UNIVERSITY OF KWAZULU-NATAL

LEARNER ERRORS AND MISCONCEPTIONS IN RATIO AND PROPORTION

A Case Study of Grade 9 Learners From a Rural KwaZulu-Natal School

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DECLARATION

I declare that 'Learner errors and misconceptions in ratio and proportion' is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of a complete reference.

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ABSTRACT

Proportionality is the content domain of mathematics that is rooted in ratio and proportion. It is believed to be vital for problem solving and reasoning, which are key cognitive domains of mathematics teaching and learning. Hence, ratio and proportion forms part of curricula for all countries. Studies carried out in different parts of the world found that while learners can do simple and routine manipulations of ratio and proportion, they struggle to solve problems that involve these concepts. Researchers apportion the blame for this to the strategies that learners use to solve the problems. Researchers found that learners use flawed strategies due to misconceptions that learners have on ratio and proportion.

The purpose of the study is to explore learner errors and misconceptions on ratio and proportion. A test that comprised of questions that are appropriate to the National Curriculum Statement (NCS), for General Education and Training (GET) band, was used to collect data. Items in the instrument were selected and adapted from a tool used in Concepts in Secondary Mathematics and Science (CSMS) study. The participants in the study are 30 Grade 9 learners from a rural school in KwaZulu-Natal (KZN).

The findings of the study are that learners have a limited knowledge and understanding of ratio and proportion, hence their performance in items on the topic is poor. A great proportion of the learners have serious misconceptions of ratio and proportion. They use incorrect strategies to solve problems on ratio and proportion that produce errors. The errors and misconceptions they exhibit are not different from those observed by similar studies conducted in other parts of the world.

The study recommends a structured focus on ratio and proportion because the topic is fundamental to proportional reasoning. It recommends clarity for teacher trainers, textbook writers and teachers on what learners need to learn on ratio and proportion. It recommends serious exploration of errors and misconceptions on ratio and proportion, and a teaching approach that considers errors and misconceptions as opportunities for learning.

Table of Contents

List of Tables

Chapter 1

Background to the study and rationale for the study

1.1 Introduction

Mathematics is one of the oldest subjects studied at school. It was initially done as two separate subjects called Arithmetic and Geometry. The term mathematics emerged among Greeks during the period 600 BC to 300 BC as a common name to describe both Arithmetic and Geometry (Burton, 2007). The subject has developed extensively, with many new fields of study emerging since the 15th century. Its role and the impact on the socio-economic development of any country cannot be overemphasised. That is why school curricula of all countries are structured such that they include mathematics. In South Africa, all learners must do mathematics up to Grade 9. For Grades 10 to 12, subject packages are structured such that learners must do either mathematics or mathematical literacy.

While the subject is held in high esteem, learner performance in it is poor. This has been the case since time immemorial and this is a global trend. Arnold (2008) cautions mathematics communities that this situation has led to a very dangerous myth of all, that:

> Mathematics is different from all the other disciplines. Its demands are far more specialised, and so it is right and just that most students of mathematics will fall by the wayside, because mathematics is for the chosen few – the majority of people are just not meant to “get” mathematics. (p. 4)

He emotionally argues that this myth justifies poor mathematics teaching. It justifies poorly developed curricula and badly written books, and puts the blame for poor performance squarely on the shoulders of the learner.

I have taught mathematics to secondary school children and teacher trainees for several years. I am currently a subject advisor for mathematics, and also a part time teacher of mathematics. I do agree with the notion that the demands of mathematics are more specialised, but I also strongly disagree that it is for the chosen few. I believe that if mathematics is properly taught, a high percentage of learners would perform well in it. In fact the Far East countries such as Singapore, Hong Kong and Korea are witness to this. In excess of eighty percent of learners from these countries performed very well in the Trends in International Mathematics and Science Study and the Programme for International Student Assessment (PISA). This suggests that mathematics teachers in the countries that are renowned for poor performance in these studies need to explore forms of intervention which could alleviate the prevalent condition.

I do acknowledge that a lot has been done, but also believe that there is more that could be done. There are teachers out there who are successfully teaching mathematics. I have come across learners who love and enjoy mathematics. I strongly believe that the reason for that is that, both those teachers and learners understand mathematics and confront its challenges with confidence. The confidence comes from the ability to construct knowledge that makes sense to them. Both those teachers and learners have a relational understanding of mathematics. They have a relational understanding of key topics to mathematics like ratio and proportion. Skemp (1976) describes people with relational understanding as people who know "both what to do and why" (p. 2); people that are characterized by a rich interconnected web of ideas.

To make a difference in this country we need to increase the number of teachers and learners with the above qualities. It is for this reason that I have conducted a study of this nature. The paragraphs that follow will first present the purpose of my study. The performance of South African learners in mathematics and the rationale for the study will then be presented. Lastly, key concepts embedded in the research topic will be put into perspective.

1.2 Purpose of the study and key research questions

This study aims to make a contribution towards exterminating the myth that mathematics is for the chosen few; it is therefore right for a large proportion of learners to fail it. It intends to contribute towards improving the performance of our beloved country in mathematics. The study will confine itself to the topic on ratio and proportion. The purpose of this study is to explore the errors and misconceptions exhibited by Grade 9 learners in a particular school in KwaZulu-Natal while they are solving problems based on ratio and proportion. The corresponding research questions are:

- How do learners in the selected school perform in assessment items based on ratio and proportion?
- What errors and misconceptions do these learners commit when they solve problems on ratio and proportion?
- Why do these learners commit the identified errors and misconceptions?

1.3 Performance of South African learners in mathematics

There is a worldwide interest in standards of learner performance in mathematics. Most countries are concerned about poor learner performance in mathematics revealed by international studies such as the Trends in International Mathematics and Science Study and the Programme for International Student Assessment. The concern justifiably arises from the fact that "mathematical knowledge is perceived to be the contributory factor in economic prosperity" (Tanner, Jones, & Davies, 2002, p. 20). For example, Engineering and Commerce are regarded as pillars of the economy and studies towards these qualifications require a good knowledge and understanding of mathematics.

The International Association for the Evaluation of International Achievements (EIA) has been running cross-national studies since 1959 (Reddy, 2006). The Trends in International Mathematics and Science Study is one of the studies that were conducted by Evaluation of International Achievements since 1995 for mathematics and science. South Africa participated in three Trends in International Mathematics and Science Study projects, and has, unfortunately, come at the bottom of the hierarchy for both mathematics and science (Simkins & Paterson, 2005). In fact, in the 2003 study, only ten percent of the South African Grade 8 learners achieved a score of 400 and above, where 400 and below, according to international benchmark indicates that a learner possesses only a basic knowledge of mathematics (Reddy, 2006). The results of the Trends in International Mathematics and Science Study are an indication that something needs to be done urgently to improve performance in mathematics.

A recent report released by the Department of Basic Education (DBE) on performance in Annual National Assessment (ANA) tasks, administered to Foundation Phase (Grades 1 to 3) and Intermediate Phase (Grades 4 to 6) learners of the General Education and Training (GET) band have revealed that learners experience serious challenges in solving elementary mathematical problems. The latest statistics presented to the Education Portfolio Committee revealed an average numeracy pass mark of 35% for Grade 3 in 2007 and an average mathematics mark of 27% for Grade 6 in 2004 (Tyobeka, 2009). A report of a similar assessment conducted in 2011 does not reflect any improvement. The average pass percentage for Grade 3 in mathematics was 28%; and 30% for Grade 6 (DBE, 2011). This once more depicts a bleak performance by the South African learners in mathematics.

1.4 Rationale for the study

The topic on which the study will focus is ratio and proportion. The study focuses on ratio and proportion because this is one of the topics included in the South African curriculum. The National Curriculum Statement for mathematics, Grades R to 9, as well as the newly released Curriculum and Assessment Policy Statement (CAPS) for Senior Phase (Grades 7 to 9) mathematics, expects Grade 9 learners to be able to solve problems on ratio and proportion (DoE, 2002). I believe that one of the reasons for the inclusion of this topic in the curriculum is that ratio is central to the development of proportional reasoning (Chick & Harris, 2007); and “proportional reasoning is the cornerstone of algebra and a wide variety of topics in mathematics” (Van de Walle, 2007, p. 353). Rhode Island Department of Education (RIDoE) regards proportional reasoning as much more than the ability to set up and solve an equation. It considers proportional reasoning to entail “being able to distinguish between proportional and non-proportional relationships” (RIDoE, 2007, p. 10).

The Department of Education and Early Childhood Development (DEECD) in the State of Victoria, Australia, maintains that the study of proportional reasoning is one of the most important areas of mathematics for everyday, workplace and scientific tasks. This Department of Education asserts that proportional reasoning “underlies much of the mathematics curriculum, including work on percentages, ratio, reading and making scales, reduction and enlargement, similar triangles, construction of pie charts, linear functions, trigonometry, etc". (DEECD, 2009, p. 1). Hart (1988) supports this view when she states that:

> Ratio and proportion have always been important topics in the mathematics curriculum. In the secondary school, science, geography and art also need the concept of proportion, and teachers expect students to be able to transfer mathematical expertise to problems in these and other disciplines. (p.198)

Although this topic is of such great importance in mathematics and education at large, extended research in the field of proportional reasoning reveals that solving ratio and proportion problems is a very difficult task for most pupils in the middle school years throughout the world. One reason for this is that, "while ratio and proportion have long been included as topics of study in the middle-grades mathematics curriculum, these topics have been treated in a cursory fashion within a unit on number" (Misailidou & Williams, 2002, p.2). Students have thus "developed a limited view of proportions rather than a rich view of proportions and their use in thinking about multiplicative relationships between quantities" (Silver, 2000, p. 22).

The above arguments suggest that learners with sound conceptions of ratio and proportion are more likely to cope with challenges of school mathematics than those who struggle with the concept. A sound conception of ratio and proportion might also have a positive impact on learner performance in other school subjects. Interventions that could enhance performance in ratio and proportion are essential. Studies that identify challenges that learners are faced with in ratio and proportion, could contribute important knowledge to these endeavors. I believe that one such intervention is to explore learner errors and misconceptions as they solve problems on this topic.

1.5 Ratio and proportion

Olivier (1992a) argues that "many traditional textbooks mainly define a ratio as a fraction and proportionality as two equal ratios, so that a study of ratio and proportion is simply reduced to an application of known fractions". He considers this approach to be totally inadequate. He then distinguishes between two different meanings and uses of ratio. Firstly, he refers to a ratio as a comparison between the elements of the same set. Secondly, he refers to it as "a rate between corresponding elements of two sets that form proportionality". He then defines a ratio as "a property belonging to two sets; a special kind of relationship between the sets, namely a relationship that is described by formulae of the type $y = kx$; where x and y are variables and k a constant" (p. 309). The formula indicates that the numbers x and y have a multiplicative relationship and y can be obtained by multiplying x with a certain number k.

Singer, Kohn and Resnick (1997) refer to the first conception of ratio by Olivier (1992a) as "a multiplicative relationship between two quantities" from the same measure space "or numbers" (p. 117). The authors thus refer to what Olivier terms 'elements of the same set' as quantities from the same measure space. They (Singer et al., 1997) refer to Olivier's second conception of a ratio, as an "intensive quantity" (p. 117). They describe an intensive quantity as a multiplicative relationship between quantities from different measure spaces, which they regard as a special type of a ratio. They also define a proportion as the "equivalence of two ratios" (p.117). Long, Dunne and Craig (2010) put ratio in the same category with fractions, rate, percentage and proportion that constitutes the multiplicative conceptual field (MCF). The authors group these concepts together because they are multiplicative, "related yet subtly distinct from one another" (p. 79). This underscores the notion that a ratio is a multiplicative relationship. Reference to ratio and proportion in this study will be in this context. While the National Curriculum Statement and South African books in general portray ratio and rate as different concepts, the study will regard rate as a special ratio.

1.6 Errors and misconceptions

Since the study mainly intends to explore errors and misconceptions, it is appropriate that the concepts be also elucidated upon. Olivier (1992b) distinguishes between slips, errors and misconceptions as follows:

> Slips are wrong answers due to processing; they are not systematic, but are sporadically carelessly made by both experts and novices; they are easily detected and are spontaneously corrected… Errors are wrong answers due to planning; they are systematic in that they are applied regularly in the same circumstances. Errors are symptoms of the underlying conceptual structures that are the cause of errors. It is these underlying beliefs and principles in the cognitive structure that are the cause of systematic conceptual errors that I shall call misconceptions (pp. 196–197).

Olivier (1992b) explains how errors differ from misconceptions, and at the same time clearly articulate how the two are related. Olivier tells us that an error may result from a misconception or other factors such as, for example, carelessness, while misconceptions are indicators of poor understanding (Spooner, 2002). He also pronounces that "misconception denote a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic and non-systematic errors" (Nesher, 1987, p. 35). Errors, unlike slips, are "systematic, persistent and pervasive patterns of mistakes performed by learners across a range of contexts" (Brodie & Berger, 2010, p. 169).

1.7 Conclusion

The state of mathematics in the country portrayed above is really appalling. It needs the urgent attention of all stakeholders involved with education of mathematics in the country. The situation warrants the exploration of learner performance in key topics like ratio and proportion. It is our responsibility as mathematics teachers to identify key topics that are problematic in mathematics. If topics like ratio and proportion, which are fundamental to the understanding of other mathematics topics and other subjects, are well taught, this may result in a turnaround in the current situation. We also need to explore pertinent errors and misconceptions in those topics because knowledge of errors and misconceptions should at all times inform teaching. The next chapter will explore in detail the state of mathematics in our country, specifically in items based on ratio and proportion. It will also look into learner errors and misconceptions identified by other research studies.

Chapter 2

Literature review and theoretical framework

2.1 Introduction

Since the standards of learner achievements in mathematics are of great concern for both the developed and the developing world, extensive studies on learner performance in mathematics have been undertaken to determine interventions that may improve the situation (Tanner et al., 2002). A literature review of some research studies on learner performance in mathematics will be undertaken hereunder. The chapter will first examine performance of South African learners in selected international mathematics assessments. It will also look into observations on the teaching of ratio and proportion; and strategies that learners use to solve problems on ratio and proportion. The chapter will also present learner errors and misconceptions on ratio and proportion identified by some research studies. The theoretical and conceptual framework for the study will then be outlined.

2.2 Performance of South African learners in international assessments

The intention of this section is to paint a vivid picture of the performance of South African learners in international assessments on mathematics. The study will only present observations on learner performance from the Southern Africa Consortium for Monitoring Educational Quality Project and the performance of South African learners in the Trends in International Mathematics and Science Study on items relevant to the research topic.

In 2000 and 2007, South Africa participated in Southern Africa Consortium for Monitoring Educational Quality II and Southern Africa Consortium for Monitoring Educational Quality III projects. Fourteen countries from southern and eastern Africa participated in the project in 2000 and 15 participated in 2007. A random sample of 3 416 Grade 6 learners from 169 South African public schools was tested in reading (literacy) and mathematics (numeracy) in 2000. For 2007, the sample size almost tripled as 9 071 learners from 392 schools were sampled for participation. The learners performed particularly poorly in mathematics. Rasch measurement techniques were used to analyse learner scores. The Rasch scale had a pre-determined mean score of 500 in 2000 and

509,5 in 2007. The mean Rasch scores of 486,2 and 494,8 for 2000 and 2007 respectively, obtained by South African learners, were below the pre-determined mean Rasch scores. The results from both Southern Africa Consortium for Monitoring Educational Quality II and Southern Africa Consortium for Monitoring Educational Quality III are shown in Table 2.1 .

Table 2.1: Performance in SACMEQ

SACMEQ	**YEAR**	**LEVELS**							
		1	2	3	4	5	6	7	8
II	2000	7.8	44.4	23.8	8.8	6.1	5.8	2.1	1.3
III	2007	5.5	34.7	29.0	15.4	7.1	5.9	1.9	0.6

Taken and adapted from SACMEQ Policy Issues Series, Number 2 (Makuwa, 2010)

A description of performance levels is given in Table 2.2.

Table 2.2: SACMEQ Levels

	SACMEQ level	**School grade**
1	Pre-numeracy	2 and lower
2	Emergent numeracy	3
3	Basic numeracy	4
4	Beginning numeracy	5
5	Competent numeracy	6
6	Mathematically skilled	7
7	Concrete problem-solving	7+
8	Abstract problem-solving	7++

Taken from a report by Moloi (2006)

The results show a minimal change in achievement levels. While 52,2% of learners performed at Level 2 and below (level of a Grade 3 learner or lower) in 2000, the percentage dropped to 40,2% in 2007. On the other hand, proportion of learners performing at Level 7 and above (level of a Grade 7 learner or above) declined from 3,4% in 2000 to 2,5% in 2007. For both years it is noted that percentages diminished up the competency ladder (Moloi & Straus, 2005; Moloi & Chetty 2010). South Africa was placed ninth, both in 2000 and 2007, an indication that our performance in mathematics is below that of most countries in the southern and south eastern region of Africa.

South Africa also participated in Trends in International Mathematics and Science Study 1995, Trends in International Mathematics and Science Study 1999 and Trends in International Mathematics and Science Study 2003, but not in Trends in International Mathematics and Science Study 2007 (Long, 2007). She (South Africa) also participated in Trends in International Mathematics and Science Study 2011, the results of which are still pending. The previous chapter generally alluded to the dismal performance by learners from South Africa in the Trends in International Mathematics and Science Study. In this chapter, I will present learner performance in one item from the Trends in International Mathematics and Science Study 1999, and performance in one item from the Trends in International Mathematics and Science Study 2003. The items were selected because they assess elementary knowledge and skills on ratio and proportion. The items are presented to show how the performance of South African learners compares with that of learners from other parts of the world. There were 38 countries who participated in Trends in International Mathematics and Science Study 1999 and 45 in Trends in International Mathematics and Science Study 2003.

Before I present the selected items, let me briefly illustrate how the Trends in International Mathematics and Science Study 1999 and the Trends in International Mathematics and Science Study 2003 were structured. The two studies consisted of multiple choice questions. The questions were categorized into domains as shown in Table 2.3.

Table 2.3: Content and cognitive domains for TIMSS 1999 and TIMSS 2003

TIMSS 1999		**TIMSS 2003**	
Content Domain	**Cognitive Domain**	**Content Domain**	**Cognitive Domain**
• Fractions and number sense • Algebra • Measurement • Geometry • Data representation, analysis, and probability	• Knowing • Using routine procedures • Investigating and problem solving • Mathematical reasoning • Communicating	• Algebra • Data • Geometry • Measurement • Number	• Knowing facts and procedures • Using concepts • Solving routine problems • Reasoning

Taken from Mathematics Concepts & Mathematics Items: Williams et al., (2006) Ratio and proportion questions are in the content domain *Fraction and number sense* in the Trends in International Mathematics and Science Study 1999 and fall under the content domain *Numbers or Algebra* in the Trends in International Mathematics and Science Study 2003. When the results are presented, the international benchmark, content domain and cognitive domains are provided for each item. The percentage of students in each participating country that answered the question correctly is also given (Williams, Jocelyn, & David, 2006).

Hereunder is a brief comparison of our performance to that of other countries. The following item on ratio was in the 8th Grade Trends in International Mathematics and Science Study 1999.

> If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food (Williams *et al*., 2006, p. 3).

The content domain for the item is *Fractions and number sense.* The cognitive domain is *Investigating and solving problems.* The international average was 69%. Only 37% of South African learners correctly answered this question. In Singapore, 90% of the learners correctly answered the question. South Africa had the least number of learners that answered the question correctly, and our average mark was far below the international average (Williams, Jocelyn, & David, 2006).

The next item was in the 8th Grade Trends in International Mathematics and Science Study 2003.

> If 12/*n* = 36/21, then *n* equals …….(Williams et al., 2006, p. 93).

The item falls into *Algebra* content domain and *Knowing facts and procedures* cognitive domain. The international average for the item was 64%. Only 26% of the learners from South Africa correctly answered the question. 93% of learners from Singapore correctly answered this question. Once again, South Africa occupied the last position in this question (Williams, Jocelyn, & David, 2006).

The results show that South Africa does have learners that perform well, however, their proportion is too small. They show that South Africa performs far below the international average. The results also show that our learners struggle even with questions of low cognitive level, questions that require simple procedural proficiency. Long (2007) in the investigation she conducted on the Trends in International Mathematics and Science Study items, found that in items which fell into the domain ratio, proportion and percentage, South African Grade 8 learners do not have the *operational* understanding of this domain. Grade 8 learners, according to Piaget's stages of cognitive development, should be at a formal operational stage. Learners at this stage have the ability to systematically solve a problem in a logical and methodical way. They can reason both inductively and deductively. Long (2007) observed a lack of such qualities in South African learners' responses to the Trends in International Mathematics and Science Study.

2.3 Ratio and proportion strategies, errors and misconceptions

To understand learner errors, one has to look at the methods or strategies that the learners use to arrive at the incorrect solutions. Some researchers argue that some methods used by children lead to error, which in turn does not automatically disappear through maturation (Hart, 1988). This implies that errors are products of incorrect strategies or products of incorrect use of correct strategies. This section will explore learner errors identified by other studies through strategies or methods used by learners. It will look into both correct and incorrect strategies.

Misailidou and Williams (2002) conducted a study on strategies used to solve ratio problems among 232 learners from four schools in the North West of England, and nine trainee teachers of mathematics. The main purpose of the study was to contribute to teachers' awareness of their pupils' strategies and misconceptions in the field of *ratio*: a topic that is difficult to teach and learn in middle school years. Three correct strategies were observed among learners and teachers, namely: *for every* strategy or *unit value* method; the multiplicative strategy (within measure space approach); and the cross multiplication method. Incorrect strategies identified were: c*onstant sum* strategy; the *constant difference* or *additive* strategy; the *incomplete strategy*; the incomplete application of *build up method*; and the *magical doubling* (Misailidou & Williams, 2002; 2003). Each of the strategies will be briefly discussed next.

The *for every* strategy or *unit value* method entails finding the simplest ratio first, and then multiplying by a factor that yields the required result. Consider the paint problem given below:

> *Sue and Jenny want to paint together.*
> *They want to use each exactly the same colour (sic).*
> *Sue uses 3 cans of yellow paint and 6 cans of red paint. Jenny uses 7 cans of yellow paint.*
> *How much red paint does Jenny need?* (Misailidou & Williams, 2002, pp. 3-4)

The strategy was applied in this problem by first simplifying the ratio of Sue's yellow paint to red paint to 1 : 2. Then both terms in the simplified ratio are multiplied by 7 to arrive at the answer. The strategy was observed among 17,2% of the participants. Chunlian (2008), also observed this use of the strategy in a study that he conducted among 1002 students from Singapore and 1070 students from China. His study investigated strategies used by learners for solving word problems on speed, which is a rate. He refers to this strategy as a *unitary method*. He describes it as a method where the learner determines a value equivalent to one unit of a quantity, and then uses that value as a factor to multiply by.

Md-Nor (1997) also identified the use of this strategy when she investigated the teaching and learning of ratio and proportion in Malaysian secondary schools. Her reason for the selection of the topic was the demand for the application and understanding of ratio and proportion across the school curriculum. She investigated learning and teaching of ratio and proportion in a group of 160 students and 5 teachers from 2 randomly selected schools. The Mr Short and Mr Tall problem was one of the items in the test that she used in the study. About 24% of the learners used the unitary strategy. Hart (1988) also observed the strategy among English learners during the Concepts in Secondary

Mathematics and Science and Strategies and Errors in Secondary Mathematics (SEMS) projects. She however, points out that most British schools teach the unitary method.

The multiplicative strategy (within measure space approach) entails determining a ratio of measures from the same space and using it as a factor. Misailidou and Williams (2002, 2003) observed the strategy in one participant. If the strategy is used in the paint problem, paints of the same colour are compared. The ratio of Jenny's yellow paint to Sue's yellow paint is 7 : 3. Therefore Jenny used 7/3 times as much paint as Sue. Thus, red paint used by Jenny is $7/3 \times 6 = 14$. Hart (1988) also identified a strategy similar to this one in her projects which she refers to as the *within or between comparison strategy*. She explains *between comparison strategy* as comparison of quantities that are measured in the same unit or quantities of the same kind or qualities to come up with a ratio. That ratio will then be used as a factor to multiply by. The *within comparison strategy* is comparison of quantities from different measure spaces. Md-Nor (1997) refers to the multiplicative strategy as the *scalar strategy* because a multiplicative scalar relationship between the two quantities in the same unit is established. For example, Jenny's red paint = $k \times$ Sue's red paint, where k is a scalar.

The cross multiplication method is based on setting up a proportion, for example, in the paint problem $3/6 = 7/x$. A variable x is used for Jenny' red paint. Thus, $3x = 42$ and $x = 14$. Chunlian (2008) refers to this strategy as *algebraic method*. He says one or more unknowns are chosen as variables and the equation(s) is (are) set up. In the paint problem the unknown is denoted by x and the equation set up is $3x = 42$. Md-Nor (1997) also identified the strategy among Malaysian children. She refers to it as the *rule of* 3 since it involves use of three known values to find the fourth value (unknown). In her study, use of the strategy was observed among seven percent of the participants. Hart (1984) and Olivier (1992a) refer to this strategy as the use of the formula, $x/a = y/b$.

In the paint problem, the *constant sum* strategy assumes that the total number of cans of paint for Sue and Jenny should be the same. Sue has 3 cans + 6 cans = 9 cans of paint. Jenny thus needs 2 cans of red paint since she already has 7 cans of yellow paint. The strategy was used by 34,5% of the learners in this item in Misailidou and Williams (2002) study.

In the *constant difference* or *additive strategy*, the relationship within the ratios is computed by subtracting one term from another, and then the difference is applied to the second ratio. The strategy is sometimes just referred to as *addition strategy*. Md-Nor (1997) identified the error among 20% of her participants. Hart (1988) refers to the strategy as the *incorrect addition strategy* and describes it as the addition of a fixed amount. In the Concepts in Secondary Mathematics and Science study, the *incorrect addition strategy* was the most common misconception identified. This misconception is linked to" incorrect reasoning that results from viewing an enlargement as requiring an operation of addition and not multiplication" (Hart, 1984, p. 8). One important observation from the Concepts in Secondary Mathematics and Science study was that the adders were very often able to correctly solve the easier items using this incorrect strategy.

Misailidou and Williams (2003) also identified the additive strategy to be the dominant erroneous strategy used by their sample of participants. The researchers point out that the "additive strategy is the most commonly reported erroneous strategy in the research literature related to ratio and proportion" (Misailidou & Williams, 2003, p. 346). In their study, 53% of the participants used this strategy to solve the problem on Mr Short and Mr Tall. Long (2007) also found that in the Trends in International Mathematics and Science Study, the significant error made by South African learners was the "use of additive reasoning where multiplicative reasoning was required", and she attributes this to a developmental issue because "additive reasoning is the precursor of multiplicative reasoning" (p. 16).

The *incomplete strategy* constitutes using the same number given for the measure space. In the paint problem we are given that Sue needs 6 cans of red paint. Learners applying this strategy conclude that Jenny also needs 6 cans of red paint (Misailidou & Williams 2002, 2003). The strategy was used by 3,4% of the participants in the paint problem. This strategy links to what Hart (1988) terms *“naïve” strategies*, which incorporates “intuition, guessing or using the data in an illogical way” (p. 201).

Md-Nor (1997, p. 34) describes the building-up strategy as establishing a relationship within a ratio and then extending it to the second ratio by addition. For example, each colour of Jenny’s paint could be doubled and then a third of each colour is added to double the amount as shown below.

	Yellow	Red	
Jenny	3	6	
Double	*6*	*12*	
Third	*1*	*2*	
Double + Third	7	14	[This is Sue’s paint]

Hart (1988) refers to the strategy as the *“addition and scaling” strategy,* because it involves a multiplicative strategy combined with an additive one. Incomplete application of *build up method* refers to an incorrect application of the build up method. An example of the use of the strategy in the paint problem is doubling Sue’s paint and adding 1 to arrive at the amount of red paint required for Jenny.

The last incorrect strategy observed was the *magical doubling* method. The method refers to doubling to obtain an answer when doubling is inappropriate. Six percent of the participants applied this incorrect strategy to solve the famous *Mr Short’s / Tall’s* height problem and 13,8% used it to solve the *printing press* problem below.

> *Printing Press item*
> *A printing press takes exactly 12 minutes to print 14 dictionaries.*
> *How many dictionaries can it print in 30 minutes?* (Misailidou & Williams, 2002, p. 8)

Since there are problems on ratio and proportion where doubling is a correct strategy, Hart (1988) incorporates this strategy under the category of *using multiplication but not by the correct factor*, which I will refer to as the incorrect use of the multiplicative method.

Chunlian (2008), also mention *arithmetic methods*; *model drawing methods*; *guess and check methods*; *looking for a pattern*; and *logical reasoning* as other strategies he observed in his study. These strategies partly overlap with some of the strategies discussed above, and the researcher attributes the strategies to the teaching of various heuristics in Singapore. His study, however, does not elaborate on errors that learners made when solving the problems. A study undertaken by The Gatsby Charitable Foundation on ratio and proportion identified the following learner misconceptions: failure to realise that 2 : 8 is the same ratio as 1 : 4; believing that increasing a map scale increases map distance; and using direct instead of proportional division. The last misconception refers to situations where, if learners are told that 4 people do a job in two hours, they then say 2 people will do the same job in one hour (Graham, 2003).

Long (2007) found that in the Trends in International Mathematics and Science Study some responses were incorrect because of "incomplete reasoning". She says that learner "reasoning took them part way towards the answer" (p. 16). Brodie and Berger (2010) categorise this error as an "error of routine", and they call it a "halting signal". The researchers say,

> A halting signal may be regarded as a trigger for premature closure of routine. For example, a learner expecting an integer answer to a question may well regard her routine as complete when she derives an integer answer to one of the steps of the procedure (p.174)

The researchers give the following example to clarify the statement.

> The speed of light in space is 299 792 458 metres per second.
> What is **half** the speed, rounded to the nearest million? (p. 174)

A learner commits a "halting signal" error if he divides 299 792 458 by 2 to get 149 896 229 which is prematurely presented as the answer. The learner does not round to the nearest million to get the correct answer of 150 000 000.

While an error may result from a misconception or other factors such as carelessness, misconceptions are a product of a lack of understanding (Spooner, 2002). Nesher (1987) argues that "the notion of misconception denotes a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic and non-systematic errors" (p. 35). The strategies explained above are a witness to this.

2.4 Teaching of ratio and proportion

Some studies attribute learner misconceptions of ratio and proportion to the teaching of this topic. Md-Nor (1997) investigated the teaching and learning of ratio and proportion in Malaysian secondary schools. Her study actually investigated the relationship between teachers' pedagogical content knowledge (PCK), instructional classroom practice and students' learning; with a particular focus on the teaching and learning of ratio and proportion. Pedagogical content knowledge refers to "subject matter knowledge for teaching" and includes "an understanding of what makes the learning of specific topics easy or difficult" (Misailidou & Williams, 2002, p. 2). One way of identifying what makes learning easy or difficult is to study learner errors and misconceptions. The study found that teachers with strong pedagogical knowledge content successfully taught ratio and proportion. It is thus important for teachers to know and understand learner errors and misconceptions.

Chick and Harris (2007) also undertook a case study on the role of pedagogical content knowledge in the teaching of ratio. Their study examined the impact of the choice of examples and the context used by the teachers in the teaching of ratio. The focus of their study was on the actions of the teacher in the classroom and the examples the teacher used to illustrate ideas on ratio. The study observed what the teacher did; and the impact of this on learning in the classroom. The study found that examples which make fluent connections with a range of mathematical topics developed a better understanding of ratio in learners. Examples that make fluent connections should definitely consider learner errors and misconceptions. The study however, acknowledges that finding such examples is a big challenge. According to the researchers, pedagogical content knowledge is essential for the teacher to understand mathematics underlying ratio as well as the cognitive demands of the topic. Therefore, it is important for the identification of learner errors and misconceptions.

Chick and Harries (2007) further maintain that to successfully teach about the ratio, the teacher must have a profound understanding of fundamental mathematics that will enable him: to know how ratios relate to fraction understanding (e.g., the idea of equivalence); to understand that it is important to distinguish what is being compared (e.g., part : part or part : whole); and to realise that ratios entail multiplicative thinking rather than additive thinking. The teacher must have the knowledge of student thinking to recognise that some students may have difficulty in identifying the components being compared, or that they may respond to a question with a particular incorrect answer because they are thinking additively. For successful teaching, they emphasise importance of "knowledge of student thinking – both current and anticipated, together with knowledge of likely misconceptions" (pp 3-4) as this plays an important role in selecting examples that will promote learning.

Misailidou and Williams (2002) inform us that, research in the field of proportional reasoning reveals that solving ratio and proportion problems is a very difficult task for most pupils in the middle school years throughout the world. They then tell us that research studies have identified common errors and misconceptions in pupils' proportional reasoning which affect their learning. The researchers believe that the starting point for the effective teaching of the topic of ratio is the teachers' awareness of these misconceptions.

2.5 Theoretical framework for the study.

"John Dewey once said that theory is the most practical of all things. Theory is the stuff by which we act with anticipation of our actions' outcomes and it is the stuff by which we formulate problems and plan solutions to them" (Thompson, 1994, p. 229). "Facts can only be interpreted in terms of some theory" because "without any appropriate theory, one cannot even state the facts" (Olivier, 1992b, p. 193). If the researcher wants to understand learners' misconceptions when they solve problems on ratio and proportion, an appropriate theory must be identified. An appropriate theory will be informed by how one views learning in general. It will be informed by looking at the role of errors and misconceptions in the learning process.

This research draws on a constructivist perspective, within which learners are viewed as actively constructing their own mathematical understanding as they participate in practices and whilst interacting with others (Cobb, Jaworski, & Presmeg, 1996). The theory of constructivism posits that students learn by actively constructing their own knowledge, knowledge is created not passively received and views learning as a social process (Clements & Battista, 1990; Jaworski, 1994). Furthermore all knowledge is seen to be constructed by individuals rather than transferred directly by an expert, such as a teacher, parent or book, to the learner. This means that the learner plays the primary role in organising input from outside into meaningful knowledge (Clements & Battista, 1990). The role of prior knowledge is crucial in constructivism, and learners' interpretation of tasks and instructional activities involving new concepts is filtered in terms of their prior knowledge (Smith, di Sessa & Roschelle, 1993).

It is a generally accepted old adage that we learn through mistakes. Lannin, Barker and Townsend (2007) describe errors from a quotation by Salvador Dali "as opportunities for deepening one's understanding and as important components of learning process". They (Lannin et al., 2007) then argue that "the view of errors as a vehicle for learning, rather than an activity to eradicate, continues to gain momentum in mathematics education" (p. 44). How do constructivist's theorists view misconceptions? Brodie and Berger (2010) cite the work of Confrey (1990), Nesher (1987), Smith et al. (1993) to explain that within a constructivist perspective, "a misconception is a conceptual structure, constructed by the learner, which makes sense in relation to her/his current knowledge, but which is not aligned with conventional mathematical knowledge" (p. 169).

Smith et al. (1993) have identified a field that they call misconceptions research, which has been largely concerned with identifying misconceptions in many science and mathematics domains. They claim that many assertions emanating from the field are inconsistent with constructivism. For example, misconceptions research has seen the results of learners' learning as flawed when learners' exhibit a particular misconception. Smith et al., point out that constructivism "characterizes the process of learning as the gradual recrafting of existing knowledge that, despite many intermediate difficulties is eventually successful" (p. 17). Thus a perception that views misconceptions as interfering with learning, needing to be confronted and replaced and resisting instruction (emanating from the misconceptions research field) is inconsistent with the constructivist view that learners build more advanced knowledge from prior learning. They argue that a constructivist theory of learning must interpret learners' prior conceptions as resources for cognitive growth.

Olivier (1992b) points out that, "from a constructivist perspective, misconceptions are crucially important to learning and teaching, because misconceptions form part of a pupil's conceptual structure that will interact with new concepts, and influence new learning" (p. 196). Lannin et al. (2007) also assert that "the view of errors as sites for learning is essential to the classroom that builds on student sense-making" (p. 45). Naidoo (2009) in her study of feedback preferred by learners, commented that "in constructivism misconceptions are seen as fundamental in learning because learners can create misconceptions in the sense making process of knowledge acquisition" (p. 11). In a similar vein, Nesher's (1987) view of misconceptions is that:

> Misconceptions are usually an outgrowth of an already acquired system of concepts and beliefs wrongly applied to an extended domain. They should not be treated as terrible things to be uprooted since this may confuse the learner and shake his confidence in his previous knowledge. Instead, the new knowledge should be connected to the student's previous conceptual framework and put in the right perspective. (pp. 38–39)

Smith et al. (1993) suggest similarly that misconceptions have their roots in productive and effective knowledge; and that the main issue is the context - where the conceptions are used and how they are used. They argue that persistent misconceptions may be viewed as "novice's efforts to extend their existing useful conceptions to instructional contexts where they turn out to be unproductive" (p.61). Smith et al. (1993) argue that misconceptions, especially the widespread ones, are associated with some contexts where they are used successfully. Brodie and Berger (2010) support the view of Smith et al. (1993) that misconceptions occur when learners overgeneralise a concept from one domain to another. They provide an example of the misconception underlying the error that 0,567 is bigger than 0,67. Brodie and Berger (2010) explain the misconception as arising "because the learner has used her knowledge of whole numbers in the fractional context". She has drawn "the conception that for whole numbers the more digits in a number, the bigger it is" (p. 170). This conception holds in the whole number domain, but does not hold any longer when the domain is enlarged to include decimal numbers.

Smith et al. (1993) argue that conceptions are embedded in complex systems, and not as single units of knowledge. This perspective makes it "easier to understand how some conceptions can fail in some contexts and play productive roles" in others. This study therefore takes the view in line with those of the authors presented above that misconceptions can serve as useful tools for further learning and the view that misconceptions occur when a concept is generalized beyond its field of applicability. The above arguments are reasons for the study to focus on errors and misconceptions. The study intends to identify the errors and misconceptions with the hope that the process will assist teaching and learning in our schools. If teachers become more aware of misconceptions they will be in a stronger position to help their learners (Brodie & Berger, 2010).

2.6 Conclusion

The studies discussed in the review explore ratio extensively. What is worth pointing out is that they all have been carried out in contexts that are different from the one in which I will conduct my study. The studies also seem to look for misconceptions with the sole purpose of suggesting appropriate intervention strategies. The study I will undertake seeks to go beyond just identifying misconceptions and suggesting intervention strategies. It intends to explore learner errors and misconceptions with the purpose of understanding these. I believe that an understanding of errors and misconceptions impacts positively on teaching. The next chapter will delve into how this research study was conducted.

Chapter 3

Research design and methodology

3.1 Introduction

The study intends to explore errors and misconceptions that Grade 9 learners from a school in KwaZulu-Natal exhibit when they solved problems on ratio and proportion. This chapter will outline the research methodology that was used to conduct this study. Cohen, Manion and Morrison (2007) assert that a research methodology does not only focus on techniques and procedures used in the process of data-gathering but also describes approaches to, kinds and paradigms of research. The chapter will thus present the problem statement, the research paradigm, research methodology, data collection techniques and details of how the data was analysed. The chapter will also look into ethical issues; reliability of and limitations for the study.

3.2 Problem statement.

The allusion to poor performance by learners in school mathematics has been made in the previous chapter. The assumption made in this study is that learners lack understanding of mathematical topics that form pillars of mathematics, one of which is ratio and proportion. As indicated in the first chapter, ratio is central to the development of proportional reasoning, and proportional reasoning is the cornerstone of algebra and a wide variety of topics in mathematics. The perspective taken in this study is that errors and misconceptions provide an opportunity for learners to improve their understanding because they often point to the source of problem, and thus indicate the specific misunderstanding that teaching needs to overcome. The study thus seeks to explore how learners in the selected school performed in assessment items based on ratio and proportion. It seeks to identify errors and misconceptions learners from the school committed when they solved problems on ratio and proportion. Lastly, the study also seeks to explore why this group of learners committed the errors and misconceptions identified.

3.3 Research paradigm

A research study is always designed within a certain paradigm because researchers position themselves within a paradigm when they conduct a research study. A paradigm is referred to as a framework within which theories are built, that fundamentally influence how the researcher sees the world, determines his perspective, and shapes his understanding of how things are connected (Voce, 2004). The role of a research paradigm is to provide a conceptual framework for the researcher to see and make sense of the problem that is explored (Williams, 1998).

In this study, errors and misconceptions are regarded as *complex and dynamic* phenomena. I view errors and misconceptions as a reality that is constructed, interpreted and experienced by people in their interactions with each other and with wider social systems. I perceive errors, not only as observable phenomena, but I perceive errors and misconceptions as subjective beliefs, values and understandings. I regard errors and misconceptions as human constructs based on the *way* in which individuals concerned make meaning of their learning (Voce 2004). The study will thus explore learner errors and misconceptions from this perspective. I therefore conduct this study from an interpretivist perspective. I intend to identify learner errors and misconceptions, and then search for patterns of meaning in the identified errors.

3.4 Research methodology

The paradigm discussed above qualifies this study as a qualitative study. I have taken a decision to pursue a qualitative case study. A case study is a research study conducted to get an in-depth understanding of a particular situation. Cohen, Manion and Morrison (2007) describe a case study as a "naturalistic inquiry" that undertakes "an investigation into a specific instance or real phenomena in its real life context" (p. 170). Nieuwenhuis (2007) refers to a case study research as a "systematic inquiry into an event or a set of related events which aims to describe and explain the phenomena of interest" (p. 75). The selected methodology therefore allows an in-depth understanding of learner errors and misconceptions as they solve problems on ratio and proportion. The methodology will enable the study to describe and explain observed learner errors. The observed errors will in turn enable the study to identify learner misconceptions.

Case studies have a relatively narrow focus. They start by specifying a case that is under investigation. Stake (2005) argues that "a majority of researchers doing case work call their work by some other names", but goes on to point out that "the name '*case study'* is emphasized by some of us because it draws attention to the question of what specifically can be learned about a single case" (p. 443). The case in this research study is *the group of learners* and the phenomenon under scrutiny is *learner errors and misconceptions.* The study is not interested in errors in general, but explores errors in ratio and proportion, one topic in the mathematics curriculum. The study intends to focus on identifying learner errors and misconceptions on ratio and proportion. The study intends to understand the identified errors and misconceptions, and reason out why learners commit theses errors.

Nieuwenhuis (2007) asserts that

> From an interpretivist perspective, the typical characteristic of case studies is that they strive towards a comprehensive (holistic) understanding of how participants relate and interact with each other in a specific situation and how they make meaning of a phenomenon under study. (p. 75)

The research study therefore intends to explore how learners solve problems on ratio and proportion. What errors do they make? Is there an observable trend in learner errors? This will hopefully assist the study in getting an in-depth understanding of the errors, and hence misconceptions that underlie them. Stake (2005) actually views a case study "as both a process of inquiry about a case and the product of that inquiry" (p. 444). Thus the study brings both the process of inquiry of *learner errors and misconceptions when solving problems on ratio and proportion* and also the product of the inquiry.

Case studies are used to describe a particularly interesting set of circumstances, from which lessons can be drawn for other organizations (Emerald Publishers, 2005). The research study also intends to describe learner errors and misconceptions as they solve problems on ratio and proportion in the selected school. According to Cohen, Manion and Morrison (2007) some of the purposes of a case study are "to portray, analyse and interpret the uniqueness of individuals and situations through accessible accounts, and to catch the complexity and situatedness of behaviour" (p. 85). Creswell (1998) gives the purpose of a case study as the "exploration of a bounded system, or case (or multiple cases), over time through detailed, in-depth data collection involving multiple sources of information rich of context" (p. 61). The research study also intends to explore and then portray the situation in the selected school and to identify the nature of evident errors, and underlying misconceptions. The use of a case study will be appropriate for the collection of detailed data over a certain period of time within the school. The purpose of the study will be to explore what it is that allows learners to commit the errors observed. It is to understand what misconceptions can be attributed to the observed errors.

A limitation of a case study is that the findings cannot be generalized easily. This view is echoed by Nieuwenhuis (2007) when he states that "criticism of case study methodology is frequently levelled against its dependence on a single case and it is therefore claimed that the case study research is incapable of providing a generalizing conclusion" (p. 77). The study has no intentions of making generalisations beyond the boundaries of this case study.

3.5 Data collection

This section will focus on how data was collected. The research is a qualitative study, hence, a test item and unstructured interviews were used to collect data. The tool used to collect data and sampling for the study will be discussed in the paragraphs that follow.

Four schools were randomly selected from a list of secondary schools in KwaZulu-Natal. The schools were selected from Districts that were easily accessible and cost effective to visit. One of the schools declined to participate (reasons were given). Two of the schools that agreed to be part of the study were used to pilot the research tool. Ultimately, 30 Grade 9 learners from the fourth school wrote the test used as the research instrument. The learners were selected by a teacher from the school on a voluntary basis. The test was administered by the teacher and marked by the researcher. There was no time restriction for the test, so as to allow learners to attempt all questions.

Guided by both the nature of errors identified and the nature of learner responses in the script, five learners were identified and selected for the interviews. Most of the responses of the selected learners did not provide enough information about how a solution was obtained. The interviews were conducted because they provided the researcher with an opportunity to ask questions, listen intentionally to learner responses in a good conversation which is likely to provide meaningful information (Eisner, 1999). They were also conducted to validate data collected using the test item (Cohen, Manion & Morrison 2007). The interviews were conducted in the language participants were comfortable with, to eliminate language barriers. The interviews conducted were semi structured because questions asked were guided by learner responses, both written in the script and verbally during the interview. The interviews were videotaped in the language of the learners and later transcribed by the researcher into English.

It has been indicated above that the pen and paper method was used to collect data. The test used to collect data comprised of eight questions (see Appendix A). The questions were taken and adapted from the Concepts in Secondary Mathematics and Science tool and all of them were free response questions. The pilot study necessitated the adaptation of some of the questions to ensure that they were using language and diagrams that were accessible to the participants. Diagrams used in the questions were simplified and the language made more accessible through the use of short, simple sentences. Only Questions 1 and Question 5 were used as they are in the original instrument. Items from the Concepts in Secondary Mathematics and Science tool in Hart (1981) were used because the tool is standardised; it has been used worldwide; and it has been validated.

3.6 Data analysis

Items in the assessment tool were categorized into four cognitive levels according to skills they assess in learners as shown in Table 3.1 below. The cognitive levels have been borrowed from Hart (1981). Key skills and knowledge assessed by the questions are briefly outlined under the column heading *description* in Table 3.1.

Table 3.1: Cognitive levels of test items

Level	Description	Test items
1	No rate needed or rate given. Multiplication by 2, 3 or halving	1(a), 1(b), 1(c), 1(d), 2(a)(i) and 2(a)(ii)
2	Rate easy to find or answer can be found by taking half an amount, then half as much again	2(b), 2(c), 2(d), 3 and 8
3	Rate must be found and is harder to find than above. Fraction operation also in this group	1(e), 2(e), 2(f)(i), 2(f)(ii) and 5
4	Must recognize that ratio is needed; the questions are complex in either numbers needed or setting	4, 6(a), 6(b), 7(a) and 7(b)

Items categorized as Level 1 either do not need a rate to solve or a rate is given. All they required was halving, doubling (multiplication by 2) or multiplication by 3. The items are easy to solve. Level 4 items are complex, and thus more challenging as learners are expected to recognize their multiplicative nature.

The first step of the analysis was to mark learner scripts. Each correct response was awarded 1 mark and no mark was awarded for an incorrect response. The framework used in the Concepts in Secondary Mathematics and Science study (Hart, 1981) was adopted and adapted for the analysis of learner performance in the test (see Table 3.2).

Table 3.2: Levels of competency

Level	Criterion
0	Learner obtains less than 4/6 (66%) in Level 1 items
1	Learner obtains 4/6 (66%) or more in Level 1 items
2	Learner obtains 3/5 (60%) or more in Level 2 items
3	Learner obtains 3/5 (60%) or more in Level 3 items
4	Learner obtains 3/5 (60%) or more in Level 4 items

There were six Level 1 questions, and these were therefore marked out of a total of six marks. If a learner scored less than four marks in this category of questions, the learner was regarded as performing at competency Level 0 (refer to Table 3.2). Similarly, questions at other levels of difficulty were marked out of a total of five marks, and the benchmark for each level was 60%. The overall performance of learners in the test was then summarised. Thereafter the responses to each item were scrutinized for errors and misconceptions. In the analysis of individual questions, I looked for the cause of the error in each incorrect answer and also explored the underlying reason that caused the learner to make the error. The categorisation of errors was informed by the strategies that the learners used to solve the problems.

Lastly, for reporting purposes, and also to observe confidentiality, learners were assigned numbers. Instead of referring to the learners by name, learners have been referred to as Learner 1, learner 2, …, up to Learner 30.

3.7 Reliability and trustworthiness of the study

Yin (2009) states that "a major strength of case study data collection is the opportunity to use … different sources of evidence". Yin elaborates further that the "rationale for using multiple sources of evidence" (p. 114) is for triangulation. Triangulation refers to the use of multiple data sources or multiple methods to confirm emerging findings (Guba & Lincoln, 1989) and is also used to identify the different realities of the participants (Stake, 2008). In this study I had two sources of data, that of the learners' written responses and the responses of learners from the individual interviews. During my analysis I used the two sources to triangulate my findings.

In order to judge the adequacy of research in a qualitative paradigm, Guba and Lincoln (1985) put forward what they call trustworthiness criteria. These are called parallel criteria because they are intended to parallel the rigour criteria usually used to judge the more conventional evaluation paradigms. The criteria suggested by Lincoln and Guba (1985) are "credibility, transferability, dependability and confirmability" (p. 233). This research addressed these criteria as indicated in the table in the next page.

Table 3.3: Trustworthiness criteria in this study

STRATEGY	CRITERIA	APPLICATION
Credibility	Prolonged engagement in the field; Member checks	Document analysis of learners' written responses, video-recordings, interviews. Participants to verify comments given.
Transferability	Dense description	Verbatim quotes from interviews, reproduction of learners work within the text.
Dependability and Confirmability	Triangulation Dependability audit	Learners written responses and learners interview responses Transcripts of recordings checked against copies of learners' work

Interviews were transcribed into English. The transcripts were then checked against the originals. Scans of original learner work are provided in the text.

3.8 Ethical Issues

A letter was written to the Head of the Department of Basic Education requesting permission to work with the schools, which was granted. Letters were also written to the school principal, subject teacher, parents and the learners requesting permission from them to participate in the study. The letters explained that participation was voluntary and participants could withdraw at any time. The participants were also guaranteed that their responses would be confidential. Anonymity was also guaranteed as their names would not be revealed in any of the reports about the study. The study was conducted after the principal, subject teacher, learners and parents signed letters of consent to participate in the study; and the Faculty of Education of the University of KwaZulu-Natal had issued an ethical clearance certificate, Ethical Approval Number HSS/1366/2010 (see Appendix B).

3.9 Limitations of the study

While learners attempted all questions in the test, during the interviews, one of the limitations was the learners' reluctance or inability to explain their solutions. Some said that they could not remember, while others said they just guessed answers, especially those who did not show their working.

3.10 Conclusion

The chapter outlined the research design and research methodology used for this study. An explanation for the choice of methodology was presented. The problem statement was outlined. The paradigm for the study was provided. The chapter also outlined the data collection and analysis. The reliability of the data collection and the data analysis has also been acknowledged. Ethical matters were dealt with and limitations of the study were highlighted. The next chapter will present an analysis of the data that was collected.

Although I do not claim that the results are applicable to all Grade 9 learners in South Africa, there will be commonalities between this group and others from similar backgrounds. By providing details of the case study, it is expected that other teachers or researchers would be able to examine the findings, and find points of commonality that might be applicable in other situations. As a researcher, I have provided readers with sufficient information to determine the applicability to other learners or other similar students.

Chapter 4
Data analysis

4.1 Introduction

The previous chapter gave a detailed outline of how data was collected. The research tool used to collect data was also elaborated on. In this chapter an analysis of the data that was collected is presented. This will be done by first looking at the general performance of learners in the test. Then the performance of learners will be examined to identify prevailing errors and misconceptions. Lastly, for each question, identified learner errors will be presented and interpreted.

4.2 Learner performance in the test

In the previous chapter an elucidation of how the test items were categorized into cognitive levels was presented. A framework of Learner Levels of competency was also provided. Level 0 refers to learners that obtained less than 66% in questions categorized as Level 1; Level 1 denotes learners that obtained 66% or more in Level 1 items; Level 2 indicates that a learner obtained 60% or more in Level 2 items; Level 3 refers to learners that obtained 60% or more in Level 3 items; and Level 4 comprise learners that obtained 60% or more in Level 4 items. The number of learners performing at each cognitive level is reflected in Table 4.1.

Table 4.1: Learner levels of competency

Level	Number of learners
0	2
1	13
2	2
3	10
4	3

n = 30

Only about seven percent of the participants performed at level 0. These are the learners that could not solve elementary problems, in which rate was given. The learners struggled with problems that required multiplication by 2, 3 or halving. About 43% of the learners performed at level 3 and above. This also means that less than half of the participants could correctly solve problems in which rate was not given and the rate was not easy to find. Almost 90% of the learners could not adequately solve level 4 problems, that is, problems that required them to recognize that a ratio was needed to get a solution.

4.3 Learner performance per question

The previous section presented the percentage of learners that performed at each level of competency. This section will present an analysis of performance in each question. It will also identify and describe learner errors.

4.3.1 Performance in Question 1

The first question was based on the onion soup recipe given below. Questions that learners had to answer using the recipe are also given.

Onion soup recipe for 8 people

8 onions
2 pints of water
4 cubes of chicken soup
2 spoons of butter
½ pint of cream

I am cooking onion soup for 4 people.

(a) How much water do I need?
(b) How many cubes of chicken soup do I need?

I am cooking onion soup for 6 people.

(c) How much water do I need?
(d) How many cubes of chicken soup do I need?
(e) How much cream do I need?

The questions required learners to calculate amounts of selected ingredients of a recipe required to make soup for 4 people in Questions 1(a) and 1(b); and for 6 people in Questions 1(c) to 1(e)). Learner responses are thus analysed separately; for Questions 1(a) and 1(b) first, and Questions 1(c) to 1(e) later. Table 4.2 below shows how learners responded to Question 1 (a) and Question 1(b), and Table 4.3 shows performance in the rest of the questions.

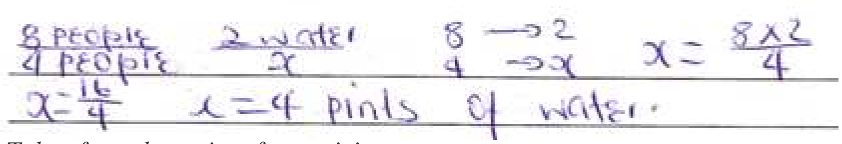
Table 4.2: Learner responses to Question 1(a) and 1(b)

Question	**Description of response**	Correct	Incomplete halving	Incorrect operation/ cross multiplication	Incorrect multiplicative strategy
1(a)	**Learner response**	1	1/2	4	16
	No. of learners	23	3	2	2
1(b)	**Learner response**	2	1/2	8	12
	No. of learners	26	1	3	0

n = 30

The solutions to Question 1(a) and Question 1(b) require halving of ingredients in the given recipe since soup is being prepared for four people. More than 70% of the participants responded correctly to both questions. Ten percent of the responses to Question 1(a) were incorrect because of incomplete halving while only three percent of the responses were erroneous due to the same incorrect strategy in Question 1(b). Incomplete halving means that learners could see that they needed to halve ingredients, but they did not do so. Learners just indicated that they needed 1/2 the ingredients given in the recipe, but did not work out the actual amounts.

About six percent and about ten percent of the learners performed an incorrect operation in Question 1(a) and Question 1(b) respectively. Learners multiplied given amounts of water and chicken soup cubes by 2, to obtain incorrect answers of 4 and 8 respectively; instead of dividing by 2. Put simply, learners doubled instead of halving. Doubling or halving is appropriate here, so the error is not a result of magical doubling (see p. 23). In the table this error is also referred to as incorrect cross multiplication as some learners obtained the solution of 4 in Question 1(a) as shown below.



Taken from the script of a participant

Learner 13 omitted an equal sign between the equivalent fractions. The ratio of people in the two recipes (given recipe for eight people and the recipe for four people) is 8 : 4, which the learner wrote as (8 *people*)/(4 *people*). The ratio of water amounts is 2 : *x*, where *x* is the amount of water needed to make soup for four people. However, Learner 13 correctly formulated ratios by comparing quantities from the same measure space. As pointed out in the first chapter, same measure space means quantities of the same kind or elements of the same set.

To find the amount of water needed (value of x), Learner 13 multiplied 8 (the number of people given in the recipe) by 2 (the amount of water needed to make the soup for eight people) and then divided the product by 4 (the number of people that soup was needed for). This is an error the study refers to as *incorrect cross multiplication.* Cross multiplication is commonly used in schools to solve problems with equivalent ratio or rate. The equivalent ratios or rates are then expressed as equivalent fractions. In this problem, $8 : 4 = 2 : x$ is written as $8/4 = 2/x$. The next step would be to multiply both sides of the equation by the LCM of the denominators which would yield the equation $8x = 8$. Cross multiplication, which entails multiplying 8 (numerator on LHS) by x (denominator on the RHS) and equating this to the product of 4 (denominator on the LHS) and 2 (numerator on the RHS), is the short cut for the process of multiplying both sides by the LCM. The learner seemed to confuse *cross multiplication* with the rule for multiplication of rational fractions; that is, multiplying numerators to get the numerator in the answer, and multiplying denominators to get the denominator in the answer.

Learner 13 was interviewed, and the interview went as follows:

> Interviewer: I can see that your answer to this question is 4 pints. Tell me how you got 4 pints.
>
> Learner : I took 8, the number of people in the recipe and wrote it down. I then took 2, the number of pints needed to make soup. I wrote 8 over 4, the number of people that I want to make the soup for. Because I do not know the number of pints of water needed, I wrote over x [*the unknown*].
> I then cross multiplied. I got $8x$ and 8. I divided by x, although I am no longer sure this is how I did the calculation [*she realises that this solution is different from the one which appears on the script*]. I then wrote x and moved 8 to the other side [*as if she has the equation* $8x = 8$].

From the interview, it is tempting to say that Learner13 made what Olivier (1992b) refers to as a slip (see Chapter 1). A perusal of the learner script revealed that the error recurs in the solutions obtained using this strategy. Brodie and Berger (2010) refer to a misconception as "a conceptual structure, constructed by the learner, which makes sense in relation to her or his current knowledge, but which is not aligned with the conventional mathematical knowledge" (p. 169). Learner 13 seems to confuse multiplication of fractions and cross multiplication. The participant thus has a misconception of the cross multiplication strategy. [Misconception of a mathematical procedure]

Learner 16 obtained the incorrect response of 16 as shown hereunder.

Taken from the script of a participant

Although some of the working is not shown, the equation shows that 4 (the number of people that soup was needed for) is multiplied by 8 (the number of people in the given recipe) and the product is divided by 2 (amount of water needed to make soup for eight people). The ratio used as a *factor* comes from different measure spaces (4 is multiplied by 8/2 or 8 : 2) hence, this error is referred to as *incorrect multiplicative strategy.* This error was observed in six percent of the participants in Question 1(a), and none of the learners in Question 1(b).

Questions 1(d) to 1(e) required learners to halve (to obtain ingredients for four people); halve again (obtain ingredients for two people) and then add the result to first halving to the result obtained after halving for the second time to get the ingredients required for 6 people. The strategy required to obtain solutions to these questions is the *build-up* strategy. For example, to determine the amount of water needed to make soup for six people, learners had to halve 2 pints of water (water needed for to make soup for eight people) to get 1 pint (amount of water needed to make soup for four people). They then had to halve 1 pint of water again to get 1/2 pint (amount of water needed to make soup for two people). The amount of water needed to make soup for six people is then 1 pint of water + 1/2 pint of water = 1½ pints of water. The performance of learners is shown in the table that follows.

Table 4.3: Learner responses to Question 1(c), 1(d) and 1(e)

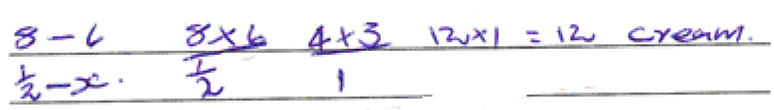
Question	**Description of response**	Correct	Incomplete build-up strategy	Incomplete strategy	Incorrect cross multiplication	Incorrect multiplicative strategy	Other incorrect strategies
1(c)	**Learner response**	1½ or 3/2	½ & 1	2	2,6	6 & 24	Miscellaneous
	No. of learners	18	2	3	1	2	4
1(d)	**Learner response**	3	2	4	5,3	12	Miscellaneous
	No. of learners	20	3	0	1	1	5
1(e)	**Learner response**	3/8	1/4	1/2	2/3	3/2	Miscellaneous
	No. of learners	3	7	4	1	1	14

n = 30

Sixty percent of the learners got the correct answer to Question 1(c), more than 66% correctly answered Question 1(d), but only 10% got the correct answer in question 1(e). This is not surprising as question 1(e) required halving and addition of fractions while Question 1(d) required halving and addition of whole numbers; and Question 1(c) required halving of whole numbers and addition of a whole number to a fraction.

Incomplete use of the build-up strategy (*incomplete build-up strategy*) was observed in all three questions. In Question 1(c), for example, six percent of the learners either halved once to get 1, or halved twice to get 1/2, Learners that halved twice did not add the first answer to the second answer. Quite an extensive proportion also used what Misailidu and William (2002, 2003) refer to as an *incomplete strategy* (using the same number given for the measure space). For example, in Question 1(e) it is stated that 1/2 pint of cream to make soup for eight people. About 13% of the learners said that 1/2 pint of cream was needed to make soup for six people too. The error was observed among six percent of the learners in Question 1(c), while none of the learners exhibited this error in Question 1(d). Three percent of the participants used an *incorrect cross multiplication* (explained for Question 1(a) above), and only six percent or less used an *incorrect multiplicative strategy* (also explained above).

Other incorrect answers obtained by the learners, categorised as other, were diverse. The solutions emanated from the incorrect performance of either basic operations, conversions from one unit to the other or both. For example, In Question 1(d), learners divided 24 by 8 and obtained 2 as the quotient (computational error). In 1(e), learners divided (1/2 × 6) by 8 and obtained 0,75 (computational error). Some learners found the product of 6 and 1/2 to be 7,2 (a computational error) and then some correctly divided 7,2 by 8 to obtain 0,9, while others divided 7,2 by 8 to obtain an incorrect answer of 9. In Question1(d) one learner incorrectly converted 1/2 to a decimal and got 1,5. To get 1/2 in Question 1(e), one Learner 15 incorrectly answered as shown below.



Taken from the script of a participant

An interview with learners did suggest that the use of *incorrect addition strategy* might also be a reason for some of the incorrect solutions obtained by learners. Hereunder, is what transpired from an interview with Learner 27.

Interviewer: You see here [*pointing at sub-questions of Question 1*] you have done very well except in this one [*pointing at* 1(e)]. Here you got 1/3. Can you explain how you got this? I see here [1(a)] that for 4 people you need 2 pints of water. Here [1(c)], for 6 people you need 3 pints of water.

In 1(e) you are told that for 8 people you need ½ pint of cream. How did you get that for 6 people we need 1/3 pint of cream?.

Learner: [*After thinking deeply for sometime*] I am not sure what happened. I think I wanted to write 1 and 1/3 [1 1/3], but ended up writing 1/3 only.

Interviewer: Do you mean that we need more cream for 6 people than we need for 8 people?[(Learner *keeps quiet for a long time.* Interviewer *probes*] Let us look at this one [1(c)], how did you get that for 6 people we need 3 cubes of chicken soup? Write down here and explain.

Learner: [*after another long pause*] I saw that for 8 people I need 4 cubes of chicken soup. For 6 people I should need 3 cubes of chicken soup because for 4 people I need 2 cubes of chicken soup. If there are 2 people, I need 1 cube of chicken soup. Therefore for 6 people I need 3 chicken soup; looking at how numbers decrease from 8 to 4.

Interviewer: Oh you saw a pattern? [*The interviewer wrote down while asking the question*] Do you mean that you saw that for 8 people we need 4 cubes, for 4 people we need 2 cubes, for 2 people we need 1 cube? The number of cubes shows the pattern 4, *space*, 2, 1? [*Learner nodded to affirm*]

Let us look at 1(e) now. Did you also see a pattern of how numbers decrease?

Learner: For 8 people I need ½ pint, if there are 6… [*he keeping quiet for a long period*].

Interviewer: [*motivating*] You said that you saw a pattern in 1 (d). Is there no pattern that you saw here?

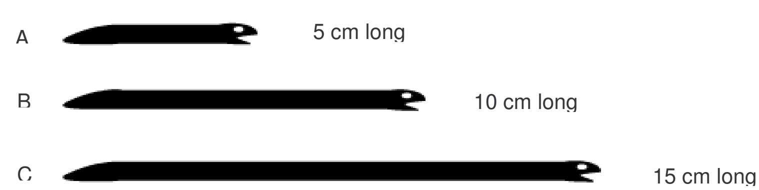
Learner: [*After long silence, not affirming the interviewer's suggestion*] I think I guessed this solution.

Looking at the pattern Learner 13 claims to have observed in 1(d), it is highly likely that for 1(e) the learner created a pattern of fractions as follows: 1/2 pint of cream for eight people (given); 1/4 pint of cream for 4 people (correct from halving). In 1(d) number decreased as follows: 4, 3, 2. Number in 1(e) should also decrease as follows: 1/2 ; 1/3 ; 1/4 . Hence, the response to 1(e) is that 6 people should get 1/3. Unfortunately, the interview could not confirm this idea. Hart (1981) did come across an error of this nature during the Concepts in Secondary Mathematics and Science project.

4.3.2 Performance in Question 2

The question below, a version of Piaget's popular eel question, was the second question in the research instrument.

Eels A, B and C are fed *sprats* according to their length.



(a) If eel A is fed 2 sprats,

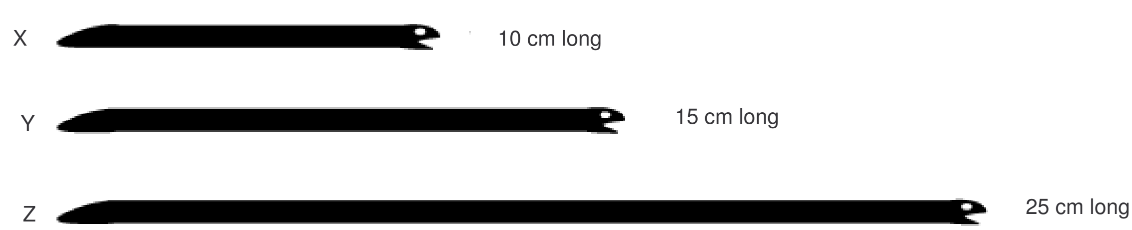
(i) How many sprats should eel B be fed?

(ii) How many sprats should eel C be fed?

(b) If eel B gets 12 sprats, how many sprats should eel C be fed?

(c) If eel C gets 9 sprats, how many sprats should eel B get?

Three other eels X, Y and Z are fed fishfingers. The mass of the fishfinger depends on the length of the eel.



(d) If eel X gets 2 grams of fishfingers, how much fishfingers should be given to eel Z?

(e) If eel Y gets 9 grams of fishfingers, how much fishfingers should be given to eel Z?

(f) If eel Z gets 10g of fishfingers,

(i) how much should eel X get?

(ii) how much should eel Y get?

Piaget's eel question is concerned with the amount of food given to eels of different lengths, the amount being proportionate to the length of the eel (Hart, 1981). Table 4.8 shows the performance of learners for each sub-question.

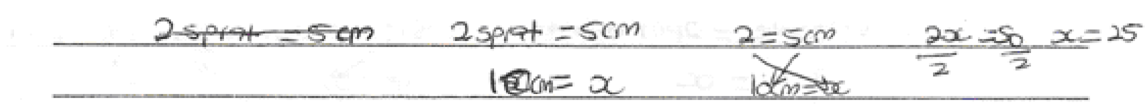
Table 4.4: Learner responses to Question 2

Question	Description of response	Correct	Incorrect cross multiplication	Incorrect addition strategy	Incorrect doubling / halving	Other incorrect strategies
2(a)(i)	**Learner response**	4	25	-	-	5
	No. of learners	27	2			1
2(a)(ii)	**Learner response**	6	37,5	-	8	4; 7 or 75
	No. of learners	24	1		2	3
2(b)	**Learner response**	18	12,5	14 or 17	24	2
	No. of learners	17	3	4	4	1
2(c)	**Learner response**	6	16,7	4 or 7	4½ (18)	Miscellaneous
	No. of learners	19	2	2	2 (1)	4
2(d)	**Learner response**	5	125	6 or 7	8	Miscellaneous
	No. of learners	11	2	5	8	4
2(e)	**Learner response**	15	41,7	11	18 (36)	Miscellaneous
	No. of learners	17	3	1	4	5
2(f)(i)	**Learner response**	4	25	6	2½	Miscellaneous
	No. of learners	15	1	2	1	11
2(f)(ii)	**Learner response**	6	37,5	5 or 8	5	Miscellaneous
	No. of learners	17	1	4	5	3

n = 30

More than 50% of the learners responded correctly to each sub-question, except in sub-Question 2(d). The performance of learners in Question 2(a), which is a level 1 question, was outstanding as 90% of the learners responded correctly to this sub-question. It is surprising to note that learners did well even in sub-questions 2(e) and 2(f) which are at level 3, while they did very poorly in sub-question 1 (e), which is at the same level. The possible reason for this might be that numbers used in Question 1 (e) are fractions.

Three to ten percent of errors in this question resulted from the *incorrect use of cross multiplication* strategy. Hereunder, it is shown how Learner 6 arrived at an incorrect answer of 25 sprats in Question 2(a)(i).



Taken from the script of a participant

Learner 6 used the equal sign incorrectly. She incorrectly expressed the rates (2 sprat/5cm and *x* sprats/10cm) as two equations. The formulation of rates is also inconsistent, that is, sprats/ length or length/sprats. In order to apply the cross multiplication strategy, two equivalent rates or ratios would be needed. Learner 6 did not do so. In fact, the result was obtained by multiplying the length of eel A (5cm) by the length of eel B (10cm) and then dividing the product by 2 (the number of sprats that A is fed with).

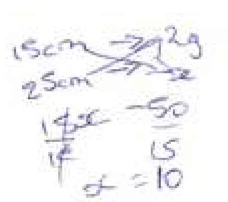
Also observed in learner responses is the *incorrect addition strategy*. Table 4.3 shows that 17% of the participants exhibited this error in Question 2(d), 10% in Question 2(b), and fewer in other sub-questions. Participants actually suggested that the eels should be given two more or less sprats or fishfingers, depending on the length of the eel. The strategy works well for Question 2(a). If eel A is fed two sprats, then eel B should be given two more sprats, because eel B is twice the length of eel A. The strategy does not work for other sub-questions. When used in Question 2(b), it yields an incorrect answer of 14 (obtained by 10% of the participants). There were instances where learners added or subtracted 5 (the difference in eel lengths). For example, an incorrect result of 17 sprats in Question 2(b) was obtained by adding 5 sprats to 12 sprats.

The *incorrect doubling* (magical doubling) or *incorrect halving* was also identified in three percent of the participants in Question 2(f)(i), and in 27% in Question 2(d). The strategy works correctly in Question 2(a)(i), but immediately fails in Question 2(a)(ii) as it gives an incorrect result of 8. Use of incorrect halving strategy is reason for the incorrect answer of 4½ in Question 2(c).

The rest of the answers (categorized as other strategy) could be attributed to *naïve strategies* (see p. 24) or just computational slips. In the solution to 2(a)(ii) given below, the lengths of eels B and C are used instead of lengths of eels A and C, which might be a slip.

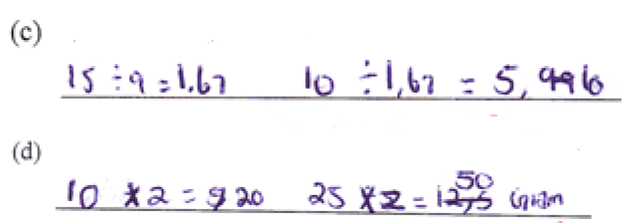
Taken from the script of a participant

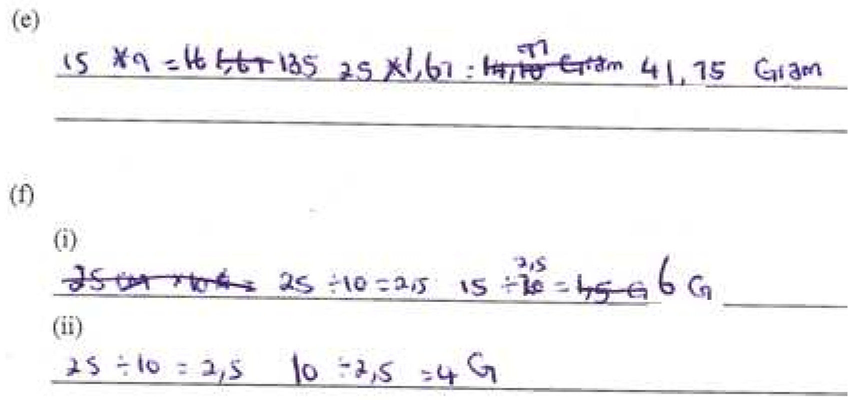
The following rough work shows how Learner 7 obtained the incorrect solution of 10 to Question 2(d). The learner divided 50 by 15 and obtained 10, a computational error. The working also shows that learners have poor skills in setting up the relationship between the quantities that form a proportion.



Taken from the script of a participant

The following incorrect solutions were obtained from the script of Learner16.




(e)

15 x 9 = 135   25 x 1,67 = 41,75 Gram

(f)

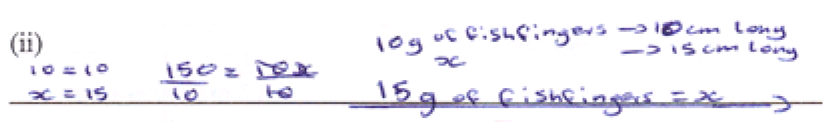
(i) 25 ÷ 10 = 2,5   15 ÷ 2,5 = 6 G

(ii) 25 ÷ 10 = 2,5   10 ÷ 2,5 = 4 G

Taken from the script of a participant

Learner 16 divided or multiplied the eel length by the number of sprats or mass of fishfingers it is fed. He divided where the number of sprats or mass of fishfingers was given for a longer eel and he wants to find how much should be given to the shorter eel. To determine how much the eel should be given, its length should be divided by the quotient obtained during division. In 2(c) the longer eel is 15cm. The eel is fed 9 sprats, hence learner P divided 15 by 9 and got 1,67. The length of eel B (10cm) was then divided by 1,67.

The solution to 2(f)(ii) below is based on an incorrect interpretation of the question. According to the question, the 25cm long eel is fed 10g of fishfinngers. Learner 20's opening statement implies that 10g of fishfingers are given to the 10cm long eel, which is incorrect.


(ii)

10 = 10   10g of fishfingers → 10cm long

x = 15   x → 15cm long

150/10 = 10x/10

15g of fishfingers = x

Taken from the script of a participant

There was additional information on learner errors obtained through interviews. The dialogue below elaborates on how Learner 8 arrived at his solutions:

Interviewer: Let us look at this one [Question 2(a)]. We are calculating the eels should be fed, right? You said that if eel A gets 2 sprats, then eel B must get 4 sprats. How did you arrive at that?

Learner: That eel A gets 2 sprats?

Interviewer: No, that if eel A gets 2 sprats, then eel B should gets 4 sprats.

Learner: If eel A gets 2 then eel B gets 4?

Interviewer: Yes, that is what you said and this is correct. How did you get the answer?

Learner: Eel A is 5 cm long, right. If eel A gets 2 sprats, then eel B gets 4 because 5 is half of 10. So if eel A gets 2, eel B gets 4.

Interviewer: If A gets 3?

Learner: Then B gets 6.

Learner 8 seems to recognize proportion very well in this problem. He realized that eel B is double the length of eel A, and therefore should get double the number of sprats given to eel A. The conversation continued as follows:

Interviewer: Ok, let us look at 2(b). The question asks that if eel B gets 12 sprats, how many sprats should eel C get? How did you get 24 sprats?
Let us start here: If eel B gets 12 sprats how many should eel A get?

Learner: If eel B gets 12 sprats, then eel A should get 6.

Interviewer: Good. If eel B gets 12 sprats, you said in your script eel C gets 24 sprats. How did you get 24?

Learner: 24? I was in a rush then, I must have said 12×2 and got 24. I should have said if B gets 12, C gets 30.

Interviewer: 30?

Learner: Yes.

Interviewer: Let us write this down. If B gets 12, write C must get how many? C should get ... [*the learner says* 30]. How did you get 30?

Learner: I said 15 x 2.

Interviewer: Write down 15 x 2. Why did you say that?

Learner Eel C eats more than eel B.

Interviewer: More?

Learner: Yes.

Interviewer: Why do you multiply by 2?

Learner: Yes, I multiplied by 2.

Interviewer: Why?

Learner: I was in a hurry and the bell was ringing.

Although nothing meaningful seems to come out of this conversation, Learner 8 multiplied 12 by 2 and got 24; multiplied 15 by 2 and got 30. He seems convinced that eel C should get twice more than eel B. Is Learner 8 using *magical doubling strategy*? The continuation of the conversation might provide an answer.

Interviewer: Let us look at this one [2(d)].You said that if eel X gets 2 gram fishfingers, eel Z must get 7 gram. How did you get 7?

Learner: I added 5.

Interviewer: Ok. You said if eel X gets 2 gram eel Z gets 2 + 5 eels?

Learner: Yes.

Interviewer: ok write down that eel Z gets....

Learner: Write Z?

Interviewer: Write eel Z gets__?___ You said 2+5

Learner: I said 2 + 5 and got 7.

Interviewer: Where did you get 5 from?

Learner I said 25/5 and got 5, then here I said 2 × 5 to get 10 [*length of eel* X]and if I multiply 5 by 5 I get 25[*length of eel* Z] over here.

Interviewer: Here [*pointing at the learner working*] you multiplied 5 by 2 and here you multiplied 5 by 5? So when you get 2 you add 5?

Learner: Yes, and I got 7.

Learner 8 is not doubling anymore. He has now shifted towards an *additive strategy*. He sees that the eel lengths are multiples of five. He might then have felt that one eel should get five more than the other. Unfortunately the information could not be obtained from the learner.

A conversation with Learner 27 on the same question went as follows:

Interviewer:: Let us look at this one [2(d)]. Eel X is given 2g of fishfingers. How did you get the answer that eel Z should get 8g of fishfingers?

Learner: If I give eel X 2g, then I must give eel Y 4g, looking at how their lengths differ [*observing a pattern of lengths*]. I saw the lengths were 10, 15 and 20. I think I did not see that eel Z is 25, I think eel Z was supposed to get 8g if its length was 20 [*seems to be trying to establish a pattern of 2, 4, 8 for eels X, Y, Z respectively*]. I was not supposed to give eel Z 8g. I was supposed to give eel Z 12g.

Interviewer: Now your pattern is 2, 4, 8 for 10, 15, 20. Why is it not 2, 4, 6, 8? An eel of length 20 would get 6g and it would make sense to give eel Z 8g.

Learner: Actually I think that is exactly what I did.

If Learner 27 thought eel Z was 20cm long, he also used a *magical doubling strategy*. After changing his mind, the method Learner 27 seems to be using suggests that he tried to establish a pattern that was arithmetic. For eel length, he saw intervals of five. For fishfinger mass the learner saw multiples of two. From the way the learner reverted to counting, one may conclude that the learner saw a ratio as somehow multiplicative (repeated addition of 2).

An analysis of scripts showed that quite a significant proportion of learners used the *incorrect doubling strategy* to solve Question 2. Interviews significantly reinforced this observation, although to an extent, they also suggest that learners do this subconsciously.

4.3.3 Performance in Questions 3

Question 3, given below, was based on dividing a quantity in a certain ratio.

> Nkosi and Rajen work on a job together.
> Nkosi works for 12 hours.
> Rajen for 8 hours.
> How should they share the payment of R350 for the job?

In this question, the ratio was not given. Learners had to first determine the ratio, and then use it to split R350 between Nkosi and Rajen. Essential for solving problems of this nature, is the knowledge of how a ratio and a fraction differ. Learners need to understand that a ratio may be a comparison of a part to a part or a part to a whole, whereas a fraction generally compares a part to a whole. The ratio 12 : 8 means that if one gets 12 parts, the other gets 8 parts. Therefore there are 20 parts in a whole. So the fraction one should get is 12/20 and the other should get 8/20.

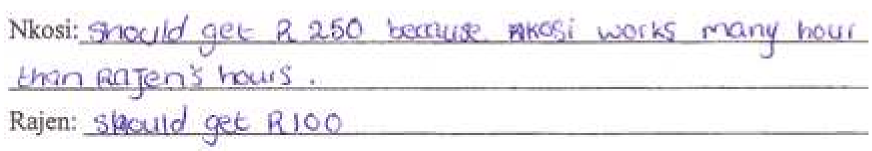
Table 4.5 below reflects learner responses to the question.

Table 4.5: Learner responses to Question 3

Description of response	Correct	Intuitive strategy	Incorrect application of formula x/a = y/b
Learner response	Nkosi: R210 Rajen: R140	Nkosi: R175, Rajen R175; Nkosi R190, Rajen R160; Nkosi R200, Rajen R150; Nkosi R200, Rajen R100; Nkosi R215, Rajen R135; Nkosi R227, Rajen R123; Nkosi 250, Rajen R100; Nkosi R300, Rajen R50; Nkosi R300, Rajen R200; Nkosi R350, Rajen Nothing	Nkosi R87,50, Rajen R87,50; Nkosi R233,33, Rajen R116,67 Nkosi R233,33, Rajen R525 Nkosi R525, Rajen R233,33 Nkosi R525, Rajen R350 Nkosi R525, Rajen R433,34
No. of learners	7	16	6

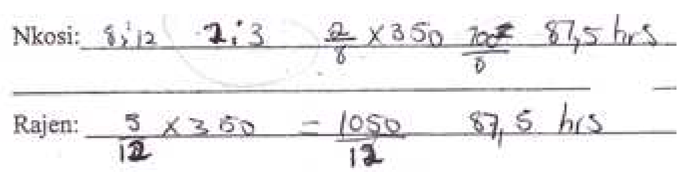
n = 30

The performance in this question was bad. Slightly more than 20% of the learners responded correctly to this question. More than 50% of the participants seemed to have guessed (used intuitive strategy) how much each person should get. Fair enough, all participants, but one, agreed that Nkosi should get more money as reflected in the example below.



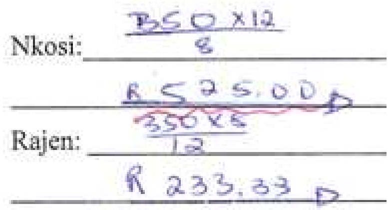
Taken from the script of a participant

The remainder of the learners, about 20%, used incorrect fractions to multiply R350. The learner below (Learner 6) found the ratio 8 hours to 12 hours and simplified it correctly to get 2 : 3. He then multiplied R350 by 2/8 and 3/12 (which looks like 5/12 in the scan copy), which are equivalent fractions, to get R87,50, which for some reason he decided to refer to as 87,5 hours.



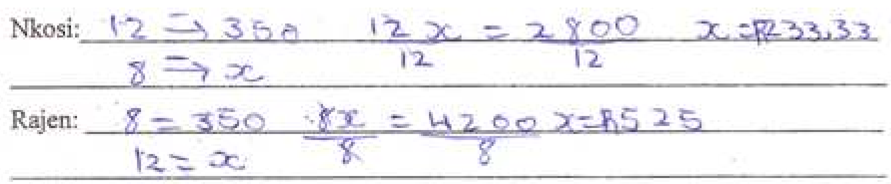
Taken from the script of a participant

In the other calculations (see example below), Nkosi was given twelve eighths (12/8) of R350 (which is more than the total amount) and Rajen was given eight twelfths (two thirds) of the amount to be shared.


Nkosi: 350 x 12 / 8
R525.00
Rajen: 350 x 8 / 12
R 233,33

Taken from the script of a participant

Some learners used the same fractions as the above learner, but did this in the reverse order. Hence, in this scheme Rajen gets a bigger share than Nkosi as seen in the calculation below.


Nkosi: 12 → 350 12x/12 = 2800/12 x = R233,33
8 → x
Rajen: 8 = 350 8x/8 = 4200/8 x = R525
12 = x

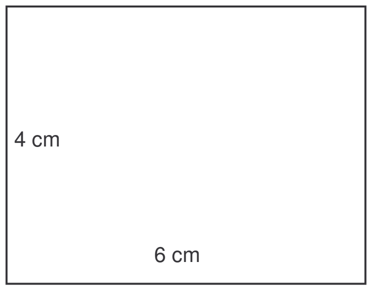
Taken from the script of a participant

Evident from the calculation is that the learners worked from an incorrect premise, that one of the people was paid R350. For example, in the calculation next to the name Nkosi, the first part indicates that Nkosi got R350 for working for 12 hours(12 → R350). The learner then worked out what amount Rajen should get, and arrived at R233,33. In the latter part, Rajen was now paid R350 for working for 8 hours, hence Nkosi would get R525. Learners seemed to be trying to create a situation in which they could use the formula *x/a = y/b*, although this was an inappropriate strategy, hence, an error of incorrect application of formula is flagged. The use of the formula only becomes appropriate once ratios in which the money should be split have been established.

4.3.4 Performance in Question 4

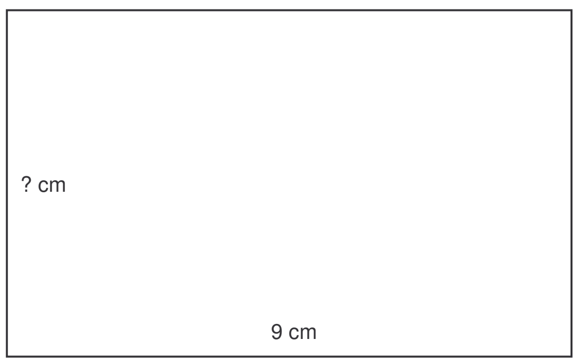
This is Question 4:

The rectangle below has a 6 cm base.



The rectangle is enlarged so that it keeps the shape (see diagram below).

The base of the enlarged rectangle is 9 cm.



What is the length of the other side?

The question was based on two similar rectangles. The rectangles were not drawn to scale. One rectangle is an enlargement of the other. Learner performance in this question is given in Table 4.6 below.

Table 4.6: Learner responses to Question 4

Description of response	Correct	Additive strategy	Incorrect arithmetic strategy
Learner response	6	7	9
No. of learners	18	11	1

n = 30

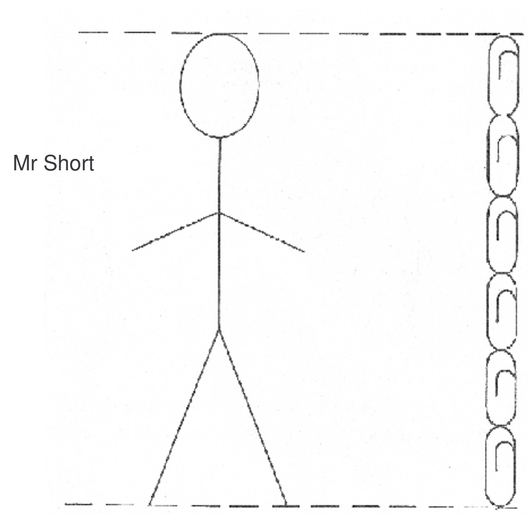
The performance of learners in this question could be described as satisfactory. The question is at level 4, as learners should determine the enlargement factor, and the enlargement factor is also not a whole number. Sixty percent of the learners responded correctly to the question. The answer 7 was outstandingly the most frequently obtained incorrect result. This incorrect response mainly reflects that learners did not see enlargement of 6cm to 9cm as multiplication by 1½. They see the enlargement as addition of 3cm to 6cm, hence, they have increased the length of 4cm to 7cm by 3cm.

The *additive strategy* was used by 36% of the learners to arrive at an incorrect answer of 7cm.

4.3.5 Performance in Question 5

Question 5 was a question also used in the Concepts in Secondary Mathematics and Science test, "adapted from Karplus (1975)" (Hart, 1981), on heights of Mr Tall and Mr Short measured in matchsticks and in paperclips given below.

Below, we are shown the height of Mr Short measured in paper-clips.



Mr Short has a friend called Mr Tall.
Mr Short's height is 4 matchsticks.
Mr Tall's height is 6 matchsticks.
What is Mr Tall's height in paper-clips?

Learners were given heights of both gentlemen in matchsticks, but given the height of Mr Short only in paperclips. They were expected to determine the height of Mr Tall in paperclips. The performance of learners in the question was poor as reflected in Table 4.7 below.

Table 4.7: Learner responses to Question 5

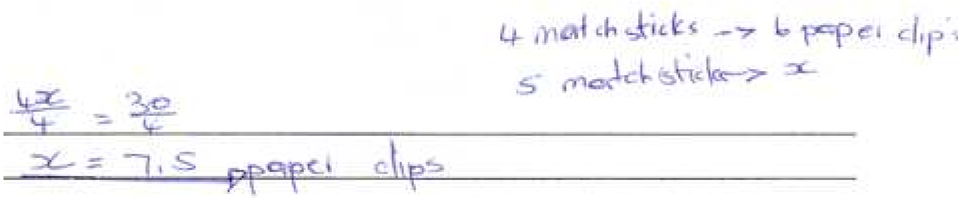
Description of response	Correct	Constant sum strategy	Incomplete strategy	Additive strategy	Magical doubling
Learner response	9	4	6	8	12
No. of learners	8	1	3	14	2

n = 30

Only 27% of the learners responded correctly to the question. The answer 8 was the most popular incorrect response. The most common error of 8 was obtained by adding 2 to 6.

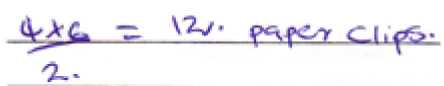
Mr Short is 4 matchsticks tall which is 6 paperclips, therefore Mr Tall, who is 6 matchsticks tall should be 2 more paperclips tall (8 paperclips). This error was identified in 47% of the learner responses. The incomplete strategy was identified in 10% of the participants and magical doubling in seven percent.

Hereunder are a few incorrect solutions that do not involve addition of 2.



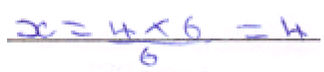
Taken from the script of a participant

This error resulted in an incorrect height of Mr Tall. The learner used 5cm as the height of Mr Tall. Otherwise the strategy used is correct. Also evident from the calculation below, is that 12 was not obtained by straight forward doubling of Mr Short's height in paperclips.



Taken from the script of a participant

Learner 14 multiplied the height of Mr Short in matchsticks by the height of Mr Tall in matchsticks and divided the product by 2. The solution below is taken from the script of Learner 17 who cross multiplied as if she was multiplying common fractions 4 matchsticks/6 matchsticks = 6 paperclipd/x paperclips, hence



Taken from the script of a participant

Learner 17 incorrectly cross multiplied, the same error she had made in 1(a) to obtain an incorrect response of 4 pints of water.

4.3.6 Performance in Question 6

Question six is given hereunder:

In a particular metal alloy (mixture of metals):

mercury : copper = 1 : 5,

tin : copper = 3 : 10, and

zinc : copper = 8 : 15.

Complete by filling in the missing numbers.

(a) mercury : tin = _______________.

(b) zinc : tin = __________________

Learners were expected to use the given ratios to suggest ratios for other alloys. This question reflected the worst performance as shown in the table below.

Table 4.8: Learner responses to Question 6

Question	Description of response	Correct	Magical doubling	Incomplete strategy
6(a)	**Learner response**	2 : 3 or 1 : 1½	2 : 6	1 : 3
	No. of learners	4	2	10
6(b)	**Learner response**	16 : 9 or 8 : 4½	16 : 6	8 : 3
	No. of learners	1	0	13

n = 30

While 13% of the participants correctly responded to Question 6(a), only three percent got the correct solution to Question 6(b). The reason may be that for 6(a), doubling the quantities forming the first ratio takes one directly to the answer. That is, 1 unit of mercury to 5 units of copper implies 2 units of mercury to 10 units of copper. But 10 units of copper require 3 units of tin. Therefore, 2 units of mercury should relate to 3 units of tin, hence the ratio of 2 : 3.

For 6(b), the relationship between tin and zinc becomes obvious after quantities in both ratios have been multiplied by 3 and 2 respectively. For example, multiplying quantities of the ratio tin : copper by 3 results in the ratio 9 : 30, which means that 9 units of tin relate to 30 units of copper. Multiplying quantities of the ratio zinc : copper by 2 results in the ratio 16 : 30, which implies that 16 units of zinc relate to 30 units of copper. Now we can conclude that the ratio zinc : tin = 16 : 9.

The most popular response to Question 6(a) was 1 : 3. This solution constituted a third of the responses to the question. Popular solutions were just made of the given first terms. For example, reasoning mainly used in 6(a) was:

> **if** 1 mercury : 5 copper and 3 tin : 10 copper,
>
> **then** 1 mercury : 3 tin.

Surprisingly enough, the incorrect strategy used to obtain this incorrect solution was used by 43% of the learners in Question 6(b). A small proportion, about six percent, doubled the quantities in this ratio to get 2 : 6.

When Learner 13 was asked how she arrived at her solution to Question 6(a) of 1 : 3, and to 6(b) of 8 : 3, this is what she said:

Interviewer:	Where did you get this 1 from?
Learner:	Here. 1 part Mercury [*pointing at mercury : copper = 1 : 5 in the question*].
Interviewer:	And the 3?
Learner:	Her, 3 parts tin [*pointing at tin:copper = 3:10 in the question*]
Interviewer:	And the first 8 in the second answer?
Learner:	I also took it from 8 parts zinc. I could not [*was failing to*] do any calculations.

This means that the learners who obtained similar solutions extracted the numbers from different ratios and formed new ratios of their own without considering the meaning of ratios. Almost all solutions provided by participants to this question were just meaningless and incorrect combinations of numbers contained in the ratios given in the question.

4.3.7 Performance in Question 7

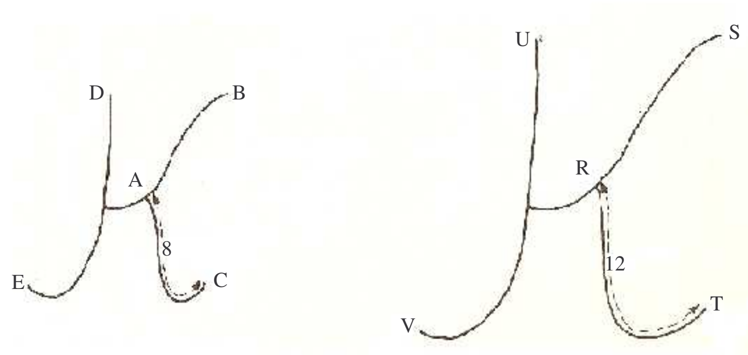
Question 7 was as follows:

The letters below are the same shape.

One is larger than the other.

The curve AC is 8 units.

The curve RT is 12 units.



(a) The curve AB is 9 units. How long is the curve RS?

(b) The curve UV is 18 units. How long is the curve DE?

Question 7, like Question 4, comprised of two diagrams, where one diagram is an enlargement of the other. While in Question 4 all dimensions were given in the smaller diagram, in this question learners had to calculate the corresponding measurements for both the enlarged diagram and the reduced diagram. The performance of learners in the question was also not good as we can see in Table 4.9 below.

Table 4.9: Learner responses to Question 7

Question	**Description of response**	Correct	Additive strategy	Incorrect doubling/ halving	Incorrect cross multiplication	Incorrect multiplicative strategy
7(a)	**Learner response**	13½	13	18	10,67	6
	No. of learners	6	16	1	1	0
7(b)	**Learner response**	12	14 (22)	9	5,3	27
	No. of learners	6	12 (1)	1	1	4

n = 30

Only 20% of the learners correctly responded to both sub-questions. The response of 13 was outstandingly the most popular incorrect response to Question 7(a). The solution was obtained by adding 4 (the difference between the lengths of RT and AC) to 9, the length of AB. This additive strategy compares corresponding parts of the two diagrams. One may also say that 13 was obtained through reasoning as follows: *9 is one bigger than 8, so the unknown part should also be 1 bigger than 12.* This additive strategy compares parts of the same diagram. More than 50% of the learners used the *additive strategy*. About 40% of the participants used the same strategy to find the solution to Question 7(b). The answer of 14 was obtained by subtracting 4 from 18. There was a learner who could not figure out that DE is shorter than UV, hence the solution given by the learner is DE = 18 + 4 = 22. Small proportions of learners used other strategies.

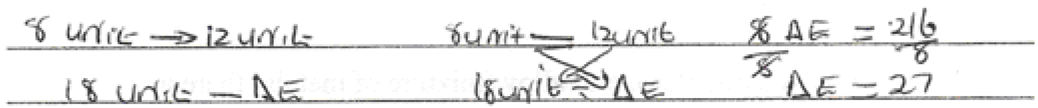
There were a few instances of meaningless manipulation of given numbers as shown in the examples below.

(a)

Taken from the script of a participant

The strategy used here is correct but Learner 6 made an error in multiplying 9 by 12. She said that 9 × 12 = 100. The next example shows how she arrived at the answer that DE = 27

(b)

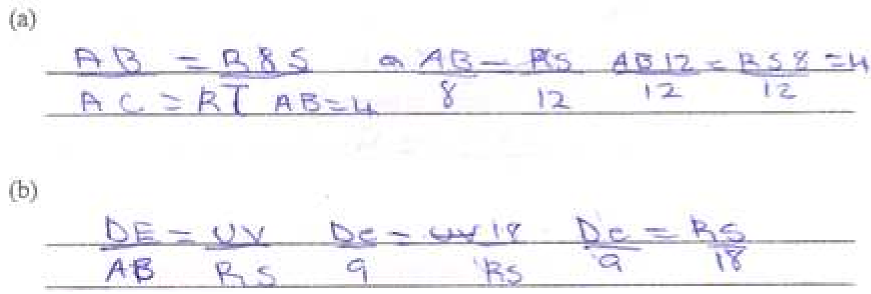


Taken from the script of a participant

When Learner 6 was interviewed, this is what she had to say:

Learner:	UV is 18 and AC is 8, these were given. We are also given that RT is 12. Because I didn't know the value of DE I put an x next to it, then I said 18/x multiplied by 8/12, I then cross multiplied and got 216 = 8x. I divided both sides by 8 and got x = 27.
Interviewer:	Which line was that?
Learner:	DE.
Interviewer:	Do you think DE is longer than UV?
Learner:	Eish! No.

In the case below, Learner 17 equated the product of the parts in the original diagram to the product of the parts in the enlargement (In 7(a) he wrote, 8 units × 12 units = 9 units × RS).



Taken from the script of a participant

He did not substitute for all the given lengths and ended up with more than one unknowns. Learner 17 thus ended up not knowing what was it that he was supposed to be looking for. Hence, for 7(a) he ended up with AB = 4 units yet it was given that AB = 9 units.

4.3.8 Performance in Question 8

Question 8 was the following Rand/Dollar exchange problem.

> On the 24 May 2010, $1 (1 US Dollar) could be exchanged for approximately R8. If I have R3 200, how much money do I have in US Dollars?

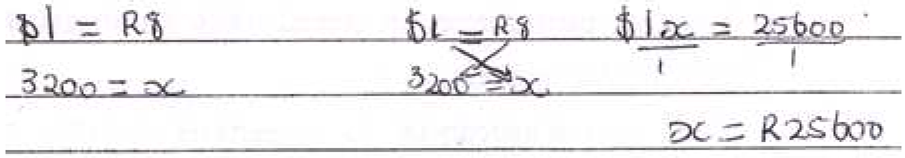
Learners were given that a Dollar is worth R8,00. They were then requested to determine how many dollars R3 200 was worth. The learner performance in this question was good as Table 4.10 below illustrates.

Table 4.10: Learner responses to Question 8

Description of response	Correct	Incorrect multiplicative strategy
Learner response	$400	$25 600
No. of learners	21	2

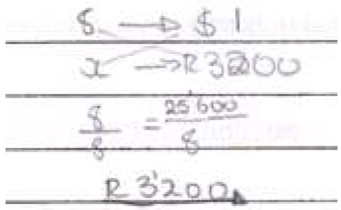
n = 30

Seventy percent of the learners correctly responded to this question. Only one response was incomplete. The incorrect solution of $25 600 was obtained through multiplying 3 200 by 8. Some learners did not realise that the number of dollars should be less, and thus divide. See the example given below.



Taken from the script of a participant

Other incorrect solutions were the result of carelessness on part of individual learners. For example, Learner 3 (in the calculation below) confused the dollar symbol with the number 8.



Taken from the script of a participant

Learner 3 also made some computational errors. In the next example, Learner 16 incorrectly multiplied 1 by 3 200 (1 × 3 200 = 3,2). The learner then divided incorrectly 3,2 by 8 (3,2 ÷ 8 = 4)

Taken from the script of a participant

4.4 Conclusion

Slightly more than 40% of the learners performed at level 3 or above. In the first part of Question 1, errors resulted mainly from the incorrect cross multiplication and the incorrect doubling. For the latter parts of Question 1, errors occurred mainly because of either the incomplete halving or the incomplete strategy. The incorrect addition strategy dominated in Question 2. In Question 3, there was also a lot of guessing and incorrect use of the formula *x/a* = *y/b*. In Questions 4, 5 and 7 the additive strategy was used by the by majority of learners. The incomplete strategy was also dominant in Question 6. Errors in Question 8 mainly resulted from the usage of the incorrect multiplicative strategy. The next chapter will elaborate on this summary when it provides answers to the research questions.

Chapter 5
Research findings and conclusion

5.1 Introduction

The purpose of the study was to explore the errors made by Grade 9 learners in a school in KwaZulu-Natal when solving problems on ratio and proportion. The research questions were: How did learners from the selected school perform in assessment items based on ratio and proportion? What errors and misconceptions do these learners commit when they solve problems on ratio and proportion? Why do these learners commit the identified errors and misconceptions? In this chapter, I will attempt to provide answers to these questions. Thereafter some implications of the findings and recommendations arising from the study will be presented.

5.2 Learner performance in items on ratio and proportion

This section addresses the first research question: how did learners from the selected school perform in the assessment items on ratio and proportion? In discussing the learners' performance, I look at their performance in terms of the different cognitive level rather than looking at the overall score obtained. A more useful picture of their performance will be painted by looking specifically at how they performed in questions at particular cognitive level rather than looking at the overall score obtained. Learner performance based on how many marks a learner obtained out of the total marks may conceal crucial information for teaching and learning.

The cognitive levels that were used to classify the assessment items have been outlined in Chapter 3. Associated with each cognitive level was a corresponding benchmark that was used to place learners at a competency level. These competency levels have also been described in Chapter 3. For example, a learner performing at competency level 1 should score four marks or more, out of six marks (75% or above), in the items that are at cognitive level 1. The analysis in Chapter 4 indicated that 2, 13, 2, 10 and 3 learners were classified as performing at Levels 0, 1, 2, 3 and 4 respectively. This shows that 50% of the learners performed at competency level 1 or lower, which means that these learners could not solve problems in which rate was not given, but easy to find. They were only able to solve problems which did not require a rate or where the rate was given.

This implies that half of the learners that participated in the study could not solve problems in which the rate was not given, but the rate was easy to find. The National Curriculum Statement stipulates that a Grade 7-9 learner should "be given ample opportunity to solve a variety of problems, using an increased range of numbers and the ability to perform multiple operations correctly and fluently". It also states that a Grade 7-9 learner should "be encouraged to sharpen the ability to estimate and judge the reasonableness of solutions, using a variety of strategies (including mental calculations, calculators and *proportional reasoning*)" (DoE, 2002, p. 62). The document then states that one of the assessment standards for the achievement of the first Grade 9 Learning Outcome is that learners should be able to solve "problems that involve ratio, rate and proportion (direct and indirect)" (DoE, 2002, p. 71). Fifty percent of the participants in the study did not show any of these qualities because they could only solve level 1 questions. These are the two learners who were placed at competency level 0 and the 13 learners performing at competency level 1.

Therefore, that half of the participants did not achieve National Curriculum Statement Grade 9 Learning Outcome for ratio and proportion. The performance of this group of learners resembles that of learners who participated in the Southern Africa Consortium for Monitoring Educational Quality tests that were discussed in Chapter 2. The Southern Africa Consortium for Monitoring Educational Quality results showed that there were Grade 6 learners who were performing at Grade 3 level or lower. The results of this study indicate that in this school there were Grade 9 learners that performed at the Grade 7 level or lower. Only six percent of the participants performed at competency level 4, that is, who could comfortably solve problems on ratio and proportion that are at Grade 9 level or beyond. This means that only six percent of the learners achieved the desired Grade 9 National Curriculum Statement Learning Outcome for ratio and proportion, which is a big concern.

5.3 Observed errors and misconceptions

In the previous chapters I referred to errors as systematic persistent patterns of mistakes performed by learners (Brodie & Berger, 2010) as they solve problems. This study revealed several mistakes that suit this definition. The errors varied with contexts and cognitive levels of the questions. Nesher (1987) pointed out that errors often lurk behind misconceptions. In this section I will outline and explain errors and misconceptions identified by the study. I will also attempt to provide explanations for why learners commit the identified errors. This section will therefore simultaneously address the second and the third research questions, which are deeply intertwined and answers to one are relevant to the other.

For Level 1 questions, most observed errors resulted from the use of an incorrect operation or incorrect use of the cross multiplication strategy. The questions in this category required multiplication by 2, 3 or halving. Use of an incorrect operation refers to, for example, doubling where the correct operation would be halving. In the recipe problem, learners who obtained a solution of 4 pints of water for Question 1(a) doubled instead of halving. Incorrect use of the cross multiplication strategy also yielded the same result of 4. As described previously in Chapter 4, the incorrect use of the cross multiplication strategy entails the incorrect application of the proportion algorithm. For example, one learner correctly set up the proportion $8 : 4 = 2 : x$ for Question 1(a), where x is the number of pints of water needed to make soup for four people. The learner then incorrectly applied the proportion algorithm and ended up with $4x = 16$. The learner ultimately ended up with $x = 4$. Errors resulting from the incorrect use of the strategy were also observed in solutions to problems at a higher cognitive level.

The above examples also demonstrate that learners do not interpret the answers they obtain to see whether they make sense in the particular context. One way to judge the reasonableness of their answers would be to compare quantities. If 4 pints of water are needed to make soup for eight people, it does not make sense to say that 8 pints of water would be needed to make soup for fewer people. The interview with one learner below shows that learners are not making any comparison of quantities concerned.

Learner: UV is 18 and AC is 8, these were given. We are also given that RT is 12. Because I didn't know the value of DE I put an x next to it, then I said $18/x$ multiplied by 8/12, I then cross multiplied and got $216 = 8x$. I divided both sides by 8 and got $x = 27$.

Interviewer: Which line was that?

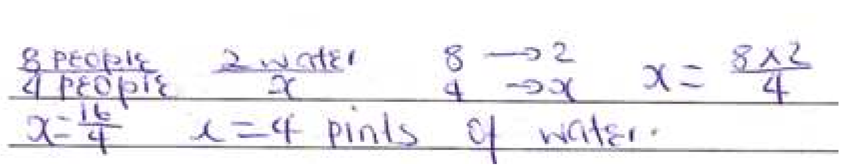
Learner: DE.

Interviewer: Do you think DE is longer than UV?

Learner: Eish! No.

The learner got DE = 27, and accepted it as the correct solution. He did not look at the size or measurements of the figures in the given diagrams. Solving the problems on ratio and proportion requires learners to have ratio sense. Ratio sense involves the ability to think flexibly in problem situations that involve ratios (RIDoE, 2007). Lack of ratio sense, such as the sense required to recognize that to make soup for fewer people one needs less water, is one reason for the errors committed by the learners. The learners who obtained results such as we need 4 pints of water to make soup for four people when given that 2 pints of water were needed to make soup for eight people, apparently did not ask the crucial question: is more or less water needed to make soup for four people? Had the learner posed questions of this nature, the learner would immediately have seen that the result that 4 pints of water was needed is incorrect. National Curriculum Statement expects teaching to ensure that learners do not just meaninglessly perform operations, but they should be able to judge the reasonableness of their solutions. This implies that teaching should aim to inculcate ratio sense in learners.

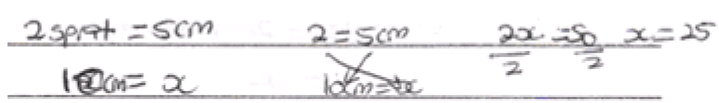
Both the working details shown on learner scripts, and the interviews often revealed incorrect conceptualizations of ratio and cross multiplication. What was often apparent was the learners' perception of a ratio as a fraction. Of course a ratio and a fraction are related, but subtly distinct from one another (Long, 2009). The following response provides an example of this misconception.



Here the learner just wrote fractions with no indication of how the fractions are related (no equal sign between them to indicate equivalence). The learner did not use other appropriate ratio notations except presenting a ratio as a fraction. The learner then *mapped* 8 to 2 and 4 to x (using arrows). The learner ultimately ended up with the equation $x = (8 \times 2)/4$, which was correctly solved to get $x = 4$. The arrows that I have referred to as a *mapping* show the learner's conception of cross multiplication, which resulted in the incorrect equation:

$$numerator \times numerator = denominator \times denominator.$$

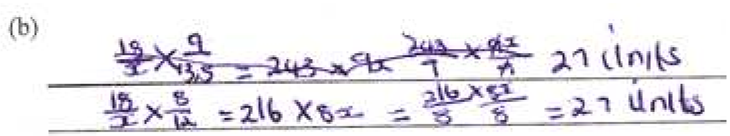
Failure to set up ratios using the appropriate notation and misconception of cross multiplication are the reasons for this incorrect result. In fact, none of the learners who were interviewed mentioned the term ratio. Consider the example below taken from a learner script.



The notation used by the learner is mathematically unacceptable. The resultant equation of $2x = 50$ is indicative of an incorrect set up of ratios. It resulted from a failure on the part of the learners to recognise that some quantities are from the same measure space, while some are not. It is clear that these students have not been able to recognize that ratio is an appropriate comparison, a recognition that Van der Walle (2007) asserts is a prerequisite to proportional reasoning.

If the cross multiplication strategy is to be used, then the order of the setup of the two ratios that are to be equated is important. If one ratio is comparing a quantity from measure space X to a quantity from measure space Y respectively, the other ratio should follow the same order for the ratios to form a proportion. In the eel problem attempted by the learner above, one measure space was *the eel length*, and the other measure space was *the number of sprats*. If the first ratio compares number of sprats that eel A got to the length of eel A, the second ratio must follow the same order for eel B. That is, 2 sprats : 5 cm = x sprats : 10 cm.

A second way to set up equal ratios would be to compare the lengths of the two eels, and equate the ratio to the comparison of the number of sprats needed for each eel, provided the order is consistent. If the first ratio compared length of eel A to the length of eel B, the second ratio must compare the number of sprats for eel A to the number of sprats for eel B. That is, 5 cm : 10 cm = 2 sprats : x sprats. Hart (1988) refers to the example of comparison in the preceding paragraph as the *within comparison strategy* and the second one in this paragraph as the *between comparison strategy*. The order of comparison of quantities forming one ratio should be consistent with that of the order in the second ratio for cross multiplication to work. Both the *within* and *between* comparison strategies could be used to generate equal ratios provided that learners recognize that quantities in a ratio covary in such a way that the relationship between them is invariant. Covary means that if one quantity changes, the other one also changes at the same rate and invariant means that the multiplicative relationship of the quantities does not change. That is, given a ratio $a : b$, if a increases so does b, but a/b or b/a is always a constant (Van der Walle, 2007). However in this study, it was quite evident that learners picked out numbers and then tried to use the cross multiplication strategy without ensuring that they were using the 'between' comparison or the 'within' comparison in a consistent manner. Here is one example of a misuse of this strategy.



In this learners' response she could have set up a *between* comparison of 8 : 12 = DE : 18 or a *within* comparison of 8 : DE = 12 : 18. Thereafter the use of the cross multiplication strategy would result in 12 × DE = 144. However, this learner carried out the strategy without first setting up the correct relationships.

Reins (2009), proposes that teaching should consider that a ratio as a multiplicative comparison of two quantities or measures. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up. It is clear that the learners in this group have moved too quickly to using the formal cross multiplication strategy without having these prerequisite understandings that Reins (2009) has outlined. Thus many of the learners carried out the cross multiplication rule without first ensuring that they have derived appropriate ratios and compared them using either a between or within approach (not a mixture of both). They also have not been concerned about the order of the quantities forming the two ratios.

Incorrect conceptualization of ratios was also observed in the incorrect answers to Question 3. Solving problems similar to this one requires learners to know partitioning. Partitioning is the process of dividing an object into a number of disjoint parts that collectively make the whole (RIDoE, 2007). In the Nkosi/Rajen salary problem some learners did not realize that the sum of the amounts given to Nkosi and Rajen should add up to R350. Some learners allocated amounts to Nkosi and Rajen which added to a total of either less than R350 or more. This is an indication that they perceive each salary as part of an unknown whole (part to whole comparison of quantities). However, there were other learners who showed an understanding of partitioning, but who could not apply proportional reasoning to do so. For example, learners who gave Nkosi R200 and Rajen R150 partitioned the R350, although they did not make the partition proportionally using the time each person worked. They realized that Nkosi should get more money because he worked longer hours.

Errors resulting from the use of the incorrect addition strategy surfaced across the various levels. An incorrect addition strategy refers to addition of the same amount to measures of one space to get the unknown measure in the second measure space. The strategy was used by a greater proportion of learners to solve the higher cognitive level (Level 3 and Level 4) questions, especially the questions on enlargement. For example, Mr Short was 4 matchsticks tall, and this is equivalent to 6 paperclips. Mr Tall was six matchsticks tall. Learners concluded that Mr Tall is 8 paperclips tall by adding 2 (the difference in Mr Short's height in paperclips and matchsticks) to 6 (Mr Tall's height in matchsticks). On average 43% of the learners used the strategy to solve Questions 4, 5 and 7, an observation similar to the findings made by other studies. The use of the strategy also results from conceptualizing ratio as an additive relationship rather than a multiplicative one.

The strategy was also observed in the level 2 questions, for example, in the eel problem. However the eel problem presented a special situation, where the learners who used the addition strategy in Question 2(a) obtained the correct answer. Recall that the eel lengths of A, B and C were 5cm, 10cm and 15cm respectively. For Question 2(a), we were given that eel A is fed 2 sprats, and learners were asked for the number of sprats that should be fed to eel B and eel C. For Question 2(b), they were given that the number of sprats fed to eel B was 12, and asked for the number of sprats that should be fed to eel C. Table 5.1 below shows the answers obtained by using the additive strategy in the two questions.

Table 5.1: Feeding sprats to eels

Question	**Sprats (Eel A)**	**Sprats (Eel B)**	**Sprats (Eel C)**	**Comment**
2(a)	2	4	6	2 more (correct)
2(b)		12	14	2 more (incorrect)

The numbers 4 and 12 belong to the same measure space (Number of sprats fed to eel B). By applying the additive strategy to Question 2(a), eel C gets two more sprats than eel B which turns out to be correct. The learner then reasoned that even in Question 2(b) eel C should get 2 more sprats than eel B. Viewed multiplicatively, in Question 2(a) eel B gets twice the number of sprats that eel A gets and eel C gets trice. The same rule should apply in Question 2(b) to arrive at the correct solution of 18 sprats. The rule of adding 2 works correctly in the first question (Question 2(a)), because in this particular situation 2×2 is equal to $2 + 2$, but the strategy does not always work correctly in Question 2(b). In the question under scrutiny (Question 2), addition or subtraction of 5, the difference in lengths of eels, was also observed in some learner scripts. For example, in Question 2(f), the longer eel Z is fed 10g of fishfingers. Some learners then said eel Y should be fed 5g less of fishfingers than eel Z since eel Y is shorter than eel Z.

The application of the additive strategy to the problems on ratio should squarely be assigned to our failure as teachers to assist learners with the transition from Additive Conceptual Field to Multiplicative Conceptual Field in the middle grades of schooling (Long, 2009). As mathematics teachers, we fail to assist learners see that a ratio is a multiplicative relationship of quantities. We present the multiplicative processes such that learners view them as addition, which limits the learners' transition from the additive to the multiplicative conceptual fields. For example, some learners could have visualised the eel problem as follows:

Some learners were be unable to see that eel length and eel feed was doubling. Learners did not see 2 + 2 as 2 × 2, nor did they see 5 + 5 as 5 × 2. An eel length of 15 cm was visualized as 5 cm + 5 cm + 5 cm, not 5 cm × 3, and hence the number of sprats given to this eel was taken as 2 + 2 + 2, which was also not seen as 2 × 3. Hence, learners did not see the multiplicative nature of a ratio.

Learner errors indicated that learners sometimes did not view ratio and proportion as concepts that belong to the *multiplicative conceptual field*. The multiplicative conceptual field entails all "situations that can be analysed as simple and multiple proportion problems and for which one usually needs to multiply or divide" (Long, 2010, p. 34). Some learners therefore conceptualised ratio as an additive relationship, which is a huge misconception. Van der Walle (2007) emphasises that instruction should emphasise that "ratios and proportions involve **multiplicative rather than additive** comparisons. Equal ratios result from **multiplication or division,** not from addition or subtraction." Van der Walle (2007) further calls for teachers to first present opportunities that help learners to "know the difference between absolute change and relative change (additive and multiplicative respectively)" (p. 353).

Use of other strategies was observed in isolated questions. In the questions that have multiple steps, or where rate needed to be found, but was harder to find (Level 3 questions), an incomplete building up strategy was observed. In Question 1(c) for example, to find the amount of water needed to make soup for six people, some learners found the amount needed for four people and stopped. Some went as far as finding water needed for two people and then stopped. This could be an error that Brodie and Berger (2010) referred to as a *halting signal error*.

Errors that resulted from an incomplete strategy were observed in Question 1 (c), where learners were given that 1/2 pint of cream was needed to make soup for eight people and they responded that the same amount was needed to make soup for six people. The incomplete strategy was also observed in Question 6. Errors such as, mercury : tin = 1 : 3 resulted from taking 1 in the ratio mercury : copper = 1 : 5 and taking 3 from the ratio tin : copper = 3 : 10. Numbers that correspond to mercury and tin were taken from the question and used as they are.

Another interesting strategy that emerged was learners' use of patterns to solve ratio problems. During an interview, one learner said that he could see in Question 2(a) that the eel lengths were multiples of 5. Since the first one was given 2 sprats, he decided to allocate sprats correspondingly according to multiples of 2. The strategy does point to the use of multiples of the given numbers. The strategy seemed to work well if the number of sprats for the shortest eel was given. If eel A was given 6 sprats, then eel B and eel C got 12 sprats and 18 sprats respectively, which are multiples of 6. In Question 2(b) learners were given that eel B got 12 sprats and had to work out the number of sprats that should be given to eel C. Use of the pattern strategy was thus not obvious in this case as it was in the scenario where eel A was given 6 sprats. But if linearity in the length of eels was expressed as a multiplicative relationship like *number of sprats for eel C* = $k \times$ *number of sprats for eel B*, where k is a constant, then the relationship would be clear.

However, there was also a student who used the pattern strategy to obtain a geometric pattern of 2 sprats, 4 sprats and 8 sprats for eels A, B and C respectively (number of sprats expressed as powers of 2). This raises a question as to whether the pattern strategy is only effective when a ratio involves a comparison of consecutive integer multiples of eel lengths, as this was the case for eels A, B, and C. In the question that involves eels X, Y and Z; should learners start by looking for the jump in eel lengths (10 cm. 15 cm and 25 cm) and then use that to work out the number of sprats to give to each eel; which might be a challenge for some learners. This also reveals another important difference between a ratio and a fraction, which is, fractions involve comparison of two quantities (a part to a whole), while a ratio may involve a comparison of two or more quantities. In Question 2(a) and Question 2(b), the ratio of eel lengths (A : B : C) was 5 : 10 : 15; which is equivalent to 2 : 4 : 6 or 6 : 12 : 18, and so on (comparison of the number of sprats); but not equivalent to 2 : 4 : 8 (a ratio that arises from the number of sprats that arise from the powers of 2). This then suggests that the strategy must be used with great caution for teaching, as it may require meaningful judgment of the reasonableness of the solution on the part of the learner. Chunlian (2008) mentioned *looking for a pattern* as one of the strategies he observed among students from Singapore and China.

Errors that resulted from slips (computational errors), guessing or use of naïve methods (Hart, 1988); and errors that resulted from magical doubling were also observed in isolated cases. Magical doubling was common in Question 1 and Question 2. In Question 2(a)(ii), a solution of 8 sprats is one example that was a result of magical doubling. Eel B was fed $2 \times 2 = 4$ sprats, and the use of magical doubling strategy resulted in eel C being fed $4 \times 2 = 8$ sprats. At least learners who use a doubling strategy have a perception of a ratio as a multiplicative relationship

5.3 Conclusion

In Chapter 2 an outline of similar studies carried out in other parts of the world was presented. Are the findings of this study the same or different from those made by the other studies? My answer to the question is that there is no significant difference between the strategies used by the participants in this study and those used by learners in the studies presented in Chapter 2, because the errors observed were similar. One strategy that emerged in this study and that has not been specifically identified in other studies is the *use of patterns*. One probable reason for the use of the strategy might be the fact that one of the curriculum learning outcomes is patterns, and learners spend much time on recognising and extending patterns in the Senior Phase.

A big concern arising from the results of this study is that learner scripts and interviews reflect superficial or no understanding of ratio in the learners. Learners did not portray the correct conceptualisation of ratio. A large proportion of the learners could barely set up a proportion correctly. In the introduction, a concern about performance in mathematics was expressed. Arnold strongly criticized the myth that mathematics is for the chosen few; therefore it is just if some learners fail it. He is of the opinion that one of the contributors to the status *quo* in mathematics is the use of ineffective and inappropriate teaching methods. I agree with the notion after carefully analyzing learner performance in the test items. Errors identified by the study point to lack of even instrumental understanding of the cross multiplication algorithm. Instrumental understanding refers to a situation where a learner knows the rule, can correctly use the rule, but does not know why the rule is used and why the rule works (Skemp, 1976). In this case, learners knew the cross multiplication algorithm, but they used it incorrectly.

Considering that the participants are in Grade 9, this confirms that learners lack an "operational" understanding of ratio (Long, 2007). The performance of the learners in this study clearly indicates that the majority of learners could not reason proportionally. Teachers should be reminded that from a constructivist perspective, errors and misconceptions result from knowledge construction by the learner using prior knowledge. This implies that errors and misconceptions are inevitable. Since errors and misconceptions cannot be avoided, they should not be treated as terrible things to be uprooted, as this may confuse the learner and shake the learner's confidence in his previous knowledge. Making errors should be regarded as part of the process of learning. Teachers should create classrooms where the atmosphere is tolerant of errors and misconceptions, and exploit them as opportunities to enhance learning (Olivier, 1992b).

Silver (2000) expressed concern about ratio and proportion being taught in a cursory fashion, to which he attributes development of "limited view of proportions rather than a rich view of proportions" (p. 22) in learners. Learners' scripts suggest that the use of cross multiplication as a strategy to solve problems on ratio and proportion was overemphasized to the detriment of developing ratio sense. Teachers need to ensure that learners can correctly set up ratios, and thus correctly set up a proportion. In fact, Van de Walle (2007) suggests that teaching should consider that "a ratio is a multiplicative comparison of two quantities or measures. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up". He further advises that "proportional thinking is developed through activities involving comparing and determining the equivalence of ratios and solving proportions in a wide variety of situations and contexts" (p. 353).

Curriculum developers need to streamline and clearly identify the knowledge and skills that learners should acquire on ratio and proportion. It is not sufficient just to state that Grade 9 learners *should be able to solve problems on ratio, rate and proportion*. A more specific breakdown of the types of problems they should be able to solve and the types of situations that they should be exposed to, will help teachers better understand what they need to do with their learners. This in turn will help authors of textbooks to produce materials that do not just focus on elementary formulation of ratios and writing ratios in the simplest form. Both authors and teacher trainers need to ensure that teachers know the similarities and differences between a ratio and a fraction. Learners need to know that fractions always compare part to whole while ratios may compare part to part. More than anything else, teachers need to emphasise the multiplicative nature of ratios.

I hope that the outlined misconceptions that were identified in this study are not unique to this particular group of learners. It may help teachers of mathematics to assess their learners for these misconceptions, as that can inform the teaching of the topic. As long as it is understood that learners construct their own knowledge using prior knowledge, mathematics teachers have an obligation to ensure learners construct the new knowledge on the correct conceptions of the topic at hand.

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APPENDIX A

Assessment Task

Mathematics

Grade 9

Name of learner: ____

Name of school: ____

Age: ____

Date: ____

1.

Onion soup recipe for 8 people

8 onions

2 pints of water

4 cubes of chicken soup

2 spoons of butter

½ pint of cream

I am cooking onion soup for 4 people.

(a) How much water do I need?

(b) How many cubes of chicken soup do I need?

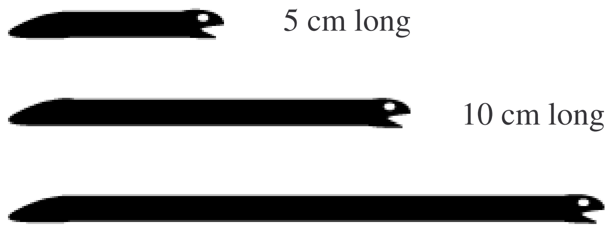
I am cooking onion soup for 6 people.

(c) How much water do I need?

(d) How many cubes of chicken soup do I need?

(e) How much cream do I need?

2. Eels A, B and C are fed *sprats* according to their length.



A — 5 cm long

B — 10 cm long

C — 15 cm long

(a) If eel A is fed 2 sprats,

(i) How many sprats should eel B be fed?

____

____

(ii) How many sprats should eel C be fed?

____

____

(b) If eel B gets 12 sprats, how many sprats should eel C be fed?

____

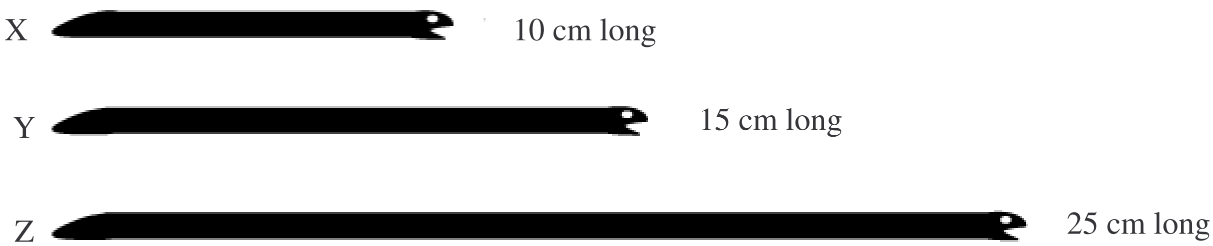
____

(c) If eel C gets 9 sprats, how many sprats should eel B get?

____

____

Three other eels X, Y and Z are fed fishfingers. The mass of the fishfinger depends on the length of the eel.



(d) If eel X gets 2 grams of fishfingers, how much fishfingers should be given to eel Z?

____

____

(e) If eel Y gets 9 grams of fishfingers, how much fishfingers should be given to eel Z?

____

____

(f) If eel Z gets 10g of fishfingers,

(i) how much should eel X get?

____

(ii) how much should eel Y get?

____

3. Nkosi and Rajen work on a job together.

Nkosi works for 12 hours.

Rajen for 8 hours.

How should they share the payment of R350 for the job?

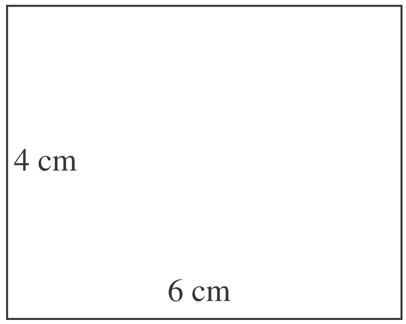
Nkosi:____________________________________

____

Rajen: ____________________________________

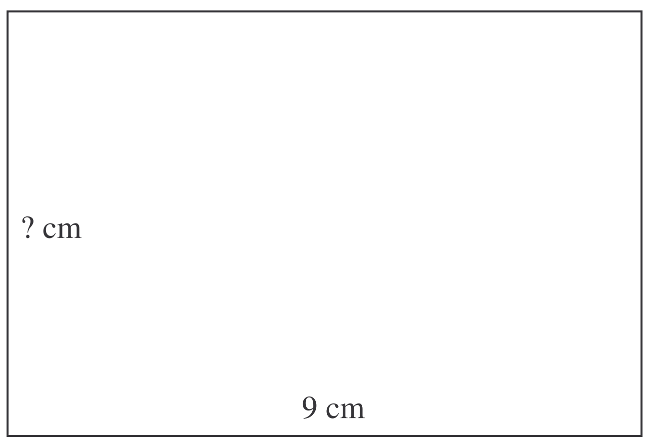
____

4. The rectangle below has a 6 cm base.



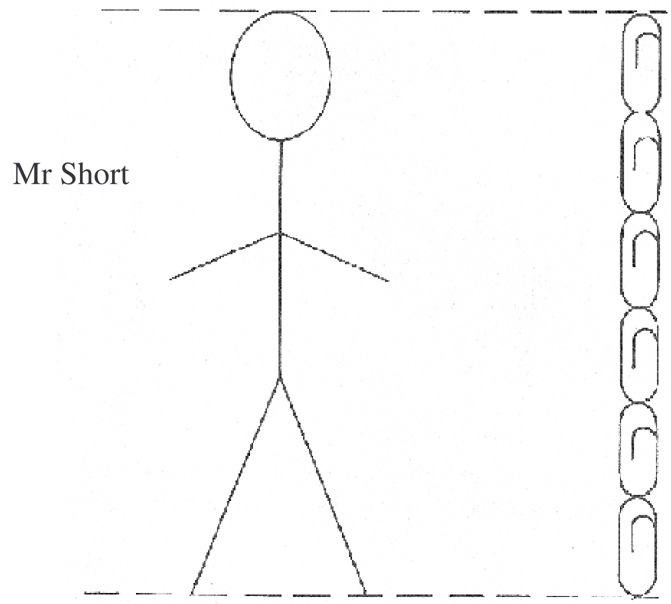
The rectangle is enlarged so that it keeps the shape (see diagram below).

The base of the enlarged rectangle is 9 cm.



What is the length of the other side?

5. Below, we are shown the height of Mr Short measured in paper-clips.



Mr Short has a friend called Mr Tall.

Mr Short's height is 4 matchsticks.

Mr Tall's height is 6 matchsticks.

What is Mr Tall's height in paper-clips?

____

____

6. In a particular metal alloy (mixture of metals):

mercury : copper = 1 : 5,

tin : copper = 3 : 10, and

zinc : copper = 8 : 15.

Complete by filling in the missing numbers.

(a) mercury : tin = ________________.

(b) zinc : tin = ___________________

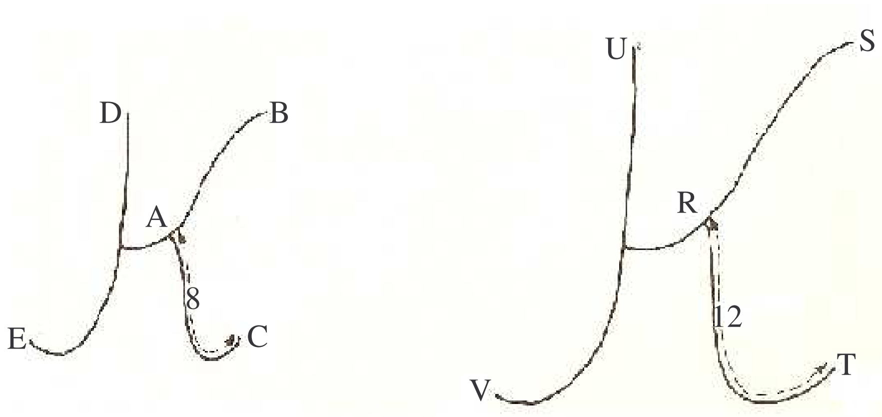
.

7. The letters below are the same shape.

   One is larger than the other.

   The curve AC is 8 units.

   The curve RT is 12 units.



(c) The curve AB is 9 units. How long is the curve RS?

____

____

(d) The curve UV is 18 units. How long is the curve DE?

____

____

8. On the 24 May 2010, $1 (1 US Dollar) could be exchanged for approximately R8. If I have R3 200, how much money do I have in US Dollars?

____

____

____

____

APPENDIX B

UNIVERSITY OF KWAZULU-NATAL

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29 November 2010

Mr P T Mahlabela
School of Mathematics Education
EDGEWOOD CAMPUS

Dear Mr Mahlabela

PROTOCOL: Learners' errors and misconceptions in ratio and proportion
ETHICAL APPROVAL NUMBER: HSS/1366/2010 M: Faculty of Education

In response to your application dated 25 November 2010, Student Number: **209530150** the Humanities & Social Sciences Ethics Committee has considered the abovementioned application and the protocol has been given **FULL APPROVAL.**

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Professor Steve Collings (Chair)
HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE

SC/sn

cc: Dr. S Bansilal (Supervisor)
cc: Mr N Memela