



MULTIVARIATE ANALYSIS OF THE BRICS FINANCIAL MARKETS

BY
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Declaration

This dissertation is submitted to the School of Mathematics, Statistics and Computer Science at University of KwaZulu-Natal, Pietermaritzburg, in fulfillment of the requirements for the degree of Master of Science in Statistics. The thesis presents the original work of the author and has not been otherwise been submitted in any form for any degree to any University. Where use has been made of the work of others it is duly acknowledged in the text.

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Dedication

This thesis is dedicated to my beloved parents Dr. and Mrs Ijumba for their support and encouragement to further my studies.

Acknowledgments

My heartfelt gratitude goes to the Almighty God. It is by his grace and unconditional love that I am writing this thesis today. I would also like to express my sincere gratitude to my supervisors Dr. Thomas Achia and Prof. Henry Mwambi for their faith in me, support and patience especially at times when I lost hope in my work. A special thanks to my family. Words cannot explain how grateful I am for all they sacrifices they made on my behalf. Last but not least I would like to extend my deepest gratitude to all my friends especially Oscar Ngesah and Robert Mathenge who supported me in many ways through the course of this study. May the almighty God bless you all abundantly.

Abstract

The co-movements and integration of financial markets has been a subject of great concern among many researchers and economists due to an interest in the impacts of stock market integration in terms of international portfolio diversification, asset allocation and asset pricing efficiency. Understanding the interdependence among financial markets is thus of immense importance especially to investors and stakeholders in making viable decisions, managing risks and monitoring portfolio performances. In this thesis, we investigated the levels of interdependence and dynamic linkages among the five emerging economies well known as the BRICS: Brazil, Russia, India, China and South Africa, using a Vector autoregressive (VAR), univariate GARCH(1,1) and multivariate GARCH models. Our data sample consisted of the BRICS weekly returns from the period of January 2000 to December 2012. We used a VAR model to examine the linear dependence among the BRICS markets. The results from the VAR model analysis provided some evidence of unidirectional linear dependencies of the Indian and Chinese markets on the Brazilian stock market. The univariate GARCH(1,1) and multivariate GARCH models were employed to explore the volatility and dynamic correlation in the BRICS stock returns respectively. The results of the univariate GARCH model suggested volatility persistence among all the BRICS stock returns where China appeared to be the most volatile followed by the Russian stock market while the South African market was found to be the least volatile. Results from the multivariate GARCH models revealed similar volatility persistence. Furthermore, we found that, the correlations among the five emerging markets varied with time. From this study, evidence of interdependence among the BRICS cannot be rejected. Moreover, it appears that there are other factors apart from the internal markets themselves that may affect the volatility and correlation among the BRICS.

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Chapter 1

Introduction

1.1 Background

The integration of financial markets in the world today is an outcome of international trade, financial flows, the exchange of technology and information and the movement of people. The dynamic linkage among nations has been of great interest to policy makers and investors through different opportunities that manifest themselves especially with the upcoming emerging key markets among them the BRICS (Brazil, China, India, Russia and South Africa).

The BRIC national economies, currently known as BRICS were originally formed by Jim O’Niel in 2001 before the official inclusion of South Africa in 2011 ([Pasumarti, 2013](#)). These emerging economies are widely recognized to be the future driving forces for global demand and supply of goods.

BRICS countries have come into focus as a symbol of the shift in world economic influence away from the G7 (the US, United Kingdom, France, Italy, Canada, Germany and Japan) economies towards the developing world. Estimates by [O’Neill and Goldman \(2001\)](#) show that BRICS economies will have outrun the G7 economies by 2050 and that, China and India would become the first and third largest economies followed by Brazil, Russia and South Africa by then. The brisk economic growth and demographics of India and China are expected to mount a large middle class base whose consumption would help energize the BRICS economic development and expansion of the

world economy. The economic growth of the BRICS nations collectively, accounted for 30% of the increase in global output during the period of 2000 to 2008 (O'Neill et al., 2009).

However, the recent global crisis is, the mid 2007 global recession which manifested itself in 2008, affected almost the whole world including the five outstanding emerging markets with potential to disrupt their recovery in the world economic array. According to O'Neill et al. (2009), Russia suffered the most from the crisis where as China, Brazil and India experienced a relatively mild effect. The 2008 financial crisis was mainly due to the downfall of the US market caused by loose credit policy of its banks (Gupta, 2011). The aftermath of the 2008 global recession has been among the major issues of attraction towards economists, traders, policy makers and academicians due to the potential destructive effects and the extreme dependency structure of financial markets.

This study seeks to investigate the dynamic relationships among the BRICS financial markets using a multivariate time-series analysis approach. The bulk of this study is based on two key concepts of interest which are interdependence and volatility patterns in the BRICS stock returns. Here interdependence refers to the behavioural pattern observed on a variable or market due to the influence of another variable or market. This feature is captured by a vector autoregressive (VAR) model. VAR models are essentially multivariate extensions of univariate autoregressive (AR) models and are useful in capturing the dynamic behaviour and interdependence of financial time series by modeling the conditional mean of return or time series data (Wang and Zivot, 2006).

Volatility is another crucial aspect especially in the fields of finance and economics use in the management of financial risks, decision making and portfolio selections among other issues. Volatility can be defined as the explosive or impulsive behaviour of financial markets (Wang and Zivot, 2006). In statistics, volatility is often measured by the variance or standard deviation of return series (Tsay, 2010). Different statistical models have been developed by different researchers that can model volatility together with its stylized facts such as volatility clustering, the dynamic patterns of fat tails and time varying volatility as often observed in financial time series.

1.2 Literature review

Several studies on methods and approaches to univariate and multivariate volatility or VAR modeling of different financial markets have been conducted by several researchers. In this section we shall review both empirical and theoretical studies that have been conducted regarding VAR and volatility models and other findings related to this study.

The work of Sims (1980) popularized the use of VAR models which thereafter became very useful tools for econometric and statistical analysis. Prior to Sims (1980) researchers using multivariate time-series data relied on large scale simultaneous equation models which had a fundamental limitation of excluding variables in the equations most often, endogenous lagged variables without any statistical or theoretical justifications (Sims, 1980). The approaches produced very poor forecasts and were plagued with numerous methodological restrictions. Over a decade later, Lütkepohl (1999) worked on vector autoregressive analysis with special emphasis on cointegration. Two or more non-stationary time series are said to be cointegrated if there exists a linear combination of them that is stationary (Wang and Zivot, 2006). Lütkepohl (1999) stipulates that, characteristics such as trending properties and seasonal fluctuations for instance, determine to some extent a suitable model for a data set. Furthermore, he argues that, for variables with stochastic trends, cointegrating relations do not appear explicitly on VAR model equations but are rather easily analyzed within a vector error correction model (VECM).

Whether a process is integrated or cointegrated of any arbitrary order, Toda and Yamamoto (1995) argue that, for as long as the order of integration of the process does not exceed the true lag length of the model, a VAR model can be estimated and general restrictions on the parameters can be tested. Their idea of restrictions on the parameters of a VAR model seeks to avoid pretests for unit roots and cointegrating ranks by simply adding extra lags in the estimation process.

Ono (2011) examined the impact of oil prices on real stock returns for the BRIC markets from 1999 to 2009 using a VAR model. With an exception of Brazil, the results of the study indicated positive significant responses to some of the oil price indicators from China, Russia and India. Furthermore, significant asymmetric effects of oil shocks were only observed in India.

Byström et al. (2013) used a VAR and a constant GARCH (CGARCH) model to estimate the volatilities and spillover effects between developed and emerging market economies. The developed

markets consisted of USA, France and Germany whereas the emerging markets consisted of Russia, China and India. Results from their study suggested that, the emerging markets with an exception of Russia were influenced by the developed markets and that volatility spillovers moved from developed to emerging markets.

Campbell (1991) conducted a study on variance decomposition for stock returns using the New York Stock Exchange monthly data. He compared the autoregressive (AR) time series model with the VAR approach for variance decomposition of stock returns. Among the results reported by Campbell (1991) was the predominant existence of negative autocorrelations in the stock returns. This result was easily captured by the VAR model while the AR model which exclusively focused on the autocovariances of the stock returns could not capture such definite results suggesting that, the VAR model performed better than the AR model. AR models can be perceived as restricted cases of VAR models because VAR models capture more data features and are more flexible compared to AR models (Brooks, 2008).

Bjørnland (2000) used a structural VAR model to analyze the sources of business cycles using real gross domestic product (GDP) data for four markets (Norway, Germany, United Kingdom and United States). After imposing a series of short and long run restrictions, the model was able to pick up different shocks that affected the four markets' economy and these shocks were in line with actual events that occurred in previous historical periods. According to Tsay (2010) structural VAR models are suitable for exploring explicit concurrent relationships between component series however, the reduced-form of VAR models is more preferable in time series analysis because concurrent correlations cannot be used in forecasting and the parameters are much easier to estimate compared to the structural VAR model parameters.

Ghirmay (2004) investigated the causal link between the level of financial development and economic growth in 13 sub-Saharan countries using a VAR model framework based on the theory of cointegration and error-correction representation. The outcome of the study revealed evidence of a long-run relationship between financial development and economic growth in 12 of the countries. Furthermore, financial development played a causal role on economic growth in 8 of the countries with bidirectional causal relationship in 6 of the countries.

Similar to the work by Ghirmay (2004), Piesse and Hearn (2002) used a VAR model in the context of

cointegration techniques on three market indices of the Southern African Customs Union to explore equity market integration. The results of their study show evidence of progress in institutional building which is considered a valuable contribution to market integration throughout financial markets in the SADC community.

Enisan and Olufisayo (2009) used a VAR model to examine the long run and causal relationships between stock market performance and economic growth from seven countries in sub-Saharan Africa. Results from their study indicated the presence of a bidirectional relationship between the stock markets' development and economic growth for Cote D'Ivoire, Kenya, Morocco and Zimbabwe.

Most traditional econometric models and statistical models such as the autoregressive moving average (ARMA) assume constant variance but, in reality most financial time series have non-constant variances. Therefore, in order to capture uncertainties in financial markets, Engle (1982) introduced a new class of stochastic processes called the ARCH model in attempt to model changes in market volatility. According to Engle (1982), ARCH processes have the characteristics of being serially uncorrelated with varying variances conditional on the past but with constant unconditional variances. To test for the presence of ARCH errors in the innovations of a time series data, Engle (1982) used the Lagrange multiplier procedure by Granger and Andersen (1978).

A few years later the ARCH model was further extended to a generalized form known as the generalized autoregressive (GARCH) model by Bollerslev (1986) so as to take into account the dynamic behaviour of the conditional variance such as leptokurtic effects and volatility clustering. In the empirical application of ARCH models, relatively long lag lengths are required in the conditional variance equation in order to capture the effects of past returns on current volatility (Danielsson, 2011). Thus, the GARCH model by Bollerslev (1986) allows for a longer memory and a more supple lag structure by including the lagged values on the conditional variance. GARCH models also provide a much better fit and a more practical learning methodology compared to ARCH models (Bollerslev, 1986). To check for the behaviour of time series in the conditional variance, Bollerslev (1986) employs the use of autocorrelation and partial autocorrelation functions on the squared residuals. Despite the fact that GARCH models capture volatility clustering and leptokurtic effects, they fail to capture features such as leverage effects (the impact of positive and negative innovations or shocks) (Danielsson, 2011). This limitation calls for more manageable volatility models that account for

asymmetric responses of positive and negative shocks such as the exponential GARCH (EGARCH) by Nelson (1991), the threshold-GARCH (TGARCH) by Glosten et al. (1993) and the asymmetric power ARCH (APARCH) models by Ding et al. (1993).

Kasman (2009) assessed the impact of sudden changes on the persistence of volatility in the BRIC markets using a GARCH model. The results of the study showed that incorporating sudden changes such as structural breaks in the GARCH model significantly reduced the persistence of volatility in each of the BRIC returns suggesting that, less weight be given to past volatility values and more weight to the current shocks in hedging strategies.

A bivariate AR(1)-EGARCH(1,1) model was used by Bhar and Nikolova (2009) to investigate on the oil price and equity returns in the BRIC countries. The results of their study showed that the volatility of the BRIC stock returns did not have any significant impact on the volatility of global oil prices despite the countries' aggressive economic growth.

Morales and Gassie (2011) conducted a research on structural breaks and financial volatility in the BRIC countries together with the US and some energy markets using GARCH and TGARCH models. From their results the Brazilian, Indian and Russian stock markets were found to be more influenced by international shocks from the US stock markets and energy market fluctuations whereas, the Chinese market appeared to be isolated from external shocks showing a higher level of stability compared to the rest of the emerging markets.

Kiani (2007) conducted a study on the determination of volatility and mean returns with evidence from the Jamaican Stock Price Index. Kiani (2007) used state space models with stable distributions that accounted for non-normality and GARCH-like effects that may be present in the return series. The results of the study revealed some significant leptokurtosis in the market together with evidence of volatility in excess returns. However, leverage effects in the Jamaican stock market appeared to be insignificant implying that, negative shocks do not necessarily imply future rise in volatility than positive shocks of the same magnitude.

Goudarzi and Ramanarayanan (2010) assessed volatility and its stylized facts on the Indian stock market daily returns from 2000 to 2009 using ARCH and GARCH model. Findings of the study suggested the GARCH(1,1) model as the most appropriate model in explaining volatility clustering, mean reverting and fat tails in the Indian stock market compared to the ARCH model which often

requires a lot of parameters to adequately describe the volatility process.

Hearn and Piesse (2010) modeled the size and illiquidity in West African equity markets using a capital asset pricing model (CAPM) and GARCH model. Their analysis revealed that investment strategies based on Francophone markets performed better than those of Anglophone markets in Africa. Moreover, there was also some evidence of limited benefits to investors due to the inclusion of the small and highly illiquid markets of Cote d'Ivoire and Ghana.

Chinzara and Aziakpono (2009) used both univariate GARCH and multivariate VAR models in investigating dynamic returns linkages and volatility transmission between South African and world major stock markets. Results of their study showed that both returns and volatility linkages existed between South Africa and the world major stock markets whereby Australia, China and the US had the most influence on the South African returns and volatility.

Kuen and Hoong (1992) compared volatility forecasts produced by the historical sample variance, GARCH and EWMA models using daily closing prices of five value-weighted indices of the Singapore stock market. Their findings strongly suggested that the EWMA model performed better than the other two models. Minkah (2007) carried out a similar study by evaluating the forecasting performance of three models which are Historical Variance, GARCH and EWMA on the Swedish and US stock markets. Results of his findings showed that, under the in-sample volatility forecasts, the GARCH and EWMA models outperformed the simple Historical Variance whereas under the out-of-sample forecast accuracy comparisons, the GARCH model performed better for shorter horizon forecasts while the simple Historical Variance model outperformed the other two models for longer horizon forecasts.

Lopez (2001) assessed the predictive accuracy of volatility models by comparing stochastic volatility models to GARCH and EWMA using four daily exchange rate series from Britain, Canada, Japan and Germany. His results found no difference on forecasting performance among the models. Danielsson (2011) on the other hand argues that, EWMA models produce poor forecasts compared to GARCH models due to their simplicity in structure.

Danielsson (2011) used the daily stock returns of IBM and Microsoft to compare the estimation efficiency of the EWMA, orthogonal-GARCH (OGARCH) and DCC models. From his findings, EWMA produced the most volatile correlation forecasts whereas the DCC and OGARCH had more stable

correlations. He suggested that the enormous swings in EWMA correlations could have been an overreaction but significant compromises are the price paid for tractability by all the three models. Understanding the correlation between financial variables is of great importance especially in the areas of financial management such as asset allocations and risk management. Engle (2002) and Tse and Tsui (2002) proposed a simple class of multivariate GARCH models known as the dynamic conditional correlation (DCC) that account for time varying correlations and their dynamics in financial data. The DCC model is an extension of the constant conditional correlation (CCC) model which has the shortcomings of the assumption of constant conditional correlation and nonstability by aggregation (Francq and Zakoian, 2010). Tse and Tsui (2002) argued that the DCC model maintains the perception and interpretation of the GARCH model and still satisfies the positive-definite condition of the variance covariance matrix. The DCC models are not linear but can be easily estimated with univariate or two-step estimation technique depending on the likelihood function (Engle, 2002). The two-step estimator is asymptotically normal and consistent (Engle and Sheppard, 2001). Bianconi et al. (2012) conducted a study on the behaviour of stocks and bond markets of the US and BRIC nations using a DCC model. The results of the study showed that all of the BRIC returns displayed significant conditional heteroskedasticity with Russia as the most responsive to conditional volatility news. Moreover the dynamic conditional correlations among the bond returns of the BRIC nations and the US financial stress appeared to have increased after the 2008 global crisis with an exception of India which appeared to be uncorrelated with the rest of the BRIC countries. Aloui et al. (2011) used the GARCH-in-mean and copula functions to examine the extent of current global crisis and contagion effects induced by conducting an empirical investigation of the extreme financial interdependences of the BRIC and markets. The results of their study revealed a strong evidence of time-varying dependence between each of the BRIC markets and the US markets with a stronger dependence for commodity-price dependent markets than for finished-product export oriented markets.

Wang and Zivot (2006) examined the existence of contagion effects between the stock markets of Thailand and Chinese Economic Area (CEA) during the 1997-98 Asian financial crisis period using a DCC model. To account for contagion effect Wang and Zivot (2006) compared the DCC mean coefficients of the post-crisis and pre-crisis periods. The results of the study revealed the existence

of contagion between the markets with a significant increase in the means of correlation coefficients across the markets between the pre and post crisis periods.

Schwert (2010) explored the forecasting capacity of a DCC-GARCH model relative to simple unconditional correlation estimate applied to extreme market environments using the US financial markets. Using five asset series the results of the study suggested that the forecasting performance of the DCC-GARCH model relative to the unconditional model was mainly determined by the asset pair involved rather than the time period.

Cappiello et al. (2006) examined the asymmetries in the conditional variances, covariances and correlations of the FTSE All-World index series and DataStream Benchmark bond returns using a DCC model. The results of their findings strongly suggested the presence of asymmetries in the conditional variance of both equity and bond returns in noticeably different manners. The study also portrayed a strong evidence of market volatility correlation for equity returns of the annualized average volatility series.

Ndako (2013) investigated on the dynamics of stock prices and exchange rates relationship for 5 sub-Saharan African financial markets which were Ghana, Kenya, Mauritius, Nigeria and South Africa using a VAR and DCC model. Results from the VAR model showed no evidence of cointegration among any of the five markets and the DCC model results indicated negative time-varying correlations for all the five markets with an exception of Ghana that indicated a positive correlation.

Engle (2007) revisited the DCC model with a new convenient estimation approach called the Mac-Gyver method. He compares the DCC with the FACTOR ARCH and the FACTOR DOUBLE ARCH models. The DCC model was then combined with the FACTOR DOUBLE ARCH model to produce the FACTOR DCC model. Using data from 18 US large cap stocks Engle (2007) compares the efficiency of the methods and the results of his findings suggested the FACTOR DCC model as the best approach followed by the FACTOR DOUBLE ARCH.

1.3 Comment on the review

Despite the fact that there is a considerable amount of literature on the analysis of volatility and interdependence between financial markets surprisingly, scarce empirical research has been conducted

on emerging markets, a clear gap of research has been found regarding the BRICS economies. Most of the studies conducted regarding volatility and interdependence of stock markets are focused on the developed economies. Furthermore, based on the available information, there has only been a few researchers such as Mallick and Sousa (2009) and Rengasamy (2012) who have conducted studies on the BRICS markets with the South African market included. To add to the body of knowledge on emerging markets and especially in the BRICS financial markets, multivariate GARCH, the vector autoregressive model and its extensions need to be utilized within the BRICS to have better understanding of the dependence structure and dynamics in the volatility and correlations within the markets. The South African economy needs to be acknowledged in the analysis of the BRICS in order to understand its influence and contribution to the five emerging markets. One of the impediments to lack of information from developing and emerging markets, has been easy access and availability of data. Probably with increased availability of data from such markets this will change.

1.4 Problem statement

Interdependence among financial markets is of substantial significance due to possible gains of international investment and portfolio diversification. In addition to that, financial stability, monetary policy and stock market efficiency are also influenced by international linkage of stock markets. Moreover, the increasing co-movement among stock markets has brought much attention to the study of dynamic correlations between financial markets. However, strong linkages of financial markets could be harmful to the world's financial stability through contagion a scenario whereby small shocks which initially affect a few financial markets, further spread to the rest of the markets. In this regard, understanding the linkage between financial markets is of great importance especially to investors and policy makers in order to effectively formulate and implement different policies that govern financial trade.

The degree to which financial prices fluctuate whereby large changes tend to be followed by large changes and small changes by small changes has attracted a great deal of research in econometrics, financial economics and statistics. This property of time series data is known as volatility clustering.

Intuitively one could say large price fluctuations reflect investor uncertainty due to fundamental uncertainties in the economy. A peculiar feature of volatility is its uncertainty a feature that has lead to the development of many models that can capture and forecast volatility. Understanding volatility together with its stylized facts is significant because it aids in decision making, asset pricing, risk management and portfolio allocation.

1.5 Objectives of the study

1.5.1 Broad objective

The primary objective of this study is to evaluate the levels of interdependence and dynamic linkage among the RBICS financial markets using appropriate univariate and multivariate time-series models.

1.5.2 Specific objectives

To achieve the broad objective we shall:

- Investigate and review the statistical properties of the main time-series models (VAR, GARCH and multivariate GARCH models) that shall be considered,
- Determine, empirically, an appropriate VAR(p) model for the BRICS return series and estimate the linear dependency among the five markets,
- Estimate and fit an appropriate GARCH(p,q) model to each of the BRICS market return series,
- Produce and evaluate the quality of forecasts arising from the GARCH model selected,
- Identify appropriate multivariate GARCH models for the BRICS returns.

1.6 Structure of the thesis

This thesis consists of five chapters. Following this introductory chapter, the second chapter provides a research methodology of the VAR model together with the model application on the BRICS returns. Chapter three, gives a research methodology of the univariate GARCH model and its application on the BRICS markets. In chapter four, we present the multivariate GARCH models' methodology and application on the BRICS financial returns and finally, in chapter five a discussion and conclusion of the study is provided. The data used in this thesis was obtained from [Yahoo! Finance](#) on 13th March 2013. The entire analysis of the data in this thesis was done using the R programming language, version 3.0.1 that was originally introduced by [Ihaka and Gentleman \(1996\)](#).

Chapter 2

Vector Autoregressive (VAR)

Modeling of BRICS stock returns

2.1 Introduction

This chapter discusses about VAR modeling with several sections that explain about; the model description, parameter estimation, diagnostic tests carried out on the selected VAR model, forecasting, strengths and weaknesses of the VAR model, application of the model unto the data and a summary of the results.

2.2 Model description

The variables in a VAR model are treated symmetrically where each variable has an equation describing its progression based on its own lags and the lags of all other variables in the model.

Consider an $N \times 1$ random vector of endogenous variables $\mathbf{y}_t = (y_{1t}, \dots, y_{Nt})'$ for $t = 1, \dots, N$. A VAR process of order p can be written as

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{y}_{t-1} + \dots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t, \quad (2.1)$$

where \mathbf{A}_i are $N \times N$ fixed coefficient matrices for $i = 1, \dots, p$, $\mathbf{c} = (c_1, \dots, c_N)$ is an $N \times 1$ vector of constants and $\mathbf{u}_t = (u_{1t}, \dots, u_{Nt})$ is an N -dimensional white noise process with $E(\mathbf{u}_t) = 0$, $E(\mathbf{u}_t, \mathbf{u}_s') = 0$ for $s \neq t$ and a time-invariant positive definite covariance matrix $E(\mathbf{u}_t \mathbf{u}_t') = \Sigma_u$.

Definition 1 (Stationary process). A stochastic process \mathbf{y}_t is said to be stationary if

i $E(\mathbf{y}_t) = \boldsymbol{\mu}$ for all t and

ii $E[(\mathbf{y}_t - \boldsymbol{\mu})(\mathbf{y}_{t-h} - \boldsymbol{\mu})'] = \boldsymbol{\Gamma}_y(h) = \boldsymbol{\Gamma}_y(-h)'$ for all t and $h = 0, 1, 2, \dots$,

where $\boldsymbol{\mu}$ is a vector of finite mean terms and $\boldsymbol{\Gamma}_y(h)$ is matrix of finite covariances. The first condition implies that all the components in \mathbf{y}_t have the same finite mean vector and the second condition calls for the autocovariances to depend only on the time period h between the vectors \mathbf{y}_t and \mathbf{y}_{t-h} . Unless stated, the return series of financial assets are assumed to be weakly stationary (Tsay, 2010). If \mathbf{y}_t in equation (2.1) is said to be covariance stationary then the unconditional mean is given by

$$\boldsymbol{\mu} = (\mathbf{I}_N - \mathbf{A}_1 - \dots - \mathbf{A}_p)^{-1} \mathbf{c},$$

where \mathbf{I}_N is an identity matrix. The mean-adjusted form of the VAR(p) process is then

$$\mathbf{y}_t - \boldsymbol{\mu} = \mathbf{A}_1(\mathbf{y}_{t-1} - \boldsymbol{\mu}) + \mathbf{A}_2(\mathbf{y}_{t-2} - \boldsymbol{\mu}) + \dots + \mathbf{A}_p(\mathbf{y}_{t-p} - \boldsymbol{\mu}) + \mathbf{u}_t.$$

Definition 2 (Stability Condition). A VAR(p) is said to be stable if the eigenvalues of

$$\det(\mathbf{I}_N - \mathbf{A}_1 z - \dots - \mathbf{A}_p z^p) \neq 0$$

have a modulus less than one that is, $|z| < 1$. Assuming that the VAR process has been initialized in the infinite past, then a stable VAR(p) process is a stationary time-series with time invariant means, variances and autocovariance structures.

Consider a lag operator L such that $L\mathbf{y}_t = \mathbf{y}_{t-1}$. We can rewrite the VAR(p) process in equation (2.1) as

$$\mathbf{A}(L)\mathbf{y}_t = \mathbf{c} + \mathbf{u}_t, \tag{2.2}$$

where $\mathbf{A}(L) = \mathbf{I}_N - \mathbf{A}_1 L - \dots - \mathbf{A}_p L^p$. Now let

$$\boldsymbol{\Phi}(L) = \sum_{i=0}^{\infty} \boldsymbol{\Phi}_i L^i$$

be an operator such that

$$\Phi(L)A(L) = I_N.$$

Multiplying $\Phi(L)$ by equation (2.2) we get

$$\begin{aligned} y_t &= \Phi(L)c + \Phi(L)u_t \\ &= c \sum_{i=0}^{\infty} \Phi_i + \sum_{i=0}^{\infty} \Phi_i u_{t-i}. \end{aligned} \quad (2.3)$$

Equation (2.3) is known as the stable VAR(p) representation of an infinite MA process with $\Phi_0 = I_N$. The Φ_i can be calculated recursively as

$$\Phi_i = \sum_{s=1}^i \Phi_{i-s} A_s, \quad (2.4)$$

where $A_s = 0$ for $s > p$.

2.3 Lag selection

Before estimating the parameters of a VAR(p) model, one has to consider the selection of an appropriate lag that is, the order of p . This step does not give information on how well the model fits the data but rather suggests a candidate model that can be tested using suitable diagnostic tests. As with the univariate autoregressive process, the lag length can be determined using information criteria such as the Akaike (AIC), Hannan and Quinn (HQ) or Schwarz (SC). These measures are defined as

$$\begin{aligned} AIC(p) &= \log \det(\tilde{\Sigma}_u(p)) + \frac{2}{T} p N^2, \\ HQ(p) &= \log \det(\tilde{\Sigma}_u(p)) + \frac{2 \log(\log(T)) p N^2}{T}, \\ SC(p) &= \log \det(\tilde{\Sigma}_u(p)) + \frac{\log(T)}{T} p N^2, \end{aligned}$$

where $\tilde{\Sigma}_u(p) = T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$, and p assigns the lag order. The selection from these criteria is based on the minimum value they produce.

2.4 Model parameter estimation

The coefficients of a VAR(p) process can be estimated efficiently by a least squares or the maximum likelihood approach. However, the two methods are asymptotically alike [Tsay \(2005\)](#). In this section we shall use the Maximum likelihood estimation (MLE) method assuming the the VAR(p) process \mathbf{y}_t is Gaussian that is to say $\mathbf{u} \sim N(0, \Sigma_u)$. T is the number of observed data that is used in the estimation of VAR and GARCH models. The matrix notation of a VAR(p)-process can be written as

$$\mathbf{Y} = \mathbf{B}\mathbf{Z} + \boldsymbol{\zeta},$$

where

$$\begin{aligned}\mathbf{Y} &= (\mathbf{y}_1, \dots, \mathbf{y}_T)', \\ \mathbf{B} &= (\mathbf{c}, \mathbf{A}_1, \dots, \mathbf{A}_p)', \\ \mathbf{Z}_t &= (\mathbf{1}, \mathbf{y}_t, \dots, \mathbf{y}_{t-p+1}) \\ \mathbf{Z} &= (\mathbf{Z}_0, \dots, \mathbf{Z}_{T-1})', \\ \boldsymbol{\zeta} &= (\mathbf{u}_1, \dots, \mathbf{u}_T)'. \end{aligned}$$

\mathbf{Y} , \mathbf{B} , \mathbf{Z} and $\boldsymbol{\zeta}$ are $(N \times T)$, $(N \times (Np + 1))$, $((Np + 1) \times T)$ and $(N \times T)$ matrices respectively. We define the following notations that will be used in the ML estimation of the VAR(p) model:

$$\begin{aligned}\mathbf{y} &= \text{vec}(\mathbf{Y}), \\ \mathbf{b} &= \text{vec}(\mathbf{B}'), \\ \mathbf{u} &= \text{vec}(\boldsymbol{\zeta}) \\ \mathbf{Y}^* &= (\mathbf{y}_{1-\mu}, \dots, \mathbf{y}_{T-\mu}), \\ \mathbf{X} &= (\mathbf{Y}_0^*, \dots, \mathbf{Y}_{T-1}^*), \\ \boldsymbol{\alpha} &= (\mathbf{A}_1, \dots, \mathbf{A}_p), \end{aligned}$$

where \mathbf{y} , \mathbf{b} , \mathbf{u} and $\boldsymbol{\alpha}$ are $(NT \times 1)$, $((N^2p + N) \times 1)$, $(NT \times 1)$ and $(N^2p \times 1)$ vectors respectively, \mathbf{Y}^* and \mathbf{X} are $(N \times T)$ and $(Np \times T)$ matrices. The probability density function of \mathbf{u} is given as

$$f_u(\mathbf{u}) = \frac{1}{(2\pi)^{\frac{NT}{2}}} |\Sigma_u|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \mathbf{u}' \Sigma_u \mathbf{u}\right), \quad (2.5)$$

where \mathbf{u} is expressed as

$$\mathbf{u} = \mathbf{y} - \boldsymbol{\mu}^* - (\mathbf{X}' \otimes \mathbf{I}_N) \boldsymbol{\alpha} \quad (2.6)$$

such that, $\boldsymbol{\mu}^* = (\mu', \dots, \mu')$. Using equation (2.6)

$$\begin{aligned} f_y(\mathbf{y}) &= \left| \frac{\partial \mathbf{u}}{\partial \mathbf{y}'} \right| f_u(\mathbf{u}) \\ &= \frac{1}{(2\pi)^{\frac{NT}{2}}} |\mathbf{I}_T \otimes \boldsymbol{\Sigma}_u|^{-\frac{1}{2}} \exp \left(-\frac{1}{2} (\mathbf{y} - \boldsymbol{\mu}^* - (\mathbf{X}' \otimes \mathbf{I}_N) \boldsymbol{\alpha})' (\mathbf{I}_T \otimes \boldsymbol{\Sigma}_u^{-1}) \right. \\ &\quad \left. (\mathbf{y} - \boldsymbol{\mu}^* - (\mathbf{I}_N \otimes \mathbf{X}') \boldsymbol{\alpha}) \right) \end{aligned}$$

Thus, the log-likelihood function

$$\begin{aligned} \log L(\boldsymbol{\mu}, \boldsymbol{\alpha}, \boldsymbol{\Sigma}_u) &= -\frac{NT}{2} \log(2\pi) - \frac{T}{2} \log(\boldsymbol{\Sigma}_u) - \frac{1}{2} (\mathbf{y} - \boldsymbol{\mu}^* - (\mathbf{X}' \otimes \mathbf{I}_N) \boldsymbol{\alpha})' (\mathbf{I}_T \otimes \boldsymbol{\Sigma}_u^{-1}) \\ &\quad \times (\mathbf{y} - \boldsymbol{\mu}^* - (\mathbf{X}' \otimes \mathbf{I}_N) \boldsymbol{\alpha}) \\ &= -\frac{NT}{2} \log(2\pi) - \frac{T}{2} \log|\boldsymbol{\Sigma}_u| - \frac{1}{2} \sum_{t=1}^T \left((\mathbf{y}_t - \boldsymbol{\mu}) - \sum_{i=1}^p \mathbf{A}_i (\mathbf{y}_{t-i} - \boldsymbol{\mu}) \right)' \boldsymbol{\Sigma}_u \\ &\quad \times \left((\mathbf{y}_t - \boldsymbol{\mu}) - \sum_{i=1}^p \mathbf{A}_i (\mathbf{y}_{t-i} - \boldsymbol{\mu}) \right) \\ &= -\frac{NT}{2} \log(2\pi) - \frac{T}{2} \log|\boldsymbol{\Sigma}_u| - \frac{1}{2} \sum_t \left(\mathbf{y}_t - \sum_i \mathbf{A}_i \mathbf{y}_{t-i} \right)' \boldsymbol{\Sigma}_u^{-1} \left(\mathbf{y}_t - \sum_i \mathbf{A}_i \mathbf{y}_{t-i} \right) \\ &\quad + \mu' \left(\mathbf{I}_N - \sum_i \mathbf{A}_i \right)' \boldsymbol{\Sigma}_u^{-1} \sum_t \left(\mathbf{y}_t - \sum_i \mathbf{A}_i \mathbf{y}_{t-i} \right) \\ &\quad - \frac{T}{2} \mu' \left(\mathbf{I}_N - \sum_i \mathbf{A}_i \right)' \boldsymbol{\Sigma}_u^{-1} \left(\mathbf{I}_N - \sum_i \mathbf{A}_i \right) \boldsymbol{\mu} \\ &= -\frac{NT}{2} \log(2\pi) - \frac{T}{2} \log|\boldsymbol{\Sigma}_u| - \frac{1}{2} \text{tr}[(\mathbf{Y}^* - \mathbf{A}\mathbf{X})' \boldsymbol{\Sigma}_u^{-1} (\mathbf{Y}^* - \mathbf{A}\mathbf{X})]. \end{aligned}$$

To obtain the maximum likelihood estimators of $\boldsymbol{\mu}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\Sigma}_u$, we take the first order partial derivative of the log-likelihood function:

$$\begin{aligned}\frac{\partial \log L}{\partial \boldsymbol{\mu}} &= \left(\mathbf{I}_N - \sum_i \mathbf{A}_i \right) \boldsymbol{\Sigma}_u^{-1} \sum_t \left(\mathbf{y}_t - \sum_i \mathbf{A}_i \mathbf{y}_{t-i} \right) - T \left(\mathbf{I}_N - \sum_i \mathbf{A}_i \right)' \boldsymbol{\Sigma}_u^{-1} \left(\mathbf{I}_N - \sum_i \mathbf{A}_i \right) \boldsymbol{\mu} \\ &= (\mathbf{I}_N - \mathbf{A}[\mathbf{k} \otimes \mathbf{I}_N])' \boldsymbol{\Sigma}_u^{-1} \left(\sum_t (\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{A} \mathbf{Y}_{t-1}^*) \right),\end{aligned}$$

$$\begin{aligned}\frac{\partial \log L}{\partial \boldsymbol{\alpha}} &= (\mathbf{X} \otimes \mathbf{I}_N)(\mathbf{I}_T \otimes \boldsymbol{\Sigma}_u^{-1})(\mathbf{y} - \boldsymbol{\mu}^* - (\mathbf{X}' \otimes \mathbf{I}_N)\boldsymbol{\alpha}) \\ &= (\mathbf{X} \otimes \boldsymbol{\Sigma}_u^{-1})(\mathbf{y} - \boldsymbol{\mu}^*) - (\mathbf{X} \mathbf{X}' \otimes \boldsymbol{\Sigma}_u^{-1})\boldsymbol{\alpha},\end{aligned}$$

$$\frac{\partial \log L}{\partial \boldsymbol{\Sigma}_u} = -\frac{T}{2} \boldsymbol{\Sigma}_u^{-1} + \frac{1}{2} \boldsymbol{\Sigma}_u^{-1} (\mathbf{Y}^* - \mathbf{A} \mathbf{X})(\mathbf{Y}^* - \mathbf{A} \mathbf{X})' \boldsymbol{\Sigma}_u^{-1},$$

where \mathbf{K} is a $p \times 1$ vector of ones. Equating the systems of derivatives to zero yields the ML estimators:

$$\begin{aligned}\hat{\boldsymbol{\mu}} &= \frac{1}{T} \left(\mathbf{I}_N - \sum_i \hat{\mathbf{A}}_i \right)^{-1} \sum_t \left(\mathbf{y}_t - \sum_i \hat{\mathbf{A}}_i \mathbf{y}_{t-i} \right), \\ \hat{\boldsymbol{\alpha}} &= ((\hat{\mathbf{X}} \hat{\mathbf{X}}')^{-1} \hat{\mathbf{X}} \otimes \mathbf{I}_N)(\mathbf{y} - \hat{\boldsymbol{\mu}}^*), \\ \hat{\boldsymbol{\Sigma}}_u &= \frac{1}{T} (\hat{\mathbf{Y}}^* - \hat{\mathbf{A}} \hat{\mathbf{X}})(\hat{\mathbf{Y}}^* - \hat{\mathbf{A}} \hat{\mathbf{X}})'. \end{aligned}$$

2.5 Diagnostic tests

Once a VAR(p) model has been estimated, it is crucial to check whether the fitted residuals obey the model's assumptions. There are three main assumptions of a VAR(p) model and these are;

1. The absence of serial correlation of errors, tested using a Portmanteau test
2. The absence of heteroscedasticity in the errors, tested using an ARCH test
3. Normality distribution of the residuals, tested using a Jarque Bera test

2.5.1 Portmanteau test

We shall use a Portmanteau test statistic by [Edgerton and Shukur \(1999\)](#) to test for the absence of serial correlation. The portmanteau test is used to test following hypothesis,

H_0 : the residuals are not serially correlated versus

H_a : the residuals are serially correlated

The test statistic is described as

$$Q_h = T^2 \sum_{i=1}^h \frac{1}{T-i} \text{tr}(\hat{C}_i' \hat{C}_0^{-1} \hat{C}_i \hat{C}_0^{-1}),$$

where $\hat{C}_i = \frac{1}{T} \sum_{t=i+1}^T \hat{\mathbf{u}}_t \hat{\mathbf{u}}_{t-i}'$. The test statistic is asymptotically distributed as a $\chi^2(N^2h - n)$ with n denoting deterministic terms of a VAR(p) model. The limiting distribution is only valid for h tending to infinity at an appropriate growing sample size rate. Thus, the trade-off is between a decent approximation to the χ^2 distribution and a loss in power of the test when the selected h is too large.

2.5.2 Multivariate ARCH test

To test for heteroscedasticity in the fitted residuals, we shall use a multivariate ARCH-LM test by [Breusch \(1978\)](#). Assuming the error vector, $\mathbf{u}_t = \mathbf{B}_1 \mathbf{u}_{t-1} + \dots + \mathbf{B}_h \mathbf{u}_{t-h} + \boldsymbol{\eta}_t$, where $\boldsymbol{\eta}_t$ is a white noise. Then, the multivariate ARCH-LM test is based on the following regression

$$\hat{\mathbf{u}}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{y}_t + \dots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{B}_1 \hat{\mathbf{u}}_{t-1} + \dots + \mathbf{B}_h \hat{\mathbf{u}}_{t-h} + \boldsymbol{\epsilon}_t, \quad (2.7)$$

where \mathbf{A}_i and \mathbf{B}_i are coefficient matrices and $\boldsymbol{\epsilon}_t$ is the regression error term. The null hypothesis is,

$H_0: \mathbf{B}_1 = \mathbf{B}_2 = \dots = \mathbf{B}_h = 0$ (absence of ARCH errors) versus

$H_a: \mathbf{B}_i \neq 0$ (presence of ARCH errors).

Under the null hypothesis, $\mathbf{u}_t = \boldsymbol{\eta}_t$. The multivariate ARCH-LM test statistic is given as

$$LM_h = T \hat{\mathbf{c}}_h' \hat{\boldsymbol{\Sigma}}_c^{-1} \hat{\mathbf{c}}_h,$$

whereas $\mathbf{c}_h = (C_1, \dots, C_h)'$ such that, $C_h = \frac{1}{T} \sum_{t=h+1}^T \mathbf{u}_t \mathbf{u}_{t-h}'$, $\hat{\boldsymbol{\Sigma}}_c$ is the covariance matrix of the residuals from equation (2.7). $LM_h \sim \chi^2(hN^2)$.

2.5.3 Jarque-Bera test

We shall follow [Lütkepohl \(2005\)](#) multivariate Jarque-Bera (JB) test approach which was initially introduced by [Jarque and Bera \(1980\)](#). The test can be computed by using residuals standardized by a Choleski decomposition of the variance-covariance matrix of a VAR(p) model ([Pfaff, 2008](#)). Moreover, the test is based on the third and fourth central moments (Skewness and kurtosis) of a Gaussian distribution. For instance, for a univariate random variable y , following a standard normal distribution, its third and fourth moments are given as $E(y^3) = 0$ and $E(y^4) = 3$. The null hypothesis for the JB test is,

H_o : the residuals are normally distributed versus

H_a : the residuals are not normally distributed.

The multivariate Jarque-Bera test statistic is defined as

$$JB_{mv} = \tau_s + \tau_k,$$

where τ_s and τ_k are calculated as

$$\tau_s = \frac{T\mathbf{b}'_1\mathbf{b}_1}{6},$$

$$\tau_k = \frac{T(\mathbf{b}_2 - \mathbf{3}_N)'(\mathbf{b}_2 - \mathbf{3}_N)}{24},$$

where \mathbf{b}_1 and \mathbf{b}_2 are third and fourth non-central moment vectors of the standardized residuals $\hat{\mathbf{u}}_t^s = \hat{P} - (\hat{\mathbf{u}}_t - \bar{\hat{\mathbf{u}}}_t)$ and \hat{P} is a lower triangular matrix with positive diagonal such that $\hat{P}\hat{P}' = \hat{\Sigma}_u$ which is the Choleski decomposition of the residual covariance matrix. The JB_{mv} test statistic is distributed as $\chi^2(2N)$ while the multivariate skewness, τ_s and kurtosis, τ_k are distributed as $\chi^2(N)$.

2.6 Forecasting

Prediction of future values based on previously observed values of a time series is one of the objectives of multivariate time series analysis. Once a VAR model has been estimated and passed the required diagnostic tests it can then be used for forecasting. Given a VAR(p), h-step ahead forecasts may be computed using the chain-rule of forecasting as

$$\mathbf{y}_{T+h|T} = \mathbf{c} + \mathbf{A}_1\mathbf{y}_{T+h-1} + \dots + \mathbf{A}_p\mathbf{y}_{T+h-p|T},$$

where $\mathbf{y}_{T+j|T} = \mathbf{t}_{T+j}$ for $j \leq 0$. The h -step prediction errors can be expressed as

$$\mathbf{y}_{T+h} - \mathbf{y}_{T+h|T} = \sum_{s=0}^{h-1} \mathbf{\Psi}_s \mathbf{u}_{t+h-s}$$

where the matrices $\mathbf{\Psi}_s$ are determined by a recursive substitution

$$\mathbf{\Psi}_s = \sum_{j=1}^{p-1} \mathbf{\Psi}_{s-j} \mathbf{A}_j,$$

with $\mathbf{\Psi}_0 = \mathbf{I}_N$ and $\mathbf{A}_j = 0$ for $j > p$. The forecasts are unbiased since all of the forecast errors have an expectation of zero and the MSE matrix of $\mathbf{y}_{t+h|T}$ is

$$\begin{aligned} \Sigma(h) &= MSE(\mathbf{y}_{T+h} - \mathbf{y}_{T+h|T}) \\ &= \sum_{j=0}^{h-1} \mathbf{\Psi}_j \Sigma \mathbf{\Psi}_j' \end{aligned}$$

The forecast confidence bands can be calculated as

$$[y_{k,T+h|T} - c_{1-\frac{\gamma}{2}} \sigma_k(h), y_{k,T+h|T} + c_{1-\frac{\gamma}{2}} \sigma_k(h)],$$

where $C_{1-\frac{\gamma}{2}}$ signifies the $(1 - \frac{\gamma}{2})$ percentage point of the normal distribution and $\sigma_k(h)$ is the standard deviation of the k th variable h -steps ahead.

2.7 Strengths and weaknesses of a VAR model

Most statistical models have both limitations and advantages regardless of their goodness of fit. This section outlines a few cons and prons of a VAR(p) model.

- Strengths

VAR models have several advantages compared to univariate time series models or simultaneous equations structural models. These include;

- (i) One does not need to specify which variables are endogenous or exogenous as all variables are endogenous. This is crucial because as a requirement for simultaneous equations structural models to be estimable, all equations in the system have to be identified.

- (ii) VAR models allow the value of a variable to depend on more than just its own lags or combinations of white noise terms that is, VAR models are more flexible than univariate AR models. Hence, VAR models can offer a very rich structure implying a capability of capturing more data features.
- (iii) VAR models provide better predictions as compared to traditional structural models. Large-scale structural models portray a poor performance in terms of their out-of-sample forecast accuracy. This could be due to the ad hoc nature of restrictions imposed on the structural models.

- Weaknesses

- (i) VAR models use very little theoretical information about the relationship between the variables to guide the specification of the model while a simultaneous structural system with valid exclusion restrictions that ensure identification of equations gives information about the structure of the model. The outcome of this is that VAR models are less responsive to theoretical analysis.
- (ii) Inclusion of too many variables (over fitting) in the system may result to poor performance of the VAR-derived forecasts. This happens when the lags of the variables in the system pick up spurious relations in the data. Therefore, unless the set of variables and lag lengths are maintained relatively small, predictions from a VAR model are likely to be inaccurate.
- (iii) In the case of relatively small sample sizes, degrees of freedom are more likely to be rapidly used up indicating the presence of large standard errors and henceforth wide confidence bounds for the model coefficients.

2.8 Application

This section provides an application of the theory discussed above on an empirical data. The data used was weekly stock returns of BRICS markets from 2000 to 2012. The stocks returns were

generated using the following formula

$$R_t = \ln \left(\frac{P_t}{P_{t-1}} \right),$$

where P_t is the price index, t is the time (weekly in this case) and R_t is the stock return. A summary of the stock returns, correlation, cross-correlation analysis and VAR modeling of the data are provided in this section.

2.8.1 Data summary

A plot of the stock returns is provided on Figure 2.1 where sd stands for the standard deviation of the mean, min and max represent the minimum and maximum values respectively. All the returns appear to be mean stationary with a lot of volatility between 2008 to 2009. This can be explained as a result of the 2008 global recession that occurred whereby many stock markets were affected as a result of the downfall of the US market. A summary of the stock returns is provided on Table 3.1. JSE appears to have the highest mean value of $0.0025(\pm 0.03)$ and IBOV with the lowest mean value of $0.00026(\pm 0.04)$. JSE appears to have the highest maximum value of -0.10 whereas RTSI appears to have the lowest minimum value of -0.24 . None of the stock market returns appear to have a normal distribution as the values of the kurtosis and skewness are non zero that is, the markets are either skewed to the left or right and they all appear to be more peaked compared to a standard normal distribution based on the values of the kurtosis.

Table 2.1: A summary of the BRICS weekly stock returns

Stock return	Mean	sd	Min	Max	Kurtosis	Skweness
IBOV	0.0003	0.04	-0.22	0.17	3.51	-0.57
RTSI	0.0024	0.05	-0.24	0.34	4.68	-0.31
BSE	0.0004	0.04	-0.17	0.13	2.52	-0.52
SSE	0.0005	0.03	-0.15	0.14	2.01	0.12
JSE	0.0025	0.03	-0.10	0.16	3.36	-0.09

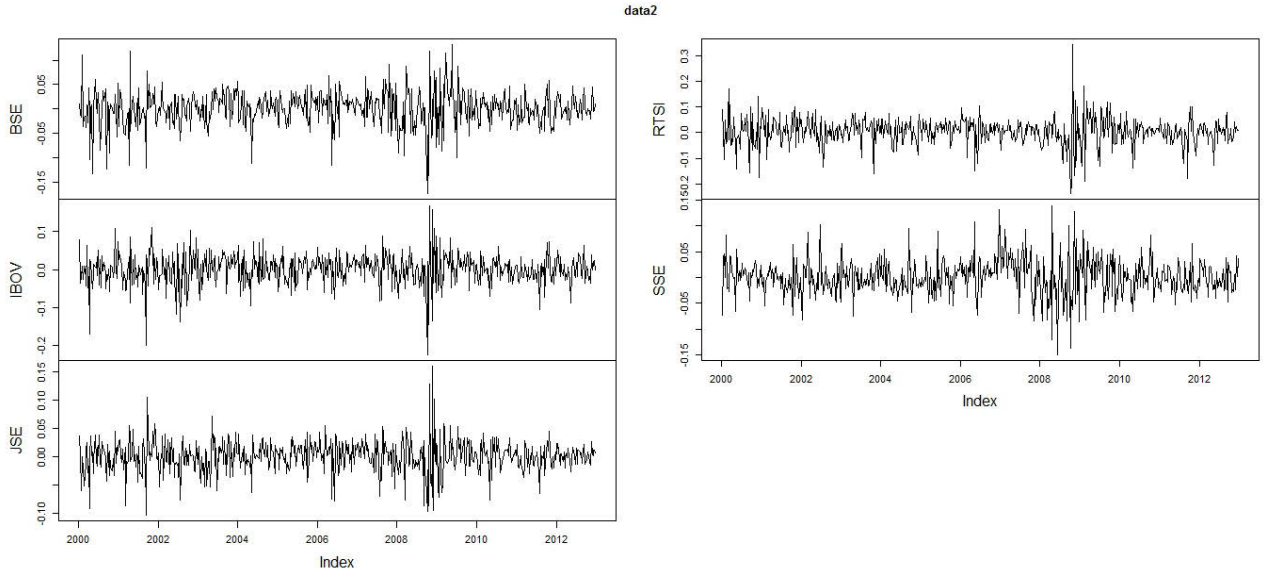


Figure 2.1: A plot of the BRICS weekly returns where BSE represents the Indian stock market, IBOV represents the Brazilian stock market, JSE represents the South African stock market, RTSI represents the Russian stock market and SSE represents the China stock market

2.8.2 Correlation analysis

A summary of correlation among the stock returns is given on Table 3.2. From the table of correlations, the strongest correlation is observed between RTSI and JSE with a correlation of 0.81 followed by RTSI and BSE with a correlation of 0.76. The Chinese stock market (SSE) appears to be negatively correlated with all the other stock markets with an exception of the JSE which however, has a very low correlation of 0.13. This can be explained by the fact that, the scale of China's economy has far out-stripped that of the other four emerging markets. The rest of the stock returns appear to be positively correlated with each other but with a relatively low correlation of ≤ 0.50 .

Table 2.2: A table of the correlation summary for the BRICS weekly returns

Stock return	IBOV	RTSI	BSE	SSE	JSE
IBOV	1.00	0.34	0.56	-0.82	-0.26
RTSI	0.34	1.00	0.76	-0.42	0.81
BSE	0.56	0.76	1.00	-0.48	0.48
SSE	-0.82	-0.42	-0.48	1.00	0.13
JSE	-0.26	0.81	0.48	0.13	1.00

2.8.3 Cross-correlation analysis

We carry out cross-correlation analysis in order to see how much of the variability of one market is predicted to influence another market at different lags. A cross-correlation analysis can have three possible outcomes which are: 1) a positive correlation where as one variable rises, the other variable is expected to rise as much, 2) zero on no correlation 3) a negative correlation whereby one variable falls while the other one rises. Figure 2.2 shows a cross-correlation plot of the BRICS stock returns. Positive lags on the plots explain past values while negative lags explain future values of the stock returns. Lag zero accounts for present values of the stock returns. From Figure 2.2, some significant correlations are observed between South Africa (JSE) and the past values of China (SSE), India(BSE) and the past values of China (SSE) and Brazil (IBOV) and the past values of China (SSE) indicating that SSE has an influence on the JSE, BSE and IBOV stock markets as the latter markets portray a linear dependence on the past values of SSE. China (SSE) and Russia (RTSI) on the other hand, show some significant correlations in the future lags suggesting that there exists a linear dependence between SSE and the future values of RTSI. The rest of the markets on Figure 2.2 show significant correlations only at lag zero implying that the stock returns have a strong correlation with their present values only. However, to have a better understanding of the dependencies among multiple return series data, a more informative approach such as building a multivariate model for the series is necessary. This is because a correctly specified model will consider simultaneously the

serial and cross-correlations among the series (Tsay, 2010).

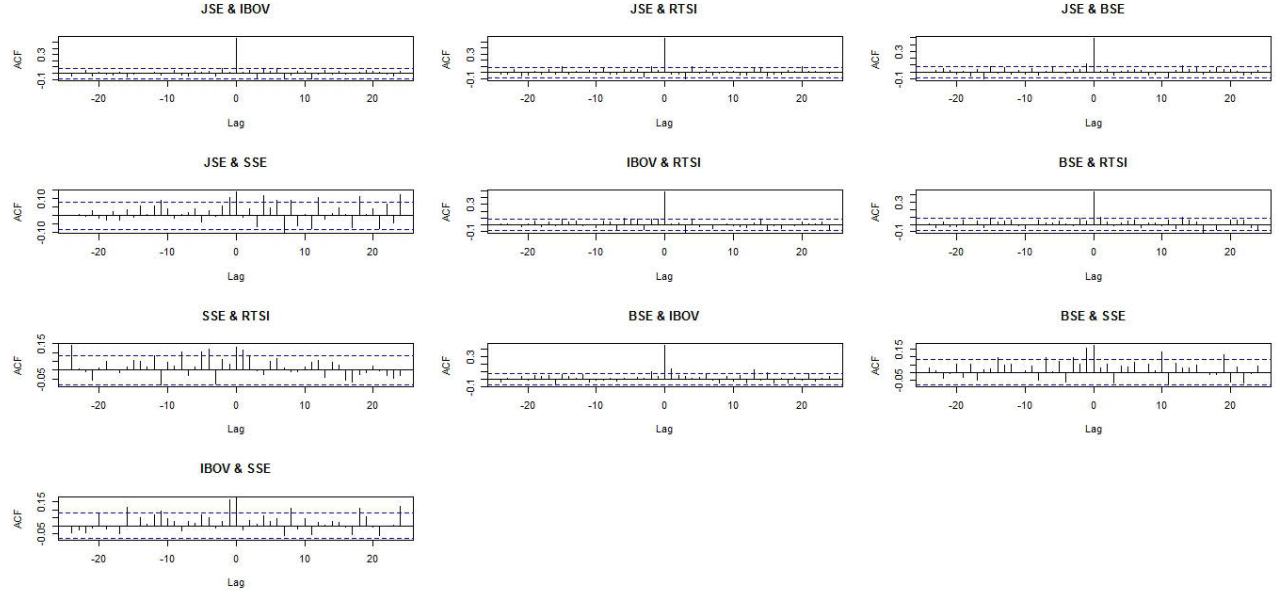


Figure 2.2: Cross-correlation plots of the BRICS weekly returns where BSE represents the Indian stock market, IBOV represents the Brazilian stock market, JSE represents the South African stock market, RTSI represents the Russian stock market and SSE represents the China stock market

2.8.4 VAR modeling

This section gives a summary of the procedures carried out while fitting a VAR model.

2.8.4.1 Lag selection

Firstly, the lag length of p for the VAR model was selected based on the three information criteria which were; the AIC, HQ and SC. These criteria aided in choosing the best lag order for p which is the fit with the lowest criterion value. Six different lags were fitted and from Table 3.3 Fit 1 corresponding to VAR with lag length one appears to have the lowest value from all the three information criteria with a value of -34.06 from the AIC, -33.97 from the HQ and -33.83 from the SC. Thus the VAR(1) model was fitted and the parameters were estimated.

Table 2.3: A summary of the VAR-model lag selection

Fit	Model	AIC	HQ	SC
1	VAR(1)	-34.06	-33.97	-33.83
2	VAR(2)	-34.02	-33.86	-33.61
3	VAR(3)	34.02	-33.78	-33.41
4	VAR(4)	-33.98	-33.67	-33.19
5	VAR(5)	-33.95	-33.57	-32.97
6	VAR(6)	-33.92	-33.46	-32.75

2.8.4.2 Parameter estimation

Before estimating the parameters of the VAR(1) model, the eigenvalues of the coefficient matrices were checked to see if their moduli are less than one. Table 2.4 gives a summary of the eigenvalues. All the eigenvalues appear to have a moduli less than one implying that the VAR(1) model is stable thus, parameter estimation of the VAR(1) model was then carried out. Table 2.5 presents a summary of the VAR(1) model parameter estimates, their residuals, t -values and their significance based on the p -values. All parameter estimates with p -values < 0.05 are significant. The parameter estimates represent the coefficient matrix Φ of the VAR(1)-model. The matrix accounts for the dynamic relationship of \mathbf{y}_t . From Table 2.5 the only autoregressive matrix coefficients that appear to be significant are: $AR(1)_{11}$, $AR(1)_{31}$, $AR(1)_{41}$, $AR(1)_{43}$ and $AR(1)_{55}$ which have the implication that, there exists a linear dependency between IBOV and its own past values, BSE and the past values of IBOV, SSE and the past values of IBOV, SSE and the past values of BSE and JSE and its own past values respectively. All these linear dependencies are unidirectional because the latter past values do not depend on the former markets' past values. Thus, the equations for the VAR(1)-model for each variable with significant parameters can now be written as

$$IBOV = -0.18(\pm 0.05)IBOV_{1,t-1} + u_{1,t} \quad (2.8)$$

$$BSE = 0.09(\pm 0.05)IBOV_{1,t-1} + u_{3,t} \quad (2.9)$$

$$SSE = 0.10(\pm 0.04)IBOV_{1,t-1} + (\pm 0.05)0.10BSE_{3,t-1} + u_{4,t} \quad (2.10)$$

$$JSE = -0.14(\pm 0.06)JSE_{5,t-1} + u_{5,t} \quad (2.11)$$

Concurrent relationships among the stock returns is explained by the off diagonal elements of the variance-covariance matrix displayed on Table 2.6. If the off diagonal elements are zero then the implication is the absence of any concurrent relationship between the variables of interest. In this case however, all the off diagonal elements appear to be non-zero implying the presence of concurrent relationships among all the stock returns.

Table 2.4: Eigenvalues of the fitted VAR(1)

Coefficient matrix	Eigenvalue
Φ_1	0.25
Φ_2	0.17
Φ_3	0.06
Φ_4	0.008
Φ_5	0.006

Table 2.5: Model Parameter Estimates of the fitted VAR(1) model

Stock return	Parameter	Variable	Estimate	SE	<i>t</i> -value	<i>p</i> -value
IBOV	$AR(1)_{11}$	$IBOV_{t-1}$	-0.18	0.05	-3.45	0.0006
	$AR(1)_{12}$	$RTSI_{t-1}$	0.03	0.04	0.78	0.44
	$AR(1)_{13}$	BSE_{t-1}	0.08	0.06	1.46	0.14
	$AR(1)_{14}$	SSE_{t-1}	-0.02	0.05	-0.44	0.66
	$AR(1)_{15}$	JSE_{t-1}	0.08	0.07	1.05	0.30
RTSI	$AR(1)_{21}$	$IBOV_{t-1}$	0.11	0.07	1.59	0.11
	$AR(1)_{22}$	$RTSI_{t-1}$	-0.01	0.05	-0.19	0.85
	$AR(1)_{23}$	BSE_{t-1}	0.03	0.08	0.44	0.66
	$AR(1)_{24}$	SSE_{t-1}	0.03	0.07	0.43	0.67
	$AR(1)_{25}$	JSE_{t-1}	0.04	0.002	1.03	0.30
BSE	$AR(1)_{31}$	$IBOV_{t-1}$	0.09	0.05	2.00	0.04
	$AR(1)_{32}$	$RTSI_{t-1}$	0.02	0.03	0.71	0.48
	$AR(1)_{33}$	BSE_{t-1}	-0.05	0.05	-1.04	0.30
	$AR(1)_{34}$	SSE_{t-1}	0.004	0.04	0.10	0.92
	$AR(1)_{35}$	JSE_{t-1}	0.08	0.07	1.14	0.25
SSE	$AR(1)_{41}$	$IBOV_{t-1}$	0.10	0.04	2.24	0.03
	$AR(1)_{42}$	$RTSI_{t-1}$	0.01	0.03	0.34	0.74
	$AR(1)_{43}$	BSE_{t-1}	0.10	0.05	2.09	0.04
	$AR(1)_{44}$	SSE_{t-1}	0.03	0.04	0.70	0.48
	$AR(1)_{45}$	JSE_{t-1}	-0.03	0.07	-0.41	0.68
JSE	$AR(1)_{51}$	$IBOV_{t-1}$	0.06	0.04	1.69	0.09
	$AR(1)_{52}$	$RTSI_{t-1}$	-0.006	0.03	-0.21	0.83
	$AR(1)_{53}$	BSE_{t-1}	0.03	0.04	0.84	0.40
	$AR(1)_{54}$	SSE_{t-1}	-0.01	0.03	-0.27	0.79
	$AR(1)_{55}$	JSE_{t-1}	-0.14	0.06	-2.60	0.01

Table 2.6: A variance-covariance matrix of the fitted VAR(1) model

Variable	IBOV	RTSI	BSE	SSE	JSE
IBOV	0.002	0.001	0.001	0.0003	0.001
RTSI	0.001	0.003	0.001	0.0002	0.001
BSE	0.001	0.001	0.001	0.0002	0.001
SSE	0.0003	0.0002	0.0002	0.001	0.0001
JSE	0.001	0.001	0.001	0.0001	0.001

2.8.4.3 Diagnostic tests for VAR(1)

Three diagnostic tests were carried out on the VAR(1) model which were; the portmanteau test which tests for serial correlation of the fitted residuals, the ARCH test which tests for the presence of ARCH errors and the Jarque-Berra test which tests for the normality of the fitted residuals and also accounts for the values of skewness and kurtosis. The hypotheses of these tests are listed under Section 2.5. Table 2.7 gives a summary of the diagnostic test results that were carried out on the VAR(1) model. All the p -values from the table appear to be less than 0.05 giving an implication of the acceptance of the alternative hypotheses for all the tests. This means that the residuals of the fitted VAR(1) model are serially correlated, contain ARCH errors and are not normally distributed. In this regard, the VAR(1) did not pass any of the diagnostic tests and therefore, can not be used to forecast future values of the stock returns.

Table 2.7: Summary Diagnostic tests of the fitted VAR(1) model

Test	Statistic	DF	<i>p</i> -value
Portmanteau	156.71	125	0.03
ARCH	2767.16	1350	< 0.001
JB	930.59	10	< 0.001
Kurtosis	837.63	5	< 0.001
Skewness	56.95	5	< 0.001

2.9 Summary

This chapter provided a discussion on the theory of a Vector autoregressive model and a summary of the model's application on the BRICS stock returns. From the application of this model we were able to determine the correlation, concurrent relationships and linear dependencies among the BRICS stock returns. The strongest correlation was observed between the JSE (South Africa) and RTSI (Russia) followed by RTSI and BSE (India) stock markets. Following the cross-correlation analysis, SSE (China) appeared to have an influence on all the stock returns with an exception of RTSI (Russia). From the VAR model estimation, we were able to observe concurrent relationships and unidirectional linear dependencies among the BRICS stock returns where BSE was observed to have an influence on the Indian (BSE) and Chinese (SSE) stock markets. However, forecasting was not carried out simply because the selected VAR model that is, the VAR(1) did not pass any of the diagnostic tests. Thus, a volatility model such as the Generalized Autoregressive Conditionally heteroscedastic (GARCH) will be necessary as an alternative model that will account for the ARCH effects present in each of the BRICS returns as seen in Section 2.8. This will be the focus of the next chapter of the thesis.

Chapter 3

Univariate GARCH Modeling of BRICS stock returns

3.1 Introduction

This chapter addresses the issue of univariate volatility modeling for each of the BRICS markets using the GARCH(p,q) model. Model description, parameter estimation, diagnostic tests, forecasting and application of the univariate GARCH model are all discussed in this chapter.

3.2 GARCH(p,q) model description

The GARCH model by [Bollerslev \(1986\)](#) is an extension of the ARCH model that was introduced by [Engle \(1982\)](#). GARCH models are the simplest and basic forms of modeling volatility. The fundamental concept behind GARCH models is the conditional variance, that is, the variance conditional on the past. The conditional variance can be expressed as a linear function of the squared past time series innovations. Consider a log return series r_t given as

$$r_t = \mu_t + \epsilon_t, \tag{3.1}$$

where ϵ_t is the innovation or shock at time t and μ is the mean.

Definition 3 (A GARCH(p,q) process). Let η_t be an iid sequence with mean 0 and variance 1. Then, the GARCH(p,q) process is satisfied if

$$\epsilon_t = \sigma_t \eta_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad (3.2)$$

where α_i and β_j are nonnegative constants, α_0 is a strictly positive constant and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$. The constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of ϵ_t is finite where as its conditional variance σ_t^2 evolves over time. Let $\zeta_t = \epsilon_t^2 - \sigma_t^2$ therefore, $\sigma_t^2 = \epsilon_t^2 - \zeta_t$. Substituting $\sigma_{t-i}^2 = \epsilon_{t-i}^2 - \zeta_{t-i}$ ($i = 0, \dots, q$) into equation (3.2), we can then rewrite the GARCH (p,q) model as

$$\epsilon_t^2 = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) \epsilon_{t-i}^2 + \zeta_t - \sum_{j=1}^q \beta_j \zeta_{t-j}. \quad (3.3)$$

Equation (3.3) is an ARMA representation of the GARCH(p,q) model for the squared innovations ϵ_t^2 . ζ_t can easily be shown to be a martingale difference sequence satisfying $E[\zeta_t] = 0$ and $cov(\zeta_t, \zeta_{t-j}) = 0$ for $j \geq 1$ but, ζ_t is generally not an iid sequence (Tsay, 2010).

Fitting a high order GARCH model is not easy. Only the lower order GARCH models such as GARCH(1,1), GARCH(1,2) and GARCH(2,1) are often used in application (Tsay, 2010). In this chapter a GARCH(1,1) model will be used in modeling each of the BRICS returns. A GARCH(1,1) model can be written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2, \quad (3.4)$$

where $\alpha_1, \beta_1 > 0$, $\alpha_0 = \gamma V_L$, where V_L is the average long run or unconditional variance and γ is the weight added to the long run variance. The condition $\alpha_1 + \beta_1 + \gamma = 1$ needs to be satisfied to obtain the average long run variance V_L given as

$$V_L = \frac{\alpha_0}{1 - \alpha_1 - \beta_1}. \quad (3.5)$$

Therefore, equation (3.4) can be rewritten as

$$\sigma_t^2 = \gamma V_L + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2. \quad (3.6)$$

3.3 GARCH (p,q) model estimation

The most common method used in the estimation of GARCH model parameters is the maximum likelihood estimation (MLE). In order to predict the volatility for a time series, one has to first fit the GARCH model to the time series of interest. This is possible only after model parameter estimation. In this section we present estimation through a likelihood approach which therefore requires distributional assumptions to be clearly understood.

3.3.1 Maximum-Likelihood Estimation (MLE)

Consider a data set of observations r_1, \dots, r_N assumed to be random from a distribution $F_r(r; \theta)$ that depends on the unknown parameters θ where in our case $\theta = [\alpha_0, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q]$ for a GARCH(p,q) model with parameter space Θ as earlier defined. R has a probability distribution function $p_r(R; \theta)$ which defines the probability that $R = r$ that is, $p(R = r)$. Suppose the probability function is known, then the unknown θ 's can be estimated using the likelihood function

$$L(\theta) = p_r(r_1; \theta) \times \dots \times p_r(r_N; \theta),$$

where $L(\theta)$ explains the probability that the values r_1, \dots, r_N are a realization of a random sample of size N from the distribution. By letting the unknown θ assume all the values in the parameter space Θ , the values of θ for which $L(\theta)$ is maximum can be estimated. These values are symbolized as θ^* . Thus the estimation of θ is chosen such that $L(\theta^*)$ is maximized for the observed r_1, \dots, r_N .

3.3.1.1 Stationarity

The concept of stationarity is crucial for statistical analysis of time series data with GARCH models. This suggests constraints on the estimated GARCH model parameters using the maximum-likelihood estimation. The two conditions that state the restrictions on the estimated parameters in the GARCH(p,q) model for stationarity are the strictly and weakly stationary conditions which are *Condition 1*: The GARCH(p,q) process with an independent and identically distributed random sequence $\epsilon_t = \sigma_t \eta_t$ such that $E[\eta_t^2] = 1$ and $E[\eta_t] = 0$, is strictly stationary with a finite variance if

the constraints in equation (3.2) are satisfied.

Condition 2: A GARCH(p,q) process is weakly stationary if and only if the constraints in equation (3.2) are satisfied with

- $E[\epsilon_t] = 0$,
- $Var(\epsilon_t) = \alpha_0 \left[1 - \left(\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j \right) \right]^{-1}$ and
- $cov(\epsilon_t, \epsilon_s) = 0$, for $t \neq s$.

3.3.1.2 Gaussian Quasi-Maximum-Likelihood Estimation (QMLE)

The likelihood estimation under the assumption of Gaussian or normal innovations is a commonly used method which is widely used for inference in time series models. However, establishing consistency and asymptotic normality of the quasi-maximum-likelihood estimator is not easy. Thus, understanding the probabilistic structure generated by the model is crucial (Straumann and Mikosch, 2006). Suppose the innovation or shock η_t , in the GARCH(p,q) model assumes a conditional normal distribution. Then, ϵ_t is a Gaussian $N(0, \sigma_t^2)$ given past values $\epsilon_{t-1}, \epsilon_{t-2}, \dots$ and conditioning argument yields the density function given as $p_{\epsilon_p, \dots, \epsilon_N}$ of $\epsilon_p, \dots, \epsilon_N$ through the conditional Gaussian densities of the ϵ_t 's given $r_1, \dots, r_N = r_N$:

$$p_{\epsilon_p, \dots, \epsilon_N}(r_p, \dots, r_N) = \frac{1}{(\sqrt{2\pi})^{n-(p+1)}} \prod_{t=p+1}^N \frac{e^{-\frac{r_t^2}{2\sigma_t^2}}}{\sigma_t} p_{\epsilon_p}(r_p).$$

Conditioning on $\epsilon_p = r_p$ and replacing $t = p+1$ with $t = 1$, the Gaussian log-likelihood of $\epsilon_1, \dots, \epsilon_N$ can be expressed as

$$l_N(\alpha_0, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q) = \frac{1}{2} \sum_{t=1}^N \log \sigma_t^2 + \frac{\epsilon_t^2}{\sigma_t^2} + \log 2\pi.$$

The likelihood function is then maximized as a function of α_i 's and β_j 's. Thus, the resulting value in the parameter space is the normal quasi maximum likelihood estimator for the GARCH(p,q) parameters.

3.3.1.3 Heavy-tailed MLE

Time series data do not always necessarily assume a normal distribution, therefore, it is important to use a distribution that reflects on the features of a time series such as skewness, long, short or fat tails and estimate the model parameters using this distribution in the likelihood function. This approach helps in dealing with non-Gaussian errors. There are many heavy tailed distributions such as the Student-t, log-normal, cauchy and the Generalized Error Distribution(GED) to mention a few. However, in this section we only consider the Student-t distribution. Consider a returns data set with observations r_1, \dots, r_N , that are assumed to follow the Student-t distribution has the following density function

$$f_R(r; v) = \frac{\Gamma[\frac{v+1}{2}]}{\sqrt{v\pi}\Gamma[\frac{v}{2}](1 + \frac{r^2}{v})^{\frac{v+1}{2}}},$$

where v denotes the degrees of freedom. Like the normal distribution, the Student-t distribution is symmetric. The mean, variance and kurtosis of the distribution are

1. $\mu = 0$ for $v \geq 2$
2. $\sigma^2 = \frac{v}{v-2}$ for $v \geq 3$
3. $kurtosis = \frac{6}{v-4}$ for $v \geq 5$.

Now, for a real-valued discrete time stochastic process ϵ_t , the log-likelihood function for a Student-t distribution is given as

$$l_N(\alpha_0, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q) = \sum_{t=1}^N \left\{ \log \Gamma\left(\frac{v+1}{2}\right) - \log \Gamma\left(\frac{v}{2}\right) - \frac{1}{2} \log(\pi(v-2)) \right. \\ \left. - \frac{1}{2} \log \sigma_t^2 - \frac{v+1}{2} \log \left[1 + \frac{\epsilon_t^2}{\sigma_t^2(v-2)} \right] \right\}.$$

The log-likelihood function is then maximized with respect to the unknown parameters of the GARCH(p,q) model and the MLE parameter estimates are obtained through iterative optimization methods.

3.4 GARCH(p,q) model diagnostics

The goodness of fit of a GARCH model is assessed by checking the significance of the parameter estimates and measuring how good it models the volatility process. There are three commonly used statistical tests that can be carried out on a fitted GARCH(p,q) model and these are

- Ljung-Box test
- ARCH-LM test
- Normality test

3.4.1 Ljung-Box test

Volatility clustering in returns is evident as autocorrelation in squared and absolute returns. The importance of these autocorrelations can be tested using the Ljung-Box Q -statistic by [Ljung and Box \(1978\)](#) given as

$$Q(p) = N(N+2) \sum_{k=1}^p \frac{\hat{\rho}_k^2}{N-k},$$

where $\hat{\rho}_k$ denotes the k -lag sample autocorrelation of the squared or absolute return and N is the sample size. The Ljung-Box tests the null hypothesis that the data are independently distributed against the alternative that they are not. If the data are white noise then the $Q(p)$ statistic will have an asymptotic chi-square distribution with p degrees of freedom. An insignificant Q statistic value with a p -value greater than 0.05 provides evidence of absence of any significant autocorrelations in the fitted residuals and vice versa.

3.4.2 ARCH-LM test

The ARCH-LM test in this section is similar to that discussed in Chapter 2 except that, it's application here is under a univariate approach. Consider an ARCH(p) regression given as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_p \epsilon_{t-p}^2. \quad (3.7)$$

Since $E(\epsilon_t) = 0$ and $E_{t-1}(\epsilon_t^2) = \sigma_t^2$, equation (3.7) can be rewritten as

$$\epsilon_t^2 = \alpha_0 + \alpha_1 \epsilon_t^2 + \dots + \alpha_p \epsilon_{t-p}^2 + u_t, \quad (3.8)$$

where $u_t = \epsilon_t^2 - E_{t-1}(\epsilon_t^2)$ is a zero mean white noise process. Equation (3.8) represents an Autoregressive (AR) process for ϵ_t^2 . Bollerslev (1986) indicated that, since an ARCH model denotes an (AR) model for the squared innovations ϵ_t^2 then, a simple Lagrange multiplier (LM) test for ARCH errors can be used based on the auxiliary regression (3.8). $\alpha_1 = \alpha_2 = \dots = \alpha_p = 0$ is the null hypothesis under which there are no ARCH effects and the alternative hypothesis is the presence of ARCH effects. The ARCH-LM test statistic given as

$$LM = N.R^2,$$

where N is the sample size and R^2 is the squared multiple correlation coefficient calculated from the auxiliary regression equation (3.8) using estimated innovations. The test statistic has an asymptotic chi-square distribution with p degrees of freedom.

3.4.3 Normality test

A GARCH model may assume several kinds of distributions depending on the nature of the time series data under consideration. The most common distribution is the normal distribution however, most financial time series data are not always consistent with this kind of distribution (Danielsson, 2011) therefore, it is crucial to determine an appropriate distribution for the data. A Jarque-Berra (JB) test is often used in the test for normality of fitted residuals. The JB test is used to test the following hypothesis

H_0 : the innovations (ϵ_t) are normally distributed versus

H_a : the innovations are not normally distributed.

The test statistic is given as;

$$S_N = N \left(\frac{\hat{\tau}^2}{6} + \frac{(\hat{k} - 3)^2}{24} \right),$$

where

$$\hat{\tau} = \frac{\frac{1}{N} \sum_{t=1}^N (\epsilon_t - \bar{\epsilon})^3}{\left(\frac{1}{N} \sum_{t=1}^N (\epsilon_t - \bar{\epsilon})^2 \right)^{\frac{3}{2}}}, \quad \hat{k} = \frac{\frac{1}{N} \sum_{t=1}^N (\epsilon_t - \bar{\epsilon})^4}{\left(\frac{1}{N} \sum_{t=1}^N (\epsilon_t - \bar{\epsilon})^2 \right)^2}, \quad \bar{\epsilon} = \frac{1}{N} \sum_{t=1}^N \epsilon_t,$$

N is the sample size, $\hat{\tau}$ and \hat{k} are the estimators of skewness τ and kurtosis k respectively. Skewness is the measure of the asymmetry of a probability distribution whereas kurtosis is the measure of the degree of peakedness of a distribution relative to the tails. However, there are a number of weaknesses that have been noted with JB test in literature. [Chen and Kuan \(2003\)](#) argue that, the JB test is not the most preferable test for normality because the test works best only if, (i) the conditional mean function has an intercept term but does not depend on past innovations and if (ii) the innovations have a conditionally constant variance. Moreover, [Thadewald and Büning \(2007\)](#) add that, the JB test is poor for short tailed distributions and that the test is biased at times. In this regard we opt for a graphical approach such as the QQ-plots (quantile quantile plots) that analyze the tails of a distribution. QQ plots assess how well a theoretical distribution fits a set of observations. If the quantiles of a theoretical and sample data distribution coincide then the QQ plot takes the shape of the line $y = x$ ([Karlsson, 2002](#)). Large deviations of the fitted points away from the prediction line imply deviations from a particular distribution. Deviations from normality could imply other distributions such as the Student-t, GED, a Skewed distribution and so on depending on the shape the QQ-plot takes. To aid in the selection of an appropriate distribution that best describes the data's features when modeling with a GARCH model, information criteria such as the AIC can be used. An AIC can be expressed as;

$$AIC = -2\log(L) + 2k,$$

where k is the number of parameters in the model, L is the maximized value of the likelihood function for the estimated model (GARCH(p,q) in this case). The distribution with the lowest AIC value is often considered as the appropriate distribution for the data.

3.5 Forecasting using a GARCH(1,1) model

Predictions of a GARCH model can be obtained recursively. Assuming the prediction origin as h , a one-step ahead prediction of a GARCH(1,1) model is given as

$$\sigma_{h+1}^2 = \alpha_0 + \alpha_1 \epsilon_h^2 + \beta_1 \sigma_h^2,$$

where ϵ_h^2 and σ_h^2 are known at time index h . For multi-step ahead predictions, we use $\epsilon_t^2 = \sigma_t^2 \eta_t^2$ and rewrite the volatility equation from equation (3.2) as

$$\sigma_{t+1}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^2 + \alpha_1\sigma_t^2(\eta_t^2 - 1).$$

For $t = h + 1$, the equation now becomes

$$\sigma_{h+2}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_{h+1}^2 + \alpha_1\sigma_{h+1}^2(\eta_{h+1}^2 - 1).$$

Since $E[\eta_{h+1}^2 | F_h] = 1$ where, F_h is the information available at time h , the 2-step ahead prediction at the origin h becomes

$$\sigma_{h+2}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_{h+1}^2.$$

Generally for an m -step ahead forecast,

$$\sigma_{h+m}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_h^2(m-1), \quad m > 1. \quad (3.9)$$

This result is similar to the predictions of an ARMA(1,1) model with an Autoregressive polynomial $1 - (\alpha_1 + \beta_1)B$, where B is a backward shift operator. With recursive substitutions in Eq(3.9), the m -step ahead prediction can be rewritten as

$$\sigma_{h+m}^2 = \frac{\alpha_0[1 - (\alpha_1 + \beta_1)^{m-1}]}{1 - \alpha_1 - \beta_1} + (\alpha_1 + \beta_1)^{m-1}\sigma_{h+1}^2.$$

Hence,

$$\sigma_{h+m}^2 \rightarrow \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \quad \text{as} \quad m \rightarrow \infty,$$

provided that $\alpha_1 + \beta_1 < 1$. Therefore, the multi-step ahead volatility predictions of a GARCH(1,1) model converge to the unconditional variance of η_t as the prediction region escalates to infinity for as long as the $Var(\epsilon_t)$ exists.

3.6 Application

3.6.1 Introduction

This section gives a summary of the univariate GARCH model application to each of the weekly BRICS stock returns. The data used in this section consist of weekly returns of the BRICS markets

from Jan 2000 to Dec 2012. The BRICS stock returns used are IBOVESPA (Brazilian stock market), RTSI (Russian stock market), BSE (Indian stock market), SSE composite (Chinese stock market) and JSE all share (South African stock market). The first section presents a summary of the exploratory data analysis, the second section, provides a summary of the GARCH(1,1) parameter estimation for each stock return, the third section gives a summary of diagnostic tests carried out on the fitted GARCH(1,1) model, the fourth section presents a summary of the predicted returns and the last section gives a summary conclusion of the GARCH(1,1) model application.

3.6.2 Exploratory data analysis

An exploratory analysis was conducted on the BRICS stock returns whereby the Autocorrelation function (ACF) of each stock return and their squared returns were plotted against several lags as shown on Figure 3.1 and Figure 3.2 respectively. From Figure 3.1 all the stock returns appear to be serially uncorrelated with an exception of a few minor lags which indicate weak serial correlation in the return series. However, it is clear from Figure 3.2 that the squared returns are serially correlated indicating the presence of some ARCH errors in the BRICS return series. To confirm these results, a Ljung-Box and an ARCH test were carried out to check for serial correlation and the presence of ARCH errors respectively in the BRICS returns. The results of these tests are given on Table 3.1. From Table 3.1 all the stock returns appear to be serially uncorrelated but with the presence of ARCH errors in all the stock returns. Under the Ljung-Box test results all the p -values appear to be greater than or equal to 0.05 implying the absence of serial correlation in the stock returns where as the p -values under the ARCH test results appear to be less than 0.05 indicating the presence of some ARCH errors in all the BRICS returns. In this regard, a model that accounts for volatility such as the GARCH model needs to be employed in the modeling of the BRICS returns.

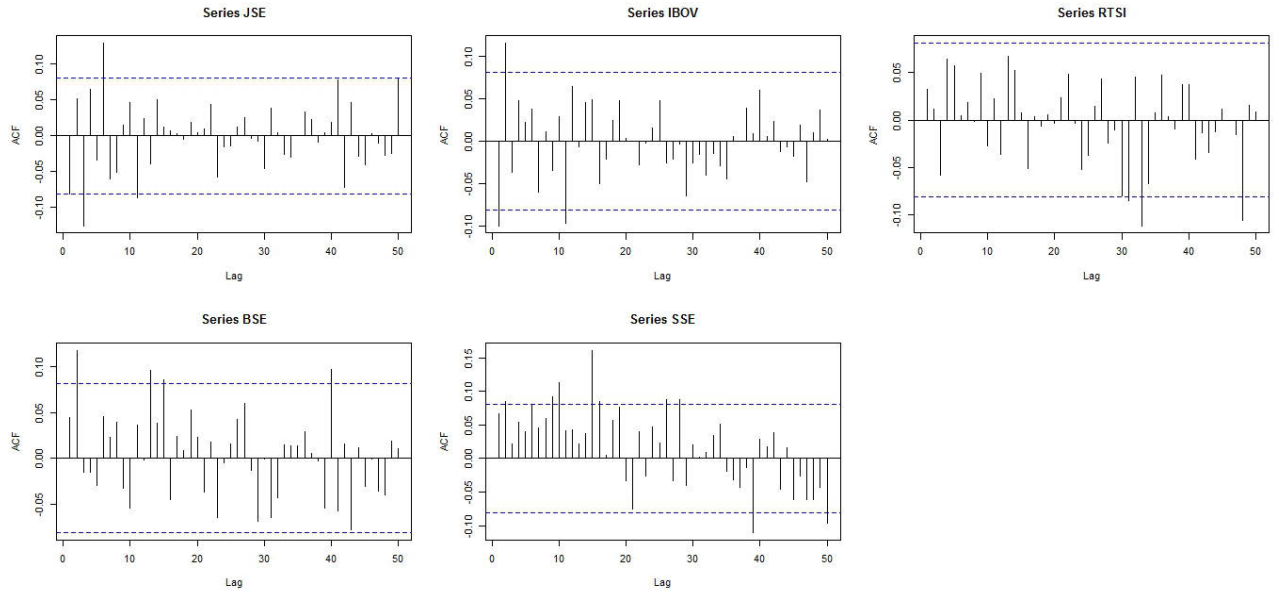


Figure 3.1: ACF plots of the BRICS returns where, JSE represents the South African stock market, IBOV represents the Brazilian stock market, RTSI represents the Russian stock market BSE represents the Indian stock market and SSE represents Chinese stock market

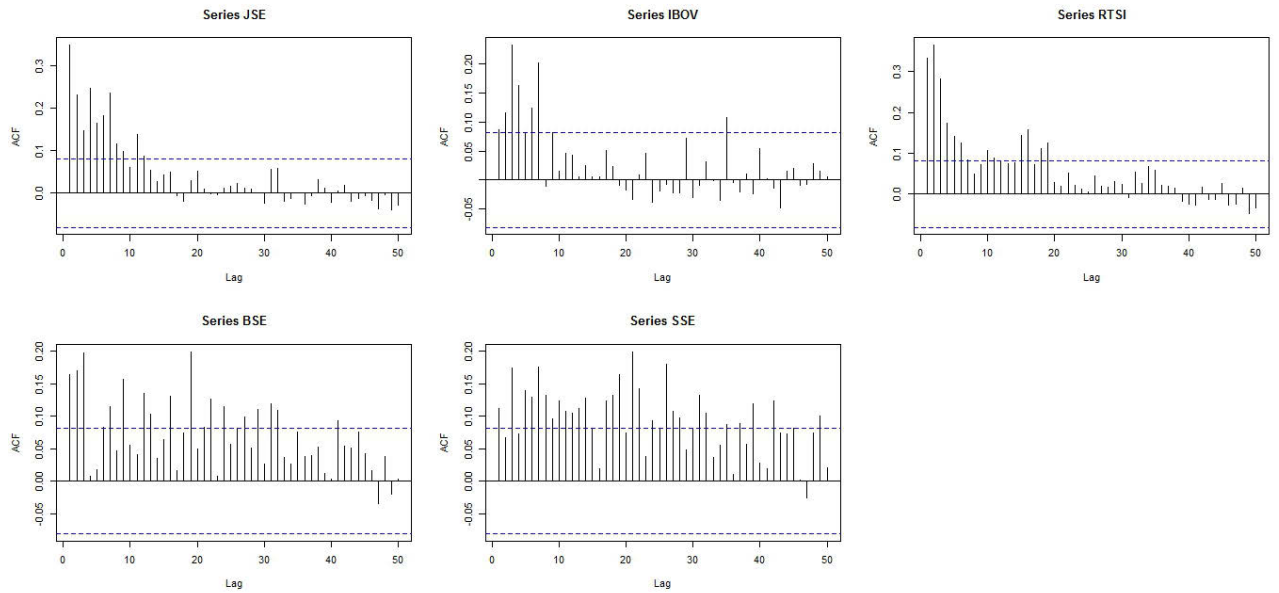


Figure 3.2: ACF plots of the BRICS squared returns where JSE represents the South African stock market, IBOV represents the Brazilian stock market, RTSI represents the Russian stock market, BSE represents the Indian stock market and SSE represents the Chinese stock market

Table 3.1: Ljung-Box and ARCH-LM tests for the BRICS stock returns diagnostics

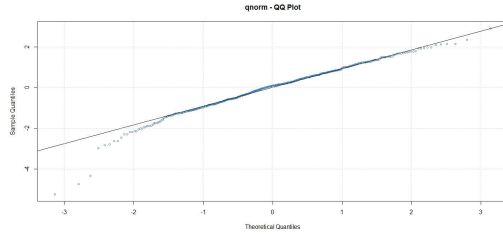
Stock return	Ljung- Box	p -value	ARCH- LM	p -value
IBOV	34.81	0.05	70.14	< 0.001
RTSI	0.60	0.42	121.17	< 0.001
BSE	1.15	0.28	68.56	< 0.001
SSE	2.61	0.12	54.97	< 0.001
JSE	5.50	0.06	119.75	< 0.001

3.6.3 GARCH(1,1)

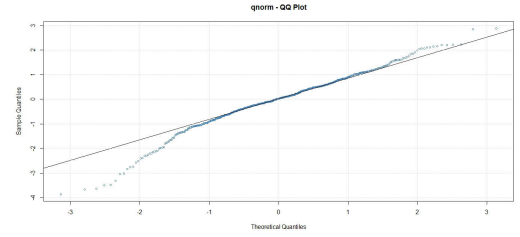
This section provides a discussion on univariate volatility modeling of the BRICS returns using a GARCH(1,1) model. As observed on the previous section, all the stock returns showed some presence of ARCH errors thus, a GARCH(1,1) model was fitted to each of the BRICS returns in order to capture volatility.

A GARCH(1,1) model may assume several conditional distributions depending on the nature of the time series data. However, the most common conditional distribution is the Gaussian normal distribution therefore, each of the BRICS returns was firstly fitted under this assumption and a Q-Q plot for each of the returns was plotted as shown on Figure 3.3. For a standard normal Gaussian distribution a theoretical Q-Q plot takes the shape of the line $y = x$ thus, an empirical data is expected to have its points lying around normal prediction line. Further deviations of the points away from this line indicate deviations from normality. From Figure 3.3, most of the points lie on the normal line however, towards the lower ends of the plots, the points of each return series appear to move relatively to the normal line indicating the presence of heavy left tails. In practice, the right and left tail distributions of returns are in most cases different it is therefore, good practice to allow the conditional distribution to be a skewed one. In this regard, two conditional distributions which were the student-t distribution (std) and the skew student-t distribution (sstd) were fitted on the GARCH(1,1) model and based on the AIC values the distribution with the lowest AIC value was

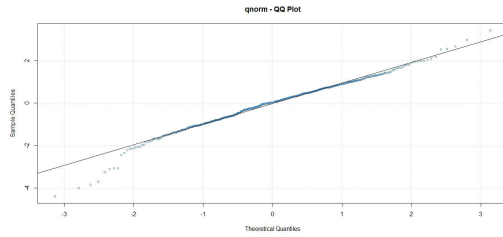
then used to fit the data. A summary of the AIC values is given on Table 3.2.



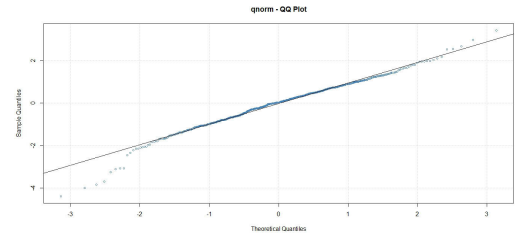
(a) IBOV



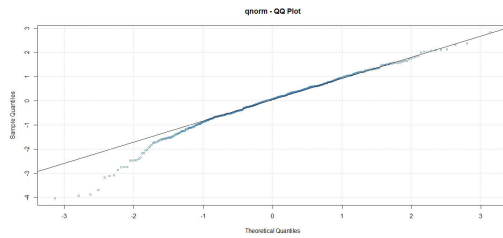
(b) RTSI



(c) BSE



(d) SSE



(e) JSE

Figure 3.3: Q-Q plots for (a) Brazil (b) Russia (c) India (d) China and (e) South African stock returns under the Gaussian normal distribution assumption

Table 3.2: AIC values of the GARCH(1,1) model under the student-t (std) and skew student-t (sstd) conditional distributions for each of the BRICS returns

Returns	AIC	
	std	sstd
IBOV	-3.69	-3.70
RTSI	-3.29	-3.30
BSE	-3.99	-4.00
SSE	-4.01	-4.02
JSE	-4.47	-4.49

From Table 3.2 the distribution that appears to have the lowest AIC values with all the stock returns is the skew student-t distribution (sstd) therefore, each stock return was then fitted with the GARCH(1,1) model under the sstd.

3.6.3.1 Parameter estimation

Based on the AIC results from Table 3.2, the estimates of the GARCH(1,1) model under the sstd assumption for each stock return are presented on Table 3.3. The parameter μ symbolizes the mean part of the model where as α_0 , α_1 and β_1 symbolize the volatility part.

Table 3.3: A summary table of the GARCH(1,1) model parameter estimates for each of the BRICS stock returns

Returns	Parameter	Estimate	SE	p-value	Longrun- variance (V_L)
IBOV	μ	0.000113	0.00155	0.94	0.000747
	α_0	0.0000714	0.0000367	0.05	
	α_1	0.00238	0.023	0.30	
	β_1	0.902	0.035	< 0.001	
RTSI	μ	0.00375	0.00186	0.04	0.00272
	α_0	0.000128	0.0000617	0.04	
	α_1	0.137	0.0429	0.001	
	β_1	0.816	0.0515	< 0.001	
BSE	μ	0.00122	0.00131	0.35	0.00112
	α_0	0.0000693	0.0000365	0.06	
	α_1	0.111	0.0404	0.01	
	β_1	0.828	0.06	< 0.001	
SSE	μ	-0.000411	0.00133	0.76	0.00114
	α_0	0.0000211	0.0000141	0.14	
	α_1	0.0603	0.0207	0.004	
	β_1	0.922	0.0266	< 0.001	
JSE	μ	0.001	0.00101	0.32	0.000456
	α_0	0.0000547	0.0000245	0.03	
	α_1	0.062	0.0654	0.34	
	β_1	0.818	0.0518	< 0.001	

From the results on Table 3.3, the GARCH(1,1) model equation for each stock return can now be written as follows;

$$\begin{aligned}
 r_t(ibov) &= 1.13e^{-4}(\pm 1.55e^{-3}) + a_t, \\
 \sigma_t^2 &= 7.14e^{-5}(\pm 3.67e^{-5}) + 2.38e^{-2}(\pm 2.30e^{-2})a_{t-1}^2 + 9.02e^{-1}(\pm 3.50e^{-2})\sigma_{t-1}^2
 \end{aligned}
 \tag{3.10}$$

$$\begin{aligned}
 r_t(rtsi) &= 3.75e^{-3}(\pm 1.86e^{-3}) + a_t, \\
 \sigma_t^2 &= 1.28e^{-4}(\pm 6.17e^{-5}) + 1.37e^{-1}(\pm 4.29e^{-2})a_{t-1}^2 + 8.16e^{-1}(\pm 5.15e^{-2})\sigma_{t-1}^2
 \end{aligned}
 \tag{3.11}$$

$$r_t(bse) = 1.22e^{-3}(\pm 1.31e^{-3}) + a_t, \quad (3.12)$$

$$\sigma_t^2 = 6.93e^{-5}(\pm 3.65e^{-5}) + 1.11e^{-1}(\pm 4.04e^{-2})a_{t-1}^2 + 8.28e^{-1}(\pm 6.00e^{-2})\sigma_{t-1}^2$$

$$r_t(sse) = -4.11e^{-4}(\pm 1.33e^{-3}) + a_t, \quad (3.13)$$

$$\sigma_t^2 = 2.12e^{-5}(\pm 1.41e^{-5}) + 6.03e^{-2}(\pm 2.07e^{-2})a_{t-1}^2 + 9.22e^{-1}(\pm 2.66e^{-2})\sigma_{t-1}^2$$

$$r_t(jse) = 1.00e^{-3}(\pm 1.01e^{-3}) + a_t, \quad (3.14)$$

$$\sigma_t^2 = 5.47e^{-5}(\pm 2.45e^{-5}) + 6.02e^{-2}(\pm 6.45e^{-2})a_{t-1}^2 + 8.18e^{-1}(\pm 5.18e^{-2})\sigma_{t-1}^2$$

r_t represents the return for each stock return whereas σ_t^2 represents the volatility part of the GARCH(1,1) model equation for each stock return. The sum of estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ is less than one for all of the BRICS return series implying that the unconditional volatility for each of the BRICS return series is finite. Furthermore, SSE is seen to have the highest volatility persistence with a value of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.982$, followed by RTSI with a volatility persistence of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.953$ whereas, JSE appears to have the lowest volatility persistence of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.88$.

Figure 3.4 presents plots of the BRICS estimated volatility. Based on the volatility scale on the plots RTSI (Russia) appears to have the highest volatility followed by BSE (India). It is clear from Figure 3.4 that, all of the BRICS returns portray a high volatility between the year 2008 to 2009. This can be attributed to the 2008 global recession.

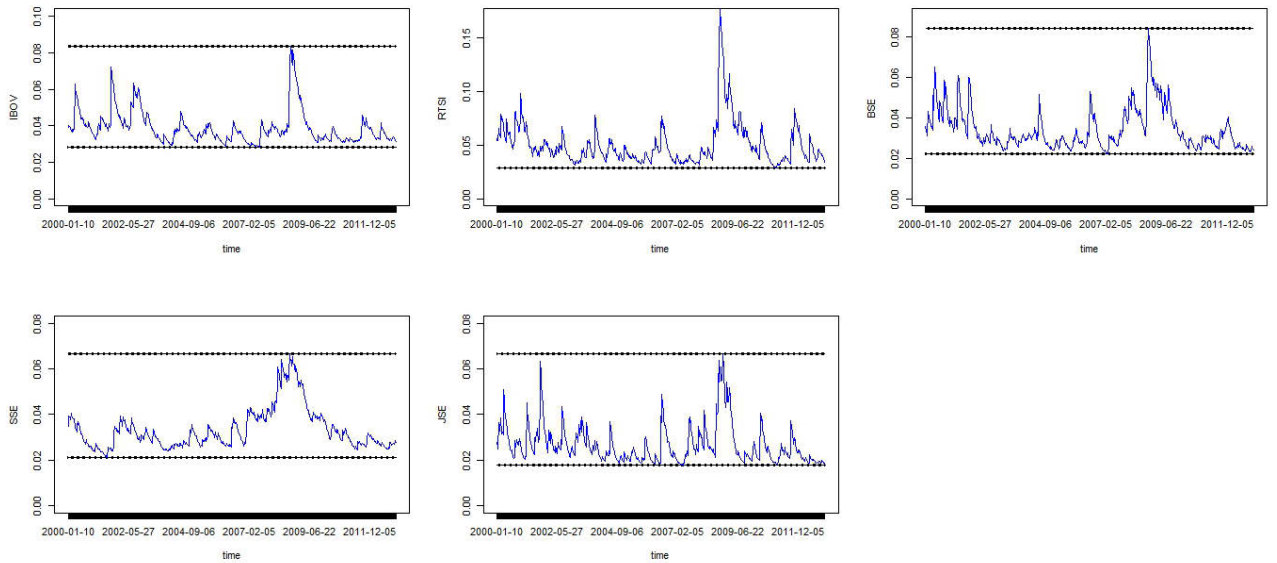


Figure 3.4: Plots of the BRICS estimated volatility where IBOV stands for the Brazilian stock market, RTSI stands for the Russian stock market, BSE stands for the Indian stock market, SSE stands for the Chinese stock market and JSE stands for the South African stock market.

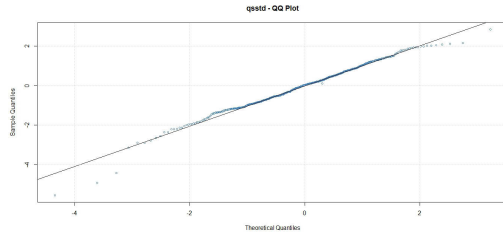
3.6.3.2 Diagnostic tests

For any model to be rendered adequate it must undergo some diagnostic assessments after being fitted. Several diagnostic tests were carried out on the fitted GARCH(1,1) model which include the Jarque-Berra (JB) test, Ljung-Box test and the ARCH-LM test. The JB test checks for the normality of fitted residuals, Ljung-Box test checks for serial correlation of the fitted residuals and the ARCH-LM test checks for the presence of ARCH errors in the fitted residuals. The Ljung-Box test was carried out on both residuals of returns and squared returns for each of the BRICS stock market. A summary of the diagnostic test results is provided on Table 3.4

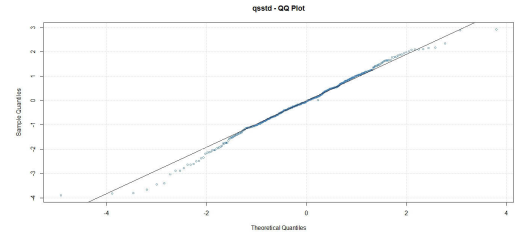
Table 3.4: Summary results of the GARCH(1,1) model diagnostic tests performed on each of the BRICS returns

Returns	Diagnostic test	Statistic	p -value
IBOV	JB	210.12	0.00
	Ljung-Box(R)	12.23	0.27
	Ljung-Box(R^2)	6.30	0.79
	ARCH-LM	6.34	0.90
RTSI	JB	93.70	0.00
	Ljung-Box(R)	9.99	0.44
	Ljung-Box(R^2)	8.42	0.59
	ARCH-LM	10.98	0.53
BSE	JB	94.31	0.00
	Ljung-Box(R)	10.34	0.41
	Ljung-Box(R^2)	7.96	0.63
	ARCH-LM	10.81	0.55
SSE	JB	44.14	0.00
	Ljung-Box(R)	27.26	0.20
	Ljung-Box(R^2)	4.42	0.93
	ARCH-LM	5.31	0.95
JSE	JB	103.62	0.00
	Ljung-Box(R)	8.32	0.60
	Ljung-Box(R^2)	3.90	0.95
	ARCH-LM	3.74	0.99

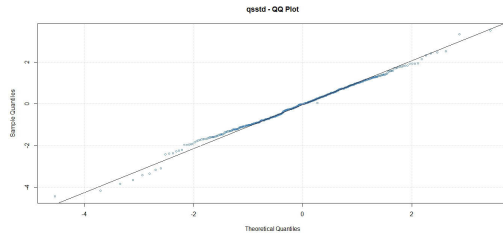
It is clear from Table 3.4 that none of the BRICS returns posses any ARCH errors nor serial correlation in the fitted residuals since all the test statistic values of the ARCH-LM and the Ljung-Box tests are significant at 5% level of significance. The JB test on the other hand rejects the property of normality on the fitted residuals of all the BRICS returns. Many studies however, have shown that the application of the JB test on conditional heteroscedastic models is weak. Therefore, a graphical assessment on normality was carried out on each of the fitted residuals of the BRICS stock return series using a normal Q-Q plots as shown on Figure 3.5.



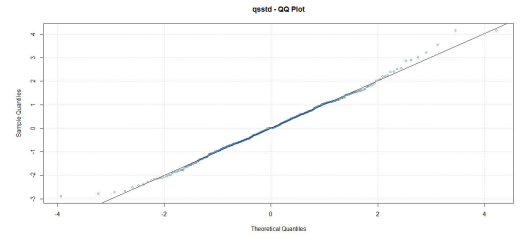
(a) IBOV



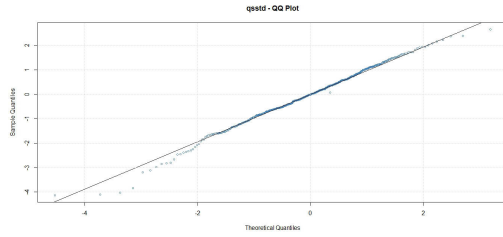
(b) RTSI



(c) BSE



(d) SSE



(e) JSE

Figure 3.5: Q-Q plots of the GARCH(1,1) fitted residuals for (a) Brazil (b) Russia (c) India (d) China and (e) South Africa stock returns under the Skewed Student-t distribution assumption

The Q-Q plots on Figure 3.5 appear to support normally distributed residuals with a few outliers since most of the points lie on the normal line or very close to it. Thus, the GARCH(1,1) model under the skew student-t distribution appears to be an adequate model for each of the BRICS returns.

3.6.3.3 Forecasting

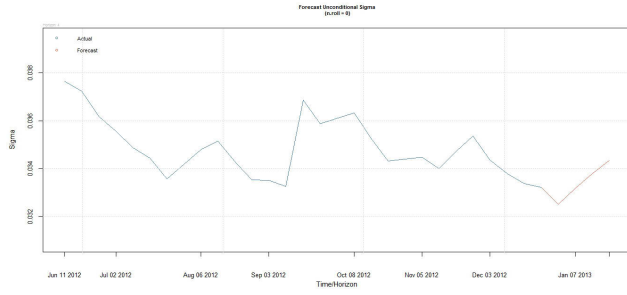
Having passed the diagnostic tests, the GARCH(1,1) model appeared to be sufficient enough to be used for forecasting the volatility of each of the BRICS returns. The volatility of each return series was forecasted 4 weeks ahead that is to say, one month ahead. Table 3.5 presents a summary of

the mean and volatility forecasts for each of the BRICS return series where CI on Table 3.5 stands for confidence intervals.

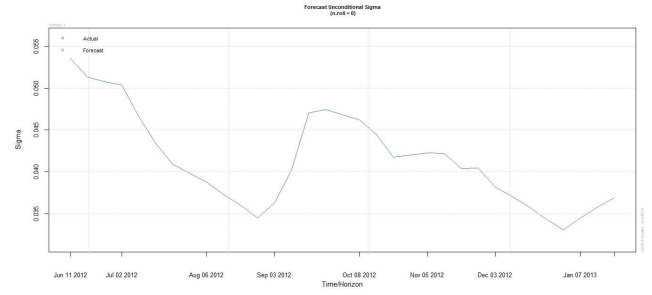
Table 3.5: A summary representation of the Mean and volatility forecasts for the BRICS weekly return series

Return	Time (weeks)	Mean forecast	Mean error	Volatility forecast	95%Lower CI	95% Upper CI
IBOV	1	0.00011	0.0309	0.03090	-0.06700	0.05539
	2	0.00011	0.0309	0.03091	-0.06700	0.05541
	3	0.00011	0.0309	0.03093	-0.06703	0.05543
	4	0.00011	0.0309	0.03093	-0.06705	0.05545
RTSI	1	0.00374	0.0329	0.03293	-0.06712	0.06382
	2	0.00374	0.0341	0.03407	-0.06958	0.06590
	3	0.00374	0.0351	0.03513	-0.07186	0.06783
	4	0.00374	0.0361	0.03610	-0.07396	0.06961
BSE	1	0.00122	0.0235	0.02347	-0.04957	0.04326
	2	0.00122	0.0242	0.02421	-0.05118	0.04497
	3	0.00122	0.0249	0.02489	-0.05265	0.04620
	4	0.00122	0.0255	0.02551	-0.05399	0.04732
SSE	1	-0.00041	0.0285	0.02847	-0.05598	0.05775
	2	-0.00041	0.0286	0.02858	-0.05620	0.05798
	3	-0.00041	0.0287	0.02869	-0.05642	0.05820
	4	-0.00041	0.0288	0.02880	-0.05663	0.05842
JSE	1	0.0010	0.0182	0.01824	-0.03298	0.03298
	2	0.0010	0.0186	0.01865	-0.04004	0.03368
	3	0.0010	0.0190	0.01899	-0.04080	0.03429
	4	0.0010	0.0193	0.01929	-0.04147	0.03482

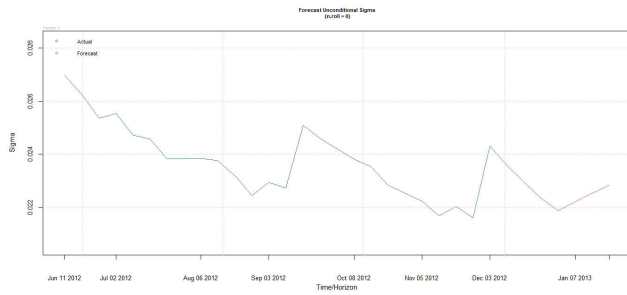
A plot of the predicted volatilities for each of the BRICS markets with a 95% confidence interval is presented on Figure 3.6. The future volatilities of IBOV, BSE, JSE and RTSI stock markets are observed to decline in the first few weeks of Jan but increase towards the end of the month. The volatility of RTSI on the other hand, increases in the first few weeks of Jan but follows a steady trend towards the end of the month.



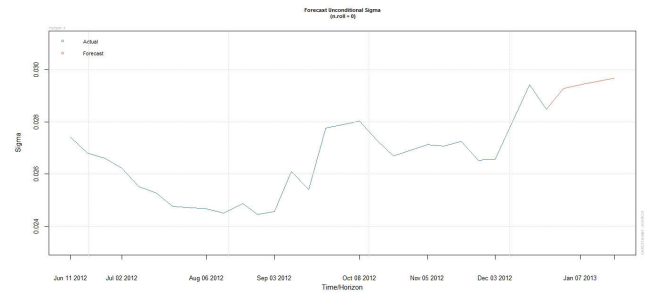
(a) IBOV



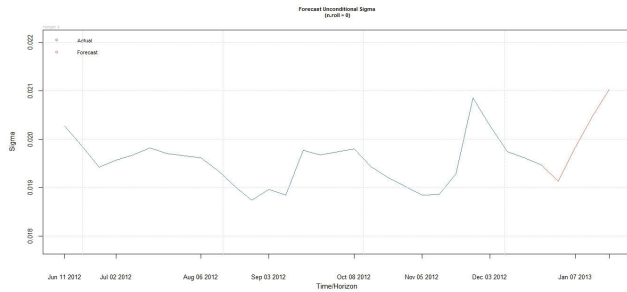
(b) RTSI



(c) BSE



(d) SSE



(e) JSE

Figure 3.6: GARCH(1,1) four weeks ahead volatility forecast plots for (a) Brazil (b) Russia (c) India (d) China and (e) South Africa stock returns

3.7 Summary

This chapter presented a discussion on the GARCH(p,q) model theory and a summary application using a GARCH(1,1) model on the BRICS weekly returns. Three distributions namely: the Gaussian normal, the student-t and the skew student-t distributions were fitted on the GARCH(1,1) model in

order to find an adequate GARCH model for each of the BRICS return series. Based on the Q-Q plots and the AIC values, the GARCH(1,1) model under the Skew Student-t distribution appeared to be the best model among the three distributions and this was justified through diagnostic tests and graphical assessments that were performed on the model.

From the estimation of the GARCH(1,1) model, volatility was observed to be persistent in all of the BRICS stock returns. Moreover, SSE appeared to have the highest volatility persistence followed by RTSI with JSE having the lowest volatility persistence. High volatility levels increase the risk of a stock market which in turn corresponds to a higher probability of a declining market. High levels of volatility have direct impacts on portfolios. This aspect raises the investors' concern on their portfolios worth as they watch them move aggressively and decline in value.

The issue of non-normality of stock returns has crucial impacts in finance especially in risk management fields. Assuming normality of returns in risk calculations may lead to underestimation of risk and poor investment choices (Greenspan, 1997). The use of non-normal methods is obscure but when used correctly leads to proper outcomes.

This chapter emphasized on the use of a univariate GARCH model in modeling the volatility of each of the BRICS markets. The next chapter focuses on multivariate volatility models and their applications on the BRICS returns.

Chapter 4

Multivariate GARCH Modeling of BRICS stock returns

4.1 Introduction

In this chapter we extend univariate volatility modeling to a multivariate approach and discuss some methods for modeling the dynamic relations in the volatility processes of the BRICS financial markets. The first section gives a general description of multivariate models, the second section discusses the exponentially weighted moving average (EWMA) model, the third section discusses the dynamic conditional correlation (DCC) model, the fourth section presents a summary of the models' application on the BRICS stock markets and the last section gives a summary of the whole chapter.

4.2 Multivariate GARCH models

Modeling the co-movement of multiple return series is of great significance as most applications deal with portfolios where it is essential to understand the dynamic relations among asset returns in order to make proper decisions on their allocations. The most important part in multivariate volatility modeling is the covariance matrix that is, the conditional covariance matrix of multiple

asset returns. The covariance matrix is essential for applications such as portfolio selection, asset allocation, contagion and risk management. However, multivariate volatility models face the obstacle of dimensionality due to excessive number of parameters in the GARCH model leading to difficulties in parameter estimation. This in turn limits the number of asset returns to be included in the model for easy estimation. Thus, many multivariate volatility models encounter complexities in real application despite their appealing structure. In this chapter we shall illustrate two of the multivariate volatility models that are useful and manageable in empirical application.

Consider a multivariate return series \mathbf{r}_t of dimension $N \times 1$ such that, $\mathbf{r}_t = (r_{1t}, \dots, r_{Nt})'$. Given a vector of innovations $\boldsymbol{\epsilon}_t = (\epsilon_{1t}, \dots, \epsilon_{Nt})'$ \mathbf{r}_t can be written as

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t, \quad (4.1)$$

where $\boldsymbol{\mu}_t = E[\mathbf{r}_t | \mathbf{F}_{t-1}]$ is the conditional expectation of \mathbf{r}_t provided the past information \mathbf{F}_{t-1} is known and $\boldsymbol{\epsilon}_t = \mathbf{H}_t^{\frac{1}{2}} \boldsymbol{\xi}_t$ where $\boldsymbol{\xi}_t$ is an $N \times 1$ sequence of independent and identically distributed variables with mean zero and an identity covariance matrix \mathbf{I}_N . $\mathbf{H}_t^{\frac{1}{2}}$ is an $N \times N$ positive definite covariance matrix whereby

$$\mathbf{H}_t = Cov[\mathbf{r}_t | \mathbf{F}_{t-1}] = \begin{pmatrix} \sigma_{11,t} & \cdots & \sigma_{1N,t} \\ \vdots & \ddots & \vdots \\ \sigma_{N1,t} & \cdots & \sigma_{NN,t} \end{pmatrix},$$

the diagonal element $\sigma_{ii,t}$ is the variance of the i^{th} return and the $(i, j)^{th}$ element is the covariance between \mathbf{r}_{it} and \mathbf{r}_{jt} .

Definition 4 (Second-order stationarity). The process \mathbf{r}_t is said to be second-order stationary if

- i $E[r_{it}^2] < \infty, \forall t \in \mathbb{Z}, i = 1, \dots, N,$
- ii $E[\mathbf{r}_t] = \boldsymbol{\mu}, \forall t \in \mathbb{Z},$
- iii $Cov[\mathbf{r}_t, \mathbf{r}_{t+h}] = E[(\mathbf{r}_t - \boldsymbol{\mu})(\mathbf{r}_{t+h} - \boldsymbol{\mu})'] = \Gamma(h), \forall t, h \in \mathbb{Z}.$

The mean function is an N -dimensional vector of unconditional expectations of \mathbf{r}_t , the function $\Gamma(h)$ is a covariance matrix of dimension $N \times N$. At lag zero, $\Gamma_{\mathbf{r}}(0) = Var(\mathbf{r}_t)$ is a symmetric matrix.

4.2.1 Exponentially Weighted Moving Average (EWMA) model

The EWMA model was popularized by Morgan (1995) whereby more emphasis is placed on the most recent returns using a decaying factor λ . The underlying principle behind the concept is that recent price fluctuations are better predictors of future movements so they should be given greater consideration.

Definition 5. The sample covariance matrix of $(\mathbf{r}_t)_{t \in \mathbb{Z}}$ from equation (4.1) is given by

$$\Sigma = \frac{1}{T-1} \sum_{t=1}^T (\mathbf{r}_t - \boldsymbol{\mu})(\mathbf{r}_t - \boldsymbol{\mu})',$$

where $\boldsymbol{\mu}$ is an $N \times 1$ of sample mean vector. In this expression the same weight ($\frac{1}{T-1}$) is applied to each term in the summation. To allow for time varying covariance matrix and to emphasize on more weight added to the most recent innovations, the covariance matrix is estimated by

$$\hat{\Sigma} = \frac{1-\lambda}{1-\lambda^{t-1}} \sum_{i=1}^{t-1} \lambda^{i-1} \boldsymbol{\epsilon}_{t-i} \boldsymbol{\epsilon}_{t-i}', \quad (4.2)$$

where $0 < \lambda < 1$ and the weights $\frac{(1-\lambda)\lambda^{i-1}}{1-\lambda^{t-1}}$ sum to one. When t is very large such that $\lambda^{t-1} \approx 0$, then equation (4.2) can be rewritten as

$$\hat{\Sigma} = (1-\lambda) \boldsymbol{\epsilon}_{t-1} \boldsymbol{\epsilon}_{t-1}' + \lambda \hat{\Sigma}_{t-1} \quad (4.3)$$

with individual elements given by

$$\hat{\sigma}_{t,ij} = \lambda \hat{\sigma}_{t-1,ij} + (1-\lambda) \epsilon_{t-1,i} \epsilon_{t-1,j}, \quad i, j = 1, \dots, N. \quad (4.4)$$

Equation (4.3) is referred to as the exponentially weighted moving average (EWMA) model. Given λ and an initial estimate Σ_1 , the time varying exponential weighted covariance matrices can be computed from equation (4.3). Correlations from the EWMA model can be obtained by

$$\rho_{t,ij} = \frac{\lambda \sigma_{t-1,ij} + (1-\lambda) r_{t-1,i} r_{t-1,j}}{\sqrt{[\lambda \sigma_{t-1,i}^2 + (1-\lambda) r_{t-1,i}^2][\lambda \sigma_{t-1,j}^2 + (1-\lambda) r_{t-1,j}^2]}}, \quad (4.5)$$

where $\sigma_{t,i}^2$ and $\sigma_{t,j}^2$ are the variances for the individual series $r_{t,i}$ and $r_{t,j}$ respectively. The EWMA model is widely used in industry especially for portfolio value at risk (VaR) and as an industry standard for market risk. The main advantage of the EWMA model is that it is tremendously easy

to estimate volatility because there are no parameters involved in estimation. However, the limitation of the EWMA model is its restrictiveness due to the simplicity of its structure and the assumption of non-estimated λ . Moreover, the fact that λ is identical to all assets and time periods is not realistic. However, one can estimate λ using a quasi-maximum likelihood approach by assuming ϵ_t in equation (4.3) follows a multivariate normal distribution with zero mean and $\Sigma_t = Cov_{t-1}(\epsilon_t)$ as the covariance of ϵ_t conditional on the past (Danielsson, 2011). The log likelihood function of the time series data can be written as

$$\log L = -\frac{NT}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T |\Sigma_t| - \frac{1}{2} \sum_{t=1}^T (\mathbf{r}_t - \boldsymbol{\mu})' \Sigma_t^{-1} (\mathbf{r}_t - \boldsymbol{\mu}).$$

Thus, given the initial value Σ_1 , the mean vector $\boldsymbol{\mu}$ and λ from can be treated as unknown and estimated using a quasi-maximum likelihood estimation method.

4.2.2 Dynamic Conditional Correlation (DCC) model

The model in this class was developed on the idea of modeling the conditional variances and correlations instead of directly modeling the conditional covariance matrix. The DCC model was suggested by Engle (2002) and Tse and Tsui (2002) as an extension of the constant conditional correlation (CCC) model by Bollerslev (1990). The DCC model was developed to attempt to overcome the shortcoming of the CCC model that is, the assumption of constant correlation which did not seem realistic in real application.

Definition 6. The DCC model by Tse and Tsui (2002) is described as

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{P}_t \mathbf{D}_t \quad (4.6)$$

where $\mathbf{D}_t = \text{diag}(h_{ii,t})^{\frac{1}{2}}$ is an $N \times N$ diagonal matrix, h_{ii} can be expressed as any univariate GARCH model and

$$\mathbf{P}_t = (1 - \theta_1 - \theta_2) \mathbf{P} + \theta_1 \mathbf{P}_{t-1} + \theta_2 \boldsymbol{\Psi}_{t-1}. \quad (4.7)$$

θ_1 and θ_2 from equation (4.7) are assumed to be positive scalars with $\theta_1 + \theta_2 < 1$, $\mathbf{P} = [\rho_{ij}]$ is a time-invariant $N \times N$ positive definite correlation matrix with unit diagonal elements and $\boldsymbol{\Psi}_{t-1}$ is an $N \times N$ correlation matrix of the standardized residuals $(\eta_{t-1}, \dots, \eta_{t-K})$ for a prespecified K .

The $(i, j)^{th}$ element of Ψ_{t-1} is given as

$$\Psi_{ij,t-1} = \frac{\sum_{h=1}^K \eta_{i,t-h} \eta_{j,t-h}}{\sqrt{(\sum_{h=1}^K \eta_{i,t-h}^2)(\sum_{h=1}^K \eta_{j,t-h}^2)}}, \quad 1 \leq i < j \leq N \quad (4.8)$$

where $\eta_{it} = \frac{\epsilon_{ij}}{\sqrt{h_{ii,t}}}$ for $i = 1, \dots, N$.

Definition 7. The DCC model by Engle (2002) is described as

$$\mathbf{D}_t \mathbf{P}_t \mathbf{D}_t, \quad (4.9)$$

where \mathbf{P}_t is the conditional correlation matrix of the standardized innovations $\boldsymbol{\eta}_t$ such that $\boldsymbol{\eta}_t = \mathbf{D}_t^{-\frac{1}{2}} \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \mathbf{P}_t)$. The conditional correlation matrix \mathbf{P}_t is described as

$$\mathbf{P}_t = (\text{diag}(\mathbf{S}_t))^{-\frac{1}{2}} \mathbf{S}_t (\text{diag}(\mathbf{S}_t))^{-\frac{1}{2}}, \quad (4.10)$$

where \mathbf{S}_t is an $N \times N$ symmetric positive definite correlation matrix defined by

$$\mathbf{S}_t = (1 - \theta_1 - \theta_2) \bar{\mathbf{S}} + \theta_1 \boldsymbol{\eta}_{t-1} \boldsymbol{\eta}_{t-1}' + \theta_2 \mathbf{S}_{t-1}. \quad (4.11)$$

As in equation (4.7) θ_1 and θ_2 are positive scalars satisfying the constraint $\theta_1 + \theta_2 < 1$. $\bar{\mathbf{S}}$ is an $N \times N$ unconditional covariance matrix estimated by

$$\bar{\mathbf{S}} = \frac{1}{T} \sum_{t=1}^T \boldsymbol{\eta}_t \boldsymbol{\eta}_t'.$$

Individual correlation elements of \mathbf{P}_t are given as $\rho_{ij,t} = \frac{s_{ij,t}}{\sqrt{s_{ii,t} s_{jj,t}}}$, where $s_{i,t}$ are elements of the matrix \mathbf{S}_t . Furthermore, \mathbf{S}_t has to be positive definite to ensure \mathbf{P}_t is positive definite.

The DCC model has an advantage of being estimated consistently using a two-step estimation technique. Engle and Sheppard (2001) show that the log-likelihood can be broken into two parts: one is for parameters accounting for univariate volatilities or univariate GARCH parameters and the other part is for parameters determining the correlations. This is an advantage because large covariance matrices can be estimated reliably with this approach without the need of too much computational power.

The limitation of the DCC model is that the parameters θ_1 and θ_2 are constants suggesting that the conditional correlations of all returns are determined by identical dynamics an assumption which is often idealistic. However, θ_1 and θ_2 are essential to guarantee \mathbf{P}_t is positive definite at all times through sufficient constraints on the parameters.

4.2.2.1 DCC Model parameter estimation

In this section we describe the two step estimation approach of the DCC model parameters using a quasi-maximum likelihood estimation. Under the assumption of multivariate Gaussian distribution of the innovations ϵ_t the likelihood function can then be expressed as

$$L(\boldsymbol{\theta}) = \prod_{t=1}^T \frac{1}{\sqrt{2\pi|\mathbf{H}_t|}} \exp\left(-\frac{1}{2}\boldsymbol{\epsilon}_t'\mathbf{H}_t^{-1}\boldsymbol{\epsilon}_t\right),$$

where T is the time period used to estimate the model, $\boldsymbol{\theta} = (\omega_1, \alpha_{11}, \dots, \alpha_{1p}, \beta_{11}, \dots, \beta_{1q}, \omega_2, \dots, \beta_{Nq}, \rho_{12}, \dots)$ is a vector of the DCC model parameters with p and q representing the order of the GARCH(p,q) model. The log-likelihood function is given as

$$\begin{aligned} \log(L(\boldsymbol{\theta})) &= -\frac{1}{2} \sum_{t=1}^T \left(N\log(2\pi) + \log(|\mathbf{H}_t|) + \boldsymbol{\epsilon}_t'\mathbf{H}_t^{-1}\boldsymbol{\epsilon}_t \right) \\ &= -\frac{1}{2} \sum_{t=1}^T \left(N\log(2\pi) + \log(|\mathbf{D}_t\mathbf{P}_t\mathbf{D}_t|) + \boldsymbol{\epsilon}_t'\mathbf{D}_t^{-1}\mathbf{P}_t^{-1}\mathbf{D}_t^{-1}\boldsymbol{\epsilon}_t \right) \\ &= -\frac{1}{2} \sum_{t=1}^T \left(N\log(2\pi) + 2\log|\mathbf{D}_t| + \log|\mathbf{P}_t| + \boldsymbol{\epsilon}_t'\mathbf{D}_t^{-1}\mathbf{P}_t^{-1}\mathbf{D}_t^{-1}\boldsymbol{\epsilon}_t \right) \quad (4.12) \end{aligned}$$

which can then be decomposed into two parts namely the volatility component which is the sum of the univariate GARCH likelihoods and the correlation component. The likelihood in the first step is the outcome of replacing \mathbf{P}_t with the identity matrix \mathbf{I}_N which results in the quasi-likelihood function while in the second step, the correlation parameters are estimated using the correctly specified log-likelihood function given the volatility parameters. The volatility component is given by

$$\begin{aligned} \log(L_v(\boldsymbol{\sigma})) &= -\frac{1}{2} \sum_{t=1}^T \left(N\log(2\pi) + 2\log|\mathbf{D}_t| + \log|\mathbf{I}_N| + \boldsymbol{\epsilon}_t'\mathbf{D}_t^{-1}\mathbf{I}_N\mathbf{D}_t^{-1}\boldsymbol{\epsilon}_t \right) \\ &= -\frac{1}{2} \sum_{t=1}^T \left(N\log(2\pi) + 2\log|\mathbf{D}_t| + \boldsymbol{\epsilon}_t'\mathbf{D}_t^{-1}\mathbf{I}_N\mathbf{D}_t^{-1}\boldsymbol{\epsilon}_t \right) \\ &= -\frac{1}{2} \sum_{t=1}^T \left(N\log(2\pi) + \sum_{i=1}^N \left[\log(h_{iit}) + \frac{\epsilon_{iit}^2}{h_{iit}} \right] \right) \\ &= \sum_{i=1}^N \left(-\frac{1}{2} \sum_{t=1}^T \left[\log(h_{iit}) + \frac{\epsilon_{iit}^2}{h_{iit}} \right] + c \right), \end{aligned}$$

where σ is a vector of parameters for the univariate GARCH component for each asset $i = 1 \dots, N$ and c is a constant. From this step the log likelihood is maximized with respect to σ to obtain the volatility estimates (σ^*). Following the first step, only the correlation parameters are unknown to the second step thus, the quasi-likelihood for the correlation component is expressed as

$$\begin{aligned} \log(L_c(\rho)) &= -\frac{1}{2} \sum_{t=1}^T \left(N \log(2\pi) + 2 \log|D_t| + \log|P_t| + \epsilon'_t D_t^{-1} P_t^{-1} D_t^{-1} \epsilon_t \right) \\ &= -\frac{1}{2} \sum_{t=1}^T \left(N \log(2\pi) + 2 \log|D_t| + \log|P_t| + \eta'_t P_t^{-1} \eta_t \right), \end{aligned}$$

where ρ denotes a vector of parameters for the correlation component. Since D_t is constant when conditioning on the parameters from the first step, we eliminate the constant terms and maximize

$$\log(L_c(\rho^*|\sigma^*)) = -\frac{1}{2} \sum_{t=1}^T \left(\log|P_t| + \eta'_t P_t^{-1} \eta_t \right).$$

In each step the constraints on θ_1 and θ_2 must be satisfied throughout the iterations otherwise P_t may become an explosive sequence. The estimates under this procedure are consistent but not fully efficient because $\hat{\sigma}^*$ and $\hat{\rho}^*$ are limited information estimators (Engle and Sheppard, 2001). Bauwens et al. (2006) suggest that, the use of one iteration of Newton-Raphson algorithm applied to equation (4.12) will yield an estimator that is asymptotically efficient.

4.2.2.2 DCC model diagnostics

To determine the adequacy of the DCC model in its estimation performance and statistical inference we use the Ljung-Box test for serial correlations and the Lagrange Multiplier (LM) test for constant correlations. In both tests, the standardized residuals are used. The Ljung-Box test in this case is a univariate one and is similar to the one described in Chapter 3. A univariate Ljung-Box test is preferred and applied on the standardized residuals of each fitted variable because its multivariate version for the squared standardized residuals is not sensitive to misspecification of the conditional correlation structure (Matteson and Ruppert, 2011).

Bauwens et al. (2006) argue that, Lagrange multiplier tests are usually more powerful compared to portmanteau tests even though they can be asymptotically equivalent under certain circumstances. Engle and Kroner (1995) and Bollerslev et al. (1988) among others have developed several LM tests

for multivariate GARCH models. However, we are going to use the LM test for constant correlations by Engle and Sheppard (2001). Let $\hat{\xi}_t = \hat{P}_t^{-\frac{1}{2}} \hat{D}_t^{-1} \hat{\epsilon}_t$ denote the $N \times 1$ vector of standardized residuals. Following Engle and Sheppard (2001) the LM test is based on the following regression

$$vech^s(\hat{\xi}_t \hat{\xi}_t' - I_N) = \beta_0^* + \beta_1^* vech^s(\hat{\xi}_{t-1} \hat{\xi}_{t-1}' - I_N) + \dots + \beta_p^* vech^s(\hat{\xi}_{t-p} \hat{\xi}_{t-p}' - I_N) + \eta_t^*, \quad (4.13)$$

where $vech^s$ is an operator that stacks a matrix as a column but it only chooses the elements above the diagonal. The null hypothesis under this test is $H_0 : P_t = \bar{P}$ tested against the alternative $H_1 : vech(P) = vech(\bar{P}) + \beta_1 * vech(P_{t-1}) + \dots + \beta_p vech(P_{t-p})$ where \bar{P} implies constant correlation. Under the null hypothesis, the coefficients of equation (4.13) are equivalent to zero. The test can be carried out as $\frac{\hat{\delta} X' X \hat{\delta}'}{\hat{\sigma}^2}$ which is asymptotically χ_{p+1}^2 , where $\hat{\delta}$ are the estimated regression parameters and X is a matrix of containing the regressors (Engle and Sheppard, 2001).

4.3 Application

4.3.1 Introduction

This section provides a summary application of EWMA and DCC models on the BRICS weekly returns from 2000 to 2012. The first section gives a summary of multivariate volatility estimation of the BRICS markets using the EWMA model, the second section provides a summary of multivariate volatility estimation of the BRICS markets using the DCC model, the third section presents a summary of the estimation comparison between EWMA and DCC models and the last section gives a summary of the chapter.

4.3.2 EWMA application results

Since the formation of the model by Morgan (1995) it has been common practice to assume $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data. However, λ is a constant therefore, one may choose any λ close to 1 to ensure that more weight is emphasized on the most recent returns. On the other hand, if $\lambda \rightarrow 0$ then more weight is given to older information thus, equation (4.3) will produce poor predictions of the future price movements. In this section we use $\lambda = 0.94$ to estimate the

volatility of the BRICS weekly returns. Based on the recursive estimation of the covariance matrix from equation (4.3) the volatilities of the BRICS returns were estimated and plotted on Figure 4.1.

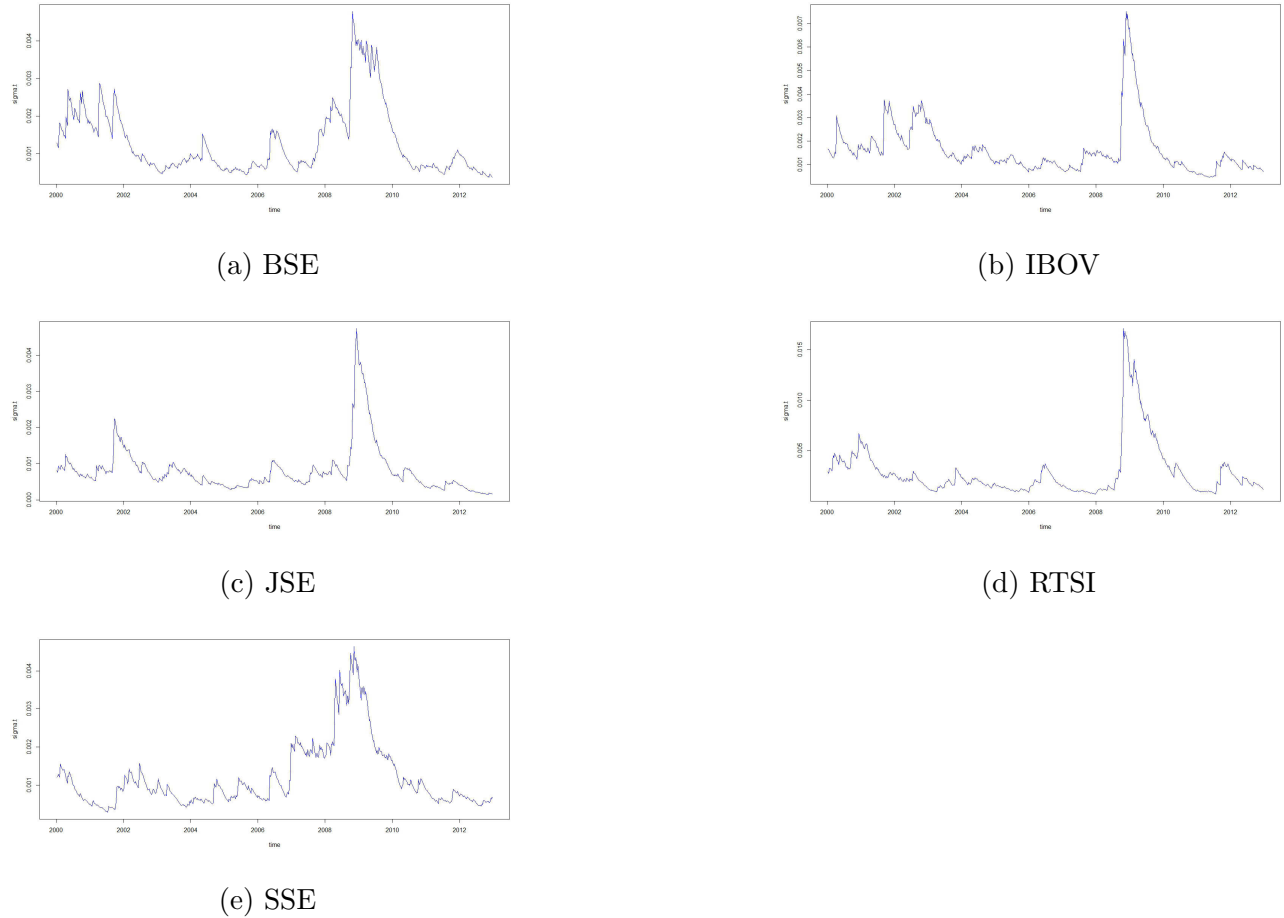


Figure 4.1: EWMA estimated volatility plots for (a) India (b) Brazil (c) South Africa (d) Russia and (e) China stock returns.

From Figure 4.1 the volatility estimates of the BRICS returns between 2003 to 2006 appear to be moving in a rather steady manner. An obvious sharp rise in volatility is observed in all the BRICS markets between 2008 to 2009 a pattern which could be explained by the 2008 global recession which caused a lot of instabilities within these markets as reported by O'Neill et al. (2009). From 2009 to 2010 a sudden drop off in volatility in all markets is observed indicating a rise in the stock markets performance during this period. Towards 2012 the volatility of the BRICS stock returns show low fluctuations indicating a stable recovery from the global crisis.

From the estimation of the covariance matrix Σ_t , correlations between the BRICS financial returns were also estimated and plotted on Figure 4.2

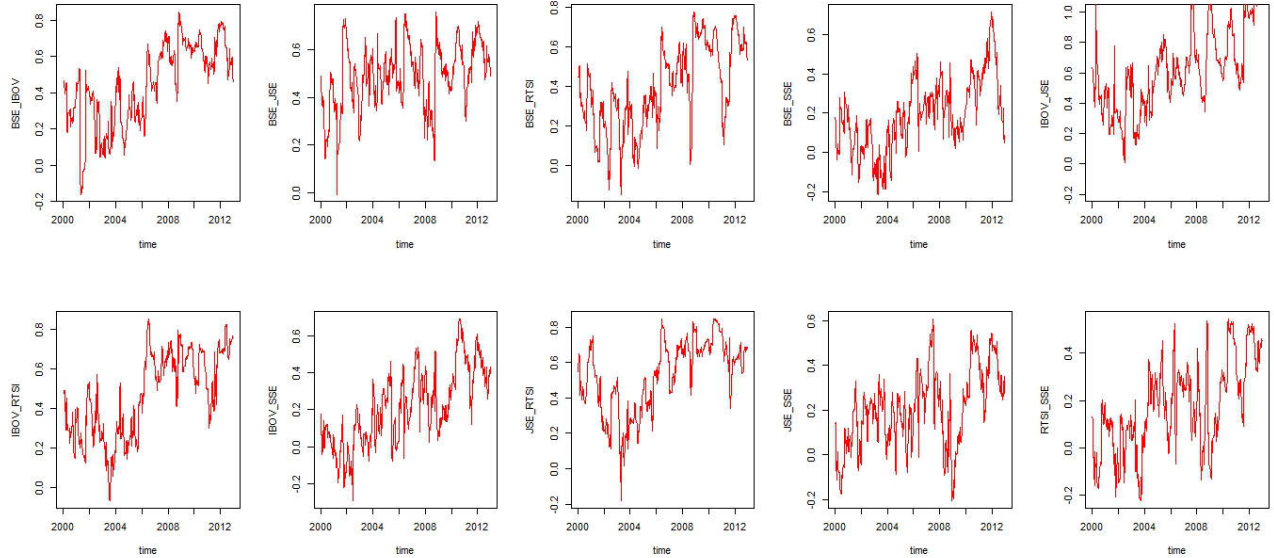


Figure 4.2: Estimated correlation plots for the BRICS stock returns using an EWMA model where BSE is the Indian stock market, IBOV is the Brazil stock market, JSE represents the South Africa stock market, RTSI represents the Russia stock market and SSE represents the China stock market.

Looking at Figure 4.2 the estimated correlation plots show a more non stable pattern although, the moving pattern appears to be centralized around an upward trend correlation. For most of the plots a decline in correlation between the markets is observed between 2008 to 2009. However, the degree of decline in correlation differs for instance the correlation between RTSI and JSE appears to be approximately 0.4 between 2008 to 2009 whereas the correlation between JSE and SSE appears to be approximately -0.2 . Again within the same period of financial turmoil, the correlation between BSE and IBOV appears to be is approximately 0.39 whereas the correlation between BSE and RTSI is approximately 0 implying absence of any correlation between the Russian and Indian stock market during that time. This is an interesting result because with a shrewd approach the correlation between two assets during a shock is expected to increase simply because the majority of asset prices are moving in the same direction. Thus, from Figure 4.2 it seems that the global financial

turmoil affected the correlations among the BRICS markets to a point of driving the correlations down to negative values for some markets implying opposite movements of these assets during the crisis.

4.3.3 DCC application results

In this section we apply the theoretical frame work of the DCC methodology on the BRICS weekly returns and present the results of the model estimation together with the diagnostic test results of the fitted model using the Ljung-Box and Langrange multiplier tests.

4.3.3.1 Parameter estimation

Table 4.1 gives a summary of the DCC model parameter estimates. The univariate GARCH(1,1) parameter estimates for each of the BRICS returns represent the diagonal elements of \mathbf{D}_t as defined in equations (4.6) and (4.9) whereas, θ_1 and θ_2 represent the DCC conditional correlation parameters. All the parameter estimates appear to be significantly different from zero at 5% level of significance which implies that they all have a significant effect. The significance of the univariate GARCH parameters α_1 and β_1 imply that the conditional volatility of the BRICS markets is highly persistent and that the stock markets are reacting to different shocks received from other markets. Moreover, the sum of the estimates ($\hat{\alpha}_1 + \hat{\beta}_1$) is less than one for all the BRICS stock returns with SSE having the highest value of 0.982, followed by RTSI and BSE with the value 0.958. The results suggest that China has the highest volatility persistence among the BRICS stock returns. The DCC correlation parameters on the other hand, are significantly different from zero implying that the correlations between the BRICS markets is dynamic. In addition, the correlation parameter estimates show adherence to the restriction imposed on them that is, $\hat{\theta}_1 + \hat{\theta}_2 = 0.952 < 1$ suggesting that the estimated correlation matrix \mathbf{P}_t is positive definite.

Table 4.1: A summary table of the DCC model parameter estimates for the BRICS stock returns

Market	Parameter	Estimate	SE	<i>p</i> -value
BSE	α_0	0.00005	0.000025	0.037
	α_1	0.100	0.037	0.007
	β_1	0.856	0.048	0.000
IBOV	α_0	0.00006	0.000037	0.043
	α_1	0.063	0.023	0.006
	β_1	0.895	0.038	0.000
JSE	α_0	0.00003	0.000020	0.027
	α_1	0.117	0.035	0.001
	β_1	0.844	0.049	0.000
RTSI	α_0	0.00006	0.000059	0.034
	α_1	0.139	0.047	0.003
	β_1	0.819	0.055	0.000
SSE	α_0	0.00002	0.000012	0.031
	α_1	0.055	0.020	0.005
	β_1	0.927	0.024	0.000
	θ_1	0.014	0.002	0.000
	θ_2	0.983	0.003	0.000

A plot of the estimated conditional correlations by the DCC model is presented on Figure 4.3. From the figure 4.3 a general impression is that most markets show a decline in correlation at the beginning of the estimated period especially from 2000 to 2003 and an upward trend with few correlations afterwards. With such trends it appears that portfolio diversification was higher between 2003 to 2004 compared to the period after, as the correlations in the former period appear to be lower than those in the latter period. Looking at Figure 4.2 the correlation estimates for EWMA appear to be highly exaggerated as some the correlations drop down to negative values. For instance, looking at the correlation plots for JSE and SSE on both figures between the period of 2007 to 2009, the estimates from Figure 4.2 show an approximate decline in correlation from 0.32 to 0.1 while on Figure 4.3 the decline in correlation during the same period is approximately from 0.6 to -0.2 suggesting an opposite movement of the two markets under the EWMA estimation during the

financial catastrophe while the movements of the same markets under the DCC estimation appear to be moving in the same direction during the crisis.

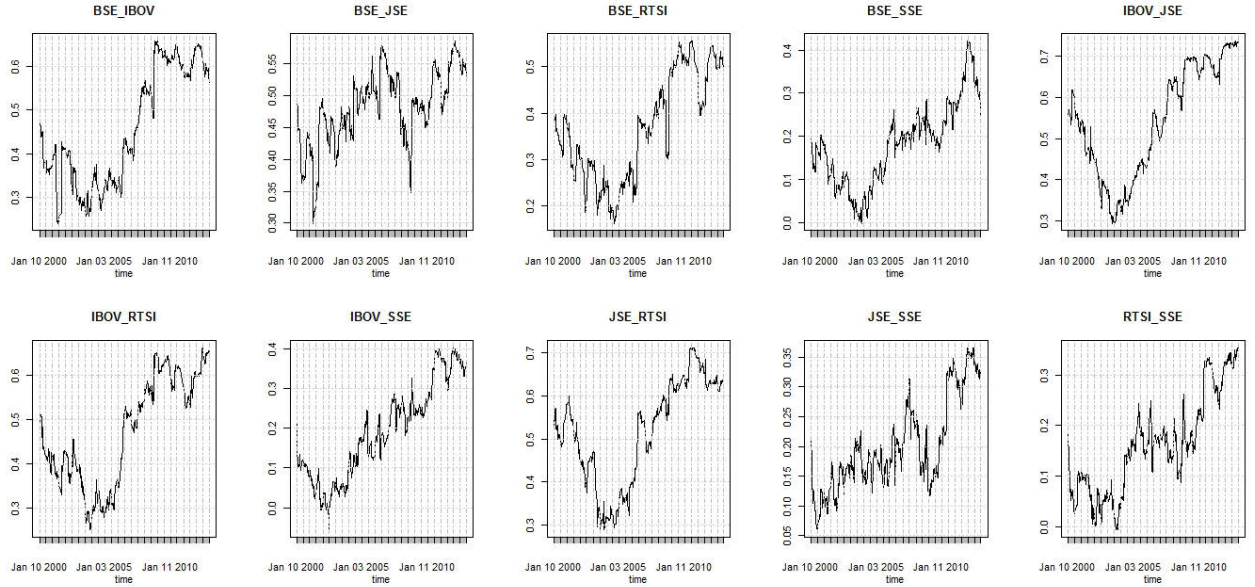


Figure 4.3: Estimated conditional correlation plots for the BRICS stock returns using a DCC model where BSE is the Indian stock market, IBOV is the Brazil stock market, JSE represents the South Africa stock market, RTSI represents the Russia stock market and SSE represents the China stock market.

4.3.3.2 Diagnostic tests

Having the parameters estimated, we then investigated the adequacy of the model using the standardized residuals. To test for serial correlation we apply the univariate Ljung-Box test on each of the BRICS standardized residuals. A summary table of the Ljung-Box statistic is presented on Table 4.2. From Table 4.2 we accept the null hypothesis of no serial correlation since all the p -values appear to be > 0.05 . To test for constant correlation we use the LM test by Engle and Sheppard (2001). The results of the LM test are summarized on Table 4.3. From Table 4.3 the residuals appear to have $p - values < 0.05$ suggesting the rejection of the null hypothesis of constant correlation. Therefore this confirms that the DCC model is a good fit and that correlations among the BRICS markets evolve over time.

Table 4.2: A summary Table for the DCC model diagnostics under the Ljung-Box test

Stock return	Q-statistic	p -value
BSE	34.28	0.50
IBOV	49.43	0.05
JSE	36.62	0.39
RTSI	42.43	0.18
SSE	114.29	0.06

Table 4.3: A summary Table for the DCC model diagnostics under the multivariate LM test

Lags	LM- Statistic	p -value
10	39.22	< 0.001
20	51.23	< 0.001
30	6.22	< 0.001
40	82.48	< 0.001

4.3.4 DCC and EWMA estimation comparison

Both EWMA and DCC models are capable of estimating the correlations of the BRICS markets however, we are interested to find out which model outperforms the other in estimation. Since the two models are not nested a likelihood ratio test approach would not be very informative in this case therefore, we are simply going to compare the plots of the estimated correlations by the two models using JSE and SSE markets as examples as shown on Figure 4.4.

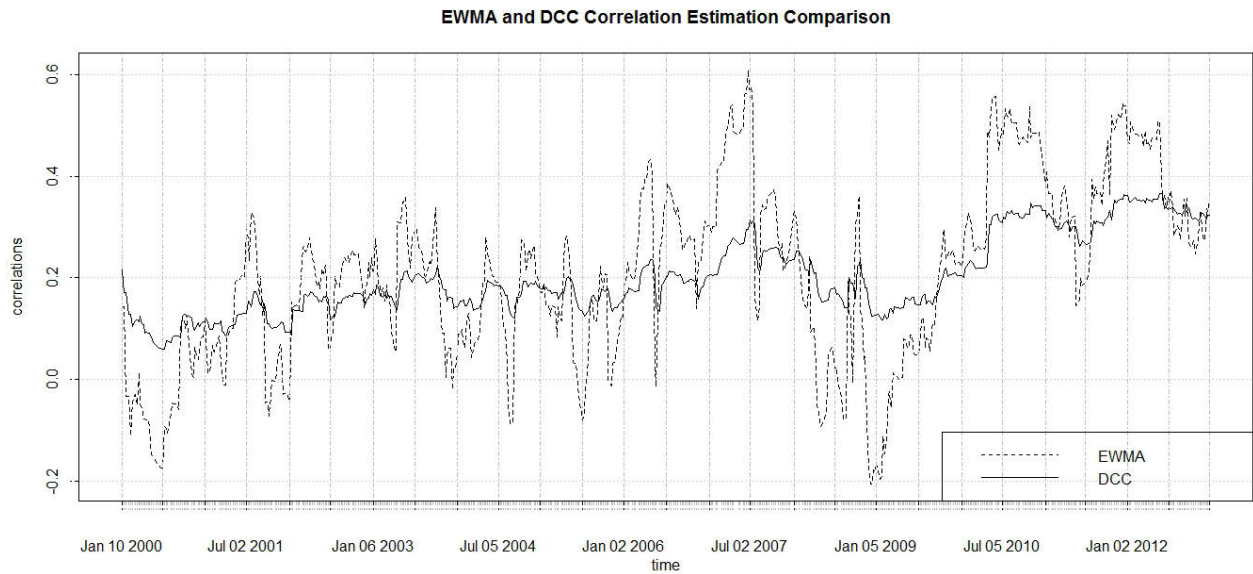


Figure 4.4: A plot of the Estimated correlations for South Africa (JSE) and China (SSE) stock markets using EWMA and DCC models.

From Figure 4.4 the correlation estimates for DCC appear to be stable with low fluctuations whereas the correlation estimates for EWMA seem to be more volatile which is not surprising since the correlations by the EWMA model are modeled in simple and direct manner as shown in equation (4.5). Looking at the recession period between 2008 and 2009 on Figure 4.4, the correlation estimates for the EWMA appear to be overestimated to a negative value while those for the DCC drop down but with a positive correlation indicating a high probability of portfolio diversification with minimal risk in expected asset returns from EWMA correlation estimates compared to the DCC estimates. From this graphical analysis, the DCC model appears to outperform the EWMA model in estimation performance. Danielsson (2011) and Matteson and Ruppert (2011) among other researchers obtained similar results when they compared the estimation performance of the EWMA and DCC models.

4.4 Summary

In this chapter we presented a discussion on multivariate volatility models with a theoretical approach on the EWMA and DCC models together with their empirical applications on the BRICS weekly returns. With both models, the conditional volatility and correlations of the BRICS returns were estimated. Both models portrayed persistence of volatility in all of the BRICS returns with China showing the highest volatility persistence whereas the Indian market appears to be the least volatile. Unlike the univariate GARCH results, the JSE in this case does not seem to be the least volatile. This could be attributed by its interaction with the other markets because a multivariate model can capture simultaneously the interaction among multiple time series. The hypothesis of constant correlation among the BRICS markets is rejected as the correlations among the BRICS returns appear to vary with time. Diagnostic tests were carried out on the DCC to check for the adequacy of the model. The results of the tests showed that the DCC model was sufficient enough to estimate the volatility and correlations of the BRICS returns. Both EWMA and DCC models played an important role in the estimation of volatility and correlations among the BRICS markets however, based on graphical analysis of correlation estimations, the DCC portrayed better correlation estimates compared to the EWMA model as the latter produced exaggerated estimates with very high or low correlation swings. Therefore, in this chapter we conclude that the DCC model outperforms the EWMA model in estimation.

Chapter 5

Discussion and Conclusion

The main focus of this study was to assess the inter-relations and dynamic linkages among the BRICS financial markets using multivariate time series models. It is well known that financial volatilities move closely together across financial markets and assets ([Orskaug, 2009](#)). An appreciate of the co-movements and levels of interdependencies among financial markets is thus of great importance especially for stakeholders and investors, such information assist in managing risk and making viable decisions regarding portfolios. In this thesis, we used a VAR, univariate and multivariate GARCH models to investigate the dynamic structure, volatility and correlations among the BRICS stock returns using weekly data from Jan 2000 to Dec 2012.

From the VAR model we were able to determine concurrent relationships among all the BRICS markets together with unidirectional linear dependencies between India (BSE) and the past values of Brazil (IBOV) and between China (SSE) and the past values of Brazil (IBOV) implying that the Brazilian stock market has an influence on the Indian and Chinese stock markets. The linear dependence obtained from the VAR model estimation was not surprising because of late, within the BRIC nations, Brazil's growth economic score was reported as the highest among the four emerging markets with India having the lowest score ([O'Neill et al., 2009](#)). This could explain the influence of Brazil on the two stock markets. Furthermore, we detected the presence of ARCH errors in the BRICS return series suggesting the presence of volatility in the BRICS markets. To the best of our knowledge, there has been very few studies, most of which are focused on the BRIC without the

inclusion of South Africa that have previously been done on the analysis of the BRICS markets using VAR models (see [Mallick and Sousa \(2009\)](#) and [Ono \(2011\)](#)). Furthermore, the results of these studies are mainly focused on external factors influencing the BRIC/BRICS financial markets such as oil prices, exchange rates and monetary policies to mention a few.

Following the ARCH error detection in the BRICS financial returns we proceeded with modeling and forecasting the volatility of each of the BRICS markets using a univariate GARCH model. Estimates from the univariate GARCH model revealed persistence of volatility in the BRICS returns with China (SSE) having the highest volatility persistence followed by Russia (RTSI). These results are similar to the study by [Kasman \(2009\)](#), where China had the highest volatility persistence among the BRIC markets. However, [Kasman \(2009\)](#) argues that, ignoring structural breaks in a return series could lead to overestimation of volatility persistence.

The EWMA and DCC models were both used to estimate the volatility and correlation among the BRICS returns. Again we obtain similar results of high volatility persistence in the Chinese (SSE) stock market from the GARCH components of the DCC model estimates. The high volatility persistence in the Chinese stock market could be explained by factors such as uncertainties in the financial policies in China or its interactions with developed markets such as the US which makes the country prone to shock transmissions (see [Kenourgios et al. \(2011\)](#) and [Byström et al. \(2013\)](#)). Moreover, the results from the DCC and EWMA models also revealed the presence of time-varying correlations among the BRICS financial markets. Surprisingly, the correlation between the BRICS market pairs appeared to be declining during the 2008 global crisis. Similar results were found by [Hartman and Sedlak \(2013\)](#) whose study was focused on other markets which were the EURO and USD exchange rates. [Hartman and Sedlak \(2013\)](#) however, did not provide any reasons to such ambiguity in their results. In my opinion, this ambiguity can be attributed to the fact that the BRICS markets are on different economic levels with different trading activities for instance, Russia is the biggest oil producer among the BRIC nations ([O'Neill et al., 2009](#)) whereas South Africa's largest trade is minerals thus, the markets tend to be decoupled during crises depending on the type of shock causing financial instabilities in the BRICS economies.

An estimation comparison between the EWMA and DCC models was carried out to compare their efficiency in estimating the correlations among the BRICS stock markets. The DCC model was

found to be a better model compared to the EWMA model as the latter produced exaggerated correlation estimates between the BRICS market pairs. Similar results were obtained by Danielsson (2011) using IBM and Microsoft stock returns.

Modeling volatility with multivariate GARCH models is often challenging due to numerical complexities that arise in estimation. Empirical applications of most multivariate GARCH models is difficult and complex due to the curse of dimensionality. Moreover, unlike the univariate GARCH models, the constraint for covariance stationarity is much more crucial with multivariate GARCH models because if, the condition is violated the covariance matrix may no longer be invertible leading to difficulties when evaluating the likelihood. Therefore, there is a great need to develop feasible multivariate GARCH models that can accommodate large numbers of assets without causing parameter explosions during estimation.

Although we were able to capture volatility clustering among the BRICS markets, the GARCH models employed had a limitation of symmetric response to positive and negative shocks. Furthermore, it is generally argued that a negative shock is likely to intensify the levels of volatility in a stock return compared to a positive shock of the same magnitude thus, a symmetric GARCH model may be incapable of capturing potential leverage effects (Brooks, 2008). For future research, we can further extend this study to examine the leverage effects in the emerging market volatilities using asymmetric models such as the EGARCH and GJR-GARCH models, investigate co-integration relationships among the BRICS financial markets if any, and employ non stationary models such as the vector error correction (VEC) model that can account for the long run relationships among the BRICS markets. Furthermore, in modeling the dynamic correlations, we could apply the orthogonal GARCH (OGARCH) model that transforms correlated returns into uncorrelated portfolios and uses a GARCH to forecast the volatilities of each uncorrelated portfolio individually. In addition to that, with an OGARCH model it is possible to build a covariance matrix for large numbers of assets.

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Appendix A

R Codes for the Vector Autoregressive (VAR), Univariate and Multivariate GARCH Models

A.1 R Codes for the VAR(1) Model

```
library(vars)

#####
#selecting an appropriate VAR(p)
#####
VARselect(data,lag.max=6,type="const")

#####
#Fitting a VAR(1)
#####
data1=data[,c("IBOV","RTSI","BSE","SSE","JSE")]
fit<-VAR(data1,p=1,type="const")
```

```
summary(fit)

#####
#Diagnostic tests for VAR(1)
#####
#portmanteau test
ser=serial.test(fit,lags.pt=6,type="PT.asymptotic")

#Jarque-bera test
norm=normality.test(fit, multivariate.only = TRUE)

#Multivariate ARCH-LM test
arch=arch.test(fit,lags.multi=6,multivariate.only=TRUE)
```

A.2 R codes for the univariate GARCH(1,1) model

```
library(rugarch)

#####
#GARCH(1,1) assuming a t distribution
#####
garch11.t.spec = ugarchspec(variance.model = list(garchOrder=c(1,1)),
                             mean.model = list(armaOrder=c(0,0)),
                             distribution.model = "sstd")
fit1 = ugarchfit(data = IBOV[,1], spec = garch11.t.spec)
plot(fit1)
fit2 = ugarchfit(data = RTSI[,1], spec = garch11.t.spec)
plot(fit2)
fit3 = ugarchfit(data = BSE[,1], spec = garch11.t.spec)
```

```

plot(fit3)
fit4 = ugarchfit(data = SSE[,1], spec = garch11.t.spec)
plot(fit4)
fit5 = ugarchfit (data= JSE[,1], spec= garch11.t.spec)
plot(fit5)
#####
#Forecasting with the GARCH(1,1) model
#####
forc1 = ugarchforecast(fit1, n.ahead=75,n.roll=0)
plot(forc1)
forc2 = ugarchforecast(fit2, n.ahead=4,n.roll=0)
plot(forc2)
forc3 = ugarchforecast(fit3, n.ahead=4,n.roll=0)
plot(forc3)
forc4 = ugarchforecast(fit4, n.ahead=4,n.roll=0)
plot(forc4)
forc5 = ugarchforecast(fit5, n.ahead=4,n.roll=0)
plot(forc5)

```

A.3 R codes for the EWMA model

```

t=length(data[,1])
lambda=0.94
#creating a matrix that holds the covariance matrix for each t
ewma=matrix(nrow=t,ncol=15)
#intial covariance matrix
s=cov(data)
ewma[1,]=c(s)[c(1,7,13,19,25,2,3,4,5,8,9,10,14,15,20)]
#loop through the sample data

```

```

for(i in 2:t){
temp=as.numeric(data[i,])
s=lambda*s+(1-lambda)*temp%%t(temp)
ewma[i,]=c(s)[c(1,7,13,19,25,2,3,4,5,8,9,10,14,15,20)]
}

```

```

#####
#extracting volatilities
#####
volat=ewma[,1:5]

```

```

#####
#extracting correlations
#####
#RTSI and SSE
ewmarho1=ewma[,15]/sqrt(ewma[,4]*ewma[,5])
#JSE and SSE
ewmarho=ewma[,14]/sqrt(ewma[,3]*ewma[,5])
#JSE and RTSI
ewmarho2=ewma[,13]/sqrt(ewma[,3]*ewma[,4])
#IBOV and SSE
ewmarho3=ewma[,12]/sqrt(ewma[,2]*ewma[,5])
#IBOV and RTSI
ewmarho4=ewma[,11]/sqrt(ewma[,2]*ewma[,4])
#IBOV and JSE
ewmarho5=ewma[,10]/sqrt(ewma[,1]*ewma[,3])
#BSE and SSE
ewmarho6=ewma[,9]/sqrt(ewma[,1]*ewma[,5])
#BSE and RTSI

```

```
ewmarho7=ewma[,8]/sqrt(ewma[,1]*ewma[,4])
#BSE and SSE
ewmarho8=ewma[,7]/sqrt(ewma[,1]*ewma[,3])
#BSE and IBOV
ewmarho9=ewma[,6]/sqrt(ewma[,1]*ewma[,2])
```

A.4 R codes for the DCC model

```
library(ccgarch)
#Univariate GARCH(1,1)specifications for each series
garch11.spec = ugarchspec(mean.model = list(armaOrder = c(0,0)),
variance.model = list(garchOrder = c(1,1),model = "sGARCH"),
distribution.model = "sstd")

#dcc specification - GARCH(1,1) for conditional correlations
dcc.garch11.spec = dccspec(uspec = multispec( replicate(5,garch11.spec)),
dccOrder = c(1,1),distribution = "mvnorm")

#estimating the DCC model
dcc.fit = dccfit(dcc.garch11.spec,data)
plot(dcc.fit)
#extracting correlations
dcc.fit@mfit$R

#extracting volatilities
dcc.fit@mfit$H
```