Teachers' questioning practices for enhancing learners' critical thinking in Grade 10 mathematics classrooms

by

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DECLARATION

I, Ayanda Sizwe Zondo, declare that:

1. The research reported in this thesis, except where otherwise indicated, is my original work.
2. This thesis has not been submitted for any degree or examination to any other university.
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As the candidate's supervisor, I agree to the submission of this thesis.

Signed: [redacted] ________________________________

Date: 27/07/2025

ABSTRACT

This study explored the questioning practices used by teachers to enhance learners' critical thinking in Grade 10 mathematics classrooms. The study was grounded in the Revised Bloom's Taxonomy and Duron et al.'s (2006) critical thinking framework. A qualitative case study design was employed within the interpretive paradigm. Six teachers – two each from three public high schools in the Pinetown District of the South African province of KwaZulu-Natal – participated in the study. Data were collected using three semi-structured instruments – a questionnaire, an observation schedule, and an interview schedule – and were triangulated and analysed thematically. The findings revealed that the teachers used various questioning practices. While all the teachers posed questions, those whose questions were restricted to the lower-order categories of the Revised Bloom's Taxonomy (Levels 1 and 2: remembering; understanding) limited learners' opportunities to develop their critical thinking. Teachers whose questions targeted the higher-order categories of the Revised Bloom's Taxonomy (Levels 3 to 6: analysing, evaluating, creating) also integrated open-ended and adaptive questioning techniques, enhancing learners' critical thinking. The study's key contribution is the development of a model that integrates the cognitive levels of the Revised Bloom's Taxonomy with Duron et al.'s structured critical thinking approach. This model offers a framework for using questioning that incorporates all six levels of the Revised Bloom's Taxonomy to systematically develop learners' critical thinking. This research addresses a gap in the literature by providing empirical evidence of the questioning practices used in mathematics classrooms that demonstrates the need for professional development to enhance teachers' pedagogical use of questioning. The study underscores the importance of teaching through questioning. A new model – 'CTF+RBT' – was developed during this study that integrates the key elements of Duron et al.'s critical thinking framework and the Revised Bloom's Taxonomy. To ensure the credibility of the CTF+RBT model, it is recommended that further research explore its applicability and its long-term impact on teachers' questioning.

Keywords: Revised Bloom's Taxonomy; critical thinking; critical thinking framework; CTF+RBT model; questioning; questioning practices; probing; adaptive questioning

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LIST OF ABBREVIATIONS

CAPS	Curriculum and Assessment Policy Statement
CTF	Critical Thinking Framework
CRM	Cognitive Regior Matrix
DBE	Department of Basic of Education
DoE	Department of Education
DOK	Depth of Knowledge
FET	Further Education and Traning
HSSREC	Humanities and Social Sciences Research Ethics Committee
NSC	National Senior Certificate
RBT	Revised Bloom's Taxonomy

CHAPTER 1: INTRODUCTION

1.1 Introduction and background to the study

The use of questioning by teachers to enhance learners' critical thinking in mathematics education has been widely recognised as an essential instructional practice. Effective questioning strategies provide learners with opportunities to engage in higher-order thinking, problem-solving, and reasoning, thereby enhancing their conceptual understanding and ability to apply mathematical knowledge in real-world contexts. In mathematics classrooms, questioning serves as a pedagogical tool that encourages learners to articulate their thoughts, justify their answers, and critically evaluate mathematical concepts.

Despite its importance, available research suggests that many teachers use lower-order questions predominantly – those situated at the 'remembering' and 'understanding' levels of the Revised Bloom's Taxonomy (RBT), limiting learners' cognitive development and problem-solving abilities. In this study, Levels 1 and 2 of the RBT are referred to as the 'lower-order category', and Levels 3 to 6 as the 'higher-order category'.

This study addresses a critical gap in the literature by providing empirical evidence on the actual implementation of teachers' questioning practices in mathematics classrooms. While numerous studies have discussed the theoretical significance of questioning in promoting critical thinking, limited research has examined how teachers use questioning practices practically in mathematics classrooms. By bridging this gap, the study contributes to the ongoing discourse on improving mathematics instruction.

This chapter outlines the background, context, and purpose of the study. Furthermore, the significance and scope of this research are discussed, and definitions of the terms used in the study are provided. The delimitations of the study are presented. Finally, the chapter concludes with an outline of the thesis.

1.2 Contexual background to the problem

South Africa's National Senior Certificate (NSC) is a high school leaving certificate awarded to a learner who has completed and passed Grade 12, in line with the Curriculum and Assessment Policy Statement (CAPS) requirements. The South African National Senior Certificate (NSC) Diagnostic Book 1 Examination Report for the year 2024 reports that from 2020 to 2024, a marginal increase was seen in the percentage of Grade 12 learners attaining

40% and above in mathematics, as shown in Figures 1 and 2 (these two figures should be read in conjunction). While the results displayed in Figure 1 show that an increasing percentage of learners achieved 40% or higher from 2020 to 2024, Figure 2 shows a declining performance distribution in the percentage of learners achieving 40% or higher. This suggests that learners continue to experience challenges in their learning of mathematics.

Figure 1

Overall achievement trends (in %) in Grade 12 final examination mathematics from 2020 to 2024 (Department of Basic Education, 2024, p. 218)

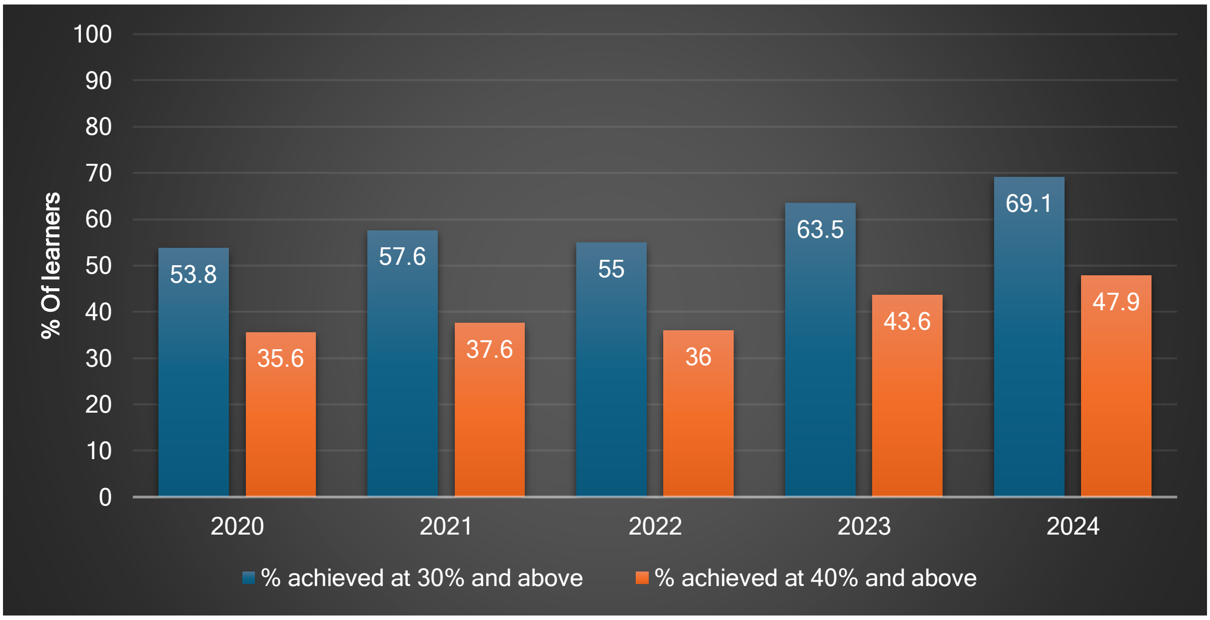
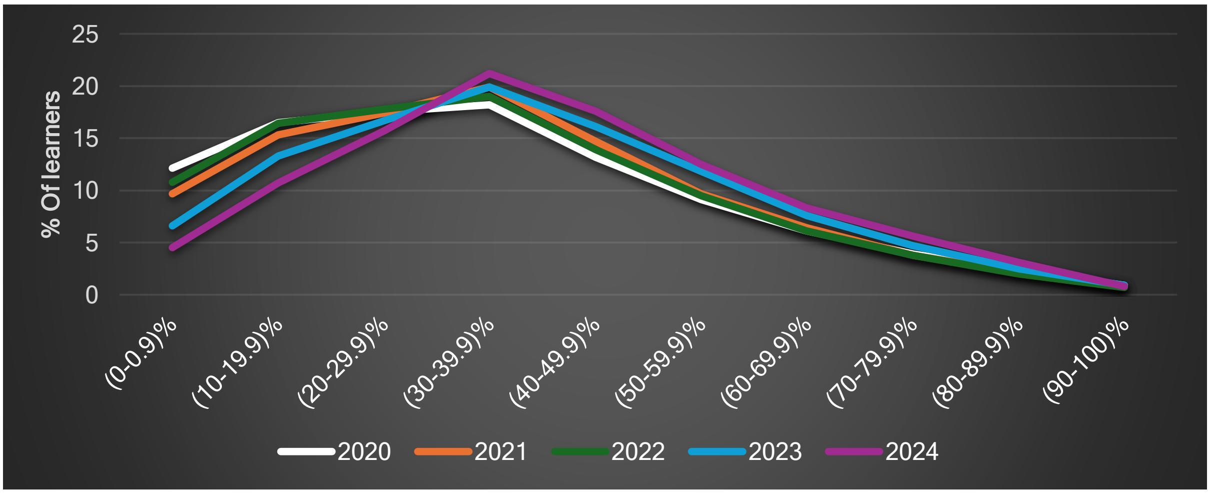


Figure 2

Distribution curves for mathematics performance (in %) from 2020 to 2024 (Department of Basic Education, 2024, p. 218)



Teachers' necessity for effective use of questioning practices can play a crucial role in enhancing critical thinking by prompting learners to move beyond the 'remember' and 'understand' levels of the Bloom’s Revised Taxonomy. These first two levels are considered the lower-order category as they require learners to recall and remember facts (Anderson et al., 2001). Questioning practices such as probing by using open-ended questions encourage learners to explore multiple problem-solving strategies (Aziza, 2021). The two final mathematics examination papers written by Grade 12 learners have questions situated at all six levels of the RBT (Department of Basic Education [DBE], 2024). The declining performance of learners indicated by the distribution curve presented in Figure 2 suggests that learners find it challenging to respond to questions situated at Levels 3 to 6 of the RBT – the levels that are considered higher-order cognitive skill levels that require learners to move beyond recall of facts. This notion is supported by Figures 3 and 4, which present my analysis of the sub-questions of the 2024 final mathematics Grade 12 examination question Papers 1 (P1) and 2 (P2) using the six levels of the RBT. This analysis determined that the sub-questions on which learners performed below 40% required learners to use higher-order cognitive skills.

Figure 3

Average performance per-sub-question of Mathematics Paper 1 for the year 2024 (National Department of Basic Education, 2024, p. 220)

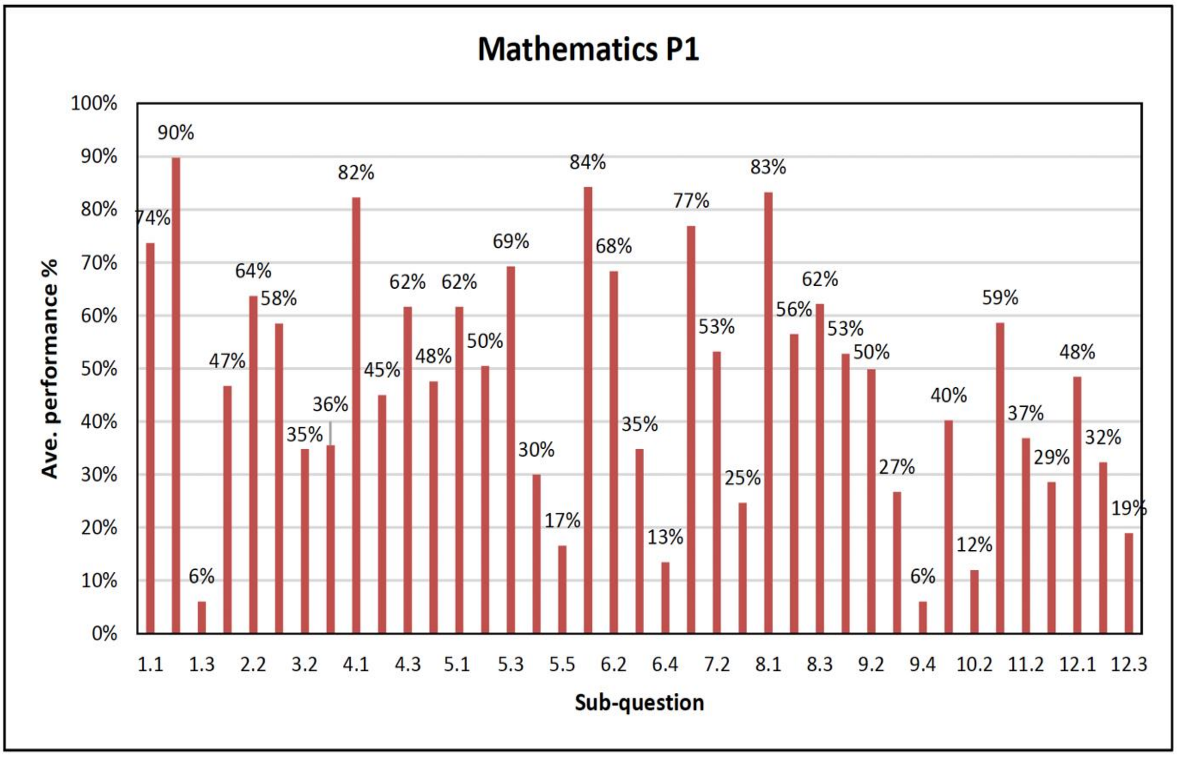
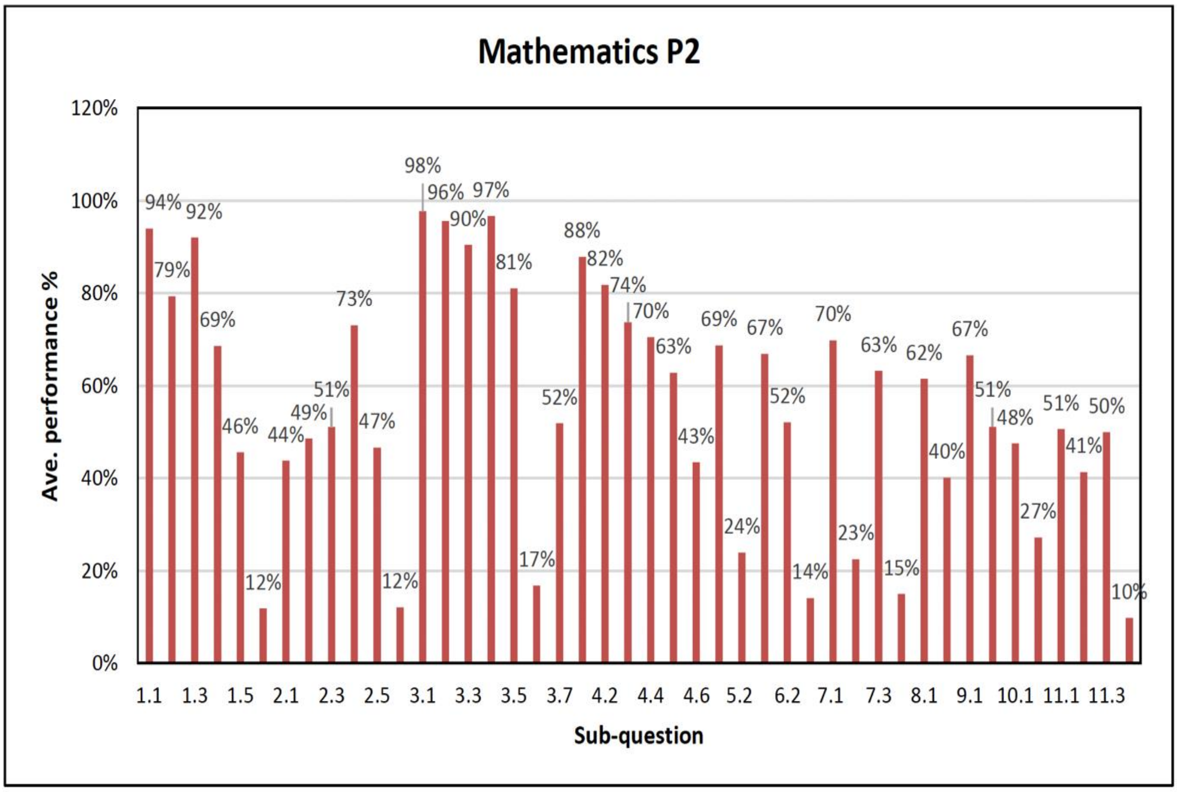


Figure 4

Average performance per-sub question of Mathematics Paper 2 for the year 2024 (Deparment of Basic Education, 2024, p. 232)



Synthesising the data presented in Figures 1 - 4, I argue that if teachers are not exposing their learners to questioning practices covering all six levels of the RBT, their learners will find it challenging to respond to questions in the higher-order category. Hence, there is a decline in the percentage of learners responding correctly to questions in the higher-order thinking skills category on both Paper 1 and Paper 2. Per the results presented in Figures 3 and 4, if learners continue to find it challenging to respond to questions in the higher-order category, this contributes to the notion that mathematics is challenging (Benidiktus et al., 2021). Hence, there was a low pass rate for mathematics (69.1%) compared to subjects such as Physical Sciences (75.6%) in 2024 (Department of Basic Education, 2024). A similar trend was observed in 2023, with a mathematics pass rate of 63.5% and physical sciences pass rate of 76.2% (DBE, 2024).

Associating the NSC performance trends to Grade 10 mathematics classrooms, the same teachers often teach Grade 10 and 12 learners (Tibane et al., 2024). In addition, there is some overlap of the concepts taught in Grades 10 and 12. Both grades cover algebra, sequence and series, finance, functions and graphs, probability, Euclidean geometry, analytical geometry, statistics, trigonometry, and probability. From Grade 10 to Grade 12, learners are expected to develop the ability to think critically about these concepts. Hence the necessity for teachers to enhance learners' critical thinking through questioning in Grade 10 mathematics lessons. I argue that there is a link between how learners are taught through questioning in their classrooms and how they perform during assessment.

According to Steyn and Adendorff (2020), some teachers in South Africa continue to find it challenging to use specific and appropriate questioning strategies situated at all six levels of the RBT. Instead, they over-emphasise teaching through questions in the lower-order category. Benidiktus et al. (2021), in their international study on mathematics instruction to promote higher-order thinking, found that for the teachers who found it challenging to teach using questioning that engaged all six levels of the RBT, the learners taught by these teachers also found it challenging to respond to higher-order questions.

The argument I present here is that all six levels of the RBT are essential for enhancing learners' critical thinking, and teachers must teach and situate their questions across all six levels. Over-reliance on questions that engage the lower-order category, while neglecting the higher-order category, could hinder learners' ability to respond to higher-order category questions during assessment. This could result in fewer learners pursuing careers in science, technology, engineering, and mathematics. If this were to happen, there would be a decline in the number of professionals in these fields in South Africa. This present study does not investigate whether learners can respond to higher-order category questions. Instead, the focus is on teachers and their practices for asking learners questions with the intention to enhance their critical thinking.

1.3 Problem Statement

While some mathematics teachers are able to teach using questioning and acknowledge that their questions should range across the six levels of the RBT, very little is known about the extent to which these teachers are able to ask learners questions at all six levels of the RBT (Dahal et al., 2019; Kastberg et al., 2019; Lim et al., 2020). Moreover, there is limited empirical evidence about the ability of South African teachers being able to ask questions pitched at all six levels of the RBT during lessons. Additionally, none of the studies I reviewed during this study that focused on the South African context investigated or explored how Grade 10 mathematics teachers ask learners questions across all six levels of the RBT or use other questioning practices to enhance learners' critical thinking.

In the last five years, some peer-reviewed research articles have recommended that mathematics teachers prioritise teaching through questioning and situate the questions they ask learners across all six levels of the RBT (Hsu et al., 2022; Khoza & Msimanga, 2022; Lim et al., 2020; Novitaningrum et al., 2020). Moreover, these studies recommended that mathematics teachers incorporate open-ended questions as a questioning practice in their mathematics classrooms. They argue that open-ended questions also have the potential to enhance learners' critical thinking when integrated with the use of the RBT.

The latest study situated in the South African context that I was able to retrieve was Mokotedi's (2023) study on the attributes of mathematics teachers' questions. However, her study focused on five teachers of Grade 9 mathematics and did not use the RBT as a theoretical framework. Before this present study, I did not know if the participants were teaching through questioning, nor the extent to which they enhance learners' critical thinking through questioning.

1.4 Rationale and significance of the study

The rationale for this study derives from my teaching experience and the findings of the research conducted for my dissertation for my Master of Education (M.Ed.) degree specialising in Mathematics, which highlighted the significance of this topic and identified this gap in the literature. The focus of my dissertation was on Grade 10 learners' cognitive engagement. I explored how the learners were responding to a mathematical task given by the teacher and whether they were cognitively engaged when responding to the questions aksed by the teacher.

As a qualified mathematics teacher with more than seven years of teaching experience in the Further Education and Training phase (Grades 10 – 12), I have developed an interest in understanding how learners develop critical thinking in mathematics classrooms. During teacher training and development workshops organised by the mathematics specialist advisor, my fellow teachers and I have discussed how our learners tend to struggle to respond to higher order questions. This deepened my interest in investigating mathematics teachers' questioning practices for enhancing learners' critical thinking in Grade 10 mathematics classrooms.

The qualitative study that I conducted for my master's research explored Grade 10 learners' cognitive engagement when responding to a Euclidean geometry task in a mathematics classroom. I found that some learners were not cognitively engaged through questioning and, thus, could not respond to higher-order questions. The teachers' over-reliance on questions that related to the lower-order category of the RBT was identified as a factor that hindered learners' ability to respond to higher-order category questions about the task. In many instances, learners responded with yes/no responses or recalled facts, thus hindering learners' ability to respond to higher-order category questions. Based on my findings, I recommended further investigation into teachers' questioning practices in Grade 10 mathematics. The present study addresses that need.

Exploring the literature on this topic, I found that there has been limited research focusing on teachers' questioning practices to enhance learners' critical thinking in South African mathematics classrooms. Previous studies have not provided compelling evidence that most teachers use questioning practices to enhance learners' critical thinking in mathematics classrooms. This study aimed to address this gap. The studies that have been done have tended to focus on the importance of questioning but do not investigate the type of questions asked and how teachers use those questions to enhance learners' critical thinking. This study contributes to addressing this gap in the literature by identifying the questioning practice used by Grade 10 mathematics teachers to enhance learners' critical thinking skills. The knowledge generated by this study may assist mathematics teachers and prospective researchers to better understand what kinds of questions effectively develop learners' critical thinking.

1.5 Research objectives of the study

In response to the problem statement, this research study pursued four main objectives. The objectives of the study are:

1. to explore the questioning practices used by teachers to enhance critical thinking in Grade 10 mathematics classes;
2. to identify questioning practices that teachers can use to enhance critical thinking in Grade 10 mathematics classes;
3. to determine the manner in which teachers use questioning practices to enhance critical thinking in Grade 10 mathematics classes; and
4. to identify the opportunities that the use of teachers' questioning practices provide for enhancing critical thinking in Grade 10 mathematics classes.

1.6 Research questions

The following four research questions guided the study:

1. What questioning practices are currently being used by teachers in Grade 10 mathematics classes?
2. What types of questioning practices can teachers use to enhance critical thinking in Grade 10 mathematics classes?
3. How do teachers use questioning practices to enhance critical thinking in Grade 10 mathematics classes?
4. Why is there a need for teachers to use questioning practices to enhance critical thinking in Grade 10 mathematics classes?

1.7 Definition of key terms

The following key terms were used in this study.

1. Questioning refers the process of asking questions to gain information, clarify understanding, stimulate thinking, or encourage discussion. It is a fundamental technique used in communication, education, research, and problem-solving (Nappi, 2017).
2. Teachers' questioning practices include when, how, and why they ask questions, the type of questions they use – for example, lower-order versus higher-order – and how they engage learners in responding (Hamel et al., 2021).
3. Critical thinking, in the context of mathematics, the ability to apply, analyse, evaluate, and create mathematical concepts, solutions, and arguments through logical reasoning, problem-solving, and reflective thinking. It also involves making reasoned judgments and applying mathematical knowledge to real-world situations (Lombardi, 2023).

1.8 Overview of the methodology

This research was conducted as a qualitative case study within the interpretive paradigm. Pathak et al. (2013) argue that a qualitative approach complements a case study well since the focus is on a humanistic approach. Teachers were purposefully and conveniently selected from three public high schools in the Pinetown District of the province of KwaZulu-Natal, South Africa, to participate in this study.

Three qualitative data collection instruments were used: a semi-structured questionnaire, a semi-structured lesson observation schedule, and a semi-structured interview schedule. The semi-structured questionnaire required teachers to respond by identifying the types of questions they used and indicating when and how they used them in their Grade 10 mathematics classrooms. The semi-structured lesson observation schedule gathered information about the step-by-step questioning practices that the participating teachers used to enhance learners' critical thinking during lessons. The semi-structured interview schedule was used to interview teachers about their views on the questioning practices that enhance learners' critical thinking.

1.9 Delimitations

Table 1

Delimitation of study

Three delimitations were identified concerning the scope of the study		
Delimitation 1	Delimitation 2	Delimitation 3
The sample constituted six teachers from three high schools in KwaZulu-Natal province.	The focus was on teachers' use of questioning practices to enhance learners' critical thinking in Grade 10 mathematics classrooms.	The manner in which learners ask questions was not investigated.

1.10 Outline of the thesis

This thesis is presented in seven chapters, as follows:

Chapter One: Introduction

This chapter has described the background and context of the study and the problem. The problem was stated and the significance of the study discussed. The four main research questions, along with the objectives, were identified. Finally, the study's delimitations were discussed.

Chapter Two: Literature Review

The literature review is presented, with a focus on teachers' questioning practices. The scope of the review covers what is already known about teachers' questioning practices to enhance learners' critical thinking and what is not known about their questioning practices. The intention is to establish context for the present study. Comparisons between scholarly findings are made. The review considers the similarities, differences, limitations and recommendations of preview studies. This enables a rich understanding of the position of this study.

Chapter Three: Theoretical framework

This chapter presents the rationale for employing the Revised Bloom's Taxonomy (RBT) developed by Anderson and Krathwohl, 2001 and the Critical Thinking Framework (CTF) developed by Duron et al. (2006), to deepen the analysis of data. The RBT was used as a lens to identify the taxonomy levels at which teachers' questions were situated. The CTF was used to explore the questioning practices employed by teachers to enhance learners' critical thinking.

Chapter Four: Research design

The research paradigm, approach, methods, design, data generating techniques, and data analysis techniques pertinent to this study are described in this chapter.

Chapter Five: Data presentation and analysis

This chapter presents and analyses the study's findings in accordance with the themes that address the research questions that guided this study.

Chapter Six: Discussion of results

This chapter discusses and interprets the findings presented in Chapter Five. It also presents and elucidates the model that was developed during the course of this study, which integrates the RBT and the CTF.

Chapter Seven: Concluding remarks

This chapter summarises the significant findings of the study in relation to the objectives and the four main research questions. The chapter further discusses the study's contribution and limitations and makes recommendations for future research.

1.11 Conclusion

This chapter has presented the contextual background to this exploration into teachers' questioning practices. Discussing the problem statement and the rationale provided grounds for the necessity and significance of conducting the present study. The research objectives, along with the research questions that guided the study, were presented. Definitions of terms were included to easily reference the central concepts used in the study. The delimitations of the study were discussed. In the next chapter, I present a review of the literature.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

The previous chapter presented the research background, aim, and purpose of the study. This chapter presents a comprehensive scholarly argument, analysis, and evaluation of the relevance of mathematics teachers' questioning practices to enhance learners' critical thinking skills as a research topic. This literature review examines the role of mathematics teachers' questioning practices in enhancing learners' critical thinking. It explores historical perspectives, the significance of questioning in mathematics classrooms, and various question types used by teachers. The chapter critically analyses existing research to identify patterns, contradictions, and gaps, demonstrating that questioning practices vary across educational settings. By critically evaluating the existing body of knowledge, this literature review establishes a foundation for the present study on teachers' questioning practices for enhancing learners' critical thinking in Grade 10 mathematics classrooms.

2.2 Components of mathematical learning through questioning

Dahal et al. (2019) define the pedagogic use of questioning in the mathematics classroom as the posing of a question by a teacher in a way that requires learners to respond. Mangwiro and Machaba (2022) substantiate this definition by stating that questioning occurs in mathematics when a teacher poses a problem or a dilemma to the learner with the expectation of an answer. Teachers’ questioning practices typically use a recitation or triadic dialogue format: a three-part exchange in which the teacher initiates by asking a question, the learner responds, and the teacher gives feedback to the learner's response (Attard & Holmes, 2022). Feedback is given verbally, in writing, or may be expressed using body language. Simply put, a question is a statement to which a response is expected.

According to DeJarnette et al. (2020), teachers use questioning for various purposes in their instructional practice in mathematics classrooms, including to encourage learners to listen carefully, to initiate an argument or discussion, or to organise specific learning activities. In their study on teachers’ efforts to develop learners’ mathematical reasoning skills, Mukuka et al. (2023) suggest that through questioning, teachers can encourage learners to explain their thoughts and investigate their reasoning. As they do so, their critical thinking is enhanced. This helps the teacher to gauge and guide the understanding of the learners. Scholars such as Amber et al., (2015); Hsu et al., (2022); Ong et al., (2016) and Zhang et al., (2024) contend that teachers use questioning as a teaching approach when they assess learners' prior knowledge of a concept at the beginning of a lesson, to develop the concept, or at the conclusion of the lesson. Hence, through questioning, teachers can challenge the thinking of learners. To do so, teachers need to be aware of the type of questions that elicit or promote learners' critical thinking in mathematics education (Ukobizaba et al., 2021).

2.2.1 Critical thinking in mathematics education

Critical thinking in mathematics is a complex and multidimensional cognitive process that extends beyond procedural fluency to encompass logical reasoning, creative problem-solving, and the ability to make meaningful connections between mathematical concepts (Benade, 2020). It is the ability to engage in reflective and independent thinking, which extends learners from responding to the lower-order category of questions to the higher-order category of the RBT by applying, analysing, evaluating, and creating information to solve problems and make decisions. According to Benade (2020), critical thinking requires learners to engage deeply with content by questioning assumptions, exploring alternative solutions, and reflecting on their reasoning processes.

In the South African Curriculum and Assessment Policy Statement (CAPS) for mathematics for the Further Education and Training (Grades 10 to 12) phase, teachers are expected to ask learners questions that promote and enhance their critical thinking using all RBT levels (Department of Basic Education, 2011. p. 16). Critical thinking is particularly important in Grade 10, a crucial year of mathematics education when learners are expected to demonstrate higher-order thinking skills (Bantwini, 2017). In the Grade 10 mathematics curriculum, foundational concepts such as algebra, geometry, and trigonometry serve as building blocks for higher-level mathematical understanding (Jojo, 2019). Jojo (2019) suggests that the development of learners' critical thinking depends significantly on teachers' instructional strategies – particularly their questioning practices, which serve as a catalyst for deeper learning and cognitive engagement.

Teachers' questioning practices are instrumental in cultivating learners' critical thinking skills by encouraging them to analyse problems, justify their reasoning, and explore different solution pathways (Bean, 2017; Hamel et al., 2021; Jacques et al., 2020). Walsh and Hodge (2018) suggest that effective questioning techniques can transform mathematics classrooms from environments focused on rote memorisation to spaces that promote inquiry and conceptual understanding. While reliance on closed or procedural questions that require only a single correct answer may limit learners' opportunities to develop reasoning skills, open-ended questions challenge learners to articulate their thought processes, fostering metacognition and deeper comprehension (Hakamata et al., 2023). This highlights the need for a deliberate and strategic approach to teachers' questioning practices in Grade 10 mathematics classrooms. The importance of questioning in enhancing critical thinking is grounded in educational theories such as the RBT and the Socratic method. The RBT categorises cognitive skills from basic recalling or remembering (lower-order), to apply, analysing, evaluating and creating (higher-order category) (Anderson & Krathwohl, 2001). According to Svanes and Andersson-Bakken (2023), teachers who strategically use phrases such as 'what', 'why', when, and 'what if' help learners progress from basic knowledge recall to deeper analytical thinking. Similarly, the Socratic method, which encourages continuous questioning and dialogue, fosters mathematical inquiry by compelling learners to justify their reasoning and refine their understanding through discussion (Marshall, 2020). This means that the Socratic method presents elements similar to the RBT for higher-order category questions. These theoretical perspectives underscore the transformative role that well-structured questioning plays in shaping learners' mathematical thinking.

Questioning practices that enhance learners' critical thinking have also been linked to improved problem-solving abilities in mathematics (Ingram & Elliot, 2019). Problem-solving is at the heart of mathematical learning, requiring learners to apply concepts to unfamiliar contexts, evaluate different strategies, and justify their solutions (Schoenfeld, 2020). When teachers pose higher-order questions that require learners to explain 'why' a particular approach is valid or 'how' different strategies compare, they enhance learners' critical thinking.

Despite the recognised benefits of questions that enhance learners' critical thinking, challenges persist in practice. Many teachers – particularly in contexts where the curriculum is rigidly structured – face constraints such as time limitations, pressure to cover the curriculum, and an overemphasis on standardised assessment, which often prioritises lower-order category questions over critical thinking (Howe et al., 2019). In Grade 10 mathematics classrooms in South Africa, where assessments frequently focus on correct answers rather than reasoning processes, teachers may feel compelled to use direct, formulaic questions that do not fully engage learners in higher-order thinking (Mangwiro & Machaba, 2022). Addressing these challenges requires a shift in pedagogical practices, as suggested by Morris and Chi (2020). Seman et al., 2017 argue that without adequate support, teachers may struggle to employ questions that promote higher-order thinking consistently.

As suggested by Hofmann and Mercer (2016), professional development is crucial to equip teachers with adequate skills to implement questioning strategies that foster critical thinking. Herbel-Eisenmann and Shah (2019) suggest that targeted teacher training on designing and delivering effective questions can significantly enhance teachers' ability to engage learners in deep mathematical discussions. They recommend workshops on questioning strategies, peer collaboration, and lesson plan approaches to help teachers refine their questioning practices and integrate them more effectively into their instructional routines.

Another critical aspect that impacts teachers' questioning practices is the classroom culture and learners' willingness to engage in mathematical discussions. A supportive classroom environment, where learners feel comfortable expressing their thoughts and exploring different solution strategies, is essential to their development of critical thinking skills (Lombardi, 2023). Teachers who create an atmosphere that encourages open-ended exploration and values multiple perspectives empower learners to take intellectual risks. This means not only asking thought-provoking questions but also giving learners adequate time to process these questions and formulate their responses. When learners are given opportunities to articulate their reasoning without fear of judgment, they develop confidence in their ability to engage critically with mathematical concepts, which promotes the development of their critical thinking.

2.2.2 Conceptual understanding

Conceptual understanding is a fundamental aspect of critical thinking in mathematics, as it enables learners to grasp the principles that govern mathematical systems and apply these principles to new contexts (Sun & Xie, 2020). Unlike procedural knowledge, which focuses on memorising steps and algorithms, conceptual understanding requires learners to internalise the 'why' behind mathematical operations, allowing them to transfer their knowledge to different problem-solving scenarios (Shanmugavelu et al., 2020). For example, a learner with a deep conceptual grasp of fractions would not only know how to perform fraction operations but would also understand why these operations work and how they relate to broader mathematical ideas, such as division and proportional reasoning (Namkung & Fuchs, 2019). Questioning is important in developing learners' conceptual understanding by prompting them to reflect on their reasoning, explore alternative methods, and justify their solutions (Ramirez, 2021).

Through questioning, teachers can guide learners toward making meaningful connections between concepts, thereby enhancing their ability to engage in critical thinking and problem-solving (Jatisunda et al., 2020). A well-structured sequence of questions should move learners from the lower-order category to the higher-order category, ensuring that they understand mathematical concepts and are able to apply them in real-life and unfamiliar contexts (Sussman et al., 2019). Despite the clear benefits that questioning as a pedagogical practice can yield in terms of developing learners' conceptual understanding, its effectiveness depends on how skilled teachers are at designing and delivering purposeful questions (Henriksen et al., 2017). Max and Welder (2020) suggest that many teachers struggle to balance questions that engage with the lower-order category of the RBT with questions that engage the higher-order category, often defaulting to questions that elicit straightforward answers rather than deeper explanations. They further argue that most teachers who cannot balance lower-order with higher-order categories tend to point to constraints – such as inadequate time to cover the curriculum.

Mangwiro and Machaba (2022) observed that effective mathematics teachers use a range of question types such as probing, follow-ups, leading prompts, and learner-specific questions as diagnostic tools, and they argue that analysing their questions and learner responses helps teachers refine their techniques. In their case study, teachers were guided to self-assess: by examining which questions elicited meaningful explanations versus rote answers, they could identify and adopt good questioning practices. This reflection is crucial because, Cevikbas and Kaiser (2022) argue that simply asking more questions is not enough. The quality of questions determines whether learners construct deep understanding or merely recall facts. Thus, professional learning that helps teachers plan clear, concept-focused questions and critically review classroom dialogue can transform questioning into a powerful lever for conceptual understanding (Lee, 2025).

Moreover, the type of question strongly influences the depth of learners’ thinking. Rapanta and Macagno (2023) indicate that classroom discourse show that factual or procedural prompts, such as asking learners the question: What is the answer? generally reveal little about learners’ conceptual grasp, whereas probing or guiding questions encourage explanation and sense-making. Suryana and Yulia (2021) found that probing questions, such as asking learners the question: How do you know that? extended learner thinking and shifted learners from low-level to higher-order reasoning, while guiding prompts steered learners to derive underlying concepts or procedures. In practice, this means a teacher who asks the questions: Why does this method work? or Can you find another way to solve this? helps learners articulate connections and justify their reasoning (Salmon & Barrera, 2021). Such open-ended questions prompt learners to explore relationships between ideas, thus fostering richer conceptual understanding. In contrast, when teachers rely mainly on closed yes/no or one-step questions, opportunities for deep reasoning are lost (Taş, 2021).

Lim et al. (2020) also emphasise how questions are posed and paced. One of their key finding is the importance of wait-time and scaffolding in questioning. Teachers should anticipate where learners may struggle and plan a sequence of questions that starts with simpler, concrete prompts and gradually moves to more complex, abstract ones. For example, a teacher might first ask: What do you notice about these shapes? and only later ask the question: How could you use that pattern to solve a new problem? This scaffolded progression gives learners a pathway into the idea. This allows learners enough thinking time. Henry et al. (2020) on effective classroom talk recommend waiting roughly three to five seconds after asking a question before speaking again. Longer pauses lead to more elaborate and accurate learner responses and encourage more learners to participate (Dale et al., 2022). By planning questions in advance with increasing difficulty and giving learners time to process each one, teachers create space for learners to connect concepts, articulate their thinking, and arrive at understanding through guided inquiry (Lee et al., 2023)

In this instance, the teacher’s knowledge and training play a crucial role in questioning quality. According to Filgona and Sakiyo (2020), eachers with strong content and pedagogical knowledge are more likely to craft and follow up on conceptual questions. Kater (2024) reports that when teachers lack familiarity with effective questioning strategies, they often default to low-level questions that do not stimulate learner thinking. Conversely, when teachers have been trained in questioning taxonomies and reflect on their practice, they become more intentional about mixing lower and higher-order questions (Bibi & Reba, 2020). For instance, Regmi and Rhee’s study participants recognised the categories of questions but used them inconsistently, suggesting that explicit instruction on question categories is beneficial. Raymond et al. (2025) similarly advocate for teacher self-assessment: by reviewing classroom transcripts or recordings, teachers can pinpoint which questions elicited meaningful dialogue and which did not. In sum, professional development that helps teachers plan purposeful questions, use appropriate wait time, and reflect on their questioning patterns is shown to enhance learners' conceptual understanding in mathematics (Evans et al., 2020).

2.2.3 Problem-solving and logical reasoning

According to Wewe (2017), logical reasoning is intertwined with problem-solving. This means that for learners to be able to solve mathematical problems, they need to be able to think logically. Problem-solving involves a structured process in which a problem is identified, a solution plan is devised, and the plan is executed, followed by reflection on the outcome (Khalid et al., 2020). The reflective component is particularly significant, as it helps learners evaluate their methods, identify potential errors, and refine their approach for future problem-solving scenarios. This iterative process enables learners to tackle mathematical challenges systematically and encourages them to engage more deeply with the subject matter.

Takahashi (2021) suggests that learners who approach complex mathematical problems with a robust ability to think critically often do not experience them as obstacles but as opportunities to apply knowledge creatively and innovatively. The challenge presented by a mathematical problem encourages learners to step beyond procedural tasks and engage in meaningful exploration of concepts, fostering resilience and adaptability (Chinofunga et al., 2024). This perspective transforms problem-solving into an intellectual pursuit where grappling with uncertainty becomes as valuable as arriving at a solution. Such experiences equip learners with the skills to navigate mathematical problems in real-world contexts, enhancing their confidence and competence (Hakamata et al., 2023).

At the heart of critical thinking in mathematics lies logical reasoning, which serves as a foundational skill for constructing and evaluating mathematical arguments (Daher, 2020). Logical reasoning involves making deductions from given premises, identifying patterns, and formulating generalisations. Through this process, learners begin to appreciate the inherent structure and coherence of mathematics, enabling them to see connections across different areas of the discipline. Engaging in logical reasoning helps cultivate their ability to communicate ideas clearly and persuasively (Arisoy et al., 2021). For learners to develop problem-solving skills and logical reasoning, they ought to have developed critical thinking involving teachers' questioning practices situated in both lower-order and higher-order categories. Over-emphasis of questions in the lower-order category and neglect of the higher-order category can hinder learners’ ability to respond to higher-order questions during assessment (Suryana et al., 2021).

2.2.4 Metacognitive strategies

Teachers' questioning practices are a pedagogical tool that directly influences learners' metacognition – their ability to reflect on, monitor, and regulate their thinking processes (Loh & Lee, 2019). Metacognition is a critical aspect of developing learners' critical thinking. By strategically designing questions, teachers can guide learners to become more aware of their own thinking processes, evaluate their reasoning, and develop independent critical thinking skills (Hwang et al., 2023). Hence, teachers' questions serve as catalysts for prompting learners to plan, monitor, and evaluate their thinking. Types of questions that foster metacognition include reflective questions, probing questions, and predictive questions. Loh and Lee (2019) concur with Hwang et al. (2023) that metacognition is an essential aspect of critical thinking and has an influence on learners' academic performance.

Chen et al. (2017) assert that learners' academic performance in mathematics classrooms is more likely to improve when teachers use effective questioning to enable them to self-regulate. They argue that learners can develop metacognitive strategies by using prompts and reflection questions. This aligns with Chikiwa and Schäfer’s (2018) notion that metacognition serves as a pathway for critical thinking and increases the likelihood that learners will perform well during assessment activities. Hence, effective questioning fosters a classroom culture that values inquiry and curiosity. Metacognition strategies are developed by consistently integrating questions that promote dialogue and critical discourse (Nobutoshi, 2023). Learners then become aware of their cognitive strengths and areas that need improvement (Namkung & Fuchs, 2019). When learners internalise the questioning strategies modelled by their teachers, they begin to ask themselves reflective and analytical questions, thus reinforcing their critical thinking and metacognitive strategies (Teng & Yue 2023).

While Shilo and Kramarski (2019) suggest strategies for developing learners’ metacognition, however, they do not explain how to design questions or recommend a framework for teachers to use to support learners in developing metacognition skills. Hence, there remains a critical gap in understanding the ways in which teachers design or use questions to enhance learners' critical thinking. Furthermore, there is still an ongoing debate regarding the specific questions that promote metacognition.

2.3 Historical perspectives on questioning practices in mathematics education

The formal education system that is in place in South Africa is still almost exclusively patterned on the education model and knowledge base imported from Europe. Thus, I will trace the use of questioning through the Western tradition in relation to our current African school system. The use of questioning in mathematics education has evolved significantly over time, influenced by philosophical, psychological, and pedagogical perspectives (Delise & Karla, 2018).

In ancient Greece, Socrates developed the Socratic method, which involved systematic questioning to stimulate critical thinking and expose contradictions in reasoning (Beversluis, 1974). Socratic questioning emphasises deep inquiry through dialogue, pushing learners to justify their answers rather than merely recall facts. This approach aligns with modern views on formative assessment and higher-order questioning (Scullion, 2024). Socratic questioning is largely informal and relies heavily on oral discourse, limiting its systematic application in diverse educational settings (Carey & Mullan, 2004).

During the medieval period, mathematical education was largely influenced by religious institutions, where rote memorisation and recitation were dominant instructional strategies. Questioning was primarily used to assess factual recall rather than to promote conceptual understanding (Walsh & Hodge, 2018). Scholasticism, which emerged in the 12th century, sought to integrate logic with theology, fostering dialectical reasoning through questioning (Mead, 2010). However, mathematical instruction remained procedural, with limited emphasis on exploratory questioning. This contrasts with Socratic approaches, which encourage learners to actively engage with problems rather than passively memorise rules.

The Renaissance and Enlightenment periods brought a shift in questioning strategies in mathematics education. Influenced by thinkers such as René Descartes and Isaac Newton, mathematics became more inquiry-based, with an emphasis on reasoning and problem-solving (Schoenfeld & Kilpatrick, 2013). Questioning strategies began incorporating deductive and inductive reasoning, enabling learners to derive general principles from specific cases (Hayes & Heit, 2018). However, formal education systems still relied on teacher-centred instruction, where questioning primarily served as a means of verifying knowledge rather than fostering deep inquiry. Compared to the medieval period, questioning during the Enlightenment allowed for more learner engagement but remained constrained by rigid curricula.

In the 19th century, the Industrial Revolution and the rise of mass schooling led to the standardisation of education. Teachers increasingly used questioning to assess learner retention, reflecting the influence of behaviourist theories emerging at the time (Roediger & Butler, 2011). This period saw the dominance of direct instruction, where teachers posed closed-ended questions to evaluate factual knowledge (Çakır et al., 2016). Although questioning strategies remained largely lower-order, thinkers like John Dewey advocated for reflective inquiry, arguing that questioning should guide learners to critically examine problems and generate solutions (Earl et al., 2016). Dewey's ideas foreshadowed later constructivist approaches, which emphasised questioning to develop independent thinking (Neimeyer et al., 2016).

The mid-20th century marked a significant transformation in questioning strategies with the emergence of cognitive psychology and constructivist theories. In 1956, Benjamin Bloom introduced his taxonomy of educational objectives, categorising cognitive skills into hierarchical levels ranging from recall to evaluation (Bloom et al., 1956). This framework provided a structured approach to questioning, encouraging teachers to design questions that progressed from basic recall to critical analysis and synthesis. Bloom's taxonomy influenced mathematics education by promoting a more deliberate use of questioning to develop problem-solving skills. However, critics argue that its rigid structure does not fully account for the fluid nature of mathematical thinking (Anderson et al., 2001).

In the late 20th century, Webb's Depth of Knowledge (DOK) model expanded on Bloom's taxonomy by focusing on the complexity of cognitive processes involved in answering questions (Barber, 2018). Webb's model classified mathematical questions into four levels, emphasising strategic thinking and extended reasoning. Compared to earlier taxonomies, the DOK provided a more nuanced understanding of the complexity of questioning, aligning with research on how learners engage with mathematical concepts. Scholars such as Vygotsky further influenced questioning strategies by introducing the concept of the Zone of Proximal Development (ZPD), suggesting that well-structured questions scaffold learners' learning by bridging the gap between what they know and what they can achieve with guidance (Goos & Bennison, 2019).

The 21st century has seen increasing emphasis on dialogic questioning and inquiry-based learning, building on previous theoretical foundations. Researchers like Boaler and Alexander argue that questioning should facilitate mathematical discourse, enabling learners to collaboratively reason, critique, and construct knowledge (Dahal et al., 2019). The shift towards learner-centred approaches contrasts with historical models that prioritised teacher-led questioning for assessment purposes. However, despite these advancements, many educational systems still prioritise standardised testing, limiting the extent to which questioning strategies can foster deep critical thinking (Kazemi & Hintz, 2023).

The historical evolution of questioning strategies in mathematics education reflects a gradual shift from rote recall to conceptual inquiry. While early methods, such as Socratic questioning, promoted deep thinking, subsequent periods often prioritised assessment over exploration (Carey & Mullan, 2004). Modern research continues to refine questioning frameworks, integrating cognitive and social perspectives to enhance critical thinking in mathematics classrooms. This historical trajectory highlights the enduring debate over procedural knowledge and conceptual understanding, demonstrating that effective questioning remains central to mathematics education.

2.4 The role of questions in mathematics classrooms

The successful teaching of mathematics relies heavily on the dynamic interaction between teachers and learners, with questioning serving as a central pedagogical tool (Tarasenkova et al., 2023). Questions stimulate critical thinking, encourage problem-solving, and promote more profound engagement with mathematical concepts (Lombardi, 2023). By framing thoughtful and purposeful questions, teachers can foster a classroom environment where learners actively participate, explore ideas, and construct their understanding of mathematical principles. This dialogue between teachers and learners highlights the transformative power of effective questioning in the teaching and learning process (Howe et al., 2019).

Moreover, questioning acts as a diagnostic tool, allowing teachers to assess learners' comprehension and identify areas requiring further clarification (Sharma & Singh, 2019). When employed strategically, questions not only test learners' retention but also encourage them to articulate their thought processes, fostering a more collaborative and interactive classroom culture (Hunter, 2021). Teachers skilled in questioning can adapt their approaches based on learners' responses, ensuring that the lesson remains accessible while challenging learners to extend their thinking. This adaptability is particularly critical in mathematics, where conceptual understanding often builds upon previous knowledge (Smith et al., 2018).

According to Pang (2022), to maximise the potential of questioning, teachers must continuously refine their questioning strategies through professional development and reflective practice. This can involve crafting open-ended and thought-provoking questions, promoting a learner-centred approach that values exploration over rote memorisation (Chikiwa & Schäfer, 2018). As teachers strive to create meaningful learning experiences, the cultivation of questioning skills becomes a cornerstone of effective teaching. In doing so, teachers not only enhance their pedagogical practices but also empower learners to become active participants in their educational journey, ultimately fostering a deeper appreciation for mathematics (Colonnese et al., 2022).

Accordin to DeJarnette and Hord (2020), the complexity of mathematical content demands that teachers employ varied questioning techniques to elicit learners' reasoning and uncover misconceptions. Carefully sequenced questions can guide learners through the logic underpinning a mathematical procedure, such as exploring why a particular method works across different contexts. Zhuang and Conner (2024), found that mathematics teachers who strategically used scaffolding questions were able to assist learners in transitioning from informal reasoning to formal mathematical justification. This deliberate use of probing questions encouraged learners to express their understanding using mathematical language, thus deepening their conceptual grasp (Yao & Manouchehri 2020). Questioning of this nature supports learners in making connections between familiar and unfamiliar problems, thereby promoting flexibility in mathematical thinking (Tañola & Lomibao, 2024).

According to Pauli and Reusser, (2024) the precision and clarity of questions asked during mathematics lessons directly influence the quality of learners' responses. When teachers pose ambiguous or narrowly focused questions, learners often provide brief, procedural answers that reveal little about their conceptual understanding (You, 2022). In contrast, well-structured questions that anticipate learner difficulties can encourage explanation, argumentation, and justification. For instance, Soysal (2021) observed that when teachers incorporated ‘why’ and ‘how’ questions during instruction, learners were more likely to elaborate on their reasoning, leading to a richer classroom discourse. Such questioning techniques not only support learners in articulating their thinking but also expose gaps in understanding that may otherwise remain hidden in routine problem-solving tasks (Ezeamuzie et al., 2022).

Effective questioning depends not solely on the type of question but also on its timing and sequencing within the lesson (Mason, 2020). Hallman-Thrasher and Spangler (2020) argue that introducing high-cognitive demand questions at strategic moments can shift the focus from simply getting the correct answer to engaging with the underlying structure of the problem. This shift enables learners to view mathematics as a coherent system rather than a collection of isolated procedures (Anthony and Walshaw, 2023). For example, introducing a comparative question after learners have solved a task, such as: What is similar and different about these two solutions? can prompt them to reflect on alternative strategies and abstract general principles. In this way, purposeful sequencing of questions transforms routine practice into opportunities for deeper engagement with mathematical content (Choy & Dindyal, 2021).

Questioning that encourages learners to evaluate and defend their solutions further supports the development of mathematical argumentation. As highlighted by Soforon et al. (2024), when teachers invite learners to critique or improve one another's reasoning through targeted questions, learners begin to internalise standards of mathematical justification. This technique fosters a culture of intellectual accountability, where learners must substantiate their answers with coherent explanations rather than relying on intuition or memorisation (Dzaiy & Abdullah, 2024). Furthermore, such questioning creates a dialogic space where multiple perspectives are considered and validated, which is particularly valuable in developing learners' confidence and competence in mathematics (Yi & Jiao, 2023). Therefore, consistent use of evaluative questions nurtures habits of critical inquiry essential for sustained mathematical development (DeJarnette et al., 2020).

2.5 Types of questions used by teachers in mathematics classrooms

Teachers use a variety of questions to facilitate mathematical discourse, encourage problem-solving, and stimulate reasoning (Jiang, 2020). The nature and type of questions posed influence learners' engagement, cognitive development, and critical thinking for conceptual understanding (Sachdeva & Eggen, 2021). Broadly, questions in mathematics classrooms can be classified into lower-order and higher-order questions, with further subcategories reflecting their pedagogical purpose (DeJarnette & Hord, 2022).

2.5.1 Lower-order questions

Lower-order questions engage with Levels 1 and 2 of the RBT, which deal with recall and factual information. These questions require learners to retrieve and reproduce previously learned information. Lower-order questions primarily assess procedural knowledge, such as recall of mathematical facts, definitions, procedures, or algorithms (Boaler & Brodie, 2004). They include phrases such as 'What is the formula for calculating…?' Recall questions help reinforce foundational knowledge. Ingram et al. (2024) argue that recall questions, when used alone, may not necessarily promote conceptual understanding. However, lower-order questions are essential for checking prior knowledge and ensuring that learners have the necessary background before progressing to more complex problem-solving tasks (Tanudjaya & Doorman, 2020). For the purpose of this study, lower-order questions are classified into two types: factual questions (recall mathematical facts and allow the correct use of mathematical language) and procedural questions (rehearse known routine procedures and use of correct mathematical language).

Reporting on their study on cognitive engagement in mathematics education, Tracey et al., 2020 argue that while lower-order questions may not necessarily enhance learners' critical thinking, they do remain important for developing learners' procedural fluency. Lower-order questions play an essential role and should not be neglected during the teaching and learning of mathematics (Hoogland & Tout, 2018). According to Jansen and Möller (2022), they need to be used in conjunction with questions targeting the higher-order category of the RBT.

Furthermore, a study by Semeraro et al. (2020) examining cognitive participation in mathematics classrooms found increased learner responses when asked lower-order questions compared to other question types. Although the study does not identify the specific lower-order category questions asked by the teachers, it could imply that these types of questions did engage learners because they felt more confident to answer lower-order questions (Anderson et al., 2023). According to Booth et al. (2017) and Hong et al. (2020), this contradicts the notion that lower-order questions, alone, may necessarily engage learners cognitively. These scholars make the argument that for learners to remain cognitively engaged, they need to be asked critical thinking questions. Hence, I concur with the argument of Tracey et al. (2020) that lower-order questions, when used alone, may not necessarily enhance learners' critical thinking.

According to DeJarnette et al. (2020), lower-order questions, particularly factual ones, remain a crucial component of mathematics instruction, especially when introducing new concepts or revisiting foundational content. These questions help learners anchor their understanding in precise terminology and definitions, which form the basis for more complex reasoning. Evidence from Bosica and MacGregor (2021) indicates that teachers often begin lessons with factual questions to assess learners' baseline knowledge, enabling them to adjust instruction accordingly. This immediate feedback loop ensures that misunderstandings are addressed early, reducing the risk of cumulative errors during the lesson. While limited in cognitive demand, such questions are vital for establishing a common ground before progressing to more abstract content (Baesalou, 2020).

According to Gradini et al. (2025), procedural lower-order questions further support the development of fluency with algorithms and computational methods. They allow learners to practise routine procedures and reinforce their application of rules. Andal and Andrade (2022) highlight that procedural fluency, when cultivated through deliberate questioning, contributes to learners' ability to approach more complex problems with confidence. However, they caution that repetitive use of procedural questions without conceptual anchoring can lead to superficial understanding. Effective teachers thus integrate procedural questions within structured practice to promote accuracy and confidence, especially when introducing new types of problems or mathematical tools (Chew & Cerbin, 2021).

Acccording to Gradini et al. (2025), in classrooms where curriculum pacing is tightly monitored, many mathematics teachers rely on lower-order questions to ensure syllabus coverage. Results from Jabali at al. (2024) demonstrates that classroom observations often show a disproportionate use of closed, factual, or procedural questions, primarily driven by time constraints and assessment pressures. While these questions efficiently elicit quick responses, their overuse may limit opportunities for learners to explain or justify their reasoning. The study underlines the need for teachers to remain critically aware of their questioning patterns and to ensure that lower-order questions serve their intended purpose on scaffolding rather than dominating classroom dialogue.

Despite their limitations, lower-order questions can play a strategic role in maintaining learner participation (Pan et al., 2024). When used deliberately at the beginning of a lesson or within a questioning sequence, they can build momentum and learner confidence. Ashcraft et al. (2022) found that learners are more likely to respond to factual and procedural questions, especially in classrooms where mathematical anxiety is prevalent. This responsiveness, in turn, can lead to increased classroom interaction and encourage learners to engage more willingly with subsequent higher-order questions. Therefore, while lower-order questions may not directly enhance critical thinking, their role in sustaining engagement and pacing instruction remains indispensable when thoughtfully employed (Barseghyan & Hovakimyan, 2024).

2.5.2 Higher-order questions

As defined by Anderson et al. (2001), higher-order questions require learners to apply, analyse, evaluate, and create new knowledge, rather than merely recall facts. These questions are critical for developing learners' reasoning and problem-solving abilities, leading them to develop their critical thinking skills. According to Silva Mangiante and Gabriele-Black (2024) and Khalid et al. (2020), higher-order questions encourage learners to justify their reasoning, make connections between concepts, and engage in mathematical discussions, leading to a deeper understanding of mathematical principles.

Worth noting is that Ong et al. (2016) and Benidiktus et al. (2021), in their separate studies focusing on higher-order thinking in mathematics classrooms, found that most of their participants – who were teachers – put less emphasis on higher-order questions and more on lower-order questions. This concurs with the results of Tracey et al. (2020) on the usage of lower-order questions compared to other question types posed by teachers in mathematics classrooms. Similar studies by Alkhatib (2019) and Abdullah et al. (2016), focusing on enhancing learners' critical thinking through higher-order questions, recommend more research on the extent to which teachers design and ask specific types of higher-order questions in mathematics classrooms. They argue that a bigger body of empirical evidence would make it more possible to develop a model for higher-order questions for teachers to use in their mathematics classrooms. The current study on teachers' questioning practices for enhancing learners' critical thinking in Grade 10 mathematics classrooms addresses this need. For the purpose of this present study, higher-order questions are classified into three types: open-ended, probing and reflective, Socratic and diagnostic.

According to Acharya (2021), higher-order questions are pivotal in promoting learners' analytical thinking by encouraging them to go beyond memorised procedures and reflect on underlying mathematical structures. When learners are prompted to compare strategies, generalise rules, or justify their problem-solving approaches, they engage more deeply with the content (DiNapoli & Miller, 2022). In an empirical study by Spangenberg and Pithmajor (2020), mathematics teachers who regularly used higher-order questions observed an improvement in learners' ability to articulate their reasoning and identify relationships across mathematical topics. Questions such as 'Can you explain why this method works in all cases?' or 'What happens if we change this condition? were shown to foster meaningful classroom dialogue that extended beyond procedural fluency.

The manner in which higher-order questions are implemented significantly affects their impact on learners' mathematical reasoning (Setyowati, 2024). When such questions are embedded purposefully within problem-solving activities, they promote sustained engagement and metacognitive awareness. Indrawati et al. (2024) that teachers who asked reflective questions during problem-based tasks saw increased learner perseverance and more thoughtful responses. Examples of reflective questions included 'What assumptions are you making here? and 'How else could this be solved?' These questions required learners to pause, evaluate their strategies, and consider alternative approaches, thereby developing a more robust understanding of the mathematical content (Fan et al., 2019).

Despite the recognised benefits of higher-order questioning, its use in mathematics classrooms remains inconsistent and often underdeveloped. In a recent investigation, Hidayat et al. (2025) found that many teachers lacked confidence in crafting open-ended and diagnostic questions, often due to uncertainty about how learners would respond or the perceived time constraints during lessons. The research highlighted that questions demanding evaluation or creativity were seldom used unless supported by specific planning or professional development. This underuse indicates the need for clearer frameworks and exemplars to help teachers systematically incorporate higher-order questions into their daily practice without disrupting curricular pacing (Liu, 2023).

Sustained use of higher-order questions requires deliberate planning, where questions are not treated as incidental but are aligned with learning objectives. In an observational study by Kee and Zhang (2025), mathematics teachers who integrated Socratic questioning techniques throughout their lessons facilitated more learner-to-learner interaction and deeper mathematical discourse (Johnson et al., 2023). Questions like: Why do you agree or disagree with this solution? and Can someone provide a counter-example? invited critical examination of ideas and promoted reasoning over answer-getting. These types of higher-order questions support the development of a classroom culture where learners view mathematics as a subject of inquiry rather than a set of fixed procedures, thus positioning them as active constructors of knowledge (Acharya et al., 2022).

2.5.2.1 Open-ended questions

Open-ended questions provide learners with opportunities to explore multiple solution pathways and justify their answers (Beldar, 2025). These questions sometimes do not have a single correct answer and often require learners to explain their reasoning or apply their knowledge in novel contexts (Aziza, 2021). Phrases that are commonly used include 'Can you find different ways to solve this equation?' or 'How does changing one variable affect the outcome?'. Other phrases used in open-ended questions include 'why' ‘when’, 'how' and ‘what if’ (Nappie, 2017).

Research by Nieminen et al. (2022) highlights that open-ended questions enhance mathematical creativity and discourse, allowing learners to articulate their thought processes while building confidence in their problem-solving abilities. Teachers follow a discourse structure commonly known as the Initiation-Response-Evaluation (IRE) (Mehan, (1979). In the IRE interaction, the teacher initiates a question, the learner responds, and the teacher evaluates the answer. While this structure provides clarity and feedback, it also places the teacher in a position of authority, determining the correctness of responses. When paired with open-ended questions, the IRE discourse structure can serve as a platform for rich discussions, where teachers guide learners towards deeper insights while fostering collaborative learning (McCarthy et al., 2016).

A study by Aziza (2021) on the use of open-ended questions in mathematics classrooms by teachers found that fewer open-ended than lower-order questions were used. In instances where open-ended questions were used, the learners did not answer the teacher. Instead, the teacher ended up providing a correct answer. A research gap exists where the use of open-ended questions remains explorable for discoveries of specific open-ended questions that can be used to suit various topics in mathematics classrooms (Weller et al., 2018).

Open-ended questions are particularly effective when designed to encourage learners to extend their responses and provide mathematical justification (Monrat et al., 2022). In a classroom-based investigation by Urrutia and Araya (2022), teachers who integrated open-ended prompts such as: What patterns do you notice in these results? observed that learners produced more elaborate written and verbal explanations. This level of engagement was attributed to the non-restrictive nature of the questions, which allowed learners to connect prior knowledge with new problems (McGarr, 2024). The findings confirmed that open-ended questions promote exploratory thinking and provide teachers with richer insight into learners' understanding than closed questions typically allow.

When used consistently, open-ended questions can promote learner autonomy in mathematical problem-solving (Russo et al., 2020). According to Septiani and Arliani (2022) mathematics learners were more willing to take intellectual risks when asked open-ended questions such as: How would you explain this concept to someone who missed the lesson? or Can you represent this situation differently? These prompts encouraged learners to explore diverse representations and to communicate their reasoning more clearly. The study further observed that learners demonstrated increased willingness to revise their answers based on peer input, indicating that open-ended questions also contributed to the development of mathematical reasoning and flexibility.

According to Aziza (2021), challenges in implementing open-ended questioning persist, particularly around managing varied responses and ensuring all learners remain engaged. In a classroom observation study by Marshall (2022), teachers reported difficulty in facilitating discussions that stemmed from open-ended questions, especially when learners' answers deviated significantly from expected outcomes. While the potential for creative responses was acknowledged, the unpredictability of learner contributions occasionally led teachers to revert to closed questioning to regain control of the lesson flow (Yu and Chen, 2021). Nevertheless, the study concluded that with appropriate planning and scaffolding, teachers could maintain the openness of questions while guiding learners toward mathematically valid conclusions.

Open-ended questions also support formative assessment by revealing the depth and structure of learners' mathematical thinking. A study conducted by Houghton (2023) indicated that learners' responses to open-ended questions served as a diagnostic tool for identifying misconceptions and gaps in understanding. For example, when prompted with the question: What happens to the graph if we change this coefficient?, learners exposed their reasoning patterns, misconceptions, and interpretive strategies. Teachers used this information to adjust instruction in real time, thus enhancing the responsiveness of teaching. This adaptive use of open-ended questions underscores their value not only in promoting engagement but also in supporting instructional decision-making grounded in learner thinking (Cheng et al., 2021).

2.5.2.2 Probing and reflective questions

Probing questions are follow-up inquiries that require learners to extend their thinking beyond their initial response (Hähkiöniemi, 2017). As Hähkiöniemi (2017) suggests, these questions often begin with phrases like 'Can you explain why?' or 'What would happen if…?' and help deepen learners' understanding by encouraging reflection. In contrast, reflective questions prompt learners to consider their own problem-solving strategies and errors, fostering metacognitive awareness. A study by Kazemi and Hintz (2023) on the usage of probing questions in mathematics classrooms suggests that these types of questions are particularly effective in formative assessment as they help teachers identify misconceptions and guide learners toward self-correction and conceptual clarity. This means that probing and reflective questions complement each other.

The link between probing and reflective questions is significant because they both aid in enhancing learners' critical thinking (Farrell & Mom, 2015). However, I did not find specific types of probing and reflective questions used in mathematics classrooms identified in the literature. While Buchbinder et al. (2021) recommend placing more emphasis on probing and reflective questions during the teaching and learning of mathematics, they do not offer a model that teachers can use when designing these question types, nor do they indicate specific phrases relevant to any of the topics covered in high school mathematics. This brings back the earlier recommendation by Alkhatib (2019) and Abdullah et al. (2016) that more research still needs to be conducted on question types that promote learners' critical thinking.

According to Aeni and Ariyani (2025), probing questions are vital in uncovering learners' reasoning and challenging them to clarify or expand upon their initial responses. In an empirical study by Tawfik et al. (2020), teachers who regularly employed probing such as: Can you prove that? or Is this always true, and why? noticed that learners developed more precise mathematical language and began to justify their answers more rigorously. These prompts allowed teachers to identify the depth of learners' understanding and encouraged learners to revisit and refine their thinking, rather than merely offering surface-level responses. The study further highlighted that probing questions facilitated more detailed classroom discussions, which contributed to improved conceptual accuracy.

Using probing questions effectively enhances learners' engagement with abstract mathematical ideas (Middendorf & Shopkow, 2023). In a study by Aziza (2021), mathematics teachers who used probing prompts during algebra and geometry lessons reported increased learner participation and improved analytical responses. Examples such as: How does your solution relate to what we did yesterday? Or What patterns do you notice here? were especially effective in encouraging learners to link prior knowledge with new concepts. The study further confirmed that learners were more inclined to reason beyond memorised steps when teachers sustained the cognitive demand of tasks through probing questions, particularly during collaborative activities and whole-class discussions.

In addition to supporting deeper reasoning, probing questions are instrumental in addressing misconceptions. Camhur and Guven (2022) investigated the role of probing follow-up questions during formative assessment. They found that learners were more likely to self-identify and resolve their errors when teachers responded to incorrect answers with further inquiry instead of correction. For example, a teacher asking learners the question: What makes you think this method works in this case? prompted learners to re-examine their procedures, often uncovering faulty assumptions or misapplied rules. This approach positioned the learner as an active problem-solver and reinforced the use of questioning as a tool for constructive dialogue rather than evaluation alone (Kabar & Tasdan, 2020).

Despite their benefits, the strategic use of probing questions remains inconsistent in many mathematics classrooms. Findings by Russo et al. (2020) suggest that while teachers recognise the value of probing, they often revert to directive questioning due to time constraints or a focus on coverage. The study further revealed that teachers who planned specific probing prompts for key tasks were more likely to use them effectively in class. Questions such as: How would you convince someone that your answer is correct? or What if the problem conditions were changed? were shown to enhance both learner engagement and depth of explanation. These findings underscore the importance of intentional planning and question design to ensure probing questions are used purposefully to promote clarity, reasoning, and conceptual depth in mathematics teaching (Kaufmann et al., 2023).

2.5.2.3 Socratic and diagnostic questions

Socratic questioning involves a structured dialogue where the teacher challenges learners' assumptions and guides them towards logical reasoning through continuous inquiry (Foster, 2020). This questioning strategy effectively encourages critical thinking and mathematical argumentation, as van den Kieboom et al. (2014) suggests. Diagnostic questions are used to identify learners' misconceptions and gaps in understanding. Phrases such as 'Why do you think this method works?' can reveal learners' reasoning processes and provide insight into their conceptual grasp of the learned topic (Thanheiser et al., 2021). These questions are essential for formative assessment and targeted instructional interventions. Socratic and diagnostic questions collectively have the ability to enhance learners' critical thinking and remain relevant when used in mathematics classrooms (Wijaya, 2019). Zhang and Lamb (2024 also supports the notion that these question types overlapping with probing and open-ended questions in the way they are phrased. For the purpose of this study, Socratic and diagnostic questions are collectively termed ‘adaptive questions’.

According to Bimantara et al. (2025), Socratic and diagnostic questions are central in uncovering the reasoning behind learners’ mathematical thinking and fostering conceptual clarity. In a study, Pokhrel et al. (2024) found that teachers who posed Socratic prompts such as: What makes this solution valid? or Can we reach the same result another way? observed richer learner engagement in mathematical reasoning tasks. These questions required learners to critically evaluate their strategies, compare alternatives, and justify their thinking. The structured inquiry created by these questions helped shift classroom talk from procedural repetition to logical justification, allowing learners to refine their mathematical arguments through dialogue (Ding et al., (2024).

The diagnostic function of questions is especially effective when teachers anticipate common misconceptions (Timothy et al., 2023). In another study by Soeharto (2021), and teachers used diagnostic prompts such as ‘What might go wrong if we apply this formula here?’ to identify and address errors in learners’ understanding of algebraic expressions. These targeted questions exposed incorrect generalisations and prompted learners to revise flawed reasoning with teacher guidance. Rather than correcting the errors directly, teachers allowed learners to uncover inconsistencies through structured questioning. The findings emphasised the importance of timing and precision when posing diagnostic questions, particularly when aligned with lesson objectives and mathematical content progression (Schons et al., 2023).

The combination of Socratic and diagnostic questioning also proved beneficial in promoting sustained mathematical dialogue among learners (Basir et al., 2024). When teachers posed questions such as ‘Why is that assumption necessary? or ‘Can anyone challenge this reasoning?, learners engaged in collaborative critique and justification. These interactions deepened learners' engagement with core concepts and highlighted the mathematical principles underpinning their procedures. The study also revealed that such questioning not only encouraged peer explanation but also helped learners articulate their uncertainties, thus creating opportunities for teachers to provide focused clarification.

While the use of Socratic and diagnostic questions is associated with improved learner outcomes, their implementation requires careful planning. In their study Gallagher et al. (2022) indicated that teachers who prepared sequences of adaptive questions in advance, tailored to key concepts in geometry and functions, were more effective in guiding learners through complex problem-solving tasks. Phrases such as: Is this always true, or are there exceptions?" and Which part of this solution might confuse someone else? led to reflective analysis and supported a deeper understanding of mathematical structure. These findings demonstrate that when used strategically, adaptive questions can strengthen both individual reasoning and whole-class discussion, ultimately enriching the quality of mathematics instruction (Song et al., 2024).

2.6 Teachers' practices (behaviours) when asking learners questions

2.6.1 Wait-time and response

Studies consistently highlight the significant impact of increasing wait time (the pause after a teacher's question before a learner responds) on learner engagement. When teachers pause for around three seconds or more, learners tend to provide longer, more thoughtful answers and even begin to ask their own questions (Yang, 2017). This extended wait time offers learners the mental space they need to process information, fostering richer classroom discussions and deeper cognitive engagement (Kyle et al., 2018). In contrast, Rosenshine (2015) argues that classroom wait times often fall below two seconds, which may not be enough for learners to engage in deeper thinking. As a result, intentionally increasing wait time is associated with higher-quality responses and more meaningful participation, particularly in subjects like mathematics.

Other scholars have explored how different wait durations influence learners' reasoning processes. For instance, Gilliam et al. (2018) used psychophysiological measures to demonstrate that learners experience an initial burst of cognitive engagement within the first two to three seconds after a question is posed. However, if the wait time extends to around eight seconds, learners often re-engage their thinking, suggesting a second wave of cognitive activity. Based on these findings, the researchers recommend providing at least three seconds of wait time for all questions, regardless of difficulty, to ensure learners have adequate time to process and formulate their responses.

For more complex problems, longer pauses may be necessary, sometimes exceeding five seconds (Fan et al., 2019). These extended wait times allow learners to revisit and refine their reasoning, leading to deeper understanding. These findings align with earlier recommendations, such as those from Rowe and Rowe's (2021) seminal work, which advocates for a minimum wait time of three seconds to support learner thinking. Longer wait times are also linked to increased participation, as they encourage more learners to volunteer answers rather than relying solely on the quickest responders (Mahmud, 2019). In secondary school science classes, Iksan and Daniel (2015) found that longer wait times have been shown to result in more detailed and complex learner responses, increased speaking time, and even more questions directed back to the teacher. This suggests that wait time can be more influential than the complexity of the question itself in driving learner engagement.

In mathematics classrooms, extended wait times similarly enable learners to articulate their reasoning and collaborate with peers. Mahmud et al. (2019) observed that generous wait times encouraged less confident learners to attempt answers and fostered more peer-to-peer discussions during problem-solving activities. This not only gave learners time to think individually but also created opportunities for collaborative dialogue, making mathematical discussions more inclusive and dynamic. Such findings highlight the role of wait time as a catalyst for broader participation and a window into learners' conceptual understanding (Nappi, 2017).

However, the benefits of wait time are not evenly distributed across all learner groups, raising important equity considerations. Some studies have found that teachers may unconsciously allocate different wait times based on factors like gender or perceived ability (Günay et al., 2017). For example, earlier research noted that male learners or those from majority groups were often given slightly more time to respond than female or minority learners (Riley, 2015). This disparity can influence who gets to answer higher-order questions, potentially disadvantaging certain groups. On the other hand, adequate wait time has been shown to particularly benefit lower-achieving or shy learners. By pausing longer, teachers signal that every learner's thinking is valuable, fostering confidence and participation (Xie & Or, 2017). In mixed-ability math classes, teachers have reported that without sufficient wait time, only the most advanced learners tend to respond, leaving others behind (Lineback, 2015). Thus, the intentional use of wait time serves as a crucial equity strategy, ensuring that all learners, regardless of gender or ability, can engage in meaningful mathematical reasoning.

To implement effective wait time strategies, teachers often employ specific techniques. One key approach is learning to embrace silence (Putnam, 2016). Teachers might silently count to five after posing a question to resist the urge to fill the pause. Many also find it helpful to communicate the purpose of the silence to learners, saying something like ‘I will give everyone a minute to think’, to set the expectation that thoughtful pauses are part of the learning process (Tharayil, 2018). According to Rutledge (2023), structured routines like Think-Pair-Share can also be valuable, as they inherently build in wait time by allowing learners to think individually or discuss with a partner before sharing publicly. This not only extends wait time but also reduces pressure on individual learners, as they have time to refine their ideas.

Additionally, teachers are encouraged to avoid calling on the first hand that goes up. Instead, scanning the room and waiting a few extra seconds can encourage more learners to participate, indicating that more have had time to formulate their responses (Farbman, 2015). By pacing their questioning and sometimes writing the question on the board while learners think, skilled teachers can maximise engagement and thinking time (Delpit, 2018).

Despite its benefits, implementing optimal wait times in real classrooms can be challenging. One common concern is the tension between wait time and lesson pacing. Teachers often worry that long pauses might disrupt the flow of instruction or lead to lost instructional time (Gilbert & Byers 2017). The average teacher waits only one to three seconds before intervening, often due to discomfort with silence or ingrained habits (Wilson, 2017). Breaking this pattern requires conscious effort and professional development. There is also an upper limit to productive wait time: pausing too long without a learner response can lead to awkwardness or disengagement (Woods, 2020). For example, an observational study in a clinical teaching context recommended waiting at least five seconds but generally no more than ten seconds on an initial question to maintain momentum (Pitt et al., 2015). Hence, teachers must also gauge when a learner is genuinely thinking versus when they are stuck, which requires careful judgment in the moment.

Learner expectations can also pose a challenge. In classrooms where rapid questioning is the norm, learners might initially find prolonged silences unusual (Sedova & Navratilova, 2020). Teachers may need to explicitly normalise the practice, reassuring learners with statements like 'It is okay to take time to think'. Classroom management concerns can also arise, as extended pauses might lead to off-task behaviour if routines for wait time are not well established (Sieberer-Nagler, 2016). Overcoming these challenges requires a delicate balance, but teachers who persist often find that the benefits enhance learner thinking, participation, and inclusivity, and thus are well worth the effort (Alam & Mohanty, 2023).

2.6.2 Feedback

The type of feedback a teacher provides after a learner responds to a question can significantly shape learning outcomes in mathematics (Gan et al., 2021). Kyaruzi et al. (2019) categorise feedback into three types: evaluative feedback (judges an answer as right or wrong), descriptive or elaborative feedback (explains why an answer is correct or how to improve an incorrect response), and scaffolding feedback (offers hints or asks follow-up questions to guide learners toward a better answer). According to Hattie and Timperley's (2007) feedback framework, effective feedback can operate at multiple levels, addressing the task itself, the process used to solve it, or even the learner's self-regulation of their learning. Instead of saying 'incorrect' (an evaluative remark at the task level), a teacher might add, 'Let us check the steps: what should we do next?' (a process-level prompt). This kind of scaffolded feedback encourages learners to think critically about their approach, making it more constructive than simple praise or criticism (Cook, 2020). Thus, by focusing on guiding learners, rather than just evaluating them, teachers can correct errors while maintaining learners' motivation to persevere in problem-solving.

According to Smit et al. (2023), feedback fosters mathematical thinking and problem-solving skills. When teachers engage with learners' reasoning rather than simply verifying answers, they encourage deeper reflection and refinement of thought processes. Empirical studies support this approach. In one study, Toker (2021) found that learners who received ongoing formative feedback during math problem-solving improved their ability to explain their thinking and made fewer calculation errors. The feedback, which included asking learners to clarify their drawings and solution steps, helped learners make their thought processes visible and improved their problem representations (Barton, 2018). Feedback focused on conceptual understanding, such as probing learners' reasoning or asking them to justify their answers, can enhance problem-solving skills and higher-order thinking (Roar & Kirsti, 2021). In essence, feedback that guides learners to think about their thinking transforms each question-and-answer exchange into a learning opportunity, deepening mathematical reasoning.

The timing of feedback, whether it is immediate or delayed, also impacts learning outcomes. While many teachers and learners prefer immediate feedback, as it can address misunderstandings immediately, recent research highlights its benefits, particularly for learning new or complex concepts. Lu et al. (2021) conducted a controlled study on feedback timing. They found that learners who received immediate feedback after attempting problems showed greater conceptual understanding gains than those who received delayed feedback. Immediate feedback helps prevent learners from internalising misconceptions and provides quick affirmation or correction, keeping their problem-solving on track.

In contrast, delayed feedback, such as corrections given the next day, can sometimes be less effective, as learners may have moved on or forgotten their original reasoning by the time they receive input (Li, 2016). However, the effects of timing can vary by context. Some studies suggest that slight delays might encourage learners to reflect or self-correct before hearing the answer, which could benefit long-term retention, in certain cases. Overall, immediate or prompt feedback is favoured to support learning and reasoning processes (Zhu & Lee, 2020).

Effective feedback practices are closely tied to the principles of formative assessment, where the goal is to use information about learner understanding to adjust teaching and support progress. Feedback serves as the primary vehicle for this information (Foster, 2022). Feedback should be specific, actionable, and focused on the task or process, helping learners identify and address gaps in their learning (Cabang & Roble, 2022). Hence, it also fosters a growth mindset by reinforcing the idea that understanding improves with effort and guidance.

Moreover, teachers who use feedback to inform their instruction, such as noticing patterns in learner errors and addressing them with follow-up questions or hints, tend to see better learner outcomes. This aligns with Duron et al.'s (2006) Critical Thinking Framework, which positions feedback as a final step in the questioning cycle to solidify learning. By integrating thoughtful feedback after each question, teachers can turn questioning into a formative dialogue that continually assesses and advances learners' mathematical understanding.

Feedback also has a significant impact on learners' motivation and attitudes toward mathematics. How a teacher responds to answers often determines whether learners feel encouraged or discouraged to participate further (Zhu & Lee, 2020). A recent study by Smit et al. (2023) found that task-level feedback, such as pointing out specific errors or next steps, positively influenced learners' interest in mathematics, which in turn improved their problem-solving performance. In other words, effective feedback not only enhances motivation but also mediates better learning outcomes. Conversely, purely evaluative feedback can undermine motivation, leading learners to view mistakes as failures rather than learning opportunities. To keep learners motivated, teachers should aim to provide supportive and constructive feedback, acknowledging what learners did well, encouraging their efforts, and offering clear guidance for improvement.

According to Cabang and Roble (2022), providing effective feedback in real-time can be challenging for teachers despite its importance. One common obstacle is the fast pace of classroom interactions during a discussion. Teachers may default to brief evaluative responses to keep the lesson moving. Overcoming this requires a conscious effort to slow down and follow up on learner answers with probing questions or explanations, which can be difficult amid time constraints (Andrews, 2024). Additionally, delivering high-quality feedback on deeper mathematical thinking is inherently complex. A classroom video study by Stovner et al. (2021) found that most math teachers' feedback focused on procedural skills, with far fewer instances targeting conceptual understanding or mathematical practices. When teachers did attempt to address conceptual understanding, it often involved real-time assessment of learner thinking and complex decision-making, all while managing the classroom environment. Practical constraints like large class sizes, limited one-on-one time, and curriculum pressures can lead to superficial feedback or missed opportunities for deeper engagement (Mantai & Huber, 2021).

2.6.3 Sequencing questions

Teachers often structure questions in a sequence from lower-order to higher-order cognitive categories, scaffolding learners' learning step by step (Toledo & Dubas, 2016). A mathematics lesson could start with simple, factual, and procedural question types to check prior knowledge and then progress to open-ended, probing and adaptive question types (Gradini & Rahmayanti, 2024). Hence, more challenging problems can be introduced once foundational concepts are established. According to Santiago and Dubas (2016), this sequential progression ensures that learners gradually build understanding and foster confidence and problem-solving skills.

The RBT guides teachers in planning question progressions, aligning questions with cognitive levels from remembering to creating. By moving up the levels of the Revised Bloom's Taxonomy, teachers encourage critical thinking. According to Purdum-Cassidy et al. (2015), teachers tend to introduce lower-order questions – which help activate prior knowledge and build confidence – during the lesson introduction, then transition to higher-order order questions – which encourage deeper connections between concepts – during the lesson development, then move back again to lower-order questions in the lesson conclusion to review key concepts. In contrast, another study found that teachers often adjusted question sequences based on learner responses. In their study, Ni et al. (2014) found that teachers would mix lower-order and higher-order questions in each of the three lesson stages.

According to Sullivan (2015), implementing effective question sequences presents challenges. Unpredictable learner responses can derail planned sequences, requiring teachers to adapt in real time. Curriculum pressures often lead to a dominance of lower-order questions, leaving higher-order questions underused. Contextual factors also influence how teachers sequence questions (Widana et al., 2018). In exam-focused environments, teachers may rely on lower-order questions to maintain control. Systemic factors, such as curriculum expectations and teacher training, also play a role. Teacher beliefs about a problem-solving discipline also shape questioning strategies. Thus, cultural norms, class size, and language factors all influence how teachers sequence questions.

Effective sequencing of questions enables teachers to guide learners through the stages of mathematical reasoning systematically (Jensen & Skott, 2022). Mathematics teachers who sequenced questions from basic recall to complex reasoning tasks reported improved learner engagement and clearer transitions between lesson phases (Herbert, 2023). For instance, lessons began with factual prompts such as: What is the definition of a linear equation? before progressing to higher-order questions like: How would you construct a linear equation to model this scenario?. This structured questioning approach allowed learners to consolidate prior knowledge before confronting abstract problem-solving tasks, which improved both participation and accuracy.

Strategic sequencing also supports differentiated instruction, allowing teachers to respond to varied learner needs within a single lesson. Findings by Ginting (2021) demonstrated that teachers who adjusted their question sequence in response to learners' comprehension levels were more successful in maintaining cognitive engagement throughout the lesson. In their study, when learners struggled with procedural steps, teachers inserted mid-level questions such as: Can we try a different approach? before advancing to conceptual analysis. This adaptability ensured that all learners were given access to higher-order mathematical thinking through scaffolded questioning paths regardless of their initial confidence (Kim et al., 2021).

Purposefully sequenced questions enhance coherence across topics and promote long-term retention. Wong et al. (2023) observed that teachers who revisited earlier lower-order questions during later stages of the lesson reinforced conceptual links and improved retention of key ideas. For example, after learners engaged with open-ended questions about graphical representations, the teacher returned to initial definitions and formulae to consolidate understanding. This deliberate revisiting of foundational questions allowed learners to reflect on how their thinking had evolved during the lesson, reinforcing their grasp of mathematical relationships and procedures (Cian & Cook, 2020).

The success of sequencing questions also depends on lesson pacing and timing. A study by Surahman and Wang (2022) indicated that time allocation for different question types significantly influenced the depth of learner responses. When higher-order questions were introduced too late in the lesson, learners often responded superficially due to time constraints. Conversely, learners disengaged when teachers introduced complex questions earlier but failed to scaffold with sufficient lower-order prompts. Effective sequencing, therefore, involves careful balancing, allocating adequate time to each cognitive stage and ensuring that transitions between question types are both logical and responsive to learner progress (Wut & Jing, 2021).

2.7 The Cognitive Rigor Matric (CRM) for enhancing learners' critical thinking

The Cognitive Rigor Matric (CRM) is a model proposed by Amber et al. (2015). The version of the model shown in Table 2 is adapted from Hess et al. (2009, p. 8) and shows a twodimensional structure that incorporates the cognitive levels of the RBT. The Cognitive Rigor Matric integrates Webb's Depth of Knowledge (DOK) and the Revised Bloom's Taxonomy, thus providing a possible framework for analysing teachers' questioning strategies. This model highlights the varying cognitive demands of questions, ranging from simple recall to complex problem-solving and reasoning.

Table 2

Webb's Depth of Knowledge (adapted from Hess et al., 2009, p. 8)

	Webbs's - Depth of Knowledge (DOK)			
Revised Bloom's Taxonomy	DOK 1: Recall and Reproduction	DOK 2: Basic Skills and Concepts	DOK 3: Strategic Thinking and Reasoning	DOK 4: Extended Thinking
Remember	Recall, recognise, and locate basic facts, ideas and principles			
Understand	Describe/explain the steps required for specified logarithm	Specify and explain relationships (explain why the procedure for a specified algorithm is reasonable)	Explain strategies and reasoning processes for solving tasks for which procedures have not been specified	Explain how concepts or ideas specifically relate to other content domains or concepts
Apply	Apply an algorithm or formula	Solve routine problems applying multiple concepts or decision points	Use concepts to solve non-routine problems	Select or devise an approach among many alternatives to solve a novel problem
Analyse	Retrieve information from a table or graph to answer a question	Compare and contrast figures or data	Generalise a pattern	Gather, analyse, and organise information
Evaluate			Verify reasonableness of results	Draw and justify conclusions
Create	Brainstorm ideas, concepts or perspectives to a topic or concepts	Generate conjectures or hypotheses based on observations or prior knowledge	Formulate an original problem	Design a model to inform and solve real-world, complex, or abstract situations

In the context of enhancing learners' critical thinking, the Cognitive Rigor Matric places emphasis on the necessity of moving beyond DOK:1 (recall and reproduction) and DOK:2 (basic skills) to higher-order questioning levels, such as DOK:3 (strategic thinking) and

DOK:4 (extended thinking). Research by Chin (2007) supports this notion, arguing that effective questioning strategies should involve scaffolding learners from lower-order to higher-order thinking by encouraging explanations, justifications, and generalisations.

A comparison between the CRM and studies on questioning techniques reveals that many teachers tend to focus on factual recall and procedural questions, which align with the lower DOK levels (DOK:1 and DOK:2). Weay et al. (2016) suggest that questions that promote deep learning and critical thinking should be situated in DOK:3 and DOK:4. Such questions require learners to analyse, evaluate, and create processes that encourage deeper engagement with mathematical concepts. For instance, instead of merely asking learners to apply a formula (DOK:1), teachers should pose questions that require them to justify why a particular formula is appropriate for a given problem, thereby fostering mathematical reasoning and critical analysis.

Moreover, the CRM's emphasis on strategic and extended thinking aligns with the constructivist perspective, which posits that learners construct knowledge through active inquiry and problem-solving. Adler et al. (2019) suggest that questioning strategies should involve open-ended prompts that require learners to make connections between concepts, formulate hypotheses, and engage in metacognitive reflection. This corresponds to DOK:4, where learners must integrate knowledge from different domains to solve complex mathematics problems.

2.8 The use of critical thinking questions by teachers in South Africa

Research conducted by South African scholars has examined the impact of various questioning strategies teachers employ in mathematics classrooms across the country. These studies, conducted in different provinces and educational contexts, provide valuable insights into how different questioning practices influence learner engagement and understanding.

In the early 2000s, Brodie (2007) conducted an observational study in the province of Gauteng focusing on teacher-learner interactions in mathematics classrooms. The research highlighted that traditional question-and-answer methods often fail to foster the interactive engagement required by contemporary curricula. Brodie observed that when teachers posed open-ended questions and encouraged learner-initiated inquiries, there was a notable enhancement in classroom dialogue and mathematical reasoning. This approach facilitated a deeper understanding of mathematical concepts among learners.

In Western Cape province, Steyn and Adendorff (2020) explored the questioning techniques of seven fourth-year Foundation Phase Education learners teaching Grade 1 mathematics. Through the analysis of six lessons, the study found that preservice teachers predominantly used closed questions aimed at eliciting specific answers, which limited opportunities for learners to develop higher-order thinking skills. The researchers emphasised the need for training programs that equip preservice teachers with skills to formulate questions that promote young learners' critical thinking and problem-solving abilities.

Mokotedi (2023) conducted a qualitative case study in North West province involving five mathematics teachers from four different schools and a single Grade 9 class. The study examined the attributes of questions developed and utilised by teachers within the Lesson Study framework. Findings indicated that while teachers considered the questions intended to facilitate learning, the predominant use of questions stimulated interactions primarily within the topical zone, suggesting a need for professional development focused on diversifying questioning strategies to enhance deeper learner engagement.

In the province of KwaZulu-Natal, Ramnarain (2011) investigated how five teachers employed questioning strategies to support Grade 9 learners during science investigations. Although centred on science education, the study's insights are pertinent to mathematics instruction. The research revealed that teachers' probing questions at various stages of the investigative process enabled learners to articulate, reflect upon, and reassess their thinking and actions. This approach promoted learner autonomy and a deeper comprehension of scientific concepts, underscoring the potential of strategic questioning in facilitating independent learning (Ngwenya & Arek-Bawa, 2020).

In the Eastern Cape, Chikiwa and Schäfer (2018) examined the role of teacher questioning in promoting critical thinking within multilingual Grade 11 mathematics classrooms. The study found that teachers predominantly posed lower-order questions, which did not sufficiently challenge learners to engage in critical analysis or develop problem-solving skills. The researchers advocated for the intentional use of higher-order questions – including in learners' home languages – to stimulate deeper cognitive engagement and enhance understanding of mathematical concepts.

Toit and Kotze (2009) conducted research in Free State province, focusing on the use of metacognitive strategies by Grade 11 mathematics learners and their teachers. The study utilised questionnaires to gather data on the prevalence of metacognitive strategies, including planning and self-evaluation. Findings indicated that both teachers and learners frequently employed planning strategies and reflective thinking. However, techniques such as journal-keeping and thinking aloud were underutilised, suggesting areas for further development in fostering metacognitive practices through effective questioning.

Khoza and Msimanga (2022) explored the nature of questioning and teacher talk moves in interactive physical sciences classrooms in South Africa. The qualitative case study involved three teachers and aimed to understand how different questions and follow-up strategies affect classroom interaction. The study found that while open-ended questions often led to extended discussion, the effectiveness of such questions depended significantly on the teacher's subsequent pedagogical choices, highlighting the complexity of fostering meaningful dialogue through questioning.

These studies underscore the critical role of strategic questioning in enhancing learner engagement and understanding in South African mathematics classrooms. The research highlights a prevalent reliance on lower-order, closed questions, which may impede the development of critical thinking and problem-solving skills. There is a clear indication that both preservice and in-service teachers would benefit from targeted professional development aimed at diversifying their questioning techniques to foster deeper cognitive engagement among learners.

2.9 The use of critical thinking questions by teachers in other countries

Studies conducted in other countries have also explored the use of critical thinking questions by teachers in mathematics classrooms, revealing significant insights into how questioning enhances critical thinking. The studies I found and reviewed were conducted between 1999 and 2024 and were conducted across diverse educational settings. These studies highlight the nuanced effects of different questioning techniques used by teachers to enhance their critical thinking.

In the United States, Franke et al. (2009) conducted a comprehensive study involving 22 elementary school teachers and their learners. The research focused on how teachers' questioning practices elicited learners' mathematical thinking. Their findings indicated that sequences of probing, specific questions were most effective in encouraging learners to elaborate on their initial responses, leading to more complete and accurate explanations. This approach not only clarified learners' thought processes but also enhanced their conceptual grasp of mathematical principles. McCarthy et al. (2016) conducted a case study involving two Grade 8 mathematics teachers. The study aimed to identify the questioning strategies employed during classroom discourse. Analysis of videotaped lessons revealed that teachers predominantly used probing and follow-up questions, leading questions, check listing, and learner-specific questioning. These strategies were instrumental in assessing learner understanding and guiding instructional decisions, thereby fostering a more interactive and responsive learning environment. Lee (2024) examined the questioning techniques of six elementary preservice teachers working with first and fourth-grade learners. The research focused on how different functional moves in teacher questioning affected learners' contextualisation during mathematical word problem-solving. Findings suggested that task clarification questions effectively enhanced contextual understanding, notably when learners demonstrated a strong sense of agency. However, the success of these questions was contingent upon the learners' ability to engage deeply with the problem (Rubel & McCloskey, 2021). According to Webb et al., (2018) it is important to explore pedagogical tensions in teachers' questioning practices within mathematics classrooms. In another study involving two mathematics teachers, focused on the balance between guiding learners toward correct answers and encouraging independent thinking (Hockings, 2018). Results indicated that while teachers aimed to promote learner autonomy, their questioning often funnelled learners toward predetermined solutions, highlighting the complexity of implementing open-ended questioning strategies in practice.

In Singapore, Liu (2021), in his study, asserts that pedagogical practices of teachers, include checking understanding and providing feedback to activating prior knowledge, arousing interest, and using questions to deepen learning. The Singaporean curriculum emphasises the effective use of questioning by teachers as a mandatory practice to enhance learners' critical thinking. Chen et al. (2017) reaffirm that the academic performance of learners in mathematics classrooms is more likely to improve when teachers use effective questioning practices.

In Australia, Way (2008) investigated the use of questioning to stimulate mathematical thinking among primary school learners. The study found that while lower-order, knowledge-based questions dominated classroom discourse, incorporating higher-order questions was essential for developing learners' critical thinking and problem-solving skills.

The research advocated for a deliberate shift toward questions that challenge learners to analyse, evaluate, and create, thereby fostering deeper mathematical understanding.

In a systematic literature review, Davoudi et al. (2015) analysed teachers' questioning strategies in mathematics classrooms across various countries. The review highlighted that effective questioning techniques, such as open-ended and probing questions, were crucial in promoting learner engagement and higher-order thinking. However, the study also noted that many teachers faced challenges in consistently implementing these strategies, often reverting to closed questions that limited learner discourse.

2.10 Factors influencing teachers' questioning practices in mathematics classrooms

According to Ukobizaba et al. (2021), various factors influence mathematics teachers' questioning practices and, subsequently, influence classroom dynamics. Teachers' questioning practices in mathematics classrooms are influenced by multiple, interrelated factors, including their pedagogical beliefs, content knowledge, professional training, classroom dynamics, curriculum expectations, and assessment requirements. Teachers' beliefs about teaching and learning mathematics significantly shape the type of questions they ask (Leikin, 2021). Teachers who view mathematics as a set of fixed procedures and rules often pose recall-based and procedural questions aligned with lower-order levels. In contrast, those who perceive mathematics as a conceptual and problem-solving discipline are more likely to use open-ended and higher-order questions that stimulate critical thinking. This distinction is supported by Gervasoni et al. (2021), who found that teachers with constructivist beliefs tended to facilitate discussions that required learners to explain, justify, and critique mathematical ideas, rather than simply providing correct answers.

Teachers' beliefs and attitudes significantly influence their classroom questioning practices, particularly if they have a preference for lower-order questions over higher-order questions. Ramnarain and Hlatswayo (2018) suggest many educators prioritise lower-order cognitive questions due to the perception that such questions are more manageable and lead to immediate responses. This reliance is often linked to the belief that learners may struggle with higher-order questions, thus prompting teachers to avoid higher-order questioning strategies (Tokan & Imakulata, 2019). Additionally, external factors such as time constraints and curriculum demand further reinforce teachers' inclination toward lower-order questioning, as these questions appear more efficient for covering content during limited instructional periods (Nortvedt & Buchholtz, 2018).

Moreover, teachers' attitudes toward their own pedagogical competence play a crucial role in their questioning patterns. Studies have found that educators who lack confidence in their ability to facilitate critical discussions often resort to lower-order questions, fearing that learners' inability to respond adequately might disrupt the lesson flow (Suh et al., 2021). Conversely, teachers who embrace constructivist beliefs about learning and view learners as active participants in knowledge construction tend to integrate higher-order questioning more effectively (Delgado-Rebolledo & Zakaryan, 2020). This discrepancy suggests that professional development focusing on fostering self-efficacy in using higher-order questioning techniques could be instrumental in shifting questioning practices (Colonnese et al., 2022).

Another critical factor is teachers' mathematical content knowledge and pedagogical content knowledge (PCK). According to Gervasoni et al. (2021), effective questioning requires deep subject knowledge and understanding of how learners conceptualise mathematical ideas. Teachers with strong PCK are more likely to craft questions that elicit reasoning, problem-solving, and connections between mathematical concepts. Conversely, those with limited content knowledge may rely on closed questions that emphasise rote memorisation. A study by Kastberg et al. (2019) corroborates this, demonstrating that teachers with a firm grasp of mathematical concepts posed more thought-provoking questions that challenge learners to justify their reasoning and consider multiple solution strategies.

Professional development and training also play a vital role in shaping teachers' questioning strategies. Research by Walsh and Hodge (2018) highlights that many teachers receive limited training in formulating and using questions effectively to promote higher-order thinking. Without targeted professional development, teachers may default to traditional questioning patterns that prioritise procedural fluency over conceptual understanding. Furthermore, Davoudi et al. (2015) argue that professional learning communities and continuous training can support teachers in refining their questioning techniques by exposing them to research-based strategies that enhance learners' critical thinking.

Classroom dynamics, including learner engagement levels and the perceived cognitive abilities of learners, also impact teachers' questioning practices (Traver, 2019). Teachers modify their questioning strategies based on their expectations of learners' abilities (Rubie-Davies et al., 2020). When teachers believe that learners are capable of high-level reasoning, they are more likely to pose open-ended questions that encourage exploration and discussion

(Hattie & Zierer, 2018). However, in classrooms where learners struggle with foundational concepts, teachers may use more closed-ended or guided questions to maintain engagement and prevent frustration. This aligns with the findings of Howe et al. (2019), who argue that effective questioning should be adaptive, considering learners' cognitive and emotional readiness.

According to Alvunger, (2018) curriculum expectations and standardised assessments further influence teachers' questioning practices. Many national and regional curricula emphasise procedural fluency and content coverage, often leading teachers to prioritise lower-order questions that align with assessment requirements (Kazemi & Hintz, 2023). Furthermore, Kazemi and Hintz (2023) suggest that focusing on exam preparation can limit opportunities for critical thinking, as teachers may feel pressured to cover extensive content rather than engage learners in deep, inquiry-based discussions. Howe et al. (2019) criticise this approach, arguing that excessive emphasis on test performance discourages rich mathematical dialogue and problem-solving, which are essential for developing critical thinking.

Lastly, the socio-cultural context of the classroom shapes questioning practices (Mulyatna, 2021). Teachers operating in environments that value learner participation and inquiry-based learning are more likely to employ dialogic questioning that fosters critical thinking (Byrd & Alexander, 2020). In contrast, in traditional, teacher-centred classrooms, questioning often serves as a means of assessing knowledge rather than stimulating deeper understanding. Research by Barnes et al. (2024) emphasises that questioning should be viewed as a tool for knowledge co-construction rather than a unidirectional mechanism for evaluating learner responses. This perspective aligns with Vygotskian theories of learning, which stress the importance of guided questioning in scaffolding learners' cognitive development.

2.11 Common themes and findings across the reviewed research studies

This review of the literature revealed several key themes related to conceptualising teachers’ questioning practices and their relationship to critical thinking in mathematics classrooms. These themes include the need for enhancing critical thinking, classification of questions, the influence of teachers' pedagogical beliefs, learner engagement and participation, the impact of classroom discourse, the challenges in implementing effective questioning strategies, cultural and contextual influences on questioning, and the role of professional development in shaping teacher questioning practices.

A predominant theme in the literature is the need for critical thinking amongst learners in mathematics classrooms, which can be achieved through a questioning approach that balances lower-order and higher-order questions. Studies indicate that when teachers use probing, open-ended, and reflective questions, learners develop critical thinking abilities (Boaler & Brodie, 2004). These questions encourage learners to articulate reasoning, justify answers, and explore multiple solution pathways. However, the literature also highlights that merely posing higher-order questions does not guarantee critical engagement; the manner in which teachers scaffold responses and manage discussions significantly impacts learning outcomes.

Another crucial theme in the literature is the classification of teacher questions, distinguishing between lower-order and higher-order questions. Lower-order questions focus on recall and procedural knowledge, while higher-order questions stimulate analysis, evaluation, and synthesis (Tanudjaya & Doorman, 2020). The review suggests that while many mathematics teachers rely primarily on lower-order questions, those who consciously integrate higher-order questioning facilitate deeper conceptual understanding. This reflects the ongoing debate between traditional and constructivist approaches to mathematics teaching, where the latter prioritises learner reasoning over rote learning.

The review also discusses the influence of teachers' pedagogical beliefs on their questioning strategies. Teachers who embrace constructivist perspectives are more likely to use questions that promote exploration and inquiry (Adler & Sfard, 2017). Conversely, those with a traditionalist orientation may emphasise factual recall and procedural fluency. This contrast underscores the broader implications of teachers' beliefs in shaping classroom dynamics and learner thinking. Moreover, the literature suggests that institutional expectations and curriculum pressures may constrain teachers' ability to employ questioning strategies that support deep mathematical inquiry.

Learner engagement and participation emerge as another significant theme, as the effectiveness of questioning is contingent upon how learners respond. The review highlights that some learners may hesitate to engage in discussions due to a fear of making mistakes or a lack of confidence in their mathematical abilities (Howe et al., 2019). Teachers who create an inclusive, non-threatening environment tend to elicit more meaningful learner responses. Additionally, the frequency and quality of follow-up questions are found to be crucial in sustaining engagement and encouraging deeper reflection.

The impact of classroom discourse on learning is another recurring theme. Research suggests that productive mathematical discussions, facilitated through questioning, enhance conceptual understanding and reasoning skills (Webb et al., 2019). Dialogic teaching, where learners interact and build on each other's ideas, contrasts with monologic approaches in which teachers dominate discourse. However, achieving a balanced and interactive classroom discourse remains challenging, particularly when learners are accustomed to passive learning (Howe et al., 2019).

The literature further identifies challenges in implementing effective questioning strategies. Time constraints, large class sizes, and rigid curricula are often cited as barriers (Kazemi & Hintz, 2023). Teachers may struggle to balance content coverage with the depth of learner inquiry, leading to an overreliance on closed questions that elicit brief responses. Additionally, the expertise of teachers in formulating and managing higher-order questions varies, highlighting the need for continuous professional development.

Cultural and contextual influences on questioning practices also receive attention in the literature. In some educational contexts, hierarchical classroom structures discourage learner questioning and critical discussion (Tanner, 2017). In contrast, inquiry-based approaches are more prevalent in classrooms where collaborative learning occurs. These variations suggest that questioning practices cannot be universally applied without considering socio-cultural factors.

This review has explored the role of professional development in enhancing teacher questioning practices. Studies indicate that targeted training can improve teachers' ability to effectively pose and scaffold higher-order questions (Alrawili et al., 2020).

2.12 Gaps, limitations and contradictions identified in the current literature

The literature that has been reviewed on teachers' questioning practices in mathematics classrooms has provided valuable insights into the role of questioning in enhancing learners' critical thinking skills. However, significant gaps remain. One of the most evident gaps is the contextual limitation of the research findings. Most of the studies I reviewed focus on specific educational settings, such as particular schools, provinces, states or countries, with distinct curriculum structures and pedagogical expectations (Ukobizaba et al., 2021b). As a result, their findings cannot be universally applied to different educational contexts where socio-cultural, economic, and institutional factors shape classroom dynamics differently. According to Rivas (2022), questioning strategies used by teachers that enhance learners' critical thinking in one setting may not yield similar results in another due to variations in learner engagement, teacher training, and curriculum priorities.

Another key limitation is the lack of consensus on which specific question types are most effective in enhancing learners' critical thinking. While scholars agree that higher-order questions contribute to deeper cognitive engagement, studies diverge in their classifications and definitions of such questions (Anderson et al., 2001; Jiang, 2020). Some emphasise open-ended questioning such as probing and reflective questions as central to stimulating learners' reasoning (Aziza, 2021; Çakır et al., 2016; Hakamata et al., 2023). The absence of a unified framework or model for structuring effective questioning strategies results in inconsistencies across research studies. Consequently, teachers may struggle to implement questioning techniques effectively due to a lack of clear, research-backed guidance on best practices.

Moreover, research findings indicate that teachers often default to lower-order questions that focus on factual recall and procedural knowledge, particularly in examination settings (Ong et al., 2016; Tracey et al., 2020). The literature suggests that curriculum pressures and time constraints compel teachers to prioritise coverage over depth, limiting opportunities for learners to engage in meaningful discussions (Mercer & Howe, 2022). However, the extent to which these constraints impact questioning practices remains underexplored. Not many of the studies provide empirical evidence on how teachers balance content coverage with critical thinking development, creating a research gap in understanding how questioning can be optimised within rigid curricular frameworks.

Contradictions also emerge regarding the impact of questioning on learner engagement and participation. Some studies report that higher-order questions lead to increased classroom discourse and critical reasoning (Schoenfeld & Kilpatrick, 2013; Walsh & Hodge, 2018), while others find that such questions can intimidate learners, particularly those who lack confidence in mathematics (Hong et al., 2020; Booth et al., 2017). In contrast, lower-order questions, despite being criticised for promoting rote learning, are found to encourage broader learner participation (Semeraro et al., 2020). This contradiction suggests that while higher-order questions may foster deeper thinking, they may not always be the most effective in engaging all learners. The literature does not sufficiently address how teachers can scaffold questioning to accommodate diverse learner abilities and confidence levels.

A further limitation concerns professional development and teacher preparedness in implementing effective questioning strategies. Several studies highlight the need for targeted training to equip teachers with skills to design and facilitate higher-order questioning (Abosalem, 2016; Alhassora et al., 2017; Benidiktus et al., 2021; Davoudi et al., 2015). However, there is limited empirical research on the effectiveness of professional development initiatives in improving teachers' questioning practices. Studies that discuss teacher training often provide general recommendations rather than specific evidence on how professional development translates into improved classroom questioning. I argue that this gap leaves uncertainty about the practical application of research findings in real-world teaching environments.

Additionally, the literature does not adequately explore the role of metacognitive questioning in fostering learners' self-regulation and critical thinking skills. While some studies suggest that reflective and probing questions can enhance learners' ability to monitor and evaluate their thought processes (Farrell & Mom, 2015; Loh & Lee, 2019), there is no clear framework for how teachers should structure such questions to maximise cognitive engagement. I argue that the lack of empirical evidence on how metacognitive questioning influences learners' problem-solving strategies creates a gap in understanding how teachers can systematically integrate these questions into daily instruction.

Also, while theoretical frameworks such as the Revised Bloom's Taxonomy and Webb's Depth of Knowledge provide useful categorisations of question types, they do not fully account for the dynamic and interactive nature of classroom discourse (Amber et al., 2015; de Oliveira et al., 2023). Real-world classroom interactions involve spontaneous questioning and varied learner responses that these models do not always capture. As a result, teachers may find it challenging to apply these frameworks effectively.

I did not find a single study from the South African context focusing specifically on teachers' questioning practices to enhance learners' critical thinking in Grade 10 mathematics classrooms. Thus, the present study addresses the gap identified in the literature by exploring teachers' questioning practices to enhance learners' critical thinking in Grade 10 mathematics classrooms.

2.13 Conclusion

This chapter has presented and discussed teachers' questioning practices and critical thinking in mathematics classrooms. Several key areas, including the classification of questions into lower-order and higher-order questions, highlighting their impact on learner engagement and critical thinking. The discussion on critical thinking explained the need for problem-solving and metacognitive strategies that can be achieved through effective teachers' questioning practices. A discussion of the historical development of mathematics education described how questioning has been used in the past. The literature review thus provided a detailed exploration of the multifaceted nature of teacher questioning and its implications for mathematics education and critical thinking. In the next chapter, I present the theoretical framework that underpins this study.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1 Introduction

Multiple theoretical frameworks and models are often used together in a study as a multifaceted lens to ensure a comprehensive and nuanced understanding of the phenomenon under investigation (Varpio et al., 2020). According to Lederman and Lederman (2015), a theoretical framework provides a distinct perspective from which researchers analyse and make sense of facts. This enables a more holistic analysis. For the present study, the Revised Bloom's Taxonomy (RBT) and Duron et al.'s (2006) Critical Thinking Framework (CTF) were used in tandem. The RBT was used as a lens to identify the taxonomy levels at which teachers' questions were situated. The CTF was used to explore the questioning practices employed by teachers to enhance learners' critical thinking. These frameworks assisted the analysis of data and presentation of the research findings.

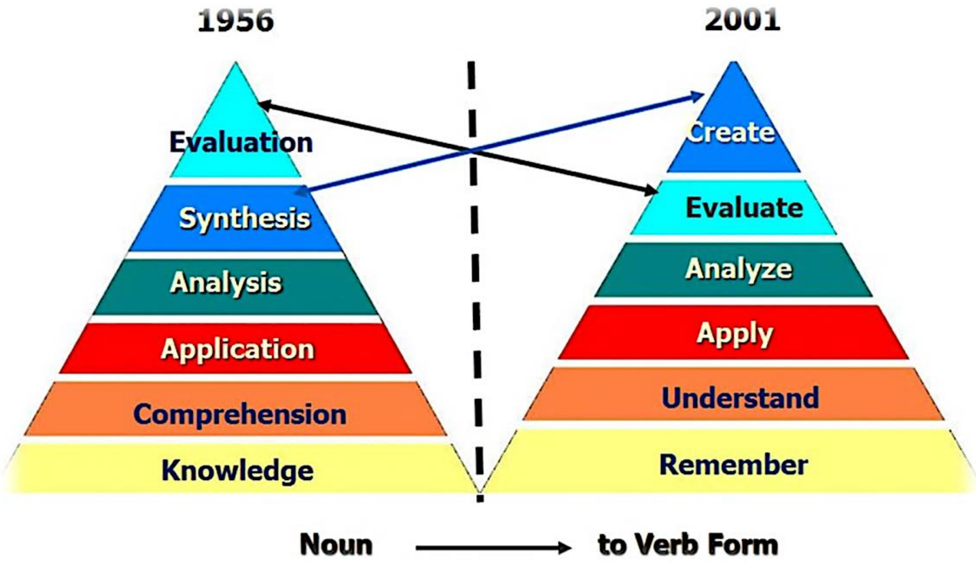
3.2 The Revised Bloom's Taxonomy (RBT) levels

The Revised Bloom's Taxonomy, developed by Anderson and Krathwohl (2001) from Bloom et al.'s (1956) original model, classifies cognitive processes into levels of complexity, ranging from lower-order thinking skills (remembering and understanding) to higher-order thinking skills (applying, analysing, evaluating, and creating). Anderson and Krathwohl's (2001) model expands the applicability of Bloom et al.'s (1956) original model to diverse educational contexts. While the original model used nouns to describe cognitive levels, Anderson and Krathwohl (2001) use verbs to describe cognitive skills. This shift emphasises active learning outcomes. The RBT is designed to help teachers design curricula and assessments that promote deeper learning and critical thinking (Kieran & Anderson, 2018). By aligning learning objectives with the cognitive levels of the RBT, teachers can create meaningful and engaging learning experiences that meet diverse learning needs.

Bloom et al.'s original model, published in 1956, identified six cognitive hierarchical levels ranked from knowledge, as the lowest level, through increasingly more complex and abstract cognitive levels, to the highest level, evaluation. In the revised model, Anderson and Krathwohl (2001) changed the names of the categories from nouns to verbs and revised the names of three levels: 'knowledge' was changed to 'remember', 'comprehension' to 'understand', and 'synthesis' to 'create'. The revised model thus classifies cognitive skills into the following hierarchical levels: remember, understand, apply, analyse, evaluate, and create (Kaur, 2023; Radmehr & Drake, 2019; Retno et al., 2019), as shown in Figure 5 below.

Figure 5

Bloom et al. 's (1956) original taxonomy (left) and Anderson and Krathwohl's (2001) revised taxonomy (right) (Köksal et al., 2023, p. 136)



The Revised Bloom's Taxonomy is continuing to be used widely for its relevance and applicability to mathematics education (Kaur, 2023; Radmehr & Drake, 2019; Retno et al., 2019). The following sub-sections describe each level of the taxonomy.

3.2.1 Remembering

In mathematics teaching and learning, learners are required to recall or recognise mathematical facts, rules, and formulas. This employs the cognitive skill 'remembering'. The learner employs their memory to recognise or recall knowledge, such as definitions, facts, or lists, or to recite previously learned information. For example, a learner might be asked to state the Pythagorean theorem or recall the formula for the area of a circle. The cognitive level of remembering provides the foundation upon which more complex skills are built.

Empirical research supports the importance of foundational knowledge in mathematics education. Webb et al. (2019) found that a strong grasp of basic arithmetic significantly predicted subsequent success in more advanced mathematical topics. Their findings also suggest that when learners cannot recall or remember facts, they find it difficult to develop critical thinking. This positions ‘remembering’ as a cognitive skill that is required for further cognitive development. Booth et al. (2017) also found that interventions to strengthen learners' memory of mathematical knowledge resulted in improved problem-solving skills.

These studies illustrate that a solid foundation in remembering mathematics knowledge is crucial for learners' overall success and progression in mathematics.

Retention of mathematical terminology and symbols is critical to remembering cognitive skills (Hiebert, 2020). Learners who consistently struggled with mathematical word problems were found to have difficulty recalling the precise meanings of common mathematical terms (Peng et al., 2020). The study revealed that frequent oral recall tasks, such as quick quizzes on definitions and notation, improved learners' ability to engage with mathematical problems confidently. This underscores that remembering supports factual recall and enables accurate interpretation of tasks across topics such as algebra, geometry, and statistics.

The consistent use of low-stakes testing has been shown to enhance memory retrieval processes in mathematics classrooms. Kunwar and Sapkota (2022) examined the effects of retrieval practice on learners' retention of mathematical formulas and procedures. When learners were routinely asked to recall key facts during lesson openings, their test performance improved over time. A question such as: What is the rule for multiplying negative numbers? prompted learners to access stored information more fluently. The study concluded that the act of retrieval strengthened learners' long-term memory, contributing to better mathematical fluency.

Visual aids and mnemonics also support the remembering skill by reinforcing information through associative techniques. Jaffe and Bolger (2023) investigated the use of diagrams and memory cues in mathematics classrooms. They found that learners who were exposed to consistent visual reinforcement, such as colour-coded steps in procedures, demonstrated higher levels of recall during assessments. These strategies enabled learners to remember isolated facts and sequential steps within operations. Such findings suggest that varied representations of mathematical knowledge can enhance learners' ability to recognise and retrieve essential content reliably.

3.2.2 Understanding

The next level of the taxonomy, 'understand', refers to the understanding of concepts and principles. This skill involves interpreting, exemplifying, classifying, summarising, inferring, comparing, or explaining various activities. At this level, learners are expected to interpret and summarise mathematical information, explain ideas in their own words, and grasp the underlying principles behind mathematical operations. Understanding enables learners to connect various mathematical concepts.

Empirical research underscores the significance of comprehension in mathematics education. Nilimaa (2023) found that learners who developed a deeper understanding of mathematical principles, rather than just procedural skills, demonstrated improved problem-solving and adaptability in mathematics. According to Ellis et al. (2019), instructional methods that focus on understanding, such as encouraging learners to explain their reasoning and explore different solution strategies, are essential for them to advance to the next cognitive level. These studies collectively highlight that fostering understanding in mathematics education forms the necessary foundation for learners to develop the skills they need to apply their knowledge effectively, adapt to new problems, and achieve long-term success.

Learners who engage with mathematical concepts through multiple representations are more likely to demonstrate understanding. In a study by Mpuangnan and Govender (2024, learners who were exposed to numerical, graphical, and symbolic forms of linear functions showed greater conceptual clarity than those taught through a single representation. The study revealed that learners who could transition between these representations were better at explaining relationships and identifying errors in their work. This reinforces that the ability to understand mathematics relies on interpreting and making sense of different formats of the same concept.

Classroom discussions centred on justification and explanation also promote understanding. Mikeska and Howell (2020) found that when learners were prompted to explain their mathematical reasoning aloud, they demonstrated a stronger grasp of concepts such as factorisation and proportional reasoning. Tasks that required learners to describe why a particular method was used, or to compare two solution strategies, encouraged them to move beyond rote application of procedures. As a result, learners developed a more coherent mental framework for solving unfamiliar problems, indicating growth at the understanding level.

The role of teacher questioning in facilitating understanding has also received empirical support. Hutagalung et al. (2025) found that questions prompting learners to make inferences and offer examples led to more meaningful engagement with mathematical content. In one classroom, the question: Can you give another example that fits this rule? helped learners generalise from specific instances. These prompts required learners to classify, exemplify, and elaborate on their understanding, aligning with the cognitive skill of understanding as defined in the Revised Bloom's Taxonomy.

Written tasks that require learners to explain their thinking have also been shown to enhance understanding. A study by Botelho et al. (2023) examined the use of open-response items in mathematics assessments. Learners were required to explain their answers rather than simply provide calculations. The findings indicated that those who regularly engaged in such written explanations performed better on conceptual items in later assessments. This suggests that expressing understanding in written form reinforces cognitive engagement and supports the internalisation of mathematical ideas.

3.2.3 Applying

At the next cognitive level of the taxonomy, 'apply', learners use the knowledge they have built from the levels 'remember' and 'understand' to solve problems. They apply the mathematical concepts and methods that they understand to unfamiliar contexts. Examples of verbs that relate to this function include apply, relate, translate, demonstrate, show, illustrate, and explain. Through teachers' questioning practices, learners may be required to explain or show their findings regarding mathematics to others, including methods used and their step-by-step process leading them towards it.

The learner explains what they know or how they think, and the teacher asks questions that go deeper into the topic or clarify understanding. Taub et al. (2020) demonstrated that learners who engaged in problem-solving learning – which emphasises applying mathematical concepts to real-world problems – showed improved critical thinking compared to those who were asked to work at the 'remember' or 'understand' level. A study conducted by Eshaq (2024) found that learners who practised applying mathematical concepts through hands-on activities and real-life problem-solving tasks developed increased critical thinking skills and greater skill proficiency. Furthermore, a study by Ngayubwiko and Andala (2024) indicated that teachers' questioning strategies that focused on applying mathematical concepts such as exploratory projects and real-world problem-solving exercises, significantly improved learners' critical thinking.

Application of mathematical knowledge often involves translating abstract principles into practical situations (Dekgado-Rebolledo, 2020). Hwang et al. (2020) explored learners solving problems related to measurement in real-life contexts, such as calculating the cost of tiling a floor or estimating paint required for a surface. Learners who were able to identify the relevant formulas and adapt them appropriately to suit the problem context demonstrated proficiency at the applying level. The findings emphasised the importance of exposure to varied tasks where learners are required to draw on conceptual knowledge and implement it effectively.

Classroom questioning that targets application skills tends to involve ‘how’ and ‘why’ prompts (Morris & Chi, 2020). Kayima and Jakobsen (2020) observed that when teachers asked learners to explain how a formula applies to a specific problem, such as determining the gradient of a line from a table of values, learners displayed more precise procedural fluency. These questions required learners not only to recall knowledge but to connect it meaningfully to the structure of the problem. As learners engaged in these applications, they also developed the ability to identify which procedures were most efficient in different scenarios.

The level of learner engagement increases when the application is integrated with practical problem contexts. A study by Jacosalem and Futalan (2025) found that learners who were asked to apply concepts of proportionality to budgeting exercises in consumer mathematics performed significantly better on application-based assessment items. These tasks required learners to translate textbook procedures into budgetary decisions, such as adjusting quantities or calculating discounted prices. This facilitated meaningful learning and improved knowledge transfer to unfamiliar situations, a hallmark of the apply level.

Question sequencing also contributes to the successful development of application skills. When lessons progressed from direct instruction to guided application and then to independent problem-solving, learners performed better in multi-step problems that required synthesis of previously learned skills (Wang et al., 2023). In one lesson on quadratic equations, learners were first reminded of the formula, then guided through an example, and finally asked to solve real-world problems involving projectile motion. This progression supported cognitive activation at the apply cognitive level and encouraged learners to take ownership of their problem-solving processes.

3.2.4 Analysing

The next level of the taxonomy, 'analyse', involves breaking down materials or concepts into parts and determining their interrelationships and overall structure. This cognitive skill involves differentiating, organising, attributing, and distinguishing between components. Examples of verbs at this level include analyse, compare, differentiate, discover, investigate, examine, and classify. Analysis is critical for solving multifaceted problems and advancing to higher levels of mathematical thinking. For example, in algebra, learners can analyse the structure of an equation to understand how changing one variable affects the entire system.

A study by Teck et al. (2023) revealed that learners who were taught to analyse mathematical problems by breaking them down into smaller, more manageable parts performed better on problem-solving tasks and showed greater mathematical comprehension. Schoenfeld (1985) demonstrated that teaching learners analytical skills, such as identifying underlying assumptions and patterns, significantly improved their ability to tackle non-routine mathematical problems.

Additionally, Tachie (2019) found that learners who engaged in metacognitive activities that involved analysing their problem-solving processes and strategies achieved higher levels of mathematical reasoning and performance. These studies reiterate that fostering analytical skills in mathematics enhances learners' critical thinking, and learners can use their analytical skills to develop their ability to evaluate – which is considered a higher cognitive skill than analysis.

Classroom instruction that encourages learners to compare mathematical structures supports the development of analytical thinking. Tondorf and Alezandra (2022), reported that when learners were tasked with comparing different solution methods for the same algebraic expression, they demonstrated a deeper understanding of equivalence and mathematical structure. Learners who engaged in comparative analysis were better able to articulate why one method might be more efficient or elegant than another, indicating mastery at the analysis level.

Questioning strategies that prompt learners to examine underlying patterns and relationships have also been shown to develop analytical thinking. Uygun (2020) found that classrooms where teachers posed questions that required learners to investigate the impact of manipulating terms within geometric sequences. Learners who responded to questions such as: What happens to the nth term if we increase the common ratio? displayed improved reasoning in identifying regularities and justifying general rules. This ability to dissect and reorganise information formed a basis for higher-level mathematical thinking.

Error analysis tasks have also been effective in fostering analytical skills. Nwoke et al. (2024) in a cases study investigated the use of deliberate error identification activities in senior secondary mathematics classrooms. Learners were asked to examine incorrect solutions and explain the source of the error. Those who successfully identified and corrected misconceptions showed an enhanced ability to evaluate logical coherence within mathematical procedures. This type of analysis encouraged learners to scrutinise reasoning pathways, rather than merely following set procedures.

Task design also plays a critical role in stimulating analytical thinking. He et al. (2022) reported that mathematical investigations requiring classification of functions based on their graphical or algebraic features led learners to identify structural characteristics, such as symmetry, intercepts, and rates of change. Learners articulated distinctions and similarities across function types when teachers supported these investigations through sequenced questioning. This process reinforced analytical thinking and supported conceptual clarity, especially in preparing learners for more abstract content in calculus and algebra.

3.2.5 Evaluating

Evaluation involves making judgements and arguments by combining various elements and concepts to form a new, coherent whole. Evaluation entails forming assessments using criteria and standards to generate critiques, recommendations, and reports. In the Revised Bloom's Taxonomy, evaluating precedes creating, often as a prerequisite. This level requires learners to evaluate ideas by integrating knowledge from different areas of mathematics. Evaluation is also involved when developing a new theorem, creating a unique problem-solving strategy, or designing a mathematical model to explain real-world phenomena. This level encourages creativity and innovation, as learners must draw upon their foundational knowledge, comprehension, and analytical skills to make a judgement.

According to Wu and Rau (2019), learners who engage in activities requiring them to evaluate mathematical and scientific knowledge to create models showed improved understanding and retention of mathematical concepts. Learners who worked well in problem-solving activities that required them to compare, classify, investigate, and synthesise various mathematical ideas developed better problem-solving skills and demonstrated higher levels of mathematical thinking than those who did not. Additionally, Cai and Leikin (2020) indicated that encouraging learners to respond to questions that require them to investigate and compare solutions significantly enhanced their knowledge and applied it creatively. These studies collectively demonstrate that 'evaluate', as a cognitive level, is important for enhancing learners' critical thinking.

According to Aliyu (2023), evaluation in mathematics classrooms also emerges through learners' engagement with multiple solution strategies. In another empirical study, Attard and Holmes (2022) demonstrated that when learners were asked to assess the efficiency and appropriateness of different methods for solving quadratic equations, they developed a deeper understanding of algorithmic choices and their justifications. Learners made evaluative judgements about each method's strengths and limitations, which sharpened their mathematical reasoning and capacity for informed decision-making.

The practice of justifying solutions during mathematical argumentation further illustrates the application of the evaluation skill (Indrawatiningsih, 2020). In their study Lee and Zeidler (2020), learners were required to present and defend their solutions in small groups before reaching a consensus. This process demanded that learners assess the validity of different responses, leading to higher-quality reasoning. Through structured questioning and peer critique, learners refined their arguments and learned to judge the soundness of mathematical claims based on logical coherence rather than surface features.

Rubric-guided evaluation tasks have also been found to support the development of evaluative thinking in mathematics. A study by Naidoo and Kapofu (2020) involved learners using predetermined criteria to assess their peers' geometric constructions and explanations. Learners internalised the same standards in their work by evaluating the precision, logical flow, and completeness of their classmates' responses. This reflective judgement process facilitated a more critical engagement with mathematical quality and accuracy.

Evaluative questioning by teachers has further contributed to the development of higher-order reasoning in mathematics classrooms. Kim (2025) examined the effects of teacher-initiated questions such as: Which method is more efficient and why? or Is this the only way to approach this problem? Learners exposed to this form of questioning exhibited enhanced evaluative thinking, demonstrated by their ability to weigh alternatives and provide reasoned justifications. This aligns evaluation with purposeful inquiry, enabling learners to assess both process and product in mathematics.

3.2.6 Creating

The highest cognitive skill in the hierarchy, 'create', involves assembling elements to create a functional whole or reorganising them into new patterns or structures. This level requires learners to synthesise existing knowledge, skills, and concepts to generate novel ideas, products, or processes. Innovation and originality are emphasised, moving beyond mere application or analysis to construct something fundamentally new. Key verbs associated with this level include design, construct, produce, invent, and develop. Learners move beyond rote computation or the application of formulae to integrate diverse mathematical domains into coherent, innovative frameworks (Jonsson et al., 2022).

Tasks are open-ended: for example, creating or designing a board game. Such activities demand iterative refinement, justification of choices (Why does this model best satisfy the conditions?), and critical evaluation of the validity and efficacy of one's work (Barana et al., 2022). This is highest mental function in the RBT. When learners are deemed to be at the 'create' level, they are considered to have developed cognitive skills at all lower cognitive levels, as well. Hence, the development of learners' cognitive ability to 'create' is as vital as the other five previous levels for their overall mathematical competence.

Creation in mathematics classrooms frequently involves formulating original problems that require learners to integrate various concepts. Maftuh et al. (2023) found that learners who were asked to design their own word problems based on real-life contexts displayed deeper conceptual understanding and enhanced creativity. This task required them to combine prior knowledge of mathematical operations, select relevant quantities, and structure the problem in a coherent manner. Such engagements positioned learners as active producers of mathematical content rather than passive recipients.

Another approach that cultivates the creation cognitive skills is the development of mathematical models. Xu (2025) examined how learners constructed mathematical models to represent population growth scenarios. Learners selected appropriate functions, justified assumptions, and modified their models based on feedback and further analysis. The study revealed that model construction demanded integration of knowledge from algebra, data handling, and functions. Learners engaged in iterative design, reflecting the complex nature of the 'create' level in the cognitive hierarchy.

Design-based learning tasks have further enhanced learners' capacity to create mathematical artefacts. Ibili et al. (2020) implemented a research project where learners were required to construct 3D geometrical shapes using recyclable materials and calculate surface areas and volumes. The process of designing, constructing, and presenting their models allowed learners to explore mathematical ideas through tangible outputs. The task also encouraged learners to justify design decisions mathematically, reinforcing the connection between creativity and precision in mathematics.

Programming-based tasks have also served as powerful platforms for developing creation skills in mathematics. Kado (2022) integrated Python-based coding activities, where learners created algorithms to solve mathematical problems. Learners combined algebraic reasoning with logic to develop, test, and refine their code. This required sustained creative effort, as learners had to devise new strategies when initial attempts failed. The findings indicated increased learner autonomy, persistence, and originality, hallmarks of the create level in mathematics education.

3.3 Using the Revised Bloom's Taxonomy to analyse and categorise the questions teachers ask

The Revised Bloom's Taxonomy provides a framework for analysing and categorising teachers' questions to enhance learners' critical thinking. The six hierarchal levels serve as a lens through which the design of the questions posed by teachers can be analysed and categorised in terms of their promotion of lower-order thinking skills and higher-order thinking skills. Lower-order thinking skills link to the two lowest cognitive levels of the RBT, while higher-order thinking skills link to four highest cognitive levels. In this study, the taxonomy facilitates an investigation into whether teachers predominantly use recall-based questions ('remember', Level 1 and 'understand', Level 2) or questions that promote the development of higher-order cognitive skills.

The taxonomy enables teachers' questions to be classified in terms of the cognitive demand they place on learners. Lower-order questions that focus on remembering and understanding typically require learners to recall definitions, rules, and procedures without engaging in deeper reasoning. In contrast, higher-order questions that require learners to apply, analyse, evaluate, and create mathematical knowledge are more likely to enhance their critical thinking.

In a Grade 10 mathematics classroom, questions that require learners to use their higher-order cognitive skills ('apply', Level 3; 'analyse', Level 4; 'evaluate', Level 5; and 'create', Level 6) are essential for enhancing learners' critical thinking. In this study, the Revised Bloom's Taxonomy thus provides a valuable theoretical lens that enables teachers' questioning strategies to be analysed for their effectiveness in fostering an environment conducive to critical thinking. The RBT also aligns with contemporary educational approaches prioritising active learning and metacognitive awareness. As mathematics education increasingly emphasises problem-solving and reasoning, integrating the RBT into teacher training programmes helps ensure that questioning practices systematically enhance learners' critical thinking. Thus, the use of the RBT in this study enables it to contribute to the broader discourse on effective teaching methodologies in mathematics education.

3.3.1 The use of the Revised Bloom's Taxonomy by researchers in South Africa

Several empirical studies in South Africa have explored the application of the Revised Bloom's Taxonomy in mathematics education, particularly focusing on teachers' questioning strategies. One notable study by Steyn and Adendorff (2020) investigated the questioning techniques employed by Foundation Phase (Grades R to 3) pre-service teachers during mathematical problem-solving lessons in mathematics classes in the Western Cape province. The researchers observed and analysed six lessons. They found that the pre-service teachers often struggled to formulate higher-order questions, highlighting a need for targeted training in this area (Steyn & Adendorff, 2020).

In another study, Ngwenya and Arek-Bawa (2020) examined the cognitive demand of assessment tasks in first-year accounting textbooks used at South African universities. Utilising a hybrid framework combining the Revised Bloom's Taxonomy and Leon's levels of difficulty, the researchers found a dominance of tasks requiring application-level thinking (Leon, 2006). This suggests a potential gap in promoting higher-order cognitive skills among learners.

Additionally, a study by McCarthy et al. (2016) analysed the questioning strategies of two Grade 8 mathematics teachers. The research identified various techniques, including probing, follow-up, leading, and learner-specific questioning. The findings emphasised the importance of guiding teachers to reflect on their questioning practices to enhance classroom discourse and learner engagement (McCarthy et al., 2016). Collectively, these studies underscore the significance of employing the Revised Bloom's Taxonomy as a framework to develop questioning strategies in mathematics education to enhance learners' critical thinking.

The RBT has also been applied to examine the cognitive levels embedded in mathematics textbooks and how these influence learners' engagement with mathematical content Zeynivandnezhad et al. (2024), conducted an analysis of mathematics textbooks widely used in schools. By categorising textbook questions according to the RBT, the study revealed an overwhelming focus on lower-order cognitive levels such as remembering and understanding. The limited presence of tasks aimed at higher-order thinking, particularly at the evaluating and creating levels, raised concerns about the adequacy of these resources in cultivating critical thinking and problem-solving skills.

Mathematics teacher education programmes in South Africa have similarly used the RBT to evaluate and improve pedagogical practices. Govender (2020) assessed the lesson plans of mathematics student teachers. Lesson plans were analysed to determine the extent to which different cognitive levels were addressed. The findings indicated that while most student teachers incorporated activities at the understanding and application levels, there was limited evidence of planning for analysing, evaluating, and creating. This pointed to the need for explicit instruction in the use of questioning frameworks aligned to the taxonomy in pre-service teacher training.

Zana et al. (2024) explored the alignment between mathematics classroom assessment tasks and the RBT across selected secondary schools. The analysis demonstrated that while application-level tasks were relatively common, tasks that required learners to engage in analytical and evaluative reasoning were rare. Teachers reported limited exposure to cognitive taxonomies during professional development workshops. The study recommended increased training on how to construct questions that address a broader range of cognitive demands to support curriculum goals.

Akinboboye (2021), offered a further contribution by investigating the use of the RBT in continuous assessment practices among mathematics teachers. By observing classroom activities and examining assessment tasks, the study found that teachers predominantly relied on procedural questioning, often neglecting the deeper cognitive processes promoted by the taxonomy. Teachers expressed uncertainty in differentiating between levels, especially between analysing and evaluating. These findings highlighted the need for sustained, context-sensitive support in embedding the taxonomy into everyday instructional practice within South African mathematics classrooms.

3.3.2 The limitations of the Revised Bloom's Taxonomy

The taxonomy's rigid hierarchical structure, organised linearly from 'remember' to 'create', may oversimplify the complexity of mathematical thinking. In practice, math questions often blend multiple cognitive levels. For instance, solving a word problem might require learners to understand the scenario, apply formulas, and analyse relationships between variables simultaneously. The RBT's emphasis on isolating questions into cognitive, misrepresents their multidimensional nature.

A second limitation is the RBT's lack of consideration for of contextual and social dimensions. Learning in mathematics classrooms may include collaborative problem-solving and dialogue, yet the taxonomy focuses solely on individual cognition (Takahashi, 2021). For example, a teacher's question prompting group discussion to explore geometric proofs may enhance critical thinking but would not be adequately analysed using the RBT's cognitive tiers alone, as they overlook the role of peer interaction in cognitive development (Widyatiningtyas et al., 2015).

Furthermore, the RBT does not account for learners' prior knowledge or affective factors. A question deemed 'analyse' for one learner might function as 'remember' for another, depending on familiarity with the content learned, yet the taxonomy treats cognitive demand as static (Barana et al., 2022). It also ignores motivational or engagement-based aspects of questioning, such as how a teacher's phrasing influences learners’ perseverance during challenging tasks. By framing questions only based on their cognitive complexity, the RBT may overlook the pedagogical strategies that are essential for fostering a growth mindset in mathematics. These gaps limit the model’s utility in holistically assessing the effectiveness of teachers’ questions in dynamic classroom environments.

When used in isolation, the RBT also lacks precision when applied to open-ended or divergent mathematical tasks. In many problem-based learning contexts, particularly those involving inquiry or exploration, learners may not follow a linear progression from lower-to higher-order skills. For instance, when learners are asked to devise multiple strategies to solve a non-routine problem, they may cycle between applying known methods, analysing outcomes, and creating novel approaches in a recursive rather than sequential manner. This non-linearity challenges the hierarchical logic of the taxonomy and suggests that real mathematical thinking often defies compartmentalisation into discrete levels.

Another limitation lies in the taxonomy's content-agnostic structure, which reduces the specificity required for mathematics education (Lin, 2021). The same cognitive verb, such as 'analyse' or 'evaluate', can hold different implications depending on the mathematical content in question. Analysing a quadratic function is fundamentally distinct from analysing a geometric transformation, yet RBT assigns the same cognitive classification regardless of mathematical domain. This abstraction limits its sensitivity to content-specific pedagogical demands and may obscure the complexity of conceptual understanding required across varied mathematical topics.

The RBT's emphasis on observable behaviours may also constrain assessment in mathematics classrooms. Its alignment with performance-based verbs encourages the framing of questions that yield measurable responses, potentially discouraging teachers from posing exploratory or philosophical questions that promote deeper mathematical reflection (West, 2023). For example, prompting learners to consider the aesthetic nature of a proof or the historical development of a concept may not fit neatly into any one cognitive category. Yet, such questions can enrich learners' appreciation of mathematics and deepen conceptual engagement.

Moreover, the taxonomy's limited applicability to formative assessment restricts its usefulness in responsive pedagogy (Grebin et al., 2020). Teachers often adjust their questioning in real time based on learners' misconceptions, emotional cues, or engagement levels—factors that fall outside the taxonomy's scope. This dynamic adaptation, crucial for effective teaching, is challenging to capture through a static framework like RBT. Consequently, while the taxonomy serves as a helpful starting point for planning cognitive demand, it falls short in accommodating the fluid, context-sensitive nature of teaching and learning in mathematics classrooms.

3.4 Duron et al.'s (2006) Critical Thinking Framework

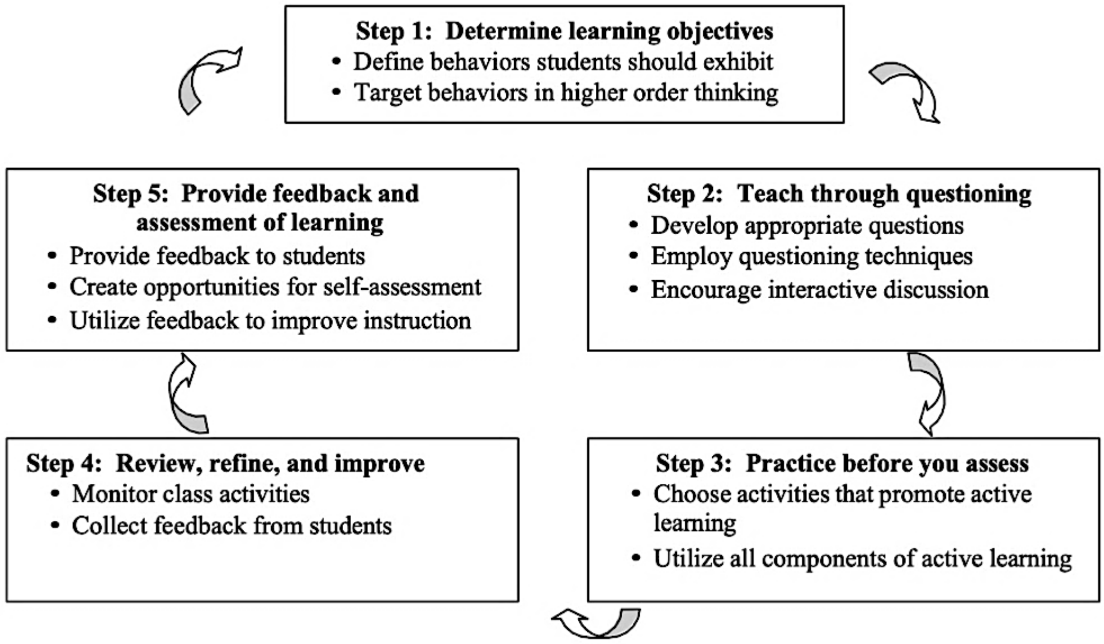
Duron, Limbach and Waugh (2006) proposed a critical thinking framework to address the need for a structured guideline for enhancing learners' critical thinking. Their framework was developed in response to concerns that many teachers were not adequately equipping learners with the ability to independently apply, analyse, evaluate, and create mathematical knowledge. Teacher-centred pedagogical strategies often relied more heavily on Levels 1 and 2 of the RBT, rather than aiming at enhancing learners' critical thinking. To bridge this gap, Duron et al. (2006) designed a systematic approach that teachers could use to integrate critical thinking into their questioning practices. The aim was to ensure that learners develop the necessary cognitive skills for problem-solving.

The Critical Thinking Framework is thus grounded in the RBT, which organises the cognitive processes into hierarchical levels. Duron et al. (2006) emphasise that effective teaching should encourage learners to progress beyond Levels 1 and 2 to the more advanced levels of thinking represented by Levels 3 to 6. Their framework provides a structured step-by-step approach to achieving this, ensuring questioning practices move beyond lower-order thinking. By aligning their model with the RBT, they have created a framework that is practically applicable across various disciplines.

Duron et al. (2006) recognise that most teachers need practical strategies to effectively enhance learners' critical thinking. By outlining five specific steps ('Step 1: determining learning objectives', 'Step 2: teach through questioning', 'Step 3: Practice before you assess', 'Step 4: Review, refine and improve', 'Step 5: Provide feedback and assessment of learning'), they offer a concrete model that can be applied in various educational settings. Hence, the framework was developed to support teachers in planning and teaching using questioning, where critical thinking is central. Figure 6 presents Duron et al.’s (2006) 5-step model to move learners toward critical thinking.

Figure 6

The five steps of Duron et al. 's (2006, p. 161) model for developing learners' critical thinking skills



The following sub-sections discuss each step of Duron et al.'s (2006) model.

3.4.1 Step 1 – Determine learning objectives

Determining learning objectives is fundamental to fostering critical thinking in learners. Duron et al. (2006) assert that educators must establish clear, measurable goals that align with higher-order cognitive skills. This step ensures that lessons are designed to move beyond Levels 1 and 2 of the RBT, thus engaging learners at Levels 3 to 6. When teachers set objectives that prioritise critical thinking, they create opportunities for learners to develop problem-solving skills, intellectual curiosity, and the ability to construct well-reasoned arguments.

The relevance of this step to teachers' questioning practices lies in its ability to guide the formulation of thought-provoking questions. Questions should align with the higher-order levels of taxonomy, encouraging learners to engage in deeper reasoning processes. When teachers frame questions based on explicit learning goals, they provide a structured approach that facilitates cognitive engagement. This allows learners to develop their reasoning abilities progressively, rather than merely responding to surface-level queries. Additionally, well-defined learning objectives assist in assessing learners' cognitive growth. If objectives emphasise critical engagement with content, teachers can employ strategic questioning techniques that require learners to apply, analyse, evaluate, and create mathematical knowledge (Duron et al., 2006).

3.4.2 Step 2 – Teach through questioning

Questioning is a crucial pedagogical tool that fosters learners' engagement with mathematical content and enhances their cognitive abilities. Duron et al. (2006) highlight that effective questioning strategies encourage learners to think independently and refine their reasoning skills. Teachers who incorporate questioning as a teaching technique create a dialogic classroom environment that promotes inquiry and reflection. The process of questioning serves as a cognitive scaffold, enabling learners to develop a deeper understanding of the subject matter.

The role of questioning in developing critical thinking is evident in how it challenges learners to apply, analyse, evaluate, and create information. When teachers pose higher-order thinking, learners are encouraged to engage in self-reflection, justify ideas, and evaluate multiple viewpoints. This process prompts learners to articulate their reasoning and interrogate the assumptions underlying their responses. By systematically questioning learners, teachers facilitate a shift from passive reception of information to active construction of knowledge. Furthermore, questioning supports formative assessment by allowing teachers to gauge learners' levels of understanding and reasoning abilities. Teachers can adjust their questioning techniques to address misconceptions and promote deeper cognitive engagement (Duron et al., 2006).

3.4.3 Step 3 – Practice before you assess

Providing learners with opportunities to practise critical thinking before formal assessment is essential in ensuring their intellectual development. Duron et al. (2006) emphasise that learners need structured practice to refine their reasoning and problem-solving skills. This step involves engaging learners in discussions, problem-based tasks, and inquiry-driven activities that reinforce critical thinking. Through consistent practice, learners become adept at applying, analysing, evaluating, and creating arguments.

Teachers' questioning practices are crucial in facilitating this step by offering learners structured engagement with complex ideas. When teachers pose higher-order questions during practice sessions, they enable learners to apply critical thinking skills. This allows learners to explore different viewpoints, challenge their preconceptions, and develop confidence in articulating reasoned responses. During practice, this creates a bridge between instruction and assessment, ensuring that learners develop cognitive resilience. Moreover, practising critical thinking through questioning prepares learners for more rigorous intellectual challenges in assessments. Learners develop a habit of reasoning beyond surface-level understanding by repeatedly engaging with higher-order questions. Teachers' questioning techniques thus serve as a cognitive scaffold that progressively strengthens learners' critical thinking abilities, ensuring they are well-equipped for critical thinking assessments (Duron et al., 2006).

3.4.4 Step 4 – Review, refine and improve

Reviewing, refining, and improving learners' critical thinking abilities is an essential component of effective teaching. Duron et al. (2006) stress that teachers must continuously monitor learners' reasoning processes and provide opportunities for deeper intellectual engagement. Reviewing learners' responses allows teachers to identify gaps in understanding, misconceptions, and areas requiring further cognitive development. Step 4 can be iterated. Through the iterative process, learners develop the ability to articulate their thoughts with clarity and precision.

Teachers' questioning practices are integral to this refinement process. By posing follow-up questions, requesting elaboration, and prompting learners to apply, analyse, create, and evaluate their reasoning, teachers facilitate learners' critical thinking. This approach ensures that learners do not simply provide superficial answers but engage in thoughtful analysis and critical evaluation. When teachers strategically refine their questioning techniques, they encourage learners to reconsider their perspectives, revise their arguments, and develop a more nuanced understanding of concepts (Duron et al., 2006). Moreover, this iterative engagement with content through questioning ensures that learners develop a disciplined approach to thinking critically, which is essential for academic progress.

3.4.5 Step 5 – Provide feedback and assessment of learning

Providing feedback and assessing learning is essential in developing learners' critical thinking skills. Effective feedback plays a crucial role in this process, as it helps learners recognise their cognitive strengths and areas requiring further development. The assessment process thus serves as a means of reinforcing critical engagement with content. Teachers' questioning practices are instrumental in providing formative feedback that enhances learners' critical thinking. When teachers use questioning as a feedback mechanism, they create opportunities for learners to refine their reasoning and articulate more coherent responses. Instead of merely indicating correctness, effective questioning prompts learners to reconsider their justifications, explore alternative perspectives, and strengthen their arguments. This ensures that feedback is not a one-way process but a dynamic engagement with learners' cognitive development.

Also, assessment through questioning fosters a reflective learning culture where learners actively evaluate their intellectual progress. Learners develop a critical awareness of their thought processes by engaging in self-assessment and peer discussions prompted by targeted questions. Teachers' questioning practices thus extend beyond assessment to become a tool for deepening learners' engagement with knowledge, reinforcing their ability to think critically and independently (Duron et al., 2006).

3.5 The synergy between the RBT and CTF

The RBT provides the essential cognitive architecture, defining the hierarchical complexity of thinking skills, forming the core processes directly associated with critical thinking. Duron et al.'s framework then builds operationally upon this foundation, particularly leveraging the higher-order levels; it translates the abstract into a practical, teachable model with distinct, actionable phases, teaching fundamental CT skills, practising them within discipline-specific contexts, and applying them through complex tasks like solving problems. Furthermore, the metacognitive knowledge dimension emphasised in the RBT aligns with Duron et al.'s focus on learners' self-assessment of their reasoning, effectively making Bloom's theoretical structure actionable for developing CT through structured classroom practice and application.

3.6 Conclusion

In this chapter, I have discussed the Revised Bloom's Taxonomy and Duron et al.'s (2006) Critical Thinking Framework. I have demonstrated their relevance to mathematics education, particularly to enhancing learners' critical thinking through teachers’ questioning practices and their usefulness as the components of the theoretical framework underpinning this study. Combining the Revised Bloom's Taxonomy with the Critical Thinking Framework underscores the role of teachers' questioning practices in facilitating critical thinking. The model's structured steps offer a systematic approach to enhancing critical thinking. The methodological design used in the current study is presented in the following chapter.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

This chapter presents the methods that were used to conduct this qualitative study and justifies the methodological choices that were made. The study was conducted by responding to the main research questions. The research paradigm, approach, and style adopted to answer the main research questions are presented. The process used to select the research site and participant sample is discussed. The data generation instruments that were used to generate data are explained. Table 3 provides an overview of the research design.

Table 3

Overview of the research design

Research approach	Qualitative
Research paradigm	Interpretive
Research style	Intrinsic Multiple Case Studies
Sampling procedure	Convenience sampling, with elements of purposive sampling
Research sites	Three public high schools in KwaZulu-Natal (Pinetown District)
Participants	Six teachers (two per school)
Data analysis technique	Thematic
Data generation instruments	• Semi-structured questionnaire • Semi-structured observation schedule • Semi-structured interview schedule

The chapter also discusses the ethical considerations involved in this study, including the methods followed to ensure the trustworthiness of the research.

Methodological choices were designed to answer the four main research questions that the study sought to answer:

1. What questioning practices are currently being used by teachers in Grade 10 mathematics classes?
2. What types of questioning practices can teachers use to enhance critical thinking in Grade 10 mathematics classes?
3. How do teachers use questioning practices to enhance critical thinking in Grade 10 mathematics classes?
4. Why is there a need for teachers to use questioning practices to enhance critical

thinking in Grade 10 mathematics classes?

4.2 Research paradigm and approach

4.2.1 Research paradigm

Kivunja and Kuyini (2017) define a research paradigm as a fundamental framework that guides how research is conducted by establishing the philosophical assumptions about reality (ontology), knowledge (epistemology), and the methodological approach to inquiry. A paradigm provides a worldview that influences the entire research process, including formulating research questions, selecting research methods, and interpreting findings (Ryan, 2018). This means that a research paradigm reflects the researcher's beliefs about how knowledge is constructed and validated, shaping the methodological choices and justifying the approaches used in a study. According to Pervin and Mokhtar (2022), research paradigms typically align with particular philosophical and methodological orientations, such as positivism, interpretivism, critical theory, and pragmatism.

A research paradigm comprises three key constituents: ontology, epistemology, and methodology, which must be explicitly aligned with the study's objectives (Abdul Rehman & Alharthi, 2016). Ontology pertains to the researcher's view of reality – whether it is objective and singular, or subjective and multiple. Epistemology concerns with how knowledge is created (Kivunja & Kuyini, 2017). Methodology refers to the strategies and procedures used to investigate research questions (Bloomfield & Fisher, 2019). Therefore, researchers must carefully articulate how these elements are integrated into their study to ensure coherence and philosophical consistency.

The current study aimed to explore teachers' questioning practices to enhance learners' critical thinking in Grade 10 mathematics classrooms for the purpose of existential understanding. For this purpose, the interpretive paradigm was determined to be the most suitable paradigm for this study, as it focuses on understanding participants’ subjective meanings and experiences within their natural setting. This paradigm assumes that reality is socially constructed, and that knowledge is best understood through the perspectives and interactions of those involved (Kivunja & Kuyini, 2017). Since teachers' questioning practices are deeply embedded in their pedagogical beliefs, classroom dynamics, and interactions with learners, an interpretive approach allowed me to explore these aspects holistically. It facilitated an in-depth understanding of how the teachers formulated and delivered their questions.

An interpretive ontological stance was chosen for this study because it assumes that reality is socially constructed, subjective, and context-dependent (Tubey et al., 2015). Teachers' questioning practices are not fixed or universal; rather, they emerge from their experiences, beliefs, and interactions with learners. The study sought to explore how teachers perceive and enact questioning strategies and how these practices enhance learners' critical thinking. Since questioning is an interactive and dynamic process that varies based on classroom culture, curriculum expectations, and individual teacher styles, reality in this context is multiple and constructed through social interactions rather than existing as an absolute truth that can be objectively measured (Rahi, 2017).

An interpretivist epistemological stance was appropriate because it assumes that knowledge is subjective, co-constructed, and context dependent. This perspective aligns with the understanding that teachers' questioning techniques and their impact on learners' critical thinking cannot be objectively measured but must be explored through teachers' lived experiences and interactions. Knowledge in this study emerged from teachers' perspectives, classroom discourse, and the meanings they assigned to questioning practices in mathematics instruction. I engaged with the teachers recognising that knowledge is not discovered but rather interpreted through social interactions and shared experiences.

This epistemological stance was justified because it values that knowledge is constructed through dialogues, reflections, and engaging with participants (Creswell & Poth, 2018). Since questioning is an interactive and discursive process, I relied on teachers' narratives and classroom observations to understand how their questioning enhances critical thinking. This approach ensured that knowledge was not treated as an external reality but as something shaped by the context, experiences, and perspectives of those involved in the teaching and learning process (Treagust & Won, 2023). By adopting an interpretive epistemology, the study acknowledged that teachers were central to understanding the effectiveness of questioning strategies.

The interpretive methodological approach was determined to be appropriate for this study because it prioritises exploratory qualitative methods that capture the complexity and contextual nature of teachers' questioning practices (Kumatongo & Muzata, 2021). Given that the study seeks to understand teachers' experiences, beliefs, and interactions in real classroom settings, exploratory methods such as classroom observations, semi-structured interviews, and a semi-structured questionnaire were determined to be suitable to enable an in-depth exploration of how questioning enhances critical thinking. These methods enabled me to gather rich, descriptive data on how teachers structure their questions and how questioning strategies contribute to deeper mathematical reasoning and problem-solving.

This methodological approach was justified because I was able to capture the lived experiences of teachers, ensuring that the study provides contextually relevant and meaningful insights rather than generalised conclusions detached from practice (Abdul Rehman & Alharthi, 2016). Data generation methods such as semi-structured interviews allowed teachers to reflect on their questioning practices, while semi-structured classroom observations provided direct evidence of how questioning unfolds in real teaching situations. Furthermore, the interpretive methodology supports thematic analysis, which helps identify patterns in teachers' questioning strategies and their impact on learners' critical thinking (Croucher & Cronn-Mills, 2018). This approach ensured that the study remained flexible, iterative, and responsive to emerging themes, making it well-suited for exploring the nuanced and interactive nature of questioning in mathematics classrooms (Kivunja & Kuyini, 2017).

4.2.2 Research approach

When conducting a research study, it is crucial that the researcher chooses an appropriate research approach to guide their exploration (Johnson & Christensen, 2025). Three broad approaches are used in research: qualitative, quantitative or mixed methods approaches. For the purpose of this study, I chose the qualitative research approach, as it enabled an in-depth exploration of teachers' experiences, perceptions, and practices within their classrooms (Mulisa, 2022). Unlike quantitative research, which seeks to measure variables numerically, qualitative research focuses on understanding social phenomena through rich, descriptive data (Creswell & Poth, 2018).

Since questioning practices are inherently interactive and context-dependent, a qualitative approach allowed me to explore how teachers construct and implement questioning techniques and how they contribute to enhancing learners' critical (Shanmugavelu et al., 2020). This approach is beneficial in educational research, where the goal is to capture the complexities of teaching and learning rather than to generalise findings across populations (Babchuk, 2016).

A key justification for using a qualitative approach in this study is its alignment with the interpretive research paradigm, which assumes that reality is socially constructed and best understood from the perspectives of those involved (Kivunja & Kuyini, 2017). Teachers' questioning practices are influenced by their individual pedagogical beliefs, classroom dynamics, and learner characteristics, making it necessary to explore these elements in-depth through qualitative methods (Svanes & Andersson-Bakken, 2023). Another significant reason for employing a qualitative approach is its capacity to capture the depth and complexity of human experiences (Denzin & Lincoln, 2008). In the context of mathematics education, critical thinking is developed through interactions, reasoning, and problem-solving, which are best understood through direct engagement with teachers and learners (Ghafar & Hazaymeh, 2024). A quantitative method, such as standardised testing, may fail to capture the subtleties of classroom discourse, while qualitative methods allowed me to explore how teachers design, phrase, and pose their questions (Coe et al., 2021).

Additionally, the qualitative research approach was justified for this study because of its capacity to explore the contextual and situational factors that shape educational practices (Peterson, 2019). Teachers' questioning techniques are influenced by curriculum demands, institutional policies, and classroom dynamics, all of which vary across different classrooms. Hence, through a qualitative approach, I was able to situate findings within specific teaching contexts, providing a holistic understanding of how questioning contributes to critical thinking in mathematics classrooms. By capturing teachers' reflections and classroom dynamics, qualitative research ensures that findings remain relevant and applicable within the specific educational environments being studied (Johnson & Christensen, 2025). Hence, prioritising participants' voices and experiences, qualitative research ensures that findings reflect authentic classroom realities rather than imposing externally defined constructs.

4.3 Research design

According to Mann (2006), a "research style is a particular perspective toward conducting educational research determined by the psychological or sociological context, not by personal preference" (p. 3). For the purpose of this study, a case study was selected as the most appropriate research style. Creswell and Poth (2018) define a case study as a qualitative approach in which the researcher explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time through detailed, in-depth data generation involving multiple sources of information such as observations, interviews, documents, and reports. The case is studied within its context, allowing for a comprehensive understanding of the phenomenon under investigation.

Creswell and Poth (2018) identify three main types of case studies: instrumental, intrinsic, and collective case studies, each serving distinct research purposes. These case study types differ in their focus, objectives, and scope, making them applicable to different kinds of research inquiries. Selecting the appropriate case study type is crucial for ensuring alignment with the study's research objectives, methodology, and philosophical assumptions (Shaw et al., 2023). On hand, a case study in which the focus is on the case itself is known as an instrumental case study. On the other hand, an intrinsic case study focuses on understanding a specific, unique case in depth rather than generalising findings or illustrating a broader phenomenon.

The unit of analysis is a critical component in intrinsic case study research as it defines the primary focus of inquiry and determines how data is collected and analysed (Stake, 2010). It refers to the 'what' or 'who' that is being studied – which could be an individual, a group, an organisation, an event, or a process. The selection of an appropriate unit of analysis ensures that the research findings remain focused, relevant, and methodologically rigorous. The unit of analysis varies based on the scope and objectives of the research. In a single-case study, the unit of analysis is typically one entity or phenomenon within a specific context. This could be an individual teacher, a single classroom, a school, or even a specific teaching method. The researcher conducts an in-depth examination of a single case to provide detailed, context-rich insights about the phenomenon under investigation (Crowe et al., 2011). The findings of a single-case study are often highly contextual, meaning they may not be readily generalisable but provide a deep, nuanced understanding of a particular instance (Creswell & Poth, 2018). In contrast, a multiple-case study involves analysing several cases, allowing the researcher to compare patterns across different settings, identify similarities and differences, and enhance the study's credibility (Stake, 2010).

Given that questioning practices are shaped by individual teacher experiences, classroom dynamics, and institutional contexts, an intrinsic case study approach allowed me to examine these elements holistically. Unlike broad, generalised studies, an intrinsic multiple case study was suitable to provide detailed insights into how questioning influences learners' critical thinking in specific mathematics classrooms, making it an appropriate choice for understanding complex pedagogical interactions (Stake, 2010). The relevance of using an intrinsic case study approach is justified in this current study because it aligns with the interpretive paradigm, which assumes that knowledge is socially constructed and context-dependent (Kivunja & Kuyini, 2017). Since teachers' questioning practices vary based on teaching styles, learner needs, and curriculum demands, the study required a research style that could capture these variations in depth (McCarthy et al., 2016). Moreover, an intrinsic multiple case study research style enables multiple forms of qualitative data to be collected – such as classroom observations, teacher interviews, and analysis of lesson transcripts – allowing for a comprehensive understanding of how questioning supports critical thinking in mathematics (Yazan, 2015). This depth of inquiry ensured that the findings reflected the authentic experiences of teachers and learners rather than being limited to surface-level descriptions.

Furthermore, an intrinsic multiple case study was determined to be an appropriate research style for this study because it supports naturalistic inquiry, which seeks to understand phenomena within their real-world settings (Creswell & Poth, 2018). Teachers' questioning techniques and their influence on learners' critical thinking cannot be isolated from the classroom environment, as they are shaped by classroom culture, learners' engagement, and pedagogical decisions. The intrinsic multiple case study research style enabled me to observed how questioning unfolds in live classroom interactions, rather than relying solely on retrospective reports or surveys. This approach ensured that the study captured genuine classroom practices and provided a rich, contextualised account of how questioning fosters mathematical reasoning and problem-solving. It was chosen because the aim was not to generalise how teachers implement their questioning during lessons but to emphasise their individual practices.

The unit of analysis chosen for this study was the individual teacher within each school. This means that while each school represented a separate case, the primary focus remained on the teachers' questioning practices, allowing for comparative analysis across different educational contexts. This approach is recommended by Yazan (2015) because it enables the researcher to identify patterns across cases while preserving the contextual richness of each setting.

4.4 Sampling of participants and site selection

Sampling, in qualitative research, involves selecting participants, settings, or events that provide rich, in-depth insights into the research problem rather than aiming for statistical representativeness (Creswell & Poth, 2018). Common qualitative sampling techniques include purposive, convenience, snowball, and theoretical sampling (Sharma, 2017). Purposive sampling is widely used in qualitative studies as it enables researchers to intentionally select participants with specific knowledge or experiences relevant to the study (Berndt, 2020). This approach contrasts with random sampling, which is typically used in quantitative research for generalizability (Sharma, 2017). Convenience sampling, on the other hand, involves selecting participants based on availability, accessibility, and willingness to participate, making it a practical and efficient approach for studies conducted in natural settings (Campbell et al., 2020). Snowball sampling is often used when participants help identify other potential subjects, while theoretical sampling is guided by emerging data patterns, rather than predetermined selection criteria (Gill, 2020). The choice of sampling technique depends on the research objectives, accessibility to participants, and the need for deep, context-specific insights (Stratton, 2021).

In this study on teachers' questioning practices, convenience sampling – with purposive elements – was determined to be the most suitable sampling method because it allowed the researcher to select accessible participants who met specific inclusion criteria (Rahman, 2018). Convenience sampling ensured practical access to mathematics teachers, particularly where time constraints, institutional approvals, and logistical limitations influenced participant selection (Miles et al., 2014). Since the study was conducted at three public high schools, the researcher had to rely on finding teachers who were available and willing to participate, making convenience sampling necessary. However, to maintain methodological rigour and alignment with the research objectives, purposive elements were incorporated to ensure that only mathematics teachers who actively used questioning techniques were included (Campbell et al., 2020) (Etikan & Bala, 2017). This hybrid approach ensured both feasibility and relevance, allowing the study to focus on teachers who could provide meaningful insights into the research problem (Campbell et al., 2020).

4.4.1 Site selection

Three public high schools in close geographical proximity to the researcher were selected as the site of the study. In addition, these three schools were also chosen based on their NSC examination results performance ranging between highest, average and lowest. The three schools offered Grades 8 through 12. All the schools are in the Pinetown District of the province of KwaZulu-Natal. Obtaining permission to conduct research at these schools was considered a fundamental principle of ethical qualitative research. Since the schools are public, they are owned by the KwaZulu-Natal provincial Department of Education; hence, permission to visit and conduct research in these schools was necessary. Permission was granted by the KwaZulu-Natal Department of Education (KZN DoE) (Appendix B) and by the school principals (Appendix C by means of formal 'permission to conduct research' letters.

4.4.2 Selection of participants

After the KZN DoE and the school principals granted me access to their schools, I approached the Grade 10 mathematics teachers at each school to request their participation in the study. Each of the three schools had two Grade 10 mathematics classes, each with its own teacher. None of the teachers had been involved in my previous research study. All six teachers agreed to participate in the study for its duration and signed informed consent forms (Appendix D). In the informed consent form, I provided the teachers with comprehensive information about the research study's purpose, procedures, and potential risks and benefits and explained that if they agreed to participate, they would be free to withdraw at any time without penalty. In this way, as a researcher, I respected the teachers' autonomy, allowing them to make informed decisions about their involvement in the research study.

4.4.3 Description of participants

All six teachers were qualified mathematics teachers. Each had a formal teaching qualification with mathematics as one of their specialised teaching subjects. They held a range of formal teacher education qualifications: a Bachelor of Education (B.Ed.), a Bachelor of Education Honours (B.Ed. Hons) or a Master of Education (M.Ed.) degree. Their teaching qualification dictated the phases (range of grades) which they were qualified to teach. In this case, each of the six teachers was qualified to teach Mathematics to Grades 7 to 12 (the Senior and FET Phases) or to Grades 10 to 12 (FET Phase). Table 4 presents a profile for each teacher, indicating their gender, highest formal teaching qualification, phase they were qualified to teach, grades for which they taught mathematics, and number of years they had been teaching at the time when data was generated.

Table 4

Profiles of the six participating teachers

School	Teacher	Gender	Age	Highest formal teaching qualification	Phase qualified to teach	Grades currently teaching mathematics	Number of years teaching
1	**Teacher A**	Male	46	Bachelor of Education (B.Ed.)	Senior and FET Phases	8, 10, 12	15
	Teacher B	Female	28	Bachelor of Education (B.Ed.)	Senior and FET Phases	9, 10	3
2	**Teacher C**	Female	38	Master of Education (M.Ed.)	Senior and FET Phases	8, 10, 11	10
	Teacher D	Male	53	Bachelor of Education Honours (B.Ed. Hons.)	FET Phase	9, 10, 11	15
3	**Teacher E**	Female	45	Bachelor of Education (B.Ed.)	FET Phase	9, 10, 12	11
	Teacher F	Male	47	Master of Education (M.Ed.)	FET Phase	10, 12	8

As depicted in Table 4, there were three male and three female participants. The ages of the participants ranged from 28 to 53, and their teaching experience ranged from 3 to 15 years. Each of the six teachers taught mathematics to more than one grade.

4.5 Data collection during the pilot study

Bertram and Christiansen (2014) describe data generation as the process of obtaining evidence from sources that allow researchers to answer research questions systematically and rigorously. Data generation involves the use of systematic methods to gather information relevant to the study objectives (Creswell & Poth, 2018). Hence, data generation is also intended to capture rich, descriptive, and contextually embedded information, often focusing on the lived experiences of participants. In this study, textual data was collected using three data generation instruments: a questionnaire, an observation schedule, and an interview schedule – all of which were semi-structured.

A pilot study was conducted before data collection began.

4.5.1 Pilot study

A pilot study, also known as a feasibility study, is a small-scale version of the main study that is conducted to test and refine the research design, methods, and procedures (Lowe, 2019). It plays a crucial role in ensuring the success of the main study. One of the primary advantages of a pilot study is that it enables research instruments such as questionnaires or interview guides to be tested (Patten, 2016). According to Malmqvist et al. (2019), testing allows researchers to identify problems or ambiguities and make necessary adjustments before conducting the main study (Varela-Candamio et al., 2018). Pilot studies can help estimate response rates for surveys or questionnaires, which is helpful in planning the sample size for the main study.

A pilot study can also provide insights into the reasons for non-response, enabling researchers to address underlying issues in the design of the main study (Lowe, 2019). The pilot study can reveal practical issues related to data collection, recruitment of participants, or allocation of resources, allowing researchers to address these challenges proactively. Pilot studies can also be used to refine data analysis techniques (Malmqvist et al., 2019). Pilot studies thus enhance the validity and reliability of the main study, save time and resources, protect participants, and increase the likelihood of success. Despite their small scale, the impact of pilot studies on the quality of research is significant (Ghazali, 2016).

A pilot study was employed in this qualitative research study to test the feasibility of using three research data generation instruments. A public high school in the Pinetown District of KwaZulu-Natal province was purposefully selected per the research design approach used in the main study. This school was situated in a similar context to the three schools selected for the main study. Two Grade 10 mathematics teachers were asked to complete a semi-structured questionnaire. Then, using the semi-structured observation schedule, I observed one lesson in each of their Grade 10 mathematics classes. Thereafter, I conducted individual semi-structured interviews with the two teachers. Through this exercise, I was able to identify some limitations in the design of the data collection instruments. All three research instruments were revised to include more descriptive elements. This enhanced the three data generation instruments, equipping them to capture a detailed description of how each teacher asked learners questions.

4.6 Collection of data

Data was collected in three stages, each using a different data collection instrument. As in the pilot study, participants were first asked to complete a semi-structured questionnaire. Then, using the semi-structured observation schedule, I observed one lesson in each of their Grade 10 mathematics classrooms. Thereafter, I conducted individual semi-structured interviews with the teachers.

4.6.1 Stage 1: Questionnaire

A questionnaire is a data generation instrument used to obtain responses from participants in a structured manner (Chowdhury et al., 2022). Questionnaires can be structured, unstructured, or semi-structured, depending on the degree of control the researcher chooses to exert over the responses. According to Bryman (2016), questionnaires are used widely in both qualitative and quantitative research, but their format determines whether they yield standardised data for statistical analysis or in-depth insights into participants' perspectives. The choice of questionnaire type depends on the nature of the study, the research paradigm, and the depth of inquiry required. While structured questionnaires aim for uniformity in responses, unstructured and semi-structured questionnaires provide greater flexibility, making them suitable for studies that explore complex social and educational phenomena (Cohen et al., 2018).

A structured questionnaire consists of closed-ended questions, where respondents choose from predefined options such as multiple-choice, Likert scale, or dichotomous 'Yes/No' responses (Oben, 2021). This type of questionnaire is primarily used in quantitative research, as it enables researchers to collect standardised data efficiently and analyse it statistically (Creswell & Poth, 2018). However, structured questionnaires are limiting in interpretive studies, as they do not allow respondents to elaborate on their experiences or opinions in detail (Chowdhury et al., 2022).

In contrast, an unstructured questionnaire consists of open-ended questions that encourage respondents to provide detailed, narrative responses. Unstructured questionnaires lack consistency and comparability, making data analysis more complex.

A semi-structured questionnaire combines elements of both structured and unstructured questionnaires, providing a balance between standardisation and flexibility (Cohen et al., 2018). It consists of predetermined key questions but allows for probing and follow-up questions based on respondents' answers (Kurzhals, 2021). This format is particularly useful in qualitative studies, as it guides the discussion while allowing for rich, contextual insights (Radhakrishna, 2020). Compared to structured questionnaires, semi-structured questionnaires offer depth and allow for clarification, but, unlike unstructured questionnaires, they maintain consistency across responses, making analysis more systematic (Silverman, 2024). This adaptability has resulted in semi-structured questionnaires being used widely in educational research, especially for studying teaching practices, pedagogical beliefs, and classroom interactions.

For the present study, a semi-structured questionnaire design was determined to be appropriate. The questionnaire was designed to obtain answers to the first and second research questions. As the study followed a qualitative approach within an interpretive paradigm, it sought to obtain deep insights into teachers' questioning strategies. A structured questionnaire would have restricted teachers' responses, preventing them from explaining how they formulated and implemented questions. On the other hand, an unstructured questionnaire could have produced disjointed or overly broad responses, making it difficult to analyse trends across different teachers. The semi-structured design, in contrast, allows the researcher to ask predetermined key questions while also allowing participants to elaborate on their experiences. In this study, the semi-structured questionnaire was designed to include predetermine questions and also allowed flexibility for the teachers to elaborate on some of their responses (Appendix E).

4.6.2 Stage 2: Observations

An observation schedule is a systematic data collection instrument used to record and analyse behaviours, interactions, and events in natural settings (Jones et al., 2022). Observation schedules vary based on the level of structure imposed on data generation and can be categorised into structured, unstructured, and semi-structured formats (Cohen et al., 2018). Observation schedules are particularly useful in qualitative educational research, where the goal is to capture real-time practices, instructional strategies, and classroom interactions (Schoenfeld et al., 2018). According to Creswell and Poth (2018), observational methods allow researchers to collect first-hand, non-verbal, and contextual data that cannot be fully captured through interviews or questionnaires.

Choosing the appropriate type of observation schedule depends on the study's objectives, the degree of flexibility required, and the need for comparability across different settings (Cohen & Goldhaber, 2016). A structured observation schedule follows a predefined checklist or coding system, systematically recording specific behaviours or events using quantifiable indicators (Clark et al., 2021). This method is commonly used in quantitative research to ensure standardisation and comparability across different observations (Noben et al., 2022). However, structured schedules limit flexibility, as they do not capture unexpected behaviours, emergent themes, or the contextual meaning behind interactions.

An unstructured observation schedule is highly flexible and open-ended, allowing the researcher to record events as they naturally unfold, without predefined categories. This approach is useful for exploratory research, enabling a rich, holistic account of classroom practices. However, data analysis may become more complex and less systematic as observations vary widely in focus and detail.

A semi-structured observation design was determined to be the appropriate choice for the observation schedule used in this study. Since the study focused on real-time questioning practices in diverse mathematics classrooms, a structured schedule would have been too rigid, failing to capture spontaneous teachers' practices, while an unstructured schedule, though flexible, might have required overly complex analysis. Hence, a semi-structured schedule ensured a systematic approach, allowing me to record specific questioning techniques while allowing room for emergent observations (Creswell & Poth, 2018). Using a semi-structured observation schedule allowed me to capture questioning strategies across three different schools while ensuring the comparability of data. By incorporating both predetermined categories and open-ended notes, the study was able to document not only the frequency of questioning techniques but also their contextual effectiveness. (Appendix F).

4.6.3 Stage 3: Interviews

An interview schedule is a structured framework researchers use to guide the process of interviewing participants, ensuring that data generation remains focused, systematic, and relevant to the research objectives (Avidan, 2017). Interview schedules vary in structure, including structured, unstructured, and semi-structured formats, each suited for different research paradigms and methodological approaches. According to Avidan (2017), qualitative research relies heavily on interviews to capture participants' lived experiences, beliefs, and perspectives, making the design of the interview schedule crucial to ensure rich and meaningful data generation (de la Croix et al., 2018). According to Deeks et al. (2019), choosing an appropriate interview schedule depends on the depth of inquiry required, the level of researcher control, and the flexibility needed to explore emerging themes.

A structured interview schedule consists of predefined, standardised questions that are asked in the same order and using the same format for all participants (Bryman, 2016). This approach is helpful in quantitative and mixed-methods research, as it allows responses to be easily compared and analysed statistically. However, structured interviews lack flexibility, preventing researchers from probing deeper to obtain valuable but unforeseen insights (Crowe et al., 2011). On the other hand, an unstructured interview schedule has no fixed set of questions, allowing for a free-flowing, open-ended conversation where participants lead the discussion based on their experiences. This approach is valuable for exploratory research but lacks consistency, making cross-case analysis difficult and potentially leading to inconsistent or irrelevant data.

For the present study, a semi-structured interview design was determined to be appropriate for the interview schedule. A semi-structured interview schedule offers a balance between a structured and unstructured designs, making it highly suitable for qualitative research (Cohen et al., 2018). It includes a set of predefined key questions, ensuring that the research remains focused, but also allows for probing and follow-up questions to explore responses in greater depth. Compared to structured interviews, semi-structured interviews enable deeper data generation by allowing participants to elaborate on their experiences while maintaining comparability across different interviews. Additionally, probing questions were used during individual interviews with the teachers, allowing me to clarify responses and enable deeper exploration. The semi-structured interview schedule was designed to obtain answers to the fourth research question (Appendix G).

4.7 Data analysis

Data analysis in research is the systematic process of organising, interpreting, and making sense of the data that has been generated to address the research objectives. In qualitative research, data analysis involves identifying patterns, themes, and meanings within the data, allowing researchers to understand participants' perspectives and experiences. Unlike quantitative analysis, which focuses on numerical data, qualitative data analysis is interpretive, iterative, and context-dependent, requiring continuous engagement with the data to ensure depth and rigour (Nowell et al., 2017). The process involves categorising responses, identifying relationships between concepts, and constructing meaningful insights from textual, observational, and documentary sources (Braun & Clarke, 2023).

Since the study follows a qualitative approach, analysing data systematically ensures that teachers' perspectives are meaningfully interpreted. Without rigorous analysis, raw data from interviews, classroom observations, and the questionnaire may remain fragmented, making it difficult to draw valid conclusions about teachers' questioning strategies that can promote learners' critical thinking (Nowell et al., 2017). Several qualitative data analysis techniques exist, each suited to different research objectives.

Content analysis involves systematically categorising text into codes and themes, making it helpful in analysing large datasets such as interview transcripts (Elo & Kyngäs, 2008). Discourse analysis examines how language is used in specific social contexts, which can be relevant when studying teacher-learner interactions and questioning patterns (Sardareh et al., 2014). Narrative analysis focuses on individual stories and experiences, making it applicable to biographical or case-based studies. Grounded theory analysis, as proposed by Glaser and Strauss (2017), involves developing theories from emerging data patterns, making it more suited for research aiming to construct new theoretical frameworks. Thematic analysis is a widely used qualitative research technique that involves identifying, analysing, and reporting patterns (themes) within data (Byrne, 2022).

Before analysis, the data collected from the semi-structured questionnaire, semi-structured lesson observation schedule, and semi-structured interview schedule were transcribed. Thematic analysis was determined to be the most appropriate approach for analysing the data generated in this study.

A six-phase process of thematic analysis was used:

Phase 1: I familiarised myself with the data. This included reading and reviewing the generated data multiple times.

Phase 2: I generated the initial codes. Coding ensures that data is categorised meaningfully, allowing for deeper analysis.

Phase 3: I searched and identified themes by grouping related codes. Themes should meaningfully address the research question, ensuring that they reflect patterns across all three data sources.

Phase 4: I reviewed and refined the themes. I checked whether they accurately represent the data. Through this process, I ensured that the themes were consistent across the three data generation instruments and that they captured meaningful insights into teachers’ questioning practices. Any themes lacking sufficient supporting data were reassessed or combined with stronger themes to maintain analytical rigour.

Phase 5: Each theme was named and defined, ensuring it contributed to the study's findings. Themes were named based on their significance.

Phase 6: I reported the findings. The final step involved writing up the research findings, where each theme was supported by direct quotes from interviews, excerpts from observations, and document analysis data. I connected findings to existing literature and the theoretical frameworks, ensuring the study contributes meaningfully to understanding questioning practices in mathematics education.

4.8 Methods to ensure the trustworthiness of the study

According to Korstjens and Moser (2018), trustworthiness in qualitative research involves specific criteria to ensure the quality and credibility of the research findings. Unlike quantitative research, which relies on post-positivist constructs like validity and reliability, qualitative research uses criteria such as credibility, transferability, dependability, and confirmability to affirm the rigour of its findings (Lincoln & Guba, 1986). These criteria help researchers, and their audiences assess whether the results are reliable and meaningful (Denzin & Lincoln, 2008). Following Loh (2013), observing these criteria is essential to ensure that findings are credible and valuable, meeting the standards necessary for acceptance in the broader knowledge community. The four criteria proposed by Lincoln and Guba (1986) are addressed below to demonstrate how this study ensured trustworthiness.

4.8.1 Credibility

Anney (2014) describes credibility as the confidence that can be placed in the truth of the research findings, emphasising that researchers must ensure alignment between participants' expressions and actions and the study's interpretation of these elements. This study strengthened credibility through triangulation, which involved collecting data through three different instruments: a semi-structured questionnaire, interviews, and lesson observations. This facilitated a nuanced understanding of teachers' questioning practices to enhance learners' critical thinking in Grade 10 mathematics classrooms. This is consistent with Natow’s (2019) notion that triangulation through multiple data generation instruments enriches the depth and reliability of the study's findings.

To further strengthen rigour, participant checking was implemented through the semi-structured interviews by returning to the six teachers to confirm that my interpretation of their observed actions aligned with their intended meanings. This method is also espoused by Tobin and Begley (2004), who argue that credibility is achieved when there is a close 'fit' between participants' views and the researcher's portrayal of those views.

Furthermore, following Lincoln and Guba (1986) recommendations, this study employed prolonged engagement and persistent observation to strengthen credibility. Having spent prolonged time with the six teachers in their classrooms, I was able to generate appropriate data, capturing a comprehensive understanding of the studied phenomenon.

4.8.2 Transferability

Transferability in research addresses the applicability of findings to other contexts (Nowell et al., 2017). Shaw et al. (2023) note that transferability in qualitative studies is generally evaluated on a case-by-case basis, making thick descriptions of contextual factors essential. Unlike quantitative studies, transferability does not assume universal applicability but empowers readers to make informed judgements about relevance. To support transferability, this study provided detailed accounts of the research site, methods, data analysis, and participants' backgrounds, allowing others to assess whether the findings could apply to contexts with similar characteristics. As Korstjens and Moser (2018) highlight, qualitative researchers enable others to judge transferability through these comprehensive descriptions. This is consistent with Babbie (2020), who further describes transferability as the extent to which findings can be applied in other settings or with different respondents.

4.8.3 Dependability

Galli et al. (2021) assert that dependability in research is achieved when the process is logical, thoroughly documented, and easily traceable. This is consistent with Anney (2014), who also asserts that dependability reflects the consistency and reliability of findings documented in a way that allows others to audit and critique the research process. Hence, this study ensured dependability by reporting the findings using a structured thesis format with clear subsections and headings, guided and confirmed by peer debriefers who verified the document's logical consistency and clarity. Lincoln and Guba (1986) further suggest that dependability requires an audit trail of the research process. I was able to document every step of the process and keep the raw data in a locked cabinet in my supervisor's office. I contend that if another researcher were to conduct the research within a similar context, they would likely reach comparable conclusions, aligning with the dependability standards outlined by Campbell et al. (2020b). To support future verification, all raw data and reflexive journals will be put in a locked cabinet of the supervisor and preserved for five years, facilitating data cross-referencing and process transparency, as Nowell et al. (2017) recommend.

My supervisor, an expert in mathematics education, also conducted an inquiry audit and reviewed the collected data, analysis, interpretations, and recommendations. This audit confirmed the coherence between the raw data and my interpretations. A detailed account of methodological procedures was provided to meet this criterion from the study's inception to its conclusion, enabling readers to evaluate the research practices followed. An audit trail was maintained, providing transparent documentation of all procedures, data generation methods, raw and analysed data, decisions made, and access protocols to the schools and participants, as detailed in the relevant appendices.

4.8.4 Confirmability

Confirmability, the final strategy for ensuring a study's trustworthiness, involves alignment between the researcher's interpretations and participants' actual expressions and actions during data generation (Babbie, 2020). To achieve this, I used a reflexive journal, audio recordings of interviews, and video recordings of classroom observations, allowing my supervisor to review the data and confirm that my interpretations matched the participants' statements and behaviours. The reflexive journal captured detailed events and notable moments ('ah' events) encountered in the research field, documenting the records of the research path transparently (Korstjens & Moser, 2018). Audio recordings allowed me to review participants' responses multiple times, ensuring accurate transcription and minimising researcher bias or figments of personal interpretation.

This study's findings, based solely on data from the six teachers as participants, were documented without researcher fabrication, allowing for external verification. According to Lincoln and Guba (1986b), confirmability can only be achieved after establishing credibility, dependability, and transferability. Throughout this research, all methodological, theoretical, and analytical choices were transparently justified to allow readers to understand the decision-making process, following Campbell et al.'s (2020) suggestion that such transparency is crucial for confirming a study's rigour.

4.9 Ethical considerations

Ensuring that ethical standards are maintained is a crucial aspect of mixed-methods research. Ethical clearance was obtained for this study from the University of KwaZulu-Natal (Appendix A). Permission was also obtained from the KwaZulu-Natal Department of Education to conduct research at the selected schools (Appendix B). These steps were taken to ensure the protection of the participants. In research, the well-being of participants is paramount. I treated the participants with respect, including respecting their autonomy to participate in the study or withdraw and protecting their privacy. The six teachers were assured of the confidentiality of their personal information during data generation. Each participant was given a pseudonym (Teachers A to F) to ensure anonymity and confidentiality (Cohen et al., 2018).

4.10 Conclusion

This chapter has presented the methodological approaches employed in this study. It covered the researcher's choice of research design, paradigm, sampling methods, data generation techniques, and data analysis procedures, providing a justification for each. Measures to enhance trustworthiness and rigour were detailed, along with strategies to uphold the quality and integrity of findings. The ethical considerations that guided the research were explained, including approval received from relevant institutions. The next chapter presents and analyses the data.

CHAPTER 5: PRESENTATION OF FINDINGS

5.1 Introduction

This chapter presents and analyses data concurrently. This process is crucial for ensuring transparency, credibility, and the generation of meaningful insights from the study (Braun and Clarke, 2022). According to Male (2016), the use of concurrent presentation and analysis allows researchers to identify and address any inconsistencies or ambiguities that may arise. Hence, the intention is to enhance the rigour and responsiveness of the research process (Green et al., 2007).

This study was guided by four research questions, which were investigated using three data generation instruments. Table 5 indicates the research questions and the instruments used to answer them.

Table 5

The four research questions and the three data generation instruments used to answer them

Research questions	Data generation instruments
1. What questioning practices are currently used by teachers in grade 10 mathematics classrooms?	Questionnaire Observation schedule
2. What types of questioning practices can teachers use to enhance critical thinking in grade 10 mathematics classrooms?	Interview schedule
3. How do teachers use questioning practices to enhance critical thinking in grade 10 mathematics classrooms?	Observation schedule
4. Why is there a need for teachers to use questioning practices to enhance critical thinking in grade 10 mathematics classrooms?	Interview schedule

Using thematic analysis, as described in Chapter 4, four themes – with corresponding subthemes – emerged from the data. These themes were used to guide the analysis and presentation of data.

The first section of this chapter describes the six participants. Thereafter, the themes and sub-themes that emerged from the data are presented. After the discussion of the themes, I present and analyse the data using each theme.

5.2 Profile of participants

Six Grade 10 mathematics teachers from three public high schools participated in the study. All three schools were located in the Pinetown District of KwaZulu-Natal province.

Each participant was given a pseudonym (Teachers A to F) to ensure anonymity and confidentiality (Cohen et al., 2018). Similarly, each school was given a pseudonym (Schools 1 to 3). Table 6 presents the teachers' profiles, by school.

Table 6

Profile of participants

Teacher	Gender	Ethnicity	Home language	Age	Highest teaching qualification	Phase qualified to teach	Mathematics grades taught	Number of years teaching	Number of Grade 10 learners taught
SCHOOL 1: QUINTILE 3									
A	Male	African	isiZulu	46	Bachelor of Education	Senior FET	8, 10, 12	15	42
B	Female	African	isiZulu	28	Bachelor of Education	Senior FET	9, 10	3	44
SCHOOL 2: QUINTILE 3									
C	Female	African	isiXhosa	38	Master of Education	Senior FET	8, 10, 11	10	56
D	Male	Coloured	Afrikaans	53	Bachelor of Education Honours	FET	9, 10, 11	15	50
SCHOOL 2: QUINTILE 4									
E	Female	African	isiZulu	45	Bachelor of Education	FET	9, 10, 12	11	65
F	Male	White	English	47	Master of Education	FET Phase	10, 12	8	67

The South African government has categorised all schools into five quintiles: 1, 2, 3, 4, and 5. Quintile 1 schools are those serving the poorest communities, while Quintile 5 schools serve the least poor communities. To determine a school’s quintile ranking, the Department of Basic Education (DBE) considers indices such as income, literacy, and unemployment levels in the surrounding community. Based on this classification, Quintiles 1 to 3 are designated as ‘no fee-paying’ schools, meaning they do not charge parents school fees, while Quintiles 4 and 5 are ‘fee-paying’ schools (DBE, 2004).

Half of the participants were men and half were women. While each teacher held the required teaching qualification as per the minimum requirements in South Africa, some had furthered their studies: Teacher D had obtained a Bachelor of Education Honours degree; Teachers C and F had obtained Master of Education degrees. In addition to teaching Grade 10, each teacher also taught at least one other high school grade.

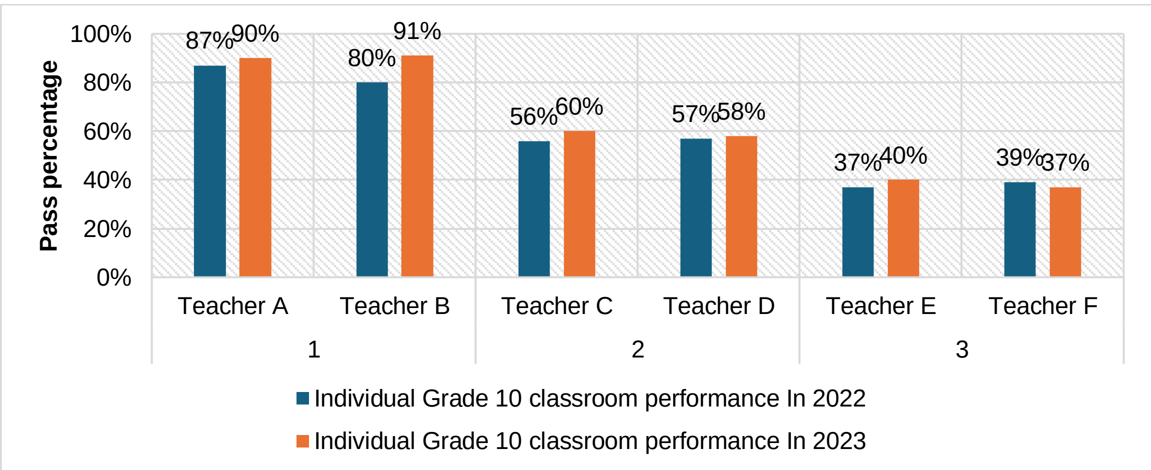
In addition, per the minimum requirements for teacher education qualification in South Africa, the Phases teachers were qualified to teach determined the actual grades they taught. In essence, teachers who taught mathematics from Grades 7 to 12 must have obtained a qualification with an endorsement to teach both the Senior Phase (Grade 7 to 9) and the FET Phase (Grade 10 to 12). Those teaching mathematics from Grades 10 to 12 must have obtained their qualification with the FET Phase endorsement. Only Teachers A, B, C and F matched the endorsement requirements, while Teachers D and E, are identified also teaching Senior phase grades instead of the FET grades only.

In terms of the number of years the teachers had taught Grade 10, Teachers A and D had taught Grade 10 (15 years), while Teacher B had taught Grade 10 for the shortest period (3 years). Of the six teachers, four were African and spoke isiZulu (3) or isiXhosa (1) as their home languages; one was Coloured and spoke Afrikaans; and 1 was White and spoke English as their home language. As English was used as the language of teaching and learning in all lessons, only one of the teachers taught in their home language. Teacher A and B, appear to be teaching fewer learners compared to the other teachers.

Error! Reference source not found., below, shows the combined performance results of Grade 10 learners taught by the participating teachers in 2022 and 2023.

Figure 7

Mathematics final exam results of Grade 10 learners taught by the participating teachers in 2022 and 2023



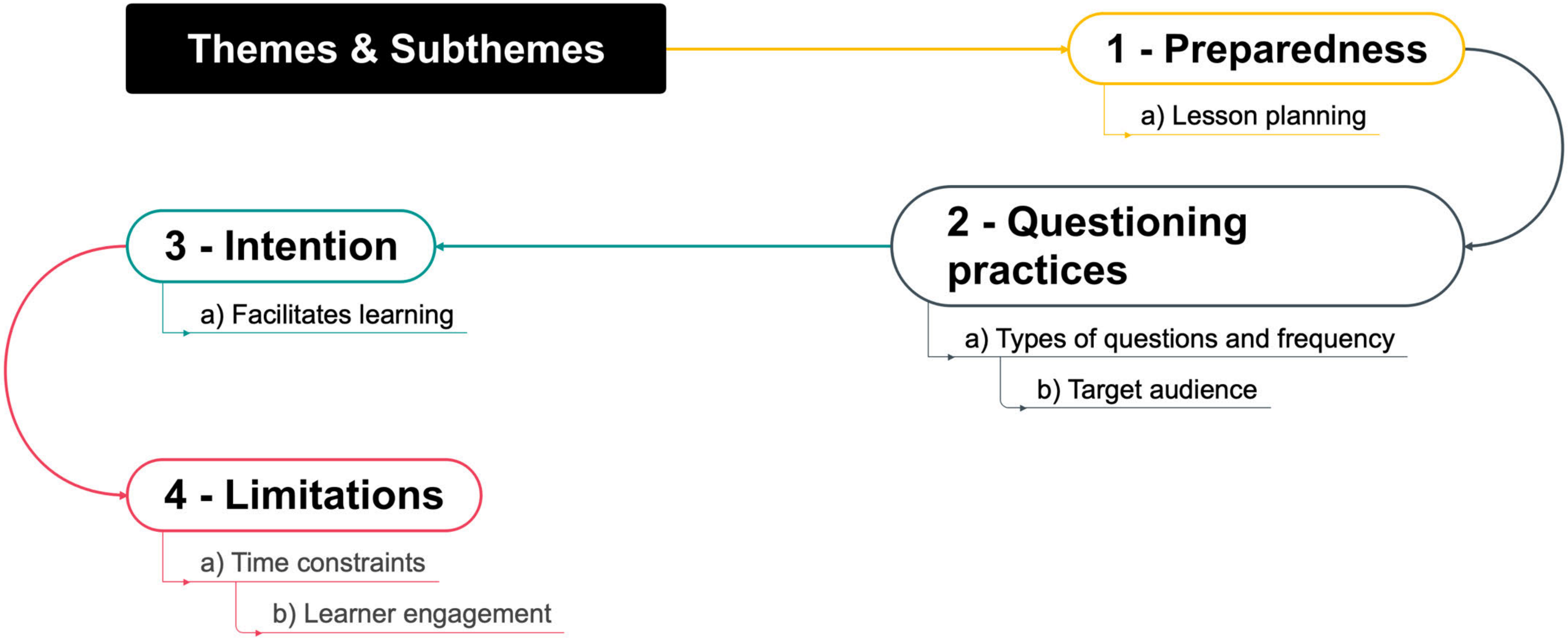
The final examination results shown in Figure 7 reveal differences in learner performance. At School 1, the learners of Teachers A and B achieved an average of 83.5% (2022) and 90.5% (2023), while at School 2, the learners of Teachers C and D achieved an average of 56.5% (2022) and 59% (2023), and at School 3, the learners of teachers E and F achieved an average of 38% (2022) and 38.5% (2023). When focusing on the results of the learners of individual teachers, there was improvement from 2022 to 2023 for all teachers' learners except for Teacher F, whose cohort of learners in 2023 performed lesser than the 2022 cohort'. The learners taught by Teachers A and B appeared to perform better than the learners taught by the other four teachers. I could not determine any relationship or correlation between the performance results and the number of years each teacher has been teaching, or their qualification.

5.3 Emergence of themes and sub-themes from the data

The five phases of the thematic analysis involved generating and refining themes. This analytic technique yielded four themes. These themes, and their sub-themes, are depicted in a thematic map, in Figure 8.

Figure 8

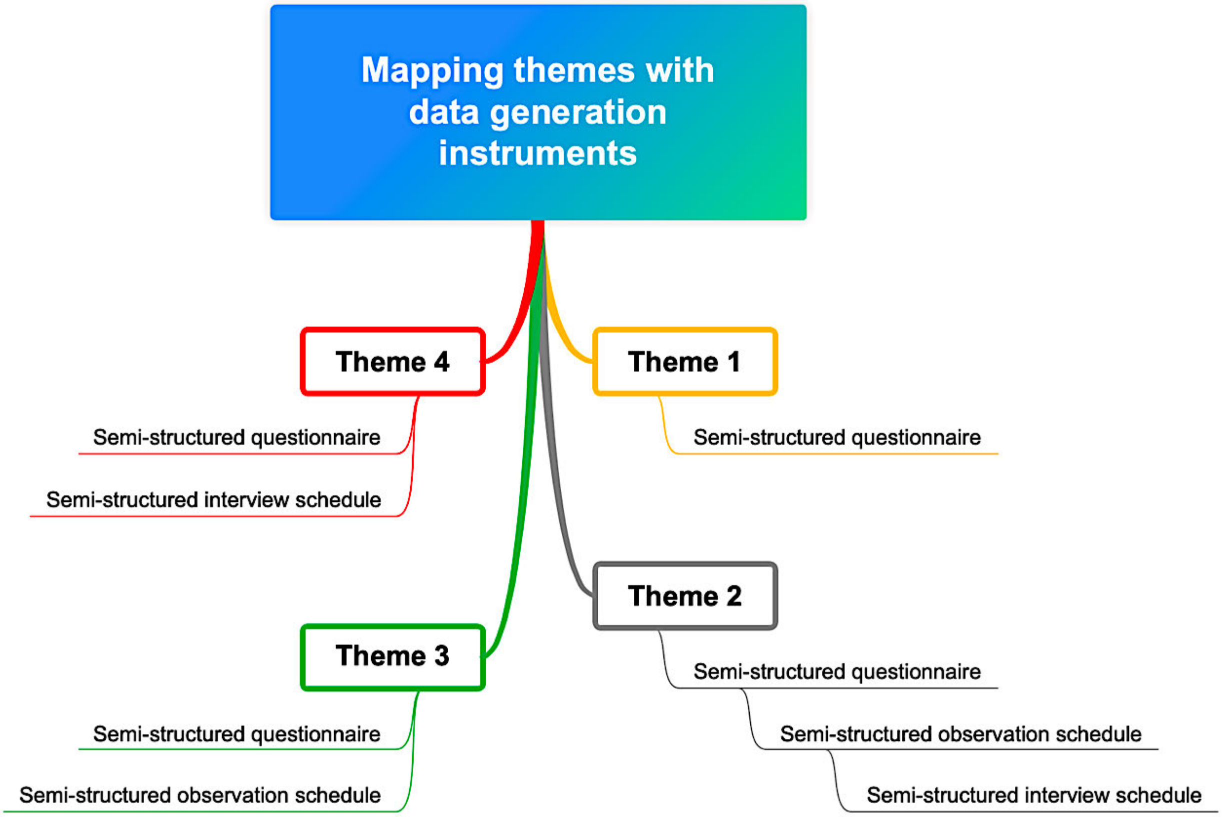
Map of themes that emerged from the data



The four themes – 'preparedness', 'questioning practices', 'intention', and 'limitations' – emerged from the three data generation instruments. The categories were determined with the aim of answering the four research questions. To some extent, a single theme might provide answers to more than one research question. Each theme was mapped to the corresponding data generation instruments for the presentation and analysis of data. For reference, Figure 9 shows how each theme is mapped to the data generation instrument.

Figure 9

Map of themes with data generation instruments



5.4 Theme 1: Preparedness

Theme 1 – along with its subtheme 'lesson planning', aims to provide answers to the third research question: What types of questioning practices can teachers use to enhance critical thinking in grade 10 mathematics classrooms?

One subtheme: lesson planning (considers teachers' views about preparing questions).

5.4.1 Lesson planning

Questions 4.1 and 4.2 of the questionnaire required teachers to respond on how they plan their questions. Four teachers (A, B, C, and D) answered that they prepare questions before, during, and after the lesson. Teachers A and C supporting their responses as follows:

> **Teacher A**: Before the lesson, I plan a set of questions included in the lesson plan. During the lesson, I usually add other questions based on learners' responses and then after, I prepare others for the next lesson to remind them what they learned the previous lesson. To see if learners do remember.

> **Teacher C**: Learner participation is maximised when questions are planned properly during all stages of lesson planning. Most important, I include some in the lesson plan...

The responses of these teachers indicated that they acknowledged that planning questions is necessary for all lesson stages. Per their reasons, Teacher A emphasised lesson planning and that questions must be included. Furthermore, he acknowledged that questions may still be added for other lesson stages. Teacher C emphasised that he included questions in his lesson plan covering all lesson stages. He also shared the same notion expressed by Teacher A, of including them in the lesson plan.

The other two teachers, Teacher E and F, responded differently to Question 4.1: They selected during and after the lesson. Teacher E explained:

> **Teacher E**: I prefer asking learners questions during the lesson because I want to draw their interest and capture their thoughts.

The response by Teacher E indicated that planning questions may not be as necessary during lesson planning as during the lesson, because that is where she can draw learners' interest.

5.5 Theme 2: Questioning practices

Theme 2 aims to provide answers to the first three research questions: What questioning practices are currently used by teachers in Grade 10 mathematics classrooms?; What types of questioning practices can teachers use to enhance critical thinking in Grade 10 mathematics classrooms?; and How do teachers use questioning practices to enhance critical thinking in grade 10 mathematics classrooms?

Two subthemes emerged: type of questions (considers the different questions teachers ask their learners) and target audience (focuses on the learner audience at which the questions are directed).

5.5.1 Types of questions

The data presented and analysed in this subsection is based on Table 7 as a framework. The aim is to identify the types of questions used by the teachers. Using the applicable RBT levels, I will also categorise the questions as either lower-order or higher-order.

Table 7

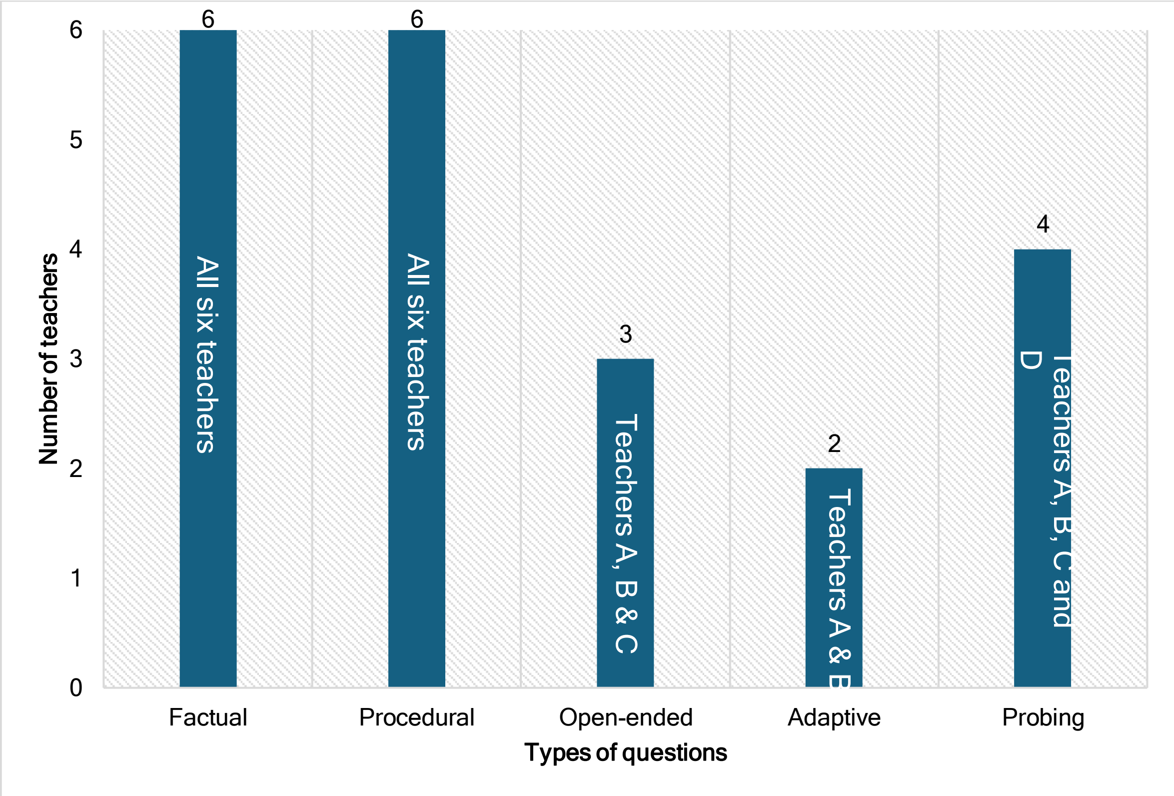
Question types adapted from Boaler and Broadie (2004, p. 777) and Hess et al., (2009, p. 8)

<table>
<tr><th>Question type and description</th><th>Some of the examples</th><th>Applicable RBT levels with verbs</th></tr>
<tr><td>Factual: Recall facts (Allows the correct use of mathematical language)</td><td>What is the equation called? What is the value of the principal amount? What is the equation called?</td><td>Remember, Understand: Recall, recognise</td></tr>
<tr><td>Procedural: Rehearse known routine procedures (Allows the use of correct mathematical language)</td><td>Can you solve this problem using the simple interest formulae?</td><td>Remember, Understand: Describe, explain the steps, apply the steps</td></tr>
<tr><td>Open-ended: Lobby contributions from the learners</td><td>What do you think about this? How did you get this answer?
Why is this different?</td><td rowspan="3">Apply, Analyse, Create and Evaluate: Compare, generalise, analyse, apply concepts to non-routine, retrieve, brainstorm, design, formulate, verify, justify, devise, organise, generate conjecture</td></tr>
<tr><td>Adaptive: Points to relationships between mathematical ideas</td><td>Where else can we apply this?
When can this formula satisfy the conditions?</td></tr>
<tr><td>Probing: Ask learners to articulate, clarify or elaborate their responses.</td><td>Can you explain your answer?
How is it different?</td></tr>
</table>

I begin by presenting and analysing the data from the semi-structured questionnaire, based on Question 6. The participants were asked to choose the specific types of questions they ask learners during mathematics lessons in their Grade 10 classrooms. Each participant was allowed to choose more than one question type that they use.

Figure 10

Specific question types teachers said they use in their Grade 10 mathematics classrooms



The data in Figure 10 shows variations among the teachers' views. There are similarities and differences in the choices they selected. For the similarities, all the participants chose factual and procedural question types. Differences were found for three question types: open-ended, adaptive and probing. While Teachers A and B selected all five types of questions, Teacher C chose four of the 5 options – excluding adaptive questions, Teacher D chose three – excluding open-ended and adaptive questions, and Teachers E and F only chose only two: factual and procedural question types.

All six participants were then observed teaching their learners lessons on financial mathematics, and data was recorded in three stages: lesson introduction, lesson development and lesson conclusion.

At the start of their lessons, each teacher introduced the lesson during the first 10 minutes of the lesson and then started asking their learners questions. Learners were also instructed to refer to their lesson notes found in their textbooks and other supplementary revision materials. Quoted below are some of the question types asked by Teachers A, B, and F during the lesson introduction:

Teacher A's set of questions:

1. Does anyone among you remember the definition of the terms simple interest and compound interest?
2. What is the formula for using calculating simple interest?
3. What is the formula for calculating compound interest?
4. Solve the problem: How many months are there annually?

(Total number of question asked by Teacher A = 7)

Teacher B's set of questions:

1. Ukhona (Is there anyone) okhumbulayo (who remembers) ukuthi (that): what do the terms accumulated amount, principal amount, interest and period mean?
2. Can you identify the difference between the compound and simple interest formulae?
3. Anyone ongangichazela (can explain) what are the steps to calculate simple interest?

(Total number of question asked by Teacher B = 6)

Teacher F's set of questions:

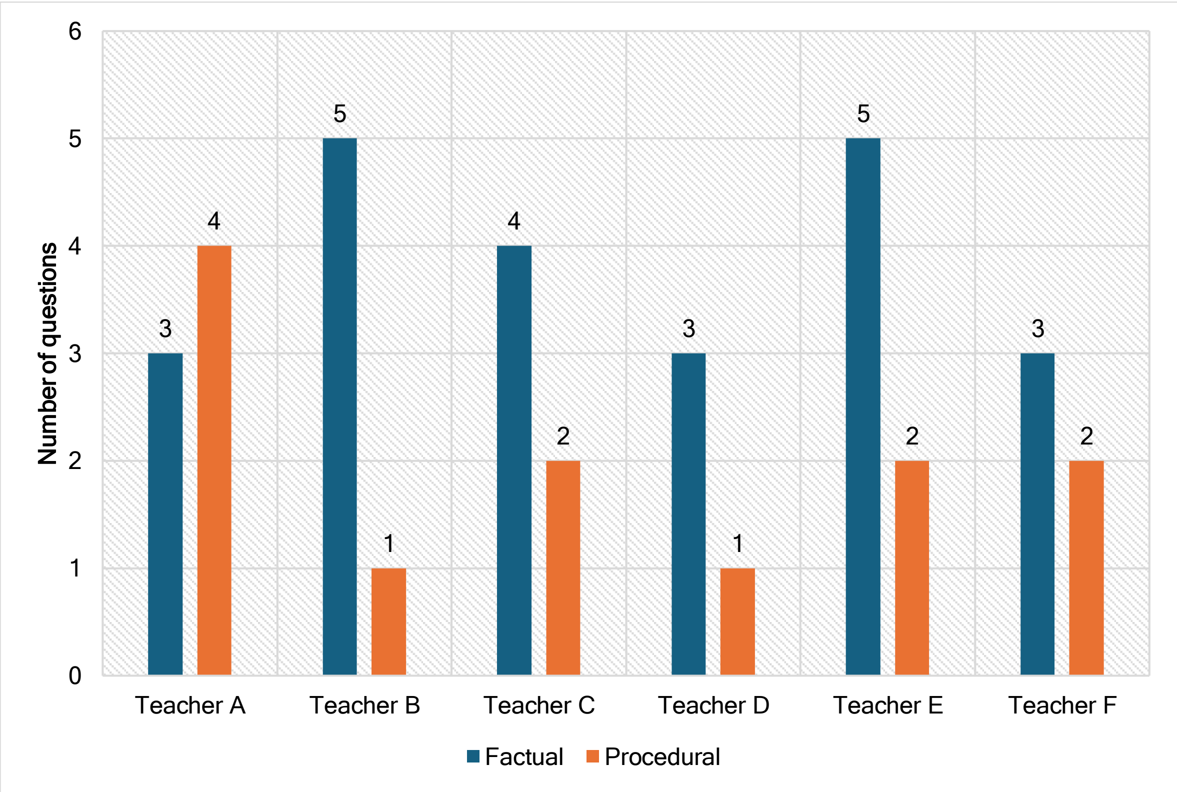
1. Steve (a learner): tell me what does the term compound interest mean? Can you tell us the definition?
2. Class: if someone says to you, 'interest is calculated annually', can you explain what the term 'annually' means?
3. Anyone who can remind me how do we convert interest?

(Total number of question asked by Teacher F = 5)

Teacher A, in his first question, required learners to recall the definition of the terms. This implies that the teacher had once defined the two terms, hence learners are now required to recall the definitions. In her third question Teacher B asked learners to explain the steps to calculate simple interest. In this question, learners were required to recall and rehearse the steps for calculating simple interest. In his third question, Teacher F required learners to remind him how to convert interest. As with Teacher B's third question, Teacher F's question requires learners to recall the procedure of converting interest. It must be noted that the other teachers asked similar questions. All question types asked by the participants required learners to recall facts and, to some extent, perform certain procedures. These question types resemble the first two levels of the RBT. Figure 11 presents a summary of the use of questions during the lesson introduction.

Figure 11

Total number of questions per question type asked by the participants during the introduction stage of the lesson



The results presented in Figure 11 show that, collectively, a total of 35 questions were asked by the participants to the learners. Teacher E asked the most questions (7); Teacher D asked the least questions (4). With the exception of Teacher A, all teachers asked more factual questions than procedural questions.

During the development stage of the lesson, which was between 10 minutes to 45 minutes in length, the participants continued to ask learners questions, which were recorded. Quoted below are some of the question types asked by Teachers A, D and F asked their learners during the development of their lessons:

Teacher A's question types and learner responses:

1. Why do banks charge interest on loans and offer interest on savings?
   (Eric): I think banks make money by lending it to people, so they charge interest.
2. Excellent Eric! Now, can anyone tell me how do you think simple interest and compound interest differ in the way they grow money?
   (Andile): Sir**,** Simple interest adds the same amount every time, while compound interest keeps increasing because it adds interest ngaphezu (over) kwe (the) interest.
3. That's insightful! Let's apply this to a scenario. Suppose you invest R500.00 at an interest rate of 8% per year, calculated using simple interest. How much will you have after three years, Siphumelele?
   (Siphumelele): That's super easy, sir, because we use the formula $A = P(1 + in)$. So I think if we plug values, it will read as: $A = 500\ (1 + 0.08 \times 3)$, which gives us R6 200.00.
4. Well done! Now, let's switch to compound interest. What if the same amount is invested at 8% per annum, compounded annually for three years? How will the calculation change? Can anyone tell me?
   Xolani: Sir! Sir! I know the answer. We use a different formula. We use $A = P(1 + i)^n$, so $A = R500.00(1 + 0.08)^3 = R6\ 200.00$

5. Good! Now tell me, Grade 10s: how does compounding influence long-term investments compared to simple interest?
   Eric: The longer you save, the more interest you get compared to simple interest.

(Overall total number of question types asked by Teacher A = 16)

Teacher A begins by asking his learners the first question using the phrase 'why'. By virtue of this phrase, this is an open-ended question requiring learners to make an evaluation. Upon Eric's response, the teacher continues with the second question, using the phrase 'how', which is another open-ended question requiring learners to make comparisons. Teacher A then adapts his third question based on the responses of the learners. He uses the phrase 'let's apply' and asks Siphumelele to calculate the total amount after three years. In the fourth question, Teacher A continues to adapt his question based on Siphumelele's response. The phrases 'what', 'how', and 'why' continued to prevail in the question types Teacher A is asking. Teacher A used both open-ended and adaptive questions during the lesson development.

Teacher D's question types and learner responses:

1. Can you explain to me the steps you used, Thulani, in your calculation?
   Thulani: I substituted all known values to the formulae that were given. $\mathrm{A} = \mathrm{P}(1 + \mathrm{in})$. $\mathrm{A} = \mathrm{R1\ 000}$, $i = 0,07$ and $n = 3$. Then I got $\mathrm{A} = \mathrm{R1\ 210.00}$
2. Class, do your agree with Thulani?
   Learners: Yes, Madam, we agree.
3. Another question. Class: what does the term accumulate mean to you?
   Thulani: Madam, I think it's when your money grows. It keeps on increasing.
4. Do you all agree, class?
   Learners: Yes!

(Overall total number of question types asked by Teacher D = 10)

Teacher D begins by asking Thulani to list the steps he used in his calculation. Thulani responds by mentioning all the steps he used. In his second question, the teacher asks learners if they agree with Thulani's response. What is significant about this question is that she does not ask learners to give a reason for their response. Even when the learners respond, there are no follow-up questions. In the third question, learners are required to define the term 'accumulate'. Thulani responds with an answer. In question four, she asked learners in the same way she did in the second question, and learners respond that they agree.

The set of questions asked by Teacher D resembles factual and procedural questions. Learners were required to recall and explain their procedures. This pattern continued for the duration of Teacher D's development of the lesson. A total of eight questions were asked by Teacher D during this stage.

Teacher F's question types and learner responses:

1. Which method do you prefer in the given example on page 21 of your notes?
   Pamella: Ma'am, method 2 is easier: there is a formula given.
2. Anyone who can list the steps for me in method 2?
   Most group of learners: Identify the formula will be used, write down using the variables in formulae what you are given and identify what is it you must calculate.
3. Do you promise me, you will not forget these steps?
   Pamella: Mina memu, ngeke ngikhohlwe (I will not forget, Ma'am**)**
4. Anyone who will forget?
   Most group of learners: Lutho memu, siyakuthembisa (No, Ma'am, we promise.)

(Overall total number of question types asked by Teacher F = 8)

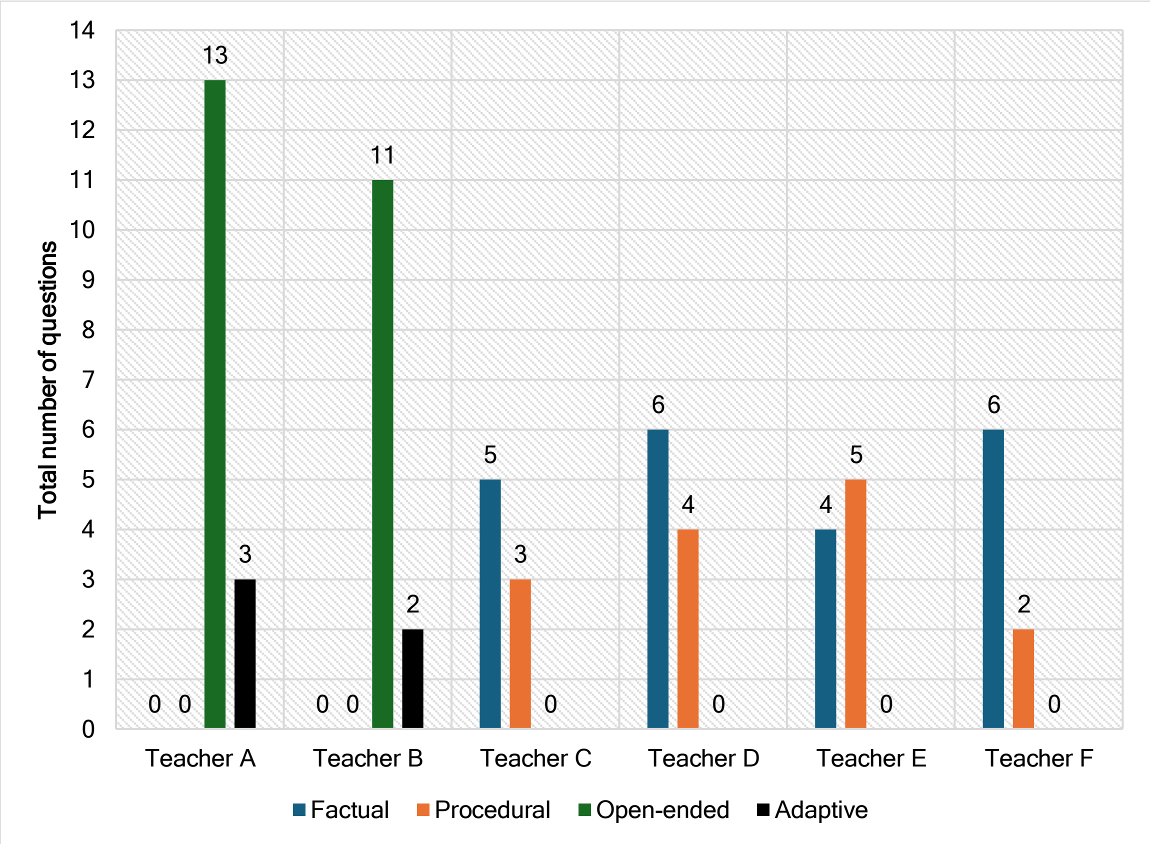
Teacher F begins by asking learners to indicate their preferred method on page 21 of their notes. Pamella responds. No comment or feedback is provided by Teacher F to Pamella's response. In question 2, Teacher F asks learners to list the steps of method 2. The learners respond by listing all three steps. Another following question requires learners to promise they will not forget the steps. Pamella responds, saying she will not forget. Teacher F redirected the question to the entire class, and the learners answer that they will not forget.

The set of questions asked by Teacher F resembles factual and procedural questions. Learners were required to recall and list procedures. This pattern continued for the remainder of Teacher F's lesson development. A total of seven questions were asked by Teacher F during this stage.

Figure 12 presents a summary of questions asked during the lesson development stage.

Figure 12

Total number of questions per question type asked by the participants during the lesson development stage



No probing questions were asked by any of the teachers during the lesson development stage. Collectively, the participants asked a total of 63 questions. **Error! Reference source not found.** shows that Teachers A and B were the only teachers who asked open-ended and adaptive questions. This set of questions resembles levels three and four of the RBT. The other teachers continued asking learners factual and procedural question types. These sets of questions resemble levels one to two of the RBT.

During the lesson conclusion stage, which constituted the last 10 minutes of the lessons, the participants continued to ask learners questions, which were also recorded. Quoted below are examples of the question types Teachers A, C, and E asked their learners during the lesson conclusion.

<u>Teacher A's question types and learner responses:</u>

1. Eric, do you still remember how Siphumelele calculated the simple interest problem from the scenario I gave earlier?
   Eric: Easy, sir: she used the formula for simple interest and substituted all values which differed from what Xolani used.
2. Tell me, Eric: how is it different from the one used by Xolani?
   Eric: With the compound interest formula, sir, the variable n is always written in exponential form, hence Xolani substituted correctly.
3. Good. Class: now that you have learned how to calculate compound and simple interest related problems, how will you advise your parents at home about investments?
   Melusi: They must first check whether the bank offers simple or compound interest. Even then they must check the interest rate applicable. A high interest rate does not always mean a good return as this depends on the whether a simple or compound interest formulae is used.

(Overall total number of question types asked by Teacher A = 9)

In his first question, Teacher A requires Eric to recall and explain how Siphumelele calculated the simple interest. Upon Eric's reply, Teacher A probes Eric to explain how it differs from Xolani's response. In question 3, the teacher ask learners an open-ended question, requiring learners to make comparisons between simple and compound interests. Teacher A continues to asking questions, switching between factual, procedural, and open-ended questions, for the remainder of the lesson conclusion. The factual and procedural questions resemble levels one and two of the RBT, while the open-ended questions resemble levels three to six of the RBT.

<u>Teacher C's question types and learner responses:</u>

1. Who can tell me why the compound interest is different from the simple interest? Anyone can tell me, Class?

Cynthia: Madam, isn't that with simple interest, inzalo (interest) stays the same for the period of the investment, okuhlukile (which is different) when we apply the compound interest formula?

2. Yes. But tell me: how is it different?

   Cythnia: With compound interest, it's interest over interest.

3. Thabani do you still remember the formulae for calculating simple interest?

   Thabani: Yes, I do.

(Overall total number of question types asked by Teacher C = 10)

In her first question, Teacher C, using an open-ended question, requires learners to compare compound and simple interest. Upon Cynthia's reply, Teacher C proceeds to question 2, probing Cynthia to explain how simple interest is different. In question 3, she switches to a recall question. This exchange continues for the remainder of the lesson conclusion. The factual and procedural questions resemble level one of the RBT, while the open-ended questions resemble levels three to six of the RBT.

<u>Teacher E's question types and learner responses:</u>

1. So today we learned about inflation, compound interest and simple interest. Anyone who can tell what was the formula for calculating simple interest?

   Learners: A is equal to P into open brackets – one plus i multiply by n – close brackets: $A = P(1 + in)$

2. You will not forget it?

   Learners: No sir!

3. Good, now tell the formula for calculating compound interest.

   Learners: A is equal to P into open brackets – one plus i – close brackets, then raise to exponent n: $A = P(1 + i)^n$

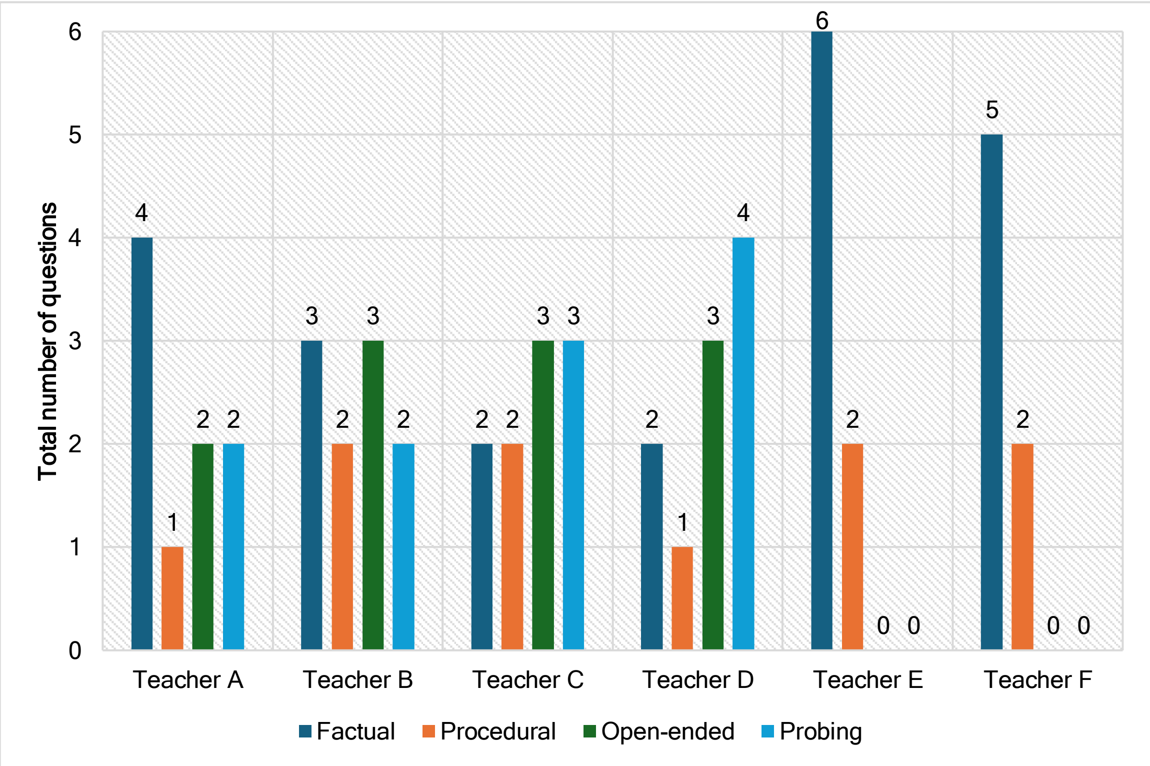
(Overall total number of question types asked by E = 7)

In her first question, Teacher E asks learners to recall the formula for calculating simple interest. In question 2, she asks if learners will not forget. In question 3, she asks a procedural question. This exchange continues for the remainder of the lesson conclusion. The factual and procedural questions resemble level one of the RBT.

Figure 13 summarises the question types used during the conclusion stage of the lessons.

Figure 13

Total number of questions per question type asked by the participants during the conclusion stage of the lessons



Collectively, the participants asked a total of 54 questions. Figure 13 shows that Teacher E asked the most factual questions, followed by Teacher F. As shown in Figures 12 and 13, Teachers E and F asked only factual and procedural questions. These question types exceeded their open-ended and probing questions during the lesson conclusion stage. Also, Figure 12 shows that Teachers C and D did not ask any open-ended or probing questions during the lesson development stage, while 13 shows that they asked more questions of these types than factual and procedural questions during the conclusion stage.

Next, a comparison was made between the question types the participants reported that they used in the questionnaire and the question types they used during the observed lessons. The summary in Table 8 shows that responses were consistent for some teachers and not for others. Teachers E and F were observed using the same types of questions in their lessons that they had indicated that they used on the questionnaires. However, Teachers A and B use adaptive questions during their observed lessons, which they had not reported, while Teachers C and D used probing questions during the lessons, which they had not reported.

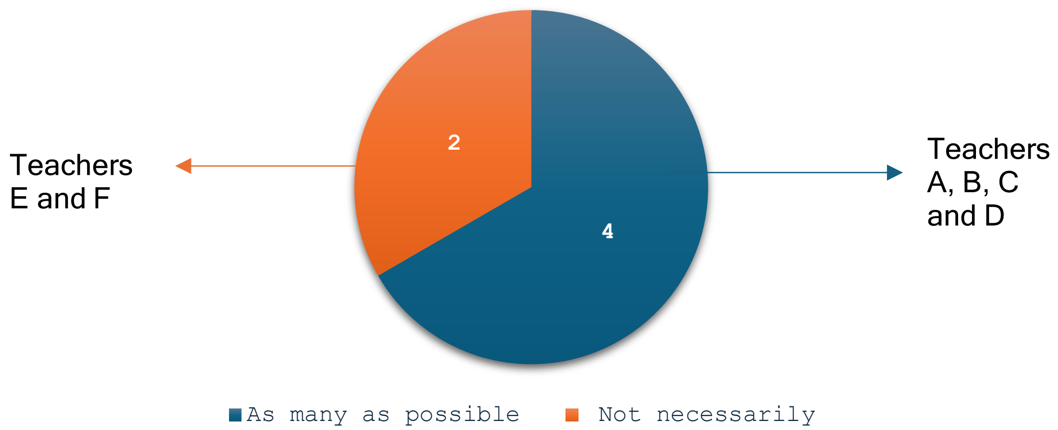
Table 8

Comparisons between the specific types of questions teachers said they ask and the current types of questions they were asking learners

Teacher	Specific question types, teachers said they ask learners	Current question types, teachers they ask learners
Teacher A	Factual, Procedural, Open-ended, Probing	Factual, Procedural, Open-ended, Adaptive, Probing
Teacher B	Factual, Procedural, Open-ended, Probing	Factual, Procedural, Open-ended, Adaptive, probing
Teacher C	Factual, Procedural, Open-ended	Factual, Procedural, Open-ended, Probing
Teacher D	Factual, Procedural, Open-ended	Factual, Procedural, Open-ended, Probing
Teacher E	Factual, Procedural	Factual, Procedural
Teacher F	Factual, Procedural	Factual, Procedural

Figure 14

The frequency with which teachers ask questions using the phrases 'how', 'why', 'when' and 'what if...



On the questionnaire, the participants were asked about the frequency with which they used the phrases 'how', 'why', 'when', and 'what if...' in their questions. The results are summarised in Figure 14. Four teachers responded that they use the phrases 'how', 'why', 'when' and 'what if...' as many as possible while the other two participants responded with 'not necessarily'. These results mean that the participants who selected as many as possible regard these phrases as necessary, while the other two participants regard them as not. The same pattern was observed in the classroom. Teachers A, B, C and D used some of the phrases, while Teachers E and F did not.

5.5.2 Target audience

Question 2 of the questionnaire asked the teachers what audience they target when asking learners questions and offered three choices: individual learners, whole class, or a certain group of learners. Teachers A, B, and C indicated that they ask individual learners. Teachers D, E, and F indicated they target the whole class. The reasons they provided varied. Teachers A and B gave the following explanations:

> **Teacher A:** I prefer asking individual learners questions rather than the entire class. I can recognise learners who struggle.

> **Teacher B**: In my teaching experience, I have seen that learners tend to respond collectively, only to find that others are echoing responses without understanding. In this way, I see which learners are struggling.

These excerpts show that both teachers expressed similar concerns about struggling learners; this informed their strategy to target individual learners.

The teachers who indicated that they target the whole class when they ask questions also shared a similar rationale for this approach: asking the whole class gave either equal opportunity to all learners to respond to the question, or saved time, as shown in these examples:

> **Teacher D:** All learners have the same opportunity to respond to the questions.

> **Teacher F**: It is easier to direct questions to a larger group and saves time.

During the lesson observations, the teachers were observed targeting different audiences with their questions. While none of the participants indicated on the questionnaire that they targeted a certain group of learners when they asked questions, I observed Teacher F posing some questions to individual learners and others to a certain group of learners. Teachers A, B, C, D and E targeted the same learners they had indicated in the questionnaire. This might indicate that these three teachers were aware of their target audiences.

5.6 Theme 3: Intention

Theme 3 aims to provide answers to the fourth research question: Why is there a need for teachers to use questioning practices to enhance critical thinking in Grade 10 mathematics classrooms?

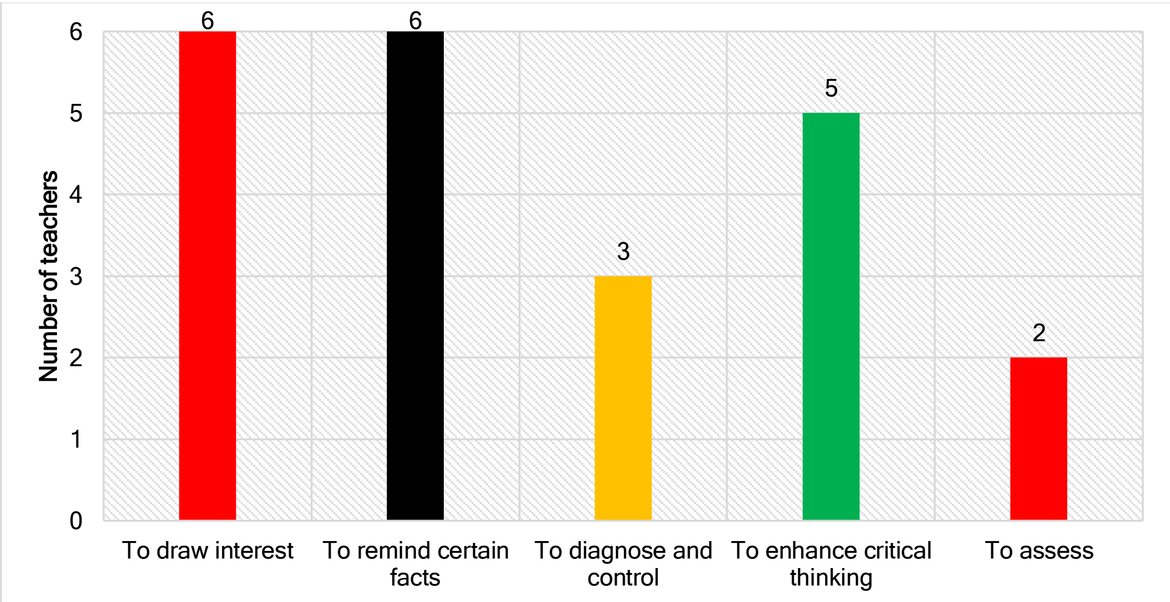
One sub-theme is considered: facilitates learning (focuses on the participants' views for asking questions).

5.6.1 Facilitates learning

Each teacher had selected more than one choice for the reasons they ask learners questions. Teachers A and B selected all the options. Teachers, D, E and F chose the same three options: 'to draw interest', 'to remind certain facts', and 'to enhance critical thinking'. These results show that each teacher identified that questioning may serve a particular purpose. The results are summarised in Figure 15.

Figure 15

Participants' stated intention for asking learners questions



For question 5 of the questionnaire, all six teachers indicated that they supported encouraging other teachers to ask questions aligned with the all levels of RBT during lessons. Of significance were the reasons they provided. The teachers emphasised the link between improved academic performance and the question types aligned per the Revised Bloom's Taxonomy levels. Following are excerpts from Teachers B, C, and F's responses:

Teacher B: Absolutely, it is the best way to measure learners' understanding. Questions must be posed at different levels.

Teacher C: In doing so, learners will gain skills to respond to questions on various levels, thus passing their examination.

Teacher D: I suppose it is important since learners are exposed to different levels of questions during an assessment. Usually, when they perform better in assignments, tests and examinations, it's because they were exposed to critical thinking questions.

These excerpts reveal that the teachers associated asking learners questions across all levels of the Revised Bloom's Taxonomy with improved academic performance. Teacher B asserts that questioning at different cognitive levels is the best way to measure learners' understanding, indicating that using all of the levels of RBT enhances learners' critical thinking. Thus, they perform well during an assessment. Teacher C supports this notion by mentioning that learners will gain skills to respond to questions on various levels, thus passing their examinations. This suggests that exposure to different question types per all levels of RBT enhances learners' ability to pass assessment tasks successfully. Teacher D further highlights the role of critical thinking questions in assessments, stating that learners who perform better in assignments, tests, and examinations often do so because they have been exposed to critical thinking questions. Teacher D's view suggests that incorporating the Revised Bloom's Taxonomy levels in questioning can translate into improved assessment performance. The teachers agreed that using questions at different levels of the RBT contributes to learners' ability to excel in assessments.

Furthermore, during interviews, the teachers reported similar views about this. Following are excerpts of the responses of Teachers B, C, and D.

Teacher B: Thank you, Mr Zondo, for that question again. I will always recommend that all teachers pose these questions during their lessons in mathematics classrooms, simply because they can measure if their learners understand a concept. Also, they will know if learners have gaps or misconceptions. In this way, teachers can better prepare their lessons to achieve the desired outcomes.

> **Teacher C**: Absolutely, the same teaching practice used by my teachers while I was still a learner worked, and I am using the same procedure, and it is yielding positive results. Hence, I will recommend the same for other teachers during their lessons in mathematics classrooms.

> **Teacher D:** I will recommend on the basis that it allows learners to provide justification and arguments on their thinking. In turn, it allows them to be confident in the subject manner.

In the observation of the teachers' lessons, however, they did not use questions matching all the levels of the Revised Bloom's Taxonomy.

Question 7 on the questionnaire asked participants relating to the number of times they think the questions with phrases 'how', 'why', 'when' and 'what if...' must be asked learners. Half of the teachers responded, 'As many as possible'. These were Teachers A, B, and C. This indicates that these teachers acknowledge the need to use the phrases when asking learners questions, based on the implication that these phrases enhance learners' critical thinking questions.

The other half of the teachers, Teachers D, E and F selected 'Not necessarily'. This differs significantly from what the previous three teachers had indicated. 'Not necessarily' is regarded as contrasting to the category 'As many as possible'. This implies that the three teachers deem asking questions using phrases unimportant.

Teacher A stated the following caveat:

> **Teacher A**: They should be asked more; however, there's a different cohort of learners each year. In other years, you may have learners who are struggling. Hence, they tend not to respond to the questions.

This response indicates a possible limitation. While Teacher A recognises that the phrases must be used more, they also argue that learners tend to struggle to respond to questions with these phrases, highlighting the different between the learner cohorts each year, which could mean certain learner cohorts are able to respond to questions using these phrases, while other learner cohorts struggle to respond to these questions. Teachers D and E gave similar responses.

Teacher D: But learners tend to respond less on these questions.

This could mean that Teacher D may experience a similar situation to that described by Teacher A. The two teachers, Teacher E and F, whose reasons seem to lean in the same position as the other four teachers, also highlight the difficulty that comes with the use of these phrases when questioning learners.

Teacher E: Most learners tend not to respond to these questions because of difficulty.

This implies that all these teachers, to some extent, held similar views on the limitations that come with the use of the phrases. Hence, based on the six teachers' responses to Question 9, I drew the same conclusion about the level of difficulty associated with these questions. Using of the 'how', 'why', 'when' and 'what if...' phrases, this time was related to assessment. A similar recording was noted, with Teachers A, B, C and D all choosing the 'Yes' choice, while Teachers E and F opted for the 'No' option.

During classroom observations, Teachers A, B, C and D did use the phrases. Of significance was that when these teachers asked learners questions with these phrases, very few learners raised their hands to respond. Most learners raised their hands when questions were asked within the lower levels of the RBT, and numbers decreased for the higher the taxonomy levels. These observations could, to some extent, confirm the challenges that the teachers were referring to, such as learners struggling to respond.

5.7 Theme 4: Limitations

Theme 4 also aimed to provide answers to the fourth research question. There were two sub-themes, time constraints (focussing on the teachers’ views on time taken when asking learners questions) and learner engagement (focussing on teachers' views about the extent to which learners engage when asked questions).

5.7.1 Time constraints

One of the teachers suggested in a response on the questionnaire that time is a crucial factor to take into account when asking learners questions.

Teacher F: It is easier to direct questions to a larger group and saves time.

In this response, Teacher F seems to be comparing the time taken when asking a large group of learners versus asking a smaller group or individual learners. In other words, based on his response, asking individual learners takes more time than asking a larger group.

5.7.2 Learner engagement

Each teacher responded to the question on learner engagement by explaining their views about opportunities that come with critical thinking. This is evident from the reasons provided by Teachers A and E in the following selected interview excerpts:

Teacher A:

> **Teacher A:** Each question posed to learners must always be used to enhance their critical thinking. Is it not that we want our learners to be critical thinkers so that solving problems is not a challenge?
>
> **Researcher probing response**: So you are saying that there is a connection between enhanced critical thinking and learners solving problems?
>
> **Teacher A**: Mr Zondo, let me make you an example: If I ask my learners a question such as 'Why will this procedure not always work? I intend to know the extent at which learners understand the restrictions of that procedure. Different if ask my learners: Do you know the formulae of this procedure? Them replying yes sir or no sir, for me, does not imply you are cognitively engaging with me. So I think with critical thinking learners become more cognitively engaged, hence they successfully solve problems encountering less to no difficulties

Teacher E:

> **Teacher E**: Mr Zondo, these days, learners tend not to respond to questions during lessons. While I know that critical thinking comes with benefits such as increased engagement and it promotes a learner centred approach, they just don't engage. Believe me, Mr Zondo, I always have encouraged them during informal discussion outside the classroom, but still they are reluctant.

In the interview excerpts, Teachers A and Teacher E discuss their views on opportunities that enhanced critical thinking provides, particularly related to learner engagement. On the one hand, Teacher A highlights that posing questions designed to stimulate critical thinking allows learners to engage cognitively, making problem-solving less difficult for learners. This is evident when Teacher A explains, *"Each question posed to learners must always be used to enhance their critical thinking" and* further illustrates this by contrasting questions that prompt deep reasoning versus those requiring simple recall. His view emphasises how critical thinking encourages learners to actively engage with content rather than passively responding with yes-or-no answers. Teacher A concludes that when learners think critically, they encounter fewer difficulties in problem-solving, suggesting a direct link between engagement and intellectual independence. Teacher E acknowledges that while critical thinking is beneficial for engagement and promotes a learner-centred approach, actual learner participation remains a challenge. Teacher E states, *"I know that critical thinking comes with benefits such as increased engagement… but they just don't engage,"* indicating a gap between the potential for engagement and the reality of learner reluctance.

Despite encouraging discussions in the classroom, Teacher E finds that some learners remain hesitant to participate, suggesting that engagement is not guaranteed simply by fostering critical thinking. While both teachers recognise the link between critical thinking and engagement, Teacher A focuses on its success in facilitating problem-solving. In contrast, Teacher E highlights the challenge of ensuring active participation. The images presented in Figures 16 and 17 show how learners engaged during the lesson observations in Teacher A and E's classrooms:

Figure 16

Learners engaging on a task in Teacher E's classroom

Figure 17

Learners engaging on a task in Teacher A's classroom

On one hand, in Teacher E's classroom, some learners appear to be working either individually or in pairs. Significantly, another learner seems to be looking backwards, focusing on what others were showing him, which could imply the learner was hesitant to engage in the task. On the other hand, in Teacher A's classroom, learners appear to be working individually, with one learner whose hand is raised, awaiting a response from Teacher A.

5.8 Conclusion

This chapter presented profiles of the participants. The four themes that emerged from the data were described and were used as a framework for the presentation and analysis of data. All the research findings that were presented were extracted from the three data generation instruments. These findings were shaped by the Revised Bloom's Taxonomy and the Critical Thinking Model. The next chapter discusses the main research findings in relationship to the four research questions.

CHAPTER 6: DISCUSSION OF FINDINGS

6.1 Introduction

This chapter presents the discussion of findings of the question types and practices used by the participants when asking learners questions. Teachers' question types are examined to determine how they align with the levels of the Revised Bloom's Taxonomy in terms of whether they classify as lower-order or higher-order questions, to determine the extent to which they promote the development of learners' critical thinking. Furthermore, Duron et al.'s (2006) Critical Thinking Framework is used to determine if the practices used by the teachers when preparing and asking learners questions enhance learners' critical thinking. For reference, Table 9 shows the categories of cognitive ability per question type.

Table 9

Category of cognitive ability for question types adapted from Boaler and Broadie (2004, p.777) and Hess et al., (2009, p.8)

Category of cognitive ability	Question type and description	Examples	Applicable RBT levels (with verbs)
Lower-order	Factual: Recall facts (Allows the correct use of mathematical language)	What is the equation called? What value for the principal amount?	Remember, Understand: Recall, recognise
	Procedural: Rehearse known routine procedures (Allows the use of correct mathematical language)	Can you solve this problem using the simple interest formula?	Remember, Understand: Describe, explain the steps, apply the steps
Higher-order	Open-ended: Lobby contributions from the learners	What do you think about this? How did you get this answer? Why is this different?	Apply, Analyse, Create and Evaluate: Compare, generalise, analyse, apply concepts to non-routine, retrieve, brainstorm, design, formulate, verify, justify, devise, organise, generate conjecture
	Adaptive: Points to relationships between mathematical ideas	Where else can we apply this? When can this formula satisfy the conditions?	
	Probing: Ask learners to articulate, clarify or elaborate their responses.	Can you explain your answer? How is it different?	

The presented Table 9 categorises cognitive abilities into lower-order and higher-order skills, mapping specific question types, examples, and the associated RBT level for each category. The lower-order category include factual and procedural questions both primarily aligned with RBT's Remember and Understand levels. The higher-order category encompass the open-ended, adaptive and probing questions. These target the higher RBT levels of 'Apply', 'Analyse', 'Create', and 'Evaluate', employing verbs like compare, generalise, analyse, justify, formulate, devise, and generate conjecture to foster deeper, non-routine thinking. The framework thus provides a structured approach for designing questions that progressively develop cognitive complexity.

By integrating insights from empirical research, this discussion presents a comprehensive analysis of teachers' questioning practices, identifying strengths and gaps in the six teachers' questioning practices. The final section introduces a new integrated model that combines the Revised Bloom’s Taxonomy with Duron et al.'s (2006) Critical Thinking Framework to offer a structured guide for improving questioning practices to enhance learners' critical thinking in mathematics classrooms.

The following discussion is presented using the themes that emerged from the data that were presented and analysed in CHAPTER 5.

6.2 Theme 1: Preparedness

Teachers A and B indicated the importance of planning questions, both prior to the lesson and during the lesson. This aligns with the first step of Duron’s et al., (2006) CTF, suggesting that teachers need to design and plan a set of questions they will ask learners per the learning objectives. Planning questions before and during a lesson is essential for effective mathematics instruction, as it allows teachers to guide learner thinking, diagnose misconceptions, and facilitate meaningful discussions. Pre-planned questions help ensure that lessons align with learning objectives and promote conceptual understanding rather than mere procedural fluency (Boaler, 2019). Research highlights that well-structured questions scaffold learning by progressively increasing cognitive demand, starting with lower-order recall and moving toward higher-order problem-solving and reasoning (Sullivan et al., 2020). Thus, by planning questions in advance, teachers can anticipate potential learner difficulties and prepare appropriate prompts to address them. However, reliance on pre-planned questions alone may limit opportunities for spontaneous exploration of learners' ideas, reinforcing the need for in-the-moment questioning adjustments during lessons (Yang, 2017).

The dynamic nature of classroom interactions requires teachers to continuously adapt their questioning strategies based on learners' responses. Within-lesson questioning allows teachers to probe learners' reasoning, clarify misconceptions, and extend discussions in real-time (Kazemi & Stipek, 2018). Research by Ingram and Elliott (2019) suggests that responsive questioning, where teachers adjust their questions based on learners' mathematical thinking, enhances learner engagement and fosters deeper conceptual understanding. This adaptive approach supports formative assessment practices, enabling teachers to gauge learner comprehension and modify instruction accordingly. Thus, while pre-planned questions provide a structured foundation for the lesson, in-the-moment planned questioning also ensures that teaching remains flexible and responsive to learners' evolving needs, ultimately leading to a richer learning experience in mathematics classrooms (Marzano, 2017).

6.3 Theme 2: Questioning practices

This section discusses the question types and practices teachers used and the audience they targeted when they asked questions. Both the RBT levels and the CTF (steps 2 to 5) are used in identifying the category of cognitive ability the questions target and the extent to which teachers' questioning practices enhance learners' critical thinking. During classroom observations, teachers were observed at different stages of the lesson.

6.3.1 Factual questions

Factual questions engage the 'remember' and 'understand' levels of the RBT. They prompt learners to retrieve or recall knowledge, definitions, and previously learned information. They are identified with the lower-order cognitive category. Before the classroom observations, the participants were asked if they ask learners factual questions. All six teachers responded that they do. All participants were observed asking factual questions during the lesson. Teachers A, B and C targeted individual learners, while teachers D, E targeted their whole class, and Teacher F targeted individual learners and a certain group of learners.

All six teachers asked factual questions to check learners' recall of key mathematical concepts such as simple and compound interest, interest rates, period, and principal amounts.

The participants asked learners a total of 66 factual questions. These were spread out between lesson stages: introduction(23), development(21) and conclusion (22). Asking factual questions during the lesson introduction aligns with a study by Booth et al. (2017) that found that factual questions helped establish foundational understanding at the start of a lesson, allowing learners to connect prior knowledge with new learning experiences.

Furthermore, if factual questions are asked during the lesson development, they reinforce concepts that were taught during the lesson introduction. For example, Teacher A asked: “Does anyone among you remember the definition of the terms simple interest and compound interest?″. This type of question necessitates memory retrieval but does not require learners to make connections or analyse concepts beyond rote recall. Similarly, Teacher B asked, “Ukhona (Is there anyone) okhumbulayo (who remembers) ukuthi (that) what do the terms accumulated amount, principal amount, interest, and period mean?”. Such questions support foundational understanding but offer limited cognitive engagement beyond remembering definitions.

Teachers A and B did not ask any factual questions during the lesson development stage. This suggests that these two teachers may have limited their learners' ability to reinforce the already learned concepts. Teachers E and F asked more factual questions (29) throughout the lesson than the other four teachers, however, they did not ask higher order questions, thus revealing an overreliance on factual questions. This aligns with the findings of Svanes and Andersson-Bakken (2023), who suggest that when teachers rely more on factual questions, learners tend to memorise information without developing conceptual understanding, thus limiting their ability to develop critical thinking.

The perceptions of the participants regarding factual questions were significant to consider. In their responses, all the teachers indicated that they ask factual questions. This was observed during their teaching of lessons. However, Teachers A and B indicated that they recognised the limitations that come with over-reliance on factual questioning. They acknowledged the importance of asking learners questions beyond recall to enhance their critical thinking. This could suggest that they did not ask factual questions during the lesson development because they did not want to rely on these and limit the development of learners' critical thinking skills.

The order in which the participants asked learners factual questions was also crucial. Teachers A and B asked factual questions during the lesson introduction and then again during the lesson conclusion. Teachers B and C asked factual questions during all three lesson stages, similar to Teachers E and F. This may suggest that this was a questioning pratice were practices followed by these teachers during lessons. A study by Walsh and Hodge (2018) suggests that factual questions are necessary for foundational learning. Also, Boaler and Brodie's (2004) argument that classroom discourse, even at the recall level, enhances learners' ability to make connections between mathematical ideas supports using factual questions during mathematics lessons.

6.3.2 Procedural questions

Factual questions and procedural questions together comprise the lower-order cognitive category. Like factual questions, procedural questions engage the ‘remember’ and ‘understand’ levels of the RBT. They prompt learners to rehearse known routine procedures and promote the use of correct mathematical language. All six teachers asked procedural questions in order for the learners to rehearse procedures, such as substituting values into the appropriate formulae for compound and simple interest, choosing a preferred method, and listing steps for using their preferred method. The participants asked learners a total of 36 procedural questions. These were used in all lesson stages: introduction (12), development (14) and conclusion (10). In their study, Booth et al., (2017) suggest that procedural questions are crucial in enabling learners to master procedures in all lesson stages.

Procedural questions can also be used by teachers to identify if learners have misconceptions. For example, Teacher F asked: “Anyone who can remind me how do we convert interest?” This question necessitated learners to remember and articulate procedural steps but did not require deeper analytical engagement. Procedural questions were also frequently employed during lesson development, as seen when Teacher D asked, “Explain to me the steps you used, Thulani, in your calculation”. Thulani's response indicated that he followed the routine steps in solving for interest: “I substituted all known values into the formula: $\mathrm{A} = \mathrm{P}(1 + in)$ $\mathrm{A} = \mathrm{R}1\ 000, i = 0{,}07$ and $n = 3$, then I got $\mathrm{A} = \mathrm{R}1\ 210.00$.”

When the participants were asked if they do ask procedural questions, and some indicated that they did. Teachers A and B did not ask procedural questions during the lesson development. This practice could suggest that these two teachers may have limited their learners' ability to rehearse procedures during the lesson development.

It was Teachers E and F asked the most procedural questions, asking learners a total of 15 related questions during the lesson. These teachers did not ask questions situated in the higher-order cognitive category, thus revealing an overreliance on procedural questions. As alluded to earlier, with the use of factual questions, Svanes and Andersson-Bakken (2023) suggest that when teachers rely more on procedural questions, learners tend to memorise procedures without developing conceptual understanding.

The order in which the participants asked learners procedural questions was significant. Teachers A and B asked procedural questions during the lesson introduction and then again at the lesson conclusion. However, Teachers A and B did not ask procedural questions during the lesson development. This practice could suggest that these two teachers may have limited their learners' ability to rehearse procedures during the lesson development. Teachers B and C asked procedural questions during all lesson stages, this was a similar approach followed by Teachers E and F. This may suggest that this was a practice followed by these teachers during lessons. A study by Walsh and Hodge (2018) suggests that procedural questions are crucial for identifying learners' misconceptions, errors, and mistakes. Also, Boaler and Brodie (2004) recommend that procedural questions be asked during mathematics lessons.

6.3.3 Open-ended questions

Open-ended questions engage Levels 3 - 6 of the RBT. They lobby contributions from the learners. Such questions are identified with the higher-order cognitive category. Before the classroom observations, when the participants were asked if they ask learners open-ended questions, only three teachers (A, B, and C) indicated that they did. During classroom observations, however, Teachers A, B, C, and D asked open-ended questions, encouraging learners to provide descriptive responses, rather than just responding with a yes or a no answer.

These questions were used by the four participants to elicit comprehensive and insightful responses on simple and compound interest mathematical concepts. They asked learners a total of 35 open-ended questions. These were asked during two lesson stages: development

(24) and conclusion (11). Teachers A and B asked most of these questions (29), Teachers C and D asked learners a total of six open-ended questions. Teachers A, B and C most often targeted individual learners, while Teachers D and E targeted their whole class, and Teacher F targeted individuals and a certain group of learners.

The use of open-ended questions aligns with a study by Hattie and Zierer (2018) that found that asking open-ended questions contributes to enhancing learners' critical thinking more than only relying on factual and procedural questions. This suggests that Teachers E and F may have limited the development of their learners’ critical thinking abilities since they did not use any open-ended questions in their lessons. These teachers relied on factual and procedural questions.

Teacher A, in his first question during the lesson development stage, asked learners the open-ended question: “Why do banks charge interest on loans and offer interest on savings?” The use of the phrase 'why' in this question requires learners to respond with an answer beyond ‘yes’ or ‘no’, thus making this question open-ended. To substantiate this case, Eric, a learner in his class, replied: “I think banks make money by lending it to people, so they charge interest.” Similarly, during a lesson conclusion, Teacher C asked learners: “Who can tell me why the compound interest is different from the simple interest?”. The use of the phrase 'why' prompted Cynthia, a learner in her class, to respond: “Madam, isn't that with simple interest, inzalo (interest) stays the same for the period of the investment, okuhlukile (which is different) when we apply the compound interest formula?”. This suggests that these teachers encouraged and promoted the development of critical thinking more effectively than Teachers E and F did. The teachers also provided feedback, which aligns with Steps 3 to 5 of the CTF.

What was significant was the order in which these participants asked these questions. Teachers A and B asked more of these questions during the lesson development than the lesson conclusion. Teachers C and D, however, only asked these questions during the lesson conclusion. This could suggest that the order in which they ask open-ended questions is a practice they follow.

Open-ended questions do serve as a vital tool in stimulating learners' critical thinking. The four participants demonstrated the use of such questions, prompting learners to justify their answers and explore alternative problem-solving methods. Such questions align with the second stage of Duron et al.'s (2006) framework, which emphasises the need for varied and thought-provoking questioning to encourage deeper reasoning. Research by Nappi (2017) supports the effectiveness of this approach, highlighting that open-ended questioning increases learner engagement and conceptual understanding.

6.3.4 Probing questions

Probing questions are closely associated with open-ended questions and also engage Levels 3 - 6 of the RBT. These questions prompt learners to articulate, clarify, or elaborate their responses. Such questions are identified with the higher-order cognitive category. Before the classroom observations, the participants were asked if they ask probing questions. Only four teachers (A, B, C, and D) indicated they did. These four teachers were observed asking probing questions during the lessons. The probing questions were asked when teachers prompted learners to elaborate on their responses. All 11 probing questions were asked during the lesson conclusions.

Teachers C and D asked more of these questions (7) than Teachers A and B, who asked a total of four probing questions. Having Teachers E and F not ask learners probing questions is discouraged by Newman (2023) who found that when learners are not prompted to clarify or elaborate their responses, teachers may be unable to identify learners' misconceptions. On the contrary, learners who are often asked probing questions tend to be cognitively engaged, thus remaining motivated and persistent to find solutions to challenging mathematics problems.

Teacher C asked a learner, Cynthia, a probing question following Cynthia’s reply that interest is calculated differently for simple and compound interest. Teacher C asked: “Yes, but tell me, how is it different?”. Cynthia responded: “With compound interest, it's interest over interest.” This exchange between Teacher C and Cynthia suggests that probing questions do not seek a ‘yes’ or ‘no’ response. Instead, a learner is required to clarify their response. Hence, Teacher C was able to determine if Cynthia knows the difference between compound and simple interest calculations.

In another exchange, Teacher A asked Eric if he still remembered how Siphumelele had calculated the simple interest problem. Eric's first reply was: “Easy, sir: she used the formula for simple interest and substituted all values, which differed from what Xolani used”. Teacher A then probed Eric: “Tell me, Eric: how is it different from the one used by

Xolani?”. Eric replied: “With the compound interest formula, sir, the variable n is always written in exponential form, hence Xolani substituted correctly”. The exchange between Teachers A and C and learners suggests that they required their learners to elaborate or clarify their responses.

6.3.5 Adaptive questions

As with open-ended questions, adaptive questions are closely associated with probing questions and engage Levels 3 - 6 of the RBT. These questions prompt learners to make links or point to relationships between mathematical ideas. Adaptive questions are identified within the higher-order cognitive category. For reference, before classroom observations, the participants were asked if they asked adaptive questions, and only two teachers (A and B) indicated that they did. These two teachers were observed using adaptive questions during their lessons. They asked learners a total of five adaptive questions – during the development stage. Teacher A asked three, while Teacher B asked two. Teachers C to F did not ask learners adaptive questions. Newman (2023) recommends that teachers need to ask adaptive questions for enhancing learners critical thinking. When teachers do not ask adaptive questions, they may find it challenging to recognise if learners respond from memorisation or conceptual understanding (Newman, 2023). In contrast, learners who are often asked adaptive questions tend to be cognitively engaged as well and remain motivated and persistent in finding solutions to challenging mathematics problems.

Teacher A first asked learners an open-ended question during the lesson development. After a learner, Eric, responded, Teacher A gave him feedback, acknowledging his accuracy. The exchange continued after Andile responded to another open-ended question. Teacher A then decided to adapt his third question based on the two responses that Eric and Andile provided. He asked Siphumelele: “Let's apply this to a scenario. Suppose you invest R500.00 at an interest rate of 8% per year, calculated using simple interest. How much will you have after three years, Siphumelele?”. While this question may be associated with the procedural question type, it followed the responses the two learners had provided prior. Siphumelele responded: “That's super easy, sir, because we use the formula $\mathrm{A} = \mathrm{P}(1 + \mathrm{in})$. So I think if we plug values, it will read as: $\mathrm{A} = 500\ (1 + 0.08 \times 3)$, which gives us R6 200.00.”

Adaptive discussions create a more interactive classroom environment where learners are encouraged to question their assumptions and explore alternative solutions (Barton, 2018).

Through adaptive questions, learners' ability to 'apply' is essential in mathematics education as they require learners to take their understanding of concepts and apply them to new and unfamiliar situations. This level of questioning helps bridge the gap between theoretical knowledge and practical application, reinforcing learners' problem-solving abilities (Foster, 2022). Teachers A and B were observed engaging learners through adaptive questions that required learners to apply, prompting them to use their understanding of financial mathematics concepts in real-world scenarios.

This aligns with research by Benade (2020), who argues that adaptive-based questions enhance learners' ability to transfer knowledge beyond the classroom. Teachers A and B asked adaptive questions during the lesson development. This may suggest that these two teachers follow this order when asking adaptive questions. Another key feature of adaptive questioning is its emphasis on reasoning through problem-solving, rather than memorising procedures. Teachers A and B encouraged learners to apply formulae to different investment scenarios, requiring them to determine financial outcomes based on given data. Their approach reflects the findings of Sun and Xie (2020), who suggest that applying knowledge in diverse contexts strengthens learners' conceptual understanding and mathematical reasoning.

6.4 Theme 3: Intention

All participants indicated that questioning serves multiple purposes, including drawing interest, diagnosing misconceptions, and enhancing critical thinking. Their views align with the findings by Traver (2019), who suggests that questioning enhances classroom discourse and cognitive engagement. The question types used by Teachers A, B, C, and D during classroom observations appear to have done more than draw interest or prompt learners to recall learned procedures involving compound and simple interest. The participants asked learners questions situated in both cognitive categories. According to DeJarnette et al., (2020) while lower-order questions assist in developing foundational knowledge, adding higher-order questions enhances learners' abilities to develop critical thinking.

Teachers A and B suggested that prompting learners through questioning using phrases 'why,' 'how,' and 'what if' extends learners' engagement. Teachers A and B used more of these phrases to encourage learners to explore alternative solutions and justify their reasoning. This practice aligns with Benade (2020), who found that questioning techniques that demand justification and explanation lead to greater cognitive engagement and a deeper understanding of mathematical concepts.

6.5 Theme 4: Limitations

Teachers E and F, who predominantly asked factual and procedural questions, reported that their learners rarely engaged in extended discussions. Teachers' reliance on factual and procedural questions often stems from learners' limited engagement in extended discussions, a challenge widely documented in educational research (Chin, 2018). Factual questions require recall of discrete information and procedural questions. These questions assess learners' understanding of methods or processes. However, these question types may not sufficiently promote higher-order thinking or conceptual understanding (Mulyatna, 2021). Learners' reluctance to engage in extended discussions may be influenced by various factors, including limited confidence, lack of prior knowledge, and classroom dynamics that do not support open-ended inquiry (Nystrand et al., 2020). Teachers may, therefore, default to factual and procedural questions as a practical response to low learner participation, reinforcing a cycle in which learners are not encouraged to articulate and defend their reasoning in depth.

In their study, Arisoy et al. (2021) suggest that despite the prevalence of factual and procedural questioning, effective classroom discourse necessitates a balance between different question types, including those in the higher-order cognitive category. Studies by Alexander (2020) and Mercer and Howe (2019) highlight that dialogic teaching, where learners are encouraged to explore ideas collaboratively, enhances critical thinking and problem-solving skills. However, such approaches require scaffolding strategies that gradually increase learners’ participation in discussions. For example, introducing structured peer discussions and teacher modelling of extended responses can foster a more interactive learning environment. If teachers rely predominantly on factual and procedural questions, without integrating strategies to elicit deeper responses, learners may view knowledge as static, rather than evolving through dialogue and reasoning (Widana et al., 2018) .

6.6 New model combining the Critical Thinking Framework with the Revised Bloom's Taxonomy

The synthesis of Bloom's Revised Taxonomy and Duron et al.'s (2006) Critical Thinking Framework provides a comprehensive model for effective questioning in mathematics classrooms. This new proposed model integrates cognitive levels of learning with structured questioning strategies that enhance learners' ability to engage in critical thinking. By systematically aligning Bloom's taxonomy levels with Duron et al.'s (2006) critical thinking framework steps (identifying learning objectives, questioning techniques, practice, feedback, and assessment), this model fosters a more dynamic and inquiry-driven approach to mathematics education. A visual representation of the model is presented in Figure 18.

Figure 18

CTF+RBT Model for enhancing learners' critical thinking in mathematics classrooms (Author's own diagram)

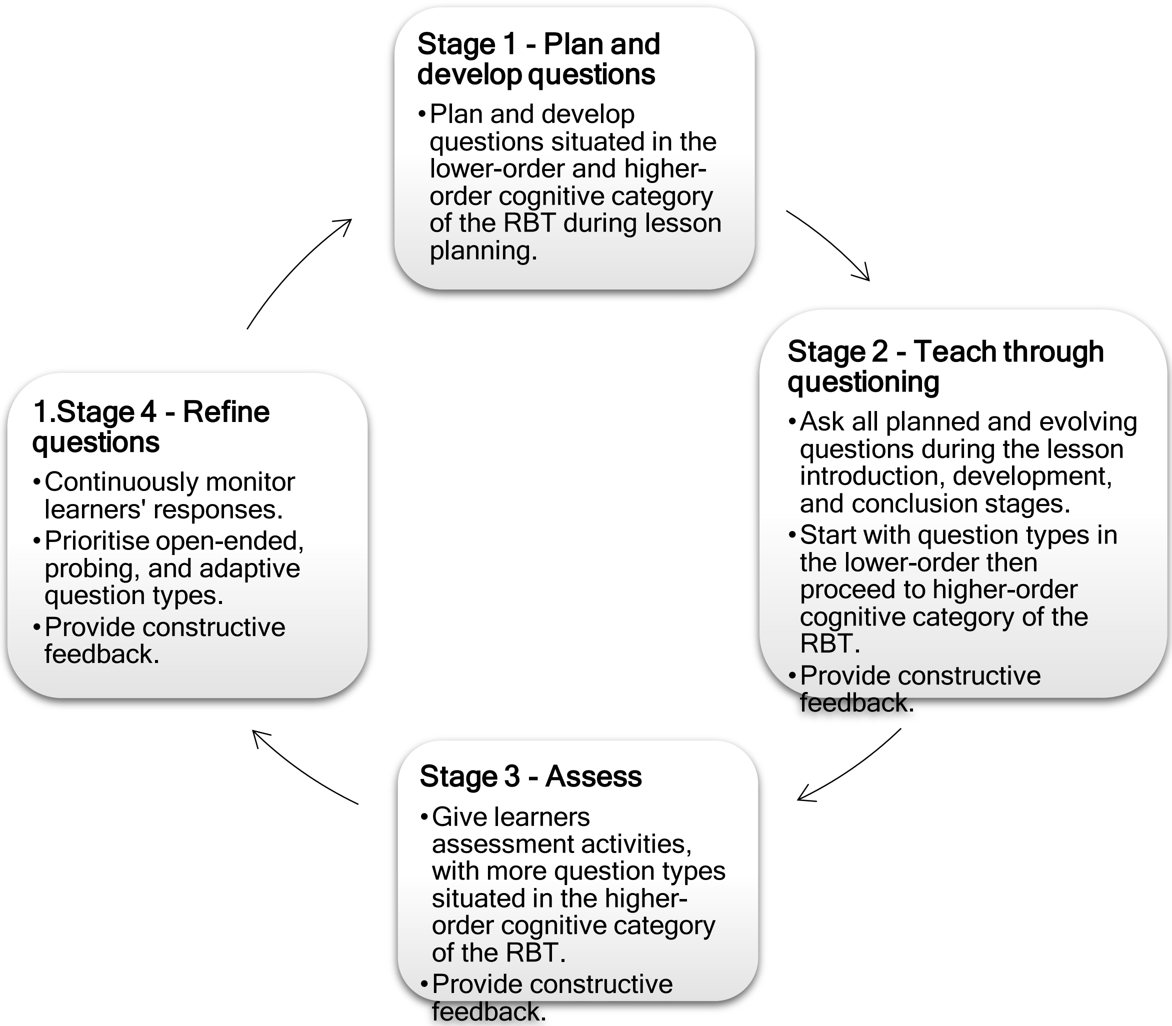


Figure 18 illustrates the cyclical nature of questioning and cognitive development. The model situates the Revised Bloom's Taxonomy within the broader critical thinking framework, demonstrating how each of the four stages supports and promotes critical thinking by systematically engaging with all levels of the RBT by incorporating a range of questioning practices.

The model involves four sequential stages:

Stage 1: Plan and develop questions. This is the foundation of this model. It emphasises the importance of thoughtful question design. At this stage, teachers design and plan questions during lesson planning. Questions should be developed, planned intentionally, and aligned with the RBT's six levels to promote both lower-order and higher-order categories of cognitive engagement. The order in which questions will be asked by teachers needs to be set and planned clearly across all three lesson stages: introduction, development, and conclusion.

Stage 2: Teach through questioning. This stage focuses on the actual practices, where teachers ask learners the planned and evolving questions. Teachers are encouraged to start with question types such as factual and procedural questions (lower-order cognitive category) involving Levels 1 to 2 of the RBT, then proceed to ask learners question types in the higher-order category involving Levels 3 to 6. This can be an iterative process, starting with lower-order and progressing to higher-order, then returning to lower-order, as often the teacher can ask questions. During the questioning process, which runs across all three lesson stages, giving feedback to learners – such as acknowledging where they have responded well – is important. Provide scaffolding techniques where learners may be facing challenges or show knowledge gaps. Through questioning classroom dialogues, collaborative learning is promoted.

Stage 3: Assess. This extends the questioning approach into formal and informal assessment activities. This step emphasises prioritising question types in the higher-order cognitive category in assessment tasks, encouraging learners to demonstrate their critical thinking skills. This does not mean that factual and procedural questions must be neglected; however, more emphasis should be placed on higher-order cognitive category questions using Levels 3 – 6 of the RBT.

Stage 4: Refine questions. This stage highlights the iterative nature of effective questioning. It emphasises the importance of continuously monitoring learners’ responses and the willingness to adjust questions and techniques based on learners’ understanding. This requires teachers to be reflective practitioners, constantly evaluating their questioning practices and their impact on learners' learning. This stage can happen at any time during the lesson and can be used to prepare for upcoming lessons.

6.6.1 Key features of the model

One of the core features of the model is its emphasis on adaptive questioning, where teachers adjust their prompts based on learners’ responses. Teachers who actively listen and modify their questions create opportunities for deeper engagement. This approach mirrors the principles outlined in Duron et al.'s (2006) model, where questioning serves as a mechanism for progressively challenging learners while maintaining accessibility to all learners.

The model also underscores the necessity of open-ended questions, particularly those using key prompts such as 'why', 'how', 'when', and 'what if'. These questions encourage learners to justify their reasoning, explore multiple solutions, and reflect on mathematical relationships. Teachers A and B effectively incorporated such prompts, fostering a learning environment where learners engaged in extended discussions and collaborative problem-solving. This aligns with findings by Hakamata et al. (2023), who argue that open-ended questioning is one of the most effective strategies for enhancing critical thinking in mathematics education.

To further enhance learners' cognitive engagement, the new model recommends integrating real-world problem-solving contexts. By asking questions that require learners to apply their mathematical understanding to practical situations, teachers help them develop transferable skills. For example, linking interest rate calculations to real-life financial decisions prompts learners to analyse and evaluate mathematical principles beyond the classroom. This strategy is consistent with Herbel-Eisenmann and Shah (2019), who advocate for problem-based learning as a means to develop reasoning and analytical skills.

In addition to incorporating question types in the high-order order cognitive category, the model highlights the importance of sequential questioning techniques, where initial responses lead to further probing questions. Teachers who utilise sequential questioning challenge learners to refine their arguments, strengthen their justifications, and explore alternative viewpoints. As outlined in Duron et al.'s (2006) model, this iterative process ensures that learners actively engage with content rather than passively receiving information.

Another key component of the model is the structured use of feedback, which plays a crucial role in reinforcing critical thinking. Effective questioning should be followed by feedback that prompts reflection and further inquiry. Teacher A, for instance, provided feedback that encouraged learners to reconsider their approaches rather than simply confirming correct or incorrect answers. This aligns with Namkung and Fuchs (2019), who emphasise the importance of feedback in guiding learners toward deeper understanding, thus enhancing their critical thinking.

Another critical aspect of the model is its emphasis on collaborative learning. Encouraging learners to discuss their responses with peers, justify their reasoning, and critique each other's approaches enhances communication skills and critical thinking. Teachers A and B incorporated peer discussions as part of their questioning strategies, leading to increased learner engagement and cognitive development. Research by Khalid et al. (2020) suggests that peer-led discussions help learners internalise concepts more effectively by exposing them to diverse perspectives.

Despite the possibilities offered by the CTF+RBT Model, it is important to acknowledge its potential limitations. Effectively using this model requires teachers to have an understanding of the Revised Bloom's Taxonomy, question types, and feedback strategies. Barlow and Brown (2020) assert that mathematics teachers often find it challenging to design and ask questions using all levels of the revised Bloom's Taxonomy. Thus, teachers need to strategically plan and teach using questions to enhance learners' critical thinking. Moreover, various contextual factors, such as class size, available resources, curriculum requirements, and school culture, can influence the model's effectiveness. For example, teachers may find it difficult to provide individual attention and feedback to all learners in large classes.

Thorough planning and implementation of question types aligned with all six Revised Bloom's Taxonomy levels, along with providing constructive feedback, can be time-consuming. Teachers working under pressure to cover large amounts of content might find it challenging to dedicate sufficient time to each stage, especially in preparing various higher-order questions and providing individualised feedback. The model's iterative nature (Stage 4) also requires ongoing reflection and adaptation, which adds to the time investment.

Ultimately, this new model serves as a guide for mathematics teachers to plan, teach and assess through questioning. The refinement of questions and constructive feedback is also considered necessary, ensuring alignment with the Revised Bloom's Taxonomy levels. Teachers can use this proposed model to enhance learners' ability to think critically and solve complex problems.

6.7 Conclusion

This chapter has comprehensively discussed the question types and practices employed by six mathematics teachers, aligning their strategies with the Revised Bloom's Taxonomy and Duron et al.'s (2006) critical thinking framework. The analysis has demonstrated that while some of the teachers incorporated a range of question types to enhance learners' critical thinking, others remained reliant on asking learners only factual and procedural questions, limiting learners' opportunities for critical thinking. The discussion highlighted the importance of progressively challenging learners through questioning and incorporating open-ended, probing and adaptive question types. Furthermore, this chapter introduced a new model that synthesises the RBT with Duron et al.'s (2006) structured CTF to enhance learners' critical thinking. This CTF+RBT model presents an integrated framework that emphasises the role of questioning in enhancing learners' critical thinking. By demonstrating how question types can be systematically implemented to enhance learners' critical thinking, the proposed model provides a structured approach for teachers seeking to enhance learners' critical thinking. In the next chapter, Chapter 7, the study concludes with a summary of key findings structured around the four main research questions. It highlights the new knowledge generated by this study, presents the study's recommendations, and outlines its limitations.

CHAPTER 7: SUMMARY, CONCLUSION AND RECOMMENDATIONS

7.1 Introduction

This final chapter presents the concluding remarks of the study, synthesising the key findings in relation to the four main research questions and objectives. The first section presents a summary of the major findings in relation to the research objectives: (i) explore teachers' questioning practices for enhancing critical thinking in Grade 10 mathematics classes; (ii) identify questioning practices that teachers can use to enhance critical thinking in Grade 10 mathematics classes; (iii) determine the manner in which teachers use questioning practices to enhance critical thinking in Grade 10 mathematics classes; and (iv) identify the opportunities that the use of teachers' questioning practices for enhancing critical thinking in Grade 10 mathematics classes provides.

The chapter also reflects on the new knowledge contributed by this study, particularly regarding the integration of the Revised Bloom's Taxonomy and Duron et al.'s (2006) Critical Thinking Framework. The study's limitations are discussed, providing avenues for future research. The chapter outlines practical recommendations for future research and curriculum practitioners. Recommendations are also made regarding the support needed for teachers to effectively implement higher-order questioning strategies. This chapter serves as a culmination of the research, offering valuable insights for teachers, policymakers, and researchers seeking to improve instructional practices in mathematics education.

7.2 Summary of key findings

7.2.1 Addressing the first three research questions

The research findings reveal that the six teachers who participated in this study employed various question types during classroom observations: factual, procedural, open-ended, probing, and adaptive questions. These question types were all aimed at enhancing learners' critical thinking. The factual and procedural questions they asked learners were situated in the lower-order cognitive category, while the open-ended, probing, and adaptive questions were situated in the higher-order cognitive category. In the case of School 1, Teachers A and B asked learners all five question types. In the case of school 2, Teachers C and D asked their learners factual, procedural, open-ended, and probing questions. In the case of School 3, Teachers E and F asked only procedural and factual questions.

The analysis revealed that teachers at Schools 1 and 2 used question types situated in both cognitive categories (lower-order and higher-order); thus, their learners stood a greater chance of developing critical thinking than learners at School 3, whose teachers asked questions situated in the lower-order cognitive category. These findings align with prior research, which suggests that some mathematics teachers still emphasise questions that engage lower-order cognitive skills and neglect questions that engage higher-order cognitive skills (Tanudjaya & Doorman, 2020). The adaptive questioning used by Teachers A and B allowed these teachers to scaffold learning effectively, ensuring that learners moved beyond rote memorisation to engage in higher-order thinking. This aligns with the work of Sahin and Kulm, (2008), who emphasise that questioning should be responsive to learners' needs to facilitate deeper learning.

This study found that the participants asked questions in a similar manner. At school 1, the teachers asked learners questions through progression and in an iterative process, starting with factual and procedural questions during the lesson introduction, then open-ended, probing and adaptive questions during the lesson development, and then returning to factual, procedural, open-ended and probing questions during the lesson conclusion. Another practice was observed at School 2, where Teachers C and D used factual and procedural questions during both the lesson introduction and development, and then introduced open-ended and probing questions during the lesson conclusion. At School 3, Teachers E and F asked their learners only factual and procedural questions throughout the lessons. Of significance is that despite the variation in participants' practices, they all asked a similar number of questions. This suggests that the participants might have followed a deliberate sequencing approach when deciding on the number of questions they ask learners. Furthermore, while questioning is a fundamental instructional tool, its effectiveness depends on how questions are structured and are asked to learners in Grade 10 mathematics classrooms.

Another essential aspect of effective questioning is the facilitation of learner-led discussions, where learners engage in peer dialogue to evaluate mathematical ideas collaboratively (Hunter, 2021). In the lessons observed at Schools 1 and 2, where learners were given opportunities to discuss their reasoning with peers they stood a greater chance of developing problem-solving abilities and a deeper understanding of mathematical relationships (Yang et al., 2022). These findings highlight that questioning should not only involve teacher-learner interactions but also encourage learner-learner engagement to maximise learning opportunities.

Additionally, the integration of real-world problem-solving contexts was found to be an essential questioning practice used by teachers at Schools 1 and 2. Teachers who linked mathematical problems to real-life applications helped learners see the relevance of their learning, improving engagement and conceptual understanding (Schoenfeld & Kilpatrick, 2013). This approach supports the findings of Jonsson et al. (2022), who argue that mathematical reasoning is strengthened when learners can relate their knowledge to authentic situations.

7.2.2 Addressing the fourth research question

The study identifies key opportunities presented by teachers' questioning practices, such as increased learner engagement, improved problem-solving abilities, and stronger conceptual understanding. Teachers who emphasised questioning as a tool for enhancing learners' critical thinking created interactive and discussion-based learning environments where learners felt encouraged to participate actively. When teachers employed open-ended and probing questions, learners responded, making attempts at various solution pathways and justifying their answers. This ability to articulate reasoning is crucial in developing critical thinking skills, enabling learners to transfer their knowledge to new and unfamiliar contexts (Tossavainen, 2022). Furthermore, classroom dialogue among learners and their teachers facilitated peer learning, where learners learned from each other's perspectives and problem-solving approaches (Boaler & Brodie, 2004). These collaborative discussions encouraged deeper exploration of mathematical concepts, reinforcing previous research study that suggests that learners' discourse plays a vital role in cognitive development (Rubie-Davies et al., 2020).

The study further revealed that questioning provided an avenue for formative assessment, allowing teachers to gauge learners' understanding in real-time and adjust instruction accordingly (Trenholm et al., 2019). Teachers who used questioning effectively were able to provide immediate feedback and guide learners toward improved problem-solving approaches. By leveraging questioning as a formative assessment tool, teachers created a dynamic learning environment that fostered continuous cognitive development.

7.3 Contribution of the study

This study makes a significant contribution to the field of mathematics education by providing empirical evidence on the role of teachers' use of questioning in enhancing learners' critical thinking. One of its key contributions is the development of an integrated model that combines the Revised Bloom's Taxonomy with Duron et al.'s (2006) Critical Thinking Framework. By synthesising these two models, the study presents a structured approach to questioning that not only enhances cognitive development but also provides a practical framework for mathematics teachers seeking to refine their instructional strategies. This model fills a research gap by offering a comprehensive guide to structuring questioning practices that progressively lead learners toward higher-order thinking and problem-solving abilities.

Another major contribution is the study's identification of the specific questioning strategies that enhance learner engagement and critical thinking. While previous studies have highlighted the importance of questioning in learning, this research provides detailed insights into how questioning practices can be effectively implemented in mathematics classrooms. By showing how the participants asked open-ended questions and adaptive questioning, the study offers actionable strategies for improving classroom instruction. These findings contribute to existing literature by bridging the gap between theory and practice, providing a more nuanced understanding of how questioning strategies translate into meaningful learning experiences. Furthermore, while much of the existing research has focused on theoretical discussions of questioning techniques, this study provides empirical data on how teachers apply these strategies in practice. The findings reveal that while some teachers effectively use questioning to stimulate critical thinking, others may experience challenges, such as learners not responding to the questions they pose.

Additionally, the study contributes to the broader discourse on mathematics education by emphasising the role of questioning in fostering learner independence and problem-solving skills. The research findings demonstrate that teachers' questioning practices that require learners to explain, justify, and explore multiple solutions, quite often promote learners' development of higher-order thinking skills. The findings provide a strong rationale for integrating questioning practices as a fundamental component of mathematics instruction.

7.4 Limitations of the study

While this study provides valuable insights into the role that questioning can play in fostering critical thinking in Grade 10 mathematics classrooms, it is important to acknowledge its shortcomings. One limitation is the relatively small sample size, which included only six teachers. This sample size restricts the richness of the qualitative data generated. The reliance on a questionnaire, classroom observations, and teacher interviews as the data generation methods also limited the richness of the qualitative data generated. While these methods provided in-depth insights into teachers' questioning practices, they did not capture learners' perspectives; neither did they capture teachers' instructional documents, such as lesson plans. Additionally, the study focused primarily on questioning practices in mathematics education, which limits its applicability to other subjects. Finally, the study was conducted within the specific educational context of South African public education, meaning that the findings may not be directly applicable to different curriculum frameworks.

7.5 Conclusion

This study has provided an in-depth examination of how questioning practices influence critical thinking in Grade 10 mathematics classrooms. Through an analysis of six teachers' questioning strategies, the study has demonstrated the varying degrees to which teachers employ open-ended, adaptive, and probing questioning techniques. While some teachers effectively implemented strategies that promote deeper reasoning, others relied primarily on lower-order questioning, limiting learners' opportunities to engage in higher-order thinking. The findings underscore the importance of well-structured questioning approaches that encourage problem-solving, reasoning, and conceptual understanding in mathematics education. A key contribution of this study is the integration of the Revised Bloom's Taxonomy with Duron et al.'s (2006) Critical Thinking Framework, resulting in a new model that provides a structured approach for designing and implementing questioning strategies. This model highlights the necessity of balancing questions aligned with lower and higher-order cognitive categories while ensuring that learners are guided through progression that fosters critical thinking. The findings also emphasise that effective questioning not only enhances mathematical understanding but also serves as a fundamental tool for formative assessment, learner engagement, and instructional improvement.

Furthermore, this study has identified several key areas for future research, including the need for broader investigations into how questioning strategies impact learners' learning across different contexts, subjects, and educational systems. The study also highlights the necessity of professional development programs to equip teachers with the skills to employ questioning as a tool for critical thinking development. Overall, this study reinforces the significance of teachers' questioning as an essential pedagogical tool for fostering learners' engagement and critical thinking. By implementing the recommendations provided, researchers, teachers and policymakers can support the development of a more inquiry-driven mathematics learning environment that encourages learners to think critically, justify their reasoning, and explore mathematical concepts at deeper levels. The findings serve as a foundation for ongoing research and practical applications in mathematics education, ultimately contributing to the enhancement of learner learning outcomes and instructional effectiveness.

7.6 Recommendations

For future research studies focusing on teachers' questioning practices, a larger sample size may provide more generalisable findings and richer insights for enhancing learners' critical thinking. In addition to focusing on teachers, future studies could incorporate interviews or surveys to understand how learners perceive and respond to different questioning techniques, thereby offering a more holistic understanding of the impact of teachers' questioning on learners' learning. A comparative study exploring teachers' question types along with their practices across multiple subjects would provide a broader perspective on how questioning fosters critical thinking in various academic disciplines. I argue that future research needs to bridge this theoretical-practical divide by examining how questioning unfolds in authentic classroom settings and how teachers can adapt questioning strategies in response to learners' evolving needs.

It is recommended that subject specialists at the education district centres organise professional development workshops that enable teachers to share their experiences, learn from each other, and offer support to each other on how to better handle challenges in their pedagogic practices. These workshop sessions could incorporate trainings on practical strategies such as designing strategic sequencing of question types situated in both cognitive categories, understanding RBT along with the CTF for facilitating adaptive discussions, and using probing techniques to enhance learners' critical thinking. It is recommended that schools support teachers' autonomy to facilitate collaborative learning communities where teachers can share experiences and allocate time for reflection and refinement of questioning strategies. Additionally, it is recommended that higher education institutions incorporate questioning practices into pre-service teacher education programs to ensure that future teachers are well-prepared to use questioning as an instructional tool.

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APPENDICES

Appendix A: Ethical clearance letter (UKZN)

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

29 September 2022

Ayanda Sizwe Zondo (210506848)
School Of Education
Edgewood Campus

Dear AS Zondo,

Protocol reference number: HSSREC/00004772/2022
Project title: Mathematics teachers' questioning practices for enhancing critical thinking in Grade 10 mathematics classrooms.
Degree: PhD

Approval Notification – Expedited Application

This letter serves to notify you that your application received on 07 September 2022 in connection with the above, was reviewed by the Humanities and Social Sciences Research Ethics Committee (HSSREC) and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

This approval is valid until 29 September 2023.
To ensure uninterrupted approval of this study beyond the approval expiry date, a progress report must be submitted to the Research Office on the appropriate form 2 - 3 months before the expiry date. A close-out report to be submitted when study is finished.

HSSREC is registered with the South African National Research Ethics Council (REC-040414-040).

Yours sincerely,

[redacted]

Professor Dipane Hlalele (Chair)

/dd

Humanities and Social Sciences Research Ethics Committee
Postal Address: Private Bag X54001, Durban, 4000, South Africa
Telephone: +27 (0)31 260 8350/4557/3587 **Email:** hssrec@ukzn.ac.za **Website:** http://research.ukzn.ac.za/Research-Ethics

Appendix B: Permission letter to conduct research in the KZN DoE institutions

KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE HEAD OF DEPARTMENT

Private Bag X9137, PIETERMARITZBURG, 3200
Anton Lembede Building, 247 Burger Street, Pietermaritzburg, 3201
Tel: 033 392 1063
Email: Phindile.duma@kzndoe.gov.za

Enquiries: Phindile Duma **Ref.**:2/4/8/4079

Mr AS Zondo
[redacted]
[redacted]
DURBAN NORTH
4051

Dear Mr Zondo

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"TEACHERS' QUESTIONING PRACTICES FOR ENHANCING CRITICAL THINKING IN GRADE 10 MATHEMATICS CLASSROOMS",** in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 30 May 2022 to 02 May 2025.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

PINETOWN DISTRICT

[redacted]

Mr GN Ngcobo
Head of Department: Education
Date: 01 June 2022

GROWING KWAZULU-NATAL TOGETHER

Appendix C: Letter to the school principal

LETTER TO THE SCHOOL PRINCIPAL

The principal / School's Governing Body

School name:______________________

Address:__________________________

Date:_____________________________

Dear Sir/Madam

Re: Permission to conduct research

I am currently a PhD learner at the University of KwaZulu-Natal. I am presently exploring teachers questioning practices for enhancing critical thinking in grade 10 mathematics classrooms.

I seek permission to engage with the teachers in the school to collect data for my research. I am willing to discuss and share data collection methods that I plan to use when engaging with the teachers. I have sent a letter requesting permission to the KwaZulu-Natal Department of Basic Education. I enclose all these forms for your perusal.

I assure you that my data collection methods will be ethically employed, ensuring the teachers' and learners' physical, emotional, and psychological well-being. My supervisor is Prof J Naidoo. She can be contacted: 031 2601127 OR naidooj2@ukzn.ac.za; to verify all information provided.

I trust that my request will be considered favourably. Yours faithfully

Mr A.S Zondo

Appendix D: Letter of informed consent

INFORMED CONSENT LETTER

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus

Dear Participant

INFORMED CONSENT LETTER

My name is Ayanda Sizwe Zondo I am a PhD candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. My study focuses on Grade 10 teachers' use of questioning practices during mathematics lessons as a strategy to enhance learners' critical thinking in the Pinetown District, KwaZulu-Natal. I am interested in gathering data for research purposes. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person but reported only as a population member opinion.
- The interview may last for about 30 minutes to 45 minutes.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalised for taking such an action.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment		
Photographic equipment		
Video equipment		

I can be contacted at the University of South Africa
Email: zondas@unisa.ac.za
Alternative Email: 210506848@stu.ukzn.ac.za
Cell: [redacted]

My supervisor is Prof J Naidoo, located at the School of Education, Edgewood campus, University of KwaZulu-Natal (UKZN).
Contact details: Room CU 118, Main Tutorial Building, Edgewood Campus, UKZN.
email: [redacted]; Phone number: 031 2601127

You may also contact the Research Office through:
[redacted]

Thank you for your contribution to this research.

Appendix E: Semi-structured questionnaire

<u>SEMI-STRUCTURED QUESTIONNAIRE</u> ON TEACHERS' QUESTIONING PRACTICES FOR ENHANCING CRITICAL THINKING IN GRADE 10 MATHEMATICS CLASSROOMS.

TEACHER:______

Grade	School	Years of experience	Qualifications	Race	Gender	Other grades teaching
10						

No of classes?	No of learners	
	Class A	Class B

QUESTION 1

Which official language do you mainly use when asking learners questions?

IsiZulu	English	Other

Explain:

………..

QUESTION 2

Which of these target audiences do you generally direct your questions toward?

Individual learner	Certain group	The whole class	Other:

Because: ……

QUESTION 3

For what reasons do you mainly use questions during lessons?

To draw interest and attention	To remind certain facts	To diagnose and control	To enhance critical thinking
To assess			

QUESTION 4

Which of the following do you do after you ask questions in your class?

I expect the learners to give only the response I expect to get.	I provide the correct answer after I ask a question.	I use probing questions to get the right answer when learners cannot answer correctly.

Other: ……

QUESTION 5

Which Bloom's taxonomy levels do you mainly use when generating and asking questions?

Applying	Evaluating	Analysis	All
Understanding	Synthesising	Remembering	

QUESTION 6

Do you think there is a need to ask learners questions during lessons to enhance their critical thinking?

YES	NO

Explain: ………

QUESTION 7

When do you prepare questions for mathematics lessons?

Before the lesson	During the lesson	After the lesson	Anytime	Other

Explain:

………

QUESTION 8

In what way do you encourage teachers to ask questions as per Bloom's taxonomy levels during mathematics lessons Explain:

……….

QUESTION 9

Which question types do you ask your learners during lessons?

FQ	PQ	CTQ	Other:

Reference

FACTUAL QUESTIONS (FQ)	PROBING QUESTIONS (PQ)	CRITICAL THINKING QUESTIONS (CTQ)
(Knowledge Questions)	**Routine and complex procedures**	**Problem Solving questions and some complex procedures**
Ask learners yes or no questions.	Ask learners to derive from the given information	Ask learners what information is given about the problem.
Ask learners to recall, list etc.	Ask learners to identify and use (after changing the subject.	Ask learners how the given information is helping them to derive solution(s).
Ask learners to identify…	Ask learners to give more details	Ask learners why they are using the planned strategy.
Ask learners to select/choose….	Ask learners to generalise	Ask learners what they would do differently next time.

QUESTION 10

Learners' critical thinking can be measured often by how learners respond to the questions by teachers with the phrases*; How, Why, When and What*. Should this be the case, how frequently do you think these questions should be asked to the learners?

As many as possible	Less	Not necessarily

Explain:

………

QUESTION 11

Do you comment on learners' responses after asking a question?

Yes	No

-

Explain:

………

QUESTION 12

Do you use what, why, when and how questions during learner assessment

Yes	No

-

Explain:

………

Appendix F: Semi-structured lesson observation schedule

<u>LESSON OBSERVATION SCHEDULE</u> ON TEACHERS' QUESTIONING PRACTICES FOR ENHANCING CRITICAL THINKING IN GRADE 10 MATHEMATICS CLASSROOMS

Date	Subject	Grade	Lesson Topic	Observer
	Mathematics			Researcher

KEYS: **A** – Assessment; **I** – Instruction; **B** – Both (A&I)

LESSON INTRODUCTION

FACTUAL QUESTIONS (Knowledge questions)	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners yes/no						*How frequently is each question asked? Describe observation…. How long were questions asked: Any other observations:*
	Asked learners to recall						
	Asked learners to identify						
	Asked learners to choose						
	Other						
	Factual questions are set out clearly,						

PROBING QUESTIONS (Problem Solving questions and some	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners to derive from the given information						*How frequently is each question asked?* *Describe observation….* *How long were questions asked: Any other observations:*
	Asked learners to identify and use (after changing the subject.						
	Asked learners to give more details						
	Asked learners to generalise						
	Other						
	Are probing questions set out clearly?						

CRITICAL THINKING QUESTIONS (Problem Solving questions and some complex procedures)	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners what information is given about the problem.						*How frequently is each question asked?* *Describe observation….*
	Asked learners how the given information is helping them to derive solution(s).						

	Asked learners why they are using the planned strategy.						*How long were questions asked:* *Any other observations:*
	Asked learners what they would do differently next time.						
	Other						
	Are critical thinking questions set out clearly?						

LESSON DEVELOPMENT

	Teacher questions	**INDICATE**		**INTENTION**			**DESCRIPTIVE OBSERVATIONS**
		YES	**NO**	**A**	**I**	**B**	
FACTUAL QUESTIONS (Knowledge questions)	Asked learners yes/no						*How frequently is each question asked?* *Describe observation….* *How long were questions asked:* *Any other observations:*
	Asked learners to recall						
	Asked learners to identify						
	Asked learners to choose						
	Other						
	Factual questions are set out clearly,						

PROBING QUESTIONS (Problem Solving questions and some complex procedures)	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners to derive from the given information						*How frequently is each question asked?* *Describe observation….* *How long were questions asked:* *Any other observations:*
	Asked learners to identify and use (after changing the subject.						
	Asked learners to give more details						
	Asked learners to generalise						

CRITICAL THINKING QUESTIONS (Problem Solving questions and some complex procedures)	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners what information is given about the problem.						*How frequently is each question asked?* *Describe observation….* *How long were questions asked:* *Any other observations:*
	Asked learners how the given information is helping them to derive solution(s).						
	Asked learners why they are using the planned strategy.						
	Asked learners what they would do differently next time.						
	Other						
	Are critical thinking questions set out clearly?						

LESSON CONCLUSION

FACTUAL QUESTIONS (Knowledge questions)	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners yes/no						*How frequently is each question asked?* *Describe observation….* *How long were questions asked:* *Any other observations:*
	Asked learners to recall						
	Asked learners to identify						
	Asked learners to choose						
	Other						
	Factual questions are set out clearly, 1						

PROBING QUESTIONS (Problem Solving questions and some complex procedures)	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners to derive from the given information						*How frequently is each question asked?* *Describe observation….* *How long were questions asked:* *Any other observations:*
	Asked learners to identify and use (after changing the subject.						
	Asked learners to give more details						
	Asked learners to generalise						
	Other						
	Are probing questions set out clearly?						

CRITICAL THINKING QUESTIONS	Teacher questions	INDICATE		INTENTION			DESCRIPTIVE OBSERVATIONS
		YES	NO	A	I	B	
	Asked learners what information is given about the problem.						*How frequently is each question asked? Describe observation….* *How long were questions asked: Any other observations:*
	Asked learners how the given information is helping them to derive solution(s).						
	Asked learners why they are using the planned strategy.						
	Asked learners what they would do differently next time.						
	Other						
	Are critical thinking questions set out clearly?						

Appendix G: Semi-structured interview schedule

SEMI-STRUCTURED INTERVIEW SCHEDULE ON TEACHERS' QUESTIONING PRACTICES FOR ENHANCING CRITICAL THINKING IN GRADE 10 MATHEMATICS CLASSROOMS.

Before the start of the interview, I will explain to the individual teacher that the following interview is based on their questioning practices for enhancing critical thinking., Also, I will provide the definitions of the following terms:

i. Questioning Practices
ii. Bloom's Taxonomy of levels
iii. Critical thinking

Then proceed to ask the following questions:

INTERVIEW QUESTIONS

1. What is your view about asking specific questions to your learners during lessons?
2. Why does it remain important to ask what, why, and how questions for enhancing critical thinking in your grade 10 mathematics classrooms?
3. What opportunities do you think using teacher questioning practices for enhancing learners' critical thinking has?
4. How do you know that asking learners critical thinking questions is associated with improved learner performance?
5. Why you will recommend that teachers use what, why, and how questions for enhancing critical thinking in their mathematics classrooms.

-

Since this is a semi-structured interview schedule, I will ask other questions to probe or clarify responses.

Appendix H: Data set for semi-structured questionnaire

Question 3: Which of these target audiences do you generally direct your questions toward?

Choices: Individual learner; whole classroom; certain group of learners

Give a reason why you chose the target audience in the Question 10 mathematics classroom.

Teacher	Target audience	Reason
A	Individual	I prefer asking individual learners questions rather than the entire class. I can recognise learners who struggle.
B	Individual	In my teaching experience, I have seen that learners tend to respond collectively, only to find that others are echoing responses without understanding. In this way, I see which learners are struggling or having challenges.
C	Individual	In many instances learners tend to respond as a group, hence I ask each learner.
D	Class	All learners have the same opportunity to respond to the questions.
E	Class	I get to know if the entire class understands what I have taught them.
F	Class	It is easier to direct questions to a larger group and saves time.

Question 4: For what reasons do you mainly use questions during lessons?

Choices: To draw interest; to remind certain facts; to diagnose and control; to enhance critical thinking; to assess

Teacher	Target audience
A	To draw interest; to remind certain facts; to diagnose and control; to enhance critical thinking; to assess
B	To draw interest; to remind certain facts; to diagnose and control; to enhance critical thinking; to assess
C	To draw interest; to remind certain facts; to diagnose and control; to enhance critical thinking
D	To draw interest; to remind certain facts; to enhance critical thinking

E	To draw interest, to remind certain facts
F	To draw interest, to remind certain facts

Question 5: Which of the following do you do after you ask questions in your class?

Choices: I expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question; I use probing questions to get the right answer when learners cannot answer correctly

Teacher	Lesson phase
A	I expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question; I use probing questions to get the right answer when learners cannot answer correctly
B	I expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question; I use probing questions to get the right answer when learners cannot answer correctly
C	expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question; I use probing questions to get the right answer when learners cannot answer correctly
D	expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question; I use probing questions to get the right answer when learners cannot answer correctly
E	I expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question
F	I expect the learners to give only the answer I expect to get; I provide the correct answer after I ask a question

Question 6.1: Which Bloom's taxonomy levels of you mainly use when generating and asking questions

Choices*:* Remember; understand; apply; analyse; evaluate; create

Teacher	RBT Levels
A	Remember; understand; apply; analyse; evaluate; create
B	Remember; understand; apply; analyse; evaluate
C	Remember; understand; apply; analyse; evaluate; create
D	Remember; understand; apply; analyse
E	Remember; understand; analyse
F	Remember; understand

Question 7.1: When do you prepare questions for mathematics lessons?

Choices*:* Before the lesson; during the lesson; after the lesson

Question 7.2: Explain your answer

Teacher	Lesson phase	Reason
A	Before the lesson, during the lesson, after the lesson	During the lesson, I usually draw interest, and then after the lesson, it is a way of recapping. To see if learners do remember.
B	Before the lesson, during the lesson, after the lesson	It is also easier to ask questions as they come during a lesson. I would see if learners do not understand a certain procedure.

C	During the lesson	Learner participation is maximised. Learners remain in charge of the lesson since it is learner-centred.
D	During the lesson	During lesson development, to ensure that learners are following through.
E	During the lesson	I prefer asking learners questions during the lesson because I want to draw their interest and capture their thoughts.
F	After the lesson	I get to measure their understanding.

Question 8: In what way can you encourage other teachers to ask questions as per the Bloom's taxonomy levels during mathematics lessons

Teacher	Reason
A	Absolutely, it is the best way to measure learners' understanding. Questions must be posed at different levels
B	All mathematics teachers must ask questions on various levels from lower-order to higher-order. This enables learners to think creatively and become problem solvers.
C	Asking different levels of questions , ensures learners are prepared for formal assessment.
D	In doing so, learners will gain skills to respond to various questions
E	Learners can be able to answer questions on various levels during assessment
F	I suppose it is important since learners are exposed to different levels of questions during assessment

Question 9: Which question types do you ask learners during lessons?

Choices*:* Factual questions; procedural questions; open-ended questions; adaptive questions; probing questions

Teacher	Reason
A	Factual questions; procedural questions; open-ended questions; adaptive questions; probing questions
B	Factual questions; procedural questions; open-ended questions; adaptive questions; probing questions
C	Factual questions; procedural questions; open-ended questions; probing questions
D	Factual questions; procedural questions; open-ended questions; probing questions
E	Factual questions; procedural questions
F	Factual questions; procedural questions

Question 10.1: Learners' critical thinking can be measured how often learners respond to the questions asked by teachers with the phrases; how; why; when; what if…?

Choices*:* Less; not necessarily; as many as possible; I do not measure

Question 10.2: Should this be the case, how frequently do you think these questions should be asked the learners?

Teacher	Measuring learners critical thinking	Reason
A	As many as possible	They should be asked more; however, there's a different cohort of learners each year. In other years, you may have learners who are struggling. Hence, they tend not to respond to the questions.
B	As many as possible	I think when learners' know-how, it is easier to apply.
C	As many as possible	The more I ask these questions, the more arguments I create in the classroom. In turn, learners participate and think creatively.
D	As many as possible	But learners tend to respond less on these questions.
E	Not necessarily	Most learners tend not to respond to these questions because of difficulty.
F	Not necessarily	It depends on the learners understanding

Question 11: Do you comment on learners' responses after asking a question?

Choices: Yes; no

Teacher	Lesson phase	Reason
A	Yes	I believe in giving feedback, so that learners will know if they are in the right track.
B	Yes	Feedback is important. It gives learners an idea of whether they are on the right path.
C	Yes	Learners want to know what my thoughts are as per their responses. So, I give feedback related to the question.
D	Yes	Yes, in instances where they give correct answers I acknowledge them.
E	Yes	Feedback is important. It gives learners an idea of whether they are on the right path.
F	Yes	When learners give a correct response, I commend them. Where they give incorrect answers, I say so and give a correct answer.

Question 12 Do you use what, why, when and how questions during learner assessment?

Choices: Yes; no

Teacher	Lesson phase	Reason
A	Yes	Yes, and instances where they give correct answers, I acknowledge them.
B	Yes	I do try, and ask learners using these phrases
C	Yes	These set of questions enhance learners thinking, instead of yes or no questions.
D	Yes	Yes, in instances where they give correct answers I acknowledge them.
E	No	Learners tend not respond to these questions.
F	No	Most questions I direct to the learners are based on their understanding.

Appendix I: Transcripts of semi-structured interviews

Question 1: What is your view about asking specific questions to your learners during lessons?

Teachers	Responses
A	It's more about knowing if my learners understand a certain procedure or part of a problem that I am teaching. If I ask them if they understand or not, they respond.
B	Mr Zondo, thank you for that question. Asking learners specific question is a way of knowing their understanding of a particular concept. Each question must be directed to a particular aspect. **Researcher Probing:** What do you mean by saying each question must be directed to a particular aspect? Let me make an example: If I ask my learner a question such as "Why will this procedure not always work?" I intend to know the extent at which learners understand the restrictions of that procedure. Differently if ask my learners "Do you understand this procedure" In this case learners are bound to respond either by a Yes/No. That doesn't tell me much if they understand or not. Interestingly about today's learners, they will forever say yes, because they don't want us teachers to keep probing them. **Researchers' Response**: Now I see.
C	I think, it is a way of getting learners to participate during a lesson. I do not want my lessons to be a one man's show. I have found that, when I am the one talking at all times during my lessons, learners hardly ever understand what I am delivering. In most cases, when I talk more without giving them a chance to respond, I am unable to pick where they do not understand or where they want me to provide examples for easier understanding.
D	For me specifically, I ask learners questions because I want them to pay attention on most important facts during a procedure.
E	It is a must that we as teachers continuously ask our learners questions during lessons. It is a way of measuring their performance and understanding. Imagine not asking them, you would think they hear what we are saying. Then comes an exam, they perform poorly. So rather we ask them before assessment, so that we train them.
F	Mr Zondo, these days these learners tend not to respond to questions during lessons. I always have encouraged them, but still there are reluctant. When I ask them why they do not ask questions, their response is: Madam we heard you, and you are always these procedures very well.

Question 2: Why does it remain important to ask what, why, and how questions for enhancing critical thinking in your grade 10 mathematics classrooms?	
Teachers	**Responses**
A	See, the challenge I have encountered when posing questions in that manner, my learners hardly respond. To answer your question, I do think it is important mainly because you can measure the extent at which learners understand a certain problem. However, with me it has become a draining exercise since my learners hardly respond.
B	Mr Zondo, those three questions are very important. It is like you are always present during my lessons. See, when you use questions such as what, why and how, you don't want learners to respond by Yes/No. You want them to explain and give reference to their reasoning. In all, these questions equip them to be able to respond to high order questions during assessment. They will be in a better position to respond to them.
C	With critical thinking questions, these are associated to higher order questions. I therefore believe that when learners are exposed to such questions, they stand a better chance to respond to higher order questions during exams. Also, remember these questions seek different responses. For an example, when you use the "how" you want learners to show you how they work out a particular problem. Differently when you use "why" In this case you may want learners to tell you more by giving reasons that substantiate their methods.
D	I suppose asking learners critical thinking questions is a way of ensuring that there are problem solvers.
E	These set of questions, encourages learners to provide arguments. In fact, they substantiate. On any response, they provide reasons behind their thinking.
F	As a mathematics teacher, it is very important to ask those set of questions. This allows learners to not always accept procedures as they are. Instead, they provide arguments. In one of my previous schools before coming to this one, I had such learners. If all learners were similar like those learners. I enjoyed teaching them because they wanted to know more.

Question 3: What opportunities do you think using teacher questioning practices for enhancing learners' critical thinking has?	
Teachers	**Responses**
A	If you say opportunities, are you referring to the benefits or advantages? **Researchers response**: Yes, I am. Well, similar how you asked me, when I ask a question, I am seeking clarity for better understanding. So, for my learners, I think when I ask them, it is equipping them for better understanding. When they understand, it is easier for them to respond to problems.
B	Mr Zondo, when I ask learners questions, they develop high thinking skills, such as knowing how to give reference and support their answers. Secondly, for us as teachers, it allows us to be in better position of measuring and knowing learners understanding. This makes us to be aware and plan accordingly. **Researcher Probing**: What do you mean by planning accordingly? With all my lessons Mr Zondo, I prepare questions in advance, besides those that will be on the spot during the lesson. In this way, it is easier to direct learners to a concept for understanding. **Researcher's response**: I see.
C	I teach for understanding. Hence, I ask my learners questions. I was also taught the same way in school. At first, I thought my teachers were putting me on a spot light in comparison to other learners whether I understand a concept or not. However, they explained that it is way for them to better prepare their lessons. If I respond correctly to your question, asking learners questions with the intention to enhance their critical thinking, it trains them to be able to respond to high order question.
D	There's' an existing correlation between asking learners critical thinking question and their academic performance. In this regard I would think, the more exposure they have on critical thinking questions, the better they perform.
E	Learners remain attentive during the lesson. In turn they perform better.

F	Improved in learner performance. I must honest Mr Zondo, with our subject in particular during results analysis we are always at the bottom since our learners find mathematics challenging.

Question 4: How do you know that asking learners critical thinking questions is associated with improved learner performance?	
Teachers	**Responses**
A	The more my learners cannot respond to critical thinking questions, they cannot answer similar questions during an assignment, test, or exam. I therefore think in my case it describes a poor pass rate.
B	Well, Mr Zondo, if my learners are unable to respond to my questions, chances are, they will be not be able to respond to them even during formal assessment. Hence, I strive and prepare them by asking a variety of questions ranging from low order to high order and probing them. Hence this explains a good pass rate I get each year.
C	I have seen from my learners over the years, the more they respond to critical thinking question during lessons, they better they perform during exams. Hence, I know that I must ask them more of these questions so that they can pass their exams without any challenges.
D	Exam question papers come with a variety of questions in different levels, ranging from low order to high order. A learner who is able to respond to a higher order question, stands a greater chance on also responding correctly to lower order questions.
E	I once tried exposing my learners to those questions, they hardly responded. However, I am aware that critical thinking questions have the potential of an improved learner performance.
F	I noted this from my previous school. Learners were performing exceptionally.

Question 5: Why you will recommend that teachers use *what, why,* and *how* questions for enhancing critical thinking in their mathematics classrooms?	
Teachers	**Responses**
A	I suppose, however it will depend on if learners will be able to respond to those questions. You know with the current learners I have, a method that I think works is to drill them on easier questions. **Researcher Probing:** Don't you think if learners, were taught how to respond to these questions, they will be in better position to respond. Wait, I haven't thought of that, interesting. I suppose so. I think I will plan around that.
B	Thank you Mr Zondo for that question again. I will always recommend that all teachers pose these questions during their lessons in mathematics classrooms, simply because they can measure if their learners understand a concept. Also, they will know if leaners have gaps or misconceptions. In this way teachers can better prepare their lessons to achieve the desired outcomes.
C	Absolutely, the same teaching practice used by my teachers while I was still a learner worked and I am using the same procedure and it is yielding positive results. Hence, I will recommend the same for other teachers during their lessons in mathematics classrooms.
D	I will recommend on the basis that; it allows learners to provide justification and arguments on their thinking. In turn it allows them to be confident in the subject manner.
E	I think I will. Remember Mr Zondo we are teaching learners not only to respond to exam questions but also to become problem solvers.
F	Yes, I will recommend that teachers use these questions. At the end of the day, we as teachers are required to pass our learners. So, asking them such questions has the potential of enabling them to respond well in higher order questions.

Appendix J: Sample lesson notes from different Grade 10 textbooks

Sample lesson from the grade 10 Mind Action Series learner's textbook, page 225

SIMPLE INTEREST

If interest is calculated at the end of each year using the original amount of money saved or borrowed, then it is called **simple interest**. For example, suppose that R2500 is invested for 3 years at 10% per annum (p.a.) simple interest.

The accumulated amount (future value) after 1 year is:

$$A_1 = 2500 + 10\% \text{ of } 2500$$
$$= 2500 + \tfrac{10}{100} \times 2500$$
$$= 2500 + 0{,}10 \times 2500$$
$$= R2750$$

(Notice that the interest made in the first year is R250)

The accumulated amount (future value) after 2 years is:

$$A_2 = 2750 + 10\% \text{ of } 2500$$
$$= 2750 + 0{,}10 \times 2500$$
$$= R3000$$

(Notice that the interest made in the second year is also R250 and R500 over the first two years)

The accumulated amount (future value) after 3 years is:

$$A_3 = 3000 + 0{,}10 \times 2500$$
$$= R3250$$

(Notice that the interest made in the third year is also R250 and R750 over the three-year period)

A useful formula which enables us to calculate the **future value** (A) of an original amount P, which has been invested for n years at a rate of $r\%$ **simple interest** is:

$$\boxed{A = P(1 + in)}$$

EXAMPLE 1

Tom invests R45 000 for 12 years at an interest rate of 13% per annum simple interest. Calculate:

(a) the accumulated amount (future value) of the investment in 12 years time.
(b) the simple interest received at the end of the 12th year.
(c) the simple interest received each year.

Solutions

(a)
$$A = P(1 + in)$$
$$A = 45\ 000(1 + 0{,}13 \times 12)$$
$$A = R115\ 200$$

(b) Interest received
$$= 115\ 200 - 45\ 000$$
$$= R70\ 200$$

(c)
$$\frac{R70\ 200}{12} = R5850 \text{ per year}$$

EXAMPLE 2

Lauren started to save money six years ago. The current value of her investment is R38 000. The interest rate for the investment was 7% per annum simple interest. How much did she invest six years ago?

Solution

$$38\ 000 = P(1 + 0{,}07 \times 6)$$
$$\therefore 38\ 000 = P(1{,}42)$$
$$\therefore \frac{38\ 000}{(1{,}42)} = P$$
$$\therefore P = R26\ 760{,}56$$

Lesson extracted from the grade 10 Everything Mathematics learners textbook on page 331

QUESTION

Carine deposits R 1000 into a special bank account which pays a simple interest rate of 7% p.a. for 3 years. How much will be in her account at the end of the investment term?

SOLUTION

Step 1: Write down known values

$$P = 1000$$
$$i = 0{,}07$$
$$n = 3$$

Step 2: Write down the formula

$$A = P(1 + in)$$

Step 3: Substitute the values

$$\begin{aligned} A &= 1000(1 + 0{,}07 \times 3) \\ &= 1210 \end{aligned}$$

Step 4: Write the final answer

At the end of 3 years, Carine will have R 1210 in her bank account.

Exercise 9 – 1:

1. An amount of R 3500 is invested in a savings account which pays simple interest at a rate of 7,5% per annum. Calculate the balance accumulated by the end of 2 years.
2. An amount of R 4090 is invested in a savings account which pays simple interest at a rate of 8% per annum. Calculate the balance accumulated by the end of 4 years.
3. An amount of R 1250 is invested in a savings account which pays simple interest at a rate of 6% per annum. Calculate the balance accumulated by the end of 6 years.
4. An amount of R 5670 is invested in a savings account which pays simple interest at a rate of 8% per annum. Calculate the balance accumulated by the end of 3 years.
5. Calculate the accumulated amount in the following situations:
   a) A loan of R 300 at a rate of 8% for 1 year.
   b) An investment of R 2250 at a rate of 12,5% p.a. for 6 years.
6. A bank offers a savings account which pays simple interest at a rate of 6% per annum. If you want to accumulate R 15 000 in 5 years, how much should you invest now?

Lesson extracted from the Western Cape learners' revision material, page 21

Financial Mathematics

Terminology:

Interest: is money earned when money is saved in the bank or it is money you have to pay on money you borrowed.
Simple Interest: Is the interest on an initial (principal) sum of money. Each year you receive or are charged the same amount of interest.
Compound Interest: Is also interest on an initial (principal) sum of money. If the interest is paid yearly for each year the principal amount, is the previous years final amount. The previous years final amount plus the interest for that year.

Simple Interest:

$$A = P(1 + i . n)$$

A: is the accumulated amount or final amount

P: is the principal amount or the original sum of money invested or borrowed

i **:** is the interest rate

n : is the number of periods, that is the number of years if interest is paid yearly or number of months if interest is paid monthly

Compound Interest:

$$A = P(1 + i)^n$$

A: is the accumulated amount or final amount

P: is the principal amount or the original sum of money invested or borrowed

i **:** is the interest rate

n : is the number of periods, that is the number of years if interest is paid yearly or number of months if interest is paid monthly

Example 1:

If you borrow R1000 at 12% simple interest for two years, how much will you owe after 2 years.

> To do this we will do two methods:
>
> 1. Reasoning
> 2. Using formulae
>
> This will show you that the formula works and could be used later.

Solution:

Method 1

12% of R1 000 is R120

After two years you will owe:

R1 000 + R120 (interest for 1st year) + R120 (interest for 2nd year)

= R1 240

Method 2:

> STEPS:
>
> - Identify which formulae will be used.
> - Write down using the variables in formulae what you are given.
> - Identify what it is you must calculate

$A = P(1 + i . n)$

$P = R1\ 000\ ,\ i = \frac{12}{100}\ ,\ n = 2$

$A???$: is the final amount which must be calculated

$A = P(1 + i . n)$

$= R1\ 000\ (1 + \frac{12}{100} \times 2)$

$= R1\ 240$

Example 2:

If you borrow R1000 at 12% compound interest for two years how much will you owe after 2 years.

> To do this we will do two methods:
>
> 3. Reasoning
> 4. Using formulae
>
> This will show you that the formula works and could be used later.

Solution:

Method 1

Year 1: 12% of R1 000 is R120

Year 2: 12% of (R1 000 + R120) is R134,40

After two years you will owe:

R1 000 + R120 (interest for 1st year) + R134,4 (interest for 2nd year)

= R1 254,40

Appendix K: Turnitin similarity report

Teachers' questioning practices for enhancing learners' critical thinking in grade 10 mathematics classrooms

ORIGINALITY REPORT

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Appendix L: Certificate of professional editing

CERTIFICATE
OF PROFESSIONAL EDITING

I, Barbara L. Louton, declare that I am a professional editor with a Bachelor of Arts in Professional Writing, a Master of Arts in Development Administration, and 19 years of experience as an editor, researcher, and writer.

I declare that I was contracted by Ayanda Sizwe Zondo, a doctoral candidate under the supervision of Dr. Prof. Jayaluxmi Naidoo, at the University of KwaZulu-Natal, to complete a professional edit of his thesis:

"Teachers' questioning practices for enhancing learners' critical thinking in Grade 10 mathematics classrooms".

I declare that I have completed a two-stage professional edit of the document. The edited addressed issues with the logic, structure, language, style (APA 7th Ed) and formatting of the thesis. Changes were tracked and comments were made in text with recommendations for the client to address.

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Responsibility for the originality and accuracy of the material presented in the proofread document lies with the student. I have not verified the originality or accuracy of statements, quotations or citations and references presented in the dissertation. Where I have detected inaccuracies, I have rectified them or reported them to the client. Please note that the client was free to make further changes to the proofread document after the proofread was complete.

I can be contacted at:
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Barbara L Louton	[redacted]	21 March 2025
Name	Signed	Date