AN INVESTIGATION INTO GRADE 12 TEACHERS' UNDERSTANDING OF EUCLIDEAN GEOMETRY

By

Sikhumbuzo Sithembiso Dhlamini

A Dissertation submitted to the Faculty of Education

Of

University of KwaZulu-Natal

In partial fulfilment of the requirement for the

Degree of Master of Education

(Mathematics Education)

Supervisor: Prof. M. De Villiers

March 2012

PREFACE

The research project described in this dissertation was carried out with ten mathematics teachers falling under Empembeni Ward, Ezibayeni Ward and KwaMsane Ward in Hlabisa Circuit, in the Obonjeni District of KwaZulu Natal. The project commenced from May 2011 to January 2012 under the supervision of Prof. M. De Villiers of the University of KwaZulu-Natal, Durban in Edgewood Campus (Mathematics Education).

This study represents the original work by the author and has not been submitted in any form for any diploma or degree to any other tertiary institution. Where the author has made use of work of other authors, it is duly acknowledged in the text.

.... Date: ....

Sikhumbuzo Sithembiso Dhlamini

Student Number: 208524873

As the candidate's Supervisor I agree/do not agree to the submission of this dissertation.

.... Date: ....

Prof. M. De Villiers

Supervisor

Durban

ACKNOWLEDGEMENTS

First and foremost, I would like to give thanks to God Almighty for keeping me alive and helping me through this dissertation especially during the times when I thought of giving up and tended to lose hope.

Among the people who helped me in completing this dissertation, particularly, I want to thank Prof. M. De Villiers for his guidance, continuous encouragement and the role he played as my supervisor. I would also like to extend my genuine appreciation to the Grade 12 mathematics teachers of Empembeni Ward, Ezibayeni Ward and KwaMsane Ward who participated in this study. Their understanding of Euclidean Geometry developed my own knowledge and understanding.

My genuine gratitude also goes to my friends Thangos (and his wife), Lean, Gazeth (N.D.), Nkosie, Gugu (G.P.), Spar and Linda, who have always been beside me, encouraging me to continue with this project. Finally, special thanks go to the rest of my family members: dad, mom, Mndeni and Tano for their continuous support.

ABSTRACT

The main focus of the research was to investigate the understanding of Euclidean Geometry of a group of Grade 12 mathematics teachers, who have been teaching Grade 12 mathematics for ten years or more. This study was guided by the qualitative method within an interpretive paradigm. The theoretical framework of this research is based on Bloom's Taxonomy of learning domains and the Van Hiele theory of understanding Euclidean Geometry.

In national matriculation examination, Euclidean Geometry was compulsory prior to 2006; but from 2006 it became optional. However, with the implementation of the latest Curriculum and Assessment Policy Statement it will be compulsory again in 2012 from Grade 10 onwards.

The data was collected in September 2011through both test and task-based interview. Teachers completed a test followed by task-based interview especially probing the origin of incorrect responses, and test questions where no responses were provided. Task-based interviews of all participants were audio taped and transcribed.

The data revealed that the majority of teachers did not posses SMK of Bloom's Taxonomy categories 3 through 5 and the Van Hiele levels 3 through 4 to understand circle geometry, predominantly those that are not typical textbook exercises yet still within the parameters of the school curriculum. Two teachers could not even obtain the lowest Bloom or lowest Van Hiele, displaying some difficulty with visualisation and with visual representation, despite having ten years or more experience of teaching Grade 12. Only one teacher achieved Van Hiele level 4

understanding and he has been teaching the optional Mathematics Paper 3. Three out of ten teachers demonstrated a misconception that two corresponding sides and any (non-included) angle is a sufficient condition for congruency.

Six out of ten teachers demonstrated poor or non-existing understanding of the meaning of perpendicular bisectors as paths of equidistance from the endpoints of vertices. These teachers seemed to be unaware of the basic result that the perpendicular bisectors of a polygon are concurrent (at the circumcentre of the polygon), if and only if, the polygon is cyclic. Five out of ten teachers demonstrated poor understanding of the meaning and classification of quadrilaterals that are always cyclic or inscribed circle; this exposed a gap in their knowledge, which they ought to know.

Only one teacher achieved conclusive responses for non-routine problems, while seven teachers did not even attempt them. The poor response to these problems raised questions about the ability and competency of this sample of teachers if problems go little bit beyond the textbook and of their performance on non-routine examination questions. Teachers of mathematics, as key elements in the assuring of quality in mathematics education, should possess an adequate knowledge of subject matter beyond the scope of the secondary school curriculum. It is therefore recommended that mathematics teachers enhance their own professional development through academic study and networking with other teachers, for example enrolling for qualifications such as the ACE, Honours, etc. However, the Department of Basic Education should find specialists to develop the training materials in Euclidean Geometry for pre-service and in-service teachers.

LIST OF ABBREVIATIONS USED

AMESA	Association for Mathematics Education of South Africa
BTC	Bloom's Taxonomy Category
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DoE	Department of Education
FET	Further Education and Training
GET	General Education and Training
HOD	Head of Department
KZN	KwaZulu Natal
NATED	National Technical Education
NCS	National Curriculum Statement
No	Number
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
SA	South Africa
SAG	Subject Assessment Guidelines
SMK	Subject Matter Knowledge
VHL	Van Hiele Level

LIST OF FIGURES

LIST OF TABLES

APPENDIXES

TABLE OF CONTENTS

CHAPTER 2 THEORETICAL FRAMEWORK AND LITERATURE REVIEW

CHAPTER 3 RESEARCH METHODOLOGY

CHAPTER 1

INTRODUCTION AND OVERVIEW

1.1. Introduction

This study sought to investigate ten mathematics teachers' understanding of Euclidean Geometry. The participants of this study were teachers who have been teaching mathematics for ten or more years in Grade Twelve.

This chapter is an outline of the study. The motivation for undertaking this research is discussed and the research questions are introduced.

1.2. Motivation for the Study

In my teaching experience of mathematics, the part that has given learners most difficulty is Euclidean Geometry. According to Mthembu (2007) it is often felt that the learners are weak and not the teachers; therefore, that is the reason this study investigated teachers' understanding of Euclidean Geometry. When teachers do not understand Euclidean Geometry, they are not likely to teach it in a way that would help learners understand it.

As from 2008, Euclidean Geometry in its traditional form of theorem recognition, solving riders and proofs construction has been optional in South African Grade 10, 11 and 12 curricula (Van Putten et al., 2010). It became part of the optional third mathematics paper, which most schools do not teach. The implementation of the Curriculum and Assessment Policy Statement (CAPS) for Grade 10 – 12 Mathematics (DBE, 2011) in South Africa is currently under way, CAPS is to be implemented in Grade 10 from 2012 onwards and Euclidean Geometry will then again be compulsory for all mathematics learners. Euclidean Geometry will form part of the second mathematics paper; therefore, there will be no optional paper. Only two papers will be written in Grade 10 as from year 2012 onwards.

One of the main reasons for Euclidean Geometry being made optional in South Africa in 2008 was the view that teachers are not familiar with the content (Bowie, 2009). Some teachers find the Euclidean Geometry section difficult, even if they studied Euclidean Geometry in high school and at tertiary level; let alone those who did not study Euclidean Geometry in high school or at tertiary level. Therefore, the question is whether the main reason for making it voluntary has actually been resolved or not. In this regard, this study specifically focuses on investigating mathematics teachers' understanding of Euclidean Geometry to identify areas of strengths and weaknesses. It aims to make recommendations on how to develop their understanding in their areas of weakness.

The Department of Basic Education (DBE) has called for a concerted effort to advance mathematics teaching and learning, in responding to the above situation (DoE, 2006). Teachers are called upon to improve their mathematical content knowledge, while being provided with better learning support materials within a frame of revised curriculum structures and assessment procedures (Brown, 2002). The Greek philosopher Socrates states that teachers should be scholars, researchers and lifelong learners, who always discover something new, since many situations do not have simple and straightforward answers (Egan, 2001).

As the Head of the Department of Science at my school, I have witnessed situations where suitably qualified mathematics teachers are scarce. However, principals are forced to appoint unqualified teachers. In the school where I teach, the Science stream (Math, Life Sciences, Physical Sciences, etc) is formed by seven teachers and I am the only qualified teacher; others are holding academic qualifications in other fields of study, for example Bachelor of Arts, Bachelor of Science, etc. These teachers did not study methods of teaching mathematics, and their understanding and knowledge of mathematics lead one to doubt their ability to teach it adequately.

This study investigated mathematics teachers' understanding of Euclidean Geometry in the situations stated above. Sampling of this study focused on experience of teaching mathematics in Grade 12 irrespective of qualifications held. Their understanding was arranged according to Bloom's Taxonomy levels of assessment and to the Van Hiele levels of understanding Euclidean Geometry. Bloom's Taxonomy has six categories of assessment levels, while Van Hiele has five levels of understanding Euclidean Geometry. This study focused on the first five categories of Bloom's Taxonomy and first four levels of Van Hiele theory.

1.3. Focus of the Study

The discipline of mathematics, especially Euclidean Geometry in particular, offers an almost seamless avenue of research prospects to the researcher (Cassim, 2006). However, time and space limit the focus and scope of this study. Teachers were the main focus of this study. The research conducted in this study documented Grade 12 mathematics teachers' understanding of Euclidean Geometry with specific reference to circle geometry.

Whilst all mathematics teachers in secondary school, that is from Grade 8 – 12, ought to have understanding of Euclidean Geometry, this study focused on Grade 12 teachers in particular. This study was further delimited by the involvement of only one District in KwaZulu-Natal (Zululand Region).

The criteria for the selection of Grade 12 mathematics teachers believed that they understand the Euclidean Geometry done on lower or previous grades. This study investigated only the understanding of teachers, not their effectiveness in teaching Euclidean Geometry in class. Due to the restricted capacity of this study, it is not possible to generalise findings, but the researcher nonetheless anticipates that curriculum planners, examiners and subject advisors will benefit from the findings and recommendations of this study. Most probably this study will contribute to the identification of teachers' areas of weakness.

1.4. Research Questions

This was an investigative study into Grade 12 teachers' understanding of Euclidean Geometry, with specific reference to circle geometry, in the Obonjeni District in KwaZulu-Natal (Zululand Region) currently known as Umkhanyakude District, in order to answer the following research questions:

1.4.1. What are the general Bloom's Taxonomy learning domains and Van Hiele levels of understanding of Euclidean Geometry of a sample of Grade 12 mathematics teachers?

1.4.2. What are Grade 12 mathematics teachers' specific conceptions and misconceptions with respect to circle geometry?

1.4.3. What is the relationship between Grade 12 mathematics teachers' understanding, conceptions and misconceptions, and their ability to apply their knowledge of circle geometry?

The above questions were the focus of this study: to explore and investigate how well Grade 12 teachers understand some aspects of the circle geometry in Euclidean Geometry. This study is underpinned by a version of the newly revised Bloom's Taxonomy categories and the Van Hiele levels of understanding geometry. These two models can be used as teaching tools as well as assessment tools. These models provided a conceptual framework that facilitated an understanding of the topic for this study; detail is set out in Chapter 2.

1.5. Research Methods

This study intended to investigate Grade 12 teachers' understanding of circle geometry in Euclidean Geometry. The qualitative research method was employed in this investigation. A qualitative research method, as described by De Vos (2002) and Cohen, Manion and Morrison (2007), presents descriptive data, such as data from language or text, or descriptions of observable behaviour. The aspects relating to research methodology such as the research design, suitability of a qualitative framework, and the development and reliability of data collection tools are set out in detail in Chapter 3. For brevity the researcher used two data collection tools:

1.5.2. Test

Each teacher was asked to answer the attached paper-and-pen test on their own (see Appendix: 4). The test involved Euclidean Geometry problems with specific reference to circle geometry.

1.5.3. Task-based interview

To increase validity, the test and task-based interview questions were piloted with five Grade 12 mathematics teachers from the Vryheid District currently known as Zululand District before the participants wrote it. The researcher used both primary and secondary sources to obtain information. The primary source for this investigation was that of test and task-based interview, while the secondary source involved information from the relevant literature. Descriptive data was collected from the sample of participants selected from the target population.

1.6. Analysis of Data

1.6.1. Test

The coding system was utilised to develop categories and identify patterns among the categories of teachers' responses.

1.6.2. Task-based Interview

A task-based interview schedule was used to conduct interviews with ten teachers individually. The task-based interview generated data on:

- Teachers' understanding of specific conceptions with respect to circle geometry.
- Teachers' misconceptions with respect to circle geometry.

The researcher transcribed the response of the task-based interview verbatim. Item analysis was undertaken by grouping all responses to a particular question. Furthermore, audiotapes of the teachers' responses on task-based interview were utilised to prepare the interview transcripts.

1.7. Study Limitations

It would have been appropriate to investigate Grade 12 teachers' understanding of Euclidean Geometry in the whole of the Obonjeni District of the KwaZulu Natal Province; however, this study was limited by finance, logistics and the time frame. Consequently this investigation was limited to three wards out of seventeen wards within Obonjeni District. Ten Grade 12 mathematics teachers were selected as participants; use of such a small sample could bring into question the external validity of the findings.

This study did not include all areas of Euclidean Geometry but focused specifically on circle geometry as that is the main focus of the Grade 11 – 12 curricula and of the matric examination at the end of Grade 12. This study was conducted within the paradigm of qualitative research, employing test and task-based interview as data collection methods. Marshall and Rossman (1995), argue that there is a weakness in qualitative research with regards to the transferability of results, as each qualitative research approach has its own unique features. Much of the interpretation of data was based on the researcher's own subjective judgement, which could result in bias in the findings, but the researcher attempted to be as 'objective' as possible to minimise this aspect.

1.8. Organisation of the Study

Whilst this chapter has outlined key reasons for the research, further reasons are advanced and clarified in detail in the subsequent chapters. This investigative study is organised into six chapters with the following content:

Chapter 1

This chapter sets the stage for the study and will therefore consist of the introduction, motivation for the study, focus of the study, research questions, research methods, analysis of data and study limitations.

Chapter 2

This chapter focuses on literature relevant to Grade 12 teachers' understanding of Euclidean Geometry. It presents the literature review which supports this study, as well as the theoretical framework which forms the basis of the analysis and arguments put forward in this report.

Chapter 3

This chapter sets the stage for the research methodology and therefore consist of an introduction, the research method, the research design, methods of data analysis, ethical considerations and a conclusion. The development of materials and processes undertaken to improve on the reliability of results and the different data collection tools used are discussed in depth.

Chapter 4

This chapter presents the analysis of the data that was collected from the field.

Chapter 5

This chapter consists of a critical discussion of key aspects of the research.

Chapter 6

This chapter deals with the summary of the research, recommendations and conclusions based on these Grade 12 teachers' understanding of Euclidean Geometry.

CHAPTER 2

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1. Theoretical Framework

2.1.1. Introduction

The traditional form of Euclidean Geometry teaching focused on the writing up of the theorems, solving riders and proof construction, which may be why learners perform poorly when it comes to Euclidean Geometry (Cassim, 2006). Freudenthal as cited in De Villiers (1997) has stated that the deductive presentation of former Euclidean Geometry has not just been a didactical success. He believes that geometry has failed because it is not taught as ‘reinvention’, as Socrates did, but is imposed ‘ready-made’ on the learners. However, research shows that other mathematical content (like Algebra, Functions, Data Handling and Trigonometry) is poorly taught too, but the poor teaching is often masked by exams which largely tests rote learning and instrumental skills which learners can be drilled to perform well in, without necessarily having much understanding (Adler & Pillay, 2006). Prospective teachers made such comments when asked why geometry is taught (Van Putten, 2008):

- “Know how to prove theorems;”
- “Know which theorems to apply;” and
- “Memorise proofs.”

Such comments from prospective mathematics teachers clearly indicate that they simply define geometry as the section dealing with theorems and proof. In the study undertaken by Pournara (2004), a group of prospective teachers at University of the Witwatersrand were asked, “Why do we teach Euclidean Geometry at school?” Some of their responses included:

- Euclidean Geometry is about theorems to attain results;
- To prove theorems;
- Perhaps to bring marks down a bit;

- I do not know but our teachers use to say that we will need skills we learn in Euclidean Geometry to apply it in our everyday lives; and
- I do not know why Euclidean Geometry is taught at high school because the vast majority of it cannot be applied to everyday life, nor does it have meaning or relevance to the learners' lives.

The comments above reflect the limited view of Euclidean Geometry held by prospective teachers who are expected to teach the subject in the not too distant future. They lead one to doubt teachers' understanding of Euclidean Geometry. Perhaps even more frightening to contemplate are those who did not learn Euclidean Geometry at their high school or tertiary level, while they are expected to teach it in the implementation of CAPS in 2012 (DBE, 2011).

It was suggested that Euclidean Geometry should be scrapped from the South African school curriculum totally, despite pronouncements by certain sceptics that geometry is alive and well and experiencing a revival in most countries throughout the world (De Villiers, 1997, p.37). Instead it ended up being optional (Mathematics Paper 3).The DoE undertook to train teachers so that they could be prepared to teach content and processes related to the optional assessment standards, with the view that in 2009 in Grade 10 Euclidean Geometry would be compulsory (Govender, 2010). Govender (2010) reports though that regrettably the envisaged training did not happen, and later the DoE announced that Paper 3 would not be compulsory.

According to Cassim (2006) there is no doubt about the importance of Euclidean Geometry not only to develop logical thinking, but also as a support in developing insights into other mathematical content as well as in other fields of study such as architecture, astronomy, engineering and physics. The Association for Mathematics Education of South Africa (AMESA) also views Euclidean Geometry as providing a more convenient "vehicle" to drive the development of both logical reasoning and deductive thinking, which helps us expand both mentally and emotionally (Govender, 2010). He further states that the exclusion of Euclidean Geometry is affecting students' performance to a negative extent at tertiary level especially in their first year.

This study is located in the midst of the sea of educational changes in South Africa and its challenges. In the introduction of National Curriculum Statement (NCS) Policy for Grade 10 – 12, Euclidean Geometry was put in an optional paper (Paper 3) (DoE, 2002; 2008). In the implementation of CAPS for Grade 10 – 12 (DBE, 2011), in Grade 10 in 2012 Euclidean Geometry will be a compulsory and no more optional paper. The DBE proposed that training of teachers on the Euclidean Geometry should be undertaken for no less than two weeks in 2011(during school holidays) since the inclusion of Euclidean Geometry will be implemented in 2012.This study is a qualitative case study, using multiple sources. The case study was explored over time through in-depth, detailed data collection involving different sources of information rich in contextual information (Creswell, 2005).

These two models (Bloom's Taxonomy categories and the Van Hiele levels) can be used as teaching tools as well as assessment tools. These models provided a conceptual framework that facilitated an understanding of the topic for this study.

2.1.2. The Revised Bloom's Taxonomy Categories

Bloom's Taxonomy was named after Benjamin S. Bloom. He was at the forefront of educational theory in United States in the 1950's. Bloom developed his famous Bloom's Taxonomy after obtaining a PhD from the University of Chicago. A committee of the college, led by Benjamin Bloom in 1948, identified three domains of educational activities (Bloom, 1956):

- **Cognitive** – intellectual skills (comprehension), consisting six levels;
- **Affective** – growth in feeling or emotional areas (mind-set), consisting five levels; and
- **Psychomotor** – manual or bodily (skills), consisting six levels.

In 1956, eight years after the committee started, work based on the cognitive learning domain was completed and a handbook commonly known as “Bloom’s Taxonomy” was published (Forehand, 2010). The small quantity anticipated for university examiners “has been transformed into a basic reference for all teachers worldwide” (Anderson & Sosniak, 1994, p.1). Over the years, Bloom’s Taxonomy has served to provide a classification of educational system objectives, especially to help administrators, examiners, subject education specialists (e.g. on SAG), and researchers to evaluate problems and discuss curricula with greater precision (Bloom, 1994). Forehand (2010, p.2) writes; “it should be noted that while other educational taxonomies and hierarchical systems have been developed, it is Bloom’s Taxonomy which remains, even after nearly fifty years, the de facto standard”.

Sample questions from Bloom’s Taxonomy provide an examiner with a layout of a variety of different types of questions an examiner can ask his/her learners (Pohl, 2000). Pohl further state that Bloom’s Taxonomy can be used to categorise test questions that may appear on an assessment. However some questions require more “brain power” and a more detailed elaboration in answers. The test questions for this study were organised within the first five categories of new revised Bloom’s Taxonomy, in an ascending order.

2.1.3. The Characteristics of the Bloom Categories

Remembering

The person can distinguish, retrieve and recall appropriate facts from long-term memory.

Understanding

The person can construct meaning from verbal, textual, and graphic meaning through classifying, exemplifying, illuminating and summarising.

Applying

The person can carry out or use a formula through executing, or implementing.

Analysing

The person can break material into constituent elements, determining how the elements relate to each other and to an overall composition or intention through attributing, differentiating and organising.

Evaluating

The person can make standards through checking, critiquing and making judgements based on criteria.

Creating

The person can put structures through planning, producing or generating parts together to form functional whole or coherent, recognising parts into a new pattern.

2.1.4. An Explanation and Illustration of the Categories

Category 1: Remembering

At this basic category of the Taxonomy the person should be able to:

- Straight recall;
- Estimate and use proper decimal numbers;
- Classify and use a correct formulae;
- Use of mathematics actuality; and
- Use proper mathematics vocabulary.

Example:

Use the formula of Pythagoras' Theorem ($r^2 = x^2 + y^2$) to calculate the radius of a circle, given the coordinates of points on the circle.

Category 2: Understanding

At this category of the Taxonomy the person should be able to:

- Simplify calculations and problems which may engage many steps; and
- Use the given information for derivation.

Example:

Complete the statement: The angle at the centre of a circle is . . . at the circumference.

Category 3: Applying

At this category of the Taxonomy the person should be able to:

- Perform well-known procedures;
- Classify and apply (after changing the subject) the correct formulae; and
- Carry out procedures generally similar to those that have been seen before.

Example:

Write down the general rule that represents the stated transformation in the form (x; y) . . .

Category 4: Analysing

At this category of the Taxonomy the person should be able to:

- Realise that there is often no simple solution for the application;
- Understand that applications need not be relevant in real life context; and
- Recognise that applications require conceptual understanding.

Example:

A, B and C are points on the circle with centre O. DA is a tangent to the circle at A. Use the sketch to prove the theorem which states that $\angle TAC = \angle B$.

Category 5: Evaluating

At this basic category of the Taxonomy the person should be able to:

- Generalise elementary axioms into proofs for congruency, line geometry and similarity;
- Make substantial connections between unlike illustrations; and
- Answer non-textbook and non-routine applications (which are not necessarily complex).

Example:

XY and ST are two parallel chords of a circle with centre O. XT and YS intersect at P. Show that $\angle$XOS = $\angle$XPS.

Category 6: Creating

At this basic category of the Taxonomy the person should be able to:

- Answer applications which comprehend difficult calculations and higher order questions;
- Involve high order knowledge and techniques; and
- Ability to break the applications into constituent elements.

Example:

Prove the theorem that the cute angle which is formed between a tangent drawn to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment of the circle.

This study uses the revised version of Bloom’s Taxonomy (Forehand, 2010) Figure 1 below shows both versions to eliminate confusion. Forehand further states that the two Bloom’s Taxonomy versions present noticeable differences and can also cause perplexity. Essentially, Bloom’s Taxonomy six major categories were transformed from noun to verb forms. For an example, the lowest level in old version, knowledge (noun), was renamed and became remembering (verb).

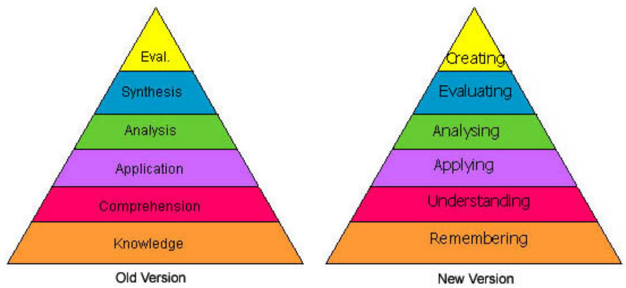


Figure 1: Bloom’s Taxonomy versions

Table 1: Bloom's Taxonomy categories key words (Verbs)

Category	Examples of Verbs
Remembering: Recall data or information	**Examples**: List mathematics formulae. Knows mathematics methods to solve an application. **Key Words**: Defines, knows, lists and recognises.
Understanding: Comprehend the meaning, conversion, interpolation, and analysis of orders and applications.	**Examples**: Recognise the ethics of assessment writing. Justify in one's particular words the patterns performed in a complex assignment. **Key Words**: Comprehends, converts, distinguishes, justifies, generalises and rewrite.
Applying: Utilize an idea into a new state or spontaneous use of a concept.	**Examples**: Use standard deviation formulae to solve for x. Apply laws of exponents (index) to simplify application. Solve x and y simultaneously. **Key Words**: Applies, calculate, computes, demonstrates, discovers, manipulates, modifies, predicts, produces, relates, shows, solves and uses.
Analysing: Separates items or concepts into constituent elements so that its organisational composition maybe understood.	**Examples**: Use the definition to differentiate (use first principle). Draw a box and whisker diagram of the information given. **Key Words**: Analyses, breaks down, compares, contrasts, diagrams, deconstructs, differentiates, discriminates, distinguishes, identifies, illustrates, infers, outlines, relates, selects and separates.
Evaluating: Construct a composition or model from various fundamentals. Position parts jointly to form a totality, with accent on creating a new significance or composition.	**Examples**: Write the coordinates of the other points of intersection. Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$. **Key Words**: Categorises, combines, creates, devises, designs, explains, generates, modifies, organises, plans, rearranges, reconstructs, relates, revises, rewrites, tells and writes.
Creating: Formulate judgments concerning the importance of information or resources.	**Examples**: Describe, in words, the transformation of f to g. Explain or justify your logical reasoning. **Key Words**: Describes, evaluates, explains, interprets, justifies and supports.

2.1.5. The Van Hiele Levels of Understanding Geometry Theory

The pioneers of the Van Hiele theory were Dutch teachers experiencing challenges in their mathematics classroom. These challenges underpinned the focus for their doctoral dissertations. The Van Hieles' intentions were to try and categorise students' cognitive thinking in geometry by levels. Pierre Van Hiele and his wife Dina Van Hiele-Geldof developed their famous Van Hiele theory in the respective doctoral dissertations at the University of Utrecht, in 1959 at Netherlands.

The Van Hieles specialised in different fields in their studies. Dina Van Hiele-Geldof focused in teaching practice on how to assist students to make good progress with the levels of learning, and described five phases of teaching within each level. Her husband, Pierre Van Hiele, was accountable for developing the model and describing these levels in depth (Lawrie & Pegg, 1997). Dina unfortunately passed on shortly after the completion of her study, and her husband developed and disseminated the theory further in later publications. The Van Hiele theory has four characteristics, which are summarised in the following order by Usiskin as cited in De Villiers (1997, p.45):

- **Fixed order** – the order in which persons grow through the thinking levels is invariant. For a example, a person should master level *n-1* to be on level *n*.
- **Adjacency** – at every level of thinking that which was essential in the previous level becomes extrinsic in the present level.
- **Distinction** – every level has its own linguistic symbols and own group of associations connecting those symbols.
- **Separation** – two individuals who reason at distinct levels cannot understand each other.

The Van Hieles discovered that the curriculum is presented at a higher level to the level of those being taught and that is the primary reason for the failure of traditional geometry (De Villiers, 2010). Monaghan (2000) assume that most learners start secondary school geometry thinking at the first or second Van Hiele Level, therefore teachers understanding ought to be above that level. De Villiers further states that the Van Hiele theory has separated the mastering of geometry into five levels, and the hypothesis is that they form a learning hierarchy. The theory states that someone cannot achieve a specific level without mastering the preceding levels. The first four levels are the most significant ones for high school geometry course and level five is significant at tertiary level geometry course.

2.1.6. The Characteristics of the Van Hiele Levels

The Van Hieles in their primary source numbered the levels from 0 to 4. American researchers, however, numbered the levels from 1 to 5; they included pre-recognition level as level 0. This study uses the 1 to 5 numbering scheme:

Level 1: Recognition/Visualisation

The person recognises figures by appearance only, frequently by comparing them to an identical sample. The properties of a figure are not perceived. At this level, person’s decisions are formulated based on visual perception, not logical reasoning.

Level 2: Analysis

The person sees figures as collections of properties. S/he can identify and label properties of geometric figures; however a person cannot see the logical relationship between these properties. When describing an object, a person functioning at this level may list all properties the person knows, however not distinguish which properties are essential and which are adequate to illustrate the object.

Level 3: Ordering/Abstraction

A person perceives relationships between properties and between figures. At this level, a person can construct significant definitions and provide casual arguments to substantiate his/her reasoning. Rational significance and category inclusions, such as squares being a category of rectangle, are understood. The function and implication of formal reasoning, however, is not understood.

Level 4: Deduction

A person can construct proofs, understand the function of axioms and definitions, and provide essential meanings and enough conditions. At this level, a person ought to be able to construct proofs such as those usually found in secondary school geometry course.

Level 5: Rigor

A person at this level understands the proper aspects of reasoning, such as comparing and establishing mathematical systems. A person at this level can comprehend the use of indirect proof and proof by contrapositive, and can understand non-Euclidean systems.

2.1.7. The Van Hiele Levels Explanation

Level 1: Recognition/Visualisation

At this basic level of the theory a person is basically aware of the space around him/her. Geometric objects are rather considered in their totality than in terms of their properties of constituent parts. A person at this basic level has the ability to identify specific shapes, reproduce them and learn the appropriate geometric vocabulary (Feza & Webb, 2005). For example, a person at this basic level may be able to identify that Figure 2 (on the left) contains rectangles and Figure 3 (on the right) contains triangles. However, a person would not be able to state that the opposite sides of a rectangle are parallel or the angles at the vertices are 90°.

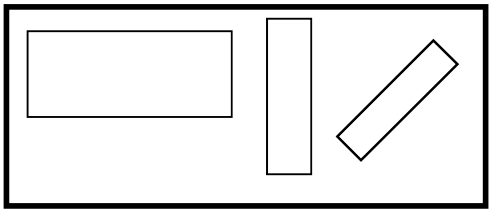


Figure 2: Rectangles of different sizes

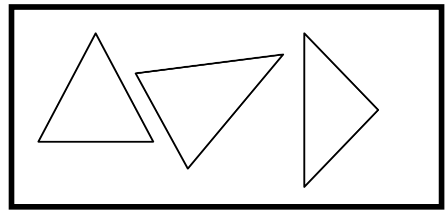


Figure 3: Triangles of different sizes

Level 2: Analysis

At this level of analysis, properties of geometric shapes are being understood by persons through experimentation and observation. These new properties are used to conceptualise classes of shapes. For instance a person is able to recognise that square is a kite, since a square has all the properties of a kite. While persons at this level are able to master the relevant technical knowledge to describe figures, they still lack the capacity of “interrelate figures or properties of figures, and make sense of definitions” (De Villiers, 1997, p.41).

Level 3: Ordering/Abstraction

At this level of understanding, persons are able to establish the interrelationships that exist between and among figures. For instance persons are able to state that in a quadrilateral, if the opposite sides are parallel, then the opposite angles are equal, as well as that a square is a trapezium as it has all the properties of a trapezium.

At this level, persons are able to deduce properties of figures and also recognise classes of figures. Persons are able to understand class inclusion. Definitions begin to make sense for persons and are understood by them. However, at this level, persons are not able to “comprehend the significance of deduction as a whole or the role of axioms” (Crowley, 1987, p.3).

Level 4: Deduction

At this level persons are able to understand “the significance of deduction, the role of axioms, theorems and proof” (De Villiers, 1997, p.41). At this level the persons have the ability to construct proofs based on their own understanding. Persons do not need to memorise readymade proofs and produce them on demand in teaching. The person is able to develop a proof in many different ways. Furthermore, “the interaction of necessary and sufficient conditions is understood; distinctions between a statement and its converse can be made” (Crowley, 1987, p.3). However, very few pre-service teachers reach this stage of “advanced” reasoning as shown in Mayberry (1983). Some mathematics educationists and some textbook authors regard axioms as self-evident truths, and do not regard axioms as the initial building blocks of a mathematical system (De Villiers, 1997).

Level 5: Rigor

The person at this level does not function only with the axiomatic system of Euclidean Geometry. The person has the potential to study non-Euclidean Geometry systems such as spherical geometry. By being exposed to other axiomatic systems, the person is able to compare similarities and differences that exist between the systems. Euclidean Geometry can be studied or seen in abstract form.

In the five levels of the theory this last level is the least developed originally. All persons who have studied Euclidean Geometry are expected to be at this level; however, it is surprising that most of the research done focuses on the lower levels of the theory. The key features of the model can be summarised as shown in Table 2.

Table 2: A summary of the Van Hiele theory of geometric reasoning

LEVELS	What is studied?	How are they studied?	Examples
1	Individual objects, e.g. triangles, squares or kite.	Visually recognised on the basis of physical appearance.	Squares of all sizes having same orientation group together on the basis of their orientation or look.
2	A class of shapes, e.g. rhombuses are kites since rhombuses have all the properties of kites.	Figures having common characteristics are grouped together, e.g. rhombuses are a subclass of kites.	A rhombus has all sides equal, perpendicular diagonals and an axis of symmetry through one pair of opposite angles.
3	A teacher begins to define figures/shapes belonging to same grouping/family.	Observing and noticing relationships between properties studied. This is done largely on an informal basis.	Through construction and measurement, it is observed that if a quadrilateral has equal sides, it will have perpendicular, bisecting diagonals, and vice versa.
4	More formal proofs are studied.	Using an axiomatic system to prove relationships. A more formal approach is adopted.	Prove challenging riders or theorems on their own such as napoleon's theorem, Vivian's theorem, etc.
5	Geometry is studied on an abstract level. There is a move between systems (e.g. using algebraic system to solve geometry riders).	As an interrelationship of different systems.	A circle in 2-dimension is extended to include a sphere in 3- or higher dimensional space. The Euclidean axioms are compared with those of the non-Euclidean geometries.

According to the Van Hieles, persons move through each level of thought, through organised instruction of five phases of learning as described below. According to Mason (1997) these five phases can be applied to move the teachers to the next level of geometric development.

Phase 1: Inquiry or Information: Through discussion the tutor recognises what persons already know about circles and quadrilaterals and the persons become familiar to this.

Phase 2: Direct or Guided Orientation: Persons investigate the objects of instruction from structured circles and quadrilaterals questions to investigate specific concepts.

Phase 3: Explication: Persons describe what they have learnt from the tutor's explanation of mathematical terms and concepts in their own words.

Phase 4: Free Orientation: Persons solve applications and investigate more open-ended questions by applying the relationships learnt.

Phase 5: Integration: Persons develop a network of objects and relations by summarising and integrating what they learnt.

2.1.8. The Role of Language

According to the Department of Basic Education, one of the aims of mathematics is for learners to "develop the ability to understand, interpret, read, speak and write mathematical language" (DBE, 2011, p.6). Language is a key component of learning. Effective learning happens as persons actively explore the objectives of study in proper contexts of geometric knowledge and as they engage in conversation and reflection using the language of the vocabulary of mathematics (Teppo, 1991). The role of language in geometry cannot be overstated. Language appropriate to the person's level of thinking, as well as the identification of suitable material, are pivotal aspects in the development of the person's geometric thinking.

According to Van Hiele, one of the primary reasons for the lack of success of traditional geometry curricula can be attributed to the communication gaps between teacher and learner (Fuys, 1985). De Villiers (1997, p.41) captures it aptly when he writes, “they could not understand the teacher nor could the teacher understand why they could not understand”. In order to enhance conceptual understanding it is important for persons to “communicate (articulate) their linguistic associations for words and symbols and that they use that vocabulary” which is very poor English (Crowley, 1987, p.13). Verbalisations call for the persons to make conscious efforts to express what may be considered vague and incoherent ideas. Verbalisation can also serve as a tool to expose persons “immature and misconceived ideas” (Crowley, 1987, p.14). At first persons should be encouraged to express their geometric thinking in their own words, e.g. “F-angles” for corresponding angles; “a square that has been kicked” for a rectangle, etc. As persons advance their geometry understanding, they should be exposed to the appropriate terminology and use it correctly.

A person’s usage of word (or term) in mathematics does not imply that the listener (the learner) and the facilitator (the teacher) share the same meaning of the word used. For example, when a teacher uses the word parm (short for parallelogram), is the listener thinking of a parallelogram or the palm of his/her hand? Another example shows this is, if a person is given a square in a standard position as shown in Figure 4, i.e., the person is able to identify the square, however if it is rotated 45° as shown in Figure 5, then it is no longer a square, maybe a kite or diamond to others. In the example the person focuses on the orientation of the figure as the determining fact of the “squareness” of the figure. By engaging persons in conversation and discussion, a person’s misconceptions and incomplete ideas as well as correct perceptions can be exposed. Thus language can cause confusion which can lead to the formation of misconceptions. As Dias (2000) states, meanings of words are different in mathematical contexts compared to their common usage.

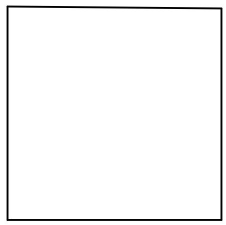


Figure 4: A Square

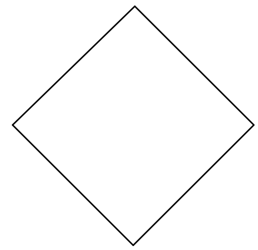


Figure 5: Rotated square

Language is an important key in learning and understanding. Every level of geometric thinking has its own vocabulary and its own translation of the similar term. Discussing and verbalising concepts are essential aspects of phases of learning, which are: Explication, Free Orientation, Guided Orientation, Information and Integration. Persons clarify and recognise their ideas through talking about them. For the persons to acquire and correctly utilize the appropriate language, the role of a tutor becomes paramount. For example, if the person is working at level 3 of the Van Hiele theory, then the tutor should be seen to be using terms such as "axioms, postulate, converse, necessary, sufficient and theorem" (Crowley, 1987). As the person progresses along the Van Hiele continuum, appropriate terms need to be used.

The type of questions posed by tutors is key to directing a person's thinking. Questions that require regurgitation of information supplied by the tutor will not foster critical thinking, which is needed for geometry. A person needs to be able to explain and justify their explanation in a critical manner. "Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations" (DoE, 2003, p.9). A person should be asked to explain "why" as well as to think about alternative approaches to their initial explanation, by posing appropriate questions, allowing sufficient waiting time and engaging persons in discussion of their answers and in methods which consider persons' level of thinking.

For growth in the person's thinking to happen, the level of instruction of tutor needs to correspond with the person's level of development. Thus the tutor must be able to ascertain the person's level of geometric thought, for each of the levels in the Van Hiele theory is characterised by its own unique vocabulary which is used to identify the concepts, structures and networks at play within a specific level of geometric thinking. "Language is useful, because by the mention of a word parts of a structure can be called up" (Van Hiele, 1986, p.23).

2.1.9. Conclusion

Bloom's Taxonomy categories and the Van Hiele theory are constructive attempts to identify a person's stage of geometric understanding and to identify the means to progress through the categories and levels. Succession through the Van Hiele levels is dependent more on the type of instruction received than on the person's physical or biological maturation level. The categories and theory have been used extensively in different research studies to assess teachers' understanding of geometry (Pandiscio & Knight, 2009; Senk, 2002; Feza & Webb, 2005).

2.2. Literature Review

2.2.1. Introduction

Van Putten (2008) states that if we accept the assumption that "Mathematics is a science of pattern and order" (NCTM, 1989, p.31), it should not preclude the notion of philosophy in terms of a search for truth. She further states that the science aspect lies in the need for inductive reasoning; to find out what is not at first clear and to define the truth to which that reasoning leads. The order aspect lies in the use of deductive reasoning which uses the truths found inductively, and applies them to that which needs to be defined and ordered. Mathematicians establish truth by using proof based on logical, deductive reasoning from axioms (Clements & Battista, 1992). In the context, it is useful momentarily to consider the basic tenets of both Bloom and Van Hiele:

- Bloom's Taxonomy categories, illustrate how thinking progresses from being random to empirically structured, and finally to being coherent and deductive.
- The Van Hiele theory, deals particularly with geometric thoughts as it progresses over numerous levels of complexity underpinned by school curricula.

Whilst many reformed-based curricula are being developed, they have not been explicitly designed to support a person's knowledge of subject matter (Van der Sandt & Nieuwoudt, 2005). They argue that in order for the new curriculum changes to be successful, persons need to learn new classroom practice. Shulman (1986) proposed a framework of knowledge areas that are necessary for science subjects (Mathematics, Physical Sciences, Life Science, etc) teaching to be successful: pedagogical knowledge (PK), pedagogical content knowledge (PCK) and subject matter knowledge (SMK). Shulman further states that in the construction of SMK the volume and organisation of comprehension is in the mind of the person.

The study done by Goulding (2007) in assessing teacher trainees' subject knowledge, argues that primary school teachers are expected to teach the content prepared by government, including concepts and processes not in the primary school curriculum, however unrelated. She identified some weakness in substantive knowledge, in terms of generalisation, reasoning and proof. Functioning with the similar obligations to assess and remediate primary United Kingdom teacher trainees' mathematical knowledge, however with different assessment instruments, Sanders and Morris (2000) discovered problems in all areas of the curriculum and Jones and Mooney (2000) discovered weaknesses specifically in geometry. This study investigated teachers' understanding of circle geometry in Euclidean Geometry, since it will be compulsory on the implementation of CAPS from 2012 in Grade 10. It is particularly relevant as some last taught it in National Technical Education (NATED) 550 curriculum in 2007. The instrument used in this study to investigate teachers' understanding included concepts and processes related to circle geometry. The researcher has looked at other studies based on Bloom's Taxonomy categories and Van Hiele theory as a foundation of this study.

2.2.2. Bloom's Taxonomy Categories

Bloom's Taxonomy of learning domains is a well-known explanation of levels of educational objectives. It may be helpful to bear in mind this Taxonomy when signifying educational objectives (Martha et al., 2001); Taxonomy objectives are useful to researchers in matching their goals and specific objectives. Although Bloom's Taxonomy has been criticised for being too simplistic (Pritchett, 1999), it provides a useful framework that can be used by members of academic staff when developing assessment (Pritchett, 1999; Krathwohl & Anderson, 2001). Bloom's Taxonomy has been recommended to be used across all levels: at school level, at tertiary level, by researchers and even at the work place, as it ensures that the level of the questions asked within assessment is sufficiently challenging (Lilley et al., 2004).

Bloom's Taxonomy involves categorising the excellence of an individual answer (Pegg, 1992). Issues such as enthusiasm, the structure of the enquiry and previous information in a particular area, may influence a person's answer. This study used cumulative hierarchy of Bloom's Taxonomy in designing test questions (Appendix 5). Bloom's Taxonomy consists of six categories that are grouped into three levels, lower, middle and higher (DoE, 2003).

Table 3: Cognitive levels

Cognitive Level	Bloom's Taxonomy	Verbs
Lower order	Categories 1 & 2	What, Who, When, Name, Mention, List, etc.
Middle order	Categories 3 & 4	Discuss, Explain, Describe, Identify, Compare, Distinguish, etc.
Higher order	Categories 5 7 6	Analyse, Examine, Apply, Synthesize, Criticize, Suggests, etc.

Newly implemented primary mathematics curricula in Turkey have been underpinned by constructivist theory in which learners are actively involved in their own learning tasks by action, experiencing and exploring (Ali & Hakan Sevkin, 2010). In order to improve high order skills, Forehand (2010) has argued that examiners should utilise both appropriate teaching and assessment tools and techniques. She further points out that one must be aware that techniques or methods used in evaluation practice may influence candidates' knowledge and progress in both bad and good ways. This study used the newly revised Bloom's Taxonomy by Krathwohl and Anderson (2001) which are: remembering, understanding, applying, analysing, evaluating and creating. According to Ferguson (2002) teachers' abilities should be investigated via diverse levels of questions, and Bloom's Taxonomy is one useful instrument to assist in this regard.

The results of the data analysis in the study by Ali and Hakan Sevkin (2010) show that there was a slight reflection of the constructivist method on the examination papers, which were set by the educators who reported that they were applying constructivist methodology. Most of the problems asked in the assessment tools required remembering or memorising capacity. It is such findings that lead the researcher of this study to investigate teachers' understanding using Bloom's Taxonomy categories excluding the category of creating. The division of the problems in examination papers shows that educators are not concerned with the comprehension and systematic practice dimensions of Bloom's Taxonomy (Ali & Hakan Sevkin, 2010).

The test used in this study applied a cumulative hierarchical framework as will be further explained in Chapter 3; each category requires mastering of the earlier skill or ability prior to the next, more complex one. Forehand further states that 'ability' can be measured by using classifications of levels of scholars' behaviour essential in learning and assessment such as Bloom's Taxonomy. Krathwohl (2002) promotes the use of Bloom's Taxonomy in research, by arguing that it can also clarify the essential questions, goals and objectives of other studies.

An investigation of the World Wide Web yields transparent proof that Bloom's Taxonomy has been useful to a range of situations (Ferguson, 2002). In the commencement the scope of Bloom's group was limited to facilitate the replacement of test objects measuring the identical educational objectives. However, when persons are designing effective lesson plans, they frequently look to Bloom's Taxonomy for assistance (Paul, 1985). The revised Bloom's Taxonomy includes precise verb and product relationship with each of the levels of cognitive procedure; therefore, it offers the persons an additional influential tool to assist in designing their lesson plan. Implementing a revised Bloom's Taxonomy in this study was the best choice as every teacher is expected to refer to it on daily basis during lesson preparation and also in designing tasks.

The researcher of this study used objective questions, which were designed in such a way that the marking process would not depend on any subjective judgement on the part of the researcher. This study was not for grading teachers, but to investigate and try and determine their level of understanding of circle geometry from both Bloom's Taxonomy and Van Hiele theory as guidelines. Remembering questions (i.e. at category 1 of the Taxonomy) were included. These were restricted to the ability to remember previously learned Euclidean Geometry. Bloom (1956) states that in remembering the only required skill is the ability to retrieve pertinent and accurate information from long-term memory. 'Understanding' questions were used to investigate teachers' ability to understand learning material. Bloom defines understanding as the ability to construct meaning from different contexts ranging from the numerical to the graphical or geometrical, to interpret material and to draw logical conclusions from given information. This intellectual skill represents a step further from simply recalling learning material, and corresponds to the lowest level of 'understanding' (Gronlund, 2000).

'Application' questions investigated the ability to use pre-knowledge and procedures in new concrete and abstract situations. Bloom (1956) defines it as the use of rules, formulas, methods, concepts, principles and theories. 'Analysis' questions investigated how teachers break information into its element parts and discover how the parts connect to each other and to a global composition and intention. 'Evaluation' questions were based on making judgements based on criteria and standards (Bloom, 1956). Lilley et al (2004) presented guidelines to be considered when designing an assessment:

- The question being asked should be relevant to the study.
- There should be a balance between the intellectual skills being assessed.
- All the components of the questions - stem, key and distracters - should be correct and clear.

The six major levels of Bloom's Taxonomy of the Cognitive Domain (Lee, 2008):

I. Remember. Recalling information.

Describe, match, name, recognise and record.

- State the relationship of co-interior angles of a cyclic quad.
- Write the equation of Pythagoras theorem.
- Complete the statement.
- Determine the derivative of the following equations.

II. Understand. Explain the meaning of information.

Describe, generalise, paraphrase, summarise and estimate.

- Estimate if $\angle$A will be less or greater than 90°.
- Describe in text what is given graphically.

III. Apply. Using abstraction in real situations.

Apply, determine, employ, expand, explain, plan and table.

- Determine the formulae of the line passing through points Q and S.
- Solve simultaneously for x and y.
- Use the diagram given to show that $m^2 = n^2 + o^2 - 2no \cos M$.

IV. Analyse. Breaking down entire application into elementary parts.

Classify, differentiate, evaluate point out and recognise.

- Illustrate the constraints on the graph paper provided in the diagram sheet.
- Compare the graphs above. Which company supplies bulbs that have a higher deviation from the mean?

V. Evaluate. Putting elements collectively to form a modern and integrated whole.

Create, design, plan, organise, generate and write.

- Investigate alternative definitions of various polygons.
- Design a house plan showing all measurements.

VI. Create. Making judgment regarding the merits of data, resources or phenomena.

Analyse, assess, choose, consider and moderate.

- Prove that for a convex, cyclic hexagon *ABCDEF* that

$\angle A + \angle C + \angle E = \angle B + \angle D + \angle F = 360°$.

- Without using a calculator, show that sin 18^0 is a solution of the cubic equation $8x^3 - 4x + 1 = 0$.

2.2.3. The Van Hiele Theory

This study is investigating Grade 12 mathematics teachers' understanding of Euclidean Geometry with specific reference to circle geometry. Other researchers have investigated teachers' level of geometric reasoning; these researchers have laid a foundation for this study. Furthermore, others have sought to classify misconceptions held by pre-service and in-service teachers. The study done by Mayberry (1983) was one of the pioneers to study the Van Hiele levels of geometric reasoning; 19 pre-service elementary teachers were interviewed. This study focused only on ten Grade 12 mathematics teachers; who were given test and task-based interview as is more elaborated on Chapter 3.

In the study of Mayberry (1983), questions were formulated at each level of Van Hiele, covering seven major topics established in the basic curriculum. The analysis of educators' responses indicated a lack of readiness for prescribed deductive geometry module (13 of them had already had secondary school geometry). The questions in this study are asked from level 1 to 4 (American order) of Van Hiele theory and assessed in category 1 to 5 of Revised Bloom's Taxonomy. Mayberry's other intention was to assess whether questions posed to the population formed a hierarchy that would relate to the Van Hiele levels. This claim was confirmed by applying the Guttman scalogram analysis and has been substantiated by other researchers (Fuys, Geddes & Tischler, 1988; Mason, 1997; Clements & Battista, 1992).

In the study of Goulding and Suggate (2001), anxieties seemed to be allayed by peer teaching and interview with tutors, where talking through errors and misconceptions obliged them to think more explicitly about mathematics. In this study, teachers wrote a test based on circle geometry, their responses were analysed and then task-based interview followed to identify specific conceptions and misconceptions about circle geometry. Teachers' subject matter knowledge in terms of the Van Hiele levels of understanding geometry was investigated further by task-based interview and the test was set according to Bloom's Taxonomy of learning domains.

Researchers such as Mason and Schell (1988) also investigated the levels of reasoning and misconceptions at hand in pre-service elementary and high school educators as well as in-service high school educators. They used Mayberry's (1983) interview procedure was used to question participants. They state that high school in-service educators tended to obtain highest of the three levels in terms of Van Hiele (at the top two levels). The results of their study showed that over 75% of high school pre-service educators were at Van Hiele level 3 or higher (based on the initial 0 – 4 numbering system).

At the same time, several misconceptions were identified with this population, for example, some did not point out that parallel lines had to lie in the same plane. For the pre-service elementary educators, 38% were functioning below Van Hiele level 3 and 8% not even at Van Hiele level 0. Several teachers also provided an incorrect description of isosceles triangle. An additional field of deficiency was characterised among sufficient and necessary conditions. Some had problems with hierarchical inclusion and incorrectly indicated that rectangles were squares. Others also had difficulty in noting the congruence of corresponding angles with similar figures.

The researchers Pandiscio and Knight (2009) tackled the matter of whether pre-service mathematics educators at both the elementary and secondary level hold sufficient understanding of geometry to teach the subject well, and whether the essential geometry course they take enhance their geometric capability. Mathematics teachers' competence and understanding of Euclidean Geometry will determine how they will teach it. Therefore, mathematics teachers should be at least a level ahead of learners for cognitive progress (Pandiscio & Knight, 2009). The study of Jacobson and Lehrer (2000) also suggests that educators should be exposed to investigations currently taking place on reasoning for both teachers and learners. In fact they recommend making teachers and learners' participants in studies, in order to expose them more to current issues of mathematics.

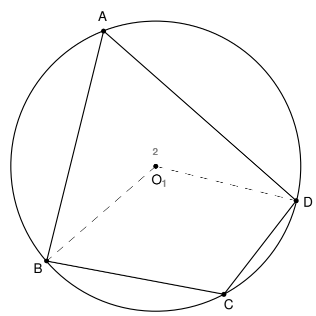
The study done by Van Putten et al (2010) investigated the attitude towards, as well as the level of understanding of Euclidean, Geometry in pre-service mathematics education students. In their study, a geometry module was offered over a semester and an interview before and after the module was conducted to discuss participants' experiences of learning geometry and analyse their attitude towards Euclidean Geometry. The geometry module offered changed the students' attitude towards geometry, but still did not bring about a sufficient improvement in their level of understanding for these students to be able to teach geometry adequately (Van Putten et al, 2010).

Usually the Russian geometry curriculum at school level is formed by two phases, namely, an ***intuitive*** phase for Grade 1 to 5 and a ***systematisation*** (deductive) phase from Grade 6 (12/13 years old). According to De Villiers (2010) a Russian study undertook a comprehensive analysis of the intuitive and the systematisation phases that showed little progress in geometry. In their analysis, the Van Hiele theory played a key component. Therefore, after that, Russians subsequently planned a very successful investigational geometry curriculum based on the Van Hiele theory. According to the Van Hiele theory, a person cannot attain one level of understanding without having mastered all the previous levels.

Theorems of Euclidean Geometry in the majority of textbooks appear as a finished product, for example Figure 6. By the term ‘product’ in mathematics, is meant here “the end-result of some mathematical activity which preceded it” (De Villiers, 1997, p.45). Tutors and many textbook authors continue providing learners with readymade content, especially in Euclidean Geometry, which they are then expected to “assimilate and regurgitate” in tests and exams (De Villiers, 1997, p.45).

Theorem:

"The opposite angles of a cyclic quadrilateral are supplementary$\angle$ Opp. $\angle$'s of cyclic quad.)"



Given : Circle O containing cyclic quad. ABCD

Required to prove : $\angle A + \angle C = 180°$

and $\angle B + \angle D = 180°$

Proof : Draw BO and DO

$\angle O_1 = 2\angle A$ ($\angle$ at centre = 2 x $\angle$ at circ.)

$\angle O_2 = 2\angle C$ ($\angle$ at centre = 2 x $\angle$ at circ.)

$\angle O_1 + \angle O_2 = 2(\angle A + \angle C)$

But $\angle O_1 + \angle O_2 = 360°$

$\angle A + \angle C = 180°$

Figure 6: Theorem

The internal moderator for the Mathematics Paper 3, 2008 NCS Examination, reported that basic cyclic quadrilateral theory and problems as well as the tan-chord theorem were not known by some candidates. In the analysis of question 8 in that examination, it appeared that some candidates performed poorly in circle geometry, which may have resulted from the fact that they were not taught circle geometry (DoE, 2008). He suggested that circle geometry, proportionality and similarity be taught in Grade 11 and revised in Grade 12. According to his 2010 report there is a lack of reasoning and logic, lack of application of theorems, axioms and corollaries appropriately and confusion around similarity and congruency (DBE, 2010). He further states that, the techniques of proving a rider were lacking in some candidates, for example, they used what they were supposed to prove in their arguments; in other words circular reasoning. None of his recommendations have been considered officially by amending work schedules or examination guidelines; his observation shows a great improvement with regards to learners' performance. Learners attempted almost all the questions as it is evident in the sample of 100 scripts of learners moderated (DBE, 2010).

There has been a growing interest in mathematics education in recent times regarding the teaching and learning of proof (De Villiers, 1990; 1997; Hanna, 2000). Traditionally proof has been seen as a primary as means to verifying the accuracy (correctness) of mathematical statements (De Villiers, 1990). According to De Villiers and other researchers, proof consists of the following aspects:

- Verification (establishing the **validity** of a conjecture);
- Explanation (providing insight into **why** it is accurate);
- Systematisation (the **categorisation** of different results into a deductive structure of axioms, key concepts and theorems);
- Discovery (the finding or development of **new** results);
- Communication (the **spread** of mathematical information).

Whilst the above five aspects will not be dealt with in any detail in this study, it is sufficient to state that proof is similar to Van Hiele's level 4 stage of reasoning. In the Van Hiele model of geometric reasoning, this level epitomises the persons' ability to understand "the interrelationship and role of undefined terms, axioms, definitions, theorems, postulates and proof" (Crowley, 1987, p.3). Another characteristic of the person at this level of geometric thinking is the person's ability to "construct not just to memorise proofs" (Crowley, 1987, p.3).

In the National Curriculum Statement (NCS) for mathematics for Grade 10 – 12, learning outcome 3 deals specifically with space, shapes and measurement. Learners are supposed to be exposed to an enquiring, investigative, developmental approach to Euclidean Geometry. The objective of secondary school geometry is the succession to Van Hiele level 4, identified as deductive.

In the South African curriculum, geometry has been a long standard module in secondary school mathematics. The decade of the 1970s emphasised including geometry at the primary school level, due to its worthiness in advanced mathematics, in universal education, and in practical problems. One consequence of this improved standard of geometry is that educators at all levels require some experience of studying geometry in order to reach the content knowledge required to be effective instructors (Usiskin, 1987; Swafford, Jones & Thornton, 1997). The researchers investigated the subject of whether pre-service mathematics educators at both the elementary and secondary level held a sufficient understanding of geometry to teach the content well, and whether the essential geometry module they took enhanced their geometric competence.

Byrnes (2001) says the linguist Vygotsky explains the process where persons have developed comprehension of concepts only when it presented within their zone of proximal growth. This proposal forms a key part of the Van Hiele theory, particularly as it explains how instruction in geometry might assist a person's progress from a particular level to the next higher level. Instruction plays an essential function in a learner's succession throughout the levels. For persons to obtain comprehension, it is not possible to miss a level. The Van Hiele model of mathematical reasoning has become a proved descriptor of the progress of someone's developmental sequences in geometry (Jaime & Gutierrez, 1995).

"The common belief is that the more a teacher knows about a subject and the way learners learn, the more effective that individual will be in nurturing mathematical understanding" (Swafford et al., 1997, p.467). The Van Hiele theory is a good predictor of a persons' achievement in a geometry module (Usiskin, 1982 & Senk, 1989). Yet according to Usiskin (1982), a majority of prospective educators do not reach an acceptable level of geometric understanding, even with a geometry module in their undergraduate preparation, much less without it. Similarly, Mayberry (1983) discovered that in-service and pre-service elementary educators have demonstrated low levels of geometric understanding. Hershkowitz & Vinner (1984) discovered that in-service educators and their learners tend to demonstrate similar patterns of misconceptions when evaluated on their comprehension of basic geometrical figures and attributes of these figures.

Most studies have shown that learners are performing poorly in the geometry section (Strutchens & Blume, 1997; Mullis et al., 2000). It has become obvious that most learners do not comprehend the meaning of proof, much less how to go about writing one, even after completing an annual course in geometry (Senk, 1989). These complications motivated the researcher of this study to investigate teachers' understanding of Euclidean Geometry with specific reference on circle geometry.

2.2.4. Conclusion

Bloom's Taxonomy categories and Van Hiele theory of geometric thinking levels were presented as the overarching theory that underpins this study. Both aspects of Bloom's Taxonomy categories and Van Hiele theory – the aspect proposing levels of geometric understanding, and the aspect concerned with instruction in geometry – were discussed, with a view to identifying and specifying frameworks for analysing the data generated in this study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1. Introduction

This chapter will introduce the research methods adopted by this study. These will be discussed and the reasons for preference will be outlined. The issues connected to the research, the problems investigated and proposed solutions will be discussed. This chapter will also include a discussion of ethical issues that were considered.

3.2. Research Method

The term ‘research methods’ refer to the approach or strategy followed in finding out more or studying a phenomenon in order to obtain information. Basically there are two research methodologies: these are the qualitative and quantitative research approaches (Cohen et al., 2007). The choice of any of these approaches depends on the purpose and fitness of the study. This study was designed to investigate Grade 12 teachers’ levels of understanding of circle geometry in Euclidean Geometry. In this investigation, the method employed in completing the study is the qualitative research method. The researcher collected the data by giving participants a test to write. To supplement the information gathered from the test, participants were also given a task-based interview.

3.2.1. Reason for Choosing Research Method

The qualitative research method presents descriptive data which involves collecting verbal or textual data and observable behaviour (Cohen et al., 2007). In a qualitative research approach, the researcher is not merely gathering data (information), but he or she is approaching the empirical world in a specific manner. The reason for the choices of qualitative method, and the researcher’s choices and actions will determine strategy (De Vos, 2002).

This investigation therefore lends itself to the qualitative approach within an interpretive paradigm, since the researcher attempted to understand this in terms of teachers' understanding of circle geometry concurrent to the new revised version of Bloom's Taxonomy categories of learning levels and Van Hiele theory. This study collected data in two forms from the person, in the form of a test and a task-based interview. Another reason why the qualitative approach was chosen is because it assists in gaining more insight about the nature of a particular phenomenon (Leedy & Ormorond, 2001).

The qualitative research method allows for a combination of different strategies that can be used together to investigate teachers' understanding of circle geometry. Conducting test and task-based interview allowed the researcher to understand teachers' intentions and reflections better. Maxwell (1996) states that the advantage of using the qualitative research method is that it allows for formative evaluations, intended to improve existing processes or programmes. The qualitative research approach allows the researcher to be able to move back and forth for the purpose of gaining different meanings, gathering diverse data and identifying various perspectives of different practice.

3.2.2. Rationale for the Qualitative Approach

The rationale for the choice of the qualitative research approach comes from the purpose of the study. Thus in order to investigate teachers' understanding of circle geometry, it was decided to collect descriptive data from person's own written and spoken words. Therefore, the qualitative approach is suitable for a research study of this nature which seeks to understand the conceptual understanding of teachers and how it can be improved in mathematics education. Furthermore, the preference of the qualitative approach for this research is due to the flexibility and uniqueness of the research design. De Vos (2002) states that there are no fixed patterns that are followed and the arrangement cannot be exactly replicated.

3.2.3. Reliability of the Qualitative Approach

There can be no reliability without validity and thus no credibility without dependability (Lincoln & Guba, 1985). Reliability refers to the degree at which a measurement instrument (a test and/or task-based interview) can yield the same results on repeated applications (Durrheim, 1999(a)). In this research the instruments used in data collections were test and task-based interview. The test was firstly administrated to five Grade 12 teachers from Vryheid District in KwaZulu-Natal before the main participants wrote it and task-based interview was conducted with the same population to increase reliability.

The final questions were then produced after some adjustments were made, based on the pilot study. The purpose of doing the trial was to distinguish whether the test instrument can be used to determine understanding. The use of the pilot study was based on the argument that pretesting what they intend investigating (Mouton, 2001) can help to refine and improve the test instrument.

3.2.4. Validity of the Qualitative Approach

Validity is the most important characteristic to consider when constructing or selecting research instruments. A valid research instrument is one which measures what it is intended to measure (Ross, 2005). He further states that there are four important types of validity in education research: content validity, construct validity, external validity and internal validity. This study used content validity, teachers subject matter content on circle geometry was investigated as stated in Chapter 2.

3.2.5. Researcher's Role in Qualitative Approach

According to De Vos (2002), in a qualitative research approach, the researcher is directly participating in the study. The researcher interacts with the participants, and he or she is the 'instrument'. In the qualitative approach, much of the data analysed is based on the researcher's subjective interpretation and also depends largely on the skills of the researcher. From the above reasons, one can conclude that a qualitative report can never exclude the researcher's own perspectives. Marshall and Rossman (1995) further acceded that the extent to which the researcher plan their participation in the study, or outlines their role should determine the extent to which his own perceptions will be reflected in the final report. This, of course, has a major contribution in how events are shaped and interpreted.

The researcher needs to establish affinity with the participants in order to gain information from them. This was necessary for this research since personal interview was conducted. Such a harmonious compatibility with the participants encouraged them to give relevant and detailed information without feeling intimidated.

3.3. Research Design

The research design, according to Durrheim (2002), is the plan of how the researcher will systematically collect and analyse the data that is required to give valid solutions to research problems. The research design refers to the logical structure of the inquiry so that emphatic conclusions are afforded (De Vos, 2002; Pillay, 2004). Pillay further states that research design proceeds in a certain order and specific manner. Thus methods should be selected to furnish the necessary information required to produce a complete research study as suggested by Bell (2005) as shown in Table 4.

Table 4: Research design flowchart

Explain the Aims and Objectives
↓
Design Methodology
↓
Design Feasibility
↓
Develop Research Instruments
↓
Representativeness/Sampling
↓
Conduct Pilot Test
↓
Revised Instruments/Tools
↓
Conduct Research
↓
Analyse Data
↓
Write Report

Durrheim (2002) defines research design as a designed and planned nature of observation that distinguishes research from any other forms of observation. From the table in the previous page, one can clearly distinguish that each category in the flow chart is dependent on the successful completion of all the previous categories. Therefore, it is essential not to pass over any single step and to ensure that each step is thoroughly completed. However, Durrheim notes that research design should not be seen as a fixed plan that proceeds in a very structured, linear way. Research is a flexible and non-linear process that is often influenced by practical considerations.

In collecting data to investigate Grade 12 mathematics teachers' understanding of circle geometry, an appropriate research design was followed. For the purpose of this proposed study, the research instruments consisted of pre-test and task-based interview schedule with Vryheid District Grade 12 mathematics teachers. Before going into any details about the research instrument, the researcher will firstly outline the techniques for data collection and the kind of sampling that was used.

3.3.1. Data Collection Instruments

In order to collect data for the study, a test and a task-based interview were conducted.

3.3.1.1. Test

The paper-and-pen test (see Appendix 4) was designed and developed by the researcher, with questions posed in English according to the university policy. Individual items in the test were designed or adapted by the researcher to comply with the requirements of this study. These are the items that reveal teachers' Bloom's Taxonomy categories of learning levels and Van Hiele levels of understanding Euclidean Geometry with specific reference to circle geometry. This test consisted of twelve questions, based on the first five categories of Bloom's Taxonomy and the first four levels of Van Hiele theory.

The same paper-and-pen test was administered prior to the intervention to the teachers from Vryheid district. It was not the aim of this study to test the validity of the Bloom's Taxonomy categories and the Van Hiele theory since this was reliably done (Gutierrez et al., 1991; Senk, 1989 and Mayberry, 1983). The test items were created and selected in such a way that no knowledge beyond what lay within the reach of the South African high school was accessed.

The test was designed with the following characteristics:

- The duration for completion was two hours.
- The test consisted of 12 questions and no marks were allocated since the study aimed to investigate teachers' levels of understanding of circle geometry, not to grade them.

According to Gronlund (1998, p.55), knowledge and comprehension items are used to "measure the degree to which previously learned material is remembered and determines whether teachers have grasped the meaning of material without requiring them to apply it". Therefore, the research instrument used an extended-response question. The statement given has the information necessary for the solution of the extended-response question, as is customary in tests dealing with geometric riders (Van Putten et al., 2010).

The items were classified according to category 1 through category 5 of Bloom's Taxonomy and level 1 through level 4 of Van Hiele theory. The researcher decided that Van Hiele level 5 would not be tested since it involved high order questions that did not fall within the school curriculum and would be more suitable perhaps at the tertiary level. Van Hiele level 1 is the most elementary of all levels and depends upon the recognition of shapes (De Villiers, 2010). Even though one might expect Van Hiele level 1 to have been attained by teachers long before their entry into the Further Education and Training (FET) phase as learners themselves, Van Hiele level 1 items were included to examine if they had really mastered that level.

The order of the items in terms of Bloom's Taxonomy categories and Van Hiele levels was not random; teachers were required to think in a sequence of Bloom's Taxonomy categories and Van Hiele levels, demonstrating their ability to rapidly access reasoning power in various categories and at various levels. Bloom's Taxonomy categories and Van Hiele levels are arranged in a cumulative hierarchy since the groups of objectives were arranged in a complex of ascending order and the level of complexity of tasks increases as each group of behaviours was gathered to include all the behaviours of the low complex groups (Krietzer & Maduas, 1994). There are more Bloom's Taxonomy category 4 items compared to Bloom's Taxonomy category 1, 2, 3 and 5; and there are more Van Hiele level 3 and 4 items in comparison to Van Hiele level 1 and 2.

3.3.1.2. Task-Based Interview

An interview is a conversation between the researcher and the respondent. However, it is different from an everyday conversation in that the researcher is the person who sets the agenda (task-based) and asks the questions (Cohen et al., 2007). It is a structured conversation where the researcher has in mind particular information that he/she wants from the respondent, and has designed particular questions (task-based interview) to be answered.

Cooper and Schindler (2003, p.323) describe an interview as a "two-way conversation initiated by an interviewer to obtain information from a participant". An interview gives the interviewer an understanding of meaning that can further probe initial responses during an interview. It seeks to collect objective information which might depend on the attitudes, beliefs and ethos of the investigation. In this study, ten mathematics teachers from nine selected secondary schools in Obonjeni District (KwaZulu-Natal) were interviewed. Participants were interviewed individually in order to understand individual levels of insight based on Bloom's Taxonomy categories and Van Hiele theory.

For one to understand the levels of Grade 12 mathematics teachers' understanding in solving geometric riders, it was necessary for the researcher to gather in-depth information from the teachers. Teachers were expected to complete the given test (Appendix 4) on their own which would then after be followed by task-based interview. The purposes for the teachers to complete the test were twofold. Firstly, the strategies which teachers employ to solve geometric riders, with specific reference to circle geometry, can be ascertained. Secondly, using the teachers' responses in the test, the researcher was able to identify task-based interview questions for each participant individually. Thus, the research instrument was structured to be flexible. This structure would assist in gathering of information on predetermined topics that the researcher regarded as valuable to the study. Flexibility, on the other hand, would aid in capturing the understanding of teachers when solving geometric riders by allowing the researcher to explore and probe unanticipated responses.

3.3.1.3. Data Collection Limitations

In qualitative research, it is often difficult to overcome certain limitations of test and interview (Cohen et al., 2007). It is significant to acknowledge and best minimise their influence on research process. Regarding bias, Bell (1987, p.43), notes that it is easier to "acknowledge the fact that bias can creep in than to eliminate it together". The limitations need to be viewed in the context of the proposed, general research design. The intention of this study is not to duplicate the research design, since each case study is unique, but to extend the results and findings to teachers' understanding of Euclidean Geometry.

3.3.2. Sampling

Sampling, according to Cohen et al (2007), is a process of decision-making about the population (community), settings, events or deeds to observe. Further they say, exactly what will be studied in a particular study depends on the unit of analysis. Representativeness is the main concern in sampling. However, the researcher must choose a sample size characteristic of the population in which conclusions will be drawn from. The targeted population for this study was all Grade 12 mathematics teachers in Obonjeni District. Since all of them cannot be reached because of time constraints and other logistical problems, a representative sample was therefore selected.

Apart from the research methodology and instrumentation, the research study also relies on the quality of the sample chosen. Questions of sampling arise directly from defining the population description on which the investigation will be based or focused on. Hence, sampling needs to be considered early on in the research plan. Cohen et al (2007) propose that judgements need to be made based on four key factors in sampling, namely, the size of the sample, the representativeness and sample parameters, sample access and the strategy of sampling. Furthermore, it remains the researcher' decision to use either a probability or non-probability sample. Within a probability sample, the chances of members of a wider population being selected are known, whereas in a non-probability sample the chances of members of the wider population being selected for the sample are unknown (Cohen et al., 2007).

In purposive sampling, the researcher comes to a decision about which population to involve in the study (Strydom et al, 2004). Therefore, this type of sampling is entirely based on the researcher's decision. In qualitative research, purposeful sampling involves the selection of participants who are knowledgeable of what is to be investigated (Creswell, 2003). Qualitative sampling techniques such as probability and purposive sampling were used in this selection process because the researcher enquiry did not aim to represent or make a generalisation of the wider population (Cohen et al., 2007).

Teachers selected in this study were from the same district, in three wards and therefore the findings will be assumed to be most representative of the entire district. The participants in the study were drawn from Empembeni Ward, Ezibayeni Ward and KwaMsane Ward in Obonjeni District. The following procedure was followed in selecting the participants:

- The sample size was ten teachers;
- Must be a Grade 12 mathematics teacher;
- Must have ten or more years of teaching Grade 12 mathematics;

- Has achieved a qualification (professional or academic) to teaching.

Teachers from schools that had more than one Grade 12 mathematics teacher were all selected to participate if they all met the requirements.

3.3.3. Seeking Permission to Conduct Test and Task-based Interview

The data collection activities of this study, which were mainly test and task-based interview, took place in Hlabisa Municipality Library and Mtubatuba Municipality Library. The researcher applied for permission to use these facilities from the respective line managers. Attached to the application letters was the schedule of dates for appointments with the participants. The permission was telephonically granted to the researcher, with the rule that the researcher will remind the librarian two days before library use at any given time.

3.3.4. Research Conditions

Library managers offered an empty and silent discussion room where the research test and task-based interview were to be conducted. This process was conducted after working hours and on Saturdays so as to ensure that school functionality was not disrupted by the participants' and researcher's absenteeism.

3.3.5. Research Language

For both data collection methods, the test and the task-based interview, questions were posed in English. The written test was strictly answered in English. During the task-based interview, some of the respondents attempted answering some of the verbal questions in their home language (IsiZulu).

3.3.6. Duration of the Study

The researcher used test and task-based interview methods to collect data for this study. The major limitation of these methods is the fact that they were conducted once and interview is time-consuming. The researcher initially arranged appointments that were convenient to the interviewer (researcher) and participants. The task-based interview itself took more than two hours in some instances because responses were both verbal and written.

The initial researchers' intended time plan was to complete the whole process of both the test and task-based interview in a period of four weeks. This was all delayed by ethical clearance and rescheduling of appointments by the participants. The task-based interview therefore continued as long as it took until all the participants were interviewed and all the required data was collected (a period of three weeks).

3.3.7. The Process Followed in Completing this Study

- The researcher applied for Ethical Clearance to the University of KwaZulu-Natal, Durban in Edgewood Campus (17 June 2011).
- The researcher applied for Permission from the Department of Basic Education to conduct research in KwaZulu Natal Department employees (17 June 2011).
- The researcher explained the study to participants and their seniors (Principals and Head of Department Mathematics) (02 August 2011).
- The researcher negotiated appointment dates with the participants individually for task-based interview (12 August 2011).
- The schedule of dates was drawn up according to appointments (16 August 2011).
- The researcher wrote letters to use the Hlabisa and Mtubatuba libraries (23 August 2011).
- On the agreed dates, the test was written and the task-based interview was audio taped (01 – 20 September 2011).

3.4. Data Analysis

Analysis of data entails breaking down the information into elements in order to obtain responses to research questions (Owusu-Mensah, 2008). De Vos (2002) further elaborates, by stating that analysis means the categorising, ordering, manipulating and summarising the information achieved in responses to research questions. The intention of analysis is therefore to ease data to an interpretable and understandable form, so that relations of research problems can be studied and tested and conclusions drawn (De Vos, 2002). In this study, qualitative data analysis mainly involved the development of categories and identifying steps between categories.

According to Henning (2004, p. 33) data analysis is “a relatively systematic process of coding, categorising and interpreting data to provide explanation of a single phenomenon of interest”. The researcher transcribed the responses of the interviews verbatim. The item analysis was done first by grouping all the responses to a particular question under that question. Then, the responses were coded in relation to Bloom’s Taxonomy and the Van Hiele theory. Therefore, the interview data was organised in order to get an overview of what it revealed and test responses were grouped into manageable themes. The typological and inductive data analysis method was used to make sense of the data collected (Hatch, 2002). This typological analysis helped to analyse questions asked in both test and interview according to Bloom’s Taxonomy categories and the Van Hiele levels. This data was used later for some interpretive work in the final analysis, when comparing the findings with other research and contesting theories.

3.5. Ethical Considerations

Ethics in research that involves animals and people are essential (Lubisi, 1998). The principles are autonomy and beneficence (Durrheim & Wassenaar, 2002). Durrheim and Wassenaar further note that researchers need to be careful with protection of groups and individual identities when publishing study results. Therefore, the ethics of research code are concerned with the researcher’s attempt to value the human rights.

The researcher applied for permission from the KwaZulu-Natal Department of Basic Education (KZN DBE) research unit and for ethical clearance from the University of KwaZulu-Natal, Faculty of Education to conduct this study. The researcher also gave explanation about the intention of this study to the principals of the nine schools, the Head of Department of Mathematics (HOD) and 10 Grade 12 mathematics teachers who were selected as participants. The researcher achieved written consent form from all participants and granted permission for the task-based interview to be audio taped.

To achieve the following acceptable codes of ethics, the researcher

- Explained to principals and the participants that the real names for both schools and teachers would not be used in the write up. As an ethical consideration the researcher acknowledged the integrity of all those who helped make this study possible by using pseudonyms, so not to reveal the true identity of the schools and participants of this study (Durrheim, 2002).
- Informed the participants about their freedom to withdraw from the research at any point without penalty (Cohen et al., 2007).

3.6. Conclusion

In this chapter the research methodology has been described. A brief statement of what the research method is has been given. The methodology used in this research has been indicated as the interpretive research paradigm and employs qualitative approach. The rationale for the choice of these research methods has been dealt with in the chapter. How validity and reliability are addressed in qualitative approach has been described. The role of the researcher and language in a qualitative research has been dealt with.

The research design, which involves a logical strategy for gathering evidence for a research, has been described in this chapter. The instruments used in the data collection for the research have also been described. A representative sample was selected for the study. The method applied in the selection of the sample has been explained. In the following chapter the data collected is analysed and interpreted and research findings given.

CHAPTER 4

RESULTS AND ANALYSIS

4.1. Introduction

The purpose of this chapter is to report on the investigation carried out in relation to the questions set out in chapter one. This chapter presents the empirical findings of this study. The information was achieved using the methods described in the previous chapter. The researcher's intention was to investigate Grade 12 mathematics teachers' understanding of Euclidean Geometry with specific reference to circle geometry. The data provided by this study was collected through two sources: a test and task-based interview. A test was administered once and an individual task-based interview was conducted once. Analysis of the task-based interview data began with the study of the transcripts, in September 2011.

In this chapter, the researcher begins by analysing the findings of the case study, based on the coding of data collected for both test and task-based interviews. The researcher provided a largely qualitative analysis of the test responses and task-based interview responses, including of the audio taped responses according to the framework and the indicator descriptors set out in the previous two chapters. Finally, attempts were made to answer the following research questions:

4.1.1. What are the general Bloom's Taxonomy learning domains and Van Hiele levels of understanding of Euclidean Geometry of a sample of Grade 12 mathematics teachers?

4.1.2. What are Grade 12 mathematics teachers' specific conceptions and misconceptions with respect to circle geometry?

4.1.3. What is the relationship between Grade 12 mathematics teachers' understanding, conceptions and misconceptions, and their ability to apply their knowledge of circle geometry?

4.2. Coding Process

To safeguard the identity of the participants (teachers) involved, codes were used when referring to participants. All participant codes start with **T**, then two letters, for example teacher **TSD**. The responses of the ten teachers were then categorised and analysed according to the following categories.

Table 5: Categories of Participant's Responses

Category Number	Category
C1	Correct response with **valid reasons** and working shown (when necessary)
C2	Correct response with **some valid** reasons and working shown (when necessary)
C3	Correct response with **no valid** reasons and working shown (when necessary)
C4	Inconclusive response with **some attempt** made to solve question
C5	No response, with **no attempt** to solve question

As discussed in Chapter 3, the test and task-based interview instruments were designed in such a way that questions were arranged according to Bloom and Van Hiele in an ascending order of categories and levels. Questions did not specialise on each category and level per question, so as not to predispose the teachers to think that they were working from the easiest to the most difficult questions, and to prompt them to demonstrate their ability to access reasoning power on various categories and levels.

The performance of the teachers in terms of Bloom's Taxonomy and the Van Hiele model is shown by the respective distributions of teacher-responses coded as C1 in Tables 6 and 8 (for participants test responses) and Tables 7 and 9 (for participants task-based interview responses). Their performance according to Bloom's Taxonomy is given in Tables 6 and 7, and demonstrates that teachers scored an average of 50% and above on category 1 through category 4, in both test and task-based interview questions. Teachers' levels of understanding reasoning according to the

Van Hiele theory are given in Tables 8 and 9. These demonstrate that teachers scored an average of 50% and above were scored on level 1 through level 3, in both test and task-based interview questions. Out of all ten participants, only one participant correctly answered test questions eight and twelve.

Table 6: Participants that achieved C1 per test question in terms of Bloom's Taxonomy categories

Questions	1	2.1	2.2	3.1 a	3.1 b	3.2 a	3.2 b	4	5.1	5.2	6	7	8	9.1	9.2	10.1	10.2	11	12	Av %
BTC 1	7	9																		80
BTC 2			9	7		8		1												63
BTC 3					6		6		7		6	10		3		3	---			59
BTC 4															1			8		45
BTC 5										5			1						1	23

Table 7: Participants that achieved C1 per task-based interview question in terms of Bloom's Taxonomy categories

Questions	1	2.1	2.2	3.1 a	3.1 b	3.2 a	3.2 b	4	5.1	5.2	6	7	8	9.1	9.2	10.1	10.2	11	12	Av %
BTC 1	10	10																		100
BTC 2			10	8		8		1												68
BTC 3					7		7		8		8	10		4		4	3			64
BTC 4															3			10		65
BTC 5										6			2						1	30

Table 8: Participants that achieved C1 per test question in terms of Van Hiele levels

Questions	1	2.1	2.2	3.1 a	3.1 b	3.2 a	3.2 b	4	5.1	5.2	6	7	8	9.1	9.2	10.1	10.2	11	12	Av %
VHL 1	7	9																		78
VHL 2			9	7		8		1	7											57
VHL 3					6		6			5	6	10		3	1	3		8		53
VHL 4													1				---		1	07

Table 9: Participants that achieved C1 per task-based interview question in terms of Van Hiele levels

Questions	1	2.1	2.2	3.1 a	3.1 b	3.2 a	3.2 b	4	5.1	5.2	6	7	8	9.1	9.2	10.1	10.2	11	12	Av %
VHL 1	10	10		8		8														90
VHL 2			10					1	8											63
VHL 3					7		7			6	8	10		4	3	4		10		66
VHL 4													2				3		1	20

For test responses the results of teachers show that about 65% were at Van Hiele level 2 or lower. This is lower than those reported by Mayberry (1983) who found that her sample of primary pre-service teachers' responses was largely at Van Hiele level 3 or higher. She used students as the sample for her study while this study focused on experienced teachers who have taught geometry before. Such performance is therefore worrying in terms of the Grade 12 teachers we have.

Although certain questions were well answered (average of 60% and above in C1 responses), questions like question 5.2, 8, 9.1, 9.2, 10.1, 10.2 and 12 were either incorrectly answered or not answered at all. However, most of the questions that were not answered are not normally incorporated into school text-books or Grade 12 external examinations in South Africa. Nonetheless, these questions were designed to test their understanding of geometric concepts and their ability to apply their knowledge in unfamiliar contexts.

Teachers' performance slowly shifted for task-based interview responses compared to test responses. However, certain questions such as question 8; 9.2; 10.2 and 12 were either incorrectly answered or still not answered at all. There is a definite descending trend in performance in Bloom's Taxonomy category 1 through to Bloom's Taxonomy category 5 and in Van Hiele level 1 through to Van Hiele level 4 which seem to support the theoretical assumption that the levels form a hierarchy. In fact, the average performance of the participants lies within 50% and below in Bloom's Taxonomy category 4 through to category 5 and within Van Hiele level 3 through level 4.

Teachers performed better in Bloom's Taxonomy category 1 through to Bloom's Taxonomy category 3 and the Van Hiele level 1 through to Van Hiele level 2. Therefore, the participants' average performance may be an indication that their ability to reason in a formal deductive way has not been developed to a point where this can be done consistently or successfully.

Table 10, shows each participants' response to each test question. Table 10 also provides some additional biographical information regarding each participant who participated in the test. Response scoring categories were formulated prior to the participants' responses to the test.

Table 10: Participants Responses According to Categories Outlined in Table 6

Participants	**Gender**	**Teaching Experience**	**Questions**																
	M/F	**Years**	**1**	**2.1**	**2.2**	**3.1**	**3.2**	**4**	**5.1**	**5.2**	**6**	**7**	**8**	**9.1**	**9.2**	**10.1**	**10.2**	**11**	**12**
TJM	M	12	C1	C1	C1	C1	C1	C4	C1	C1	C4	C1	C4	C1	C5	C1	C4	C1	C5
TMM	M	15	C1	C1	C1	C4	C4	C1	C1	C1	C1	C1	C4	C4	C1	C4	C4	C1	C4
TDM	M	20	C1	C1	C1	C1	C1	C4	C4	C4	C1	C1	C5	C4	C5	C3	C4	C1	C5
TSD	M	16	C1	C1	C1	C1	C1	C4	C1	C1	C4	C1	C1	C1	C4	C1	C4	C1	C1
TMB	M	10	C4	C1	C1	C4	C4	C5	C4	C5	C1	C1	C5	C5	C5	C1	C5	C1	C5
TLD	F	12	C4	C4	C4	C4	C3	C4	C1	C5	C4	C1	C5	C5	C5	C4	C5	C5	C5
TWB	M	10	C1	C1	C1	C1	C1	C4	C4	C4	C4	C1	C5	C5	C5	C5	C5	C5	C5
TTB	F	11	C4	C1	C1	C3	C3	C4	C1	C1	C1	C1	C5	C1	C5	C4	C4	C1	C5
TTN	F	10	C1	C1	C1	C1	C1	C4	C1	C1	C1	C1	C5	C5	C5	C5	C5	C1	C5
TDS	M	14	C1	C1	C1	C1	C1	C4	C1	C4	C1	C1	C5	C5	C5	C4	C4	C1	C5

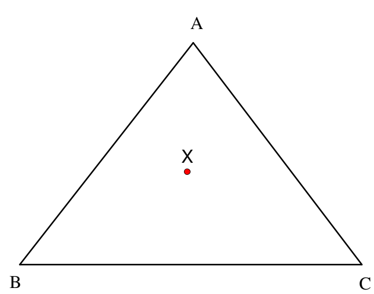
4.3. Analysis

A selection of some teachers' responses to test and task-based interview questions will be analysed shortly. The analysis sample was extracted in both test and task-based interview, irrespective of response categories (C1 – C5). However, the task-based interview was conducted mostly with teachers who did not achieve the correct responses in test questions. The rationale was to provide a spectrum of responses as well as information on teachers' reasoning when answering the test and task-based interview questions.

The rationale was also to investigate teachers' ability to answer the task-based interview questions in a formal deductive way. The task-based interview questions were similar to test questions, however, during the task-based interview teachers were requested to explain their reasoning in those questions in which they achieved C2 (correct response with some valid reasons) through C5 (no response, with no attempt). Participants were also required to provide clarity to some of the question responses where they achieved category C1 (correct response with valid reasons) on the test questions.

Question One (BTC 1 and VHL 1)

Draw perpendicular line from *X* to AB.



For the teachers to be able to answer this question, they needed to be able to visualise and know the properties of perpendicular lines:

- Perpendicular lines are straight lines (even if they are drawn roughly).
- Perpendicular lines intersect or meet at right angles to each other.

A correct test answer (1 = C1)

Viewing teachers' responses based on this question, 70% more or less drew a correct perpendicular line. Only 70% of these teachers indicated the right angle where two lines intersect. Even though the question did not ask them to indicate the right angle, it is probably more appropriate, from a pedagogical point of view, to show the right angle as teacher TDM did as shown in Figure 7, even though his 'perpendicular' clearly is not very close to 90 degrees.

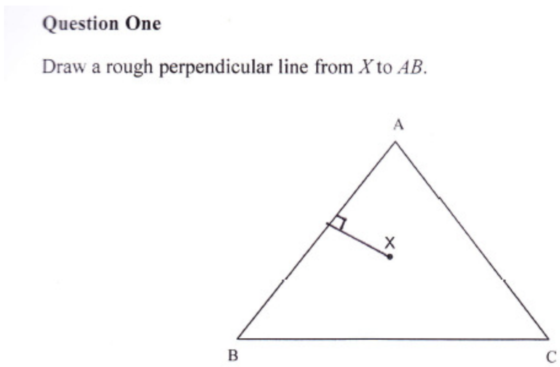


Figure 7: Response by teacher TDM

Inconclusive test answers (1 = C4)

Most teachers achieved a correct response to this test questions, even though their drawings were generally poor, so that one can visually see that the angle is not a right angle, but reasonably close. However, responses such as that of teacher TMB were way off from being correct. The angle in Figure 8 is clearly far from 90 degrees.

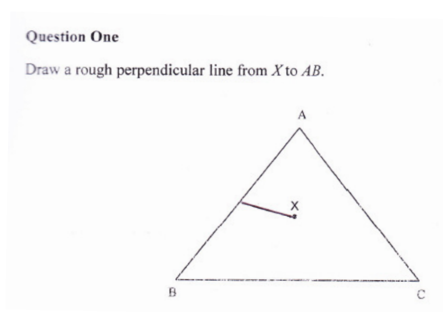


Figure 8: Response by teacher TMB

The researcher has found that 3 out of 10 (30%) teachers responded as teacher TLD as shown in Figure 9 where the angle is even further from 90 degrees, and drawn almost parallel to the base BC. This seems to indicate a lack of visualisation, a characteristic of Van Hiele level 1. For senior secondary teachers to draw such poor representations is clearly problematic as they are likely to pass the same misconception on to their learners.

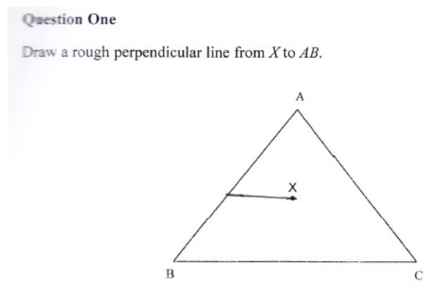


Figure 9: Response by teacher TLD

One might of course speculate that these three teachers might have visualised the drawing differently. Perhaps instead of visualising it as flat and in the plane, teachers who responded like this might have thought that perhaps the plane was slanted in three dimensions with BC the closest side (edge) of the plane and point *A* far away in the distance. When visualised this way, it is possible, as is often done in 3D trigonometry problems, that a perpendicular line in a plane seen from the side does not make a right angle. However, this is pure speculation at this point and the precise cause of this misconception observed in this small sample of teachers needs to be further and more deeply investigated with larger samples of both teachers and learners.

Correct task-based interview answers (1 = C1)

Teacher TLD's response to this test question was incorrect.

Interviewer : "Your response to this question was incorrect."

Teacher TLD : "Oh, no .... really? Was it incorrect? Wow, ok let's read the question again."

Teacher TLD then correctly drew a perpendicular line from X to AB as shown in Figure 10. Teacher TLD thought the question was asking for the straight line not perpendicular line.

Teacher TLD : “I read the question incorrect that is why I did not even indicate 90 degrees angle.”

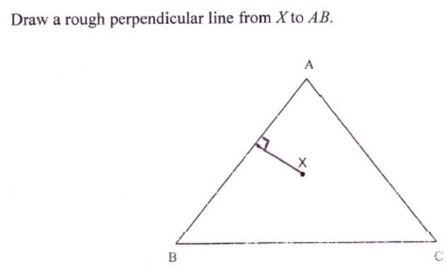
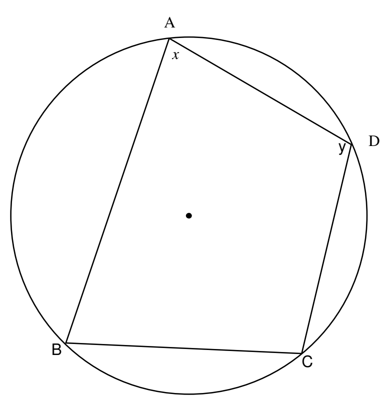


Figure 10: Response by teacher TLD

During the task-based interview, teacher TLD, teacher TTB and teacher TMB demonstrated an understanding of properly drawn perpendicular lines. The angle size where two lines meet was illustrated, and the ability to visualise the perpendicular lines was demonstrated. It does therefore appear that they somehow misinterpreted the question in the test.

Question Two (BTC 1 & 2 and VHL 1 & 2)



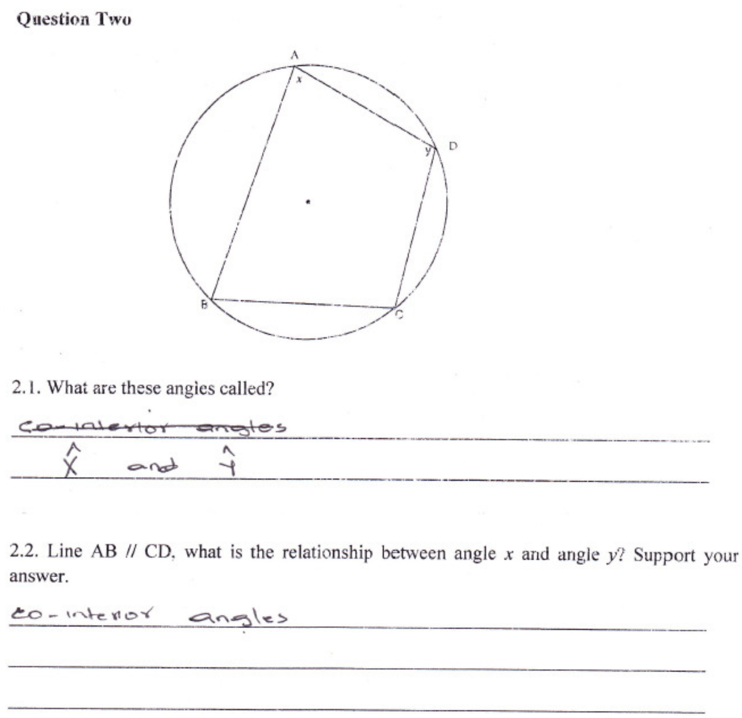
2.1. What are these angles called?

2.2. Line *AB* // *CD*, what is the relationship between angle *x* and angle *y*? Support your answer.

The names of angles formed when lines intersect, and the relationships between them when some of the lines are parallel, is taught in Grade 8. A Grade 12 mathematics teacher is expected to know how to answer this question and distinguish it from other names and relationships. In order to answer question 2.1, a teacher needs to understand and visualise the shape formed by these angles, an operation which is at Van Hiele level 1. This shape is a [-shape, therefore, these angles are adjacent or co-interior. The answer for question 2.2 is depends on 2.1 responses, co-interior angles sum up to 180° (supplementary angles); when the two lines are opposite to each other, they are parallel.

Inconclusive test answers (2.1 = C4 and 2.2 = C4)

Only one out of the ten teachers did not give the expected response, and this one failure was due to the vagueness of the question. Teacher TLD's response as shown in Figure 11, highlighted how she interpreted it differently. With this question the interviewer expected the naming of the marked angles, *x* and *y*; and the relationship between them. However, teacher TLD just named the angles *x* and *y* in 2.1 and called them co-interior. This describes a positional relationship rather than a propositional relationship (i.e. that they are supplementary). It therefore seems that this teacher was able to visualise, a characteristic of Van Hiele level 1 but provided inconclusive evidence of her knowledge of the propositional relationship between *x* and *y*, a, characteristic of Van Hiele level 2.


Question Two

2.1. What are these angles called?

~~co-interior angles~~

$\hat{X}$ and $\hat{Y}$

2.2. Line AB // CD, what is the relationship between angle *x* and angle *y*? Support your answer.

co-interior angles

Figure 11: Response by teacher TLD

Correct task-based interview answers (2.1 = C1 and 2.2 = C2)

Interviewer : “What are these marked angles called?”

Teacher TLD : “Mmmhhh .... *x* and *y*? They are co-interior angles.”

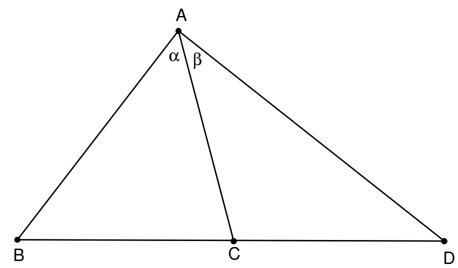
Interviewer : “What is the relationship between these angles if line *AB* // *CD*?”

Teacher TLD : “They are supplementary since *x* and *y* adds up to 180 degrees.”

In terms of the Van Hiele levels, the teacher was able to use the analysis and identification of properties of a shape type, and make a correct conclusion regarding their relationships.

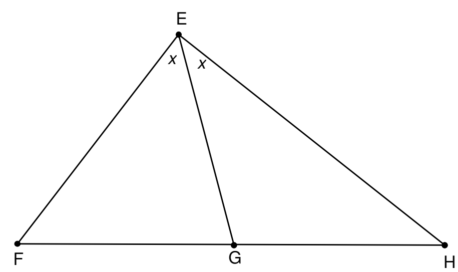
Question Three (BTC 2 & 3 and VHL 1 & 3)

3.1. Given that *BC* = *CD*.



Do you think the two marked angles α and β are equal? Why or why not? Please explain your answer as best you can.

3.2.



If $\angle FEG = \angle GEH$, do you think *FG* = *GH*? Why or why not? Please explain your answer as best you can.

Test question three consists of two parts. The first part assessed the teachers' assumption and the second part required teachers to substantiate their assumption. This question was assessing the application of Bloom's Taxonomy category 3, but in order to be able to answer this question correctly competence in the previous Bloom's Taxonomy categories is a prerequisite. Furthermore, teachers needed to know the properties of triangles and understand the interrelationship between triangles. The information given in 3.1 does not respectively make $\angle$FEG = $\angle$GEH and BC = CD. For the angles and line segments in each case to be equal, triangles ABC and ACD must be congruent, or equivalently the median and angle bisector will have to coincide, which is not generally true (except if given that triangle ABC is isosceles with the respective sides adjacent to angles A and E equal).

Correct test answers with no valid reasons (3.1 = C3 and 3.2 = C3)

Teachers, who achieved correct responses with valid reasons in this question, explained their responses differently. During the task-based interview, these teachers were asked if they could think of any possible supporting statements as to why these angles and lines segments would not necessarily be equal. Teacher TTB gave correct responses to both test questions 3.1 and 3.2 but gave no reason for 3.1. In 3.2 the teacher gave the reason "since the triangle is not congruent", which presumably refers to the two triangle EFG and EGH. In order to determine what she meant, she was further interviewed.

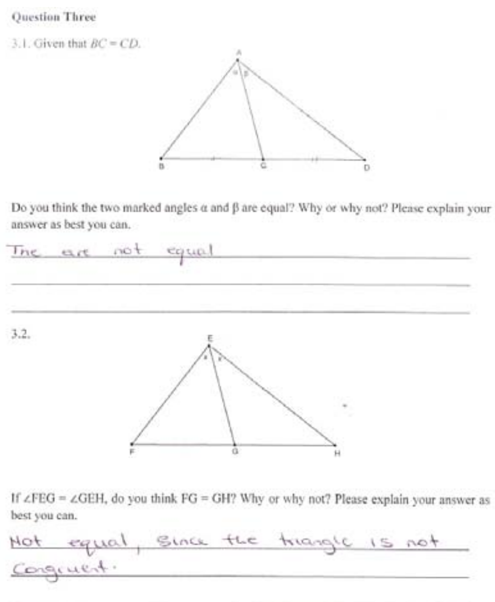


Figure 12: Response by teacher TTB

Interviewer : "Why did you said angle α and β are not equal?"

Teacher TTB: "The triangle is not congruent".

Interviewer : "What you meant by the triangle is not congruent?"

Teacher TTB: "Oh let us see, there is no S∠S or ∠S∠ between the triangles"

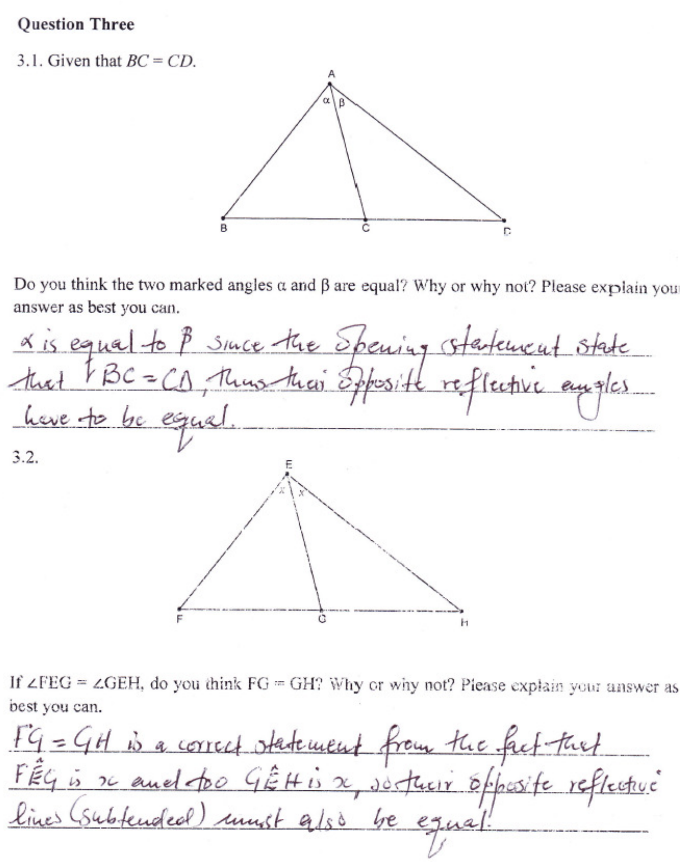
Interviewer : "In triangle ABC we have side BC, angle α and side AC; isn't this triangle congruent?"

Teacher TTB: “Oh no, the answer should say triangles are not congruent. Mathematically, congruency is based on the comparison of two triangles.”

Teacher TTB then recognised that her written justification in 3.2, as shown in Figure 12, was incorrect since she referred to a single triangle and not triangles.

Teacher TMB did not apply the properties of triangles (Van Hiele level 3). In 3.1 the teacher used the question to substantiate his answer, while in 3.2 the teacher used the information given to substantiate his answer. Also teacher TMM as shown in Figure 13 appears to have the misconception that the “angle bisector” is equivalent to the “median” of a triangle.

Inconclusive responses (3.1 = C4 and 3.2 = C4)


Question Three

3.1. Given that $BC = CD$.

Do you think the two marked angles α and β are equal? Why or why not? Please explain your answer as best you can.

α is equal to β since the opening statement state that BC = CD, thus their opposite reflective angles have to be equal.

3.2.

If ∠FEG = ∠GEH, do you think FG = GH? Why or why not? Please explain your answer as best you can.

FG = GH is a correct statement from the fact that FÊG is x and too GÊH is x, so their opposite reflective lines (subtended) must also be equal.

Figure 13: Response by teacher TMM

Interviewer : "What is the meaning of opposite reflective angles or lines?"

Teacher TTM: "Opposite reflective angles mean opposite sides to angles are equal and vice versa."

Interviewer : "Are these triangles congruent?"

Teacher TTM: "Yes, they are. *AC* is a common side, *BC* = *CD*; and α = β; is given, therefore these triangles are S∠S or SS∠."

The response of teacher TMM shows a misconception that having two corresponding sides and any (non-included) angles is a sufficient condition for concurrency. For two triangles to be congruent in the condition (S, ∠, S) the angle has to be included between the sides. According to the data collected, two teachers out of ten have this misconception. This is quite alarming given that they are Grade 12 teachers, and though this misconception is common among learners, as has been found by internal moderators, one would not expect that among teachers. These two teachers were not able to establish the interrelationships that exist between and among the two triangles and their related properties, a characteristic of Van Hiele level 3. According to Tables 7 and 9, only six out of ten teachers were able to deduce the properties of triangles and also recognise the classes of triangles.

Question Four (BTC 2 and VHL 2)

Represent the following statement with suitable rough sketches (drawings), as you might do in class with learners.

The angle subtended by a chord at the centre of a circle is equal to twice the angle subtended by the same chord at the circumference.

This question is based on the theorem known as "angle at centre is twice angle at circumference" in Euclidean Geometry curricula at school level. Seemingly theorems are presented as a finished product (see Chapter two); teachers normally do not prove it but only present it in sketches. In Grade 11, where this theorem is taught, some text-books represent it in three different cases (like Classroom Mathematics for Grade 11 textbook), therefore the researcher expected teachers to sketch three drawings to represent the three different cases. As is shown on the table above, only one teacher (10%) of the teachers represented the given theorem in the three different cases, the others in less than three sketches.

A correct test answer (4 = C1)

Teacher TMM shows an understanding of this theorem in different representations; and this teacher represented it in three different ways. However, in his first representation, he incorrectly wrote ½ $\angle EOC$ = $\angle EBC$ which is correct, but not an example of the theorem, which refers to $\angle AOB = 2\angle ACB$. Out of 6 teachers who achieved inconclusive responses with some attempt, 5 of them sketched only one diagram. This question required plural sketches (drawings) to represent the different cases of the theorem. It therefore appeared that the majority of these teachers only expose their learners to the standard representation of this theorem (e.g. the 2nd one in Figure 14), which is likely to seriously disadvantage their learners should they need to recognise and apply the theorem in one of the other two possible cases.

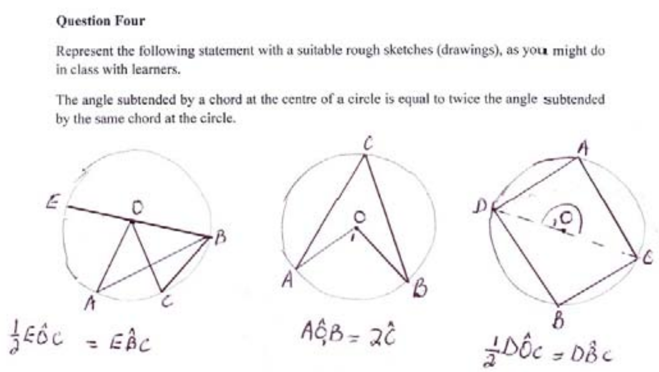


Figure 14: Response by teacher TMM

Inconclusive test response (4 = C4)

Teacher TLD, as shown in Figure 15, only drew the standard representation. This question constitutes Bloom's Taxonomy category 2 and Van Hiele level 2, where teachers start to analyse technical terminology to interrelate figures or properties of figures.

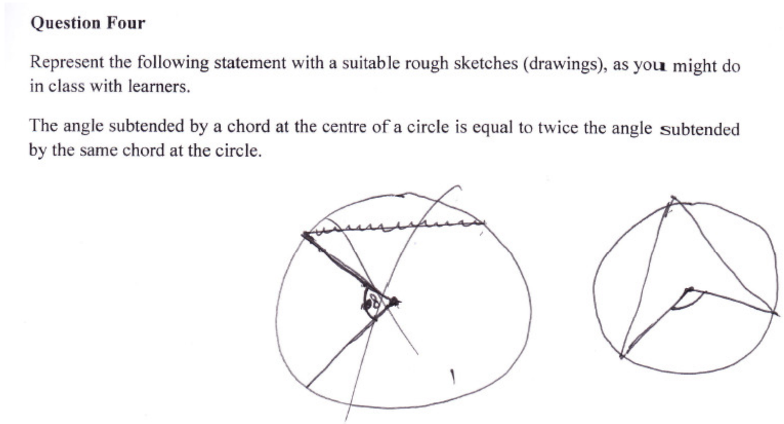


Figure 15: Response by teacher TLD

Teacher TWB, as shown in Figure 16, achieved an incorrect response in this question. He confused the theorem with another theorem known as “perpendicular from centre to chord”, and one can speculate that it might be due to carelessness or some language difficulty since he is an English second language speaker. Either way, this is problematic for a Grade 12 teacher if s/he is not to even be able to carefully read the formulation of a theorem and represent different cases.

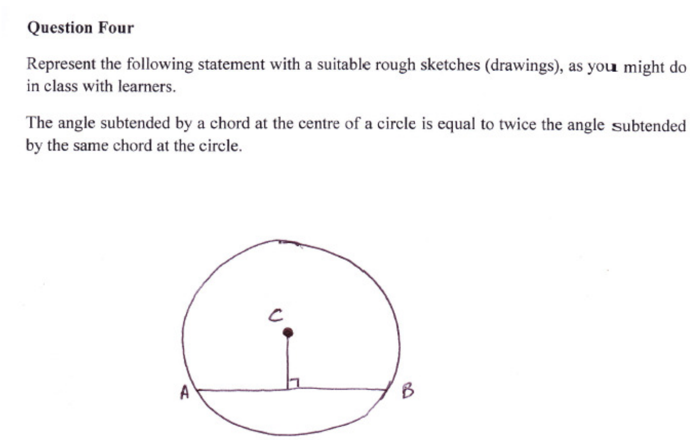
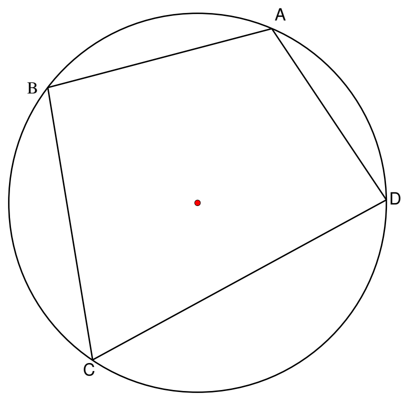


Figure 16: Response by teacher TWB

Question Five (BTC 3 & 5 and VHL 2 & 3)

5.1. If angle *A* was a right-angle, would *ABCD* necessarily be a square?



5.2. Justify your answer by logical reasoning or providing a suitable counter example.

This question required teachers to know properties of a square, which are:

- All angles are equal to 90°;
- All sides are equal; and
- Opposite sides are parallel

For teachers to obtain a correct response with valid reasons and working shown, they needed to have some understanding of necessary and sufficient conditions for a square. Teachers who achieved C2 (correct response with some valid reasons) through C5 (no response, with no attempt), may demonstrate understanding of necessary yet insufficient conditions of a square. According to the data, teacher TLD gave the correct response in 5.1 but could not justify why *ABCD* is not necessarily be a square, as shown in Figure 17. These teachers applied the information given to answer 5.1 but could not justify it.

A correct test answer, but no justification (5.1 = C1 and 5.2 = C5)

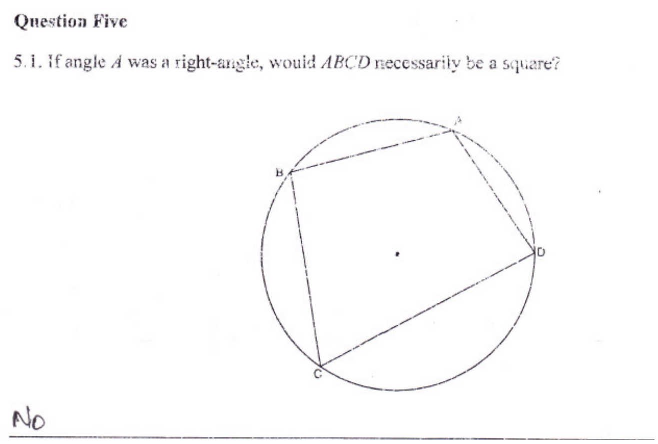


Figure 17: Response by teacher TLD

A correct test answer and justification (5.1 = C1 and 5.2 = C1)

Teacher TTN's response is given in Figure 18 and shows an understanding of conditions of a square when justifying test question 5.1 response. The teacher considered what is given to build up her logical argument. The teacher further declared that, even if $\angle A = \angle C = 90°$, that does not mean $\angle D$ will be 90°, but $\angle B$ and $\angle D$ can be any degree; the key is that they will be supplementary.

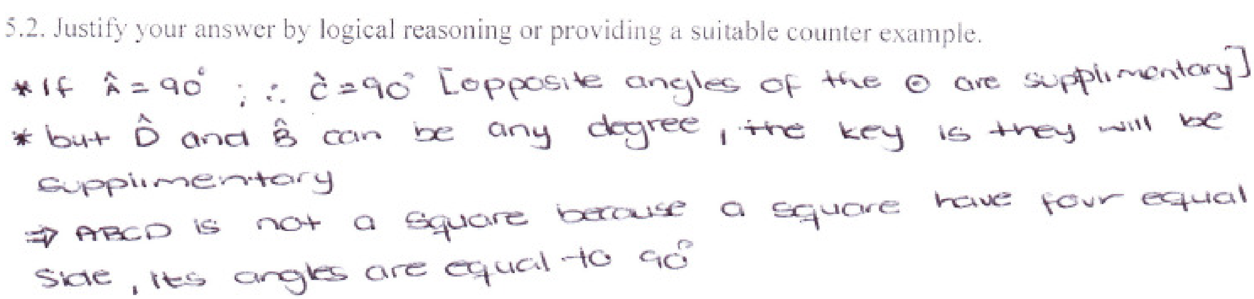
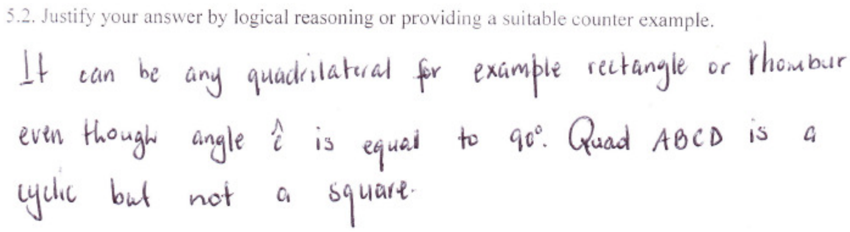


Figure 18: Response by teacher TTN

Inconclusive justification (5.2 = C4)

Teacher TDS justified his correct response in 5.1 differently from TTN, as shown in Figure 19. Teacher TDS claims that it can be any quadrilateral, for example, a rectangle, which is correct, but incorrectly claims that it could also be a rhombus which is impossible since a rhombus is not cyclic.

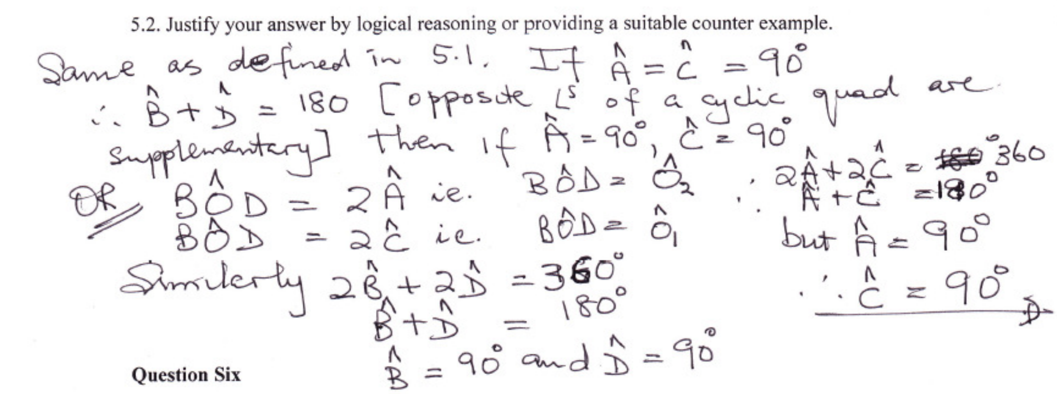

5.2. Justify your answer by logical reasoning or providing a suitable counter example.

It can be any quadrilateral for example rectangle or rhombur. even though angle $\hat{C}$ is equal to 90°. Quad ABCD is a cyclic but not a square.

Figure 19: Response by teacher TDS

Incorrect test answer and justification (5.1 = C4 and 5.2 = C4)

Teacher TDM's response is given in Figure 20. In contrast to teacher TTN, teacher TDM thought that it would be a square, although teacher TDM also arrived at the correct conclusion that $\angle B + \angle D = 180°$, but then incorrectly deduced that this implies that $\angle B = \angle D = 90°$. It is, however, not clear why teacher TDM thought this to be the case and it might have been interesting to pursue this aspect further during an interview.


5.2. Justify your answer by logical reasoning or providing a suitable counter example.

Same as defined in 5.1. If $\hat{A} = \hat{C} = 90°$ $\therefore \hat{B} + \hat{D} = 180$ [opposite ∠s of a cyclic quad are supplementary] then if $\hat{A} = 90°$, $\hat{C} = 90°$

OR $B\hat{O}D = 2\hat{A}$ ie. $B\hat{O}D = \hat{O}_2$ $\quad \therefore 2\hat{A} + 2\hat{C} = 360°$

$B\hat{O}D = 2\hat{C}$ ie. $B\hat{O}D = \hat{O}_1$ $\quad \hat{A} + \hat{C} = 180°$

but $\hat{A} = 90°$

Similarly $2\hat{B} + 2\hat{D} = 360°$ $\quad \therefore \hat{C} = 90°$

$\hat{B} + \hat{D} = 180°$

$\hat{B} = 90°$ and $\hat{D} = 90°$

Question Six

Figure 20: Response by teacher TDM

Teacher TWB also thought *ABCD* would be a square and tried to justify his response logically as shown in Figure 21. As with teacher TDM, teacher TWB seemed to assume that if two opposite angles in a cyclic quadrilateral are equal to 90° each, then, that will mean that the other two angles are 90° also, making all four sides equal.

5.2. Justify your answer by logical reasoning or providing a suitable counter example.

Because all chords would be equal and
the angles also equel to 90°, 4 side will be equel and
4 angles equel,

Figure 21: Response by teacher TWB

It is worrying that six of the ten teachers were initially unable to give a correct response, not realising that there was insufficient information to logically conclude that ABCD was a square. This shows a lack of propositional reasoning located at Van Hiele level 3. They seemed to either have difficulty in logically determining that insufficient information was given or an inability to simply provide a counter-example, such as a rectangle. According to the curriculum learners ought to learn how to refute false statements, and textbooks traditionally do not provide such experiences to learners. They only have to accept given statements as true or to prove they are true. Moreover, as this example shows, teachers themselves do not appear competent in refuting statements. This seems to be an area that might need attention in the mathematics education of future mathematics teachers, and for the professional development of current mathematics teachers.

A correct task-based interview answer and justification (5.1 = C1 and 5.2 = C1)

Interviewer : “What is the relationship of opposite angles of the cyclic quadrilateral?”

Teacher TDM: “They are supplementary or added up to 180°.”

Interviewer : “Does that made these angles equal to 90°.”

Teacher TDM: “Mmmhhh ....no, it depends on the size of each angle but these must add up to 180°.”

Teacher TDM was randomly asked to mention properties of the square and then he was asked to answer question 5. Teacher TDM responded as shown in Figure 22.

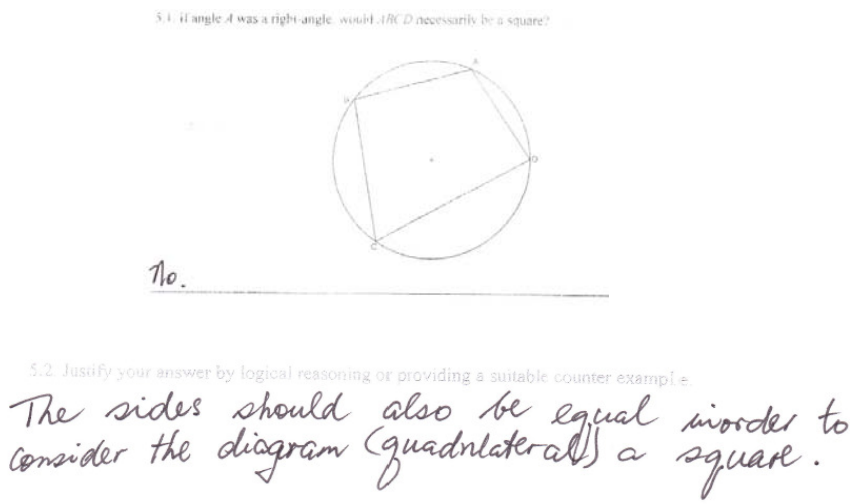


Figure 22: Response by teacher TDM

Inconclusive task-based interview justification (5.2 = C4)

Interviewer : “Why do you believe that a quadrilateral ABCD can be a rectangle or rhombus?”

Teacher TDS: “The rectangle and rhombus are cyclic.”

Interviewer : “What about a square, is it not cyclic?”

Teacher TDS: “It is, but all sides and angles must be equal. In quad ABCD, sides are not equal.”

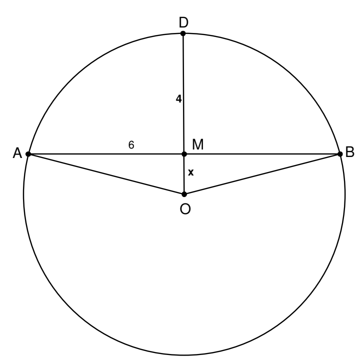
Interviewer : “What about the sides of the rhombus?”

Teacher TDS: "Oh yes, they are equal. The square has combined properties of the rectangle and rhombi (e.g. all angles and sides are equal). Therefore, it can either be a rectangle or a rhombus".

Teacher TDS clearly displayed a misconception that a rhombus is a cyclic quadrilateral; while in fact it is a circumscribed quadrilateral. Since the classification of quadrilaterals forms part of Grade 9 Mathematics curriculum, one would expect Grade 12 teachers to know this. A rhombus cannot be a cyclic quadrilateral as this would means its vertices would fit on a circle, which is impossible as its diagonals are not necessarily the same length. Though teacher TDS can recognise and name the properties of a rectangle and a rhombus, he could not recognise the relationships between these properties, this is characteristic of Van Hiele level 2. He could not discern which properties are necessary and which are sufficient to describe the object.

Question Six (BTC 3 and VHL 3)

AB is a chord of a circle centre O and 12 *cm* long. M is the midpoint of AB. MD ⊥ AB cuts ·circle centre O at D. Calculate the radius of the circle if MD = 4 *cm*.



This is a fairly standard type question in textbook exercises and a variation of this type of question is also used in AMESA Mathematical Challenges and the SA Mathematics Olympiad. This question is a traditional application of the theorem known as line through centre and midpoint or Midpoint chord. In order to answer this question, a teacher also needed to discern the Pythagoras Theorem stated as x^2 +

$y^2 = r^2$ and that radii are equal. The question posed required the radius of the circle and not the value of *x*. This circle has three radii namely *OA*, *OB* and *OD*; therefore a teacher can calculate any of the three but firstly, the *x*-value has to be determined.

A correct test answer (6 = C1)

The response of teacher TDM as shown in Figure 23 indicates that the teacher understood the question but could not reach the final product at first attempt. Teacher TDM applied correct methods of mathematics by applying Pythagoras Theorem and substitution, although the teacher ended up with two unknowns MD and *x*. The teacher recognised that MD is 4*cm* but partially substituted.

$$MD\,(2MD + x) = 36$$

$$MD\,(2\,(4) + x) = 36$$

$$MD\,(8 + x) = 36$$

However at a later stage, teacher TDM corrected and completed his reasoning as shown in Figure 23.

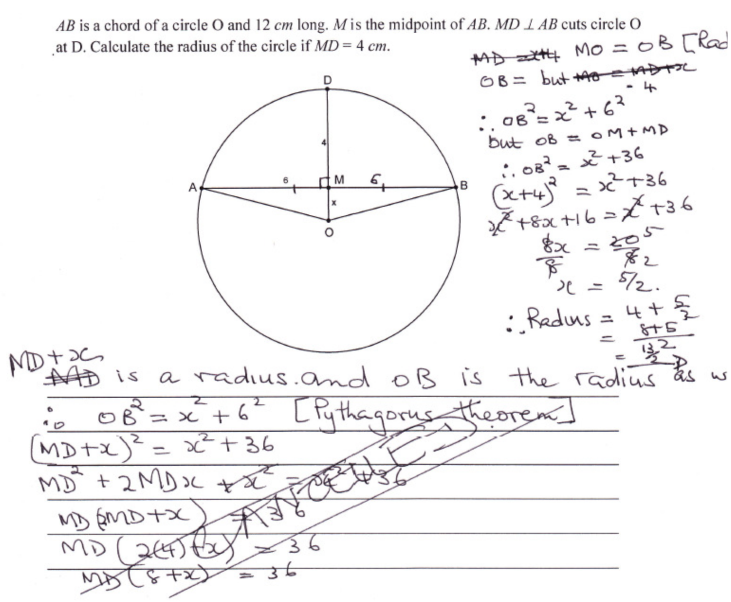


Figure 23: Response by teacher TDM

Inconclusive test answers with some attempt (6 = C4)

Some of the teachers, including teacher TSD, as shown in Figure 24, calculated the value of *x* and not the value of a radius as requested and that is why their responses were categorised as inconclusive response (C4).

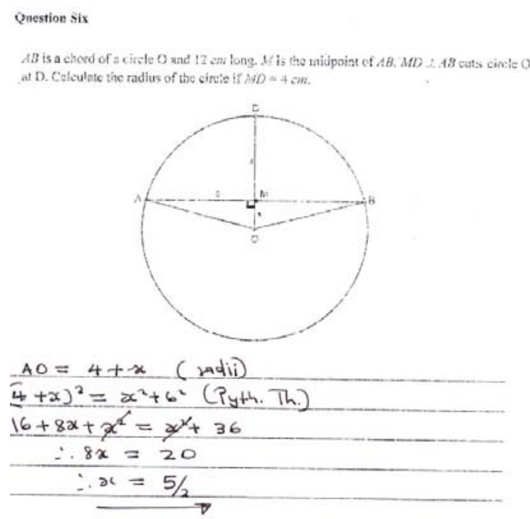
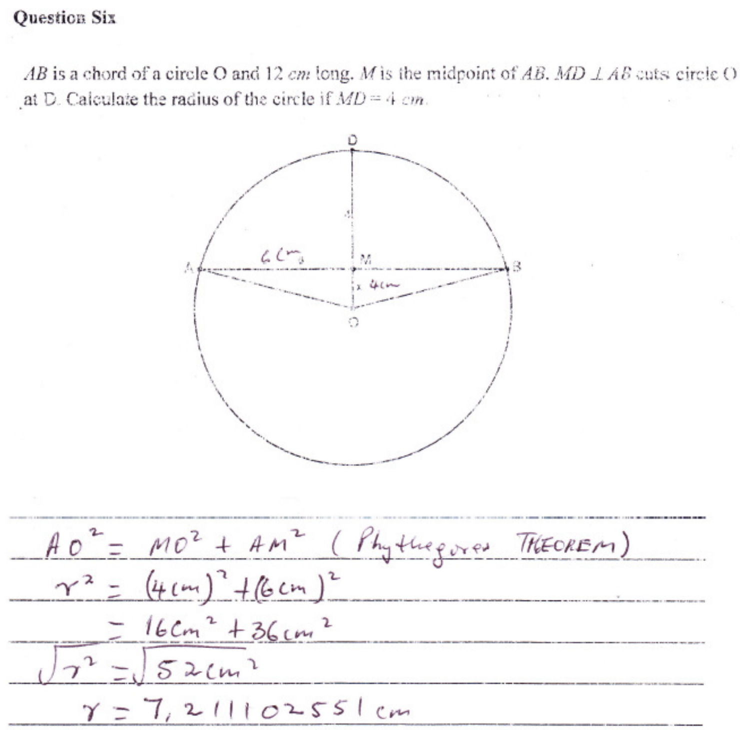


Figure 24: Response by teacher TSD

Incorrect test answer (6 = C4)

Teacher TWB achieved an incorrect response as shown in Figure 25. Apparently he assumed that the value of *x* was 4*cm* perhaps assuming that M was also a midpoint of OD, even though the diagram clearly shows that it is not and is not implied by the given data. Such a response raises a question, either in terms of the teacher's mastery of Van Hiele level 1 (visualisation), or that the teacher might have not read carefully and somehow confused OM and MD and perhaps thought that *x* was given as 4*cm*. However, without interviewing the teacher further about this error, it is difficult to ascertain the source of this particular error.



Question Six

AB is a chord of a circle O and 12 *cm* long. *M* is the midpoint of *AB*. $MD \perp AB$ cuts circle O at D. Calculate the radius of the circle if $MD = 4$ *cm*.

$AO^2 = MO^2 + AM^2$ (Phythegores THEOREM)

$r^2 = (4cm)^2 + (6cm)^2$

$= 16cm^2 + 36cm^2$

$\sqrt{r^2} = \sqrt{52cm^2}$

$r = 7,211102551\ cm$

Figure 25: Response by teacher TWB

A correct task-based interview answers (6 = C1)

During the task-based interview, the researcher asked teacher TSD, teacher TJM and teacher TWB to have another look at this question in order to try and determine their understanding and reasoning. Both teachers TSD and TJM achieved correct responses with valid reasons (C1) unlike in test question responses where they both achieved inconclusive responses (C4) for this question. Since these teachers were able to reach a conclusion during the task-based interview, there is most likely a possibility that they did not carefully read the question or fully understood it during the test.

Inconclusive task-based interview answer (6 = C4)

Teacher TWB still considered M as the midpoint for line OD and AB, therefore, OM = MD = 4 *cm*. Teacher TWB concluded by adding up OM and MD to give OD, the radius, which is equal to 8 *cm*.

Interviewer : "Why you think OM = 4 cm?"

Teacher TWB: "We are told that M is a midpoint, therefore, it cuts the lines into two equal pieces."

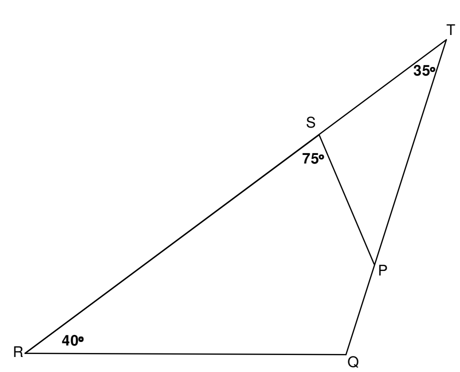
Interviewer : "Is M the midpoint of both AB and OD?"

Teacher TWB: "Mmmhhh, yes it is. Line AB and OD intersect each other at M, at 90°. Since these lines are perpendicular to each other, they are cut into two equal pieces."

Such a response from teacher TWB shows a misconception of intersecting lines. When lines intersect at a certain point, it does not mean that these lines are always divided into two equal parts. Seemingly, this teacher has not mastered either Van Hiele level 1 (visualisation) nor Van Hiele levels 2 or 3. This is supported by his statement stating that OM = OD. He has not created meaningful definitions or supplied informal arguments to justify his reasoning which is characteristic of Van Hiele level 3. This is a common error among learners; however, if it is also found among mathematics teachers, one begins to doubt the ability of such Grade 12 mathematic teachers' ability to teach circle geometry. With the impending return of Euclidean Geometry to the compulsory exams, this is cause for concern and perhaps an area of further large scale research among mathematics teachers.

Question Seven (BTC 3 and VHL 3)

Prove that *PQRS* is a cyclic quadrilateral in the following diagram.



This question required a simple proof of a cyclic quadrilateral and it is a very standard textbook or examination question. Teachers demonstrated an understanding of proving this question, and expressed their responses in diverse approaches.

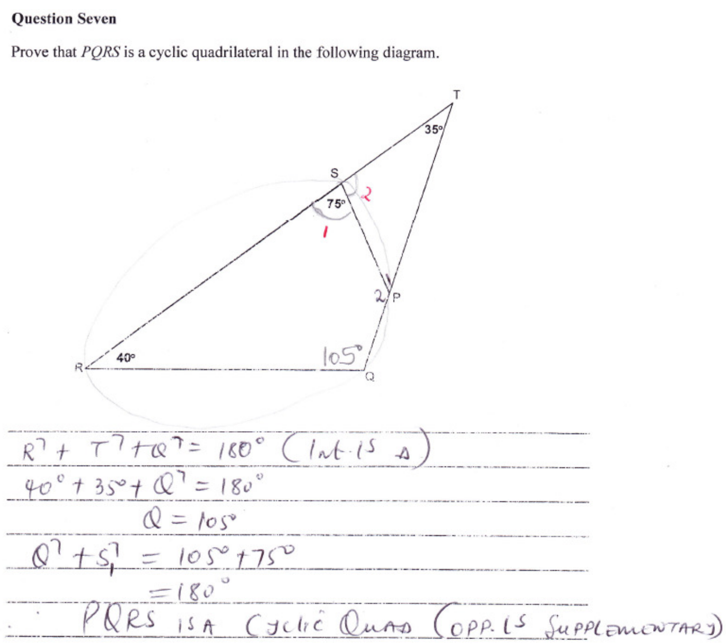
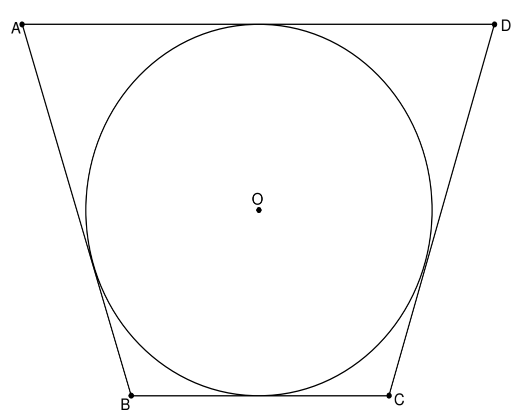


Figure 26: Response by teacher TJM

Question Eight (BTC 5 and VHL 4)

In the figure, O is the centre of the inscribed circle of quadrilateral ABCD. The perimeter of the quadrilateral is 25 *cm* and AD = 8 *cm*. Find the length of BC.



In order to answer this question, teachers needed to know the properties of a circle. Teachers firstly needed to identify that the sides of the quadrilateral were tangents to the circle. Then, they needed to consider the relationship between the tangent and the radius, namely that the tangent is perpendicular to a radius drawn from the centre to the tangent point. Finally, teachers needed to recall and apply the theorem which declares that tangents from the same point are equal. After considering this, they would then have been able to obtain the length of BC from the information provided.

This problem was chosen as it is not a routine textbook problem, to check whether the teachers would be able to solve a non-routine problem that they have not seen before, by analysing and applying their knowledge of circle geometry. One would expect good geometry teachers to be able to solve problems that they have not seen before.

Inconclusive test answer (8 = C4)

Teacher TMM and teacher TJM gave inconclusive response with some attempt made to solve question by constructing radii to tangents. Teacher TJM also indicated as shown in Figure 27 that tangents from the same point are equal, yet could not progress further with the question.

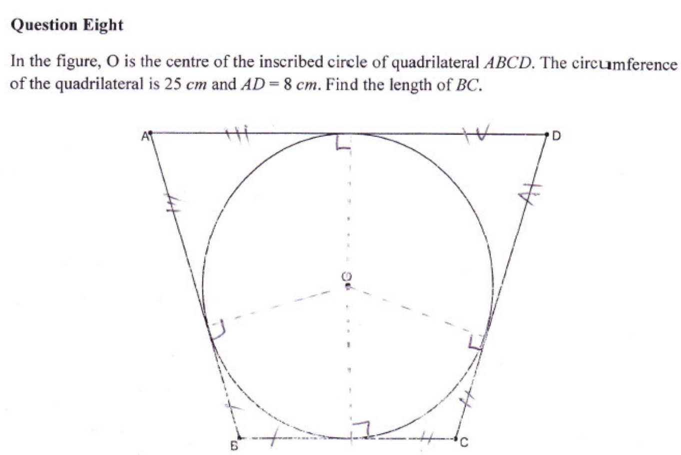


Figure 27: Response by teacher TJM

Correct test answer (8 = C1)

Only teacher TSD achieved a correct response with some valid reasons and working shown (when necessary) as shown in Figure 28, but made unwarranted assumptions in doing so. He managed to arrive at the correct answer, namely, that BC = 4 ½ *cm*, but incorrectly assumed from the symmetrical appearance of the diagram that P and R bisect AD and BC respectively, even though it was not given that this was a case.

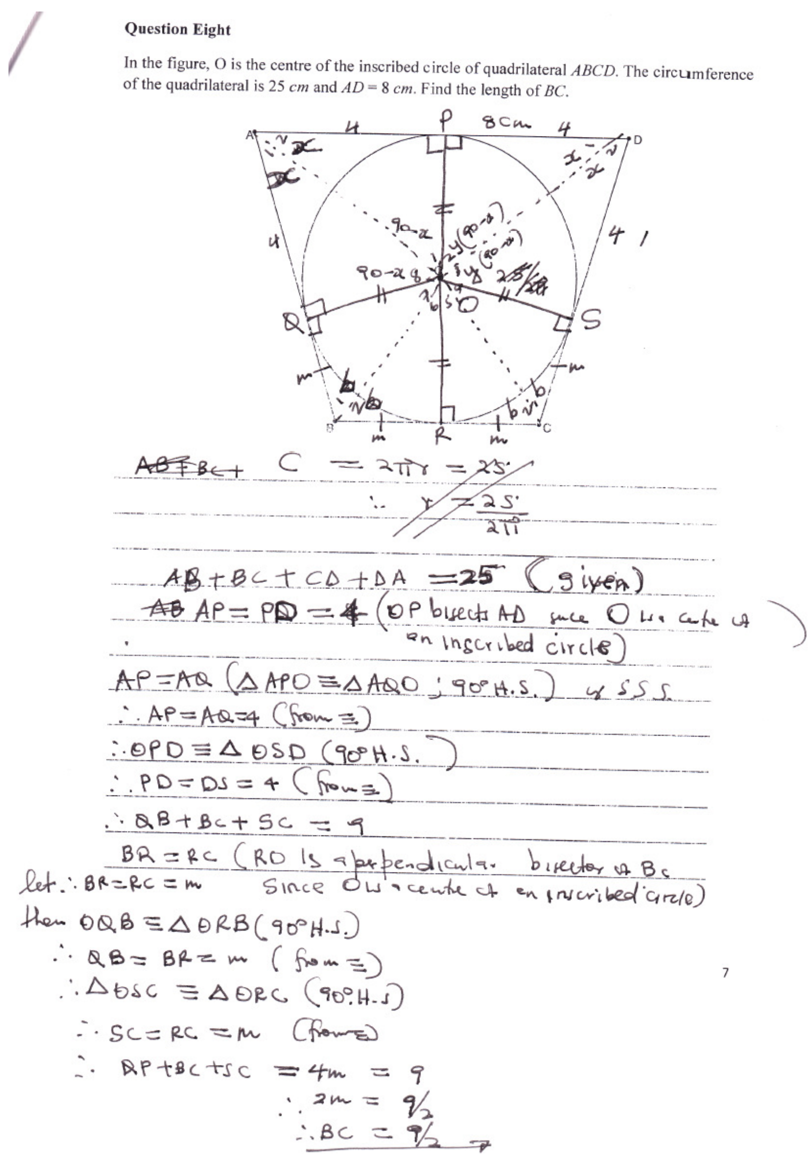


Figure 28: Response by teacher TSD

An inconclusive task-based interview answer (8 = C4)

During the task-based interview, the researcher firstly asked teachers to read question eight question at least twice before answering it, change the wording ‘circumference’ to ‘perimeter’. Most teachers did not even want to attempt to answer this question. “Oh no, lets skip this one, I have never seen it and could not even try to answer it. I left it unanswered in the test”. This shows a reluctance of teachers to attempt problems they have not seen before.

After reading the question, teacher TTN initially constructed radii, as shown in Figure 29.

Teacher TTN: “Oh, the perimeter of the circle is 25 cm. Since the radius is perpendicular to the tangent, it will then divide the tangent into two equal parts”.

Like teacher TSD she assumed or argued incorrectly that the perpendicular radii to the two parallel sides bisect them. This is probably attributable to the appearance of the sketch, or perhaps even confusion in thinking that a line from the centre of a circle perpendicular to a chord bisects it. Teacher TTN further indicated, in her sketch, that all sides are equal.

Interviewer : “Is the given perimeter of a circle or of a quadrilateral?”

Teacher TNN: “The perimeter is that of a circle and not a quadrilateral. Quadrilateral is only formed by lines or chords”.

Her mention of ‘chords’ here indicate her lack of distinction between a chord of a circle and a tangent and support the earlier conjectured explanation of why she might have thought the tangents were bisected by the radii.

Interviewer : “Does a square or rectangle have a perimeter?”

Teacher TNN: “No, it is only the circle that has a perimeter. Oh, but the opening statement mentions the perimeter of the quadrilateral. Wow, hay' Dhlams, angisazi. Asidlule kulesi (I

do not know now, let's continue)."

It is obvious that this teacher had an idea that the term 'perimeter" is restricted only to circles, which is in keeping with the classical meaning of the circumference as the perimeter of a circle or circular objects.

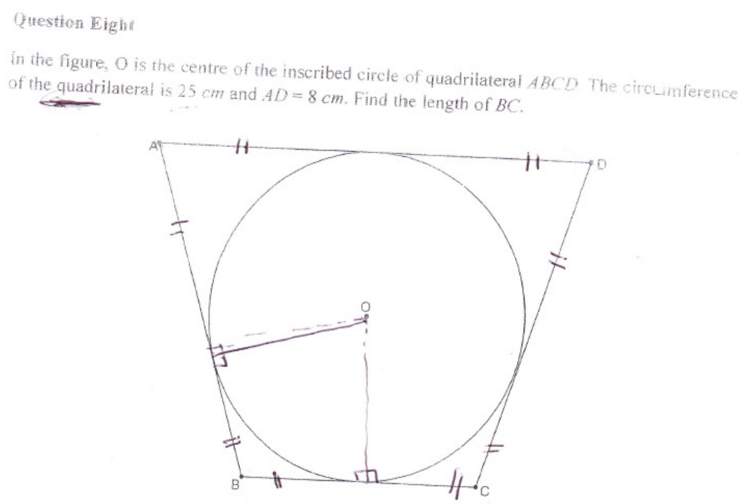


Figure 29: Response by teacher TTN

A correct task-based interview answer (8 = C1)

Teacher TJM indicated that the tangents to a circle from the same exterior point are equal, as shown in Figure 30.

Interviewer : "Why the quadrilaterals were marked as sides *a*, *b*, *c* and *d*?"

Teacher TJM: "This is because tangents from the same point are equal."

Interviewer : "Why did you done constructions?"

Teacher TJM: "The radii from circle centre O are perpendicular to the perimeter of the quadrilateral." He proceeded by saying "Mmmhhh, what's next now? Let me see. We are told that the perimeter of the circle is 25 cm, and then??? The question is asking for the length of BC on this quad. Ay! There is nothing more that I can do here."

Interviewer : “May you please explain if the given perimeter is that of a circle or of a quadrilateral?”

Teacher TJM: “Mmmmh... of a quad. Ok, let’s see what’s next now.”

Teacher TJM worked his way to the final answer, as shown in Figure 30.

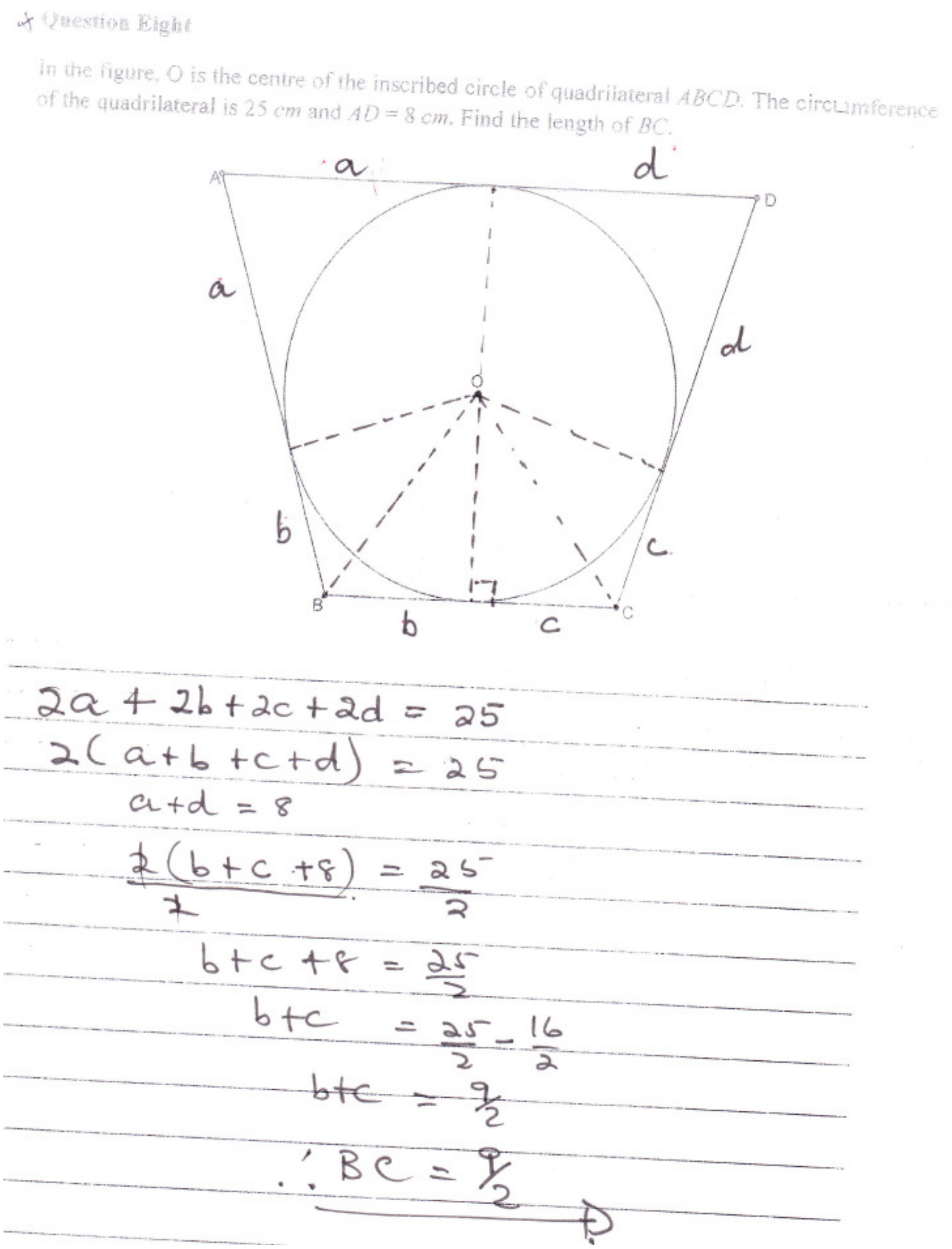


Figure 30: Response by teacher TJM

The poor response to this problem raised questions about the ability and competency of this sample of teachers if problems go a little bit beyond the textbook and of their performance on non-routine examination questions. One can only wonder, somewhat fearfully, what the situation is like among the total population of Grade 11 – 12 teachers in South Africa. This type of non-routine problem is typical of questions that have been asked at the Senior SA Mathematics Olympiad level, and if teachers themselves are not able to tackle and solve problems like these, how would they be able to assist and prepare their stronger learners for such challenges? As a country, South Africa cannot afford not to identify and develop our mathematical talent, as it often requires a good teacher to spark the interest of precocious learners, and ignite the required intellectual curiosity that transcends the narrow bounds of the textbook, the examinations and the curriculum.

Question Nine (BTC 3 & 4 and VHL 3)

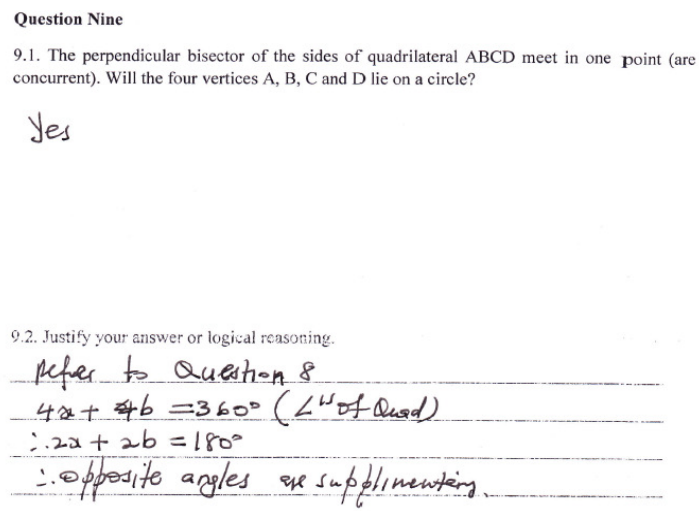
9.1. The perpendicular bisectors of the sides of quadrilateral ABCD meet in one point (are concurrent). Will the four vertices A, B, C and D lie on a circle?

9.2. Justify your answer or logical reasoning.

This question consists of two parts. To answer the first part, teachers were required to know that four vertices will lie on a circle, provided the perpendicular bisector of sides of the quadrilateral meet in one point. The second part needed teachers to justify their responses or to give logical reasoning to their answers. This entire question requires understanding of the meaning of a perpendicular bisector as the path of all points equidistant from the endpoints of a line segment. For example, if the perpendicular bisectors of a quadrilateral meet at point P, this will mean P is equidistant from all four vertices, and is therefore the centre of the circumscribed circle. This means one can put one leg of one's compass at P, and open the other leg to any one of the vertices, and because the distance from P to all the vertices is the same, one can then draw a circle through all four vertices.

A correct test answer, but incorrect justification (9.1 = C1 and 9.2 = C4)

Teacher TSD's answer is correct, but his justification is not sufficiently general, as he falsely assumed that ABCD (a figure of which was not given) was an isosceles trapezium such as shown in Figure 31.



Question Nine

9.1. The perpendicular bisector of the sides of quadrilateral ABCD meet in one point (are concurrent). Will the four vertices A, B, C and D lie on a circle?

Yes

9.2. Justify your answer or logical reasoning.

Refer to Question 8

$4a + 4b = 360°$ ($\angle$s of Quad)

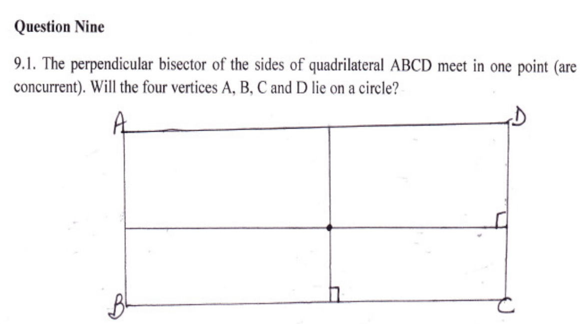
$\therefore 2a + 2b = 180°$

$\therefore$ opposite angles are supplimentary.

Figure 31: Response by teacher TSD

Non-interpretable test answer and no justification (9.1 = C4 and 9.2 = C5)

Some teachers stated that these four vertices will lie on a circle, but could not provide a valid justification or give logical reasoning to this. Teacher TDM just drew a rectangle as shown in Figure 32, but did not give any response to either 9.1 or 9.2, and was therefore coded as non-interpretable.



Question Nine

9.1. The perpendicular bisector of the sides of quadrilateral ABCD meet in one point (are concurrent). Will the four vertices A, B, C and D lie on a circle?

Figure 32: Response by teacher TDM

The poor response of teachers in this question indicates that the textbooks and perhaps the curriculum as well as their own past mathematical education, have not given attention to the meaning of a perpendicular bisector in relation to the important and fundamental concept of equidistance. These teachers seemed to be unaware that the basic result is that the perpendicular bisectors of a polygon are concurrent (at circumcentre of the polygon) if, and only if the polygon is cyclic. If the polygon is cyclic the general proof can be written as: the circumcentre is equidistant from all the vertices (radii are equal), but each perpendicular bisector is the locus of all the points equidistant from the endpoints (vertices) of each side. Therefore each perpendicular bisector must pass through the circumcentre. The converse result follows in the same way if the perpendicular bisectors are concurrent, then the polygon is cyclic.

A correct task-based interview answer, but incorrect justification (9.1 = C1 and 9.2 = C4)

Teacher TDM did not give any test question responses to either 9.1 or 9.2 and was therefore coded as non-interpretable, as shown in Figure 33. During the individual, task-based interview, the researcher gave teacher TDM the opportunity to re-answer this question. Though teacher TDM gave a correct response, he incorrectly assumed that ABCD was a rectangle (which was not given), as shown in Figure 33. He clearly did not understand the problem.

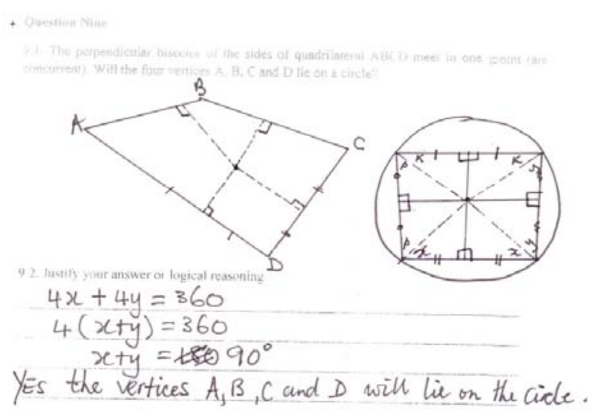


Figure 33: Response by teacher TDM

Initially, teacher TTN read the question and underlined the two consecutive terms “perpendicular” and “bisector”. Though she answered correctly to 9.1, she really had no understanding of why this was the case. Like teacher TDM, it seems she also incorrectly assumed that ABCD was a rectangle, as shown in Figure 34. It does appear from her comment “perpendicular bisectors will cut the lines and leave two parts equal” that she understands that a perpendicular bisector of a side bisects it, but not its meaning as a path of points, equidistant from the vertices of a side.

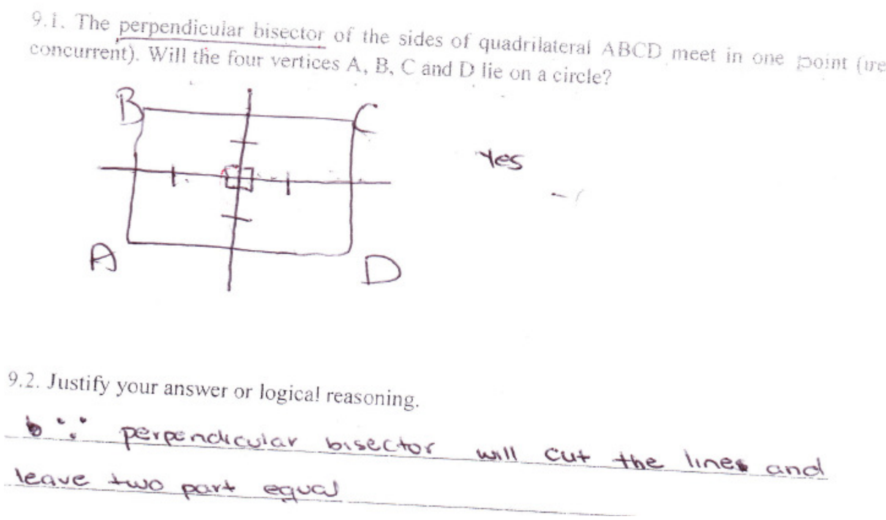


Figure 34: Response by teacher TTN

Question Ten (BTC 3 and VHL 3 & 4)

10. Square, rhombus, rectangle, parallelogram, kite or trapezium.

10.1. Which of the above quadrilaterals are always cyclic? Please explain your reasoning.

10.2. Which of the above one quadrilaterals always have an inscribed circle? Please explain your reasoning.

Question 10 consists of two questions. The first part required teachers to identify the cyclic quadrilaterals and substantiate their response of choice. The second part of this question required teachers to identify quadrilateral with an inscribed circle (circumscribed) and validate their choice of responses using logical reasoning. To master these questions, teachers needed to distinguish between inscribed circle (circumscribed) and cyclic quadrilaterals.

The perpendicular bisector is the locus of all points equidistant from the vertices of a line segment, and an angle bisector is the locus (path) of all points equidistant from the rays that form an angle. So in order for a polygon to have an inscribed circle, it has to have a point equidistant from the sides, so if the angle bisectors are concurrent; then there is an equidistant point from the sides. The angle bisectors are obviously concurrent from symmetry for a kite, rhombus and a square; hence they have inscribed circles.

Correct test answers and justifications (10.1 = C1)

Only teachers TSD, TMB and TJM were able to accomplish a conclusive response in identifying that of these quadrilaterals only squares and rectangles are always cyclic quadrilaterals.

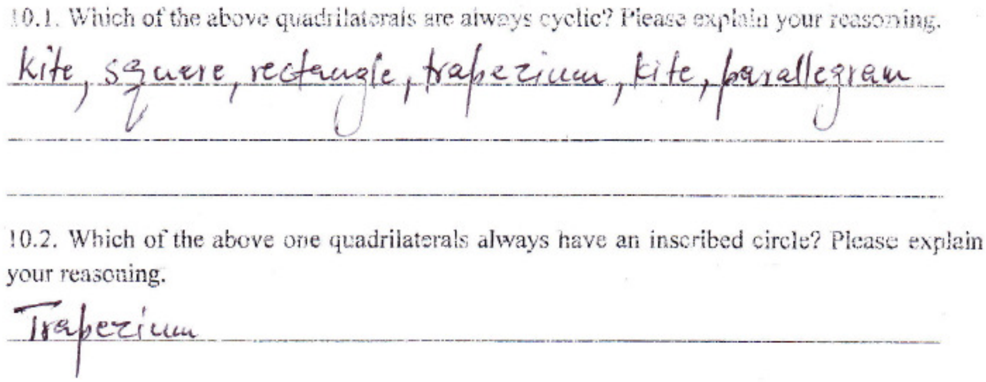
One correct test answer, but no justification (10.1 = C3 and 10.2 = C4)

Teacher TDM achieved a correct response with no valid reasons (C3) in 10.1. In other words, he was able to identify the quadrilaterals that are always cyclic, but did not provide a justification to this. However, teacher TDM achieved an inconclusive response with no valid reasons (C4), in 10.2, by including a trapezium as an inscribed circle quadrilateral, which is incorrect.

Inconclusive test answers and justification (10.1 = C4 and 10.2 = C4)

Teacher TMM achieved an inconclusive response with an attempt on question 10.1 and an incorrect response to 10.2. This teacher thought all of the given quadrilaterals were cyclic and only the trapezium was an inscribed quadrilateral but gave no reasons for these responses as shown in Figure 35. Also, teacher TDM response was classified as C4 also included a trapezium as an inscribed circle quadrilateral which is incorrect.

10.1. Which of the above quadrilaterals are always cyclic? Please explain your reasoning.


kite, square, rectangle, trapezium, kite, parallegram

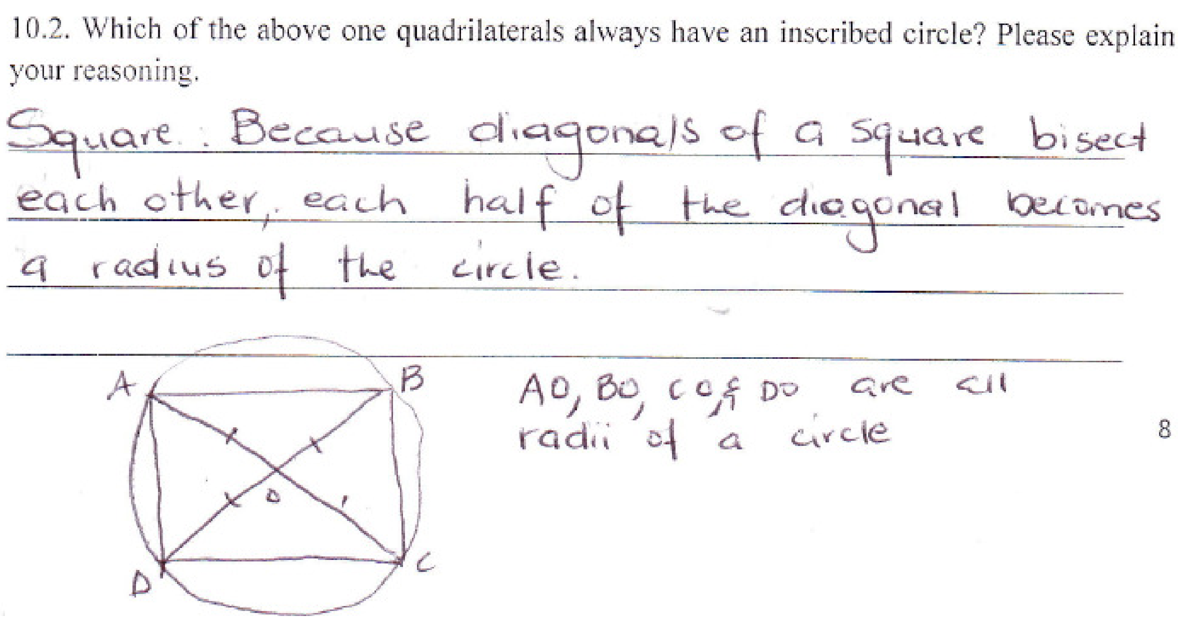
10.2. Which of the above one quadrilaterals always have an inscribed circle? Please explain your reasoning.

Trapezium

Figure 35: Response by teacher TMM

Inconclusive test answer and incorrect justification (10.2 = C4)

Out of all six given quadrilaterals, six teachers who achieved C4, thought the square is the only quadrilateral with an inscribed circle. Out of these five teachers, only two teachers (teacher TDS and teacher TSD) tried to justify why a square has an inscribed circle. However, the justification of teacher TDS was completely incorrect as he was referring to the circumscribed circle, not the inscribed circle.

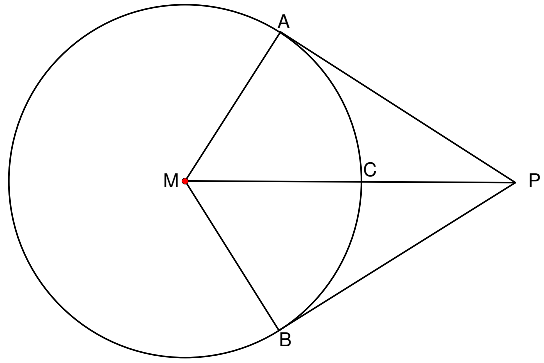

10.2. Which of the above one quadrilaterals always have an inscribed circle? Please explain your reasoning.

Square. Because diagonals of a square bisect each other, each half of the diagonal becomes a radius of the circle.

Figure 36: Response by teacher TDS

The poor responses of teachers in this question indicate that the textbooks and perhaps the curriculum as well, do not give enough attention to the meaning of quadrilaterals that are always cyclic or inscribed circle. The researcher has concluded that teachers do not pay much attention to the previous grades' curriculum, since the classification of quadrilaterals is taught in Grade 9. Quadrilaterals that are always cyclic are: square, rectangle, isosceles trapezium and cyclic quadrilateral. Quadrilaterals that are always inscribed circles are: square, rhombus, kite and circumscribed quadrilateral. This performance is exposing a gap in teachers' knowledge, which they ought to know.

Question Eleven (BTC 4 and VHL 3)



If PA and PB are tangents to circle M, will the kite PAMB be cyclic? Justify your answer by saying why or why not.

Since radii are constructed to tangents, teachers needed to recall the theorem which declares that the tangent is perpendicular to the radius. Therefore, application of this theorem to the given diagram would validate that, $\angle MAP = \angle MBP = 90°$, thus these angles are supplementary. Hence, from the theorem "if opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic", it follows that PAMB is a cyclic. In the NATED 550 curricula, this theorem was for Higher Grade learners only.

A correct test answer (11 = C1)

Out of the ten teachers only eight teachers utilised this theorem to answer this question to obtain correct responses. Teacher TTN's response in Figure 37 is a typical example of a correct response that was given.

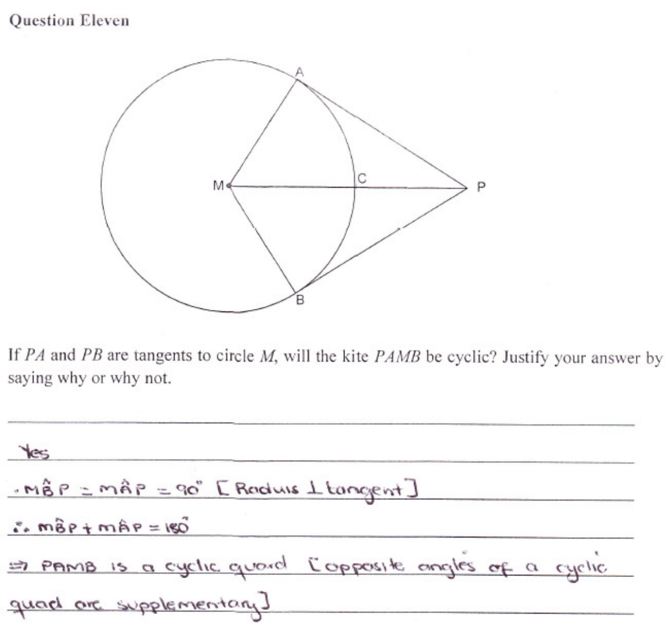


Figure 37: Response by teacher TTN

A correct task-based interview answer (11 = C1)

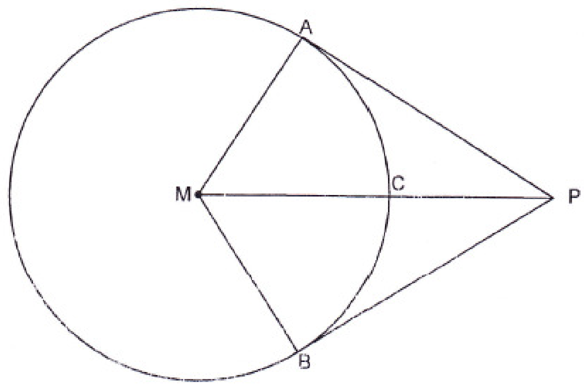
Teacher TLD and teacher TWB did not get this question correct in the test. In order to determine their understanding or difficulty with the problem, teacher TLD and teacher TWB were interviewed.

Interviewer : "What is the relationship between the radius and tangent?"

Teacher TWB: "Wait, let me see ... AM = MB because both of these are radii, then AP and BP are tangents, isn't so? Oh, you said relationship? Radius is perpendicular to the tangent, therefore, $\angle MAP = \angle MBP = 90°$. ... got it now ... These angles are supplementary, therefore, *PAMB* will be cyclic."

Teacher TWB and teacher TLD demonstrated an understanding of this question.

Question Eleven

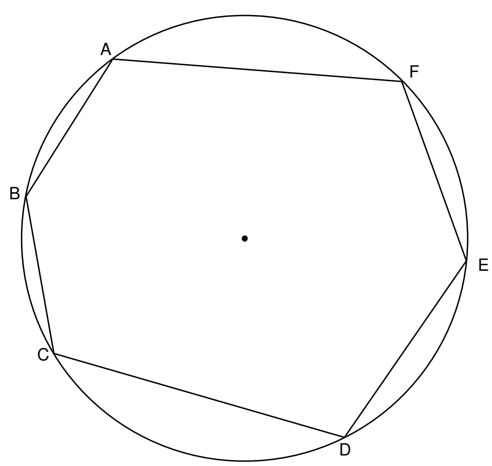


If *PA* and *PB* are tangents to circle *M*, will the kite *PAMB* be cyclic? Justify your answer by saying why or why not.

Yes, because $\hat{A} = \hat{B}$ [both = 90°] meaning that opposite angles are supplementary

Figure 38: Response by teacher TWB

Question Twelve (BTC 5 and VHL 4)



Prove that $\angle A + \angle C + \angle E = \angle B + \angle D + \angle F = 360°$

This problem was chosen as it is not a routine textbook problem, to check whether teachers would be able to solve a non-routine problem that they have not seen before by analysing and applying their knowledge of circle geometry. One would expect good geometry teachers to be able to solve problems that they have not seen before, irrespective of whether they teach Mathematics Paper 3 or not. This non-routine problem could be solved long-windedly as TSD did as shown in Figure 40, or more elegantly and economically by drawing a main diagonal, and then just applying their knowledge of cyclic quadrilaterals.

This question required teachers to be at a level of knowing all the Bloom's Taxonomy categories and to master all Van Hiele levels used in this study to obtain a correct response with a valid reason (C1). Teachers also needed to construct a structure or a pattern from diverse elements, and put parts together to form a whole with emphasis on creating a new meaning or structure. At this level, teachers started to develop longer sequences of statements and began to understand the significance of deduction and the role of axioms. The diagram provided is a cyclic hexagon of which the two sums of alternate angles have to be proven to be equal to 360 degrees.

An inconclusive test answer (12 = C4)

Teacher TMM constructed radii, which did not assist in attaining a conclusive response to the question, except showing that the angle at a centre summed up to 360° revolution.

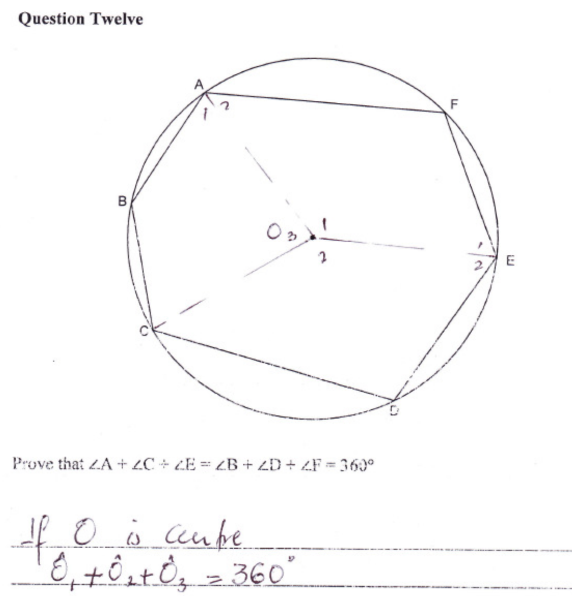
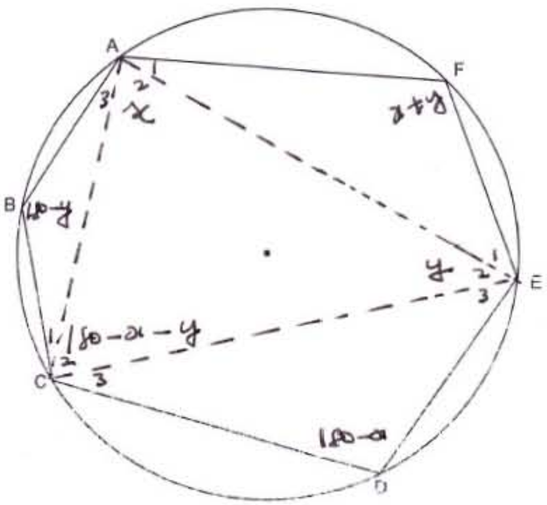


Figure 39: Response by teacher TMM

A correct test answer (12 = C1)

Only teacher TSD demonstrated an understanding in attaining a conclusive response to this question. This is the only participant who achieved a correct response with valid reasons (C1) in this question.

Question Twelve



Prove that $\angle A + \angle C + \angle E = \angle B + \angle D + \angle F = 360°$

let $\hat{A}_1 = x$

then $\hat{D} = 180° - x$ (opp. ∠'s of cyclic Quad)

let $\hat{E}_2 = y$

then $\hat{B} = 180° - y$ (opp. ∠'s of cyclic Quad)

~~let~~ ∴ $\hat{C}_2 = 180° - x - y$ (∠'s of $\triangle ACE$)

∴ $\hat{F} = x + y$ (opp. ∠'s of cyclic Quad)

∴ $\hat{A}_1 + \hat{E}_1 = 180° - x - y$ (∠'s of △)

$\hat{A}_3 + \hat{C}_1 = y$ (∠'s of △)

∴ $\hat{C}_3 + \hat{E}_3 = x$ (∠'s of $\triangle CDE$)

∴ $\hat{B} + \hat{D} + \hat{F} = 180° - y + 180° - x + x + y = 360°$

∴ $(\hat{A}_1 + \hat{A}_2 + \hat{A}_3) + (\hat{C}_1 + \hat{C}_2 + \hat{C}_3) + (\hat{E}_1 + \hat{E}_2 + \hat{E}_3) = \hat{A}_1 + x + \hat{A}_3 + \hat{C}_1 + 180 - x - y + \hat{C}_3 + \hat{E}_1 + y + \hat{E}_3$

$= (\hat{A}_1 + \hat{E}_1) + (\hat{C}_1 + \hat{A}_3) + (\hat{E}_3 + \hat{C}_3) + 180° - x - y + x + 180 + y$

$= (180 - x - y) + y + (x) + 180°$

$= 360°$

∴ $\hat{A} + \hat{C} + \hat{E} = \hat{B} + \hat{D} + \hat{F} = 360°$

Figure 40: Response by teacher TSD

Inconclusive task-based interview answer (12 = C4)

The majority of teachers did not demonstrate an understanding of this question even though they were given an oral hint to draw a diagonal to divide the hexagon into two cyclic quadrilaterals. Most teachers constructed lines that did not assist them in answering this question. None of the nine interviewed teachers interviewed afterwards achieved a correct response (C1) for this question, as shown in Figure 41.

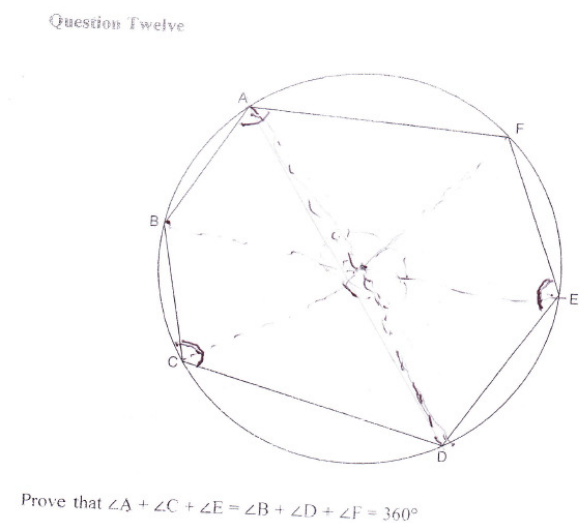


Figure 41: Response by teacher TMB

The researcher of this study observed that most teachers performed poorly or did not answer non-textbook and non-routine examination questions. The ability and competence of this study population raised questions since they are experienced Grade 12 mathematics teachers.

4.4. Conclusion

This chapter depicted the knowledge possessed by participants who took part in test and task-based interview. The study revealed that most teachers could not demonstrate an understanding of Bloom's Taxonomy category 5 and the Van Hiele level 4 in responding to questions. The next chapter summarises the findings and link them to previous studies that were discussed in Chapter two.

CHAPTER FIVE

DISCUSSION

5.1. Introduction

The researcher presented and discussed the data collected from both test and task-based interview in the previous chapter. In this chapter, the researcher will present an interpretation of the data collected. However, rather than making interpretations for each data collected separately, the researcher organised the information thematically and also made use of the framework on the principle of pedagogical content knowledge (PCK), as the data showed some overlapping in the responses by the teachers, Bloom's Taxonomy of categories and the Van Hiele theory (Shulman, 1986; Forehand, 2010; De Villiers, 2010).

5.2. Pedagogical Content Knowledge

As discussed in Chapter 2, SMK refers to a deep understanding of both the subject matter and the pedagogical knowledge of teaching and learning. The data revealed that the majority of teachers do not process SMK of Bloom's Taxonomy category 3 through 5 and the Van Hiele level 3 through 4 to understand circle geometry, predominantly those that are not in typical textbooks yet still within the parameters of the school curriculum. In some cases, this challenge could have a direct impact on learners' academic performance in Euclidean Geometry (Young, 2006).

Shulman (2004) argues that teachers ought to be in possession of a deep SMK; therefore, teachers ought to be above learners' levels in terms of Bloom and the Van Hiele theory. Teachers therefore need to understand the central concepts of circle geometry and understand how best to present and communicate specific concepts. Similarly, Freire (1989) as cited in Howard & Aleman (2008) points out the need for teachers to be knowledgeable in their field and to apply and implement a challenging curriculum. The findings are that teachers lack this type of knowledge, especially in high order questions.

Kahan et al (2003) concluded that learners would learn more mathematics if their educators deeply understand mathematics and that pedagogical knowledge in the subject area alone does not suffice for excellent teaching. The content of pedagogical content knowledge is content-specific and covers both understanding the mathematics as well as the pedagogical issues involved with the teaching and learning of the subject matter. It therefore goes further than knowledge and understanding of mathematics, and even a mathematician might not posses PCK if s/he lacks with regard to the pedagogical dimension, as shown in Figure 42 (An et al., 2004, p. 147)

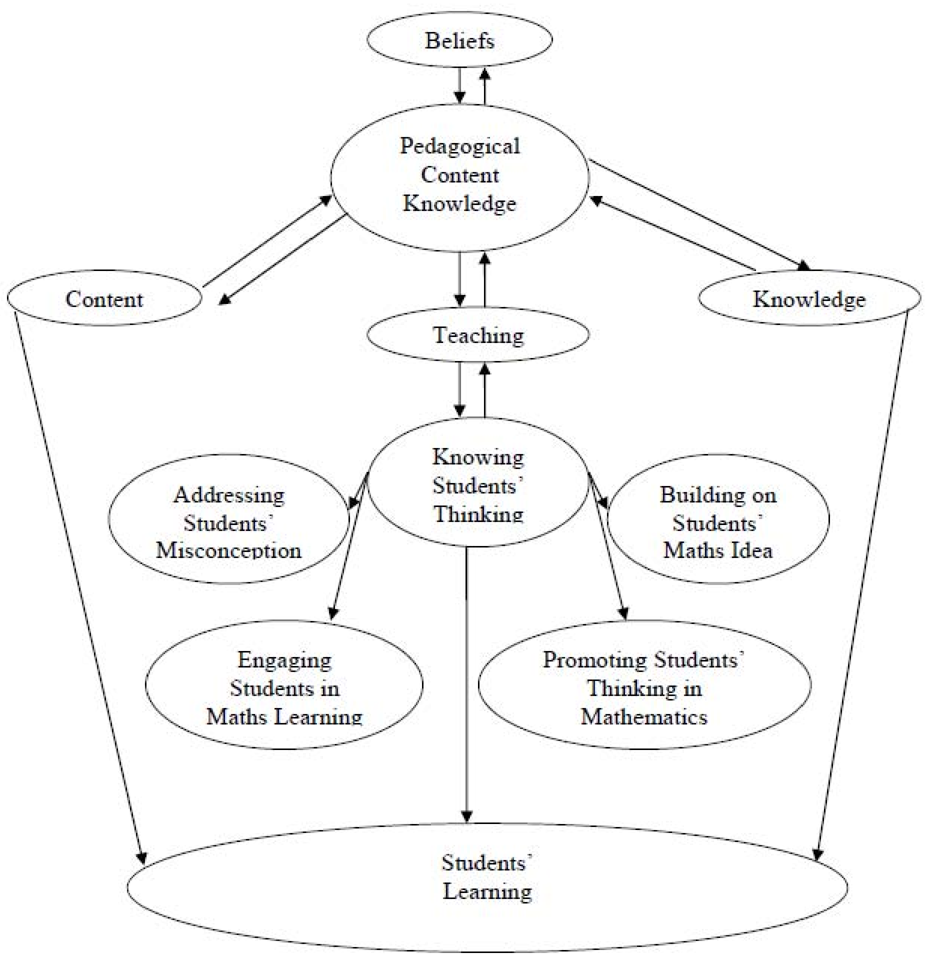


Figure 42: A system of Pedagogical Content Knowledge

5.3. Teacher Profiles of Bloom and Van Hiele Levels of Achievement

The newly revised Bloom's Taxonomy categories and Van Hiele levels of understanding geometry theory were used to investigate Grade 12 mathematics teachers' understanding of Euclidean Geometry with specific reference to circle geometry. The researcher used Bloom's Taxonomy to categorise test and task-based interview questions.

The questions for this study were organised within the first five categories as stated in Chapter 2, however, they required "brain power" and logical reasoning or justification. Bloom's Taxonomy categories illustrate how thinking can progress from being random to empirically structured, and finally to being coherent and deductive. Monaghan (2002) assumes that most learners start secondary school geometry module thinking at the first or second Van Hiele level. One would expect teachers' level of understanding to be at least one level above those of learners.

Table 11: Number of testing items according to Bloom's Taxonomy categories

Category	Category 1	Category 2	Category 3	Category 4	Category 5
Number of Items	2	3	6	2	3

Table 12: Number of testing items according to Van Hiele levels

Levels	Level 1	Level 2	Level 3	Level 4
Number of Items	3	3	7	3

According to the researcher, for the teacher to demonstrate understanding of each category or level, one should obtain 60% or more in that specific category or level. For an example, in Bloom's Taxonomy category 3, the teacher should obtain five or more correct testing items to demonstrate an understanding of this category. The below table represents each teacher's performance based on researcher's grading.

Table 13: Teachers understanding of Bloom's Taxonomy category and Van Hiele level

Teachers	Bloom's Taxonomy Category (BTC) or Van Hiele Level (VHL)								
	BTC 1	**BTC 2**	**BTC 3**	**BTC 4**	**BTC 5**	**VHL 1**	**VHL 2**	**VHL 3**	**VHL 4**
TJM	√	√	√	x	x	√	√	√	x
TMM	√	x	x	√	x	x	√	x	x
TDM	√	√	x	x	x	√	x	x	x
TSD	√	√	√	√	x	√	√	√	√
TMB	x	x	x	x	x	x	x	x	x
TLD	x	x	x	x	x	x	x	x	x
TWB	√	√	x	x	x	√	√	x	x
TTB	x	x	√	x	x	x	√	√	x
TTN	√	√	√	x	x	√	√	√	x
TDS	√	√	√	x	x	√	√	√	x

Table 13 was further represented in Figures 43 and 44, the highest Bloom's Taxonomy categories and Highest Van Hiele levels that teachers achieved. In Figure 43, Bloom's Taxonomy category is represented as BTC, while in Figure 44, Van Hiele level is represented as VHL. The researcher introduced BTC 0 and VHL 0, for teachers that could not obtain 60% and above in each category or level used in this study.

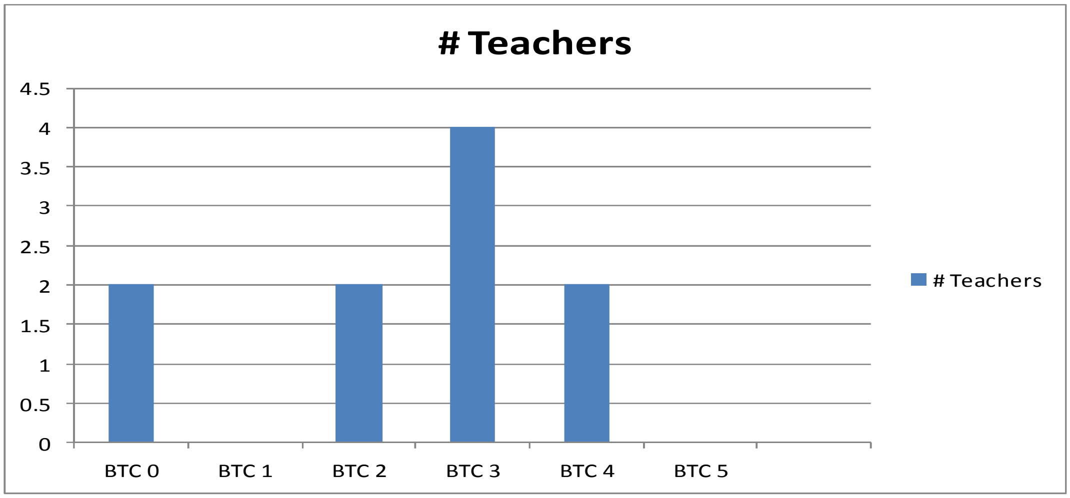


Figure 43: Teachers' highest Bloom's Taxonomy category

According to Figure 43, two teachers could not obtain even lower category (BTC 1). This is worrying since these teachers have an experience of ten years or more and teaching Grade 12. However, every teacher is expected to use Bloom in designing assessment tasks (DoE, 2003), meaning they ought to be familiar with the demands of the categories. One would expect a Grade 12 learner at least to master Bloom's Taxonomy category 3 questions, and then a teacher should be above, although Bloom is not hierarchically structured. Two teachers achieved category 2 and their achievement is hierarchic (BTC 1-2).

Four out of ten teachers achieved category 3, including one teacher that did not obtain the previous categories. This may have been the result of category 3 (eight items) having more testing items than other categories. With few items findings cannot be generalised, except to be directly drawn to the sample. Two teachers achieved category 4, including one teacher that is an outlier. The outlier did not obtain categories 2 and 3. None of the participants in this sample achieved category 5; the results may be that two out of the three testing items were non-routine problems or non-examination questions. Only two teachers were outliers and eight teachers followed a hierarchy to achieve the highest Bloom's Taxonomy category. In summary, eight teachers demonstrated an understanding of BTC 0 through BTC 3, two in BTC 4 and none in BTC 5.

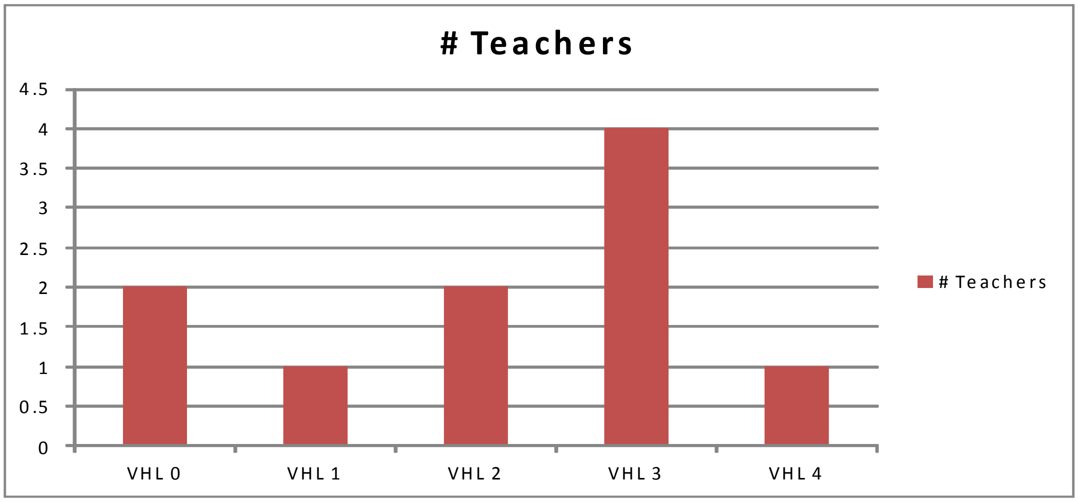


Figure 44: Teachers' Highest Van Hiele Level

The Van Hiele theory deals particularly with geometric thoughts as it progress over numerous levels of complexity underpinned by school curricular. Two out of ten teachers achieved VHL 0 meaning they could not achieve even the lower level. It is alarming for a Grade 12 mathematics teachers to be unable obtain 60% or above in visualising testing items. One out of eight teachers achieved level 1, which is a fundamental level of Van Hiele theory.

Two teachers achieved level 2, including one outlier. Testing items for Van Hiele level 2 were more based on properties of geometric shapes and logical reasoning of the responses, but it only involved three out of nineteen items. Four teachers achieved level 3, including an outlier that did not obtain level 1. This may be the result of level 3 having nine out of 19 testing items and including secondary school curriculum problems. An important hypothesis of the Van Hiele theory is that the levels form a hierarchy (De Villiers, 2010); one cannot be at a specific level without having passed through the preceding levels. Two out of the ten are outliers, in terms of the theory. One teacher achieved level 4 hierarchical and he is currently teaching Mathematics Paper 3. Two out of three items were non-routine, typical questions asked in Senior SA Mathematics Olympiad level. Our teachers are experienced, however they could only attempt problems within textbooks or the curriculum of their grades.

5.4. Research Questions

What are the general Bloom's Taxonomy learning domains and Van Hiele levels of understanding of Euclidean Geometry of this sample of Grade 12 mathematics teachers?

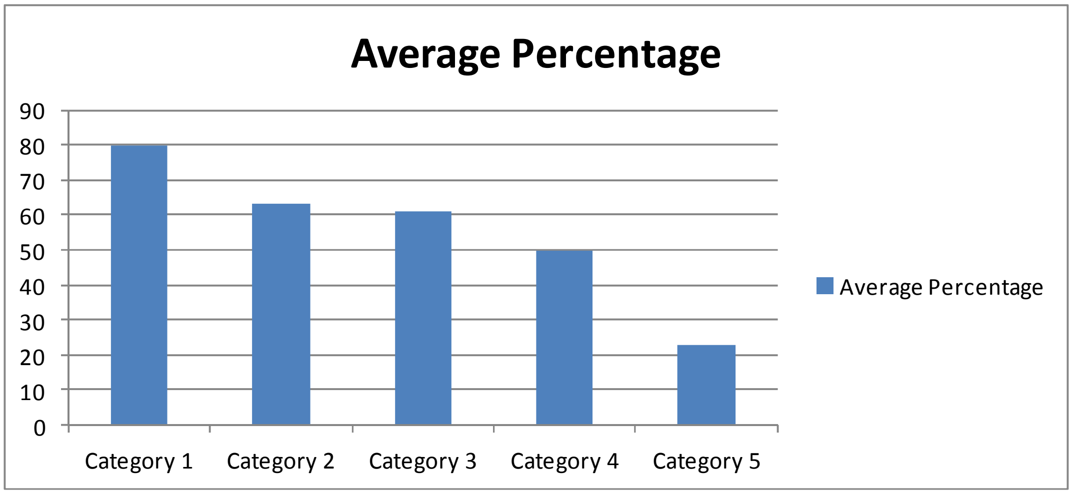


Figure 45: Average percentage of teachers that achieved C1 per Bloom's Taxonomy category in test

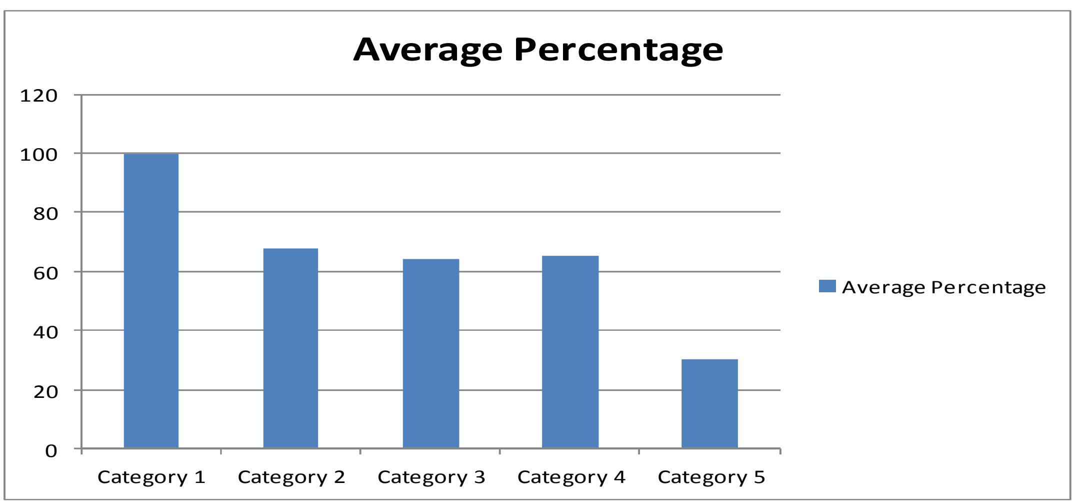


Figure 46: Average percentage of teachers that achieved C1 per Bloom's Taxonomy category in the task-based interview

Teachers demonstrated a reasonably in-depth understanding of Bloom's Taxonomy categories 1 through 3. This was supported by the data presented in Tables 6 and 7 for ten out of 14 items testing Bloom's Taxonomy category 1 or 2 or 3, teachers got 60% or above. However, for three out of five items testing Bloom's Taxonomy category 4 or 5, teachers got below 50%. During the task-based interview, all teachers correctly answered Bloom's Taxonomy category 1 items; this suggests that they developed some further understanding from reflection on their test responses.

Question four was one of the five items testing Bloom's Taxonomy category 2; only one out of ten teachers achieved a conclusive response in this question. This question required a presentation of the theorem known as "angle at centre is twice angle at circumference". Teachers' demonstrated limited knowledge, as shown in their different sketches in representing the stated theorem. When responding to this question in the test, five teachers drew only one sketch, namely the traditional representation of the theorem from mathematics textbooks. This is likely to limit their learners should other configurations be asked in Grade 12 external examination. During the task-based interview, two out of five teachers were, then, able to sketch two out of three acceptable/relevant diagrams.

There was a noticeable decrease in the average percentage of teachers who achieved conclusive responses, based on Bloom's Taxonomy category 1 through category 5. The purpose of the task-based interview was to better understand their reasoning and understanding of geometry. After the task-based interview two out of eight teachers still believed that ABCD will be a square in question 5. In their argument to support, they thought that if angles are supplementary then that will mean each angle is equal to 90 degrees. However, irrespective of angles, participants needed to visualise the diagram (VHL 1) and remember square properties (BTC 1) in order to answer this question on Bloom's Taxonomy category 5 and Van Hiele level 3. One teacher incorrectly argued that ABCD could not be a square but could be any quadrilateral such as a rhombus, which is impossible since a rhombus is not cyclic.

Three out of eight teachers achieved conclusive responses in 10.1, but only two of the three were able to give logical reasoning for this. And in question 10.2, all six teachers who attempted this question achieved inconclusive responses. This shows that teachers do not pay attention to the curriculum of the previous grades, since content related to question 10 is taught in Grade 9. Therefore, the general Bloom's Taxonomy learning domain of this sample of Grade 12 mathematics teachers is at lower (remembering and understanding) and middle (applying and analysing) orders.

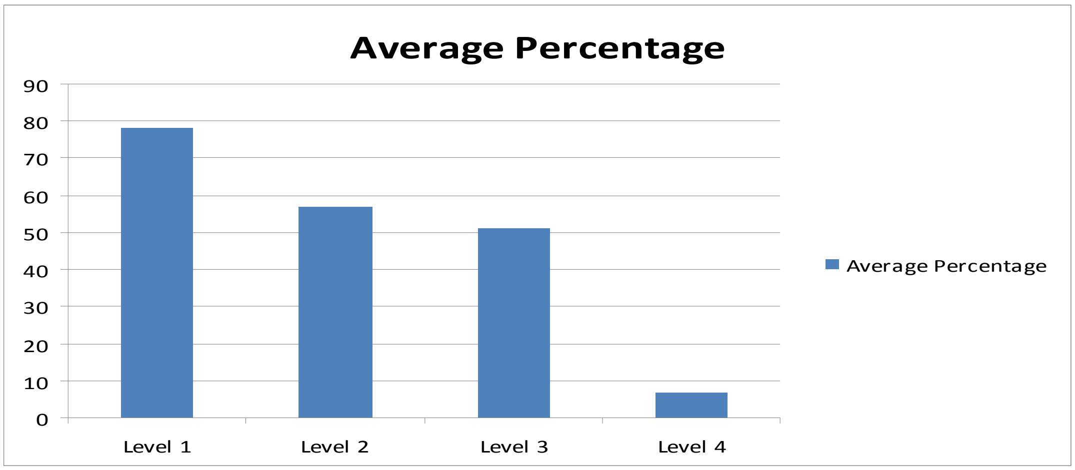


Figure 47: Average percentage of teachers that obtained C1 per Van Hiele level in test

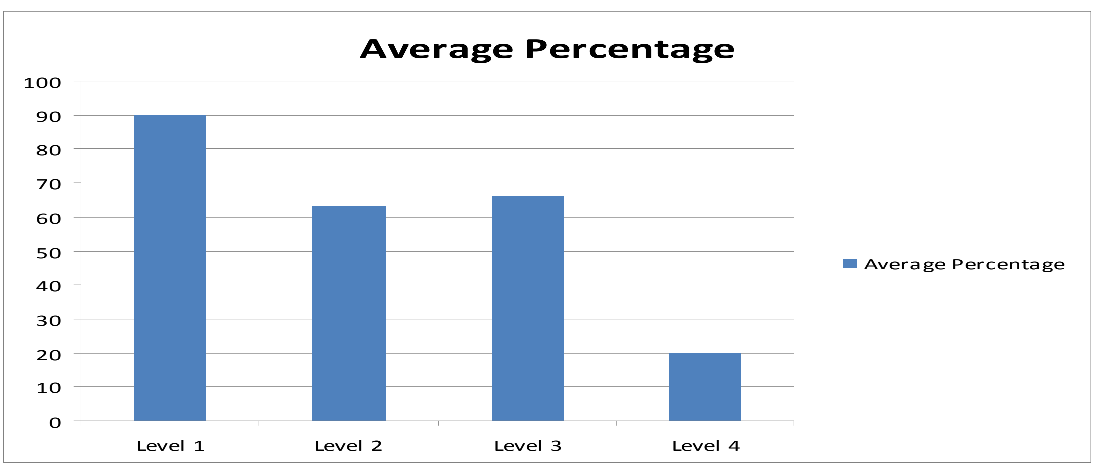


Figure 48: Average percentage of teachers that obtained C1 per Van Hiele level in task-based interview

There was a similar decrease of the average percentage of correct responses achieved in both the test and task-based interview from Van Hiele level 1 to level 4. For six out of seven items testing Van Hiele level 1 or 2, at least seven or more teachers achieved correct responses in these items, except for one out of seven items, only one teacher achieved a conclusive response. This item was theorem presentation. Teachers seemed to be careless or having some language difficulty; however, one teacher confused the theorem known as “angle at centre is twice angle at circumference” with another known as “perpendicular from centre to chord”. Teachers demonstrated more understanding of Van Hiele level 1 (visualisation) compared to any other Level, however, some could not for example, correctly respond in items 3.1 (a) and 3.2 (a).

The average percentage of teachers who achieved conclusive responses in Van Hiele level 1 and level 3 improved by 10% or more in the task-based interview in comparison to the test responses. This can probably be attributed to the interview providing an opportunity for the researcher to probe their understanding deeper and teachers having afforded more time to further reflect on various items. Teachers demonstrated limited reasoning and understanding in both items testing Van Hiele level 4. These items (question 8 and 12) are non-routine in the textbooks and non-routine in the external examinations in Grade 12. Only one out of ten teachers demonstrated reasoning and understanding of these two problems; he is the only one who is currently teaching Mathematics Paper 3 amongst all the study participants.

The main aim of this study was to investigate teachers’ understanding of geometry, irrespective whether they teach Mathematics Paper 3 or not, since geometry is experiencing a revival in the implementation of CAPS in 2012. During the task-based interview, teachers could recognise figures and compare them to known prototypes. The general levels of understanding of Euclidean Geometry of this sample of Grade 12 mathematics teachers were at Van Hiele level 1 and level 2. These are informal levels of understanding that provide “conceptual substructures” for formal activities at the next level (De Villiers, 2010). Though some teachers were able to respond to questions related to informal activities, they could not justify or provide a logical reasoning for their responses, which were located at the more formal Van Hiele levels 3 and level 4.

Not all teachers who achieved correct informal responses could justify or provide logical reasoning for their responses in items like question 3.1; 3.2; 5.2; 9.1; 9.2; 10.1 and 10.2. In question 9, one teacher falsely assumed that ABCD was an isosceles trapezium, one teacher presented a non-interpretable justification and two teachers drew a rectangle which was not given (VHL 3). In question 3, some teachers applied properties of a triangle (VHL 3) with given information to justify their answers.

What are Grade 12 mathematics teachers' specific conceptions and misconceptions with respect to circle geometry?

This study is formed by 12 questions, with 19 items in total testing Bloom's Taxonomy category 1 through category 5 and Van Hiele level 1 through level 4. Teachers demonstrated reasonable understanding of Bloom's Taxonomy category 1 through category 3 and Van Hiele level 1 through level 2; and limited understanding of Bloom's Taxonomy category 4 through 5 and Van Hiele level 3 through 4. However, specific conceptions and misconceptions with respect to circle geometry were identified in all the Bloom's Taxonomy categories and Van Hiele levels used in this study. In questions such as question 1 and 2; etc, teachers demonstrated adequate reasoning and understanding but some misconceptions were identified, that resulted on inconclusive responses. For example, seven out of ten teachers showed a specific conception in visually estimating the angle in question 1, several teachers clearly did not draw it very close to 90 degrees. The researcher believes that dynamic geometry can help teachers and learners to visualise, provided it is used in conjunction with physical measuring with a protractor as well.

More than half (50%) of the teachers substantiated their explanation as to why two marked angles are not equal, in question 3.1. Teachers provided justifications why the angles were not congruent or giving propositional relationships between angles (conclusive responses). They further correctly applied properties of a triangle (BTC 3 and VHL 3) in their justification. In question 5.2, six teachers demonstrated adequate reasoning or understanding of supplementary angles. They realised that angles B and D would be supplementary but that did not mean each angle was necessarily 90 degrees. Therefore, these six teachers understood that the logical conclusion that angles B and D were supplementary did not imply they were each right angles.

Teachers were required to prove if PQRS and PAMB are cyclic quadrilaterals in question seven and eleven, on Van Hiele level 3. After the task-based interview, all teachers achieved conclusive responses in these questions. They expressed responses in diverse approaches; however, responses were all mathematically correct. In question seven, they proved that PQRS is a cyclic quadrilateral since opposite angles are supplementary. In question eleven, teachers applied the theorems known as “tangent perpendicular to radius” and “tangents from the same point are equal” to justify their logical reasoning.

The researcher of this study has further identified misconceptions with this sample, in questions three and nine. In question three, two teachers justified their responses by using what they were supposed to prove in their arguments. The internal moderator of Mathematics Paper 3 (KZN-DBE) has identified similar misconceptions from learners that one should not expect from teachers. This is quite alarming given that they are Grade 12 teachers, since the external examination have a similar questioning style like question 3 and 5. Other misconception observed in this sample in question 3, was that four of the teachers seemed to view “an angle bisector” as equivalent to the “median” of a triangle.

In question five, two teachers demonstrated a misconception about quadrilateral ABCD. On their argument to support why ABCD will be a square, they thought if angles are supplementary then that will mean each angle is equal to 90 degrees. However, irrespective of angles, participants needed to visualise the diagram (VHL 1) and remember square properties (BTC1) in order to answer this question on Bloom's Taxonomy category 5 and Van Hiele level 3. One teacher claimed that ABCD could not be a square but any quadrilateral such as a rhombus, which is impossible, since a rhombus is not cyclic.

Some teachers in this sample showed a misconception about the midpoint of a chord. This was demonstrated by one teacher in question five and four teachers in question nine. In question six, it was given that M is the midpoint of AB and that OD passed through M. Though this was not given, these two teachers assumed (believed) that M was also a midpoint of OD, even though it did not even visually appear to be the case. More than 50% these six teachers show a misconception of the intersecting lines in question 9. They assumed that when two lines intersected each other perpendicular, they are necessarily cut into two equal pieces. For example, "perpendicular bisectors will cut the lines and leave two parts equal" was one of the responses showing this apparent misconception.

What is the relationship between Grade 12 mathematics teachers' understanding, conceptions and misconceptions, and their ability to apply their knowledge of circle geometry?

Grade 12 mathematics teachers' notion of understanding involves knowing more than just isolated facts; instead they ought to have an organised knowledge system and be able to connect new ideas to pre-existing theory. A personal understanding takes place when s/he creates and shapes his/her conceptual frameworks. Research conducted by Swan (2001) shows that erroneous conceptions are so stable because they are not always incorrect. Therefore, misconceptions relate to understanding and/or lack thereof, and are thus important indicators of what teachers understand in circle geometry.

Several teachers confused necessary with sufficient conditions in question five. Though the majority of teachers correctly argued that when opposite angles of a cyclic quadrilateral are supplementary, they incorrectly concluded that each angle would be 90 degrees. Having two angles supplementary is not a necessary condition for each of them to be a right angle, which they seemed to interpret as a sufficient condition. Some teachers believed that an angle bisector is equivalent to the median of a triangle, and this misconception led to correct responses in question 3.

The way that geometry has been taught in South Africa has been largely text-book dependent. This was confirmed by the findings of this study as most teachers could not answer non-routine problems or non-routine Grade 12 examination problems. Most South African textbooks present problems that can be solved without thinking about the underlying mathematics, but by blindly applying the procedures that have just been studied. This research shows that in high order questions teachers have unquestionably been found seriously lacking in terms of their ability to apply the content knowledge of circle geometry. Teachers commonly experienced what refers to as an impasse, where they just simply came to a standstill in constructing a proof and could not continue.

5.5. Conclusion

A person can achieve Bloom's Taxonomy category 5 without passing through the previous categories, especially to routine textbooks and examination problems. However, one cannot achieve a higher level of Van Hiele without passing through the previous levels. Advancement from one Van Hiele level to the next is more dependent on educational experiences than on age or maturation, and certain types of experiences can impede (or facilitate) advancement within a level and to a higher level.

This study found sufficient evidence among the ten Grade 12 mathematics teachers to support the theoretical claim for the hierarchical nature of the Van Hiele levels. A teacher who seemed to have achieved Van Hiele level 4 had little difficulty with the lower level items. However, most of the mathematics teachers investigated in this study functioned only at the bottom two levels of the Van Hiele hierarchy.

CHAPTER SIX

SUMMARY, RECOMMENDATIONS AND CONCLUSION

6.1. Introduction

This Chapter is a final section of the research project where the investigation is summarised, recommendations are given and conclusion reached. The aim of this study was to explore Grade 12 mathematics teachers' understanding of Euclidean Geometry with specific reference to circle geometry. The primary focus of this study was to investigate what cognitive levels and levels of understanding Euclidean Geometry teachers have mastered in order to solve and show understanding of geometrical riders.

6.2. Summary of the Research

In **Chapter two**, the Bloom's Taxonomy categories and Van Hiele levels of geometric thought were discussed and provided the theoretical framework for collecting, analysing, interpreting and reporting Grade 12 mathematics teachers' understanding of circle geometry. The findings, by other researchers, about mathematics teachers and curriculum changes in mathematics provided justification for this study. The reasons for making Euclidean Geometry optional and its compulsory re-implementation in 2012 were important for this study focus on analysing this group of Grade 12 mathematics teachers' understanding of the circle geometry.

In **Chapter three**, the choice of a case study as an appropriate research tool was explained. Using a case study allowed the researcher to explore the strategies of teachers' reasoning, discover important questions to ask during the task-based interview and try to understand teachers' thinking processes. The choices of test and task-based interview in the case study design allowed for the rich description and analysis of data.

In **Chapter four**, teachers' responses were analysed, coded and categorised as described in section 4.2 in Table 5. The following key aspects were identified in an attempt to understand teachers' reasoning skills, namely:

- Teachers performed poorly in Bloom's Taxonomy category 4 - 5.
- Teachers performed poorly in Van Hiele level 3 - 4.
- There was poor ability to visually estimate angle size.
- The majority of teachers expose their learners to the standard representation of a particular theorem.
- Teachers that have not been teaching Mathematics Paper 3 could not answer non-routine problems

These five aspects facilitated a detailed analysis of teachers understanding of Euclidean Geometry. In **Chapter five**, each of the five key aspects identified in Chapter four were discussed in more generic terms.

6.3. Implications and Recommendations

It has been argued in this study that Euclidean Geometry poses several challenges to Grade 12 teachers' reasoning abilities. Adopting an alternative strategy for Euclidean Geometry will imply that many teachers would be removed from their present comfort zone of presenting theorems as a finished product. In the context of this study, the following recommendations can be put forward as means to enhance teachers' reasoning ability in terms of Euclidean Geometry.

Teachers of mathematics, as key elements in the assuring of quality in mathematics education, should possess an adequate knowledge of subject matter beyond the scope of the secondary school curriculum. Any new curriculum is bound to fail unless the teachers who are to implement it are well trained in content and in instructional approaches (Laridon, 1993). The current degree courses for mathematics teachers to train more and better qualified mathematics teachers are therefore recommended, to include Euclidean Geometry.

As is stated in the literature review section, one of the main reasons for making Euclidean Geometry optional in South Africa in the period from 2008 in Grade 12 was that the teachers were not familiar with the content (Bowie, 2009). Govender (2011) says the majority of teachers in South African schools are not ready to teach geometry content. He further says, even experienced teachers have a backlog in their geometry content knowledge as well as their pedagogical and curriculum specific knowledge of geometry. However, it does not seem as if the problem has been adequately addressed even though the DBE has offered professional development courses on a continuous basis.

It is therefore recommended that mathematics teachers enhance their own professional development through academic study and networking with other teachers, for example enrolling for qualifications such as the ACE, BEd, etc. In addition to a focus on content knowledge and the method of teaching, mathematics workshops, seminars and conferences must also focus on the teachers' ability to blend technique and content. This also includes understanding of how the given topics are related to one another and how they are most effectively organised. AMESA also recommends that thorough continuous in-depth training should be provided to teachers in geometry. However, the DBE should find specialists to develop the training materials in Euclidean Geometry so that training and learning could be effectively accomplished at all levels with much understanding and confidence.

The DBE ought to organise compulsory education for all teachers and no one should be left behind. It is recommended that those who are engaged as providers of education and training (like subject education specialists) should themselves receive some form of education to update them on the techniques, methodology and required knowledge to impart to participants. To develop teachers' ability, non-routine problems should be dealt with during seminars and workshops. AMESA, other mathematics associations and teachers unions should be involved in training since they support the DBE's recommendation that Euclidean Geometry should be included in Mathematics Paper 2.

Since learners' and students' ability to measure angle sizes (DBE, 2010), and even some of the teachers showed difficulty in this study estimating approximate right angles; measuring ability with protractors should be taught in early Grades (GET Phase). Text-books give limited information about geometric constructions and perhaps dynamic geometry can help learners and teachers to better visualise, provided it is used in conjunction with measurements. The pedagogical advantages of employing a reconstructive approach are:

- It allows the scholars to become actively engaged in the measurement and construction of the content;
- Its implementation accentuates the meaning of the content.

Therefore, employing a reconstructive approach is thus characterised by not presenting content as a finished product (De Villiers, 1998), but instead by focusing on the real mathematical activities through which the content is to be developed.

The researcher also recommends a workable alternative to the rigid axiomatic approaches to Euclidean Geometry by utilising computer programmes such as Geometer's Sketchpad, GeoGebra, etc to facilitate and enhance learners' and students' ability to the making and testing of conjectures. Several researchers have argued in favour of dynamic software programmes since "The main advantage of computer exploration of topics . . . is that it provides powerful visual images and intuitions that can contribute to a person's growing mathematical understanding" (De Villiers as cited in Yushan et al., 2005, p.17). Using computer programmes to visualise a problem, it helps mathematicians to have a global picture of the problem to be solved.

6.4. Recommendations for Further Study

- The sample size should be increased in order to generalise the finding;
- Teachers at different grade levels and from different type of schools (e.g. township, ex-model C) should be included;
- Teachers proof schemes use should be investigated for different types of questions;

- The use of more non-routine problems in data collection instruments; and
- Developing and evaluating appropriate curricula and text-book materials in association with the Van Hiele levels and/or the Bloom categories.

6.5. Limitation of the Study

Shortcomings in the research process could affect the credibility and dependability of the data. The sample size should be large enough to generate sample data (Cohen et al., 2007). The sample used in the study was small and that was the main limitation that impacts on the generalising of the data. It is not certain that all teachers teaching Grade twelve especially in the Hlabisa Circuit of Obonjeni District have the reasoning ability like those who participated in this study. Another limitation was that the schools, where participants teach, are situated in rural areas or informal settlements.

This study was not a prolonged study. Prolonged engagement on the field will ensure trustworthiness of the research process. The type of questions on the test and task-based interview might not be totally representative of circle geometry concepts. Further the researcher is a mathematics teacher, and bias and subjectivity could have affected the task-based interview process, such as asking leading questions. The ability of the researcher to use relevant prompts and probes to allow the teachers to explain, clarify and elaborate, produces rich and first hand information (Maree, 2007). This is one of the shortcomings experienced in the interview process. At certain instances the task-based interview took on a question and answer format. The researcher could have pursued the responses to the questions further in order to acquire thick data.

6.6. Conclusion

The intention of this study was to make a meaningful contribution to the body of knowledge related to teachers' understanding of Euclidean Geometry. During the task-based interview some of the teachers gave expressions of frustration, demotivated and indifferent towards Euclidean Geometry because they felt incompetent in dealing with it, especially in non-routine problems. Geometry researchers such as De Villiers (1997) have argued strongly that geometry has experienced a strong resurgence in the last decade or two, and as a field of mathematical research is alive and well. Consequently, Euclidean Geometry is experiencing a renaissance in South Africa and other countries, at all levels of education. It is important for many careers such as pure and applied architecture, art, engineering, mathematics etc. Recent curriculum changes (such as CAPS) in South Africa demonstrate a marked shift from the traditional approach to a constructivist approach to geometry.

For the curriculum reform initiatives to be of any significance, there also needs to be a radical re-look at the teachers' education courses at both pre-service and in-service levels. Only one out of ten teachers who participated in this study teach Mathematics Paper 3. Higher institutions of learning offering teacher education courses need to have compulsory modules in Euclidean and non-Euclidean Geometry for both primary and secondary teachers. Much of the 'failure' of geometry in the past could perhaps be attributed to its neglect in the primary school – so you need to emphasize the importance of primary geometry education.

This study focused primarily on teachers' reasoning when solving geometric problems; the researcher believes that the investigation into:

- how they reason on pedagogical choices and mathematically reason when teaching geometry,
- the relevance and role of the language used by teachers and its suitability to the conceptual understanding in geometry.

The Van Hiele theory offers an opportunity to broaden and deepen one understands of the model. The researcher also believes that such a study would be able to inform the effective teaching of Euclidean Geometry. There is a need to understand geometry teaching practice at the chalk face: how teachers teach geometry, how they use the language of geometry, and to investigate the extent to which their use of the language of geometry takes into consideration learners' level of development in terms of the Van Hiele theory. We need to explore further the extent to which providing pre-service and in-service teachers with opportunities to engage in activities that "require classifying answers by Van Hiele levels" (Feza and Webb, 2005, p. 45) might contribute to effective practice in Euclidean Geometry.

References

Adler, J. & Pillay, V. (2006). *An investigation into Mathematics for Teaching: Insight from a case*. In Press.

Ali, T. & Hakan Sevkin, A. (2010). Analysing "Science and Technology Course Exam Questions" According to Revised Bloom Taxonomy. *Journal of Turkish Science Education*, *7*(1), 26-29.

An, S., Kulm, G. & Wu, Z. (2004). The pedagogical content knowledge of middle school mathematics teachers in China and the U.S. *Journal of Mathematics Teachers Education*, 7, 145-172.

Anderson, L.W. & Sosniak, L.A. (1994). *Bloom's Taxonomy: A forty-year retrospective. 93rd year book of the National Society for the Study of Education*. Chicago: University of Chicago.

Babbie, E. & Mouton, J. (2001). *The practice of social research*. Cape Town: Oxford University Press.

Bell, J. (1987). *Doing your research project: A guide for first-time researchers in education and social science*. Philadelphia: Open University Press.

Bell, J. (2005). *Doing your research project: A guide for first time researchers in education, health and social science.* Philadelphia: Open University Press.

Bloom, B.S. (1956). *Taxonomy of educational objectives. Handbook I: The Cognitive Domain*. New York: David McKay Co Inc.

Bloom, B.S. (1994). Reflection on the development and use of the taxonomy. In L. Anderson & L. Sosniak (Eds.), *Bloom's Taxonomy: A Forty-Year Retrospective* (pp. 1-8). Chicago: The National Society for the Study of Education.

Bowie, L. (2009). *What is Mathematics Paper 3 for*? Marang Centre for Mathematics and Science Education, Marang News, Issue 5, June 2009.

Brown, B. (2002). Learning Maths in South Africa. *Achiever*, 3, 42-43.

Byrnes, J.P. (2001). *Theories of cognitive development and learning. Cognitive development and learning in instructional contexts* (2nd ed.) Needham Heights, MA: Allyn and Bacon.

Cassim, I. (2006). *An exploratory study into grade 12 learners' understanding of Euclidean Geometry with special emphasis on cyclic quadrilateral and tangent theorems*. Unpublished Master's thesis. University of the Witwatersrand, Johannesburg.

Clements, D. & Battista, M. (1992). Geometry and spatial reasoning. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning, A project of the National Council of Teachers of Mathematics* (pp. 420-464). New York: Macmillan Publishing Company.

Cohen, L. Manion, L. & Morrison, K. (2007). *Research methods in education*. (6th ed.) London and New York: Routledge

Cooper, D.R. & Schindler, P.S. (2003). *Business research methods*. (8th ed.) New York: McGraw Hill.

Creswell, J.M. (2003). *Research design: A qualitative, quantitative and mixed method approaches*. Thousand Oaks CA: Sage Publications.

Creswell, J.M. (2005). *Educational research: Planning, conducting and evaluating quantitative and qualitative research*. Thousand Oaks NJ: Pearson Prentice Hall.

Crowley, M.L. (1987). The Van Hiele model of geometric thought. In M.M. Lindquist & A.P. Shuklte (Eds.), *Learning and Teaching Geometry, K-12. 1987 Year Book.* Reston, VA: *National Council of Teachers of Mathematics*.

Department of Education, (2002). *Revised National Curriculum Statement Grade 10 – 12: Mathematics*. Pretoria: National Department of Education.

Department of Education. (2003). *National Curriculum Statement Grade 10 – 12 (General) Policy: Mathematics*. Pretoria: National Department of Education.

Department of Education. (2006). *Government Gazette Republic of South Africa*, *539*(3136), 1-20.

Department of Education. (2008). *Internal moderators report for Mathematics Paper 3 – KZN*. KZN: Examination Section.

Department of Education, (2008). *Subject Assessment Guidelines Grade 10 – 12: Mathematics*. Pretoria: National Department of Education.

Department of Education. (2010). *Internal moderators report for Mathematics Paper 3 – KZN*. KZN: Examination Section.

Department of Basic Education. (2011). *Curriculum and Assessment Policy Statement Grades 10-12: Mathematics*. Pretoria: National Department of Education.

De Villiers, M.D. (1987). *Research evidence on hierarchical thinking, teaching strategies, and the Van Hiele theory: Some critical comments*: Paper presented at Learning and teaching geometry: Issues for research and practice working conference, Syracuse,NY, Syracuse University

De Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras*, 24, 17-23.

De Villiers, M. (1996). *Why proof in dynamic geometry? Proceedings of topic group 8: Proofs and proving: Why, When and How*? Durban: Centrahil, AMESA.

De Villiers, M. (1997). *Alternative instructional strategies for geometry education: A theoretical and empirical study*. http://mzone.mweb.co.za/residents/profmd/homepage4.html

De Villiers, M. (1997). The future of secondary school geometry. *Pythagoras*, 44, 37-54.

De Villiers, M. (1998). An alternative approach to proof in dynamic geometry. Chapter in R. Lehrer & D. Chazan (Eds) *New directions in teaching and learning geometry*. (pp. 369 - 393). Lawrence & Erlbaum.

De Villiers, M. (2010). *Some reflections on the Van Hiele theory*. Retrieved 22 January, 2011, from http://mzone.mweb.co.za/residents/profmd/homepage4.html

De Vos, A.S. (2002). *Research at grass root for the social sciences and human service professions*. Pretoria: Van Schaik Publishers.

Dias, A.L.B. (2000). Overcoming algebraic and graphic difficulties. *Proceedings of the 7th International Conference on Adult Learning Mathematics*. Boston: MA.

Durrheim, K. (1999a). Qualitative measurements. In M. Terre Blanche & K. Durrheim (Eds.), *Research in Practice: Applied methods for the social sciences*. (pp. 29 - 53). Cape Town: University of Cape Town Press.

Durrheim, K. (2002). Research Design. In M. Terre Blanche & K. Durrheim (Eds.) *Research in practice: Applied methods for the social sciences*. (2nd ed.). Cape Town: University of Cape Town Press.

Durrheim, K. & Wassenaar, D. (2002). Putting design into practice: Writing and evaluating research proposals. In M. Terre Blanche & K. Durrheim (Eds.). *Research in Practice: Applied methods for the social sciences*. (2nd ed) Cape Town: University of Cape Town Press.

Egan, K. (2001). “Why education is so difficult and contentious.” *Teachers College Record*, *103*(6), 923-941.

Ferguson, C. (2002). Using the Revised Taxonomy to plan and deliver team-taught, integrated, thematic units. *Theory into Practice*, *41*(4), 239-244.

Feza, N. & Webb, P. (2005). Assessment standards, Van Hiele levels and grade seven learners' understanding of geometry. *Pythagoras*, 62, 36-47.

Forehand, M. (2010). *Bloom's Taxonomy – emerging perspectives on learning, teaching and technology*. University of Georgia, Athens, United States.

French, D. (2002). *Teaching and learning algebra*. London: Continuum International Publisher Group.

Fuys, D. (1985). Van Hiele levels of thinking in geometry. *Educational and Urban Society*. *17*(4), 447-462.

Fuys, D., Geddes, D. & Tischler, R. (1988). The Van Hiele model of thinking in geometry among adolescents. *Journal of Research in Mathematics Education Monograph 3*. Reston, VA: National Council of Teachers of Mathematics.

Geddes, D. (1992). *An investigation of the Van Hiele model of thinking in geometry among adolescents*. Paper presented at the Annual Meeting of the American Educational Research Association, March 1992. New York.

Goulding, M. (2007). *How do prospective primary teachers assess their own mathematical knowledge*? York, UK: University of York.

Goulding, M. & Suggate, J. (2001*). Opening a can of worms: Investigating primary teachers' subject knowledge in mathematics*, 13, 41–54.

Govender, R. (2010). *AMESA position on paper 3.* Retrieved 3 February, 2011, from http://www.amesa.org.za

Gutierrez, A., Jaime, A. & Fortuny, J.M. (1991). An alternative paradigm to evaluate the acquisition of the Van Hiele levels. *Journal for Research in Mathematics Education*, *22*(3), 237-251.

Gronlund, N.E. (1998). *Assessment of student achievement*. Massachusetts: Allyn & Bacon.

Gronlund, N.E. (2000). *How to Write the Use Instrumental Objectives*. (6th ed.) Upper Saddle River, NJ: Merrill.

Hanna, G. (2000). Proof, explanation and exploration: an overview. *Educational Studies in Mathematics*, 44, 5-23.

Hatch, J.A. (2002). *Doing qualitative research in educational settings*. New York: SUNY.

Henning, E. (2004). *Finding your way in qualitative research.* Pretoria: W. & Smith, B. Van Schaik Publishers.

Hershkowitz, R. & Vinner, S. (1984). Children's concepts in elementary geometry: A reflection of teachers' concepts? In Southwell, B., Eyland, R., Cooper, M., Conroy, J. & Collis, K. (Eds.), *Proceedings of English Conference for the Psychology of Mathematics Education* (pp. 63-69). Mathematical Australia: Association of New South Wales.

Howard, T.C. & Aleman, G.R. (2008). Teacher capacity for diverse learners: What do teachers need to know? In M. Cochran-Smith, S. Feiman, N. Nemser, D.J. McIntyre and K.E. Demers (Eds.), *Handbook of research on teacher education: Enduring questions in changing contexts* (pp. 157-174). Florence, KY: Taylor and Francis Group.

Jacobson, C. & Lehrer, R. (2000). Teacher appropriation and student learning of geometry through design. *Journal for Research Mathematics Education*, *31*(1), 71-88.

Jaime, A. & Gutierrez, A. (1995). Guidelines for teaching plane isometries in secondary schools. *The Mathematics Teacher*, *88*(7), 591-597.

Jones, K. & Mooney, C. (2000). Trainee primary teachers' knowledge of geometry for teaching. *Proceedings of the British Society for Research into Learning Mathematics*, *22*(2), 95-100.

Kahan, J., Cooper, D. & Bethea, K. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*, 6, 223-252.

Krathwohl, D.R. (2002). A revision of Bloom's Taxonomy: An overview. *Theory into Practice*, *41*(4), 212-218.

Krathwohl, D.R. & Anderson, L.W. (2001). *A taxonomy for learning, teaching and assessing: A revision of Bloom's Taxonomy of educational objectives*. New York: Longman.

Kreitzer, A. & Maduas, G. (1994). Empirical investigations of the hierarchical structure of the Taxonomy. In L. Anderson & L. Sosniak (Eds.), *Bloom's Taxonomy: A forty-year retrospective* (pp. 64-81). Chicago: The National Society for the Study of Education.

Laridon, P. (1993). Towards transforming mathematics education in South Africa. *Pythagoras*, *31*(1), 41-68.

Lawrie, C. & Pegg, J. (1997). *Some issues in using Mayberry's test to identify Van Hiele levels*. Unpublished paper.

Lee, V.S. (2008). Benjamin Bloom's Taxonomy on cognitive behaviours. *The National Teaching and Learning Forum*, *8*(4), 18-23.

Leedy, P.D. & Ormorond, J.E. (2001). *Practical research planning and design*. (7th ed.) Upper Saddle River: New Jersey Columbus.

Lilley, M., Barker, T. & Maia, M. (2004). *The use of objective questions in higher education: Potential and limitations*. United Kingdom: University of Hertfordshire.

Lincoln, Y.S. & Guba, E.G. (1985). *Naturalistic inquiry*. London: SAGE Publications.

Lubisi, C. (1998). Validity and reliability: A summary of key ideas. In C. Lubisi (Ed.), *Assessment in education: Principles, practice and critique*. Pietermarzburg: University of Natal Press.

Maree, K. (2007). *First steps in research*. Pretoria: Van Schaik.

Marshall, C. & Rossman, G.B. (1995). *Designing a qualitative research*. (2nd ed.) Thousand Oaks, CA: Sage.

Martha, L.A., Kathryn, D. & Mya, P. (2001). *Program-based review and assessment tools and techniques for program improvement*. President's Reserve: University of Massachusetts.

Mason, M.M. (1997). The Van Hiele model of geometric understanding and mathematically talented students. *Journal of Education of the Gifted*, *21*(1), 38-53.

Mason, M.M. & Schell, V. (1988). *Geometric understanding and misconceptions among preservice and in- service mathematics teachers*. Paper presented at the International Group for the Psychology of Mathematics Education. North American Chapter, DeKalb, Illinois.

Maxwell, J.A. (1996). *Qualitative research design: An interactive approach*. London: Sage Publications.

Mayberry, J.W. (1981). *An investigation of the Van Hiele levels of geometric thought in undergraduate preservice teachers*. Unpublished dissertation, University of Georgia.

Mayberry, J. (1983). The Van Hiele levels of geometric thought in undergraduate pre-service teachers. *Journal for Research in Mathematics Education*, *14*(1), 58-69.

Monaghan, F. (2000). What difference does it make? Children's view of the difference between some quadrilaterals. *Educational Studies in Mathematics*, 42, 179-196.

Mouton, J. (2001). *How to succeed in your master's and doctorial studies: A South African guide and resources book*. Pretoria: Van Schaik.

Mthembu, S.G. (2007). *Instructional approaches in teaching Euclidean Geometry in Grade 11*. Unpublished Master's thesis. University of KwaZulu-Natal, Durban.

Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Gregory, K.D., Garden, R.A., O'Connor, K.M., et al. (2000). *TIMSS 1999 International Mathematics Report*. Chestnut Hill, MA: The International Association for the Evaluation of Educational Achievement.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, V.A: National Council of Teachers of Mathematics.

Owusu-Mensah, J. (2008). *A critical investigation into the nature and quality of in-service education and training programmes for further education and training (FET) mathematics teachers in the greater Taung area*. Unpublished masters' thesis. University of Pretoria, Pretoria.

Pandiscio, E.A. & Knight, K.C. (2009). An investigation into the Van Hiele levels of understanding geometry of preservice mathematics teachers. *Journal of Research in Education*, *21*(1), 45-53.

Paul, R.W. (1985). Bloom's taxonomy and critical thinking instruction. *Educational Leadership*, *42*(3), 36-43.

Pegg, J. (1992). *Students' understanding of geometry: Theoretical perspectives*. University of New England.

Pillay, T. (2004). *Below the surface: African learner's experiences of schooling in a predominantly Indian school in KwaZulu-Natal*. Unpublished master's thesis. University of KwaZulu-Natal, Durban.

Pohl, M. (2000). *Learning to think, thinking to learn: Models and strategies to develop a classroom culture of thinking*. Cheltenham UK, Vic: Hawker Brownlow.

Pournara, C. (2004). Geometry for preservice teachers: some lessons learnt from a pilot module. In S. Nieuwoudt, S. Froneman & P. Nkhoma (Eds.), *Proceedings of the Tenth National Congress of the Association for Mathematics Education of South Africa, 1, 207-215*. Potchefstroom: Advert Square.

Pritchett, N. (1999). Effective Question Design. In S. Brown, P. Race & J. Bull (Eds.), *Computer-Assisted Assessment in Higher Education*. London: Kogan Pege.

Ross, K.N. (2005). *Educational research: some basic concepts and terminology*. France: UNESCO.

Sander, S. & Morris, H. (2000). Exposing students teachers' content knowledge: empowerment or debilitation? *Educational Studies*, *26*(4), 397-408.

Senk, S. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*. *20*(3), 309-321.

Senk, S.L. (2002). The effects of the UCSMP secondary school curriculum on students' achievement. In S.L. Senk & D.R. Thompson (Eds.), *Standards-based school mathematics curricula: What are they? What do students learn?* Mahwah, NJ: Erlbaum.

Shulman, L.S. (1986). Those who understand knowledge growth in teaching. In L. Shulman (Ed.), *Educational Research*, *15*(2), 4-14.

Shulman, L.S. (2004). *The wisdom of practice: Essays on teaching, learning and learning to teach*. San Francisco: Jossey Bass.

Strutchens, M.E. & Blume, G.W. (1997). What do students know about geometry? In P.A. Kenney & E.A. Silver (Eds.), *Results from the sixth mathematics assessment of the National Assessment of Educational Progress* (pp. 165-193). Reston, VA: National Council of Teachers of Mathematics.

Strydom, H., Fouche, C.B. & Delport, C.S.L. (2004). *Research at grass roots: For social sciences and human service professions*. Pretoria: Van Schaik Publisher.

Swafford, J.O., Jones, G.A. & Thornton, C.A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education*. *28*(4), 467-483.

Swan, M. (2001). Dealing with misconception in mathematics. In P. Gates (Ed.), *In Issues in Mathematics Teaching* (pp. 147-165). London: Routledge Falmer.

Teppo, A. (1991). Van Hiele levels of geometric thought revisited. *Mathematics Teachers*, 13, 210-221.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school Geometry*. Chicago: University of Chicago.

Usiskin, Z. (1987). Resolving the continuing dilemmas in school geometry. In M.M. Lindquist & A.P. Shulte (Eds.), *Learning and teaching geometry, K-12: 1987 yearbook*. Reston, VA: NCTM.

Van der Sandt, S. & Nieuwoudt, H.D. (2005). Geometry content knowledge: Is pre-service training making a difference? *African Journal of Research in SMT Education*, *92*(2), 109-120.

Van Hiele, P.M. (1986). *Structure and insight: A theory of mathematics education*. Orlando FL: Academic Press.

Van Putten, S. (2008*). Levels of thought in Geometry of pre-service mathematics educators according to the Van Hiele Model*. Unpublished Master's Thesis. University of Pretoria, Pretoria.

Van Putten, S., Howie, S. & Stols, G. (2010). Making Euclidean Geometry compulsory: Are we prepared? *Perspectives in Education*, *28*(4), 22-31.

Young, V.M. (2006). Teacher's use of data: Loose coupling, agenda setting and team norms. *American Journal of Education*, *112*(4), 521-548.

Yushan, B., Mji, A. & Wessels, D.C.J. (2005). The role of technology in fostering creativity in the teaching and learning of mathematics. *Pythagoras*, 62, 12-22.

Appendix 1

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

Research Office, Govan Mbeki Centre
Westville Campus
Private Bag x54001
DURBAN, 4000
Tel No: +27 31 260 3587
Fax No: +27 31 260 4609
mohunp@ukzn.ac.za

1 August 2011

Mr SS Dhlamini (208524873)
School of Maths, Science, Computer & Technology Education
Faculty of Education
Edgewood Campus

Dear Mr Dhlamini

PROTOCOL REFERENCE NUMBER: HSS/0664/011M
PROJECT TITLE: An investigation into Grade 12 teachers' understanding of Euclidean Geometry

In response to your application dated 26 July 2011, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment /modification prior to its implementation. In case you have further queries, please quote the above reference number.
PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Professor Steven Collings (Chair)
HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE

cc. Supervisor: Prof M de Villiers
cc. Ms T Mnisi, Faculty Research Office, Faculty of Education, Edgewood Campus

1910 - 2010
100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

Appendix 2

Letter of Consent

To: Participant(s)

Research Project:

An investigation into Grade 12 teachers' understanding of Euclidean Geometry.

Year: 2011

I, **Sikhumbuzo Sithembiso Dhlamini** (Mathematics Education Student) am doing a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** with Prof. M. De Villiers**.** His contact number is 031 – 260 7252 (work). We want to investigate Grade 12 teachers' understanding of Euclidean Geometry in KwaZulu-Natal: South Africa.

Teachers are asked to help by taking part in this research project as it would be of benefit to our community and interested researchers in the field of medical science. However, participation is completely voluntary and has no impact on their employment. Participants will be asked to take part in the test and task-based interviews after the presentation has been completed. The whole session of the interview will be tape-recorded. All participants will be noted on transcripts and data collections by a *pseudonym* (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secured password and not been used for any other purpose except for the research.

Participants may leave the study at any time by telling the researcher. Participants may review and comment on any parts of the researchers' written reports.

____________________________ ______________________

(*Researcher's Signature*) (*Date*)

DECLARATION

I,________________________________ (*Participant's NAME)*_________________(*Signature)*

☐ Agree.

N.B. Tick ONE

☐ Disagree.

To participate/allow participation in the research being conducted by **Sikhumbuzo Sithembiso Dhlamini** concerning: *An investigation into Grade 12 teachers' understanding of Euclidean Geometry.*

Appendix 3

kzn education
Department:
Education
KWAZULU-NATAL

Enquiries: Sibusiso Alwar | Tel: 033 341 8610 | Ref.: 2/4/8/63

Mr. Sithembiso Sikhumbuzo Dhlamini
P.O. Box 424
Emondlo
3105

Dear Mr. Dhlamini

PERMISSION TO CONDUCT RESEARCH IN THE KZNDoE INSTITUTIONS

Your application to conduct research entitled: **An investigation into Grade 12 teachers' understanding of Euclidean Geometry,** in the KwaZulu Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.

2. The researcher must ensure that Educator and learning programmes are not interrupted.

3. Interviews are not conducted during the time of writing examinations in schools.

4. Learners, educators, schools and institutions are not identifiable in any way from the results of the research.

5. A copy of this letter is submitted to District Managers, Principals and Head of Institutions where the intended research and interviews are to be conducted.

6. The period of investigation is limited to the period: From 01 July 2011 to 31 July 2012.

7. Your research and interviews will be limited to the schools you have proposed and approved by the Superintendent General. Please note that Principals, Educators, Departmental Officials and Learners **are under no obligation to participate or assist you in your investigation.**

8. Should you wish to extend the period of your survey at the school(s), contact Mr Alwar at the contact numbers below.

...dedicated to service and performance
beyond the call of duty.

KWAZULU-NATAL DEPARTMENT OF EDUCATION

POSTAL: Private Bag X9137, Pietermaritzburg, 3200, KwaZulu-Natal, Republic of South Africa

PHYSICAL: Office G 25, 188 Pietermaritz Street, Metropolitan Building, Pietermaritzburg, 3201

TEL: Tel: +27 33 341 8610/11 | Fax: +27 33 341 8612 | email:sibusiso.alwar@kzndoe.gov.za

9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Address to: The Director: Resource Planning; Private Bag X9137; Pietermaritzburg; 3200

The Department of Education in KwaZulu Natal fully supports your commitment toward research and wishes you well in your endeavours. It is hoped that you will find the above in order.

Nkosinathi SP Sishi, PhD
Head of Department: Education

2011 — 07 — 25
Date

...dedicated to service and performance beyond the call of duty.

KWAZULU-NATAL DEPARTMENT OF EDUCATION

POSTAL: Private Bag X9137, Pietermaritzburg, 3200, KwaZulu-Natal, Republic of South Africa

PHYSICAL: Office G 25, 188 Pietermaritz Street, Metropolitan Building, Pietermaritzburg, 3201

TEL: Tel: +27 33 341 8610/11 | Fax: +27 33 341 8612 | email:sibusiso.alwar@kzndoe.gov.za

Appendix 4

UNIVERSITY OF KWAZULU-NATAL

MATHEMATICS EDUCATION RESEARCH (MASTERS STUDY)

RESEARCH INSTRUMENT

TEACHERS' UNDERSTANDING OF CIRCLE GEOMETRY

Name of the Participant: ____

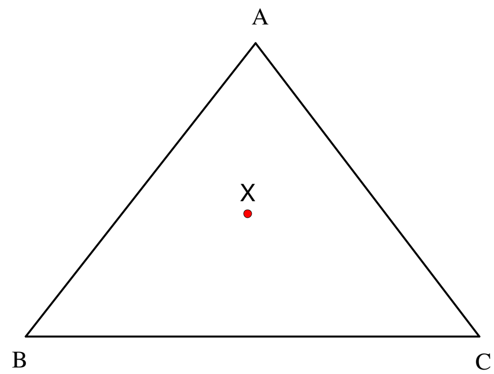
Participant Pseudonym Name: ____

Focus and Purpose of the Study

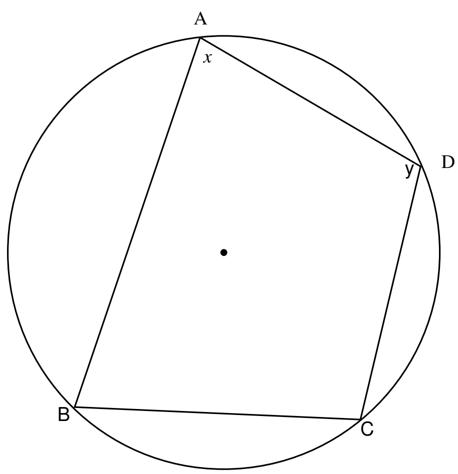
- This study is based at Obonjeni District in KwaZulu-Natal (Zululand Region).
- This study investigates the sample of Grade 12 mathematics teachers' understanding in Circle Geometry.
- The study is underpinned by Bloom's Taxonomy of Learning and Van Hiele Levels of understanding Euclidean Geometry.
- The conclusion of this study will make recommendations not generalisation, since the study is based on one District and sample size is small.

Question One

Draw perpendicular line from *X* to *AB*.



Question Two



2.1. What are these angles called?

____

____

2.2. Line AB // CD, what is the relationship between angle *x* and angle *y*? Support your answer.

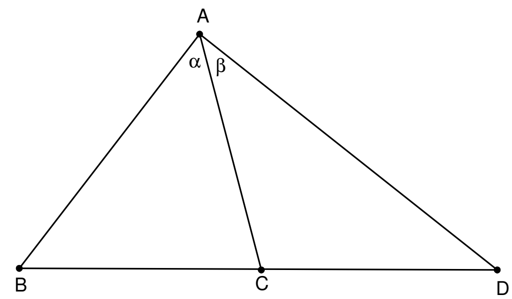
____

____

____

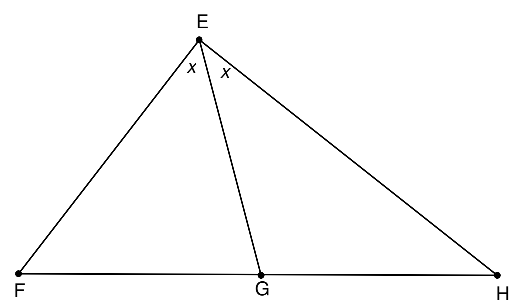
Question Three

3.1. Given that *BC* = *CD*.



Do you think the two marked angles α and β are equal? Why or why not? Please explain your answer as best you can.

3.2.



If ∠ FEG = ∠ GEH, do you think FG = GH? Why or why not? Please explain your answer as best you can.

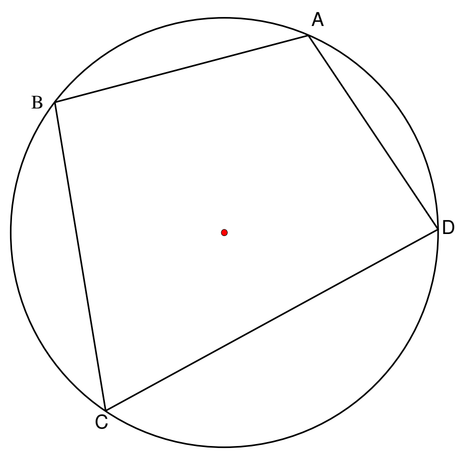
Question Four

Represent the following statement with suitable rough sketches (drawings), as you might do in class with learners.

The angle subtended by a chord at the centre of a circle is equal to twice the angle subtended by the same chord at the circumference.

Question Five

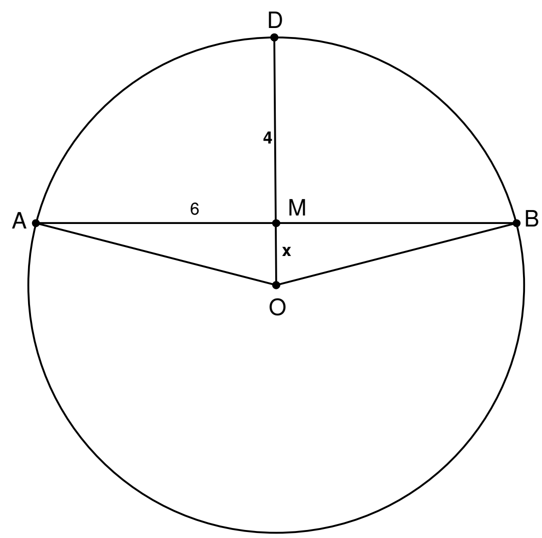
5.1. If angle *A* was a right-angle, would *ABCD* necessarily be a square?



5.2. Justify your answer by logical reasoning or providing a suitable counter example.

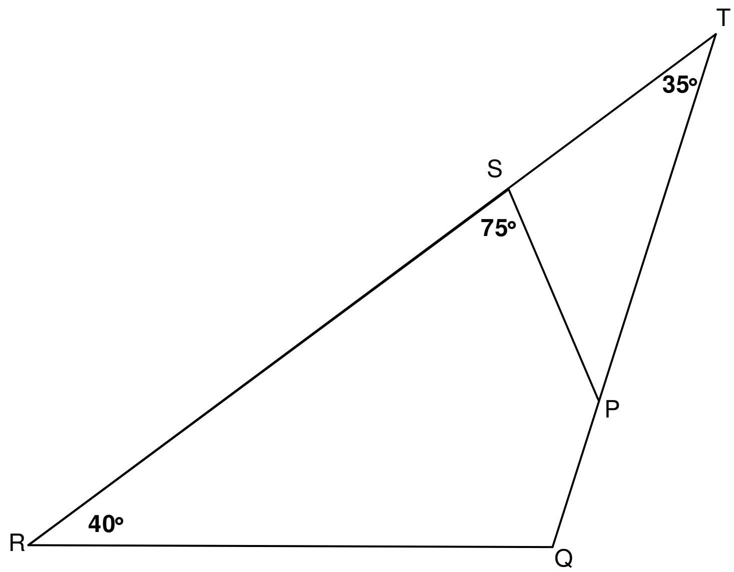
Question Six

AB is a chord of a circle centre O and 12 *cm* long. *M* is the midpoint of *AB*. $MD \perp AB$ cuts circle centre O at D. Calculate the radius of the circle if *MD* = 4 *cm*.



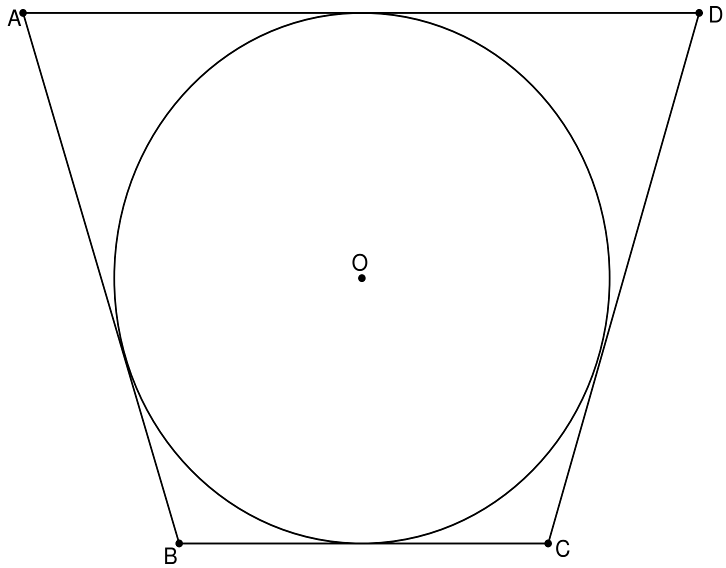
Question Seven

Prove that *PQRS* is a cyclic quadrilateral in the following diagram.



Question Eight

In the figure, O is the centre of the inscribed circle of quadrilateral *ABCD*. The perimeter of the quadrilateral is 25 *cm* and *AD* = 8 *cm*. Find the length of *BC*.



Question Nine

9.1. The perpendicular bisector of the sides of quadrilateral ABCD meet in one point (are concurrent). Will the four vertices A, B, C and D lie on a circle?

9.2. Justify your answer or logical reasoning.

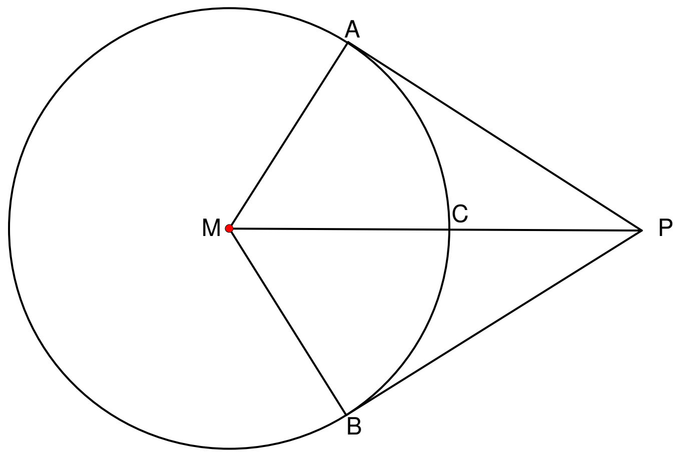
Question Ten

10. Square, rhombus, rectangle, parallelogram, kite or trapezium.

10.1. Which of the above quadrilaterals are always cyclic? Please explain your reasoning.

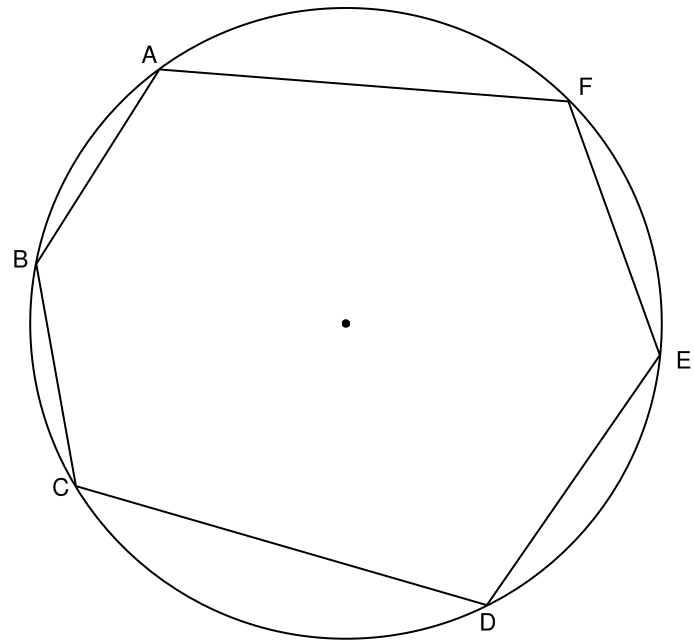
10.2. Which of the above one quadrilaterals always have an inscribed circle? Please explain your reasoning.

Question Eleven



If *PA* and *PB* are tangents to circle *M*, will the kite *PAMB* be cyclic? Justify your answer by saying why or why not.

Question Twelve



Prove that $\angle A + \angle C + \angle E = \angle B + \angle D + \angle F = 360°$

Appendix 5:

Questions, Categories and Levels:

Question	BTC	VHL
1	1	1
2	1 and 2	1 and 2
3	2 and 3	1 and 3
4	2	2
5	3 and 5	2 and 3
6	3	3
7	3	3
8	5	4
9	3 and 4	3
10	3	3 and 4
11	4	3
12	5	4

Appendix 6

P.O. Box 424

Emondlo

3105

01 August 2011

Att: Hlabisa Library Manager

Municipality Park Drive

Hlabisa

3937

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN A LIBRARY

Dear sir or madam

My name is Sikhumbuzo Sithembiso Dhlamini, and a Master of Education (Mathematics Education) student at the University of KwaZulu – Natal (UKZN) in Durban at Edgewood, student number 208524873. The research I am conducting for my Master's thesis involves an investigation into Grade 12 Mathematics teachers' understanding of Euclidean Geometry. This project is conducted under the supervision of Prof. M. De Villiers (UKZN, South Africa).

I am hereby seeking your consent to use the discussion room for research tests and interviews. I have provided you with a copy of my thesis proposal which includes copies of the measure and consent and assent forms to be used in the research process, as well as a copy of ethical clearance which I received from the UKZN Research Ethics Committee.

Upon completion of the study, I undertake to provide the University of KwaZulu – Natal Senate with a bound copy of the full research report. If you require any further information, please do not hesitate to contact me on 083 3354 474/ 072 4846 198, skhmbzsithembiso@yahoo.com.

Thank you for your time and consideration in this matter.

Yours sincerely,

Mr S.S. Dhlamini

University of KwaZulu-Natal (Edgewood)

Appendix 7

P.O. Box 424

Emondlo

3105

01 August 2011

Att: Mtubatuba Library Manager

Civic Centre Park

Hlabisa

3937

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN A LIBRARY

Dear sir or madam

My name is Sikhumbuzo Sithembiso Dhlamini, and a Master of Education (Mathematics Education) student at the University of KwaZulu – Natal (UKZN) in Durban at Edgewood, student number 208524873. The research I am conducting for my Master's thesis involves an investigation into Grade 12 Mathematics teachers' understanding of Euclidean Geometry. This project is conducted under the supervision of Prof. M. De Villiers (UKZN, South Africa).

I am hereby seeking your consent to use the discussion room for research tests and interviews. I have provided you with a copy of my thesis proposal which includes copies of the measure and consent and assent forms to be used in the research process, as well as a copy of ethical clearance which I received from the UKZN Research Ethics Committee.

Upon completion of the study, I undertake to provide the University of KwaZulu – Natal Senate with a bound copy of the full research report. If you require any further information, please do not hesitate to contact me on 083 3354 474/ 072 4846 198, skhmbzsithembiso@yahoo.com.

Thank you for your time and consideration in this matter.

Yours sincerely,

Mr S.S. Dhlamini

University of KwaZulu-Natal (Edgewood)

Appendix 8

Meetings Schedule

Hlabisa Library				Mtubatuba Library			
Dates	**Time**	**Type of Meeting**	**Number of Participants**	**Dates**	**Time**	**Type of Meeting**	**Number of Participants**
01 Sep 11	15h00	Test	Five	03 Sep 11	9h30	Test	Two
02 Sep 11	14h30	Test	One	05 Sep 11	14h30	Test	Two
07 Sep 11	13h30	Task-based Interview	One	10 Sep 11	8h30	Task-based Interview	One
07 Sep 11	16h00	Task-based Interview	One	10 Sep 11	13h00	Task-based Interview	One
12 Sep 11	14h00	Task-based Interview	One	17 Sep 11	9h00	Task-based Interview	One
14 Sep 11	16h00	Task-based Interview	One	17 Sep 11	13h00	Task-based Interview	One
20 Sep 11	13h00	Task-based Interview	One				
20 Sep 11	15h00	Task-based Interview	One				

Appendix 9

Turn-it-in Report

AN INVESTIGATION INTO GRADE 12 TEACHERS' UNDERSTANDING OF EUCLIDEAN GEOMETRY

ORIGINALITY REPORT

13%	12%	3%	4%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	www.lib.ncsu.edu *Internet Source*	1%
2	nwlink.com *Internet Source*	1%
3	math.kennesaw.edu *Internet Source*	1%
4	sigmaa.maa.org *Internet Source*	1%
5	Submitted to Walden University *Student Paper*	1%
6	www.mcdougallittell.com *Internet Source*	1%
7	mzone.mweb.co.za *Internet Source*	1%
8	hied.uark.edu *Internet Source*	1%
9	www.terc.edu *Internet Source*	< 1%
10	Submitted to University of KwaZulu-Natal *Student Paper*	< 1%
11	www.coe.uga.edu *Internet Source*	< 1%
12	www.umaine.edu *Internet Source*	< 1%
13	online.fiu.edu *Internet Source*	< 1%
14	www.k-12prep.math.ttu.edu *Internet Source*	< 1%
15	www.mdot.state.ms.us *Internet Source*	< 1%

Appendix 10

Editor's Report

Crispin Hemson

15 Morris Place

Glenwood

Durban

South Africa 4001

hemson@ukzn.ac.za

C: 082 926 5333
H: 031 206 1738

10th March 2012

TO WHOM IT MAY CONCERN

This is to record that I have carried out a language editing of pages 1-142 of the dissertation by Sikhumbuzo Sithembiso Dhlamini, entitled **An Investigation Into Grade 12 Teachers' Understanding Of Euclidean Geometry**.

Crispin Hemson