.762	0.330	-0.600	1.000					
AGRICEMPLO~R	-0.706	0.182	0.768	-0.180	-0.765	-0.295	0.628	-0.684	1.000				
MINEMPLOY_R	-0.610	-0.056	0.521	0.057	-0.546	-0.291	0.519	-0.587	0.760	1.000			
NGDPCAP	0.997	-0.496	-0.725	0.495	0.791	0.354	-0.625	0.988	-0.741	-0.640	1.000		
TAX_REV	0.979	-0.527	-0.664	0.526	0.719	0.515	-0.567	0.972	-0.679	-0.596	0.976	1.000	
SOCBEN_GOVT EXP	0.765	-0.116	-0.602	0.115	0.696	0.272	-0.499	0.761	-0.709	-0.724	0.784	0.732	1.000

Appendix A.2: Correlation Matrix for the MIMIC model.

Variable	TAX_BUR	MIN_NGDP	MINEMPLOY_R	AGRIC_NGDP	AGRICEMPLOY_R	SOC_BEN	UNEMPLOY_R	SELFEMPLOY_R	GOVTEMPLOY_R	CPI	EMPLOY_CAP	EMPLOY_R	TEMPLOY	TAX_REV	GOVTEXP_NGDP
TAX_BUR	1.0000														
MIN_NGDP	0.2677	1.0000													
MINEMPLOY_R	-0.2908	-0.6459	1.0000												
AGRIC_NGDP	-0.1009	0.5977	-0.4169	1.0000											
AGRICEMPLOY_R	-0.2597	-0.6657	0.6661	-0.4319	1.0000										
SOC_BEN	0.2720	0.7908	-0.7243	0.5876	-0.6350	1.0000									
UNEMPLOY_R	0.0869	0.5050	0.0569	0.3296	-0.2144	0.1153	1.0000								
SELFEMPLOY_R	-0.1752	-0.7121	0.5197	-0.4798	0.7623	-0.6000	-0.3679	1.0000							
GOVTEMPLOY_R	0.1603	0.7763	-0.5464	0.5250	-0.6982	0.6956	0.2089	-0.8394	1.0000						
CPI	0.3298	0.9653	-0.5869	0.6635	-0.6247	0.7610	0.5552	-0.6875	0.7615	1.0000					
EMPLOY_CAP	0.4440	0.3830	-0.4662	0.1833	-0.2668	0.4268	-0.2590	-0.1999	0.1998	0.3249	1.0000				
EMPLOY_R	-0.0867	-0.5057	-0.0556	-0.3301	0.2178	-0.1155	-1.0000	0.3701	-0.2108	-0.5561	0.2587	1.0000			
TEMPLOY	0.4540	0.9173	-0.6602	0.5905	-0.6231	0.7720	0.3253	-0.6374	0.6936	0.9200	0.6658	-0.3262	1.0000		
TAX_REV	0.5145	0.9377	-0.5957	0.5567	-0.6178	0.7316	0.5263	-0.6622	0.7188	0.9716	0.4301	-0.5270	0.9453	1.0000	
GOVTEXP_NGDP	0.3031	0.7660	-0.4591	0.4754	-0.4605	0.5839	0.4704	-0.5158	0.6161	0.7973	0.1623	-0.4708	0.6890	0.7653	1.0000

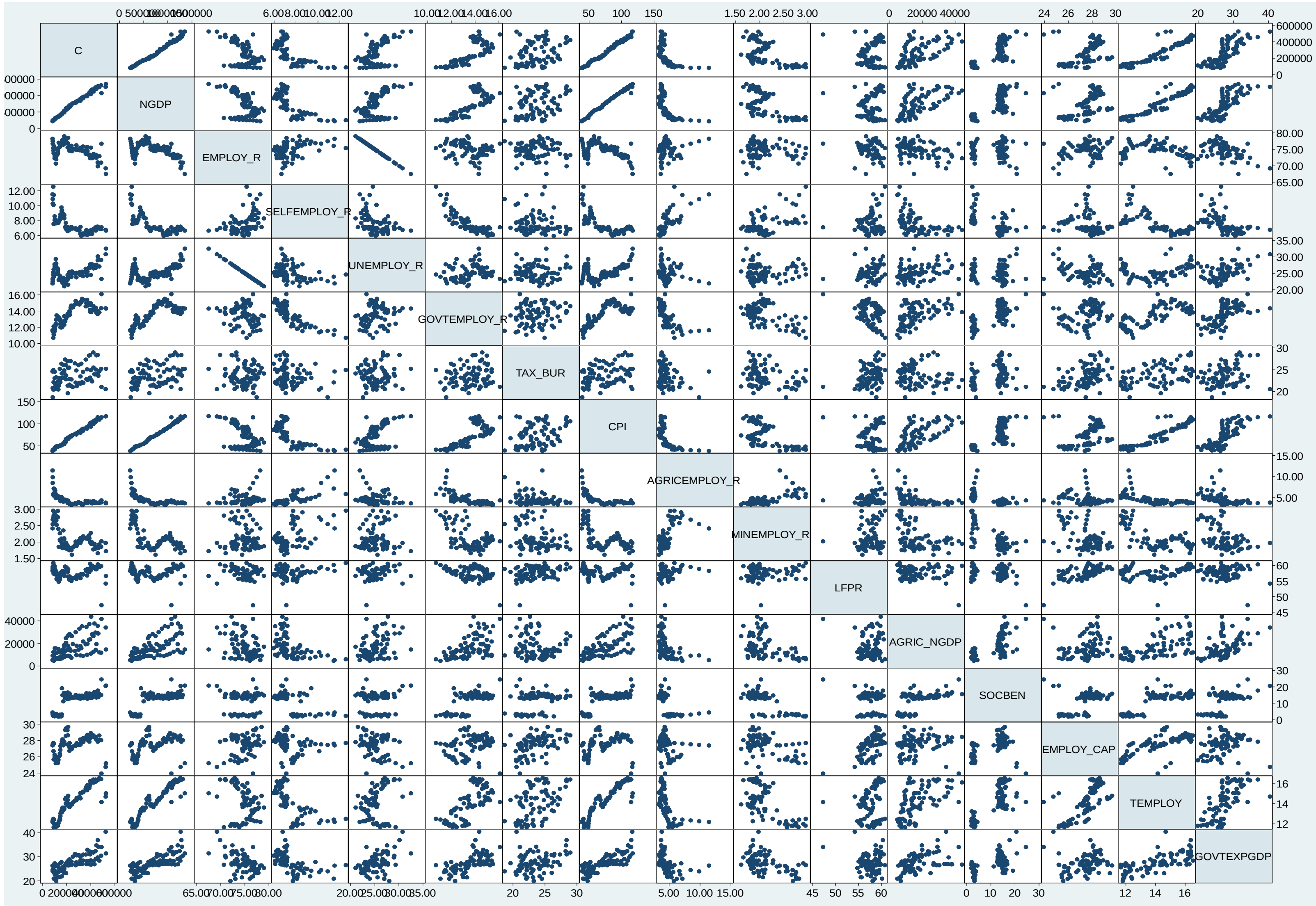
Appendix A.3: Description of variables and data sources.

Variable	Description	Source
Currency in circulation outside the banks	Notes and coins in circulation.	SARB
Currency in circulation outside the banks per NGDP	Currency outside banks normalised by NGDP.	Own ratio calculated from SARB and STATSSA data
Currency growth rate	Growth rate in notes and coins in circulation.	Own calculation derived from SARB data
Nominal GDP	Gross Domestic Product at current prices.	STATSSA
Real GDP	Gross Domestic Product at constant prices.	STATSSA
NGDP growth rate	The growth rate of NGDP.	Own calculation derived from STATSSA data
NGDP per capita	GDP at current prices normalised by the population size.	Own calculation derived from STATSSA and World Bank data
Labour Force Participation rate	The number of persons in the labour force relative to the population between 15 and 64 years.	STATSSA
Total Employment	The total number of employed individuals.	STATSSA
Employment rate	The number of employed individuals relative to the total labour force.	STATSSA
Employment per capita	The number of employed individuals normalised by the total population.	Own calculation derived from STATSSA and World Bank data
Self-employment rate	These are own account workers with one or more partners and have not engaged on a continuous basis any employees to work for them	Own calculation derived from STATSSA data

Variable	Description	Source
	during the reference period. The rate is calculated as a proportinof the total labour force.	
Unemployment rate	The number of unemployed people relative to the total labour force.	STATSSA
Government Employment rate	The number of people employed in the public service as a proportion of total labour force.	Own calculation derived from STATSSA data
Agriculture, Forestry & Fishing NGDP	The nominal GDP of the agriculture, forestry and fishing sector.	STATSSA
Agricultural employment rate	The number of people employed in the agriculture, forestry and fishing sector as a proportion of total labour force.	STATSSA
Mining share in employment	The number of people employed in the mining sector as a percentage of total labour force.	STATSSA
Mining NGDP	The NGDP of the mining sector.	STATSSA
Tax Burden	Tax revenue normalised by NGDP.	Own calculation derived from SARB and STATSSA data
Tax Revenue	Total net national government tax revenue	SARB
Deposit interest rate	The rate paid by commercial banks on cash deposits of account holders.	IMF
CPI headline index	The official measure of inflation in South Africa.	STATSSA

Variable	Description	Source
Social benefits paid by the government as a percentage of total government expenditure	These are payments received by households from government. The amount is expressed as a proportion of total government expenditure.	SARB

Appendix A.4: Scatterplot of MIMIC variables.



Appendix B.1: Augmented Dickey-Fuller test.

Currency

Level (0)

```
. dfuller lncu, trend lags(5)

Augmented Dickey-Fuller test for unit root         Number of obs   =        78

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -1.436           -4.088            -3.472            -3.163
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.8500
```

Level (1)

```
. dfuller d.lncu, trend lags(5)

Augmented Dickey-Fuller test for unit root         Number of obs   =        77

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -3.730           -4.091            -3.473            -3.164
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0204
```

NGDP

Level (0)

```
. dfuller lnNGDP, trend lags(4)

Augmented Dickey-Fuller test for unit root         Number of obs   =        79

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)             1.619           -4.086            -3.471            -3.163
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 1.0000
```

Level (1)

```
. dfuller d.lnNGDP, trend lags(4)

Augmented Dickey-Fuller test for unit root         Number of obs   =        78

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -3.997           -4.088            -3.472            -3.163
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0089
```

SOCIAL BENEFITS AS PERCENTAGE OF TOTAL GOVERNMENT EXPENDITURE.

Level (0)

```
. dfuller lnsocben, trend lags(2)

Augmented Dickey-Fuller test for unit root         Number of obs   =        81

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -2.053           -4.082            -3.469            -3.161
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.5724
```

Level (1)

```
. dfuller d.lnsocben, trend lags(2)

Augmented Dickey-Fuller test for unit root         Number of obs   =        80

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -6.863           -4.084            -3.470            -3.162
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0000
```

UNEMPLOYMENT RATE.

Level (0)

```
. dfuller lnunempl, trend lags(4)

Augmented Dickey-Fuller test for unit root         Number of obs   =        79

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -1.309           -4.086            -3.471            -3.163
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.8856
```

Level (1)

```
. dfuller d.lnunempl, trend lags(4)

Augmented Dickey-Fuller test for unit root         Number of obs   =        78

                              ——— Interpolated Dickey-Fuller ———
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
─────────────────────────────────────────────────────────────────────────────
 Z(t)            -3.985           -4.088            -3.472            -3.163
─────────────────────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0092
```

TAX BURDEN.

Level (0)

```
. dfuller lntb, noconstant lags(5)

Augmented Dickey-Fuller test for unit root         Number of obs   =        78

                               ———————— Interpolated Dickey-Fuller ————————
                  Test         1% Critical       5% Critical      10% Critical
                Statistic          Value             Value             Value
——————————————————————————————————————————————————————————————————————————————
 Z(t)              0.138            -2.609            -1.950            -1.610
```

Level (1)

```
. dfuller d.lntb, noconstant lags(5)

Augmented Dickey-Fuller test for unit root         Number of obs   =        77

                               ———————— Interpolated Dickey-Fuller ————————
                  Test         1% Critical       5% Critical      10% Critical
                Statistic          Value             Value             Value
——————————————————————————————————————————————————————————————————————————————
 Z(t)             -3.888            -2.609            -1.950            -1.610
```

INTEREST RATE.

Level (0)

```
. dfuller lnint, trend lags(2)

Augmented Dickey-Fuller test for unit root         Number of obs   =        81

                               ———————— Interpolated Dickey-Fuller ————————
                  Test         1% Critical       5% Critical      10% Critical
                Statistic          Value             Value             Value
——————————————————————————————————————————————————————————————————————————————
 Z(t)             -2.851            -4.082            -3.469            -3.161
——————————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 0.1788
```

Level (1)

```
. dfuller d.lnint, trend lags(2)

Augmented Dickey-Fuller test for unit root         Number of obs   =        80

                               ———————— Interpolated Dickey-Fuller ————————
                  Test         1% Critical       5% Critical      10% Critical
                Statistic          Value             Value             Value
——————————————————————————————————————————————————————————————————————————————
 Z(t)             -4.080            -4.084            -3.470            -3.162
——————————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 0.0068
```

SELF EMPLOYMENT RATE.

Level (0)

```
. dfuller lnselfe, trend lags(1)

Augmented Dickey-Fuller test for unit root         Number of obs   =        82

                               ———————— Interpolated Dickey-Fuller ————————
                  Test         1% Critical       5% Critical      10% Critical
                Statistic          Value             Value             Value
——————————————————————————————————————————————————————————————————————————————
 Z(t)             -3.295            -4.080            -3.468            -3.161
——————————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 0.0671
```

Level (1)

```
. dfuller d.lnselfe, trend lags(1)

Augmented Dickey-Fuller test for unit root         Number of obs   =        81

                               ———————— Interpolated Dickey-Fuller ————————
                  Test         1% Critical       5% Critical      10% Critical
                Statistic          Value             Value             Value
——————————————————————————————————————————————————————————————————————————————
 Z(t)             -8.736            -4.082            -3.469            -3.161
——————————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 0.0000
```

Appendix B.2: VECM / CDA results.

Vector error-correction model

Sample: 2000q4 - 2020q4				Number of obs	= 81
				AIC	= -21.43052
Log likelihood = 985.9362				HQIC	= -20.03101
Det(Sigma_ml) = 6.31e-20				SBIC	= -17.94231

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_lnCU	16	.019358	0.8659	419.8843	0.0000
D_lnNGDP	16	.027981	0.6266	109.0914	0.0000
D_lnINT	16	.064406	0.4365	50.35247	0.0000
D_lnUNEMPL	16	.047339	0.4530	53.83284	0.0000
D_lnTB	16	.04239	0.8972	567.4312	0.0000
D_lnSELFEMPL	16	.051816	0.3290	31.87467	0.0104
D_lnSOCBEN_GOV~X	16	.271173	0.3939	42.23765	0.0004

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_lnCU						
_ce1						
L1.	-.0289003	.0087761	-3.29	0.001	-.0461011	-.0116995
lnCU						
LD.	-.6091183	.0834129	-7.30	0.000	-.7726046	-.4456319
L2D.	-.2000367	.1196142	-1.67	0.094	-.4344762	.0344027
lnNGDP						
LD.	-.1329877	.0995469	-1.34	0.182	-.3280961	.0621207
L2D.	.2875549	.1077669	2.67	0.008	.0763357	.4987741
lnINT						
LD.	-.0561861	.0444321	-1.26	0.206	-.1432714	.0308993
L2D.	.0065632	.0409711	0.16	0.873	-.0737387	.0868652
lnUNEMPL						
LD.	-.1187741	.0611961	-1.94	0.052	-.2387163	.0011681
L2D.	-.2274102	.0700509	-3.25	0.001	-.3647075	-.0901129
lnTB						
LD.	-.2677764	.0459545	-5.83	0.000	-.3578456	-.1777071
L2D.	-.2962727	.0389783	-7.60	0.000	-.3726688	-.2198767
lnSELFEMPL						
LD.	.0597879	.0482026	1.24	0.215	-.0346874	.1542632
L2D.	-.0241031	.0492864	-0.49	0.625	-.1207027	.0724965
lnSOCBEN_GOVTEX						
LD.	-.0035551	.0096262	-0.37	0.712	-.022422	.0153119
L2D.	.0179147	.0086264	2.08	0.038	.0010072	.0348222
_cons	.0175973	.0065648	2.68	0.007	.0047305	.0304641

Appendix B.3: SEM /MIMIC results.

```
lavaan 0.6-9 ended normally after 17 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         7

                                                  Used
  Number of observations                            83

Model Test User Model:

  Test statistic                                 0.074
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.964

Model Test Baseline Model:

  Test statistic                                42.437
  Degrees of freedom                                 7
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.190

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)               -213.087
  Loglikelihood unrestricted model (H1)       -213.050

  Akaike (AIC)                                 440.174
  Bayesian (BIC)                               457.106
  Sample-size adjusted Bayesian (BIC)          435.026

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent confidence interval - lower         0.000
  90 Percent confidence interval - upper         0.000
  P-value RMSEA <= 0.05                          0.970

Standardized Root Mean Square Residual:

  SRMR                                           0.008

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured


Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  n =~
    dNGDPn           -1.000                               -0.518   -0.522
    dLFPRn            0.606    0.253    2.391    0.017    0.314    0.316

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  n ~
    dTAXBURn          0.334    0.082    4.075    0.000    0.645    0.641
    dMINEMPLRn        0.189    0.074    2.550    0.011    0.365    0.363
    dGOVTEXPGDPn      0.275    0.079    3.491    0.000    0.532    0.528

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .dNGDPn ~~
   .dLFPRn            0.230    0.091    2.527    0.011    0.230    0.289

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .n                 0.000                                0.000    0.000
   .dNGDPn            0.716    0.111    6.442    0.000    0.716    0.728
   .dLFPRn            0.886    0.138    6.442    0.000    0.886    0.900

R-Square:
                   Estimate
    n                 1.000
    dNGDPn            0.272
    dLFPRn            0.100
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