PREPARING MATHEMATICS TEACHERS FOR THE 21ST-CENTURY CLASSROOM

BY

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DECLARATION

I, Celestine Francis Ofili George, declare this dissertation's original work. It does not contain copied data, tables, graphs, or writings of other authors. Every piece of information I sourced from other persons was referenced accordingly.

[redacted]

26th July 2023

Candidate: Celestine Francis Ofili George

Date

I have given consent for this dissertation to be submitted.

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Supe[redacted]molan Mudaly

26th July 2023

Date

DEDICATION

I dedicate this dissertation to God Almighty for His unfailing love and mercy, my strong pillar, my source of inspiration, wisdom, knowledge, and understanding. He has been the source of my strength throughout this program, and I soared on His wings.

I also dedicate this work to my lovely parents, who gave birth to me and so much of themselves to raise me and nurture my spirituality.

Words are insufficient to express my gratitude for their love and sacrifice.

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ABSTRACT

This qualitative study explored pre-service mathematics teachers' preparation for teaching in the 21st-century classroom during their B.Ed. Programme at a university. This purpose was explored mainly from the perspective of a set of 4th year pre-service mathematics teachers' contextual understanding of the effectiveness of the theories and practices that comprise their training. The researcher employed a qualitative approach because it allowed for multiple perspectives and permitted flexibility in questions and probes. This study utilised an open-ended questionnaire and semi-structured interviews to elicit the required answers to the research questions. Hence, a purposeful sampling technique was utilised to select 6 participants to represent the entire population. The participants' responses were organised using the coding process to generate categories and themes for analysis and interpretation of the findings.

From the analysis of the data, the following findings emerged: Participants identified lesson planning and enactment and the use of appropriate teaching methods as some of the most critical training that was helpful in their teaching practice: question and answers, group work, concrete and visual representations were effective teaching strategies used by the participants to engage learners actively; the independent practice was used to allow each learner to work towards mastery of the subject content knowledge and progress at their own pace, while guided practice was offered to support learners with difficulties, but these were shot lived because participants high jacked the process to provide quick answers due to shortage of time**;** real-world problems were used to prove that mathematics is about reasoning and connecting abstract ideas to improve our daily lives, rather than memorising the procedures. Above all, participants demonstrated they could explain mathematical concepts better by using the 4Cs.

The study also identified some factors that inhibit preservice mathematics teachers' ability to teach effectively, including frustration due to lack of resources, inadequate time, learner mindset, and language barrier. The following recommendations were made to serve as a remedy to the above challenges: it is recommended that priority be given to quality assessment rather than quantity; provide a Mathematics remediation programme, which focuses on reviewing and building foundational mathematics skills; and provide adequate learning resources. These findings affirm the efficacy of using 21st-century skills to actualise effective teaching and learning in our classrooms, and the researcher recommends its application in schools.

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND OF THE STUDY

Several nations have made significant curriculum and assessment modifications to better prepare learners for the more rigorous educational requirements of the twenty-first century. What abilities do educators and learners need to succeed in this world that is quickly changing? (Nwagwu et al., 2021). The 4Cs of 21st-century learning, collaborative problem-solving, creativity, communication, and critical thinking, are crucial for instructors' learners to flourish in the classroom (Rusdin & Ali, 2019). The emphasis was on utilising 21st-century abilities to equip future teachers for mathematics instruction and learning in Nigeria's Junior secondary schools.

For instance, pre-service mathematics teachers can use collaborative problem-solving to engage learners in joint activities in which everyone plays a role in finding answers to the problem, enhancing each other abilities, fostering critical discussion and research by questioning presumptions, and analysing how they fit into society and the classroom (Nilimaa, 2023). Communication skills prepare pre-service teachers to teach mathematics using a variety of representations such as tables/graphs/pictures, symbols, and technology to enhance learners' mathematical and analytical skills and overall higher-order thinking (Zulkarnain et al., 2021).

Creativity enables mathematics pre-service teachers to analyse a given problem in multiple ways, observe patterns, avoid stereotypes, unravel relationships, and provide alternate ideas on how to solve a mathematical problem (Adedeji, 2018; Nilimaa, 2023). Critical thinking skills help pre-service mathematics teachers encourage learners to use their inquiry skills to probe and understand the relationships between concepts, recognise common errors in reasoning, and utilise knowledge gained to construct new ideas and develop meaningful and coherent arguments (Chiketa & Okigbo, 2021; Adebiyi & Akintoye, 2020). ICT has helped teachers explain mathematical concepts better through projectors, graphic calculators, computerised graphing, and spreadsheets (Aliyu, 2021; Ukah & Odey, 2018; Ameen et al., 2019).

Preparing teachers for the 21st-century classroom refers to the professional training given to teachers as facilitators of learning and creativity, using current technology to work with learners to achieve learning outcomes and to prepare them for the future (Adedeji, 2018; Chiketa & Okigbo, 2021). The need to promote quality preparation of pre-service teachers is of national interest because acquiring new knowledge is usually considered a criterion for capacity advancement, increasing income and asset base, and an alleyway to achieving excellence.

Suppose the impact of education is so recognised. In that case, the value of any school system is primarily contingent on the effectiveness of the training the instructors have acquired, given that no educational outcome could be superior to the dedication and value of its teachers. Teachers' preparation can improve through the provision of 21st-century training and retraining programmes for those studying towards a teaching qualification (pre-service) and those already teaching (in-service) (Adedeji, 2018; Ogunyinka et al., 2015).

1.2 Statement of the Problem

Education is the most efficient means to accomplish social and economic growth. Despite this knowledge, research has revealed much criticism directed towards newly employed teachers who in Nigeria have low levels of mathematical literacy. In Nigeria, instructors still in training must be adequately informed to apply the interactive skills and 21st-century pedagogy that have been taught into practice (Ameen et al., 2019; Otun & Olaoye, 2019). Through better pre-service and in-service training, the government at all levels must take action to raise the calibre of teachers (Otun & Olaoye, 2019).

The challenge for mathematics teachers in the 21st century requires a shift from the traditional ways of teaching mathematics in an abstract mathematical world that discourages learners who do not see its relevance to developing a knowledge of mathematics basic ideas, developing an ability to convert a situation they are in into a format that demonstrates the use of maths (Rusdin & Ali, 2019). In the past, it was sufficient to receive and memorise information by heart to find answers to a question (Willingham, 2021). Still, in the 21st century, learning involves self-direction, constructing one's own mental model and synthesis of knowledge, cultivating an inquiry-based attitude, and analysing and resolving incongruent data (Uddin, 2019; Szabo et al., 2020).

The justifications mentioned above highlight the necessity of an effective system in Nigeria for teacher training. There needs to be more adequate literature that draws attention to understanding the challenges of preparing teachers for the 21st-century classroom. The available works of literature are primarily quantitative studies that do not provide an occasion for teachers and pre-service teachers as stakeholders in education to have a say (Okoli et al., 2015; Adedeji, 2018). Many admirable programs have failed, primarily due to failing to consider instructors' viewpoints. Mathematics teachers should be trained to improve teaching quality and increase teacher productivity (Adedeji, 2018; Ameen et al., 2019).

Programmes for pre-service teacher education are how educators are equipped to support efficient instruction and learning in 21st-century classrooms. In doing this, pre-service teachers face challenges such as the disconnection between university coursework and practice, lack of adequate mathematical pedagogical knowledge, and low levels of ICT integration in schools (Adedeji, 2018; Ameen et al., 2019). These challenges impede pre-service teachers in their preparation to become better teachers. This study aims to comprehend pre-service teachers' preparation, educational experience, and the efficacy of the theories and practices that comprise their training program at one university. The findings boost our efforts to provide pre-service teachers with the best possible training and increase their ability to translate theory into practice (Darling-Hammond & Oakes, 2019).

1.3 The Rationale of the Study

Three issues serve as the foundation for this study's rationale. The study will start with personal experiences: My extensive experience teaching in many schools has made me interested in how pre-service mathematics teachers are prepared for the classroom of the twenty-first century. Notice that some teachers need a solid understanding of instructing learners by curricular objectives, test criteria, and assessment standards appropriate for a 21st-century classroom. As a result, most mathematics teachers needed help comprehending the curriculum's content as a vital tool for providing context for a particular set of outcomes. This deficit induces demotivation, weakens dedication and self-assurance, and reduces their ability to respond positively to new curriculum reforms that call for advanced technology.

Secondly, the theory-practice divide: The global concern on the "theory-practice divide," identified in several works of literature regarding the seeming challenges of pre-service teachers to translate theory into action in the workplace, catalyses this study (Mavhunga & Merwe, 2020). Thirdly, national interest: The need to promote quality education for pre-service teachers should be seen in the national interest because education is fundamental to development and growth. Teachers are vital in developing the necessary human capital for sustained economic progress. Additionally, they contribute to fostering equity, fairness, and social justice while helping reduce poverty. There is little doubt that pre-service teacher education has a significant role to play in assisting African societies in overcoming the socioeconomic and technological difficulties they face (Ogunyinka et al., 2015; Ntuli et al., 2018).

1.4 Purpose of the study

This qualitative study investigates how pre-service mathematics teachers are prepared for teaching in the 21st-century classroom during their B.Ed. Programme at a particular institution. This purpose was primarily investigated from the perspective of pre-service mathematics teachers' contextual understanding of the efficacy of the theories and practices that make up their training programme.

1.5 Literature Review

1.5.1 Curriculum for 21st-century schools

A multifaceted approach to teaching and learning is necessary for today's environment. Instead of emphasising material memorisation and subject acquisition, a 21st-century skill-based curriculum concentrates on the abilities that will best protect the minds of the next generation of learners. Instead of just providing knowledge, active participation and learner interaction are encouraged. For learners to achieve academic achievement and a bright future, pre-service teachers in the 21st-century classroom must master several abilities. These abilities include collaborative thinking, creativity, clear communication, critical thinking, and ICT (Rusdin & Ali, 2019).

1.6 The Study Objectives

The objectives of this study are

i. To explore how pre-service mathematics teachers are being trained to teach in 21st-century classrooms.
ii. Critically analyse their training experience to identify the existing challenges between theory and professional practice.
iii. To establish how their training and experience can be used to develop greater competence as teachers of the 21st century.

1.7 Critical Research Questions

The following research questions, derived from the objectives, guided the study:

1. What are the training experiences of pre-service mathematics teachers currently being trained to teach in the 21st century?

2. What are the existing challenges pre-service mathematics teachers experience when they learn to teach or translate theory into practice?
3. How do these preservice teachers intend to develop greater competence as teachers of the 21st century?

1.8 Organisation of Study

The approach for this study has been organised as follows.

Chapter One of this study provided an overview. The study's background, the problem statement, the rationale and reason for undertaking the study, the purpose of conducting the study, a brief literature review, study objectives, the critical research questions, and the organisation of the study are all presented in chapter one.

Chapter Two conducts an extensive literature review to place this work within the context of more general research views on pre-service teacher education, as well as expounding the theoretical framework with which the specific viewpoint of this study is examined.

Chapter Three discussed the study's paradigm, approach, design, and methodology. The qualitative approach within the critical paradigm was highlighted. The study was a case study conducted at a tertiary institution in Lagos, Nigeria, where the six participants had been purposely selected. These prospective teachers were chosen because they were responsible for teaching the JSS3 (Grade 9) learners at the time of the study. The researcher generated data using partially structured interviews and an open-ended questionnaire. In addition, chapter three presented the data analysis, trustworthiness, ethical concerns, and the study's limitations.

Chapter Four presents the data analysis for the two data generation tools, semi-structured interviews and open-ended questionnaires. The themes and categories that emerged from the data were used as a basis for data analysis.

Chapter Five discusses the summary, findings, recommendations, and implications concerning future research.

CHAPTER 2: LITRATLITERATURE REVIEW

2.1 INTRODUCTION

To effectively comprehend the research topic, this chapter focuses on a survey of related literature. This section begins with the pre-colonial mathematics school curriculum, the development of the mathematics school curriculum in Nigeria, and the 21st-century school curriculum, as it provides insight into the work of teacher educators. Literature on critical mathematics and teaching mathematics for social justice are reviewed as analytical tools to understand the inequities in schools and social life and how to reconfigure them. Teacher preparation with professional development is assessed as it brings to the fore the impact of the changes in teacher education pedagogy. The theory-practice divide and the relationship between the university and secondary school are critical concerns in mathematics teacher education that were also discussed. Case studies of pre-service mathematics teachers' education in an international context were also reviewed. The chapter ends with a theoretical framework.

2.2 Pre-colonial Mathematics School Curriculum

The corporeal, communal, and religious state of pre-colonial African communities was the bedrock of the pre-colonial curriculum. The external surroundings influenced the learning plan and what learners are to be taught, given that the lesson's purpose was to enable the learner to conform, exploit, and gain from his environment. In pre-colonial education, there was almost no dichotomy between theory and practice. The realm of exploitative labour was virtually non-existent (Anyor & Abah, 2014; Mosimege, 2020; Adeyeye & Mason, 2020).

This section debunks the myth that Africans did not know mathematics till the coming of the Europeans. Despite research indicating that the pre-colonial counting system, African arts, architecture, games, riddles, and puzzles can be used to promote and establish a connection between classroom activities and real-life contexts, this connection has not been sufficiently explored for it to become a reality in many mathematics classrooms. (Mosimege, 2020; Adeyeye & Mason, 2020). Mudaly (2018) states that most mathematics instruction occurs in unfamiliar settings. In his view, mathematics cannot be separated from indigenous knowledge, which enables learners to grasp mathematics better by placing it in a familiar context.

2.2.1 The Yoruba counting system

Pre-colonial Yoruba, Igbo, Efik, and other Nigerian societies had an organised counting system. The numbers 1-10 in Yoruba have distinctive names: 1 is Ikan, 2 is Meji, 3 is Meta, 4 is Meri, 5 is Marun, 6 is Mefa, 7 is Meje, 8 is Mejo, 9 is Mesan, and 10 is Mewa. More so, the Yoruba counting system is a sophisticated system centred on 20 (vigesimal) that expresses numbers by subscriptions. Numbers 11-14 were written additively, while 15-19 were represented by subtracting from twenty. The pattern keeps on till the number 30. After thirty, every number is expressed as a multiple of twenty, plus or minus tens and units. The pattern is also repeated up until 200. The number becomes irregular after 200. Trade was carried out through numerical words; for example, 20 cowries equal one bag or ten heads, 40 cowries equal one string, and 2000 cowries equal one head or fifty strings ((Ẹlẹ́sẹ̀moyọ̀ & Ọdẹ́jọbí, 2022).

2.2.2 Ifa is a Compendium of Scientific and Mathematical Systems

Kelani (2016) asserts that mathematics and science are ingrained in Yoruba oral tradition and religion, much like classical philosophy. According to the Ifa (Yorba divination system), the counting unit needs to be binary. Stated differently, there are always good and bad things. That is well known in Western philosophy, but the positive is the antithesis of the negative. They have to be kept apart at all times. However, Ifa recognizes it is impossible to separate. Take the head and tail of the coin. Is it possible to take off the head and just have the bottom? They cannot exist apart. That is the reason the Yoruba did not count 1, 2, 3, and 4 when the Europeans did. 01 is one figure in Yoruba culture. There must always be two, whether it is 11 or 22. This was not binary at first; it was only recently discovered in Western mathematics (Kelani, 2016).

Another example demonstrates how materialism is among the most widely accepted theories in the West. In other words, everything in the world is made of matter or material, and we can use our five senses to confirm that anything is material. This is accomplished by tasting, touching, etc. They do acknowledge the existence of something non-material, which they refer to as spirituality. They assert that it is unreal. However, materiality and its antithesis, the spiritual, are inextricably linked, according to ancient Yoruba philosophy. In modern science, a related idea known as particle physics has emerged (Kelani, 2016).

2.2.3 Geometry – Art and Symmetry

The investigation of symmetries in African art, which include stripe patterns, plane patterns, and bilateral and rotational symmetries, has primarily attracted the attention of mathematicians.

Geometry can be seen or used in Benin bronze, Yoruba Adire clothing (Nigeria), Mozambique's purse, hat, and broom manufacturing. Examining symmetry in African art highlights and confirms the ingenuity and mathematical problem-solving skills of the craftsmen and artisans involved (Abah, 2020).

2.2.4 Architecture

In terms of architecture, the Great Walls of Benin built between 800 AD and the middle of the fourteenth century, were mathematically precise in their design. The walls were four times as big as China's Great Wall. It is believed that the length of the inner and external walls is close to 16000 kilometres. It is regarded as the most remarkable archaeological phenomenon on earth and is said to have required locals to excavate for 150 million hours. According to this structure, the Benin wall builders may have been mathematicians and used a mathematical design to guide their work over 600 years to complete it (Think Africa, 2018).

2.2.5 Games, Riddles, and Puzzles

Ayo (Yoruba), Okwe (Igbo), Dara (Hausa), Shisima (Kenya), Achi (Ghana), Mancala 2-row version (Ethiopia), and Sega (Egypt) are a few of the popular gaming genres played in Africa, including alignment and mathematical principles. These games suggest that the creators of counting rhymes, board games and dice games were mathematicians. It further implies that it might be significant in reconstructing the origin of mathematical thought in Africa. Researchers are examining the possibility of applying answers to the mathematical problems posed by these games to address real-world issues. It also reveals that the rules built into the rounds suggest that the inventors had an empirical understanding of probability, which may inspire further study in mathematics (Abah, 2020; Mudaly, 2018; Ojo, 2022).

The Morabaraba, a pre-colonial game played in South Africa that incorporates mathematical concepts, epitomises some indigenous games that positively influenced many communities. This game is briefly explained below.

2.2.5.1 The Indigenous Game of Morabaraba

The degree to which mathematical notions are incorporated into a game can be determined through a mathematical analysis (using mathematical concepts, principles, and methods). The examination of the Morabaraba game reveals the following mathematical ideas:

1. Recognising different quadrilaterals (squares) and comparing and contrasting them.

2. Symmetry: At least several instances of symmetry may be seen on the four sides of the board due to the repetitive movement of the chips on the board.
3. Rationally making inferences while carrying out multiple calculations inherent in the game.
4. Adding, accumulating and deducting the chips till the number of chips remaining determines the winner of the game.

According to Mosimege (2020), more than 200 variations of this count and capture game are played throughout Africa, each with minor variations in the rules. These variations are all played in North and West Africa. In East and Southern Africa, they play with four rows, while Ethiopians use three rows. After each board, stores are present in some games but not others. In Kenya, this game's variant is known as Bao. To release an animation of the game, UNESCO is collaborating with the National Museums of Kenya (Mosimege, 2020). Particularly among young people in Kenya, the UNESCO-led digitalisation of regional traditional games has promoted and nurtured innovative thinking, teamwork, and creative thinking.

Modernising or digitising African traditional mathematical principles rooted in African ways of life can also aid in increasing public consciousness and positively affect the preservation of traditional knowledge to improve the development of the mathematics curriculum in Nigeria innovatively in the 21st century.

2.3 Nigerian Mathematics Curriculum Development

According to Ali (as cited in Iyekekpolor & Uveruveh, 2023), Curriculum refers to the organised materials or instruction that learners require to meet certain societal goals or modify their behaviour. It is provided by educators and administrators in schools and is built upon an organised system of ongoing assessment. There can be no training in education without a curriculum because it provides the blueprint for organising and guiding the instructional experiences that learners encounter in educational institutions. Education and the crucial curriculum development process play a significant role in Nigeria.

About 1842, close to the end of the first part of the nineteenth century, colonial missionaries arrived, marking the beginning of a formal educational tool. Reading, writing, mathematics, and religion were all taught in Western-style primary schools. As a result, the basic version of the arithmetic curriculum turns out to be the pioneering mathematics curriculum (Anyor & Abah, 2016). According to Alade (2011), several socioeconomic reasons caused a reassessment of the arithmetic curriculum, which gave rise to the supplementary mathematics subjects of geometry and trigonometry. Since 1859, these have been taught in Grammar and High Schools.

The First Education Ordinance, which outlined educational policy, was created in 1888. To evaluate the African educational system seriously, the Phelps-Stokes Commission was founded in 1920. The Commission produced a plan that paid particular attention to education so that what was taught was relevant to societal demands. Later, it was discovered that the Commission's arguments needed to be revised since its suggested changes to the school curriculum left out the importance of including science classes and vocational and technical education. Between 1910 and 1925, additional progress led to Core Mathematics and Applied Mathematics introduction. These initial educational initiatives mainly relied on the British educational system (Anyor & Abah, 2016). Formal schools were formed, but their curricula were limited and designed to create teachers and administrative personnel who could advance the interests of the British Colonials.

The National Curriculum Conference was held in 1969 due to several proposals by Nigerian educationists during the post-colonial era and specific issues addressed by the pre-independence Ashby Commission. The National Policy on Education (NPE) was partly drafted due to this conference. The former Western Region of Nigeria developed the Universal Primary Education (UPE) system in 1955 in response to the discrepancies of the British-style academic curriculum that must reflect current circumstances (Anyor & Abah, 2016). With the introduction of the UPE, an entirely new mathematics curriculum was created that was very different from previous versions. The Eastern Region adopted the policy of the UPE in February 1957. The official debut of the UPE as a national program on September 6, 1976, underlines the emergence of an educational paradigm shift in Nigeria. The National Policy on Education (NPE) was published in 1977, soon after its launch. In essence, the UPE supports a 6-3-3-4 method of education.

Permanent literacy, numeracy, and good communication were goals of the UPE curriculum. Understanding how mathematics connects to other academic disciplines was highlighted in the UPE mathematics programme. This outstanding work of literature covered many topics, including number and Numeration, Basic Operations, Measurement, Algebraic Processes, Mensuration and Geometry, and Statistics. The core components of the UPE curriculum served as the cornerstone for subsequent revisions, updates, and improvements (Anyor & Abah, 2016).

The Federal Government introduced Universal Basic Education (UBE) in 1999 following the Declaration on Education for All (EFA) as suggested by the Jomtien Conference in 1990. In September 2008, all primary and junior secondary schools in Nigeria began implementing the new primary education mathematics curriculum, designed to be completed over nine years of instruction (Anyor & Abah, 2016). In essence, the demand for creative teaching and learning strategies that support 21st-century abilities like creativity and critical thinking, cooperation, and effective communication drove the development of the new curriculum (NERDC, 2012).

According to NERDC (2012), a distinctive aspect of the nine-year Basic Education Mathematics Curriculum focuses on emotive domain and quantitative reasoning to enhance learners' cognitive and psychomotor abilities. It also offered the most significant amount of assistance for teaching by recommending themes, learning outcomes, activities for learners and teachers, and assessment guidelines, e.g., The curriculum's specific goals are to:

1. Help learners develop the mathematical understanding they need to thrive in the digital age.
2. Develop the knowledge and use of mathematics concepts and abilities needed to prosper in the rapidly evolving technological world.
3. Encourage the development of critical math study skills of communication, connection, and problem-solving.
4. Take advantage of the many professional options that mathematics offers.
5. Prepare for additional mathematics and related subjects (NERDC, 2012, p. vi).

The current mathematics curriculum's restructuring and realignment confirm that a vibrant, dynamic, and living curriculum that embraces 21st-century skills is crucial for any educational system to effectively address the challenges of education for the sustainable development of society (Anyor & Abah, 2016; Sodangi & Adamu, 2023).

2.4 21st-century school curriculum

Employers and educators concur that due to changes in the world's economy, learners entering college and the workspace must have excellent knowledge and experience in mathematics (Ojo, 2019). Fusing mathematical content and mathematical practices with 21st-century Skills is one of the most crucial strategies to help learners attain this mastery (Partnership for 21st Century Skills, 2011). To prepare learners for the future, it is essential to emphasise the 4Cs: creativity, critical thinking, communication, and collaboration (Partnership for 21st Century Skills, 2011;

Ojo, 2019). These abilities should be taught in 21st-century classrooms as preparation for the outside world because they are appreciated in all professions.

2.4.1 Creativity and problem-solving

Creativity involves discovering new perspectives, making connections between seemingly unrelated ideas, and coming up with creative solutions to challenging issues. The development and application of fresh, novel, and valuable ideas by teachers and learners to enhance the effects of mathematics teaching and learning are examples of creativity in a mathematics classroom (Nilimaa, 2023; Joklitschke, 2022). According to Nilimaa (2023), creativity in mathematics can take many different forms. Some examples include the following:

- Provide several approaches or solutions to a problem;
- Seek out and investigate connections, relationships, and patterns among mathematical concepts;
- Use your imagination and modify tried-and-true strategies to address new or unusual issues;
- Promoting a secure and motivating atmosphere for learning that promotes taking risks,
- experimenting, and learning from errors;
- allowing learners to participate in open-ended assignments, practical problem-solving, and cooperative learning experiences;
- Motivating learners to analyse information critically, examine their ideas, and adopt a growth mindset; and
- Modelling and appreciating creativity in how they teach and communicate with learners.

From the above, it can be argued that Mathematical creativity is the capacity to view a problem from various angles, spot patterns, differences, and parallels, develop numerous solutions, and select the best approach to handle challenging mathematical situations. To help learners succeed when faced with new obstacles, mathematics teachers must find creative and inspiring ways to teach the subject (Nilimaa, 2023; Joklitschke, 2022).

The teachers ' expertise and input in creative mathematics are crucial for effectiveness when presenting creative mathematical exercises for the learners to develop mathematical thinking skills. Therefore, it is vital to support pre-service teachers' training to provide them with the skills necessary to plan and administer a learning environment that fosters creativity in a 21st-century mathematics classroom. The pre-service teacher should give the learners plenty of opportunities to learn to investigate, make conjectures, modify strategies, justify conclusions, and reflect on the experience gathered during the learning processes. Collaboration becomes crucial as learners are exposed and receptive to new, diverse perspectives in the atmosphere described above. Teamwork is encouraged by incorporating group input and sharing creative thoughts and feedback (Ojo, 2022; Li, 2021; Nilimaa, 2023).

When tackling a suggested lesson topic, the mathematics pre-service teacher considers various cutting-edge idea-generation strategies (like brainstorming) to help him and the learners get the most out of the learning process and meet predetermined goals. He presents innovative and valuable ideas using resources that can be developed from the environment or culture the learners are already familiar with. The focus will be on providing activities that are relevant to the learners' prior experience to understand the new information better, get learners to develop, analyse, and evaluate their ideas to maintain their interest and help them discover and enhance their mathematical creativity (Li, 2023; Ojo, 2022).

Creating a mathematics laboratory is one of the most innovative ways to encourage creativity in mathematics teaching and learning. By employing a variety of tasks, different resources, and the application of new concepts in a particular setting, learners can acquire basic mathematical ideas in a mathematics laboratory, as well as verify mathematical facts and theorems (Ojo, 2022; Central Board for Secondary Education; CBSE, 2014). For instance, the pre-service teacher is encouraged to clarify and reinforce abstract mathematical concepts with the help of concrete objects, charts, graphs, and drawings (Ojo, 2022).

The following are some inventive mathematical concepts that work best in a laboratory setting:

The topic: Pythagoras' theorem

Grade: JSS 3

Theorem: A right-angled triangle's hypotenuse has a square area equal to the sum of the square regions of the other two sides.

Objective: Use paper folding, cutting, and pasting to demonstrate the Pythagoras theorem.

Previous knowledge:

- The idea of a square's area is one that learners are familiar with.

- They can cut and paste paper, build a right-angled triangle, and take basic measurements.

Teaching aids

- Scissors, gum, pencils, mathematical sets, and cardboard paper.

Procedure:

- Use cardboard of standard size.
- Draw a right-angled triangle ABC on the cardboard with sides a, b, and c, with b as the hypotenuse.
- On each side of the triangle, create a square whose sides are the same length as the corresponding side of the triangle.
- The squares on sides and c should be cut out.
- Each square should be divided into unit squares and then cut out.
- Collect all the unit squares, then carefully arrange them in a neat row inside the square on the right-angled triangle's base (side b).

Observation:

The Pythagoras theorem is verified, and the learner's understanding is established when the unit squares made from the squares on sides a and c of the right-angled triangle fill the squares on side b of the same triangle. A creative mathematics lesson can simplify the supposedly complex mathematical formula A right triangle ABC has a hypotenuse whose square is equal to the sum of the squares of the other two legs (Sharma, 2022).

Example 2

Grade: JSS 3

The topic: Area of a rectangle.

Definition: The area of a rectangle is the number of unit squares contained in it.

Objective: Create a formula to calculate the area of a rectangle.

Previous knowledge:

- Learners are accustomed to rectangles
- They have cutting and pasting capabilities.

Teaching aids

- Scissors, mathematical sets, and cardboard paper.

Procedure:

- Draw a rectangle of any size on the cardboard paper, such as 8 cm by 6 cm.
- Mark the opposite side of each side at a 1 cm interval, then connect a perpendicular from each mark to that side.
- A network of 1-by-1-cm square units is created.
- Count the unit squares and record the number. This is the rectangle's area.
- Multiply the dimensions (length and width) used as a scale for drawing and note the result (8cm × 6cm = 48cm2).
- Are the outcomes from (4) and (5) above comparable or different?
- If the two findings are comparable, the area of a rectangle is calculated using the formula given by the multiple of the length and width (l w) in square units, which is equal to

The formula for the area of a rectangle that the learners themselves created has been absorbed and valued.

Creative techniques used in the lessons

- Learners would have the chance to imbibe mathematical ideas through practical activities.
- Learners could use models or paper-cutting and folding procedures to test and uncover various mathematical characteristics and facts.
- Since learners would have many opportunities to show originality and creativity in their work and comprehend the connection between mathematical principles and their everyday lives, they could develop motivation and confidence in learning the subject.
- Individual involvement in learning and developing as independent learners might be expanded, aiding cognition.

- Trial-and-error exercises help learners see failure as a teaching opportunity and comprehend that creativity and innovation are long-term, cyclical processes of little successes and repeated errors.
- The procedures would encourage learners to reflect, interact with one another and the pre-service mathematics teacher, and digest ideas more successfully.
- The pre-service mathematics teacher would have plenty of opportunities to use concrete objects and models to illustrate, clarify, and reinforce abstract mathematical concepts **(**Uwaezuoke & Charles-Ogan, 2016**).**

Thus, learners could compare several approaches to solving conventional mathematics issues and develop original solutions from the above creative lessons, utilising real-world examples when necessary. Providing the right kind of feedback could inspire learners to reverse their arguments and ways of thinking. They can learn from their errors and try again and again to find solutions. The ability of learners to search for patterns that suggest original shortcuts or frames of reference that are more straightforward is improved. They can conclude the general population by observing patterns in repeated calculations. Learners are empowered to recognise that mathematics is a creative endeavour by gaining new insights and interacting with others to build on prior learning (Nilimaa, 2023; Joklitschke, 2022;).

2.4.2 Critical Thinking Skill

According to Celik and Ozdemir (2020), thinking is a cognitive skill that can be learned to increase one's knowledge. This view is supported by Adebiyi and Akintoye (2020) and Olanrewaju and Ogundip (2021) who said that critical thinking skills have a positive correlation with learners' performance in mathematics.

Everybody needs to think a lot to get through their daily lives, and applying critical thinking to mathematics is crucial to comprehending our world. Therefore, pre-service mathematics educators must organise and carry out their instructional activities to enhance the dispositions of their learners' problem-solving, reasoning, and critical-thinking skills. Because memorisation is the primary mental activity for non-thinking learners, they cannot apply the knowledge they already know (Arifin et al., 2020; Ogunniyi, et al., 2019). Duru and Obasi, (2023) claim that being critical entails questioning and examining our thought processes and those of others to understand the things and people around us.

Fonga et al. (cited in Celik and Ozdemir, 2020) defined critical thinking as a self-regulatory judgment that is intentional and results in interpretation, analysis, evaluation, and inference, as well as justifications of the factors that led to that judgment. According to research, the learner's academic curriculum should include instruction in critical thinking skills. Although this responsibility should fall on pre-service teachers, this skill has not yet been fully developed (Duru & Obasi, 2023). However, aspiring mathematics teachers have many responsibilities. Both aptitude and ability are necessary for critical thinking. In other words, critical thinking is a multifaceted concept that encompasses both cognitive and affective tendencies (Celik & Ozdemir, 2020).

However, aspiring math teachers have many responsibilities. Both aptitude and ability are necessary for critical thinking. So, according to Celik and Ozdemir (2020), critical thinking is a multifaceted concept encompassing cognitive and affective tendencies. In a review on critical thinking, Lai (cited in Nwosu et al., 2015) identified three aspects of disciplines—philosophy, psychology, and education—on which the origin of critical thinking could be traced, with each discipline having its approach to the concept. The philosophical approach emphasises the qualities of ideal essential thinkers and the standards of good thought; the psychological process concentrates on how people think critically and the kinds of actions and behaviours that critical thinkers can exhibit; and the educational approach focuses on the Bloom taxonomy of information processing skills (Nwosu et al., 2015).

Regardless of their theoretical backgrounds, experts in the field of critical thinking tend to concur that critical thinking involves particular abilities like analysing arguments, drawing conclusions through inductive and deductive reasoning, interpreting and evaluating information, solving problems, and making reflective decisions ((Setyawati et al., 2022; Duru & Obasi, 2023).

According to Hitchcock (2018), who agreed with those above, critical thinking is thinking with a goal and includes both cognitive skills (mental abilities involved in critical thinking) and dispositions (habits of the mind). The definitions vary but typically involve logical, sceptical, and objective analysis or evaluation of factual evidence. The topic is complex, and there are many different definitions available. Self-directed, self-disciplined, self-monitored, and self-corrective are all critical thinking characteristics (Clarke, 2019).

The following mathematics lessons on critical thinking are most appropriate for JSS 3.

Example 1.

Grade: JSS 3.

Objectives: Learners should be able to recognise critical mathematical questions, create their ideas, and analyse one another's responses.

Prior knowledge:

- Circles are well-known to learners.
- Learners can conduct basic measurements.

The following are the prerequisites:

There are two groups in the class. The circumference (c) and diameter (d) of circular objects, such as the lid of a jar, the face of a clock, or a pie plate, are measured by one group of learners using a piece of string and a ruler. They determine c/d for each object they measure. The average of each result is then calculated to determine a rough estimate of pi. With inscribed and circumscribed polygons, the learners in the other group employ the Archimedes method. Learners contrast the outcomes of the two groups. They understand that pi cannot be accurately measured because it is irrational. After that, they look into various attempts to calculate pi throughout history by people from multiple cultures.

There is widespread agreement that critical thinking is crucial for problem-solving and mental operations like problem-building and reasoning. It is essential for mathematics education. Effective methods that encourage learners to consider new approaches while solving mathematical problems can help learners develop critical thinking in mathematics teaching as well as help them improve their mathematical communication skills (Belecina & Ocampo, 2018).

2.4.3 Communication

This section emphasised the importance of developing solid mathematical communication skills to prepare future math teachers for the classroom of the twenty-first century. According to Zulkarnain et al. (2021), reading, writing, speaking, and listening are all parts of literacy, and effective communication is the capacity to use these skills to facilitate teaching, learning, and development. Communication is crucial in mathematics because many mathematical symbols and concepts are too complex for learners to understand.

Communication is also known as mathematical communication. Through communication, pre-service teachers receive training on clearly expressing their ideas through oral, written, and nonverbal communication techniques in various settings. They listen carefully to understand the meaning of every information, attitudes, values, and intentions of others.

Effective mathematical communication gives pre-service teachers the knowledge and abilities to instruct, guide, and inspire students to comprehend mathematical ideas. Acknowledging the multicultural setting in which the education of learners occurs can be accomplished in various ways (Diwa et al., 2023; Zulkarnain et al., 2021). Mathematical communication skills allow pre-service teachers to comprehend, interpret, respond to, and use mathematical symbols, pictures, graphs, computer games, and words to explain concepts in oral and written form in a language that learners can relate to, facilitating easy understanding during the learning process (Diwa et al., 2023).

The following mathematics communication lesson would be most appropriate For JSS3.

Example 1

Grade: JSS 3

Topic: Algebra Block and Its Use.

Definition: Algebra block refers to a prop consisting of square and rectangle pieces of flat paper that the specified algebraic form will arrange.

Objective: Using the paper folding method and pasting to verify algebraic factorisation using algebra block.

Previous knowledge:

- They can cut and paste paper, build a rectangle and a square, and take basic measurements.
- The idea of recognising each variable's and constant's coefficients in algebraic form is well-known to learners.

Teaching aids:

Pencils, mathematical sets, gum, scissors, and coloured cardboard paper.

Procedure:

The following are the steps involved in building algebraic blocks.

- Recognize the algebraic coefficients for each variable and constant, positive or negative.
- Choose or arrange the paper according to the constants and coefficients.
- Arrange the paper pieces into squares or rectangles.
- Measure the dimensions of the resulting square or rectangle.
- The length and width are decided after arranging the algebraic building blocks into squares or rectangles.

For instance, in an algebraic form x2 + x – 6, the likely arrangement of algebraic blocks is the following:

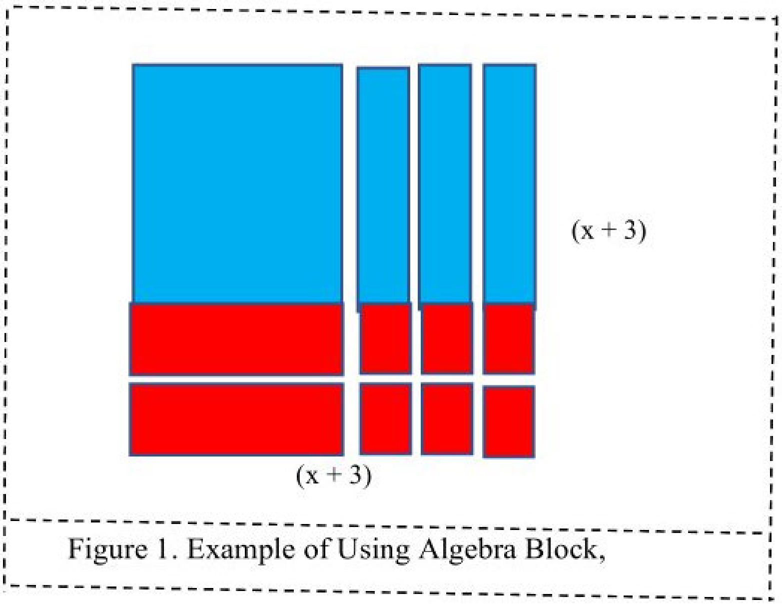

Figure 1. Example of Using Algebra Block (Disasmitowati & Utami, 2017).

Figure 1 illustrates that the length dimension is (x + 3), and the width is (x – 2). Therefore, the algebraic factorization of $x^2 + x - 6$ equals (x – 2) (x + 3).

If you can visualise the formed algebraic blocks, factoring algebraic form using the distributive will be more straightforward. Building geometry (square or rectangle) is depicted in Figure

1. There is one large blue square, which indicates the representation of x2, followed by three blue rectangle pieces, which represent 3x, two red rectangle pieces, which represent -2x, and six small red squares, which represent constants at a value of - 6.

x 2 + x - 6 = x 2 + 3x - 2x - 6.

= (x2 + 3x) - (2x + 6).

= (x2 + 3x) - (2x + 6).

= (x + 3) (x – 2).

Observation:

Through effective communication, the learners understood algebraic factorisation. Block Algebra was then used to factor the algebraic form to make learning easier without conventional methods. The learner's mathematical communication abilities improve with prior solid knowledge, skills, and reasoning. (Disasmitowati and Utami, 2017). To improve the learner’s mathematics communication abilities, pre-service mathematics teachers must be aware of social injustices and other issues that might obstruct effective math instruction. The crucial mathematical investigation is therefore covered in the following section.

2.5 Critical Mathematical Inquiry

According to Boldt (2019), radical teaching is the focus of critical education. He argued that for education to be necessary for both practice and research, it must discuss the fundamental conditions for learning, be conscious of social issues such as inequality and repression, and work to transform education into a force for social progress. When instructional and educational activities concentrate on issues of equity, plurality, the rule of law, and justice for all, this is the region of the spectrum where lowercase critical thinking (e.g., making reasonable decisions) meets uppercase critical thinking (e.g., assessing various forms of power and oppression).

In light of how oppression, power, and access interact, the educator comprehends the political aspect of education (Boldt, 2019). Educators must encourage and support students' investigations into social inequalities and injustices to change society so that social justice is possible (Boldt, 2019). These are the goals of critical education. Critical mathematical enquiry is one of the techniques for teaching mathematics for social justice, an essential illustration of education.

2.5.1 Mathematical Instruction for Social Justice

Many people who teach mathematics for social justice have linked the philosophical foundations of their work to Freire. In contrast, many individuals who teach mathematics for social justice cite Rico Gutstein's 2006 book Reading and Writing the World with Mathematics as their primary philosophical inspiration. Reading is a process that helps one understand the political and socio-cultural circumstances that impact a person's existence and society (Boldt, 2019; Register et al., 2022).

Thus, writing is an example of Freirean education, which focuses on changing the globe via deliberate, beneficial activity (Boldt, 2019). The two components of praxis are reading (reflection) and writing (exercise). Teaching mathematics for social justice (TMSJ) is an approach developed by Gutstein that has both mathematical and social justice pedagogical goals (see Figure 2).

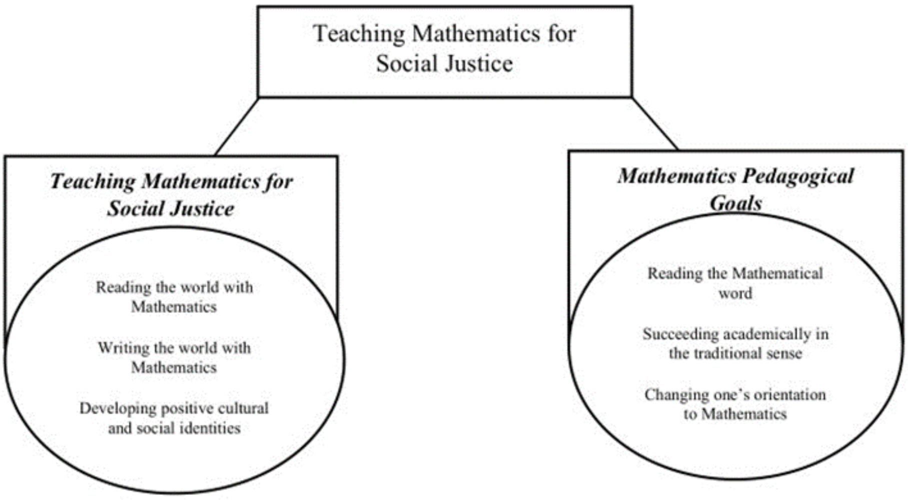

Figure 2. Eric Gutstein's model of mathematics education for social justice (Bolt, 2019).

The three main objectives of Gutstein's (2006) mathematics curriculum are reading mathematics, achieving academic excellence in the conventional sense, and changing one's attitude toward mathematics. Last but not least, shifting students' perspectives on mathematics requires them to stop seeing it as a collection of disjointed procedures to be memorised and remembered and start seeing it as a potent tool for investigating and comprehending intricate, everyday occurrences.

Gustein's model has social significance. It also fulfils pedagogical goals and reveals its unique strength in this regard. These involve creating meaningful socio-cultural identities and interpreting and writing the world using math. Reading the world with mathematics entails utilising math to better understand the dynamics of power, unequal distributions of resources, and unequal opportunities among individuals and groups, as well as to understand visible prejudice based on inequalities as a result of ethnicity, socioeconomic status, sexuality, language, and other factors (Register, 2022; Atta & Bonyah, 2023; Boldt, 2019).

Describing the world in terms of mathematics means using mathematics to change the world while creating a feeling of being an agent of change and believing that one has the potential to bring about transformation (Boldt, 2019; Govender et al., 2023). Gutstein's social justice pedagogy aims to help learners build strong cultural and social identities and ground mathematics instruction in their native tongues and cultures.

2.6 Teacher Preparation and Professional Development (PD)

According to OECD (2023), more and more people are viewing teacher preparation from the standpoint of lifelong learning. While initial education and induction programmes, lay the foundation for future success, ongoing staff development offers a way to raise the standard of workers and keep productive employees on board over the long term. It may facilitate the transition of new instructors into the workforce and make up for inadequate initial training. Professional development implies an activity requiring special training or skills, while an occupation is regularly executed by someone with special skills for remuneration, as opposed to a voluntary activity (Iroegbu & Ogbodo, 2019). This explains why professional development is necessary for teachers.

No one in any career can guarantee that what you have learned during your education will remain helpful. Things are changing faster, new technologies are evolving, and new socio-psychological situations affect classroom situations. Teachers must also continually develop their skills (OECD, 2019). This underscores why there is a growing need to train prospective teachers with the necessary 21st-century skills to facilitate the effective implementation of the mathematics school curriculum. Initial preparation alone cannot prepare teachers for all the challenges they will face throughout their careers. Therefore, continuous professional development is crucial for teachers to expand their knowledge and keep up with changing research, practices and learner needs (OECD, 2019; Schwarz & Kaiser, 2019).

The need for professional development of mathematics teachers may arise as a result of the following: The increasing demand concerning a knowledge-based economy, nurturing teachers' psychomotor and cognitive skills and developing teachers' competence in mathematical subject content knowledge and pedagogical skills that is expanding. Other needs include learning to adapt to new curriculum concepts or adopt a unique learning methodology, embracing new ICT skills, and adopting the best practices for the benefit of learners (Singh, 2020; Schwarz & Kaiser, 2019; Weli & Nyimejie, 2021; Tatto, 2021). Professional growth is a lifelong, ongoing process you must engage in. You need to learn new things, and you need to unlearn older items, and you need to relearn a few things that you have learned earlier but now need them again to use somewhere (Sodangi & Adamu, 2023; Weli & Nyimejie, 2021).

Concerning the professional development of mathematics teachers (Schwarz & Kaiser, 2019) distinguished the following categories: subject matter knowledge, pedagogical content knowledge, and curricular knowledge—figure 3—teachers' knowledge developing in context Fennema and Franke (cited in Schwarz & Kaiser, 2019).

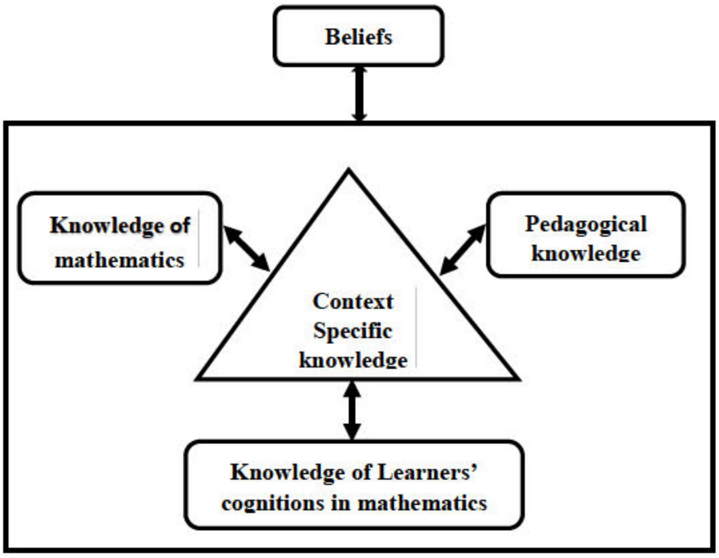

Figure 3. Teachers' knowledge developing in context Fennema and Franke (cited in Schwarz & Kaiser, 2019:331).

Fennema and Franke (cited in Schwarz & Kaiser, 2019) described their model. According to their model, the central triangle represents the teachers' knowledge and beliefs within a specific context. The context acts as a framework that determines the elements of knowledge and ideas involved. In a given context, the teacher's knowledge of the subject matter interacts with their understanding of pedagogy and the learners' cognitive abilities, combining with their beliefs to form a distinctive body of knowledge that influences classroom behaviour in the 21st century.

In Nigeria, aspiring teachers and current teachers who need retraining are educated at standard Teacher Training Colleges, Educational Institutes, National Teachers' Institutes, and Teacher

Training Universities. Those who enrol at Colleges of Education spend at least three years there to obtain the Nigeria Certificate in Education (NCE). It takes at least four years to get a bachelor's degree in pedagogy (Federal Ministry of Education 2014; Agi, 2020; Sodangi & Adamu, 2023).

Despite certificates and degrees awarded by teacher education institutions, learner mathematics performance still needs to improve on internal and external examinations (Idowu, 2016). To support this view, a report of WAEC results from 2009 to 2018 showed a decline in secondary school students' performance in core subjects (including mathematics).

Specifically, in 2009, only 25.99% of the learners had earned five credits and above, including mathematics, and a decrease to 23.36% was seen in 2010. According to Suleiman and Hammed (2019), the performance dropped to 36.57% in 2013 and 31.28% in 2014. The WAEC Results Statistics of 2018, as reported by the National Bureau of Statistics (NBS) (2019), estimate that 48.15 per cent of candidates in 2018 received five credits or more in subjects like mathematics and English language. Whereas in 2021 only 32.02% of Nigeria West African Examinations Council (WAEC) candidates obtained credit in mathematics (WAEC, 2021).

Adequate teacher preparation and professional development can help learners learn mathematics through content-focused experiences, incorporate active learning, support collaborative learning, use a model of effective practice, provide coaching and expert support, Etc. (Darling-Hammond et al., 2017).

Content-focused: This component deliberately emphasises creating discipline-specific curricula in areas like mathematics. Preservice teachers may benefit from this professional development by examining their learners' work, experimenting with various curricula, and learning effective teaching strategies for a multicultural society.

Incorporates active learning: Active learning engages preservice teachers/teachers with "sense-making" interactive activities and authentic artefacts, incorporates technology, and uses various tactics to give highly contextualised professional learning in striking contrast to sitting through lectures (Darling-Hammond et al., 2017).

Supports collaboration: For learners to feel that they belong and connect with their classmates, pre-service teachers/teachers can help interpersonal relationships in the classroom by encouraging active collaboration and exchanging ideas (Darling-Hammond et al., 2017).

Uses models of effective practice: Teachers can get an excellent illustration of optimal methods through instruction modelling. Teachers can use models such as written or video instruction cases, peer observations, curriculum materials with examples of assessments, and student work.

Provides coaching and expert support: Coaching and assistance from experts include scaffolding, knowledge sharing on subjects and evidence-based techniques, and direct attention to the needs of teachers and students. As a result, pre-service teachers can investigate how to use technology in various circumstances to increase learners' mathematical understanding and confidence (Darling-Hammond et al., 2017).

Offers feedback and reflection: High-quality professional learning typically offers time for instructors to reflect, get feedback, and adjust how they teach. Despite feedback and reflection being two separate fields, they complement each other to support instructors toward attaining the best ideals in practice (Darling-Hammond et al., 2017).

Sustained duration: Research has shown that consistent and ongoing professional learning, which provides teachers with multiple opportunities to engage in education, has a higher likelihood of positively impacting teaching methods and learners' learning outcomes (Darling-Hammond et al., 2017).

From the above, it is clear that the professional growth of teachers plays a crucial role in equipping them with the necessary knowledge and skills to support their colleagues, contribute to the collective growth of the teaching profession, and establish trust and self-confidence in carrying out their work professionally. To support teacher professional development at the national level, the Nigerian Federal government has pledged financial assistance to Bachelor of Education undergraduate students, with a promise of N75,000 per semester, and NCE students, with a commitment of N50,000 (Onyedinefu, 2021). This commendable incentive aims to enhance pre-service teachers' expertise and learners' performance and can be further reinforced through practical evaluation and assessment methods.

2.7 Assessment in Mathematics

Assessment in mathematics education during the 20th century focused primarily on testing low cognitive-demanding knowledge, especially memorised arithmetic procedures. In the twenty-first century, we need to emphasise higher-order thinking (HOT), and math assessments should reflect this. The assessment process will require significant revision and training, as teachers must carefully consider how to get the best learners ready for assessment styles that stimulate HOT (Radmehr & Vos, 2020).

Thus, an assessment is one of the natural methods for deciding who moves forward to the next phase of learning, which is one of the reasons why assessment is seen as an essential part of secondary education in Nigeria (Longjohn & Audu, 2020). Assessment in education generally is the systematic process of observing, documenting and using empirical data on knowledge, skill, and attitudes to evaluate programs and improve learners' learning (Longjohn & Audu, 2020). According to Longjohn and Audu (2020), assessment is the process of organising, measuring, and interpreting data so that conclusions or decisions can be drawn from it. In his view, the assessment answers such questions as: How well did the learners perform this year? How did the male learners perform with female learners? With the realisation of the importance of assessment in the school system, many countries have embraced two primary assessment levels, namely school-based assessment and certificate examinations.

2.7.1 School-Based Evaluation

A school-based assessment is described as student assessments frequently arranged and carried out by each educational establishment created in a nation (UNESCO, 2020). Teachers typically make the assessment instruments. The findings are used to improve the teaching-learning process, control the classroom, and give parents and students immediate feedback. The results of these tests are used in some nations to determine a student's graduation or selection. A school-based evaluation is a transformation system emphasising less exam-oriented teaching and learning (Siti et al., 2019). According to Longjohn and Audu (2020), school-based assessment essentially calls for teachers to keep tabs on their learners' learning progress, assess their performance, document their progress, and offer feedback.

2.7.2 Certificate Examinations

The second category of assessment is called certificate examinations. This form of assessment is carried out to award a certificate to indicate the mastery level of the examinee. It is known as an external examination, which is organised by experts outside an examinee's school, college or university. Individual schools have no control over it because it is superficial, and it produces a summative evaluation of candidates (Longjohn & Audu, 2020).

External exams are being administered in Nigeria by local testing organisations like the West African Examinations Council (WAEC). West African nations administer the WASSCE, a specific kind of standardised test. Students who pass the test are given a certificate attesting to completing secondary school. The West African Senior School Certificate is the academic school-learning credential granted upon exam completion. English, mathematics, integrated science, social studies, and three or four elective courses are tested in the WASSCE. The letters A to F are used in the current WAEC marking and grading system to denote how well a result is. The WAEC grading scale is illustrated as follows:

Table 1: Grading Criteria and Definition of the WAEC.

Grades	Definition	Interpretation	Equivalent
A1	Excellent	75%-100%	1
B2	Very good	70%-74%	2
B3	Good	65%-64%	3
C4	Credit	60%-64%	4
C5	Credit	53%-59%	5
C6	Credit	50%-54%	6
D7	Pass	45%-49%	7
E8	Pass	40%-44%	8
F9	Fail	0%-39%	9

According to Atsumbe and Raymond (2012), no consistent standards are being used to implement assessments in many schools. It is well known that some schools uphold stricter requirements than others. These schools frequently expose learners to difficult exams, which leads to a scenario wherein learners study primarily for their exams rather than comprehending the learning process. These contradictions inform the need for the harmonisation of national assessment practices.

Jäder (2015) surveyed mathematics textbooks for lower secondary students from 12 different nations (USA et al., Etc.) and discovered that 79% of the tasks could be accomplished by following the instructions exactly, 13% by using standard techniques with slight adjustments and only 9% required innovative problem-solving.

Classroom-based assessments, national exams, and international comparative studies use the low cognitive demand mathematics assessment methods indicated above. They have their roots in behaviourist and psychometric viewpoints (Radmeh & Vos, 2020). Digital platforms have perpetuated these traditional examinations in the modern era. Today's tests are administered digitally, with pupils filling in their responses into an input field in computers rather than using pencils in local, national and international tests and examinations (Drijvers, 2018). Several tasks can be created and distributed to learners in these settings, and their responses can be automatically graded. In the digital environment, learners can use clickable calculators, dynamic animations, YouTube resources, and gamification features for correct answers (stars, jingle bells, or other rewards).

The crucial point is that the emphasis on procedural knowledge has remained unchanged due to digital assessment. Learners are expected to work alone, under time constraints, to replicate processes they have learned in class, perform standard calculations, and produce the right results. Modern mathematics assessments must be updated, and we can already see some changes.

For instance, tasks about fraction representation diagrams can now be used in addition to fraction computations. This innovation challenges the conventional idea that mathematics is all about calculations. Concerning assessment's organisational components, there has been a second substantial development. There are now mathematics assessment formats that allow for group collaboration and give students plenty of time to reflect by giving them a project task that they may work on for a half-day or more (Radmehr & Vos, 2020). These forms are an improvement over having students work alone on short tasks under time pressure. A presentation of the research done by learners may be required, and it may also be graded.

Some recommended evaluation tasks that support HOT are provided in the following paragraphs.

2.7.3 Problem-posing tasks

A strategy to activate HOT involves learners in problem-posing activities (Radmehr & Drake, 2018; Cai & Hwang, 2023). To solve a mathematical problem, learners must infer, compare, interpret, and understand the problem provided to generate a mathematical solution. According to research, problem-posing exercises help learners learn better by enhancing their conceptual understanding, creativity, problem-solving, and critical thinking abilities (Radmehr & Vos, 2020). Let us take a scenario where learners may be asked to provide possible responses for the equation 2(x) 2-5x+3 = 0; learners may offer suggestions such as plotting a graph, creating a table, factoring, or identifying a situation that the equation might be used to illustrate.

2.7.3.1 Mathematical Puzzle-Problem Tasks

According to Radmehr & Vos, (2020) and Radmehr et al. (2021), involving learners in puzzle problems can help develop their problem-solving techniques, conceptual understanding, and motivation to learn mathematics related to the real world. Let us look at an illustration of a riddle taken from the Universal Mathematics book (Darling, 2004): 100 pounds of potatoes are brought home by Fred, each of which is made up of 99 per cent water; they are left outside all through the night so that they are 98% water. Their new weight, according to Fred, is 50 pounds. Is he accurate? Justify your response, please.

This question is paradoxical because it appears that losing 1% of water causes the potatoes to lose 50% of their body weight. To correctly solve this problem, learners should understand that 99% of the potatoes were made up of water and 1% of potato matter before dehydration. In other words, one pound is equivalent to 1% potato substance. Dehydration did not modify the amount of potato substance, which remains at 1 pound. Nevertheless, this 1-pound potato matter is now 2% of the potato.

$$\frac{2}{100} = 1 \Rightarrow 2x = 100 \Rightarrow x = 50.$$

2.7.3.2 Modelling Mathematical Tasks

We frequently encounter "bare tasks" in conventional mathematics assessment, which are problems composed solely of mathematical symbols preceded by a few words or phrases like "solve," "find," or "calculate" (Vos, 2020, p. 35). Research on task formats that give learners a valuable activity that connects mathematics with real-life circumstances is needed; this is in line with academics who contend that such bare tasks obscure the relevance of mathematics (Klymchuk, 2017). The term "mathematical modelling tasks" currently refers to these tasks.

As an illustration, consider the Rubber Band assignment from the study by Harmon et al. (cited in Vos, 2020), in which learners were given ten rings, a rubber band with a hook, and a line gauge. They were then instructed to calculate how much further the band would stretch if given 12 rings. The absence of the two extra rings posed a challenge for learners to extrapolate. Although this was an activity they had never been taught, many of them were able to resolve it by extending a graph of the rubber band's length versus the number of rings, as shown in Figure 4.

Ring 1	11.5cm
Ring 2	12cm
Ring 3	12.2cm
Ring 4	12.7cm
Ring 5	13cm
Ring 6	13cm
Ring 7	12.8cm
Ring 8	13.1cm
Ring 9	13.4cm
Ring10	13.4cm

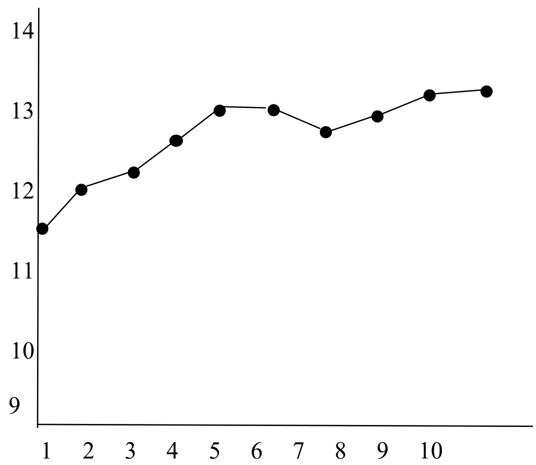

Figure 4: Grade 8 students' tabular-graphical approach to the task Rubber Band modified from a model by Radmehr and Vos (2020, p. 12).

2.7.3.3 Cloze Paragraphs

In this test type, learners must fill in the blanks of a provided text. According to Radmehr & Vos (2020), a Cloze Paragraph is a task that comes after learners have finished a modelling task and in which learners are requested to recreate and reflect on what was done. An outsider would not recognise these objects as mathematical since the required solutions are words rather than numbers. Learners may find this style challenging because the necessary terms are not listed on the side of the text. To fill in the gaps, one must recall definitions and relationships included in the mathematical model, have a mental grasp of the underlying mathematical ideas, and be aware of the model's limits as illustrated in Figure. 5 (Lewis et al., 2019; Radmehr & Vos, 2020).

Cloze Paragraph

Fill in the blanks with an appropriate phrase or words.

When scaling, the surface area will vary as the ______________ of a linear dimension, and the _______________ will change as the curb of the linear dimension. All _______________ sizes in a scale model are to the same scale. On a human, the skin grows proportionally to the _______________ of the scale factor. The weight increases proportionally to the _________ of the scale factor. Bone strength is proportional to the _____________ area of the bone. A perpendicular cross-section is a surface found by cutting an object ______________ to its _____________. Thus, the cross-sectional area of a leg bone can be thought of as an irregular _____________ with a hole in the middle.

Figure 5: Cloze Paragraph to the modelling project "Could King Kong exist?" modified from a model by Radmehr and Vos (2020).

Figure 5 shows the gaps in the modelling Cloze Paragraph "Could King Kong exist?" (Radmehr & Vos, 2020, p. 16). In this two-day project, learners examine different creatures' surface area, volume, and bone strength to see if an animal as large as King Kong may exist. The task is accomplished when the fill-in-the-blank paragraph is completed. Learners are prompted to employ a robust mathematical vocabulary in this assessment approach. They must finish sentences using words like square, volume, dimension, perpendicular, etc. Learners must articulate their reasoning in this activity by first comprehending the sentences and their possible meanings, then matching the text to their knowledge of the mathematical qualities resulting from the scaling. These tasks are mostly related to memory (recognition and recall), comprehension (summarisation and description), and analysis (Radmehr & Vos, 2020).

The objectives of mathematics education must be considered while developing appropriate evaluation methods. According to Radmehr and Vos (2020) and Lewis et al. (2019), a successful assessment should address the following issues: what mathematical concepts are crucial, how mathematics is acquired, how mathematics is being taught, and what applications mathematics has in the real world. Although mathematics assessment emphasises developing students' higher-order thinking (HOT), there must be a match between teachers' knowledge and their capacity to use theory to accomplish HOT in the classroom, necessitating a further understanding of the gap between theory and practice.

2.8 Theory practice divide

According to Mavhunga and Merwe (2020) and Barendsen and Henze (2019), the gap between theory and practice is a persistent problem that is frequently linked to teachers' knowledge gaps between knowing what to do (planning) and doing what you know (enacting in class). Even a century after Dewey publicly protested the issue (cited in Mavhunga & Merwe, 2020), the gap between theory and practice in teacher education remains problematic. It was referred to as the "Achilles' heel of teacher education" by Darling Hammond (Darling-Hammond, 2013, p. 3).

Recognising the mismatch between teaching preparation and actual teaching is at the core of the theory-practice split. This contradiction highlights the lack of harmony between what Aristotle (384–322 BC) called phronesis and episteme in Book VI of the Nicomachean Ethics (Pitman & Kinsella, 2019). As a result, knowledge as episteme assumes the existence of a substantial corpus of knowledge, which is both abstract and general and constant and independent of context. Phronesis refers to a grasped abstract principle in a particular context, wrapped with the relevant ethical concerns, that is, the proper course of action in a specific circumstance.

Pitman and Kinsella (2019) state that Phronesis uses intellect and sound judgment. It is frequently referred to as "practical common sense" and emphasises practical ethics by adding a moral purpose and the intention to behave in the best interests of the situation.

This theory-practice split is particularly pronounced in teacher education for many reasons, according to specific theories. The ongoing division of teacher education duties between universities and schools is a significant element that has prolonged the "theory-practice divide" (Pitman & Kinsella, 2019, p.58). One of the considerable issues in mathematics teacher education is creating congruence between theory and practice. Thus, the main elements of prospective teachers' programs contain contradictions and inconsistencies. Stronger relationships between universities and schools are required to address the perceived disconnect between campus and field- or school-based teacher education (Cochran-Smith & Reagan, 2021).

According to Burroughs et al. (2020), many of the reported flaws in many on-campus teacher preparation programs are due to insufficient cooperation among partners. Such inadequate channels of the corporation have resulted in better-defined roles and responsibilities of the parties involved. As a result, teaching practice monitoring personnel in schools and higher education institutions are frequently not sure of how they should support pre-service teachers and interact with one another whilst doing so.

Academics in mathematics education have pondered whether theory and practice coexist harmoniously for years. And if so, how?" According to Burroughs et al. (2020), one of the most influential and successful methods for bridging the theory-practice gap in training mathematics teachers for the 21st-century classroom is to link properly designed practicum experiences with on-campus courses. For Mavhunga and Merwe (2020), the theoretical construct of pedagogical content knowledge (PCK), along with topic-specific pedagogical content knowledge (TSPCK), allows educators to move from knowing what to do (knowledge manifesting in planning) to doing what they know (knowledge manifesting in implementation), to enhance learners' understanding within their local context.

Cochran-Smith and Reagan (2021), in one of the most comprehensive national studies on teacher education, highlighted many ways that best practice might be attained by fusing theory and practice in the practicum. These methods include active participation from faculty and staff at the school and university and well-planned on-the-ground activities that create a clear link between theory and practice using, for instance, performance evaluations and action research. While the theory-practice split exposes the existing gap in pre-service teacher education, it is crucial to consider how a connection between theory and practice (i.e., a relationship between the university and the school) could make preparing mathematics instructors for the 21st-century classroom easier.

2.8.1 Linking Theory with Practice

In discussing linking theory with practice, the focus will be on South Africa, Finland, Nigeria, and the United States (US). The prioritisation of the learning experiences of student teachers in higher education is currently under the spotlight, with debates on 21st-century work-integrated learning and the theory/practice connection. Hence, this section may provide opportunities for aspiring teachers to explore what is being done in other countries to improve pre-service mathematics teachers' education.

2.8.1.1 South Africa

2.8.1.1.1 Structure of Teacher Education

South Africa's present statutory standard for a trained teacher is the M+4 requirement, which consists of a matric (school-leaving) certificate and the completion of a four-year initial teacher education program. According to the National Qualifications Framework Act 67 of 2008 and the MRTEQ policy (cited in Centre for Development and Enterprise, 2015), a teacher must either complete a four-year Bachelor of Education (B.Ed.) degree, which consists of 480 credits with a practical component worth 120 credits at level 7, or an appropriate first degree followed by a one-year Advanced Diploma in Education. The policy outlines the skills that a qualified teacher must have. For instance, graduates must show they can apply 21st-century skills and abilities in specific phases and contexts (Taylor, 2021).

2.8.1.1.2 Teacher Supply

According to the Centre for Development and Enterprise (CDE), South Africa will require 456,000 teachers by 2025 to provide high-quality instruction. The Department of Basic Education reports that 410,000 teachers work in South Africa's public school system (Maphalala & Mpofu, 2019, p. 1). These educators instruct 12.9 million learners across 25,000 schools nationwide (Maphalala & Mpofu, 2019). There is a significant increase in students enrolling in and graduating from ITE programs. For instance, in the years 2009 and 2012, the total number of new teacher graduates increased from 6,978 in 2009 to 13,708 in 2012, indicating there were more than 6730 recent teacher graduates (NTGs) than there were in 2009 (CDE, 2015). On the other hand, others argue that South Africa does not produce enough teachers to balance the demand and supply equation for the profession. Recently, the initial teacher's education programmes in the country had 15,000 new teachers yearly. This falls short of the 25,000 benchmarks needed to keep an efficient teacher-to-learner ratio. In addition, between 18,000 and 22,000 teachers quit their jobs each year (Maphalala & Mpofu, 2019).

2.8.1.1.3 Contextual challenges

There were approximately 444.9 thousand teachers in South Africa as of 2019 (Galal, 2019). Their contribution is strategically significant for young people's ethical, cognitive, and cultural development. The unequal demand for effective teachers across the different school stages constitutes one of the biggest challenges facing teacher education in South Africa. The graph below in Figure 6 provides insight into the context of teacher supply in different school phases in South Africa.

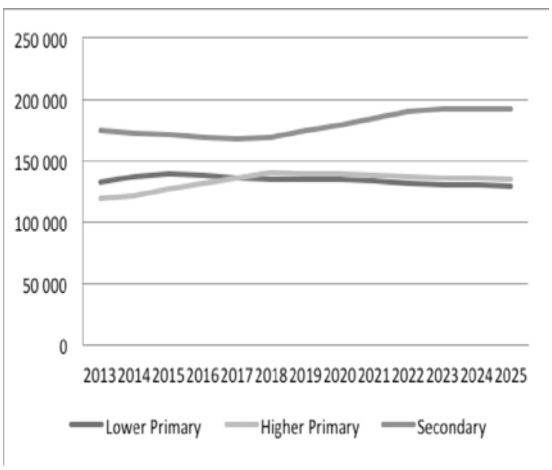

Figure 6: The context of teacher supply in different school phases in South Africa (CDE, 2015).

The result of the above graph shows that if the thirteen years of learning are divided into lower primary (Grades R–3), higher primary (Grades 4–7), and secondary (Grades 8–12), by 2025, there will be a 3% decrease in the need for teachers in the lower primary sector a 13% increase in the market for teachers in the upper primary, and a 10% increase in the demand for teachers in the secondary school section. The prevailing recruitment strategy of the Department of Higher Education and Training (DHET) does not account for this unequal demand.

Secondly, it appears that some subjects have an excess of teachers, whilst some essential subject areas suffer from severe shortages of mathematics in the Intermediate and Senior phases, mathematical literacy in the Further Education and Training (FET) Phase and languages in all grades, (CDE, 2015; Amnesty International, 2020). Given these circumstances, it has been revealed that South Africa has one of the most unequal educational systems in the world in terms of learners' performance. Learners in the best 200 schools obtain better mathematics grades than those in the remaining 6,600 schools. In 2019, there was a drop in matriculants' performance in mathematics. This decline can be measured in terms of what percentage of learners were successful: about 54% of the learners who took the exam were successful, indicating a drop in performance. From 58% in 2018, the percentage of successful candidates decreased to 54% in 2019 (Maphalala & Mpofu, 2019).

The effectiveness of initial teacher education (ITE) is adversely impacted by several issues, according to research done by CDE in 2015. These concerns include ineffective recruitment procedures, low admission standards, a dearth of program coherence and intellectual demand, substandard 21st-century educational resources, minimal parental involvement, lack of motivation, and excessive policies (Maphalala & Mpofu, 2019). It would be necessary for the nation to offer greater inducements to mathematics instructors, such as improved salaries and remunerations, security of tenure (protection against victimisation), motivation and provision of safety against work-related hazards and risks to address this teacher shortage (Maphalala & Mpofu, 2019). In light of the teacher shortage, it is crucial to look at the steps being taken to prepare mathematics teachers for the classroom of the twenty-first century.

2.8.1.1.4 The Proposed Model of Teaching Mathematics

The aims of South Africa's educational policies and curricula are not achieved through teaching and learning mathematics in South African classrooms (DoE, 2018). To meet these goals in the twenty-first century, a long-lasting initiative that modifies how mathematics is taught is needed. According to the Curriculum and Assessment Policy Statement (CAPS), in South Africa, mathematics is a language that uses notations and symbols to explain numerical, geometric, and graphical relationships. It is a human endeavour that entails identifying, visualising, and analysing quantitative links between mathematical objects and physical and social occurrences. It facilitates the development of measures that improve accuracy, logical and critical reasoning, and problem-solving abilities that will aid in proffering a well-informed decision or assessing alternative solutions (DoE, 2018).

The above definition of mathematics offers some guidelines on how mathematics teachers should design and deliver lessons encouraging learners to think conceptually about mathematical concepts. This framework does not substitute for the CAPS or constitute a new curriculum. On the contrary, it helps teachers alter how they teach by offering a model that will support the application of the present curriculum. To provide teachers with advice to transform their teaching, the framework's document recommended a four-dimensional model to support practitioners (DoE, 2018).

If the present South African curriculum statement, the CAPS, were to be executed as it was designed, it could equip South African learners with the required knowledge for the demands of the 4th Industrial Revolution, which emphasises cyber-physical production systems as advocated by the World Economic Forum, and also fully prepare them for the demands of the twenty-first-century labour market. These needed abilities include adopting, adapting, and efficiently using digital technology and applications, utilising multiple concepts simultaneously, and mathematical reasoning and problem-solving skills (DoE, 2018).

It is not a gain saying that poor learning results in mathematics are a problem. Nevertheless, implementing the mathematics curriculum (CAPS) in most South African schools is at odds with this. The aim of drafting the Mathematics Framework was to address this gap to help instructors focus on essential curriculum elements so they may take the required actions to change how mathematics is imparted (and subsequently mastered). Kilpatrick's five strands of mathematical competency have an impact on the conceptualisation of the five-part framework that is described here. The five-part framework shows how the strands have been contextualised and adjusted for the South African context (DoE, 2018). The following initiatives need to be taken to transform mathematics education in South Africa:

- Impart conceptual knowledge in mathematics to help learners grasp mathematical ideas, procedures, and relationships;
- Ensuring that learners are taught to become procedurally fluent, which involves the ability to carry out procedures effectively, quickly, and responsibly;
- Increase learners ' analytical competency, or their capacity to design, represent, and select the best approaches to solving mathematical issues;
- Offer learners numerous and diverse opportunities to hone their mathematical reasoning abilities, which include the ability to reason logically, reflect, clarify and justify answers.
- Encourage instructors to engage with students in ways that prioritise mathematics learning for all, resulting in a learner-centred classroom that supports the abovementioned activities.

One could argue that a greater priority is needed to be placed on understanding mathematical concepts since they serve as the basis upon which all other dimensions are developed. The five critical dimensions listed above depend on one another and should be appropriately integrated to optimise successful mathematics teaching. By prioritising conceptual understanding, the difficulties of learning mathematics without understanding how mathematical concepts were developed, why they function, or when they are beneficial are adequately addressed (DoE, 2018).

2.8.1.1.5 University-School Partnership

A document named The Integrated Strategic Planning Framework for Teacher Education and Development in South Africa (also referred to as the Framework) has caused a current need to encourage cooperation between universities and schools. Through creating Teaching Schools and Professional Practice Schools, the Framework aims to reinforce the practical classroom experience of teacher education programs Department of Basic Education and Higher Education and Training (2011 as cited in Gravett et al., 2019).

Teaching schools in this context are called teaching laboratories, where pre-service teachers participate in the practical implementation of theories by engaging in micro-teaching (using small groups) and observing exemplary mentors. In comparison, professional practice schools are considered specially designated practicum centres where pre-service teachers can receive excellent assistance during their teaching practice, irrespective of their background. The Framework also suggests that professional practice schools and teaching schools support one other in how teachers are educated. To achieve the above objectives, the South African Government have encouraged public-private partnerships to collaborate with universities and schools. Examples of university-school collaboration include the University of Cape Town School Improvement Initiative Programme and the partnership between the University of Johannesburg (UJ), the Gauteng Department of Education and the European Union to support schools in Gauteng (Gravett & Ramsaroop, 2019).

2.8.1.2 Finland

In Finland, primary enrolment is 99.7%, and the literacy rate is 99%. The eighth-most educated nation in the world is Finland. Finland is ranked third among other countries in education, with a total score of 1.631K in 2021. Finland has the best-developed education system, according to the World Economic Forum's Global Competitive Survey (Leverage Edu, 2022; OECD, 2019).

2.8.1.2.1 Educational Equity

The education policies covered by the equity policy are designed to serve all learners fully while preventing dropouts. The education is modified to match the unique needs of each learner rather than treating all of their pupils as a single unit (OECD, 2019; Burg, 2018).

2.8.1.2.2 Education System of Finland

Finland has nine years of mandatory primary education, early childhood care and education, upper secondary education, higher education for colleges and universities, and adult education. Below is a description of each of these stages.

- Early Childhood Education and Care (offered to pupils before the start of compulsory education).
- Pre-Primary Education (for 6-year-olds, one year)
- Basic Education (a 9-year mandatory program for kids ages 7 to 16).
- Upper secondary education, which encompasses both vocational education and training as well as general upper secondary education, is the focus of this information.
- Higher education refers to the educational programs of universities and universities of applied sciences.

- Adult Education is an area of study that aims to provide knowledge and skills to adults in various subjects. This field of education focuses on unique needs (Leverage Edu, 2022).

2.8.1.2.3 Higher Education (Universities/Universities of Applied Sciences)

In conventional universities, a bachelor's degree can be earned in three years, while a master's program takes two years. The UAS Bachelor's and UAS Master's degrees are given to students pursuing higher education at Finland's Universities of Applied Sciences. A major in university mathematics is often completed in five or six years in secondary school teacher education at the University of Helsinki and other Finnish universities (Aarrevaara & Pietilainen 2021; Krzywacki et al., 2016).

2.8.1.2.4 Local and National Curriculum

Finland's educational cycle lasts roughly ten years. The National Core Curriculum for Basic Education (NCCBE) was modified in 2014 to replace the one from 2004. It is expected to go into effect in 2016. Three leading causes are responsible for Finland's current national curriculum improvement:

- A description of general goals aligned with core civil values such as democracy, human rights, equality, and cultural diversity, and a discussion of 21st-century capabilities.
- Local government has the independence to provide and organise education so that the local curriculum becomes the instructional document at the local level.
- Adopting multiple methods of teaching (Korhonen & Lavonen, 2017).

These key factors aid instructors in creating a quality curriculum and study plans for their students.

2.8.1.2.5 University - school cooperation

Universities, schools, and businesses/industries collaborate under the supervision of the national LUMA Resource Centre, which the University of Helsinki administers. This collaboration fosters and advances mathematics, natural sciences, computer science, and technology education in Finland (Leverage Edu, 2022; OECD, 2019).

2.8.1.2.6 Themes Characterising Mathematics Education

This section highlights four universal themes that epitomise Finnish universities' pre-service teacher education programs and characterise the essence of mathematics education.

- Improving emotional qualities, such as having a positive mindset and an interest in mathematics, is crucial for understanding mathematics.
- The use of concrete resources and manipulatives to enhance students' understanding of mathematical concepts.
- Prioritising problem-solving creativity and critical thinking processes in preparing for teacher training.
- Recognising and providing maximum assistance to students with unique needs for learning mathematics and learning challenges (Krzywacki et al., 2016; Leverage Edu, 2022).

2.8.1.2.7 Principles of Finland's Education Success

- It is based on the tenets of "Lifelong Learning" and Free Education (from Pre-Primary to Higher) for Citizens of Finland and other EU/EEA nations.
- The emphasis on quality, equity efficiency, and internationalisation is the primary goal of the education policy.
- There are no standardised tests because teachers grade each student according to their developed method.
- The criteria for recruitment of teachers are very high, meaning that only individuals with master's degrees are eligible to apply for teaching positions. In addition, every teacher is given a personal principal to monitor their development.
- Emphasizes group work and learner collaboration to promote cooperation over competitiveness in the classroom (Krzywacki et al., 2016; Leverage Edu, 2022; OECD, 2019).

2.8.1.2.8 Twenty-Century Skills

According to Vivekanandan (2019), 21st-century skills can be defined as the re-evaluation of educational objectives and methods for arranging skills that are universally applicable to improve learning and working in response to the needs of the modern era. The key components of this process are critical thinking, the ability to recognise and comprehend the crucial function of mathematics to society as a whole, the ability to draw conclusions based on evidence, and the ability to engage with mathematics as a means to satisfy one's needs as a positive, caring, and responsible citizen. Table 2 lists the skills people in Finland need to influence their lives and direct them towards attaining their goals and the skills society needs to meet its current and future problems.

Table 2. 21st-century Competencies as Ways of Thinking and Working.

Competences needed in the 21st-century	**Examples of competencies**
Ways of thinking	Creative and critical thinking
	Use of knowledge and information interactively
	Learning to learn, use of metacognition
Ways of working	Communication, collaboration, and networking (teamwork in a heterogeneous group)
	Competence to act autonomously
	Identifying issues (questioning), arriving at conclusions based on information, explaining phenomena, and organising information
	Competence to use both creative and critical thinking in problem-solving and decision-making
	Use of ICT tools interactively
	Managing and resolving conflicts
Tools for working	Literacy: knowledge (network of concepts), nature of knowledge, and attitude (willingness to engage)
	ICT literacy
	Skills needed in inquiry and problem-solving
	Moral and ethical code
Context for working	Personal, citizenship
	Social, local
	Working life, career
	Global
Attitude needed for working.	Willingness to use knowledge (motivation)
	Self-efficacy

It can be challenging to select instructional strategies that can help students learn these competencies because of the students' wide range of backgrounds and academic abilities. Hence, employing several teaching strategies that enable students to develop their understanding by working together on practical activities with classmates is critical. In the twenty-first century, it is necessary to be able to grasp a wide range of valuable skills in learning, creativity, technology, and school-community collaboration (SCC) to contribute meaningfully to society as useful citizens, using the required job skills for life and business (OECD, 2019; Leverage Education, 2022). Students should be taught new skills that advance creativity and invention to prepare them to develop new occupations and goods that still need to be made.

2.8.1.3 Nigeria

The National University Commission (NUC) (2019) states that mathematics teacher education programs were specifically developed to provide mathematics education to pre-service, new and current teachers and empower them with the requisite subject matter knowledge of mathematics as well as the efficiency and drive needed for providing efficient teaching in the classroom.

The national policy on teacher education's objective for mathematics teacher education training seeks to achieve the following (NUC, 2019):

- It makes it possible for learners to learn the many mathematical ideas, rules, theories, and laws;
- empowering learners to learn the essential teaching techniques and other components of mathematics teaching methods;
- Assisting learners in becoming proficient classroom instructors;
- Develop professional teaching ethics;
- become a certified professional mathematics instructor;
- spread knowledge of mathematics across society;
- Acquire virtues and attitudes that will enable them to perform their job of teaching effectively.

The Nigerian mathematics teacher education training is focused on achieving the goals of the National Policy Programme of the National University Commission (NUC). However, the available data revealed that less than 50% of learners routinely passed mathematics and science exams. As interest in mathematics subjects declines, Nigeria has yet to reach the 60:40 enrolment target in science and technology compared to the arts and humanities (Federal Ministry of Education, 2018). For a teacher education program to succeed, Ntuli et al. (cited in Ibrahim et al., 2020) asserted that it must concentrate on the overall objectives of education and incorporate pedagogical knowledge and 21st-century skills that will enable shifts in mindset, competence, and capacity. This might give teachers the information and abilities necessary to impact learners' behaviour (Ibrahim et al., 2020).

2.8.1.3.1 Nigerian Mathematics Teachers' Professional Development

Certain institutions provide professional training for teachers to carry out the objectives of the NUC and develop qualified mathematics teachers. These include:

- **Universities' faculties and educational institutions:** These universities and institutes provide Bachelor of Education degree programs to prepare pre-service teachers and in-service teachers for careers as senior secondary school teachers. Additionally, they provide doctoral and master's degree programs in mathematics instruction.
- **Colleges of Education:** These institutions provide tertiary NCE training courses, which prepare mathematics instructors for use in elementary and junior secondary schools. Since 1998, the NCE has been the primary school teaching prerequisite.
- **The National Teachers' Institute (NTI):** This institute was founded to offer professional teachers opportunities for promotional courses and to acquire new knowledge related to their discipline through seminars, symposiums and training sessions and to create policies and programs designed to enhance the standard and scope of mathematics education throughout the nation.
- **Faculty of Education in Polytechnic:** Polytechnics have schools of education that provide certificate programs for the development of mathematics instructors for technical and vocational courses.
- **National Mathematical Centre and the National Institutes of Nigerian Languages:** For mathematics teachers currently in the classroom, the National Mathematical Centre and the National Institutes of Nigerian Languages arrange special training in subject-specific content and pedagogical techniques. In addition, they create educational resources for use in classrooms (Kyari et al., 2018).

Despite the stated goals of the training and the national policy objectives, it is challenging to find qualified secondary school mathematics teachers prepared to deliver high-quality instruction. Concerns have been raised about the calibre of recently graduating pre-service mathematics instructors (Ibrahim et al., 2020).

2.8.1.3.2 Disconnection between university and school

According to some scholars in education, the inability of teacher preparation institutions to link the university training curriculum with the school mathematics subject matter that pre-service teachers have been trained to teach was responsible for some school mathematics teachers' lack of subject matter expertise (Ibrahim et al., 2020). Society expects that pre-service teachers in mathematics will have good pedagogical content knowledge and competence, allowing them to impact their students and improve the standard of education (Ibrahim et al., 2020).

According to Adedeji (2018), the existing education system needs to be in an acceptable state regarding its outcome in terms of appeal, teachers' quality, subject mastery, professional competence, and pedagogical wit, among others.

2.8.1.3.3 Challenges in Nigerian Mathematics Education

Despite the intrinsic worth of mathematics, many problems appear to plague Nigerian mathematics education. As a result, learners need to improve on mathematics in the senior school certificate examination (SSCE) (Duru & Obasi, 2023). The following problems stand out among them, according to STAN (as cited in Kyari et al., 2018):

- Learners ' lack of interest in mathematics
- the severe shortage of experienced professional mathematics teachers
- and the overemphasis on covering mathematics syllabus rather than using them in real-world situations.
- The difficulty of curriculum and inadequate financing
- Insufficient mathematics laboratories and infrastructure at our educational institutions

Since the beginning of contemporary mathematics in Nigeria, the mathematics curriculum has been a contentious issue for teachers. They believed the curriculum was foreign and needed more capacity to meet Nigerians' demands adequately. This list of challenges does not foster an environment where mathematics education can flourish in this nation. It is necessary to improve mathematical education for mathematics to establish a solid foundation in our society. Simplifying the subject matter in Nigerian classrooms and using improved 21st-century methodologies could help attain this admirable objective (Sodangi & Adamu, 2023; Kyari et al., 2018).

2.8.1.3.4 Steps to Transforming Nigerian Schools into 21st-century Learning Classrooms

- **Digitalization**

Digital technologies must be allowed in classrooms and other school settings in Nigerian schools. The usage of ICT in education is a must because it is evolving into a natural aspect of human existence. It can be as easy as enabling internet access, allowing cell phones, and setting up digital hubs to enable students to access resources on a size and depth that no school library could ever match (Aliyu et al., 2021; Ibok et al., 2023).

- **Collaborative Learning Spaces**

Yang et al. (2022) describe 21st-century learning spaces as a shift from arranged rows facing the teacher to students working in groups and collaborating on projects. Nigerian teacher preparation institutions need to imbibe these concepts in planning schools to meet the 21^{st}-century needs of education. They must re-think classroom designs to encourage project-based learning and promote agility, continuous self-reconfiguration and good interactions between teachers and learner as well as between learners (Bature & Atweh, 2019; Yang et al., 2022)

- **Flexible Curriculum and Student Inclusion**

The curriculum is a dynamic instrument designed to enhance mathematics teaching in the twenty-first-century classroom. Teachers are expected to create an atmosphere where the curriculum may be implemented through their enthusiasm for creativity and problem-solving, critical thinking, and competency in using effective communication and ICT resources to build self-efficacy (Akudo & Eziuzo, 2023; Think Strategic Group, 2019).

The curriculum needs to be adaptable to suit various types of learners (Udosen, n.d.). A school must have a supportive atmosphere and provide time and other resources to provide all personnel with excellent professional training from the time they begin their careers. To ensure that the former is created so that what is taught at the university is stably integrated into the school's everyday practice, it must connect work-based learning and external learning (workshops or university courses).

Transforming our universities and schools into 21st-century learning classrooms is a task before the Nigerian government and all stakeholders if our educational system is to reveal to the learners the true nature of mathematical concepts as they apply to real life. For Nigeria to produce graduates and efficient mathematic teachers that can compete on a global scale and contribute to national development, there is an urgent need to rework our classrooms into learning spaces, our students into creators, and our teachers into good communicators and collaborators (Udosen, n.d.; Akudo & Eziuzo, 2023).

2.8.1.4 United States of America (USA)

2.8.1.4.1 States Have Responsibility for Education

In the United States, the first ten amendments to the Constitution, known as the Bill of Rights, were passed in 1789, which stated explicitly that the federal government would devolve numerous powers and duties about governance and civic welfare to the states: The ability to regulate schools and education was one of the powers granted to the states. This means that pre-service teacher training differs from state to state in terms of its features and organisational frameworks, depending on the state legislatures' chosen educational policies. The achievement gap between learners from less affluent backgrounds and those from more affluent backgrounds has been a critical concern for educators and politicians in the 21st century. These worries led to federal legislation that increased school funding to assist underprivileged learners.

2.8.1.4.2 Model and length of training in Texas, USA

Texas's most popular type of teacher preparation requires pre-service secondary teachers to complete a four-year bachelor's degree program. Learners take classes in a variety of academic fields throughout their first two years, with an emphasis on the area for which they will eventually receive certification. Learners apply for admission to the teacher training program at a college of education after their second year. After being accepted, students enrol in pedagogy courses for the following two years. Learners spend one semester taking classes and spending time in classrooms throughout the second of these two years. After that, students must complete a practicum during their final semester, commonly known as student teaching. In this model, learners attend one-year pedagogy courses in education after earning a bachelor's degree, in the case of learners pursuing their teacher certifications through a postbaccalaureate program. In contrast, a student teaching and supervision semester is required (Ries, 2016).

2.8.1.4.3 Special Situations and Alternative Paths for Teachers

Here are some unique circumstances and alternate routes enabling you to obtain a certificate or licence to teach temporarily without going through the rigours of getting a diploma. These include Private and Charter Schools, Accelerated Certification Programs, and Emergency Teaching Credentials (All Education Schools. Com).

2.8.1.4.4 Standards for Mathematical Practice

The United States has implemented Common Core State Standards for Mathematics to actualise focus and coherence in mathematics education in the twenty-first century.

Mathematics teachers at all levels should aim to help their students build various competencies, according to the Standards for Mathematical Practice in the USA. These procedures are supported by crucial "processes and proficiencies" that have long been well-established in mathematical instruction. The first is the NCTM process standards for problem-solving, justification and proof, communication, representation, and connections. The second is the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, practical, and worthwhile, coupled with a belief in diligence and one's efficacy) (Common Core State Standards Initiative, 2010).

The Standards establish grade-specific requirements but must outline the remediation strategies or resources required for learners who perform significantly less than or more than needed for the grade. The Common Core State Standards Initiative, 2010, provide the following benchmarks for teaching mathematics by the twenty-first-century competencies for mathematical practice:

- **Analyse issues and persist in finding solutions**

Effective mathematics teachers equip learners with twenty-first-century abilities to define a problem for themselves and search for potential solutions. Mathematics teachers should be able to prepare learners to think about related issues, try special situations, and take the original problem in simpler forms to obtain insight into how to solve it. Using natural objects or visuals to assist younger learners in understanding and solving a problem should be promoted (Common Core State Standards Initiative, 2010).

- **Think abstractly and quantitatively**

Instead of replicating mathematical procedures as standard practice in a twenty-first-century classroom, skilled and experienced mathematics teachers concede that the world is constantly evolving and that learners must be trained to understand quantities and their relationships in solving a mathematical problem. Such training should empower students to be able to decontextualise. In order words, they should be able to simplify an abstract problem, represent it symbolically, manipulate the symbols as if they have a life of their own, and contextualise to pause as necessary during the manipulation process to probe into the referents for the symbols involved—both of which are complementary skills that they bring to bear on problems involving quantitative relationships (Common Core State Standards Initiative, 2010)

- **Create convincing arguments and criticise the logic of others**

Teachers of mathematics in the twenty-first century prepare their students to comprehend and apply given assumptions and well-established results in argument construction. They formulate hypotheses and develop a logical chain of reasoning to test the integrity of their hypotheses. They can deconstruct issues into individual cases and can identify and apply counterexamples. They defend their conclusions, share them with others, and address counterarguments. Teachers are urged to let their students devise solutions to issues and then have them justify them (Common Core State Standards Initiative, 2010).

- **Use mathematical models**

According to Ferrazzo (2017), mathematical modelling is a method of teaching and learning that builds models to represent real-world scenarios and explain them. Teachers and learners are thus prepared to use what they know about mathematics to address issues that arise in society and work. This might be as easy as creating an addition equation representing a scenario in the early grades. In the middle grades, learners may use proportional reasoning to organise a school function or assess a social issue. A learner might solve problems involving abstract designs and shapes in high school. More so, students can use mathematical modelling to identify and analyse significant quantities in a real-world setting and map their relationships by using diagrams or graphs.

- **Use the proper equipment effectively**

Mathematics teachers prepared for the twenty-first-century classroom are taught to give high school learners the analytical abilities to interpret graphs of functions and solutions derived from a graphing calculator. They proactively employ estimating and other mathematical expertise to identify potential mistakes. They know technology may help them see the outcomes of different assumptions, investigate the implications, and contrast forecasts with facts while developing mathematical models. Pencil and paper, a protractor, a calculator, and a spreadsheet are suitable tools for teaching and learning mathematics effectively.

2.8.1.4.5 Challenges of Teaching Mathematics in the USA

In one of the most recent scores of an examination for international teens, the USA was placed ninth in reading and 31st in mathematics literacy out of 79 countries. The percentage of top-performing Mathematics pupils in America is lower than the global average, and mathematics test scores have virtually remained unchanged for the past 20 years (Richards, 2020; Balingit & Van Dam, 2019). One potential explanation is that rather than training students to think creatively about solving complicated problems, U.S. high schools frequently place more emphasis on formulae and processes. Because of this, it is more difficult for students to compete internationally, whether on an international exam or in universities and professions that place a high premium on complex reasoning and data science (Richards, 2020). Although mathematics teachers face various difficulties Ferlazzo (2017) argued that prerequisite skills, student mindsets, and resources are the three significant challenges confronting teachers in the classroom. According to Danielson and Powell (2016), Mathematics teachers in the 21st-century face three main problems: their attitudes about teaching and learning, their understanding of the subject and pedagogy, and the availability of time for reflection.

2.8.1.4.6 Strategies for Improvement

An increasing number of mathematicians are proposing ways to update the American curriculum to make it more comparable to what students learn in nations with higher test scores. Making mathematics more engaging, applicable, and inclusive can be achieved in the following ways:

- **Instilling the required knowledge**

Most mathematics teachers concur that learners need more prerequisite knowledge and skills for their mathematics classes. Most curricula assume that the student completed their last course the day before. What, then, can educators do? Teachers can provide the following possibilities for parents and students: 1) Outlining a set of required knowledge that students must master to begin the next course on the level. 2) Identifying learning materials and opportunities for parents and learners to practice such skills (Ferlazzo, 2017).

- **Teacher /learner mindsets and beliefs**

The mathematical mindsets of learners often remain unchanged. They have a misconception that learners are either born with mathematical prowess or are not. Teaching mathematics can become even more complicated when you add this locked mindset and the absence of required abilities. Teachers must first examine their attitudes toward mathematics to combat this. Pre-service teachers must approach mathematics with a growth mentality. Our classroom may become a mix of those who can and cannot if we have a fixed perspective, as learners will sense that (Ferlazzo, 2017).

- **Resources**

Implementing the Common Core Standards nationally has caused teachers to change how mathematics is taught. Common Core Math requires rigour, depth, coherence across classes and application to real-life circumstances. Thus far, no textbook is 100% aligned with the standards (Ferlazzo, 2017). With this knowledge, teachers must become seekers, Googling resources and specific activities for the standards. For example, EngageNY has lesson plans and videos that teachers can use as a starting point. NCTM Illuminations, Illustrative Mathematics, and Learn Zillion are just a few more sites with great resources (Ferlazzo, 2017).

- **Make more room for data science and computer-based math**

The world is at a point where things are changing, and we need to help learners navigate that new world. Estonian students ranked first among European countries in mathematics, reading, and science on the 2018 Programme for International Student Assessment. Many factors may have helped: The government offers high-quality early childhood education to all kids, class sizes are small, and there is little high-stakes testing, leaving more time for instruction. Unlike other countries, Estonia teaches computer programming at all grade levels – a strategy started in the upper grades in the late '90s and extended to elementary schools around 2012 (Richards, 2020). Similarly, Data Science courses were introduced in 2020 in the USA, and about 3,300 learners in 15 Southern California school districts took a new Introduction to Data Science course that featured data and statistics, real-life data collection and coding to analyse the data. These efforts must be sustained (Richards, 2020).

2.9 Theoretical framework

Effective mathematics instruction requires pre-service mathematics teachers to be familiar with appropriate learning theories that complement the qualities of their learners. That is to say, pre-service mathematics teachers can use learning theories as a guide to address issues in mathematics education and improve learners' achievement.

Definition of theory and theoretical framework

According to Varpio et al. (2019, p.9), "A theory is an abstract description of the relationships between concepts that help us to understand the world". Put succinctly, theories are developed to explain, predict, and comprehend phenomena as well as, to challenge and extend known knowledge within the bounds of crucial bounded assumptions or behavioural predictions (USC, 2020). The etymology of the word theory is traceable to the Latin phrase the½ria, which is derived from the Greek word the½ros which means spectator (noun) or speculative (verb). In other words, theory can be effectively applied to explain a phenomenon, draw connections, and make predictions. This ability to predict or speculate could help guide researchers in choosing appropriate research questions.

According to Grant & Osanloo (2014, cited in Ahmad, 2019), the theoretical framework serves as the dissertation's blueprint. It provides the plank on which the researcher will approach the dissertation as a whole from a philosophical, epistemological, methodological, and analytical standpoint. Essentially, the need for a scientific basis where all research findings are relevant, acceptable, and generalizable is one of the main goals of the theoretical framework (Adom et al., 2018, Cohen et al., 2018).

This section focuses on the constructivist theory that underpins this research study. It addresses the development of constructivist theory, drawing on the works of Piaget (as cited in Wadsworth, 1989) and Vygotsky (1978). The principles of these theories were discussed as well as their relevance to the research. The low level of mathematical literacy among educators coupled with the objectives of the New Secondary Mathematics Curriculum, as set forth by the Universal Basic Education Commission UBEC (2008, as cited in NERDC, 2012) motivated the decision to base this study on the works of Piaget (as cited in Wadsworth, 1989) and Vygotsky (1978), which have had an impact on the teaching and learning of mathematics. These objectives encourage the training and retraining of mathematics teachers to update their technological competence and acquire more teaching skills, which will enable both teachers and learners to overcome their classroom challenges and realise their full potential (Afolabi, 2015). In addition, the constructivist theory was chosen for this study because Piaget's theories of cognitive development highlight how the constructivist theory is closely related to the phenomena of preparing mathematics teachers for the 21st-century classroom.

A Brief History of Constructivism

The constructivist theory is a product of the fast-growing field of cognitive science. It draws its source mainly from the works of Jean Piaget, Lev Vygotsky's socio-historical work, and Jerome Bruner's constructivist explanation of discovery learning (Drew, 2023).

Jean Piaget

The original concept of constructivism was influenced by the educational psychology research of Jean Piaget (1896–1980). Piaget's research focuses on how individuals combine ideas and experiences to create meaning. The central idea of constructivism is that learners acquire knowledge by active participation as opposed to passive observation. Learners are expected to utilize their past knowledge in the learning environment where they must continuously reexamine their understanding. Piaget, articulated mechanisms by which learners internalise knowledge. According to Piaget (cited in Drew, 2023), there are six key components to learning:

- Learning through experience: Whenever we encounter a new experience, we process it in our minds. We will use that experience to understand our worlds. Piaget called learners 'lone scientists' who go out into the world and investigate to learn.
- Prior knowledge: Each new experience is compared to previous experiences. We will look at our new experiences and use them to understand what we are looking at
- Cognitive Schema: Piaget stated that a cognitive schema is a packet of knowledge in our minds. We can add to a cognitive schema (assimilation) or change it (accommodation).
- Assimilation: Piaget used this term to explain 'adding new knowledge' to our knowledge bank (cognitive schemata). I will place new knowledge into a cognitive schema if I encounter new knowledge to add to my knowledge bank. For example, if I see a new dog breed, I will recognise it as a dog to add it to my 'dog' cognitive schema.
- Cognitive disequilibrium: When we come across a new experience, it may contradict our prior knowledge. A learner will be confused and not understand what they can see. At this point, a learner is in a state of cognitive disequilibrium. We always want to be in a cognitive equilibrium where everything makes sense. Thus, the preservation of the status quo to prevent new ideas from conflicting with preexisting ones is known as equilibrium.
- Accommodation: The process of adjusting cognitive systems to a novel idea that contradicts preexisting mental models is known as accommodation. To overcome cognitive disequilibrium (confusion), we must 'fix' old or broken prior knowledge. We will need to recall a cognitive schema and repair it. For example, if I see a horse for the first time, I may think it is a dog because it has four legs. Then, someone will tell me it is a different animal called a 'horse'. Now, I need to fix my 'dog' schema by breaking

it up so I do not bank all horses into the dog schema. I might create two new schemata: one for dogs and one for horses.

The central idea of Piaget's theory is that a child's cognitive development develops in phases until it approaches adulthood. According to Piaget, there are four phases:

1. The sensorimotor phase, which lasts from birth to age two
2. The preoperational phase lasts two to seven years
3. The concrete-operational phase which lasts from seven to twelve years.
4. The formal operating phase that separates adolescents from adults. This is one of Piaget's core theories which states "Early intellectual development results primarily from the child's interactions with objects in the environment" Kuiper and BEP (as cited in Chand, 2023, p.276).

Lev Vygotsky

Lev Vygotsky (1896–1934) is another proponent of the constructivist theory, who advocated that we learn through social interaction (Drew, 2023; Main, 2021; Chand, 2023). Vygotsky and Piaget believed that experience is essential for our learning, but while Piaget thought of learners as lone scientists or individualistic, Vygotsky thought of learners as social beings.

By interacting with others, we can hear other perspectives and think versatilely. These social experiences are essential for helping us come to solid and logical conclusions (Drew, 2023). Zankov (1975, as cited in Guseva & Solomonovich, 2017), expounding on Vygotsky's view of constructivism, posited that if the learning process does not include overcoming the difficulties, i.e., does not involve any mental tension, students' development moves slowly. The teacher has to teach at the optimal level of difficulty because only teaching, which systematically supplies material for complex mental activities, stimulates the intensive development of children.

Vygotsky identified this process as the More Knowledgeable Other and the Zone of Proximal Development (ZPD). Vygotsky defines ZPD as the distance between the developmental level determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers (Akpan et al., 2020; Main, 2021). In other words, the ZPD is the difference between what a learner can do without help and what a learner can do with help.

Based on the analogy of Piaget and Vygotsky one can argue that constructivism in mathematics education is founded on a learning philosophy that knowledge is not passively received from the world or authoritative sources but constructed by individuals or groups reflecting and making sense of their experiential worlds (Taber, 2019; Oppong-Gyebi, et al., 2023; Nyamekye, et al., 2023).

2.9.1 The Constructivist Teaching Methods

2.9.1.1 The Zone of Proximal Development

The zone of proximal development helps mathematics teachers identify the target difficulty levels for their lessons. ZPD explains creating a lesson at the perfect difficulty level. This is sometimes referred to as the Goldilocks principle:

- A lesson that is too easy will not help students progress;
- A lesson that is too hard will not stimulate learning;
- A challenging but achievable lesson will help students progress (Drew, 2023; Main, 2021**).**

That skill level that's challenging but achievable is called the zone of proximal development. This approach involves getting a teacher to connect unlearned theories to familiar materials. Then, the pre-service mathematics teacher creates lessons just a step harder than the student's current abilities. The pre-service mathematics teacher must provide support as the student moves into the zone of proximal development. That is where scaffolding comes in.

2.9.2 Scaffolding

Scaffolding involves providing guided support to a student. Guidance is removed when a student's abilities improve. The term scaffolding in education is a metaphor emphasising that, like scaffolding in construction, it is a temporary structure removed upon attaining the teaching objectives (Drew, 2023). In the 21st-century mathematics classroom, scaffolds may include manipulatives, games, prompts, partial solutions and think-aloud modelling (Guseva & Solomonovich, 2017; Darling-Hammond et al., 2020). For example, the teacher may scaffold a multiplication problem by relating the problem to an activity that is familiar to the students, reviewing skills needed to solve the problem, providing tools for students to work with and offering support while allowing the students to find their solutions **(**Guseva & Solomonovich, 2017**).**

2.9.3 Problem and Inquiry-Based Learning

Problem-based learning is an approach where the pre-service mathematics teacher presents students with a problem. Learners then must address the problem by discussing things, conducting experiments, and going through a discovery process. Inquiry-based learning focuses on using systematic and scientific methods to collect data and come up with answers to problems. Both approaches (used in conjunction) help students learn through student-led and student-initiated discovery. Learners use their cognitive skills and trial-and-error to reach their conclusions and construct new knowledge in their minds (Drew, 2023; Akudo & Eziuzo, 2023).

2.9.4 Cooperative Learning

A cooperative learning approach will involve a teacher getting students to work in groups to complete a task. One way to do this is to get learners to work in groups (let them name their group) based on ability level or learning style. Another approach may be to get each learner to develop expertise in one specific topic area. Then, the students get together to share what they learned. In this model, every learner in the group is an expert on one aspect of the task. (Drew, 2021; Guseva & Solomonovich, 2017; Bature & Atweh, 2019).

2.9.5 Play-Based Learning

A play-based approach to learning involves getting students to discover new things through play. While often reserved for early childhood, play-based learning could be suitable for any age! Constructivism is about learning through exploration, interaction, discovery and thinking things through. That is precisely what we do when we play. We make mistakes, learn new ways to do something, think creatively, and gain new experiences (Drew, 2023).

Generally, constructivist mathematics teaching and learning are geared towards linking the formal mathematical theorems in the curriculum to have a bearing on the learners' intuitive understanding of materials and previous mathematical knowledge prevalent in the learners' culture and environment (Uddin, 2019; Smith & Chao, 2018). For example, pre-service teachers can use symmetries in African sculptures, such as the Benin bronzes and Yoruba Adire cloths in Nigeria, to introduce the teaching of geometry (Dele-Ajayi et al., 2019). In the same way, African games, puzzles, and natural things like a tree and the branching patterns involving mathematical principles can be used to trigger cognitive development in teaching mathematics. The preceding views contradict traditional classroom practice, as illustrated below.

2.9.6 Tenets of Constructivist Learning and its Difference from Traditional Learning

Main (2021), provides insight into constructivist learning and its difference from traditional approaches to learning.

Table 3. The Distinction between Constructivist and Traditional Learning Methods.

Traditional Classroom	Constructivist Classroom
Begins with parts of the whole emphasising basic skills.	It starts with the whole - and expands to parts.
Strict adherence to the fixed curriculum.	Pursuits of children's questions/ interests.
Textbooks and workbooks are used.	Primary sources/ manipulative materials are used.
The instructor gives/ students receive.	Learning is interaction-building on what students already know.
The instructor assumes a directive/ authoritative role.	The instructor interacts/ negotiates with students.
Assessment via testing/ correct answers.	Assessment via student works, observations, points of view, and tests. The process is as important as the product.
Knowledge is inert.	Knowledge is dynamic changes with experiences.
Students work individually.	Students work in groups.

In the constructivist classroom, the focus shifts from the teacher to the learners. In the constructivist model, the learners are urged to participate actively in their learning process. The mathematics teacher functions more as a facilitator who dialogues, mediates, prompts, and guides learners to develop and assess their understanding. In the constructivist classroom, teachers and learners think of knowledge not as inert factoids to be memorised but as a dynamic, ever-changing view of our world and the ability to successfully stretch and explore that view (Main, 2021; Oppong-Gyebi, et al., 2023).

There is no one constructivist theory of learning, but most constructivist theories agree on the following central ideas: Learners are active in constructing their knowledge; new learning is built on prior knowledge, the social interactions process makes it a useful learning theory for learning mathematics in context, and meaningful learning develops through "authentic" tasks (Taber, 2019; Main, 2021). Contrary to criticisms by some traditional teachers, constructivism does not dismiss the teacher's active role. Constructivism modifies that role, so pre-service teachers emphasise the learner and the learning process, focusing on problem-solving and how one arrives at a specific answer rather than reproducing a series of facts. The constructivist pre-service teacher is equipped to provide tools such as:

- Problem-solving and inquiry-based learning (IBL) activities with which learners formulate and test their ideas,
- Allow multiple interpretations and expressions of learning (multiple intelligences) and
- Encourage group work and use peers as resources (collaborative learning). (Bada & Olusegun, 2015).

Having outlined the tenets of the constructivist theory, it's critical to consider how the theory is put into practice in mathematics classrooms.

2.9.7. Implications for the Mathematics Classroom

According to Educational Broadcasting Corporation, (2019, as cited in Chand, 2023) in a constructivist mathematics classroom learning is: constructed, active reflective, collaborative, inquiry-based, and evolving.

Constructed: constructivism recognises that learners are not empty vessels onto which mathematical information is etched. They should be encouraged to bring their preexisting ideas, knowledge, and understanding to the learning situation. The new knowledge they will produce will be built upon this past knowledge.

Active: The learner is encouraged to experiment, ask questions, and try things that don't work, but the teacher coaches, moderates, and offers suggestions. Every learning activity (including practical experiments) requires the learners' active participation. Use a variety of symbols to help learners communicate effectively and clearly.

Communications: Use a variety of symbols and representations to help explain mathematical concepts to solve problems so learners can understand.

Reflective: Learners take the initiative to reflect on their experiences because they are responsible for their learning. They become experts at self-learning through this process. The mathematics instructor should also create exercises that inspire learners to raise questions think critically and consider their past learning and experiences. Talking about what was learned and how it was learned is crucial.

Collaborative: Learners' collaboration plays a major role in the constructivist mathematics classroom. The benefits of collaboration for learning are manifold. One of the main reasons constructivism is so popular is that learners learn about learning from both themselves and their peers. Learners can learn new tactics and techniques from one another when they review and reflect on their learning processes as a group.

Inquiry-based: The main goal of a constructivist mathematics classroom is problem-solving. Learners are encouraged to ask questions and use a range of resources to find the answers by using inquiry techniques. Learners make conclusions as they research a subject, which they then go back to review. The investigation of one question raises another.

Evolving: Learners may formulate theories they may later realise are unsound, inaccurate, or insufficient to fully account for novel experiences. These ideas represent transitory stages in the knowledge integration process. For instance, a child might think that all trees lose their leaves in the fall until she comes across an evergreen forest. The constructivist approach to teaching and learning mathematics builds on the learner's prior knowledge while also taking it into account.

The implication of constructivist theory can be summed up as follows: instruction must direct each learner toward making the adjustments and assimilations necessary to regain equilibrium and learning of mathematics can be enhanced through social interaction.

2.9.8. Application of constructivism in this study

The 21st century's challenges for mathematics teachers require a shift from the traditional methods of teaching abstract mathematics to a more modern approach that entails teachers facilitating the learning process, getting learners involved in classroom activities, developing an inquiry-based mindset, encouraging learners to develop their own mental model and synthesis of knowledge (Chand, 2023; Uddin, 2019; Szabo et al., 2020).

The researcher considers the contributions of Piaget's adaptation process of learning which involves assimilation, accommodation and equilibrium important in this study because they have greatly improved the teaching and learning process in schools, by placing the learner at the centre of the learning process. According to Piaget's assimilation theory, learners are not empty canvases waiting to be filled with knowledge. They bring preexisting concepts, knowledge, and understandings to learning situations (Drew, 2023). Given the theory of assimilation, the researcher will look at how participants evaluate learners' past knowledge to identify misconceptions and provide context for their lessons. The assimilation process would provide information about how well-prepared mathematics teachers are for their classroom roles. To engage learners in a practical, hands-on activity that proves Pythagoras' theory for instance, participants will demonstrate how they used the learners' prior knowledge and the teaching materials (e.g., learners can cut and paste paper, build a right-angled triangle, and take basic measurements). The lesson plan and delivery by the participants will show how a creative mathematics lesson can break down an ostensibly complex mathematical theory such that the learners' comprehension is confirmed and that assimilation has occurred.

According to Chand (2023) and Drew (2023) learners are seen as active knowledge builders who actively engage in mathematics education and develop their comprehension of mathematical concepts through their experiences. Thus, this theoretical framework will enable the researcher to evaluate the participants' feedback regarding the use of the Piaget accommodation process to determine whether or not instructors design tasks that encourage learners to be active, experiment, and reconstruct new knowledge to overcome cognitive disequilibrium (confusion).

The research participants' responses will show the degree to which they used problem-based learning, creative thinking, and teamwork to inspire learners. To bridge the existing gap between the new topic and learners' prior knowledge, research participants must provide appropriate reinforcement to address cognitive confusion that arises when switching from two similar but different topics, such as simple interest to compound interest. To form the proper schemata and facilitate successful assimilation, learners are guided to experiment and provide solutions during the reinforcement process. This allows for the gradual achievement of the accommodation process, which ultimately leads to equilibration.

Furthermore, by using the equilibrium theory, the study will provide evidence of how well participants use problem-based exercises to push learners to think critically and creatively, interact with real-world issues, and solve problems rather than giving learners familiar questions that promote memorization**.** The framework will also help in providing evidence regarding the degree to which Vygotsky social constructivist theory impacted the training of the research participants.

The works of Bada & Olusegun (2015) and Drew (2023) provide us with an insight into some of the benefits and critique of Constructivism; they include:

2.9.9 Benefits of the constructivist theory include:

- Learners are considered capable learners and encouraged to exercise creative, critical and independent thinking.
- Educators recognise that students require targeted, differentiated lessons matching their cognitive needs and ability levels based on age.
- Constructivism gives students ownership of what they learn since learning is based on learners ' questions and explorations.
- Constructivism promotes social and communication skills by creating a classroom environment emphasising collaboration and exchanging ideas.

2.9.9.1 Critiques of the constructivist theory include:

- Learning through trial and error is a time-consuming process. In the age of a crowded curriculum, teachers often do not have the time to organise sustained problem-based learning lessons.
- International testing regimes that push standardised curricula encourage conformity and memorisation over inquiry-based critical thinking.
- Constructivism requires differentiation so that students learn at the optimal cognitive level. Differentiation for each child is challenging and often impractical for educators.

2.9.10 Chapter Summary

The chapter reviewed the relevant literature on preparing mathematics teachers for the 21st-century classroom. The chapter begins by reviewing the literature on the pre-colonial mathematics school curriculum, the development of mathematics school curriculum in Nigeria, the 21st-century school curriculum, teacher preparation and professional development, etc. The chapter also discussed constructivism as the theoretical framework underpinning the study. It helps students to develop 21st-century skills such as collaboration, cooperation and creativity. Central to this theory is learning by 'mulling over' new ideas and coming to our conclusions through logic and reasoning. Learners must engage in active learning by doing and personal experiences to achieve this learning. The chapter also discussed the theoretical framework of the study. The next chapter (Chapter 3) discusses the research methods that underpin the study.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

The previous Chapter focused on situating this study in the broader perspective of existing research, with a comprehensive literature review on preparing mathematics teachers for the 21st-century classroom and clarifying the theoretical framework underpinning the study. This chapter articulates the research design and methodology used to fulfil the objectives and research questions mentioned in the first chapter. The chapter deliberated on the following: research paradigm (critical paradigm), research approach (qualitative), and research design (case study). It highlights how the participants were sampled and the instruments used to generate and analyse the data. The ethical considerations observed and limiting factors to the study conclude the chapter. A chapter summary was included, providing information on what has been discussed in the entire chapter and what will follow in Chapter Four.

3.2 Research Paradigm

There is a consensus among researchers that a research paradigm is defined as a set of principles and coherent assumptions, pursuing what problems are to be investigated and how and proffering solutions that are generally acceptable (MacMillian, 2021; Marshall, et al., 2021). Within a paradigm, it is specified what kind of questions should shape the researcher's thinking. What can be observed and investigated? How do you generate well-informed data? Moreover, how to interpret the findings (Maree, 2020; Creswell & Creswell, 2018). This proves that the research paradigm plays a crucial role in determining the choice of data generation and interpretation.

Cohen et al. (2018) and Maree (2020) posit that it is essential that every research work explicitly define the research paradigm that is most appropriate to achieve its set objectives. It is assumed that each paradigm expresses a particular idea about the nature of truth. Several research paradigms are grouped according to their ideological standpoint on ontology, epistemology, methodology, and axiology, which help shape the researcher's actions and thinking patterns (Creswell & Creswell, 2018; Bertram & Christiansen, 2019). That is to say, an essential characteristic of a paradigm is its incommensurability with each other.

According to Lather (cited in Cohen et al., 2018), four paradigms guide research work in education: prediction (positivism), understanding (interpretive approaches), emancipatory (critical theoretical approaches), and deconstruction (post-structuralist). Denzin and Lincoln

(2018) proposed four classic examples of paradigmatic claims: positivism, post-positivism, critical theory, constructivism and participatory. This shows the controversy on the paradigms that underpin research in education. The critical paradigm was deemed most appropriate for exploring pre-service mathematics teachers' preparation for teaching in the 21st-century classroom because the critical paradigm in education is described as an emancipatory paradigm that is used in studies that aim to empower and provide a fairer education to all learners (Cohen et al., 2018; Ankomah, 2020).

The Critical paradigm is an analysis of society intent on understanding how society is structured so that injustices and structural inequalities are understood as created and sustained by robust systems. This paradigm differs from others because it seeks praxis as the overarching goal. Praxis is the combination of theory and action. Rather than seeking to understand society or the educational system, the critical paradigm actively seeks to change them positively (Fleming, 2018). Mathematics is traditionally seen as the most neutral of disciplines and detached from politics and social life controversy. The critical paradigm in mathematics education opposes these assumptions of neutrality. It proposes that history and society have shaped mathematics in terms of its applications and moulding its concepts and methods (Skovsmose, 2020).

As an alternative to traditional mathematics, proponents of critical mathematics education contend that the values of shared dialogue and empowerment of the learner are essential dimensions of teaching mathematics if it is to contribute toward social justice (Skovsmose, 2020; Smith & Chao, 2018). Research has shown that school mathematics serves as a 'gatekeeper' in enabling society to select individuals because mathematical ability is a valued source of human capital. What is more of a concern in this process to stakeholders is the issue of achievement gaps (between rich and poor), which arguably is said to reproduce further social and educational inequalities (Uddin, 2019). On this strength, the critical paradigm in mathematics aims to encourage delivering high-quality mathematics education to all learners in non-discriminatory ways within universities and schools, which ought to be considered centres of social transformation (Avei, 2020; Uddin, 2019).

The need to integrate critical mathematics education in the 21st-century classroom is an excellent way to discuss how an alternative for future education might work to reverse some of the features of negative capitalist education, such as students experiencing learning Western mathematics as enculturation that focuses on the individual-competitive mathematics exam.

The system produces 'losers' and 'dropouts" who accept their failure as objective qualities of themselves rather than reinforcing acculturation in which students selectively modify some currently held ideas due to a positive influence from the mathematics taught in the classroom (Aikenhead, 2021).

We can observe within the critical paradigm the equivalent of many of the demands of an educational reformist. For example, we see pedagogies that aim to foster mathematical tasks that have the potential to provide intellectual challenges, rejection of school mathematics that promotes social screening to maintain a privileged status quo, teaching for understanding rather than for grades, an emphasis on inquiry-based collaboration rather than individually competitive learning; encouraging learners to assess and evaluate what they see and read, and redefining the teaching and learning of mathematics from the vestiges of Eurocentric models to more meaningful mathematics connected to the learner's local environment where their cultural identities form and evolve (Mosimege, 2020; Smith & Chao, 2018; Aikenhead, 2021).

The critical paradigm teaches that knowledge is power. This means that the researcher creates an effort to get inside the participants to appreciate them from within regarding their reflective knowledge in evaluating applications of mathematics (as it relates to a real-life context). Thus, rather than the mere recording of observation by the researcher, he empowers participants by giving them a voice, and participants' understanding of the ways one is oppressed (in terms of inequity or any other bias) sensitises them to take action to change. This investigative component and critically informed research in practice can resist dogma and make this paradigm reformative (Aikenhead, 2021; Skovsmose, 2020).

3.3 Research Approach

Researchers use three research approaches to facilitate inquiry and give direction to a research study, namely qualitative, quantitative and mixed-method research approaches (Creswell & Creswell, 2018; Creswell & Poth, 2018). For Maree (2020), the unique feature of the qualitative approach is that it relies more on linguistic and meaning-based words rather than statistical forms of data analysis. Marshall et al. (2021) strongly recommend qualitative research as a genre becoming an increasingly important mode of inquiry for the social sciences and fields such as education. In this research study, I have used the qualitative research approach to solicit participants' opinions in a way that allows for the generation of rich non-numerical data from the perspective of a small number of participants to contribute to a more general understanding of how pre-service mathematics teachers are currently being trained to teach in the 21st-century classroom (Creswell & Poth, 2018; McCombes, 2019).

Historically, qualitative researchers cited two significant purposes of a study: to 'describe' and 'explore' and to 'describe' and 'explain' (Maree, 2020). This infers that the qualitative approach chosen for this study was appropriate. It allowed the researcher to explore pre-service mathematics teachers' preparation for teaching. According to MacMillian (2021**),** qualitative research is naturalistic; it highlights subjective social values and attempts to study the everyday life of different groups and communities in their natural settings and is mainly convenient to give voices to participants and investigate issues that lie beneath the surface of educational behaviours and processes. The study was carried out in a naturalistic setting in a university, with the researcher attempting to make sense of the phenomena of how pre-service mathematics teachers were currently being trained to teach in the 21st-century classroom in terms of the meanings the participants brought to bear through their responses to the research questions during the interviews (Creswell & Creswell, 2018; Cohen et al., 2018).

The researcher in this study preferred a qualitative research approach because it allowed for multiple perspectives in a short period, permitted flexibility in questions and probes, and encouraged a critical and dialogic approach that provided greater insight and detailed information concerning the research questions. Qualitative data was obtained through the administration of a questionnaire and a semi-structured interview, which enabled the researcher to present a narrative of the research setting (Maree, 2020; MacMillian, 2021).

Cohen et al. (2018) lament that the qualitative research approach is expensive and requires a skilled facilitator. The above limitation did not limit my study. I transcribed the data on my own, which helped reduce cost, and in so doing, I was able to unravel recurring categories and patterns of concepts used to analyse the data. The qualitative approach allowed the researcher to ensure that the data were without bias. The researcher had to overcome the problem of stripping out the context to obtain themes without destroying the heart of the qualitative approach (Creswell & Guetterman, 2019). These procedures significantly augmented the deficit of not utilising numerical research.

3.4 Research Design

A research design or strategy is a plan to answer a set of questions within which research is conducted (McCombes, 2019). The research design describes how the researcher systematically investigates the central problem and influences the type of data to be gathered, analysed and interpreted (Creswell et al., 2018). The case study research design was adopted to achieve the research objectives of this study. Yin (2018) argued that a case study design provides an in-depth analysis of one or more real-life entities—social groups, individuals, or other bounded systems in their natural context.

The case study applies to this study because it focused on a specific phenomenon, which was, in this case, to explore pre-service mathematics teachers' preparation for teaching in the 21st-century classroom at a university in Nigeria (Yin, 2018; MacMillian, 2021). Case study design can be used in studying any phenomenon and multiple cases (Maree, 2020). The unit of analysis is the focus of the study upon which a researcher makes inferences based on evidence and reasoning at the end of the study. My unit of analysis was the preservice teachers from whom I obtained answers to the research questions. The observation unit understood how pre-service mathematics teachers were being trained to teach in the 21st-century classroom. Research questions played a critical role in determining this study's analysis unit, while the data generation methods determined the unit of observation.

There are several types of case studies. The several types can be determined by their purposes. Cohen et al. (2018) identified three types in terms of outcomes: descriptive, exploratory and explanatory. The descriptive case study provides a narrative account of a phenomenon in a research study and the contextual factors in which the phenomena occurred (Yin, 2018; Cohen et al., 2018). Exploratory case study research is a methodological approach that investigates research questions that have not previously been studied in-depth or are not clearly understood and could probably lead to further research (Marshall et al., 2021)**.** This study is an exploratory case study to explore pre-service mathematics teachers. Cohen et al. (2018) described explanatory case study design as testing theories explaining how or why a particular phenomenon occurs with limited data. It may generate operational definitions to aid a better research model, increase researchers' understanding of a given topic, verify the existing theory and predict future occurrences.

In addition, case study design can also be classified according to the number of cases involved. Yin (cited in Cohen et al., 2018) identifies four main case study designs: I) The single-case study can focus on a critical case, an extreme case, or a unique case, where only one phenomenal setting is considered. II) The embedded single-case study incorporates more than one unit of analysis (may also use subunits); III) The multiple-case study explores real-life multiple bounded cases using in-depth multiple data generation methods. IV) The embedded multiple-case study utilises sub-units in each of the different cases, and a range of instruments might be used for each sub-unit, and each is kept separate from each case. This study was a single case study as only the pre-service mathematics teachers who are currently being trained to teach at one research site were used to find answers to the research questions.

According to Cohen et al. (2018), a case study methodology has many strengths and weaknesses for the researcher and contextual settings. Table 4 below provides a concise insight into the strengths and weaknesses of a case study.

Table 4: Strengths and Weaknesses of the Case Study.

S/N	Strengths	Weaknesses
1	The results are more easily understood by a broad audience (including non-technical academics) as they are frequently written in everyday, non-technical language.	The results may only be generalisable where other readers/researchers see their application.
2	They are immediately intelligible; they speak for themselves	They are not readily open to cross-checking; hence, they may be selective, biased, personal and subjective
3	They catch unique features that may otherwise be lost in larger-scale data (e.g., surveys); these unique features might hold the key to understanding the situation.	They are prone to observer bias problems despite attempts to address reflexivity.
4	They are firm on reality.	
5	They provide insights into other similar situations and cases, thereby assisting the interpretation of similar cases.	
6	A single researcher can undertake them without needing an entire research team.	
7	They can embrace and build in unanticipated events and uncontrolled variables.	

The case study uses unique features that may otherwise be lost in large-scale data. These unique features promoted collaboration between the researcher and the participants, which helped the researcher generate rich data within natural settings about the unit of study. The case study design also provides insights into similar cases, allowing the researcher to understand the research problem more fully and recommend practical solutions for solving it.

Paradoxically, case study data are vital in reality but challenging to organise. This strength, in reality, enabled the researcher to be more concerned with practical issues related to the research questions while seeking a balance with theories. As a result, the researcher was able to captivate the participants' attention as they shared their experiences, thus providing a natural basis for more subjective evaluation and analytic generalisation (Yin, 2018; MacMillian, 2021). The study began at 2:00 p.m. - 3:00 p.m. daily for a week, an extensive period during which I could generate data from multiple sources**.** These provided in-depth information on the research questions, making the data intelligible as they spoke for themselves**.** Though the study period was extensive, it was undertaken by a single researcher without needing an entire research team.

Critics of the case study are cynical that case studies lack reliability and external validity, with which generalisability is frequently associated (Marshall et al., 2021). This implies that the conclusions drawn from a particular case may not be transferable to other settings (Mcleod, 2019). For this study, the selection of one university to explore pre-service mathematics teachers' preparation for teaching was not representative of all universities, for the aim of the study was not to generalise my findings to a greater populace. Also, case studies are difficult to replicate; therefore, the extent to which re-study and obtaining consistent results across studies is made feasible cannot easily be validated. Given that case studies are based on the analysis of qualitative data, there is possibly a tendency for observer bias and the subjective judgment of the researcher may interfere with the assessment of what the data truly represents (Mcleod, 2019; Marshall et al., 2021).

Yin (2018) posited that utilising case studies can be associated with possible weaknesses. However, he averred that these weaknesses could be surmounted if the research is well thought out. He proposed that researchers utilising case study design should report honestly without bias, as case studies are not conducted to generalise results but to seek particularisation. In this study, I abided by trustworthiness to ensure all explanations were supported by evidence from the field.

3.5 Target Population/Sampling

Target population

The target population for this study were the 130 registered 4th-year B.Ed. Mathematics pre-service teachers registered for mathematics degrees at a public university.

Sampling

Engaging in a sampling process requires the researcher to follow some basic steps for proper sampling. First, complete the first two steps in Figure 7 to find the target population and identify the sample frame.

Basic steps on how to sample properly

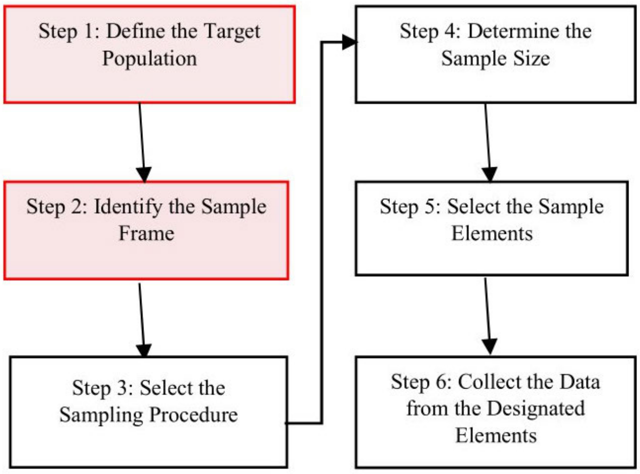

Figure 7: Basic guidelines for appropriate sampling, modified from Baker (2020).

The phrase population is often used in the context of sampling. The population is the entire group intended to be covered by the current study. The sample is the set of units selected to represent the population of interest (Baker, 2020; Zou et al., 2023). Baker (2020), describes the sample as a subset of the group that should be representative of the entire group, whereas sampling is defined as the procedure utilised in choosing a particular portion of the whole population to generate data from the selected portion (Marshall et al., 2021).

Step 1. A sampling procedure is to define the target population**.** The target population for this study were the 130 registered 4th-year B.Ed. Mathematics prospective students registered for a mathematics degree at a public university.

Step 2. This requires the researcher to define the appropriate target population and identify the sampling frame. The sampling frame is the literal physical tool used to draw the sample. Examples in education research include lists of learners' email addresses, maps from which areas will be sampled and lists of pre-service teachers. The sampling for this study consists of an administrative list of 4th-year registered mathematics pre-service teachers at the JSS form 3 level collected from the head of the mathematics department. To avoid issues related to the errors of omission and inclusion in the sampling, the researcher cross-checked the list of pre-service mathematics teachers collected from the head of the mathematics department with the list provided by other administrators to ensure the sample frame was drawn from only 4th-year pre-service mathematics teachers, who were registered and active (Baker, 2020; Lohr, 2021).

Step 3. This requires the researcher to select the sampling procedures. There are two fundamental procedures for selecting a sample from a given population: probability sampling and non-probability sampling as shown in Table 5.

Table 5. The distinction between Probability Sampling and Non-Probability Sampling.

Probability Sampling	**Non-Probability Sampling**
• Sample selected using a random method. • Mainly used in quantitative research	The sample was selected in the non-random method. They are used in qualitative and quantitative analysis.

Probability sampling focuses on the higher population, and equal chances are given to select a sample using a random method. Probability sampling helps ensure that the researcher's sampling is representative and unbiased. With this sample type, the researcher can use generalisation statistics (Maree, 2020; Cohen et al., 2018). The non-probability sampling or purposive sampling technique uses non-randomized methods to draw the sample, and participants are selected because they are convenient and easy to achieve. Still, they have more risk of bias (Maree, 2020).

Cohen et al. (2018) and MacMillian (2021**),** expound that purposive sampling gives a researcher the privilege to handpick which participants or cases are to be selected for the study based on unique qualities sought by the demands of the study. The researcher, therefore, chose to employ purposive sampling techniques for this research study since the study's main focus was on the richness of data rather than on working towards generalizability (Cohen et al., 2018)—pre-service mathematics teachers at B.Ed. Level 4 was selected, believing they had more experience with the teacher training process.

Step 4. Sample size denotes the number of participants included in a study **(**Maree, 2020**).** As stated above, the target population was 130 4th-year B.Ed. Mathematics pre-service teachers, based on this number, a sample size of six (6) candidates training for the Junior Secondary School JSS form 3 (grade 9) level was initially targeted. Still, out of this number, only 6 participants met the requirements and fully participated. Purposeful sampling was utilised to ensure that pre-service teachers were best suited to help the researcher find a rich resource to understand the problem and the research question. The sample size comprises four males and two females to represent both genders.

Step 5. The sampling element is the unit of analysis or case in a population (Cohen et al., 2018). The selected sample element to achieve the richness of data was in two categories—first, six pre-service teachers at B.Ed. Level 4 was set as a single case, believing they had undergone the teacher training process. Second, I requested that participants be drawn from a mixture of academically achieving pre-service mathematics teachers who had achieved 70% and above and those performing below the average of 40%. This process helped to provide a balanced view regarding how participants had been trained and the challenges confronting them. The sample was drawn with the assistance of the head of the mathematics department and representatives of the National Association of Nigerian Students (NANS) in a teacher training faculty at a university.

Inclusion and exclusion selection criteria.

It could be argued that a researcher would have a sampling frame that perfectly overlaps with his population of interest in a perfect world, but this is hardly the case. Instead, there may be inconsistencies between what a researcher may have in a sampling frame and what the population of interest comprises. This leads to errors of inclusion and omission in the selection criteria (Baker, 2020; Lohr, 2021).

The errors of omission occur when members of the population of interest are included in the list so that they have no chance of being sampled. Although a non-probability sampling was used, which deals with a minimal number of investigation units, caution was taken to check errors. To deal with the errors of omission, the researcher cross-checked the list of pre-service mathematics teachers with the Head of the Mathematics Department and administrators to ascertain if errors were relevant enough to warrant executing a new sample frame. In this scenario, there were no errors found that could impact the findings and conclusion.

Errors of inclusion are people in the sampling frame who are supposed to be absent (Baker, 2020; Lohr, 2021). To avoid issues related to inclusion errors, the researcher ensured that a purposeful sample was used, allowing a limited number of participants to be used. More so, the sample frame was drawn from only 4th-year pre-service mathematics teachers who are registered and active.

Finding and recruiting participants

Recruitment is generally a dialogue between the researcher and the prospective participant (through paper-based or online announcements, media communication or face-to-face interaction). It is a prelude to the informed consent process (Manohar et al., 2018). The purposive sampling method was intentionally used to recruit participants because it allowed me to choose which category of pre-service mathematics teachers would be included in the sample (Bertram & Christiansen, 2019). Thus, the participants recruited for this study were the 4th-year group of pre-service mathematics teachers. The recruitment process involved identifying and enlisting prospective participants through a list provided by the Head of the mathematics department and collaborated by the administrative staff, which closely represented the target population and met the sample size. I requested that participants be drawn from both genders, including a mixture of academically achieving pre-service mathematics teachers who had achieved 70% and above and those performing below the average of 40%.

A detailed poster about the study was created and posted in the mathematics lab and classrooms to create awareness. Pre-service teachers who showed interest were then contacted via email and telephone follow-up. Upon contact, I shared the focus and purpose of the study to establish their interest in the proposed study. I allowed prospective participants to raise concerns about their discomforts if any (Manohar et al., 2018). Eligible participants were contacted again for a preliminary meeting, where I presented detailed information, including ethical considerations about the study and formally asked them to participate. Upon agreeing to participate, participants were asked to sign consent forms to formalise their readiness to participate in the study (Creswell & Guetterman, 2019; McCombes, 2019).

Regarding logistics, I facilitated participants' involvement by scheduling the interview venue at a location that is easily accessible to participants and close to public transport. I also thought it expedient to use individuals with whom participants can relate (e.g., those with the same ethnic/social background as the sample size, using an insider approach) (Creswell & Poth, 2018). Hence, the assistance of the National Association of Nigerian Students (NANS) representatives was used as a backup to mobilise participants. These enabled participants to freely communicate without fear in their language and rhythm of expression, thereby improving recruitment by establishing a good rapport and building trust (Manohar et al., 2018).

3.6 Geographical Location of the Study

Many factors contributed to the criteria for identifying the most suitable location for the research study. The choice of an institution that offered Mathematics for undergraduate students was a prerequisite. The institution had to have 21st-century facilities such as the internet, computers, projectors, and a mathematics lab available on-site. Although internet facilities are available at the institution, the number of computers and other mathematical accessories must be increased to serve all pre-service mathematics teachers. This research study was conducted in an institution located in one of the densely populated Local Government Areas of Lagos State, Nigeria. It is a public tertiary institution comprising students from different multiethnic backgrounds. The research setting has a landmass of 180sq km, and about 30% of it can be described as a riverine as shown in Figure 8.

Figure 8: An illustration of the Nigeria map showing Lagos State, where the research site is located.

Lagos is a state located in the southwestern part of Nigeria; it is portrayed with a red spot on the Nigerian map in Figure 7. Lagos is the smallest state among Nigeria's 36 states, with a current metro area population of about 15,945,912, in 2033 which has increased by 558,273 over the past year 2022, indicating an annual change of 3.63% (World Population Review, 2023). Lagos was the former capital of Nigeria and arguably had the country's highest internally generated revenue (IGR). In terms of industrial investment, it accounts for over 70% of Nigeria's industrial and commercial establishments, as well as international seaports and airports. In addition, it has 70% of the nation's banking institutions and 90% of foreign nations' foreign embassies. Currently, Lagos contributes 25% of Nigeria's GDP. Lagos State is bounded on the north and east by Ogun State. In the west, it shares borders with the Republic of Benin. Its southern borders are with the Atlantic Ocean. 22% of its 3,577 km2 are Lagoons and creeks (World Population Review, 2023).

3.7 Data Generation Methods

Creswell and Guetterman (2019**),** describe a research method as "a way of finding empirical data of the world and notes action research, ethnography, case study research, and grounded theory as examples of qualitative research methods. Choosing such a research method in a qualitative study influences how the researcher generates data. The data generation steps include setting the boundaries for the study through sampling and recruitment, generating information through interviews and observations, documents, and visual materials, and establishing the protocol for recording information (Creswell & Creswell, 2018; Goldkuhl, 2019).

This study utilised an open-ended questionnaire and semi-structured interviews to elicit the information required to answer the research questions. According to Cohen et al. (2018, p. 475), "an open-ended question is used to elicit an honest, personal comment from participants". The open-ended questionnaire enabled participants to write a free account in their own words, reveal their thoughts, provide answers within their frame of reference and narrate the reasons for their responses (Cohen et al., 2018). The semi-structured interview is a data generation method initiated by the researcher who has prepared questions, allowing room for continuous probing or follow-up questions after the participants have answered a question (Cohen et al., 2018).

3.7.1 Open-ended Questionnaire

According to Cohen et al. (2018, p. 475), "an open-ended question is used to elicit an honest, personal comment from participants". Open-ended responses may contain the 'gems' of information the questionnaire might miss. More so, it puts the responsibility for and ownership of the data into the participants' hands. The responses to questions can be used as a process of naturalistic inquiry that seeks in-depth information regarding the research problem. Whereas closed-ended questions elicit yes/no quantitative responses, participants may feel constrained by the restricted number of options (Creswell & Creswell, 2018). In this study, an open-ended questionnaire was used to generate data and their feedback was immediately retrieved to minimise bias (Creswell & Poth, 2018). The open-ended questionnaire empowered participants to narrate their thoughts freely, provide answers within their frame of reference and explain the reasons for their responses (Cohen et al., 2018).

3.7.2 Semi-structured interviews

According to Cohen et al. (2018, p. 475), a semi-structured interview has "a clear structure, sequence, focus, but the format is open-ended, which enables the respondents to reply in their own words." A semi-structured interview was used to generate data because it allowed for flexibility and a balance between the main question, follow-ups, and probes (Creswell & Poth, 2018). During the interviews, a digital recorder was used to record the discussions. This enhanced the dialogue and enabled the researcher to pay full attention to the interviewees with steady eye contact to motivate them to provide adequate feedback. The use of a digital recorder also facilitated easy transcription and analysis of the data. Field notes were taken to record the vocal but non-verbal communication elements during the interview, such as facial expressions, gestures, sighs, tone and stress of speech and other paralinguistic cues (Mashall et al., 2021).

Creswell and Poth (2018) cautioned that data generated from semi-structured interviews might be biased. I addressed this critique by reiterating the aim of the research to the participants and imploring them to be honest in their feedback. Integrating an open-ended questionnaire and a semi-structured interview helped check the occurrence of biased and deceptive data.

3.7.3 The Semi-Structured Interview Protocol

In qualitative research, the researcher is the research-gathering instrument (Creswell & Poth, 2018; Maree, 2020). The researcher used a written interview protocol to provide the order in which the research instrument was applied. The interview protocol included a header comprising spaces to record the interviewee's name, date, location, and background information (Creswell & Poth, 2018). The header included an introduction read to the interviewee, clarifying the purpose of the study, what will be done with the data to protect the anonymity of the interviewees and how long the interview will last (Marshall et al., 2021). The consent form was given to the interviewees to sign once it was obvious that they understood the terms by which the interview was to be conducted (Maree, 2020; McCombes, 2019).

The questions were presented to allow the interviewee maximum flexibility to respond. For example, the first question sought to relax the interviewee and create rapport to motivate them to participate. The questions were easy to understand, which helped to stimulate the interviewees' ability to reflect on experiences they could discuss (Creswell & Poth, 2018). The questions were constructed carefully to inspire an inquiry-based conversation. The interview protocol allowed the interviewer to anticipate logical gaps in the data generation and close them (Creswell & Poth, 2018; Cohen et al., 2018). The interview protocol was tested on three members of the selected participants before the primary interview (Marshall et al., 2021). The communication medium was English because all participants commonly understood it. Table 6 shows the data generation plan for this study.

Table 6. The Data Generation Plan

The Data Production Plan Questions	First Research Question	Second Research Question	Third Research Question
Research questions	What are the training experiences of pre-service mathematics teachers currently being trained to teach in the 21st century?	What are the existing challenges pre-service mathematics teachers experience when they learn to teach or translate theory into practice?	How do these pre-service teachers intend to develop greater competence as teachers of the 21st century?
Why is the data being generated?	To explore how pre-service mathematics teachers are being trained to teach in 21st-century classrooms.	Critically analyse their training experience to identify the existing challenges between theory and professional practice.	To establish how their training and experience can be used to develop greater competence as teachers of the 21st century.
What are the data generation methods?	Open-ended questionnaire/ Semi-structured interview. (Participants were allowed to choose their preferred method during the interview .)	Open-ended questionnaire/ Semi-structured interview. (Participants were allowed to choose their preferred method during the interview .)	Open-ended questionnaire/ Semi-structured interview. (Participants were allowed to choose their preferred method during the interview .)
Who (What) were the sources of data?	Six pre-service mathematics teachers were teaching JSS3 (grade 9): 4 males and two females. Participants were selected from the 130 registered 4th-year B.Ed. Mathematics pre-service teachers.	Six pre-service mathematics teachers were teaching JSS3 (grade 9): 4 males and two females. Participants were selected from the 130 registered 4th-year B.Ed. Mathematics pre-service teachers.	Six pre-service mathematics teachers were teaching JSS3 (grade 9): 4 males and two females. Participants were selected from the 130 registered 4th-year B.Ed. Mathematics pre-service teachers.
Sound reasons for this plan to fit data generation.	The open-ended questionnaire empowered participants to freely provide in-depth information in their own words to research questions. The semi-structured interviews allowed for flexibility and a balance between the main questions, follow-ups, and probes. These two methods enabled me to triangulate and understand how pre-service mathematics teachers are currently being trained to teach in the 21st-century classroom. A digital recorder was used to transcribe the participants' responses to ensure the study's validity.	The open-ended questionnaire empowered participants to freely provide in-depth information in their own words to research questions. The semi-structured interviews allowed for flexibility and a balance between the main questions, follow-ups, and probes. These two methods enabled me to triangulate and understand how pre-service mathematics teachers are currently being trained to teach in the 21st-century classroom. A digital recorder was used to transcribe the participants' responses to ensure the study's validity.	The open-ended questionnaire empowered participants to freely provide in-depth information in their own words to research questions. The semi-structured interviews allowed for flexibility and a balance between the main questions, follow-ups, and probes. These two methods enabled me to triangulate and understand how pre-service mathematics teachers are currently being trained to teach in the 21st-century classroom. A digital recorder was used to transcribe the participants' responses to ensure the study's validity.

3.8 Data Analysis

Creswell and Creswell (2018) describe data analysis, especially in qualitative studies, as a researcher's intent to make sense of text and images generated by participants. For Maree (2020)**,** data analysis is a systematic set of procedures that a researcher indulges in after generating information, which splits a chunk of data into smaller segments to make meanings. There is no single way to analyse and present qualitative data; how one does the fitness of the purpose determines it. Since this is qualitative, data analysis presupposes identifying and interpreting patterns and themes in non-numerical words rather than statistics (Cohen et al., 2018). Content analysis was used to analyse data. Content analysis is defined as the systematic reduction of content, analysed with particular attention to the context in which it was created, to identify themes and extract meaningful interpretations of the data (Roller, 2019).

The intention of qualitative content analysis, argue Gläser and Laudel (as cited in Cohen, 2018), is to deliberately move from the original text to analysis of the information extracted from it, focusing on the meanings and the reduction of texts into summary form, through the use of pre-existing categories and emergent themes to generate or test a theory. Essentially, I used content analysis that utilised inductive reasoning. Inductive analysis means categories were not predetermined but occurred when interacting with the actual generated data (Maree, 2020;). This process can be achieved manually or as enabled by computer-assisted analysis. In this study, data analysis was carried out simultaneously during the data generation process and the write-up of findings (Creswell & Creswell, 2018). The researchers adopted a manual approach to analyse the data. The researcher adopted only five of the seven essential steps as recommended by Creswell (2009, 2014). An overview of the data analysis process is shown in Figure 9.

Data Analysis in Qualitative Research.

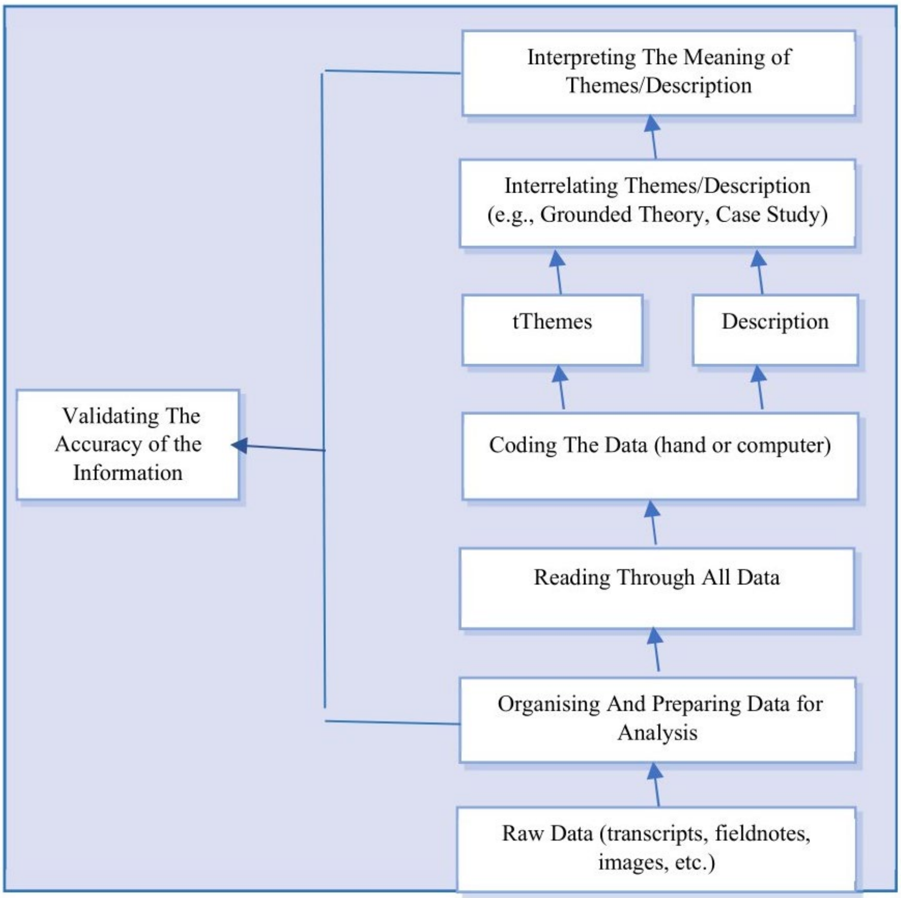

Figure 9: A brief overview of the procedure for data analysis modified from Creswell (2014:247).

Step 1. Organise and prepare the data for analysis. Data generated from the open-ended questionnaire and semi-structured interviews were first transcribed. This involved listening to each participant's full audio recording, transcribing the taped interview verbatim through an audio-to-text converter and typing field notes into digital text. Transcription notation symbols and comments were used to capture non-transcribable text to gain as much of the complete picture as possible (Creswell & Poth, 2018). The researcher also catalogued all of the research materials by sorting the data into different types depending on the sources of information.

Step 2. Reading through all the data, this step enabled me to understand what each participant was saying, reflected on the underlying principle that informed their idea, and how rich and credible the information they provided was. My impression of the content of the transcribed text provided me with the impetus to revise and make developmental edits (by recording my thoughts and deleting errors). This idea is collaborated by Creswell (2014), who asserted that qualitative researchers write notes and make sketches in the margins of transcripts.

Step 3. Start coding all of the data. First, while revising the transcript, I used a highlighter pen to mark salient features and recurring phrases. I wrote notes in the margins to recognise parts of the text containing relevant information for answering the research questions (What are the training experiences of pre-service mathematics teachers currently being trained to teach in the 21st century?). Second, I inscribed labels on sticky pads to generate an index of terms, which helped me connect with the relevant experience of the participants and determine which category of file the labelled information belongs to (create categories about the data). The coding process was appropriate because it helped to answer the question of why a piece of particular information is relevant.

The coding process for the field notes and transcripts consisted of three steps described by Creswell (2014): open coding, axial coding and selective coding.

- Open coding involves labelling concepts, breaking down and naming data segments to generate meaning from the transcripts about the research questions. The open coding procedure was on terminology, context, uniformity, rate of occurrences, specificity and extensiveness of comments. The segments of meaning from the field notes and transcripts were marked (highlighted) and labelled descriptively.
- Axial coding was done by critically reflecting on my thoughts on the initial coding. During reflection, I noted the interrelationship between emerging codes and adapted some ideas, which helped me generate more coherent information. Categories and patterns were identified during this step and organised in terms of causality, context and coherence.
- Selective coding was done by carefully scanning all codes and categories identified for comparison and relationship to the research questions and for a central theme or critical linkage that occurred and validating those relationships. The codes were eventually evaluated, while related codes were listed in categories according to their relevance to the research aims and theoretical framework.

Step 4. Use the coding process to generate a description and themes for analysis. This stage was about refocusing by merging codes into some order or group to generate an overarching theme. I created a long list of codes by grouping those codes that were similar to derive themes that encapsulated all of those codes. The next step was for me to consider how those themes may interact or interlink and how they may have different levels of hierarchy and order. A detailed description of multiple perspectives from participants was supported by diverse quotations and specific evidence from the data, as discussed in Chapter Four.

Step 5. The final step in data analysis involves interpreting qualitative research findings or results. Asking what lessons were learned captures the essence of this idea (Creswell, 2014). Data interpretation focused on the meaning the researchers derived from comparing the findings with information from literature and theories.

3.9 Trustworthiness

The trustworthiness of any study is very significant because it assists in demonstrating the validity and reliability of that study (Korstjens & Moser, 2018). Validity is one of the strengths of qualitative research, which conveys the procedures taken during the investigation and determines whether the research findings are credible and transferable from the perspective of the researcher, the participant, or the readers. Qualitative reliability denotes how the researcher's approach is consistent across different researchers and among different projects and convinces readers that its findings are ethically conducted and can be trusted (Creswell & Creswell, 2018; Maree, 2020).

For Cohen et al. (2018), trustworthiness in a research study can be established by generating rich data, rigorous, long-term participation, participant validation, and triangulation. Researchers utilise triangulation to establish the validity of their studies. Crewell and Creswell (2018) described triangulation as referring to the generation of data by a researcher using a diversity of data generation methods. In this study, trustworthiness was guaranteed by multiple data generation methods, namely open-ended questionnaires and semi-structured one-on-one interviews. Yin (2018) and Creswell and Poth (2018) identify four trustworthiness indicators that apply to qualitative studies: dependability, confirmability, credibility, and transferability. These indicators were considered, which helped me gather and analyse data.

3.9.1 Dependability

Dependability refers to the consistency and reliability of the research findings and the degree to which research procedures are documented, allowing the reader to follow, audit, and critique the research process (Lincoln et al., 2018). Creswell and Guetterman (2019) describe dependability as the degree to which a research study is likely to produce similar results if the study is carried out again in the same context using the same methods. Dependability in qualitative research focuses on data stability over time and conditions. In this study, the researcher appraised the process of formulating the research problem and its questions, the details of data generation, and the data analysis and theory generation decisions, which were documented for dependability.

MacMillian (2021) argued that if a researcher wants to achieve dependability for a study, the researcher should go back to the participants to check if what was recorded in the field is correct. Thus, I took the transcribed data to the six participating pre-service teachers to authenticate if what I transcribed reflected their opinions. According to Erickson (2018), dependability involves transparently providing the correct information. Hence, I quoted verbatim from the six participating pre-service teachers to buttress transparency and evade being biased.

3.9.2 Confirmability

Creswell (2014) describes confirmability as the steps a researcher takes to ensure that the data presented represents the correct information generated from the participants. Confirmability ensures that interpretations of data are not in any way an invention of the researcher. To ensure confirmability for this study, I ensured that the research findings resulted from the participants' experiences by recording such responses verbatim (Maree, 2020; Denzin & Lincoln, 2018). To tackle the issue of biases, I used an independent peer reviewer to reappraise the inferences I made. I also used member checking by sending the findings to participants to confirm if I captured their experiences correctly. I used probing questions to go into greater depth where a response was unclear, incomplete or raised significant issues that had not been explored before (Denzin & Lincoln, 2018; MacMillian, 2021)

This study enhanced confirmability by safely keeping the data used for interpretation to enable other interested parties to access the exact data for inspection and confirmation. I provided a clear and detailed research methodology so the reader could decipher how adequate and meaningful the data and findings emerged from the study. Critical to this process, I kept the audit trail, which enabled me to trace the procedures followed in data generation, data analysis and discussion of findings (McCombes, 2019).

3.9.3 Credibility

Credibility, a parallel to internal validity under quantitative design, is concerned with the researcher verifying the accuracy of the results of qualitative research (Creswell & Guetterman, 2019). It is the most crucial aspect in establishing trustworthiness as it links the findings with reality. Credibility can be strengthened by the use of triangulation (Creswell & Creswell, 2018; Roller, 2019). In this study, triangulation was strengthened through multiple data generation sources, such as an open-ended questionnaire and a semi-structured interview, which led to one credible, reliable and diverse source of construction of meanings (McMillan, 2021). To maintain credibility, interviews were audio-recorded and personally transcribed verbatim. Participants were allowed to read the transcripts to validate accuracy, which helped minimise the researcher's subjectivity (Denzin & Lincoln, 2018; Creswell & Guetterman, 2019).

3.9.4 Transferability

Creswell and Poth (2018) argued that transferability is the extent to which results from a research study can be applied to research in another context. For a researcher to ensure transferability in a study, thick descriptions of context and research methods should be provided, as well as evidence of raw data to allow readers of the study to consider the applicability of the results to their research contexts. I enhanced transferability by providing a thick description of the context of the study, with a comprehensive explanation of the findings, which made the results richer and more realistic (Creswell & Poth, 2018). In addition, the two data generation methods were well described with direct quotations from the participants. As this was a qualitative study, no substantive generalisations were expected, but the focus was on exploring how pre-service mathematics teachers were currently being trained to teach in the 21^{st}-century classroom rather than a replication of the same to the broader population.

3.10 Ethical Issues

To some extent, human science is regulated by principles, standards of behaviour or societal values when conducting research (Morse, 2018; McCombes, 2019). Research ethical committees govern how research is performed at most research institutions, such as universities, and how findings are to be published in a way that protects the rights and dignity of human and animal participants and the environment (Marshall et al., 2021).

Research ethical considerations are essential for the following reasons: they hold researchers accountable for their actions; they uphold community moral values and standards; they build public trust in research work, give meaning to collaborative work through mutual respect and fairness, ensure informed consent is obtained, providing confidentiality and anonymity and preventing harm to participants (Marshall et al., 2021; Cohen et al., 2018; Maree, 2020), The University of KwaZulu-Natal's Humanities and Social Sciences Research's Ethics Committee approved the ethical clearance application. When the preservice teachers were recruited, an informed consent letter was provided to the participating volunteers. Consent to audio-tape interviews was duly obtained from each participant. The consent forms and matters of harm, adverse effects and personal gain explained voluntary participation in the study. It was only after an agreement was signed that participants were included in the study. The names of organisations and participants remained anonymous throughout the study to ensure that the data was kept confidential**.** The data concerning each participant was kept on documents with password protection. The data was not kept on online platforms to ensure it would not leak. The audio of the participants was also kept on a password-protected computer. Participants were numbered in the order of the data that was generated. I intend to keep the data for five years after the research. To destroy it, I plan to contact my supervisor and the institution where it will be stored for safekeeping and assistance. After five years, data will be incinerated if it is in paper form, and all electronic gadgets will be reset to factory settings.

3.11 Limitation of the study

Research has shown that only a study is complete with flaws and inclusive of all aspects. Logistic and time management were limitations regarding meeting appointments using public transport to the research setting because public transport could have used more time. For example, I had to wait about 30 to 1 hour before the BRT commuter Bus was filled. I overcame this obstacle by hiring an Uber or Bolt taxi until I finished my studies. While an Uber safes time, it was, however, more expensive.

I tackled the issue of time management by scheduling appointments at a time convenient for the participants, particularly after their regular classes. While it was convenient to interview participants after classes, I was confronted with the long traffic associated with Lagos roads after office hours. The use of an Uber enabled me to use alternative routes with ease.

The interviews intimidated the participants, and others felt uncomfortable being voice-recorded. I overcame this by allowing the participants to provide feedback freely without my interference. I guaranteed participants anonymity and that their responses were only used for research purposes, which were not linked to evaluating their academic work. I assured them that the findings would be kept safe with my supervisor at the university safe house, that when writing my research, I would use pseudonyms and that they should feel free to respond to the questions in the language they are comfortable with**.** Hence, they wrote and communicated freely when responding to questions. This proves that they were not under pressure to please me.

A further limitation was that the study used a small sample size because only one university in Nigeria was included. Consequently, the results needed to be more generalisable to Nigeria's broader population of universities. The small-scale study is mainly criticised by scholars who argue that case studies, being idiographic, have limited generalisability, and their findings may be subjective (Yin, 2018; Denzin & Lincoln, 2018).

3.12 Chapter Summary

Chapter 3 has outlined the research paradigm, research design and methodology. It highlighted how the participants were sampled, including the sample size, the instruments used to generate data, and its analysis. Trustworthiness (dependability, conformability, credibility, and transferability) and ethical considerations were also presented as how they were observed during the research process and the limiting factors to the study concluded the chapter. The subsequent chapter (Chapter Four) focused on data presentation and analysis using the open-ended questionnaire and semi-structured interviews that were reviewed. The data analysis also utilised the literature review and the theoretical framework presented in the preceding chapter.

CHAPTER 4: INTERPRETATION AND ANALYSIS OF DATA

4.1 INTRODUCTION

The study utilised two methods to generate data namely the semi-structured interview and the open-ended interview. A total of six pre-service mathematics teachers were selected from a university to participate in the interviews. This chapter focuses on interpreting and analysing the findings from the semi-structured interview which was one of the primary data generation methods used to probe the responses by engaging participants in a more detailed conversation. The details of the criteria for the selection of participants were discussed. The data revealed that several practices influenced pre-service teachers' preparation for teaching in the 21st-century classroom, which include pre-service mathematics teacher lesson planning, classroom discussion, evaluation, and the 4Cs. All the data were read, categorised, and analysed with content analysis. The interpretation of the analysis of results from the open-ended questionnaires, was, however, discussed in Chapter 5.

4.2 Data Presentation and Analysis

This section provides a general insight into how participants were selected. The criterion for selecting participants began with determining the sample population for this study, which was 130 4th-year B.Ed. Mathematics pre-service teachers. Based on this number, a sample size of twelve (12) candidates training for the Junior Secondary School JSS form 3 (grade 9) level were initially targeted to be selected amongst the academically achieving pre-service mathematics teachers who had achieved 70% and above and those performing below the average of 40%, but only 6 participants met the requirements and fully participated. Purposeful sampling was utilised to select participants to help the researcher find a rich resource to understand the problem and the research question.

In this chapter, the sample comprises two males who volunteered to participate in the semi-structured interview.

4.3 Semi-Structured Interview

This section analyses the data presented by the two participants engaged in this phase of the semi-structured interview schedule. The semi-structured interview schedule was the first instrument to be administered. Upon commencement of the interview, the participants and I began a casual conversation regarding several practices influencing pre-service teachers' preparation for teaching. This provided a leeway for me to formally discuss preparing mathematics teachers for the 21st-century classroom. It was an opportunity for the participants to gain confidence and gather their thoughts regarding the research topic.

This conversation led to the first question for discussion.

4.3.1 Preservice Teachers' Teaching Practice Experience

The semi-structured interview analysis revealed that the two participants who engaged in this interview phase used teaching strategies/activities to help learners process information and understand the lesson. For example, Participant 1 indicated he used various teaching strategies, like classroom discussion and evaluation, to encourage learners' participation in the teaching and learning process. Participant 6 stated that he used concrete and visual representations to facilitate teaching so learners are actively involved in solving real-world problems.

The findings showed that participants 1 and 6 recognised the importance of using appropriate teaching strategies to actualise their learning objectives. Participant 1's (P1's) response implied that he identified classroom discussion and interaction as an effective teaching strategy that permits learners to exchange ideas and learn from one another. Participant 6 (P6) thought that using concrete and visual representations in teaching mathematics helped provide clarity so that learners could derive better meaning and understanding of the mathematical concept. The data also revealed the type of instructional strategies both participants utilised. Not only were the teaching strategies stated, but they alluded to the fact that they used them in a way that learners were actively involved. The use of classroom discussion and evaluation by P1 and the use of concrete and visual representation by P6 aligns with the constructivist theoretical framework in which learning is enhanced when learners are active in constructing their own knowledge and meaningful learning develops through "authentic" discussions and tasks (Taber, 2019; Main, 2021). In order words, one can assume that the teaching strategies of both participants emphasise learners' active role in creating their mental representation and building their understanding.

4.3.2 Feeling Prepared to Begin the Task of Teaching

Participants were asked to indicate their responses to questions relating to their feelings on being prepared to begin the task of teaching. Participant 1 stated that he was ready to begin the task of teaching because he has learned to use multiple teaching strategies, including classroom discussion (teacher-learner discussion and learner-learner discussion) and evaluations like direct questions and answers, a project or group tasks that engage students with hands-on experience to connect theory in the classroom to real-world situations and providing appropriate feedback. Participant 6 explained that he was prepared to begin the task of teaching because he learned to utilise concrete objects and visual representations like cylinders, pictures, diagrams, and charts to teach, which enables learners to ask questions and explore real-world problems.

The data analysis revealed that both participants believed they felt prepared to begin the teaching task because they had learned to use relevant teaching strategies that could assist them in improving their teachings, such as using classroom discussion and concrete objects and visual representations. The similarity in both participants' responses was that the classroom discussion and concrete and visual representation adopted by participants were a learner-centred approach, which aligns with the constructivist theory that says learners construct their knowledge by reflecting and making sense of their experiential worlds rather than just taking in passive information (Taber, 2019; Main, 2021).

4.3.3 Undergraduate Programme Preparation for Teaching

4.3.3.1 Lesson planning

Participants were asked if their B.Ed. The degree programme has helped them in their preparation to teach in school. The data analysis shows that the participants spoke more about the importance of lesson planning than their experience while enacting their lesson plan. For example, Participant 1 indicated that the lesson planning helped him to select topics and determine his instructional objectives and the appropriate teaching aids, procedures and evaluation instruments to use to interact with learners in the classroom effectively. However, Participant 6 explained that the lesson planning helped him to outline beforehand the lesson objectives and the most appropriate teaching aids, procedures, learning activities, assessment and feedback mechanisms to be used to achieve the lesson objectives effectively.

Both participants acknowledge that lesson planning has helped them know which teaching aids would be needed so that they are collected before the class. They also shared a common perspective that lesson planning guides what learning activities and evaluation strategies would be used to facilitate the teaching. Participants were silent on their experiences during and after the lesson plan's enactment. This setback did not allow for a thorough analysis of the effectiveness of the lesson plan in the classroom.

4.3.3.2 Feedback

Participants acknowledged feedback as an essential classroom practice that helped them prepare to teach in school. This view was supported by Participant 1, who stated that one of the classroom practices that enabled him to improve his teaching was to use both one-on-one and group feedback to support learners in their areas of need and consider their criticism so he could adjust his method of instruction and the materials used accordingly. Similarly, Participant 6 averred that feedback played an essential role in his training. It enabled him to provide learners with adequate advice based on their classroom activities, take-home assignments and tests to determine if the lesson should be reviewed before moving on.

The analysis of the above responses indicated that both participants agreed that feedback provided learners with accurate information concerning what they understand and can do and areas in which they need improvement to develop their capacity. Given this scenario, I argue that feedback was a form of scaffolding participants used to correct errors and reinforce learning, which is one of the constructivist teaching methods propounded by Vygotsky. In addition, feedback could also be used to support learning, which serves as a catalyst for assimilation and equilibration in the learning process. This implies that if feedback is effectively provided by the pre-service teacher and utilised by the learner**,** it will enable the learner to improve their subsequent performance. It is my view that participants' knowledge about the feedback aspect of the assessment was thorough since their feedback provided information to the learners regarding their strengths and weaknesses and what they are expected to do next to help them improve their learning.

4.3.3.3 Participants' Most Interesting Experiences During Training

4.3.3.3.1 Paper Folding and pictorial representation

What the participants considered the most exciting experiences during their training formed the basis for this analysis. All participants identified the use of mathematical representation as an essential experience during their teachings. However, the type of representation they used differed. For example, Participant 1 explained that one of his most exciting experiences was using cardboard paper to prove Pythagoras' theorem and other mathematical concepts. Participant 6 indicated that one of his most meaningful experiences was using concrete objects, pictures, diagrams, and drawings to illustrate mathematical concepts and procedures.

The above responses showed that both participants valued the use of experiential learning to develop their learners' knowledge and skills. Participants' use of experiential learning is a constructivist learning strategy because it involves learning by doing, which enables learners to interpret the new concept within the framework of their prior knowledge, which Piaget referred to as assimilation. On the other hand, experiential learning may also allow the learner to modify their cognitive schemas to accommodate new information and experiences, which Piaget referred to as accommodation. The participants demonstrated that they considered the learners active in the educational process by engaging them with the abovementioned activities.

4.3.3.3.2 Peer learning

Participants also indicated that peer learning was meaningful in their teaching practice. Participant 1 stated that he used peer learning to help learners consolidate their knowledge by allowing them to exchange their ideas in group work. This view was collaborated by Participant 6, who explained that one of the most important experiences during his teaching practice was to peer learners in group work to share their insight and reasoning, which helped to boost their confidence.

Participants' responses denote that sharing ideas amongst learners helps consolidate their knowledge and boost their confidence. Peer learning facilitates cognitive equilibrium during the learning process to achieve mental balance, where learners can adapt to the new knowledge (see 4.5). However, it must be said that participants needed to provide more evidence of peer learning in their class activities. Still, on a few occasions, they indicated that two learners were called upon to answer questions on the board for the benefit of their classmates. Both participants showed that group task consumes much time; hence, they rarely utilise them.

4.3.3.4 Participants' Classroom Activities

As earlier indicated, both Participants 1 and 6 seemed to acknowledge the importance of using mathematical representation as classroom activities to help their learners construct ideas and make meaning. However, the type of classroom activities they choose varies from each other. For example, Participant 1 engaged learners with classroom activities to solve mathematical problems using paper folding and cutting to prove the Pythagoras theorem; Participant 6 preferred a combination of concrete objects like cylinders, rulers, string, and diagrams to illustrate the circumference and diameter of circles.

In the above responses, one can infer that participants' use of concrete materials, diagrams and charts makes it easier for the learners to grasp complex abstract concepts and enables them to learn new ideas and relate them to what they have already learned. This aligns with Uddin

(2019) and Smith and Chao (2018), who argued that constructivist mathematics teaching and learning is geared towards linking the formal mathematical theorems in the curriculum to have a bearing with the learners' intuitive understanding of materials and previous mathematical knowledge, prevalent in the learners' culture and environment.

4.3.3.5 Description of Participants' Involvement

The participants' specific role in promoting classroom activities formed the basis for this analysis. In this section, all participants provided opportunities for learners to experiment with their hands and to learn by doing. For example, Participant 1 demonstrated the proof of the Pythagoras theorem during the interview using paper cutting and pasting, as illustrated below.

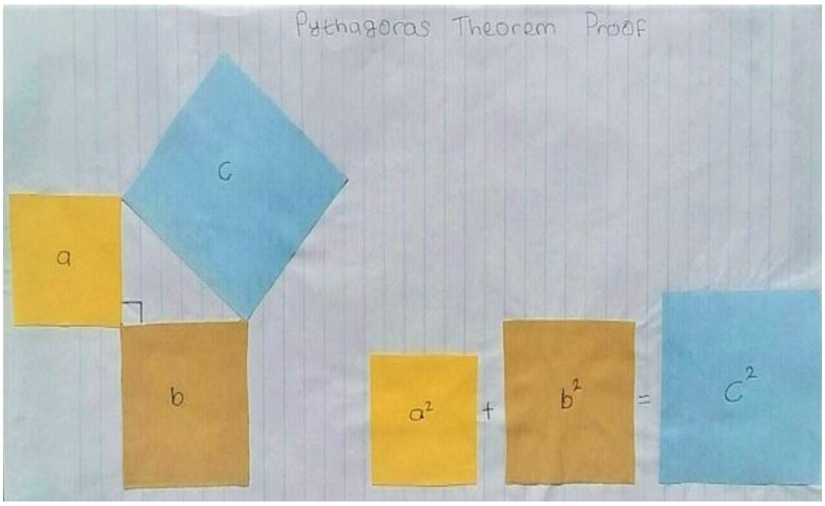

Figure 10: Proof of Pythagoras' theorem using paper cutting and pasting.

Participant 1 began his classroom activities by presenting the relevant teaching materials to prove the Pythagoras theorem. With an illustration, he directed the learners to cut out three squares from cardboard paper; each square must be larger than the other size. Learners were advised to name each of the squares in order of size: A, B and C. Then, they were directed to stack the three squared papers on top of one another in the following order: Take square A and place the vertex horizontally on square B; next, take square C and place the vertices such that each vertex of the square C is on the vertices of the squares A and B. Hence, a right-angled triangle has been constructed, with the hypotenuse of the right-angled triangle at square C, the adjacent side at square B and the opposite side at square A. This proves the Pythagoras theorem that a squared equals b and c squared. He encouraged each learner to ask questions and take turns replicating the processes while walking around to verify learners' understanding and misconceptions.

Participant 6 indicated that learners were asked to come to class with a round-shaped object, ruler, scissors, and strings. The first thing he did was to choose a cylinder object and wrap the string around the object. He picked the string and used it as a measuring tool. After that, he used the ruler to measure the length of the string. The length of the string should be the circumference of the object. Next, he also took a ruler to measure the diameter of the cylinder object. Once he had demonstrated the process of obtaining the circumference and diameter of the cylinder, he divided the students into groups. He asked the learners to practice the exercise with the materials they had come with. The participant engaged learners with a short exercise: 1) he drew a blank circle on the board. 2) he asked two volunteers to come to the board and label the circumference and diameter.

A glance at participant responses showed that both participants acknowledged using mathematical representation in their classroom activities. At the same time, one of the participants opted for using cardboard paper folding and cutting to explain Pythagoras' theorem that a squared equals b and c squared. The other participant chose a combination of the measurement of string and drawing of diagrams on the board to explain the circumference and diameter of a circle. More so, both participants encouraged their learners to seek a better understanding of their lesson through questions and answers. The participants' area of divergence was that one was more focused on helping learners work independently to solve problems. In contrast, the other participant demonstrated an inclination for group work/cooperative learning and the use of diagrams to aid his explanation.

Keeping in line with the constructivist theoretical framework, it is worth noting that the teaching strategies of both participants were learner-centred because participants served as facilitators while learners were actively involved in the learning process. The summation of participants' responses indicated that the classroom activities were done with suitable teaching strategies like tangible materials, group work and open discussion so that the mathematical concepts are easily fitted into the learners' knowledge bank for the process of assimilation to be successful.

4.3.3.6 Appreciation of Classroom Activities

The data analysis verifying participants' appreciation of their classroom activities showed that participants appreciated students' engagement in their classroom activities because it made the concepts easier to understand and allowed learners to take ownership of the learning process. To support this finding, the participant shared the following sentiments: Participant 1 explained that he was satisfied because the paper folding activity explained the Pythagoras theorem that a right-angled triangle's hypotenuse has a square area equal to the sum of the square regions of the other two sides which can be simplified through a creative mathematics lesson. In the same vein, Participant 6 explained that he appreciated his participation in the classroom activities because it enabled learners to take ownership of the learning process through their active participation in classroom activities.

Considering participants' responses, I noticed they did not dominate the lesson process, which encouraged learners to be actively involved in the learning process. However, none of the participants provided evidence of their learners' performance in these classroom activities, which would have helped to show the extent to which their learning objectives were achieved ipso facto, justifying their claim for appreciating their classroom activities.

4.3.3.7 Significant Aspects of Participants' Classroom Activities

Participants in this study noted that they found it significant to adopt experiential learning in their classrooms. However, the method they choose for their experiential learning tends to differ. Participants expressed the following views in this regard: P1 explained that he found the learners using their hands to fold and cut papers into different sizes to prove the Pythagoras theorem significant because it helped him to clear learners' misconceptions and empowered them to learn by doing and reflecting on their experiences. P6 stated that he asked two volunteers to come to the board and label the circle's circumference**.** I consider this significant because it allows learners to take ownership of the learning and see mistakes as catalysts to improve their understanding.

The Participants' accounts implied that hands-on activities made their teachings understandable while learning by doing enabled the learners to make sense of their experiences and think analytically about the outcome. The act of asking volunteers to answer questions on the board implied that learners were motivated, actively engaged, and provided with a sense of ownership in the learning process. This encapsulates the constructivist theory that says that knowledge is constructed by individuals or groups reflecting and making sense of their experiential worlds (Taber, 2019; Main, 2021). The findings also revealed that while one of the participants considered the use of concrete materials as a significant aspect of his teaching, the other participant thought it was also significant to expand his teaching strategies/activities to include the use of concrete objects, verbal questions and answers, and the use of diagrams to create a blackboard activity to elicit learners' interest and participation.

4.3.4 Using the 4CS for Teaching

For this section, participants were expected to explain how using the 4Cs (creative, critical thinking, collaborative, and communication skills) affected their teaching experience. The findings indicated that learners have a fair knowledge of the meaning of the 4Cs but chose to use one specific component of the 4Cs in their lesson. Creative/innovative skills and critical thinking skills were the two frequent skills participants utilised during their teaching.

4.3.4.1 Innovative skills

Findings from the data indicated that participants considered using innovative skills as an essential aspect in their preparation for teaching because it enabled them to create a new way of teaching different from the traditional ways to improve learners' understanding. For example, P1 explained that using creative and innovative skills enabled him to improvise teaching materials, allowing learners to experiment with practical tools and apply their imagination to uncover new ways or alternative possibilities to solve problems and justify their arguments. Similarly, Participant 6 indicated that the use of the 4Cs was helpful during his teaching practice because it enabled him to use innovative ways of solving mathematical problems, which include practical hands-on activities with natural objects (cylinder, ruler and cord) visual representation (diagrams and charts) and using real-life questions.

The analyses of the participants' responses showed that participants were more amenable to using creative skills that support both concrete and virtual learners who may find it easier to relate to objects and pictures to make sense of new concepts and provide reference points as they work. This aligns with Vygotsky and Piaget's constructivist theory, which suggests that experience is essential for learning. Piaget puts it succinctly by saying that the role of the teacher in the teaching and learning process involves providing suitable learning experiences and materials that stimulate learners to foster their thinking (Drew, 2021; Oppong-Gyebi et al., 2023, Chand, 2023).

4.3.4.2 Problem-Solving and Inquiry-Based Learning

The findings also showed that participants utilised problem-solving and inquiry-based learning approaches in their teaching and learning process. Participant 1 indicated that using critical thinking skills enabled him and his learners to raise questions and use a task that promoted reasoning to prove mathematical concepts through activity in the classroom, e.g., A sprinkler sprays water on the field. The radius of the spray is 12.6m. What area of the field is being watered? Participant 6 explained that using critical thinking skills enabled him to facilitate learners' problem-solving ability by focusing less on teaching learners how a problem is solved. Instead, he concentrated more on having them discover solutions independently through multiple approaches in classroom activities and problem-solving exercises.

The above data analysis indicated that participants utilise problem-based exercises to encourage learners to think hard, using their existing understanding to engage with real-life problems and work out solutions, instead of giving learners familiar questions that promote memorisation. The problem-based/inquiry-based learning used by participants is a learning approach that focuses on enhancing the learner's ability to think critically and test ideas. Despite this, participants failed to explain how they integrated the other components of the 4Cs, such as collaborative and communicative skills, in the actual planning and enactment of their lesson to enable learners to work as a team and efficiently convey ideas.

4.3.5 Feeling Confident to Translate 21st -Century Teaching Skills into Practice

Participants indicated they were confident in their ability to translate 21st-century teaching skills into practice in the classroom. Participants expressed the following views: Participant 1 indicated he feels confident because he has learned to prepare his lesson plan in advance and use practical activities to motivate learners to solve unfamiliar problems. Participant 6 explained that he feels confident because he has learned to use hands-on activities and group tasks to inspire learners' creativity and thinking.

Considering participants' responses, their confidence was boosted by their ability to utilise creative and critical thinking skills to motivate learners to solve diverse kinds of unfamiliar mathematical problems. However, while the participants' responses at this stage gave an impression that they were conversant with the integrated approach to using the 4Cs in their lesson., their lesson plan indicated they focused only on one particular component of the 4Cs rather than simultaneously utilising all component of the 4Cs as they implied.

4.3.6 Factors Constituting a Hindrance to Teach

The factors constituting a hindrance for participants to teach in the classroom informed the basis for this analysis. The participants identified four significant factors inhibiting their ability to teach effectively: Pre-service teachers' workload, lack of basic elementary knowledge of Mathematics, insufficient lesson time and learner-aggressive behaviour.

4.3.6.1 Work overload

Both participants 1 and 6 identified pre-service mathematics teachers' work overload as an impediment to their teaching ability. To support this claim, the participant shared the following sentiments: Participant 1 explained that he teaches JSS3 learners consisting of classes A, B, C and D with a minimum of 30 learners in each class. However, he must give notes and assignments to each class at the end of the lesson, mark all scripts and provide feedback/report the next day. Participant 6 identified too much paperwork (e.g., preparing a lesson plan and marking learners' assignments and test scripts) and extended working hours as obstacles to his teaching.

The data analysis revealed that participants felt that their classroom size compromised their ability to focus on teaching and understanding their learners, which impacted the volume of feedback/ reports they were required to provide. Although participants did not provide clarity on how the impact of the size of their classroom exceeded their time and resources available to teach, their responses, however, implied that preparing the lesson plan, marking and grading of the learner's assessment may not be accomplished during teaching hours, but may require extended working hours which makes the job more cumbersome.

4.3.6.2 Lack of basic elementary knowledge of Mathematics

Participants indicated that some learners need to possess the basic elementary knowledge of mathematics they should have known through their previous lessons or life experiences. To support this claim, participants shared the following views: Participant 1 explained that learners lacked the basic elementary knowledge of mathematics that could catalyse the introduction of more complex concepts. Participant 6 explained that learners come into the classroom needing more knowledge and remembering the essential skills for the next mathematics class.

The findings showed that participants were unanimous in their view that learners lacked the relevant background knowledge they ought to have acquired in their previous grade level, lesson or daily experiences necessary to facilitate the learning of new concepts. These positions align with Ferlazzo (2017), who argued that most Mathematics teachers would agree that learners come into the classroom needing to remember prerequisite skills for their math courses. This implies that learners' inability to practice mathematical skills for weeks between a previous topic and a new topic may result in deficits in retaining basic mathematical knowledge.

The findings are also supported by Piaget's constructivist learning theory, which states that prior knowledge is one of the critical components of learning. Piaget argued that each new experience is compared to previous experiences to enable us to understand what we are looking at through assimilation (fitting new experiences into existing mental models) and accommodation (revising existing mental models to fit new experiences (Chand, 2023). In my view, this constructivist learning theory implies that to teach well, we must seek to understand the mental models learners come within the classroom and the assumptions they make to support those models and provide the link they need to understand mathematical concepts.

Notwithstanding the unanimity among participants that students lacked the relevant fundamental knowledge of mathematics, they failed to provide evidence of learners' errors and misconceptions in their classroom activities to justify their claims as required.

4.3.6.3 Insufficient Lesson Time

Participants indicated that more lesson time could have improved their teaching ability. While participants agreed that limited time constituted an impediment to their ability to teach, they had differing views on the issues of the time needed to improve. The following are the participants' views: Participant 1 indicated that the time allocated for each lesson was insufficient to effectively engage students with hands-on activities, group discussions and project presentations. Participant 6 stated that pressure to cover the scheme of work and syllabus did not allow enough time to engage learners with reflective activities.

Literature suggests constructivist teaching and learning as it relates to the theoretical framework involves difficulties such as limited time associated with learning through trial and error, lack of time for teachers to organise problem-based learning lessons in a crowded curriculum, and the lack of time for inquiry-based learning due to the pressures of a standardised curriculum (Main, 2021; Drew, 2023). Consequently, using hands-on activities, group discussions, project presentations, and pressure to cover the syllabus, as implied by the participants, is convincing and could be considered time-consuming.

Literature from other countries doing well in mathematics, like Finland, negates participants' responses, which shows that the teacher has enough time to focus on the needs of the learners because there is no standardised testing system as learners are graded individually with a grading system created by their teacher (Vainikainen & Harju-Luukkainen, 2020**;** Leverage Edu, 2022). Based on this analysis, I believe that what a pre-service teacher can do to address issues related to insufficient time is limited. Still, they are concerned that the time allotted for teaching mathematics in schools has not produced the desired outcome and should be improved.

4.3.7 Learner's Aggressive Behaviour

Both participants 1 and 6 experienced learner-aggressive behaviour that was different. The following are the participants' views: Participant 1 explained that verbal and physically aggressive behaviour occasionally occurs, mainly at the last roll of the classroom. This aggression could be caused by abuse or insult, which may lead to a verbal reaction like screaming or a violent confrontation like pushing and hitting another learner. Participant 6 indicated that some learners get emotional or ashamed if they fail to answer a question correctly in the classroom. Sometimes, such emotions become overwhelming and are expressed through non-verbal means such as angry facial expressions, unwillingness to respond to further questions and withdrawal from active participation in group activities.

Participants demonstrated their understanding of the challenges posed by learner aggressive behaviour as it relates to the differences in aggression and types of it. The data analysis showed that participants identified three categories of aggressive learner behaviour: verbal, physical, and emotional. Participants' descriptions of verbal and physically aggressive learner behaviour could be appropriate because the participants gave the impression that verbal aggressions are words learners use to trigger conflict. In contrast, physical aggressions are physical behaviours that could cause bodily harm to others.

Participants' description of emotional learner-aggressive behaviour was weak, even though they provided good examples. Participants failed to point out that emotional learners' aggressive behaviour is an impulsive aggression or emotion that occurs with minimal consideration for thinking. Despite this shortfall, I am convinced participants drove home that emotional aggression does not involve physical acts of violence or vulgar language. Nevertheless, they constitute a type of disruptive behaviour in the learning process.

Bearing in mind the constructivist theoretical framework underpinning this study, I argue that learners with emotional instability related to health may benefit from constructivism because constructivist teaching and learning recognise that learners require targeted, differentiated lessons that match their cognitive needs and ability levels based on their age. Although constructivism has the potential to Foster critical thinking and problem-solving abilities, it might not fully attend to the learners' holistic needs. The pre-service teacher needs to strike a balance between the advantages of constructivism and the significance of attending to the disadvantages to boost every facet of the student's growth (Main, 2021).

4.3.8 Discussing with Teachers the Challenges Encountered

Both participants indicated they did not return to their lecturers to discuss challenges during their teaching practice. This implies they should have considered it essential to assess whether their strategies were adequate to achieve their goals and receive feedback.

4.3.8.1 Benefits of Participant's Discussion with Teachers

Participants indicated no benefits because they did not return to their lecturers to discuss their challenges during their teaching practice.

4.3.9 Participant Recommendations

For this section, the participants were expected to suggest what should be abandoned, retained and improved to address issues and constraints identified in the study. The following were the views of participants in this regard: P1 indicated placing more emphasis on examination-based assessment over the assessment that promotes authentic learning should be abandoned, innovative skills and inquiry-based learning should be retained, the previous topic should be reviewed, and resources that parents and learners can use to improve learner's knowledge of mathematics should be provided, content delivery and time should be improved through the use of audio-visual aids, document the classroom rules and create a reward system to reinforce discipline.

Participant 6 explained that too many assessments should be abandoned, problem-solving skills, group work, and positive feedback should be retained, and lack of basic elementary knowledge in mathematics should be improved by giving assignments, cutting down on insufficient time by delegating different activities to learners during the free time such as the writing of notes on the board, reduce work overload by cutting down on the number of assessment, model the ideal behaviour and give tangible rewards to mitigate aggressive behaviours.

Considering practices that should be abandoned, participants recommended that emphasis on one assessment over the other should be discouraged and that too many assessments should be abandoned. Participants' views implied that they were perhaps raising attention concerning schools that subject learners to rigorous tests, creating a situation whereby learners study mainly to pass their examinations without internalising the learning experience. These contradictions call for harmonising standards in the country's assessment practice. The call for cutting down on the number of assessments is also apt because assessments should be based on quality rather than quantity. Participants were also unanimous in their views that innovative skills, problem-solving skills, group work and positive feedback should be retained. Participants' perspectives were similar to the extent that they were both the learners-centred approach to teaching that allows learners to conduct experiments through trial and error to reach their conclusions and construct new knowledge in their minds (Drew, 2023).

4.3.10 Chapter Summary

This chapter presented the participants' responses and analyses generated by the two pre-service mathematics teachers who participated in the semi-structured interview. According to the interview results regarding the participants' teaching practice experiences in section 4.3.1, both participants recognised the value of utilising effective teaching techniques, like classroom discussion and interaction as well as concrete and visual representations, to help learners understand mathematical concepts and provide clarity. When participants were asked in section 4.3.2 if they felt prepared to begin teaching, both participants responded in the affirmative. However, participants did not share any details about their experiences before, during, or after the lesson plan was implemented. This obstacle prevented a comprehensive evaluation of the lesson plan's efficacy in the classroom.

An analysis of the two participants' most memorable training experiences in section 4.3.3.3 showed they cherished the use of experiential learning to advance their learners' knowledge and skills. According to the subsequent data analysis verifying the participants' appreciation of their classroom activities, participants valued learners' participation in their classroom activities because it helped learners have a better grasp of what was being taught, and equally provided a sense of ownership over what they were learning. Nevertheless, none of the participants offered proof of their learners'' performance in these classroom exercises, which would have demonstrated the degree to which their learning objectives were met.

When asked whether they felt comfortable putting their 21st-century teaching skills into practice, participants through their lesson plan demonstrated they concentrated on just one of the 4Cs, rather than using all of them at once as they implied. The participants identified four key factors that were impeding their ability to teach effectively: Pre-service teachers' workload, lack of basic elementary knowledge of Mathematics, insufficient lesson time, and learner-aggressive behaviour. The interpretation and analysis of the data acquired from the open-ended questionnaire were covered in detail in Chapter 5. The data was examined in light of the literature reviewed and the constructivist theoretical framework underpinning the study.

CHAPTER 5: DISCUSSION OF RESULTS

5.1 Introduction

In the previous chapter, the analysis of the information derived from the semi-structured interview was provided. This chapter begins with the interpretation and analysis of the results from the open-ended questionnaires. A discussion of the results in light of the literature and theoretical framework was also discussed. The key ideas in the findings are summarized in the chapter summary, which concludes the chapter.

5.2 Open-Ended Questionnaire

A sample size of twelve (12) candidates was initially targeted to be selected for the research study. Participants were selected through purposeful sampling, out of which only six fully participated, where two participated in the semi-structured interview, while four participated in the open questionnaire (see 3.5 and 4.2). The open-ended questionnaire was the second instrument to be administered. This section analyses the data submitted by the four participants who responded to the open-ended questionnaire. This was done to eliminate biases by generating data from different sources, and conformability was prioritised.

Ten questions had to be answered.

5.3 Preservice Teachers' Teaching Practice Experience

Participants' viewpoints on the most exciting experiences during their training formed the basis for this analysis. The data analysis revealed that participants recognised the importance of lesson planning and lesson plan enactment and the use of appropriate teaching methods and activities as factors that have been helpful in their teaching experience. The following were the participants' views: Participant 2 indicated that she has learned that learners learn differently; a few students are fast learners, and others are moderate. Some learn by experience or hand, and others prefer audio-visual support. Acknowledging learners' differences and abilities has helped her to support learners who are in need so they can make progress at their own pace and attain their full potential in the classroom.

Participant 3 explained that his experience helped prepare him to plan and enact his lesson plan, manage classroom disturbances, and utilise the relevant teaching methods and activities to enhance his teaching. For example, he asked those disrupting the lesson to stand up; on other occasions, he would adopt a flexible teaching strategy that engages learners with hands-on activities and assigns group tasks, which helped to reduce disruption and improve learners' concentration.

Participant 4 claimed that his teaching practice experience showed that using appropriate lesson plans and effective teaching strategies like hands-on activities and classroom discussions helped him simplify abstract mathematical computations. He emphasised that learners enthusiastically used paper folding to prove Pythagoras’ theorem and circles.

Participant 5 explained that she had both an exciting and challenging experience. She claimed it was interesting because she had learned to prepare and enact her lesson plan, use suitable teaching methods and administer assessments effectively. She also indicated it was challenging because the school needed ICT and basic instructional materials, and the learners needed a better understanding of the English language and mathematical terminologies.

Based on the above responses, three out of the four participants identified lesson planning, lesson plan enactment and the use of teaching methods and activities as the most occurring factors that have been helpful in their teaching experiences. There were areas in which their views differed. For example, the P2 response highlighted the importance of recognising learners’ differences and learning abilities in the classroom to provide alternative strategies to support learners' progress at their own pace. P3 emphasised the relevance of using a stand-up technique as a justifiable consequence to discipline disruptive learners. At the same time, P4 identified hands-on activities as a flexible approach that helped learners understand abstract mathematical ideas. Unlike the other three participants, only Participant 5 indicated she had exciting and challenging experiences. She shared a realistic experience that highlighted exciting and challenging moments, which was lacking in the experiences shared by the others.

In constructivist learning, the teacher’s role is to serve as a facilitator and to arrange the conditions of learning so that learners will learn what is intended in the lesson plan. The findings showed that participants were aware of the objectives and teaching methods of the lesson, which they enacted because, from that understanding, they would teach the correct content to the learners.

5.3.1 Feeling Prepared to Begin the Task of Teaching

Most participants felt they were prepared to begin the task of teaching. Participant 2 submitted four reasons why she feels prepared to begin the task of teaching: consideration for learners’ different abilities, patience to listen to learners' views, and use of innovative skills to push the limit of the learner's current understanding.

Participant 3 indicated that he feels prepared to begin the teaching task because he has learned how to prepare and present his lesson plan, understand the subject content, manage his classroom, time and goals effectively and be flexible in his teaching methods.

Participant 4 provided three reasons he feels ready to begin the teaching task: innovative skills like hands-on activities to explain the logic behind an answer, encouraging classroom discussion through group tasks and utilising multilingual skills to translate content. On the other hand, the participant also raised two reservations about why he may not be as prepared as he should be; these include the excessive workload and lack of ICT equipment and adequate instructional materials in the classrooms.

Participant 5 stated two reasons why she does not feel prepared to begin the task of teaching: lack of adequate subject content knowledge and the lack of computers and internet facilities to facilitate teaching and learning in the classroom.

The above responses revealed that different factors determined whether participants were prepared to begin the task of teaching. Two participants explained that they were not confident of their readiness to teach because they needed more technological facilities to promote a better understanding of mathematics in the classroom. This was evident in the P4 statement: "*I still have some fears because of the ... lack of ICT equipment and adequate instructional materials in the classrooms"*. This view was corroborated by the P5 statement: "*The lack of computers and internet facilities in my school hampered my ability to use technology as a medium of instruction in the classroom*". These assertions were indicators that participants experienced inadequate access to the relevant technology, which implies that the provision of ICT in schools plays a vital role in shaping the pre-service teachers' level of preparedness to teach. The above perspectives were collaborated by Aliyu et al. (2021) and Kyari et al. (2018) while recommending steps to transform Nigerian schools into 21st-century learning classrooms, stating that Nigerian schools need to fund digital tools in classrooms and school environments.

Furthermore, the data from two participants also highlighted the importance of subject content knowledge in determining if participants were prepared to teach. For instance, Participant 3 acknowledged having a better understanding of the subject content as one of the reasons he feels prepared to teach, and this view was different from Participant 5, who said that her lack of adequate subject content knowledge contributed to her unpreparedness. This implies that the participant's lack of subject content knowledge may lead to inadequate academic knowledge, which may negatively impact providing effective teaching and learning for learners. This finding aligns with the constructivist theoretical framework, which advocates that teachers should provide multiple perspectives on the content and encourage learners to acquire knowledge through involvement with content as they construct their meaning rather than imitation (Main, 2021).

5.3.2 B.Ed. programme preparation for teaching

5.3.2.1 Lesson Planning and Lesson Plan Enactment

Participant 2 identified five areas in lesson planning that have helped her to prepare to teach in a school, which include clarifying the lesson objectives, identifying questions that are aimed to determine learners' prior knowledge, introducing the lesson and determining the type of teaching method and instructional material that is most appropriate for the lesson. During the enactment of the lesson plan, the participant explained that it helped her introduce the lesson by asking learners questions about their previous knowledge and connecting it to the new topic. It also helped keep her focused and adequately utilise the time allocated to each aspect of the lesson plan, following the outlined teaching methods and activities with precision, such as classroom discussion, group work/presentations, and using visuals and concrete materials. In concluding the lesson, the participant indicated she summarised the lesson, highlighted vital facts, entertained questions and gave assignments. After the enactment of the lesson, the participant stated she reflected on the outcome and sought improvement using self-directed learning and collaboration with colleagues and her lecturers on areas she feels challenged.

Participant 3 indicated that lesson planning had taught him how to clearly outline the objectives he wants learners to achieve at the end of the lesson, introduce the lesson by raising questions that link the learner's indigenous knowledge to the new topic and choose the right resources either digital or concrete material to facilitate learning. During the presentation of the lesson plan, the participant indicated the lesson plan helped him to use questions to connect learners' previous knowledge with the new topic; digital aids like a projector helped learners gain better insight into abstract ideas and accelerated the teaching and learning process than he had expected, while, group work enabled him to monitor learners' activities and clear misconceptions.

After the enactment of the lesson plan, the participant noted that the hands-on activities and group work take much time, which sometimes delays the effective implementation of the lesson plan. To save time and complete the scheme of work, he resorted to teaching using questions and answers to evaluate learners most of the time. At the same time, he encouraged students seeking further clarity to meet him during break time or during their free period.

Participant 4 indicated that the lesson planning enabled him to organise the information to be presented and determine the teaching strategies and reflective activities used for teaching. During the enactment of the lesson plan, the participant indicated that the lesson plan enabled him to use multiple teaching strategies like group discussion, questions and answers to reinforce learning, monitor and identify learners' differences and abilities and provide needed support. The participant noted that group work sometimes leads to unnecessary noise and delay if poorly handled. After the lesson enactment, the participant indicated he had an open-door policy so that learners who needed a further explanation on specific aspects of the topic could be supported. He also collaborated with the resident's mathematics teacher on areas where he needed support.

Participant 5 explained that the lesson planning helped him to determine the lesson objectives, the most suitable teaching materials/activities, and the type of assessment relevant to the lesson. The participant indicated she started the enactment of her lesson plan with a recap of the previous lesson; checking on learners' previous knowledge often takes more time than expected. This was followed by a brief introduction of the current topic and a detailed discussion of the procedures. She indicated that learners showed more interest when she used creative skills to communicate geometric ideas by drawing on the board and presenting pictures to validate an argument. However, learners were not given sufficient examples commensurate with the planned assessment problems due to a lack of time.

The data analysis in this study revealed that the four participants utilised almost the same teaching aids in their lesson planning. The common aids used by P3 and P4 were digital and concrete materials, while P5 and P2 utilised visual and concrete materials. This perspective aligns with the findings of Nilimaa (2023) and Ojo, (2022), who argued that one of the most creative ways to spur creativity in mathematics teaching and learning is to reinforce abstract mathematical ideas by using concrete objects, charts, graphs, and pictures. This agrees with Joklitschke (2022) and Ojo, (2022), who said that raw data and primary sources such as manipulatives and interactive and physical materials promote learning in a constructivist classroom.

During the enactment of the lesson plan, the findings showed that participants commonly used questions and answers to link learners' previous knowledge to the new topic. This was manifest in the P2 statement: *"Participants… identify questions to determine students' prior knowledge"*. Similarly, P3 said, *"Introduce the lesson by raising questions that link the student's indigenous knowledge to the new topic"*.

The findings imply that it is impossible to assimilate new knowledge without using the learners' previous knowledge as a foundation and building on it with new things they learn. That is to say, the more we ask questions, the more we establish the connection between what learners already know and what they need to know. Thus, the participants' attempt to connect their lesson to the learner's previous knowledge was apt because it sought to create a pathway into the new topic for the learners by recognising their own experiences and perceptions.

The view of Participant 3 aligns with Mudaly's (2018), idea that mathematics teaching and learning is geared towards linking the formal mathematical theorems in the curriculum to have a bearing with the learners' intuitive understanding of materials and previous mathematical knowledge by providing familiar contexts that are prevalent in the learners' culture and environment. This implies that once a teacher does not know the exact prior knowledge of learners, the newly imparted knowledge may cause learners to experience challenges such as misconceptions or cognitive disequilibrium (Main, 2021).

Another area of common interest among the participants was in concluding the lesson. Two participants agreed that the lesson plan allowed them to review essential information that needed consolidation. The data also revealed that effective lesson planning and enactment stimulate participants to evaluate their instructional strategies and reflect on improving their teaching. This was manifest in the statement of P2, *"He also reflected on the outcome of the teaching, seeks improvement using self-directed learning, collaboration with colleagues and his lecturers"*. The same sentiment was shared by P4, who said that he *"reflects on his teaching and collaborates with the resident mathematics teacher on areas he needed support"*.

The above responses imply that the participants recognised the need to assess their learning outcomes and strategies during their lessons. Still, in a constructivist classroom, the emphasis is on learners. The aim is to create lesson plan(s) and activities that promote meta-cognition, self-regulation, reflection and awareness that allow learners to develop skills that enable them to set clear goals for themselves and monitor their progress. However, despite the thoroughness of the lesson planning and lesson plan enactment narrated by the participants, P5 explained that, due to lack of time, learners needed to be provided with more examples that matched the planned assessment problems. This implied that learners might be demoralised if they struggle and take longer to accomplish classroom activities.

5.3.2.2 Participants 'Most Interesting Experiences During Training

This discussion explores participants 'most exciting experiences during their training. Participants' interests were grouped into the following categories: the questions and answers/visuals and tangible materials and digital aids/group work. Participants' views on these components were explained in the following sub-sections.

5.3.2.3 Questions and answers/visuals and tangible materials

Participant 2 explained that the exciting experiences for her include classroom discussions involving questions and answers because they promoted effective communication and dialogue between her and the learners and the use of tangible materials like cardboard papers, scissors, and rulers to explain concepts. After all, it provided innovative means for the learners to understand mathematical ideas and apply them in real-life contexts.

Participant 4 stated that using visuals like diagrams, tables, and graphs was an essential experience in his training because it makes data presentable and easy to understand. In contrast, using tangible materials like cardboard papers, scissors, and rulers helped motivate and engage students in group work, increasing their creativity and understanding of mathematical ideas.

Participant 5 indicated it was interesting to see learners who had no interest in mathematics doing their best to bisect lines and draw segments with tools like cardboard paper, compass and protractors because it helped to increase their reasoning. In contrast, questions and answers helped learners improve their confidence and consolidate the knowledge gained.

5.3.2.4 Digital aids/group work

Participant 3 indicated that using digital aids like a projector was an exciting experience in his training because it enhanced communication with learners, provided better insight into abstract ideas and accelerated the teaching and learning process than he had expected. At the same time, group work enabled him to provide hands-on activities, monitor learners' work and clear misconceptions.

The data analysis showed that most participants found using multiple teaching aids/resources as an essential aspect of their training that boosted their teaching. This assertion was seen in P2's statement that using visuals and tangible materials provided innovative means for the learners to understand mathematical ideas and apply them in a real-life context. Likewise, P5 indicated that using cardboard paper, a compass, and a protractor helped increase learners' reasoning. These responses suggested that the classroom activities were multi-faceted and well-planned. At the same time, learners were treated as capable learners and encouraged to exercise their creative, critical and independent thinking skills (Drew, 2023).

The data analysis also disclosed that two participants identified group work and questions and answers as essential teaching methods that aided their training. According to P2, using questions and answers was significant because it facilitated effective communication and dialogue, while P4 indicated it helped learners increase their confidence and consolidate the knowledge gained. Participants' responses showed that a reasonable attempt was made to use group work, questions, and answers to maximise learners' involvement in their lessons.

5.3.2.5 Participants' Classroom Activities

This discussion elicited responses regarding the activities participants engaged in exactly. The following were the views of participants in this regard: Participant 2 indicated she engaged learners in a guided practice that divided learners into four homogenous groups to solve a mathematical problem using a compass and a ruler to construct a perfect circle and a ruler to make an accurate measurement of a radius. E.g., What is the area of a circle with a radius of 3cm?

Participant 3 engaged learners with origami-style activities using scissors, ruler and cardboard paper to prove the Pythagoras theorem. E.g., Pythagoras' theorem and proof. Participant 4 indicated he used classroom discussion to explain calculating the median of a data set from a list or chart. For example, eight pupils received the following marks on their homework assignments: 15, 17, 15, 18, 16, 20, 20, and 15. What is the median of their scores? Participant 5 indicated she used a group task to engage students with hands-on activities using a compass, protractor and a ruler to bisect any angle, e.g., angle 90º.

The data analysis showed that three out of the four participants agreed that using group/hands-on activities was an essential aspect of their training that enabled learners to be actively involved in their learning process. In contrast, only one participant identified visual activities as essential to enhancing his classroom teaching. The questions and the objectives for the classroom activities were well-formulated and clear—participants' choice of classroom activities incorporated group input and feedback mechanisms into the learning process.

5.3.2.6 Description of Participants' Involvement

The exact role played by the participants to promote the classroom activities formed the basis for this analysis. The following were participants' views: Participant 2 indicated she used 10 minutes for guided practice. She started by writing a question on the board, e.g., What is the area of a circle with a radius of 3cm? Each group was asked to work with their compass and ruler to solve the problem while she moved around the classroom to clear misconceptions. Next, she asked two group leaders to volunteer to share their answers on the board while the rest of the other groups compared their work with the presentation.

Participant 3 indicated he used cardboard paper and scissors to construct a proof of the Pythagoras theorem. Next, he asked each learner to construct based on what they had demonstrated earlier. Subsequently, he set a question on the board and asked each learner to work independently to find a solution. Learners were given few minutes to work; however, for the sake of time, he could not monitor and attend to the learner's individual needs but provided the answer on the board while the rest of the class checked their work. Below is an excerpt of the problem-based question and the solution he presented.

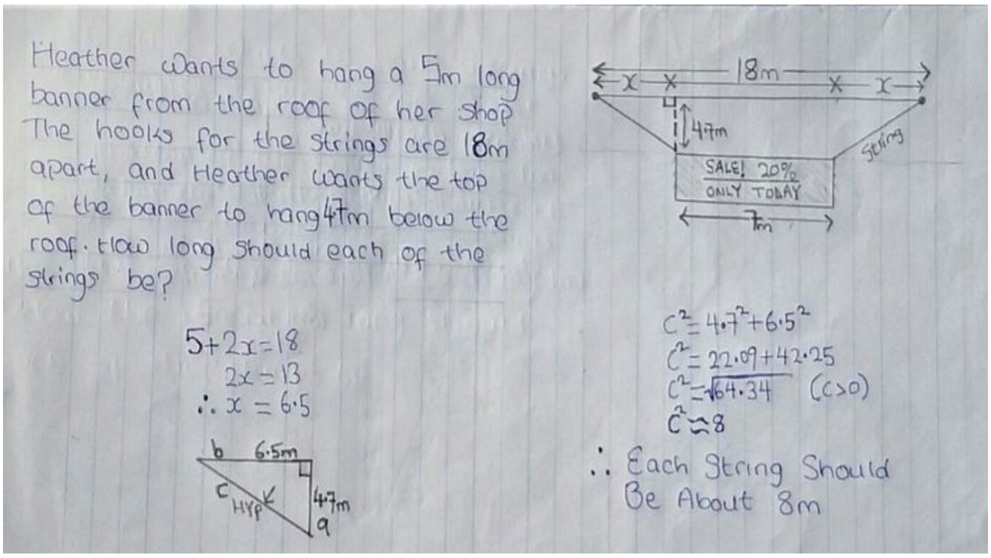

Figure 11: Problem-based question to prove Pythagoras' Theorem.

Participant 4 indicated he started by writing a question on the board, e.g., eight pupils received the following marks on their homework assignments: 15, 17, 15, 18, 16, 20, 20, and 15. What is the median of their scores? Compare it to the mean of the scores. Learners were asked to work independently to solve the problem. Next, he reminded the class that there is no middle number when we have an even number of items. We look at the two middle numbers and find their mean. We add the two numbers and divide by 2. He moved around the classroom to monitor a few individuals' work due to a shortage of time, provided the answers and entertained questions.

Participant 5 indicated she started her classroom activity by writing the following question on the board: Construct angles of 90 and 45 degrees using a ruler and compass only. Learners were asked to work with their partners to solve the problem in their exercise books. She reminded learners that a straight line should be drawn before constructing any angle, and the position of the compass must not shift from the original spot when bisecting an angle. She moved around the classroom and helped learners when there was a need. Two volunteers were asked to share their answers with the class while the rest of the learners checked their work.

The data analysis showed that two participants used independent practice that allowed each learner to work towards mastery of the knowledge/skill presented in the lesson. However, they needed to enable the learner to share their ideas due to limited time. Instead, they provided the answers to the questions on the board. This implied that the participants adopted more of a teacher-centred approach to teaching than allowing the learners to take centre stage.

The findings showed that two other participants utilised a collaborative guided practice as part of their involvement to enhance their teaching. This was evident in P2 and P5 statements. Both asked each group to work together using their tools and later asked two group leaders to volunteer to share their answers on the board. This view agrees with the work of Drew (2021), who asserted that in the constructivist model, the learners are urged to be actively involved in their learning process by interacting with others. The data analysis also showed that most participants used guided practice to enhance their teachings. This was manifest in the views of P2, who said that she moved around the classroom to monitor and clear misconceptions. Similarly, P5 stated she moved around the classroom to assess learners' progress and provided support when needed. This aligns with Vygotsky's view of scaffolding that provides guided support for learners, which may include reviewing skills required to solve the problem, providing tools for learners to work with and offering support while allowing the students to find their solutions (Oppong-Gyebi, et al., 2023; Chand, 2023).

5.3.2.7 Participant Appreciation of Classroom Activities

In this discussion, participants were expected to explain if they appreciated their participation in the classroom activities. The following were the participants' views: Participant 2 indicated she appreciated these activities because her participation provided guidance. It gave learners ownership of what they learned, empowered them to demonstrate their potential to work diligently with different groups and valued the contributions made by their peers.

Participant 3 indicated that he and the learners appreciated the activities because they provided opportunities for learners to understand the mathematical ideas contained in the activities, making them critical-minded and creative—however, the time for these activities needed to be.

Participant 4 indicated he cherished these activities because through learners' participation in independent classroom practice, they were encouraged to think critically, and he was able to identify each learner's ability and area of difficulty and provide support, but what he did not appreciate was that he could only attend to few learners due to insufficient time.

Participant 5 indicated she appreciated the guided group activities because they enabled her to manage the class effectively, have an insight into the learners' mindset, interrogate their misconceptions and help them develop their creative abilities.

The participants' responses revealed the importance of using classroom activities to help the learners become critical-minded and creative. This can be seen in the reaction of P3, who said that *"it provided opportunities for students to reason and understand the mathematical ideas ...making them critical-minded and creative"*. This experience was collaborated by P5, who said class activities enabled "*her* to *have an insight into the student's mindset, interrogate their misconceptions and help them to develop their creative abilities"*. These responses implied that through classroom activities, participants could assist learners in using their cognitive skills and trial-and-error to reach their conclusions and construct new knowledge in their minds (Drew, 2023; Li, 2022). The data analysis also showed that two participants felt that limited time impeded the effective implementation of their classroom activities. This implies that participants feel their classroom activities could be more effective if enough time were allocated for teaching mathematics. The findings showed that the participants either used a guided or independent practice, whereas both classroom activities could be jointly used in a single lesson; however, these were lacking in the participants' responses.

5.3.2.8 Significant Aspects of Participants' Classroom Activities

In this discussion, participants explained why they found the classroom activities significant. Participants expressed the following views in this regard: Participant 2 explained she found the learners taking ownership of what they learned interesting because the activities were centred on them. It also allowed learners to ask questions and improve their communication skills.

Participant 3 indicated he found the hands-on activities significant because they enabled him to engage the learners' minds creatively, making teaching and learning mathematics fun, marking a radical departure from the traditional way of teaching. Participant 4 indicated that independent practice was significant because it allowed each individual to think critically and find solutions. In addition, it encourages real-world problems that enable students to understand that mathematics is about reasoning and connecting abstract ideas to improve our daily lives rather than memorising the procedures only. Participant 5 indicated that using guided group activities was significant because it allowed learners to freely share their ideas and knowledge and take ownership of the learning process.

Constructivist learning generally begins with a question, a case, or a problem. In typical constructivist classrooms, as learners work on a problem, the teacher intervenes only as required to guide learners in the right direction. This was seen in the participants' responses because they indicated that independent practice and guided practice constituted a significant aspect of their training experience. It allowed learners to ask questions, think critically, and share their ideas freely. The data analysis also revealed that the use of hands-on activities was significant because it allowed the participants to engage the minds of the learners creatively and, in doing so, improve learners' communication skills. This implies that effective learner engagement enabled learners to take ownership of the learning process in line with constructivist teaching methods.

5.3.4 Using the 4Cs for teaching

For this section, participants were expected to explain how using the 4Cs (creative, critical thinking, collaborative, and communication skills) affected their teaching experience. The findings indicated that learners have a fair knowledge of using the integrated teaching approach with the 4Cs. Participants expressed the following views in this regard:

Participant 2 indicated that although she had planned to emphasise creative skills in the lesson, the fact remains that other skills were utilised simultaneously. For example, creative skills helped her use tangible materials like cardboard papers, rulers and pencils to prove the Pythagoras theorem that a squared equals b and c squared. The learning scenarios encourage her learners to reflect critically to understand, collaborate, and communicate to discover creative solutions.

Participant 3 explained that he often integrates two or more 4Cs skills in his lesson. For instance, the use of innovative skills helped him use teaching materials like paper, compass, marking pen and scissors to illustrate how to transform a circle into diverse shapes and validate the formula for the area of a circle, while he used group tasks to stimulate critical thinking and for learners to work as a team to find a solution to a problem. This is evident in Figure 12, which illustrates the transformation of a circle into a parallelogram:

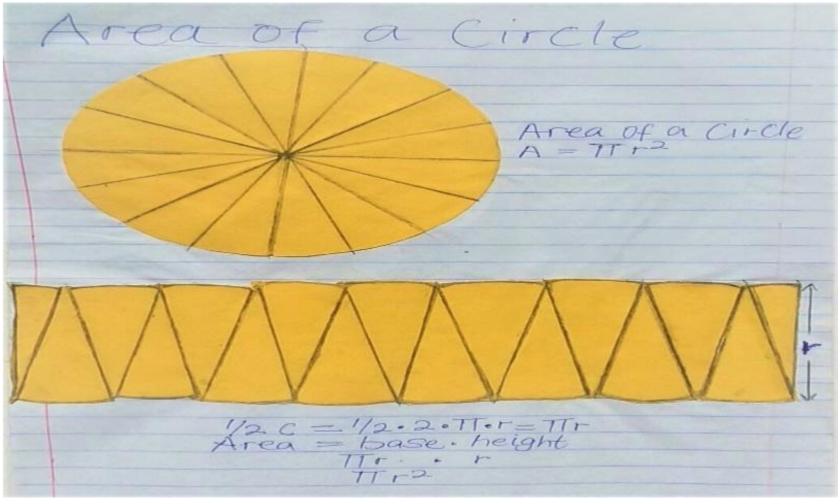

Figure 12: A circle is transformed into a parallelogram.

Here are some verbatim quotations:

> *"I folded the circle into two along its diameter four times. "I unfolded all I did and displayed the crease formed for all students. I numbered all the creased parts…. I took scissors and cut out each part. After cutting, I arranged all the cut-out parts and displayed them serially. The circle was transformed into a parallelogram".*

Participant 4 stated that he likes to focus on one of the 4Cs in a lesson, noting that others are being reinforced in teaching a particular skill. He stated, "I create classroom activities that allow students to make inferences o*r judgments as they solve problems".* Figure 13 below illustrates an application of critical thinking skills in mathematics to help us understand the world around us:

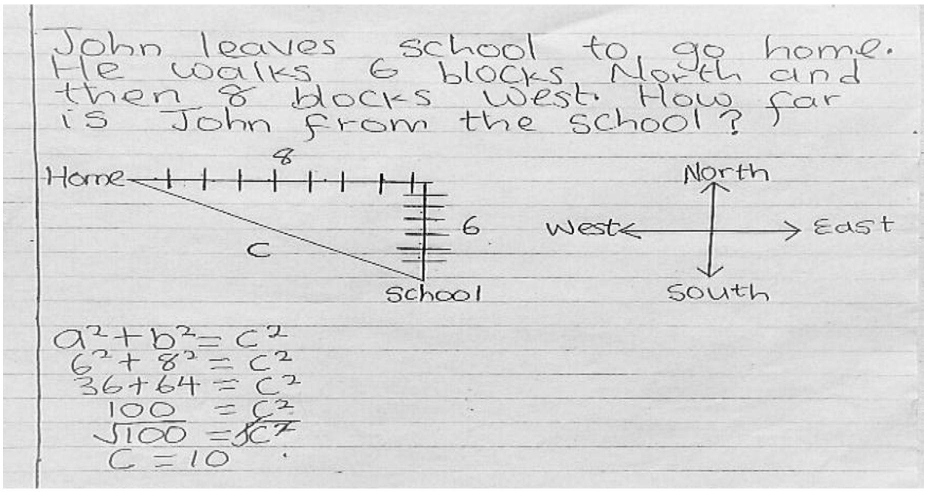

Figure 13: Utilising critical thinking abilities to demonstrate Pythagoras' Theorem.

Participant 5 explained that her B.Ed. Degree preparation was relevant to what she taught in the classroom because it enabled her to elicit critical thinking and creative behaviour by providing problem-based tasks to inspire learners to apply their imagination to solve problems.

The data analysis revealed that 3 out of four participants were acquainted with the integrated approach to teaching with the 4Cs**.** These perspectives are manifest in the views of Participant 2, who indicated That she had planned to emphasise creative skills in the lesson. The learning scenarios for these activities encouraged her learners to reflect critically to understand, collaborate, and communicate to discover creative solutions. Likewise, participant 3 explained that the use of innovative skills helped him use teaching materials like paper to illustrate how to transform a circle into diverse shapes and validate the formula for the area of a circle, while he uses group tasks to stimulate critical thinking and for learners to work as a team. One of the objectives of introducing the 4Cs in mathematics is to simplify mathematical concepts using multiple perspectives to engage the learners, which was encouraged in the classroom.

As a theory of learning, constructivism is relevant in this study as the researcher wished to establish how the participants utilised the 4Cs within the constructivist context. Examples of the constructivist classroom activities that participants utilised can be grouped into the following: Creativity: Participants provided activities that engaged learners' minds and hands. Inquiry-based Learning (IBL): Some of the participants' questions were more reflective while others were not, consisting of higher-order and lower-order thinking questions. However, not much was seen on how the participants helped the learners generate their content-related questions and how they were guided through the investigation. IBL emphasises learner-led and learner-initiated discovery, which was minimal in this case.

Problem-based learning: Participants provided critical thinking problems, motivated their students and intervened only as required to guide learners in the appropriate direction. Collaborative learning: Though opportunities were provided for learners to work in groups and dialogue with the pre-serve teachers and with one another, the motive of exposing the learners to alternative viewpoints was short-lived as the participants high-jacked the learner's involvement and provided quick solutions to save time. Despite this shortcoming, when analysed from the constructivist settings, the participants' learning activities and using the 4Cs showed active engagement, invention and meaning-making inquiry during learning.

5.3.5 Feeling Confident to Translate 21st-Century Teaching Skills into Practice

Participants were asked to indicate their responses to questions relating to their feelings on being prepared to begin the task of teaching. The following were the views of participants regarding this. Participant 2 feels confident because she has learned to prepare and present her lesson plan, demonstrate creative skills to understand mathematical ideas, recognise learners' abilities, and use various types of reasoning (inductive and deductive).

Participant 3 stated he feels confident because he has learned to use concrete and visual representation efficiently to translate mathematical ideas, use group tasks to stimulate teamwork and articulate his ideas effectively using oral, written, and nonverbal communication skills.

Participant 4 indicated he felt confident to teach because he had learnt to use different creative techniques like hands-on activities and visuals to explain mathematical concepts. He encouraged group tasks and classroom discussions, allowing learners to interpret information and draw conclusions. He also explained his fears concerning the excessive workload and the lack of ICT equipment and instructional knowledge.

Participant 5 indicated she was less confident than she should be because despite acquiring practical experience that elicits critical thinking and creativity in learners, she still needs more subject content knowledge to facilitate effective teaching and learning in the classroom.

The data analysis revealed that participants' confidence was boosted through 21st-century teaching skills, which enabled them to analyse a given problem using concrete objects and visual representation in many ways. These perspectives are in harmony with the views of Main (2021), who declared that in constructivist learning, the teacher should provide activities that help learners develop their metacognition and interpret the multiple perspectives of the world. This constructivist approach was manifest in the participants' responses because they provided evidence that they used different creative techniques to explain mathematical concepts, recognise learners' different abilities, and use various types of reasoning (inductive and deductive). The findings also showed that two out of the four participants expressed doubt that they were not as confident as they ought to be because they lacked the necessary ICT equipment and instructional knowledge and had poor mastery of subject content knowledge.

5.3.6 Factors Constituting a Hindrance to Teach

The factors hindering the participants from teaching in the classroom informed the basis for this analysis. The participants identified four significant factors inhibiting their ability to teach effectively: frustration due to lack of resources, inadequate time, learner mindset, and language barrier. Participants' perspectives on these issues were discussed in the following sections.

5.3.6.1 Frustration due to lack of resources

Participant 3 indicated that the school needed a board compass, projector, or ruler to help him effectively teach geometry construction. In addition, most learners need their mathematical set to attend class, which hinders effective learner participation in class activities. Participant 4 revealed there was no multi-functional classroom with computer applications, videos and projectors, e-library/e-lesson preparation rooms, internet facilities and an interactive whiteboard for effective teaching and learning to occur in the classroom. Participant 5 explained that the factors that hindered her teaching included the lack of ICT and basic instructional materials like a board ruler or board compass and a functional e-library and mathematical laboratory.

The data analysis denotes that some participants need more support and adequate technology and training to meet learners' needs effectively. This is in harmony with the findings by STAN (cited in Kyari et al., 2018), who stated that the lack of adequate teaching facilities and mathematics laboratories in our schools constitutes a challenge to mathematics education in Nigeria. P3 rightly identified the lack of teaching materials but needed to provide the specific topic the materials were meant for; instead, he provided the chapter topic.

5.3.6.2 Inadequate time

Participant 2 indicated that the time allocated for themes/topics, guided practice and independent practice in a lesson needed to be increased to achieve learning objectives. Participant 3 stated insufficient reflective activities in the classroom due to the limited time allocated to teaching mathematics. Participant 5 explained that there needed to be more time for her to attend to slow learners and engage her learners with group discussions and presentations.

Three of the participants could address by name the specific areas in their teaching time that were inadequate. For example, Participant 2 indicated that time allocated for guided practice and independent practice in a lesson was insufficient, whilst Participant 5 explained that there was not enough time for her to attend to slow learners and also engage her learners with group discussions and presentations. Literature from other countries adhering to constructivist learning in mathematics, like Finland, negates participants' responses, which shows that the teacher has enough time to focus on the needs of the learner because there is no standardised testing system as learners are graded individually with a grading system created by their teacher, which shows learning is learner-centred (Vainikainen & Harju-Luukkainen, 2020; Aarrevaara & Pietilainen, 2021; Leverage Edu, 2021). (See 4.3.6.3)

5.3.6.3 Learners Mindset

Participant 2 explained that the misconception among learners that math is complicated to understand constitutes a barrier to teaching and learning because learners, from the start of the lesson, do not attempt to link what is being taught to their knowledge structure to make meaning. Participant 3 stated that learners with the wrong belief that mathematics is difficult to understand are easily distracted. They do not try to question procedures and answers; instead, they are quick to end the class. Participant 5 indicated that learners come into the classroom with the wrong mindset that mathematics is difficult to understand; as a result, they need to concentrate on class activities.

The data analysis showed that participants were unanimous in their experience that most learners come with a predetermined mindset that mathematics is difficult to understand. This aligns with the views of Ferlazzo (2017), who argued that learners' mindsets about Mathematics tend to be fixed. To overcome this, teachers must first evaluate their mindsets about mathematics.

Piaget's theory of equilibrium and disequilibrium helps to explain why some learners come with the predetermined mindset that mathematics is difficult to understand. During equilibrium, new information easily fits into an existing schema. When new information is learned and does not fit into an existing schema, disequilibrium occurs. During this state of cognitive disequilibrium, a learner is confused and thinks mathematics is difficult to understand (Mcleod, 2023; Waite-Stupiansky, 2022). Research has shown that the brain processes where the new information should reside during the struggle. Learning occurs during this stage and can create a new model in which the current knowledge can exist, creating a new state of equilibrium (Kusmaryono et al., 2022; Boaler, 2019). The best way to respond to learners' wrong mindsets is for the pre-service teacher and learners to be resilient.

5.3.6.4 Language Barrier

Participant 4 indicated his inability to interpret mathematical terms in the learner's language disrupted the easy flow of the lessons since learners were always asking for translations in their home language. Participant 5 explained that learners' inability to understand and communicate with the English language effectively impeded their effective translation of meaning and hindered their effective participation in classroom discussions verbally and in writing.

Participants were unanimous in their views that the inability of some learners to understand and communicate with the English language obstructed the effective translation of mathematical concepts. This implied that the language barrier may have created confusion and prevented easy comprehension of instructions. In other words, participants needed help conveying the mathematical concepts' actual ideas. This perspective is collaborated by Urbani et al. (2017), who said that language and communication are essential in the 21st-century mathematics classroom because there are several symbols and mathematical concepts that learners hardly understand.

Piaget's theory of assimilation and accommodation, through which the individual constructs knowledge, was ineffective due to language and reasonable communication constraints. To overcome these constructivist challenges, pre-service teachers must consider a linguistically responsive teaching style, which includes considering the learners' background and using cognitive tools like a web-based language instruction system as a scaffold for learning to make the academic content in English comprehensible.

5.3.7 Discussing with Teachers the Challenges Encountered

The data analysis indicated that only one participant discussed her challenges during her teaching practice with her lecturers, whereas the other three did not. The participants who did not return to their lecturers to discuss their challenges did not explain their inactions. This implied that some participants engaged in the teaching practice to fulfil the university requirement rather than as a matter of interest.

5.3.8 Benefits of Participants' Discussion with Teachers

Only one participant indicated that her discussion with her lecturer promoted a dialogue that enabled her to find a better insight into addressing her challenges.

5.3.9 Participant Recommendations

In this discussion, participants were expected to make recommendations on what should be done to address issues and constraints identified in the study. The following were the views of participants in this regard: Participant 2 indicated that learners should be encouraged to abandon the wrong belief that mathematics is complex, increase the time allotted for the teaching of mathematics so that reflective activities can be strengthened, and schools should pay more attention to rewarding creativity rather than rewarding speed during assessment which leads to root memorisation. Participant 3 indicated the school should provide digital technology and teaching materials relevant to mathematical concepts like a board compass and projector, board ruler, etc., provide sufficient time to accommodate reflective activities, reduce the number of assignments to scale down on the workload, etc.

Participant 4 indicated that the 35 minutes allocated for teaching mathematics should be abandoned. The number of teaching times increased, helped learners improve their understanding of the English language and provided a multi-functional classroom equipped with computer applications, videos and projectors, e-library/e-lesson, etc. Participant 5 indicated that learners should be reoriented to abandon the misconception that mathematics is complex, improve learners' multi-lingual skills, and provide adequate ICT/instructional materials and mathematical laboratory.

The data analysis showed that two participants agreed in their response that learners should be reoriented to abandon the wrong belief that mathematics is complex. Three participants unanimously agreed that multi-functional classroom equipment like computer applications, videos and projectors, and e-library should be provided to enhance their teaching. Likewise, two participants agreed that the number of minutes provided for teaching mathematics should be increased so teachers and learners could engage in creative activities. These findings imply that participants know the importance of ICT and the need to increase the time allocated for teaching as inputs that could help improve their preparation for teaching.

5.4 An examination of the findings through the lens of the theoretical framework.

This study is located within the constructivist theoretical framework. Constructivist learning strategies are founded on the premise that knowledge is constructed by individuals or groups reflecting and making sense of their experiential world (Taber, 2019**;** Main, 2021). (See details in 2.9). Analysing the data through the lens of the constructivist theoretical framework and the relevant literature indicated specific trends. Piaget's theory of assimilation, accommodation, equilibrium and disequilibrium are used to analyse the findings of this study. In terms of the study, each of these theories uniquely explains the phenomena of preparing mathematics teachers for the 21st-century classroom. Each of the theories is inextricably connected to the findings. Based on these theories, Table 7 captures the essential features of Piaget's critical components of constructivist learning relevant to this study.

Table 7. Piaget Components of Constructivist Learning Modified from a Model by Booth, 2014.

Processes	Instructional Principles	Components of 21st-Century Learning
Assimilation	Assess the prior knowledge of each learner to find misconceptions and give background to the new topic.	Pre-service teachers brainstorm and prepare learners to accept the new knowledge by provoking their cognitive ability.
	Guide learners to learn to work with a cylinder, ruler, scissors, and strings to prove the circumference and diameter of a circle.	Ask a challenging and facilitative question during the lesson.
	Support the learners to express problems in a friendly way and get feedback from others.	Have non-grade activities during the lesson. Encourage group discussion and peer learning.
Accommodation	Design tasks and learning situations in a way that provokes learners to reconstruct new knowledge individually or collaboratively for deeper understanding.	Present quizzes/puzzles for reinforcement. Compare ideas and find common or uncommon solutions. Ask the learner to write a report after each session or briefly summarise the session.
	Encourage learners to proffer suggestions in problem-based learning and be innovative.	Do hands-on exercises after each session. Ask learners to design exercises for each other.
Equilibrium	Design authentic tasks to make an active classroom for learners and prepare learners to learn through problem-solving.	Team working Discussion in the group Evaluation of teamwork
	Allow learners to criticise or reflect on the learning process and learning content.	Feedback forms The freedom of learners to evaluate their mentor.
Disequilibrium	Provide learners with permission to change, redo and improve exercises.	Flexibility in the learning process to correct or change the answer.
	Challenge problems.	

Assimilation: Piaget used this term to explain the reinterpreting of new experiences to fit into the already existing cognitive structures (schemas) (Mcleod, 2023; Waite-Stupiansky, 2022; Drew, 2023). To discuss the theory of assimilation related to this study, the researcher examines how participants assess learners' prior knowledge to find misconceptions and give background to the new topic, as indicated in Table 7. The findings regarding the entry behaviour of most participants indicated they considered the learner's prior knowledge, intellectual ability and misconception. However, they did not provide details of their brainstorming discussions, while some did not consider learners' prior knowledge.

The findings on issues related to participants' guided practice indicated that using concrete materials, diagrams, and charts makes it easier for the learners to grasp complex abstract concepts and relate them to what they have already learned. For example, Participant 6 provided evidence that he used a cylinder, ruler, scissors, and strings to guide learners to prove the circumference and diameter of a circle. To ensure assimilation, Participant 6 used the string to measure the boundaries of the base of the cylinder. After that, he used the ruler to measure the length of the string. The length of the string was interpreted as the circumference of the object. Next, he also took a ruler to measure the diameter of the cylinder object to prove the circumference and diameter of a circle; the participant indicated that the learners had already acquired the concept of a circle in the previous lesson. They know that a circle is any round shape or spherical object. However, in grasping the concept of a circle, the learners only experienced concrete examples like the base of a cylinder, a football, and a wall clock. Now that the pre-service teacher has introduced a new topic titled "The circumference of a circle", the idea of a circumference of a circle is linked and consolidated into their idea of the bottom of a cylinder, without effectively changing the idea of a circumference and the diameter of a circle in any significant way (see, 4.3.3.5).

Thus, through these hands-on activities, learners were spurred to fit the new topic into their knowledge or add it to their cognitive schema. To ensure that the lesson is fully assimilated, Participant 6 used non-graded activities during the lesson to engage learners with a short exercise by drawing a blank circle on the board and asking two volunteers to come and label the circumference and diameter. Regarding encouraging group discussion and peer learning to enhance assimilation, participants needed to provide more evidence of peer learning in their lesson plans and class activities, but on a few occasions.

The summation of the findings indicates that the use of tangible objects to link the learning of the circumference of a circle with real-life context makes the mathematical concept easy to remember in the cognitive map of the learners so that the process of assimilation is successful.

The implication of these findings will augur well for pre-service mathematics teachers yearning to use concrete materials, diagrams and charts to simplify abstract mathematical concepts.

Accommodation: This is modifying our cognitive schemas to accommodate new information and experiences (Waite-Stupiansky, 2022; Drew, 2021). Although this section focused on the accommodation theory, it integrates the process of assimilation and equilibrium to shed light on how accommodation occurs during the learning process. It underscores the fact that these theories do not necessarily operate in isolation. To discuss the accommodation theory related to this study, the researcher examines how participants designed tasks and learning situations to provoke learners to reconstruct new knowledge individually or collaboratively for deeper understanding, as indicated in Table 7. The findings showed that participants used problem-based exercises to encourage learners to think critically, engage with real-life problems, and work out solutions instead of giving learners familiar questions that promote memorisation. For instance, Participant 3 designed a task to make learners think critically for a more profound understanding; John left school to go home. He walks six blocks North and then eight blocks West. How far is John from the school? This task could create cognitive disequilibrium in the learners when they attempt to reconcile the question with their existing schemata. For the P3 question, the participant had to access the learner's prior knowledge about the basic concepts of the cardinal points north, south, west and east and use it as a stimulus since this knowledge is required for learners to develop appropriate strategies to solve the question. Suppose the instruction fits into the learners' schema. In that case, assimilation occurs, wherein a new idea has contributed to the development of an existing schema of the learner by expanding their existing concept of the cardinal points, which was initially limited to 6 blocks North and eight blocks West, to create a proper diagram now to apply the formula of Pythagoras theorem to determine how far John was from the school.

The expansion above can help remove the earlier cognitive disequilibrium the learners have experienced, restoring equilibrium within the cognitive structures. Furthermore, since the assimilation of the distance John walked from school to home into the cardinal point schema (the original assimilatory schema) was successful, the original assimilatory schema must be transformed into a new one (i.e., modifications must occur). Once this modification of the cardinal points schema has occurred, we say that accommodation of how to determine the distance John has walked from school to home has occurred in the cognitive structure (Waite-Stupiansky, 2022; Drew, 2023; Mcleod, 2023).

The example used on this is that conceptual change has occurred in the learners' minds about the learners' previous knowledge and that equilibrium has been established. The implication of the conceptual change and associated equilibrium is that in the future when the learner sees a similar question, he will most likely recognise it and be able to develop an appropriate diagram or visual representation to solve it. Nevertheless, in cases where a learner is faced with a new experience that is very much incongruent with existing schema(s), a cognitive impasse will undoubtedly arise. In such instances of disequilibrium, the process of accommodation and not assimilation has to 'kick' in first to reinstate equilibrium, i.e., schema (or schemata) must be reconstructed and re-organised to realise the desired equilibrium within the cognitive structures (Waite-Stupiansky, 2022; Drew, 2023; Mcleod, 2023).

From the above analysis, one can argue that the participants designed classroom tasks to encourage learners to use their cognitive skills to reconstruct new knowledge collaboratively for a more profound understanding (Drew, 2023). However, their inability to utilise quizzes to reinforce learning and the high jack of learners' involvement in group tasks to save time did pose a threat to the development of the learners' schemas to adapt to the instructions for accommodation to occur. From this perspective, Piaget's theory of assimilation, accommodation and equilibrium could be seen as a solid theoretical framework for preparing mathematics teachers for teaching.

Equilibrium/Disequilibrium: Piaget's cognitive theory suggests that the human need for cognitive equilibrium propels cognitive development – that is, equilibrium is a state of mental balance where everything makes sense (Main, 2021; Drew, 2023). To discuss the equilibrium theory, the researcher examines how participants designed authentic tasks to create an active classroom for learners and prepare them to learn through problem-solving. The findings showed that participants used authentic tasks and classroom activities with suitable teaching strategies like group work and open discussion and evaluation of the group work in line with the constructivist theoretical framework (see 4.3.3.5). Let us take a cue from the P3 problem-based question, which states: Heater wants to hang a 5m long banner from the roof of her shop, the hooks for the strings are 18m apart, and Heater wants the top of the banner to hang 4.7m below the roof. How long should each of the strings be?

This task could create cognitive dissonance in the learners as they attempt to reconcile the question with their existing schemata. For the P3 question, the participant had to translate the question into a diagram to illustrate the banner from the shop roof that learners can easily relate to and use as a stimulus to create a right-angle triangle to solve the question. Suppose the participant diagram and explanations fit into the learners' cognitive structure. In that case, assimilation has occurred, but if his explanation does not integrate with the learners' existing mental schema, confusion or an imbalance is created to trigger cognitive disequilibrium.

To fix the cognitive disequilibrium, the pre-service teacher has to provide appropriate reinforcement that bridges the gap between the problem-based question and the learners' prior knowledge. In the reinforcement process, learners are guided to experiment and provide their solutions, forming the right schemata so that the assimilation process is successful and the accommodation process can be achieved gradually, leading to equilibration.

To analyse participants teaching strategies from the lens of equilibrium/disequilibrium, the findings showed that classroom activities were done with suitable teaching strategies like the use of group work/peer learning, open discussion, evaluation of group work and the use of feedback to reinforce learning which serves as a catalyst for equilibration in the learning process (see, 4.3.3.3.1; 4.3.3.2; 4.3.3.5). Despite these efforts, participants needed to provide more opportunities for learners to criticise or reflect on the learning process and learning content due to a shortage of time, which could create disequilibrium amongst slow learners.

Finally, although the above table and resultant analysis focused on Piaget's theories of assimilation, accommodation, equilibrium and disequilibrium, it must be said that there were several other instances participants used elements of constructivist teaching strategies such as guided practice, cultural tools, scaffolding, etc. that suggest participants were also amenable to Vygotsky social constructivist theory. Where this occurred, it was, however, indicated during the data analysis.

5.5 Chapter Summary

This chapter discussed the responses and analyses generated by the four pre-service mathematics teachers who completed the open-ended questionnaire. The chapter begins the discussion in section 5.3 by reviewing the findings regarding participants, and teaching practice experience. This showed that the majority of participants identified lesson planning, instructional strategies, and practical exercises as some of the most helpful strategies in their teaching experiences. Section 5.3.1 discussed if participants were ready to begin the task of teaching. The majority of participants attributed their readiness to the following factors: flexibility, grasp of the subject matter, understanding of the learners' varied abilities, and the application of creative skills to create and deliver lesson plans.

Section 5.3.2 2 discussed the training experiences that the participants considered the most memorable. The data analysis revealed that the majority of them thought that utilising a variety of teaching tools, such as tangible materials and visual aids, provided learners with creative ways to comprehend mathematical concepts and apply them in practical settings. The data analysis also disclosed that participants identified group work and questions and answers as essential instructional strategies that aided their training because it boosted effective communication and dialogue,

The successive data analysis that examined the participants' appreciation of their classroom activities revealed that they all agreed that utilising group and hands-on activities were a crucial part of their training because it allowed learners to use their cognitive skills to create new knowledge in their minds. It was also evident from the data analysis that participants believed that having a limited amount of time made it difficult to carry out their classroom activities effectively. This suggests that if adequate time was set aside for teaching mathematics, participants believe their classroom activities could be more successful.

An analysis of the participants' learning activities using the 4Cs settings revealed active engagement, inventiveness, and meaning-making inquiry during the learning process. The data analysis further revealed that participants' confidence increased when asked if they felt confident in putting 21st-century teaching skills into practice. This was because the use of 4Cs teaching techniques allowed participants to analyse a given problem in a variety of ways using tangible objects and visual representation. The next chapter (Chapter 6) discusses this result, given the research questions, concluding remarks, and recommendations.

CHAPTER 6: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

This chapter summarises the whole research study, discusses significant research findings about the research questions, makes recommendations, proposes areas for future research studies and concludes.

6.2 Summary of the Study

Given the challenges of Nigeria's initial teacher education programs, it is impossible to guarantee that preservice teachers in Nigeria possess all the skills required to teach mathematics in classrooms once they have finished their undergraduate degrees. This emphasizes how important it is that Nigeria establish a strong framework for teacher preparation. This study investigated the experiences of pre-service mathematics teachers and how their current B.Ed. teacher education program equips them to teach in the 21st-century classroom. It aims to critically evaluate their training experience, identify the current gaps between theory and professional practice, and offer relevant recommendations.

Participants in the study were therefore limited to fourth-year B.Ed. mathematics pre-service teachers who were approaching the end of their teacher education programs at one university. This study is significant because it emphasizes the necessity of assisting pre-service mathematics teachers to improve their teaching abilities. Chapter One of this study provides an overview of the study. It starts with the background, which provides the study's context, this is followed by the problem statement, rationale for carrying out the investigation, critical research questions, etc.

Three research questions underlined this study, which were:

4. What are the training experiences of pre-service mathematics teachers currently being trained to teach in the 21st century?
5. What are the existing challenges pre-service mathematics teachers experience when they learn to teach or translate theory into practice?
6. How do these preservice teachers intend to develop greater competence as teachers of the 21st century?

Chapter two begins by reviewing the literature on several topics like the 21st-century school curriculum, assessment in mathematics and linking theory with practice**.** The works of researchers, such as Nilimaa (2023), and Duru & Obasi (2023), were in agreement that one of the most important ways to assist learners in becoming ready for the outside world is to combine mathematical practices with 21st-century skills, focusing on the 4Cs. Their perspectives were very helpful in giving this study more context and clarity.

The literature concerning assessment made it clear that teachers must reevaluate the assessment process to prepare learners for assessment styles that stimulate higher-order thinking. This argument was supported by the review of several assessment types, such as problem-posing tasks, modelling mathematical tasks, and cloze paragraphs, that were considered appropriate for improving the performance of pre-service mathematics teachers (Radmehr & Vos, 2020)

The literature also covered the topic of linking theory with practice with a focus on South Africa, Finland, Nigeria, and the United States (US). These case studies demonstrated, how crucial it is to have a partnership between universities and schools to strengthen the practical classroom experience of teacher education programmes. This is something that other developing nations lacking effective university-school partnerships can learn from as well (Krzywacki et al., 2016; Leverage Edu, 2022; OECD, 2019). (see 2.8.1.1.6; 2.8.1.2.5).

Following the review of the literature, the theoretical framework was introduced, which functioned as a portal for data analysis and the discussion of results. Piaget and Vygotsky's theory of constructivism played a critical role in shaping the ideological basis on which this study was built. Both Piaget and Vygotsky believed that experience is crucial to learning. By placing the learner at the centre of the learning process, their contributions to constructivism have greatly enhanced the teaching and learning process in schools. The results of this investigation were analysed using Piaget's theories of assimilation, accommodation, equilibrium, and disequilibrium. The data analysis highlighted the close relationship between each of these theories and the results.

The research design provided further details about the research paradigm and the study's methodology. This study used a qualitative research design. A single, in-depth case study of preservice teachers at a university was examined to support the qualitative research design. This study's use of the case study research methodology was suitable because it focused on the 21st-century classroom and investigated a specific population of preservice teachers.

The sample of the study was composed of six pre-service mathematics teachers in their fourth year doing a Bachelor of Education degree programme at an institution of higher learning in Nigeria. The purposive sampling method was used to select the sample. Ethical clearance was obtained before data collection for both the research site and the academic institution where the study was registered. Furthermore, all six research participants provided informed consent before the semi-structured and open-ended questionnaires were administered.

The data analysis was divided into three chapters due to the large amount of data. Chapter 4. covered the interpretation and presentation of the findings derived from the Semi-structured interview. The interpretation and presentation of the findings derived from the open-ended questionnaire and the results were discussed in Chapter 5 within the context of the literature review and the theoretical framework. In Chapter 6, the research conclusion which addressed the research questions was thoroughly explained. The three steps of the analysis were coherent, data presentation influenced the discussion of the result, which in turn influenced the research conclusion.

6.3 Discussion of Findings

The data collected in this study was used to investigate and answer the three critical research questions.

1. What are the training experiences of pre-service mathematics teachers currently being trained to teach in the 21st century?
2. What are the existing challenges pre-service mathematics teachers experience when they learn to teach or translate theory into practice?
3. How do these preservice teachers intend to develop greater competence as teachers of the 21st century?

In answering the first research question based on the semi-structured interview and the open-ended questionnaire used in this study, most participants identified lesson planning, enactment, and appropriate teaching methods and activities as the most critical factors that helped shape their training experiences. Most participants acknowledged that the use of lesson planning provided a guide on what learning activities and evaluation strategies would be most appropriate to facilitate their teaching, while classroom discussion, questions and answers, and the use of concrete and visual representations were some of the teaching methods and activities that helped them to improve in their teaching abilities.

The majority of the participants feel that question-and-answer was an effective instructional strategy during their lesson plan delivery because it enabled them to introduce the lesson by raising questions that linked the learner's indigenous knowledge to the new topic, facilitated effective communication and dialogue, helped learners to increase their confidence and consolidate the knowledge gained. In addition, the majority of the participants feel the use of concrete materials, diagrams and charts enabled learners to be actively engaged in the learning process, making it easier for the learners to grasp complex abstract concepts and enabled them to learn new concepts and relate them to what they have already learned. This perspective aligns with the findings of Mudaly (2018), who argued that one of the most creative ways to spur creativity in mathematics teaching is to reinforce abstract mathematical ideas by providing contexts that are familiar to the learners.

The evidence from the data analysis showed that participants did not only use the above teaching strategies but also used independent practice that allowed each learner to work towards mastery of the knowledge presented. However, they should have encouraged the learners to share their ideas because they thought it was time-consuming; instead, they provided quick answers to the questions. In this instance, participants were more inclined to use a teacher-centred approach to teaching than allowing the learners to take centre stage. There were also scenarios where some other participants utilised a collaborative guided practice that allowed participants to monitor and clear misconceptions and encouraged volunteers to share their answers on the board.

The findings on the use of creative skills showed most participants were able to demonstrate that they could engage the minds as well as the hands of learners by using diverse concrete objects like cardboard paper, scissors, cylinder, string, and ruler to prove the Pythagoras theorem, and Area of a Circle than teaching it abstractly (see. 4.3.3.5; 5.3.2.6;). These findings concurred with the works of Ojo (2022), who says one of the most creative ways to spur mathematics teaching is to establish a mathematics classroom where pre-service teachers are encouraged to explain abstract mathematical ideas using different learning materials. In instilling critical thinking skills, the findings showed that participants used real-world problems to illustrate that mathematics is about reasoning and connecting abstract ideas to improve learners' daily lives rather than memorising the procedures. This finding is supported by the participants' responses and the literature used (See 5.3.2.6; 5.3.4). Most participants also provided opportunities for learners to work in groups and dialogue with one another. However, participants' eagerness to save time sometimes disrupted the motive of exposing the learners to alternative viewpoints. The findings also revealed that participants were unanimous in their response that communication in the mathematics classroom enhances learners' interpersonal skills and provides insight into our learners' thinking. This view is also supported by Ried (2018), who contended that learners learn more when they can talk to one another and are actively involved in taking greater responsibility for their learning as they become the writers, speakers, listeners and thinkers within our classrooms.

Thus, most participants indicated they feel prepared to begin the teaching task because they have learned to use relevant teaching skills that could assist them in improving their teachings. In contrast, only some feel prepared to begin the task of teaching due to the need for adequate technological facilities. Lack of subject content knowledge to promote a better understanding of mathematics in the classroom. This implies that the participant's lack of ICT and subject content knowledge may lead to inadequate academic knowledge and poor teaching. This result leads to the second research question highlighting some challenges pre-service mathematics teachers experience when teaching or translating theory into practice.

The evidence revealed that participants feel their ability to concentrate on teaching and understanding their learners was compromised by their work overload arising from the classroom size, the volume of feedback, extra mural activities, and marking and grading of the learner's assessment after school hours, which makes the job difficult. Secondly, evidence from this study showed that most participants agreed that learners lacked the prerequisite background knowledge they ought to have in their previous lessons necessary to facilitate the learning of new concepts. This view underscores Piaget's constructivist learning theory, which states that each new experience is compared to previous experiences, enabling the learner to imbibe new knowledge through assimilation and accommodation. This theory implies that to teach well; we must seek to understand the mental models learners possess in the classroom and the assumptions they make to support those models and provide the link they need to understand mathematical concepts.

Thirdly, participants raise concerns that the time allotted for teaching mathematics in schools has yet to produce the desired outcome and should be increased. Literature from other countries doing well in mathematics, like Finland, contradicts participants' responses, which shows that the teacher has enough time to focus on the needs of the learners because there is no standardised testing system as learners are graded individually with a grading system created by their teacher (Vainikainen & Harju-Luukkainen, 2020; Leverage Edu, 2022).

Other observable challenges showed that participants are not fully equipped with adequate technology and training to meet learners' needs effectively. This is in harmony with the findings by STAN (cited in Kyari et al., 2018), who stated that the lack of adequate teaching facilities and mathematics laboratories in our schools constitutes a challenge to mathematics education in Nigeria. Further evidence showed that participants were unanimous in their experience that most learners come with a fixed mindset that mathematics is difficult to understand. This view is collaborated by Ferlazzo (2017), who contended that learners' attitudes about Mathematics tend to be fixed. To overcome this, teachers must first evaluate their mindsets about mathematics. In addition, participants agreed that the inability of some learners to understand and communicate with the English language obstructed the effective translation of mathematical concepts. This implied that the language barrier may have created confusion and prevented easy comprehension of instructions.

Evidence from this study helped answer the third research question, which focused on how pre-service teachers intend to develop greater competence as teachers of the 21st century. In addressing this question, participants responded to what should be abandoned, retained and proffer recommendations. Considering practices that should be abandoned, participants suggested that emphasis on assessment over the other should be discountenanced and that too many assessments should be abandoned. In other words, assessments should focus on quality rather than quantity. These contradictions bring to the fore the need for harmonisation of standards in the practice of assessment in Nigeria (Vainikainen & Harju-Luukkainen, 2020). The majority of the participants were also unanimous in their views that innovative skills, problem-solving skills, group work and positive feedback should be retained in schools, whilst teachers should abstain from resorting to traditional ways of teaching whenever they feel under pressure.

Other vital recommendations were that assessments should focus on learners' reorientation to abandon the misconception that mathematics is complex, computer applications, videos and projectors, and e-library should be provided to enhance teaching. It is my view that pre-service teachers should always find a balance to fulfil the needs of the learners within the demands of the curriculum and seek ways to understand 21st-century teaching and learning strategies, particularly integrating the 4Cs into their lessons, while not fulfilling the demands of the curriculum at the expense of the learners.

6.4 Recommendation

To address the pre-service mathematics teachers' challenges, such as insufficient lesson time, the learners' aggressive behaviour, lack of English language proficiency, etc. The following steps are recommended as easy ways to save pre-service mathematics teachers' teaching time: concentrate on teaching what is most important, modify instructional activities to address learning gaps, empower learners by assigning them specific tasks to help organise materials, copy notes, and solve problems in small groups and above all increase the number of minutes for the teaching of mathematics.

The following recommendations address learners' aggressive behaviour: modelling appropriate behaviour, offering praise and handshake, documenting rules, allowing learners to make input and creating a reward system to reinforce discipline are indicators of classroom management skills that can elevate the learners' positive behaviour. These views align with Carr (2023), who argued that teachers used behavioural management strategies to remediate learners not meeting expectations so that they will be more likely to meet those expectations in the future.

I recommend that schools prioritise acquiring teaching and learning resources to address the issue of scarce resources and lack of ICT and basic instructional materials. The Federal and State Ministry of Education should hold schools financially accountable and ensure that all schools have a financial support system, particularly for disadvantaged schools, to sustain the continuous provision of basic educational needs for mathematics education.

Lack of English language proficiency should be recognised as a disadvantage and remedied as it has become apparent that such learners cannot benefit from participating in classroom activities. Another remedy could be to provide an intervention programme for learners to acquire multi-lingual skills incorporated into the school programs. I also recommended that mathematics teaching commences with concrete objects and visual representations that rely on the learners' spatial-temporal reasoning. This can help learners expand their understanding of the formal mathematics vocabulary.

To address the gap between theory and practice in preparing mathematics teachers for the 21st-century classroom, I recommend that the Federal and State Ministries of Education ensure that all teacher training colleges and universities include a comprehensive module on 21st-century skills in their teacher training programmes. Such 21st-century skills modules must comprise a practical component that will expose pre-service teachers to real-life experiences in diverse classrooms under the guidance of mentors and tutors before they can be ready to handle classes.

Curriculum planners and designers should incorporate the 4Cs and the integrated approach into reviewed textbooks and instructional materials. These changes must be felt along the teaching and learning curve continuum as they must be included in teacher training and retraining programmes. There is also a need to encourage learners to express their 4C skills to improve their potential. This quest should be unrestricted due to a need for more time.

6.5 Generating New Insight

Research has shown that there needs to be more criticism concerning the competence of newly appointed teachers with low mathematical literacy skills in Nigeria (Otun & Olaoye, 2019). The findings on issues related to participants' classroom activities indicated that using concrete materials and visual representations such as diagrams and charts makes it easier for the learners to grasp complex abstract concepts and relate them to what they have already learned. Given that most participants use either independent practice or guided practice, not as an integrated approach in their classroom activities, I unravelled that participants did not provide enough opportunities for learners to engage in meaningful discussion, criticise or reflect on the learning process and learning content due to a shortage of time, lack of subject content knowledge, language barrier etc.

It makes sense to present a novel approach in which both independent and guided practice can flourish as an integrated approach to learning simultaneously. To actualise these, I propose a model that synergies Piaget's and Vygotsky's theories, whereby where one of the theories (Piagetian, i.e., independent practice) is in use, the other theory (Vygotsky, i.e., guided practice) is silently or simultaneously reinforced. This harmonises the interplay of both Piaget's and Vygotsky's theories in contributing to the development of the pre-service mathematics teacher's preparation for teaching. This happens at the intersection of the two theories of Piaget and Vygotsky, as illustrated in the interlocking circles in Figure 14.

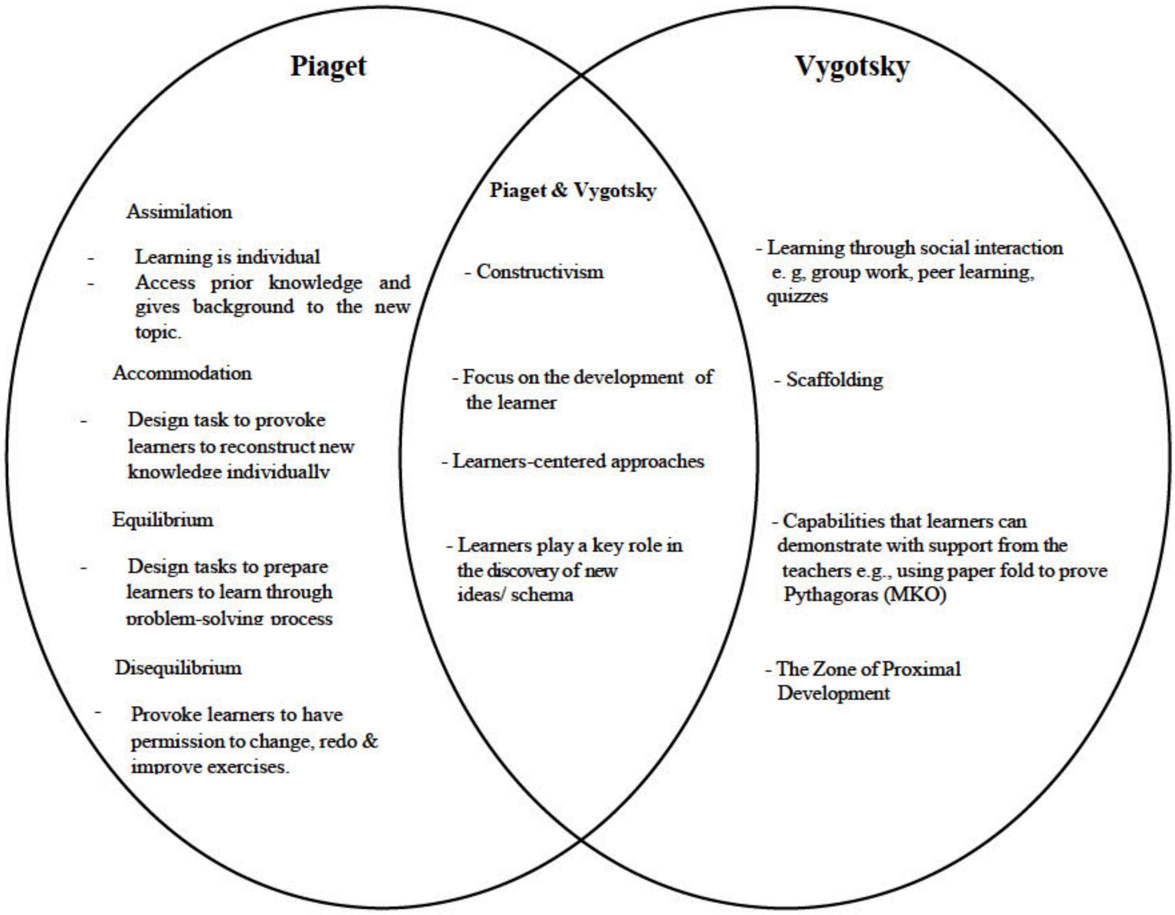

Figure 14: Interlocking circle showing the intersection of Piaget's and Vygotsky's theories.

Examining the interlocking circles from the bottom, we see the link between Piaget's Cognitive Disequilibrium (CD) and Vygotsky's Zone of Proximal Development (ZPD). See details of CD and ZPD in 2.9. Using Piaget's theory to fix the cognitive disequilibrium, the pre-service teacher has to provide appropriate reinforcement that bridges the gap between the problem-based question and the learners' prior knowledge. According to the findings, learners were guided to experiment and provide solutions in the reinforcement process. This resulted in forming the right schemata so that the assimilation process was successful and the accommodation process could be achieved gradually, leading to equilibration, as shown in Figure 13. As the study progressed, it became evident that the notion of effective teaching was closely integrated with Vygotsky's ZPD; hence, the use of scaffolding provided guided support to learners, ensuring that unfamiliar skills were broken into smaller, easily accessible ideas, such that guidance was removed when a learner's abilities improved. I argue therefore, that mental pictures derived from the use of paper folding to prove Pythagoras' theorem and the transformation of a circle into a parallelogram, such as the ones in Figure 10 and Figure 12, have the potential to promote the process of internalisation which, according to Vygotsky, is an essential part of cognitive development (Vygotsky, 1978).

Further investigation found that effective teaching was only attainable if the learning was situated within a social context, where learning is facilitated through social interaction, e.g., group work, peer learning, and quizzes. Through these interactions, learners can bridge the gap between what they know and what they need to know, mainly when a more knowledgeable person facilitates the process of learning facilitates the learning process, as indicated in Figure 14.

I envisaged that the interplay of Piaget's and Vygotsky's theories will create an ideal constructivist mathematics teacher that is focused on the development of the learner and allows the learner to play a vital role in discovering new ideas or schemata. The model has important implications for education. By understanding the role of Piaget and Vygotsky's theories in shaping learner's thinking and learning, the proposed model will benefit pre-service mathematics teachers, learners, heads of schools and teachers and curriculum experts in the following ways: enables teachers to design lessons and activities that are developmentally appropriate for their learners; provide hands-on learning for learners to actively explore and experiment; promote peer-to-peer learning and collaborative problem-solving to internalise new information; use language to think abstractly, reflect, and understand complex ideas; avoid rote learning and design curriculum based on learner's interests. This model is not exhaustive but proposes a way to coherently utilise Piaget and Vygotsky's learning theories to strengthen independent practice and guided practice in the classroom.

6.6 Implications for Future Research

There is a need to investigate why pre-service mathematics teachers still experience weak performance in school mathematics despite spending four years during their B.Ed. Degree programme. Future studies could investigate in-service mathematics teachers' preparation for teaching and then contrast it with the pre-service mathematics teachers' preparation for teaching in the 21st-century classroom. Since this study focused on 21st-century skills, namely learning and innovative skills or the 4Cs, future researchers can focus on Information, Media and Technological Skills. Finally, because this study was not representative of all the higher education institutions in Nigeria, more research could be conducted focusing on other Nigerian higher education institutions to broaden the scope of studies like these.

6.7 Conclusion

This chapter concludes this investigation. It outlined the study summary, and the findings were organised according to the three critical research questions. The chapter also highlighted the recommendations of the study and listed implications for future research, and the study's conclusion was stated.

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Appendix A. The University of Kwazulu-Natal Ethics Clearance Certificate

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

12 October 2022

Celestine Francis Ofili George (215079128)
School Of Education
Edgewood Campus

Dear CF Ofili George,

Protocol reference number: HSSREC/00004723/2022
Project title: Preparing mathematics teachers for the 21st-century classroom
Degree: Masters

Approval Notification – Expedited Application

This letter serves to notify you that your application received on 07 September 2022 in connection with the above, was reviewed by the Humanities and Social Sciences Research Ethics Committee (HSSREC) and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. **PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.**

This approval is valid until 12 October 2023.
To ensure uninterrupted approval of this study beyond the approval expiry date, a progress report must be submitted to the Research Office on the appropriate form 2 - 3 months before the expiry date. A close-out report to be submitted when study is finished.

HSSREC is registered with the South African National Research Ethics Council (REC-040414-040).

Yours sincerely,

[redacted]

Professor Dipane Hlalele (Chair)

/dd

Humanities and Social Sciences Research Ethics Committee
Postal Address: Private Bag X54001, Durban, 4000, South Africa
Telephone: +27 (0)31 260 8350/4557/3587 **Email:** hssrec@ukzn.ac.za **Website:** http://research.ukzn.ac.za/Research-Ethics

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

INSPIRING GREATNESS

Appendix B. Letter of Informed Consent

UNIVERSITY OF
KWAZULU-NATAL

Curriculum and Educational Studies
Cluster School of Education
College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,
18 October 2023

Dear respondent/s,

Letter of informed consent

I am a curriculum studies master's candidate with the University of Kwazulu-Natal, Edgewood Campus (UKZN). My research is titled: Preparing mathematics teachers for the 21st-Century Classroom. The purpose of this qualitative study is to explore pre-service mathematics teachers' preparation for teaching in the 21st-century classroom during their B.Ed. Programme at one university. The objectives of the study include: exploring the experiences of pre-service mathematics teachers, on how their current teacher education programme at the B.Ed. degree level prepares them to teach in the 21st-century classroom, to critically analyse their experience to identify the existing challenges between theory and their professional practice and to suggest appropriate recommendations based on the findings that will help to develop the competence of pre-service teachers in addressing the challenges in the 21st-century classroom.

Please note that:

- The information you provide will be treated with absolute confidentiality and the researcher will not share the information with any third party.

- Your anonymity is guaranteed as your name and other identifying information will not appear on the study report. Instead, your input will be reported only as a student population member view. Codes such as A, B, C, or X, Y, and Z will be used to represent participants' names and identities.

- If you are filling out a questionnaire, the questionnaire may last for an approximate period of 30 minutes to a maximum of 1 hour.

- If you prefer to be interviewed, the interview may last about 45 minutes. The time frame may be split depending on your preference.
- Information provided by you cannot be used against you and the collected data will be used for academic purposes only.

- Data collected will be preserved in a safe location and after 5 years, data will be incinerated if it is in paper form and all electronic gadgets will be reset to the factory settings.

- Participation in the interview is voluntary as a result you have a right to choose to participate, not to participate or freely withdraw your participation at any time without the fear of any negative consequence.

- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the discussion proceeding to be recorded using the following equipment.

Recording equipment to be used in the study	**I am willing**	**I am not willing**
Audio equipment		

I appreciate your cooperation and the time you have put aside to help me with this research project. Thank you for your contribution to this research.

Kind regards,
Celestine Francis Ofili George.

Curriculum and educational studies Cluster, School of Education, Edgewood campus, University of KwaZulu-Natal.

Cell: +2[redacted]
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My supervisor is Prof. Vimolan Mudaly
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You may also contact the Research Office through:

Simangele Shezi
HSSREC Research Office,
Tel: 27 31 2604557- Fax: 27 31 2604609
Email: HSSREC@ukzn.ac.za

DECLARATION

I ____ (**Full name of participant/designation)** hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participate in the research project. I understand that I can withdraw from the project at any time, should I so desire.

______________________________ ______________________

SIGNATURE OF PARTICIPANT **DATE**

X

______________________________ ______________________

SIGNATURE OF RESEARCHER **DATE**

X

- ❖ The signature of a **participant** in this document indicates agreement to participate in this study.
- ❖ The signature of the **researcher** on this document indicates agreement to include the participant in the research and attestation that the participant has been fully informed of his/her rights.

Appendix C. Interview Schedule

Introduction

I am a curriculum studies master's candidate with the University of Kwazulu-Natal, Edgewood Campus (UKZN). My research is titled: Preparing Mathematics Teachers for the 21st Century Classroom: A Case Study. The purpose of this qualitative study is to explore pre-service teachers' preparation for teaching in the 21st-century classroom during their B.Ed. Programme at one university. The objectives of the study include: exploring the experiences of pre-service teachers, on how their current teacher education programme at the B.Ed. degree level prepares them to teach in the 21st-century classroom, to critically analyse their experience to identify the existing challenges between theory and their professional practice and to suggest appropriate recommendations based on the findings that will help to develop the competence of pre-service teachers in addressing the challenges in the 21st-century classroom.

RESEARCH INTERVIEW GUIDE QUESTIONS

<<OPENING>>

Establish Rapport: [shake hands, smiles] My name is Celestine Francis Ofili George. Thank you for taking the time to meet with me in person. As stated earlier in my email, I am currently a curriculum studies master's candidate with the University of Kwazulu-Natal, Edgewood Campus (UKZN), under the supervision of Prof. Vimolan Mudaly. I have been granted permission to conduct my research with the 4th year B.Ed. degree students of the University of Lagos (UNILAG). This research explores pre-service teachers' preparation for teaching in the 21st-century classroom during their B.Ed. Programme.

Purpose: I would like to ask you some questions regarding your experiences with how your B.Ed. degree studies prepare you to teach in the 21st-century classroom.

Motivation: The prioritisation of the learning experiences of student teachers in higher education is currently under the spotlight, with several countries in the world undertaking curriculum/educational reforms aimed at improving learners for the higher educational demands of life and work in the 21st century. Communication, collaboration, creativity, and critical thinking are some of the learning skills relevant for today and the future. Hence this study will provide opportunities for aspiring teachers to have a better insight into some of the factors that promote or impede their ability to translate theory into practice, re-evaluate their beliefs about teaching, and gain confidence in addressing the challenges of the real 21^{st}-century classroom (Ronald, Gonzales, Ronnie & Ilagan 2020).

Duration: The interview comprises 11 questions and will take between 30-45 minutes of your time. Is there anything you would like to clarify before we begin?

<<<**ASK PERMISSION BEFORE RECORDING**>>>

Please be informed that a tape recorder will be used to record the discussion. You are however free to provide information off the record when and where you choose to. To ensure confidentiality, you will be provided with a code name at the time of writing the research report. For transcription could you please state clearly your names and titles?

<<<Begin interview>>>

Research questions

1. Name and background- what is your name? Tell us a little about yourself.
2. How long have you been a pre-service mathematics teacher? Where? What disciple? What subjects?
3. What is your present teaching practice experience in the 21^{st}-century classroom? school? Subjects? [probe: why do you say x]
4. Do you feel prepared to begin the task of teaching? Why?
5. Some of the underpinning objectives of the B.Ed. the degree programme is to prepare you for classroom practices (such as lesson plans, course materials, classroom management, evaluation and assessment, and teaching methods to prepare you to teach), have these classroom practices helped you in your preparation to teach in school?

5.1 What were 2 or 3 of the most interesting or important experiences for you during your training? Why do you find these aspects interesting or significant? [Focus on each aspect].

5.2 What kind of activities did you engage in exactly?

5.3 Describe some of the activities you did as part of your involvement.

5.4 Did you appreciate your participation in these activities? [Why?]

5.5 Why do you find these aspects interesting or significant? [Focus on each aspect].

6. Did you find your B.Ed. degree theoretical preparation (for example, using the 4Cs collaborative problem solving, creativity, communication, critical thinking and ICT) for teaching relevant to what you were teaching in class? [How was it relevant?] [Explain by way of using an example if you can to demonstrate how it was useful to you in your work with learners].

7. Did you or did you not feel confident in your ability to translate teaching principles into teaching practice? [Why?]

8. What factors did you identify that hinder or promote your ability to teach in the 21st-century classroom?

9. Did you have opportunities to discuss with your teachers the challenges you encountered during your teaching practice? If so, how often?

10 What were the benefits of such discussion?

11 If you were to make a recommendation, what would you want to be done differently? [What should be abandoned?] [What should be retained?] [What should be improved?].

12 Is there anything else you think would be helpful for me to know?

<<CLOSING>>

Thank you very much for participating in this interview. The findings of this research will be made available after it is published.

Appendix D. Questionnaire to be Administered to B.Ed. Students

Preparing Teachers for the 21st Century Classroom: A Case Study

Dear respondent/s,

I am currently researching the above topic. As part of a master of education study, this questionnaire seeks your view on student teachers' preparation for teaching in the 21st-century classroom during their B.Ed. Programme at one university. Your response will be treated with absolute confidentiality in keeping with research ethics. Kindly answer the questions as fully and honestly as possible. Thank you for your voluntary participation.

Research questions

1. Name and background- what is your name? tell us a little about yourself.

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1 How long have you been a pre-service mathematics teacher? Where? What disciple? What subjects?

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3 What is your present teaching practice experience in the 21st-century classroom? school? Subjects? [probe: why do you say x]

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4 Do you feel prepared to begin the task of teaching? Why?

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5 Some of the underpinning objectives of the B.Ed. the degree programme is to prepare you for classroom practices (such as lesson plans, course materials, classroom management, evaluation and assessment, and teaching methods to prepare you to teach), have these classroom practices helped you in your preparation to teach in school?

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5.1 What were 2 or 3 of the most interesting or important experiences for you during your training? Why do you find these aspects interesting or significant? [Focus on each aspect].

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5.2 What kind of activities did you engage in exactly?

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5.3 Describe some of the activities you did as part of your involvement.

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5.4 Did you appreciate your participation in these activities? [Why?]

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5.5 Why do you find these aspects interesting or significant? [Focus on each aspect].

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6 Did you find your B.Ed. degree theoretical preparation (for example, using the 4Cs collaborative problem solving, creativity, communication and critical thinking and ICT) for teaching relevant to what you were teaching in class? [How was it relevant?] [Explain by way of using an example if you can to demonstrate how it was useful to you in your work with learners].

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7 Did you or did you not feel confident in your ability to translate 21st-century teaching skills into practice in the classroom? [Why?]

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8 What factors did you identify that constitute a hindrance or promote your ability to teach in the 21st-century classroom?

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9 Did you have opportunities to discuss with your teachers the challenges you encountered during your teaching practice? If so, how often?

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10 What were the benefits of such discussion?

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11 If you were to make a recommendation, what would you want to be done differently? [What should be abandoned?] [What should be retained?] [What should be improved?].

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12 Is there anything else you think would be helpful for me to know?

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<<CLOSING>>

Thank you very much for participating in this interview. The findings of this research will be made available after it is published.

Ends…

Appendix E. Turnitin Certificate

PREPARING MATHEMATICS TEACHERS FOR 21ST CENTURY CLASSROOM

ORIGINALITY REPORT

18%	14%	6%	6%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

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