SECONDARY SCHOOL LEARNERS' INTUITIVE MODELLING STRATEGIES FOR SOLVING PROBLEMS IN KINEMATICS

by

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To my daughters….

Presheen, Judisha & Sandrika

"Life is full of challenges, put your best foot forward and you will succeed."

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DECLARATION

I declare that the entire thesis unless otherwise indicated to the contrary is my own work, and has not been submitted previously for any degree at any university.

Pritha Devi Chetty

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ABSTRACT

The post-apartheid period focuses much attention on education. One of the major concerns is that of the poor Grade 12 examination results, especially in Physical Science and Mathematics. There is reason to believe that traditional methods for teaching Physical Science at schools are inefficient and that substantial improvement in instruction methods can be achieved by a vigorous program of pedagogical research and development.

The intention of this research was to gather and document qualitative data regarding problem-solving strategies used by Secondary School Physical Science learners when solving real-life problems. Specifically the purpose was to investigate whether it was possible for learners to model and formulate a particular kinematic equation on their own, without receiving any prior formal instruction in kinematics. Secondly the purpose was to investigate the intuitive mathematical strategies that they used.

Seven Physical Science learners from Grade 11 participated in this study. Their problem-solving behaviour was documented using think-aloud sessions, paper-and-pen solutions of problems and interviews. The learners individually solved two problems in kinematics dealing with speed, time and acceleration. The problems were presented in verbal form together with an incomplete table and they were required, in stages, to eventually develop a symbolic equation.

The representation of the problem-solving process in terms of intuitive modelling strategies provided insight into what conceptual and intuitive knowledge learners bring to bear in a problem situation. The insight obtained on the fundamental aspects of problem solving in terms of appropriate strategy use, inappropriate strategy use and misconceptions could help inform the structuring, representation, and access of knowledge.

Evidence obtained from this research shows that five out of thirteen (40%) correct responses were obtained without prior instruction nor any form of guidance. This shows

that learners have the potential to model strategies of real-life problem situations, formulate verbal relationships and translate these into symbolic form. Thus this study indicates that if learners are provided with appropriate guidance from educators, they might not only be able to formulate the equations on their own, but they might also be able to recall the equations with ease and apply them correctly in novel situations. However, this is a matter of further research not covered in this study.

Since no other research has addressed problem-solving strategies in Physical Science as it has in this study, it is hoped that the findings of this study would contribute substantially to the teaching of kinematics at Secondary School level.

CHAPTER ONE

INTRODUCTION

One of the aims of education, according to Ernest (1991:199), is the empowerment of an individual through education, providing "tools for thought" enabling that person to take control of his/her life, and to participate fully and critically in a democratic society. Learning is an individual and a life-long process requiring the capacity to judge when understanding has been achieved and to draw conclusions and make inferences from acquired knowledge (Arons 1997:364).

A number of philosophers have identified problems and problem solving as lying at the heart of the educational enterprise (Ernest 1991:281). Problem solving motivates, stimulates interest and creativity and provides enjoyment. It puts decision-making into the hands of the problem solver and this moves him/her towards increased self-confidence and personal satisfaction. It thus improves the quality of life not only for the individual but society as a whole. (Ernest 1991; Wheatley 1995; Watts 1994).

This study focuses on problem solving and considers the solving of problems as central to the teaching and learning of physical science in the secondary school classroom. However, it is not possible to divide mathematics and science into separate categories as according to Fremont (undated:173), mathematics is an abstract system of ideas which provides the scientist with a powerful tool for the study of the real world.

Students who enter higher levels of education particularly science and mathematics courses, do so with abilities and strategies that handicap them in achieving success. (McKeachie 1988:5). Novak & Gowin (1984), McDermott (1991), Clement (1980) and Arons (1997) have shown that while learners performed well on familiar problems, attempts to solve novel problems showed that serious conceptual misunderstandings exist. Thus such research amongst others shows that the traditional approach to teaching problem-solving has not enjoyed much success.

The learner memorizes an algorithm for getting the "right answer," but lacks an understanding of the concepts and propositions of physics that explain the phenomenon. According to Arons (1997:363), learners are being crushed into the "*flatness of equation-grinding automatons and forcing them into blind memorization of problem-solving procedures. We do not even give them a chance to begin to understand what 'understanding' really means.*"

Thus there exists a large gap between the "protoconcepts" with which most learners come to the study of kinematics and their grasp of the physical constructs in the face of conventional instruction (Arons 1997:45). In many cases one of the problems is that neither home background nor present day education has made learners aware of the alternative ways of approaching the problem-solving situation. In general these are teacher-directed and the learner learns to conform to the teacher's directions without any conscious thought about why the teacher directs them to carry out certain activities. (McKeachie 1988:4). In addition learners are not aware when they do not fully comprehend the meaning of words and phrases in the context in which they occur and this underlies substantial portions of "illiteracy" that we find currently deplored in many disciplines and not science alone (Arons 1997:19).

To develop genuine understanding of concepts and theory that underlies operational knowledge, learners must engage in deductive and inductive mental activity coupled with interpretation of personal observation and experience. This can be nurtured, developed and enhanced in the majority of learners provided it is experientially rooted and not too rapidly paced and providing the mind of the learner is actively engaged. (Arons 1997:347.)

This study employs a constructivist approach, of which the basic premise is that knowledge is constructed as a result of cognitive processes within the human mind and that the learner is an active participant in the construction of his/her knowledge. The learners come to science lessons already holding ideas about natural phenomena which they use to make sense of of everyday experiences. (Driver & Oldham 1986; Leonard [undated]; McDermott 1991; Haeney 1988 and Scott 1987.)

Meaningful learning, which implies the ability to interpret and use knowledge in situations not identical to those in which it was acquired, requires deep mental engagement by the learner. The learner's mind is not a blank slate on which new information can be written without regard to what is already there. If the educator does not make a conscious effort to guide the learner into making the modifications needed to incorporate new information correctly, the learner may do the rearranging. In that case, the message inscribed on the slate may not be the one the educator intended to deliver. (McDermott 1991:305.)

The intention of this research is to gather and document qualitative data regarding problem-solving strategies used by grade 11 learners when solving real-life problems they have not seen before. Specifically, the purpose was to investigate whether it is possible for learners to formulate some relationships/equations in kinematics on their own without prior instruction. This would be made available to the classroom educator, subject advisors of Physical Science, curriculum planners, pre-service and in-service education and training colleges.

The implementation of this transformed form of educating/learning cannot, at a stroke, solve the educator's (and learner's) problems, but may foster a more acute awareness for both the educator and learner of the nature of involvement in the teaching and learning process and it is hoped that, by degrees, it may prove to be successful.

Learners from a secondary school in Durban participated in this study in 1999. The problem-centred approach adopted aimed to build on learner's informal knowledge and facilitate the development of their conceptual and procedural knowledge through the solution of real life and other problems. The learners involved in this study were representative of those in a "normal" class.

RESEARCH QUESTIONS

Four main research questions will be addressed in this study:

1. What strategies do learners use to complete a table of a real-life problem situation involving speed and time ?

2. What are the strategies used by the learners in recognizing and formulating a functional relationship between the variables speed and time of a real life problem situation?

3. Are the learners able to translate the functional relationship into symbolic form (e.g. mathematical formula) ?

4. What is the feasibility of using modelling as a teaching approach for teaching speed and time relationships ?

OVERVIEW

This thesis is organized into five chapters. The first two chapters describe the research focus, theoretical framework and related research. Chapter Three deals with the research methodology employed in this study. The findings are discussed in Chapter Four under the headings: interpretation of terminology; results and strategies used in question one; results and strategies used in question two; and analysis and discussion of common strategies used in question one and two. Chapter Five summarizes the conclusion of the study; its strengths and limitations; the implications of the findings; and suggestions for further research.

RATIONALE

In this study, the learner was required to construct explicitly a meaning for a functional relationship derived from a real life problem, thus allowing him/her to grasp the concept intuitively before it became formalized symbolically. The underlying assumption is that such a construction would anchor the concept of functional relationships and give meaning to a real life problem. The basic approach is from a mathematical and scientific standpoint and relies on the research in both these areas.

Mathematics with its abstract system of ideas provides science with a powerful tool for the study of real life. "*Algebra is the language of the scientist*" because it adds to our ability to understand and describe relationships between objects in the physical world as well as between the algebraic symbols themselves. (Fremont undated:172.)

A scientific law, says Hestenes (1987:442), is a relation among descriptive variables which is presumed to represent a relationship among properties of real objects, because it has been validated in some empirical domain by the testing of models. Most of the laws of physics are expressed as mathematical equations.

While the evidence for the value of problem solving is strong, too little is known about how one becomes an effective problem-solver and what types of experiences will provide the potential opportunities for the learner to develop ways of coping with novelty. In particular we do not know much about problem-solving in practice. (Wheatley 1995:90.) It is hoped that evidence from this study would be useful to learners, educators, science and mathematics superintendents, curriculum planners, policy makers, in-service training and pre-service training colleges and the educational community in developing effective problem-solving skills in learners.

Educators often have this to say about their learners: "I taught them how to solve this just the other day and to-day, it is as if they have never seen it before". Generally educators unwittingly are quick to blame learners for their poor performance saying that they are either lazy or not scientifically inclined. However, in my experience as a physical science educator, I have found that learners struggle with problems in science and mathematics, in particular context-rich, real-life problems. When learners are asked about their poor performance in tests and examinations, the response that I usually get is: "I don't know? I studied really hard." Parents also are very concerned and reiterate what their children have to say. According to Heller et al (1992:627), this problem is peculiar not only to secondary school science but extends to college students as well who experience the following difficulties: "I can follow the examples in the textbook, but your test problems are too different" or "I understand the material, but I just can't solve the problems."

Reflection on the teaching of any subject, theme or unit should never become static, but should be continuously rethought in the light of new insights with regard to learners and the subject:

Figure 1.1

Not only the performance of learners should be evaluated, but also and especially the desired aims and objectives as well as the effectivity of the teaching methods that are employed. (De Villiers 1985:2, Schwebel & Ralph 1974:xi).

It appears that schooling is too often an assault on learner's ego because the rote, arbitrary, verbatim instruction so common in classrooms has few intrinsic rewards. Learners who do see meaning in such instruction often fail. For them school is at best frustrating and at worst an ordeal in which they must suffer the ridicule of the educator, classmates and sometimes parents. The cost of these factors, both to the individual and to society is enormous. (Novak & Gowin 1984:xi.)

The study of problem-solving strategies involves an attempt to understand how to help learners improve their ability in problem-solving, ability to remember and to think. This is based on an emerging cognitive theory of human learning and memory. One of the reasons for research in problem-solving been undertaken is because of the large numbers of learners who have demonstrated severe deficiencies in their knowledge of these processes and how to use what they know to their advantage. Studies discussed by McKeachie have shown that improvement in achievement occurs when learners are taught to use more effective problem-solving and studying strategies. (McKeachie 1988:328.)

The following paragraphs outline the need for educators to be aware of problem-solving strategies and the advantages of the modelling approach as opposed to the traditional approach of selecting appropriate formulae to solve problems; the need for learners and the educational community being aware of problem-solving strategies.

Kamii (1974:224) explains the poor performance as follows: when educators explain to a learner, demonstrate to him/her, or program according to their adult commonsense, what they think they are teaching and what the learner actually learns may turn out to be different things. This is because what we teach is received by the learner not directly but always through his/her cognitive structure. Therefore the accent should be on what and how the learner learns rather than on what we think we are teaching. Children are not miniature adults. Learners come to class with their own problem-solving strategies and what the educator does is to impose his/her methods onto the learner. We want learners to move on eventually to new and more powerful strategies, but, if these are forced upon learners regardless of their methods, they feel ashamed and defensive about their own. (Hughes 1986:177.)

According to Greer (1967:60), learner's informal solution methods should be respected and the introduction of formal methods (the ultimate need for which is not disputed) should be carried out much more gradually and sensitively.

Educators and researchers struggle for greater understanding of how learners learn and of what the educator ought to do to facilitate these processes (Schwebel & Raph 1974:xi). Their concern is the developing of knowledge, not skills or information. It is easy to look up a fact in a book, it is even fairly easy to memorize it, but to teach the underlying framework that alone would give meaning to the information is a total different matter. (Sinclair 1974:41.)

An educator who knows how learners think and the strategies used when solving problems will modify his/her perception and understanding of what is taking place in his/her classroom, and consequently, the quality and goals of his/her interventions (De Meuron 1974:232; Marshall 1994:23).

According to Arons (1997:384), a small portion (less than one third) of learners is ready for critical reasoning. The rest, lacking the steady, supportive help and explicit exercises required, resort in desperation, to memorization of end results and procedures. Failing to develop the processes underlying critical thinking, they fail to have experience of genuine understanding and come to believe that knowledge is inculcated by educators. Rief (1986:51) describes these conventional teaching practices in Physical Science as being "deleterious" because of excessive emphasis on mathematical symbolism and quantitative problem formulation. Skinner (1974:126) similarly regards rule-following as the "veneer" of civilization.

For physical science educators the most obvious implications of this research is that our traditional view about teaching is inaccurate, where learners lack deep processing that is, they do not understand the meaning of what they are learning. The learner is not an empty vessel in which the educator pours knowledge of physics equations, and functional relationships and problem-solving strategies. (Fuller 1982:44.)

Learners acquire knowledge best through recreating the process for themselves (Modell 1985:20). In the traditional approach to problem solving learners come to see the selection of correct formulae as the key to problem solving. Thus they tend to develop a problem statement for a list of formula-centered problem solving strategy as follows: select a problem statement for a list of given and unknown variables; search a list of formulae for an equation which involoves those variables alone; solve the equation for the unknown. This strategy is especially effective for homework problems where the necessary formulae can be found in the chapter from which the problems were assigned. Dedicated learners learn this strategy well by working a number of assigned problems for they know that "practice makes perfect." Indeed they may become quite adept at formula hunting. However, the formula-centered approach fails when learners are faced with problems requiring a deeper conceptual understanding.

Problem-centered learning, which leads to developing rules has often been found to facilitate transfer and problem-solving skills when compared with expository learning. According to Andre (1986:194), the problem-centered method forces learners to search for generality, expository methods may or may not require the learner to possess the generality. There is an important difference between being told what a definition or formula is and deriving one's own. The former requires only memorization and the latter emphasizes thinking.

To Polya (1945:16), the advantage of learners deriving formulae on their own from "experimental evidence" is that it acquires new significance and has a better chance of being remembered and appropriately used. It has been noted by a number of educators that when the lessons are "process" orientated i.e. focussing on the learners' thinking process rather than "answer" orientated i.e. focussing on performing skills and getting the right answer, learners' performance improves substantially. (James 1992:157.)

It is to be expected that an improvement in learner's problem solving performance would come by a deeper understanding of the nature of this process. We need to study what learners' natural approaches to problems are and what the actual obstacles to success are. It seems clear that teaching general heuristics in the context of a suitable collection of problems improves performance and leads to some consciousness of the strategies. (Bell 1979:363.)

In addition, educators need to know about strategies that learners apply when solving problems so that they can assist learners in developing these strategies. By virtue of our cognitive capacities, humans have discovered logic and have learned how to use it in drawing valid inferences from premises and data. It is seldom the case that an individual approaches a problem with no general hypothesis whatsoever to direct the interpretation of data. (Ausubel et al 1978:570.) Learners choose strategies that make sense in their scheme of things (Biggs 1984:130).

Reflecting on and finding out how one learns and thinks makes the process concerned more accessible and usable. As learners begin to understand how they can learn for meaning, it gives them confidence to try when they previously would not. As confidence develops through greater awareness of how learning occurs, the process of conceptual learning and thinking grows, so that there is a constantly changing and developing body of meaning. Metacognition helps a learner to become autonomous, but not isolated. The learners take charge of their own learning, because they know what is happening and can manage the process. Concepts and principles are learned, have meaning and are usable, and are not simply memorized as a set of words. (Biggs 1984:241; Duell 1986:237.)

If these strategies are based on lack of understanding, it is the role of the educator who is knowledgeable of the strategies, to develop these effective strategies in the learner rather than simply impose correct strategies. The more educators know about learners' individual conceptual frameworks, the better educators can design situations/learning experiences for the learners and the more likely learners will be able to rethink their conceptual schemas of their world-views.

Van Heuvelen (1991:893) has quoted many studies on problem-solving which indicate that learners do not understand the meaning of basic quantities. When given a problem, learners identify some structural feature described in the problem. They search randomly for and inappropriately use an equation they associate with that feature. Understanding must come before learners start using mathematics in problem-solving, otherwise equations become crutches that short-circuit attempts at understanding.

The reason for learners lack of understanding is that they find it difficult to directly relate their science and mathematics knowledge to the real world. The root of the problem is that science and mathematics is mostly taught in a decontextualized fashion, completely devoid of any relationship to the real world (De Villiers 1983; McDermott 1991; Kowleswar 1992; Brookes 1994). A possible reason could be that physical science and mathematics has developed in the mind of the educator as pure theoretical separate discipline. Mathematics and science is not viewed as a quantitative language which plays a role across many disciplines and within many real-life situations. Once mathematics and science skills and content have been taught completely divorced from any real world interpretation and meanings, attempts to do so at a later stage, are mostly futile. (De Villiers 1983; Kowlesar 1992.) According to Blignaut, Ladewig & Oberholzer (1985), the modelling approach is not being consciously dealt with in mathematics and science teaching.

Learners' own theories of learning and problem-solving often involve the notion that failure to succeed academically is a result of low innate ability, they attribute their failures to stable, unchangeable factors which they can do nothing about. Their motivation is low because they feel that it is useless to try changing attributions and self-concepts to include the idea that needed skills can be developed and may have a significant effect upon motivation. (McKeachie 1988:5.) Learners are often not aware of their lack of understanding or misunderstanding of concepts. They are often willing to accept wildly erroneous answers without questioning why or how such claims could be valid. They seldom test their claims against those that relevant concepts or principles would suggest or even against common sense. (Novak & Gowin 1984:74.)

Performance will be enhanced and with minimum anxiety by knowledge of one's memory processes, capacity limitations and repertoire of strategy skills, monitoring and evaluation of strategy effectiveness, organization of activities to changing task demands. Reflection on and knowing how one solves problems makes the process concerned more accessible and useable, it helps learners become autonomous, they take charge of their own learning because they know what is happening and can manage the process. (Biggs 1984; Andre 1986; Novak & Gowin 1984; Phyle & Andre 1986; Duell 1986.) Schoenfeld (1985:207), has shown that the conscious application of problem-solving strategies does indeed have a positive impact on learners problem-solving performance.

Superintendents in education are concerned about the poor results especially in science and mathematics but they have not been able to ascertain the cause of this. Articles on these topics are very rarely published and if they are, they are very general in content and lack much insight into research. (McKeachie 1988:327.)

Parents and School Governing Body members in most cases are not familiar with problem-solving and learning strategies. Parents may verbalize that their child does not know how to study, resulting in poor mathematics and science test and examination results but they do not understand why. Much research is not readily available to those outside the field. It is important to find a way to help educate these individuals about learning skills and strategies and why they are important. Good theories are needed which if implemented well will help to make a difference to results obtained by many learners. (McKeachie 1988:328.) Further, as we continue to increase the awareness of the educational community there is an increased probability that we can establish the conditions under which study and problem-solving strategies can be effective, identify the variables that influence the effectiveness of particular strategies and assess the kinds of changes in cognitive processing that results from learners' application of study habits. The results from this research will be translated into a language, format and programme that can be usable by schools.

The purpose of this research is to yield some insights into thinking processes of learners when solving problems. Furthermore it offers the prospect that these insights can be used to teach the learner such skills.

Once knowledge of the intuitive strategies used by learners together with the specific conceptual stumbling blocks is identified, it can be made available to stakeholders involved in education, and hence more appropriate educational strategies can be designed for teaching and learning in science and mathematics.

CHAPTER TWO

REVIEW OF LITERATURE

INTRODUCTION

The review of literature will be dealt with under the following headings:

THEORIES OF LEARNING RELATED TO SCIENCE AND MATHEMATICS

The respective rationale underlying two major learning theories that had a major impact on the learning and teaching of science will be discussed in some detail. They are the traditional approach whose underlying theory is Behaviourism and the alternative approach whose learning theory is Constructivism.

PROBLEM-SOLVING

An effective way of learning is by solving problems. It challenges the curiosity of the learner and stimulates him/her to develop new knowledge and concepts. Problem-solving dates back to the beginning of mankind forming, an integral part of a person's everyday life. However, it is only recently that educational researchers have focussed on strategies used by learners when solving problems in mathematics and science. This discussion includes a brief historical development of problem-solving strategies and also outlines learner's personal attributes that contribute to problem-solving.

PROBLEM-CENTERED APPROACH

This approach to teaching commences with a problem, query or puzzle that a learner has to solve. In this way learners are motivated to get actively involved in the learning process. The discussion that follows advocates the problem-centered approach and outlines some difficulties affecting its manipulation.

ALGEBRAIC FUNCTIONS AND SYMBOLS

Mathematics is mankind's greatest intellectual achievement. Although it may have changed in some ways since its origin, the basic concepts or ideas remain the same. The origin of mathematics, the nature of functions and the interpretation of symbols will be discussed.

MODELLING

Educators and researchers have found that the traditional way of "*getting knowledge into the heads of students*" is not very successful. Most cognitive mathematicians and scientists now believe in a constructivist model of knowledge, and they are now considering modelling, which is based on a constructivist view of teaching and learning, as a more successful method of teaching. The steps involved in the modelling process and the main categories of modelling will be discussed in some detail.

THEORIES OF LEARNING RELATED TO SCIENCE AND MATHEMATICS

Over the past decades the views of and approaches to how learners understand and learn mathematics and science have varied. Various research perspectives on the learning and teaching of mathematics and science have been adopted. Learning theories such as Behaviourism, Piagetian theories, Cognitive Psychology and Constructivism have been dominant influences in education. Two of these theoretical approaches, which have differing principles and approaches will be discussed. They are Behaviourism and Constructivism and they seem to have had the greatest impact on the teaching and learning of Mathematics and Science.

The Behaviourist Theory

Behaviourism is a philosophy of the science of human behaviour. This theory, which is also referred to as the traditional approach or connectionist theory, relates to the empiricist philosophy of science that all knowledge originates in experience. The traditional empiricist motto is "*there is nothing in the mind that was not first in the senses.*" Hence a person can obtain knowledge of any reality, because the senses, the image of that reality, corresponds exactly to the reality (a replica or photocopy). (Olivier 1992:194).

A child develops "mental" traits from the environment (Watson 1925:75). In mentalistic formulations the physical environment is moved into the mind and becomes experience. Behaviour is moved into the mind as purpose, intention, ideas and acts of will. Perceiving the world and profiting from experience becomes "*general-purpose cognitive activities.*" (Skinner 1974:102.)

Behaviourists assume that learners learn what they are taught, therefore it is assumed that knowledge can be transferred intact from one person to another. The leaner is viewed as a passive recipient of knowledge, an "empty vessel" to be filled. Behaviourists believe that knowledge is taken directly from experience and that a learner's current knowledge is unnecessary to learning. (Olivier 1992:195.)

Behaviourists claim that there is a response to every effective stimulus and that the response is immediate. By effective stimulus, is meant that the stimulus must be strong enough to overcome the normal resistance of the passage of the sensory impulse from sense organs to muscles. Habit-forming has to come in before certain stimuli can be effective. It starts in all probability in embryonic life and even the human young environment shapes behaviour so quickly that all of the older ideas about what types of behaviour are inherited and what are learnt, breaks down. (Watson 1925:79). The more times a stimulus-induced response is elicited, the longer the learning will be retained. Positive reinforcement by way of success and reward strengthens any behaviour and negative reinforcement results from failure. (Mackenzie 1977; Olivier 1992; Skinner 1974; Watson 1925.)

The educator is "*to cultivate the mind as a farmer cultivates his field, and the intellect is to be trained as a vine is trained in a vineyard.*" In the absence of any adequate account of the development or growth of a person's exposure to an environment, the almost inevitable result is that important aspects of thinking are assigned to genetic endowment. (Skinner 1974:115.) Consequently learning must proceed from the simple to the complex, short sequences of small items of knowledge and exercise of these in turn through drill and practice (Olivier 1992:195). According to Bouvier (1987, cited in Olivier 1992:195), learners learn by stockpiling, by accumulation of ideas.

From a behaviourist perspective, errors and misconceptions are not important because it does not consider learners' current concepts as relevant to learning. This perspective is succinctly put by Gagne (1983): "*The effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules ... This means that teachers would best ignore the* *incorrect performance and set about as directly as possible teaching the rules for correctness.*" (cited in Olivier 1992:195.)

Critique

Although the view has been powerful and influential it does have significant short-comings. Behaviourism was based largely on an empiricist approach. It never managed to produce a significant body of lasting scientific knowledge comparable to what can be found in many other less endowed sciences. Partly as a result of this lack, behaviourism's grand theories have almost all been abandoned. (Mackenzie 1977:1.) Educators adopting the behaviourist method of teaching, have at some time or other expressed their concern that concepts taught are never remembered even over a short period of time (Wilkerson & Hundert 1991:161). It has long been known by researchers and educators, that learners forget concepts and make mistakes in computations, despite these very careful teaching methods. Sinclair (1974:41) contends that it is easy to look a fact up in a book, it is even fairly easy to memorize it, but to teach the underlying framework that alone will give meaning to the information is a totally different matter.

Children do not think like adults and their fundamental knowledge is differently structured. (Sinclair 1974:41). Piaget's theory presents knowledge, not as something imposed on humans by hereditary or the environment, but as freely created from within (Furth 1980:8). Scott (1987:3), like many other researchers who oppose the traditional view, strongly believe that in the long run this direct teaching has drastic negative effects on children's ability to learn mathematics and science. The rationale/explanation offered is that if the educator shows the learner how to solve a problem, this knowledge is not properly constructed from within. Instead the learner memorizes bits and pieces, which s/he frequently assembles and applies in the wrong order for an unsuitable situation. (Murray 1992:11.)

According to the behaviourist approach it is necessary to first present/teach the theory and then involve the learners in practical problems. In other words, learners are first taught the rules (skills) which they are expected to apply to solve problems. From this perspective, doing mathematics and science involves calculating answers and mastering the procedures and rules to do so. Thus learning mathematics and science is thought to consist of absorbing pieces of knowledge with an emphasis on acquiring skills rather than on conceptual understanding and operations. This form of practice acts to perpetuate the educators' and learners' beliefs that school mathematics and science is about finding ways for calculating correct answers rather than for developing numerical or functional relationships through their own reasoning. Learners do well on straightforward calculations, but experience difficulties when the problem becomes complex.

As the educator is seen as the dispenser of knowledge, the learners are considered to be passive receivers/learners. The responsibility of learning according to this approach lies with the educator and not the learner. The learners are therefore dependent on the educator.

A theory that seems to be a powerful source for an alternative to direct instruction is that of constructivism. These theorists believe that the behaviourist theory has missed the richness of human behaviour. Constructivists believe that one has to examine the problem-solving ability of learners, that is, the higher mental processes that they use to deal with problems. They speculate on what people are thinking, on what strategies they are using and that these processes are not observable. Educators cannot simply transmit information to their learners and assume that it will be learned. For learners to understand new information, they must be given the opportunity to engage in the process of coming to know, through problem-solving, exploration, observation and practice, with direction and assistance from the educator. (Weade 1994:87.) This is in direct contrast with the behavioural learning theory.

In support of the constructivist view, Wood, Cobb & Yackel (1994:178), consider mathematics as a "*science of pattern and order*" that relies on logic rather than on observation as its standard truth. It does however incorporate a scientific approach in learning the truth by means of observation, stimulation, and experimentation. Mathematics and science then, becomes a human activity in which individual meaning is constructed through sensorimotor and conceptual activity.
This approach will be discussed in more detail.

The Constructivist Theory

A constructivist theory of learning guides the theoretical orientation of this research, which aims to understand the learning process from a cognitive point of view. In the past few decades, particularly in the 1970's, constructivism rapidly gained recognition. Constructivism is a theory of knowledge with roots in philosophy, psychology and cybernetics (Ernest 1991:70).

The development of knowledge is a process different from physiological maturation, different also from mere accumulation of new knowledge content, which can be called learning from outside (related to accommodation). Rather, development is a construction on the part of the child, i.e., from within. It is a progressive restructuring and results in the child's acquisition of new knowledge capacities. (Furth 1980:3).

For Piaget, says Furth (1980:8), development is intrinsic to knowledge. Piaget's theory presents knowledge not as something imposed on humans by heredity or environment, but as freely created from within. The first principle drawn from Piaget's theory (Kamii 1974:199) is the view that learning has to be an active process, because knowledge is constructed from within. Physical knowledge can be built by discovery, but logico-mathematical knowledge cannot. It can be built only by the child's own invention. All logico-mathematical structures have to be invented, or created by a child's own cognitive activity. In the logico-mathematical realm, the role of the educator is not to impose and to reinforce the "correct" answers but to strengthen the learner's own process of reasoning.

Piaget was the first constructivist in the sense that his view that knowledge was constructed in the mind of the learner was based on research on how children acquire knowledge. Piaget believed that knowledge is acquired as the result of a lifelong constructive process in which we try to organize, structure, and restructure our experiences in light of existing schemes of thought and thereby gradually modify and expand these schemes. Indeed, his definition of knowledge "*invariance under transformation*" has no meaning outside of the constructivist perspective. Piaget argues that objects appear "*permanent*" or "*invariant*" as the result of the individual's coordination or experiential data and the subsequent projection of these co-ordinations into the world that lies beyond our senses. (Bodner 1986:874.)

According to constructivists (Ernest 1991; Olivier 1992; Heaney & Watts 1988 and others), knowledge is made by us and our way of experiencing, rather than given by an independently existing objective world. This knowledge arises from the interaction between experience and our current knowledge structures and does not simply arise from experience. This view accounts for the development of subjective knowledge of the external world. It explains how an individual constructs subjective knowledge, notably a theoretical model of a portion and how this knowledge or model develops (Ernest 1991:71).

The individual from the moment of birth, receives sense impressions from, and interacts with the external and social worlds. They also formulate subjective theories to account for, and hence guide, their interactions with these realms. These theories are continually tested through interaction with the environment, animate and inanimate and they survive in a pragmatic and instrumental sense only as long as they are useful. (Ernest 1991:71; Bodner 1986:875.)

From a constructivist perspective knowledge is not passively received by the learner but actively built by the cognising subject from within. Each of us builds our own view of reality by trying to find order in the chaos of signals that impinge on our senses. The only thing that matters is that whether the knowledge we construct from the information functions satisfactorily in the context in which it arises. (Bodner 1986:874.)

It is not possible that knowledge can be transferred ready-made and intact from one person to another (Olivier 1992:196). Children construct personal models to explain their experiences and their environment. They are the architects of their knowledge, rather than simply passive recipients of second-hand knowledge from their educators. Personal constructs are tentative models of the environment. These models are there to predict and control events and are continually being evaluated against personal criteria. A construct will only be abandoned or modified if it no longer seems to be useful if it no longer successfully predicts and controls events. Thus, such personal constructs are the basic intellectual tools brought to bear in problem-solving. Problem-solving is a way of challenging these personal constructs (Heaney & Watts 1988:9).

Human (1999:2) regards the constructing of knowledge as a synonym for learning thereby defining learning as the (personal) construction of (personal) knowledge without disclaiming that other people may have a profound influence on the knowledge a learner constructs as a result of interpreting a given representation.

To help the learner assimilate abstract concepts, it is essential to engage the learner's mind in active use of concepts in concrete situations. The concepts must be explicitly connected with immediate, visible, or kinaesthetic experience. Furthermore, the learner should be led to confront and resolve the contradictions that result from his/her own misconceptions.

The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality. Thus reality is personal not absolute (Ernest 1984; Haeney & Watts 1988; Kamii 1974; Wood, Cobb & Yackel 1994; Bodner 1986).

Learners not only interpret knowledge, but they organise and structure this knowledge into large units of interrelated concepts, which Skemp (1971:59) called 'schema.' A schema is a conceptual structure stored in memory. For a given input, different schemas may be activated for different people. Even for the same person and the same input, at different times, different schemas may be activated.

Learning basically involves the interaction between a learner's schemas and new ideas. This interaction involves two interrelated processes, assimilation and accommodation. Assimilation occurs when a new recognisably familiar idea is encountered. This new idea is incorporated directly into an existing schema, thus expanding existing concepts. When this does not work, when our experiences do not fit our ideas, equilibration can occur by adjusting our schema to fit the sensory data we perceive and this process is called accommodation (Bodner 1986:875). Accommodation involves the restructuring and extension of current knowledge (Olivier 1992; Human 1996). While assimilation and accommodation are twin processes of knowing, they are obligatory occasions for further development insofar as present activity feeds back into the child's structure (Furth 1980:8). Piaget believes that a child is more likely to accommodate his behaviour to solve a problem when the new behaviour that is required differs only slightly from those already in his repertoire (Birns & Golden 1974:128).

The way in which a learner embeds specific information (that is assigns meanings, seeks and senses purposes and justifications, experiences epistemological nature and relates information to possessed knowledge) depends on a variety of factors other than the nature of the explicit representations of syntactic information that may be available to him/her. These factors include the knowledge already possessed by the person, the messages conveyed by the context in which the information is encountered, the demands (obligations) experienced by learners, the social relationships impacting on learning, other information that is simultaneously available, in short the social and cognitive contexts in which information is encountered. Hence different learners in different situations (or even in the same situation) may construct quite different imbedments for the same piece of syntactic information, they may come to know the same thing in substantially different ways (Human 1999:3). Emphasis is therefore placed on the learner's own thinking and judgement and a constructivist environment provides opportunities for the learner to construct his/her own knowledge and moral standards through his/her own reasoning (Kamii 1974:213).

The constructivist model assumes that there is a purpose to learning and that learners do perceive the purpose and actively engage in constructing the meaning to be gained by bringing their ideas to that experience (Evertson & Murphy 1994:300). The implications of the constructivist model for education require a subtle shift in perspective for the educator. A shift from someone who "teaches" to someone who tries to facilitate learning, and a shift from teaching from imposition to teaching by negotiation. (Bodner 1986:876.)

Learning science and mathematics according to Scott (1987:7), involves learners not only in adopting new ideas, but also in modifying or abandoning their pre-existing ones. The following are a few key points in this view that need mentioning:

- What is already in the learner's mind matters.
- Individuals construct their own meaning.

- The construction of meaning is a continuous and active process.
- Learning may involve conceptual change.
- The construction of meaning does not always lead to belief.
- Learners have the final responsibility for their learning.
- Some constructed meanings are shared.

The adoption of a constructivist view of learning necessarily has fundamental implications for classroom activity. Some of these are: teaching commences with orientation, elicitation of ideas, restructuring of ideas, application of ideas and reviewing change in ideas. (Scott 1987:10.)

Thus the research approach adopted by constructivist researchers and educators is quasi-ethnographic, which means that it includes clinical and task-based interviews, participant observation and descriptive case studies. Researchers need to look beyond what learners say, i.e. explicitly for knowledge that is implicit in what they do. In current research on children's understanding of mathematics and science, researchers look at the invented procedures children develop, as one kind of evidence of their implicit knowledge. A similar stance has been adopted in this study. Research evidence indicates that prescriptive teaching methods of computation and problem-solving (based on Behaviourism) necessarily induce in a learner a receptive, passive, dependent attitude towards learning, whereas a constructivist approach induces an active, self-reliant, creative reflective, attitude towards learning. (Penchaliah 1997:9.)

PROBLEM SOLVING

Problem solving is an integral part of life, an everyday activity, a process in which everybody engages at varying degrees of sophistication every time they carry out a task or make a decision. The aim of a progressive educator is to contribute to the overall development of the growing human being, to develop the learners' creativity and self-realization through experience of learning mathematics and science. This involves the learners' active response to the environment, autonomous inquiry by the learner, seeking out relationships and creating artefacts and knowledge. (Ernest 1991:192.) Problem solving is a very sophisticated cognitive skill, hence understanding and teaching scientific problem-solving is both practically important and intellectually challenging.

Leser (1980) defines a problem as, "*a situation in which an individual or a group is called upon to perform a task for which there is no readily accessible algorithm which determines completely the method of solution*" (cited in Ernest 1991:287). There are four components to problems: the goal(s), the givens, the obstacles and methods. Problem-solving consists of the mental and behavioural activities that are involved in dealing with problems. It may involve cognitive, emotional and/or motivational components. (Andre 1986:170.)

While science and mathematics educators and researchers acknowledge the importance of problems in scientific progress, many (for example Skinner, 1974:111; Munson, 1988:12) are of the view that problems should focus on the justification of theory or understanding rules by exposure to natural contingencies. However, to me, learners from solving real life problems, acquire and develop concepts and processes by creating scientific theories and by applying these theories to appropriate problems. Nott (1988:45), called problem-solving in such context creative problem solving.

According to Schroender & Leser (1989:31) the theme of school mathematics and science has shifted from "back to basics" to "problem solving." A large number of problem-solving resources have been developed for classroom use in the form of collections of problems, lists of strategies to be taught, suggestions for activities, and guidelines for evaluating problem-solving performance. Much of this material has helped educators in making problem-solving a focus of their instruction. However, it has not provided the sort of coherence and clear direction that is needed, primarily because to date little agreement has been reached on how this goal can be achieved. There are vast differences among educators of what it means to make problem solving the focus of school science and mathematics. One of the ways of coming to grips with these differences is to distinguish among three approaches to problem-solving instruction:

- Teaching *about* problem solving: An educator who teaches about problem solving highlights Polya's (1945) model of problem solving (or some minor variation of it). Learners are explicitly taught the phases in problem solving.

Teaching about problem solving also includes experiences with actually solving problems, but it always involves a great deal of explicit discussions of, and teaching about, how problems are solved.

- Teaching *for* problem solving: The educator concentrates on ways in which science and mathematics being taught can be applied in the solution of both routine and non-routine problems. A strong adherent to this might argue that the sole reason for learning science and mathematics is to be able to use this knowledge gained to solve problems.
- Teaching *via* problem solving: The learning of science and mathematics in this way can be viewed as a movement from the concrete (a real-world problem that serves as an instance of the mathematical concept or technique) to the abstract (a symbolic representation of a class of problems and techniques for operating with these symbols).

This research advocates teaching via problem solving.

It is Schroender & Leser's (1989:39) belief that mathematics instruction should focus on the understanding as their goal. By doing so they will shift from the narrow view that mathematics is simply a tool for solving problems to the broader conception that mathematics is a way of thinking about and organizing one's experiences. Thus the role of problem solving will change from being an activity that learners engage in after they have acquired certain concepts and skills to being both a means of acquiring new knowledge and a process for applying what has been learned previously.

The primary advantage of self-generated knowledge is that it is tied to what the learner already knows. Furthermore, when learners construct new knowledge for themselves, they learn not only concepts, facts, skills, etc., but also how to manage and regulate the application of this new knowledge. That is, they are in charge of this knowledge (and of their learning in general), thereby making it more useful to them in solving problems and in learning new concepts and skills. (Schroender & Leser 1989:39.)

Nott (1988:44) advocates problem solving because it is a more effective method of learning. He believes that when solving problems, learners will be active (learning is better through doing), they will be able to participate in choosing/selecting/ formulating problems and by encountering problems learners can be challenged to find things out, learn skills or processes or question their own beliefs about the world (strong motivation promotes better learning). However the problems should be relevant in that they must relate to the learner's knowledge and experiences (learning should start with the familiar before it moves to the unfamiliar). Hobden (1998:227) adds that learners should be encouraged to bring their contexts to the classroom. Then, collaboratively with the educator, they can construct problems that "*touch their lives.*" Learning in science will than involve the reconstruction of personal experiences.

According to Nott (1988:44) and Heaney (1988:9), if learners are actively involved in solving relevant problems that they have participated in formulating, they will be highly motivated, they will enjoy the experience and so they will learn whatever it is we want them to learn better than by learning with other teaching methods. Polya's (1945:16) contention is that when the learner develops a formula s/he is convinced that it is correct because s/he derived it carefully himself/herself from 'experimental evidence.' The details of the formula acquire new significance and are linked up with various facts. The formula has therefore a better chance of being remembered and the knowledge of the learner is consolidated. Finally, these questions can be easily transferred to similar problems. After some experience with similar problems, a learner may perceive the underlying general idea that makes use of relevant data, variation of data, symmetry and analogy. If s/he gets onto the habit of directing his/her attention to such points, his/her ability to solve problems may definitely profit. Polya's (1945:v) view is that when learners are given a problem proportionate to their knowledge, it challenges their curiosity and may give them a taste for, and some means of independent cognitive strategies of their own.

Using problem solving to acquire and develop concepts and processes by creating scientific and mathematical theories and by applying these theories to appropriate problems, learners acquire a mathematical and scientific education. Vocational arguments also advocate problem-solving because it is a process that people will need at work. It prepares them to be creative and innovative, to be able to work with others and to appreciate technological and economic relevance. (Nott 1988:45).

Success in problem-solving enhances self-confidence and disposition to improve, whereas a mild degree of failure may prove salutary by increasing drive, attentiveness and willingness to consider other alternatives (Ausubel et al 1978:581).

The possession of a sound cognitive structure, particularly if clear, stable, and discriminable, facilitates problem solving. Without such knowledge, no problem-solving is possible irrespective of the learner's degree of skill in discovery learning; without it the learner could not even begin to understand the nature of the problem confronted. (Ausubel et al 1978:571.) Personal heuristics are constructed by the individual as they reflect on their experiences, not as a result of direct instruction and practice (Wheatley 1995:2). Creativity is the highest degree of problem-solving involving novel or original transformation of ideas and the generation of new integrative and explanatory principles (Ausubel et al 1978:566).

According to Phyle & Andre (1986:144), problem-solving consists of a set of highly practiced sequences (schema, frame, plans, etc.) that provides the basis for an integration of processing components. So what is implied is a highly specific mode of information processing that requires little or no conscious monitoring. These highly developed schema, rules, procedures, plans, etc. involve sequences of processing stored in long-term memory storage that, along with controlled processing in short-term working memory, provides that basis for skilled performance. This view is supported by Scheider & Shiffrin (1977).

Problem-Solving Strategies

Looking at problem-solving strategies historically, it started with the Behavioural approach which was based on "trial-and error" learning. Such learning occurs when a stimulus situation demands a response, but the correct response is not dominant in response hierarchy for that situation. The learner tries out responses in their order of dominance. Incorrect responses are extinguished, and the correct response reinforced until it becomes dominant in that situation. This view leaves little room for thought and planning in problem-solving. (Andre 1986:172)

Next followed the Gestalt approach involving thinking about a situation and rearranging the mental elements into a structure that provides a solution to the problem. Wallis in 1926, described the stages of problem-solving as a preparation stage entailing an analysis of the problem, followed by an incubation stage involving subconscious consideration of the problem while the problem-solver is relaxing or considering something else, next is the inspiration stage where the solution comes to the learner unexpectedly, and finally the verification stage which involves checking the solution and working out the details. (Andre 1986:172.)

This was followed by the Piagian approach which focussed on mental logic.

Since the early 1960's, the dominant position in American psychology had moved from a behavioural position to a cognitive information-processing position. Problem-solving was viewed as the processing of information by an information processing system (the brain) such that information in the initial state is transformed into the information of the desired end state. (Andre 1986:175)

Although Dewey outlined his problem-solving strategies in 1910, it has not been appreciably improved upon by recent researchers. Polya (1945), Rief et al (1976), Schoenfeld (1985), Andre (1986), Haeney (1988), Mestre (1991), Van Heuvelen (1991), Heller et al (1992), Watts (1994), Dhillon (1998), Ausubel et al (1978) and Snowman (1986) have analysed problem-solving strategies and their views can be basically categorized as following:

- Identify, understand and translate the problem:

  This step involves describing the problem, exploring it verbally and pictorially, and considering the given conditions, assumptions, goals and obstacles. At this stage, according to Mestre (1991:58), the problem-solvers must use their knowledge to analyse the problem qualitatively before resorting to mathematical manipulations. While van Heuvelen (1991:891) agrees with the qualitative analysis of a problem, he places emphasis on the problem being seen as describing a physical process.
- Plan a solution:

  This stage of building a model is a creative, brainstorming stage. It draws on existing knowledge and experience. The existing cognitive structure plays a key role in problem-solving as is evident from the fact that the solution of any given problem involves a reorganization of the residue of past experiences so as to fit the particular requirements of the current problem situation. Since ideas in cognitive structure constitute the raw materials of problem solving, whatever transfer occurs, positive or negative, obviously reflects the nature and influence of cognitive structure variables. (Ausubel 1978:571). Frequently this step may call forth some algebraic relationship and equation that gives relationships among the various quantities in the problem.

- Implementation strategy:

  The problem is analysed by using ones ability to do science and mathematics, i.e. substituting numerical values into algebraic equations and scientific formulae and computing numerical results.

- Interpretation of the results:

  The solution is translated back into the language of practice to decide whether it is plausible.

The above four steps were also considered by Blignaut et al (1992:1) and de Villiers (1994:34) in the "modelling" approach which involves starting any problem at the natural origin, the real-life situation.

Many textbooks present learners with what the authors think are sensible, systematic schemes for approaching the solutions. The first step is to draw a diagram of the physical situation, set up the position line, translate from the verbal statement to symbolic so as to firstly tabulate the known quantities together with the symbols and secondly to list the symbols of the unknown quantity, select the equation that gives the most efficient solution, make the necessary calculations and finally interpret the results. If the textbook does not provide this systematic approach then the educator does. (Arons 1997:38).

The central difficulty of problem-solving is the need to make judicious decisions to find among the many reasoning paths that lead nowhere, one that leads to the desired goal. Although there are no prescriptions guaranteeing good decisions, there are systematic methods that can make the search for a solution much easier. (Reif 1986:51.) Research evidence according to Schoenfeld (1982:31), strongly concurs the correlation between the use of heuristics and competent problem-solving performance thus suggesting that problem instruction via heuristics will enhance learner's problem-solving performance.

According to Ausubel et al (1978:566), problem-solving refers to any activity in which both the cognitive representation of prior experience and the components of a current problem situation are reorganized in order to achieve a designated objective. Such activity may consist of more or less trial-and-error variation of variable alternatives or of a deliberate attempt to formulate a principle or discover a system of relations underlying the solution of a problem (insight). The trial-and-error approach consists of random or systematic, approximation and correction of responses until a variant emerges.

The insightful approach, implies a "set" that is orientated towards discovery of a meaningful means-end relationship underlying the solution of a problem. It may involve either simple transposition of a previous learned principle to an analogous new situation or mere fundamental cognitive restructuring and integration of prior and current experience to fit the demands of a designated goal. (Ausubel et al 1978:567.)

Interpersonal factors influence problem-solving strategies. Intelligence is one of the most important determinants of problem-solving ability. Reasoning power, comprehension, memory, information processing and the ability to analyse also affect problem-solving. Other cognitive traits such as open-mindedness, flexibility, capacity for generating multiple and novel hypotheses, attentiveness, problem sensitivity, intellectual curiosity and the ability to integrate ideas influence problem-solving in rather self-evident ways. Cognitive style is obviously a relevant factor, particularly with respect to general strategies of problem-solving. Many temperamental and personality traits such as high kinetic level, decisiveness, self confidence and self-critical ability facilitate problem-solving when present in a moderate to high level. (Ausubel et al 1978:579.)

In conclusion, if problem solving is to become the focus of the curriculum, then it must be central to the way educators teach. Learners cannot be expected to solve problems unless they are helped to build their mathematics and science knowledge from problems rather than taught procedures that later will be applied to problems. (Schroender & Leser 1998:8.) The educational community needs to be aware of the changing view of science and mathematics and the implications for schooling (Lindquist 1998:12).

THE PROBLEM-CENTERED APPROACH

The problem-centred approach is a learning strategy based on constructivism, the basic premise of which is that learners actively build up their knowledge based on their own experience. Problem-based learning is the most significant innovation in education (Boud & Feletti 1991:13).

The principal idea behind problem-based learning is that the starting point for learning is not an exposition of disciplinary knowledge, but a problem, a query, or a puzzle that the learner wishes to solve. It is a way of constructing and teaching courses using problems as the stimulus and focus for learner activity. Learners are moved towards the acquisition of knowledge and skills through a staged sequence of problems presented in context, together with associated learning material and support from educators. (Coles 1991:301; Boud &Feletti 1991:13.)

The problem-centred approach uses stimulus material to engage learners in considering a problem, which as far as possible, is presented in the same context, as they would find it in "real-life," this often means that it crosses traditional disciplinary boundaries. Information in how to tackle the problem is not given, although resources are available to assist learners to clarify what the problem consists of and how they may deal with it. Learning that has occurred from this process is summarized and integrated into the learners' existing knowledge and skills.

Traditional teachings have been criticized for their lack of attention to issues such as the relevance of subjects, for placing poor attempts at developing skills of inquiry in learning, emphasizing on memorization and for inadequate portrayal of context of major issues and problems. Problem-based learning addresses these criticisms head-on and uses such deficiencies as the foundation of its approach. (Boud & Feletti 1991:15). The important components of a problem-based curriculum are cumulative, integrated and progressive learning and consistency in learning (Engel 1991:29).

According to Margetson (1991), Ernest (1991), Boud & Feletti (1991) problem-based learning:

- Takes into consideration how learners learn. It is becoming increasingly apparent that learning takes place most effectively when learners are actively involved in learning in the context in which knowledge is used.
- Encourages open-mindedness, reflective, critical and active learning.
- Is morally defensive in that it pays due respect to both learner and educator as persons with knowledge, understanding, feelings and interests who come together in a shared educational process.
- Reflects the nature of knowledge, i.e., knowledge is complex and changes as a result of responses by communities of persons to problems they perceive in their worlds.
- Prepares learners to be able to learn quickly, efficiently and independently when particular knowledge base or information is required rather than to have assimilated all information which their educator believes is desirable i.e. as Engel (1991:21) suggests, developing learners for capacity rather than learning for the sake of acquiring knowledge.

This approach to learning raises the question of discovery and/or invention, for the learner appears to have to discover knowledge or invent knowledge that is new to the him/her, even though it is well known to experts (Margetson 1991:46). Thus a problem-centred pedagogy represents a powerful emancipatory teaching approach, and when successfully implemented, empowers learners epistemologically. That is, it encourages active knowing and the creation of knowledge by learners and it legitimates knowledge as mathematical and scientific, at least in the school context.

Coles (1991:297), from his studies on "deep processing," a process which occurs when learners understand the meaning of what they were learning, provided evidence that deep processing learners who elaborate their knowledge, i.e. who see the interconnections and links between different knowledge areas not only gain the highest scores in examinations which test their knowledge but, are better able to retrieve and use in some novel situation the information they have learnt.
The major difficulties affecting the implementation of problem-based learning are:

- The role change of the learner changes to active participant, constructing, interpreting, making and exploring meaning and taking responsibility for their own learning. Their habits and expectations of learning are in conflict with traditional methods although they do see the relevance and benefits of problem-based learning.
- The role change of the educator. Since the role of the learner changes, the educator's role changes accordingly. The problem-based educator is responsible for designing appropriate activities, observing learners at work and guiding their progress through questioning and feedback. To be successful the educator must trust the learners to do the work of learning. Given the range of topics that any one case can stimulate, educators cannot be the authority on every topic under discussion. In addition, the educator needs to possess a broad set of specific teaching skills to use in responding to learners' needs.
- The negative reaction of colleagues. The teaching of process skills is difficult for many teachers who may not have developed these skills fully themselves. If they feel inadequate they may not put enough on the learners developing and practising these important processes. Without the direction, support and confidence which problem-solving strategies and process skills give the learners, they can often flounder.

Successful implementation of problem-based learning does not come easily. All the strengths and skills of educators are required. Their behaviour and beliefs are challenged. Complex difficulties may arise, and educators will have to possess the ability to explore options and generate creative solutions in co-operative contexts. Commitment, determination and teamwork are essential, and above all self-knowledge and considerable understanding of the learning process. (Todd 1991:135.)

Despite the challenges facing the problem-centred approach, many educators are attempting this approach in the hope of creating active, interdependent and independent learners; holistic, divergent and creative thinkers; and people who can solve problems or improve situations (Drinan 1991:315).

ALGEBRAIC FUNCTIONS, EQUATIONS AND SYMBOLS

Mathematics is the language of the scientist. It not only aids in the understanding of our environment, but also provides the language with which scientists communicate. Algebra adds to our ability to understand and describe relationships between objects in the physical world, as well as between the algebraic symbols themselves. Thus, if there is a single theme that runs through all of algebra, it is the mathematical notion of function. (Fremont undated:172.)

The Origin of Mathematical Knowledge

The oldest written records of mankind's mathematical knowledge was found in and around Egypt and dates back to some 4 000 years before the birth of Christ. These documents showed mankind with so comparatively advanced a mathematical system, that it is clear that a great break-through had taken place much earlier, which dates back to as far as 100 000 years before Christ. The documents represented one of the huge achievements of the human mind and held an important clue to the nature of numbers. It also illustrated the interplay between practical needs and deep ideas that are characteristic of the history of mathematics. (Morgan 1972:11.)

The investigation of motion and mechanics, which was regarded as intermediate between mathematics and physics, was first investigated by a Greek philosopher-mathematician, Archytas in the year 400 AD. He is regarded as the founder of "mechanics." The Babylonian's exploration with numbers took them towards the discovery of relations that held true whatever the numbers used.

It was only much later, at the beginning of the seventeenth century that scientists like Descartes took up the challenge of translating data involving motion and mechanics into formulae and laws. Descartes, a French mathematician and scientist who was born in 1596 and died in 1650, was the first philosopher to describe the physical universe in terms of matter and motion. He was a pioneer in the attempt to formulate simple, universal laws of motion that govern physical changes. He was also the first to use the algebraic language of functions virtually identical to the modern use, as well as the concept of a function which has been called the most important idea in all mathematics. Thus the concept of a function evolved from the application of mathematics to the study of the physical universe. Many functions occur throughout the sciences. (Morgan 1972; World Book Encyclopedia 1994.)

In 1637 Descartes published his discovery of analytical geometry, known as "Cartesianism," proposing mathematics as the perfect model for reasoning. Scientists like Galileo and Copernicus investigated motion, but it was only as the seventeenth century opened did they start thinking to translate such data into formulae and laws. (Morgan 1972:90.)

The problem of notation was not fully solved until nearly 1000 AD. In 1500, Europeans were still setting out their arguments, largely in words. Later in 1600 a group of pioneers, such as Steven, expressed their ideas in algebraic form using symbols. In their attempt to solve various types of equations, the later mathematicians acquired considerable skill in handling symbols and, perhaps began not to worry too much about what these symbols stood for. The "=" sign was introduced in 1557 by an English mathematician. (Morgan 1972:83.)

Mathematics is one of mankind's greatest intellectual achievements. Numerical knowledge is present from a very early point in ontogeny. Knowledge entailed by numerosity perception and correspondence construction is present during infancy. Counting and rudimentary arithmetical algorithms emerge during the transitional period between infancy and early childhood, and arithmetical knowledge develops during early childhood. (Klein & Starkey 1987:13.)

Algebraic Functions and Equations

Knowledge about the nature of a function and interpretation of symbols is essential for the construction of algebraic expressions during the modelling of practical situations (De Villiers 1985:3).

The mathematical term, function, is a special kind of relation between sets of elements that are usually, although not always, numbers. A function *f*, is defined as a "a set of ordered pairs (x,y) where x is an element of set X; y is an element of set Y and no two pairs in *f* have the same first elements. A function indicates the relation between the dependent and the independent variable(s) as expressed in the form of an algebraic formula. The set of values belonging to the argument is the domain of the function, and the set of values belonging to the function value is the range of the function.

Learners can first be introduced informally to the concept of functions by means of incomplete flow diagrams, which they are required to complete. There are three types of flow diagram exercises, one where the input numbers and rule are given and the learner has to produce the output number; the other type is where the rule and output numbers are given and the learner has to produce the input numbers, and the third type is where the input numbers and output numbers are given and the learner has to identify the rule. The same three exercise types may be presented in the form of tables, using algebraic expressions to describe the rules, or using a real life statement to describe the rule and usc of a table to formulate an equation. Exercises like these show the relationship between variables and specifically of the dependence of one variable (the output set) on another (the input set). Most of high school algebra is a study of such relationships (functions). (Human et al 1989:25.)

There are three principle ways of expressing a function: namely by using a table, a graph, or an equation. The most common means of expressing a function is a formula that gives the function values by a rule embodied in an equation. A general definition of an equation, according to Herscovics & Kieran (1980:577), is "*any algebraic expression of equality containing a letter or letters.*" The concept of equation can reach a good number of learners by it being introduced through problem-solving.

In this study, learners were required to construct an equation based on arithmetic identities (from a table). According to Herscovics and Kieran (1980:576), it seems to make good pedagogical and psychological sense to introduce initially mathematical forms that can be anchored in some specific content. However, the use of arithmetic identities can be used as an intermediary step in the acquisition of the concept of equations and it can be abandoned once the learner has constructed meaning for the purpose involved.

An integral part of a variable is its domain, that is, the set of values that can be assigned to it. The nature of the domain is influenced by the situation. The situation suggests the units and the form of the variable. Context is another characteristic of a variable that is interconnected to the domain. A variable similar to a situation can either be abstract or contextualized. Abstract variables are usually discrete (e.g. natural numbers). Contextualized variables are most often continuous. Of the continuous variables, the following are used most frequently in graphing and functions: time, length, speed, temperature, weight and age. (Kowlesar 1992:10.)

Rooy (1988:36) noted that an extremely important strand in elementary algebra is the construction of formulae and equations as mathematical models of real situations, and specifically of construction of formulae fitting given sets of data representing the relationship between two variables.

The Use of Symbols

A symbol system is a set of symbols corresponding to a set of concepts, together with relationships between the symbols, corresponding to relationships between the concepts, thus symbolic understanding is a mutual assimilation between a symbol system and an appropriate conceptual structure. (Skemp 1971:60.)

We use written symbols to aid our memory, both short and long-term. Symbols make our schema and concepts more available to our consciousness and that of others, which can be done only by the further use of symbols. (Skemp 1972:203.) Letters in equations may stand for a label referring to a concrete entity or, as variables standing abstractly for a number of things (Rosnick 1981:418). Letter symbols in mathematics and science are used to represent numbers although it is often used inappropriately as a shorthand for objects.

Letter symbols are used with several different meanings by the adult mathematician in algebra, including:

- Letter symbols as place holders for specific unknown numbers in equations
- As representative of many values and as holders for arbitrary numbers from a certain set to describe the general properties of these numbers
- As place holders for numerical variables in algebraic expressions or formulae. (De Villiers 1985.)
- Letter as object can be used to cope successfully with items like 'simplify $2a+5b+a$.' Here no content is given, but learners can solve the item by inventing one. (Kuchemann 1982:48.)

Hart (1981:104), from her research, categorized learners' interpretations of the letters as follows:

- Letter evaluated. The letter is assigned a numerical value from the outset.
- Letter not used. The letter is ignored or, at best it is acknowledged but without meaning.
- Letter used as an object. The letter is regarded as shorthand for an object or as an object in its own right.
- Letter used as specific unknown. The letter is regarded as a specific but unknown number and can operate upon it directly.
- Letter used as a generalized number. The letter is seen as representing, or at least as being able to take, several values rather than just one.
- Letter used as a variable. The letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values.

Several researchers e.g. Rosnick (1981), Nickerson (1985), Kuchemann (1982), etc., besides making reference to their own similar research, have also made reference to

Clement's (1980) study focussing on students' ability to translate English sentences into algebraic expressions. The results showed that students find these translations difficult. The difficulties that learners have in understanding mathematical symbolism cannot be overlooked, says Skemp (1971:59).

A common problem encountered was that of 'reversed equation,' which occurred when letters were used as a shorthand for an object, rather than as a number. Reversed equations, claims Clement (1980), reflect misconceptions about the meaning about variables and equations. The two sources of the reversed equation error are "word-order matching," which involves a direct mapping of two words into the symbols of an equation. The second source of error is the application of what is called "static comparison." In this case, the learner appears to understand what the sentence implies. However, s/he believes that the correct way to express the relationship is to take the larger number with the symbol representing the larger group.

Learners, being influenced by textbooks and classroom instruction, interpret letters in several ways. Commonly the letter as an object is intended only as an analogy. Learners not only use letters as objects, but objects (or pictures of objects) seem to be used as numbers. (Kuchemann 1982:49.) Herscovics & Kieran (1980:578) have confirmed from their studies that learners have a problem in performing arithmetic operations on algebraic expressions, which they say is partly due to the difficulties the learners have in thinking of the letter as representing a number. Textbooks have displayed an uneasy mixture of the letter as objects and the letter as numbers. In addition, some have shifted from the letter as object to using the same letters to describe a numerical property or quantity of object in the same problem. This leads to confusion in the mind of the learner and the formulation of vague of expressions.

Throughout mathematics, the terms and expressions in the formal notation have both referential and formal functions. As referential symbols they refer to objects or cognitive entities external to the formalism. As formal symbols they are elements in a system that obeys rules of its own, and they can function without continuous reference to the mathematical objects they name. As referential functions, algebraic expressions and equations can be used to represent the relationships that hold in situations. If one understands the referential linkage between situations and algebraic formalism, one is in a position to construct the equations that correctly 'mathematize' a situation and thus use formal algebra to reach particular problem solutions. (Resnick et. al. 1987:200.)

In order to understand and meaningfully manipulate the function concept, it is necessary to view letter symbols as variables. Besides letter symbols assuming several values, they also describe the behaviour of functional relationships. There is some sort of relationship between the letters, and the functional values change in a systematic way, i.e. the one is dependent on the other and further more, how the values of one are dependent on the other. (De Villiers 1985:3.)

One great power of algebra is that it allows extensive manipulation of relationships among variables within a completely reliable system that does not require continuous attention to the referential meaning to the intermediate expressions that are generated. Potentially unbearable demands on processing capacity would be placed on individuals who tried to reason through some of the complex problems for which algebra is used if, at each step, they were considering physical, situational, or specific quantitative referents for the transformations they produced. Algebra is not only a device for reducing capacity demands, its very abstraction away from the situations, quantities, and relationships that are its referential meaning is part of what permits certain mathematical deductions to be made.

Learners will engage in the necessary mental construction with whatever knowledge they have that they construe as relevant. This means that learners' representations of the learning problem, along with the specific knowledge they have, will control the kinds of constructions they make. (Resnick et al. 1987:201.)

According to Booth (1989:58), a major part of learners' difficulty in algebra stems precisely from their lack of understanding arithmetical relations. The ability to work meaningfully in algebra, and thereby handle the notational conventions with ease, requires that learners first develop a semantic understanding of arithmetic. Research done by Liebenberg et al (1999:173), suggests that learners are not aware of the underlying strategies of arithmetic operations and their properties and that this situation is most likely due to a predominately computational focus in the early grades.

The distinction between content (concepts, rules, relationships) and mathematical form (the notation and symbolism used to express the content) is reflected in this study. Learners can use these equations to construct meaning for the concept of nontrivial equation. In participating in the construction of equations, learners acquire a level of formal understanding of the topic. (Herscovics 1980:579.)

MODELLING

Modelling is firstly a mathematical and scientific method/process/skill which entails the construction of a mathematical model describing, representing or idealizing a certain practical situation. The model may be in the form of number sets, geometric figures, equations, tables, graphs, formulae, etc., to describe the property or properties of the real object or phenomenon under study (Sadovskii & Sadovskii 1993:8).

The mathematical world is a world of abstract ideas, whereas the physical world is a three-dimensional reality. Mathematical answers to real-life problems do not always fit the physical situation. Thus, we begin to intuitively develop the idea that mathematics is a "model," which matches reality remarkably well. As such, mathematics provides much information, but results have to be checked in the physical situation to be sure that they "work." (Fremont undated:188.)

Arithmetic story problem solutions depend upon constructing mental models of the situation that constrain the relations among the given and unknown numbers. These models rather than the particular words of the story are used to infer the arithmetic operations that are required.

Hestenes (1987:441) describes a model as "*a surrogate object, a conceptual representation of a real thing which behaves in accordance with physical laws.*" Oberholzer (1992:144) says that modelling could also be viewed as a teaching style and can be thought of as mathematics used "*in context of everyday life.*"

There is nothing new, nor unusual about modelling. In fact, it is obvious to scientists, since they have learned to follow it automatically in the analysis of physical situations and problems (Hestenes 1987:444). Using the means and the language of mathematics in industrial, economic, or any other design, is in effect, mathematical modelling (Sadovskii & Sadovskii 1993:8).

Modelling as a teaching approach begins with a real-life problem, because the laws of nature are built at the interface between our sensory experiences of the external world and our reasoning about these experiences. Piaget suggested the interaction model of assimilation-accommodation-equilibration as the way knowledge and problem-solving strategies are constructed. In this model of the dynamic interaction between the minds of the learners and the internal experiences, the time when they are most likely to develop new understandings and new strategies is when their present experiences do not fit their mental preconceptions. This period of disequilibration, of being slightly confused, is the time when they are most likely to make intellectual growth. (Fuller 1982:47.)

In traditional teaching practices, theory comes before practice but with the modelling approach, theory is developed after solving practical problems. Modelling is the process of taking any problem which, at the time of doing it, does not have a clear-cut algorithm or any other standard way that can or may be used, to obtain an answer. Thus instead of thinking of the problem as an effort to determine some unknown quantity, Van Heuvelen (1991:898), adopted the modelling approach by encouraging his students to think of the problem statement as describing a physical process, the objective of which is to represent that process or event in ways that lead to qualitative and quantitative understanding.

Learners create their own computational algorithm. Solving the problem is mainly the responsibility of the learner and the educator's role is merely to help and not to provide a cut-and-dried method of solution, since this would not enhance learners' conceptual understanding of science and mathematics (Madell 1985:22).

The modelling approach is based on the Constructivist view of teaching and learning, it acknowledges that learners interpret instructional situations in profoundly different ways (De Villiers 1992:2). When a learner perceives an idea, s/he considers ways in which sense can be made of it by asking questions that are intellectually appealing and induce the learner to seek answers and to reflect, for example, on its usefulness, interest or significance. Thus the content and nature of knowledge is constructed. (Human 1999:4.)

The Process of Modelling

Four distinct steps can be isolated in the modelling process:

- Know your problem and understand it. This includes amongst others, determining the important characteristics, deciding on data to use and ignore, and assumptions to be made.
- Build your model. This is a creative process requiring the development of mathematical relationships. When you build a model, you move away from the real world. The model must be adequate to the real object under study, i.e., it must correctly describe the object based on certain characteristics. This is a matter of primary importance. Hestenes (1987:446) advocates the slogan "*The model is the message.*"
- Solve your model, which requires application of mathematical techniques. With the advent of technology, appropriate software can assist in solving the model, however, human ingenuity and understanding is absolutely essential for the other three processes. New computer software can greatly assist with routine manipulation involved in this step. Human ingenuity and understanding is absolutely essential. If a model is inappropriate the computer may produce an answer which is senseless. (De Villiers 1994:34.) Modern developments in mathematics and in computer studies have made available new analytical models,

which have resulted in a revolution of modelling as part of a scientific method (Ost 1987:363).

- Interpret your results. Check if it is realistic by critically comparing it with a real-life situation.

The modelling process together with some of the different mathematical processes at various stages has been represented by de Villiers (1989:25) as follows:-

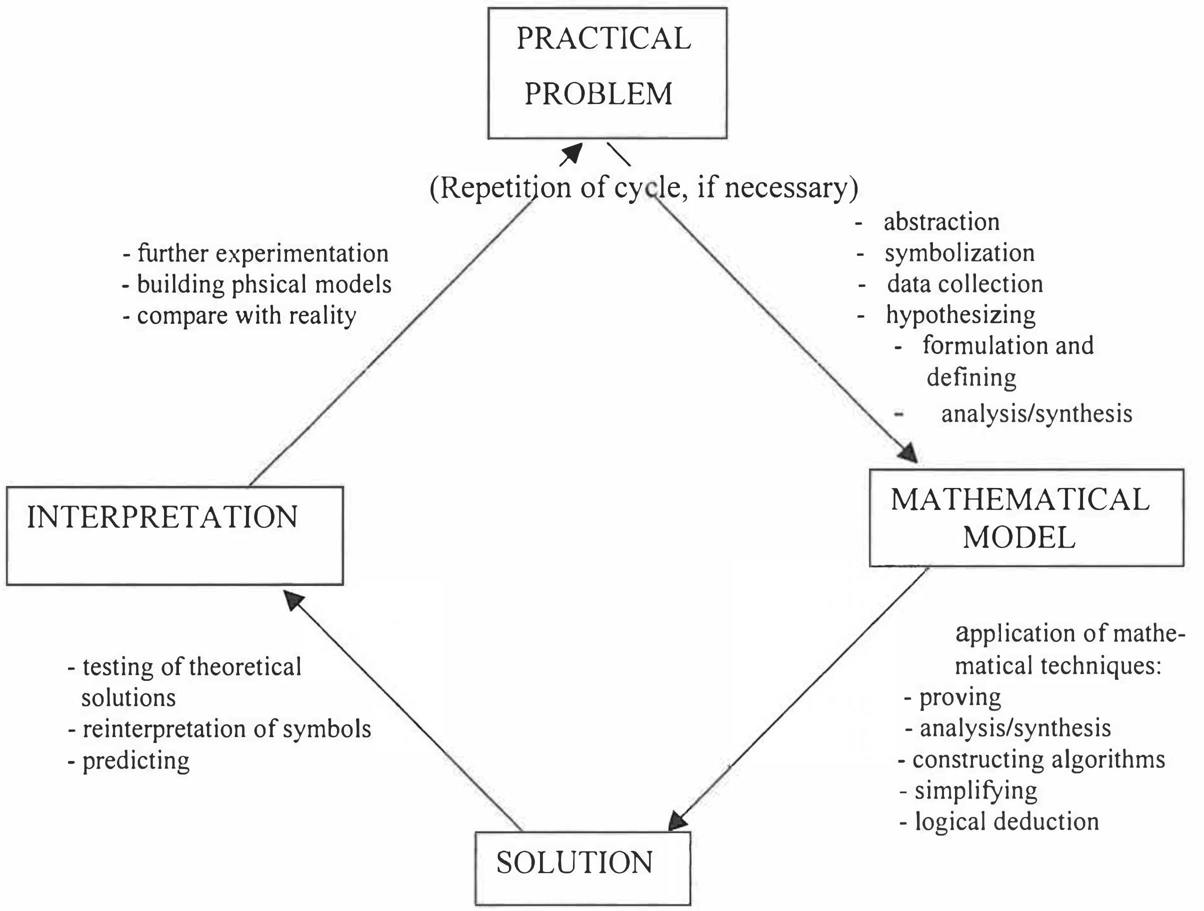

Figure 2.1

Since a model is in a sense, simpler than the object itself, it does not usually stimulate all the features of the object, but only those of most importance to the investigation, so that it can be studied more conveniently. It all depends on the object (phenomenon, situation) under study and on the characteristics the model takes into account. (Sadovskii &

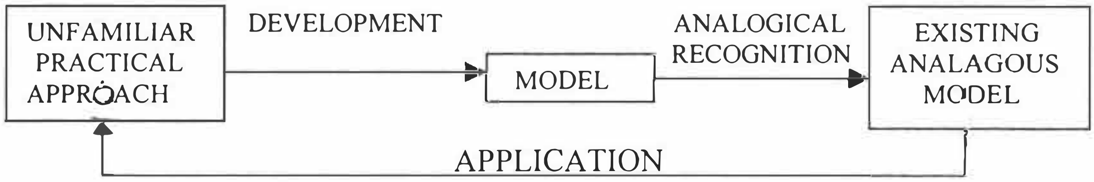

Figure 2.3

Creative Application

The creation of a previously unknown model consisting of totally new concepts and techniques, to represent an unfamiliar situation. This is one of the many ways in which theory in mathematics and physics is created.

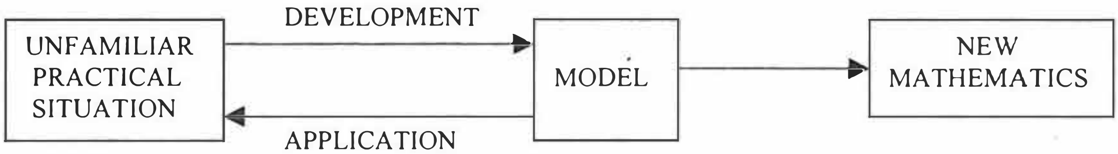

Figure 2.4

These three categories roughly increase in level of difficulty from category 1 to category 3. Category 3 suggests a powerful teaching strategy, which is especially important if science is at all to be represented to learners as a useful tool by which humans strive to understand the world.

This study adopts category 3 as a teaching strategy whereby functional relationships are derived directly from practical situations. When concepts, formulae, equations, etc., are directly abstracted from practical situations and problems, it is likely that the links between such content and the real world is stronger than when it is attempted to create such links only after the content has been presented in a vacuum. (De Villiers 1993:2; Kowlesar 1992:16.)

Modelling can be used to develop skills of explanation, interpretation, predictions and analysis. Theoretical models and modelling are inexpensive to use in the classroom or laboratory. Perhaps one of the greatest values of the computer is its availability as a sophisticated tool to work with data and to develop important skills of modern scientific enquiry. The relationships among variables can be readily observed. Modelling is an effective method to provide learners with experiences in hypothesis formulation. (Ost 1987:367.)

Although the modelling approach is a time consuming teaching method, the long-term benefits could be manifold. The learner could develop a thorough grasp of the subject matter which has a better chance of being remembered and it is more accessible and usable, even in novel situations; rapid progress is made and there is no need for endless repetition before mastery of subject matter is gained; it taps on the learners' natural creativity and ability to solve problems rather than the learner becoming preoccupied with trying to comply to the rules of the educator that they lose sight of the actual problem; it instils in the learner a positive attitude towards solving problems knowing that they can and should solve computational problems relying on their own intellectual thinking ability.

Modelling theory tells us that a situation in the real world is accounted for "physically" by constructing a mathematical model to represent it. In Hestenes' (1987:50) view there are two kinds of representations for the model: an external (objective) representations in terms of mathematical symbols, maps, diagrams, etc. and an internal (subjective or mental) representation in the brain of someone who understands it, i.e. to understand a mathematical model one needs a corresponding mental model. Evidently to construct such mental models is what physicists mean by " physical intervention." Cognitive development could best be described as development of mental modelling skills.

Modelling is an ideal way to introduce decision-making as it is used in science, mathematics, technology and society. In addition to developing important skills for use in science and mathematics, the learner trained in modelling, will have gained an important general education. (Ost 1987:368.)

The hurdle that the educator has to overcome in adopting the modelling approach is "weaning" the learners from a formula-centered problem solving strategy that proves to be successful in dealing with examples from the textbooks. They need to be confronted with situations where the formula-centered strategy clearly fails, and recognize that a better strategy is available. To facilitate the transition to a powerful model-centered strategy, the educator needs a clear understanding of the modelling theory and a systematic method for teaching it. (Hestenes 1987:449.)

Mathematical modelling of the physical world should be the central theme of physics and mathematics instruction, it calls for severe reorganization of priorities in physics and mathematics teaching, which can be justified on strong epistemological and psychological grounds (Hestenes 1987:453). After all, says van Heuvelen (1991:898), a physicist depends on qualitative analysis and representations to understand and help construct a mathematical representation of a physical process.

Understanding models, knowing how to model, and recognizing the use of modelling is certainly part of "technological literacy." In Hestenes' (1987:446) view, a problem cannot be fully understood until a model has been constructed. Models and modelling can be effective and efficient ways to relate science, mathematics and technology to society (Ost 1987:368).

OTHER STUDIES ON MATHEMATICAL MODELLING AND PROBLEM-SOLVING STRATEGIES

While a considerable number of studies have been carried out on problem-solving strategies, very few have addressed the issue of learners' intuitive modelling strategies used to solve real life problems in kinematics prior to formal instruction.

According to researchers, some of whom are Dhillon (1998), Mestre (1991), McDermott (1984), Larkin & Reif (1979) and Heller et al (1992), research on problem solving, based on real life problems has focused on "*what mathematics could be extracted*" from the problem (Salzano 1983:7). When learners solve real-life problems in kinematics, they are expected to select and apply appropriate equations. However, in this study of real-life problems, the learners were provided with the relevant variables i.e. speed and time, and they were required to derive and formulate their own equation. They did not have access to any equations since the study was conducted prior to any formal instruction in kinematics.

Salzano's Study on the Teaching of Skills in Mathematical Modelling

The study by Salzano (1983) focussed on the extent to which the teaching strategies developed on the modelling process were effective in helping learners develop their modelling ability. By analysing learners' intuitive modelling strategies and their misconceptions, Salzano was able to devise a successful teaching program. Similarly it is hoped that from the analysis of this present study of learners' intuitive modelling strategies in solving problems, educators and curriculum planners can implement a 'problem-solving' teaching program based on the modelling approach.

Salzano conducted her study with average and below average, fifteen year old learners from "Design and Technology" classes and "Physics" classes.

In order to develop a teaching strategy for the modelling process, Salzano first conducted a study with forty Design and Technology learners. She presented them with open problems with which they were familiar. They were required to write down their analysis of these problems and this was followed by individual interviews. This was done to elucidate both how they were able to articulate their thoughts and what steps they had taken in the resolution of the problem.

A detailed analysis of the scripts showed the following problem areas: the learners had a tendency to effect analysis without following an effective plan; their solutions seemed to be derived more from a 'guess' rather than from a reasoned argument; they tended to be 'impulsive' rather than 'reflective'; they were reluctant to make assumptions or to express personal judgements; they did not appear to have difficulty in generating variables but rather in distinguishing the most important ones and creating relationships between them.

From the analysis, Salzano developed her teaching strategies which aimed at taking into account the needs of the learners; to be used for the solution of open problems; and to help learners overcome the difficulties which they encounter in the modelling process.

The strategy developed included activities which tended to make learners' thoughts explicit and to organize them in a way such that the correct problem to be solved could emerge from the initial open one. The lessons were conducted for three terms with 40 learners from the Design and Technology course. In these lessons learners were encouraged to make use of open diagrams as an aid to analysing the problem and to recognize the key areas of it. This was followed by class discussions facilitated by the educator and these discussions were helpful in generating new approaches to the problem and developing detailed sub-problem charts.

The second phase of the research aimed to investigate to what extent the teaching strategy developed was valid. To this end a comparison was effected between the two groups: one group of forty learners who had experience in the resolution of open problems which the strategy and materials developed and another group of forty learners from the Physics course who had not. The comparison was done by using the results of three tests given at an interval of three months. The learners were required to work individually and write between two to five pages about each problem, to present structured arguments rather than subjective opinion and to recognize the problem as being open-ended. They were told that it was the quality of the argument used rather than the specific content of the answers which was being marked.

There were no substantial differences on the mean scores obtained on the tests, however, analysis of the scripts and interviews revealed significant differences between the two groups.

Analysis of the question: "Terry is soon to go to secondary school. The bus trip to school costs 25p and Terry's parents are considering the alternative of buying a bicycle. Help

Terry's parents decide what to do by carefully working out the relative merits of the two alternatives," revealed the following: both groups tended to qualify the variables by assumption and rarely by reasoned calculation; few of the Technology group considered an economic model to be a sufficient answer even if they seemed equally concerned with some of the non-economic factors; and the Physics group appeared to have more difficulty in relating the variables to each other and hence the solution phase was rarely reached. Most of the scripts of the Physics group appeared to be little more than a list of unquantifiable facts such as convenience, safety, social facts etc.

Another problem used was: "A school wishes to purchase notebooks for the pupils to use for recording and correcting their spelling errors. One of the three notebooks supplied can be chosen. Which is the best buy? Write a short report (1 to 2 pages) which explains which notebook should be bought and how you have come to your descision."

The analysis of the Technology group on this problem showed that their work was systematic and linear as compared with the Physics group, whose work for the most part, was large and dispensed. The Technology group tried to qualify the variables mathematically and their decisions seemed to be based on reasoned calculations and assumptions. The Physics group on the other hand tended to get lost in amplified analysis without managing to reach the essence of the problem and since the variables were rarely related to one another, their decisions often appeared to be a flat guess.

The third problem was: "Planning a car park: Merchant Ltd. Are planning to construct a consumer car park on the vacant land next to their supermarket. If the land measures 54.2m by 158.9m, design the best layout for the car park."
A study of the scripts to problem three revealed more pronounced differences in the techniques used to arrive at a solution to the problem. Whereas the scripts of the Physics group examined the different possibilities for eventual layouts, those of the Technology group consisted of a written analysis of the problem itself and the design of the best layout followed as a conclusion to the analysis. Most of the Technology group learners started their analysis by "asking questions," something that was emphasized during the

lessons. This account to Salzano was a success bearing in mind the general ability level of this group and the difficulty which the resolution of open problems provided to them.

The overall analysis of the work of the Technology group showed the application of the techniques used in class for the resolution of open problems. They did however, appear to have difficulty in generating relationships between variables. These learners enjoyed working with "real" problems more that the Physics group.

The fact that the mean scores of the Physics groups were slightly higher than that of the Technology group was attributed to physics being a problem solving subject. Physics is taught with large numbers of problem elements, so problem solving elements are developed through physics courses, and the real problem solving elements in Technology instruction often is new to the learners.

Salzano's study has shown that besides learner's performance in problem solving improving after receiving instruction in mathematical modelling, learners also enjoy working with real-life problems.

Dhillon's Study of Individual Differences Within Problem-Solving Strategies Used in Physics

Dhillon (1998) carried out a comparative study of the problem-solving strategies used by experts and novices in solving problems on rotational dynamics, focussing specifically on the activities within the strategy with a view to influencing the representation of knowledge for a problem-solving program in rotational dynamics. Evidence of the strategies and activities discussed by Dhillon are present in the learners' performance in the present study. In addition both these studies are based on a constructivist perspective on learning.

Within the process of problem-solving Dhillon identified six commonly used strategies which are reported in literature and five general strategies or methods. Within each of these he identified and described fourteen activities, constituting physical and cognitive actions. These activities were used to describe and compare the fundamental aspects of expert and novice problem-solving styles. The representation of the problem-solving process in this alternative way using activities provides novel insight into the minute processes taking place within the general methods and problem-solving strategies. The relationship between the activities and the general methods and strategies authenticates this alternative way of viewing and representing problem solving. Representation using the activities can help inform and accurately pinpoint student difficulties. The insight obtained on the fundamental aspects of problem solving through the activities and in relation to the strategies has helped to inform the structuring, representation, and access of knowledge. These have in turn informed the development of a problem-solving programme for use by high school and first-year university students.

Strategies give an overview of the learners' plan of action and reasoning employed, but do not provide the specific information or the type of help required by learners. They do however, supply insight into sequencing and representing large blocks of problem-solving knowledge. In Dhillon's opinion providing appropriate help to learners and having a meaningful interaction with them requires knowledge of the activities performed within the macroscopic problem-solving strategies. A strategy commences with an activity, constitutes a series of activities, and is concluded with an activity. Within each strategy an activity or set of activities can be performed repeatedly. The same problem may be solved in varying ways by different individuals due to dissimilar problem-solving steps, strategies and knowledge. This implies the need to obtain insight into the activities that the solver performs at the fundamental level.

The purpose of Dhillon's study was to:

- Identify the problem-solving activities within the broader strategies used.
- Depict the expert and novice and problem-solving styles using these activities.
- Relate the activities to general problem-solving methods used by the participants.
- Relate the activities to the problem-solving strategies reported in the literature.

Six questions were chosen for the problem-solving data collection. The study was conducted on nine novices and four experts.

Dhillon identified five general methods or strategies namely, assessing information; transformation of information; attempting one-step solution; generalizing information application and solution assessment. Six commonly used strategies namely, generate-and-test; means-end analysis; problem decomposition; forward strategy; analogy and envisioning and fourteen activities namely, checking; pictorial representation; quantitative representation; question reading; relating quantities; reference; symbol usage; clarifying; comparison; declaring quantities; qualitative analysis; recapitulating; and resolving difficulties were identified as well. Each of the general methods and strategies was linked to some or all of the fourteen activities. He noted that these activities were interrelated in the problem-solving process. For example, when resolving a difficulty, participants might recapitulate, perform quantitative representations, and qualify work being done. Furthermore, there is an overlap between the activities. For example, in the process of recapitulating, the participant would also relate quantities.

The following is a brief comparative discussion of the activities performed by experts and novices. The activities and codes used by Dhillon were:

- Checking (Check): The experts performed checking to determine the logic of the solution steps as an inherent part of their problem solving. They used their practical knowledge and experience in their checks. The novices generally performed checking when they encountered difficulties or when the expression obtained failed to make sense. They used their intuitive knowledge as a check.
- Clarifying (Clarify): This activity was done by all the participants except one novice to explain, simplify, and refine the work currently being considered by themselves. It was used to create clarity and transparency of the information being processed or coming to mind at that time of the work currently under consideration.
- Comparison (Compare): None of the experts made any comparisons with other examples. The experts knew the information required to solve the problem

because they were either lecturing or tutoring the subject. Six of the novices referred to the examples in the text.

- Declaring quantities (DecQuant): This activity involved mentioning a principle or quantity, an equation, an expression using one or more quantities, or merely stating a value for a quantity.
- Pictorial representation (PictRep): The diagrams drawn by the experts contained precise information. The novices had incomplete information of quantities involved. Novices failed to successfully perform the problem solution because they had incorrect or incomplete representations of the information pertaining to problem solution.
- Resolving difficulties (ResDiff): This activity often occurred when participants reached an expression or answer that did not make sense or which they had difficulty in attempting to simplify. Resolving difficulties was performed by all the participants.
- Qualitative analysis (QualAnal): Novices used surface features whereas experts used physical entities to categorize physics problems.
- Qualifying (Qualify): This activity involved using content knowledge and principles to make deductions and to provide supportive information to connect the solution steps. All the participants performed this to some extent.
- Quantitative representations (QuantRep): This activity involved using numbers or algebraic symbols to represent quantities. This was an activity that all participants performed to a large extent.
- Question reading (Read): Generally the novices read the question verbatim before beginning the next problem-solving step. Two experts processed the information provided as they read the question.
- Recapitulating (Recap): The experts recapitulated when relating solution steps and when considering the plausibility of the answer. The novices recapitulated mainly when they found an error in their calculations or logic or, primarily, when they were at a loss in knowing how to proceed.
- Reference (Ref): All the novices except one, made reference to the text.

- Relating quantities (RelQuant): This activity involved using previous experience and knowledge to recall information needed to proceed in the solution of the problem. The knowledge recalled was usually in the form of equations and algebraic relations. All the participants performed this activity.
- Symbol usage (Symbol): Symbols were used to algebraically describe physical quantities used in the problem solution. All the participants performed this activity.

On the average, the total number of activities used to solve the problem was much less for the experts than the novices. The experts with the largest frequency of activities of 100 took a couple of wrong paths before obtaining a solution. The only successful novice had a frequency of 130.

When problem solving, the participants used a variety of general methods and common strategies as outlined in literature, performing all or some of the fourteen activities in each of the strategies. An example of the activities performed by a novice, within the 'generate-and-test' strategy is given by the following excerpt in table 2.1.

Table 2.1: Generate-and-Test Strategy

Activities	Excerpt	Comments
DecQuant Qualify QuantRep	At point A I can write down that the, I will have lost all my potential energy assuming that it is zero at that point so that I can write that the kinetic energy at that point is going to be half m v squared plus its kinetic angular part which is going to be a half omega squared. ... which is seven tenths m v squared ...	(Writes an expression for the total energy at the bottom of the loop.) $E = ½mv^2 + ½\ Iw^2$

These finding by Dhillon suggested that the understanding of problem solving, in terms of activities in relation to strategies, provides insight into knowledge structuring and representation. A programme has been successfully developed from the findings of his study. It is currently being expanded to include more problems. Many of the activities outlined by Dhillon were carried out by the learners in the present study, often identifying with those activities carried out by the novices.

Clement's Study of Algebra Word Problem Solutions

In mathematics and science, one way of presenting data of a real-life problem situation is in the form of word problems, tables or graphs and learners are required to translate this to formulae. Formal algebra is difficult for learners, even more so if they have not worked with numerical values.

Clement's (1980) research focussed on translation of word problems to formulae, and an investigation into the cognitive processes leading to correct answers or misconceptions. His intention was to make it easier to design more effective strategies for teaching algebraic skills.

Clement presented 150 freshman engineering students with four word problems, two of these involved numerical values and the other two involved symbols. Each of the numerical problems was similar to a word problem involving symbols. He found that the contrast between the numbers of students who correctly solved the numerical versus the algebraic problem indicates that students have specific difficulties in translating from words to algebraic equations.

One of the algebraic problems was: "Write an equation using the variables S and P to represent the following statement: 'There are six times as many students as professors at this university.' Use S for the number of students and P for the number of professors."

63% of the students responded correctly. The results revealed that 68% of the errors were reversal errors, "6S = P." It was most unlikely that these errors were due to carelessness, since the students were warned to "be careful."

From clinical interviews and think-aloud sessions while students worked through the problems, Clement was able to identify two conceptual sources of reversal errors, a syntactic, "word order matching" strategy, and a semantic, "static comparison" strategy.

In the syntactic type, the student simply assumes that the order of key words in the problem statement will map directly into the order of symbols appearing in the equation. This is a syntactic strategy in the sense that it is based on rules for rearranging symbols in an expression which does not depend on the meaning of the expression. An example of this incorrect strategy is evident in one student's response to the Student and Professor problem. He immediately wrote "6S = P," and said "*Well, the problem states it right off: '6 times students.' So it will be 6 times S is equal to professors.*" Here there is no evidence for any more complicated strategy than that of mechanically matching the order of symbols in the equation to the order of words in the problem statement.

An example of the second incorrect strategy, the "static comparison" strategy, was provided by a student solving the England problem: "Write an algebraic equation for the statement: 'There are 8 times as many people in China as there are in England.'"

A typical incorrect response was "8C = 1E." The explanation given by a student was "*There is a larger number of Chinese than there are Englishman; therefore the number of Chinese to Englishman should be larger... 8C = 1E.*" Here he clearly indicated that he had comprehended the relative sizes of the two groups in the problem that there are more people in China. This indicated that he had gone beyond a syntactic approach that is independent of the meaning of the problem. But his intuitions about how to symbolize the relationship were to place the multiplier (eight) next to the letter associated with the larger group (China). He apparently did this to indicate that that group is larger. There is some semblance of reason in this approach as an intuitive attempt at symbolization, but the approach is an extremely literal attempt to compare the relative sizes of the two groups in a static manner. Thus Clement labelled this as the "static comparison" approach. This is a semantic strategy in the sense that is a "meaning based" strategy which takes into account the meaning of the expression produced. The student has an accurate picture of the relative sizes of groups in the practical situation, but still fails to translate his understanding correctly to an equation. Such students appear to have had a misconception concerning the actual meaning of the equations they are generating, rather than a misconception of the practical situation described by the problem.

A response from a successful student on the Student and Professor problem was, "*S equals the number of students and P equals the number of professors.*" He correctly considered S and P to be numbers and he verified his answers by substituting numbers for S and P.

Analysis of protocols from successful students indicates that the key to understanding correct translations lies in the ability to conceive a mental action that produces an equivalence, and to realise that it is precisely this action that is symbolized in, for example, the equation, "6S = P." Clement called this the "*operative approach*" to signify the fact that it involves viewing the equation as an active operation on a variable quantity, not just a static comparison of the two groups.

Some students obtained incorrect answers by simply making a direct syntactic translation via word order matching from an English statement to the written equation, for example, the response from one student was: "*Six times as many students as professors ...So let's use S for students and P for professors ...6S equals P.*" This type of translation might be likened to the simple act of paraphrasing a long sentence in short hand form by copying the main elements, in the order in which they appear, and dropping out the inessential words. Such a translation might be performed with little or no understanding of the meaning of the sentence. The student simply assumes that the order or contiguity of key words will map directly into the order of symbols appearing in the equation.

Many students were found to move back and forth between the approaches. Clement calls this "*shifting between approaches*" and hypothesizes that it refects an unobservable internal process of shifting between cognitive schemes used to deal with the problem. This provides one more piece of evidence for the notion that human cognition is not always based on consistent processes, schemes which lead to contradictory results can apparently exist fairly autonomously an independently in the same individual. One scheme may become active and dominate for a time, only to be superceded by the other.

Evidence from Clement's work has shown that learners experience difficulties in translating word problems involving symbols, to formulae, however they perform better on word problems with numerical values. A large proportion of the students were unable to solve a simple algebraic word problem. Writing equations with more than one variable exposes a number of common misconceptions that were previously invisible. The error appears in writing equations where a multiplying factor is placed on the wrong side of the equation.

CHAPTER THREE

METHODOLOGY

INTRODUCTION

This research is essentially a descriptive/interpretative study of grade 11 learners' ability to use intuitive modelling strategies in answering questions in kinematics. The methodology involved was think-aloud, pen-and-paper and clinical interview. These sessions were audiotaped.

This research design was considered appropriate for this study because it enabled the researcher to directly examine the intuitive modelling strategies learners use in solving problems involving real-life situations with no formal instruction in these standard algorithms. Although it is acknowledged that these problems are not 'real-life' problems in the concrete sense and in that it is already an abstraction, it is a situation that learners are familiar with.

Specifically, the following critical questions were addressed:

1. What strategies do learners use to complete a table of a real-life problem involving speed and time ?

2. What are the strategies used by the learners in recognizing and formulating a functional relationship between the variables speed and time of a real life problem?

3. Are the learners able to translate the functional relationship into symbolic form (e.g. mathematical formula) ?

4. What is the feasibility of using modelling as a teaching approach for teaching speed and time relationships ?

Details of the procedure and type of instruction, selection of problem types, selection of learners, collection of data and analysis of data are discussed in this chapter. Also included in this chapter is a theoretical discussion of clinical interviews and qualitative analysis.

PROCEDURE/TYPE OF INSTRUCTION

The following methods were used to ensure reliability, validity and richness of data collected.

- Think-aloud sessions as a means of recording the problem-solving behaviour of learner. Problem-solving tasks which require a learner to assess and manipulate his/her mathematical and physics knowledge in the solution of a problem are particularly useful if the learner is encouraged to "think aloud" as s/he solves the problem. The transcripts have been used to capture the train of thought of the solver, and the problem-solving knowledge and style used. According to Piaget (1929) (cited in Posner & Gertzog 1982:195), when the learner is allowed to talk about what s/he is thinking and doing, it gives the researcher an opportunity to notice how the thoughts unfold themselves. The novelty lies not only in being content simply with the written answers given by the learner, but in letting him/her talk about what s/he is doing. One benefit of these think-aloud protocols, according to Clement (1980:3), is that a researcher can learn whether errors are simply due to carelessness or to deeper conceptual problems.

- Pen-and-paper sessions to explore the information assessed and used to solve a problem, and to supplement the think aloud-data collected.

- Clinical interview. This is a tool often used by mathematics and science education researchers to investigate learners' thinking processes. Clinical interviews were first used in educational research to study children's language and logic. Piaget modelled his interviews on the methods used by psychiatrists in the beginning of the twentieth century. (Posner & Gertzog 1982:196.) This kind of interview is a face-to-face meeting between two people, but it is not an ordinary interview (Bingham et al 1959:3). This method allows the researcher to document the multiple ways by which learners understand mathematics and science (Zietmann undated).

A brief discussion of clinical interviews follows:
The written solution of a problem may reveal little about the underlying thought processes that generated it. Although a solution shows a particular reasoning pattern that leads to a desired goal, it does not indicate how this particular path was found, how it was chosen over alternative paths, or how impasses were avoided. (Reif 1986:50.) The researcher in a clinical interview is trying to look at the learners' cognitive structure and ascertain not only what concepts and propositions are there, but also how these concepts are structured and how they can be evoked for problem-solving. The concern is with the learners' individual framework of knowledge and reasoning strategies. (Novak & Gowin 1984:121.)

When a learner is allowed to talk about what s/he is thinking and doing, it gives a researcher an opportunity to notice how the thoughts unfold themselves. A researcher can find out about a learner's knowledge about a certain subject matter by letting him/her talk, by analysing the way the child's thought unfold and by not sticking with the immediate answers, but probing and clarifying the interviewee's ideas and written material carefully (Zietmann undated).

A skilful researcher is able to probe the areas of the knowledge domain of particular interest and let the child speak freely, while constantly checking his or her spontaneous remarks for those that will prove genuinely revealing. The clinical interviewer has to be continually alert, ready for the unexpected and able to respond to it, for it is that very response which may lead to something unique and essential in the thought of the subject. (Posner & Gertzog 1982:198.)

The clinical interview method of eliciting verbal explanations was felt to be a more valid and "*revealing*" indicator of a learner's cognitive structure than selection- or production-type written assessment instruments (Posner & Gertzog 1982:199).

It is Posner & Gertzog's (1982:194) contention that clinical interviewing is directed toward the information-gathering function. Its chief goal is to ascertain the nature and extent of the learner's knowledge about a particular domain by identifying the relevant conceptions s/he holds and the perceived relationships among these conceptions. Another aspect that emerges from interviewing studies is the complexity of learner's thinking.

Often students reach invalid conclusions from sound reasoning based on false premises or even valid conclusions from unsound reasoning based on false premises. In the clinical interview it is important to identify any mistake the learners are making regardless of the validity of their conceptions. (Posner & Gertzog 1982: 206.)

CHOICE OF PROBLEMS

Learners usually perform well in textbook type problems on kinematics dealing with equations of motion, but they flounder when problems require deeper conceptual understanding. Physical science questions in kinematics are generally presented in the form of verbal statements and learners are required to model mathematical strategies to find the solution. Most learners, according to Hestenes (1992:747), are blind to the structure of physics and its insights into the structure of the physical world. An alternate approach which attempts to address this problem is the "model-centered instruction."

Various ways of introducing the concepts of the equation 'v = at' and 'v = u+at' confront the educator with the perennial question of choosing between a general or more specific presentation. General approaches tend to be more formal and as such reach fewer learners, whereas restricted approaches are relatively more concrete and thus accessible to more learners. Using the latter approach, it is felt that if the learners modelled these mathematical formulae on their own, it would be better understood and more appropriately applied.

According to Greenman(1973), (cited in Salzano 1983:33), the problems to be modelled should be familiar to the learner, s/he should already have some intuition for that situation, the mathematics should be sufficiently simple and familiar so that the discussion is not dominated by mathematics, and the situation should be sufficiently open-ended so that there is a variety of ways of building the model.

Two verbal problems involving real-life situations dealing with time, speed and acceleration were chosen since the learners could identify with them and these problems could provide motivation for theoretical work. The learners were required to model strategies and eventually establish equations dealing with speed and time for the problems.

Because, according to Herscovics & Kieran (1980:572), translating word problems into equations is tantamount to "*translating it into a language unknown to them*" the learners were given some guidance in the form of incomplete tables in order to facilitate formulating the equations.

This research was conducted prior to any form of formal instruction in the section on kinematics. Although the problems were novel to the learners, they could identify with them. The problems presented to the learners were meaningful, but ones that they could not solve with ease using routine procedures or drilled responses. Learners were required to explain, discuss, critique and justify their interpretations and solutions. The two problems varied in their degree of complexity.

My role as a researcher, the intention of the research and the instructions, were clearly stated to the learners before they commenced with solving of the problems. According to Rich (1971:32), it is important in any research for a researcher to start out by informing the learners of his/her own obligations and intentions.

The following statements were provided:

1. The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.
2. The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

The learners were required to fill in the blanks in the tables from the information given in the statements.

Table for statement 1:

Time (s)	0	1	2	3	4	5	10	50	120			560
Speed (m/s)										700	1200	

Table for statement 2:

Time (s)	0	1	2	3	4	5	6	10	50	80		
Speed (m/s)											410	1000

The questions asked were:

1. What does 'initial' mean?
2. What does 'speed' mean?
3. What do you understand by 'metres per second (m/s)?'
4. What do you understand by '5 metres per second every second (5 m/s)?' A more appropriate question would have been: "What do you understand by an increase in 5 metres per second every second?"
5. How is it possible for an object to undergo an increase in speed?

After answering the questions the learners were required to carry out the following tasks for each of the questions:

- Fill in the given table.
- Establish a relationship between speed and time.
- Formulate this relationship between speed and time in their own words.
- Write down the above formula or equation in symbolic form given that 'v' is the speed and 't' the time.

SELECTION OF LEARNERS

The sample chosen for this research was seven grade 11 learners form Durban Girls' Secondary School. They were randomly chosen, and they varied in their intellectual abilities.

Each learner was observed as she worked through the problems on paper and was encouraged to explain aloud what she was doing. The sessions were conducted individually and the subjects were not allowed to communicate with the other subjects before the session. There was no time limit to the problems.

A pilot study was conducted on two learners prior to the actual research in order to ascertain the validity and suitability of the questions.

COLLECTION OF DATA

The data in this study was gathered by qualitative research methodologies, including observation and interaction with individuals in the form of interviews on a one-to-one basis. The core of data collection in qualitative design consists of results gathered from apparently simple behaviour – watching, listening, asking questions and collecting things. The principal instrument in qualitative research is the researcher working face-to-face with those studied. In contrast, the quantitative researcher is usually distanced from research participants, often by paper-and-pencil, administered by interchangeable research assistants.

ANALYSIS OF DATA

Qualitative Analysis

The results of this study were analysed qualitatively. According to Preissle-Goetz & LeCompte (1991:61), qualitative researchers generally study fewer people, than do quantitative researchers, but they study those few more intensively. As a consequence, they are often able to pursue patterns in the behaviour, belief, and knowledge of individuals across settings, circumstances, time, and other variations. Investigators can substantiate hunches, collect multiple points of view and establish ranges for discerned patterns.

Research methods can be broadly divided into quantitative methods taking a positivist approach and qualitative methods which are relativist in their perspective. Relativist researchers postulate that the world may look different to other people. Hitchock & Hughes (1995:296) regard such researchers as naturalistic, interpretive, and qualitative researchers. Their purpose is not to obtain a set of facts, but to gain insight into a perspective. (Johnson 1994:182.) Naturalistic qualitative enquiry is concerned with the description and explanation of phenomena as they occur in routine, ordinary natural environments. It deals in words and meanings, seeking to maximize understanding of events and facilitating the interpretation of data.

Qualitative research is based on and grounded in descriptions of observations. Some methodologists object to the name "qualitative research." They believe it to be imprecise, misleading, and implying a lack of concern with quantity. Among the synonyms used are interpretative research, naturalistic research, phenomenological research and descriptive research. Because of its preoccupation with complete, detailed, and concrete depictions, some people call qualitative research descriptive research. (Preissle-Goetz & LeCompte 1991:56.)

Quantitative data can be integrated into qualitative studies. Two kinds of research designs often assumed by educational researchers to be quantitative are surveys and observational studies. This is usually a legitimate assumption, but there are exceptions. In many classroom observational studies, standardized protocols are used. Observational researchers may compile data-bases partially or entirely consisting of qualitative data. Likewise, most survey designs use pre-coded responses, but many surveys include open-ended questions and some are composed entirely of open-ended requests and tasks. Hence those survey and observational studies based on sensory data recorded in narrative form are qualitative. (Preissle-Goetz & LeCompte 1991:57.)

Philosophical frameworks, one of which is constructivism, has an influence on qualitative work. The purpose of qualitative research with its rich descriptive and subjective/introspective character of data produced using qualitative techniques, together make qualitative data analysis a very different enterprise than statistical analysis (Hitchock & Hughes 1995:296).

Sherman et al (1988:7) says that the aim of qualitative research is not verification of a predetermined idea, but discovery that leads to insights. Thus qualitative research focuses on natural settings, not abstract or theoretical settings. Qualitative researches employ an interpretive frame of reference in order to bring meaning to experience. In this sense, qualitative inquiry is not merely a search for knowledge for knowledge sake, but a search for significance of knowledge.

The formal goals or purposes of research fall into three broad areas: descriptive, analytical, and theoretical. These in turn are linked to the kind of question addressed in any given study. The purposes and kind of question affect the type of research design chosen.

Researchers whose purpose is theoretical address the question, "How can it be explained and understood?" These researchers apply their findings to the generation, refinement, and verification of generalizations about some area of human experience. Qualitative investigations are usually more concerned with generating theory than with confirming already established explanations. As a result, qualitative research is more often inductive than is quantitative research. (Preissle-Goetz & LeCompte 1991:60.)

Qualitative analysis is inductive and recursive, allowing investigators to trace through what actually does happen to something like a social studies innovation, rather than merely reporting the degree to which what was expected occurred or failed to occur.

Qualitative research is a loosely refined category of research designs or models, all of which elicit verbal, visual, tactile, olfactory, and gustatory data. These data take the form of descriptive narratives like field notes, recordings, and other transcripts from audio- and videotapes, and other written records, as well as pictures or films. Qualitative researchers also may collect artefacts - products or things people use – such as objects people make and records of what they do, say, produce, or write.

Qualitative designs differ according to their own history and their links to human science and enquiry. The designs most frequently considered to be qualitative are ethnographies, field studies, community studies, case studies biographical or life history investigations, and document analysis.

Qualitative paradigms are characterized by the assumptions that reality is ever changing and only incompletely knowable, that knowledge consists of tentatively held understandings, and that research designs and results are inevitably permeated by values – those of the researcher, the research participants, and the research audience.

Among the strategies used to confirm patterns are cross-checking information and inferences with data from several individuals, looking at a given phenomenon with data obtained in different ways, enlisting as a confederate another researcher or an informed participant, and soliciting from participants reactions and interpretations of patterns developed by the researcher. All of these are means of triangulation. Most qualitative investigators triangulate with data collection methods and sources, they use two or more different kinds of data or data sources to get differing perspectives on the same phenomena. (Preissle-Goetz & LeCompte 1991:61.)

Qualitative researchers record their observations manually and mechanically. Audio- and videotape recorders, still and motion cameras, laptop computers, and wireless microphones are among the mechanical devices used. Nearly all qualitative researchers also produce field notes. These typically consist of records of observations and a commentary on what was observed, how it was observed and what it might mean. (Preissle-Goetz & LeCompte 1991:60.)

The more formal process of qualitative research will involve the researcher breaking down the data where a fairly inductive approach is taken – data being explored in terms of both the general and particular units of meaning displayed within them. The qualitative researcher is looking for patterns, themes, consistencies and exceptions to the rule. Codes and categories can therefore emerge from the data and become formally identified by the researcher. (Hitchock & Hughes 1995:296.)

According to Guba & Lincoln (1994:106), qualitative data can provide rich insight into human behaviour. In this study, there are two sets of records: the written transcript for each problem solved, and the audiotaped record of the learners talking and working through the problem incorporated with the interview. The audiotaped interviews were transformed into written transcripts. A collection of such data in Larkin & Reif's (1979:192) view is quite detailed.

A disadvantage of qualitative methods is that it is slow and may be anxiety creating to the learners because of the lack of structure. Moreover, since the research question is being developed and refined during, rather than prior to, the research, it is more difficult to plan the research programme as a whole. (Johnson 1994:183.)

Although investigators have some analytical approaches in mind as they begin, each analysis is developed to fit its accumulating database. Consequently, qualitative approaches possess a flexible, evolving character that contrasts with fixed and linear approaches of much quantitative research. Rather than a one-shot process at the end of data collection, qualitative analysis is ongoing and recursive - the investigator reviews and reanalyses previous material as new material is developed. Analysis involves discovering and deriving patterns in the data, looking for general orientations in the data, and in short, trying to sort out what the data are about, why and what kinds of things might be said about them.

Qualitative researchers build system and rigour into ways to observe classroom lessons, and educator-learner interactions – an invaluable tool for revealing what occurs there. A major emphasis is placed on the perspective of the participants involved: what are the learners making of this situation?; what is going on?; and how are their understanding affecting what they and others know and do in the social world.

Qualitative researchers attempt to construct holistic views of events, permitting analysis of the complex relationships among such factors as learners, educators, classrooms, and curricula. This holism typically extends beyond the borders of the school itself, taking into account communities and their subgroups and the general socio-cultural context within which they are embedded.

Qualitative data analysis involves making sense of the data. The task is initially one of sorting the data into manageable units. The researcher seeks to organize the data in such a way so as to facilitate understanding of their meaning and significance. This will involve breaking the data down into units of meanings, topics or categories which the researcher can then subsume under a general heading bringing together diverse activities. The researcher's task is to put some kind of order on to the data without distortion. The use of codes and categories helps to break the data down into manageable pieces, it allows for the identification of relationships between units of meaning and to begin initial analysis. Although qualitative research can make use of observer-generated codes and categories, these must at some point in the process, be related to the participant's codes and categories and more importantly their coding and categorizing system. In this way the qualitative researcher ensures a movement from description to explanation.

The complexity of the coding and the categories generated will depend upon the nature of the study. In this study, there was no need for a coding system since only seven learners participated in the study, and in addition, the number of strategies used was limited.

Theoretical sensitivity is the ability to recognize what is important in data and to give it meaning. It helps to formulate theory that is faithful to the reality of the phenomena under study. Theoretical sensitivity has two sources. First, it comes from being well groomed in the technical literature as well as from professional and personal experience. This complex knowledge is brought into the research situation. However, theoretical sensitivity is also acquired during the research process through continual interactions with the data. (Hitchock & Hughes 1995:298.)

The process of analysis will also need to consider the question of validity. In the analysis of the data, the researcher will be concerned to validate or verify the kinds of analysis made and explanations offered. This will mean constantly moving backwards and forwards between data and analysis, and between data and any theories and concepts developed, and between the data and other studies or literature. Once a series of relationships is observed, the researcher attempts to formulate a series of insights or hunches in the light of the relationships observed, into a theory which will cover and account for all the cases as far as possible. (Hitchock & Hughes 1995:297.)

The learners' solution strategies in this study is analysed according to a more specific scheme of mathematical strategies, namely horizontal additive, horizontal multiplicative, vertical multiplicative, counting on, ratio and proportion, a sophisticated form of skip counting, a functional rule, a proportionate functional rule, and backward strategy. A detailed analysis of these strategies is discussed in Chapter Four. The specific and general use of strategies for questions one and two will be discussed, and compared with findings in literature.

Educational researchers study socially constructed phenomena when they study classroom interaction, or school academic climate, or student test performance (Cherryholmes 1991:42). Preissle-Goetz & LeCompte (1991:64) believe that qualitative research is an effective way of studying society and culture because they are charged with teaching the young about human world. Qualitative approaches can also increase the level of understanding of the inside world of learners, educators, administrators, parents, and others involved in education.

Qualitative researchers are able to provide feedback in a way that participants find productive and encouraging. Its success depends on cooperation and commitment from participants, from whom researchers require much, and to whom little may be returned. Qualitative research calls into question the existence of correct, absolute solutions to human problems and treats knowledge in tentative, sceptical and relative ways. For educators whose lives and research have been devoted to improving the human condition, using qualitative approaches means settling for the possibility that there are no quick fixes.

CHAPTER FOUR

RESULTS AND ANALYSIS

What follows is firstly a summary of the learners' interpretation of the terms used in the problem. This is followed by an analysis of the strategies used by the learners in filling in the table, formulating an equation relating speed and time and writing down the equation in symbolic form.

The learners were provided with novel problems involving real-life situations dealing with motion. They were required to complete the tables that were provided from the information given in the statements and hence formulate functional relationships between the variables speed and time on their own without receiving prior instruction.

The end result expected of the learners was for them to develop a rule for calculating the speed when the time and acceleration were given. By supplying the learners with a table of time and speed for each problem, the "*problem was broken into a collection of smaller parts*." By making use of the solutions of these smaller parts, the final rule or algorithm could be derived. Rolston (1988) (cited in Dhillon 1988:382) called this strategy "*problem decomposition.*"

The learners who completed these tasks successfully would have developed their own algorithms and thus developed a better understanding of the concepts and propositions of the physics involved in kinematics. Salzano (1983:10) said that understanding and success depends on the knowledge of the problem situation rather than the use of mathematical knowledge. Completing the table required an interpretation and analysis of the given statement. It also required the development of a relationship between the variables speed and time. Thus it is virtually impossible to separate the mathematics from the physics in dealing with these problems at the initial stages. However, once the algorithm is developed by the leaner, it could be used with understanding and confidence in solving novel problems dealing with kinematics in physical science.

When solving the problems, the learners used a variety of methods and strategies. The learners were not always aware of the strategies they were using, however these were inferred by the researcher from the written data, think-aloud data and interviews which were conducted while the learners solved the problems.

According to Dhillon (1998:385), a strategy is a plan of action. It is the representation of a block of knowledge, procedural or declarative, used to move from the initial stage to the goal stage. The intuitive modelling strategies used by the learners in this study were horizontal additive, horizontal multiplicative, vertical multiplicative, ratio and proportion, counting on, a sophisticated form of skip counting, proportionate functional rule, and backward strategy.

This study was also influenced by the ten commonly used strategies reported in literature and that which was used by Dhillon (1998:381). They are analogy, brainstorming, envisioning, forward strategy, generate-and-test, heuristic search, means-ends analysis, problem abstraction, problem decomposition and working backwards. Each of these strategies according to Dhillon (1998:385) consists of a series of sequenced application of activities. Thus, activities may be considered as forming an internal structure of a strategy. Dhillon identified fourteen such activities. While evidence of these activities were present in the learners responses, details would not be specifically referred to as it goes beyond the scope of this dissertation.

The following instructions were given to the learners:
"I have a statement that relates to a real-life situation. I want you to read through it and then answer questions based to it. This exercise is to ascertain how you interpret the situation, what you are thinking, your plan of action, your reasoning, how you obtain relationships etc. Besides writing down anything you feel is necessary, you must verbalize what you are thinking and doing. This exercise is by no means a judgemental exercise, so you should see me as someone trying to understand what goes on in your mind when you interpret the statement and answer the questions."

It is hoped that the results obtained from this research would contribute to improving not only my teaching but also those of other educators and hence impact positively on learners' performance.

The statements were:

1. The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.
2. The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

The learners were required to fill in the blanks in the tables from the information given in the statements.

Table for statement 1:

Time (s)	0	1	2	3	4	5	10	50	120			560
Speed (m/s)										700	1200	

Table for statement 2:

Time (s)	0	1	2	3	4	5	6	10	50	80		
Speed (m/s)											410	1000

The learners were required to explain the terms in the statement before filling in the table. The reason for this was, since the intention of this research was to ascertain the learners' intuitive modelling strategies for solving the problems, the learners should not be disadvantaged due to incorrect interpretations of the terms used in the statement. After all says Polya (1945:6), it is foolish to answer a question that you do not understand. First of all the verbal statement must be understood. According to Janvier (1978), it is highly informative to a learner to provide sub-questions i.e. simpler questions so as to minimize errors in the answers. Thus a reasonable understanding of the terminology was required.

The questions asked were:

1. What does 'initial' mean?
2. What does 'speed' mean?
3. What do you understand by 'metres per second (m/s)'?
4. What do you understand by '5 metres per second every second (5 m/s)'? A more appropriate question would have been: "What do you understand by an increase in 5 metres per second every second?"

5. How is it possible for an object to undergo an increase in speed?

After answering the questions the learners were required to carry out the following tasks for each of the questions:

- Fill in the given table. Establish a relationship between speed and time.
- Formulate this relationship between speed and time in their own words.
- Write down the above formula or equation in symbolic form given that 'v' is the speed and 't' the time.

Calculators were used, where necessary.

The seven learners who were part of this research study were Shireen, Nancy, Wendy, Fatima, Irene, Dinesha and Erica. Their names have been changed for the purpose of confidentiality.

Unfortunately the tape-recorded data of Fatima's interview for statement 1 could not be captured due to an electrical fault with the tape recorder. However, sufficient written data was collected so as to make reasonable conclusions to the strategies she used.

The results and analysis will be discussed as follows:

1. Interpretation of the terminology.
2. Results and analysis of statement 1 which will be subdivided:
   2.1. Introduction.
   2.2. Speed at zero seconds.
   2.3. Filling in the table.
   2.3. Establishing the relationship between speed and time and hence formulation of an equation.
   2.4. Summary.
3. Results and analysis of statement 2 which will be subdivided:
   3.1. Introduction.
   3.2. Filling in the table.

3.3. Establishing the relationship between speed and time and hence formulation of an equation.

In the analysis of question one and question two each learner's response will be considered separately because, although there was a fair degree of overlap in the choice of their strategies, their idiosyncratic use of these strategies needed to be recognized and discussed separately. A general discussion of the strategies will follow at the end of this chapter.

INTREPRETATION OF TERMINOLOGY

Initial

All the learners had a reasonable understanding of the term 'initial' and were able to apply it appropriately when filling in the table. Shireen, Wendy and Erica gave a general meaning while Nancy, Dinesha and Erica were more specific, relating it to the statement.

Shireen, Wendy and Erica correctly understood 'initial' to mean "*at the beginning*" or "*at first.*" Erica elaborated further by relating it to the statement and said that it started with a speed of zero metres per second. Nancy interpreted 'initial' as being the speed "*right now*" i.e. at that point in time, meaning at the start. Nancy's and Dinesha's views were similar to Erica's having taken the problem in context and stating that the initial speed was zero. They also considered initial to be the point when nothing happens, with some change taking place thereafter which in Dinesha's words were "*at the beginning, before anything happens*" and in Nancy's words were "*the speed was going to increase every second thereafter.*" Irene, a second language learner, explained 'initial' as being the "*permanent speed.*" Unfortunately what she meant by this was not followed up and so her interpretation could not be ascertained. However, while filling in the table it was apparent that 'initial' to her meant at the start because, she, correctly stated "*the object has an initial speed of zero, so it starts with a speed of zero.*"

Speed

All the learners associated speed with movement. With the exception of Nancy all the learners considered speed to be "*how fast*" the object was going. Nancy associated speed with "*very fast movement,*" thus likening 'speed' with 'speeding'. Dinesha's interpretation of speed was the "*rate of travel,*" probably meaning the rate at which distance was covered per unit time. This point was not pursued because of time constraints and the fact that the focus of this study was the strategies used in solving the problem and from her understanding of speed she was able to arrive at appropriate answers. The idea of rate as the distance covered in a given time had not been explicitly stated by any of the learners.

Metres per second (m/s)

All the learners interpreted it as being the physical quantity 'speed,' yet 'm/s' featured nowhere in the discussion of speed above. With the exception of Erica, all the learners conveyed the idea of distance travelled in 1 second. Nancy gave the following examples: "*1 metre a second, 2 metres a second.*"

Shireen assumed that the number preceding 'm/s' was necessarily '1' as if it was an algebraic variable, for example, 'x' implies '1x.' She elaborated further by saying that "*If there was a 2 in front, it means that the object is travelling at 2 metres every second, if it had a 6, then it's travelling at 6 metres every second.*"

Dinesha, Wendy and Irene felt that it was more appropriate to explain 'm/s' in terms of 'metres' and 'seconds', considering 'm' and 's' to be the symbolic abbreviations for metres and seconds respectively. Dinesha and Wendy both described it as "*how many metres it travels per second*" while Irene's explanation was "*in 1 second it covers so many metres.*"

Erica, on the other hand described it as the "*distance travelled over a certain period of time.*"

All the learners considered 'm/s' to be the physical quantity speed. None of them recognized it as the unit in which the physical quantity speed is measured. This is a very common misconception where learners regard the physical quantity and the unit for the physical quantity as being synonymous with each other. This is probably due to the fact that when these concepts are first introduced to learners, a clear distinction is not adequately illustrated. The following comments made respectively by Erica and Dinesha elsewhere in their discussions illustrates this point: "... *seconds is time*" and "*m/s is the speed ...*"

Another commonly associated problem is that of assigning incorrect units to physical quantities, for example, Irene referred to speed as "*5 metres.*"

5 Metres per second every second (5 m/s)

This question initially led to confusion because it should have been: "What do you understand by the phrase: increases at 5 m/s every second?" The initial response received from all the learners except Irene was "*5 metres for every second*". Erica and Wendy, having realized that this term pertained to the statement, read the question again and correctly responded as "*for every second the speed increases by 5 metres per second.*" Irene's immediate response was "*every second its speed goes higher into 5 metres.*"

How is it possible for the speed to increase?

All the responses were, by "*accelerating*". Erica elaborated further by relating this to the given statement by adding "*so 5 metres per second every second is the acceleration.*" Shireen discussed acceleration by comparing the speed of a car on a freeway with the speed in a public area. She was given 'a car' only as an example, but she felt most comfortable discussing acceleration in terms of something she was familiar with. In this way of working from a familiar to an unfamiliar situation she was able to grasp a better understanding of acceleration. Thus supporting Kamii's (1974:199) view that knowledge is constructed from within and Heaney & Watts' views that knowledge is made by us and our of experience.

The learners conveyed a reasonable understanding of the all the terminology, except for 'm/s', used in the statement. Thus it would have appeared that completing the rest of the tasks would not have been hindered by the inadequate interpretation of the terminology. However, in the process of completing the table it became evident that the meanings of some of the terms e.g. 'initial' was disregarded or changed. In question two the learners considered initial speed to be zero metres per second although it was specified as 10 m/s. Thus the meaning of 'initial' was disregarded. This was probably due to the influence of question one where the initial speed was per second. This is in keeping with Davies' (1984) view that learners create separate, different 'frames'. It is possible that the frame developed earlier persists and is sometimes inappropriately retrieved.

RESULTS AND ANALYSIS OF QUESTION ONE

Introduction

The question was: "The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second."

Most solvers used a combination of problem-solving strategies. The strategies depended on the amount of factual and procedural information available and the experience of the problem solver. Many solvers used strategies without realizing the style adopted. In general, search processes dominate much of the problem-solving behaviour of novices. (Chi, Feltovich & Glaser(1981) cited in Dhillon 1998:383.) Problem abstraction, done by concentrating on the most important elements of the statement, was a strategy used by all the learners.

When filling in the table, all the learners looked for a pattern, either with time only, or speed only, or between speed and time. All of them, at some stage of filling in the table, realized that a relationship between speed and time had to be developed.

Determining the speed at zero seconds

All the learners had obtained a speed of 0 m/s at the time 0 seconds.

While some learners were able to obtain a speed of zero metres per second immediately, others deliberated before obtaining the correct answer.

Shireen's immediate response after reading the statement was to establish a pattern between the times. She engaged in a heuristic search, trying to develop a horizontal pattern between the times. The following excerpt illustrates this:

"Looking at time 0, 1, 2, 3, 4, 5, 10, 50, 120." Read statement again. *"It's 10, 50, 120, it's 1, 2, 3, 4, 700, 1200 umm… 5 x2 is 10; 10 x 5 is 50; 50 x 6 equals 120; 120 x 6 equals oh no! I was going along with time. You gave me zero, then 1, 2, 3, 4, 5, 10*

then 50, then 120. So looking at how time is going about and the seconds, time is 1 second, 2 seconds, 3 seconds etc."

Shireen's response was similar to Schoenfeld's (1985:73) findings that learners are generally inefficient in their strategy selection. They will often make their first attempt on a problem using a complicated and time-consuming strategy, without checking to see whether simpler and faster techniques might be appropriate. As a result they waste much time and effort.

Shireen generated and tested a number of possibilities. Throughout her work she had shown good problem solving skills in that she persevered and she evaluated her work as she went along. Persistence, according to Wheatley (1992:532) is a necessary factor in problem solving. However, since she was spending too much time on trying to develop a horizontal pattern, she was asked to try and determine the speed, which she did after reading the question again. Dinesha too, initially thought of developing a horizontal pattern between the times but abandoned the idea immediately because she was not sure how to do it. Shireen and Dinesha made use of "*an initial speed of zero metres per second*" from the statement to obtain a speed of 0 m/s at a time of zero seconds. The latter was the immediate response of Wendy, Irene, Nancy and Fatima. Thus in determining the speed at zero seconds, all the learners except Erica concentrated on the most important elements of the problem, a strategy that Rolston (1988) (cited in Dhillon 1998:382), called 'problem abstraction.' Erica did not start from zero seconds. She filled in the rest of the table and then came back to zero seconds, thus adopting a backward strategy. Details of this will be discussed under Erica's response to filling in the table.

Filling in the table

Introduction

The initial problem description was used to generate solutions, for example all the learners started with the "*speed increases at 5 m/s every second*" in order to obtain the speed at zero seconds. The values in the table were obtained by considering the previous calculations i.e. a forward strategy was used. To calculate the speed at the initial stages of the motion, Dinesha and Erica used a horizontal multiplicative strategy while the others used a horizontal additive strategy.

All the learners with the exception of Wendy, engaged in a process of brainstorming and generating-and-testing their solutions with a view to developing an algorithm which Newell & Simon (1972) (cited in Dhillon 1998:382), called 'heuristic search'. A vertical functional rule was developed and the algorithm, 'speed = time x 5' was used by all the learners except Irene. Despite generating and testing possibilities, Irene was unable to develop a relationship.

Since each learner displayed uniqueness in their initial selection and application of the strategies, the details of each learner's response including their completed table will be discussed separately.

Shireen

Time (s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	~~10~~	15	60	~~300~~				700	1200	2800
			10	15	20	25	50	250m	600			

Figure 4.1.

Shireen initially used a horizontal additive strategy to determine the speed. However, her calculations were correct only up to 15 m/s at time 3 seconds. A possible reason for her not obtaining correct answers thereafter is that she used a calculator to 'add on' so when determining the speed at 4 seconds, on the display of the calculator was the previous speed of '15' and since she was calculating the speed at 4 seconds, she simply entered 'x 4' although she verbalized "*15 then another 5.*" This could possibility be due to the fact that she was intuitively thinking also about the functional rule 'v = 5t' or the vertical multiplicative strategy with time equal to 4, but realized that 15, being the previous speed had something to do with the answer. In effect what Shireen did was to combine both the rules, i.e. 'v = 5 x **4**,' and 'v = **15** + 5,' and obtained '15 x 4.' An interesting point to note here is that what she did and what she verbalized was different which shows that the mind does not always 'verbalize' what it is thinking. Nancy in her discussion had displayed a similar behaviour. The researcher preferred not to probe at this point because she did not want to interfere with nor did she want to influence Shireen's thought processes.

In calculating the next value i.e. for 5 seconds, she similarly entered 'x 5', thus it was '60 x 5.' This '5' could have been the time 5 seconds or the increase in speed of 5 m/s every second. However, for the next calculation which was at 10 seconds, she had '300 x 5' and not '300 x 10' thus indicating that the '5' that she was considering was the acceleration. In the five calculations that Shireen had done thus far, she used three different strategies, namely, a horizontal additive strategy for 1 second to 3 seconds then a multiplicative strategy multiplying time by the previous speed and finally a multiplicative strategy again but this time multiplying the previous speed by the acceleration.

She realized her mistake only after obtaining an "*unrealistic value*" of 1 500 m/s as the speed at 10 seconds. This is in keeping with Dhillon's (1998:387) findings that a novice generally performed checking when the expression obtained failed to make sense. Again she followed what she had done earlier but realized that a speed of 60 m/s after 15 m/s at times 4 seconds and 3 seconds respectively, was too large. She noted that she had wrongly related the quantities and attempted to resolve her difficulties by considering "*it travelled 5 metres in 1 second, in 4 seconds it's 5 x 4 equals 20*" thus she changed her strategy to a vertical multiplicative rule. Using this rule she correctly completed the speed in the rest of the table.

In order to calculate the time when the speed was given she simply divided the speed by 5. Shireen displayed good problem-solving skills because she constantly evaluated her work and checked whether her answers were reasonable as she went along.

Wendy

Time(s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	~~30~~	35x	600	700	1200	112
							50	250	600x			

Figure 4.2.

Wendy made use of the fact that "*every second it increased its speed by 5 metres per second*" and went directly into the horizontal additive strategy. She carried on adding 5's without realizing the gap in times after 5 seconds. Irene and Fatima had also done the same. Wendy realized her mistake when she arrived at a speed of 35 m/s at 50 seconds, which was much smaller than the given speed of 700 m/s. After ascertaining that 6 seconds was not given she intuitively switched to a vertical multiplicative strategy i.e. time x 5. The following is an excerpt of how she developed the rule and then proceeded to complete the rest of the table:

"*6 was not given. So then if at zero seconds the speed was zero then at 10 seconds, the speed increases at 5 so it will be 50, am I right? So at 4 seconds it was 20, 5 will be 25 so at 10 seconds – every second it increases 5, that means it will be 50 metres per second and if it's 50 seconds and every second it increases 5, so it will 250 metres per second and then it's 120 seconds it increases 5 so 120 x 5 equals 600. So if 700, I want time now, and then every time ... I'll divide by 5 from the ratio and proportion of 700 divided by 5 equals 240 and 560 divided by 5 equals 112.*"

In the time that she paused before calculating the time, she was probably thinking about another strategy i.e. the relationship between speed and time in terms of a ratio. She subsequently made use of ratio and proportion to complete the rest of the table. It would have made no difference to the calculation in this particular example if she used either the vertical multiplicative rule or the proportional rule, since the initial speed was zero.

Wendy seemed quite confident about the strategies that she used and the calculations that she had done so she did not check her work like the others had done, thus she displayed poor problem-solving skills. The only time that she did check her work was when she noticed a big discrepancy between the speeds 35 m/s and 700 m/s. She made a careless error, which went unnoticed, when calculating the speed at 560 seconds. She simply followed the trend of dividing by 5 and because this was the last value to be calculated, there were no other values to compare her answer with. However, had she checked the answers she would have realized that the speed of 112 m/s was much smaller than the previous speed of 1200 m/s because she was aware that the trend was for the speed to increase with time.

Fatima

Time (s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	~~30~~ 50	250	600	700	1200	2800

Figure 4.3.

Fatima started her work with a pictorial representation, which was inappropriate and served no purpose. Fatima's response to filling in the table was very similar to Wendy's except that she realized her mistake earlier i.e. at 10 seconds. She realized her error because she was a careful worker and constantly checked her work. Fatima, like Wendy immediately switched to a vertical multiplicative strategy and completed the rest of the table correctly. Some of her calculations were done as follows:

$5 \times 4 = 20$
$50 \times 5 = 250$

$560 \times 5 = 2800$

Figure 4.4.

Nancy

Time (s)	0	1	2	3	4	5	10	50	120	~~140~~	~~160~~	560
Speed (m/s)	0	5	10	15	20	25	50	250	~~288~~	700	1200	500

Figure 4.5.

Nancy also started with a horizontal additive strategy and after the time of 4 seconds switched to the vertical multiplicative strategy. She worked with a 'ratio and proportion' strategy like Wendy did, although she did not explicitly state it as Wendy did. Her ratio and proportion computations were done as follows:

$\frac{1}{10} \times \frac{5}{x}$ $\qquad$ $\frac{1}{120} \times \frac{5}{600}$

$50 \quad x$
$x = 5$

Figure 4.6.

She correctly calculated the speed using ratio and proportion, at 120 seconds as 600 m/s, using the argument *"for 1 second its speed increases by 5 metres per second so for 120 seconds the speed will increase 5 times, 600 m/s."* On reflection she changed 5 to 2,4 while still using the strategy of ratio and proportion. Her argument now was *"The speed didn't increase by 5 over here* [from 50 to 120 seconds] *the speed increased by 2,4* [i.e. 120/50] *so it will be, for 1 second the increase is 2,4 and then cross multiply to get 288."*

1 2,4

120

288

Figure 4.7.

She incorrectly multiplied 120 seconds, rather than 250 m/s, by 2,4. She was however aware of the proportionate functional rule as was evident in her following comment: "*When the time increases by 2,4 times, speed increases by 2,4*" which she mentioned and also made use of in a later discussion. In addition, when she checked her work she used the correct principle, dividing 288 by the previous speed, 250 i.e. she went back to the horizontal multiplicative rule. She temporarily abandoned this strategy after checking and seeing that it *"didn't work."*

Nancy correctly calculated the times when the speeds were 700 m/s and 1200m/s. Thereafter she went into a lengthy verification process verifying the speed and then time by firstly multiplying by 5 and then working backwards by dividing by 5 in order to verify the time e.g. *"5 times 10 equals 50 so 50 divided by 5 is 10."* She was so intent on creating clarity and transparency in the calculations that what she was thinking and what she verbalized was different. She was very focussed on '5'. The following verbal transcript illustrates these points:

"*... 50 divided by 5 I'm getting 5* [meaning 10] , *and if I say 10* [meaning 25] *divided by 5 I'm getting 5, then I say 5* [meaning 20] *divided by 20* [meaning 4] *then I should be getting 5, then I say 5* [meaning 15] *divided by 15* [meaning 3] *then I'll get 5, then I say 5* [meaning 10] *divided by 10* [meaning 2] *then I'll get 5 ...*"

Schmalz (1988:36) described this phenomenon as the most fundamental discipline in developing the faculty of the intuition as sustained attention, where the learner focuses attention.

She was asked to verify the calculation for 5 divided by 10 and her response was "*5 divided by 10 will give 2*". It is not uncommon for errors like these to be made, especially when the mind is intently engaged in a particular thought process, that what one says is different from what one is thinking, although these may be very closely related. One is often not aware of this difference.

Although she had calculated the times correctly for the given speeds of 700 m/s and 1200 m/s, she was now not sure whether to divide the speed by 5 or by 2,4." After much debate, she finally convinced herself: "*but we are not moving 5 anymore*," so she divided the respective speeds by 2,4 in order to determine the times.

When calculating the speed at 560 seconds, she worked out the increase in the time from 500 seconds to 560 seconds as 1,12 [560/500]. In keeping with the trend of dividing for the previous two calculations, she incorrectly divided by 1,12 and got 500. Wendy had adopted a similar strategy in simply following the trend for calculating the speed at 560 seconds. Neither of them had realized that it was the speed that had to be calculated and not the time. Instead of dividing the time, Wendy should have multiplied the time by 5 and Nancy, according to her calculation, should have multiplied by 1,12.

Although Nancy was able to get only 7 of the 11 calculations correct, she showed very good problem-solving skills by constantly checking her work and ascertaining the validity of her strategy use and making changes accordingly. She used many different strategies. At times she doubted her ability to assess her own performance, by directing comments such as: "*Is it right*?" to me as the authority. Confrey (1985:478) who worked on problems learners experience in mathematics encountered similar problems with her learners. Nancy shifted between correct and incorrect answers e.g. the time when the speed was 7 m/s and 1200 m/s was correctly calculated as 140 seconds and 240 seconds respectively. After subsequent strategy changes, she abandoned the correct answers. According to Clement (1980:18), shifting between correct and incorrect strategies indicates that contradictory schemes may continue to exist independently in the same individual.

Irene

Time (s)	0	1	2	3	4	5	10	50	120	500		560
Speed (m/s)	0	5	10	15	20	25	30	35	40	700	1200	

Figure 4.8.

Irene had also used the horizontal additive strategy without noticing the 'gaps' in the time intervals. However, she, unlike the other learners did not recognize anything unusual in a speed of 40 m/s being followed by a speed of 700 m/s. In calculating the time when the speed was 700 m/s, she assumed the speed to have been 45 m/s i.e. following the additive rule. She calculated the time as: "*700 – 45 = 655*". This value of 655 seconds was according to Irene too large a jump from 120 seconds and so she discarded this strategy. Irene was able to recognize the big jump with the time but failed to recognize such a jump in the speed from 40 m/s to 700m/s.

Irene tried establishing a pattern for the time increase by looking at the difference between 50 seconds and 10 seconds, 120 seconds and 50 seconds, which were 40 and 70 seconds respectively. The problem with these differences, according to her, was that they were not equal. She then subtracted 120 seconds from 700 m/s and obtained 580 seconds, which to her was too large a value to follow a time of 120 seconds, so this too, was disregarded. Irene did not know that subtracting numerical values of two different physical quantities was not possible. She simply considered the values for the variables as ordinary numbers. Thus showing the need for possession of sound cognitive structure in order to facilitate problem solving (Ausubel et al. (1978:571).

She then multiplied 40 by 5 and subtracted the answer from 700 m/s and obtained a time of 500 seconds. She abandoned filling in the rest of the table because the large values that she obtained did not *"follow the trend."*

While Irene may not have been able to fill in all the correct values nor complete the table, she did show some good problem solving skills in that she was able to recognize some answers as being "*unrealistic.*"

Dinesha and Erica

Time (s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	50	250	600	700	1200	2800

Figure 4.9: Dinesha

Time(s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	50	250	600	700	1200	2800

Figure 4.10: Erica

The strategy that Dinesha and Erica employed was different from that of the others. They said that since the time increase from 1 second to 2 seconds was by a factor of 2 so too the speed must increase by a factor of 2, thus using a proportionate functional rule. Dinesha's argument was " *At 1 second it should be 5 metres, and at 2 seconds it* [previous speed] *should be multiplied by 2 so speed is 10* [2 x 5]. " Erica's argument whilst being similar to Dinesha's was somewhat clearer. She said: "*It increases at 5 metres per second every second so for every second there's 5 metres and here there's 2 seconds so it will be 2 times the amount in 1 second which will give me 10.*"

They both used a proportionate functional rule, which they without being aware of it changed, Dinesha at 3 seconds and Erica at 4 seconds. Dinesha changed to a vertical multiplicative rule after considering "*It's increasing at 5 metres per second every second so then at 3 it should be 15 – by multiples of 5.*" Although Erica intended to "*... do the same with 4 and 5*" as she had done with 2 and 3 seconds she, without being aware of it, switched strategies to a vertical multiplicative rule by "*4 times 5 would be 20 and 5 times 5 would be 25 ...*". Thus both of them changed from a proportionate functional rule to a vertical multiplicative strategy without any apparent reason being provided for doing so. Neither of them was aware of having changed her strategy. Unlike Shireen, Wendy, Nancy and Fatima who deliberately changed strategies due to problems that they experienced in filling in the table, Dinesha and

Erica had experienced no such problems. Using the vertical multiplicative strategy the table was correctly completed by Dinesha and all but the speed at zero seconds by Erica.

Erica debated about what the speed at zero seconds was. Eventually, working with a backward strategy on the number line as shown in figure 4.4, she correctly determined the speed as 0 m/s. The following excerpt shows the argument she used:

"Researcher: *Why haven't you filled in zero?*

Erica: *Because at the time zero I don't know what the speed is because they say the speed increases at 5 m/s every second. Now there's no time so I think that the speed is zero because there's no time so ... As time increases by 1 second, speed increases by 5 metres per second so I can get speed at time equal to zero seconds.* [Read question]. *No I can't get speed at time equal to zero.*

Researcher: *Why not?*

Erica: *Speed at which it started before it started increasing ... so at time equal to zero, speed will remain zero because if we work from the question, one can see the speed increases by 5 so that will follow the pattern so probably speed will be 1 because we can see 0, 1, 2, 3, 4, 5 – if we have all these on the number line we have to count from 5 – 1, 2, 3, 4, 5 – we will have to start from zero in order to get 5 metres per second every second so I think this will be zero.*

Researcher: *But you said that it was 1 earlier?*

Erica: *Yes, but I had to draw the number line to see the pattern.* "

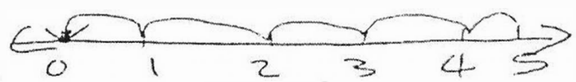

Figure 4.11.

The fact that the general rule, that she had formulated to fill in the rest of the table, could be applied to zero seconds as well, did not occur to Erica.

THE RELATIONSHIP BETWEEN SPEED AND TIME AND FORMULATION OF AN EQUATION IN VERBAL AND SYMBOLIC FORM

All the learners with the exception of Irene were able to develop the correct relationship between speed and time. Although they all used the relationship 'speed =

time x 5,' in filling in the table, their verbal and symbolic interpretations varied and were often inappropriate. While learners experience no problems when dealing with numerical relationships, they find it difficult moving from the numerical rule to the verbal rule to the symbolic rule. Herscovics & Kieran (1980:572) and Clement (1980) amongst others discussed the difficulties experienced by learners in translating from a verbal statement to an equation. The individual responses will be discussed separately.

Wendy

The relationship given by Wendy was stated explicitly as a ratio: "*Time is to speed equals 1 is to 5,*" thus displaying a clear understanding of the relationship.

For every second the speed increases 5 sec so it implies that the ratio 1 : 5 → time : speed
1 : 5.

Figure 4.12.

Writing down a formula or an equation created no problem for Wendy and her immediate response was: *"Speed equals time times 5 because it increases 5 metres every second."* She qualified the statement that she made thus showing that she had a clear verbal and symbolic perception of the situation. The verbal and symbolic forms of the equation given by Wendy were as follows:

Speed = Time x 5
V = 5t

Figure 4.13.

When Wendy was asked if '5' represented any physical quantity, her reply was "*The object travels 5 metres faster every second. 5's add up.*" Thus she showed an understanding of 5 metres per second every second as a rate or as the acceleration.

Dinesha

Dinesha had also considered the relationship in terms of a ratio. Her response was "*at every second the speed increases by 5 metres per second so they are* *proportional."* After some hesitation she came up with the following equation in words:

speed = time multiplied by 1 rate of increase of speed.

Figure 4.14.

She too seemed to have some intuitive idea of the rate of change of speed as the acceleration. Denisha experienced difficulty translating the verbal rule into the formal, symbolic form. While she was able to translate speed and time into 'v' and 't' respectively, 'the rate of increase of speed' seemed to create a problem. She said that "*metres per second is the speed*" thus considering the unit 'metres per second' to be the same as the physical quantity 'speed,' a common error referred to on page 78. The equations provided were:

$$V = t \times m/s$$

$$V = t \times 5\ m/s$$

Figure 4.15.

Because she considered 'm/s' to be speed and 'v' as the speed as well, she was puzzled at having speed on both sides of the equation. She said "*It does not make sense to me because you calculated speed and how can you already have speed*?"
Denisha could not make sense of it, what she did know though, was "*This* [5 m/s] *doesn't change but these* [speeds in the table], *do change*" and that time was multiplied by this constant 5. This problem could have been obviated had she had a clear understanding of the physical quantities, their symbols and their respective units.

The equation, '*v = t x m/s*' was incorrect because it violates the rules of mathematics and science in that it related physical quantities and units. It appears that the letter symbol "m/s" is interpreted here as a variable representing speed. The equation '*v = t x 5 m/s*' is also incorrect because an equation representing physical quantities has no units. It is clear that she has some difficulty understanding the precise meaning of algebraic symbols and equations as representing quantitative variables and relationships.

Dinesha was not too confident about the right hand side of the equation having time appearing twice. Her interpretation was that the speed was, "*5 metres per second,*" and that incorporated in this was "*per second*" which to her was the time. Hence her comment, that speed and time "*intertwine*" because "*time is part of the speed. Like you say metres per second.*"

Fatima

Fatima did what Dinesha had done in formulating the equation. She 'converted' the numbers into physical quantities as follows:

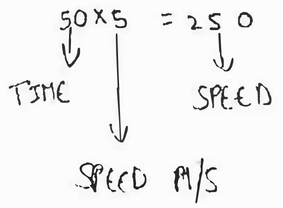

Figure 4.16.

Having obtained the equation, "*time x speed m/s = speed,*" Fatima was initially confused at having speed on both sides of the equation. Upon reflection she qualified that the speed (V) on the left hand side of the equation "$T \times V = V$," was the constant and equal to 5 metres per second. She used the same symbol to represent different physical quantities, which to her mind, although different i.e. one being the constant speed and the other the changing speed, represented the same physical quantity speed. This confirms Herscovics & Kieran's (1980:573) view that symbolism and notation may not carry the same meaning for both learners and educators. The CSMS algebra test study shows that to learners different symbols cannot have the same value (Kucheman 1982:489). However, Fatima regarded the same symbol as having different values.

A point worth noting is that Fatima included the unit for speed on the left hand side of the equation and not on the right hand side thus emphasising the fact that speed on the left hand side was the one from the given statement i.e. '5 m/s.' Although both Fatima and Dinesha had identified '5 m/s' as being the constant that time was multiplied by, their initial thoughts on it were different. Here too the units were not appropriately used.

Shireen

Shireen experienced difficulty with formulating a verbal equation although she had established a correct numerical rule between speed and time in filling in the table. The following excerpt from her written work shows the use of the functional rule:

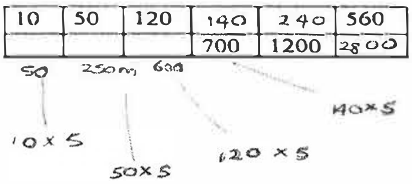

Figure 4.17.

She was able to work with the numbers but experienced difficulties with translating the numerical equation into a verbal equation. Fatima had experienced a similar problem. Shireen was 'assisted' by directed questions being asked e.g.

"By looking at the table, can you see a relationship between speed and time?"

"*How did you get these* [speed] *values in the table?"*

"How did you get the speed knowing the time?"

The following is an excerpt of the interview that followed:

Researcher: *Write down a formula or equation relating speed and time.*

Shireen: *speed = time x 5*

Researcher: *Can you use this formula to get the speed at any time?*

Shireen: *Yes, as long as I can get the constant, which in this case is 5.*

Researcher: *If 'v' represents the speed and 't' the time, write down a formula or equation relating speed and time in symbolic form.*

Shireen:

t x c = speed.

[Figure 4.18.]

V = t x constant, can I use any alphabet for the constant?

Researcher: *Whatever you want to.*

Shireen: *V = t x a.*

Researcher: *Why have you used 'a'?*

Shireen: *I thought I had to give it an alphabet and acceleration is 5.*

With guidance, she was able to establish the equation: *"speed = time x 5"* which written in general form was: *"v = t x constant".* She recognized this constant as the rate, which was the acceleration equal to 5. Her final equation in symbolic form was *"V = t x a."* It is obvious from Shireen's initial response that she experienced a problem translating from a numerical equation to a verbal equation. However, with guidance she was able to obtain the verbal equation. Clement's (1980) study clearly shows that students perform well in translating word problems involving numbers into equations but experience difficulties when the word problems involved symbols which in this case also involved the physical quantities speed and time.

Nancy

It was evident from Nancy's response to filling in the table that the speed "*increases by 5 but not all the time.*" For her there were three rates of increase, namely, from zero seconds to 50 seconds, the increase was 5 then from 120 seconds to 700 seconds the increase was 2,4 and at 560 seconds it was 1,12. She was asked to formulate a relationship for zero seconds to 50 seconds since this part formed the major part of the table with a common relationship.

Nancy interpreted *"5 metres every second"* as *"5 metres"* being the distance and "*every second*" as the time so *"you move a distance of 5 metres for 1 second"* so speed according to her could be found by multiplying the distance by the time. To Nancy the distance was a constant 5 metres. This is not possible since the object was in motion and therefore the distance covered by it could not be constant. However, her misinterpretation arose because she considered the 'rate of change of speed' to be the distance. She did, however, have an idea of the 'distance' 5 metres, as she interpreted it, being the acceleration. This is evident in the following comment that she made: *"I used 5 from the statement but this is not always the case, what if we move 10 metres per second or 20 metres a second? But it's just the distance that gives the speed, distance travelled times time will give speed."* Thus the formula:

$$\underset{\text{Speed}}{V} = \underset{\text{Time}}{T} \times \underset{\text{Dis}}{D}$$

Figure 4. 19.

which written in symbolic form was

$$V = T \times D$$

Figure 4.20.

This problem with 'rate' and 'amount' is peculiar not only to Nancy, but is evident with tertiary level students as well as is highlighted by Nickerson (1985:205).

Nancy verified the correctness of this formula by substituting numerical values:

$$\begin{array}{lcl} 0 & = & 0 \times 5 \\ 5 & = & 1 \times 5 \\ 10 & = & 2 \times \$5 \end{array}$$

Figure 4.21.

She considered these three cases as being sufficient for her to make a generalization.

Erica

She attempted to proportionally formulate the relationship. Although she had carried out the proportional rule in filling in only two values in the table, and the vertical multiplicative rule for the rest of the table, the rule that she implied for the relationship was based on the proportionate functional rule. The relationship given by Erica was:

As time ↑ by a certain factor
speed ↑ by 5 x that (same) factor

Figure 4.22.

She did not realize that by including "*5 times*" into her statement it resulted in the rule being mathematically incorrect. A reason for her including '5 times' into the relationship could be that she had initially started with the horizontal multiplicative strategy so the speed at 3 seconds, which increased 3 times from 1 second to 3 seconds, would also increase 3 times from 5 m/s to 15 m/s, calculated as 5 x 3. At this stage she 'automatically' progressed into a vertical multiplicative functional rule, considering the '5' that she had considered in the above calculation to be the speed at 1 second, to be the rate of increase in the speed.

The subsequent speeds were calculated using the rule: 'time x 5.' Erica 'merged' these two strategies i.e. she took the rule "*as time increases by a certain factor, speed increases by the same factor*" and merged it with, "*as the time increases, the speed increases by a factor of 5.*"

The formula that she presented was: "*Speed = 5 x Time*", thus she experienced no difficulty translating from numerical to verbal. Her problem was interpreting the verbal equation into symbolic form. She initially wrote it in terms of the units. While she had stated that m/s was the unit for speed, she also likened m/s to the speed, hence the formula:

$$\text{Speed} = 5 \times \text{Time}$$
$$m/s = 5(\cancel{[illegible]}\,s)$$

Figure 4.23.

On reflection she changed the above formula to: "$V = 5(T)$".

Irene

Irene used the additive rule for correctly calculating the first five values in the table. The rule was inappropriately applied for the next two calculations thereafter she was confused. In order to complete the rest of the table, a rule had to be established between the two variables time and speed. Such a rule could not be established by Irene.

SUMMARY OF RESPONSES TO QUESTION ONE

The learners used different intuitive strategies in answering question one. The appropriate strategies used were: horizontal additive, ratios, vertical multiplicative, the functional rule and the proportionate rule. The latter three strategies were interchanged by some of the learners who used them. However, this did not affect the answers as in this particular case, the initial speed was zero. In problem two, where the initial speed was not zero, these rules could not be interchanged.

All through the working the learners were mindful of the fact that the as the time increased the speed increased as well.

Five out of seven (72%) of the learners filled the table in correctly, one out of seven (14%) of the learners obtained correct answers for 67% of the table and the remaining learner (14%) obtained correct answers for 50% of the table.

Six out of seven (86%) of the learners, when determining the verbal form of the relationship used the relationship of speed as being the product of time and 5 or the product of time and a physical quantity. Three of the seven learners (43%) gave the relationship as the product of time and 5, three (43%) gave the relationship as the product of time and a physical quantity. The physical quantity to Fatima was the "*constant speed which is equal to 5 metres per second,*" to Dinesha was "*the rate of increase of speed,*" and to Nancy was the "*distance which equals to 5.*"

Table 4.1: Strategies Used

STRATEGY	LEARNERS	PERCENTAGE
Horizontal Additive	All except Erica & Dinesha	71%
Vertical Multiplicative	All except Irene	86%
Ratio	Nancy, Wendy	29%
Horizontal Multiplicative	Nancy	14%
Proportionate Rule	Erica, Dinesha	29%
Backward Strategy	Erica	14%

Table 4.2: Responses to Filling in the Table

CALCULATION OF THE:	PERCENTAGE
Speed up to 5 seconds	100%
Speed at 10 seconds	86%
Speed at 50 seconds and 120seconds	71%
Speed at 560 seconds	57%
Time	71%

Table 4.3: Correct Responses to the Relationship Between Speed and Time

RELATIONSHIP	PERCENTAGE CORRECT
Formulation	86%
Verbal Relationship	71%
Symbolic Equation	43%

RESULTS AND ANALYSIS OF QUESTION TWO

Introduction

The learners were provided with the following statement:

'The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.'

The learners were required to fill in the blanks in the following table from the information given in the statement.

Time (s)	0	1	2	3	4	5	6	10	50	80		
Speed (m/s)											410	1000

They were then required to formulate the relationship between speed and time and then translate it into symbolic form.

The results were analysed in terms of the strategies used. The learners used a variety of strategies. The common and relevant strategies used were: horizontal additive, horizontal multiplicative, the functional rules: $v = 10 + 2t$ and the functional proportionate rule, counting on, and skip counting. Since each learner adopted different overall approaches to this question, it is appropriate to discuss each learner's response individually. An overall summary of the responses and strategies is provided at the end of the discussion.

Six learners attempted question two. Nancy was excluded from this exercise since she had spent sixty-five minutes on question one and the researcher felt that if she attempted this exercise, she would not have done justice to it.

Each learner's response to filling in the table and formulating the relationship between speed and time will be discussed separately and each discussion will be preceded by the learner's completed table. This will be followed by a summary of the common and or relevant strategies used in question two.

Shireen's response to filling in the table

Time(s)	0	1	2	3	4	5	6	10	50	80	205	500
Speed (m/s)	10	20	40	120	480					~~28~~	410	1000
		~~12~~	14	16	18	20	22	24	26	28		
			~~24~~	~~36~~	~~48~~	60	72	120	600			
		12	24	6	8	10	12	20	100	160		

Figure 4.24.

Shireen's immediate response was to try and establish a relationship between speed and time from the given statement. Since she made very little progress, she was asked to fill in the table.

Shireen assigned a speed of 0 m/s for the time 0 seconds, although she, while analysing the statement had recognized the initial speed to be at the "*beginning*." The reason for this was that she compared this problem with problem one where the initial speed was 0 m/s. However, when asked to justify the speed of 0 m/s, she reread the question and changed the speed to 10 m/s.

In order to calculate the speed at 1 second, she adopted a horizontal multiplicative strategy, multiplying the previous speed by 2, since according to her "*it increases at 2 metres per second.*" The speed was calculated as '10 x 2' and '20 x 2', At this point she subconsciously changed her strategy to 'previous speed x time' i.e. '40 x 3 = 120, 120 x 4 = 480. The fact that she was not aware of having changed her strategy stems from her comment but "*I was multiplying by 2.*" Having obtained a speed of 480 m/s preceding 410 m/s she realized that this strategy had to be reviewed. Her initial strategy i.e. 'previous speed x 2' did not take into consideration the time but did consider '2' because according to her reasoning '2' had to feature in the calculation. However, while calculating the speed at 3 seconds, she subconsciously realized that a functional relationship between the speed and time had to be considered hence the switch in the strategy to include time.

With the aid of a pictorial representation, Shireen established that in 1 second "*It's* [the speed] *10 metres plus 2 metres per second.*" Bell (1979:417) provided evidence that diagrams are valuable in clarifying a problem. She then switched to the horizontal additive strategy, from 0 seconds to 80 seconds, adding 2 to the previous speed. She did not consider the gap after 6 seconds and having obtained 28 m/s preceding 410 m/s her comment was: *"... but it doesn't follow there's no relationship between 28 and 410 because it is supposed to increase at the same speed at each time."*

Shireen changed her strategy for 2 seconds to 50 seconds to '12 x time' e.g. "*12 x 2 = 24, 12 x 3 = 36, etc.*" She realized that a functional relationship existed between the time, the initial speed and acceleration. She first added together the initial speed and the acceleration to get 12 so she worked with the functional rule '$v = (10 + 2)t$' instead of '$v = 10 + 2t$'. Shireen was the only learner who used this strategy.

To calculate the time, given the speed, she tried establishing a relationship between the speed 410 m/s and 1000 m/s. After comparing this problem with problem 1, she changed the above strategy to a functional rule that she had applied in the previous problem i.e. 'speed equals rate of increase of speed x time.' She obtained the following speed values 2, 4, 6 ... 160 m/s up to 80 seconds. Working with this strategy, she calculated the time when the speed was 410 m/s and 1000 m/s as an inverse relationship i.e. 'speed divided by 2' and thus obtained 205 seconds and 500 seconds respectively. It was possible in the previous problem to have worked with the functional rule "speed equals rate of increase of speed x time.' However in this problem the above rule could not be applied because the initial speed was not zero. When asked to verify the speed of 2 m/s at 1 second, she realized that it should have been 12 m/s. She went back to the formula '$(10 + 2)t$' but only for 2 seconds and 3 seconds after which she switched to 'previous speed x time' for 4 seconds and 5 seconds. Having realized that obtaining a speed of 720 m/s preceding 410 m/s was not in keeping with the trend, Shireen abandoned this strategy.

Shireen went back to the horizontal additive strategy that she used previously. Here too, she disregarded the jump after 6 seconds and obtained 28 m/s as the speed at 80 seconds as before. This time she abandoned this strategy because "*80 divided by 28*" i.e. 't divided by speed' yielded a "*point value.*" Learners are often sceptical about obtaining fractions as answers. This is probably due to the fact that when they were taught arithmetic at their early stages, fractions were not included thus conditioning them to obtain non-fractions as answers. In this respect Confrey (1985:477) outlined one of the problems in mathematics education as learners' mathematics knowledge being limited and rigid. They focus on answers which they expect to be whole numbers. Confrey considers their powers of flexibility as being weak. Shireen applied a horizontal additive rule to obtain a speed of 28 m/s for 80 seconds but she checked this answer by using the inverse of a vertical multiplicative rule, a rule that she had abandoned in her previous calculation. It is possible to apply one rule when filling in the table and another rule to verify the use of the strategy. Both the strategies that Shireen used were inappropriate. Shireen subsequently reviewed the situation and recognizing the jump after 6 seconds, she used a 'counting on' strategy and obtained 30 m/s as the speed at 10 seconds.

In calculating the speed at 50 seconds, she used the following argument: "*If time increases by 10, speed increases by 10.*" Thus assuming that the speed increased at 1 metre per second every second. Her calculation was as follows:

10 - 30
20 - 40
30 - 50
40 - 60
50 - 70
60 - 80
70 - 90
80 - 110

Figure 4.25.

She verified the speed of 110 m/s at 80 seconds by dividing the given speed, 410 m/s by the previous calculated speed, 110 m/s i.e. she used the inverse of a horizontal multiplicative strategy. She had not used a horizontal multiplicative strategy initially to obtain the answer of 110 m/s. While different strategies may be used to verify answers, in this case the use of the strategies were inappropriate. She disregarded this calculation because she obtained a decimal fraction as the answer, which to her was unacceptable. She accepted the previous calculation i.e. 'speed divided by 2' as the acceptable strategy for calculating the time.

Shireen switched strategies a number of times because she realized that a trend had to be followed and that decimal fractions were unacceptable. At times she was inconsistent in the use of the strategies in that she started with one and along the line changed it, often subconsciously.

She displayed typical problem-solving habits where learners rally back and forth modelling, evaluating and discarding different strategies until eventually they obtain an acceptable answer or they give up. Clement (1980:16) called this "*shifting between approaches*." Shireen also showed signs of not always actively taking advantage of what she knew. Rather, she seemed to run into things that might be useful and then followed them up in some way. Responses like these were also obtained by Schoenfeld (1985:212) in his work.

Shireen, however, persevered until she had obtained, what to her mind, were reasonable answers. Eventually she settled for the following strategies: 'horizontal additive' up to 6 seconds, 'counting on' up to 10 seconds, 'skip counting' which was inappropriately used for 50 seconds and 80 seconds and an 'inverse vertical multiplicative' rule which was used to calculate the time. She realized that in order to complete the rest of the table, a relationship between speed and time had to be developed, hence the rule 'if time increases by 10 then speed increases by 10'. It was unfortunate that she had formulated the correct relationship 'speed is equal to time times 2 plus 10' only after she had completed filling in the table and did not use the formula to go back and verify all her calculations.

Shireen's response to determining the relationship between speed and time

Shireen was confused about whether to write the relationship in terms of speed or time because she was asked to relate the speed to time. Since she had just calculated the time with the speed given, she settled for the relationship, 'time is equal to speed divided by 2' where 2 was 'a' which was identified as the acceleration. She was influenced by problem 1 in the choice of this strategy. The equation written in symbolic form was as follows:

$$t = \frac{v}{2/a}$$

$$\Rightarrow \frac{410}{2}$$

$$\Rightarrow 205$$

Figure 4.26.

Considering her symbolic equation would mean that the denominator was 2 divided by 'a,' but what Shireen meant was that the denominator was 2, which in general form was represented by the letter 'a.' While the equation was mathematically incorrectly represented, Shireen was clear about its use as was evident in her numerical calculation.

The use of this equation was verified by substituting 410 m/s for the speed and obtaining 205 seconds for the time.

Her response to whether this formula could be used to calculate the speed when the time was given was, "*Yes. Speed is equal to time times a. You have to remember that 10 is in your mind so I'm just working with the 2 only.*" She verified this rule by substituting for 2 seconds as follows: "*t is 2 seconds times 2 equals 4, right, plus the 10 will give me 14.*" However, when writing down the equation in symbolic form she omitted '10' but included it when checking the validity of the formula as is evident in the following:

$$
\begin{aligned}
v &= t \times a \\
&= 2s \times 2 \\
&\Rightarrow 4, + 10 \\
&\Rightarrow 14
\end{aligned}
$$

Figure 4.27.

Thus showing that while Shireen may have had no problem in the numerical calculation and expressing it as a verbal statement, she experienced a problem expressing this in the form of a symbolic equation.

It had not occurred to Shireen that when calculating the time she would have to use the inverse strategy that she had used in calculating the speed. The strategy that she had adopted in question 1 seemed to have had a greater influence here. According to Schoenfeld (1985:140), this happens when a learner loses sight of the problem-solving repertoire and becomes locked into one approach.

It was quite clear from her discussions that the initial speed was important and therefore had to be included in calculating the speed. However, having not reflected on this sufficiently, she did not see the need for its inclusion in calculating the time. Had she checked whether the time was correctly calculated by using the inverse of the 'speed' formula, she would have realized that she had made a mistake. A point to note is that she had formulated the relationship '$v = 10 + 2t$' after the table was filled.

Wendy's response to filling in the table

Time(s)	0	1	2	3	4	5	6	10	50	80	137	333
Speed (m/s)	10	12	14	16	18	20	22	30	150	240	410	1000

Figure 4.28.

Wendy debated about whether the speed at zero seconds should be 0 m/s or 10 m/s. Her arguments were: "*If the time was zero it wouldn't have been travelling at all, so the speed was zero*" and "*In the table if time equals zero seconds and the initial speed is 10 metres per second then it started at 10.*" After reading the statement a number of times she settled for the speed as 10 m/s.

Wendy started with a horizontal additive strategy to calculate the speed up to 6 seconds. She noticed the jump after 6 seconds and like Shireen used the 'counting on' strategy to obtain a speed of 30 m/s at 10 seconds.

To calculate the remaining speed she felt it necessary to develop a proportional rule between speed and time. She chose not to work with zero seconds and 10 m/s because "*I can't find the ratio and proportion here because nothing works.*" She considered other possibilities like "*divided by 3, divided by 4 ... no.*" She finally settled for the following, "*At 10 seconds the speed was 30 so at time 50 seconds the speed would be ... let's cross multiply.*" In this way she obtained 150 m/s as the speed at 50 seconds and using this proportional relationship calculated the speed at 80 seconds to be 240 m/s.

When the time was required, she realized that she had to work the proportional relationship the "*other way*" around and obtained answers of 137 seconds and 333 seconds when the speed was 410 m/s and 1000 m/s respectively.

Wendy used two different strategies. She used a horizontal additive strategy for zero seconds to 6 seconds and thereafter "*For the last five it works out the ratio is 1 is to 3.*" She used a proportional rule to calculate the time and checked it by dividing by 3. Wendy checked her numerical calculation and not the validity of the strategy used.

Wendy's response to determining the relationship between speed and time

Wendy's response to formulating a relationship was, "*The ratio is 1 is to 3. Every second the object increases its speed by 2 metres.*" She used two different strategies namely, the speed increases at 2 m/s for every second for the first part of the table, and the ratio 1 is to 3 for the latter part of the table. Because of this she could not formulate a single relationship between the speed and time for the entire table. However for the latter part of the table she gave the equation "*Time is equal to three speed.*" She had developed the ratio 'time is to speed is equal to 1 is to 3' from which she concluded that time was equal to three speed i.e. '$T = 3V$.'

Wendy simply considered the letters as shorthands for the physical quantities time and speed and wrote the letters and the numbers in the order in which they appeared. Such an error has been described by Clement (1980:12) as a 'reversed equation' error which to him has a deeper cognitive source. Had she taken the 'T' as 'the *number* of seconds' and 'V' as 'the *number* of metres per second' her error could have been detected. Davies (1984) (cited in Nickerson 1985:213) attributes the reversed equation phenomenon to faulty retrieval from memory when learners retrieve the incorrect frame from two acquired frames, i.e. 'verbal based frame,' and the 'numerical variable frame.' Such errors are based on deeply seated misconceptions regarding the meaning of variables that are difficult to remediate by training. The best time to make this distinction clear presumably occur long before one encounters high school physics, and probably when one is first introduced to the concept of multiplication. (Nickerson 1985:211.)

When asked about the relationship for the first six values, her response was: "*There's no common relationship here. Over here for zero it's 10 then it's 1 – 12, 2 – 14 … nothing I can work with.*"

Salzano (1983:12) explains a response like Wendy's as the learner realizing that there is a problem but not being able to recognize it and not having the "*machinery*" that would enable her to get started. She considers the building up of this "*machinery*" as a slow process with each child developing his/her own method of how to proceed.

Wendy was focussed on obtaining a ratio and because she could not see a proportional relationship at the beginning of the table, she gave up. Wendy is the type of learner who gives up easily. She was often content with her work and showed little signs of perseverance. Wendy's determination wavered and she could not persevere. This happens, says Polya (1945:88), when we do not see any way out of a difficulty. We are depressed when the way we have followed is suddenly blocked.

Dinesha's response to filling in table

Time (s)	0	1	2	3	4	5	6	10	50	80	205	500
Speed (m/s)	10	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	30	~~150~~	~~240~~	410	1000

~~5~~ 12 14 16 18 20 22 110 170

2 4 6 8 10 12 20 100 160

Figure 4.29.

Dinesha's initial assumption was that the initial speed was 0 m/s at zero seconds and that the speed was 10 m/s right up to 6 seconds. After reading the question again she realized that these values were incorrect. She changed the initial speed to 10 m/s. To calculate the speed at 1 second, she simply divided the numbers that were given in the statement i.e. 10 divided by 2. Her response was similar to that found by Schoenfeld (1985:370), where learners do not carefully analyse the problems they have to solve. One of the problems is that they try to use all the given numbers in the problem statement in their calculation without regard for the relationship of either the given numbers or the resulting numbers to the problem situation.

Dinesha immediately realized that this was incorrect and switched to a horizontal additive strategy but only for 1 second and 2 seconds because she was not too confident about this strategy since dividing speed by time did not yield a constant, as was the case with problem 1. She needed to get a constant, 2, which she identified as the acceleration. Like Shireen, Dinesha used the horizontal additive strategy to obtain the answers and inappropriately used the inverse of the vertical multiplicative rule to check her answers. After reading the question again she reverted to the horizontal additive strategy. She recognized the gap after 6 seconds and used the 'counting on' strategy in order to get 30 m/s at time 10 seconds.

In calculating the speed at 50 seconds and 80 seconds she did exactly what Wendy had done i.e. used ratios and obtained 150 m/s and 240 m/s.

When calculating the time she developed the following strategy by referring to the table, when the time increases from zero seconds to 10 seconds then the speed increases from 10 m/s to 30 m/s which she explained as *"If you take 10 units for your time you go 20 units for your speed."* This rule was applied as follows:

10 = 30
20 = 50
30 = 70
40 = 90
50 = 110
60 = 130
70 = 150
80 = 170

Figure 4.30.

Denisha used a more sophisticated 'counting on' strategy, counting in terms of 10's and 20's. She subsequently corrected the speed for 50 seconds and 80 seconds to 110 m/s and 170 m/s respectively.

In order to calculate the time she debated once again about the way in which she could get a constant. She argued that in the previous problem she was able to get a constant by dividing the speed by time so this should also be the case in this problem. Since this was not the case here she contemplated reviewing the strategy that she had used in determining the speed.

She changed to a vertical multiplicative rule i.e. time x 2 throughout the table because in this way dividing the speed by time would yield the constant 2. Thus to calculate the time she divided the speed by 2 and got the answers as 205 seconds and 500 seconds respectively. She did however have reservations about the speed being 2, 4, 6 ...160 m/s for the times 1, 2, 3 ... 80 seconds respectively, which she expressed as "*That would work, but then, it says an initial speed of 10 metres per second which puts everything off. But if the speed at the beginning is 10 we would have to add this* [i.e. 2, 4, 6 ...160] *to the 10.*" According to the given statement Dinesha was convinced that the additive rule should apply but according to the rule developed in the previous problem the vertical multiplicative rule should apply. She realized that only one strategy would apply. She was confused because like Shireen she failed to consider this problem for what it was worth. They constantly compared it with the previous problem. Dhillon (1998:387) would consider learners like these as novices because in his studies he found that novices frequently compared questions. Denisha did not commit herself to one strategy at this point.

Dinesha's response to determining the relationship between speed and time

Dinesha was focussed on getting a constant by dividing speed by time. She also changed her view in that the constant could have any value and that the constant could be different for different sets of values as long as it was a whole number. Therefore since 12 divided by 1 and 14 divided by 2 yielded the whole numbers 12 and 7 respectively the rule was followed. However thereafter the rule failed because 16 divided by 3 gave a fraction. She doubted yet again as to the correct strategy, trying to convince herself that "*according to the statement it makes sense*" for the speed to be 10, 12, 14, etc, but the fact that 'speed divided by time' does not yield a constant threw some doubt onto this strategy. Eventually she settled for the additive strategy for calculating the speed for 1 second to 10 seconds. However she was unable to develop a relationship because according to her only a vertical multiplicative rule would be acceptable, and this rule was not the only rule in filling in the table. Of the learners who used different strategies when filling in the table, Dinesha was the only one who realized that only one rule would apply to the table, although what she did do in her numerical calculation contradicted her view. The following excerpt shows the state of confusion that she was in and because of this she was unable to formulate a relationship.

"Using the multiples of 2, 2 times 7 equals 14, 1 times 12 equals 12, 3 times what will give 16? Not possible, I know that 3 times 5 equals 15 not 16. I have no idea how to

do this. I can't see a relationship. There is a relationship but in this case I can't see it."

In Dinesha's case there existed conflicting 'frames,' and according to Davies (1984), the frame that develops earlier normally persists and is sometimes retrieved inappropriately if at all.

Irene's response to filling in the table

Time(s)	0	1	2	3	4	5	6	10	50	80	~~708~~ 394	984
Speed (m/s)	~~0~~ 10	~~2~~ 2	4	12	48	240				96	410	1000
		12	14	16	18	20	22	~~24~~ 26	28 66	30		

Figure 4.31.

Although Irene recognized the starting speed as the initial speed, she filled in 0 m/s as the speed at 0 seconds. After reading the question again she changed the speed to 10 m/s. She filled in a speed of 2 m/s at time 1 second because the speed increases at 2 m/s every second. Irene did not consider the situation realistically because if she did she would have realized that if an object starts with a speed of 10 m/s and its speed increases then it's speed after 1 second cannot be less than that which it started with.

To calculate the speed up to 6 seconds, she used the functional rule 'previous speed times time.' She realized her mistake when she got a speed of 1440 m/s for 6 seconds and this value was much larger than 410 m/s. Since a trend was being followed these values were unacceptable and the strategy was abandoned.

Irene was asked to verify the speed of 2 m/s at 1 second. She read the question again and having realized that she had made a mistake, she corrected it using the additive horizontal strategy. The gap after 6 seconds went unnoticed initially. Subsequently she noticed the jump from 6 seconds to 10 seconds as 4 seconds so she reasoned that the speed would also change by 4 m/s from 22 m/s to 26 m/s. Similarly she reasoned that the speed would increase by 40 m/s and 30 m/s at 50 seconds and 80 seconds to yield 66 m/s and 96 m/s respectively. She worked on the assumption that when the time increased by a certain value the speed increased by the same value.

In keeping with this trend, she calculated the time by first considering the increase in speed from 96 m/s to 410 m/s. She then added this difference of 314 to 80 seconds. Similarly she calculated the increase in speed for the next value as 590 m/s and added this difference to the previous time of 314 seconds.

Irene used two different strategies to fill in the table. She started with a horizontal additive strategy when calculating the speed up to 6 seconds thereafter she switched to a basic counting on strategy counting in terms of 1's and 1's.

Irene's response to determining the relationship between speed and time

Irene said that the time and speed increased equally. She did not consider the strategy applied in the first part of the table thus showing that she reflected only on her most recently developed strategy. A phenomenon peculiar not only to Irene, but to Dinesha as well.

The application of a horizontal additive rule does not lend itself to the formulation of a functional relationship between speed and time. Hence Irene was unable to develop a relationship.

Irene's responses were similar to those found by Biggs (1984:130) where low achieving learners make incongruent strategy choices and use these strategies in ways that would seem to be inappropriate.

Fatima's response to filling in the table

Time (s)	0	1	2	3	4	5	6	10	50	80	200	495
Speed (m/s)	~~0~~ 10	20 12	14	16	18	20	22	30	110	170	410	1000

Figure 4.32.

Fatima's immediate response to determining the speed at zero seconds was 0 m/s which she, on reflection, changed to 10 m/s. In order to get the speed at 1 second, she intended applying the same strategy that she had used in the previous problem. Unfortunately she recalled it incorrectly as the 'previous speed multiplied by 5' instead of 'previous speed plus 5' and so she incorrectly multiplied the previous speed, 10 m/s, by 2 to get 20 instead of '10 plus 2' as the speed at 1 second. On reflection she realized that this problem was different from the previous one. This was evident in the following comment: "*In the other example I multiplied time by 5, but here I can't do that because the speed at the beginning is 10 not zero...so I'm going to have to add it.*" This is in keeping with Cobb & Merkel's (1989:79) view that learners invent strategies that make sense to them. They develop increasingly powerful arithmetic concepts that they build on in subsequent learning.

After reading the question again Fatima still maintained that the speed at 1 second was 20 m/s except that this time she calculated it as "*10 times 2 times 1.*" She provided a reason for multiplying by 2 by saying that every second the speed "*increases by 2*" but she was not sure why she had multiplied by 1. She intuitively recognized that time had to feature in the calculation.

Having reread the question she changed to a horizontal additive strategy to calculate the speed but used this strategy to calculate the speed at 1 second only. Thereafter, starting with calculating the speed for 2 seconds she switched to the functional relationship '$v = 10 + 2t$.' She multiplied the time by 2 because "*it says for every second it increases by 2*" and then she added 10 because the speed "*is increasing from 10.*" Although she had qualified the choice of this strategy she did not seem too confident after calculating the speed at 3 seconds because she reflected on the strategy used in problem 1. When asked, "*What makes you think it's wrong?*" She replied, "*I just feel so*" but tried convincing herself by qualifying the choice of the strategy. Unlike Dinesha who had also reached a state of confusion after being influenced by problem one, Fatima qualified and convinced herself of this strategy. Once she was confident that this was the correct strategy, she carried out the following computations:

~~10 x 2 = 20~~

(1 x 2) + 10 = 12

(2 x 2) + 10 = 14

(3 x 2) + 10 = 16

(4 x 2) + 10 = 18

(5 x 2) + 10 = 20

(6 x 2) + 10 = 22

(10 x 2) + 10 = 30

(50 x 2) + 10 = 110

(80 x 2) + 10 = 170

410 − ~~30~~ 10 = 400

Figure 4.33.

When calculating the time she realized that the inverse operation had to be carried out. She did this by trial and error, first dividing the speed by 2 then subtracting 10 and after working backwards to check, she abandoned this inverse strategy since it did not yield the given speed. She then tried subtracting 10 first then dividing by 2 and accepted this inverse strategy as being correct since by working backwards she was able to get the given speed. Fatima had displayed good problem solving skills because she qualified, evaluated and checked all her working.

Fatima seemed to be assessing her own performance which was clear by the following comment that she made: "*I know it's wrong.*"

Fatima's response to determining the relationship between speed and time

The verbal form of the relationship was formulated as, "*speed = speed x time + initial speed of 10*" which translated in symbolic form was *"V = V x T + U."* The 'V' on the left hand side of the equation was the speed that was required and the 'V' on the right hand side was the speed that was given which she subsequently identified as the acceleration equal to 2. The fact that she had speed on both sides of the equation shows that she experienced a problem with the verbal form, although this was subsequently ratified. Her symbolic representation shows an inadequate understanding of the meaning of algebraic symbols by using the same symbol to represent two different variables.

Fatima was the only learner who, with the exception of the first calculation in the table, developed the correct functional relationship immediately and intuitively and used it appropriately in completing the rest of the table.

Erica's response to filling in the table

Time (s)	0	1	2	3	4	5	6	10	50	80	200	495
Speed (m/s)	10	18									410	1000

12 14 16 18 20 22 30 ~~70~~ 170

110

Figure 4.34.

Erica's response after reading the statement was, *"So from 10 metres it increases, add on 2 metres every second."* Thus she immediately recognized the additive strategy. She assigned a value of 10 m/s as the speed at zero seconds but after comparing it with problem 1 she changed it to 0 m/s. Erica's view was that "*speed at zero can be anything, there is no such thing as zero time it can be worked from anywhere.*" She was the only learner who considered time to be relative. In responding to the question "*What does initial mean*?" she changed the initial speed to 10 m/s *"Because that's when we started timing it.*"

When determining the speed at 1 second she immediately embarked on a horizontal additive strategy. She recognized the jump after 6 seconds and used the counting on strategy to determine the speed at 10 seconds as follows, " *... and at 10 it will be ...I'll have to work this out ...7, 8, 9 – 24, 26, 28, 10 will be 30. "*

To calculate the speed at 50 seconds she realized that the additive strategy would fall away and that a functional relationship between speed and time had to be formulated. She considered "*If you look at zero to 10, for every 5 it* [i.e. the speed] *increased by 10, therefore at zero it was 10 and then at 5 it was 20 and at 10 it was 30. "* She tried developing a pattern between the time and speed. Thus, a more sophisticated 'skip counting' strategy similar to Dinesha's was developed. The only difference between their strategies was that Dinesha skip counted in terms of 10's and 20's and Erica skip counted in terms of 5's and 10's. Both the learners developed their respective relationships from the table although this could have been gleaned from the problem statement, a thought that had not occurred to either of them. Emily inappropriately added on in terms of 10's and 10's but on reflection realized her mistake and changed it as follows:

10 - 30
15 - 40
20 - 50
25 - 60
30 - 70
35 - 80
40 - 90
45 - 100
50 - 110
55 - 120
60 - 130
65 - 140
70 - 150
75 - 160
80 - 170

Figure 4.35.

Erica had displayed good problem solving skills in that she corrected her work by reflecting on it.

In order to determine the time Erica conceived that she would have to *"Work out a shorter method,"* and to do this she needed to know "*What's the relationship between speed and time?*" She realized that it now had to be worked the other way around so she tried establishing the inverse rule stating that, "*If the speed increases by 10 then time will decrease by 5.*" This was inappropriately stated because the speed of the object increased as the time increased. What she meant was that an inverse operation had to be carried out. She reasoned that replacing 'decrease' by 'increase' would justify the inverse operation.

By dividing the speed by 5, Erica used an inappropriate strategy because she was working with a time increase of 5 seconds resulting in the speed increasing by 10 m/s. On reflection she realized that a time of 82 seconds was incorrect. She reasoned that at 80 seconds the speed was 170 m/s so at 85 seconds the speed would be 180 m/s so at 82 seconds the speed could not be 410 m/s. This strategy was abandoned.

In attempting to establish a relationship, Erica went back to 2 seconds and tried to figure out how the speed of 14 m/s could be obtained by application of a functional relationship. This was a strategy used only by Erica. In doing this she showed that although 14 m/s was obtained by the application of an additive strategy, the application of a functional rule would also be applicable. This was a good way of establishing the relationship because if the rule was incorrectly formulated it could be verified using an alternative strategy. No other learner appropriately used two different strategies to verify the correctness of the strategy used.

Without much deliberation Erica reasoned that, "*For 2 seconds the speed is equal to 10 plus 4, then the speed is equal to 10 plus 6. It goes in multiples of 2 so speed equals 10 plus time times 2. So we have for example, time equal to 6 then the speed will equal to 10 plus 6 times 2 equals 10 plus 12 equals 22. It works well!*"
After she developed the rule using two sets of values she verified the application of the rule by calculating the speed at 6 seconds, thus showing good problem solving skills. An excerpt of her reasoning is as follows:

speed = 10 +
speed = 10 + 2
speed = 10 + 4 × 2
speed = 10 + 6
speed = 10 + (time × 2)
speed = 10 + 6 × 2
= 10 + 12
= 22

Figure 4.36.

Erica switched strategies to: "*Speed equals 10 plus time times 2*" to develop the inverse rule and this time it was correctly applied. She reasoned that "*If we multiply time by 2 and add it to 10 to get the speed then our speed minus 10 divided by 2 ...*" will give the time. Before applying the functional rule to get the required time she verified its application by using the following example in which the speed had already been calculated using the additive rule:

eg. time = 20 − 10 ÷ 2
= 10 ÷ 2
= 5

Figure 4.37.

Erica was not too particular about the use of brackets because she knew that she would have subtracted before dividing. According to Herscovics & Kieran (1980:574), learners would write down their operations, one by one, as they were thinking of them. To then be asked to evaluate them according to the conventions conflicted with the more natural tendency of evaluating them in the order in which they were written, which was the order in which they had been conceived. To others the operations would have been carried out in the reverse order according to the rule that division precedes subtraction. However, Erica was aware of the significance of the brackets as is evident in the following figure:

speed = 10 + (time x 2)

Figure 4.38.

Although here, using the mathematical rule for the order of operations, multiplication would precede addition.

Erica's response to determining the relationship between speed and time

Erica formulated the relationship "*Speed equals 10 plus time times 2*" which in the form of a verbal equation was given as "*speed = 10 + (time x 2).*" Symbolically she represented it as "$v = u + (T \times 2)$."

Three different strategies were used by Erica to fill in the table. She started with a horizontal strategy followed by a counting on strategy and to determine the time she used a functional relationship.

Erica showed good problem solving skills because she constantly evaluated and qualified her work. She was the only learner who displayed confidence in her work.

SUMMARY OF RESPONSES TO QUESTION TWO

Filling in the table

It was evident that all the learners used a trial and error method. All of them tried different strategies, tested them and persevered until what to them seemed reasonable answers were obtained.

In determining the speed at zero seconds all the learners eventually obtained 10 m/s. Some of them obtained the correct speed only after reading the question again while the others needed some guidance. Four of the learners compared this question with question 1 and considered the speed to be zero metres per second.

After obtaining a speed of 10 m/s, all the learners embarked on a horizontal additive strategy. With the exception of Fatima all of them used this strategy to correctly calculate the speed up to 6 seconds. Fatima used this strategy for only the first calculation thereafter she switched to the functional rule '($v = 10 + 2t$)' to correctly fill in the rest of the table.

All the learners with the exception Irene (83%) obtained the correct speed at 10 seconds. Fatima used the functional rule, '$v = 10 + 2t$' to do this. The remaining four learners recognized the jump after 6 seconds and they used the counting on strategy to determine the speed at 10 seconds. Irene counted on '1' to the speed instead of '2' as the others had correctly done.

Only Fatima, Erica and Denisha (50%) responded correctly to the speed at 50 seconds and 80 seconds. While Fatima (17%) used the functional rule to do this, Shireen, Erica and Direshni (50%) used a sophisticated counting on strategy. Erica and Denisha counted on correctly in terms of 5's and 10's, and 10's and 20's respectively while Shireen counted on in terms of 10's and 10's which was incorrect. Wendy (17%) used a proportionate rule. Irene (17%) used a basic counting on rule counting in terms of 1's.

Erica and Fatima (33%) were the only learners who obtained the correct values for the time. They both (33%) used the functional rule, '$v = 10 + 2t$.' Shireen (17%) who had also formulated this relationship did not make use of it because she had unfortunately developed this rule only after her calculations were completed and it did not occur to her to go back and use it to verify her calculations. Shireen and Denisha (33%) who compared this problem with problem 1 used the rule 'time equals speed divided by 2' Shireen, Denisha, Fatima and Erica were all influenced to some extent or the other by problem 1. Erica and Fatima realized that this problem was different and so abandoned the idea of comparing them. Denisha ended up confused due to the influence of question one, hence she was unable to complete the table correctly nor was she able to formulate a relationship.

Relationship between speed and time

Erica, Fatima and Shireen (50%) formulated the functional rule, '$v = 10 + 2t$'. While Fatima and Erica expressed this relationship explicitly and made use of it in their calculations, Shireen formulated it only after the calculations and unfortunately she did not make use of it in the calculations neither did it occur to her to verify her calculations using this formula. Wendy, Denisha and Irene could not establish a relationship.

Since only Shireen, Fatima and Erica were able to formulate a verbal relationship only they were asked to represent this in symbolic form. Fatima initially experienced some problems translating from numerical to verbal but this was immediately ratified. Shireen's problem was translating from verbal to symbolic and she provided an incomplete symbolic equation. This exemplifies problems experienced by learners in translating relationships from one form to another e.g. numerical to verbal, verbal to an equation and expressing this as a symbolic equation. Erica had experienced no problems with the translations.

All the learners had at some point in their calculation used different strategies to fill in the table. While Fatima and Erica used the different strategies appropriately, the rest of the learners did not always make appropriate use of them because they ignored the underlying concepts given in the statement.

Table 4.4: Strategies Used

STRATEGY	LEARNERS	PERCENTAGE
Horizontal Additive	All	100%
Functional Rule	Erica, Fatima, Shireen	50%
Proportionate Rule	Shireen, Wendy	33%
Counting on	Shireen, Fatima, Erica, Dinesha	67%
Skip Counting	Erica, Shireen, Dinesha	50%

Table 4.5: Responses to Filling in the Table

CALCULATION OF THE:	PERCENTAGE
Speed up to 6 seconds	100%
Speed at 10 seconds	83%
Speed at 50 seconds and 80seconds	50%
Time	33%

Table 4.6: Correct Responses to the Relationship Between Speed and Time

RELATIONSHIP	PERCENTAGE CORRECT
Formulation	50%
Verbal Relationship	50%
Symbolic Equation	33%

ANALYSIS AND DISCUSSION OF COMMON STRATEGIES USED IN QUESTION ONE AND QUESTION TWO

In the process of completing the table, the learners considered pure mathematics in search of an applicable model. There are various techniques, says Human (1983:8), for finding functional rules, their relative suitability depending on the nature of the given data about the function. The technique used in completing the task in this study, was initially by inspection, which eventually led to the formulation of an equation. It was clear from the results obtained that the learners' understanding varied in degrees or completeness, and that their understanding depended on the amount of knowledge they possessed in the concepts involved. For example, learners' poor understanding of 'rate' resulted in the inability or incorrect formulation of symbolic equations. This is in keeping with the view of Nickerson (1985:217).

Some learners rushed into calculations without any plan or general idea. Their heuristic reasoning was usually a provisional guess as was the case with Dinesha, who in question two, assigned a value of zero m/s for the initial speed and 10 m/s as the speed up to 6 seconds, but on reflection realized that these were incorrect.

What Polya (1945:5) had to say about problem-solving was quite evident in the way the learners tackled the problems. In trying to solve the problem, they repeatedly, some more than others, changed their points of view. Their positions were shifted again and again. This was especially evident in Shireen's response to question two and Nancy's response to question one.

This use of different strategies, and moving back and forth between strategies, Clement (1980:16) called "*shifting between approaches.*" While this phenomenon may be observable, it reflects an unobservable internal process of shifting between cognitive schemes used to deal with the problem. This provides one more piece of evidence for the notion that human cognition is not always based on consistent processes; schemes which lead to contradictory results apparently exist fairly autonomously and independently in the same individual. One scheme may become active and dominate for a time, only to be superceded by the other. This was quite clearly the case with Wendy, in question two, where she could not establish a relationship for the first six values. She had worked with one rule in determining the numerical values, but had subsequently changed her rule for the rest of the table. The shift between correct and incorrect strategies further indicates that contradictory schemes may continue to exist independently in the same individual. Dinesha, in question two, displayed similar behaviour. This according to Clement (1980:17) implies that teaching a learner a standard method is no guarantee that another intuitive method will not "*take over*" in a later problem-solving situation. On the other hand, Fatima in answering question two initially regarded the initial speed of 10 m/s as being problematic. However, she had the ability to adapt the technique used in question one to question two and thus generated a plausible approach (Schoenfeld 1982:43).

From a learner's point of view, it was often convenient to switch from one interpretation to another in the course of solving a problem, which may make it difficult for the individual herself to disentangle the real meaning being used, as was the case with Wendy in question two, where she used two different strategies and could not develop a relationship; Shireen in question two, by moving back and forth between strategies; and Erica in question one, where she did not realise that the rule that she had developed could be applied at zero seconds as well.

It was also clear that some learners, like Nancy in question one, Irene in question one and two and Shireen in question two were not always aware of the inconsistencies between the processes they were using.

At certain points in their working, the learners engaged in a trial and error strategy, which according to Ausubel et al (1978:567), is inevitable in problems where no meaningful pattern or relationship exists or is discernable by the learner. When the learners tried out a new strategy, they tested and checked their answers. Learners checked the validity of their numerical calculations and the validity of the strategies used. This emergence of hypothesis in the learner's repertoire is a fundamental conceptual issue.

Strategies were reviewed when learners ended up with answers that did not "*follow a trend,*" for example, they realised that in the tables the speed was increasing with time, so when a speed was greater than the preceding one, they reviewed the strategy. However, in some cases the inappropriate choice of strategies was not recognized. From a constructivist perspective, misconceptions are crucially important to teaching and learning, because misconceptions form part of a learner's conceptual structure that will interact with new concepts, and influence new learning, mostly in a negative way, thus misconceptions generate errors. (Olivier 1992:196.)

From the different strategies and approaches that were employed by the learners, it is apparent that understanding can vary in degrees or completeness. Understanding in every day life, according to Nickerson (1985:229), is enhanced by the ability to build bridges from one conceptual domain to another and that a major aspect of this ability is a sensitivity to similar relationships in different contexts. According to Birns & Golden (1974:128) this view is supported by Piaget who believes that a learner is more likely to accommodate his/her behaviour to solve a problem when the new behaviour that is required differs only slightly from those already in his/her repertoire.

In their choice of the strategies for question two, five of the learners were influenced by question one. Some of them, like Fatima, recognized question two as being different, however. Dinesha and Shireen who also tried different strategies, found it difficult to view question two in any other way, but to liken it to question one. This view expressed by Schoenfeld (1985:140) that a failure while executing a procedure forces a departure from the simple execution of algorithms into a more strategic mode; a learner is forced to decide what other goal to resume or pursue. When a local failure is wrongly assessed as a global failure, like the failure of a particular procedure to produce a desired result, some learners quit instead of looking for another approach. Similarly, when perspectives are at too low a level, learners can lose sight of the problem-solving repertoire, locking themselves into one approach.

Generally, the learners started with a horizontal additive strategy for both questions. They realized that a functional rule had to be developed when faced with a gap after 5 seconds, in question one. Six of the seven learners were able to correctly adopt either a vertical functional strategy, or a proportionate functional rule to complete part of the table. Problem two seemed to be initially problematic, because of the initial speed being 10 m/s. This was evident in Fatima's comment, "*This 10 is confusing me.*"

The next stumbling block for them was the gap after 6 seconds and the learners used a counting on strategy. For the next part of the table, they realized that a functional rule had to be adopted. While three of the six learners used a sophisticated counting on strategy, one of them switched to a proportionate rule.

While different strategies could be used to obtain correct answers for the different parts of the table, only one rule or formula was correct for each problem. Learners inappropriately used different strategies in the same table. In question one, Dinesha used a proportionate functional rule, a horizontal multiplicative rule, and a vertical multiplicative rule. In question two she used an additive rule and being influenced by the vertical multiplicative strategy used in question one, she thought it necessary to formulate the relationship in terms of this rule. Problems of this nature may arise due to learners not having a sound understanding of functions. Vinner (1983:302) explained this as, if the correspondence between the numbers looks arbitrary to a learner s/he might speak of infinitely many functions as if each number has its own rule of correspondence.

The key to understanding correct translating, according to Clement (1980:6), lies in the ability to conceive a mental action that produces an equivalence. He called this the "operative approach" to signify the fact that it involves viewing the equations as an active operation on a variable quantity, not just as a static comparison of two group of variables. The concept of a variable clearly implies some kind of understanding of an unknown as its value changes, and if this is to go beyond the ideas already present in seeing a letter as a specific unknown and generalized number, it would seem reasonable to argue that the concept implies, in particular, some understanding of how the values of an unknown changes. One reason why the concept is so elusive is because many items that might be thought to involve variables can nonetheless be solved at a lower level of interpretation. (Hart 1981:110.)

In question one the number of learners who were able to formulate the symbolic equation from the verbal relationship was halved, thus exemplifying problems learners experience with translating from verbal to symbolic form. While it may be true that learners experience difficulty in thinking of a letter as a number, the reverse is also true where the learners were able to do numerical manipulations, but experience difficulties representing these numerical manipulations by means of general equations. According to Clement (1980:2), asking learners to write equations in more than one variable, exposes a number of misconceptions that were previously invisible. The contrast between the number of students who correctly solve the numerical versus algebraic problem, indicate that the learners have a specific difficulty in translating from words to algebraic equations.

Learners were confused with the concepts speed, distance and acceleration. This kind of confusion with 'rate' and 'amount' was highlighted by Nickerson (1985:205) who found very high error rates among college-level, science-orientated students, in dealing with 'acceleration,' In question one, Nancy identified distance as equal to "*rate of change of speed.*" The suggestion offered by Lochhead (1980) (cited in Nickerson 1985:205), who found error rates of 80%-90% in similar questions, was for greater facilitation in teaching of rate concepts. This may be obtained by using computer graphics to illustrate dynamically various ways in which functionally related variables may change altogether. However, this facility may not be easily accessible to all schools.

changed to speed. Problems like these are common to many learners as can be vouched by science educators. These problems need to be addressed at the initial stages of introduction to these concepts. Nickerson (1985:235) has provided evidence that learners often get through many years of formal education without acquiring a sufficiently deep understanding of some of the fundamental concepts that they have studied and they are not able to apply these concepts effectively in new contexts.

Learners displayed all the stages that are present in the constructive process, which Herscovics & Bergeron (1984:192) called "Model of Understanding." The learners displayed intuitive understanding e.g. Fatima's immediate response to question 2, using the calculation "$(1 \times 2)+10 = 12$," etc.; procedural understanding e.g. counting on strategy; mathematical abstraction e.g. calculating the time as a reverse operation of the rule that was applied earlier; and formulation e.g. relating speed and time in the form of an equation.

Nickerson (1985:211) has found that word problems that involve a narrative description of quantitative relationships among variables seem to give learners the greatest difficulty. However, this study has shown that with relevant guidance given to learners, numerical, verbal, and symbolic relationships can be developed and ultimately the equations '$v = 5t$' and '$v = 10+2t$' can be developed.

CHAPTER FIVE

CONCLUSIONS

INTRODUCTION

The main research problem investigated in this study was Secondary School learners' intuitive strategies and the intuitive models that they use to solve real-life problems in kinematics. The investigation involved physical science, grade 11 learners of varying ability levels. This chapter summarizes the main findings, strengths and limitations of the study implication of these findings and suggestions for future research.

SUMMARY OF MAIN FINDINGS

This summary will be discussed with reference to the research questions described in Chapter One and the analysis of results discussed in Chapter Four.

Learners were provided with a verbal problem of a real-life problem situation together with an incomplete table and they were required to model appropriate mathematical strategies, formulate a verbal relationship and finally formulate a symbolic equation.

Many of the learners embarked on a 'trial-and-error' strategy. Hence they tended to move back and forth between strategies, some more than others. The correct strategies used were, horizontal additive, functional rule, proportionate functional rule, counting on, and skip counting. Most of learners displayed good problem-solving skills because they constantly checked the validity of their strategies and answers and they changed their strategies when the need arose.

In question one, five out of seven (71%) learners filled in the table correctly while in question two, two out of six (33%) learners filled in the table correctly. In question one six out of seven (86%) of the learners had some idea of the relationship between speed and time, as 'speed is the product of 5 and time,' however only three out of seven (43%) stated this explicitly. In question two, five out of six (83%) of the learners correctly calculated values in the table using numbers alone i.e. up to 10 seconds. However, all except two out of six (33%), experienced difficulties when a relationship was needed to be formulated to complete the rest of the table. An overall result of five out of thirteen (40%) total correct responses was obtained for questions one and two.

Evidence from this study has shown that while the majority of the learners experienced no problem working with numerical relationships, they experienced difficulties translating these into verbal relationships and symbolic equations. For example only two out of six (33%) of the learners in question one and three out of seven (43%) of the learners in question two, obtained correct symbolic equations. This provides enough evidence that learners can model real-life problem situations using intuition, and that it is possible for learners to formulate specific equations '$v = 5t$' and '$v = 10 + 2t$.' It is conceivable that with more examples of a similar type that they could eventually formulate the general formula '$v = u + at$.' Thus it would appear that with the educator taking on the role of a facilitator and providing appropriate guidance to the learners, they could not only be led to formulate the equations on their own but they could also be able to recall it with ease and apply it correctly and with ease in novel situations. However this is a matter of further research not covered in this study.

By considering the intuitive mathematical modelling strategies used by learners and also addressing the misconceptions and specific problem areas that have emerged from this study, it is hoped that this study together with similar studies could contribute towards reviewing the traditional approach to teaching of physical science at schools and produce results that are rewarding to the educator and more importantly to the learner and society at large.

STRENGTHS AND LIMITATIONS

This investigation confirmed that a modelling approach can be successfully implemented and specifically, relationships between speed and time can be derived from real-life problem situations by learners themselves.

There were five main limitations in this study:

1. Learners were only observed individually, and there was no opportunity to investigate any interaction between learners.

2. The learners were not observed in their natural environment, i.e. their own classrooms.
3. This study was restricted to one school with female learners. It was convenient for me, being an educator at the school and in addition the learners were comfortable with me. According to Rich (1971:25), fear, suspicion, or hostility will stop learners from being motivated to communicate and the need to solve the problem will be of secondary importance. In addition, I knew the learners to be cooperative and committed, qualities outlined by Preissle-Goetz & LeCompte (1991:63) as contributing to the success of a research of this nature in which a researcher requires much more than is returned to the learner.
4. The two problems only provided opportunity for modelling specific numerical formulae '$v = 5t$' and '$v = 10 + 2t$' suitable for these particular contexts. It should be noted, however that in a teaching situation learners would have to be given many different problems not only two, from which to model the general formulae, '$v = at$' and '$v = u + at$.' This is in keeping with James' (1992:157) view that a number of specific examples have to considered before a general rule is established.
5. This study did not look at the graphical representation of functional relationships in kinematics, and learners' ability to translate between tables, graphs and formulae adequately.

IMPLICATIONS OF FINDINGS

The evidence from this research clearly indicates that learners are able to intuitively model different mathematical strategies that can be used to successfully complete a table of speed and time of a real-life problem situation. Hence they are able to formulate a verbal relationship between the variables, time and speed, which can then be written as a functional relationship in symbolic form. However, since not all learners were able to complete the whole task successfully, directed guidance from an educator could prove to be successful.

SUGGESTIONS FOR FUTURE RESEARCH

1. Although this research analysed learner's individual modelling strategies, it did not address strategies developed from collaborative discussions in small

groups. Social interaction is viewed as an alternative way of facilitating conceptual development (Olivier 1990:7). Heller et al (1991:627) have found that better problem solutions emerged through collaboration than when achieved by individuals working on their own. They found that in a well-functioning group, learners share their conceptual and procedural knowledge as they solve problems together. Future research could address the impact that social interaction has on the problem-solving strategies of learners in a similar context as this study. Clinical interviews could be conducted with small groups of a class and whole classes.

2. This study was restricted to only two problems, future research could involve giving learners many different problems (e.g. different starting values, positive and negative acceleration) in order to investigate whether they could model the general formula '$v = u + at$.'
3. Longitudinal studies on learners' understanding of these formulae, that they have formulated themselves, in kinematics, and their ability to apply them to novel real-life problems.
4. Studies of learners' ability to model:
   - From genuine real-life situations, for example actual experimental work like Newton's Law – experiments carried out in the laboratory on falling bodies, balls rolling down a ramp, etc.
   - Other formulae in kinematics. For example '$v^2 = u^2 + 2as$' and '$s = ut + 1/2\ at^2$, etc.
   - Situations from Physics and Chemistry. For example Boyle's Law, Ohm's Law, reaction rates, etc.
5. A similar study could be carried out with male learners to establish whether male learners display the same or different intuitive modelling strategies as female learners do, and compare the level of competency.
6. A study could be conducted on learners' understanding of physical concepts, quantities and their respective units, and the use of symbols in this respect.

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LEARNERS' WRITTEN RESPONSE: QUESTION ONE

Shireen

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed fast or slow.

m/s 1 metre per second

5 m/s every second — acceleration – to speed . increase

Time (s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	~~10~~	15	60	~~300~~				700	1200	2800
			10	15	20	25	50	250m	600			

140 x 5

2 s

10 x 5.

50x 5

120 x 5

1 sec 5 m
2sec 10m
3sec 15m
4sec 20m
5sec 25 m
10 sec 50 m
50 sec 250 m
120 sec 600m

700 speed constant $\boxed{5}$ ↑ 5m/s

$\frac{700}{5}$ = 140 s t ?

t x c = speed
5

$\frac{\text{speed}}{\text{constant}}$ = time

speed = t x $\boxed{5}$

v = t x 5

$t = \frac{v}{5}$

Nancy

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed - movement

m/s

5 m/s every second

2 91,7

500 240 1,12

×5 ×5 ×2,4 ×2,4

Time (s)	0	1	2	3	4	5	10	50	120	~~140~~	~~160~~	560
Speed (m/s)	0	5	10	15	20	25	50	250	~~288~~	700	1200	500

5 10+5 5 +5 ×5 ×5 ~~600~~ 288

10

1 5 × 10 x

50 x

x = 5

1 5 × 120 600

1 2,4 × 120

288

1 2,4

700

700

$\frac{700}{2,4}$

1 5

~~8~~

2 10 × 5

3 15 × 5

4 20 × 5

5 25 × 5

10 50 × 5

50 250 × 5

120 × 2,4

$$\overset{V}{\text{speed}} = \overset{T}{\text{Time}} \times \overset{D}{\text{Dis}}$$

0 = 0 × 5

1 = 1 × 5

10 = 2 × ~~5~~ 5

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed

m/s

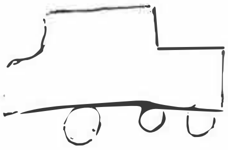
5 m/s every second – accelerate.

Time(s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	~~30~~	35x	600	700	1200	112
							50	250	600x			

For every second the speed increases 5 sec so it implies that the ratio 1 : 5 ⇒ time : speed
1 : 5

~~Time~~ Speed = Time × 5

$$V = 5t$$

Fatima

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed

m/s

5 m/s every second ACC

Time (s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	~~30~~ 50	250	600	700	1200	2800

SPEED = TIME

SPEED = PER SECOND

5X4 = 20

50X5 = 250 　　　 50 X 5 = 250

TIME 　　 SPEED 　　 560 x 5 = 2800

SPEED M/S

Irene

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed

m/s

5 m/s every second - accelaro

Time (s)	0	1	2	3	4	5	10	50	120	500		560
Speed (m/s)	0	5	10	15	20	25	30	35	40	700	1200	

700
x5
3500

3500
700
-

40 X5
=200

700
-200

as ↑ time ↑
the speed ↑

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed

m/s

5 m/s every second

Time (s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	50	250	600	700	1200	2800

~~S = T x~~

Speed = time multiplied by 1 rate of increase of speed.

$V = t \times m/s$

$V = t \times 5\ m/s$

$a = \cancel{V \times t}\ \frac{V}{t}$

$5\ m/s = \cancel{5 \times 4\ t +}\ \frac{5}{1}$

$5\ \cancel{+0}\ m/s = \cancel{5 \times 2}\ \frac{10}{2}$

Emily

The speed of an object with an initial speed of 0 m/s increases at 5 m/s every second.

initial

speed

m/s

5 m/s every second accelarate

Time(s)	0	1	2	3	4	5	10	50	120	140	240	560
Speed (m/s)	0	5	10	15	20	25	50	250	600	700	1200	2800

As time ↑ by a certain factor speed ↑ by 5 x that (same) factor

Speed = 5 x Time

m/s = 5(s)

Speed = V

Time = T

$V = 5(T)$

TRANSCRIPTIONS OF INTERVIEWS: QUESTION ONE

After reading the statement, the learner's responses to the questions asked were as follows:

LEARNER : SHIREEN (S)

R: What does initial mean?

S: At first, at the beginning.

R: What does speed mean?

S: How fast or how slow something is.

R: Is there any other way that you could describe speed?

S: Not really. I don't know. The movement of something. At what pace it goes, whether it's going at 60 km/h or 25 km/h.

R: What does m/s mean?

S: Metres per second. For one second it passes 1 metre. This stands for 1 metre per second. If there was a 2 in front it means that the body is travelling at 2 metres every second, if it had a 6 then it's travelling at 6 metres every second.

R: What does "5 m/s every second" mean?

S: 5 metres per second, 5 metres per second. ...... Every second right it's going at 5 metres per second. I would say that every second .... every single second not one missing like every alternate second means that every single second it travels 5 metres in every second.

R: Where does 5 m come from?

S: There's it here [*i.e. 5 m/s in the statement*]

R: How is it possible for an object e.g. a car to undergo a change in speed?

S: Like driving on a highway one won't be travelling at about 60 km/h, you'd be travelling at 100 km/h to 120 km/h. and so you use your accelerator to speed up or slow down your speed because if you were travelling in a public area you will have to go at slow paces.

R: So how can the speed be changed?

S: By accelerating. Acceleration is to speed up something. If we are talking about a car, how fast or how slow it is moving , to make it go even faster than what it's going at.

R: Fill in the blanks in this table that was drawn up from the above statement.

S: *Read question.* Looking at time 0, 1, 2, 3, 4, 5, 10, 50, 120. *Reread the question.* It's 10, 50, 120, it's 1, 2, 3, 4, 700, 1200 amm... 5 x 2 is 10; 10 x 5 is 50; 50 x 6 equals 120; 120 x 6 equals oh no!

R: Is something wrong?

S: I was going along with time. You gave me zero, then 1, 2, 3, 4, 5, 10 then 50, then 20. So looking at how time is going about and the seconds, time is 1 second, 2 seconds, 3 seconds etc.

Too much time was spent on figuring out how the time changes.

R: Can you fill in any of the speeds?

S: To find the speed ... *Read question.* Speed at the beginning is zero metres.

R: The speed then is zero metres?

S: Yes, zero metres.

R: What does this [*pointing to 0 m/s*] mean?

S: Metres per second. At first the speed of the object was zero metres per second. It increases at 5 metres per second. So at one second it moves it should have increased at 5. The second second should be another 5 so 10. Then 15 then another 5 will be 60 [*15 x 4 seconds*] and another 5 will be 300 [*60 x 5 seconds*] then 30 x 5 will be 1500, oh boy!

R: What's the problem?

S: I made a mistake. Unrealistic value. For every second that it took I added 5, I added 5 every 5 m that it moved. It increases at 5 m every time it moves so 5 m per second every second for 1 second it moves 5 metres, so this is 1 second the speed would be 5 metres no ... 1 second – 5 then 1 second is moved at 5, second second it should have moved twice that, 10 metres, that's the way I was working it out. Then 3 will be 15, 4 will be [*15 x 4*] 60 but then it's not following – I'm getting more. See it travelled 5 metres in 1 second, in 4 seconds it's 5 x 4 equals 20. Another 5 will be 25. Then 10 seconds will be 50.

R: How did you get 50?

S: You gave me 10 seconds and 5 metres in every second. So in 10 seconds ... in 1 second the object travelled 5 metres then in 10 seconds it will be 50 metres, 50 will be 50 x 5 equals 250, 120 will be 120 x 5 equals 600. Then 700 seconds divide by 5 is 140 and then 1200 divide by 5 is equal to 140. Then 560 x 5 equals 2 800.

R: What is the relationship between speed and time?

S: As time increases, speed increases according to time. For every second it will increase at so much say at 5 seconds it will increase so much. At 1 second it moves 5 metres so as time increases the speed increases. The faster you go the quicker you will get somewhere.

R: Write down a formula or an equation relating speed to time.

Pause.

R: Is speed related to time?

S: Yes, the faster you go, the quicker you get there. As time increases, speed increases.

R: How does the speed increase?

S: By accelerating. By moving at a faster rate.

R: Is there any particular value(s) you could get from the statement?

S: Eh .... Not really.

R: By looking at the table can you see any relationship between speed and time?

S: Yes. I proved it. To get speed you need to work with time. The faster you ... The more you increase time , the more you increase speed.

R: How did you get these values in the table?

S: When the object was travelling for 1 second the question said it moved at 5 m/s every second. When it travelled for 1 second it covered 5 metres. When it travelled for 2 seconds then it moved twice that, then at 3 seconds it will be thrice that, at 4 seconds 5, 5, 5, 5 that's 20. Another 5 seconds 5 x 5 equals 25, then another 10, it will move at another 5 metres then 50.

R: How did you get 50?

S: In 1 second, it moved 1 metre so for every 1 second move 5 metres now you are moving for 10 seconds so it will be 5 x 10 which will be 50.

R: How did you get 250?

S: 50 x 5. Then 120, its 600.

R: How did you get the speed knowing the time?

S: The increase in speed is constant, if the increase remains constant then the speed will increase with a certain pattern, like this.

R: If you know the time, can you get the speed?

S: You will divide by the constant. For example the speed is 700, you would divide by 5 to get 140.

R: What did you get in this way?

S: The time.

R: But if you wanted the speed, how would you get it?

S: Then time is multiplied by the constant … will give you the constant.

R: What is the constant?

S: 5.

R: Write down a formula or equation relating speed and time.

S: speed = time x 5.

R: Can use this formula to get the speed at any time?

S: Yes, as long as I can get the constant, which in this case is 5.

R: If "v" represents the speed and "t" the time, write down a formula or equation relating speed and time in symbolic form.

S: V = t x constant, can use any alphabet for the constant?

R: Whatever you want to.

S: V = t x a.

R: Why have you used "a"?

S: I thought I had to give it an alphabet and acceleration is 5.

LEARNER : NANCY (N)

R: What does initial mean?

N: Initial means the speed at which it is right now and it's going to increase every second.

R: What do you understand by speed?

N: Speed is movement and speed can also be very fast movement. Because you can say a car is moving and you can also say a car is speeding, so speed to me means movement.

R: What does m/s mean?

N: Movement per second … metres per second. How many metres it moves in a second, like 1 metre a second, 2 metres a second.

R: What does 5 m/s every second mean?

N: 5 metres per second every second so it moved 5 metres every second.

R: How is it possible for the speed of an object e.g. a car to increase?

N: More power, the acceleration changes.

R: Fill in the table.

N: They tell us the time 0, 1, 2, 3, 4 and so on and the ... so if it increased 5 metres for 1 second so zero seconds the speed will increase mmm...I don't know because our time is zero that's like no... and they want to know what our speed is going to be and in our statement they say the initial speed was zero metres per second and then it increased at 5 metres per second every second. So our speed here should also be zero because for each time there's a speed and here also zero.

Our speed hasn't increased yet only when our time increases then our speed will increase and ... this is 1 second it will move 5 metres and for 2 seconds it will move 10 metres per second because for 1 second it moved 5 then over another 5 it will be 10 ... and then 15 and then 20 ... 25 wait a minute that's 50 ... I'm checking with the ...you gave us the two speeds taken so what I would try and do is multiply 120 x 5 equals 600.

By how many are we increasing the time by 1, by 1, by 1, by 1 and then by 5 then by another 5 ... 50, 60, 70, 80, 90, 100, 110, 120 okay ... 5 x 50 equals 250 so now how did it go from 50 to 120? 120 divided by 5 [*meaning 50*] equals 24 so the speed from 50 to 120 increased not 5 times but 24 times because 50 x 24 equals 1 200 no, no, no!

For 10 seconds the speed increases by, ...okay for 1 second the speed increases by 5 so for 10 seconds the speed will increase 5 times 10 is 50, x times so that will give me x, and x equals 50. So we worked out that the speed increased for 2,4 seconds because we took 120 and divided it by 50. I said that from 10 to 50 it increases 5 times because 50 divided by 10 equals 5 so from 50 to 120 I took 120 and divided by 50 to see what time and it increased by 2,4 seconds.

So for 1 second its speed increases by 5 metres per second so for 120 the speed will increase by 5 times 600 yes metres per second. I cross multiply and over here we don't have ... The speed didn't increase by 5 over here, the speed increased by 2,4 so it will be, for 1 second it increases by 2,4 and then cross multiply to get 288, check 288 divided by 250 equals 1,152 and not 2,4 ... no that didn't work checking answer by dividing here we jump every 5 metres per second so I was trying to work out if my answer is right.

R: Is it right?

N: I don't think so because over here it was 700 divided by 5 gives 140. First we going every 1 then we go every 5 now we going every 2, 2 seconds – see 1 to 2 is 1, 4 to 5 that's 1, 10 to 50 that's 5 then from 50 to 120 that's 2,4 so I said 700 divided by 5 because that's what we were working with only when we came here [*i.e. time 50 to 120*] it's slightly different so I said 700 divided by 5 gives me 140 now we are jumping every 2 so here we should get …

R: How did you get 2?

N: I said 700 divided by 5 because that was our speed at which we were going and then to work out this [*time for speed 1 200*] I'll say 1 200 divided by 5 equals 240 here I'm supposed to be getting 240 and not 160 so it means that this answer [*i.e.160*] is not correct, rises as time increases as well so for every 1 second it increases by 5 and for every 2 seconds by10 if I multiply by 5.

R: What did you multiply by 5?

N: 4 by 5; 5 by 5; 10 by 5; 50 by 5 but when I came here [*i.e.120*] I said that this didn't move 5 because if I say 50 divided by 5 [*meaning 10*] I'm getting 5 and if I say 10 [*meaning 25*] divided by 5 I'm getting 5 then I say 5 [*meaning 20*] divided by 20 [*meaning 4*] then I should be getting 5, then I say 5 [*meaning 15*] divided by 15 [*meaning 3*] then I'll get 5 then I say 5 [*meaning 10*] divided by 10 [*meaning 2*] then I'll get 5 then I'll say 5 divided by 5 then I'll get 1.

R: What is the answer for 5 divided by 10?

N: Oh no! 5 [*meaning 10*] divided by 10 [*meaning 5*] will give 2; 5 divided [*meaning multiplied*] by 3 equals 15. I'm using my 5 from the top where they told us … See, 5 divided … we getting our time to check if our speed is right, so if you moved every 5, so 5 times 10 equals 50 that's what we said. So 50 divided by 5 is 10, then 5 times 5 equals 25, and 25 divided by 5 equals 5, then 5 times 4 equals 20 so 20 divided by 5 equals 4 then 5 times 3 equals 15, and 15 divided by 5 equals 3, then 5 times 2 equals 10 and 10 divided by 5 equals 2, then 5 times 1 equals 5 and 5 divided by 5 equals 1, that's what I was trying to do here. 5 times 50 equals 250 then 250 divided by 5 equals 50 then when I came here I said 120 divided by 5 will give me …

R: Why are you dividing, is this what you've done before?

N: No I just divided by 5. 50 times 5 equals 250 to check that 250 equals 50. I'm checking my speed as well as my time. I said 250 divided by 5 equals 50, 120 times 5 equals 600 then 600 divided by 5 equals 120.

R: How did you get 288 for the time 120 seconds?

N: For every second, I moved 5 metres so from 120 divided by 50 is 2,4 because 50 divided by 5 equals 10 and 10 divided by 5 equals 5 [*meaning 2*], that's just the relationship here. When you start counting in 5's because we didn't move 1 any more we moved up 5. All over here we moved 1, 1, 1, 1, when we came here [*5 to 10*] we moved up 5 so I said for 1 second we moved 5 times for this one. So every time I divided this to check my answer to see how many times we moved, it was 5 times each time so when I came to 120 then I said 50 [*meaning 120*] divided by 120 [*meaning 50*] which gives 2,4 that's how much we move, we didn't move 5 only 2,4, so I said in 1 second we moved 2,4 metres then in 120 seconds we moved 288 metres, so then I'll take 120 then I'll say 700 because I divided this by that then I'll say …

Pause

R: Is there a pattern between speed and time?

N: They will move in 5's until 50 no until 10, well they are all multiples of 5.

R: Is the speed related to time? Can you get the speed by looking at the time?

N: Yes because we are moving 5 … no we are moving 1, if we look at our speed we can get our time.

R: If you have the time can you get the speed by looking at the table?

N: For 1 second we move 5 metres, at 2 moved another 5 which will give 10.

R: Could you have got the speed from the time i.e. not looking at the previous speed?

N: Yes we could have said 2 times 5?

R: Why?

N: Because we move 5 metres every second.

R: Could you get the speed knowing the time?

N: That will be times 5 that is 15.

R: How did you get 15?

N: I said 10 plus 5; 20 - 15 + 5 then 25 equals 20 + 5; 25 to 50? That's when there's a change jumping in 5's and not in 1's. First we were moving in 1's now its 5 seconds, got 50. I said 25 times 5 [*meaning 2*] because we weren't moving 1 anymore we were moving 5. I didn't say plus 5 same thing for 120. [*Time for speed 700*] here we don't move 5 anymore we don't know our time so we have to work with our speed so we have 700 and we know that … so we have to work opposite to this so we say that … we want to know our time there is a difference in time so I don't know whether to take it as 5 or we jump more than 5 or 2,4 so I'll try to see if there's a relationship between speed divided opposite way, speed for last one taken divided by time so 288 divided by 700 to see what I got or 700 divided by 288 i.e. 2,4 so jumped 2,4 again I took 288 and 700 divided by 288 to see how many times we jumped because we don't have any speed so I'm going to try and do the same thing for the one before that 288 divided by 250 equals 1,15 that's not right so I can't do it this way.

R: What way?

N: To use 2,4.

R: Is there any other way? Is there any pattern you see between time and speed?

N: 1 – 5; 3 -15; 4 – 20; 5-25; 10-50; 50- 25; 120 times 2,4

R: Is there a pattern between the two columns?

N: Yes I multiplied by 5 so 2 – 10 (x5); 3-15 (x5); 4-20 (x5); 5-25 (x5); 10 –50 (x5); 50 – 250 (x5) then 120 x 5 I'll get 600 but we are not moving 5 anymore.

R: Where did these 5's come from?

N: The question says when time increases then speed increases by 5.

R: Can you see any pattern between the speed and time?

N: For every 1 second, I moved 5 here [*speed for 0 second to 5 seconds*] but when time increases by 2,4, speed increases by 2,4. This 600 [*speed for 120 seconds*], is not right its 120 x 2,4 equals 288.

R: Can you complete the rest of the table?

N: [*Working for the time when the speed is 700*]. Now moving once every 2,4 seconds that 700 divided by 2,4 because 288 divided by 2,4 equals 120 and that's what was given. So I want 700 divided by 2,4 equals 291,66, take it as 291,7. Same thing for the next one. I'll say 1 200 divided by 2,4 equals 500. The time increases from 500 to 560 i.e. 560 divided by 500 which equals 1,12 times so to get the speed for time 560 seconds it's 560 divided by 1,12 which equals 500.

R: Write down a formula or an equation relating speed and time,

N: Every time the time increases by 1, the speed increases by 5, every time the time increases by 5, the speed increases by 5, but when it's less than 5, …

mm.. okay ... *read question again* ... oh okay! so this is 5 seconds. When time increases, speed increases as well.

R: Does it increase in any particular way?

N: Yes it increases by 5 but not all the time. There were 3 different rates of increase, from zero seconds to 50 seconds the increase was by 50, for 120 seconds, 700 m/s and 1200 m/s the increase was 2,4 and at 560 seconds, the increase was 1,12.

R: Consider the time zero seconds to 50 seconds, write down a formula relating speed to time.

N: We can say that time is equal to speed times ... they don't give us the distance, they only tell us 5 metres every second. Oh yes, every second that's your time and speed increases 5 metres so you move a distance of 5 metres for that 1 second so you to find out the speed it's equal to time times distance. Let's try it out. If t =0 then 0 x 5 =0; next 1 x 5 = 5; 2 x 5 = 10.

I used 5 from the statement but this in not always the case, what if we move 10 metres per second or 20 metres a second? But it's just the distance that gives the speed – distance travelled times time will give speed, say if I walked from here to the desk, the distance at which I travel times time gives me my speed.

R: You have written the formula in words, now write the formula in symbolic form using "v" to represent the speed and "t" to represent the time.

N: V = T x D.

R: What does D represent?

N: The distance travelled i.e. 5.

LEARNER: WENDY (W)

R: What does initial mean?

W: Initial, at first.

R: What does speed mean?

W: Speed, how fast it's going.

R: What does m/s mean?

W: How many metres it travels per second.

R: What do you understand by 5 m/s every second?

W: Every second the object travels 5metres … [*read question*] per second. Every second the object increases its speed by 5 metres per second.

R: How is it possible for an object, e.g. a car to increase its speed?

W: By accelerating.

R: Fill in the following table.

W: So initial at the beginning speed was zero, … and every second, it increased its speed by 5 metres per second, so every second it increases by 5 so at 1 second it will be 5 metres per second, at 2 it will be 10 and then it will carry 3 seconds will be 15, 4 will be 20, 5 will be 25, and 6 will be 30 then 35 oh no! …

R: Is there a problem?

W: 6 was not given. So then if at zero seconds the speed was zero then at 10 seconds, the speed increases at 5 so it will be 50, am I right? So at 4 seconds it was 20, 5 will be 25 so at 10 seconds – every second it increases, 5 that means it will be 50 metres per second and if its 50 seconds and every second it increases 5, so it will be 250 metres per second and then if it's 120 seconds it increases 5 so 120 x 5 = 600 so if 700, I want time now and then every time … I'll divide by 5 from the ratio and proportion 700 divided by 5 equals 240 and 560 divided by 5 equals 112.

R: Why did you use 5?

W: It follows the pattern of 5.

R: What is the relationship between speed and time?

W: It's ratio and proportion, 1 is to 5. For every second the speed increases 5 seconds so it implies that the ratio 1 is to 5, time is to speed equals 1 is to 5.

R: Write down a formula or an equation relating speed and time.

W: Time, let's say speed equals time times 5 because it increases 5 metres every second.

R: If "v" represents the speed and "t" represents the time, write down a formula or an equation relating speed and time in symbolic form.

W: $V = 5\,t$.

R: Does 5 represent any physical quantity to you?

W: Yes, the object travels 5 metres faster every second. 5's add up.

LEARNER: FATIMA (F)

It was not possible to capture the data due to an electrical fault with the tape recorder. However, sufficient written data was collected so as to make reasonable conclusions regarding the strategies she used.

LEARNER: IRENE (I)

R: What does initial mean?

I: It's there it's already there it's the permanent speed.

R: What does speed mean?

I: Speed is the fastness in which the object moves.

R: What does m/s mean?

I: In 1 second it covers so many metres.

R: What does 5 m/s every second mean?

I: Every second its speed goes higher into 5 metres.

R: How is this possible in a real-life situation e.g. car?

I: If you press the accelerator, the car moves faster.

R: Fill in the table.

I: We said that the object has an initial of speed of zero so it starts with a speed of zero. Then in 1 second it moves 5 metres and then as it moves into 2 seconds it will increase to 10 and then it moves into 15 and 20, 25 30, 35 and then 40. Now ... [*working out the time when the speed is 700*] I can take 700 and subtract 45. I'm taking 700 minus 45.

R: Why did you subtract 45?

I: Because I would assume that as it moves 5 seconds all the time I will add 5 seconds to the speed of 40. Now I've got 655 which is too much, it does not coincide with the time. Now the time moves in terms of 5, 10, 50 – the difference of 40, then 120 .... 560. I'm looking at the difference in the time so 150 minus 120.

R: Where did you get 150 from?

I: Sorry I took the wrong figure, it is supposed to be 50. So it's 120 minus 50 equals 70. So the difference here is 70 and the difference there [*50 – 10*] was 40 ... The speed now is at 700 ... I've taken the time and I've tried to find the difference in time I found that it's not equal so it won't give you the same because the time difference is not the same and then 700 minus 120 ... if I take my speed and minus 120 from your time I would assume that I'll get my time 580 seconds, it's too large.

R: Why do you say that it's too large?

I: It's because it' going from smallest to largest so it would be very unnatural for the time to jump higher than 560.

700 to 1 200 moves higher, time also moves higher so it will be obvious for me to get a time which is increasing. The question says increase at 5 metres per second. I'm trying to see if I had to multiply 40 times 5.

R: Is there any relationship that exists between speed and time?

I: So far the only relationship that I can see is that they increase simultaneously. Speed is in metres. How did I get my speed, since the object has an initial speed of 5 metres per second, as the object moves in 1 second the object will move at a speed of 5 as it moves for 2 seconds it moves at 10 for 120 it has been moving for 40, ...50 then 120 seconds. I'm trying to estimate how fast time is moving.

R: Is there a pattern with the time change?

I: No, not that I can see.

R: Is there a pattern with the speed change?

I: Yes, it moves 5, 5, 5 ...

R: Is there a relationship between the speed and time?

I: [*Working the time when the speed is 700*] I'm taking 40 times 5 equals 200, which means the time here had changed that means I'll take this 200 here ... I'm taking this 40 times 5 seconds because the speed is 5 metres per that much minutes 40 times 5 gives 200 then to get my time I will take 700 and subtract 200 because I assume that since it's moving at 5, I take the 5 and multiply it by 40 because the speed is moving at 5. Now I get 200 I'm trying to get a difference from the speed so it will give me that [*700*] so I'm going to be assuming that it's going to be 500, again we have 700 [*previous speed*] there 700 times 5 gives 3 500. Take 3 500 minus 700 equals 2 800 then it becomes unnatural because it doesn't follow the trend.

R: What trend?

I: Time increases from smallest to biggest.

R: Looking at the values that you were able to get, is there a relationship between speed and time?

I: The object will move 5 metres in 1 second. The object will move 5 seconds faster so in every 5 seconds the object will move 5 faster than before.

R: Write down a formula or equation relating speed and time.

I: As the time increases, the speed increases.

R: If "v" represents the speed and "t" represents the time, write down a formula or an equation in symbolic form relating speed and time.

I: It's not possible.

LEARNER: DINESHA (D)

R: What does initial mean?

D: At the beginning, before anything happens.

R: What does speed mean?

D: Rate at which something takes on speed. The rate at which something travels, how fast it goes.

R: What do you understand by m/s?

D: The measure of speed. It just says how many metres something travels per second.

R: What does 5 m/s every second mean?

D: It travels 5 metres for every second.

R: How is it possible for an object, e.g. a car to increase its speed?

D: By accelerating.

R: Fill in the table.

D: Time – 1, 2, 3, 4, 5, 10. I'm not sure how this increases but then I can work it out. At zero the speed should be zero, because the question says initial speed is zero. 5 metres per second – so at one second it should be 5 metres, and at 2 it [*i.e. the previous speed*] should be multiplied by 2 so speed is 10 (2 x 5). It's increasing at 5 metres per second every second so then at 3 it should be 15 – by multiples of 5. 20, 5 – 25, 10 – 50, at 50 (50 x 5) yes 250, at 120 (120 x 5) gives 600. To work speed to time, divide by 5. Divide 600 by 5 to get 120 basic multiplication and division. Divide 700 by 5 equals 140 and 1 200 divided by 5 gives you 240, 560 times 5 gives 2 800.

R: What is the relationship between speed and time?

D: At every second the speed increases by 5 metres per second so they are proportional.

R: Write down a formula or equation relating speed and time.

D: Speed is equal to time multiplied by … I don't know how to put this? equals time multiplied by the rate of … I don't know this word – when something gathers forces … increase of speed.

R: If "v" representatives the speed and "t" the time, write down a formula or an equation relating to speed and time in symbolic form.

D: V= t x so many m/s i.e. speed though I'm not so sure how to word this because it's kind of difficult because both intertwine so much.

R: What intertwines?

D: Time and speed because time is part of the speed. Like you say metres per second.

R: Looking at the table can you say what the speed is equal to?

D: Time multiplied by metres per second which is speed.

R: What value(s) will you give for this speed?

D: This [*i.e. 5 m/s*] doesn't change but these [*pointing to speed in the table*] do change.

R: From the table can you write down a relationship between the speed and time?

D: Time multiplied by 5 because it increases with time, every 1 second it increases by 5 metres per second.

R: If "v" represents the speed and "t" the time, write down a formula or an equation relating speed and time in symbolic form.

D: Then it will be V = t x m/s.

R: What is m/s?

D: m/s is the speed but it doesn't make any sense to me because you calculated speed and how can you already have speed?

R: If you know the time how can you get the speed?

D: Speed increases 5 metres every second, just multiply the number that is time by 5 metres per second.

R: You've written here 'rate of change of speed', what does it mean?

D: The speed increases every second and 5 metres per second is its value.

R: Write down the equation in symbolic form.

D: V = t x 5 m/s.

LEARNER: ERICA (E)

R: What does initial mean?

E: I think it means previously like the speed firstly. The speed of the object was zero metres per second and that is the speed we start off with.

R: What does speed mean?

E: Fast, how fast object is travelling.

R: What does m/s mean?

E: What distance it's travelling over a period of time over seconds how many metres.

R: What do you understand by 5 m/s every second?

E: It means that the object is travelling 5 metres for every second. Measure of the distance that it moves every second. 5 metres per second then they repeat and say every second so 5 metres per second is speed. *Read the question* – so for every second the speed increases at 5 metres per second.

R: How is it possible for an object e.g. a car to increase its speed?

E: By going faster. By pressing on the accelerator object goes faster so 5 metres per second every second is the acceleration.

R: Fill in the following table.

E: For every second means1 second so second is time so if there's no time there's no speed. I'll look for a pattern it says 5 metres every second your speed will increase. Do I have to start with zero?

R: Not necessarily.

E: I'll start at 1. It increases 5 metres per second every second so at 1 second I think that it's 5 metres per second and at 2 I think it's 2 times 5 which is 10 and then it's 3 times 5, then it's multiples of 5.

R: What makes you say that it's multiples of 5?

E: Because it says that it increases at 5 metres per second every second so for every second there's 5 metres and here there's 2 seconds so it will be 2 times the amount in 1 second which will give me 10 – a speed of 10 metres per second and I would do the same throughout so 3 seconds – 3 times 5 which is 15 the same with 4 and 5 – 4 times 5 would be 20, and 5 times 5 would be 25 and now it's 10 times 5 equals 50 and 5 times 50 is 250 and then 5 times 120 is 600, and then at 700 divided by 5 because since I was saying the time times 5 will give speed now we've got the speed I'd say 700 divided by 5 gives 140 and the same with 1 200 divided by 5 which gives 240 and then we've got time again, times 5 – 560 times 5 gives 2 800.

R: Why haven't you filled in zero?

E: Because at the time of zero I don't know what the speed is because they say the speed increases 5 metres per second every second. Now there's no time so I think that the speed is zero because there's no time so … As time increases by 1 second, speed increases by 5 metres per second so I can get speed at time equal to zero seconds. *Read question* – no I can't get speed at time equal to zero.

R: Why not?

E: Speed at which it started before it started increasing – so at time equal to zero, speed will remain zero because if we work from the question one can see the speed increases by 5 so that will follow the pattern so probably speed will be 1 because we can see 0, 1, 2, 3, 4, 5 – if we have all these on the number line we are jumping 5 spaces all the time.

For every second we jump 5 spaces so if we have to count from 5 – 1, 2, 3, 4,5 – we will have to start from zero in order to get 5 metres per second for every second so I think this will be zero.

R: But you said it was 1 earlier?

E: Yes, but I had to draw the number line to see the pattern.

R: What is the relationship between speed and time?

E: There is a relationship because in the given statement if we didn't have time we wouldn't be able to work out speed because they said for every second and second is time so if we don't know that it increases at 5 metres per second for every second then we wouldn't be able to work out the speed because we wouldn't know where to start from.

R: What is the relationship between speed and time?

E: …. x's and y's?

R: In words.

E: As time increases by a certain factor, speed increases 5 times that same factor.

R: Write down a formula or an equation relating speed and time.

E: Speed equals 5 multiplied by time.

R: If “v” represents the speed and “t” represents the time, write down a formula or an equation relating speed and time in symbolic form.

E: m/s because speed is measured in m/s and time is in seconds so m/s equals 5 times seconds and V = 5 (T).

Shireen

The speed of an object with an initial speed of 10 m/s increases at 2 m/s every ~~second~~ beginning.

10 m in 1s

2m in 1s

Time(s)	0	1	2	3	4	5	6	10	50	80	205	500
Speed (m/s)	10	20	40	120	480					~~28~~	410	1000

~~12 — 14 — 16 — 18 — 20 — 22 — 24 — 26 — 28~~

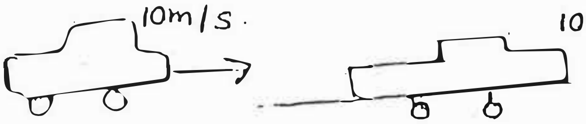
~~24 36 48 60 72 120 600~~

rease → 12

24 6 8 10 12 20 100 160

10m/s

10m + 2m/s



t	0	1	2	3	4	5	6	10	50	80	205	500
speed	10	12	24	36	144	720					410	100
		12	14	16	18	20	22	~~30~~ 24	60			

30 ~~70~~ 100

70 110

7s 8s 9s 10s

24 26 28 30

10 - 30

20 - 40

30 - 50

40 - 50

50 - 70

60 - 80

70 - 90

80 = 110

$\frac{410}{2} = 205$

~~$t = \frac{v}{2/a}$~~

~~$t = v$~~

$t = \frac{v}{2/a}$

$\Rightarrow \frac{410}{2}$

$\Rightarrow 205$

$V = t \times 2$

$\Rightarrow 2s \times 2$

$\Rightarrow 4s + 10$

$\Rightarrow 14$

The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

Time(s)	0	1	2	3	4	5	6	10	50	80	~~708~~ 394	984
Speed (m/s)	10 ~~0~~	2 ~~20~~	4	12	48	240				96	410	1000

12 14 16 18 20 22 ~~24~~ 28 30

26 66

314
506

540

10 m
2sec

50 4

2 280

The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

Time (s)	0	1	2	3	4	5	6	10	50	80	200	495
Speed (m/s)	~~0~~ 10	~~20~~ 12	14	16	18	20	22	30	110	170	410	1000

$\cancel{10 \times 2 = 20}$

$(1 \times 2) + 10 = 12$

$(2 \times 2) + 10 = 14$

$(3 \times 2) + 10 = 16$

$(4 \times 2) + 10 = 18$

$(5 \times 2) + 10 = 20$

$(6 \times 2) + 10 = 22$

$(10 \times 2) + 10 = 30$

$(50 \times 2) + 10 = 110$

$(80 \times 2) + 10 = 170$

$410 - \cancel{50}\,10 = 400$ $\quad 2\overline{)400}$

$\quad 200$

195

SPEED = SPEED x TIME + INITIAL SPEED OF 10

Wendy

The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

Time(s)	0	1	2	3	4	5	6	10	50	80	137	333
Speed (m/s)	10	12	14	16	18	20	22	30	150	240	410	1000

1 : 3

t : V

t → 50 Sp. →

T → 7 Sp → 24

8 → 26

9 → 28

T → 10 Sp. 30

T → 50 x

$10x = 1500$

$x = \frac{1500}{10}$

$= 150$

T → ~~8~~50 Sp. 150

80 x

T 50 ~~80~~ 150

T. ?x Sp. 410

$50x = 12000$

$x = 240$

$150x = 20500$

$x = 137.$

150 50

Sp. 1000 T x

$150x = 50000$

$x = 333$

The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

Time (s)	0	1	2	3	4	5	6	10	50	80	205	500
Speed (m/s)	10	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	30	~~150~~	~~210~~	410	1000

12 14 16 18 20 22 110 170

2 4 6 8 10 12 20 100 160

~~a =~~

10 = 30

20 = 50

30 = 70

40 = 90

50 = 110

60 = 130

70 = 150

~~90~~ 80 = 170

Erica

The speed of an object with an initial speed of 10 m/s increases at 2 m/s every second.

Time (s)	0	1	2	3	4	5	6	10	50	80	200	495
Speed (m/s)	10	18									410	1000

12 14 16 18 20 22 30 ~~70~~ 110 170

7 - 24
8 - 26
9 - 28
20 - 40
30 - 50
40 - 60
50 - 70
60
70

10 - 30
15 - 40
20 - 50
25 - 60
30 - 70
35 - 80
40 - 90
45 - 100
50 - 110
55 - 120
60 - 130
65 - 140
70 - 150
75 - 160
80 - 170

speed = 10 +
speed = 10 + 2
speed = 10 + 4 ×2
speed = 10 + 6
speed = 10 + (time × 2)
speed = 10 + 6 × 2
= 10 + 12
= 22

speed = 10 + (time × 2)
time = speed − 10 ÷ 2.
eg. time = 20 − 10 ÷ 2
= 10 ÷ 2
= 5
= 410 − 10 ÷ 2
= 200

time = 1000 − 10 ÷ 2

V = 10 + (T × 2)

= 10 + 5 × 2
= 10 + 10
= 20.

TRANSCRIPTIONS OF INTERVIEWS: QUESTION TWO

After reading the statement, the learner's responses to the questions were as follows:

LEARNER: SHIREEN (S)

R: Read the statement and then fill in the table.

S: Initial speed i.e. at the beginning is 10 metres per second and it increases at 2 metres per second, so every 10 metres it travels it increases by 2 metres for every second, mmm .... So that's 10 metres per second and it increases at 2 metres per second for every second. So every second it travels 10 metres ..no .. [*Read question*] every 10 metres per second that it travels, it increases at 2 metres for that 1 second. Okay, let me try this out, time is zero, 10 metres that's the time ... 1 second travels at ... relationship between the time and the speed using the statement ...

R: Fill in the table.

S: When the time is zero, the speed will be zero because every second that it travels, it will increase at 2 metres.

R: Do you agree with the speed at time zero seconds?

S: [*Read question*] No, speed at the beginning is 10 metres per second. Every second it increases at 2 metres per second for every second so at 1 second, 2 metres will be 20, at 2 seconds it will increase at 2 metres then it will be at 40 [*20 x 2 i.e. previous speed x time for which the speed was being calculated*], at 3 seconds it will increase and be 120, for 4 seconds it will be 480 oh no! ... I was multiplying by 2 [*however the operation she carried out was different*]. Can I draw a diagram?

R: Yes.

S: This is my car and it is travelling at 10 metres per second, in 1 second it's going at 10 metres, now at every second its going at 2 metres, it's gone 10 metres plus 2 metres per second ... no... this doesn't make sense.

R: Why not?

S: I don't know? 10 metres and it increases at 2 metres in 1 second.

R: Why did you add 2 metres?

S: Just to make my diagram easy, because it's 10 metres in 1 second and it's moving and it's increasing at 2 metres. So I'm trying to work with something .... It starts with 10 metres and then it increases at ... for every second that it moves it increases at 2, 2, 2, 2 and so on. If I had to move at 10 metres per second, at 2 then it will move at 10 plus that 2 so it will be at 12, so that another 1 second it will move, if it is at 12 then it will increases at another 2 seconds it will be 14 metres, and so on. It increases at 3 – 16, 4 – 18, 20, 22, 24, 26, 28 amm ...doesn't work out.

R: What doesn't work out?

S: I was adding 2 metres every time that it increased at but it doesn't follow because there's no relationship between 28 and 410 because it supposed to increase at the same speed at each time.

[*Read question*] So every second that it goes it goes at 10 metres and increases at 2. It had a speed of 10 metres when it started and then it travelled at 10 metres and increased at 2 so that the speed [*for 1 second*] will be 12, 12 x 2 = 24, 12 x 3 = 36, 12 x 4 = 48, 12 x 5 = 60, 12 x 6 = 72, 12 x 10 = 120 and 12 x 50 = 600.

R: Why are you multiplying by 12?

S: Because if it increases at 2 metres it still had to have the initial speed of 10 but it looks like this doesn't work. I need to find a relationship between these two i.e. 410 and 1000. Can I have a look at the previous sheet. 1000 ÷ 410 = ... but this doesn't work.

R: Is there any way that the time and speed are related?

S: This is tricky for me. 2 metres per second every second is not for every metre, 2 seconds its 2 metres for every second and then the 10 metres as well. It starts at 10 and increases at 2. If it increases at 2, 4, 6, 8, 10, 12 ...

R: How did you get these values?

S: I'm trying to see whether it increases at 2 at every second. So it's 20, 100, 160, 410 ÷ 2 = 205 and then 1000 ÷2 = 500

R: Is the speed after one second, 2 metres per second?

S: [*Read question*]. It should be 12 because at 1 second it increases at 2 and it started off with 10 metres per second. That 10 added on to 2 will be 12. So in

1 second it's travelling at 12 metres and in another second it will be travelling at 24.

R: How did you get 24?

S: 12 x 2 because in 1 second it's 12, in 2 seconds 24. Then 12 x 3 = 36;... No it doesn't work ... I tried this at the beginning so 12 x 3= 36, 36 x 4 =144, 144 x 5 =720. No ... that's too much because we can't have 720 before 410.

[*Read question*]. Starts at 10 metres increases by 2 every second. So it's12, 14, 16, 18, 20, 22, 24, 26, 28 [*back to strategy 2*]. But 28 won't be right?

R: Why?

S: If I had to divide it by 80, 80 ÷ 28 gives me a point value. I went back to what I had where I said it increases by 2 metres for every second. I got 12, 14, 16, 18, 22 but when I came up to 10 seconds ... it's 24.

R: How did you get 24?

S: Because I was following this pattern. Because ... at 7 seconds it will move at 24 at 8 seconds it will move at 26, 9 seconds it will move at 28 and 10 seconds it will be 30 and then now at 50 seconds ... would, be I'll have to work this whole thing out.

R: How did you get this values?

S: I just increased the speed by 10 because of the 2 metres that it moves every second, so if time increases by 10, speed increases by 10. So 10 will be 30, 20 – 40, 30 – 50, 40 – 60, 50 – 70, 60 will be 80, 70 – 90, 80 –110.

[*Calculated time when speed equals 410*]. 410 ÷ 110 = 3,72, still doesn't work, 410 ÷2 =205 and 1000 ÷ 2 = 500. I did do this at the beginning.

R: Is there any relationship between speed and time?

S: Yes, because it's increasing at 2 metres per second every second. So it started off with 10 initially and you have to carry on with the10 throughout. So you keep in mind 10, at every second it's moving at 2, 2, 2 so you add on that 10 to that 2 but you carrying on adding 2 not 12. 2, 2, 2 and then that's how you get your speed. If you had to take away that, you can get your time by saying that divided by the 2 that it increased by and you get your time.

R: Write down a formula relating speed and time.

S: Speed [*meaning time*] is equal to time [*meaning speed*] divided by 2 which is 'a'.

R: If 'u' represents the initial speed, 'v' the final speed and 't' the time, write down a formula or an equation relating 'u', 'v' and 't'.

S: v is equal to v divided by 2/a where a is equal to 2. No, it's ... To get the time you say speed divided by 2, that's how I got 205, so $t = v \div 2/a$ where 2 equals a, so it's 410 divided by 2 equals 205.

R: Will this formula work when the time is given and the speed has to be determined?

S: Yes. Speed is equal to time times a. You have to remember that 10 is in your mind so I'm just working with the 2 only. t times what is t? ... t is seconds 2 seconds times 2 equals 4, right plus the 10 will give me 14.

LEARNER : IRENE (I)

R: Read the statement and then fill in the table.

I: You gave us the initial speed as 10 metres then the object will move at 2 seconds. So the initial speed is zero. [*read question*]

R: What is the initial speed?

I: Initial is the speed that has been put into the object, that starts the speed and the time we start at is zero, so the speed is 10 and it increases 2 metres per second every second so here it will be 2 seconds. [*speed at time 1 second*].

R: What does this 2 seconds represent?

I: The time.

R: Which row are you filling?

I: The time.

R: So is 2 seconds correct?

I: Yes, because they say it increases at 2 metres at every second so ...[*read question*]. The speed starts at 10 so in 1 second it will be 2 metres more then I'll take this 2 [*speed at 1 second*] and multiply it by that 2 [*time equal to 2 seconds*] to get the speed, reason being ... Can I work on a trial and error basis?

R: Yes.

I: 4 times 3 equals 12 and then 12 times 4 equals 48, 48 times 5 equals 240. For every 6 seconds it will move by the previous speed multiplied by the time, 240 times 6 equals 1 440 ... No ... It supposed to moving in a trend. The time is increasing so the speed has to increase all the time but then I find that the speed for 6 seconds is much higher than the speed of 410. [*read statement*] They say that the object has a starting speed of 10 metres per second and this speed increases at 2 minutes in every second, so they are saying in every 2 seconds per minute.

R: What does this mean? [*2m/s*]

I: In 2 metres it will move 1 second.

R: Do you think the speed at one second is equal to 2 metres per second is correct?

I: [*Read question*] No, it has to be 10 and then 12 and then 14, 16, 18, 20, 22, 24, 28.

R: How did you get the speed?

I: I increased speed by 2 all the time.

R: Why?

I: Because it increases at 2 in one second.

R: How does the time change?

I: From zero to 6 the time increases by 1 second and from 6 to 10 it's 4, next it increased by 40 and then by 30.

R: Why is this 2? [*speed at a time of 1 second*]

I: Because at 1 second the object will move 2 metres higher.

R: What about from 6 seconds to 10 seconds?

I: The object moved in a time of 6 to 10 seconds it had a difference of 4 seconds so because they increase as they go, I increased the speed by 4 also.

R: Is this 24 correct?

I: No, it's a mistake it's 22 to 26, and then from 10 to 50, a difference of 40 so my speed will increase by 40 so it's 26 plus 40 equals 66 and then from 50 to 80 there's a difference of 30 so it's 30 plus 66 equals 96. Now I'll try to find the difference ... See ... I've been working on a trend because as the time increases the speed has to increase.

Now I'm trying to find the time, I would also use the same principle of time moving along with the speed. I'll find the difference between 410 and 96 and that difference which is 506 [*she actually added*] ... it's 410 minus 96 equals 314. This is the difference in which the time has moved. To 80 seconds I will add 314 because that's how the speed changed so it's 314 plus 80 equals 394 and then I'll do the same thing, I'll take 1 000 minus 410 equals 590. My time will increase with 590 seconds so I add 394 to 590 and I'll get 984. So what I've deduced from that is that when the speed increases ... Okay ... they gave me 2 seconds for the speed so I realised that in 1 minute it only moves 2 seconds. Then I found the point where it doesn't increase by 2 so I realised I had to find the difference in which they moved at that point in order to make the time increase with that speed at that time.

R: Is there a relationship between speed and time?

I: They increase equally i.e. if they gave me initial speed of 50 and they told me this speed increases by 4 per second every second, at this speed I must increase 4 because it increases by 4 and now if they gave me a speed of 280 I must add the same 4 because time increased by 4. So it makes sense to say that time increases with speed.

LEARNER : FATIMA (F)

R: Read the statement and then fill in the table.

F: At time zero the speed will be zero. The speed increases by 2 metres per second. So for 1 second it will be ... [*read question*] No, initial speed is 10 metres per second according to the statement so at times zero second the speed will be 10. For every 1 second it will be 2, 10 times 2 equals 20. [C*ompared with previous statement*] In the other example I multiplied speed [*meaning time*] by 5, but here I can't do that because the speed at the beginning is 10 not zero. When it increases ... 10 metres per second ... [*read question*]. Oh! for every second it increases by 2 ... [*read question*] 10 times 2 times 1.

R: Why did you do this?

F: Because it increases by 2 for every second. I'm not sure why I multiplied by 1. Let me do this ... For every second it's 10, ya, it's 20 [*read question*] For every second that it takes it's increasing by 2. Oh! it's increasing by 2 so it's 10 plus 2. For 2 seconds it will increase. I multiplied by 2 because it says for every second it increases by 2 and I added 10.

R: Why did you add 10?

F: It's increasing from 10 so for 1 second it will be 12, for 2 seconds it will be 14, for the other example it was zero and for this one it is 10 so I'm going to have to add it. In the other one I did not add it but ... I don't know ... For 3 seconds it will be 3 times 2 plus 10 equals 16. I know it's wrong but ...

R: What makes you think it's wrong?

F: I just feel so but I multiplied by 2 because it increases by 2 and then I added 10. 4 times 2 plus 10 equals 18, 5 times 2 plus 10 equals 20, 6 times 10 plus 10 gives 22, 10 times 2 plus 10 equals 30, 50 times 2 plus 10 equals 110, 80 times 2 plus 10 equals 170. Err ...now the other way around. Okay, I'll have to divide 410 divided by 2 equals 205, then 205 minus 10 equals 195. Working backwards to see if I get 410, 195 times 2 plus 10 equals 400 so that's not right. Check by calculation it's the same thing. So I have to work backwards again. This 10 is confusing me because for the previous one I multiplied by 2, if I divided by 2 it doesn't work out, unless I subtract 10 first, I'm just trying. I'll work backwards 410 minus 10 equals 400 divided by 2 equals 200. Let's check 200 times 2 minus 10 equals 410, yes. For 1 000 it's

1 000 minus 10 divided by 2 equals 495 and if I work backwards I'll get 1 000.

R: What is the relationship between speed and time?

F: Speed equals speed times time. We have 10 so speed equals speed times time plus initial speed of 10.

LEARNER : WENDY (W)

R: Read the statement and then fill in the table.

W: At the first second the object is going at 10 metres per second then it accelerates at 2 metres per second every second. In the table if time equals zero and the initial speed is 10 metres per second so it started at 10. If the time was zero. [R*ead question*] So if the time was zero, at zero the object would have been travelling what? ... At first it was 10 then ... and if the time was zero it wouldn't have been travelling at all, so the speed was zero. Let's start at 1 ... [*read question*] ... Till 10 metres per second ... all right! So it's

10 because it started at 10. When time was zero, speed was 10 metres per second.

If every second it increases to metres that means when the seconds was 1 then the speed will increase 2 so it will be 12 and then a second after that it will increase another 2 so it will be 14 and one second after that it will increase another 2 so it will be 16 and a second after that another 2 that's 18 and it will carry on and then it will be 20, 22.

The seconds was 10 then the speed will be ... [*read question*]. So we saw that it started at 10 then every second we added 2, then when it's 10 we'll be adding ... by 7 it will be, doing it the long way, if time was 7 speed will be 24 and if the time was 8, 9, then the speed would have been 26, 28. At time 10 speed will be 30.

If time was 50 speed would have been ... Let's work out a ratio and proportion here... [*pointing to 10*] I won't be able to do that ... I can't find the ratio and proportion here because nothing works ... I can't have 1 is to 12 ... No ... Divided by 3, divided by 4 ... no. Started at 10 by the time it went to 10 seconds the speed was 30 so at time 50 seconds the speed would be, we are adding 2 to every 1 second so ... I can't find another way ... No I'll do it like this but it'll take long. If the time is 10 and it's speed is 30, if the time is 50, the speed would be what? That's a good way of doing it so let's cross multiply, we get 10x equals 1 500, x equals 1 500 over 10 which equals 150 and so the speed at 80 seconds will be worked out the same way. The time here is 80 and then the speed ... If the time was 50 then the speed was 150 and if the time is 50 then the speed will be 50x equals 150 times 80 equals 1 200 then x equals to 40, so if the time is 80 then the speed would be 240. [*Calculating the time when the speed equals 410m/s*] Work this out the other way if the time was 80 and the speed was 240, and then now we have the speed is 410 and we want the time, then we'll say ... Okay ... time was 50, speed 150, we'll work with this. What is the time if the speed is 410. I'll multiply 410 by 50 equals 2 500 so 150x equals 20 500 divided by 150 equals137, ya ... I think it makes sense. So if the speed is 1 000 what is the time. The speed was 1 000 and the time was x so if the speed was 150 and the time 50 then 150 x equals 1 000 times 50 which is 50 000 and then x equals divided by 150 is 333. For the last 5 it works out the ratio is 1 is to 3.

Let's check 410 divided by 3 equals 137, ya, the ratio is 1 is to 3 for time is to speed.

R: What is the relationship between speed and time?

W: The ratio is 1 is to 3. Every second, the object increases it's speed by 2 metres.

R: Write down a formula or an equation relating speed and time.

W: Time is equal to 3 speed but that's only for the last 5. It doesn't work like that for all, how come? [*Read question*] So we said the initial speed was 10 and every second we will add 2. I can't get a relationship or formula because 1 is to 3 doesn't work for all only for the last few.

R: What about a relationship for the first 6?

W: There's no common relationship there. Over here for zero it's 10 then it's 1 – 12, 2 –14 ... nothing I can work with.

LEARNER : DINESHA (D)

R: Read the statement and fill in the table.

D: Speed is zero for time zero and for one second right up to 6 seconds it's all 10 metres per second ... oh no ... it increases at 10 metres per second, no, 2 metres per second every second ... mmm ... initial speed of 10

R: Why did you change this speed? [*i.e. speed at time equal to zero seconds*].

D: Because it says so, the speed of an object with an initial speed of 10 metres per seconds.

R: Why did you have zero at first?

D: Because I just assumed that at zero seconds there won't be any speed, acceleration what so ever. Then I thought about it again, which is what I do most of the time. I just assumed that there was no acceleration.

R: Fill in the rest of the table.

D: At 1 second it's 10 over 2 which equals 5, which is not really right.

R: Why is it incorrect?

D: Because in order to get ... okay ... wait, I have to think about this ... [*read question*] ... No, it increases at 2 metres per second and if it increases at 2 metres per second, at 1 second it would be 12, wouldn't it? And at 2 seconds, 2 metres per second, 2 seconds it would be 14, but then it doesn't really work out.

R: What makes you say so?

D: Because if you had to work out ... if you want to get a constant they all divide and give you the same thing which doesn't really happen.

R: What constant do you want to get?

D: I'm trying to get the acceleration right.

R: What value are you looking for?

D: Mmm ... I still have to determine the value. ... Mmm ... I don't like this one. ... 10 metres per second ... 2 metres every second which would be ... I have to get a constant but I'm not sure how to.

R: What constant do you have to get?

D: The acceleration which increases at 2 metres per second every second. I'm highly confused. Because it has an initial speed of 10 metres per second and it increase at 2 metres per second every second but if you add it using my other equation acceleration equals speed divided by time ... [*read question*] ... How do I get the constant which is 2 metres per second which is the acceleration? Speed divided by time will have to give me that. If it increases at 2 metres per second every second then it will be adding on 2 metres every second. So for 2 seconds it will be 14 metres per second and at 3 it will be 16, 4 – 18, 5 – 20, 6 – 22, 7 – 24, 8- 26, 9 – 28, 10 is 30 ... and it's kind of difficult to calculate 50 because you can't go at that pace all the time ... If 10 is 30 using my times tables 30 divided by 10 is 3 so it will be 50 times 3 which equals 150 and 80 times 3 equals 240.

Now to calculate time. 10 is 30 and using the 2 times table 20 would be 50, 30 would be 70.

R: Why are you using the 2 times table and how are you making use of it?

D: Because 2 times 5 equals 10, and 10 times 2 equals 20 just add 20 and then 30 equals 70, 40 equals 90, 50 equals 110.

R: Explain how you got these values?

D: Speed at zero is 10. If speed was 1 it would help a whole lot, but anyway, if we calculate speed and you count from 30 upwards you would get those values.

R: How did you count from 30 upwards?

D: Using the 2 times table 32 is 11, 34 is 12, 36 is 13, 38 is 14 and so on. If you take 10 units for your time you go 20 units for your speed. This value [*speed equal to 150 at time equal to 50 seconds*] changes it's 110, 60 it would be 130, 70 would be 150 and 90, no, 80 would be 170. To calculate the time 410 is divided by … how do I calculate time? … I have no idea, 30 divided by 10 is 3 but that's not right. … mmm … [*read question*] …that's all right, but I don't know how to calculate time. I can't see a relationship between speed and time. I feel stupid because there should be a relationship between speed and time.

R: What makes you say that there should be a relationship between speed and time?

D: Because the rate at which it increases and the time should be corresponding. Speed is the measure of the rate of increase. If I divided speed by time it should give a constant but in this case I'm not getting a constant … Maybe something's wrong with my speed?

R: Why do you have to get a constant?

D: Because for any speed time relationship there is a constant e.g. in the other example there was an acceleration and that acceleration was constant. In this example there is an acceleration which is 2 metres per second every second and if 2 is a constant then 1 should be 2 and 2 should be 4 for time … and in this way I'll get a constant. 2, 4, 6, 8, 10, 12, 10 times 2 is 20, 50 times 2 is a 100, 80 times 2 is 160 so if I divide 410 by 2 it's 205 and 1000 equals 500. That would work, but then, it says an initial speed of 10 metres per second which puts everything off. But if the speed at the beginning is 10 we would have to add this [*2, 4, 6 …*] to the 10.

R: What is the relationship between speed and time?

D: The formula acceleration equals speed over time does work but then here it goes off, 16 divided by 3 gives you a weird number.

R: What is the number and makes you say it's weird?

D: Because it's 5,33.

R: Why can't this be the speed?

D: Because it's weird, it doesn't correspond with the table because you're supposed to get a constant not 5,33, 18 divided equals 4,5, it doesn't correspond at all. One is 5,33 and the other is 4,5 but the acceleration is supposed to be a constant.

R: Which are the correct speed values?

D: I'm not too sure whether it's the top or bottom answers.

R: Can you see the relationship between speed and time?

D: With this I do, [*2, 4, 6, 8, …*] but with those [*10, 12, 14*, …] I don't. If the initial speed was zero I can get a relationship, but now the initial speed is 10, I can't.

R: Could these values 2, 4, 6, etc. be used for this given statement?

D: No, these values [*10, 12, 14, etc*] are what I'll use but I'm not sure if they are correct because when I divide them I don't get a constant. But according to the statement it makes sense.

R: What makes you think it is incorrect?

D: My reasoning abilities.

R: What is the relationship between speed and time?

D: Using the multiples of 2, 2 times 7 equals 14, 1 times 12 equals 12, 3 times what will give 16? Not possible, I know 3 times 5 equals 15 not 16. I have no idea how to do this. I can't see a relationship. There is a relationship but in this case I can't see it.

LEARNER : ERICA (E)

R: Read the statement and then fill in the table.

E: [*Read the question again*]. So from 10 metres, it increases, add on 2 metres every second. At 1 second it will be 10 because they say it increases by 2 for every second but the initial speed that it started with was 10 metres per second and then it increases.

R: What is the speed at the time zero second?

E: Like with the previous example, I think it would be … If I had to work with this table it would follow a certain pattern and I have to start at zero. The speed at zero can be anything there is no such thing as zero time, it can be worked from anywhere. …

R: What does initial mean?

E: Initial to me means the speed of the object before you started timing, it was 10 metres per second, but when you stared timing it you concluded that the speed increased every second so I'll have 10 here [*at time equal to zero seconds*] because that's when we started timing it.

For every second it increases at 2 metres per second so 10 plus 2 equals 12, and then 2 seconds it will increase by another 2 which is 14 and for 3 seconds it will increases by another 2 which will be 16 and then at 4 seconds it will be 18 and at 5 seconds it will increase to 20, at 6 to 22, and at 10 it will be ... I'll have to work this out ... 7, 8, 9 – 24, 26, 28, 10 would be 30 and then you work out if 10 was 30 then if you look at zero to 10, for every 5 it increased by 10, because at zero it was 10 and then at 5 it was 20 and at 10 it was 30. So at 20 it will be 40 and at 30 it will be 50, at 40 it will be 60, 50 – 70, at 60 ... no we must increase in fives.

10 was 30 ... so we'll have 15, 20 ,25, 30, 35, 40, 45, 50, we started of with 10 being 30 so that will be 40, 50, 60, 70, 80, 90, 100, 110. So ... as it increases by 5, 55, 60, 65, 70, 75, 80, ...120, 130, 140, 150, 160, 170.

[*Working the time when speed was 410 m/s*]. What I'm saying for every 5 it increases by 10. So I was adding ... for every increase of 10 in speed time will decrease by 5. I'm trying to work out a shorter method – what's the relationship between speed and time?

R: Do you see any relationship between the speed and time?

E: As the time increases by 5, the speed increases by 10. But now I'm trying to find it the other way around, speed to time. If time increases by 5 and speed increases by 10 then, for every 5 then, if the speed decreases by 10 then time will decrease by 5. Let's see 410 divided by 5 equals 82, it's not right.

R: Why is it not right?

E: Because for every 5 it increases, the speed increases by 10 so if at 80 the speed is 170, then at 82 the speed can't be 410. You see a pattern is followed because at 85 it will be 180 so for 410 it can't be 82 that's why it didn't work out. I'm trying to figure out how to get from speed to time but I know how to work from time to speed.

Speed equals to time times what? ... I'm trying to figure out how I got 14 at time equal to 2 seconds. I added it to 10. If I had to say speed is equal to 10, that's the initial speed we started of at, 10 plus your time ... no ... speed is equal to ... 10 ... I don't know how to write this. Speed is equal to 10 plus 2 for every second. Speed is equal to 10, initial speed plus 2 metres per second. For 2 seconds the speed is equal to 10 plus 4, then, speed is equal to 10 plus 6. So it increases in multiples of 2, but this doesn't happen with the whole table, only for time equal to 1 second to time equal to 6 seconds and from 6 seconds to time equal to 10 seconds. We also see that for every 5 it increases by 10 ... I'm trying to see if I subtract the speed from the time if I'll get a certain number. I'm trying to calculate time. As the time increases by 1, the speed increases by 10 plus 2 ... It all goes in multiples of 2 so speed equals to 10 plus time times 2, so we have for example, time equal to 6 then the speed will equal to 10 plus 6 times 2 equals 10 plus 12 equals 22, it works out!

Speed equals 10, because it is the initial speed times 2 because it increases by multiples of 2 so I took an example, as time equals to 6 seconds and I got 22. The final equation is speed equals 10 plus time times 2, because I'm working in multiples of 2 because it increases by 2 all the time.

R: Complete the rest of the table.

E: To work out time, time equals speed divided by 2 because if we multiply time by 2, and add it to 10 to get speed then our speed minus 10 divided by 2 for example, to see if my calculation us correct, take the speed of 20, 20 minus 10 divided by 2 equals 10 divided by 2 equals 5 and it works out that way. If I take 410 minus 10 divided by 2, I'll get 200. Check: 200 times 2 plus 10, it works out! Similarly with 1 000, take 1 000 minus 10 divided by 2, is equal to 495. Check: 495 times 2 plus 10 is 1 000.

R: What is the relationship between speed and time?

E: Speed is equal to 10 plus time times 2.

R: Write down the formula or an equation relating speed and time.

E: Speed is equal to 10 plus brackets time times 2.