UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

INVESTIGATING GRADE 10 LEARNERS' UNDERSTANDING OF QUADRILATERALS IN EUCLIDEAN GEOMETRY

BY

STANLEY IZUCHUKWU NKWOJI

217016276

A dissertation submitted to the University of KwaZulu-Natal (UKZN), Edgewood Campus, School of Education, in fulfilment of the academic requirement for the award Master's Degree in Mathematics Education

SUPERVISOR

PROFESSOR SARAH BANSILAL

5 FEBRUARY 2025

DEDICATION

This academic project is dedicated to the Almighty God, from whom wisdom, knowledge and understanding comes.

Secondly, to my beloved wife, Mrs Kenalemang Nkwoji, who never ceases to support, and encourage me during this journey with words of strength and courage, alongside my two children- Peace and Joshua Nkwoji.

ACKNOWLEDGMENTS

The completion of this dissertation would not have been a reality without acknowledging certain individuals who played one role or the other, towards the completion, amidst the enormous challenges I encountered on the way.

Although I eventually submitted this dissertation, although the challenges I encountered in the process, could have derailed me from seeing this a reality. At some point, I almost lost the inner drive, feeling demoralized due to demands on my table, at some point, to mention but a few. Nonetheless, in spite of these challenges, I triumphed eventually, but not without the supports and help I received from different angles as highlight below:

First and foremost, I remain eternally grateful to God, my Maker, Sustainer, and Keeper, who, among others, graciously restored the lost drive and focus, I almost lost, by granting me the strength and tenacity required, toward seeing the dream come through. Hence, I was able to see light beyond the tunnel.

My gratitude also goes to my beloved spouse, Mrs Kenalemang Nkwoji, who has also gone through the rigor of obtaining a Master's degree, even as a mother of two, with other responsibilities. She stood with me through thick and thin, especially when she saw that my attention to my studies was not as it should have been. Her words of encouragement, prayers and emotional support immensely contributed to the realisation of this dream She is indeed a help-meet in every ramification. Thank you, my beloved wife.

I cannot forget my supervisor, Professor Sarah Bansilal, whom I first encountered during my honours degree program, as my supervisor. She is not just a renowned scholar in every ramification, but someone who has a heart for the students she supervises. Despite the challenges I had at some point, especially, when it looked as if I was not going to make it, she did not make me feel hopeless, rather, she was readily available to provide the needed support from time to time. She is indeed a supervisor anyone would appreciate to have. Professor Bansilal, I am very and deeply grateful for your encouragement and unwavering role you played in this journey.

My two growing children, Peace and Joshua, you are dearly cherished and loved. Thank you for coping, even when I had little or no time for you, due to My Masters' pursuit, yet you stood and reasoned liked adults without complaining much of my absenteeism.

To grade 10 learners and the Management of the School where I collected my data, you are highly appreciated for the support and accommodation you accorded me during the data collection.

I also want to acknowledge Apostle Busisiwe Thebehali for her relentless spiritual and moral support; you are a indeed a blessing to me and my family. Furthermore, I want to appreciate my siblings, especially, those who had gone through this journey in the past. You truly inspired me. Finally, to all individuals, who I may not have mentioned, I am grateful.

DECLARATIONS

I, STANLEY IZUCHUKWU NKWOJI, (217016276), declare that;

This dissertation, titled, *"An Investigation of Grade 10 Learners' Understanding of Quadrilaterals in Euclidean Geometry",* has not been previously submitted, to any other tertiary institution, as a requirement for the award of either a diploma or degree, by anyone. As such, it is not a copy of another person's work.

All relevant information used in this study from different sources, were acknowledged accordingly. This includes, all cited authors, images, and direct quotes.

No part of this dissertation was a copy of another person's work without being either duly acknowledged or rephrased.

This dissertation was solely supervised by Professor Sarah Bansilal, from the school of Education, mathematics and Computer Science Education Cluster, University of KwaZulu-Natal, Edgewood Campus.

That, in the course of this study, compliance to the University's regulations and policies, regarding conducting research of this nature, were duly adhered to.

Signed: ___[redacted]___________ Signed: ___[redacted]_

Researcher's name: S Nkwoji Supervisor's name: S Bansilal ___________________

Date: ____14 August 2025___ Date: ____14 August 2025__

ABSTRACT

With the growing focus on geometry education and the challenges South African learners face in understanding Euclidean geometry with reference to quadrilaterals, it is imperative, therefore, to examine the understanding and reasoning of learners, specifically in quadrilaterals.

This study focused on investigating Grade 10 learners' understanding and reasoning of quadrilaterals in Euclidean geometry, within the South African context. Being the entry grade into the Further Education and Training (FET) phase, a sound foundation in geometric reasoning at this FET stage is important for academic progression in mathematics. The objectives of the study seek to identify key factors that influence the understanding of learners in quadrilateral concepts, unearth common misconceptions they hold, and gain insights into learners' reasoning in terms of the van Hiele model of geometric thought.

A qualitative research approach was adopted within an interpretivist paradigm, which aids in understanding the phenomenon being studied. Data was collected from 21 participants from Grade 10 learners, by means of questionnaires, (multiple-choice and open-ended tasks) and semi-structured interviews. The data were analysed using a thematic approach aimed at interpreting the responses of learners to the given tasks. The analysis focused on learners' reasoning abilities, conceptual understanding, and challenges with properties of geometry, proofs, and classification of quadrilateral shapes.

The research findings reveal that learners experience significant challenges, such as misidentification of parallelograms and trapeziums, difficulty in classification of quadrilaterals, and mixing up 'necessary' and 'sufficient' conditions in geometric-related proofs. Further findings reveal that learners depend excessively on visual cues over analytical reasoning. Procedural errors in constructing proof and misappropriation of geometric terminology were also evident. Most learners' reasoning in terms of the van Hiele level of geometry is still at the lower levels of Visualization and Informal Deduction, which impedes progression towards the higher level of deductive reasoning. The emerging critical factors that hinder the development of learners' geometric reasoning were cognitive challenges, language barriers, and inadequate instructional support.

The study highlights the importance of considering the use of structured, level-appropriate instructional approaches, in line with the van Hiele model. These include the use of

reinforced vocabulary, visual aids, and guided informal proof tasks, which improve the geometric reasoning skills of learners. This will ensure that the identified challenges are addressed, and recommendations for curriculum development, classroom practices, and teacher development are provided as well.

TABLE OF CONTENTS

Table of Contents

LIST OF FIGURES (DIAGRAMS)

LIST OF TABLES

LIST OF ACRONYNMS

DBE	Department of Basic Education
CAPS	Curriculum and Assessment Policy Statement
FET	Further Education and Training
VHL	van Hiele Levels
NSC	National School Certificate
TIMSS	Trends in International Mathematics and Science Study
ANA	Annual National Assessments
PCK	Pedagogical Content Knowledge
SCK	Subject Content Knowledge (SCK)
TCK	Teacher Content Knowledge
ZPD	Zone of Proxima Development
CPS	Collaborative Problem-Solving
NCTM	National Council of Teachers of Mathematics
MCQ	Multiple Choice-Question

LIST OF TASKS- QUESTIONNAIRES A AND B

Questionnaire A

Questionnaire B

LIST OF RESPONSES OF LEARNERS TO THE TASKS

Findings from learners’ response to Questionnaire A

Findings from learners’ response to Questionnaire B

CHAPTER 1

INTRODUCTION TO THE STUDY

1.1 Introduction and Overview of the Study

Mathematics is one of the primary subjects in the educational curriculum, and performance in the subject is vital for students to be admitted into scientific and technological professions and make a difference within the scientific sphere (Rikhotso, 2015). Thompson (2003) postulates that whether it is science, engineering, or technology, it is pivotal that students be well-equipped mathematically if they are going to have diverse options in their lives. The criticality of Mathematics carries with it the assumption that the knowledge of the subject is essential for all members of society, especially students and mathematics teachers. Adequate skills and knowledge in Mathematics, especially in geometry, are vital machineries of successful contemporary life and socio-economic development Department of Basic Education (DBE, 2003).

Geometry is an important topic in mathematics. It is argued that the study of geometry has been one of the integral topics in the mathematics curricula of most high schools, likewise, Euclidean geometry (Van Putten et al., 2010). The study of quadrilaterals, which is an aspect of Euclidean geometry, is an integral part of the South African curriculum, known as the Curriculum and Assessment Policy Statement (CAPS), for high schools. Geometry deals with the study of shape and space from a mathematical point of view (Van Putten et al., 2010). Geometry seeks to develop spatial reasoning, problem-solving skills, and logical thinking necessary for mathematics and day-to-day applications.

Although geometry focuses on the study of shape and space, Euclidean geometry, on the other hand, consists of knowledge structures bound by statements that follow from proofs leading from axioms (Kotzé, 2007). The foundation for logical and spatial reasoning, as well as problem-solving skills, requires understanding geometry concepts. Hence, within the South African context, Euclidean geometry, with reference to quadrilaterals, forms a vital part of the CAPS curriculum in the Further Education and Training (FET) Phase.

Despite its importance, learners face challenges consistently in grasping geometric concepts, leading to poor performance in mathematics, particularly, in understanding quadrilateral-related properties, classification, and reasoning processes (DBE, 2023).

The teaching and learning of quadrilaterals requires learners, not only to be acquainted with the ability to recognise quadrilateral shapes, but also to demonstrate understanding of properties, proof-related reasoning, and relationships. However, studies (Abakah & Brijlall, 2023; Maqoqa, 2024), show that many Grade 10 learners still depend on identifying quadrilateral shapes through visual recognition, instead of through formal deductive reasoning in identifying quadrilaterals. This dependence results in recurring misconceptions, as evidenced by incorrect classification and application of properties of quadrilaterals. Furthermore, difficulties in advancing through the Van Hiele model of geometric thought (VHL) also militate against their ability to meaningfully engage in proofs and reasoning, geometrically.

Having an understanding of the geometric reasoning of learners through the lens of the Van Hiele levels of geometric thought is vital, aimed at diagnosing the difficulties learners experience and providing structured instructional approaches that support the development of quadrilateral concepts. This study, therefore, seeks to investigate Grade 10 learners' understanding of quadrilaterals in Euclidean geometry, centring on their reasoning levels, the misconceptions they hold, and the factors influencing their geometric thinking. It is hoped that through this study, insights will be gained in relation to the nature of the difficulties learners face, which will contribute to enhanced teaching approaches and learning experiences in Euclidean geometry.

1.2 Study Background/Motivation

As part of the ongoing process of curriculum revision, which started in 1994 in South Africa. The Department of Education DBE (2011) in South Africa introduced the Curriculum and Assessment Policy Statement (CAPS) in January 2012, which brought Euclidean geometry back as part of the compulsory mathematics curriculum. According to CAPS expectations, all learners passing through the Further Education and Training (FET) phase (Grade 10-12) are expected to acquire skills and functioning knowledge of mathematics, which are vital for successful contemporary life and socio-economic development (2003 DBE, 2003), and geometry forms a key part of the mathematics curriculum. The mathematics curriculum with reference to geometry also requires learners to apply transformations and use symmetry to analyse mathematical situations; use visualization, spatial reasoning, and geometric modeling to solve problems (Alex & Mammen, 2014; Mamali, 2015). Therefore, for meaningful teaching of mathematics in general, particularly in geometry, to take place, developing learners' understanding is critical (Alex & Mammen, 2014).

The South African Curriculum Assessment and Policy Statement (CAPS) states that it is vital for learners in mathematics to be able to question, examine, conjecture, and experiment (DBE, 2011). These attributes enhance logical thinking in learners and engage them in rigorous and analytical thinking (Naidoo & Kapofu, 2020). Despite its importance, poor performance of learners in geometry, as a result of persistent difficulties among learners, particularly in classification of quadrilaterals and reasoning, still remains a concern to educators, parents, and government (Adolphus, 2011; DBE, 2023; Naidoo & Kapofu, 2020).

The Grade 12 mathematics result of 2014 in South Africa indicates that 22.3% of the group who wrote mathematics achieved a pass rate of 50% and above (DBE, 2015). Five years later, according to the National Senior Certificate (NSC) Examination diagnostic reports in Mathematics, the Department of Basic Education still expressed concerns following a decline in performance in some subjects, including mathematics (DBE, 2019). The poor performance in the 2019 examination, with reference to geometry, was mainly attributed to a deficiency in the understanding of basic concepts across some topics in the curriculum (DBE, 2019). Hence, teachers, and scholars are constantly in search of reasons learners are challenged with learning geometry, which is aimed at developing pedagogic approaches to lessen these challenges (Naidoo & Kapofu, 2020).

Furthermore, the challenge faced by many learners in understanding geometric concepts persists, as evidenced by the 2023 Annual Diagnostic Report of the National Senior Certificate (DBE, 2023), as shown in Table 1.1. The table provides quantitative evidence of these challenges, which reveals variation in the pass rate of mathematics, showing difficulties in Euclidean geometry.

Table 1.1: Performance Trends in Mathematic- NSC Examinations Diagnostic Report Data- 2000-2023 (DBE, 2023)

Year	Number that wrote	Number that achieved at 30% and above	% achieved at 30% and above	Number achieved at 40% and above	% achieved at 40% and above
2020	233,315	125,526	53.8%	82.964	35.6%
2021	259,143	149,177	57.6%	97.561	37.6%
2022	269,734	148,346	55.0%	97,041	36.0%
2023	262, 016	166,337	63.5%	114,311	43.6%

From the data, an improvement compared to the previous years was noted. Nonetheless, the overall outcome, coupled with some of the suggestions made in (DBE, 2023), for improvement clearly shows the need for further intervention. For instance, given the DBE 2023 report, almost half of learners who wrote mathematics in the 2023 NSC exam, which is 43.6% attained above 40%, while more than half, which is 63.5% achieved a pass mark of 30% and above, which still reveals some eminent gaps. The education department (DBE, 2023), attributed the performance on geometry to several noticeable misconceptions regarding vital concepts in geometry. Erroneous assumptions were also highlighted, as well as skipping of vital steps when handling proof-related problems and incorrect application of theorems. All these identifiable difficulties show gaps in both content knowledge and reasoning abilities. Some of the suggestions for improvement, according to (DBE, 2023), state "Teachers should focus on developing learners' skills to analyse the question and the diagram for clues on which theorems are required to answer the question correctly. Learners need to be made aware that writing correct statements that are irrelevant to the answer in Euclidean Geometry will not earn them any marks in an examination" (p. 233).

The data above also agrees with the findings of Maqoqa (2024) regarding conceptual and procedural challenges in geometry education. Maqoqa (2024) showed that many students face difficulties with basic concepts in geometry, including the identification and differentiation of properties of shapes. The study unpacks the over-dependence on visual signs without adequate comprehension of features that differentiate one quadrilateral from another, which also confirms the challenges linked to insufficient basic understanding, which can hinder the progression of learners through the van Hiele levels of geometric reasoning. Agreed with the view based on a similar study conducted. These challenges are also in agreement with a similar study by Abakah and Brijlall (2023), whose study identified incompetency among high school learners in basic geometric concepts, such as identification and application of geometric concepts. Remedial interventions, such as the use of visual aids, and an interactive instructional approach, were suggested, as these will aid understanding of learning.

Although, several studies have been conducted within this field with the focus on exploring learners' understanding of geometry (Alex & Mammen, 2014; Borji et al., 2021; Maqoqa, 2024; Ngirishi, 2015). There is still a paucity within the ambience of understanding of geometric concepts among FET learners, with particular focus on grade 10, being the first FET grade in that phase, aimed at establishing how prepared they are to embark on the 3-year geometric cognitive developmental journey ahead, before they sit for their National Senior Certificate Examination (NSC).

It is therefore from this background that this study is informed to investigate grade 10 learners' understanding of quadrilaterals in Euclidean geometry, owing to the fact that poor understanding of the expected geometric concepts at the early stage of the FET phase may have an effect at a later stage as they advance in the phase.

1.3 Problem Statement

In spite of the reintroduction of Euclidean geometry more than a decade ago, following the establishment of CAPS, and its focus on improving the spatial reasoning, logic, and analytical abilities of learners, there is still evidence of underperformance in geometry among learners in the FET phase, with reference to grade 10. In the diagnostic report (DBE, 2023), based on the past 4-year performance trend, (2023 to 2022), it was indicated that many learners still face difficulties when it comes to understanding geometric concepts, particularly, with emphasis on identification, classification, and quadrilateral reasoning. The challenge of understanding and identifying fundamental qualities of quadrilaterals by learners, together with prevalent misconceptions, including incorrect application of theorems, and overreliance on visual signs, shows the existence of gaps in fundamental geometric concepts, which might affect their progression in geometric reasoning. This informs the need to specifically conduct this study with grade 10 learners as the unit of investigation, as a lack of conceptual understanding may hurt their later success in mathematics as a subject through their FET phase experience.

Given these difficulties, the study seeks to investigate the grade 10 learners' understanding of quadrilaterals in Euclidean geometry, aimed at identifying misconceptions held by learners in relation to the concept of quadrilaterals, examining the factors influencing reasoning development in geometry, and gaining insight about Grade 10 learners' understanding of quadrilaterals in geometry in terms of the van Hiele levels of geometry thought.

1.4 Research Objectives

This study aims to:

1. Investigate the vital factors influencing Grade 10 learners' understanding and reasoning regarding quadrilaterals in Euclidean geometry.
2. Identify and analyse common misconceptions held by Grade 10 learners regarding quadrilateral concepts- properties and classification of quadrilaterals, and,
3. Gain insight about Grade 10 learners' geometric understanding and reasoning in terms of the van Hiele levels of geometry thought.

1.5 Research Questions:

In view of the objectives, the study therefore seeks to answer the following research questions:

1. What are the vital factors influencing Grade 10 learners' understanding and reasoning in quadrilaterals in Euclidean geometry?
2. What are the common misconceptions held by Grade 10 learners regarding quadrilateral concepts- properties and classification of quadrilaterals?
3. What insight can we get about Grade 10 learners' understanding and reasoning of quadrilaterals in geometry, in terms of the van Hiele levels of geometry of thought?

It is hoped that these objectives and questions will provide a structured framework for the phenomenon being investigated.

1.6 Rationale of the Study

The rationale for the study serves the purpose of educational insight, which is aimed at providing an in-depth understanding of Grade 10 learners' level of geometric reasoning, regarding quadrilaterals. This is to analyse their misconceptions and patterns of reasoning, through the van Hiele levels of geometric thought, which will enhance the study's contribution to valuable insights regarding the challenges learners face cognitively in Euclidean geometry. The second purpose centres on using the insight provided from the findings to provide valuable information for the purpose of instructional approaches, which will help in closing the gaps in the conceptual understanding of learners. Furthermore, by identifying common misconceptions and challenges in reasoning, educators can be well informed regarding the suitable targeted interventions, aimed at improving learners' geometric understanding and reasoning skills in their developmental journey in geometric reasoning through their FET phase.

1.7 Location of the Study

This study was conducted at one of the Secondary Schools in Umlazi District, in the city of Ethekwini, KwaZulu-Natal Province, South Africa. The location is based on accessibility and familiarity with the school as a pre-service mathematics student at the time. Another reason for selecting the location is that, although the school is a government school rated as a quintile 5 school, the school has learners from different socio-economic backgrounds, which also makes it fit for purpose.

1.8 Organisation of the Thesis

This dissertation adopts the following structure, which briefly summarises the content of each proceeding chapter.

Chapter 1 provides an overview of the study, such as the background, problem statement, research questions, rationale for the study, and the location of the study.

Chapter 2 provides the literature review and the theoretical frameworks of the study.

Chapter 3 outlines the methodological approach adopted for the study, including the research design, methods of data collection, followed by the ethical considerations procedure.

Chapter 4 presents the findings and analysis of the data collected, which will be interpreted through the lens of the van Hiele model.

Chapter 5 presents the discussion of the findings of the study in relation to the research questions. It also provides the recommendations, limitations, and suggestions for further research, before a conclusive note.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL CONSIDERATIONS

2.1 Introduction

The study of geometry, particularly Euclidean geometry, holds an important place in mathematics curricula, globally, and especially in South Africa. Euclidean geometry is concerned with the study of shapes, sizes, and positioning of figures (Chigonga et al., 2017a). It also deals with properties of shapes: hence, it enhances human cognition (such as spatial consciousness, logical reasoning, and analytical abilities), which is vital for handling real-life and mathematical problems (Chigonga et al., 2017b). It is believed that accomplishments in science, engineering, and technology are made possible through the study and understanding of geometry, particularly, Euclidean geometry (Maqoqa, 2024).

However, most students have challenges in handling Euclidean geometry as they do not seem to value its importance (Žilková, 2015). Quadrilaterals, as an integral part of Euclidean geometry, present significant challenges among learners, especially, in relation to understanding their properties, relationships, and classifications. Several studies have shown that misconceptions regarding learners' reasoning in quadrilaterals continue across different educational settings, which necessitate the need for further investigation into the factors militating against the understanding of learners (Jones, 2001; van Hiele, 1986b).

This chapter provides a literature review relevant to understanding Grade 10 learners' understanding of quadrilaterals in Euclidean geometry. This chapter started by underscoring geometry education and its relevance, emphasizing its role in the development of cognition and evolution, particularly within the South African educational curriculum. Challenges in geometry learning, including misconceptions, coupled with quadrilateral classification difficulties, are also discussed. This is followed by factors influencing learners' understanding of quadrilaterals.

The theories upon which the study is framed are also presented and explored, contextualising the stages of learners' development in geometry. The chapter further reviews the application of the van Hiele model within the CAPS curriculum, its implications for teaching and learning, as well as strategies for addressing challenges in geometry education. These insights form the basis for understanding the study.

This chapter provides a comprehensive review of relevant literature and theoretical frameworks that underpin this study. First, the chapter engages in discourse on global and local perspectives on geometry education and conceptual challenges with quadrilaterals. In addition, the chapter further examines common misconceptions, the van Hiele model of geometric thought, and educational methodologies for the improvement of the reasoning skills of learners. Through the synthesis of the existing relevant studies, the basis for the investigation of Grade 10 learners' understanding of quadrilaterals in Euclidean geometry will be established. Gaps in existing studies will also be reviewed, which will necessitate the need for further research into instructional approaches, aimed at supporting geometric reasoning at different cognitive levels. The theoretical considerations will be discussed.

As noted by Shongwe (2019), theories provide a systematic way to explain and predict behaviors and relationships between variables. In this case, it is a principle that frames the investigation of learners' progression in geometric thinking.

2.2 Purpose of a literature review

Literature review is a platform that aids in reviewing scholarship in the same field of study, consequently, helping the researcher to be aware of the findings made in terms of investigating the problem they are interested in (Mouton, 2001). As the name implies, it reviews and provides an essential framework in research, presenting insights from previous studies and contextualising the current work within the wider academic discussion. Authors (Cooper, 2015; Marshall & Rossman, 2014) assert that literature reviews serve many essential functions: they clarify findings from relevant studies, contextualize new research within established scholarship, identify gaps, and improve current knowledge. Creswell and Creswell (2017) further clarify that a literature review establishes the relevance of the study and provides a standard for juxtaposing its conclusions with preexisting findings. Furthermore, a literature review provides insights for readers in relation to previous studies done within the same field of interest by the reader (Bakare, 2021).

2.3 The relevance of Geometry

Many scholars have extensively and widely conducted several studies on geometry-related subjects, focusing on different aspects, from different angles, aimed at addressing different gaps, based on different rationales: (Bansilal et al., 2014; Bansilal & Ubah, 2019; Chigonga et al., 2017b; Henderson, 1973; Maqoqa, 2024; Marchiş, 2008; Mwadzaangati, 2017; Naidoo & Kapofu, 2020; Ngirishi & Bansilal, 2019; Patkin & Levenberg, 2012; Shongwe, 2019; Usiskin, 2008). The global and local interest, as evidenced by the wide range of studies aimed at understanding the impact of geometry on learners' cognitive and spatial development, illustrates the need for today's study on geometry, with a focus on quadrilaterals.

Geometry is defined as a part of mathematics that has different applications in science and technology (Bassarear & Moss, 2012). In another view, geometry is defined as "The branch of mathematics concerned with properties and relations of points, lines, surfaces, solids, and higher dimensional analogues (Soanes, Stevenson, & Hawker, 2004, *p.603*). It is an exploratory field of mathematics that connects with the real world (Chambers, 2008).

The Department of Basic Education (DBE, 2011) of South Africa defines Mathematics as a subject comprising several branches, involving symbols, graphs, notations, and geometrical figures, and geometry is a core discipline of mathematics that significantly contributes to the development of learners' cognitive, spatial, and problem-solving abilities. Being an important topic in mathematics, the study of geometry improves the abilities of learners in reasoning logically, with spatial perception skills, and critical reasoning, aimed at applying it in our world. Hence, embarking on this study, which aims to explore how learners engage with the properties of quadrilaterals, can help us understand more about learners' geometric reasoning competencies.

The vital role geometry plays in mathematics education seeks to promote essential cognitive skills such as logical reasoning, spatial visualization, and problem solving. Within the global space, geometry is regarded as a basis for the development of analytical reasoning and conceptual understanding, especially regarding quadrilateral properties (Clements & Battista, 1992; Jones, 2002). Spatial reasoning is a skill that is not only used in mathematics but also contributes to the development of scientific reasoning because it involves the manipulation and presentation of shapes and forms (Bassarear & Moss, 2012). Proficiency, both in mathematics and intellectual growth, is fostered by the application of geometry. Learners become more aware of their environment, which enables them to integrate and engage meaningfully within the global space. This agrees with the CAPS curriculum, whose part of the objectives is to promote the cultivation of logical and spatial competence in handling both academic and real-world issues. By linking the concepts of development of the learners' spatial reasoning skills and curriculum objectives in geometry, the study situates itself within the contexts of policy-based and theoretical contexts.

Geometry goes beyond the development of spatial cognition, but also aids in the development of skills of reasoning and stimulation of the mind, which enables handling the many challenges (French, 2004). Authors Chang and Beilock (2016), comment that geometry enhances the development of our cognitive reasoning process, which improves our critical and logical thinking, and decision-making abilities that go beyond mathematics as a high school subject. It develops cognitive processes that improve logical and critical thinking, precision, and decision-making abilities that go beyond mathematics as a school subject.

Significantly, this also explains why geometry is not only instrumental to the development of abstract thinking abilities, but also provides the required and needed abilities in dealing with real-life applications. (Darling-Hammond et al., 2015) describe mathematics, and by extension geometry, as a language employing symbols and notations to articulate numerical, graphical, and spatial connections. By studying geometry, learners see, depict, and investigate patterns, thereby improving their understanding of spatial relationships. The capacity to understand and engage with spatial dimensions and shapes is crucial in fields such as engineering, architecture, and computer science (Chambers, 2008). Hence, promoting the study of quadrilateral relationships and properties of shapes cannot be ignored in high schools. Hence, the need for this study, as it positions itself in terms of practical relevance, is about learners' understanding of quadrilaterals, a foundation for applying geometric concepts.

In South Africa, the Curriculum and Assessment Policy Statement (CAPS) highlights the necessity for learners to develop spatial and logical thinking abilities, among others (DBE, 2011). The Department of Basic Education's view (DBE, 2011) recognises the relevance of geometry, particularly quadrilateral shapes and properties, as essential elements in geometry, and in the mathematics curriculum as a whole, with the intention to equip learners for practical applications in everyday life and many professions. Furthermore, complex spatial relationships and structures can be pictured through the application of geometric concepts, which will result in greater efficacy in perception and evaluation of the setting (French, 2004). Through spatial cognizance, scientific reasoning, as it relates to the representation and handling of shapes and forms, can be enhanced by geometry education (Bassarear & Moss, 2012).

The importance of geometry, especially quadrilaterals, in the South African context has been emphasised within the Curriculum and Assessment Policy Statement (CAPS). The policy seeks to promote and improve the logical reasoning skills of learners and prepare them for advanced mathematical thinking. Hence, need for the consideration of the reinstatement of Euclidean Geometry. According to (French, 2004), the reasons for the inclusion of geometry in teaching and learning are, firstly, that it develops the reasoning skills, and secondly, that it infuses spatial awareness. Consequently, the focus of the study, which seeks to understand learners' cognitive competence in quadrilaterals, aligns with the primary focus of the curriculum.

2.4 Re-instatement of Euclidean Geometry in the CAPs Curriculum

The foundation for educational reform in South Africa was laid in alignment with the provisions of the Constitution (Act 108 of 1996). The constitution placed much emphasis on equitable access to quality education as a fundamental right of the people of South Africa. As an aftermath of the foundation laid, the National Curriculum Statement (NCS) was established for the purpose of redressing the past inequalities, in order to foster inclusivity, as stipulated in the constitution. NCS was initially structured separately as distinct frameworks- Grades R-9 and 10-12.

However, both frameworks were later amalgamated as NCS Grades R-12, which eventually led to the establishment of what we know today as the Curriculum Assessment Policy Statement (CAPS) in 2012. CAPS curriculum provided an explicit curriculum that is well organised and prescriptive, which also includes guidelines on how the academic year (terms) is to be run, term-by-term. This initiative by CAPS was to improve proficiency in topics and learner performance, ultimately establishing a healthier groundwork for academic success (Maddock & Maroun, 2018).

Following the implementation of CAPS in 2012, Euclidean geometry was reintroduced in the mathematics curriculum, this time as a compulsory component in the Further Education and Training (FET) phase (Grades 10-12), after it was relegated to optional paper 3 (DBE, 2011).

This revision emphasises enhancing vital mathematical concepts that will improve the abilities of learners' analytical and spatial reasoning. Brijlall (2017) agrees with the fact that CAPS promotes a cumulative learning approach, wherein topics taught in prior grades are reviewed and expanded upon in the proceeding or higher grade. This systematic, cumulative method, as recommended by CAPS, focuses on knowledge advancement and facilitates the enhancement of learners' logical thinking and spatial analytical abilities, essential for academic and practical problem-solving.

Nonetheless, despite the provision of CAPS and what it promotes, challenges in understanding geometric principles, particularly quadrilateral concepts, still remain a challenge among many learners, and it is viewed that misconceptions and challenges with proofs can hinder learners' progress in mathematics. As indicated earlier, geometry, which is an integral part of mathematics, is perceived to be the most difficult aspect of the mathematics curriculum, and students' perceptions often seem not to be aware of the importance of geometry in reality (Patkin & Levenberg, 2012). National and International Assessments, such as the Trends in International Mathematics and Science Study (TIMSS) and Annual National Assessments (ANA), show that many South African students have difficulties in comprehension and application of geometric concepts in an effective way (Mason, 2010).

2.5 Challenges in Geometry

Geometry education is characterised by several challenges. It is true that geometry, as an important element in mathematics, is vital for the development of spatial reasoning, logical thinking, and problem-solving, yet it is widely believed to be one of the most difficult aspects of mathematics for learners to grasp confidently. Students, based on performance, have consistently experienced difficulties in grasping the abstract aspects of geometric concepts, which require both logical reasoning as well as visual and spatial abilities (Clements & Battista, 1992; Jones, 2003).

A comparative analysis within the global space and South African contexts underscores the persistent challenges faced by learners in advancing through the levels of van Hiele levels of geometric thought. Studies reveal that many learners operate at lower levels of the van Hiele levels (Visualization and analysis), as they struggle to transit to higher levels of geometric reasoning, which is informal and formal, which is essential for advanced geometric reasoning (Siyepu, 2013). Even on the global front, learners still encounter difficulties with the abstract characteristic aspect of geometry and the need to acquire the cognitive skills needed for spatial and logical thinking. That is why students in general see geometry as a distinct topic that has no bearing on real-life applications and, as a result, their interest is affected, with no sign of motivation (Sinclair & Bruce, 2015). Some of the common challenges are often linked to insufficient teacher preparation, limited exposure to inquiry-based learning approaches, and learners' dependence on memorization, instead of conceptual reasoning in geometry (Venkat & Adler, 2020). Other areas of challenges faced by learners include having an understanding of how geometric shapes relate, visualizing 3-dimensional constructs, and formulating formal proofs, particularly quadrilaterals in Euclidean geometry.

Furthermore, conventional teaching techniques that emphasize rote memorization of theorems without promoting conceptual understanding led to superficial learning, restricting students' capacity to apply concepts in problem-solving scenarios (Jones, 2003). Clements and Battista (1992), assert that effective learning in geometry requires developing both visualization abilities and logical reasoning; nevertheless, several educational institutions find it challenging to achieve this equilibrium. As a result, students have difficulties in progressing from shape recognition to comprehending their qualities and relationships, as emphasised by the van Hiele model of geometric reasoning (van Hiele, 1986a), which outlines five developmental levels that students generally progress through.

2.6 Geometry Education Challenges in the South African Context

Although the focus of the CAPS policy modification in 2012 enhanced its focus on teaching Euclidean geometry, as highlighted earlier, nonetheless, students continue to have difficulties with understanding its geometric concepts and how to apply them. The reintroduction of Euclidean geometry in the curriculum did not come without challenges, intensified by vital concerns, such, as limited resources, congested classes, and varying levels of teacher expertise, particularly in rural schools, where underfunding is prevalent. Lack of alignment between instructional approaches and learners' developmental levels of geometric thought, as provided by the Van Hiele model, seems to be one of the identifiable challenges in geometry education (Feza & Webb, 2005). This existing gap between the developmental stages of learners and teaching delivery is especially noteworthy in this study, because of its direct impact in terms of how learners approach and comprehend the properties of quadrilaterals. As alluded to by Mamali (2015), in spite of the importance embedded in geometry and the emphasis by CAPS in fostering critical thinking and problem-solving skills, difficulty in achieving proficiency due to misconceptions, procedural errors, and lack of conceptual understanding keeps affecting learners from acquiring the necessary skills.

The teaching approaches adopted by teachers often contribute to the challenges in the adequate dissemination of the required knowledge to students. It is noted that teachers often focus on rote memorization kind of learning of theorems, instead of conceptual understanding, and as such, students struggle to handle vital tasks that may require deductive reasoning and proof construction (Abakah & Brijlall, 2023). A study by Chigonga et al., (2017b), indicates that the teachers often promote rote memorisation because some of them lack sufficient subject content knowledge (SCK) and pedagogical content knowledge (PCK), to properly and effectively teach students to the point of understanding. Alex and Mammen (2014) further note that these deficiencies exacerbate the gap in learners' understanding, especially with reference to hierarchical relationships and properties of quadrilateral shapes.

Recurring and deepening misconceptions in approaching problems in Euclidean geometry are identified as a major challenge students face, as alluded to in the 2023 diagnostic report (DBE, 2023). The report highlighted misconceptions as one of the major ongoing challenges faced by many students, even to the point of struggling with shape identification. This also goes with struggling to differentiate their properties as well. They often find it even difficult to provide properties distinct to each of the quadrilateral shapes. These concerns reiterate the need for this study, which seeks to probe the underlying cause of such misconceptions and their implications for understanding geometry.

According to the CAPS's mathematics curriculum, at grade 9, students are expected to have mastered the properties of quadrilateral shapes in preparation for grade 10, which is a required foundational knowledge ahead of their entrance to the FET phase. However, it is not the case. As a result of this deficiency in possessing the required identification skills, it shows significant gaps in their understanding of vital quadrilateral properties, for instance, when identifying the properties of a rectangle and a rhombus (Luneta, 2015). This further highlights the possible gap between what the curriculum expects and the actual performance of learners- a gap this study intends to investigate. According to Luneta (2015), the deficiency of not possessing the required property identification and differentiation skills will hinder their developmental growth in cognitive and spatial reasoning and also affect their progression through higher van Hiele levels of geometric reasoning. As a result, it will negatively impact them in carrying out proof-related problems or solving complex problems in Euclidean geometry as well (Feza & Webb, 2005).

Systematic matters, such as adopting old-fashioned instructional approaches and language barriers, also constitute a part of the challenge students face in Euclidean geometry education, as it has an adverse effect in the grasping of concepts and students' performance as a whole. As part of CAPS curriculum, much emphasis is placed on advancing analytical and spatial reasoning actualisation. Nonetheless, the traditional way of teaching militates against the chances of achieving this goal, as these challenges of language barriers and the use of unconventional teaching approaches limit students from being explorative and interactive in the course of their learning (Mamali, 2015). These systemic obstacles exacerbate the conceptual difficulties, thus making proficiency a daunting task.

In addition, South Africa has 11 official languages, and this plays a role in making it harder for learners since geometry has its own terminology, which is usually expressed and communicated in English as a medium of communication, which in most cases is not their first language. In view of this, students often either misunderstand or misinterpret some of the terminology during instructional practices (Abakah & Brijlall, 2023). To address these systemic gaps, the need for strategic and regular professional development programs is suggested and encouraged. This is aimed at enhancing teachers' teaching skills, and promoting innovative instructional approaches, through the use of modern and relevant teaching technology and manipulatives (Alex & Mammen, 2014). If such interventions can be implemented, they will aid in enhancing the teaching of the concepts of quadrilaterals, and could even alleviate the identified gaps in learner understanding.

2.7 Quadrilaterals in Euclidean Geometry

Quadrilaterals is an important aspect of Euclidean geometry that still pose conceptual and cognitive challenges among learners, globally, including South Africa. Understanding quadrilaterals involves having familiarity with their characteristics, classifications, and logical proof frameworks - abilities that can prove notably difficult due to the abstract characteristics of geometry (Clements, 1992; Clements & Battista, 1992; Jones, 2003). Quadrilateral shapes generally are of a four-sided nature, called polygons, which include parallelograms, rectangles, kites, squares, trapezoids, and rhombuses. These shapes are characterised by distinct properties in relation to angles, sides, and parallelism. Furthermore, a characteristic acquaintance with these shapes aids students in building their geometric reasoning and problem-solving skills. Despite their importance in Euclidean geometry, students still struggle to classify and understand these shapes principally as a result of their abstract nature and being used to the visual prototypes. Consequently, these challenges are exacerbated by misconceptions students have regarding hierarchical relationships among quadrilaterals and the emphasis that is placed on rote memorization rather than conceptual understanding (Clements & Battista, 1992; Fujita & Jones, 2007).

2.8 Properties and Classification of Quadrilaterals: A Hierarchical Approach

As highlighted above, quadrilaterals, as four-sided polygons, have different kinds of classifications, such as squares, rectangles, rhombuses, parallelograms, kites, and trapezoids; each one has its unique characteristics, such as angles, length of sides, and parallelism (Clements & Battista, 1992; Van de Walle et al., 2014). The ability of learners to correctly identify and distinguish these features is vital for the appropriate classification of shapes and progress in their geometric reasoning and the understanding of their relationships in mathematics. Nonetheless, studies show that learners often experience difficulties with the classification of quadrilaterals as a result of interrelating properties of shapes, and hierarchical classification. (Clements & Battista, 1992; Feza & Webb, 2005; Fujita, 2008; Usiskin et al., 2008) These challenges stem often from over-reliance on visual prototypes and difficulties in understanding hierarchical (categorization of quadrilateral) inclusion. For instance, where students struggle to recognise that a square is also a rectangle and a parallelogram due to the quadrilateral family they belong to (Jones, 2001; Mwadzaangati, 2017). This is also supported by the van Hiele model, attributing these challenges as the reasons many learners remain at a lower level of geometric reasoning, due to reliance on visual shape appearances, rather than relational properties of the various shapes (van Hiele, 1986b). In order to address these issues, a structured, hierarchical approach known as the hierarchical class inclusion model, designed by Usiskin et al. (2008), aids in distinguishing quadrilateral types, based on their defining properties. Figure 2.1 shows the Usiskin et al.'s (2008) hierarchical class inclusion model, taken from (Mbatha, 2022).

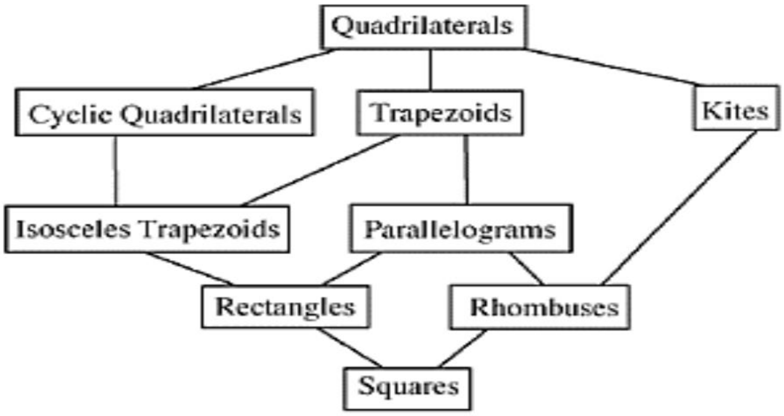


Figure 2.1: Hierarchical class inclusion (model) of quadrilaterals **(Usiskin et al., 2008,** *p. 60***)**

2.9 Hierarchical Classification of Quadrilaterals and Class Inclusion

The structured framework provided by hierarchical classification of quadrilaterals, for categorising quadrilateral shapes, is aimed at helping learners to understand relationships between shapes, properties, and distinctions. The model classifies quadrilaterals based on their defining properties, and also highlights the properties each set shares, based on sub-categorical arrangements. For instance, the class of parallelograms includes shapes, such as rectangles, rhombuses, and squares. The model aids students to understand that once they see a square, they know that although a square has its own properties that define its uniqueness, however, in terms of class inclusion, a square is both a rectangle and a parallelogram, the reason being that they all belong to the same family or class of parallelograms according to the model. On the other hand, that does not necessarily imply that all rectangles are squares, as rectangles also have their distinctive properties, as earlier indicated. This model helps in having a holistic understanding of quadrilaterals, indicating their similarities quality-wise based on multiple categorizations, rather than perceiving them as though they do not co-exist. This structural arrangement fosters learner-advancement in grasping quadrilateral concepts, hence enhancing their cognitive skills, which also facilitates in solving problems related to geometry (NCTM, 1988; Türnüklü et al., 2013).

2.9.1 Hierarchical arrangement of Quadrilaterals

Learners' reasoning about shapes is enhanced when they have an understanding of quadrilateral relationships based on hierarchical inclusion. This model aligns with the van Hiele model, which emphasises advancement from shape identification (which is Level 0- Visualization) to understanding properties and how they relate, which aligns with Levels 1 (Analysis) and 2 (Informal Deduction), respectively. Nonetheless, there is still a struggle among many learners in advancing beyond level 1 (visualization), which is the identification or recognition stage. This implies that learners may easily recognise shapes by visual appearance, but struggle to recognise them by the definition of their properties. This difficulty may be attributed to insufficient exposure to the hierarchical inclusion model, which can reinforce learners' understanding of how these shapes relate (Jones, 2003).

Although the hierarchical model aims to support students in familiarising themselves with the properties of quadrilateral shapes and the existing relationships between the shapes, thereby developing their cognitive skills with ease and in a well-structured manner, studies indicate that many students struggle to understand the class inclusion of quadrilaterals, leading to notable misconceptions. Below is a review of common misconceptions in the classification of quadrilaterals.

2.9.2 Misconceptions in Quadrilaterals Classification

A common misconception among learners is their difficulty in articulating properties and reasoning logically. Many learners struggle to recognise quadrilaterals and to distinguish the unique characteristics that set one quadrilateral apart from another. Mwadzaangati (2017) views that, although students may visually identify a rhombus, they often struggle to articulate its distinguishing characteristics, such as equal side lengths and opposite equal angles. Türnüklü et al. (2013) suggest that understanding hierarchical categories requires reasoning abilities to logically connect attributes. However, many learners encounter difficulties, especially with less common quadrilaterals.

Another misconception is overgeneralisation (assumptions) and incorrect calculations. This is where a student may assume all quadrilaterals are either parallelograms or rectangles, simply on the basis of observation, thereby ignoring the unique attribute criteria associated with each shape (Atebe & Schafer, 2010; Feza & Webb, 2005). Often, students erroneously assume that a particular quadrilateral is a parallelogram, based on a visual scan, whereas it is another quadrilateral. This conclusion is simply based on visualization by assumption, without checking if the figure meets the requirements to be recognised as a parallelogram, such as the fact that the opposite sides need to be parallel. Misconception by overgeneralisation can be minimised if teachers enable students to master the distinct characteristics of each of the quadrilateral shapes.

Many students are also caught up with prototypical shape misconceptions, which are when learners rely on the standard representation of shapes. For instance, students may expect a rectangular shape to be in a horizontal orientation for easy recognition. However, the same learner may struggle to recognise the same rectangular shape when it is in a different orientation. This dependence on visual prototypes for recognition may hinder their ability to identify quadrilaterals by their properties, instead of visually looking for the prototypical orientation of the shape (Fujita, 2012; Van de Walle et al., 2014). Similarly, students struggle to recognise shapes when they are rotated or presented in an unconventional way or angle. This misconception focuses on spatial difficulty faced by students once they see a shape appearing in a position they are not used to, like a rotated square at an angle of 45^0, appearing like a diamond. Rectifying these misconceptions requires learners to be aware that the defining properties of each shape is unchanging irrespective of their orientation. This also suggests a need for a specific instructional approach aimed at reinforcing the invariance of properties of shapes irrespective of their positioning on a 2-dimensional platform (Jones, 2001; Marchiş, 2008).

Understanding quadrilaterals involves recognition of quadrilateral properties and classification of shapes. However, the identified common misconceptions through the literature review, provide valuable insights into existing misconceptions, which will guide the study. Nonetheless, since the study seeks to investigate grade 10 learners' understanding of quadrilaterals in Euclidean geometry, it becomes pertinent to delve into further literature review on factors that could be affecting students' understanding of quadrilaterals, which will further provide insights and guide the study.

2.10 Factors Influencing Quadrilateral Understanding

Exploring the literature review on factors influencing learners' understanding of quadrilaterals provides an essential basis for the study. By aligning the findings of this study, with the existing literature, it will enhance the narrative and the credibility of the study. Although the findings of the study will provide specific insights, rooted in the data collected and analysed, the literature review seeks to provide an academic foundation vital for meaningful discussion and realistic recommendations (Creswell & Creswell, 2017).

The views of Cooper (2015); Mouton (2001) assert that such a review not only provides insights into already existing challenges based on the phenomenon being investigated in a general perspective, but also situates the focus of this study in the context of wider academic discourse. Furthermore, the literature review will reveal possible existing gaps, which will inform the theoretical framework of the study, thereby providing a guide regarding the research design and, importantly, the interpretation of the findings. The review will further serve as a yardstick for comparison purposes between the existing factors, and the factors that will emerge from the study, hence providing a clear indication as to whether or not the findings of the study align with the existing literature.

Having insights into the existing influencing factors will also aid in coming up with valuable and targeted recommendations, which will enhance the relevance of the study, especially in the areas of curriculum design and targeted instructional approaches (Alex & Mammen, 2018). Specifically, the review will reinforce the significance of the focus, particularly having stepped into the FET phase, where foundational knowledge of quadrilaterals will be critical (Brijlall, 2017; Naidoo & Kapofu, 2020).

2.10.1 Curriculum Development and Framework

The structure of the mathematics curriculum plays a central role in determining students' understanding of geometric concepts, including quadrilaterals. The reintroduction of geometry as a core mathematical component (Grades 10 -12), in the South African education curriculum, following the establishment of CAPS, aimed at strengthening learners' spatial reasoning and logical thinking skills (DBE, 2011). Although grade 10 learners are expected to be operating at level 3 (Informal Deduction) of the van Hiele model of geometric thought, however, deficiencies in the education (curriculum) incoherence can affect this from being a reality. Inadequate foundational exposure to basic shapes often affects learners' understanding of quadrilaterals at this stage, as alluded to by Mthethwa (2015). A well-structured curriculum design, with progressive scaffolding, is crucial to avoid fragmented knowledge, and, on the other hand, promote deeper relational thinking among learners (Kilpatrick & Swafford, 2002). This view is endorsed by Alex and Mammen (2018), asserting that the role of curriculum design can support learners' understanding of geometry terminology, which is an important factor in the conceptual grasp of the properties of quadrilaterals.

2.10.2 Pedagogical Methods and Instructional Techniques

The adopted strategies in teaching and learning influence learners' understanding of geometry concepts. Effective strategies entail hands-on activities, manipulatives, and dynamic geometry software, which allow learners to actively investigate relationships in geometry (Fujita and Jones, 2007). Tools that engender interaction, such as GeoGebra, are believed also to be beneficial in enhancing the spatial reasoning and prompting deeper conceptual understanding of geometric principles (Brijlall, 2017). However, a shortage of resources often discourages educators from embarking on or adopting these approaches, particularly in schools that are underfunded, resulting in many schools tending to rely on textbook-based representations. Maqoqa (2024) emphasises that such practice tends to support memorisation at the expense of conceptual understanding (Maqoqa, 2024).

2.10.3 Time Constraints Limitations and Congested Classrooms

Time limitations and congested classrooms, especially in some South African schools, pose another significant challenge, creating pressure on teachers. As a result of these constraints, teachers end up focusing on rote memorisation, rather than conceptual and inquiry-based learning (Naidoo & Kapofu, 2020). Approaches, such as, inquiry-based learning, foster analytical reasoning and relational understanding in geometry, which support students in moving from the recognition level to logical deduction (Fujita & Jones, 2007). Group learning and peer-based instruction are also highlighted as effective methods that enable students to share their views and, in the process, construct geometric knowledge together (Gerdes, 1999). By adopting these methods in a balanced approach, which combines inquiry methods, visual tools, and group techniques, teachers can be in a position to create a conducive learning environment that fosters a deeper understanding of the concepts of geometry.

2.10.4 Educator Subject Matter Expertise and Professional Development

Subject matter expertise and professional development (regularly) play a major role in guaranteeing quality learning, in this case, understanding of quadrilateral concepts and connections. Effectiveness in teaching geometry, with a focus on quadrilaterals, requires competent teachers with a deeper and adequate understanding of the subject matter, as it correctly positions educators to confidently address misconceptions as they arise among learners, fostering a deeper understanding of geometric relationships. Naidoo and Kapofu (2020), assert that several South African teachers do not have adequate knowledge in geometry. As a result, the required objectives to teach the needed concepts in geometry, such as properties in geometry and hierarchical classification with understanding, become seemingly unachievable and unrealisable. Owing to this non-achievement by teachers, learners face difficulty in understanding the required quadrilateral characteristics, causing a hindrance in conceptual growth in mathematics.

On the other hand, the developmental aspect of educators in their field, is also very instrumental towards effective teaching and learning of quadrilaterals. Bansilal and Rosenberg (2011), highlight the need for specific or specialised professional development programs in geometry, citing that educators with no adequate training platforms, for professional developmental purposes, may end up with no other option but to centre their teaching approaches on procedural-based, rather than fostering deeper conceptual knowledge. Students, being the recipients, also end up not advancing in their conceptual understanding of the subject- in this case, quadrilateral concepts, which will enable them to handle geometry-related problems.

The term "Pedagogical Content Knowledge" (PCK), established by Shulman (1986), implies that educators are expected to acquire both subject-matter know-how (widely known as Teacher-Content-Knowledge, TCK) and the ability to communicate the knowledge efficiently. By implication, it means that educators must acquire the knowledge of both the characteristics of particular quadrilateral shapes and the logical links among them, as well as the ability to communicate the content knowledge in a manner that will foster substantial progress in grasping concepts among learners. When there is an insufficient regular teacher-training platform, teachers may not be able to properly guide learners beyond rote memorisation. It is believed that such professional regular training program efforts will aid teachers in bridging the gaps between learners' visual and analytical understanding of geometry.

2.10.5 Language and Terminology

The significant role language plays in mathematics education, particularly in a multilingual environment like South Africa, is of utmost importance, especially in relation to understanding geometry. Understanding geometric terminology, which is tied to language, can be challenging, especially in the FET phase, beginning from grade 10, where learners are gradually exposed to the rigor and the complexity of Euclidean geometry, particularly in quadrilaterals. It is more critical in a multilingual country like South Africa, where there are 11 official languages, and learners have to circumnavigate multiple languages, especially when pedagogical practices are carried out in a second or even third language (Gorgorió & Planas, 2001).

Grasping geometric terminology can be challenging, especially in the FET phase, beginning from grade 10, where learners are gradually exposed to the rigor and complexity of Euclidean geometry, which necessitates the need for students in FET to understand the basic terms such as "parallel," "perpendicular" and even "diagonal". However, Bishop (1986), notes that students tend to embrace and practice rote memorisation more, rather than conceptual understanding, because these terms have no direct translations in their respective home languages, which often also lead to misconceptions, thereby affecting their performance and competence in geometry education.

This is what Setati (2005b), describes as the "Language of Learning and Teaching" (LoLT) challenge, which is especially distinctive in geometry education, where distinct terminology is crucial. The barriers associated with language, therefore, often contribute to the struggle learners have in relating to recognition and application of properties in hierarchical classification of shapes.

Chiphambo and Feza (2020), further emphasise how challenges rooted in language result in alternative misconceptions and procedural errors, with a negative effect on learners' ability in terms of accuracy in identification and classification of geometric figures. In order to address this, the adoption of instructional strategies is needed. This will incorporate simplified language, such as visual aids, and relevant examples that students can relate to based on their culturally lived experiences, aimed at bridging the divide caused by language and terminology complexity. With this suggested approach, it is believed that it will enhance students' engagement and also enhance their understanding of geometric concepts.

2.10.6 Socio-economic and Resource Limitations

Socioeconomic and limited resource factors further deepen the gap in learners' understanding of geometric concepts, especially in the under-resourced schools. Many schools in South Africa are faced with shortages of fundamental educational resources, such as geometry sets and manipulatives, which consequently, limiting the quality of geometry instruction (DBE, 2019, 2023)
The gap in learning geometric concepts, especially as a result of socioeconomic and limited resource factors, is real and factual among many under-resourced rural schools. It is noted that shortages of fundamental resources for education, such as geometry sets, and manipulatives, in many schools, (particularly in township schools), hamper the effectiveness of geometry instruction.

Feza and Webb (2005), examine how resource constraints indirectly intensify learning challenges, particularly in geometry, when teachers have access to fewer educational instruments that will aid in facilitating and enhancing varied teaching methods in geometry, as well as creating an interactive environment for learning. This constraint often compels teachers to depend on instructional styles or approaches that are traditional in nature, as well as textbook-centred instruction, which fosters rote memorisation over conceptual understanding.

Khaliq et al. (2016), pointed out that there is a significant relationship between the socio-economic status of parents and the academic performance of learners, especially in geometry. Sonali (2016), similarly highlighted that learners from low economic backgrounds perform more poorly academically than learners from more economically well-to-do backgrounds. Soudien (2016), attributed this disparity to the legacy of the apartheid system, the negative impact of which still has a direct effect in terms of the performance of learners, particularly at under-resourced schools. In addition, Adebayo et al. (2020), argued that not only low economic status of parents contributes to poor performance of learners in mathematics, with geometry as a reference, but also a lack of resources, play a key role particularly in affecting academic attainment and quality. Visser et al. (2015), further attest that both home and school resources predict the outcome of learners' performance in mathematics.

Resource constraints in most cases lead to a shortage of teaching materials. When this happens, teachers will have access to fewer educational instruments that will aid in facilitating and enhancing varied teaching methods in geometry, as well as creating an interactive environment for learning.

In another study, Naidoo and Kapofu (2020), asserted that the difficulties posed by socio-economic factors contribute to the success gap in geometry, affecting especially students in rural and township schools, as they are often deprived of the opportunities to explore geometry through learning experiences that are interactive in nature. Naidoo and Kapofu (2020), further indicated that the lack of access to the required and needed resources and tools, such as manipulatives and visual aids, should be avoided, as students end up settling with fewer opportunities to deeply explore quadrilaterals, hence, making it difficult for them to develop spatial reasoning as well as understanding the hierarchies of quadrilateral shapes. As noted, and contained in DBE reports, these constraints widen learners' performance gap in geometry, leaving learners unempowered to grasp geometric concepts that are complex, consequently hindering their advancement in geometric reasoning stages (DBE, 2019, 2023).

Although the study recognises broader influences on geometry learning, such as socio-economic factors, the scope of the study remains classroom-based and instructional factors, which is evident from the objectives of the study whose focus centre cognitive development in terms of learners' understanding of quadrilaterals at grade 10 level, and identifiable misconceptions they hold, and the level they operate according to van Hele level of geometric model of though. However, it is also important to briefly provide a review of other supplementary influences, such as learner attitudes and motivation and external systems.

Attitudes of learners towards the subject of mathematics, particularly in geometry, are a major determinant of their commitment, performance and success. Studies show that approaching geometry education with the mindset that it's an abstract or difficult topic often results in low motivation, which hampers understanding and difficulty in applying geometric concepts (Naicker, 2021). On the other hand, approaching the topic with a positive mindset, and eager to learn, coupled with seeing geometry as part of our environment practically, can improve learners' enthusiasm, with the inner drive and passion to tackle geometry-related concepts, the general perception, notwithstanding.

Outside (External) School support, such as, the involvement of parents/guardians, the community, and access to extramural lessons, can be of tremendous help towards learners' understanding. This is especially in the context of socio-economically less privileged schools, where the provision of supplementary assistance can bridge gaps based on classroom experience, by reinforcing foundational skills and making available resources that may not have been provided by the schools or stakeholders, for independent learning (Bishop, 1986; Feza & Webb, 2005).

The following section will now discuss the theoretical framework that unpins this study.

2.11 Theoretical Considerations

2.11.1 Theoretical Framework: Constructivism and the van Hiele Model of Geometric Thought

In view of the phenomenon being investigated, which is 'investigating grade 10 learners' understanding of quadrilateral in Euclidean geometry', this study is theoretically framed within constructivism theory, and underpinned by the van Hiele model of geometric thought. The theory is principally pioneered by two constructivists- Piaget (1970) and Vygotsky (1978), which underpins most of the contemporary educational research, highlighting that learners construct their own understanding through lived experiences and interactions. This theory is particularly relevant to geometry education, because of its developmental role in spatial reasoning and conceptual understanding, instead of rote memorization.

Piaget's theory of constructivism specifically focuses on the cognitive development of learners, which entails the learning process of learners to be active and progressive, through different developmental stages, to ensure that learners' geometric reasoning is emerging at the concrete operational and formal operational stages. Piaget and Inhelder (1967) suggested that learners must be actively engaged in problem-solving, in line with the area of focus, in this case, learning geometric figures- such as manipulating models, drawing, and exploring relationships, in order to develop meaningful knowledge. Piaget and Inhelder (1967) further viewed that meaningful construction is relational to active engagement of hands-activities.

Vygotsky's socio-cultural constructivism theory acknowledges the impact of social-cultural and scaffolding aspects in the cognitive development of learners. His ideology of the Zone of Proxima Development (ZPD) underscores the gap between what a learner can grasp, without guidance, compared to what the learner can grasp when properly guided (Vygotsky, 1978). The concept of Zone of Proxima Development (ZPD) reveals the gap between what a learner can understand when learning unguided, and when properly guided in the learning process. This concept of Zone of Proxima Development (ZPD) will go a long way in supporting learners, in view of the difficulties they face in transitioning beyond lower levels of the van Hiele geometric model of thought, as this concept of Zone of Proxima Development (ZPD) will enhance their progression.

In view of the two concepts of the theorists, constructivism theory, in general, remains a suitable learning theory that explains how learning happens within the context of the classroom; a reflection of their lived experiences, interactions, and previous cognitive processes (Mbili, 2011; Piaget, 2005). Klir and von Glasersfeld (1991), highlighted that learning should be an active process of mental construction and adjustment of schema, rather than passive transfer of knowledge. Constructivists, such as, Piaget (2005), linked constructivism with instructional approaches that foster active learning. In a constructivist classroom, learners have the platform to explore, question, and form connections, which enables them to deepen their understanding of the subject matter, thus making them active participants in the course of learning (Klir & von Glasersfeld, 1991).

Constructivism theorists are of the view that learners learn when they construct their own knowledge from their own world of realities, their thinking pattern shows the true reflection of their day-to-day experiences, including what they believe and their way of life (Van De Walle, 1994). In constructivism, a learner may actively engage and even proactively and productively argue with the teacher about content being taught; in the process, either new knowledge is constructed, or mental adjustment occurs. This implies that in constructivism, the instructional approach is learner-centred, not teacher-centred, as such, it is the learners' responsibility to ensure learning is taking place (Ngirishi, 2015).

Recent studies affirm the advantages of adopting a constructivist (learning) approach, which promotes collaborative problem-solving (CPS) in a classroom setting (Çibukçiu, 2025; Pöysä-Tarhonen et al., 2022; Tursynkulova et al., 2023). The studies further showed that learning through a problem-based approach improves the cognitive and creative abilities of learners through their involvement in complex geometry-related tasks, which require some form of analysis and collaboration. It shows how group interaction and collective reasoning contribute to deeper comprehension of the concepts in geometry. The studies further highlighted that in addition to the strategies of adopting a constructivist learning approach, which enhance understanding and problem-solving, it also promotes critical reasoning, autonomy, and the ability to interconnect concepts or ideas in mathematics.

The constructivist method in South Africa corresponds with the CAPS framework, which prioritises the cultivation of critical thinking, problem-solving, and relational comprehension

(DBE, 2011). Von Glasersfeld (1984) argues that within a constructivist paradigm, learning entails the construction and ongoing modification of one's cognitive models. This method promotes learner-centred activities in which the instructor (educator) assists, while the learner assumes responsibility for their educational journey (Ngirishi, 2015).

Studies have shown that engaging constructivist instructional methods, such as inquiry-related learning, the use of dynamic software, and collaborative problem-solving, considerably improve the geometric understanding of students. Sinclair & Bruce (2015) are of the view that incorporating digital tools in geometric education will enhance learners' dynamic learning of geometric concepts, and provide a platform for engaging with multiple representations of quadrilaterals. Research also shows that teaching that is guided by constructivist theory enhances learners' higher-order thinking skills in terms, of allowing learning to establish connections between different properties of quadrilateral shapes and applying reasoning that goes beyond rote learning (Battista, 2007; Fujita & Jones, 2007).

Therefore, in view of the phenomenon being investigated, which entails active learner-participation, learner-centred, and guided scaffolding, constructivist learning theory provides a strong theoretical basis for this study, underpinned by van Hiele's model of geometric thinking, which is based on the principles of constructivism. The Van Hiele theory of geometry thinking is now discussed.

2.11.2 The van Hiele Model of Geometric Thought- (VHL)

The model of the van Hiele theory of geometry thinking, was designed for teaching and learning geometry (Crowley, 1987). It was developed by Pierre van Hiele and Dina van Hiele-Geldof. The theory, explains and seeks to account for the process learners go through in developing or constructing geometry concepts. The theory postulates the existence of five-level geometric thought (Crowley, 1987), which explains how learners advance in developing their geometric reasoning skills from basic shape recognition to abstract deductive reasoning. Each level builds based on the skills already acquired in the previous stage or level (van Hiele, 1986a). The model provides a framework that outlines five levels of geometric reasoning, and each of the levels reveals a distinct stage of understanding that must be developed sequentially, through well-defined instructional guidance. The levels are highlighted below:

Level 0: Recognition (Visualization)

Usiskin (1982, *p*.2) states, "the students can learn names of figures and recognise shapes as a whole." at the recognition level. This level enables the learners to visually differentiate shapes such as triangles, squares, and rectangles, without identifying their differences through their respective properties. For instance, learners may recognise a square based on its general form but not distinguish it from a rectangle. It is viewed that this stage is a foundational stage, as it provides the platform for property-based analysis in the proceeding stages.

Level 1: Analysis

At the analysis level, Usiskin (1982), states, that students can identify properties of figures. Learners are able to analyse shapes based on their parts and properties. This level expects learners to identify each shape's properties on their own, such as squares, rectangles, angles, without linking one shape's properties to another shape's properties. For instance, learners can identify a square as having four right angles, but may not be able to connect its properties to classification as a parallelogram. Hence, Mason, (2010), argues that learners are still not able at this level to establish connections between shapes. This level marks a critical step toward thinking that is relation-based, as learners start analysing shapes based on their attributes.

Level 2: Abstraction (Informal Deduction, Ordering or relational) level

At this level, learners have the ability to logically order figures (shapes) and relationships (Usiskin, 1982). Learners can understand the relationships existing between the properties of figures (shapes) and the relationships between figures (shapes). This stage involves the ability to organise and categorise shapes within a hierarchy. For instance, learners can recognise that all squares are rectangles because they meet the defining properties of rectangles, but not all rectangles are squares. Informal reasoning is emphasised at this level according to Fujita and Jones (Fujita & Jones, 2007). At this level, learners recognise that certain properties imply others and group shapes, based on their shared characteristics. However, learners are still developing the formal-proofs construction ability, but only depend on informal logic, at this stage, rather than rigorous deduction.

Level 3: Deduction

This is the stage where formal proofs, definitions, and theorems are used to validate geometric statements. (Usiskin, 1982) postulates that learners, at this level, understand the importance of formal deduction and the functions of theorems, and proofs. This is supported by Senk's, (1989) assertion, that learners operate at this stage with the ability to work within an axiomatic system, by using deductive reasoning to validate connections between shapes and properties. For instance, learners can prove that the diagonals of a rectangle are the same (congruent) on the basis of the properties of a parallelogram. So, learners can now carry out proofs with an understanding of the importance of geometric terminology, definitions, axioms, and theorems in Euclidean geometry. It is noted that this level aligns with the curriculum's focus for high school on deductive reasoning and proof in Euclidean geometry, especially for the FET phase students.

Level 4: Rigor

At level 4, learners understand abstract geometric systems, such as non-Euclidean geometry (van Hiele, 1986a). Learners understand the importance of rigor, which implies that learners can make abstract deductions (Usiskin, 1982). In other words, learners can work with abstract geometric systems, such as comparing Euclidean and non-Euclidean geometries. By implication, learners possess a high-level understanding of geometric structures and evaluate different axiomatic systems. De Villiers (1999) notes that it is generally rare among school learners, adding that it requires an advanced understanding of formal mathematical proofs and abstract reasoning skills beyond what is obtainable in the high school curriculum. Hence, Ngirishi (2015) agrees that learners are also expected to work with axioms, comparing different systems, hence, geometry is now seen as abstract.

2.11.3 Justification for Using VHL

The Van Hiele theory is a widely known vital theory which can be used to determine the levels at which learners operate in geometric reasoning, based on well structurally designed given tasks. Nonetheless, given its important role in assessing geometric reasoning, the model has been subjected to several criticisms, based on its stage-based approach.

Singh (2006), asserts that the model is not sufficient enough to account for understanding at a higher level. Burger and Shaughnessy (1986), argue also that the development of cognition in geometry does not adopt a strict straight line or one-way kind of progression, adding that learners (in the course of knowledge construction), may be compelled to revisit the previous stage or stages, especially when they encounter new concepts during the learning process.

Burger and Shaughnessy (1986), further make it clear that the model's potential rigidity seems not to recognise that learning patterns should not be seen as a linear or straightforward process. De Villiers (1999) also alluded that the model focuses heavily on formal deductive reasoning, which may weaken the importance of alternative reasoning styles, such as inductive and intuitive approaches, which are also beneficial in geometry-related problem-solving.

Although, van Hiele model's limitations are noted, the model plays a vital role in the evaluation of the geometric thought process or pattern of learners, which invariably provides a systematic framework on which teachers stand to design instructional approaches that will address the geometry developmental needs of their learners.

2.11.4 Application of the van Hiele Model within the South African Educational Context

The underlying concept of the van Hiele level of geometric thought agrees with the main objectives of the South African Curriculum and Assessment Policy Statement (CAPS), based on the framework structure it provides towards the implementation of the Curriculum. The South African educational curriculum places emphasis on the development of cognitive reasoning, such as spatial, and deductive reasoning, and conceptual understanding in Euclidean geometry, which makes the van Hiele level of geometric thought a vital tool that provides measures for assessing and guiding the geometric progress of learners.

According to the South African curriculum, learners, especially in the FET phase (Grades 10-12), are expected to advance from the recognition level, which is shape identification to formal deductive reasoning, which is outlined in the CAPS. The van Hiele model of geometric thought serves as a tool that determines the geometric reasoning levels of learners, as this helps teachers to structure their teachings and approaches to meet the needs of learners developmentally, based on the levels each learner operates at a time.

For instance, although, the CAPS curriculum promotes conceptual understanding, studies show that many learners struggle to move beyond the lower levels of van Hiele, because they focus more on rote memorization and procedural tasks (Abakah & Brijlall, 2023). Hence, the need for teachers to promote relational and deductive reasoning, by structuring their approach to align with the van Hiele model.

Prevailing imbalances in South Africa, such as limited resources and socio-economic factors in many schools, especially rural schools, also further disturb the application of the model. For instance, although, van Hiele model recommends specialised instructional approaches for the realisation of the goal of the model, however, this recommendation is often not realizable as a result of overcrowded classes, a lack of sufficient teachers, and insufficient resources in some schools. Abakah and Brijlall (2023) also note that limited access to relevant instructional tools, such as GeoGebra, which could have facilitated progression through van Hiele levels, in many rural schools, also restrict the potential impact. The multilingual nature of South African classrooms also seems to contribute to promoting effective application of the van Hiele model, as it presents language-related barriers, which hinder the ability of learners to comprehend geometric terminology- a vital element to advancing to higher levels of geometric thought (Setati, 2005b).

Although, the challenges highlighted above may seem to water down the van Hiele model, the model remains an effective framework within the CAPS curriculum objectives, which can enhance understanding and support geometric reasoning in South African classrooms. By finding out the levels at which each learner operates, educators can design suitable target intervention programs, such as collaborative learning and exploratory activities, aimed at addressing misconceptions, and improving progression through the levels. Importantly, the model also provides a basis for designing classroom assignments and assessments that are appropriate developmentally, which aligns pedagogical practices with the abilities of learners.

2.12 Implications for Teaching and Learning Geometry

The Van Hiele model has proven its relevance over the years, in relation to providing the foundational framework in guiding instructional activities in the classroom, and also improving understanding of geometry. It’s geometric thought, which is sequence-based, highlights the importance of systemic and appropriate instructional approaches, in a developmental manner, that align with the progression of learners. Below are the reviewed key implications of the Van Hiele model for teaching and learning geometry in South Africa, taking cognizance of the challenges and contextual factors.

2.12.1 Curriculum Development and alignment with Cognitive Progression

Based on the objectives outlined in the Curriculum and Assessment Policy Statement (CAPS) document, for deductive reasoning and relational understanding, particularly at the Further Education and Training (FET) phase, Fujita and Jones (2007), and Naidoo and Kapofu (2020), noted that the procedural emphasis of CAPS often limits the progression of learners beyond the lower levels of van Hiele levels of geometric reasoning. In order to address this, De Villiers (1999), suggested that the CAPS design must align with the order of instructional sequences, outlined by the van Hiele levels, as this will enable learners' transition from visualization to abstract deductive reasoning. De Villiers (1999) further noted that organised activities, which enhance relational thinking and hierarchical classification, should be given priority, as this recommendation can provide the needed learner support in understanding quadrilateral properties of quadrilaterals.

2.12.2 Teacher Training and Pedagogical Knowledge

The relevance of teacher-training and Pedagogical knowledge in relation to learners' advancement through the van Hiele levels is very critical. Shulman (1986), highlights the importance of pedagogical content knowledge (PCK), as well as the acquisition of sound pedagogical content knowledge (PCK) by teachers, as it enables teachers to effectively provide the needed guidance as learners journey in their geometric reasoning stages. As pointed out by Naidoo and Kapofu (2020), learners' struggles in understanding complex geometric concepts can be attributed to gaps in teachers' content knowledge. Hence, professional development programs designed to align with the van Hiele model are essential for equipping teachers with both the required subject-content knowledge and the pedagogical approach as a scaffolding measure in providing the assistance needed by learners in enhancing their understanding. A further suggestion underscores that training programs should place emphasis on strategies that will enable teachers to address misconceptions and foster a deeper understanding of geometric relationships.

2.12.3 Interactive and Inquiry-Based Instruction

Brijlall (2017), asserts that learners' ability to visualize and manipulate geometric concepts can be enhanced by the use of dynamic tools such as GeoGebra. Furthermore, inquiry-based learning and collaborative problem-solving methods, as recommended by Fujita and Jones (2007), provide learner support in developing their relational understanding. Integrating group exercises, interactive tools, coupled with scaffolded learning assignments, assists learners in actively exploring and constructing knowledge in an active way. By doing so, the gap between theoretical concepts and practical application will be bridged.

2.12.4 Addressing Socio-Economic and Resource Constraints

Teaching and learning can be adversely affected by socio-economic inequalities, South Africa being a typical case. The most affected schools are under-resourced schools as they have little or no access to resources such as geometry kits and digital tools, which hinder effective instruction in the classrooms (Feza & Webb, 2005). Government-funded resource allocation and the adoption of freely accessible geometry software can also help mitigate these challenges. The strategy of group learning can also rectify shortages of resources by fostering peer-based knowledge sharing (Brijlall & Ndlovu, 2013).

2.12.5 Language and Terminology

The role of language in the mathematics subject in general is very significant, especially in a country like South Africa, known for multilingualism. Setati (2005b) emphasises the challenges encountered by learners in understanding geometric terminology as a result of language difficulties. Understanding and fostering of inclusivity can be enhanced by simplifying the geometric terms and contextualising them. This will enable learners to make sense of the geometric terms based on their existing knowledge. Furthermore, educators are encouraged to employ accessible language and facilitate clear articulation of abstract geometric concepts as they enhance the understanding of learners.

Having examined the implications of the van Hiele model within the educational context of South Africa, and the ‘teaching and learning’ implications of the model, the next section focuses on the specific strategies for addressing the persistent challenges in geometry education, although the ‘implications for teaching and learning geometry’ broadly highlighted the instructional and curricular-based considerations. The next section examines strategies that will be based on both the van Hiele model and the South African instructional practice experiences. The review will further strengthen this study by providing a foundation for recommendations that are realistic and context-sensitive.

2.13 Strategies for Addressing Challenges in Geometry Education

In addressing the challenges faced by learners in geometry education, especially in the context of quadrilaterals, multifaceted strategies that are research-based, which also align with pedagogical best practices, will be required. The presented strategies are based on existing literature, combining both the theoretical underpinnings of the van Hiele model and practical classroom applications.

2.13.1 The use of Visual and Interactive Tools

Learners' understanding of quadrilaterals in geometry gains deeper insight when visual and interactive tools such as dynamic software, (e.g., GeoGebra), visual aids, and manipulatives are in use. It will help learners in visualizing and interacting with shapes, which enhances understanding in the process (Clements & Battista, 1992). Scholars highlight that technology-supported visualization enhances engagement and understanding in a situation where there are limited resources, especially in rural schools in South Africa (Alex & Mammen, 2018). The availability of manipulatives and visual aids enables teachers to bridge the gap between abstract concepts and the real-world experiences of learners. Furthermore, such tools support differentiated instruction by providing various styles of learning. When learners have the opportunity to handle manipulatives and transform shapes, the chances are that it leaves lasting mental pictures, invariably enhancing retention and constructing spatial reasoning skills, which are critical for geometric progression.

2.13.2 Sequential Instruction Supported by van Hiele Levels

Van Hiele model of geometric reasoning, which provides a structured, level-based teaching approach, is viewed as essential, as it supports learners' gradual understanding. Fujita and Jones (2007), advocate for an instructional strategy in a progressive manner where learners, for instance, build foundational knowledge at a lower level before advancing to the next levels of the model. A step-by-step approach can ensure learners, particularly FET students, build foundational knowledge, in classifying quadrilaterals and having an understanding of the existing relationships among diverse shapes. Moreover, integrating assessing learners using a formative approach, aligns with the model as it will aid educators to track the learning process, detect misconceptions on time, and come up with a suitable framework that will be able to advance learners within their Zone of Optimal Development.

2.13.3 Rectifying Misconceptions Through Hierarchical Classification

Common misconceptions about hierarchical classification relationships can be tackled by placing emphasis on explicit teaching on hierarchical classification of quadrilaterals (Usiskin et al., 2008). Introducing diagrams and examples of hierarchical classification of quadrilaterals also enhances learners' understanding of relationships between quadrilaterals, such as recognising that squares are also rectangles and parallelograms (Usiskin et al., 2008). Van de Walle et al. (2014), also suggest that incorporating these diagrams in the classroom practice, enhances relational understanding, thereby reducing error-related concepts, and enhancing analytical reasoning. In addition, the engagement of learners in a group-based discussion platform, where they have the opportunity to validate classifications or defend their arguments, can also further reinforce the concept of hierarchical classification of quadrilaterals and reduce the misconceptions they have regarding properties of shape. Structure can also enhance the creation of a learner-peer-discussion platform.

2.13.4 Collective learning and Peer Instruction

Group learning promotes conversation, helping learners to articulate their geometric reasoning, and also promoting relational understanding. Within the South African context, peer-assisted learning, is perceived to be effective, under-resourced schools (Brijlall & Ndlovu, 2013). As learners engage and refine each other's reasoning, through discourse and shared inquiry, understanding gaps are bridged in the process. An environment that promotes a collaborative learning approach also provides opportunities for exposure of learners to several viewpoints and strategies in problem-solving, particularly in geometry, where various methods can still arrive at the same solution. Teaching learners as a group provides some form of confidence and cultivates a supportive learning culture, which helps learners to feel safe to express their thoughts, make mistakes without feeling bad, and learn from one another in the process, which in turn reduces dependence on teachers and promotes independence.

2.13.5 Contextualisation in Real-World Settings and Relevant culturally-based Examples

Connecting geometry education to real-life situations has been shown to help promote students' engagement. Culturally pertinent examples in geometry classes help learners in making sense of the abstract concepts based on their lived experiences (Gerdes, 1999). Engaging this contextualised approach bridges the existing gap between theory and practical knowledge, which is of benefit especially for students in rural areas (Mutodi & Ngirande, 2014).

Furthermore, integrating objects, patterns, and environmental features that are alike- such as agricultural plots, housing structures, or African designs, can provide real contexts that will aid in understanding concepts such as angles, spatial relationships, and symmetry. Such an application sustains the interest of learners and also authenticates their identity culturally, thus promoting inclusivity and meaningfulness in their learning experience.

2.13.6 Structured Exercises for Deductive Reasoning Development

Regular practice with well-thought-out activities that require deductive reasoning skills is vital for learners progressing to the deductive level, as this approach is vital for enhancing formal proofs and logical reasoning. Senk (1989) is of the view that scaffolded proof activities should be incorporated in instructions, especially for students at the Deduction level, adding that students benefit from scaffolded proof activities, particularly at the initial stage of developing deductive reasoning. It guides students through the process of formal geometric proofs reasoning construction. This approach agrees with the CAPS curriculum in South Africa, which focuses on deductive reasoning, where students are initially provided with scaffolding support at first, for the purpose of gaining more conceptual insights. Later, the support is gradually reduced, aimed at encouraging independence in problem-solving and allowing them to independently construct proofs on their own.

2.14 Conclusion

This chapter provided a literature review relevant to understanding Grade 10 learners' understanding of quadrilaterals in Euclidean geometry. This chapter started by underscoring geometry education and its relevance, emphasizing its role in the development of cognition and evolution, particularly within the South African educational curriculum. Challenges in geometry learning, including misconceptions, coupled with quadrilateral classification difficulties, were also discussed. This was followed by factors influencing learners' understanding of quadrilaterals.

The theories upon which the study is framed were also presented and explored, contextualising the stages of learners' development in geometry. The chapter further reviewed the application of the van Hiele model within the CAPS curriculum, its implications for teaching and learning, as well as strategies for addressing challenges in geometry education. These insights form the basis for understanding the study.

The next chapter will provide a complete methodological approach used for the study.

CHAPTER 3

METHODOLOGICAL APPROACH- RESEARCH DESIGN

The approach adopted for this study was a qualitative study, conducted within an interpretative paradigm, and will adopt a case study approach.

The methodological approach of this research is an important aspect of the study. It outlines the strategy and methods employed to investigate the research questions and achieve the objectives of the study. Creswell and Creswell (2017), Creswell and Poth (2016), and Merriam and Tisdell (2015), assert that the choice of an appropriate research design ensures the validity and reliability of the findings in educational research, such as the study of learners' understanding of geometry.

This chapter, therefore, outlines the research paradigm and approach, followed by the style of research adopted for this study. It further details the data collection process, the instruments employed, the selection of participants, and the data analysis procedures. Additionally, ethical considerations and the trustworthiness of the research will be discussed to affirm the rigor and integrity of the study.

3.1 Research Paradigm

Creswell and Creswell (2017) describe a research paradigm as the philosophical foundation that underpins the methodology of a study. It provides guidance in terms of how the research is conducted and how the knowledge is interpreted. It encompasses assumptions about reality (ontology), how knowledge is acquired (epistemology), and the procedures employed in obtaining the data (methodology) (Creswell & Creswell, 2017). Furthermore, Guba and Lincoln (1994) allude that research paradigms serve as philosophical frameworks that influence the researcher's approach to inquiry. Guba (1990, p.17) further states that a research paradigm "is a basic set of beliefs that guides action". Scholars' approaches to knowledge development in educational research have historically been influenced by the interpretivist and critical paradigms (Opie, 2004).

The view of the interpretivist paradigm, in particular, emphasises the subjective understanding of human experiences and social phenomena, which seeks to explore context-specific-based insights, rather, than universal truths (Guba & Lincoln, 1994).

This study, therefore, considers the interpretivist paradigm suitable as the study seeks to investigate grade 10 learners' lived experiences, in a bid to investigate their understanding of quadrilaterals in Euclidean geometry within a real-world classroom context.

Interpretivism aligns with the goals of this research because it grants the researcher the opportunity to explore the learners' experiences, perceptions, and constructions of geometric concepts from their unique perspectives (Habermas, 1988). The interpretivist paradigm enables a deeper exploration of these subjective realities since learners' understanding of geometry is likely influenced by their personal, social, and educational backgrounds.

Since the study adopted a qualitative method, instruments such as task-based worksheets, and interviews provided a platform for detailed and relevant data that reflected how learners conceptualize and engage with quadrilaterals in a natural setting (Cresswell, 2013).

The relevance of the interpretivist paradigm in this study is also evident in its emphasis on contextualising understanding. By situating the research in a specific school in the Umlazi district of KwaZulu-Natal province, this paradigm enabled the researcher to focus on the local, historical, and social realities that influence learners' understanding of Euclidean geometry (Guba & Lincoln, 1994). The interpretivist approach emphasises that knowledge is co-constructed by the researcher and participants, and is not fixed or objective (Merriam & Tisdell, 2015). The co-construction of knowledge is in line with the study's objectives, which are to determine how learners in grade 10 interpret geometric concepts through their experiences and learning processes.

Additionally, the interpretivist paradigm aligns with educational research, as it seeks to explore how learners construct knowledge, by focusing on uncovering deep, meaningful insights, instead of having findings that can be generalized in the context of a wider population (Cohen et al., 2011). The researcher, based on the paradigm, hopes to gain a nuanced understanding of the learners' specific cognitive and social processes that shape their grasp of quadrilaterals, based on their natural educational environment.

This paradigm also aligns with the use of van Hiele's model of geometric thought as a framework for analysing the learners' level of understanding, providing insights into their cognitive development and conceptual challenges they may face in geometry (van Hiele, 1986b).

3.2 Research Approach

A research approach is the comprehensive strategy and methodology employed by researchers to address their study questions or objectives. A research strategy, includes the full framework, comprising the general design, technique, and theoretical perspectives, that directs the collection and analysis of data (Terre Blanche et al., 2014). Research approach establishes the framework for the whole research process, specifying the kinds of data that are analysed, how they are collected, and the findings that are drawn from them. Williams (2011) further adds that research approaches are shaped by the nature of the research problem, emphasizing that different approaches, whether quantitative or qualitative, offer distinct advantages depending on the objectives of the study. This study adopted a qualitative research approach to investigate Grade 10 learners' understanding of quadrilaterals in Euclidean geometry, allowing the researcher to explore the issue in-depth.

McMillan and Schumacher (2001) view qualitative research as a suitable research approach for studies that focus on understanding phenomena in their natural settings and from the perspectives of those involved. Adoption of a qualitative approach compels the researcher to focus on the "what," "how," and "why," of human experiences and seeks to capture the complexity of the phenomenon under study. Unlike the quantitative research, which typically relies on statistical analysis, qualitative research provides a detailed understanding of social phenomena based on rich, contextualized data. Teherani et al. (2015), assert that qualitative research is particularly suitable for studies where not much is known about the subject, as it generates a deep understanding of the meaning and lived experiences that individuals attach to their actions. In this case, the adopted research approach allows for a thorough examination of learners' conceptual frameworks alongside the challenges they face in understanding geometric concepts, focusing on quadrilaterals.

The view of Manning and Stage (2015) about qualitative research stipulates that qualitative research allows researchers to gain insights into how individuals experience and interpret phenomena. The view suggests that adopting a qualitative approach helps to uncover subjective meanings and perceptions of individuals within specific contexts. Hence, qualitative enquiry is regarded as an ideal approach for educational research, which seeks to understand learners' cognitive processes. This again reaffirms its relevance for this study as it focuses on the cognitive and social dimensions of learners' interactions with geometry. Furthermore, the qualitative approach will accord the research the opportunity to investigate the learners' thought processes, strategies for problem-solving, and the challenges they encounter in understanding quadrilateral concepts, providing a richer understanding of the learning process.

The aim of adopting a qualitative research approach for this study is also underpinned by the goal of generating rich, descriptive data through the use of task-based questionnaires, semi-structured interviews, and one-on-one interviews. The view of Nieuwenhuis (2010), also asserts that qualitative research focuses on the subjective experiences of participants to unearth the quality of the phenomena being studied. Employing this approach, the researcher will investigate how Grade 10 learners at a specific high school in the Umlazi district, in KwaZulu-Natal, engage with Euclidean geometry, focusing on quadrilaterals. This method is also consistent with Worku et al.'s (2023), assertion that qualitative research allows researchers to record "the participants' voices" and "construct a detailed narrative" regarding their experiences. This objective is crucial for comprehending learners' geometric reasoning and the variables affecting their conceptual understanding.

By adopting a qualitative approach, this study recognises that learners' understanding of quadrilaterals cannot be reduced to simple scores or numerical data, and it does so by taking a qualitative approach. The objective is to identify the underlying social and cognitive mechanisms that influence how students approach geometric-related concepts or problems. This also aligns with Maree (2020), who asserts that qualitative research in the educational field provides a platform to fully understand learners' learning process, through ideas, perceptions, and lived experiences of learners. Creswell and Poth's (2018), qualitative approach provides the needed opportunity to gain insights into the nuances of learners' experiences and interactions within educational contexts.

These authors assert that qualitative approaches improve understanding of learners' interactions with complex education phenomena, which is crucial for addressing particular challenges in teaching and learning. Merriam and Tisdell (2015), argue that qualitative research offers critical insights that quantitative approaches may overlook, underscoring its significance in educational contexts.

3.3 Style of Research-Case Study

A case study is a style of research that seeks to make sense of a phenomenon, based on a particular location in a real-world context. A case study could involve an individual, a school, a group, an organisation, or a community. It can also involve an event, movement or a geographical unit Neuman (2007).

The main feature of case studies is their emphasis on investigating and understanding the uniqueness of the case under study. Cohen et al. (2011) assert that case studies are commonly used in qualitative research because they provide an in-depth analysis of specific cases. Case studies provide the potential to get deep insights into the phenomena being investigated by collecting extensive and contextually rich data. This agrees with Rule and John (2011), who define a case study as a systematic and thorough investigation into a particular situation within the context of nature, intended to produce relevant data.

This research used a case study technique to investigate grade 10 learners' understanding of quadrilaterals in Euclidean geometry at a particular school in the Umlazi District, in the KwaZulu-Natal province in South Africa. This approach is especially appropriate for educational research, as it enables an extensive understanding of learners' conceptualizations and experiences within a real classroom setting. The case study will allow the researcher to investigate both learners' understanding of geometric ideas and the environmental circumstances that may affect their learning process.

Bertram and Christiansen (2014), view the case study approach as a framework (within an educational context) capable of investigating the "how" and "why" regarding a phenomenon. In the context of the study, the researcher, through investigation of learners' understanding of quadrilaterals, will gain insight into the phenomenon being investigated.

In the South African context, case studies provide a method for investigating localized educational difficulties by socioeconomic, cultural, and policy-related aspects (Bertram & Christiansen, 2014). This study, centred on a singular school in the Umlazi District, aims to generate insights that are context-specific and might apply to other educational settings experiencing similar issues. Furthermore, Stake (1995) stresses the exploratory and descriptive nature of case studies, making them a suitable approach for investigating phenomena that need a detailed understanding of participant experiences. This study employs the case study technique to facilitate an in-depth investigation of learners' misconceptions, reasoning processes, and geometric knowledge levels, which are vital to guiding future pedagogical strategies.

The case study method offers an effective framework for investigating grade 10 learners' understanding of quadrilaterals in their real educational context. By narrowing the study to a single school, the case study will provide an in-depth and contextually enriched investigation of the learners' experiences and issues, providing significant insights into the larger educational ramifications for the South African context.

3.4 Sample Population, Sampling, and Recruitment Process

The study's target population is drawn from Grade 10 learners of a particular school in the Umlazi District, in KwaZulu-Natal, in South Africa. The selection process was guided by individual consent, alongside their parents, and voluntary participation.

This study, being qualitative and interpretative, will employ purposive and convenience sampling methods to recruit participants. Purposive sampling was used in order to ensure the inclusion of participants capable of providing necessary data to address the study questions (Cohen et al., 2007). This approach ensures the participants of learners with varying degrees of understanding of quadrilaterals, since it corresponds with the objectives of the study of investigating varied learner experiences.

Convenience sampling was used to determine learners who were easily accessible and willing to participate, providing a practical approach for participant selection in an educational context (Teddlie & Yu, 2007). Convenience sampling, is a method of non-probability, as the participants' recruitment is based on accessibility of learners within the school, which assures resourceful data collection (Kumar, 2018).

At the selection process, a total of 20 learners were initially recruited, on the basis of their willingness to be part of the study and its objectives. However, the final recruited number increased by 2, totalling 22 participants. Being qualitative research, the increased number will contribute to the richness of the data, both in-depth and in scope. This will also enhance the transferability of the findings to similar educational contexts.

Ethically speaking, Bertram and Christiansen (2014) stress the importance of ethical consideration in education-based research. Adherence to ethical norms was followed. Participants were thoroughly briefed on what the study is all about and the implications, also assuring them of confidentiality and willingness to withdraw at any time. Thereafter, since the study involves minors, the informed consent forms, were made available to the willing learners, as well as their parents or guardians.

3.5 Data generation- Data collection instruments, and procedures

Data generation in qualitative research refers to the systematic collection of comprehensive information from participants within their natural environment. Creswell and Poth (2018), along with Merriam and Tisdell (2015), define qualitative data generation as the methodological collection of substantial data from participants within the context of their everyday lives, to investigate their lived experiences and views about the subject matter issue. In contrast to quantitative research, qualitative data collection methods provide an in-depth analysis of the subject matter, allowing for a thorough understanding of participants' subjective experiences within their natural setting (Creswell & Creswell, 2017; Denzin et al., 2023; Patton, 2014). This study comprised two qualitative instruments for data generation, which will be discussed below: task-based worksheets (questionnaires) and semi-structured interviews, to investigate Grade 10 learners' comprehension of quadrilaterals in Euclidean geometry. These instruments align with Bertram & Christiansen's (2014), suggestion for employing diverse qualitative methods to provide thorough insights into educational contexts.

3.5.1 Task-based Worksheet (Questionnaires)

In qualitative research, questionnaires are widely used to gather detailed, and subjective data from participants, particularly when the questions have been designed to prompt thinking and explanation. A task-based worksheet served as the primary instrument for data collection in this study. The worksheet had two parts, aimed at assessing learners' reasoning and understanding of quadrilaterals:

Questionnaire A: This section consisted of 20 multiple-choice questions (MCQ) divided into four subgroups, based on van Hiele's geometric thinking levels *(see Table 3.1 on the next page).* The questions were designed to evaluate learners' reasoning and conceptual understanding of geometry, particularly their attainment of van Hiele Levels (VHL) 0, 1, 2, and 3 as detailed below:

Questions 1-5 were structured to assess learners' understanding at van Hiele Level 0 (Visualization). At this stage, learners are required to recognise shapes, particularly quadrilaterals, by their visual characteristics rather than their properties.

Questions 6-10 focus on van Hiele Level 1 (Analysis), whereby learners commence the recognition and description of specific characteristics of quadrilaterals, including the number of sides and angles.
Questions 11-15 were designed to assess learners' thinking at van Hiele Level (VHL) 2 (informal Deduction). The questions necessitated that learners analyse the interrelations among several aspects of quadrilaterals, namely the correlation between the properties of a square and those of a rectangle.
Questions 16-20 were designed to assess learners' understanding at van Hiele Level 3 (Forma Deduction), wherein learners were expected to use logical thinking and establish connections among diverse geometric concepts.
The fifth level of the van Hiele Model (Rigor) was not considered, as it entails a level of abstract reasoning that is above the Grade 10 curriculum, in alignment with CAPS standards. The questions were modified from Ngirishi (2015) and Usiskin (1982)'s instruments, to fit with the (CAPS) expectations for Grade 10 learners in line with curricular requirements.

The learners' responses to the multiple-choice questions were eventually marked and assessed according to the van Hiele Model, to ascertain their level of geometric reasoning at different cognitive levels (refer to chapter 4).

Table 3.1: A description of how questions in questionnaire A were structured in line with the classification of the van Hiele Levels of geometric thinking

Questions (Qs)	Question type/Expected Reasoning level	Expected Operating van Hiele level (VHL)
Qs: 1-5	Identification/recognition of 2D shapes	level 0- Visual level
Qs: 6-10	Identification of 2D shapes Properties via analytical reasoning	level 1- Analysis level
Qs: 11-15	Identifying interrelationships between and among 2D shapes' properties	level 2- Order level
Qs: 16-20	Testing for geometric deductive proofs and differentiations with necessary and minimum conditions; (knowledge of proofs, based on awareness of the role of definitions, theorems and axioms.)	level 3- Deduction level

Questionnaire B had eight open-ended questions with four groups of two questions each. Each group was structured to assess learners' understanding of geometric ideas in accordance with the van Hiele Model of geometric reasoning. The questions were designed to assess learners' advancement through van Hiele Levels 0, 1, 2, and 3, with two questions structured in accordance with each level (refer to Table 3.2 below).

Questions related to Rigor level 5 were also not considered based on CAPS learning outcomes for grade 10. Hence, Level 5 of the model (Rigor), was skipped, just like in questionnaire A. The structure of questionnaire B was designed to gain more insight in terms of the possible and common misconceptions learners hold in geometric reasoning, based on their responses, according to the van Hiele levels of geometric thought (Bertram & Christiansen, 2014). This approach will provide an in-depth understanding of their reasoning skills, which is vital to actualise the main objective of the study.

The questionnaire, before being administered, was carefully reviewed to ensure that it was fit for purpose and suitable for the learners in line with curricular requirements for grade 10. Below is the summary of the objectives of questionnaire B, which has open-ended questions, followed by the tabular presentation of the same, highlighting the objectives of each question.

Questions 1 and 2 were designed to assess possible misconceptions they hold while attempting the questions, which is to visually recognise certain quadrilateral shapes, given sufficient reasons for their identification, and correctly name each shape based on visualization. Questions 3 and 4 were set to assess learners' analytical reasoning and identify possible misconceptions they hold in their reasoning process. Learners were required to recognise and articulate properties of quadrilaterals, by applying the same in proving given parallelogram shapes as parallelograms. In questions 5, and 6, learners were to apply their logical reasoning, to the relationships between properties of geometric shapes in solving the problems. Lastly, questions 7, and 8, were designed to assess learners' ability to evaluate their understanding of their formal geometric reasoning and identify possible misconceptions they hold as well. Table 3.2 shows the tabular presentation of the structure of the questions, aimed at identifying common misconceptions learners hold in their geometric reasoning.

Table 3.2: A description of how questions in questionnaire B were structured

Questions (Qs)	Question type/Expected Objectives
Qs 1-2	Leaners were expected to describe/identify shapes by appearance and name shapes/other geometric figures
Qs 3-4	Learners were required to identify equal angles between parallel lines with reasons.
Qs 5-6	Learners were required to know about the transitivity property and knowledge of relationships between the properties of shapes, equal angles of an isosceles triangle and vertically opposite angles.
Qs 7-8	Learners were required to use analytical means (calculations) for proof purposes

In addition to the eight (8) open-ended questions in questionnaire B, which were structured to assess the understanding of learners and identify possible and common misconceptions, 8 impromptu supplementary or conceptual assessment questions were presented to the participants. The eight (8) supplementary questions were integrated into questionnaire B to further provide qualitative diagnostic insights in relation to learners' perceptions, conceptual struggles, as well as foundational understanding of quadrilateral concepts and terms. Secondly, the responses from the supplementary questions were also intended to gain further insight in terms of their misconceptions, to strengthen and deepen the depth of the analytical aspect of questionnaire B, aimed at ensuring a comprehensive understanding of the geometric reasoning processes of the participants and the common misconceptions.

3.5.2 Semi-Structured Interviews

Semi-structured interviews were also employed as a data collection tool in this study. They are widely accepted instruments in qualitative research as they combine predetermined questions, at the same time providing flexibility that enables researchers to investigate participants' perspectives while simultaneously addressing certain study questions throughout the interview process (Cohen et al., 2011). This also aligns with the views of Gill et al. (2008) that semi-structured interviews allow researchers to obtain the depth of participants' experiences, rendering it a suitable strategy for collecting qualitative data on learners' understanding of difficult topics such as geometry. Kvale (2009) defines a semi-structured interview as “an interview aimed at acquiring descriptions of the interviewee’s life world to interpret the significance of the described phenomena.”

The total number of participants randomly selected for the semi-structured interviews increased by three (3), from the initially planned seven (7), according to the approved proposal. The researcher is also of the view that the addition will enrich the quality of the data as well.
The increase in the number of participants for the interview will enhance an in-depth assessment of participants’ understanding of quadrilaterals and identify common misconceptions, aimed at addressing the research questions. More participants to be interviewed will also enrich the study’s insights into the impact of differing levels of geometric reasoning on responses, just like in the case of the increase in the total number for the study, from 20 to 22.

Furthermore, allowing an additional number of interviewees would enhance the study’s credibility and trustworthiness, by providing a broader representation of the sample. Lastly, it will reflect the adaptable features of qualitative research, permitting adjustments to sample sizes to enhance data richness and facilitate comprehensive exploration of emerging themes. The interviews lasted for an average of 25 minutes for each participant, enabling the researcher to explore the learners’ understanding of quadrilaterals in Euclidean geometry with greater depth. This adaptability plays an important role in qualitative research, as it allows the interviewer to investigate the intriguing topics provided by participants (Taylor et al., 2015).

The semi-structured interview format was used to allow a balance between structured guidance and open-ended question (open-ended) formats. While the interviewer followed a list of predetermined questions, there was also flexibility to ask follow-up questions based on the learners’ responses. The approach allowed for an investigation of learners’ reasoning, misunderstandings, and cognitive processes related to geometric ideas.

In a study on qualitative health research, DeJonckheere and Vaughn (2019) explore how effective the use of a semi-structured interview instrument is with reference to quality and in-depth data collection. The view asserts that this method enables the researcher to further probe sensitive or complex topics to collect open-ended, rich data, by the exploration of the thoughts, beliefs, and lived experiences of the participants, in a flexible and guided manner. This approach aids the researcher in gaining a greater understanding of learners’ perspectives, in this case, the phenomenon being investigated.

This study, therefore, utilised interviews to provide the researcher with broad insights into learners' approaches to the challenges in the task-based worksheets. This conversation highlighted both the learners' understanding of concepts and the difficulties they had in understanding and applying geometric ideas. All interviews were recorded and fully transcribed for analysis, with pseudonyms allocated to participants to maintain confidentiality.

3.5.3 Pre-Data Collection Procedure: School Management Involvement

Before the data collection, the school management of the location of the study facilitated the mass printing of copies of the task-based questionnaires and administered them to the participants. The researcher was there during the assessment to ensure proper administration and to address any queries from the learners. The researcher was also accompanied by his research assistant.

During the data collection, the researcher observed that question number 7 in Questionnaire A was wrongly presented, hence, it was not considered across the board. Secondly, it was also noted that one participant's script was incomplete, with 12 out of 20 multiple-choice questions missing, and also in Questionnaire B, with another 3 questions missing (questions 3, 4 and 8), from the same participant's script. This disparity could have been attributed to possible neglect or omission in the printing process. Consequently, the participant's remaining (responses) data from both questionnaires A and B were excluded from the study. This is to ensure the trustworthiness (integrity and credibility) of the study was maintained all through.

Furthermore, it was observed and important to mention that Question 5 in Questionnaire B was also missing from the scripts of three (3) other participants, presumably due to errors in printing. These disparities highlight the difficulties encountered during data collection. The researcher was of the view that the omission of question 5 would not significantly affect the study's credibility, since the aim of Questionnaire B was mainly to identify the common misconceptions among learners. Besides, question 6 was also set at the same van Hiele level as question 5. The responses to question 6 were analysed to assess the 3 participants' level of understanding and misconceptions held as well.

Consequently, in view of the glitches encountered during data collection, out of 20 multiples questions in Questionnaire A, 19 questions were considered. Similarly, in terms of the total number of participants, data from 21 participants were eventually considered for the entire study, as opposed to 22 students who participated. Table 3.3 below provides a summary of the methods utilised in data collection in relation to the research questions.

Table 3.3: Data collection procedure in relation to the research questions and participants

Research Questions	Participants	Research Instruments
What are some of the factors that influence Grade 10 learners' understanding of quadrilaterals in Euclidean geometry?	Grade 10 Learners	Task-based Worksheet (Questionnaires A & B)
What are some misconceptions held by Grade 10 learners in relation to the concept of quadrilaterals?	Grade 10 Learners	Task-based Worksheet (Questionnaires A & B) Semi-Structured Interviews
What insight can we get about grade 10 learners' understanding of quadrilaterals in geometry in terms of the van Hiele levels (VHL) of geometry thought?	Grade 10 Learners	Task-based Worksheet (Questionnaire A only)

3.5.4 Data Organisation and Procedures for the Analysis

Data organisation is the systematic compilation and structuring of data for analysis, ensuring that the information is accessible and readily interpretable (Gibbs, 2012). Data for this study was collected by task-based questionnaires, semi-structured interviews, and additional note-taking during interviews. Interview recordings were transcribed and responses to questionnaires were compiled and anonymized with pseudonyms to protect participants' identities.

The data were subsequently categorised according to learners' responses, in line with the research questions, enabling efficient coding and analysis in accordance with the van Hiele geometric Model.

Creswell and Creswell (2017) view data analysis as a process, involving the systematic examination, categorization, and interpretation of data aimed at providing important insights in relation to the research questions. The analysis of the data aimed to identify patterns and emerging themes, that reflect learners' understanding of quadrilaterals. The inductive thematic analysis approach was guided by the principles of Braun and Clarke (2006), was used. Inductive analysis is described as "a process of making sense of field data by sorting data into categories to establish patterns and themes" (Clark & Creswell, 2008). The six-phase thematic analysis process of Braun and Clarke (2006), which was followed in this study, includes:

Familiarisation with the data: This first stage entailed the researcher thoroughly going through responses from the task-based questionnaires to understand the content, followed by going through the second data generated from the semi-structured interviews, aimed at connecting with the data.

Generating Initial codes: The second phase entailed going through the data again to identify significant information that would help the researcher to address the research questions. This information helped to initiate codes that represent that information. After getting in touch with the data, learners' geometric thinking-related important facts were labelled using initial codes.

Searching for themes: Codes were reviewed and grouped into broader themes that responded to the research's questions, concentrating on learners' approaches to and understanding of quadrilaterals.

Reviewing themes: the suggested themes were refined by reviewing how well the themes captured the data and ensuring that they correctly reflected the participants' responses, in relation to the research questions.

Defining and naming the themes: The themes were then defined and assigned specific names that captured the core of learners' understanding and misconceptions regarding quadrilaterals.

Writing up: The last stage of the analysis entailed correlating the themes with the study objectives and theoretical framework to explain the findings of the learners' geometric reasoning.

3.5.5 Task-Based Questionnaires Explained: A & B

The data obtained from the participants using task-based questionnaires (Questionnaire A and Questionnaire B) were carefully analysed, aimed at addressing the research questions. In order to properly present the performance trend of participants in an analytical form, Microsoft Excel, as an analysis tool, was used to put the data (scores) in a tabular form, organise the responses, and calculate scores.

Questionnaire A consisted of 20 multiple-choice questions (excluding one question), with one mark allocated to each question, totalling 29 marks (excluding one question). Eight open-ended questions totalling 80 marks, made up Questionnaire B. The marks were distributed according to the contents and nature of each question. In Questionnaire A (Multiple Choice Questions), learners' responses were marked as either correct 'C' or incorrect 'IC' since it was a multiple-choice kind of questionnaire. The basic dichotomous marking of questionnaire A provided an initial indication of learners' understanding of geometric concepts across various van Hiele's Levels.

Questionnaire A responses were recorded, and studied to understand the distribution of correct and incorrect answers in relation to different van Hiele levels. This step aimed at identifying patterns in participants' performance, including common errors, or areas of competence, which could provide insight into their understanding of geometric concepts and help in addressing the research questions in the subsequent chapter. The process of scoring agreed with the reasoning of learner-interpretation in a qualitative approach by linking the patterns of the learners' responses to the conceptual understanding of the van Hiele levels.

3.5.6 Questionnaire B (analysis of Open-Ended questions)

The analysis for questionnaire B was more comprehensive and rigorous since it entailed open-ended questions. Besides checking the solution to each question, the conceptual reasoning demonstrated, alongside the procedural steps undertaken by learners, was carefully captured. The emphasis was on identifying both the final response (answer) and the thinking steps and rationale utilised by learners to arrive at their answers. Common errors, misconceptions and unusual methodological approaches were captured to provide a more profound knowledge of learners' understanding and difficulties. This was essential for actualising the research objectives, since it showed how learners constructed geometric concepts and identified the difficulties they faced in the process.

3.5.7 Link to the Geometric Thinking Levels of van Hiele (VHL)

The van Hiele geometric thinking levels were then used for the analysis of Questionnaire A only. Every question was matched to one of the van Hiele levels (0 to 3), and the amount (level) of reasoning demonstrated by the participants in their responses was used to categorise them. This process provided the pattern of understanding among the learner group, indicating which topics or aspects of quadrilaterals were grasped well and where learners had difficulties.
(See Chapter 4)

3.5.8 Semi-Structured Interviews Analysis

Following the marking of the task-based questionnaires, ten (10) participants were later selected as described in the selection process and interviewed in a semi-structured manner for an average of 25 minutes each, to substantiate the information they had written. The interviews provided participants another opportunity to think through their responses to the task-based problems and to evaluate if their understanding of quadrilaterals had remained unchanged or improved. During the interviews, learners were allowed to clarify the most suitable approaches to the problems, hence, providing further insight into their understanding of quadrilaterals. Note-taking also took place (by the researcher's research assistant) during the interview, aimed at capturing key points in their responses, which also served as supplementary data. The interviews were audio recorded (in English, which was their preferred medium of engagement) and fully transcribed in English. Extensive use of probing questions was employed to get additional insight and confirm learners' understanding of concepts. This method enabled the researcher to delve deeper into participants' cognition. The interview transcripts and written responses were subsequently analysed to fully evaluate the learners' understanding of quadrilaterals (see Chapter 4).

3.6 Ethical Considerations

Every researcher is obligated to comply with ethical issues, as it is a concept that seeks to protect the rights of participants. It also guides data integrity, and improves the credibility of the study findings. Resnik (2018), states that ethical issues are the framework that ensures compliance in relation to a step-by-step procedure for providing informed consent, ensuring confidentiality among all intending participants, voluntary participation, and managing the risk factor.

Ethical issues are essential in research. It's a concept that protects the rights of participants, maintains data integrity, and enhances the credibility of findings. It entails compliance with guidelines that provide informed consent, confidentiality, voluntary participation, and risk reduction (Resnik, 2018). Ethical practices protect participants, foster trust, ensure adherence to statutory requirements, and avert harm, all of which are crucial to the integrity of the research process (Israel & Hay, 2006).

This study adhered to ethical standards before, during, and after data collection, in accordance with the University Code of Conduct as it relates to ethical matters. Before data collection, gatekeeper permission was obtained from the appropriate authorities (the ethical Committee), and informed consent was obtained from all participants and their legal guardians, clearly outlining the goal, procedures, and voluntary nature of participation (Cohen et al., 2002). *(Please see appendices B, C, D,& E)*

Participants' names were anonymised in line with the confidentiality clause throughout the study, prior to data collection, during, and post-data collection. Participants were informed of their right to withdraw from the study, at any time, during the course of the study. This also aligns with the ethical considerations. Confidentiality of information was also maintained in line with ethical standards. Safe-keeping of the generated data (in hard-copy format) is stored in a repository and afterwards destroyed after five years, alongside the data in digital form, discarded after five years. Participants were made aware of their right to view the material before the official publication of the project. Proper arrangements were put in place for the interviews to ensure the avoidance of any disturbance to the standard instructional routine of the participants.

3.6.1 The Trustworthiness of the study

In qualitative research, trustworthiness, credibility, transferability, dependability, and confirmability are crucial for demonstrating the study's validity, reliability and rigor (Lincoln & Guba, 1985). These parameters (as explained below) ensure that the findings appropriately reflect the participants' experiences and research context, providing an adequate foundation for data interpretations. Trustworthiness has to do with ensuring the integrity and reliability of the research methodology. It involves ensuring that the findings of the study are credible, authentic, and reliably represent the perspectives of the participants. This study attained the required credibility based on the methodological approach employed for data collection, analysis and interpretation of the data.

3.6.2 Credibility

The credibility of a qualitative study ensures that the findings of the study are a reflection of the lived experiences of the participants. The credibility of the study was enhanced by employing triangulation, which implies the use of multiple data collecting instruments; in this case, the use of questionnaires, structured in two different forms- A and A, and semi-structured interviews. Credibility was also enhanced by member checking, where participants were allowed to check how they performed, to confirm that the study's findings are a reflection of their lived experiences. This approach validated that the results aligned with the participants' experiences, therefore ensuring the data validity (Creswell & Poth, 2018).

3.6.3 Transferability

Transferability of any study focuses on whether its findings may apply in another environmental setting or different circumstances. Transferability was taken into cognizance, by making available clear and detailed descriptions of the circumstances in which the research was conducted, including the provision of the demographics of the participants. This would give room for any reader who would want to contextualise the findings in different circumstances, acknowledging the fact that though the findings of the study are situated in a particular location, however, the findings may provide relevant insights to similar situations (Guba & Lincoln, 1994; Lincoln, 1980).

3.6.4 Dependability

Dependability was attained, by having records of the trail in the course of the study, implying that records of steps, decisions and changes prior to and during the study are in place; promoting consistency and reliability of the methodology employed in the study. This is vital to ensuring dependability in qualitative research (Shenton, 2004).

3.6.5 Confirmability

Confirmability was achieved by the implementation of reflexivity approaches, wherein the researcher consistently mediated his biases and the possible effect on data interpretation. An audit train was maintained to ensure that the conclusions were based on the views of the participants rather than researcher bias (Guba & Lincoln, 1994).

3.7 Conclusion

This Chapter provided a detailed methodological approach, alongside the research design adopted in this qualitative study. The preferred paradigm was also discussed. The chapter further discussed the case study approach adopted, which is essential based on the phenomenon being investigated. The participants' selection procedure was also discussed, alongside challenges encountered during data collection. The methods and instruments employed for data collection were also discussed. Ethical considerations associated with qualitative research of this nature were also considered. The next chapter presents the findings of the study.

CHAPTER 4

RESEARCH FINDINGS

The preceding chapter outlined the methodological approach adopted. It entails the research design, data collection techniques, and ethical considerations relevant to this study.

This chapter presents the findings of the study; first, the overview of the overall results in Questionnaire A, based on the selected 21 participants for the study. This will be followed by the analysis of the items in Questionnaire A. The classification of participants in relation to the van Hiele level of geometric thought, based on Questionnaire A, will be presented and discussed. The overall results in Questionnaire B, will then be presented and discussed. The chapter will finally present the findings from participants' responses to Questionnaire B items.

4.1 Summary of results from questionnaires A (Multiple-choice questions- MCQ) and B (Open-Ended questions)

As highlighted in Chapter 3, Questionnaire A of the task-based problems comprised 20 Multiple-Choice Questions (MCQ). The 20 questions (*with the exclusion of question 7 due to an error found*), were divided into four subgroups, based on van Hiele's geometric thinking levels 0 (visualization) to 3 (Formal Deduction).

Table 4.1 on the next page provides Individual Performance in Questionnaire A, set at different van Hiele Levels and their marks, (*with the exclusion of one participant (P22) whose questions in both Questionnaires were incomplete.* The structure of Table 4.1 is explained below:

Demographics:

The first column (Ps') represents the participants, identified as, P1 to P21 *(Excluding a participant's entire data that were not considered in the study as earlier indicated)*

The second column ("Gender") represents the participant's respective gender

Questions:

The remaining columns, labelled Q1 to Q20, denote the 20 multiple-choice questions. *(Q7 was excluded due to an error found)*

The underlined letter beneath each question (Q) number indicates the correct option for that question. L0, L1, L2 and L3 connote the van Hiele Levels; each set of questions was set at Levels 0, 1, 2, and 3, respectively.

Participants Responses

Each cell under the question columns (Q1 to Q20) contains a letter representing the participant's selected answer; where 'C' In black colour refers to as 'Correct', and 'IC' in red colour denotes 'Incorrect '.

Question marks ('?') denote, 'question' not attempted by the participant.

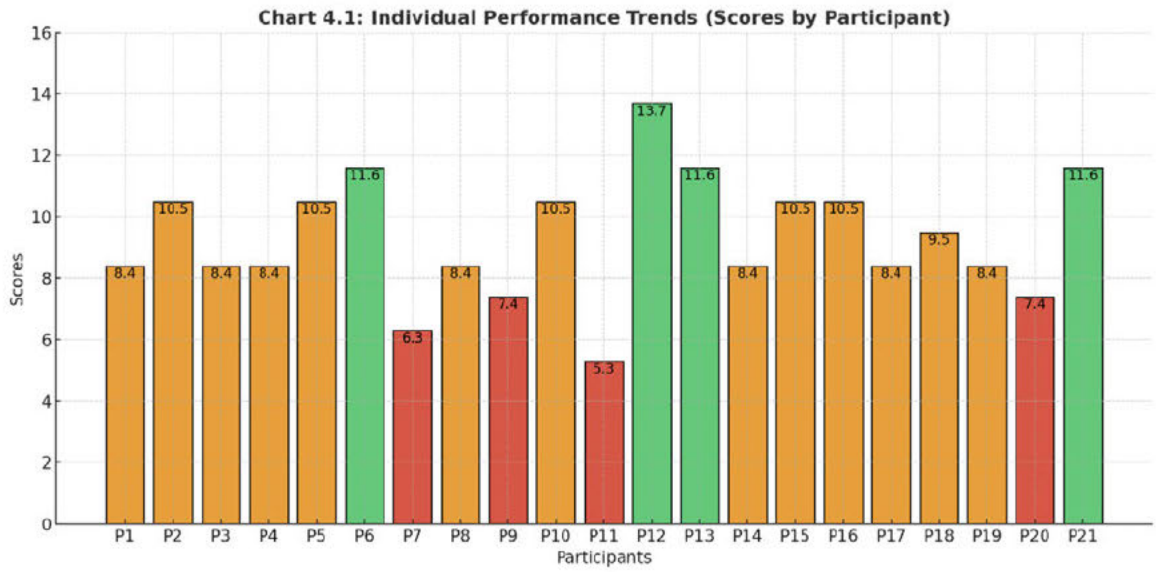
Table 4.1: Overall Individual Performance in Questionnaire A and their marks.

Ps'	Gender	Q 1 C	Q 2 B	Q 3 C	Q 4 D	Q 5 B	Q 6 B	Q 8 B	Q 9 C	Q10 E	Q11 E	Q12 A	Q13 B	Q14 D	Q15 D	Q16 D	Q17 B	Q18 B	Q19 B	Q 20 C	Out of 100 %
		L0	L0	L0	L0	L0	L1	L1	L1	L1	L2	L2	L2	L2	L2	L3	L3	L3	L3	L3	
P1	F	C	C	IC	C	IC	IC	C	IC	C	IC	IC	IC	C	IC	IC	C	IC	C	IC	42.1
P2	M	C	IC	C	C	C	C	C	IC	C	C	IC	IC	IC	IC	IC	C	IC	C	IC	52.7
P3	M	C	C	IC	C	C	C	C	C	?	IC	?	IC	IC	IC	IC	IC	IC	C	IC	42.1
P4	M	C	C	C	C	C	IC	C	IC	C	C	IC	IC	IC	IC	IC	IC	IC	IC	IC	42.1
P5	M	C	C	C	C	IC	C	C	C	C	IC	IC	IC	IC	C	IC	C	IC	IC	IC	52.6
P6	M	C	C	C	C	C	C	C	C	IC	IC	C	IC	IC	C	IC	IC	IC	IC	C	57.9
P7	F	IC	IC	C	C	IC	IC	IC	IC	C	IC	IC	IC	C	IC	IC	IC	C	C	IC	31.6
P8	F	C	C	C	C	?	IC	C	C	?	?	IC	C	IC	?	IC	C	IC	IC	IC	42.1
P9	F	C	C	C	C	IC	IC	C	IC	C	IC	IC	IC	IC	IC	IC	IC	C	IC	IC	36.8
P10	F	C	C	C	C	IC	IC	IC	C	C	C	C	IC	C	IC	IC	IC	C	IC	IC	52.6
P11	F	C	C	C	C	IC	IC	?	IC	C	IC	?	?	?	?	?	?	?	?	?	26.3
P12	F	C	C	C	C	C	C	C	C	IC	IC	C	IC	C	IC	IC	C	IC	C	C	68.4
P13	F	C	C	C	C	IC	IC	C	IC	IC	IC	C	IC	C	C	C	IC	?	C	C	57.9
P14	F	C	C	C	C	C	IC	C	C	IC	IC	C	IC	IC	IC	IC	IC	IC	IC	IC	42.1
P15	M	C	C	C	C	C	IC	C	C	C	C	IC	IC	IC	C	IC	IC	IC	IC	IC	52.6
P16	F	C	C	C	C	C	IC	C	IC	C	C	C	IC	IC	IC	C	?	IC	IC	IC	52.6
P17	M	C	C	C	C	IC	IC	C	IC	IC	IC	IC	IC	C	C	IC	IC	IC	C	IC	42.1
P18	M	C	C	IC	C	IC	?	C	C	IC	IC	IC	C	C	IC	IC	C	IC	C	IC	47.4
P19	F	IC	C	C	C	C	IC	C	IC	IC	IC	IC	IC	C	C	IC	IC	IC	IC	C	42.1
P20	F	C	C	C	C	IC	IC	C	IC	IC	C	IC	IC	IC	IC	IC	C	IC	IC	IC	36.8
P21	~~M~~	C	C	C	C	IC	C	C	IC	C	IC	IC	IC	C	C	IC	C	IC	C	IC	57.9

Figure 4.1 below shows the visual representation of the individual Performance Trends alongside their scores. Three colors are used to represent different performance ranges: Red bars represent learners with scores ≤ 7.49 (39.42%); Orange bars denote scores between

between 7.5 (39.47%) and 10.95 (57.63%); whereas green bars denote scores $\geq$ 10.99 (57.84%).

Figure 4.1: Individual Performance Trends (Scores by Participant)



Going further in terms of the items set within specific van Hiele levels, Table 4.2 below shows the number of responses for each item, for the 21 participants. (*with the exclusion of one participant whose data were not considered, due to omission of a significant number of items in both Questionnaires of the script, as previously mentioned).*

Table 4.2: Summary of Group Performance and Responses Across VHLs, in Questionnaire A.

QUESTIONS (Qs)	C (Correct)	IC (Incorrect)	No Attempt	Percentage of correct answers for each item (*n*=21 Respondents)	General Performance Comments based on VHLs.
	Qs 1 to 5:	Set at VHL 0			
Q1	19	2	0	*90.48*	*High responses to shape recognition*

Q2	19	2	0	*90.48*	*High responses on shape recognition*
Q3	18	3	0	*85.71*	*High responses on shape recognition*
Q4	21	0	0	*100.00*	*Highest responses on shape recognition*
Q5	9	11	1	*42.86*	*Poor responses on shape recognition*
	Qs 6 to 10:	Set at VHL 1			
Q6	6	14	1	*28.57*	*Poor responses on the knowledge of shape properties*
Q8	18	2	1	*85.71*	*High responses on the knowledge of shape properties*
Q9	9	12	0	*42.86*	*low responses on the knowledge of shape properties*
Q10	11	8	2	*52.38*	*Moderate responses on knowledge of shape properties*
	Qs 11 to 15	Set at VHL 2			
Q11	<u>6</u>	<u>14</u>	1	*28.57*	*Poor responses on the knowledge of making connection between properties*
Q12	6	13	2	*28.57*	*Poor responses on the knowledge of making connection between properties*
Q13	2	18	1	*9.52*	*Very Poor responses on the knowledge of making connections between properties*
Q14	9	11	1	*42.86*	*Average responses on knowledge, making a connection between properties*
Q15	7	12	2	*33.33*	*poor responses on the knowledge of making connections between properties*
	Qs 16 to 20	Set at VHL 3			
Q16	2	18	1	*9.52.*	*Very poor responses on formal geometric reasoning*
Q17	8	11	2	*38.10*	*Poor responses on formal geometric reasoning*

Q18	3	16	2	*14.29*	*Very poor responses on formal geometric reasoning*
Q19	9	10	2	*42.86*	*Average responses on formal geometric reasoning*
Q20	4	16	1	*19.05*	*Very poor responses on formal geometric reasoning*

Figure 4.2 below provides the visual presentation version of the group performance of the items and their responses, set within specific VHLs, based on questionnaire A.

Figure 4.2: Visual Presentation of the Group Performance in Percentages

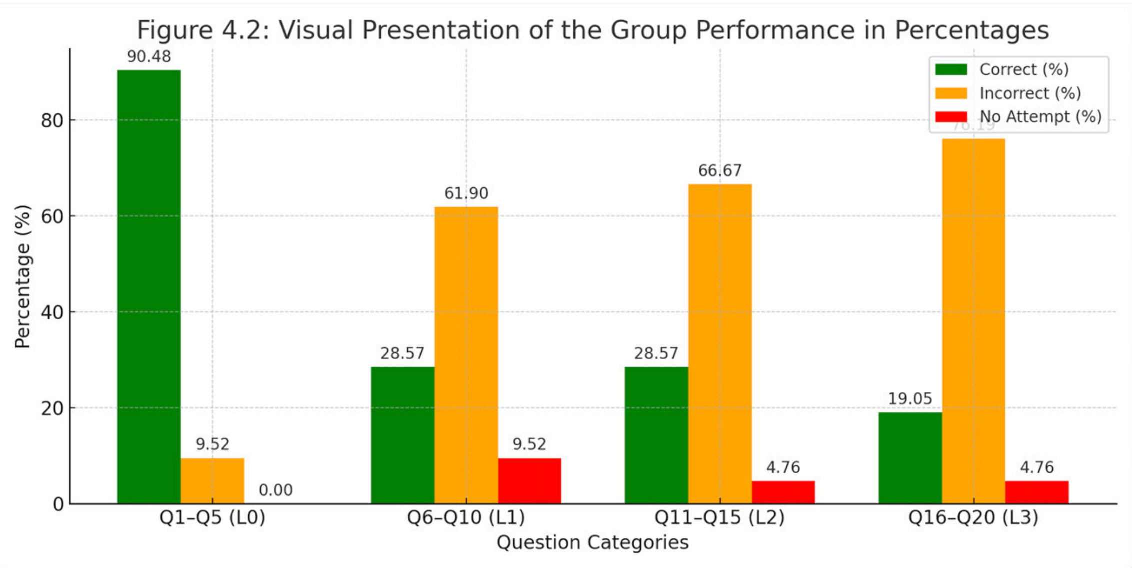
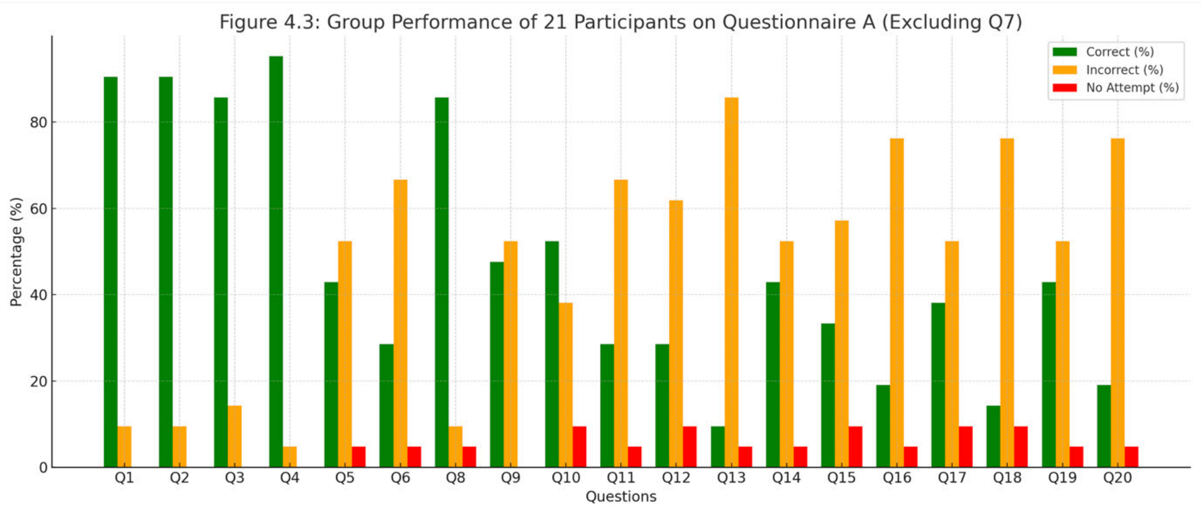


Figure 4.3 shows the visual presentation of the Performance of Participants on Questionnaire A, showing the percentages of ‘correct’ and ‘incorrect’ answers, as well as ‘no attempt’ for each question. This also aids in identifying areas where learners performed well and areas where they struggled.

Figure 4.3: Group Performance of 21 Participants on Questionnaire A



The data in tables 4.1 and 4.2 reveal high responses at questions set at the visualization level, aimed at assessing participants' ability to recognise quadrilateral shapes through visualization, as shown by the high correct response rates. However, question 5 showed a slightly below-average performance based on the percentage of correct answers recorded. A significant decline, as evidenced in the 'correct response rate' for the remaining questions (Q6 to Q20), was observed. The set of questions with the lowest marks occurred in the last set of five questions (Q16 to Q20), where formal deductive reasoning was assessed, which corresponds to level 3 of the van Hiele geometric model. Most learners seem to have struggled to attain level 3 of VHL.

Below provides further analysis of each question (in **Questionnaire A**), supported by responses from either the interviews or responses from the supplementary questions, where possible.

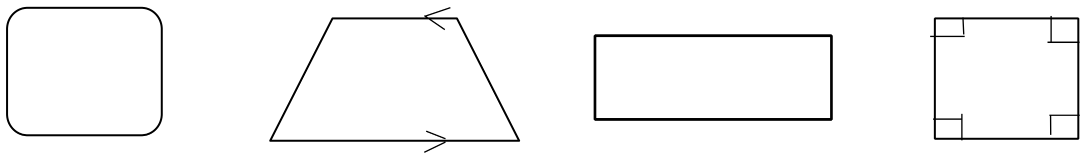
At van Hiele level 0 (Questions 1 to 5), learners were required to recognise shapes based on visual characteristics.

4.1.1 Questionnaire A and Analysis of Learners’ Responses

Question 1:

Which of these is a trapezium?

(a) (b) (c) (d)



None of these are squares

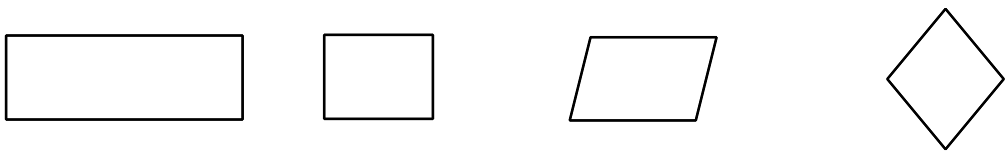
a. d only
b. b only
c. b and d only
d. All are squares

In question 1, responses were high, as evidenced by the high percentage of 90.48% (19 learners). Most learners demonstrated strong recognition skills by visual appearance, which is a requirement at this level. Specifically, most learners were able to easily recognise a trapezium by visual appearance. This could be attributed to the fact that trapeziums, just like squares and rectangles, are common quadrilateral shapes learners are exposed to, even outside the classroom in various forms like objects, signs. Secondly, the easy identification could also be based on the fact that the shapes are isolated; they are not integrated within another quadrilateral shape, which would have probably made it difficult for some of them, like in question 2 of questionnaire B. This familiarity could strengthen learners’ accuracy of recognition even without formal geometric understanding.

Question 2:

Which of these are rectangles?

(a) (b) (c) (d)



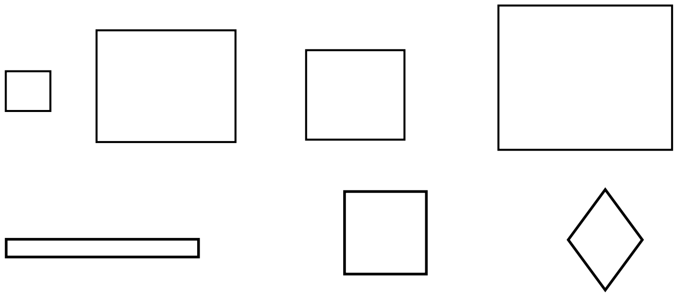
None of these are rectangles

a. a only-
b. a and b only
c. b and d only
d. All are triangles

In *question 2*, responses were also high, with the same percentage of 90.48% (19 learners), indicating a strong demonstration of recognition skills. The high percentage could also have been attributed to learners' familiarity with visual attributes of quadrilateral shapes and frequent exposure to familiar quadrilateral shapes, such as, rectangles and squares, in their daily lives. Similarly, a visualization-based question in questionnaire B, also indicated that most participants easily identified shapes, like squares and rectangles. Reliance on visual appearance is evident at this level, rather than analytical reasoning. The responses show that most learners operate above the visualization level.

Question 3:

The figures below are examples of a figure called a square.



Q R V

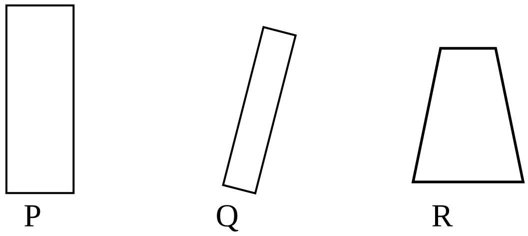
Which of these shapes can be called rectangles?

(a) V only
(b) Q only
(c) R only-
(d) All are squares
(e) R and V only

In *question 3*, learners were required to identify a square, based on specific visual features of a square. Correct responses were also high with a percentage of 85.71% (18 learners), although slightly less than the percentages in questions 1 and 2. The participants relied on visual indications due to their ease. For instance, P6 was able to visually differentiate between a rectangle and a square. Most learners also demonstrated the required visual geometric reasoning, as evidenced by the high response rate to this question, just like in questions 1 and 2. The high score on question 3 could also be linked to learners' reliance on visual prototypes, which aligns with visualization reasoning. The remaining 3 participants who marked option 'e' as the reason for selection, may have based their answer on the assumption of shape 'V' as a square, without taking cognizance of hierarchical class inclusion of quadrilateral shapes. This reflects confusion with similar shapes, and reliance on visual prototypes.

Question 4:

Which of these shapes can be called rectangles?



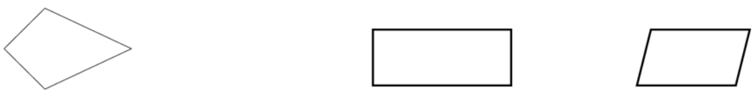
All can be called rectangles

a. Q only
b. R only
c. P and Q only-
d. Q and R only

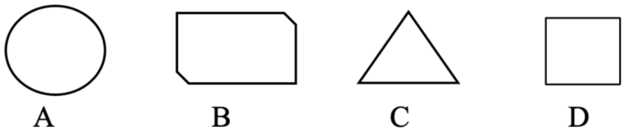
In *question 4,* which was also set at level 0, all learners easily and correctly identified the two rectangular shapes. One of the rectangles is in a prototyped position, whereas the other is in a 'slanted' positioning, yet all of them were able to identify it as a rectangle. The high correct responses, as indicated by the 100% response, could also be attributed to the simplicity of the shape, which also confirms learners' proficiency in identifying familiar and simple quadrilateral shapes. The proficiency of the visualization level is significant.

Question 5:

These are examples of figures called quadrilaterals:



Which of these figures below are quadrilaterals?



a. A
b. D
c. B
d. B and D
e. D and C

There was a significant decline in *question 5,* based on the correct response rate of 42.86% (9 learners). This was a concern, as item 5 was also set at visualization level. Learners were required to identify quadrilateral shape(s) among the four listed shapes, of which three (3) of the shapes are not quadrilateral shapes. A total of 10 learners incorrectly selected either option 'c' or 'd' as their answer. This implies that almost half of the participants mistook shape B, which is a rounded rectangle as a quadrilateral, probably as a (rectangular shape). This reflects misconception, which also makes it difficult for such learners to advance to van Hiele Level 1 (Analysis). It also reveals confusion about their understanding of what defines a quadrilateral, as a four straight-sided polygon. Shape like a rounded rectangle does not fall under that criterion. Secondly, visual prototype of shapes also seems to have dominated the reasoning of some learners, hence, they struggle to correctly identify the only quadrilateral shape, which is figure D- a square.

In general, most learners (at visualization level) correctly identified the required quadrilateral shapes, implying strong recognition skills at the visualization level, which is foundational in geometric reasoning. Below were some of the relevant comments during the interviews and some responses to the supplementary questions the learners were asked:

P16: *"I know a quadrilateral has four sides… and there are different types of quadrilaterals… parallelogram, square, rhombus, rectangles, and trapezium."*

"A quad is defined by 4 vertices and 4 sides."

P15: "*A quad shape consists of 4 sides. Interior angles of a quad add up to 360 degrees."*

P6: *"You will know a quadrilateral shape by proving that opposite sides are equal."*

P12: *"it has 4 vertices and 4 sides."*

From the above responses, learners' understanding of "4 vertices and 4 sides" helped them to correctly identify various quadrilateral shapes at the visualization level. Secondly, their knowledge of "four sides," "angles summing to 360^0 or "parallel sides," also strengthened their visualization knowledge of recognition of shapes.

At van Hiele Level 1 (Questions 6 to 10), learners were required to demonstrate their analytical reasoning in identifying properties of shapes, which is beyond the visual recognition level. Below provides further analysis of each of the questions set at the analysis level.

Question 6:

What properties do all rectangles have that some parallelograms do not have?

a. Opposite sides equal
b. Diagonals equal-
c. Opposite sides parallel
d. Opposite angles equal
e. None of (a) –(d)

In *question 6,* the analytical reasoning ability of learners was assessed through this item. Learners were to exhibit an understanding of geometric relationships among quadrilateral shapes, which define each of the quadrilateral shapes, through the identification of quadrilateral properties, such as equality of sides, angles' types, as well as diagonal properties. The low correct response rate of 28.57% (6 learners) could suggest that several learners were probably unable to apply the required reasoning in differentiating a rectangular shape from a parallelogram shape, using their defining properties. The way the question was set could have also confused them. In any case, there seems to be a lack of relational understanding of shapes among most learners, which would have assisted learners in connecting and differentiating shapes with their defining properties, beyond visual similarities.

The frequency of selection of incorrect responses recorded indicated prevailing misconceptions about the equality of sides and angles, as well as the parallelism factor in most quadrilateral shapes. 4 learners from the group of incorrect responses selected option 'e', (None of a-e), indicating that none of the options apply. In other words, none of the 4 quadrilateral properties (listed options) applies to a rectangular shape. This is an indication of a lack of understanding of the properties of either all or most quadrilateral shapes, particularly the defining property of a rectangle. Another 6 learners from the group of incorrect responses indicated option a (Opposite sides equal) as the correct answer, while another 4 selected option ‘c’ as their correct answer. This also shows confusion among some learners in the proper differentiation of properties of shapes. They forgot that properties, such as, ‘opposite sides equal’ and ‘opposite sides parallel’ also apply to both a rectangle and a parallelogram. One of the learners did not attempt to select any of the options, which also shows a lack of understanding of the defining property of a rectangle.

Below were some of their responses from the interviews;

P13: “*I can identify shapes, but when you ask for reasons or proof, that’s where I get stuck.”*

P13’s response shows a struggle transitioning to analysis level (Level 1), which requires analytical reasoning and understanding of properties

P15: *“I have to learn the basics of the quads and all their properties and also attempt various questions.”*

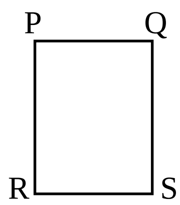
The response from P15 indicates admission of having difficulty with analysing and applying the properties of shapes, as required in question 6.

(Question 7 was omitted due to an error as previously indicated.)

Question 8:

PQRS is a rectangle.

Which relationship is true in all rectangles?



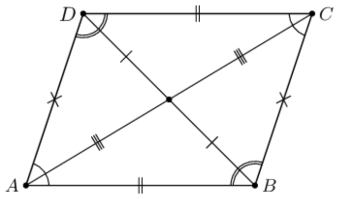
a. $\overline{PR}$ and $\overline{RS}$ have the same length.
b. $\overline{PQ}$ and $\overline{SR}$ have the same length-
c. $\overline{PS}$ and $\overline{QR}$ are perpendicular.
d. $\overline{PS}$ and $\overline{QS}$ have the same length.
e. $\overline{QS}$ and $\overline{PR}$ are perpendicular

In *question 8*, a high number of learners got the answer correctly, as evidenced by the percentage of the correct answer recorded, 85.71% (18 learners). Unlike in question 6, which was a bit tactical, question 8 appears to be a straightforward question that could have contributed to the correct response rate. The simplicity of the question, and the type of shape that learners are exposed to in their everyday life, may have also contributed to the high score. Secondly, the question primarily tested the learners’ knowledge of well-defined properties, such as diagonal, sides, and angles, which makes it the easiest question compared to other questions set at the analysis level. For instance, P6 demonstrated a clear understanding of the properties of a diagonal, which could have contributed to getting the answer correct:

P6: *"Diagonals of rectangles are equal, and those of parallelograms bisect each other."*

Question 9:

If ABCD is a parallelogram, which of the following statements is true about a parallelogram?



Triangle ABD is congruent to triangle ABC

The diagonals are congruent.

Both pairs of opposite sides are equal in length-

The perimeter of ABCD is four times the length of DB

AC is the same length as BD

In *question 9,* learners were required to identify the statement among the given options that only applies to a parallelogram. 42.86% (9 learners) identified the correct option. 12 learners were unable to correctly apply the knowledge of properties of a parallelogram; hence the 12 learners selected option '*a* (*Triangle ABD is congruent to triangle ABC*) as their correct answer, implying a fundamental misunderstanding of geometric concepts of shapes within quadrilateral shapes, such as a parallelogram. Reliance on visual appearance may have also contributed to the low performance, instead of the application of analytical reasoning, in this case, using the geometric properties in the verification of congruency. Gaps in awareness of hierarchical relationships within quadrilaterals and logical reasoning may have equally contributed to the performance.

Question 10:

Consider the following properties of a quadrilateral:

1. Opposite sides of one pair are parallel
2. The pair of opposite sides that are known are also equal in sides

3. No information is available about the other pair of sides

For which type (of types) of figure have such properties as always true?

a. Square
b. Kite
c. Parallelogram
d. Rectangle
e. a, c and d-

In *question 10,* learners' ability to identify quadrilateral shapes that share certain common properties was tested. 11 learners responded correctly, exhibiting a good understanding of the hierarchical class of quadrilateral shapes sharing common properties, such as square, parallelogram, and rectangle. However, 8 learners did not get the correct option. 2 learners selected option 'a', which is incorrect. 3 participants from the same 8 participants selected option (b), which is also incorrect, while 2 learners incorrectly selected option (c). The remaining 2 learners did not attempt the item. This shows a struggle among the 10 learners to identify shapes that share certain common properties based on class inclusion. There still seems to be a need for learners to be acquainted with quadrilateral definitions and the quadrilateral hierarchical classification of shapes. Hence, some learners confuse shapes that are similar, and are unable to apply the concept of hierarchical order of classification of shapes in their analytical reasoning process.

Van Hiele level 2 (Questions 11 to 15); learners were required to use informal deductive reasoning to make connections between properties of shapes.

Question 11:

Which of these is true?

a. All properties of squares are properties of all parallelograms
b. All properties of a parallelogram are properties of all rhombuses
c. All properties of squares are properties of all rectangles
d. All properties of rectangles are properties of all parallelograms
e. None of the (a)-(d) is true

Question 11 seems to be one of the trickiest questions among the items set within level 2, which requires informal deductive reasoning ability. Only 6 (28.57%) learners out of 21 learners,

correctly identified the correct option, which is (e). This implies significant evidence of struggle among learners in making connections between properties of shapes. It also suggests that learners struggle to operate at an informal deductive level, which is level 2, which requires logical connections and understanding of hierarchical relationships existing among quadrilateral shapes.
The selection of option (b) by 5 learners, seems to suggest that a rhombus inherits all the properties associated with a parallelogram, without additional defining properties, which is incorrect. It is a misconception. A rhombus has additional defining properties that do not apply to all parallelograms, such as, equality of all four sides, and perpendicularity of the diagonals. So, option (b) is incorrect. Learners must have assumed this selection, without proper differentiation between general and unique properties among the shapes.

The same applies to option ‘d’, which was selected by another 5 learners, reflecting another misconception regarding the two shapes- rectangles and parallelograms. Although a rectangular shape satisfies the properties of a parallelogram at a general level, such as, equality of opposite sides and parallelism. However, the reverse is not the case, as may have been assumed by the learners. Parallelograms do not have the defining properties of rectangles, such as, equality of right angles for all four angles and congruency of diagonals. Hence, this option is also incorrect.

The 4 learners who selected one of the options, (a) or (c) seem to have made their decisions of selection based on either using their visual reasoning or based on viewing the properties of each of the pair of shapes, in isolation, in this case, ‘squares and ‘parallelogram’, for option (a), as well as ‘squares’ and ‘rectangles’, for option (c). They did not apply the required relational reasoning. They misinterpreted the unique and inter-relational characteristic concepts among the shapes. For instance, the 2 learners who selected option (c) mistook all the properties of squares as applicable to rectangles, forgetting the defining property of a square, which is equality of all sides. Similarly, the other 2 learners who selected option (a), seem to have assumed that all properties of squares are the same of parallelograms, without recognising the defining properties of squares that differentiate them from parallelograms, such as equality of all four sides and the perpendicular nature of the diagonals. So, the only available and valid correct option is option (e): None of the (a)-(d) is true.

The observations, both from the interviews and responses from the supplementary questions, highlight misconceptions learners have regarding class inclusion relationships. There is an assumption by learners that the properties of shapes apply in both ways. For instance, *"if squares are parallelograms, parallelograms must also have all the properties of squares*". This seems to explain why 10 of the learners opted for either option (d) or (b).

Question 12:

All rectangular shapes have the following properties in common with properties of all parallelograms except;

a. Diagonals are congruent-
b. Opposite angles are congruent
c. The opposite sides are congruent
d. The diagonals bisect each other
e. None of the (a) –(d)

In *Question 12,* learners were required to differentiate rectangles from parallelograms with a defining property applicable only to rectangles, 'congruency of diagonals.' Only 6 learners identified the correct option, (a)- 'Diagonals are congruent', which does not hold for parallelograms. The low correct response rate of 28.57% shows a lack of understanding of the defining property of rectangles. 7 learners, (33.33%), incorrectly selected option (d), "diagonals bisect each other," supposedly by assumption, revealing confusion with regards the defining or differentiating properties of parallelograms and their relationship to rectangles. Their selection of option (d) also seems to suggest a possible misconception that rectangles have 'diagonals bisect each other' as a unique rectangular property, whereas, it is a property that applies to all parallelograms, including rectangles.

This misconception seems to stem from a lack of understanding of the classification of quadrilateral shapes, as they struggle to recognise and understand hierarchical relationships between shapes. Hence, the confusion and assumption, without substantial informal deductive reasoning application. The same applies to another 2 learners who selected option (b), 'opposite angles are congruent', assuming such property as the defining property that applies only to rectangles, whereas, both rectangles and parallelograms have that property in common.

Another learner, who selected option (c), made his/her selection possibly based on similar misunderstandings, taking such property as a property unique to rectangles only. The selection of option (e) by 3 other learners still points to the fact that learners seem uncertain or do not have the required informal deductive reasoning level to determine the defining or unique property of rectangles in this instance. The remaining 2 learners who did not attempt the item possibly demonstrated a lack of confidence or a lack of understanding of the topic in its entirety. The performance trends in this item suggest that most learners still operate below the informal deductive reasoning, which is an important requirement to advance to a higher van Hiele's level. This is based on misconceptions regarding diagonals, as evidenced by the frequency of selection of the wrong options and limited relational reasoning. They also seem to have difficulty with statements that involve diagonals, assuming all shapes in the class of parallelograms have identical characteristics.

Question 13:

What type of a figure can be called both a rectangle and a rhombus?

a. Rhombus
b. Square-
c. Rectangle
d. No figure
e. Parallelogram

Question 13 performance was poor, based on the low correct response rate of 9.52%, (2 learners), who identified the correct option (b)- square, being a figure that can be called both a rectangle and a rhombus, or a figure whose properties satisfy both a rhombus and a rectangle. The poor performance reveals that learners encounter challenges in understanding the concept of quadrilateral hierarchical relationships.

This is also evidenced by a significant number of repeated selections of the incorrect option (d) "No figure" as their answer, by 10 learners, (47.62%). It suggests that the learners may not be aware that a square satisfies the properties for both a rhombus (with equality of all sides) and a rectangle (with equality of all angles, 90^0). Furthermore, this reconfirms how learners confuse and overlap the characteristic definitions in the quadrilateral hierarchy, as a result of a lack of informal deductive reasoning that aids in logically deducing relationships, consequently, operating at van Hiele level 2 appears to be a struggle.

Similarly, another 8 learners who incorrectly selected option (e) also reflect a fundamental misunderstanding, as parallelograms do not meet certain required unique characteristic criteria for both a rectangle and a rhombus. No selection of either option (a) or (c) was recorded as answers. Although their possible reason(s) for ruling out these 2 options may not be based on their knowledge of geometric principles. It could have been based on the fact that since the question wanted learners to identify the figure that satisfies the properties of both a rectangle and a rhombus, logically, the right option couldn't have been any of the 2 options; (a) 'rhombus and (c) 'rectangle.

Overall, the performance in question 13 reveals misconceptions regarding relationships in the quadrilateral hierarchy. The number of learners who selected options (d and e) also shows deficiencies in informal deductive reasoning, as most learners had difficulty in deducing existing relationships between categories of quadrilateral shapes. Most of the learners may have selected their options based on looking at these quadrilateral shapes as unrelated, based on their properties, instead of applying the needed relational reasoning, which is an important and required skill for advancing to a higher level of the van Hiele model of geometric thought- Level 3 (formal Deduction).

Question 14:

Consider a four-sided figure shape PQRS, diagonal PR bisecting angle SPQ.
Given PQ =PS= 10. What type of figure is that?

a. Square
b. Rhombus
c. Rectangle
d. Either A or B
e. Not enough information to determine this

Question 14, the correct response rate was 42.86% (9 learners), who identified the correct option- 'Either A or B'. learners who identified the correct answer could have recognised that any figure, whose adjacent sides are equal, and with diagonals bisecting angles, can be either a rhombus or a square. So, the 9 learners must have demonstrated a good level of understanding of the properties of quadrilaterals, especially about the behaviour of the diagonal. Another 8 learners (38.10%) had the notion that there was not enough information to determine this', hence they incorrectly selected option (e), showing a lack of understanding of how specific quadrilaterals are determined through diagonal behaviours and properties of sides. They also exhibited geometric reasoning difficulty regarding connecting conditions given (based on the question) to geometric relationships. A few learners selected option (b), taking a rhombus as

the only figure that satisfies the given condition, not knowing that even a square also meets the given condition.
The reasoning gaps underlined based on the wrong selection of options, suggest that learners may have challenges with deducing relationships when it comes to questions involving multiple conditions, such as side lengths, and the behaviour of diagonals. Also, the misconceptions regarding the defining characteristics of quadrilaterals, as shown in the previous items, are still evident, especially, the interrelating properties between rhombus and squares.

The performance in general on question 14 reveals reasoning challenges at the informal deduction Level, as evidenced by the struggle learners faced in relation to combining the properties of diagonals (in bisecting angles), alongside the equality of sides, in an attempt to deduce the classification of figures. Misconceptions regarding the properties of quadrilaterals were also observed, based on the incorrect selection of wrong options. Lack of confidence in combining geometric information was also noted, which is critical for advancing to formal deduction- van Hiele Level 3.

Question 15:

Here are two statements.
Statement 1: Figure P is a parallelogram
Statement 2: Figure R is a rectangle

Which one of these is wrong?

a. If opposite angles in Figure P are congruent, then the opposite angles in Figure R are also congruent
b. If the diagonals of figure P bisect each other, then diagonals of Figure R also bisect each other
c. If the opposite sides in Figure P are congruent, then the opposite sides of Figure R are also congruent
d. If the diagonals of figure P are not equal, then the diagonals of figure R also should not be equal-
e. None of (a) – (d) is wrong

In question 15, being the last question, set at an informal Deductive level 3, learners were required, using logical deductive reasoning, to compare and analyse the properties of two (2) given figures: Figure P and Figure R, a parallelogram and a rectangle, respectively, and eventually recognise the correct option with a logical inconsistency. To identify this inconsistency logically, learners were expected to apply an informal deductive reasoning in order to deduce relationships and exceptions among the listed options.

Learners needed to understand the trickiness of the question, about the diagonal behavior or difference (in terms of their diagonal properties), between parallelograms (bisection of diagonals, with inequality of diagonals) and rectangles (having equality of diagonals). Some learners did not take this into cognizance, hence the selection of incorrect options. For instance, 19.05%, (4 learners) selected option (b), probably based on the overgeneralisation of properties both shapes share, without taking into cognizance the diagonal differences between the two shapes. So, misconceptions regarding diagonal properties were apparent.

Although the question may not have appeared to be complex, however, it put learners to task, to be able to distinguish between the properties parallelograms and rectangles share in common, as well as their defining property, in this case, the diagonal property difference. Learners who have yet to develop relational reasoning, that is, the ability to compare, contrast, and integrate information effectually, may struggle to fully understand the intricacy or the abstract nature of such a question. Without this skill, identifying the correct option (answer) would be difficult, as learners would focus on the shapes in an isolated manner, instead of looking at the geometric interactive nature of the two shapes.

This is evident by the low correct response rate of 33.3%, (7 learners), who selected the correct option, (d). It implies that the 7 learners were able to identify the logical inconsistency, which is parallelograms, having diagonals that bisect each other, but with inequality of diagonals in lengths, unlike in rectangles with equality of diagonals, in addition to diagonals bisecting each other. This probably explains why many learners, (accounting for 57.14%), did not get the correct option (answer), because of the misleading similarity between options (a), (b), and (e), as a result of not recognizing the distinctive diagonal property associated with rectangles. This shows a gap in grasping the hierarchical relationships existing among quadrilaterals. 3 learners who selected option (a) must have assumed the congruence of opposite angles, distinguish between parallelograms and rectangles, whereas 'parallelograms' and 'rectangles' have that property in common. The 3 learners who selected option (e) must have exhibited no confidence or uncertainty about any of the rest of the options, to avoidance of selecting a particular incorrect statement, seemingly.

In general, the low response to the correct option shows that learners have difficulty in reasoning at van Hiele level 2, based on difficulty in logical reasoning, confusion regarding the properties parallelograms and rectangles share in common, and their respective unique properties. There was also a tendency towards overgeneralisation of properties among the 2 shapes, parallelograms and rectangles, without taking cognizance of one of the unique properties that exempts them, which is the diagonal property.

Questions 16 to 20 were set at Van Hiele Level 2 (formal deduction), primarily to assess the ability of learners to transition their reasoning from informal deductive reasoning to formal deductive reasoning.

This specifically, implies possessing knowledge of geometric proofs, counterexamples and logical implications. It also entails the ability to apply formal logic to establish or disprove relationships between geometric properties, as well as a demonstration of mastery of higher-level geometric thinking, beyond recognition or relational reasoning. The set of questions was also aimed at testing the understanding of learners' knowledge of how geometric properties apply across different quadrilaterals, such as parallelograms, rhombuses, and rectangles.

Question 16:

Here are two statements.
If a figure is a rectangle, then its diagonals bisect each other
If the diagonals of a figure bisect each other, the figure is a rectangle

Which is correct?

a. To prove i is true, it is enough to prove that ii is true
b. To prove ii is true, it is enough to prove that i is true
c. To prove ii is true, it is enough to find our rectangle whose diagonals bisect each other
d. To prove ii is false, ii is enough to find one non-rectangle whose diagonals bisect each other-
e. None of (a) – (d) is correct

In question 16, learners were required to determine if the truth of one statement (regarding diagonals of rectangles) could imply or disprove the truth of another, (that is, analysing and comparing statements i and ii). To establish this, learners were to apply deductive reasoning, by recognizing that not proving statement ii would require them to find another shape, (which is a parallelogram), whose diagonals bisect each other, but not a rectangle. The question also assessed the ability of the learners to distinguish between the properties of quadrilateral shapes have in common, such as diagonals bisecting, and unique properties of specific figures, such as rectangles.

Based on the responses, only two learners, (9.52%) selected the correct option, (d). The two learners were able to apply the required deductive reasoning to prove that statement ii is false, by finding another non-rectangular shape whose diagonals bisect each other, which is a parallelogram.

Another three learners who selected option (b) reveal a misconception, assuming that proving statement i invariably justifies statement ii. This is a possible confusion of logical implications with equivalence. *The option* with the highest number of selections was option (c).

11 learners had a wrong notion that finding a rectangle (with diagonals bisecting each other is sufficient to prove the second statement, forgetting that statement ii provides the necessary evidence for statement i.

The frequency of selection of option (c) reveals a struggle in understanding the concept of counterexamples, as well as the rigor of geometric proof. The three learners who selected option (e) ‘None of (a)-(d), must have exhibited either a lack of confidence in selecting any of the other options or uncertainty. The one learner who did not attempt an answer may have chosen not to answer because of a lack of conceptual clarity or confidence in attempting the question.

In general, learners’ performance on this question reveals gaps in understanding of logical structures in geometry, especially the importance of counterexamples in disproving statements. This invariably indicates that most learners face underlying challenges in formal deductive reasoning, which is what informs van Hiele Level 3. This is supported by the interview responses:

P13: *“I can identify shapes, but when you ask for reasons or proof, that’s where I get stuck.”*

This shows difficulty in transition from visualization to logical reasoning. In another related question asked, P15: *"I have to learn the basics of the quads and all their properties and also attempt various questions."*

This reveals the need to reinforce the basics before tackling deductive reasoning-related questions. In another question asked, P6 responded, thus:

"Diagonals of rectangles are equal, and those of parallelograms bisect each other." The response shows knowledge properties of diagonals; however, it does not imply having an understanding of counterexamples.

Question 17:

If we have what is assumed as (given) and what is to be shown as (proved) in the following statement:

A quadrilateral with diagonals forming four isosceles right triangles.

Which one is correct?

a. Given: A square
   Prove: the diagonals form four different triangle angles
b. Given: A quadrilateral with diagonals forming four right isosceles triangles
   Prove: the figure is a square-
c. Given: A parallelogram with diagonals perpendicular bisectors of each other.
   Prove: the figure is a rectangle
d. Given: A quadrilateral with diagonals are congruent
   Prove: the figure is a rhombus
e. Given: A quadrilateral forming triangles with the same angles on the base
   Prove: the figure is a parallelogram

Question 17 was designed to assess learners' understanding of geometric proofs, and also their ability to recognise how a quadrilateral is defined by given conditions. Learners were to apply deductive reasoning, by establishing a link between specific conditions of diagonals that form four isosceles right triangles, to the unique properties of a square. The question also assesses learners' ability to distinguish between the properties that quadrilaterals share in common and their unique properties.

Eight learners (38.10%) selected the correct option (b), implying that they were able to identify the diagonals that form four isosceles right triangles in a given quadrilateral is peculiar to the square property. The eight learners demonstrated an understanding of geometric proofs, that link the given condition to a specific quadrilateral property- square.

The selection of option (a) by five learners reveals a misunderstanding of the diagonal condition, by assuming it results in different angles of a triangle, which, in this case, is not correct for squares. The four learners who selected option (e) are a reflection of misinterpretation that identical triangle angles on the base are satisfactory to define a parallelogram, while the only learner who selected option (c), must have confused the two properties of squares and rectangles, in relation to their diagonal relationships. The percentage of 9.52% (two learners) that did not attempt the question is an indication of of lack of understanding of the structure of proof and the given conditions.

The performance trends identify deductive reasoning gaps and misconceptions regarding the properties of quadrilaterals. It also highlights gaps in construction of proof and logical reasoning; misconceptions regarding quadrilateral properties, and difficulties in linking given conditions to specific figures (shapes).

Question 18:

A figure has the following properties.
Property D: It has diagonals whose length are equal
Property S: It is a square.
Property R: It is a rectangle.

Which of these is correct?

a. D implies R which implies S
b. S implies R which implies D-
c. R implies D which implies S
d. R implies S, which implies D
e. D implies S which implies R

Question 18, was intended to assess the ability of learners in comprehending and applying logical implications within the hierarchy of quadrilateral properties, about the relationships between rectangles, squares, and their diagonal properties. The question was designed to evaluate the learners' formal deductive reasoning at van Hiele Level 3, focusing on deductive logic and the flow of properties of geometry. 14.29% (learners), which accounted for the correct response rate for option (b), demonstrated an understanding of the hierarchical relationship between rectangles, squares, and their diagonal properties. 15 learners, (76.19%) who selected any of the options (a), (c), and (e) reveal different misconceptions. There seems to be a misunderstanding of the flow of implications between the given properties. Learners either must have assumed that equality of diagonal lengths directly implies that a figure is a square (in the case of options a and e), or may have misinterpreted the role of rectangles in the quadrilateral hierarchical order, like in the case of option (c). No attempt response of three learners could be attributed to uncertainty or lack of understanding of the required abstract nature of the question demands.

In general, the challenges learners faced in tackling this question seem to be centred on misconceptions regarding quadrilaterals' properties, difficulty with logical implications and dependence on intuition or non-logical connections between properties, instead of applying formal reasoning at van Hiele Level 3.

Question 19:

Consider the statements below:
Statement A: A rectangle is a parallelogram with right angle
Statement B: A rectangle with perpendicular diagonals is a square.
Which of the these is true?

a. A and B are definitions
b. A is a definition; B is a theorem-
c. A and B are theorems
d. A and B are postulates
e. A is a postulate, B is definition

Question 19 assessed the ability of learners to distinguish between definitions and theorems in geometry. In specificity, the question directly assessed whether learners could identify that Statement A's definition, describes a rectangular property. Secondly, to also assess whether learners could understand that Statement B is a theorem that requires proof, as it establishes a condition, (perpendicular diagonals) which qualifies a rectangle to be a square.

The correct response rate of 42.87%, (nine learners) demonstrated an understanding of the distinction between definitions (that is, basic properties that define a shape), and theorems, (that is, statements requiring proof). Four learners' selection of option (a) suggests a reflection of confusion as to whether statement B is a definition or not.

The four learners may have assumed that all property descriptions qualify as definitions. Meanwhile, the five learners who selected option (c) misclassified the two Statements as theorems, which shows difficulty in differentiating between basic definitions and geometric statements that can be proved. One learner selected option (e), which appears to be a separate misunderstanding of 'definition' versus 'postulates. Two learners decided not to attempt the question, which again could suggest either no confidence or difficulty in understanding the abstract nature of the question. The performance of question 19 was characterised by an inadequate understanding of definitions and theorems, difficulty with knowledge of the hierarchy of quadrilaterals, and conceptual confusion.

Question 20:

Which of the statements (a) to (c) is another correct way of making this statement?

"A quadrilateral whose diagonals form four isosceles right triangles is a square. bisect each other is a parallelogram."

a. If a quadrilateral is a square, then the diagonals form four isosceles right triangles
b. If the diagonals of a square form four isosceles right triangles, then the figure is a quadrilateral
c. If the diagonals of a quadrilateral form four isosceles right triangles, then the figure is a square-
d. Both (a) and (c) are accurate restatements.
e. All of the above are accurate restatements

Q*uestion 20*, which was also set at van Hiele Level 3, was designed to assess the ability of learners in two main folds: to analyse and identify equivalent statements that are logical, based on a geometric condition (diagonals that form four isosceles right triangles), and secondly, to test their ability to distinguish between logically sound implications and non-equivalent or overgeneralised statements. To accurately identify and restate a geometric theorem, one will require the ability to apply deductive reasoning.

Four learners (19.05%) selected option (c), by demonstrating the ability to correctly identify that "a quadrilateral whose diagonals form four isosceles right triangles is a square, and bisecting each other is a parallelogram." It can only be a square. This is correct because a square is the only quadrilateral shape whose diagonals are perpendicular, equal, and also bisect each other, thereby forming four isosceles right triangles. It is a unique defining property of a square. So, option (c) is the only option that provides a logically equivalent statement to the original statement.

Their selection is a reflection of their understanding of logical equivalence and the hierarchical relationships between quadrilateral shapes. The incorrect responses of options (a), (b), (d), and (e), which accounted for 76.19% (16 learners), were characterised by confusion and misunderstanding. Learner one, who selected option (a), shows confusion between a necessary condition and a restatement. Three learners who selected option (b) seemed to have had a misunderstanding that a property of squares (in this case, isosceles triangles) directly proves that all quadrilaterals with these properties are squares. Nine learners' option of (d) reflects confusion about assuming multiple statements as correct restatements, without differentiating their logical equivalence. Three learners' choice of selection of option (e) indicates overgeneralisation and an inability to identify that not all statements are logically equivalent.

The remaining percentage of 4.76% (one learner) was unable to demonstrate the required abstract reasoning, like most of the learners, due to a lack of confidence, as there is no way option (e) could be the correct option. The performance of this question shows a struggle in understanding logical implications in geometry. Secondly, overgeneralisation of properties was observed, especially among those learners who selected options (d) and (e). This implies that advancing to the formal deductive reasoning of van Hiele Level 3 is a difficulty.

4.1.2 Summary of Analysis of items in questionnaire A, (Based on 21 respondents)

The analysis of questionnaire A, with the aid of the visual representations, indicate performance trends among learners in the understanding of quadrilaterals based on van Hiele levels.
Learners demonstrated high proficiency in shape recognition (level 0), with correct responses surpassing 85% in the majority of questions.
At level 1, knowledge of shape properties varied, as evidenced by the percentages, while others indicated a deficiency in the required knowledge level. At levels 2 and 3, which are informal

deduction and formal deduction levels respectively, most learners had difficulties, as they were unable to establish connections between shape properties and deductive reasoning. This was evidenced by the low percentage of correct responses (below 50%). This is except question number eight, where over 80% of participants identified the correct option. This pattern, though may reflect easy identification of shapes, however, learners seem to have difficulties in grasping deeper geometric concepts and the required geometric reasoning level in handling questions set at higher levels of the van Hiele level of geometric model. Below provides the classification of participants across van Hiele levels of geometric thoughts based on questionnaire A and the rationale for the classification.

4.1.3 Classification of Participants in relation to Van Hiele levels of geometric thoughts based on Questionnaire A

Since the study is theoretically framed in van Hiele model of geometric thought, and one of the primary objectives of the assessment (questionnaire A) seeks to gain insight in terms of accessing learners' geometric reasoning, from visualization to formal deductive reasoning levels of the van Hiele model, the suitable method adopted in classifying learners across the levels will be based on consistency in reasoning approach (across Levels 0 to 3). This is in accordance with the van Hiele model of geometric thought, which highlights the sequential (geometric) development of learners, through the levels, instead of classification on the basis of the highest level of proficiency. This aims to ensure learners are accurately classified based on their progression of reasoning, rather than having high scores in an isolated manner, at higher levels, without progression of reasoning through the levels.
The rationale for the classification of learners based on consistency in reasoning across van levels required learners to achieve the threshold of 60% of the total marks for each set of questions set at each level. For instance, for a learner to be classified at Analysis Level 1, the learner must obtain 60% (three out of five of the questions) or higher, in Level 1 (Questions 6 to 10). In addition, the learner must meet the geometric reasoning proficiency threshold at the preceding Level- Visualization Level (Questions 1 to 5). This is to ensure sequential progression in their geometric reasoning. For instance, a learner cannot be classified at Level 3 (formal Deductive reasoning) level, if the learner fails to meet or satisfy the thresholds for the preceding levels- 0, 1 and 2: For instance, assuming a learner has the following scores:

four out of five at Visualization, (threshold met)
three out of five questions at Analysis, (threshold met)

two out of five questions at Informal Deductive level, (below threshold)
four out of five questions at Formal Deduction level, (threshold met)

The learner will be classified at Analysis Level 1 (Analysis) because his/her inconsistency at the Informal Deductive level disallows him/her from advancing beyond that level.
This approach ensures that a solid foundational reasoning is laid, which also aids the learner in advancing to higher geometric thoughts or levels, in accordance with the van Hiele model's sequential progression.

The choice of selecting 60% as the threshold was considered reasonable and justifiable. Its selection is essentially on the basis of practical and theoretical considerations. It aligns with educational standards, as it reflects the level of competence without having an unrealistic threshold. Secondly, a 60% threshold allows or accommodates the complexity of questions set at higher levels, such as Levels 2 and 3 (Informal Deduction and Formal Deduction), so that the benchmark will not be so difficult for the achievement of proficiency for questions involving abstract reasoning and formal proofs. Thirdly, the selection of 60% reflects basic proficiency, which requires an understanding of essential concepts at every level.

The classification of learners across VHLs, based on their performance, as shown in Table 4.1, is presented in Table 4.3 below: The structure of Table 4.3 is explained below:

Participant (Ps): Identifier for each participant (e.g., P1, P2, P3).
Total Marks Obtained at each level-L0, L1, L3 and L3 (based on five Marks/each set of questions set at each level)
Percentage (%) obtained by each learner per level
Meets Proficiency Threshold (60%)?- To determine whether the learner's mark meets the required threshold percentage.
Learners' classification across VHL, based on a 60% threshold

Table 4.3: Classification of Learners According to VHL (Questionnaire A)

Participants (Ps)	Total Marks Obtained at L0-	Percentage (%) Obtained	Meets Proficiency Threshold (60%)?	Total Marks Obtained at L1-	Percentage (%)	Meets Proficiency Threshold (60%)?	Total Marks Obtained at L2-	Percentage (%)	Meets Proficiency	Total Marks Obtained at L3-	Percentage (%)	Meets Proficiency	Learners Classification

P1	3/5	60	Yes	2/4	50	No	1/5	20	No	2/5	40	No	L0
P2	4/5	80	Yes	3/4	75	Yes	1/5	20	No	1/5	20	No	L1
P3	4/5	80	Yes	2/4	50	No	0/5	0	No	1/5	20	No	L0
P4	5/5	100	Yes	2/4	50	No	1/5	20	No	0/5	0	No	L0
P5	4/5	80	Yes	4/4	100	Yes	1/5	20	No	1/5	20	No	L1
P6	5/5	100	Yes	3/4	75	Yes	2/5	40	No	1/5	20	No	L2
P7	2/5	40	No	1/4	25	No	1/5	20	No	2/5	40	No	Below L0
P8	4/5	80	Yes	2/4	50	No	1/5	20	No	1/5	20	No	L1
P9	4/5	80	Yes	2/4	50	No	0/5	0	No	1/5	20	No	L0
P10	4/5	80	Yes	2/4	50	No	3/5	60	Yes	1/5	20	No	L0
P11	4/5	80	Yes	1/4	25	No	0/5	0	No	0/5	0	No	L0
P12	5/5	100	Yes	3/4	75	Yes	2/5	40	No	3/5	60	Yes	L1
P13	4/5	80	Yes	1/4	25	No	3/5	60	Yes	3/5	60	Yes	L0
P14	5/5	100	Yes	2/4	50	No	1/5	20	No	0/5	0	No	L0
P15	5/5	100	Yes	3/4	75	Yes	2/5	20	No	0/5	0	No	L1
P16	5/5	100	Yes	2/4	50	No	2/5	40	No	1/5	20	No	L0
P17	4/5	75	Yes	1/4	25	No	2/5	40	No	1/5	20	No	L0
P18	3/5	60	Yes	2/4	50	No	2/5	40	No	2/5	40	No	L0
P19	4/5	80	Yes	1/4	25	No	2/5	40	No	1/5	20	No	L0
P20	4/5	80	Yes	1/4	25	No	1/5	20	No	1/5	20	No	L0

P21	4/5	80	Yes	3/4	75	Yes	2/5	40	No	2/5	40	No	L1

Going further in terms of the classification of the learners across the van Hiele levels, Table 4.4 below provides the total number of learners operating at each van Hiele level.

Table 4.4: Summary of the Participants' Classification Across van Hiele Levels in Questionnaire A:

Van Hiele Level	Level Description	Number of Participants	Percentage of Participants (%)
Below Visualisation Level	Struggling to identify shapes based on appearances, without properties	1	4.8
Level 0 (Visualisation	Recognition of shapes based on appearances, without the properties	13	61.9
Level 1 (Analysis)	Understanding and identifying properties of shapes, such as lengths and angles	6	28.6
Level 2 (Informal Deduction)	Logical reasoning with properties, such as identifying parallelograms based on features	1	4.8
Level 3 (Formal Deduction)	Formal proofs and understanding relationships between properties	0	0
Total		21	100

The classification of learners across the van Hiele Levels, as shown in Table 4.3, indicates a significant percentage of learners operating at Level 0- visualisation, 61.9% (13 out of 21), indicating limited exposure to geometric reasoning to shape recognition, based on visual appearances. This implies that more than 50% of learners struggle to advance beyond the

visualization level due to foundational misconceptions or over-dependence on visual prototypes and methods. One learner struggled to demonstrate the ability to recognise shapes based on visualisation.
At level 1 (Analysis), six learners (28.6%) were able to demonstrate the ability to identify and analyse shapes using their properties, beyond visual recognition. The low percentage of learners at this level underlines a gap among learners in applying the required analytical reasoning to comprehend the relationships between properties of shapes.

At Level 2 (Informal Deductive reasoning), only one learner (4.8%) geometrically reasoned beyond analysis level. The learner showed some ability to connect properties and informally reason about how shapes relate characteristically. By implication, it shows that most learners struggle to advance from analytical reasoning to informal deductive reasoning.
No learner exhibited the required formal deductive reasoning, which includes proofs and abstract relationships. This indicates that learners are generally not conceptually equipped to demonstrate the required higher van Hiele level of geometric proficiency.

4.1.4 Presentation of the findings in Questionnaire B

The analysis of learners' responses to questionnaire B provided more insight in terms of their experiences and feelings about the quadrilateral in Euclidean geometry. Unlike questionnaire A, where a participant could get the correct answer by assumption, questionnaire B compelled the participants to demonstrate their understanding through the application of their geometric concepts (knowledge) to possibly gain more insight regarding their misconceptions. As indicated in Chapter 3, questionnaire B had eight open-ended questions with four groups of two questions each.

The presentation of the findings of learners' written responses for each question follows, based on their performance as shown in Table 4.5 below. This will also be supported by relevant interview responses, as it may be necessary.

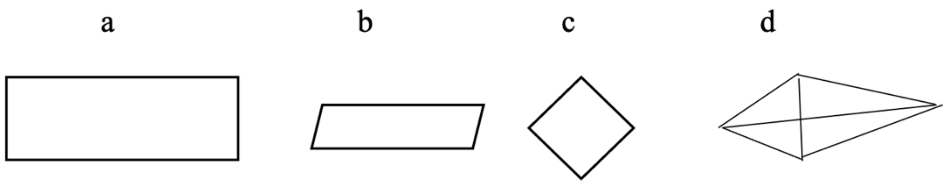
Table 4.5: Individual Performance and their percentages in Questionnaire B

Participants (Ps)	Q1 (4 Marks)	Q2 (11.5 Marks)	Q3 (7.5 Marks)	Q4 (10 Marks)	Q5 (10 Marks)	Q6 (11 Marks)	Q7 (13 Marks)	Q8 (13 Marks)	Total Marks (out of 80 Marks)	Total marks out of (100%)	
P1	3.0	1.0	1.0	0	Q5	9.0	0	0	16.0	20.0	
P2	4.0	5.5	7.5	2	10	5.5	0	0	32.5	40.6	
P3	3.5	5.0	1.0	0	10	5.5	1	2	28.0	35.0	
P4	2.0	4.5	0	2	10	2.5	0	0	22.0	27.5	
P5	3.0	2.0	0	0	10	0.5	0	0	15.5	19.4	
P6	4.0	6.5	0	0	10	5.5	0	0	26.0	32.5	
P7	1.0	0.0	0	0	Q5	0.5	0	0	1.71	2.1	
P8	3.5	3.5	0	0	Q5	5.5	0	0	14.3	17.9	
P9	3.0	3.0	3	0	10	0.5	2	2	21.5	26.9	
P10	4.0	3.5	0	0	9	2.5	0	0	19.0	23.8	
P11	4.0	4.0	0	0	10	5.5	0	0	23.5	29.4	
P12	4.0	5.0	0	0	10	10.5	0	0	29.5	36.9	
P13	2.0	2.0	0	0	5	0.0	3	3	15.0	18.8	
P14	4.0	3.5	0	0	10	0.5	2	0	20.0	25.0	
P15	4.0	6.5	0	0	10	10.5	2	0	33.0	41.3	
P16	3.0	1.5	7.5	0	5	10.5	0	0	27.5	34.4	
P17	3.0	1.0	0	0	7.5	0.0	0	0	11.5	14.4	
P18	4.0	0.0	0	0	Q5	0.0	0	0	4.6	5.8	
P19	1.5	0.0	0	0	10	0.5	0	0	12.0	15.0	
P20	3.5	4.0	0	0	Q5	3.0	0	0	12.0	15.0	
P21	2.0	1.5	0	0	7	0.0	1	0	11.5	14.4	

4.1.5 Questionnaire B and Analysis of Learners' Responses

Question Number 1: (4 marks)

The figures below are quadrilateral shapes. Describe what each figure looks like with reason and identify each shape by their names. (4 marks)



Question 1 assessed the recognition skill of the participants. Learners were required to identify/describe shapes by appearance (visual characteristics) and name shapes with reasons. Below provides a summary of the expected response:
Figure a is a rectangle, with equal opposite sides in length and parallel. All four angles are right angles (90 degrees). The defining property of a rectangle is that it has four right angles, and the opposite sides are equal and parallel.

Figure b is a parallelogram with opposite sides equal in length and parallel. The opposite angles are also equal. The reason is that it has two pairs of parallel sides.

Figure c is a square in a rotated position, not a rhombus. A square is a special type of rhombus because all rhombus properties apply to a square, and a square has right angles. However, not all rhombuses are a square, unless the four angles are right angles. A square and a rhombus have all sides equal. All four angles are right angles (90 degrees) in the case of a square. The diagonals are equal and bisect each other at right angles (90 degrees), unlike a rhombus, whose diagonals are not equal, but bisect each other at right angles.

Figure d is a kite with two distinct pairs of adjacent sides equal. One pair of opposite angles is equal, which are the angles between the pairs of equal-length sides. The defining property of a kite is that it has two pairs of adjacent sides equal in length, and one pair of opposite angles equal. Below are findings (analysis) of learners' responses to question (item) 1:

4.1: Findings from P21's response to question 1

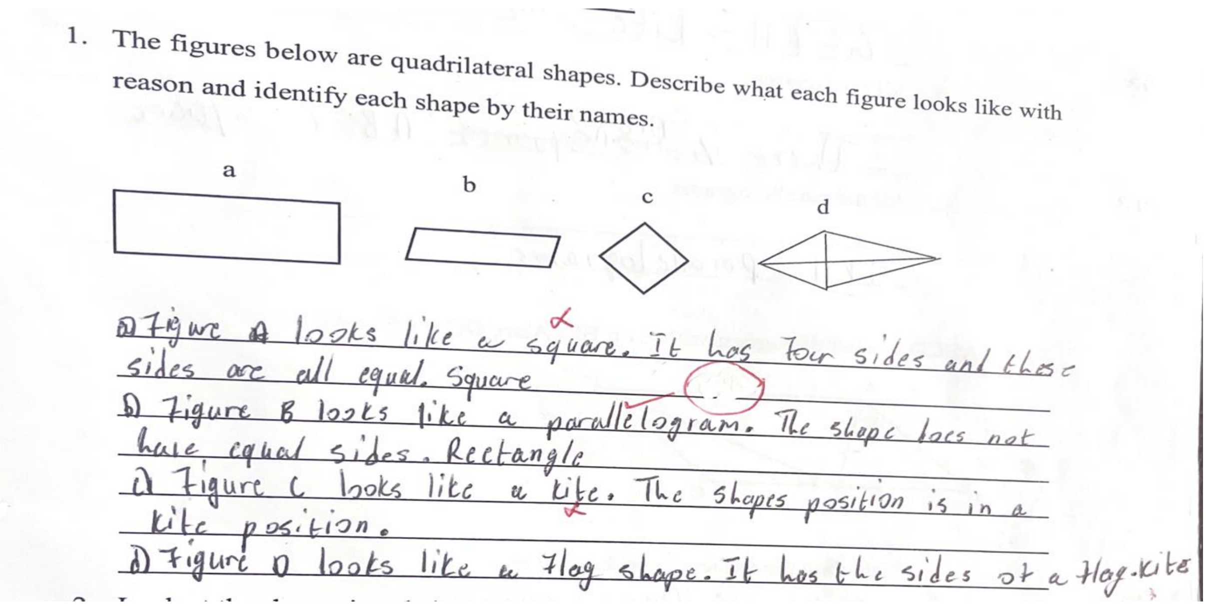


Figure a (Rectangle):

P21's response: P21 identifies Shape A as a "square," stating that '*Figure a look like a square. It has four sides and the sides are all equal. Square."* P21 was unable to differentiate between a "square" and a "rectangle". P21 was unable to identify shape A as a rectangle, having its opposite sides equal, with all sides not congruent. P21's response is incorrect. A rectangle is mistaken for a square, as the reason given is a defining property for a square, not a rectangle "*It has four sides and the sides are all equal. Square*." This is a misconception.

Figure b (Parallelogram):

P21's response: "*Figure b looks like a parallelogram. The shape does not have equal sides. Rectangle."* P21 first stated Shape B as a parallelogram, but concluded by identifying the same shape as a "rectangle", demonstrating a lack of confidence and uncertainty in terms of how rectangles and parallelograms visually relate, as well as how they relate by hierarchy and definitions. This depicts a reflection of a misconception about the distinction of different quadrilateral shapes.

Figure c (square)

P21's response: *'Figure c looks like a kite. The shape's position is in a kite position."* The response of P21 is incorrect. P21 seems to have perceived the shape based on the conventional positioning of shapes on paper, without knowing that the paper positioning of shapes does not affect the defining property of shapes. By implication, confusion between a square and a kite might indicate that the participant focused more on the visual position of the shape (like the “kite position” mentioned), rather than the actual visual characteristic nature of the shape based on the visualisation level of van Hiele level of geometric thought. P21 reflects deeply rooted misconceptions, by assuming that Shape C is a kite based merely on its visual orientation.

Figure d (Kite)
P21’s response: *“Figure d looks like a flag shape. It has the sides of a flag-kite.”*
Interpretation: The description somewhat matches, as the correct shape is indeed a kite, having two pairs of adjacent sides equal. But the learner struggled to provide a reason using the required geometric terminology. By describing shape D as “flag shape) or “a flag kite” reveals a lack of conceptual understanding of diagonals and symmetry. Over-reliance on visual indications was noted, and confusion in distinguishing specific categories of quadrilaterals- e.g., parallelogram, rectangle, and square.

P21’s understanding of quadrilaterals seems to have been mainly influenced by over-dependence on visual models, as evidenced by the way Figure ‘a’ was classified as a ‘square’ instead of a ‘rectangle’. Limited conceptual understanding of properties of shape was also evident in misidentifying Figure ‘b’ as a parallelogram, without taking cognizance of the defining properties of a parallelogram, such as equality of opposite sides and their parallelism. Confusion between the categories of shapes was also evident in the labelling of Figure ‘c’ as a kite, showing difficulty in differentiating shapes that have related characteristics. The inability to provide valid reasons for their classifications indicates a lack of depth and accuracy in geometric reasoning, especially at the recognition (foundational) stage. Factors such as overreliance on visual cues, underdeveloped reasoning skills, and inadequate understanding of geometry-related definitions, are evident and consistent with difficulties faced.

4.2: Findings from P1's response to question 1

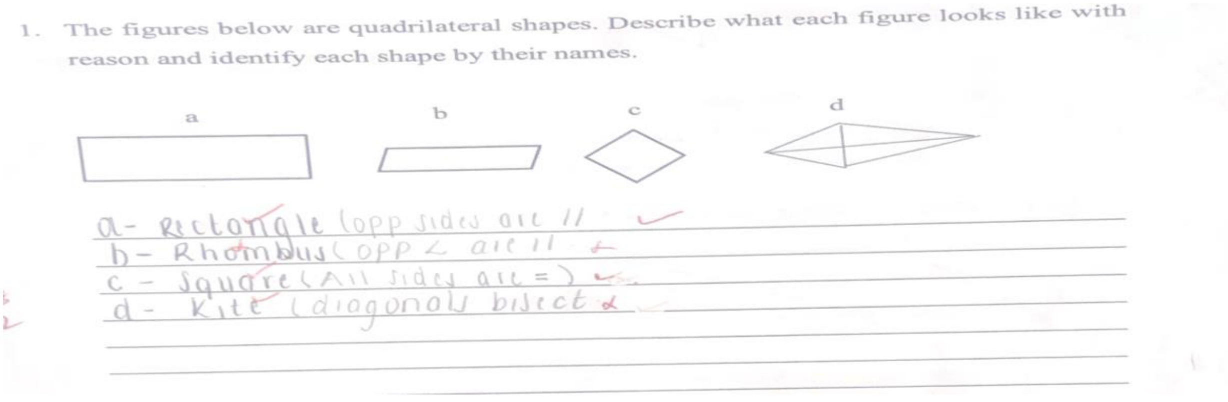


Figure a (Rectangle)

P1's response: "*Rectangle (Opp sides are ||)"* The learner correctly identified shape A as a rectangle with the correct reason, which is a key defining property. It shows the participant can recognise the shape based on its visual appearance.

Figure b (Parallelogram)

P1's response: (*"Rhombus (opp ∠ are ||)"* The learner's response is incorrect as the shape is a parallelogram. P1's response does not show evidence of recognitive skills, which is key at the visualization stage of the Hiele levels of geometric reasoning. The reason given does not suggest characteristic knowledge of the shape. The reasoning shows a misconception of the defining properties of a rhombus (equality of all sides and diagonals bisecting at right angles). Rather, P1 confuses it with a parallelogram in terms of equality of opposite angles, not necessarily parallel. This mix-up suggests evidence of demonstration of lack of clarity in differentiating between the two shapes- parallelograms and rhombus, which is likely influenced by misconceptions in grasping the quadrilateral properties and hierarchy.

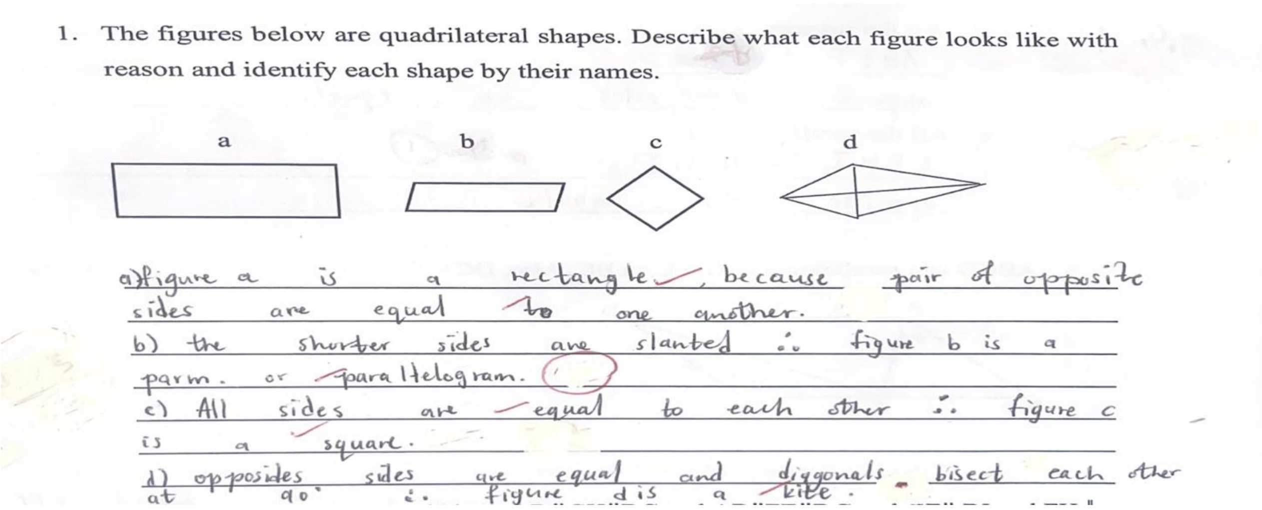
Figure c (Square)

P1's response: *"Square (all sides are =)"* P1's response is correct based on identification by visualisation. P1 also mentioned one of the defining properties of a square, even though not sufficient, because a rhombus also has the same property. P1's response demonstrates some familiarity with basic quadrilateral properties.

Figure d (Kite)

P1's response: Kite "diagonals bisect". The response to Figure 'd' is correct, however, with a misleading reason, "diagonals bisect." A kite does have two bisecting diagonals, which is a generic property, as shapes like a square also have such a property. P1 fails to provide the defining property of kites. It would have been appropriate if the learner had said, a kite has two pairs of adjacent sides equal and with intersecting diagonals at right angles. P1 cannot apply specific geometric reasoning. Over-generalisation of properties of shapes seems to be evident (e.g., "diagonals bisect" in the case of shape 'd' and "opposite angles are parallel", in the case of shape 'b'), without taking into consideration the specific defining properties.

4.3: Findings from P3's response to question 1


1. The figures below are quadrilateral shapes. Describe what each figure looks like with reason and identify each shape by their names.

a b c d

a) figure a is a rectangle, because pair of opposite sides are equal to one another.
b) the shorter sides are slanted ∴ figure b is a parm. or parallelogram.
c) All sides are equal to each other ∴ figure c is a square.
d) opposides sides are equal and diagonals bisect each other at 90° ∴ figure d is a kite.

Figure a (Rectangle)

P3’s response: *“Figure ‘a’ is a rectangle because a pair of opposite sides are equal to one another.”* P3 correctly identified Figure a as a rectangle. However, P3 provided unclear expression as this property, “*pair of opposite sides are equal”* is not only applicable to a rectangular shape. P3 did not highlight the differentiating properties of a rectangle, such as right angles. Based on the unclear expression “*pair of opposite sides are equal,”* there is evidence of confusion between properties, which is an identified misconception.

Figure b (Parallelogram)

P3’s response: *“The shorter sides are slanted in figure ‘b’. Figure ‘b’ is a parallelogram.”* The learner correctly identified the shape as a parallelogram. The expression reflects an understanding of how a parallelogram looks, although the explanation could be clearer, using the correct geometric language, rather than, “shorter sides are slanted”, which could be seen as a misclassification of shapes.

Figure c (Square)

P3’s response: *“All sides are equal to each other; therefore, figure ‘c’ is a square.”* The participant is correct in identification. Though the reason given was limited to “all sides are equal”, P3 ignored the vital defining properties of a “square”, such as equality of diagonals or perpendicular bisectors. This is another indication of confusion between properties.

Figure d (Kite)

P3's response: *"Opposite sides are equal, and diagonals bisect each other; figure 'd' is a kite."* The pupil correctly identified the shape, but lacked reasoning clarity, hence, there is a possible misclassification of shapes. Stating, *"Opposite sides are equal"* is incorrect, rather, "sides adjacent to each other are equal". P3 was correct based on the statement, "diagonals bisect,", however, insufficient to differentiate a kite from other quadrilaterals with the same diagonal properties, such as the parallelogram family- rhombuses, rectangles, and squares. In a kite, only one diagonal, which is the longer one, bisects the other diagonal. "Diagonals are perpendicular" is a more defining property of kites, instead of the diagonals bisecting each other.

Several factors influencing P3's understanding were identified, such as difficulty in language comprehension. At the interview, P3 admitted that the language used in some of the questions was "very complicated," revealing that language barriers seem to have influenced reasoning. P3 also admitted not being conversant with properties, and having difficulty in the differentiation of shapes, especially similar quadrilaterals, such as rectangles and parallelograms.

> P3*: "The English is very complicated…..I'm not used to this type of questions"*
> *"I think I have to put more effort in understanding the properties, because most of the time I do read them, they do not make too much sense"*

4.4: Findings from P15's response to question 1

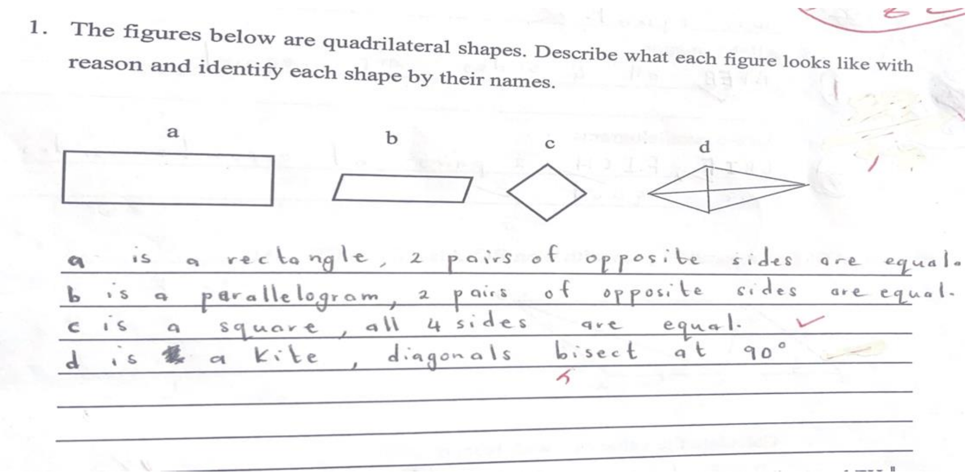


Figure a (Rectangle)

P15's response: P15 correctly identified the shape with a non-exclusive property for a rectangle, *"two pairs of opposite sides being equal,"* as it also applies to a parallelogram. This shows a misconception of the generalisation of properties of properties, without stating the defining property of each shape.

Figure b (Parallelogram)

P15's response: P15 correctly identified the shape as a parallelogram, but repeated the same reason used for the rectangle ("two pairs of opposite sides are equal"). This indicates an inability to differentiate between parallelograms and rectangles using their defining property, which is another misconception.

Figure c (Square)

P15's response: Correct identification of the shape as a square with a valid justification, "all four sides are equal." However, Rhombus also has that property, thus making that property non-exclusive to the square. P15 fails to justify the reason with a complete set of defining properties of a square- equality of angles and/or equality of diagonals, bisecting each other.

Figure d (Kite)

P15's response: Correct identification of Figure 'd' as a 'kite'. Though, the given property "diagonals bisect at 90^0" is true, and applicable to a kite, but not exclusive to kites. This suggests that P15 depends on using properties in an isolated manner without the knowledge of how these properties differentiate a kite from other quadrilateral shapes, such as a square and a rhombus.

P15's response reveals influencing factors affecting P15's understanding, such as, the repetition or overgeneralisation of properties, limited knowledge of defining properties, and inability to distinguish similar quadrilateral shapes in terms of similarity of attributes- parallelograms and rectangles. Lack of knowledge of hierarchical relationships was evident.

4.5: Findings from P14's response to question 1

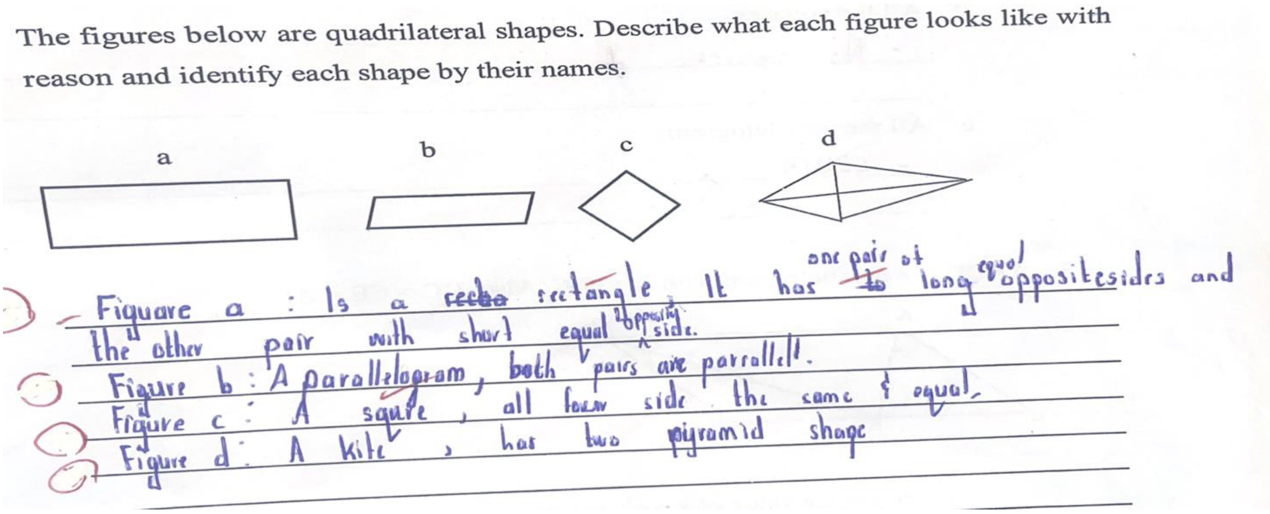


Figure a (Rectangle)

P14's response: P14 correctly identified the shape as a rectangle. The description is also valid, but with evidence of limited conceptual understanding of the right terminologies. Stating "*one pair of long opposite sides"* and another pair of *"short equal opposite sides"* is valid, but seems not justifiable enough, because such a description also applies in the case of a parallelogram. P14 lacked understanding of how the parallelism property and the property of right angles define a rectangle.

Figure a (Parallelogram)

P14's response: Correct identification of the shape as a parallelogram, however, P14 oversimplified the reason without taking cognizance of additional defining properties- equality of opposite angles. There is evidence of generalisation of properties and a lack of knowledge of the peculiarity of properties that distinguish different shapes.

Figure c (Square)

P14's response: P14 correctly identified Figure c as a square. Although P14 mentioned a key property "all four sides are the same and equal," but fails to mention a key defining property of a square- equality of diagonals or right angles, which is an indication of inadequate conceptual knowledge, as "all four sides are the same and equal," also applies to a rhombus.

Figure d (Kite)

P14's response: Correct identification of Figure d as a kite, stating an unclear statement, *"having two pyramid shapes."* It reflects a misconception of the definition of the properties of a kite, such as equality of adjacent sides, and intersection of diagonals at right angles, with one diagonal bisecting the other.

A number of influencing factors to understanding of quadrilaterals include limited conceptual understanding. At the interview, P14 did admit to being familiar with quadrilaterals, however, they struggled with recalling their properties, especially when differentiating between similar shapes, such as a rectangle and a parallelogram. The reasoning of P14 suggests to be on the basis of visualization, as seen in the descriptions provided. This shows an inability to abstractly conceptualise properties beyond identification by visualization. P14 also faces the challenge of proof-related geometric tasks, as a result of insufficient knowledge of properties. P14 stated, *"I couldn't prove…..I'm used to working with numbers."*

This shows over-reliance on procedural knowledge- working with numbers- rather than conceptual geometric reasoning. It also reveals having challenges with analysing and describing shapes based on their properties, which implies gaps in conceptual knowledge.

4.6: Findings from P16's response to question 1

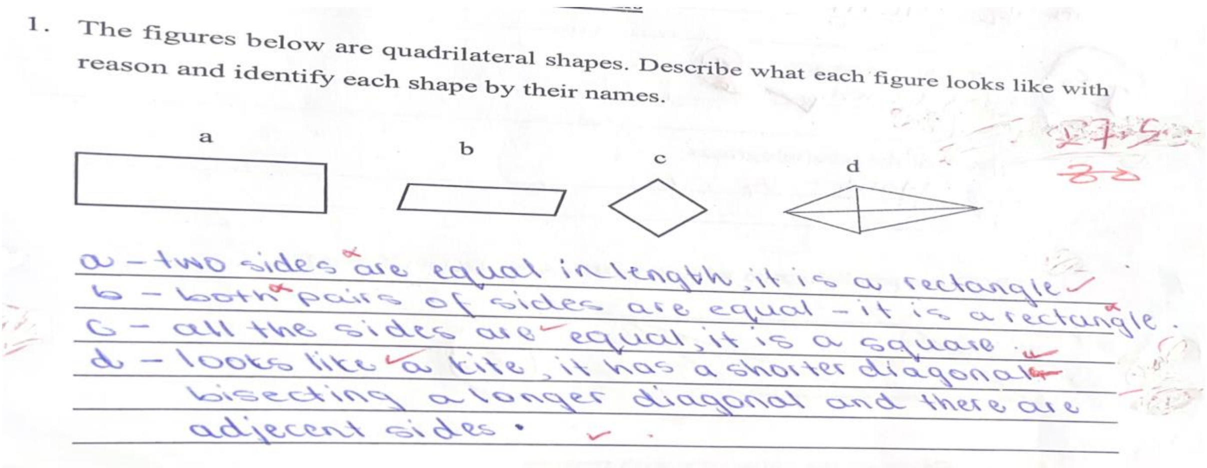


Figure a (Rectangle)

P16's response: Although, P16 identified the shape correctly as a rectangle with insufficient justification in the accurate definition of a rectangle: "two sides are equal in length". P16 fails to recognise that one major rectangular defining property is, opposite sides are equal and parallel, with all interior angles as right angles. This reflects poor conceptual knowledge of the defining properties of a rectangle.

Figure b (Parallelogram)

P16's response: P16 incorrectly identified the shape as a rectangle, whereas, it is a parallelogram. The response of P16 confuses the properties of both shapes- rectangles and parallelograms. By saying, "both pairs of sides are equal," it does not sufficiently communicate correct justification. Omission of the word, "opposite" in the statement, "both pairs of sides are equal" does not convey an accurate defining property. Even if P16 had stated, "both pairs of opposite sides are equal," it would not have been a defining property, as it applies also to a rectangle. This still reflects an inability to correctly differentiate similar shapes with their unique properties.

Figure c (square)

P16's response: Correct identification with a valid, but insufficient justification: "all the sides are equal," as such a property applies to a square and a rhombus as well. P16 omits the defining properties of a square, such as equality of all interior angles at 90^0 and bisection of two diagonals at right angles, revealing a lack of conceptual knowledge and inability to differentiate similar shapes with their defining properties, hence, overgeneralisation of properties seems to be the way out for learners.

Figure d (Kite)

P16's Response: Correct identification of the Figure as a kite, with a valid kite properties justification, by mentioning "adjacent sides" and "diagonals". However, P16 fails to mention a key characteristic of kites- the perpendicular nature of the diagonals.

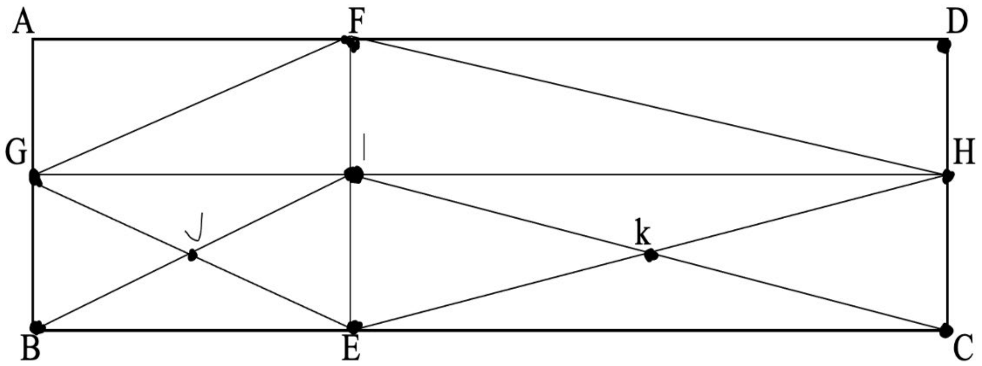
Notable misconceptions from P16's response include overgeneralisation of properties. P16 applies properties of rectangles (equal sides) to a parallelogram (Figure B), which is a sign of wrong classification of shapes using their properties. Although P16 was able to correctly identify some shapes, but also failed to provide their vital defining properties, which shows limited conceptual clarity. Difficulty in differentiating properties of shapes with similar features seems to be evident, especially the confusion between rectangles and parallelograms in Figure b. At the interview, P16 confirms the difficulty in differentiating shapes using their defining properties, which reveals overreliance on visual similarity instead of using their definitional identity.

P16: *"I find it hard to remember which shapes has angles or sides that must be the same. Sometime they look similar."*

In response to a question asked from supplementary conceptual questions, regarding differentiating rectangles and parallelograms, P16 stated:
"*A rectangle and a parallelogram are almost the same, but may be one has all sides equal and the other does not*."
This is a reflection of a misconception, as noticed in Figure b, where P16 incorrectly applies the rectangular properties to a parallelogram.

Question Number 2: (11.5 marks)

Look at the shape given below. AD∥GH∥BC and AB∥FE∥DC and GF∥ BI and FH ∥ IC. Name all the shapes listed below with reasons:



a. All the trapeziums
b. All the rectangles
c. All the kites
d. Squares
e. All the parallelograms

In question (item) 2, learners were basically required to identify and list all the listed shapes in (a to e), found in the given image above, with reasons:

Below are the listed shapes (using their letters), based on the overall appearance of the figure:

Trapeziums:

GJIF	GBFE	AGEF
FIKH	FEHC	
AGEH	FEHD	
CDFI	ABIF	

Rectangles:

ABCD ADGH

GBCH AGIF

FIHD GBIE

IECH ABEF and FEDC

Kites

EFGH

Squares

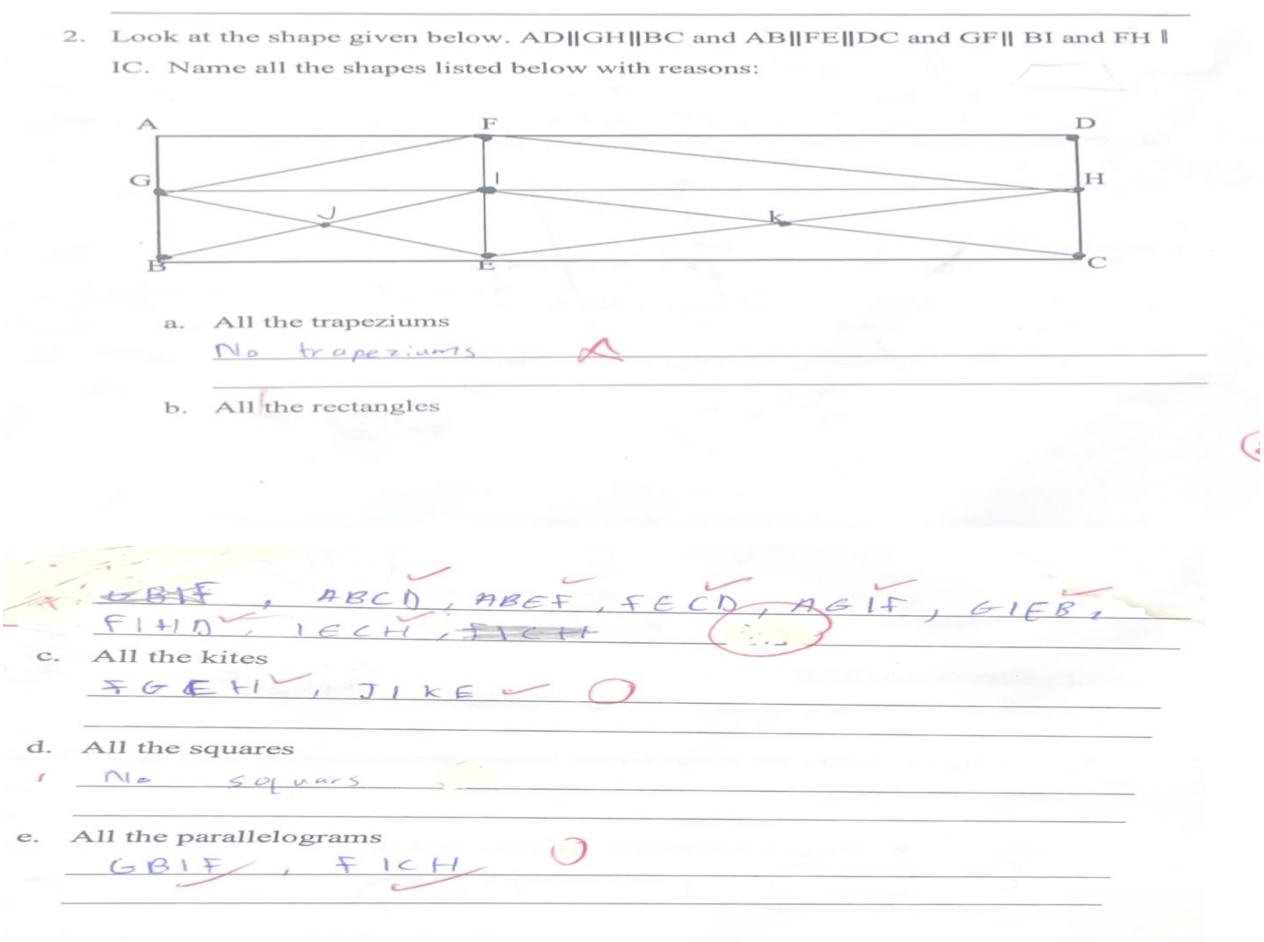
ABEF

Parallelograms

FICH GBIF

Below are findings from learners' responses to question 2

4.7: Findings from response of P2 to question 2



P2 struggled to identify trapeziums, stating, “no trapeziums.” However, when probed at the interview, the participant managed to identify one trapezium out of 9 trapeziums:

Researcher: “I see you did not name any shape that looks like a trapezium?”

(P2)*: “I realized later.”*

Researcher: “Okay, mention a shape that looks like a trapezium…”

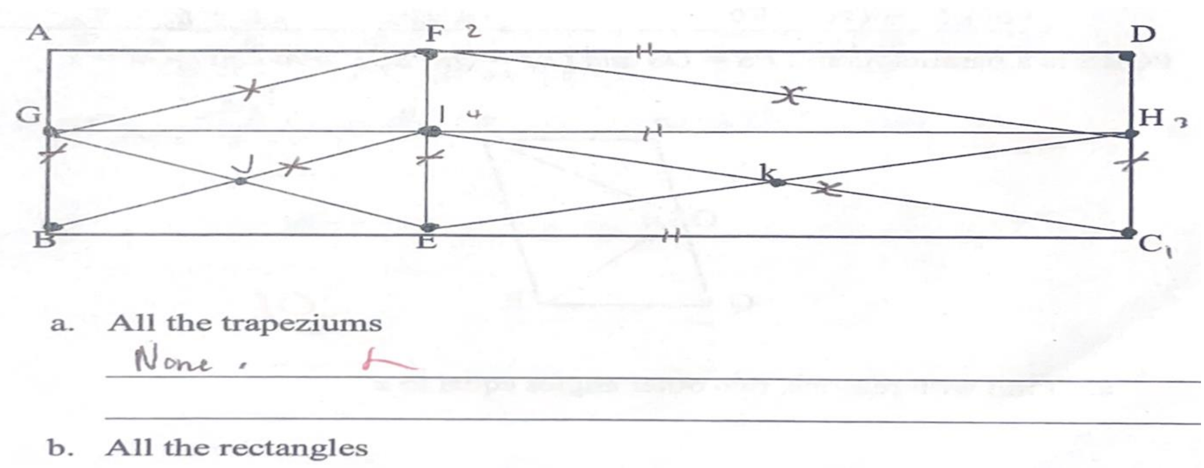
(P2): *“Ehh…GJIF…”*

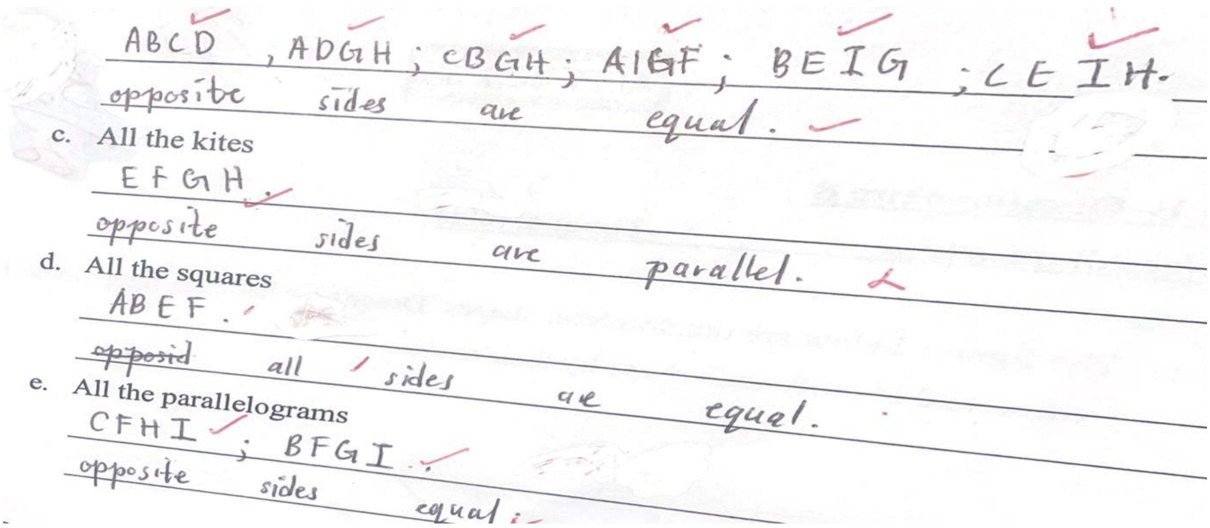
Researcher: Alright.

Though P2 seems to have had an initial difficulty in identifying a trapezium. However, P2’s performance still suggests that the learner has not completely mastered the required conceptual proficiency of identification of shapes like a trapezium, using its defining properties, especially within a complex diagram like the one given, which does not appear like a standard diagram. P2, upon reflection, following guided interaction, later recognised, and was able to identify one trapezium. P2 still lacks the required deep conceptual understanding to identify trapeziums, irrespective of the form in which it is presented.

Out of nine rectangles in the given figure, P2 demonstrated reasonable recognition skills by correctly listing seven of the shapes. P2 asserted that there was no square, whereas there is a square-ABEF. The same applies to the correct identification of both the ‘kites’ and the ‘parallelograms'; however, with a lack of conceptual clarity of properties of shapes, which reflects another common misconception.

4.8: Findings from the response of P3 to question 2





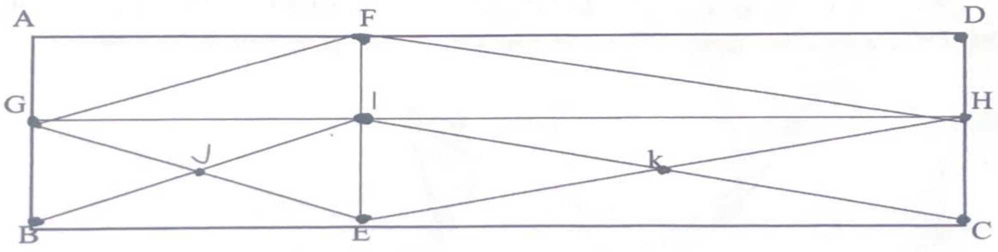
The learner (P3) couldn't identify any trapezium in the given diagram. When asked in the interview, P3 stated, "*I couldn't find any, to be honest. I didn't see any, but when you showed me, then it makes sense…yes..*" This misconception is related to difficulty in visually identifying trapeziums in the given seemingly complex diagram, which can be classified as a foundational misconception. The learner correctly identified multiple rectangles, with a valid property, though not sufficient property for correct justification: "opposite sides are equal.", as this property also applies to shapes such as, a square, and a parallelogram.

The learner also correctly identified the only square (ABEF), and one of the kite shapes- EFGH, though with no valid or defining properties for the respective shapes. This suggests a lack of conceptual understanding of the key defining properties of 'kite'.

P3 might have focused solely on diagonals intersecting at right angles, without considering the defining characteristic of adjacent equal sides. P3 also demonstrated the ability to visually identify the two (2) parallelograms, correctly- CFHI and BFGI. However, the same reason applies in the case of rectangular shapes. This suggests misconceptions of over-generalisation of properties, especially between rectangles and parallelograms, incomplete definitions of properties, and proper differentiation of shapes using their properties.

4.9: Findings from the response of P12 to question 2

2. Look at the shape given below. AD||GH||BC and AB||FE||DC and GF|| BI and FH || IC. Name all the shapes listed below with reasons:



a. All the trapeziums

None

b. All the rectangles

AFIG, GIEB, FDHI, IHEC, FDEC, ADCB good identification; no reasons.

c. All the kites

GFHE

d. All the squares

AFEB 1/2

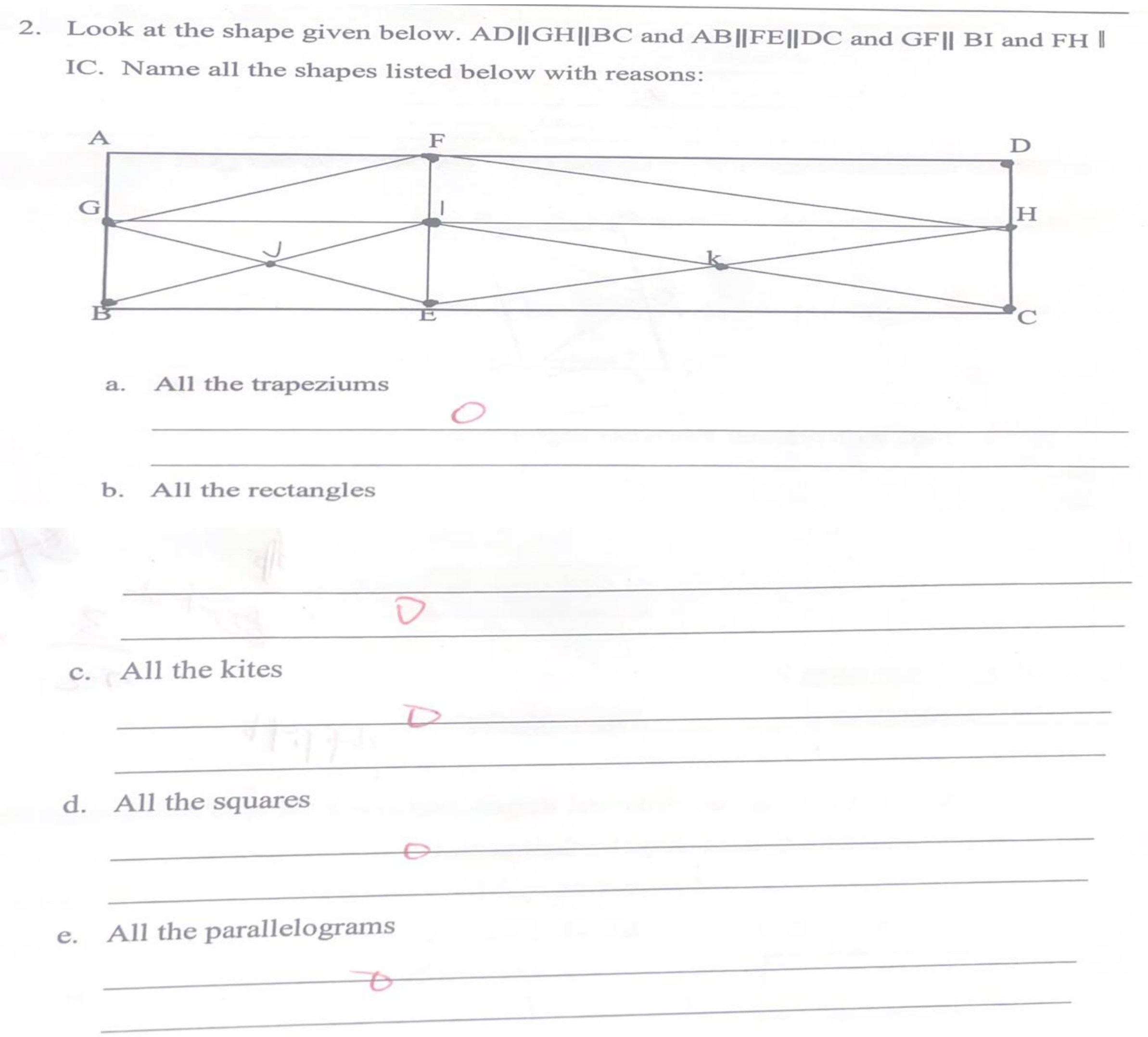
e. All the parallelograms

GFIB, FHCI

P12's response to part 'a' of the question stated, "*none*" under trapeziums. This suggests a lack of the required knowledge about the defining properties of a trapezium for easy identification. The learner correctly identified several rectangular shapes, but without justifying the identification. This omission implies dependence on visual identification, instead of application of conceptual knowledge of the defining properties of rectangles, such as equality of opposite sides and parallel sides. P12 correctly identified one kite (GFHE) out of the existing two, however, without any justification, such as 'adjacent equal sides and perpendicular diagonals, which are defining properties of a kite.

The learner also identified the only square, (AFEB), also without any justification, which reveals an over-reliance on visual cues. There is a lack of the required geometric reasoning for identification. The learner was able to correctly identify the two parallelograms (GFIB, and FHCI), although, without any justification. P12's responses demonstrate a lack of conceptual geometric reasoning at a foundational level and difficulty in articulating geometric properties, and probably depend on intuition. There is still a struggle in recognizing less familiar shapes, such as trapeziums, especially in a complex diagram given.

4.10: Findings from the response of P7 to question 2


2. Look at the shape given below. AD||GH||BC and AB||FE||DC and GF|| BI and FH || IC. Name all the shapes listed below with reasons:

a. All the trapeziums

b. All the rectangles

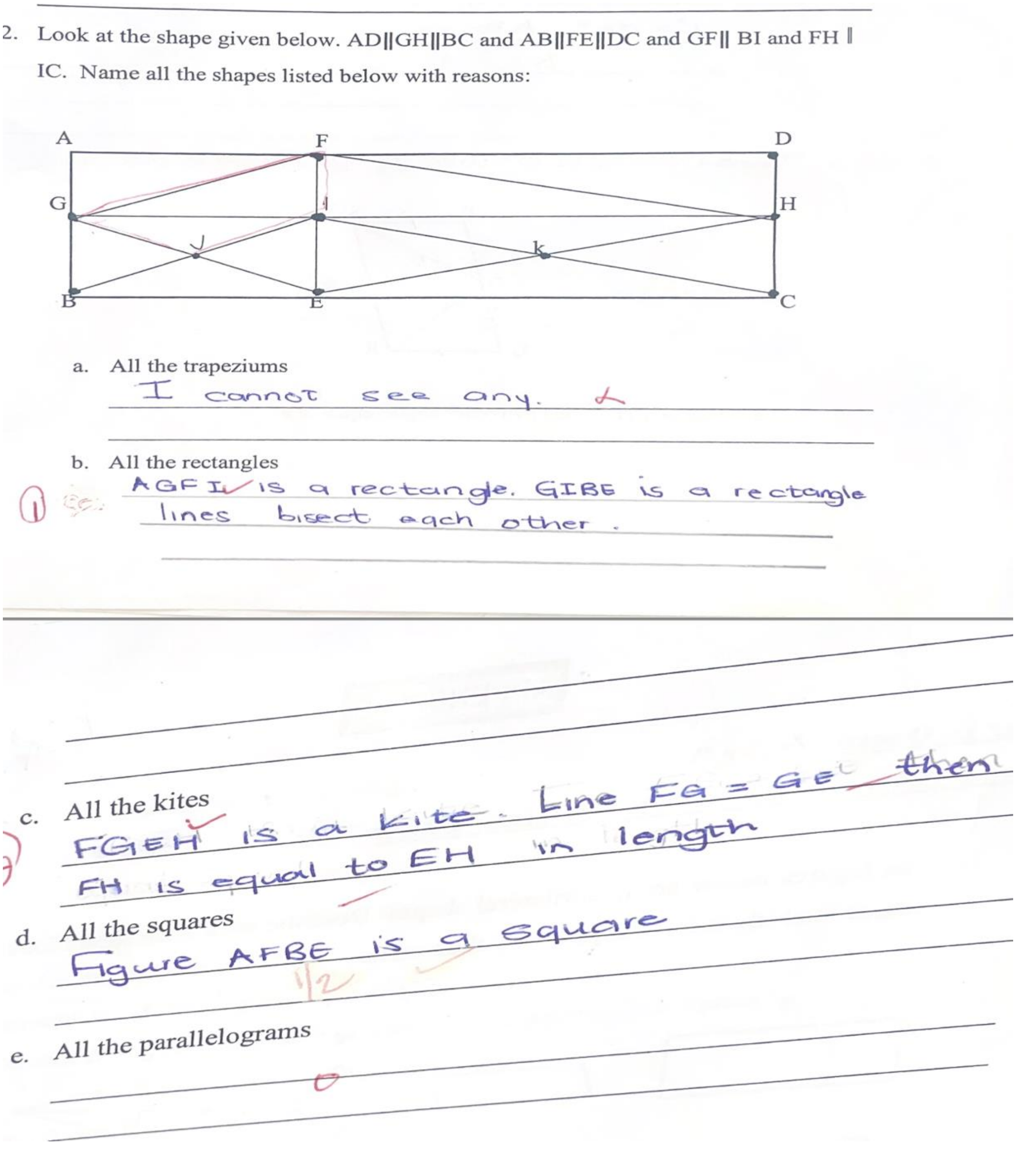
c. All the kites

d. All the squares

e. All the parallelograms

Learner P7 struggled to identify any shapes across the entire categories-trapeziums, rectangles, kites, or parallelograms. This suggests notable gaps in understanding geometric knowledge. No naming of any shape suggests either a lack of familiarity with the quadrilateral properties, or having difficulty in application, based on the given diagram. Probably, the complexity of the diagram, and its intricate nature could have overwhelmed the learner to process information systematically, hence, no attempt was made. No attempt could also be attributed to a lack of confidence in his or her response, hence, there was no demonstration of geometric reasoning, even at the foundational level.

4.11: Findings from the response of P13 to question 2



Learner P13's response to category 'a' of the question, under trapezium, stated, "I cannot see any." This indicates a lack of geometric familiarity with the properties of the shapes, such as the trapezium. It also suggests limited visual perception or conceptual understanding of trapeziums. From the interview, P13 did admit struggling to identify trapeziums:

Researcher: *"Okay, I see here in question number 2, in paper B, you said I cannot see any trapezium?*

P13: *"Trapezium…yes, I didn't ..it took my time"*

Researcher: *"But some of your classmate got it…."*

P13: *"They told me that they saw one."*

Researcher: *"Let's see….this is a trapezium; GJIF."*

P13: *"Oh my God…I couldn't see it."*

The learner correctly identified one of the rectangles, with an unclear reason; "lines bisect each," which does not align with the defining properties of a rectangle. It is a reflection of an incorrect definition of properties.

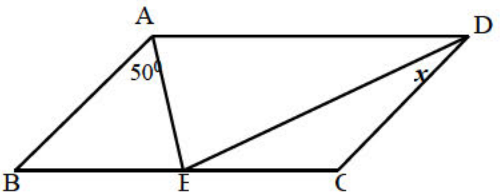
P13 correctly identified one of the two (2) kites, FGEH, with some justification ("line FG = GE; FH equals EH is length"). The learner demonstrated an attempt to provide side properties as the justification, though, it does not show deeper conceptual knowledge of the defining properties of a kite, such as, perpendicular diagonals, and one pair of equal adjacent sides. communicate property expression.

In the case of identifying squares, P13 correctly identified the only square shape, "AFBE", although without any justification. This could be based on a misconception or assumption, instead of the application of conceptual knowledge of the defining properties- equality of all sides, angles, and diagonals bisecting each other at 90^0. No parallelogram was identified, which shows difficulty or lack of application of the knowledge of the required properties of a parallelogram- equality of opposite sides, angles, and parallel.

P13's responses recorded several misconceptions, such as difficulty in the identification of shapes, such as, trapeziums and parallelograms. This could also be attributed to challenges in identifying shapes within a complicated diagram, such as the given diagram. Lack of property knowledge and proper application of the same properties across the shapes. There is evidence of limited foundational understanding of the properties of quadrilaterals.

Question 3

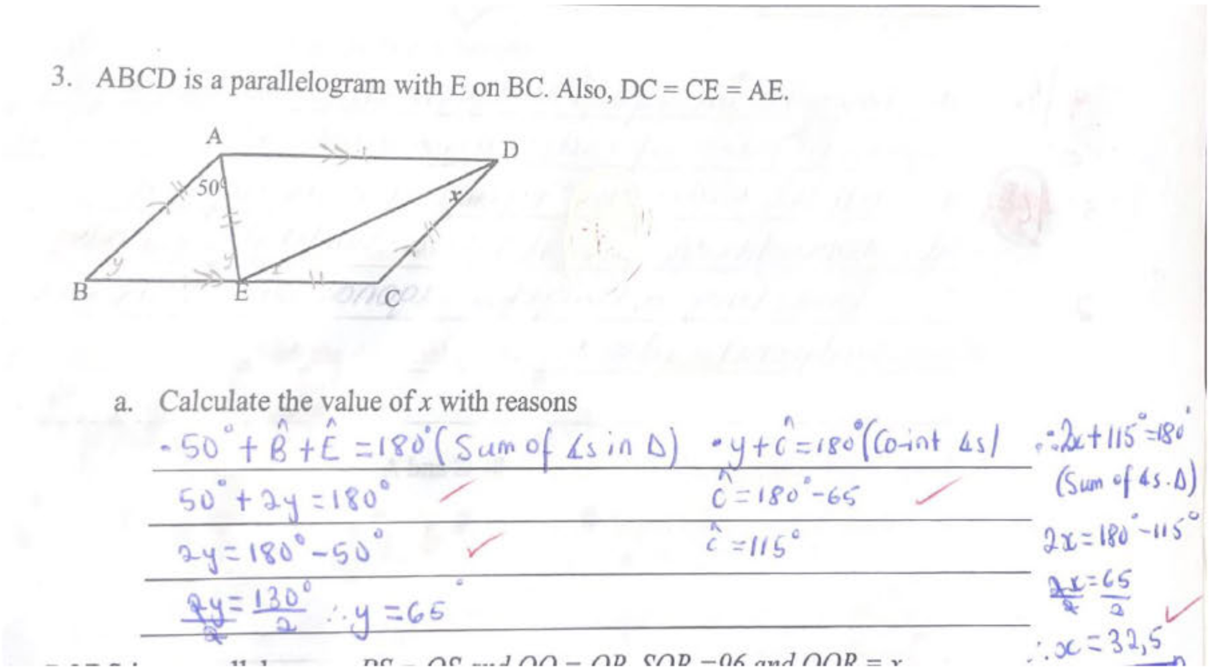
ABCD is a parallelogram with E on BC. Also, DC=CE=AE (8 Marks)



Calculate the value of x with reasons

Question 3 requires learners to have adequate geometric skills in recognising and articulating quadrilateral properties in solving quadrilateral-related problems. In this case, with the knowledge of properties of a parallelogram and that of an Isosceles triangle, learners were required to find the required angle x with reasons. Most learners (16) did not attempt this question. Only three learners demonstrated the required analytical proficiency in tackling the problem. Below is the response of one of the three learners- P13

Figure 4.12: Findings from the response of P16 to question 3



P16 was among the three that correctly calculated the value of x=32.5^0, through the application of the properties of a parallelogram. The learner demonstrated an understanding of geometric knowledge of parallelogram properties. Hence, P16's reasoning indicates good familiarity with the sum of angles in a triangle, which is 180^0, and understanding of complementary angles, which was factored systematically.

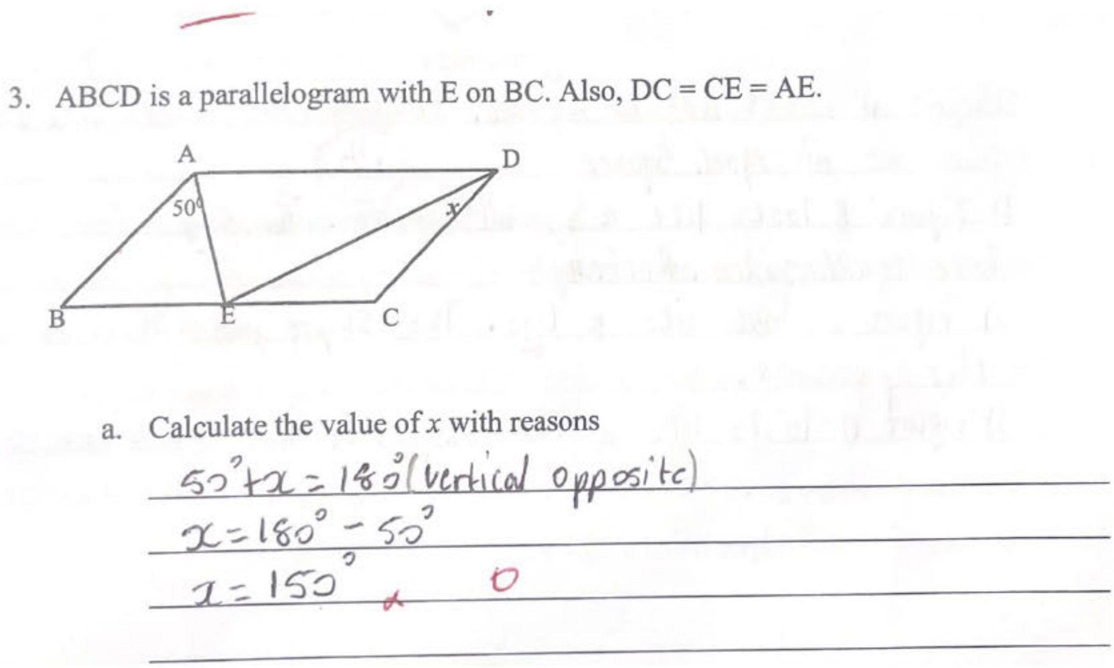
The learner may have adopted procedural skills in calculations, demonstrating an understanding of the geometric properties of parallelograms and relationships between angles, such as equality of opposite angles and supplementary angles within triangles:

$2y = 180^0 - 50^0 \Longrightarrow \quad y = 65^0$

$2x + 115^0 = 180^0 \Longrightarrow \quad x = 32.5^0$

However, although the calculations are correct, misconceptions, such as, the mismanagement of symbols or the inability to label the angles for clearer explanations and full conceptual clarity, were evident. The learner's responses did not provide a detailed justification of the properties that are being applied in the calculation process (e.g., parallelogram's angle properties). P16 must have relied strongly on procedural skills, by following a systematic formula-based approach, instead of a conceptual-based approach or understanding. Hence, the limited conceptual explanation and lack of clear reasoning are evident.

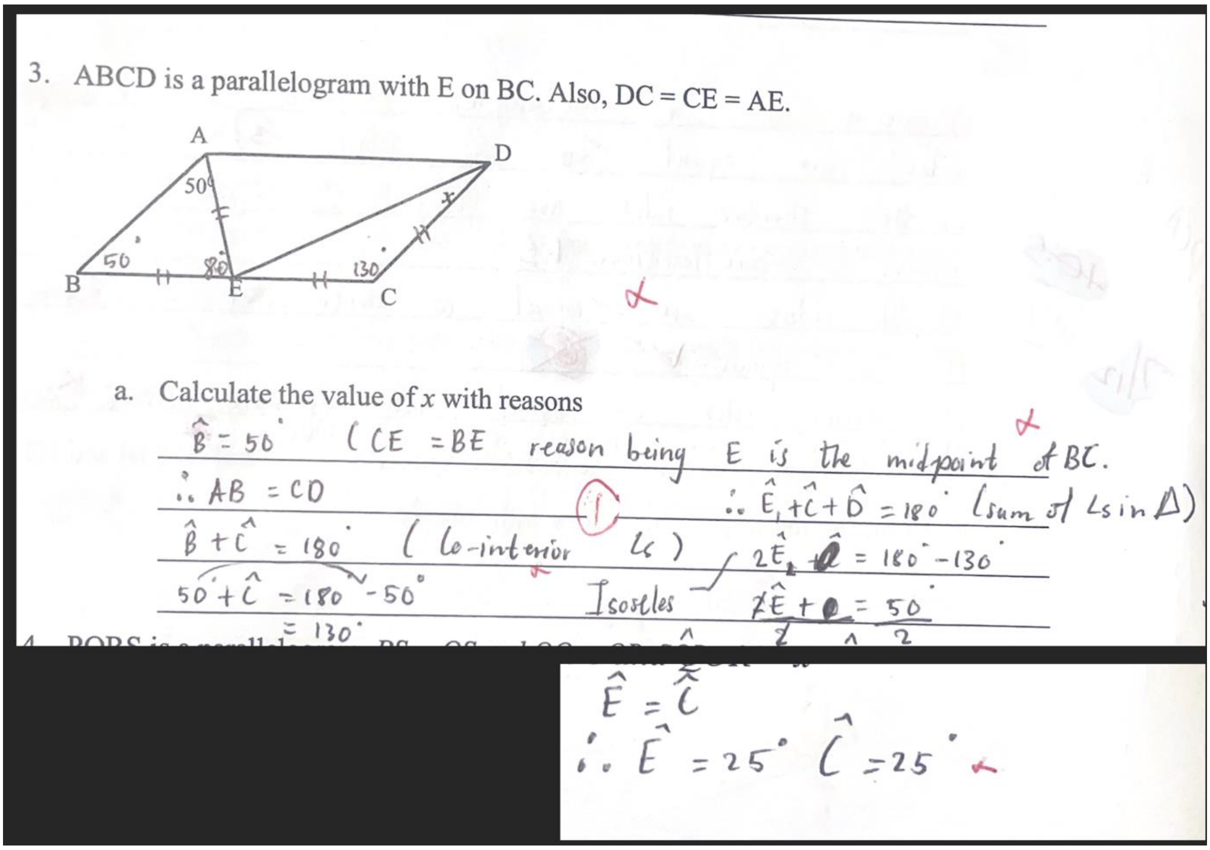
4.13: Findings from the response of P21 to question 3



First, the value of x as 150^0 is incorrect. P21 misapplied the property (concept) of vertical opposite angles, which indicates a fundamental misconception of the relationship of angles within parallelograms.

By inappropriately applying the 'vertical opposite angle' rule in the context of a parallelogram, it shows not sure of the specific geometric properties of parallelograms to use. P21 confused angle types, by failing to recognise that in parallelograms, adjacent angles are supplementary, which should have been applied in this context (problem). P21's response did not reveal conceptual knowledge of broader properties of a parallelogram, to ensure the very correct and applicable property in finding the value of x. Rather, there was reliance on isolated procedural knowledge, (e.g., vertical angles). In general, P21's response seems to have depended on a procedural approach rather than geometric reasoning, hence the confusion of not knowing the correct angle property to use within a parallelogram. This reveals a gap between mastering geometric concepts and their applications.

4.14: Findings from the response of P3 to question 3


3. ABCD is a parallelogram with E on BC. Also, DC = CE = AE.

a. Calculate the value of x with reasons

$\hat{B} = 50°$ (CE = BE reason being E is the midpoint of BC.
∴ AB = CD
$\hat{B} + \hat{C} = 180°$ (co-interior ∠s)
$50 + \hat{C} = 180° - 50°$
$= 130°$
∴ $\hat{E}_1 + \hat{C} + \hat{D} = 180°$ (sum of ∠s in Δ)
$2\hat{E}_1 + \hat{C} = 180° - 130°$ Isosceles
$\frac{2\hat{E} + \hat{C}}{2} = \frac{50°}{2}$

[redacted]

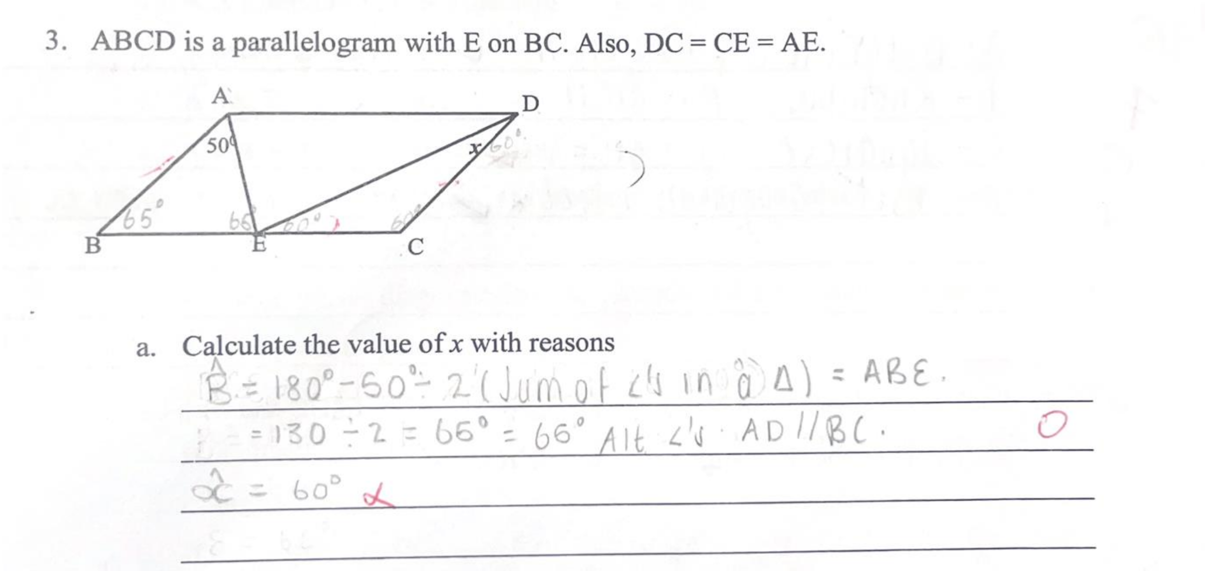
$\hat{E} = \hat{C}$
∴ $\hat{E} = 25°$ $\hat{C} = 25°$

P3's calculations for the value of x are not correct. The solution, ($x = 25^0$) is also incorrect. P3's approach reveals procedural errors and wrong reasoning, regarding the use of properties of angles in parallelograms and triangles.

The key misconceptions here were the lack of understanding of the angle relationships in a parallelogram. This is based on the wrong application of the co-interior angles concept found in a parallelogram, with a wrong interpretation of the supplementary angle aspect of it. Misinterpretation of isosceles triangles was also evident as another misconception. This is also evidenced by the assumption of P3 taking angles DEC and ECD is triangle DEC to be equal, based on the reasoning isosceles triangle property. Although, P3 marked E as the midpoint of BC, the learner fails to appropriately use this information to reason through the problem.

Overall, procedural knowledge over conceptual understanding is evident. P3 seems to have been guided by isolated rules, such as properties of co-interior angles, and isosceles triangles, without taking cognizance of or full understanding of their applications in relation to parallelograms. Deficiency in mastering the properties of a parallelogram, such as, supplementary nature of adjacent angles or relationships between diagonals and vertices, is evident.

4.15: Findings of the response of P1 to question 3



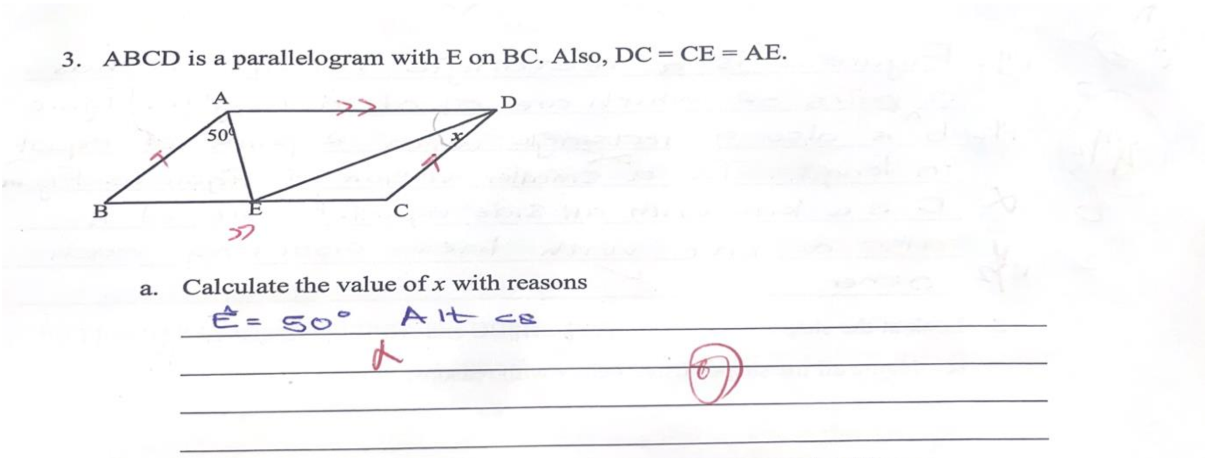
The value of x given as 60^0 by P1 is incorrect. Two major misconceptions are evident: The reasoning process, which involves the incorrect application of the triangle properties, (angle sum of a triangle), and the corresponding relationships of the parallelogram's angles. The incorrect application of the alternate interior angles property, "Alt ∠'s AD || BC", shows the learner's misunderstanding regarding how alternate angles relate in the question, based on the parallelogram and triangle. The referencing of the sum of angles in △ABE is correct, but the learner's inconsistent approach in calculating the angles shows difficulty in relational reasoning between angles within the given shape (parallelogram) and its diagonals.

The response of P1appears to have depended on applying property rules in isolation, but faces difficulty in incorporating these concepts (rules), such as the application of the properties of angle sum in a triangle and that of alternate angles, in a logical approach. P1 also seems to have presumed equality of angles without seeing the need for mathematical authentication, which shows reliance on memorization of rules, instead of a deeper understanding of the required concepts. Below transpired when probed at the interview:

> Researcher: *"In question 3, how did you get angles ABE and AEB as 65^0?"*
> P1: *"I subtracted 50^0 from 180^0, since the sun of a triangle is 180^0, and then divide the result by 2."*
> *Researcher: "But, on what basis did you do that? Do you know what kind of triangle is ABE, based on the question?"*
> P1: *'Oh no, I assumed it is triangle ABE, I equilateral. No, not equilateral. It could be an isosceles triangle that has two equal sides."*

Incorrect assumptions and wrong applications of shape properties were observed, which revealed a lack of knowledge of angle relationships and properties, which were the main misconceptions.

4.16: Findings from the response of P13 to question 3

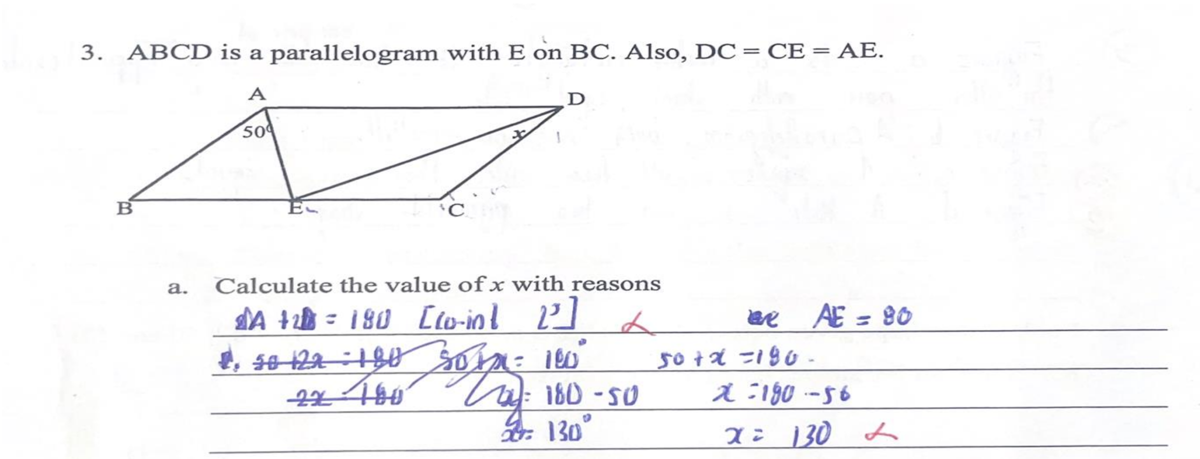


P13's response did not provide a correct value for x. Besides, the calculation was left incomplete, stating, ∠E $=50^0$. This incompletion is a reflection of a lack of conceptual understanding of how to use the information given, (regarding ∠ABE and the parallelogram properties efficiently), in finding the value of x. When probed at the interview, P13's response reflects a lack of confidence in the given information. The learner expressed doubt and uncertainty regarding providing parallelogram properties. Below transpired at the interview:

> Researcher: *"Okay, and also in question number 3, paper B?"*
> P13*: "I was just starting…but I was not sure about it…. because these lines are not parallel… that was my problem…."*
> Researcher: *"Which lines?"*
> P13: *"These ones". (referring to this symbol: >>)*
> Researcher: *"AD and BC?"*
> P13*: "Yes, that's why I wasn't sure if they're parallel… v.* Researcher: *"Who said they are not parallel?"*
> P13: *"Because I didn't see those parallel sign."*
> Researcher: *"But the question already tells us that the shape is a parallelogram…you don't have to look for the sign to confirm whether the shape is a parallelogram or not, okay?*
> P13: *"Okay,"*

The above responses show that the learner was not sure as to whether AD||BC, as a result of the absence of the symbols in the diagram, which is often used to denote parallel lines. This reveals P13's dependence on visual signs (symbols), rather than using the information provided in the question. P13's reasoning implies a lack of conceptual understanding of the implications of the properties of parallelograms, in calculating the value of x; such as opposites sides/angles are equal, as well as parallel, or supplementary consecutive angles, with or without the presence of parallel symbols. P13 lacks the conceptual ability to identify the relevant geometric properties from the question in calculating the value of x. The learner did identify angle 50^0, which shows an initial understanding of angles in the given triangle. However, he struggled to proceed further as a result of conceptual and procedural gaps.

4.17: Findings from the response of P14 to question 3



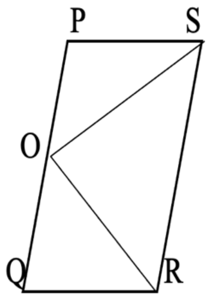
P14's calculation for the value of x as 130^0 is incorrect, which shows a lack of understanding of the relationships between parallelograms and triangles. A key error in the response was P14's assumption of ∠A + ∠D =180^0, which was not relevant based on the given task. Ignorance of vital information that would have been instrumental to calculating the value of x, where evident, such as ∠ADE and the sides DC, CE and AE, which are congruent.

The reasoning of P14 indicates both overgeneralisation of properties and reliance on procedural rules, rather than focusing on the clarification of concepts. The learner struggled to incorporate parallelogram properties, such as, equality of opposite angles or triangle relationships, resulting in procedural mistakes. Notable misconceptions are misidentification of angle relationships and overlooking the information provided in the question.

P14 may have identified the importance of angle relationships, but was unable to apply the required geometric concepts, which reveals the necessity for deeper conceptual understanding of the properties of the two shapes in question- the parallelogram and triangles.

Question 4:

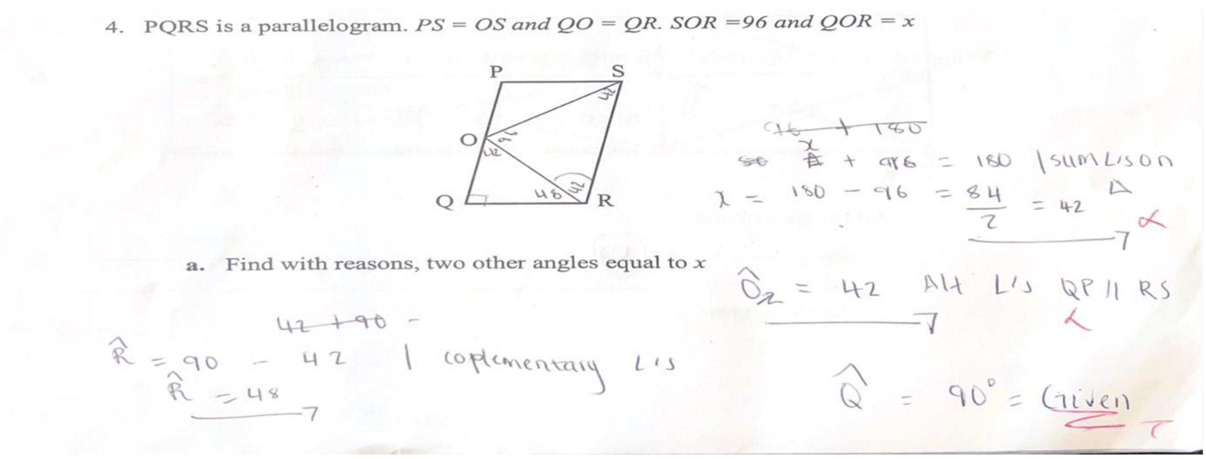
PQRS is a parallelogram. PS = OS and QO = QR. SOR =96 and QOR = x (10 marks)



Find with reasons, two other angles equal to x

The question basically requires recognition of basic geometric relationships, such as the Isosceles triangle property and alternate angles in a parallelogram. Learners, therefore, were to recognise the properties of a parallelogram, analyse their relationships with the embedded triangles, and logically apply them, aimed at finding two *other angles, having the same value as x.* None of the learners' responses was correct due to several notable misconceptions. Below is Figure 4.19, indicating the response of P11.

4.18: Findings from the response of P11 to question 4



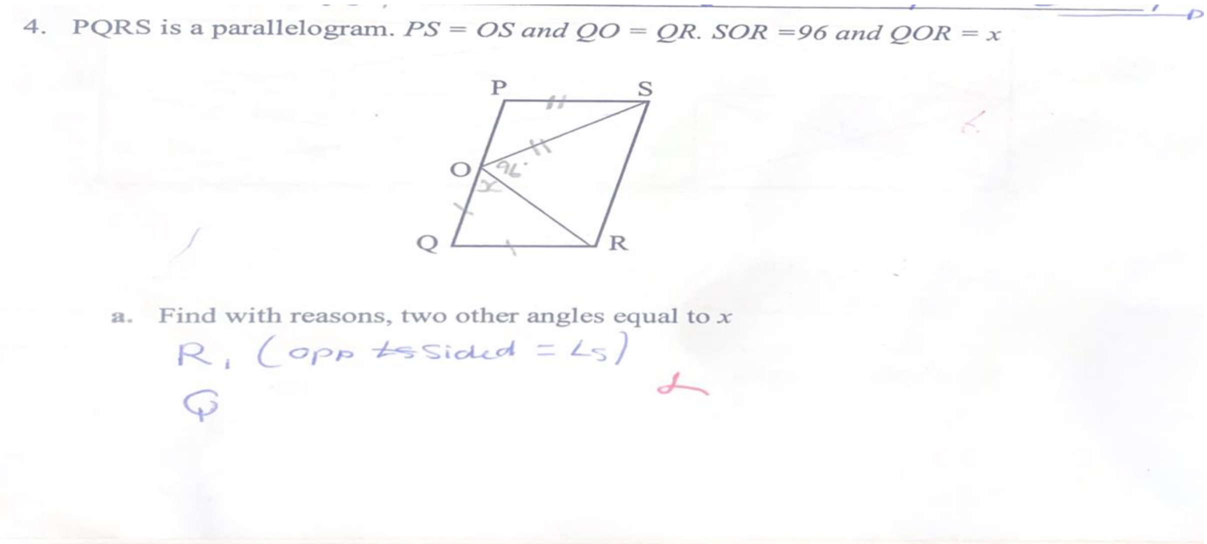
Notable misconceptions were evident in P11'resposne to item 4, in relation to angle relationships and properties of a parallelogram. P11 incorrectly assumed angle relationships. It seems P11 involved external angles from △QOR. Secondly, P11 erroneously used supplementary angle relationships. Although adjacent angles are supplementary in a parallelogram, however, P11 failed to recognise that ΔQOR is part of a parallelogram, with its angle summed to 180^0, based on the property that says that the three angles of a triangle sum to 180^0, instead of supplementary relationships.

Another possible error was that P11 may have wrongly believed that ∠QOR and another angle (possibly ∠Q or ∠R) had to be added to sum to either 90^0 or 180^0, which was not the case in this instance. Hence, errors were noticeable in both the calculations and reasoning.

Since QO=QR, (given), it means that △QOR is an isosceles triangle, which is the key to finding the two other angles equal to *x*. P11 should have focused solely on △QOR and recognised that the base angles ∠QOR and ∠QRO are equal to *x,* without involving any external angles from the parallelogram. Thereafter, the 'alternate angle property' applies to the second angle whose value is *x;* ∠QOR = ∠ORS (*alt angle*).

P11's approach suggests dependence on assumptions and disjoined reasoning, instead of making use of the known parallelogram properties, like equality of opposite angles and adjacent angles, which are supplementary. In addition, the use of the term "complementary angles" was not necessary in this instance, which also shows a lack of conceptual understanding and evidence of struggle in involving multiple principles of geometry in a coherent solution.

4.19: Findings from the response of P12 to question 4



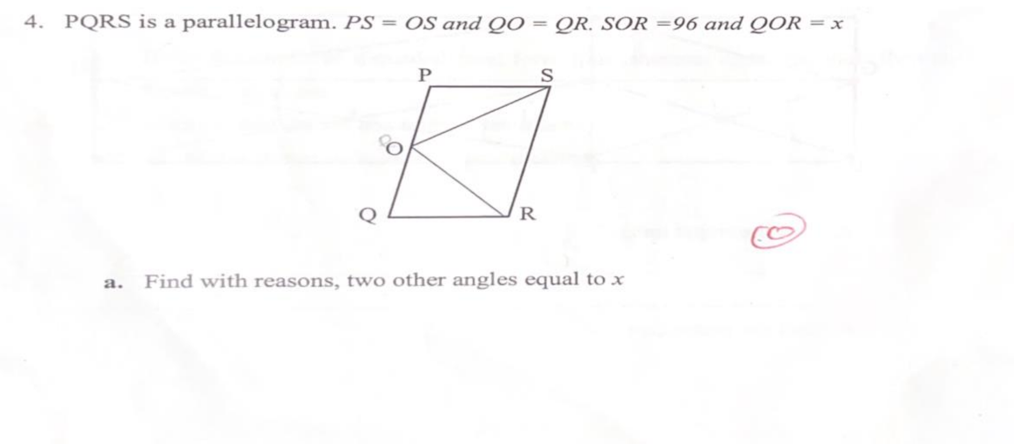
P12's responses reveal notable misconceptions. The learner's reasoning was incorrect, as evidenced by the incorrect identification of angle R_1 (opposite angles), as equal to x, with no justifiable reason or calculation. The reasoning given has no bearing on the properties of a parallelogram, especially, the relationships between opposite and adjacent angles. Lack of detailed justification, as evidenced by the single phrase P12 provided, "opposite sides = angles," also reveals a lack of depth in geometric reasoning, hence, P12 was unable to provide reasoning grounded in the given geometric relationships. P12's reasoning also reflects confusion between opposite and adjacent angles. This is evidenced by the incorrect assumption P12 holds, that opposite angles automatically are equal without taking into cognizance the parallelogram properties. The learner, cannot approach geometric relationships logically, as attested in the interview:

> Researcher: *"I see you got the answer in question 3 in section b with the workings…What about question 4a?*

P12: *Yes sir….I have challenges with proving…that is why I didn't attempt those questions that require proof, and questions 7 and 8…*

P12's inability to engage with this proof-related question logically, shows no confidence and acquaintance with the required process of logical reasoning in tackling proof-related questions. Briefly stating, "opposite sides = angles," shows evidence of dependence on rote learning, rather than conceptual reasoning and understanding of properties and their relationships.

4.20: Findings from the response of P7 to question 4



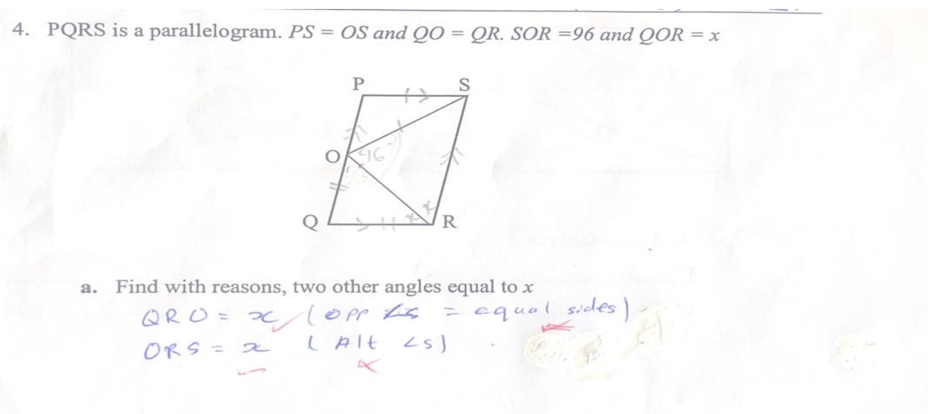
P7 made no effort in attempting question 4. The learner was unable to identify and geometrically reason regarding angles in a parallelogram, indicating either lack of confidence in approaching such questions or uncertainty. P7's response at the interview reveals an important difficulty in terms of geometric comprehension and application of the concepts. This is evidenced by the learner's response, when asked about the reason for not attempting question 4. P7 stated, *"Yes, it was quite challenging…I didn't know what to do."* When further probed, below was the response:

Researcher: *But you have been taught this topic…have you not been taught about it in class?*
P7: *yes…we have*

Although, P7 admitted to having been taught, yet, the learner struggled to apply the concept taught, which also shows that the challenges faced are not necessarily as a result of a lack of exposure, but rather unable to either apply or internalise the concepts efficiently. The notable misconceptions and influencing factors include a lack of conceptual clarity. P7 was unable to establish vital parallelogram properties, such as, equality of opposite angles and supplementary adjacent angles, in solving the problem. No attempt shows either an inability to recall these principles or identify their relevance to the given question.

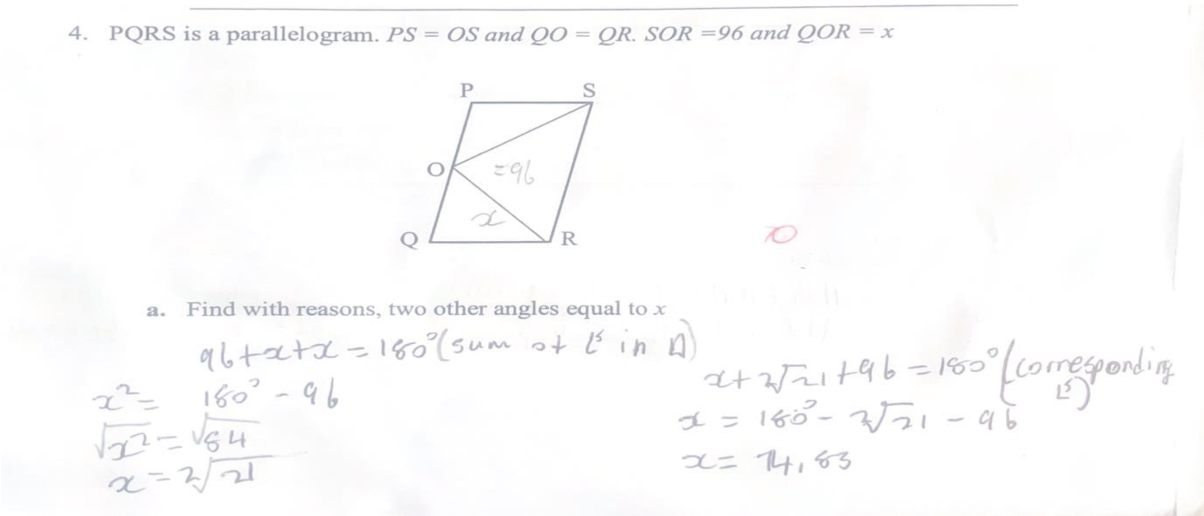
P7's response also reveals a deficiency in proof-based reasoning, as the learner was unable to apply logical reasoning and geometric properties in deducing the two angles with the same value as x. Indeed, P7 was not comfortable attempting the proof-related question (such as question 4), due to a lack of familiarity. P7's lack of confidence in attempting a proof-related question shows a lack of analytical reasoning. In general, P7 struggles with the application of geometric knowledge, which is compounded by a lack of confidence and reasoning difficulty.

4.21: Findings from the response of P2 to question 4



P2 demonstrated the ability to establish (recognising and articulating) the other two angles with same value as x, using geometric properties in solving the problem, but without clear reason or justifications. The incomplete use of geometric terminologies was evident as one of the misconceptions, hence, the reason given was not clear. Where P2 stated QRO= *x* ("opp. ∠s = equal side." The correct reason should have been, *"opp base ∠QRO and ∠*ORS *of the Isosceles triangle △QRO are =).* Secondly, stating, *"∠ORS = x (alt ∠*s)" lacked clarity. The correct reason should have been, ∠QOR = ∠ORS, *(Alt interior ∠s between parallel PQ and SR with transversal RO). So,* ∠ORS = *x.* P2's reasoning lacked depth and specificity, although, there was evidence of identification of the two angles.

4.22: Findings from the response of P21 to question 4



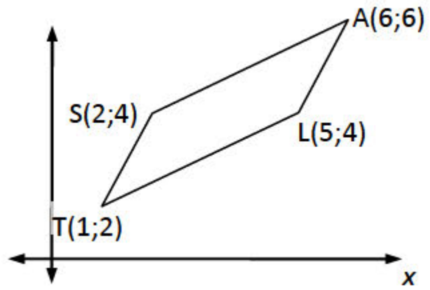
P21's response to the question highlights significant flaws and misconceptions in light of the interpretations given, as evidenced by the misapplication of square roots (√x), which is irrelevant in this instance. This shows that P21 lacked the required geometric principles (concepts) with regard to geometric properties, specificity, and angle relationships within parallelograms.

P21 demonstrated a lack of conceptual understanding of the properties of a parallelogram, (that is, the relationships in angles, within a parallelogram, and the rules of parallelism). The learner's misinterpretation of the sum of angles to 180^0, by assuming the equation 98 + x + x = 180^0 is a reflection of a major misunderstanding of the relationship of angles in a triangle within a parallelogram. The decision to apply this formula $x + 2\sqrt{21} + 96 = 180^0$ in this instance highlights a lack of geometric concepts and principles of the sum of angles in a parallelogram, and alongside geometric relationships.

P21 also ignored the properties of parallel lines, hence, struggled to apply by reasoning, the concepts of either alternate angle or opposite angle properties, reflecting a lack of conceptual understanding of the geometric properties, particularly, the parallelogram.

Question 5:

Quadrilateral ABCD with vertices S(2;4), A(6;6), L(5;4) And T(1;2) is shown in the figure alongside. Show that SALT is a parallelogram. (10 marks)

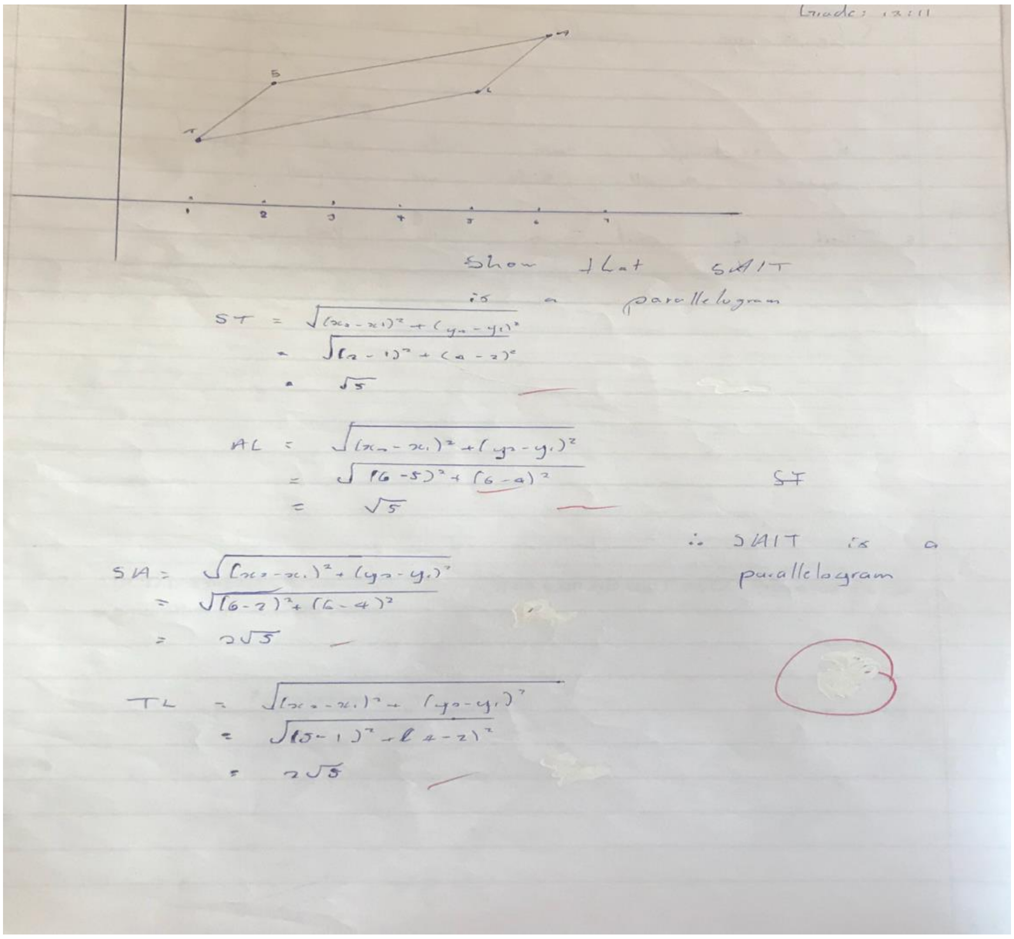


In question 5, learners were required to demonstrate that quadrilateral SALT is a parallelogram by applying their understanding of geometric properties through logical reasoning.
To prove that SALT is a parallelogram, participants were required to verify one of the sufficient defining properties of a parallelogram, namely, both pairs of opposite sides are equal, both pairs of opposite sides are parallel, or both diagonals bisect each other.

Learners can use the Distance formula to determine equality of both pairs of opposite sides, specifically SA and LT, as well as ST and AL. Alternatively, learners can also verify parallelism (another important parallelogram property) by calculating the slopes of both pairs of opposite sides, which must be of the same value: SA and TL, as well as ST and AL. Thirdly, learners can also apply the midpoint formula to show that the diagonals bisect each other, by finding the midpoint of both diagonals. Although, verifying one of the mentioned conditions is sufficient, however, validating additional properties, such as parallel sides or bisection of diagonal, will strengthen the verification.

As indicated in Table 4.5, most learners generally demonstrated significant procedural geometric proficiency, as evidenced by their responses to the question. This could be attributed to the simplicity and straightforwardness of the question, which requires both the distance formula and gradients. Out of 16 participants whose question script had item number 5, scored 70%, or above, except one learner, who scored 50%. No participants left the question unanswered. Figure 4.24 below shows the response from P5 to question 5.

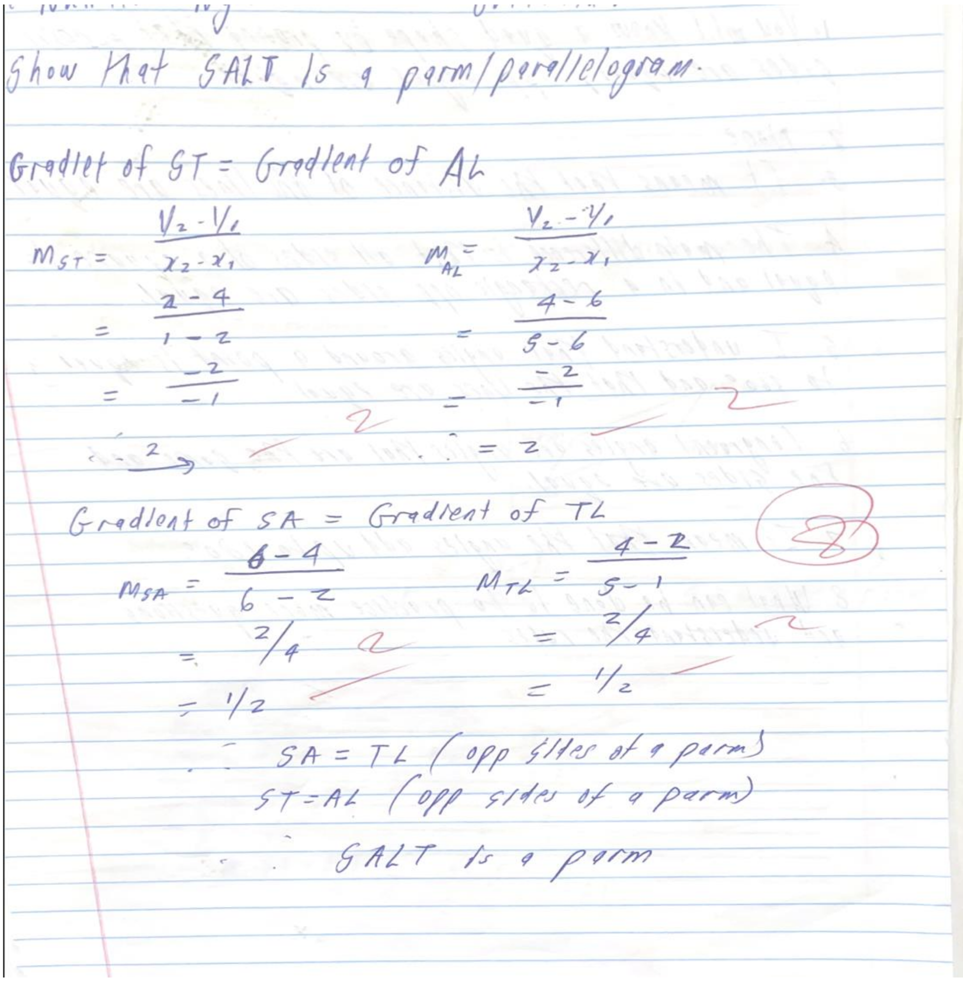
4.23: Findings from the response of P5 to question 5



P5's response reveals proficiency procedurally. The learner used the distance formula in determining the equality of sides: ST = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

P5 correctly calculated sides ST and AL as equal to $\sqrt{5}$, as well as sides SA and LT as equal to $2\sqrt{5}$. P5 concluded SALT as a parallelogram based on the values of the congruency of both pairs of opposite side lengths – a key defining property of a parallelogram. Nonetheless, determining the equality of opposite sides' lengths would have been further strengthened by verifying additional properties, such as the parallelism of opposite sides or the bisection of the diagonals. P5 failed to further establish another property to clearly verify that SALT is a parallelogram. In addition, P5 failed to verify or justify why side lengths alone are enough to justify SALT as a parallelogram. The minimum condition was met. There is evidence of demonstration of procedural understanding of the distance formula used for the sides, but the lack of conceptual reasoning beyond the use of the formula was evident.

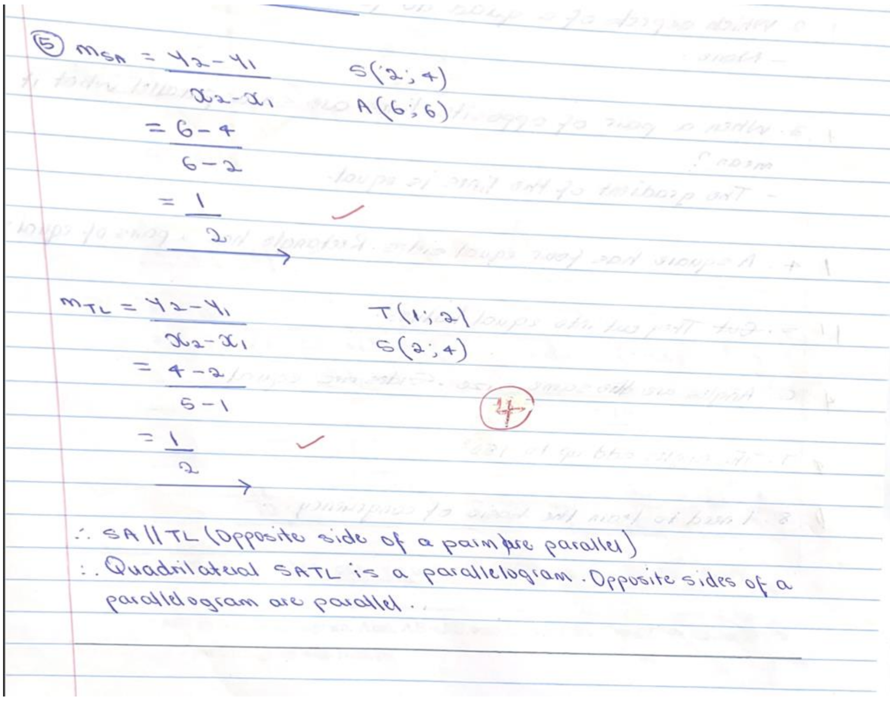
4.24: Findings from the response of P6 to question 5



P6's response established that SALT is a parallelogram by confirming the parallel nature, through calculating the gradients (slopes) of both pairs of opposite sides. According to P6, the gradient value, of ST, is equal to the gradient value of AL. Likewise, the gradient value of SA is equal to the gradient value of TL. On this premise, P6 confirmed that SALT is a parallelogram, since the opposite sides have equal gradients, supposedly confirming that the opposite sides are parallel.

However, P6 failed to demonstrate deeper conceptual understanding, by further providing the values sides lengths using the distance formula. P6's conclusive statement does not truly verify or justify SALT as a parallelogram. There was no explicit statement to justify the calculation. Furthermore, P6 seems to have assumed that the gradient values also imply that the opposite sides must be equal as well, without calculating the side lengths using the distance formula. The learner must have relied only on the gradient formula for verification.

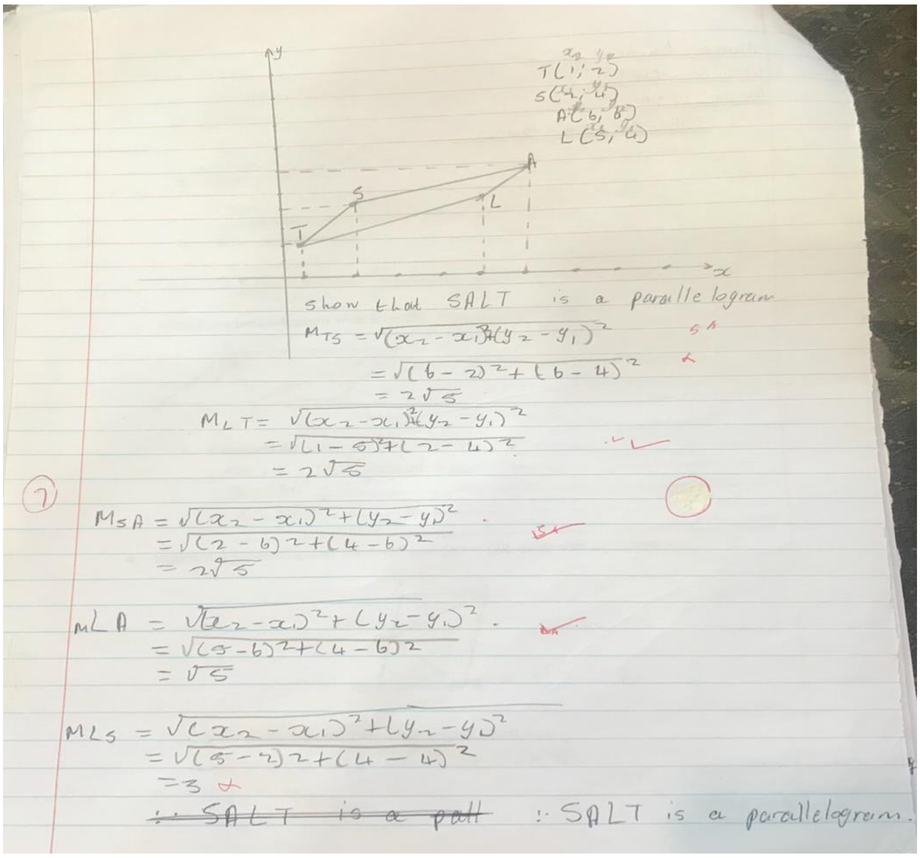
4.25: Findings from the response of P16 to question 5.



P16’s response to question 5 did not demonstrate the required sound proficiency of geometric reasoning in terms of establishing the conditions to prove that SALT is a parallelogram. P16 correctly calculated the gradients of one pair of the opposite sides- ST and AL, but failed to calculate the other pair of the opposite sides- SA and TL to correctly confirm the parallel nature, which is a key requirement to fully prove that a quadrilateral is a parallelogram.

Both pairs of opposite sides must be proved as parallel, or one pair of opposite sides must be proved as both parallel and congruent. Hence, confirming the parallelism condition is still not sufficient to prove SALT to be a parallelogram. Omitting the side calculation (using the distance formula) is also an indication that although one pair of the gradients was correctly calculated, P16 did not fully meet the necessary condition to prove SALT to be a parallelogram. Overreliance on a formula suggests a lack of a deeper understanding of the required defining geometric properties of a parallelogram.

4.26: Findings from the response of P21 to question 5.



P21 made an attempt to prove that SALT is a parallelogram, by calculating the side lengths, using the distance formula. This shows that P21 has some procedural understanding of the principle. The learner correctly calculated one pair of opposite sides; SA, and TL, as $2\sqrt{5}$. However, reviewing the length, TS shows it was wrongly calculated. Hence, SALT could not be proved to be a parallelogram as the required condition must either establish the parallelism on both pairs of opposite sides or verify one pair of opposite sides parallel and equal. In other words, P21 wrongly calculated the length of TS, although the other corresponding opposite side of TS, which is LA, was correctly calculated as $\sqrt{5}$.

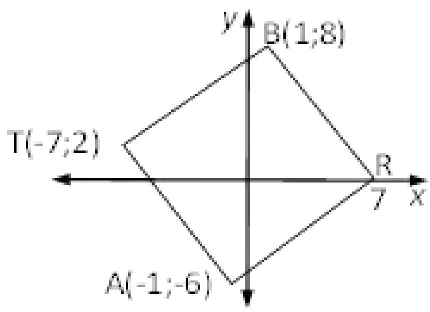
Although P21 applied the distance formula correctly for the verification of opposite sides equal, however, the learner failed to verify the parallelism, using the gradient formula, which thus, makes the proof not fully conclusive. Most learners' response seems to have a common misconception that once they prove the equality of opposite sides, it automatically implies that the quadrilateral is a parallelogram. This is not necessarily correct as quadrilateral shapes, such as rectangles and rhombuses, also share the same property- equal opposite sides.

Hence, additional property will be needed to differentiate a parallelogram from the other quadrilateral class members. Generally, the learner demonstrated procedural accuracy in relation to the calculation of distances, however lacked conceptual understanding to be able to fully meet the sufficient condition of labelling a quadrilateral parallelogram.

Question 6:

Below is a quadrilateral BRAT. Use the quadrilateral to answer the following questions. (11 marks).

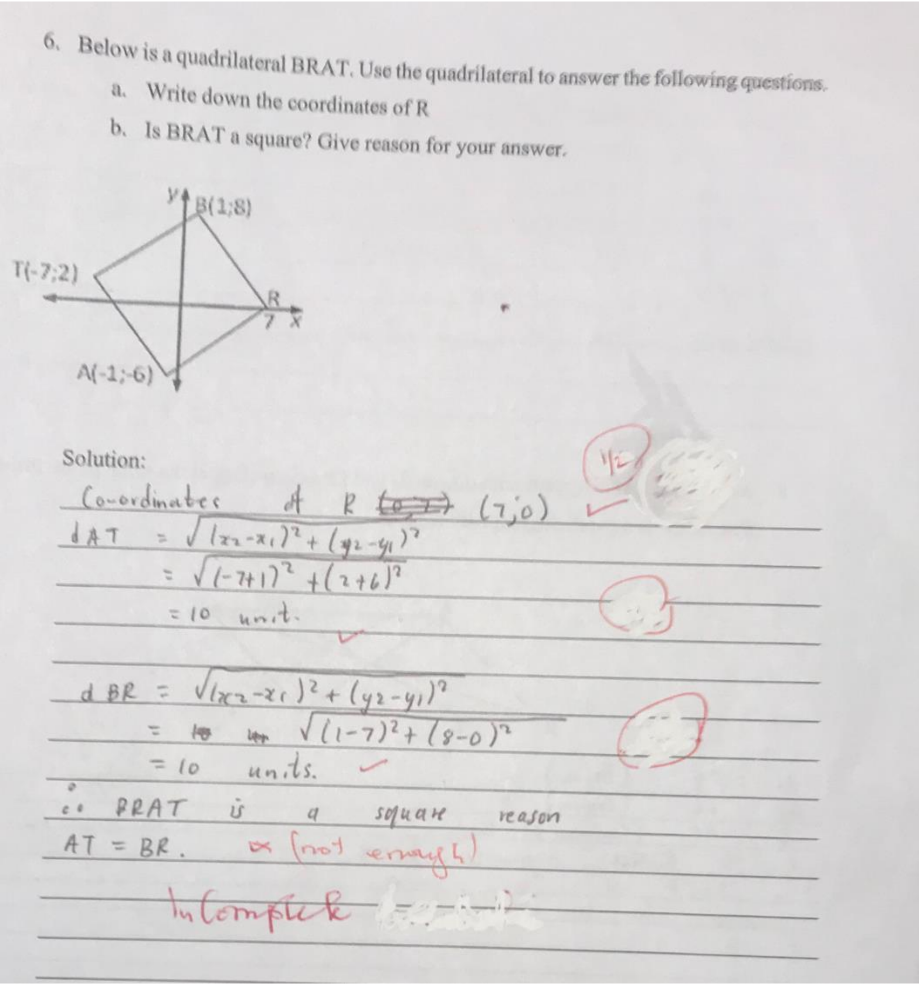
Write down the coordinates of R (7; y), where *y* lies on the x-axis. (1 mark)
Is BRAT a square? Give a reason for your answer. (10 marks)



Question 6 requires analytical and logical reasoning to establish relationships between the properties of geometric figures. In question 6a, learners were asked to find the missing value of *y*, why y lies on the x-axis. All that was required here was for learners to say that the value of *y* is 0 because *y* lies on the x-axis. Question 6b requires learners to prove that the figure in the coordinates, BRAT, is a square. To prove it, the application of square properties is required.

To establish this, participants were to either use the *distance formula* to calculate all the side lengths for equality; (BT, TA, AR, and BR), which will also confirm the equality of the diagonals, or use the slope formula to confirm that the adjacent sides are perpendicular.
According to Table 4.5, out of 21 responses, nine learners (42.90%) scored 50% and above. Out of the nine learners who scored 50% and above, three demonstrated significant geometric proficiency as evidenced by their scores. Below are the findings from the response of P3 to question 6.

4.27: Findings from the response of P3 to question 6



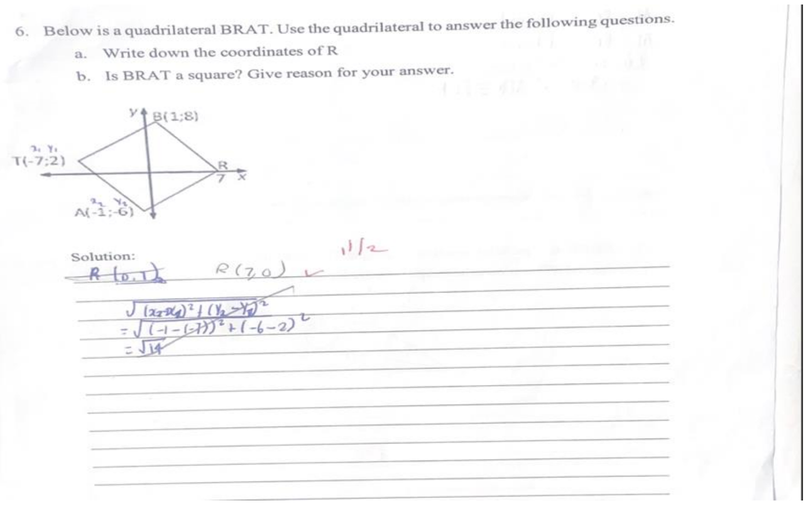
P3 correctly identified the coordinates of R as (7, y), but did not provide any reason or justification. When probed, the learner admitted to not being sure whether the value of y is 0:

> Researcher: *"Yes, how did you find the missing coordinates of R?"*
> (P3): *"Usually, when you find the coordinate, you use the midpoint of x and y."*
> Researcher: *"So, what's the value of y since x is 7?"*
> (P3): *"I think it's '0'."*
> Researcher: *"You think?"*
> (P3): *"I'm not sure."*

The engagement above shows that although the learner may have correctly identified the coordinates, he lacked the conceptual understanding, hence, the expression of uncertainty, despite the correct answer.

In question 6b, P3 failed to calculate the distances of BT and AR, after calculating the sides of the other opposite side, AT and BR. Hence, BRAT could not be proved as a square because the conditions were not met. The learner was unable to check for the quality of diagonals or perpendicular slopes, a necessity for proving that a quadrilateral is a square. P3, out of oversight, concluded that BRAT is a square on the basis of establishing one pair of opposite sides, which is not sufficient in this regard, as this is a misconception, assuming that equality of sides verification is enough proof to define a square. BRAT is proved to be a square if all sides are proved to be the same in length, and the four angles are verified as 90^0.
Overall, the response of P3 underscores a common misconception where learners depend only on equality of side lengths without taking cognizance of other defining properties, such as diagonals and perpendicularity.

4.28: Findings from the response of P14 to question 6



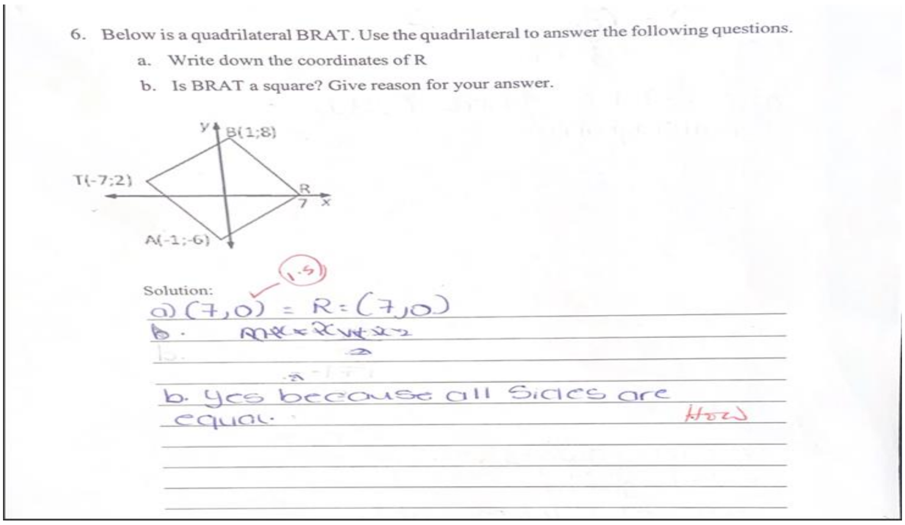
P14's response to question 6a was correct, but no justification- a common error by learners. Additionally, no conceptual proficiency was exhibited in solving question 6b, as P14 started the calculations, but got stuck, because of no conceptual understanding. This could indicate a procedural knowledge gap, as evidenced by the wrong application of the distance formula, which led to an incorrect response.

The learner attempted to calculate side AT, which is a requirement or a condition based on the question, but struggled to correctly calculate the length in question, which shows a lack of geometric understanding of the concepts and application of geometric theorems.
The verification of the four sides or proof for perpendicularity was a struggle, as evidenced by either the learner's inability to effectively use the distance formula or not being sure of what to

do. Furthermore, P14 struggled to incorporate several geometric properties for the classification of quadrilaterals, like a square. When probed, shown below is what transpired in the interview:

> Researcher: *"You were unable to prove that BRAT is a square in question 6b? Any reason? Hove you done the distance formula in analytical geometry?"*
>
> P14: *"Yes, in analytical geometry."*
>
> Researcher: *"So, how come you couldn't attempt the question?"*
>
> P14: *"I didn't know how to start with this one."*

4.29: Findings from the response from P9 to question 6

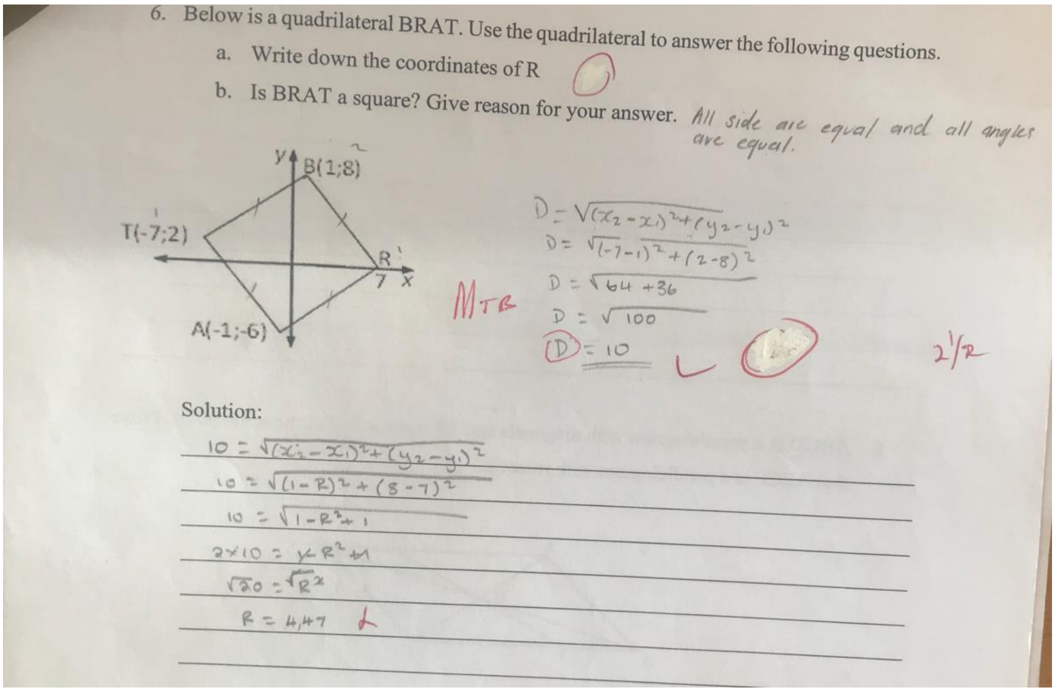


Several participants got question 6a correct, however, with no justification for their response. The same with P9, with a correct response, but no justification.

In question 6b, P9 attempted to prove that BRAT is a square, however, based on an assumption. There was no procedural demonstration, geometric reasoning, or calculation in that regard, to confirm BRAT as a square. Stating, "*Yes, because all sides are equal*.", does not make BRAT a square as the conclusion is based on an assumption.

This is a common misconception, as learners seem to lack understanding of defining properties of quadrilaterals generally, and also struggle to differentiate them, especially those that share similar properties, including a square in this case. The response of P9 does not indicate any proficiency in geometric reasoning. The claim, "*Yes, because all sides are equal*.", further suggests confusion between a square and a rhombus, as both shapes share the same property, besides, equality of sides as a property does not determine a square. P9 also lacked the conceptual knowledge of angle verification.

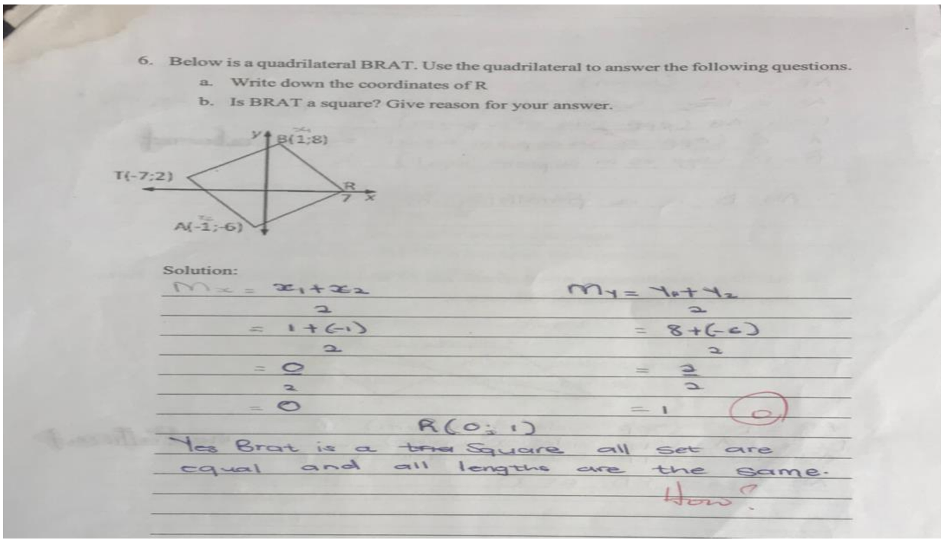
4.30: Findings from the response of P10 to question 6



P10 applied the distance formula in calculating the *y*-coordinate of R in question 6a, but got it wrong, which could be a result of a miscalculation in computation and reasoning. The miscalculation of the *y*-coordinate of R, as 4.47, instead of (7,0), is traceable to misapplication of the distance formula, which suggests procedural misconception and difficulty in handling algebraic equations.

In the second part of the question, the learner presumptuously asserted that *"all sides are equal and all angles are equal,"* hence, BRAT is a square. No complete verification, including the angles, was given. Also, P10 failed to check for perpendicular slopes or use geometric reasoning. This could be attributed to overgeneration of square properties, without differentiation of square and rhombus, which share the same property, in terms of equality of sides, but not necessarily angles.

4.31: Findings from the response of P13 to question 6



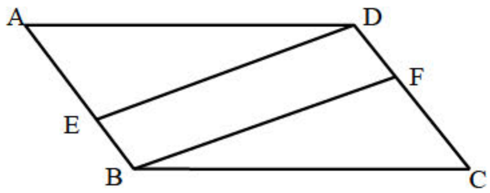
The response of 13 to question 6 reveals two vital misconceptions, namely, incorrect coordinate determination and incorrect classification of quadrilateral BRAT. In question 6b, the learner incorrectly identified R coordinate as (0,1), rather than (7,0), which could be confusion about symmetry and x-axis limitations.

P13 assumed BRAT to be a square simply based on 'all sides are equal.' Overgeneralisation of equality of side lengths seems to be evident among learners. This appears to be a common misconception, as it also reflects a lack of knowledge of defining properties of quadrilaterals, such as a square, having equality of all angles and bisection of diagonals.

P3 failed to recognise that a rhombus also shares the 'equality of side lengths.' P13's approach also confirms the lack of justification in proofs, as evidenced by the inability to correctly provide the essential calculations, to confirm whether BRAT meets the sufficient conditions to be called a square. This reflects a possible challenge in formal reasoning, that is, writing and deductive reasoning within geometric space.

Question 7:

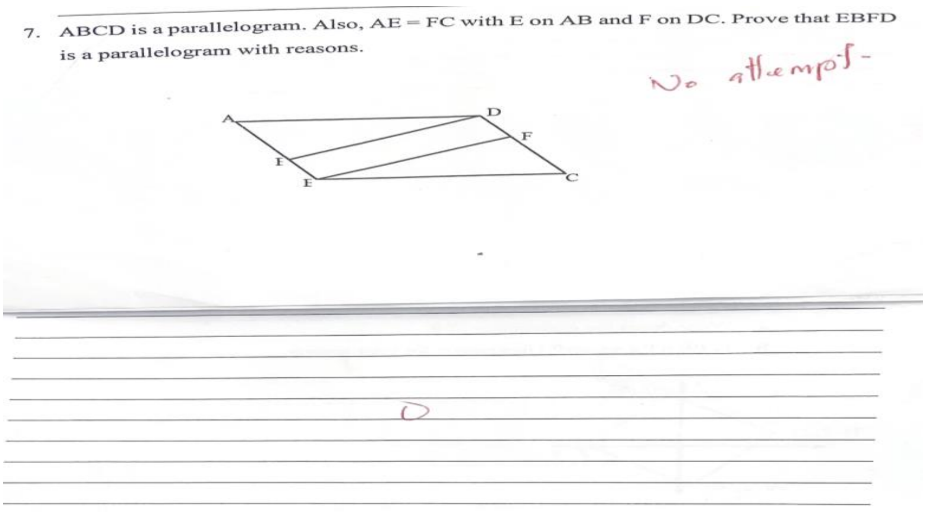
ABCD is a parallelogram. Also, AE = FC with E on AB and F on DC. Prove that EBFD is a parallelogram with reasons. (13 marks)



In question 7, deductive reasoning is required, using postulates, theorems, and axioms to establish geometric relationships. Participants were asked to prove that EBFD is a parallelogram, given other information to help in reasoning and proving. In proving that EBFD is a parallelogram, recognition of the existing relationships within the parallelogram ABCD is required. Secondly, employing the properties of a parallelogram, such as ‘parallelism’ and ‘opposite sides equal’, is also to be considered in the process. Furthermore, learners were expected to apply the required formal deductive reasoning, based on an acceptable sequence, through the application of the relevant theorems to support each step.

Virtually all learners failed to demonstrate the required formal deductive reasoning required in proving that EBFD is a parallelogram. The responses highlight significant struggles faced by learners in handling proof-related geometry problems. Notably, Table 4.5 shows that, out of 21 participants, 80.9% of learners, (17) did not make any effort in attempting the problem for possibly different reasons. The rest made a slight attempt but still struggled to provide the proof. Below highlights some of the responses from some learners:

4.32: Findings from the response of P7 to question 7


7. ABCD is a parallelogram. Also, AE = FC with E on AB and F on DC. Prove that EBFD is a parallelogram with reasons.

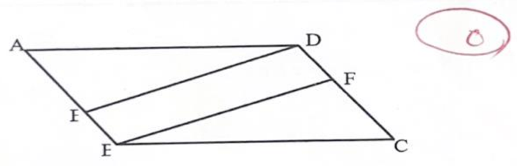
P7 was one of the learners who did not attempt the question. This could mean a lack of conceptual understanding or confidence in tackling formal geometric-related questions. Below shows what transpired during the interview with P7:

> Researcher: *"Any part of the question most challenging?"*
> P7: *Questions 7, and 8, in fact also 6."*
> Researcher: *"Okay…, so based on your experience, it seems you are still struggling with how to prove?*
> P7: *Yes sir..*

P7's non-attempt regarding the question shows a challenge in identifying the required main properties for the proof. This is evidenced by the interview response. The response from the learner seems to have revealed uncertainty about how to structure proof-related geometric questions- a common challenge faced by many learners. This goes to say, that many learners avoid proof-related questions as a result of a lack of adequate exposure to problems that require formal geometric reasoning.

4.33: Findings from the response of P12 to question 7

7. ABCD is a parallelogram. Also, AE = FC with E on AB and F on DC. Prove that EBFD is a parallelogram with reasons.



P12 also did not attempt to prove the proof. The reason(s) for non-attempt could have been as a result of similar difficulties faced by other learners in proof-related problems- lack of confidence, lack of conceptual difficulties, or lack of understanding of what is required. When probed at the interview, below is what transpired:

Researcher: *"…what happened in question number 4?*

P12: *".I have challenges with proving…that is why I didn't attempt those questions that require proof, that is questions 7 and 8…"*

P12's interview response shows that the lack of formal deductive reasoning in solving this type of question remains the underlying factor. P12's challenge seems to be a common challenge among learners, in terms of proof-related, as proofs in geometry go beyond knowing or understanding the quadrilateral's properties. It also requires formal deductive reasoning (constructive arguments), based on the conditions given.

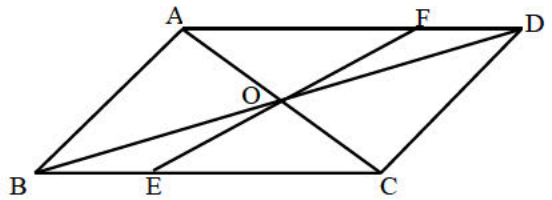
Some of the possible misconceptions held among learners in handling proof-related problems include, difficulty in recognising the defining properties of quadrilaterals, particularly a parallelogram.

Another possible challenge is utilizing the given conditions to prove what is needed. For instance, in this question, confusion with the given conditions could have also affected their understanding, as some learners may have assumed that E and F are midpoints of AB and DC, respectively, which is incorrect. Rather, the question states that AE = FC with E on AB and F on DC, which does not necessarily imply that E and F are midpoints.

P12's response in the interview also shows that most learners either overgeneralize common properties like 'opposite sides equal' or 'dependence on given condition, in this case, parallelism, hence, reasoning through, using what is given to logically establish what is required, becomes a challenge to demonstrate. For instance, according to the question, AB and DC were given as parallel. Now, P12 probably may have demonstrated uncertainty on how to apply the given information to prove that EBFD is a parallelogram.

Question 8:

ABCD is a parallelogram with diagonals EF intersecting at the same point. Prove that AECF is a parallelogram with reasons (13 marks)

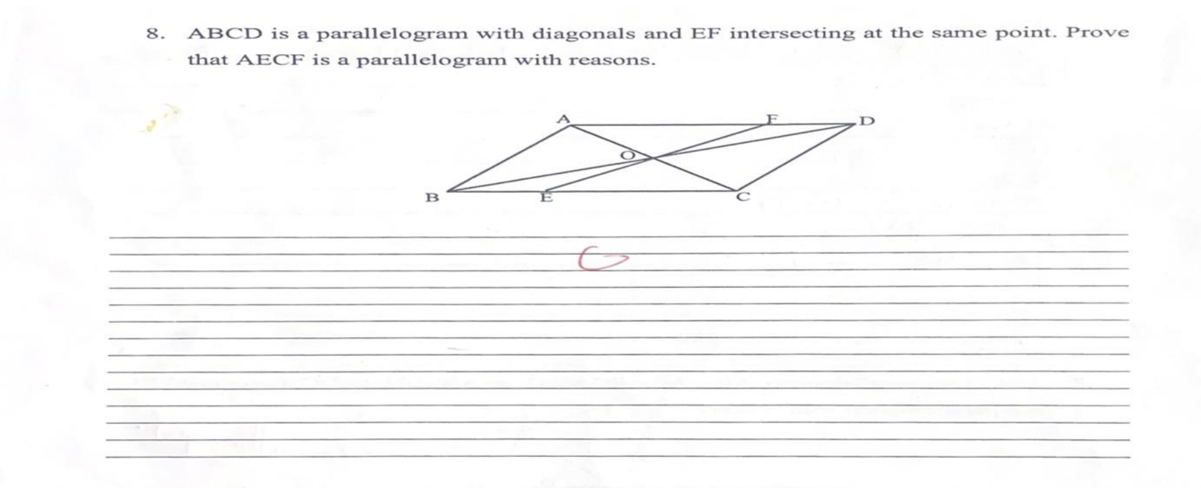


Question 8 also requires formal deductive reasoning to prove. The question was designed to evaluate the ability of learners in applying their logical formal proofs reasoning in establishing what was asked, which is verifying the existing relationships within the given shapes' structure. It required learners to show that AECF is a parallelogram, by using the parallelogram properties, such as 'both pairs of opposite sides' in AECF being either parallel or equal in length.

Since ABCD is a parallelogram as given, learners were required to leverage the parallelogram properties, in terms of parallelism, as well as the factor of 'diagonals bisecting each other'. A critical point of relationship, to be recognised, is provided by the fact that the intersection of EF with the diagonals bisects at point O. Based on that given information, learners were expected by logical reasoning, use this intercession to establish: AE∥FC and AF∥EC. Alternatively, they can also establish the 'opposite-sides-equal' as a key property of a parallelogram.

Just like in question 7, almost all learners failed to demonstrate the formal deductive reasoning required in proving that AECF is a parallelogram. The responses also highlight significant struggles, learners face in handling proof-related geometry problems. Notably, Table 4.5 shows that, out of 21 participants, 85.7% of learners, (18) did not make any attempt in tackling the problem for possible different reasons. The rest made a slight attempt but still struggled to provide the proof. Below highlights some of the responses from some learners:

4.34: Findings from the response from P7 to question 8



As indicated previously, the majority of participants did not attempt this question, and P7 was one of them. When probed during the interview, shown below is the response in relation to this question:

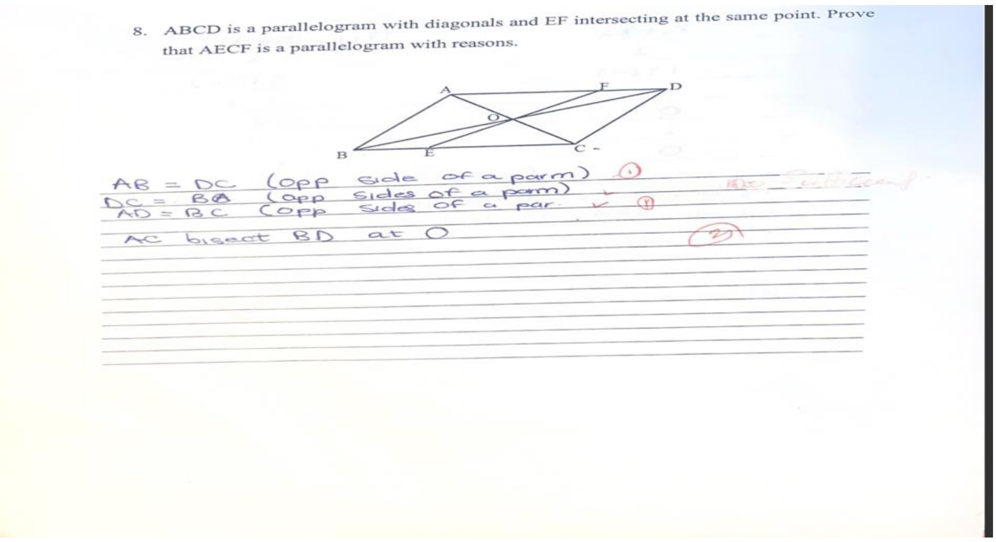
Researcher: *"Any part of the question most challenging?*
P7: *"..question 7 and 8, in fact also 6.."*
Researcher: *"I see most of you did not attempt the last question. Why?"*
P7: *"Time!"*
Researcher: *"Time? But you were given more than one hour."*
P7: *"I was still busy with question 6."*
Researcher: *"Oh"*
P7: *"Yes"*
Researcher*: "And I see also you did not attempt question 4, even question 3"*
P7: *Yes, it was quite challenging. I didn't know what to do.."*
Researcher: *But you have been taught this topic...have you not been taught about it in*

class?"

P7: *"Yes, we have"*

From the interview, several misconceptions and influencing factors emerged as possible causes for non-attempt by P7. First, the learner claimed a time constraint. When further probed, based on additional time given, the learner stated, "I was busy with question 6". Further analysis of the interview reveals that, although the learner claimed a lack of time management or poor time allocation, the interview also indicates challenges in proof-related questions, such as in question 8. The learner admitted that questions 7 and 8 were the most difficult of all, an indication of a lack of conceptual confidence in attempting complex geometry proofs or any proof-related questions, like questions 3 and 4 as well. Although the learner did admit that this topic has been taught, and yet conceptual understanding and application of the same geometric concepts remain a struggle.

4.35: Findings from the response of P13 to question 8



P13 showed a partial understanding of the properties of a parallelogram, based on the given figure- ABCD, but was unable to establish a logical step-by-step approach; hence, P13 got stuck on the way. P13 could not use the initial statements established to prove that AECF is also a parallelogram as required.

P13 was among the few who attempted the question; however, common misconceptions were also noted. P13 demonstrated understanding of the properties of a parallelogram, but was unable to logically apply the properties in proving the required proof- proving AECF as a parallelogram. The learner, just highlighted the parallelogram ABCD properties, instead of using it to establish the defining characteristics of AECF. Also, P13 failed to incorporate the role of line EF, an important property in establishing the proof. P13 must have depended on a procedural approach, instead of conceptual understanding. Hence, no further advancement was made in proving. Definitions are also recalled, yet they were not conceptually applied in the proving process.

There seems to be a significant gap between the instruction of concepts and the application of the concepts. To further confirm this and the challenge trend, below also highlight several responses from some of the participants who did not attempt the proof-related questions, such as question 8.

Researcher: *"You did not attempt question 8, what happened?"*

P16: *"Emm sir, as I have mentioned it, I'm not good in proving."*

Researcher: *"What do you thinking that can be done to help you improve your understanding?"*

P16: *"I think I need more clearer understanding from teachers."*

Researcher*: "So, which part, based on this test was most difficult for you?"*

P3: *"emmm…most definitely the..when you have to prove…yes, very ..ya….always challenging…*

Researcher: *"does it mean you have not been taught?"*

P3: no, no, no, definitely, I have been taught proving several times..to be honest yes, but it always gives me a hard time, to prove, but today, the question that gave me a hard time was the last one, number 8…"

Researcher: *"I also noticed that most of you did not attempt the last question in section B…"*

P14: *"Yes, question 8 in section b, where do you start? They haven't given enough information."*

Researcher: *" I see you got the answer in question 3 in section b with the workings..what about question 4a?"*

P12: *"Yes sir…I have challenges with proving…that is why I didn't attempt those questions that require proof, that is questions 7 and 8."*

Researcher: *"Tell me, how did you find the exercise generally?"*
P1: *"Yeah, it's difficult. It was very difficult."*
Researcher: *"..all or which area?"*
P1: *Eish, the last part…"*
Researcher: *"The questions where you are required to prove?"*
P1: *"Yeah"*

Researcher: *"Okay, you didn't attempt the last question."*
P15: *"Yes…proving is my weak point.."*

Researcher: *"Most of you didn't attempt proof-related questions; have you done proof?"*
P21: *"Yes, we have done proof"*

Understanding and application of geometric proofs seem to be the underlying reasons for the no-attempt trend in question 8.

4.2 Summary of Findings: Key Misconceptions, based on Participants' Responses to items in Questionnaire B

The summary provides the key misconceptions identified from the responses of participants, based on Questionnaire B. It underlines errors based on reasoning, understanding, and how geometric principles were applied in relation to quadrilaterals and proof-related questions.

The classification of quadrilaterals reoccurred as one of the key misconceptions held by learners. The geometric reasoning pattern of many learners showed that they are faced with difficulty in identifying quadrilaterals correctly. Misclassification of shapes was evident, particularly parallelograms and trapeziums. There is an assumption that a quadrilateral with equal opposite sides, is either a square or a rectangle, automatically, without minding or considering the possibility of a parallelogram, being one of the quadrilateral shapes that also shares the same property. Some are also of the view that a shape with one pair of parallel sides, must be a parallelogram, instead of a trapezium.

The second notable misconceptions were difficulty or confusion in differentiating between necessary and sufficient conditions, particularly in the case of parallelograms. Some learners assume that ‘equality of opposite sides alone’ was sufficient to justify that a quadrilateral must be a parallelogram, without taking cognizance of the need to verify another key defining property- parallelism. Some even ignore the diagonal property, which is also essential in substantiating the nature of quadrilaterals, rather, they seem to depend solely on the equality of side-length.

Extreme dependence on visual clues over analysis of proof was observed as another misconception, as evidenced by several responses. Many learners depended on the visual-based interpretation, rather than applying analytical reasoning, through the use of relevant properties, or mathematical methods with geometric alignment (principles). Misidentification of parallel sides was also identified among some learners, solely on the basis of shape appearance, instead of the application of algebraic concepts in proof. Difficulty in clear labelling when identifying key properties of shapes was also missing from diagrams, which also suggests reliance on the annotations of the diagram.

Misinterpretation of given conditions was another common misconception among learners. Learners either misinterpreted the given information or made incorrect assumptions. Hence, some misapplied known quadrilateral properties. They either see all quadrilaterals with equality of sides as squares or assume all parallelograms have equal diagonals. The overgeneralisation of certain quadrilateral properties often occurs as a result of overreliance on the visual look of shapes rather than reasoning through analysis, hence the persistent and common errors in relation to the classification of shapes and problem-solving.

Several procedural errors and incomplete justifications in proof-related problems were noted. For instance, most learners will state a property, such as “opposite sides are equal”, without clearly verifying it. Other learners tried to prove with only one condition, instead of incorporating multiple relevant properties. Some learners will provide incomplete justifications, without clarity in proving them. In addition, learners attempted to use the distance formula but incorrectly applied it. Many were unable to properly make justifiable conclusions in their arguments, hence, they left the problems incomplete.

This explains why most learners avoided proof-related questions due to lack of confidence and uncertainty, especially in questions 7 and 8, primarily due to logical reasoning difficulty.

Interview responses showed that although learners had been taught, they still struggled to apply the required geometric reasoning in proof-related problems. Other learners acknowledged their deliberate skipping of any proof-related questions because they found such questions unapproachable

4.3 Conclusion

This chapter presented the overview of the overall results in Questionnaire A, based on the selected 21 participants for the study, followed by the analysis of the items in Questionnaire A. The classification of participants in relation to the van Hiele level of geometric thought, based on Questionnaire A, was also presented and discussed. This was followed by the presentation and discussion of the overall results in Questionnaire B, and finally, the presentation of the findings from participants' responses to Questionnaire B items was given.

The findings will provide in-depth insights in the next chapter, where an in-depth discussion of the findings will be presented, aimed at answering the research questions.

CHAPTER 5
DISCUSSION AND CONCLUSION

The previous chapter provided the study findings. This chapter discusses the key findings by providing an interpretation with reference to the objectives of the study, rooted in the research questions, reviewing relevant existing literature, and the theoretical frameworks used in the study. The discussion will underscore the contribution and impact of factors influencing learners' understanding of quadrilaterals, their misconceptions, and their reasoning in geometry, based on van Hiele levels, to the broader understanding of geometry education. The implications of teaching Euclidean geometry will also be discussed. This will be followed by the limitations of the study, recommendations, and conclusion.

The discussion is centred on the research questions:

1. What are the vital factors influencing Grade 10 learners' understanding and reasoning in quadrilaterals in Euclidean geometry?
2. What are the common misconceptions held by Grade 10 learners regarding quadrilateral concepts- Properties and classification of quadrilaterals?
3. What insight can we get about Grade 10 learners' understanding and reasoning of quadrilaterals in geometry, in terms of the van Hiele levels of geometry of thought?

Addressing these research questions will aid in unpacking insights in relation to how learners' quadrilateral concepts and alignment of their reasoning with the geometric learning theories.

5.1 Discussion of findings

As indicated above, this section discusses in depth, the findings in Chapter 4, by analysing the findings regarding the objectives of the study, literature, and theoretical framework that guides the study. The discussion centres on learners' understanding and quadrilateral reasoning, their misconceptions, and their classification in line with van Hiele levels.

5.1.1 What are some of the vital factors that influence grade 10 learners' understanding of quadrilaterals in Euclidean geometry?

This section investigates the factors influencing grade 10 learners' understanding of quadrilaterals, in relation to the findings from questionnaires and semi-structured interviews conducted. Through the analysis of the tasks and the interviews, major themes emerged in relation to the first research question. These major themes focus on the pedagogical, cognitive, and environmental factors that impact learners' understanding of quadrilaterals in Euclidean geometry. Each of the main identified themes was examined in the context of the existing literature and as well as the van Hiele model of geometric reasoning, aimed at providing an inclusive understanding.

As indicated in Chapter 3, the data analysis technique adopted was the Six-Phase Thematic Analysis principle by Braun & Clarke (2006). The first phase necessitated familiarisation with the data, to get to know the data, and in the process, understand possible unique challenges learners face in learning quadrilaterals. This familiarisation phase paved the way for the initial coding process, which helped to identify the patterns that recurred. The recurring patterns include, properties of shape, difficulties with geometry-related terminology, and proofs. These patterns provided some insights as to the prevailing issues learners face in learning quadrilaterals in grade 10.

In the succeeding phases of the thematic analysis process, broader themes emerged by grouping related codes into coherent categories, aimed at addressing the research questions on possible factors that could be influencing leaders' understanding of quadrilaterals in Euclidean geometry. This process eventually led to the emergence of five key influences: Challenges with proving and Reasoning, Inconsistent Understanding of Quadrilateral Properties, Language and Cognitive barriers, need for Instructional and Practice-based support, and spatial Visualization Difficulties. Each theme reveals an aspect of learners' cognitive and educational experiences rooted directly in the data. Explanation of each theme follows:

Challenges with Proving and Reasoning

Proof-based related questions remain a challenge among many learners. For instance, P16 indicated that proving was very challenging. P16 stated: *"I'm really having difficulties in*

Euclidean geometry. Proving actually…. I don't understand proving". This implies that learners often demonstrate unpreparedness for any task that requires formal reasoning.

Cassim (2006), agrees with proving and reasoning challenges faced by learners, adding that learners often struggle with geometric logical reasoning, due to the logical sequence entailed in the process of geometric proof-related questions. Hence, most learners' reasoning, in most cases, is based on the visual appearance rather than on rigorous deductive reasoning. Cassim (2006), highlights the need for instructional approaches, aimed at assisting in bridging the gap between learners' procedural knowledge and their conceptual understanding.

Inconsistency in Understanding of Quadrilateral Properties

Learners often confuse shapes and their respective defining properties. Hence, they exhibited inconsistency in demonstrating their understanding of quadrilateral properties. For instance, when interviewing P2, there was uncertainty when differentiating a parallelogram and a rectangle: *Rectangle angles, they are 90°, each angle is 90°, because they sum up to 360°... and a parallelogram it can differentiate…."*. This response not only indicates confusion in relation to particular shapes, but also shows a lack of confidence in applying properties of shapes as a means of classifying quadrilateral shapes.

This inconsistency in understanding of quadrilateral properties aligns with Naidoo and Kapofu (2020), who noted that learners often forget these geometric properties, hence they mix them up property-wise, particularly, shapes such as, parallelograms and rectangles, resulting in misconceptions when it comes to classification. As part of the intervention, Naidoo and Kapofu (2020), advocate the need for consistent exposure of learners to these properties to reinforce the skills of classification of these shapes to avoid persistent misconnections

Language and Cognitive Barriers

The language of geometry education poses an additional barrier, as many learners still have difficulty in understanding the geometry-related terminology or in accurately recalling it when necessary. P16 admitted struggling with terms like "supplementary" and "complementary," adding that, *"there is supplementary and complementary; I don't know how to differentiate between the two"*. Interviews further showed that some learners struggled to conceptually differentiate which angle is "supplementary" and "complementary". Other learners admitted to being conversant with the terms but still struggled to link them correctly.

This shows that challenges such as language-related barriers can hinder learners from understanding basic concepts and also applying them in solving problems. This barrier brings into line with the notion of Setati (2005a), who highlighted how proficiency in language impacts understanding in mathematics and reasoning, calling it "Language of Learning and Teaching" (LoLT) barrier.

Iftanti et al. (2021), further emphasise this issue, stating that there is a correlation and a negative effect of semantic, syntactic, pragmatic, and cognitive barriers towards the accuracy of Geometry proof. Hence, both the complexity of language and cognitive reasoning limitations significantly affect learners' ability to grasp geometric concepts.

This resonates with the work of Bansilal and Naidoo (2012), who show that language in mathematics often acts as a cognitive barrier, especially, when learners are not familiar with the terminology that is associated with a specific subject. They suggest that clear, accessible language is crucial for understanding, especially in South Africa, where multilingualism is evident and prominent. Understanding the terms is vital for advancing through the van Hiele levels, where precise language underpins learners' ability to identify and reason with geometric properties.

Moreover, Gorgorió and Planas (2001), align with the view of Bansilal and Naidoo (2012), that the relationship between 'language' and 'cognitive' barriers can complicate the learning of geometric concepts in multilingual classrooms, especially when learners experience terms in mathematics that they are not used to. Gorgorió and Planas (2001), added that such can negatively impact learners who have the tendency to disengage from learning geometry, with the notion that it is not accessible.

Integrating certain strategies, such as examples that appeal to learners in contexts of what they are used to; visual aids, and collaborative learning settings, promote both fluency of language and conceptual understanding. Such a remedial approach is crucial as it enables learners to overcome these two-in-one challenges, thereby developing deeper confidence in their geometric reasoning.

Need for Instructional and Practice-Based Support

More support from teachers was vividly echoed by several learners as their desire, noting that additional and regular practice with clarity, could enhance their understanding, particularly with proofs. P13 echoed this as a concern, adding that more instructional practice would enhance their understanding of concepts in geometry.

This response not only reveals shortfalls in regular pedagogical practice, but also shows the need for a well-structured and tailored teaching style, aimed at addressing the conceptual gaps as well as providing structured guidance. Furthermore, providing planned support such as step-by-step modelling of proof construction, piloted examples, and a platform for repeated practice will aid in reinforcing and building confidence among learners.

Suleiman et al. (2025), indicated in a quasi-experimental study that adopting scaffolding instructional strategies significantly enhanced the success of learners in geometry education compared to using conventional approaches. The study further asserts that learners who are taught with scaffolded-based activities, such as guided tasks, have a better understanding in terms of application of geometric concepts than learners taught conventionally. This reaffirms the importance of the use of instructional and practice-oriented support approaches in mathematics education. This agrees with the CAPS curriculum that encourages steady reinforcement of geometric concepts, which helps learners to advance through van Hiele levels of geometric thought.

The response from learners, by implication, also reflects the inconsistency of providing learners with mathematical problems, coupled with a lack of exposure to a variety of types of problems, which can help them apply what they have learnt. In addition, Brijlall and Ndlovu (2013), advocate for problem-solving in a collaborative manner and interactive instruction approaches, adding that such approaches can also improve and promote engagement among learners and foster deeper understanding

Spatial Visualization Difficulties

Spatial visualization factor was also one of the emerging themes, as one of the influencing factors. This is based on responses from several learners, as it affected their ability to recognise and distinguish shapes within diagrams. For instance, P13 admitted by stating, *"I have a big problem with a rhombus and a square. Yeah, differentiating two. I can only see them when I calculate them…but I can't see the difference by seeing them."* This reveals reliance on calculation instead of identification by visualization or intuition, which is the first and very important stage in advancing in the van Hiele levels of geometric reasoning. Dependency on calculation could suggest an underdeveloped spatial skill and reasoning ability.

Ngirishi (2015), emphasises that learners who depend mainly on calculation in identifying shapes, without having the fundamental knowledge of spatial visualization skills, are likely to experience challenges when involved in geometric-related exercises. Ngirishi (2015), further suggests the importance of visualization skills because they aid in internalising geometric properties and differentiating shapes based on their properties. Furthermore, Ngirishi (2015), suggests that early development of spatial reasoning skills, enhances interpretation and effective manipulation of geometric diagrams mentally, which is critical for progressing through the van Hiele levels of thought.

The South African CAPS curriculum also stresses the need for learners to develop spatial visualization skills, based on its foundational importance in Euclidean geometry in general DBE (2011). However, Brijlall and Ndlovu (2013), note that realising this goal of spatial development among learners might be hindered, due to resource constraints, such as limited access to interactive tools like GeoGebra, by under-funded schools. Suggested interventions, like the incorporation of dynamic geometric software and hands-on activities, are said to provide opportunities for learners to enhance their spatial reasoning, and also reduce their dependency on traditional calculation, fostering a more all-inclusive understanding of relationships in geometry.
Learners may be in a better position to interpret geometric properties and be able to classify quadrilaterals through visualization, which will eventually improve their ability to solve complex problems in geometry.

5.1.2 What Are Some Misconceptions Held by Grade 10 Learners in Relation to Concepts of Quadrilaterals?

This section seeks to examine the misconceptions Grade 10 learners hold about quadrilaterals, as evidenced by their responses. The analysis adopted the Six-Phase Thematic analysis approach by Braun & Clarke (2006). Being guided by the Six-Phase Thematic analysis, it facilitated the emergence of the themes related to misconceptions, in relation to properties of quadrilaterals, definitions, and processes of reasoning. Responses from learners also provided critical insights into their challenges with understanding, reasoning about quadrilaterals in the framework of Euclidean geometry, and differentiating.

Confusion in Understanding and Differentiating Quadrilateral Properties.

Learners' confusion in understanding and differentiating between the properties of different quadrilaterals is one of the common misconceptions identified. This is traceable to over-reliance on visual prototypes, and struggling to recognise hierarchical relationships between shapes. For instance, in question 1 of questionnaire B, P2 was unable to differentiate between rectangles and parallelograms, stating that a parallelogram has: *"slanting opposite sides"* rather than clearly identifying the properties that differentiate it, such as 'all four angles in a rectangle are 90^0. In another interview, P1 commented, *"A square and a rectangle are the same because they both have four sides, so I don't know why they are called different shapes."*. Such responses show confusion regarding shapes and also a lack of understanding of properties some shapes share in common and those that differentiate them.

The findings of Clements and Battista (1992), agree with the confusion learners often have in understanding and differentiating between the properties of different quadrilaterals, adding that learners tend to classify shapes in groups by artificial features, instead of critical characteristics. This is as a result of no focused teaching with emphasis on vital distinctions. Naidoo and Kapofu 2020 discovered that learners habitually misconstrue the classification of quadrilateral shapes by hierarchical order. They fail to understand that, for instance, a square is a specific type of rectangle, owing to its extra properties, like equality of all sides. This deficiency of knowledge is often caused by not much emphasis being placed on relational reasoning during pedagogical instruction.

Burger and Shaughnessy (1986), further argue that when learners are consistently exposed to the hierarchical structure of shapes, it enhances the reinforcement of the required geometric knowledge. Absence of consistent exposure to the hierarchical structure of shapes, reliance on superficial visual characteristics is inevitable, resulting in the inability to recognise critical connections, such as rectangles within parallelograms. To lessen this misconception, clear teaching approaches, like the use of Venn diagrams and tasks that are associated with relationships of shapes, are recommended, aimed at helping learners navigate the hierarchical framework of quadrilaterals. This is also in alignment with the van Hiele model, which highlights the importance of relational reasoning at an advanced stage of the geometric model of thought.

Misconceptions in Quadrilateral Definitions and Hierarchies

Misunderstandings regarding the quadrilateral definitions and the quadrilateral hierarchical classification are often exhibited by learners. In question one of questionnaire B, P14'resposne implies misunderstanding on the part of the learner. P14 mistook a square for a rectangle, *"just a special rectangle."* P14 failed to recognise the defining features of a square, hence, the learner mistook the shape for a rhombus, since both shapes have certain shared attributes. The findings of Fujita and Jones (2007), noted that learners often struggle to understand the hierarchical relationships that exist among quadrilateral shapes; for instance, understanding that all squares are rectangles, but, not all rectangles are squares.

Van Hiele (1986b), even pointed out that learners at the lower stage of the model may only identify shapes on the basis of their appearances, without taking cognizance of their intrinsic properties and connections in terms of hierarchy. Owing to this deficiency of hierarchical understanding may hinder learners from advancing in geometric reasoning, especially where they can engage in abstract connections between different types of quadrilaterals.

The research of Usiskin (1982), agrees with this, adding that learners often fail to understand that similar shapes, such as rectangles and squares, can belong to several categories simultaneously, as a result of the properties they have in common. For instance, a learner may, by visual identification, correctly identify a square of a shape with four right angles and equal sides, at the same time, be unable to acknowledge the fact that a square shares certain attributes in common with both a rectangle and a rhombus based on hierarchical classification of quadrilaterals. Following the reality of these misconceptions, there is a need for instructional methods that reinforce the knowledge of the hierarchical nature of quadrilaterals, adding that specific properties can position a shape into several categories within a hierarchy of geometry.

Misapplying Reasoning in Proofs

Another aspect of misconception identified was in misapplying logical reasoning in proof-related questions. Several learners had challenges with proof-related questions. For instance, P12, admitted to having difficulties with proofs, especially in questions where understanding of quadrilateral properties is assessed or involved. P12 stated: *"I have challenges with proving…that is why I didn't attempt those questions that require proof."* This clearly shows a major misconception of the misapplication of reasoning in proofs.

In a study by Senk (1989), it was noted that learners commonly experience proof-related difficulties as a result of the reasoning required, which is abstract in nature. Knowing properties is not enough to handle abstract geometry-related problems, such as proof. Hence, there is a need for a high order of geometric reasoning of which several learners see as an overwhelming task.

Reasoning abstractly and logically, as noted by Brijlall (2017), is best enhanced through regular practice that entails guided reasoning instead of isolated factual recall. This suggests the need for instructional styles that enhance and promote understanding of reasoning itself, instead of attempting the result of abstract reasoning. When learners are provided with the required organised learning platforms, with proper supervision and guidance in terms of the reasoning procedures involved, it can aid in bridging the existing gap between knowledge of geometric properties and the application of such properties in a proof-related question.

In view of these highlights, it establishes the need to integrate reasoning practice into geometry classrooms, by allowing learners to build a stronger foundation in relation to thinking logically, which is vital for competence in handling geometric problems in proofs.

Definitions and Terminology Challenges

The two-in-one barrier was also identified as a factor- challenges with definitions and terminology. Learners often fall into the problem of confusing terms, not knowing which one has which definition. The geometry terminology might be known to them, but they struggle to differentiate them correctly. For instance, P13 did not hesitate to acknowledge this challenge during the interview, in relation to terms, such as, "supplementary" and "complementary." P13 acknowledged*: "I cannot distinguish between the two..,"* This reveals a fundamental insufficiency that may affect more complex geometric exercises in the future if left unaddressed. It is noted by Piaget and Inhelder (1967), that constant reinforcement is needed for further development of mathematical language, adding that when this is not in place, learners end up articulating either incomplete or erroneous definitions, as evidenced by the outcomes of the assessments.

The effect of misunderstanding geometry terminology on learners' geometric reasoning and interpretation can adversely hinder learners' progression in geometric reasoning. McLachlan and Essien (2022), examined the vital role language plays in understanding mathematical concepts, adding that language poses a barrier in the learning of concepts in mathematics education, especially where precise terminology is vital, such as in geometry. Furthermore, the language barrier is exacerbated in multilingual settings like South Africa, where learners frequently grapple with mathematics terminology in English, which may not be their mother tongue language as a result of being taught and compelled to learn these terms and concepts in a foreign language. This challenge not only explains the confusion learners have around terminology, but also how these terms shape their understanding of the required geometric concepts, thus making this two-in-one barrier an aspect that needs to be further addressed, aimed at reducing misconceptions.

The findings of McLachlan and Essien (2022), agree with Duval (1999), who highlighted that proficiency in mathematics language is critical for reasoning, as terminology operates as the basis upon which cognitive reasoning anchors, as this concept (order) helps learners to build upon their prior knowledge with no confusion. When there are no clear, and reinforced definitions, it results in misconceptions, which hinder the ability of learners to correctly understand geometric concepts as well as apply them. This language gap invariably affects the ability of learners in the correct application of geometric principles, particularly where there is no sound foundation in the understanding of quadrilateral-related terms. Hence, the challenge of language and terminology is vital to be addressed in order to improve learners' reasoning and comprehension in geometry.

5.1.3 What insight can we get about grade 10 learners' understanding of quadrilateral geometry in terms of van Hiele levels of geometry thought?

This section seeks to investigate the quadrilateral understanding, in relation to the van Hiele model of geometric reasoning, with the intention of addressing Research Question 3. The van Hiele model, which is significantly utilised in geometry instructional practices in classrooms, classifies learners' thinking into hierarchical phases, which advance in a sequential manner: Visualisation, Analysis, Informal Deduction, Deduction, and Rigor.

Following the analysis of the responses of learners, through thematic analysis, thematic patterns that situate learners at the initial stages of the van Hiele model, emerged: Visualization, Analysis, and Informal Deduction. This implies that the highest level in the van Hiele level of geometric model of thought, (which is Rigor) did not emerge as a theme. The analysis provides insights in relation to the current reasoning stages of learners, identifying misconceptions or barriers in cognition that prevent progress.

The study identifies critical areas of instruction to enhance learners' advancement in Euclidean geometry by evaluating their responses across different levels. Below, outlines in detail each identified theme, explored in depth, which aligns learners' responses with the van Hiele levels for the purpose of clarifying their current understanding and areas where additional support may be needed.

Level 0: Visualization

At the Visualization level, learners correctly identify shapes and objects on the basis of the general appearances of the shapes, instead of their properties. By implication, learners can identify and name quadrilateral shapes, like, rectangles, squares, and triangles. However, their understanding at this stage is mainly based on visualization. This is evidenced by the percentage of learners who were able to identify shapes visually.

In Table 4.5 (participants' VHLs in Questionnaire A), in Chapter 4, 42.9% of the participants, which is nine out of the 21 participants, attained level 0, which is almost half of the group. However, in Table 4.6 (participants' VHLs in Questionnaire B), this percentage increased, slightly to 47.2% (10 out of 21). This shows that several learners exhibited a basic understanding of identifying quadrilateral shapes by appearance, but were unable to identify the shapes by their defining properties. This still aligns with the description by van Hiele (1986a). For instance, P7 was asked in the interview to differentiate between a rectangle and a parallelogram through their defining property. P7 stated, *"Oh…not that I can remember of them."*. In another instance, P14 easily identified a square and a rectangle, but was unable to relate this characteristic accordingly.

Level 1: Analysis

At the Analysis level, learners begin to recognise shapes beyond the visualization stage, which is Analysis. This implies having the ability to differentiate shapes based on their properties. Several learners, (though not compared to the number at level 0), demonstrated emerging ability to identify quadrilateral shapes through their defining properties.

In a response from P3, stating, *"I get confused between a parallelogram and a rectangle because sometimes they look the same,"* reveals that although certain properties can be listed, the challenge of consistent application of those defining properties remains as a challenge. In another response from P2, *"A rectangle has opposite sides equal and four right angles,"* revealing a step beyond the visualization stage to more analysis.

However, inconsistency in applying these properties is still a struggle, which indicates the need for further instruction. This is evidenced by the records in tables 4.5 and 4.6, where 28.6% of participants, six out of 21 participants, attained this level in questionnaire A. However, in questionnaire B, where learners were to actually demonstrate their geometric knowledge through an open-ended questionnaire, the percentage significantly dropped to 4.8% (one out of 21). This shows that, although some could identify properties, however, the difficulty of inconsistent application of this knowledge is still an issue.

This emerging yet fragmented thinking corresponds with van Hiele's (1986) findings, which show that learners at Level 1 tend to view properties as isolated rather than interrelated. Burger and Shaughnessy (1986), further assert that the analysis stage is a vital stage for advancing to higher stages, as it provides learners the opportunity to transit from viewing shapes as wholes to viewing them as collections of properties. Although this level emerged as a theme, however, the reasoning of learners remains disjointed with a deficiency of relational knowledge, which is vital for geometric advancement. The statistics in tables 4.5 and 4.6 show the need for targeted exercises in order to promote relational thinking among learners, which will enhance their ability to establish links between features of various quadrilateral shapes.

Level 2: Informal Deduction

At the informal deduction level, which is also another emerging theme, learners have knowledge of relationships between properties, with the ability to start reasoning about shapes in a more abstract way. Learners begin by making arguments at an informal level, about the relationships between different shapes and their properties.

Few learners attained this level, indicating their initial capabilities, in engaging in informal deductions. This is evidenced by the data in Table 4.5, which indicates that 14.3% of participants, (three out of 21) demonstrated some informal deductive reasoning at this level. The, percentage, though, increased to 28.6% (six out of 21) in Questionnaire B according to

Table 4.6, implying an improvement in logical thinking concerning knowledge of interrelationships between shapes.

Senk (1989), highlights that learners at this stage are transiting, as they start to recognise logical linkages among characteristics but still need further assistance to cultivate the formal proof abilities necessary for advancement to higher levels. This theme highlights that while certain learners have the ability for deductive reasoning, their comprehension remains informal and necessitates organised lessons to enhance their confidence in constructing logical arguments and establishing connections between geometric properties. This focused assistance might facilitate the transition from informal deduction to the formal reasoning necessary at the subsequent level in the van Hiele model.

5.2. Summary of Key Findings Across Research Questions

Building on the analysis of each research question, this section synthesises key findings across themes, highlighting overall patterns and relationships between factors, misconceptions, and reasoning stages that influence learners' understanding of quadrilaterals, based on the study of each research question. This synthesis reveals the cognitive, linguistic, and pedagogical obstacles affecting learners' geometric reasoning.

In Research Question 1, through thematic analysis, several factors influencing learners' understanding of quadrilaterals, were identified, such as, Challenges with Proving and Reasoning, Inconsistent understanding of quadrilaterals, Language & cognitive barriers, Need for Instructional & practice-based support, and Spatial visualization difficulties.

Research Question 2, identified prevailing misconceptions learners often make in quadrilaterals, such as confusion in understanding quadrilaterals, misconceptions in quadrilaterals, Definitions & Hierarchies, Misapplication with Reasoning in Proofs and Definitions and Terminology. These factors centre on difficulties in differentiating similar shapes and misconceptions related to geometric characteristic features such as angles and sides.

Research Question 3 assessed the level of learners in relation to the van Hiele levels of geometric reasoning. The placement indicated that most of the learners were operating at the lower levels of van Hiele levels of geometric thought, within levels 0 (Visualization) to 2 (Informal Deduction), with a few indicating a sign of readiness for level 3 (Deduction).

Bringing these findings together, provides a broad perspective in relation to difficulties faced by learners in understanding Euclidean geometry concepts.

A recurring theme across all three research questions is the role of language and terminology as a barrier to geometric understanding. A significant number of learners demonstrated uncertainty regarding fundamental words such as “supplementary,” “bisect,” and “congruent,” which hindered their capacity to correctly understand and analyse the features of quadrilaterals. Bansilal and Naidoo (2012), assert that difficulties associated with language in mathematical terminology are common in South African classrooms, especially where learners are learning concepts or terminology in a language different from the language they are used to.

Barriers from a linguistic perspective align with the van Hiele model of geometric thought, wherein terminology in understanding geometry is crucial for advancement from the visualization stage (level 0) to the Analysis stage (Level 1), since recognition and articulation of shapes by learners require knowledge of properties, and not just mere appearance. Duval (1999), further argues that proficiency in geometric language serves as the foundation upon which cognitive development occurs, as it facilitates learners to navigate challenging properties and improve their progression through levels of reasoning.

In Research Question 2, some of the misconceptions identified include learners’ difficulties in differentiating between rectangles and parallelograms, which reinforces the findings from Research Question 1 regarding the insufficient basic understanding of quadrilateral properties. For example, P2’s failure to distinguish properties specific to rectangles or parallelograms illustrates the compartmentalized thinking outlined in the van Hiele (1986b) model, where learners at level 1 may recognise properties but lack relational understanding across shapes. Burger and Shaughnessy (1986), assert that learners at this level often perceive properties in isolation, a viewpoint that limits their analytical abilities and hinders advancement to Level 3 (Informal Deduction). The findings indicating 14.3% to 28.6% of learners attained level 2 in the van Hiele model (as shown in tables 4.5 and 4.6, respectively) imply that the majority of learners are restricted to visualization and basic analysis, because they encounter difficulties in constructing logical arguments regarding the relationships among quadrilateral properties.

The proof-related themes and deductive reasoning difficulties are also linked to the findings from Research Questions 2 and 3. The findings show that many learners exhibited unwillingness to tackle proof-based questions, implying insufficient deductive reasoning skills, which is a critical requirement for operating at level 3 (Deduction) in the van Hiele model. For instance, P13 admitted skipping proof-related questions totally as a result of a deficiency in geometric deductive reasoning required to tackle such problems. This indeed reveals a gap in the logical structure and the required deductive reasoning at this level. Learners' deductive reasoning remains underdeveloped, as noted by Senk (1989), due to no opportunity to practice questions that could develop such reasoning at that level. Brijlall (2017) is of the view that learners' deductive reasoning skills can be enhanced once the instructional practices emphasise structured reasoning tasks, which involve more substantive geometric reasoning than depending on procedural tasks all the time.

Another emerging factor in relation to understanding and analysing quadrilaterals, reflecting across the three research questions, is spatial reasoning. Difficulties in understanding complex geometric relationships are traceable to learners with difficulties with spatial perception or reasoning. Battista (2007), asserts that spatial thinking is crucial for progress in geometry, as it supports learners' capacity to cognitively manipulate shapes as well as understand how they work in connection to one another. Naidoo and Kapofu (2020), indicate that spatial reasoning problems often hinder learners' performance in geometry within the South African context, especially with the understanding of transformations and congruence. For the majority of participants in this study, progression beyond Level 0 (Visualization) requires specific interventions to enhance spatial reasoning abilities, which are crucial for moving to relational understanding within the van Hiele hierarchy.

The findings from the research questions highlight the interrelatedness of language, cognition, and instructional factors that influence learners' understanding of geometry. The impact of language limitations, persistent misconceptions, proof-related difficulties, and restricted spatial reasoning skills collectively explains why the majority of learners remain at the initial van Hiele levels. Addressing these interrelatedness issues necessitates an extensive instructional strategy that integrates language support, conceptual scaffolding, and reasoning practice into the curriculum. In total, these findings indicate that specific support at each van Hiele level, tailored to the learners' current reasoning levels, might enhance their understanding of Euclidean geometry and facilitate advancement up the van Hiele hierarchy.

5.3 Practical Implications for Teaching and Curriculum Design & Review

The findings of this study identify critical areas where targeted instructional strategies, curricular adjustments, and professional development might assist learners in addressing barriers to geometric understanding. This practical application includes overcoming language difficulties, enhancing conceptual understanding, and directing learners' advancement through the van Hiele levels in accordance with their cognitive development phases.

5.3.1 Practical Teaching Implications in relation to van Hiele Levels (VHL)

The VHL model provides a systematic framework that will aid educators in structuring their training to suit the current VHL level that learners operate in.

Level 0 (Visualization): Instructional practice at this level for learners should give priority to visual aids and manipulatives. Including tools such as, models or interactive software can help learners in identifying and articulating shapes on the basis of their visual properties, as this will improve the analytical reasoning or ability of the learners. Clements (1992), suggests visual resources that will stimulate spatial thinking among learners at this level, as this approach will help learners to associate the image they are seeing with the properties of such shapes.

Level 1 (Analysis): Educators are to emphasise buttressing terminology as well as improving the understanding of the properties of shapes. The use of structured worksheets can be helpful as it will prompt learners to identify, and classify quadrilateral shapes according to their properties, such as their lengths, sides, and angles. Naidoo and Kapofu (2020), assert that organised, property-oriented exploration activities enhance learners' analytical reasoning and facilitate their progression towards relational understanding.

Level 2 (Informal Deduction): Informal proof activities are suggested for learners prepared to investigate the relationships among properties. Educators may employ collaborative tasks that necessitate students to rationalize the validity of particular attributes in various quadrilaterals. Senk (1989), asserts that informal proofs facilitate a shift to deductive reasoning, assisting learners in progressing to Level 3 by engaging in logical structuring activity.

5.4 Targeted Interventions for Misconceptions and Language Barriers

Another important learning roadblock identified in the study is the misconceptions and language barriers.

Clarifying Misconceptions: Rectification of misconceptions in relation to properties, through tasks that target on clarification of concepts. Comparative tasks may be employed, to enable learners to discern what is similar and different between the quadrilaterals (e.g, rectangles versus parallelograms), aimed at reducing confusion. Applying this approach improved the capacity of learners in differentiating shapes based on their characteristics, instead of depending on visual resemblance (Burger & Shaughnessy, 1986).

Overcoming language barriers: Language barriers hamper understanding; hence, resources such as visual dictionaries can explain geometric terminology. Initiating a dictionary that can visually explain terms such as “supplementary” and “congruency” would be helpful for learners in making sense of the terms with their properties. Bansilal and Naidoo (2012), stress the importance of such initiatives as they will reinforce the terminology, and enhance their language in mathematics, especially in multilingual classrooms, where terminology may not be a means of translation.

5.5 Recommendations for Curriculum Development

The structure of the curriculum determines how geometric reasoning classroom practices are shaped. Below are highlights and suggestions for possible considerations:

Structured Progression through van Hiele Levels

Curriculum designers are to ensure instructional practices on geometric topics in the classrooms align with van Hiele levels, methodologically, for the purpose of presenting ideas or concepts in a way that will improve learners’ current understanding level of geometric reasoning. For instance, early grade learners should focus on the visualization stage, to be grounded in spatial reasoning, before advancing to the next stage, which is analytical reasoning at a higher grade. Van Hiele (1986a) is of the view that this systematic approach will facilitate a more identical way of thinking abilities throughout their educational levels.

Emphasis on Spatial and Deductive thinking

Addressing gaps in spatial reasoning requires the curriculum to integrate extra geometry-related activities in the classrooms. This includes shape transformation, exploration and construction. Such integrated activities will enhance the formal reasoning abilities of learners who are advancing to Level 2 or 3. In a study, Battista (2007), emphasises that emphasis is placed on long-term advantages associated with synthesising spatial and logical activities, adding that it will equip learners for complex mathematical reasoning.

Teacher Professional Development

Facilitating qualitative teacher-training programs regularly can help identify learners' van Hiele levels in geometry and respond to their needs accordingly.
Assessment of van Hiele Levels: The training of teachers should include guidelines on learner-evaluation procedures in relation to van Hiele levels should be conducted. Identifying the stage of each learner helps educators tailor their lessons to the current geometric understanding level of learners. Clements and Battista (1992), agree with the use of assessment instruments by educators for the purpose of assessing learners' stages of geometric reasoning, as this practice will enhance the alignment between what is taught and what is expected.

Approaches for Addressing Language Challenges and Misconceptions

Programs that are aimed at developing educators in a professional way can provide useful approaches on how to address language barriers and address prevailing misconceptions. Such programs, in the form of workshops, can feature demonstrations on how visual aids, organised worksheets, and vocabulary supports, can assist educators in the implementation of these resources during instructional practices. Brijlall (2017), asserts that professional development in these domains enables educators to establish inclusive and accessible learning environments, particularly in multicultural classrooms.

The above highlighted practical implications, provide tangible strategies for addressing the identified challenges in this study, which offers educators and curriculum designers clear pathways to enhance learners' development in geometric reasoning. Through the implementation of targeted instructional and curriculum-based interventions, educators may cultivate a more robust foundation for learners as they go through the van Hiele stages.

5.6 Study Limitations

The researcher acknowledges several limitations that may have affected the study findings, directly or indirectly. Below highlights the limitation factors.

Scope and Sample Size: The study focused on one secondary school, situated in a particular geographic area. While the sample of 21 learners yielded significant insights, the researcher acknowledges that the findings may not apply to other schools or locations due to the particular setting of the location of the study. Maree (2020) notes that a qualitative researcher with a restricted sample size can provide profound insights, yet may lack the generalisability characteristic of bigger, quantitative studies. Subsequent studies need to consider a bigger, larger, and more diverse sample for enhanced generalisability.

Self-Reported Data: The data collected was on the basis of self-reported information from learners through the questionnaires administered and the interviews conducted. This method, which is subject to biases, such as social desirability bias and reputational bias, may have affected the quality of the responses from learners, based on what they believe instead of their true thoughts or lived experiences. Considering other data collection methods, such as checking their previous assessment worksheets or records, could help to gain more insight into their understanding level.

Data Collection Constraints: The participation of the school management assisted with the preliminary data collection processes, - the large-scale printing and distribution of the task-based questionnaires. Nonetheless, certain practical challenges came up, such as incomplete questionnaire printouts and missing questions in particular scripts. For example, the responses of three participants were influenced by a printing problem, leading to data omission. These limits emphasise the difficulties of data collecting logistics, which agrees with Creswell and Poth's (2016), assertion that logistical limitations might affect data quality, highlighting the necessity for precise preparation in further and subsequent studies.

Time Constraints: The limited data collection period did not allow for a longitudinal approach, which could have provided more insights in terms of the progression of learners' understanding of quadrilaterals over time. Creswell and Poth (2016), assert that longitudinal studies provide distinct insights into developmental changes, hence improving understanding of learning progressions in geometry. Extending the period of study could be helpful in terms of the quality assurance of the study.

Focus on Quadrilaterals: Focusing on quadrilaterals in this study has provided insights, although, it may have limited the applicability of the findings regarding other geometric concepts. Similar studies in the future should consider extending the scope of the study to other geometry aspects for a wider understanding of learners' geometric reasoning.

5.7 Proposals for Future Research

In view of the inevitability of limitations, the researcher provides recommendations for future research.

Broadened Scope and Diverse Sample: Future studies should factor in a broader range of schools, districts, and socioeconomic backgrounds for the purpose of enhancing the quality of the study, and ensuring generalisability of the study across diverse education settings

Longitudinal Studies: Creswell and Poth (2016), assert that longitudinal studies provide distinct insights into developmental changes, hence improving understanding of learning progressions in geometry. Extending the period of study could be helpful in terms of quality assurance of the study.

Enhanced Data Collection Methods: Considering the incorporation of online data collection, in addition to on-site, with a more cautious approach could enhance the quality of the data, both in content and in-depth. It will also ensure the elimination of misprints of any kind, as experienced in the study.

Comparative Studies: Conducting studies involving more than one school, could enrich the quality of the study findings, both in depth and in content, based on the phenomenon being investigated- investigating grade 10 learners' understanding of quadrilaterals in Euclidean geometry.

5.8 Conclusion

This study investigated Grade 10 learners' understanding of quadrilaterals in Euclidean geometry. It focused on conceptual abilities in quadrilaterals, challenges, common misconceptions, and progression through the van Hiele levels of geometric reasoning, using grade 10 learners as the unit of analysis. Grade 10 is significant in the South African educational system, as it is the entry level in the FET phase, which sets the stage for exposure to higher levels in geometric reasoning. As such, the need for a solid foundation at this entry-level stage is critical, hence the need for the study. This study provides useful insight into the cognitive, linguistic, and instructional components influencing Grade 10 learners' understanding, therefore aiding them in achieving the fundamental knowledge necessary for advanced geometry.

The findings indicated that learners face several significant challenges in understanding quadrilaterals, such as specific misconceptions regarding properties of shapes, geometric vocabulary, and difficulty in advancing beyond level 2 of the van Hiele levels of geometric thought. The investigation further showed the restrictive nature of language in relation to spatial reasoning and unfamiliarity with proof-based tasks, which hinders the advancement of the learners to higher geometric reasoning. They also stress the need for structured instructional methods that align with the geometric needs of learners' current skills, to bridge the gaps and, in the process, enhance a deeper understanding of Euclidean geometry.

The practical and significant inputs from the study provide specific recommendations for instructional methods and curriculum design that could improve the geometric reasoning of learners. Furthermore, understanding the role the van Hiele model plays, as a key instrument of assessing learners' geometric progression, will enable educators to be in a position to embrace its guidelines to effectively assist learners in overcoming fundamental challenges, especially in progressing from visualization to analysis, and finally, to deduction in geometric reasoning.

Suggestions such as the utilisation of visual aids for Level 0 learners, the reinforcement of vocabulary at Level 1, and the introduction of informal proof tasks for Level 2 are practical measures that educators may adopt to improve learners' engagement with geometry.

Furthermore, this study enhances the subject of geometry instruction by highlighting the particular problems learners have in cultivating reasoning abilities during the FET foundation year of Grade 10. These insights can guide policy and practice, helping curriculum designers and educational authorities in developing supportive learning environments that address cognitive and language challenges.

The alignment of the study with the goals of CAPS was distinct, as evidenced by the objectives of the study, methodological approach adopted, and the recommendations provided.
In conclusion, the study places emphasis on the vital role structured instruction and curricular alignment with the van Hiele model of geometric thought can play in ensuring the progression of learners in geometric concepts as they advance in the FET phase.

By addressing the identified challenges, learners can be empowered with the necessary abilities for achieving in advanced mathematics, through the collaboration of educators and curriculum developers, in terms of cultivating an enabling learning environment. The study provides insights that are foundational towards further investigation, prompting future scholars to improve on these findings, and enlarge the comprehension of the progress of learners in geometric reasoning in South Africa.

Reference List

Abakah, F., & Brijlall, D. (2023). *Exploring High School Learners' Proficiency in Euclidean Geometry*. In *The European Conference on Education 2023: Official Conference Proceedings* (Vol. I, pp. 73–89). https://doi.org/10.22492/issn.2188-1162.2023.7

Adebayo, K. A., Ntokozo, N., & Grace, N. Z. (2020). Availability of educational resources and student academic performances in South Africa. *Universal Journal of Educational Research*, *8*(8), 3768-3781.

Adolphus, T. (2011). Problems of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. *International Journal of Emerging Sciences*, *1*(2), 143-152.

Alex, J., & Mammen, K. (2014). An assessment of the readiness of grade 10 learners for geometry in the context of curriculum and assessment policy statement (CAPS) expectation. *International Journal of Educational Sciences*, *7*(1), 29-39.

Alex, J., & Mammen, K. J. (2018). Students' understanding of geometry terminology through the lens of Van Hiele theory. *Pythagoras*, *39*(1), 1-8.

Atebe, H. U., & Schafer, M. (2010). Beyond teaching language: towards terminological primacy in learners' geometric conceptualisation. *pythagoras*, *2010*(71), 53-64.

Bakare, O. M. (2021). *An exploration of a visualization intervention in a Grade 7 mathematics classroom in the Pinetown District* (Master's thesis, University of KwaZulu-Natal, Durban). UKZN Research Space. https://hdl.handle.net/10413/20304

Bansilal, S., Mkhwanazi, T., & Brijlall, D. (2014). An exploration of the common content knowledge of high school mathematics teachers. *Perspectives in Education*, *32*(1), 34-50.

Bansilal, S., & Naidoo, J. (2012). Learners engaging with transformation geometry. *South African Journal of Education*, *32*(1), 26-39.

Bansilal, S., & Rosenberg, T. (2011). South African rural teachers' reflections on their problems of practice: Taking modest steps in professional development. *Mathematics Education Research Journal*, *23*, 107-127.

Bansilal, S., & Ubah, I. (2019). The use of semiotic representations in reasoning about similar triangles in Euclidean geometry. *Pythagoras*, *40*(1), 1-10.

Bassarear, T., & Moss, M. (2012). *Mathematics for elementary school teachers* (5th ed.). Brooks/Cole, Cengage Learning.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843–908). National Council of Teachers of Mathematics.

Bertram, C., & Christiansen, I. (2014). *Understanding research: An introduction to reading research*. Van Schaik Publishers.

Bishop, A. J. (1986). What are some obstacles to learning geometry. *Studies in mathematics education*, *5*, 141-159.

Borji, V., Radmehr, F., & Font, V. (2021). The impact of procedural and conceptual teaching on students' mathematical performance over time. *International Journal of Mathematical Education in Science and Technology*, *52*(3), 404-426.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in psychology*, *3*(2), 77-101.

Brijlall, D. (2017). Exploring outcomes of a post graduate mathematics curriculum. *Journal for New Generation Sciences*, *15*(1), 207-223.

Brijlall, D., & Ndlovu, Z. (2013). High school learners' mental construction during solving optimisation problems in Calculus: a South African case study. *South African Journal of Education*, *33*(2), 1-18.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, *17*(1), 31-48.

Cassim, I. (2006). *An exploratory study into grade 12 learners' understanding of Euclidean geometry with special emphasis on cyclic quadrilateral and tangent theorems* (Master's thesis, University of KwaZulu-Natal, Durban). UKZN Research Space. https://hdl.handle.net/10413/3166

Chambers, P. (2008). *Teaching mathematics*. Sage.

Chang, H., & Beilock, S. L. (2016). The math anxiety-math performance link and its relation to individual and environmental factors: A review of current behavioral and psychophysiological research. *Current Opinion in Behavioral Sciences*, *10*, 33-38.

Chigonga, B., Kahle, G., & Chuene, K. (2017a). Exploring Grade 9 Learners' Knowledge of Properties of Quadrilaterals. ISTE international Conference at Kruger National Park, Mopani Camp,

Chigonga, B., Kahle, G., & Chuene, K. (2017). Exploring Grade 9 learners' knowledge of properties of quadrilaterals. In *Proceedings of the 25th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE)* (pp. 173–182). SAARMSTE.

Chiphambo, S., & Feza, N. (2020). Students' alternative conceptions and misunderstandings when learning geometry: A South African perspective. In *EDULEARN20 Proceedings: 12th International Conference on Education and New Learning Technologies* (pp. 8899–8907). IATED. https://doi.org/10.21125/edulearn.2020.2207

Çibukçiu, B. (2025). The impact of constructivist methods on students' mathematical problem-solving. *Discover Education*, *4*(1), 1-8.

Clark, V. L. P., & Creswell, J. W. (2008). *The mixed methods reader*. Sage.

Clements, D. H. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420–464). Macmillan.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420–464). National Council of Teachers of Mathematics.

Cohen, L., Manion, D., & Morrison, K. (2007). *Research methods in education* (6th ed.). Routledge.

Cohen, L., Manion, L., & Morrison, K. (2002). *Research methods in education* (5th ed.). Routledge.

Cohen, L., Manion, L., & Morrison, K. (2011). Observation. In *Research methods in education* (7th ed., pp. 456–475). Routledge.

Cooper, H. (2015). *Research synthesis and meta-analysis: A step-by-step approach* (Vol. 2). Sage publications.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). SAGE Publications.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches* (5th ed.). SAGE Publications.

Creswell, J. W., & Poth, C. N. (2016). *Qualitative inquiry and research design: Choosing among five approaches* (4th ed.). SAGE Publications.

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design (International student edition): Choosing among five approaches* (4th ed.). SAGE Publications.

Crowley, M. L. (1987). The van Hiele model of the development of geometric thought. *Learning and Teaching Geometry, K–12, 1*(1), 16–21.

Darling-Hammond, L., Barron, B., Pearson, P. D., Schoenfeld, A. H., Stage, E. K., Zimmerman, T. D., Cervetti, G. N., & Tilson, J. L. (2015). *Powerful learning: What we know about teaching for understanding*. John Wiley & Sons.

Department of Basic Education. (2003). *National curriculum statement grades 10–12 (General): Mathematics*. Department of Basic Education.

Department of Basic Education. (2015). *Annual report*. https://nationalgovernment.co.za/department_annual/84/2015-department:-basic-education-(dbe)-annual-report.pdf

Department of Basic Education. (2019). *DBE diagnostic report 2019: Book 1 (Chapter 10 Mathematics).* https://mgonline.mgslg.co.za/pluginfile.php/34228/mod_resource/content/0/DBE%20

Diagnostic%20Report%202019%20-%20Book%201%20%28Chapter%208%20Life%20Sciences%29.pdf

Department of Basic Education. (2023). *NSC examinations 2023: Diagnostic report Book 1*. https://www.education.gov.za/Portals/0/Documents/Reports/Diagnostic%20Reports%202022/Diagnostic%20Report%202023%20Book%201.......pdf?ver=2024-02-27-144522-073

Department of Basic Education. (2011). *Curriculum and assessment policy statement: English as a second additional language*. Department of Basic Education.

De Villiers, M. (1999). *Rethinking proof with the Geometer's Sketchpad*. Key Curriculum Press.

DeJonckheere, M., & Vaughn, L. M. (2019). Semistructured interviewing in primary care research: A balance of relationship and rigour. *Family Medicine and Community Health, 7*(2), e000057. https://doi.org/10.1136/fmch-2018-000057

Denzin, N. K., Lincoln, Y. S., Giardina, M. D., & Cannella, G. S. (2023). *The SAGE handbook of qualitative research* (5th ed.). SAGE Publications.

Duval, R. (1999). *Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for learning*. Springer.

Feza, N., & Webb, P. (2005). Assessment standards, Van Hiele levels, and grade seven learners' understandings of geometry. *Pythagoras, 62*, 36–47. https://doi.org/[if

French, D. (2004). *Teaching and learning geometry: Issues and methods in mathematical education* (1st ed.). Continuum.

Fujita, T. (2008). Learners' understanding of the hierarchical classification of quadrilaterals. *Proceedings of the British Society for Research into Learning Mathematics*, *28*(2), 31-36.

Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon. *The Journal of Mathematical Behavior, 31*(1), 60-72.

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education, 9*(1), 3–20

Gerdes, P. (1999). *Geometry from Africa: Mathematical and educational explorations* (Vol. 10). Cambridge University Press.

Gibbs, G. R. (2012). Different approaches to coding. *Sociological Methodology, 42*(1), 82-84.

Gill, P., Stewart, K., Treasure, E., & Chadwick, B. (2008). Methods of data collection in qualitative research: interviews and focus groups. *British dental journal*, *204*(6), 291-295.

Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational studies in mathematics*, *47*(1), 7-33.

Guba, E. G. (1990). *The paradigm dialog*. Paper presented at the Alternative Paradigms Conference, Indiana University, School of Education, San Francisco, CA, United States, March 1989.

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. *Handbook of qualitative research*, *2*(163-194), 105.

Habermas, J. R. (1988). *On the logic of the social sciences*. Polity Press.

Henderson, K. B. (Ed.). (1973). *Geometry in the mathematics curriculum* (36th Yearbook). National Council of Teachers of Mathematics.

Iftanti, E., Zahroh, U., & Musrikah, M. (2021). Correlation Among Semantic, Syntactic, Pragmatic, And Cognitive Barriers Towards Accuracy Geometry Proofs. *English Review: Journal of English Education*, *10*(1), 309-322.

Israel, M., & Hay, I. (2006). *Research ethics for social scientists*. Sage.

Jones, K. (2001). Issues in the teaching and learning of geometry. *Aspects of Teaching Mathematics*.

Jones, K. (2003). Issues in the teaching and learning of geometry. In *Aspects of teaching secondary mathematics* (pp. 137-155). Routledge.

Khaliq, A., Baig, I., Ameen, M., & Mirza, A. (2016). Socio-economic status and students' achievement score at secondary level: A correlational study. *International Journal of Research in Education and Social Science, 1*(2), 1-7.

Kilpatrick, J., & Swafford, J. (2002). *Helping children learn mathematics*. National Academy Press.

Klir, G. J., & von Glasersfeld, E. (1991). *An exposition of constructivism: Why some like it radical*. Springer.

Kotzé, G. (2007). Investigating shape and space in mathematics: a case study. *South African Journal of Education*, *27*(1), 19-35.

Kumar, R. (2018). *Research methodology: A step-by-step guide for beginners* (5th ed.). SAGE Publications Ltd.

Kvale, S. (2009). *Interviews: Learning the craft of qualitative research interviewing* (2nd ed.). SAGE Publications.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. SAGE Publications.

Luneta, K. (2015). Understanding students' misconceptions: An analysis of final Grade 12 examination questions in geometry. *Pythagoras, 36*(1), 1–11.

Maddock, L., & Maroun, W. (2018). Exploring the present state of South African education: Challenges and recommendations. *South African Journal of Higher Education*, *32*(2), 192-214.

Mamali, N. R. (2015). *Enhancement of learners' performance in geometry at secondary schools in Vhembe District of the Limpopo Province* (Unpublished master's thesis). [Name of University], South Africa.

Manning, K., & Stage, F. K. (2015). What is your research approach? In *Research in the college context* (pp. 19-44). Routledge.

Maqoqa, T. (2024). *An exploration of learners' understanding of Euclidean geometric concepts: A case study of secondary schools in the OR Tambo Inland District of the Eastern Cape* (Unpublished master's thesis). Walter Sisulu University, South Africa.

Marchiş, I. (2008). Geometry in primary school mathematics. *Educatia 21*(6), 131-139.

Maree, K. (2020). *First steps in research* (Third edition ed.). Van Schaik Publishers.

Marshall, C., & Rossman, G. B. (2014). *Designing qualitative research*. Sage publications.

Mason, J. (2010). Attention and intention in learning about teaching through teaching. In J. Mason (Ed.), *Learning through teaching mathematics: Development of teachers' knowledge and expertise in practice* (pp. 23–47). Springer.

Mbatha, M. M. (2022). *Exploring Pre-service Teachers' Understanding of Similarity and Proofs in Euclidean Geometry* University of KwaZulu-Natal, Edgewood].

Mbili, L. A. (2011). *Learners' Conceptual Understanding of Congruent Triangles in Transformation Geometry* (Doctoral dissertation, University of KwaZulu-Natal, Edgewood).

McLachlan, K., & Essien, A. A. (2022). Language and multilingualism in the teaching and learning of mathematics in South Africa: A review of literature in Pythagoras from 1994 to 2021. *Pythagoras, 43*(1), 1-11.

McMillan, J. H., & Schumacher, S. (2001). *Research in education: A conceptual introduction*. Longman.

Merriam, S. B., & Tisdell, E. J. (2015). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Mouton, J. (2001). *How to succeed in your master's and doctoral studies: A South African guide and resource book.* Van Schaik.

Mthethwa, M. (2015). *Application of GeoGebra on Euclidean geometry in rural high schools: Grade 11 learners* (Unpublished master's thesis). University of Zululand, South Africa.

Mutodi, P., & Ngirande, H. (2014). The impact of parental involvement on student performance: a case study of a South African secondary school. *Mediterranean Journal of Social Sciences,* *5*(8), 279-289.

Mwadzaangati, L. E. N. (2017). An exploration of mathematical knowledge for teaching geometric proofs. *Unpublished doctoral dissertation, University of Malawi, Zomba, Malawi. Retrieved from* http://repository*. cc. ac. mw, 8080*.

Naicker, K. (2021). *An exploration of the barriers to effective geometric thought in the Further Education and Training phase of selected secondary schools in the Umlazi District* (Unpublished master's thesis). University of KwaZulu-Natal, South Africa.

Naidoo, J., & Kapofu, W. (2020). Exploring female learners' perceptions of learning geometry in mathematics. *South African Journal of Education*, *40*(1), 1-11.

National Council of Teachers of Mathematics, Research Advisory Committee. (1988). NCTM curriculum and evaluation standards for school mathematics: Responses from the research community. *Journal for Research in Mathematics Education, 19*(4), 338–344.

Neuman, W. L. (2007). *Basics of social research*.

Ngirishi, H. (2015). *An exploration of FET mathematics learners' understanding of geometry*

Ngirishi, H., & Bansilal, S. (2019). An exploration of high school learners' understanding of geometric concepts. *Problems of Education in the 21st Century, 77*(1), 82.

Nieuwenhuis, J. (2010). Analysing qualitative data (pp. 72-80). *First steps in research. Pretoria: Van Schaik.*

Opie, C. (2004). *Doing educational research: A guide to first-time researchers* (2nd ed.). SAGE Publications.

Patkin, D., & Levenberg, I. (2012). Geometry from the world around us. *Learning and Teaching Mathematics, 2012*(13), 14-18.

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. Sage publications.

Piaget, J. (2005). *The psychology of intelligence*. Routledge.

Piaget, J., & Inhelder, B. (1967). The coordination of perspectives. *The child's conception of space*, *8*, 209-246.

Pöysä-Tarhonen, J., Häkkinen, P., Tarhonen, P., Näykki, P., & Järvelä, S. (2022). "Anything taking shape?" Capturing various layers of small group collaborative problem solving in an experiential geometry course in initial teacher education. *Instructional science, 50*(1), 1-34.

Resnik, D. B. (2018). *The ethics of research with human subjects: Protecting people, advancing science, promoting trust* (Vol. 74). Springer.

Rikhotso, S. B. (2015). *Primary school learners' attitudes on Mathematics learning in Mathematics*. University of South Africa (South Africa).

Rule, P., & John, V. (2011). *Your guide to case study research*. Van Schaik Pretoria.

Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, *20*(3), 309-321.

Setati, M. (2005). Power and access in multilingual mathematics classrooms. In *Proceedings of the fourth international mathematics education and society conference* (pp. 7-18). Brisbane: Centre for Learning Research, Griffith University.

Setati, M. (2005b). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education*, *36*(5), 447-466.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for information*, *22*(2), 63-75.

Shongwe, B. (2019). *Exploring grade 11 learners' functional understanding of proof in relation to argumentation in selected high schools* (Doctoral dissertation, University of KwaZulu-Natal, Edgewood).

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM*, *47*, 319-329.

Singh, R. I. (2006). *An investigation into learner understanding of the properties of selected quadrilaterals using manipulatives in a grade eight mathematics class* (Doctoral dissertation, University of KwaZulu-Natal).

Siyepu, S. (2013). The zone of proximal development in the learning of mathematics. *South African Journal of Education*, *33*(2), 1-13.

Sonali, S. (2016). Role of socio-economic status in academic stress of senior secondary students. *Int J Adv Educ Res*, *12*, 44-50.

Soudien, C. (2016). South Africa: The struggle for social justice and citizenship in South African education. In *The Palgrave international handbook of education for citizenship and social justice* (pp. 571-591). Springer.

Stake, R. (1995). *Case study research*. Springer.

Suleiman, G. S., Christopher, D., & Daniel, A. G. (2025). Effect of Scaffolding Instructional Strategy on Secondary School Students' Achievement in Circle-Geometry in Keffi, Nasarawa State, Nigeria. *ATBU Journal of Science, Technology and Education*, *12*(4), 522-528.

Taylor, S. J., Bogdan, R., & DeVault, M. L. (2015). *Introduction to qualitative research methods: A guidebook and resource*. John Wiley & Sons.

Teddlie, C., & Yu, F. (2007). Mixed methods sampling: A typology with examples. *Journal of mixed methods research, 1*(1), 77-100.

Teherani, A., Martimianakis, T., Stenfors-Hayes, T., Wadhwa, A., & Varpio, L. (2015). Choosing a qualitative research approach. *Journal of graduate medical education*, *7*(4), 669-670.

Terre Blanche, M. J., Durrheim, K., & Painter, D. (2014). *Research in practice : applied methods for the social sciences* (Second edition ed.). Juta and Company Ltd.

Thompson, K. M. (2003). *Geometry students' attitudes toward mathematics: An empirical investigation of two specific curricular approaches*. California State University, Dominguez Hills.

Türnüklü, E., Akkaş, E. N., & Alaylı, F. G. (2013, February). Mathematics teachers' perceptions of quadrilaterals and understanding the inclusion relations. In *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 705-714).

Tursynkulova, E., Madiyarov, N., Sultanbek, T., & Duysebayeva, P. (2023, December). The effect of problem-based learning on cognitive skills in solving geometric construction problems: a case study in Kazakhstan. In *Frontiers in Education* (Vol. 8, p. 1284305). Frontiers Media SA.

Usiskin et al., U., Z., Griffin, J., Witonsky, D., & Willmore, E. . (2008). *The classification of quadrilaterals: A study in definition.* . Information Age Publishing. https://books.google.co.za/books?hl=en&lr=&id=ff0nDwAAQBAJ&oi=fnd&pg=PR1&dq=Usiskin,+Z.,+Griffin,+J.,+Witonsky,+D.,+%26+Willmore,+E.+(2008).+The+classification+of+quadrilaterals:+A+study+in+definition.+Charlotte,+NC:+Information+Age+Publishing.&ots=dpB6s04HVk&sig=jFmvf0rp7frxpQ78murSHS_cMxE#v=onepage&q&f=false

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry*. CDASSG Project.

Usiskin, Z. (2008). *The classification of quadrilaterals: A study in definition*. IAP.

Van De Walle, J. A. (1994). *Elementary school mathematics: Teaching developmentally* (2nd ed. ed.). Longman.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2014). *Elementary and middle school mathematics*. Pearson.

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.

Van Putten, S., Howie, S., & Stols, G. (2010). Making Euclidean geometry compulsory: Are we prepared? *Perspectives in Education, 28*(4), 22–31.

Venkat, H., & Adler, J. (2020). Mediating mathematics in instruction: Professional development and knowledge of mathematics teachers. In S. Zehetmeier, D. Potari, & M. Ribeiro (Eds.), *Professional development and knowledge of mathematics teachers* (pp. 5–23). Routledge.

Visser, M., Juan, A., & Feza, N. (2015). Home and school resources as predictors of mathematics performance in South Africa. *South African Journal of Education, 35*(1), Article 1.

Von Glasersfeld, E. (1984). An introduction to radical constructivism. *The invented reality, 1740*, 28.

Williams, C. (2007). Research methods. *Journal of Business & Economics Research, 5*(3), 65–72.

Worku, M., Alemayehu, H., Ashebir, L., & Debela, K. L. (2023). "An all-inclusive, user-friendly resource": A Review of Taylor, Bogdan, and DeVault's Introduction to Qualitative Research Methods: A Guidebook and Resource (4th Edition) [Book Review]. *Qualitative Report*, *28*(7), 1895-1901. https://doi.org/10.46743/2160-3715/2023.6488

Žilková, K. (2015). Misconceptions in pre-service primary education teachers about quadrilaterals. *Journal of Education, Psychology and Social Sciences, 1*(1), 1–10.

APPENDIX A

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

02 November 2022

Stanley Izuchukwu Nkwoji (217016276)
School Of Education
Edgewood Campus

Dear SI Nkwoji,

Protocol reference number: HSSREC/00003090/2021
Project title: An Investigation of Grade 10 learners understanding of quadrilaterals in Euclidean Geometry

Approval Notification – Recertification Application

Your request for Recertification dated 27 October 2021 was received.

This letter confirms that you have been granted Recertification Approval for a period of one year from the date of this letter. This approval is based strictly on the research protocol submitted and approved in 2021.

Any alteration s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study must be reviewed and approved through the amendment /modification prior to its implementation. Please quote the above reference number for all queries relating to this study.

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years

HSSREC is registered with the South African National Research Ethics Council (REC-040414-040).

Yours sincerely,

[redacted]

Professor Dipane Hlalele (Chair)

/dd

Humanities & Social Sciences Research Ethics Committee
UKZN Research Ethics Office Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Tel: +27 31 260 8350 / 4557 / 3587
Website: http://research.ukzn.ac.za/Research-Ethics/

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

INSPIRING GREATNESS

APPENDIX B

Informed Consent letter for parents (legal guardians) whose learners have been invited to participate in the research Project

School of Education,
College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,
INFORMED CONSENT LETTER

Dear Sir/Madam,

My name is Stanley I. Nkwoji. I am a master's student from the Mathematics Education cluster, School of Education, College of Humanities, University of KwaZulu-Natal. My student number is, 217016276. I am conducting research, titled **'An investigation of grade 10 learners' understanding of quadrilaterals in Euclidean geometry'.**

Severally studies notably suggest that poor achievement in mathematics, particularly in geometry by leaners remains a concern and challenging in South Africa. This is mainly attributed to deficiency in the understanding of basic geometric concepts in the curriculum. As a result, many learners particularly in FET do not possess the proficiencies on critical thinking, problem-solving and high levels of geometric thinking skills as required by the Curriculum Assessment Policy statement (CAPS).

In view of the forgoing, I intend to investigate grade 10 learners' understanding Quadrilaterals in Euclidean geometry, with focus on quadrilaterals.
This letter therefore serves to request for your kind permission for your child to participate in this study as a grade 10 learner. To gather the information for the study, I am interested in requesting for your permission for your child to participate in this project.

Whenever research is conducted with minors (teenagers), the researcher is expected to ask for the permission of the guardian first. After you have been fully acquainted with what the study is all about and agree, then the next thing I will do is to ask for a similar permission from your child you are a guardian to. Both of you have to independently agree. You are at liberty to ask any question you deem fit and I will take the time to answer.

Participation is voluntary and participants are free to withdraw from the study at any stage, for any reason, whether personal or not. Your child's participation is completely on a voluntarily basis and confidentiality and anonymity will be ensured.

Any information given by your child cannot be used against your child, and the collected data or information will be used for purposes of this research only. In the course of the study, participants will be given a task in form of questions to attempt based on quadrilaterals related questions. The task will last for a total of 60 minutes. Thereafter, there will be an individual interview of 25-30 minutes duration based on their experiences during the exercise of attempting the questions. The interviews will be recorded and stored at a safe place together with their answer scripts and destroyed by a shredder after five (5) year, which is in line with the UKZN University data storage and disposal policy. Your child also has the right to decide to stop participating in the study at any time of your child's wish, or your wish.

The research is mandatory according to UZKN, as a major requirement for the completion of the degree I am currently working towards having. The involvement of your child is purely for academic purposes only, and there are no financial benefits involved.

I also hereby undertake that the details of your child and that of the school will not be mentioned in the study. I will also make sure that normal learning and teaching will not be disrupted in any way whatsoever whilst the study is in progress. In addition, the necessary protocols as it relates to the current Covid-19 regulations will also be strictly adhered to.

I will also share my findings and feedback on this research with you and staff members and the principal of the school. I hope that the information gained from this research will be of great help to your child and as we work together aimed at finding solutions for the current challenge we face in the teaching and learning of geometry.

For further information patterning to this study, feel free to contact any of our research or relevant offices of UKZN Edgewood Campus as highlighted below.

Office	Name	Contact	Email address
Researcher	Stanley Nkwoji	[redacted]	[redacted]
Supervisor	Prof Sarah Bansilal	031-260 3451	
Research Administrator	Sabelo Nkululeko Mthembu	+27-312603919/ [redacted]	
Research and Higher Degrees Office, Edgewood Campus	Nontobeko Dlamini	+27 31 260 3895	
Research and Higher Degrees Office, Edgewood Campus	Aveesha Seerpath	+27 31 260 3440	

Thank you for your time

Your kind consent to allow your child to participate will greatly be appreciated

Thank you for taking the time to read about this research.

Stanley Nkwoji (Mr.)
Contact Number: [redacted]
Email: [redacted]

APPENDIX C

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

INFORMED CONSENT FORM: PARTICIPANT LETTER

Informed Consent letter for Participating learners in the Project

School of Education,

College of Humanities,

University of KwaZulu-Natal,

Edgewood Campus,

INFORMED CONSENT LETTER

Dear Prospective Participant,

My name is Stanley I. Nkwoji. I am a master's student from the Mathematics Education cluster, School of Education, College of Humanities, University of KwaZulu-Natal. My student number is, 217016276. I am conducting research, titled **'An investigation of grade 10 learners' understanding of quadrilaterals in Euclidean geometry'.**

Severally studies notably suggest that, poor achievement in mathematics, particularly in geometry by leaners remains a concern and challenging in South Africa. This is mainly attributed to deficiency in the understanding of basic geometric concepts in the curriculum. As a result, many learners particularly in FET do not possess the proficiencies on critical thinking, problem-solving and high levels of geometric thinking skills as required by the Curriculum Assessment Policy statement (CAPS).

In view of the forgoing, I intend to investigate grade 10 learners' understanding in Euclidean geometry, focusing on quadrilaterals. The primary objectives of the research is to investigate the grade 10 learners' understanding of quadrilaterals in Euclidean geometry.

This letter therefore serves to invite you to participate in this study as a grade 10 learner.

Whenever research is conducted especially with minors (teenagers), the researcher is expected to ask for the permission of both the guardian first and you as the learner.

After you are fully familiar with what the study is all about and agree, both you and your parent/guardian will indicate acceptance by signing the consent. You are at liberty to ask any question you deem fit and I will take the time to answer.

Participation is voluntary and participants are free to withdraw from the study at any stage, for any reason, whether personal or not. Your participation is completely on a voluntarily basis and confidentiality and anonymity will be ensured.

Any information you give in the course of the project cannot be used against you. The collected data or information will be used for purposes of this research only. In the course of the study, the selected participants who have consented will be given a task in form of questions to attempt, based on quadrilaterals related questions. The task will last for a total of 60 minutes. Thereafter, there will be an individual interview of 25-30 minutes duration, based on their experiences during the exercise of attempting the questions.

The interviews will be recorded and stored at a safe place together with your answer scripts and destroyed by a shredding machine after five (5) year, which is in line with the UKZN University data storage and disposal policy. You have the right to decide to stop participating in the study at any time of your wish.

The research is mandatory according to UZKN, as a major requirement for the completion of the degree I am currently working towards having. Please note that your involvement is purely for academic purposes only, and there are no financial benefits involved.

I also hereby undertake that your details and that of the school will not be mentioned in the study. I will also make sure that normal learning and teaching will not be disrupted in any way whatsoever whilst the study is in progress. I will also share my findings and feedback on this research with you, your teacher and the school authority. In addition, the necessary protocols as it relates to the current Covid-19 regulations will also be strictly adhered to.

It is expected that the information gained from this research will be of great help to you, and as we work together, aimed at contributing towards improving the performance of Grade 10 learners' understanding of quadrilaterals in Euclidean geometry.

For further information patterning to this study, feel free to contact any of our research or relevant offices of UKZN Edgewood Campus as highlighted below.

Office	Name	Contact	Email address
Researcher	Stanley Nkwoji	[redacted]	[redacted]
Supervisor	Prof Sarah Bansilal	031-260 3451	
Research Administrator	Sabelo Nkululeko Mthembu	+27-312603919/ [redacted]	
Research and Higher Degrees Office, Edgewood Campus	Nontobeko Dlamini	+27 31 260 3895	
Research and Higher Degrees Office, Edgewood Campus	Aveesha Seerpath	+27 31 260 3440	

Thank you for your time

Your kind consent to participate will greatly be appreciated

Thank you for taking the time to read about this study.

Stanley Nkwoji (Mr.)
Contact Number: [redacted]
Email: [redacted]

APPENDIX D

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

CERTIFICATE OF CONSENT: PARENTS/GUARDIANS

I have been asked to give consent for the child I am the parent/guardian of to participate in this research study, which involves writing a task-based questionnaire and being involved in an interview. **I have read the forgoing information. I consent voluntarily for my child to participate as a participant in this study.**

Informed consent – permission to interview. *Please note that this document is produced in duplicate – one copy to be kept by the respondent, and one copy to be retained by the researcher.*	
Signed consent	
• I understand that the purpose of this interview is solely for academic purpose. The findings will be published as a thesis, and may be published in academic journals.	**Yes** ☐ **No** ☐
• I understand that my child will remain anonymous.	**Yes** ☐ **No** ☐
• I understand that my child will not be paid for participating but a souvenir will be given.	**Yes** ☐ **No** ☐
• I understand that my child reserves the right to discontinue and withdraw her participation any time.	**Yes** ☐ **No** ☐
• I understand that my child will not be coerced into commenting on issues against her will, and that she may decline to answer specific questions.	**Yes** ☐ **No** ☐
• I understand my child reserves the right to schedule the *time* and *location* of the interview.	**Yes** ☐ **No** ☐
• I consent to have this interview recorded.	**Yes** ☐ **No** ☐
*** By signing this form, I consent that I have duly read and understood its content.**	

________________	________________	________________
Name of Guardian	**Signature**	**Date**
Stanley Nkwoji	________________	________________
Name of Researcher	**Signature**	**Date**

APPENDIX E

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

CERTIFICATE OF ASSENT: PARTICIPANT

I have read this information (or had the information read to me). I have had my questions answered and know that I can ask questions later if I have them. I consent voluntarily to participate in the task and one interview

Informed assent – Permission to interview. *Please note that this document is produced in duplicate – one copy to be kept by the respondent, and one copy to be retained by the researcher.*		
Signed assent		
• I understand that the purpose of this interview is solely for academic purpose. The findings will be published as a thesis, and may be published in academic journals.	**Yes** ☐	**No** ☐
• I understand I will remain anonymous.	**Yes** ☐	**No** ☐
• I give permission for photos of my body map to be taken and appear in the thesis. Your name will be removed to ensure you remain anonymous	**Yes** ☐	**No** ☐
• I understand that I will not be paid for participating but a souvenir will be given.	**Yes** ☐	**No** ☐
• I understand that I reserve the right to discontinue and withdraw my participation any time.	**Yes** ☐	**No** ☐
• I consent to be frank to give the information.	**Yes** ☐	**No** ☐
• I understand I will not be coerced into commenting on issues against my will, and that I may decline to answer specific questions.	**Yes** ☐	**No** ☐
• I understand I reserve the right to schedule the *time* and *location* of the interview.	**Yes** ☐	**No** ☐
• I consent to have this interview recorded.	**Yes** ☐	**No** ☐
*** By signing this form, I consent that I have duly read and understood its content.**		

Name of Participant	**Signature**	**Date**
Stanley Nkwoji		
Name of Researcher	**Signature**	**Date**

APPENDIX F

Extract of Turnitin report

Turnitin Originality Report

Processed on: 25-Aug-2025 4:39 PM CAT
ID: 2735013244
Word Count: 48632
Submitted: 1

thesis draft By Stanley Nkwoji

Similarity Index	Similarity by Source	
2%	Internet Sources:	1%
	Publications:	1%
	Student Papers:	N/A

< 1% match ()
Mudhefi, Fungirai. "An exploration of learning difficulties experienced by grade 12 learners in euclidean geometry : a case of Ngaka Modiri Molema district", 2022

< 1% match ()
Ngirishi, Harrison.. "An exploration of FET mathematics learners' understanding of geometry.", 2015

< 1% match ()
Crowley, Mary Lora Noffsinger. "THE DESIGN AND EVALUATION OF AN INSTRUMENT FOR ASSESSING MASTERY VAN HIELE LEVELS OF THINKING ABOUT QUADRILATERALS", 1989

< 1% match ()
"Massachusetts mathematics curriculum framework", Massachusetts Department of Education, 2000

< 1% match ()
Mateya, Muhongo. "Using the van Hiele theory to analyse geometrical conceptualisation in grade 12 students: a Namibian perspective", Faculty of Education, Education, 2009

< 1% match ()