AN APPLICATION OF SOME INVENTORY CONTROL TECHNIQUES

BY

CAROL ANNE SAMUELS

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SUPERVISORS: DR W.H. MOOLMAN
PROFESSOR K.C. RYAN

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

There are numerous concepts and techniques that are available to large companies which could assist in the planning and control of inventories.

The purpose of this study is to investigate, with the aid of generally accepted concepts and techniques, possible inventory policies for a particular large company.

1.1 TERM OF REFERENCE FOR AN APPLICATION OF SOME INVENTORY CONTROL TECHNIQUES

The company has at the moment a stock policy which is described in Chapter 2. "An application of some inventory control techniques", will adapt some existing techniques to improve on the decision making regarding the present inventory policy and also, increase the profit of the company.

1.2 DESCRIPTION OF THE PROBLEM

The company purchases perishable goods (mostly foods), keeps it in cold storage (hopefully for a short period of time) and sells these goods to its customers.

The ideal situation for the company would be, when the demand (sales) equals the quantity of goods ordered by the company and that they do not have to wait for the goods to be delivered, that is, a smooth transfer of goods from the point of purchase to the point of sale with the goods kept in storage for a minimum period of time.

However, this ideal situation does not occur in practice. Some of the reasons why this is so, are:

1. Delays when ordering goods.
2. A volatile consumer market which makes sales forecasting rather difficult.

The management of the company must decide how many goods to purchase and when to purchase them.

The two extreme decisions are:

1. To purchase more goods than needed (liberal policy) in order to make sure that they do not run out of stock.

   The penalties for such a policy would be

   (a) that a lot of money (on which a high interest rate is charged) would be tied up in stock;

   (b) considerable strain is placed on their storage facilities which might result in some of the products being damaged as well as an increase in the costs of maintaining the cold storage facilities.

2. To purchase a little less goods than needed (conservative policy). This will lead to a considerable reduction of the strain placed on the storage facilities and less money tied up in stock, but would result in the

company being out of stock at certain times.

In such a case the company's customers would become dissatisfied which in turn will lead to the company losing business.

The ideal inventory control policy would be something in between these two extremes. The accurate forecasting of sales is vital to a formulation of a policy that will lead to a smooth transfer of goods (from the point of purchase to the point of sale).

1.3 OBJECTIVE OF THE STUDY

The objectives of this study are:

(i) To examine the demand pattern of some of the goods of the company covering the period from November 1988 to September 1989. Two forecasting techniques will be used on each of the products to establish whether a forecasting technique would improve the present system of inventory control.

(ii) To find ordering strategies for various policies and to do a test run on the data that became available in the next month, i.e., October 1989.

(iii) To suggest a general inventory control policy that results in the total cost related to stock holding being less than the corresponding cost for the current policy. This should convince the manager that the solution presented will result in a considerable reduction in costs under varying conditions.

(iv) To explain the solution to management, in a language that they (who are non-statisticians) can understand. This must include rules of thumb that can easily be applied. A flow diagram that explains the "best" policy will be given.

1.4 SUMMARY OF THE DATA, GENERAL ASSUMPTIONS AND COST STRUCTURE

At the request of the management of the company that is being investigated, the name of the company is not disclosed. Thus, for the purpose of this study, the fictitious name, "XYZ (Pty) Ltd" will be used and brand-names of products will be kept confidential by using the notation Y_1, Y_2, Y_3, etc. The demand unit for each of the products is in kilograms.

Since the company purchases, stores and sells a few hundred different products, a complete study involving all the products could not be undertaken. This study involves only the three highest selling products which account for about 6% of their total sales. Since the general assumptions vary only slightly for different products, an inventory policy that would be successful for the three selected products, would also be successful for the whole company. The demand for the three selected products, (Y_1, Y_2 and Y_3) over the eleven month period is found in Appendix 1.

The present company inventory policy and the other inventory policies under consideration are implemented by using the demand data that became available during the next month.

The demand for products Y_1, Y_2, and Y_3 during the next month is found in Appendix 2.

The general assumptions used for the investigation of the various inventory control policies are:

1. The demand for the products are probabilistic.

2. The average annual demand remains constant over time.

3. The system under consideration uses transaction reporting, i.e., all transactions of interest are recorded as they occur, and the information is immediately made known to the decision maker.

4. The leadtimes L_i, $i = 1, 2, 3$ are assumed to be fairly deterministic, some products have a leadtime of one week while others have a leadtime of two weeks. Products Y_1, Y_2, and Y_3 have leadtimes of one week. The leadtime is independent of the demand rate and the quantity ordered.

5. The entire quantity is delivered as a single package, that is, it never happens that an order is split so that part of it arrives at one time and part at another time.

6. The unit cost of each product is independent of the quantity ordered.

7. The cost of operating the information processing system is independent of the quantity ordered and the reorder point.

8. The company's inventory control policy allows for lost sales. The lost sales include the lost profit only.

   The period during which lost sales occur is small enough to be neglected, so that the average number of cycles per year is independent of the length of the lost sales period.

The inventory systems under consideration have been defined as systems in which only the following three types of costs are significant, and in which any two or all three are subject to control:

(1) the carrying cost.

(2) the shortage cost.

(3) the replenishing cost.

The corresponding costs can be defined as follows:

C_1: the average carrying cost per year.
C_2: the average shortage cost per year.
C_3: the average replenishing cost per year.

In the systems under study, the unit cost of carrying inventory is Rc_1 per kilogram per year; the unit cost of incurring a shortage in inventory is Rc_2 per kilogram per year; the unit cost of replenishment is Rc_3 for each replenishment; and c_1, c_2 and c_3 are constants for all products.

Thus for the systems under study we have

$$C_1 = c_1 x_1 \tag{1.4.1}$$
$$C_2 = c_2 x_2 \tag{1.4.2}$$
$$C_3 = c_3 x_3 \tag{1.4.3}$$

where x_1 is the average amount carried in inventory, x_2 is the average shortage in inventory, and x_3 is the average number of replenishments per year.

Hence, the total cost per year of the system will be calculated by

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= c_1 x_1 + c_2 x_2 + c_3 x_3 \end{aligned} \tag{1.4.4}$$

The following information was obtained from the manager of the company and is necessary for the analysis of the inventory control policies under study.

The Demand Rate

If the demand of size S units occurs over a period of t years, the demand rate is given by

$$\lambda = \frac{S}{t}$$

Note, t is the largest possible time period for which it is believed that the demand rate is representative of the current demand rate.

If we let $\lambda_i = \dfrac{S_i}{t}$ for products Y_i, $i = 1, 2, 3$

$$S_i = \sum_{j=1}^{231} S_j, \quad \text{where } S_j \text{ is the demand for the } j\text{th day}$$

$$\text{and } t = 231 \text{ days} = \underline{0,916 \text{ years}}$$

Then it follows from totalling the demands in the table in Appendix 1, that

$$\begin{aligned}
\lambda_1 &= \frac{S_1}{t} \\
&= \frac{82\ 872}{0,916} \\
&= \underline{90\ 406} \\
\lambda_2 &= \frac{S_2}{t} \\
&= \frac{403\ 442}{0,916} \\
&= \underline{440\ 119} \\
\lambda_3 &= \frac{S_3}{t} \\
&= \frac{83\ 646}{0,916} \\
&= \underline{91\ 250}
\end{aligned}$$

The Inventory Carrying Charge

The inventory carrying charge will be denoted by the letter I. Since it varies for the different products the carrying charge for product i will be I_i , $i = 1, 2, 3$ where $I_1 = 0,005$, $I_2 = 0,0044$, and $I_3 = 0,009$. The physical dimension of I is cost per year per rand invested in inventory.

The procurement cost per kilogram

The cost per kilogram per year of all products stored is, $C' = \text{R}2,52$.

The inventory holding cost per kilogram per year

$$Y_1 \;:\; c_1 = I_1 C' = (0,005)(\text{R}2,52) = \text{R}0,0126$$

$$Y_2 \;:\; c_1 = I_2 C' = (0,0044)(\text{R}2,52) = \text{R}0,011088$$

$$Y_3 \;:\; c_1 = I_3 C' = (0,009)(\text{R}2,52) = \text{R}0,02268$$

Lost Sales Cost:

For products Y_1, Y_2, and Y_3, the lost sales cost c_2 is R0,03 per kilogram per year.

The Replenishment Cost:

The replenishment cost c_3 is R1,19 for each replenishment. The cost is the same for products Y_1, Y_2, and Y_3.

The Leadtime

The leadtime is 5 days, i.e. $L = \frac{5}{252}$ years $= 0,0198\ 412$ years for each of the three products Y_1, Y_2, and Y_3.

When the inventory control policies under study are implemented, the following information regarding the stock on hand and on order (which will arrive five days later) is available:

Table 1.1: Stock on hand and on order at the start of the implementation of the inventory control policies

PRODUCT	STOCK ON HAND	QUANTITY ON ORDER
Y_1	1812	3500
Y_2	6178	9000
Y_3	1271	3000

CHAPTER 2

AN EVALUATION OF THE PRESENT INVENTORY POLICY

2.1 DESCRIPTION OF THE PRESENT POLICY

The XYZ (Pty) Ltd has at present an inventory policy where the demand for each product for the next day is predicted according to the previous week's demand. The inventory controller always makes sure that there is enough stock for the demand during the leadtime (which is 5 days) and the following week, that is, enough stock for ten days. Everytime a demand is made, a decision with respect to a replenishment is made.

The predicted demand is used for the establishment of what is called an inventory bank. This system is discussed in detail by Naddor (1966).

The company determines the average demand as of the end of day i, by finding the mean demand over a period of M days immediately preceding day i:

$$\bar{S}_i = \frac{1}{M} \sum_{j=i-M+1}^{i} S_j \tag{2.1.1}$$

where S_j = demand during day j.

The company's analysis is concerned only with the bank B_i which is subject to control by a decision maker. The bank is viewed as composed of N days of average demand, that is,

$$B_i = N\bar{S}_i \tag{2.1.2}$$

where N is the number of days that the stock is in the bank.

The inventory on hand at the end of day i is q_i, where

$$q_i = Q_i - S_i\ , \tag{2.1.3}$$

where

$$Q_i = q_{i-1} + R_{i-1}\ , \tag{2.1.4}$$

is the inventory on hand at the beginning of day i,

$$R_i = P_{i-L} \ , \tag{2.1.5}$$

is the replenishment added to the inventory at the end of the day, available at the beginning of day $i+1$ and P_i is the quantity ordered for replenishment on day i.

The quantity P_i is formally given by

$$P_i = \max[B_i - A_i, 0] \tag{2.1.6}$$

where

$$A_i = \begin{cases} q_i & L = 0 \\ q_i + \sum\limits_{j=i-L}^{i-1} P_j & L > 0 \end{cases} \tag{2.1.7}$$

The amount to be replenished on day i raises the available inventories A_i to a bank B_i. No returns are allowed.

The cost calculation will be demonstrated in the next section.

2.2 EVALUATION OF THE PRESENT POLICY

The available demand data for the next month is used for the evaluation of the present policy. In the calculations to follow, the leadtime L is 5 days, M is 5 days and N is 10 days. The following results are obtained by using the formulae in the previous section.

TABLE 2.1 Implementation of the Present Policy for product Y_1

i	Q_i	S_i	q_i	S_i	B_i	A_i	$B_i - A_i$	P_i	R_i
.		350						0	0
.		411						0	0
.		490						0	0
.		376						3500	0
1	1812	415	1397	408	4080	4897	-817	0	0
2	1397	221	1176	383	3830	4676	-846	0	0
3	1176	249	927	350	3500	4427	-927	0	0
4	927	296	631	311	3110	4131	-1021	0	3500
5	4131	344	3787	305	3050	3787	-737	0	0
6	3787	312	3475	284	2840	3475	-635	0	0
7	3475	309	3166	302	3020	3166	-146	0	0
8	3166	362	2804	325	3250	2804	446	446	0
9	2804	238	2566	311	3110	3012	98	98	0
10	2566	323	2243	307	3070	2787	283	283	0
11	2243	264	1979	298	2980	2806	174	174	0
12	1979	275	1704	291	2910	2705	205	205	446
13	2150	320	1803	282	2820	2590	230	230	98
14	1901	260	1641	288	2880	2533	347	347	283
15	1924	304	1620	285	2850	2576	274	274	174
16	1794	324	1470	297	2970	2526	444	444	205
17	1675	274	1401	296	2960	2696	264	264	230
18	1631	240	1391	280	2800	2720	80	80	347
19	1738	413	1325	311	3110	2387	723	723	274
20	1599	281	1319	306	3060	2829	231	231	444
21	1762	388	1374	319	3190	2672	518	518	264

A cycle is defined as the time between the placement of two successive orders.

The number of cycles for this month is 14.

The average inventory held during cycle:

$\underline{\text{One}}$: $\dfrac{\sum_{i=1}^{8} Q_i}{8} = \dfrac{19871}{8} = 2484$ $\quad$ $\underline{\text{Two}}$: $Q_9 = 2804$

$\underline{\text{Three}}$: $Q_{10} = 2566$ $\quad$ $\underline{\text{Four}}$: $Q_{11} = 2243$

$\underline{\text{Five}}$: $Q_{12} = 1979$ $\quad$ $\underline{\text{Six}}$: $Q_{13} = 2150$

$\underline{\text{Seven}}$: $Q_{14} = 1901$ $\quad$ $\underline{\text{Eight}}$: $Q_{15} = 1924$

$\underline{\text{Nine}}$: $Q_{16} = 1794$ $\quad$ $\underline{\text{Ten}}$: $Q_{17} = 1675$

$\underline{\text{Eleven}}$: $Q_{18} = 1631$ $\quad$ $\underline{\text{Twelve}}$: $Q_{19} = 1738$

$\underline{\text{Thirteen}}$: $Q_{20} = 1599$ $\quad$ $\underline{\text{Fourteen}}$: $Q_{21} = 1762$

Thus, the average inventory held is,

$$\frac{\substack{8(2484)+2804+2556+2243+1979+2150+1901 \\ +1924+1794+1675+1631+1738+1599+1762}}{8+1+1+1+1+1+1+1+1+1+1+1+1+1}$$

$$= \underline{2173}$$

Since 14 orders are made during this month, it is assumed that the average number of orders made for the year is 168.

Thus the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,0126)(2173) + 0 + (1,19)(168) \\ &= \underline{\text{R}227,30} \end{aligned}$$

When implementing the present policy for products Y_2 and Y_3, we obtain

TABLE 2.2

Summary of the Present Policy for products Y_2 and Y_3

PRODUCT	AVERAGE INVENTORY	AVERAGE NO. OF ORDERS	AVERAGE YEARLY COST
Y_2	7072	192	R306,89
Y_3	1782	132	R197,50

*Detailed information for the above table is found in Appendix 3.

CHAPTER 3

CLASSICAL INVENTORY POLICIES

3.1 INTRODUCTION

A brief study of the assumptions used by many inventory control policies was undertaken. The assumptions used by the deterministic-lot size policy with no stockouts and the lot size-reorder point policy with stochastic demand, best suited the problem at hand. A forecasting demand policy was also chosen since it will be sensitive to demand fluctuations and will, hence, improve the present system of inventory control.

A short summary of the various formulae involved for these methods will be given in the next section.

3.2 SUMMARY OF THE FORMULAE TO BE USED

3.2.1 DETERMINISTIC-LOT SIZE MODEL WITH NO STOCKOUTS

The deterministic lot size model with no stockouts was discussed by Hadley and Whitin (1963).

The assumptions made are the same as those made in section 1.3 except that the rate of demand for the item is deterministic and that assumption 8 is not applicable, i.e. this policy does not allow for lost sales. Since lost sales is negligible, the deterministic lot-size model is appropriate to the case at hand.

The two terms involved in calculating the average cost for the year include the average yearly carrying cost and the average yearly ordering cost.

The quantity ordered each time the system orders replenishment stock is denoted by P.

Thus, the time T between the placement of orders is $T = \frac{P}{\lambda}$. Similarly, the time between the arrival of successive procurements is T.

Since there are λ demands per year and since all demands are met, the number of orders placed per year must average to $\frac{\lambda}{P}$, and the fixed procurement costs per year average to $\frac{\lambda}{P}c_3$.

The average inventory is one half the sum of the maximum inventory $P + q$ and the minimum inventory q, i.e., $\frac{P}{2} + q$, where q is the on hand inventory in the system at the time of arrival of a procurement.

Hence, the relevant average annual variable cost, which is the sum of ordering and inventory carrying costs is,

$$C = c_1 \left[\frac{P}{2} + q\right] + 0 + c_3 \frac{\lambda}{P} \tag{3.2.1}$$

Examination of equation (3.2.1) shows that the only term which depends on the reorder rule is $c_1 q$. This term is minimized by having $q = 0$, so that the system just runs out of stock as a new procurement arrives. The requirement that $q = 0$ results in equation (3.2.1) being a function of P only, i.e.

$$C = c_1 \frac{P}{2} + c_3 \frac{\lambda}{P} \tag{3.2.2}$$

Using calculus we obtain

$$P^* = P_w = \sqrt{\frac{2\lambda c_3}{c_1}} \tag{3.2.3}$$

An optimal reordering rule for any given P value can be determined as follows:

Let m be the largest integer less than or equal to L/T, where L is the procurement leadtime. Then, if we place an order when the on hand inventory reaches the level

$$\begin{aligned} r_h &= \lambda(L - mT) \\ &= \lambda L - mP \\ &= \mu - mP \end{aligned} \tag{3.2.4}$$

where $\mu = \lambda L$ is the leadtime demand (i.e., the number of units demanded from the time an order is placed until it arrives), the on hand inventory will be zero at the time the order arrives.

The number r_h is called the reorder point. each time the on hand inventory in the system reaches r_h an order for P units is placed.

The reorder point, given by equation (3.2.4) (with p^* replacing P) tells us when an order should be placed. The quantity to be ordered is given by equation (3.2.3).

3.2.2 LOT SIZE-REORDER POINT MODEL WITH STOCHASTIC DEMAND AND LOST SALES

A heuristic approach to solving this model was discussed by Hadley and Whitin (1963).

The assumptions used are those made in section 1.3. So, the lot size-reorder point model with stochastic demand and lost sales is appropriate to the case at hand.

The terms used in calculating the average daily cost include the cost of carrying inventory, the cost of a lost sale, and the ordering cost.

Because of assumption (6) it is unnecessary to include the cost of the units, since the unit cost C_1 is independent of P. The average daily cost of units procured is independent of the order quantity and the reorder point.

If the reorder point r is based on the inventory position or net inventory, then

$$\epsilon(s,r) = \begin{cases} r-s & r-s \geq 0 \\ 0 & r-s < 0 \end{cases} \tag{3.2.5}$$

is the on hand inventory when the procurement arrives when the leadtime demand is s.

The expected amount on hand when a procurement arrives is

$$
\begin{aligned}
q &= \int_0^\infty \epsilon(s,r)h(s)ds \\
&= \int_0^r (r-s)h(s)ds \ .
\end{aligned}
\tag{3.2.6}
$$

where $h(s)$ represents the marginal distribution of leadtime demand.

From equation (3.2.6) it follows that,

$$
\begin{aligned}
q &= \int_0^\infty (r-s)h(s)ds - \int_r^\infty (r-s)h(s)ds \\
&= r - \mu + \int_r^\infty sh(s)ds - rH(r) \ .
\end{aligned}
\tag{3.2.7}
$$

Thus, the average yearly cost of carrying inventory is

$$
c_1\left[\frac{p}{2} + r - \mu\right] + c_1\left[\int_r^\infty sh(s)ds - rH(r)\right] \ . \tag{3.2.8}
$$

The expected number of lost sales per period $\bar{\eta}(r)$ is,

$$
\begin{aligned}
\bar{\eta} &= \int_0^\infty \eta(s,r)h(s)ds \\
&= \int_0^\infty (s-r)h(s)ds \\
&= \int_r^\infty s\ h(s)ds - rH(r) \ ,
\end{aligned}
\tag{3.2.9}
$$

where $H(s)$ is the distribution function of the leadtime demand.

It follows that the average yearly variable cost for the reorder point model with stochastic demand and lost sales is,

$$
\begin{aligned}
C &= c_1\left[\frac{P}{2}+r-\mu\right]+\left(c_1+c_2\frac{\lambda}{P}\right) \\
&\quad \left[\int_r^{\infty} sh(s)ds - rH(r)\right]+\frac{\lambda}{P}c_3 \,.
\end{aligned} \tag{3.2.10}
$$

As before we wish to determine the values of P and r which minimize C.

Using calculus, we obtain

$$P=\sqrt{\frac{2\lambda[c_3+c_2\bar{\eta}(r)]}{c_1}} \tag{3.2.11}$$

and

$$H(r)=\frac{Pc_1}{c_2\lambda+Pc_1}\,. \tag{3.2.12}$$

The reorder point given by equation (3.2.12) is found by using the distribution function and ordinates of the Standard Normal Density. To compute P_2 equation (3.2.9) is used and then the r_2 value is calculated from equation (3.2.12). The procedure is repeated until there is no change in the r value.

If $h(s)$ is a normal distribution, then the equation for the lost sales case is

$$
\begin{aligned}
C &= c_3 \frac{\lambda}{P} + c_1 \left[\frac{P}{2} + r - \mu \right] \\
& \left(c_1 + c_2 \frac{\lambda}{P} \right) \left[(\mu - r)\Phi(\tfrac{r-\mu}{\sigma}) + \sigma \phi(\tfrac{r-\mu}{\sigma}) \right] .
\end{aligned} \tag{3.2.13}
$$

3.3 EVALUATION OF THE CLASSICAL INVENTORY POLICIES

The information given in section 1.3 is used for the calculation of the classical inventory policies when applied for each of the products Y_1, Y_2 and Y_3.

For each of the products approximate theoretical average costs will be calculated and a test run done using the demand data for the next month.

3.3.1 THE LOT-SIZE MODEL WITH NO STOCKOUTS PRODUCT Y_1

The quantity to order each time an order is made is,

$$
\begin{aligned}
P_w^* &= \sqrt{\frac{2\lambda_1 c_3}{c_1}} \\
&= \sqrt{\frac{2(90\,406)(1,19)}{0,0126)}} \\
&= \underline{4\,132}
\end{aligned}
$$

The time between placement of orders is

$$\begin{aligned} T^* &= \frac{P_w^*}{\lambda_1} \\ &= \frac{4132}{90\,406} \\ &= 0,0457049 \text{ years} \end{aligned}$$

The leadtime demand is

$$\begin{aligned} \mu &= \lambda_1 L \\ &= (90\,406)(0,0198412) \\ &= \underline{1794} \end{aligned}$$

The reorder point based on the on hand plus on order inventory level is then $r^* = 1794$.

The reorder point based on the on hand inventory level is

$$r_h^* = \mu - mP \ ,$$

where

$$\begin{aligned} m &= \left[\frac{L}{T}\right] \\ &= \left[\frac{0.0198412}{0,0457049}\right] \\ &= [0,4341153] \\ &= \underline{0} \end{aligned}$$

From the above it follows that

$$r_h^* = \underline{1794}$$

The average yearly cost per cycle is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 0,0126\left(\frac{4132}{2}\right) + 0 + 1,19\left(\frac{90406}{4132}\right) \\ &= \underline{R52,07} \end{aligned}$$

In the following table, $P^* = 4132$ and $r^* = 1794$ is used.

TABLE 3.1 Implementation of the Deterministic-Lot Size Model with no stockouts for Product Y_1.

Day	Available Inventory	Demand	On hand	Order quantity	Arrival
1	1812	415	1397	0	0
2	1397	221	1176	0	0
3	1176	249	927	0	0
4	927	296	631	0	3500
5	4131	344	3787	0	0
6	3787	312	3475	0	0
7	3475	309	3166	0	0
8	3166	362	2804	0	0
9	2804	238	2566	0	0
10	2566	323	2243	0	0
11	2243	264	1979	0	0
12	1979	275	1704	4132	0
13	1704	320	1384	0	0
14	1384	260	1124	0	0
15	1124	304	820	0	0
16	820	324	496	0	0
17	4628	274	4354	0	0
18	4354	240	4114	0	0
19	4114	413	3701	0	0
20	3701	281	3420	0	0
21	3420	388	3032	0	0

The number of cycles during this month is 1.

The average inventory held during this cycle is,

$$\frac{\sum_{i=0}^{12} Q_i}{12} = \frac{29463}{12} = 2455$$

Since 1 order is made during the month, it is assumed that 12 orders will be made on average for the year.

Thus, the average yearly costs is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,0126)(2455) + 0 + (1,19)(12) \\ &= \underline{\text{R}45,21} \end{aligned}$$

Following the same procedure above, for products Y_2 and Y_3 we obtain,

TABLE 3.2

SUMMARY OF THE DETERMINISTIC POLICY WITH NO STOCKOUTS FOR PRODUCTS Y_2 AND Y_3

Product	P_w^*	r_n^*	Average Inventory	Average No. of orders	Average yearly cost
Y_2	9720	8732	7444	24	R111,10
Y_3	3094	1811	2095	12	R61,79

3.3.2 THE LOT SIZE-REORDER POINT MODELS WITH NORMALLY DISTRIBUTED STOCHASTIC DEMANDS

Product Y_1

The expected leaadtime demand and standard deviation of the leadtime demand is estimated by finding the standard deviation of the weekly demand from Table A.1 found in Appendix 1.

For product Y_1 , $\mu = 1794$ and $\hat{\sigma} = 274,55$,

$$\begin{aligned} P_w^* &= \sqrt{\frac{2\lambda_1 c_3}{c_1}} \\ &= \sqrt{\frac{2(90406)(1,19)}{0,1026}} \\ &= \underline{4132} \end{aligned}$$

and

$$\begin{aligned} H(r) = \Phi\left(\frac{r - 1794}{274,55}\right) &= \frac{P_1 c_1}{c_2\lambda_1 + P_1 c_1} \\ &= \frac{(4132)(0,0126)}{3(90406) + 4132(0,0126)} \\ &= \underline{0,0188345} \end{aligned}$$

From tables provided by Johnson (1974) it follows that,

$$\frac{r_1 - 1794}{274,55} = 2,08$$

therefore

$$\begin{aligned} r_1 &\approx 1794 + 571 \\ &= \underline{2365} \end{aligned}$$

To compute P_2 we need

$$\begin{aligned} \bar{\eta}(r_1) &= (\mu - r_1)\Phi\left(\frac{r_1 - \mu}{\sigma}\right) + \sigma\phi\left(\frac{r_1 - \mu}{\sigma}\right) \\ &= (-571)(0,0188345) + 274,55(0,045861) \\ &= -10,7545 + 12,591138 \\ &= \underline{1,8366376} \end{aligned}$$

$$\begin{aligned} P_2 &= \sqrt{\frac{2\lambda_1[c_3 + c_2\bar{\eta}(r)]}{c_1}} \\ &= \sqrt{\frac{2(90406)[1,19 + 3(1,8366)]}{0,0126}} \\ &= \underline{4227} \end{aligned}$$

$$\begin{aligned} \Phi\left(\frac{r_2 - 1794}{274,55}\right) &= \frac{P_2 c_1}{c_2\lambda_1 + P_2 c_1} \\ &= \frac{(4227)(0,0126)}{(0,03)(90406) + 4227(0,0126)} \\ &= \underline{0,0192593} \end{aligned}$$

Hence,

$$\frac{r_2 - 1794}{274,55} = 2,07$$

therefore

$$\begin{aligned} r_2 &\approx 1794 + 568 \\ &\approx \underline{2362} \end{aligned}$$

To compute P_3 we need

$$\begin{aligned} \bar{\eta}(r_2) &= (\mu - r_2)\Phi\left(\frac{r_2 - \mu}{\sigma}\right) + \sigma\phi\left(\frac{r_2 - \mu}{\sigma}\right) \\ &= (-568)(0,0192593) + 274,55(0,046823) \\ &= -10,939282 + 12,855255 \\ &= \underline{1,9159727} \end{aligned}$$

$$\begin{aligned} P_3 &= \sqrt{\frac{2(90406)[1,19 + 0,03(1,9159727)]}{0,0126}} \\ &= \underline{4231} \end{aligned}$$

$$\begin{aligned} \Phi\left(\frac{r_3 - 1794}{274,55}\right) &= \frac{P_3 c_1}{c_2\lambda_1 + P_3 c_1} \\ &= \frac{(4231)(0,0126)}{(0,03)(90406) + (4231)(0,0126)} \\ &= \underline{0,019277} \end{aligned}$$

Hence,

$$\frac{r_3 - 1794}{274,55} = 2,07$$

therefore

$$\begin{aligned} r_3 &\approx 1794 + 568 \\ &= \underline{2362} \end{aligned}$$

Since there has been no change in safety stock, additional iterations are not needed since the changes will be negligible.

The optimal values are $P^* = 4231$ and $r^* = 2362$.

The expected time between placement of orders is

$$T = \frac{P}{\lambda} = \frac{4231}{90406} \quad = \quad 0,0467999 \text{ years}$$

The average annual cost of carrying inventory, lost sales, and ordering, is easily computed from equation (3.2.13).

$$\begin{aligned} C &= \frac{90406}{4231}(1,19) + 0,0126\left[\frac{4231}{2} + 2362 - 1794\right] \\ &\quad + \left(0,0126 + 0,03\frac{3(90406)}{4231}\right)(1,9159727) \\ &= \underline{\text{R}60,49} \end{aligned}$$

Using the above mentioned policy with $P^* = 4231$ and $r^* = 2362$, the following table is obtained.

TABLE 3.3 Implementation of the Lot Size-Reorder Point Model with normally distributed stochastic demands for Product Y_1

Day	Available Stock	Demand	On hand	Order	Arrival
1	1812	415	1397	0	0
2	1397	221	1176	0	0
3	1176	249	927	0	0
4	927	296	631	0	3500
5	4131	344	3787	0	0
6	3787	312	3475	0	0
7	3475	309	3166	0	0
8	3166	362	2804	0	0
9	2804	238	2566	0	0
10	2566	323	2243	4231	0
11	2243	264	1979	0	0
12	1979	275	1704	0	0
13	1704	320	1384	0	0
14	1384	260	1124	0	0
15	5355	304	5051	0	0
16	5051	324	4727	0	0
17	4727	274	4453	0	0
18	4453	240	4213	0	0
19	4213	413	3800	0	0
20	3800	281	3519	0	0
21	3519	388	3131	0	0

The number of cycles for this month is 1.

The average inventory held is,

$$\frac{\sum_{i=1}^{10} Q_i}{10} = \frac{25241}{10} = \underline{2524}$$

Since 1 order is made during this month, it is assumed that 12 orders will be made on average for the year.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,0126)(2524) + 0 + (1,19)(12) \\ &= \underline{\text{R}46,08} \end{aligned}$$

Following the same procedure above, for products Y_2 and Y_3 we obtain,

TABLE 3.4

SUMMARY OF THE LOT-SIZE REORDER POINT MODEL WITH NORMALLY DISTRIBUTED STOCHASTIC DEMAND FOR PRODUCTS Y_2 AND Y_3

Product	P_w^*	r^*	Average Inventory	Average no. of orders	Average yearly cost
Y_2	10162	12140	10943	36	R164,18
Y_3	3434	2486	3438	24	R106,53

*Detailed information for the above table is found in Appendix 5.

CHAPTER 4

AN INVENTORY POLICY BASED ON THE BOX-JENKINS FORECASTING TECHNIQUE

4.1 DESCRIPTION OF THE POLICY

Forecasts of demand for a product is needed to regulate inventories for that product. Forecasting involves analyzing past data and projecting it into the future, usually by employing some appropriate mathematical model by assuming that the underlying demand pattern continues as it has been in the recent past.

Forecasts for the demand for the next week for each of the products Y_1, Y_2, and Y_3 will be obtained by the Box- Jenkins model.

When a demand occurs, the inventory controller will place an order when the amount on day i plus the amount on order is less than the forecasted demand for the week plus the safety stock, i.e.,

$$q_i + \sum_{j=i-4}^{i-1} P_j < \sum_{j=1}^{5} \hat{S}_{i,j} + k\hat{\sigma}_\epsilon \ ,$$

where $\hat{S}_{i,j}$ is the forecasts made on day i for the jth day into the future and $\hat{\sigma}_\epsilon$ the estimated standard deviation of the residuals.

The value of k is obtained from the standard normal distribution such that

$$P(Z < k) = 1 - \alpha \ ,$$

where α can be seen as the probability of not satisfying a demand and k is defined as the safety factor, i.e., the number of standard deviations used to provide safety stock to result in a given service level.

If an order is placed the inventory controller will order

$$\left(\sum_{j=1}^{5} \hat{S}_{i,j} + k\hat{\sigma}_{\epsilon}\right) - \left(q_i + \sum_{j=i-4}^{i-1} p_j\right)$$

$$\text{i.e.} \quad \begin{pmatrix}\text{forecasted demand for the} \\ \text{week plus the safety stock}\end{pmatrix} - \begin{pmatrix}\text{stock on hand plus} \\ \text{the amount on order}\end{pmatrix} \quad .$$

4.2 SUMMARY OF THE BOX-JENKINS FORECASTING TECHNIQUE

The following is a summary of the Box-Jenkins forecasting technique as discussed by Johnson (1974).

In order to apply the Box-Jenkins forecasting technique, the series must be stationary. One way of obtaining stationarity from a nonstationary series is to difference the series. Once the series is stationary, the three-step iterative procedure of model building may begin.

First, a tentative model is identified from actual data. Then, the unknown parameters in the model are estimated. Finally, diagnostic checks are performed to determine the adequacy of the model, or to indicate possible improvements.

1. IDENTIFICATION

Tentative identification of a time series model is done by analysis of historical data. The primary tool used in this analysis is the estimated autocorrelation function. As a supplementary aid the estimated partial autocorrelation function also proves useful.

From the estimated autocorrelation and partial auotcorrelation function, which can be conveniently exhibited by a graph, a tentative model is selected by comparison with the theoretical autocorrelation and partial autocorrelation function patterns.

These theoretical patterns are shown in Table 4.1 as shown by Johnson and Montgomery (1974).

TABLE 4.1 Behaviour of theoretical autocorrelation and partial autocorrelation functions for stationary models.

MODEL	AUTOCORRELATION FUNCTION	PARTIAL AUTO-CORRELATION FUNCTION
AR (P)	Tails off	Cuts off after lag p.
MA (q)	cuts off after lag q	Tails off
ARMA (p,q)	Tails off; exhibits damped sine wave after $(q-p)$ lags	Tails off; exhibits damped sine wave after $(p-q)$ lags

By "tailing off", we mean that the function has an approximately exponential or geometric decay, with a relatively large number of nonzero values. By "cutting off" we mean that the function truncates abruptly, with only a few nonzero values.

2. ESTIMATION

Once the time series has been tentatively identified, the procedure is to obtain estimates of the model parameters. There are quite a number of computer packages that can calculate these estimates. In this study the STATGRAPHICS package will be used to do these calculations.

3. DIAGNOSTIC CHECKING

Model diagnostics, is concerned with testing the goodness-of-fit of a model and, if the fit is poor, suggesting appropriate modifications. Two complementary approaches will be presented: analysis of the overparametised models and analysis of the residuals from the fitted model.

In the analysis of the overparameterised models a general model that contains the identified model which is believed to be an adequate model is fitted.

The identified model would be confirmed if:

1. the estimate of the additional parameter is not significantly different from zero, and

2. the estimates of the parameters in common, do not change significantly from their original estimates.

In the analysis of the residuals the autocorrelation function of the residuals are considered. The residual autocorrelations must be within plus or minus two standard deviations of zero to confirm the adequacy of the fitted model.

Once it has been verified that a time series model is valid, this model may be used to generate optimal (in a minimum mean square error sense) forecasts.

4.3 DESCRIPTION OF NOTATION USED FOR A BOX-JENKINS MODEL

The data under study appears to be nonstationary, since differencing is applied, the Box-Jenkins (abbreviated as an ARIMA model) is referred to as a (p, d, q) model, where

p	=	order of nonseasonal autoregressive term.
d	=	order of nonseasonal differencing.
(1)q	=	order of nonseasonal moving-average term.

For the The seasonal Box-Jenkins (abbreviated as a SARIMA model) is referred to as a $(p, d, q) \times (P, D, Q)_s$ model, where

(1)P	=	order of seasonal autoregressive term.
D	=	order of seasonal differencing.
(1)Q	=	order of seasonal moving-average term.
(1)S	=	length of seasonality.

[1] The symbols P, q, Q and s were used earlier to denote different entities. However, these symbols are standard symbols associated with specifying the Box- Jenkins model and therefore, their use should not cause any confusion.

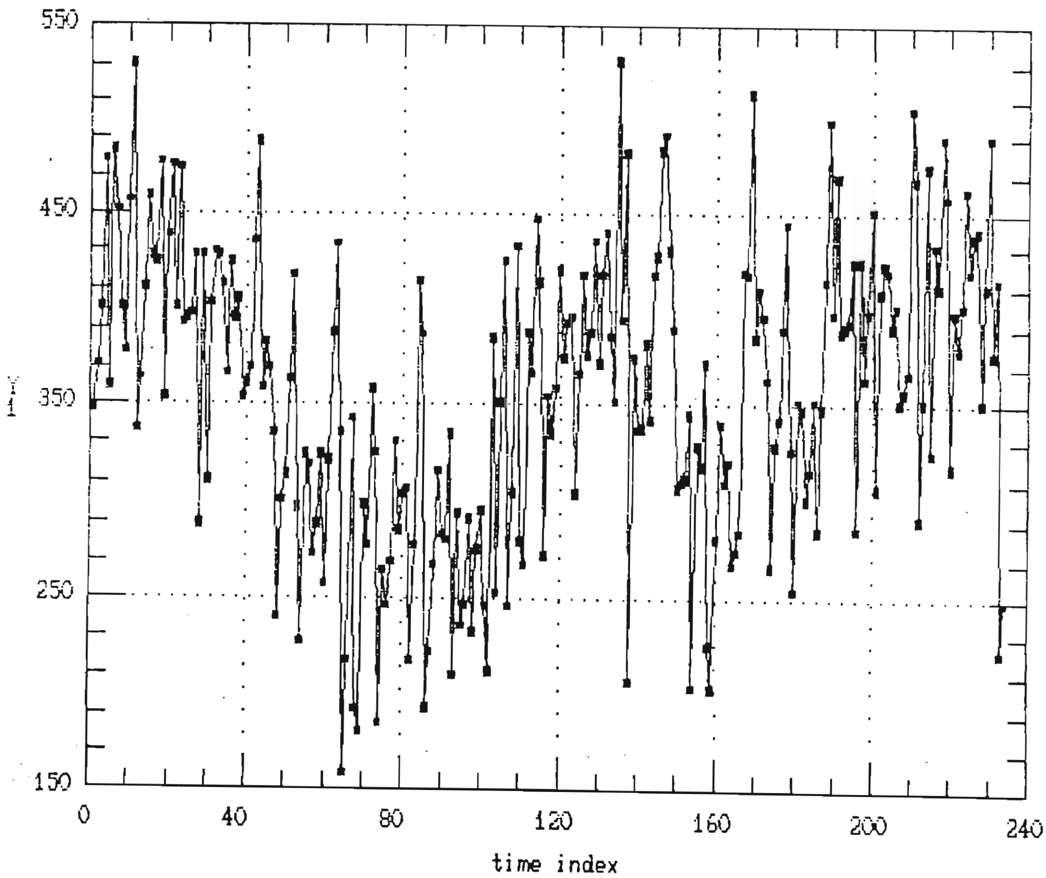
4.4 IMPLEMENTATION OF THE BOX-JENKINS TECHNIQUE

PRODUCT Y_1

The first step before identifying a tentative model, is to check the stationarity of the time series.

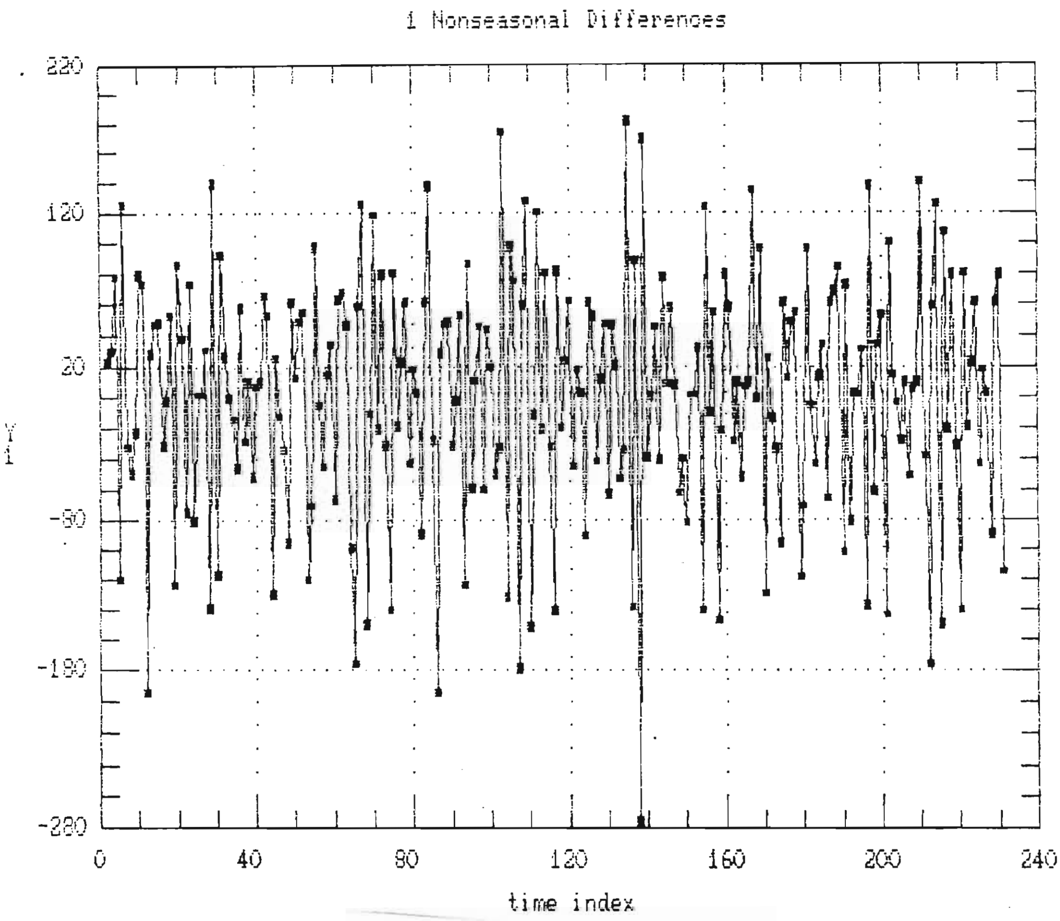
The time series in the figure below, indicates that the series appears to be nonstationary, since it does not appear to have a constant mean over the time period. The series must therefore be differenced.

Fig. 4.1: A Plot of the Original Series of Product Y_1



After differencing once, the series appears to be stationary. See figure 4.2 below.

Fig. 4.2: A Plot of the First Difference of the Series of Product Y_1



1. IDENTIFICATION

In seeking a tentative model, we examine the autocorrelation and partial autocorrelation functions of the differenced series of product Y_1. See figures 4.3 and 4,4 below.

Fig. 4.3: A Plot of the Autocorrelation Function of the Original Series

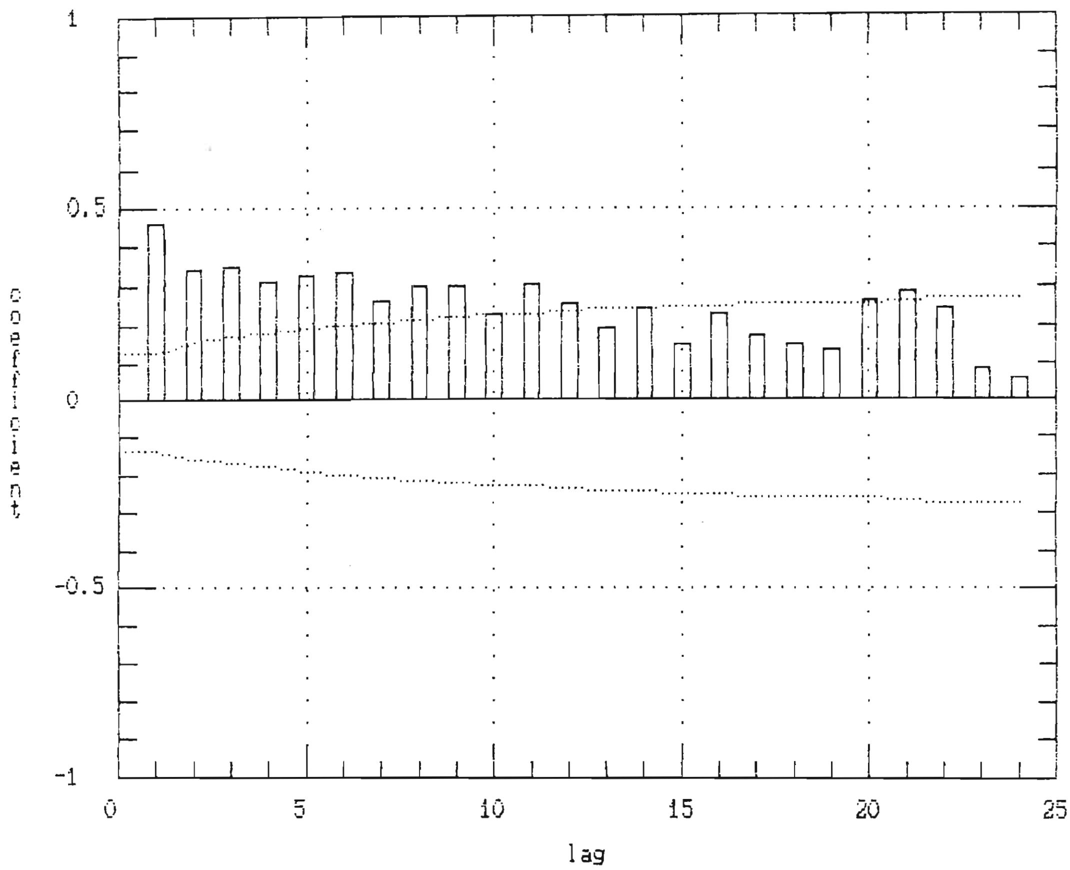


Fig. 4.4: A Plot of the Partial Autocorrelation Function of the Original Series

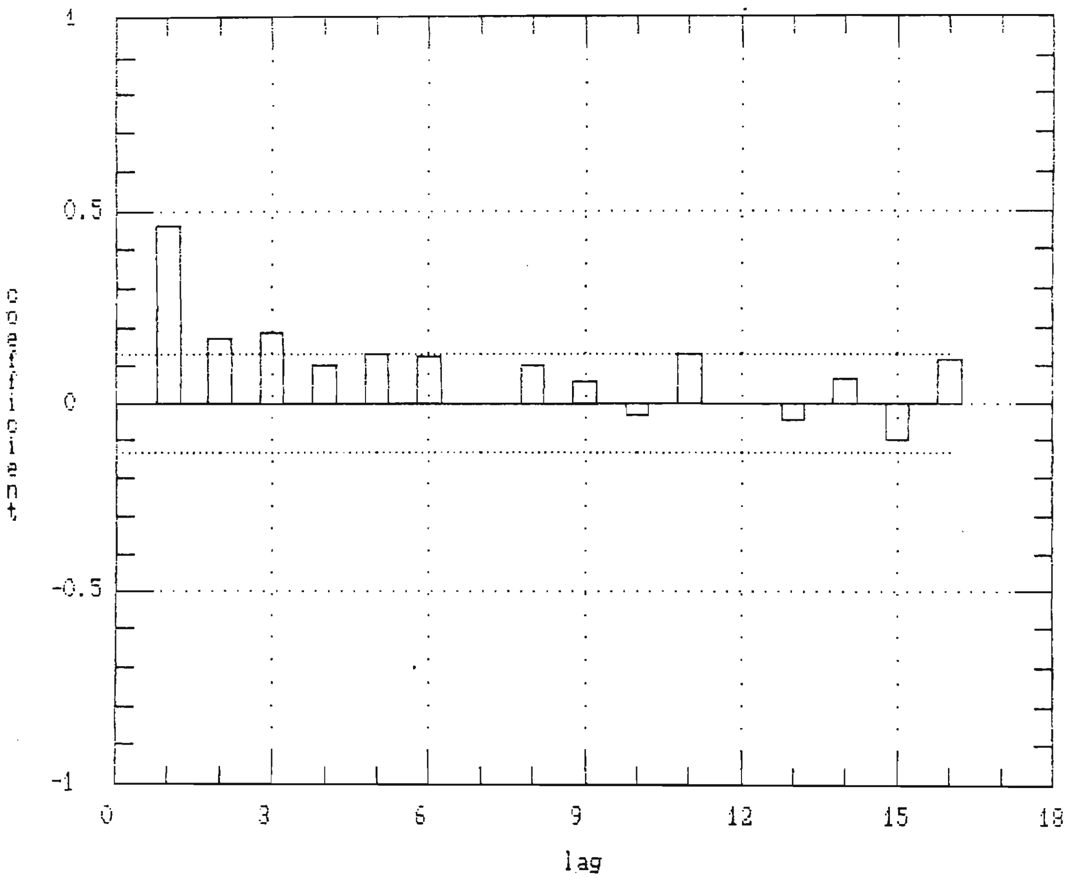


Fig. 4.5: A Plot of the Estimated Autocorrelations of the First Difference of the Series for Product Y_1

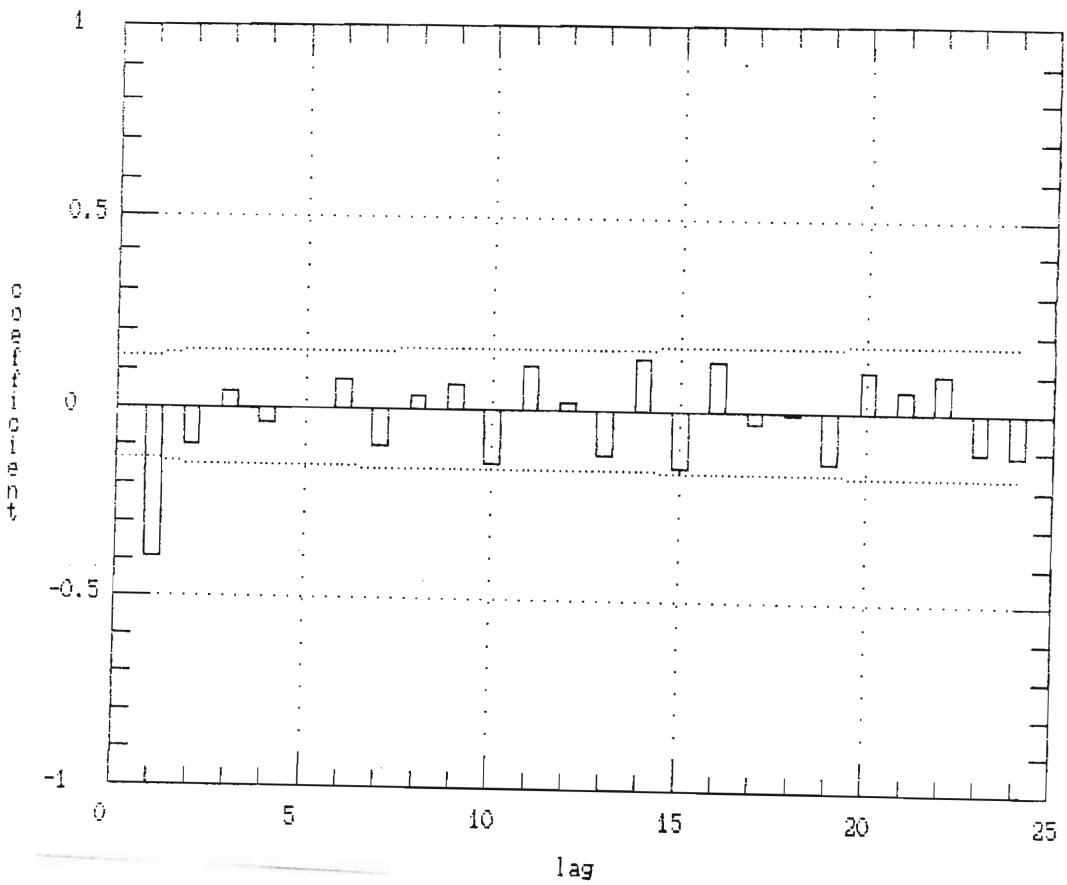
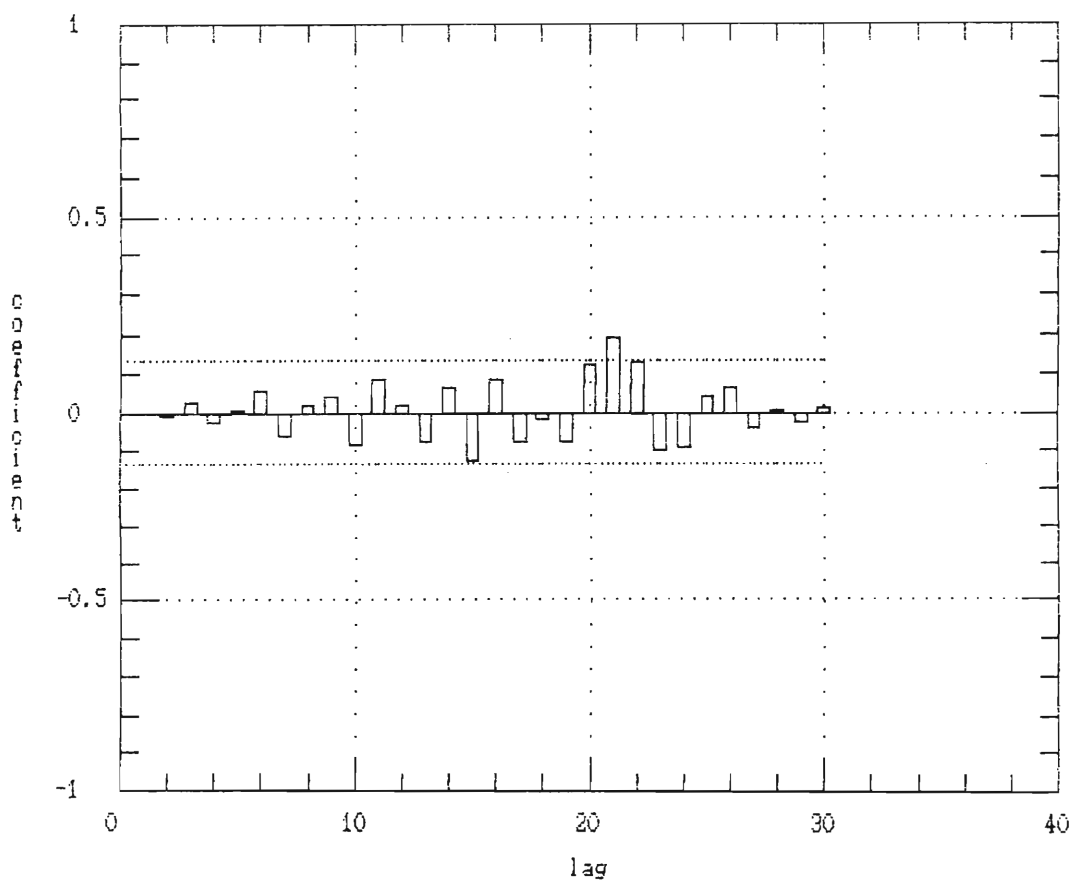


Fig. 4.6: A Plot of the Estimated Partial Autocorrelations of the First Difference of the Series for Product Y_1



The estimated autocorrelations suggest a $MA(1)$ process, since it cuts off after lag 1. The estimated partial autocorrelations seem compatible with this.

2. ESTIMATION

The parameters are estimated using the minimum least squares method, and are shown in the table below.

TABLE 4.2 Products with their respective models and estimated parameters

PRODUCT	PARAMETER	ESTIMATE	STANDARD ERROR
Y_1	MA(1)	0,69540	0,06554
Y_2	MA (1)	0,56744	0,05526
Y_3	MA (1)	0,40553	0,06163

The service level used is chosen by management. In the implementation of the Box-Jenkins Technique to follow, a 95% service level is chosen. Thus, the safety factor $k = 1,65$ is used.

The estimated standard deviation of the residuals were obtained from the fitted models and are shown in the table below. These are used to calculate the order point, i.e., $\sum_{j=1}^{5} \hat{S}_{i,j} + k\hat{\sigma}_{\epsilon}$.

TABLE 4.3 Estimated standard deviation of the residuals for the three products

PRODUCT	$\hat{\sigma}_{\epsilon}$
Y_1	65
Y_2	212
Y_3	68

3. DIAGNOSTIC CHECKING

Although an MA(1) was identified, an MA(2) and an ARMA(1,1) overfit was processed.

TABLE 4.4

PARAMETER ESTIMATES OF THE OVERFIT MODELS.

Model	Parameter Estimates	Standard Error of Estimates	$\hat{\sigma}_a^2$	chi^2
MA(1)	$\hat{\theta} = 0,80393$	0,04148	4213	21,4701
MA(2)	$\hat{\theta}_1 = 0,69540$	0,06554	4169	20,90
	$\hat{\theta}_2 = 0,13336$	0,06586	4169	20,90
ARIMA(1,1)	$\hat{\theta} = 0,85279$	0,04367	4176	20,49
	$\phi = 0,13798$	0,08105	4176	20,49

For the ARIMA(1,1) overfit

The $\hat{\phi}$ is not significantly different from zero. Therefore, the ARIMA(1,1) overfit is not justified.

For the MA(2) overfit

The $\hat{\theta}_2$ is just significantly different from zero and the parameters in the MA(2) overfit are significantly different from those of the MA(1) model. Also, the shock variance is smaller for the MA(2) overfit and the χ^2 value for the MA(2) overfit is nonsignificant. Therefore, the MA(2) overfit is justified.

In the analysis of the residuals of the MA(1) process, the estimated residual autocorrelations and partial autocorrelations lie within plus or minus two standard deviations, hence, the MA(1) model is justified. See Figures 4.5 and 4.6.

But, since the MA(2) overfit was justified in the analysis of overparameterisation, the estimated residuals of the MA(2) overfit are examined. See figures 4.7 and 4.8. The estimated residual autocorrelations and partial autocorrelations lie within plus or minus two standard deviations, hence, the MA(2) model is justified.

Fig 4.7: A Plot of the Estimated Residual Autocorrelations of Product Y_1 for the MA(1) Process

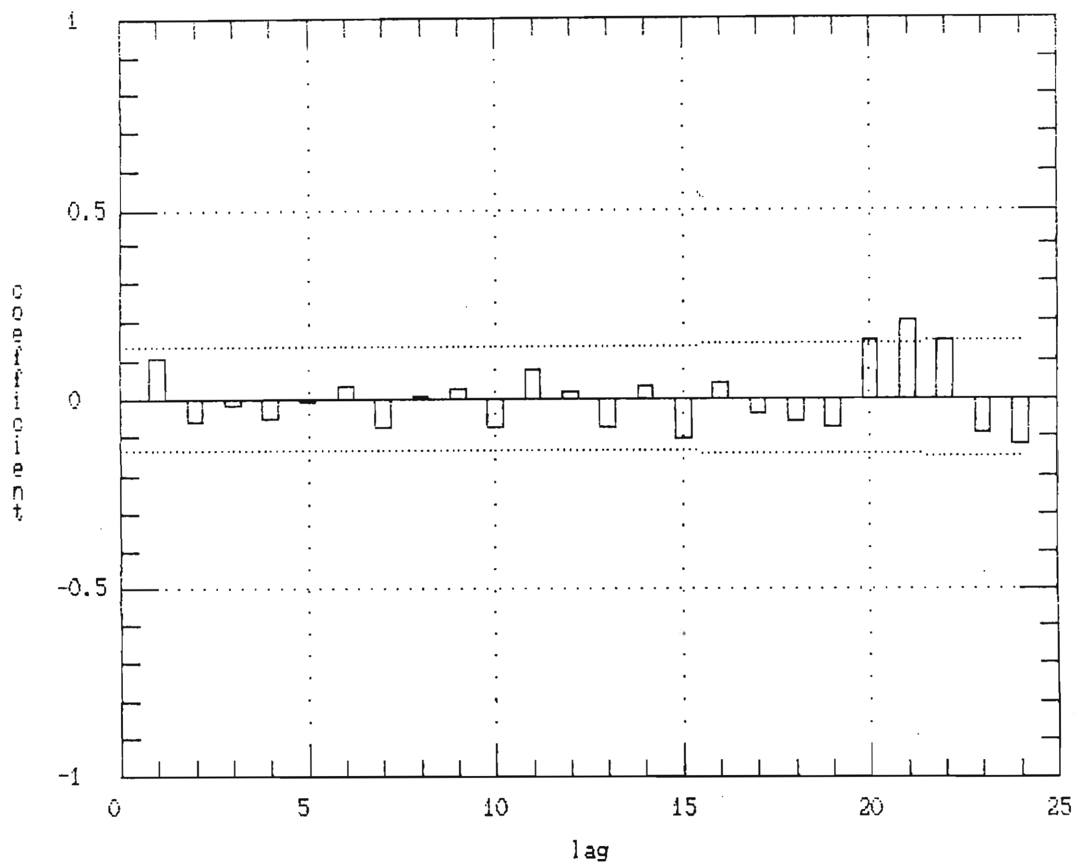


Fig 4.8: A Plot of the Estimated Residual Partial Autocorrelations of Product Y_1 for the MA(1) Process

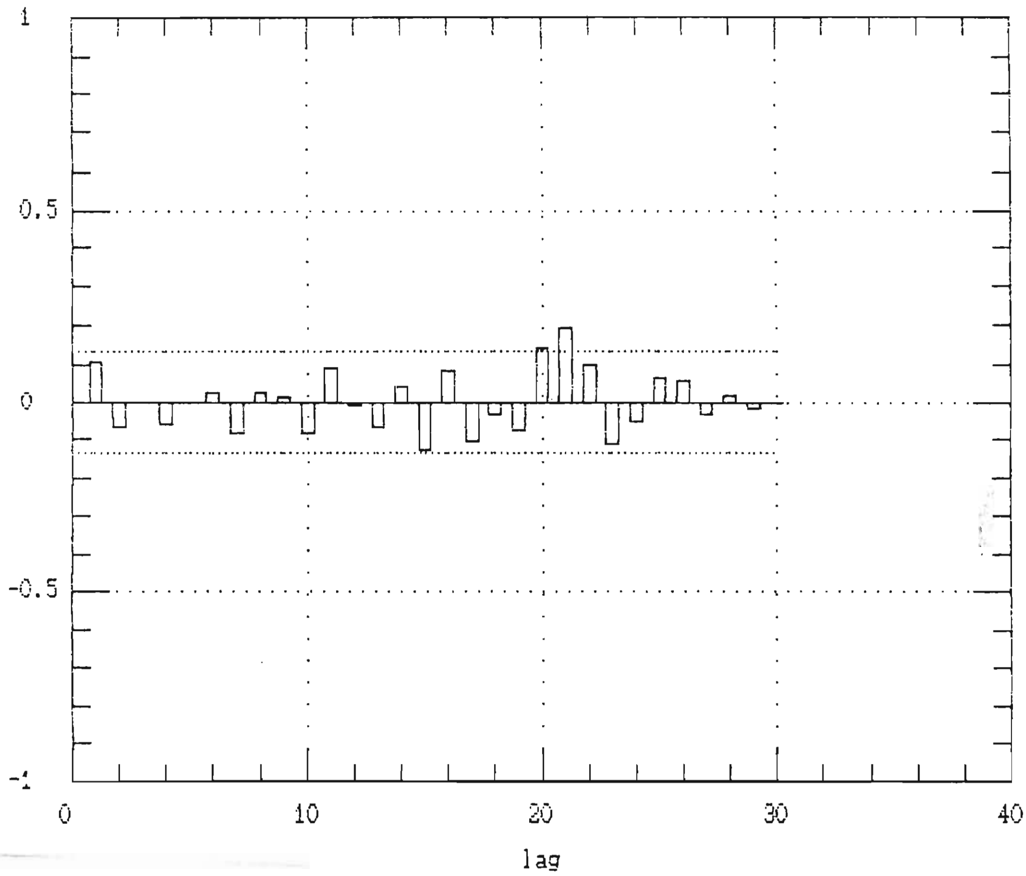


Fig. 4.9: A Plot of the Estimated Residual Autocorrelations of Product Y_1 for the MA(2) Process

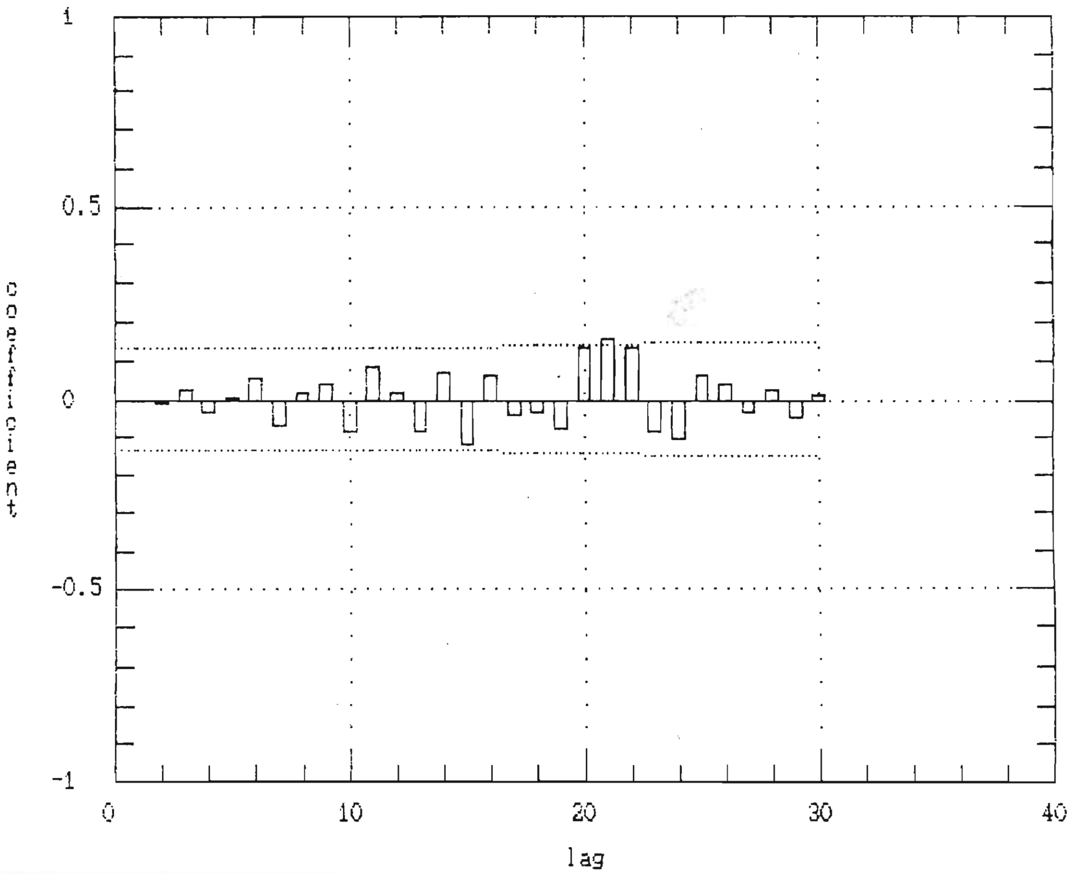
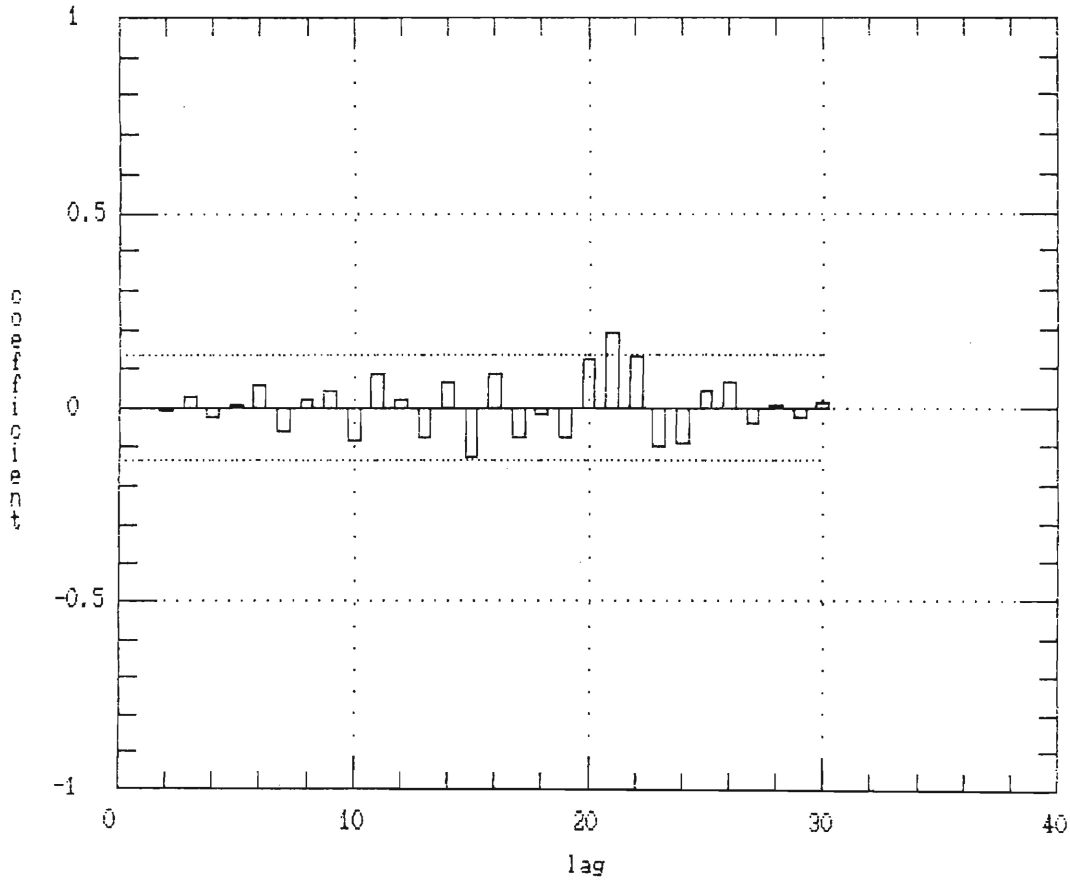


Fig. 4.10: A Plot of the Estimated Residual Partial Autocorrelations of Product Y_1 for the MA(2) Process



At the end of each day the demand is updated and the statgraphics package is used to forecast the five $\hat{S}_{i,j}$ values for the next five days.

Table 4.4 demonstrates the implementation of the inventory control policy based on Box-Jenkins forecasting technique.

TABLE 4.5 Implementation of the inventory control policy based on Box-Jenkins forecasting technique for Product Y_1

Day	Available Stock	Demand	On Hand	Forecasted Demand	Forecasted Demand for the week plus safety stock	Order Quantity	Arrival
1	1812	415	1397	416 415 415 415 415	2183	0	0
2	1397	221	1176	357 382 382 382 382	1992	0	0
3	1176	249	927	349 364 363 363 363	1909	0	0
4	927	296	631	346 353 353 353 353	1865	0	3500
5	4131	344	3787	353 353 353 352 352	1870	0	0

Day	Available Stock	Demand	On Hand	Forecasted Demand	Forecasted Demand for the week plus safety stock	Order Quantity	Arrival
6	3787	312	3475	339 345 344 344 344	1823	0	0
7	3475	309	3166	334 338 338 338 338	1793	0	0
8	3166	362	2804	348 344 343 343 343	1828	0	0
9	2804	238	2566	308 322 321 321 321	1700	0	0
10	2566	323	2243	327 324 324 324 324	1730	0	0
11	2243	264	1979	304 311 311 311 310	1654	0	0
12	1979	275	1704	302 305 305 304 304	1627	0	0
13	1704	320	1384	311 308 308 308 307	1649	265	0

Day	Available Stock	Demand	On Hand	Forecasted Demand	Forecasted Demand for the week plus safety stock	Order Quantity	Arrival
14	1384	260	1124	292	1587	198	0
				298			
				297			
				297			
				296			
15	1124	304	820	302	1606	323	0
				300			
				299			
				299			
				299			
16	820	324	496	307	1627	345	0
				304			
				303			
				303			
				303			
17	496	274	222	293	1586	233	265
				297			
				297			
				296			
				296			
18	487	240	247	280	1529	183	198
				286			
				286			
				285			
				285			
19	445	413	32	328	1678	562	323
				311			
				311			
				311			
				310			
20	355	281	74	297	1610	213	345
				302			
				302			
				301			
				301			
21	419	388	31	405	2164	942	233
				413			
				413			
				413			
				413			

The number of cycles during this month is 9.

The average inventory held during this cycle is:

One: $\frac{\sum_{i=1}^{13} Q_i}{13} = \frac{27667}{13} = 2128$

Two: $Q_{14} = 1384$

Three: $Q_{15} = 1124$

Four: $Q_{16} = 820$

Five: $Q_{17} = 496$

Six: $Q_{18} = 487$

Seven: $Q_{19} = 445$

Eight: $Q_{20} = 355$

Nine: $Q_{21} = 419$

The average inventory held is,

$$\frac{13(2128) + 1384 + 1124 + 820 + 496 + 487 + 445 + 355 + 419}{13 + 1 + 1 + 1 + 1 + 1 + 1 + 1} = \underline{1581}$$

Since 9 orders are made during this month, it is assumed that the average number of orders made for the year is 108.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 0,0126(1581) + 0 + 1,19(108) \\ &= \underline{R148,44} \end{aligned}$$

Following the same procedures above, for products Y_2 and Y_3 we obtain

TABLE 4.6

SUMMARY OF THE INVENTORY CONTROL POLICY BASED ON BOX-JENKINS TECHNIQUE FOR PRODUCTS Y_2 AND Y_3

Product	Average Inventory	Average No. of Orders	Average Yearly cost
Y_2	4009	156	R232,04
Y_3	1504	84	R134,16

*Detailed information for the above table is found in Appendix 7.

CHAPTER 5

AN INVENTORY POLICY BASED ON BROWN'S EXPONENTIAL SMOOTHING TECHNIQUE

5.1 DESCRIPTION OF THE POLICY

The inventory policy is exactly like the one used in section (4.1) except that the forecasts are obtained by using Brown's Exponential Smoothing Technique.

5.2 SUMMARY OF BROWN'S EXPONENTIAL SMOOTHING TECHNIQUE

The following is a summary of the exponential smoothing technique as discussed by Wheelwright and Makridakis (1980). Brown's exponential smoothing technique was chosen since it can forecast five values ahead when using the statgraphics package. Also, Brown's Exponential Smoothing Technique is capable of handling a trend pattern. Another advantage of this technique is that is can also handle the horizontal pattern just as well as the simple exponential smoothing can. Even when there is a step change horizontally, Brown's Exponential Smoothing Technique can adjust it rapidly.

Exponential Smoothing is very similar to the Moving Averages approach but does not use a constant set of weights for the N most recent observations. Rather, an exponentially decreasing set of weights is used so that the more recent values receive more weight than older values. Additionally, the computational characteristics of this method make it unnecessary to store all of the past values of the data series being forecast. The only data required are the weight that will be applied to the most recent value (often called alpha), the most recent forecast and the most recent actual value.

The equations used in implementing Brown's one-parameter linear exponential smoothing are shown below as in (5.1) through (5.5).

$$S'_t = \alpha S_t + (1-\alpha)S'_{t-1} \tag{5.1}$$

$$S''_t = \alpha S'_t + (1-\alpha)S''_{t-1} \tag{5.2}$$

where

S_t is the actual demand
S'_t is the single exponential smoothed value
S''_t is the double exponential smoothed value

$$\begin{aligned} a_t &= S'_t + (S'_t - S''_t) \\ &= 2S'_t - S''_t \end{aligned} \tag{5.3}$$

$$b_t = \frac{\alpha}{1-\alpha}(S'_t - S''_t) \tag{5.4}$$

$$F_{t+m} = a_t + b_t m \tag{5.5}$$

where

α is the exponential smoothing constant,
m is the number of periods ahead to forecast.

5.3 IMPLEMENTATION OF BROWN'S EXPONENTIAL SMOOTHING TECHNIQUE

The first step, is to choose alpha. Alpha is chosen by comparing the M.S.E. (mean square error) values for different alpha values. The alpha that gives the minimum mean square error is chosen. The minimum mean square error is determined through trial and error. A value for alpha is chosen, the mean square error is computed over a test set, and then another α value is tried. The MSE's are then compared to find the α value that gives the minimum MSE.

In the table below, using all past data for product Y_1, MSE = 4336,17 when $\alpha = 0,2$, is the minimum MSE. The table below was computed by the stats-graphics package through Brown's Exponential Smoothing Technique.

TABLE 5.1 FORECAST SUMMARY FOR PRODUCT Y_1

Data: Y1

Percent: 100

Forecast summary	M.E.	M.S.E.	M.A.E.	M.A.P.E.	M.P.E.
Simple: 0.1	1.32807	4500.93	51.7069	15.9953	-3.64386
Simple: 0.2	0.08042	4336.17	50.3252	15.6039	-3.67215
Simple: 0.3	-0.32141	4390.43	50.9247	15.8135	-3.66387

At the end of each day the demand is updated and the statgraphics package is used to forecast the five $\hat{S}_{i,j}$ values for the next five days. These forecasted values are used in the implementation of the inventory control policy based on Brown's Exponential Smoothing Technique. See Table 5.1 below.

TABLE 5.2

IMPLEMENTATION OF THE BROWN'S EXPONENTIAL SMOOTHING TECHNIQUE FOR PRODUCT Y_1

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FORECASTED DEMAND	FORECASTED DEMAND FOR WEEK & SAFETY STOCK	ORDER QUANTITY	ARRIVAL
1	1812	415	1397	415	2184	0	0
				415			
				415			
				415			
				415			
2	1396	221	1176	415	2184	0	0
				415			
				415			
				415			
				415			
3	1176	249	927	376	1989	0	0
				376			
				376			
				376			
				376			
4	927	296	631	351	1864	0	3500
				351			
				351			
				351			
				351			
5	4131	344	3787	340	1809	0	0
				340			
				340			
				340			
				340			
6	3787	312	3475	341	1814	0	0
				341			
				341			
				341			
				341			
7	3475	309	3166	335	1784	0	0
				335			
				335			
				335			
				335			
8	3166	362	2804	330	1759	0	0
				330			
				330			
				330			
				330			
9	2804	238	2566	336	1789	0	0
				336			
				336			
				336			
				336			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FORECASTED DEMAND	FORECASTED DEMAND FOR WEEK & SAFETY STOCK	ORDER QUANTITY	ARRIVAL
10	2566	323	2243	315	1684	0	0
				315			
				315			
				315			
				315			
11	2243	264	1979	317	1694	0	0
				317			
				317			
				317			
				317			
12	1979	275	1704	306	1639	0	0
				306			
				306			
				306			
				306			
13	1704	320	1384	300	1609	225	0
				300			
				300			
				300			
				300			
14	1384	260	1124	304	1629	280	0
				304			
				304			
				304			
				304			
15	1124	304	820	295	1584	259	0
				295			
				295			
				295			
				295			
16	820	324	496	297	1594	334	0
				297			
				297			
				297			
				297			
17	496	274	222	302	1619	299	225
				302			
				302			
				302			
				302			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FORECASTED DEMAND	FORECASTED DEMAND FOR WEEK & SAFETY STOCK	ORDER QUANTITY	ARRIVAL
18	447	240	207	297 297 297 297 297	1594	215	280
19	487	413	74	285 285 285 285 285	1534	353	259
20	333	281	52	311 311 311 311 311	1664	411	334
21	386	388	-2	305 305 305 305 305	1634	358	299

The number of cycles during this month is 9.

The average inventory held during cycle:

<u>One</u>: $\frac{\sum_{i=1}^{13} Q_i}{13} = \frac{31\,167}{13} = \underline{2397}$

<u>Two</u>: $Q_{14} = 1384$

<u>Three</u>: $Q_{15} = 1124$

<u>Four</u>: $Q_{16} = 820$

<u>Five</u>: $Q_{17} = 496$

<u>Six</u>: $Q_{18} = 447$

<u>Seven</u>: $Q_{19} = 487$

Eight: $Q_{20} = 333$

Nine: $Q_{21} = 386$

The average inventory held is,

$$\frac{13(2397) + 1384 + 1124 + 820 + 496 + 447 + 487 + 333 + 386}{13 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1}$$

$$= \underline{1745}$$

Since 9 orders are made during this month, it is assumed that the average number of orders made for the year is 108.

The number of lost sales made during this month is 2. Therefore, the expected number of lost sales for the year is 24.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,0126)(1745) + (0,03)(24) + (1,19)(108) \\ &= \underline{R151,23} \end{aligned}$$

When implementing Brown's Exponential technique for products Y_2 and Y_3, we obtain

TABLE 5.3

SUMMARY OF RESULTS OF BROWN'S EXPONENTIAL TECHNIQUE FOR PRODUCTS Y_2 AND Y_3

PRODUCT	AVERAGE INVENTORY	AVERAGE NO. OF ORDERS	AVERAGE YEARLY COST
Y_2	4039	156	R474,86
Y_3	1508	84	R147,48

Detailed information for the above table is found in Appendix 8.

CHAPTER 6

IMPLEMENTATION

6.1 COMPARISON OF POLICIES

The following table gives a summary of the individual and the total costs for the various inventory control policies:

TABLE 6.1 **Comparison of the individual and total costs for the policies under study**

POLICIES	Y_1	Y_2	Y_3	TOTAL
PRESENT POLICY	R227,30	R306,89	R197,50	R731,69
DETERMINISTIC WITH NO STOCKOUTS	R45,21	R111,10	R61,79	R218,10
REORDER POINT WITH STOCHASTIC DEMAND	R46,08	R164,18	R106,53	R316,79
FORECASTING DEMAND POLICY	R148,44	R718,25	R159,99	R1026,68
BROWN'S EXPONENTIAL TECHNIQUE	R151,23	R474,86	R147,48	R773,57

When comparing the total costs of the different policies we see that the determinsitic policy with no stockouts is the best, in the sense that it gives the lowest cost of all the policies considered, for the three products.

It costs the company an average of R585,48 per year, i.e. (R199, 92 + R228, 48 + R157, 08 on average), for ordering products Y_1, Y_2, and Y_3 respectively, when using the present policy. Yet, it only costs the company an average of R57,12 per year, i.e. (R14, 28 + R28, 56 + R14, 28 on average), for ordering products Y_1, Y_2, and Y_3 respectively, when using the deterministic policy. It is, therefore, clear that the present policy causes the inventory controller to order more times than necessary.

With the above points in mind, it is suggested that the company implement the determinstic policy with no stockouts.

6.2 IMPLEMENTATION OF THE "BEST" POLICY

Flow charts are used to simplify the procedure for implementing the deterministic policy with no stockouts. The flow charts should be used to program the procedures for the implementation of the deterministic no stockouts inventory control policy. A description of the operation of the flow charts is given after the flow charts.

Fig. 6.1 Flow diagram for the calculation of P_w^* and r_h^*

PROGRAM: $P_w^* r_h^*$

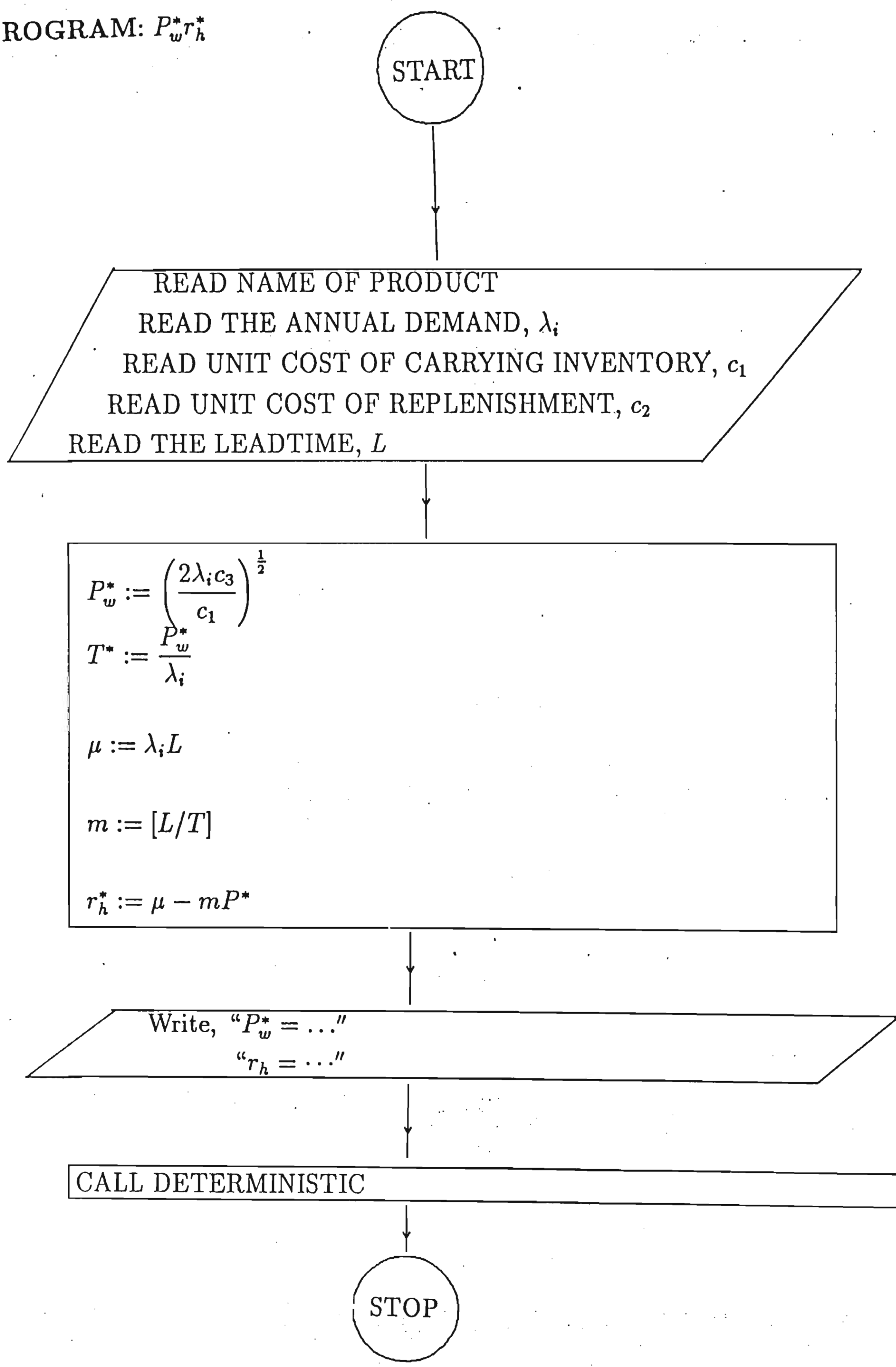
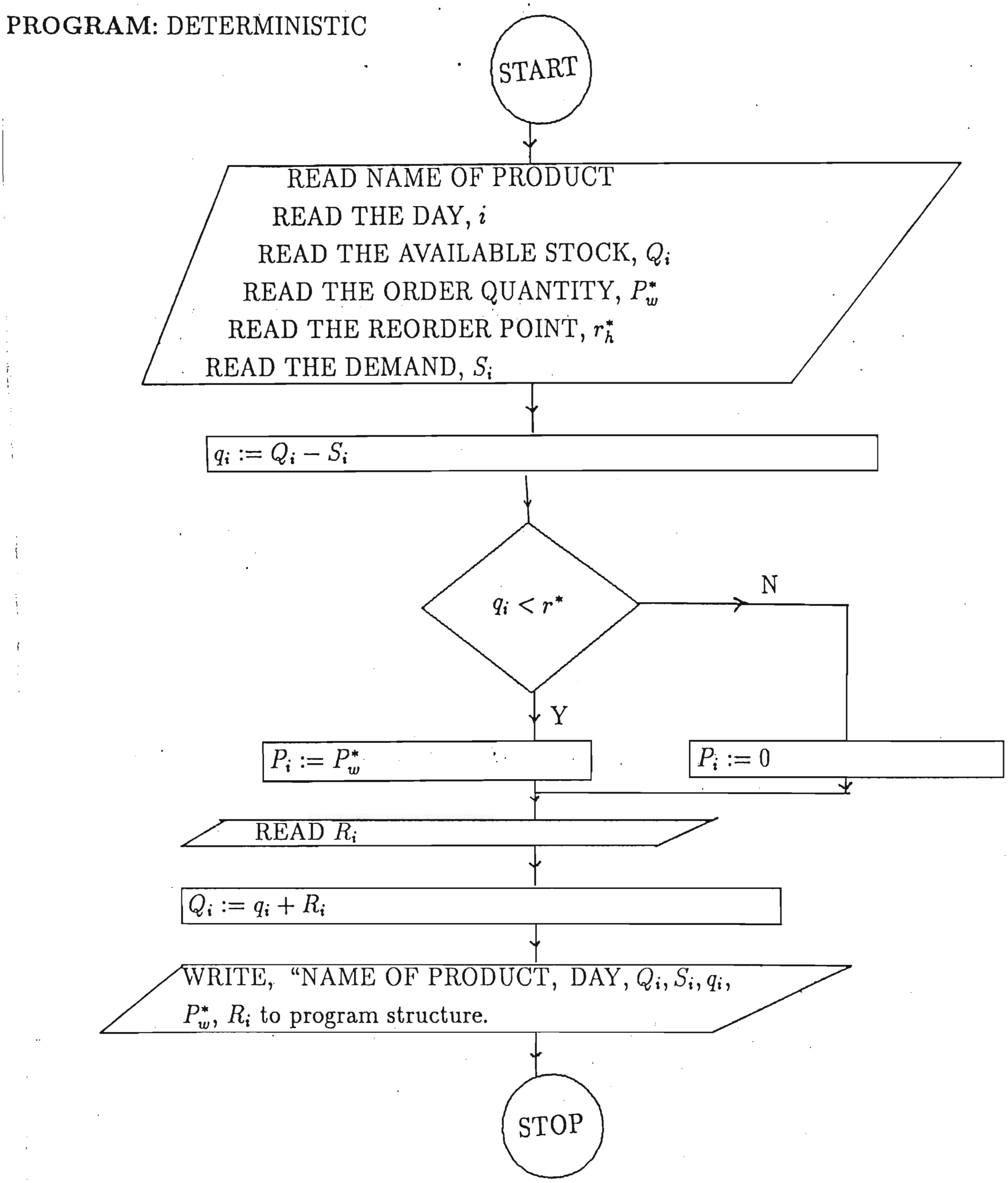
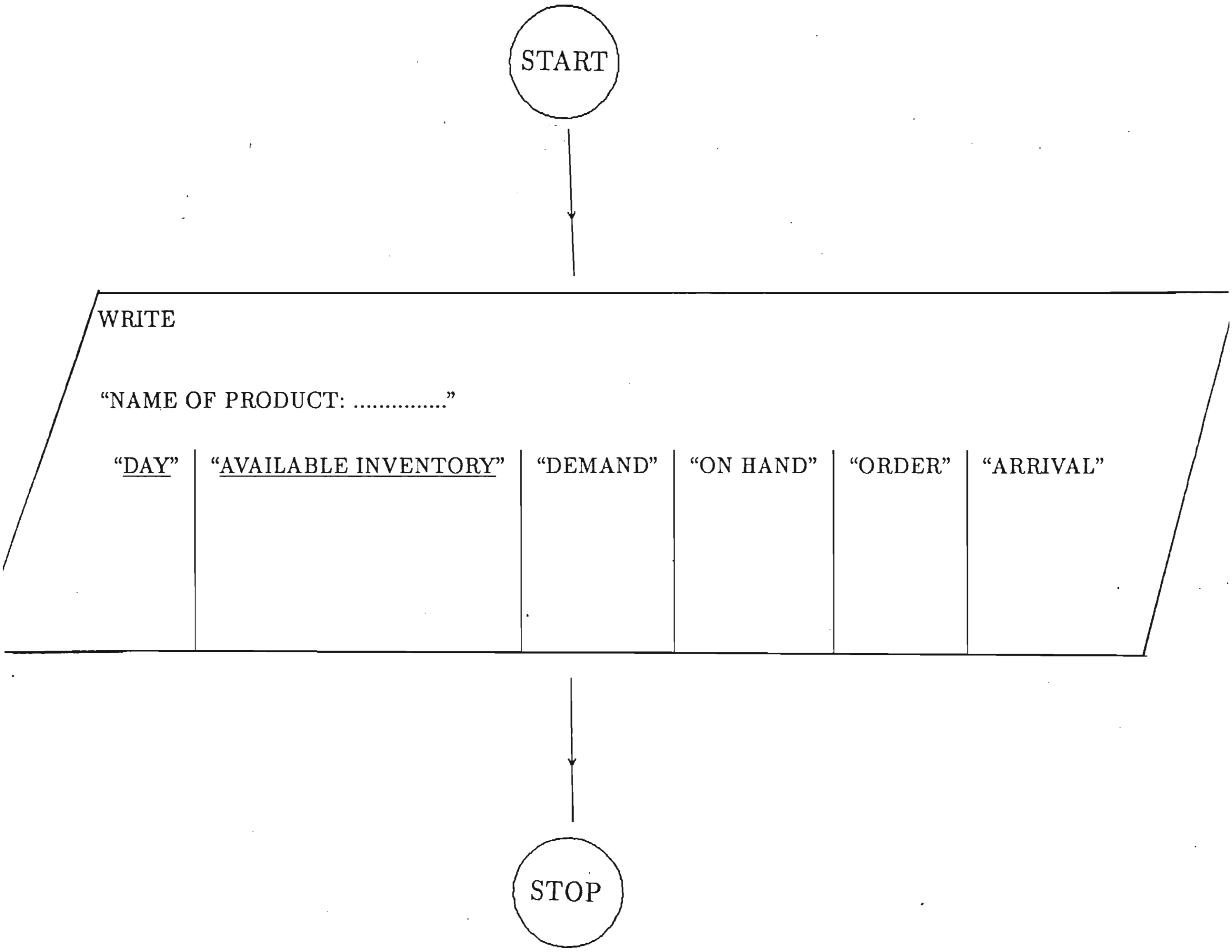


Fig. 6.2 Flow diagram for implementing the deterministic lot-size model with no stockouts.



6.3 Flow diagram of the presentation of the information of the "best" policy

PROGRAM: STRUCTURE



Firstly, all past available data are used to calculate the annual demand rate. The annual demand rate is calculated by dividing the total demand by the time period, and then inputting its value together with the unit cost of carrying inventory and the unit cost of replenishment so that P_w^* and r_h^* can be calculated.

As soon as a demand occurs, and if, the inventory controller does not know P_w^* and r_h^* for that particular product, he calls program "$P_w^* r_h^*$" and determines these values.

Once P_w^* and r_h^* are known, the inventory controller will call program "deterministic" and input the required information. The stock on hand will then be calculated and compared with the reorder point. If the stock on hand is less than the reorder point, the inventory controller will order a quantity of P_w^*. If the stock on hand is more than the reorder point, the inventory controller will not order.

When goods that were ordered arrive, the inventory controller updates the available stock. Program "deterministic" will then output the necessary information for the screen as shown in Fig. 6.3.

The advantages of implementing the deterministic no stockouts policy are:

1. The policy operates smoothly without human intervention.

2. The technique described is reasonably simple and fast in operation and will not consume excessive calculation time.

The users will be trained by myself on how to handle the deterministic policy. Since there is a change from the existing system, the basic principles will be taught before implementation takes place. Once all seems well, the tests are okay, the users are satisfied it does work and does what they want, then it will be time to implement the deterministic policy on all the products.

6.3 CONCLUSION

The main objective of this study was to examine the present inventory control policy and other inventory control policies to see whether costs for the company could be minimised.

The present policy and the inventory control policies under study were implemented by using the demand data of the recent month. Results showed that the deterministic policy would minimise the costs of the company.

Since there is a difference of R513,55 per kg, i.e., (R731,65 - R218,10) per kg between the total costs for the present policy and the determinstic policy for only the three products studied, it is clear that when all products are used in the implementation of the deterministic policy, the company's inventory control costs would be minimized considerably.

APPENDIX 1

TABLE A.1: DEMAND SUMMARY OF PRODUCTS[1]

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
1	347	2402	184
2	371	2015	194
3	401	2090	230
4	479	2163	181
5	359	1945	217
6	484	1662	234
7	451	1845	193
8	401	1884	165
9	378	2031	253
10	457	2191	198
11	530	1840	267
12	336	1627	53
13	364	1914	184
14	411	1719	153
15	459	1765	154
16	427	1812	136
17	424	1897	278
18	477	1914	217
19	353	1902	225
20	438	2105	269
21	476	2166	318
22	401	1814	350
23	474	1784	248
24	393	1876	268
25	395	2111	257
26	397	1749	230
27	428	2126	156
28	289	2026	204

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
29	428	1874	214
30	311	1805	253
31	403	2045	242
32	429	2266	202
33	428	1679	147
34	413	1765	221
35	367	1906	271
36	424	2051	183
37	395	1953	195
38	405	1945	217
39	353	2104	170
40	360	1732	218
41	370	1987	221
42	435	2002	314
43	488	1901	371
44	358	1938	234
45	383	2073	280
46	370	1892	377
47	335	2025	259
48	240	2087	251
49	301	1934	344
50	314	1833	327
51	363	2356	388
52	418	1955	364
53	298	2208	254
54	227	1417	317
55	324	1902	302
56	319	1791	320
57	274	1921	368
58	289	2254	332

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
59	324	2026	347
60	258	2051	350
61	321	2216	280
62	388	2273	327
63	434	2347	346
64	335	2207	386
65	159	2416	286
66	217	2181	305
67	342	2189	294
68	192	2194	267
69	181	2286	193
70	299	1887	241
71	278	2284	251
72	357	2285	290
73	325	2209	331
74	185	2366	240
75	265	1950	185
76	246	2240	259
77	269	2109	208
78	330	2270	220
79	286	2188	233
80	304	2216	255
81	307	2069	258
82	217	2089	256
83	278	2119	309
84	415	2062	299
85	387	1337	322
86	193	1171	489
87	221	1305	534
88	268	1125	631

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
89	316	1258	514
90	284	1219	506
91	281	1167	597
92	334	1066	581
93	210	1487	642
94	295	1188	618
95	236	1440	509
96	247	1870	571
97	292	1135	556
98	232	1022	574
99	276	1025	622
100	296	1054	586
101	246	1385	600
102	212	1258	603
103	385	1184	534
104	253	1448	581
105	350	1340	903
106	425	1583	929
107	246	1618	831
108	305	1755	856
109	432	1571	839
110	280	1707	813
111	268	1769	741
112	388	1614	787
113	367	1510	798
114	447	2044	835
115	414	1635	875
116	273	1893	888
117	354	1087	733
118	334	1581	906

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
119	358	1465	805
120	420	1468	767
121	375	1600	881
122	393	1939	801
123	396	1707	755
124	305	1734	802
125	366	1902	852
126	418	1918	540
127	376	1766	398
128	388	1856	452
129	435	1708	392
130	372	1747	394
131	418	1753	419
132	439	1846	395
133	386	1442	316
134	351	1843	419
135	532	1844	362
136	394	1768	399
137	482	1926	439
138	207	1506	322
139	375	1799	407
140	336	1666	373
141	337	1829	428
142	382	1746	394
143	341	1774	400
144	418	1625	418
145	427	1646	375
146	484	1676	368
147	492	1934	445
148	430	1715	276

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
149	390	1324	285
150	308	1399	320
151	310	1358	272
152	312	1253	308
153	344	968	324
154	204	1153	284
155	328	1191	257
156	318	1340	395
157	372	1502	289
158	225	1147	357
159	203	933	148
160	282	1222	276
161	339	1210	245
162	310	1071	246
163	320	1118	281
164	268	1206	368
165	275	1026	308
166	285	1222	315
167	419	1416	357
168	418	1578	331
169	514	1871	255
170	385	1366	312
171	410	1513	248
172	396	1893	250
173	362	1436	277
174	267	1403	251
175	328	1762	168
176	341	1700	277
177	390	1938	217
178	444	1843	256

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
179	326	1416	298
180	255	1660	185
181	351	1600	264
182	346	1672	229
183	302	1858	273
184	316	1852	250
185	351	1776	257
186	286	1685	276
187	347	1514	233
188	415	1585	223
189	499	1865	304
190	397	2139	459
191	469	1954	359
193	388	2030	379
193	391	1988	368
194	393	1884	341
195	424	1602	267
196	287	1785	315
197	424	1823	325
198	363	1971	363
199	398	2233	405
200	451	1881	314
201	308	1668	259
202	408	1854	333
203	423	2146	382
204	419	1806	294
205	391	1754	307
206	401	1939	328
207	350	1658	282
208	356	1842	329

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
209	366	1853	332
210	506	1802	375
211	468	1513	274
212	292	1483	283
213	350	1575	319
214	475	1810	271
215	325	1448	306
216	432	1825	323
217	411	1725	282
218	490	1573	255
219	458	1504	394
220	318	1744	287
221	398	1965	355
222	379	1378	145
223	402	1564	274
224	463	1605	244
225	419	1750	245
226	437	1652	279
227	440	1744	367
228	350	1803	306
229	411	1431	314
230	490	1686	358
231	376	1498	300

A month consists of 21 working days.

APPENDIX 2

TABLE A.2: DEMAND SUMMARY OF PRODUCTS FOR THE NEXT MONTH

DAY	PRODUCT Y_1	PRODUCT Y_2	PRODUCT Y_3
232	415	1368	173
233	221	829	191
234	249	1008	275
235	296	1389	214
236	344	931	182
237	312	897	278
238	309	1258	253
239	362	1195	214
240	230	1435	197
241	323	1339	258
242	264	911	314
243	275	1156	164
244	320	1095	186
245	260	1167	222
246	304	1355	259
247	324	1349	234
248	274	1272	273
249	240	1282	178
250	413	1009	243
251	281	1181	235
252	388	1186	256

APPENDIX 3

TABLE A.3.1: IMPLEMENTATION OF THE PRESENT POLICY FOR PRODUCT Y_2

Day	Available Inventory	Demand	On hand	Average Demand	Bank	On hand plus on order	(Bank) - (on hand plus on order)	Order Quantity	Arrival
.		1803						0	0
.		1431						0	0
.		1686						0	0
.		1498						9000	0
1	6178	1368	4810	1557	15570	13810	1760	1760	0
2	4810	829	829	1362	13620	14741	-1121	0	0
3	3981	1008	1008	1278	12780	13733	-953	0	0
4	2973	1389	1389	1218	12180	12344	-164	0	9000
5	10584	931	9653	1105	11050	11413	-363	0	1760
6	11413	897	10516	1011	10110	10516	-406	0	0
7	10516	1258	9258	1097	10970	9258	1712	1712	0
8	9258	1195	8063	1134	11340	9775	1565	1565	0
9	8063	1435	6628	1143	11430	9905	1525	1525	0
10	6528	1339	5289	1225	12250	10091	2159	2159	0
11	5289	911	4378	1228	12280	11339	941	941	1712
12	6090	1156	4934	1207	12070	11124	946	946	1565
13	6499	1095	5404	1187	11870	10975	895	895	1525
14	6929	1167	5762	1134	11340	10703	637	637	2159
15	7921	1355	6566	1137	11370	9985	1385	1385	941
16	7507	1349	6158	1224	12240	10021	2219	2219	946
17	7104	1272	5832	1248	12480	10968	1512	1512	895
18	6727	1282	5445	1285	12850	11198	1652	1652	637
19	6082	1009	5073	1253	12530	11841	689	689	1385
20	6458	1181	5277	1219	12190	11349	841	841	2219
21	7496	1186	6310	1186	11860	11004	856	856	1512

The number of cycles is 16.

The average inventory held during cycle:

One: $Q_1 = 6178$	Two: $\frac{\sum_{i=2}^{7} Q_i}{6} = \frac{44277}{6} = 7380$
THree: $Q_8 = 9258$	Four: $Q_9 = 8063$
Five: $Q_{10} = 6628$	Six: $Q_{11} = 5289$
Seven: $Q_{12} = 6090$	Eight: $Q_{13} = 6499$
Nine: $Q_{14} = 6929$	Ten: $Q_{15} = 7921$
Eleven: $Q_{16} = 7507$	Twelve: $Q_{17} = 7104$
Thirteen: $Q_{18} = 6727$	Fourteen: $Q_{19} = 6082$
Fifteen: $Q_{20} = 6458$	Sixteen: $Q_{21} = 7496$

Thus, the average inventory held is

$$\frac{1(6178) + 6(7380) + 9258 + 8063 + 6628 + 5289 + 6090 + 6499 + 6929 + 7921 + 7507 + 7104 + 6727 + 6082 + 6458 + 7496}{1+6+1+1+1+1+1+1+1+1+1+1+1+1+1+1} = \underline{7072}$$

Since 16 orders are made during the month, it is assumed that the average numbers of orders for the year is 192.

Thus the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,011088)(7072) + 0 + (1,19)(192) \\ &= \underline{\text{R}306,89} \end{aligned}$$

TABLE A.3.2: IMPLEMENTATION OF THE PRESENT POLICY FOR PRODUCT Y_3

Day	Available Inventory	Demand	On hand	Average Demand	Bank	On hand plus on order	(Bank) - (on hand plus on order)	Order Quantity	Arrival
.		306						0	0
.		314						0	0
.		358						0	0
.		300						3000	0
1	1271	173	1098	290	2900	4098	-1198	0	0
2	1098	191	907	267	2670	3907	-1237	0	0
3	907	275	632	259	2590	3632	-1042	0	0
4	632	214	418	231	2310	3418	-1108	0	3000
5	3418	182	3236	207	2070	3236	-1166	0	0
6	3236	278	2958	228	2280	2958	-678	0	0
7	2958	253	2705	240	2400	2705	-305	0	0
8	2705	214	2491	228	2280	2491	-211	0	0
9	2491	197	2294	225	2250	2294	-44	0	0
10	2294	258	2036	240	2400	2036	364	364	0
11	2036	314	1722	247	2470	2086	384	384	0
12	1722	164	1558	229	2290	2306	-16	0	0
13	1558	186	1372	224	2240	2120	120	120	0
14	1372	222	1150	229	2290	2018	272	272	364
15	1514	259	1255	229	2290	2031	259	259	384
16	1639	234	1405	213	2130	2056	74	74	0
17	1405	273	1132	235	2350	1857	493	493	120
18	1252	178	1074	233	2330	2172	158	158	272
19	1346	243	1103	237	2370	2087	283	283	259
20	1362	235	1127	233	2330	2135	195	195	74
21	1201	256	945	235	2350	2074	276	276	493

The number of complete cycles is 11.

The average inventory held during cycles:

<u>One</u>: $\frac{\sum_{i=1}^{10} Q_i}{10} = \frac{21010}{10} = 2101$

<u>Two</u>: $Q_{11} = 2036$

<u>Three</u>: $\frac{\sum_{i=12}^{13} Q_i}{2} = \frac{3280}{2} = 1640$

<u>Four</u>: $Q_{14} = 1372$

<u>Five</u>: $Q_{15} = 1514$

<u>Six</u>: $Q_{16} = 1639$

<u>Seven</u>: $Q_{17} = 1405$

<u>Eight</u>: $Q_{18} = 1252$

<u>Nine</u>: $Q_{19} = 1346$

<u>Ten</u>: $Q_{20} = 1362$

<u>Eleven</u>: $Q_{21} = 1201$

Thus the average inventory held is,

$$\frac{10(2101) + 2036 + 2(1640) + 1372 + 1514 + 1639 + 1405 + 1252 + 1346 + 1362 + 1201}{10 + 1 + 2 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1}$$
$$= \underline{1782}$$

Since 11 orders are made during the month, it is assumed that the average number of orders for the year is 132.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,02268)(1782) + 0 + (1,19)(132) \\ &= \underline{\text{R}197,50} \end{aligned}$$

APPENDIX 4

PRODUCT Y_2:

The quantity to order each time an order is made is

$$\begin{aligned} P_w^* &= \sqrt{\frac{2\lambda_2 c_3}{c_1}} \\ &= \sqrt{\frac{2(440\,119)(1,19)}{0,011088}} \\ &= \underline{9720} \end{aligned}$$

The time between placement of orders is

$$T^* = \frac{P_w^*}{\lambda_2} = \frac{9720}{440119} = \underline{0,0220849 \text{ years}}$$

The leadtime demand is

$$\begin{aligned} \mu &= \lambda_2 L \\ &= (440119)(0,0198412) \\ &= \underline{8732} \end{aligned}$$

The reorder point based on the on hand plus on order inventory is then $r^* = 8732$.

The reorder point based on the on hand inventory level is

$$\begin{aligned} r_h^* &= \mu - mP^* \\ m &= [L/T] \\ &= \left[\frac{0,0198412}{0,0220849}\right] \\ &= [0,8984057] \\ &= \underline{0} \end{aligned}$$

therefore $r_h^* = \underline{8732}$.

The average yearly cost C is calculated by

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 0,011088\left(\frac{9720}{2}\right) + 0 + 1,19\left(\frac{440119}{9720}\right) \\ &= \underline{\text{R}107,77} \end{aligned}$$

In the following table, $P^* = 9720$ and $r^* = 8732$ is used.

TABLE A.4.1: IMPLEMENTATION OF THE DETERMINISTIC LOT-SIZE MODEL WITH NO STOCKOUTS FOR PRODUCT Y_2

Day	Available Inventory	Demand	On hand	Order Quantity	Arrival
1	6178	1368	4810	0	0
2	4810	829	3981	0	0
3	3981	1008	2973	0	0
4	2973	1389	1584	0	9000
5	10584	931	9653	0	0
6	9653	897	8756	0	0
7	8756	1258	7498	9720	0
8	7498	1195	6303	0	0
9	6303	1435	4868	0	0
10	4868	1339	3529	0	0
11	3529	911	2618	0	9720
12	12 338	1156	11 182	0	0
13	11 182	1095	10 087	0	0
14	10 087	1167	8920	0	0
15	8920	1355	7565	9720	0
16	7565	1349	6216	0	0
17	6216	1272	4944	0	0
18	4944	1282	3662	0	0
19	3662	1009	2653	0	9720
20	12 372	1181	11 192	0	0
21	11 192	1186	10 006	0	0

The number of cycles in the month is 2.

<u>One</u>: $\frac{\sum_{i=1}^{7} Q_i}{7} = \frac{46935}{7} = 6705$

<u>Two</u>: $\frac{\sum_{i=8}^{15} Q_i}{8} = \frac{64725}{8} = 8091$

Thus the average inventory held is,

$$\frac{7(6705) + 8(8091}{7+8} = \underline{7444}$$

Since two orders are made during the month, it is assumed that 24 orders will be made on average for the year.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,011088)(7444) + 0 + (1,19)(24) \\ &= \underline{R111,10} \end{aligned}$$

<u>PRODUCT Y_3</u>:

The quantity to order each time an order is placed is

$$\begin{aligned} P_w^* &= \sqrt{\frac{2\lambda_3 c_3}{c_1}} \\ &= \sqrt{\frac{2(91250)(1,19)}{0,02268}} \\ &= \underline{3094} \end{aligned}$$

The time between placement of orders is

$$T^* = \frac{P_w^*}{\lambda_3} = \frac{3094}{91250} = \underline{0,0339068 \text{ years}}$$

Leadtime is

$$\begin{aligned} \mu &= \lambda_3 L \\ &= (91250)(0,0198412) \\ &= \underline{1811} \end{aligned}$$

The reorder point based on the on hand plus on order inventory level is then $r^* = 1811$.

The reorder point based on the on hand inventory level is

$$r_h^* = \mu - mP$$

and

$$\begin{aligned} m &= [L/T] \\ &= \left[\frac{0,0198412}{0,0339068}\right] \\ &= [0,5851687] \\ &= \underline{0} \end{aligned}$$

Therefore $r_h^* = 1811$.

The average annual cost is

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 0,02268\left(\frac{3094}{2}\right) + 0 + 1,19\left(\frac{91250}{3094}\right) \\ &= \underline{R70,19} \end{aligned}$$

In the following table, $P_w^* = 3094$ and $r^* = 1811$, is used.

TABLE A.4.2: IMPLEMENTATION OF THE DETERMINISTIC LOT-SIZE MODEL WITH NO STOCKOUTS FOR PRODUCT Y_3

Day	Available Inventory	Demand	On hand	Order Quantity	Arrival
1	1271	173	1098	0	0
2	1098	191	907	0	0
3	907	275	632	0	0
4	632	214	418	0	3000
5	3418	182	3236	0	0
6	3236	278	2958	0	0
7	2958	253	2705	0	0
8	2705	214	2491	0	0
9	2491	197	2294	0	0
10	2294	258	2036	0	0
11	2036	314	1722	3094	0
12	1722	164	1558	0	0
13	1558	186	1372	0	0
14	1372	222	1150	0	0
15	1150	259	891	0	3094
16	3985	234	3751	0	0
17	3751	273	3478	0	0
18	3478	178	3300	0	0
19	3300	243	3057	0	0
20	3057	235	2822	0	0
21	2822	256	2566	0	0

The number of cycles in the month is 1.

The average inventory held is.

$$\frac{\sum_{i=1}^{11} Q_i}{11} = \frac{23046}{11} = \underline{2095}$$

Since 1 order is made during this month, it is assumed that 12 orders will be made on average fo the year.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,02268)(2095) + 0 + (1,19)(12) \\ &= \underline{\text{R}61,79} \end{aligned}$$

APPENDIX 5

PRODUCT Y_2:

The expected leadtime demand and standard deviation of the leadtime demand is estimated by finding the mean and standard deviation of the weekly demand from Table A.1.

$$\begin{aligned} P_1 &= P_w^* = \sqrt{\frac{2\lambda_2 c_3}{c_1}} \\ &= \sqrt{\frac{2(440119)(1,19)}{0,011088}} \\ &= \underline{9720} \end{aligned}$$

For product Y_2, $\mu = 8732$ and $\hat{\sigma} = 1426$

$$\begin{aligned} H(r) = \Phi\left(\frac{r - 8732}{1426}\right) &= \frac{P_1 c_1}{c_2\lambda_2 + P_1 c_1} \\ &= \frac{(9720)(0,011088)}{(0,03)(440119) + (9720)(0,011088)} \\ &= \underline{0,0080965} \end{aligned}$$

Hence,

$$\frac{r_1 - 8732}{1426} = 2,40$$

It follows that,

$$\begin{aligned} r_1 &= 8732 + 3422 \\ &= \underline{12154} \end{aligned}$$

To compute P_2 we need

$$\begin{aligned}
\eta(r_1) &= (\mu - r_1)\Phi\left(\frac{r_1-\mu}{\sigma}\right) + \sigma\phi\left(\frac{r_1-\mu}{\sigma}\right) \\
&= (-3422)(0,0080965) + (1426)(0,022395) \\
&= \underline{4,229047} \\
P_2 &= \sqrt{\frac{2\lambda_2[c_3 + c_2\bar{\eta}(r_1)]}{c_1}} \\
&= \sqrt{\frac{2(440119)[1,19 + 0,03(4,229047)]}{0,011088}} \\
&= \underline{10225} \\
\Phi\left(\frac{r_2 - 8732}{1426}\right) &= \frac{P_2 c_1}{c_2\lambda_2 + P_2 c_1} \\
&= \frac{(10225)(0,011088)}{(0,03)(440119) + (10225)(0,011088)} \\
&= \underline{0,0085135}
\end{aligned}$$

and

$$\frac{r_2 - 8732}{1426} = 2,39$$

It follows that,

$$\begin{aligned}
r_2 &= 8732 + 3408 \\
&= \underline{12\,140}
\end{aligned}$$

To compute P_3 we need

$$\begin{aligned}
\eta(r_2) &= (\mu - r_2)\Phi\left(\frac{r_2-\mu}{\sigma}\right) + \sigma\phi\left(\frac{r_2-\mu}{\sigma}\right) \\
&= (-3408)(0,0085135) + (1426)(0,022937) \\
&= \underline{3,694154} \\
P_3 &= \sqrt{\frac{2\lambda_2[c_3 + c_2\bar{\eta}(r_2)]}{c_1}} \\
&= \sqrt{\frac{2(440119)[1,19 + 0,03(3,694154)]}{0.011088}} \\
&= \underline{10162} \\
\Phi\left(\frac{r_3 - 8732}{1426}\right) &= \frac{P_3c_1}{c_2\lambda_2 + P_3c_1} \\
&= \frac{(10162)(0,011088)}{(0,03)(440119) + (10162)(0,011088)} \\
&= \underline{0,0084615}
\end{aligned}$$

and

$$\frac{r_3 - 8732}{1426} = 2,39$$

Hence,

$$\begin{aligned}
r_3 &= 8732 + 3408 \\
&= \underline{12\,140}
\end{aligned}$$

Since there has been no change in safety stock, additional iterations are not needed since the changes will be negligible.

The optimal values are $P^* = 10162$ and $r^* = 12140$.

The expected time between placing of orders is

$$\begin{aligned}
T = \frac{P}{\lambda} = \frac{10162}{440119} &= 0,0230892 \text{ years} \\
&\approx \underline{6 \text{ days}}
\end{aligned}$$

The average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= \frac{(440119)(1,19)}{10162} + 0,011088\left[\frac{10162}{2} + 12140 - 8732\right] \\ &= \left(0,011088 + 0,03(\frac{440119}{10162})\right)(3,694154) \\ &= \underline{\text{R}150,51} \end{aligned}$$

Using the above mentioned policy with $P^* = 10162$ and $r^* = 12140$, the following table is obtained.

TABLE A.5.1: IMPLEMENTATION OF THE LOT-SIZE REORDER POINT MODEL WITH NORMALLY DISTRIBUTED STOCHASTIC DEMAND FOR PRODUCT Y_2

Day	Available Stock	Demand	On hand	Order	Arrival
1	6178	1368	4810	0	0
2	4810	829	3981	0	0
3	3981	1008	2973	10162	0
4	2973	1389	1389	0	9000
5	10389	931	9458	0	0
6	9458	897	8561	0	0
7	8561	1258	7303	0	10162
8	17465	1195	16270	0	0
9	16270	1435	14835	0	0
10	14835	1339	13496	0	0
11	13496	911	12585	0	0
12	12585	1156	11429	10162	0
13	11429	1095	10334	0	0
14	10334	1167	9167	0	0
15	9167	1355	7812	0	0
16	7812	1349	6463	0	10162
17	16625	1272	15353	0	0
18	15353	1282	14071	0	0
19	14071	1009	13062	0	0
20	13062	1181	11881	101162	0
21	11881	1186	10695	0	0

The number of cycles in the month is 3.

The average inventory held during cycle:

<u>One</u>: $\dfrac{\sum_{i=1}^{3} Q_i}{3} = \dfrac{14969}{3} = \underline{4990}$

<u>Two</u>: $\dfrac{\sum_{i=4}^{12} Q_i}{9} = \underline{11781}$

<u>Three</u>: $\dfrac{\sum_{i=13}^{20} Q_i}{8} = \underline{12232}$

Thus, the average inventory held is

$$\frac{3(4990) + 9(11781) + 8(12323)}{3 + 9 + 8}$$
$$= \underline{10943}$$

Since three orders are made during the month, it is assumed that 36 orders will be made on average for the year.

Thus, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,011088)(10943) + 0 + (1,19)(36) \\ &= \underline{\text{R}164,18} \end{aligned}$$

<u>PRODUCT Y_3</u>

The expected leadtime demand and standard deviation of the leadtime demand is estimated by finding the mean and standard deviation of the weekly demand from Table 1.1.

$$\begin{aligned} P_1 &= P_w^* = \sqrt{\frac{2\lambda_3 c_3}{c_1}} \\ &= \sqrt{\frac{2(91250)(1,19)}{0,02268}} \\ &= \underline{3094} \end{aligned}$$

For product Y_3, $\mu = 1811$ and $\hat{\sigma} = 872,38$

$$H(r) = \Phi\left(\frac{r - 1811}{872,38}\right) = \frac{P_1 c_1}{c_2\lambda_3 + P_1 c_1}$$

$$= \frac{(3094)(0,02268)}{(0,03)(91250) + (3094)(0,02268)}$$

$$= \underline{0,024984}$$

and

$$\frac{r_1 - 1811}{872,38} = 1,96$$

From the above it follows that,

$$r_1 = 1811 + 1710 = \underline{3521}$$

To compute P_2 we need,

$$\bar{\eta}(r_1) = (\mu - r_1)\Phi\left(\frac{r_1 - \mu}{\sigma}\right) + \sigma\phi\left(\frac{r_1 - \mu}{\sigma}\right)$$

$$= (-1710)(0,024984) + (872,36)(0,058441)$$

$$= \underline{8,26}$$

therefore

$$P_2 = \sqrt{\frac{2\lambda_3[c_3 + c_2\bar{\eta}(r_2)]}{c_1}}$$

$$= \sqrt{\frac{2(91250)[1,19 + 0,03(8,26)]}{0,02668}}$$

$$= \underline{3401}$$

It follows that,

$$\Phi\left(\frac{r_2 - \mu}{\sigma}\right) = \frac{P_2 c_1}{c_2\lambda_3 + P_2 c_1}$$

$$= \frac{(3401)(0,02268)}{(0,03)(91250) + (3401)(0,02268)}$$

$$= \underline{0,0274048}$$

and

$$\frac{r_2 - 1811}{872,38} = 1,92$$

From the above it follows that,

$$r_2 = 1811 + 1675$$

$$= \underline{3486}$$

To compute P_3 we need

$$\begin{aligned}
\bar{\eta}(r_3) &= (\mu - r_2)\Phi\left(\frac{r_2-\mu}{\sigma}\right) + \sigma\phi\left(\frac{r_2-\mu}{\sigma}\right) \\
&= (-1675)(0,0274048) + (872,38)(0,063157) \\
&= \underline{9,1938637}
\end{aligned}$$

Therefore,

$$\begin{aligned}
P_3 &= \sqrt{\frac{2\lambda_3[c_3 + c_2\bar{\eta}(r_3)]}{c_1}} \\
&= \sqrt{\frac{2(91250)[1,19 + 0,03(9,1938637)]}{0,02268}} \\
&= \underline{3434} \\
\Phi\left(\frac{r_3 - 1811}{872,38}\right) &= \frac{P_3 c_1}{c_2\lambda_3 + P_3 c_1} \\
&= \frac{(3434)(0,02268)}{(0,03)(91250) + (3434)(0,02268)} \\
&= \underline{0,0276634}
\end{aligned}$$

and

$$\frac{r_3 - 1811}{872,38} = 1,92$$

From the above it follows that

$$\begin{aligned}
r_3 &= 1811 + 1675 \\
&= \underline{3486}
\end{aligned}$$

Since there has been no change in safety stock, additional iterations are not needed since the changes will be negligible.

The optimal values are $P^* = 3434$ and $r^* = 3486$.

The expected time between placing of orders is

$$\begin{aligned}
T = \frac{P}{\lambda} = \frac{3434}{91250} &= 0,0376328 \text{ years} \\
&\approx \underline{9 \text{ days}}
\end{aligned}$$

The average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= \frac{(1,19)(91250)}{3434} + 0,02268\left[\frac{3434}{2} + 3486 - 1811\right] \\ &= \left(0,02268 + \frac{(0,03)(91250)}{3434}\right)(9,1938637) \\ &= \underline{\text{R}116,09} \end{aligned}$$

Using the above mentioned policy with $P^* = 3434$ and $r^* = 3486$, the following table is obtained.

TABLE A.5.2: IMPLEMENTATION OF THE REORDER POINT MODEL WITH NORMALLY DISTRIBUTED STOCHASTIC DEMAND FOR PRODUCT Y_3

Day	Available Stock	Demand	On hand	Order	Arrival
1	1271	173	1098	0	0
2	1098	191	907	0	0
3	907	275	632	0	0
4	632	214	418	0	3000
5	3418	182	3236	3434	0
6	3236	278	2958	0	0
7	2958	253	2705	0	0
8	2705	214	2491	0	0
9	2491	197	2294	0	3434
10	5728	258	5470	0	0
11	5470	314	5156	0	0
12	5156	164	4992	0	0
13	4992	186	4806	0	0
14	4806	222	4584	0	0
15	4584	259	4325	0	0
16	4325	234	4091	0	0
17	4091	273	3818	0	0
18	3818	178	3640	0	0
19	3640	243	3397	3434	0
20	3397	235	3162	0	0
21	3162	256	2906	0	0

The number of cycles in the month is 2.

The average inventory held during cycle:

<u>One</u>: $\frac{\sum_{i=1}^{5} Q_i}{5} = \frac{7326}{5} = \underline{1465}$

<u>Two</u>: $\frac{\sum_{i=6}^{19} Q_i}{14} = \frac{58000}{14} = \underline{4143}$

Thus, the average inventory held is

$$\frac{5(1465) + 14(4143)}{5+14} = \underline{3438}$$

Since there are 2 orders during the month, it is assumed that 36 orders will be made on average for the year.

Thus, the average yearly cost is,

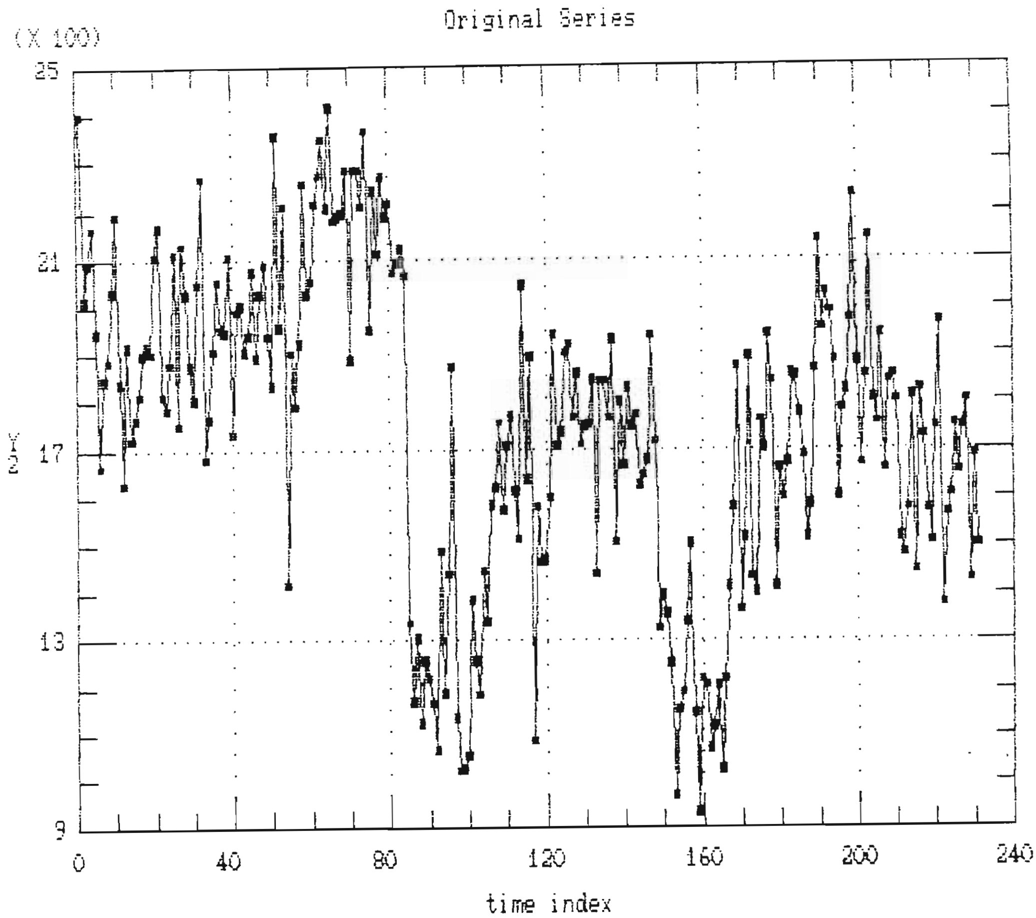
$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,02268)(3438) + 0 + (1,19)(24) \\ &= \underline{\text{R}106,53} \end{aligned}$$

APPENDIX 6

PRODUCT Y_2

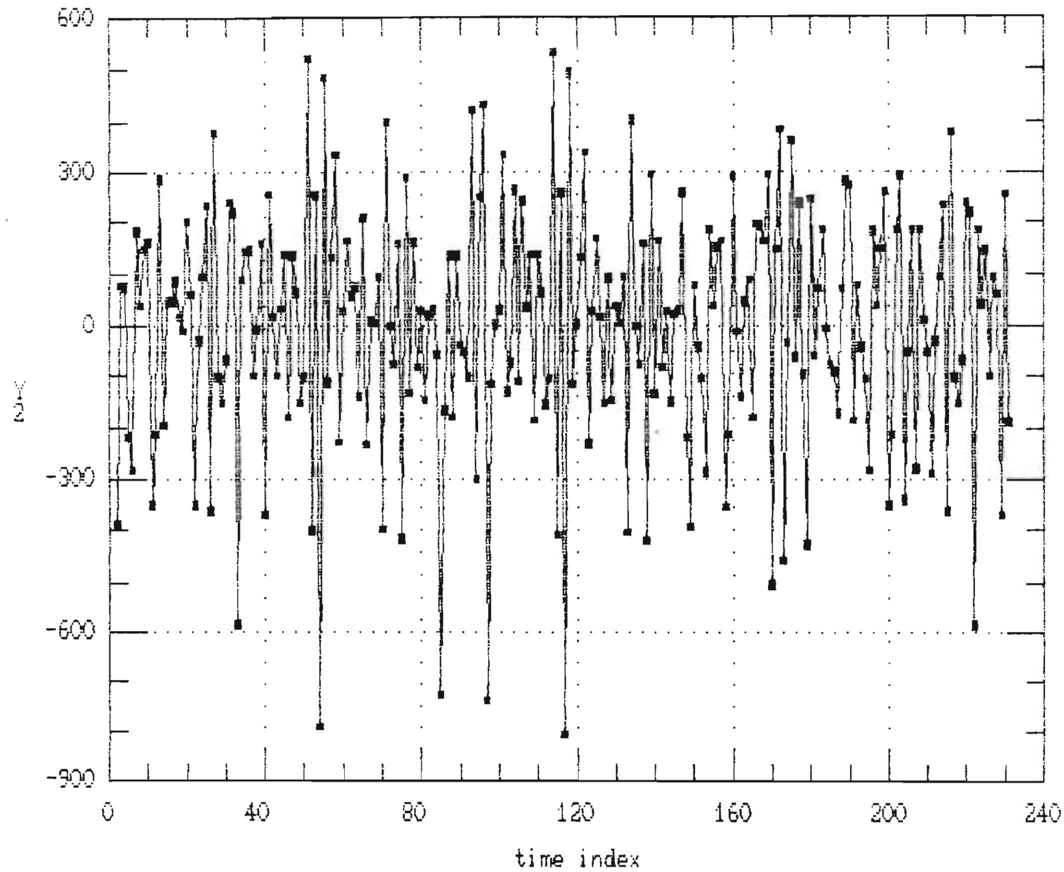
The time series in the figure below, indicates that the series is nonstationary. The series must therefore be differenced.

FIG. A.6.1: A PLOT OF THE ORIGINAL SERIES OF PRODUCT Y_2



After differencing once, the series appears to be stationary. See Figure A.6.2.

FIG. A.6.2: A PLOT OF THE FIRST DIFFERENCE OF THE SERIES OF PRODUCT Y_2.



IDENTIFICATION

In seeking a tentative model, we examine the autocorrelation and partial autocorrelation functions of the differenced series of product Y_2. See figures A.6.3 and A.6.4.

FIG. A.6.3: A PLOT OF THE AUTOCORRELATIONS FUNCTION OF THE ORIGINAL SERIES OF PRODUCT Y_2

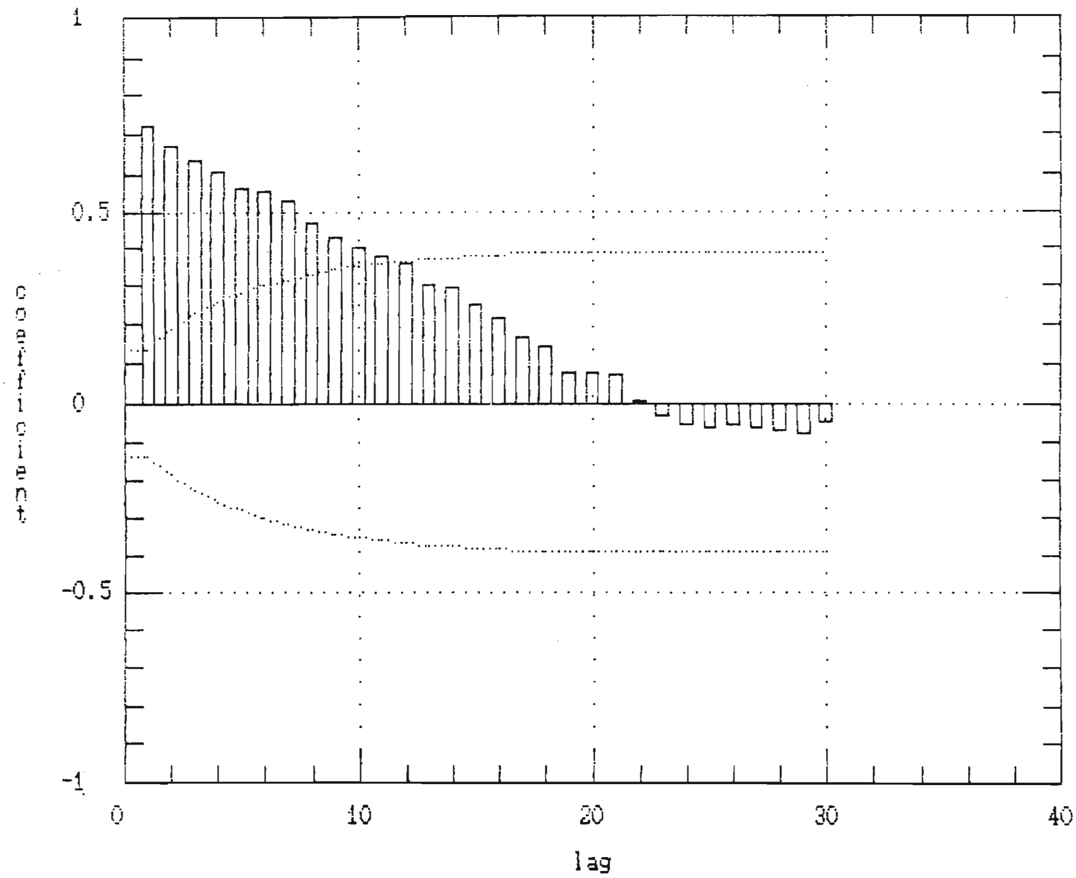
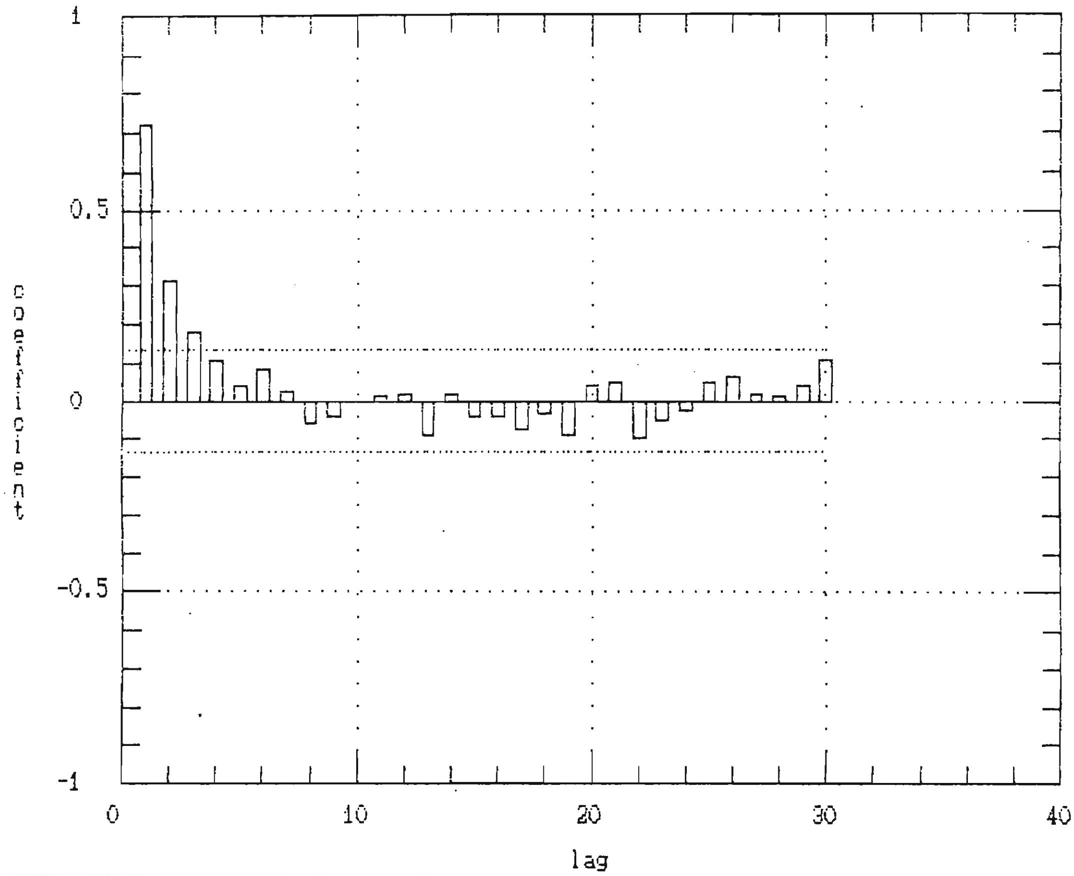


FIG. A.6.4: A PLOT OF THE PARTIAL AUTOCORRELATION FUNCTION OF THE ORIGINAL SERIES OF PRODUCT Y_2



The estimated autocorrelations suggest a MA(1) process, since it is after lag 1 that it cuts off. The estimated partial autocorrelations seem compatible with this.

2. ESTIMATION

The parameters are estimated by statgraphics using the minimum least squares method, and are shown in Table 1.

3. DIAGNOSTIC CHECKING

Although an MA(1) was identified, an MA(2) and an ARMA(1,1) overfit was processed.

TABLE A.6.1

PARAMETER ESTIMATES OF THE OVERFIT MODELS

MODEL	PARAMETER ESTIMATES	STANDARD ERROR OF ESTIMATES	$\hat{\sigma}_a^2$	χ^2
MA(1)	$\hat{\theta} = 0,56744$	0,05526	44798	8,237
MA(2)	$\hat{\theta}_1 = 0,54346$	0,06631	44928	7,889
	$\hat{\theta}_2 = 0,04090$	0.06652	44928	7,889
ARIMA(1,1)	$\hat{\theta} = 0,61082$	0,09096	44931	7,912
	$\hat{\phi} = 0,06301$	0,11466	44931	7,912

The ARIMA(1,1) Overfit

Since $\hat{\phi}$ is not significantly different from zero and $\hat{\theta}$ in the overfit is not significantly different from $\hat{\theta}$ in the MA(1) process, the ARIMA(1,1) overfit is not justified.

The MA(2) Overfit

Since $\hat{\theta}_2$ is not significantly different from zero and $\hat{\theta}_1$ in the overfit is not significantly different from $\hat{\theta}$ in the MA(1) model, the MA(2) overfit is not justified.

In the analysis of the residuals of the MA(1) process, the estimated residual autocorrelations and partial autocorrelations lie within plus or minus two standard deviations, hence, the MA(1) model is justified. See Figures A.6.5 and A.6.6.

FIG. A.6.5: A PLOT OF THE ESTIMATED RESIDUAL AUTOCORRELATIONS OF PRODUCT Y_2 FOR THE MA(1) PROCESS.

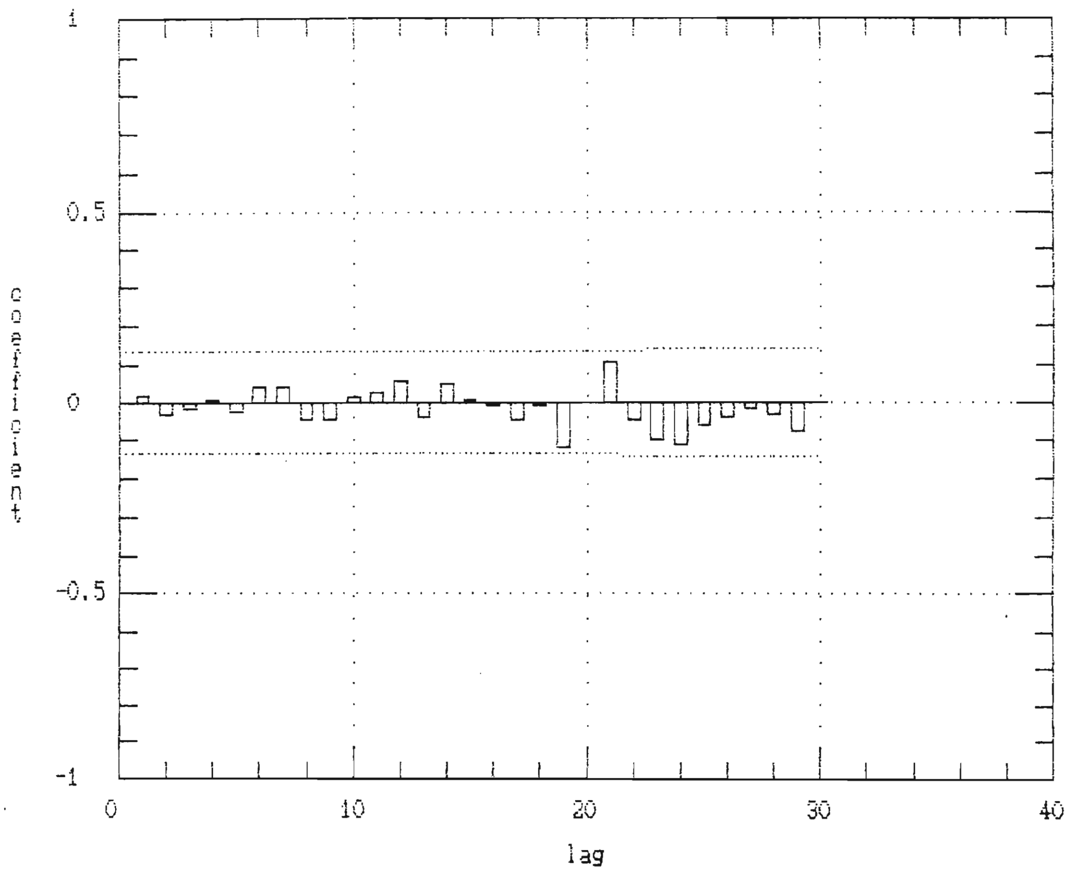
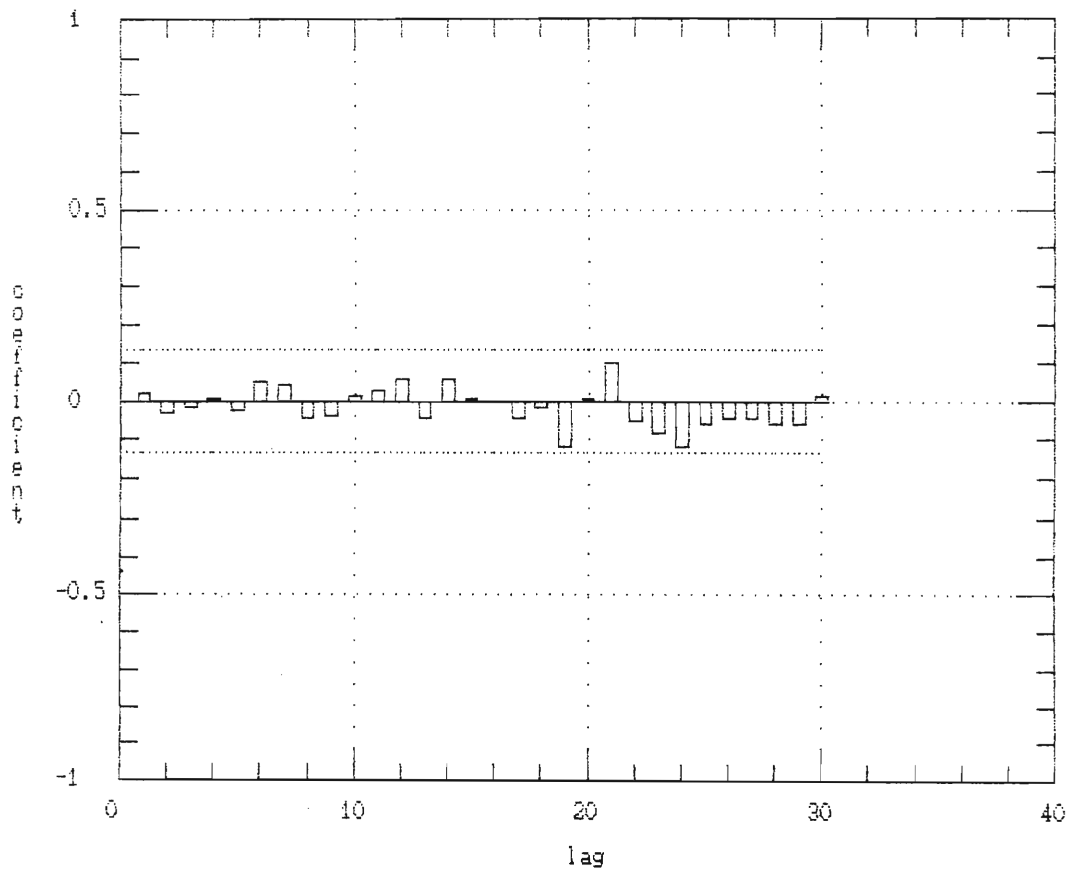


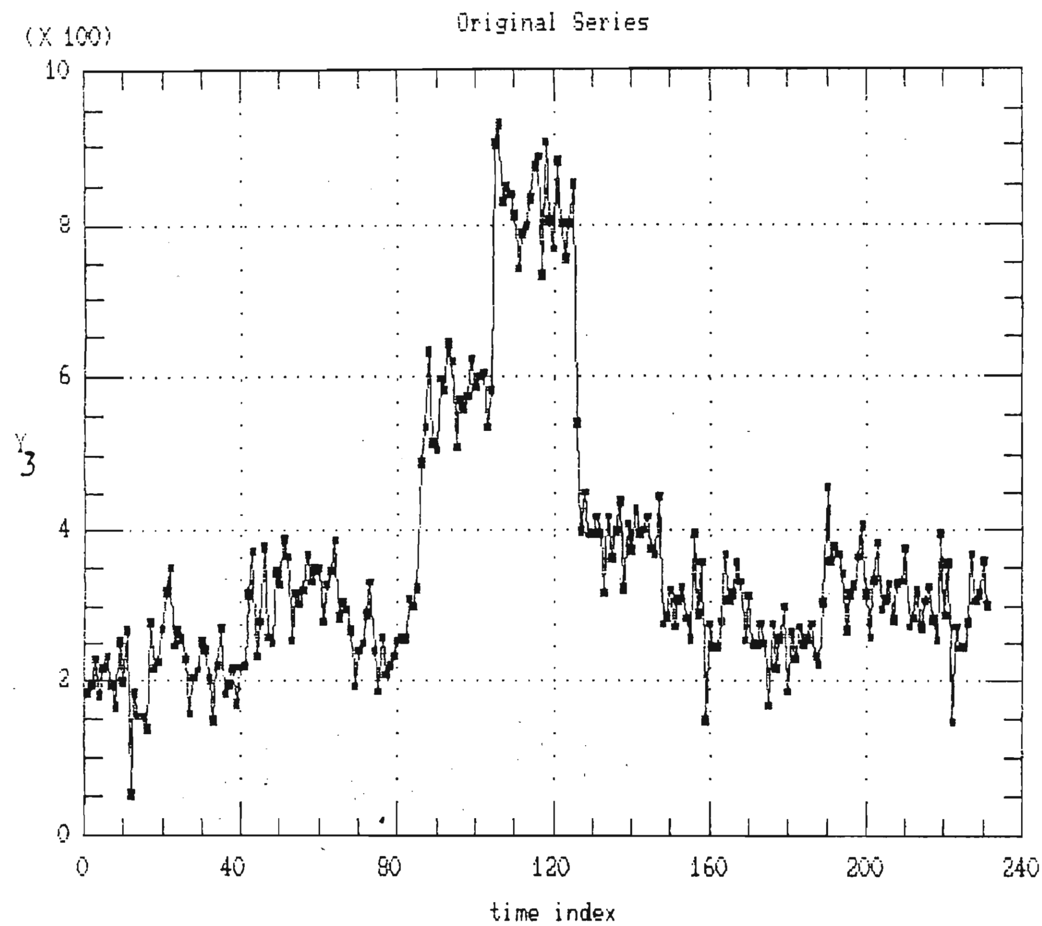
FIG. A.6.6: A PLOT OF THE ESTIMATED RESIDUAL PARTIAL AUTOCORRELATIONS OF PRODUCT Y_2 FOR THE MA(1) PROCESS.



PRODUCT Y_3

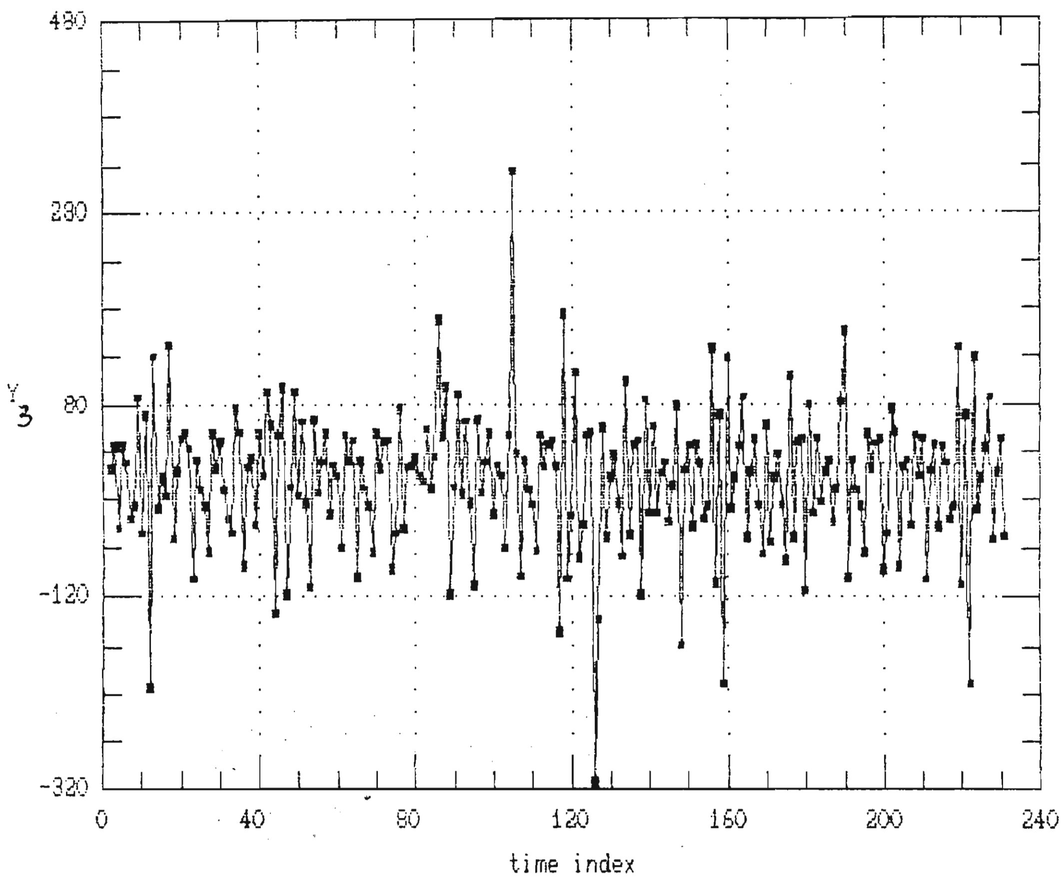
The time series in the figure below, indicates that the series is nonstationary. The series must therefore be differenced.

FIG A.6.7: A PLOT OF THE ORIGINAL SERIES Y_3



After differencing once, the series appears to be stationary. See Figure A.6.8.

FIG A.6.8: A PLOT OF THE DIFFERENCED SERIES AT LAG 1 FOR Y_3.



IDENTIFICATION

In seeking a tentative model, we examine the autocorrelations and partial autocorrelations of the differenced series of product Y_3. See figures A.6.9 and A.6.10.

FIG. A.6.9: A PLOT OF THE ESTIMATED AUTOCORRELATIONS FOR 1 NONSEASONAL DIFFERENCED SERIES OF PRODUCT Y_3

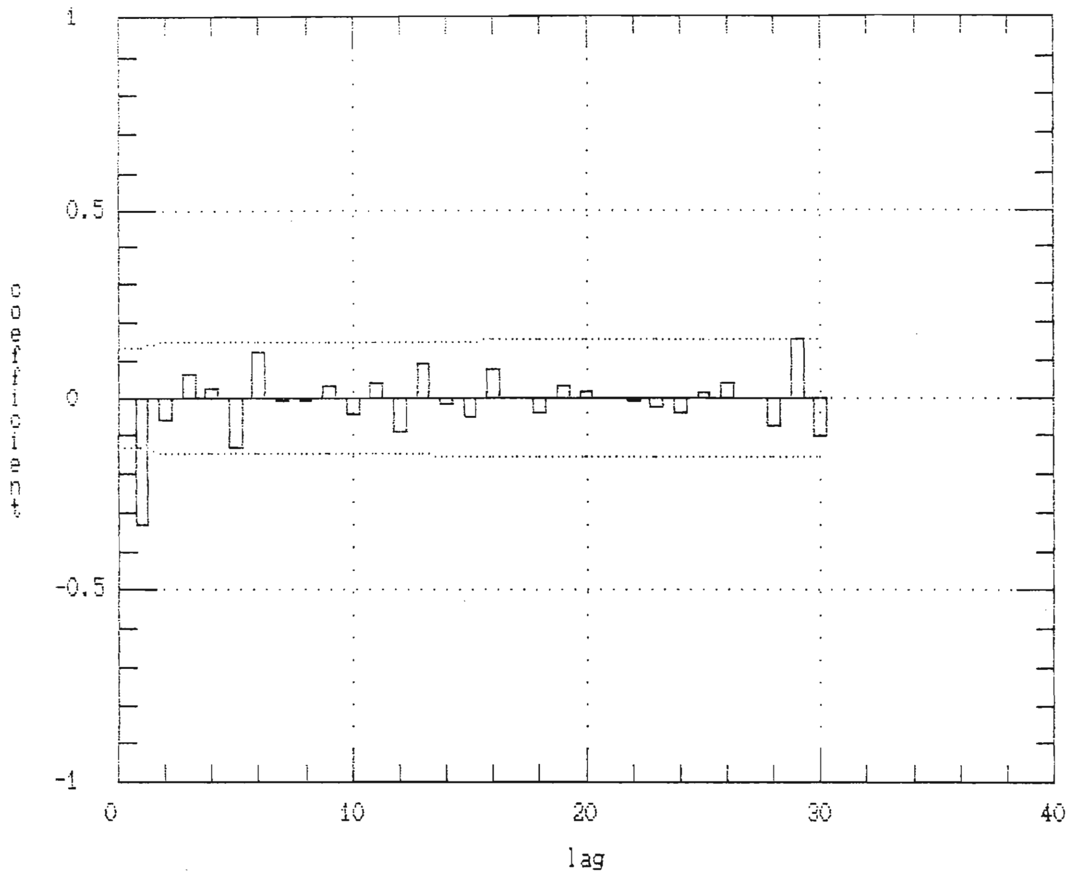
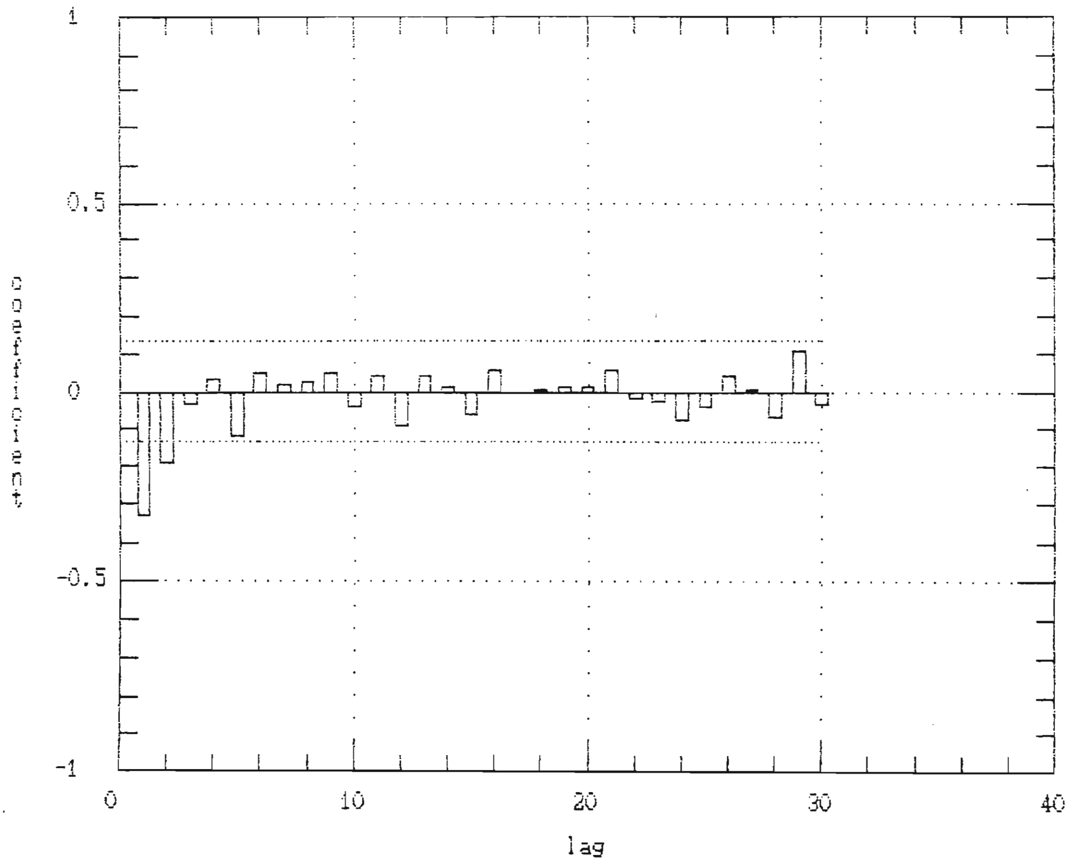


FIG. A.6.10: ESTIMATED PARTIAL AUTOCORRELATIONS FOR 1 NONSEASON DIFFERENCED SERIES OF PRODUCT Y_3



The estimated autocorrelations suggest an MA(1) process, since it is after lag 1 that it cuts off. The estimated partial autocorrelations are not compatible with this, and suggest an AR(2) process. Here, an ARIMA(1,1) model could also be considered.

ESTIMATION

The parameters are estimated by statgraphics using the minimum least square method, and are shown in Table A.6.2.

DIAGNOSTIC CHECKING

Although an MA(1), AR(2) and ARIMA(1,1) were considered in the identification stage, AR(3) and MA(2) overfit were processed.

TABLE A.6.2

PARAMETER ESTIMATES OF THE OVERFIT MODELS

MODEL	PARAMETER ESTIMATES	STANDARD ERROR	$\hat{\sigma}_a^2$	χ^2
MA(1)	$\hat{\theta} = 0,40553$	0,06163	4659	9,637
MA(2)	$\hat{\theta}_1 = 0,38877$	0,6626	4677	9,689
	$\hat{\theta}_2 = 0,03521$	0,06646	4677	9,689
AR(2)	$\hat{\phi}_1 = -0,39285$	0,06520	4664	8,609
	$\hat{\phi}_2 = -0,19180$	0,06525	4664	8,609
AR(3)	$\hat{\phi}_1 = -0,39918$	0,06653	4680	8,42
	$\hat{\phi}_2 = -0,020478$	0,07037	4680	8,42
	$\hat{\phi}_3 = -0,03303$	0,06660	4680	8,42
ARIMA(1,1)	$\hat{\theta}_1 = 0,43094$	0.14754	4677	9,541
	$\hat{\phi}_1 = 0,02676$	0,16045	4677	9,541

The MA(2) Overfit
$\hat{\theta}_2$ is not significantly different from zero and the parameter $\hat{\theta}_1$ in the MA(2) overfit is not significantly different from $\hat{\theta}$ in the MA(1) model. So, the overfit is not justified.

The AR(3) Overfit
Since $\hat{\phi}_3$ is not significantly different from zero and the AR(3) overfit has parameters which are not significantly different from those of the AR(2) process, the overfit is not justified.

The ARIMA(1,1) Overfit
Since $\hat{\phi}$ is not significantly different from zero, and $\hat{\theta}$ is not significantly different from $\hat{\theta}$ in the MA(1) process, the ARIMA(1,1) overfit not justified.

AR(2) and MA(1) process
Since the $\hat{\phi}$'s are highly significant in the AR(2) process and also the $\hat{\theta}$ in the MA(1) process is significant, by the principle of parsimony, the MA(1) process is suggested for the series Y_3 in the analysis of the parameterised models.

In the analysis of the residuals of the MA(1) process, the estimated residual autocorrelations and partial autocorrelations lie within plus or minus two standard deviations, hence, the MA(1) model is justified. See figures A.6.11 and A.6.12.

FIG. A.6.11: A PLOT OF THE ESTIMATED RESIDUAL AUTOCORRELATIONS OF PRODUCT Y_3 FOR THE MA(1) PROCESS

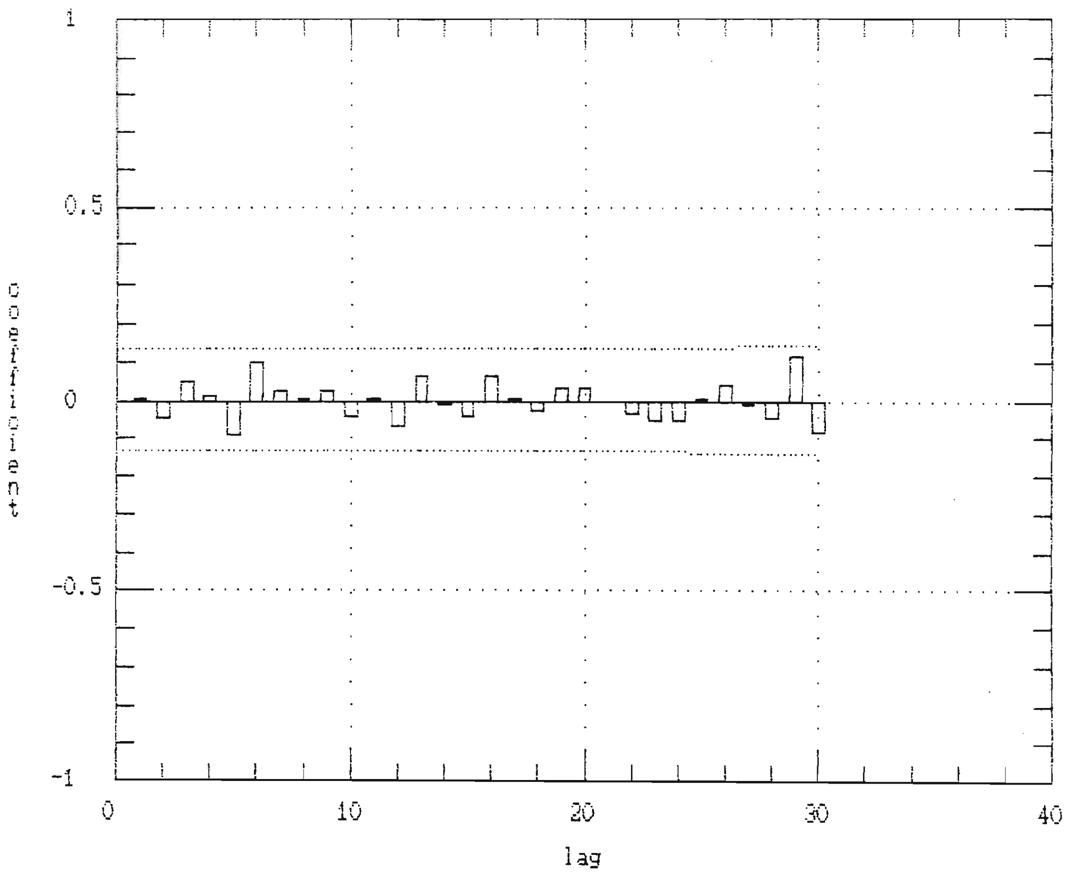
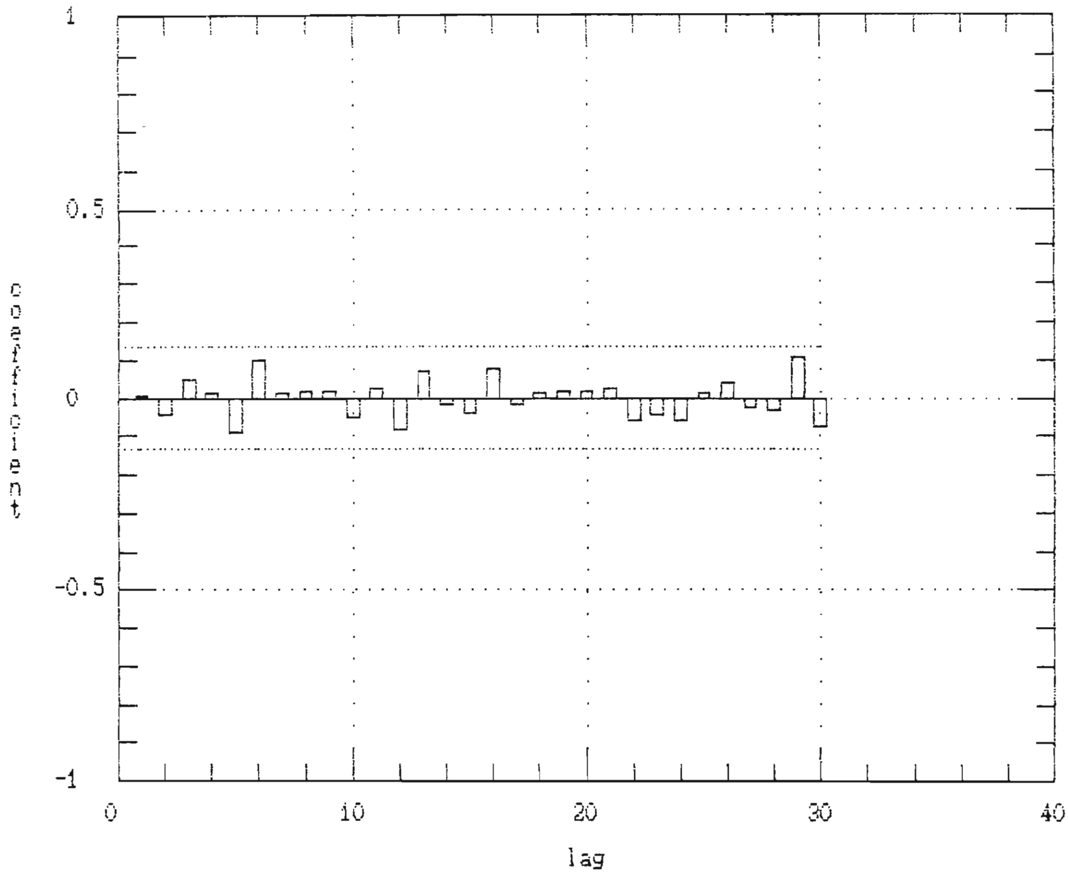


FIG. A.6.12: A PLOT OF THE ESTIMATED RESIDUAL PARTIAL AUTOCORRELATIONS OF PRODUCT Y_3 FOR THE MA(1) PROCESS



APPENDIX 7

TABLE A.7.1

IMPLEMENTATION OF THE INVENTORY CONTROL POLICY BASED ON BOX- JENKINS FORECASTING TECHNIQE FOR PRODUCT Y_2

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
1	6178	1368	4810	1572	8179	0	0
				1569			
				1566			
				1563			
				1560			
2	4810	829	3891	1479	7711	0	0
				1476			
				1472			
				1469			
				1466			
3	3981	1008	2973	1179	6197	0	0
				1174			
				1170			
				1165			
				1160			
4	2973	1389	1584	1091	5754	0	9000
				1086			
				1081			
				1076			
				1071			
5	10584	931	9653	1224	6425	0	0
				1220			
				1215			
				1211			
				1206			
6	9653	897	8756	1088	5736	0	0
				1083			
				1077			
				1072			
				1067			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
7	8756	1258	7498	995	5270	0	0
				990			
				984			
				979			
				973			
8	7498	1195	6303	1109	5844	0	0
				1104			
				1099			
				1094			
				1089			
9	6303	1435	4868	1143	6015	1147	0
				1138			
				1133			
				1128			
				1124			
10	4868	1339	3529	1270	6657	1981	0
				1266			
				1262			
				1257			
				1253			
11	3529	911	2624	1297	6798	1046	0
				1293			
				1289			
				1289			
				1281			
12	2624	1156	1468	1120	5900	258	0
				1115			
				1110			
				1105			
				1101			
13	1468	1095	373	1131	5985	1153	1147
				1127			
				1122			
				1117			
				1112			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
14	1520	1167	353	1110	5853	1062	1981
				1106			
				1101			
				1096			
				1091			
15	2334	1355	979	1131	5956	1458	1046
				1126			
				1121			
				1117			
				1112			
16	2025	1349	676	1226	6437	1830	258
				1222			
				1218			
				1213			
				1209			
17	934	1272	-338	1277	6694	1529	1153
				1273			
				1269			
				1265			
				1261			
18	815	1282	-467	1271	6664	1252	1062
				1267			
				1263			
				1259			
				1255			
19	595	1009	-414	1272	6669	1014	1458
				1268			
				1264			
				1260			
				1256			
20	1044	1181	-137	1150	6055	567	1830
				1146			
				1141			
				1137			
				1132			
21	1693	1186	507	1160	6103	1234	1529
				1155			
				1151			
				1146			
				1142			

The number of cycles during this month is 13.

The average inventory held during cycle:

One: $\frac{\sum_{i=1}^{9} Q_i}{9} = \frac{60736}{9} = 6748$

Two: $Q_{10} = 4868$

Three: $Q_{11} = 3529$

Four: $Q_{12} = 2624$

Five: $Q_{13} = 1468$

Six: $Q_{14} = 1520$

Seven: $Q_{15} = 2334$

Eight: $Q_{16} = 2025$

Nine: $Q_{17} = 934$

Ten: $Q_{18} = 815$

Eleven: $Q_{19} = 595$

Twelve: $Q_{20} = 1044$

Thirteen: $Q_{21} = 1693$

The average inventory held is,

$$\frac{9(6748) + 4868 + 3529 + 2624 + 1468 + 1520 + 2334 + 2025 + 934 + 815 + 595 + 1044 + 1693}{9 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1}$$
$$= \underline{4009}$$

Since 13 orders are made during this month, it is assumed that the average number of orders made for the year is 156.

The number of lost sales for the month is 338 + 467 + 414 + 137 = 1356. The expected number of lost sales for the year is 16272.

Therefore, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 0,011088(4009) + 0,03(16272) + 1,19(156) \\ &= \underline{\mathrm{R}718,25} \end{aligned}$$

TABLE A.7.2

IMPLEMENTATION OF THE INVENTORY CONTROL POLICY BASED ON BOX- JENKINS FORECASTING TECHNIQUE FOR PRODUCT Y_3

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
1	1271	173	1098	317	1659	0	0
				318			
				318			
				319			
				319			
2	1098	191	907	232	1228	0	0
				232			
				232			
				232			
				232			
3	907	275	632	207	1103	0	0
				207			
				207			
				207			
				207			
4	632	214	418	248	1310	0	3000
				248			
				248			
				248			
				248			
5	3418	182	3286	228	1209	0	0
				228			
				228			
				228			
				229			
6	3286	278	2958	201	1073	0	0
				201			
				201			
				201			
				201			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
7	2958	253	2705	247	1304	0	0
				247			
				247			
				247			
				248			
8	2705	214	2491	251	1324	0	0
				251			
				251			
				251			
				251			
9	2491	197	2294	229	1216	0	0
				229			
				230			
				230			
				230			
10	2294	258	2036	210	1120	0	0
				210			
				210			
				211			
				211			
11	2036	314	1722	239	1263	0	0
				239			
				239			
				239			
				239			
12	1722	164	1558	284	1490	0	0
				284			
				284			
				284			
				284			
13	1558	186	1372	214	1138	0	0
				214			
				214			
				214			
				214			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
14	1372	222	1150	197	1057	0	0
				198			
				198			
				198			
				198			
15	1150	259	891	212	1128	237	0
				212			
				212			
				212			
				212			
16	891	234	657	240	1268	374	0
				240			
				240			
				240			
				240			
17	657	273	384	237	1253	258	0
				237			
				237			
				237			
				237			
18	384	178	206	258	1361	286	0
				258			
				259			
				259			
				259			
19	206	243	-37	212	1128	10	237
				212			
				212			
				212			
				212			
20	200	235	-35	230	1219	326	374
				230			
				230			
				230			
				231			
21	339	256	83	233	1235	272	258
				233			
				233			
				234			
				234			

The number of cycles is 7.

The average inventory held during cycle:

<u>One</u>: $\frac{\sum_{i=1}^{15} Q_i}{15} = \frac{28898}{15} = 1927$

<u>Two</u>: $Q_{16} = 891$

<u>Three</u>: $Q_{17} = 657$

<u>Four</u>: $Q_{18} = 384$

<u>Five</u>: $Q_{19} = 206$

<u>Six</u>: $Q_{20} = 200$

<u>Seven</u>: $Q_{21} = 339$

The average inventory held is,

$$\frac{15(1927) + 1(891) + 1(657) + 1(384) + 1(206) + 1(200) + 1(339)}{15 + 1 + 1 + 1 + 1 + 1 + 1 + 1}$$

$$= \underline{1504}$$

Since there are 7 made during this month, it is assumed that the average number of orders made for the year is 84.

The number of lost sales for the month is $37 + 35 = 72$. Therefore, the expected number of lost sales for the year is 864.

Therefore, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,02268)(1504) + 0,03(864) + 1,19(84) \\ &= \underline{\text{R}159,99} \end{aligned}$$

APPENDIX 8

For product Y_2, $\alpha = 0,4$ is chosen since the M.S.E. = 44 331 is the minimum M.S.E. See Table A.8.1 below.

TABLE A.8.1 FORECAST SUMMARY FOR PRODUCT Y_2

Data: Y2 — Percent: 100

Forecast summary	M.E.	M.S.E.	M.A.E.	M.A.P.E.	M.P.E.
Simple: 0.1	-32.3876	66229.5	196.005	12.5894	-4.14835
Simple: 0.2	-16.8912	50590.0	169.793	10.7006	-2.66251
Simple: 0.3	-11.5339	45708.0	162.951	10.1635	-2.11339
Simple: 0.4	-8.84290	44331.0	159.899	9.90914	-1.83538
Simple: 0.5	-7.21657	44520.2	160.005	9.86768	-1.66566

TABLE A.8.2

IMPLEMENTATION OF THE BROWN'S EXPONENTIAL SMOOTHING TECHNIQUE FOR PRODUCT Y_2

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
1	6178	1368	4810	1585 1585 1585 1585 1585	8272	0	0
2	4810	829	3891	1498 1498 1498 1498 1498	7837	0	0
3	3981	1008	2973	1230 1230 1230 1230 1230	6497	0	0
4	2973	1389	1584	1141 1141 1141 1141 1141	6052	0	9000
5	10584	931	9653	1241 1241 1241 1241 1241	6552	0	0

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
6	9653	897	8756	1117	5932	0	0
				1117			
				1117			
				1117			
				1117			
7	8756	1258	7498	1029	5492	0	0
				1029			
				1029			
				1029			
				1029			
8	7498	1195	6303	1120	5947	0	0
				1120			
				1120			
				1120			
				1120			
9	6303 1435	4868	1150	6097	1229	0	
				1150			
				1150			
				1150			
				1150			
10	4868	1339	3529	1264	6667	1909	0
				1264			
				1264			
				1264			
				1264			
11	3529	911	2624	1294	6817	1055	0
				1294			
				1294			
				1294			
				1294			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
12	2624	1156	1468	1141	6052	391	0
				1141			
				1141			
				1141			
				1141			
13	1468	1095	373	1145	6072	1115	1229
				1145			
				1145			
				1145			
				1145			
14	1602	1167	435	1126	5977	1072	1909
				1126			
				1126			
				1126			
				126			
15	2334	1355	989	1142	6057	1435	1055
				1142			
				1142			
				1142			
				1142			
16	2044	1394	695	1227	6482	1774	391
				1227			
				1227			
				1227			
				1227			
17	1086	1272	-186	1276	6727	1517	1115
				1276			
				1276			
				1276			
				1276			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
18	929	1282	-335	1274	6717	1272	1072
				1274			
				1274			
				1274			
				1274			
19	719	1009	-290	1277	6732	1024	1435
				1277			
				1277			
				1277			
				1277			
20	1145	1181	-36	1170	6197	646	1774
				1170			
				1170			
				1170			
				1170			
21	1738	1186	552	1174	6217	1206	1517
				1174			
				1174			
				1174			
				1174			

The number of cycles during this month is 13.

The average inventory held during cycle:

<u>One</u>: $\frac{\sum_{i=1}^{9} Q_i}{9} = \frac{60736}{9} = 6748$

<u>Two</u>: $Q_{10} = 4868$

<u>Three</u>: $Q_{11} = 3529$

<u>Four</u>: $Q_{12} = 2624$

<u>Five</u>: $Q_{13} = 1468$

<u>Six</u>: $Q_{14} = 1602$

<u>Seven</u>: $Q_{15} = 2344$

<u>Eight</u>: $Q_{16} = 2044$

<u>Nine</u>: $Q_{17} = 1086$

<u>Ten</u>: $Q_{18} = 929$

<u>Eleven</u>: $Q_{19} = 719$

<u>Twelve</u>: $Q_{20} = 1145$

<u>Thirteen</u>: $Q_{21} = 1738$

The average inventory held is,

$$\frac{9(6748) + 4868 + 3529 + 2624 + 1468 + 1602 + 2344 + 2044 + 1086 + 929 + 719 + 1145 + 1738}{9+1+1+1+1+1+1+1+1+1+1+1+1} = \underline{4039}$$

Since 13 orders are made during this month, it is assumed that the average number of orders made for the year is 156.

The number of lost sales for the month is 353 + 290 + 36 = 679. Therefore, the expected number of lost sales for the year is 8148.

Therefore, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= 0,011088(4039) + (0,03)(8148) + 1,19(156) \\ &= \underline{\text{R}474,86} \end{aligned}$$

For products Y_3, $\alpha = 0,6$ is chosen since the M.S.E. = 4599,62 is the minimum M.S.E. See Table A.8.3 below.

TABLE A.8.3 FORECAST SUMMARY FOR PRODUCT Y_3

Data: Y3

Percent: 100

Forecast summary	M.E.	M.S.E.	M.A.E.	M.A.P.E.	M.P.E.
Simple: 0.1	5.12344	10606.9	72.1935	21.2265	-4.39972
Simple: 0.2	2.67206	6613.66	55.5653	17.5307	-4.03563
Simple: 0.3	1.87042	5371.69	50.1744	16.4327	-3.73180
Simple: 0.4	1.43931	4858.98	48.0177	16.0678	-3.60186
Simple: 0.5	1.15647	4647.80	47.5070	16.1041	-3.54692
Simple: 0.6	0.95363	4599.62	48.3358	16.4855	-3.52912
Simple: 0.7	0.80074	4659.84	49.3939	16.9435	-3.53182

TABLE A.8.4

IMPLEMENTATION OF THE BROWN'S EXPONENTIAL SMOOTHING TECHNIQUE FOR PRODUCT Y_3

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
1	1271	173	1098	316 316 316 316 316	1692	0	0
2	1098	191	907	230 230 230 230 230	1262	0	0
3	907	275	632	207 207 207 207 207	1147	0	0
4	632	214	418	248 248 248 248 248	1352	0	3000
5	3418	182	3286	227 227 227 227 227	1247	0	0
6	3286	278	2958	200 200 200 200 200	1112	0	0

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAI
7	2958	253	2705	247	1347	0	0
				247			
				247			
				247			
				248			
8	2705	214	2491	251	1367	0	0
				251			
				251			
				251			
				251			
9	2491	197	2294	229	1257	0	0
				229			
				230			
				230			
				230			
10	2294	258	2036	210	1162	0	0
				210			
				210			
				211			
				211			
11	2036	314	1722	239	1307	0	0
				239			
				239			
				239			
				239			
12	1722	164	1558	284	1532	0	0
				284			
				284			
				284			
				284			
13	1558	186	1372	212	1172	0	0
				212			
				212			
				212			
				212			

DAY	AVAILABLE STOCK	DEMAND	ON HAND	FOREASTED DEMAND	FORECASTED DEMAND FOR A WEEK PLUS SAFETY STOCK	ORDER QUANTITY	ARRIVAL
14	1372	222	1150	196	1092	0	0
				196			
				196			
				196			
				196			
15	1150	259	891	212	1172	281	0
				212			
				212			
				212			
				212			
16	891	234	657	240	1312	374	0
				240			
				240			
				240			
				240			
17	657	273	384	236	1292	253	0
				236			
				236			
				236			
				236			
18	384	178	206	258	1402	288	0
				258			
				258			
				258			
				258			
19	206	243	-37	210	1162	3	281
				210			
				210			
				210			
				210			
20	244	235	9	230	1262	335	374
				230			
				230			
				230			
				230			
21	383	256	127	234	1282	276	253
				234			
				234			
				234			
				234			

The number of cycles is 7.

The average inventory held during cycle:

One: $\frac{\sum_{i=1}^{15} Q_i}{15} = \frac{28898}{15} = 1927$

Two: $Q_{16} = 891$

Three: $Q_{17} = 657$

Four: $Q_{18} = 384$

Five: $Q_{19} = 206$

Six: $Q_{20} = 244$

Seven: $Q_{21} = 383$

Thus the average inventory held is,

$$\frac{15(1927) + 891 + 657 + 384 + 206 + 244 + 383}{15 + 1 + 1 + 1 + 1 + 1 + 1 + 1}$$
$$= \underline{1508}$$

Since there are 7 orders made during this month, it is assumed that the average number of orders made for the year is 84.

The number of lost sales for the month is 37. Therefore, the expected number of lost sales for the year is 444.

Therefore, the average yearly cost is,

$$\begin{aligned} C &= C_1 + C_2 + C_3 \\ &= (0,02268)(1508) + (0,03)(444) + (1,19)(84) \\ &= \underline{\text{R}147,48} \end{aligned}$$

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