$\mathbb{F}$ischer $\mathbb{C}$lifford $\mathbb{M}$atrices and $\mathbb{C}$haracter $\mathbb{T}$ables of $\mathbb{C}$ertain $\mathbb{G}$roups $\mathbb{A}$ssociated with $\mathbb{S}$imple $\mathbb{G}$roups $\mathbb{O}_{10}^{+}(2)$, $\mathbb{H}S$ and $\mathbb{L}y$

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Abstract

The character table of any finite group provides a considerable amount of information about a group and the use of character tables is of great importance in Mathematics and Physical Sciences. Most of the maximal subgroups of finite simple groups and their automorphisms are extensions of elementary abelian groups. Various techniques have been used to compute character tables, however Bernd Fischer came up with the most powerful and informative technique of calculating character tables of group extensions. This method is known as the Fischer-Clifford Theory and uses Fischer-Clifford matrices, as one of the tools, to compute character tables. This is derived from the Clifford theory. Here $\overline{G}$ is an extension of a group N by a finite group G, that is $\overline{G} = N.G$. We then construct a non-singular matrix for each conjugacy class of $\overline{G}/N \cong G$. These matrices, together with *partial* character tables of certain subgroups of G, known as the *inertia* groups, are used to compute the full character table of $\overline{G}$.

In this dissertation, we discuss Fischer-Clifford theory and apply it to both split and non-split extensions. We first, under the guidance of Dr Mpono, studied the group $2^7{:}S_8$ as a maximal subgroup of $2^7{:}SP(6,2)$, to familiarize ourselves to Fischer-Clifford theory. We then looked at $2^6{:}A_8$ and $2^8{:}O_8^+(2)$ as maximal subgroups of $2^8{:}O_8^+(2)$ and $O_{10}^+(2)$ respectively and these were both split extensions. Split extensions have also been discussed quite extensively, for various groups, by different researchers in the past. We then turned our attention to non-split extensions. We started with $2^{4\cdot}S_6$ and $2^{5\cdot}S_6$ which were maximal subgroups of HS and $HS{:}2$ respectively. Except for some negative signs in the first column of the Fischer-Clifford matrices we used the Fisher-Clifford theory as it is. The Fischer-Clifford theory, is also applied to $5^{3\cdot}L(3,5)$, which is a maximal subgroup of the Lyon's group Ly. To be able to use the Fisher-Clifford theory we had to consider *projective* representations and characters of inertia factor groups. This is <u>not</u> a simple method and quite some smart computations were needed but we were able to determine the character table of $5^{3\cdot}L(3,5)$. All character tables computed in this dissertation will be sent to GAP for incorporation.

Preface

The work described in this thesis was carried out under the supervision and direction of Professor Jamshid Moori, Department of Mathematics and Applied Mathematics, University of KwaZulu-Natal, Pietermaritzburg, from August 2007 to October 2011.

The thesis represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any other University. Where use has been made of the work of others it is duly acknowledged in the text.

Signed:

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Dedication

I together with my dear wife Malehlohonolo and my three sons Mafa, Tefo and Kutlwano dedicate this dissertation to my mother Madinko, as our present, in appreciation of her kindness and steadfast support, for her 75th birthday. This is just but, a humble effort, to follow the tracks of the footsteps left by my father Bobby.

Acknowledgements

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Notation and conventions

Throughout this thesis all groups will be assumed to be finite, unless otherwise stated. We will use the notation and terminology from the $\mathbb{ATLAS}$ [23] and $\mathbb{ATLAS}$ $\mathbb{V}3$ [124].

$\mathbb{N}$	natural numbers
$\mathbb{Z}$	integers
$\mathbb{Q}$	rational numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
G, N, H, K	groups
1_G	the identity element of G
$H \leq G$	H is a subgroup of G
$H \cong G$	H is isomorphic to G
$\mathbb{F}$	a field
$\mathbb{F}^*$	$\mathbb{F} - \{0\}$
$\langle x, y \rangle$	the subgroup generated by x and y
$N.G$	an extension of N by G
$N{:}G$	a split extension of N by G
$N^{\cdot}G$	a non-split extension of N by G
h^g	conjugation of h by g
nX	a general conjugacy class of G with representatives of order n
$g_1 \sim g_2$	g_1 is conjugate to g_2
$o(g)$	order of $g \in G$

$C_G(g)$	the centralizer of g in G
$[g]$	a conjugacy class of G with representative g
$N_G(H)$	the normalizer of a subgroup H in G
Hg	the right coset of H in G
gH	the left coset of H in G
X, Y, Ω	sets
$\|\Omega\|$	the cardinality of the set Ω
$1^{\alpha}2^{\beta}3^{\gamma}\ldots$	cycle structure of a permutation
$Irr(G)$	the set of ordinary irreducible characters of G
I_G	the identity character of G
$\chi(G\|H)$	the permutation character of G on H
χ_H	the restriction of the character χ of G to the subgroup H
ψ^G	the induction of the character ψ of subgroup H to G
na, nb, $\ldots$	irreducible characters of G of degree n
$\langle \chi_i, \chi_j \rangle$	the inner product of the characters χ_i and χ_j
$dim(V)$	the dimension of a vector space V
A_n	the alternating group on n symbols
S_n	the symmetric group on n symbols
$GF(q)$	the Galois field of q elements
$V(n, q)$	a vector space of dimension n over $GF(q)$
$Sp_{2n}(q)$	symplectic group of dimension $2n$ over $GF(q)$
$O^{+}_{2n}(q)$	the orthogonal group consistent with the form f^{+} on $V = V(2n, q)$ invariant
$O^{+}_{8}(2)$	the simple orthogonal group of dimension 8 over $GF(2)$, $\|O^{+}_{8}(2)\| = 2^{12} \times 3^{5} \times 5^{2} \times 7$
$O^{+}_{10}(2)$	the simple orthogonal group of dimension 10 over $GF(2)$, $\|O^{+}_{10}(2)\| = 2^{20} \times 3^{5} \times 5^{2} \times 7 \times 17 \times 31$
$L(n, q)$	the projective special linear group $(PSL(n, q))$ on $V = V(n, q)$
$L(3, 5)$	the projective special linear group of dimension 3 over $GF(5)$, $\|L(3, 5)\| = 2^{5} \times 3 \times 5^{3} \times 31$
HS	the Higman-Sims group
Ly	the Lyons group
p^n	an elementary abelian group of order p^n

Contents

1

Introduction

The classification of finite simple groups theorem states that every finite simple group is isomorphic to exactly one of the following :

- cyclic groups of prime order,
- alternating groups A_n, where $n \geq 5$,
- groups of Lie type and
- the 26 sporadic groups.

Groups of Lie type can also be divided into two types namely the classical groups and exceptional groups that include twisted groups. Classical groups are the projective special linear groups, the sympletic groups, the unitary groups and the orthogonal groups. Groups of exceptional type are groups of the form $G_n(q), F_n(q), E_n(q)$ and the Tits group ${}^2F_4(2)'$. The sporadic groups are the five Mathieu groups, seven Leech lattice groups, three Fischer groups, five Monster groups and the six pariahs.

The completion of the classification of all simple groups might have led to some naive thought of the death of group theory. On the contrary this has led to very serious studies on the classification of maximal subgroups of simple groups. One way of classifying finite groups was by character theory. Isaacs provides an extensive study of character theory in [60]. In the classification of maximal subgroups of simple groups, calculating character tables of these groups, has been one of the methods used a lot. Quite a large number of maximal subgroups of simple groups are extensions of elementary abelian groups. In this dissertation all groups studied are extensions of elementary abelian groups. In the past most of these character tables were calculated using methods that did not provide a lot of insight about the structure of the group concerned. In this dissertation we use the Fischer-Clifford theory, for computing character tables, which is a method used by Bernd Fischer. In this dissertation we apply the Fischer-Clifford method to both split and, with some amendments if needed, to non-split extensions. We follow from Ali [1] also Ali and Moori [2] who not only applied it to split extensions but also to non-split extensions. This follows on the tracks of a large number of researchers who applied this method to split extensions. For further reading, on Fischer-Clfford theory, one can go to Ali and Moori [3], Almestady [4], Darafsheh and Iranmanesh ([26], [27]), Fischer ([34], [36], [37]), List [75], List and Mohammed [76], Moori and Mpono ([90], [91], [92]), Mpono [99], Pahlings [103], Saleh [112], Schiffer [113] and Whitely [120].

In Chapter 2 we give some preliminary results on group extensions and group characters which will be used in later chapters. In section 2.2 we define group extensions and discuss some basic results. In section 2.3 we discuss the conjugacy classes of group extensions. We briefly discuss the technique of *coset analysis* for computing conjugacy classes of a group extension $\overline{G}$ of N by G where N is an elementary abelian normal subgroup of $\overline{G}$. The technique of coset analysis was developed by Moori [81] which he also used in [82] and this has been widely used for computing conjugacy classes of group extensions. Analogous to the programmes developed in MAGMA by Ali [1] and in Cayley by Mpono [99] we developed Programmes A and B in GAP [41] which we used to compute the conjugacy classes of the groups $2^7{:}S_8$, $2^6{:}A_8$ and $2^8{:}O_8^+(2)$ that we studied in this dissertation in Chapters 6, 7 and 8 respectively. In section 2.4 we studied preliminary results on representation and characters which is used in later chapters. In section 2.5 we look at induced characters, section 2.6 deals with permutation characters and in section 2.7 we develop Programme C in GAP [41] that we use to compute orbit lengths of orbits of conjugacy classes and irreducible characters. For further reading on group extensions, representations and character of groups readers can also look at the following [5], [7], [10], [11], [14], [20], [21], [30], [48], [56], [60], [61], [62], [63], [71], [74], [100], [105], [110], [116] and [117].

In Chapter 3 we discuss projective representations and projective characters. The first step in obtaining the projective representations of a group G is to compute its *Schur multiplier*. In section 3.2 we discuss the Schur multiplier of a group and methods of computing the Schur multiplier. In section 3.3 we discuss the projective representations of G. We prove that for a projective representation P with factor set α of degree n, then $o([\alpha]) \mid n$. We show how a projective representation of G can be obtained from an ordinary representation of a "representation group" of G. We also look at three different methods of constructing a projective representation of a group G. We then study the projective characters of G in section 3.4. For further reading on projective representations and projective characters one can read [11], [47], [51], [55], [58], [60], [95], [96], [97], [98], [100], [107], [108] and [109].

In Chapter 4 we study the Clifford theory for ordinary and projective representations of a group $\overline{G} = N.G$, where $N \trianglelefteq G$. Here we discuss how the groups N and G are related to $\overline{G}$ and the consequences thereof. In section 4.2 we study Clifford Theory and normal subgroups. In section 4.3 we discuss Clifford Theory and projective representations. In Section 4.4 we look at group action and how to use GAP in our computations where the dimension of N is not the same as the permutation degree of G. Last in Section 4.5 we look at irreducible constituents and conjugacy classes. This is all required for the construction of Fischer-Clifford matrices which we discuss in Chapter 5. For further reading on Clifford Theory, one can go to [11], [40], [58], [60], [66], [67] and [100].

Chapter 5 is devoted to one of the most important tools used in this dissertation namely the Fischer-Clifford matrices. If $\overline{G} = N.G$ is an extension of N by G where N, which is normal in $\overline{G}$, is an elementary abelian group. We compute a non-singular matrix for each conjugacy class of $\overline{G}/N \cong G$. Then we use these matrices, fusion maps and the partial character tables of inertia factor subgroups to compute the full character table of $\overline{G}$. In this dissertation we apply this technique to both split and non-split extensions. This technique has been used mostly in split extensions by Almestady [4], Darafsheh and Iranmanesh ([26], [27]), Fischer ([34], [35], [36]), [37], List [75], List and Mohammed [76], Moori and Mpono ([90], [91], [92]), Mpono [99], Pahlings [103], Saleh [112], Schiffer [113] and Whitely [120]. With, the necessary adjustments, Ali [1] used this method for both split and non-split extensions. Ali and Moori [2, 3] also used it for non-split extensions.

In Chapter 6 we look at the group $2^7{:}S_8$ as a maximal subgroup of $2^7{:}SP(6,2)$ which in turn is a maximal subgroup of $\overline{Fi}_{22}$, the full automorphism group of the smallest Fischer sporadic simple group Fi_{22}. In section 6.1 we compute the generators of $Sp(6,2)$ which we use in section 6.2 to compute the generators of S_8. In section 6.3 we compute and discuss the conjugacy classes of $2^7{:}S_8$ and then in section 6.5 we look at the power maps of $2^7{:}S_8$ which we use in computing the fusion of $2^7{:}S_8$ into $2^7{:}SP(6,2)$. We conclude this chapter by determining the character table of $2^7{:}S_8$.

Chapter 7 is based on a group of the form $2^6{:}A_8$ as an inertia factor group of $2^8{:}O_8^+(2)$. Here we use two methods to compute A_8 with 6×6 matrix generators. In section 7.2 we use combinotorics to compute these generators. In section 7.3 we use GAP to compute the generators of A_8 inside $O_8^+(2)$. In section 7.4 we compute and discuss the conjugacy classes of $2^6{:}A_8$. In 7.5 we compute and discuss the Fischer-Clifford matrices of $2^7{:}A_8$. In section 7.6 we finish off the chapter by calculating the character table of $2^6{:}A_8$.

Chapter 8 is concerned with a group of the form $2^8{:}O_8^+(2)$ as a maximal subgroup of $O_{10}^+(2)$. The group $2^{10+16}{\cdot}O_{10}^+(2)$ in turn is a maximal subgroup of $F_1 = M$, the monster. In section 8.1 we define the bilinear forms from which we use to define orthogonal forms which are in turn used to define orthogonal groups in section 8.2. In section 8.3 we look at the action of $O_8^+(2)$ on the elementary abelian group 2^8. We then compute the inertia factor groups of $2^8{:}O_8^+(2)$ and their fusion into $O_8^+(2)$ in section 8.4.1. In section 8.4.2 we compute the Fischer-Clifford matrices while section 8.4.3 deals the power maps and the character table of $2^8{:}O_8^+(2)$.

Chapter 9 is focused on groups of the form $2^4{\cdot}S_6$ and $2^5{\cdot}S_6$ which are both non-split extensions, with $2^4{\cdot}S_6$ and $2^5{\cdot}S_6$ maximal subgroups of the Higman-Sims HS, and its full automorphism group $HS{:}2$ respectively. In order to be able to use the Fischer-Clifford theory on these extensions, we use the methods that are used by Ali [1]. In section 9.2 we discuss the Higman Sims group in relation to the Conway groups. We use the $\mathbb{ATLAS}$ $\mathbb{V}3$ to construct HS in section 9.1.2. In section 9.1.3 again using programmes from the $\mathbb{ATLAS}$ $\mathbb{V}3$ we construct the subgroups $2^4{\cdot}S_6$ and $2^5{\cdot}S_6$ of HS

and $HS{:}2$ respectively. In the group $2^{5\cdot}S_6$ we constructed, we found that there were three distinct groups of order 11 520. One of them was $2^{4\cdot}S_6$, the second was the split extension $2^4{:}S_6$ and the third we determined as $2^5{:}A_6$. In section 9.2.1, inside $HS{:}2$, we constructed $2^{4\cdot}S_6$ isomorphic to the one which was a maximal subgroup of HS. We then used the GAP method used in Chapter 7 to construct the generators of S_6 as 4×4 matrices. In section 9.2.2 and we let S_6 act on 2^4 and we then computed the conjugacy classes and inertia factor groups of $2^{4\cdot}S_6$. In section 9.2.3 we compute the Fischer-Clifford matrices and full character table of $2^{4\cdot}S_6$. In section 9.3 we look at the group $2^5.S_6$, constructed as an automorphism group of $2^{4\cdot}S_6$. In section 9.3.1, similar to the methods in section 9.2.1, we construct the generators of S_6 as 5×5 generators in $GF(2)$. In section 9.3.2 we let S_6 act on 2^5 and we then compute the conjugacy classes and inertia factor groups of $2^{5\cdot}S_6$. We finished off the chapter in section 9.3.3 by computing the Fischer-Clifford matrices and character table of $2^{5\cdot}S_6$.

The main focus of Chapter 10 is on a group of the form $5^{3\cdot}L(3,5)$ as a maximal subgroup of the Lyons group Ly. Again as in Chapter 9, using the $\mathbb{ATLAS}$ $\mathbb{V}3$ we construct the group $5^{3\cdot}L(3,5)$ inside the group Ly. This was done in section 10.2. In section 10.3 we use the methods of Chapter 6, which were used again in Chapter 9, to construct the group $L(3,5)$. In 10.6 we let $L(3,5)$ act on 5^3 and then we compute the inertia factor groups of $5^{3\cdot}L(3,5)$. To be able to adopt the Fischer-Clifford method we compute the Schur multiplier and to do this we need the projective character table of $5^2{:}2.A_5$. In section 10.6 we compute the Fischer matrices, in section 10.7 we deal with the power maps of $5^{3\cdot}L(3,5)$. The computation of the character table of $5^3.L(3,5)$ is done in section 10.8. A programme from GAP, I developed with the help of F. Ali, was used to check the accuracy and consistency of the character tables computed.

2

Coset Analysis

2.1. Prologue

In this chapter we give preliminary results on group extensions and group characters that will be required in later chapters. In section 2.2 we present definitions and some basic results on group extensions. In section 2.3 we discuss the conjugacy classes of elements of group extensions. We describe the technique of *coset analysis* for computing the conjugacy classes of group extension $\bar{G}$ of N by G where N is an abelian normal subgroup of $\bar{G}$. This technique was developed and first used by Moori in [81, 82] and has since been widely used for computing the conjugacy classes of group extensions in all cases where it is applicable. For example, it has been used in Ali [1], Mpono [99], Saleh [112] and Whitely [120]. We also develop two GAP Programmes A and B analogous to the programmes developed by Mpono [99] for CAYLEY and Ali [1] for MAGMA to compute the conjugacy classes and the orders of the class representatives for the split extensions $\bar{G} = N{:}G$ where N is an elementary abelian p-group. We use these programmes to compute the conjugacy classes of the group extensions $2^7{:}S_8$, $2^6{:}A_8$ and $2^8{:}O_8^+(2)$ which will be studied in Chapters 6, 7 and 8 respectively. In Section 2.4 we present some theory on representations and characters of groups by concentrating on those results which would be useful in later chapters. Section 2.5 deals with the relationship between the characters of a group G and the characters of a subgroup H of G. In this section we will first study restriction of characters and then go on to study induced characters. Finally in Section 2.6 we give some results on permutation characters. For further information and readings on group extensions, group representations and characters readers are encouraged to consult [5, 7, 10, 11, 14, 56, 60, 61, 62, 63, 71, 74, 100, 105, 110, 116, 117] and many other relevant sources.

2.2. Group Extensions

Definition 2.2.1. *Let N and G be groups. An* **extension** *of N by G is a group $\bar{G}$ that satisfies the following properties* *(i)* $N \trianglelefteq \bar{G}$,

(ii) $\bar{G}/N \cong G$.

We say that $\bar{G}$ *is a* **split extension** *of* N *by* G *if* $\bar{G}$ *contains subgroups* N *and* G_1 *with* $G_1 \cong G$ *such that*

(i) $N \trianglelefteq \bar{G}$,

(ii) $NG_1 = \bar{G}$,

(iii) $N \cap G_1 = \{1_{\bar{G}}\}$.

In this case $\bar{G}$ *is also called a* **semi-direct product** *of* N *and* G*, and identify* G_1 *and* G.

Following $\mathbb{ATLAS}$ [23], we denote an arbitrary extension of N by G by $N.G$. A split extension of N by G is denoted by $N{:}G$ and a non-split extension is denoted by $N{\cdot}G$.

Definition 2.2.2. *The* **automorphism group** *of a group* G*, denoted by* $Aut(G)$*, is the set of all automorphisms of* G *under the binary operation of composition.*

For $\bar{G}$, a semidirect product of N by G, every element in $\bar{G}$ can be uniquely expressed in the form ng, where $n \in N$ and $g \in G$ and the multiplication of elements of $\bar{G}$ is given by

$$(n_1g_1)(n_2g_2) = n_1n_2^{g_1}g_1g_2 \quad ,$$

where $n^g = gng^{-1}$. Also there is a homomorphism $\theta : G \longrightarrow Aut(N)$ given by $\theta(g) = \theta_g$, where $g \in G$, $\theta_g : N \longrightarrow N$ is defined by $\theta_g(n) = gng^{-1}$ and θ_g is an automorphism of N. Hence G acts on N.

Definition 2.2.3. *Let* $\bar{G}$*,* N *and* G *be as defined above and* $\theta : G \longrightarrow Aut(N)$*. The semidirect product* $\bar{G}$ *of* N *by* G *is said to* **realize** θ *if* $\theta_g(n) = n^g \; \forall \, n \in N, g \in G$.

Remark 2.2.4. For $\bar{G}$ a semidirect product of N by G, then $\bar{G}$ is isomorphic to a semidirect product of N by G that realizes θ for some $\theta : G \longrightarrow Aut(N)$.

If $\bar{G}$ is a split extension of N by G, then $\bar{G} = NG = \bigcup_{g \in G} Ng$. So G may be regarded as a right transversal for N in $\bar{G}$ (that is, a complete set of right coset representatives of N in $\bar{G}$). Now suppose $\bar{G}$ is any extension of N by G, not necessarily split, then since $\bar{G}/N \cong G$, there is a homomorphism $\lambda : \bar{G} \longrightarrow G$ that is onto, with kernel N. For $g \in G$ define a lifting of g as an element $\bar{g} \in \bar{G}$ such that $\lambda(\bar{g}) = g$. By choosing a lifting of each element of G, we get the set $\{\bar{g} : g \in G\}$ that is a transversal for N in $\bar{G}$.

We now show that even for a non-split extension of N by G, if N is abelian , G acts on N.

Lemma 2.2.5. *([1, 99, 111, 120]) Let $\bar{G}$ be an extension of N by G where N is abelian. Then there is a homomorphism $\theta : G \longrightarrow Aut(N)$ such that $\theta_g(n) = \bar{g}n(\bar{g})^{-1}, n \in N$ and θ is independent of the choice of liftings $\{\bar{g} \ : \ g \in G\}$.*

PROOF. Let $a \in \bar{G}$ and γ_a denote conjugation by a. Since N is a normal subgroup of $\bar{G}$, $(\gamma_a)_N \in Aut(N)$ and the function $\mu : \bar{G} \longrightarrow Aut(N)$ defined by $\mu(a) = (\gamma_a)_N$ is a homomorphism. If $a \in N$, then since N is abelian we have $\mu(a) = I_N$. Thus there is a homomorphism $\mu^* : \bar{G}/N \longrightarrow Aut(N)$ which is given by $\mu^*(Na) = \mu(a)$. However $G \cong \bar{G}/N$ and for any lifting $\{\bar{g} \ : \ g \in G\}$, the function $\phi : G \longrightarrow \bar{G}/N$ defined by $\phi(g) = N\bar{g}$ is an isomorphism. If $\{\bar{g}_1 \ : \ g \in G\}$ is another choice of liftings, then $\bar{g}\ \bar{g}_1^{-1} \in N$ for every $g \in G$ and thus $N\bar{g} = N\bar{g}_1$. Therefore the isomorphism ϕ is independent of the choice of liftings. Let $\theta : G \longrightarrow Aut(N)$ be the composition $\mu^* \circ \phi$. For $g \in G$ and $\bar{g}$ a lifting of g, then $\theta(g) = \mu^*(\phi(g)) = \mu^*(N\bar{g}) = \mu(\bar{g}) \in Aut(N)$ and thus for $n \in N$, we have $\theta_g(n) = \mu(\bar{g})(n) = \bar{g}n(\bar{g})^{-1}$. Hence the result. ■

Remark 2.2.6. *[120]* Let $\bar{G}$ be an extension of N by G where N is abelian and for each $g \in G$ let $\bar{g}$ be a lifting of g. We identify G with $\bar{G}/N$ under the isomorphism $g \longmapsto N\bar{g}$. Thus $\{\bar{g} \mid g \in G\}$ is a right transversal for N in $\bar{G}$ and thus every $x \in \bar{G}$ has a unique expression of the form $x = n\bar{g}$ where $n \in N$ and $g \in G$.

2.3. Conjugacy Classes of Group Extensions

In this section we discuss the technique of *coset analysis*, which was initially introduced by Moori [81], to determine the conjugacy classes of group extensions but first we state the following two results.

Theorem 2.3.1. *Let G be a finite group*

(i) Suppose that C_1 and C_2 are two conjugacy classes of G such that $C_1 \neq [1_G]$ and $C_1^n = C_2$ for some integer $n \geq 2$, where

$$C_1^n = \{x_1x_2 \cdots x_n \mid x_i \in C_1,\ 1 \leq i \leq n\} \ .$$

Then there exists some normal subgroup N of G and $g \in G - N$ such that C_1 is the coset Ng and the map $x \mapsto x^n$ is a bijection from C_1 onto C_2.

(ii) If G has a normal subgroup N and $g \in G - N$ such that the coset Ng is a single conjugacy class of G, and such that for some $n \in \mathbf{Z}$ the map $x \longmapsto x^n$ for $x \in Ng$ is a monomorphism, then Ng^n is a conjugacy class of G and $(Ng)^n = Ng^n$.

PROOF. See [12]. ■

Proposition 2.3.2. *Let $\bar{G} = N.G$, $\bar{g} \in \bar{G}$ a lifting of $g \in G$, C be the centralizer of $N\bar{g}$ in G and $\bar{C}$ be the complete pre-image in $\bar{G}$ of C. Then* *(i) the union of the cosets $N\bar{x}$ which are conjugate in G to $N\bar{g}$, is the union of the conjugacy classes $L_1, L_2, \ldots, L_r$ of $\bar{G}$,*

(ii) $\bar{C}$ acts on the coset $N\bar{g}$ by conjugation,

(iii) $\bar{C}$ has r orbits in its action on $N\bar{g}$ and the orbit representatives $\bar{g}_1, \bar{g}_2, \ldots, \bar{g}_r$ are representatives of the conjugacy classes $L_1, L_2, \ldots, L_r$ of $\bar{G}$,

(iv) the centralizer $C_{\bar{G}}(\bar{g}_i)$ for $1 \leq i \leq r$ is the stabilizer of $\bar{g}_i$ in $\bar{C}$ in its action on $N\bar{g}$.

PROOF. See [16]. ■

We now briefly discuss the technique of *coset analysis* to determine the conjugacy classes of elements of group extensions $\bar{G} = N.G$ where N is an abelian normal subgroup of $\bar{G}$. For detailed information about this technique we encourage readers to consult F Ali [1], Moori [81, 82] and Mpono [99].

For each conjugacy class $[g]$ in G with representative $g \in G$, we analyze the coset $N\bar{g}$, where $\bar{g}$ is a lifting of g in $\bar{G}$ and

$$\bar{G} = \bigcup_{g \in G} N\bar{g} \quad .$$

To each class representative $g \in G$ with lifting $\bar{g} \in \bar{G}$, we define

$$C_{\bar{g}} = \{x \in \bar{G} \ : \ x(N\bar{g}) = (N\bar{g})x\} \quad .$$

Then $C_{\bar{g}}$ is the stabilizer of $N\bar{g}$ in $\bar{G}$ under the action by conjugation of $\bar{G}$ on $N\bar{g}$, and hence $C_{\bar{g}}$ is a subgroup of $\bar{G}$.

Remark 2.3.3. It is not difficult to see that N is a normal subgroup of $C_{\bar{g}}$.

Lemma 2.3.4. *[120] $C_{\bar{g}}/N = C_{\bar{G}/N}(N\bar{g})$.*

PROOF. Consider Nk, where $k \in \bar{G}$. Then

$$\begin{aligned}
Nk \in C_{\bar{G}/N}(N\bar{g}) \quad &\Leftrightarrow \quad Nk(N\bar{g})(Nk)^{-1} = N\bar{g} \\
&\Leftrightarrow \quad NkN\bar{g}Nk^{-1} = N\bar{g} \\
&\Leftrightarrow \quad NkN\bar{g}k^{-1} = N\bar{g} \\
&\Leftrightarrow \quad NkNn\bar{g}k^{-1} = N\bar{g} \ \ \forall n \in N \\
&\Leftrightarrow \quad Nkn\bar{g}k^{-1} = N\bar{g} \ , \ \forall n \in N \\
&\Leftrightarrow \quad kn\bar{g}k^{-1} \in N\bar{g} \ , \ \forall n \in N \\
&\Leftrightarrow \quad k \in C_{\bar{g}} \\
&\Leftrightarrow \quad Nk \in C_{\bar{g}}/N \ .
\end{aligned}$$

Thus we obtain that $C_{\bar{g}}/N = C_{\bar{G}/N}(N\bar{g})$. ■

Remark 2.3.5. Using Remark 2.3.4 and Lemma 2.3.5 we deduce that $C_{\bar{g}} = N.C_{\bar{G}/N}(N\bar{g})$. For $\bar{g}$ a lifting of $g \in G$ in $\bar{G}$, we can identify $C_{\bar{G}/N}(N\bar{g})$ with $C_G(g)$ and write $C_{\bar{g}} = N.C_G(g)$ in general. If $\bar{G} = N{:}G$ then we can identify $C_{\bar{g}}$ with $C_g = \{x \in \bar{G} \;:\; x(Ng) = (Ng)x\}$, where the lifting of g in $\bar{G}$ is g itself since $G \leq \bar{G}$ in the case of a split extension.

Corollary 2.3.6. *If $\bar{G} = N{:}G$, then $C_g = N{:}C_G(g)$.*

PROOF. We have that N is a normal subgroup of C_g. Now we show that $C_G(g) \leq C_g$ and that $N \cap C_G(g) = \{1\}$. Let $x \in C_G(g)$. Then we obtain $(Ng)^x = x(Ng)x^{-1} = xNgx^{-1} = Nxgx^{-1} = Ng$. Thus $x \in C_g$ and hence $C_G(g) \leq C_g$. Since $N \cap C_G(g) \leq N \cap G = \{1_G\}$, then we have that $N \cap C_G(g) = \{1_G\}$. Hence the result. ■

The conjugacy classes of $\bar{G}$ (where N is abelian) will be determined by the action by conjugation of $C_{\bar{g}}$, for each conjugacy class $[g]$ of G, on the elements of $N\bar{g}$. To act $C_{\bar{g}}$ on the elements of $N\bar{g}$, we first act N and then act $\{\bar{h} \;:\; h \in C_G(g)\}$, where $\bar{h}$ is a lifting of h in G. We outline this action in two steps as follows:

STEP 1: *The action of N on $N\bar{g}$*: Let $C_N(\bar{g})$ be the stabilizer of $\bar{g}$ in N. Then for any $n \in N$ we have $x \in C_N(n\bar{g}) \;\; \Leftrightarrow \;\; x \in C_N(\bar{g})$. Thus $C_N(\bar{g})$ fixes every element of $N\bar{g}$. Now let $|C_N(\bar{g})| = k$. Then under the action of N, $N\bar{g}$ splits into k orbits $Q_1, Q_2, \ldots, Q_k$, where

$$|Q_i| = [N : C_N(\bar{g})] = \frac{|N|}{k} \quad ,$$

for $i \in \{1, 2, \ldots, k\}$.

STEP 2: *The action of $\{\bar{h} \mid h \in C_G(g)\}$ on $N\bar{g}$*: Since the elements of $N\bar{g}$ are now in the orbits $Q_1, Q_2, \ldots, Q_k$ from Step 1 above, we need only act $\{\bar{h} \mid h \in C_G(g)\}$ on these k orbits. Suppose that under this action f_j of these orbits $Q_1, Q_2, \ldots, Q_k$ fuse together to form one orbit Δ_j, then the f_j's obtained this way must satisfy

$$\sum_j f_j = k$$

and we have

$$|\Delta_j| = f_j \times \frac{|N|}{k} \quad .$$

Thus for $x = d_j\bar{g} \in \Delta_j$, we obtain that

$$|[x]_{\bar{G}}| = |\Delta_j| \times |[g]_G| = f_j \times \frac{|\overline{G}|}{k|C_G(g)|}$$

and thus we obtain that

$$|C_{\bar{G}}(x)| = \frac{|\bar{G}|}{|[x]_{\bar{G}}|} = |\bar{G}| \times \frac{k|C_G(g)|}{f_j|\bar{G}|} = \frac{k|C_G(g)|}{f_j} \quad .$$

Thus to calculate the conjugacy classes of $\overline{G} = N.G$, we need to find the values of k and the f_j's for each class representative $g \in G$.

Remark 2.3.7. However in the case of $\bar{G} = N{:}G$ a split extension, we analyze the coset Ng instead of $N\bar{g}$ since in this case $G \leq \bar{G}$. Under the action of N on Ng, we always assume that $g \in Q_1$. Also instead of acting $\{\bar{h} \ : \ h \in C_G(g)\}$ on the k orbits $Q_1, Q_2, \ldots, Q_k$ we just act $C_G(g)$ on these orbits. Since $g \in Q_1$, then $C_G(g)$ always fixes Q_1 and thus we will always have $f_1 = 1$. Hence

$$k = \sum_j f_j = 1 + \sum_m f_m \quad ,$$

where the sum is taken over all m such that $g \notin Q_m$.

We now prove and discuss techniques that are useful in the determination of the orders of the elements of $\bar{G} = N{:}G$.

Theorem 2.3.8. *Let $\bar{G} = N{:}G$ and $dg \in \bar{G}$ where $d \in N$ and $g \in G$ such that $o(g) = m$ and $o(dg) = k$. Then m divides k.*

PROOF. We have that

$$1_{\bar{G}} = (dg)^k = dd^g d^{g^2} d^{g^3} \ldots d^{g^{k-1}} g^k \quad .$$

Since G acts on N and $d \in N$, we have $d, d^g, d^{g^2}, \ldots, d^{g^{k-1}} \in N$. Hence $dd^g d^{g^2} \ldots d^{g^{k-1}} \in N$. Thus we must have that $dd^g d^{g^2} \ldots d^{g^{k-1}} = 1_N$ and $g^k = 1_G$. Hence m divides k. ■

Theorem 2.3.9. *Let $\bar{G} = N{:}G$ such that N is an elementary abelian p-group, where p is prime. Let $dg \in \overline{G}$ where $d \in N$ and $g \in G$ such that $o(g) = m$ and $o(dg) = k$. Then either $k = m$ or $k = pm$.*

PROOF. See ([1],[99]). ■

Remark 2.3.10. Let $\bar{G} = N{:}G$, where N is an elementary abelian p-group. Let $dg \in \bar{G}$ with $d \in N$, $g \in G$ such that $o(g) = m$ and $o(dg) = k$, then we observe that

$$(dg)^m = d.d^g.d^{g^2}.\ldots.d^{g^{m-1}} g^m \quad .$$

Since $g^m = 1_G$, we obtain that $(dg)^m = w$, where $w \in N$ and it is given by

$$w = d.d^g.\ldots.d^{g^{m-1}} \quad .$$

By Theorem 2.3.9 above, we have that if $w = 1_N$ then $k = m$ and if $w \neq 1_N$ then $k = pm$.

We use the method of coset analysis discussed above (outlined in Steps 1 and 2) together with Theorems 2.3.8 and 2.3.9 and Remark 2.3.10 in developing Programmes A and B in GAP [41] (analogous to the programmes developed by Ali [1] for MAGMA and Mpono [99] for CAYLEY) which are applied for the computation of conjugacy classes and the orders of the class representatives of the extension $\bar{G} = N{:}G$ where N is an elementary abelian p-group for prime p on which a linear group G acts.

Programme A

```
V:=FullRowSpace(GF(q), n);
gr[1]:=(OneGF(q))*[n × n matrix group generators];
gr[2]:=(OneGF(q))*[n × n matrix group generators];
⋮
gr[k]:=(OneGF(q))*[n × n matrix group generators];
grp:=Group(gr[1], gr[2], ··· , gr[k]);
Ccl:=ConjugacyClasses(m);
O:=Union(Orbits(grp,V));
for i in [1..n(Ccl)] do
Print(Representative(Ccl[i]));
w:=One(GF(q))*[0, 0, ··· , 0];
e:=[ ];
while Difference(O,e) <> [ ] do
d:=[ ];
for x in O do;
y:=[x+w+(x*(Representative(Ccl)[i]))];
d:=Union(d,y);
od;
Print(d);
e:=Union(d,e);
if Difference(O,e) <> [ ] then
w:=Representative(Difference(O,e));
fi;
od;
r:=[ ];
u:=One(GF(q))*[0, 0, ··· , 0];
while Difference(O,e) <> [ ] do
m:=[ ];
for g in Centralizer(grp,Representative(Ccl)[i]) do
l:=[u*g];
m:=Union(m,l);
od;
Print("A block for the vectors under the action of a centralizer");
Print(m);
r:=Union(m,r);
if Difference(O,r) <> [ ] then
u:=Representative(Difference(O,r));
fi;
od;
Print("*********************************");
od;
```

Programme B

```
V:=FullRowSpace(GF(q), n);
m[1]:=(OneGF(q))*[n × n matrix group generators];
m[2]:=(OneGF(q))*[n × n matrix group generators];
⋮
m[k]:=(OneGF(q))*[n × n matrix group generators];
```

```
m:=Group(m[1], m[2], ··· , m[k]);
c:=ConjugacyClasses(m);
g:=Representative(c[i]);
d:=One(GF(q))*[α_1, α_2, ··· , α_n];
w:=d + d * g + d * g^2 + ··· + d * g^{m-1};
Print(w);
```

In Programme B we have $o(g) = m$ and $g \in S$ is a class representative, for $1 \leq j \leq n, \alpha_j \in GF(q)$, $d * g = d^g$, and $+$ signifies the operation in V and $dg \in \overline{G}$ is a class representative from the coset Ng.

2.4. Representations and Characters

In this section we give some preliminary results on representations and characters of groups which will be needed in later chapters. For further reading one can go to [1, 99].

Definition 2.4.1. *Let G be a group, $\mathbb{F}$ a field and $GL(n, \mathbb{F})$ the general linear group which is the multiplicative group of all nonsingular $n \times n$ matrices over F for some integer n. Then a homomorphism $\rho : G \longrightarrow GL(n, \mathbb{F})$ is called a* **representation** *of G over $\mathbb{F}$ or simply an $\mathbb{F}$-representation. The representation ρ is said to have degree n. The function $\chi : G \longrightarrow \mathbb{F}$ given by $\chi(g) = trace(\rho(g))$ is called the $\mathbb{F}$-character of G afforded by the F-representation ρ. The degree of χ is the same as that of ρ.*

Two $\mathbb{F}$-representations ρ_1 and ρ_2 of G are said to be **equivalent** if there exists $P \in GL(n, \mathbb{F})$ such that $\rho_1(g) = P\rho_2(g)P^{-1}$ for all $g \in G$. An $\mathbb{F}$-representation ρ of G is said to be **reducible** if it is equivalent to a representation α which is given by

$$\alpha(g) = \begin{pmatrix} \beta(g) & \gamma(g) \\ 0 & \delta(g) \end{pmatrix}$$

for all $g \in G$, where β, γ, δ are $\mathbb{F}$-representations of G. If ρ is not reducible, then it is said to be **irreducible**. Since similar matrices have the same trace, then it follows that equivalent representations afford the same character. The character afforded by an irreducible representation is called an **irreducible character**. Sums and products of characters are themselves characters.

We now give a famous result of Schur [114] which provides an assessable approach to group characters.

Theorem 2.4.2. *(***Schur's Lemma***) Let $\rho_1 : G \longrightarrow GL(n, \mathbb{F})$ and $\rho_2 : G \longrightarrow GL(m, \mathbb{F})$ be two irreducible representations of a group G over a field $\mathbb{F}$. Assume that there exists a matrix P such that $P\rho_1(g) = \rho_2(g)P$ for all $g \in G$. Then either P is the zero matrix or P is nonsingular so that $\rho_1(g) = P^{-1}\rho_2(g)P$.*

PROOF. See Theorem 1.8 of [89]. ∎

Corollary 2.4.3. *[89] If $\rho : G \longrightarrow GL(n, \mathbb{F})$ is an irreducible representation of a group G over an algebraically closed field $\mathbb{F}$, then the only matrices which commute with all matrices $\rho(g)$, $g \in G$ are scalar matrices aI_n, where $a \in \mathbb{F}$ and I_n is the $n \times n$ identity matrix.*

PROOF. Let P be an $n \times n$ matrix such that $P\rho(g) = \rho(g)P$ for all $g \in G$. Then for any $a \in F$ we have that

$$(aI_n - P)\cdot\rho(g) = \rho(g)\cdot(aI_n - P), \ \forall g \in G \ . \quad (1)$$

Let $m(x) = det(xI_n - P)$ be the characteristic polynomial of P. Since $m(x)$ is a polynomial over F and F is algebraically closed, then there exists $a_1 \in F$ such that $m(a_1) = 0_F$. Hence $det(a_1 I_n - P) = 0_F$ and thus $a_1 I_n - P$ is singular. Then from relation (1) above and Schur's Lemma, we obtain that $a_1 I_n - P = 0$ and hence $a_1 I_n = P$. ■

Definition 2.4.4. *Let G be a group, $\mathbb{F}$ a field and $\phi : G \longrightarrow \mathbb{F}$ be a function which is constant on conjugacy classes of G. Then ϕ is called a* **class function** *of G.*

From the above definition, we observe that every character is a class function. From now on, we will consider representations and characters of a finite group G over the complex field $\mathbb{C}$. We shall use the notation $Irr(G)$ to denote the set of all irreducible characters of the group G. These irreducible characters are presented in a table, called the **character table** of G. In this table, the columns correspond to the conjugacy classes of G and the rows to the irreducible characters, with entry a_{ij} being the value of the i-th irreducible character on an element of the j-th conjugacy class.

We can show that every class function ϕ of G can be uniquely expressed in the form $\phi = \sum_{\chi \in Irr(G)} b_\chi \chi$, where $b_\chi \in \mathbb{C}$. Moreover ϕ is a character if and only if all $b_\chi \in \mathbb{N} \cup \{0\}$ and $\phi \neq 0$. We can also show that the following properties hold:

(i) Two representations of G have the same character if and only if they are equivalent.

(ii) The number of irreducible characters of G is equal to the number of conjugacy classes of elements of G.

(iii) Any character of G can be written as a sum of irreducible characters.

Definition 2.4.5. *Let G be a group, χ a character of G and $Irr(G) = \{\chi_1, \chi_2, \ldots, \chi_r\}$ such that $\chi = \sum_{i=1}^{r} n_i \chi_i$, where $n_i \in \mathbb{N} \cup \{0\}$. Then those χ_i for which $n_i > 0$ are called the* **irreducible constituents** *of χ. In general, if ψ is a character of G such that $\chi - \psi$ is a character or is zero, then ψ is a constituent of χ.*

Orthogonality relations for characters are the cornerstone of character theory. Among other applications, they allow us to express an arbitrary class function in terms of irreducible characters and to determine instantaneously whether or not any given character is irreducible.

Theorem 2.4.6. *(***Generalized Orthogonality Relation***) Let G be a group and $Irr(G) = \{\chi_1, \chi_2, \dots, \chi_r\}$. Then the following holds for every $h \in G$:*

$$\frac{1}{|G|}\sum_{g\in G}\chi_i(gh)\chi_j(g^{-1}) = \delta_{ij}\frac{\chi_i(h)}{\chi_i(1_G)} \quad .$$

PROOF. See Theorem 2.13 of [60]. ■

Theorem 2.4.7. *Let χ be a character of G afforded by a representation ρ of degree n. Then for $g \in G$, $\rho(g)$ is similar to a diagonal matrix $diag(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_r)$ where each ε_i is a complex root of unity. Then $\chi(g) = \sum_i^r \varepsilon_i$ and $\chi(g^{-1}) = \overline{\chi(g)}$, where $\overline{\chi(g)}$ is the complex conjugation of $\chi(g)$.*

PROOF. This is the Lemma 2.15 in [60]. ■

Definition 2.4.8. *Let χ and ψ be class functions of a group G. Then the* **inner product** *of χ and ψ is defined by*

$$\langle \chi, \psi \rangle = \frac{1}{|G|}\sum_{g\in G}\chi(g)\overline{\psi(g)} \quad .$$

The following theorems are derived from the generalized orthogonality relation and are called the first and second orthogonality relations respectively.

Theorem 2.4.9. *[60](***First Orthogonality Relation***) Let G be a group and $Irr(G) = \{\chi_1, \chi_2, \dots, \chi_r\}$. Then*

$$\frac{1}{|G|}\sum_{g\in G}\chi_i(g)\overline{\chi_j(g)} = \delta_{ij} = \langle \chi_i, \chi_j \rangle \quad .$$

PROOF. Using the generalized orthogonality relation and taking $h = 1_G$, then the result follows immediately. ■

Theorem 2.4.10. *[60](***Second Orthogonality Relation***) Let G be a group and $Irr(G) = \{\chi_1, \chi_2, \dots, \chi_r\}$ and $\{g_1, g_2, \dots, g_r\}$ be a set of representatives of the conjugacy classes of elements of G. Then*

$$\sum_{\chi\in Irr(G)}\chi(g_i)\overline{\chi(g_j)} = \delta_{ij}|C_G(g_i)| \quad .$$

PROOF. Let X be the character table of G. Then viewed as a matrix, X is an $r \times r$ matrix whose (i,j)-th entry is given by $\chi_i(g_j)$. Let C_i be the conjugacy class which contains g_i and D be the diagonal matrix with entries $\delta_{ij}|C_i|$. Then by the first orthogonality relation, we obtain that

$$|G|\delta_{ij} = \sum_{g\in G}\chi_i(g)\overline{\chi_j(g)} = \sum_{t=1}^{r}|C_t|\chi_i(g_t)\overline{\chi_j(g_t)} \quad .$$

Then we obtain a system of r^2 equations which can be written as a single matrix equation as follows

$$|G|I = XD\overline{X}^T \quad ,$$

where I is the identity $r \times r$ matrix and $\overline{X}^T$ is the transpose of $\overline{X}$. Since X is a nonsingular matrix, then we obtain that

$$|G|I = D\overline{X}^T X \quad .$$

Rewriting the above matrix system as a system of equations yields

$$|G|\delta_{ij} = \sum_{t=1}^{r} |C_i| \overline{\chi_t(g_i)} \chi_t(g_j) \quad .$$

Hence we obtain that

$$\sum_{\chi \in Irr(G)} \chi(g_j)\overline{\chi(g_i)} = |C_G(g_i)|\delta_{ij} \quad .$$

■

Let G be a group and χ be a character of G afforded by a representation ρ. Then we define

$$ker(\chi) = \{g \in G \mid \chi(g) = \chi(1_G)\} \quad .$$

It can be shown that $ker(\chi) = ker(\rho)$ and hence $ker(\chi)$ is a normal subgroup of G. If $Irr(G) = \{\chi_1, \chi_2, \ldots, \chi_r\}$, then every normal subgroup of G is the intersection of some of the $ker(\chi_i)$.

Theorem 2.4.11. *Let G be a group and N be a normal subgroup of G. Then*

(a) If χ is a character of G and $N \subseteq ker(\chi)$, then χ is constant on the cosets of N in G and the function $\hat{\chi}$ defined on G/N by $\hat{\chi}(Ng) = \chi(g)$ is a character of G/N.

(b) If $\hat{\chi}$ is a character of G/N, then the function χ defined by $\chi(g) = \hat{\chi}(Ng)$ is a character of G.

(c) In both (a) and (b) above, $\chi \in Irr(G)$ if and only if $\hat{\chi} \in Irr(G/N)$.

PROOF. See Theorem 2.2.2. of [120]. ■

If N is a normal subgroup of G and ρ is a representation of G such that $N \subseteq ker(\rho)$, then there exists a unique representation $\hat{\rho}$ of G/N defined by $\hat{\rho}(Ng) = \rho(g)$. Thus knowing ρ, we can obtain $\hat{\rho}$ and vice versa. We also obtain that ρ is irreducible if and only if $\hat{\rho}$ is irreducible. Hence ρ and $\hat{\rho}$ can be identified. If ρ affords a character χ of G, then $\hat{\rho}$ affords a character $\hat{\chi}$ of G/N and also χ and $\hat{\chi}$ can be identified. Under this identification, we obtain that

$$Irr(G/N) = \{\chi \in Irr(G) \mid N \subseteq ker(\chi)\} \quad .$$

Thus the irreducible characters of G/N are precisely those irreducible characters of G which contain N in their kernels.

Definition 2.4.12. *Let G be a group, N a normal subgroup of G and $\hat{\chi}$ be a character of G/N. Then the character χ of G defined by*

$$\chi(g) = \hat{\chi}(Ng)$$

is called a **lifting** *of $\hat{\chi}$ to G.*

Thus given characters of G/N, we can obtain some characters of G by the lifting process. The character $\hat{\chi}$ and its lifting χ have the same degree.

2.5. Induced Characters

In this section we look at the ways of relating the representations of a group to the representations of its subgroups.

Definition 2.5.1. *Let G be a finite group and $H \leq G$. If ρ is a representation of G, then the* **restriction** *of ρ to H is a representation of H. This representation is denoted by ρ_H. If χ is a character of G afforded by ρ, then the restriction of χ to H is denoted by χ_H and is a character of H afforded by the representation ρ_H such that*

$$\chi_H = \sum_{\psi \in Irr(H)} k_\psi \psi \quad ,$$

where $k_\psi \in \mathbb{N} \cup \{0\}$.

The characters χ_H and χ take on the same values on the elements of H. If χ_H is irreducible, then χ is irreducible in G but the converse is not true in general.

Karpilovsky in [67] proves a theorem (Theorem 23.1.4) due to Gallagher that if $H \leq G$, $\chi \in Irr(G)$ such that $\chi(g) \neq 0 \ \forall \ g \in G - H$, then χ_H is irreducible, and for any $g \in G - H$, $\chi(g)$ is a root of unity. We also observe that (see [63]) every irreducible character of H is a constituent of some irreducible character of G restricted to H.

Theorem 2.5.2. *[63] Let G be a group, $H \leq G$, $\chi \in Irr(G)$ and $Irr(H) = \{\psi_1, \psi_2, \ldots, \psi_r\}$. Then*

$$\chi_H = \sum_{i=1}^{r} k_i \psi_i \quad ,$$

where $k_i \in \mathbb{N} \cup \{0\}$ satisfy the following relation

$$\sum_{i=1}^{r} k_i^2 \leq [G : H] \quad .$$

Moreover, equality in the above relation holds if and only if $\chi(g) = 0$ for all $g \in G - H$.

PROOF. See [99] ■

Theorem 2.5.3. *Let G be a group, H be a normal subgroup of G and $\chi \in Irr(G)$. Then all the constituents of χ_H have the same degree.*

PROOF. See Proposition 20.7 of [63]. ■

Let G be a group and $H \leq G$ such that the set $\{x_1, x_2, \ldots, x_r\}$ is a transversal for H in G. Let ϕ be a representation of H of degree n. Then we define ϕ^* on G as follows:

$$\phi^*(g) = \begin{pmatrix} \phi(x_1 g x_1^{-1}), \phi(x_1 g x_2^{-1}), \ldots, \phi(x_1 g x_r^{-1}) \\ \phi(x_2 g x_1^{-1}), \phi(x_2 g x_2^{-1}), \ldots, \phi(x_2 g x_r^{-1}) \\ \vdots \\ \phi(x_n g x_1^{-1}), \phi(x_n g x_2^{-1}), \ldots, \phi(x_n g x_r^{-1}) \end{pmatrix}$$

where $\phi(x_i g x_j^{-1})$ are $n \times n$ sub-matrices of $\phi^*(g)$ satisfying the property that

$$\phi(x_i g x_j^{-1}) = 0_{n\times n} \;\; \forall\, x_i g x_j^{-1} \notin H \quad .$$

Then we can show that ϕ^* is a representation of G of degree nr.

Definition 2.5.4. *Let G, H, ϕ and ϕ^* be as above. Then the representation ϕ^* is called the representation of G* **induced** *from the representation ϕ of H and we denote this by writing $\phi^* = \phi^G$.*

If ψ is a representation of H which is equivalent to ϕ, then it can be shown that ψ^G is equivalent to ϕ^G. Thus the induction process preserves equivalence between representations.

Definition 2.5.5. *Let G be a group and $H \leq G$. Let χ be a class function of H. Then we define χ^G as follows:*

$$\chi^G(g) = \frac{1}{|H|}\sum_{x\in G} \chi^\circ(xgx^{-1}) \quad ,$$

where

$$\chi^\circ(h) = \begin{cases} \chi(h) & \text{if } h \in H \\ 0 & \text{otherwise} \end{cases} \quad .$$

Then χ^G is a class function of G, called the **induced class function** *of G induced from χ. Also we have that $deg(\chi^G) = [G : H]deg(\chi)$.*

Theorem 2.5.6. *[60](***Frobenius Reciprocity Theorem***) Let G be a group, $H \leq G$ and suppose that χ is a class function of H and ϕ is a class function of G. Then*

$$\langle \chi, \phi_H \rangle = \langle \chi^G, \phi \rangle.$$

PROOF. We obtain that

$$\langle \chi^G, \phi\rangle = \frac{1}{|G|}\sum_{g\in G}\chi^G(g)\overline{\phi(g)} = \frac{1}{|G|}\frac{1}{|H|}\sum_{g\in G}\sum_{x\in G}\chi^\circ(xgx^{-1})\overline{\phi(g)} \quad .$$

Putting $y = xgx^{-1}$ and since ϕ is a class function, then we obtain that $\phi(y) = \phi(g)$. Hence we have

$$\begin{aligned} \langle \chi^G, \phi\rangle &= \frac{1}{|G|}\frac{1}{|H|}\sum_{g\in G}\sum_{x\in G}\chi^\circ(xgx^{-1})\overline{\phi(g)} = \frac{1}{|G|}\frac{1}{|H|}\sum_{y\in G}\sum_{x\in G}\chi^\circ(y)\overline{\phi(y)} \\ &= \frac{1}{|H|}\sum_{y\in H}\chi(y)\overline{\phi(y)} = \langle \chi, \phi_H\rangle. \end{aligned}$$

Hence the result. ■

Let $H \leq G$ and ϕ be a representation of H that affords a character χ of H. Then χ^G is a character of G afforded by the induced representation ϕ^G of G. The character χ^G is called the *induced character* of G. The induction and restriction processes do not necessarily preserve irreducibility of characters. For further reading on induced characters, readers are encouraged to consult [8], [9], [62], [101] and many other relevant sources.

Theorem 2.5.7. *Let G be a group and $H \leq G$. Let χ be a character of H, $g \in G$ and $\{x_1, x_2, \ldots, x_m\}$ be a set of representatives of the conjugacy classes of elements of H which fuse into $[g]$ in G. Then we obtain that*

$$\chi^G(g) = |C_G(g)| \sum_{i=1}^{m} \frac{\chi(x_i)}{|C_H(x_i)|} \quad ,$$

where we have that $\chi^G(g) = 0$ whenever $H \cap [g] = \emptyset$.

PROOF. We have that

$$\chi^G(g) = \frac{1}{|H|} \sum_{x \in G} \chi^\circ(xgx^{-1}) \quad .$$

If $H \cap [g] = \emptyset$, then $xgx^{-1} \notin H$ and thus $\chi^\circ(xgx^{-1}) = 0 \ \ \forall\, x \in G$ and hence $\chi^G(g) = 0$. Now if $H \cap [g] \neq \emptyset$, then let $h \in H \cap [g]$. Then as x runs over elements of G, we have $xgx^{-1} = h$ for exactly $|C_G(g)|$ values of x. Hence we obtain that

$$\chi^G(g) = \frac{1}{|H|} \sum_{x \in G} \chi(xgx^{-1}) = \frac{|C_G(g)|}{|H|} \sum_{h \in H \cap [g]} \chi(h) = |C_G(g)| \sum_{i=1}^{m} \frac{\chi(x_i)}{|C_H(x_i)|} \quad .$$

Hence the result. ■

2.6. Permutation Characters

Knowledge of the permutation characters of a group leads to information about the subgroup structure of the group. In this section we discuss permutation characters.

We say that a group G *acts* on a set X if there is a homomorphism $\phi : G \longrightarrow S_X$, where S_X is the symmetric group on X. We say that G acts *faithfully* on X if ϕ is a monomorphism. In this case G can be identified with a subgroup of S_X and G becomes a *permutation group* on X. In this section we assume that X is a finite set.

Definition 2.6.1. *Let G be a group acting on a set X such that for any two k-tuples $(x_1, x_2, \ldots, x_k)$ and $(y_1, y_2, \ldots, y_k)$ of k distinct elements of X, there exists $g \in G$ for which $x_i^g = y_i$ for $i = 1, 2, \ldots, k$. Then we say that G is k-**transitive** on X.*

If G is 1-transitive on X, then we say that G is *transitive*. In this case G has only one *orbit* on X.

If G acts on X, we define a representation $\pi : G \longrightarrow GL(n, \mathbb{C})$, where $n = |X|$. Let $X = \{x_1, x_2, \dots, x_n\}$. For each $g \in G$ we define $\pi_g = (a_{ij})$ by

$$a_{ij} = \begin{cases} 1 & \text{if } x_i^g = x_j \\ 0 & \text{otherwise} \end{cases} \quad .$$

Then π_g is a permutation matrix of the action of g. The representation π defined above is called the *permutation representation* of G obtained from the action of G on X.

Definition 2.6.2. *Let G be a group and X be a set such that G acts on X. Then we denote the character afforded by the permutation representation π by $\chi(G|X)$. This character is called the* **permutation character** *of G associated with the action of G on X. It is not difficult to show that for $g \in G$ we have*

$$\chi(G|X)(g) = |\{x \in X \mid x^g = x\}| = \textit{ the number of points of } X \textit{ fixed by } g.$$

Suppose that G acts transitively on X and G_x is the stabilizer of $x \in X$. Then the action of G on X is the same as the action of G on the cosets of $H = G_x$. Hence $\forall\, g \in G$, $\chi(G|X)(g)$ also gives the number of cosets of $H = G_x$ that are fixed by $g \in G$ and in this case we denote this number by $\chi(G|H)(g)$. Due to the fact that the action of G on X is the same as the action of G on the cosets of H, then we can write $\chi(G|H) = \chi(G|X)$.

Theorem 2.6.3. *Let G be a group acting transitively on a set X. Let $\alpha \in X$, $H = G_\alpha$ and $\chi(G|H)$ be the permutation character of this action. If I_H is the identity character of H, then $\chi(G|H) = (I_H)^G$.*

PROOF. We have that

$$(I_H)^G(g) = \frac{1}{|H|} \sum_{x \in G, xgx^{-1} \in H} I_H(xgx^{-1}) = \frac{1}{|H|} \sum_{x \in G, xgx^{-1} \in H} 1 \quad .$$

Now if $xgx^{-1} \in H$, then $xg \in Hx$. Thus $Hxg = Hx$ and hence Hx is fixed by $g \in G$. However the summation is taken over all $x \in G$ such that $xgx^{-1} \in H$. Hence the summation is taken over all $x \in G$ for which the coset Hx is fixed by $g \in G$. But $\forall\, y \in Hx$, $Hx = Hy$ and thus we obtain that

$$\sum_{x \in G, xgx^{-1} \in H} 1 = |H||\{Hx \mid Hxg = Hx\}|$$

so that

$$(I_H)^G(g) = \frac{1}{|H|}|H||\{Hx \mid Hxg = Hx\}| = |\{Hx \mid Hxg = Hx\}| = \chi(G|H)(g).$$

Hence the result. ■

Theorem 2.6.4. *[60] Let G be a group acting on a set X with $\chi(G|X)$ as the permutation character of the action. If X splits into k orbits under the action of G, then*

$$\langle \chi(G|X), I_G \rangle = k.$$

PROOF. Suppose that the k orbits of X under the action of G are $\{X_1, \ldots, X_k\}$. Then we obtain that

$$X = \bigcup_{i=1}^{k} X_i.$$

Let $x_i \in X_i$ and H_i be the stabilizer of $x_i \in X_i$. Also let $\chi_i(G|H_i)$ be the permutation character of G on the cosets of H_i. Then we obtain that

$$\chi(G|X) = \sum_{i=1}^{k} \chi_i(G|H_i) \quad \text{where} \quad \chi_i(G|H_i) = (I_{H_i})^G.$$

By the Frobenius reciprocity theorem, we obtain that

$$\langle \chi_i(G|H_i), I_G \rangle = \langle (I_{H_i})^G, I_G \rangle = \langle I_{H_i}, I_{H_i} \rangle = 1 \quad .$$

Hence we obtain that

$$\langle \chi(G|X), I_G \rangle = \sum_{i=1}^{k} \langle \chi_i(G|H_i), I_G \rangle = \sum_{i=1}^{k} 1 = k.$$

Hence the result. ■

The following result will be used in later calculations to determine the conjugacy class fusions of subgroups of G.

Corollary 2.6.5. *Let $H \leq G$. Let $g \in G$ and let $x_1, x_2, \ldots, x_m$ be representatives of the conjugacy classes of H that fuse to $[g]$. Then*

$$\chi(G|H)(g) = \sum_{i=1}^{m} \frac{|C_G(g)|}{|C_H(x_i)|}.$$

PROOF. This follows from Theorem 2.4.7. ■

In the following we present some properties of the permutation characters.

Theorem 2.6.6. *Let G be a group, $H \leq G$ and $\chi = \chi(G|H)$.*

(i) $deg(\chi)$ divides $|G|$.

(ii) $\langle \chi, \psi \rangle \leq deg(\psi)$ for all $\psi \in Irr(G)$.

(iii) $\langle \chi, I_G \rangle = 1$.

(iv) $\chi(g) \in \mathbb{N} \cup \{0\}$ *for all* $g \in G$.

(v) $\chi(g) \leq \chi(g^m)$ *for all* $g \in G$ *and* $m \in \mathbb{N} \cup \{0\}$.

(vi) $\chi(g) = 0$ *if* $o(g)$ *does not divide* $|G|/deg(\chi)$.

(vii) $\chi(g)\frac{|[g]|}{deg(\chi)}$ *is an integer for all* $g \in G$.

PROOF. This is Theorem 2.5.6 in [120].

Let ϕ be a representation of G and α an automorphism of G. Then ϕ^α is a representation of G given by

$$\phi^\alpha(x) = \phi(x^\alpha) \quad \text{and} \quad \phi^\alpha(xy) = \phi^\alpha(x)\phi^\alpha(y)$$

for $x, y \in G$. If the representation ϕ affords a character χ of G, then the representation ϕ^α affords a character χ^α of G which is given by $\chi^\alpha(x) = \chi(x^\alpha)$ for $x \in G$. Then the representation ϕ^α and the character χ^α are called the *algebraic conjugates* of ϕ and χ respectively induced by the automorphism α. Let $X = (\chi_i(x_j))$ be the character table of G, where $\chi_i \in Irr(G)$, $1 \leq i \leq n$ and x_j, $1 \leq j \leq n$ are representatives of the conjugacy classes of elements of G. Then the automorphism α of G induces a permutation on the conjugacy classes of G and thus induces a permutation on the columns of X. For each $\chi_i \in Irr(G)$, we deduce that $\chi_i^\alpha \in Irr(G)$. Hence α induces a permutation on the irreducible characters χ_i of G and thus induces a permutation on the rows of X. Moreover since $\chi_i^\alpha(x_j) = \chi_i(x_j^\alpha)$, then the matrices obtained from X by these two operations are identical. Hence we obtain the following theorem known as Brauer's Theorem.

Theorem 2.6.7. *[43](***Brauer's Theorem***) Let N be a group and $G \leq Aut(N)$. Then the number of orbits of G as a group of permutations acting on the irreducible characters of N is the same as the number of orbits of G as a group of permutations acting on the conjugacy classes of N.*

PROOF. Let X be the character table of N. Then as a matrix, X is square and nonsingular. Let α be an automorphism of N such that $\alpha \in G$. Then α induces a permutation on the conjugacy classes of N and thus induces a permutation on the columns of X. Hence G acts on the conjugacy classes of N. Since $\alpha \in G$, then to each character χ of N, we obtain a character χ^α of N such that $\chi^\alpha \in Irr(N)$ whenever $\chi \in Irr(N)$. For $y \in N$, we obtain that $\chi^\alpha(y) = \chi(y^\alpha)$. Thus α induces a permutation on the rows of X. Hence G acts on the irreducible characters of N. Let X^α denote the image of X under α. Then we obtain that

$$P(\alpha)X = X^\alpha = XQ(\alpha),$$

where $P(\alpha), Q(\alpha)$ are appropriate permutation matrices which are uniquely determined by $\alpha \in G$. Suppose that $\alpha, \beta \in G$. Then we obtain that $X^{\alpha\beta} = (X^\alpha)^\beta$. Also we have that

$$P(\alpha\beta)X = X^{\alpha\beta} = (X^\alpha)^\beta = (P(\alpha)X)^\beta = P(\beta)P(\alpha)X$$

and hence $P(\alpha\beta) = P(\beta)P(\alpha)$, since X is non-singular. We also have that $X^{\alpha\beta} = XQ(\alpha\beta)$ and $(X^\alpha)^\beta = (XQ(\alpha))^\beta = XQ(\alpha)Q(\beta)$. Since $X^{\alpha\beta} = (X^\alpha)^\beta$, we obtain that $XQ(\alpha\beta) = XQ(\alpha)Q(\beta)$. Again the non-singularity of X implies that $Q(\alpha\beta) = Q(\alpha)Q(\beta)$. Define mappings π_1 and π_2 on N by $\pi_1(\alpha) = (P(\alpha))^t$ and $\pi_2(\alpha) = Q(\alpha)$, where t denotes the transpose operation on matrices. Then π_1 and π_2 are permutation representations of N. Let θ_1 and θ_2 be the permutation characters afforded by π_1 and π_2 respectively. Since $X^{-1}P(\alpha)X = Q(\alpha)$, $P(\alpha)$ and $Q(\alpha)$ are similar and thus have the same trace. Since $trace(P(\alpha))^t = trace(P(\alpha))$, we have that $trace(P(\alpha))^t = trace(Q(\alpha))$. Hence $\theta_1 = \theta_2$ and π_1 and π_2 are equivalent. Let d_1, d_2 be the number of orbits of G on the irreducible characters and on the conjugacy classes of N respectively. Thus we observe that d_1 is the number of orbits of $\pi_1(G)$ in its action as a group of permutations. Also d_2 is the number of orbits of $\pi_2(G)$ in its action as a group of permutations. Since θ_1 is the permutation character of G acting on the irreducible characters of N, we obtain that $\langle\theta_1, I_G\rangle = d_1$. Also for θ_2, we obtain that $\langle\theta_2, I_G\rangle = d_2$. However $\theta_1 = \theta_2$ and thus $\langle\theta_1, I_G\rangle = \langle\theta_2, I_G\rangle$ and hence $d_1 = d_2$. Hence the result. ■

2.7. Orbit Lengths

Brauer's theorem states that when G acts on an automorphism group N in our case an elementary abelian group, then the number of orbits of G as a group of permutations on the conjugacy classes of N is equal to the number of orbits on the irreducible characters of N. However Brauer's theorem does not apply to orbit lengths, as orbit lengths of permutations on conjugacy classes may not be the same as orbit lengths of permutation on irreducible classes. To get the orbit lengths of the irreducible character and the conjugacy classes of N, we use Programme C which we developed in GAP.

Programme C

```
V:=FullRowSpace(GF(q),n);
gen[1]:=(OneGF(q))*[n × n matrix group generators];
gen[2]:=(OneGF(q))*[n × n matrix group generators];
⋮
gen[k]:=(OneGF(q))*[n × n matrix group generators];
G:=Group(gen[1],gen[2],··· ,gen[k]);
O:=Orbits(G,V);;
k:=OrbitLengths(G,V);
l:= OrbitLengths(Group(List(G,TransposedMat)),V);
Print(k);
Print(l);
```

We use Programme C to compute the orbit lengths of both the conjugacy classes and the irreducible characters. If G is an $n \times n$ matrix group, we would wish to let G act on our elementary abelian group N. To be able to do this we rewrite N as an n-dimensional row vector space V over $GF(q)$. The action of G on V is multiplication of V from the right by G. This gives us the orbits of G as a group of permutations on the conjugacy classes of N. The action of G^t, the transpose of G, on V, is multiplication on the right of V by G^t. This is equivalent to the multiplication of the column vectors of V from the left by G. This multiplication gives us the orbits of G as a group of permutations on the irreducible characters of N. We give an example that shows that the orbit lengths of the conjugacy classes need not be equal to the orbit lengths of irreducible classes.

Example 2.7.1. Let $\overline{G} = 2^5{:}S_6$ and $V =$ FullRowSpace$(GF(2), 5)$ where

$G = S_6 =<$ matrix group with 2 generators $>$.

Using Programme C above, we get

OrbitLengthsOfConjugacyClasses = **[1,6,15,10]**

OrbitLengthsOfIrreducibleCharacters = **[1,15,15,1]**.

We have four orbits for both the irreducible characters and the conjugacy classes but the orbit lengths differ.

3

Projective Representation

3.1. Prologue

In this chapter we study the projective representations and characters which will be required in the subsequent chapters. We follow very closely the work of Ali [1]. We refer to the group representations and group characters that we defined in Chapter 2 as ordinary representations and ordinary characters respectively. The Schur multiplier of a group G plays an important role in the study of projective representations of G. We have therefore devoted Section 3.2 to the study of Schur multiplier of G. In Section 3.3 we are dealing with projective representations of G. We study the relationship of projective representations with the ordinary representations. We discuss that how projective representations of G can be constructed using three different approaches. We also show that how projective representations of G can be determined from the ordinary representations of a so-called representation group of G. Finally in Section 3.4 we discuss projective characters and study the orthogonality relations analogous to the ones for ordinary characters. For further readings on projective representations and projective characters readers are referred to [11, 47, 51, 55, 58, 29, 60, 95, 96, 97, 98, 100, 107, 108, 109].

3.2. Schur Multiplier

The first step in obtaining the projective representations of a group G is to compute its Schur multiplier. In this section we discuss results useful in finding the Schur multiplier of a group.

Definition 3.2.1. *A function $\alpha : G \times G \longrightarrow \mathbb{C}^*$ is called a* **factor set** *of G if*

$$\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z) \quad \textit{for all } \; x, y, z \in G.$$

Two factor sets α and α' are said to be *equivalent* if there exists a function $\rho : G \longrightarrow \mathbb{C}^*$ such that $\alpha'(x, y) = \frac{\rho(x)\rho(y)}{\rho(xy)}\alpha(x, y)$ for all $x, y \in G$. This is an equivalence relation and we denote the equivalence class of the factor set α by $[\alpha]$. For factor sets α and α', let $(\alpha\alpha')(x, y) = \alpha(x, y)\alpha'(x, y)$ for all $x, y \in G$. Then $\alpha\alpha'$ is a factor set, as is α^{-1} defined by $\alpha^{-1}(x, y) = (\alpha(x, y))^{-1}$.

Definition 3.2.2. *The set of all equivalence classes of factor sets forms a group by defining* $[\alpha][\alpha'] = [\alpha\alpha']$. *The identity of this group is* $[1]$ *where* 1 *is the factor set* $1(x, y) = 1$ *for all* $x, y \in G$, *and* $[\alpha]^{-1} = [\alpha^{-1}]$. *This group is called the* **Schur multiplier** *of* G *and we denote it by* $M(G)$.

Theorem 3.2.3. *(i)* $M(G)$ *is a finite abelian group.*

(ii) If G *is a cyclic group, then* $M(G) = 1$.

PROOF. See [100]. ■

Lemma 3.2.4. *Suppose that* N *is a normal subgroup of a finite group* G. *If* $M(G) = 1$, *then* $M(G/N) \cong (N \cap G')/[N, G]$. *In general,* $|(N \cap G')/[N, G]|$ *divides* $|M(G/N)|$.

PROOF. See [66]. ■

Theorem 3.2.5. *Let* G *be a finite group and* H *be a subgroup of index* n. *Then the group* $(M(G))^n$ *of all* n*-th powers of* $M(G)$ *is isomorphic to a subgroup of* $M(H)$.

PROOF. See [66]. ■

Schur [114] reduced the problem of finding $M(G)$ to obtaining the Schur multiplier of the Sylow p-subgroups of G. The following theorem describes the Schur multiplier of G in terms of the subgroup structure of G.

Theorem 3.2.6. *[114] Let* S *be a Sylow* p*-subgroup of* G. *Then the Sylow* p*-subgroup of* $M(G)$ *is isomorphic to a subgroup of* $M(S)$.

PROOF. See [66]. ■

Theorem 3.2.7. *A group* G *has trivial Schur multiplier if and only if it has a set of subgroups with trivial Schur multipliers and relatively prime indices.*

PROOF. See [66]

Schur investigated the group $G = SL(2, q)$. He discovered that $M(G)$ is trivial unless $q = 4$ or $q = 9$. In Chapter 10 we look at the case $5^2{:}2.A_5$. Noting that $2.A_5 \cong SL(2, 5)$, the Schur multiplier for $2.A_5$ is trivial but for $5^2{:}2.A_5$ we have that $M(G)$ is a cyclic group of order 5.

For any group G we follow the methods of Ali [1] to test if we need an ordinary representation or we need a projective representation. In chapter 10 we need a projective representation for $5^2{:}2.A_5$ which is a split extension. To get the Schur multiplier for this group we use MAGMA for Programme J .

Programme J

```
> G:=Group;
> M:=GModule(G);
> X:=CohomologyModule(G,M);
> E:=SplitExtension(X);
> Eperm:=DegreeReduction(CosetImage(E,sub<E|>));
> pMultiplicator(Eperm,$p_i$);
> exit;
```

However the group $5^2{:}2.A_5$ is a unique *perfect* group, recall a group is said to be perfect if it is equal to its derived group, that is $G' = G$. We use the following Programme J' in GAP.

Programme J′

```
gap> gg:=PerfectGroup(|G|,1);

gap> AbelianInvariantsMultiplier(gg);
```

3.3. Projective Representations

The notion of projective representation, due to Schur, was suggested by the study of relations between linear representations of a group and its factor groups over a central subgroup.

Definition 3.3.1. *Let G be a group and $\mathbb{F}$ be a field. Consider the map $P : G \longrightarrow GL(n, \mathbb{F})$ such that*

(i) $P(1_G) = I_n$, where I_n is the identity $n \times n$ matrix.

(ii) For all $x, y \in G$, there exists a map $\alpha : G \times G \longrightarrow \mathbb{F}^*$ such that

$$P(x)P(y) = \alpha(x, y)P(xy) \quad \text{where} \quad \alpha(x, y) \in \mathbb{F}^* \quad .$$

Then P is called a **projective representation** *of G over $\mathbb{F}$ of degree n. The map α is called the* **factor set** *associated with P.*

From the above definition, we observe that

$$\alpha(x, y) = P(x)P(y)(P(xy))^{-1} \quad .$$

Thus for the factor set α associated with P, if $\alpha(x, y) = 1_F$ for all $x, y \in G$, then we obtain that $P(xy) = P(x)P(y)$ and hence P becomes an ordinary representation of G. Sometimes a pair (P, α) is used to indicate a projective representation P and its associated factor set α.

There is another way of looking at projective representations. The group $PGL_n(\mathbb{F}) = GL_n(\mathbb{F})/Z(GL_n(\mathbb{F}))$ is called the projective general linear group where $Z(GL_n(\mathbb{F}))$ is the center of $GL_n(\mathbb{F})$ which consists of all non-zero scalar matrices. If P is a projective $\mathbb{F}$-representation of G then the composition of P with the natural homomorphism $G \longrightarrow PGL_n(\mathbb{F})$ is a homomorphism $G \longrightarrow PGL_n(\mathbb{F})$. Conversely, if $\pi : G \longrightarrow PGL_n(\mathbb{F})$ is any homomorphism, a projective representation P of G can be defined by setting $P(g)$ equal to any element of the coset $\pi(g)$ of $Z(GL_n(\mathbb{F}))$ in $GL_n(\mathbb{F})$. Thus the projective $\mathbb{F}$-representations of G can be identified with the homomorphisms of G into the projective general linear group.

We now consider the associated factor sets of the projective representations.

Lemma 3.3.2. *Let α be the associated factor set of a projective representation P of G. Then α satisfies $\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z)$ for all $x, y, z \in G$.*

PROOF. By associativity we have

$$P(x)P(y)P(z) = \alpha(x, y)P(xy)P(z) = \alpha(x, y)\alpha(xy, z)P(xyz)$$

and

$$P(x)P(y)P(z) = \alpha(y, z)P(x)P(yz) = \alpha(y, z)\alpha(x, yz)P(xyz).$$

Now the result follows since $P(xyz)$ is invertible. ■

As with ordinary representations, we now define equivalence and irreducibility of projective representations. We will consider projective representations over the complex field $\mathbb{C}$ from now on.

Definition 3.3.3. *Two projective representations P_1 and P_2 of G are* **equivalent** *if there is a non-singular matrix T such that for all $g \in G$, $P_1(g) = c(g)TP_2(g)T^{-1}$ for some $c(g) \in \mathbb{C}^*$. If $c(g) = 1$ for all $g \in G$ then P_1 and P_2 are* **linearly equivalent***. A projective representation P is* **irreducible** *if it is not linearly equivalent to a projective representation of the form*

$$\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}.$$

Lemma 3.3.4. *If two projective representations are equivalent then they have equivalent factor sets; if they are linearly equivalent they have equal factor sets.*

PROOF. Let P_1 and P_2 be equivalent projective representations with factor sets α_1 and α_2 respectively. Suppose T is a non-singular matrix and $c : G \longrightarrow \mathbb{C}^*$ such that $P_1(g) = c(g)TP_2(g)T^{-1}$ for all $g \in G$. Now for $g, h \in G$,

$$\begin{aligned}
\alpha_1(g, h) &= P_1(g)P_1(h)(P_1(gh))^{-1} \\
&= c(g)TP_2(g)T^{-1}c(h)TP_2(h)T^{-1}(c(gh))^{-1}T(P_2(gh))^{-1}T^{-1} \\
&= c(g)c(h)(c(gh))^{-1}TP_2(g)P_2(h)(P_2(gh))^{-1}T^{-1} \\
&= c(g)c(h)(c(gh))^{-1}\alpha_2(g, h),
\end{aligned}$$

so α_1 and α_2 are equivalent. If P_1 and P_2 are linearly equivalent, then $c(g) = 1$ for all $g \in G$ in the above expressions, so $\alpha_1 = \alpha_2$. ∎

Let $F[G, \mathbb{C}]$ be the set of all functions $\lambda : G \longrightarrow \mathbb{C}$. If P is a projective representation of G with factor set α and $\lambda \in F[G, \mathbb{C}]$, then $P' = \lambda P$, where $P'(g) = \lambda(g)P(g)$ for all $g \in G$, is a projective representation of G with factor set α', and

$$\alpha'(x, y) = \lambda(x)\lambda(y)(\lambda(xy))^{-1}\alpha(x, y) \tag{3.1}$$

for all $x, y \in G$.

Remark 3.3.5. *It follows from (3.1) that $\alpha \sim 1$ if and only if there exists $\lambda \in F[G, \mathbb{C}]$ such that for all $x, y \in G$*

$$\alpha(x, y) = \lambda(x)\lambda(y)(\lambda(xy))^{-1}.$$

The following result provides a close connection between the degrees of the irreducible projective characters with factor set α and the $o([\alpha])$.

Lemma 3.3.6. *[11] Let P be a projective representation of G with factor set α and $deg(P) = n$. If $o([\alpha]) = m$ then m divides n.*

PROOF. We know that

$$P(x)P(y) = \alpha(x, y)P(xy).$$

Taking determinant we obtain

$$\begin{aligned} det(P(x))det(P(y)) &= det(\alpha(x, y)P(xy)) \\ &= \alpha(x, y)^n det(P(xy)) \end{aligned}$$

which implies

$$\alpha(x, y)^n = det(P(x))det(P(y))(det(P(xy))^{-1}.$$

By Remark 3.3.5 we obtain $[\alpha]^n = 1$. Hence m divides n. ∎

Projective representations of a group G can be obtained by three different ways. Firstly, we may obtain the projective representations of a group G by considering a central extension of G. Now we show that how the projective representations of a group G can be constructed from the ordinary representations of a so-called representation group of G.

Definition 3.3.7. *A* **central extension** *of G is a group H together with a homomorphism π of H onto G such that $ker(\pi)$ lies in the center of H.*

Lemma 3.3.8. *Let (H, π) be a central extension of G with $A = ker(\pi)$. Let X be a set of coset representatives for A in H, and write $X = \{x_g : g \in G\}$, where $\pi(x_g) = g$. Define $\alpha : G \times G \longrightarrow A$ by $x_g x_h = \alpha(g, h)x_{gh}$. Then α is an A-factor set of G and the equivalence class of α is independent of the choice of X.*

PROOF. See Isaacs [60]. ■

Corollary 3.3.9. *Let H be a central extension of G with A, X and α as in the previous lemma. Let T be an ordinary representation of H such that the restriction T_A is the scalar representation λI for some $\lambda \in Hom(A, \mathbb{C}^*)$, that is*

$$T(a) = \begin{pmatrix} \lambda(a) & & & \\ & \lambda(a) & & \\ & & \ddots & \\ & & & \lambda(a) \end{pmatrix}_{n\times n} \quad \forall a \in A,$$

where $n = deg(T)$. Define $P(g) = T(x_g)$ for $g \in G$. Then P is a projective representation of G with factor set $\lambda(\alpha)$, where $\lambda(\alpha)(g,h) = \lambda(\alpha(g,h))$. Furthermore, P is irreducible if and only if T is and the equivalence class of P is independent of the choice of coset representatives X.

PROOF. See [60].

Remark 3.3.10. *Note that if T is an ordinary irreducible representation of H then the condition that T_A be scalar representation is satisfied by the Schur's lemma (Theorem 3.3.2), since A lies in the center of H.*

Definition 3.3.11. *A projective representation of G that can be constructed from an ordinary representation of a central extension H of G as in Corollary 3.3.8 is said to be* **lifted** *to H. A* **representation group** *of G is a central extension H of G such that every projective representation of G can be lifted to H.*

Every group has a representation group by the following result which is due to Schur [114].

Theorem 3.3.12. *Let G be a finite group of order n. Then G has at least one representation group H of order mn where $m = |M(G)|$ and the kernel of the extension is isomorphic to the Schur multiplier $M(G)$ of G.*

PROOF. See, for example, [60]. ■

Secondly, projective representations of G can also be obtained by the generalization of Clifford's method of constructing representations of G using representations of a normal subgroup N of G.

Finally, third approach to obtain projective representations involve a natural generalization of the group algebra which plays such an important role in ordinary representation theory. Interested readers are encouraged to consult Morris [95] and other relevant sources.

The projective representations of a group are often constructed by using a combination of the above mentioned three techniques. Interested readers are referred to a series of articles by Morris [96, 97, 98] and Read [107, 108, 109].

3.4. Projective Characters

Definition 3.4.1. *Let P be a projective representation of G with factor set α. Define $\xi(g) = Trace(P(g))$ for all $g \in G$. Then ξ is called a* **projective character** *of G. We say that ξ is* **irreducible** *if P is, and ξ has factor set α, where α is the factor set of P.*

Definition 3.4.2. *Given a factor set α of G, an element $g \in G$ is said to be* **α-regular** *if $\alpha(g,x) = \alpha(x,g)$ for all $x \in C_G(g)$.*

If g is α-regular, so is every conjugate of g, and an element g is α-regular if and only if g is α'-regular for every factor set α' equivalent to α. So we can define a conjugacy class of G to be α-regular if each of its elements is α-regular.

An important feature of ordinary characters is that they are class functions. However, this no longer true for projective characters. For projective characters we have

Proposition 3.4.3. *Let ξ be the projective character of G with factor set α. If for any α-regular element x in G and for any y in G, $\alpha(x,y) = \alpha(y, y^{-1}xy)$ then ξ is a class function.*

PROOF. This is Proposition 2.2(iii) in [66]. ■

Theorem 3.4.4. *Two projective representations P_1 and P_2 with factor set α are linearly equivalent if and only if they have the same projective character.*

PROOF. See Theorem 4.4 in [95]. ■

The projective characters of G can be determined from the ordinary characters of a representation group (H,π) of G. Let $\pi : H \longrightarrow G$ be defined by the extension H of G, and let $\{x_g : g \in G\}$ be a set of coset representatives for $ker(\pi)$ in H. If P is a projective representation of G with projective character ξ then there is an ordinary representation T of H such that $P(g) = T(x_g)$ for $g \in G$. Let χ be the character of H afforded by T, then $\xi(g) = \chi(x_g)$ for all $g \in G$.

Projective characters also satisfy the usual orthogonality relations. We have analogues to ordinary characters.

Theorem 3.4.5. *(i) The number of irreducible projective characters of G with factor set α is equal to the number of α-regular conjugacy classes of G.*

(ii) Let $\xi_1, \xi_2, \ldots, \xi_t$ be the projective characters of G with factor set α, and let $C_1, C_2, \ldots, C_t$ be the α-regular conjugacy classes of G with g_i a representative of C_i for $i = 1, 2, \ldots, t$. Then

$$\sum_{i=1}^{t} \xi_i(g_j)\overline{\xi_i(g_k)} = \delta_{jk}|C_G(g_j)| \quad \textit{for } j,k \in \{1, 2, \ldots t\}.$$

(iii) An element g of G is α-regular if and only if there is an irreducible projective character ξ of G with factor set α such that $\xi(g) \neq 0$.

PROOF. See [47]. ■

Let G^0 be the set of all α-regular elements of the group G. Then we have the following.

Theorem 3.4.6. *Let Let $\xi_1, \xi_2, \ldots, \xi_t$ be the projective characters of G with factor set α, and let $C_1, C_2, \ldots, C_t$ be the α-regular conjugacy classes of G with g_i a representative of C_i for $i = 1, 2, \ldots, t$. Then*

$$\sum_{g \in G^0} \xi_i(g)\overline{(\xi_j(g)} = |G|\delta_{ij}.$$

PROOF. See [66] ■

Haggarty and Humphreys [47] showed that it is possible to determine the projective characters of G with a given factor set without the full representation group G. Suppose α is a factor set of G, with $[\alpha]$ having order e in the Schur multiplier $M(G)$. Let ω be an e^{th} root of unity and let α' be a representative of $[\alpha]$ whose values are powers of ω. For $g, h \in G$ define $\alpha'(g, h)$ by $\alpha'(g, h) = \omega^{a(g,h)}$. Let L be the group generated by an element x of order e and elements $x_g \quad (g \in G)$ with multiplication $x^i x_g x^j x_h = x^{i+j} x^{a(g,h)} x_{gh}$. Then L is a quotient of the representation group H and any projective representation of G with factor set α can be lifted to an ordinary representation of L. Thus the projective characters of G with factor set α can be determined from the ordinary character table of L.

4

Clifford Theory

4.1. Prologue

An important method for constructing irreducible representations of groups consists in the application of three basic operations:

(i) Restriction to a subgroup,

(ii) Extension from a subgroup,

(iii) Induction from a subgroup.

The theory attains particular richness when the underlying subgroup is a normal subgroup of its extension. This is the content of the Clifford theory, originally developed by *Clifford* in 1937 [20] for ordinary representations and extended by *Mackey* in 1958 [79] to projective representations. In this chapter, we study the Clifford theory and its related consequences which are required to describe the Fischer-Clifford matrices in the next chapter. In Section 4.2, we study the relation between the characters of a group $\bar{G}$ and its normal subgroup N. We will give various sufficient conditions for the extendibility of an irreducible character θ of N to $\bar{G}$. In Section 4.3, we engage the Clifford theory of projective representations. We will study, how it is always possible to extend an irreducible character of a normal subgroup N to a projective character of its inertia group $\bar{H}$. Finally in Section 4.4, we will study the problem which asserts that if χ is a $\bar{G}$-invariant irreducible character of a normal subgroup N of a finite group $\bar{G}$, then the number of distinct irreducible constituents of $\chi^{\bar{G}}$ is equal to the number of χ-regular conjugacy classes of $\bar{G}/N$. We also show that the number of irreducible constituents of $\chi^{\bar{G}}$ is equal to the number of conjugacy classes of $\bar{G}/N$ if and only if χ extends to a character of each subgroup $N{<}x, y{>}$ of $\bar{G}$ with $[x, y] \in N$.

4.2. Clifford Theory and Normal Subgroups

In this section we study the important connection between characters of group $\bar{G}$ and a normal subgroup N of $\bar{G}$.

Definition 4.2.1. *Let $\bar{G}$ be a group, $N \leq \bar{G}$ and θ be a character of N. Then for $\bar{g} \in \bar{G}$, we define $\theta^{\bar{g}} : \bar{g}^{-1}N\bar{g} \longrightarrow \mathbb{C}$ by $\theta^{\bar{g}}(t) = \theta(\bar{g}t\bar{g}^{-1})$ for all $t \in \bar{g}^{-1}N\bar{g}$. Then $\theta^{\bar{g}}$ is said to be a* $\bar{G}$**-conjugate** *of θ. If N is a normal subgroup of $\bar{G}$ and $\theta^{\bar{g}} = \theta$ for all $\bar{g} \in \bar{G}$, then θ is said to be* $\bar{G}$**-invariant**.

Theorem 4.2.2. *Let G be a group, $K, H \leq G$ such that $K \leq H \leq G$ and χ be a character of K. Then for all $g \in G$ we have*

(i) $(\chi^H)^g = (\chi^g)^{g^{-1}Hg}$

(ii) $(\chi^g)^G = \chi^G$.

PROOF. See [67] ■

Remark 4.2.3. *If $N \leq \bar{G}$ and $\bar{g} \in \bar{G}$, then $\theta^{\bar{g}}$ is a character of $\bar{g}^{-1}N\bar{g}$. However if N is normal in $\bar{G}$, $\theta^{\bar{g}}$ becomes a character of N.*

Let $\bar{G}$ be a group, N a normal subgroup of $\bar{G}$ and $\theta \in Irr(N)$ then we define

$$Irr(\bar{G}, \theta) = \{\chi \mid \chi \in Irr(\bar{G}), \langle \chi_N, \theta \rangle > 0\} \ .$$

Observe that $\langle \chi_N, \theta \rangle_N = \langle \chi, \theta^{\bar{G}} \rangle_{\bar{G}}$.

Definition 4.2.4. *Let $\bar{G}$ be a group, N a normal subgroup of $\bar{G}$ and $\theta \in Irr(N)$. Then*

$$I_{\bar{G}}(\theta) = \{\bar{g} \in \bar{G} \mid \theta^{\bar{g}} = \theta\}$$

is the **inertia group** *of θ in $\bar{G}$.*

Since $I_G(\theta)$ is the stabilizer of θ in the action of $\bar{G}$ on $Irr(N)$, it follows that it is a subgroup and that $I_G(\theta) \supseteq N$.

Lemma 4.2.5. *[58] Let $\bar{G}$ be a group, N a normal subgroup of $\bar{G}$ and $\theta \in Irr(N)$. Then*

(a) For $\bar{g}_1, \bar{g}_2 \in \bar{G}$ we have $\theta^{\bar{g}_1\bar{g}_2} = (\theta^{\bar{g}_1})^{\bar{g}_2}$. In particular $N \leq I_{\bar{G}}(\theta) \leq \bar{G}$. If

$$\bar{G} = \bigcup_{i=1}^{m} I_{\bar{G}}(\theta)\bar{g}_i$$

with $[\bar{G} : I_{\bar{G}}(\theta)] = m$, then $\{\theta^{\bar{g}} | \bar{g} \in \bar{G}\} = \{\theta^{\bar{g}_1}, \theta^{\bar{g}_2}, \ldots, \theta^{\bar{g}_m}\}$, and $\theta^{\bar{g}_1}, \theta^{\bar{g}_2}, \ldots, \theta^{\bar{g}_m}$ are pairwise distinct.

(b) If ψ_1, ψ_2 are any characters of N and $\bar{g} \in \bar{G}$, then $\langle \psi_1^{\bar{g}}, \psi_2^{\bar{g}} \rangle_N = \langle \psi_1, \psi_2 \rangle_N$. In particular $\theta^{\bar{g}} \in Irr(N) \quad if \quad \theta \in Irr(N)$.

(c) If ψ is a character of $\bar{G}$ and θ of N, then $\langle \psi_N, \theta \rangle_N = \langle \psi_N, \theta^{\bar{g}} \rangle_N$ for all $\bar{g} \in \bar{G}$.

PROOF. See [58]. ∎

We now state a fundamental theorem, which is due to Clifford [20] but we give a proof from Huppert [58].

Theorem 4.2.6. *[58]* (Clifford Theorem) *Suppose $\bar{G}$ is a group, N a normal subgroup of $\bar{G}$, $\theta \in Irr(N)$ and $\chi \in Irr(\bar{G}, \theta)$. Let $\langle \chi_N, \theta \rangle = e > 0$. Assume also that*

$$\bar{G} = \bigcup_{i=1}^{m} I_{\bar{G}}(\theta)\bar{g}_i \quad \text{and} \quad m = [\bar{G} : I_{\bar{G}}(\theta)].$$

Then we have

(a) $(\theta^{\bar{G}})_N = |I_{\bar{G}}(\theta)/N| \sum_{i=1}^{m} \theta^{\bar{g}_i}$.

(b) $\langle \theta^{\bar{G}}, \theta^{\bar{G}} \rangle_{\bar{G}} = |I_{\bar{G}}(\theta)/N|$. In particular $\theta^{\bar{G}} \in Irr(\bar{G})$ if and only if $I_{\bar{G}}(\theta) = N$.

(c) $\chi_N = e \sum_{i=1}^{m} \theta^{\bar{g}_i}$. In particular,

$$\chi(1) = em\theta(1) \quad \text{and} \quad \langle \chi_N, \chi_N \rangle_N = e^2 m.$$

Also

$$e^2 \leq |I_{\bar{G}}(\theta)/N| \quad \text{and} \quad e^2 m \leq |\bar{G}/N|.$$

PROOF. (a) For $x \in N$ we have by the previous lemma

$$\theta^{\bar{G}}(x) = \frac{1}{|N|} \sum_{\bar{g} \in \bar{G}} \theta(x^{\bar{g}^{-1}}) = \frac{1}{|N|} \sum_{\bar{g} \in \bar{G}} \theta^{\bar{g}}(x) = \frac{|I_{\bar{G}}(\theta)|}{|N|} \sum_{i=1}^{m} \theta^{\bar{g}_i}(x).$$

(b) By Frobenius reciprocity (Theorem 2.5.6) and part (a) we obtain

$$\langle \theta^{\bar{G}}, \theta^{\bar{G}} \rangle_{\bar{G}} = \langle (\theta^{\bar{G}})_N, \theta \rangle_N = |I_{\bar{G}}(\theta)/N|.$$

(c) For all $\bar{g} \in \bar{G}$, we have by Lemma 4.2.4(c)

$$\langle \chi_N, \theta^{\bar{g}} \rangle_N = \langle \chi_N, \theta \rangle_N = e.$$

Hence

$$\chi_N = e \sum_{i=1}^{m} \theta^{\bar{g}_i} + \psi,$$

where ψ is a character of N or zero. As

$$\langle \theta^{\bar{G}}, \chi \rangle_{\bar{G}} = \langle \theta, \chi_N \rangle_N = e,$$

we obtain

$$\theta^{\bar{G}} = e\chi + \dots .$$

Restriction to N shows by part (a)

$$e\chi_N + \dots = (\theta^{\bar{G}})_N = |I_{\bar{G}}(\theta)/N| \sum_{i=1}^{m} \theta^{\bar{g}_i}.$$

Hence in χ_N there does not appear any irreducible character of N different from the $\theta^{\bar{g}_j}$. Therefore

$$\chi_N = e \sum_{i=1}^{m} \theta^{\bar{g}_i},$$

which implies immediately

$$\chi(1) = em\theta(1) \quad \text{and} \quad \langle \chi_N, \chi_N \rangle_N = e^2 m.$$

By part (b) we have

$$|I_{\bar{G}}(\theta)/N| = \langle \theta^{\bar{G}}, \theta^{\bar{G}} \rangle_{\bar{G}} = \langle e\chi + \dots, e\chi + \dots \rangle_{\bar{G}} \geq e^2$$

and hence

$$|\bar{G}/N| = |\bar{G} : I_{\bar{G}}(\theta)||I_{\bar{G}}(\theta)/N| \geq me^2.$$

■

Remark 4.2.7. *It can be shown that the number e in the above theorem is the degree of an irreducible projective representation of $\bar{G}/N$, hence it divides $|\bar{G}/N|$. See Huppert [58].*

As a consequence of Clifford theorem we have the following result, which is of fundamental importance in the character theory of normal subgroups.

Theorem 4.2.8. *[60] Let $\bar{G}$ be a group, N a normal subgroup of $\bar{G}$, $\theta \in Irr(N)$ and $T = I_{\bar{G}}(\theta)$. Let*

$$A = \{\psi \in Irr(T) \mid \langle \psi_N, \theta \rangle \neq 0\},$$

$$B = \{\chi \in Irr(\bar{G}) \mid \langle \chi_N, \theta \rangle \neq 0\}.$$

Then

(a) If $\psi \in A$, then $\psi^{\bar{G}} \in Irr(\bar{G})$.

(b) If $\psi^{\bar{G}} = \chi$ and $\psi \in A$, then $\langle \psi_N, \theta \rangle = \langle \chi_N, \theta \rangle$.

(c) If $\psi^{\bar{G}} = \chi$ and $\psi \in A$, then ψ is the unique irreducible constituent of χ_T which sits in A.

(d) The map $\psi \longmapsto \psi^{\bar{G}}$ is a bijection of A to B.

PROOF. See Isaacs [60]. ■

Remark 4.2.9. *From the previous theorem we deduce that induction to $\bar{G}$ maps the irreducible characters of T that contain θ in their restriction to N faithfully onto the irreducible characters of $\bar{G}$ that contain θ in their restriction to N.*

An important task of the Clifford theory is to examine when irreducible characters of normal subgroups are extendible to their respective inertia groups.

Definition 4.2.10. *Let $\bar{G}$ be a group, H a subgroup of $\bar{G}$, $\theta \in Irr(H)$ and $\chi \in Irr(\bar{G})$ such that $\chi_H = \theta$. Then θ is said to be* **extendible** *to an irreducible character of $\bar{G}$.*

If θ is extendible to an irreducible character of $\bar{G}$, we will simply say that θ is extendible to $\bar{G}$. There are various conditions which have to be satisfied in order that θ can be extended to $\bar{G}$. Readers can also consult [11, 38, 39, 60, 65, 58] for further reading and information on extendibility of characters.

Theorem 4.2.11. *[67] Let N a normal subgroup of $\bar{G}$, $\chi \in Irr(N)$, where χ is $\bar{G}$- invariant and let Γ be a matrix representation of N which affords χ. Then*

(i) there exists a projective representation ρ of $\bar{G}$ such that $\Gamma(n) = \rho(n)$ and $(\rho(\bar{g}))^{o(\bar{g})} = I$, for all $n \in N, \bar{g} \in \bar{G}$ where I is the identity matrix,

(ii) if $\bar{G} = NH$ for some $H \leq \bar{G}$ and if ρ_H is an ordinary representation of H, then χ can be extended to $\bar{G}$.

PROOF. (i) Let $\bar{g} \in \bar{G}$. Since χ is $\bar{G}$-invariant, then the representations Γ and $\Gamma^{\bar{g}}$ of N are equivalent. Hence there is an invertible matrix $\theta(\bar{g})$ such that $(\theta(\bar{g}))^{-1}\Gamma(n)\theta(\bar{g}) = \Gamma^{\bar{g}}(n)$, for all $n \in N$. We may assume that $\theta(n) = \Gamma(n)$ for all $n \in N$. We have that $\theta : \bar{G} \longrightarrow GL(k, \mathbb{F})$, where $k = deg(\Gamma)$, and that $\theta_N = \Gamma$. Now let $\bar{g_1}, \bar{g_2} \in \bar{G}$. Then we obtain that

$$\begin{aligned}(\theta(\bar{g_1}\bar{g_2}))^{-1}\Gamma(n)\theta(\bar{g_1}\bar{g_2}) &= \Gamma^{\bar{g_1}\bar{g_2}}(n) = (\Gamma^{\bar{g_1}})^{\bar{g_2}}(n) = (\theta(\bar{g_2}))^{-1}\Gamma^{\bar{g_1}}(n)\theta(\bar{g_2}) \\ &= (\theta(\bar{g_2}))^{-1}(\theta(\bar{g_1}))^{-1}\Gamma(n)\theta(\bar{g_1})\theta(\bar{g_2}).\end{aligned}$$

So that

$$\theta(\bar{g_1})\theta(\bar{g_2})(\theta(\bar{g_1}\bar{g_2}))^{-1}\Gamma(n) = \Gamma(n)\theta(\bar{g_1})\theta(\bar{g_2})(\theta(\bar{g_1}\bar{g_2}))^{-1} \quad .$$

Thus for all $n \in N$, $\theta(\bar{g_1})\theta(\bar{g_2})(\theta(\bar{g_1}\bar{g_2}))^{-1}$ commutes with $\Gamma(n)$ and thus by the Corollary 2.3.3, we can define a function $\alpha : \bar{G} \times \bar{G} \longrightarrow \mathbb{C}^*$ such that $\theta(\bar{g_1})\theta(\bar{g_2}) = \alpha(\bar{g_1}, \bar{g_2})\theta(\bar{g_1}\bar{g_2})$. Since Γ

is a representation of N, then we obtain that $\theta(1_N) = \Gamma(1_N) = I$. Hence θ is a projective representation of $\bar{G}$ with associated factor set α. Let $o(\bar{g}) = m$ and if $\bar{g} \in N$, then we obtain that $(\theta(\bar{g}))^m = I$. However if $\bar{g} \in \bar{G} - N$, then since $\theta(\bar{g}^m) = \theta(1_{\bar{G}}) = I$, there exists $\lambda(\bar{g}) \in \mathbb{C}^*$ such that $(\theta(\bar{g}))^m = \lambda(\bar{g})I$. Now let $\mu(\bar{g}) \in \mathbb{C}^*$ such that $(\mu(\bar{g}))^m = (\lambda(\bar{g}))^{-1}$ and let $\mu(n) = 1$ for all $n \in N$. Then the projective representation ρ of $\bar{G}$ given by $\rho(\bar{g}) = \mu(\bar{g})\theta(\bar{g})$ is such that $\rho(n) = \mu(n)\theta(n) = \theta(n) = \Gamma(n)$ for all $n \in N$. Also we have that

$$(\rho(\bar{g}))^m = (\mu(\bar{g})\theta(\bar{g}))^m = (\mu(\bar{g}))^m(\theta(\bar{g}))^m = (\lambda(\bar{g}))^{-1}\lambda(\bar{g})I = I \quad .$$

Hence property (i) is established.

(ii) Let T be a transversal for $N \cap H$ in H containing 1_H. Then every $\bar{g} \in \bar{G}$ has a unique expression of the form $\bar{g} = tn$, where $t \in T, n \in N$. Now let $\bar{g_1} \in \bar{G}$, $\bar{g_1} \neq \bar{g}$ be given by $\bar{g_1} = t_1n_1$, where $t_1 \in T, n_1 \in N$. Since $t, t_1 \in T$, then $t, t_1 \in H$ and hence $tt_1 \in H$. Now let $tt_1 = t_2n_2$, where $t_2 \in T$ and $n_2 \in N \cap H$. Define ψ on $\bar{G}$ by $\psi(\bar{g}) = \rho(t)\rho(n)$. Since $n_2t_1^{-1}nt_1n_1 \in N$, we obtain that

$$\psi(\bar{g}\bar{g_1}) = \psi(tnt_1n_1) = \psi(tt_1t_1^{-1}nt_1n_1) = \psi(t_2n_2t_1^{-1}nt_1n_1) = \rho(t_2)\rho(n_2t_1^{-1}nt_1n_1) \quad .$$

Also we have

$$\begin{aligned}\psi(\bar{g})\psi(\bar{g_1}) &= \rho(t)\rho(n)\rho(t_1)\rho(n_1) = \rho(t)\rho(t_1)(\rho(t_1))^{-1}\rho(n)\rho(t_1)\rho(n_1) \\ &= \rho(t)\rho(t_1)[(\rho(t_1))^{-1}\rho(n)\rho(t_1)]\rho(n_1).\end{aligned}$$

However from the proof of part(i) above we have that $(\rho(\bar{g}))^{-1}\Gamma(n)\rho(\bar{g}) = \Gamma^{\bar{g}}(n)$ and $\rho(n) = \Gamma(n)$ for all $n \in N, \bar{g} \in \bar{G}$. Since $t_1^{-1}nt_1 \in N$, then we obtain that

$$\rho(t_1^{-1}nt_1) = \Gamma(t_1^{-1}nt_1) = \Gamma^{t_1}(n) = (\rho(t_1))^{-1}\Gamma(n)\rho(t_1) = (\rho(t_1))^{-1}\rho(n)\rho(t_1).$$

Since by the assumption ρ is an ordinary representation on H we have $\rho(tt_1) = \rho(t)\rho(t_1)$ since $tt_1 \in H$. We deduce that

$$\begin{aligned}\psi(\bar{g})\psi(\bar{g_1}) &= \rho(t)\rho(t_1)\rho(t_1^{-1}nt_1)\rho(n_1) = \rho(tt_1)\rho(t_1^{-1}nt_1)\rho(n_1) \\ &= \rho(t_2n_2)\rho(t_1^{-1}nt_1)\rho(n_1) = \rho(t_2)\rho(n_2t_1^{-1}nt_1n_1).\end{aligned}$$

Hence we obtain that $\psi(\bar{g}\bar{g_1}) = \psi(\bar{g})\psi(\bar{g_1})$. Therefore ψ is an ordinary representation of $\bar{G}$. However $\forall\, n \in N$, we obtain that $\psi(n) = \rho(n) = \Gamma(n)$ and thus the character afforded by the representation ψ of $\bar{G}$, extends χ to $\bar{G}$. Hence the result. ■

Theorem 4.2.12. *[67] Let $\bar{G} = NG$ where N is a normal subgroup of $\bar{G}$, and $G \leq \bar{G}$ such that $N \cap G \subseteq N'$. If θ is an irreducible $\bar{G}$-invariant character of N such that $(deg(\theta), |G|) = 1$, then θ can be extended to $\bar{G}$.*

PROOF. For a detailed proof which uses the previous theorem, see Corollary 27.1.2 of [67] ■

Theorem 4.2.13. *[24, 120, 99] (***Mackey's Theorem***) Let N be a normal subgroup of $\bar{G}$ and θ be a $\bar{G}$-invariant irreducible character of N. If N is abelian and $\bar{G}$ splits over N, then θ can be extended to $\bar{G}$.*

PROOF. Let $\bar{G} = N{:}G$. Since $\bar{G}$ is a semidirect product of N by G, then any $x \in \bar{G}$ can be expressed uniquely as $x = ng$, where $n \in N, g \in G$. Define χ on $\bar{G}$ by $\chi(ng) = \theta(n)$. Since N is abelian, θ has degree 1 and thus is linear. The invariance of θ in $\bar{G}$ implies that $\theta(n) = \theta(xnx^{-1})$ for all $x \in \bar{G}$. Now let $x_1 = n_1g_1, x_2 = n_2g_2$ be elements of $\bar{G}$. Then we obtain that

$$\begin{aligned}\chi(x_1x_2) &= \chi(n_1g_1n_2g_2) = \chi(n_1n_2^{g_1}g_1g_2) = \theta(n_1n_2^{g_1}) \\ &= \theta(n_1)\theta(n_2^{g_1}) = \theta(n_1)\theta(n_2) = \chi(x_1)\chi(x_2).\end{aligned}$$

Therefore χ is a linear character of $\bar{G}$ such that $\chi_N = \theta$. ■

Remark 4.2.14. Mackey*'s theorem has been proved differently in Ali [1] and Mpono [99] by applying Theorem 4.2.11.*

Theorem 4.2.15. *Let N be a normal subgroup of a finite group $\bar{G}$ and θ be an irreducible character of N which is invariant in $\bar{G}$, then θ is extendible to a character of $\bar{G}$ if $([\bar{G} : N], \frac{|N|}{deg(\theta)}) = 1$.*

PROOF. See [38]. ■

Theorem 4.2.16. *Suppose $\bar{G}$ is a splitting extension of a normal subgroup N, then any linear character $\theta \in Irr(N)$ can be extended to its inertia group $I_{\bar{G}}(\theta)$.*

PROOF. See [1, 99]. ■

Note that Mackey's theorem is reinforced by the Theorem 4.2.15 since for N abelian, all its irreducible characters are linear and hence are extendible to their respective inertia groups.

Theorem 4.2.17. *[39, 59, 120](***Gallagher's Theorem***) Let N a normal subgroup of $\bar{G}$, $\theta \in Irr(N)$ and $\bar{H} = I_{\bar{G}}(\theta)$. If θ can be extended to $\psi \in Irr(\bar{H})$ then as β ranges over all the irreducible characters of $\bar{H}$ which contain N in their kernels, $\beta\psi$ ranges over all the irreducible characters of $\bar{H}$ which contain θ in their restriction to N.*

PROOF. Since $\bar{H} = I_{\bar{G}}(\theta)$, then θ is self-conjugate in $\bar{H}$ and thus by Clifford's theorem we obtain that $(\theta^{\bar{H}})_N = f\theta$ for some positive integer f. Comparing degrees we have $(\theta^{\bar{H}})_N = [\bar{H} : N]\theta$ and so $\langle \theta^{\bar{H}}, \theta^{\bar{H}} \rangle = \langle \theta, (\theta^{\bar{H}})_N \rangle = [\bar{H} : N]$. Now we claim that $\theta^{\bar{H}} = \sum_\beta \beta(1_{\bar{G}})\beta\psi$, where β ranges over all the irreducible characters of $\bar{H}$ that contain N in their kernels. Both $\theta^{\bar{H}}$ and $\sum_\beta \beta(1_{\bar{G}})\beta\psi$ are zero off N since for $g \notin N, xgx^{-1} \notin N$ for all $x \in \bar{G}$ and thus $\theta^{\bar{H}}(g) = 0$. Also for $g \notin N$, by the orthogonality of the columns of the character table of $\bar{H}/N$ we have that $\sum_\beta \beta(1_{\bar{G}})(\beta\psi)(g) = [\sum_\beta \beta(1_{\bar{G}})\beta(g)]\psi(g) = 0$. We also have that $(\theta^{\bar{H}})_N = [\bar{H} : N]\theta = (\sum_\beta \beta(1_{\bar{G}})\beta\psi)_N$ since for $g \in N$,

$\sum_\beta \beta(1_{\bar{G}})\beta(g)\psi(g) = \sum_\beta(\beta(1_{\bar{G}}))^2\psi(g) = [\bar{H} : N]\psi(g) = [\bar{H} : N]\theta(g)$. Hence we obtain that $\theta^{\bar{H}} = \sum_\beta \beta(1_{\bar{G}})\beta\psi$. So we have

$$[\bar{H} : N] = \langle \theta^{\bar{H}}, \theta^{\bar{H}} \rangle = \langle \sum_\beta \beta(1_{\bar{G}})\beta\psi, \sum_\tau \tau(1_{\bar{G}})\tau\psi \rangle = \sum_{\beta,\tau} \beta(1_{\bar{G}})\tau(1_{\bar{G}})\langle \beta\psi, \tau\psi \rangle \quad .$$

The diagonal terms contribute at least $\sum(\beta(1_{\bar{G}}))^2 = [\bar{H} : N]$, so the $\beta\psi$ are irreducible and distinct, and are all the irreducible constituents of $\theta^{\bar{H}}$ and so are all the irreducible characters of $\bar{H}$ that contain θ in their restriction to N. For $\phi \in Irr(\bar{H})$ such that $\langle \phi_N, \theta \rangle \neq 0$, we obtain that $\langle \phi_N, \theta \rangle = \langle \phi, \theta^{\bar{H}} \rangle$ which implies that ϕ is an irreducible constituent of $\theta^{\bar{H}}$ and hence is of the form $\beta\psi$. ■

Remark 4.2.18. *Let $\bar{G}$ be an extension of N by G. If every irreducible character of N can be extended to its inertia group in $\bar{G}$, then by application of Theorem 4.2.7 and Remark 4.2.8, the characters of $\bar{G}$ can be obtained as follows:*

Let $\theta_1, \theta_2, \ldots, \theta_t$ be representatives of the orbits of $\bar{G}$ on $Irr(N)$. For each i, let $\bar{H}_i = I_{\bar{G}}(\theta_i)$ and let $\psi_i \in Irr(\bar{H}_i)$ with $(\psi_i)_N = \theta_i$. Now each irreducible character of $\bar{G}$ contains some θ_i in its restriction to N by Clifford's theorem. So by Theorem 4.2.7 and Remark 4.2.8 we have

$$Irr(\bar{G}) = \bigcup_{i=1}^{t} \{(\beta\psi_i)^{\bar{G}} : \beta \in Irr(\bar{H}_i), N \subset ker(\beta)\}.$$

Hence the characters of $\bar{G}$ fall into t blocks, with each block corresponding to an inertia group.

Finally in this section, we give a result due to *Isaacs* about the value of an extension χ of θ to G. For $N \trianglelefteq G$, $\theta \in Irr(N)$ has an extension χ to G if $I_G(\theta) = G$. We prove that the values of χ are equally distributed over the cosets of N.

Theorem 4.2.19. *Suppose $N \trianglelefteq G$, $\chi \in Irr(G)$ with $\chi_N \in Irr(N)$. Then*

$$\frac{1}{|N|} \sum_{y \in Ng} |\chi(y)|^2 = 1$$

for all $g \in G$.

PROOF. See Theorem 21.5 of [58]. ■

4.3. Clifford Theory and Projective Representations

The projective representations of a group are closely related to Clifford theory. In this section we study the Clifford theory for projective representations.

Definition 4.3.1. *Let $N \trianglelefteq \bar{G}$. If Y is an irreducible (ordinary) representation of N then for $\bar{g} \in \bar{G}$, $Y^{\bar{g}}$ defined by $Y^{\bar{g}}(n) = Y(\bar{g}n\bar{g}^{-1})$, $n \in N$, is a representation of N, called a* **conjugate** *of Y. The* **inertia group** *of Y, $T(Y)$, is the set of all $\bar{g} \in \bar{G}$ such that Y is equivalent to $Y^{\bar{g}}$. Note that $T(Y) = I_{\bar{G}}(\theta)$ where θ is the character of N afforded by Y.*

Now let Y be an irreducible representation of N, where $N \trianglelefteq \bar{G}$ and let $\bar{H} = T(Y)$, so Y is equivalent to all its conjugates in $\bar{H}$. The following theorem shows that Y can always be extended to a projective representation of $\bar{H}$ and gives a necessary and sufficient condition for Y to be extendible to an ordinary representation of $\bar{H}$.

Theorem 4.3.2. *Let $N \trianglelefteq \bar{G}$, Y an irreducible representation of N and $\bar{H}$ be as above. Then Y extends to a projective representation X of $\bar{H}$ with factor set $\bar{\alpha}$ such that $\bar{\alpha}$ is constant on cosets of N in $\bar{H}$. Therefore $\bar{\alpha}$ can be regarded as a factor set α of $H = \bar{H}/N$ defined by $\alpha(Nh, Nk) = \bar{\alpha}(h,k)$. Also, α satisfies $\alpha^{d|N|} \sim 1$ where d is the degree of Y. Furthermore, Y extends to a linear representation of $\bar{H}$ if and only if $\alpha \sim 1$. In particular, if $H^2(\bar{G}, \mathbb{C}^*) = 1$, then Y always extends to a linear representation of G.*

PROOF. See Nagao and Tsushima [100]. ■

Theorem 4.3.3. *Let $N \trianglelefteq \bar{G}$, Y be an irreducible representation of N with $\bar{H} = T(Y)$ and $H = \bar{H}/N$. Extend Y to a projective representation X of $\bar{H}$ as in Theorem 4.3.2 with factor set $\bar{\alpha}$. Then*

1. *If W is an irreducible representation of H that has Y as one of its irreducible constituents in its restriction to N then there exists an irreducible projective representation Z of H with factor set α^{-1} such that W is equivalent to the representation $\bar{Z} \otimes X$ of $\bar{H}$, where α is the factor set of H obtained from $\bar{\alpha}$, and $\bar{Z}$ is the representation of $\bar{H}$ obtained naturally from Z.*
2. *If, conversely, Z is any irreducible projective representation of H with factor set α^{-1}, then $\bar{Z} \otimes X$ is an irreducible representation of $\bar{H}$ which is equivalent to some representation that contains Y in its restriction to N.*

PROOF. See Nagao and Tsushima [100]. ■

Theorem 4.3.4. *[113] Let $N \lhd \bar{H}$, $\varphi \in Irr(N)$ be invariant under $\bar{H}$ and let $\bar{\varphi}$ be a projective extension of φ to $\bar{H}$ with factor set α. Then*

$$Irr(\bar{H}, \varphi) = \{\bar{\varphi}\psi \mid \psi \quad \textit{is an irreducible}\, \alpha^{-1}\textit{−projective character of} \quad \bar{H}/N\}.$$

In particular, the number of irreducible α^{-1}- projective characters of $\bar{H}/N$ is equal to the number of α- regular classes of $\bar{H}$.

PROOF. See [113]. ■

Now we restate the results from Theorems 4.3.2 and 4.3.3 in the form in which we will be using them, in terms of projective and ordinary characters.

Corollary 4.3.5. *[82] Let $\bar{G} = N{\cdot}G$, where $N \trianglelefteq \bar{G}$ and $\bar{G}/N \cong G$. Let $\theta \in Irr(N)$ and $\bar{H} = I_{\bar{G}}(\theta)$.*

(i) There exists a projective character φ of $\bar{H}$ with factor set $\bar{\alpha}$ such that $\varphi_N = \theta$ and $\bar{\alpha}$ is constant on cosets of N, so $\bar{\alpha}$ can be regarded as a factor set α of $H = \bar{H}/N$.

(ii) *If* $\theta(1_N) = d$, *then* $\alpha^{d|N|} \sim 1$.

(iii) *If* η *runs over all the irreducible projective characters of* H *with factor set* α^{-1}, *then* $\varphi\bar{\eta}$ *runs over all irreducible characters of* $\bar{H}$ *that contains* θ *in their restrictions to* N *where* $\bar{\eta}$ *is the projective character of* $\bar{H}$ *obtained naturally from* η.

PROOF. See [82]. ■

Remark 4.3.6. *In the above theorem, if* θ *extends to an ordinary character of* $\bar{H}$, *then we show that* $\alpha \sim 1$. *In this case* η*'s are the ordinary irreducible characters of* H. *Hence Theorem 4.2.16 is a special case of the above corollary.*

Remark 4.3.7. *Now by Remark 4.2.8 and Corollary 4.3.5, the characters of* $\bar{G} = N{\cdot}G$ *can be obtained as follows:*

Let $\theta_1, \theta_2, \ldots, \theta_t$ *be the representatives of the orbits of* $\bar{G}$ *on the set* $Irr(N)$. *Let* $\bar{H}_i = I_{\bar{G}}(\theta_i)$, φ_i *be a projective character of* $\bar{H}_i$ *with factor set* $\bar{\alpha}_i$ *such that* $\theta_i = \varphi_N$. *Then*

$$Irr(\bar{G}) = \bigcup_{i=1}^{t}\{(\eta\varphi_i)^{\bar{G}} \mid \eta \in IrrProj(\bar{H}_i),\ \text{with factor set}\quad \alpha_i^{-1}\},$$

where α_i *is obtained from* $\bar{\alpha}_i$ *as in Corollary 4.3.5.*

Hence the characters table of $\bar{G}$ *is partitioned into* t *blocks* $\triangle_1, \triangle_2, \ldots, \triangle_t$ *where* $\triangle_i$ *is produced from the inertia subgroup* $\bar{H}_i$.

4.4. Irreducible Constituents and Conjugacy Classes

This section treats two topics. The first concerns the number of irreducible constituents of induced characters, and the second the number of conjugacy classes. Using some properties of extensions of characters, we will study the problem which asserts that if χ is a $\bar{G}$-invariant irreducible character of a normal subgroup N of a finite group $\bar{G}$, then the number of distinct irreducible constituents of $\chi^{\bar{G}}$ is equal to the number of χ-regular conjugacy classes of $\bar{G}/N$. We also show that the number of irreducible constituents of $\chi^{\bar{G}}$ is equal to the number of conjugacy classes of $\bar{G}/N$ if and only if χ extends to a character of each subgroup $N{<}x, y{>}$ of $\bar{G}$ with $[x, y] \in N$.

Most of the results in this section are from *Gallagher* [40] but we give proofs from [67].

Lemma 4.4.1. *Let* N *be a normal subgroup of* $\bar{G}$ *such that* $\bar{G}/N$ *is cyclic of order* n. *If* χ *is* $\bar{G}$*-invariant irreducible characters of* N, *then there exists precisely* n *irreducible characters of* $\bar{G}$ *extending* χ *and their sum is* $\chi^{\bar{G}}$.

PROOF. See Lemma 23.3.2 of [67]. ■

Let N be a normal subgroup of a group $\bar{G}$ and, for each $\bar{g} \in \bar{G}$, let the group $C_{\bar{g}}$ containing N be as defined in section 2.3, where

$$C_{\bar{g}}/N = C_{\bar{G}/N}(N\bar{g}).$$

Let χ be a $\bar{G}$-invariant irreducible character of N. From the proof of Lemma 4.3.1, χ extends to a character $\chi_{\bar{g}}$ of the subgroup $N\langle\bar{g}\rangle$ with $\bar{g} \in \bar{G}$. We say that $\bar{g}$ is χ-*regular* if $(\chi_{\bar{g}})^x = \chi_{\bar{g}}$ for all $x \in C_{\bar{g}}$. Note that Gallagher [40] uses the term *goodness* instead of χ-regular.

Remark 4.4.2. *In [67] it was proved that the notion of χ -regularity is independent of the choice of $\chi_{\bar{g}}$ and depends only on χ and the conjugacy class of $N\bar{g}$ in $\bar{G}/N$.*

We say that the conjugacy class of $N\bar{g}$ in $\bar{G}/N$ is χ-regular if $\bar{g}$ is χ-regular. By the above Remark, this notion is well defined.

Theorem 4.4.3. *[40]. Let N be a normal subgroup of a group $\bar{G}$ and let χ be a $\bar{G}$-invariant irreducible character of N. Then the number of distinct irreducible constituents of $\chi^{\bar{G}}$ is equal the number of χ-regular conjugacy classes of $\bar{G}/N$.*

PROOF. See [67]. ∎

Corollary 4.4.4. *[40] Let N be a normal subgroup of a group $\bar{G}$ and let χ be a $\bar{G}$-invariant irreducible character of N. Then the number of distinct irreducible constituents of $\chi^{\bar{G}}$ is at most the number of conjugacy classes of $\bar{G}/N$ with equality if and only if χ extends to a character of each subgroup $N\langle x, y\rangle$ with $[x, y] \in N$.*

PROOF. See [67]. ∎

Now we provide some information on the number of conjugacy classes of G by using certain character-theoretic facts. In what follows $r(G)$ denotes the number of conjugacy classes of G. Then $r(G)$ is also the number of irreducible complex characters of G.

Lemma 4.4.5. *[67] The following formula holds:*

$$r(G) = \frac{1}{|G|} \sum_{g \in G} |C_G(g)|.$$

PROOF. The group G acts on itself by conjugation. If χ is the corresponding permutation character, then

$$\chi(g) = |C_G(g)| \quad \text{for all} \quad g \in G$$

and the G-orbits are precisely the conjugacy classes of G. Hence,

$$r(G) = \langle \chi, 1_G \rangle = \frac{1}{|G|} \sum_{g \in G} \chi(g) = \frac{1}{|G|} \sum_{g \in G} |C_G(g)|,$$

as desired. ∎

Theorem 4.4.6. *[40]. Let N be a normal subgroup of $\bar{G}$. Then*

(i) $r(\bar{G}) \leq r(\bar{G}/N)r(N)$;

(ii) The following conditions are equivalent:

(a) $r(\bar{G}) = r(\bar{G}/N)r(N)$,

(b) $C_{\bar{g}} = C_{\bar{G}}(\bar{g})N$ *for all* $\bar{g} \in \bar{G}$,

(c) each irreducible character of N extends to a character of each subgroup $N<x, y>$ with $[x, y] \in \bar{G}$.

PROOF. See Theorem 28.2.3 of [67]. ■

5

Fischer - Clifford Matrices

5.1. Prologue

The character table of a group provides considerable information about the group, and hence it is of importance in the physical sciences as well as in pure mathematics. Character tables of finite groups can be constructed using various techniques. For example, the Schreier-Sims algorithm, Todd-Coxeter coset enumeration method, the Burnside-Dixion algorithm and various other techniques. However *Bernd Fischer* studied a technique for constructing the character tables of group extensions. This technique, which is known as the technique of *Fischer-Clifford Matrices*, derives its fundamentals from the Clifford theory and provides very powerful information for constructing character tables. If $\bar{G} = N.G$ is an appropriate extension of N by G, the method involves the construction of a nonsingular matrix for each conjugacy class of $\bar{G}/N$. In this dissertation we apply this technique to both split and non-split extensions. This technique has also been discussed and used (mainly to split extensions) in Almestady [4], Darafsheh and Iranmanesh [26, 27], Fischer [34, 35, 36, 37], List [75], List and Mohammed [76], Lux and Pahlings [77], Moori and F. Ali [2], Moori and Mpono [90, 91, 92], Mpono [99], Pahlings [103], Saleh [112], Schiffer [113] and Whitely [120]. For the Fischer-Clifford matrices and their properties, although we shall note the work of Mpono [99], Schiffer [113] and Whitely [120], we follow the work of F. Ali [1] closely as he discussed both split and non-split extensions.

In Section 5.2 we define Fischer-Clifford matrices in general. In Subsection 5.2.1 we shall discuss the properties of the Fischer-Clifford matrices which are helpful in their computation. In Subsection 5.2.2 we study a special case of Fischer-Clifford matrices of a $\bar{G} = N.G$ with the property that every irreducible character of N can be extended to an irreducible character of its inertia group in $\bar{G}$. Sections 5.3 and 5.4 deal with the Fischer-Clifford matrices for the split cosets and non-split extensions respectively.

5.2. Definition and General Theory

Let $\bar{G} = N{\cdot}G$ be an extension of N by G, where N is normal subgroup of $\bar{G}$ and $\bar{G}/N \cong G$. Let $\bar{g} \in \bar{G}$ be a lifting of $g \in G$ under the natural homomorphism $\bar{G} \longrightarrow G$ and $[g]$ be a conjugacy class of elements of G with representative g. Let $X(g) = \{x_1, x_2, \ldots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\bar{G}$ from the coset $N\bar{g}$ whose images under the natural homomorphism $\bar{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. Let $\{\theta_1, \theta_2, \ldots, \theta_t\}$ be a set of representatives of the orbits of $\bar{G}$ on $Irr(N)$ such that for $1 \le i \le t$, we have $\bar{H}_i = I_{\bar{G}}(\theta_i)$ with the corresponding inertia factors H_i and let ψ_i be a projective character of $\bar{H}_i$ with factor set $\bar{\alpha}_i$ such that $(\psi_i)_N = \theta_i$. By Remark 4.3.7 we have

$$Irr(\bar{G}) = \bigcup_{i=1}^{t} \{(\psi_i\bar{\beta})^{\bar{G}} \mid \beta \in IrrProj(H_i), \text{ with factor set } \alpha_i^{-1}\},$$

where α_i is obtained from $\bar{\alpha}_i$ and $\bar{\beta}$ from β as in Remark 4.3.6. Without loss of generality suppose that $\theta_1 = 1_N$ is the identity character of N. Then $\bar{H}_1 = \bar{G}$ and $H_1 = G$. Now choose $y_1, y_2, \ldots, y_r$ to be the representatives of the α_i^{-1}-conjugacy classes of elements of H_i that fuse to $[g]$ in G. Since $y_k \in H_i$ for $1 \le k \le r$, then we define $y_{lk} \in \bar{H}_i$ such that y_{lk} ranges over all representatives of the conjugacy classes of elements of $\bar{H}_i$ which map to y_k under the homomorphism $\bar{H}_i \longrightarrow H_i$ whose kernel is N. Now by using the formula for induced characters given in Theorem 2.5.7, we have

$$\begin{aligned}
(\psi_i\bar{\beta})^{\bar{G}}(x_j) &= \sum_{1 \le k \le r} \sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|} \psi_i\bar{\beta}(y_{\ell_k}) \\
&= \sum_{1 \le k \le r} \sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|} \psi_i(y_{\ell_k})\bar{\beta}(y_{\ell_k}) \\
&= \sum_{1 \le k \le r} (\sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|} \psi_i(y_{\ell_k}))\beta(y_k)
\end{aligned}$$

where $\sum_{\ell}{}'$ is the summation over all ℓ for which $y_{\ell_k} \sim x_j$ in $\overline{G}$. Now we define a matrix $M_i(g)$ by $M_i(g) = (a_{uv})$, where $1 \le u \le r$ and $1 \le v \le c(g)$, and

$$a_{uv} = \sum_{\ell}{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H}_i}(y_{\ell_k})|} \psi_i(y_{\ell_k}) \quad .$$

Then we obtain that

$$(\psi_i\beta)^{\overline{G}}(x_j) = \sum_{1 \le k \le r} a_{uv}\hat{\beta}(y_k) \quad .$$

By doing so for all $1 \le i \le t$ such that H_i contains an element in $[g]$ we obtain the matrix $M(g)$ given by

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix} \quad ,$$

where $M_i(g)$ is the submatrix corresponding to the inertia group $\bar{H}i$ and its inertia factor H_i. If $H_i \cap [g] = \emptyset$, then $M_i(g)$ will not exist and $M(g)$ does not contain $M_i(g)$. The size of the matrix $M(g)$ is $l \times c(g)$ where l is the number of α_i^{-1}-regular conjugacy classes of elements of the inertia factors H_i's for $1 \leq i \leq t$ which fuse into $[g]$ in G and $c(g)$ is the number of conjugacy classes of elements of $\bar{G}$ which correspond to the coset $\bar{g}N$. Then $M(g)$ is the *Fischer-Clifford matrix* of $\bar{G}$ corresponding to the coset $\bar{g}N$. We will see later that $M(g)$ is a $c(g) \times c(g)$ nonsingular matrix. Let

$$R(g) = \{(i, y_k) \mid 1 \leq i \leq t \ , \ H_i \cap [g] \neq \emptyset \ , \ 1 \leq k \leq r\}$$

and we note that y_k runs over representatives of the α_i^{-1}-conjugacy classes of elements of H_i which fuse into $[g]$ in G. Following the notation used in Fischer [33], Mpono [99] and Whitely [120] we denote $M(g)$ by writing $M(g) = Cl(N\bar{g}) = (a_j^{(i,y_k)})$, where

$$a_j^{(i,y_k)} = {\sum_{\ell}}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i(y_{\ell_k}) \quad ,$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$. Then the partial character table of $\bar{G}$ on the classes $\{x_1, x_2, \ldots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}$$

where the Fischer-Clifford matrix $M(g)$ is divided into blocks with each block corresponding to an inertia group $\bar{H}_i$ and $C_i(g)$ is the partial projective character table of H_i with factor set α_i^{-1} consisting of the columns corresponding to the α_i^{-1}-regular classes that fuse into $[g]$ in G. We obtain the characters of $\bar{G}$ by multiplying the relevant columns of the projective characters of H_i with factor set α_i^{-1} by the rows of $M(g)$. We can also observe that the number of irreducible characters of $\bar{G}$ is the sum of numbers of projective characters of the inertia factors H_i's with factor set α_i^{-1}, for all i, $1 \leq i \leq t$.

5.2.1 Properties of Fischer-Clifford Matrices

In this section we shall discuss some properties of the Fischer-Clifford matrices which are useful in their computation. These properties have been discussed in [1, 26, 27, 35, 36, 75, 76, 90, 91, 92, 99, 112, 120].

Let K be a group and $A \leq Aut(K)$. Then by Brauer's theorem (Theorem 2.6.7) A acts on the conjugacy classes of elements of K and on the irreducible characters of K resulting in the same number of orbits.

Lemma 5.2.1. *Suppose we have the following matrix describing the above actions:*

$$\begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_t \end{array} \begin{array}{c} \begin{array}{cccccc} 1=l_1 & l_2 & \cdots & l_j & \cdots & l_t \end{array} \\ \left(\begin{array}{cccccc} 1 & 1 & \cdots & 1 & \cdots & 1 \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2t} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{it} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{tj} & \cdots & a_{tt} \end{array} \right) \end{array}$$

where $a_{1j} = 1$ *for* $j \in \{1, 2, \ldots, t\}$*,* l_j*'s are lengths of orbits of* A *on the conjugacy classes of* K*,* s_i*'s are lengths of orbits of* A *on* $Irr(K)$ *and* a_{ij} *is the sum of* s_i *irreducible characters of* K *on the element* x_j*, where* x_j *is an element of the orbit of length* l_j*. Then the following relation holds for* $i, i' \in \{1, 2, \ldots, t\}$*:*
$\sum_{j=1}^{t} a_{ij}\overline{a_{i'j}}l_j = |K|s_i\delta_{ii'}$.

PROOF. This result has been proved as Lemma 2.3.2 in [112] and as Lemma 4.3.2 in [120]. ■

For arithmetical properties weights are important. We present $M(g)$ with corresponding *weights*. Let $x_j \in X(g)$. For a fixed coset $X = \bar{g}N \in \bar{G}/N$, we define $m_j = [N_{\bar{G}}(X) : C_{\bar{G}}(x_j)]$.

The Fischer-Clifford matrix $M(g)$ is partitioned row-wise into blocks, where each block corresponds to an inertia group. The columns of $M(g)$ are indexed by $X(g)$ and for each $x_j \in X(g)$, at the top of the columns of $M(g)$, we write $|C_{\bar{G}}(x_j)|$ and at the bottom we write m_j. The rows of $M(g)$ are indexed by $R(g)$ and on the left of each row we write $|C_{H_i}(y_k)|$, where y_k fuses into $[g]$ in G. Then in general we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks corresponding to the inertia groups are separated by horizontal lines.

Table 5.1

$$
\begin{array}{c|cccc|}
 & |C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\
|C_G(g)| & a_1^{(1,g)} & a_2^{(1,g)} & \cdots & a_{c(g)}^{(1,g)} \\
\hline
|C_{H_2}(y_1)| & a_1^{(2,y_1)} & a_2^{(2,y_1)} & \cdots & a_{c(g)}^{(2,y_1)} \\
|C_{H_2}(y_2)| & a_1^{(2,y_2)} & a_2^{(2,y_2)} & \cdots & a_{c(g)}^{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_i}(y_1)| & a_1^{(i,y_1)} & a_2^{(i,y_1)} & \cdots & a_{c(g)}^{(i,y_1)} \\
|C_{H_i}(y_2)| & a_1^{(i,y_2)} & a_2^{(i,y_2)} & \cdots & a_{c(g)}^{(i,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_t}(y_1)| & a_1^{(t,y_1)} & a_2^{(t,y_1)} & \cdots & a_{c(g)}^{(t,y_1)} \\
|C_{H_t}(y_2)| & a_1^{(t,y_2)} & a_2^{(t,y_2)} & \cdots & a_{c(g)}^{(t,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & m_1 & m_2 & \cdots & m_{c(g)}
\end{array}
$$

Remark 5.2.2. Fischer [36] has shown that the Fischer-Clifford matrix $M(g)$ satisfies complex conjugation.

The following result gives the orthogonality relation for $M(g)$. Its proof was obtained from Whitley [120], Proposition 4.3.3.

Proposition 5.2.3. *[1, 99, 120](*Column orthogonality*) Let $\overline{G} = N{\cdot}G$, then*

$$\sum_{(i,y_k)\in R(g)} |C_{H_i}(y_k)| a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} = \delta_{jj'} |C_{\overline{G}}(x_j)| \quad .$$

`PROOF`. The partial character table of $\overline{G}$ at classes $x_1, \ldots, x_{c(g)}$ is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}.$$

By column orthogonality of the character table of $\overline{G}$, we have

$$\begin{aligned}
|C_{\overline{G}}(x_j)|\delta_{jj'} &= \sum_{i=1}^{t} \sum_{\beta_i \in IrrProj(H_i)} \big(\sum_{y_k:(i,y_k)\in R(g)} a_j^{(i,y_k)} \beta_i(y_k)\big)\big(\overline{\sum_{y'_k:(i,y'_k)\in R(g)} a_{j'}^{(i,y'_k)} \beta_i(y'_k)}\big) \\
&= \sum_{i=1}^{t} \sum_{\beta_i \in IrrProj(H_i)} \big(\sum_{y_k} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y'_k)}} \beta_i(y_k)\overline{\beta_i(y_k)} \ + \\
&\quad \sum_{y_k} \sum_{y'_k \neq y_k} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y'_k)}} \beta_i(y_k)\overline{\beta_i(y'_k)}\big) \\
&= \sum_{i=1}^{t} \big(\sum_{y_k} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} \sum_{\beta_i \in IrrProj(H_i)} \beta_i(y_k)\overline{\beta_i(y_k)} \ + \\
&\quad \sum_{y_k} \sum_{y'_k \neq y_k} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y'_k)}} \sum_{\beta_i \in IrrProj(H_i)} \beta_i(y_k)\overline{\beta_i(y'_k)}\big) \\
&= \sum_{i=1}^{t} \big(\sum_{y_k} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} |C_{H_i}(y_k)| \ + \ 0\big) \\
&= \sum_{(i,y_k)\in R(g)} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} |C_{H_i}(y_k)|.
\end{aligned}$$

■

Theorem 5.2.4. $a_j^{(1,g)} = 1$ *for all* $j \in \{1, 2, \ldots, c(g)\}$.

PROOF. For $y_{\ell_k} \sim x_j$ in $\overline{G}$, we have $|C_{\overline{G}}(x_j)| = |C_{\overline{H}_1}(y_{\ell_k})|$. Thus we obtain that

$$a_j^{(1,g)} = {\sum_{\ell}}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H}_1}(y_{\ell_k})|} \psi_1(y_{\ell_k}) = {\sum_{\ell}}' 1 = 1 \quad .$$

Hence the result. ■

Proposition 5.2.5. *[75, 120] The matrix $M(1_G)$ is the matrix with rows equal to the orbit sums of the action of $\overline{G}$ on $Irr(N)$ with duplicate columns discarded. For this matrix we have $a_j^{(i,1_G)} = [G : H_i]$, and an orthogonality relation for rows:*

$$\sum_{j=1}^{t} \frac{1}{|C_{\overline{G}}(x_j)|} a_j^{(i,1_G)} a_j^{(i',1_G)} = \frac{1}{|C_{H_i}(1_G)|} \delta_{ii'} = \frac{1}{|H_i|} \delta_{ii'} \quad .$$

PROOF. See [99]. ■

As a consequence of Lemma 5.3.1, Proposition 5.3.3 and from Fischer [36], we have the following properties:

(a) $|X(g)| = |R(g)|$,

(b) $\sum_{j=1}^{c(g)} m_j a_j^{(i,y_k)} \overline{a_j^{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \frac{|C_G(g)|}{|C_{H_i}(y_k)|}|N|$,

(c) $\sum_{(i,y_k)\in R(g)} a_j^{(i,y_k)} \overline{a_{j'}^{(i,y_k)}} |C_{H_i}(y_k)| = \delta_{jj'}|C_{\overline{G}}(x_j)|$,

(d) $M(g)$ is square and nonsingular.

5.2.2 Fischer-Clifford Matrices (Special Case)

Let $\bar{G} = N.G$ be an extension of N by G such that every irreducible character θ of N can be extended to its inertia group $\bar{H} = I_{\bar{G}}(\theta)$. Now we define the Fischer-Clifford matrices in the same way as the general case. Let $\bar{g} \in \bar{G}$ be a lifting of $g \in G$ under the natural homomorphism $\bar{G} \longrightarrow G$ and $[g]$ be a conjugacy class of elements of G with representative g. Let $X(g) = \{x_1, x_2, \ldots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\bar{G}$ from the coset $N\bar{g}$ whose images under the natural homomorphism $\bar{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. Let $\{\theta_1, \theta_2, \ldots, \theta_t\}$ be a set of representatives of the orbits of $\bar{G}$ on $Irr(N)$ such that for $1 \le i \le t$, we have $H_i = I_{\bar{G}}(\theta_i)$ with $H_i = \bar{H}_i/N \le G$ and that $\psi_i \in Irr(\bar{H}_i)$ is an extension of θ_i to $\bar{H}_i$. Then without loss of generality suppose that $\theta_1 = I_N$ is the identity character of N. Then $\bar{H}_1 = \bar{G}$ and $H_1 = G$. Now choose $y_1, y_2, \ldots, y_r$ to be the representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Since $y_k \in H_i$ for $1 \le k \le r$, then we define $y_{\ell_k} \in \bar{H}_i$ such that y_{ℓ_k} ranges over all the representatives of the conjugacy classes of elements of $\bar{H}_i$ which map to y_k under the homomorphism $\bar{H}_i \longrightarrow H_i$ whose kernel is N. Let $\beta \in Irr(\bar{H}_i)$ such that $N \subseteq ker(\beta)$. Then β is a lifting of $\hat{\beta} \in Irr(H_i)$ such that $\beta(y_{\ell_k}) = \hat{\beta}(y_k)$ for any lifting $y_{\ell_k} \in \bar{H}_i$ of $y_k \in H_i$. Now by using Theorem 2.4.7, as in the general case, we obtain that

$$(\psi_i\beta)^{\bar{G}}(x_j) = \sum_{1\le k\le r} (\sum_{\ell}{}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{\ell_k})|}\psi_i(y_{\ell_k}))\hat{\beta}(y_k)$$

where $\sum_\ell{}'$ is the summation over all ℓ for which $y_{\ell_k} \sim x_j$ in $\overline{G}$. We define a matrix $M_i(g)$ by $M_i(g) = (a_{uv})$, where $1 \le u \le r$ and $1 \le v \le c(g)$, and

$$a_{uv} = \sum_{\ell}{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{H_i}(y_{\ell_k})|}\psi_i(y_{\ell_k}).$$

Then we obtain that

$$(\psi_i\beta)^{\overline{G}}(x_j) = \sum_{1\le k\le r} a_{uv}\hat{\beta}(y_k).$$

By doing so for all $1 \le i \le t$ such that H_i contains an element in $[g]$ we obtain the matrix $M(g)$ given by

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix} ,$$

where $M_i(g)$ is the submatrix corresponding to the inertia group $\bar{H}_i$ and its inertia factor H_i. Then as in the previous section, $M(g)$ is the *Fischer-Clifford matrix* of $\overline{G}$ corresponding to the coset $\bar{g}N$. Let

$$R(g) = \{(i, y_k) \mid 1 \le i \le t\ ,\ H_i \cap [g] \ne \emptyset\ ,\ 1 \le k \le r\}$$

and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Again we denote $M(g)$ by writing $M(g) = (a_j^{(i,y_k)})$, where

$$a_j^{(i,y_k)} = {\sum_{\ell}}' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}i}(y_{\ell_k})|}\psi_i(y_{\ell_k}) \quad ,$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$. Then we obtain the irreducible characters of $\bar{G}$ by multiplying the relevant columns of the irreducibles characters of H_i by the rows $M(g)$.

Remark 5.2.6. *All our results of Section 5.2.1 are applicable with irreducible projective characters are replaced by ordinary irreducible characters.*

5.3. Split Cosets

From now on suppose that N is an elementary abelian normal p-subgroup of $\bar{G}$ and $\bar{g}N = X$ is a fixed coset of $\bar{G}/N \cong G$. Let $M = C_{\bar{g}} = N_{\bar{G}}(X)$. We define

$$N_{\bar{g}} := < [\bar{g}, n],\ n \in N > .$$

With these notations we have the following lemma.

Lemma 5.3.1. *(i)* $N_x = N_{\bar{g}}$ *for all* $x \in X$ *and*
$[\bar{g}, u_1].[\bar{g}, u_2] = [\bar{g}, u_1u_2]$ *for all* $u_1, u_2 \in N$.

(ii) $N_{\bar{g}} \lhd M$ *and* $N_{\bar{g}} \le N$.

(iii) If $\varphi \in Irr(N)$, *then* $N_{\bar{g}} \le ker(\varphi)$ *or* $I_{\bar{G}}(\varphi) \cap \bar{g}N = \emptyset$.

PROOF.

(i) Let $x = \bar{g}n \in \bar{g}N$ and $u \in N$, then

$$\begin{aligned} [x,u] &= [\bar{g}n, u] = n^{-1}(u^{-1})^{\bar{g}}nu \\ &= (u^{-1})^{\bar{g}}u \qquad \text{since } N \text{ is abelian} \\ &= \bar{g}^{-1}u^{-1}\bar{g}u = [\bar{g}, u] \end{aligned}$$

which implies that

$$N_x = N_{\bar{g}} \quad \text{for all} \quad x \in \bar{g}N.$$

Also since N is abelian, we obtain for all $u_1, u_2 \in N$

$$\begin{aligned}[\bar{g}, u_1].[\bar{g}, u_2] &= (u_1^{-1})^{\bar{g}} u_1 (u_2^{-1})^{\bar{g}} u_2 \\ &= (u_1^{-1} u_2^{-1})^{\bar{g}} u_1 u_2 \\ &= [\bar{g}, u_1 u_2] \ .\end{aligned}$$

Hence

$$[\bar{g}, u_1].[\bar{g}, u_2] = [\bar{g}, u_1 u_2] \quad \text{for} \quad u_1, u_2 \in N.$$

(ii) Since $[\bar{g}, u] = (u^{-1})^{\bar{g}} u \in N$, we obtain $N_{\bar{g}} \leq N \leq M$. Conversely, let $m \in M$ then

$$\begin{aligned}[\bar{g}, u]^m &= m^{-1}[\bar{g}, u]m \\ &= (\bar{g}^{-1})^m (u^{-1})^m \bar{g}^m u^m \\ &= (\bar{g}^m)^{-1} (u^m)^{-1} \bar{g}^m u^m \\ &= [\bar{g}^m, u^m] \in N_{\bar{g}} \ .\end{aligned}$$

Hence $N_{\bar{g}} \lhd M$.

(iii) Let $\varphi \in Irr(N)$ be fixed. Then

$$\begin{aligned}N_{\bar{g}} \leq Ker(\varphi) &\Leftrightarrow \varphi([\bar{g}, u]) = \varphi(1) = 1 \quad \text{for all} \quad u \in N \\ &\Leftrightarrow \varphi(\bar{g}^{-1} u^{-1} \bar{g} u) = \varphi((u^{-1})^{\bar{g}} u) = 1 \\ &\Leftrightarrow \varphi((u^{-1})^{\bar{g}}) = (\varphi(u))^{-1} = \varphi(u^{-1}) \\ &\Leftrightarrow \varphi^{\bar{g}}(u^{-1}) = \varphi(u^{-1}) \\ &\Leftrightarrow \varphi^{\bar{g}} = \varphi \\ &\Leftrightarrow \bar{g}N \subseteq I_{\bar{G}}(\varphi) \\ &\Leftrightarrow \bar{g}N \cup I_{\bar{G}}(\varphi) \neq \emptyset \ .\end{aligned}$$

■

Remark 5.3.2. *We can easily show that* $<X>/N_{\bar{g}}$ *is abelian and* $X/N_{\bar{g}}$ *is a coset of* $<X>/N_{\bar{g}}$.

Lemma 5.3.3. *[36] The rows of the Fischer-Clifford matrix* $Cl(X)$ *can be identified with restrictions of* M*-invariant characters of* $<X>/N_{\bar{g}}$ *to* $X/N_{\bar{g}}$.

PROOF. This is Lemma 5.3 in [36]. ■

Remark 5.3.4. *In the above lemma, the rows of* $Cl(X)$ *will be an independent set of orbit sums, under the action of* M *on* $<X>/N_{\bar{g}}$. *This observation was first given in Fischer [34].*

Definition 5.3.5. *A coset* X *is said to be a* **split coset** *if it contains an element* x *such that* $M = N.C_{\bar{G}}(x)$.

Note that we do not require $<x> \cap N = <1>$ in the above definition.

Lemma 5.3.6. *[113] If the extension split, then every coset is a split coset.*

PROOF. Let $X = \bar{g}N$ and $h \in C_{\bar{G}}(\bar{g})$ then $h(\bar{g}n)h^{-1} = (h\bar{g}h^{-1})(hnh^{-1}) = \bar{g}hnh^{-1} = \bar{g}n^h \in \bar{g}N$. Now since $N \leq M$ and $C_{\bar{G}}(\bar{g}) \leq M$ then $M \geq N.C_{\bar{G}}(\bar{g})$. Let C be the complement of N in $\bar{G}$ such that $\bar{g} \in C$. Let $m \in M$ then $m = n.k$, for some $k \in C$. Since $M = N_{\bar{G}}(\bar{g}N)$, $(\bar{g}N)^m = \bar{g}N$. Hence

$$\begin{aligned} \bar{g}N &= (\bar{g}N)^m = m(\bar{g}N)m^{-1} \\ &= n(k\bar{g}Nk^{-1})n^{-1} = n(k\bar{g}Nk^{-1})n^{-1} = n(\bar{g}N)^k n^{-1}. \end{aligned}$$

So that $n^{-1}(\bar{g}N)n = (\bar{g}N)^k$ and $n^{-1}\bar{g}N = (\bar{g}N)^k$. Hence $\bar{g}N = (\bar{g}N)^k$. It follows that $\bar{g}N = (\bar{g}N)^k = \bar{g}^k N$, which implies that $\bar{g}^k \in \bar{g}N$. Hence $\bar{g}^k \in C \cap \bar{g}N = \{\bar{g}\}$ and so $k \in C_{\bar{G}}(\bar{g})$, which implies that $m = n.k \in N.C_{\bar{G}}(\bar{g})$ and so $M \leq N.C_{\bar{G}}(\bar{g})$. Thus $M = N.C_{\bar{G}}(\bar{g})$. ■

The following result is of fundamental importance and very helpful to fill the entries of Fischer-Clifford matrices.

Lemma 5.3.7. *[36] Let X be a split coset then the rows of $Cl(X)$ can be identified with M-invariant characters of $N/N_{\bar{g}}$ multiplied by a p-th root of unity.*

PROOF. See [36] and [113]. ■

Lemma 5.3.8. *Let $X = \bar{g}N$ be a split coset and $N_{\bar{G}}(X) = NC_{\bar{G}}(x)$ for $x \in X(g)$. Then we have the following:*

(i) $a_1^{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$,

(ii) $|a_j^{(i,y_k)}| \leq |a_1^{(i,y_k)}|$ *for all* $1 \leq j \leq r$,

(iii) If $|N| = p^w$, *then* $a_j^{(i,y_k)} \equiv a_1^{(i,y_k)} (mod\ p)$.

PROOF. See [113].

5.4. Non-Split Extensions

Let $\bar{G} = N^{\cdot}G$ be a non-split extension, where N is an elementary abelian normal p-subgroup of $\bar{G}$. Let $\bar{g}N$ be a conjugacy class representative of $\bar{G}/N$ and φ be a representative of $\bar{G}$-orbit irreducible characters of N with the projective extension $\bar{\varphi}$ to $\bar{G}$. We consider the groups $<\bar{g}>N \leq \bar{G}$ and $<\bar{g}N> \leq \bar{G}/N$.

Lemma 5.4.1.

$$<\bar{g}>N/N = <\bar{g}N> .$$

PROOF. Let $x \in <\bar{g}>N/N$, then $x = \bar{g}^m nN = \bar{g}^m N$ for some $m \in \mathbb{Z}$. So that $x = (\bar{g}N)^m \in <\bar{g}N>$. Hence $<\bar{g}>N/N \leq <\bar{g}N>$. Conversely, let $x \in <\bar{g}N>$. Then $x = (\bar{g}N)^m = \bar{g}^m N$ for some $m \in \mathbb{Z}$. Hence $x = (\bar{g}^m N) \in <\bar{g}>N/N$. Thus $<\bar{g}N> \leq <\bar{g}>N/N$. Therefore $<\bar{g}>N/N = <\bar{g}N>$. ■

Lemma 5.4.2. *With the above notations, we have the following:*

(a) $<\bar{g}>N \leq M$.

(b) $(<\bar{g}>N)' = N_{\bar{g}}$ *where* $(<\bar{g}>N)'$ *denotes the commutator subgroup of* $<\bar{g}>N$.

(c) $<\bar{g}>N \leq I_M(\varphi)$ *where* $\varphi \in Irr(N)$.

(d) Given $\varphi \in Irr(N)$ *there exists an extension* $\eta\beta$ *to* $<\bar{g}>N$ *where* $\eta = (\bar{\varphi})_{<\bar{g}>N}$ *and* β *is a projective character of* $<\bar{g}N>$.

PROOF.

(a) Let $x \in <\bar{g}>N$ then $x = \bar{g}^m N$ for some $m \in \mathbb{Z}$. Now

$$\begin{aligned} x(\bar{g}N) &= \bar{g}^m n(\bar{g}N) = \bar{g}^m n\bar{g}N = \bar{g}^m nN\bar{g} \quad (\text{since} \quad N \trianglelefteq \bar{G}) \\ &= \bar{g}^m N\bar{g} = N\bar{g}^{m+1}. \end{aligned}$$

Similarly, $(\bar{g}N)x = N\bar{g}^{m+1}$. Hence $x \in M = N_{\bar{G}}(\bar{g}N)$ and so $<\bar{g}>N \leq M$.

(b) First suppose that $[\bar{g}, n] \in N_{\bar{g}}$ then $[\bar{g}, n] \in (<\bar{g}>N)'$ and thus $N_{\bar{g}} \leq (<\bar{g}>N)'$. Also, for $n \in N$, by the definition of $N_{\bar{g}}$, we have

$$(\bar{g}N_{\bar{g}})(nN_{\bar{g}}) = (nN_{\bar{g}})(\bar{g}N_{\bar{g}}).$$

Therefore $(<\bar{g}>N/N_{\bar{g}})$ is abelian, and hence $(<\bar{g}>N)' \leq N_{\bar{g}}$ and we deduce that $(<\bar{g}>N)' = N_{\bar{g}}$.

(c) Let $\varphi \in Irr(N)$ then $N_{\bar{g}} \leq Ker(\varphi)$. Now by Lemma 5.3.1, we have $\bar{g}N \cap I_M(\varphi) \neq \emptyset$. Therefore $\bar{g}$ lies in $I_M(\varphi)$ and so $<\bar{g}> \leq I_M(\varphi)$. Hence $<\bar{g}>N \leq I_M(\varphi)$.

(d) Notice that by part (c), $W = <\bar{g}>N$ is a subgroup of $I_M(\varphi)$. Hence φ is invariant under W. So we can apply the Theorem 4.3.3 to φ and W (see Theorem 5.8 in [100]). Let $\chi \in Irr(<\bar{g}>N, \varphi)$ then by the Clifford theorem (Theorem 4.3.3) we obtain $\chi = ((\bar{\varphi})_{<\bar{g}>N})\beta = \eta\beta$ where β is an $\bar{\alpha}^{-1}$-projective character of $<\bar{g}>N/N = <\bar{g}N>$ and $\bar{\alpha}$ is the factor set of $<\bar{g}>N \times <\bar{g}>N$ obtained from α. If N is abelian, then χ is linear since $\chi_N = \varphi$ is linear (because $deg(\chi) = deg(\varphi) = 1$).

■

Theorem 5.4.3. *[35] Let $\bar{g} \in \bar{G}$ so that $<\bar{g}>N$ is abelian. Then $<\bar{g}N> \leq Z(\bar{G}/N)$ and the rows of Fischer-Clifford matrix of $\bar{g}N$ for regular classes of the inertia group of φ in $\bar{G}$ can be regarded as restrictions to $\bar{g}N$ of the $\bar{G}$-orbit sums of the (projective) extension $\eta\beta$ to $<\bar{g}>N$ of φ.*

PROOF. See [35] and [113]. ■

5.5. Character Table and GAP

Using Fischer-Clifford matrices and partial character tables, we are able to compute the full character table of $\overline{G}$. Since the character tables have been computed manually, in order to detect errors we tested their validity using GAP. For doing so we developed and used Programme E to rewrite the character tables in GAP format.

Programme E

gap>ct:=fuction()local ct;ct:=rec();
>ct.SizesCentralizers:=[n Centralizer Orders];
>ct.OrdersClassRepresentatives:=[n Class Representatives Orders];
>ct.Irr:=[[$n \times n$ irreducibles]];
>ct.UnderlyingCharacteristic:=0;ct.Id:=$\overline{G}$;
>ConvertToLibraryCharacterTable NC(ct);return ct;end;ct:=ct();
gap>SetInfoLevel(InfoCharacterTable,2);
gap>IsInternallyConsistent(ct);
gap>PossiblePowerMaps(ct,p); (p-prime divisor of $\overline{G}$).

I would like to acknowledge F. Ali who helped me to develop Programme E. The Programme E also computes the power maps.

6

A group $2^7{:}S_8$ in $\overline{Fi}_{22}$

Prologue

The group $S_8 \cong O_6^+(2)$ is a maximal subgroup of $SP_6(2)$ of order 40320 and index 36. It can be generated by two elements of orders 2 and 7 respectively inside of $SP_6(2)$. The group $2^7{:}S_8$ is a maximal subgroup of $2^7{:}SP_6(2)$ of index 36. However $2^7{:}SP_6(2)$ is itself a maximal subgroup of the full automorphism group $\overline{Fi}_{22}$ of the smallest Fischer sporadic simple group Fi_{22}, of index 694980. The object of this chapter is to compute the Fischer-Clifford matrices of $2^7{:}S_8$ which can then be used together with the ordinary character tables of the inertia factors of S_8 to compute its full ordinary character table. One can look at [1, 18, 84, 85, 90, 91, 99] for further reading. The notation used is taken from the $\mathbb{ATLAS}$ of finite groups [23] which we denoted $\mathbb{ATLAS}$ and $\mathbb{ATLAS}$ of finite group representation [124] denoted $\mathbb{ATLAS}$ $\mathbb{V}3$.

6.1. Introduction

The group $\overline{Fi}_{22} = Fi_{22}.2$ is the full automorphism group of the smallest Fischer sporadic simple group Fi_{22}. It has a maximal subgroup $2^7{:}SP_6(2)$ of order 185794560 and index 694980 which has been discussed in [91]. More details about this maximal subgroup can be obtained from [99]. This maximal subgroup contains a group of the form $2^7{:}S_8$ as a subgroup of order 5160960 and index 36. The group S_8 is a maximal subgroup of $SP_6(2)$ of order 40320 and index 36. There are two orthogonal groups sitting maximally in $SP_6(2)$ viz. $O_6^+(2) \cong S_8$ and $O_6^-(2) \cong U_4(2){:}2$ of orders 40320 and 51840 and indices 36 and 28 respectively.

We used the computer algebra system of GAP [41] running on a SUN GX2 computer at the University of KwaZulu-Natal in Pietermaritzburg. The naming of the conjugacy classes of elements will be consistent with that used in the $\mathbb{ATLAS}$ [23].

We generate $SP_6(2)$ as a matrix group by two 7×7 matrices α and β over $GF(2)$, where α and β

are given by

$$\alpha = \begin{bmatrix} 0&1&0&1&1&1&0\\ 0&1&1&1&0&1&0\\ 0&1&1&1&0&0&0\\ 1&0&1&0&0&0&0\\ 1&1&1&0&1&0&0\\ 0&1&1&0&1&0&0\\ 0&0&0&0&0&0&1 \end{bmatrix}, \quad \beta = \begin{bmatrix} 0&0&1&1&0&0&0\\ 1&1&1&1&0&0&0\\ 1&1&0&0&1&1&0\\ 0&1&0&0&1&1&0\\ 0&1&1&1&1&0&0\\ 1&0&0&1&1&0&0\\ 1&0&0&1&1&1&1 \end{bmatrix},$$

with $o(\alpha) = 5$ and $o(\beta) = 2$.

6.2. The action of S_8 on 2^7

We generate S_8 as a matrix group inside of $SP_6(2)$ by two 7×7 matrices α_1 and β_1 of orders 2 and 7 respectively as follows:

$$\alpha_1 = \begin{bmatrix} 0&1&0&1&1&1&0\\ 1&0&0&1&1&1&0\\ 1&1&1&1&1&1&0\\ 0&0&0&1&0&0&0\\ 1&1&0&1&0&1&0\\ 1&1&0&1&1&0&0\\ 0&0&0&0&0&0&1 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 0&0&1&1&1&0&0\\ 0&1&1&0&0&0&0\\ 0&1&1&0&0&1&0\\ 1&0&1&1&1&1&0\\ 1&1&1&0&1&1&0\\ 0&1&0&0&0&1&0\\ 0&0&1&1&0&1&1 \end{bmatrix}$$

When S_8 acts on 2^7, we obtain six orbits of lengths 1, 1, 28, 28, 35, 35 with corresponding point stabilizers $S_8, S_8, S_6 \times 2, S_6 \times 2, (S_4 \times S_4){:}2, (S_4 \times S_4){:}2$ of orders 40320, 40320, 1440, 1440, 1152, 1152 respectively. These point stabilizers can be generated inside of S_8 as groups of 7×7 matrices over $GF(2)$ as follows:

$$S_6 \times 2 = \langle \alpha_2, \beta_2 \rangle \ , \ (S_4 \times S_4){:}2 = \langle \alpha_3, \beta_3 \rangle$$

where

$$\alpha_2 = \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0\\ 0&1&0&0&1&0&1 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 1&1&0&1&1&0&0\\ 1&1&1&0&1&1&0\\ 0&0&1&1&1&0&0\\ 1&0&1&0&0&1&0\\ 1&0&1&1&1&0&0\\ 1&0&1&1&0&0&0\\ 0&1&0&0&0&0&1 \end{bmatrix}$$

$$\alpha_3 = \begin{bmatrix} 1&1&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&1&1&0&0&0&0\\ 0&1&0&1&0&0&0\\ 0&0&1&1&1&1&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix}, \quad \beta_3 = \begin{bmatrix} 1&1&1&0&1&1&0\\ 1&1&0&0&0&0&0\\ 1&1&0&0&1&1&0\\ 0&1&0&1&0&0&0\\ 1&0&1&0&1&0&0\\ 0&0&1&1&1&1&0\\ 1&0&1&0&1&1&1 \end{bmatrix}$$

We have that $o(\alpha_2) = 2, o(\beta_2) = 6, o(\alpha_3) = 2, o(\beta_3) = 12$. The six orbits resulting from the action of S_8 on 2^7 have the following representatives $(0,0,0,0,0,0,0), (1,1,0,1,0,0,1), (1,0,1,0,1,0,1), (0,0,0,1,0,0,0)$, $(1,1,1,1,1,1,1), (1,0,0,0,0,0,0)$ respectively.

6.3. The conjugacy classes of $2^7{:}S_8$

The action of S_8 on 2^7 produces six orbits of lengths 1, 1, 28, 28, 35, 35 with corresponding point stabilizers $S_8, S_8, S_6 \times 2, S_6 \times 2, (S_4 \times S_4){:}2, (S_4 \times S_4){:}2$ of orders 40320, 40320, 1440, 1440, 1152, 1152 respectively. Let $\chi(S_8 \mid 2^7)$ be the permutation character of S_8 acting on 2^7. Then, from methods that were developed in [99] and also used in [1], we obtain that

$$\begin{aligned}\chi(S_8 \mid 2^7) &= 1 + 1 + 2I_{S_6\times 2}^{S_8} + 2I_{(S_4\times S_4):2}^{S_8}\\ &= 1a + 1a + 2(1a + 7a + 20a) + 2(1a + 14a + 20a)\\ &= 6\times 1a + 2\times 7a + 2\times 14a + 4\times 20a\end{aligned}$$

where $I_{S_6\times 2}^{S_8}, I_{(S_4\times S_4):2}^{S_8}$ are the identity characters of $S_6\times 2$ and $(S_4\times S_4){:}2$ induced to S_8 respectively. Thus $\chi(S_8 \mid 2^7)$ will give the number k of points of 2^7 fixed by each $g \in S_8$ such that $k = 2^m$, where $m \in \mathbb{N}$ satisfies $1 \le m \le 7$. These are given in Table 6.1 below.

Table 6.1:

$[g]_{S_8}$	1a	2a	2b	2c	2d	3a	3b	4a	4b	4c	4d
k	128	64	32	32	16	32	8	16	16	8	8

$[g]_{S_8}$	5a	6a	6b	6c	6d	6e	7a	8a	10a	12a	15a
k	8	16	4	4	8	4	2	4	4	4	2

We used GAP for programmes A and B (see Appendix A), which can also be found in [1] and [99], written in MAGMA and CAYLEY. We also used coset analysis, which is also discussed in chapter 2, to compute the conjugacy classes of elements of $2^7{:}S_8$. These conjugacy classes are given in Table 6.2 and the descriptions of the parameters used can also be found in [1, 99, 120]. We give programmes A and B and the table for conjugacy classes:

PROGRAMME A for $2^7{:}S_8$

```
gap>V:=FullRowSpace(GF(2),7);
gap>gr1:=(OneGF(2))*[7 × 7 matrix group generators];
gap>gr2:=(OneGF(2))*[7 × 7 matrix group generators];
gap>grp:=Group(gr1,gr2);
gap>Ccl:=ConjugacyClasses(grp);
gap>O:=Union(Orbits(grp,V));
gap>for i in [1..22] do
>Print(Representative(Ccl[i]));
>w:=One(GF(q))*[0,0,··· ,0];
>e:=[ ];
>while Difference(O,e) <> [ ] do
>d:=[ ];
>for x in O do;
>y:=[x+w+(x*(Representative((Ccl)[i]))];
```

```
>d:=Union(d,y);
>od;
>Print(d);
>e:=Union(d,e);
>if Difference(O,e) <> [ ] then
>w:=Representative(Difference(O,e));
>fi;
>od;
>r:=[ ];
>u:=One(GF(2))*[0, 0, ··· , 0];
>while Difference(O,e) <> [ ] do
>m:=[ ];
>for g in Centralizer(grp,Representative(Ccl[i])) do
>l:=[u*g];
>m:=Union(m,l);
>od;
>Print("A block for the vectors under the action of a centralizer");
>Print(m);
>r:=Union(m,r);
>if Difference(O,r) <> [ ] then
>u:=Representative(Difference(O,r));
>fi;
>od;
>Print("**********************************");
>od;
```

PROGRAMME B for $2^7{:}S_8$

```
gap>V:=FullRowSpace(GF(2), 7);
gap>m1:=(OneGF(2))*[7 × 7 matrix group generators];
gap>m2:=(OneGF(q))*[7 × 7 matrix group generators];
gap>m:=Group(m1,m2);
gap>c:=ConjugacyClasses(m);
gap>g:=Representative(c[i]);
gap>d:=One(GF(2))*[α1, α2, ··· , α7];
gap>w:=d + d * g + d * g^2 + ··· + d * g^(k-1);
gap>Print(w);
```

Table 6.2: Conjugacy Classes of $2^7{:}S_8$

$g \in S_8$	k	f_j	d_j	w	$[x]_{2^7:S_8}$	$\lvert C_{2^7:S_8}(x)\rvert$
$1A$	2^7	1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$1A$	5 160 960
		1	$(1,1,0,1,0,0,1)$	$(1,1,0,1,0,0,1)$	$2A$	5 160 960
		28	$(1,0,1,0,1,0,1)$	$(1,0,1,0,1,0,1)$	$2B$	184 320
		28	$(0,0,0,1,0,0,0)$	$(0,0,0,1,0,0,0)$	$2C$	184 320
		35	$(1,1,1,1,1,1,1)$	$(1,1,1,1,1,1,1)$	$2D$	147 456
		35	$(1,0,0,0,0,0,0)$	$(1,0,0,0,0,0,0)$	$2E$	147 456
$2A$	2^6	1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2F$	92 160
		1	$(1,0,0,1,1,0,1)$	$(1,1,0,0,1,1,0)$	$4A$	92 160
		6	$(1,0,0,0,0,1,1)$	$(0,1,0,0,0,0,0)$	$4B$	15 360
		6	$(1,0,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$2G$	15 360
		10	$(1,1,1,1,1,1,1)$	$(0,0,1,0,0,0,0)$	$4C$	9 216
		10	$(0,0,1,1,1,0,0)$	$(0,0,0,0,0,0,0)$	$2H$	9 216
		15	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$2I$	6 144

continued on next page

Table 6.2 (continued from previous page)

$g \in S_8$	k	f_j	d_j	w	$[x]_{2^7:S_8}(x)\|$	
		15	$(0,0,0,1,0,0,0)$	$(0,1,0,1,1,0,0)$	$4D$	6 144
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2J$	12 288
		1	$(0,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$2K$	12 288
		1	$(1,1,0,1,0,0,1)$	$(0,0,0,0,0,0,0)$	$2L$	12 288
$2B$	2^5	1	$(1,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	2M	12 288
		2	$(0,0,1,1,0,0,1)$	$(0,0,0,0,0,0,0)$	$2N$	6 144
		2	$(1,1,1,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2O$	6 144
		12	$(1,0,1,0,1,0,1)$	$(0,1,1,0,1,1,0)$	$4E$	1 024
		12	$(0,0,0,1,0,0,0)$	$(0,1,1,0,1,1,0)$	$4F$	1 024
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2P$	6 144
		1	$(1,1,0,0,1,0,1)$	$(1,1,1,1,1,1,0)$	$4G$	6 144
		1	$(1,0,0,0,0,1,1)$	$(1,0,0,0,0,1,0)$	$4H$	6 144
		1	$(0,1,0,0,1,1,0)$	$(0,0,0,0,0,0,0)$	$2Q$	6 144
$2C$	2^5	3	$(1,0,1,0,1,0,1)$	$(1,1,1,1,1,1,0)$	$4I$	2 048
		3	$(1,0,0,0,1,1,0)$	$(0,0,0,0,0,0,0)$	$2R$	2 048
		3	$(0,1,0,1,0,0,1)$	$(1,1,0,1,1,0,0)$	$4J$	2 048
		3	$(1,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2S$	2 048
		8	$(0,1,1,1,0,1,1)$	$(1,1,0,1,1,0,0)$	$4K$	768
		8	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,1,0)$	$4L$	768
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$2T$	1 536
		1	$(1,0,0,0,0,0,1)$	$(0,1,0,0,0,0,0)$	$4M$	1 536
		1	$(0,0,0,1,1,1,1)$	$((0,0,1,1,0,0,0)$	$4N$	1 536
$2D$	2^4	1	$(0,1,0,1,1,1,0)$	$(0,1,1,1,1,1,0)$	$4O$	1 536
		3	$(1,0,1,0,1,0,1)$	$(0,1,1,0,0,0,0)$	$4P$	512
		3	$(0,1,0,0,1,0,1)$	$(0,1,1,1,0,0,0)$	$4Q$	512
		3	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$2U$	512
		3	$(0,1,0,0,1,0,0)$	$(1,1,0,0,1,1,0)$	$4R$	512
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3A$	11 520
		1	$(1,0,1,0,1,0,1)$	$(1,1,0,0,1,0,1)$	$6A$	11 520
$3A$	2^5	5	$(0,1,1,1,0,1,1)$	$(1,1,1,1,0,0,0)$	$6B$	2 304
		5	$(0,0,0,1,0,0,0)$	$(0,1,1,1,0,1,0)$	$6C$	2 304
		10	$(1,0,0,0,0,1,1)$	$(0,1,1,0,0,0,1)$	$6D$	1 152
		10	$(0,1,0,0,1,1,0)$	$(0,1,1,0,1,0,0)$	$6E$	1 152
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$3B$	288
		1	$(1,0,1,0,1,0,1)$	$(1,0,1,0,1,0,1)$	$6F$	288
$3B$	2^3	1	$(0,1,1,1,0,1,1)$	$(0,1,1,1,0,1,1)$	$6G$	288
		1	$(0,0,0,1,0,0,0)$	$(1,0,0,1,0,1,0)$	$6H$	288
		2	$(1,0,0,0,0,1,1)$	$(0,1,1,0,0,0,1)$	$6I$	144
		2	$(1,0,1,1,1,1,0)$	$(1,0,1,1,1,1,0)$	$6J$	144
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4S$	1 536
		1	$(1,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$4T$	1 536
$4A$	2^4	3	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$4U$	512
		3	$(1,0,0,0,1,1,0)$	$(0,0,0,0,0,0,0)$	$4V$	512
		4	$(0,1,1,1,0,1,1)$	$(1,0,1,0,0,1,0)$	$8A$	384
		4	$(0,0,0,1,0,0,0)$	$(1,1,0,1,1,1,0)$	$8B$	384
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4W$	512
		1	$(1,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$4X$	512
		1	$(1,1,0,0,1,1,1)$	$(0,0,0,0,0,0,0)$	$4Y$	512
$4B$	2^4	1	$(0,1,0,0,1,0,0)$	$(0,0,0,0,0,0,0)$	$4Z$	512
		2	$(1,0,1,1,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AA$	256
		2	$(1,1,0,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4AB$	256
		4	$(1,0,1,0,1,0,1)$	$(1,1,0,1,1,1,0)$	$8C$	128

continued on next page

Table 6.2 (continued from previous page)

$g \in S_8$	k	f_j	d_j	w	$[x]_{2^7:S_8}(x)\|$	
		4	$(0,0,0,1,0,0,0)$	$(1,0,1,0,0,1,0)$	$8D$	128
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AC$	256
		1	$(0,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$4AD$	256
$4C$	2^3	1	$(1,1,0,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AE$	256
		1	$(1,0,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4AF$	256
		2	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AG$	128
		2	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AH$	128
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$4AI$	128
		1	$(1,0,1,0,1,0,1)$	$(0,0,0,1,0,0,1)$	$8E$	128
		1	$(0,1,1,1,0,1,1)$	$(0,1,0,1,0,0,0)$	$8F$	128
$4D$	2^3	1	$(0,1,0,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$4AJ$	128
		1	$(0,1,1,0,0,1,1)$	$(0,0,0,0,0,0,0)$	$4AK$	128
		1	$(0,0,0,1,0,0,0)$	$(0,1,0,0,0,0,0)$	$8G$	128
		1	$(1,0,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$4AL$	128
		1	$(1,0,0,0,0,1,0)$	$(0,0,1,0,1,1,0)$	$8H$	128
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$5A$	240
$5A$	2^3	1	$(0,1,0,0,1,0,1)$	$(0,0,0,1,1,0,0)$	$10A$	240
		3	$(1,0,1,0,1,0,1)$	$(1,0,1,1,1,0,0)$	$10B$	80
		3	$(0,0,0,1,0,0,0)$	$(1,0,0,0,1,1,0)$	$10C$	80
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6K$	576
		1	$(1,0,1,1,1,0,1)$	$(1,0,0,0,0,1,0)$	$12A$	576
		1	$(0,0,0,0,1,1,1)$	$(0,0,0,0,0,0,0)$	6L	576
$6A$	2^4	1	$(1,0,0,1,0,0,0)$	$(1,1,1,1,1,0,0)$	$12B$	576
		3	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$6M$	192
		3	$(1,0,0,0,0,1,1)$	$(1,1,1,0,0,0,0)$	$12C$	192
		3	$(0,0,0,1,0,0,0)$	$(1,0,0,0,0,1,0)$	$12D$	192
		3	$(1,0,0,0,1,1,0)$	$(0,0,0,0,0,0,0)$	$6N$	192
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6O$	192
		1	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$6P$	192
$6B$	2^3	1	$(1,1,1,1,1,1,1)$	$(1,1,1,1,1,0,0)$	$12E$	192
		1	$(0,1,0,0,1,1,0)$	$(1,1,1,1,1,1,0)$	$12F$	192
		2	$(0,1,1,1,0,1,1)$	$(1,0,1,0,0,1,0)$	$12G$	96
		2	$(0,0,0,1,0,0,0)$	$(1,1,1,1,1,0,0)$	$12H$	96
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6Q$	144
$6C$	2^2	1	$(1,0,1,0,1,0,1)$	$(1,1,0,0,1,1,0)$	$12I$	144
		1	$(1,0,0,0,0,1,1)$	$(1,1,1,0,0,0,0)$	$12J$	144
		1	$(1,0,1,1,1,1,0)$	$(0,0,0,0,0,0,0)$	$6R$	144
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6S$	96
		1	$(0,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$6T$	96
$6D$	2^3	1	$(1,1,0,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$6U$	96
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6V$	96
		2	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$6W$	48
		2	$(1,0,0,0,1,1,0)$	$(0,0,0,0,0,0,0)$	$6X$	48
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$6Y$	48
$6E$	2^2	1	$(1,0,1,0,1,0,1)$	$(1,1,0,0,0,1,0)$	$12K$	48
		1	$(0,1,0,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$6Z$	48
		1	$(0,0,0,1,0,0,0)$	$(0,0,1,0,0,1,0)$	$12L$	48
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$7A$	14
$7A$	2	1	$(1,1,1,1,1,1,1)$	$(1,1,0,1,0,0,1)$	$14A$	14
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$8I$	32
$8A$	2^2	1	$(1,0,1,0,1,0,1)$	$(0,0,0,0,0,0,0)$	$8J$	32
		1	$(0,1,1,1,0,1,1)$	$(0,0,0,0,0,0,0)$	$8K$	32

continued on next page

Table 6.2 (continued from previous page)

$g \in S_8$	k	f_j	d_j	w	$[x]_{2^7:S_8}(x)\|$	
		1	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$8L$	32
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$10D$	40
$10A$	2^2	1	$(1,0,1,0,1,0,1)$	$(1,1,0,1,0,0,1)$	$20A$	40
		1	$(0,1,0,0,1,0,1)$	$(0,0,1,0,0,1,0$	$20B$	40
		1	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$10E$	40
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$12M$	48
$12A$	2^2	1	$(1,0,1,0,1,0,1)$	$(1,1,0,0,1,0,1)$	$24A$	48
		1	$(0,1,1,1,0,1,1)$	$(1,1,0,1,0,0,1)$	$24B$	48
		1	$(0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0)$	$12N$	48
		1	$(0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0)$	$15A$	30
$15A$	2	1	$(1,1,1,1,1,1,1)$	$(1,1,1,1,1,0,0)$	$30A$	30

We obtain that $2^7{:}S_8$ has altogether 128 conjugacy classes of elements.

6.4. The Fischer-Clifford matrices of $2^7{:}S_8$

From the above section, we obtained 128 conjugacy classes of elements of $2^7{:}S_8$. From [89] since we have 128 conjugacy classes of elements, we have 128 irreducible characters of $2^7{:}S_8$. By [14] when S_8 acts on $Irr(2^7)$ we get six orbits and we use programme C (see Appendix A) to show that these orbits are of lengths $1, 1, 28, 28, 35$ and 35 respectively. The inertia factor groups H_i, $i = 1, \cdots, 6$ are subgroups of S_8 of index $1, 1, 28, 28, 35$ and 35 respectively and from the 𝔸𝕋𝕃𝔸𝕊 [23] we get that $H_1 = S_8, H_2 = S_8, H_3 = S_6 \times 2, H_4 = S_6 \times 2, H_5 = (S_4 \times S_4){:}2$ and $H_6 = (S_4 \times S_4){:}2$. The fusions of the inertia factor groups $S_6 \times 2$ and $(S_4 \times S_4){:}2$ into S_8 are given in Table 6.3 below

Table 6.3: The fusion of $S_6 \times 2$ and $(S_4 \times S_4){:}2$ into S_8

$[g]_{S_6\times 2}$	$\longrightarrow$	$[y]_{S_8}$	$[g]_{(S_4\times S_4):2}$	$\longrightarrow$	$[y]_{S_8}$
$1A$		$1A$	$1A$		$1A$
$2A$		$2A$	$2A$		$2B$
$2B$		$2A$	$2B$		$2D$
$2C$		$2B$	$2C$		$2A$
$2D$		$2B$	$2D$		$2C$
$2E$		$2C$	$2E$		$2B$
$2F$		$2C$	$2F$		$2D$
$2G$		$2D$	$3A$		$3A$
$3A$		$3A$	$3B$		$3B$
$3B$		$3B$	$4A$		$4A$
$4A$		$4A$	$4B$		$4C$
$4B$		$4B$	$4C$		$4B$
$4C$		$4B$	$4D$		$4D$
$4D$		$4C$	$4E$		$4D$
$5A$		$5A$	$4F$		$4C$
$6A$		$6A$	$6A$		$6B$

continued on next page

Table 6.3 (continued from previous page)

$[g]_{S_6\times 2}$	$\longrightarrow$	$[y]_{S_8}$	$[g]_{(S_4\times S_4):2}$	$\longrightarrow$	$[y]_{S_8}$
$6B$		$6A$	$6B$		$6A$
$6C$		$6B$	$6C$		$6E$
$6D$		$6C$	$8A$		$8A$
$6E$		$6D$	$10A$		$10A$
$6F$		$6E$	$12A$		$12A$

From the above fusions, we are now able to obtain the Fischer-Clifford matrices of $2^7{:}S_8$. According to [99], all these Fischer-Clifford matrices will have integer entries. These Fischer-Clifford matrices are thus given in Table 6.4 below

Table 6.4: The Fischer-Clifford Matrices of $2^7{:}S_8$

M(g)	M(g)
$M(1A)=\begin{bmatrix}1&1&1&1&1&1\\1&-1&1&-1&1&-1\\28&-28&4&-4&-4&4\\28&28&4&4&-4&-4\\35&35&-5&-5&3&3\\35&-35&-5&5&3&-3\end{bmatrix}$	$M(4A)=\begin{bmatrix}1&1&1&1&1&1\\1&-1&1&-1&1&-1\\6&-6&-2&2&0&0\\6&6&-2&-2&0&0\\1&1&1&1&-1&-1\\1&-1&1&-1&-1&1\end{bmatrix}$
$M(2A)=\begin{bmatrix}1&1&1&1&1&1&1&1\\1&-1&1&-1&1&-1&-1&1\\1&1&-1&-1&-1&-1&1&1\\15&-15&5&-5&-3&3&1&-1\\1&-1&-1&1&-1&1&-1&1\\15&15&5&5&-3&-3&-1&-1\\15&-15&-5&5&3&-3&1&-1\\15&15&-5&-5&3&3&-1&-1\end{bmatrix}$	$M(3A)=\begin{bmatrix}1&1&1&1&1&1\\1&-1&-1&1&-1&1\\5&-5&3&-3&-1&1\\5&5&-3&-3&1&1\\10&-10&-2&2&2&-2\\10&10&2&2&-2&-2\end{bmatrix}$
$M(2B)=\begin{bmatrix}1&1&1&1&1&1&1&1\\1&-1&-1&1&-1&1&-1&1\\4&4&4&4&-4&-4&0&0\\4&-4&-4&4&4&-4&0&0\\3&-3&-3&3&-3&3&1&-1\\8&-8&8&-8&0&0&0&0\\3&3&3&3&3&3&-1&-1\\8&8&-8&-8&0&0&0&0\end{bmatrix}$	$M(4C)=\begin{bmatrix}1&1&1&1&1&1\\1&1&-1&-1&1&-1\\1&1&1&1&-1&-1\\2&-2&2&-2&0&0\\1&1&-1&-1&-1&1\\2&-2&-2&2&0&0\end{bmatrix}$
$M(2C)=\begin{bmatrix}1&1&1&1&1&1&1&1&1&1\\1&-1&-1&1&-1&1&-1&1&1&-1\\2&2&-2&-2&2&2&-2&-2&0&0\\6&-6&6&-6&2&-2&-2&2&0&0\\2&-2&2&-2&-2&2&2&-2&0&0\\6&6&-6&-6&-2&-2&2&2&0&0\\1&1&1&1&1&1&1&1&-1&-1\\6&-6&-6&6&2&-2&2&-2&0&0\\1&-1&-1&1&-1&1&-1&1&-1&1\\6&6&6&6&-2&-2&-2&-2&0&0\end{bmatrix}$	$M(3B)=\begin{bmatrix}1&1&1&1&1&1\\1&-1&-1&1&-1&1\\1&-1&-1&1&1&-1\\1&1&1&1&-1&-1\\2&-2&2&-2&0&0\\2&2&-2&-2&0&0\end{bmatrix}$

continued on next page

Table 6.4 (continued from previous page)

M(g)	M(g)
$M(2D) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\ 3 & 3 & 3 & 3 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 3 & -3 & -3 & 3 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\ 3 & -3 & 3 & -3 & -1 & -1 & 1 & 1 \\ 3 & 3 & -3 & -3 & 1 & -1 & 1 & -1 \end{bmatrix}$	$M(5A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 3 & -3 & 1 & -1 \\ 3 & 3 & -1 & -1 \end{bmatrix}$
$M(4B) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 2 & -2 & 0 & 0 \\ 2 & 2 & 2 & 2 & -2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 4 & -4 & 4 & -4 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 4 & 4 & -4 & -4 & 0 & 0 & 0 & 0 \end{bmatrix}$	$M(6C) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$
$M(4D) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$	$M(6B) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 0 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$
$M(6A) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ 3 & -3 & 3 & -3 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 \\ 3 & 3 & -3 & -3 & 1 & -1 & -1 & 1 \\ 3 & -3 & -3 & 3 & -1 & 1 & -1 & 1 \\ 3 & 3 & 3 & 3 & -1 & -1 & -1 & -1 \end{bmatrix}$	$M(6D) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 0 & 0 \end{bmatrix}$
$M(6E) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$	$M(7A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
$M(8A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$	$M(10A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$
$M(12A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$	$M(15A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

6.5. The power maps of $2^7{:}S_8$

The power maps of $2^7{:}S_8$ are given in Table 6.5 below

Table 6.5: The Power Maps of elements of $2^7 : S_8$

$[g]_{S_8}$	$[x]_{2^7:S_8}$	2	3	5	7	$[g]_{S_8}$	$[x]_{2^7:S_8}$	2	3	5	7
	$1A$						$2F$	$1A$			
	$2A$	$1A$					$4A$	$2C$			
	$2B$	$1A$					$4B$	$2C$			
$1A$	$2C$	$1A$				$2A$	$2G$	$1A$			
	$2D$	$1A$					$4C$	$2C$			
	$2E$	$1A$					$2H$	$1A$			
							$2I$	$1A$			
							$4D$	$2C$			
	$2J$	$1A$					$2P$	$1A$			
	$2K$	$1A$					$4G$	$2C$			
	$2L$	$1A$					$4H$	$2C$			
	$2M$	$1A$					$2Q$	$1A$			
$2B$	$2N$	$1A$				$2C$	$4I$	$2C$			
	$2O$	$1A$					$2R$	$1A$			
	$4E$	$2C$					$4J$	$2C$			
	$4F$	$2C$					$2S$	$1A$			
							$4K$	$2C$			
							$4L$	$2C$			
	$2T$	$1A$					$3A$		$1A$		
	$4M$	$2C$					$6A$	$3A$	$2A$		
	$4N$	$2C$					$6B$	$3A$	$2A$		
$2D$	$4O$	$2C$				$3A$	$6C$	$3A$	$2C$		
	$4P$	$2C$					$6D$	$3A$	$2B$		
	$4Q$	$2C$					$6E$	$3A$	$2C$		
	$2U$	$1A$									
	$4R$	$2C$									
	$4S$	$2P$					$3B$		$1A$		
	$4T$	$2P$					$6F$	$3B$	$2B$		
$4A$	$4U$	$2P$				$3B$	$6G$	$3B$	$2C$		
	$4V$	$2P$					$6H$	$3B$	$2A$		
	$8A$	$4J$					$6I$	$3B$	$2C$		
	$8B$	$4J$					$6J$	$3B$	$2A$		
	$4W$	$2P$					$4AC$	$2J$			
	$4X$	$2P$					$4AD$	$2L$			
	$4Y$	$2P$					$4AE$	$2J$			
$4B$	$4Z$	$2P$				$4C$	$4AF$	$2L$			
	$4AA$	$2P$					$4AG$	$2L$			
	$4AB$	$2P$					$4AH$	$2L$			
	$8C$	$4J$									
	$8D$	$4J$									
	$4AI$	$2P$					$6K$	$3A$	$2F$		
	$8E$	$4J$					$12A$	$6C$	$4C$		
	$8F$	$4J$					$6L$	$3A$	$2H$		
$4D$	$4AJ$	$2R$				$6A$	$12B$	$6C$	$4A$		
	$4AK$	$2R$					$6M$	$3A$	$2I$		

continued on next page

Table 6.5 (continued from previous page)

$[g]_{S_8}$	$[x]_{2^7:S_8}$	2	3	5	7	$[g]_{S_8}$	$[x]_{2^7:S_8}$	2	3	5	7
	8G	4J					12C	6C	4B		
	4AL	2P					12D	6C	4A		
	8H	4J					6N	3A	2G		
	5A			1A			6O	3A	2P		
	10A	5A		2A			6P	3A	2Q		
5A	10C	5A		2C		6B	12E	6C	4H		
	10C	5A		2C			12F	6C	4G		
							12G	6C	4K		
							12H	6C	4L		
	6Q	3B	2F				6S	3B	2J		
	12I	6I	4D				6T	3B	2K		
6C	12J	6I	4C			6D	6U	3B	2L		
	6R	3B	2H				6V	3B	2M		
							6W	3B	2L		
							6X	3B	2M		
	6Y	3B	2T				7A				1A
6E	12K	6G	4L			7A	14A	7A			2A
	6Z	3B	2U								
	12L	6G	4M								
	8I	4AD					10D	5A		2F	
8A	8J	4AE				10A	20A	10B		4D	
	8K	4AE					20B	10B		4B	
	8L	4AD					10E	5A		2H	
	12M	6Q	4S				15A		5A	3A	
12A	24A	12E	8A			15A	30A	15A	10A	6A	
	24B	12E	8B								
	12N	6O	4S								

6.6. The fusion of $2^7{:}S_8$ into $2^7{:}SP_6(2)$

The group $2^7{:}SP_6(2)$ is a maximal subgroup of $\overline{Fi}_{22}$ containing a maximal subgroup of the form $2^7{:}S_8$. To determine the fusion of $2^7{:}S_8$ into $2^7{:}SP_6(2)$, we shall use the fusion of S_8 into $SP(6,2)$, Theorem 7.5.1 from [99] and the power maps of both groups, listed in Table 6.1 above and in [41] respectively. The fusion of S_8 into $SP(6,2)$ is given in Table 6.6.

Table 6.6: The Fusion of S_8 into $SP(6,2)$

$[g]_{S_8}$	$\longrightarrow$	$[y]_{SP(6,2)}$	$[g]_{S_8}$	$\longrightarrow$	$[y]_{SP(6,2)}$
$1A$		$1A$	$2A$		$2A$
$2B$		$2C$	$2C$		$2D$
$2D$		$2B$	$3A$		$3A$
$3B$		$3C$	$4A$		$4C$
$4B$		$4E$	$4C$		$4D$
$4D$		$4B$	$5A$		$5A$
$6A$		$6A$	$6B$		$6D$
$6C$		$6E$	$6D$		$6F$
$6E$		$6G$	$7A$		$7A$
$8A$		$8B$	$10A$		$10A$
$12A$		$12B$	$15A$		$15A$

Thus the fusion of $2^7{:}S_8$ into $2^7{:}SP_6(2)$ is given in Table 6.7 below.

Table 6.7: The fusion of $2^7{:}S_8$ into $2^7{:}SP(6,2)$

$[g]_{S_8}$	$[x]_{2^7:S_8}$	$\longrightarrow$	$[y]_{2^7:SP(6,2)}$	$[g]_{S_8}$	$[x]_{2^7:S_8}$	$\longrightarrow$	$[y]_{2^7:SP(6,2)}$
$1A$	$1A$		$1A$	$2A$	$2F$		$2D$
	$2A$		$2B$		$4A$		$4C$
	$2B$		$2A$		$4B$		$4A$
	$2C$		$2C$		$2G$		$2F$
	$2D$		$2B$		$4C$		$4B$
	$2E$		$2C$		$2H$		$2F$
					$2I$		$2E$
					$4D$		$4C$
$2B$	$2J$		$2G$	$2C$	$2P$		$2K$
	$2K$		$2H$		$4G$		$4F$
	$2L$		$2I$		$4H$		$4H$
	$2M$		$2J$		$2Q$		$2M$
	$2N$		$2I$		$4I$		$4G$
	$2O$		$2J$		$2R$		$2L$
	$4E$		$4D$		$4J$		$4H$
	$4F$		$4E$		$2S$		$2M$
					$4K$		$4I$
					$4L$		$4J$
$2D$	$2T$		$2N$	$3A$	$3A$		$3A$
	$4M$		$4K$		$6A$		$6A$
	$4N$		$4L$		$6B$		$6A$
	$4O$		$4O$		$6C$		$6C$
	$4P$		$4M$		$6D$		$6B$
	$4Q$		$4N$		$6E$		$6C$
	$2U$		$2O$				
	$4R$		$4P$				
$3B$	$3B$		$3C$	$4A$	$4S$		$4Y$
	$6F$		$6E$		$4T$		$4Z$
	$6G$		$6F$		$4U$		$4AA$
	$6H$		$6G$		$4V$		$4AB$

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Table 6.7 (continued from previous page)

$[g]_{S_8}$	$[x]_{2^7:S_8}$	$\longrightarrow$	$[y]_{2^7:SP(6,2)}$	$[g]_{S_8}$	$[x]_{2^7:S_8}$	$\longrightarrow$	$[y]_{2^7:SP(6,2)}$
	6I		6F		8A		8C
	6J		6G		8B		8D
	4W		4U		4AC		4AC
	4X		4V		4AD		4AD
	4Y		4W		4AE		4AE
4B	4Z		4X	4C	4AE		4AF
	4AA		4W		4AG		4AG
	4AB		4X		4AH		4AH
	8C		8A				
	8D		8B				
	4AI		4AI		5A		5A
	8E		8E		10A		10A
	8F		8F		10B		10B
4D	4AJ		4AJ	5A	10C		10C
	4AK		4AK				
	8G		8G				
	4AL		4AL				
	8H		8H				
	6K		6H		6O		6O
	12A		12A		6P		6P
	6L		6J		12E		12F
6A	12B		12C	6B	12F		12G
	6M		6I		12G		12H
	12C		12B		12H		12I
	12D		12C				
	6N		6J				
	6Q		6U		6R		6Q
	12I		12J		6S		6R
6C	12J		12K	6D	6T		6S
	6R		6V		6U		6T
					6V		6S
					6W		6T
	6X		6W		7A		7A
6E	12K		12L	7A	14A		14A
	6Y		6X				
	12M		12M				
	8I		8I		10D		10D
8A	8J		8J	10A	20A		20A
	8K		8K		20B		20B
	8L		8L		10E		10E
	12N		12P		15A		15A
12A	24A		24C	15A	30A		30A
	24B		24D				
	12O		12Q				

We use Fisher-Clifford matrices and partial character tables of inertia factor groups to compile the character table. We rewrite the character table in the GAP format and we then use programme E (see Appendix A) to check its validity and its consistency concerning the character table orthogonalities. This character table is given in Table 6.8

PROGRAMME E for $2^7{:}S_8$

```
gap>ct:=fuction()local ct;ct:=rec();
>ct.SizesCentralizers:=[128 Centralizer Orders];
>ct.OrdersClassRepresentatives:=[128 Class Representatives Orders];
>ct.Irr:=[[128 × 128 irreducibles]];
>ct.UnderlyingCharacteristic:=0;ct.Id:=G̅;
>ConvertToLibraryCharacterTable NC(ct);return ct;end;ct:=ct();
gap>SetInfoLevel(InfoCharacterTable,2);
gap>IsInternallyConsistent(ct);
gap>PossiblePowerMaps(ct,p); (p-prime divisor of G̅).
```

Note: Although this was a good exercise to begin with, on closer inspection, we found out that the group $2^7{:}S_8$ was actually the group $(2^6{:}S_8) \times 2$ which is a direct product.

Table 6.8: Character table of $2^7{:}S_8$

	$1A$						$2A$							
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$4A$	$4B$	$2G$	$4C$	$2H$	$2I$	$4D$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	7	7	7	7	7	7	-5	-5	-5	-5	-5	-5	-5	-5
χ_4	7	7	7	7	7	7	5	5	5	5	5	5	5	5
χ_5	14	14	14	14	14	14	-4	-4	-4	-4	-4	-4	-4	-4
χ_6	14	14	14	14	14	14	4	4	4	4	4	4	4	4
χ_7	20	20	20	20	20	20	-10	-10	-10	-10	-10	-10	-10	-10
χ_8	20	20	20	20	20	20	10	10	10	10	10	10	10	10
χ_9	21	21	21	21	21	21	-9	-9	-9	-9	-9	-9	-9	-9
χ_{10}	21	21	21	21	21	21	9	9	9	9	9	9	9	9
χ_{11}	28	28	28	28	28	28	-10	-10	-10	-10	-10	-10	-10	-10
χ_{12}	28	28	28	28	28	28	10	10	10	10	10	10	10	10
χ_{13}	35	35	35	35	35	35	-5	-5	-5	-5	-5	-5	-5	-5
χ_{14}	35	35	35	35	35	35	5	5	5	5	5	5	5	5
χ_{15}	42	42	42	42	42	42	0	0	0	0	0	0	0	0
χ_{16}	56	56	56	56	56	56	-4	-4	-4	-4	-4	-4	-4	-4
χ_{17}	56	56	56	56	56	56	4	4	4	4	4	4	4	4
χ_{18}	64	64	64	64	64	64	-16	-16	-16	-16	-16	-16	-16	-16
χ_{19}	64	64	64	64	64	64	16	16	16	16	16	16	16	16
χ_{20}	70	70	70	70	70	70	-10	-10	-10	-10	-10	-10	-10	-10
χ_{21}	70	70	70	70	70	70	10	10	10	10	10	10	10	10
χ_{22}	90	90	90	90	90	90	0	0	0	0	0	0	0	0
χ_{23}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1
χ_{24}	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{25}	7	-7	7	-7	7	-7	-5	5	-5	5	-5	5	5	-5
χ_{26}	7	-7	7	-7	7	-7	5	-5	5	-5	5	-5	-5	5
χ_{27}	14	-14	14	-14	14	-14	-4	4	-4	4	-4	4	4	-4
χ_{28}	14	-14	14	-14	14	-14	4	-4	4	-4	4	-4	-4	4
χ_{29}	20	-20	20	-20	20	-20	-10	10	-10	10	-10	10	10	-10
χ_{30}	20	-20	20	-20	20	-20	10	-10	10	-10	10	-10	-10	10
χ_{31}	21	-21	21	-21	21	-21	-9	9	-9	9	-9	9	9	-9
χ_{32}	21	-21	21	-21	21	-21	9	-9	9	-9	9	-9	-9	9
χ_{33}	28	-28	28	-28	28	-28	-10	10	-10	10	-10	10	10	-10
χ_{34}	28	-28	28	-28	28	-28	10	-10	10	-10	10	-10	-10	10
χ_{35}	35	-35	35	-35	35	-35	-5	5	-5	5	-5	5	5	-5
χ_{36}	35	-35	35	-35	35	-35	5	-5	5	-5	5	-5	-5	5
χ_{37}	42	-42	42	-42	42	-42	0	0	0	0	0	0	0	0
χ_{38}	56	-56	56	-56	56	-56	-4	4	-4	4	-4	4	4	-4
χ_{39}	56	-56	56	-56	56	-56	4	-4	4	-4	4	-4	-4	4
χ_{40}	64	-64	64	-64	64	-64	-16	16	-16	16	-16	16	16	-16
χ_{41}	64	-64	64	-64	64	-64	16	-16	16	-16	16	-16	-16	16
χ_{42}	70	-70	70	-70	70	-70	-10	10	-10	10	-10	10	10	-10
χ_{43}	70	-70	70	-70	70	-70	10	-10	10	-10	10	-10	-10	10
χ_{44}	90	-90	90	-90	90	-90	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$2B$								$2C$									
	$2J$	$2K$	$2L$	$2M$	$2N$	20	$4E$	$4F$	$2P$	$4G$	$4H$	$2Q$	$4I$	$2R$	$4J$	$2S$	$4K$	$4L$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3	3	3	3	3
χ_4	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3	3	3	3	3
χ_5	6	6	6	6	6	6	6	6	2	2	2	2	2	2	2	2	2	2
χ_6	6	6	6	6	6	6	6	6	2	2	2	2	2	2	2	2	2	2
χ_7	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
χ_8	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
χ_9	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	1	1	1	1
χ_{10}	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	1	1	1	1
χ_{11}	-4	-4	-4	-4	-4	-4	-4	-4	4	4	4	4	4	4	4	4	4	4
χ_{12}	-4	-4	-4	-4	-4	-4	-4	-4	4	4	4	4	4	4	4	4	4	4
χ_{13}	3	3	3	3	3	3	3	3	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
χ_{14}	3	3	3	3	3	3	3	3	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
χ_{15}	-6	-6	-6	-6	-6	-6	-6	-6	2	2	2	2	2	2	2	2	2	2
χ_{16}	8	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0
χ_{17}	8	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{20}	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2	2	2	2	2	2	2
χ_{21}	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2	2	2	2	2	2	2
χ_{22}	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
χ_{23}	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{24}	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{25}	-1	1	1	-1	1	-1	1	-1	3	-3	-3	3	-3	3	-3	3	3	-3
χ_{26}	-1	1	1	-1	1	-1	1	-1	3	-3	-3	3	-3	3	-3	3	3	-3
χ_{27}	6	-6	-6	6	-6	6	-6	6	2	-2	-2	2	-2	2	-2	2	2	-2
χ_{28}	6	-6	-6	6	-6	6	-6	6	2	-2	-2	2	-2	2	-2	2	2	-2
χ_{29}	4	-4	-4	4	-4	4	-4	4	4	-4	-4	4	-4	4	-4	4	4	-4
χ_{30}	4	-4	-4	4	-4	4	-4	4	4	-4	-4	4	-4	4	-4	4	4	-4
χ_{31}	-3	3	3	-3	3	-3	3	-3	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{32}	-3	3	3	-3	3	-3	3	-3	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{33}	-4	4	4	-4	4	-4	4	-4	4	-4	-4	4	-4	4	-4	4	4	-4
χ_{34}	-4	4	4	-4	4	-4	4	-4	4	-4	-4	4	-4	4	-4	4	4	-4
χ_{35}	3	-3	-3	3	-3	3	-3	3	-5	5	5	-5	5	-5	5	-5	-5	5
χ_{36}	3	-3	-3	3	-3	3	-3	3	-5	5	5	-5	5	-5	5	-5	-5	5
χ_{37}	-6	6	6	-6	6	-6	6	-6	2	-2	-2	2	-2	2	-2	2	2	-2
χ_{38}	8	-8	-8	8	-8	8	-8	8	0	0	0	0	0	0	0	0	0	0
χ_{39}	8	-8	-8	8	-8	8	-8	8	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	-2	2	2	-2	2	-2	2	-2	2	-2	-2	2	-2	2	-2	2	2	-2
χ_{43}	-2	2	2	-2	2	-2	2	-2	2	-2	-2	2	-2	2	-2	2	2	-2
χ_{44}	-6	6	6	-6	6	-6	6	-6	-6	6	6	-6	6	-6	6	-6	-6	6

The character table of $2^7{:}S_8$(continued)

	$2D$								3A						3B	
	$2T$	$4M$	$4N$	$4O$	$4P$	$4Q$	$2U$	$4R$	$3A$	$6A$	$6B$	$6C$	$6D$	$6E$	$3B$	$6F$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
χ_3	-1	-1	-1	-1	-1	-1	-1	-1	4	4	4	4	4	4	1	1
χ_4	1	1	1	1	1	1	1	1	4	4	4	4	4	4	1	1
χ_5	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	2	2
χ_6	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	2	2
χ_7	-2	-2	-2	-2	-2	-2	-2	-2	5	5	5	5	5	5	-1	-1
χ_8	2	2	2	2	2	2	2	2	5	5	5	5	5	5	-1	-1
χ_9	3	3	3	3	3	3	3	3	6	6	6	6	6	6	0	0
χ_{10}	-3	-3	-3	-3	-3	-3	-3	-3	6	6	6	6	6	6	0	0
χ_{11}	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1	1
χ_{12}	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
χ_{13}	3	3	3	3	3	3	3	3	5	5	5	5	5	5	2	2
χ_{14}	-3	-3	-3	-3	-3	-3	-3	-3	5	5	5	5	5	5	2	2
χ_{15}	0	0	0	0	0	0	0	0	-6	-6	-6	-6	-6	-6	0	0
χ_{16}	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-1	-1
χ_{17}	4	4	4	4	4	4	4	4	-4	-4	-4	-4	-4	-4	-1	-1
χ_{18}	0	0	0	0	0	0	0	0	4	4	4	4	4	4	-2	-2
χ_{19}	0	0	0	0	0	0	0	0	4	4	4	4	4	4	-2	-2
χ_{20}	2	2	2	2	2	2	2	2	-5	-5	-5	-5	-5	-5	1	1
χ_{21}	-2	-2	-2	-2	-2	-2	-2	-2	-5	-5	-5	-5	-5	-5	1	1
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	1	-1
χ_{24}	-1	1	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	1	-1
χ_{25}	-1	1	-1	1	-1	-1	1	1	4	-4	-4	4	-4	4	1	-1
χ_{26}	1	-1	1	-1	1	1	-1	-1	4	-4	-4	4	-4	4	1	-1
χ_{27}	0	0	0	0	0	0	0	0	-1	1	1	-1	1	-1	2	-2
χ_{28}	0	0	0	0	0	0	0	0	-1	1	1	-1	1	-1	2	-2
χ_{29}	-2	2	-2	2	-2	-2	2	2	5	-5	-5	5	-5	5	-1	1
χ_{30}	2	-2	2	-2	2	2	-2	-2	5	-5	-5	5	-5	5	-1	1
χ_{31}	3	-3	3	-3	3	3	-3	-3	6	-6	-6	6	-6	6	0	0
χ_{32}	-3	3	-3	3	-3	-3	3	3	6	-6	-6	6	-6	6	0	0
χ_{33}	-2	2	-2	2	-2	-2	2	2	1	-1	-1	1	-1	1	1	-1
χ_{34}	2	-2	2	-2	2	2	-2	-2	1	-1	-1	1	-1	1	1	-1
χ_{35}	3	-3	3	-3	3	3	-3	-3	5	-5	-5	5	-5	5	2	-2
χ_{36}	-3	3	-3	3	-3	-3	3	3	5	-5	-5	5	-5	5	2	-2
χ_{37}	0	0	0	0	0	0	0	0	-6	6	6	-6	6	-6	0	0
χ_{38}	-4	4	-4	4	-4	-4	4	4	-4	4	4	-4	4	-4	-1	1
χ_{39}	4	-4	4	-4	4	4	-4	-4	-4	4	4	-4	4	-4	-1	1
χ_{40}	0	0	0	0	0	0	0	0	4	-4	-4	4	-4	4	-2	2
χ_{41}	0	0	0	0	0	0	0	0	4	-4	-4	4	-4	4	-2	2
χ_{42}	2	-2	2	-2	2	2	-2	-2	-5	5	5	-5	5	-5	1	-1
χ_{43}	-2	2	-2	2	-2	-2	2	2	-5	5	5	-5	5	-5	1	-1
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$3B$				$4A$						$4B$					
	$6G$	$6H$	$6I$	$6J$	$4S$	$4T$	$4U$	V	$8A$	$8B$	$4W$	$4X$	$4Y$	$4Z$	$4AA$	$4AB$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	1	1	1	1	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_4	1	1	1	1	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1
χ_5	2	2	2	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2
χ_6	2	2	2	2	-2	-2	-2	-2	-2	-2	2	2	2	2	2	2
χ_7	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_8	-1	-1	-1	-1	2	2	2	2	2	2	2	2	2	2	2	2
χ_9	0	0	0	0	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_{10}	0	0	0	0	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1
χ_{11}	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
χ_{12}	1	1	1	1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{13}	2	2	2	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{14}	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1
χ_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{18}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{19}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{20}	1	1	1	1	4	4	4	4	4	4	0	0	0	0	0	0
χ_{21}	1	1	1	1	-4	-4	-4	-4	-4	-4	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
χ_{24}	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
χ_{25}	-1	1	-1	1	-3	3	-3	3	-3	3	1	-1	-1	1	-1	1
χ_{26}	-1	1	-1	1	3	-3	3	-3	3	-3	-1	1	1	-1	1	-1
χ_{27}	-2	2	-2	2	2	-2	2	-2	2	-2	-2	2	2	-2	2	-2
χ_{28}	-2	2	-2	2	-2	2	-2	2	-2	2	2	-2	-2	2	-2	2
χ_{29}	1	-1	1	-1	-2	2	-2	2	-2	2	-2	2	2	-2	2	-2
χ_{30}	1	-1	1	-1	2	-2	2	-2	2	-2	2	-2	-2	2	-2	2
χ_{31}	0	0	0	0	-3	3	-3	3	-3	3	1	-1	-1	1	-1	1
χ_{32}	0	0	0	0	3	-3	3	-3	3	-3	-1	1	1	-1	1	-1
χ_{33}	-1	1	-1	1	2	-2	2	-2	2	-2	2	-2	-2	2	-2	2
χ_{34}	-1	1	-1	1	-2	2	-2	2	-2	2	-2	2	2	-2	2	-2
χ_{35}	-2	2	-2	2	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
χ_{36}	-2	2	-2	2	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	2	-2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	2	-2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	-1	1	-1	1	4	-4	4	-4	4	-4	0	0	0	0	0	0
χ_{43}	-1	1	-1	1	-4	4	-4	4	-4	4	0	0	0	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$4B$		$4C$						$4D$							
	$8C$	$8D$	$4AC$	$4AD$	$4AE$	$4AF$	$4AG$	$4AH$	$4AI$	$8E$	$8MF$	$4AJ$	$4AK$	$8G$	$4AL$	$8H$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
χ_4	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
χ_5	-2	-2	2	2	2	2	2	2	0	0	0	0	0	0	0	0
χ_6	2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0
χ_7	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{10}	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{11}	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{14}	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{15}	0	0	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{20}	0	0	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0
χ_{21}	0	0	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0
χ_{22}	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2
χ_{23}	-1	1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
χ_{24}	1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
χ_{25}	-1	1	-1	-1	1	1	-1	1	1	-1	1	-1	1	-1	1	-1
χ_{26}	1	-1	-1	-1	1	1	-1	1	1	-1	1	-1	1	-1	1	-1
χ_{27}	2	-2	2	2	-2	-2	2	-2	0	0	0	0	0	0	0	0
χ_{28}	-2	2	2	2	-2	-2	2	-2	0	0	0	0	0	0	0	0
χ_{29}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	-1	1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
χ_{32}	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
χ_{33}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	1	-1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1
χ_{36}	-1	1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1
χ_{37}	0	0	2	2	-2	-2	2	-2	-2	2	-2	2	-2	2	-2	2
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	-2	-2	2	2	-2	2	0	0	0	0	0	0	0	0
χ_{43}	0	0	-2	-2	2	2	-2	2	0	0	0	0	0	0	0	0
χ_{44}	0	0	2	2	-2	-2	2	-2	2	-2	2	-2	2	-2	2	-2

The character table of $2^7{:}S_8$(continued)

	$5A$				$6A$								$6B$			
	$5A$	$10A$	$10B$	$10C$	$6K$	$12A$	$6L$	$12B$	$6M$	$12C$	$12D$	$6N$	$6O$	$6P$	$12E$	$12F$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_3	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0
χ_4	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0
χ_5	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_6	-1	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
χ_7	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_8	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_9	1	1	1	1	0	0	0	0	0	0	0	0	-2	-2	-2	-2
χ_{10}	1	1	1	1	0	0	0	0	0	0	0	0	-2	-2	-2	-2
χ_{11}	-2	-2	-2	-2	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_{12}	-2	-2	-2	-2	1	1	1	1	1	1	1	1	1	1	1	1
χ_{13}	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_{14}	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_{15}	2	2	2	2	0	0	0	0	0	0	0	0	2	2	2	2
χ_{16}	1	1	1	1	2	2	2	2	2	2	2	2	0	0	0	0
χ_{17}	1	1	1	1	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0
χ_{18}	-1	-1	-1	-1	2	2	2	2	2	2	2	2	0	0	0	0
χ_{19}	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0
χ_{20}	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{21}	0	0	0	0	1	1	1	1	1	1	1	1	-1	-1	-1	-1
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	1
χ_{24}	1	-1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
χ_{25}	2	-2	-2	2	-2	2	2	-2	-2	2	-2	2	0	0	0	0
χ_{26}	2	-2	-2	2	2	-2	-2	2	2	-2	2	-2	0	0	0	0
χ_{27}	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{28}	-1	1	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1
χ_{29}	0	0	0	0	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
χ_{30}	0	0	0	0	1	-1	-1	1	1	-1	1	-1	1	-1	-1	1
χ_{31}	1	-1	-1	1	0	0	0	0	0	0	0	0	-2	2	2	-2
χ_{32}	1	-1	-1	1	0	0	0	0	0	0	0	0	-2	2	2	-2
χ_{33}	-2	2	2	-2	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
χ_{34}	-2	2	2	-2	1	-1	-1	1	1	-1	1	-1	1	-1	-1	1
χ_{35}	0	0	0	0	1	-1	-1	1	1	-1	1	-1	1	-1	-1	1
χ_{36}	0	0	0	0	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1
χ_{37}	2	-2	-2	2	0	0	0	0	0	0	0	0	2	-2	-2	2
χ_{38}	1	-1	-1	1	2	-2	-2	2	2	-2	2	-2	0	0	0	0
χ_{39}	1	-1	-1	1	-2	2	2	-2	-2	2	-2	2	0	0	0	0
χ_{40}	-1	1	1	-1	2	-2	-2	2	2	-2	2	-2	0	0	0	0
χ_{41}	-1	1	1	-1	-2	2	2	-2	-2	2	-2	2	0	0	0	0
χ_{42}	0	0	0	0	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1
χ_{43}	0	0	0	0	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$6B$		$6C$				$6D$						$6E$			
	$12G$	$12H$	$6Q$	$12I$	$12J$	$6R$	$6S$	$6T$	$6U$	$6V$	$6W$	$6X$	$6Y$	$12K$	$6Z$	$12L$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_3	0	0	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_4	0	0	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1
χ_5	-1	-1	2	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_6	-1	-1	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_7	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
χ_8	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_9	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1
χ_{12}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{13}	1	1	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{14}	1	1	2	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{15}	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{17}	0	0	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
χ_{18}	0	0	2	2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{20}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_{21}	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	-1	1	1	1	-1	-1	1	-1	-1	1	-1	1	1	1	-1	-1
χ_{24}	-1	1	-1	-1	1	1	-1	1	1	-1	1	-1	1	1	-1	-1
χ_{25}	0	0	1	1	-1	-1	-1	1	1	-1	1	-1	-1	-1	1	1
χ_{26}	0	0	-1	-1	1	1	1	-1	-1	1	-1	1	-1	-1	1	1
χ_{27}	1	-1	2	2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{28}	1	-1	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{29}	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	1	1	-1	-1
χ_{30}	-1	1	1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1
χ_{31}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	-1	-1	1	1
χ_{34}	-1	1	1	1	-1	-1	-1	1	1	-1	1	-1	-1	-1	1	1
χ_{35}	-1	1	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{36}	-1	1	2	2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{37}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	-1	-1	1	1	-1	1	1	-1	1	-1	-1	-1	1	1
χ_{39}	0	0	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1
χ_{40}	0	0	2	2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0
χ_{42}	1	-1	-1	-1	1	1	-1	1	1	-1	1	-1	1	1	-1	-1
χ_{43}	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	1	1	-1	-1
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$7A$		$8A$				$10A$				12A				15A	
	$7A$	$14A$	$8I$	$8J$	$8K$	$8L$	$10D$	$20A$	$20B$	$10E$	$12M$	$24A$	$24B$	$12SN$	$15A$	$30A$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1
χ_3	0	0	1	1	1	1	0	0	0	0	0	0	0	0	-1	-1
χ_4	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	-1	-1
χ_5	0	0	0	0	0	0	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_6	0	0	0	0	0	0	-1	-1	-1	-1	1	1	1	1	-1	-1
χ_7	-1	-1	0	0	0	0	0	0	0	0	1	1	1	1	0	0
χ_8	-1	-1	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0
χ_9	0	0	-1	-1	-1	-1	1	1	1	1	0	0	0	0	1	1
χ_{10}	0	0	1	1	1	1	-1	-1	-1	-1	0	0	0	0	1	1
χ_{11}	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1
χ_{12}	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_{13}	0	0	1	1	1	1	0	0	0	0	-1	-1	-1	-1	0	0
χ_{14}	0	0	-1	-1	-1	-1	0	0	0	0	1	1	1	1	0	0
χ_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
χ_{16}	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1
χ_{17}	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	1	1
χ_{18}	1	1	0	0	0	0	-1	-1	-1	-1	0	0	0	0	-1	-1
χ_{19}	1	1	0	0	0	0	1	1	1	1	0	0	0	0	-1	-1
χ_{20}	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0
χ_{22}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	-1
χ_{24}	1	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1
χ_{25}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	-1	1
χ_{26}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	-1	1
χ_{27}	0	0	0	0	0	0	1	1	-1	-1	-1	-1	1	1	-1	1
χ_{28}	0	0	0	0	0	0	-1	-1	1	1	1	1	-1	-1	-1	1
χ_{29}	-1	1	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{30}	-1	1	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{31}	0	0	-1	-1	1	1	1	1	-1	-1	0	0	0	0	1	-1
χ_{32}	0	0	1	1	-1	-1	-1	-1	1	1	0	0	0	0	1	-1
χ_{33}	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	-1
χ_{34}	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	1	-1
χ_{35}	0	0	1	1	-1	-1	0	0	0	0	-1	-1	1	1	0	0
χ_{36}	0	0	-1	-1	1	1	0	0	0	0	1	1	-1	-1	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1
χ_{38}	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	1	-1
χ_{39}	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	1	-1
χ_{40}	1	-1	0	0	0	0	-1	-1	1	1	0	0	0	0	-1	1
χ_{41}	1	-1	0	0	0	0	1	1	-1	-1	0	0	0	0	-1	1
χ_{42}	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{44}	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$1A$						$2A$							
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$4A$	$4B$	$4C$	$4D$	$4E$	$4F$	$4G$
χ_{45}	28	-28	4	-4	-4	4	16	-14	4	-6	-4	2	2	0
χ_{46}	28	-28	4	-4	-4	4	-16	14	-4	6	4	-2	-2	0
χ_{47}	28	-28	4	-4	-4	4	14	-16	6	-4	-2	4	0	-2
χ_{48}	28	-28	4	-4	-4	4	-14	16	-6	4	2	-4	0	2
χ_{49}	140	-140	20	-20	-20	20	-40	50	-20	10	4	-14	2	8
χ_{50}	140	-140	20	-20	-20	20	50	-40	10	-20	-14	4	8	2
χ_{51}	140	-140	20	-20	-20	20	-50	40	-10	20	14	-4	-8	-2
χ_{52}	140	-140	20	-20	-20	20	40	-50	20	-10	-4	14	-2	-8
χ_{53}	140	-140	20	-20	-20	20	-20	10	0	10	8	2	-6	-4
χ_{54}	140	-140	20	-20	-20	20	10	-20	10	0	2	8	-4	-6
χ_{55}	140	-140	20	-20	-20	20	-10	20	-10	0	-2	-8	4	6
χ_{56}	140	-140	20	-20	-20	20	20	-10	0	-10	-8	-2	6	4
χ_{57}	252	-252	36	-36	-36	36	-36	54	-24	6	0	-18	6	12
χ_{58}	252	-252	36	-36	-36	36	54	-36	6	-24	-18	0	12	6
χ_{59}	252	-252	36	-36	-36	36	-54	36	-6	24	18	0	-12	-6
χ_{60}	252	-252	36	-36	-36	36	36	-54	24	-6	0	18	-6	-12
χ_{61}	280	-280	40	-40	-40	40	-40	20	0	20	16	4	-12	-8
χ_{62}	280	-280	40	-40	-40	40	20	-40	20	0	4	16	-8	-12
χ_{63}	280	-280	40	-40	-40	40	-20	40	-20	0	-4	-16	8	12
χ_{64}	280	-280	40	-40	-40	40	40	-20	0	-20	-16	-4	12	8
χ_{65}	448	-448	64	-64	-64	64	16	16	-16	-16	-16	-16	16	16
χ_{66}	448	-448	64	-64	-64	64	-16	-16	16	16	16	16	-16	-16
χ_{67}	28	28	4	4	-4	-4	16	14	4	6	-4	-2	-2	0
χ_{68}	28	28	4	4	-4	-4	-16	-14	-4	-6	4	2	2	0
χ_{69}	28	28	4	4	-4	-4	14	16	6	4	-2	-4	0	-2
χ_{70}	28	28	4	4	-4	-4	-14	-16	-6	-4	2	4	0	2
χ_{71}	140	140	20	20	-20	-20	-40	-50	-20	-10	4	14	-2	8
χ_{72}	140	140	20	20	-20	-20	50	40	10	20	-14	-4	-8	2
χ_{73}	140	140	20	20	-20	-20	-50	-40	-10	-20	14	4	8	-2
χ_{74}	140	140	20	20	-20	-20	40	50	20	10	-4	-14	2	-8
χ_{75}	140	140	20	20	-20	-20	-20	-10	0	-10	8	-2	6	-4
χ_{76}	140	140	20	20	-20	-20	10	20	10	0	2	-8	4	-6
χ_{77}	140	140	20	20	-20	-20	-10	-20	-10	0	-2	8	-4	6
χ_{78}	140	140	20	20	-20	-20	20	10	0	10	-8	2	-6	4
χ_{79}	252	252	36	36	-36	-36	-36	-54	-24	-6	0	18	-6	12
χ_{80}	252	252	36	36	-36	-36	54	36	6	24	-18	0	-12	6
χ_{81}	252	252	36	36	-36	-36	-54	-36	-6	-24	18	0	12	-6
χ_{82}	252	252	36	36	-36	-36	36	54	24	6	0	-18	6	-12
χ_{83}	280	280	40	40	-40	-40	-40	-20	0	-20	16	-4	12	-8
χ_{84}	280	280	40	40	-40	-40	20	40	20	0	4	-16	8	-12
χ_{85}	280	280	40	40	-40	-40	-20	-40	-20	0	-4	16	-8	12
χ_{86}	280	280	40	40	-40	-40	40	20	0	20	-16	4	-12	8
χ_{87}	448	448	64	64	-64	-64	16	-16	-16	16	-16	16	-16	16
χ_{88}	448	448	64	64	-64	-64	-16	16	16	-16	16	-16	16	-16

The character table of $2^7{:}S_8$(continued)

	2B								2C									
	$2J$	$2K$	$2L$	$2M$	$2N$	20	$4E$	$4F$	$2P$	$4G$	$4H$	$2Q$	$4I$	$2R$	$4J$	$2S$	$4K$	$4L$
χ_{45}	4	4	4	4	-4	-4	0	0	8	-4	4	-8	4	0	-4	0	0	0
χ_{46}	4	4	4	4	-4	-4	0	0	8	-4	4	-8	4	0	-4	0	0	0
χ_{47}	-4	-4	-4	-4	4	4	0	0	4	-8	8	-4	0	-4	0	4	0	0
χ_{48}	-4	-4	-4	-4	4	4	0	0	4	-8	8	-4	0	-4	0	4	0	0
χ_{49}	4	4	4	4	-4	-4	0	0	0	-12	12	0	-4	-8	4	8	0	0
χ_{50}	-4	-4	-4	-4	4	4	0	0	12	0	0	-12	8	4	-8	-4	0	0
χ_{51}	-4	-4	-4	-4	4	4	0	0	12	0	0	-12	8	4	-8	-4	0	0
χ_{52}	4	4	4	4	-4	-4	0	0	0	-12	12	0	-4	-8	4	8	0	0
χ_{53}	-12	-12	-12	-12	12	12	0	0	8	-4	4	-8	4	0	-4	0	0	0
χ_{54}	12	12	12	12	-12	-12	0	0	4	-8	8	-4	0	-4	0	4	0	0
χ_{55}	12	12	12	12	-12	-12	0	0	4	-8	8	-4	0	-4	0	4	0	0
χ_{56}	-12	-12	-12	-12	12	12	0	0	8	-4	4	-8	4	0	-4	0	0	0
χ_{57}	-12	-12	-12	-12	12	12	0	0	0	-12	12	0	-4	-8	4	8	0	0
χ_{58}	12	12	12	12	-12	-12	0	0	12	0	0	-12	8	4	-8	-4	0	0
χ_{59}	12	12	12	12	-12	-12	0	0	12	0	0	-12	8	4	-8	-4	0	0
χ_{60}	-12	-12	-12	-12	12	12	0	0	0	-12	12	0	-4	-8	4	8	0	0
χ_{61}	-8	-8	-8	-8	8	8	0	0	-8	16	-16	8	0	8	0	-8	0	0
χ_{62}	8	8	8	8	-8	-8	0	0	-16	8	-8	16	-8	0	8	0	0	0
χ_{63}	8	8	8	8	-8	-8	0	0	-16	8	-8	16	-8	0	8	0	0	0
χ_{64}	-8	-8	-8	-8	8	8	0	0	-8	16	-16	8	0	8	0	-8	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	4	-4	-4	4	4	-4	0	0	8	4	-4	-8	-4	0	4	0	0	0
χ_{68}	4	-4	-4	4	4	-4	0	0	8	4	-4	-8	-4	0	4	0	0	0
χ_{69}	-4	4	4	-4	-4	4	0	0	4	8	-8	-4	0	-4	0	4	0	0
χ_{70}	-4	4	4	-4	-4	4	0	0	4	8	-8	-4	0	-4	0	4	0	0
χ_{71}	4	-4	-4	4	4	-4	0	0	0	12	-12	0	4	-8	-4	8	0	0
χ_{72}	-4	4	4	-4	-4	4	0	0	12	0	0	-12	-8	4	8	-4	0	0
χ_{73}	-4	4	4	-4	-4	4	0	0	12	0	0	-12	-8	4	8	-4	0	0
χ_{74}	4	-4	-4	4	4	-4	0	0	0	12	-12	0	4	-8	-4	8	0	0
χ_{75}	-12	12	12	-12	-12	12	0	0	8	4	-4	-8	-4	0	4	0	0	0
χ_{76}	12	-12	-12	12	12	-12	0	0	4	8	-8	-4	0	-4	0	4	0	0
χ_{77}	12	-12	-12	12	12	-12	0	0	4	8	-8	-4	0	-4	0	4	0	0
χ_{78}	-12	12	12	-12	-12	12	0	0	8	4	-4	-8	-4	0	4	0	0	0
χ_{79}	-12	12	12	-12	-12	12	0	0	0	12	-12	0	4	-8	-4	8	0	0
χ_{80}	12	-12	-12	12	12	-12	0	0	12	0	0	-12	-8	4	8	-4	0	0
χ_{81}	12	-12	-12	12	12	-12	0	0	12	0	0	-12	-8	4	8	-4	0	0
χ_{82}	-12	12	12	-12	-12	12	0	0	0	12	-12	0	4	-4	-4	8	0	0
χ_{83}	-8	8	8	-8	-8	8	0	0	-8	-16	16	8	0	8	0	-8	0	0
χ_{84}	8	-8	-8	8	8	-8	0	0	-16	-8	8	16	8	0	-8	0	0	0
χ_{85}	8	-8	-8	8	8	-8	0	0	-16	-8	8	16	8	0	-8	0	0	0
χ_{86}	-8	8	8	-8	-8	8	0	0	-8	-16	16	8	0	8	0	-8	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^7{:}S_8$(continued)

	$2D$								3A						3B	
	$2T$	$4M$	$4N$	$4O$	$4P$	$4Q$	$2U$	$4R$	$3A$	$6A$	$6B$	$6C$	$6D$	$6E$	$3B$	$6F$
χ_{45}	4	2	2	4	-2	0	0	-2	10	-10	-2	2	2	-2	1	-1
χ_{46}	-4	-2	-2	-4	2	0	0	2	10	-10	-2	2	2	-2	1	-1
χ_{47}	-2	-4	-4	-2	0	2	2	0	10	-10	-2	2	2	-2	1	-1
χ_{48}	2	4	4	2	0	-2	-2	0	10	-10	-2	2	2	-2	1	-1
χ_{49}	4	2	2	4	-2	0	0	-2	20	-20	-4	4	4	-4	-1	1
χ_{50}	2	4	4	2	0	-2	-2	0	20	-20	-4	4	4	-4	-1	1
χ_{51}	-2	-4	-4	-2	0	2	2	0	20	-20	-4	4	4	-4	-1	1
χ_{52}	-4	-2	-2	-4	2	0	0	2	20	-20	-4	4	4	-4	-1	1
χ_{53}	0	-6	-6	0	-2	4	4	-2	-10	10	2	-2	-2	2	2	-2
χ_{54}	-6	0	0	-6	4	-2	-2	4	-10	10	2	-2	-2	2	2	-2
χ_{55}	6	0	0	6	-4	2	2	-4	-10	10	2	-2	-2	2	2	-2
χ_{56}	0	6	6	0	2	-4	-4	2	-10	10	2	-2	-2	2	2	-2
χ_{57}	0	6	6	0	2	-4	-4	2	0	0	0	0	0	0	0	0
χ_{58}	6	0	0	6	-4	2	2	-4	0	0	0	0	0	0	0	0
χ_{59}	-6	0	0	-6	4	-2	-2	4	0	0	0	0	0	0	0	0
χ_{60}	0	-6	-6	0	-2	4	4	-2	0	0	0	0	0	0	0	0
χ_{61}	8	4	4	8	-4	0	0	-4	10	-10	-2	2	2	-2	1	-1
χ_{62}	4	8	8	4	0	-4	-4	0	10	-10	-2	2	2	-2	1	-1
χ_{63}	-4	-8	-8	-4	0	4	4	0	10	-10	-2	2	2	-2	1	-1
χ_{64}	-8	-4	-4	-8	4	0	0	4	10	-10	-2	2	2	-2	1	-1
χ_{65}	0	0	0	0	0	0	0	0	-20	20	4	-4	-4	4	-2	2
χ_{66}	0	0	0	0	0	0	0	0	-20	20	4	-4	-4	4	-2	2
χ_{67}	4	-2	-4	2	0	0	-2	2	10	10	2	2	-2	-2	1	1
χ_{68}	-4	2	4	-2	0	0	2	-2	10	10	2	2	-2	-2	1	1
χ_{69}	-2	4	2	-4	-2	2	0	0	10	10	2	2	-2	-2	1	1
χ_{70}	2	-4	-2	4	2	-2	0	0	10	10	2	2	-2	-2	1	1
χ_{71}	4	-2	-4	2	0	0	-2	2	20	20	4	4	-4	-4	-1	-1
χ_{72}	2	-4	-2	4	2	-2	0	0	20	20	4	4	-4	-4	-1	-1
χ_{73}	-2	4	2	-4	-2	2	0	0	20	20	4	4	-4	-4	-1	-1
χ_{74}	-4	2	4	-2	0	0	2	-2	20	20	4	4	-4	-4	-1	-1
χ_{75}	0	6	0	-6	-4	4	-2	2	-10	-10	-2	-2	2	2	2	2
χ_{76}	-6	0	6	0	2	-2	4	-4	-10	-10	-2	-2	2	2	2	2
χ_{77}	6	0	-6	0	-2	2	-4	4	-10	-10	-2	-2	2	2	2	2
χ_{78}	0	-6	0	6	4	-4	2	-2	-10	-10	-2	-2	2	2	2	2
χ_{79}	0	-6	0	6	4	-4	2	-2	0	0	0	0	0	0	0	0
χ_{80}	6	0	-6	0	-2	2	-4	4	0	0	0	0	0	0	0	0
χ_{81}	-6	0	6	0	2	-2	4	-4	0	0	0	0	0	0	0	0
χ_{82}	0	6	0	-6	-4	4	-2	2	0	0	0	0	0	0	0	0
χ_{83}	8	-4	-8	4	0	0	-4	4	10	10	2	2	-2	-2	1	1
χ_{84}	4	-8	-4	8	4	-4	0	0	10	10	2	2	-2	-2	1	1
χ_{85}	-4	8	4	-8	-4	4	0	0	10	10	2	2	-2	-2	1	1
χ_{86}	-8	4	8	-4	0	0	4	-4	10	10	2	2	-2	-2	1	1
χ_{87}	0	0	0	0	0	0	0	0	-20	-20	-4	-4	4	4	-2	-2
χ_{88}	0	0	0	0	0	0	0	0	-20	-20	-4	-4	4	4	-2	-2

The character table of $2^7{:}S_8$(continued)

	$3B$				$4A$						$4B$					
	$6G$	$6H$	$6I$	$6J$	$4AA$	$4AB$	$8A$	$8B$	$8C$	$8D$	$4AC$	$4AD$	$8E$	$8F$	$8G$	$8H$
χ_{45}	-1	1	1	-1	6	-6	-2	2	0	0	2	-2	-2	2	2	-2
χ_{46}	-1	1	1	-1	-6	6	2	-2	0	0	-2	2	2	-2	-2	2
χ_{47}	-1	1	1	-1	6	-6	-2	2	0	0	-2	2	2	-2	-2	2
χ_{48}	-1	1	1	-1	-6	6	2	-2	0	0	2	-2	-2	2	2	-2
χ_{49}	1	-1	-1	1	-6	6	2	-2	0	0	-2	2	2	-2	-2	2
χ_{50}	1	-1	-1	1	6	-6	-2	2	0	0	-2	2	2	-2	-2	2
χ_{51}	1	-1	-1	1	-6	6	2	-2	0	0	2	-2	-2	2	2	-2
χ_{52}	1	-1	-1	1	6	-6	-2	2	0	0	2	-2	-2	2	2	-2
χ_{53}	-2	2	2	-2	6	-6	-2	2	0	0	2	-2	-2	2	2	-2
χ_{54}	-2	2	2	-2	-6	6	2	-2	0	0	2	-2	-2	2	2	-2
χ_{55}	-2	2	2	-2	6	-6	-2	2	0	0	-2	2	2	-2	-2	2
χ_{56}	-2	2	2	-2	-6	6	2	-2	0	0	-2	2	2	-2	-2	2
χ_{57}	0	0	0	0	6	-6	-2	2	0	0	2	-2	-2	2	2	-2
χ_{58}	0	0	0	0	-6	6	2	-2	0	0	2	-2	-2	2	2	-2
χ_{59}	0	0	0	0	6	-6	-2	2	0	0	-2	2	2	-2	-2	2
χ_{60}	0	0	0	0	-6	6	2	-2	0	0	-2	2	2	-2	-2	2
χ_{61}	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	2	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	2	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	1	1	-1	-1	6	6	-2	-2	0	0	2	2	2	2	-2	-2
χ_{68}	1	1	-1	-1	-6	-6	2	2	0	0	-2	-2	-2	-2	2	2
χ_{69}	1	1	-1	-1	6	6	-2	-2	0	0	-2	-2	-2	-2	2	2
χ_{70}	1	1	-1	-1	-6	-6	2	2	0	0	2	2	2	2	-2	-2
χ_{71}	-1	-1	1	1	-6	-6	2	2	0	0	-2	-2	-2	-2	2	2
χ_{72}	-1	-1	1	1	6	6	-2	-2	0	0	-2	-2	-2	-2	2	2
χ_{73}	-1	-1	1	1	-6	-6	2	2	0	0	2	2	2	2	-2	-2
χ_{74}	-1	-1	1	1	6	6	-2	-2	0	0	2	2	2	2	-2	-2
χ_{75}	2	2	-2	-2	6	6	-2	-2	0	0	2	2	2	2	-2	-2
χ_{76}	2	2	-2	-2	-6	-6	2	2	0	0	2	2	2	2	-2	-2
χ_{77}	2	2	-2	-2	6	6	-2	-2	0	0	-2	-2	-2	-2	2	2
χ_{78}	2	2	-2	-2	-6	-6	2	2	0	0	-2	-2	-2	-2	2	2
χ_{79}	0	0	0	0	6	6	-2	-2	0	0	2	2	2	2	-2	-2
χ_{80}	0	0	0	0	-6	-6	2	2	0	0	2	2	2	2	-2	-2
χ_{81}	0	0	0	0	6	6	-2	-2	0	0	-2	-2	-2	-2	2	2
χ_{82}	0	0	0	0	-6	-6	2	2	0	0	-2	-2	-2	-2	2	2
χ_{83}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{86}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0

The character table of 2^7:S_8(continued)

	$4B$		$4C$						$4D$							
	$8I$	$8J$	$4AE$	$4AF$	$4AG$	$4AH$	$8K$	$8L$	$4AL$	$8L$	$8M$	$8N$	$8O$	$8P$	$8Q$	$8R$
χ_{45}	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2	0
χ_{46}	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2	0
χ_{47}	0	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2
χ_{48}	0	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2
χ_{49}	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2	0
χ_{50}	0	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2
χ_{51}	0	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2
χ_{52}	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2	0
χ_{53}	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2	0
χ_{54}	0	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2
χ_{55}	0	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2
χ_{56}	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2	2
χ_{57}	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2	0
χ_{58}	0	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2
χ_{59}	0	0	0	0	0	0	0	0	0	-2	0	2	0	-2	0	2
χ_{60}	0	0	0	0	0	0	0	0	2	0	-2	0	2	0	-2	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2	-2
χ_{68}	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2	0
χ_{69}	0	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2
χ_{70}	0	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2
χ_{71}	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2	0
χ_{72}	0	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2
χ_{73}	0	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2
χ_{74}	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2	0
χ_{75}	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2	0
χ_{76}	0	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2
χ_{77}	0	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2
χ_{78}	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2	0
χ_{79}	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2	0
χ_{80}	0	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2
χ_{81}	0	0	0	0	0	0	0	0	0	-2	0	-2	0	2	0	2
χ_{82}	0	0	0	0	0	0	0	0	2	0	2	0	-2	0	-2	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

7

A group of the form $2^6{:}A_8$ as an inertia factor group of $2^8{:}O_8^+(2)$

Prologue

The group $\overline{G} = 2^6{:}A_8$ is an inertia factor group of $2^8{:}O_8^+(2)$. This group is also a maximal subgroup of $O_8^+(2)$ of index 135 and order 1290240. As an inertia factor group, our group $\overline{G}$ plays an essential role in the construction of the character table of $2^8{:}O_8^+(2)$ as there is a block of irreducible characters in the character table of $2^8{:}O_8^+(2)$ corresponding to $\overline{G}$. In this chapter we look at two ways of constructing $\overline{G}$. In the first method, we use combinatorics and the natural action of A_8 on 2^6. In the second method, we use GAP and we construct $\overline{G}$ inside $O_8^+(2)$. We then compute the Fischer-Clifford matrices of $\overline{G}$ which can then be used together with the ordinary character tables of the inertia factors of A_8 to compute its full ordinary character table. For more reading, on the methods used, one can also go to [1, 2, 3, 26, 37, 81, 82, 99, 120, 126, 94].

7.1. Introduction

The group $\overline{G} = 2^6{:}A_8$ is a maximal subgroup of $O_8^+(2)$ of index 135 and order 1290240. It is also an inertia factor of $O_8^+(2)$. As an inertia factor it plays an essential role in the construction of the character table of $2^8{:}O_8^+(2)$ as there is a block of irreducible characters in this table that corresponds to $\overline{G}$. In the construction of $\overline{G}$, A_8 acts on the elementary abelian group 2^6. The action on 2^6 is multiplication on the right of the six dimensional row vector space $N = 2^6$. This requires A_8 to be represented by 6×6 matrices. It then becomes necessary to reconstruct A_8 from a 8×8 representation to a 6×6 representation. Here we look at two ways to do this.

Although it is much simpler and natural to consider the embedding of $2^6{:}A_8$ into $O_8^+(2)$ (see Section 7.3), but it is interesting to construct this group combinatorially and this is our main reason for discussing the first method. In our first method, we first take an 8-dimensional module V on which S_8 acts naturally by permuting its basis elements. We then obtain two submodules of V, namely

M_1 and M_2 of dimensions 1 and 7 respectively. Let $W = M_2/M_1$, then $dim(W) = 6$ and W is a G-invariant where $G = S_6$ or A_8 (see Theorem 7.2.2). Let α and β be two permutation cycles of orders 7 and 3 respectively where $A_8 =< \alpha, \beta >$. We then, by the action of α and β on the generators of W, get a matrix representation of both α and β. These are 6×6 matrix representations. We are then able to represent A_8 by 6×6 matrices. Letting this A_8 act on W, we obtain three orbits of lengths $1, 28$ and 35 respectively. These have corresponding point stabilizers which we obtain from the $\mathbb{ATLAS}$ [23] . We are then able to construct $\overline{G}$.

For the second method we use GAP . We first construct $O_8^+(2)$ from the general orthogonal group $GO_8^+(2)$. We then construct, $\overline{G} = 2^6{:}A_8$, inside $O_8^+(2)$. This has only one proper normal subgroup, namely 2^6, which we can always obtain from $\overline{G}$. We then obtain the 6 generators of 2^6 which are 8×8 matrices. From the generators of $\overline{G}$, we are able to get two, 8×8 matrix generators of A_8 namely, a and b each of order 4. We then let a and b act on the generators of 2^6 by conjugation. Since $2^6 \trianglelefteq \overline{G}$ the result of these actions are elements of 2^6. We get a 6×6 matrix representation of both. This leads us to a 6×6 representation of A_8. We then let this A_8, using GAP, to act on 2^6. Using the representatives of resulting orbits, we obtain corresponding point stabilizers of A_8. These turn out to be the same as those obtained by combinatorics above.

The two groups constructed have the same character table, and through GAP, one can confirm that they are in deed isomorphic. We compute the Fischer-Clifford matrices which together with the character tables of the inertia factor groups of A_8 we use to compute the full character table of $2^6{:}A_8$. Note that there might be other easier methods to achieve this but our aim is to use the Fischer-Clifford theory to compute the character table.

7.2. The Combinatorics Method

The combinatorics method can also be found in [1] and [99] and is used extensively in [94] and [126]. The group S_8 acts naturally on a module of dimension 8 by permuting the basis elements which generate the module. Let V be the 8-dimensional natural module of S_8 over $GF(2)$, where $V =< e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 >$, and $e_i^2 = 1$ for $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$. We regard V as a multiplicative elementary abelian 2-group of order 2^8.

Theorem 7.2.1. Let V be the natural module of S_8 over $GF(2)$. Then there exist S_8 submodules M_1 and M_2 of V such that $V \supset M_2 \supset M_1 \supset 0$ and that

$$dim(M_2) = 7 \qquad \text{and} \qquad dim(M_1) = 1.$$

PROOF. Let $V =< e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 >$, and $e_i^2 = 1$ for $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$. Then S_8 acts naturally on V and this natural action results in the following orbits :

1. $O_0 = \{1_V\}$ and $|O_0| = 1$.

2. $O_1 = \{e_i | 1 \leq i \leq 8\}$ and $|O_1| = 8$.

3. $O_2 = \{e_ie_j | 1 \leq i, j \leq 8, i \neq j\}$ and $|O_2| = \binom{8}{2} = 28$.

4. $O_3 = \{e_ie_je_k | 1 \leq i, j, k \leq 8,$ distinct $i, j, k\}$ and $|O_3| = \binom{8}{3} = 56$.

5. $O_4 = \{e_ie_je_ke_l | 1 \leq i, j, k, l \leq 8,$ distinct $i, j, k, l\}$ and $|O_4| = \binom{8}{4} = 70$.

6. $O_5 = \{e_ie_je_ke_le_m | 1 \leq i, j, k, l, m \leq 8,$ distinct $i, j, k, l, m\}$ and $|O_5| = \binom{8}{5} = 56$.

7. $O_6 = \{e_ie_je_ke_le_me_n | 1 \leq i, j, k, l, m, n \leq 8,$ distinct $i, j, k, l, m, n\}$ and $|O_6| = \binom{8}{6} = 28$.

8. $O_7 = \{e_ie_je_ke_le_me_ne_o | 1 \leq i, j, k, l, m, n, o \leq 8,$ distinct $i, j, k, l, m, n, o\}$ and $|O_7| = \binom{8}{7} = 8$.

9. $O_8 = \{e_ie_je_ke_le_me_ne_oe_p | 1 \leq i, j, k, l, m, n, o, p \leq 8,$ distinct $i, j, k, l, m, n, o, p\}$ and $|O_8| = \binom{8}{8} = 1$.

Thus S_8 produces 9 orbits on V. Set $M_1 =< e_1e_2e_3e_4e_5e_6e_7e_8 >$. Then M_1 is an S_8 - invariant submodule of V with $dim(M_1) = 1$. Now set $M_2 = O_0 \cup O_2 \cup O_4 \cup O_6 \cup O_8$. Then $|M_2| = 128$, so we have $dim(M_2) = 7$. Since $M_1 = O_0 \cup O_8$, we obtain that $V \supset M_2 \supset M_1 \supset 0$. This implies that M_2 is a reducible S_8 - invariant submodule of V. ■

Since S_8 is 8-transitive, A_8 is 6-transitive on $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$. It is clear that $O_0, O_1, O_2, O_4, O_5, O_6$ are also orbits under the action of A_8. Now since A_8 does not have a proper subgroup of index less than 8, O_7 remains as an orbit of length 8. Obviously O_8 also remains as an orbit of length 1.

Theorem 7.2.2. Let $W = M_2/M_1$, then $dim(W) = 6$. Also W is a G - invariant module where $G = S_8$ or A_8.

PROOF. It is clear that $dim(W) = 6$, since $dim(M_1) = 1$ and $dim(M_2) = 7$. If $g \in G$ and $\alpha \in M_2$, then since M_2 is G invariant, $g(\alpha M_1) = g(\alpha)M_1 \in M_2/M_1 \;\forall\; g \in G$ and $\alpha \in M_2$. So W is S_8 (A_8) invariant. ■

Let $W =< e_1e_2M_1, e_1e_3M_1, e_1e_4M_1, e_1e_5M_1, e_1e_6M_1, e_1e_7M_1 >$. The set $B = \{e_1e_2, e_1e_3, e_1e_4, e_1e_5, e_1e_6, e_1e_7\}$ is a linearly independent set. Let

$$\gamma_1 = e_1e_2M_1, \gamma_2 = e_1e_3M_1, \gamma_3 = e_1e_4M_1, \gamma_4 = e_1e_5M_1, \gamma_5 = e_1e_6M_1, \gamma_6 = e_1e_7M_1.$$

Also, if $\alpha = (1\ 2\ 3\ 4\ 5\ 6\ 7)$ and $\beta = (6\ 7\ 8)$ then $A_8 =< \alpha, \beta >$.

We obtain

$$\alpha : \gamma_1 \to \gamma_1\gamma_2,\ \gamma_2 \to \gamma_1\gamma_3,\ \gamma_3 \to \gamma_1\gamma_4,\ \gamma_4 \to \gamma_1\gamma_5,\ \gamma_5 \to \gamma_1\gamma_6,\ \text{and} \quad \gamma_6 \to \gamma_1.$$

We give two examples for the action of α. Under the action of α we have

$$\gamma_2 = e_1e_3M_1 \to e_2e_4M_1 = e_1e_2e_1e_4M_1 = \gamma_1\gamma_3$$

That is $\alpha(\gamma_2) = \gamma_1\gamma_3$. Also

$$\gamma_6 = e_1e_7M_1 \to e_2e_1M_1 = \gamma_1.$$

That is $\alpha(\gamma_6) = \gamma_1$. Hence α can be represented by the following matrix

$$\alpha = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

with $o(\alpha) = 7$.

Similarly for β we have

$$\beta : \gamma_1 \to \gamma_1,\ \gamma_2 \to \gamma_2,\ \gamma_3 \to \gamma_3,\ \gamma_4 \to \gamma_4,\ \gamma_5 \to \gamma_6,\ \gamma_6 \to \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\gamma_6$$

As an example we see that

$$\gamma_6 = e_1e_7M_1 \to e_1e_8M_1 = e_2e_3e_4e_5e_6e_7M_1 = \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\gamma_6.$$

That is $\beta(\gamma_6) = \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\gamma_6$. Here we obtain

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix},$$

with $o(\beta) = 3$. We are now able to write all the elements of A_8 as 6×6 matrices. By acting A_8 directly on W, using the orbits of A_8 on M_2 and the fact that $M_1 = \{1, e_1e_2e_3e_4e_5e_6e_7e_8\}$, we can see that A_8 has 3 orbits namely

$$\Delta_0 = \{O_0M_1\} = \{O_8M_1\} = \{M_1\},$$

$$\Delta_1 = \{O_2M_1\} = \{O_6M_1\} = \{e_ie_jM_1|\text{distinct}\ \ e_i, e_j\},$$

$$\Delta_2 = \{O_4M_1\} = \{e_ie_je_ke_lM_1|\text{distinct}\ \ e_i, e_j, e_k, e_l\}.$$

Clearly $|\Delta_0| = 1,\ |\Delta_1| = 28,\ \ |\Delta_2| = \frac{70}{2} = 35$ and $W = \Delta_0 \cup \Delta_1 \cup \Delta_2$.

Theorem 7.2.3. A_8 acts irreducibly on W.

PROOF. Let $U \leq W$ be such that $U \neq 0$ and U is A_8 invariant. Since $U \neq 0\ \ \exists\ x \in U$ such that $x \neq 0$. Since $x \neq 0$ and $U \leq W$ we have two cases.

Case 1 : Suppose $x \in \Delta_1,\ \ x = e_ie_jM_1$ for distinct i, j. Hence $g(x) \in U\ \ \forall\ \ g \in A_8$. However

$$\{g(x)|g \in A_8\} = \Delta_1 \quad \Rightarrow \Delta_1 \subseteq U \quad \Rightarrow e_ie_jM_1 \in U\ \ \forall\ i, j.$$

Hence we have $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6 \in U$.

Case 2: Suppose $x \in \Delta_2$, then $x = e_ie_je_ke_lM_1$ for some distinct e_i, e_j, e_k, e_l. Hence $g(x) \in U\ \ \forall\ g \in A_8$. Now

$$\{g(x)|g \in A_8\} = \Delta_2 \quad \Rightarrow \Delta_2 \subseteq U.$$

Since $\Delta_2 \subseteq U,\ e_ke_le_me_rM_1$ and $e_ke_le_me_iM_1$ are in U for distinct k, l, m, r, i. Since U is closed we get

$$(e_ke_le_me_rM_1)(e_ke_le_me_iM_1) = e_ie_r \in U\ \ \forall\ \ \text{distinct}\ \ k, l, m, r, i.$$

This shows that $U \subseteq W$. So similar to case 1, we have $U = W$.

Hence W is a unique 6-dimensional $GF(2)$ module that A_8 acts irreducibly on. ■

By methods of coset analysis that can be found in chapter 2, when $G = A_8$ acts on W we obtain three orbits of lengths $1, 28$ and 35 respectively. These have corresponding point stabilizers K_1, K_2 and K_3 of indices $1, 28$ and 35 respectively. One can immediately see that $K_1 = G$ and K_2, K_3 must each sit in a maximal subgroup of G. However any maximal subgroup of G which contains K_i must have an order divisible by $|K_i|$ and its index in G must divide 28 and 35 respectively. From the $\mathbb{ATLAS}$ [23], we get that up to isomorphism and conjugacy there is only one maximal subgroup of G, in each case, that would contain K_2 and the other K_3 and these are the symmetric group S_6 and the group $2^4{:}(S_3 \times S_3)$ respectively. However since $|K_2| = |S_6|$ we have $K_2 \cong S_6$. Similarly we have $K_3 \cong 2^4{:}(S_3 \times S_3)$. For each $g \in G$, the number of fixed points $g \in G$ in N is equal to $k = |C_N(g)|$. Since the zero vector of N is fixed by every $g \in G$ we have

$$k = 1 + \chi(G|K_2)(g) + \chi(G|K_3)(g) = 1 + (\chi(G|K_2) + \chi(G|K_3))(g).$$

From this we determine, $\chi = \chi(A_8|2^6)$, the permutation character of A_8 on 2^6. We have

$$\chi = 1a + I_{S_6}^{A_8} + I_{2^4:(S_3 \times S_3)}^{A_8} = 3 \times 1a + 7a + 14a + 2 \times 20a,$$

where $I_{S_6}^{A_8} = 1a + 7a + 20a$ and $I_{2^4:(S_3 \times S_3)}^{A_8} = 1a + 14a + 20a$, are the characters of A_8 induced from the identity characters of S_6 and $2^4{:}(S_3 \times S_3)$ respectively. Since $C_N(g) \leq N$, we must have $k = 2^n$ where $n \in \{1, 2, 3, 4, 5, 6\}$. Hence we obtain the values of the $k's$ in Table 7.1 .

Table 7.1:

$[g]_{A_8}$	1a	2a	2b	3a	3b	4a	4b	5a	6a	6b	7a	7b	15a	15b
$\chi(A_8\|S_6)$	28	4	8	10	1	0	2	3	1	2	0	0	0	0
$\chi(A_8\|2^4:(S_3 \times S_3))$	35	11	7	5	2	3	1	0	2	1	0	0	0	0
k	64	16	16	16	4	4	4	4	4	4	1	1	1	1

7.3. The GAP Method

In this section for all our computations we use GAP [41]. We first construct $O_8^+(2)$ inside the general orthogonal group $GO_8^+(2)$. This we do by getting the maximal normal subgroup of $GO_8^+(2)$ and this is a group of 8×8 matrices of size 174182400 over $GF(2)$. We then construct $\overline{G} = 2^6{:}A_8$ inside $O_8^+(2)$ by first constructing an 8-dimensional row vector space U, over $GF(2)$. We then let $O_8^+(2)$ to act on U and we get three orbits of lengths $1, 120$ and 135. Using the $\mathbb{ATLAS}$ [23] and Programme C (see Appendix A), given below, the maximal subgroup of index 135 is $2^6{:}A_8$ which corresponds to the third orbit. We then get the stabilizer of a representative of this orbit in $O_8^+(2)$, which gives us a group of 8×8 matrices of size 1290240 which is our $2^6{:}A_8$. We are now ready to construct 2^6 and A_8 inside our $\overline{G}$. Note that we use $\mathbb{ATLAS}$ [23] for character tables of A_8 and S_6. The character table of $2^4{:}(S_3 \times S_3)$ is given in the GAP Library [41], but it is also presented in the table below.

Character Table of $2^4{:}(S_3 \times S_3)$

	1A	2A	2B	2C	4A	4B	3A	6A	3B	3C	2D	2E	4C	4D	6B	6C
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_3	1	1	1	-1	-1	-1	1	1	1	1	-1	1	-1	1	1	-1
χ_4	1	1	1	-1	-1	-1	1	1	1	1	1	-1	1	-1	-1	1
χ_5	2	2	2	0	0	0	-1	-1	-1	2	0	2	0	2	-1	0
χ_6	2	2	2	0	0	0	-1	-1	-1	2	0	-2	0	-2	1	0
χ_7	2	2	2	0	0	0	-1	-1	2	-1	-2	0	-2	0	0	1
χ_8	2	2	2	0	0	0	-1	-1	2	-1	2	0	2	0	0	-1
χ_9	4	4	4	0	0	0	1	1	-2	-2	0	0	0	0	0	0
χ_{10}	6	2	-2	2	-2	0	3	-1	0	0	0	0	0	0	0	0
χ_{11}	6	2	-2	2	-2	0	3	-1	0	0	0	0	0	0	0	0
χ_{12}	9	-3	1	1	1	-1	0	0	0	0	3	3	-1	-1	0	0
χ_{13}	9	-3	1	1	1	-1	0	0	0	0	-3	-3	1	1	0	0
χ_{14}	9	-3	1	-1	-1	1	0	0	0	0	3	-3	-1	1	0	0
χ_{15}	9	-3	1	-1	-1	1	0	0	0	0	-3	3	1	-1	0	0
χ_{16}	12	4	-4	0	0	0	-3	1	0	0	0	0	0	0	0	0

We first pay our attention to A_8. We first obtain the generators of $\overline{G}$. We get four of these and from the four we pick two, call them a and b that generate A_8 . We give a, b and their inverses below. Note that $o(a) = o(b) = 4$.

$$a = \begin{bmatrix} 1&0&0&0&0&1&0&0\\ 0&1&0&1&0&1&0&1\\ 1&0&1&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 1&1&0&0&1&1&0&0\\ 0&0&0&0&0&1&0&0\\ 1&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}, \quad b = \begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&1&1&1&1&1&0\\ 1&0&0&1&0&0&0&1\\ 1&0&1&0&0&0&0&1\\ 1&1&0&0&0&1&1&1\\ 1&1&1&1&0&1&0&1\\ 0&1&0&0&0&0&1&0\\ 1&1&1&1&0&0&0&0 \end{bmatrix}$$

$$a^{-1} = \begin{bmatrix} 1&0&0&0&0&1&0&0\\ 0&1&0&1&0&1&0&1\\ 1&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&1&0&1&1&1&0&1\\ 0&0&0&0&0&1&0&0\\ 1&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}, \quad b^{-1} = \begin{bmatrix} 0&0&0&0&1&1&1&1\\ 0&0&1&1&1&1&1&0\\ 1&0&0&1&1&0&1&0\\ 1&0&1&0&1&0&1&0\\ 1&1&0&0&1&1&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&1&1&1&1&0&0\\ 1&0&0&0&0&1&0&1 \end{bmatrix}.$$

The group $N = 2^6$ is the only proper normal subgroup of $\overline{G}$ and we use GAP [41] to obtain this normal subgroup. We then obtain its generators, which are given below.

$$\gamma_1 = \begin{bmatrix} 1&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}, \quad \gamma_2 = \begin{bmatrix} 1&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&1 \\ 1&1&0&0&1&1&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix},$$

$$\gamma_3 = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&1 \\ 1&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}, \gamma_4 = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 1&0&0&1&0&1&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix},$$

$$\gamma_5 = \begin{bmatrix} 1&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 0&1&0&1&0&1&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}, \quad \gamma_6 = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0&1 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&1 \\ 0&0&1&1&1&1&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}.$$

Noting that the generators of 2^6 and A_8 are both 8×8 matrices . Computing the conjugate of each γ_i with respect to a, that is $a\gamma_i a^{-1}$ and noting that 2^6 is normal in $2^6{:}A_8$ we get that $a\gamma_i a^{-1} = \gamma_{j_1}\gamma_{j_2}\cdots\gamma_{j_k}$, where $\gamma_{j_r} = \gamma_j$ or 1 for some $j_r = 1, \cdots, 6$. We denote this as $\gamma_i \rightarrow \gamma_{j_1}\gamma_{j_2}\cdots\gamma_{j_k}$. We then get

$$\gamma_1 \rightarrow \gamma_1,\ \gamma_2 \rightarrow \gamma_2\gamma_5,\ \gamma_3 \rightarrow \gamma_2\gamma_4\gamma_5\gamma_6,\ \gamma_4 \rightarrow \gamma_4,\ \gamma_5 \rightarrow \gamma_1\gamma_4\gamma_5,\ \gamma_6 \rightarrow \gamma_1\gamma_2\gamma_3\gamma_4.$$

Similarly with b we get

$$\gamma_1 \rightarrow \gamma_1\gamma_2\gamma_3\gamma_4\gamma_6,\ \gamma_2 \rightarrow \gamma_2\gamma_3\gamma_4\gamma_5\gamma_6,\ \gamma_3 \rightarrow \gamma_1\gamma_2\gamma_3\gamma_5\gamma_6,\ \gamma_4 \rightarrow \gamma_1\gamma_3\gamma_4,$$

$$\gamma_5 \rightarrow \gamma_4,\ \gamma_6 \rightarrow \gamma_2\gamma_3\gamma_4.$$

Representing this information in matrix form, where the i-th row will correspond to the i-th conjugate, we get a 6×6 matrix representation of $G = A_8$. Hence we have $A_8 =< a', b' >$, where

$$a' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}, \quad b' = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

We now turn our attention to the inertia factor groups of $2^6{:}A_8$. Here we use Programme C below, to act G on $Irr(N)$, $N = 2^6$. To be able to do this we need to rewrite N as a row vector space V of dimension 6 over $GF(2)$, that is V:= FullRowSpace(GF(2),6). We have two procedures at our disposal. First we can act G on V from right and this action gives us the orbits of G acting as a permutation group on the conjugacy classes of N. Secondly we act G^t, that is the set consists of transpose of elements of G, on V from right. This action is equivalent to multiplying the column vectors of V on the left by G. This action gives the orbits of G acting as a permutation group on the irreducible characters of N.

PROGRAMME C for $2^6{:}A_8$

```
gap>V:=FullRowSpace(GF(2), 6);
gap>m1:=(OneGF(2))*[6 × 6 matrix group generators];
gap>m2:=(OneGF(2))*[6 × 6 matrix group generators];
gap>m:=Group(m1,m2);
gap>k:=OrbitLengths(m,V);
gap>l:= OrbitLengths(Group(List(m,TransposedMat)),N);
```

From the above, the action of G on $Irr(N)$ produces three orbits of lengths $1, 28$ and 35 respectively. We then take representatives of the orbits of lengths 28 and 35. For each of the orbit representative we find its stabilizer in G. For the representative of the orbit of length 28, the corresponding stabilizer is a group of 6×6 matrices of size 720 isomorphic to S_6. For the orbit of length 35 the corresponding stabilizer is a group of 6×6 matrices of size 576 isomorphic to $2^4 : (S_3 \times S_3)$. This is the same result which we got in Section 2. We use GAP to check our calculations for the number of fixed points using Programme F (see Appendix A) and we list the values of the k's in Table 7.2.

Table 7.2:

$[g]_{A_8}$	1a	2a	2b	3a	3b	4a	4b	5a	6a	6b	7a	7b	15a	15b
k	64	16	16	16	4	4	4	4	4	4	1	1	1	1

Since the two $2^6{:}A_8$ constructed are isomorphic we use one of them to compute the Fisher-Clifford matrices and its character table.

7.4. The Conjugacy Classes of $2^6{:}A_8$

We first give the representatives of the conjugacy classes of A_8 in Table 7.3.

Table 7.3: Conjugacy classes of A_8

$[g]_G$	6×6 matrix	$\lvert[g]_G\rvert$	$[g]_G$	6×6 matrix	$\lvert[g]_G\rvert$
1A	$\begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	1	2A	$\begin{pmatrix} 0&0&0&1&0&0\\ 1&0&1&0&0&1\\ 0&1&0&1&0&1\\ 1&0&0&0&0&0\\ 0&1&1&1&1&1\\ 0&0&0&0&0&1 \end{pmatrix}$	105
2B	$\begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&1\\ 1&0&1&0&1&0 \end{pmatrix}$	210	3A	$\begin{pmatrix} 0&1&1&1&1&0\\ 0&1&0&0&0&0\\ 1&1&0&0&0&0\\ 1&1&1&1&0&0\\ 0&0&0&1&0&0\\ 0&0&0&1&1&1 \end{pmatrix}$	112
3B	$\begin{pmatrix} 1&1&1&0&0&0\\ 1&0&1&1&0&1\\ 1&0&1&0&0&1\\ 0&1&1&1&0&0\\ 0&1&0&0&0&0\\ 0&0&1&1&1&1 \end{pmatrix}$	1 120	4B	$\begin{pmatrix} 0&0&1&1&1&1\\ 0&0&1&1&0&0\\ 0&0&0&0&1&0\\ 1&0&1&0&1&1\\ 0&1&0&1&0&1\\ 1&1&1&0&0&0 \end{pmatrix}$	1 260
4A	$\begin{pmatrix} 0&0&1&1&1&0\\ 0&0&1&1&0&0\\ 0&0&0&1&0&0\\ 1&0&1&1&0&1\\ 0&1&0&1&0&1\\ 1&1&1&1&1&1 \end{pmatrix}$	2 520	5a	$\begin{pmatrix} 0&1&0&1&0&1\\ 1&0&1&0&0&1\\ 0&0&0&1&0&0\\ 0&0&0&1&1&1\\ 0&0&0&0&0&1\\ 0&1&1&1&1&0 \end{pmatrix}$	1 344
6A	$\begin{pmatrix} 0&0&0&1&0&0\\ 1&0&1&0&0&1\\ 1&0&1&0&1&1\\ 0&1&1&1&1&0\\ 1&0&0&0&0&0\\ 1&1&1&1&1&1 \end{pmatrix}$	1 680	6B	$\begin{pmatrix} 0&0&1&1&1&1\\ 0&0&1&0&0&0\\ 1&0&1&1&0&1\\ 1&0&1&0&1&1\\ 0&1&1&1&1&0\\ 1&0&0&0&0&0 \end{pmatrix}$	3 360
7A	$\begin{pmatrix} 0&0&0&0&1&0\\ 0&0&1&1&0&0\\ 1&0&0&1&0&1\\ 1&0&1&1&0&1\\ 0&1&1&0&0&0\\ 1&0&0&0&0&0 \end{pmatrix}$	2 880	7B	$\begin{pmatrix} 1&0&1&0&0&1\\ 1&0&0&1&0&1\\ 1&1&0&0&0&0\\ 1&1&1&1&1&1\\ 0&0&0&1&1&1\\ 0&1&1&0&0&0 \end{pmatrix}$	2 880
15A	$\begin{pmatrix} 1&0&1&0&1&1\\ 1&0&1&0&0&1\\ 1&1&1&1&0&0\\ 1&1&1&1&1&1\\ 0&0&0&1&1&1\\ 0&1&1&0&0&0 \end{pmatrix}$	1 344	15B	$\begin{pmatrix} 0&0&1&1&1&0\\ 0&0&1&1&1&1\\ 1&1&1&1&1&1\\ 1&0&0&0&0&0\\ 0&1&1&0&0&0\\ 1&0&1&1&0&1 \end{pmatrix}$	1 344

From the methods of coset analysis, which can also be found in chapter 2 and by Programmes A and B (see Appendix A), we are able to compute the conjugacy classes of $2^6{:}A_8$ which are given in Table 7.4. We give a very brief summary of coset analysis. We look at the action of $\overline{G}$ on $N\overline{g}$, for the split extension it is suffices to look at the coset $Ng, g \in G$. First N acts on Ng and we get k orbits. Then we act $C_G(g)$ on these orbits and f_j of these orbits, fuse to form one orbit with $\sum f_j = k$, and d_j a representative of these fused orbits. For this we use Programme A (see Appendix A).

PROGRAMME A for $2^6{:}A_8$

```
gap>V:=FullRowSpace(GF(2), 6);
gap>gr1:=(OneGF(2))*[6 × 6 matrix group generators];
gap>gr2:=(OneGF(2))*[6 × 6 matrix group generators];
gap>grp:=Group(gr1,gr2);
gap>Ccl:=ConjugacyClasses(grp);
gap>O:=Union(Orbits(grp,V));
gap>for i in [1..14] do
>Print(Representative(Ccl[i]));
>w:=One(GF(q))*[0, 0, ··· , 0];
>e:=[ ];
>while Difference(O,e) <> [ ] do
>d:=[ ];
>for x in O do;
>y:=[x+w+(x*(Representative((Ccl)[i]))];
>d:=Union(d,y);
>od;
>Print(d);
>e:=Union(d,e);
>if Difference(O,e) <> [ ] then
>w:=Representative(Difference(O,e));
>fi;
>od;
>r:=[ ];
>u:=One(GF(2))*[0, 0, ··· , 0];
>while Difference(O,e) <> [ ] do
>m:=[ ];
>for g in Centralizer(grp,Representative(Ccl[i])) do
>l:=[u*g];
>m:=Union(m,l);
>od;
>Print("A block for the vectors under the action of a centralizer");
>Print(m);
>r:=Union(m,r);
>if Difference(O,r) <> [ ] then
>u:=Representative(Difference(O,r));
>fi;
>od;
>Print("*********************************");
>od;
```

Let $o(dg) = k$ and $o(g) = m$. If $w = (dg)^m$, then if $w = (0, 0, 0, 0, 0, 0),\ k = m$. On the other hand if $w \neq (0, 0, 0, 0, 0, 0)$, then $k = 2m$. To get w we use Programme B (see Appendix A).

PROGRAMME B for $2^6{:}A_8$

gap>V:=FullRowSpace($GF(2),6$);
gap>m1:=(OneGF(2))*[6×6 matrix group generators];
gap>m2:=(OneGF(q))*[6×6 matrix group generators];
gap>m:=Group(m1,m2);
gap>c:=ConjugacyClasses(m);
gap>g:=Representative(c[i]);
gap>d:=One(GF(2))*[$\alpha_1, \alpha_2, \cdots, \alpha_6$];
gap>w:=$d + d * g + d * g^2 + \cdots + d * g^{k-1}$;

gap>Print(w);

We obtain that $2^6{:}A_8$ has altogether 41 conjugacy classes which are given in Table 7.4 below.

Table 7.4: Conjugacy Classes of $2^6{:}A_8$

$g \in A_8$	k	f_j	d_j	w	$[x]_{2^6:A_8}$	$\|C_{2^6:A_8}(x)\|$
$1A$	2^6	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$1A$	1 290 240
		28	$(0,0,0,0,0,1)$	$(0,0,0,0,0,1)$	$2A$	46 080
		35	$(0,0,0,1,0,1)$	$(0,0,0,1,0,1)$	$2B$	36 864
$2A$	2^4	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2C$	3 072
		1	$(0,0,1,0,0,1)$	$(0,0,0,0,0,0)$	2D	3 072
		1	$(0,0,1,1,1,1)$	$(0,0,0,0,0,0)$	2E	3 072
		1	$(0,0,1,1,1,1)$	$(0,0,0,0,0,0)$	2F	3 072
		12	$(0,0,0,0,0,1)$	$(1,1,1,0,0,1)$	4A	256
$2B$	2^4	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2G$	1 536
		1	$(0,0,1,1,1,1)$	$(0,0,1,0,0,0)$	4B	1 536
		3	$(1,0,1,0,1,0)$	$(0,0,0,0,0,0)$	2H	512
		3	$(0,0,1,1,0,1)$	$(0,0,1,0,1,0)$	4C	512
		8	$(0,0,0,0,0,1)$	$(0,0,0,1,1,0)$	4D	192
$3A$	2^4	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$3A$	2 880
		5	$(1,1,0,0,1,0)$	$(0,0,1,1,1,1)$	6A	576
		10	$(0,1,0,0,1,1)$	$(0,0,1,1,0,0)$	6B	288
$3B$	2^2	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$3B$	72
		1	$(1,1,0,1,0,0)$	$(1,0,0,0,0,1)$	$6B$	72
		1	$(1,0,1,0,1,0)$	$(1,0,1,0,1,0)$	$6C$	72
		1	$(0,0,0,1,0,0)$	$(1,1,1,1,1,1)$	$6D$	72
$4A$	2^2	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4E$	64
		1	$(1,1,0,1,0,0)$	$(0,0,0,0,0,0)$	$4F$	64
		1	$(1,0,1,0,1,0)$	$(0,0,0,0,0,0)$	$4G$	64
		1	$(0,0,0,1,0,0)$	$(0,0,0,0,0,0)$	$4H$	64
$4B$	2^2	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4I$	32
		1	$(1,1,0,1,0,0)$	$(0,0,1,1,1,1)$	$8A$	32
		1	$(1,0,1,0,1,0)$	$(0,0,0,0,0,0)$	$4J$	32
		1	$(0,0,0,1,0,0)$	$(1,1,1,0,1,0)$	$8B$	32
$5A$	2^2	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$5A$	60
		3	$(0,1,0,1,0,0)$	$(0,0,0,1,0,1)$	$10A$	20
$6A$	2^2	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6F$	48
		1	$(1,0,1,0,1,0)$	$(0,1,1,0,1,0)$	$12A$	48
		2	$(1,0,1,0,1,0)$	$(1,0,1,0,1,0)$	$12B$	24
$6B$	2^2	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6G$	24
		1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,1)$	$6H$	24

continued on next page

Table 7.4 (continued from previous page)

$g \in AS_8$	k	f_j	d_j	w	$[x]_{2^7:A_8}(x)\|$	
		1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6I$	24
		1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6J$	24
$7A$	1	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$7A$	7
7B	1	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$7B$	7
15A	1	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$15A$	15
15B	1	1	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$15B$	15

7.5. The Fischer-Clifford matrices of $2^6{:}A_8$

The inertia factor groups are $H_1 = A_8$, $H_2 = S_6$ and $H_3 = 2^4{:}(S_3 \times S_3)$, as we discuss in section 7.3. We construct H_2 and H_3 inside A_8 in terms of 6×6 matrices. Their conjugacy classes are given in Table 7.5 and Table 7.6 respectively. The fusions of the inertia factor groups into A_8, which can also be done using Programme D (see Appendix A), are given in Table 7.7.

PROGRAMME D for $2^6{:}A_8$

```
gap>g:=Group(H1);
gap>T1:=CharacterTable(g);
gap>h:=Group(H2);
gap>T2:=CharacterTable(h);
gap>k:=Group(H3);
gap>T3:=CharacterTable(k);
gap>FusionConjugacyClasses(h,g);
gap>FusionConjugacyClasses(k,g);
```

Using these fusions, and properties of Fischer-Clifford matrices which can be found in chapter 5.2.1, we are now able to obtain the Fischer-Clifford matrices of $2^6{:}A_8$. For each class representative $g \in A_8$, we construct a Fischer-Clifford matrix $M(g)$ which are given in the Table 7.8.

Table 7.5: Conjugacy classes of S_6

$[g]_{S_6}$	6×6 matrix	$\lvert [g]_{S_6} \rvert$	$[g]_{S_6}$	6×6 matrix	$\lvert [g]_{S_6} \rvert$
1A	$\begin{pmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	1	2A	$\begin{pmatrix} 1&0&1&0&1&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&1 \\ 0&0&0&1&0&1 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&1 \end{pmatrix}$	45
2B	$\begin{pmatrix} 1&0&0&0&0&0 \\ 1&1&0&1&1&0 \\ 0&0&1&1&1&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	15	2C	$\begin{pmatrix} 1&0&0&1&1&0 \\ 0&1&1&1&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	15
3A	$\begin{pmatrix} 1&0&1&1&0&0 \\ 0&0&1&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	40	3B	$\begin{pmatrix} 1&0&0&1&1&0 \\ 0&0&1&1&0&0 \\ 1&1&1&1&1&0 \\ 1&0&0&0&1&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	40
4A	$\begin{pmatrix} 1&0&1&1&0&0 \\ 1&1&0&1&1&0 \\ 0&0&1&1&1&1 \\ 0&0&0&1&0&1 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&1 \end{pmatrix}$	90	4B	$\begin{pmatrix} 1&0&1&1&0&0 \\ 0&0&1&1&0&0 \\ 0&0&1&1&1&0 \\ 0&1&0&1&0&1 \\ 0&0&0&1&0&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	90
6A	$\begin{pmatrix} 1&0&1&1&0&0 \\ 1&1&0&1&1&0 \\ 0&1&1&1&1&0 \\ 0&1&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	120	6B	$\begin{pmatrix} 1&0&0&1&1&0 \\ 0&0&1&1&0&0 \\ 0&0&1&0&0&1 \\ 0&1&0&1&0&1 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&1 \end{pmatrix}$	120
5A	$\begin{pmatrix} 1&0&1&1&0&0 \\ 1&1&0&0&0&1 \\ 0&1&1&1&1&1 \\ 0&1&0&1&0&1 \\ 0&0&0&1&0&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	144			

7.6. The Character Table of $2^6{:}A_8$

We use the Fischer-Clifford matrices (Table 7.8) and the character tables of the inertia factors groups A_8, S_6, $2^4{:}(S_3 \times S_3)$ together with the fusions of these inertia factors into A_8 which are given in Table 7.7 to obtain the full character table of $2^6{:}A_8$. The Fischer-Clifford matrix $M(g)$ will be partitioned row-wise into blocks, where each block corresponds to an inertia group $\overline{H}_i$. Then given the character table of the inertia factor group H_i of $\overline{H}_i$, we therefore take the columns of this character table which correspond to the classes of H_i which fuse to class $[g]$ in A_8 and multiply these columns by the rows of the Fischer-Clifford matrix $M(g)$ which correspond to $\overline{H}_i$

Table 7.6: Conjugacy classes of $2^4 : (S_3 \times S_3)$

$[g]_{2^4:(S_3\times S_3)}$	6×6 matrix	$\lvert[g]_{2^4:(S_3\times S_3)}\rvert$	$[g]_{2^4:(S_3\times S_3)}$	6×6 matrix	$\lvert[g]_{2^4:(S_3\times S_3)}\rvert$
1A	$\begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	1	2A	$\begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&1&0\\ 0&0&0&1&1&0\\ 0&0&0&0&1&0\\ 1&1&1&1&1&1 \end{pmatrix}$	6
2B	$\begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&1&0\\ 0&0&1&0&1&0\\ 0&0&0&1&1&0\\ 0&0&0&0&1&0\\ 1&1&1&0&1&1 \end{pmatrix}$	9	2C	$\begin{pmatrix} 1&0&0&1&0&0\\ 0&1&1&1&1&0\\ 0&0&1&0&1&0\\ 0&0&0&1&1&0\\ 0&0&0&0&1&0\\ 1&1&1&0&1&1 \end{pmatrix}$	36
2D	$\begin{pmatrix} 1&0&0&0&0&0\\ 1&1&0&1&1&0\\ 0&0&1&1&1&0\\ 0&0&0&1&1&0\\ 0&0&0&0&1&0\\ 1&1&1&0&1&1 \end{pmatrix}$	12	2E	$\begin{pmatrix} 1&0&0&1&1&0\\ 1&1&1&1&0&0\\ 0&0&1&1&1&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	12
3A	$\begin{pmatrix} 0&1&0&1&1&0\\ 0&1&0&0&1&0\\ 1&1&0&1&0&0\\ 1&1&1&0&0&0\\ 0&0&0&0&1&0\\ 1&1&1&0&1&1 \end{pmatrix}$	16	3B	$\begin{pmatrix} 1&1&1&0&1&0\\ 0&1&1&1&0&0\\ 1&1&1&1&1&0\\ 1&0&1&1&1&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	32
3C	$\begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&1&1&0\\ 0&1&0&0&1&0\\ 0&1&1&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	32	4A	$\begin{pmatrix} 0&1&0&1&1&0\\ 0&1&0&0&0&0\\ 1&1&0&1&0&0\\ 1&1&1&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	36
4B	$\begin{pmatrix} 1&0&0&1&1&0\\ 0&0&1&1&0&0\\ 0&0&1&0&0&1\\ 0&1&0&1&0&1\\ 0&0&0&0&1&1\\ 0&0&0&0&0&1 \end{pmatrix}$	72	4C	$\begin{pmatrix} 1&0&0&1&0&0\\ 0&1&1&1&1&0\\ 0&0&1&0&1&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 1&0&1&1&0&1 \end{pmatrix}$	36
4D	$\begin{pmatrix} 1&0&0&1&0&0\\ 1&1&1&1&1&0\\ 0&0&1&1&1&0\\ 0&0&0&1&1&0\\ 0&0&0&0&1&0\\ 1&1&1&0&1&1 \end{pmatrix}$	36	6A	$\begin{pmatrix} 1&0&0&1&1&0\\ 0&0&1&1&0&0\\ 1&1&1&1&1&0\\ 1&0&0&0&1&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	48
6B	$\begin{pmatrix} 1&0&0&1&1&0\\ 0&1&1&1&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	96	6C	$\begin{pmatrix} 0&1&0&1&1&0\\ 1&1&0&1&1&0\\ 0&1&0&0&1&0\\ 1&0&1&1&1&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$	96

Table 7.7: Fusion of S_6 and $2^4{:}(S_3 \times S_3)$ into A_8

$[x]_{S_6}$	$\longrightarrow$	$[g_1]_{A_8}$	$[x]_{2^4:(S_3\times S_3)}$	$\longrightarrow$	$[g_1]_{A_8}$
$1A$		$1A$	$1A$		$1A$
$2A$		$2B$	$2A$		$2B$
$2B$		$2B$	$2B$		$2A$
$2C$		$2A$	$2C$		$2B$
$3A$		$3A$	$2D$		$2A$
$3B$		$3B$	$2E$		$2A$
$4A$		$4B$	$3A$		$3A$
$4B$		$4B$	$3B$		$3B$
$5A$		$5A$	$3C$		$3B$
$6A$		$6A$	$4A$		$4A$
$6B$		$6B$	$4B$		$4B$
			$4C$		$4A$
			$4D$		$4A$
			$6A$		$6A$
			$6B$		$6B$
			$6C$		$6B$

and then fill the portion of the character table of $2^6{:}A_8$ which is in the block corresponding to $\overline{H}_i$ for the classes of $2^6{:}A_8$ which come from the coset Ng. The set of irreducibles characters of $2^6{:}A_8$ will be partitioned into three blocks B_1, B_2, and B_3 corresponding to the inertia factors A_8, S_6 and $2^4{:}(S_3 \times S_3)$ respectively. In fact $B_1 = \{\chi_i|\ 1 \le i \le 14\}$, $B_2 = \{\chi_i|\ 15 \le i \le 25\}$ and $B_3 = \{\chi_i|\ 25 \le i \le 41\}$. Note that the centralizers of elements of $2^6{:}A_8$ were listed in the last column of Table 7.4. We use Fischer-Clifford matrices and partial character tables of inertia factor groups and computed the character table of $\overline{G}$. This character table is given in Table 7.10. We then convert the character table to the GAP format and used Programme E (see Appendix A) to test its validity and to compute the possible power maps. We list the power maps of $2^6{:}A_8$ in Table 7.9 .

Table 7.9: Power maps of elements of $2^6{:}A_8$

$[g]_{A_8}$	$[x]_{2^6:A_8}$	2	3	5	7	$[g]_{A_8}$	$[x]_{2^6:A_8}$	2	3	5	7
1A	1A					2A	2F	1A			
	2A	1A					2C	1A			
	2B	1A					2D	1A			
							2E	1A			
							2F	1A			
							4A	2A			
2B	2G	1A				3B	3A	1A			
	4B	2B					6C	3B	2B		
	2H	1A					6D	3B	2B		
	4C	2B					6E	3B	2A		
	4D	2B									
3A	3B	1A				4A	4E	2C			
	6A	3B	2B				4F	2D			

continued on next page

Table 7.9 (continued from previous page)

$[g]_{A_8}$	$[x]_{2^6:A_8}$	2	3	5	7	$[g]_{A_8}$	$[x]_{2^6:A_8}$	2	3	5	7
	6B	3B	2A				4G	2E			
							4H	2F			
4B	4I	2G				5A	5A			1A	
	8A	4B					10A	5A		2A	
	4J	2H									
	8B	4C									
6B	6H	3A	2C			6A	6G	3A	2G		
	6I	3B	2D				12A	6A	4B		
	6J	3B	2E				12B	6A	4C		
	6K	3B	2F								
7A	7A				1A	7B	7B				1A
15A	15A		5A	3B		15B	15B		5A	3B	

Table 7.8: Fischer-Clifford matrices of $2^6{:}A_8$

$M(1A) = \begin{bmatrix} 1 & 1 & 1 \\ 28 & 4 & -4 \\ 35 & -5 & 3 \end{bmatrix}$	$M(4A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$
$M(2A) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & -4 & 4 & 0 \\ 3 & 3 & 3 & 3 & -1 \\ 4 & -4 & 4 & -4 & 0 \\ 4 & 4 & -4 & -4 & 0 \end{bmatrix}$	$M(4B) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$
$M(2B) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & -2 & 2 & -2 & 0 \\ 6 & 6 & -2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & -6 & -2 & 2 & 0 \end{bmatrix}$	$M(6B) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$
$M(6A) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$	$M(3B) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$
$M(3A) = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 2 & -2 \\ 5 & -3 & 1 \end{bmatrix}$	$M(5A) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(7A) = \begin{bmatrix} 1 \end{bmatrix}$	$M(15A) = \begin{bmatrix} 1 \end{bmatrix}$
$M(7B) = \begin{bmatrix} 1 \end{bmatrix}$	$M(15B) = \begin{bmatrix} 1 \end{bmatrix}$

Table 7.10: Character table of $2^6{:}A_8$

	$1A$			$2A$					$2B$				
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$4A$	$2G$	$4B$	$2H$	$4C$	$4D$
χ_{1}	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	7	7	7	-1	-1	-1	-1	-1	3	3	3	3	3
χ_{3}	14	14	14	6	6	6	6	6	2	2	2	2	2
χ_{4}	20	20	20	4	4	4	4	4	4	4	4	4	4
χ_{5}	21	21	21	-3	-3	-3	-3	-3	1	1	1	1	1
χ_{6}	21	21	21	-3	-3	-3	-3	-3	1	1	1	1	1
χ_{7}	21	21	21	-3	-3	-3	-3	-3	1	1	1	1	1
χ_{8}	28	28	28	-4	-4	-4	-4	-4	4	4	4	4	4
χ_{9}	35	35	35	3	3	3	3	3	-5	-5	-5	-5	-5
χ_{10}	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_{11}	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_{12}	56	56	56	8	8	8	8	8	0	0	0	0	0
χ_{13}	64	64	64	0	0	0	0	0	0	0	0	0	0
χ_{14}	70	70	70	-2	-2	-2	-2	-2	2	2	2	2	2
χ_{15}	28	4	-4	4	-4	-4	4	0	8	4	0	-4	0
χ_{16}	28	4	-4	-4	4	4	-4	0	4	8	-4	0	0
χ_{17}	140	20	-20	-4	4	4	-4	0	12	0	4	-8	0
χ_{18}	140	20	-20	4	-4	-4	4	0	0	12	-8	4	0
χ_{19}	140	20	-20	12	-12	-12	12	0	4	8	-4	0	0
χ_{20}	140	20	-20	-12	12	12	-12	0	8	4	0	-4	0
χ_{21}	252	36	-36	-12	12	12	-12	0	0	12	-8	4	0
χ_{22}	252	36	-36	12	-12	-12	12	0	12	0	4	-8	0
χ_{23}	280	40	-40	8	-8	-8	8	0	-16	-8	0	8	0
χ_{24}	280	40	-40	-8	8	8	-8	0	-8	-16	8	0	0
χ_{25}	448	64	-64	0	0	0	0	0	0	0	0	0	0
χ_{26}	35	-5	3	11	3	3	-5	-1	7	-5	-1	3	-1
χ_{27}	35	-5	3	3	11	-5	3	-1	-5	7	3	-1	-1
χ_{28}	35	-5	3	-5	3	3	11	-1	7	-5	-1	3	-1
χ_{29}	35	-5	3	3	-5	11	3	-1	-5	7	3	-1	-1
χ_{30}	70	-10	6	14	-2	14	-2	-2	2	2	2	2	-2
χ_{31}	70	-10	6	-2	14	-2	14	-2	2	2	2	2	-2
χ_{32}	70	-10	6	14	14	-2	-2	-2	2	2	2	2	-2
χ_{33}	70	-10	6	-2	-2	14	14	-2	2	2	2	2	-2
χ_{34}	140	-20	12	12	12	12	12	-4	4	4	4	4	-4
χ_{35}	210	-30	18	-6	-6	-6	-6	2	-10	14	6	-2	-2
χ_{36}	210	-30	18	-6	-6	-6	-6	2	14	-10	-2	6	-2
χ_{37}	315	-45	27	3	27	-21	3	-1	-9	3	-1	-5	3
χ_{38}	315	-45	27	-21	3	3	27	-1	3	-9	-5	-1	3
χ_{39}	315	-45	27	3	-21	27	3	-1	-9	3	-1	-5	3
χ_{40}	315	-45	27	27	3	3	-21	-1	3	-9	-5	-1	3
χ_{41}	420	-60	36	-12	-12	-12	-12	4	4	4	4	4	-4

Character table of $2^6{:}A_8$(continued)

	$3A$			$3B$				$4A$				$4B$			
	$3A$	$6A$	$6B$	$3B$	$6C$	$6D$	$6E$	$4E$	$4F$	$4G$	$4H$	$4I$	$8A$	$4J$	$8B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	4	4	4	1	1	1	1	-1	-1	-1	-1	1	1	1	1
χ_3	-1	-1	-1	2	2	2	2	2	2	2	2	0	0	0	0
χ_4	5	5	5	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_5	6	6	6	0	0	0	0	1	1	1	1	-1	-1	-1	-1
χ_6	-3	-3	-3	0	0	0	0	1	1	1	1	-1	-1	-1	-1
χ_7	-3	-3	-3	0	0	0	0	1	1	1	1	-1	-1	-1	-1
χ_8	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_9	5	5	5	2	2	2	2	-1	-1	-1	-1	-1	-1	-1	-1
χ_{10}	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
χ_{11}	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
χ_{12}	-4	-4	-4	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{13}	4	4	4	-2	-2	-2	-2	0	0	0	0	0	0	0	0
χ_{14}	-5	-5	-5	1	1	1	1	-2	-2	-2	-2	0	0	0	0
χ_{15}	10	2	-2	1	-1	-1	1	0	0	0	0	2	-2	0	0
χ_{16}	10	2	-2	1	-1	-1	1	0	0	0	0	0	0	2	-2
χ_{17}	20	4	-4	-1	1	1	-1	0	0	0	0	0	0	-2	2
χ_{18}	20	4	-4	-1	1	1	-1	0	0	0	0	-2	2	0	0
χ_{19}	-10	-2	2	2	-2	-2	2	0	0	0	0	0	0	-2	2
χ_{20}	-10	-2	2	2	-2	-2	2	0	0	0	0	-2	2	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2
χ_{23}	10	2	-2	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{24}	10	2	-2	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{25}	-20	-4	4	-2	2	2	-2	0	0	0	0	0	0	0	0
χ_{26}	5	-3	1	2	0	0	-2	3	-1	-1	-1	1	1	-1	-1
χ_{27}	5	-3	1	2	0	0	-2	-1	3	-1	-1	-1	-1	1	1
χ_{28}	5	-3	1	2	0	0	-2	-1	-1	3	-1	1	1	-1	-1
χ_{29}	5	-3	1	2	0	0	-2	-1	-1	-1	3	-1	-1	1	1
χ_{30}	-5	3	-1	1	3	-3	-1	2	-2	-2	2	0	0	0	0
χ_{31}	-5	3	-1	1	3	-3	-1	-2	2	2	-2	0	0	0	0
χ_{32}	-5	3	-1	1	-3	3	-1	2	2	-2	-2	0	0	0	0
χ_{33}	-5	3	-1	1	-3	3	-1	-2	-2	2	2	0	0	0	0
χ_{34}	5	-3	1	-4	0	0	4	0	0	0	0	0	0	0	0
χ_{35}	15	-9	3	0	0	0	0	2	-2	2	-2	0	0	0	0
χ_{36}	15	-9	3	0	0	0	0	-2	2	-2	2	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	-1	-1	-1	3	1	1	-1	-1
χ_{38}	0	0	0	0	0	0	0	3	-1	-1	-1	-1	-1	1	1
χ_{39}	0	0	0	0	0	0	0	-1	3	-1	-1	1	1	-1	-1
χ_{40}	0	0	0	0	0	0	0	-1	-1	3	-1	-1	-1	1	1
χ_{41}	-15	9	-3	0	0	0	0	0	0	0	0	0	0	0	0

Character table of $2^6{:}A_8$(continued)

	$5A$		$6A$			$6B$				$7A$	$7B$	$15A$	$15B$
	$5A$	$10A$	$6F$	$12A$	$12B$	$6G$	$6H$	$6I$	$6J$	$7A$	$7B$	$15A$	$15B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	2	2	0	0	0	-1	-1	-1	-1	0	0	-1	-1
χ_3	-1	-1	-1	-1	-1	0	0	0	0	0	0	-1	-1
χ_4	0	0	1	1	1	1	1	1	1	-1	-1	0	0
χ_5	1	1	-2	-2	-2	0	0	0	0	0	0	1	1
χ_6	1	1	1	1	1	0	0	0	0	0	0	A	/A
χ_7	1	1	1	1	1	0	0	0	0	0	0	/A	A
χ_8	-2	-2	1	1	1	-1	-1	-1	-1	0	0	1	1
χ_9	0	0	1	1	1	0	0	0	0	0	0	0	0
χ_{10}	0	0	0	0	0	0	0	0	0	B	/B	0	0
χ_{11}	0	0	0	0	0	0	0	0	0	/B	B	0	0
χ_{12}	1	1	0	0	0	-1	-1	-1	-1	0	0	1	1
χ_{13}	-1	-1	0	0	0	0	0	0	0	1	1	-1	-1
χ_{14}	0	0	-1	-1	-1	1	1	1	1	0	0	0	0
χ_{15}	3	-1	2	-2	0	1	-1	-1	1	0	0	0	0
χ_{16}	3	-1	-2	2	0	-1	1	1	-1	0	0	0	0
χ_{17}	0	0	0	0	0	-1	1	1	-1	0	0	0	0
χ_{18}	0	0	0	0	0	1	-1	-1	1	0	0	0	0
χ_{19}	0	0	-2	2	0	0	0	0	0	0	0	0	0
χ_{20}	0	0	2	-2	0	0	0	0	0	0	0	0	0
χ_{21}	-3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	-3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	2	-2	0	-1	1	1	-1	0	0	0	0
χ_{24}	0	0	-2	2	0	1	-1	-1	1	0	0	0	0
χ_{25}	3	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	0	0	1	1	-1	2	0	0	-2	0	0	0	0
χ_{27}	0	0	1	1	-1	0	2	-2	0	0	0	0	0
χ_{28}	0	0	1	1	-1	-2	0	0	2	0	0	0	0
χ_{29}	0	0	1	1	-1	0	-2	2	0	0	0	0	0
χ_{30}	0	0	-1	-1	1	-1	1	-1	1	0	0	0	0
χ_{31}	0	0	-1	-1	1	1	-1	1	-1	0	0	0	0
χ_{32}	0	0	-1	-1	1	-1	-1	1	1	0	0	0	0
χ_{33}	0	0	-1	-1	1	1	1	-1	-1	0	0	0	0
χ_{34}	0	0	1	1	-1	0	0	0	0	0	0	0	0
χ_{35}	0	0	-1	-1	1	0	0	0	0	0	0	0	0
χ_{36}	0	0	-1	-1	1	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	1	1	-1	0	0	0	0	0	0	0	0

$A = -E(15)^7 - E(15)^{11} - E(15)^{13} - E(15)^{14}$

$B = E(7) + E(7)^2 + E(7)^4$

8

A Group of the Form $2^8{:}O_8^+(2)$ as a maximal subgroup of $O_{10}^+(2)$

Prologue

The group $\overline{G} = 2^8{:}O_8^+(2)$ is a group of order 44590694400. It is also a maximal subgroup of index 527 of $O_{10}^+(2)$. In turn $2^{10+16}{\cdot}O_{10}^+(2)$ is a maximal subgroup of the monster $M = F_1$. The group $\overline{G}$ has three inertia factor groups namely, $O_8^+(2)$, $SP(6,2)$ and $2^6{:}A_8$ of index 1, 120, and 135 respectively in $O_8^+(2)$. We first give a detailed definition of $O_8^+(2)$ and we then compute the Fischer-Clifford matrices of $\overline{G}$ which together with the partial character tables of inertia factor groups are used to compute the full character table of $\overline{G}$.

8.1. Bilinear Forms

Definition 8.1.1. A *symmetric bilinear form* f is a function $f : V \times V \to \mathbb{F}_q$ which first satisfies *linearity* in x that is

$$f(\lambda_1 x_1 + \lambda_2 x_2, y) = \lambda_1 f(x_1, y) + \lambda_2 f(x_2, y).$$

If linearity is also satisfied in y, then we say f is *bilinear*. We say f is *symmetric* if $f(x, y) = f(y, x)$. If f is bilinear and symmetric we say f is in *symmetric bilinear form*.

Definition 8.1.2. We define the *quadratic form* $Q : V \to \mathbb{F}_q$ to be a function satisfying

$$Q(\lambda x + \mu y) = \lambda^2 Q(x) + \lambda\mu f(x, y) + \mu^2 Q(y),$$

for some symmetric bilinear form f, called the *associated bilinear form*.

Definition 8.1.3. The *kernel* of f is the subspace of all x such that $f(x, y) = 0 \ \ \forall\, y$. Also the kernel of the quadratic form Q, is the set of all $x \in \ker(f)$ such that $Q(x) = 0$.

We can now define nullity, rank, singular and isotropic subspaces.

Definition 8.1.4. The *nullity* and *rank* of f is the dimension and codimension of its kernel respectively. We say f is *non-singular* if the nullity of f is zero.

A subspace W of V is said to be *totally isotropic* for f if $f(x, y) = 0 \; \forall \; x, y \in W$.

We also define the *Witt index* of a quadratic form Q as the greatest dimension of any totally isotropic subspace for Q.

Note that if any two non-singular quadratic forms Q_1 and Q_2 over $\mathbb{F}_q$ have the same Witt index, then they are equivalent to a scalar multiple of each other. We now define the *general orthogonal group*.

8.2. Orthogonal Groups

Definition 8.2.1. The *general orthogonal group* $GO(V, f)$, where f is a bilinear form, is defined as the group of linear maps g satisfying $f(u^g, v^g) = f(u, v) \;\; \forall \; u, v \in V$. We write it as $GO(n, q)$ or $GO_n(q)$, where V is an n-dimensional vector space over $GF(q)$.

The elements of a general orthogonal group $GO(V, f)$ have determinants ± 1. This is since for if M is the matrix of the form and $g \in G(V, f)$, then $gMG^t = M$ and so $\det(g) = \det(M(G^t)^{-1}M^{-1}) = \det(g)^{-1}$. Hence we have $[\det(g)]^2 = 1$ thus $\det(g) = \pm 1$. The elements of the group that have determinant 1 is a subgroup of index 2 called the *special orthogonal group* $SO(n, q)$. The *projective special orthogonal group* $PSO(n, q)$ is the group obtained from $SO(n, q)$ by factoring it by the group of scalar matrices they contain. When $n = 2m + 1$, that is when n is odd, all non-singular quadratic forms on a space of dimension n over $\mathbb{F}_q$ have Witt index m and are equivalent up to scalar forms. If $n = 2m$, then n is even and we, up to equivalence, get two types of quadratic form namely, the *plus* type with Witt index m and the *minus* type with Witt index $m - 1$. Hence if n is odd we get $GO(n, q)$ and when n is even $GO^{\epsilon}(n, q)$ with $\epsilon = +$ or $\epsilon = -$. We are more interested in $n = 2m$ and in particular $\epsilon = +$.

For an orthogonal group we define a *reflection* r_v, for each vector $v \in V$ for which $f(v, v) \neq 0$ as elements of $GO(n, q)$ defined by

$$r_v : x \to x - 2\frac{f(x, v)}{f(v, v)}v.$$

In characteristic 2 if we let $\frac{1}{2}f(v, v) = Q(v)$ in the equation above, then for each vector v of norm 1 we define the *orthogonal transvection* t_v by

$$t_v : w \to w + f(w, v)v.$$

This is a linear map and preserves the quadratic form since

$$
\begin{aligned}
Q(w+f(w,v)v) &= \frac{1}{2}f(w+f(w,v)v, w+f(w,v)v) \\
&= \frac{1}{2}[f(w,w)+f(w,f(w,v)v)+f(f(w,v)v,w)+f(f(w,v)v,f(w,v)v)] \\
&= \frac{1}{2}[f(w,w)+f(w,f(w,v)v)+f(w,f(w,v)v)+f(f(w,v)v,f(w,v)v)] \\
&= \frac{1}{2}[f(w,w)+2f(w,f(w,v)v)+f(f(w,v)v,f(w,v)v)] \\
&= \frac{1}{2}f(w,w)+0+\frac{1}{2}f(f(w,v)v,f(w,v)v) \\
&= Q(w)+Q(f(w,v)v).
\end{aligned}
$$

Orthogonal groups of dimension ≥ 6 can be generated by these transvections. The *quasi-determinant* of an element x is defined to be $+1$ or -1 depending on whether x can be written as a product of even or odd number of of orthogonal transvections.

Thus the quasi-determinant of an element of $GO_{2m}^+(q)$ is the sign of the permutation describing its action on this set. The kernel of the quasi-determinant map is a subgroup of index 2 in $GO_{2m}^+(q)$ which we denote $\Omega_{2m}^+(q)$. These are simple for all $m \geq 3$ and all q. We define $\Omega_{2m}^\epsilon(q)$ as a subgroup of index 1 or 2 in $SO_{2m}^\epsilon(q)$. The image of $P\Omega_{2m}^\epsilon(q)$ in $PSO_{2m}^\epsilon(q)$ is denoted $O_{2m}^\epsilon(q)$ and is the commutator subgroup of $SO_{2m}^\epsilon(q)$. Also the group $Sp_{2m-2}(q)$ is a maximal subgroup of both groups $O_{2m}^\epsilon(q)$. Looking at the orders of orthogonal groups in particular those of even dimension and $\epsilon = +$, we have that

$$
\begin{aligned}
|GO_{2m}^+(q)| &= \Pi_{i=1}^m (q^{i-1})(q^{i-1}+1)q^{2i-2} \\
&= 2q^{m(m-1)}(q^m-1)\Pi_{i=1}^{m-1}(q^{2i}-1).
\end{aligned}
$$

For further reading one can also go to [19, 23, 64, 52, 60, 74, 103, 106, 125].

In our case we have $O_8^+(2)$ is of index 2 in $GO_8^+(2)$ and hence $P\Omega_8^+(2) = O_8^+(2) \cong PSO_8^+(2)$. Also for the order $O_8^+(2)$ since it is of index 2 in $GO_8^+(2)$ we have

$$
|O_8^+(2)| = 2^{12}(2^4-1)\Pi_{i=1}^3(2^{2i}-1).
$$

We are interested in the group $2^8{:}O_8^+(2)$ which is a maximal subgroup of $O_{10}^+(2)$. The group $O_{10}^+(2)$ has nine conjugacy classes of maximal subgroups. It has exactly four conjugacy classes of involutions presented in the $\mathbb{ATLAS}$ [23] by $2A, 2B, 2C$ and $2D$ respectively. In $O_{10}^+(2)$, we have $N_{O_{10}^+(2)}(2^8) = 2^8{:}O_8^+(2)$ and using the list of maximal subgroups of $O_{10}^+(2)$ given in the $\mathbb{ATLAS}$ [23], we can see that $2^8{:}O_8^+(2)$ is a maximal subgroup of $O_{10}^+(2)$. There are three non-equivalent 8-dimensional 2-modular representations of the group $O_8^+(2)$, with corresponding vector spaces V_1, V_2 and V_3. Here we have

$$
\dim_{GF(2)}(V_1) = 8, \quad \dim_{GF(4)}(V_2) = \dim_{GF(4)}(V_3) = 8.
$$

We also have that $V_1 = 2^8$ is irreducible over $GF(2)$ and here we are concerned with the group $V_1{:}O_8^+(2)$. We also note that V_2 and V_3 are irreducible over $GF(4)$. Hence $O_{10}^+(2)$ has only one class for the maximal subgroups representatives of type $2^8{:}O_8^+(2)$

Let $\overline{G} = N{:}G$, where $N = 2^8$ is the vector space of dimension 8 over $GF(2)$ and $G = O_8^+(2)$ acts irreducibly on N. We use the method of coset analysis, which was discussed in chapter 2 , to determine the conjugacy classes of $\overline{G}$. We then construct the complete character table using Fischer-Clifford matrices and partial character tables of inertia factor groups. The complete fusion of $2^8{:}O_8^+(2)$ into $O_{10}^+(2)$ will also be fully determined. Our computations were done using GAP [41]
.

8.3. The action of $O_8^+(2)$ on 2^8

We generate $O_8^+(2)$ as a matrix group as the only proper normal subgroup of $GO_8^+(2)$ by three 8×8 matrices α, β and γ of orders $15, 15$ and 4 as follows.

$$\alpha = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\gamma = \begin{pmatrix} 1&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&1&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&1&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 1&0&0&0&1&0&0&1 \end{pmatrix}.$$

We give the class representatives for each $g \in O_8^+(2)$ in terms of 8×8 matrices over $GF(2)$ in Table 8.1 where $[g]_G$ is the class containing g and M is the matrix that represents that particular class. This is written in GAP format [41].

Table 8.1: Conjugacy Classes of $O_8^+(2)$

$[g]_G$	M	$\lvert[g]_G\rvert$	$[g]_G$	M	$\lvert[g]_G\rvert$
1a	$\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$	1	2a	$\begin{pmatrix} 1&0&0&1&1&1&1&1\\ 0&1&1&1&1&0&1&1\\ 0&0&1&0&0&0&1&0\\ 0&0&0&1&1&0&1&0\\ 1&1&0&1&1&1&0&1\\ 0&1&1&1&0&0&1&1\\ 0&1&0&0&1&0&0&0\\ 1&1&1&1&1&0&0&1 \end{pmatrix}$	3780
2b	$\begin{pmatrix} 0&1&0&1&0&1&1&0\\ 0&0&1&0&0&1&0&0\\ 0&0&0&0&0&1&1&0\\ 0&0&1&0&0&1&0&1\\ 0&1&0&1&1&0&0&1\\ 0&0&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0 \end{pmatrix}$	1575	2c	$\begin{pmatrix} 1&0&1&0&0&1&1&0\\ 0&0&0&1&0&1&1&0\\ 0&1&0&1&1&1&1&1\\ 1&0&1&1&1&1&1&0\\ 0&1&0&1&1&0&0&1\\ 0&0&1&1&1&1&1&0\\ 1&0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0&1 \end{pmatrix}$	3780
2d	$\begin{pmatrix} 0&0&1&0&1&0&0&0\\ 0&1&0&1&1&0&0&0\\ 0&1&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 1&0&1&0&1&0&1&0\\ 0&1&1&1&1&1&0&0\\ 0&0&1&1&1&1&0&0\\ 1&0&1&0&0&0&1&0 \end{pmatrix}$	3780	2e	$\begin{pmatrix} 0&0&1&0&0&0&0&0\\ 1&1&0&1&1&1&0&0\\ 1&1&0&0&1&1&0&1\\ 1&0&1&0&0&0&0&0\\ 1&0&0&0&0&1&0&0\\ 1&0&0&0&1&0&0&0\\ 1&0&1&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$	56700
3a	$\begin{pmatrix} 1&0&0&0&0&1&0&1\\ 0&0&1&0&1&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&1&0&0&0&0&1&1\\ 1&0&0&0&0&0&0&1\\ 1&0&1&1&1&1&1&0\\ 1&0&1&0&0&0&0&1\\ 0&0&0&1&0&1&0&0 \end{pmatrix}$	89600	3b	$\begin{pmatrix} 1&1&1&1&1&0&0&0\\ 1&0&0&0&1&0&0&1\\ 1&1&0&1&1&1&0&1\\ 1&1&1&0&1&1&0&1\\ 0&1&0&0&0&1&0&0\\ 1&1&1&1&0&0&1&0\\ 0&1&0&0&0&1&0&1\\ 1&0&0&0&1&1&1&1 \end{pmatrix}$	2240

continued on next page

Table 8.1 (continued from previous page)

$[g]_G$	M	$\lvert[g]_G\rvert$	$[g]_G$	M	$\lvert[g]_G\rvert$
3c	$\begin{pmatrix} 0&0&0&1&0&0&0&0 \\ 0&0&1&1&0&1&1&1 \\ 1&1&0&0&0&0&1&1 \\ 1&0&0&0&0&0&0&0 \\ 1&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 1&0&0&1&0&1&1&0 \\ 1&0&0&1&0&0&0&1 \end{pmatrix}$	268800	3d	$\begin{pmatrix} 1&0&0&1&1&1&1&1 \\ 1&1&1&0&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \\ 0&1&0&0&0&0&0&1 \\ 1&0&0&0&0&1&1&0 \\ 1&0&0&0&1&0&1&0 \\ 0&0&1&1&1&1&0&1 \\ 1&0&0&0&1&1&1&1 \end{pmatrix}$	2240
3e	$\begin{pmatrix} 1&1&1&1&1&0&0&1 \\ 0&1&0&0&0&0&0&1 \\ 1&1&0&1&1&0&1&1 \\ 0&0&1&0&0&1&0&1 \\ 1&1&0&0&1&1&1&0 \\ 0&1&1&0&0&0&0&0 \\ 1&0&1&1&0&0&1&1 \\ 0&1&0&0&0&1&0&0 \end{pmatrix}$	2240	4a	$\begin{pmatrix} 0&0&0&0&0&0&0&1 \\ 0&0&0&1&1&0&1&0 \\ 0&0&0&1&1&0&0&0 \\ 0&0&0&1&0&1&0&1 \\ 0&0&0&1&0&0&0&0 \\ 0&1&1&1&1&1&0&0 \\ 0&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0 \end{pmatrix}$	37800
4b	$\begin{pmatrix} 1&1&1&1&1&0&0&0 \\ 1&1&1&0&0&0&1&1 \\ 0&1&1&0&0&0&1&0 \\ 0&1&1&0&0&0&0&1 \\ 1&1&1&0&0&1&1&1 \\ 1&1&0&0&1&0&1&1 \\ 1&1&0&0&0&0&1&1 \\ 0&1&1&1&0&1&1&1 \end{pmatrix}$	907200	4c	$\begin{pmatrix} 1&0&0&0&0&1&1&0 \\ 0&0&0&1&0&0&0&1 \\ 0&1&1&1&0&1&1&1 \\ 1&0&1&0&0&1&1&0 \\ 0&0&1&0&0&1&1&0 \\ 1&1&1&0&1&1&1&0 \\ 0&0&1&0&0&0&1&0 \\ 0&0&0&0&1&0&0&1 \end{pmatrix}$	340200
4d	$\begin{pmatrix} 0&0&1&0&0&1&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&1&0&0&1&0&1&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&1&0&1&0&1&0 \\ 1&0&1&0&0&0&0&1 \\ 0&0&1&0&0&0&0&0 \\ 1&0&1&1&0&0&1&1 \end{pmatrix}$	907200	4e	$\begin{pmatrix} 0&1&0&0&1&0&0&0 \\ 0&0&0&1&0&1&0&0 \\ 0&1&0&0&0&0&0&0 \\ 1&0&1&1&1&1&1&0 \\ 0&0&1&0&1&0&0&0 \\ 0&0&0&1&0&0&0&1 \\ 0&1&0&0&0&0&0&1 \\ 0&0&1&0&1&0&1&0 \end{pmatrix}$	907200
4f	$\begin{pmatrix} 1&1&0&1&1&1&1&0 \\ 1&0&1&0&1&0&0&0 \\ 1&0&1&0&0&0&1&0 \\ 1&0&1&0&1&0&0&1 \\ 0&1&1&0&0&1&0&1 \\ 0&0&0&1&0&1&0&1 \\ 1&1&1&0&1&1&0&1 \\ 0&0&1&1&1&1&0&1 \end{pmatrix}$	2721600	5a	$\begin{pmatrix} 1&0&0&1&0&1&0&1 \\ 1&1&1&1&0&1&1&0 \\ 0&0&0&1&0&1&0&1 \\ 0&1&0&1&0&1&1&0 \\ 1&0&1&0&1&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&0&0&1 \\ 1&0&1&0&0&1&1&0 \end{pmatrix}$	580608
5b	$\begin{pmatrix} 1&0&0&1&0&1&0&1 \\ 1&0&0&1&1&0&0&0 \\ 0&1&0&0&0&1&1&0 \\ 0&0&0&0&1&0&1&0 \\ 1&1&1&1&1&0&1&0 \\ 1&1&0&1&0&1&1&1 \\ 0&1&1&0&0&0&0&0 \\ 1&1&0&0&0&1&1&1 \end{pmatrix}$	580608	5c	$\begin{pmatrix} 0&1&1&0&1&0&0&1 \\ 0&0&1&0&0&1&0&0 \\ 0&0&0&0&0&1&1&0 \\ 0&0&0&1&1&0&1&0 \\ 1&0&0&0&0&0&1&0 \\ 0&1&1&0&0&1&1&0 \\ 0&0&1&0&0&1&1&0 \\ 1&1&1&1&1&0&0&1 \end{pmatrix}$	580608

continued on next page

Table 8.1 (continued from previous page)

$[g]_G$	M	$\lvert[g]_G\rvert$	$[g]_G$	M	$\lvert[g]_G\rvert$
6a	$\begin{pmatrix} 0&0&0&0&0&1&0&1\\ 0&0&1&1&0&0&1&1\\ 1&0&1&0&0&1&0&1\\ 1&0&0&1&0&1&0&1\\ 0&1&1&1&1&0&1&1\\ 0&0&0&0&0&1&0&0\\ 1&1&1&1&0&1&0&0\\ 1&0&0&0&0&1&0&0 \end{pmatrix}$	2419200	6b	$\begin{pmatrix} 1&0&0&1&1&0&1&0\\ 0&0&1&0&1&1&1&1\\ 0&1&1&0&1&0&1&0\\ 1&0&0&0&1&1&1&1\\ 1&0&1&0&1&0&0&1\\ 1&1&1&1&0&0&1&0\\ 1&0&1&0&0&1&1&0\\ 1&1&1&1&1&0&0&0 \end{pmatrix}$	806400
6c	$\begin{pmatrix} 1&0&1&1&1&1&0&1\\ 1&0&1&0&1&0&1&0\\ 1&0&1&1&1&1&0&0\\ 0&0&0&0&1&0&1&0\\ 1&1&1&1&1&0&0&0\\ 1&1&1&1&0&1&0&1\\ 1&1&0&1&1&1&0&1\\ 0&1&0&1&0&0&0&0 \end{pmatrix}$	2419200	6d	$\begin{pmatrix} 0&0&1&0&0&0&0&1\\ 1&0&0&0&0&0&1&0\\ 0&0&1&0&1&0&0&0\\ 0&1&1&1&1&0&1&1\\ 0&0&0&0&1&0&0&0\\ 0&0&1&1&1&1&1&0\\ 0&1&1&0&0&0&0&1\\ 1&0&1&0&1&0&0&0 \end{pmatrix}$	806400
6e	$\begin{pmatrix} 1&1&1&1&0&1&0&1\\ 1&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&1\\ 1&0&1&1&0&0&1&1\\ 0&1&0&1&1&1&1&1\\ 1&0&0&1&0&1&0&1\\ 1&0&0&1&0&0&0&1\\ 1&1&1&1&1&0&1&0 \end{pmatrix}$	806400	6f	$\begin{pmatrix} 0&1&0&0&0&1&0&1\\ 0&0&0&1&0&1&1&0\\ 1&0&0&0&0&1&1&0\\ 0&1&0&0&1&1&1&1\\ 1&0&0&0&0&0&1&0\\ 1&1&1&1&1&0&0&0\\ 1&1&1&0&0&1&1&1\\ 1&0&0&1&0&0&1&0 \end{pmatrix}$	100800
6g	$\begin{pmatrix} 1&1&1&1&1&0&0&0\\ 0&0&0&1&1&0&0&0\\ 1&0&0&1&0&0&0&1\\ 0&1&1&1&1&1&1&0\\ 1&0&0&1&0&0&0&0\\ 0&1&0&1&1&0&1&0\\ 0&1&1&1&1&1&0&0\\ 1&0&0&1&1&0&0&1 \end{pmatrix}$	100800	6h	$\begin{pmatrix} 0&1&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&1&0&0&1\\ 1&1&0&0&0&1&1&1\\ 1&1&1&0&1&1&0&0\\ 1&1&1&0&0&0&1&1\\ 1&1&0&0&1&1&1&0 \end{pmatrix}$	806400
6i	$\begin{pmatrix} 1&0&0&1&0&0&0&0\\ 1&1&0&0&1&0&1&1\\ 1&1&1&1&0&1&0&1\\ 0&0&0&0&1&0&0&0\\ 1&1&1&0&0&0&1&1\\ 0&0&0&1&1&0&0&0\\ 1&0&0&0&0&0&1&0\\ 1&0&0&0&1&0&0&1 \end{pmatrix}$	604800	6j	$\begin{pmatrix} 0&0&0&1&0&1&1&0\\ 0&0&1&0&0&1&0&0\\ 0&1&0&0&0&0&1&0\\ 0&0&0&1&1&1&1&1\\ 0&0&0&0&0&1&1&0\\ 1&0&1&0&0&0&0&1\\ 0&1&0&0&0&1&1&0\\ 1&0&0&1&1&1&1&1 \end{pmatrix}$	100800
6k	$\begin{pmatrix} 0&1&0&0&1&0&0&1\\ 0&0&1&1&0&0&1&1\\ 1&0&1&0&0&0&1&0\\ 1&0&0&1&0&0&1&0\\ 0&0&1&1&0&1&1&1\\ 0&1&0&0&1&1&1&1\\ 1&1&1&1&0&1&0&0\\ 1&0&0&0&0&1&0&0 \end{pmatrix}$	2419200	6l	$\begin{pmatrix} 1&0&1&1&1&1&0&1\\ 1&1&0&0&1&1&0&0\\ 1&0&0&0&0&1&1&0\\ 0&1&0&1&0&1&1&0\\ 1&0&1&0&0&1&0&0\\ 1&0&1&0&1&0&0&1\\ 1&0&1&1&1&0&1&1\\ 0&1&1&0&1&0&1&0 \end{pmatrix}$	604800

continued on next page

Table 8.1 (continued from previous page)

$[g]_G$	M	$\lvert[g]_G\rvert$	$[g]_G$	M	$\lvert[g]_G\rvert$
6m	$\begin{pmatrix} 0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&1&1&0 \\ 1&1&0&0&1&1&0&0 \\ 1&0&0&0&0&1&1&0 \\ 1&0&1&0&1&0&1&0 \\ 1&1&1&0&1&1&0&0 \\ 1&0&0&1&0&1&1&0 \end{pmatrix}$	604800	6n	$\begin{pmatrix} 1&1&0&0&1&0&1&1 \\ 0&0&1&1&1&1&0&0 \\ 1&0&0&0&1&1&1&1 \\ 0&0&0&1&1&0&0&0 \\ 0&1&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&1 \\ 1&1&0&1&1&0&1&1 \\ 0&1&1&0&0&1&0&0 \end{pmatrix}$	7257600
8a	$\begin{pmatrix} 0&0&0&1&0&1&1&0 \\ 1&1&0&1&1&1&1&0 \\ 0&0&1&0&1&0&0&0 \\ 1&1&1&1&1&0&1&0 \\ 0&1&0&1&1&1&1&1 \\ 1&0&0&1&1&1&1&1 \\ 1&1&1&0&0&0&1&1 \\ 0&0&1&0&0&0&0&1 \end{pmatrix}$	5443200	8b	$\begin{pmatrix} 1&0&1&0&1&0&0&1 \\ 0&0&0&1&0&1&1&0 \\ 1&0&0&1&0&0&1&0 \\ 1&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1 \\ 1&1&0&1&1&0&1&1 \\ 1&0&1&1&0&0&1&1 \\ 1&1&0&0&1&0&1&1 \end{pmatrix}$	5443200
9a	$\begin{pmatrix} 0&0&0&1&0&0&0&0 \\ 1&1&1&0&1&1&0&1 \\ 1&0&0&0&1&0&0&1 \\ 1&0&0&0&0&1&0&1 \\ 1&0&0&0&1&1&1&1 \\ 1&1&0&0&1&1&0&0 \\ 1&1&0&0&1&1&0&1 \\ 0&0&0&1&1&1&1&1 \end{pmatrix}$	89600	9b	$\begin{pmatrix} 0&0&0&1&1&0&0&0 \\ 0&0&0&1&1&1&1&1 \\ 1&1&0&0&0&0&1&1 \\ 0&0&0&0&1&0&1&0 \\ 1&0&0&0&1&0&1&0 \\ 1&1&1&0&0&1&1&1 \\ 1&0&0&1&1&0&1&0 \\ 1&0&1&1&1&1&0&1 \end{pmatrix}$	6451200
9c	$\begin{pmatrix} 1&1&0&0&1&1&0&0 \\ 0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&1&0&1&0&1&0 \\ 1&0&0&0&1&0&0&0 \\ 1&0&0&1&0&1&0&0 \\ 1&0&1&0&1&0&0&0 \\ 1&1&0&1&1&1&0&1 \end{pmatrix}$	6451200	10a	$\begin{pmatrix} 0&1&1&0&0&0&0&1 \\ 1&0&1&0&0&1&0&1 \\ 1&1&1&1&0&1&0&0 \\ 0&0&1&0&0&1&0&1 \\ 1&0&0&0&1&1&1&1 \\ 0&0&0&0&0&1&0&0 \\ 0&0&1&1&0&0&1&1 \\ 0&0&0&0&0&1&0&1 \end{pmatrix}$	8709120
10b	$\begin{pmatrix} 0&1&0&0&0&1&0&1 \\ 0&1&1&0&1&0&0&0 \\ 0&1&0&0&1&0&1&0 \\ 0&1&1&1&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \\ 1&1&1&0&1&1&0&0 \\ 1&0&0&0&0&1&1&0 \\ 0&1&0&1&1&1&1&1 \end{pmatrix}$	8709120	10c	$\begin{pmatrix} 1&0&1&0&0&0&1&0 \\ 1&1&0&1&1&1&0&1 \\ 0&1&0&0&1&1&1&1 \\ 1&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&1&0 \\ 1&0&0&0&0&1&0&0 \\ 1&1&1&1&1&0&0&0 \\ 1&0&0&0&0&0&1&0 \end{pmatrix}$	8709120
12a	$\begin{pmatrix} 0&1&0&1&0&1&1&0 \\ 1&0&1&1&0&1&1&1 \\ 1&0&1&0&1&1&1&1 \\ 0&0&0&1&1&1&1&1 \\ 0&1&0&1&1&0&0&1 \\ 0&0&1&1&1&1&1&0 \\ 1&1&0&1&0&0&1&1 \\ 0&0&0&0&1&0&0&1 \end{pmatrix}$	4838400	12b	$\begin{pmatrix} 0&0&0&1&0&1&0&0 \\ 1&1&1&1&0&1&1&0 \\ 1&1&1&0&1&1&0&0 \\ 1&0&0&1&0&1&1&0 \\ 1&1&1&1&1&0&0&1 \\ 1&1&0&1&0&1&1&1 \\ 0&1&0&0&0&1&0&1 \\ 0&1&1&0&1&0&0&1 \end{pmatrix}$	907200

continued on next page

Table 8.1 (continued from previous page)

$[g]_G$	M	$\lvert[g]_G\rvert$	$[g]_G$	M	$\lvert[g]_G\rvert$
12c	$\begin{pmatrix} 0&1&1&0&0&1&1&0\\ 0&0&0&0&1&0&0&0\\ 0&1&0&1&0&0&0&1\\ 1&0&0&0&1&0&0&0\\ 0&0&0&1&1&0&0&0\\ 0&0&1&1&0&0&1&1\\ 1&0&0&1&0&0&1&0\\ 0&0&0&1&0&0&0&1 \end{pmatrix}$	1209600	12d	$\begin{pmatrix} 1&1&0&0&1&1&0&0\\ 0&0&0&0&0&1&0&1\\ 1&0&1&0&0&1&0&0\\ 1&1&1&1&1&1&1&1\\ 1&0&1&0&0&1&0&1\\ 1&0&0&1&1&0&1&0\\ 1&1&1&1&0&0&0&0\\ 0&0&1&0&0&0&0&1 \end{pmatrix}$	7257600
12e	$\begin{pmatrix} 0&1&1&1&1&0&1&1\\ 0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&1&0\\ 1&1&1&1&1&0&1&0\\ 1&0&1&0&0&0&0&0\\ 0&1&1&0&0&0&1&0\\ 0&0&1&0&0&1&0&0\\ 0&0&0&0&1&0&0&0 \end{pmatrix}$	7257600	12f	$\begin{pmatrix} 1&0&0&1&1&0&0&0\\ 1&0&0&0&0&1&0&1\\ 1&0&0&1&0&0&0&0\\ 1&1&0&1&1&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&1&1&0&0&0&1&0\\ 0&0&0&1&0&0&0&1\\ 1&1&1&1&1&0&0&0 \end{pmatrix}$	1209600
12g	$\begin{pmatrix} 0&0&1&1&0&1&1&1\\ 1&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&1\\ 0&1&1&1&1&1&0&1\\ 0&1&0&1&1&0&0&0\\ 1&0&1&0&0&1&1&0\\ 0&0&1&0&1&1&1&1\\ 1&1&1&1&1&0&0&1 \end{pmatrix}$	7257600	15a	$\begin{pmatrix} 0&1&1&0&0&1&0&0\\ 1&0&0&1&1&0&1&0\\ 1&0&0&0&1&0&0&0\\ 1&1&0&0&1&0&1&1\\ 1&1&1&0&1&1&0&1\\ 1&0&0&0&1&0&1&0\\ 0&1&0&0&0&1&0&0\\ 0&0&0&1&1&0&0&0 \end{pmatrix}$	11612160
15b	$\begin{pmatrix} 0&1&0&1&1&1&1&1\\ 0&1&1&0&0&1&0&1\\ 0&1&1&1&1&1&0&1\\ 1&1&0&0&1&1&0&1\\ 0&0&1&1&0&1&1&1\\ 0&1&0&1&1&0&0&1\\ 0&0&1&1&1&1&0&1\\ 1&1&0&1&1&0&1&1 \end{pmatrix}$	11612160	15c	$\begin{pmatrix} 0&0&0&1&1&1&1&1\\ 0&1&1&0&0&1&0&1\\ 0&0&0&0&1&1&1&1\\ 0&1&1&0&0&0&0&1\\ 1&1&0&0&0&0&1&1\\ 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0&1\\ 0&1&0&1&0&1&1&0 \end{pmatrix}$	11612160
7a	$\begin{pmatrix} 1&0&1&1&1&0&1&1\\ 1&1&0&0&1&1&0&1\\ 1&0&0&1&0&0&1&0\\ 1&0&1&1&1&1&1&0\\ 1&0&0&1&0&1&0&0\\ 0&0&1&0&0&0&0&1\\ 1&1&0&1&1&0&1&1\\ 0&0&1&1&1&0&1&1 \end{pmatrix}$	24883200			

We obtain that $O_8^+(2)$ has 53 conjugacy classes. The action of $O_8^+(2)$ on 2^8 gives rise to three orbits of lengths $1, 120$ and 135 with corresponding point stabilizers that we get from the $\mathbb{ATLAS}$ [23] namely, $O_8^+(2), Sp_6(2)$ and $2^6{:}A_8$. Let ρ_1 and ρ_2 be permutation characters of $O_8^+(2)$ of degrees 120 and 135. Then from $\mathbb{ATLAS}$ [23], we deduce that $\chi_{\rho_1} = 1a+35a+84a$ and $\chi_{\rho_2} = 1a+50a+84a$.

Suppose $\chi = \chi(O_8^+(2)|2^8)$ is the permutation character of $O_8^+(2)$ on 2^8. Then we get

$$\chi = 1a + I_{Sp_6(2)}^{O_8^+(2)} + I_{2^6:A_8}^{O_8^+(2)}$$

$$= 3 \times 1a + 35a + 50a + 2 \times 84a,$$

where $I_{Sp_6(2)}^{O_8^+(2)}$ and $I_{2^6:A_8}^{O_8^+(2)}$ are the characters of $O_8^+(2)$ induced from the identity characters of $Sp_6(2)$ and $2^6{:}A_8$ respectively.

For each class representative $g \in O_8^+(2)$,we calculate the values of $\chi(Sp_6(2)|2^8)$, $\chi(2^6{:}A_8|2^8)$ on g and $k = \chi(g)$ on 2^8, the number of fixed points of g on 2^8. These are given in Table 8.2.

Table 8.2:

$[g]_{O_8^+(2)}$	1a	2a	2b	2c	2d	2e	3a	3b	3c	3d	3e	4a	4b	4c
$\chi(Sp_6(2)\|2^8)$	120	32	24	8	0	0	36	0	0	3	6	12	4	8
$\chi(2^6{:}A_8\|2^8)$	135	31	39	7	15	15	27	0	0	0	9	3	11	7
k	256	64	64	16	16	16	64	1	1	4	16	16	16	16

$[g]_{O_8^+(2)}$	4d	4e	4f	5a	5b	5c	6a	6b	6c	6d	6e	6f	6g	6h
$\chi(Sp_6(2)\|2^8)$	0	0	1	10	0	0	12	0	0	8	0	0	0	6
$\chi(2^6{:}A_8\|2^8)$	3	3	2	5	0	0	3	0	0	7	0	0	3	9
k	4	4	4	16	1	1	16	1	1	16	1	1	4	16

$[g]_{O_8^+(2)}$	6i	6j	6k	6l	6m	6n	7a	8a	8b	9a	9b	9c	10a
$\chi(Sp_6(2)\|2^8)$	0	0	2	0	0	2	1	2	2	3	0	0	2
$\chi(2^6{:}A_8\|2^8)$	3	3	1	3	3	1	2	1	1	0	0	0	1
k	4	4	4	4	4	4	4	4	4	4	1	1	4

$[g]_{O_8^+(2)}$	10b	10c	12a	12b	12c	12d	12e	12f	12g	15a	15b	15c
$\chi(Sp_6(2)\|2^8)$	0	0	3	0	0	3	2	0	0	1	0	0
$\chi(2^6{:}A_8\|2^8)$	0	0	0	0	0	0	1	0	0	2	0	0
k	1	1	4	1	1	4	4	1	1	4	1	1

We can also check our calculations for the values of k using Programme F (see appendix). Let $k_1 = \chi(Sp_6(2)|2^8)(g)$, $k_2 = \chi(2^6{:}A_8|2^8)(g)$ then $k = k_1 + k_2 + 1$, where $k_1 := cut[1] + cut[3] + cut[7]$, $k_2 := cut[1] + cut[6] + cut[7]$, then $k := 3 * cut[1] + cut[3] + cut[6] + 2 * cut[7]$. Here $cut[i]$ is the i-th row of the character table of $O_8^+(2)$ as shown in the ATLAS [23].

Having obtained the values of the $k's$ for the various classes of G, then we need to calculate the $f_j's$ corresponding to the various $k's$. For this purpose we use Programme A (see appendix) For a class representative $dg \in \overline{G}$, where $d \in 2^8$ and $g \in O_8^+(2)$ and $o(g) = m$, by Theorem 3.3.10 [99] we have

$$o(dg) = \begin{cases} m & \text{if } \; w = 1_N \\ 2m & \text{otherwise} \end{cases}$$

To calculate the orders of the class representatives $dg \in \overline{G}$, we use Programme B (see appendix)

Here if $o(g) = m$ and $w = 1_N$ then $o(dg) = m$ otherwise if $w \neq 1_N$, then $o(dg) = 2m$. Table 8.3 gives detailed information about the conjugacy classes of $\overline{G}$

Table 8.3: Conjugacy Classes of $2^8{:}O_8^+(2)$

$g \in O_8^+(2)$	k	f_j	d_j	w	$[x]_{2^8:O_8^+(2)}$	$\|C_{2^8:O_8^+(2)}(x)\|$
$1A$	2^8	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$1A$	44 590 694 400
		120	$(0,0,0,0,0,0,1,0)$	$(1,0,1,0,1,0,1,0)$	$2A$	371 589 120
		135	$(0,0,0,0,0,0,0,1)$	$(1,0,0,1,0,0,0,1)$	$2B$	330 301 440
$2A$	2^6	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2C$	7 077 888
		6	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	2D	1 179 648
		9	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	2E	786 432
		48	$(0,0,0,1,0,0,0,0)$	$(1,0,0,0,0,1,1,1)$	4A	147 456
$2B$	2^6	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2F$	2 949 120
		6	$(0,0,0,0,0,0,1,0)$	$(0,1,0,1,0,0,1,0)$	4B	491 520
		10	$(0,0,0,0,0,0,0,1)$	$(0,1,0,0,0,0,0,0)$	4C	294 912
		15	$(0,0,0,1,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	2G	196 608
		32	$(0,0,1,0,0,0,0,0)$	$(1,0,0,0,0,1,1,1)$	4D	92 160
$2C$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2H$	737 280
		15	$(0,0,0,0,0,0,1,0)$	$(1,1,1,1,1,1,0,0)$	4E	49 152
$2D$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2I$	737 280
		15	$(0,0,0,0,0,0,1,0)$	$(1,1,0,1,0,0,1,0)$	$4F$	49 152
$2E$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2J$	49 152
		8	$(0,0,0,1,0,0,0,0)$	$(0,0,0,1,0,0,0,0)$	$4G$	6 144
		1	$(0,0,0,0,0,0,1,0)$	$(1,1,0,1,0,0,1,0)$	$4H$	49 152
		6	$(0,0,0,0,0,0,0,1)$	$(1,0,1,0,1,0,1,0)$	$4I$	8 192
$3A$	2^6	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3A$	4 976 640
		36	$(0,0,0,0,0,0,0,1)$	$(1,0,1,0,0,1,0,1)$	$6A$	138 240
		27	$(0,0,0,0,0,0,1,0)$	$(0,0,0,1,0,1,1,1)$	$6B$	184 320
$3B$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3B$	77 760
$3C$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3C$	77 760
$3D$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3D$	7 776
		3	$(0,0,0,0,0,0,0,1)$	$(0,0,0,1,0,0,0,0)$	$6C$	2 592
$3E$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3E$	10 368
		6	$(0,0,0,0,0,0,1,0)$	$(1,1,0,1,0,0,1,0)$	$6D$	1 728
		9	$(0,0,0,0,0,0,0,1)$	$(0,0,0,1,0,0,0,0)$	$6E$	1 152
$4A$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4J$	73 728
		12	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	$4K$	6 144
		3	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	$4L$	24 576
$4B$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4M$	8 192
		4	$(0,0,0,1,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	4N	2 048
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	4O	8 192
		2	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	4P	4 096
		8	$(0,0,0,1,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	4Q	1 024
$4C$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4R$	3 072
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	4S	3 072
		3	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	4T	1 024
		3	$(0,0,0,1,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	4U	1 024
		4	$(0,0,0,1,0,0,0,1)$	$(0,0,0,0,0,1,1,1)$	8A	768
		4	$(0,0,1,1,0,0,0,0)$	$(1,1,1,1,1,0,0,0)$	8B	768
$4D$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4V$	768
		3	$(0,0,0,0,0,0,1,0)$	$(1,1,0,1,0,0,1,0)$	$8C$	256
$4E$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4W$	768
		3	$(0,0,0,0,0,0,1,0)$	$(1,0,0,1,0,0,1,0)$	$8D$	256
$4F$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4X$	256
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,1,0,0,0,0)$	8E	256

continued on next page

Table 8.3 (continued from previous page)

$g \in O_8^+(2)$	k	f_j	d_j	w	$[x]_{2^8:O_8^+(2)}$	$\lvert C_{2^8:O_8^+(2)}(x)\rvert$
		2	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,1,1,1)$	8F	128
$5A$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$5A$	4 800
		5	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,1,1,1)$	10A	960
		10	$(0,0,0,0,0,0,0,1)$	$(0,1,0,1,0,0,1,1)$	10B	480
$5B$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	5B	300
$5C$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	5C	300
$6A$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6F$	27 648
		12	$(0,0,0,0,0,0,1,0)$	$(1,0,1,1,1,0,0,0)$	12A	2 304
		3	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	6G	9 216
$6B$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6H$	1 728
$6C$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6I$	1 728
$6D$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6J$	4 608
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,1,1,1)$	12B	4 608
		3	$(0,0,0,1,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	6K	1 536
		3	$(0,0,0,1,0,0,0,1)$	$(0,0,0,1,1,0,1,1)$	12C	1 536
		8	$(0,0,1,1,0,0,0,0)$	$(,0,1,0,0,1,0,1)$	12D	576
$6E$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	6L	288
$6F$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	6M	288
$6G$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6N$	864
		3	$(0,0,0,0,0,0,1,0)$	$0,0,0,0,0,0,0,0)$	6O	288
$6H$	2^4	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	6P	3 456
		6	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	6Q	576
		9	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	6R	384
$6I$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	6S	864
		3	$(0,0,0,0,0,0,1,0)$	$(1,0,1,0,0,1,0,0)$	12E	288
$6J$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6T$	864
		3	$(0,0,0,0,0,0,1,0)$	$(1,0,1,1,1,0,0,0)$	12F	288
$6K$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6U$	288
		1	$(0,0,0,0,0,0,1,0)$	$(1,0,1,0,1,0,1,0)$	12G	288
		2	$(0,0,0,0,0,0,0,1)$	$(0,1,0,1,0,1,0,1)$	12H	144
$6L$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6V$	288
		3	$(0,0,0,0,0,0,1,0)$	$(0,0,0,1,1,0,1,1)$	12I	96
$6M$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	6W	288
		3	$(0,0,0,0,0,0,1,0)$	$(,0,1,0,0,1,0,1)$	12J	96
$6N$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6X$	96
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,1,1,0,0,0)$	12K	96
		2	$(0,0,0,0,0,0,0,1)$	$(1,0,1,1,1,0,0,0)$	12L	48
$7A$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	7A	28
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,1,0,0,0,1)$	14A	28
		1	$(0,0,0,0,0,0,0,1)$	$(1,0,1,0,1,0,1,0)$	14B	28
		1	$(0,0,0,1,0,0,0,0)$	$(0,0,0,0,0,1,1,1)$	14C	28
$8A$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	8G	128
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	8H	128
		2	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	8I	64
$8B$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	8J	128
		1	$(0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0)$	8K	128
		2	$(0,0,0,0,0,0,0,1)$	$(0,0,0,0,0,0,0,0)$	8L	64
$9A$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	9A	108
		3	$(0,0,0,0,0,0,1,0)$	$(1,0,1,0,0,1,0,0)$	18A	36
$9B$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	9B	27
$9C$	1	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	9C	27
$10A$	2^2	1	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	10C	80

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Table 8.3 (continued from previous page)

$g \in O_8^+(2)$	k	f_j	d_j	w	$[x]_{2^8:O_8^+(2)}$	$\vert C_{2^8:O_8^+(2)}(x)\vert$
		1	(0,0,0,0,0,0,1,0)	(1,1,1,1,1,1,0,0)	20A	80
		2	(0,0,0,0,0,0,0,1)	(0,0,0,1,1,1,0,0)	20B	40
$10B$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	10D	20
$10C$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	10E	20
$12A$	2^2	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12M	576
		3	(0,0,1,0,0,1,0,0)	(0,0,0,0,0,0,0,0)	12N	192
$12B$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12O	144
$12C$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12P	144
$12D$	2^2	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12Q	144
		3	(0,0,1,0,0,1,0,0)	(0,0,0,0,0,0,0,0)	12R	48
$12E$	2^2	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12S	96
		1	(0,0,0,0,0,0,1,0)	(1,0,1,1,0,0,0,1)	24A	96
		1	(0,0,0,0,0,1,0,0)	(1,0,1,0,1,0,1,0)	24B	96
		1	(0,0,0,0,1,0,0,0)	(0,0,0,0,0,0,0,0)	12T	96
$12F$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12U	24
$12G$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	12V	24
$15A$	2^2	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	15A	60
		1	(0,0,0,0,0,0,0,1)	(1,0,1,0,1,1,1,1)	30A	60
		1	(0,0,0,0,0,0,1,0)	(1,0,1,0,1,0,1,0)	30B	60
		1	(0,0,0,1,0,0,0,0)	(0,0,0,0,1,1,1,1)	30C	60
$15B$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	15B	15
$15C$	1	1	(0,0,0,0,0,0,0,0)	(0,0,0,0,0,0,0,0)	15C	15

8.4. The Character Table of $2^8{:}O_8^+(2)$

8.4.1 Inertia factor groups $Sp(6,2)$ and $2^6{:}A_8$ and their fusion into $O_8^+(2)$

When $O_8^+(2)$ acts on 2^8 we get three orbits of conjugacy classes so that by Brauer [14] when $O_8^+(2)$ acts on $Irr(2^8)$ we also get three orbits of irreducible characters. In this case the orbit lengths of the irreducible characters are also $1, 120$ and 135. These have corresponding point stabilizers H_1, H_2 and H_3 of indices $1, 120$ and 135 respectively. From the $\mathbb{ATLAS}$ [23] the corresponding inertia factor groups are $H_1 = O_8^+(2)$, $H_2 = Sp_6(2)$ and $H_3 = 2^6{:}A_8$. We have seen from Table 8.3 that $2^8{:}O_8^+(2)$ has 124 conjugacy classes and hence it has 124 irreducible characters.

We give the fusion of the conjugacy classes of $2^6{:}A_8$ and $Sp_6(2)$ into $O_8^+(2)$ respectively in two tables, namely Tables 8.5 and 8.6.

The work required for the fusions of $2^6{:}A_8$ into $O_8^+(2)$ are listed in Table 8.4. We use programme D (see appendix) to compute the fusion of $SP(6,2)$ into $O_8^+(2)$.

Table 8.4:

Class of $O_8^+(2)$	1a	2a	2b	2c	2d	2e	3a	3e	4a	4b	4c	4d	4e	4f
Class of $2^6{:}A_8$														
1a	135													
2a			1											
2b		3												
2c		36												
2d				15										
2e					15									
2f						1								
2g			30											
2h						6								
3a							27							
3b								9						
4a										1				
4b									3					
4c										2				
4d											1			
4e										8				
4f														1
4g												3		
4h													3	
4i											6			
4j														2
$\chi(O_8^+(2)\|2^6{:}A_8)$	135	39	31	15	15	7	27	9	3	11	7	3	3	3

Class of $O_8^+(2)$	5a	5b	5c	6a	6b	6c	6d	6e	6f	6g	6h	6i	6j	6k	6l	6m	6n
Class of $2^6{:}A_8$																	
5a	5																
6a				3													
6b							1										
6c													3				
6d												3					
6e														1			
6f							6										
6g											9						
6h																	
6i															3		
6j																	1
$\chi(O_8^+(2)\|2^6{:}A_8)$	5	0	0	3	0	0	7	0	0	0	9	3	3	1	3	0	1

Class of $O_8^+(2)$	7a	8a	8b	10a	12a	12b	12c	12d	12e	12f	12g	15a	15b
Class of $2^6{:}A_8$													
7a	1												
7b	1												
8a		1											
8b			1										
10a				1									
12a					3								
12b									3				
15a												1	
15b												1	
$\chi(O_8^+(2)\|2^6{:}A_8)$	2	1	1	1	3	0	0	0	3	0	0	2	0

Table 8.5: The fusion of $2^6{:}A_8$ into $O_8{}^+(2)$

$[x]_{2^6:A_8}$	$\longrightarrow$	$[g_1]_{O_8^+(2)}$	$[x]_{2^6:A_8}$	$\longrightarrow$	$[g_1]_{O_8^+(2)}$
$1A$		$1A$	$2A$		$2B$
$2B$		$2A$	$2C$		$2A$
$2D$		$2C$	$2E$		$2D$
$2F$		$2E$	$2G$		$2B$
$2H$		$2E$	$3A$		$3A$
$3B$		$3E$	$4A$		$4B$
$4B$		$4A$	$4C$		$4B$
$4D$		$4C$	$4E$		$4B$
$4F$		$4F$	$4G$		$4D$
$4H$		$4E$	$4I$		$4C$
$4J$		$4F$	$5A$		$5A$
$6A$		$6A$	$6B$		$6D$
$6C$		$6J$	$6D$		$6I$
$6E$		$6K$	$6F$		$6D$
$6G$		$6H$	$6H$		$6M$
$6I$		$6L$	$6J$		$6N$
$7A$		$7A$	$7B$		$7A$
$8A$		$8A$	$8B$		$8B$
$10A$		$10A$	$12A$		$12A$
$12B$		$12E$	$15A$		$15A$
$15B$		$15A$			

Table 8.6: The fusion of $SP(6,2)$ into $O_8^+(2)$

$[x]_{SP(6,2)}$	$\longrightarrow$	$[g_1]_{O_8^+(2)}$	$[x]_{SP(6,2)}$	$\longrightarrow$	$[g_1]_{O_8^+(2)}$
$1A$		$1A$	$2A$		$2B$
$2B$		$2A$	$2C$		$2B$
$2D$		$2E$	$3A$		$3A$
$3B$		$3D$	$3C$		$3E$
$4A$		$4A$	$4B$		$4C$
$4C$		$4C$	$4D$		$4B$
$4E$		$4C$	$6A$		$6D$
$5A$		$5A$	$6B$		$6A$
$6C$		$6G$	$6D$		$6D$
$6E$		$6K$	$6F$		$6H$
$6G$		$6N$	$7A$		$7A$
$8A$		$8A$	$8B$		$8B$
$9A$		$9A$	$10A$		$10A$
$12A$		$12E$	$12B$		$12E$
$12C$		$12D$	$15A$		$15A$

8.4.2 The Fischer-Clifford Matrices of $\overline{G}$

By using the the fusions of the inertia factor groups and the properties of the Fischer-Clifford matrix from chapter 5, we computed the Fischer-Clifford matrices. These are given in Table 8.7.

Table 8.7: The Fischer-Clifford Matrices of $2^8{:}O_8^+(2)$

M(g)	M(g)
$M(1A) = \begin{bmatrix} 1 & 1 & 1 \\ 120 & 8 & -8 \\ 135 & -9 & 7 \end{bmatrix}$	$M(2C) = \begin{bmatrix} 1 & 1 \\ 15 & -1 \end{bmatrix}$
$M(2A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 24 & 8 & -8 & 0 \\ 3 & 3 & 3 & -1 \\ 36 & -12 & 4 & 0 \end{bmatrix}$	$M(2D) = \begin{bmatrix} 1 & 1 \\ 15 & -1 \end{bmatrix}$
$M(2B) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & -2 & -2 & 2 & 0 \\ 30 & 10 & -6 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 30 & -10 & 6 & -2 & 0 \end{bmatrix}$	$M(3D) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(2E) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 0 & -8 & 0 \\ 1 & -1 & 1 & 1 \\ 6 & 0 & 6 & -2 \end{bmatrix}$	$M(4D) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(3A) = \begin{bmatrix} 1 & 1 & 1 \\ 36 & -4 & 4 \\ 27 & 3 & -5 \end{bmatrix}$	$M(4E) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(3E) = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{bmatrix}$	$M(6G) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(4A) = \begin{bmatrix} 1 & 1 & 1 \\ 12 & 0 & -4 \\ 3 & -1 & 3 \end{bmatrix}$	$M(6I) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(4B) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 0 & 4 & -4 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 2 & -2 & 2 & 2 & 0 \\ 8 & 0 & -8 & 0 & 0 \end{bmatrix}$	$M(6J) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(4C) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 6 & -6 & -2 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 6 & 6 & -2 & -2 & 0 & 0 \end{bmatrix}$	$M(6L) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(4F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{bmatrix}$	$M(7A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$
$M(5A) = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 2 & -2 \\ 5 & -3 & 1 \end{bmatrix}$	$M(6M) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

continued on next page

Table 8.7 (continued from previous page)

M(g)	M(g)
$M(6A) = \begin{bmatrix} 1 & 1 & 1 \\ 12 & 0 & -4 \\ 3 & -1 & 3 \end{bmatrix}$	$M(9A) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(6D) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & -2 & 2 & -2 & 0 \\ 6 & 6 & -2 & -2 & 0 \\ 1 & 1 & 1 & 1 & -1 \\ 6 & -6 & -2 & 2 & 0 \end{bmatrix}$	$M(12A) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(6H) = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{bmatrix}$	$M(12D) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(6K) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$	$M(3B) = M(3C) = \begin{bmatrix} 1 \end{bmatrix}$
$M(6N) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$	$M(5A) = M(5B) = \begin{bmatrix} 1 \end{bmatrix}$
$M(8A) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$	$M(6B) = M(6C) = M(6E) = M(6F) = \begin{bmatrix} 1 \end{bmatrix}$
$M(8B) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$	$M(9B) = M(9C) = \begin{bmatrix} 1 \end{bmatrix}$
$M(10A) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$	$M(10B) = M(10C) = \begin{bmatrix} 1 \end{bmatrix}$
$M(12E) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$	$M(12B) = M(12C) = M(12F) = M(12G) = \begin{bmatrix} 1 \end{bmatrix}$
$M(15A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$	$M(15B) = M(15C) = \begin{bmatrix} 1 \end{bmatrix}$

We use programme D (see appendix) to give the fusion of $2^8{:}O_8^+(2)$ into $O_{10}^+(2)$ in Table 8.8.

Table 8.8: The fusion of $2^8{:}O_8^+(2)$ into $O_{10}^+(2)$

$[g]_{2^8:O_8^+(2)}$	$\longrightarrow$	$[y]_{O_{10}^+(2)}$	$[g]_{2^8:O_8^+(2)}$	$\longrightarrow$	$[y]_{O_{10}^+(2)}$
$1A$		$1A$	$2A$		$2B$
$2B$		$2A$	$2C$		$2A$
$2D$		$2D$	$2E$		$2C$
$2F$		$2B$	$2G$		$2D$
$2H$		$2C$	$2I$		$2C$
$2J$		$2D$	$3A$		$3A$
$3B$		$3C$	$3C$		$3C$

continued on next page

Table 8.8 (continued from previous page)

$[g]_{2^8:O_8^+(2)}$	$\longrightarrow$	$[y]_{O_{10}^+(2)}$	$[g]_{2^8:O_8^+(2)}$	$\longrightarrow$	$[y]_{O_{10}^+(2)}$
$3D$		$3D$	$3E$		$3B$
$4A$		$4B$	$4B$		$4A$
$4C$		$4B$	$4D$		$4C$
$4E$		$4D$	$4F$		$4D$
$4G$		$4F$	$4H$		$4D$
$4I$		$4E$	$4J$		$4A$
$4K$		$4G$	$4L$		$4E$
$4M$		$4B$	$4N$		$4I$
$4O$		$4E$	$4P$		$4D$
$4Q$		$4H$	$4R$		$4C$
$4S$		$4G$	$4T$		$4I$
$4U$		$4F$	$4V$		$4H$
$4W$		$4H$	$4X$		$4I$
$5A$		$5A$	$5B$		$5B$
$5C$		$5B$	$6A$		$6B$
$6B$		$6A$	$6C$		$6G$
$6D$		$6I$	$6F$		$6A$
$6E$		$6F$	$6G$		$6D$
$6H$		$6H$	$6I$		$6H$
$6J$		$6B$	$6K$		$6D$
$6L$		$6L$	$6M$		$6L$
$6N$		$6E$	$6O$		$6M$
$6P$		$6C$	$6Q$		$6K$
$6R$		$6J$	$6S$		$6I$
$6T$		$6F$	$6U$		$6I$
$6V$		$6J$	$6W$		$6J$
$6X$		$6K$	$7A$		$7A$
$8A$		$8B$	$8B$		$8A$
$8C$		$8C$	$8D$		$8C$
$8E$		$8C$	$8F$		$8D$
$8G$		$8A$	$8H$		$8D$
$8G$		$8D$	$8H$		$8A$
$8I$		$8E$	$8J$		$8B$
$8K$		$8D$	$8L$		$8F$
$9A$		$9A$	$9B$		$9B$
$9C$		$9B$	$10A$		$10A$
$10B$		$10B$	$10C$		$10B$
$10D$		$10C$	$10E$		$10C$
$12A$		$12B$	$12B$		$12B$
$12C$		$12A$	$12D$		$12D$
$12E$		$12E$	$12F$		$12E$
$12G$		$12E$	$12H$		$12G$
$12I$		$12I$	$12J$		$12I$
$12K$		$12I$	$12L$		$12K$
$12M$		$12A$	$12N$		$12F$
$12O$		$12H$	$12P$		$12H$
$12Q$		$12C$	$12R$		$12J$
$12S$		$12D$	$12T$		$12F$
$12U$		$12L$	$12V$		$12L$
$14A$		$14A$	$14B$		$14B$
$14C$		$14C$	$15A$		$15C$
$15B$		$15E$	$15C$		$15E$

continued on next page

Table 8.8 (continued from previous page)

$[g]_{2^8:O_8^+(2)}$	$\longrightarrow$	$[y]_{O_{10}^+(2)}$	$[g]_{2^8:O_8^+(2)}$	$\longrightarrow$	$[y]_{O_{10}^+(2)}$
$18A$		$18A$	$20A$		$20A$
$20B$		$20B$	$24A$		$24A$
$24B$		$24B$	$30A$		$30C$
$30B$		$30F$	$30C$		$30D$

We use programme E (see appendix) to give the power maps of elements of $2^8{:}O_8^+(2)$ in Table 8.9.

Table 8.9: The Power Maps of elements of $2^8{:}O_8^+(2)$

$[g]_{S_8}$	$[x]_{2^8:O_8^+(2)}$	2	3	5	7	$[g]_{S_8}$	$[x]_{2^8:O_8^+(2)}$	2	3	5	7
1A	1A	1A				2A	2C	1A			
	2A	1A					2D	1A			
	2B	1A					2D	1A			
							2E	1A			
							4A	2A			
2B	2F	1A				2E	2J	1A			
	4B	2B					4G	2A			
	4C	2B					4H	2B			
	2G	1A					4I	2B			
	4D	2A									
2C	2H	1A				2D	2I	1A			
	4E	2B					4F	2B			
3A	3A	3A	1A			3D	3D	3D	1A		
	6A	3A	2A				6C	3D	2B		
	6B	3A	2B								
3B	3B	3B	1A			3C	3C	3C	1A		
3E	3E	3E	1A			4A	4J	2C			
	6D	3E	2A				4K	2D			
	6E	3E	2B				4L	2C			
4B	4M	2C				4C	4R	2F			
	4N	2D					4S	2G			
	4O	2C					4T	2G			
	4P	2C					4U	2F			
	4Q	2E					8A	4C			
							8B	4B			
4D	4V	2H				4E	4W	2I			
	8C	4E					8D	4F			
4F	4X	2E				5A	5A	5A	5A	1A	
	8E	4H					10A	5A	5A	2B	
	8F	4G					10B	5A	5A	2A	
5B	5B	5B	5B	1A		5C	5C	5C	5C	1A	
6A	6F	3A	2C			6D	6J	3A	2F		
	12A	6B	4A				12B	6A	4C		
	6G	3A	2D				6K	3A	2G		
							12C	6A	4B		
							12D	6B	4D		

continued on next page

Table 8.9 (continued from previous page)

$[g]_{S_8}$	$[x]_{2^8:O_8^+(2)}$	2	3	5	7	$[g]_{S_8}$	$[x]_{2^8:O_8^+(2)}$	2	3	5	7
6B	6H	3B	2C			6C	6I	3C	2C		
6E	6L	3B	2H			6F	6M	3C	2I		
6H	6P	3E	2C			6G	6N	3D	2C		
	6Q	3E	2D				6O	3D	2D		
	6R	3E	2E								
6I	6S	3E	2C			6J	6T	3E	2C		
	12E	6E	4A				12F	6E	4A		
6K	6U	3E	2F			6L	6V	3E	2H		
	12G	6E	4C				12I	6E	4E		
	12H	6D	4D								
6N	6X	3E	2J			6M	6W	3E	2I		
	12K	6D	4G				12J	6E	4F		
	12L	6E	4H								
7A	7A	7A	7A	7A	1A	9A	9A	9A	3D		
	14A	7A	14A	14A	2A		18A	9A	6C		
	14B	7A	14B	14B	2B						
	14C	7A	14C	14C	2B						
9B	9B	9B	3D			9C	9C	9C	3D		
8A	8G	4J				8B	8J	4M			
	8H	4K					8K	4N			
	8I	4L					8L	4P			
10A	10C	5A	10C	2F		12A	12M	6F	4J		
	20A	10A	20A	4B			12N	6G	4L		
	20B	10B	20B	4D							
10B	10D	5B	10D	2H		10C	10E	5C	10E	2I	
12B	12O	6H	4J			12C	12P	6I	4J		
12D	12Q	6N	4J			12E	12S	6J	4R		
	12R	6N	4L				24A	12B	8A		
							24B	12C	8B		
							12T	6K	4S		
12F	12U	6L	4V			12G	12V	6M	4W		
15A	15A	15A	5A	3A		15B	15B	15B	5B	3B	
	30A	15A	10B	6B							
	30B	15A	10A	6A		15C	15C	15C	5C	3C	
	30C	15A	10A	6A							

To compute the character table of $2^8{:}O_8^+(2)$, as an example consider the following. Let $C_1(2B), C_2(2B), C_3(2B)$ be the partial character tables of the inertia factors for the classes that fuse to $2B \in O_8^+(2)$. The portions of the character table of $\overline{G} = 2^8{:}O_8^+(2)$ corresponding to the coset $2B$ are :

$$C_1(2B)M_1(2B) = \begin{bmatrix} 1 \\ 4 \\ 11 \\ -5 \\ -5 \\ 10 \\ 20 \\ 4 \\ 4 \\ 15 \\ 26 \\ 10 \\ 10 \\ 20 \\ -10 \\ 5 \\ 39 \\ -9 \\ -9 \\ 20 \\ 60 \\ -20 \\ -20 \\ 24 \\ -40 \\ -40 \\ 36 \\ 50 \\ -30 \\ -30 \\ 64 \\ 0 \\ 0 \\ 40 \\ 15 \\ 15 \\ 15 \\ -60 \\ 20 \\ 20 \\ 64 \\ 0 \\ 0 \\ 12 \\ -36 \\ -36 \\ 51 \\ -45 \\ -45 \\ 0 \\ 0 \\ 40 \\ -45 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 & 4 \\ 11 & 11 & 11 & 11 & 11 \\ -5 & -5 & -5 & -5 & -5 \\ -5 & -5 & -5 & -5 & -5 \\ 10 & 10 & 10 & 10 & 10 \\ 20 & 20 & 20 & 20 & 20 \\ 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 & 4 \\ 15 & 15 & 15 & 15 & 15 \\ 26 & 26 & 26 & 26 & 26 \\ 10 & 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 & 10 \\ 20 & 20 & 20 & 20 & 20 \\ -10 & -10 & -10 & -10 & -10 \\ 5 & 5 & 5 & 5 & 5 \\ 39 & 39 & 39 & 39 & 39 \\ -9 & -9 & -9 & -9 & -9 \\ -9 & -9 & -9 & -9 & -9 \\ 20 & 20 & 20 & 20 & 20 \\ 60 & 60 & 60 & 60 & 60 \\ -20 & -20 & -20 & -20 & -20 \\ -20 & -20 & -20 & -20 & -20 \\ 24 & 24 & 24 & 24 & 24 \\ -40 & -40 & -40 & -40 & -40 \\ -40 & -40 & -40 & -40 & -40 \\ 36 & 36 & 36 & 36 & 36 \\ 50 & 50 & 50 & 50 & 50 \\ -30 & -30 & -30 & -30 & -30 \\ -30 & -30 & -30 & -30 & -30 \\ 64 & 64 & 64 & 64 & 64 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 40 & 40 & 40 & 40 & 40 \\ 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 \\ -60 & -60 & -60 & -60 & -60 \\ 20 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 20 \\ 64 & 64 & 64 & 64 & 64 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 12 & 12 & 12 & 12 & 12 \\ -36 & -36 & -36 & -36 & -36 \\ -36 & -36 & -36 & -36 & -36 \\ 51 & 51 & 51 & 51 & 51 \\ -45 & -45 & -45 & -45 & -45 \\ -45 & -45 & -45 & -45 & -45 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 40 & 40 & 40 & 40 & 40 \\ -45 & -45 & -45 & -45 & -45 \end{bmatrix}$$

$$C_2(2B)M_2(2B) = \begin{bmatrix} 1 & 1 \\ -5 & 3 \\ -5 & 3 \\ 9 & 1 \\ -11 & 5 \\ 15 & 7 \\ -5 & -5 \\ 15 & 7 \\ -24 & 8 \\ -10 & 6 \\ 4 & 4 \\ -35 & 5 \\ 25 & 9 \\ 5 & -3 \\ 40 & 8 \\ 40 & 8 \\ 21 & -11 \\ -51 & 13 \\ -39 & 1 \\ 50 & 2 \\ 10 & 10 \\ -24 & 8 \\ -40 & -8 \\ 40 & 8 \\ -45 & 3 \\ -16 & -16 \\ -30 & 2 \\ 45 & -3 \\ 20 & -12 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 & -2 & 2 & 0 \\ 30 & 10 & -6 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 32 & 8 & -8 & 0 & 0 \\ 80 & 40 & -8 & -16 & 0 \\ 80 & 40 & -8 & -16 & 0 \\ 48 & -8 & -24 & 16 & 0 \\ 128 & 72 & -8 & -32 & 0 \\ 240 & 40 & -72 & 16 & 0 \\ -160 & -40 & 40 & 0 & 0 \\ 240 & 40 & -72 & 16 & 0 \\ 192 & 128 & 0 & -64 & 0 \\ 160 & 80 & -16 & -32 & 0 \\ 128 & 32 & -32 & 0 & 0 \\ 80 & 120 & 40 & -80 & 0 \\ 320 & 40 & -104 & 32 & 0 \\ -80 & -40 & 8 & 16 & 0 \\ 320 & 0 & -128 & 64 & 0 \\ 320 & 0 & -128 & 64 & 0 \\ -288 & -152 & 24 & 64 & 0 \\ 288 & 232 & 24 & -128 & 0 \\ -48 & 88 & 72 & -80 & 0 \\ 160 & -80 & -112 & 96 & 0 \\ 320 & 80 & -80 & 0 & 0 \\ 192 & 128 & 0 & -64 & 0 \\ -320 & 0 & 128 & -64 & 0 \\ 320 & 0 & -128 & 64 & 0 \\ 0 & 120 & 72 & -96 & 0 \\ -512 & -128 & 128 & 0 & 0 \\ 0 & 80 & 48 & -64 & 0 \\ 0 & -120 & -72 & 96 & 0 \\ -320 & -160 & 32 & 64 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3(2B)M_3(2B) = \begin{bmatrix} 1 & 1 \\ 7 & 3 \\ 14 & 2 \\ 20 & 4 \\ 21 & 1 \\ 21 & 1 \\ 21 & 1 \\ 28 & 4 \\ 35 & -5 \\ 45 & -3 \\ 45 & -3 \\ 56 & 0 \\ 64 & 0 \\ 70 & 2 \\ 4 & 8 \\ 4 & 4 \\ -5 & 7 \\ -5 & 7 \\ -5 & -5 \\ -5 & -5 \\ -10 & 2 \\ -10 & 2 \\ -10 & 2 \\ -10 & 2 \\ 20 & 0 \\ 20 & 12 \\ -20 & 4 \\ 20 & 8 \\ 20 & 4 \\ -30 & 14 \\ -30 & -10 \\ 36 & 0 \\ 36 & 12 \\ 40 & -16 \\ 40 & -8 \\ -45 & 3 \\ -45 & 3 \\ -45 & -9 \\ -45 & -9 \\ -60 & 4 \\ 64 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & -1 \\ 30 & -10 & 6 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 31 & -9 & 7 & -1 & -1 \\ 97 & -23 & 25 & 1 & -7 \\ 74 & -6 & 26 & 10 & -14 \\ 140 & -20 & 44 & 12 & -20 \\ 51 & 11 & 27 & 19 & -21 \\ 51 & 11 & 27 & 19 & -21 \\ 51 & 11 & 27 & 19 & -21 \\ 148 & -12 & 52 & 20 & -28 \\ -115 & 85 & 5 & 45 & -35 \\ -45 & 75 & 27 & 51 & -45 \\ -45 & 75 & 27 & 51 & -45 \\ 56 & 56 & 56 & 56 & -56 \\ 64 & 64 & 64 & 64 & -64 \\ 130 & 50 & 82 & 66 & -70 \\ 244 & -76 & 52 & -12 & -4 \\ 124 & -36 & 28 & -4 & -4 \\ 205 & -75 & 37 & -19 & 5 \\ 205 & -75 & 37 & -19 & 5 \\ -155 & 45 & -35 & 5 & 5 \\ -155 & 45 & -35 & 5 & 5 \\ 50 & -30 & 2 & -14 & 10 \\ 50 & -30 & 2 & -14 & 10 \\ 50 & -30 & 2 & -14 & 10 \\ 50 & -30 & 2 & -14 & 10 \\ 20 & 20 & 20 & 20 & -20 \\ 380 & -100 & 92 & -4 & -20 \\ 100 & -60 & 4 & -28 & 20 \\ 260 & -60 & 68 & 4 & -20 \\ 140 & -20 & 44 & 12 & -20 \\ 390 & -170 & 54 & -58 & 30 \\ -330 & 70 & -90 & -10 & 30 \\ 36 & 36 & 36 & 36 & -36 \\ 396 & -84 & 108 & 12 & -36 \\ -440 & 200 & -56 & 72 & -40 \\ -200 & 120 & -8 & 56 & -40 \\ 45 & -75 & -27 & -51 & 45 \\ 45 & -75 & -27 & -51 & 45 \\ -315 & 45 & -99 & -27 & 45 \\ -315 & 45 & -99 & -27 & 45 \\ 60 & -100 & -36 & -68 & 60 \\ 64 & 64 & 64 & 64 & -64 \end{bmatrix}$$

We used the Fischer-Clifford matrices and partial character tables of inertia factor groups and computed the character table of $\overline{G}$. This character table is given in Table 8.10. We converted this character table to the GAP format and used Programme E (see appendix) to test its validity and to compute the power maps.

8.4.3 The Character Table of $\overline{G}$

Table 8.10: The Character Table of $2^8{:}O_8^+(2)$

	$1A$			2A				2B					$2C$		2D	
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$2F$	$4B$	$4C$	$2G$	$4D$	$2H$	$4E$	$2I$	$4F$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	28	28	28	-4	-4	-4	-4	4	4	4	4	4	4	4	4	4
χ_3	35	35	35	3	3	3	3	11	11	11	11	11	-5	-5	-5	-5
χ_4	35	35	35	3	3	3	3	-5	-5	-5	-5	-5	11	11	-5	-5
χ_5	35	35	35	3	3	3	3	-5	-5	-5	-5	-5	-5	-5	11	11
χ_6	50	50	50	18	18	18	18	10	10	10	10	10	10	10	10	10
χ_7	84	84	84	20	20	20	20	20	20	20	20	20	4	4	4	4
χ_8	84	84	84	20	20	20	20	4	4	4	4	4	20	20	4	4
χ_9	84	84	84	20	20	20	20	4	4	4	4	4	4	4	20	20
χ_{10}	175	175	175	-17	-17	-17	-17	15	15	15	15	15	15	15	15	15
χ_{11}	210	210	210	-14	-14	-14	-14	26	26	26	26	26	10	10	10	10
χ_{12}	210	210	210	-14	-14	-14	-14	10	10	10	10	10	26	26	10	10
χ_{13}	210	210	210	-14	-14	-14	-14	10	10	10	10	10	10	10	26	26
χ_{14}	300	300	300	12	12	12	12	20	20	20	20	20	20	20	20	20
χ_{15}	350	350	350	-2	-2	-2	-2	-10	-10	-10	-10	-10	-10	-10	-10	-10
χ_{16}	525	525	525	45	45	45	45	5	5	5	5	5	5	5	5	5
χ_{17}	567	567	567	-9	-9	-9	-9	39	39	39	39	39	-9	-9	-9	-9
χ_{18}	567	567	567	-9	-9	-9	-9	-9	-9	-9	-9	-9	39	39	-9	-9
χ_{19}	567	567	567	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	-9	39	39
χ_{20}	700	700	700	92	92	92	92	20	20	20	20	20	20	20	20	20
χ_{21}	700	700	700	-4	-4	-4	-4	60	60	60	60	60	-20	-20	-20	-20
χ_{22}	700	700	700	-4	-4	-4	-4	-20	-20	-20	-20	-20	60	60	-20	-20
χ_{23}	700	700	700	-4	-4	-4	-4	-20	-20	-20	-20	-20	-20	-20	60	60
χ_{24}	840	840	840	8	8	8	8	24	24	24	24	24	-40	-40	-40	-40
χ_{25}	840	840	840	8	8	8	8	-40	-40	-40	-40	-40	24	24	-40	-40
χ_{26}	840	840	840	8	8	8	8	-40	-40	-40	-40	-40	-40	-40	24	24
χ_{27}	972	972	972	108	108	108	108	36	36	36	36	36	36	36	36	36
χ_{28}	1050	1050	1050	58	58	58	58	50	50	50	50	50	-30	-30	-30	-30
χ_{29}	1050	1050	1050	58	58	58	58	-30	-30	-30	-30	-30	50	50	-30	-30
χ_{30}	1050	1050	1050	58	58	58	58	-30	-30	-30	-30	-30	-30	-30	50	50
χ_{31}	1344	1344	1344	64	64	64	64	64	64	64	64	64	0	0	0	0
χ_{32}	1344	1344	1344	64	64	64	64	0	0	0	0	0	64	64	0	0
χ_{33}	1344	1344	1344	64	64	64	64	0	0	0	0	0	0	0	64	64
χ_{34}	1400	1400	1400	-72	-72	-72	-72	40	40	40	40	40	40	40	40	40
χ_{35}	1575	1575	1575	-57	-57	-57	-57	15	15	15	15	15	15	15	15	15
χ_{36}	1575	1575	1575	-57	-57	-57	-57	15	15	15	15	15	15	15	15	15
χ_{37}	1575	1575	1575	-57	-57	-57	-57	15	15	15	15	15	15	15	15	15
χ_{38}	2100	2100	2100	52	52	52	52	-60	-60	-60	-60	-60	20	20	20	20
χ_{39}	2100	2100	2100	52	52	52	52	20	20	20	20	20	-60	-60	20	20
χ_{40}	2100	2100	2100	52	52	52	52	20	20	20	20	20	20	20	-60	-60
χ_{41}	2240	2240	2240	-64	-64	-64	-64	64	64	64	64	64	0	0	0	0
χ_{42}	2240	2240	2240	-64	-64	-64	-64	0	0	0	0	0	64	64	0	0
χ_{43}	2240	2240	2240	-64	-64	-64	-64	0	0	0	0	0	0	0	64	64
χ_{44}	2268	2268	2268	-36	-36	-36	-36	12	12	12	12	12	-36	-36	-36	-36

The character table of $2^8{:}O_8^+(2)$(continued)

	1A			2A				2B					2C		2D	
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$2F$	$4B$	$4C$	$2G$	$4D$	$2H$	$4E$	$2I$	$4F$
χ_{45}	2268	2268	2268	-36	-36	-36	-36	-36	-36	-36	-36	-36	12	12	-36	-36
χ_{46}	2268	2268	2268	-36	-36	-36	-36	-36	-36	-36	-36	-36	-36	-36	12	12
χ_{47}	2835	2835	2835	-45	-45	-45	-45	51	51	51	51	51	-45	-45	-45	-45
χ_{48}	2835	2835	2835	-45	-45	-45	-45	-45	-45	-45	-45	-45	51	51	-45	-45
χ_{49}	2835	2835	2835	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	51	51
χ_{50}	3200	3200	3200	128	128	128	128	0	0	0	0	0	0	0	0	0
χ_{51}	4096	4096	4096	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	4200	4200	4200	-24	-24	-24	-24	40	40	40	40	40	40	40	40	40
χ_{53}	6075	6075	6075	27	27	27	27	-45	-45	-45	-45	-45	-45	-45	-45	-45
χ_{54}	120	8	-8	24	8	-8	0	32	8	-8	0	0	0	0	0	0
χ_{55}	840	56	-56	-24	-8	8	0	80	40	-8	-16	0	0	0	0	0
χ_{56}	1800	120	-120	168	56	-56	0	80	40	-8	-16	0	0	0	0	0
χ_{57}	2520	168	-168	-72	-24	24	0	48	-8	-24	16	0	0	0	0	0
χ_{58}	2520	168	-168	120	40	-40	0	128	72	-8	-32	0	0	0	0	0
χ_{59}	3240	216	-216	72	24	-24	0	240	40	-72	16	0	0	0	0	0
χ_{60}	4200	280	-280	72	24	-24	0	-160	-40	40	0	0	0	0	0	0
χ_{61}	4200	280	-280	264	88	-88	0	240	40	-72	16	0	0	0	0	0
χ_{62}	6720	448	-448	-192	-64	64	0	192	128	0	-64	0	0	0	0	0
χ_{63}	8400	560	-560	-240	-80	80	0	160	80	-16	-32	0	0	0	0	0
χ_{64}	10080	672	-672	480	160	-160	0	128	32	-32	0	0	0	0	0	0
χ_{65}	12600	840	-840	24	8	-8	0	80	120	40	-80	0	0	0	0	0
χ_{66}	12600	840	-840	-168	-56	56	0	320	40	-104	32	0	0	0	0	0
χ_{67}	12600	840	-840	408	136	-136	0	-80	-40	8	16	0	0	0	0	0
χ_{68}	14400	960	-960	-192	-64	64	0	320	0	-128	64	0	0	0	0	0
χ_{69}	20160	1344	-1344	192	64	-64	0	320	0	-128	64	0	0	0	0	0
χ_{70}	22680	1512	-1512	-72	-24	24	0	-288	-152	24	64	0	0	0	0	0
χ_{71}	22680	1512	-1512	-72	-24	24	0	288	232	24	-128	0	0	0	0	0
χ_{72}	22680	1512	-1512	504	168	-168	0	-48	88	72	-80	0	0	0	0	0
χ_{73}	25200	1680	-1680	48	16	-16	0	160	-80	-112	96	0	0	0	0	0
χ_{74}	25200	1680	-1680	-336	-112	112	0	320	80	-80	0	0	0	0	0	0
χ_{75}	25920	1728	-1728	576	192	-192	0	192	128	0	-64	0	0	0	0	0
χ_{76}	33600	2240	-2240	-192	-64	64	0	-320	0	128	-64	0	0	0	0	0
χ_{77}	33600	2240	-2240	576	192	-192	0	320	0	-128	64	0	0	0	0	0
χ_{78}	37800	2520	-2520	-504	-168	168	0	0	120	72	-96	0	0	0	0	0
χ_{79}	40320	2688	-2688	384	128	-128	0	-512	-128	128	0	0	0	0	0	0
χ_{80}	45360	3024	-3024	-144	-48	48	0	0	80	48	-64	0	0	0	0	0
χ_{81}	48600	3240	-3240	-648	-216	216	0	0	-120	-72	96	0	0	0	0	0
χ_{82}	50400	3360	-3360	96	32	-32	0	-320	-160	32	64	0	0	0	0	0
χ_{83}	61440	4096	-4096	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	135	-9	7	39	-9	7	-1	31	-9	7	-1	-1	15	-1	15	-1
χ_{85}	945	-63	49	-15	33	17	-7	97	-23	25	1	-7	-15	1	-15	1
χ_{86}	1890	-126	98	258	-30	66	-14	74	-6	26	10	-14	90	-6	90	-6
χ_{87}	2700	-180	140	204	12	76	-20	140	-20	44	12	-20	60	-4	60	-4
χ_{88}	2835	-189	147	-45	99	51	-21	51	11	27	19	-21	-45	3	-45	3

The character table of $2^8{:}O_8^+(2)$(continued)

	$1A$			2A				2B					$2C$		2D	
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$2F$	$4B$	$4C$	$2G$	$4D$	$2H$	$4E$	$2I$	$4F$
χ_{89}	2835	-189	147	-45	99	51	-21	51	11	27	19	-21	-45	3	-45	3
χ_{90}	2835	-189	147	-45	99	51	-21	51	11	27	19	-21	-45	3	-45	3
χ_{91}	3780	-252	196	-60	132	68	-28	148	-12	52	20	-28	-60	4	-60	4
χ_{92}	4725	-315	245	213	69	117	-35	-115	85	5	45	-35	45	-3	45	-3
χ_{93}	6075	-405	315	27	171	123	-45	-45	75	27	51	-45	-45	3	-45	3
χ_{94}	6075	-405	315	27	171	123	-45	-45	75	27	51	-45	-45	3	-45	3
χ_{95}	7560	-504	392	456	72	200	-56	56	56	56	56	-56	120	-8	120	-8
χ_{96}	8640	-576	448	192	192	192	-64	64	64	64	64	-64	0	0	0	0
χ_{97}	9450	-630	490	138	234	202	-70	130	50	82	66	-70	-30	2	-30	2
χ_{98}	3780	-252	196	132	-60	4	4	244	-76	52	-12	-4	-60	4	-60	4
χ_{99}	3780	-252	196	-156	36	-28	4	124	-36	28	-4	-4	60	-4	60	-4
χ_{100}	4725	-315	245	405	-123	53	-3	205	-75	37	-19	5	45	-3	45	-3
χ_{101}	4725	-315	245	-171	69	-11	-3	205	-75	37	-19	5	45	-3	45	-3
χ_{102}	4725	-315	245	117	-27	21	-3	-155	45	-35	5	5	-75	5	165	-11
χ_{103}	4725	-315	245	117	-27	21	-3	-155	45	-35	5	5	165	-11	-75	5
χ_{104}	9450	-630	490	-54	42	10	-6	50	-30	2	-14	10	-30	2	210	-14
χ_{105}	9450	-630	490	-54	42	10	-6	50	-30	2	-14	10	210	-14	-30	2
χ_{106}	9450	-630	490	522	-150	74	-6	50	-30	2	-14	10	-30	2	210	-14
χ_{107}	9450	-630	490	522	-150	74	-6	50	-30	2	-14	10	210	-14	-30	2
χ_{108}	18900	-1260	980	84	-108	-44	20	20	20	20	20	-20	-60	4	-60	4
χ_{109}	18900	-1260	980	-204	-12	-76	20	380	-100	92	-4	-20	60	-4	60	-4
χ_{110}	18900	-1260	980	468	-108	84	-12	100	-60	4	-28	20	180	-12	180	-12
χ_{111}	18900	-1260	980	-492	84	-108	20	260	-60	68	4	-20	180	-12	180	-12
χ_{112}	18900	-1260	980	372	-204	-12	20	140	-20	44	12	-20	-180	12	-180	12
χ_{113}	28350	-1890	1470	-162	126	30	-18	390	-170	54	-58	30	-90	6	-90	6
χ_{114}	28350	-1890	1470	-162	126	30	-18	-330	70	-90	-10	30	-90	6	-90	6
χ_{115}	34020	-2268	1764	-540	36	-156	36	36	36	36	36	-36	180	-12	180	-12
χ_{116}	34020	-2268	1764	324	-252	-60	36	396	-84	108	12	-36	-180	12	-180	12
χ_{117}	37800	-2520	1960	168	-216	-88	40	-440	200	-56	72	-40	-120	8	-120	8
χ_{118}	37800	-2520	1960	-408	-24	-152	40	-200	120	-8	56	-40	120	-8	120	-8
χ_{119}	42525	-2835	2205	-675	333	-3	-27	45	-75	-27	-51	45	45	-3	45	-3
χ_{120}	42525	-2835	2205	1053	-243	189	-27	45	-75	-27	-51	45	45	-3	45	-3
χ_{121}	42525	-2835	2205	189	45	93	-27	-315	45	-99	-27	45	405	-27	-315	21
χ_{122}	42525	-2835	2205	189	45	93	-27	-315	45	-99	-27	45	-315	21	405	-27
χ_{123}	56700	-3780	2940	-324	252	60	-36	60	-100	-36	-68	60	-180	12	-180	12
χ_{124}	60480	-4032	3136	-192	-192	-192	64	64	64	64	64	-64	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	$2E$				3A			3B	$3C$	3D		3E			4A		
	$2J$	$4G$	$4H$	$4I$	$3A$	$6A$	$6B$	$3B$	$3C$	$3D$	$6C$	$3E$	$6D$	$6E$	$4J$	$4K$	$4L$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-4	-4	-4	-4	10	10	10	10	10	1	1	1	1	1	8	8	8
χ_3	3	3	3	3	14	14	14	5	5	-1	-1	2	2	2	7	7	7
χ_4	3	3	3	3	5	5	5	14	5	-1	-1	2	2	2	7	7	7
χ_5	3	3	3	3	5	5	5	5	14	-1	-1	2	2	2	7	7	7
χ_6	2	2	2	2	5	5	5	5	5	-4	-4	5	5	5	-2	-2	-2
χ_7	4	4	4	4	21	21	21	-6	-6	3	3	3	3	3	4	4	4
χ_8	4	4	4	4	-6	-6	-6	21	-6	3	3	3	3	3	4	4	4
χ_9	4	4	4	4	-6	-6	-6	-6	21	3	3	3	3	3	4	4	4
χ_{10}	-1	-1	-1	-1	-5	-5	-5	-5	-5	13	13	4	4	4	-1	-1	-1
χ_{11}	2	2	2	2	39	39	39	-15	-15	-6	-6	3	3	3	6	6	6
χ_{12}	2	2	2	2	-15	-15	-15	39	-15	-6	-6	3	3	3	6	6	6
χ_{13}	2	2	2	2	-15	-15	-15	-15	39	-6	-6	3	3	3	6	6	6
χ_{14}	12	12	12	12	30	30	30	30	30	3	3	-6	-6	-6	8	8	8
χ_{15}	-2	-2	-2	-2	35	35	35	35	35	-1	-1	-1	-1	-1	26	26	26
χ_{16}	-19	-19	-19	-19	30	30	30	30	30	12	12	3	3	3	-7	-7	-7
χ_{17}	-9	-9	-9	-9	81	81	81	0	0	0	0	0	0	0	15	15	15
χ_{18}	-9	-9	-9	-9	0	0	0	81	0	0	0	0	0	0	15	15	15
χ_{19}	-9	-9	-9	-9	0	0	0	0	81	0	0	0	0	0	15	15	15
χ_{20}	-4	-4	-4	-4	-20	-20	-20	-20	-20	-2	-2	7	7	7	0	0	0
χ_{21}	12	12	12	12	55	55	55	10	10	7	7	4	4	4	-4	-4	-4
χ_{22}	12	12	12	12	10	10	10	55	10	7	7	4	4	4	-4	-4	-4
χ_{23}	12	12	12	12	10	10	10	10	55	7	7	4	4	4	-4	-4	-4
χ_{24}	8	8	8	8	-24	-24	-24	30	30	3	3	3	3	3	16	16	16
χ_{25}	8	8	8	8	30	30	30	-24	30	3	3	3	3	3	16	16	16
χ_{26}	8	8	8	8	30	30	30	30	-24	3	3	3	3	3	16	16	16
χ_{27}	12	12	12	12	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	-6	-6	-6	-6	15	15	15	15	15	-3	-3	6	6	6	-10	-10	-10
χ_{29}	-6	-6	-6	-6	15	15	15	15	15	-3	-3	6	6	6	-10	-10	-10
χ_{30}	-6	-6	-6	-6	15	15	15	15	15	-3	-3	6	6	6	-10	-10	-10
χ_{31}	0	0	0	0	84	84	84	-24	-24	-6	-6	-6	-6	-6	0	0	0
χ_{32}	0	0	0	0	-24	-24	-24	84	-24	-6	-6	-6	-6	-6	0	0	0
χ_{33}	0	0	0	0	-24	-24	-24	-24	84	-6	-6	-6	-6	-6	0	0	0
χ_{34}	-8	-8	-8	-8	50	50	50	50	50	-4	-4	5	5	5	-16	-16	-16
χ_{35}	-9	-9	-9	-9	90	90	90	-45	-45	9	9	0	0	0	11	11	11
χ_{36}	-9	-9	-9	-9	-45	-45	-45	90	-45	9	9	0	0	0	11	11	11
χ_{37}	-9	-9	-9	-9	-45	-45	-45	-45	90	9	9	0	0	0	11	11	11
χ_{38}	4	4	4	4	75	75	75	-60	-60	-6	-6	3	3	3	12	12	12
χ_{39}	4	4	4	4	-60	-60	-60	75	-60	-6	-6	3	3	3	12	12	12
χ_{40}	4	4	4	4	-60	-60	-60	-60	75	-6	-6	3	3	3	12	12	12
χ_{41}	0	0	0	0	-4	-4	-4	-40	-40	-10	-10	2	2	2	0	0	0
χ_{42}	0	0	0	0	-40	-40	-40	-4	-40	-10	-10	2	2	2	0	0	0
χ_{43}	0	0	0	0	-40	-40	-40	-40	-4	-10	-10	2	2	2	0	0	0
χ_{44}	12	12	12	12	81	81	81	0	0	0	0	0	0	0	-12	-12	-12

The character table of $2^8{:}O_8^+(2)$(continued)

	$2E$				3A			3B	$3C$	3D		3E			4A		
	$2J$	$4G$	$4H$	$4I$	$3A$	$6A$	$6B$	$3B$	$3C$	$3D$	$6C$	$3E$	$6D$	$6E$	$4J$	$4K$	$4L$
χ_{45}	12	12	12	12	0	0	0	81	0	0	0	0	0	0	-12	-12	-12
χ_{46}	12	12	12	12	0	0	0	0	81	0	0	0	0	0	-12	-12	-12
χ_{47}	3	3	3	3	-81	-81	-81	0	0	0	0	0	0	0	3	3	3
χ_{48}	3	3	3	3	0	0	0	-81	0	0	0	0	0	0	3	3	3
χ_{49}	3	3	3	3	0	0	0	0	-81	0	0	0	0	0	3	3	3
χ_{50}	0	0	0	0	-40	-40	-40	-40	-40	14	14	-4	-4	-4	0	0	0
χ_{51}	0	0	0	0	64	64	64	64	64	-8	-8	-8	-8	-8	0	0	0
χ_{52}	8	8	8	8	-30	-30	-30	-30	-30	15	15	-3	-3	-3	-8	-8	-8
χ_{53}	-21	-21	-21	-21	0	0	0	0	0	0	0	0	0	0	-9	-9	-9
χ_{54}	8	0	-8	0	36	-4	4	0	0	3	-1	6	2	-2	12	0	-4
χ_{55}	-8	0	8	0	144	-16	16	0	0	-6	2	6	2	-2	36	0	-12
χ_{56}	-8	0	8	0	0	0	0	0	0	-9	3	18	6	-6	-12	0	4
χ_{57}	-24	0	24	0	216	-24	24	0	0	9	-3	0	0	0	60	0	-20
χ_{58}	-24	0	24	0	216	-24	24	0	0	9	-3	0	0	0	12	0	-4
χ_{59}	24	0	-24	0	324	-36	36	0	0	0	0	0	0	0	36	0	-12
χ_{60}	24	0	-24	0	180	-20	20	0	0	-3	1	12	4	-4	84	0	-28
χ_{61}	24	0	-24	0	180	-20	20	0	0	-3	1	12	4	-4	-12	0	4
χ_{62}	0	0	0	0	396	-44	44	0	0	6	-2	12	4	-4	0	0	0
χ_{63}	-16	0	16	0	-180	20	-20	0	0	21	-7	6	2	-2	24	0	-8
χ_{64}	32	0	-32	0	-216	24	-24	0	0	9	-3	18	6	-6	48	0	-16
χ_{65}	8	0	-8	0	540	-60	60	0	0	-9	3	-18	-6	6	60	0	-20
χ_{66}	8	0	-8	0	0	0	0	0	0	18	-6	18	6	-6	-36	0	12
χ_{67}	-56	0	56	0	0	0	0	0	0	18	-6	18	6	-6	-36	0	12
χ_{68}	0	0	0	0	540	-60	60	0	0	-18	6	0	0	0	0	0	0
χ_{69}	64	0	-64	0	216	-24	24	0	0	18	-6	-18	-6	6	0	0	0
χ_{70}	-24	0	24	0	324	-36	36	0	0	0	0	0	0	0	108	0	-36
χ_{71}	-24	0	24	0	324	-36	36	0	0	0	0	0	0	0	-36	0	12
χ_{72}	-24	0	24	0	324	-36	36	0	0	0	0	0	0	0	-36	0	12
χ_{73}	-48	0	48	0	540	-60	60	0	0	9	-3	0	0	0	-24	0	8
χ_{74}	16	0	-16	0	-540	60	-60	0	0	-18	6	18	6	-6	72	0	-24
χ_{75}	0	0	0	0	-324	36	-36	0	0	0	0	0	0	0	0	0	0
χ_{76}	64	0	-64	0	360	-40	40	0	0	30	-10	6	2	-2	0	0	0
χ_{77}	0	0	0	0	-180	20	-20	0	0	-24	8	-12	-4	4	0	0	0
χ_{78}	24	0	-24	0	0	0	0	0	0	-27	9	0	0	0	-60	0	20
χ_{79}	0	0	0	0	216	-24	24	0	0	-18	6	0	0	0	0	0	0
χ_{80}	-48	0	48	0	-324	36	-36	0	0	0	0	0	0	0	72	0	-24
χ_{81}	-24	0	24	0	0	0	0	0	0	0	0	0	0	0	-36	0	12
χ_{82}	32	0	-32	0	0	0	0	0	0	-9	3	18	6	-6	-48	0	16
χ_{83}	0	0	0	0	-576	64	-64	0	0	24	-8	-24	-8	8	0	0	0
χ_{84}	7	-1	7	-1	27	3	-5	0	0	0	0	9	-3	1	3	-1	3
χ_{85}	17	1	17	-7	108	12	-20	0	0	0	0	9	-3	1	9	-3	9
χ_{86}	18	-6	18	2	-27	-3	5	0	0	0	0	18	-6	2	6	-2	6
χ_{87}	28	-4	28	-4	135	15	-25	0	0	0	0	-9	3	-1	12	-4	12
χ_{88}	3	3	3	-5	162	18	-30	0	0	0	0	0	0	0	3	-1	3

The character table of $2^8{:}O_8^+(2)$(continued)

	2*E*				3A			3B	3*C*	3D		3E			4A		
	2*J*	4*G*	4*H*	4*I*	3*A*	6*A*	6*B*	3*B*	3*C*	3*D*	6*C*	3*E*	6*D*	6*E*	4*J*	4*K*	4*L*
χ_{89}	3	3	3	-5	-81	-9	15	0	0	0	0	0	0	0	3	-1	3
χ_{90}	3	3	3	-5	-81	-9	15	0	0	0	0	0	0	0	3	-1	3
χ_{91}	20	4	20	-12	27	3	-5	0	0	0	0	9	-3	1	12	-4	12
χ_{92}	-27	-3	-27	13	135	15	-25	0	0	0	0	18	-6	2	-15	5	-15
χ_{93}	-21	3	-21	3	0	0	0	0	0	0	0	0	0	0	-9	3	-9
χ_{94}	-21	3	-21	3	0	0	0	0	0	0	0	0	0	0	-9	3	-9
χ_{95}	8	-8	8	8	-108	-12	20	0	0	0	0	-9	3	-1	0	0	0
χ_{96}	0	0	0	0	108	12	-20	0	0	0	0	-18	6	-2	0	0	0
χ_{97}	10	2	10	-6	-135	-15	25	0	0	0	0	9	-3	1	6	-2	6
χ_{98}	4	-4	4	4	270	30	-50	0	0	0	0	9	-3	1	12	-4	12
χ_{99}	-28	4	-28	4	270	30	-50	0	0	0	0	9	-3	1	24	-8	24
χ_{100}	-11	5	-11	-3	135	15	-25	0	0	0	0	18	-6	2	-15	5	-15
χ_{101}	5	-11	5	13	135	15	-25	0	0	0	0	18	-6	2	-15	5	-15
χ_{102}	21	-3	21	-3	135	15	-25	0	0	0	0	18	-6	2	21	-7	21
χ_{103}	21	-3	21	-3	135	15	-25	0	0	0	0	18	-6	2	21	-7	21
χ_{104}	26	-14	26	10	-135	-15	25	0	0	0	0	9	-3	1	6	-2	6
χ_{105}	10	2	10	-6	-135	-15	25	0	0	0	0	9	-3	1	6	-2	6
χ_{106}	10	2	10	-6	-135	-15	25	0	0	0	0	9	-3	1	6	-2	6
χ_{107}	-44	-4	-44	20	540	60	-100	0	0	0	0	-9	3	-1	36	-12	36
χ_{108}	20	4	20	-12	540	60	-100	0	0	0	0	-9	3	-1	0	0	0
χ_{109}	36	-12	36	4	135	15	-25	0	0	0	0	-36	12	-4	12	-4	12
χ_{110}	-12	12	-12	-12	-270	-30	50	0	0	0	0	18	-6	2	12	-4	12
χ_{111}	-12	-12	-12	20	-270	-30	50	0	0	0	0	18	-6	2	24	-8	24
χ_{112}	-18	6	-18	-2	405	45	-75	0	0	0	0	0	0	0	-30	10	-30
χ_{113}	30	6	30	-18	405	45	-75	0	0	0	0	0	0	0	42	-14	42
χ_{114}	-60	12	-60	4	0	0	0	0	0	0	0	0	0	0	36	-12	36
χ_{115}	36	-12	36	4	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	8	-8	8	8	270	30	-50	0	0	0	0	9	-3	1	-24	8	-24
χ_{117}	40	8	40	-24	270	30	-50	0	0	0	0	9	-3	1	-48	16	-48
χ_{118}	-3	-27	-3	37	0	0	0	0	0	0	0	0	0	0	-27	9	-27
χ_{119}	-51	21	-51	-11	0	0	0	0	0	0	0	0	0	0	-27	9	-27
χ_{120}	-27	21	-27	-19	0	0	0	0	0	0	0	0	0	0	-3	1	
χ_{121}	-3	-3	-3	5	0	0	0	0	0	0	0	0	0	0	9	-3	9
χ_{122}	-3	-3	-3	5	0	0	0	0	0	0	0	0	0	0	9	-3	9
χ_{123}	12	12	12	-20	-405	-45	75	0	0	0	0	0	0	0	12	-4	12
χ_{124}	0	0	0	0	-540	-60	100	0	0	0	0	-18	6	-2	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	4B					4C						4D		4E		4F		
	$4M$	$4N$	$4O$	$4P$	$4Q$	$4R$	$4S$	$4T$	$4U$	$8A$	$8B$	$4V$	$8C$	$4W$	$8D$	$4X$	$8E$	$8F$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	-1	-1	-1	-1	-1	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1
χ_4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	-1	-1	-1	-1	-1
χ_5	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	-1	-1	-1
χ_6	6	6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2	2
χ_7	4	4	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0
χ_8	4	4	4	4	4	0	0	0	0	0	0	4	4	0	0	0	0	0
χ_9	4	4	4	4	4	0	0	0	0	0	0	0	0	4	4	0	0	0
χ_{10}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3
χ_{11}	-2	-2	-2	-2	-2	2	2	2	2	2	2	-2	-2	-2	-2	2	2	2
χ_{12}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	-2	-2	2	2	2
χ_{13}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2	2
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2
χ_{16}	1	1	1	1	1	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1
χ_{17}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	-1	-1	-1
χ_{18}	-1	-1	-1	-1	-1	3	3	3	3	3	3	-1	-1	3	3	-1	-1	-1
χ_{19}	-1	-1	-1	-1	-1	3	3	3	3	3	3	3	3	-1	-1	-1	-1	-1
χ_{20}	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{21}	-4	-4	-4	-4	-4	4	4	4	4	4	4	0	0	0	0	0	0	0
χ_{22}	-4	-4	-4	-4	-4	0	0	0	0	0	0	4	4	0	0	0	0	0
χ_{23}	-4	-4	-4	-4	-4	0	0	0	0	0	0	0	0	4	4	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	-2	-2	-2	-2	-2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2
χ_{29}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	-2	-2	-2	-2	-2
χ_{30}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	-2	-2	-2
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{36}	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{37}	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{38}	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	0	0	0	0	0	0	0
χ_{39}	-4	-4	-4	-4	-4	0	0	0	0	0	0	-4	-4	0	0	0	0	0
χ_{40}	-4	-4	-4	-4	-4	0	0	0	0	0	0	0	0	-4	-4	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	4	4	4	4	4	-4	-4	-4	-4	-4	-4	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	$4B$					$4C$						4D		4E		4F		
	$4M$	$4N$	$4O$	$4P$	$4Q$	$4R$	$4S$	$4T$	$4U$	$8A$	$8B$	$4V$	$8C$	$4W$	$8D$	$4X$	$8E$	$8F$
χ_{45}	4	4	4	4	4	0	0	0	0	0	0	-4	-4	0	0	0	0	0
χ_{46}	4	4	4	4	4	0	0	0	0	0	0	0	0	-4	-4	0	0	0
χ_{47}	3	3	3	3	3	-5	-5	-5	-5	-5	-5	3	3	3	3	-1	-1	-1
χ_{48}	3	3	3	3	3	3	3	3	3	3	3	-5	-5	3	3	-1	-1	-1
χ_{49}	3	3	3	3	3	3	3	3	3	3	3	3	3	-5	-5	-1	-1	-1
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	-8	-8	-8	-8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	-1	-1	-1	-1	-1	3	3	3	3	3	3	3	3	3	3	3	3	3
χ_{54}	4	0	4	-4	0	8	-6	0	2	0	-2	0	0	0	0	0	0	0
χ_{55}	-4	0	-4	4	0	4	-10	-4	-2	4	2	0	0	0	0	0	0	0
χ_{56}	12	0	12	-12	0	4	-2	-4	6	-4	2	0	0	0	0	0	0	0
χ_{57}	4	0	4	-4	0	-4	10	4	2	-4	-2	0	0	0	0	0	0	0
χ_{58}	4	0	4	-4	0	0	-6	-8	2	0	6	0	0	0	0	0	0	0
χ_{59}	-4	0	-4	4	0	12	-2	4	6	-4	-6	0	0	0	0	0	0	0
χ_{60}	-4	0	-4	4	0	-8	6	0	-2	0	2	0	0	0	0	0	0	0
χ_{61}	12	0	12	-12	0	12	-10	4	-2	4	-6	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	-8	0	-8	8	0	0	0	0	0	0	0	0
χ_{63}	8	0	8	-8	0	-8	12	8	-4	0	-4	0	0	0	0	0	0	0
χ_{64}	16	0	16	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	4	0	4	-4	0	-12	2	-4	-6	4	6	0	0	0	0	0	0	0
χ_{66}	-12	0	-12	12	0	0	-6	-8	2	0	6	0	0	0	0	0	0	0
χ_{67}	4	0	4	-4	0	-4	2	4	-6	4	-2	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	8	0	8	-8	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	4	0	4	-4	0	8	-6	0	2	0	-2	0	0	0	0	0	0	0
χ_{71}	-12	0	-12	12	0	8	-6	0	2	0	-2	0	0	0	0	0	0	0
χ_{72}	4	0	4	-4	0	-12	10	-4	2	-4	6	0	0	0	0	0	0	0
χ_{73}	-8	0	-8	8	0	-8	12	8	-4	0	-4	0	0	0	0	0	0	0
χ_{74}	-8	0	-8	8	0	-16	12	0	-4	0	4	0	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	8	0	8	-8	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	-8	0	-8	8	0	0	0	0	0	0	0	0
χ_{78}	12	0	12	-12	0	0	6	8	-2	0	-6	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	-8	0	-8	8	0	16	-12	0	4	0	-4	0	0	0	0	0	0	0
χ_{81}	20	0	20	-20	0	0	-6	-8	2	0	6	0	0	0	0	0	0	0
χ_{82}	-16	0	-16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	11	-1	-5	3	-1	7	5	-1	-3	-1	1	3	-1	3	-1	3	-1	-1
χ_{85}	-7	5	9	1	-3	9	3	1	-5	-3	3	-3	1	-3	1	1	-3	1
χ_{86}	30	-10	-2	14	-2	2	-2	2	-2	-2	2	6	-2	6	-2	2	2	-2
χ_{87}	12	-4	12	12	-4	4	-4	4	-4	-4	4	0	0	0	0	0	0	0
χ_{88}	3	7	-13	-5	-1	-5	-7	3	1	-1	1	3	-1	3	-1	-1	3	-1

The character table of $2^8{:}O_8^+(2)$(continued)

	$4B$					$4C$						4D		4E		4F		
	$4M$	$4N$	$4O$	$4P$	$4Q$	$4R$	$4S$	$4T$	$4U$	$8A$	$8B$	$4V$	$8C$	$4W$	$8D$	$4X$	$8E$	$8F$
χ_{89}	3	7	-13	-5	-1	-5	-7	3	1	-1	1	3	-1	3	-1	-1	3	-1
χ_{90}	3	7	-13	-5	-1	-5	-7	3	1	-1	1	3	-1	3	-1	-1	3	-1
χ_{91}	-4	12	-4	-4	-4	4	-4	4	-4	-4	4	0	0	0	0	0	0	0
χ_{92}	-7	-11	9	1	5	-11	-1	-3	7	5	-5	-3	1	-3	1	-3	1	1
χ_{93}	-1	3	-17	-9	3	3	9	-5	1	3	-3	3	-1	3	-1	3	-1	-1
χ_{94}	-1	3	-17	-9	3	3	9	-5	1	3	-3	3	-1	3	-1	3	-1	-1
χ_{95}	16	-16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	-18	6	14	-2	-2	2	-2	2	-2	-2	2	-6	2	-6	2	-2	-2	2
χ_{98}	-4	-4	-4	-4	4	12	12	-4	-4	0	0	0	0	0	0	-4	4	0
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	25	5	-23	1	-3	5	7	-3	-1	1	-1	-3	1	-3	1	1	-3	1
χ_{101}	-7	5	9	1	-3	5	7	-3	-1	1	-1	-3	1	-3	1	5	1	-3
χ_{102}	-11	1	5	-3	1	-7	-5	1	3	1	-1	-3	1	9	-3	-3	1	1
χ_{103}	-11	1	5	-3	1	-7	-5	1	3	1	-1	9	-3	-3	1	-3	1	1
χ_{104}	-18	6	14	-2	-2	-2	2	-2	2	2	-2	-6	2	6	-2	2	2	-2
χ_{105}	-18	6	14	-2	-2	-2	2	-2	2	2	-2	6	-2	-6	2	2	2	-2
χ_{106}	14	6	-18	-2	-2	-2	2	-2	2	2	-2	-6	2	6	-2	-2	-2	2
χ_{107}	14	6	-18	-2	-2	-2	2	-2	2	2	-2	6	-2	-6	2	-2	-2	2
χ_{108}	4	4	4	4	-4	-12	-12	4	4	0	0	0	0	0	0	4	-4	0
χ_{109}	-8	-8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	-4	12	-4	-4	-4	-4	4	-4	4	4	-4	0	0	0	0	0	0	0
χ_{111}	-4	-4	-4	-4	4	-12	-12	4	4	0	0	0	0	0	0	4	-4	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	-6	2	26	10	-6	-2	2	-2	2	2	-2	6	-2	6	-2	-2	-2	2
χ_{114}	18	-6	-14	2	2	-2	2	-2	2	2	-2	-6	2	-6	2	2	2	-2
χ_{115}	4	4	4	4	-4	12	12	-4	-4	0	0	0	0	0	0	-4	4	0
χ_{116}	-8	-8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	8	8	8	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	21	1	-27	-3	1	-3	-9	5	-1	-3	3	-3	1	-3	1	-3	1	1
χ_{120}	-11	1	5	-3	1	-3	-9	5	-1	-3	3	-3	1	-3	1	1	5	-3
χ_{121}	-15	-3	1	-7	5	9	3	1	-5	-3	3	-3	1	9	-3	1	-3	1
χ_{122}	-15	-3	1	-7	5	9	3	1	-5	-3	3	9	-3	-3	1	1	-3	1
χ_{123}	12	-4	12	12	-4	-4	4	-4	4	4	-4	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	5A			5B	5C	6A			6B	6C	6D					6E	6F
	5A	10A	10B	5B	5C	6F	12A	6G	12B	12C	6H	6I	6J	12D	6K	12E	12F
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	3	3	3	3	3	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2
χ_3	5	5	5	0	0	6	6	6	-3	-3	2	2	2	2	2	1	1
χ_4	0	0	0	5	0	-3	-3	-3	6	-3	1	1	1	1	1	2	1
χ_5	0	0	0	0	5	-3	-3	-3	-3	6	1	1	1	1	1	1	2
χ_6	0	0	0	0	0	-3	-3	-3	-3	-3	1	1	1	1	1	1	1
χ_7	4	4	4	-1	-1	5	5	5	2	2	5	5	5	5	5	-2	-2
χ_8	-1	-1	-1	4	-1	2	2	2	5	2	-2	-2	-2	-2	-2	5	-2
χ_9	-1	-1	-1	-1	4	2	2	2	2	5	-2	-2	-2	-2	-2	-2	5
χ_{10}	0	0	0	0	0	-5	-5	-5	-5	-5	3	3	3	3	3	3	3
χ_{11}	5	5	5	0	0	7	7	7	1	1	-1	-1	-1	-1	-1	1	1
χ_{12}	0	0	0	5	0	1	1	1	7	1	1	1	1	1	1	-1	1
χ_{13}	0	0	0	0	5	1	1	1	1	7	1	1	1	1	1	1	-1
χ_{14}	0	0	0	0	0	6	6	6	6	6	2	2	2	2	2	2	2
χ_{15}	0	0	0	0	0	-5	-5	-5	-5	-5	-1	-1	-1	-1	-1	-1	-1
χ_{16}	0	0	0	0	0	6	6	6	6	6	2	2	2	2	2	2	2
χ_{17}	7	7	7	-3	-3	9	9	9	0	0	-3	-3	-3	-3	-3	0	0
χ_{18}	-3	-3	-3	7	-3	0	0	0	9	0	0	0	0	0	0	-3	0
χ_{19}	-3	-3	-3	-3	7	0	0	0	0	9	0	0	0	0	0	0	-3
χ_{20}	0	0	0	0	0	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
χ_{21}	0	0	0	0	0	-1	-1	-1	2	2	3	3	3	3	3	-2	-2
χ_{22}	0	0	0	0	0	2	2	2	-1	2	-2	-2	-2	-2	-2	3	-2
χ_{23}	0	0	0	0	0	2	2	2	2	-1	-2	-2	-2	-2	-2	-2	3
χ_{24}	-5	-5	-5	0	0	8	8	8	-10	-10	0	0	0	0	0	2	2
χ_{25}	0	0	0	-5	0	-10	-10	-10	8	-10	2	2	2	2	2	0	2
χ_{26}	0	0	0	0	-5	-10	-10	-10	-10	8	2	2	2	2	2	2	0
χ_{27}	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	0	0	0	0	0	-17	-17	-17	7	7	-1	-1	-1	-1	-1	3	3
χ_{29}	0	0	0	0	0	7	7	7	-17	7	3	3	3	3	3	-1	3
χ_{30}	0	0	0	0	0	7	7	7	7	-17	3	3	3	3	3	3	-1
χ_{31}	-1	-1	-1	4	4	4	4	4	-8	-8	4	4	4	4	4	0	0
χ_{32}	4	4	4	-1	4	-8	-8	-8	4	-8	0	0	0	0	0	4	0
χ_{33}	4	4	4	4	-1	-8	-8	-8	-8	4	0	0	0	0	0	0	4
χ_{34}	0	0	0	0	0	-6	-6	-6	-6	-6	-2	-2	-2	-2	-2	-2	-2
χ_{35}	0	0	0	0	0	-6	-6	-6	3	3	-6	-6	-6	-6	-6	3	3
χ_{36}	0	0	0	0	0	3	3	3	-6	3	3	3	3	3	3	-6	3
χ_{37}	0	0	0	0	0	3	3	3	3	-6	3	3	3	3	3	3	-6
χ_{38}	0	0	0	0	0	-5	-5	-5	4	4	3	3	3	3	3	-4	-4
χ_{39}	0	0	0	0	0	4	4	4	-5	4	-4	-4	-4	-4	-4	3	-4
χ_{40}	0	0	0	0	0	4	4	4	4	-5	-4	-4	-4	-4	-4	-4	3
χ_{41}	-5	-5	-5	0	0	-4	-4	-4	8	8	4	4	4	4	4	0	0
χ_{42}	0	0	0	-5	0	8	8	8	-4	8	0	0	0	0	0	4	0
χ_{43}	0	0	0	0	-5	8	8	8	8	-4	0	0	0	0	0	0	4
χ_{44}	-2	-2	-2	3	3	9	9	9	0	0	-3	-3	-3	-3	-3	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	$5A$			$5B$	5C	6A			6B	6C	6D					6E	6F
	$5A$	$10A$	$10B$	$5B$	$5C$	$6F$	$12A$	$6G$	$12B$	$12C$	$6H$	$6I$	$6J$	$12D$	$6K$	$12E$	$12F$
χ_{45}	3	3	3	-2	3	0	0	0	9	0	0	0	0	0	0	-3	0
χ_{46}	3	3	3	3	-2	0	0	0	0	9	0	0	0	0	0	0	-3
χ_{47}	5	5	5	0	0	-9	-9	-9	0	0	3	3	3	3	3	0	0
χ_{48}	0	0	0	5	0	0	0	0	-9	0	0	0	0	0	0	3	0
χ_{49}	0	0	0	0	5	0	0	0	0	-9	0	0	0	0	0	0	3
χ_{50}	0	0	0	0	0	8	8	8	8	8	0	0	0	0	0	0	0
χ_{51}	-4	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	-6	-6	-6	-6	-6	-2	-2	-2	-2	-2	-2	-2
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	10	2	-2	0	0	12	0	-4	0	0	8	4	0	-4	0	0	0
χ_{55}	20	4	-4	0	0	24	0	-8	0	0	-4	4	-4	4	0	0	0
χ_{56}	0	0	0	0	0	-24	0	8	0	0	-4	4	-4	4	0	0	0
χ_{57}	10	2	-2	0	0	0	0	0	0	0	-12	-12	4	4	0	0	0
χ_{58}	10	2	-2	0	0	24	0	-8	0	0	8	16	-8	0	0	0	0
χ_{59}	20	4	-4	0	0	36	0	-12	0	0	12	0	4	-8	0	0	0
χ_{60}	0	0	0	0	0	-36	0	12	0	0	8	4	0	-4	0	0	0
χ_{61}	0	0	0	0	0	-12	0	4	0	0	12	0	4	-8	0	0	0
χ_{62}	10	2	-2	0	0	12	0	-4	0	0	-12	0	-4	8	0	0	0
χ_{63}	0	0	0	0	0	-12	0	4	0	0	16	20	-8	-4	0	0	0
χ_{64}	-10	-2	2	0	0	24	0	-8	0	0	-16	-8	0	8	0	0	0
χ_{65}	0	0	0	0	0	12	0	-4	0	0	-4	-8	4	0	0	0	0
χ_{66}	0	0	0	0	0	-48	0	16	0	0	8	-8	8	-8	0	0	0
χ_{67}	0	0	0	0	0	24	0	-8	0	0	4	-4	4	-4	0	0	0
χ_{68}	0	0	0	0	0	12	0	-4	0	0	-4	-8	4	0	0	0	0
χ_{69}	-20	-4	4	0	0	24	0	-8	0	0	8	16	-8	0	0	0	0
χ_{70}	-10	-2	2	0	0	-36	0	12	0	0	0	12	-8	4	0	0	0
χ_{71}	-10	-2	2	0	0	-36	0	12	0	0	0	12	-8	4	0	0	0
χ_{72}	-10	-2	2	0	0	36	0	-12	0	0	12	0	4	-8	0	0	0
χ_{73}	0	0	0	0	0	-12	0	4	0	0	-8	-4	0	4	0	0	0
χ_{74}	0	0	0	0	0	12	0	-4	0	0	8	4	0	-4	0	0	0
χ_{75}	10	2	-2	0	0	-36	0	12	0	0	-12	0	-4	8	0	0	0
χ_{76}	0	0	0	0	0	-24	0	8	0	0	-8	-16	8	0	0	0	0
χ_{77}	0	0	0	0	0	-36	0	12	0	0	-4	-8	4	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	10	2	-2	0	0	-24	0	8	0	0	16	8	0	-8	0	0	0
χ_{80}	-20	-4	4	0	0	36	0	-12	0	0	0	-12	8	-4	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	48	0	-16	0	0	-8	8	-8	8	0	0	0
χ_{83}	20	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	5	-3	1	0	0	3	-1	3	0	0	7	-5	-1	3	-1	0	0
χ_{85}	10	-6	2	0	0	12	-4	12	0	0	4	4	4	4	-4	0	0
χ_{86}	-5	3	-1	0	0	-3	1	-3	0	0	-7	5	1	-3	1	0	0
χ_{87}	0	0	0	0	0	15	-5	15	0	0	11	-1	3	7	-5	0	0
χ_{88}	5	-3	1	0	0	18	-6	18	0	0	-6	18	10	2	-6	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	$5A$			$5B$	5C	6A			6B	6C	6D					6E	6F
	$5A$	$10A$	$10B$	$5B$	$5C$	$6F$	$12A$	$6G$	$6H$	$6I$	$6J$	$12B$	$6K$	$12C$	$12D$	$6L$	$6M$
χ_{89}	5	-3	1	0	0	-9	3	-9	0	0	3	-9	-5	-1	3	0	0
χ_{90}	5	-3	1	0	0	-9	3	-9	0	0	3	-9	-5	-1	3	0	0
χ_{91}	-10	6	-2	0	0	3	-1	3	0	0	7	-5	-1	3	-1	0	0
χ_{92}	0	0	0	0	0	15	-5	15	0	0	11	-1	3	7	-5	0	0
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	5	-3	1	0	0	-12	4	-12	0	0	-4	-4	-4	-4	4	0	0
χ_{96}	-5	3	-1	0	0	12	-4	12	0	0	4	4	4	4	-4	0	0
χ_{97}	0	0	0	0	0	-15	5	-15	0	0	-11	1	-3	-7	5	0	0
χ_{98}	15	-9	3	0	0	6	-2	6	0	0	10	-14	-6	2	2	0	0
χ_{99}	15	-9	3	0	0	6	-2	6	0	0	-14	10	2	-6	2	0	0
χ_{100}	0	0	0	0	0	-9	3	-9	0	0	7	-5	-1	3	-1	0	0
χ_{101}	0	0	0	0	0	-9	3	-9	0	0	7	-5	-1	3	-1	0	0
χ_{102}	0	0	0	0	0	-9	3	-9	0	0	7	-5	-1	3	-1	0	0
χ_{103}	0	0	0	0	0	-9	3	-9	0	0	7	-5	-1	3	-1	0	0
χ_{104}	0	0	0	0	0	9	-3	9	0	0	-7	5	1	-3	1	0	0
χ_{105}	0	0	0	0	0	9	-3	9	0	0	-7	5	1	-3	1	0	0
χ_{106}	0	0	0	0	0	9	-3	9	0	0	-7	5	1	-3	1	0	0
χ_{107}	0	0	0	0	0	9	-3	9	0	0	-7	5	1	-3	1	0	0
χ_{108}	0	0	0	0	0	12	-4	12	0	0	-4	-4	-4	-4	4	0	0
χ_{109}	0	0	0	0	0	12	-4	12	0	0	-4	-4	-4	-4	4	0	0
χ_{110}	0	0	0	0	0	-9	3	-9	0	0	7	-5	-1	3	-1	0	0
χ_{111}	0	0	0	0	0	-6	2	-6	0	0	14	-10	-2	6	-2	0	0
χ_{112}	0	0	0	0	0	-6	2	-6	0	0	-10	14	6	-2	-2	0	0
χ_{113}	0	0	0	0	0	-27	9	-27	0	0	-3	9	5	1	-3	0	0
χ_{114}	0	0	0	0	0	-27	9	-27	0	0	-3	9	5	1	-3	0	0
χ_{115}	-15	9	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	-15	9	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	6	-2	6	0	0	10	-14	-6	2	2	0	0
χ_{118}	0	0	0	0	0	6	-2	6	0	0	-14	10	2	-6	2	0	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	0	0	0	27	-9	27	0	0	3	-9	-5	-1	3	0	0
χ_{124}	15	-9	3	0	0	-12	4	-12	0	0	4	4	4	4	-4	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	$6G$		$6H$			$6I$		6J		6K			6L		6M	
	$6N$	$6O$	$6P$	$6Q$	$6R$	$6S$	$12E$	$6T$	$12F$	$6U$	$12G$	$12H$	$6V$	$12I$	$6W$	$12J$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	5	5	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1
χ_3	3	3	0	0	0	0	0	0	0	2	2	2	-2	-2	-2	-2
χ_4	3	3	0	0	0	0	0	0	0	-2	-2	-2	2	2	-2	-2
χ_5	3	3	0	0	0	0	0	0	0	-2	-2	-2	-2	-2	2	2
χ_6	0	0	3	3	3	3	3	3	3	1	1	1	1	1	1	1
χ_7	-1	-1	5	5	5	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_8	-1	-1	-1	-1	-1	5	5	-1	-1	1	1	1	-1	-1	1	1
χ_9	-1	-1	-1	-1	-1	-1	-1	5	5	1	1	1	1	1	-1	-1
χ_{10}	1	1	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	0
χ_{11}	-2	-2	-5	-5	-5	1	1	1	1	-1	-1	-1	1	1	1	1
χ_{12}	-2	-2	1	1	1	-5	-5	1	1	1	1	1	-1	-1	1	1
χ_{13}	-2	-2	1	1	1	1	1	-5	-5	1	1	1	1	1	-1	-1
χ_{14}	3	3	0	0	0	0	0	0	0	2	2	2	2	2	2	2
χ_{15}	7	7	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
χ_{16}	0	0	3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1
χ_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{20}	2	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{21}	-1	-1	-4	-4	-4	2	2	2	2	0	0	0	-2	-2	-2	-2
χ_{22}	-1	-1	2	2	2	-4	-4	2	2	-2	-2	-2	0	0	-2	-2
χ_{23}	-1	-1	2	2	2	2	2	-4	-4	-2	-2	-2	-2	-2	0	0
χ_{24}	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	-1	-1	-1	-1
χ_{25}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	-1	-1
χ_{26}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	1	1	4	4	4	-2	-2	-2	-2	2	2	2	0	0	0	0
χ_{29}	1	1	-2	-2	-2	4	4	-2	-2	0	0	0	2	2	0	0
χ_{30}	1	1	-2	-2	-2	-2	-2	4	4	0	0	0	0	0	2	2
χ_{31}	-2	-2	4	4	4	-2	-2	-2	-2	-2	-2	-2	0	0	0	0
χ_{32}	-2	-2	-2	-2	-2	4	4	-2	-2	0	0	0	-2	-2	0	0
χ_{33}	-2	-2	-2	-2	-2	-2	-2	4	4	0	0	0	0	0	-2	-2
χ_{34}	0	0	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	1
χ_{35}	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	-2	-2	1	1	1	1	1	1	1	3	3	3	-1	-1	-1	-1
χ_{39}	-2	-2	1	1	1	1	1	1	1	-1	-1	-1	3	3	-1	-1
χ_{40}	-2	-2	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	3	3
χ_{41}	2	2	-4	-4	-4	2	2	2	2	-2	-2	-2	0	0	0	0
χ_{42}	2	2	2	2	2	-4	-4	2	2	0	0	0	-2	-2	0	0
χ_{43}	2	2	2	2	2	2	2	-4	-4	0	0	0	0	0	-2	-2
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	$6G$		$6H$			$6I$		6J		6K			6L		6M	
	$6N$	$6O$	$6P$	$6Q$	$6R$	$6S$	$12E$	$6T$	$12F$	$6U$	$12G$	$12H$	$6V$	$12I$	$6W$	$12J$
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	2	2	-4	-4	-4	-4	-4	-4	-4	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	3	3	3	3	3	3	3	3	3	1	1	1	1	1	1	1
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	3	-1	6	2	-2	0	0	0	0	2	-2	0	0	0	0	0
χ_{55}	6	-2	-6	-2	2	0	0	0	0	2	-2	0	0	0	0	0
χ_{56}	3	-1	6	2	-2	0	0	0	0	2	-2	0	0	0	0	0
χ_{57}	9	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	-3	1	12	4	-4	0	0	0	0	-4	4	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	9	-3	0	0	0	0	0	0	0	-4	4	0	0	0	0	0
χ_{61}	-3	1	12	4	-4	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	-6	2	-12	-4	4	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	-3	1	-6	-2	2	0	0	0	0	-2	2	0	0	0	0	0
χ_{64}	-3	1	-6	-2	2	0	0	0	0	2	-2	0	0	0	0	0
χ_{65}	3	-1	6	2	-2	0	0	0	0	2	-2	0	0	0	0	0
χ_{66}	6	-2	-6	-2	2	0	0	0	0	2	-2	0	0	0	0	0
χ_{67}	6	-2	-6	-2	2	0	0	0	0	-2	2	0	0	0	0	0
χ_{68}	-6	2	-12	-4	4	0	0	0	0	-4	4	0	0	0	0	0
χ_{69}	6	-2	-6	-2	2	0	0	0	0	2	-2	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	-3	1	12	4	-4	0	0	0	0	4	-4	0	0	0	0	0
χ_{74}	-6	2	6	2	-2	0	0	0	0	2	-2	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	-6	2	6	2	-2	0	0	0	0	-2	2	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	-4	4	0	0	0	0	0
χ_{78}	9	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	-6	2	-12	-4	4	0	0	0	0	4	-4	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	3	-1	6	2	-2	0	0	0	0	-2	2	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	9	-3	1	3	-1	3	-1	1	1	-1	3	-1	3	-1
χ_{85}	0	0	-9	3	-1	3	-1	3	-1	1	1	-1	-3	1	-3	1
χ_{86}	0	0	0	0	0	6	-2	6	-2	2	2	-2	0	0	0	0
χ_{87}	0	0	9	-3	1	-3	1	-3	1	-1	-1	1	3	-1	3	-1
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	6G		6H			6I		6J		6K			6L		6M	
	6N	6O	6P	6Q	6R	6S	12E	6T	12F	6U	12G	12H	6V	12I	6W	12J
χ_{89}	0	0	0	0	0	63	21	63	-21	0	0	0	63	-21	63	-21
χ_{90}	0	0	9	-3	1	84	-28	84	-28	1	1	-1	84	-28	84	-28
χ_{92}	0	0	18	-6	2	105	-35	105	-35	2	2	-2	105	-35	105	-35
χ_{93}	0	0	0	0	0	135	-45	135	-45	0	0	0	135	-45	135	-45
χ_{94}	0	0	0	0	0	135	-45	135	-45	0	0	0	135	-45	135	-45
χ_{95}	0	0	-9	3	-1	168	-56	168	-56	-1	-1	1	168	-56	168	-56
χ_{96}	0	0	-18	6	-2	192	-64	192	-64	-2	-2	2	192	-64	192	-64
χ_{97}	0	0	9	-3	1	210	-70	210	-70	1	1	-1	210	-70	210	-70
χ_{98}	0	0	9	-3	1	84	-28	84	-28	-1	-1	1	84	-28	84	-28
χ_{99}	0	0	9	-3	1	84	-28	84	-28	-1	-1	1	84	-28	84	-28
χ_{100}	0	0	-18	6	-2	105	-35	105	-35	0	0	0	105	-35	105	-35
χ_{101}	0	0	-18	6	-2	105	-35	105	-35	0	0	0	105	-35	105	-35
χ_{102}	0	0	-18	6	-2	105	-35	105	-35	0	0	0	105	-35	105	-35
χ_{103}	0	0	-18	6	-2	105	-35	105	-35	0	0	0	105	-35	105	-35
χ_{104}	0	0	-9	3	-1	210	-70	210	-70	3	3	-3	210	-70	210	-70
χ_{105}	0	0	-9	3	-1	210	-70	210	-70	3	3	-3	210	-70	210	-70
χ_{106}	0	0	-9	3	-1	210	-70	210	-70	3	3	-3	210	-70	210	-70
χ_{107}	0	0	-9	3	-1	210	-70	210	-70	3	3	-3	210	-70	210	-70
χ_{108}	0	0	-9	3	-1	420	-140	420	-140	1	1	-1	420	-140	420	-140
χ_{109}	0	0	-9	3	-1	420	-140	420	-140	1	1	-1	420	-140	420	-140
χ_{110}	0	0	-9	3	-1	420	-140	420	-140	1	1	-1	420	-140	420	-140
χ_{111}	0	0	-9	3	-1	420	-140	420	-140	1	1	-1	420	-140	420	-140
χ_{112}	0	0	-9	3	-1	420	-140	420	-140	1	1	-1	420	-140	420	-140
χ_{113}	0	0	0	0	0	630	-210	630	-210	0	0	0	630	-210	630	-210
χ_{114}	0	0	0	0	0	630	-210	630	-210	0	0	0	630	-210	630	-210
χ_{115}	0	0	0	0	0	756	-252	756	-252	0	0	0	756	-252	756	-252
χ_{116}	0	0	0	0	0	756	-252	756	-252	0	0	0	756	-252	756	-252
χ_{117}	0	0	9	-3	1	840	-280	840	-280	-1	-1	1	840	-280	840	-280
χ_{118}	0	0	9	-3	1	840	-280	840	-280	-1	-1	1	840	-280	840	-280
χ_{119}	0	0	945	-315	945	-315	0	0	0	945	-315	945	-315	-1	-1	1
χ_{120}	0	0	945	-315	945	-315	0	0	0	945	-315	945	-315	-1	-1	1
χ_{121}	0	0	945	-315	945	-315	0	0	0	945	-315	945	-315	-1	-1	1
χ_{122}0	0	945	-315	945	-315	0	0	0	945	-315	945	-315	-1	-1	1	
χ_{123}0	0	0	0	0	1260	-420	1260	-420	0	0	0	1260	-420	1260	-420	
χ_{124}0	0	-18	6	-2	1344	-448	1344	-448	2	2	-2	1344	-448	1344	-448	

The character table of 2^8:$O_8^+(2)$(continued)

	6N			7A				8A			8B		
	$6X$	$12K$	$12L$	$7A$	$14A$	$14B$	$14C$	$8G$	$8H$	$8I$	$8J$	$8K$	$8L$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	0	0	0	0	-2	-2	-2	2	2	2
χ_3	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_4	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_5	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_6	-1	-1	-1	1	1	1	1	0	0	0	0	0	0
χ_7	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_8	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_9	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{10}	2	2	2	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{11}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{12}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{13}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	-1	-1	-1	-1	2	2	2	-2	-2	-2
χ_{15}	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{16}	-1	-1	-1	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{17}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{18}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{19}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{20}	-1	-1	-1	0	0	0	0	2	2	2	-2	-2	-2
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{24}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{25}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{26}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{27}	0	0	0	-1	-1	-1	-1	-2	-2	-2	2	2	2
χ_{28}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_{36}	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_{37}	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_{38}	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{39}	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{40}	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	6N			7A				8A			8B		
	$6X$	$12K$	$12L$	$7A$	$14A$	$14B$	$14C$	$8G$	$8H$	$8I$	$8J$	$8K$	$8L$
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{48}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{49}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{50}	0	0	0	1	1	1	1	0	0	0	0	0	0
χ_{51}	0	0	0	1	1	1	1	0	0	0	0	0	0
χ_{52}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	-1	-1	-1	-1	1	1	1	1	1	1
χ_{54}	2	-2	0	1	-1	1	-1	2	-2	0	2	-2	-2
χ_{55}	-2	2	0	0	0	0	0	-2	2	0	2	-2	-2
χ_{56}	-2	2	0	1	-1	1	-1	2	-2	0	-2	2	2
χ_{57}	0	0	0	0	0	0	0	-2	2	0	2	-2	-2
χ_{58}	0	0	0	0	0	0	0	-2	2	0	-2	2	2
χ_{59}	0	0	0	-1	1	-1	1	2	-2	0	-2	2	2
χ_{60}	0	0	0	0	0	0	0	2	-2	0	2	-2	-2
χ_{61}	0	0	0	0	0	0	0	-2	2	0	2	-2	-2
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	2	-2	0	0	0	0	0	2	-2	0	-2	2	2
χ_{66}	2	-2	0	0	0	0	0	-2	2	0	-2	2	2
χ_{67}	-2	2	0	0	0	0	0	2	-2	0	-2	2	2
χ_{68}	0	0	0	1	-1	1	-1	0	0	0	0	0	0
χ_{69}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	-2	2	0	-2	2	2
χ_{71}	0	0	0	0	0	0	0	2	-2	0	2	-2	-2
χ_{72}	0	0	0	0	0	0	0	-2	2	0	2	-2	-2
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	-1	1	-1	1	0	0	0	0	0	0
χ_{76}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	-2	2	0	-2	2	2
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	-1	1	-1	1	2	-2	0	2	-2	-2
χ_{82}	2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	1	-1	1	-1	0	0	0	0	0	0
χ_{84}	1	1	-1	2	0	-2	0	1	1	-1	1	1	1
χ_{85}	-1	-1	1	0	0	0	0	1	1	-1	1	1	1
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	1	1	-1	-2	0	2	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1

The character table of $2^8{:}O_8^+(2)$(continued)

	6N			7A				8A			8B		
	$6X$	$12K$	$12L$	$7A$	$14A$	$14B$	$14C$	$8G$	$8H$	$8I$	$8J$	$8K$	$8L$
χ_{89}	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{90}	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{91}	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{92}	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{93}	0	0	0	-1	$-A+\overline{A}$	1	$A-\overline{A}$	1	1	-1	1	1	1
χ_{94}	0	0	0	-1	$A-\overline{A}$	1	$-A+\overline{A}$	1	1	-1	1	1	1
χ_{95}	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	2	0	-2	0	0	0	0	0	0	0
χ_{97}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{98}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{99}	-1	-1	1	0	0	0	0	-2	-2	2	2	2	2
χ_{100}	-2	-2	2	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{101}	2	2	-2	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{102}	0	0	0	0	0	0	0	1	1	-1	1	1	1
χ_{103}	0	0	0	0	0	0	0	1	1	-1	1	1	1
χ_{104}	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{105}	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{106}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{107}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{108}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{109}	-1	-1	1	0	0	0	0	2	2	-2	-2	-2	-2
χ_{110}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	2	2	-2	-2	-2	-2
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	-2	-2	2	2	2	2
χ_{117}	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{118}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{119}	0	0	0	0	0	0	0	1	1	-1	1	1	1
χ_{120}	0	0	0	0	0	0	0	1	1	-1	1	1	1
χ_{121}	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{122}	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	9A		9B	9C	10A			10B	10C	12A		12B	12C
	$9A$	$18A$	$9B$	$9C$	$10C$	$20A$	$20B$	$10D$	$10E$	$12M$	$12N$	$12O$	$12P$
χ_1	9450	3780	3780	4725	4725	4725	4725	9450	9450	9450	9450	18900	18900
χ_2	-630	-252	-252	-315	-315	-315	-315	-630	-630	-630	-630	-1260	-1260
χ_3	490	196	196	245	245	245	245	490	490	490	490	980	980
χ_4	138	132	-156	405	-171	117	117	-54	-54	522	522	84	-204
χ_5	234	-60	36	-123	69	-27	-27	42	42	-150	-150	-108	-12
χ_6	202	4	-28	53	-11	21	21	10	10	74	74	-44	-108
χ_7	-70	4	4	-3	-3	-3	-3	-6	-6	-6	-6	20	20
χ_8	130	244	124	205	205	-155	-155	50	50	50	50	20	380
χ_9	50	-76	-36	-75	-75	45	45	-30	-30	-30	-30	20	-100
χ_{10}	82	52	28	37	37	-35	-35	2	2	2	2	20	92
χ_{11}	66	-12	-4	-19	-19	5	5	-14	-14	-14	-14	20	-4
χ_{12}	-70	-4	-4	5	5	5	5	10	10	10	10	-20	-20
χ_{13}	-30	-60	60	45	45	-75	165	-30	210	-30	210	-60	60
χ_{14}	2	4	-4	-3	-3	5	-11	2	-14	2	-14	4	-4
χ_{15}	-30	-60	60	45	45	165	-75	210	-30	210	-30	-60	60
χ_{16}	2	4	-4	-3	-3	-11	5	-14	2	-14	2	4	-4
χ_{17}	10	4	-28	-11	5	21	21	26	26	10	10	-44	20
χ_{18}	2	-4	4	5	-11	-3	-3	-14	-14	2	2	-4	4
χ_{19}	10	4	-28	-11	5	21	21	26	26	10	10	-44	20
χ_{20}	-6	4	4	-3	13	-3	-3	10	10	-6	-6	20	-12
χ_{21}	-135	270	270	135	135	135	135	-135	-135	-135	-135	540	540
χ_{22}	-15	30	30	15	15	15	15	-15	-15	-15	-15	60	60
χ_{23}	25	-50	-50	-25	-25	-25	-25	25	25	25	25	-100	-100
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	9	9	9	18	18	18	18	9	9	9	9	-9	-9
χ_{29}	-3	-3	-3	-6	-6	-6	-6	-3	-3	-3	-3	3	3
χ_{30}	1	1	1	2	2	2	2	1	1	1	1	-1	-1
χ_{31}	6	12	24	-15	-15	21	21	6	6	6	6	36	0
χ_{32}	-2	-4	-8	5	5	-7	-7	-2	-2	-2	-2	-12	0
χ_{33}	6	12	24	-15	-15	21	21	6	6	6	6	36	0
χ_{34}	-18	-4	0	25	-7	-11	-11	-18	-18	14	14	4	-8
χ_{35}	6	-4	0	5	5	1	1	6	6	6	6	4	-8
χ_{36}	14	-4	0	-23	9	5	5	14	14	-18	-18	4	-8
χ_{37}	-2	-4	0	1	1	-3	-3	-2	-2	-2	-2	4	-8
χ_{38}	-2	4	0	-3	-3	1	1	-2	-2	-2	-2	-4	8
χ_{39}	2	12	0	5	5	-7	-7	-2	-2	-2	-2	-12	0
χ_{40}	-2	12	0	7	7	-5	-5	2	2	2	2	-12	0
χ_{41}	2	-4	0	-3	-3	1	1	-2	-2	-2	-2	4	0
χ_{42}	-2	-4	0	-1	-1	3	3	2	2	2	2	4	0
χ_{43}	-2	0	0	1	1	1	1	2	2	2	2	0	0
χ_{44}	2	0	0	-1	-1	-1	-1	-2	-2	-2	-2	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	9A		9B	9C	10A			10B	10C	12A		12B	12C
	$9A$	$18A$	$9B$	$9C$	$10C$	$20A$	$20B$	$10D$	$10E$	$12M$	$12N$	$12O$	$12P$
χ_{45}	-6	0	0	-3	-3	-3	9	-6	6	-6	6	0	0
χ_{46}	2	0	0	1	1	1	-3	2	-2	2	-2	0	0
χ_{47}	-6	0	0	-3	-3	9	-3	6	-6	6	-6	0	0
χ_{48}	2	0	0	1	1	-3	1	-2	2	-2	2	0	0
χ_{49}	-2	-4	0	1	5	-3	-3	2	2	-2	-2	4	0
χ_{50}	-2	4	0	-3	1	1	1	2	2	-2	-2	-4	0
χ_{51}	2	0	0	1	-3	1	1	-2	-2	2	2	0	0
χ_{52}	0	15	15	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	-9	-9	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	3	3	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	-15	6	6	-9	-9	-9	-9	9	9	9	9	12	12
χ_{58}	5	-2	-2	3	3	3	3	-3	-3	-3	-3	-4	-4
χ_{59}	-15	6	6	-9	-9	-9	-9	9	9	9	9	12	12
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	-11	10	-14	7	7	7	7	-7	-7	-7	-7	-4	-4
χ_{63}	1	-14	10	-5	-5	-5	-5	5	5	5	5	-4	-4
χ_{64}	-3	-6	2	-1	-1	-1	-1	1	1	1	1	-4	-4
χ_{65}	-7	2	-6	3	3	3	3	-3	-3	-3	-3	-4	-4
χ_{66}	5	2	2	-1	-1	-1	-1	1	1	1	1	4	4
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	9	9	-9	18	-18	0	0	9	9	-9	-9	9	-9
χ_{72}	-3	-3	3	-6	6	0	0	-3	-3	3	3	-3	3
χ_{73}	1	1	-1	2	-2	0	0	1	1	-1	-1	1	-1
χ_{74}	3	-3	-3	0	0	0	0	9	-9	-9	9	3	3
χ_{75}	-1	1	1	0	0	0	0	-3	3	3	-3	-1	-1
χ_{76}	3	-3	-3	0	0	0	0	-9	9	9	-9	3	3
χ_{77}	-1	1	1	0	0	0	0	3	-3	-3	3	-1	-1
χ_{78}	1	1	1	-2	-2	-2	-2	-1	-1	-1	-1	-1	-1
χ_{79}	1	1	1	-2	-2	-2	-2	-1	-1	-1	-1	-1	-1
χ_{80}	-1	-1	-1	2	2	2	2	1	1	1	1	1	1
χ_{81}	3	-3	3	0	0	-6	6	3	-3	3	-3	-3	3
χ_{82}	-1	1	-1	0	0	2	-2	-1	1	-1	1	1	-1
χ_{83}	3	-3	3	0	0	6	-6	-3	3	-3	3	-3	3
χ_{84}	-1	1	-1	0	0	-2	2	1	-1	1	-1	1	-1
χ_{85}	1	1	-1	-2	2	0	0	-1	-1	1	1	1	-1
χ_{86}	1	1	-1	-2	2	0	0	-1	-1	1	1	1	-1
χ_{87}	-1	-1	1	2	-2	0	0	1	1	-1	-1	-1	1
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	9A		9B	9C	10A			10B	10C	12A		12B	12C
	$9A$	$18A$	$9B$	$9C$	$10C$	$20A$	$20B$	$10D$	$10E$	$12M$	$12N$	$12O$	$12P$
χ_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{92}	0	0	-2	-1	-1	1	1	0	0	0	0	0	2
χ_{93}	0	0	-2	-1	-1	1	1	0	0	0	0	0	2
χ_{94}	0	0	2	1	1	-1	-1	0	0	0	0	0	-2
χ_{95}	0	0	2	-1	-1	1	1	0	0	0	0	0	-2
χ_{96}	0	0	2	-1	-1	1	1	0	0	0	0	0	-2
χ_{97}	0	0	-2	1	1	-1	-1	0	0	0	0	0	2
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	0	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{104}	0	1	1	0	0	0	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	-3	-6	6	3	3	3	3	-3	-3	-3	-3	0	0
χ_{108}	1	2	-2	-1	-1	-1	-1	1	1	1	1	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	-1	0	0	-1	-1	-1	-1	1	1	1	1	0	0
χ_{114}	-1	0	0	-1	-1	-1	-1	1	1	1	1	0	0
χ_{115}	1	0	0	1	1	1	1	-1	-1	-1	-1	0	0
χ_{116}	1	0	0	1	1	1	1	-1	-1	-1	-1	0	0
χ_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	12D		12E				12F	12G	15A				15B	15C
	$12Q$	$12R$	$12S$	$24A$	$24B$	$12T$	$12U$	$12V$	$15A$	$30A$	$30B$	$30C$	$15B$	$15C$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	1	1	0	0	0	0	-1	-1	-1	-1	-1	-1	0	0
χ_4	1	1	-1	-1	-1	-1	0	-1	0	0	0	0	-1	0
χ_5	1	1	-1	-1	-1	-1	-1	0	0	0	0	0	0	-1
χ_6	-2	-2	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_7	1	1	1	1	1	1	0	0	1	1	1	1	-1	-1
χ_8	1	1	0	0	0	0	1	0	-1	-1	-1	-1	1	-1
χ_9	1	1	0	0	0	0	0	1	-1	-1	-1	-1	-1	1
χ_{10}	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_{11}	0	0	-1	-1	-1	-1	1	1	-1	-1	-1	-1	0	0
χ_{12}	0	0	1	1	1	1	-1	1	0	0	0	0	-1	0
χ_{13}	0	0	1	1	1	1	1	-1	0	0	0	0	0	-1
χ_{14}	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	-1	-1	1	1	1	1	1	1	0	0	0	0	0	0
χ_{16}	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	0	0	-1	-1	-1	-1	0	0	1	1	1	1	0	0
χ_{18}	0	0	0	0	0	0	-1	0	0	0	0	0	1	0
χ_{29}	0	0	0	0	0	0	0	-1	0	0	0	0	0	1
χ_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{21}	-1	-1	1	1	1	1	0	0	0	0	0	0	0	0
χ_{22}	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0
χ_{23}	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0
χ_{24}	1	1	0	0	0	0	0	0	1	1	1	1	0	0
χ_{25}	1	1	0	0	0	0	0	0	0	0	0	0	1	0
χ_{26}	1	1	0	0	0	0	0	0	0	0	0	0	0	1
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	-1	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{29}	-1	-1	1	1	1	1	-1	1	0	0	0	0	0	0
χ_{30}	-1	-1	1	1	1	1	1	-1	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1
χ_{32}	0	0	0	0	0	0	0	0	1	1	1	1	-1	1
χ_{33}	0	0	0	0	0	0	0	0	1	1	1	1	1	-1
χ_{34}	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	-1	-1	2	2	2	2	-1	-1	0	0	0	0	0	0
χ_{36}	-1	-1	-1	-1	-1	-1	2	-1	0	0	0	0	0	0
χ_{37}	-1	-1	-1	-1	-1	-1	-1	2	0	0	0	0	0	0
χ_{38}	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	1	1	1	1	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	1	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{44}	0	0	-1	-1	-1	-1	0	0	1	1	1	1	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	12D		12E				12F	12G	15A				15B	15C
	$12Q$	$12R$	$12S$	$24A$	$24B$	$12T$	$12U$	$12V$	$15A$	$30A$	$30B$	$30C$	$15B$	$15C$
χ_{45}	0	0	0	0	0	0	-1	0	0	0	0	0	1	0
χ_{46}	0	0	0	0	0	0	0	-1	0	0	0	0	0	1
χ_{47}	0	0	1	1	1	1	0	0	-1	-1	-1	-1	0	0
χ_{48}	0	0	0	0	0	0	1	0	0	0	0	0	-1	0
χ_{49}	0	0	0	0	0	0	0	1	0	0	0	0	0	-1
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_{52}	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	3	-1	2	-2	0	0	0	0	1	-1	1	-1	0	0
χ_{55}	0	0	-2	2	-2	2	0	0	-1	1	-1	1	0	0
χ_{56}	-3	1	-2	2	2	-2	0	0	0	0	0	0	0	0
χ_{57}	-3	1	2	-2	2	-2	0	0	1	-1	1	-1	0	0
χ_{58}	3	-1	0	0	0	0	0	0	1	-1	1	-1	0	0
χ_{59}	0	0	0	0	2	-2	0	0	-1	1	-1	1	0	0
χ_{60}	3	-1	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{61}	-3	1	0	0	-2	2	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	2	-2	0	0	1	-1	1	-1	0	0
χ_{63}	-3	1	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{64}	3	-1	0	0	0	0	0	0	-1	1	-1	1	0	0
χ_{65}	-3	1	0	0	-2	2	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	2	-2	-2	2	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	-2	2	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0
χ_{70}	0	0	2	-2	0	0	0	0	-1	1	-1	1	0	0
χ_{71}	0	0	2	-2	0	0	0	0	-1	1	-1	1	0	0
χ_{72}	0	0	0	0	2	-2	0	0	-1	1	-1	1	0	0
χ_{73}	3	-1	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	0	-2	2	0	0	1	-1	1	-1	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	2	-2	0	0	0	0	0	0	0	0
χ_{78}	3	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0
χ_{80}	0	0	-2	2	0	0	0	0	1	-1	1	-1	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	-3	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0
χ_{84}	0	0	1	1	-1	-1	0	0	2	0	0	-2	0	0
χ_{85}	0	0	0	0	0	0	0	0	-2	0	0	2	0	0
χ_{86}	0	0	-1	-1	1	1	0	0	-2	0	0	2	0	0
χ_{87}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{88}	0	0	-2	-2	2	2	0	0	2	0	0	-2	0	0

The character table of $2^8{:}O_8^+(2)$(continued)

	12D		12E				12F	12G	15A				15B	15C
	$12Q$	$12R$	$12S$	$24A$	$24B$	$12T$	$12U$	$12V$	$15A$	$30A$	$30B$	$30C$	$15B$	$15C$
χ_{89}	0	0	1	1	-1	-1	0	0	-1	$B-\overline{B}$	$-B+\overline{B}$	1	0	0
χ_{90}	0	0	1	1	-1	-1	0	0	-1	$-B+\overline{B}$	$B-\overline{B}$	1	0	0
χ_{91}	0	0	1	1	-1	-1	0	0	2	0	0	-2	0	0
χ_{92}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	0	0	0	0	0	2	0	0	-2	0	0
χ_{96}	0	0	0	0	0	0	0	0	-2	0	0	2	0	0
χ_{97}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{101}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{102}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{103}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{104}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{105}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{106}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{107}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{114}	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$A = E(7) + E(7)^2 + E(7)^4$

$B = -E(15)^7 - E(15)^{11} - E(15)^{13} - E(15)^{14}$

9

$2^{4}{\cdot}S_6$ and $2^{5}{\cdot}S_6$ as maximal subgroups of HS and $HS{:}2$ respectively

Prologue

The group $HS{:}2$ is a full automorphism group of the Higman-Sims group HS. The groups $2^{4}{\cdot}S_6$ and $2^{5}{\cdot}S_6$ are maximal subgroups of HS and $HS{:}2$ respectively. The group $2^4.S_6$ is of order 11520 and $2^{5}{\cdot}S_6$ is of order 23040 and each of them is of index 3850 in HS and $HS{:}2$ respectively. The aim of this chapter is to compute $\overline{G} = 2^5.S_6$ as a group of the form $2^{4}{\cdot}S_6.2$, that is $\overline{G} = \overline{G_1}.2$. We then compute the Fischer-Clifford matrices of $\overline{G_1}$ and $\overline{G}$ respectively. These together with the partial character tables of the inertia factors of $\overline{G_1}$ and $\overline{G}$ are used to compute the full character tables of $\overline{G_1}$ and $\overline{G}$ respectively. We then fuse $\overline{G_1}$ into $\overline{G}$.

9.1. Introduction

The Higman-Sims group, HS is a sporadic simple group of order $2^9.3^2.5^3.7.11 = 44352000$. This is a group that was discovered in 1967 by Higman and Sims [49]. This is a simple group of index two in the group of automorphisms of the Higman-Sims graph. Higman and Sims were attending a presentation by Marshall Hall on the Hall-Janko group J_2, which is a permutation group on a hundred points with the stabilizer of a point a subgroup with the other two orbits of length 36 and 63. They then thought of a group of permutations on a 100 points containing the Mathieu group M_{22}, which has a permutation representation on 22 and 77 points . From these two ideas they found HS, with one-point stabilizer isomorphic to M_{22}. Higman, in 1969 [50], independently discovered this group as a doubly transitive group acting on a certain "geometry" on 176 points. In his classical paper Conway [22] showed that HS is a subgroup of each of the Conway groups Co_1, Co_2 and Co_3. This group is also one of the seven sporadic groups found in Co_1 but not in the Mathieu groups and this set of groups is also known as *second generation* of sporadic groups. The group $HS{:}2$ is of order $88704000 = 2^{10}.3^2.5^3.7.11$ and it is the full automorphism group of HS.

The aim of this chapter is to compute the Fischer-Clifford matrices of $\overline{G_1}$ and $\overline{G}$. We use these matrices and the partial character tables of each inertia factor group to compute the full character table of each group. The notation and method used is taken from F. Ali [1]. For more information on Fischer-Clifford theory and projective characters of non-split extensions one is encouraged to read [1, 2, 55, 66, 107, 108, 109].

We follow the work of Conway leading up to the computations of the groups HS and $HS{:}2$ in the subsections below.

9.1.1 The Conway Groups

Leech created a lattice that gives the tightest lattice packing of spheres in 24 dimension [72]. Conway analyzed the symmetry of this lattice in detail in [22] and discovered three previously unknown sporadic groups namely the Co_1, Co_2 and Co_3. Let us give the definition of a Leech lattice which is given as Theorem 5.1 in [125].

Definition 9.1.1. A *Leech lattice* Λ is a 24 dimensional even integral lattice containing no vectors of norm 2, 196560 vectors of norm 4, 16773120 vectors of norm 6 and 398034000 vectors of norm 8.

We first construct the biggest Conway group $Aut(\Lambda) = .O = 2.Co_1$ as a group of 24×24 matrices and simultaneously we construct a Leech lattice . All the vectors of norm 8 in the Leech lattice fall into congruence classes of 48 pairs of mutually perpendicular vectors called the *crosses* and we get 8292375 such crosses. When $.O$ acts on crosses, the stabilizer of a cross is a group $2^{12}{:}M_{24}$, which is a maximal in $.O$. So $.O$ is a group of order $8292375.2^{12}.|M_{24}|$. The group $.O$ is a perfect group with $Z(.O) = 2$. The quotient of this group by the center is a group denoted by $.1 = Co_1$ and is of order

$$|Co_1| = 4157776806543360000 = 2^{21}.3^9.5^4.7^2.11.13.23.$$

Note that the action of $.O$ on crosses is transitive and Co_1 is a simple group.

Also $.O$ acts transitively on vectors of norm 4 having the products ± 4 or 0. These three orbits of $2^{12}{:}M_{24}$ on vectors of norm 4 are fused into a single orbit under $2.Co_1$. The stabilizer of a vector of norm 4 is denoted by Co_2, where

$$|Co_2| = 42305421312000 = 2^{18}.3^6.5^3.7.11.23.$$

Lastly $.O$ is transitive on vectors of norm 6. The stabilizer of a vector of norm 6 is denoted by Co_3 and is of order

$$|Co_3| = 423054213122000 = 2^{10}.3^7.5^3.7.11.23.$$

From the $\mathbb{ATLAS}$ [23] we see that $Co_3 \leq Co_2 \leq Co_1$ with Co_2 and Co_3 both maximal subgroup of Co_1 and Co_3 a maximal subgroup of Co_2.

9.1.2 The Higman-Sims Group

We get the Higman-Sims group HS by showing that Co_3 acts transitively on the set S of 11178 vectors of norm 4 which have inner product -2 with vector v, when $v = (-2^{12}, 0^{12})$. The monomial group $2 \times M_{12}$ fixes v and has six orbits on S. When $u = (-5, -1^{23})$, the group M_{23} fixes u and has five orbits on S . The only way for both these sets of orbits to fuse into orbits for Co_3 is a single orbit of length 11178. Thus the stabilizer in Co_3 of such a vector in S, is a subgroup of index 11178. This is the Higman-Sims group HS of order

$$|HS| = 44352000 = 2^9.3^2.5^3.7.11 \; .$$

Moreover if we let $w = (5, 1, 1^{22})$ and $x = (-1, -5, -1^{22})$ the stabilizer of the set $\{w, x\}$ is the monomial group M_{22}:2 and we get an involution of the group which interchanges the two vectors. This results in HS extending to HS:2 which is a full automorphism group of HS. A complete list of maximal subgroups of the Conway groups is provided in Table 5.3 of [125]. For further reading one can also go into [22, 72, 89, 125].

We use [124] to find two 20×20 matrices a and b with a from class $2A$, b from class $5A$ and $HS =< a, b >$. Again using [124] we find two 20×20 matrices c, d from classes $2C$ and $5C$ of HS:2 respectively, with $HS{:}2 =< c, d >$. From the HS computed, HS:2 is an automorphism group of an isomorphic copy of it.

9.1.3 The Groups $2^{4\cdot}S_6$ and $2^{5\cdot}S_6$

The group $2^{4\cdot}S_6$, is a group of order 11520, and a maximal subgroup of HS.. There is a group $2^4{:}S_6$ which is maximal in $\overline{M_{22}}$ and hence sits inside HS:2. Our group $2^{4\cdot}S_6$ is a subgroup of HS and hence is also a subgroup of HS:2. For further reading on $2^4{:}S_6$ as a maximal subgroup of $\overline{M_{22}}$ one can read [81] and [120]. The group $2^{5\cdot}S_6$ is a group of order 23040 and is a maximal subgroup of the automorphism group of the Higman-Sims HS:2. The groups $2^{4\cdot}S_6$ and $2^{5\cdot}S_6$ are unique maximal subgroups of their form in HS and HS:2 respectively. Using generators a and b of HS and Programme G (see Appendix A) we obtain elements a_1' and b_1' with $o(a_1') = 2$, $o(b_1') = 5$ and $\overline{G_1'} =< a_1', b_1' >= 2^{4\cdot}S_6$. Similarly using generators c and d from HS:2 and Programme H (see Appendix A) we obtain two elements c' and d' with $o(c') = 2$, $o(d') = 5$ and $\overline{G'} =< c', d' >= 2^5.S_6$. Both Programme G and Programme H are obtained from [124]. Our aim is to compute $\overline{G} = 2^5.S_6$ as $\overline{G_1}.2$, where $\overline{G_1} = 2^{4\cdot}S_6$ and we compute these inside HS:2. Since $\overline{G_1'}$ is in HS we seek for its isomorphic copy $\overline{G_1}$ in HS:2 . The extension of $\overline{G_1}$ is $\overline{G_1}.2 = \overline{G}$ and $\overline{G} \cong \overline{G'}$.

Having obtained $\overline{G'}$, using GAP [41], we get three of its subgroups of order 11520. By methods of coset analysis (see Chapter 2), we determine that each of these three subgroups is of the form $2^{4\cdot}S_6$. From these three subgroups only one, $\overline{G_1}$, is isomorphic to $\overline{G_1'}$ in HS. The group $\overline{G_1}$ has seven generators of which five are of order two, one of order 5 and one of order 6. To this list of seven generators we add one of the generator of $HS{:}2$ of order two namely c. The group generated by these eight elements is $\overline{G} = 2^{4\cdot}S_6.2 = 2^5.S_6$.

The groups $2^{4\cdot}S_6$ and $2^{5\cdot}S_6$ will be discussed fully in Sections 9.2 and 9.3 respectively.

9.2. The Group $\overline{G_1} = 2^{4\cdot}S_6$

9.2.1 Construction of $G_1 \cong S_6$

From [124] we get two 20×20 matrices a and b over $GF(2)$ with $o(a) = 2$, $o(b) = 5$, $o(ab) = 11$ and $HS =< a, b >$. Again from [124] we get Programme G (see Appendix A).

Let $a_1' = output[1]$ and $b_1' = output[2]$. Then we have $o(a_1') = 2$, $o(b_1') = 5$, $o(a_1'b_1') = 6$ and $\overline{G_1'} =< a_1', b_1' >= 2^{4\cdot}S_6$. Up to isomorphism, there is only one group of the type $2^{4\cdot}S_6$ that is a maximal subgroup of HS and this has 21 conjugacy classes of which two are classes of involutions. Going back to [124], we get two 20×20 matrices c and d with $o(c) = 2$, $o(d) = 5$ and $HS{:}2 =< c, d >$. Again from [124] we get Programme H (see Appendix A).

Let $c_1' = output[1]$ and $d_1' = output[2]$. We get that $o(c_1') = 2$, $o(d_1') = 10$, $o(c_1'd_1') = 6$ and $\overline{G'} =< c_1', d_1' >= 2^5.S_6$. Using GAP [41], we get eight normal subgroups of $\overline{G'}$. Three of these groups (we call them $S1, S2, S3$) are of order 11520 and for each group the conjugacy class $2A$ has 15 elements and when S_6 acts on 2^4 we get two orbits of length 1 and 15 hence all these groups are of the form $2^{4\cdot}S_6$. One of them ($S2 = 2^4{:}S_6$), however has 24 conjugacy classes and is thus not a maximal subgroup of HS. The other one ($S3$, a split extension of 2^5 by A_6) has five classes of involutions and again is not a maximal subgroup of HS, this group from [81] and [120] is actually a maximal subgroup of $\overline{M_{22}}$. This leaves us with the group $S1 = \overline{G_1} \cong \overline{G_1'}$. See Remark 9.2.1 for more details on groups $S1$, $S2$ and $S3$. The group $\overline{G_1}$ has seven generators $a_1, a_2, a_3, a_4, a_5, a_6$ and a_7 with a_1 of order 2, a_2 of order 5 and a_3 of order 6 and the rest of order 2. We use GAP to compute normal subgroups of $\overline{G_1}$ and it has only one proper normal subgroup, the elementary abelian group $N_1 = 2^4$. Our aim is to act $\overline{G_1}$ on N_1 and to do this we use Programme C (see Appendix A) and this requires us to consider N_1 as a full row space V_1 of dimension four over $GF(2)$. The action of $\overline{G_1}$ on V_1 is multiplication of V_1 from the right . This then requires us to rewrite $\overline{G_1}$, from a 20×20 representation to a 4×4 one. To do this we act $\overline{G_1}$ on N_1 by acting the seven generators $a_i,\ i = 1, \cdots, 7$ of $\overline{G_1}$ on the four generators $\lambda_i,\ i = 1, \cdots, 4$ of N_1.

Writing this action as maps we get.

$$a_1 : \lambda_1 \to \lambda_2,\ \lambda_2 \to \lambda_1,\ \lambda_3 \to \lambda_1\lambda_3\lambda_4,\ \lambda_4 \to \lambda_1\lambda_2\lambda_4;$$

$$a_2 : \lambda_1 \to \lambda_2,\ \lambda_2 \to \lambda_4,\ \lambda_3 \to \lambda_1\lambda_2,\ \lambda_4 \to \lambda_2\lambda_3\lambda_4;$$

$$a_3 : \lambda_1 \to \lambda_2\lambda_3\lambda_4,\ \lambda_2 \to \lambda_4,\ \lambda_3 \to \lambda_1\lambda_2\lambda_3,\ \lambda_4 \to \lambda_2;$$

For the rest that is a_4 to a_7 we get

$$a_i : \lambda_1 \to \lambda_1,\ \lambda_2 \to \lambda_2,\ \lambda_3 \to \lambda_3,\ \lambda_4 \to \lambda_4.$$

Writing this in matrix form we get :

$$\alpha_1 := \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}; \quad \alpha_2 := \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}; \alpha_3 := \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

For the rest α_4 to α_7 we get

$$\alpha_i := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Let $G_1 =< \alpha_1, \alpha_2, \alpha_3 >\cong S_6$, that is the action of $\overline{G_1}$ on N_1 is isomorphic to S_6.

Remark 9.2.1. Note that $N_1 = 2^4$ is generated by 4 commuting involutions from the class $2A$ of HS. From $\mathbb{ATLAS}$ we can see that $S1 = \overline{G_1} = N_{HS}(N_1)$, $S2 = N_{\overline{M}_{22}}(N_1)$, and $N_{HS:2}(N_1) = 2^{5\cdot}S_6 = \overline{G}$. As observed, $S1, S2, S3$ are non-isomorphic maximal subgroups of $\overline{G}$ and that $S2$ and $S3$ do not sit inside HS. Our computations show that

$$\begin{aligned} S1 &= 2^{4\cdot}S_6 = \overline{G} \cap HS \leq_{max} HS \leq_{max} HS{:}2 \\ S2 &= 2^4{:}S_6 = \overline{G} \cap \overline{M}_{22} \leq_{max} \overline{M}_{22} \leq_{max} HS{:}\ 2; \\ S3 &= 2^{4\cdot}(A_6 \times 2) \cong 2^5{:}A_6 \leq_{max} \overline{G} \leq_{max} HS{:}\ 2; \\ S1 \cap S2 &= S2 \cap S3 = S1 \cap S3 = N_{M_{22}}(N_1) = 2^4{:}A_6 \leq_{max} M_{22} \leq_{max} HS. \end{aligned}$$

In addition to the character table of $S1 = \overline{G_1}$ we also give the character tables of $S2$ and $S3$, in Table 1 and Table 2 of Appendix B respectively. It is also interesting to note that except for the conjugacy classes, the character tables of $S1$ and $S2$ are the same. A pictorial view of Remark 9.2.1 is given in Figure 9.1, where $A = 2^4.A_6$.

We compute the permutation characters of $HS{:}2$ when acting on $S1, S2$ and $S3$. For interest sake we also include $\chi(HS|S1)$ and $\chi(2^5.S_6|Si),\ i = 1, 2, 3$. We use GAP[41] for our computations.

$$\begin{aligned} \chi(HS|S1) &= 1a + 22a + 77aa + 154a + 175a + 693a + 770a + 825a + 1056a = \chi(HS{:}2|2^5.S_6) \\ \chi(2^5.S_6|S1) &= 1a + 1b, \\ \chi(2^5.S_6|S2) &= 1a + 1c, \\ \chi(2^5.S_6|S3) &= 1a + 1d. \end{aligned}$$

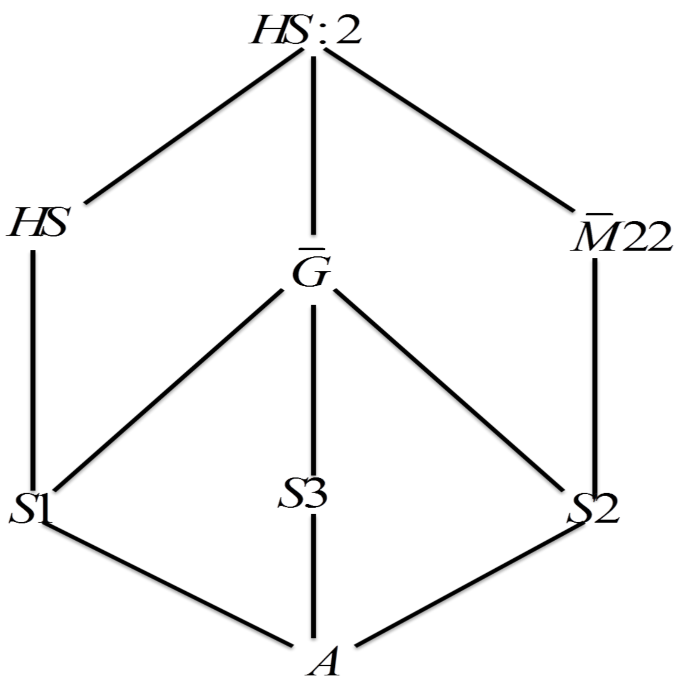

Figure 9.1: S1,S2 and S3

$\chi(HS{:}2|S1) = 1a+1b+22a+22b+77aa+77bb+154a+154b+175a+175b+693a+693b+770a+770b+825a+825b+1056a+1056b$,

$\chi(HS{:}2|S2) = 1a+22aa+77aaa+154a+175a+231a+693a+770aa+825aa+1056a+1925a$,

$\chi(HS{:}2|S3) = 1a+22a+22b+77aa+77b+154a+175a+231a+693a+770a+770b+825a+825b+1056a+1925b$.

Lemma 9.2.2. $\overline{G} = S1 \cup S2 \cup S3$.

Proof: First we see that $\overline{G} \supseteq S1 \cup S2 \cup S3$. But we also have $S1 \cup S2 \cup S3 = (S1 - A) \cup (S2 - A) \cup (S3 - A) \cup A$. Hence

$$\begin{aligned}
|S1 \cup S2 \cup S3| &= |S1 - A| + |S2 - A| + |S3 - A| + |A| \\
&= (16 \times 6! - 16 \times \frac{6!}{2}) + (16 \times 6! - 16 \times \frac{6!}{2}) + (16 \times 6! - 16 \times \frac{6!}{2}) + (16 \times \frac{6!}{2}) \\
&= 3(16 \times 6! - 16 \times \frac{6!}{2}) + (16 \times \frac{6!}{2}) \\
&= 3 \times 16 \times 6! - 2 \times 16 \times \frac{6!}{2} \\
&= 2 \times 16 \times 6! \\
&= |2^5.S_6|.
\end{aligned}$$

Thus $2^5.S_6 = S1 \cup S2 \cup S3$.□

Theorem 9.2.3. $HS{:}2$ has only three conjugacy classes of subgroups of type $2^4.A_6.2$. In particular $S1$ and $S2$ are of type $2^4.A_6.2_1$ and $S3$ is of type $2^4.(A_6 \times 2)$.

Proof: From the $\mathbb{ATLAS}$ we can see that if $H \leq HS{:}2$ is of type $2^4.A_6.2$, then H must sit in one of the maximal subgroups of $HS{:}2$ of type $HS, \overline{M}_{22}$ or $2^5.S_6$. Also since $N_{HS:2}(S1) \supseteq N_{\overline{G}}(S1) = \overline{G}$ and $\overline{G}$ is maximal but not normal in $HS{:}2$, we have $N_{HS:2}(S1) = \overline{G}$. Hence $[HS{:}2 : N_{HS:2}(S1)] = [HS{:}2 : \overline{G}] = 3850$. Similarly since $N_{HS:2}(S2) = N_{HS:2}(S3) = \overline{G}$, we have $[HS{:}2 : N_{HS:2}(S2)] = [HS{:}2 : N_{HS:2}(S3)] = 3850$. Hence we have 3 conjugacy classes for the subgroups of type $2^4.A_6.2$ in $HS{:}2$. Thus the total number of subgroups of type $2^4.A_6.2$ in $HS{:}2$ is $3 \times 3850 = 11550$.□

9.2.2 Conjugacy Classes and Inertia Factors of $\overline{G_1}$

Using GAP [41], we compute the conjugacy classes of $2^{4\cdot}S_6$. The action of $\overline{G_1}$ on N_1 is viewed as the action of of G_1 on V_1. If G_1 acts on N_1, we get two orbits of length 1 and 15. From the $\mathbb{ATLAS}$ [23], by checking on the indices of maximal subgroups of S_6, we can see that there are two inertia factor groups namely S_6 and $S_4 \times 2$. The full inertia groups are of the form $\overline{H_i} = 2^4.H_i$ of indices 1 and 15 in $2^{4\cdot}S_6$ respectively. We note that $H_1 \cong S_6$ and $H_2 \cong S_4 \times 2$. The character tables of H_1 and H_2 are easy to compute. The fusion of $S_4 \times 2$ into S_6 is given in Table 9.1.

Table 9.1: The fusion of $S_4 \times 2$ into S_6

$[x]_{S_4\times 2}$	$\longrightarrow$	$[g_1]_{S_6}$
$1A$		$1A$
$2A$		$2C$
$2B$		$2B$
$2C$		$2B$
$2D$		$2A$
$2E$		$2A$
$3A$		$3A$
$4A$		$4A$
$4B$		$4B$
$6A$		$6A$

We computed the conjugacy classes of $2^{4\cdot}S_6$ by using GAP [41] and then fused them into HS. Having the length of each coset, we use the fusion map to convert the conjugacy classes of $2^{4\cdot}S_6$ into the form that is required for the computation of Fischer-Clifford matrices (that is into a form normally obtained by coset-analysis). We give the conjugacy classes of $2^{4\cdot}S_6$ in Table 9.2.

9.2.3 Fischer-Clifford Matrices of $\overline{G_1}$

Again from the inertia factors and fusions we compute the Fischer-Clifford matrices.

Table 9.2: Conjugacy Classes Of $2^4{\cdot}S_6$

$[g]_{S_6}$	$[x]_{2^4{\cdot}S_6}$	$C_{2^4{\cdot}S_6}(x)$	$\longrightarrow HS$
1A	1A	11520	1A
	2A	768	2A
2A	2B	96	2B
	4A	384	4A
	4B	128	4B
2B	2C	64	2A
	4C	64	4B
	4D	32	4B
2C	2D	192	2A
	4E	64	4B
3A	3A	192	3A
	6A	24	6B
3B	3B	18	3A
4A	4F	16	4B
	8A	16	8A
4B	4G	16	4C
	8B	16	8A
5A	5A	5	5C
6A	6B	12	6A
	12A	12	12A
6B	6C	6	6B

From the fusions we get $M(1A) = \begin{bmatrix} 1 & 1 \\ 15 & -1 \end{bmatrix}$.

Let $Irr(HS) = \{\psi_i : 1 \leq i \leq 24\}$ as given in the $\mathbb{ATLAS}$. We get :

$[x]_{HS}$	1A	2A
ψ_2	22	6
ψ_3	77	13

Let r and s be the rows of the Fischer-Clifford matrix $M(1A)$. Since $< (\psi_2)_N, 1_N >= 7$ we get the following decomposition, $22 = 7 + 15s$. Thus $s = 1$ and this shows that the partial character table of H_2 comes from the ordinary table of H_2. Hence we use the ordinary character table of $S_4 \times 2$. We use the properties of the Fischer - Clifford matrices and the fusion of $S_4 \times 2$ into S_6, the centralizer orders of $2^4{\cdot}S_6$, the fusion of $\overline{G}$ into HS, together with restriction of HS to $\overline{G}$ that forces the signs of the Fischer-Clifford matrices. We give these in Table 9.3

Note the change of sign in $M(2A)$. For example we calculate the partial character table corresponding to coset $2A \in S_6$. From $M(2A)$ we get $M_1(2A) = [1\ 1\ 1]$, $\quad M_2(2A) = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -6 & 2 \end{bmatrix}$

Let $C_1(2A), C_2(2A)$ be the partial character tables of the inertia factors for the classes that fuse to $2A \in S_6$. Then the portions of the character table of $\overline{G} = 2^4{\cdot}S_6$ corresponding to the coset $2A$ are :

Table 9.3: The Fischer-Clifford matrices of $2^{4\cdot}S_6$

$M(1A) = \begin{bmatrix} 1 & 1 \\ 15 & -1 \end{bmatrix}$	$M(2A) = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -6 & 2 \end{bmatrix}$
$M(2B) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{bmatrix}$	$M(2C) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$
$M(3A) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$	$M(3B) = \begin{bmatrix} 1 \end{bmatrix}$
$M(4A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(4B) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
$M(6A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$M(5A) = M(6B) = \begin{bmatrix} 1 \end{bmatrix}$

$$C_1(2A)M_1(2A) = \begin{bmatrix} 1 \\ -1 \\ -3 \\ 3 \\ -1 \\ 1 \\ -3 \\ 3 \\ -2 \\ 2 \\ 0 \end{bmatrix} [1\ 1\ 1] = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ -3 & -3 & -3 \\ 3 & 3 & 3 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ -3 & -3 & -3 \\ 3 & 3 & 3 \\ -2 & -2 & -2 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_2(2A)M_2(2A) = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -2 & 0 \\ 2 & 0 \\ -3 & -1 \\ 3 & -1 \\ -3 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 0 & -6 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -5 & 3 \\ 1 & 5 & -3 \\ -1 & 7 & -1 \\ 1 & -7 & 1 \\ 2 & -2 & -2 \\ -2 & 2 & 2 \\ 3 & 3 & -5 \\ -3 & 9 & 1 \\ 3 & -9 & -1 \\ -3 & -3 & 5 \end{bmatrix}.$$

We get the character table of $2^{4\cdot}S_6$ in Table 9.4 which can be compared to the one in GAP.

Table 9.4: The character table of $2^{4\cdot}S_6$

	1A		2A			2B			2C		3A		3B	4A	
	1a	2a	2b	4a	4b	2c	4c	4d	2d	4e	3a	6a	3b	4f	8a
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
χ_3	5	5	-3	-3	-3	1	1	1	1	1	2	2	-1	-1	-1
χ_4	5	5	3	3	3	1	1	1	-1	-1	2	2	-1	1	1
χ_5	5	5	-1	-1	-1	1	1	1	3	3	-1	-1	2	1	1
χ_6	5	5	1	1	1	1	1	1	-3	-3	-1	-1	2	-1	-1
χ_7	9	9	-3	-3	-3	1	1	1	-3	-3	0	0	0	1	1
χ_8	9	9	3	3	3	1	1	1	3	3	0	0	0	-1	-1
χ_9	10	10	-2	-2	-2	-2	-2	-2	2	2	1	1	1	0	0
χ_{10}	10	10	2	2	2	-2	-2	-2	-2	-2	1	1	1	0	0
χ_{11}	16	16	0	0	0	0	0	0	0	0	-2	-2	-2	0	0
χ_{12}	15	-1	-1	-5	3	3	-1	-1	3	-1	3	-1	0	1	-1
χ_{13}	15	-1	1	5	-3	3	-1	-1	-3	1	3	-1	0	-1	1
χ_{14}	15	-1	-1	7	-1	-1	3	-1	3	-1	3	-1	0	-1	1
χ_{15}	15	-1	1	-7	1	-1	3	-1	-3	1	3	-1	0	1	-1
χ_{16}	30	-2	2	-2	-2	2	2	-2	-6	2	-3	1	0	0	0
χ_{17}	30	-2	-2	2	2	2	2	-2	6	-2	-3	1	0	0	0
χ_{18}	45	-3	3	3	-5	1	-3	1	3	-1	0	0	0	1	-1
χ_{19}	45	-3	-3	9	1	-3	1	1	-3	1	0	0	0	1	-1
χ_{20}	45	-3	3	-9	-1	-3	1	1	3	-1	0	0	0	-1	1
χ_{21}	45	-3	-3	-3	5	1	-3	1	-3	1	0	0	0	-1	1

The character table of $2^{4\cdot}S_6$ (continued)

	4B		5a	6A		6B
	4g	8b	5a	6b	12a	6c
χ_1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	-1
χ_3	-1	-1	0	0	0	1
χ_4	-1	-1	0	0	0	-1
χ_5	-1	-1	0	-1	-1	0
χ_6	-1	-1	0	1	1	0
χ_7	1	1	-1	0	0	0
χ_8	1	1	-1	0	0	0
χ_9	0	0	0	1	1	-1
χ_{10}	0	0	0	-1	-1	1
χ_{11}	0	0	1	0	0	0
χ_{12}	1	-1	0	1	-1	0
χ_{13}	1	-1	0	-1	1	0
χ_{14}	-1	1	0	1	-1	0
χ_{15}	-1	1	0	-1	1	0
χ_{16}	0	0	0	1	-1	0
χ_{17}	0	0	0	-1	1	0
χ_{18}	-1	1	0	0	0	0
χ_{19}	1	-1	0	0	0	0
χ_{20}	1	-1	0	0	0	0
χ_{21}	-1	1	0	0	0	0

9.3. The Group $\overline{G} = 2^{5\cdot}S_6$

Having completed the computation of the full character table of $2^{4\cdot}S_6$, we now turn our attention to $2^{5\cdot}S_6$. We compute $2^{5\cdot}S_6 = 2^{4\cdot}S_6.2$, this we do by adding the generator c of HS:2, that is from $\overline{G_1}$ we get $\overline{G} =< a_1, a_2, a_3, a_4, a_5, a_6, a_7, c >$. Since $2^{5\cdot}S_6$ is the only group of its type that is a maximal subgroup of HS:2, we have $\overline{G} \cong \overline{G'}$, where $\overline{G'}$ was computed using Programme H. Our aim is to compute the full character table of $2^{5\cdot}S_6$. We first want to let $\overline{G}$ act on the elementary abelian group $N = 2^5$. We use GAP [41] to compute $N = 2^5$ as a normal subgroup of $\overline{G}$.

9.3.1 Construction of $G \cong S_6$

For the action of $\overline{G}$ we use Programme C (see Appendix A). We consider N as a full row vector space V of dimension 5 over $GF(2)$. For us to be able to act on a five dimensional vector space V it becomes necessary to rewrite $\overline{G}$ from 20×20 to a 5×5 representation. To do this we first take the eight generators of $\overline{G}$ namely a_1 to a_7 and c. We let these act on generators $\gamma_i,\ 1 = 1, \cdots, 5$ of our elementary abelian group $N = 2^5$.

Writing these as maps we get :

$$a_1 : \gamma_1 \to \gamma_1,\ \gamma_2 \to \gamma_3\gamma_4,\ \gamma_3 \to \gamma_1\gamma_3,\ \gamma_4 \to \gamma_1\gamma_2\gamma_3,\ \gamma_5 \to \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5;$$

$$a_2 : \gamma_1 \to \gamma_2\gamma_3\gamma_4,\ \gamma_2 \to \gamma_3,\ \gamma_3 \to \gamma_1\gamma_3,\ \gamma_4 \to \gamma_2,\ \gamma_5 \to \gamma_2\gamma_5;$$

$$a_3 : \gamma_1 \to \gamma_1\gamma_2,\ \gamma_2 \to \gamma_1\gamma_2\gamma_3\gamma_4,\ \gamma_3 \to \gamma_4,\ \gamma_4 \to \gamma_1\gamma_4,\ \gamma_5 \to \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5;$$

$$c : \gamma_1 \to \gamma_3,\ \gamma_2 \to \gamma_2,\ \gamma_3 \to \gamma_1,\ \gamma_4 \to \gamma_4,\ \gamma_5 \to \gamma_5;$$

For the rest a_4 to a_7 we get

$$a_i : \gamma_1 \to \gamma_1,\ \gamma_2 \to \gamma_2,\ \gamma_3 \to \gamma_3,\ \gamma_4 \to \gamma_4,\ \gamma_5 \to \gamma_5.$$

Writing this in matrix form we get :

$$\beta_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix},$$

$$\beta_3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \beta_4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

For the rest β_5 to β_8 we get that $\beta_i = I_5$.

Let $G =< \beta_1, \beta_2, \beta_3, \beta_4 >$, then $G \cong S_6$ which means that the action of $\overline{G}$ on N is isomorphic to S_6.

9.3.2 Conjugacy Classes and Inertia Factors of $\overline{G}$

The action of $\overline{G}$ on N is reflected by the action of G on V. We use Programme C (see Appendix A). When G acts on V we get four orbits of conjugacy classes of lengths $1, 6, 10$ and 15. Let G^t be the set of all transpose of elements of G. The group G^t can also be generated by transpose matrices of each generator of G. When G^t acts on V, which is the equivalent of G acting on $Irr(N)$, by Brauer's Theorem [14] we get four orbits but these are of lengths $1, 1, 15$ and 15. These have corresponding point stabilizers H_1, H_2, H_3 and H_4. Let the full inertia groups be $\overline{H_i} = 2^5.H_i,\ i = 1, 2, 3, 4$. From the 𝔸𝕋𝕃𝔸𝕊 [23], the corresponding inertia factor groups are $S_6,\ S_6,\ S_4 \times 2$ and $S_4 \times 2$. Where we get $H_1 \cong H_2 \cong S_6$ and $H_3 \cong H_4 \cong S_4 \times 2$. The character tables of S_6 and that of $HS{:}2$ are obtained from the 𝔸𝕋𝕃𝔸𝕊 [23] . We give also, the fusion of $S_4 \times 2$ into S_6 in Table 9.5.

Table 9.5: The fusion of $S_4 \times 2$ into S_6

$[x]_{S_4 \times 2}$	$\longrightarrow$	$[g_1]_{S_6}$
$1A$		$1A$
$2A$		$2C$
$2B$		$2B$
$2C$		$2B$
$2D$		$2A$
$2E$		$2A$
$3A$		$3A$
$4A$		$4A$
$4B$		$4B$
$6A$		$6A$

We computed the conjugacy classes of $2^{5}{\cdot}S_6$ by using GAP [41] and then fused them into $HS{:}2$. Having the length of each coset, we use the fusion map to convert the conjugacy classes of $2^{5}{\cdot}S_6$ into the form that is required for the computation of Fischer-Clifford matrices (that is into a form normally obtained by coset-analysis). We give the conjugacy classes of $2^{5}{\cdot}S_6$ in Table 9.6

Table 9.6: Conjugacy Classes Of $2^{5\cdot}S_6$

$[g]_{S_6}$	$[x]_{2^{5\cdot}S_6}$	$C_{2^{5\cdot}S_6}(x)$	$\longrightarrow$	HS:2
	1A	23040		1A
1A	2A	3840		2D
	2B	2304		2C
	2C	1536		2A
	2D	768		2C
	4A	768		4A
2A	2E	256		2D
	4B	256		4B
	4C	192		4A
	2F	192		2B
	2G	128		2A
	2H	128		2D
2B	4D	128		4D
	4E	128		4B
	4F	64		4C
	4G	64		4A
	2I	384		2A
2C	2J	384		2C
	4H	64		4B
	4I	64		4D
	3A	144		3A
3A	6A	144		6C
	6B	48		6E
	6C	48		6B
3B	3B	36		3A
	6D	36		6A
	4J	32		4A
4A	8A	32		8C
	4K	32		4B
	8B	32		8A
	4L	32		4C
4B	8C	32		8A
	4M	32		4D
	8D	32		8D
5A	5A	10		5C
	10A	5		10D
	6E	24		6D
6A	6F	24		6A
	12A	24		12A
	12B	24		12B
6B	6G	12		6E
	6H	12		6A

9.3.3 Fischer Clifford Matrices of $\overline{G}$

From the fusions and orbit lengths and centralizer orders, we compute the Fischer-Clifford matrix $M(1A)$.

$$M(1A)=\begin{bmatrix}1&1&1&1\\1&-1&-1&1\\15&5&-3&-1\\15&-5&3&-1\end{bmatrix}$$

Having computed $M(1A)$ we want to determine the type of partial character tables we are going to use for our computations. We follow the methods used by F.Ali [1]. We use the character table of $HS{:}2 =< a, b >$. Let $Irr(HS : 2) = \{\Psi_i : 1 \leq i \leq 39\}$, the notation is the same at the one used in the 𝔸𝕋𝕃𝔸𝕊 [23].

$C_{\overline{G}}(x)$	23040	3840	2304	1536
$[x]_{HS:2}$	1A	2A	2B	2C
Ψ_2	1	-1	-1	1
Ψ_3	22	0	8	6
Ψ_4	22	0	-8	6
Ψ_5	77	5	21	13
Ψ_6	77	-5	-21	13

Let $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ be the rows of the Fischer - Cliford matrix $M(1A)$. First we get

$$< (\Psi_2)_N, 1_N >= \frac{1}{32}(1-6-10+15) = 0,$$

$$< (\Psi_3)_N, 1_N >= \frac{1}{32}(22\times1+6\times0+10\times8+15\times6) = \frac{1}{32}(22+80+90) = 6,$$

$$< (\Psi_4)_N, 1_N >= \frac{1}{32}(22\times1+6\times0+10\times(-8)+15\times6) = \frac{1}{32}(22-80+90) = 1,$$

$$< (\Psi_5)_N, 1_N >= \frac{1}{32}(77\times1+6\times5+10\times21+15\times13) = \frac{1}{32}(77+30+210+195) = 16.$$

Restricting the character Ψ_3 to N, since $< (\Psi_3)_N, 1_N >= 6$, we get the following equations, where a, b, c represent coefficients of $\gamma_2, \gamma_3, \gamma_4$ respectively.

$$\begin{aligned}22 &= 6+a+15b+15c,\\ 0 &= 6-a+5b-5c,\\ 8 &= 6-a-3b+3c,\\ 6 &= 6+a-b-c.\end{aligned}$$

Solving we get : $a = 1, b = 0$ and $c = 1$. So we have the following decomposition.

$$(\Psi_3)_N = 6\gamma_1 + \gamma_2 + \gamma_4.$$

Considering the coefficients of γ_2 and γ_4 we deduce that we have irreducible characters χ_2 and $\chi_4 \in Irr(\overline{G})$ with $deg(\chi_2) = 1$ and $deg(\chi_4) = 15$. Since $deg(\chi_2) = 1$, we only need to use the

Table 9.7: The Fischer-Clifford matrices of $2^{5\cdot}S_6$

$M(1A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 15 & 5 & -3 & -1 \\ 15 & -5 & 3 & -1 \end{bmatrix}$	$M(2A) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ -6 & 6 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ -6 & -6 & 2 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix}$
$M(2B) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -2 & 2 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ -2 & -2 & 2 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix}$	$M(2C) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ -3 & 3 & 1 & -1 \end{bmatrix}$
$M(3A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & 1 & -1 \\ 3 & 3 & -1 & -1 \end{bmatrix}$	$M(3B) = M(5A) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
$M(4A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$	$M(4B) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$
$M(6A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$	$M(6B) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

ordinary character table of H_2. For $deg(\chi_4) = 15$, if $[x_1, x_2, \cdots, x_t]$ is the transpose of the partial entries for the projective characters of H_4 on $1A$, then $C_4(1A)M_4(1A)$ is a $t \times 4$ matrix with first set entry $15x_1 = 15$, hence $x_1 = 1$. This shows that the partial character table of H_4 that we used contains a character of degree 1. Thus the partial character table is an ordinary character table of H_4. Similarly, one can show that $< (\Psi_3)_N, \gamma_2 >= 6$. This gives us $(\Psi_3)_N = \gamma_1 + 6\gamma_2 + \gamma_3$. So again H_1 and H_3 have partial character tables that each contains a character of degree 1. Thus the partial character tables of H_1 and H_3 are ordinary character tables of S_6 and $S_4 \times 2$ respectively. Using fusions, centralizer orders of $\overline{G}$ and properties of Fischer-Clifford matrices that can also be found in [1], [99] and [120] , we complete Table 9.8 of Fischer - Clifford matrices.

To compute the character table of $\overline{G}$, as an example consider the following. Let $C_1(2A), C_2(2A)$, $C_3(2A), C_4(2A)$ be the partial character tables of the inertia factors for the classes that fuse to $2A \in S_6$. Then the portions of the character table of $\overline{G} = 2^{5\cdot}S_6$ corresponding to the coset $2A$ are :

$$C_1(2A)M_1(2A) = \begin{bmatrix} 1 \\ -1 \\ -3 \\ 3 \\ -1 \\ 1 \\ -3 \\ 3 \\ -2 \\ 2 \\ 0 \end{bmatrix} [1\ 1\ 1\ 1\ 1\ 1] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 & -3 & -3 \\ 3 & 3 & 3 & 3 & 3 & 3 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ -3 & -3 & -3 & -3 & -3 & -3 \\ 3 & 3 & 3 & 3 & 3 & 3 \\ -2 & -2 & -2 & -2 & -2 & -2 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_2(2A)M_2(2A) = \begin{bmatrix} 1 \\ -1 \\ -3 \\ 3 \\ -1 \\ 1 \\ -3 \\ 3 \\ -2 \\ 2 \\ 0 \end{bmatrix} [-1\ 1\ -1\ 1\ -1\ 1] = \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 3 & -3 & 3 & -3 & 3 & -3 \\ -3 & 3 & -3 & 3 & -3 & 3 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 3 & -3 & 3 & -3 & 3 & -3 \\ -3 & 3 & -3 & 3 & -3 & 3 \\ 2 & -2 & 2 & -2 & 2 & -2 \\ -2 & 2 & -2 & 2 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3(2A)M_3(2A) = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -2 & 0 \\ 2 & 0 \\ -3 & -1 \\ 3 & -1 \\ -3 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -6 & 6 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 7 & 3 & -1 & -1 & -1 \\ 5 & -7 & -3 & 1 & 1 & 1 \\ 7 & -5 & -1 & 3 & -1 & -1 \\ -7 & 5 & 1 & -3 & 1 & 1 \\ -2 & -2 & -2 & -2 & 2 & 2 \\ -2 & -2 & -2 & -2 & 2 & 2 \\ 3 & -9 & -5 & -1 & 3 & 3 \\ 9 & -3 & 1 & 5 & -3 & -3 \\ -9 & 3 & -1 & -5 & 3 & -3 \\ -3 & 9 & 5 & 1 & -3 & -3 \end{bmatrix}.$$

$$C_4(2A)M_4(2A) = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -2 & 0 \\ 2 & 0 \\ -3 & -1 \\ 3 & -1 \\ -3 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -6 & -6 & 2 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -7 & 3 & -1 & 1 & -1 \\ 5 & 7 & -3 & -1 & 1 & -1 \\ 7 & 5 & -1 & -3 & -1 & 1 \\ -7 & -5 & 1 & 3 & 1 & -1 \\ -2 & 2 & -2 & 2 & 2 & 2 \\ 2 & -2 & 2 & -2 & -2 & 2 \\ 3 & 9 & -5 & 1 & 3 & -3 \\ 9 & 3 & 1 & -5 & -3 & 3 \\ -9 & -3 & -1 & 5 & 3 & -3 \\ -3 & -9 & 5 & -1 & -3 & 3 \end{bmatrix}.$$

The fusion of $\overline{G}$ into $HS{:}2$ together with the restriction of characters of $HS{:}2$ to $\overline{G}$ forces the signs of the Fischer-Clifford matrices and the order of the elements of the conjugacy classes of $\overline{G}$. Hence we give the character table in Table 9.8.

Table 9.8: The character table of $2^{5\cdot}S_6$

	$1A$				$2A$						$2B$					
	1a	2a	2b	2c	2d	4a	2e	4b	4c	2f	2g	2h	4d	4e	4f	4g
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_3	5	5	5	5	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_4	5	5	5	5	3	3	3	3	3	3	1	1	1	1	1	1
χ_5	5	5	5	5	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_6	5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
χ_7	9	9	9	9	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_8	9	9	9	9	3	3	3	3	3	3	1	1	1	1	1	1
χ_9	10	10	10	10	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{10}	10	10	10	10	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2
χ_{11}	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1
χ_{13}	1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
χ_{14}	5	-5	-5	5	3	-3	3	-3	3	-3	1	-1	1	-1	1	-1
χ_{15}	5	-5	-5	5	-3	3	-3	3	-3	3	1	-1	1	-1	1	-1
χ_{16}	5	-5	-5	5	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
χ_{17}	5	-5	-5	5	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1
χ_{18}	9	-9	-9	9	3	-3	3	-3	3	-3	1	-1	1	-1	1	-1
χ_{19}	9	-9	-9	9	-3	3	-3	3	-3	3	1	-1	1	-1	1	-1
χ_{20}	10	-10	-10	10	2	-2	2	-2	2	-2	-2	2	-2	2	-2	2
χ_{21}	10	-10	-10	10	-2	2	-2	2	-2	2	-2	2	-2	2	-2	2
χ_{22}	16	-16	-16	16	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	15	5	-3	-1	-5	7	3	-1	-1	-1	-1	3	3	-1	-1	-1
χ_{24}	15	5	-3	-1	5	-7	-3	1	1	1	-1	3	3	-1	-1	-1
χ_{25}	15	5	-3	-1	7	-5	-1	3	-1	-1	3	-1	-1	3	-1	-1
χ_{26}	15	5	-3	-1	-7	5	1	-3	1	1	3	-1	-1	3	-1	-1
χ_{27}	30	10	-6	-2	-2	-2	-2	-2	2	2	2	2	2	2	-2	-2
χ_{28}	30	10	-6	-2	2	2	2	2	-2	-2	2	2	2	2	-2	-2
χ_{29}	45	15	-9	-3	3	-9	-5	-1	3	3	-3	1	1	-3	1	1
χ_{30}	45	15	-9	-3	9	-3	1	5	-3	-3	1	-3	-3	1	1	1
χ_{31}	45	15	-9	-3	-9	3	-1	-5	3	3	1	-3	-3	1	1	1
χ_{32}	45	15	-9	-3	-3	9	5	1	-3	-3	-3	1	1	-3	1	1
χ_{33}	15	-5	3	-1	-5	-7	3	1	-1	1	-1	-3	3	1	1	1
χ_{34}	15	-5	3	-1	5	7	-3	-1	1	-1	1	3	3	1	-1	1
χ_{35}	15	-5	3	-1	7	5	-1	-3	-1	1	3	1	-1	-3	-1	1
χ_{36}	15	-5	3	-1	-7	-5	1	3	1	-1	3	1	-1	-3	-1	1
χ_{37}	30	-10	6	-2	-2	2	-2	2	2	-2	2	-2	2	-2	-2	0
χ_{38}	30	-10	6	-2	2	-2	2	-2	-2	2	2	-2	2	-2	-2	2
χ_{39}	45	-15	9	-3	3	9	-5	1	3	-3	-3	-1	1	3	1	-1
χ_{40}	45	-15	9	-3	9	3	1	-5	-3	3	1	3	-3	-1	1	-1
χ_{41}	45	-15	9	-3	-9	-3	-1	5	3	-3	1	3	-3	-1	1	-1
χ_{42}	45	-15	9	-3	-3	-9	5	-1	-3	3	-3	-1	1	3	1	-1

The character table of 2^5:S_6(continued)

	$2C$				$3A$				3B		$4A$			
	2i	2j	4h	4i	3a	6a	6b	6c	3b	6d	4j	8a	4k	8b
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1
χ_3	1	1	1	1	2	2	2	2	-1	-1	-1	-1	-1	-1
χ_4	-1	-1	-1	-1	2	2	2	2	-1	-1	1	1	1	1
χ_5	3	3	3	3	-1	-1	-1	-1	2	2	1	1	1	1
χ_6	-3	-3	-3	-3	-1	-1	-1	-1	2	2	-1	-1	-1	-1
χ_7	-3	-3	-3	-3	0	0	0	0	0	0	1	1	1	1
χ_8	3	3	3	3	0	0	0	0	0	0	-1	-1	-1	-1
χ_9	2	2	2	2	1	1	1	1	1	1	0	0	0	0
χ_{10}	-2	-2	-2	-2	1	1	1	1	1	1	0	0	0	0
χ_{11}	0	0	0	0	-2	-2	-2	-2	-2	-2	0	0	0	0
χ_{12}	1	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	1	1
χ_{13}	-1	1	-1	1	1	-1	1	-1	1	-1	1	1	-1	-1
χ_{14}	1	-1	1	-1	2	-2	2	-2	-1	1	1	1	-1	-1
χ_{15}	-1	1	-1	1	2	2	2	-2	-1	1	-1	-1	1	1
χ_{16}	3	-3	3	-3	-1	1	-1	1	2	-2	-1	-1	1	1
χ_{17}	-3	3	-3	3	-1	1	-1	1	2	-2	1	1	-1	-1
χ_{18}	-3	3	-3	3	0	0	0	0	0	0	-1	-1	1	1
χ_{19}	3	-3	3	-3	0	0	0	0	0	0	1	-1	-1	1
χ_{20}	2	-2	2	-2	1	-1	1	-1	1	-1	0	0	0	0
χ_{21}	-2	2	-2	2	1	-1	1	-1	1	-1	0	0	0	0
χ_{22}	0	0	0	0	-2	2	-2	2	-2	2	0	0	0	0
χ_{23}	3	3	-1	-1	3	-3	-1	1	0	0	-1	1	-1	1
χ_{24}	-3	-3	1	1	3	-3	-1	1	0	0	1	-1	1	-1
χ_{25}	3	3	-1	-1	3	-3	-1	1	0	0	1	-1	1	-1
χ_{26}	-3	-3	1	1	3	-3	-1	1	0	0	-1	1	-1	1
χ_{27}	-6	-6	2	2	-3	3	1	-1	0	0	0	0	0	0
χ_{28}	6	6	-2	-2	-3	3	1	-1	0	0	0	0	0	0
χ_{29}	3	3	-1	-1	0	0	0	0	0	0	-1	1	-1	1
χ_{30}	-3	-3	1	1	0	0	0	0	0	0	-1	1	-1	1
χ_{31}	3	3	-1	-1	0	0	0	0	0	0	1	-1	1	-1
χ_{32}	-3	-3	1	1	0	0	0	0	0	0	1	-1	1	-1
χ_{33}	-3	3	1	-1	3	3	-1	-1	0	0	1	-1	-1	1
χ_{34}	3	-3	-1	1	3	3	-1	-1	0	0	-1	1	1	-1
χ_{35}	-3	3	1	-1	3	3	-1	-1	0	0	-1	1	1	-1
χ_{36}	3	-3	-1	1	3	3	-1	-1	0	0	1	-1	-1	1
χ_{37}	6	-6	-2	2	-3	-3	1	1	0	0	0	0	0	0
χ_{38}	-6	6	2	-2	-3	-3	1	1	0	0	0	0	0	0
χ_{39}	-3	3	1	-1	0	0	0	0	0	0	1	-1	-1	1
χ_{40}	3	-3	-1	1	0	0	0	0	0	0	1	-1	-1	1
χ_{41}	-3	3	1	-1	0	0	0	0	0	0	-1	1	1	-1
χ_{42}	3	-3	-1	1	0	0	0	0	0	0	-1	1	1	-1

The character table of 2^5:S_6(continued)

	$4B$				$5A$		6A				$6B$	
	4l	8c	4m	8d	5a	10a	6e	6f	12a	12b	6g	6h
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_3	-1	-1	-1	-1	0	0	0	0	0	0	1	1
χ_4	-1	-1	-1	-1	0	0	0	0	0	0	-1	-1
χ_5	-1	-1	-1	-1	0	0	-1	-1	-1	-1	0	0
χ_6	-1	-1	-1	-1	0	0	1	1	1	1	0	0
χ_7	1	1	1	1	-1	-1	0	0	0	0	0	0
χ_8	1	1	1	1	-1	-1	0	0	0	0	0	0
χ_9	0	0	0	0	0	0	1	1	1	1	-1	-1
χ_{10}	0	0	0	0	0	0	-1	-1	-1	-1	1	1
χ_{11}	0	0	0	0	1	1	0	0	0	0	0	0
χ_{12}	1	1	-1	-1	1	-1	1	-1	1	-1	-1	1
χ_{13}	1	1	-1	-1	1	-1	-1	1	-1	1	1	-1
χ_{14}	-1	-1	1	1	0	0	0	0	0	0	-1	1
χ_{15}	-1	-1	1	1	0	0	0	0	0	0	1	-1
χ_{16}	-1	-1	1	1	0	0	-1	1	-1	1	0	0
χ_{17}	-1	-1	1	1	0	0	1	-1	1	-1	0	0
χ_{18}	1	1	-1	-1	-1	1	0	0	0	0	0	0
χ_{19}	1	1	-1	-1	-1	1	0	0	0	0	0	0
χ_{20}	0	0	0	0	0	0	1	-1	1	-1	1	-1
χ_{21}	0	0	0	0	0	0	-1	1	-1	1	-1	1
χ_{22}	0	0	0	0	1	-1	0	0	0	0	0	0
χ_{23}	-1	1	-1	1	0	0	-1	1	1	-1	0	0
χ_{24}	-1	1	-1	1	0	0	1	-1	-1	1	0	0
χ_{25}	1	-1	1	-1	0	0	-1	1	1	-1	0	0
χ_{26}	1	-1	1	-1	0	0	1	-1	-1	1	0	0
χ_{27}	0	0	0	0	0	0	-1	1	1	-1	0	0
χ_{28}	0	0	0	0	0	0	1	-1	-1	1	0	0
χ_{29}	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{30}	-1	1	-1	1	0	0	0	0	0	0	0	0
χ_{31}	-1	1	-1	1	0	0	0	0	0	0	0	0
χ_{32}	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{33}	1	-1	-1	1	0	0	1	1	-1	-1	0	0
χ_{34}	1	-1	-1	1	0	0	-1	-1	1	1	0	0
χ_{35}	-1	1	1	-1	0	0	1	1	-1	-1	0	0
χ_{36}	-1	1	1	-1	0	0	-1	-1	1	1	0	0
χ_{37}	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{38}	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{39}	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{40}	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{41}	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{42}	-1	1	1	-1	0	0	0	0	0	0	0	0

9.4. Fusion of $2^{4\cdot}S_6$ into $2^{5\cdot}S_6$

We give the fusion of $\overline{G_1}$ into $\overline{G}$ in Table 9.9

Table 9.9: The fusion of $2^{4\cdot}S_6$ into $2^{5\cdot}S_6$

$[x]_{2^{4\cdot}S_6}$	$\longrightarrow$	$[g_1]_{2^{5\cdot}S_6}$	$[x]_{2^{4\cdot}S_6}$	$\longrightarrow$	$[g_1]_{2^{5\cdot}S_6}$
$1A$		$1A$	$4E$		$4H$
$2A$		$2C$	$4F$		$4K$
$2B$		$2F$	$4G$		$4L$
$2C$		$2G$	$5A$		$5A$
$2D$		$2I$	$6A$		$6B$
$3A$		$3A$	$6B$		$6F$
$3B$		$3B$	$6C$		$6G$
$4A$		$4A$	$8A$		$8C$
$4B$		$4B$	$8B$		$8B$
$4C$		$4E$	$12A$		$12A$
$4D$		$4F$			

10

A group of the form $5^{3\cdot}L(3,5)$ as a maximal subgroup of the Lyons Group Ly

Prologue

The group $\overline{G} = 5^{3\cdot}L(3,5)$ is a subgroup of order 46500000 and of index 1113229656 in Ly. The group $\overline{G}$ in turn has $L(3,5)$ and $5^2{:}2.A_5$ as inertia factors. The group $5^2{:}2.A_5$ is of order 3000 and is of index 124 in $L(3,5)$. The aim of this chapter is to compute the Fischer-Clifford matrices of $\overline{G}$, which together with associated partial character tables of the inertia factor groups, are used to compute a full character table of $\overline{G}$. We will obtain that the partial projective character table corresponding to $5^2{:}2A_5$ is required, hence we have to compute the Schur multiplier and projective character table of $5^2{:}2A_5$.

10.1. Introduction

The Lyons group was discovered in 1970 by Richard Lyons [78], using the concept of classifying simple groups with an involution centralizer $2.A_n$. The smallest value of n for which $2.A_n$ has non-central involutions is $n = 8$, for which the McLaughlin group M^cL, has an involution centralizer $2.A_8$. The only other case that arises is $n = 11$ which is in the Lyons group Ly that is the Lyons group has an involution centralizer $2A_{11}$. Moreover, a 3-cycle in $2.A_{11}$ centralizes $2.A_8$ and the full centralizer of this 3-cycle in Ly is the triple cover $3.M^cL$ of the McLaughlin group. The Lyons group Ly, is a sporadic simple group of order $2^8.3^7.5^6.7.11.31.37.67 = 51765179004000000$.

The existence of this group and its uniqueness up to isomorphism was shown by C C Sims [118, 119], using *coset enumeration* and it is often referred to as the "Lyons-Sims" group. The group Ly has elements of order 37 and 67 which cannot be found in the monster and is one of the six sporadic simple groups called the "pariahs" which are not subgroups of the monster. The other five "pariahs" being $J_1, J_3, J_4, O'N$ and Ru, the last to be determined in [122] being J_1. The group $5^{3\cdot}L(3,5)$ is also maximal in the baby monster B. The group Ly has nine conjugacy classes of maximal subgroups. One of the maximal subgroups of the form, $\overline{G} = N.G$ is a group of order $46500000 = 2^6.3.5^6.31$, where $N \cong 5^3$ and $G \cong L(3,5)$. The aim of this chapter is to compute the Fischer-Clifford matrices which together with partial character tables will be used to compute a character table for $\overline{G}$. The notation used is consistent with that of the $\mathbb{ATLAS}$ [23] and method used is taken from [1, 2]. One can read more on Fischer-Clifford theory and projective characters from [99, 120] and [55, 66, 107, 108, 109] respectively. For the theory of characters one can also read Character Theory of Finite Groups [60].

10.2. Construction of $\overline{G} \cong 5^{3\cdot}L(3,5)$

From the $\mathbb{ATLAS}$ of group representation [124] we get two 111×111 matrices a, b, with $o(a) = 2,\ o(b) = 5,\ o(ab) = 14$ and $Ly =< a, b >$. Again from [124] we get Programme I (see Appendix A), where if we use $a = input[1]$ and $b = input[2]$ and we get $\bar{x} = output[1]$ and $\bar{y} = output[2]$, where $o(\bar{x}) = 2,\ o(\bar{y}) = 3,\ o(\bar{x}\bar{y}) = 31$ and $\overline{G} =< \bar{x}, \bar{y} >$. From [124] we see that $o(\bar{x}\bar{y}\bar{x}\bar{y}^2) = 25$ and if we let $gen[1] = (\bar{x}\bar{y}\bar{x}\bar{y}^2)^5$, then $o(gen[1]) = 5$, we also get that gen[2] $= \bar{y}gen[1]\bar{y}^{-1}$, gen[3] $= \bar{x}gen[2]\bar{x}^{-1}$ and $N = 5^3 =< gen[1], gen[2], gen[3] >$. Let $\lambda_i = gen[i],\ i = 1, 2, 3$. We use GAP to compute the conjugacy classes of $5^{3\cdot}L(3,5)$ and also the fusion of $5^{3\cdot}L(3,5)$ into Ly and these are given in Table 10.1.

Table 10.1: Conjugacy Classes Of $5^{3\cdot}L(3,5)$

$[g]_{L(3,5)}$	$[x]_{5^{3\cdot}L(3,5)}$	$C_{5^{3\cdot}L(3,5)}(x)$	$\longrightarrow$	Ly
1A	1A	46500000		1A
	5A	375000		5A
2A	2A	2400		2A
	10A	600		10A
3A	3A	120		3A
	15A	30		15B
4A	4A	480		4A
4B	4B	480		4A
4C	4C	80		4A
	20A	20		20A
	5B	2500		5A
5A	5C	1250		5B
	5D	1250		5B
5B	25A	25		25A
6A	6A	120		6B
	30A	30		30B
8A	8A	24		8B
8B	8B	24		8B
	10B	100		10A
10A	10C	50		10B
	10D	50		10B
12A	12A	24		12B
12B	12B	24		12B

continued on next page

Table 10.1 (continued from previous page)

$[g]_{L(3,5)}$	$[x]_{5^3\cdot L(3,5)}$	$C_{5^3.L(3,.5)}(x)$	$\longrightarrow$	Ly
20A	20B	20		20A
20B	20C	20		20A
24A	24A	24		24C
24B	24B	24		24B
24C	24C	24		24B
24D	24D	24		24C
31A	31A	1		31B
31B	31B	31		31A
31C	31C	31		31E
31D	31D	31		31D
31E	31E	31		31C
31F	31F	31		31B
31G	31G	31		31A
31H	31H	31		31E
31I	31I	31		31D
31J	31J	31		31C

10.3. Construction of $G \cong L(3,5)$

Our aim in this section is to let $\overline{G}$ act on the elementary abelian group N. We use the method discussed in chapters 8 and 9. In this method N is considered as a vector space V, of dimension 3 over $GF(5)$. For us to be able to act on a three dimensional vector space V it becomes necessary to rewrite $\overline{G}$ from 111×111 to a 3×3 representation. To do this we have to act $\overline{G}$ on N by letting the two generators of $\overline{G}$, $\bar{x}$ and $\bar{y}$ to act, on the generators of N, $\lambda_i,\ i = 1,2,3$ by conjugation, using GAP [41]. Writing these as maps we get :

$\bar{x} : \lambda_1 \rightarrow \lambda_1^4$
$\lambda_2 \rightarrow \lambda_3$
$\lambda_3 \rightarrow \lambda_2$;

$\bar{y} : \lambda_1 \rightarrow \lambda_2$
$\lambda_2 \rightarrow \lambda_1\lambda_2\lambda_3^4$
$\lambda_3 \rightarrow \lambda_2^2\lambda_3^4$.

Writing these in 3×3 matrix form, over $GF(5)$ we get

$$x = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, y = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 4 \\ 0 & 2 & 4 \end{pmatrix}$$

and $G =< x, y >$. Then $G \cong L(3,5)$ which means that the action of $\overline{G}$ on N is isomorphic to

$L(3,5)$.

10.4. Inertia Factors of $\bar{G}$

We use GAP [41] to compute the permutation character of Ly acting on $5^{3\cdot}L(3,5)$. That is

$\chi(Ly|5^{3\cdot}L(3,5)) = 1a + 45694a + 381766a + 1534500aa + 3028266a + 4226695aa + 11834746a + 18395586abc + 19212250a + 21312500ab + 22609664abc + 27252720aabbcd + 28787220aa + 29586865a + 33813560aa + 38734375a + 43110144abcde + 45648306b + 45694000ab + 56022921a + 64906250a + 71008476a$

We then use Programme C (see Appendix A) to compute the orbit lengths of the action on the orbits on N and on $Irr(N)$. We let G act on a full row vector space V of dimension 3 over $GF(5)$. We get two orbits of conjugacy classes of lengths 1 and 124. By Brauer's Theorem [14] when G acts on $Irr(N)$, we get two inertia groups of index 1 and 124. Using the 𝔸𝕋𝕃𝔸𝕊 [23], we see that $5^2{:}GL(2,5)$ is of index 31 in $L(3,5)$ and $|GL(2,5)| = 480$. Since we are looking for the maximal subgroup of index 124 in $L(3,5)$, we have to get a group of index 4 in $GL(2,5)$ and the group should be of order 120. So with the help of the 𝔸𝕋𝕃𝔸𝕊 [23] this can only be the group $2^{\cdot}A_5$. Thus the group of index 4 in $5^2{:}GL(2,5)$ is $5^2{:}2.A_5$. The full inertia groups are $\bar{H}_i = 5^3.H_i,\ i = 1,2$, where $H_1 = L(3,5)$ and $H_2 = 5^2{:}2.A_5$. We used GAP [41] to calculate the character table of H_2. We give the fusion of H_2 into $L(3,5)$ in Table 10.2.

Table 10.2: The fusion of $5^2{:}2.A_5$ into $L(3,5)$

$[x]_{5^2:2.A_5}$	$\longrightarrow$	$[g_1]_{L(3,5)}$
$1A$		$1A$
$2A$		$2A$
$3A$		$3A$
$4A$		$4C$
$5A$		$5A$
$5B$		$5B$
$5C$		$5B$
$5D$		$5A$
$5E$		$5B$
$5F$		$5B$
$5G$		$5A$
$6A$		$6A$
$10A$		$10A$
$10B$		$10A$

10.5. Projective Character Table of $5^2{:}2.A_5$

From the fusions and orbit lengths and centralizer orders, we compute the Fischer-Clifford matrix $M(1A)$ of $\bar{G}$.

$$M(1A) = \begin{bmatrix} 1 & 1 \\ 124 & -1 \end{bmatrix}$$

Having computed $M(1A)$ we want to determine the type of partial character tables we are going to use for our computations. We will show that the partial projective character table of H_2 is required. We follow the methods used in [1, 2] and we use the character table of $Ly =< a, b >$. Let $Irr(Ly) = \{\Psi_i : 1 \le i \le 53\}$, where the notation is the same at the one used in the $\mathbb{ATLAS}$ [23]. From the list we take the values of $\Psi_2,\ \Psi_3,\ \Psi_4$ on $1A$ and $5A$.

$C_{\overline{G}}(x)$	46500000	375000
$[x]_{Ly}$	1A	5A
Ψ_2	2480	-20
Ψ_3	2480	-20
Ψ_4	45694	69

Let γ_1, γ_2 be the rows of the Fischer-Cliford matrix $M(1A)$. Then

$$< (\Psi_2)_N, 1_N >= \frac{1}{125}(2480 - 20.124) = 0.$$

Since $< (\Psi_2)_N, 1_N >= 0$, we get that $2480 = 0 + 20.124$, so that $(\Psi_2)_N = 0.\gamma_1 + 20.\gamma_2$. Let $[x_1, \cdots, x_t]$ be the transpose of the partial entries for the ordinary characters of $H_2 = 5^2{:}2.A_5$ on $1A \in L(3,5)$. Then $C_2(1A)M(1A)$ is a $t \times 2$ matrix with entries on the first column $124x_1 = 2480$. Hence $x_1 = 20$. From the ordinary character table of $H_2 = 5^2{:}2.A_5$, there is no character of degree 20. Similarly

$$< (\Psi_4)_N, 1_N >= \frac{1}{125}(45694 - 69.124) = 434,$$

which gives us $x_1 = 365$ and this is a very large character degree which is not possible for $H_2 = 5^2{:}2.A_5$, and this holds for the remaining characters. Hence we have to use the projective character table of H_2. There are three primes dividing the order of H_2 namely $2, 3$ and 5. Using MAGMA Programme K , or GAP Programme K' (see Appendix A), we determine the Schur multiplier of H_2. The p - Sylow subgroups corresponding to $p = 2$ and 3 are cyclic , using methods from [1, 2] the Schur multipliers of both p-Sylow subgroups are trivial. Hence the Schur multiplier of H_2 is the cyclic group of order 5. The projective characters of H_2 with factor set α^{-1} where $\alpha^5 \sim 1$ is given in Table 10.3. Note that from the table we can see that $5a, 5b, 5c, 5e, 5f$ are all not α regular classes and we have a total of 9 α regular classes.

Let $A = -E(5) - E(5)^4$, and $A^* = 1 - A = -E(5)^2 - E(5)^3$. Then $A + A^* = 1$, $A^*A = A(A^*) = -1$, $A^2+(A^*)^2 = 3$, $A^3+(A^*)^3 = 4$. In fact we get a Fibonacci sequence, with $f_{i+1} = f_i+f_{i-1}$, $i \geq 2$, where $f_i = A^i + (A^*)^i$. This helps us to compute the Fischer-Clifford matrices and character table of $\overline{G} = 5^{3\cdot}L(3,5)$.

Table 10.3: The projective character table of $5^2{:}2.A_5$ with factor set α^{-1}

	1a	5a	2a	4a	3a	6a	5b	5c	5d	10a	5e	5f	5g	10b
χ_1	5	0	1	1	1	1	0	0	1	1	0	0	1	1
χ_2	15	0	3	-1	0	0	0	0	A	A	0	0	A^*	A^*
χ_3	15	0	3	-1	0	0	0	0	A^*	A^*	0	0	A	A
χ_4	20	0	4	0	1	1	0	0	-1	-1	0	0	-1	-1
χ_5	20	0	4	0	1	-1	0	0	-1	1	0	0	-1	1
χ_6	25	0	5	1	-1	-1	0	0	0	0	0	0	0	0
χ_7	10	0	2	0	-1	1	0	0	-A	A	0	0	-A^*	A^*
χ_8	10	0	2	0	-1	1	0	0	-A^*	A^*	0	0	-A	A
χ_9	30	0	6	0	0	0	0	0	1	-1	0	0	1	-1

10.6. Fischer-Clifford Matrices of $\overline{G}$

Having computed the projective character table of H_2, Table 10.3, we get the α-regular conjugacy classes. These together with the fusions of $5^2{:}2.A_5$ into $L(3,5)$ in Table 10.2 help us to compute the sizes of the Fischer-Clifford matrices of $\overline{G}$. We use these the projective characters, the fusions, the centralizer orders of $\overline{G}$ and properties of Fisher-Clifford matrices discussed in section 5.12.1, to compute Table 10.4 of Fischer - Clifford matrices.

To compute the character table of $5^{3\cdot}L(3,5)$, as an example consider the following. Let $C_1(5A)$ and $C_2(5A)$ be the partial character tables of the inertia factors for the classes that fuse to $5A \in L(3,5)$. The the portions of the character table of $\overline{G} = 5^{3\cdot}L(3,5)$ corresponding to the coset $5A$ are :

Table 10.4: The Fischer-Clifford matrices of $5^{3\cdot}L(3,5)$

$M(1A) = \begin{bmatrix} 1 & 1 \\ 124 & -1 \end{bmatrix}$	$M(2A) = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$
$M(3A) = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$	$M(4C) = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$
$M(5A) = \begin{bmatrix} 1 & 1 & 1 \\ 10 & -5A^{*} & -5A \\ 10 & -5A & -5A^{*} \end{bmatrix}$	$M(10A) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -A & -A^{*} \\ 2 & -A^{*} & -A \end{bmatrix}$
$M(6A) = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$	All Others $= \begin{bmatrix} 1 \end{bmatrix}$

$$C_1(5A)M_1(5A) = \begin{bmatrix} 1 \\ 5 \\ 6 \\ 6 \\ 6 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 5 \\ 5 \\ 5 \\ 11 \end{bmatrix} [1\ 1\ 1] = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 5 & 5 \\ 6 & 6 & 6 \\ 6 & 6 & 6 \\ 6 & 6 & 6 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -4 & -4 & -4 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \\ 11 & 11 & 11 \end{bmatrix}$$

$$C_2(5A)M_2(5A) = \begin{bmatrix} 1 & 1 \\ A & A^{*} \\ A^{*} & A \\ -1 & -1 \\ 0 & 0 \\ -A & -A^{*} \\ -A^{*} & -A \\ -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 10 & -5A^{*} & -5A \\ 10 & -5A & -5A^{*} \end{bmatrix} = \begin{bmatrix} 20 & -5 & -5 \\ 10 & 10 & -15 \\ 10 & -15 & 10 \\ -20 & 5 & 5 \\ 0 & 0 & 0 \\ -10 & -10 & 15 \\ -10 & 15 & -10 \\ -20 & 5 & 5 \\ 20 & -5 & -5 \end{bmatrix}$$

The fusion of $\overline{G}$ to Ly together with the restriction of characters of Ly to $\overline{G}$ forces the signs of the Fischer-Clifford matrices and the orders of the elements of the conjugacy classes of $\overline{G}$.

10.7. PowerMaps of $\overline{G}$

We then give the power maps of elements of $5^{3\cdot}L(3,5)$ in Table 10.5.

Table 10.5: The Power Maps of elements of $5^{3\cdot}L(3,5)$

$[g]_{L(3,5)}$	$[x]_{5^{3}\cdot L(3,5)}$	2	3	5	31	$[g]_{L(3,5)}$	$[x]_{5^{3}\cdot L(3,5)}$	2	3	5	31
1A	1A	1A	1A	1A	1A	2A	2A	1A	2A	2A	2A
	5A	5A	5A	1A	5A		10A	5A	10A	2A	10A
3A	3A	3A	1A	3A	3A	4A	4A	2A	4A	4A	4A
	15A	15A	5A	1A	15A	4B	4B	2A	4A	4A	4A
						4C	4C	2A	4C	4C	4C
5A	5B	5B	5B	1A	5B	5B	25A	25A	25A	5A	25A
	5C	5C	5C	1A	5C						
	5D	5D	5D	1A	5D						
6A	6A	3A	2A	6A	6A	10A	10B	5A	10B	2A	10B
	30A	15A	5A	6A	30A		10C	5A	10C	2A	10C
							10D	5A	10D	2A	10D
8A	8A	4A	8A	8A	8A	8B	8B	4B	8B	8B	8B
12A	12A	6A	4A	12A	12A	12B	12B	6A	4B	12B	12B
20A	20A	10A	20A	4A	20A	20B	20B	10A	20B	4B	20B
24A	24A	12A	8A	24A	24A	24B	24B	12B	8B	24B	24B
24C	24C	12A	8A	24C	24C	24D	24D	12B	8B	24D	24D
31A	31A	31A	31A	31A	1A	31B	31B	31B	31B	31B	1A
31C	31C	31C	31C	31C	1A	31D	31D	31D	31D	31D	1A
31E	31E	31E	31E	31EA	1A	31F	31B	31F	31F	31F	1A
31G	31G	31G	31G	31G	1A	31H	31H	31H	31H	31H	1A
31I	31I	31I	31I	31I	1A	31J	31J	31J	31J	31J	1A

The character table of $5^{3\cdot}L(3,5)$ is given in Table 10.6.

10.8. Character Table of $\overline{G}$

Table 10.6: The character table of $5^{3\cdot}L(3,5)$

	$1A$		$2A$		$3A$		$4A$	$4B$	$4C$		$5A$			5B
	1a	5a	2a	10a	3a	15a	4a	4b	4c	20a	5b	5c	5d	25a
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	30	30	6	6	0	0	6	6	2	2	5	5	5	0
χ_3	31	31	7	7	1	1	-5	-5	-1	-1	6	6	6	1
χ_4	31	31	-5	-5	1	1	A	/A	1	1	6	6	6	1
χ_5	31	31	-5	-5	1	1	/A	A	1	1	6	6	6	1
χ_6	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_7	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_8	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_9	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{10}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{11}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{12}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{13}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{14}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{15}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{16}	124	124	4	4	1	1	4	4	0	0	-1	-1	-1	-1
χ_{17}	124	124	4	4	1	1	4	4	0	0	-1	-1	-1	-1
χ_{18}	124	124	4	4	1	1	-4	-4	0	0	-1	-1	-1	-1
χ_{19}	124	124	4	4	1	1	-4	-4	0	0	-1	-1	-1	-1
χ_{20}	124	124	-4	-4	-2	-2	B	-B	0	0	-1	-1	-1	-1
χ_{21}	124	124	-4	-4	-2	-2	-B	B	0	0	-1	-1	-1	-1
χ_{22}	124	124	-4	-4	1	1	-B	B	0	0	-1	-1	-1	-1
χ_{23}	124	124	-4	-4	1	1	B	-B	0	0	-1	-1	-1	-1
χ_{24}	124	124	-4	-4	1	1	-B	B	0	0	-1	-1	-1	-1
χ_{25}	124	124	-4	-4	1	1	B	-B	0	0	-1	-1	-1	-1
χ_{26}	125	125	5	5	-1	-1	5	5	1	1	0	0	0	0
χ_{27}	155	155	11	11	-1	-1	-1	-1	-1	-1	5	5	5	0
χ_{28}	155	155	-1	-1	-1	-1	C	/C	1	1	5	5	5	0
χ_{29}	155	155	-1	-1	-1	-1	/C	C	1	1	5	5	5	0
χ_{30}	186	186	-6	-6	0	0	6	6	-2	-2	11	11	11	1
χ_{31}	620	-5	4	-1	-4	1	0	0	-4	1	20	-5	-5	0
χ_{32}	1860	-15	12	-3	0	0	0	0	4	-1	10	10	-15	0
χ_{33}	1860	-15	12	-3	0	0	0	0	4	-1	10	-15	10	0
χ_{34}	2480	-20	16	-4	-4	1	0	0	0	0	-20	5	5	0
χ_{35}	3100	-25	20	-5	4	-1	0	0	-4	1	0	0	0	0
χ_{36}	1240	-10	-8	2	4	-1	0	0	0	0	-10	-10	15	0
χ_{37}	1240	-10	-8	2	4	-1	0	0	0	0	-10	15	-10	0
χ_{38}	2480	-20	-16	4	-4	1	0	0	0	0	-20	5	5	0
χ_{39}	3720	-30	-24	6	0	0	0	0	0	0	20	-5	-5	0

The character table of $5^3L(3,5)$(continued)

	$6A$		$8A$	$8B$	$10A$			$12A$	$12B$	$20B$	20C	$24A$	$24B$	$24C$	$24D$
	6a	30a	8a	8b	10b	10c	10d	12a	12b	20b	20c	24a	24b	24c	24d
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	0	0	0	0	1	1	1	0	0	1	1	0	0	0	0
χ_3	1	1	-1	-1	2	2	2	1	1	0	0	-1	-1	-1	-1
χ_4	1	1	D	-D	0	0	0	-1	-1	F	/F	D	-D	D	-D
χ_5	1	1	-D	D	0	0	0	-1	-1	/F	F	-D	D	-D	D
χ_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	1	1	2	2	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
χ_{17}	1	1	-2	-2	-1	-1	-1	1	1	-1	-1	1	1	1	1
χ_{18}	1	1	E	-E	-1	-1	-1	-1	-1	1	1	-D	D	-D	D
χ_{19}	1	1	-E	E	-1	-1	-1	-1	-1	1	1	D	-D	D	-D
χ_{20}	2	2	0	0	1	1	1	-E	E	-D	D	0	0	0	0
χ_{21}	2	2	0	0	1	1	1	E	-E	D	-D	0	0	0	0
χ_{22}	-1	-1	0	0	1	1	1	-D	D	D	-D	G	/G	-G	-/G
χ_{23}	-1	-1	0	0	1	1	1	D	-D	-D	D	/G	G	-/G	-G
χ_{24}	-1	-1	0	0	1	1	1	-D	D	D	-D	-G	-/G	G	/G
χ_{25}	-1	-1	0	0	1	1	1	D	-D	-D	D	-/G	-G	/G	G
χ_{26}	-1	-1	-1	-1	0	0	0	-1	-1	0	0	-1	-1	-1	-1
χ_{27}	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1
χ_{28}	-1	-1	-D	D	-1	-1	-1	1	1	D	-D	-D	D	-D	D
χ_{29}	-1	-1	D	-D	-1	-1	-1	1	1	-D	D	D	-D	D	-D
χ_{30}	0	0	0	0	-1	-1	-1	0	0	1	1	0	0	0	0
χ_{31}	4	-1	0	0	4	-1	-1	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	2	-3	2	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	2	2	-3	0	0	0	0	0	0	0	0
χ_{34}	4	-1	0	0	-4	1	1	0	0	0	0	0	0	0	0
χ_{35}	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	4	-1	0	0	2	-3	2	0	0	0	0	0	0	0	0
χ_{37}	4	-1	0	0	2	2	-3	0	0	0	0	0	0	0	0
χ_{38}	-4	1	0	0	4	-1	-1	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	-4	1	1	0	0	0	0	0	0	0	0

The character table of $5^{3\cdot}L(3,5)$(continued)

	$31A$	$31B$	$31C$	$31D$	$31E$	$31F$	$31G$	$31H$	$31I$	$31J$
	$31a$	$31b$	$31c$	$31d$	$31e$	$31f$	$31g$	$31h$	$31i$	$31j$
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	0	0	0	0	0	0	0	0	0	0
χ_4	0	0	0	0	0	0	0	0	0	0
χ_5	0	0	0	0	0	0	0	0	0	0
χ_6	H	/H	L	/L	K	/K	J	/J	I	/I
χ_7	/H	H	/L	L	K	/L	/K	/K	/I	I
χ_8	I	/I	H	/H	L	/L	K	/K	J	/J
χ_9	/I	I	/H	H	/L	L	/K	K	/J	J
χ_{10}	J	/J	I	/I	H	/H	L	/L	K	/K
χ_{11}	/J	J	/I	I	/H	H	/L	L	/K	K
χ_{12}	K	/K	J	/J	I	/I	H	/H	/L	L
χ_{13}	/K	K	/J	J	/I	I	/H	H	/L	L
χ_{14}	L	/L	K	/K	J	/J	I	/I	H	/H
χ_{15}	/L	L	/K	K	/J	J	/I	I	/H	H
χ_{16}	0	0	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0	0
χ_{20}	0	0	0	0	0	0	0	0	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	0	0
χ_{26}	1	1	1	1	1	1	1	1	1	1
χ_{27}	0	0	0	0	0	0	0	0	0	0
χ_{28}	0	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0

A = -1+6*E(4) = -1+6*ER(-1) = -1+6i

B = 4*E(4) = 4*ER(-1) = 4i

C = -5+6*E(4) = -5+6*ER(-1) = -5+6i

D = E(4) = ER(-1) = i

E = 2*E(4) = 2*ER(-1) = 2i

F = -1+E(4) = -1+ER(-1) = -1+i

G = $-E(24)^{11} + E(24)^{19}$

H = $E(31) + E(31)^5 + E(31)^{25}$

I = $E(31)^3 + E(31)^{13} + E(31)^{15}$

J = $E(31)^8 + E(31)^9 + E(31)^{14}$

K = $E(31)^{11} + E(31)^{24} + E(31)^{27}$

L = $E(31)^2 + E(31)^{10} + E(31)^{19}$

A

Programmes

Programme A

```
V:=FullRowSpace(GF(q), n);
gr[1]:=(OneGF(q))*[n × n matrix group generators];
gr[2]:=(OneGF(q))*[n × n matrix group generators];

    ⋮

gr[k]:=(OneGF(q))*[n × n matrix group generators];
grp:=Group(gr[1], gr[2], ··· , gr[k]);
Ccl:=ConjugacyClasses(grp);
O:=Union(Orbits(grp,V));
for i in [1..n(Ccl)] do
Print(Representative(Ccl)[i]);
w:=One(GF(q))*[0, 0, ··· , 0];
e:=[ ];
while Difference(O,e) <> [ ] do
d:=[ ];
for x in O do;
y:=[x+w+(x*(Representative(Ccl)[i]))];
d:=Union(d,y);
od;
Print(d);
e:=Union(d,e);
if Difference(O,e) <> [ ] then
w:=Representative(Difference(O,e));
fi;
od;
r:=[ ];
u:=One(GF(q))*[0, 0, ··· , 0];
while Difference(O,e) <> [ ] do
m:=[ ];
for g in Centralizer(grp,Representative(Ccl)[i]) do
l:=[u*g];
m:=Union(m,l);
od;
Print("A block for the vectors under the action of a centralizer");
Print(m);
r:=Union(m,r);
```

```
if Difference(O,r) <> [ ] then
u:=Representative(Difference(O,r));
fi;
od;
Print("*********************************");
od;
```

Programme B

```
V:=FullRowSpace(GF(q), n);
m[1]:=(OneGF(q))*[n × n matrix group generators];
m[2]:=(OneGF(q))*[n × n matrix group generators];

    ⋮

m[k]:=(OneGF(q))*[n × n matrix group generators];
m:=Group(m[1], m[2], ··· , m[k]);
c:=ConjugacyClasses(m);
g:=Representative(c[i]);
d:=One(GF(q))*[α_1, α_2, ··· , α_n];
w:=d + d * g + d * g^2 + ··· + d * g^(m−1);
Print(w);
```

PROGRAMME C

```
gap>V:=FullRowSpace(GF(q), n);
gap>m[1]:=(OneGF(2))*[n × n matrix group generators];
gap>m[2]:=(OneGF(2))*[n × n matrix group generators];

    ⋮

m[k]:=(OneGF(q))*[n × n matrix group generators];
gap>m:=Group(m[1], m[2], ··· , m[k]);
gap>k:=OrbitLengths(m,V);
gap>l:= OrbitLengths(Group(List(m,TransposedMat)),N);
```

PROGRAMME D

```
gap>g:=Group(Main Group);
gap>T1:=CharacterTable(g);
gap>h:=Group(Inertia Group 1);
gap>T2:=CharacterTable(h);
gap>k:=Group(Inertia Group 2);
gap>T3:=CharacterTable(k);
gap>FusionConjugacyClasses(h,g);
gap>FusionConjugacyClasses(k,g);
```

PROGRAMME E

```
gap>ct:=fuction()local ct;ct:=rec();
>ct.SizesCentralizers:=[n Centralizer Orders];
>ct.OrdersClassRepresentatives:=[n Class Representatives Orders];
>ct.Irr:=[[n × n irreducibles]];
>ct.UnderlyingCharacteristic:=0;ct.Id:=G̅;
>ConvertToLibraryCharacterTable NC(ct);return ct;end;ct:=ct();
gap>SetInfoLevel(InfoCharacterTable,2);
gap>IsInternallyConsistent(ct);
```

gap>PossiblePowerMaps(ct,p); (p-prime divisor of $\overline{G}$).

PROGRAMME F

gap>ct:=CharacterTable(G);
gap>SetInfoLevel(InfoCharacterTable,2);
gap>ct;
gap>cut:=Irr(ct){[i..j]};;
gap> $k := k_1 * \text{cut}[i_1] + k_2 * \text{cut}[i_2] + ... + k_r * \text{cut}[i_r]$;

Programme G

```
work := [ ];;
output := [ ];;
work[1] := a;;
work[2] := b;;
work[3] := work[1] * work[2];;
work[4] := work[3] * work[2];;
work[5] := work[3] * work[4];;
work[6] := work[3] * work[5];;
work[7] := work[6] * work[3];;
work[8] := work[7] * work[4];;
work[9] := work[3] * work[8];;
work[2] := work[1] * work[9];;
work[4] := work[3] * work[3];;
work[5] := work[3] * work[4];;
work[4] := work[5]^-1;;
work[3] := work[4] * work[2];;
work[2] := work[3] * work[5];;
work[6] := work[7] * work[7];;
work[5] := work[7] * work[6];;
work[4] := work[6] * work[5];;
work[3] := work[4]^-1;;
work[5] := work[3] * work[1];;
work[1] := work[5] * work[4];;
output[1] := work[1];;
output[2] := work[2];;
```

Programme H

work := [ ];;

output := [ ];;

work[1] := c;;

work[2] := d;;

work[3]:= work[1] * work[2];;

work[4] := work[3] * work[2];;

work[5] := work[4] * work[3];;

work[6] := work[2] * work[2];;

work[7] := work[4] * work[6];;

work[4] := work[5] * work[7];;

work[6] := work[2] * work[5];;

work[5] := work[6] * work[3];;

work[2] := work[5]$^{-}$1;;

work[3] := work[2] * work[4];;

work[2] := work[3] * work[5];;

output[1] := work[1];;

output[2] := work[2];;

Programme I

work := [ ];;

output := [ ];;

work[1] := a;;

work[2] := b;;

work[3] := work[1] * work[2];;

work[4] := work[3] * work[2];;

work[5] := work[3] * work[4];;

work[6] := work[3] * work[5];;

work[7] := work[6] * work[3];;

work[5] := work[3]$^{-1}$;;

work[9] := work[5] * work[1];;

work[1] := work[9] * work[3];;

work[2] := work[7]^{3};;

work[6] := work[4]^{12};;

work[5] := work[6]$^{-1}$;;

work[3] := work[5]* work[2];;

work[2] := work[3] * work[6];;

output[1] := work[1];;

output[2] := work[2];;

$\overline{G}$:= Group(output[1],output[2]);

Programme J

```
> G:=SL(2,5);
> M:=GModule(G);
> X:=CohomologyModule(G,M);
> E:=SplitExtension(X);
> Eperm:=DegreeReduction(CosetImage(E,sub<E|>));
> pMultiplicator(Eperm,2);
[ 1 ]
> pMultiplicator(Eperm,3);
[ 1 ]
> pMultiplicator(Eperm,5);
[ 5 ]
> exit;
```

Programme J′

```
gap> gg:=PerfectGroup(3000,1);

 A5 2^1 5^2

gap> AbelianInvariantsMultiplier(gg);

[ 5 ]
```

B

Character Tables of S2 and S3

Table 1

	1a	2a	4a	2b	2c	3a	6a	4b	2d	4c	4d	8a	6b	12a	2e	4e	4f	8b	5a	6c	3b
2	8	8	5	7	7	3	3	6	6	5	4	4	2	2	6	6	4	4	.	1	1
3	2	1	1	.	1	2	1	.	.	.	.	.	1	1	1	.	.	.	.	1	2
5	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	-1	1
X.3	5	5	3	3	3	2	2	1	1	1	1	1	.	.	-1	-1	-1	-1	.	-1	-1
X.4	5	5	-3	-3	-3	2	2	1	1	1	-1	-1	.	.	1	1	-1	-1	.	1	-1
X.5	5	5	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	3	3	-1	-1	.	.	2
X.6	5	5	1	1	1	-1	-1	1	1	1	-1	-1	1	1	-3	-3	-1	-1	.	.	2
X.7	9	9	-3	-3	-3	.	.	1	1	1	1	1	.	.	-3	-3	1	1	-1	.	.
X.8	9	9	3	3	3	.	.	1	1	1	-1	-1	.	.	3	3	1	1	-1	.	.
X.9	10	10	-2	-2	-2	1	1	-2	-2	-2	.	.	1	1	2	2	.	.	.	-1	1
X.10	10	10	2	2	2	1	1	-2	-2	-2	.	.	-1	-1	-2	-2	.	.	.	1	1
X.11	15	-1	-1	3	-5	3	-1	3	-1	-1	-1	1	1	-1	3	-1	-1	1	.	.	.
X.12	15	-1	1	1	-7	3	-1	-1	3	-1	-1	1	-1	1	-3	1	1	-1	.	.	.
X.13	15	-1	1	-3	5	3	-1	3	-1	-1	1	-1	-1	1	-3	1	-1	1	.	.	.
X.14	15	-1	-1	-1	7	3	-1	-1	3	-1	1	-1	1	-1	3	-1	1	-1	.	.	.
X.15	16	16	.	.	.	-2	-2	.	.	.	.	.	.	.	.	.	.	.	1	.	-2
X.16	30	-2	-2	2	2	-3	1	2	2	-2	.	.	-1	1	6	-2	.	.	.	.	.
X.17	30	-2	2	-2	-2	-3	1	2	2	-2	.	.	1	-1	-6	2	.	.	.	.	.
X.18	45	-3	-3	1	9	.	.	-3	1	1	-1	1	.	.	-3	1	-1	1	.	.	.
X.19	45	-3	3	-1	-9	.	.	-3	1	1	1	-1	.	.	3	-1	-1	1	.	.	.
X.20	45	-3	-3	5	-3	.	.	1	-3	1	1	-1	.	.	-3	1	1	-1	.	.	.
X.21	45	-3	3	-5	3	.	.	1	-3	1	-1	1	.	.	3	-1	1	-1	.	.	.

Table 2

```
     2  8  8   7   7  3  3  3  3  5  6  6  6  6  5  4  4  4  4  1   1  1 1
     3  2  1   1   2  2  1  1  2  .  .  .  .  .  .  .  .  .  .  .   .  .  .
     5  1  .   1   .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  1   1  1   1

       1a 2a  2b  2c 3a 6a 6b 6c 4a 4b 2d 4c 2e 4d 8a 4e 8b 4f 5a 10a 5b 10b

X.1  1  1   1   1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1 1 1 1

X.2  1  1  -1  -1  1  1 -1 -1 -1 -1 -1  1  1  1 -1 -1 1  1 1 -1 1 -1

X.3  5  5   5   5  2  2  2  2  1  1  1  1  1 1 -1 -1 -1 -1 . . . .

X.4  5  5   5   5 -1 -1 -1 -1  1  1 1  1 1 1 -1 -1 -1 -1 .   . . .

X.5  5  5  -5  -5  2  2 -2 -2 -1 -1 -1 1  1 1 1 1 -1 -1  . . . .

X.6  5  5  -5  -5 -1 -1 1 1 -1 -1 -1 1 1 1 1 1 -1 -1  . . . .

X.7  8 8-8 -8 -1 -1 1  1  . . .  . . . . .  .  .  .  A -A*A -*A

X.8  8 8  -8 -8 -1 -1  1  1 . .  .  . . . . .  .  .  . *A -*A A  -A

X.9  8  8 8 8 -1 -1 -1 -1 .  .  .  . . . .  . .  .  A A*A  *A

X.10  8  8 8   8 -1 -1 -1 -1  .  .  . . . . .  .  . . *A*A A   A

X.11  9  9   9   9 .  . .  .  1  1 1 1  1 1 1 1  1  1 -1  -1 -1  -1

X.12  9  9 -9  -9 .  .  . . -1 -1 -1 1 1  1 -1 -1  1  1 -1 1 -1 1

X.13  10 10  10 10 1  1  1  1 -2 -2 -2 -2 -2 -2  .  .  . . .   .  ..

X.14  10 10 -10 -10  1 1 -1 -1 2 2 2 -2 -2 -2  . .  . .  .   . . .

X.15  15 -1  -5 3  3 -1  1 -3 -1 -1  3  3 -1 -1 1 -1 1 -1  . .  . .

X.16  15 -1  -5   3 3 -1  1 -3 -1 3 -1 -1 3 -1 -1  1 -1 1  . .  . .

X.17 15 -1   5 -3  3 -1 -1  3 1 -3 1 -1  3 -1  1 -1 -1  1  . . ..

X.18 15 -1 5  -3  3 -1 -1 3 1  1 -3  3 -1 -1 -1  1 1 -1 .   . . .

X.19 30 -2 -10   6 -3 1 -1  3 -2  2 2 2  2 -2  .  . .  .  . . . .

X.20 30 -2  10 -6 -3  1 1 -3  2 -2 -2  2 2 -2 .  . .  .  .   . . .

X.21 45 -3 -15   9 .  . . .  1 -3  1 1 -3  1 -1  1 -1  1  . . . .

X.22 45 -3 -15 9 .  .  .  .  1  1 -3 -3  1  1  1 -1  1 -1 . .  . .

X.23 45 -3 15 -9 .  .  .  . -1 -1 3 -3  1  1 -1  1  1 -1 . .  . .

X.24 45 -3 15 -9  .  .  . . -1  3 -1  1 -3  1  1 -1 -1 1  . . . .

     2  1  1
     3  2  2
     5  .  .

       6d 3b

X.1     1  1

X.2    -1  1

X.3    -1 -1

X.4     2  2

X.5     1 -1

X.6    -2 2
```

```
X.7       1 -1

X.8       1 -1

X.9      -1 -1

X.10     -1 -1

X.11      .  .

X.12      .  .

X.13      1  1

X.14     -1  1

X.15      .  .

X.16      .  .

X.17      .  .

X.18      .  .

X.19      .  .

X.20      .  .

X.21      .  .

X.22      .  .

X.23      .  .

X.24      . .

A = -E(5)-E(5)^4
  = (1-ER(5))/2 = -b5
```

****

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