A description of mathematical proficiency, in number skills, of grade ten learners in both the Mathematics and Mathematics Literacy cohorts at a North Durban school

by

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ABSTRACT

The main objective of this study was to determine the progress made in the development of mathematical proficiency, in the domain of number skills, by learners in the two cohorts (namely: Mathematics and Mathematical Literacy) during Grade ten. The research was conducted at Temple Valley Secondary School, which is situated at Verulam, north of Durban in KwaZulu-Natal. Furthermore, this research study was questionnaire-based employing basic descriptive statistics as well as qualitative analyses. Data was collected through numeracy (number) skill tests, a questionnaire and focus group interview responses of grade ten mathematics learners. A total of two skill tests were administered and 3 focused group interviews were conducted with six grade ten learners each. The first test and first attitudinal questionnaire was administered in May 2006 when learners had branched off into the two paths of mathematics. Thereafter, a second test and second attitudinal questionnaire was administered, during October 2006; when learners had completed most of the school year. The October-test was followed by the focus group interviews. A convenient sample was used in selecting the learners for this research study and a purposeful sampling technique was used for the focus group interviews. All grade ten learners at Temple Valley Secondary School were selected as the sample. The results showed that the mathematics learners slightly outperformed the mathematical literacy learners in the development of mathematical proficiency for number skills. The findings from this research could inform: Teachers of grade ten learners with an interest in improving the mathematical proficiency, in number skills, of learners and; curriculum developers and materials development specialists who prepare mathematical material for grade ten classrooms. The results showed that none of the grade ten learners, from either cohort, were mathematically proficient in May or October. From the study, it was noted that the female grade ten learners generally outperformed their male counterparts. The focus group interviews revealed that learners had a positive attitude to the learning of the subject mathematics, despite the poor test results of this study.

PREFACE

The work described in this thesis was carried out in the School Mathematics Education, University of KwaZulu-Natal, from **May 2006 to October 2006** under the supervision of **Mrs Sally Hobden** (Supervisor).

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

The University of KwaZulu-Natal Research Office granted ethical clearance for this project. The Ethics Clearance Approval number is HSS/06124A.

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ABBREVIATIONS

LO: Learning outcome

GET: General Education and Training

FET: Further Education and Training

ML: Mathematical literacy

RQ: Research question

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Last but by no means least, my wife and two children, Zelda, Tyra and Tyler; for their support, commitment and patience with my mental and physical preoccupation during the months of writing this dissertation.

Dedication

This thesis is dedicated to my loving family and the 2006 grade ten learners of Temple Valley Secondary.

CHAPTER 1

In this chapter, I will outline the research process. First, I will discuss what motivated this research study leading to the discussion of the purpose of this study. I will then state the research questions which I will attempt to answer through the research process. Further, I will clarify the concepts of the critical question being researched and briefly describe the methodology used in this study. Finally, I will indicate what occurs in the following chapters of this dissertation.

1.1 MOTIVATION FOR STUDY

I am currently a mathematics educator at a secondary school in Verulam. This research grew out of the concern for the poor performance of learners in the General Education and Training phase (grade seven-nine) of mathematics education. In recent years, I have met many mathematics teachers, mathematics examiners and mathematics subject advisors from different parts of Durban in workshops and seminars; who echoed my sentiments exactly, about the poor performance of learners especially in senior mathematics. My concern was that if learners performed very poorly at the end of grade nine, they cannot drop mathematics but are forced to continue with it. Both the pure mathematics and mathematical literacy learners who have been promoted to grade ten, have the same starting point, a pass in grade nine, but are exposed to different content and context during the course of their grade ten year therefore I wanted to see whether they were adequately prepared for pure mathematics and mathematical literacy. There was a need to examine what lies behind the difficulty experienced by my learners in coping with the study of mathematics.

1.2 BACKGROUND TO THE STUDY

In the past, successful mathematics learning meant fluency in the computational skills of arithmetic (Kilpatrick, Swafford and Findell, 2001). Recently, there has been a debate about what success in mathematics really means. Today, many educators and curriculum developers (Government) agree that skills are needed, but in addition agree that there should be more emphasis on students learning procedures with understanding (Ibid). All stakeholders; including parents, teachers, state and community leaders; need to create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields (Ibid).

One of the major questions that needed an answer was whether learners in South African (S.A) schools became sufficiently mathematically proficient by the end of the compulsory phase of schooling. The Department of Education (DoE) in South Africa believed that this was not the case and therefore introduced the new FET curriculum for senior secondary learners, where learners chose either formal mathematics or mathematical literacy (ML) to help improve their mathematical proficiency (Department of Education, 2003). South Africa was one of fifty countries participating in the Third International Mathematics and Science Study (TIMSS) 2003, with 255 schools and 8952 learners. What was shocking was that South Africa had the lowest performance score for mathematics and science of the fifty TIMSS participants. According to Reddy (2006), South Africa had the largest variation in scores, ranging from mostly very low, to a few very high scores, meaning this score distribution was not normal but skewed to the left. This implies that certain learners performed above the average and others far below. TIMSS results sets the foundation for my study in that the majority of South African learners are not mathematically proficient. My research wants to determine whether the introduction of the new curriculum (FET) will make a difference to these learners proficiency in mathematics. Therefore, my research was based on whether these two paths of mathematics actually improved the mathematical proficiency (number skills) of grade ten learners.

1.3 DERIVING THE RESEARCH TOPIC

Mathematical Literacy, a new subject in the South African FET curriculum, was introduced for the first time in 2006. Mathematics was divided into two paths, namely: mathematical literacy and mathematics. Learners had a choice of either path. Through my eleven years of teaching mathematics, I have noticed that the development of mathematical proficiency in number skills; which incorporate Learning Outcome One (LO), *Number and Operations in Context* and *Number and Number Relationships,* in Mathematical Literacy and Mathematics respectively; was seriously lacking in learners at this senior level. Therefore, I have decided to describe the development of mathematical proficiency (in LO 1) of the grade ten learners in both the mathematics and mathematical literacy cohorts.

Why did I choose the number domain? Kilpatrick et al (2001) suggests that number sense is the foundation of all later number work. Familiarity and fluency with numbers, number sequences and estimation can be developed from a very early age. Number is fundamental to describing and understanding the world in which we live and yet we take it for granted that children will unravel its complexities. The mathematics curriculum during the preschool, primary school, and high school years has many components. However, at the heart of mathematics in these years is the mathematical domain of number. Furthermore, the domain of number both supports and is supported by other branches of mathematics, including algebra, measure, space, data, and chance (Ibid).

Some reasons for choosing the particular questions

This study is framed by the following four research questions:

1. What is the mathematical proficiency of grade ten Mathematical Literacy learners in the domain of numbers?
2. How does the mathematical proficiency of the grade ten Mathematical Literacy learners compare with the mathematical proficiency of the grade ten Pure Mathematics learners?
3. How did the mathematical proficiency change over a four-month period in both cohorts for these grade ten learners?
4. How did gender and misconceptions influence the achievements of the mathematical proficiency of these grade ten learners?

Research question one is included to determine the initial mathematical proficiency with which the mathematical literacy learners will enter grade ten. In research question two, I found it useful to compare the mathematical literacy learners with the pure mathematics learners in terms of their mathematical proficiency at entry into grade ten so that I could gauge which cohort performed better. In question three it is important and necessary to this research study to assess whether there was a change in mathematical proficiency over a period of about four months in both cohorts. Finally, I wanted to determine what factors may influence the development of mathematical proficiency of these grade ten learners, namely gender and misconceptions.

1.4 OUTLINE OF THE RESEARCH

This research study involved both quantitative and qualitative methods of data analysis. According to Denzin and Lincoln (2003), quantitative studies have their emphasis on the measurement and analysis of causal relationships between variables, where the researcher is objective and the participant’s responses are value-free. Qualitative studies involve the researcher locating her/himself, as a part, in the participant’s world (Ibid). Furthermore, they argue that qualitative researchers study things in their natural settings, attempting to interpret the results in terms of the meanings people (learners) bring to them. For the purpose of this study, qualitative data was collected in focus group interviews, reflective writings, and questionnaires; whereas quantitative data was collected in two tests.

The following tools were used for data collection:

1) A May-test and October-test to assess mathematical knowledge and skills
2) A May and October questionnaire to assess attitudes to mathematics
3) Reflective writing, and
4) Semi-structured focus group interviews.

This research represented an analysis of the numeracy (number) skill tests, questionnaires and interview responses of grade ten learners.
A May-test was administered shortly after learners had branched off into the two paths of mathematics. Thereafter, the October-test was administered towards the end of the year, while learners were undertaking either mathematics or mathematical literacy. The May-test and October-test were followed by a piece of reflective writing.
Focus group interviews were conducted in between the May-test and October-test.

A convenience sampling technique was used in selecting grade ten learners for this study. In addition, three focus groups were interviewed in order to obtain rich data. Each group consisted of six learners either from the mathematical literacy and/or from mathematics class. I selected the learners according to their performance scores in the May-test.

This research applied the five strands for mathematical proficiency (Kilpatrick et al, 2001) in analyzing the data collected. The data were analysed using the theoretical framework of mathematical proficiency, which is fully described in Chapter 2.

1.5 CONCEPT CLARIFICATION

At the outset, it is helpful to clarify some key concepts, and meaning they have in the context of this study.

1.5.1 Mathematical proficiency

Kilpatrick et al (2001) suggests that mathematical proficiency empowers learners with the competence, expertise, knowledge, and facility in mathematics; which is necessary for any learner to learn mathematics successfully. It is also believed that this proficiency enables learners to cope with mathematical challenges in everyday life. Mathematical Proficiency has five strands, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. These five strands are interdependent and interwoven, which has implications for how learners acquire mathematical proficiency.

1.5.2 Mathematics

For the purpose of this study, mathematics refers to a knowledge constructed through the establishment of numerical, descriptive and symbolic relationships. The Department of Education (2003b), states that mathematics encourages logical and creative reasoning in the real world.

1.5.3 Mathematical literacy

For the purpose of this study, mathematical literacy refers to the learner relating mathematics to real life applications. The Department of Education (2003a) defines mathematical literacy as being "… driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems" (p. 10).

1.5.4 Number skills

For the purpose of this study, number skills refers to the ability of learners to recognise, describe, represent, estimate, calculate, check solutions, and confidently investigate a range of different contexts; which included financial aspects (Department of Education, 2003b; Department of Education, 2003a).

According to Department of Education (2003b), “The focus of this learning outcome is on the investigation and solution of problems that require a sound understanding of numbers and their use in calculations, especially in financial contexts, ranging from personal to international issues…. Learners should develop sound estimation and mental calculation skills and a facility in using equivalent forms to simplify calculations. Proper conceptual understanding will be required to enable learners to use calculators appropriately and effectively” (p. 11). In the above, I refer to Learning Outcome One: Number and Number Relationships.

1.6 OVERVIEW OF THE THESIS

In Chapter 1, I introduce the research study. It highlights the reasons why I chose this topic and gives a brief background to the study. This chapter also explains from whence the research topic was derived and gives an outline of the tools and time-line for this study. I conclude this chapter with some concept clarification to help readers better understand the content of this research.

Chapter 2 in which I discuss the literature pertaining to my research topic from an international and local perspective follows this introductory chapter; and in particular Mathematical Proficiency, which provides a theoretical framework for this study.

In Chapter 3, I document the research design and methodology followed during the fieldwork. This chapter will begin with the design of the study. This will be followed by a discussion of the research methodology, including the research instruments, structured tests, questionnaires, semi-structured interviews, and pilot study. Next, the ethical issues, administration of instruments, limitations of design and methodology, and data analysis will be discussed.

I include the presentation and discussion of the results for this research in Chapter 4. The main trends and patterns in the data will be discussed with reference to the four research questions. Furthermore, the results will be interpreted in terms of the literature/theory reviewed in Chapter 2.

In the final chapter, I include the conclusions and recommendations arising from this study. This chapter will firstly provide a summary of this study. Secondly, the conclusions for each proficiency strand will be discussed. Thirdly, the larger significance and value of the study will be shown. This chapter will conclude with recommendations regarding further research, the implementation of the findings and the possible policy implications.

CHAPTER 2

This study has attempted to describe mathematical proficiency, in number skills, of grade ten learners in both the Maths and Maths Literacy cohorts (within the NCS) at a school. In order to provide answers for the above statement, I have read many local and international authors who presented different or various perspectives about the links between mathematical proficiency, mathematics and mathematical literacy. The five strands of mathematical proficiency namely, Procedural fluency, Adaptive reasoning, Conceptual understanding, Strategic competence and Productive disposition have been used in trying to explain and understand the role of mathematics and mathematical literacy in promoting mathematical proficiency amongst school learners. It was decided that the literature would be demarcated into local and international literature.

2.1 UNDERSTANDING MATHEMATICAL LEARNING

I will describe two theoretical perspectives from which mathematical learning can be viewed. The first is the more recent notion of mathematical proficiency, which offers a means to describe the understanding of mathematics learning and the second, the constructivist perspective, which is a theory that describes how learning takes place. I will then discuss how the two perspectives can be linked to create a theoretical framework for the analysis of the research data.

2.1.1 Mathematical proficiency

The mathematical proficiency of South African citizens, in the past apartheid era, was of a very low level due to the poor quality or lack of education (HSRC, 2000). In addition, many learners who found mathematics difficult in their junior secondary level of schooling usually terminated their studying in mathematics; thus decreasing the levels of mathematical proficiency in South African schools and society. The TIMSS (2003) indicated that learners in South Africa have performed very poorly as compared to learners from developed or developing countries. In order to remedy this problem of innumeracy, the Department of Education (2003a) has included Mathematical Literacy as a subject in the Further Education and Training curriculum to ensure that the citizens of the future are highly proficient in mathematics. Furthermore, they state that Mathematical Literacy will help develop the basic mathematical skills and ability to solve everyday problems in various realistic situations.

In this study, I have assessed the development of numeracy skills through the framework of mathematical proficiency. This framework comprises five components or strands. The five strands of mathematical proficiency are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al, 2001). Conceptual understanding is the learners understanding of mathematical concepts and operations. Procedural fluency is the learner carrying out skills (practice) accurately, efficiently and appropriately. However "… practice on computational procedures should be designed to build on and extend understanding." (Kilpatrick, Swafford, and Findell, 2001, p. 423). As mentioned above, understanding of procedures is of vital importance for the success of the learner in mathematics. Strategic competence is the learners' ability to formulate and solve problems. It must also be noted, "Problem solving should be the site in which all of the strands of mathematics proficiency converge." (Kilpatrick et al, 2001, p. 421). On the contrary, too much emphasis is placed on problem solving in mathematics and as a result, the skills and knowledge components are lagging behind. The new curriculum needs to find a balance between context, skills and knowledge. Adaptive reasoning is the learners' capacity for logical thinking, explanation and justification of action. Lastly, productive disposition is the learners' attitude to see mathematics as sensible, useful and worthwhile in and outside of school. From my teaching experience, this aspect was seriously lacking in the past educational system.

Conceptual understanding

The first strand is conceptual understanding where learners' comprehension of mathematical concepts, operations and relations are determined. Conceptual understanding refers to an interconnected and functional comprehension of mathematical ideas. A learner with conceptual understanding does not learn facts or methods in isolation because they understand why these mathematical ideas are important and how it is useful in different contexts. Milgram (2004) states that a student should not simply memorize concepts and repeat it verbatim but should understand why it is stated the way it is. In order to verify whether students understand the concepts, the following three questions should be asked: What does the statement include?; what does the statement exclude?; and what would happen if the concepts were changed and why is the changed concept not used? (Ibid). In contrast, primary and secondary students generally learn mathematics without any understanding and then expect to apply their knowledge in solving problems. As a result, I believe the performance is dismal for these primary and secondary learners in mathematics as compared to other subjects. Therefore, these learners must organize their new knowledge into existing schemas in a meaningful way, which enables them to retain, use, remember and reconstruct mathematical ideas when forgotten. In addition, students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. According to the NCTM (1989) and Department of Education (2003a) conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural fluency

The second strand is called procedural fluency where the learners’ skill in carrying out procedures flexibly, accurately, efficiently and appropriately is assessed. Students demonstrate procedural knowledge in mathematics when they select and apply appropriate procedures correctly; verify or justify the correctness of a procedure using concrete models or symbolic methods; or extend or modify procedures to deal with factors inherent in problem settings. Procedural knowledge encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform non-computational skills such as rounding and ordering. Procedural knowledge is often reflected in a student's ability to connect an algorithmic process with a given problem situation, to employ that algorithm correctly, and to communicate the results of the algorithm in the context of the problem setting (NAGB, 2005).

Many tasks involving mathematics in everyday life require general procedures for performing calculations either mentally or in writing. In addition, some procedures are important as concepts, which show the link between conceptual understanding and procedural fluency. Moreover, a certain level of skill is required to learn many mathematical concepts with understanding and using procedures can help strengthen and develop that understanding. When learners' practice procedures they do not understand, there is a danger they will practice incorrect procedures, thereby making it more difficult to learn correct ones. Another major danger of children learning without understanding is that they separate what happens in school from what happens outside. In other words, these learners have different sets of procedures for solving problems in and outside of school. This separation limits the learners' ability to apply what they learn in school to solve real-life problems.

Strategic competence

The third strand of mathematical proficiency is strategic competence where the learners' ability to formulate; represent and solve mathematical problems are considered. Strategic competence involves students learning how to formulate mental pictures of the problems, detect mathematical relationships and then devise a mathematical solution when needed. Strategic competence is mutually interwoven with both conceptual understanding and procedural fluency. For example, the development of strategy for the solving of non-routine problems depends on the learners understanding the quantities involved and fluency in solving routine problems. According to the NCTM (1989) and Department of Education (2003a), students demonstrate problem solving in mathematics when they determine the consistency of data; use strategies, data, models; generate, extend, and modify procedures; use reasoning in new settings; and judge the reasonableness and correctness of solutions. Moreover, strategic competence comes into play at every step in developing procedural fluency in calculations.

Adaptive reasoning

The fourth component is adaptive reasoning where the learners' capacity for logical thought, reflection, explanation and justification is determined. In mathematics, adaptive reasoning is the central core to learning. Adaptive reasoning includes informal explanations, justification, intuition and inductive reasoning. Learners' are able to display reasoning ability when three conditions are met: They have a sufficient knowledge base, the task is understandable and motivating and the context is familiar.

One of the functions for adaptive reasoning should be to determine whether the procedure is appropriate.

Another component of adaptive reasoning includes knowledge of ways to estimate the result of a procedure. Estimation problems utilized in adaptive reasoning require a judgement to be made or a justification for a certain procedure. Kilpatrick et al (2001) states that estimation is another neglected area of the development of number sense. When children first arrive at school, they make many informal guesses and estimates of quantity. They do not seem too concerned about 'right answers'. This state of affairs does not persist though as all too often children begin to seek the exact answer and will often be seen rubbing out an incorrect estimate and replacing it with the actual answer arrived at after the estimate. In real life, estimations are frequently used. When shopping for instance it is quite a common practice to round prices up and down as items are added to the shopping trolley. A precise total is not needed but keeping within a budget or being able to know that the prices have been entered correctly is. Sometimes when cooking, estimation is important too, for instance knowing how many potatoes to chop for French Fries or how many carrots to chop is not usually treated as a precise mathematical activity. The important thing is that everyone has sufficient food without too much waste. Estimation develops with practice and experience. Only after concrete experiences do the judgements about quantity, size or chance develop. These experiences can be built into the mental routines. If from an early age children, expect to estimate a ballpark figure before actually computing mentally, with paper and pencil or a calculator they will begin to expect to find realistic solutions to those computations and will spot any errors that occur and hopefully stop and look for reasons for the differences between the estimates and the computed answers.

Productive disposition

The final strand of mathematical proficiency is called productive disposition where learners are inclined to see mathematics as sensible, useful and worthwhile. If the learners are to develop conceptual understanding, procedural fluency, strategic competence and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that, with great effort, it can be learned and used; and that they are capable of figuring it out. There are three aspects to productive disposition. First, the usefulness of the subject mathematics, where the learner appreciates the usefulness of mathematics in their lives. Second, the attitude to the subject mathematics, that is whether the learner has a positive or negative attitude to the subject and lastly, the ability to do mathematics, where the learner believes that he/she has the ability to do mathematics. The teacher of mathematics also plays a critical role in encouraging learners' to maintain positive attitudes towards mathematics. In summary, Table 2.1 provides the characteristics of a learner having proficiency in each strand.

Table 2.1: Classification of the characteristics of learners having proficiency in each strand.

Strand	A proficient learner possesses:
Procedural Fluency	A knowledge of procedures; knowledge when and how to use them appropriately; and skill in performing them flexibly, accurately and efficiently. The ability to estimate the result of a procedure.
Conceptual Understanding	Knowledge of isolated facts and methods. An understanding of why a mathematical idea is important and the kinds of contexts in which it is useful. The ability to represent mathematical situations in different ways, of knowing that different Representations can be useful for different purpose
Strategic Competence	The ability to: Formulate mathematical problems, Represent them, and Solve them. The ability to solve non-routine problems.
Adaptive Reasoning	The capacity to think logically about the relationships Among concepts and situations. The knowledge of how to justify the conclusions. The knowledge to include deductive reasoning; inductive reasoning and intuitive reasoning based on pattern, analogy and metaphor. The understanding to determine whether the procedure is appropriate.
Productive Disposition	The tendency to see mathematics as a coherent subject. The tendency to perceive mathematics as both useful and worthwhile. The tendency to perceive their ability to do mathematics positively.

Relationship between strands

It is important to note that these five strands are not independent but are interwoven and interdependent in the development of proficiency in mathematics. The framework seeks to describe successful mathematics learning. As a result, this proficiency should help them to cope with the mathematical challenges of daily life and help them to continue their study of mathematics in high school and beyond. These five strands will provide me with a framework for discussing the knowledge, skills, attitudes and values that make up mathematical proficiency in the learners. Furthermore, ideas contributing to this framework are supported by research and theories in the cognitive sciences (Olivier, 1989). Mental representation is fundamental in understanding the theory of the five strands. In other words, how learners represent and connect pieces of knowledge is a key factor in whether they will understand it deeply and can use it in problem solving. According to cognitive scientists, ability in an area of learning depends not only on how that knowledge is stored but more importantly on how it is connected and structured that will aid appropriate retrieval and application (Constructivism). Therefore, "learning with understanding is more powerful than simply memorizing because the organization improves retention, promotes fluency, and facilitates learning related materials." (Kilpatrick et al, 2001, p. 118) When learners possess deep understanding, they are able to connect pieces of knowledge, which is necessary for competent problem solving. This highlights the need for the strands to be interwoven. If one or more strands are undeveloped, then learners should not be thought of as having proficiency.

2.1.2 Constructivist perspective

Literature on the constructivist theory

The development of the above strands for mathematical proficiency is best understood through the constructivist perspective. According to this learning theory, concepts are not taken directly from experience, but rather a person's ability to learn from and what he/she learns from an experience depends on the quality of the ideas that he/she is able to bring to that experience (Piaget, 1970 and Skemp, 1979). Therefore, knowledge does not simply arise from experience but it arises from the interaction between experience and the learner's current knowledge structures. As a result, a learner is not a passive recipient of knowledge from the environment and knowledge cannot be transferred intact from one person to another. Therefore, although instruction clearly affects what children learn, it does not determine it, because the child is an active participant in the construction of his/her own knowledge (Olivier, 1989). Here construction takes place through the interaction of a learner's existing ideas and new ideas. The new ideas are also interpreted and understood in the light of the learner's own current knowledge, which is constructed from his/her previous experiences. Thus, the challenge is how to create classroom experiences so that a student's understanding grows over time. As stated in Donovan, M., Bransford, J., and Pellegrino, J. (1999), "students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classrooms" (p. 10)

Assimilation and Accommodation

Learners do not only interpret knowledge, but they also organize and structure this knowledge into larger units of interrelated concepts (schemas) (Olivier, 1989). Therefore, learning is simply the interaction between the child's existing schemas and the new ideas. This interaction involves two processes: Assimilation and Accommodation. Assimilation is the direct incorporation of new ideas encountered into the child's existing schema (Ibid). Through this process, the new idea contributes to the schemas by expanding existing concepts. Accommodation, is when a new idea is very different from the existing schemas; therefore, there is a need for the child to re-organize/restructure her/his existing schemas. It is important to note that this process leaves the previous knowledge intact, that is, previous knowledge is not erased.

Rote versus understanding

Understanding is vital for learners to become proficient in mathematics. According to the constructivist theory understanding an idea means to incorporate it into an appropriate existing schema. The problem arises in mathematics when the new idea may be so different from any available schema, that it is impossible to link it to any existing schemas, that is assimilation or accommodation is impossible. As a result, the learner creates a new "schema" and tries to memorize the idea. This is rote learning because it is not linked to any previous knowledge, it is not understood; it is isolated knowledge therefore it is difficult to remember (Olivier, 1989). This rote learning is the cause of many mistakes in mathematics as learners try to recall partially remembered and distorted rules. Because knowledge cannot be transferred intact and to support a learner to construct his/her own knowledge; communication, discussion, reflection and negotiation should be an essential components of our mathematics curriculum in South Africa. In this way, it is envisaged that our learners can become more proficient mathematically. How learners represent and connect pieces of knowledge is also a key in whether they will understand it deeply and can use it in problem solving. According to the constructivist theory, competency in mathematics depends upon knowledge that is not merely stored but represented mentally (schemas) and organized (assimilation or accommodation) in ways that facilitate appropriate retrieval and application. Therefore, learning with understanding is more powerful than simply memorizing because the organization improves retention, promotes fluency, and facilitates learning related material (Kilpatrick al, 2001).

Link between strands

The conceptual understanding strand is the most important of all strands because without proper understanding the link to the other strands will be flawed, thus resulting in misconceptions, which lead to errors in mathematics. According to research, misconceptions generate errors (Olivier, 1989). This is crucial to the learning and teaching of mathematics because misconceptions form part of a learner's conceptual structure that will interact with new concepts and definitely influence new learning, mostly in a negative way. In order for the five strands of mathematical proficiency to be useful/effective, they must be interwoven. Thus, a deep understanding is required to connect pieces of knowledge, and this connection is vital in whether learners can use what they know effectively in solving problems. "Metacognition can be defined as the knowledge about one's own thinking and ability to monitor one's own understanding and problem-solving activity" (Kilpatrick et al, 2001, p. 118) The preceding quote supports the two strands, Strategic Competence and Adaptive Reasoning. Lastly, learning is also influenced by motivation, which is a component of Productive Disposition (Hiebert and Carpenter, 1992). The above learning theory (constructivist) supports the kinds of cognitive changes necessary in mathematics learners so they can be successful (proficient) in learning mathematics.

As can be seen from the above constructivism is consistent with mathematical proficiency as opposed to other learning theories such as Behaviourists. This research is concerned with the end product of learning in the classroom where learners make their own knowledge therefore the constructivists theory best explains this type of learning.

2.2 THE MATHEMATICS/ MATHEMATICAL LITERACY DEBATE

In the next section, I will define and describe a key component of my research study namely number skills. This will be followed by a brief discussion on the two types of mathematics (that is pure mathematics and mathematical literacy) within the South African curriculum framework. Lastly, this section will focus on the international and local perspectives of mathematical literacy.

2.2.1 Terminology

Stoesigger (2003) defines number skills as the type of mathematical skills needed to function in everyday life, in the home, workplace and community. It is also interesting to note that internationally, Mathematical Literacy is referred to as Quantitative Literacy (in the United States) and Numeracy (in the England). Numeracy has similar definitions all around the world, but Stoesigger (2003) terms it as critical numeracy, which he claims is politically loaded. In other words, it is not only about numbers but also is culturally based and socially constructed. In the South African context, numeracy is often related to Adult Basic Education necessary for coping with workplace demands, for further formal training and handling practical tasks (at home; shopping; etc.). However, today mathematical literacy has a much broader application including all forms of mathematics relating to real-life.
In this regard, it is also important to note that numeracy and numeracy skills are not one and the same thing. Numeracy (ML) is a subject/course that one can study, whereas numeracy (number) skills are the procedures that one can achieve within the subject of Numeracy (Stoesigger, 2003).

2.2.2 Pure mathematics and mathematical literacy

According to Madison (2004), there are two types of mathematics, that is, the rigorous mathematics that real mathematicians study, appreciate and extend and the contextualized mathematics of everyday life. He believes that mathematical literacy (ML) is the combination of arithmetic within a social context. ML is different from mathematics in that in mathematics students learn abstract concepts before learning to apply them, whereas in ML the learner looks at a number of applications and then extracts the abstractions, something that is common in business and engineering. Within the South African context, learners undertake the school subject mathematics for nine formal years. Thereafter, learners have a choice in either choosing pure mathematics or mathematics literacy in their final phase of schooling. This final phase is known as the Further Education and Training phase in South Africa.

The literature on mathematical literacy supports the idea that this discipline focuses on the functional use of mathematics in real life (Madison, 2004). In the following section, I have decided to discuss mathematical literacy, using international and local literature.

2.2.3 International and local researchers' perspectives on mathematical literacy

In this section, the first four aspects are arguments for the introduction of mathematical literacy and the fifth is an argument for assessment reform in mathematics. Firstly, there is a need for curriculum transformation, in order to help learners cope with their ever-changing environment. Secondly, there needs to be a new way learners are taught mathematics at school. Thirdly, mathematical literacy is necessary for all South African citizens and not just an elect few. Fourthly, school mathematics must help learners solve non-routine everyday problems and lastly, if a new mathematics curriculum is introduced, then it is necessary to also introduce a new assessment strategy.

The need for curricular reform

The mathematics students need to learn today is not the same mathematics that their parents and grandparents needed to learn. When today's students become adults, they will face new demands for mathematical proficiency that school mathematics should attempt to anticipate. Brombacher (2001), a South African scholar, does not see mathematical literacy as developing a lot of new mathematical knowledge, but rather applying and analyzing General Education and Training (GET) mathematics in different contexts. However, mathematics is a subject no longer restricted to a select few. All students, young and old, must learn to think mathematically; and they must think mathematically to learn (Kilpatrick et al, 2001). In primary school, the mathematics curriculum should prepare students for the study of senior algebra, so it should also include attention to other domains of mathematics. Students need to learn to make and interpret measurements and to engage in geometric reasoning. They also need to gather, describe, analyze, and interpret data and to use elementary concepts from probability. Instruction that emphasizes more than a single strand of proficiency has been shown to enhance students' learning about space and measure and shows considerable promise for helping students learn about data and chance (Ibid).

According to Schoenfield (2001), formal mathematics provided him an impoverished education because it has excluded real-life concerns. As opposed to formal mathematics, ML must be practiced within a variety of contexts that will aid repetition and experience, which eventually leads learners to develop habits rather than rote learning or application with no meaning. He further states that on tests of conceptual understanding and problem solving, students who learn from reform curricula consistently outperform students who learn from traditional curricula by a wide margin. On tests of basic skills, there are generally no significant differences between students who learn from traditional or reform curricula. (Schoenfeld, 2001). The following extracts support Schoenfield argument:

> Students who memorize facts or procedures without understanding often are not sure when and how to use what they know and such learning is often quite fragile. (Bransford, Brown, and Cocking, 1999, cited in NCTM, 2000, p. 20)

> There is a long history of research, going back to the 1940s and the work of William Brownell, on the effects of teaching for meaning and understanding in mathematics. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention, and an increased likelihood that the ideas will be used in new situations. These results have also been found in studies conducted in high-poverty areas (Grouws and Cebulla, 2000, p. 13)

> Instructional programs that emphasize conceptual development, with the goal of understanding, can facilitate significant mathematics learning without sacrificing skill proficiency. (Hiebert, 2003, p. 16)

> Students who develop conceptual understanding early perform best on procedural knowledge later. (Grouws and Cebulla, 2000, p. 15)

From my experience, some teachers do not see the distinction between the two types of mathematics. As a result, there is much debate about the compatibility or incompatibility of mathematics and mathematical literacy. Therefore, as a solution, Madison (2004) proposes that these two cohorts can be integrated, where the mathematics taught can be contextual and apparently more relevant to the learners' real life society.

According to Labaree (1999), during the past century calls for reform have had remarkably little effect on the character of teaching and learning in American classrooms. Furthermore, in traditional classrooms the mathematical content is separated from practical problem situations and taught in isolation from other subjects where students are differentiated by ability and sequenced by age and instruction is grounded in textbooks and delivered in a teacher-centred environment. Therefore, the quality of student performance should be judged in terms of whether students are mathematically literate. This means information should be gathered about what concepts and procedures students know with understanding and how students can use such knowledge to mathematize a variety of problem situations. Only then can one judge whether student performance meet society's needs.

The need for effective teaching

Teaching that fosters the development of mathematical proficiency over time—has a variety of forms, each with its own possibilities and risks. All forms of instruction can best be examined from the perspective of how teachers, students, and content interact in contexts to produce teaching and learning (Kilpatrick et al, 2001). The effectiveness of mathematics teaching and learning is a function of teachers' knowledge and use of mathematical content, of teachers' attention to and work with students, and of students' engagement in and use of mathematical tasks. Furthermore, effective programs of teacher preparation and professional development help teachers understand the mathematics they teach, how their students learn that mathematics, and how to facilitate that learning. In these programmes, teachers are not given prescriptions for practice or readymade solutions to teaching problems. Instead, they adapt what they are learning to deal with problems that arise in their own teaching. Unfortunately, schools offering the Further Education and Training band might not be ready to cope with the teaching of Mathematical Literacy because the teaching of mathematics (both cohorts) is compulsory; therefore, the majority will probably choose Mathematical Literacy (Brombacher, 2001). This is a cause for concern because there are unlikely to be enough qualified mathematics teachers available to teach Mathematical Literacy and as a result the majority of these schools might employ the services of under or unqualified mathematics teachers. This will definitely have an adverse effect on the progress of these learners.

Mathematical literacy for all

Schields (2002) notes that learners/citizens that lack this literacy (ML) are in fact illiterate and thus not productive and informed workers. When the mathematics is based on the context, students are stimulated to think in a complex manner, expand their understanding and link content to practice in the real world. Students (both primary and secondary) are lacking proficiency in “number sense” skills, that is, attaching meaning to numbers. The majority of adults need skills to be able to plan and handle the use of resources (such as money, supplies or time). Kilpatrick et al (2001) argues that even from the youngest age children should be encouraged to enjoy playing with numbers, exploring how they work in a variety of situations, and developing fluency and flexibility in their use. It is likely for instance that some difficulties with place value and subtraction may have their roots in lack of fluency with counting on and counting back and with number sequences and patterns. Many children when asked to perform a subtraction, such as, "I had six lollies and I gave 2 to my friends. How many lollies do I have left?" will solve the problem by counting on from 4 rather than counting back from 6. Most early subtraction is done by counting on and adding and for some students the understanding of take away is very difficult to carry out. If we relate this back to number and number sense, it seems likely that more time needs to be spent exploring counting on and back from different starting numbers and in different amounts so that fluency and flexibility is achieved. Mental routines can target these areas effectively and with fun and understanding. Unfortunately, these skills greatly differ from those needed to solve word problems, which simulate real-world issues.

In this regard, The Programme for International Student Assessment (PISA) document defines mathematical literacy as “… the capacity to identify, understand and engage in mathematics, and to make well founded judgements about the role that mathematics plays in an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen (OECD, 2001, p. 22). Therefore, the PISA document is proposing that mathematical literacy not only involves solving problems but is also a subject that will help learners comprehend their world. This idea is supported by another international scholar, Wallace (2000), who states that for adequate knowledge and skills in mathematics, "it is necessary for nearly everybody to be able to read, do basic arithmetic, and even understand what a function is and how it can express information" (p. 31). However, it is unreasonable to state that almost everybody must understand the complexities of logarithms or calculus. In other words, South Africa does not need everybody proficient/literate in pure mathematics, but rather only in contextual mathematics called mathematical literacy.

However, mathematical literacy that involves a non-routine pattern of instruction that allows students to become mathematically literate is not easy to create. According to Brombacher (2001), the majority of learners, completing grade nine (NQF 1) in South African schools, are not proficient enough to enter mathematical literacy. In this regard, he states that mathematical literacy could be interpreted by the public as the new mathematics standard grade. He is emphatic about this. It is not! Mathematical literacy is a different kind of mathematics, not a different, lower level of mathematics. Mathematical literacy will be at least as demanding as mathematics to teach and certainly as challenging for pupils to learn. The above misconception of mathematical literacy will thus compromise the successful implementation of this subject. Nevertheless, it is possible, and with appropriate guidance from teachers, students can learn to mathematize. Unfortunately, the problem with the vision of school mathematics is that they are ideas put forward by educational leaders, policymakers, and professors about what mathematical content, pedagogy, and assessments should be. Therefore, according to Brombacher (2001), one of the most significant concerns regarding mathematical literacy is that the DoE (in South Africa) assumes that learners beginning at the grade ten level are proficient in the mathematics at NQF level 1.
A number of factors can undermine implementation of such ideals. For example, not everyone agrees with the goal of mathematical literacy for all; some influential persons believe that the traditional course of study for school mathematics works reasonably well (Romberg, 2001).

Social function of mathematical literacy – mathematising

According to Romberg (2001), in mathematical literacy the emphasis is on the use of contexts in varied, reflective and creative ways. However, for such use to be possible and viable, a great deal of fundamental mathematical knowledge and skills are needed. In a broader sense, the term "literacy" refers to the human use of language. In fact, one's ability to read, write, listen, and speak a language is the most important tool we have through which human social activity is mediated. In this regard, mathematics as a language implies that students not only must learn the concepts and procedures of mathematics, but they must learn to use such ideas to solve non-routine problems and learn to mathematize in a variety of situations (its social functions). The above was well illustrated by this example put forward by Organization for Economic Co-operation and Development, (2001 p.3):

> The Town Council has decided to construct a streetlight in a small triangular park so that it illuminates the whole park. Where should it be placed? This social problem can be solved following general strategy used by mathematicians, mathematizing, that can be characterized as having five aspects:
>
> 1. Starting with a problem situated in reality (Locating where a street light is to be placed in a park).
> 2. Organizing it according to mathematical concepts (The park can be represented as a triangle and equal illumination from a light as a circle with the street light at its centre).
> 3. Gradually trimming away the reality through processes such as making assumptions about what are the important features of the problem, generalizing and formalizing (in this case the problem is transformed to locating the centre of a circle that circumscribes the triangle).
> 4. Solving the mathematical problem (Using the fact that the centre of a circle that circumscribes a triangle lies at the point of intersection of the perpendicular bisectors of the triangle's sides, construct the perpendicular bisectors of two sides of the triangle. The point of intersection of the bisectors is the centre of the circle).
> 5. And, making sense of the mathematical solution in terms of the real situation (Relating this finding to the real park. Reflecting on this solution and recognizing that if one of the three corners of the park was an obtuse angle, this solution would not be reasonable since the location of the light would be outside the park).

It is these processes that characterize how mathematicians often do mathematics, how people use mathematics in a variety of current and potential occupations, and how informed and reflective citizens should use mathematics to fully and competently engage with the real world (Romberg, 2001). In fact, learning to mathematize should be the primary educational goal for all students.

New assessments required

The current instruments, in America, commonly used to assess student performances in mathematics were not designed to assess mathematical literacy (Kilpatrick et al, 2001). This also holds true for the new South African mathematics curriculum (NCS) because the tests used by schools typically measure the number of correct answers "a student can give to questions about knowledge of facts, representing and recognizing equivalents, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, manipulating expressions containing symbols and formulae in standard form, and doing calculations." (Romberg, 2001, p. 8) At best, these tests measure a student's knowledge of some of the procedures associated with mathematical literacy. It is questionable as to whether such instruments measure understanding of such procedures. Moreover, the majority fail to make any serious attempt to assess student capability to mathematize. Thus, to assess the intended impact of these tests in mathematics a new assessment system will need to be developed.

2.3 CONTEXTUALISING NUMBER SKILLS WITHIN THE SOUTH AFRICAN SCHOOL CURRICULUM

In this section, I will briefly discuss the historical background, post democratic initiatives and the curriculum: Learning Outcomes for mathematics and mathematical literacy as stipulated in the South African National Curriculum Statement (Department of Education, 2003a, 2003b) with particular focus on the assessment standards related to number skills.

2.3.1 Historical background

One source of information about mathematical learner performance in S.A is the Third International Mathematics and Science Study (TIMSS), an assessment of learners' knowledge and skills in mathematics. TIMSS 1995 and 1999 included surveys on the mathematical proficiency of Grade seven/eight pupils from 28 countries. This survey was repeated in 2003 in 50 countries, which included two developing African countries. Disappointingly, South Africa was ranked last in both studies (HSRC, 2000). This is a serious problem for S.A. because fewer pupils choose mathematics than mathematical literacy and in addition, there is a decrease in mathematically proficient pupils entering the workplace (Ibid). Furthermore, the TIMSS study revealed that our pupils had trouble with the interpretation of tables and figures, and were unable to express themselves in writing. Pupils also found difficulty in comprehending word problems and to solve problems in writing because the majority of learners writing the test in English would be attending African schools and English would not be their first language. In addition, pupils struggled in communicating the answers in the language of the test and they lacked the basic mathematical knowledge expected at the Grade 8 level (HSRC, 2000).

The HSRC (2000) also revealed that significant gender differences were found only at school leaving age, at which point males outperformed females. Furthermore, while females achieved at higher levels than boys at grade six did, males achieved to a slightly higher level in mathematics literacy at school leaving age.
South African pupils performed poorly when compared to other participating countries. The mean score of 275 is well below the international mean of 487. The result is significantly below the mean scores of all other participating countries, including the two other African countries of Morocco and Tunisia. Only 1% of South African pupils reached the International Upper Quarter benchmark – the average score achieved by the top 25% of pupils internationally – which corresponds to a score of 555 points. This is a great contrast to the Asian countries where the benchmark was reached by more than 60% of pupils from Japan, Hong Kong, Korea and Chinese Taipei and 75% of Singapore pupils. The top 25% of South Africa's pupils achieved 337 out of 800 (42%). Overall, the South African results appear very low in comparison to all the other countries participating. South African pupils' performance was relatively low in every content area. A detailed analysis shows that pupils have trouble with the interpretation of tables, figure and illustrations. They struggle with complex questions requiring more than one-step and appear unable to express themselves in writing. Difficulties were noted where pupils were required to comprehend word problems and to articulate and solve problems in writing. Pupils also had considerable difficulty dealing with fractions

and with geometry questions regarding calculating "area". In general, when faced with multiple-choice questions pupils resorted to guessing the answer and in some cases were successfully distracted by questions testing misconceptions.

2.3.2 Post democracy iniatives

One of the issues the first democratic government in South Africa had to tackle was to transform the racially based school education system. The new system should reflect the values being striven for by the new society and should prepare the students to graduate from schools better prepared to participate in the economic development of the country. Through a series of consultations and negotiations with a variety of stakeholders ranging from school student organizations to the labour movement, it was concluded that the best way to realise the goals of the “new” society was to organise the curriculum around outcomes-based education (OBE). It is important to note that the Learning Outcomes of OBE were used in order to improve on the past educational outcomes. The traditional system in the apartheid era was replaced by the Outcomes Based Education (OBE) system, which was criticised for containing too many outcomes (Brombacher, 2001). As a result, OBE failed leading to the development of the National Curriculum Statement. This research on mathematical proficiency is at a different time (as compared to ten years ago), where context supersedes content/knowledge and rote learning of skills is underplayed. From my experience, the new curriculum tends to concentrate on high skills, knowledge, attitudes and values but to varying degrees. Therefore, we need to investigate whether our learners will become mathematically proficient. This is what this research will try to answer.

2.3.3 The curriculum

For the purpose of this research, I focused on the development of mathematical proficiency of grade ten learners in LO1 (number senses), for both cohorts. Table 2.2 displays the Learning Outcomes in both the GET and FET phases. In this research, I have considered just four outcomes.

Table 2.2: Learning outcomes in GET and FET phases

PHASES			
LO	**GET**	**FET**	
		MATHS	**ML**
LO 1	Numbers, operations and relationships	Number and number relationships	Number and operation in context
LO 2	Patterns, functions and algebra	Functions and algebra	Functional relationships
LO 3	Space and shape (geometry)	Space, shape and measurement	Space, shape and measurement
LO 4	Measurement	Data handling and probability	Data handling
LO 5	Data handling		

The NCS described LO 1 for Mathematics as: When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions, and the NCS has described LO 1 for Mathematical Literacy as: The learner is able to use numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues. In order for learners to successfully achieve LO 1, in both cohorts, all five strands of mathematical proficiency must be developed.

2.4 FACTORS INFLUENCING THE ACHIEVEMENT OF MATHEMATICAL PROFICIENCY

Mathematical proficiency can be influenced by many factors. In this section, I will discuss three of these factors namely; misconceptions and inadequate basic skills, a lack of motivation on part of the learner, parent and mathematics teacher and finally the influence of the gender factor on achievement of mathematical proficiency.

2.4.1 Inadequate basic skills and entrenched misconceptions

First, one of the most challenging tasks encountered by educators in grade one to grade eight is to see that children are making progress along every strand and not just one or two (Kilpatrick et al, 2001). This is a major reason for inadequate/poor basic skills in our learners approaching secondary schooling. For example the simple concept of even and odd require an integration of several ways of thinking such as grouping items by twos, grouping items into two groups, choosing alternative points on the number line and looking at only the last digit of the number (Ibid). A child at grade one may know one or two of these interpretations, but at an older age a much deeper understanding of even and odd means all four interpretations are connected and can be reasoned one based on the other. If these connections cannot be made, the child is disadvantaged with having inadequate basic skills for even and odd numbers in their senior years.

Second, from a constructivist perspective misconceptions form part of a learner's conceptual structure that will interact with new mathematical concepts and influence new learning, mostly in a negative way, because misconceptions generate errors (Olivier, 1989).

2.4.2 Lack of motivation

According to the Mathematics Association of America (2005), the fear of mathematics that is often called "math phobia" or "math anxiety" stunts the cognitive development of those who suffer from it. It is usually learned, not inborn, and a curricular component devoted to promoting quantitative literacy, if competently and compassionately taught, can be powerfully therapeutic against it.

In addition, the negative attitudes of parents and teachers (including guidance counsellors) toward mathematics are all too easily picked up by the next generation. Statements like "Oh, I never was good at math myself" or "Just get this math out of the way and then forget it; you'll never need it again" or "For punishment, you will have to do thirty extra math problems" can do enormous amounts of damage to the learner's motivation. The above is supported by Kilpatrick et al (2001) who stated that if learners' are to develop mathematical proficiency, they must believe that mathematics is understandable and with great effort, it can be learned and used. In addition, he comments on the vital role of all stakeholders namely; parents, teachers and policy-makers, in positively influencing the learner in mathematics. The factor, lacking in motivation, overlaps with Production disposition, which is the fifth strand of mathematical proficiency. Therefore, it is imperative for learners to view mathematics as useful, sensible and worthwhile for their lives (Ibid).

2.4.3 Gender

It is important for us to know that eighteen years ago females begin in high school to perform less well than males on mathematical problem-solving tasks (Kimball, 1989). During the past century, attempts for change have had remarkably little effect on the character of teaching and learning in the classroom (Labaree, 1999). The mathematics teachers must take more responsibility in the teaching of problem solving, both because it is an important area of mathematics and because it is an issue of gender equity. Boys may have more access to problem-solving experiences outside the mathematics classroom than do girls, creating boys' pattern of better performance (Ibid). In explaining the lesser presence of women in college-level mathematics courses and in mathematics-related occupations, we must look to other factors, such as internalized belief systems about mathematics, external factors such as sex discrimination in education and in employment (Kimball, 1989), and the mathematics curriculum at the primary and secondary level.

The results of TIMSS for 9 and 13 year old students (Lokan, J. Ford, P. and Greenwood, L (1997), TIMSS data for Australian Grade 12 students showed that boys were significantly ahead of the girls in mathematical literacy (Mullis, I. V. S., Martin, M. O., Fierros, E. G., Goldberg, A. L. and Stemler, S. E (2000). Traditionally, females have found advanced mathematics achievement elusive. Girls' mathematic achievement in the primary school is equal to boys' but decreases in the junior high school (Ibid). Research evidence suggests that boys’ and girls’ experience of mathematics is still influenced by the idea — disproven by achievement statistics — that males do better at mathematics then females (Leder and Forgasz, 1997). However, males see mathematics as a male domain rather more than do than females. This has led some researchers to speculate that it may be the behaviour of males that deters females from full participation in mathematics (Frost, L. A., Hyde, J. S., and Fennema, E, 1994). However, the TIMSS study in South Africa for 2000 showed no significant difference in achievement for mathematics for either males or females (HSRC, 2000).

2.5 CONCLUSION

Proficiency in mathematics is an important foundation for further teaching in mathematics as well as for future education in areas that require mathematical competence. For learners to be able to compete in today and tomorrow's world, they need to be able to adapt the knowledge they are acquiring; apply mathematical reasoning to problems and view maths as a useful tool. In other words, they need to be mathematically proficient. In South Africa, citizens lack this literacy (ML) which leads to unproductive and uninformed workers. In my opinion, these citizens have greatly disadvantaged themselves and society, because they are not Mathematically Literate.

This chapter has provided an overview of the literature concerned with mathematical proficiency and its links to schools and society. In the next chapter, the research design and methodology employed in the research will be discussed.

CHAPTER 3

The main objective of this study was to describe mathematical proficiency, in the domain of number skills, of grade ten learners in both the Maths and Maths Literacy cohorts. This chapter will describe the research design and methodology including the sample, research instruments used, method of data collection and data analysis. Issues of ethics and the limitations of the study will also be discussed.

3.1 RESEARCH METHOD

This research (quantitative) aims is to determine the relationship between one thing (mathematical proficiency) and another (May/October-test) in a population (grade ten learners).

A May-test/October-test comparison was used for the purpose to allow relatively straightforward assessment of a pedagogical intervention by detecting differences in learning outcomes between two points in time – before and after it (RQ3). With regards to this research, the description of the May/October-test comparisons consisted of the following simple steps: decided what learning outcomes are of interest (learning outcome 1 – number sense), found or created the measures to capture those (May-test and October-test instruments), assigned students to groups (all grade ten learners), administered the May-test, administered the October-test, and then analyzed. The May-test/October-test comparison design for this research was a prospective case study design in which students were assigned to groups (either pure mathematics or mathematical literacy) and identical measures were used to assess the learning outcome of each group. The case study design provided the most reliable information on the effectiveness of the pedagogical intervention.

This research also employed a delayed test design (October-test), which enabled the researcher to assess the more prolonged effects of their course. Changes in fundamental reasoning or beliefs are good candidates as such (Campbell and Stanley, 1963).

3.2 PHILOSOPHICAL ORIENTATION OF THE STUDY

This study has drawn on two paradigms namely, the positivist and interpretive paradigms. Positivists claim that the truth, obtained from facts, can override the individual's/researcher's opinions and their biases (Denzin and Lincoln, 2003). According to positivists, research is value-free with respect to the researcher (myself) and researched (grade ten learners). They also believe that the culture, traditions and attitudes of the community (learners) has no influence on the truth. They claim that they have a stable and unchanging reality; which can be researched by using empirical methods. This study is orientated towards the Positivist approach. I have statistically contrasted and compared my research data for the May-test and October-test with respect to mathematical proficiency in number sense. According to Usher (1996), in any research, all human action is meaningful and therefore has to be interpreted and given meaning within the context of social practices. The difference between the Positivist and Interpretive approaches is that in any research study both the researcher and other people have the same characteristics of being meaning makers. Also the construction of knowledge is perceived as being circular rather than linear in nature, as conceived by the Positivists. Furthermore, the attitudinal aspect of this research is interpretive because the researcher is interested in the subjective worlds of his research participants.

3.3 CASE STUDIES

According to Cohen and Manion (1989) a case study is where the "researcher typically observes the characteristics of an individual unit – a child, a clique, a class, a school or a community. The purpose of such observation is to probe deeply and to analyse intensively the multifarious phenomena that constitutes the life cycle of the unit with a view to establishing generalisations about the wider population to which that unit belongs." (pp. 124-125) (cf Bassey, 1999). This research design is a descriptive case study. A descriptive case study is a complete description of a phenomenon within its context (Ibid). The case (unit of analysis) in my study was all the grade ten learners in a particular school. Furthermore, it was bounded in time, from May to October 2006, as well as place, one school. The results from this study can be generalised to other schools with similar characteristics as to my school. My research study involved both the quantitative and qualitative methods for data collection. Quantitative methods were based on scientific considerations such as accuracy, objectivity, reliability and credibility. I adopted a descriptive case study method, which involves an in-depth examination of a single instance or event (all grade ten learners in a specific secondary school). This method provided a systematic way of looking at events, collecting data, analyzing information, and reporting the results. As a result, I gained a sharpened understanding of why the instance happened as it did, and what might become important to look at more extensively in future research. This descriptive research design consisted of a baseline May-test and a follow-up October-test conducted at the predetermined follow-up time. In this follow-up study, the baseline results are compared to the observed outcomes of the follow-up process, and thus the results is evaluated.

3.4 RESEARCH CONTEXT AND SAMPLING

The research was conducted at Temple Valley Secondary School, located on the outskirts of a small town in the north of Durban. There were fourteen classes ranging from grade 8 to 12. The school was in a not-so-well-to-do community, where the majority of learners came from a disadvantaged background. The pupil composition was made up of Indian, Black and Coloured learners, where Blacks were in the majority implying an uneven nature of the sample.

In order to obtain data, a convenient sampling technique was used in selecting all grade ten learners in the school. However, to obtain rich qualitative data, a purposeful sampling technique was employed for the focus group interviews. Creswell (1998) argues that the purposeful selection of participants represents a key decision point in qualitative research and the researcher needs clear criteria and rationales for decisions. My questionnaire was administered to 181 learners of whom 28 studied mathematics and the remainder (153), mathematical literacy. This research sample consisted of 111 females and 70 males in total. The racial breakdown is about 161 Black pupils and the remaining 20 Indians. There also were three interview focus groups, comprising of six learners each.

3.5 RESEARCH DESIGN AND FIT TO RESEARCH QUESTIONS

The following four questions frame this study:

1) What is the mathematical proficiency of grade ten Mathematical Literacy learners in the domain of numbers?
2) How does the mathematical proficiency of the grade ten Mathematical literacy learners compare with the mathematical proficiency of the grade ten Pure Mathematics learners?
3) How did the mathematical proficiency change over a four month period in both cohorts for these grade ten learners?
4) How did gender and misconceptions influence the achievement of the mathematical proficiency of these grade ten learners?

The purpose of this research was to answer the research questions (RQ) and this is how I went about trying to achieve this. Table 3.1 depicts the match of data collection instruments to the four research questions.

Table 3.1: Match of data collection instruments to Research Questions.

Instruments	RQ 1	RQ 2	RQ 3	RQ 4
May-test	X	X		X
Attitude (May)	X	X		X
October-test			X	
Attitude (Oct.)			X	
Interviews	X	X		X

Data was collected through structured tests, questionnaires and focus group interviews. I chose to assess Learning Outcome 1: Numbers, Operations and Relationships. Therefore, I only choose question that pertained to this specific Learning Outcome. The qualitative data collected (questionnaires and interviews) helped in representing the data in descriptive qualitative ways. Quantitative methods (May-test and October-test) helped in collecting data in quantified form. Due to the nature of this study, it was only possible to study the attitudes of learners towards mathematics through qualitative methods, namely questionnaires and focus group interviews. Although the Likert scale in the questionnaire (quantitative method) was also useful in supplying valuable data about the participants, there was a need for not only objective methods but also subjective methods, which provided me with ‘rich’ data on the participants. Thus, this is broadly interpretive research because I am interested in the subjective worlds of my research participants, that is, their meanings, feelings and attitudes; which I will try to understand by interpreting their responses according to the five-strand framework of mathematical proficiency.

3.6 DATA GENERATION AND ANALYSIS

In this subsection, I will discuss the sampling procedure and the data collection instruments used in the research process.

3.6.1 <u>The research instruments</u>

Pilot study

The May-test and October-test were tested on a group of grade 9 mathematics students from the same secondary school under study. The questionnaire was also tested on my wife, a friend and my supervisor. The results from these pilot studies helped me to rephrase and clarify questions where necessary. It also helped me establish the amount of time needed to complete the tests and questionnaires respectively. This pilot study was very important because I have learnt not to take the meaning of certain questions for granted. In other words, if the researcher understands a question in a certain way, this does not mean that others (grade ten students especially) would. This has thought me to avoid ambiguity. Initially there were too many questions included in the tests and this pilot study helped me identify this problem. If this problem were not identified, the entire research results would have been jeopardised.

Structured tests (Instrument one)

Construction of test:

The May-test (see Appendix A) and October-test (see Appendix B) where each consisted of twenty-four multiple-choice questions and three extended response questions. The May-test and October-test were similar with respect to the first twenty-four multiple-choice questions but in the October-test, these multiple-choice questions were re-ordered to bring about some variation from the May-test. However, the extended response questions, in the May-test and October-test, were different. These open questions required learners to explain/justify their responses. I used TIMSS

(HSRC, 2000) and NAEP (Department of education, 2003) question banks to draw some important questions that assisted me in better understanding the proficiency of these learners. Mathematics educators and mathematicians developed these questions. When the questions were selected, I arranged the questions from easiest to most difficult for both tests. This was necessary to help give pupils the confidence needed in attempting these tests. Furthermore, the pupils would find the incentive to continue answering the tests. These questions were reviewed to ensure that potential sources of bias were identified and eliminated. In this research, a May/October-test comparison was used to detect differences in learning outcomes between two points in time. This assessment strategy is very common in educational research since its implementation is relatively non-intrusive and its analysis does not normally require more advanced statistical procedures (Campbell and Stanley, 1963). Refer to Appendix F for the classification and description of these questions.

The test items were chosen for their match to the GET curriculum (LO1) which was what was being tested at the beginning of grade ten. Although the TIMSS test items were intended to be for grade 8 learners, they constitute a bank of standardized questions. I have also included some open-ended questions into my research instruments because these questions will promote imagination and creativity and assess higher order skills of mathematics learners. Furthermore, it will help promote the testing of the learners' ability to analyse and solve problems, to communicate clearly in writing and ability to express their own opinion (Criticos, 2002). The May and October tests only assessed the four content dependent strands of mathematical proficiency: Procedural Fluency, Conceptual Understanding, Adaptive Reasoning and Strategic Competence.

The test questions measure number sense, properties, and operations. The content area focused on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, and complex numbers), operations, estimation, and applications to real-world situations. Students were expected to demonstrate an understanding of numerical relationships as expressed in ratios, proportions, and percents. Students were also expected to understand properties of numbers and operations, generalize from numerical patterns, and verify results. Number sense includes questions that address a student's understanding of relative size, equivalent forms of numbers, and use of numbers to represent attributes of real-world objects and quantities. The tests will contain all the necessary biographical information needed for my research.

I used the Revised National Curriculum Statement Grades R-9 (schools) policy document for Mathematics reference to place each question into its correct category, Learning Outcome and Assessment Standard. In order to categorize each question into a specific cognitive strand, I used Kilpatrick's (2001) strands of mathematics proficiency. I applied the five strands of mathematical proficiency for the construction of the tests (Kilpatrick et al, 2001).

I outlined the criterion for categorizing each question into its respective chief targeted strand (See Appendix F).

Table 3.2: Classification rules for questions for May and October-test

Strand	Criteria
Procedural Fluency	The problem must be context free It must involve a calculation to arrive at an answer An operation is given (or is obvious)
Conceptual Understanding	The problem must rely on in-depth knowledge of concepts more than learnt procedures It must involve translations between representations
Adaptive Reasoning	The problem must involve logical reasoning about numbers in context It must also involve the justifying of answers There must be no computations necessary to arrive at the answers
Strategic Competence	The problem must involve the extracting of mathematics from words/problems It must involve the choice of an operation/strategy It must include non-routine problems

I used Table 3.2 for consistency in the selection criteria. Conceptual Understanding (CU): I assessed the learners' conceptual understanding of numbers, in part, by asking them about the properties of the number system. For example with rational numbers, I asked them to list the numbers from smallest to largest. These types of questions helped me understand whether learners really understand the numbers they calculated/manipulated. I also included a few non-routine problems to assess learners' conceptual understanding.

Procedural Fluency (PF): I assessed the learners' procedural fluency of whole numbers by asking them to add/subtract two-and three-digit whole numbers. These questions assessed the proficiency of procedural fluency in the easiest context. Research has shown that learners are less fluent in operating with rational numbers, both common and decimal fractions (Kilpatrick et al., 2001). Therefore, I included problems involving the addition, subtraction, multiplication and division of these fractions.

Strategic Competence (SC): I included basic whole number and rational operations and concepts in numerical and simple applied contexts to assess problem-solving abilities of learners. I also included a more complex problem solving-question to differentiate how learners respond to these and the easier context problems. The complex problem included more than one-step or extraneous information. These problems had small changes, as compared to the more complex problems, with respect to wording, context and presentation because I wanted to assess learners' abilities in simple/typical problem-solving situations.

Adaptive Reasoning (AR): Seven questions were chosen to measure the learners' proficiency in adaptive reasoning, but in conjunction with other strands. For example I asked learners to estimate which circle has approximately the same fraction shaded as that of the rectangular blocks (see May-test question 5). This question only required that basic understanding and reasoning be connected. Furthermore, I included a question that asked learners to justify and explain their solutions. These types of questions also measured adaptive reasoning.

It is important to note that these five cognitive strands (Kilpatrick et al, 2001) are interwoven and interdependent in the development of proficiency in mathematics. I decided on the chief targeted strand of each question; that is the strand which best describes the knowledge, skills, abilities and beliefs that constitute the question. To understand the column ‘Reason’ (Appendix F), please refer to Table 3.2.

The column ‘Complexity’ (Appendix F) reveals the level of difficulty of each question in the question bank. I categorized each question according to their mathematical complexity, which describes the cognitive demands of the questions. There are three levels of mathematical complexity (Department of Education, 2003):

1) Low complexity (LC): This category relies heavily on the recall and recognition of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution.

2) Moderate complexity (MC): Items in the moderate-complexity category involve more flexibility of thinking and choice among alternatives than do those in the low-complexity category. They require a response that goes beyond the habitual, is not specified, and ordinarily has more than a single step. The student is expected to decide what to do, using informal methods of reasoning and problem-solving strategies, and to bring together skill and knowledge from various domains.

3) High complexity (HC): High-complexity items make heavy demands on students, who must engage in more abstract reasoning, planning, analysis, judgment, and creative thought. A satisfactory response to the item requires that the student think in an abstract and sophisticated way.

I had decided to select twenty-four multiple-choice questions and three short-response questions. This gave me a total of twenty-seven questions for my May and October tests. I balanced the distribution of questions in the following manner: Procedural Fluency – six questions (3 LC + 2 MC + 1 HC), Conceptual Understanding -Seven questions (3 LC + 3 MC + 1 HC), Adaptive Reasoning - Seven questions (3 LC + 3 MC + 1 HC), Strategic Competence - Seven questions (3 LC + 3 MC + 1 HC).

The three short-response questions were selected from the strands Conceptual Understanding, Adaptive Reasoning and Strategic Competence. These questions were designed to assess the pupils' mathematical knowledge and skills. The free response questions consisted of two types of constructed-response questions, namely: short constructed-response questions that required pupils to provide answers to computation problems or to describe solutions in one or two sentences, and extended response questions that require pupils to provide longer responses.

Analysis:

Once the data was collected, I read the data; checked for any spoilt or incomplete data; and organised the data into meaningful 'chunks' by considering items in each strand separately. Thereafter, I coded the test questions and biographical data, using the SPSS software. I set up a codebook to help in analysing the data. I also coded data from the multiple-choice questions and free response questions. A correct multiple-choice response was coded 1 and an incorrect response with a 0. The free response questions were coded as follows: 1 – for a correct response (2 marks); 2 – partially correct response (1 mark); 3 – incorrect response (0 mark); 4 – don't know response and 5 – blank. Thereafter, I analysed the data, using SPSS, for the following: the mean score for each question, the mean score for each strand, the frequency of each response and the overall mean score. Mean scores for each strand were compared for significant differences both within and between the mathematical and ML cohorts.

I also did a frequency run, using the SPSS program, to check whether the data captured was reasonable thus minimising error.

Administration of test:

Both tests were administered during the normal mathematics lessons under strict invigilation conditions. I administered the first test on the 3rd May 2006 and the second test in October 2006. I invigilated for both tests in all classes except for one class, which was invigilated by a relief teacher. Participants did not encounter any problems when answering either test.

Limitations of test:

First, it was not possible to administer the tests simultaneously due to insufficient personnel. I administered the first test in May 2006. This date was rather late due to the delay in the ethical clearance. This delay prevented me from assessing these grade ten learners over a longer period, which limited this research from determining whether the new FET phase was effective or not. Second, the sample group also varied slightly due to pupils taking transfer and absentees over a period from May to October. This might be overcome by absenteeism and new arrivals sitting for the October-test. Third, I would have preferred administering the October-test at the end of the year in order to get a more complete picture of the learners' gained knowledge in numeracy skills. However, due to the time constraints in the fourth term (from May to October, only five months), this was not feasible. Fourth, these tests consisted of twenty-four multiple-choice questions and learners generally tend to guess when answering multiple-choice questions, therefore I overcame this by providing the learners with sufficient space to show all their workings. Fifth, the sample was restricted to one school with a poor balance of gender and mathematics/ML and not a variety of schools in the region. A greater sample would provide more generalisability to the results of my research, but as a Master's student time and resources were limiting factors. Last, if you test a child at the end of grade ten, he/she should perform better than at the beginning of grade ten. This might be true but not in all cases because proficiency does not depend on maturation/age and the development of proficiency might not be even across the strands.

Attitude questionnaires (Instrument two)

Construction of questionnaire:

The questionnaire (refer to Appendix C) provided the second instrument for obtaining data for this study. I developed one questionnaire, which was divided into two parts namely, questions and reflective writing. These questionnaires were also designed to obtain the biographical data that required details regarding the participant's gender, age, grade and name. In the reflective writing section, I asked my participants to: Write three sentences about their experiences of maths so far. Here learners did a reflective piece at the end of May and then again, at the end of the third term (October) where learners were asked to include their grade ten experiences. The learners' free responses from the reflective writings were coded for themes and patterns by reading through the organized data and considering which data could be grouped together.

In order to measure the abstract concept of attitude, I chose to use a Likert scale in this study. The Likert scale employed in this research required participants to choose between a number of categories, thus indicating the degree of agreement or disagreement with the given statement. In this scale the scores range from 1 to 5 (1 being for strongly disagree and 5 for strongly agree). The statements were drawn from modified scales as described by Doepken (2006).

Included in the questionnaire was a short-response question, on the learners' mathematical experience from grade 1 to grade ten. The questionnaire statements were chosen to assess whether the pupils see mathematics in a positive sense in line with the notion of productive disposition; perceive mathematics as both useful and worthwhile; and whether they view themselves as an effective doer of mathematics. Using the above, I divided the questionnaire into themes namely, *usefulness of the subject, attitude to the subject* and *ability to do mathematics*.

Administration of questionnaire:

The May/October-tests were both followed by questionnaires on the pupils' disposition. Learners were required to do a reflective piece at the end of May and then again at the end of the third term (October) where learners will be asked to include their grade ten experiences. I originally planned to administer the questionnaire and test together, but after some deliberation, I changed my mind. After administering these instruments, I realized that my decision was a good one because the majority of learners used the full one hour to complete the test and therefore would not have had the time to finish the questionnaire if given together. These questionnaires were administered in a similar fashion to the tests.

Analysis of questionnaire:

This survey assessed the pupils' attitudes/values towards mathematics, which was categorized in the last strand, Productive Disposition (refer to Table 2.1). I used the tests questions and questionnaires to obtain the quantitative information about the pupils whilst the essay responses provided important qualitative information. The rubrics in Tables 2.1 and 3.2 serve as my assessment guide. Since the majority of participants in this study were all grade ten learners, who speak and understand English as a second language, simple language was used in order to minimise misunderstanding. In order to overcome the effect of negative statements, I used reverse coding. I coded -2 for strongly disagree, -1 for disagree, 0 for not sure, 1 for agree and 2 for strongly agree. Then I used the above coding to determine the mean agreement. I also coded the positive version of the originally negative statements with a lower case “p”, for example S23p (statement 23 positive).

Limitations of questionnaires:

The items in the questionnaire could become very monotonous; therefore, learners could get tired of answering them. As a result, learners would pen responses without thinking.

Semi-structured interview

I used focus group interviews in order to collect data regarding the mathematical proficiency of grade ten learners in a secondary school. According to Best and Kuhn (1986), “an interview is often superior to other data gathering devices as people are more willing to talk than to write” (p. 186). Therefore, I had chosen to include interviews in this study.

Although focus group interviews were first used in the private sector as a tool for conducting market research, the technique has gained popularity with evaluators as a means of assessing program implementation and outcomes (Krueger, 1986). De Vos (1998) claims that “focus group interviews are used in order to understand how people think or feel about an issue or products or services” (p. 305).

Construction of interview

In the semi-structured interviews used in this research, all interviewees were asked the same questions, but with variation in the order of questions according to how respondents answered. This technique was used to collect qualitative data by setting up a situation (the interview) that allows a respondent the time and scope to talk about their opinions on the research topic. The objective was to understand the respondent's point of view rather than generalise about behaviour. The interviews were based on open-ended questions, some suggested by the researcher and some arose naturally during the interview. In the interviews, I provided learners with a few questions relating to mathematics, in general, and their personal attitudes and feelings. My interviews included some of the following questions: Do you view mathematics as useful for solving everyday problems? Explain; Do you think it is important for you as an individual to know a lot of mathematics? Why? (Refer to Appendix H) These questions helped my research focus on the learners' attitudes towards mathematics; beliefs about their own ability and their beliefs about the usefulness of mathematics, which helped in assessing the productive disposition strand of mathematical proficiency. I provided learners with a few questions relating to mathematics in general, and their personal attitudes and feelings (see Appendix H). These questions helped my research focus on the learners' *attitudes* towards mathematics, beliefs about their own *ability* and their beliefs about the *usefulness* of mathematics, which aided me in assessing the productive disposition strand. I used the above three values as the themes for analysing the interviews. These three themes were very closely linked to the attitude questionnaire. All the items in the questionnaire were developed with these themes in mind.

Data collection

There were three groups comprising six pupils each. One group consisted of mathematics pupils; the other of mathematical literacy pupils; and the last group a combination of both. Each group consisted of learners from the high, average, and low achievers with respect to the May-test scores. These groups comprised a mixture of Black (80%) and Indian (20%) students. The three focus group interviews took place on four different days during the breaks. In consultation with the interviewees, all interviews were conducted in my classroom. I also organized a video camera to record the interviews for future referencing. Interviewees were informed in advance that these interviews will be video taped, and none objected. I began each interview by stating the purpose for the study.

Analysis of interviews:

Firstly, I transcribed the interviewees' responses from a Dictaphone. Secondly, these free responses from the interviews (qualitative analysis) were coded for themes and patterns by reading through the organized data and considering which data could be grouped together.

Limitations of interviews:

At times learners would try to impress me and thus say what I want to hear. It also happens that certain learners are shy or afraid to say what they really want to say.

The above instruments assessed the development of mathematical proficiency in skills, knowledge, attitudes and values of the grade ten learners, depending on the school context.

3.7 ETHICAL CONSIDERATIONS

I considered the ethical issues when conducting this research. The University of KwaZulu-Natal sent a letter to the Department of Education, seeking permission for me to conduct my research at this secondary school. Furthermore, I have sent consent letters (refer to Appendix E) to the parents of all learners involved in this research, seeking their permission for the involvement of their children in this research. I was teaching at the school at the time this research study was conducted. The research participants were assured that their relationship with the researcher would not be jeopardised due to their participation in this study. Furthermore, these learners were allowed to write these tests under normal school (non-threatening) and bias free conditions with no compulsion on any student. The management team and mathematics colleagues of the researcher monitored the research process to ensure that no learner was victimised or forced to participate in this research programme. As a result, the power relationship between teacher (as researcher) and pupil was taken into consideration, although one has to acknowledge the power differential that exists. In addition, when the information was examined, confidentiality was maintained and anonymity was ensured. When the tests and interviews were completed, all the material was safely stored in my cupboard under lock and key. These research materials will be under safekeeping for a few years, thereafter destroyed by fire.

3.8 CONCLUSION

In this chapter, I have identified the research design as a case study, explained how both quantitative and qualitative methods were used for the collection of data. I also focused on some ethical issues that pertain to this study. The next chapter will include the presentation and discussion of the findings for this study.

CHAPTER 4

In this chapter, I will present and discuss the findings of the research according to the four research questions. I will also analyse the data and draw meaning from it, in terms of the five strands of mathematical proficiency. The first four strands were assessed from the test items, which were selected specifically for this purpose, and the fifth strand was assessed from qualitative data derived from the questionnaire and interviews. Firstly, in answer to RQ1 I will present the data relating to the mathematical proficiency of grade ten ML learners and discuss the results in terms of the five strands described in section 2.1.2 and represented in the test as detailed in section 3.7.1.

4.1 THE MATHEMATICAL PROFICIENCY OF GRADE TEN MATHEMATICS LITERACY LEARNERS IN THE DOMAIN OF NUMBERS

Evidence from the May test indicates that the overall mathematical proficiency of grade ten learners in the mathematical literacy cohort was generally poor with an overall score of 27% obtained in the test. The performance in the procedural fluency strand was slightly better than in the other strands (see Figure 4.1 below) but the difference was not statistically significant. The productive disposition strand was found to be strong in the area of usefulness of mathematics but weaker in terms of beliefs in ability to do mathematical literacy. In the ensuing sections I will present and discuss the results of the May test separately in terms of the five strands of mathematical proficiency.

4.1.1 Procedural Fluency strand of proficiency

Procedural fluency refers to grade ten learners' skill in carrying out procedures flexibly, accurately, efficiently and appropriately. Questions 1, 2, 3, 14, 16 and 24; in the May-test; involved the strand of procedural fluency (refer to Appendix A).

The average score of the mathematics literacy learners in the strand procedural fluency was 32% (see Figure 4.1). These learners performed better in this strand than the other four strands.

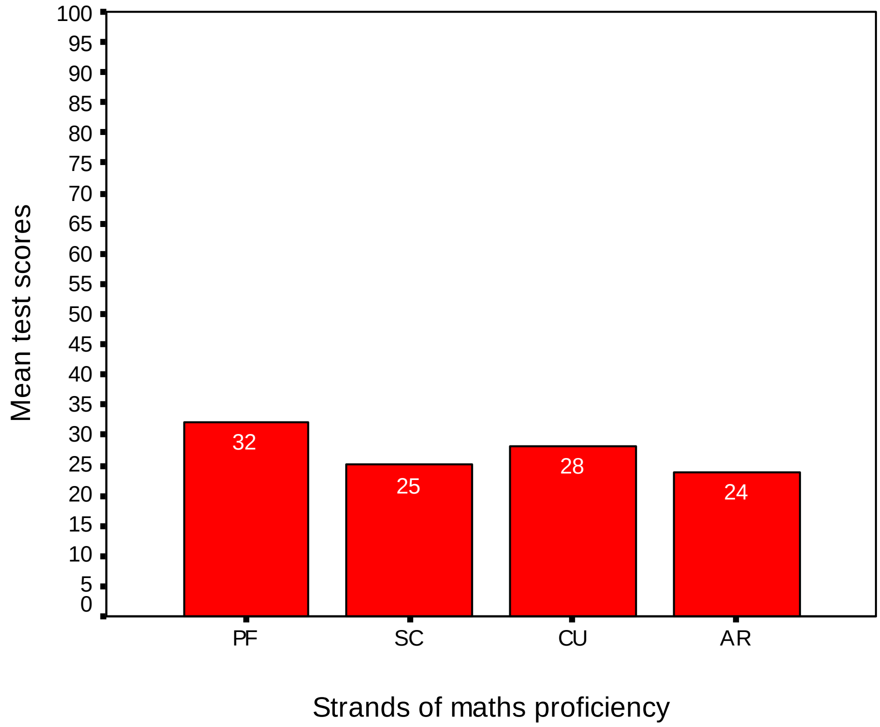


Figure 4.1 Mathematical proficiency of Grade ten mathematics literacy learners in each of the four content strands

The six questions in the test that assessed procedural fluency were not equally well answered. Figure 4.2 shows the mean score obtained by the learners for each question. A mean of one would indicate that all learners had that item correct. Even in Question two, which was the best answered, only half the learners got the correct answer. This means that only half the learners in the Grade ten mathematics literacy class can correctly subtract decimal numbers with three decimal places, a competence prescribed for Grade 7 in the RNCS – LO 1, AC 7.4 (Department of Education, 2002). Question 1 presented some interesting information. This question, involving the subtraction of 2369 from 6000, showed some common error patterns. This is discussed in section 4.3. In figures 4.2 to 4.5, the horizontal axis contains the question number, a short description of the question and its complexity in brackets.

Question 16 was classified as a low complexity item because the multiplication of integers was taught in the GET phase. Therefore, this question was a relatively simple and straightforward problem.

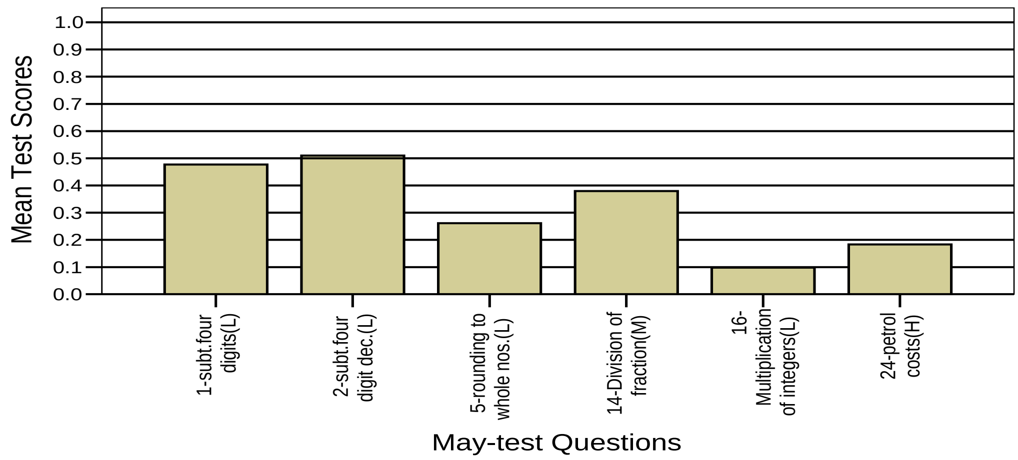


Figure 4.2 Mathematical proficiency of Grade ten mathematics literacy learners in procedural fluency

Then why did so few mathematics literacy learners choose the correct answer? (see Figure 4.2) I think the problem lies with the concept of estimation. The poor performance can be best summarized by the following extract:

> If from an early age children, expect to estimate a ballpark figure before actually computing mentally, with paper and pencil or a calculator they will begin to expect to find realistic solutions to those computations and will spot any errors that occur and hopefully stop and look for reasons for the differences between the estimates and the computed answers. (Kilpatrick et al, 2001)

These grade ten learners skill in estimation was very low; thus impacting negatively on the percentage of learners choosing the correct answer.

4.1.2 Adaptive reasoning strand

Adaptive reasoning refers to these grade ten learners' capacity for logical thought, reflection, explanation and justification. In mathematics, adaptive reasoning is the central core to learning. Adaptive reasoning includes informal explanations, justification, intuition and inductive reasoning. Questions 8, 11, 13, 15, 21 and 26; in the May-test; involved the strand of adaptive reasoning. The average score of the mathematics literacy learners in the strand adaptive reasoning was 24%. The level of proficiency in adaptive reasoning (refer to Table 2.1 on p. 13) for the mathematics literacy learners was very low (lowest of all four strands). This implies that these learner's have a low capacity for logical thinking, reflection, explanation and justification (Kilpatrick et al, 2001).

These learners performed the worst in this strand than the other four strands. Similar to procedural fluency above, the six questions that assessed adaptive reasoning were also not equally well answered. Figure 4.3 shows the mean score obtained by the learners for each question. The number of mathematics literacy learners that had questions 8, 11, 13, 15, 21 and 26 correct were 65, 46, 23, 48, 23 and 4 respectively.

Although question 8 (see Appendix A) was a low complexity item, which was the best answered, less than half the learners got the correct answer. The mean score of 0.42 implies that just over 40% of grade ten learners can correctly use numbers and their relationships to investigate a range of different contexts, which is a competence prescribed for Grade 9 in the RNCS – LO 1, AC 4 (Department of Education, 2003a). Question 26, which tested the learners' ability to reason about multiplication between whole numbers and fractions, was most poorly answered with only 3% of the 161 mathematics literacy learners responding correctly.

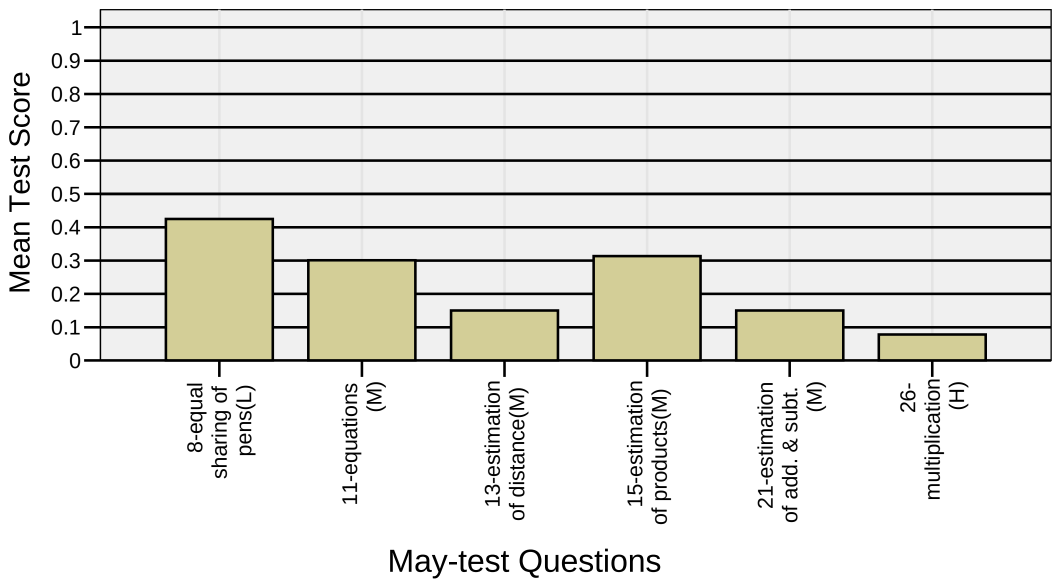


Figure 4.3 Mathematical proficiency of Grade ten mathematics literacy learners in adaptive reasoning

4.1.3 Conceptual understanding strand

In this context, Conceptual understanding refers to the grade ten learners' comprehension of mathematical concepts, operations and relations. Conceptual understanding also refers to an interconnected and functional comprehension of mathematical ideas. Questions 4, 7, 10, 12, 18, 20, 23 and 25; in the May-test; involved the strand of conceptual understanding. The average score of the mathematics literacy learners in the strand conceptual understanding was 28%. The mathematics literacy learners performed the second best for the strand conceptual understanding with an average score of 28% in the May-test. Figure 4.4 shows the mean score obtained by the learners for each question.

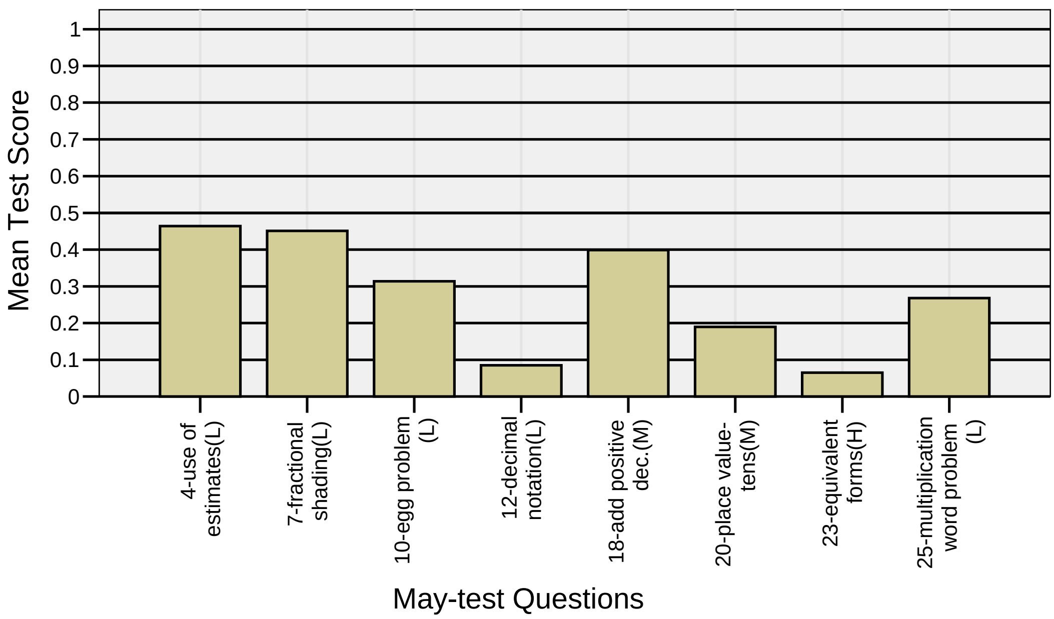


Figure 4.4 Mathematical proficiency of Grade ten mathematics literacy learners in conceptual understanding

Even in Question 4 relating to the best time to use estimation, which was the best answered, less than half the learners got the correct answer. This means that less than half the learners in the Grade ten mathematics literacy class can correctly identify suitable times to use estimation, a competence prescribed for Grade 9 in the RNCS – LO 1, AC 5 (Department of Education, 2002).

The grade ten mathematics literacy learners performed the worst, for conceptual understanding, in question 23 which involves the listing of a common fraction and decimals from smallest to largest. Only 6% of the grade ten mathematics literacy were able to arrange the sequence in ascending order and convert common fractions to decimal form (a competence prescribed for Grade 9 in the RNCS – LO 1, AC 2 (Department of Education, 2002).

The grade ten mathematics literacy learners performed most poorly in question 23, which required the conversion of a common fraction to a decimal. This implies that these learners could not convert a common fraction to a decimal successfully and therefore they lacked the conceptual understanding for this mathematical concept. The majority of these learners failed to make an interconnection and functional comprehension (Kilpatrick et al, 2001) for the conversion of common fractions to decimals.

4.1.4 Strategic competence strand

In the context of this study, strategic competence refers to the grade ten mathematics literacy learners' ability to formulate; represent and solve mathematical problems Questions 3, 6, 9, 17, 19, 22 and 27; in the May-test; involved the strand of strategic competence. The average score of the mathematics literacy learners in the strand strategic competence was 25%. Although the percentage between strands was not significantly different, the mathematics literacy learners performed second worst for the strand strategic competence. As before, the seven questions in the test that assessed strategic competence was very unevenly scored. Figure 4.5 shows the mean score obtained by the learners for each question.

Question 6 stated, *"Siphiwe, Thandi and their mother were eating a cake. Siphiwe ate ½ of the cake. Thandi ate ¼ of the cake. Their mother ate ¼ of the cake. How much of the cake is left?"* Although Question 6 was the best answered, only slightly more than half the learners got the correct answer. This means that just over half the learners in the Grade ten mathematics literacy class can correctly add common fractions, a competence prescribed for Grade 7 in the RNCS – LO 1, AC 7.3 (Department of Education, 2002).

In contrast, these grade ten literacy learners performed most poorly for strategic competence in questions 19 and 27. Question 19 stated, *"Fifteen boxes each containing 8 radios can be repacked in 10 larger boxes each containing how many radios?"*

Only 11% of the grade ten mathematics literacy learners had this question correct. This implies that just over 10% of this grade ten mathematics literacy learners were able to formulate a mental picture of the above problem, detect the mathematical relationships and then devise a mathematical solution competently to repack the radios into ten larger boxes, a competence prescribed for Grade 9 in the RNCS – LO 1, AC 3.1 (Department of Education, 2002).

Question 27, which stated, *"One store, Edgars, reduces the price each week of a R100 stereo by 10 percent of the original price. Another store, Woolworths, reduces the price each week of the same R100 stereo by 10 percent of the previous week's price. After 2 weeks, how will the prices at the two stores compare?"*, involved an extended response with no learner obtaining a fully correct answer. Only 10% of the mathematics literacy learners had this question, which involved the calculation and comparison of percentages, partially correct.

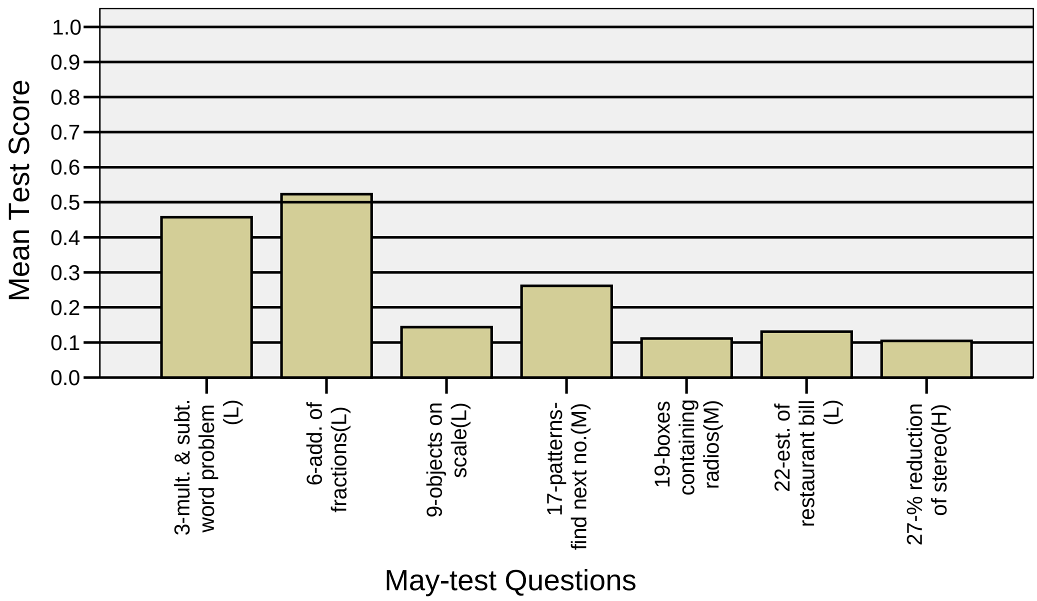


Figure 4.5 Mathematical proficiency of Grade ten mathematics literacy learners in strategic competence

The learners performed most poorly in questions 9, 19, 22 and 27. All these questions above required the learners to be competent in comprehending the quantities and properties in the problem. Furthermore, from my experience as an educator at this school these learners do English as their second language and therefore find difficulty in understanding word problems. This fact is supported by the TIMSS study done in South Africa:

> The grade 8 learners (do English as their second language) struggle with complex questions requiring more than one-step and appear unable to express themselves in writing. Difficulties were noted where pupils were required to comprehend word problems and to articulate and solve problems in writing (HSRC, 2000).

4.1.5 Productive disposition strand

Productive disposition refers to the grade ten mathematics literacy learners' inclination to see mathematics as sensible, useful and worthwhile to do. Items were chosen from the May-questionnaire and grouped according to themes. These themes included the usefulness of the subject, attitude toward subject and ability to do mathematics. Table 1 (see Appendix G) describes the usefulness of mathematics literacy in the learner's everyday life. The identified items assessed the learner's attitude to the usefulness of the subject. The agreement with the nine statements in the questionnaire that assessed the usefulness of mathematics literacy was relatively balanced. Table 1 is a frequency table depicting the distribution of the grade ten mathematical literacy learners' feelings about the usefulness of their subject (refer to Appendix C for a summary of the questions in Tables 1 to 3 in Appendix G). Note that S23p is the positive version of the original statement 23.

It is clear from the table 1 (see Appendix G) that the majority of the mathematical literacy learners saw their subject as being useful in their life because the majority of learners either agreed or strongly agreed with the respective items above. These grade ten learners had the strongest mean agreement for question 13 closely followed by items 19 and 29. Item 13 states: *"Knowing maths will help me get a good job"* ; item 19: *"I'll need maths for my future work"* and item 29: *"I'll need a good understanding of maths for my future work"*.

The items in Table 2 (refer to Appendix G) assessed the learners' attitude to mathematics literacy. The strongest mean agreement was recorded in item 31 which states: *"I wish I was better at maths"* followed by questions 21p and 16 which state: *"I would want to study maths again"* and "*I am sure that I can learn maths"* respectively. It is interesting to note that only for 4p the number of learners that chose either strongly disagreed or disagreed were relatively the same to those that chose strongly agreed or agreed. Item 4p stated: *"I never feel nervous when I am asked to solve maths problems"*.

Table 3 (see Appendix G) represented the grade ten learners' perceptions of their ability to do mathematical literacy. The majority of these learners agreed that if they work hard at maths, their results would improve (S28). Interestingly, about 90 grade ten mathematics literacy learners each agreed with statement 15p and 10p which stated: *"I do understand what I learn in maths"* and *"I don't think I can learn maths"* respectively. These items are contradictory, because when a learner understands mathematics, they generally have a positive attitude toward learning mathematics.

The majority of grade ten learners choosing mathematical literacy had seen the usefulness of studying mathematics literacy. Many of the grade ten interviewees agreed that their subject was useful. As examples, I selected two interviewees who stated the following:

> It also helps when you doing the budget thing and you know how to budget money. It also teaches us about SMS's and cell phones and what is free- talking (learner 219, focus group interviewee), and
>
> Mathematical literacy teaches us on budgeting, on how to approximate; it teaches us how to be money-wise and how to calculate when you are in a shop or something. When I go to the shop, I approximate the amount of money I need (learner 307).

Again, another interviewee supported the fact that most of the grade ten mathematics literacy learners are positive about their subject:

> Maths is good and I like maths because it is a very important subject that can make my life better and very well. I like maths very much (learner 112).

Three grade ten ML learners responded as follows:

> Sometimes I find maths very difficult but I catch up at last (learner 211);
>
> Maths is one of the hardest subjects I ever came across. It leaves me feeling stressful and also it confuses my mind (learner 213); and
>
> I think maths is very difficult but I am trying to understand it (learner 222).

There seem to be a contradiction between the learners' responses above and table 3, but this is not the case. The data in Table 3 is related to their ability to do mathematics literacy this year. Here these learners indicated that they found mathematics literacy relatively easy. On the other hand, the responses above dealt with these grade ten learners' feelings about the subject mathematics from grade one to nine. The fact is that these learners did find the mathematics difficult therefore chose mathematics literacy in 2007.

4.2 THE MATHEMATICAL PROFICIENCY OF THE GRADE TEN MATHEMATICS LITERACY COHORT COMPARED WITH THE MATHEMATICAL PROFICIENCY OF GRADE TEN MATHEMATICS COHORT

Information from the May test indicates that the mathematical proficiency of pure mathematics grade ten learners was slightly higher, than the mathematical literacy learners, with an overall score of 38% obtained in the test. The performance of the mathematics learners in the adaptive reasoning strand was slightly better than in the other strands but the difference was not statistically significant ($p = 0.4$). The mathematical literacy learners performed the worst in this strand. The productive disposition strand was found to be strong in the area of attitude and usefulness of mathematics for both cohorts but weaker in terms of beliefs in ability to do mathematical literacy. In the following sections I will compare the results of mathematics and ML for the May test separately in terms of the five strands of mathematical proficiency.

4.2.1 Procedural Fluency strand of proficiency

The average score of the mathematics literacy and mathematics learners in the strand procedural fluency was 32% and 37% respectively. The comparison between the two cohorts for procedural fluency was found to be statistically significant. The mathematical literacy learners performed the better in this strand than the other four strands, which was not the case for the mathematics learners.

Figure 4.6 shows the mean scores obtained by the mathematics and mathematical literacy learners for the four strands of mathematical proficiency. In May-test question one, I have noted that the mathematics and mathematics literacy learners performed relatively well with the mathematics learners slightly out performing the mathematics literacy learners. Their success rates for mathematics and mathematics literacy were 46% and 45% respectively (see Appendix I).This means that less than half the grade ten learners can correctly subtract four digit whole numbers, a competence prescribed for Grade 3 in the RNCS – LO 1, AC 8.1 (Department of Education, 2002). However, in May-test question two the mathematics literacy learners out performed the mathematics learners with a score of 48% to 43%. These scores are not statistically significant ($p = 0.1$) but included due to big variation. These scores imply that less than 90 grade ten learners, from 181, responded correctly to May-test question 2.

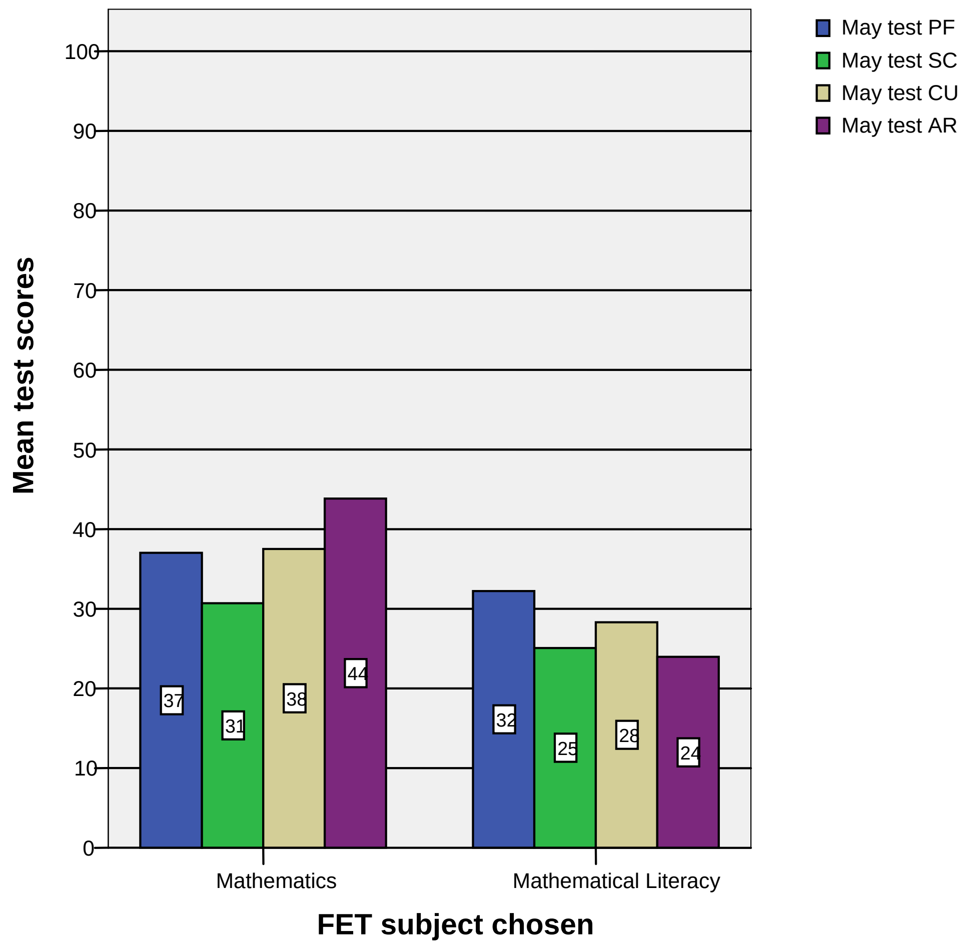


Figure 4.6 Comparison of mathematical proficiency of Grade ten mathematical literacy and Mathematics learners in the May test

It is surprising to note that the mathematics literacy learners performed better at May-test question 2. I would have expected the mathematics learners to perform better than the mathematics literacy pupils, which was not the case because the learners currently doing mathematics have out performed learners taking mathematics literacy, with respect to previous year mathematics marks. In other words, these students are considered ‘brighter’ than the mathematics literacy pupils, which were hinted at by students in the focus group interviews. For example, consider the following two statements from the May questionnaire:

> “In the other grades I didn't find maths difficult but I felt that in grade ten maybe it would be difficult, that is why I chose something that I would manage” (learner 150),
> “I was failing maths in grade 8 so I decided I won’t make it so I chose mathematical literacy” (learner 143).

4.2.2 Adaptive reasoning strand

The average score of the mathematics literacy and mathematics learners in the strand adaptive reasoning was 24% and 44% respectively. These mean scores were found to be statistically significant ($p = 0.04$). The mathematics learners performed the better in this strand than the other four strands while the mathematical literacy learners performed the worst in this strand.

Question 11 (If *39 + 93 = 132 is true, which of the following is true?*) tested the grade ten’s competency in the transposing of equations where 71% and 29% of mathematics and mathematics literacy learners respectively chose the correct answer, a competence prescribed for Grade 9 in the RNCS – LO 2, AC 4 (Department of Education, 2002).

Question 21(*Estimate the value of: 11/12 + 6/7*) tested grade 8 competence of estimation in addition of common fractions. Here the mathematics learners performed better with a 25% success rate as compared to 14% with the mathematics literacy learners (see appendix I). This means that less than a quarter of the learners in Grade ten can correctly estimate the addition of two common fractions, a competence prescribed for Grade eight in the RNCS – LO 1, AC 5 (Department of Education, 2002).

The pure mathematics learners outperformed the mathematics literacy learners by 20% in the May-test for adaptive reasoning. The greatest difference between the mathematics and mathematics literacy learners, with respect to test scores, was in the strand adaptive reasoning. In question 11, a very low percentage of mathematics literacy learners had this question correct (29%) as compared to the pure mathematics learners (71%). Many of the mathematics literacy learners chose option A (43%) as their correct answer. This could indicate a degree of misconception relating to equations. These misconceptions will be discussed further under adaptive reasoning for Research Question 4. In question 21 both the mathematics (25%) and the mathematics literacy (14%) learners attained very low scores. This question involved the competency to estimate. From my experience of teaching these grade ten learners, the low scores could be attributed to the lack of emphasis to the concept of estimation, during my teaching process. Therefore, these grade ten learners are disadvantaged and cannot use common sense to estimate.

4.2.3 Conceptual understanding strand

The average scores of the mathematics literacy and mathematics learners in the strand conceptual understanding were 28% and 38% respectively. The mathematics grade ten learners either performed better or matched the mathematics literacy learners in all questions testing conceptual understanding except questions 20, which stated, *"By how much would 217 be increased if the digit 1 were replaced by a digit 5?"* See Figure 4.7, which shows the mean scores obtained by both cohorts for the strand conceptual understanding. This question tested the competency of the grade ten learners with respect to their conceptual understanding of the 'tens' digit, a competence prescribed for Grade four in the RNCS – LO 1, AC 4 (Department of Education, 2002).

The mathematics learners performed well in questions 7 and 25 (see Appendix A for questions) with a mean score of more than 0.5. The complexities of these two questions were classified as low. The majority of the mathematics learners seemed not to understand the concept of 'tens' with reference to question 20. This question was classified as a medium complexity item.

A greater number of mathematics and mathematics literacy learners had question 1 (see Appendix A for question) correct (classified as a low complexity) which involved the understanding of the concept of 'ten and units' as compared to the other conceptual understanding questions. It is possible that these pupils learnt the facts about 'ten and units' in isolation. A learner with conceptual understanding does not learn facts or methods in isolation because they need to understand why these mathematical ideas are important and how it is useful in different contexts (Kilpatrick et al, 2001).

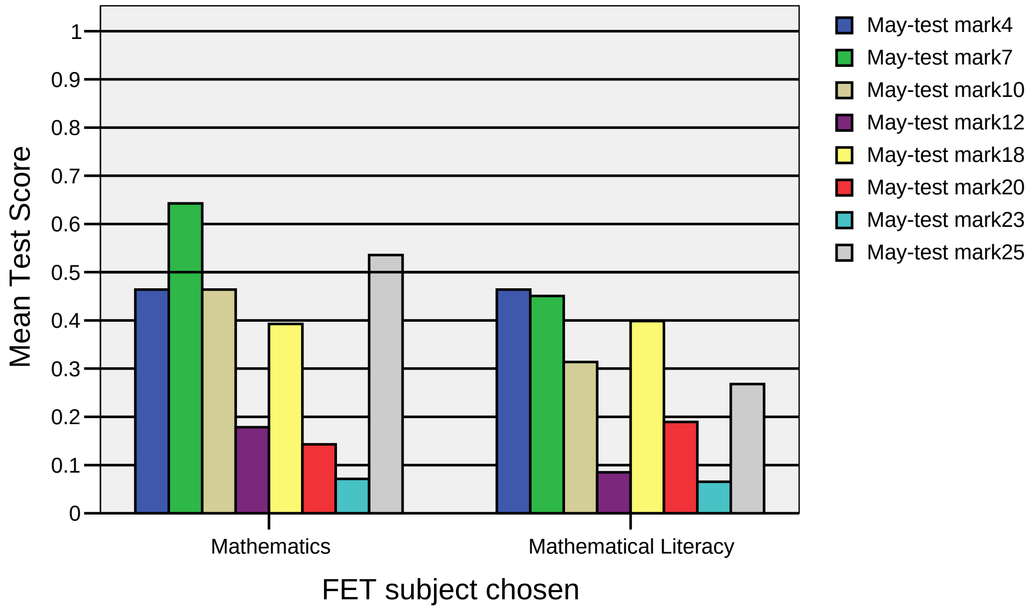
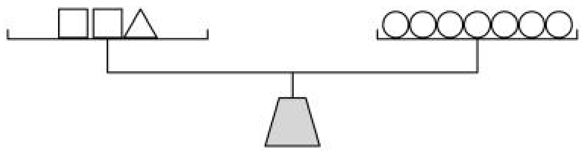


Figure 4.7 Mathematical proficiency of Grade ten mathematical literacy and mathematics learners in conceptual understanding

4.2.4 Strategic competence strand

The average score of the mathematics literacy and mathematics learners in the strand strategic competence was 31% and 25% respectively.

The mathematics grade ten learners out performed the mathematics literacy learners in questions 9 and 19 (see figure 4.9). Question 19 has been discussed in section 4.1.5, therefore let us consider question 9.



The objects on the scale above make it balance exactly. According to the scale, if △ balances ○○○, then □ balances which of the following?

Figure 4.8 May-test question 9

This question tested the learners' ability to reason about the mathematical relationships between quantities of different shapes, a competence prescribed for Grade 9 in the RNCS – LO 2, AC 6.4 (Department of Education, 2002).

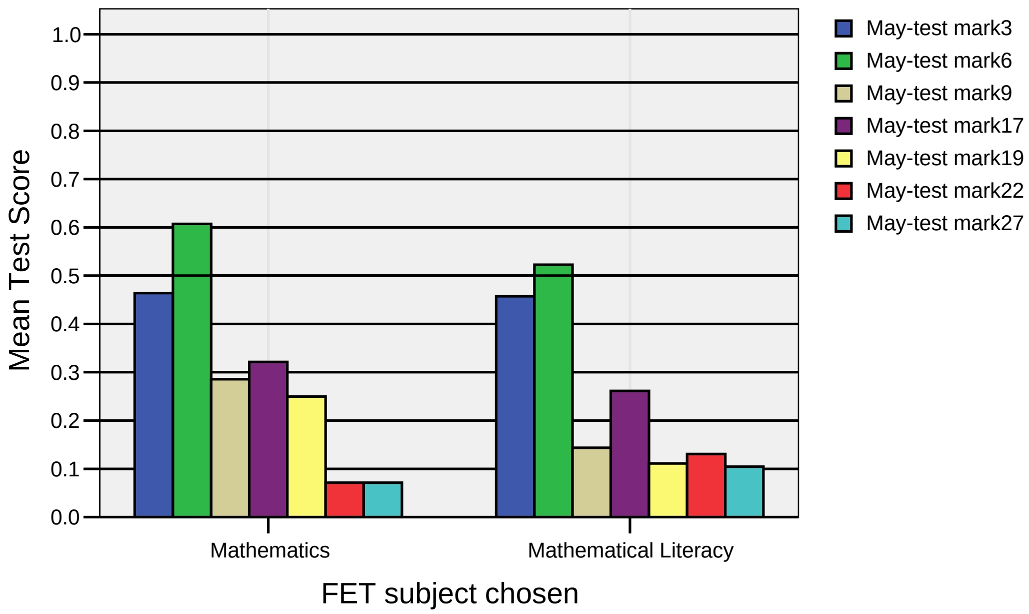


Figure 4.9 Mathematical proficiency of Grade ten mathematical literacy and mathematics learners in strategic competence

The overall performance of the mathematics cohort was better than the mathematical literacy learners for the strands conceptual understanding and procedural fluency. Since these two strands are very closely interwoven with the strand strategic competence (Kilpatrick et al, 2001), it follows that the pure mathematics learners would outperform the mathematics literacy learners and this was what exactly happened. It therefore appears that the mathematics grade ten learners are better able to formulate mental pictures and detect mathematical relationships than the mathematics literacy learners.

4.2.5 Productive disposition strand

It is evident from Tables 2 and 4 (refer to Appendix G) that the majority of both the mathematics and mathematics literacy grade ten learners had a positive attitude to their subject. The items on the questionnaire assessed the grade ten learner's attitude (positive) to their respective subject.

Consider item 30 where more mathematics learners disagreed than agreed with this item, which implied that the majority of these learners do not believe that they need a maths brain to do well in maths. When compared to item number 30 in table 2, the majority of the mathematical literacy learners tended to agree with this statement. Although both cohorts had seen the usefulness of their subject, the mathematics learners had seen their subject as more useful (refer to table 1 for mathematical literacy frequency table in Appendix G).

With reference to item 4p in table 4, which displays the coded data (see section 3.7.1) for the positive attitude of the mathematics learners to their subject, the mathematics learners had a more positive attitude to the subject at the beginning of the year therefore; they were not nervous/afraid when asked to solve a mathematical problem. The mathematics literacy learners chose mathematics literacy because they feared or felt that they cannot perform at mathematics (evidence for this was obtained from the interviews) therefore, they naturally felt afraid/nervous towards solving mathematical problems.
Research has shown that a vast majority of learners taking mathematical literacy have been advised by their teachers to do so based on their grade nine mathematics results (Graven and Venkatakrishnan, 2007). Thus, most of these grade ten mathematics literacy learners were 'forced' to choose mathematics literacy because it was a compulsory subject in the new FET curriculum. Therefore, these learners did not see the subject as useful when compared to the mathematics learners who premeditatedly chose mathematics for a career/job or because they simply like the subject. As a result,

these grade ten mathematical literacy classes have few students with strong levels of confidence and competence in mathematics (Ibid).

4.3 THE MATHEMATICAL PROFICIENCY CHANGING OVER THE COURSE OF THE GRADE TEN YEAR

Evidence from the October test indicates that the mean score for mathematical proficiency of pure mathematics grade ten learners improved from 38%, in May to 44% in October. The ML learners' mean score improved from 28%in May to 29% in October. Surprisingly, this difference was also statistically significant (p = 0.01, see Appendix J). Although the mathematical literacy learners improved on their score from the May to October tests, it was only marginal and still far below the expected mathematical proficiency at this level.

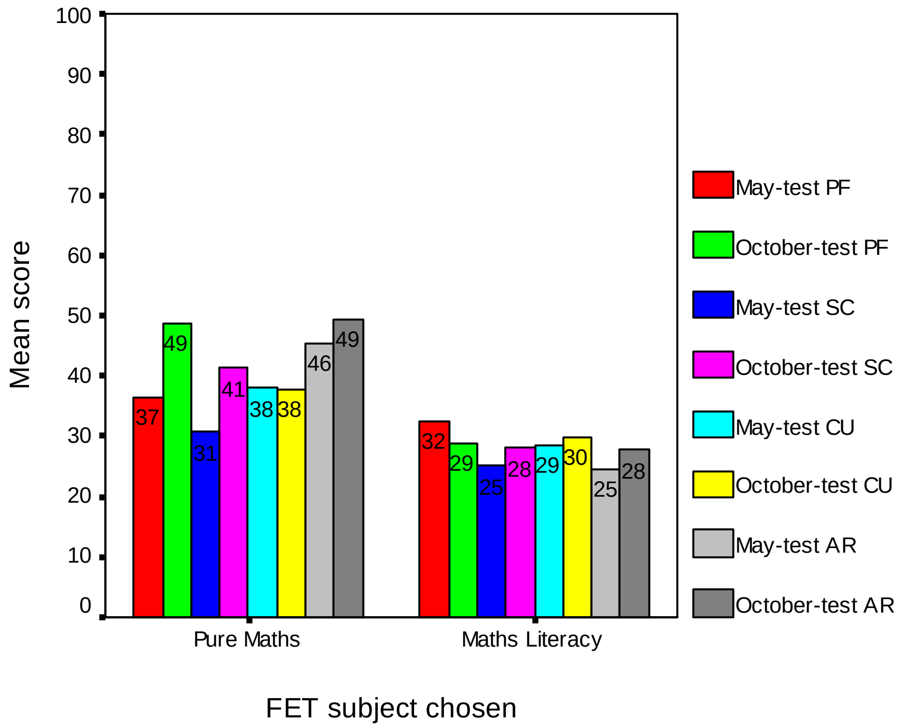


Figure 4.10 Comparison of mathematical proficiency between Grade ten mathematics and mathematical literacy learners over the course of the year

There was an overall improvement in the performance of the pure mathematics learners in all strands except for conceptual understanding. The mathematical literacy learners performed only marginally better in all strands except in procedural fluency. The productive disposition strand was found to be strong in the area of attitude and ability to do mathematics for both cohorts but weaker in terms of usefulness of the subject. In the following sections, I will present and discuss the results of the May and October tests separately in terms of the five strands of mathematical proficiency.

4.3.1 Procedural fluency strand

In question 5, less than half of the mathematics grade ten learners got the correct answer. This means that only 32% of the learners in the Grade ten mathematics class can correctly round off decimal numbers to one decimal place, a competence prescribed for Grade seven in the RNCS – LO 1, AC 7.1 (Department of Education, 2002). Although the scores for the mathematics learners were low (i.e. 32%) they were consistent as compared to 25% to 11% for the mathematics literacy learners. It seems that the mathematical literacy learners probably guessed on the May-test for this question.

With respect to question 14, the mathematics literacy learners did better in the May-test by about 4%. Interestingly, the mathematics learners improved in their achievement for the October-test but the mathematics literacy learners performed more poorly in this question for the October-test. This question required the competency in division of common fractions, a competence prescribed for Grade 8 in the RNCS – LO 1, AC 6.2 (Department of Education, 2002).

The mathematics literacy learners obtained only a 9% as compared to 39% for the mathematics learners in the May-test for question 16. Again, less than half (39%) of the mathematics grade ten learners got the correct answer. However, there was a slight improvement with these scores in the October-test, namely 13% and 54% for mathematics literacy and mathematics learners respectively (see Appendix H). In both the May and October-tests for question 24, the majority of learners chose A (28/100) as their correct answer as opposed to C (7/25) implying that they either cannot interpret the question or they cannot simplify fractions.
The overall score of procedural fluency for grade ten learners in the May and October-test was 33%, which implies low levels of proficiency in these contexts. Learners

performed the best when asked to subtract four-digit whole/decimal numbers, with an average of 46% in both May and October-test. I found that the grade ten learners are less fluent with integers, common and decimal fractions where the average was 26%. The mathematics literacy learners did not improve, in the October-test, for the strand procedural fluency. A possible reason for this is that the new mathematics literacy course, as stipulated by the Department of Education (2003a), was to focus more (even exclusively) on context rather than skills and knowledge and as a result these learners are more poorly equipped in procedural fluency than the pure mathematics learners.

A conclusion that I can draw is that these grade ten learners have many misconceptions and thus have not fully developed procedural fluency. A few learners can compute with whole numbers in simple contexts but the majority (about 68%) still have difficulty computing with rational (common and decimal) numbers.

4.3.2 Adaptive reasoning strand

Question 11 required the learners to reason about numbers and their properties, a competence prescribed for Grade 9 in the RNCS – LO 2, AC 4 (Department of Education, 2002). In addition, it assessed their conceptual understanding. Only 35% of grade ten's chose the right answer in the May-test and 34% in the October-test, which is definitely lower than the percentage of learners who can actually solve this problem. Interestingly 39% and 29% of grade ten's chose A in the May-test and October-test respectively; that is 39 = 93 + 132 (in May-test) or 37 = 73 + 110 (in October-test). The mathematics learners performed more poorly (64%) and the performance of the mathematics literacy learners remained constant for this question. The majority of mathematical literacy learners chose A (31%). Another item involving adaptive reasoning is question 21. Sixty eight percent, of grade ten's chose either 17 or 19 as their correct response in the May-test and about 44% chose A (23) in the October-test. Only 17% of learners reasoned correctly in the May-test and 15% in the October-test. Yet another item involving adaptive reasoning is May-test question 26, which is contextual in nature. It is surprising to note that 62% of grade ten learners chose 'Mary' (incorrect) as their correct answer, while a meagre 4% agreed with Jack (correct) and only 3% provided a partially correct response.

In the October-test, question 25 (contextual) involved adaptive reasoning where 85% of these learners chose either 'before' or 'after Victor's van' (incorrect) as their correct answer and only 5% of grade ten's chose the correct answer and gave the correct explanation. About one percent of learners got this question partially correct. This clearly shows that these grade ten learners have a serious problem with adaptive reasoning within contextual problem solving because both the May and October-tests results are proof of this.

Questions 4, 8, 13 and 15 in the May and October-tests also involve adaptive reasoning. May and October-tests questions 4, 13 and 15 all dealt with estimation. The average mark for these three questions in the May and October-tests were 31% and 35% respectively. From my experience, I believe learners performed poorly because the teaching of mathematics does not emphasize the process of estimation.

4.3.3 Conceptual understanding strand

I assessed the grade ten learners' conceptual understanding of numbers by asking them about the properties of the number system. As mentioned earlier about 49% (cumulative) of learners had May-test question one and two correct (i.e. subtraction of multidigit numbers), but only 18% of them could determine 'By how much would 217 be increased if the digit 1 were replaced by a digit 5?' (May-test question 20). These same grade ten learners performed more poorly on October-test question one and two, with an average of 43% as compared to 49%, but had a slight improvement on October-test question 20 (22%).

Only 7% of grade ten learners correctly ordered four decimals, including one common fraction in the May-test and an even less 5% in the October-test and only 10% asked for four hundredths written in decimal notation, chose the right answer in the May-test with a slight increase of 15% choosing the correct answer in October-test question 12.
As mentioned earlier about 49% and 43% of learners had May-test and October-test question 1 and 2 correct respectively (i.e. subtraction of multidigit numbers), but only 18% of them could determine *'By how much would 217 be increased if the digit 1 were replaced by a digit 5?'*; and only 22% of grade ten learners chose the correct answer for October-test question 20. However, this is a common trend because according to research (Olivier, 2001) more learners can work successfully with numbers than work with the properties of those same numbers. Judging from my results, the same is true for rational numbers. Only 7% and 5% of grade ten learners correctly ordered four

decimals, including one common fraction in the May-test and October-test respectively; and only 10% and 15% asked for four hundredths and six hundredths respectively written in decimal notation, chose the right answer. This suggests to me that these learners are working with numbers that they do not really understand (Olivier, 2001). It is also interesting to note that the mathematics learners did not improve in the strand conceptual understanding. The reason for this could lie in the fact that mathematics, in general, focuses a lot on skills and procedural development and lacks in promoting understanding of concepts of basic numbers.

4.3.4 Strategic competence strand

I have noticed that the grade ten learners performed comparatively better on the May and October-test for questions (3, 6, 10 and 17) about basic number operations and concepts in simple applied contexts. However, these learners had great difficulty with more complex problem-solving contexts. For example, when asked what fraction of a cake was left after dividing the cake into different common fractions; 57% (average score of May and October-tests for question 6) of grade ten's gave the correct answer. However, on a multi-step word problem (question 22), only 12% (average score) of grade ten's gave the correct answer. In May-test question 27, learners were required to construct an extended response, where only 10% of learners obtained a partially correct answer. Furthermore, in October-test question 26 and 27 (contextual question with extended response required) only 16% (cumulative) of grade ten learners achieved a partially correct answer and only one learner obtained full marks for these questions.

Throughout my twelve years of experience as a mathematics teacher, I have noticed that learners often tend to have great difficulty in problem solving. This was supported by my research results for problems based on strategic competence. On a multistep word problem (question 22), only 12% of grade ten's gave the correct answer in the May and October-test. In May and October-test question 27, learners were required to construct an extended response, where only about 14% of learners obtained a partially correct answer and only one grade ten learner obtained full marks for this question in the October-test. I also noticed through my results that when extra information is included into a word problem, the performance of the learners decline dramatically. Furthermore, the majority of grade ten learners are second language English pupils, which has a direct impact on their interpretations of the problem. In a broader sense, the term “literacy” refers to the human use of language. In fact, one’s ability to read, write, listen, and speak a language is the most important tool we have through which human social activity is mediated (Romberg, 2001). Nevertheless, the results indicate that these grade ten learners are seriously lacking in their problem-solving abilities.

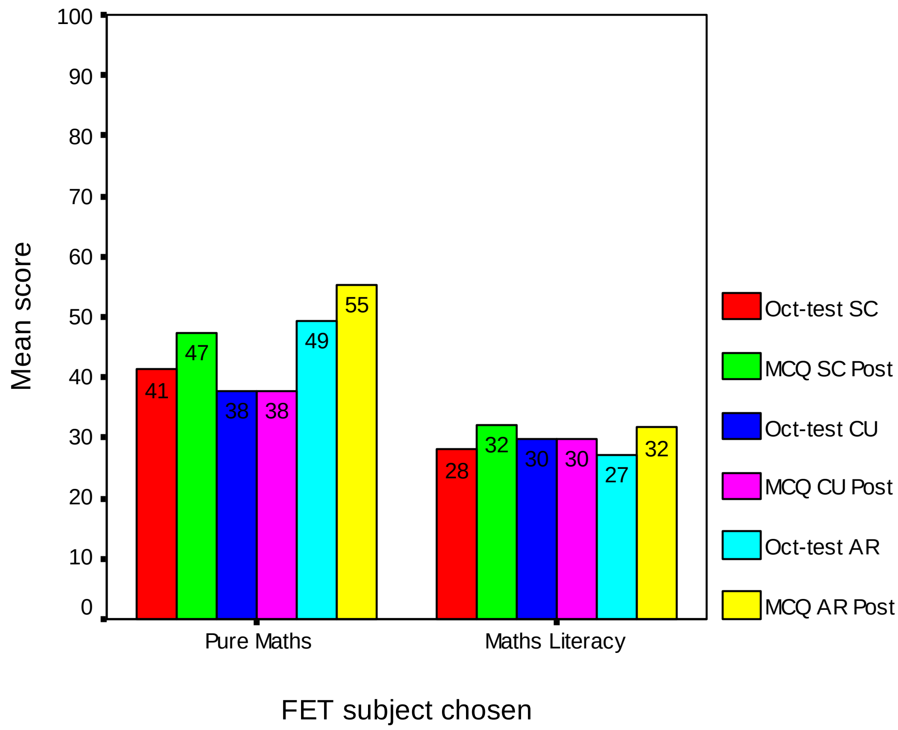


Figure 4.11 Comparison of mathematical proficiency between the two cohorts with respect to multiple-choice and contextual questions

Figure 4.11 represents a comparison between the October-test strand marks that include the last three questions, which required an extended response (contextual) from the learners, and the October-test strand marks that do not include them (i.e. the latter only contain the multiple choice questions). When the above two graphs are compared (Figures 4.10 and 4.11), one would notice that there was no decline in performance in any of the strands for the October-test, whereas in Figure 4.10 there was a decline in the procedural fluency strand for the mathematics literacy learners. This strongly suggests that the ML learners found some difficulty in problem solving involving contexts with an extended response. Schoenfield (2001) states that on tests of conceptual understanding and problem solving, students who learn from reform curricula (like the new Mathematics curricula in South Africa – October-test results) consistently outperform students who learn from traditional curricula (May-test results) by a wide margin. Therefore, these learners can be hopeful that over a few years of schooling, they would be more proficient in solving contextual problems with extended responses.

4.3.5 Productive disposition strand

The mathematics literacy learners' attitude of the usefulness to the subject changed negatively over the course of the year. Similarly, this was also the case with the mathematics learners. This indicates that these grade ten learners had a negative change in attitude towards their respective subjects from May to October. The values from Tables 6 to 11 (see Appendix G) represent the number of grade ten learners agreeing with the respective items.

What is striking from table 4.7 is that item 23p had the same mean agreement for both May and October. In every other question, the number of learners in the May-test was higher than the October-test. Q23p stated: *"Maths is important for my everyday life"*. This implies that the mathematics learners remained constant in their belief that mathematics is important for their lives.

Tables 8 and 9 compare the attitude of the grade ten learners to their subject from May to October. These values were made up of the combination of values from the strongly agreed and agreed categories. Except for questions 22p and 25p in Table 4.8, it is evident from the tables that for both cohorts the attitude of these grade ten learners towards their subject had become less positive from May to October. Questions 22p and 25p stated, *"I do not find my maths class very stressful"* and *"I am intelligent enough to do maths"* respectively. This seems to indicate that the mathematics learners became more stressful and less confident towards their subject over the period May to October.

Tables 10 and 11 compare the perceptions of the grade ten learners of their ability to do their subject from May to October. As mentioned above these values (weighted according to strong agreement) were made up of the combination of values from the strongly agreed and agreed categories. The overall perception of these grade ten learners, for both cohorts, on their ability to do their subject diminished from May to October. With exception of items 10p and 15p for mathematical literacy and mathematics respectively, all other items had a smaller mean agreement for October.

Questions 10p and 15p stated, *"I think I can learn maths"* and *"I understand what I learn in maths"* respectively. This means that more mathematical literacy learners felt that they could learn mathematics over the period from May to October. Similarly, more mathematics learners (higher mean agreement) felt that they could understand what they learnt in mathematics over the same period.

The mathematics and mathematical literacy learners had a negative change in attitude towards their subject over the course of the year. Many of these learners agreed from May to October, with respect to item 14, which states: "*I am often nervous/afraid when I enter the maths class"*. According to the interview responses, many learners stated that their subject had become more difficult throughout the course of the year. For example, learners 132 and 133 stated, *"Like graphs and the work we are doing now (geometry) are very difficult"* and "*it was not easy in the past and this year is even harder"*. Another reason for the above negative attitudes of grade ten mathematics learners could be the long syllabus with the inclusion of grade twelve work into the grade ten syllabus which involves more critical thinking thereby making the subject more difficult and discouraging for these learners. Secondly, the mathematics learners' attitude to the usefulness of the subject changed negatively from May to October. Many grade ten learners became negative (in attitude) towards the subject because they considered it very difficult. For examples: learner 115 stated, *"I find maths very difficult this year especially solving for x binomials and squaring of terms I find very easy but graphs and geometry we are doing now I find very difficult*" and learner 219 stated, *"if you don't like something, most people don't like maths and if you don't like something you wont do well in it, I think it is difficult"*

A reason for this could be that pure mathematics and mathematical literacy (FET) were taught for the first time in 2006 and therefore the educators (at my school) still (in 2006) focused heavily on skills and knowledge thereby depriving these learners of seeing mathematics in context. Lastly, the perceptions of the grade ten learners' ability to do the subject also changed negatively over the course of the year for both cohorts. This could be related to the fact that many of these learners had negative feeling towards the subject. For example learner 307 stated, *"I hate maths because it has numbers. I do not like the fact of counting. I can add, multiply, subtract and divide but doing it everyday in school, I don't like it"* and learner 150 stated, *"it is boring, just* *stressful, it takes all your time and you have to stress"*. In addition, the grade ten learners for both cohorts lacked in the basic knowledge and skills required at this level. As a result, this compounded their fears about mathematics.

4.3.6 Comparison between the two cohorts for each strand over the course of the year

Procedural fluency

In question five the mathematics learners performed better than the ML learners especially with respect to the May-test where differences in scores were about 21%. while the mathematics learners outperformed them in the October-test by about 15%. The mathematics learners outperformed the ML learners for procedural fluency in both the May and October-test question 16.

Adaptive reasoning

It is interesting to note that the mathematics learners performed better in question 21 with a 25% success rate as compared to 14% with the ML learners. However, the ML learners improved tremendously in the October-test and even out performed the mathematics learners with a score of 36% to a meagre 12% for mathematics learners. The October-test results are surprising because the ML learners greatly improved on their results while the mathematics learners performed more poorly.

The overall performance of adaptive reasoning for grade ten's shows an improvement for both cohorts. Although both cohorts improved by about 3% each, the mathematics learners out performed the ML learners by about 20%. This seems to indicate that the mathematics learners are better able to reason with numbers than mathematics literacy learners. Learners are able to display their reasoning ability when three conditions are met: (1) the learners must have a sufficient knowledge base; (2) the task must be understandable and motivating; and (3) the context must be familiar and comfortable (Olivier, 2001). It therefore suggests that the mathematics literacy learners' are lacking on some or all of the above conditions because their overall (correct) performance is a low 30% in the May and October-test. In the new grade ten curriculum, mathematics literacy emphasizes more on context while pure mathematics more on knowledge and skills; which could be the cause for the poorer results from the mathematics literacy learners. This could indicate that mathematics literacy learners are either lacking in sufficient knowledge or understanding. In this regard, I agree with Milgram (2004) in that a student should not simply memorize the mathematics and repeat it verbatim but should understand why it is stated the way it is. In order to verify whether students understand the statements in the mathematics, the following three questions should be asked: (a) What does the statement include?; (b) what does the statement exclude? and (c) what would happen if the definitions/statements were changed and why are the changed definitions/statements not used? I strongly agree with Milgram (2004) because the results of the mathematics literacy grade ten learners indicate that they generally learn mathematics without any understanding and then expect to apply their knowledge in solving problems. As a result, the performance is dismal for learners in mathematics literacy as compared to pure mathematics.

Conceptual understanding

The overall performance of conceptual understanding for grade ten's shows that the mathematics learners' performed slightly better than the mathematics literacy learners over the course of the year. This implies that the mathematics learners are better able to conceptualize numbers than the mathematics literacy learners.

4.4 FACTORS INFLUENCING THE ACHIEVEMENT OF MATHEMATICAL PROFICIENCY

I have identified two factors, namely gender and misconceptions, that influenced the achievement of the mathematical proficiency of these grade ten learners. Evidence from both tests indicates that the performance in mathematical proficiency of both cohorts were influenced by misconceptions and gender to a certain degree. In keeping with recent (HSRC, 2000) research on gender, the female grade ten learners generally performed better than their male counterparts for both cohorts.

Through the study, I also noticed that language, a third factor, had an influence on the achievement of the mathematical proficiency of these learners. The language factor will not be discussed in detail in this research study, but will make interesting study for the future research.

4.4.1 Misconceptions

Misconceptions appear to play a significant role in the procedural fluency strand but not in the other strands.

It concerned me that 44% of the total number of learners (mathematics and mathematical literacy) chose A in May-test question 1 while 49% chose B in October-test question 1; and about 30% chose C and D in May-test question 2 while 35% chose C and D in October-test question 2 as their correct answer (refer to Appendix G). The reason for these incorrect answers is due to misconceptions in the process of subtraction. Refer to May and October-test questions 1 and 2 (see Appendix A and B). It was also a surprise to note a higher percentage of mathematics pupils chose C and D as their correct answer in May-test question 2. These options could have contained misconceptions, which will be discussed in the next section.

It is more interesting to note that 50% of mathematics and 41% of mathematics literacy learners chose A for May-test question 1 and 46% of mathematics and 46% of mathematics literacy chose C and D (cumulative percentage). This implies that more mathematics learners have a misconception with subtraction than mathematics literacy learners do. It also means that there are more learners in both cohorts that struggle with these misconceptions. However, 32% of mathematics and 40% of mathematical literacy learners chose A for October-test question 1, which indicates that learners from both cohorts improved on their misconceptions.

It is also quite striking from the tables (see Appendix G) that the females outperformed the males, in both cohorts, for question 14. This question was included to test grade 8 competencies for the division of common fraction, a competence prescribed for Grade 8 in the RNCS – LO 1, AC 6.2 (Department of Education, 2002). Furthermore, a large proportion of learners chose A in May and October-test question 14 which implies a misconception on the operation of division with whole numbers. The number of mathematics learners increased from 25% to 32% in choosing A in the May-test to the October-test, which indicates that these learners carried their misconceptions through to the October-test whereas with the mathematical literacy learners there was a slight improvement by about 9%.

In both the May and October-tests for question 24, which tested the grade ten’s ability to simplify a common fraction, the majority of learners chose A (28/100) as their correct answer as opposed to C (7/25) implying that they either cannot interpret the question or they cannot simplify fractions, a competence prescribed for Grade 9 in the

RNCS – LO 1, AC 2 (Department of Education, 2002). Once again, it shows that learners are carrying their errors/misconceptions through without it being rectified.

According to Van Lehn (1982), many learners in the primary and secondary schools make frequent and persistent errors (cf Olivier, 1989), for example:

374	657
- 239	- 486
145	231

Many teachers will try to address the above error by re-teaching place value, ‘borrowing’ and number combinations. Yet the problem only recurs repeatedly. What is the problem?

Although the above problem is categorized within the strand of Procedural Fluency, as discussed earlier, all five strands are interwoven, that is, dependent on one another. Here, the problem lies with Conceptual Understanding namely the over generalising of operations. In other words, the teacher needs to analyse what knowledge previously learnt is influencing the current problem. Research (Davis, 1984) has suggested that the problem is in the learners’ erroneous conception that subtraction is commutative. This implies that the order does not matter, so 9 – 3 and 3 – 9 are the same.

Now the question: Why do learners think that subtraction is commutative? The problem is not that these learners have learnt incorrect concepts but that previously learnt concepts (correct) are now influencing this new knowledge (subtraction). One major problem is that in the system of whole numbers primary school children work only with 9 – 3 and 3 – 9 in grade 8 when negative numbers are introduced (Olivier, 1989). In addition, research (Ibid) has shown that although learners know that 9 – 3 and 3 – 9 have different meanings, they reason that the method to get the answer of 3-9 is to calculate 9 – 3. Personally, I have also noticed this misconception with my senior learners (grades ten to twelve), in geometry, who frequently write 70 – 180 = 110. According to Olivier (1989), the main contributory factor for learners seeing subtraction as commutative is due to them having extensive experience of the commutativity of addition. In other words, they are over generalising over operations. Once learners have learned procedures without understanding, it can be difficult to get them to engage in activities to help learners understand the actual reasoning behind the procedure (Kilpatrick et al, 2001). Furthermore, when learners practice procedure without understanding, there is a danger that they will practice incorrect procedures. As a result, they make it more difficult for themselves to learn the correct procedures.

When a learner tries to learn a new concept without understanding, she/he is learning by rote because it is not linked to any previous knowledge. As a result, the new concept (subtraction) is not understood; and becomes isolated knowledge; therefore, it is difficult to remember. Such (rote) learning is the cause of many mistakes/misconceptions in mathematics as learners try to recall partially remembered and distorted rules (Olivier, 1989). Thus, it is important and necessary that teachers in primary and secondary schools help identify and eradicate such misconceptions because these mistakes will influence new learning, in a negative way, and generate errors (as in May and October-tests questions 1 and 2). Research has shown the advantages of mathematics literacy as opposed to formal mathematics, which must be practiced within a variety of contexts that will aid repetition and experience, which eventually leads learners to develop habits rather than rote learning or application with no meaning (Schoenfield, 2001).

Another problem of learning without understanding is that learners separate what they learnt in school from what happens in the real world. Madison (2004) also echoes similar sentiments, where pure mathematics students learn abstract concepts before learning to apply them in the real world. Therefore, these learners will be limited in using what they learned at school to solve real-life problems. He adds that in mathematics literacy the learner looks at a number of applications and then extracts the abstractions, something that is common in business and engineering.

Only 35% (May-test) and 34% (October-test) of grade ten’s chose the right answer for question 11, which is definitely lower than the percentage of learners who can actually solve this problem. Interestingly, 39% (May-test) and 29% (October-test) of grade ten’s chose A; that is 39 = 93 + 132 or 37 = 73 + 110 respectively. Why? The problem could be that some learners view the equal sign as a symbol separating two expressions; that

is an expression on either side of the equal sign (Kieran, 1981). The choice of A (39 = 93 + 132 or 37 = 73 + 110) clearly indicates that the majority, if not all, of the 34% (cumulative percentage) of learners do not see the equal sign as a symbol of equivalence between the left and right sides of the equation. Furthermore, learners view the equal sign as a need to ‘solve’ the equation. From my own experience, learners are also too involved in computing with numbers rather than seeing the equal sign as a symbol of equivalence between left and right hand side of an equation.
How does the teacher help such learners overcome these problems? Definitely not by trying to reteach the same thing over and over again because the problem (misconception) is deeply rooted in the learner’s mind and learners will therefore still cling to their misconception (Hewson, 1996). The best way to help these learners see their mistakes is through carefully planned examples, together with discussions and arguments amongst learners about their patterns of thinking. What is of crucial importance is that the teacher must provide opportunities for the learners to rectify their own problem/misconceptions independently.

The contextual item involving adaptive reasoning was May-test question 26. It was surprising to note that 62% of grade ten learners chose Mary (incorrect) as their correct answer, while a meagre 4% agreed with Jack (correct) and only 3% provided a partially correct response. The major misconception here that causes the error is that ‘multiplication makes bigger and division makes smaller’. Therefore, the learners who chose Mary reasoned that multiplication will give an answer of six or more, which was driven by the misconception that ‘multiplication makes bigger’. The problem lies with the working of whole numbers where multiplication always makes bigger (except for special cases of zero and one). The concept holds true for whole numbers but is not generally true for decimals numbers, fractions and integers.

It is important to note that children do not make mistakes because they are silly, but that they are rationally trying to cope with the mathematics (Olivier, 1989). We as teachers should not look negatively upon misconceptions, like the above, because they are difficult to avoid and should be seen as part of the process of learning. The repetition of teaching concepts that were missed or misapplied will not do to help overcome this problem. In addition, teachers need to create an ethos within the mathematics class that will tolerate these errors and misconceptions and use them to promote learning of mathematics rather than inhibit it.

4.4.2 Gender

Gender seems to play a slight role in all the strands. The difference between mean scores of males and females is not statistically significant ($p = 0.65$).

Procedural fluency:

I have noticed from my teaching experience that boys were generally considered to be better performers at mathematics than girls but it is interesting to note that the female grade ten learners out performed their male counterparts by about 18% in May-test question 1 and about 9% in October-test question1; and 20% in May-test question 2 and about 13% in October-test question 2. Consider the following figures on the performance of gender in the two cohorts:

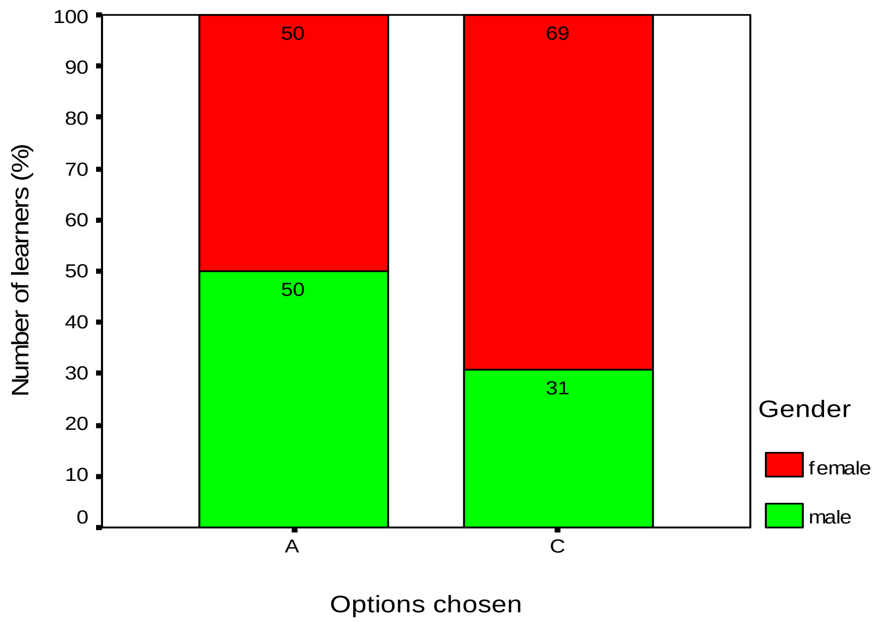


Figure 4.12 Gender comparisons of Grade ten mathematics learners for May-test question 1

It is clear from Figures 4.12 and 4.13 that the females doing mathematics outperformed the males in both the May and October-tests for question 1. What is also of interest is that a large number of grade ten learners chose either A or B in the May and October-test respectively, which implies that a large number of these learners have misconceptions with respect to this question. In figure 4.12, C is the correct option and D for figure 4.13.

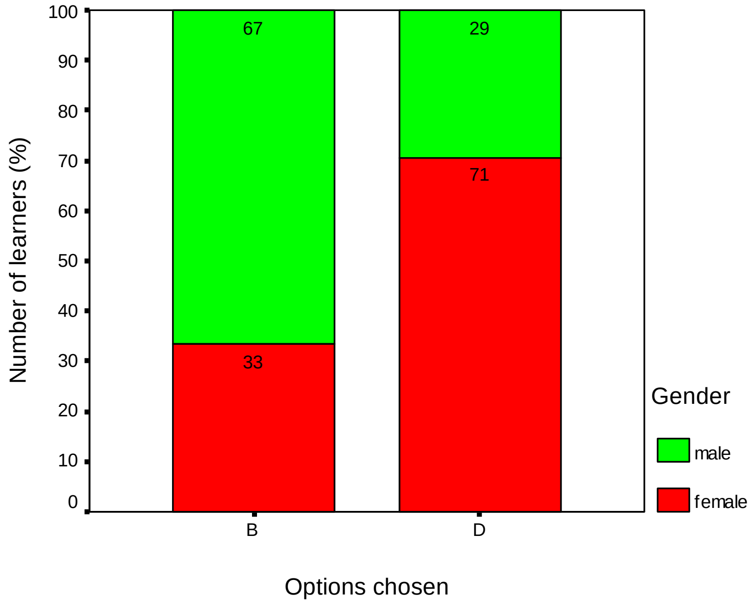


Figure 4.13 Gender comparisons of Grade ten mathematics learners for October-test question 1

The performance of mathematics literacy learners, according to gender, for question 1 were very similar to the mathematics learners above, i.e. the female grade ten learners outperformed the males in this question.

Adaptive Reasoning:

The female grade ten learners performed better than the males in question 11 for both the May and October-tests. This is clearly observable in table 4.1, which compares the gender performance (percentage) of both the mathematics and mathematical literacy learners for question 11 in both the May and October-tests.

This is in keeping with the recent ‘norms’ in the mathematical world, where girls generally outperform the boys (Department of Education, 2003).

Table 4.1 Gender comparisons of both cohorts for question 11 in the May and October-tests

	Pure Mathematics			Maths Literacy		
Gender	Male	female	Total	Male	Female	Total
May-test	29	43	72	14	15	29
Oct-test	27	42	69	11	26	37

In question 21 the female learners performed better than the males however there were more females who chose 17 and 19 as their correct answer in the May-test and 23 in the October-test, probably implying that females struggle more with the concept of estimation with fractions than their male counterparts. Sixty-eight (68%) percent of grade ten's chose either 17 or 19 (in the May-test) and 76% chose 23 or 21 (in the October-test) as their correct response. Here learners were asked to estimate 11/12 + 6/7. Only 16% (cumulative percentage of May and October-test) of learners reasoned correctly. This implies that the simple levels of reasoning at this stage is seriously lacking. The simple reasoning required was that 11/12 and 6/7 are both numbers less than one and therefore their sums will be less than two, thus making 17 and 19 (in May-test) and 21 and 23 (in October-test) unreasonable answers. It is clear that for many learners the connection between basic understanding and reasoning is not being made (Kilpatrick et al, 2001).

The females performed better at questions 4, 8, 11, 13, 15 and 26 while the males only slightly beat the females at questions 25 for the October-test, which is the contextual item assessing adaptive reasoning in the October-test. This implies that the grade ten female learners are better performers at problems involving adaptive reasoning than grade ten male learners.

Conceptual Understanding:

The female grade ten learners' best performance against the males was in question 25 (see Figure 4.14) which stated, *"A high school orders 11 buses to transport 418 students. If each bus can seat 35 students will the number of buses ordered be enough to provide a seat for each student?"*

Question 25 required a short response from the learners and tested their competence in understanding the concepts represented in the problem above, a competence prescribed for Grade 9 in the RNCS – LO 1, AC 3 (Department of Education, 2002). The male grade ten learners' performed best against the females in May-test question 12, which tested the grade 8 competence, related to the conversion of common fractions to decimals (LO 1, AC 3). Question 12 stated, *"What is four hundredths written in decimal notation?"*

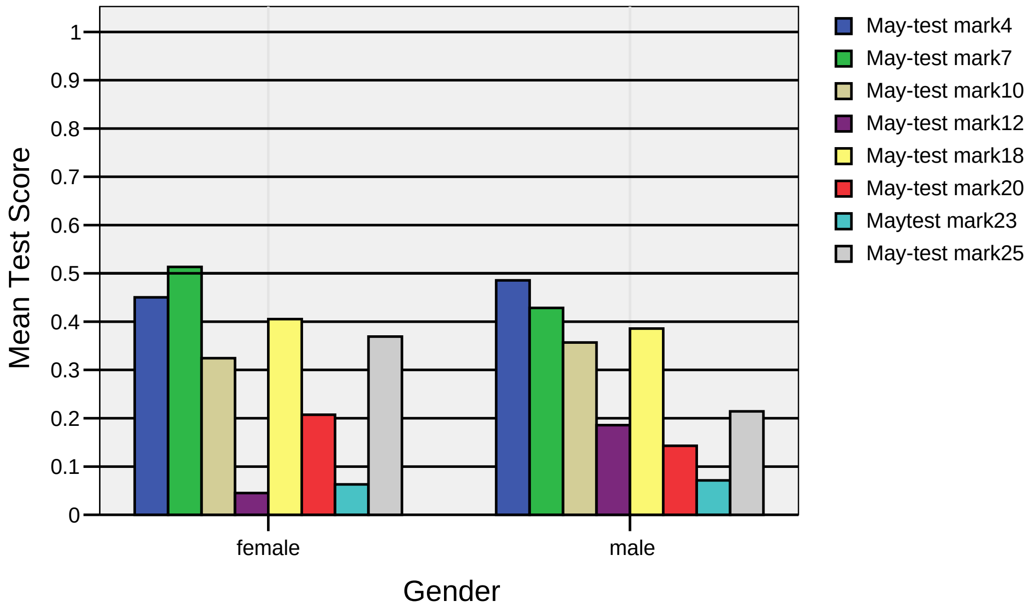


Figure 4.14 Gender comparisons of the mathematical proficiency of Grade ten learners in conceptual understanding (May-test)

Although the males did better at questions 4, 10, 12 while females did better at questions 7, 18 and 25, there was no significant difference between them. This is in keeping with the literature: the TIMSS data revealed that significant gender differences were found only at school leaving age, at which point males outperformed females in 18 of the 21 countries.

Strategic Competence:

The male grade ten learners improved on their competencies in the questions dealing with strategic competence from the May to October-tests, (see figures 4.15 and 4.16). Their most successful improvement was in October question 9 (discussed above).

The males either matched their May scores or improved on them. But the female grade ten learners performed most poorly in question 22 in the October-test.

Question 22: *"Mr. Moodley and two friends ate at a restaurant. The bill was R57. In addition, they left a R13 tip. Approximately what percent of the total bill did they leave as a tip?"*

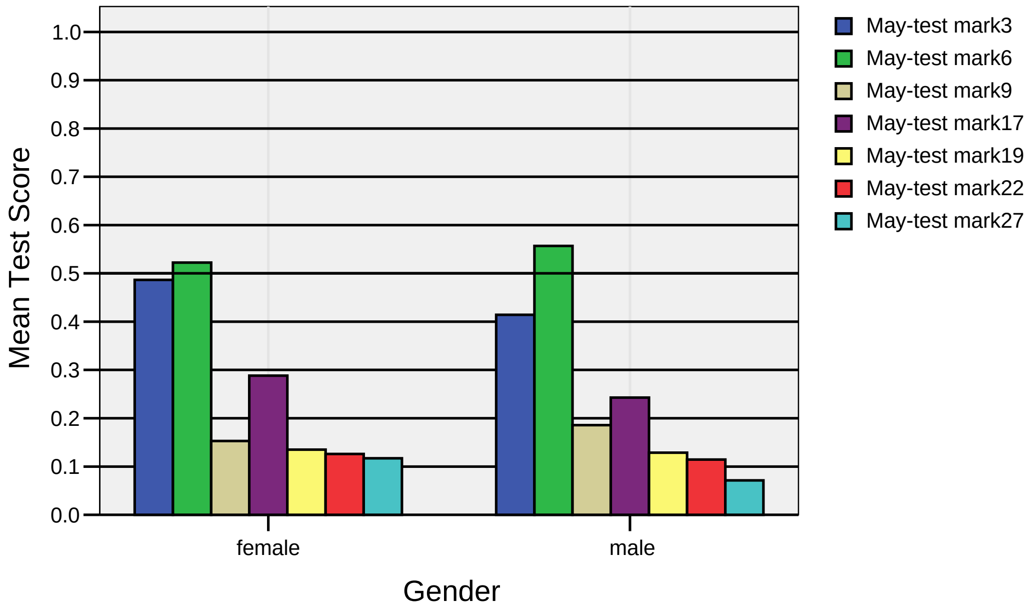


Figure 4.15 Gender comparisons of the mathematical proficiency of Grade ten learners in strategic competence (May-test)

The above question was testing the grade ten learners' ability to estimate a restaurant bill, a competence prescribed for Grade 9 in the RNCS – LO 1, AC 5 (Department of Education, 2002). The overall performance for both male and female were poor for question 22. I believe learners performed poorly because the teaching of mathematics (at my school) does not emphasize the process of estimation. Thus, learners view mathematics as a subject strictly made up of right or wrong answers. Through my personal experience as a pupil and teacher (and teachers I work with) of mathematics, I have noticed that the culture of teaching and learning of mathematics lies in the emphasis of procedures for operations and no checking was done or encouraged, especially through estimation.

According to research done by Stage, E. K., Kreinberg, N., Eccles, J. R., and Becker, J. R. (1985), boys perform slightly better than girls on tests of mathematical reasoning (primarily solving word problems). However, the grade ten girls performed better than the males on questions 3 and 27 (see Figure 4.16 above), both of which involve a word problem and the ability to reason. I therefore feel that the success rates between male and females depends more on the motivation given by the teacher as well as the cultural background of the learners rather than mere generalization of research results.

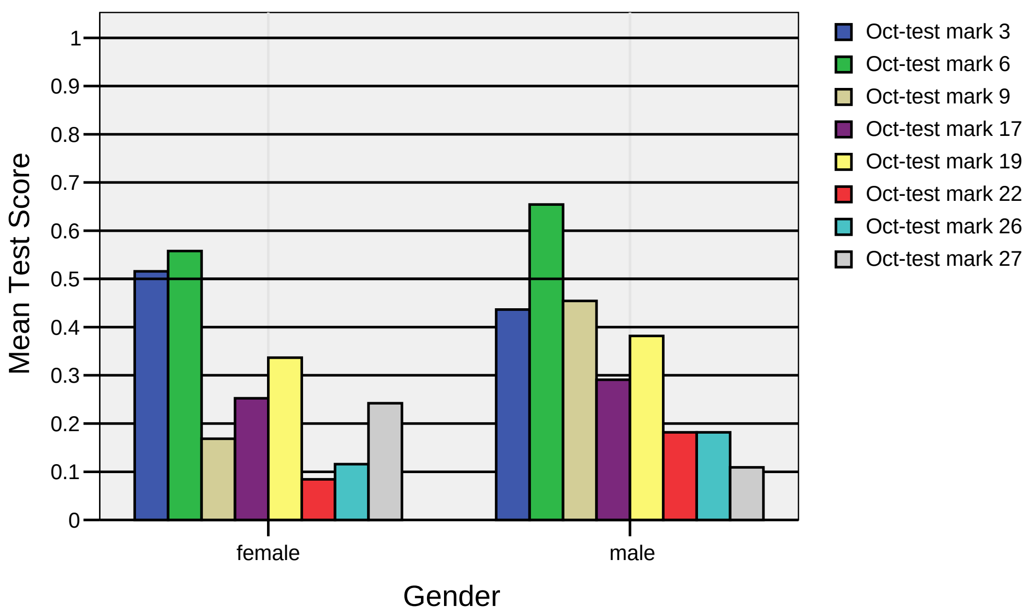


Figure 4.16 Gender comparisons of the mathematical proficiency of Grade ten learners in strategic competence (October-test)

Productive Disposition:

Table 13 (see Appendix G) is a gender comparison of the usefulness of the subject from May to October. These scores represent the mean agreement. The items assessed the grade ten learner's disposition to the usefulness of the subject mathematics in their everyday and future life. The nine items in the May and October questionnaires that assessed the usefulness of the subject mathematics was not responded to equally.

Table 13 shows that the majority of both the male and female grade ten learners had seen the usefulness of their subject in the May and October questionnaires. From the May to the October questionnaire, there has been a decrease mean agreement for each item. The greatest differences in mean scores, in descending order, were in items 29, 13, 17, 19 and 26. It is interesting to note that all these items dealt with the subject mathematics being related to a job/career. This implies that these grade ten learners have changed their minds over the course of the year on whether to follow a career/job involving mathematics. Table 14 (refer to Appendix G) is a gender comparison of the grade ten learners' attitude to their subject over the period from May to October. Except in items 1 and 20p for the females and males respectively, it is evident from the table that the male and female grade ten learners' attitude towards their subject had changed negatively from May to October. Table 15 (see Appendix G) depicts the gender comparison of grade ten learners' ability to do mathematics. Again, it is very noticeable that both the male and female grade ten learners have a general apathy to their ability to do maths.

The negative attitude of the learners could be linked to their poor performance in the subject. It is interesting to note that the mean agreement for the female grade ten learners decreased from May to October, with respect to items dealing with future work or career opportunities, such as question 19, which stated, *"I'll need maths for my future work"*.

There has been a general decline in these grade ten learners' attitude, for both cohorts, over the period from May to October. This was because these learners found their subject very difficult. Consider the following statements:

Maths is one of the hardest subjects I ever came across. It leaves me feeling stressful and it confuses my mind. Since primary school, maths has been a problem to my work. I would like some help in this subject because it seems so difficult to me to learn maths I found it very complicated to understand maths since I was in primary school I could not earn good marks for maths. To me this subject is very difficult (learner 213-female);

Maths is not important in my life. I do not like maths in my life. I don't like to learn maths because I don't understand it (learner 338) and

I like maths but I don't understand maths. It is difficult for me. Maths is very hard; you have to learn hard in maths because I can't have a good job without maths. I'm afraid when I have to do maths because I have problems with maths (learner 236).

4.4.3 Language

The grade ten learners for both cohorts had performed very poorly in the extended response questions, which required more language comprehension. I anticipate serious problems, especially with the mathematical literacy learners, because of the increased language and comprehension required by mathematical literacy due to its more applied, contextualised and ‘real life’ problem-solving nature (Graven and Venkatakrishnan, 2007).

4.5 SUMMARY AND CONCLUSION

In this chapter, I have presented and discussed the findings of the research according to the four research questions. The ML learners performed poorly in the May-test, indicating that their levels of mathematical proficiency were very low. When comparing the scores for the two cohorts on the May-test, I found that the mathematical proficiency of the mathematics learners were higher than the ML learners. Considering the evidence from the October-test, I have discovered that the mean score of the Mathematical proficiency for the mathematics learners has increased more substantially than the mean scores for the ML learners. The difference in mean scores for mathematics and ML were about 6% and 1% respectively. Lastly, I have identified two factors (gender and misconceptions) that have an influence on the mathematical proficiency of these grade ten learners. The second factor, misconceptions, seemed to play a role only in the procedural fluency strand. Concerning gender, I found that the female grade ten learners generally performed slightly better than their male counterparts for both cohorts, in each strand, although the difference was not significant. Unfortunately, I did not find any major differences, worth emphasizing. In the next chapter, I will present recommendations and conclusions for the study. Similar to the results of the TIMSS study, these grade ten learners (for both cohorts) lacked the basic mathematical knowledge expected at the Grade 8 level (Department of Education, 2003).

CHAPTER 5

In this chapter, the research study will be summarized and conclusions and recommendations for further research studies will be given. The findings and recommendations are organized using the framework of the four research questions.

5.1 SUMMARY OF THE STUDY

This research study focused on the development of mathematical proficiency, in number skills, of grade ten learners in both the Mathematics and Mathematical Literacy cohorts. It involved both quantitative and qualitative methods for data collection. The test items used in this study came from the NAEP and TIMSS study, both of which formed part of an international study testing the mathematical proficiency of Grade 8 learners. The data was collected for this study by means of the following research instruments: structured tests (May and October-tests), questionnaires, structured interviews and a piece of reflective writing. This research was conducted at Temple Valley Secondary School, selecting all grade ten learners in the school as a sample, for the purpose of this study. This school was selected for this study because of its convenience (the researcher teaches at this school). As a school, they offered two streams of grade ten mathematics, namely: pure mathematics and mathematical literacy as described in the new FET curriculum.

The analysis was structured according to the five strands of mathematical proficiency. The data from the structured tests were analyzed using a software package called SPSS, which helped to determine the level of mathematical proficiency attained in these mathematics classrooms. The attitude questionnaires were analyzed thematically. Transcriptions of the grade ten learners’ interviews were analyzed to determine the attitudes and feelings these learners’ experienced, in their respective mathematics classes during the course of the year.

The findings of this study are summarised according to the four research questions.

5.1.1 Mathematical proficiency of grade ten mathematical literacy learners in the domain of numbers

The mathematical proficiency of the ML learners was generally poor. The ML learners performed the best in the procedural fluency strand and the worst in the adaptive reasoning strand, indicating that these learners have a lower capacity for logical thinking and reasoning than for doing routine procedures.

5.1.2 Mathematical proficiency of grade ten mathematical literacy compared with the mathematical proficiency of grade ten mathematics learners for the May test

Results from the May test indicate that the mathematical proficiency of the mathematics learners was slightly higher than the ML learners. The mathematics learners also performed better than the ML learners in all four strands, for the May test. It is interesting to note that learners from both cohorts had positive attitudes to the subject mathematics at the beginning of the year.

5.1.3 The mathematical proficiency changing over the course of the grade ten year

This research found that there was a greater improvement, in the mean scores for mathematical proficiency, for the pure mathematics learners than the ML learners. Over the course of four to five months, I found that the mathematics learners improved by about 6% as compared to only 1% for the ML learners. The research revealed that the proficiency levels of procedural fluency are high in the easiest contexts, that is, when asked to add or subtract three-digit whole or decimal numbers in the standard format. However, these grade ten learners were less fluent in working with rational numbers, both common and decimal fractions. The first conclusion that can be drawn is that, at the grade ten level, many learners may have procedural fluency with respect to particular concepts, but may lack procedural fluency in others. This study also assessed the grade ten learners' conceptual understanding of numbers by asking them about the properties of the number system. The second conclusion drawn was that more grade ten learners could calculate successfully with numbers than could work with the properties of the same numbers. The same was true for rational numbers. A third conclusion emanating from the research was that these learners were working with numbers that they really did not understand. These grade ten mathematics learners had performed fairly well on questions about basic whole number operations and concepts in numerical and simple applied contexts. However, they had difficulty with more complex problem solving situations. It appears that the performance on word problems drops drastically when additional features are included, such as more than one-step or extraneous information. Furthermore, any small changes in the wording of the problem, context or presentation yielded a dramatic change in these grade ten's success rates. One can conclude that the problem solving abilities of the grade ten mathematics learners is very weak, together with the fact that the language of the questions seems to affect their success.

In this research study, the grade ten learners of both cohorts developed proficiency among the five strands in a very uneven way. They were most proficient in aspects of procedural fluency and less proficient in conceptual understanding, strategic competence, adaptive reasoning and productive disposition. The results from the research showed that many grade ten learners made few connections among these strands. The results showed that none of the grade ten learners, from both cohorts, was mathematically proficient because they did not seem empowered enough with the competence, expertise and knowledge in mathematics.

The learners' proficiency in adaptive reasoning was assessed using several kinds of items, often in conjunction with other strands. The reasoning in question 21 did not require procedural fluency or any additional proficiency. Therefore, this task was less demanding than the computational tasks and required only that basic understanding and reasoning be connected. This research suggests that for many grade ten mathematics learners this connection was not being made. Another kind of item (question 26 in October test) that measured adaptive reasoning was one that asked learners to justify and explain their solutions. It is apparent that these grade ten learners had trouble justifying their answers even in relatively simple cases. The research revealed that the proficiency levels of adaptive reasoning are low even in the simple contexts.

In terms of the strand of productive disposition, this research study has focussed on usefulness of mathematics, attitude toward mathematics and ability to do mathematics. In general, both the male and female grade ten learners had a positive attitude toward mathematics. However, the pure mathematics learners had developed a negative attitude toward their subject over the course of the year. The research also found that many of the grade ten learners view mathematics as useful in their everyday life and for a future career or job.

5.1.4 Factors influencing the achievement of mathematical proficiency

For the purposes of this study, I selected two factors to investigate, namely misconceptions and gender.

Misconceptions affected the results in the procedural fluency strand more than any other strand. The results from this research showed that these grade ten learners had misconceptions with the process of subtraction. It was interesting to note that more mathematics learners had misconceptions with subtraction than ML learners did. Research has shown that males performed better than females on problems involving mathematical reasoning (Stage et al, 1985). However, my research has found that girls perform better on problems involving words and the ability to think or reason. Although, mathematics has been made a compulsory subject in South African schools, I am concerned that too few females, compared to males (in the majority of South African schools), are developing mathematical proficiency and furthering their study of mathematics.

5.2 RECOMMENDATIONS

This study can alert teachers and curriculum developers to help learners overcome some of their misconceptions, discussed in this research study. It can also aid these professionals in developing more mathematically proficient grade ten learners. A possible way to overcome misconceptions will be for teachers in the secondary school to identify the deeper levels of misconceptions and help correct these. Here teachers will need to help develop the learner's conceptual understanding and in this way the learner is less likely to forget the critical steps and will be able to reconstruct them independently when needed. Furthermore, it is important that pupils in primary schools learn with understanding because this will make for more efficient learning in the secondary school (or future).

The state and major companies will need to invest in our female learners. This is currently in practice, but on a very limited scale. More incentives must be provided from the corporative world so our learners would want to persue a career in mathematics.

This research could inform both teachers and curriculum developers in the mathematics education field: Firstly, *Teachers* of grade ten learners with an interest in improving the mathematical proficiency, in number skills, of learners. Numeracy skills (LO1) is the foundational requirement for LO3 (Space, Shape and Measurement) and LO4 (Data Handling) in both cohorts. If learners are not highly efficient in number skills then this will be an obstacle for the Learning Outcomes mentioned above. Consequently, teachers will need to design intervention programs to support the development of mathematical proficiency in numeracy skills in these learners. Learners who perform poorly in May and October-tests scores for proficiency in number skills will participate in an intervention program (in 2007) where remediation of learners' weaknesses can be focussed on and dealt with. Secondly, this study is of interest to *Curriculum developers and materials development specialists* who prepare mathematical material for grade ten classrooms. Curriculum developers and materials development specialists could design/develop a 'do it yourself' series, focussing on mathematical proficiency skills development. These series could be designed as separate alongside skills booklets or as additional sections in the prescribed books. Also in keeping with technology, compact discs (CD's) containing additional basic exercisers could be purchased to aid learners improve their proficiency in number skills.

This research has shown that the five strands of mathematical proficiency were unevenly developed. The mathematics currently taught has developed some procedural fluency, but it clearly has not helped these grade ten learners develop the other strands very far, nor has it helped them connect the strands. Therefore, all strands with respect to number skills have suffered. The grade ten learners need enough time to engage in activities about a specific mathematical topic if they are to become proficient with it (Kilpatrick et al, 2001). Therefore, these curriculum and materials development specialists must take into consideration that mathematical proficiency can only be developed over time, where the curriculum will need to be restructured so that extra time is allocated for this purpose. Furthermore, universities and colleges will need to include the study of these five strands into the curriculum for future mathematics educators to become efficient in teaching for mathematical proficiency.

Historically, mathematics in South Africa has been a discipline for a select group of learners. However, more recently the government of South Africa has insisted on mathematics for all learners. The problem is that only a few learners have access to high-quality mathematics education. The state needs to pump in more funds to train and upgrade mathematics educators.

Arising from this research, I recommend that future studies be conducted that will pay attention to:

- The development of materials in OBE and FET that will encourage mathematical proficiency within the school context
- Instruction that would support the development of mathematical proficiency for all grades
- The development of learner support materials that integrate the five strands of mathematical proficiency.

We as a new democracy are far from developing grade ten learners that are mathematically proficient. Mathematical proficiency cannot be achieved through isolated efforts. All interested stakeholders, including parents, teachers, administrators and policy makers, must work together to improve the mathematics at school.

References

Bassey, M. (1999). Case study research in educational settings. Buckingham: Open University Press.

Best, J.W. and Kuhn, J.V. (1986). *Research in education*. New Jersey: Prentice Hall.

Bransford, John D., Brown, Ann L., and Cocking, Rodney R. (Eds.) (1999). *How People Learn: Brain, Mind, Experience, and School.* Washington, D.C.: National Academy Press.

Brombacher, A. (2001). AMESA submission to SAQA on the proposed Unit Standards for Mathematical Literacy (NQF levels 2, 3, and 4). AMESA Curriculum Committee: Cape Town.

Campbell, D. T. and Stanley, J. C. (1963). *Experimental and quasi-experimental designs for research.* Chicago: Rand McNally and Company.

Creswell, J.W. (1998). *Research inquiry and research design: choosing among five traditions.* Thousands Oaks: Sage.

Criticos, C. (2002). *Getting Practical About Outcomes-bases Teaching: Learning Guide.* South Africa: SAIDE and Oxford University Press. (Chapter 3: Assessing learning and teaching).

Davis, R. B. (1984). *Learning mathematics. The Cognitive Science Approach to Mathematics Education.* Croom Helm, London.

Denzin, N and Lincoln, Y (2003) 'Introduction: the discipline and practice of qualitative research' in *The landscape of qualitative research: theories and issues* edited by Norman Denzin and Yvonna Lincoln. Thousand Oaks, CA.: Sage Publications. pp. 1-17.

Department of Education. (2003). National Centre for Education Statistics. *Institute of Education Sciences.* Washington, D.C.: United States.

Department of Education. (2003a). *National Curriculum Statement Grades 10-12* *(general) Mathematical Literacy*. Pretoria: Department of Education.

Department of Education. (2003b). *National Curriculum Statement Grades 10-12 (general) Mathematics*. Pretoria: Department of Education.

Department of Education. (2002). *Revised National Curriculum Statement Grades R-9 (Schools) Mathematics.* Pretoria: Department of Education.

De Vos, A.S. (Ed). (1998). *Research at grass roots.* Pretoria; Van Schaik.

Doepken, D., Lawsky, E., and Padwa, L. Modified Fennema – Sherman attitude scales. Accessed from http://woodrow.org/teachers/math/gender/08scale.html on 23 September 2006.

Donovan, M., Bransford, J., and Pellegrino, J. (1999). *How People Learn: Bridging Research and Practice.* Washington, DC: National Academy Press.

Frost, L. A., Hyde, J. S., and Fennema, E. (1994). Gender, mathematics performance, and mathematics-related attitudes and affect: A meta-analytic synthesis. *International Journal of Educational Research, 21*, 373-384.

Graven, M. and Venkatakrishnan, H. (2007). Emerging successes and tensions in the implementation of mathematical literacy. *Learning and Teaching Mathematics*, 4, 5-9.

Grouws, Douglas A. and Cebulla, Kristin J. (2000). *Improving Student Achievement in Mathematics.* Geneva, Switzerland: International Academy of Education.

Hewson, P.W. (1996). Educating prospective teachers of biology: Introduction and research methods. *Science Education,* 8 (3): 247-273.

Hiebert, J. (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 5-23). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., and Carpenter, T.P. (1992). Learning and Teaching with Understanding. In

D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Human Science Research Council (2000). International study finds South Africa remains behind other countries. Issued by Corporate Communications, HSRC, Pretoria (6 December 2000). Accessed from http://www.hsrc.ac.za/Document-540.phtml on 24 August 2005.

Kieran, C. (1981). Concepts Associated with the Equality Symbol. *Educational Studies in Mathematics*, 12 (3), 317-326.

Kilpatrick, J.; Swafford, J.; Findell, B (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, D.C.: The National Academies Press.

Kimball, M. M. (1989). A new perspective on women's math achievement. *Psychological Bulletin, 105*, 198-214.

Krueger, R. A. (1986). *Focus Group Interviews As An Evaluation Tool.* Adapted from a presentation at the 1984 Joint Meeting of the Evaluation Research Society and the Evaluation Network, San Francisco, California.

Labaree, D. (1999). The Chronic Failure of Curriculum Reform: With Recurring Waves of Rhetoric and Glacial Change in Practice, Little That Is Truly New Has Trickled Down to Students. *But Why? Education Week*, May 19, 1999, pp. 42-44.

Leder, G., and Forgasz, H. (1997). Single-sex classes in a co-educational high school: Highlighting parents perspectives. Mathematics Education Research Journal, 9 (3), 274–291.

Lokan, J. Ford, P. and Greenwood, L. (1997). *Maths and Science on the line: Australian middle primary school students' performance in the Third International Mathematics and Science Study.* Melbourne: ACER.

Madison, B.L. (2004). Two Mathematics. Ever then Twain Shall Meet? *Peer Review*, 6 (4): 9-12.

Milgram, J.R. (2004). *What is Mathematical Proficiency.* Stanford University. Accessed from: www.math.stanford.edu/ftp/milgram/msri-presentation.pdf on the 25 August 2006

Mullis, I. V. S., Martin, M. O., Fierros, E. G., Goldberg, A. L. and Stemler, S. E. (2000). *Gender differences in achievement: IEA's Third International Mathematics and Science Study (TIMSS).* Chestnut Hill MA: International-Association-for-the Evaluation-of-Educational-Achievement (IEA), TIMSS International Study Center, Boston College. Accessed from: http://isc.bc.edu/timss1995i/gender.html

Mathematics Association of America. *Why Quantitative Literacy? Part 1*. Accessed from: http://www.maa.org/past/ql/qlpart1.html on 22 August 2005

National Assessment Governing Board. *Mathematics Framework for the 2005 National Assessment Progress.* Washington, DC: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: Author.

Olivier, A.I. (1989). Handling Pupils' Misconceptions. *Pythagoras*, No.21, pp.10-19.

Organisation for Economic Co-operation and Development (OECD). (Ed). (2001). Knowledge and Skills for Life. First Results from the OECD Programme for International Student Assessment (PISA) (2000). Paris: OECD.

Piaget, J. (1970). *Genetic Epistemology.* University of Chicago Press, Chicago.

Reddy, V. (2006). *Mathematics and Science Achievement at South African Schools in TIMSS 2003.* Cape Town: HSRC Press.

Romberg, T. A. (2001). Mathematical Literacy: What Does It Mean for School Mathematics? *Wisconsin School News*: October, pp. 1-5.

Schields, M. (2002). Three Kinds of Statistical Literacy: What Should We Teach? In Phillips, B (Ed), *Proceedings of the Sixth International Conference on Teaching Statistics.* Cape Town: International Statistical Institute.

Schoenfeld, A. (2001). Reflections on an Impoverished Education. In Steen, L.A (Ed), *Mathematics and Democracy.* Princeton, New Jersey: National Council on Education and the Disciplines.

Skemp, R.R. (1979). *Intelligence, Learning in Action.* John Wiley and Sons, New York.

Stage, E. K., Kreinberg, N., Eccles, J. R., and Becker, J. R. (1985). Increasing the participation and achievement of girls and women in mathematics, science, and engineering. In S. S. Klein (Ed.), *Handbook for achieving sex equity through. Education* (pp. 237-269). Baltimore: Johns Hopkins University Press.

Stoessiger, R. (2003). An Introduction to Critical Numeracy. Paper presented at the VC 2003: Springsboards into Numeracy, Tasmania.

Usher, R (1996) ‘A critique of the neglected epistemological assumptions of educational research’ in *Understanding educational research* edited by David Scott and Robin Usher. London: Routledge. pp. 9-32.

Wallace, D. (2000). The Many Roads to Numeracy. In Burke, M.J. and Curcio, F.R (Eds.), *Learning Mathematics for a New Century* (pp. 28-36). Reston Va: The National Council of Teachers of Mathematics.

APPENDICES

APPENDIX A MAY-TEST AND SCORING GUIDE

HOW ARE YOUR BASIC MATHS SKILLS?

Name: ___________________________________

Gender: _______________ **Grade: 10_____** **Age: ___________**

Note: **Circle** the correct answer!

Questions:

1. Subtract: 6000

   –2369

   A. 4369

   B. 3742

   C. 3631

   D. 3531

2. Subtract: 2, 201 – 0, 753 =

   A. 1, 448

   B. 1, 458

C. 1, 548

D. 1, 558

3. How much change will John get back from R5 if he buys 2 notebooks that cost R1, 80 each?

A. R1, 40

B. R2, 40

C. R3, 20

D. R3, 60

4. James had R5 to buy milk, bread, and eggs. When he got to the shop he found that the prices were those shown below:

R1, 50 R1, 29 R1, 44

At which of these times would it make sense to use estimates rather than exact numbers?

A. When James tried to decide whether R5 was enough money

B. When the shop keeper entered each amount into the cash register

C. When James was told how much he owed

D. When the shop keeper counted James's change.

5. Justin needed to know about how much the sum of 19, 6; 23, 8; and 38, 4 is. He correctly rounded each of these numbers to the nearest whole number. What three numbers did he use?

A. 19; 23; 38

B. 19; 24; 38

C. 20; 24; 38

D. 20; 24; 39

6. Siphiwe, Thandi and their mother were eating a cake. Siphiwe ate ½ of the cake. Thandi ate ¼ of the cake. Their mother ate ¼ of the cake. How much of the cake is left?

A. ¾

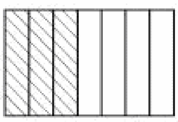
B. ½

C. ¼

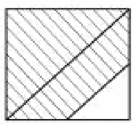
D. None

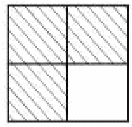
7. Which shows ¾ of the picture shaded?

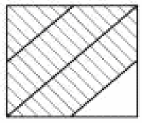
A.



B.



C. 

D. 

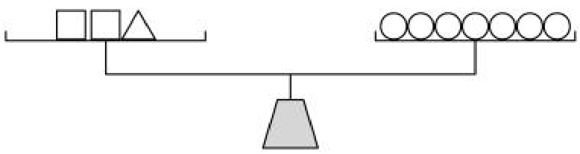
8. Six students bought exactly enough pens to share equally among themselves. Which of the following could be the number of pens they bought?

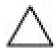
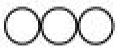
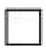
A. 46

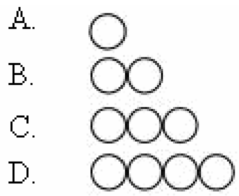
B. 48

C. 50

D. 5



9. The objects on the scale above make it balance exactly. According to the scale, if  balances , then  balances which of the following?



10. Carl has 3 empty egg cartons and 34 eggs. If each carton holds 12 eggs, how many more eggs area needed to fill all 3 cartons?

A. 2

B. 3

C. 4

D 6

11. If 39 + 93 = 132 is true, which of the following is true?

A. 39 = 93 + 132

B. 39 + 132 = 93

C. 132 – 39 = 93

D. 93 - 132 = 39

12. What is 4 hundredths written in decimal notation?

A. 0, 004

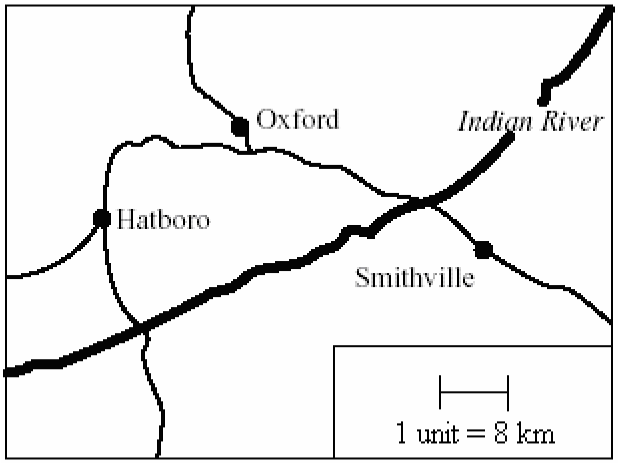
B. 0, 04

C. 0, 400

D. 4, 00

E. 400, 0

13. One unit on the map represents 8 kilometres on the land.



About how far apart are Oxford and Smithville on the land?

A. 4 km

B. 16 km

C. 35 km

D. 50 km

14. **Divide:** $^{8}/_{35} \div {}^{4}/_{15}$

A. $^{32}/_{525}$

B. $^{6}/_{7}$

C. $^{525}/_{32}$

D. $^{7}/_{6}$

15. Zuma's garden has 84 rows of cabbages. There are 57 cabbages in each row. Which of these gives the BEST way to estimate how many cabbages there are altogether?

A. 100 X 50 = 5000

B. 90 X 60 = 5400

C. 80 X 60 = 4800

D. 80 X 50 = 4000

16. (-5) (-7) =

A. -35

B. -12

C. -2

D. 12

E. 35

17. 42**;** 51**;** 49**;** 58**;** 56**;** …

If the pattern in the list above continues, what will be the next number after 56?

A. 54

B. 63

C. 64

D. 65

E. 67

18. A chemist mixes 3, 75 millilitres of solution A with 5, 625 millilitres of solution B to form a new solution. How many millilitres does the new solution contain?

A. 9, 370

B. 8, 700

C. 8, 375

D. 9, 375

19. Fifteen boxes each containing 8 radios can be repacked in 10 larger boxes each containing how many radios?

A. 8

B. 10

C. 12

D. 80

E. 120

20. By how much would 217 be increased if the digit 1 were replaced by a digit 5?

A. 4

B. 40

C. 44

D. 400

21. Estimate the value of: $^{11}/_{12} + {}^{6}/_{7}$

A. 1

B. 2

C. 17

D. 19

22. Mrs. Moodley and 3 friends ate at a restaurant. The bill was R67. In addition, they left a R13 tip. Approximately what percent of the total bill did they leave as a tip?

A. 10 %

B. 13 %

C. 15 %

D. 20 %

E. 25 %

23. Which list shows the numbers from smallest to largest?

A. 0,345 ; 0,19 ; 0,8 ; $^{1}/_{5}$

B. 0,19 ; $^{1}/_{5}$; 0,345 ; 0.8

C. 0,8 ; 0,19 ; $^{1}/_{5}$; 0,345

D. $^{1}/_{5}$; 0,8 ; 0,345 ; 0,19

24. Write 0, 28 as a fraction reduced to its lowest terms.

A. $^{28}/_{100}$

B. $^{14}/_{50}$

C. $^{7}/_{25}$

D. $^{7}/_{50}$

25. A high school orders 11 buses to transport 418 students. If each bus can seat 35 students, will the number of buses ordered be enough to provide a seat for each student?

A. Yes

B. No

Explain your answer.

____

____

____

____

____

26. Jack said, “I can multiply 6 by another number and get an answer that is smaller than 6. “ Mary said, “No, you can’t. Multiplying 6 by another number always makes the answer 6 or larger.” Who is correct? Give a reason for your answer.

27. One store, Edgars, reduces the price each week of a R100 stereo by 10 percent of the original price.

Another store, Woolworths, reduces the price each week of the same R100 stereo by 10 percent of the previous week’s price.

After 2 weeks, how will the prices at the two stores compare?

A. The price will be cheaper at Edgars.

B. The price will be the same at both stores.

C. The price will be cheaper at Woolworths.

Explain your reasoning

I understand that the information I have given will be used only for academic research purposes in a project investigating mathematical proficiency of grade ten learners and that this information will be kept confidential and my anonymity will be assured.

Sign: ______________________ Date: __________________

May-test

Table: Scoring Guide for questions 1-24 (May-test):

Questions	Solution
1	C
2	A
3	A
4	A
5	C
6	D
7	C
8	B
9	B
10	A
11	C
12	B

13	C
14	B
15	C
16	E
17	D
18	D
19	C
20	B
21	B
22	C
23	B
24	C

Scoring guide for Questions 25-27:

Question 25

Solution:

418 ÷ 11 = 38 per bus which is 3 more students than 35, or 3 more students would have to fit in each bus

OR

418 ÷ 11 = 38 is 3 more students than can fit in a bus

OR

418 ÷ 35 = $11^{33}/_{35}$ buses (must include $^{33}/_{35}$)

OR

11 × 35 is less than 418

Score and Description

Correct

Correct response

Note: Explanation must indicate that 11 buses will only seat 385 students. This may also be illustrated by an example such as

$11 \times 35 = 385$

OR $418 \div 35 = 11$ with a remainder of 33

OR needs 33 more seats

Incorrect

Any incorrect or incomplete response

An incorrect response includes an incorrect computation

In this question the student needed to apply multiplication or division to solve a word problem, and then interpret the answer in the context of the question. To earn full credit the student needed to explain either that there were not enough seats on 11 buses for 418 students or that 418 students would require more than 11 buses.

<u>Question 26</u>

Score and Description

Correct #1

Jack, with correct reason given.

Examples of correct reasons:

· If you multiply by a number *smaller* than 1 the result is less than 6.

· $6 \times 0 = 0$

· 6 × 1/2 = 3

· 6 × -1 = -6

Correct #2

No name stated but reason given is correct.

Incorrect #2

Jack, with no reason or an incorrect reason.

Incorrect #1

Any response that states that Mary is correct

OR

No name stated and reason given is incorrect.

<u>Question 27</u>

Solution:

A. Cheaper at Edgars

At Edgars the stereo would be R80 after 2 weeks.

At Woolworths, it would cost R81.

OR

Successive 10% reductions of the original price will yield greater savings than successive reductions of 10% of the reduced price.

Score and Description

Correct

Correct response cheaper at Edgars with an explanation that compares price at each store after 2 weeks (R80 vs. R81)

OR

Cheaper at Edgars with an explanation that generalizes as described in solution above

NOTE: Score CORRECT if incorrect answer is B or C with a clear statement that Edgars is cheaper and explanation is correct and complete.

Partial

Cheaper at Edgars with anything less than a complete explanation

OR

Computes the correct amount for at least 2 weeks for either Edgars or Woolworths, but conclusion is missing, incomplete, or incorrect (if the store is not identified the score is still a 2)

Incorrect

Incorrect response

In this question the student was asked to compare the sale price of a stereo, after 3 weeks, based on two different ways for reducing the price. In one store, the price was reduced each week by a fixed amount (10% of R100, or R10). In the other store the price was reduced each week by a varying amount (10% of the current price, which is less each week). To earn full credit, the student needed to indicate that the price would be less at the first store after 3 weeks and explain how the solution was obtained.

APPENDIX B OCTOBER-TEST AND SCORING GUIDE

HOW ARE YOUR BASIC MATHS SKILLS?

Name: ___________________________________

Gender: ________________ **Grade: 10**_____ **Age:** ____________

Note: **Circle** the correct answer!

Questions:

1. Subtract: 7000

   \- 4479

   A. 2432

   B. 3479

   C. 2421

   D. 2521

2. Subtract: 3, 302 – 0, 853 =

   A. 2, 459

B. 2, 449

C. 2, 549

D. 2, 559

3. How much change will John get back from R10 if he buys 2 pencils that cost R1, 90 each?

A. R7, 20

B. R8, 10

C. R6, 20

D. R3, 80

4. James had R5 to buy milk, bread, and eggs. When he got to the shop he found that the prices were those shown below:

R1, 40 R1, 39 R1, 44

At which of these times would it make sense to use estimates rather than exact numbers?

A. When James tried to decide whether R5 was enough money

B. When the shop keeper entered each amount into the cash register

C. When James was told how much he owed

D. When the shop keeper counted James's change.

5. Justin needed to know about how much the sum of 17, 6**;** 25, 8**;** and 36, 4 is. He correctly rounded each of these numbers to the nearest whole number. What three numbers did he use?

A. 17**;** 25**;** 36

B. 17**;** 26**;** 36

C. 18**;** 26**;** 37

D. 18**;** 26**;** 36

6. Siphiwe, Thandi and their mother were eating a cake. Siphiwe ate ½ of the cake. Thandi ate ¼ of the cake. Their mother ate ¼ of the cake. How much of the cake is left?

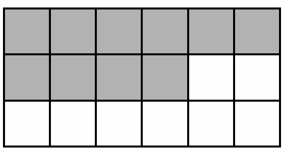
A. ¼

B. ½

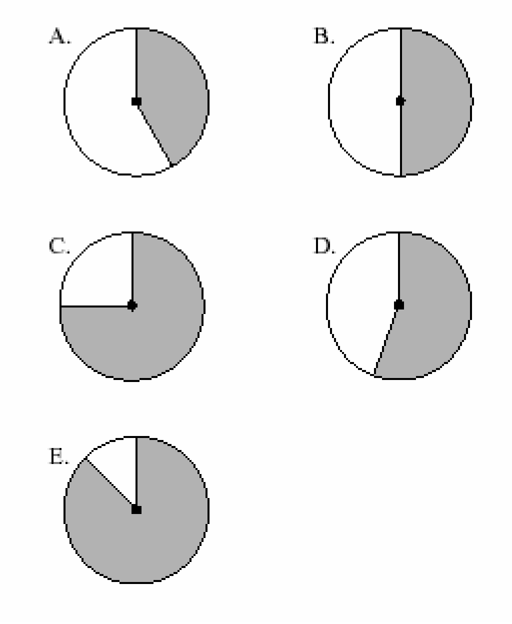
C. ¾

D. None

7.

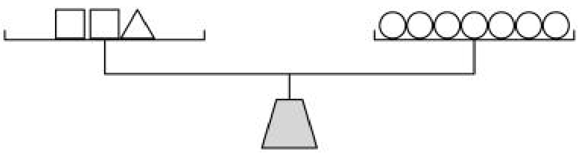


Which circle has approximately the same fraction shaded as that of the rectangle above?

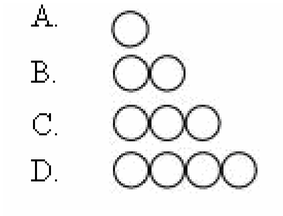


8. Eight students bought exactly enough pens to share equally among themselves. Which of the following could be the number of pens they bought?

A. 38
B. 48
C. 50
D. 58



9. The objects on the scale above make it balance exactly. According to the scale, if △ balances ○○○, then □ balances which of the following?



10. Carl has 4 empty egg cartons and 44 eggs. If each carton holds 12 eggs, how many more eggs area needed to fill all 4 cartons?

A. 2

B. 4

C. 5

D 6

11. If 37 + 73 = 110 is true, which of the following is true?

A. 37 = 73 + 110

B. 37 + 110 = 73

C. 110 – 37 = 73

D. 73 - 110 = 37

12. What is 6 hundredths written in decimal notation?

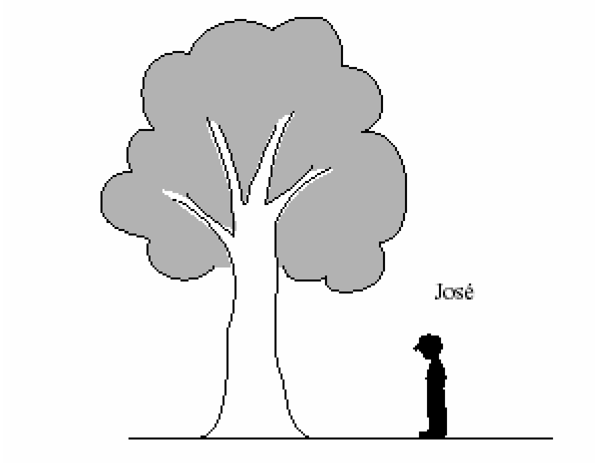
A. 0, 006

B. 0, 06

C. 0, 600

D. 6, 00

E. 600, 0



13. José is 1.5 m tall. About how tall is the tree?

A. 4 m

B. 6 m

C. 8 m

D. 10 m

14. **Divide:** $^{4}/_{25} \div {}^{8}/_{15}$

A. $^{32}/_{375}$

B. $^{10}/_{3}$

C. $^{375}/_{32}$

D. $^{3}/_{10}$

15. Zuma's garden has 84 rows of cabbages. There are 57 cabbages in each row. Which of these gives the BEST way to estimate how many cabbages there are altogether?

A. 100 X 50 = 5000

B. 90 X 60 = 5400

C. 80 X 60 = 4800

D. 80 X 50 = 4000

16. (-6) (-8) =

A. -14

B. 14

C. -2

D. 48

E. -48

17. 52**;** 61**;** 59**;** 68**;** 66**;** …

If the pattern in the list above continues, what will be the next number after 66?

A. 64

B. 73

C. 74

D. 75

E. 77

18. A pharmacist mixes 2, 75 millilitres of solution A with 6, 625 millilitres of solution B to form a new solution. How many millilitres does the new solution contain?

A. 9, 370

B. 9, 375

C. 8, 375

D. 8, 700

19. Fourteen boxes each containing 6 radios can be repacked in 7 larger boxes each containing how many radios?

A. 6

B. 7

C. 12

D. 42

E. 84

20. By how much would 257 be decreased if the digit 5 were replaced by a digit 1?

A. 4

B. 40

C. 44

D. 200

21. Estimate the value of: $^{8}/_{9} + {}^{13}/_{14}$

A. 23

B. 21

C. 2

D. 1

22. Mr. Moodley and 2 friends ate at a restaurant. The bill was R57. In addition, they left a R13 tip. Approximately what percent of the total bill did they leave as a tip?

A. 10 %

B. 13 %

C. 15 %

D. 20 %

E. 25 %

23. Which list shows the numbers from largest to smallest?

A. 0,345 ; 0,19 ; 0,8 ; $^{1}/_{5}$

B. 0,19 ; $^{1}/_{5}$; 0,345 ; 0.8

C. 0,8 ; 0,345 ; $^{1}/_{5}$; 0,19

D. $^{1}/_{5}$; 0,8 ; 0,345 ; 0,19

24. Write 0, 36 as a fraction reduced to its lowest terms.

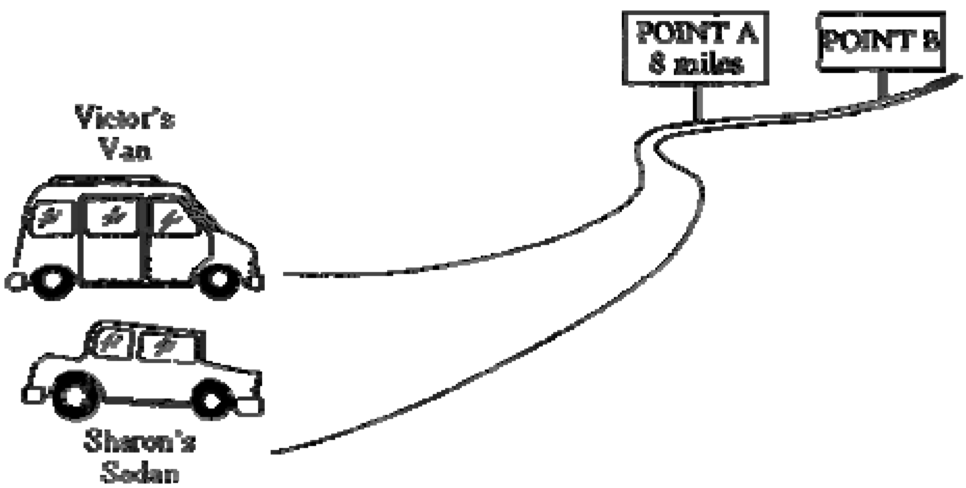
A. $^{36}/_{100}$

B. $^{18}/_{50}$

C. $^{9}/_{25}$

D. $^{9}/_{50}$

25.

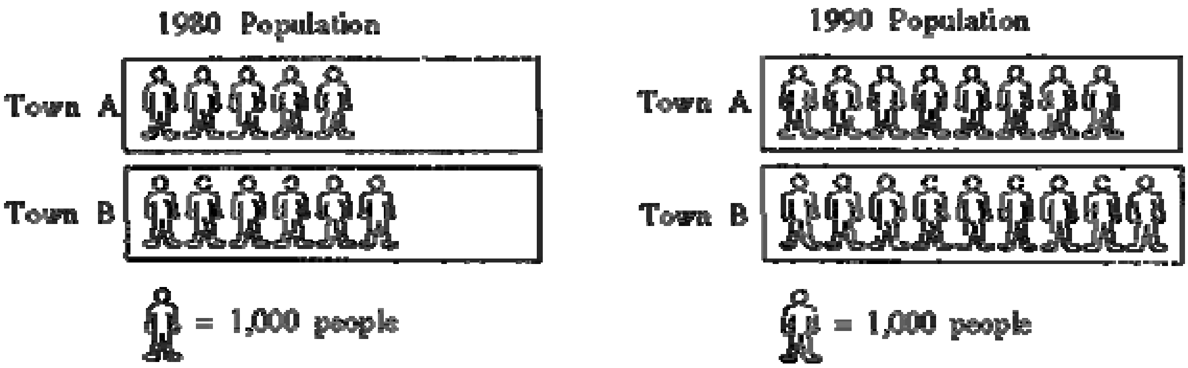


Victor's van travels at a rate of 8 miles every 10 minutes. Sharon's sedan travels at a rate of 20 miles every 25 minutes.

If both cars start at the same time, will Sharon's sedan reach point A, 8 miles away, before, at the same time, or after Victor's van?

Explain your reasoning.

26.



In 1980, the populations of Town A and Town B were 5,000 and 6,000, respectively. The 1990 populations of Town A and Town B were 8,000 and 9,000, respectively.

Brian claims that from 1980 to 1990 the populations of the two towns grew by the same amount. Use mathematics to explain how Brian might have justified his claim.

Darlene claims that from 1980 to 1990 the population of Town A had grown more. Use mathematics to explain how Darlene might have justified her claim.

---

27. One store, Edgars, reduces the price each week of a R100 stereo by 10 percent of the original price.

Another store, Woolworths, reduces the price each week of the same R100 stereo by 10 percent of the previous week's price.

After 2 weeks, how will the prices at the two stores compare?

A. The price will be cheaper at Edgars.

B. The price will be the same at both stores.

C. The price will be cheaper at Woolworths.

Explain your reasoning

I understand that the information I have given will be used only for academic research purposes in a project investigating mathematical proficiency of grade ten learners and that this information will be kept confidential and my anonymity will be assured.

Sign: ______________________ Date: __________________

October-test

Table: Scoring Guide for questions 1-24 (October-test):

Questions	Solution
1	D
2	B
3	C
4	A

5	D
6	D
7	D
8	B
9	B
10	B
11	C
12	B
13	B
14	D
15	C
16	D
17	D
18	B
19	C
20	B
21	B
22	D
23	C
24	C

Scoring Guide for Questions 25 and 26:

Question 25

Solution:

They will both reach points A and B at the same time because their rates are equal.

Scoring Guide

In this question, a student needed to use proportional thinking to solve this problem. The student should have reasoned that both Victor's rate and Sharon's rate are equal. To earn full credit the student needed to answer both parts of the question correctly with the correct justification for each. To earn partial credit, a student could have just answered one part correctly with the correct justification or answered both parts with no justification or incorrect justifications.

Score and Description

Correct

Correct response

Partial

Answers either part with correct justification.

OR

Answers both parts correctly with no justification

Incorrect

Incorrect response

<u>Question 26</u>

Solution:

Brian (constant difference)

Town A $8{,}000 - 5{,}000 = 3{,}000$ OR $6{,}000 - 5{,}000 = 1{,}000$

Town B $9{,}000 - 6{,}000 = 3{,}000$ $\quad$ $9{,}000 - 8{,}000 = 1{,}000$

Town A $\quad$ Town B

$\frac{5}{8} = .625$ with $\frac{6}{9} = .66\overline{6}$

$$\therefore \quad \begin{array}{r} 1.000 \\ .625 \\ \hline .375 \end{array} > \begin{array}{r} 1.000 \\ .66\overline{6} \\ \hline .33\overline{3} \end{array}$$

Darlene (Proportional)

Town A $\frac{8{,}000 - 5{,}000}{5{,}000} \times 100\% = 60\%$ $\quad$ $8 \div 5 = 1.6$

OR

Town B $\frac{9{,}000 - 6{,}000}{6{,}000} \times 100\% = 50\%$ $\quad$ $9 \div 6 = 1.5$

OR

	1980	1990	
$\frac{\text{Town A}}{\text{Town B}}$	$\frac{5}{6} = .8\overline{3}$	$\frac{8}{9} = .8\overline{8}$	shows Town A has grown by MORE

Scoring Guide

In this question, a student has to use mathematical reasoning to construct different justifications that support Darlene's and Brian's claims. In this real-world problem, students have to construct an appropriate method for supporting each claim in the question. One such method uses proportional thinking as well as percentages to explain how a conclusion could be reached, while the other involves comparing differences in growth by subtraction. Partial credit is earned if a student just gives a correct justification for Brian's or Darlene's claim, but not both, or shows a limited understanding of how to proceed with the problem.

Score and Description

Correct

Correct response

Partial

Gives correct reason for Brian or Darlene but not both. (Reason must include mathematics illustrated in above solution or an appropriate reason using proportion.)

OR

uses 1000 or 3000 (without additional mathematics) for Brian's claim and uses 60% and 50% (or their equivalents) without additional mathematics for Darlene's claim

Incorrect

Incorrect response

APPENDIX C QUESTIONNAIRE

<u>WHAT DO YOU THINK ABOUT MATHS?</u>

Name: ___________________________________

Gender: ______________ **Grade: 10**_____ **Age:** ___________

Below is the scale which you will use to rate your responses to the questions.

	Strongly	Disagree	Neither agree	Agree	Strongly
	1	**2**	**3**	**4**	**5**
eg. I love sport.					**X**

Please place only **one** cross in the appropriate column.

	1	**2**	**3**	**4**	**5**
1. I know I can do well in maths.					
2. I am no good in maths.					
3. I am sure that a steady effort in learning maths will benefit me.					
4. I always feel nervous when I am asked to solve maths problems.					
5. I understand what I learn in maths.					
6. I don’t find maths too difficult.					
7. I see myself applying maths in my everyday life.					
8. I can make sense of what I learn in maths.					
9. I am afraid when I open my book and see a page full of maths problems.					
10. I don’t think I can learn maths.					

11. I study maths because I know how useful it is.					
12. Maths makes me feel confident and I'm good at maths.					
13. Knowing maths will help me get a good job.					
14. I am often nervous/afraid when I enter the maths class.					
15. I don't understand what I learn in maths.					
16. I am sure that I can learn maths.					
17. I would like to further my studies in maths.					
18. Studying maths is a waste of time.					
19. I'll need maths for my future work.					
20. I do not think that a steady effort in learning maths will benefit me.					
21. I would want never to study maths ever again.					
22. I find my maths class very stressful.					
23. Maths is not important for my everyday life.					
24. I will use maths in many ways as an adult.					
25. I am not intelligent enough to do maths.					
26. I would like to follow a career in maths/science.					
27. Maths is not important for my life.					
28. If I work hard at maths, my results will improve.					
29. I'll need a good understanding of maths for my future work.					
30. To do well in maths, you need a maths brain.					
31. I wish I was better at maths.					

Write **three** sentences about your experiences of maths so far.

I understand that the information I have given will be used only for academic research purposes in a project investigating mathematical proficiency of grade ten learners and that this information will be kept confidential and my anonymity will be assured.

Sign: ______________________ Date_______________________

APPENDIX D CONSENT FORM

CONSENT FORM: Research studies in Masters of Education

This research involves the development of mathematical ability, in number skills, of the grade ten learners in both the mathematics and mathematical literacy.

The name of the researcher is **Mr. V. G. Moodley** who is a mathematics teacher at Temple Valley Secondary school. The supervisor for this research is **Mrs. Sally Hobden**, who is a lecturer at the University of Kwa-Zulu Natal.

Our **contact details** are: Mr. V.G.Moodley - (032)5331734 (W).
Mrs. Sally Hobden – (031)2603435 (W), and
E-mail: hobdens1@ukzn.ac.za

The participation of any learner is voluntary in this research programme. In other words, the learners' are not compelled/forced to participate in this research. If the learner does participate, his/her responses will be treated in a confidential manner. Furthermore, the anonymity of participants will be ensured where appropriate. The participants are free to withdraw from the research at any time without any negative or undesirable consequences to themselves.
The research involves a basic **mathematics test** based on the G.E.T (grade 7, 8 and 9) work and a **reflective piece writing** on their mathematics experiences thus far. Therefore, my subjects are all grade ten learners. These learners will write one test, the duration of which will be one hour. The results of these tests will be used for research purposes. This information will be useful for the development of mathematical materials in the future. Once the academic work is completed, the original data collected will be destroyed.

I, ____ (full name) the parent/guardian of ____________________________(learner's name) in grade ten _______ hereby confirm that I understand the contents of this document and the nature of the research project, and give consent for my child/ward to participate in this research program.

______________________ ___________________ ________________

Parent/Guardian Principal Date

APPENDIX E CLEARANCE LETTER

APPENDIX F FIT OF QUESTIONS TO CURRICULUM

Item	Description of Problem	Source	GET Phase	Chief Targeted Strand	Reason	Complexity	Order: Qu	May-test	Oct.-test
1	Subtraction of four digits	TIMSS	Gr.3:LO1, AC 8.1	Procedural Fluency	1b); c)	Low	1	1-PF	1-PF
2	Subtraction of four digits decimals	TIMSS	Gr.7: LO1, AC 7.4	Procedural Fluency	1b); c)	Low	2	2-PF	2-PF
3	Addition of common fractions	TIMSS	Gr.7: LO1, AC 7.3	Strategic Competence	4a); b); c)	Low	41	3-SC	3-SC
4	Estimation of distance	TIMSS	Gr.9: LO1, AC 5	Adaptive Reasoning	3a); c)	Medium	6	4-CU	4-CU
5	Approx. of fraction on figures	TIMSS	Gr.9: LO1, AC 6	Conceptual Understanding	2a) ;b)	Medium	29	5-PF	5-PF
6	When to use estimates	TIMSS	Gr.9: LO1, AC 5	Adaptive Reasoning	3a); c)	Low	3	6-SC	6-SC
7	Estimation of product	TIMSS	Gr.9: LO1, AC 5	Adaptive Reasoning	3a); c)	Medium	54	7-CU	7-CU
8	Addition of positive decimals	TIMSS	Gr.7: LO1, AC 7.4	Conceptual Understanding	2a)	Medium	37	8-AR	8-AR
9	Estimation of tree height	TIMSS	Gr.9: LO1, AC 5	Adaptive Reasoning	3a); c)	Medium	28	9-SC	9-SC
10	Division of whole and common fraction	TIMSS	Gr.9: LO1, AC 5	Strategic Competence	4a) ;b); c)	High	47	10-CU	10-CU
11	Rounding off- two decimal places	TIMSS	Gr.7: LO1, AC 7.1	Procedural Fluency	1a) ;b); c)	High	22	11-AR	11-AR
12	Subtraction of three common fraction	TIMSS	Gr.7: LO1, AC 7.3	Procedural Fluency	1a) ;b); c)	Medium	30	12-CU	12-CU
13	Car and fuel consumption	TIMSS	Gr.9: LO1, AC 4	Strategic Competence	4a) ;b); c)	High	4	13-AR	13-AR
14	Estimation of water consumption	TIMSS	Gr.9: LO1, AC 5	Strategic Competence	4a) ;b); c)	High	19	14-PF	14-PF
15	Addition of three common fractions	TIMSS	Gr.7: LO1, AC 7.3	Procedural Fluency	1a) ;b); c)	High	7	15-AR	15-AR
16	Equivalent forms-ascending order	TIMSS	Gr.9: LO1, AC 2	Conceptual Understanding	2a) ;b)	High	51	16-PF	16-PF

17	Division of positive decimals	TIMSS	Gr.8: LO1, AC 6.2	Procedural Fluency	1a) ;b); c)	High	42	17-SC	17-SC
18	(X) and (+) of common fractions	TIMSS	Gr.8: LO1, AC 6.2	Procedural Fluency	1a) ;b); c)	High	8	18-CU	18-CU
19	Division of common fraction	TIMSS	Gr.8: LO1, AC 6.2	Procedural Fluency	1a) ;b); c)	Medium	27	19-SC	19-SC
20	Equivalent-decimal and fractions	TIMSS	Gr.9: LO1, AC 2	Procedural Fluency	1a) ;b)	High	32	20-CU	20-CU
21	Cost of cell phones	My question	Gr.9: LO1, AC 4	Adaptive Reasoning	3a) ;b)	High	23	21-AR	21-AR
22	Equations	My question	Gr.9: LO2, AC 4	Adaptive Reasoning	3a); c)	Medium	25	22-SC	22-SC
23	Estimation-addition of fractions	My question	Gr.9: LO1, AC 5	Adaptive Reasoning	3a); c)	Medium	16	23-CU	23-CU
24	Petrol cost	My question	Gr.9: LO1, AC 3.1	Adaptive Reasoning	3a) ;b)	High	20	24-PF	24-PF
25	Estimation-restaurant bill	NAEP	Gr.9: LO1, AC 5	Strategic Competence	4a) ;b); c)	Low	46	25-CU	25-AR
26	Baseball cards	NAEP	Gr.9: LO1, AC 3.1	Adaptive Reasoning	3a) ;b); c)	Medium	39	26-AR	26-SC
27	Boxes containing radios	NAEP	Gr.9: LO1, AC 3.1	Strategic Competence	4a) ;b); c)	Medium	55	27-SC	27-SC
28	Objects on a scale	NAEP	Gr.9: LO2, AC 6.4	Conceptual Understanding	2 b)	Low			
29	Rounding off decimals to whole nos.	NAEP	Gr.7: LO1, AC 7.1	Procedural Fluency	1a) ;b); c)	Low			
30	Decimal notation	NAEP	Gr.8: LO1, AC 3	Conceptual Understanding	2a) ;b)	Low			
31	Distributive law	NAEP	Gr.8: LO1, AC 8.3	Procedural Fluency	1a) ;b)	Low			
32	Place values-tens	NAEP	Gr.4: LO1, AC 4	Conceptual Understanding	2a)	Medium			
33	Decimal notation	NAEP	Gr.8: LO1, AC 3	Conceptual Understanding	2a)	Low			
34	Equivalent- decimal	NAEP	Gr.9: LO1,	Conceptual Understanding	2a) ;b)	High			

	to common fractions		AC 2			
35	Distance between two towns	NAEP	Gr.9: LO1, AC 4	Adaptive Reasoning	3a); c)	High
36	Scholarship problem	NAEP	Gr.9: LO1, AC 4	Strategic Competence	4a) ;b); c)	High
37	Equal sharing of pens	NAEP	Gr.9: LO1, AC 4	Adaptive Reasoning	3a); c)	Low
38	A game-subtraction sums	NAEP	Gr.9: LO1, AC 6	Adaptive Reasoning	3a) ;b)	High
39	(X) of six by another no.	NAEP	Gr.9: LO1, AC 5	Adaptive Reasoning	3a) ;b); c)	High
40	Determine weight of twelve boxes	NAEP	Gr.9: LO1, AC 3 and 4	Strategic Competence	4a); b); c)	Low
41	(x) and Subtraction word problem	NAEP	Gr.9: LO1, AC 3.1	Strategic Competence	4a) ;b); c)	Low
42	Pattern: Find the next number	NAEP	Gr.9: LO2, AC 1	Strategic Competence	4a) ;b); c)	Medium
43	(X) and division word problem	NAEP	Gr.9: LO1, AC 3.1	Strategic Competence	4a) ;b); c)	High
44	Hamburger problem	NAEP	Gr.9: LO1, AC 3.1	Strategic Competence	4a) ;b); c)	Low
45	Distributive law-illustration	NAEP	Gr.8: LO1, AC 7.1	Conceptual Understanding	2a) ;b)	Medium
46	Multiplication word problem	My question	Gr.9: LO1, AC 3	Conceptual Understanding	2a) ;b)	Low
47	Egg problem	NAEP	Gr.9: LO1, AC 3	Strategic Competence	4a) ;b); c)	Low
48	Comparing populations in two towns	NAEP	Gr.9: LO1, AC 4	Adaptive Reasoning	3a) ;b); c)	High
49	Equations	NAEP	Gr.9: LO1, AC 4	Procedural Fluency	1a) ;b)	Medium
50	Equivalent fractions	NAEP	Gr.9: LO1, AC 3	Conceptual Understanding	2a) ;b)	Medium
51	Multiplication of integers	NAEP	Gr.8: LO1, AC 6.2	Procedural Fluency	1a) ;b); c)	Low

52	Rounding off decimals	NAEP	Gr.7: LO1, AC 7.1	Procedural Fluency	1c)	Low
53	Speed, distance, time problem	NAEP	Gr.9: LO1, AC 3.2	Adaptive Reasoning	3a) ;b); c)	High
54	Shaded picture	NAEP	Gr.7: LO1, AC 4.1	Conceptual Understanding	2a) ;b)	Low
55	Percentage reduction of stereo	NAEP	Gr.9: LO1, AC 3.1	Strategic Competence	4a),b);c)	High

APPENDIX G PRODUCTIVE DISPOSITION TABLES

Table 1 Mathematical proficiency of Grade ten mathematics literacy learners in productive disposition – usefulness of mathematics literacy

	Q7	Q11	Q13	Q17	Q19	Q23p	Q24	Q26	Q29
	Count	Count	Count	Count	Count	Count	Count	Count	Count
strongly disagree	1	1	1		1	11	1	1	1
disagree	39	19	6	21	16	12	15	36	11
neutral	33	26	10	28	18	18	27	28	18
agree	44	53	51	60	45	106	65	55	52
strongly agree	34	50	81	40	70	1	40	29	68

Table 2 Mathematical proficiency of Grade ten mathematical literacy learners in productive disposition – attitude to mathematical literacy

	Q1	Q4p	Q9p	Q12	Q14p	Q16	Q18p	Q20p	Q21p	Q22p	Q25p	Q27p	Q30	Q31
	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count
strongly disagree		14	10	1	13		11	7	11	9	12	14		
disagree	15	47	28	49	22	15	4	16	4	27	32	14	47	9
neutral	39	26	31	43	25	19	15	46	19	34	42	15	28	18
agree	74	61	81	38	89	64	119	77	118	71	60	104	35	36
strongly agree	22	1	1	20	1	50	1	1		1	1	1	39	88

Table 3 Mathematical proficiency of Grade ten mathematical literacy learners in productive disposition – beliefs in ability to do mathematical literacy

	Q2p	Q3	Q5	Q6	Q8	Q10p	Q15p	Q28
	Count	Count	Count	Count	Count	Count	Count	Count
strongly disagree	4		1	1	1	14	6	1
disagree	34	9	20	39	23	17	15	4
neutral	49	30	40	41	34	26	36	13
agree	62	65	56	46	69	90	90	35
strongly agree		45	31	23	22		1	97

Table 4 Mathematical proficiency of Grade ten mathematics learners in productive disposition – attitude to subject

	Q1	Q4p	Q9p	Q12	Q14p	Q16	Q18p	Q20p	Q21p	Q22p	Q25p	Q27p	Q30	Q31
	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count	Count
strongly disagree		1	2		2		2	1		5	1			
disagree	1	5	2	3	3	2		1	1	1	4	1	12	1
neutral	10	6	5	13	5	1	4	2		9	6	1	6	3
agree	13	16	19	10	18	11	22	23	26	13	16	26	6	9
strongly agree	4			2		14							4	15

Table 5 Mathematical proficiency of Grade ten mathematics learners in productive disposition – usefulness of subject

	Q7	Q11	Q13	Q17	Q19	Q23p	Q24	Q26	Q29
	Count	Count	Count	Count	Count	Count	Count	Count	Count
strongly disagree						3			
disagree	2	3		3	2	2	2	6	2
neutral	6	1	1	3	2	2	2	3	
agree	12	9	4	4	6	20	11	7	8
strongly agree	8	15	23	18	17		13	12	18

Table 6 Comparison of Mathematical proficiency of Grade ten mathematical literacy learners in productive disposition for May/Oct – usefulness of subject

	Q7	Q11	Q13	Q17	Q19	Q23p	Q24	Q26	Q29
May	78	103	132	100	115	107	105	84	120
Oct	64	86	98	71	84	90	87	53	84

Table 7 Comparison of Mathematical proficiency of Grade ten mathematics learners in productive disposition for May/Oct – usefulness of subject

	Q7	Q11	Q13	Q17	Q19	Q23p	Q24	Q26	Q29
May	20	24	27	22	23	20	24	19	26
Oct	17	19	21	16	20	20	19	16	20

Table 8 Comparison of Mathematical proficiency of Grade ten mathematical literacy learners in productive disposition for May/Oct – attitude to subject

	Q1	Q4p	Q9p	Q12p	Q14p	Q16	Q18p	Q20p	Q21p	Q22p	Q25p	Q27p	Q30	Q31
May	96	62	82	58	90	114	120	78	118	72	61	105	74	124
Oct	85	48	76	51	81	92	101	76	90	57	57	87	39	93

Table 9 Comparison of Mathematical proficiency of Grade ten mathematics learners in productive disposition for May/Oct – attitude to subject

	Q1	Q4p	Q9p	Q12p	Q14p	Q16	Q18p	Q20p	Q21p	Q22p	Q25p	Q27p	Q30	Q31
May	17	16	19	12	18	25	22	23	26	13	16	26	10	24
Oct	19	12	15	12	11	21	22	22	19	15	18	19	4	20

Table 10 Comparison of Mathematical proficiency of Grade ten mathematical literacy learners in productive disposition for May/Oct – ability to do subject

	Q2p	Q3	Q5	Q6	Q8	Q10p	Q15p	Q28
May	62	110	87	69	91	90	91	132
Oct	70	95	73	59	80	97	74	106

Table 11 Comparison of Mathematical proficiency of Grade ten mathematics learners in productive disposition for May/Oct – ability to do subject

	Q2p	Q3	Q5	Q6	Q8	Q10p	Q15p	Q28
May	23	26	19	15	22	24	15	27
Oct	13	20	16	15	17	19	18	22

Table 12 Gender comparisons of the mathematical proficiency of Grade ten learners in productive disposition – usefulness of subject

	Q7		Q11		Q13		Q17		Q19		Q23p		Q24		Q26		Q29	
	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F
May	41	57	53	74	62	97	46	76	54	84	49	78	50	79	44	59	58	88
Oct	29	52	37	68	45	74	28	59	41	63	39	71	38	68	25	44	41	63

Table 13 Gender comparisons of the mathematical proficiency of Grade ten learners in productive disposition – attitude to subject

	Q1		Q4p		Q9p		Q12		Q14p		Q16		Q18p	
	M	F	M	F	M	F	M	F	M	F	M	F	M	F
May	47	66	28	50	41	60	25	45	41	67	55	84	58	84
Oct	36	68	19	41	33	58	21	40	34	58	41	72	46	77
	Q20p		Q21p		Q22p		Q25p		Q27p		Q30		Q31	
	M	F	M	F	M	F	M	F	M	F	M	F	M	F
May	30	71	54	90	29	90	29	48	48	83	28	56	56	92
Oct	31	67	37	72	26	46	28	47	38	68	18	25	43	70

Table 14 Gender comparisons of the mathematical proficiency of Grade ten learners in productive disposition – ability to do the maths

	Q2p		Q3		Q5		Q6		Q8		Q10p		Q15p		Q28	
	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F
May	34	51	55	81	46	60	29	55	48	65	50	64	39	67	63	96
Oct	33	50	43	72	34	55	28	46	33	64	43	73	38	54	45	83

APPENDIX H INTERVIEW QUESTIONS

1) Let's begin by talking about why you decided to choose ML/maths this year?

2) Now tell me, why do you think you found maths (in grade 9 or earlier), easy/difficult?

3) So, what are your experiences of maths/ML so far?

   Are you finding the work manageable this year in grade ten?

4) What would you say are the challenges that we face in our daily lives where we have to use numbers/maths. Will you say that the mathematics that you study at school, this year, prepares you to meet these challenges?

5) When you hear the word "mathematics" mentioned to you, tell me how you do feel?

   If you performed better at maths/ML, then what would your feelings/thoughts on mathematics be? (Relate to June marks where ML learners outperformed maths learners).

6) Grade ten learners, in general, performed very poorly on the Maths Skills Test, which was testing basic number skills at grade 8 level. Why do you think this was so?

What factors do you think would have affected your maths results, in the past and currently, (Eg. Language, lack of resources, socio-economically background, laziness, etc.).

Focus Groups

There were 3 groups-comprising of 6 pupils each. One group consisted of mathematics pupils; the other of Mathematics Literacy pupils; and the last group a combination of both mathematics and Mathematics Literacy pupils. Each group consisted of learners from the high, average, and low achievers with respect to May-test scores. I organized a video camera to record the interviews.

APPENDIX I SUMMARY TABLE OF PERCENTAGE CORRECT FOR MAY/OCT-TESTS

		Maths Literacy		Pure Mathematics	
Qu. No.	Description	May % Correct	Oct. % Correct	May % Correct	Oct. % Correct
1	Subtraction of 4 digits whole nos.	45, 3	39, 3	46, 4	65, 4
2	Subtraction of 4 digits decimals	48, 4	37, 9	42, 9	65, 4
3	(X) and Subtraction word problem	43, 5	46, 0	46, 4	61, 5
4	When to use estimates	44, 1	46, 0	46, 4	46, 2
5	Rounding off decimals to whole nos.	24, 8	14, 5	32, 1	34, 6
6	Addition of common fractions	49, 7	55, 6	60, 7	76, 9
7	Shaded picture	42, 9	44, 4	64, 3	42, 3
8	Equal sharing of pens	40, 4	46, 0	71, 4	84, 6
9	Objects on a scale	13, 7	22, 6	28, 6	50, 0
10	Egg problem	29, 8	43, 5	46, 4	69, 2
11	Translation of equation	28, 6	37, 1	71, 4	69, 2
12	Decimal notation	8, 1	12, 9	17, 9	26, 9
13	Estimation of distance/height	14, 3	23, 4	21, 4	46, 2
14	Division of common fraction	36, 0	40, 3	32, 1	50, 0
15	Estimation of product	29, 8	40, 3	39, 3	46, 2
16	Multiplication of integers	9, 3	16, 9	39, 3	57, 7
17	Pattern: Find the next number	24, 8	21, 0	32, 1	53, 8

18	Addition of positive decimals	37, 9	37, 9	39, 3	46, 2
19	Boxes containing radios	10, 6	34, 7	25, 0	38, 5
20	Place values-tens	18, 0	19, 4	14, 3	34, 6
21	Estimation-addition of fractions	14, 3	12, 1	25, 0	30, 8
22	Estimation-restaurant bill	12, 4	13, 7	7, 1	3, 8
23	Equivalent forms-ascending/descending	6, 2	5, 6	7, 1	0
24	Equivalent- decimal and fractions	17, 4	25, 8	21, 4	19, 2
25	Multiplication/speed, time word problem	4, 3	1, 6	21, 4	0
26	(X) of six by another no./population	2, 5	0	10, 7	0
27	Percentage reduction of stereo	0	0	7, 1	3, 8

APPENDIX J SUMMARY T-TABLES

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	May-test PF	33.22	141	22.669	1.909
	Oct-test PF	32.51	141	22.566	1.900
Pair 2	May-test SC	26.14	141	18.147	1.528
	Oct-test SC	30.59	141	18.805	1.584
Pair 3	May-test CU	30.32	141	18.312	1.542
	Oct-test CU	31.41	141	19.259	1.622
Pair 4	May-test AR	28.49	141	20.465	1.723
	Oct-test AR	31.80	141	21.994	1.852

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	May-test PF and Oct-test PF	141	.430	.000
Pair 2	May-test SC and Oct-test SC	141	.287	.001
Pair 3	May-test CU and Oct-test CU	141	.311	.000
Pair 4	May-test AR and Oct-test AR	141	.380	.000

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	May-test PF and Oct-test PF	.71	24.142	2.033	-3.31	4.73	.349	140	.728
Pair 2	May-test SC and Oct-test SC	-4.45	22.073	1.859	-8.12	-.77	-2.391	140	.018
Pair 3	May-test CU and Oct-test CU	-1.09	22.060	1.858	-4.76	2.58	-.586	140	.559
Pair 4	May-test AR and Oct-test AR	-3.31	23.673	1.994	-7.25	.63	-1.660	140	.099

Paired Samples Statistics(a)

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	test solutions	10.12	26	3.871	.759
	Oct-test Solutions	11.85	26	4.496	.882
Pair 2	May-test PF	36.54	26	25.394	4.980
	Oct-test PF	48.72	26	22.570	4.426
Pair 3	May-test SC	30.77	26	20.481	4.017
	Oct-test SC	41.35	26	23.656	4.639
Pair 4	May-test CU	37.98	26	21.645	4.245
	Oct-test CU	37.91	26	22.110	4.336
Pair 5	May-test AR	45.51	26	23.832	4.674
	Oct-test AR	49.36	26	20.806	4.080

a FET subject chosen = Mathematics

Paired Samples Correlations(a)

		N	Correlation	Sig.
Pair 1	test solutions and Post-test Solutions	26	.601	.001
Pair 2	May-test PF and Oct-test PF	26	.608	.001
Pair 3	May-test SC and Oct-test SC	26	.439	.025
Pair 4	May-test CU and Oct-test CU	26	.334	.096
Pair 5	May-test AR and Oct-test AR	26	.375	.059

a FET subject chosen = Mathematics

Paired Samples Test(a)

		Paired Differences					t	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	May- solutions - Oct-test Solutions	-1.73	3.779	.741	-3.26	-.20	-2.335	25	.028
Pair 2	May-test PF - Oct-test PF	-12.18	21.374	4.192	-20.81	-3.55	-2.906	25	.008
Pair 3	May-test SC - Oct-test SC	-10.58	23.533	4.615	-20.08	-1.07	-2.292	25	.031
Pair 4	May-test CU - Oct-test CU	.07	25.261	4.954	-10.13	10.27	.014	25	.989
Pair 5	May-test AR – Oct-test AR	-3.85	25.081	4.919	-13.98	6.28	-.782	25	.442

a FET subject chosen = Mathematics

Paired Samples Statistics(a)

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	May-solutions	71.38	124	244.617	21.967
	Oct-test Solutions	7.73	124	3.353	.301
Pair 2	May-test PF	32.46	115	22.058	2.057
	Oct-test PF	28.84	115	20.980	1.956
Pair 3	May-test SC	25.09	115	17.506	1.632
	Oct -test SC	28.15	115	16.709	1.558
Pair 4	May-test CU	28.59	115	17.108	1.595
	Oct -test CU	29.94	115	18.343	1.710
Pair 5	May-test AR	24.64	115	17.570	1.638
	Oct -test AR	27.83	115	20.319	1.895

a FET subject chosen = Mathematical Literacy

Paired Samples Correlations(a)

		N	Correlation	Sig.
Pair 1	May-solutions and Oct-test Solutions	124	-.011	.904
Pair 2	May-test PF and Oct -test PF	115	.386	.000
Pair 3	May-test SC and Oct -test SC	115	.204	.029
Pair 4	May-test CU and Oct -test CU	115	.274	.003
Pair 5	May-test AR and Oct -test AR	115	.240	.010

a FET subject chosen = Mathematical Literacy

Paired Samples Test(a)

	Paired Differences				t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error	95% Confidence Interval of the			

					Difference				
				Mean	Lower	Upper			
Pair 1	May-solutions - Oct-test Solutions	63.65	244.677	21.973	20.16	107.15	2.897	123	.004
Pair 2	May-test PF - Oct - test PF	3.62	23.856	2.225	-.78	8.03	1.629	114	.106
Pair 3	May-test SC - Oct - test SC	-3.06	21.596	2.014	-7.05	.93	-1.519	114	.132
Pair 4	May-test CU - Oct - test CU	-1.35	21.385	1.994	-5.30	2.60	-.677	114	.499
Pair 5	May-test AR - Oct test AR	-3.19	23.456	2.187	-7.52	1.14	-1.458	114	.148

a FET subject chosen = Mathematical Literacy

Overall Samples Test for Gender

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Pretest Sol. Perc.	Equal variances assumed	.028	.866	.849	179	**.397**	1.727	2.034	-2.286	5.741
	Equal variances not assumed			.865	155.587	.388	1.727	1.997	-2.217	5.672
Post-test Sol. Perc.	Equal variances assumed	1.426	.234	-1.701	148	**.091**	-4.125	2.425	-8.917	.666
	Equal variances not assumed			-1.663	105.239	.099	-4.125	2.480	-9.044	.793
MCQ AR Post	Equal variances assumed	.888	.348	-1.913	148	.058	-7.675	4.012	-15.603	.254
	Equal variances not assumed			-1.922	114.609	.057	-7.675	3.992	-15.583	.234