The use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts

By

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DECLARATION OF ORIGINALITY

I declare that this dissertation is my own work and that all the sources I have used or quoted have been indicated and acknowledged by means of complete references.

D. Ramiah
Durban
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I would like to express my appreciation and gratitude to:

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ABSTRACT

South Africa over the past decade has developed a comprehensive set of policies in the field of education with a vision to transform education in order to make amends for the past injustices related to apartheid education.

According to the Department of Education (2005) a National Framework for teacher education was recommended so that it could articulate improvement, consistency and track a more reasonable way forward for the teachers and the South African education system. Furthermore, an important area of priority was that teachers be developed so that transformation can take place. One of the goals of mathematics education according to the Department of Education is to prepare student teachers and current educators to become proficient in their endeavours. Therefore, various teacher educators were observed, with a view to understand the various strategies used to prepare future mathematics teachers.

Many studies have been conducted in the field of visualisation of mathematics; visual strategies used at university level have been a neglected part in mathematics teaching. In this study I examine the effects of visual strategies at a university in Durban, KwaZulu-Natal. This study examines the multiple uses of visual strategies by educators at university level such as:

1. multiple visual strategies used

2. various multiple representations of visual strategies

3. use of visual strategies to enhance students' understanding

4. use of visual strategies' influence on student's mathematical understanding and educators and students' perception of visual strategies

Data was collected by means of video to analyse the visual strategies used by educators, and questionnaires were given to educators and a focus group using a semi structured interview for student teachers. The conclusions from the data analysis have shown that visual strategies certainly play a pivotal role in developing mathematical concepts. Literature shows that the link between enhancing mathematical conceptual thinking in students can be done by means of using visual strategies. The literature suggests that visualisation assists students in developing their mathematical abilities as it allows them an opportunity to show their interpretation and understanding of mathematical concepts. Through the use of visual strategies

in mathematics, students make connections in mathematics and employ appropriate strategies to apply themselves mathematically.

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CHAPTER ONE

Introduction

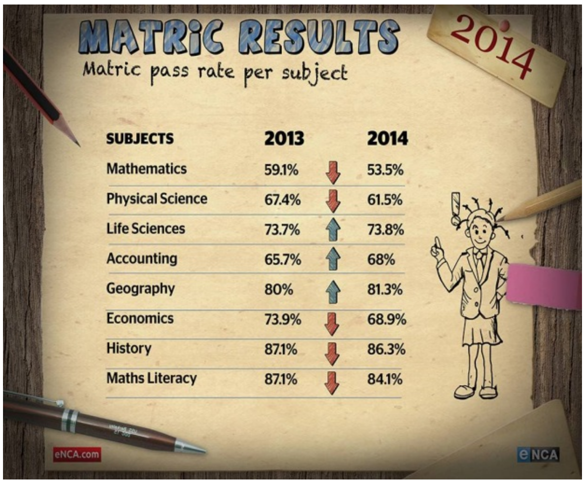
1.1. Introduction

It may be true that by seeing something, one may remember it for a longer time. Perhaps, it is better to see mathematics, than to only hear about it. Mathematical intricacy has been viewed as a sizeable challenge for decades, the use of visuals however appears to be a possible key that helps resolve this problem. Visualisation has been a window to the mathematics world by allowing students an opportunity to delve deeper in an attempt to understand mathematics. Ho (2009: 249) states that, "seeing is believing, or so the saying goes. We depend on our sight for many things in life, using a map to find our way, using a picture to aid recognition, or using diagrams to better describe what our words fail to communicate". In the mathematics classroom, sometimes the solution to a problem is right before our eyes. The contribution of visualisation to mathematics and mathematics education has raised a number of questions of an epistemological nature (Giaquinto, 2009: 1). Drawing from the mathematics CAPS document, the occurrence of visual strategies in teaching and learning of mathematics appears to be encouraged. It states that mathematics "aims to produce learners that are able to communicate effectively using visual, symbolic and/or language skills in various modes" (CAPS: 5). The incorporation of images and visual representations found in lessons is gradually becoming an important requirement for the new curriculum, which appears to be more spatially and visually inclined. A specific skill in the curriculum is that the learner ought "to develop essential mathematical skills and the learner should: communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams" (CAPS: 9). Euclidean Geometry and probability are added sections in the CAPS curriculum; these sections were previously optional in the NSC curriculum, but have now become compulsory. Drawing from my experience in teaching mathematics, these are the sections that are largely influenced by means of visuals.

In past years, many studies on visualisation were very much pessimistically stereotyped (Dreyfus & Eisenburg, 1991; Lean & Clement, 1981; Presmeg, 1992). Mathematics research education has long debated the relative presence and value of visualisation. Visualisation in mathematics has gained a positive review over recent years. In addition, Linda et al. (2010: 45) state that “Mathematical visualisation objects may have important contributions beyond the introduction of ideas to beginning students”. This has contributed to mathematics by helping students make meaning of information that may have appeared previously to have been incomprehensible. The Open University states that, “Imagery is a powerful force for perception and understanding. Being able to “see” something mentally is a common metaphor for understanding it. An image may be of some geometrical shape, or of a graph or diagram, or it may be some set of symbols or some procedure” (The Open University, 1988: 10). This research study will provide pertinent literature and evidence on visual strategies used at one university. The study firstly looks at visualisation, and visual strategies used at the university level. Secondly this research study looks at controversial issues of whether or not to use visual strategies when teaching mathematics which is discussed at length. The study also shows pertinent literature of various paradigms, models and theories underlying visualisation in mathematics as discussed in Chapter Two. Chapter Three outlines the methodological design of the study. Subsequently, a description is given of the research design, the development of the research instruments, and provides the rationalization for the use of these instruments. Chapter Four represents the data generated and finally Chapter Five outlines the conclusion and recommendations.

1.2. South African Perspective

Drawing from the South African perspective, it seems that South African students have struggled with mathematics. According to the national news provider eNCA, the mathematics results showed one of the greatest declines at the matric level (eNCA, 2015: 1).



SUBJECTS	2013		2014
Mathematics	59.1%	↓	53.5%
Physical Science	67.4%	↓	61.5%
Life Sciences	73.7%	↑	73.8%
Accounting	65.7%	↑	68%
Geography	80%	↑	81.3%
Economics	73.9%	↓	68.9%
History	87.1%	↓	86.3%
Maths Literacy	87.1%	↓	84.1%

Figure 1. 1: Math results decline (Adapted from: www.eNCA.com, 5 January 2015)

Figure 1.1 shows results of the first batch of 'CAPS matriculants' of 2014. The statistics show a decline of 5.6 percent since 2013. It is unfortunate that only 50 percent of the student population is mathematically literate. Teaching strategies used in South Africa's education system should certainly be scrutinized. It is possible that there are ineffective strategies used in our new and improved curriculum, which would require new and improved methods to fulfil a spatially and visually dominant curriculum. Recently released statistics also published in the TIMSS report stated that South Africa's mathematics position from 42 countries appeared to take position 40. Figure 1.2 shows South Africa's struggle in mathematics and very disappointingly appearing second last (V Reddy, 2011: 4).

Country	Average Scale Score	SE
Korea, Rep. of	613	2,9
Singapore	611	3,8
Chinese Taipei	609	3,2
Hong Kong SAR	586	3,8
Japan	570	2,6
Russian Federation	539	3,6
Israel	516	4,1
Finland	514	2,5
United States	509	2,6
England	507	5,5
Hungary	505	3,5
Australia	505	5,1
Slovenia	505	2,2
Lithuania	502	2,5
TIMSS Scale Centerpoint 500		
Italy	498	2,4
New Zealand	488	5,5
Kazakhstan	487	4
Sweden	484	1,9
Ukraine	479	3,9
Norway	475	2,4
Armenia	467	2,7
Romania	458	4

United Arab Emirates	456	2,1
Turkey	452	3,9
Lebanon	449	3,7
Malaysia	440	5,4
Georgia	431	3,8
Thailand	427	4,3
Macedonia, Rep. of	426	5,2
Tunisia	425	2,8
Chile	416	2,6
Iran, Islamic Rep. of	415	4,3
Qatar	410	3,1
Bahrain	409	2
Jordan	406	3,7
Palestinian Nat'l Auth.	404	3,5
Saudi Arabia	394	4,6
Indonesia	386	4,3
Syrian Arab Republic	380	4,5
Morocco	371	2
Oman	366	2,8
Ghana	331	4,3
Ninth Grade Participants		
Botswana	397	2,5
South Africa	352	2,5
Honduras	338	3,7

Figure 1.2: TIMSS Statistics for mathematics achievement (Adapted from: www.hsrc.ac.za, 12 August 2015)

Figure 1.2 displays the results of an independent and international assessment study of the mathematics and science knowledge of Grade 9 learners, released by the Human Sciences Research Council (HSRC). TIMSS was conducted in 45 countries. Of these, 42 countries participated at the Grade 8 level, and three countries, namely Botswana, South Africa and

Honduras, participated at the Grade 9 level. These three countries (Botswana, South Africa and Honduras) continued to perform at the lowest end in both mathematics and science. In addition Reddy articulates, "A striking feature of the mathematics and science scores is that the best performing South African learners matched the average performance of the top performing countries of Singapore, Chinese Taipei, the Republic of Korea, Japan, Finland, Slovenia and the Russian Federation" (Reddy, 2011: 4). According to Reddy (2011: 3), the three top performing provinces in both mathematics and science in TIMSS 2011 were the Western Cape, Gauteng and Northern Cape. The three lowest performers were KwaZulu-Natal, Limpopo and the Eastern Cape. The greatest improvement was among learners who can be described as "the most disadvantaged" and who scored lowest initially. In analysing the top-end performers against the TIMSS international performance standard, the average scores for independent, former House of Assembly and Quintile five schools, all performed below the middle score of 500. Regarding a comparison of the curriculum for these two disciplines, it was found that the Revised National Curriculum Statements that guided instruction and learning of mathematics and science at schools during 2002 and 2011 covered more than 90% of the TIMSS assessment framework on which the learners were tested. Reddy explains that "This implies that the curriculum for Grade 9 schools in South Africa is on par with the international standard, but there are many other factors that shape achievement at school level".

Perhaps the use of visual strategies in a classroom would be an attempt in transforming mathematics in South African Education. Drawing from figures 1.1 and 1.2, it is evident that mathematics is a struggle for South African learners. Visualisation in mathematics has appeared to be a channel of hope to students. There is pertinent literature by Linda et al. (2010: 3) which concludes “Visualisation objects plausibly can be used to assist in the interpretation of mathematical problems”. Many South African students still encounter problems with mathematics as a subject, which is generally deemed only for the intelligent or the more elite class of society. Valdez (2005: 1) argues that, “mathematics and science have suffered from the stereotype that only a few people can and in fact need to be highly proficient in science and mathematics”. Looking at the university, there exists a multicultural, multilingual and multi-abilitied student population. Learning mathematics from a second language perspective may be a task at hand that is most challenging. Visual strategies have become a stunning tool which allows mathematics to be displayed using pictures, images, tables, graphs, and so on and “students are able to grasp concepts easier when engaging with dynamic images” (Mudaly, 2013:

36). Visualisation in mathematics helps second language learners cut across the difficulty of mathematics that appears to be an added language to the student.

1.3. Visual Strategies

Apart from student's normal behaviour and their easy-going outlook toward mathematics, educators need to use methodologies that would get their attention and tools that can sustain good teaching in mathematics. Visual strategies have the ability to transform mathematics such that students can begin to comprehend and understand, and also where educators can find teaching mathematics with minimum difficulty. Visual strategies allow for mathematics to be understood easily (Rapp, 2009: 2). Visuals have the ability to communicate to all irrespective of colour, race or language. The language of pictures has the potential to break all barriers whether it is language or cognitive ability. Diagrams or symbols on paper often help, as do physical tools. They aid in trying to say what can (or cannot) be seen. Visualisation in mathematics education ought to become a universal communication tool for students attempting to understand mathematics. Dreyfus (1991: 33) states that visual strategies allow students to see mathematics in colour, animation and the ability to view mathematical objects three-dimensionally. They help to develop students' understanding and may allow for concepts to be grasped.

1.4. Visuals in a real world context

Many people regard mathematics as the number one subject in the hierarchical ladder owing to its levels of complexity and nature of higher order thinking that it requires. The standards and perceptions of which students and educators consider mathematics are high (Berry & Bol, 2005: 33). Mathematics commands a prominent status universally. It is a subject that holds the key to successful business people, architects, and engineers. All of these highly paid jobs have one thing in common; in the type of work they do, all require the skill of working with 'visuals'. This is further expanded in the literature review chapter. If visuals are used in jobs of the working world, surely they should be used in the classroom where student teachers and university educators need to obtain the necessary skills to prepare pupils for the working world? For example, architects work with many beautiful artefacts (blocks to make tiny buildings). Architects also work with many visuals; they construct, create, and craft visuals of which many are shown by means of visual strategies. This study attempts to understand how mathematics student teachers and university educators can be sufficiently trained using visual strategies. These educators have a task to prepare students for the work environment and using visual strategies is a suitable approach to prepare students into a largely dominant spatial working world. Chapter Two also extensively discusses how visualisation prepares them a step further and epitomizes the many various rich roles it can and should play in the learning and the doing of mathematics. At the same time, the limitations and possible sources of difficulties of visualisation may pose for students and teachers are considered.

1.5. Visual Technology

Visual Technology is a fundamental component of visual strategies used in a classroom. A visually stimulating environment based on technology can allow students to become immersed in their own knowledge construction. There are many new technological modes of visual strategies that have added a new facet to teaching. The interactive whiteboard is the most common visual strategy used in most classrooms today. “Technological change has struck a revolution with teaching. The smartboard and projector keeps me much more focused and interested. It’s much better than someone standing at the front and droning on in a monotone voice while furiously scribbling away on the board” (Iyer, 2009: 19). The benefits of this type of technology are many and can be set up without any complications. Drawing from my experience with the Smartboard visual strategy in my classroom, it has many benefits such as it allows for projecting of effectual mathematical software such as sketchpad and GeoGebra, planning lessons which undoubtedly enhance my teaching and learning. There are many new technological visual strategies used similarly to develop resources conducive to new teaching techniques that keep up with the fast paced technological world. Chapter Two embodies the benefits of using interactive whiteboards and many more technological visual strategies that can be used when teaching mathematics to students. Visual strategies have enhanced visual presentations in the classroom, improved student focus and concentration, increased enjoyment of lessons and more importantly have improved learning in classrooms. The literature review chapter shows exactly these benefits. Drawing from my experience with the new CAPS curriculum, teaching of topics such as Functions, Data Handling, Analytical Geometry, Calculus, and Trigonometry, have been made easier with software programs like Geometers

Sketchpad and GeoGebra. Teaching time traditionally required to complete a section such as Functions can be been reduced by half.

1.6. Visual Learning Style

Research studies shown in the literature and theoretical framework chapter discuss learning styles. The chapter mentions two specific learning styles (dominant visual and individual differences in visual processing in mathematics) by which visual strategies play an integral role. This dominant visual learning style lends itself to visual students; this study therefore examines the manner in which visual strategies help visual students. This learning style in this study draws on many other paradigms and theories based on visualisation in mathematics. Figure 1.3 shows the various characteristics of being a visual student. Visual students learn purely by means of 'seeing'. Learning occurs by making meaning with what they see and visual strategies are ways in which students 'see' mathematics, especially visual students. This research places some importance on visual students and referring to figure 1.3, it states that visual students learn by means of visual strategies such as pictures, charts, videos, illustrated textbooks and hand-outs.

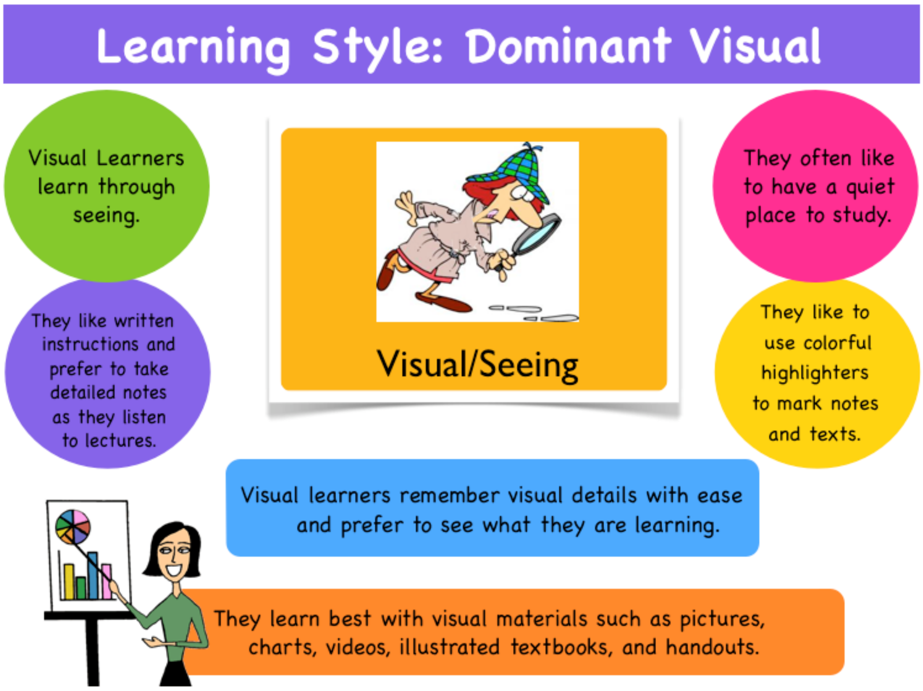


Figure 1. 3: learning styles with visual being dominant (Adapted from: https://www.google.co.za/search?q=visual+learning+style&rlz, 02 January 2017)

In the late 1970s, many mathematics educators became aware of the issue of individual student differences in mathematical processing. It is true that not all students receive information in the same way. There are differences in the way different students think and learn. Visual strategies provide a methodology to assist teachers to provide for the different ways in which different students think and learn. Kruteskii's (1976) research pointed out that there is not just one, mathematical ability, different abilities lead to different thinking styles in learning mathematics (Presmeg, 1986: 42). Kruteskii's research however also contradicts and states that it is quite possible to do mathematics with minimum visual processing, as long as the verbal-logical component of thinking is present. He also claims that many students who are capable of using visual imagery in their mathematical thinking prefer not to do so if given a choice. Visualisers really need diagrams on paper or the chalkboard, and visual images in their minds, when they learn mathematics. Thus it may be seen that there is a distinction between abilities and preferences in learning mathematics, and given the ability to do so, preference for using visuals by the use of visual strategies may determine the mathematical thinking and learning style of a student. These learning styles are discussed in greater detail in Chapter Two.

1.7. Semiotics

This research study also lends itself to another important theory called the theory of semiotics. This theory is discussed at length in Chapter Two and focuses on gestures, signs, symbols, icons and indexes used in mathematics. It can be regarded as a visual strategy used every day in the classroom. Semiotics assists in helping educators and students in making sense of how mathematics functions as a tool for problem-solving in the real world. Lemke states, "A semiotic perspective helps us understand how natural language, mathematics, and visual representations form a single unified system for meaning-making" (Lemke, 2003: 215). Lemke also argues that, it is often difficult to point to this or that sign and say whether it is mathematical or linguistic, mathematical or diagrammatic. Some linguistic signs are also mathematical, and many mathematical signs are also linguistic ones. Some diagrams are mathematical and some mathematical signs are diagrammatic. The semiotic theory can be viewed from different perspectives but this research study will only focus on Pierce's theory of signs. Pierce believed that "signs are the matter, or the substance of the thought" and said that life itself "is a train of thought", that is, life and signs are fundamentally related and inseparable for all humans" (Houser, 1987: 270).

This research study will finally look at visualisation in mathematics at university level by looking at university teacher educators' use of multiple representations of visual strategies. The purpose of this research study is to investigate the use of these visual representations (such as chalkboard strategies, computer representations, simple charts or worksheets) in getting students to understand mathematical concepts. This study will firstly analyse, tertiary educators' use of visuals and secondly, student teachers' understanding of mathematical concepts by the use of visual strategies. The aim of this investigation is to establish whether any relationship exists between the use of visual strategies and students' understanding of mathematics at university level. One of the aims of this study is to explore the relationship between educators' pedagogical and epistemological beliefs on visual strategies and their intended pedagogical practices.

RESEARCH PROBLEM

This study looks at the use of visual strategies by educators at tertiary level and their influence on student teachers' development of mathematical concepts. In order to explore the manner in which visual strategies are used at university level, I wish to seek answers to the following critical questions:

1. Are multiple visual strategies used at university level?

2. What are the various multiple representations of visual strategies that are used by educators at university level?

3. Does the use of visual strategies enhance students' understanding?

4. How does the use of visual strategies influence students' mathematical understanding through concept development?

5. What are educators and students' perceptions of visual strategies?

6. To what extent can visual strategies be used in a classroom?

RATIONALE

There are a number of dilemmas that are faced by universities as well as schools in an attempt to develop future mathematics teachers. Students however, show a lack in understanding many simple concepts in mathematics which presents a huge problem when they are asked to formally teach learners at a school. Shulman (1987: 1) highlights that knowledge of mathematics and pedagogy are not sufficient in order to become a mathematics teacher. There seems to be a gap between understanding mathematics at school and understanding mathematics at university, which presents a problem when student teachers are asked to teach mathematics. There could be several reasons for this, such as student teachers come from an ever changing curriculum and lack many basic computational skills of mathematics that tertiary educators may take for granted. Evidence provided by many tertiary educators at the university, of this research study show a large decline in the students' mathematical understanding. Despite many students being promoted to the next level, there is still a disjuncture in their mathematical performance and understanding. Research however shows that with the implementation of CAPS, there seems to be no talk about training pre-service teachers about the new CAPS curriculum. Research presented shows that training from the Department of Education is limited only to teachers. "The responsibility for the training and support of teachers in the implementation of the CAPS in the classroom from 2012 onwards rested with the provincial departments and the districts. It is clearly stated in the CAPS implementation plan that orientation and training of teachers and managers is fundamental for the effective implementation of CAPS" (Olivier, 2013: 1). The aim of investigating visual strategies at university level therefore sets out to find possible teaching strategies to alleviate the challenging nature of teaching mathematics when students are asked to teach in a formal situation. Educators seem to be teaching mathematics governed by methods, procedures and rules. Whilst these may be good to enhance mathematical thinking, students may need a little more in order to understand mathematical concepts. Students are comprised of the intelligent, the mediocre, and the weak. Gardner (1999: 1) identifies seven intelligences, and visual / spatial intelligence which is often ignored in the mathematics classrooms. Many educators teach mathematics rather than showing mathematics. Visual strategies allow for students of all intelligences and cognitive abilities to understand mathematics. Visual strategies have also catered for students in this technologically advancing society to accept mathematics as fresh and new.

There are extreme changes in society of the 21st century which place undue pressure on an educator. Expectations of educators now greatly differ from what they were a generation ago. The mathematics curriculum has changed once again lending itself to the need for a paramount change in methodologies for an educator and content for a student. The changes inclusively show that mathematics is expanding and basing itself more toward real context of everyday situations. Teaching methods should therefore differ and require useful interesting tools to sustain the mathematics CAPS curriculum. Visual strategies used in a classroom appear to be the most suitable tool to cater for the class of today and seem to have the ability to allow for the necessary skills of a student. Souhrada (2001: 1) states that the "mathematics of the twentieth century is not serving the needs of learners entering the twenty-first century". These changes in society have led to the need of developing visually literate students. We are living and teaching in a globally challenging and technologically evolving period in which visual strategies are required to instil visualisation skills. Many 21st century educators are realising that visuals are crucial to life and are assisting students to develop skills of visualisation in order to communicate in a vastly complex world. Diezmann (1995: 2) states that visual literacy has become an increasingly significant component of communication and problem solving in everyday life and that visual literacy is now essential for extracting information, constructing knowledge and building successful educational outcomes.

When prior knowledge in mathematics is low, visual representations allow for an improved form of learning mathematical facts rather than lessons by means of only verbal communication. Chanlin (1998: 166) reports how lessons with no visuals, still visuals, or animated visuals influence students with different prior knowledge levels as they attain procedural and descriptive knowledge. This report shows that students with a high level of prior knowledge of the subject responded better with the animated form of visuals in learning descriptive facts, but students responded even better with visuals of artefacts and objects especially when learning procedural knowledge. Artefacts and objects play a crucial function in mathematics education and will be discussed more in Chapter Two. Chanlin's (1998: 167) study suggests differently, he states that students with different prior knowledge levels respond differently to contrasting presentation forms for achieving learning tasks, and that the effectiveness of visual design in learning is related to the prior knowledge of the students. Animated visuals are not superior to still visuals and may even be distracting to learning if the motions are inconsistent with how students process the visual information.

Anecdotal evidence shows that underachievement in mathematics is an on-going issue in universities across South Africa. Many students are not motivated in mathematics and perform poorly. Part of the reason for this problem may be due to poor attitudes towards mathematics and poor teaching strategies in mathematics. In order to begin to remedy this problem of poor mathematics motivation and achievement, educators need to be aware and implement the best teaching strategies. Research validates the best teaching practices including the use of artefacts, real life application, and integrating technology which are all components of visual strategies, into mathematics instruction. Educators can now begin to implement these visual strategies in their own classrooms which may be a remedy to the problem of low mathematics motivation and achievement among students throughout South Africa. Furthermore, this research will show university educators highlighting the best visual strategies used in their instruction of teaching. Student teachers can then model this type of instruction in their teaching.

Findings from this research can positively influence the way students and tertiary educators see visual strategies and their connection to mathematics. It will find new meaning to visualisation that can broadly open up mathematics. There is so much more to mathematics which can be synthesised through the use of visuals. This can be done by firstly understanding whether tertiary educators use visual strategies in their lecture rooms, what type of visuals are used, and whether these influence students' development of mathematical knowledge positively or negatively. This research study will look specifically at these multiple representations of visualisation in mathematics. This research study is valuable as it can contribute extensively in the quest to produce better mathematics teachers.

STRUCTURE OF THE STUDY

This study includes five chapters, referencing and appendices. The chapters in this study are as follows:

Chapter One introduces the background to this study on visual strategies by firstly highlighting the definition of visualisation and secondly it presents a brief underlying meaning of visual strategies. It discusses the relationship between visualisation in mathematics and visual strategies as a mathematical didactic. It scrutinizes the nature and relevance of visual strategies used in classrooms of a university. It also introduces the theories that will be used in this study such as the theory of multiple intelligences, the theory of semiosis, etc. The rationale is also included in this chapter controversially describing the need for visuals in mathematics together with the purpose of why this study of visual strategies and the need for this study are to be carried out at a university. It motivates the relevance of visual strategies and their influence on students' conceptual knowledge. It presents the key research questions which are fundamental items in this study.

Chapter Two presents the pertinent literature on the areas under investigation namely multiple representations of visual strategies. The study looks at past and present literature. It shows that visualisation was a huge component since the time of Euclid and is currently obtaining much recognition again in the classroom. This chapter makes a brief study on the historical background and nature of visualisation and visual strategies used in mathematics in terms of the relevance of this study. It also provides reasons and is motivated by evidence why students should be taught using visual strategies. It presents the literature and theoretical framework for this study. The theoretical frameworks extensively show the various theories; paradigms and models used by means of visualisation and in certain instances visual strategies. This chapter gives an indication of the relevance of visualisation in mathematics by using the following theories of multiple intelligences, semiosis, Pierce's triadic model, etc.

Chapter Three presents the research design, the research methodology and procedures used to complete this study. It also discusses the research instruments used to conduct this study. It introduces the participants of three educators showing whether or not visual strategies are used in their classroom. It also introduces the student teachers as participants of the research study. It finally shows the characteristics of this study being a qualitative research approach.

Chapter Four deals with the findings and analysis of the data obtained from videos by means of observation, semi structured interviews of students and questionnaires given to educators. It displays the results for students' reaction to the visual strategies used as well as their effect on their mathematical conceptual build up. It also includes the types of visual strategies observed from university educators. This is an important chapter in this thesis, with qualitative data. It includes the actual findings, report and discussion.

Chapter Five is the final chapter, which presents the conclusion to this study, recommendations, limitations and further research. It also draws up conclusions from the results of this study. This chapter discusses the data in a summarized format. In this chapter the main points were articulated with clarity; to reiterate, summarize, and perhaps re-sequence the findings.

CHAPTER TWO

Literature Review and Theoretical Framework

2.1. Introduction

Visual strategies utilized in a mathematics classroom were purposed to initiate imagery, spatial ability and intuition in a student. As advocated by Duval (1999: 22) "There is no understanding without visualisation". Despite mathematicians adding value to visualisation in mathematics and positive links toward visual strategies, there are still many aspects shown to be controversial. In mathematics research, Presmeg (1985) started her doctoral investigation on the role of visually mediated processes in high school mathematics. She mentions that there were only a few reported studies in this field and similar to this research project, there are very few studies presented with regard to visual strategies at tertiary level. Visualisation has always made its mark in the field of mathematics education but it only became a highlighted aspect in the late 1980s. Visualisation in mathematics education consists of many theories, models and paradigms which will be discussed in the following paragraphs.

As stated by Aristotle "without image, thinking is impossible". (Stokes, 2002: 10). Visualisation has been the centre of mathematics for many years. According to a research study "Visualisation of Mathematics education has been a recent phenomenon in that it has only begun its extensive focus; however research also shows that visualisation of mathematics has been in existence since Euclid". Much research states that the use of visuals in mathematics initiated from the 18th century, gaining much popularity but lost its status in the 19th century when the use of diagrams in mathematics was said to be misleading in problem solving (Lemke, 2003: 215). Despite the rejection of visual strategies used in many publications, research shows that mathematicians still used visual reasoning in their own work (Lean & Clements, 1981; Presmeg, 1992; Dreyfus and Eisenburg, 1991). Hadamard (1954: 519) claims that even Einstein and Poincare showed the importance of visualisation by placing emphasis on visual intuition. According to Halmos (1987: 400), to be a scholar of mathematics you must be born with the ability to visualise. In addition, Bishop (1973: 271) argues "The aspect of visualisation in Mathematics education has not attracted much research attention in the recent past". Drawing from the recent history of visualisation, visual strategies certainly played an integral part in attaining mathematics knowledge in the mind of the mathematics student. In order to ascertain the integral uses of visual strategies, it is firstly vital to understand the meaning of its concept. Fosset (2004: 24) purports that "Visual strategies include the use of photographs, and visual strategies can be defined as any strategy that brings forth a representation. In the context of mathematics education visual strategies allow, for students' development of forming an internal image in their minds or external image that can be seen by the eye to convey meaning to the brain." This research study will focus primarily on tertiary educators' use of visual strategies. Visual strategies could include any representation that allows for an internal or external image developed by the student. Visual strategies could comprise of a model, a PowerPoint presentation, drawing or writing on a chalkboard, a gesture, or anything that can produce a visual representation. The methodology chapter shows exactly what visual strategies were identified in this South African research study.

South Africa looks back to an ever changing curriculum making the task of mastering teaching strategies a tedious challenge. A recent study done in South African universities shows what lecturers felt about an ever changing curriculum. Lecturers in the same university of this research study expressed their concern about the burden of programme review which may be thrown away by another possible curriculum review. In addition, Gumbo (2014: 12) stated that for educators, it would appear that the taking over by a new minister of education comes with a possibility for change or review of the curriculum. Constant curriculum change deepens the challenge to master teaching strategies. It appears that more time is focused on how to master policy documents rather than focusing on the actual implementation of what is in the curriculum. A serious look at the use of teaching strategies should be constituted as part of an approach to quality teaching and learning in educational policies. There are very few teaching strategies officially by guided documents and in some educational policies there are no guidelines to teaching strategies at all. This study looks at visual strategies from a university perspective since the context of this study originated from a university. In the milieu of a university with students attempting to master mathematics education, visual strategies should be a crucial approach used for the impartation of knowledge. Anecdotal evidence shows that tertiary educators have a lack of skill to disseminate mathematics content by the use of teaching strategies. Leu argues "Professional development of teachers has been neglected because of budget constraints and heavy emphasis on pre-service education, but when it is provided, the cascaded approach is popular for reaching many participants in a short time" (Leu, 2004: 1). Research shows that in contradiction, mathematics educators especially at university level have a deep urge and struggle to produce the best results especially since mathematics is regarded as one of the most demanding subjects (Badger, 2012: 1). The need for tertiary educators to want to improve their strategies and teaching methods are present, however there seems to be a lack of training and skill given to tertiary educators to improve their teaching strategies. Dreyfus (1991: 4) argues that “Mathematics educators seem to have recognized the potential power and the promise of visual reasoning; but in spite of this, implementation is lagging: Students tend to avoid visual reasoning. It seems that teachers continue emphasizing their instruction on non-visuals method”. At university level, mathematics can be seen as a content based module and most educators lack the ability to correlate mathematics and visuals as a teaching approach. Whilst it is done easily in primary schools and high schools, at university level there seems to be a negative cognisance in seeing the link between using visual strategies especially in mathematics owing to their cumbersome nature. Looking at South African education it is clear there is a lack of good teaching approaches used in the mathematics classroom.

In this section, a review of South African and International literature will be discussed. The objective in analysing this literature from both stances is to get different perspectives on visual strategies used at university level. This research study focuses firstly on interpreting the true meaning and function behind using visual strategies, therefore some models, theories and paradigms were analysed in this chapter. I then, extensively reviewed past and present research that suggests that multiple representations of visuals used in a classroom at university level provide an important position to mathematics. I also analysed its intense connection to the history of mathematics education. I specifically looked at how multiple forms of visual representations and their articulation impeded on the grasping of mathematical concepts with respect to tertiary education. Finally I also considered why research pointed to the effectiveness of multiple representations of visual strategies in mathematics show that all too often they do not achieve their desired educational goal and consider what can be done to overcome these problems.

2.2. Visualisation in mathematics

To adopt a definition of visualisation is crucial to the nature of this study to enhance the true purpose of visual strategies. The definition given by Zazkis et al. (1996: 435) describes visualisation as "any mental construction of objects or processes that an individual associates with objects or events perceived by her or him externally". It is impossible to look at visualisation of mathematics in isolation to visual strategies. Whilst these two are regarded broadly in the field of mathematics education, visual strategies seem to lend themselves to a deeper demand. He uses a good example to show the interrelation between visualisation and visual strategies. He also describes how educators at university use a regular hexagon as an example of dihedral groups and symmetries of cubes which were explained by the use of just a chalkboard. The result of trying to explain such a weighted section in mathematics by means of just a chalkboard turned out to be dismal. The educator then allowed these students to make these cubes and encouraged play in another lesson. The ability to visualise plays an important role in comprehending in mathematics. However, the ability to visualise at abstract level becomes a challenge but the use of visual strategies used to enhance abstract thinking eliminates this challenge for educators. Visual strategies are used to assist in enhancing the ability to visualise especially at an abstract level. Zazkis shows that the use of Venn diagrams, tables and flow diagrams are types of visual strategies that can be used in attaining a mathematical concept visually. Whilst learners are in need of using and creating images either externally or internally, it is not always easy for students to construct these representations with ease, hence the educator assists by the use of visual strategies.

Visualisation is a common metaphor used for understanding something that we simply 'see'. According to primary magazine, issue 22 "'Visualising means being able to summon a mental image of something, seeing it in your mind. The image may be of some geometrical shape, or of a graph or diagram, or it may be some set of symbols or some procedure". (p1.). Visualisation could occur by means of closing your eyes and seeing a picture, although some researchers state it has more to do with imagining. There are many aids to visualisation in reference to mathematics. The primary magazine used graph paper and a cube represented in figure 1 and figure 2 respectively to demonstrate the ability to visualise mathematically.

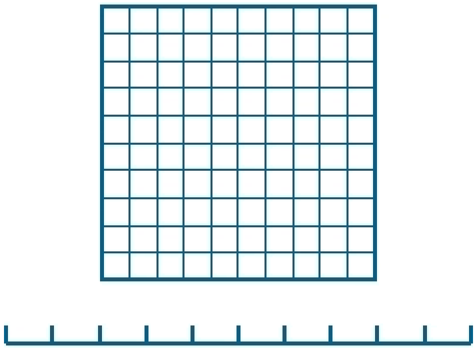


Figure 2. 1: Graph Paper (Adapted from primary magazine, 2013, 20 January 2016)

The magazine states that “there is a great deal of visualisation in mathematics. If you really want to grasp a concept or idea, struggling to visualise it is worthwhile”. (p1.). The empty number line is a useful image for students to use to support their manipulation of numbers, but they are also visualising the order of the numbers as they use it. All the various models and images encourage students to support their mathematical visualisations.

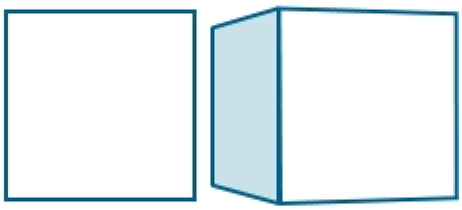


Figure 2.2: Cubes (Adapted from primary magazine, 2013, 20 January 2016)

We use visualisation in almost every area of mathematics. Figure 2 displays blocks which refer to visualisation in relation to 2D and 3D shapes. This aids students in envisaging mathematics internally and externally. Visualisation in mathematics is usually associated with drawing pictures or diagrams as an approach to getting started on mathematics problems. Visualisation however has a much greater role to play in mathematics. It includes development of ideas that is envisaged by an individual and much more. In this sense it is not just about pictures and diagrams. The visualisation process has two main elements: an internal model of visualisation or an external representation (Crapo et al., 2000, p.220).

They also emphasise the importance of the interplay between these internal and external representations which support the development of an effective model.

In as much as the ability to solve problems is at the heart of mathematics, visualisation is at the heart of mathematical problem solving. Visualisation is the ability to see and understand a problem situation. Visualising a situation or an object involves “mentally manipulating various alternatives for solving a problem related to a situation or object without benefit of concrete manipulatives” (MOE, 2001, p. 51).Visualisation can be a powerful cognitive tool in problem solving. In the revised Primary Mathematics syllabus (MOE, 2007: 13), it is highlighted as an important skill “essential in the learning and application of mathematics.” This ability to reason visually is increasingly important in the information age. Thus, the role that visualisation plays in students’ mathematical thinking and problem-solving experiences has become more significant. Piggott (2009: 1) questions, “Are there other ways in which we visualise when solving mathematical problems and if so how can we encourage, value and develop visualising in our classrooms?”

2.3. Multiple representations of visual strategies

Multiple representations of visual strategies used in a classroom are pivotal as they cater for students of various learning abilities. Multiple representations of visual strategies are different ways used to symbolize, to describe and to refer to the same mathematical entity by means of various representations. Jao states that “Many researchers have discussed the strength of using multiple representations as a vehicle to construct students’ mathematical knowledge and to support a deeper, more abstract understanding of mathematics” (Jao, 2009: 22). Multiple representations of visual strategies are used to initiate understanding, to develop, and to communicate different mathematical features, sometimes from the same object or differently. Jao adds that multiple representations may include: graphs, diagrams, tables, grids, gestures, videos, models, manipulative and pictures. “Students can represent their mathematical understanding in a variety of modes, for example: manipulatives, pictures, diagrams, spoken languages, and written symbols” (Jao, 2009: 23). Representations are thinking tools for doing mathematics. The use of multiple representations in general is an important part of teachers’ knowledge of mathematics and can play an important role in the explanation of mathematical ideas (Leinhardt et al., 1991: 87). In addition, Brophy states that “Skilled teachers have a repertoire of such representations available for use when needed to elaborate their instruction in response to student comments or questions or to provide alternative explanations for students who were unable to follow the initial instruction" (Brophy, 1991: 352).

Researchers have conversed about the notion that mathematical ideas can be represented externally and internally (Putnam, Lampert, & Peterson, 1990: 57). There are two types of representation in which mathematics can be depicted, that is, internal and external representations. External representations include manipulatives, pictures, diagrams, spoken languages, and written symbols (Lesh, Post, & Behr, 1987: 33) and internal representations include mental models and cognitive representations of the mathematical concept (Putnam et al., 1990: 140). External representations can highlight specific aspects of mathematical concepts therefore supporting this process of explanation (Kaput, 1991: 53; Ainsworth, 1999: 131). In addition, the ability to draw on multiple representations is an important aspect of pupils' mathematical understanding (Hiebert & Carpenter, 1992: 65; Greeno & Hall, 1997: 361). Visual representations enable pupils to make connections between their own experience and mathematical concepts (Post & Cramer, 1989: 221), and therefore gain insight into these abstract mathematical ideas (Duval, 1999: 3; Flevares & Perry, 2001: 330). Figure 3 below depicts an image representing learning styles. The learning styles consist of three types of learners; visual, auditory and tactile/kinaesthetic, using more than one type of representation of teaching and learning are crucial.

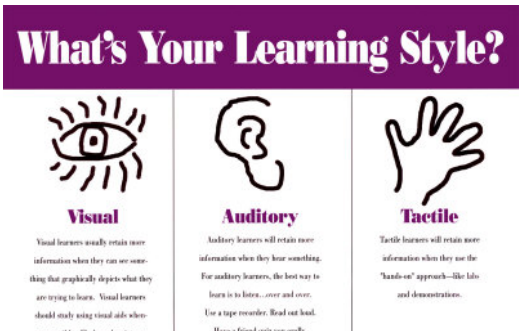


Figure 2. 3: Learning Styles

(Adapted from: http://studentblogs.le.ac.uk/management/files/2012/10/Learning-Styles-1.jpg), 25 August 2014

Perhaps a visual learner holds much importance in the above learning styles. A visual learner obtains mathematical developments not only by sight but through hearing and a sense of touch. Presmeg (1992: 595) shows that there are different forms of imagery, “continuum from specific to more general”. In contrast, the concrete image can be seen as more visual whilst the abstract demands more spatial skills. Kozhevnikov et al. (2002: 47) argue that whilst some visualisers use images and suffer the challenges; others succeed using more spatial ability. Problems of visualisers may occur because of the lack of balance between their visual and verbal understanding. It is therefore important to scrutinize the visualisers in class to improve mathematics performance.

2.3.1. Learning Styles

The past few years have projected an increasing number of research done on learning styles. Literature has shown that students gain knowledge in diverse ways. Research has shown that “Within the last three decades, the proposition that students learn and study in different ways has emerged as a prominent pedagogical issue” (Claxton & Murrell, 1987; Coffield, Moseley, Hall, & Ecclestone, 2004a, 2004b). The methodology of teaching mathematics is crucial and educators should realise, especially at university level that one approach to teaching does not produce skilful students. Research has proven that the incorporation of multiple learning styles in teaching can produce a much desired result of what educators expect in students of mathematics. A variety of teaching and learning approaches has the potential to enhance the learning and performance for a wider range of university students in mathematics.

2.3.1.1. The theory of Multiple Intelligences

The theory of multiple intelligences initiated by the Harvard psychologist Howard Gardner (1983) has captured the attention of numerous researchers, authors, and educators. The theory has been on vigorous growth since its inception. He developed the theory of eight multiple intelligences. Gardner discusses the link between these eight theories of multiple intelligences, however only the visual learner will be discussed in detail. According to Blink (2015:172) “A visual learner has the ability to perceive the visual. These learners tend to think in pictures and need to create vivid mental images to retain information. They enjoy maps, charts, videos.” The researchers above showed the crucial need for visual strategies in the classroom for the benefit of the visual learner. According to recent statistics by Krabbe (2005:5), one out of three learners constitutes the category of a visual learner. Drawing from this statistic there is a need to use visual strategies in the classroom for visual spatial learners.

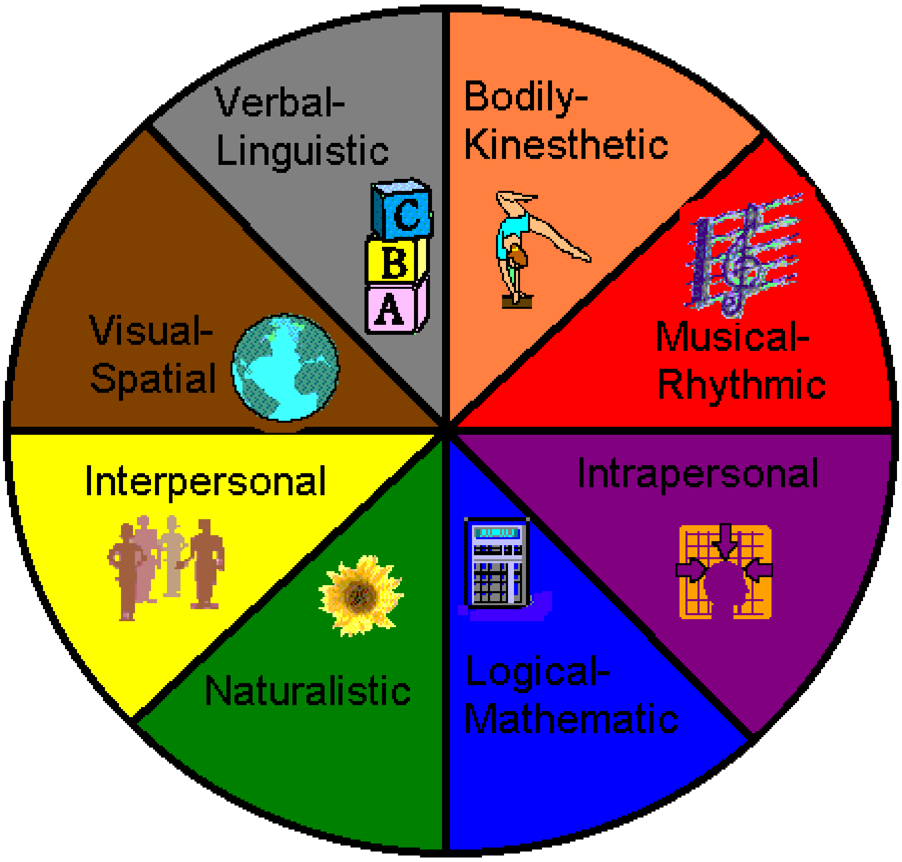


Figure 2.4: Gardener's eight multiple intelligences theory (Adapted From: *http://www.google.co.za/imgres?imgurl=http://expectumf.umf.maine.edu/piechart.gif&imgrefurl)*, 20 July 2013

According to many researchers, to speak about what makes a real mathematician is to be asserted that "to be a scholar of mathematics you must be born with the ability to visualize." Halmos (1987:400). Many mathematicians and researchers have accentuated the importance of visual reasoning in mathematics through the use of visual strategies such as Bishop (1979: 44) and Presmeg (2008: 83) in their history of visual research in mathematics. Some mathematicians such as Lemke (2000: 235) agree that visuals can aid learners, however she also claims that "Mathematics is more powerful than visualisation, it can represent a pattern that cannot be visualized." This notion of visual strategies used in the mathematics classroom to benefit the student, is controversial. Research studies done, showed that some students are perplexed by visuals. They claimed that instead of focusing on the actual mathematics, their focus was distracted by the colour and visuals. Some researchers along with Gardner believe that it takes a linking of not just visual/spatial intelligence, but other intelligence such as that mentioned in figure 4 to combine, with visual intelligence to achieve the optimum rationalization of mathematics.

2.3.1.2. Gregorc's Learning Style Model

The Gregorc Learning and Teaching Style Model are based on phenomenological research as "distinctive and observable behaviours that provide clues about the mediation abilities of individuals and how their minds relate to the world and, therefore, how they learn" (Gregorc, 1979: 19). This model is suited to students who have natural tendencies for learning. It attempts to analyse the mental intrinsic worth of a student by means of association with the environment. The four bipolar elements that will be scrutinized are abstract and concrete perception, sequential and random ordering, deductive and inductive processing along with its separative and associative relationships.

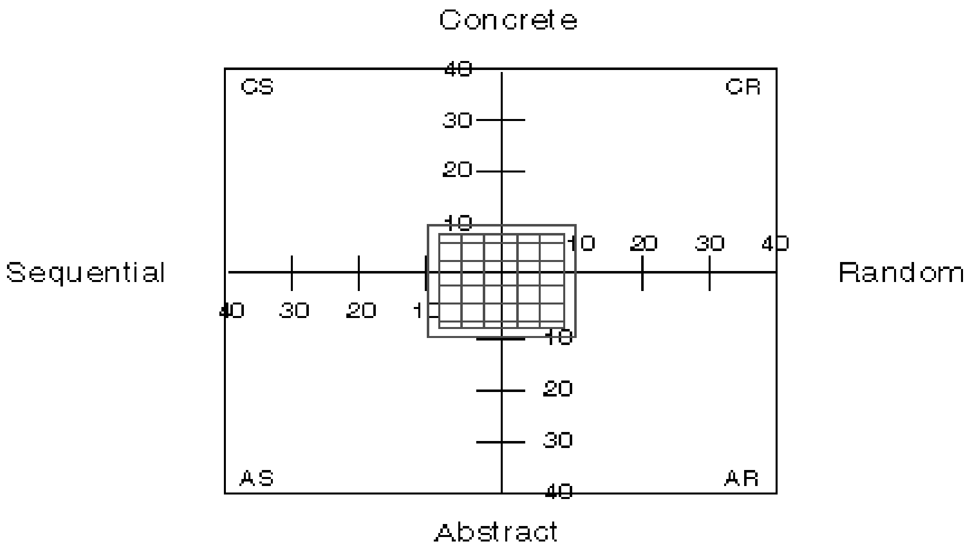


Figure 2. 5: Learning styles (Adapted from Gregorc, 1979, 25 August 2014)

The diagram in figure 5 indicates a further four learning styles, which are, Concrete-Sequential (CS), Abstract-Sequential (AS), Abstract-Random (AR), and Concrete-Random (CR). The CS students have a preference for straight less complex, hands-on experience. These students need organization and an ordered sequence to activities given to them. They follow directions and instructions well. The CS learner relates best to the concrete world with hands-on experience, prefers a structured, step-by-step learning process using all senses, and needs explicit and clear directions. The AS learner creates ideas and uses symbols to make meaning. They are reasonable students and are chronological in thinking in that they enjoy focusing on a job without distractions. The AS student uses the mind to explore, and enjoys researching and investigating, and is very analytical and evaluative. The CR learner is tentative and an explorer. He/she likes to investigate problems and makes spontaneous jumps in solving, and uses trial and error to work out solutions. The CR learner also relates well to the concrete world, prefers a nonlinear order, looks for the big picture, uses experience to investigate, and is intuitive, creative, and a risk taker. The AR learner relates best to the world of emotions and the spirit, prefers a nonlinear order that is harmonious, wants personal experiences and supportive relationships, and works for good communication. The AR student centres attention on the surroundings and would rather have discussions and conversations that are of an array that is broad. He/she requires time to reflect on experiences. Figure 6 shows the types of visual strategies and other activities needed for these types of students. Some visual strategies are cartoons, maps, diagrams and flowcharts.

Activities that accommodate Gregorc learning styles.

Concrete Sequential	Abstract Sequential	Abstract Random	Concrete Random
Checklists	Lectures	Mapping	Brainstorming
Worksheets	Outlines	Group Work	Creating Possibilities
Outlines	Documenting	Cartoons	Case Studies
Charts	Lengthy Reading	Music	Hands-on Experience
Maps	Audio Tapes	Humor	Mapping
Demonstrations	Writing Reports	Discussion	Optional Reading
Field Trips	Doing Research	Role Play	Simulations
Diagrams	Term Papers	Interviewing	Investigations
Flowcharts	Instructional Media	Keeping Journals	Problem Solving

Figure 2. 6: Activities to accommodate Gregorc's learning style (Adapted from Gregorc, 1979, 25 August 2014)

In Concrete Sequential, some visual strategies mentioned in Table 2 constitute of charts, maps, diagrams, flowcharts, and cartoons. Students have the ability to 'see' demonstrations and because of the image it sources, the student has the ability to make mathematical sense. Abstract Sequential allows for more spatial visualisation. For example, students have the ability to think and write which requires one to visualise thoughts and images in the mind before writing a term paper or a report. Abstract Random uses cartoons and role play which most certainly allows students to visualise. Finally Concrete Random which facilitates brainstorming and simulations which is also a main component of a visual strategy used to enhance mathematical development.

2.3.1.3. The VARK Model

The VARK Model makes reference to a sensory model (Eicher, 1987). The acronym VARK stands for Visual (V), Aural (A), Read/Write (R), and Kinesthetic (K). Fleming (2001: 100) defines learning style as "an individual's characteristics and preferred ways of gathering, organizing, and thinking about information. VARK is in the category of instructional preference because it deals with perceptual modes. It is focused on the different ways that we take in and give out information." The VARK shows four perceptual modes, with students having preferences for anywhere from one to all four. I will, however, only concentrate on the visual mode as the context of this study is based on visual strategies. Individual students have relative preferences along each of the four perceptual modes but can learn to function in the other modes. Figure 7 presents the VARK model (adapted from Fleming, 2001). Fleming (2001: 120) uses the VARK model to explain the visual component by stating that visual learners prefer maps, charts, graphs, diagrams, brochures, flow charts, highlighters, different colours, pictures, word pictures, and different spatial arrangements.

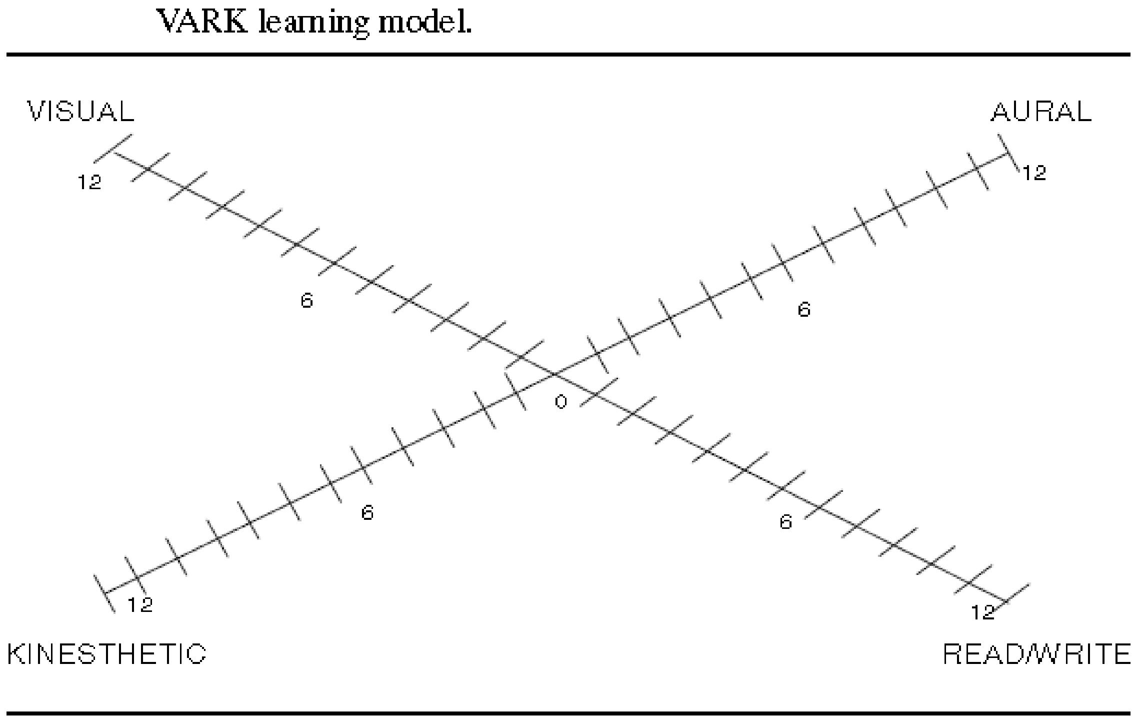


Figure 2. 7: VARK Learning Model (Adapted from Fleming, 2001, 25 August 2014)

Aural learners like to explain new ideas to others, discuss topics with other students and their teachers, use a tape recorder, attend lectures and discussion groups, and use stories and jokes. Read/Write learners prefer lists, essays, reports, textbooks, definitions, printed hand-outs, readings, manuals, Web pages, and taking notes. Kinaesthetic learners like field trips, trial and error, doing things to understand them, laboratories, recipes and solutions to problems, hands-on approaches, using their senses, and collections of samples. Fleming (2001) offers extensive suggestions for classroom approaches for matching teaching styles and learning styles. Figure 2.8 summarizes a number of learning activities to support each learning style.

Activities that accommodate VARK learning styles.

Visual	Aural	Read/Write	Kinesthetic
Diagrams	Debates, Arguments	Books, Texts	Real-Life Examples
Graphs	Discussions	Handouts	Examples
Colors	Conversations	Reading	Guest Lecturers
Charts	Audio Tapes	Written Feedback	Demonstrations
Written Texts	Video+Audio	Note Taking	Physical Activity
Different Fonts	Seminars	Essays	Constructing
Spatial Arrangement	Music	Multiple Choice	Role Play
Designs	Drama	Bibliographies	Working Models

Figure 2. 8: Activities that accommodate VARK learning styles (Adapted from Fleming, 2001, 20 January 2016)

Figure 2.8 shows the visual strategies that educators can adopt to their methodology of teaching that are shown in this learning style are proposed to constitute as diagrams, graphs, colours, charts, written texts, different fonts, spatial arrangements and design.

2.3.1.4. Felder–Silverman Learning/Teaching Style Model

The Felder–Silverman Learning and Teaching Style Model (Felder & Silverman, 1988) defines learning style as "the characteristic strengths and preferences in the ways individuals take in and process information" (Felder & Silverman, 1988: 674). It asserts that individuals have preferences "the Active-Reflective, the Sensing-Intuitive, the Verbal-Visual, the Sequential-Global, and the Intuitive-Deductive." These are represented in Figure 9: Felder–Silverman Model (Felder & Silverman, 1988). I will make secondary reference to the visual component of this learning style.

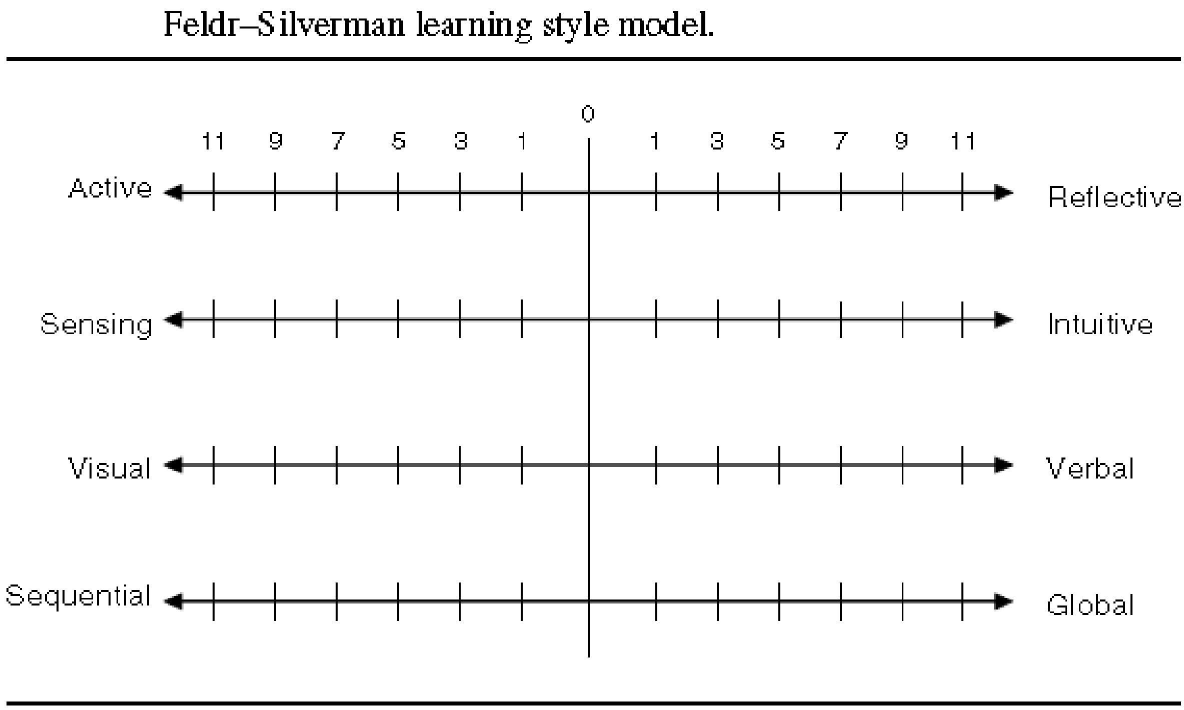


Figure 2. 9: Felder-Silverman learning style Model (Adapted from Felder & Silverman, 1988, 20 May 2016)

Individual students have preferences along each of the four processes but can learn to function in more than one. Intuiting students prefer ideas and theories, chiefly when they can take hold of new ideas and innovation. Verbal learners like to hear their information and engage in discussion, especially when they can speak and hear their own words. Visual learners like words, pictures, symbols, flow charts, diagrams, and reading books. Sequential learners prefer linear reasoning, step-by-step procedures, and material that come to them in a steady stream. Global learners are strong integrators and synthesizers, making intuitive discoveries and connections to see the overall system or pattern. Felder and Silverman (1988) discuss a number of teaching approaches useful to match the learning preferences. Visual learners want to see pictures, diagrams, flow charts, films, and demonstrations. Verbal learners like hearing and discussing information, taping lectures, and explaining themselves. Sequential learners like to move step-by-step through the material, progress logically to the solution to a problem. Global learners want to see the big picture, take in information randomly before putting it all together, and work intuitively.

2.4. The uses of visual strategies are controversial

At university level, students learn various complicated mathematical concepts, interrelating with multiple forms of representation such as diagrams, graphs, and any image associated with mathematics can bring exceptional benefits to the student. Unfortunately, there is considerable research to show that students often fail to exploit these benefits, and in the worse cases inappropriate amalgamation of representations can completely hinder learning. In other words, multiple representations are powerful tools but like all powerful tools they need careful handling if students are to use them productively. Dreyfus and Eisenburg (1990) showed that university students were very disinclined to visuals used in the classroom. They seemed resilient when taught by the use of visual strategies. Whilst many researchers show that visual strategies used in a classroom may benefit students, Dreyfus and Eisenburg show students' reluctance in the use of visuals. Many students prefer visuals however there are just as many students who shy away from visuals used in their classroom. Many students prefer simple and less complex methods when being taught such as, limited colour usage and not too many visuals as research shows that it can be confusing. "The wide use of visual images by students is not always effective in problem solving and can lead to erroneous solutions" (Lean & Clement, 1981: 6; Presmeg, 1992: 596). Mathematics as a subject discipline already poses a problem for students to master, the use of visuals to understand mathematics could be a problem added on a problem. Learners now have to understand the visuals to understand the mathematics behind it. The use of visuals can therefore bring forth erroneous solutions as stated by Lean & Clement. This poses a predicament especially if students do not understand the visuals displayed used to teach mathematics.

Healy and Hoyles (1997: 67) advocate that "Students of mathematics, unlike mathematicians, rarely exploit the considerable potential of visual approaches to meaningful learning. Where the mathematical agenda is identified with symbolic representations, students are reluctant to engage with visual modes of reasoning." Healy and Hoyles show much relevance in saying that students of mathematics are not mathematicians. Students do not automatically take a liking to visuals of symbolic representations. Many lack the ability to reason visually. Anecdotal evidence shows that in a university population of students, the majority of students lack basic reasoning ability and skill to perform simple tasks. Most students who are placed in the tertiary system have obtained below the standard performing rate in mathematics and do not have the ability to reason well mathematically. Recent studies (Vogel, 1997: 75; Orhun, 2005: 399; Britton, 2005 , New, Sharma & Yardley, 2005: 7; Lazarowitz & Lieb, 2006: 741) have shown that most senior secondary learners around the world function at below the average of expected basic mathematical competencies. This problem persists through to their tertiary education, and Britton, et al. (2005: 13) highlight that “lecturers complain that students either do not have sufficient mathematics or are unable to apply it in context.” Zimmermann (1991: 173), states that conceptually, the role of visual thinking is so fundamental to the understanding of mathematics and used calculus to show the importance of visual strategies in mathematics. He states that when doing calculus it is difficult to imagine a successful calculus course which does not emphasize the visual elements of the subject. Students of previous years in university showed a lack of mathematical ability especially when required to bring about a visualisation either externally on internally. Visuals lend themselves to a special acquisition skill to mathematics and can be burdensome for students.

Visualisation amongst many researchers can be regarded as a powerful tool to explore mathematics. It certainly allows for minimizing the complex nature when dealing with the magnitude of conceptual understanding in mathematics. However there also seems to be many limitations and difficulties around the use of visual strategies and the reluctance to visualise has also been a huge debate. “Visual techniques which are not always procedurally safe routines are considered to be cognitively demanding than analytical techniques”. (Arcavi, 2003: 235). In contradiction, visual strategies used to promote visualisation in a classroom are regarded as an important conceptual development to mathematics. According to Rosken and Rolka (2006: 456), “to understand a formal mathematical concept requires the learner to generate a concept image for it.” However according to Vinner (1997: 63) nevertheless, the intuitive mode of thinking just misleads us. It is convincingly true that some concepts in visualisation seem inherently important such as the use of visual strategies used in a classroom. According to Rosken & Rolka (2006: 457) on the one hand, visualisation proves to be a useful strategy used in a classroom and the common proverb ‘a picture says a thousand words’ seems to be inherently true. However on the other hand, it is true to say that a picture is only a thousand words. A chosen visualisation may only represent a selected concept thereby limiting a student. They also argue that even if students use visuals in a classroom they may not use the visuals in an attempt to even get the correct solutions to mathematics. I do however disagree with Rosken and Rolka in that attempting a methodology of visual strategies is better than no attempt. According to Krutetskii (1976: 200), “Visualisation is distinct from the use of logical reasoning in mathematics, which defines mathematical ability”. He claims that visualisation is often useful but it may not essentially be to the highest achievement in mathematics. He says that it may even hinder mathematical thinking if not used carefully. Presmeg (1986, 1997) found that high school students who were identified by their teachers as outstanding students in their mathematics achievement were almost always non-visualisers, while students who showed a preference for the use of visualisation often experienced difficulties in mathematics. Presmeg (1986) suggested that many students who have a tendency to visualize face difficulties in transcending the one case concreteness of an image or a diagram difficulty of which their teachers are not aware. Presmeg’s observations (1986: 44) suggest that students often get caught up by "the one-case concreteness of an image or diagram which may tie thought to irrelevant details, or may even introduce false data."

2.5. Artefacts and objects

Wartofsky (1979) posed an intriguing question: “What is it that makes human cognition distinctive?” His answer to the question was “the ability to make representations”. Wartofsky’s study was not restricted to tables, drawings and formulas but also included artefacts, objects and ideas that students envisage in their thoughts whilst the process of mathematical development occurs. Artefacts and objects appear to be one of the primary visual strategies that can develop the students’ mathematical conceptual process. It is articulated by Rap that, “Visual students will visualize different problems in different ways, so one type of manipulative will not be optimal for all students or all tasks. Here are some to start with: unifix cubes, legos, drinking straws, paper clips, buttons, geoboards and rubber bands, peg boards, beads and strings, checkers, and coins.” (Rapp, 2009: 5). According to Einsburg (1994: 105), “artefacts can include rods of varying lengths to represent whole numbers; balance beams to provide physical intuitions about multiplication; clock faces to illustrate modular arithmetic; and pegboards to introduce notions of geometric shapes.” Einsburg also shows these beautiful representations of artefacts but he also makes valid arguments, “Artefacts are by no means “royal roads” to sure fire mathematical understanding, the implied connection between a manipulative and its abstract referent may itself be a difficult task, requiring explicit instruction and practice.” Einsburg (1994: 111). The use of artefacts as a visual strategy tends to complicate mathematical understanding especially at university level. The student needs to firstly attain the skill of working with artefacts and objects which can be a real challenge in trying to master the skill behind working with artefacts. Students need to thereafter attempt to understand the mathematics around it, which amounts to holding much weight and burden on the student because the student is now required to learn the skill of mastering the use of artefacts and objects as well as the mathematics surrounding it. It would be much simpler to do without the complex nature of artefacts.

It is the aim of every mathematics educator to make mathematics as simple and clear for the student to be motivated enough to attempt the subject and understand mathematics rather then become overwhelmed by the various methods of doing mathematics. This can however be controversial as many researchers argue for the benefits of artefacts especially as a visual strategy which will be looked at in the following paragraph. It appears that not all visual strategies at university level are of benefit to the student. According to the theory of constructivism by Piaget, the use of concrete objects is linked with an earlier stage of cognition, a stage that precedes facility with abstract concepts. The use of artefacts at university level requires much more thinking power on the part of the student. According to Piaget the use of objects becomes functional at an early age in comparison to university students working with artefacts. Einsburg (1994: 115) also discusses "The culture of higher-level mathematics is undeniably more focused on issues such as proof and symbol manipulation", hence the use of semiotics is discussed later on. Vygotsky showed that people use artefacts in attaining goals that would not have been otherwise recognised. He states that mental activities are supported and developed by signs which he refers to as an internalisation process which he called psychological tools. In a Vygotskian perspective there seems to be unfathomable correlation between signs and artefacts. "The invention and use of signs as auxiliary means of solving a given psychological problem". Rabardel articulates the profound connection between signs and artefacts. Rabardel (2002: 18) states "that an artefact is a material or symbolic object per se." When artefacts are exposed to the process of solving, a double semiotic link is recognized, which is discussed later on.

2.6. Gestures

2.6.1. Definition of gestures

A gesture can be considered as part of a resource used in a mathematics classroom. Gestures include speech, inscriptions, and artefacts. According to Hit et al. (2009: 137), “Gesture offers students a second window into the task, one that students do take advantage of. If gestures were to become recognized as an integral and inevitable part of conversation in a teaching situation, it could perhaps be harnessed, offering teachers an excellent vehicle for presenting to their students a second perspective on the task at hand”. Gestures can be seen as one of the semiotic tools used by teachers and students in mathematical teaching and learning. A gesture used in a classroom may seem rather natural and irrelevant but research shows that a simple gesture can be the most valuable information transmitter to the student. In the context of this study a gesture can consist of speech or the manner in which an educator expresses his/her words. It could be an artefact as mentioned earlier, the presentation of an object or even an inscription. An inscription could be the manner in which an educator presents a message or writing. Sfard (2009: 39) explains that “Gestures are facial expressions, tone of voice, sound production, eye motion, body poise, gaze”. Gestures can be considered as a fundamental visual strategy needed for the interpretation and communication between the educator and student. According to Roth (2001: 365), teachers employ many gestural resources crucial for understanding a concept. Students depend on the speech of teachers and their hand movements or facial expressions to gain knowledge and understanding. According to Hit et al. (2009: 138), “Gestures are seen as one of the semiotic tools used by students and educators in mathematics teaching and learning.” Gestures can be seen as a component of semiotics.

2.6.2. The role of gestures

The role of gestures in a mathematics classroom can be taken for granted, though holds much significance as a visual strategy. It can be regarded as a crucial semiotic resource. According to Radford (1998: 14), gestures are part of an explanatory model, proposed by neurophysiologists as an information process theory. The use of gestures allows students to mathematically interpret much more compared to eliminating the use of gestures used in a classroom. Radford (1998: 14) states, “gestures are part of those means that allow the students to objectify knowledge that is to become aware of conceptual aspects. In addition to gestures, they include signs, graphs, formulas, tables, drawings, words, calculators, rules and so on.”

Gestures are a component of semiotics and play an integral function in the mathematical cognitive process of the student. Gestures also help the educator as a visual strategy by making their intentions of mathematics apparent to the student. Radford also explains that "Gestures help the student to notice abstract mathematical relationships and to become aware of conceptual aspects of mathematical objects".

2.6.3. Gestures and multimodality

Bjuland, et al. (2008: 271) state that "We conceive mathematics as a semiotic system; we use the multimodal approach to analyse the pupils dialogues when solving a task. The theoretical approach combines elements from the semiotic and multiple-representation framework". Bjuland et al., describe the multimodal process as a process that is used to discuss gestures. Multimodal components are considered to be expressions, speech, gestures, and written inscriptions that develop synchronically and are a paradigm that specifically analyses gestures, in particular, speech, gestures and inscriptions and their relationship to each other. Research over the years has shown that there is a paradigm of multimodality which has developed in many fields and has been shown to play a crucial role in visualisation in mathematics. According to Hit et al. (2009: 139) "language is inherently multimodal in this sense, that is, it uses many modalities linked together like sight, hearing, touch, motor actions." In addition, Radford (2003: 18) discusses the major components of the objectification process. This process observes deictic gestures which are defined by Mc. Neil as "pointing movements, which are prototypically performed with the pointing finger" (2006: 8). These kinds of gestures have an important function pedagogically. Similarly, Edwards (2005) reported that almost all gestures produced in the solution of a problem, related to fractions, by prospective teachers were of benefit to the student in aiding understanding. Nemirovsky and Ferrara (2009: 159), term multimodality as 'utterance'. It encompasses multimodal aspects especially with regard to body activity or as commonly referred to as gestures. They define 'utterance' as including multimodal aspects such as "facial expressions, gesture, tone of voice sound production, eye motion, body poise, and gaze"

2.7. The theory of Semiosis

The theory of Semiosis is based on symbols which include signs such as icons and index (Kaput, 1991: 53). Semiotics has its roots from both European tradition founded by de Saussure and the American tradition based on the works of Pierce and Moriss (Anderson, 1990). Kaput brings forth a simple yet true meaning behind the visual strategies used to develop skills in mathematics. He claims that "It is true that if learners can understand the language of symbols in mathematics, there may exist an increase in knowledge" (Kaput, 2009: 212), whilst Radford (2003: 21) argues that "Semiosis are not just icons, symbols and index, it may include something as simple as a gesture". According to Sfard (2008: 40) "facial expressions, tone of voice, eye motion, body poise, gaze all impact the students' internal image and hence the development of mathematics". Lemke (2003: 217) argues that "Mathematics can be best learned and taught as an integral component of a larger sense making resource system which also includes natural language, mathematics, and visual representations". There are many semiotic systems that explain the concept of Semiosis, however drawing from researchers; Semiosis helps comprehend the meaning behind natural language, mathematics and visual representation. Semiotics helps educators and students understand how mathematics functions as a strategy for doing mathematics in conjunction with natural language. Lemke (2003:218) refers to mathematics as a semiotic beast.

Mathematical Symbolism originated as abbreviations for the Greek, Latin and modern European words and phrases (Cajori, 1928). Moreover, Pierce (1998), defined semiotics as "signs that can be iconic, indexical or symbolic." An icon can represent an object that imitates its purpose and structure. An index represents the sign which is the effect produced of the object and a symbol refers to the object by virtue of law. Semiotics used at tertiary level is one of the most common and frequent visual strategies used today. Many mathematicians avoided the use of symbols shown over the years by previous pieces of work and are also mentioned by past researchers. According to Stylianou (2002: 310) "mathematicians avoid not only the use of words but also algebraic symbols that have a preference for vague images". Mathematicians would write their arguments out in words, sometimes aided by diagrams as opposed to using symbols. Lemke (2003:216) states that "It is perfectly possible to have mathematics without mathematical symbols, and there was a long tradition in which mathematicians, particularly in geometry, from Euclid to the early 18th century that avoided symbols and wrote their arguments out in words." Lemke (2003: 216) argues that abbreviations for words and mathematical sentences were about kinds of meanings that natural language has trouble articulating. Presmeg used an interesting example to make meaning of Pierce's definition of semiotics. She used an introduction of trigonometry. A right angle triangle was said to be both iconic and symbolic by use of a diagram. Learners memorising the acronyms SOH CAH TOA would fall under symbolic. Representing a coordinate by separation of a semi colon would be indexical. Pierce distinguishes these signs not in isolation but as part of an on-going process of Semiosis in order to make meaning.

Presmeg (2006: 205) asserts that semiotics is related to the activity of Semiosis or the study of Semiosis which is a term that comprises of any action or sign process. Lemke (2003: 216) argues that "what makes it mathematics, wherever we find it, is its characteristic ways of doing things: calculating, symbolizing, deriving, and analysing". The last 30 years has created a paradigm into which we are clustered by the use of symbols today, in mathematics. In most mathematical research before the modern age, symbolic language was uncommon. Mathematics was merely verbal texts in which mathematicians were meant to read out in words. Mathematical symbolism originated as abbreviations for the Greek. According to Lemke, " the use of language and words has trouble articulating what the use of symbols in math can do, it is just this dual nature of mathematical symbolism as a semiotic that makes it both so powerful and so difficult to understand unless the connections to more familiar natural language and to visual gestural semiotic made clear". There lies the dual nature of semiotics, whilst on one hand semiotics can be meaningful, it is important to understand the underlying meaning behind these symbols. There are many difficulties of learning symbols in mathematics and one of the reasons for poor mathematical results is due to poor understanding of mathematical symbols. This can be regarded as semantic in nature, that is, the meaning behind these signs and symbols used in mathematics. Semiotics was initially revealed by Volshinov (1884: 10). In addition, Radford (2000: 19) states that "The understanding of a sign is, after all, an act of reference between the sign apprehended and other, already known signs; In other words, understanding is a response to a sign with signs and this chain of understanding, moving from sign to sign and then to a new sign, is perfectly consistent and continuous: from one link of a semiotic nature we proceed uninterruptedly to another link of exactly the same nature". On the one hand, the use of semiotic representation for mathematical thinking is crucial because unlike other fields of knowledge, there is no other way of gaining access to mathematical objects but to produce semiotic representations. On the other hand, the understanding of mathematics requires not confusing the mathematical objects with the used representations. Bjuland (2008: 272) also states “Thinking and mathematical reasoning are not just mental but are realized through semiotic activity.”

History shows that the progress in mathematics has been linked to the development of several semiotic systems. For example, semiotic notations stemmed from written language have led to algebraic writing since the 19^{th} century. For imagery there was the construction of plane figures with tools, then the graphs in order to translate curves into equations. Each new semiotic system provided specific means of representation and processing for mathematical thinking. Lemke (2003: 217) historically views the evolution of symbolism and states that it may have evolved from natural language because mathematical texts were initially written in the form of verbal ‘rhetorical algebra’. These texts contained detailed verbal instructions about what was to be done for the solution of a problem. He states that in later research these appeared as abbreviations. Among the types of representations described, Kaput (1991: 53) presented mathematical representations using one mathematical structure to represent another, and external symbolic representation using concrete objects to represent abstract ideas. Thus Kaput suggested that different representation forms may ease communication of mathematical ideas, the words “one-half”, the symbolic notation of “1/2”, and a picture of half of an object all represent the same concept and more specifically, that concrete objects are a legitimate format for communicating abstract concepts. The variety of representation forms that exist opens the possibilities of how students can communicate their mathematical understanding (Putnam et al., 1990: 57; Sternberg & Grigorenko, 2004: 274).

2.8. Semantics

There is a close relationship between semiotics and semantics in the study of mathematics education. “The history of mathematical speaking and writing is a history of the gradual extension of the semantic reach of natural language into new domains of meaning.” Interest in mathematical language has steadily grown over the last decade. As Lemke (1998: 1175) explains, semantically mathematical symbolism exceeds the potential of language. O’Halloran (1996) states that the following role of natural language in mathematics means that semantic extensions must have also taken place in semiotics to integrate the development of mathematical symbolism. The interaction between the two semiotic systems of mathematical symbolism has resulted in further semantic expansions that would have otherwise been possible. Lemke (2003: 218) states that mathematical symbolism may have been a bridge that gapped between linguistic description and perceptual reality. The link however between the text based descriptions and in the form of symbolic statements to visual representations meant mathematical symbolism rapidly developed as an integrated semiotic system. It is therefore not possible to separate semantics from semiotics. Semantic extensions in mathematics perhaps first took place with the early development of numerical systems where textual processes were gradually replaced with arithmetical processes. Kaput argues "Instead of teaching syntax which would produce student alienation we should be teaching semantics" (Kaput, 1987: 19).

2.9. Pierce's Semiotic Triads.

2.9.1. Triadic Model

Semiotics is the study of signs (Colapetro, 1993). Pierce discusses basic components that are designated as an 'object'. Pierce's Semiotic Triads make reference to iconic, indexical and symbolic signs. A 'representamen' is standing for this object and an 'intepretant' is the result of interpreting the relationship as shown in figure 10. In mathematics, the objects referred to could be mere vocabulary in context, for e.g. 'point', 'line', 'plane'. We see these objects and communicate to others about them. It is this interpreted relationship between a sign and its object that constitutes a specific mathematical function. Pierce (1998) discusses a useful trichotomy, and states that signs may be iconic, indexical, or symbolic. He also states that these types are not inherent in the signs themselves but depend on the interpretations of their constituent relationships between signs and objects. In simple terms, an iconic, sign and the object share physical similarities, for example, a photograph of a person representing the actual person. Pierce states that "the nature of symbolic signs is that there is an element of convention in relation to a particular sign."

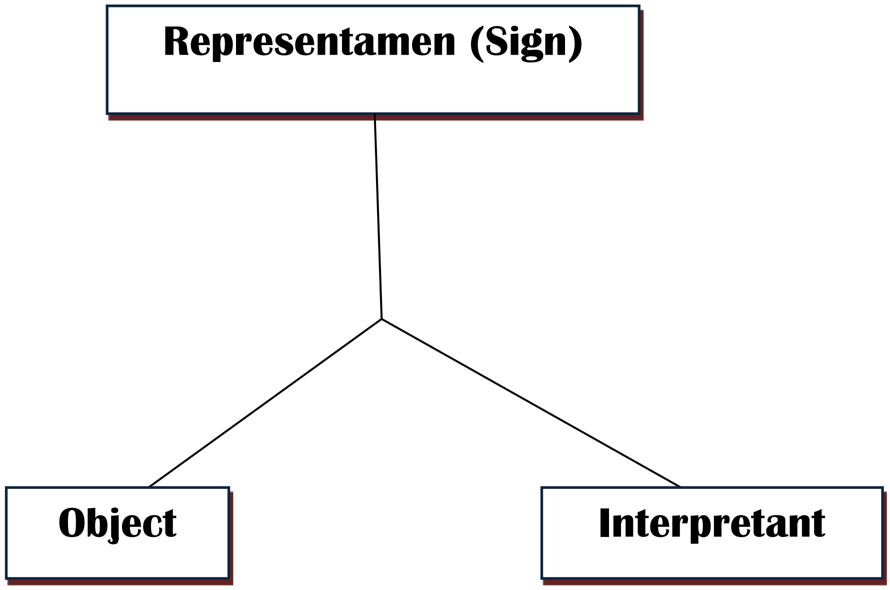


Figure 2. 10: Pierce's Semiotic Model

Cunningham (2005: 24) discusses the sign mediation between an object and interpretant. A sign is not the object itself, but is an incomplete representation of the object. A sign can only represent certain aspects of the object of which some aspects can even be irrelevant. Cunningham also states that sign theory offers a significant advantage to the analysis of the mind and says that systems of signs act as a code for some systems of objects. He identifies a special inference called abduction which is a type of thinking or reasoning that invents signs to make sense of new experiences.

Lemke (2003: 218) identifies two types of semiotics in mathematics which he refers to as Topology and Typology. Typological semiotics represent meanings by categories such as spoken words, written words and mathematical symbols which are discrete, point-like and distinctive signs. In contrast, topological semiotics makes meaning by degree such as size, shape, position, colour spectrum and quantitative representations in mathematics. The differences between topological and typological semiosis are shown in figure 11.

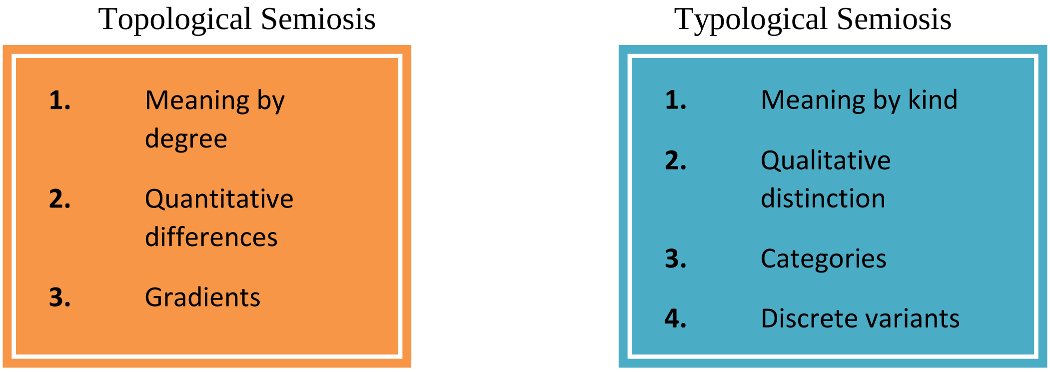


Figure 2. 11: Topological versus Typological – Semiosis by Lemke (2001)

2.9.2. A Taxonomy of inscriptions

Marcou & Gagatsis (2003: 247) examine Pierce’s triad of iconic, indexical and symbolic signs by further developing their internal and external representations which included the following pairs of characteristics: descriptive and depictive, polysemic and monosemic, autonomous and auxillary. These dichotomies of opposing strengths to semiotics will be carefully analysed.

2.9.2.1. Descriptive and depictive sign agents

Descriptive and depictive sign agents aim to communicate with words and pictures respectively, or to symbols used in mathematics. Words and symbols work in harmony therefore the descriptive relationship to mathematical objects may be interpreted as symbolic. In contrast depictive visual representations are iconic because they share physical resemblance to mathematical objects. This characterization lends itself to a theoretical classification of symbolic representation which makes reference to a descriptive sign agent and iconic which makes reference to the depictive sign agent. Presmeg argues that in actual fact these characterizations would appear more subtle in the context of a classroom as interpretations are made more by the student. Presmeg uses a good example to illustrate this. He uses the quadratic formula to explain these agents in term of his triad.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Owing to its symbolic nature, “the interpreted relationship of this inscription with its mathematical object may be characterized as symbolic” (Pierce, 1998). It is dependent however in classifying because the sign could be characterised as iconic or indexical.

2.9.2.2. Polysemic and monosemic sign agents

Owing to differing interpretants, mathematical sign agents are rarely monosemic. The symbols $\sum$ or $\int$ may be understood in two different ways. According to Presmeg (1998), it can be regarded as a conventional symbol for a sum and an integral respectively, or that can be recognized as indices pointing to the need to perform the operations of finding a sum or integrating. These can be regarded as polysemic in nature.

2.9.2.3. Autnomous and Auxillary sign agents

Symbols such as $\sum$ or $\int$ that are polysemic can be regarded as autonomous rather than auxillary. According to Presmeg(1998), “When diagrams are drawn in attempting to make sense of a mathematical problem in which no diagram is given, such diagrams are auxillary if the problem could have been solved without them”.

2.9.2.4. Semiotic means of objectification

To attempt to master the true semiotic means of objectification, the works of Vygotsky will be scrutinized. Vygotsky (1978) draws from phenomenology and works closely with a semiotic cultural perspective. The term objectification is categorized firstly in the word ‘object’ which means to throw something in the way, whilst ‘tification’ means to do or make. According to Radford (1998: 21), objectification relates to throwing something in front of somebody making something visible. In this context, it is a sign or symbol used when teaching mathematics. Kant states, “The only way that an object can be given to us by the mind is by representations of the object.” Hussel (1958), the founder of phenomenology held a similar meaning stating that conceptual thinking can be given to us in “ways of appearance”. Duval (1999: 3) states that a semiotic representation is produced with signs and rules of use that can play a fundamental role in objectification and the process of knowledge. There exists a pedagogical and epistemological role of representations in the semiotic means of objectification. Rules and signs are taken here as a semiotic means of objectification. A good example would be when an educator uses a ruler and marker for the purpose of measuring and writing, which is of a meaningful function. Signs used in mathematics however play different roles in the objectification process of knowledge. A semiotic analysis may help understand the various sign systems such as speech, writings and gestures to be discussed in the following paragraphs. Semiotic means of objectification, bearers of an embodied intelligence, can also be cross examined from a cultural perspective. Radford (1998: 21), states that “The alluded cultural signifying forms account for the ways in which the individuals enter into contact with the use of objects, tools, signs, and other means of objectification.” Radford (1998: 21) states that the semiotic means of objectification also appear embedded in a socio-psycho-semiotic meaning making process framed by cultural modes of knowing that encourage and legitimize forms of signs and tools. He also looks at it from an epistemological view and states that the semiotic means of objectification are already culturally endowed with specific ways to use.

2.9.3. The Semiotic Bundle

The semiotic bundle is a theoretical model. According to Azarello et al. (1994a: 15), “A semiotic bundle is a system of signs made of the signs that are produced by a student or by a group of students while solving a problem and/or discussing a mathematical question. Possibly, the teacher too participates in this production, and so the semiotic bundle may include also the signs produced by the teacher.” Azarello describes the semiotic bundle as a dynamic structure, as there is a deep relationship amongst signs and also refers to the dual nature of certain signs. The semiotic bundle allows students to describe the multimodal semiotic activity in their various modules and the change of these signs as well as the association these signs have with other signs. The semiotic structure is a meaningful model with respect to the relationship amongst signs and the purposeful function it creates amongst students. Azarello states, the semiotic bundle can be analysed in two parts: a synchronic analysis and diachronic analysis. In addition, Azarello et al. (1994a: 16) state, “A Synchronic analysis considers the relationships amongst different semiotic resources simultaneously activated by the subjects at a certain moment”. For example, drawing a tangent to the graph of a function and sketched by students at different points of the graphs. Azarello (1994a: 16) further adds, “The diachronic analysis focuses on evolution of signs activated by subjects in short or long periods of time.” For example a student may be asked to find the derivative of a trinomial, but may have difficulty. Consequently, the educator assists by using a bundle of semiotics or formulae to assist the student. The synchronic and diachronic analysis therefore together show the roles that the different types of signs such as gestures, speech and inscriptions have especially on the students’ cognitive processes. Semiotics has the tendency to play a dual role and support thinking processes of students enhancing the ability to grasp these dual roles amongst signs.

2.9.4. Semiotic Nodes

The uses of semiotics are remarkably useful in the mathematics classroom especially as a visual strategic tool; however they do not work in isolation. According to Radford et al. (2003: 55), knowledge objectification can be regarded as a multi-semiotic mediated activity. He describes this as a semiotic node. He explains that a semiotic node is a piece of the students' semiotic activity where action and diverse signs (e.g. gestures, words, formula) work together to achieve knowledge objectification. Drawing from the above, it is clear that the aim of Radford's definition of gestures is the developmental tool of mathematical knowledge. A semiotic node has the ability to generate a deepening understanding of how speech, gestures and the various different relationships between signs especially if we are to adopt Radford's theory is that knowledge objectification is a multi-semiotic mediated phenomenon.

2.9.5. Radford's Cultural-semiotic approach.

Radford (2008: 218) adopts a 'socio-cultural and phenomenological' view to semiotics which characterises activities that are reflexive. This is termed 'reflexive mediated activity'. This can be regarded as an essential process of cognition. It also constitutes the materialization of mathematical objects. This approach is associated with semiotic resources and students' cognition, within a social practice and a cultural and historical facet. Mathematical objects can be as a set blueprint that emanate from the reflexive mediated activity and are intertwined within the mediated activity which seek a form of ideality. Learning is considered an objectification process accomplished through a reflexive activity, a meaning making process that allows students to become aware of the mathematical objects that exist in the culture that the student does not recognize. The complexity of the objectification process requires broadening the concept of a sign and going beyond its representational role, since signs culturally mediate activity and direct the individual's intention towards the mathematical object. Signs are termed as semiotic means of objectification and they include artefacts, gestures, language and rhythm. Semiotic means of objectification stratify the mathematical object into levels of generality according the reflexive activity they mediate.

Meaning is no longer a mere relation sign-object, but is deeply interwoven with the reflexive activity, with intentional acts culturally mediated by semiotic means of objectification.

Meaning is a double sided procedure with a personal and a cultural aspect. The personal aspect refers to the individual's intentional acts directed towards a cultural unitary object. The cultural dimension refers to cultural and historical features that are condensed in the general and interpersonal mathematical object brought to the individual by teaching activities. The expected outcome of learning as an objectification process is the alignment of the personal meaning with the cultural meaning.

2.9.6. Vygotskian Semiotic Conception

The Vygotskian semiotic conception states that "sign operations are not simply invented by children or transmitted from adults; they appear from something which initially is not a sign operation and which becomes a sign operation after a succession of qualitative conversion" (Vygotsky & Luria, 1994: 99). Vygotsky emphasizes that the sign operation is the result of a complex developmental process. He also comprehensively discusses the 'natural history of sign' which is extensively analysed below. Vygotsky also argues that, "at the beginning of its development every high psychological function inevitably carries out the character of external activity. At the beginning, a sign represents, as a rule, an external auxiliary stimulus, an external mean of auto stimulation" (p. 00). Vygotsky further interprets the semiotic conception and affirms "in order to become a sign of the thing (word), the stimulus must have a support in qualities of a signified object" The preparation of the stage of a child's crucial point in time is the natural history of a sign as a relationship between an object, and sign is shown in Figure 12.

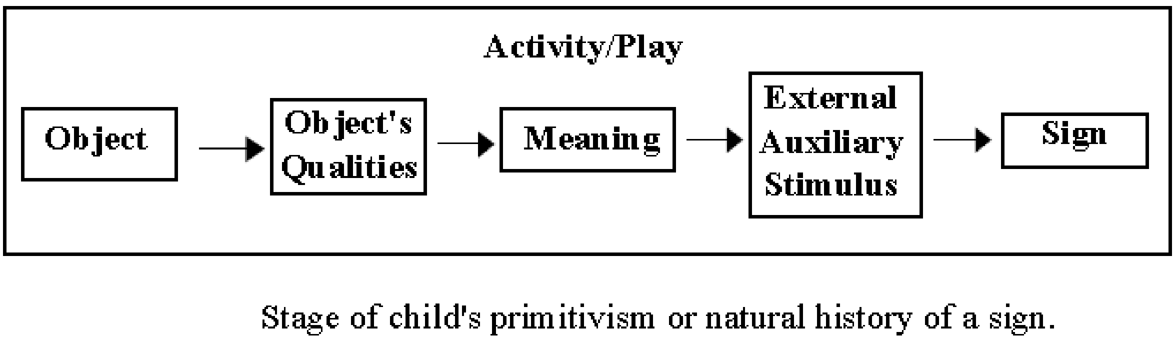


Figure 2. 12: History of a sign (Adapted from Vygotsky, 1994, 10 February 2016)

Through a sequence of qualitative alteration brought about through activities of children's development connections between an object, its meaning, and a sign that represent the object (Figure 13).

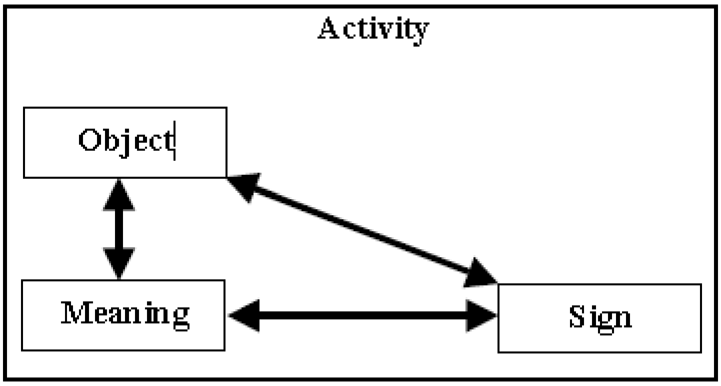


Figure 2. 13: The relationship between an object, its meaning, and a sign. (Adapted from Vygotsky, 1994, 19 May 2016)

According to Vygotsky, the "natural history of a sign" is giving a title or the development of a connection between an object, its meaning and name. Naming is a very hard psychological act for a child. To exemplify the stage of a child's primitivism Vygotsky uses this example: A child sees a word as a connection to an object by the characteristics it possesses and its overall structure.

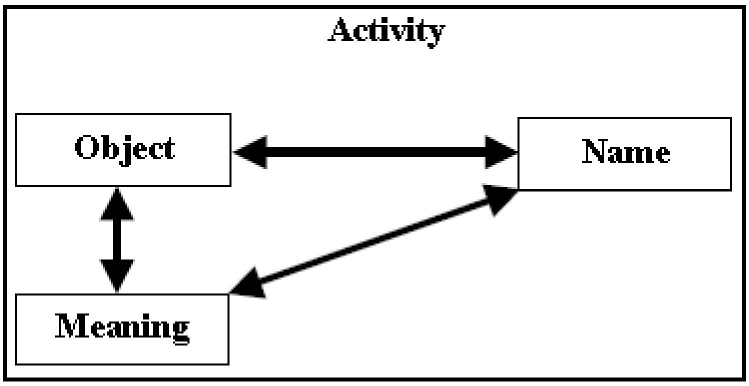


Figure 2. 14: The relationship between an object, its meaning, and a name (Adapted from Vygotsky, 1994, 19 May 2016)

This semiotic chain clenches on the connection made between an object or signified and is referred to as a sign/signifier. An informal type of activity such as play is a vital part of a child's semiotic development. During playing and interacting socially, a child changes normal external forms of semiotic behaviour into internal structures. According to Vygostsky, "The internalization of cultural forms of behaviour involves the reconstruction of psychological activity on the basis of sign operation" (Vygotsky, 1978: 57). The sign becomes a mediator between external social interaction and a child's internal being by means of cognition.

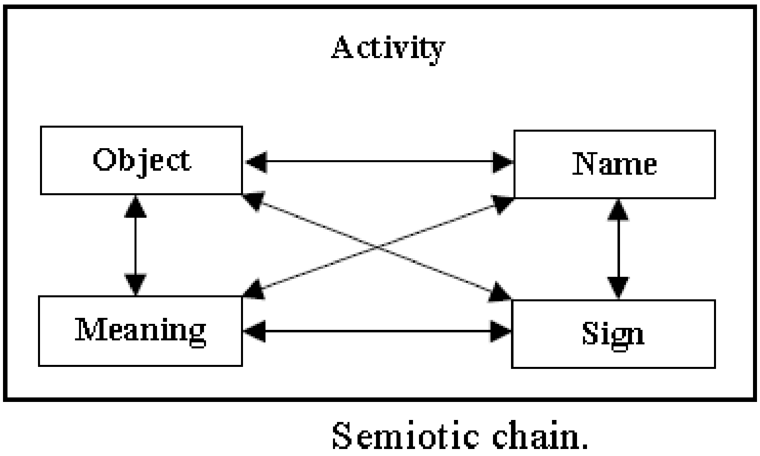


Figure 2. 15: Semiotic Chain (Adapted from Vygotsky, 1994, 19 May 2016)

2.10. Visual Students

There is much difference in being a visual student as compared to an average student; to be a visual student operates by linking the brain to what is seen and thereby connecting thought processes in pictorial representations. Arcavi states. "The largest part of the cerebrum is involved in vision and in the visual control movement, the perception and elaboration of words, and the form of colour objects. The optic nerve contains over 1 million fibres, compared to the 50 000 in the auditory nerve" (Arcavi, 2003: 215). A visual student is referred to as such by visual thoughts he or she envisages when learning occurs. In the context of mathematics, a visual student is diagnosed as a 'visual student' owing to thinking and doing mathematics by picture representations or visuals. Visual students often understand complex mathematical problems much more easily than simple ones. The explanation for this involves the roles of the two hemispheres of the brain, which deal with information processing in very different ways. A student's brain consists of a left and right hemisphere. Silverman states, "The left hemisphere better handles recall, memorization tasks, verbal fluency, syntax and grammar, time, and sequence. This is the visual student's weaker hemisphere. The right hemisphere of the brain better handles visualisation, synthesis, spatial orientation, and broader concept formation. This is the visual student's stronger hemisphere." (Silverman, 2002:10). A straightforward mathematical activity relating to knowledge recall is usually associated with the left hemisphere of the brain, so the visual student is functioning at a disadvantage. Silverman argues that when a mathematical activity becomes more complex, requiring application, synthesis or evaluation of information, both hemispheres of the brain are engaged together. The visual student however only uses his preferred and stronger hemisphere, so the complex task is easier to accomplish.

In a classroom context, research shown earlier shows that there are more visual students which could possibly be biological more than psychological. Presmeg (2008: 83) stated "A visual image is taken to be a mental construct depicting visual or spatial information, and a visualiser is a person who prefers to use visual methods when there is a choice." A visual student gains knowledge by means of images. Most visual students are defined from an early stage; one of the first indications to identify a visual student can be the love of television, playing with puzzles, computer games and board games such as chess. However since the context of this study deals with university students, at a much later stage a student can initially be detected as a visual student by reading a map easily or remembering directions, driving to a place once and easily remembering its route the next time. There are many types of students with different 'learning styles' defined by Gardner. He mentions there are visualisers, verbalizers and mixers as mentioned earlier. These learning styles show that not all learners acquire information in the same way. Visualisers are distinct individuals who study and gain knowledge only by the use of visuals. Visual students learn best by using real life examples or better yet placing them in the context of a mathematics problem. For example, when teaching visual students financial mathematics (hire purchase), exposing students to a store that sells appliances on hire purchase gives them a visual representation of what this really means. Visual students always learn best by understanding the motive behind knowing what they are learning. Most visual students are taught using auditory and sequential methods of teaching which can be regarded as ineffective methods of teaching. Visual students may learn best by using of various visual strategies to promote understanding especially in mathematics. Visual learners find this baffling. They much prefer to learn from whole-to-part, rather than the other way around. They also really need to understand why they are learning something, before they can find the motivation to give it their best. Visual students tend to think in a unique manner. Everything done in mathematics needs a motive and reasoning behind it. Mathematics can be seen as a 'problem solving based' subject which does not really require a motive. Therefore, at tertiary level, visual students still need much colour to help understand mathematics so that even if the motive does not become clear, the outward look of mathematics becomes pleasing to the eye. In addition, Silverman (2002: 10) shows crucial aspects of a visual student. He states visual spatial students are right hemispheric students, who think primarily in pictures and relate well to space but not to time. They are whole concept students, who read maps well, have a unique method of organisation and also learn best by seeing relationships or patterns and learn complex concepts easier than simpler ones.

Possible visual strategies used in a classroom could be the use of PowerPoint's animations to highlight symbols when teaching mathematics. At tertiary level the use of multimedia software to represent mathematics in many forms like 3 D cartoons showing mathematics could be highly effective. Jones articulates, "Visual learners (and kinaesthetic learners) learn basic math facts much easier when they can represent them with 3D manipulative. Blocks, cubes, play money (or real money), and dice can all be invaluable tools for helping the visual learner to "see" how all the math facts work together" (Jones, 2014). Students at this stage love technology and would take an interest in animations and multimedia software used, especially if the student is regarded as a visualiser. Visual students learn best with these artefacts and objects. Whilst the previous paragraphs showed some negative aspects of artefacts, it is shown that with visual students artefacts and objects are extremely beneficial for the students' absorption of the knowledge process.

Visual students, according to Presmeg (1985) do not tend to be the most successful academic performers. They are regarded as low achievers because there is a definite mismatch between a preferred learning style and verbal teaching. Woolner (2004: 269) states "Visualisers may even fail in mathematics because of a mismatch between their preferred learning style and predominance of verbal teaching." It is apparent that visual strategies do influence the way an educator teaches. Kozhevnikov et al. (2002: 48) argue that some visualisers tend to use pictorial images and suffer difficulties, while others succeed through using more abstract spatial representations. Visualisers, according to this theory have either high or low spatial ability which leads to low mathematical academic performance. Problems may also arise because there is a lack of balance between visual and verbal understanding. There seems to be a parasitic relationship amongst visual and verbal understanding toward mathematics. According to Rapp (2009: 3) "Visual students tend to acquire the disadvantage of missing the underlying mathematical concepts. They may not be able to recall math facts, nor readily be able to memorize the steps to complete the multiplication equations. Thus, visual students are not likely to get correct answers to the homework problems (academic stumbling block), subsequently leaving them with a lowered self-esteem and a perceived deficit in mathematical ability (emotional stumbling block)." Silverman (2000: 11) uses a flowchart to show exactly what happens in the mind of a visual learner.

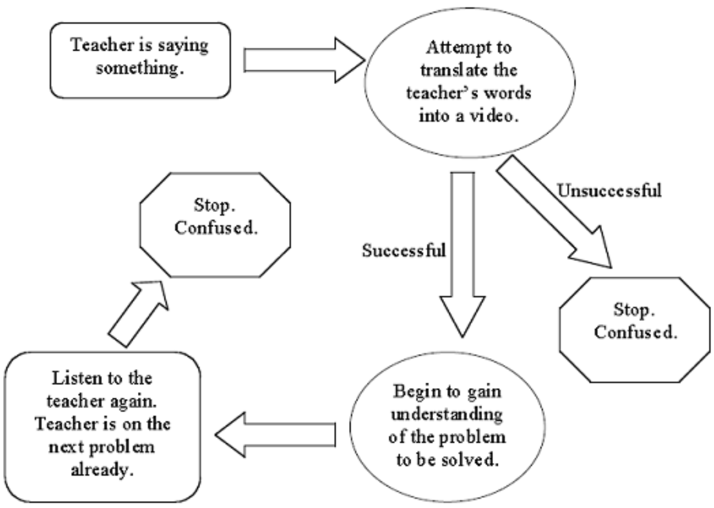


Figure 2. 16: Visual Spatial Learner Processing (Adapted from Silverman, 2002, 24 February 2016)

The visual student needs to view the information rather than hear it in order to make sense of it. The student can make sense of auditory input, but needs to interpret into visual images if any knowledge is to be gained and application of this knowledge is to occur. When an educator is presenting information verbally, the visual student is listening to the words but as mentioned by these researchers, he/she then is "actively creating a video, photograph, icon, or other image in her brain often while doodling, twirling her hair, or fiddling with an object at her desk which helps her with this translation process" (Haas, 2003: 42; Silverman, 2002: 3). This takes additional processing time, which leaves the visual student behind as compared to the rest of the students. Most visual students may be accused of daydreaming rather than paying attention to the lesson, when in fact, the visual student is actively involved in acquiring knowledge from the lesson as compared to the rest of the students in the class.

Visual students grasp concepts holistically rather than in parts. (Haas, 2003: 42; Silverman, 2002: 3).What may seem to the other students in a class like a sequence to solving a mathematics problem, to a visual student everything seems to appear muddled and erratic. If the mathematical problem is presented in a spatially meaningful way so that the visual student sees a real-world connection for finding a solution to the problem as shown in the 'hire purchase problem' earlier on, the student will arrive at a solution and therefore build a larger mathematical conceptual understanding. It may be very difficult for the visual student to display their work after this process because a visual student does not have definite set steps to follow in a specific order. It is an overall understanding of a problem and there are multiple ways of arriving at the solution. According to Rapp (2009: 1), many students will say to educators "I can't tell you how I know it. I just know it." By this the students are referring to how they have obtained an answer. This poses problems for the students when placed in a class where the educator insists that the answer is correct only when the work is shown, step-by-step. The academic impact of this is misunderstanding of mathematical concepts leads to declining grades. The emotional and psychological impact may however be much greater.

Many visual students have difficulty with writing, whether it is copying from the board or showing the steps to their work. This is owing to part of motor difficulties that accompany right-brain dominance. Freed, Kloth, & Billett (2006: 6), explain this by saying, "the very act of writing requires tremendous concentration, which takes away from the ability to focus on the task at hand. When students write, it's more difficult for them to visualize because they are looking down at the page." There are areas of mathematics that tend to be strong for visual students because they lend themselves well to visual processing and spatial reasoning. Some examples as stated by Rapp (2009: 2) are "geometry, money, Roman numerals (still a symbol system but based on a pattern and positioning of the figures), fluids, and maps". As Haas (2003) states, “this teaching strategy often works against visual student.” (p.31). However the use of visual strategy differs from all other teaching strategies as it caters for visual students.

Paivio (1971: 327) argues that most information can be encoded visually or verbally; the use of visual or verbal tendency of mathematics has created visualisers and verbalisers. Stylianou (2002: 303) emphasises the importance of visual reasoning and spatial ability to thinking for mathematical success. Therefore there is certainly an immense need for visuals in a classroom especially a university classroom as opposed to schools. Universities need to adopt visual strategies for the mere purpose of understanding visualisers and how they think. Many researchers may refer to this as visual spatial thinking. However there is also plenty of debate about the beneficial uses of visual strategies.

2.11. Visual Learner-spatial Student Model

According to (Silverman, 2002: 2) “the visual learner spatial student model is based on the newest discoveries brain research about the different functions of the hemispheres. The left hemisphere is sequential, analytical, and time-oriented. The right hemisphere perceives the whole, synthesizes, and apprehends movement in space. We only have two hemispheres, and we are doing an excellent job teaching one of them.” Silverman created an interesting tool used to detect or diagnose a student being a visual student. He called it the ‘Visual-Spatial Identifier’. There are two forms of the identifier: a self-rating questionnaire and an observer form, which was completed by parents or teachers. Results showed that, one-third of the school population emerged as strongly visual-spatial. An additional 30% showed a slight preference for the visual-spatial learning style. Only 23% were strongly auditory-sequential. This suggests that a substantial percentage of the school population would learn better using visual-spatial methods.

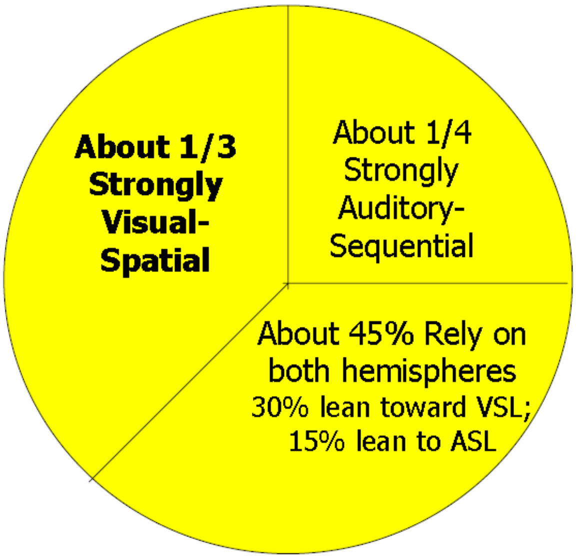


Figure 2. 17: Visual-Spatial Learner (Adapted from Silverman, 2005: www.visual-spatial.org, 10 February 2016)

According to Silverman (2002: 4), spatial and sequential dominance are two different mental organizations that affect perceptions of the student. However sequential methods of teaching take dominance in the classroom as opposed to spatial or visual methods of teaching. Visual-spatial abilities constitute the sphere of the right hemisphere whilst sequential abilities constitute the sphere of the left hemisphere. Visual students show an indication of remarkable strength in the right-hemisphere, and less capability with the left-hemisphere. It is therefore deemed necessary to teach mathematics using activities that access their right hemispheres. Silverman (2002: 5) states, "This can be done through humour, use of meaningful material, discovery learning, whole/part learning, rhythm, music, high levels of challenge, emotion, interest, hands-on experiences, fantasy and visual presentations."(p.5).

Silverman (2002: 5) uses seventeen elements to show various visual strategies that can be used in a classroom. The following strategies have been found to be effective in teaching children with visual-spatial strengths. These were some of the highlighted elements to show the effectiveness of using visual strategies in the classroom: "Use visual aids, such as overhead projectors, and visual imagery in lectures, Use manipulative materials to allow hands-on experience, Use a sight approach to reading rather than phonics, Use a visualisation approach, Group gifted visual-spatial learners together for instruction, Allow them to construct, draw, or otherwise create visual representations of concepts, Use computers so that material is presented visually.”

2.12. Heuristics

Pedagogical and didactical issues of visual strategies in mathematics used at tertiary level have not been heuristic to have received attention in recent history. George Polya (1945) was regarded as one of the most famous mathematicians to be intimately associated with visual images. He compiled a list of heuristics for successful problem solving in mathematics by offering his students to draw a figure. Mathematicians have been aware of the value of diagrams and other visual strategies both for teaching and as a heuristic for mathematical discovery. Many of the complex representational systems used to support mathematics education have a distinct number of heuristics. According to Ainsworth (2006: 183), the first heuristic is to use only the minimum number of representations that you can. If you can use one representation do so. But, if you cannot consider whether the representations that you think are necessary really are required. Secondly, carefully assess the skills and experience of the intended learners. For example, do they need support of constraining representations to stop misinterpretation of unfamiliar representations or would this extra representation not provide any new insight without a great deal of work by the student? Alternatively, they may be so experienced that the constraining representation is not needed and just adds additional work for no tangible benefit. Thirdly, consider how to sequence representations in such a way to maximise their benefits. Even if you have eight informative ways to visualise a concept, do not introduce all eight simultaneously. Allow learners to gain knowledge and confidence with fewer representations before introducing more. A fourth heuristics is to consider what extra support you need to help learners overcome all the cognitive tasks associated with learning with multiple representations. Finally, consider what pedagogical functions the multi-representational system is designed to support. If the primary goal is to support complementary functions, then it may be sufficient that learners understand each representation without understanding the relation between them (p.183).

The challenge exists for educators to identify when to select particular representations or visual strategies. Learning may be hindered if students spend a considerable time and effort in relating representations unnecessarily. According to Ainsworth (2006: 184) “If the goal is for learners to construct a deeper understanding of a domain, if they fail to relate representations, then processes like abstraction cannot occur”. Consequently, it is difficult to recommend a solution to this dilemma. Multiple representations are powerful tools to help students expand complex mathematical knowledge. But like all powerful tools, they require careful management and often considerable experience before educators can use them to their maximum effectiveness.

2.13. The Visualiser/Analyser (V/A) model

The Visualize/Analyzer model was based on Piaget’s work of perception which can be regarded as a helpful pedagogical endeavour. The model attempts to show that visualisation in mathematics works as a process to enhance distinct forms of thinking. It is categorised in forms of thinking for the process of mathematical problem solving. According to Zazkis et al., “Visualisation and analysis are two interacting and mutually supporting modes of thinking” (p.435). It is shown in terms of discrete levels of visual and analytical thinking. This model describes successive acts of visualisation in steps of analytical reasoning. The thinking as it is described in the V/A model begins with an act of visualisation which can be the actual drawing or an expression of a mental image. The V/A model has been used as a framework for the analysis of the problem solving process in mathematics. The V/A model then went through a refining process. Zazkis et al. (1996: 435) described the refined V/A model as an ordered sequence of discrete levels of visual and analytical thinking. The model shows that some students could be more visual than analytical and some more analytical than visual which can be regarded as a personal learning style. Mathematicians often follow an ordered pattern when utilizing visual representations. Visualisation and analysis can be considered as two interacting modes of thinking which support each other in the development of the understanding of mathematical concepts, rather than as a dichotomy. This model shows specific processes that advanced problem solvers’ use during this interaction between visualisation and analysis. This analysis suggests that mathematicians’ visual representation used in problem solving can be very structured when mathematicians explored the visual representations they constructed in a systematic manner. Mathematicians tend to construct visual representations in steps. The above research suggests a perspective on the issues of analysis and visualisation, presenting their interrelationship as a symbiosis rather than a rivalry of opposite poles.

2.14. Technology used as a visual strategy

The South African government has made a pledge to advance the information and communication technology (ICT) skills of individuals. “Our national government has recognized the importance of mathematics and the integration of technology in our present classrooms.” (Duncan, 2011:1). It is therefore an obligation for every educator to impart technological skills to students to prepare them for a working world that is technologically subjugated. Drawing from a broader spectrum of a real world context, visualisation of information most certainly makes the working world a smoother operational one. Engineers, doctors and analysts use visual technology to express information acquired in their frame of work. For example a doctor uses a scanning machine to detect problems found in a person. In comparison, the use of technology used in a classroom by which visuals are presented makes the life of an educator a much easier one as a scanning machine makes the life of a doctor in a real life context so much easier. If technology is a necessity in the real working world then surely it should be used as a teaching strategy at tertiary level which is a step closer before students are exposed to the real working world where technology dominates? Students must be exposed to technology in the classroom as we are emerging into a transforming world of technology of which many are visually orientated; hence universities should become visually and technologically advanced in the same regard.

The use of visual technology is an alternative strategy to improve student achievement and motivation in mathematics at university level. There is a variety of research studies done to support the effectiveness of technology used in the subject of mathematics (Cavanagh, 2007; Woodward, 2006; Whitehurst, 2003). Integrating technology into a mathematics classroom will allow for an easier channel of the transferral of skills, lower anxiety, promote fluency of basic mathematics computational skills, and will help develop higher order mathematical skills. Technology can serve a number of roles in increasing knowledge of basic skills in mathematics. For example, the use of calculators can allow for a much easier task by increasing a student’s ability to solve problems. Bowes (2010:1) states, “Technology supports achievement, enabling learners to be independent, competent and creative thinkers, as well as effective communicators and problem solvers.” Hudson, Siobhan, & Lavin (2010: 19) state “the researcher/teachers used power points, web-based games, the internet, projectors, Smart boards, Elmos, calculators, videos, DVDs, and music to enhance their mathematics instruction.” Hudson, Siobhan, & Lavin (2010: 20) state, “Overall, targeted students improved their understanding of basic math skills by using technology”.

Interactive white boards are one of the most common pieces of visual technology found in a classroom today. Developed and introduced by SMART in 1991, the interactive whiteboard is connected to an LCD projector and computer, and provides touch control of computer applications. Marr (2011: 31) states this form of interaction creates a connection between the user and the application that personalizes the learning experience. The manipulation features an interactive whiteboard which enables students to reach the board to connect with the material, because on the whiteboard you can present it in a way you could not do with a chalkboard or overhead projector. The ability to present multimedia material that is verbal, visual, auditory, and interactive is essential to draw today’s students into the subject matter.

2.15. Limitations of visual strategies used in a classroom

To determine whether or not visualisation used as a visual strategy in a classroom is valuable, I will draw from the economic model by Wijk J (2006: 1). He looks at the profitability of using visuals in a classroom. He claims that new visualisation strategies are expensive and claims that there are alternative methods to go about teaching a cost free lesson in mathematics that can bring out visuals. The use of visual strategies especially visual technology used in a classroom can be regarded as expensive and is unattainable to every classroom. For example not all universities’ classrooms have a smart board or visual projector to bring out images. There are other attempts without the use of expensive technology to use visuals in a classroom. For example when teaching 3-D trigonometry, using a tree outside perpendicular to the ground could also elicit the same achievement as you would in showing learners a picture by means of a projected screen. Educators therefore must be creative and do the best they can within their budgetary restrictions. It is also controversial because information computer technology has benefits to project what other visuals can in no way attempt or come close to explaining what can be shown especially with 4-Dimensional figures in geometry. Technology is also useful in animations and gaining interest by use of these animations to enhance mathematics however there are many limitations of using visual technology. Smartboards, Data projector, Ipads, Laptops, etc. are costly and even more costly to maintain. If broken or if there is a shortage of electrical supply it could interrupt the progress of learning.

2.16. Conclusion

Research has shown that the use of visual strategies used in a mathematics classroom is certainly beneficial to students at university level. Drawing from a South African context, research shows that although 24 years have passed since the first democratic elections in South Africa, schools are still unequally resourced (Chisholm & Sujee, 2006: 141; Soudien, 2004: 1). This affects the effectiveness and efficiency of visual strategies amongst all schools and universities. Schools and universities differ largely in terms of human resources (the number of personnel at each school/university), physical resources (infrastructure, desk, chairs, classrooms, etc.) and teaching resources (equipment, textbooks, black or white boards, etc.). Spatial arrangement was one of the visual strategies mentioned above. It is shown whilst visual strategies might work perfectly as a methodology in teaching mathematics, there are still many limitations that hinder the process of mathematical knowledge being imparted to students. Mathematics teachers, especially, are sceptical about the feasibility of teaching the same curriculum within the same time frame to all learners, regardless of the inequitable distribution of resources (Adler, 2001: 186; Reddy, 2005: 125).

Visual strategies referred to in this study were diagrams, pictures, transparencies, mathematics manipulatives, gestures, and the use of colour. The literature above showed that educators often used visual strategies unknowingly in their classes, for example when they resorted to the use of gestures, colour, lines and symbols. Additionally, this study indicated that educators often used visual strategies with the intention of assisting learners to grasp abstract concepts in order to support and improve mathematical conceptual knowledge development. This is supported by Elia and Philippou (2004: 327), who claimed that visual strategies play an important role in communicating mathematical ideas and supporting the process of reflection. It also confirms the fact that teachers' tacit knowledge, professional development and beliefs concerning the teaching and learning of mathematics influence the way in which they teach mathematics (Remillard, 2005: 211).

Roodt and Conradie (2003: 265) showed that the use of different approaches to the same problem enriches both learners and teachers. Good teachers often use symbols, colour, diagrams and gestures in the classroom as an alternative to the routine approach of ‘talk and chalk’ teaching. The use of colour and other visual tools creates an exciting and interesting mathematics classroom (Naidoo, 2011a: 23). More approaches which encourage learners to be active and allow them the opportunity to demonstrate the extent of their thinking and creativity are therefore needed (Barnes, 2005: 42). Stokes (2000: 10) suggested that the use of visual tools assists in uncovering the role that visual reasoning plays in solving problems in mathematics. This leads to interesting results in the teaching and learning of mathematics. Visual tools may also be used as a starting point to achieve interactive and stimulating learning environments (Breen, 1997: 97). In these learning environments, learners are able to interact easily with abstract concepts.

CHAPTER THREE

Research Methodology

3.1. Introduction

In Chapter Two, I reviewed the use of visual strategies used as pedagogy for mathematics that has gained much popularity over recent years. Visual strategies lend themselves to lucidity on gaining mathematical concepts, which has the tendency to complicate, one's ideas on mathematical concepts (Dreyfus & Eisenburg, 1991: 33). This study's aim was to explore the use of visual strategies used by lecturers at tertiary level and its purpose was to gather a detailed understanding of the influence that visual strategies had on pre-service teachers' development of mathematical concepts. Firstly, this chapter outlines the methodological design of the study. Then, a description is given of the research design, the development of the research instruments, and provides the rationalization for the use of these instruments. It begins with reasoning for the choice of a specific methodology mainly qualitative in nature. A qualitative design was chosen because meaning was assigned to a problem that affected either an individual or a group of people. Thirdly, in the context of this study, the researcher sought to understand the influence of visual strategies on tertiary educators (lecturers) and pre-service teachers (students). Therefore, this study was underpinned by the interpretative paradigm. This chapter will explain the chosen research instruments: observation by means of video analysis (primary method), focus group and questionnaires (secondary method). Lastly, insights are then offered on sampling preference of the research participants, data collection and analysis, and issues relating to trustworthiness, triangulation and ethical considerations. The methodological approach was based on the description shown in Figure 3.1adapted from Creswell et al. (2007: 124).

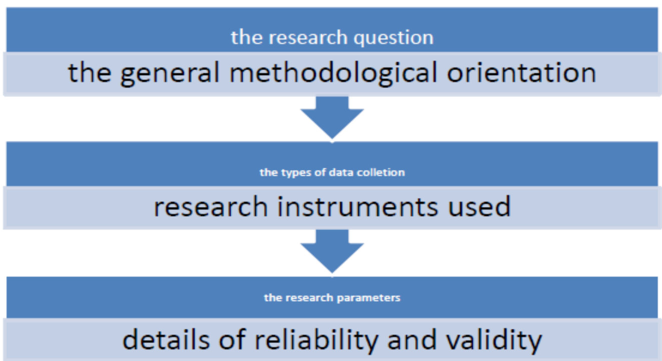


Figure 3. 1: Phases of data generated by (Creswell et al., 2007, Adapted from: www.nebraska.pure.elsevier.com, 15 March 2016)

3.2. Profiles of Research Participants

This section gives a profile of each research participant in tabular form as well as their academic and teaching narratives. The profiles enabled the researcher to understand the background of participants and shed light on their mathematical experience.

Table 3.2.1: Profiles of Lecturer Participants

Table 3.2 1: Representing the three University Lecturers

LECTURER	GENDER	TEACHING QUALIFICATION	TEACHING EXPERIENCE IN A UNIVERSITY	MODULES OBSERVED
Lecturer A	Female	BSc BEd (Hons) BEd (MEd) DEd	12 years	Primary Mathematics Education 210
Lecturer B	Female	BPaed BSc (Hons) MSc DEd	19 years	Mathematics Education 410
Lecturer C	Female	BSc HDE (old PGCE) BEd (BEd Hons) MEd PhD	22 years	Primary Mathematics Education 210

Once the process began, only female lecturers volunteered to participate in the research but it was evident that all three lecturers had substantial experience of teaching at tertiary level. All lecturers were specifically experienced in the training of pre-service teachers to become future educators. Often pre-service teachers (students) themselves are influenced by the kind of experiences provided by their school teachers and university lecturers. All three lecturers were in a very influential position and had the potential to create highly effective teachers. These lecturers were fairly versatile in their teaching at the university and had all been part of previous teacher training colleges. They showed tremendous passion in their work and participated in teacher conferences and workshops. One of these lecturers is titled professor. Another had also received her doctorate and the last lecturer was currently completing her PhD degree and had also received a teacher of the year award. It was evident that all three of them were highly qualified. It is perhaps also important to indicate that two of these lecturers worked predominantly with pre-service teachers (primary phase) being trained for the primary school and the other worked mainly with high school pre-service teachers (FET phase). This did not impact the research in anyway. It was only necessary to look at how they used multiple visual representations in their classrooms. As was stated already it is possible that their students may model a similar type of teaching in their own classrooms.

Table 3.2.1 is a representation of the three university lecturers as participants of this research study. Each lecturer is identified as lecturer A, lecturer B and lecturer C and is given a short profile about their academic calibre. All lecturers were female which was not intentional in any way. Table 3.2.2 shows the teaching qualifications and teaching experience of participants from the university. These participants are used for obtaining the researcher’s primary source of data.

Table 3.2 2: Representing the Pre-service teachers (Students)

LECTURER	STUDENT	GENDER	AGE	UNIVERSITY EXPERIENCE	MODULES
Group A	Student 1	Female	20	2nd Year	Primary Mathematics Education 210
	Student 2	Female	23	2nd Year	Primary Mathematics Education 210
	Student 3	Male	21	2nd Year	Primary Mathematics Education 210
Group B	Student 1	Female	20	4th Year	Mathematics Education 410
	Student 2	Male	21	4th Year	Mathematics Education 410
	Student 3	Male	22	4th Year	Mathematics Education 410
Group C	Student 1	Female	21	2nd Year	Primary Mathematics Education 210
	Student 2	Male	21	2nd Year	Primary Mathematics Education 210
	Student 3	Male	20	2nd Year	Primary Mathematics Education 210

Table 3.2.2 represents the biographical details of students who participated in the focus group interviews. It is a representation of nine pre-service student teachers who constitute just a part of the class of the three university lecturers respectively. Each student was identified as student one, student two and student three respectively belonging to the class of each lecturer A, lecturer B and lecturer C together comprising of nine students as participants of this research study. Students comprised of five males and four females all aged in their early twenties which ranged from twenty years old to twenty three years old. There was a good balance of gender between male and female in this research however gender had no impact in the context of this study. It was also good that the age difference was close between students due to similar maturity levels in interpreting the respective lessons. Table 3.2.2 shows that six students are

2nd year primary mathematics education students belonging to lecturer A and lecturer C classes. The remaining three students are 4th year mathematics education 410 students belonging to lecturer B class.

3.3 The Critical Questions

Many qualitative researchers such as Saldnana (2015), Seidman (2013) and Creswell (2007) see a question as a beginning point for their research. Moreover, Agee (2009: 431) confirms that it is an important part in research to understand participants' views, as this process tends to be continuous. Although research has been done on the use of visual strategies in teaching and learning, little research was found on the use of visual strategies used at tertiary level by lecturers and pre service teachers. It was for this reason that the researcher chose to explore how the use of visual strategies enhances mathematical understanding at tertiary level. This study addresses the following six critical questions (Figure 3.2).

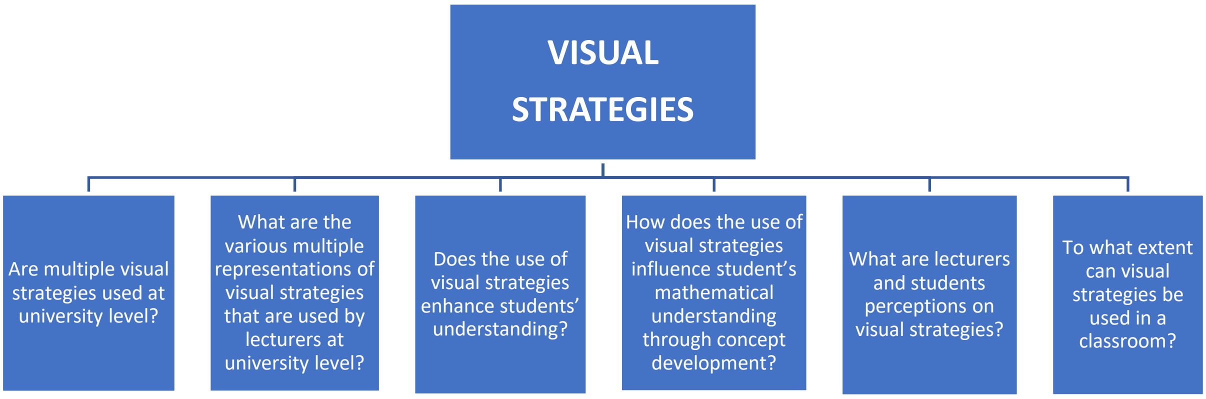


Figure 3. 2: Critical questions of the study

The rationale for these questions is as follows: the first and second questions attempted to establish the types of visual strategies used in lecture rooms. The second and third questions tried to establish a deeper understanding of whether these visual strategies enhanced understanding of students' mathematical concepts. Finally the last two questions looked at tertiary educators' (lecturers') views on visual strategies used when teaching.

3.4. Methodological Approach

The research methodology's purpose was to support the researcher to develop a research approach or plan. Moreover, it also supported the researcher to recognize the process of data collection and to have been able to analyse the data. According to Cohen, Manion and Morrison (2000: 446) the underlying principle of research assists in deciding the methodology and design of the research study and the methods, which are a range of approaches, used to gather data in research. This research study was underpinned by the intepretivist paradigm.

> Interpretive paradigm is underpinned by observation and interpretation, thus to observe is to collect information about events, while to interpret is to make meaning of that information by drawing inferences or by judging the match between the information and some abstract pattern (Aikenhead,1997: 217).

The researcher utilized the interpretive paradigm because the researcher wanted to investigate the lecturers' use of visual strategies at tertiary level. This study was also used because the researcher wanted to determine the influence visual strategies had on pre-service teachers in understanding mathematical concepts. The interpretive paradigm enabled the researcher to engage in amalgamating and critically analysing the data collected, and to draw inferences about visual strategies used in lecture rooms. This research study allowed the researcher to create meaning from the data which was seen (observed) and heard. This research study was underpinned by observations that are made by analysing the multiple representations of visual strategies used by lecturers whilst teaching. The interpretive paradigm entails understanding the world as it is from subjective experiences of individuals.

> "They use meaning (versus measurement) oriented methodologies, such as interviewing or participant observation, that rely on a subjective relationship between the researcher and subjects" (Cohen, Manion & Morrison, 2011: 459).

The rationale for this research study, being framed within the interpretive paradigm was due to the methodical analysis of educators' behaviour in teaching by the use of visual strategies used at tertiary level. Interpretations of semiotic observations were carefully conducted. An interpretative paradigm was also chosen due to the dialogic and in depth considerations of pre-service teachers' collaborative discussions in the focus group. Cole (2006: 156) and Weaver and Olson (2006: 459) argue that a qualitative methodology shares its philosophical foundation with the interpretive paradigm which supports the view that there are many truths and multiple realities. Additionally, they state that the interpretive paradigm is associated more with methodological approaches that provide an opportunity for the voice, concerns and practices of research participants to be heard.

This research study analysed data qualitatively. The research study was based on a qualitative analysis owing to the nature of understanding of how visual strategies function at tertiary level. Qualitative research is a form of enquiry that explores occurrences usually in their normal settings and uses multiple methods to interpret, explain and bring meaning to the research. Denzin and Lincoln (1994: 575) define qualitative research similarly as "multi-method in focus, involving an interpretive, naturalistic approach to its subject matter." The model depicted in Figure 3.3 is adapted from Maykut and Morehouse (1994: 92).

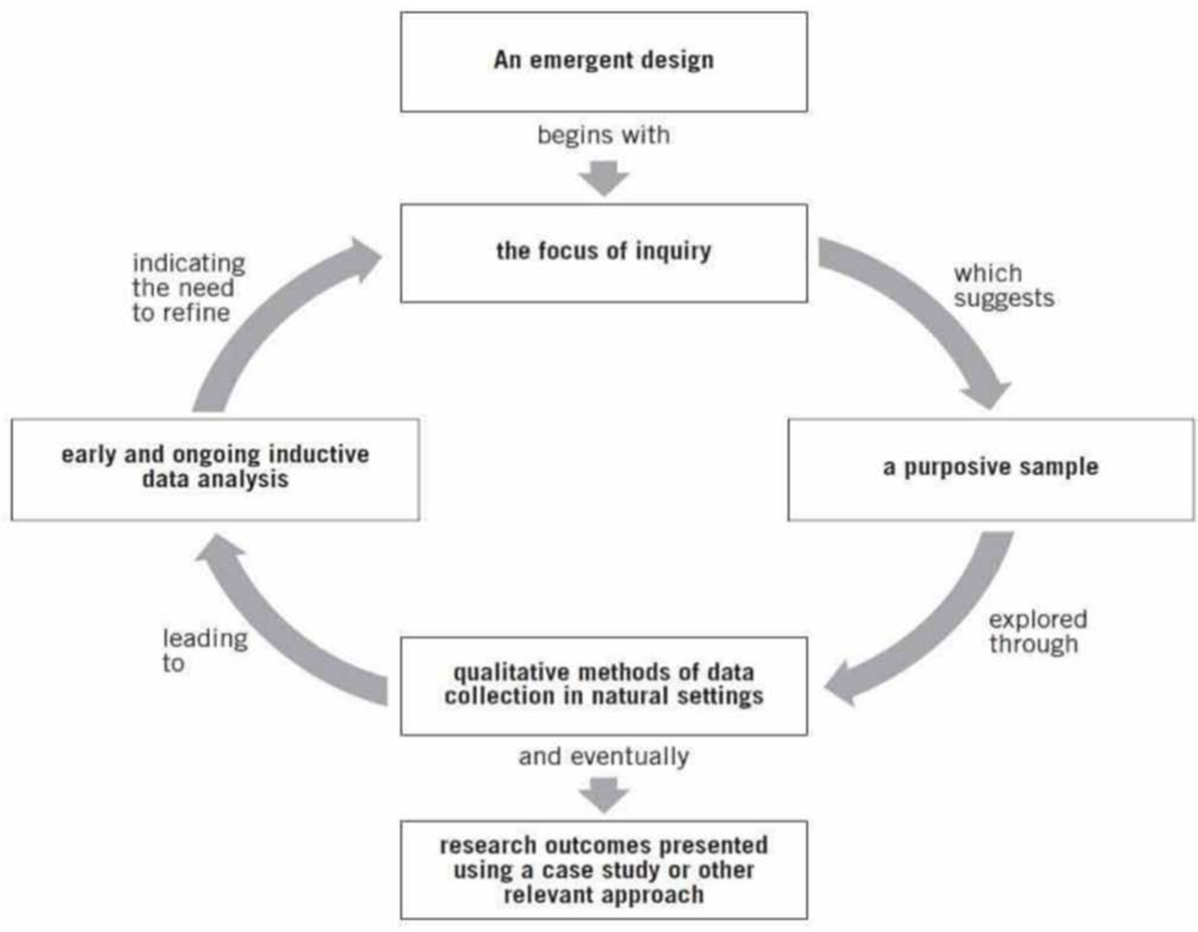


Figure 3. 3: Qualitative research design (adapted from Maykut and Morehouse, 1994, 20 March 2016)

3.5. Research Design

Parahoo (1997: 403) describes a research design as “a plan that describes how, when and where data are to be collected and analysed”. The researcher used observation by means of video analysis, focus group interviews of students and questionnaires given to lecturers (See appendices A, B and C). These instruments were used to elicit in-depth knowledge.

3.6. Observations

Observation can be viewed as a way of gathering data by watching the behaviour. Cohen et al. (2000: 447) assert that observation refers to the gathering of fresh data as they occur and access is gained to personal knowledge and happenings at the site where they occur. The rationale for the use of observation by means of video recordings allowed for real life situations to take place as they afforded the researcher the opportunity to gather 'live' data from 'live' situations" and important recordings were made and thus enabled the researcher to identify with the situation that was described. Roschelle (2000: 709) suggests that video is becoming the medium of choice for collecting data. She also stated that video recording allowed the researcher to preserve aspects of interaction including talking, gesture, eye gaze, manipulatives, and computer displays and viewing the events repeatedly to analyse many subtle cues that may be missed on first observation and analysis. This affirmation provided the incentive for the researcher to use videography of the lessons as an observation tool to enable the researcher to deeply analyse the subtleties of the interactions later.

As a research tool, observation needs proper planning, implementation and adequate recording. According to Koul (1988: 168) “The planning for observation must take into consideration the subjects to be observed, the activities to be observed, the length of each observation and the tools to be used to observe and record”. The researcher chose three lessons consisting of three mathematics lecturers who taught three different lessons to mathematics education pre-service teachers (students). These lessons were observed within a period of one month. At the site of study (the university), there were many units of mathematics education modules and many students consisted of first year up to fourth year pre-service teachers. The institution also had six other lecturers. Three lecturers volunteered to participate in the research although all lecturers were invited to participate. The three lecturers who agreed to be part of the study nominated the class in which they wished to be observed. The pre-service teachers comprised of two classes of primary mathematics education students in their second year and one class of FET mathematics education students in their fourth year. Observations were made of the three lecturers' teaching methodologies, in particular the use of visual strategies. The behaviour of the pre-service teachers (students) was also carefully analysed, in particular, their reaction to being taught through the use of visual strategies.

The lecturers were observed using a structured observation schedule (Appendix C). Koul (1988: 142) states that "it is advisable to develop an observation form while making observations". Specific behaviour (teaching style) was observed and recorded on the schedule. Field notes were kept during each lesson based on observations, pre-service teachers' interaction in the classroom, chalkboard work and the nature of resources (worksheets, artefacts, textbooks and so on) used by lecturers and pre-service teachers during the lessons. The pre – service teachers (students) were observed whilst they engaged with their lesson and in certain instances the manner in which they used visuals themselves. This took place concurrently with the observation of the lecturers. The students' books were examined to determine the type of mathematical visuals used. These observations took place three times according to the three lessons of the three different lecturers and these observations were spread out systematically during the course of one month's duration. These observations are necessary as advocated by Cohen et al. (2000: 447) who stated that "The greater the number of observations, the greater the reliability of the data might be". The lessons observed were all of one hour's duration. Lecturers were happy to co-operate as they felt it would be beneficial to the pre-service students to get exposure to this study whilst one lecturer felt it was not worth the study as she could only relate to a bare minimum of visuals being used in her classroom.

The researcher used an observational schedule (Appendix C) when monitoring the use of visual strategies used by lecturers and pre-service teachers. The observational schedule consisted of definite categories for observation. The focus was on how lecturers used visual strategies in their lessons. It was also to analyse pre-service teachers' responses to these visual strategies being used. The observed data was recorded immediately after the observation as they were still fresh in the observer's mind. The observation report was written as soon as the lesson was completed thus giving the researcher an opportunity to describe exactly what had transpired in the classroom. There were also important points recorded during the lesson. In addition, "While recording the observation, the observer must minimise the influence of his/her biases on the observation report and must objectively record the relevant data” (Koul, 1988: 170). Recording objectively is a crucial element to overcome bias to showcase reliable data. Moreover, Koul (1988:172) states that the observational method has the following advantages. It provides a system for studying various aspects of human behaviour, which may be the only effective way to gather data in a particular situation. It enables the observer to cipher and record doings at the time of their occurrence.

3.7. Questionnaire

Data sometimes lay buried deeply within the minds or within the attitudes of individuals. One common instrument used to probe below the surface and to observe data beyond the somatic reach of the observer is the questionnaire. Moreover, Koul (1988: 142) describes a questionnaire "as a systematic compilation of questions that are administered to a sample of population from which information is desired". Questionnaires are popular means of collecting all kinds of data since it was through this medium that data are generated. The main consideration involved in the selection of participants for answering a questionnaire was to contact the participants who were able to supply the crucial information that was needed for the research and to have gained rapport with the participants. In reference to this research study, questionnaires were administered to three mathematics education lecturers from the university (Appendix A).

Questionnaires were selectively given to the three mathematics education lecturers owing to the nature of this study in visualisation of mathematics. The reasons for the questionnaires were twofold: one was to establish a rapport with these lecturers when attempting to hand over the questionnaires and the other was to arrange an observation session when handing over the questionnaires for their lessons. Leedy (1974); Best (1977); Lydeard (1991); Maree (2007); Mitchell and Jolley (2007) all state that questionnaires should be designed to fulfil a specific research objective. The significance of the questionnaire was clearly and carefully stated since it sought only that information which was relevant to the study. The questionnaire obtained the views and experiences of the mathematics education lecturers and their methodical approach to using visual strategies in a mathematics classroom when teaching students. The questionnaire was brief, simple to read and easy to respond to thus allowing the participants to examine their own thinking. It was hoped that it would solicit only that data essential for the study in question. There were both closed and open-ended questions in the questionnaire. According to Maree (2007: 161), the advantages of using open-ended questions allow for greater depth of responses. The lecturers were given the freedom to reveal their opinions, experiences and to clarify their responses in the open ended questions given to them. Open-ended questions allow the participants "to explain and qualify their responses" (Cohen et al., 2000: 447). This gave the researcher a deeper understanding of the area under investigation.

The researcher used a questionnaire in this study and found it beneficial to gather data regarding a wide range of issues relevant to the use of visual strategies. Another aim in using a questionnaire was to obtain an in-depth understanding of the uses of visual strategies and methodologies as demonstrated by the lecturers. “In using questionnaires, researchers rely totally on the honesty and accuracy of the participants' response”. In addition, Gay (1992: 444) also states that careful thought must therefore be given to both the content and format of this instrument. The questionnaire was efficient in that it required less time, it was less expensive and it contained standardized questions. The questionnaires given to lecturers were handed over personally and were returned to the researcher either personally or emailed. This allowed the researcher the opportunity to again establish rapport with the participants, explain the purpose of the study and clarify individual items.

3.8. Focus group Interviews

A focus group according to Lederman (as cited in Thomas et al.,1995: 206) is a technique involving the use of in-depth group interviews in which participants are selected because they are a purposive, although not necessarily representative, sampling of a specific population, this group being ‘focused’ on a given topic’. A focus group interview is a qualitative data collection tool. The reason for choosing a focus group interview was to establish participants’ views on visual strategies being used in lecture rooms. The researcher wanted to understand the way of thinking by the selected participants and the meaning accorded by them about visual strategies. Moreover, Rabiee (2004: 656) shows that focus groups can provide so much more to the research. He states “Focus groups provide information about a range of ideas and feelings that individuals have about certain issues, as well as illuminating the differences in perspective between groups of individuals". In particular, this research sought to understand the manner in which pre-service teachers gain concepts of mathematics by means of visual strategies via a focus group interview. One of the distinct features of focus group interviews is its group dynamics; hence the type and range of data generated through the social interaction of the group are often deeper and richer than those obtained from one-to-one interviews (Thomas et al., 1995: 207). Focus groups can generate large amounts of data in a relatively short time span, and the findings may be used to pave the way for qualitative measures. The researcher had initially intended to interview the lecturers but all three indicated that they would not be available for interviews. It was hoped that the focus groups of the students would fill in the gaps created by the unavailability of the lecturers. The lecturers were given the interview questions as a questionnaire.

The researcher used a semi-structured interview schedule when conducting the focus group interview with the pre-service teachers. The semi-structured interview questions allowed for in depth data to be collected by the researcher and allowed for interaction involving the interviewer and interviewee. The focus group interview assisted the researcher in obtaining a better understanding of pre-service teachers' perceptions of visual strategies. The focus group interview process involved a face-to-face interview with pre-service teachers as it provided greater flexibility. This was done immediately after the classroom observations. According to Mitchell and Jolley (2007: 500), these kinds of interviews allow additional questions to be asked to follow up on interesting or unexpected answers. The interviewer is at liberty to "rephrase the questions, modify them, and add some new questions to his list." This interview consisted of predetermined questions to guide the process of the focus group. The pre-service teachers were asked questions pertaining to visual strategies used at tertiary level. The researcher probed their responses by asking additional questions to get a better understanding of what the students were trying to express.

There were three focus groups A, B, and C as a representation of pre-service teachers chosen from lesson 1, 2 and 3. Krueger (1994: 63) suggests that for a simple research question the number of focus groups necessary may only be three or four. There were three students chosen per focus group. These interviews were limited to 20 minutes owing to the venue being used for other lectures. The focus group interview was video recorded with the consent of the pre-service teachers and their lecturers. When all the interviews were completed, the video recordings were transcribed and copies of the students' questionnaires were merged with each transcript. This was done to maintain the authenticity of the data and to ensure there was no misinterpretation. Recording the focus group interview is a vital aspect as it helps to eradicate oversights, misrepresentation and other alterations. The recording of the focus groups also "provides an objective basis for evaluating the adequacy of the interview data" (Koul, 1988: 175). Notes were taken to support the recordings. In addition, Koul (1988: 176) also states that "the notes should include unusual and significant behaviour as well as the responses to questions of the interviewees". To ensure validity of the focus group interview the responses of the interviewees must be corroborated with other sources of data. The corroboration of the evidence obtained from the lesson observations and the focus group interviews enabled the researcher to develop a comprehensive understanding of pre-service experiences. Moreover, Koul (1988: 176) states that the interview process has advantages as it allows for an opportunity to extensively probe certain areas of inquiry, permits greater depth of response, which is not possible through other means of inquiry and enables the interviewer to obtain information concerning attitudes to certain questions. Interviews also have the following advantages. It is a useful way to obtain large amounts of data quickly and where more than one person is interviewed, a wide variety of information.

3.9. Data Analysis

In the context of this study, rich data materialized. As stated by Maree (2007: 295), "data analysis is the process of observing patterns in the data, asking questions of those patterns, asking additional questions, seeking more data, furthering the analysis by sorting, questioning, thinking, constructing and testing conjecture." The analysis of data required careful study and categorisation. In an attempt to address the critical questions, several interpretations emerged on the nature of visual strategies, making the process very complex and time-consuming. Yin (2009: 256) points out that data analysis consists of a number of stages, i.e. examining, categorising and tabulating or otherwise recombining the evidence, in order to address the initial goal of a study. Data was collected by using video observations, questionnaires and focus group interviews. This qualitative data was coded and classified into specific categories. The data was collected, processed, condensed and then interpreted using triangulation in order to make this study trustworthy, reliable and valid (Maree, 2007: 296). Data was analysed by a rich stream of evidence and was well documented and presented (See Appendices A, B and C). The researcher safeguarded the findings by means of recorded videos for observations and focus group interviews and questionnaires. The process of data analysis began during the data collection, by facilitating discussions and producing rich data from the interviews, accompanying them with the observational notes and typing the recorded information. This stage was followed by familiarisation with the data, which was achieved by listening and watching the videos, reading the transcripts in their entirety several times and reading the observational notes taken during interview and notes written during the focus group interview and immediately after the focus group interview.

3.10. Sampling

The sampling methods involved in this research study were based according to the 'people, setting, events and behaviours' that were observed (Understanding research, 2010: 41). As advocated by Singh, sampling is a process by which a relatively small number of individuals or events is selected and analysed in order to find out something about the entire population from which it was selected (Singh, 2010: 82). The researcher needed to identify who was taking part in the research that is the nature of the participants, their status and roles in the study. This pre-requisite for this study opted for three mathematics education lecturers and three students to be sampled for the questionnaires and focus group interviews. There were three lecturers and nine pre-service students who comprised the sample in the secondary participants. Each class consisted of 20, 30, and 60 students. Pre-service teachers comprised one class of 20 second year students, 60 second year students and 30 fourth year students. It was not possible to get every member of the study population involved i.e. getting every pre-service teacher in the class to be a participant leading to the nine sampled students.

In this study, the questionnaire and observation were meant specifically for the mathematics lecturers. This study was based on a relatively new investigative area in mathematics education and the researcher required in-depth knowledge from a wide range of experienced mathematics lecturers. As a matter of convenience, the questionnaire was distributed to the lecturers in the university where the researcher presently studies. For the purpose of this study, the researcher used probability sampling which includes random sampling. The participants who were randomly selected were those participants of the pre-service teachers who were representative of the entire population. There were two methods of sampling chosen in this research study of the pre-service teachers and tertiary educators namely convenience sampling and purposive sampling.

3.10.1. Convenience Sampling

A focus group interview was conducted by means of voluntary participation of students for each lesson. According to Marshall (1996: 92), "This is the least rigorous technique, involving the selection of the most accessible subjects. It is the least costly to the researcher, in terms of time, effort and money". Firstly, a convenient method of sampling was selected for the pre-service teachers owing to who voluntarily wanted to participate. Three students per lesson were selected. The first lesson consisted of 20 students of which three students were randomly chosen, similarly the second lesson consisted of 30 students of which three were randomly chosen and finally the third lesson consisted of 60 students of which three were randomly chosen and in total nine students participated by being randomly selected. Participants were made up of all races and gender. These participants were aged between nineteen and twenty three which did not impact on my findings as my research is based purely on cognitive ability.

3.10.2. Purposive sampling

Purposive sampling purposely seeks to choose participants. As stated by Barbour (2001: 155), "With purposive sampling, researchers deliberately seek to include "outliers" conventionally discounted in quantitative approaches. It allows for such deviant cases to illuminate, by juxtaposition, those processes and relations that routinely come into play, thereby enabling the exception to prove the rule." Purposive sampling was used with the mathematics education lecturers from the university who were selected owing to the nature of this study which lends itself only toward visualisation in mathematics. The participants of the research included three female lecturers which was not intentional. Three other lecturers were asked to participate in the research but declined owing to availability. The lecturers' teaching experience ranged between twelve years and twenty two years (See table 3.2.2.). This will not impact on my findings as my research is based purely on cognitive ability.

3.11. Ethical Issues

Ethical issues are a vital part of a research study (Setati, 2000; Terre Blanche and Durrheim, 2002; Maree, 2007). Protecting the participants' rights to confidentiality and privacy is a central precept to any research study. In addition, Bak (2004: 2) states that ethical guidelines must be considered by the researcher whose overall responsibility is to firstly design, conduct and report research in agreement with recognised standards of scientific aptitude and ethical research. The researcher acquired appropriate permission of the relevant parties from the university. Protecting the rights and interests of participants was a crucial prioritised element to the researcher. The researcher then took the initiative to protect the identities and interests of those involved and guaranteed the confidentiality of the information given to the researcher. Finally the university ethics committee was consulted about any unclear ethical issues. Written consent was obtained from the dean of the university to use the site (Appendix E). Ethical clearance was obtained from the institution through which the researcher is studying (Appendix G). The researcher had to take into consideration the three ethical principles namely "*autonomy, non-maleficence and beneficence*" (Terre Blanche and Durrheim, 2002: 148*).*

3.11.1. Autonomy

In the first principle, *autonomy* had to be considered (Terre Blanche and Durrheim, 2002: 148). Informed written and signed consent was obtained from the lecturers and all students participating in the research study (Appendix D). Diener and Crandall cited in Cohen et al., (2000: 448) define informed consent as the procedures in which individuals choose whether to participate in an investigation after being informed of facts that would be likely to influence their decisions. A letter was given to participants (Appendix F). In this letter, the researcher introduced herself, the purpose of the study, indicating that the respondents were under no obligation to participate and if they did they had the right to withdraw without any consequences; ensuring their confidentiality; anonymity and non-traceability in the case of publications as no identification was needed (Cohen et. al., 2000; Maree, 2007).

3.11.2. Non-maleficence

The second principle of *non-maleficence*: the researcher had to ensure that no physical, emotional, social or other harm came to the participants (Terre Blanche and Durrheim, 2002: 449).

3.11.3. Beneficence

The third principle of *beneficence* (Terre Blanche and Durrheim, 2002: 449): the researcher had to ensure that the research benefited the participants and the university. Quality education has become a critical area of focus (DoE, 2012) and the results obtained from using visual techniques in this study will address critical issues that will assist lecturers and the university fraternity at large in aiding their students becoming proficient teachers of mathematics. All ethical issues were considered for this study.

3.12. Measures to Ensure Trustworthiness of Research Instruments

3.12.1. Triangulation

Triangulation involves the amalgamation of more than two approaches used when obtaining data. The sequence of data collection in this study was observation, focus group interview and questionnaires (See Appendices A, B and C).

> "This use of multiple methods or triangulation is an attempt to secure an in-depth understanding of the phenomena in question. The process of using multiple methods to acquire knowledge of the same phenomenon using different research measures is known as triangulation" (Maree, 2007: 200).

According to Flick cited in Denzin and Lincoln (1998: 575), this combination of multiple methods, empirical methods, perspectives and observers is a strategy that adds rigour, breadth and depth to any investigation. In the research process on which this dissertation is based, triangulation was achieved through multiple data collecting sources, procedures and strategies. The researcher used observation as the primary research instrument. The focus group interviews and questionnaires were used as secondary research instruments. The use of these methods was also an attempt to triangulate and therefore verify the data collected. In addition, Leedy (2005: 95) states that "It is generally accepted that the inclusion of multiple sources of data collection in a research project is likely to increase the reliability." Triangulation also aims to enhance the credibility and validity of the results which is discussed later on in this chapter. The amalgamation of different methods and methodological perspectives strengthens this research study. Moreover, Patton (2001) advocates the use of triangulation by stating "triangulation strengthens a study by combining methods." However, Barbour (1998: 156) takes a different stance in triangulation. Barbour argues while mixing paradigms can be possible but mixing methods within one paradigm, such as qualitative research, is problematic since each method within the qualitative paradigm has its own assumption in "terms of theoretical frameworks we bring to bear on our research." Even though triangulation is used in quantitative paradigm for confirmation and generalization of a research, Barbour (1998: 156) does not disregard the notion of triangulation in the qualitative paradigm and she states the need to define triangulation from a qualitative research's perspective in each paradigm.

3.12.2. Validity and Reliability

Validity and *reliability* are key aspects of all research. Acknowledgement of these two aspects can make a difference between good research and poor research and can help to assure the researcher's findings as credible and trustworthy. This is particularly vital in a qualitative paradigm, where the researcher's subjectivity can so readily haze the analysis of the data, and where research findings are often questioned or viewed with uncertainty by the researcher's community.

Validity and Reliability

Validity looks at the end results of measurement in data. According to Cook and Beckman (2006: 166), "*Validity* is the process of collecting and analysing evidence to support the inferences based on the instrument used". It is used to determine whether the instruments are relevant in attaining the required results for the research. In this research, validity was ensured by engaging in video observations, focus group interviews and questionnaires to support the relevant data. According to Rubin (2011: 68), validity in qualitative data might be addressed through honesty, depth, richness and scope of the data achieved. The researcher adopted validity in the approach of multiple instruments undergoing the data collection process multiple times. In addition, Golafshani (2003: 603) states that the participants' approaches, the extent of triangulation and the objectivity, provide triangulation of the research data which also contributes to the validity of the test. Moreover, Creswell and Miller (2000: 126) suggest that the validity is affected by the researcher's perception of validity in the study and his/her choice of paradigm assumption. As a result, many researchers have developed their own concepts of validity and have often generated or adopted what they consider to be more appropriate terms, such as, quality, rigour and trustworthiness (Davies & Dodd, 2002; Lincoln & Guba, 1985; Mishler, 2000; Seale, 1999; Stenbacka, 2001). There are many types of validity and many names have been used to define the different types of validity. Historically, Campbell and Stanley (1966) have defined two major forms of validity that encompass the many types. They refer to "internal" and "external" validity which is further clarified later on in this chapter. According to Brink (1993: 36), internal validity is the term used to refer to the extent to which research findings are a true reflection or representation of reality rather than being the effects of extraneous variables. External validity addresses the degree or extent to which such representations or reflections of reality are legitimately applicable across groups (Brink, 1993: 36).

Reliability relates to the concept of good quality research in qualitative studies and has the purpose of generating understanding. Stenbacka (2001: 551) describes the notion of reliability as one of the quality concepts in qualitative research which is "to be solved in order to claim a study as part of proper research." It refers to the ability of a research method to yield consistently the same results over repeated testing periods.

Patton (2001) as cited in Golafshani (2003: 604) states that “validity and reliability are two factors which any qualitative researcher should be concerned about while designing a study, analysing results and judging the quality of the study.” In addition, Guba and Lincoln (1985: 300) introduced the concept of trustworthiness in qualitative studies as a similar concept to reliability and validity. Trustworthiness has four components, namely, *credibility, transferability, dependability, and confirmability* (Morse et al., 2008: 14). These terms have gained prominence in qualitative enquiry and have become the support for the overall significance in the finalized research.

13.12.2.1. Credibility

According to De Vos et al. (2005: 347), *credibility* refers to the research being conducted accurately. The term triangulation is sometimes used as a validation strategy as discussed earlier in the chapter. This implies that the research is thorough and the constraints of a credible qualitative study are adhered to. Furthermore, the study must be in depth, closely integrating the data. This study allowed for triangulation by using a mix of methods such as video observations, focus group interviews and questionnaires. The multiple methods assured the data collected was accurate and credible.

13.12.2.2. Dependability

In qualitative studies, *Dependability* refers to the process of identifying acceptable process of conducting the enquiry so that the results are consistent with the data (Cohen et al., 2001: 120). Dependability is fulfilled if the research process is subject to triangulation, member checks and respondent validation (Cohen et al., 2001: 120). In addition, Lincoln and Guba (1985: 317) use "dependability" in qualitative research. They further emphasize "inquiry audit" as one measure which might enhance the dependability of qualitative research. This can be used to examine the process and end results of the research and show consistency. Similarly, Clont (1992) and Seale (1999) endorse the concept of dependability with the concept of consistency or reliability in qualitative research.

13.12.2.3. Transferability

De Vos et al. (2005: 347) assert that choosing a case strategically can enhance a study's generalizability**.** *Transferability* is an alternative to *external validity* or generalizability. In the qualitative research paradigm, external validity is not a priority.

13.12.2.4. Confirmability

Confirmability, according to De Vos et al. (2005: 347), replaces the traditional concept of objectivity, where the influence of the researcher is removed and the data itself is examined as objective. In qualitative studies however, as described, there cannot be a situation of total objectivity as the researcher is integrated in the research context. However the study tries to remove situations of observer bias which could influence participant responses.

The terms *reliability and validity* are associated with *credibility, transferability, dependability, and confirmability* and assert that, without rigour, research is worthless, becomes fiction, and loses its utility. (Morse, Barrett, Mayan, Olson, and Spiers, 2008: 14). A great deal of attention is therefore applied to reliability and validity in all research methods. Stenbacka (2001: 552) argues that the concept of validity should be redefined for qualitative research. In searching for the meaning of rigour in research, Davies and Dodd (2002: 279) find that the term rigour in research appears in reference to the discussion about reliability and validity. The idea of discovering truth through measures of reliability and validity is replaced by the idea of trustworthiness which is “defensible” (Johnson 1997: 282) and establishing confidence in the findings (Lincoln & Guba, 1985: 316). If the issues of reliability, validity, trustworthiness, quality and rigour are meant differentiating a 'good' from 'bad' research then testing and increasing the reliability, validity, trustworthiness, quality and rigour will be important to the research in any paradigm.

In a qualitative paradigm the terms *Credibility, Neutrality or Confirmability,* Consistency or *Dependability* and Applicability or *Transferability* are to be the essential criteria for quality research (Lincoln & Guba, 1985: 317). To be more specific with the term of reliability in qualitative research, Seale (1999: 266) establishes good quality studies through reliability and validity in qualitative research, he also states that the “trustworthiness of a research report lies at the heart of issues conventionally discussed as validity and reliability.” To widen the spectrum of conceptualization of reliability and revealing the congruence of reliability and validity in qualitative research, Lincoln and Guba (1985: 316) state that: "Since there can be no validity without reliability, a demonstration of the validity is sufficient to establish the reliability". Patton (2001) as cited from Golafshani (2003: 605) with regard to the researcher's ability and skill in any qualitative research also states that reliability is a consequence of the validity in a study.

3.13. Access to University

Access to an institution of any research is deemed important. According to Kondowe and Booyens (2014: 146), gaining entry to a research site involves a combination of planning, perseverance and luck. In this study, the university was chosen as a needed jurisdiction to determine the calibre of pre-service mathematics teachers before they leave to become future educators. The research involved pre-service teachers and lecturers from the institution; hence this necessitated written consent from all stakeholders of the university. The researcher needed permission from the dean of the university, lecturers of the mathematics education faculty as well as pre-service teachers to seek permission to appear in the observational video recorded lesson at the site of the university. It was vital to seek authorization early from all stakeholders to carry out this investigation. The dean of the university, lecturers and pre-service teachers were approached by the researcher with a form of a letter describing the research study. This was personally done by the researcher which allowed for discussions and further aspects of the research (Appendix A). A background to the research and rationale thereof was discussed. The dean of the chosen university thereafter granted permission. Permission was sought from the university via the institution of the study to conduct research in the University of KwaZulu-Natal in Pinetown, Durban.

3.14. The Researcher as an Instrument

Many researchers adopt the role of an instrument in studies to obtain data. According to Poggenpoel and Myburgh (2003: 255), "In such a manner investigators become the instruments through which data for their studies are collected or generated." The researcher had chosen observation (video recordings) as the primary instrument, questionnaire and focus group interview as a secondary instrument. The researcher was under no illusion about her own role in this study notwithstanding the understanding that there is no such concept as total objectivity, only varying degrees of subjectivity. The researcher therefore deemed it necessary to comment on her objectivity. Moreover, it is stated that "Central to conducting research and more specifically qualitative research is the researcher as a research instrument" (Denzin and Lincoln, 2000: 576; Marshall and Rossman, 1995: 59). The researcher was the key person in obtaining data from respondents. It was through the researcher's facilitative collaboration that a context was created where respondents share rich data regarding their understanding of situations. "It is the researcher that facilitates the flow of communication, who identifies cues and it is the researcher that sets respondents at ease. This also contributes to a therapeutic effect for the respondents because they are listened to" (Poggenpoel and Myburgh, 2003: 256). The researcher has already alluded to her experiences as a recent graduate from the same university as well as the researcher's experience as a current educator.

3.15. Conclusion

This chapter described the methods that were used in this study. It served as a synopsis of how this study was shown in accordance with the relevant methods and procedures. The research methodology therefore aids as a guideline with respect to data collection and procedures followed. The choice of the interpretive paradigm and the qualitative approach was justified as well as a motivation for the observations, interviews and focus group, as a research methodology, was also presented. The chapter also outlined the process of sampling that was used in the study and briefly explained the data collection instruments. Issues relating to trustworthiness were discussed. Before considering the issue of research ethics, the process of data analysis was outlined. The analysis of the research data is discussed in Chapter Four, the empirical evidence gathered from the methods is presented and analysed.

CHAPTER FOUR

Data analysis

4.1. Introduction

The information presented in this chapter represents the data collated as a result of an investigation into the use of visual strategies at tertiary level and their influence on student teachers' development of mathematical concepts. The information will be presented in terms of the research instruments that have been utilized that are mainly: observations by means of video recordings (primary research method), focus group interviews of students and questionnaires of lecturers (secondary research method).

4.2. Overview of this Study

This study begins with an overview on observations of lecturers' use of visual strategies in their classrooms. Secondly there is a discussion on the comparisons between the different visual strategies used in each lecturer's lessons. Next, explanations about gestures are given. Fourthly, a discussion on artefacts used by lecturers in their classrooms is presented. Evidence of visual students is then outlined. Sixthly, semiotics is discussed followed by focus group interviews that were analysed. Lastly questionnaires are deliberated over.

4.3. Observations of tertiary educators' classroom practice.

The three lesson observations will be discussed in detail in order to capture the visual strategies used by these educators of pre-service teachers. It was necessary to scrutinize all actions with the intention of understanding how their words, actions and diagrams were used pedagogically to promote greater understanding. There was also the prospect that, the activities and strategies modeled by these lecturers influenced pre-service teachers in their practice as future educators. These observations were conducted in conjunction with the materials that the educators used, the gestures they utilized to emphasize concepts, the activities given to the students, the artefacts they produced and all other visual items they thought were useful in the classroom.

Each lesson will be discussed separately in order to present a deeper understanding of what individual lecturers did. Hence, a discussion of its comparisons will then follow. The three lessons have been recorded over a period of one month. Multiple representations of visual strategies were carefully observed by video recordings. The educators' use of visual strategies and students' development of mathematical concepts by means of visual strategies used were observed by the use of video recordings. The three educators will be represented as lecturer A, B and C as per table 3.2.1 in Chapter Three. The pictures depicted in images 4.3 are snap shots taken from the video recordings which analysed the visual strategies used, such as gestures, artefacts, classroom set, use of the chalkboard or whiteboard and booklets. An analysis of the semiotics used was scrutinized as this was one of the theoretical frameworks adopted in this research study. In the next section there will be a discussion on the overall analyses of the lessons taught by the three lecturers.

4.4.1. Analyses of lessons:

4.4.1.1. Lesson by Lecturer A

Lecturer A taught a lesson based on unifying ideas to a class of 2^{nd} year primary mathematics education 210 students. During this lesson, numerous visual strategies were observed. The visual strategies used in this lesson appeared to arouse interest amongst students and appeared to have played a positive role in developing the mathematical understanding and thinking of students. Lesson one showed that an adequate number of visual strategies was used in the classroom. The use of visuals used as a pedagogical strategy in lesson one contributed to a vast amount of knowledge gained by students. Additionally, it was observed that, the use of words and the manner in which explanations occur, plays a pivotal role. The visual strategies used in lesson one constituted of artefacts such as paper, beans, the chalkboard and overhead projector. Observation of lesson one demonstrates meaningful explanations which worked hand in hand with the artefacts used and all other visual strategies. Lecturer A used semiotics abundantly, which were used well since she did not overpower the lesson with their purpose yet was using them meaningfully. Students showed dislike when viewing many symbols in mathematics but lecturer A gained their confidence in the use of semiotics by carefully explaining at each point what each symbol was used for. Regarding other visual strategies used in the classroom, she used just enough, and mastered their role in the classroom. Next I discuss lesson B by lecturer B.

4.4.1.2. Lesson B by Lecturer B

Lecturer B taught a lesson based on 'set models and recursive sequences' to a class of 4^{th} year FET mathematics student teachers. Lesson two did exhibit visual strategies, but not as many in comparison to lessons A and C. The uses of visual strategies were very limited during the study of this lesson and were not pre-meditated. Visual strategies were not a prioritized pedagogical element used in this lesson and were in no way used to arouse interest amongst students and appeared to have played a negative role in developing the mathematical understanding and thinking of students. The use of words and the manner in which explanation occurs play an essential role. In view of this, explanation appeared to have been poorly done and students seemed very uneasy. The lack of visual strategies used in this lesson showed students disinterest in the mathematics lesson. The visual strategies used in lesson two constituted of a whiteboard, data projector, chalkboard and booklets. Gestures were also a very prominent part in this lesson. Observation of lesson two also demonstrated that meaningful explanation, works hand in hand with artefacts. Lecturer B used many semiotics. Students showed dislike when viewing symbols in mathematics and were not confident enough to fully expose themselves to semiotics. By means of observation this was probably, owing to students' misunderstanding of mathematical symbols and their function in the lesson. There were too many semiotics used thereby overpowering the lesson. Representing mathematics with a bare minimum of visuals strategies appeared to be empty and dull. A section such as recursive sequences where it is largely semiotics based can be regarded as a challenging section for students to assimilate. The use of some sort of visual strategies should be needed to assist in teaching a section like this. It was also noted that a simple visual strategy such as writing on a chalkboard, 'effectively' is crucial in a lesson. The presentation of mathematics when teaching is integral, and it is the core of any lesson. The gesture of the lecturer is also crucial in teaching. Students need eye contact, yet lecturer B continuously looked and referred to her notes, consequently focus was not on teaching which had a negative impact on students. Semiotics cannot solely be represented especially in a section like recursive sequences. The use of semiotics is heavy on students and many do not understand the meaning behind these symbols being used together, it is therefore necessary to teach and explain simply especially if the lesson is predominantly semiotic in nature. It was observed that many students have a tendency of giving up when confronted by too many symbols.

4.4.1.3. Lesson C by Lecturer C

Lecturer C taught a lesson based on ‘Number relations and operations’ to 2nd year primary mathematics education student teachers. A great number of visual strategies were observed during this lesson. Lesson three presented many interesting and exciting visual strategies used to arouse interest amongst students and played a positive role in developing the mathematical understanding and thinking of students. Lesson three showed that visual strategies constitute a crucial part with potential to easily attract students’ attention and channel a weighted amount of mathematical knowledge to them. Additionally, the tone of the lecturer and the manner in which explanation occurs plays an essential role. The visual strategies used in lesson three constituted of artefacts such as coloured pegs, number line and Russian dolls. An overhead projector, data projector and booklets and worksheets were also used as visual strategies in this lesson. By means of observation, these artefacts and other visual strategies played the most attractive role in capturing students’ attention. Observation of lesson three demonstrated meaningful explanation which worked hand in hand with artefacts. Lecturer C used many semiotics. Students showed dislike when viewing symbols in mathematics but lecturer C gained their confidence in the use of semiotics by carefully explaining at each point what each symbol was used for. She used semiotics but she did not overpower the lesson by their function and role in mathematics. She used few visual strategies and mastered their role in the classroom positively.

4.5. Comparison of Visual Strategies used:

The three lessons discussed above have given insight into the multiple visual strategies used by lecturers A, B and C. The observation schedule was drawn up to compare the visual strategies namely (gestures, classroom setup, artefacts used, etc.). Table 4.5.1 shows the comparison of visual strategies observed.

Table 4.5 1: Comparison of visual strategies

Lecturer	**A**	**B**	**C**
Focus group	1	2	3
Visual Strategies used	**1.** Paper as a manipulative **2.** Booklets **3.** Overhead projector **4.** Beans as a manipulative **5.** Chalkboard **6.** Flash cards	**7.** Whiteboard **8.** Data projector **9.** Chalkboard **10.** Booklets	**11.** Coloured Pegs **12.** Number line **13.** Data projector **14.** Booklets/ worksheets **15.** Russian dolls **16.** Overhead projector
Gestures Used	**1.** Hand movement used appropriately to enhance concepts **2.** Body movement varies appropriately **3.** Facial expression was used to emphasise points **4.** Voice tone high to emphasise points **5.** Eye contact to emphasise points	**6.** Hand movement varied too much **7.** Body movement: pacing too much **8.** Facial expression used to emphasise points **9.** Voice Tone - monotonous. **10.** Eye contact made on lecturer's preparation rather than students	**11.** Hand movement used appropriately but could be accentuated more **12.** Limited body movement. **13.** Facial expression was very limited **14.** Voice tone - monotonous explaining **15.** Eye contact made on lecturer's preparation rather than students
Semiotics	Index, symbols, Icons	Index, symbols, Icons	Index, symbols, Icons
Evidence of visual Learner	Yes	Yes	Yes
Other Observations	Class discussion played an intricate role in lesson	Explanation played an intricate role in lesson	

4.6. Gestures

4.6.1. Lecturer A/ Group ‘A’

Lecturer A used a rich amount of appropriate gestures. She had good energy levels which enhanced her body movement when explaining concepts. She made use of hand gestures efficiently and her body movement varied along with the tone of her voice as she explained. Eye content was also made to emphasize points. Image 1.1 – 1.3 are good examples of deictic gestures. According to Roth (2001: 365), “Deictic gestures are used in concrete and abstract pointing.” In the context of this lesson, gestures used by lecturer A created a warm and friendly approach toward students. This lesson was the very first lesson of students for this semester and lecturer A created an easy going student centered environment. Students initially appeared to be uneasy and scared, however lecturer A’s use of gestures added some humour to the lesson creating an easier comfortable environment for students to work in.

Image 1.1.

Image 1.2

Image 1.3

Image 2.1

Image 2.2



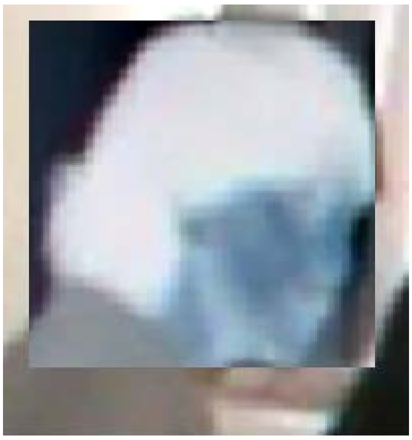


Image 3.1 Image 3.2 Image 3.3

Images 1.1-3.3 show lecturer A's use of hand gestures. She uses her hands and her fingers to emphasize points and learners visualize mathematically by the use of her tone and hand movement. Image 2 shows lecturer B explaining fractions becoming big, bigger and biggest. She used her hands to emphasize bigger fractions. She also raised her voice to emphasize big fractions and as she spoke about smaller fractions the tone of her voice lowered. She made her eyes bigger in relation to bigger fractions as demonstrated in image 3.1, 3.2 and 3.3. This gave students a good direction in visualising mentally. She also took bigger strides whilst moving from one point to another and made sudden turns when emphasizing points creating interest for students.

4.6.2. Lecturer B/ Group 'B'

Gestures were used during this lesson. The gestures used however were not intended for a purpose and were rather basic and natural. Hand gestures were used; however they were continuous hand gestures which did not hold much intention or purpose. They became very distracting to students as they focused on the hand movements rather than the actual lesson as depicted in image 4.1. and 4.2. Her movement was also either very still or she paced from one end of the chalkboard to the other end which became very distracting to students. The tone of her voice was very monotonous with little regard to whether students listened or not, the lecturer still continued. Her eye contact was with the chalkboard, and little on students was considered. Images 5.1 and 5.2 show her incorrect use of gesture in a classroom whilst she continuously flicked her hair back or played with her hair in what appeared to be an unknowing way. Body image is evidently very important to maintain a student's attention on the actual lesson. Images 4.1 and 4.2 also show lecturer B pacing about in the classroom in contrast to image 6 standing still with her notes scattered on the table limiting body movement.

Image 4.1 Image 4.2

Image 5.1 Image 5.2

Image 6

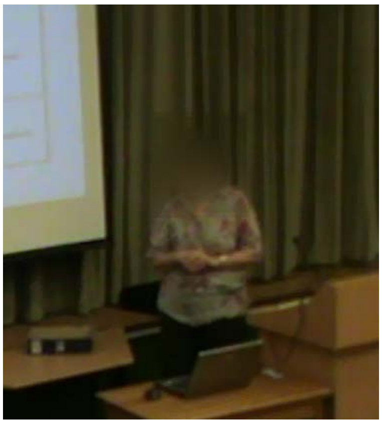
Images 4.1-4.6 show that gestures can also be used negatively. In this context it shows how body movement can both capture students' interest and allow focus or in this instance by contrast how quickly attention can be lost due to inappropriate body gestures. Not all gestures used in this context of teaching for lecturer B were unhelpful. There were plenty instances where students understood purely owing to the correct gestures used for explanation. Unfortunately there were more unhelpful gestures used than helpful ones in this lesson.

4.6.3. Lecturer C/ Group 'C'

The use of gestures in this lesson was evident however it was not a primary visual strategy that was observed. Hand movements were used but were very limited. Her body movement was almost non-existent. She moved around if she really needed to but not movement that was useful for the benefit of enhancing a concept for the lesson. Her voice was monotonous from the beginning of the lesson to the end. There was no change to the pace or levels of sound in her voice. Her eye content was to students but showed very little expression when explaining.

Image 7.1 Image 7.2

Image 7.1 shows an appropriate gesture being used. Pointing and making reference to a lesson are excellent gestures used as a visual strategy.



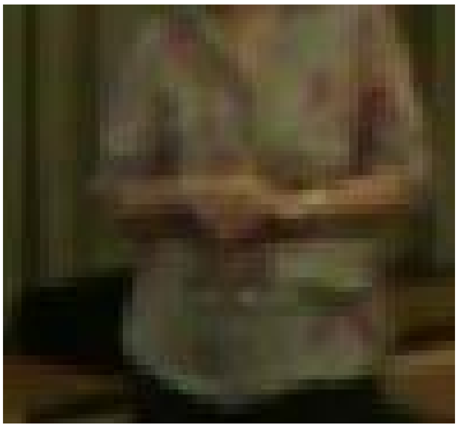


Image 8.1 Image 8.2

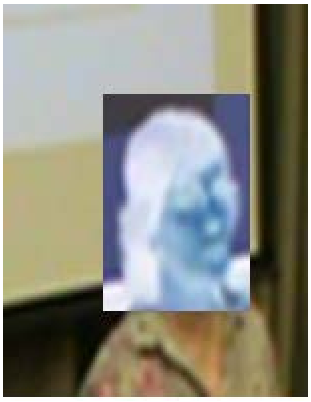


Image 9.1 Image 9.2 Image 9.3

Images 7.1 and 7.2. show lecturer C using hand gestures to indicate important concepts shown on the smart board. Lecturer C did make very good use of hand gestures when she needed to, however these hand gestures were often not used timeously. Hand gestures should be congruent with facial expression and body movement. Whilst this was no doubt an interesting lesson, the presentation of the lecturer was poor at times. Her body language was almost always too static as shown in image 8.1 and 8.2. Images 9.1- 9.4. also shows her lack of body movement along with her monotonous facial expression.

4.7. Use of Artefacts

4.7.1. Lecturer A/ Group 'A'

Lecturer A incorporated many fascinating artefacts in the duration of her lesson. The strategies used were interesting as most of them were cost effective. She used strips of paper, coloured board, beans, an overhead projector (O.H.P) was used to showcase the estimation of beans and the chalkboard was used to showcase the coloured board as flash cards. These will be further discussed in 5.6.

4.7.1.1. Strategy 1 (Strips of paper):

The lecturer based her lesson on ‘fractions’. Using strips of paper was an innovative cost effective visual strategy to demonstrate to students the concepts of fractions. Students were asked to use an A4 page and cut it in half, then a quarter and so on to demonstrate the concept of fractions. Students were able to see one out of two $(\frac{1}{2})$ or two out of eight strips of paper $(\frac{2}{8})$ and so on. By means of observation it was noticed that many students appreciated this teaching strategy as many really understood ‘equivalent fractions’ by using this artefact. This is depicted in images 10, 11 and 12. Lecturer A used all the old paper thrown out from incorrect duplication and photocopying. She also explained to them they could use chocolate which could also make an excellent visual strategy to demonstrate fractions and when done learners would have a reward of a chocolate.

Image 10

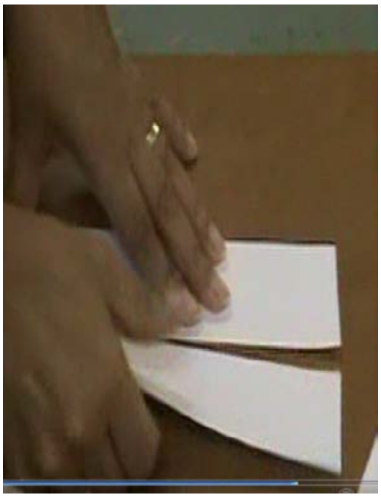


Image 11

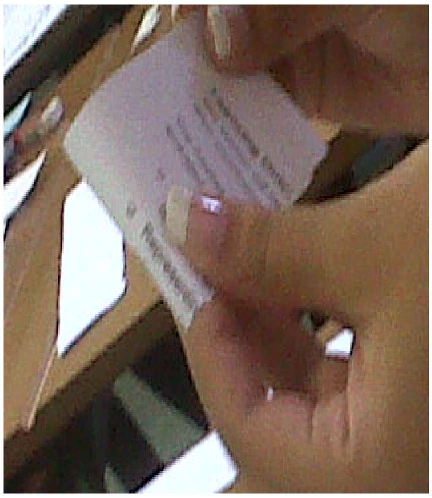


Image 12

4.7.1.2. Strategy 2 (coloured board):

After students had demonstrated for themselves what they perceived a half, one thirds and a quarter, the lecturer used ‘flash cards’ to show these equivalent fractions on the chalkboard. Students were now able to see that two halves made a whole and three one thirds also made a whole. This had given many students insight as to what is really meant by two thirds and so on. Seeing it placed on the chalkboard as demonstrated below really assisted students in understanding fractions. Lecturer A modelled this visual strategy perfectly enabling pre-service student teachers to find a suitable way to teach this. Image 13 shows that two halves make a whole and three one thirds make a whole. This gave many students the ability to visualize fractions by means of understanding their true nature as opposed to looking at them as just a concept. The use of coloured board also played a crucial role in obtaining students' attention. The coloured board became an attraction to the eye and immediately captured students' attention

Image 13

4.7.1.3. Strategy 3 (beans):

Lecturer A then moved on to teaching the next section called approximation and estimation. She used beans that she placed on an overhead projector to ask students to estimate the approximate number of beans that appear. Lecturer A then quickly turned off the O.H.P and asked students to estimate the number of beans. Students guessed random digits of 50, 38, 20, etc. There were 20 beans. This was a good method of showing estimation which many students enjoyed. Images 14 and 15 show the beans placed on the OHP.

Image 14 Image 15

4.7.1.4. Strategy 4 (Chalkboard):

The chalk board shown in image 16 was also used to explain concepts such as writing half as a fraction, decimal and percentage. It was impressively used as a tool for displaying the cardboard on fractions. It was noted when writing on a chalkboard it was very much easier for students to understand the method of working out problems. As the lecturer wrote mathematics and solved problems at task, the students watched and learnt and solved problems in a precise manner.

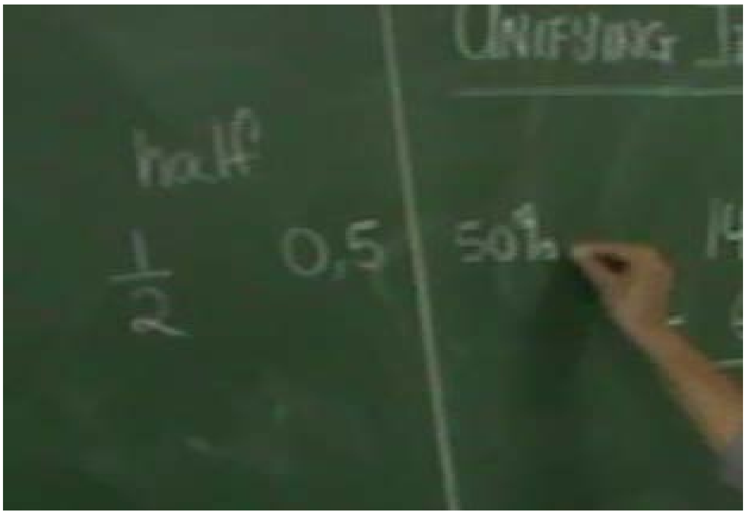


Image 16

4.7.1.5. Strategy 5 (Booklets):

Students were also given booklets to refer to as shown in image 17. The booklet also showed evidence of many visuals used to supplement students' understanding of the fractions taught in this lesson. The picture below showed that students were asked to draw five eighths. It was interesting to see the different types of visuals students used to interpret five eighths.

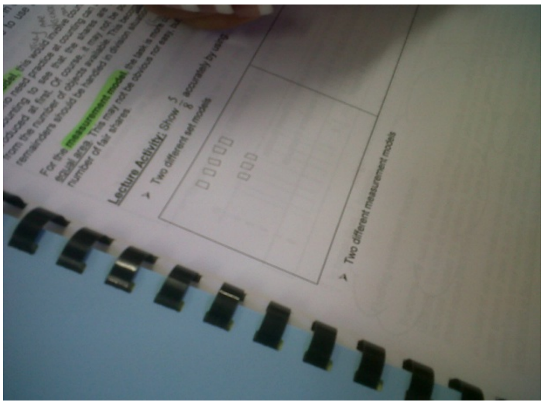


Image 17

4.7.2. Lecturer B/ Group ‘B’

Lecturer B had not incorporated many artefacts in her lesson. I will however show the strategies used by her to teach.

4.7.2.1. Strategy 1 (Whiteboard):

Lecturer B used the whiteboard to explain recursive sequences. As depicted in image 18 and 19, coloured whiteboard markers were used on the whiteboard however they appeared to make the writing very light and it became a blur to students who sat on the far end of the classroom. Lecturer B tried to incorporate the use of three different coloured markers but it was very hard to see from the far end of the classroom. Students in turn complained and got agitated because it was difficult to see. Many students gave up and just did not do anything at all because it was hard to see and comprehend what was being said. Some students had even walked out of the lesson.

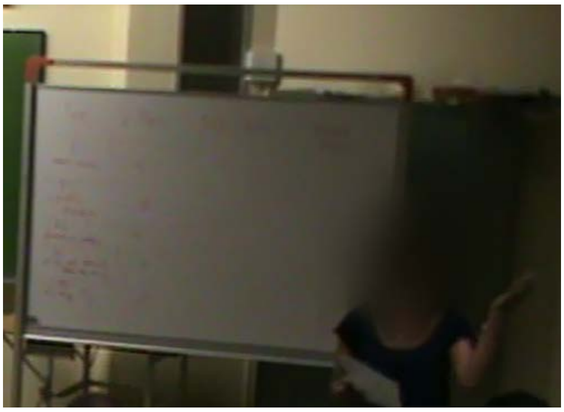


Image 18

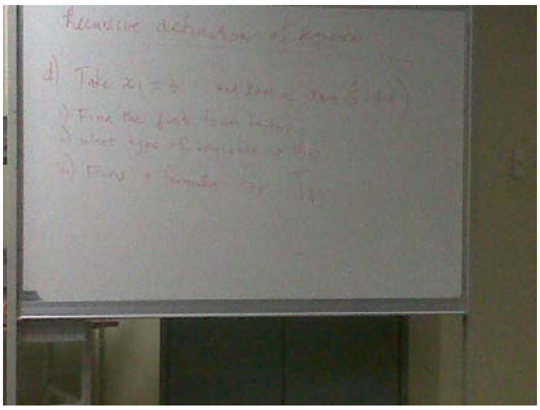


Image 19

4.7.2.2. Strategy 2 (Chalkboard):

Lecturer B used the chalkboard to show how problems of recursive sequences can be done. She also incorporated many colours which appeared to have a good visual strategy intention, however the writing presented was very shabby on the chalkboard as shown in image 19. She had smudged some writing with her hand on the chalkboard so presentation was unclear. Students were confused as they could not read off the chalkboard. The chalkboard is a major tool that supports visual learning especially accommodative to visual students. When utilized incorrectly as with any other visual strategy used incorrectly it can result in a disaster as in this context. Students not having the ability to read through all the fuzziness of the handwriting experienced much confusion and disinterest in the lesson. Chalkboard strategy was one of the very first visual strategies to have been acknowledged. According to UNESCO (1981) "The chalkboard supports visual learning. The teacher must therefore adapt his style of chalkboard presentation of the level of the 'visual vocabulary' of the learners, (or at least to their mean level)"

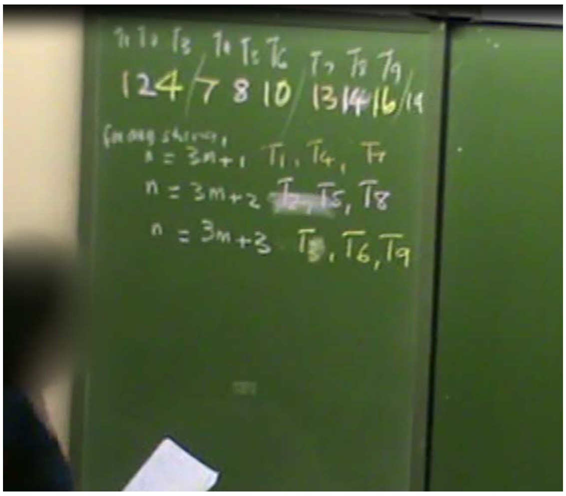


Image 20

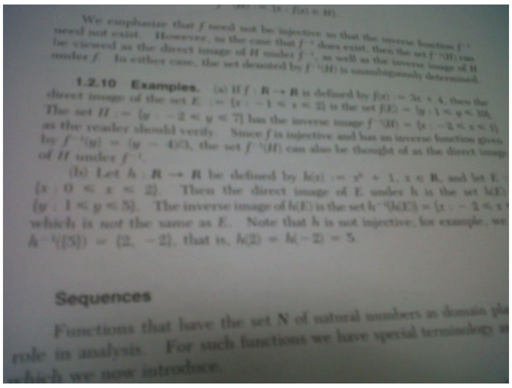
4.7.2.3. Strategy 3 (Data projector):

The data projector was placed on as displayed in image 21. Lecturer B attempted to use it but the class was noisy and uneasy so it was put off. There was no true purpose or function for it being placed on. The data projector would have been a good visual strategy used to display recursive sequences. Students would have been able to see more clearly in this venue as opposed to the chalkboard.

Image 21

4.7.2.4. Strategy 4 (Booklets):

All students were given a booklet. The booklet showed evidence of some visuals used to supplement students' understanding of mathematics. The booklet was an excellent visual strategy in reference to this lesson and it was an effective visual strategy and resource used to introduce recursive sequences. It provided good examples which students used effectively as a guide. The booklet did however contain many complicated visuals that were shown to have perplexed the majority of the students with the exception of a few students who embraced this as a challenge. Some students were discouraged by the appearance of too many semiotics used in these notes. It seemed to have shifted students' apathy to develop mathematically.


We emphasize that f need not be injective so that the inverse function f^{-1} need not exist. However, in the case that f^{-1} does exist, then the set $f^{-1}(H)$ can be viewed as the direct image of H under f^{-1}, as well as the inverse image of H under f. In either case, the set denoted by $f^{-1}(H)$ is unambiguously determined.

1.2.10 Examples. (a) If $f : \mathbf{R} \to \mathbf{R}$ is defined by $f(x) := 3x + 4$, then the direct image of the set $E := \{x : -1 \leq x \leq 2\}$ is the set $f(E) = \{y : 1 \leq y \leq 10\}$. The set $H := \{y : -2 \leq y \leq 7\}$ has the inverse image $f^{-1}(H) = \{x : -2 \leq x \leq 1\}$ as the reader should verify. Since f is injective and has an inverse function given by $f^{-1}(y) = (y - 4)/3$, the set $f^{-1}(H)$ can also be thought of as the direct image of H under f^{-1}.

(b) Let $h : \mathbf{R} \to \mathbf{R}$ be defined by $h(x) := x^2 + 1$, $x \in \mathbf{R}$, and let $E := \{x : 0 \leq x \leq 2\}$. Then the direct image of E under h is the set $h(E) = \{y : 1 \leq y \leq 5\}$. The inverse image of $h(E)$ is the set $h^{-1}(h(E)) = \{x : -2 \leq x \leq 2\}$, which is not the same as E. Note that h is not injective; for example, we have $h^{-1}(\{5\}) = \{2, -2\}$, that is, $h(2) = h(-2) = 5$.

Sequences

Functions that have the set $\mathbf{N}$ of natural numbers as domain play a ... role in analysis. For such functions we have special terminology ... which we now introduce.

Image 22 Image 23

4.7.3. Lecturer C/ Group 'C'

Lecturer C incorporated many attractive and motivating artefacts in her lesson. The strategies used encouraged much interest from students. Owing to the visual strategies used, students' attention was maintained throughout the lesson. Lecturer C used a number line, coloured pegs, Russian dolls, Smartboard and the O.H.P. These will be discussed further.

4.7.3.1. Strategy 1: (Number line and Pegs):

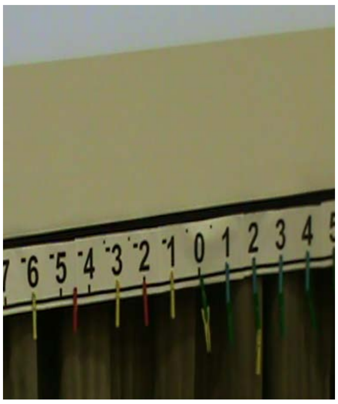
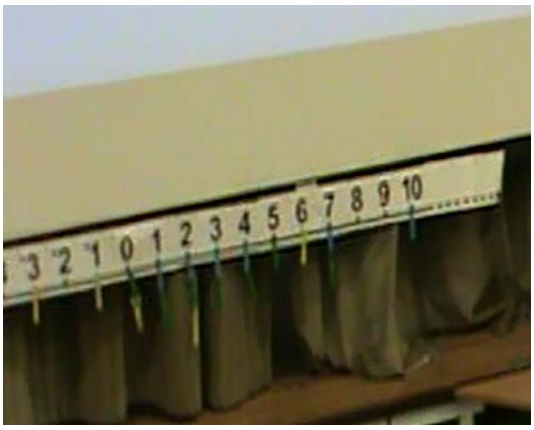


Image 24.1 Image 24.2 Image 25

This was an effective visual strategy. As students entered the room she asked a student to hand out random colours of pegs as shown in Image 25. Students were given blue, green, yellow and red pegs. She then had a huge number line from negative ten to positive ten pinned on the bottom of the white screen. She recapped with students the various definitions of natural numbers, whole numbers, integers and rational numbers. Each coloured peg represented a natural number or a whole number. For e.g. all students who had the blue peg were asked to come to the front and place the peg on numbers that represented natural numbers shown in image 24. All students who had the green peg were then asked to come in front and peg numbers that represented whole numbers. Similarly yellow pegs represented integers and red pegs represented rational numbers. Images 26, 27 and 28 and 29 show the various colours of pegs being placed on the number line demonstrating the type of category in which these pegs fall under.

Image 26

Image 27



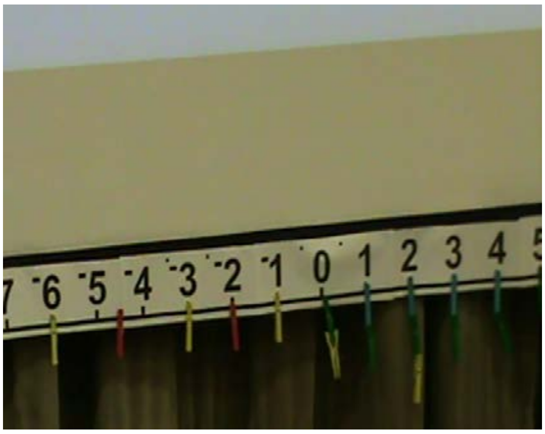


Image 28

Image 29

4.7.3.2. Strategy 2 (Russian Dolls):

Lecturer C had used a very interesting and unique artefact in her lesson. She had used Russian dolls to represent the number system showing that one system is within another system as shown in image 29-32. The smallest Russian doll was a representation of natural numbers; the next Russian doll was a representation of whole numbers. As demonstrated by image 33 lecturer C then placed the natural number Russian doll in the whole number Russian doll to show that natural numbers are a constituent of whole numbers. She did the same thing with the next Russian doll which represents integers, the other two Russian dolls i.e. the natural number and the whole number in Russian doll were placed in the integer Russian doll to become one Russian doll. This shows that natural numbers and whole numbers are constituents of integers. Similarly the same occurred between the next two Russian dolls that is rational numbers and real numbers.

Image 30

Image 31

Image 32

Image 33

4.7.3.3. Strategy 3 (Booklets):

Students were also given booklets to refer to. These booklets appeared to be a very useful tool which students used frequently in the lessons. The booklet also showed evidence of many visuals and semiotics used to supplement learners' understanding of mathematics. The booklets were also very user friendly with many tables, diagrams and cartoons making provision for visual students. Students appeared enthusiastic to write in those books as they always referred to those booklets, which were also in colour which made them pleasing to the eye of the student.

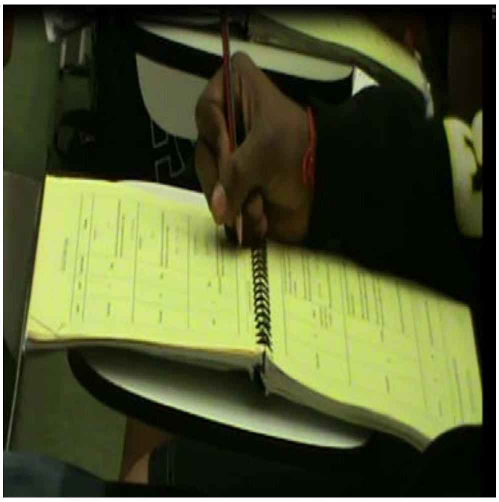


Image 34 Image 35

4.7.3.4. Strategy 4 (Data projector):

Lecturer C used the document reader most efficiently. She used it effectively as she had the booklet students used as a programme so students could see exactly what she wanted from them. She also had a Smartboard programming tool where she made markings on her laptop depicted in images 36 and 37. Students could see well since it was projected on the screen.

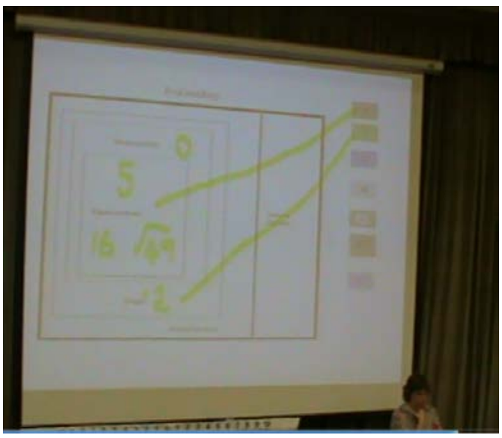


Image 36 Image 37

4.7.3.5. Strategy 5 (Overhead projector):

Lecturer C used the overhead projector by placing a blank transparency on which she wrote using different coloured markers displayed from image 38 - 41. She used this to teach students how to multiply, add and subtract using different methods. She used different coloured markers to write on the blank transparency. This was used effectively as it catered firstly for the large classroom accommodating such an enormous number of students to see from afar what was happening.

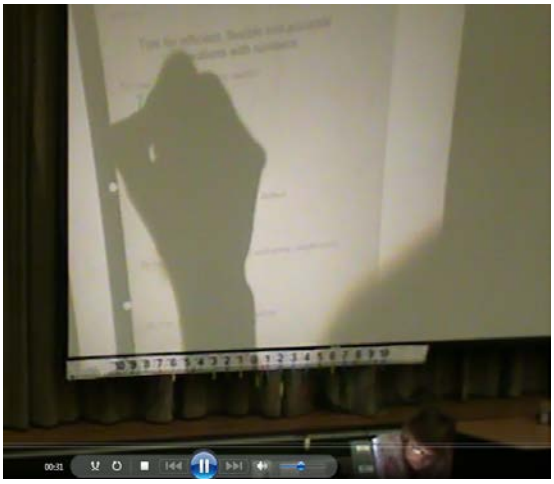


Image 38

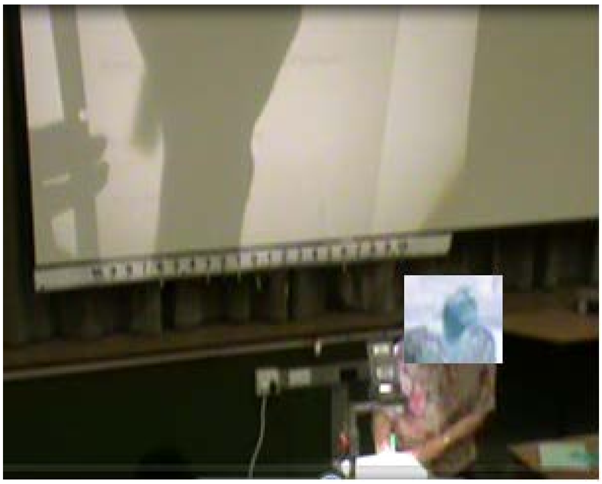


Image 39

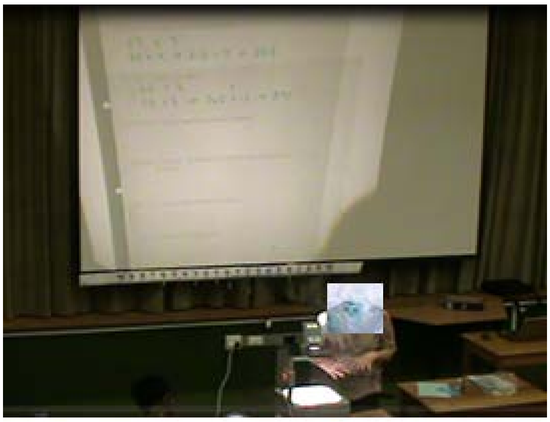


Image 40

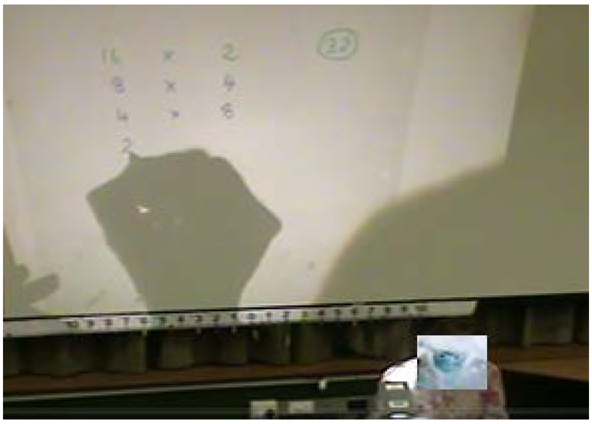


Image 41

4.8. Classroom Set-up:

4.8.1. Lecturer A/ Group ‘A’

Students were seated in three rows with each row set such that students face each other. The row is continuous along the aisle preventing students from meaningfully interacting and communicating. Students are also easily distracted by other students as they faced each other. Bags and pencil cases became a distraction to many students. Many students at the far end of the class had difficulty seeing as they became lost amongst the people in front of them. It was also noted to be uncomfortable looking at the chalkboard sideways as opposed to facing forward. Many students became tired straining their necks to see the chalkboard and some just did not look at the chalkboard anymore and faced forward downwards. Classroom setup should be regarded as a very important visual strategy however it appears to be a visual strategy that is often ignored. The aim is for students to be comfortable at what they see and to not want to keep their eye away from the lesson of the lecturer.

Image 42

4.8.2. Lecturer B/ Group ‘B’

Classroom setup was observed to be a visual strategy used to place students in place of interest. In reference to this lesson, students were seated in pews. Each row is aligned so that if one learner needs to get up the entire row needs to get up to allow one particular student to move from his/her seat. The row is continuous preventing students meaningfully communicating and interacting. Lecturer B had asked students to work in groups but students were not able to turn around easily. Students are also easily distracted by other students as it is evident what each other is doing. Classroom setup is very often ignored but can dramatically affect students' attitudes toward the lesson and developing habits of learning. Students need an environment that is organized, stimulating, and comfortable in order to learn effectively. Observing the classroom setup with reference to this lesson had shown to be a much disorganized environment. Students walked in and out as they pleased and continuously spoke sometimes above the lecturer. In this regard it would appear that classroom setup as a visual strategy ties in with classroom discipline. Creating such an environment entails arranging a practical physical layout, as was done in this lecture venue; however the lecturer should have aided in some method of organisation to the classroom setup. Students need to have a sense of belonging and ownership in a classroom and from observation many students did not feel a part or belonging to this classroom environment. The lecturer could have also aided in discipline of students. Visual strategies are meaningless if the discipline of students in a lesson is lost. The gaining of interest of students is particularly important which can be achieved by the appropriate use of visual strategies as shown in lesson 1 of lecturer A.

Image 43

Image 44

4.8.3 Lecturer C/ Group 'C'

This was also observed to be a visual strategy. Students were placed in a traditional lecture room setup facing forward. The lecture theatre was too large and the lecturer seemed to be lost in such a big room. It allowed for students to work independently and in pairs but could not accommodate efficiently for group work. It prevented learners from meaningfully communicating with one another when working in groups.

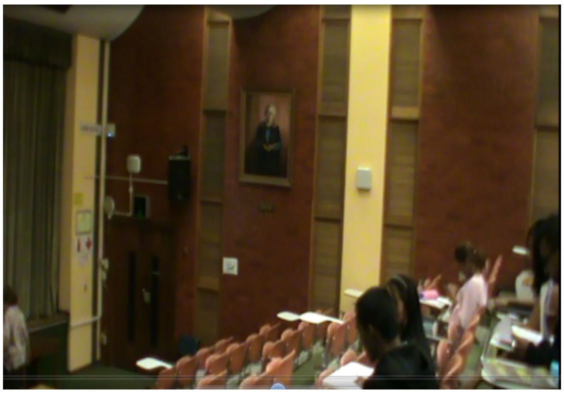


Image 45.1

Image 45.2.

As shown by images 45.1 and 45.2, it can be seen that there are many empty seats. The large lecture venue appeared to be a huge barrier in creating a conducive learning environment. Learners who were selected to come to the front of the venue took approximately 3 minutes to get there. This venue was very large for this lesson. It took about 1 minute for each student to walk from the back to the front to peg on the number line was much too long thereby wasting time. This venue created many barriers. Students and lecturer found it very hard to move about which proved that this venue was not suited to a lesson that included pair and group work.

4.9. Visual Students

4.9.1. Lecturer A/ Group 'A'

It was evident that there were visual students. Students highlighted and drew diagrams to make meaning which is also illustrated below. Students also only seem to understand the meaning behind 'one third' when they were asked to partition the paper, when they saw the coloured cardboard on the chalkboard or when asked to draw it. Many students were drawing in the air, some were sketching on a page and others took out their many colours.

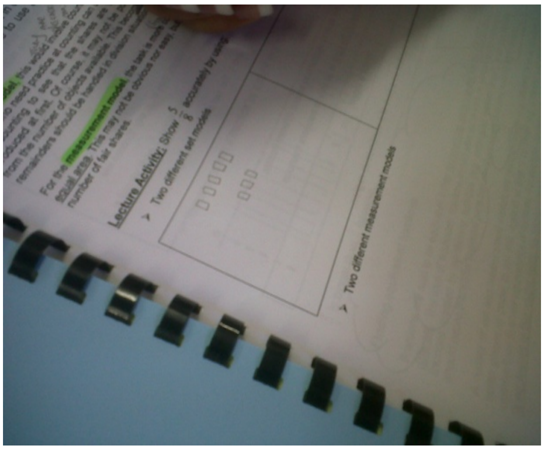


Image 46

Image 47

Image 46 depicts a student who drew fractions and is trying to make sense of what 1/8th is. She used a highlighter to emphasize key notes. Image 44 shows the various colours used by certain pre-service student teachers. According to Flevares and Schiff (2013: 330), it is stated that a visual student can be identified by the constant drawing to understand mathematics, the colours, etc.

4.9.2. Lecturer B/ Group 'B'

In this lesson there were evidently many visual students who were present. Students highlighted and drew diagrams to make mathematical meaning as illustrated below. According to Mortensen (2016:8), evidence shows that a visual student often loses focus during long verbal lectures, especially if these are not accompanied by drawings and illustrations. This lesson was a good indication of many visual students being present. Lecturer B spent many minutes just talking about recursive sequences. Students lost focus. The frequent interruptions in this lesson possibly originated from the lack of understanding from visual students leading to students leaving the lecture and walking in as they pleased. According to Mortensen (2016:10), the visual learner takes mental pictures of information given, so in order for this kind of learner to retain information, oral or written, presentations of new information must contain diagrams and drawings, preferably in color. The visual learner cannot concentrate with a lot of activity around him and will focus better and learn faster in a quiet study environment. Mortensen also explains perfectly the context of this lesson. There was too much happening in this venue with students walking in and out. Talking occurred amongst students and visual students lost focus very quickly. Image 45.1 shows that a visual student did try to understand. The worksheet depicted by image 45.1 shows highlighted diagrams and pencil work. It also shows drawing as shown in image 45.2.

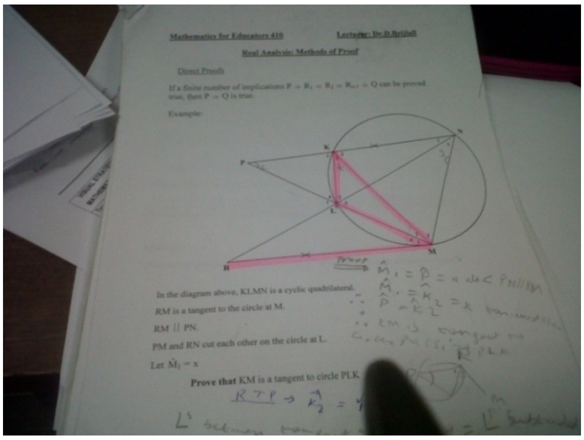


Image 48.1

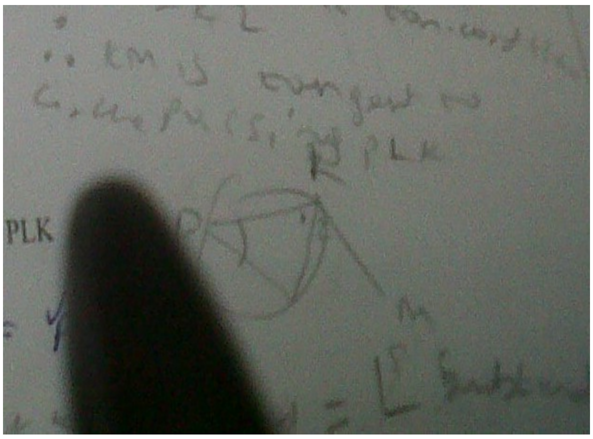


Image 48.2

4.9.3. Lecturer C/ Group ‘C’

It is evident that there were many visual students present in this class. Students highlighted and drew diagrams to make meaning which was the very first indication of visual students. Students also only seem to understand the meaning behind integers and rational numbers once they were shown by the number line and Russian dolls. Students seemed to enjoy the colours of the Russian dolls. Many students initially had a challenging time understanding the number system just by explanation but when students were asked to demonstrate using the coloured pegs, the number system became clear. Evidence shows that many were visual students because they began visualising by means of demonstration of the number system. Image 46 shows students’ books in colour amongst diagrams placed in the book. Students seemed very pleased by the colours in the book as well as the diagrams.

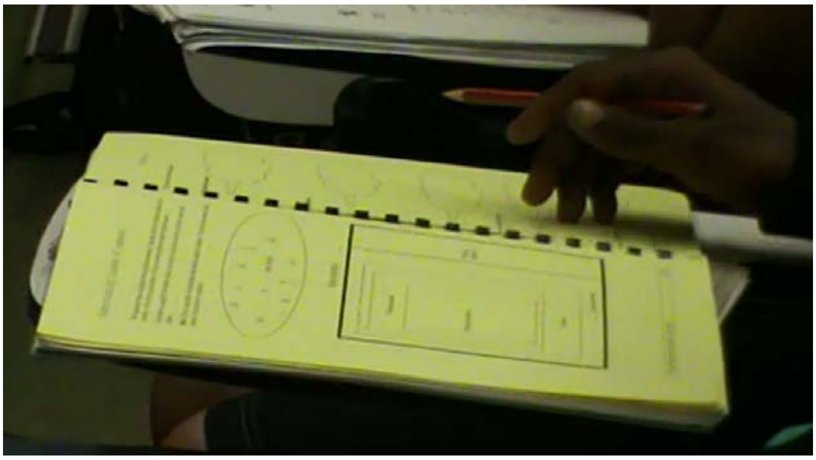


Image 49

4.10. Semiotics

4.10.1. Lecturer A/ Group 'A'

The use of semiotics was also generously used in this lesson. Students appeared to frown as they saw division signs or decimals. They seemed to have disliked the use of semiotics. Lecturer A however explained very clearly the various uses of these symbols used in mathematics. Her use of words to explain mathematics also seemed to have gained learners' understanding and confidence when using semiotics. She used semiotics but with much caution. She used semiotics but did not over use them. As shown below many symbols and index were used. Image 47.1 and image 47.2 shows the various semiotics used such as 0.5 and the fraction signs as well as the percentage symbol.

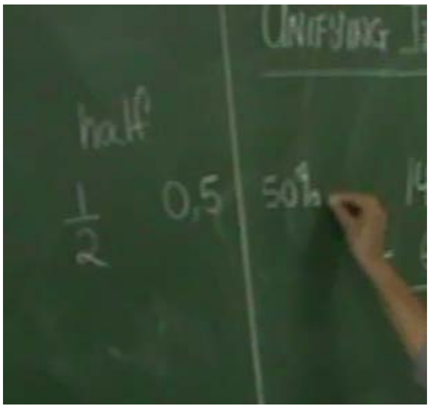
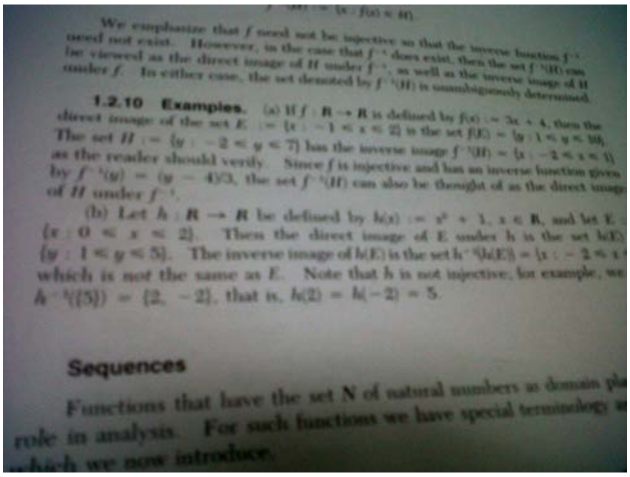


Image 50.1

Image 50.2

4.10.2. Lecturer B/ Group ‘B’

This lesson exhibited a vast amount of semiotics. The semiotics used in this lesson was overpowering owing to the nature of the topic taught, ‘recursive sequences’. Students showed signs of lack of interest immediately from the lesson when they were exposed to the large amount of semiotics used in this lesson. Image 48 displays the worksheet students had to refer to during this lesson. On this little section displayed, it appears as if there is almost every type of symbol, index and icon used. Image 48 shows the greater than sign, different arrows, the element sign, the real number symbol, colons and semicolons, the inverse sign and so on. This did appear to be a lesson much overloaded with semiotics. Semiotics and mathematics work side by side, however using semiotics in moderation is crucial for a successful lesson. This lesson was not successful and one of the reasons could have been students’ over exposure to a plethora of semiotics.



1.2.10 Examples.

(b) Let $h : R \rightarrow R$ be defined by $h(x) := x^2 + 1$, $x \in R$,

which is not the same as E. Note that h is not injective; for example,

$h^{-1}(\{5\}) = \{2, -2\}$, that is, $h(2) = h(-2) = 5$.

Sequences

Functions that have the set N of natural numbers as domain

role in analysis. For such functions we have special terminology

Image 51

4.10.3. Lecturer C/ Group ‘C’

The use of semiotics was also generously used in this lesson. Students appeared to frown as they saw symbols representing rational numbers and whole numbers. They seemed to have disliked the use of semiotics. Lecturer C however explained very clearly the various uses of these symbols used in mathematics. Her use of words to explain mathematics also seemed to have gained students’ understanding and confidence when using semiotics. She used semiotics but with much caution and was careful not to over use them. Image 50 depicts the various number systems such as natural numbers, rational numbers and their symbols along with their meaning.

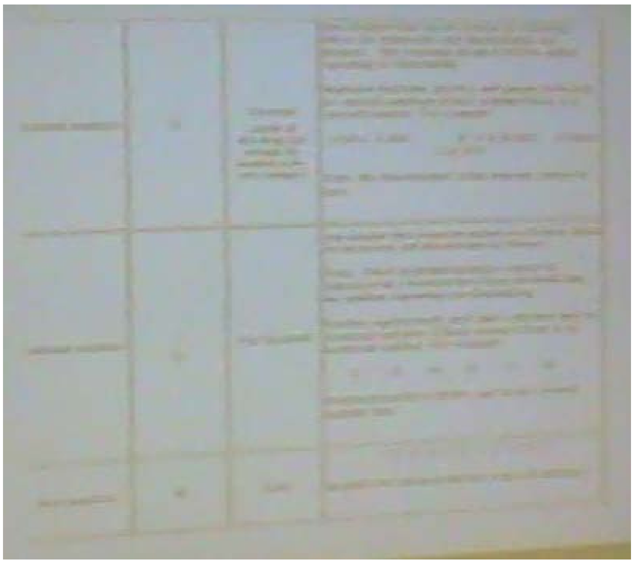


Image 52

4.11. Visual strategies used in Lesson A

4.11.1. Lecturer A/ Group 'A': Table 4.11.1

Paper as a manipulative	Overhead projector	Chalkboard
Booklets	Beans as a manipulative	Flash cards/Coloured cardboard

4.11.2. Lecturer B/ Group 'B': Table 4.11.2

Whiteboard	**Data projector**	**Chalkboard**
Booklets		

4.11.3. Lecturer C/ Group 'C': Table: 4.11.3

Coloured Pegs	**Number line**	**Data projector**
Booklets/ worksheets	Russian dolls	Overhead projector

Tables 4.11.1, 4.11.2 and 4.11.3 display the visual strategies used in lessons 1, 2, and 3 used by lecturers A, B and C respectively. Lecturer B appears to have used the least amount of visual strategies. It could be that lecturer B was disadvantaged to have taught FET students of the senior phase. Teaching the higher phase of students could limit the resources used however this

could be controversial. Lecturer A and lecturer C used many visual strategies most probably owing to teaching of students at a primary phase. It usually appears that there are many more visual strategies used with exciting innovative ideas from primary phase students as opposed to senior phase students; however this could be much debated.

4.12. Focus group interview

At the end of each lesson, students were selected randomly by volunteering to participate. They were asked questions by means of a semi structured interview. A questionnaire was prepared for this semi structured interview where students were asked to record their thoughts as questions were asked and a discussion occurred. The following table 4.12.1 represents their transcribed responses to the questions asked.

Table 4.12. 1: Lecturer A – FOCUS GROUP 1

1. What are visual strategies?					
STUDENT:	AGE:	GENDER:		YEAR OF STUDY:(e.g. 1st, 2nd, etc)	RESPONSE:
		Male	Female		
Student 1	23		√	2nd	“I understand that visual strategies are used to enhance a lesson as well as a learner’s understanding of different concepts and ideas when it comes to learning”
Student 2	20		√	2nd	“ Visual Strategies are strategies that are used to encourage the learners to engage in the lecture by seeing e.g. the overhead projector and hand gestures because some learners learn best by seeing”

Student 3	21	√		2nd	"It is the use of an overhead projector and other resources such as videos, posters and visual examples"

Student 1 stated that visual strategies are used to "enhance a lesson as well as learners understanding". Further discussions with student 1 explained that a visual strategy helps in learners understanding of "different concepts and ideas". Student 2 stated that visual strategies encourages "learners to engage in the lesson". She also went much deeper into stating that her idea of a visual strategy could be the overhead projector or hand gestures. She explains that by seeing what is projected by the OHP helps "learners learn best by seeing". Student 3 states that visual strategies is the use of an "overhead projector and other resources as videos, posters and visual examples".

Table 4.12. 2: Lecturer B – FOCUS GROUP 2

1. What are visual strategies?					
STUDENT:	AGE:	GENDER:		YEAR OF STUDY:(e.g. 1st, 2nd, etc.)	RESPONSE:
		Male	Female		
Student 1	20		√	4TH	visual strategies are seen as artefacts,gestures,pictures, images and all visual information used to translate and communicate ideas mathematically
Student 2	21	√		4th	"They are what learners see, when you teaching. The teaching strategies (writing on a board), they are the signs you use when making functions, e.g. the therefore sign ∴ "

Student 3	22	√		4th	"Visual strategies are the strategies to teaching that use visual objects"

Student 1 states that visual strategies are seen as "artefacts, gestures, pictures, images and all visual information". She explained further by saying that these visual strategies help to "translate and communicate ideas mathematically". Student 2 explained that visual strategies are what "learners see when you are teaching". Student 2 further elaborated by stating that it is the teaching strategies used like "writing on a board". She expressed that visual strategies are "signs" used for teaching like the "therefore sign". Student 3 stated that visual strategies are strategies in teaching that use "visual objects".

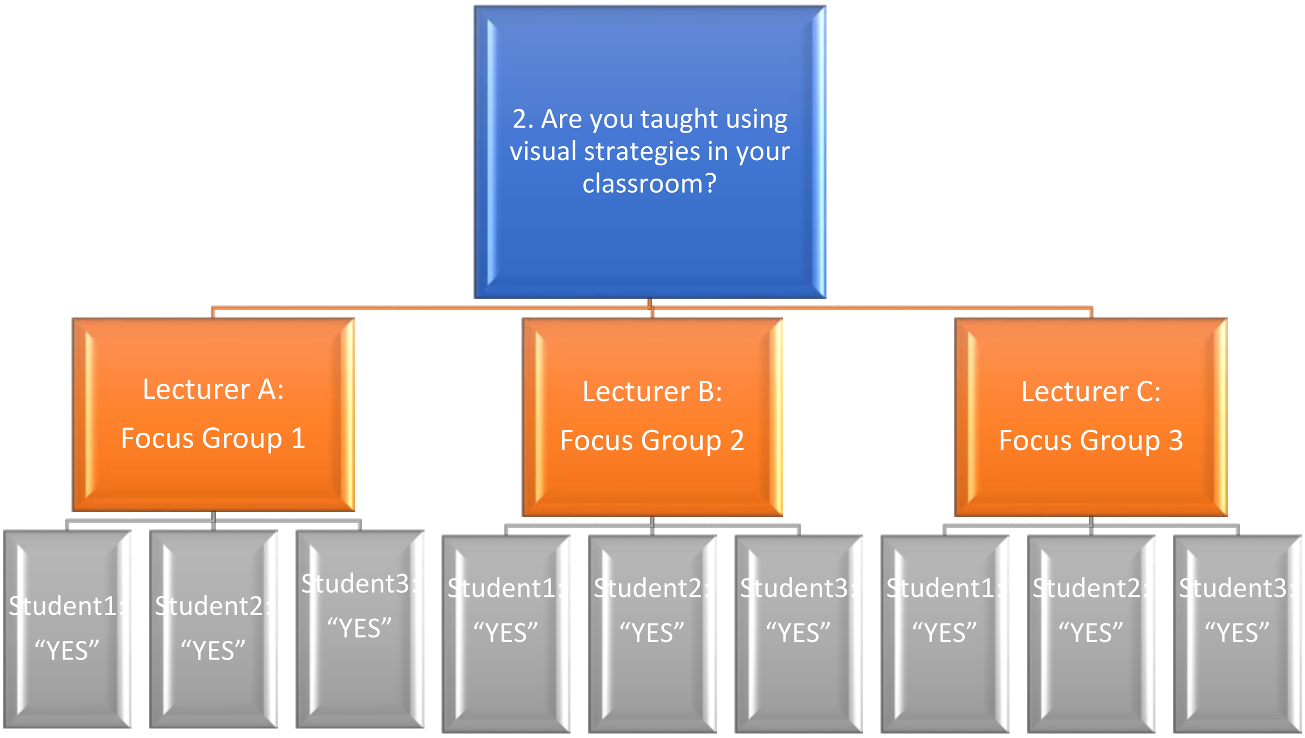
Table 4.12. 3: Lecturer C – FOCUS GROUP 3

1. What are visual strategies?					
STUDENT:	AGE:	GENDER:		YEAR OF STUDY:(e.g. 1st, 2nd, etc)	RESPONSE:
		Male	Female		
Student 1	20	√		2nd	"Visual strategies are the resources that are utilized which allow for a better understanding of concepts. These include objects, pictures, models as well as the bodily gestures of the educator"
Student 2	21		√	2nd	"Visual strategies enhance learners' abilities to understand their work better "
Student 3	21	√		2nd	"These are the things such as number lines which are used to get students involved in the lesson"

Student 1 stated that visual strategies are the "resource's utilized to allow for better understanding". She went on further to describe these resources. She stated that these include "objects, pictures, models as well as bodily gestures". Student 2 simply explained that visual strategies "enhance learner's abilities to understand". Student 3 used the actual lesson of Lecturer C to bring forth her definition of visual strategies. She explains that a visual strategy can be a number line which can get learners involved.

The table 4.12.1-3 is an analysis of the first question asked to students. The question asked was the basis of the research which is "What are visual strategies?" Drawing from the responses above most students understood a little about visual strategies. Focus groups 1, 2 and 3 students understood a little. In focus groups 2 and 3, all students understood what visual strategies were in the terminology used as circled. In total 7/9 students understood what visual strategies entailed yet their definitions varied. Students said that visual strategies were used to enhance a lesson and referred to them as engaging the lecturer to allow students to 'see'. They included resources such as a projector, posters and gestures to visual strategies.

The second question given to the students was, "Are you taught using visual strategies in your classroom?" All students responded with yes from focus groups 1, 2 and 3.

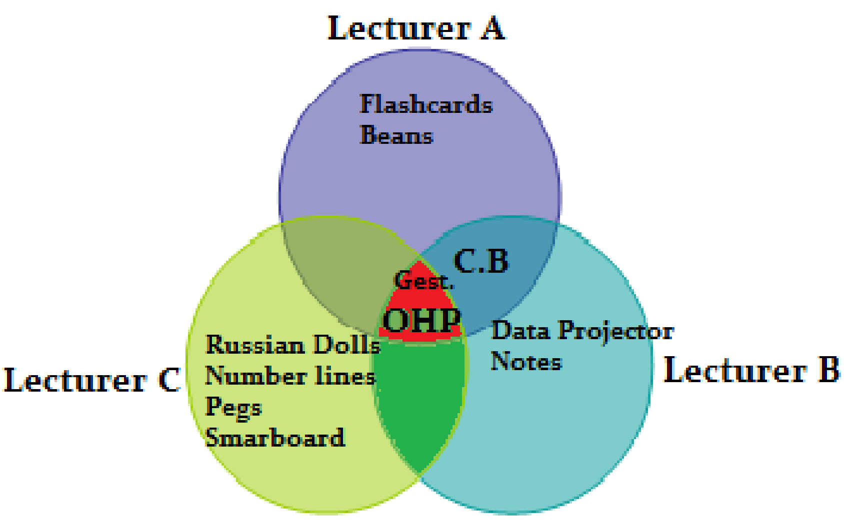


Flowchart showing students responses using visual strategies in their classrooms

Students from focus groups A, B and C said that they were taught using visual strategies in their classrooms.

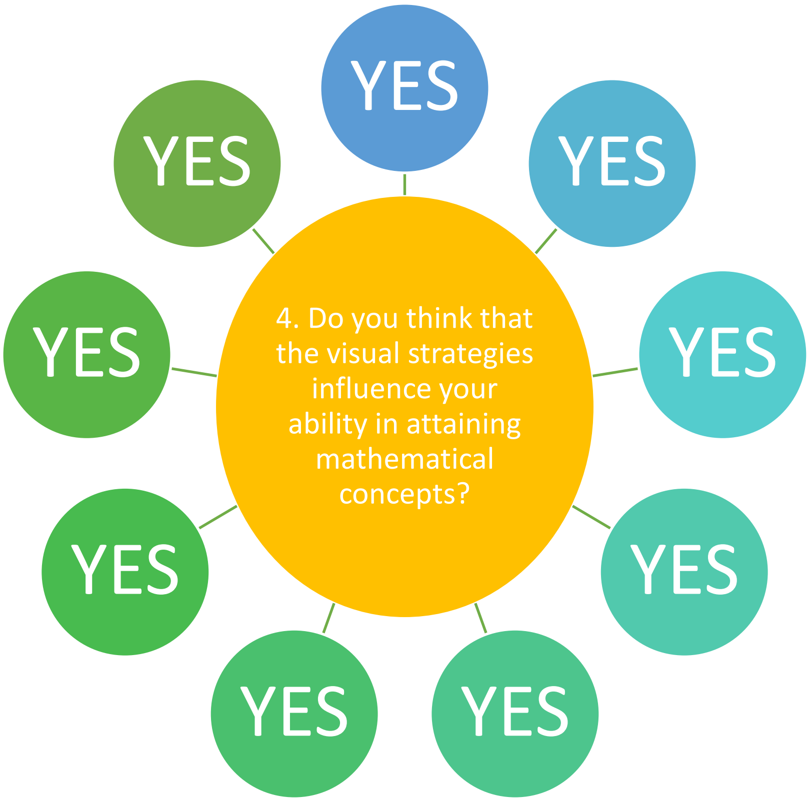
The third question students were asked was to identify the visual strategies used in their classrooms. The Venn diagram shows the visual strategies that students identified being used in their lecture rooms.

The Venn diagram shows all three lecturers used an OHP, and students' understanding of only lecturer A and lecturer B using gestures, whilst only lecturer A and lecturer B used a data projector and notes. Students' understanding of visual strategies of lecturer C were shown as the Russian dolls, number lines, pegs and Smartboard which were correct, however students' understanding for lecturer A might have been narrowed down. Lecturer A used beans, and coloured paper which students may have not identified as being a visual strategy. Similarly with lecturer B and lecturer C there were many visual strategies used as shown in the observation section above, that were eliminated from what students perceived the visual strategies used in their lecture rooms.



Venn diagram showing students responses to ‘attaining mathematical concepts using visuals’

The fourth question posed to learners was if they taught that that the visual strategies influenced their ability in attaining mathematical concepts.



All students answered “yes”, to visual strategies influencing their ability in attaining mathematical concepts.

The fifth question presented to students was if they thought that visual strategies perplex or confused them when used in their mathematics classroom.

Lecturer A – Focus group 1

Student 1: "It makes it easy to understand, because it becomes more real, also assisting the people who cannot work out things mentally"

Student 2: "They are very useful in maths lessons as you can easily visualise the different concepts and proportions"

Student 3: "No, they help to create better understanding of concepts especially in a mathematical classroom"

Student 1 deliberated that its assists "people who cannot work things out mentally". Whilst student 2 stated that it can help one to "easily visualise". Both student 1 and 2 agreed that visual strategies do not perplex or confuse a learner's understanding in mathematics. They also go on to say why it is beneficial for the learner.

Lecturer B – Focus group 2

Student 1: "No, our notes are used as a copy when we are taught; visual strategies used show what is needed from us"

Student 2: "They help me understand better, they make the notes from the course pack clear and understandable"

Student 3: "Sometimes when notes are given in a booklet form using small writing it tends to confuse us and makes learners afraid, but when a chalkboard or projector is used and the work is spaced and easy to see it is better to understand.

Student 1 attempted to explain that her notes are a form of a visual strategy which is used to help rather than used to confuse a student. This led to a further discussion to student 2 who also explained that the notes used helped in making one understand better. Student 3 however debated that the notes are in "small writing and it tends to confuse learners and makes them afraid". He stated that the chalkboard and projector was a better visual strategy to use.

Lecturer C – Focus group 3

Student 1: "No, it helps us to understand to understand the concepts, it helps us to see what is being taught

Student 2: "No, it is something we can see and what we see is believable"

Student 3: "No, I feel they enhance concept development and improve mathematical knowledge"

Student 1 stated that it helps them to understand whilst student 2 stated that when they see it is more "believable".

Most students from focus group 1, 2 and 3 agreed that visual strategies help learners to understand better rather than confuse them.

The final question posed to learners was what they understood by gestures when teaching and as a future mathematics educator, if they saw themselves using many gestures when teaching.

"Body language"

"Facial expression"

"When pointing to something, when being a statue and when using your voice"

Students described gestures as the use of body language, some also explained that it was an educator's facial expression. Most learners described gestures to be pointing at something or manipulating your voice to gain understanding.

4.13. Questionnaires

A questionnaire was given to each educator after the recorded videos.

4.13.1. Participants' understanding of the notion of visual strategies

The term visual strategies needed to be defined in the context in which it was being used. Although there are several definitions of this concept, it has become clear in the literature review, Chapter Two of this study that it is important a more common understanding of the concept was arrived at in order to use it as a basis to analyze the responses of the participants. Visual strategies may be defined as:

> Visual Strategies are things that we see. Body movements, environmental cues, pictures, objects and written language can all be used to support communication. Our environment is full of signs and logos and objects and other visual information that supports communication (Hodgon, 2017, p.1).

This definition is directly related to this study because of its relevance to mathematics education and the mathematics classroom. In the context of this study, visual strategies are seen as artefacts, gestures, pictures, images and all visual information used to translate and communicate ideas mathematically. This definition has been similarly adopted by Alibali and Goldin-Meadow (2013: 468) and Roth (2001: 366).

The discussion that follows, addresses the meaning of visual strategies as it was understood by the participants of the research study. This will become evident by focusing on their understanding of visual strategies and examining their use of them in their classroom practice.

Table 4.13.1: Lecturers' responses to question1 (What are visual strategies?)

1.	What are visual strategies?
LECTURER A	"Literally I think this includes all the strategies that are visibly demonstrated and/or used in this case, during maths teaching! So it's those that are explicit (clear) and clearly seen by all in that particular learning space (e.g. desk where one works with a group). Overall I think strategies to explain are model concept so students can visualise these better e.g. relative sizes of fractions (models help)"
LECTURER B	"I have no idea"
LECTURER C	"I would imagine that they are what we have for years used and referred to as teaching using visual aids. Any strategy that employs more than talk."

Lecturer A was of the view that visual strategies referred to the notion of those that are explicit (clear) and clearly seen by all in that particular learning space (e.g. desk where one works with a group). Overall I think strategies to explain are model concepts so students can visualize these better e.g. relative sizes of fractions (models help). She also believed that it was taking something that is practical and visual and turning it into something that is almost a rule as depicted in her lesson. In her lesson she asked her learners to use paper strips to represent fractions. This visual representation of paper allowed her students to model the concept of halving and making relative sizes of fractions using paper strips. She referred to other examples in her lesson by using chocolate to represent fractions. She also used beans as a visual strategy to show estimation.

Lecturer B's response to her understanding of visual strategies differed from the views expressed by the other participants. Whilst the other participants immediately responded to the notion of visual strategies, she indicated that she did not fully comprehend the meaning of the concept. In the context of her lesson, there were a huge amount of gestures, semiotics and images used.

Lecturer C's, response was of a positive notion in light of the use of visual strategies. She confidently asserts that it is what she would imagine they are what she has used for years and referred to as teaching using visual aids. She also defines them as any strategy that employs more than talk.

Table 4.6. 1: Lecturers' responses to question 2 (Do you use visual strategies in your classroom?)

2.	Do you use visual strategies in your classroom?
LECTURER A	"Yes"
LECTURER B	"No"
LECTURER C	"Yes"

Participants' response to this question would depend on their responses to define visual strategies as per the previous question. All three participants were asked if they used visual strategies in their classrooms. Depending on their notion of their perceptions of visual strategies, the responses of lecturers A, B and C respectively were yes, no and yes. All three participants used visual strategies in their lessons by observation. Lecturer B was not aware of the visual strategies used in her classroom. The OHP, gestures and semiotics were some of the visual strategies observed in her class.

Table 4.6 2: Lecturers' responses to question 3 (What visual strategies do you use?)

3. What visual strategies do you use?	
LECTURER A	"Physical models are used e.g. paper strips, discs, countable objects, commercially bought models, drawings, O.H.P, chalkboard and lots of gestures!"
LECTURER B	"I use visual representations"
LECTURER C	"Well at the very least I would have a PowerPoint or OHP transparency to provide a point of focus for what I am talking about. I often use examples of learner work, relevant pictures, occasionally and cartoons."

Participants shared their visual strategies used in the classroom. Lecturer A claimed that she uses physical models e.g. paper strips, discs, countable objects, commercially bought models, drawings, O.H.P, chalkboard and many gestures. Lecturer B however claimed that she uses visual representation. This was contradictory as she said she does not use visual strategies in her classroom. Overall it all depended on what her notion of visual strategies and visual representations are. Lecturer C stated, at the very least she would have a PowerPoint or OHP transparency to provide a point of focus for what she is talking about. She often uses examples of learner work, relevant pictures, and cartoons. The types of visual strategies used by all three lecturers broaden the view respectively.

Table 4.6 3: Lecturers' responses to question 4 (Do you think the use of visual strategies influences students in attaining mathematical concepts?)

4. Do you think the use of visual strategies influences students in attaining mathematical concepts?	
LECTURER A	"Yes"
LECTURER B	"I don't think of it as a visual strategy. I often would try to present a visual representation of a symbolic statement or abstract description, so that students may be able to understand certain properties that may not be as evident in the formal definition. For example the definition of surjection is hard to understand, but when one uses a mapping representation or a graphical representation it is easier for students to 'see' the meaning of the definition "for all b ϵ B a ϵ A ϶ f(a) =b" "
LECTURER C	Yes

Lecturer A and Lecturer C asserted that visual strategies influence students' understanding in gaining mathematical concepts. Lecturer B still maintained her notion on visual strategies rather making reference to visual representation. She does allude to the use of semiotics as a visual representation by means of abstract to allow students to gain concepts in mathematics.

Table 4.6 4: Lecturers' responses to question 5 (Do you use gestures? If yes, what gestures? Do you think gestures are important when teaching?)

5. Do you use gestures? If yes, what gestures? Do you think gestures are important when teaching?	
LECTURER A	"Oh! I thought these were part of my visual strategies. Yes I use gestures e.g. portioning actions, showing dimensions (length, breadth, depth, height), showing outlines in shapes, etc. I probably could not teach without gesturing!"
LECTURER B	"I think so- But I think this is a silly question. The issue is about how the teacher tries to convey meaning of the mathematics."
LECTURER C	"I think anybody who talks uses gestures. I do however try to demonstrate concepts such as length by extended linear gestures, perimeter by a gesture that indicates going around a shape and so on."

All three participants were asked about the gestures used in their classrooms. Lecturer A stated, that she uses gestures e.g. apportioning actions, showing dimensions (length, breadth, depth, height), showing outlines in shapes, etc. Her statement showed that gestures used in her classroom were quite significant. Lecturer C also supported Lecturer A's notion in claiming that, anybody who talks uses gestures. She claimed that she tries to demonstrate concepts such as length by extended linear gestures, perimeter by a gesture that indicates going around a shape and so on. Lecturer C however had a different perspective, she claimed uncertainty in using gestures in her classroom by saying she thinks she uses them. She also claimed that the issue was not about gestures, rather it is about how a teacher tries to convey meaning of mathematics.

Table 4.6 5: Lecturers' responses to question 6 (Do you think visual strategies perplex or confuse students when used in a classroom? Explain.)

6. Do you think visual strategies perplex or confuse students when used in a classroom? Explain.	
LECTURER A	"Generally I think they could be very useful to explain concepts but they are not an end in themselves. I like to think of them as scaffolds to developing conceptual understanding."
LECTURER B	"Depends what they are and what they are used for, the level of the mathematics, the level of the students and the expertise of the teacher"
LECTURER C	"All depends on how well the teachers themselves understand the underlying concept they are trying to portray. If the visual aid is well chosen and learners are given time to make sense of it, it is helpful. Flashing an animated computer explanation or demonstration past them is unlikely to be helpful."

Lecturer A and lecturer C confidently indicated that visual strategies can be used to build knowledge. This alludes to the question of whether or not visual strategies perplex or confuse students' understanding in gaining mathematical concepts. Lecturer C however takes more precaution when using visual strategies in the classroom. She states that it all depends on what they are used for, the level of mathematics and expertise of the teacher.

Table 4.6 6: Lecturers' responses to question 7 (What is your understanding about semiotics? Does it hold a significant purpose in the math class?)

7. What is your understanding about semiotics? Does it hold a significant purpose in the math class?	
LECTURER A	"Honestly (without looking in a dictionary!), I think it has to do with the use of signs (that's what I remember from someone giving me an "on the spot" explanation). So I think it would have links to gestures and yes significant in a math class."
LECTURER B	"Depends who is doing it, for what purpose and the level of understanding of the learners."
LECTURER C	"Signs and symbols. Major role in math's which abounds with symbols to which learners have to attach correct meanings."

Lecturer A and lecturer C had a good understanding of semiotics. They both claim that semiotics play a crucial role in the mathematics classroom. Lecturer B alludes to the fact that the significance of semiotics used in the classroom again depends on who is using them and the level of the students' understanding. Lecturer B however does not state what her understanding of semiotics is.

CHAPTER FIVE

Conclusion and Recommendations

5.1 Introduction

The objective of this study was to investigate the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts. This research attempted to explore the manner, in which visual strategies are used at university level, I sought answers to the following critical questions:

1. Are multiple visual strategies used at university level?

2. What are the various multiple representations of visual strategies that are used by educators at university level?

3. Does the use of visual strategies enhance students’ understanding?

4. How does the use of visual strategies influence students’ mathematical understanding through concept development?

5. What are educators and students’ perception of visual strategies?

6. To what extent can visual strategies be used in a classroom?

5.2 Findings and Conclusion

The first question focused on whether or not multiple visual strategies were used at university level. All educators used some form of visual strategy in their classroom. Owing to the broad definition of visual strategies, it was hard to separate any educator in a classroom from the use of visual strategies. This led to the next question of identifying what are the visual strategies used in the classroom.

The second question focused on the types of visual strategies that are used by the educators at tertiary level. It was crucial to firstly adopt a definition for visual strategies in the context of this research study. The definition adopted was from Liu (2012: 21), “Multiple representations may include: graphs, diagrams, tables, grids, gestures, videos, models, manipulative and pictures.” As depicted in Chapter Four, the findings show that some of the visual strategies used at university level consisted of: gestures, artefacts and semiotics.

The third question was a crucial question to this study. It focused on whether or not visual strategies enhance students’ understanding. The solution to this also answered the research question four. Literature shown found this question to be largely controversial, whilst some researchers showed that multiple representations of visuals enhance students' understanding, it was also shown that too much can perplex students' understanding. According to Krutetskii (1976: 350), "Visualisation is distinct from the use of logical reasoning in mathematics, which defines mathematical ability". He claims that visualisation is often useful but it may not essentially be to the highest achievement in mathematics. He says that it may even hinder mathematical thinking if not used carefully. After careful observation of the participants' behaviour of the students and educators, it was found that visual strategies are useful in the classroom. It does enhance students' mathematical thinking. The use of visual strategies however has to be used with caution. The uses of multiple representations are good for students but too many visual strategies used in a classroom lend themselves to perplexing students' understanding.

The fourth question was also crucial to this study. It focused on how the use of visual strategies influences students' mathematical understanding through concept development. The use of gestures, artefacts and semiotics played a large role in influencing students' understanding of mathematics. Chapter Five shows clips from videos of the gestures used by lecturer A. The use of hands enhanced concepts of fractions, such as big, bigger and biggest, whilst lecturer B showed that gestures not used appropriately can apprehend students negatively such as pacing back and forth when teaching. The artefacts played a very important role in developing students' concepts. This was shown by lecturer C when she used the Russian dolls to explain the number system. And finally lecturer B showed that too much semiotics used can perplex students' development of concepts. She also showed that visual strategies have to be used with caution. It is good to use visual strategies in a classroom provided that you use them with the correct purpose.

The fifth question aimed to investigate what educators and students' perception of visual strategies were. After interviewing the students and handing out questionnaires to the educators, it was found that most students embraced the notion of visual strategies used in the classroom and also stated that they would definitely use visual strategies in their classrooms. Two of the three educators stated that they could not imagine not using visual strategies in a classroom whilst one debated that visuals are not such an important component in her teaching. This is shown in the findings from Chapter Four.

The last question focused on the extent to which visual strategies can be used in a classroom. All questions asked prior to this, formed the answer to this question. According to Ainsworth and Labeke (2004: 241), "The first heuristic is to use only the minimum number of representations that you can." Ainsworth sums up what was highlighted in all questions answered. Multiple representations of visual strategies are good, but must be used accordingly.

5.3 Recommendations

According to the Department of Education (2005) a National Framework for teacher education was recommended so that it could articulate improvement, consistency and track a more reasonable way for the teachers and the South African education system. South Africa lends itself to an ever changing curriculum. Policy change appears to be on a constant rise. The control for these policy changes, from educators are limited. The only constant approach to good teaching and learning now is to focus on strategies used in classrooms. With huge emphasis now being placed on visualisation in mathematics, strategies of teaching by the use of visuals should also be adopted in the curriculum. Too many policy documents address all areas of teaching and learning, however the need to include good strategies in a classroom should be considered as this will place emphasis on attaining communication and reasoning skills across curricula. A policy with good practices and strategies to teaching mathematics by the use of visuals should be adopted.

Visual strategies have become a striking tool which allows mathematics to be displayed using pictures, images, tables, graphs, etc. and "students are able to grasp concepts easier when engaging with dynamic images" (Mudaly, 2008: 36). Visualisation in mathematics helps second language learners cut across the difficulty of mathematics. The use of visual strategies in a classroom allows for any learner of any nationality or choice of language to understand what words otherwise fail to do. Universities and schools should consider adopting resources to facilitate the effective role of visual strategies used.

Semiotics assists in helping educators and students in making sense of how mathematics functions as a tool for problem-solving in the real world. "A semiotic perspective helps us understand how natural language, mathematics, and visual representations form a single unified system for meaning-making" (Lemke, 2003, p1). The process of this research study found that many educators lacked the skill of attaining a good 'semiotic perspective.' A possible consideration would be sufficient training for educators at university to increase their semiotic skill in mathematics.

5.4 Limitations

During the data collection process of this study, where the researcher had to attain video data by recording lecturers in university, there were many interruptions. The first appointment set with lecturer A had to be postponed owing to the number of strikes in the university. This caused delays and gaps of weeks from the observation of one lecturer to another. The researcher had to wait for the strike to settle down. The next limitation was finding a convenient time to interview lecturers. Lecturers had a tight schedule as this was the time they had to leave the university premises to observe students teaching at schools. This lead to replacing the interviewing of lecturers to handing out interview questions as a questionnaire.

5.5 Further Research

The TIMSS study, mentioned in Chapter One displayed South Africa's status and ranking amongst all other participating countries in mathematics. This is a cause for concern because it affected the number of students who are eligible for the entry of tertiary institutions, in particular the caliber of students chosen to study mathematics education. This in turn leads to a shortage of suitably qualified and skilled mathematics teachers. Korea, Singapore and Hong Kong were rated amongst the highest in comparison to other countries and South Africa. It would be significant to investigate the pedagogical strategies and approaches used by these countries in teaching mathematics.

5.6 Conclusion

Visualisation in mathematics plays a crucial role in the development of students' mathematics thinking. South African Education should provide universities with resources, programmes and other possible interventions that facilitate visual thinking. This will enable educators and students to access doors of opportunities and succeed in education.

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Appendices:

Appendix A – Questionnaire for Math educators (Template)

A questionnaire for mathematics educators

Teacher A/B/C:

1. What are visual strategies?

____

____

____

2. Do you use visual strategies in your classroom? Yes/ No

3. What visual strategies do you use?

____

____

4. Do you think that the use of visual strategies influence learners attaining mathematical concepts? Yes/ No

5. Do you use gestures? If yes, what gestures?

____

____

6. Do you think visual strategies perplex or confuse learners when used in the classroom? Explain.

____

____

Questionnaire (Lecturer A)

Teacher (A)/B/C: **A questionnaire for mathematics educators**

Overall, I think strategies to explain/model concepts so students can visualise these better e.g. relative sizes of fractions (models help).

1. What are visual strategies?

Literally I think this includes all the strategies that are visibly demonstrated &/or used – in this case, during maths. teaching. So it's those that are explicit (clear) & clearly seen by all in that particular learning space (e.g. desk when one works with a group).

2. Do you use visual strategies in your classroom? Yes/ No YES.

3. What visual strategies do you use?

Physical models are used e.g. paper strips/discs countable objects, commercially bought models Drawings on OHP/c/board & lots of gestures!

4. Do you think that the use of visual strategies influence learners attaining mathematical concepts? Yes/ No YES.

5. Do you use gestures? If yes, what gestures? Do you think gestures are important when teaching

Oh! I thought these were part of my visual strategies. Yes I use gestures e.g. partitioning actions; showing dimensions (length, breadth depth/height); showing outlines of shapes, etc. I probably could not teach without gesturing!

6. Do you think visual strategies perplex or confuse learners when used in the classroom? Explain.

Generally I think they could be very useful to explain concepts BUT they are not an end in themselves – I like to think of them as scaffolds to develop conceptual understanding

7. What is your understanding about semiotics? Does it hold a significant purpose in the math class?

Honestly (without looking in a dictionary!) I think it has to do with the use of signs (that's what I remember from someone giving me an "on the spot" explanation). So I think it would have links to gestures & yes! significant in maths class

Questionnaire (Lecturer B)

A questionnaire for mathematics educators

Teacher A/B/C:

1. What are visual strategies?

   __I have no idea

2. Do you use visual strategies in your classroom? Yes/ No

3. What visual strategies do you use?

   ______I use visual representations.

4. Do you think that the use of visual strategies influence learners attaining mathematical concepts? Yes/ No. I don't think of it as a visual strategy. I often would try to present a visual representation of a symbolic statement or abstract description, so that students may be able to understand certain properties that may not be as evident in the formal definition. For example the definition of surjection is hard to understand, but when one uses a mapping representation or a graphical representation it is easier for students to 'see' the meaning of the definition "for all $b \in B \exists a \in A \ni f(a) = b$".

5. Do you use gestures? If yes, what gestures?

   _I think so- But desiree I think this is a silly question. The issue is about how the teacher tries to convey meaning of the mathematics.

6. Do you think visual strategies perplex or confuse learners when used in the classroom? Explain.

   _______depends what they are and what they are used for, the level of the mathematics, the level of the students and the expertise of the teacherWhat do you understanding of semiotics?

7.Do you think it plays a significant role in the math classroom

Same as my answer fro previous question- depends who is doing it , for what purpose and the level of understanding of the learners.

Questionnaire (Lecturer C)

A questionnaire for mathematics educators

Teacher A/B/C:

1. What are visual strategies?

I would imagine that they are what we have for years used and referred to as teaching using visual aids. Any strategy that employs more than talk.

2. Do you use visual strategies in your classroom? Yes

3. What visual strategies do you use?

Well at the very least I would have a Powerpoint or OHP transparency to provide a point of focus for what I am talking about. I often use examples of learner work, relevant pictures, occasionally cartoons.

Where appropriate I show maths manipulatives and demonstrate their use.

4. Do you think that the use of visual strategies influence learners attaining mathematical concepts? Yes/ No

5. Do you use gestures? If yes, what gestures?

I think anybody who talks uses gestures. I do however try to demonstrate concepts such as length by extended linear gestures, perimeter by a gesture that indicates going around a shape and so on.

6. Do you think visual strategies perplex or confuse learners when used in the classroom? Explain.

All depends on how well the teachers themselves understand the underlying concept they are trying to portray. If the visual aid is well chosen and learners re given time to make sense of it, it is helpful. Flashing an animated computer explanation or demonstration past them is unlikely to be helpful.

7. What do you understanding of semiotics? Do you think it plays a significant role in the math classroom

Signs and symbols. Major role in maths which abounds with symbols to which learners have to attach correct meanings.

Interview Schedule for Focus group (Template)

Focus group 1/2/3

1. What are visual strategies?

2. Are you taught by the use visual strategies in your classroom? Yes/ No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? Yes/ No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

6. What do you understand by gestures when teaching? and as a future math educator do you see yourself using a lot of gestures when teaching?

Student 1

NAME: Sherazel Francis
AGE: 20
GENDER: Female

1

Mrs Rosenburg

Interview schedule for focus group

Focus group 1/2/3

1. What are visual strategies?
Visual strategies are strategies that are used to encourage the learners to engage in the lecture by seeing eg the overhead projector and hand gestures because learners learn best by seeing.

2. Are you taught by the use visual strategies in your classroom? (Yes) No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)
overhead projectors, hand gestures and the chalkboard as well as white boards.

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? (Yes) No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.
They are very useful in maths lessons as you can easily visualise the different concepts and the proportions.

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?
hand Gestures are, different signs used by the educator to maybe be used in an underresourced school. They can be used to show different proportions and also to illustrate different meanings.

Interview Schedule for Focus group A

Student 2

NAME : KERSHIA MUNGAROO

AGE : 23

GENDER: FEMALE

2

Mrs Rosenburg

Interview schedule for focus group

Focus group ①/2/3

1. What are visual strategies?

   I UNDERSTAND THAT VISUAL STRATEGIES ARE USED TO ENHANCE A LESSON AS WELL AS A LEARNERS UNDERSTANDING OF DIFFERENT CONCEPTS AND IDEAS WHEN IT COMES TO LEARNING.

2. Are you taught by the use visual strategies in your classroom? Yes/ ~~No~~

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   ALL OF THE ABOVE, CHALKBOART, OHP.

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? Yes/ ~~No~~

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   NO THEY HELP TO CREATE BETTER UNDERSTANDING OF CONCEPTS ESPECIALLY IN A MATHEMATICAL CLASSROOM

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   YES. IT WOULD GRAB THE ATTENTION OF LEARNERS AND ENABLE THEM TO BECOME MORE ENGAGED.

Interview Schedule for Focus group A

Student 3

NAME: Sibusiso
AGE: 21
GENDER: male

3

Mrs Rosenburg

Interview schedule for focus group

Focus group (1)/2/3

1. What are visual strategies?

   It is the use of over head projectors and the order resources which are videos, posters and visual examples

2. Are you taught by the use visual strategies in your classroom? Yes/ No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   O.H.P, chalkboard

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? Yes/ No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   its makes it easy to understand, because it becomes more real, also assisting the people who cannot work out things mentally

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   yes, because we are now living in the modern society when learners get entertained by things like projectors, which will summarise the notes and making it easy for them to understand

Interview Schedule for Focus group B

Student

NAME: Aadila A. Moolla
AGE : 20
GENDER: Female

Dr. Bansilal

Interview schedule for focus group

Focus group 1/2/3

1. What are visual strategies?

   Visual strategies are those strategies used by the teacher to help learners understand concepts that are being taught. Visual understanding of the learner co-incides with visual strategies used by the teacher. i.e the learner visualises concepts in his/her mind

2. Are you taught by the use visual strategies in your classroom? Yes/ No

   Yes

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   Chalkboard and data projector. Solutions are sometimes written on the chalkboard and projector is used to show learners step-by-step solutions with highlighted notes that are important

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? (Yes) No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   Sometimes. When notes are given in a booklet form using small writing it tends to confuse and make learners afraid but when a chalkboard or projector is used and the work is spaced and easy to see it is better to understand.

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   Gestures could be the body language, facial expression or even hand signals that the teacher may use in a classroom. I see myself using ~~gest~~ gestures in an appropriate manner. For example to make learners understand that a graph moves from left to right by moving my finger over the graph etc.

Interview Schedule for Focus group B

Student 2

NAME: Khanyile Phulani N
AGE : 21
GENDER: Male

2

Dr. Bansilal

Interview schedule for focus group

Focus group 1/2/3

1. What are visual strategies?

   They are what learners see when you teaching, The teaching strategies, (writing on the board). basically, they are the signs you use when making functions. e.g ∴ (therefore), does they understand the sign.

2. Are you taught by the use visual strategies in your classroom? (Yes) / No   Sometimes

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   Chalkboard, ~~Smartboard~~, gestures; when the lecturer emphasizes something she uses her body or highlight on the board.

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? (Yes) / No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   They help me understand better, they make the notes from the coursepack clear and understandable in more than typed sheet.

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   gestures when teaching are when you using your body parts and when seeking for attention. I will use gestures in my class because learners understanding when you pointing at something using your hands rather than being statue and using only your voice.

Interview Schedule for Focus group B

Student 3

NAME: Sibusiso
AGE: 21
GENDER: male

3

Mrs Rosenburg

Interview schedule for focus group

Focus group (1)/2/3

1. What are visual strategies?

   It is the use of over head projectors and the order resources which are videos, posters and visual examples

2. Are you taught by the use visual strategies in your classroom? Yes/ No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   O.H.P, chalkboard

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? Yes/ No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   its makes it easy to understand, because it becomes more real, also assisting the people who cannot work out things mentally

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   yes, because we are now living in the modern society where learners get entertained by thing like projectors, which will summarise the notes and making it easy for them to understand

Interview Schedule for Focus group C

Student 1

AYESHA RAHIM
21
FEMALE

Dr. Hobden

Interview schedule for focus group

Focus group 1/2/3

1. What are visual strategies?
   Visual strategies enhance learners abilities to understand their work better.

2. Are you taught by the use visual strategies in your classroom? (Yes) No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)
   O.H.P Pegs - (different colours).
   Russian dolls
   Number lines

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? (Yes) No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.
   No, because it is visual, it is something that we can see and what we see is believable

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?
   Gestures are body language, yes.

Interview Schedule for Focus group C

Student 2

2

Yaaseen Shaik Noor / MALE / 21

Dr. Hobden

Interview schedule for focus group

Focus group 1/2/3

1. What are visual strategies?

   These are things ~~set~~ such as number lines which are used to get ~~for~~ students involved in the lesson

2. Are you taught by the use visual strategies in your classroom? (Yes)/ No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   - OHP / Smart board
   - Russian dolls
   - number lines, Pegs, gestures

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? (Yes)/ No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   No, it helps us to better understand the concepts, it helps to see what is being taught.

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   Yes. it aids to enhance the lesson

Interview Schedule for Focus group C

Student 3

Steffan Govender
age - 20
male.

3

Dr. Hobden

Interview schedule for focus group

Focus group 1/2/3

1. What are visual strategies?

   Visual strategies are resources that are utilized which allow for a better understanding of concepts. These include objects, pictures, models as well as the bodily gesture of the educator

2. Are you taught by the use visual strategies in your classroom? Yes/ No

3. What visual strategies are used? (O.H.P, Chalkboard, Smartboard, gestures if so say what gestures)

   OHP, Russian dolls, model of a numberlines, pegs, gestures and facial expression.

4. Do you think that the use of visual strategies influence your ability in attaining mathematical concepts? Yes/ No

5. Do you think visual strategies perplex or confuse you when used in the mathematics classroom? Explain.

   I feel they enhance concept development and improve mathematical knowledge.

6. What do you understand by gestures when teaching and as a future math educator do you see yourself using a lot of gestures when teaching?

   A gesture is body language and facial expression or tone of voice. Yes I will use gestures to enhance my teaching.

Observational Schedule - Template

VISUAL STRATEGIES OF A MATHEMATICS CLASSROOM	OBSERVATION
Teacher A/B/C	A/B/C
Focus group 1/2/3	1/2/3
Visual strategies used	O.H.P/ Chalkboard/ Smartboard/ Worksheets/ video/powerpoint __________ __________ __________
Gestures used	facial expressions:__________ tone of voice:__________ eye motion:__________ body poise:__________
Semiotics	Index, Symbols, Icons __________
Evidence of visual learner	Yes/No. Explain: __________ __________
Other observations	

Observational Schedule – Lecturer A

PRIMARY MATH EDUCATION 210

Unifying ideas

29/04/13

Computational Fluency/Fractions

Observation Schedule

VISUAL STRATEGIES OF A MATHEMATICS CLASSROOM	OBSERVATION
Teacher A/B/C	(A)/B/C
Focus group 1/2/3	(1)/2/3
Visual strategies used	O.H.P/ Chalkboard/ Smartboard/ Worksheets/ video/powerpoint Booklets, representation of many diagrams Use of chalkboard, charts Manipulatives – Paper partioning Models of diagram representation Beans placed on OHP. Coloured pens to emphasis fractions ($\frac{1}{8}$ of 16)
Gestures used For emphasised points	Hand movement: Pointed finger use. facial expressions: Many expressions tone of voice: louder. eye motion: bigger eyes body poise: Actions
Semiotics	Index, Symbols, Icons ← ; — Division, decimals, percentage, subtraction
Evidence of visual learner	Yes/No. Explain: learners only understood that halves can be represented differently or more than one way when asked to tear of with a piece of paper
Other observations	Semiotics used. % 0,5 learners unknowingly use semiotics Set models – Beans

Observational Schedule – Lecturer B

Mathematics for Educators 410
Recursive definitions of sequences

Observation Schedule

VISUAL STRATEGIES OF A MATHEMATICS CLASSROOM	OBSERVATION
Teacher A(B)C	A(B)C
Focus group 1(2)3	1(2)3
Visual strategies used	O.H.P/ Chalkboard/ Smartboard/ Worksheets/ video/powerpoint Coloured Chalk, white board, points circled / underlined to emphasise points. Theorems/ formulas were boxed. Coloured pens. Data projector. Booklets.
Gestures used	Hand gestures to emphasise alternate points /value facial expressions: tone of voice: Monotone. eye motion: body poise: Not of walking /Pointing to learner Books to show eny
Semiotics	Index, Symbols, Icons = {; , : Focus entirely on Semiotics
Evidence of visual learner	Yes/No. Explain:
Other observations	highlighters lecturer walked around to students to show problems

Observational Schedule – Lecturer C

Primary Math for Educator 210
Number System
22/05/13

Observation Schedule

VISUAL STRATEGIES OF A MATHEMATICS CLASSROOM	OBSERVATION
Teacher A/B/C	A/B/C
Focus group 1/2/3	1/2/3
Visual strategies used	O.H.P/ Chalkboard/ Smartboard/ Worksheets/ video/powerpoint Tablea, Powerpoint S Prop s – Pegs (Blue) – N Green Pegs (Wh) Yellow Pegs (integers), Red Pegs (Rational) Russian dolls, Flashcards / Smartboard, Booklets
Gestures used	facial expressions: tone of voice: High tone to emphasis points eye motion: body poise: Hands
Semiotics	Index, Symbols, Icons
Evidence of visual learner	Yes/No. Explain: Yes.
Other observations	Discussion

Appendix D – A letter of consent to participants (Template)

A letter of consent to participant

Dear participant

Thank you so much for voluntarily agreeing to participate in this research study. Your participation is highly appreciated. You will participate in the following, research Study: "An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts"

All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

I ______________________have read and understand the letter of invitation to take part in the research study: A Qualitative Research Study Investigating the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts. I have received adequate information regarding the nature of the study and understand what will be requested of me. I am aware of my right to withdraw at any point during the study without penalty.

Tick appropriate box:

1. **Student:** I am aware that the lessons I am seated at will be video recorded. I agree to be interviewed along with my peers.
2. **Educator:** I am aware that my lesson is video recorded and I agree to fill a short questionnaire

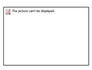
I hereby consent to participate in this research study.

Participants Signature: _______________________

Date: __________

Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah: 

Appendix D – A letter of consent to participants

Mrs Rosenburg

A letter of consent to participant

Dear participant

Thank you so much for voluntarily agreeing to participate in this research study. Your participation is highly appreciated. You will participate in the following, research Study: "An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts"

All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

I Mrs Rosenburg have read and understand the letter of invitation to take part in the research study: A Qualitative Research Study Investigating the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts. I have received adequate information regarding the nature of the study and understand what will be requested of me. I am aware of my right to withdraw at any point during the study without penalty.

Tick appropriate box:

- ☐ **Student:** I am aware that the lessons I am seated at will be video recorded. I agree to be interviewed along with my peers.
- ☑ **Educator:** I am aware that my lesson is video recorded and I agree to fill a short questionnaire

I hereby consent to participate in this research study.

Participants Signature: [signature]

Date: 29/04/13

Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah: [signature]

Appendix D – A letter of consent to participants

Dr. Bansilal

A letter of consent to participant

Dear participant

Thank you so much for voluntarily agreeing to participate in this research study. Your participation is highly appreciated. You will participate in the following, research Study: "An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts"

All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

I Dr. Bansilal have read and understand the letter of invitation to take part in the research study: A Qualitative Research Study Investigating the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts. I have received adequate information regarding the nature of the study and understand what will be requested of me. I am aware of my right to withdraw at any point during the study without penalty.

Tick appropriate box:

- ☐ **Student:** I am aware that the lessons I am seated at will be video recorded. I agree to be interviewed along with my peers.
- ☑ **Educator:** I am aware that my lesson is video recorded and I agree to fill a short questionnaire

I hereby consent to participate in this research study.

Participants Signature: ________

Date: 2/5/2013.

Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah:

Appendix D – A letter of consent to participants

Dr. Hobden

A letter of consent to participant

Dear participant

Thank you so much for voluntarily agreeing to participate in this research study. Your participation is highly appreciated. You will participate in the following, research Study: "An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts"

All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

I Dr. Hobden have read and understand the letter of invitation to take part in the research study: A Qualitative Research Study Investigating the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts. I have received adequate information regarding the nature of the study and understand what will be requested of me. I am aware of my right to withdraw at any point during the study without penalty.

Tick appropriate box:

- ☐ **Student:** I am aware that the lessons I am seated at will be video recorded. I agree to be interviewed along with my peers.
- ☑ **Educator:** I am aware that my lesson is video recorded and I agree to fill a short questionnaire

I hereby consent to participate in this research study.

Participants Signature: SHobden

Date: 22/05/13

Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah: [signature]

Appendix D – A letter of consent to participants (Sample from a student)

A letter of consent to participant

Dear participant

Thank you so much for voluntarily agreeing to participate in this research study. Your participation is highly appreciated. You will participate in the following, research Study: "An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts"

All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

I Pamela. have read and understand the letter of invitation to take part in the research study: A Qualitative Research Study Investigating the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts. I have received adequate information regarding the nature of the study and understand what will be requested of me. I am aware of my right to withdraw at any point during the study without penalty.

Tick appropriate box:

- ☑ **Student:** I am aware that the lessons I am seated at will be video recorded. I agree to be interviewed along with my peers.
- ☐ **Educator:** I am aware that my lesson is video recorded and I agree to fill a short questionnaire

I hereby consent to participate in this research study.
Participants Signature: [signature]
Date: 29/04/2013

Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah: [signature]

Appendix E

A letter to the Dean of the University

Re: Research Study: An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts.

Dear Dr./Prof ____________________________

I am currently undertaking a BEd master's degree at UKZN, Edgewood campus. As part of my assessment I am required to submit a research study on an area of interest within my professional scope of practice that is 'visualisation in mathematics'. The study I have selected is "the investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts" I am hoping to conduct this study with undergraduate students and mathematics educators of the university. I am writing to you to seek your permission to gain access to participants from the university, in particular the math faculty.

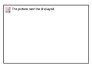
Although past research has documented visual strategies, to date little research has been carried out in this specific area and could possibly benefit from it in the future. This study will involve the participation of 3 educators and 9 students who are voluntarily selected. I request permission to video record each lesson of the three educators and to conduct a focus group interview with three learners selected from each class under voluntary basis of educators. All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

It is envisaged that this study will benefit mathematics educators practice by the awareness of multiple visual strategies. A letter of invitation will be issued to all potential participants along with a consent form. All participants will then sign a consent form which explaining the nature of this study. All participants maintain the right to withdraw from the study at any time without penalty.

This study is awaiting ethical approval from the faculty of mathematics education research ethics committee by written a letter applying for approval.

Thank you for taking the time to read this letter. I would be grateful for your permission to carry out this study within the university and access to participants from the mathematics faculty. Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah: 

Appendix F

A letter to mathematics educators of the university

Re: Research Study: An investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts.

Dear Dr./Prof. ______________________________

I am currently undertaking a BEd master's degree at UKZN, Edgewood campus. As part of my assessment I am required to submit a research study on an area of interest within my professional scope of practice that is 'visualisation in mathematics'. The study I have selected is "the investigation into the use of visual strategies by educators at tertiary level and its influence on student teachers' development of mathematical concepts". I am hoping to conduct this study with undergraduate students and mathematics educators of the university. I am writing to you to seek your permission for your participation in this research study.

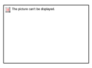
Although past research has documented papers on visual strategies of mathematics education, to date little research has been carried out in this specific area and could possibly benefit from it in the future. This study will involve the participation of 3 educators and 9 students who are voluntarily selected. I request permission to video record just one lesson and analyse your methods of visual strategies and its influence on students understanding of mathematical concepts. I would also like to conduct a focus group interview with three learners selected under voluntary basis.

All ethical issues will be considered and addressed before and during the study. Anonymity and confidentiality will be protected at all times. Pseudonyms will be used as a method of coding to distinguish information and all information transcribed will be stored on an external hard drive which will be securely locked away to ensure confidentiality and destroyed after a period of five years

It is envisaged that this study will benefit you and your students by the awareness of multiple visual strategies and the possible development of students mathematical concepts. A letter of invitation will be issued to all potential participants along with a consent form. All participants will sign a consent form explaining the nature of this study when agreeing voluntarily to participate in this research study. All participants maintain the right to withdraw from the study at any time without penalty.

Thank you for taking the time to read this letter. I would be grateful for your permission to carry out this study within the university and access to participants from the mathematics faculty. Should you have any queries please feel free to contact me at 0736632564 or email desirayr@hotmail.com at any stage. I look forward to hearing from you.

Kind Regards

D Ramiah: 

Appendix G

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

3 August 2017

Dr Vimolan Mudaly 26607
School of Education
Edgewood Campus

Dear Dr Mudaly

Protocol Reference Number: HSS/0112/013
Project Title: An investigation into the role and use of visual skills and strategies in mathematics education by trainee and practicing teachers as well as staff at UKZN and schools in Durban

Recertification Approval

This letter confirms that you have been granted Recertification Approval for a period of one year from the date of this letter and the inclusion of Desiray Ramiah (205525395) as a co-investigator. This approval is based strictly on the research protocol submitted in 2013.

Any alteration s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study must be reviewed and approved through the amendment /modification prior to its implementation. Please quote the above reference number for all queries relating to this study. PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years

Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Shenuka Singh (Chair)
Humanities & Social Sciences Research Ethics Committee

/pm

cc. Supervisor/Project Leader: Dr Vimolan Mudaly
cc. Academic Leader Research: Dr SB Khoza
cc. School Administrator: Ms T Khumalo

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
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Website: www.ukzn.ac.za