

**EXPLORING MASTER TEACHERS’
USE OF VISUALS AS TOOLS IN
MATHEMATICS CLASSROOMS**

By

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Dedication

This work is dedicated to

My Swami, Bhagavan Sri Sathya Sai Baba.

My late grandparents Mr and Mrs Kisten Naidoo & Mr M. Moodley.

Among all professions, the teaching profession carries the greatest responsibility. Teachers have to mould the young of today, so that they will grow up as worthy citizens of tomorrow.

Bhagavan Sri Sathya Sai Baba

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Declaration

I, Jayaluxmi Naidoo declare that

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Signed.....

As the candidate's Supervisor I agree to the submission of this thesis.

Signed.....

Abbreviations and Acronyms

AMESA	Association for Mathematics Education of South Africa
DMECLT	Diversity for Mathematics Education Center for Learning and Teaching
DoE	Department of Education
FET	Further Education and Training
GET	General Education and Training
KZN	KwaZulu-Natal
LOLT	Language of learning and teaching
MKO	More knowledgeable other
NCS	National Curriculum Statements
NCTM	National Council of Teachers of Mathematics
OHP	Overhead projector
OHT	Overhead transparency
SAARMSTE	Southern African Association for Research in Mathematics, Science and Technology Education
TIMSS	Third International Maths and Science Study
UKZN	University of KwaZulu-Natal
VA	Visualiser/Analyser
ZPD	Zone of proximal development

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Coding used in the study*

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Master Teacher	Code	Interviews	Observations
Alan	A	AI	AO
Sam	S	SI	SO
Dean	D	DI	DO
Karyn	K	KI	KO
Penny	P	PI	PO
Maggie	M	MI	MO

1.2. Focus Group Interviews with learners

Learner Number	School	Code
1 - 6	Daisy Secondary	L 1- 6 DFG
1 - 10	Tulip Secondary	L 1- 10 TFG
1 - 8	Rose Secondary	L 1- 8 RFG
1 - 6	Lily Secondary	L 1- 6 LFG
1 - 9	Orchid Secondary	L 1- 9 OFG
1 - 7	Carnation secondary	L 1- 7 CFG

Key

Examples of the coding used in this study are illustrated below.

For example

SO2 – Sam Observation, response to question 2

AIC2- Alan Interview Section C, response to question 2

L6 RFG 7 – Learner number 6, Rose Secondary Focus Group Interview, Response to Question 7

L 1-3 OFG 10 – Learner number 1, 2 and 3. Orchid Secondary Focus Group Interview, Response to Question 10.

* A more detailed description of the coding used in the study may be found in

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Abstract

The teaching and learning of mathematics has presented a great challenge for mathematics educationalists over many decades. Researchers have been searching for new strategies and techniques for improving the understanding of abstract mathematical concepts. With the current changes in the mathematics curriculum in South Africa, it is important to ensure that no learner is left behind in the pursuit to produce mathematically literate learners nationally. Teachers are encouraged to teach a common curriculum so that all learners have equal opportunities of attaining success in a democratic society in any chosen field. Some teachers achieve mathematical success easily while others struggle to achieve similar outcomes.

Whilst we acknowledge that teachers ought to emulate the practices of other good teachers, we often do not seek explanations of what makes a teacher effective and how they achieve success in a classroom. As can be conceived, apart from probing teachers' content knowledge, it is necessary to know how this knowledge can be used for optimal results in the course of teaching within the diverse South African classroom. In other words, it becomes necessary to interrogate the teacher's pedagogical content knowledge because of the uniqueness of the South African context. It is for this reason that an in-depth study was done to explore Master teachers' use of visuals as tools within mathematics classrooms.

This study focused on six experienced mathematics teachers or Master mathematics teachers. These teachers were selected from six Dinaledi schools located in KwaZulu-Natal. The schools catered for learners from multicultural and multiracial backgrounds.

Activity theory was used as a framework to locate the study. Each activity system was interrogated within an interpretivist paradigm. Data was collected using six methods and five research instruments.

The analysis revealed that all the Master teachers used visuals, albeit differently, as tools in their classrooms. Visual tools that are referred to in this study incorporated diagrams, symbols, pictures, transparencies, graphs, the use of colour, shapes, mathematics manipulatives, gestures and any other visual that was considered as a trigger or catalyst that prompted the need for interpretation of mathematical concepts or ideas.

A key finding of this research showed that using visuals as scaffolding tools was paramount for the effective teaching and learning of mathematics. These tools were conscious, predetermined, intuitive and relevant. The findings exhibited that each Master teacher generally found techniques of making the mathematical concepts easier to understand. These findings may have wide-ranging influence in the education sector, including teacher and curriculum development.

Chapter One: Introduction

“The students of today are the teachers of tomorrow.”
Bhagavan Sri Sathya Sai Baba

Schoenfeld (2000) proposed that research in mathematics education has two main purposes. The first is to know the nature of mathematical thinking, teaching and learning and the second is to use this knowledge to improve mathematics instruction. This study focuses on both purposes. The study explores how Master mathematics teachers teach, with the ultimate aim of using these findings to assist in improving mathematics instruction through innovations that may support teachers.

Giving teachers the support they need to succeed is crucial (Alliance for Excellent Education, 2008). This support ought to be provided before they enter the classroom. The idea of professional development is that teachers need to be provided with continued support and opportunities to develop their skills and competencies (Stipek, 2004). When this support is provided, opportunities need to be created for teachers to learn subject matter to improve their content knowledge and assist with pedagogy. Teachers ought to be able to effectively use what they know in varied situations and contexts (Ball & Cohen, 1999).

This study focused on exploring Master mathematics teachers’ use of visuals as tools in the mathematics classrooms. The Master mathematics teachers that were selected as participants for the study were identified by the KwaZulu-Natal (KZN)¹ Department of Education. Master teachers are generally knowledgeable, competent teachers who have a record of excellent achievement in the Senior Certificate Examinations (HEDCOM Secretariat, 2006). These teachers are highly performing successful teachers who have the skill and knowledge to teach mathematics and can share experience and expertise with other teachers (Department of Education, 2006).

¹ KwaZulu-Natal (KZN) is one of the nine provinces of South Africa. This is the province that the study is located in.

The reasons for working with Master mathematics teachers were twofold, firstly Master mathematics teachers view the classroom differently from novices and allow their learners the opportunity to get on with what they think their learners need (Ainley & Luntley, 2005). Secondly, these Master teachers are recognised as good teachers within the South African context. Good teachers have been studied since Plato's Meno dialogue (Beishuizen & Hof, 2001), where Plato described how Socrates taught by asking questions. This suggests that to be effective in the classroom a teacher is required to ask deep probing questions which in turn stimulates interactive discussions. These discussions allow for meaningful engagement with the material being studied.

Similarly Stipek (2004) stated that good teachers make all the difference and are crucial to the success of their learners (Alliance for Excellent Education, 2008). A teacher's personality as well as a teacher's experience, skills, content, pedagogic and curricular knowledge is paramount (Beishuizen & Hof, 2001; Bunting, 2006) for effective teaching and learning.

1.3. Teaching and learning

Nicoll & Harrison (2003) maintained that teaching and learning has presented a great challenge for researchers over many decades. Teachers are always trying to find that perfect method or that ideal strategy to ensure that effective learning takes place. Teaching is more than just standing at the front of the class and imparting knowledge. The science of teaching has developed immensely, and it is certainly not a career for those who cannot engage in depth with the didactics involved.

“Teaching can be totally absorbing but there always seems to be more to do, more that you want to do, more that you have to do, than the time allows. It's a constant struggle to fit it all in ...” (Pollard & Triggs, 1997, p. 5).

Through a teacher's commitment and quest to find the right formula for success in class, teaching becomes frustrating, physically and emotionally draining as well as intellectually

challenging (Harden & Crosby, 2000; Tatel, 1999). A good teacher continuously reflects on practice within the classroom. Reflection is a process of reviewing the experience of teaching in order to describe, analyse, evaluate and inform learning about teaching (Preen, 2007). Schön defined two types of reflection: reflection-in-action and reflection-on-action (Schön, 1983, 1987). Reflection-in-action refers to the teacher's ability to reflect during a specific lesson rather than after the lesson. This presents a more dynamic approach in the teaching and learning process. In contrast reflection-on-action involves thinking about and reviewing the lesson after the lesson has concluded. This allows the teacher the prospect of evaluating and commenting on the lesson. Options with respect to future practice are initiated through this process. Essentially teachers ought to learn from their own practical experience. In doing so, they can either engage in shallow problem solving processes entrenched in traditional norms or demands of work, or preferably engage in a deeper level of problem solving which is more meaningful and challenging (Schön, 1983).

Through this reflective process, a good teacher recognises that teaching is not a display of knowledge but rather it is a process which includes identifying an area of learning and deciding on interventions that will foster this learning (Ursano, Kartheiser, & Ursano, 2007). For example apart from acquiring the necessary knowledge, with respect to the mathematics concepts and ideas, the mathematics teacher is also required to know how to teach these concepts and ideas effectively. They are required to know how these concepts are related to each other as well as how they are linked to each other. In addition, mathematics teachers are required to comprehend how concepts progress from one grade to the next. For this to be done effectively, mathematics teachers need to possess good pedagogic content knowledge (Kazima, Pillay, & Adler, 2008). Pedagogic content knowledge entails the merging of content and pedagogy (Ball Loewenberg, Thames, & Phelps, 2008). This merger occurs as a mechanism to understand how topics are represented with the aim of teaching these topics effectively. Shulman defines pedagogical content knowledge as "... the category most likely to distinguish the understanding of the content specialist from that of the pedagogue..." (Shulman, 1987, p. 4). Orton (1992) also

claimed that the aim of teaching was to encourage learning. Pedagogical content knowledge embraces the comprehension of what makes the understanding of certain subject matter effortless or complicated. Hence when a Mathematics teacher possesses good pedagogic content knowledge this allows their learners easy access to the learning of mathematics.

Argyris and Schön have both done much research (Argyris, 1976, 1980, 1982, 1985; Argyris, Putnam, & Smith, 1985; Argyris & Schön, 1974, 1978) on social interactions, on learning, and on reflective practitioners. This research was instrumental in developing the theory of action that views individuals as designers of their own actions. Two concepts of learning arise from Argyris and Schön's theory of action. The concepts are single-loop and double-loop learning. Single-loop learning refers to the processes that occur when the teacher realises that the method or approach s/he is using is not achieving the goal of the lesson. Based on the teacher's reflection on personal action, another method to achieve the same goal is selected. However, in instances of double-loop learning, the teacher reflects on the values and rules that were the deciding factors in choosing different methods of teaching. The social structures and contexts which ensure that these methods are meaningful and instrumental are interrogated (Greenwood, 1998). Thus, double-loop learning occurs as a result of the teacher's reflection on rules, values and social interactions. These in turn support the teacher's actions.

The choice of substantial and applicable contexts when teaching mathematics is necessary to ensure effective teaching and learning. Learning is seen as a way of developing knowledge within meaningful contexts (Handal & Bobis, 2004). For a context to be meaningful, the learners within the context ought to be able to relate to and be familiar with these contexts. Essentially, social contexts that are conducive to learning need to be created, because the process of learning is itself social (Putnam & Borko, 2000; Wentzel, 2002). I agree with the researchers mentioned above but add on that whilst learning is social it is also cognitive.

1.4. The classroom context

The classroom is a site of many overlapping, complex practices that can sometimes be confusing (Cobb, 2000). The confusion arises because everyone entering the classroom maintains their own agendas. For example, the teacher's goal may be to initiate learners into mathematical ways of thinking and acting whilst the learners' goals may be quite different (Lerman, 2000). The learner may be coming in to the classroom because this is what society anticipates. This is what is considered to be socially acceptable. With this in mind, a teacher ought to be innovative enough to ensure that constructive learning occurs within the classroom context. A teacher also needs to be cognisant of the fact that learners are not 'empty vessels', but rather, learners bring into the classroom rich experiences and prior knowledge. This knowledge and experiences are assets that ought to be utilised for optimal results.

With the changes introduced in the mathematics curriculum, one of the critical outcomes of the curriculum encourages learners to "communicate effectively using visual, symbolic and/or language skills in various modes" (Department of Education, 2003a, p. 2). Whilst teachers are using visual approaches in teaching, not much is known about the use of visuals as tools in the mathematics classroom. Apart from not knowing how these visual tools are used, we also do not know much about the value of their use. Furthermore, whilst there are many new technologies to aid in the teaching and learning of mathematics, based on personal experience, not many teachers are aware of these cutting edge technologies that are available or how to use them effectively. Thus it was the intention of this study to uncover, explore and interrogate the use of visual as tools in mathematics classrooms. Visual tools that are referred to in this study incorporated diagrams, symbols drawn on the board, pictures, transparencies, graphs, the use of colour, shapes, mathematics manipulatives², gestures and any other visual tool that was considered as a trigger or catalyst that prompted the need to interpret mathematical concepts or ideas.

² Manipulatives are concrete objects that are commonly used in teaching mathematics. They include attribute blocks, geometric shapes of different colors, plastic counting cubes, paper models, dynamic models, wooden

The classrooms that were observed in this study were complex multicultural spaces, providing for mixed ability learners of different races. The Master teachers in the sample exhibited their wealth of experience and tacit knowledge by working effectively using different methods and strategies. Based on the Master teacher interview, these teachers were knowledgeable of the learning styles and ability levels of their learners. This knowledge allowed them to utilise their contact time with their learners profitably. Teachers ought to be cognisant of their learners' needs and focus on the potential of each learner rather than their limitations (Thomas, Place, & Hillyard, 2008). As can be conceived, apart from probing what teachers ought to know with respect to content knowledge, it is necessary to know how this knowledge may be used for optimal results in the course of teaching (Ball Loewenberg & Bass, 2000) within the diverse South African classroom.

1.5. Diversity in classrooms

With diversity in the classroom, multiple ways of knowing and teaching are necessary. Teachers ought to use multiple instructional methods to cater for the different needs of their learners. Likewise Bowe, Ball and Gold (1992) proclaimed that knowing how to teach well to different ability learners in a mathematics classroom is important. A good teacher ought to be able to adapt to modifications in teaching methods (Mooij, 2008), particularly in view of the shifting classroom contexts in South Africa. In cases of a diverse range of learners, the teacher ought to be able to teach for diversity and operate successfully within available resources (Harden & Crosby, 2000; Nicoll & Harrison, 2003).

The word diverse is used to mean the learners' ability levels as well as their home and cultural backgrounds. In South Africa, years after the transition from apartheid to democracy, social class inequalities have endured and a highly class conscious system of education has become embedded (Chisholm & Sujee, 2005; Soudien, 2004). Generally, in

structures, charts and everyday objects (coins, stones, beads, buttons, ice cream sticks and bottle caps) that could be used as teaching tools.

most schools, mathematics classrooms are not streamed. Thus, schools in South Africa cater for learners from diverse social class backgrounds, diverse race groups, diverse cultures and diverse ability levels (Naidoo, 2006). Whilst this is true, learners from working class backgrounds suffer the disadvantages associated with being poorly located within educational hierarchies (Cremin & Thomas, 2005).

These learners find themselves in socially mixed schools often not having the necessary requirements and resources for their lessons. This creates an additional stumbling block in 'levelling out the educational playing field'. Apart from these disadvantages emerging at schools, many schools in present day South Africa are lacking in both human as well as material resources. This discrepancy has stagnated and stunted learners' educational advancement which defeats the purpose of education. Education is meant to bring about change and ought to assist the learner to see the world differently (Zimmermann, 1991). Education ought to liberate a person and afford the individual the power to succeed.

To disseminate the power of education to all learners, a teacher is required to be adept at transmitting knowledge in other ways. Teachers are required to be good communicators and transmitters of knowledge. For example, this may be achieved by using visual tools, non-verbal gestures, providing nods of agreement or simply maintaining good eye contact whilst allowing learners to feel safe in their learning environment (Roman & Kay, 2007). The use of different teaching methods, strategies and tools allow learners the access to participate unconditionally within their learning environment.

1.6. The use of visuals as tools

Having worked as a teacher and researcher within the mathematics field for the past 15 years I have encountered many frustrated teachers and learners. Many learners have claimed that mathematics is too difficult and abstract. On the other hand, I have been intrigued by how some teachers manage to assist their learners in grasping these abstract concepts in mathematics, with ease, as compared to other teachers.

This exploration focussing on Master teachers' use of visuals as tools in mathematics classrooms arose as a means of attempting to make mathematics more accessible to more learners. The reason for the focus on the Master teacher and the use of visual tools served a dual purpose. Firstly a teacher's effect on academic growth dwarfs the effect of other factors such as class size (Stipek, 2004). Secondly the presence of visual elements in today's teaching and learning materials has increased with the integration of images and visual presentations within text (Branton, 1999). We live in a technological era where vision is one of our most important senses that enables us to have contact with the outside world (Verstraelen, 2005). Thus there is a need for teachers to encourage learners to engage with visual ways of thinking and learning.

“No soul thinks without a mental image” as argued by Aristotle (Benson, 1997, p. 146; Zazkis, Dautermann, & Dubinsky, 1996, p. 437). The emphasis is on the word ‘thinks’, one basic aspect of visual thinking is the ability to move back and forth between the graphical and analytical representation of a problem. In mathematics, a learner is continuously exposed to both analytical and graphical information. Knowing how to work with graphical and analytical information would allow the learner to gain access to the mathematics inherent in this information. Since learners develop their visual abilities through use (Stokes, 2000), it may mean that in order to teach visual thinking the teacher must engage in conscious raising exercises, so that learners become aware of the knowledge required to interpret mathematical graphics (Zimmermann, 1991). Teachers need to remember that they are teaching to the entire body and all of its senses with multiple ways of processing (Rose, 2004). They need to prepare the next generation with high level critical thinking skills (Cepni, 2003), more so because we live in an increasingly complex, highly visual and interconnected society (Rezabek, 2008).

1.7. Introducing the critical research questions

Consequently, the purpose of this study was to explore Master teachers use of visuals as tools in mathematics classrooms. This study draws on activity theory to assist the reader in understanding the use of visual tools within an activity system. The study serves to answer three critical questions. The first question identifies what visuals Master teachers use as tools in mathematics classrooms. The second question explores how Master teachers use visuals as tools in mathematics classrooms. The third question interrogates why Master teachers use visuals as tools in mathematics classrooms.

The visual tools employed by the Master teachers within the sample population were under focus in attempting to answer the above questions. Data were collected using a teacher questionnaire, lesson observations, a field diary, a semi-structured teacher interview schedule and a semi-structured focus group interview schedule for learners. All discussions with the Master teachers and learners were audio taped with the permission of each participant. A coding system was implemented, this system was exercised in order to code, analyse and then discuss the data collected.

1.8. The scope of this study

Since we have no coherent account of what good teachers are Masters of and how they achieve what they achieve (Tatel, 1999), this in-depth study has been done to explore Master teachers' use of visuals as tools within mathematics classrooms in South Africa.

This research study was limited to a group of six Master mathematics teachers teaching in six different Dinaledi schools in KwaZulu-Natal, South Africa. Dinaledi schools are schools that were selected by the National Department of Education. Dinaledi schools were intended to increase the participation and performance of Black learners and female learners in mathematics and science. The Dinaledi project was intended as a short term project providing teaching and learning resources to a limited number of schools. Schools that were selected were regarded as 'Star' schools in South Africa.

There were three male teachers and three female teachers in the sample. The teachers were of different races and of different cultures. Their teaching experience ranged from 13 years to 26 years.

The following table provides a list of schools and research participants³:

Teacher	Gender	Teaching experience (years)	Name of School
Dean	M	26	Daisy Secondary
Penny	F	24	Tulip Secondary
Karyn	F	15	Rose Secondary
Maggie	F	18	Lily Secondary
Alan	M	13	Orchid Secondary
Sam	M	25	Carnation Secondary

Table 1: Master teachers in the study

1.9. The contribution of this study to mathematics education

The literature on Master teachers' use of visual tools will reveal that this is a relatively unexamined phenomenon in South African classrooms. This perspective, namely, on teaching and learning through the use of visual tools in mathematics classrooms, recasts the relationship between what teachers teach and how they teach. It foregrounds the fact that the ways in which teachers teach and the ways in which learners learn are inextricable parts of the classroom culture. The study draws attention to the dilemma faced by many teachers in South Africa. Teachers are compelled to teach the same curriculum within the same timeframe nationally, regardless of the inequitable distribution of both human and material resources.

Methodologically, this research study embraced the complexity and challenge, through the adoption of an interpretive stance within this qualitative study. An analysis of the

³ All the Master teacher's names and names of each school are pseudonyms.

methodological challenges in gaining access and acceptance in the field is presented. The theoretical work of researchers (Bellamy, 1996; Bodker, 1996; Christiansen, 1996; Engeström, 1987, 1993, 1994, 1999, 2000, 2001; Jonassen & Rohrer-Murphy, 1999; Kaptelinin, 1996; Kaptelinin & Nardi, 1997; Kuutti, 1996; Morf & Weber, 2000; Nardi, 1996; Rivers, Calic, & Tan, 2009; Slay, 2002; Uden, 2007; Zinchenko, 1996) within the field of activity theory were explored. The study also highlights the usefulness of this theoretical framework in the reconceptualising of learner learning as interaction and activity instead of the traditional teacher/learner dyad of chalk and talk. At the same time, it offers a critique of the feasibility and appropriateness of using activity theory as a framework for analysing a learning community. Yarnit (2000) defined a learning community as follows:

“A learning community addresses the learning needs of its locality through partnership ... Learning communities explicitly use learning as a way of promoting social cohesion regeneration and economic development which involves all parts of the community.” (Yarnit, 2000, p. 11)

I concur with Yarnit’s definition because the learning communities in this study included teachers, parents, community members and learners who shared common beliefs and values. They were engaged in attaining a common goal of teaching and learning. The study also draws attention to the challenges of applying activity theory as a framework within this study.

The data analysis will reveal that the Master teachers in the study used diverse visual tools to teach in the mathematics classrooms. Whilst they used diverse tools, their reasons for using these tools were similar. Learning had occurred along comparable trajectories with respect to each activity system. Each teacher and learner within the activity system had specified roles to play; they negotiated and mediated their tools with the intention of achieving a common outcome. Each learning community successfully applied both explicit and implicit rules to achieve this outcome within each observed mathematics lesson.

1.10. Overview of this study

In determining an appropriate approach to this study, the following structure has been used. This study comprises nine chapters, the bibliography and the appendices. The chapters in this study are as follows:

Chapter one introduces the background to the study as well as the motivation for this study. In addition this chapter provides the overview of the study. Furthermore the critical research questions are also introduced in **Chapter one**.

Chapter two presents the relevant literature based on the area of investigation. Mathematics, teachers, teaching within the South African context and visualisation are explored.

Chapter three presents the theoretical framework for this study. The theory that impacts on this study is discussed. More specifically activity theory forms the framework for this research study. The relevance of this theory to the study is clearly indicated.

Chapter four presents the research design, the research methodology, and procedures undertaken to conduct this study. This chapter outlines and summarises the design of the study and the research instruments employed. The preliminary process involved with respect to the pilot study, gatekeepers, gaining access and the method of sampling is also discussed.

Chapter five concentrates on the Master teacher. Each Master teacher is introduced. The visual tools used by each teacher are discussed.

Chapter six, seven and eight explores the findings and implications of this research study. Furthermore, these chapters aim to explore and respond to the critical questions of this study. The questions that were addressed in the teacher questionnaire, teacher interview,

learner focus group interviews as well as the observations made during the video recording of lessons are discussed in these chapters.

Chapter nine is the final chapter in this research study. This chapter presents the conclusions that were drawn based on the overall research study. Some limitations and recommendations of the study are also discussed.

1.11. Conclusion

This chapter has provided an outline of the purpose, scope and rationale for this research study. The chapter concluded with an overview of the research study where a brief preview of the chapters to follow is provided. In the next chapter, the literature that has been reviewed is discussed.

Chapter Two: Literature Review

*“The end of wisdom is freedom; The end of culture is perfection;
The end of knowledge is love; The end of education is character.”
Bhagavan Sri Sathya Sai Baba*

2.1. Introduction

In the previous chapter, the rationale for this study was offered and an overview of this study was presented. This chapter aims to explore the literature that informed the study. Comparisons with research across the globe within related fields of study have been made. This assisted in discussing key aspects of the study. This chapter commences with a discussion of the importance of mathematics in general and a discussion of relevant issues in the teaching and learning of mathematics. A review of mathematics in South Africa and the importance of context when teaching mathematics follow.

This section leads to a discussion of mathematics in South Africa and the practice of teaching. The chapter progresses with a discussion of visualisation and of key concepts integrated within this section. The chapter is concluded with a discussion on the use of visual tools whilst teaching and the implications of the literature review for this study.

2.2. A global perspective of mathematics

Mathematics encompasses knowledge that is centred on concepts such as quantity, structure, space, and change. It is about theorems, axioms, and methods. According to Lerman (2000), mathematics is used throughout the world in many fields and disciplines. This includes natural science, engineering, medicine, and social science. I concur with Lerman (2000) and add that mathematics is the science of patterns, whereby mathematicians are tasked with exploring patterns found in numbers, space, science, computers and imaginary abstractions. These patterns are explored with the intention of formulating new conjectures and establishing their truth by rigorous deduction from appropriately chosen axioms and definitions. Patterns are thus seen as the essence of mathematics and the language in which it is expressed (Samson, 2007) and thought about.

It is for this reason that one envisages mathematicians as problem solvers and independent thinkers. Similarly, Mendick (2005) asserted that mathematicians are self-directed individuals who follow their own path of illumination and triumph in the end.

Likewise, Samson (2007) and Tanner and Jones (2000) maintained that thinking mathematically is a process through which we make our own decisions. We make sense of the world around us, regardless of whether we are at leisure or at work. Furthermore, we use mathematics to negotiate our existence whether we are out shopping (calculating prices, budgeting or bargaining), cooking (weighing or counting ingredients, estimating time for cooking or calculating number of portions), travelling (calculating distance or estimating the next fuel stop) or at work. Thus, mathematics is a field of knowledge in which mathematicians work to discover truths about the natural world (Herzig, 2004), it is universal and context-independent (Dorfler, 2003). Fundamentally, Otte (2006) purported that mathematics is thought about as an activity.

I concur with Otte's claims because meanings in mathematics are socially constructed through action and interaction. Actions and interaction are necessary requisites for an activity. The notion of social construction assumes that learners work together with other learners and the teacher within the classroom context to achieve a shared understanding or meaning. Along similar lines, Vygotsky emphasised that social interaction was essential for learning (Vygotsky, 1978). In the mathematics classroom these interactions revolve around quantity and numbers (Keijzer & Terwel, 2001). Whilst I agree with Keijzer and Terwel, it is important to remember that mathematics classroom interactions also incorporates the space and shape strand. The learner and teachers form their own learning community whilst using their own culturally contrived tools. Activity theory emphasises that an activity system ought to be comprised of a learning community who collectively aspire to share a common understanding with the intention of accomplishing a common goal. This suggests that mathematics can be reshaped in schools to give all learners more access to its concepts and wealth. Schools can become a place for teachers and learners to change and modify

what is expected and accepted in society (Perry, 2006). This ensures that the power of mathematics exists in its utilitarian value and in certain instances the abstract nature of mathematics may be removed.

However, whilst meanings are made, interpreted, shared and remade through many different representational modes, meaning is also sometimes lost in this transformation (Cooney, Shealy, & Arvold, 1998). It is through these miscommunications that feelings of inadequacy and notions of mathematics being difficult emerge. These notions ought to be addressed since mathematics is an essential part of our culture whereby learners are exposed to mathematics on a daily basis (Tanner & Jones, 2000).

Learners are exposed to mathematics on a daily basis; this suggests that we live in a society that values mathematical knowledge. Knowledge refers to an individual's proficiencies and skills obtained through experience and education. To ensure that learners acquire suitable mathematical knowledge, Roodt and Conradie (2003) proposed that quality education in mathematics is critical. To achieve this aim, we need to prepare our learners adequately to function appropriately and efficiently on a global platform. This means that we need to prepare our learners to be able to participate critically yet effectively in society. Learners ought to be encouraged to become independent thinkers in a democratic society. To attain critical citizenship, mathematics is a necessary component (Adler, Ball, Krainer, Lin, & Novotna, 2005). So whilst mathematics is well used, Gates (2001, p. 17) asserted that "Mathematics plays its part in keeping the powerless in their place and the strong in positions of power..."

Gates' view is crucial because mathematical knowledge is recognised as an important educational goal (Keijzer & Terwel, 2001), and hence it serves as a critical factor for learners who need to follow different careers. As can be imagined, with this new focus, mathematicians face many new challenges in the discipline (Kyle, 2004). The aspirations of the current mathematical curriculum ought to be addressed. Stylianou (2002) claimed

that the fundamental aim of teaching mathematics is to equip learners with strategies, skills and knowledge. Barnes (2004, 2005) arguments concur with those of Stylianou and adds that the confidence to use mathematics to solve real world problems is essential. However, what transpires at schools is that mathematics is being "massified" (Adler, et al., 2005, p. 360) and not much time is spent on encouraging the understanding of concepts and ideas in mathematics.

This time pressure is due to syllabus coverage, time constraints and the fact that mathematics is always associated with textbooks and curriculum material (Remillard, 2005) and not necessarily with the real world. Easy access is inhibited by this abstract nature of mathematics. Anecdotal experience confirms this notion to be true; mathematics is being taught in many schools in a way that allows sections to be ticked off on a checklist. In essence, not much quality time is spent on areas that prove its relevance to real life.

Knowledge of mathematics is essential to all members of society in order for one to participate fully in a democratic society and to reap the benefits of unrestricted career choices (Reyes & Stanic, 1988). This is a common notion in South Africa; mathematics is recognised as the gateway to a better future and is one of the key areas of study within formal educational institutions. Zevenbergen (2001a) concurs with this notion and claims that parents who realise this and grasp the importance of being educated, encourage their children to discover ways to attain access to a better education. This implies that if a learner is not proficient at mathematics, the learner cannot enter the most sought after career paths. This potentially relegates the learner who does not perform well in mathematics, to a lower paying job. This provides little or no room for learners to gain access to the more affluent lifestyles, which is synonymous with being a part of the higher socio-economic class.

Whilst this scenario reflects the status quo in society (Diversity for Mathematics Education Center for Learning and Teaching, 2007), it also intimates that one can follow some higher

paying careers only if one is well qualified in mathematics. Whilst mathematics may be used to gain access to privilege, mathematics also has the power to be used as a tool to end social exclusion. It is partly for this reason that there has been a focus on the teaching and learning of mathematics (Kyle, 2004) in South Africa.

Due to the prejudicial system of education and disadvantages during the apartheid era, teachers are currently tasked with preparing their learners to succeed in a democratic society where the education system is expected to be equitable. Another reason for this focus is to enable mathematics to be used to create a more objective and more productive society.

2.2.1. The teaching and learning of mathematics in South Africa

Reddy (2005) asserted that extreme levels of poverty and considerable inconsistencies epitomise South African society evolving from the historical policy of apartheid. I agree with Reddy because apartheid (meaning separateness) was an inhuman system of legal racial segregation. The previous governments in South Africa since 1948 enforced this system. The democratic election in 1994 signified the death of the apartheid era and the birth of a new era of democracy within South Africa. It was the first time in South African history that a government was sanctioned to plan the development of an education system for the benefit of the country and its people irrespective of race or colour.

This implied that the school system would not be divided along racial lines, although in practice this still occurs. Previously we had a tri-cameral parliamentary system which consisted of three 'houses', namely: House of Delegates (HOD); House of Assembly (HOA) and House of Representatives (HOR). These three houses were implemented in 1983. The houses were based along racial lines and were used to provide some form of self-government for the respective race group. For example, affairs of people from the Indian population were administered and managed in Parliament by the HOD. These affairs included matters regarding education, health and other community issues. The HOA was at

the top of the social ladder, followed by the HOR and then by the HOD in parliament. Although the Blacks made up the majority of the population, this racial group was poorly represented, which led to them being disadvantaged in many areas. Whilst they were not represented in parliament, the Department of Education and Training (DET) handled Black children's schooling in city areas.

The advent of democracy brought huge changes especially with respect to education, since the full impact of apartheid was directed at learners who were not White⁴. During the apartheid era, schools were racially separated and unjustly resourced (Kahn, 2004). This meant that learners who were not White were denied education that would assist them in forming a professional class. This was enforced so that learners who were Black, Indian or Coloured⁵ would not threaten the hegemony of the South Africans of European ancestry (Saul, 2001). According to Atweh, Bleicher and Cooper (1995), the concept of hegemony is the process by which the dominant classes of society use their own ideology to impose their values on the subordinate groups within society. Thus, hegemony is the political, economic, or cultural power exercised by a dominant group over other groups, regardless of how the other groups feel.

During the apartheid era, the dominant group of society was White and the subordinate groups were of Black, Coloured and Indian descent. The dominant group of society decided that people who were not White did not deserve to be treated equitably and hence they were subjected to an education system that was substandard. With a substandard education system, learners who were of Black, Coloured and Indian origin, would not be admitted into the 'elite' professions. What was evident here was that in South Africa the fundamental undertaking of apartheid education was to make sure those learners who were not White, did not have any access to the social ladder. White children's schools were

⁴ The official group classification under apartheid.

⁵ The official group classification under apartheid.

known as Model C schools. To this day former Model C schools still typically have the best facilities, best teachers and best educational opportunities for children.

With respect to the mathematics taught, the content and skills that were being taught as part of the non-White curriculum had very little relevance to one's economic, social and political activities (Bopape, 1998). Learners were taught very basic concepts and ideas preparing them to be unskilled or semi-skilled employees. Learners were conditioned to believe that they were not worthy of excelling or that they were not worthy of equal education. Moreover since the classroom is a complex social environment (Watkins, 2005), educational exclusion also meant social exclusion. Thus being excluded from mathematics also meant being excluded from the possibility of advancement in society. The effects of the divided and disparate education policies of the apartheid era were and still are appalling (Kahn, 2004).

During the apartheid era, children who came from the same or similar households, race groups and communities went to the similar schools. All the learners within these similar communities were guaranteed similar job opportunities. It is in this manner that mathematics has contributed largely to inequalities of access and income (Reddy, 2005). There often was no hope of progression and little hope of obtaining higher paid jobs because the access to mathematics was seen as a badge of eligibility for the privileges of society (Fennema & Leder, 1990). Thus, mathematical learning was viewed primarily as a process of acculturation (Barnes, 2005). Mathematics was embedded in a broader "culture of mathematics" that privileged and privileges a distinct few, according to the DMECLT ⁶ (2007, p. 415) .

Along similar lines to the DMECLT's view, Saul (2001) claimed that in South Africa during the apartheid era, 90% of the education budget was allocated to White children; the other 10% was to be shared amongst the children of the other race groups (Coloured,

⁶ Diversity for Mathematics Education Center for Learning and Teaching.

Indian and Black). Fortunately, with the advent of democracy this disparity is in the process of being reassessed and rectified. Whilst there still exist some prejudice due to decades of discrimination, we are on the right path albeit a slow track to a non-discriminatory education system. Education reformers in South Africa are now concerned about the comparisons of South African learners with those of other nations, with respect to the apparent inability of South African youth to successfully participate in the mathematical global market place (Howie, 2003; Reddy, 2006). This concern emanates from the reality that society requires this mathematical knowledge in order to survive and South African society is far behind in attaining this knowledge.

The TIMSS⁷ study in 2003 ranked South Africa as number 50 out of 50 countries (Reddy, 2005) with respect to Grade 8 learners proficiency in mathematics and science. More recently South Africa has been ranked as number 120 in an international study on proficiency in mathematics and science (Mail and Guardian, 2008). South Africa was also ranked last out of 133 countries when an international study was conducted on the quality of mathematics and science education (Dutta & Mia, 2010). Recognising that education can contribute to equality of students (Freire, 1985), learners in South Africa are urged by the Department of Education (DoE) in South Africa to become “critical citizens” in a mathematically democratic society (Department of Education, 2003a, pp. 1 - 7). To this end, schools play a crucial role in preparing learners from different social backgrounds to meet the needs of an unequal society (Atweh, et al., 1995).

Today the majority of schools cater for multicultural and multiracial learners whereby learners are encouraged to employ mathematics to analyse their world critically. The aim of this effort is to advance a democratic society in which all have an opportunity to participate completely (Diversity for Mathematics Education Center for Learning and Teaching, 2007). Knowledge of mathematics is seen as an important asset for the progression of

⁷ Trends in International Mathematics and Science Study (TIMSS). The TIMSS study tests the mathematics and science proficiency of learners at Grade 8 level.

South African society. Similarly, Reddy (2005, p. 125) maintained that “mathematics and science are key areas of knowledge and competence for the development of an individual and the social and economic development of South Africa in a globalising world.”

To assist in acquiring these aims, all South African learners are required to select Mathematics or Mathematics Literacy in grade 10 and continue until the end of their schooling life. This change in curriculum was implemented in January 2006 in order for all learners to have access to some form of mathematics. Previously if learners found pure mathematics (Higher Grade or Standard Grade) difficult, they did not choose a mathematics course. These learners were not afforded the prospect to take a mathematics course, neither were they provided with the necessary access to learn mathematics that would prepare them to succeed in our increasingly mathematical world. They were sent out into the world after completing their schooling careers with a limited and limiting education.

Now however, learners are provided with the potential to excel and be the best that they could be. It is the intention of both the Mathematics and Mathematics Literacy curriculum to permit each learner to work with mathematics in real world contexts. To be mathematically literate suggests that learners are able to identify, understand and engage in mathematics as well as to make sound judgements about the role that mathematics plays (Kotze & Strauss, 2006). In my view, the drawback of this constructive intention is that the present Mathematics Literacy curriculum is based predominantly on contextual knowledge with very little emphasis on real mathematics. By real mathematics I mean mathematics that all learners regardless of their social context would be exposed to.

Nevertheless, with the introduction of this new curriculum, it is the hope of the DoE that this curriculum will foster a critical outlook to enable learners to engage with issues that concern their lives individually, in their communities and beyond (Department of Education, 2003a). Furthermore, it is desired that this curriculum be instrumental in

developing critical thinking, including how social inequalities concerning race, gender and class are created and perpetuated (Department of Education, 2003b) in society. Moreover, it is a premise of this curriculum that it would prepare learners with the necessary mathematics skills required by individuals to function as critical, democratic citizens in modern society.

2.2.2. Contextualised mathematics

The consideration of the classroom with its diverse players, agendas, expectations, and rituals is imperative to the construction of school mathematical knowledge. The classroom context is intrinsically related to the general socio-cultural context that gives rise to the culturally constructed and esteemed knowledge called mathematics.

“What has this to do with mathematics learning? The learning of mathematics cannot be divorced from the social context in which that learning takes place...” (Isaacson, 1992, p. 160)

Since classrooms and teaching are multidimensional (Watkins, 2005) and there are interrelationships between the classroom and social contexts (Bishop, 1988), we need to give attention to the context within which learning takes place. These contexts may include the classroom, community centres and the home. This implies that mathematics ought to be presented and taught within a social context (Watson, 1994) in order to illustrate relevance and accessibility. A social context is perceived as a complex and dynamic setting (Atweh, Bleicher, & Cooper, 1998) that impinges on performance. Performance includes the attitudes and participation of learners.

To improve performance within a social context, the teaching of mathematics ought to relate to a learner’s immediate experience of their social and physical environment in addition to the wider society of which he or she is a part of (Joseph, 1993). Likewise, Mudaly (2010b) proclaimed that learners learn best from their own and not others’ experiences. This implies that a teacher ought to take into account the context within which

the school is located to try to determine how this would influence the quality of teaching and learning within the classroom. Context for the use of mathematics includes both the internal (an individual's life and everyday activity) and external (situations and the world) contexts (Kotze & Strauss, 2006). However, due to varying class sizes and dissimilar resource bases in different classrooms, it is not transparent what it might mean to enable quality teaching in different contexts. In light of this, the teacher ought to be cognisant of the fact that context does matter.

Le Roux (2001) proposed that educational success at school is affected by factors such as race, culture, gender, environmental influence, socio-economic class and genetic ability. One's socio-economic class is dependent on factors such as parent educational level, material possessions, income, and the quality of the home environment (Kotze & Strauss, 2006). Thus, to assist in achieving a high success rate in classrooms, teachers ought to be conscious of the differences within the classroom. Teachers ought to know how these differences affect learning, behaviour (Tileston, 2005) and teaching. In a country like South Africa still scarred by apartheid, when addressing issues pertaining to racial segregation and socio-economic stratification, schools and the districts in which they are located ought to build on theories about political power related to race and class (Stambach & Becker, 2006).

Watkins (2005) also claimed that classrooms do not function as separate entities; one of the crucial influences on them is the culture of the school. The community in which the school is located influences the culture of a school. I agree with Watkins, in that based on my experience in the education sector, I have observed that South African society consists of people of different cultures, and mathematics classrooms are not excluded. Therefore learners' ideas of their world are influenced by who they are, what they believe, what they value, how they see themselves in the world, their cultural background, and what they are exposed to and their interests (Perry, 2006). Fundamentally, much of what an individual

learns is gathered through interactions within various communities (Cooney, et al., 1998) within a social context.

It is for this reason that the classroom can be viewed as a social context in which mathematical knowledge is negotiated and constructed. The mathematics classroom is informed by the construction of mathematics in a wider socio-cultural context (Atweh, et al., 1998). In many locations and social milieus, mathematics classrooms embrace a range of learners who bring with them diverse practices, languages and mathematical competences. This diversity adds to the challenge of providing quality teaching, since in South Africa, the majority of learners have to learn mathematics in English, a language that is not their first language or mother tongue. Thus, the ability to embrace difference and to view diversity positively is a crucial ingredient (Watkins, 2005) in the teaching and learning of mathematics. To allay teachers' uneasiness we need to prepare teachers to engage and mediate the increasing diversity of their learners in addition to making the best possible use of available resources.

Diversity offers learners multiple ideas, perspectives and solutions to problems. Respecting and accepting diversity in the mathematics classroom would be beneficial to both the teacher and learners. Additionally, knowledge would be more valuable if built on the experiences of learners, since more formal ways of knowing may fail if not related to experience (Tanner & Jones, 2000). I agree with Tanner and Jones (2000) in that I believe that teachers ought to elicit from learners examples and real world situations. From my experience, using a learner's social context to unearth ideas for problem solving in mathematics assists in the comprehension of mathematical concepts. Teachers need to know and appreciate that learners enter classrooms with unique sets of cultural influences, life experiences, prior learning, attitudes and personalities (Brownlee Griffith, 2004).

School mathematics ought to offer opportunities for learners to appreciate its cultural links and reflect on its significance in real life situations (Chronaki, 2000). Learners come into

classrooms with a wealth of knowledge which ought to be utilised and linked with new knowledge by the teacher. Deep learning depends on cognitive activity such as selecting relevant information from a lesson, mentally organising it into a coherent structure and integrating the new knowledge with existing knowledge (Moreno & Mayer, 2007). It is in this way that designing multiple paths to a learning goal is important for both learners (Carolan & Guinn, 2007) and teachers.

No matter what strategy is used by the teacher in his/her classroom, it ought to be conceded that content takes precedence. In addition, content delivery must be effective or else the information and its quality cannot be absorbed and remembered. The teacher ought to use strategies that would enable learners to link new information with old, and the learner must be able to recall this information when necessary. Based on anecdotal experience, my belief is that the most common strategy used in the South African classroom is the ‘chalk and talk’ method and the use of textbooks to teach important mathematics concepts and ideas.

2.3. A broad view of teachers in South Africa

As discussed earlier in Chapter 2.2, apartheid education provided learners who were not White with a substandard education. Besides attending schools that were under resourced and overcrowded, the content of the curriculum was designed to equip low-income workers. This education system was instituted in order to keep people who were not White within their perceived social bracket as defined by the apartheid government. This second-rate education attempted to ensure that learners who were not White did not succeed in reaching the professional class reserved for the more privileged race group. Learners in South African schools came from diverse social, economic and political backgrounds (Kotze & Strauss, 2006). The way in which diversity was handled during the apartheid era in South Africa communicated that only a particular group of people could enter prestigious vocations. Thus, the formal job reservation of the apartheid era was foreshadowed by the expectations that carried weight at school level.

Moreover, teachers in South Africa were victims of this apartheid education as well. Brodie (2004) argued that teachers' personal mathematical practices shape their individual teaching practice. Venkat, Adler, Rollnick, Setati and Vhurumuku (2009) concur with this notion by maintaining that the damages of apartheid teacher education in mathematics education are evident when one considers the fact that teachers taught in a manner in which they themselves were instructed. This implied that the prejudices that teachers were exposed to within society were reproduced in their own classrooms intuitively. Similarly, Breen (1994) argued that teachers teaching learners who were not White studied at disadvantaged colleges of education whereby the framework used to train these teachers called for complete and trusting reverence. These teachers were trained not to retaliate against or question their substandard education. These sentiments are epitomised by the following excerpt taken from a political debate during the apartheid era

“People who believe in equality are not desirable teachers for natives⁸...What is the use of teaching the Bantu⁹ mathematics when he cannot use it in practice? The idea is quite absurd.” (Verwoerd, 1953, p. 3585)

This illustrates just one of the many reasons why more than half of the mathematics and science teachers in South Africa are unqualified (Mail and Guardian, 2008). With the crumbling of the apartheid system in 1994, learners of colour sought access into other well-resourced schools, both in terms of human as well as material resources. This trend was about parents wanting the best education for their children. Parents were of the opinion that the much yearned for education could be obtained at schools demarcated for the once privileged race group/s¹⁰. Additionally, these schools were well resourced and encouraged a high level of work ethic. The result of South Africa's racial politics and poverty is that the majority of the Blacks are the largest and poorest faction of South African society. Black schools have extreme backlogs with regards to the provision of infrastructure, learning materials and qualified teachers (Reddy, 2005).

⁸ Term used to refer to the Black people of South Africa during the apartheid era.

⁹ Term used to refer to the Black people of South Africa during the apartheid era.

¹⁰ Following the social hierarchy, Whites were more privileged than Coloureds who were more privileged than Indians who were more privileged than Blacks.

With the advent of democracy, many Black learners sought access into White, Coloured or Indian schools. This trend necessitated a change in schools with respect to policy and practice. Schools and classrooms were moving towards inclusivity and diversity as claimed by Jacobs (2001). Learner diversity may incorporate race, religion, ethnicity, disabilities and exceptional abilities (Kotze & Strauss, 2006). To complement these changes, teachers ought to have the pedagogic knowledge with respect to adapting activities, materials and resources so that each learner can participate unconditionally in a classroom. The change in schools also resulted in changes in the curriculum. Changes in the mathematics curriculum had a decisive impact on teachers in South Africa. The new curriculum required teachers to teach in a way that would heal the divisions of apartheid.

Teachers were now obligated to produce mathematically literate learners regardless of the conditions or contexts in which they were located. Compounding these issues further, was that teachers were given a limited amount of time to deliver the most crucial elements of the curriculum (Rose, 2004). Learners were also compelled to write the same examinations irrespective of the conditions or contexts in which they were situated. Thus, the changing role of the teacher has led to uneasiness for some (Harden & Crosby, 2000); this discomfort was also partly due to the fact that teachers were not equipped to meet these demands.

Similarly, Parker (2003) argued that due to the previous inferior education many practising educators lacked the mandatory knowledge required to teach. Good teachers are those that display strong knowledge of their content area and this enhances their teaching as purported by Fraser and Tobin (1989). When talking about the knowledge teachers ought to have, I am referring to procedural knowledge, propositional knowledge; practical knowledge; tacit knowledge and skills. These forms of knowledge are related to contexts of learning, which influences the teachers' interpretation, personalisation and incorporation into conceptual frameworks (Jaworski, 2003). There is considerable research into teachers' mathematical knowledge (content knowledge) and its relationships (or lack of

them) with effective teaching (e.g. Grossman, Wilson and Shulman 1989; Askew, Brown, Rhodes, Johnson and William, 1997; Rowland, Martyn, Barber and Heal, 2000) as cited in (Jaworski, 2003, p. 254). Studies indicate that beyond possessing content knowledge, good teachers understand how learners come to know their subject, where learners may stumble, what preconceptions learners may have, and how to match content with instruction methods (Carolan & Guinn, 2007). Teachers ought to be prepared to encourage, support and stimulate their learners.

More importantly though, many teachers were challenged in terms of their pedagogic content knowledge (Polk, 2006). Pedagogic content knowledge involves teachers' reasoning and thinking as well as their thinking about teaching methodologies regarding content instruction. This is crucial because it is through teaching that knowledge is distributed and reconceptualised for learners. Mathematics pedagogic content knowledge is the blending of mathematics content knowledge and pedagogic knowledge (Kazima, et al., 2008). It is the specialised knowledge that teachers possess that connects content knowledge with the practice of teaching (Ball Loewenberg, et al., 2008). It is through this specialised knowledge that teachers acquire the skills to be effective in the practice of teaching. It is in this way that learners are taught logical reasoning and critical thinking skills. If mathematics teaching does not provide learners with these skills, then an important part of their preparation for life is missing.

To clarify, some of the mathematics skills that a learner is required to develop are those of computation, problem solving, the ability to think rationally, reasoning skills, critical thinking skills and the ability to justify and prove. Due to learners not succeeding in developing these imperative skills, there is a need for an improvement in proficiency in mathematics. Teachers should begin to focus on conceptual issues in the classroom (Mudaly, 2010b). Similarly, a recent report (Mail and Guardian, 2008) indicated that teachers were not adequately equipped to prepare their learners for the examinations because they could not teach the new sections in the curriculum themselves. This indicated

that teachers do not have the adequate subject matter knowledge for teaching (Ball Loewenberg, et al., 2008). This further exemplified the need for proficiency in mathematics.

As can be conceived, educational change has consequences for social justice, equity and democracy (Ball & Cohen, 1999). The most recent changes to the mathematics curriculum (January 2006) in South Africa ushered in a new era for South African teachers. The recent changes expected teachers to play a more active role in creating learner centred instruction. Teachers were seen as the central players in bringing about this change in practice, which is similar to what Feldman and Denti (2004) proposed. Adler et al. (2005) similarly emphasised that more teachers and better mathematics teaching are needed if mathematical proficiency is to improve. The teacher ought to possess mathematics knowledge that is both “useful and usable for teaching” (Kazima, et al., 2008, p. 283). For this to be possible teachers require more professional support to reach this goal, they ought to be kept abreast of latest developments with respect to both content and pedagogical issues (Mudaly, 2010b). On the contrary, whilst the job of the teacher has changed considerably, the assistance teachers receive does not meet these demands. According to Polk (2006), professional communities, associations and membership in organisations attempt to provide an avenue for this much needed support and teacher development. In South Africa, we have two such organisations that provide such support for mathematics teachers AMESA¹¹ and SAARMSTE¹².

The purpose of mathematics teacher education is to prepare and support teachers to teach mathematics effectively and efficiently (Kazima, et al., 2008). In a similar vein, Vithal (2008) emphasised that valuing teachers’ vested interests and reasons for learning may be relevant in education systems characterised by diversity and inequalities. This thus makes teacher education relevant.

¹¹ Association for Mathematics Education of South Africa

¹² Southern African Association for Research in Mathematics, Science and Technology Education

Crouch (2003) also argued that the teacher workforce in South Africa has undergone turbulent changes in the last few years due to various factors. There seems to be much substance to his argument because, since the crumbling of the apartheid era, learners have been moving into schools that were once demarcated for the more privileged groups. This led to a diverse group of learners within each school. By diversity, I include diversity in ability level, culture, ethnicity and race. According to Sujee (2004), there has been a 25% migration of Black learners from Black schools to other department and independent schools for the period 1996 to 2002. This influx of learners into more resourced schools has led to an overpopulation of classes. Apart from the strain of teaching to larger groups of learners, many teachers also need to cater for learners who do not speak English as their first language. In a similar vein Adler et al. (2005) maintained that many mathematics teachers are teaching in large (over 40 learners) classrooms often lacking essential resources. Amongst all these exacerbating conditions, it remains the responsibility of the teacher to engage, encourage, and motivate learners to learn mathematics during these lessons.

Motivation refers to the complex forces and needs, which energise and sustain human behaviour in carrying out particular action (Schulze & Steyn, 2003). From my experience, motivating learners to learn concepts that many learners cannot see the relevance of is a mammoth and difficult task. Similarly, Feldman and Denti (2004) asserted that finding ways to enhance teaching and motivate learners to learn presents a challenge for teachers. What could be of assistance is the use of notions of developmental psychology. MacNab (2000) suggested that the use of child-centered developmental psychology by teachers, is a fundamental requirement for the successful teaching and learning of mathematics. Teaching learners reasoning skills and critical thinking skills, in essence the ability to think mathematically is important for learners to be able to process mathematical knowledge. In order to be able to teach learners to think mathematically, teachers themselves are required to be proficient in thinking mathematically.

One way of ensuring that this transpires, is by encouraging teachers to take responsibility for their own personal development in becoming critical practitioners within mathematics education. Teachers are expected to continue to educate and empower themselves (Hindle, 1997; Polk, 2006; Rose, 2004). Teachers are also expected to update their own content knowledge in accordance with curriculum changes and additions. New content knowledge may be linked to alternative approaches and strategies to teaching these new concepts and ideas. Brodie (2004) proposed that the importance of content knowledge has been complemented by the fact that pedagogic knowledge is also important. Teachers ought to be proficient in their content knowledge as well as know which methods and strategies are more effective in delivering the content. It is necessary for teachers to be effective in the classroom with the aim of maintaining interest and improving the understanding of mathematics. Teachers may use their tacit and explicit knowledge to assist in this regard.

Schön (1983, 1987) stated that tacit knowledge may be made explicit through reflecting on practice. Likewise, Frade (2005) asserted that all knowledge is tacit or constructed from tacit knowledge. Tacit or implicit knowledge refers to the knowledge one already has, it is unspoken and does not require thinking about, and tacit knowledge is inherent. The definition of tacit knowledge was instigated by the work of Michael Polanyi. He described how one could identify a familiar face in a crowd (Polanyi, 1967); since we cannot verbalise how the identification is done, our knowledge is implicit or tacit. Explicit knowledge refers to knowledge that one codifies precisely and expresses formally. It is possible to store and share explicit knowledge by using books, journals and data bases (Connell, Klein, & Powell, 2003). Regardless of whether teachers tap into their explicit or implicit knowledge bases, the focus is to sustain the effectiveness of the teacher in the mathematics classroom.

Along similar lines, Nye, Konstantopoulos and Hedges (2004) stressed that methods used to improve the effectiveness of teachers, enhance the level of learner achievement. One avenue of exploration relates to the fact that teacher content knowledge is developed when teachers' solve non-routine problems and reflect on how they may be used in classrooms

(Olson & Kirtley, 2005). Reflection includes reflecting-in and reflecting-on action. This exhibits a directly proportional relationship between content knowledge and pedagogic knowledge. Thus, reiterating what was discussed earlier in that content knowledge is crucial to the teaching and learning of mathematics (Venkat, et al., 2009). In the same vein, Tanner and Jones (2000) purported that teaching approaches must include two elements: relational understanding and practice. Teachers ought to understand how concepts and ideas relate to each other in order for them to teach these concepts and ideas effectively.

Similarly the manner in which teachers develop and maintain active engagement in their pupils is a measure of their professionalism as emphasised by MacNab (2000). In order to be able to reach and meet the demands of multi-literate children, teachers have to understand the possibilities and limitations of this challenging target group. To assist in improving content knowledge of learners, it is important for teachers to model the use of strategies that elicit talk about thinking. Learners ought to be encouraged to discuss their thoughts and ideas. Teaching mathematics ought to be aimed at easing the transition from exploratory talk to discourse specific talk (Gorgorio' & Planas, 2001). Learners need to verbalise their thinking process so that gaps or inaccuracies in their thinking processes can be rectified. Strategies such as asking questions about the content, summarising essential information, clarification, and hypothesising may be modified for use in mathematics classroom discussions (McNair, 2000).

Along similar lines, Carolan and Guinn (2007) proposed that diversity may be nurtured by maximising the potential of each learner. This was evident in this study where the Master teachers first identified the strengths and weakness of their learners and used pairing or group work to maximise the potential of learners within the classroom. Based on this, I found it appropriate to use activity theory as my theoretical framework. Activity theory emphasises that learning and activity are related and interdependent entities.

2.4. Teaching mathematics in schools

Jacobs (2005) stated that learning is experiential and this in fact sits snugly within the ambits of social constructivism and contextualised teaching and learning. Learners understand the contexts within which they exist and any relation to their context may make more sense than high levels of abstract mathematics. However, using contextual factors to teach is often difficult for teachers to adapt to because of their having had no or very little training. Similarly, Brownlee Griffith (2004) asserted that since knowledge is socially determined, it is a teacher's willingness to provide an atmosphere that encourages interaction in the classroom that would be of benefit to the teaching and learning process. This interaction ought to involve interaction with content and resources. This would encourage learners to formulate connections and identify links with their own cultural or gender needs. It is in this way that the teacher and learners socially construct an interactive environment with the primary goal of promoting the learning (Atweh, et al., 1998) and teaching of mathematics.

The practice of teaching mathematics requires teachers to be both knowledgeable in mathematics as well as to be proficient in problem solving in mathematics (Kazima, et al., 2008). However, since mathematics teaching is mainly procedural in nature (Brodie, 2004), teachers generally teach the different types of rules, axioms and processes learners ought to employ to arrive at solutions to problems. This type of teaching could lend itself to creating a negative attitude towards the learning of mathematics. These attitudes become more predominant in the classroom if learners do not succeed in grasping the rule or procedure that ought to be applied. Mathematicians thrive when different methods and modes are used to unpack and dissect a mathematics concept or idea. Teachers unpack or decompress mathematical ideas and concepts in order to provide access to the mathematical content for their learners (Kazima, et al., 2008). Likewise, different teaching styles and activities suit different learners' preferred cognitive styles. Good teachers recognise the different learning styles of their learners and develop instructional approaches that will accommodate these

styles (Montgomery, 2001). In addition, Griffin (2009) suggested that teachers can be trained to use certain styles of activities and to teach in a specific manner.

This training would be beneficial since most classrooms are diverse with respect to ability levels and learning styles. Teachers are required to design instruction that provides just enough scaffolding for learners to be able to participate in tasks that are beyond their reach (Montgomery, 2001). Scaffolding is thus referred to as “supported practice” involving the linking of new information to a learner’s prior knowledge (Mudaly, 2010b, p. 2). In a similar manner, books with pictures offers many opportunities for adding further dimensions to teaching and learning (Van Renen, 2008). This could serve as a scaffolding technique to use in the classroom to support the teaching and learning of mathematics.

Additionally, teachers need to model, coach and then fade to promote learner’s development of an area of interest (Wepner & Tao, 2002). This is how the teacher scaffolds learning in mathematics. Along similar lines, Jacobs (2001) maintained that when using scaffolding techniques in the classroom, the teacher ought to supply learners with tools that they may require to learn. This support is then slowly removed, as learners are able to do more on their own. The challenge for the teacher is to identify the problem solving tools that individual learners would require in the mathematics class (Niess, 2005). In essence teachers actively shape the classroom environment using tools that are available in order to attain their teaching goals (Remillard, 2005). In addition to the tools that teachers use, we must be aware that knowledge, skills and values are inspired by the interactions that take place between the learners’ personal characteristics and their social environment (Kotze & Strauss, 2006). To meet this challenge we require proficient, motivated and determined teachers who are driven to making a positive difference in the mathematics classrooms.

Motivated teachers demonstrate a constructive attitude towards the teaching and learning of mathematics and exhibit characteristics of good role models. The motivation of teachers has a significant impact on learning since teachers determine the learning experiences that

occur in schools (Schulze & Steyn, 2003). Stipek (2004) supported this view, and suggests that having well motivated teachers could remove the average achievement gap between learners from a lower social class and their peers from a higher social class. Removing this achievement gap could be instrumental in the crumbling of the social class structure entirely.

Apart from having motivated teachers, teachers also need to be aware of the needs of their learners. A guide to meeting the needs of a learner was initially documented in 1970 by A. H. Maslow.

Maslow's hierarchy of needs provides teachers with an illustrated version of basic needs that ought to be met for successful teaching and learning to occur. The diagram that follows depicts Maslow's hierarchy of basic needs.

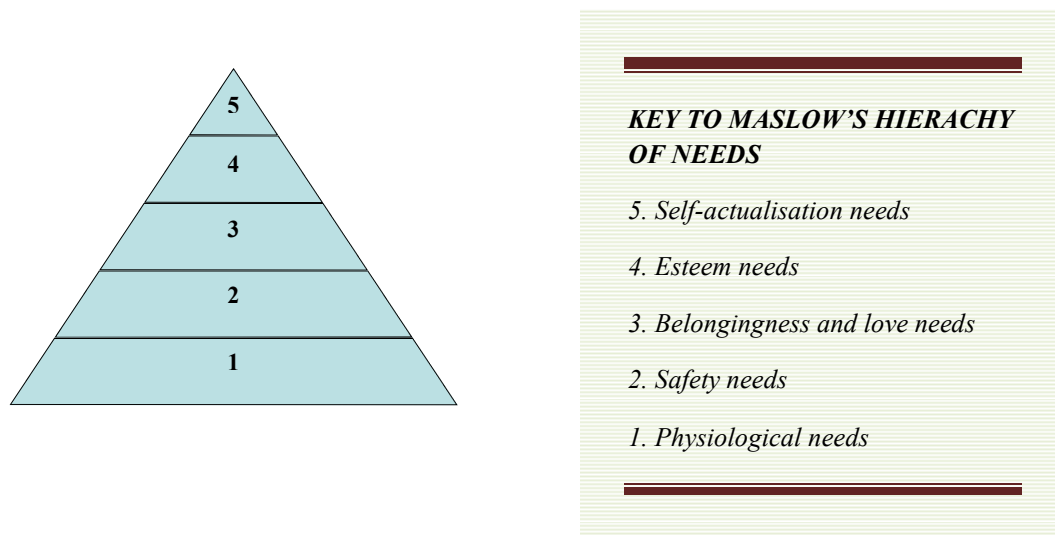


Figure 1: Maslow's hierarchy of needs
Adapted from Pollard and Triggs (1997, p. 201)

The lowest level, Level 1 includes the need for food, shelter, oxygen and water. Level 2 embraces the notions of safety and freedom. Level 3 encompasses feelings of love and

belonging; learners should feel love and respected in order to reach their potential. Level 4 refers to one's esteem needs, where the focus is on the ability to be successful and to feel good about one. Lastly, Level 5 is seen as the final aim of education. It is necessary for learners to realise and accomplish their full potential.

Additionally hand in hand with the needs of the learner; teachers should take cognisance of the different levels of thinking. To assist in this respect teachers may peruse the van Hiele levels of geometric thinking. Two Dutch educators Pierre van Hiele and Dina van Hiele-Geldof encouraged the classification of the different levels of geometric understanding. They identified the sequential levels as visualisation, analysis, informal deduction, deduction and rigor. The van Hiele levels are discussed in detail later on in Chapter 7.2.

2.5. Master teachers internationally

Internationally the need for Master teachers has been recognised. This recognition has led to a widespread certification of Master teachers (Han, 2009). Master teachers are professionals who exhibit strong content knowledge, concrete pedagogic skills and are highly motivated. Motivated teachers improve the quality of teaching and encourage effective learning (Schulze & Steyn, 2003). Master teachers are capable of providing high quality training and support to other teachers (Physics Teacher Education Coalition, 2009; State Board for Educator Certification, 2009). For this task Master teachers receive a state salary every month (Texas Education Agency, 2009). To qualify as a Master teacher in the United States of America (USA) for example, teachers are subjected to a Master teacher preparation programme. Once teachers qualify on this programme, they then need to pass a Master teacher exam. This exam incorporates content material that is subject specific (State Board for Educator Certification, 2009).

Once successful, a Master mathematics teacher is then placed in a school that is designated by the state as a high need school (Texas Education Agency, 2009). These high need schools are equivalent to our underperforming, under resourced schools in typically

disadvantaged parts of South Africa. Master mathematics teachers in the USA observe other teachers in the classroom and provide support to colleagues. The support includes assisting with content and methodologies, assisting with the use of technology in the classroom and providing motivation for colleagues to move from the traditional approach of teaching mathematics to a more inquiry oriented method (Ramses et al., 1999). Master mathematics teachers also research causes of learner failure to prepare themselves and others with ways and means of improving learner performance and pass rates. Master teachers are also found in many other countries like China, Australia and England. They have similar job descriptions as the Master teachers in the USA. Essentially a Master teacher is a highly motivated, determined individual with one pointed goal of making a positive difference within the education sector.

2.6. Master teachers in South Africa

In 2006 the Department of Education (KZN) announced that in terms of teacher development, 120 Master teachers would be appointed and an additional 2400 Master teachers would be trained (Makapela, 2007). This announcement was made due to the realisation that many schools in South Africa lacked qualified mathematics and science teachers. An urgent intervention needed to be put in place in order to rectify this crippling problem. To date only a fraction of this number have been trained and appointed. Possible reasons for this are that there were various criteria used to identify these Master teachers, and many different titles for the same job description existed. These criteria are discussed in Chapter four of this study.

In South Africa, Master teachers serve the same purpose as a mentor or expert teacher. They are senior teachers with the potential to mentor new teachers. Expert teachers provide an invaluable resource (Carolan & Guinn, 2007), since they assist with providing support to novice teachers and can provide strategies and methodologies that may improve or allow for the effective teaching and learning of mathematics. Effective teaching means that the teacher has to manage the social images and stereotyping that are prevalent in the

multicultural class with great care and empathy (Le Roux, 2001). Rather than seeing these differences as hurdles, Master teachers turn these differences into assets (Carolan & Guinn, 2007).

Ten basic characteristics of good teachers were identified by Polk (2006, p. 23) “good personal academic performance, effective communication skills, professionalism, creativity, pedagogic knowledge, appropriate learner evaluation and assessment, lifelong learning, personality, content area knowledge, and the ability to model concepts in this content”. Whilst some of these criteria were used by the DoE to identify these Master/Mentor/Expert teachers, the Master teachers in this study also exhibited determination and a commitment to improving the teaching and learning of mathematics within their classroom. The Master teachers in this study all taught at Dinaledi schools.

2.6.1. Dinaledi Master teachers

Due to the global demand for an improvement in mathematics and science, the DoE in South Africa developed a national strategy to improve participation and performance in mathematics and science education (Reddy, 2005). Following the recommendations of the national strategy discussed in 2000, the flagship programme in the strategy was the identification of 102 schools in the Dinaledi project (Kahn, 2004). As discussed earlier on in Chapter one, Dinaledi schools were intended to increase the participation and performance of Black learners and female learners in mathematics and science.

The table that follows exhibits the Dinaledi schools across the nine provinces from the time of inception of the Dinaledi project up until 2010. There were four phases during this period.

Province	2001 - 2004	2005 - 2006	2007	2008-2010
Eastern Cape	15	60	60	60
Free State	06	30	35	36
Gauteng	11	70	101	101
KwaZulu- Natal¹³	23	50	84	88
Limpopo	23	50	51	51
Mpumalanga	07	30	44	46
Northern Cape	04	10	17	17
North West	07	40	51	53
Western Cape	06	40	45	48
Total	102	400	488	500

Table 2: Number of Dinaledi Schools in each province per phase
Adapted from Department of Education (2008, p. 19)

The DoE selected these ‘Star’ schools based on criteria that identified schools that were most likely to benefit from a range of educational interventions. These interventions included resources and workshops for learners and teachers in mathematics, science and technology. The schools were provided with basic resources required for effective teaching and learning to take place, these resources include scientific calculators, textbooks, study material and computer resources (Department of Education, 2008). The teachers at these schools were also provided with professional development workshops in Mathematics, Physical Science and Life Science. Subject specialists employed by the DoE trained Master trainers in mathematics and science in order to provide this support to teachers teaching at Dinaledi schools (ibid, p. 23). These trainers included subject advisors and more experienced teachers from Dinaledi schools. The teachers in this study belong to this cohort of Master trainers.

The role of these Master trainers was to provide mentoring support to teachers at selected Dinaledi schools. Teacher training programmes (totalling 100 hours) focussed on improving the teacher’s content knowledge, teacher’s pedagogic knowledge and enhancing learner performance in mathematics and science (ibid, p. 23). Dinaledi teachers were also

¹³ The highlighted row indicates the province in which this study was conducted, i.e. KwaZulu-Natal.

provided with computer based assistance at least twice every year. Incentives and rewards were provided to individual schools based on successes and pass rates.

One could argue that one of the shortcomings of this project was that some of the schools (20%) had an unfair advantage in reaping the benefits of rewards and incentives. Selected schools already had an uneven distribution of resources. When resources were provided to the KZN Dinaledi schools, some of the schools (20%) were better resourced in terms of material and human resources. These schools would have provided good results in both mathematics and science even without the additional assistance from the DoE. This could have tainted the Grade 12 pass rates in mathematics and science. One would not be able to state undoubtedly that it was through the DoE interventions that these schools performed successfully.

The table that follows represents the average pass rates for mathematics in the Senior Certificate Examination¹⁴ for KZN Dinaledi schools during the last phase (2008 – 2009) of the Dinaledi project. The table displays a comparison of statistics for the under resourced and resourced Dinaledi schools.

Year	Under resourced schools pass rates	Resourced schools pass rates
2008	71 %	89 %
2009	69 %	86 %

Table 3: KZN Dinaledi under resourced and resourced schools average pass rates in mathematics

Based on the results it is evident that schools that were better resourced initially, performed better in mathematics. However, one can only speculate about the factors that led to this

¹⁴ The Senior Certificate Examination is a common national examination that is written by all matric (grade 12) learners in South Africa. To progress to a Higher Education Institution, learners are expected to pass the Senior Certificate Examination.

performance. The table below represents the average pass rates in mathematics for all Dinaledi schools in KZN for the 2008 and 2009 Senior Certificate Examination.

Year	Average pass rate in mathematics (N = 88)
2008	80 %
2009	78 %

Table 4: KZN Dinaledi schools average pass rate for mathematics

Whilst all the Master teachers that I have worked with in my study were based at Dinaledi schools, to avoid any lurking variables in my data collection, I selected the Master teachers from different types of schools with respect to resources (originally under resourced and resourced), location and class sizes. I discuss this selection in detail in Chapter four of this study.

2.7. Visualisation

Chiappini and Bottino (1999) declared that visualisation refers to the intricate phenomena of visual imagery that play a fundamental role in all meaning, understanding and reasoning. Visualisation essentially means the ability to form and negotiate a mental image necessary for problem solving in mathematics. Aristotle, as cited in Zazkis et al.(1996, p. 437), believed that one could not think without having an image in one's mind. Similarly, Brown, Hewitt and Mason (1994) stated that seeing comes before words, e.g. the child looks and recognises before speaking. This implies that when one is thinking, a mental image is induced intimating that we see with our minds. In a mathematics classroom, by allowing learners the opportunity to voice what they see in their minds promotes active discussion and interpretation. This enables others within the community of practice, to see what the learner is seeing, thus allowing for active communication. Learners learn a great deal through communication with their peers and teachers (Steer, de Vila, & Eaton, 2009). This assists in making the mathematics being taught in the classroom more accessible. In a

similar vein, researchers (Mason, 1992; Tall, 1991; Wheatley & Brown, 1994) have stressed that mental images are important in the construction of mathematical ideas and concepts.

Johnston-Wilder and Mason (2005) claimed that working on mental images is important in developing a learner's ability to imagine. The ability to imagine improves a learner's reasoning and thinking skills. Similarly, McLeay (2006) purported that imagery is useful for problem solving. Thus it is important for learners to organise representations into a logical mental framework and integrate this information within and in addition to their prior knowledge. There needs to be a link between what the learner is learning now with what the learner has been exposed to previously. This is so because visualisation encourages the use of the concrete to conceptualise abstract concepts and ideas (McLoughlin, 1997; Presmeg, 1997; Solano & Presmeg, 1995). Whilst mathematics encompasses many abstract notions, with visualisation, these abstract notions are made more accessible to the learner.

Since visualisation encompasses the concretisation of abstract concepts and relations (Guzman, 1996); learning mathematics involves the learners' awareness of pictures, diagrams and graphs (Orton, 1992). This makes it possible for learners to think and engage with ideas that might otherwise be labelled "new" as discussed by Davis and Maher (1997, p. 114). It is through the use of visualisation, that the learner discovers the ability to mentally manipulate objects, which in turn develops into good techniques for learning (Cotter, 2000) mathematics.

In the 1990's Davis and Maher (1997) and Presmeg (1993, 1995, 1997) have conducted much research on why learners use visualisation in mathematics. More recently researchers (Arcavi, 2003; Diezmann & English, 2001; McLeay, 2006; Verstraelen, 2005) articulated many different reasons and benefits for how and why learners use visualisation in mathematics. Among these reasons is the perception that visualisation is an act in which an

individual establishes a strong connection between an internal construct and something external. This could involve any mental construction which an individual associates with objects or events perceived as external. It is partly due to these beliefs that Elliot, Hudson and O'Reilly (2000) argued that visualisation is increasingly being recognised as a fundamental aspect of mathematical reasoning.

Along similar lines, Nakin (2003) purported that visualisation is the cornerstone in the learning of mathematics in that mathematics depends on what is being visualised and the spatial abilities embedded in such visualisation. However, whilst it is an important skill, this skill receives little attention in the school curriculum. Furthermore, in order for mathematics teachers to promote the development of visualisation skills in their learners, the teachers themselves need to be aware of the development of visualisation. Changes in teacher education in support of these developments need to be addressed. Curriculum planners are not likely to ignore these sentiments, since South Africa is a country whose education system is tainted with injustices and bias, and any attempt at an alleviation of prejudice would be fervently welcomed. Additionally, since the effective use of visualisation requires an intensive discussion on how to interpret diagrams and figures (Zimmermann, 1991), this calls for teacher support and professional development in the area of visual literacy.

Likewise, previous research (Battista & Clements, 1996; Bishop, 1978, 1980, 1989; Elliot, et al., 2000; Presmeg, 1986, 1989, 1993, 1995) have indicated that the possession of strong visualisation skills enhances a learner's learning of certain subjects particularly mathematics. The processes of visualisation entails the processes of forming images with paper and pencil, chalkboard, technology or even mentally, to investigate, discover and understand objects with which an individual relates (McLoughlin, 1997; Zazkis, et al., 1996). It is in this manner that visualisation becomes an essential component in mathematical activity (Hershkowitz, Arcavi, & Bruckheimer, 2001). Moreover, in contrast to other sciences, objects in mathematics do not necessarily have tangible existence and are

not directly accessible to perception (Fagnant, 2005). Thus, visualisation plays an important role for countless people in their construction of the symbolic form of the answer (Hershkowitz, et al., 2001).

I agree with the idea that visualisation enhances a learners' ability to learn mathematics because it becomes a powerful cognitive tool (McLoughlin, 1997), hence the focus in this study on the visual approach to teaching and learning mathematics. The reason for my bias on the use of visual tools in mathematics is what I have observed based on anecdotal experience as a mathematics teacher and a mathematics teacher educator. Learners use visual, kinaesthetic or auditory modes for learning (Slack & Norwich, 2007). Gardner (1993) in proposing his theory of multiple intelligences, argued for the special status of visual intelligence in contrast to verbal intelligences (McLoughlin, 1997). This is so because some learners who are ungifted at symbolical thinking and reasoning need to visualise a given phenomenon. These learners need to see or have a mental picture in order to understand. It is because of this that visual ways of thinking and reasoning are important for such learners (Hershkowitz, Parzysz, & Van Dor-Molen, 1996). Without these visual ways of thinking and reasoning, these learners would not be able to participate at the same level as all the other learners within the classroom. These learners will not have access to the mathematics.

It is also common that some learners prefer an integration of modes whilst others prefer a visual or algebraic approach. I believe that in order to accommodate different types of learning styles, visualisation in mathematics classrooms ought to be encouraged. A visual learner may be defined as someone who views a system visually as a whole rather than examining it in terms of different components (Chmela-Jones, Buys, & Gaede, 2007). This implies that a visual learner comprehends everything visually; the learner does not divide the problem into procedures and structure but rather sees everything structurally. This learner communicates via the visual. Visual learning is an approach to helping learners communicate through imagery (Chmela-Jones, et al., 2007). This implies that the incorporation of visual education in the curriculum may contribute to reducing an equity

problem (Goldenberg, Cuoco, & June, 1998), since, for learners who thrive when a visual approach is used, visualisation allows them access to participate.

2.7.1. Diagram literacy

McLoughlin (1997) claimed that by acquiring visual literacy skills the teaching and learning process is enhanced. Representations that encompass visual literacy skills to support the understanding of mathematics include concept mapping, mind mapping and the use of technology. Concept maps and mind maps are represented by using different and various diagrams in order to convey information (Copperman, Catriel, & Ben-Zvi, 2007) about the concept in question. This signifies that diagrams become tools to turn to as mathematicians articulate, clarify, justify and communicate their reasoning to others. Likewise, Hanna (2000) declared that diagrams and visual tools have been used quite efficiently to facilitate understanding.

A good diagram is one where the viewer needs to be able to reconstruct the situation it depicts by identifying relevant information to build up a full representation. This implies that the viewer needs to be diagram literate. Despite this view, limited research has focused on the drawing of diagrams as instructional tools (Woleck, 2001) or the use of visual tools. It is for this reason that I believe that this study is important. Furthermore, if, for many, drawing a diagram and thinking about the diagram is a natural and intuitive step in the solution process (Iwasaki, Tessler, & Law, 1995; Sloman, 2002), we need to encourage this aspect in our mathematics classrooms. After all, most diagrams are drawings that may be made with a pencil and paper (Lindsay, 1995). These tools are affordable and available to most learners. Drawing diagrams assists the learners in visualising concepts that are linked; this assists in the learners' ability to reason. This type of reasoning is suggested in Fischbein's theory of 'figural concepts'. The main idea behind Fischbein's theory was that geometry involved geometrical figures which encompass concurrent conceptual and figural characters (Fischbein, 1993). To elaborate, the conceptual nature of concepts includes

completeness, abstraction and generalisation, while the figural nature includes characteristics such as colour, size and shape as argued by Dvora and Dreyfus (2004).

Whilst there does exist rich literature about the role of diagrams and reasoning in assisting learning in mathematics, much less is known about how individuals use diagrams (Whiteley, 1999) in mathematics. It is the goal of this study to illuminate possible notions about this. We do know that diagrams are often constructed in the act of solving a problem. They allow individuals to free up more mental space for new imaging and to create new relationships. This implies that diagrams do not necessarily communicate meaning, but rather, they evoke (Wheatley, 1997) meaning. A diagram is therefore capable of showing exactly what we are talking about, which in verbal statements is left implicit (Skemp, 1971).

Barwise and Etchemendy (1991) claimed that mathematicians have long been aware of the value of diagrams and other visual tools both for teaching and mathematical discovery. If we are required to use diagrams and other visual tools for communication and shared problem solving, it becomes imperative to recognise whether or not the individuals concerned in this meaning making process are seeing the same thing. Social involvement in problem solving activities is imperative for individual development (Mercer & Sams, 2006). One way of engaging all participants equally is by incorporating personal experiences and culture in the mathematics classroom. For example, in a complicated drawing from another culture the learner may see patterns and therefore identify geometric shapes and lines of symmetry (Presmeg, 1989). This may provide the foundation for engaging learners in the classroom. Involving or borrowing from another culture whose members are in the classroom affords all learners the opportunity to enhance their understanding of the concepts under discussion. By understanding what the problem entails, the mathematical reasoning process is supported. Learners can then progress in a sequential manner to arrive at the solution.

Similarly, Zimmermann (1991) pointed out that it is important in the development of visual thinking that we develop the learners' ability to perceive mathematical meaning in all types of diagrams. Diagrams have an advantage over language because diagrams convey both structure and function at the same time. This means that by using diagrams learners can identify the procedural and structural ideas within the problem. They are allowed the opportunity to participate in the problem solving process regardless of what their inclination to learning is, whether they are visual or algebraic learners. They are faced with both an analytical and visual representation of the problem. The learners are exposed to both the diagram and words whereas in language they only have the benefit of words.

Furthermore, Diezmann (2000) proposed that general-purpose diagrams assume an important role in mathematics. She further classified diagrams as networks, matrices, hierarchies and part-whole diagrams. When sketching a diagram to represent different concepts, the first step to drawing a diagram necessitates understanding what the problem demands (Zimmermann, 1991). Researchers (Jones & Mooney, 2003; Simpson & Tall, 1998; Tall & Gray, 2001) argued that poorly designed diagrams have been shown to reduce the effective teaching and learning of mathematics. Thus, apart from drawing a diagram, the ability to use a diagram correctly is a powerful tool for mathematical thinking. It is essential for learners to be aware of why a diagram is useful as well as how to use the diagram. This allows them to gain confidence to reflect on what they are doing and make sense of the activity for themselves (Griffin, 2009). This means that in order for learners to unpack and comprehend these diagrams, they ought to be diagram literate.

Diagram literacy is a part of visual literacy. It refers to knowing about diagram use and being able to visualise this knowledge appropriately (Diezmann & English, 2001). In order for learners to attain diagram literacy, their teachers ought to design instructional activities (Pantziara, Gagatsis, & Pitta-Pantazi, 2004). These instructional activities ought to be designed so that all learners in the mathematics classroom can participate within the lesson unconditionally. Once a teacher values diagram literacy as an indispensable skill in

mathematics, there is much that the teacher can do to enable learners to engage with pictures (Arbuckle, 2004) and diagrams. These skills are essential in solving problems in mathematics as proposed by Presmeg (1989).

2.7.2. Visual imagery

Images are composed of both visual and spatial images. Visual images refer to the representation of the visual appearance of an object, e.g. its shape, colour and size, whilst spatial images refers to the representations of the spatial relationships between the parts of an object its location and movement in space (Van Garderen, 2006). Researchers in education are converging toward the importance of using visual approaches (McLoughlin, 1997). This is because researchers are searching for a different approach to teaching and learning in an increasingly visual environment. The use of visual images enhance teaching in multiple ways and provides a rich setting for the understanding and development of complex and abstract ideas (Thomas, et al., 2008).

Visual images also play a decisive role in promoting critical thinking (Rezabek, 2008), learning and communication in the mathematics classroom. However, even though visual education is important for learners to successfully interact with shapes (Freudenthal, 1971) visual education is an ignored area in the national curriculum (Hershkowitz, et al., 1996). Whilst the Hershkowitz et al. (Hershkowitz, et al., 1996) study was an international study the idea that visual education is an ignored area of the curriculum still holds true in South Africa (Naidoo, 2006). Similarly, McLoughlin (1997) lamented that although visual images are a part of human cognition, they tend to be marginalised and undervalued in education. Visual images include images that learners see; this could include mathematical symbols, the use of different colours and highlighters on the board, the use of technology, pictures and gestures. Visual images in mathematics are concerned with particular forms of existential meanings, they include abstract and statistical graphs, diagrams and computer generated graphics (O' Halloran, 2005). Graphics refer to tables, graphs, and other visual aids that occur in texts to represent and complement verbal information. Graphic

information forms an integral part of mathematics texts in that the ability to understand and communicate information in the text, form a crucial part of mathematics reading comprehension (Bohlmann & Pretorius, 2002).

As discussed earlier on in this section, learners have different biases with respect to how they learn. The focus on this study is on learning mathematics using visual images. In this study, the Master teachers used visual images as visual tools in the mathematics classroom. The Master teachers used the visual tools to assist with their teaching of concepts and ideas in mathematics; they turned to these visual tools because the tools supported the teaching of mathematics. This was so because these tools provided an avenue for individuals to associate concepts in mathematics (Place, Hillyard, & Thomas, 2008). The visual tools allowed access to the mathematics because it provided a link between the known and unknown knowledge. Along similar lines, van Garderen (2006) added that the use of visual images in mathematics has been found to be advantageous with respect to mathematics achievement.

Additionally, visual images are also found to be effective as an aid for second language learners in allowing for successful engagement in natural discussion and the building of self confidence (Arbuckle, 2004). This is particularly useful in South African schools. The language of learning and teaching (LOLT) in South African schools is English. The majority of the learners at schools are English second or third language learners (Setati & Adler, 2001). Since language is the main mode of communication in classrooms, for effective communication to occur one must possess the ability to say the right thing at the right time. Language therefore plays a significant role in learners performance in mathematics and science (Venkat, et al., 2009). If one is not exposed to, or does not have the experience of certain words and concepts one would not be able to communicate effectively within a classroom context. The language practices that the learners bring to school inescapably affect how and what they learn (Gorgorio' & Planas, 2001). Thus, the

use of visual images assists in making the language of mathematics more comprehensible and hence more accessible for many of these learners.

2.7.3. The use of visual tools in schools

Visual tools are typically conceived as dealing with the ‘concrete real world’ rather than the ‘abstract world’ of symbolism. As visual tools increasingly take their place alongside mathematical symbolism, this semiotic resource may be seen to offer more than an intuitive understanding of the phenomena and a means for experimentation and synthesis. It reinforces understanding of various mathematical skills, subject matter and procedures (Holton, Ahmed, Williams, & Hill, 2001). In the global forum, intelligent activities often depend on how individuals use resources such as tools and notational systems. Many of these tools transform and distribute understanding in order to create opportunities for innovation and invention (Putnam & Borko, 2000). The most widely accepted tools are those that fit snugly within the existing social and conceptual structure of the classroom. These tools include visual representations.

Teachers often use visual representations unknowingly in class, for example, when they resort to the use of gestures, graphs, shapes, lines and diagrams. A gesture is any physical body movement (Maschietto & Bartolini Bussi, 2009; Roth & Lawless, 2002) that assists in a communication function (Sfard, 2009). They are a variety of movements (Roth, 2001) that accompany or are tied to speech (Goldin-Meadow, 2004). Narrators often use gestures that refer to the world they are talking about, some of the gestures are iconic and depict things and others are deictic and locate things (Clark & Van der Wege, 2001). Gestures have been classified into different types. This includes beats, deictic, iconic and metaphoric gestures. Beats are interactive gestures that regulate the coordination of speaking turns or signify acknowledgment of understanding. Deictic gestures are used in concrete or abstract pointing whilst iconic gestures illustrate a perceptual relationship with concrete objects. The shape of the iconic gesture is the same as the content or concept that they convey. Metaphorical gestures are similar to iconic gestures but visual demonstrations rather than

concrete objects are provided (Edwards, 2009; Pozzer-Ardenghi & Roth, 2005; Roth, 2001; Roth & Lawless, 2002).

Experience indicates that mathematics teachers often use visual tools with the intention of assisting learners to grasp a concept or problem in order to improve mathematical conceptual knowledge. This affirms that the teacher's tacit knowledge and beliefs concerning the teaching and learning of mathematics influence how they teach mathematics (Remillard, 2005). Mathematics teachers also use visual tools to make mathematics more concrete and accessible to learners. Roodt and Conradie (2003) supported this claim and highlighted that the use of different approaches to the same problem enriches both learners and teachers.

Good teachers often use symbols, signs, colour, diagrams, gestures and pictures in the classroom as an alternative to the mundane approach of 'chalk and talk' teaching. Lather (2004) maintained that good teachers are those who realise that no learner ought to be left behind. Since nonverbal learners do not present much interest in words, it becomes important for teachers to present an interesting, colourful and exciting classroom environment that stimulates curiosity (Rose, 2004) and assists in the teaching and learning process. The use of colour, pictures and other visual tools creates an exciting and interesting mathematics classroom.

Thus, more approaches, which encourage learners to be active, productive and allow them the opportunity to demonstrate the extent of their thinking and creativity, are needed (Barnes, 2005). For this to be possible, a multimodal learning environment ought to be created. Multimodal learning environments are learning environments that encompasses different modes of teaching to represent both verbal and non-verbal content knowledge. According to Jacobs (2005), the use of multiple perspectives reinforces learning by providing multiple connections. This implies that there ought to be an integration of various sign systems such as language, gestures, mathematics and music, as suggested by

Lemke (2005). This strategy was evident in most of the observed Master teachers' lessons during this study.

Stokes (2000) suggested that the use of visual tools assists in uncovering the role that visual reasoning plays in solving problems in mathematics. This presents encouraging results in the teaching and learning of mathematics. Visual tools may also be used as a starting point to achieve interactive and stimulating learning environments (Breen, 1997). Learning environments that incorporate technology add value to a mathematics lesson. In these learning environments, learners are able to interact easily with concepts that were once considered abstract.

2.7.4. The use of technology in classrooms

With the growth of the use of technology, visual images may now be manipulated and synthesised (O' Halloran, 2005). For example, teachers teaching transformations in geometry can use technology to manoeuvre rotations and reflect images. Learners are now able to see these transformations, allowing them to concretise these once abstract mathematics concepts. Likewise Roodt and Conradie (2003) maintain that learning opportunities that extend the possibilities of technology and teacher-centred approaches that emphasise role learning by individual learners ought to be created. Technology infused classrooms support Vygotsky's emphasis that social interaction is essential for learning (Steer, et al., 2009; Wepner & Tao, 2002). Technological tools like the smart board and the calculator enable teachers and learners to display ideas and allow for multiple interpretations. These interpretations may be discussed, interpreted and revised based on feedback from peers (McLoughlin, 1997). A smart board is similar to a traditional black board, but the heart of the system is a computer and the screen is similar to a touch screen (Starkings & Krause, 2007).

In an attempt at looking at the uses and benefits of technology, research on the use of electronic books was conducted by Wepner and Tao (2002). The findings indicated that

learners can be taught to read visual information in a similar fashion as they are taught to read textual information. For both, they bring in their prior knowledge to assist in improving their conceptual knowledge. Likewise, Berger (2004) states that learners do not develop meaning for concepts and words independent of their meaning in their social world. Both the social context in which learning transpires and the social contexts that the learners carry to their learning milieu are of great consequence. This implies that the learner's social background and experiences are important because the physical environment influences the visual experiences available (Cooper, 2008).

In mathematics there are various computer-based packages (e.g. *Geometer's Sketchpad*; *Cabri Geometry*; *GeoGebra*; *GeoProof*; *Cinderella* and *Graphmatica*) available for the teaching and learning of mathematics. Mathematics teachers have recognised the potential power and the promise of visual learning, however, despite this observation, implementation is lacking (Dreyfus, 1991). Teachers ought to use these software packages to make geometry as well as other sections in mathematics more visible to learners. By using software programmes like *Geometer's Sketchpad*, the teacher frees up more time to interact with the learners and to ask conceptual, probing questions (Steer, et al., 2009). This is ideal for occasions when the learner cannot see or understand what the teacher is talking about within a mathematical context. In addition, these tools have the added benefit of allowing learners to discover rules and generalisations for themselves (ibid, 2009). It is in this way that engaging learners and teachers in different approaches to learning mathematics instil a love for mathematics.

Therefore, whilst the availability of technology is important it depends on the willingness and the initiative shown by the teacher. Both the teaching and technology ought to have a symbiotic relationship. This means that the teaching strategy and technology ought to be compatible with each other; the one should complement the other. Whilst this is a gracious idea, one has to concede that in most schools, technological resources are not readily available, hence teachers have no option but to use traditional methods of teaching.

Teachers using the traditional approach to teaching may find it difficult to create an engaging learning environment (Holton, et al., 2001), unless they research potential, non electronic but visual ways of teaching.

2.8. Implications of the literature review for this study

The literature review afforded a basis for exploring, clarifying, interpreting and illuminating various concepts and ideas dealing with the present research study with respect to the teaching of mathematics. The literature review:

- Highlighted the nature of mathematics and the nature of mathematical thinking.
- Drew attention to the scope and range of problems experienced by mathematics teachers more specifically mathematics teachers in South Africa.
- Revealed the need to integrate the use of visual tools in the mathematics classroom.

Drawing on the literature, the present study set about exploring Master teachers' use of visuals as tools in mathematics classrooms. The study explored six Master mathematics teachers teaching in six KwaZulu-Natal Dinaledi schools in South Africa.

2.9. Conclusion

This chapter began by providing an overview of the literature on mathematics in general and mathematics in South Africa. Issues of contextualising mathematics have also been explored. I then went on to discuss teachers in South Africa and the practice of teaching mathematics. The concept of Master teachers has been examined both locally and internationally. Following this discussion, I went on to discuss the impact of visualisation on teaching mathematics. Key aspects related to visualisation and the research study was interrogated. The chapter concluded with a reflection on the implication of the literature review for this study. The next chapter, Chapter three focuses on the theoretical framework informing this research study.

Chapter Three: Theoretical Framework

“Teachers are reservoirs from which, through the process of education, students draw the water of life.”
Bhagavan Sri Sathya Sai Baba

3.1. Introduction

This study explored Master teachers’ use of visuals as tools within mathematics classrooms. In the previous chapter, literature informing this study was introduced and discussed. In this chapter, I discuss the theoretical framework within which this study was located. This chapter begins by presenting the theoretical orientation of this research study with the intention of creating the context for the theoretical framework that is later established. An explanation of the coherence between the methodological orientation and the theoretical framework is presented. Subsequently, I provide a discussion of the origin and development of activity theory. This is achieved by examining the background and principles of the theory. Next, a description of the heart of the theoretical framework used in this research study, namely, the different generations of activity theory, is presented together with an outline of previous research conducted using activity theory. Finally, because activity theory provided the theoretical framework for the presentation and analysis of data in Chapters five and six respectively, a detailed account of how activity theory has emerged in this study, is presented.

Activity plays an important role in mathematics learning and development (Groves & Dale, 2004), however, very little activity of significance is accomplished individually (Jonassen & Rohrer-Murphy, 1999). Activity embraces interaction and social cohesion amongst the participants. I wanted to exhibit these sentiments in this study; I wanted to approach the study using an interactive, social approach. This research study was embedded within an interpretive, qualitative paradigm. This approach provided opportunities to unearth meanings within their research context. Qualitative research provides an enhanced agency of reflecting social reality as compared to quantitative research. The discourse of qualitative research is that of interpretation (Neuman, 2000). Qualitative researchers are concerned with meaning and sense making. They attempt to uncover how individuals make

meaning of their lives, what they experience, how they construe their experiences and construct their social world (Merriam, 1988; Neuman, 2000). Research founded within an interpretivist paradigm assumes that human action is naturally noteworthy. For example, in order to understand a specific social action, it is important for the researcher to grapple with the meanings that encompass that action. The meanings of actions can only be understood in terms of the scheme of meanings to which they belong (Schwandt, 2003).

Fundamentally this study uses a qualitative approach within which the theoretical framework is activity theory.

“Activity theory provides an ideal theoretical framework for describing the structure, development and human work and praxis, that is, an activity in context.” (Uden, 2007, p. 90)

This theory focuses on complex, collaborative interaction within a social context. I used this theory to explore the relationship between the Master teacher, the learners, and the visual tools within the classroom environment. A qualitative interpretation was made of the relationships evidenced within the social context of each classroom.

Based on my experience, an effective mathematics environment is one that encourages and supports actions, group activity, communication, creativity, democracy, mediated meaning, and facilitation. This for me suggests that, in order for effective teaching and learning to occur, the premise that learning is a social activity whereby meanings are socially constructed ought to be adopted. Activity theory conforms to similar principles of social construction of knowledge and ideas. In addition, this theory illustrates how actions and processes are divided and shaped by a larger community interested in accomplishing a common goal (Waite, 2005).

This activity theory framework assumed that all human actions are mediated by artefacts or tools and cannot be separated from the social milieu in which action is carried out. This was evident in this research study, given that the Master teachers used visual tools to teach

mathematics. These tools differed based on the context within which each school was located. For example, during the teaching of transformation geometry, one Master teacher used the smart board; another Master teacher used a stick with different coloured elastic bands and the next Master teacher used paper folding and gestures. These diverse tools were used to teach the concept of rotation and reflection in transformation geometry. This is discussed in detail in Chapters six, seven and eight.

It was evident in this study that activity theory provided a socio-cultural lens to analyse human behaviour. This includes human behaviour within the classroom environment. When individuals interpret educational situations, the notion of the social-cultural context becomes imperative (Kofod-Petersen & Cassens, 2005). Since this study focused on the formal educational context of the classroom, activity theory was appropriate to the study.

3.2. Activity theory

“The fundamental assumption of activity theory is that tools mediate or alter the nature of human activity and, when internalised, influence humans’ mental development.” (Jonassen & Rohrer-Murphy, 1999, pp. 66 - 67).

3.2.1. Background

Activity theory surfaced in the 1920s and 1930s. The founders of the cultural-historical school of Russian psychology fathered this theory. These revolutionary psychologists included L.S. Vygotsky, A. N. Leontiev and A.R. Luria (Engeström, 2001; Fitzsimons, 2005; Kuutti, 1996; Uden, 2007). They formulated an advanced theoretical concept to go beyond the prevalent conceptions of psychology. A new method was devised for investigating thought and consciousness. This emerged at a time when psychology was dominated by psychoanalysis and behaviourism, as pointed out by Engeström and Miettinen (1999).

Cultural-Historical Activity Theory (CHAT) commonly referred to as Activity theory, rebelled against what was then the norm in psychology. Vygotsky examined the tools of

psychology (maps, language and writing) and claimed that since they were social, they were contrived. This was Vygotsky's criticism of the views of theorists working within the field of psychology. Activity theory was therefore the outcome of a greater effort to formulate an unconventional psychology. The theory was developed to illustrate the role of society in influencing the mind of the individual. Likewise, Kuutti (1996) maintained that to advance the notion of social interaction and associations, different forms of human practices were studied as developmental processes. This type of research was in total dissonance with principles underpinning psychoanalysis and behaviourism. In the course of implementing this theory, it was held that one's thought processes were developed in response to the environment, through activities.

Whilst activity theory provided a theoretical framework for examining human practice, including the practice of teaching and learning, it demonstrated most relevance in situations that had significant historical and cultural contexts. Since within these contexts, participants and their tools are generally in a state of change, activity theory accentuated the importance of culture and history (Nardi, 1996) on human behaviour. Activity theory infiltrated the global arena and was recognised internationally in the 1980's and 1990's. This acknowledgement of activity theory took place in the midst of changes in the political and economic structures globally.

I believe that activity theory is relevant within the South African context since South Africa also underwent rapid change during this period. South Africans were now allowed to interact within different social milieus regardless of race, culture and class. Community members within activity systems created new rules and took on different roles with activity systems, the object of each activity system evolved in the midst of political change. For example when parents scrutinised the opportunities that were now available to their children, they quickly sought spaces at more resourced schools. These schools were once forbidden to learners who were not white. Thus, rapid change influences how individuals and groups think and behave within activity systems. Change affects interaction and

communication amongst the community members within each activity system. Change also influences the invisible hierarchical structures that exist in most communities.

In South Africa, this rapid change commenced most noticeably with the release of Nelson Mandela in 1994. Dr. Mandela's release signified the demise of the apartheid era and the birth of democracy. This was of paramount importance to the majority of the population. This was so because the majority of the population were Black, and they occupied the lowest rung of the social ladder within South Africa society. They were the disadvantaged masses desperate for change. As discussed earlier in Chapter two, many changes were necessary to establish full democracy. The education department was a key site for redress and reform. Vast changes were made within the education sector in order to benefit the previously disadvantaged groups in South Africa. These included changes in curriculum, pedagogy, infrastructure and policy. Thus the object of the activity system governing the education sector changed drastically.

3.2.2. The principles of activity theory

Activity theory assimilates the ideas of planning, negotiation, history and cooperation with the intention of understanding that consciousness and activity are interrelated and integrated (Nardi, 1996; Uden, 2007). To further this aim, Kuutti (1996) proposed that by using activity theory there existed the potential to discuss issues belonging to various levels within the framework. These levels include operations, actions and activities. Additionally, activity theory consisted of a set of rudimentary principles that may be used as the basis for more specific theories. These principles include:

- The hierarchical structure of an activity
- Object-orientatedness
- Externalisation and internalisation
- Mediation
- Development

Whilst these principles are discussed individually in the next section, it is important that these principles are regarded as an integrated scheme. Each principle cannot exist in isolation.

3.2.2.1. The hierarchical structure of an activity

Activity takes place at different levels at the same time. They are not rigid but undergo transformation and progression (Kuutti, 1996; Uden, 2007) on a continuous basis. Activities consist of a series of actions and actions in turn consist of a series of operations. These ideas may be represented diagrammatically as in Figure 2.

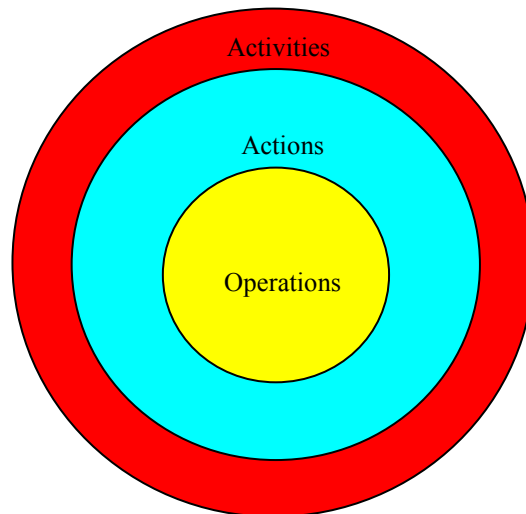


Figure 2: Levels of activities

On interrogating these levels, we find that whilst they are hierarchical in nature, they do not necessarily have a sequence. At the top level we have activities, at the second level, we have actions and at the last level, we have operations. As depicted in Figure 2, the innermost circle representing operations is subsumed by the next level which is called actions. This is so because Figure 2 epitomises a nested level of activities. Actions are dependent on the activity within which it occurs and all operations are dependent on the actions that ought to be carried out.

As an overview it must be stated that on these levels we find that operations are actions when they are first carried out; this implies that actions are a chain of operations. The actions that are at the second level are directed at specific goals (Fitzsimons, 2005; Rivers, et al., 2009; Uden, 2007). Subsequently, if we had to represent the three levels diagrammatically to illustrate their association in a hierarchical manner, the representation would appear as in Figure 3.

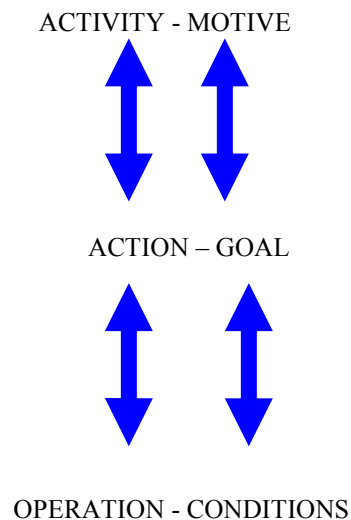


Figure 3: The hierarchical structure of an activity
Adapted from Daniels (2008, p. 119)

Zinchenko (1996) explained that activity and action are explanatory principles for all human psychology. The border between activity and action is always indefinite, mainly due to the fact that there is a possibility of movement in both directions. Whilst Figure 3 represents the hierarchical nature of an activity, it also intimates the reciprocal nature of activity, action and operation. This implies that an activity may become an action and an action may become an operation (Kuutti, 1996). However, this hierarchy is not permanent; all levels have the potential to be changed. To clarify and attach meaning to these hierarchical levels, Table 5 demonstrates two hypothetical situations.

The purpose of the following examples is to provide the reader with an initial grasp of how the levels work. The use of these examples is merely meant to concretise the theoretical

ideas of activity theory for the reader. It defines characteristics of motivation, goals and conditions within a contextual scenario.

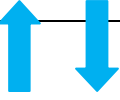
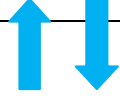
Activity Level (Motivation)	Planning a party	Developing material for a university mathematics course
 Action Level (Goal)	Decorating the venue with balloons and decorations. Ordering the food. Transporting the food by car.	Arranging a meeting with other lecturers. Programming a template for typesetting material. Writing out the content.
 Operation Level (Conditions)	Blowing up balloons. Pinning up decorations. Telephoning the caterer to order the food. Changing gears when driving to pick up the food.	Telephoning or emailing lecturers with the venue and time for meeting. Typing out minutes of the meetings. Using operating system commands for generating the template. Using appropriate wording for the course material.

Table 5: Examples of activities, actions and operations

3.2.2.2. Object-orientatedness

Researchers (Kuutti, 1996; Morf & Weber, 2000) pointed out that an activity involves individual/s who are focussed on attaining a specific goal or object. In a mathematics classroom, the individuals within the activity system would be the mathematics teachers and the learners. The individuals are focussed on the teaching and learning of specific concepts in mathematics. Activities differ based on the object in question. In the classroom, activities could encompass the active engagement of learners in group work to achieve adequate understanding of the rules for reflection in transformation geometry. Jonassen and Rohrer-Murphy, (1999) concurred with Kuutti (1996) and elaborated further that an object may be something physical (e.g. a library that is being built), software, (e.g. a computer programme to calculate volume of 3D shapes), or even something conceptual (e.g. a theory of an activity that is being discussed). The object guides how individuals interact with the

world, but they do not characterise human activity. In addition, objects manipulate how the subject and object relate to each other. Thus, due to the versatility of an object (they can be changed during the course of the activity) activities are not fixed, but are always varying and developing (Kuutti, 1996). The presence of objectives allows for object-orientatedness to become more tangible.

Activities are dynamic and an activity is defined with the aid of a concept of the object. This implies that the activity cannot be understood without considering the role of the artefact holistically. To clarify, one needs to comprehend the role of the artefact in everyday existence with respect to its (the artefact's) assimilation into social practice (Nardi, 1996). The activity in question precedes any other process because abstract notions arise due to individuals performing tasks. Before an action is performed in the real world, it is planned in the consciousness using a model. This implies that the more user friendly the model, the more successful the action. If for some reason the model does not prove to be suitable, thought is given to the reformulation and reconstruction of the model. This phase is called orientation; this implies that models are not rigid, but tentative resources (Kuutti, 1996).

For example, in mathematics, the manipulation of a geometric shape according to the rule of reflection about the y-axis normally materialises after the learner has a chance to contemplate the shape, and the learner visualises what the shape looks like. S/he then moves on to the application of the rule for reflection, it is at this stage that the process of analysis occurs. Once this manipulation (according to the rule of reflection) is visualised mentally, the learner then 'sees' the new image (the reflected image). This can be considered as the next stage of visualisation. This image is then concretised on a piece of paper or chalkboard. For this action to be successful, visualisation and analysis need to be considered as two interacting modes of thinking that support each other in the development of one's conceptualising and contextualising of mathematics.

This notion was investigated by researchers (Naidoo & Bansilal, 2010a, 2010b) within transformation geometry. The researchers investigated strategies used by learners to assist in answering problems based on transformation geometry. Learners were found to use either visual strategies, analytical strategies or a combination of both to solve problems in transformation geometry. Those learners that achieved most success were those that used the combination of visual and analytical strategies. What was established, based on these findings in both studies, is that mathematics teachers may find it beneficial if they incorporated both the visual and analytical process of teaching and learning mathematics. This would encourage learners to use visualisation and analysis as complementary strategies rather than isolated strategies.

Likewise, researchers (Zazkis, et al., 1996) suggested that the strongest mathematical thinking was a synthesis of visual and analytical thinking. In doing so, they proposed the Visualiser/Analyser or VA model. The VA model views visual thinking and analytical thinking as complementary rather than contrary. As discussed earlier in Chapter two, visualisation is defined as the connection that an individual makes between a visual, concrete mathematics manipulative, picture, diagram, gesture or cut out and something in his/her mind. Analysis may be defined as any mental manipulation of objects or processes with or without the aid of symbols (Zazkis, et al., 1996). Learners need to acquire the ability to move back and forth between the graphical and analytical representation of a problem (Zimmermann, 1991).

Figure 4 that follows is a good way of explaining this model.



Figure 4: The VA model
Adapted from Zazkis, Dautermann et al. (1996, p. 447)

Based on this model, at first, the individual sees acts of visualisation and analysis (V_1 and A_1) as different acts. The movement from one to another may pose a problem, but gradually the two kinds of thoughts become more interrelated and the movement between them becomes less of a concern. This implies that the process that occurs internally is in harmony with what is projected externally.

3.2.2.3. Externalisation and internalisation

The principle of externalisation and internalisation is one of five principles that underpin activity theory. Internalisation is the conversion of external activities into internal ones. Internalisation provides a means for individuals to use mental simulation, imagination and the consideration of alternative plans. This provides the individual with an opportunity of using virtual objects or images before interacting with the real object. The process of internalisation is mediated by tools. Language is one of the most important of tools (Swain, Brooks, & Tocalli-Beller, 2002) for this mediation. In mathematics, whilst research in South Africa indicates that a learner's main language is a resource (Setati, 2006), a common finding indicates that the more proficient a learner is in the English language, the better they are at understanding problems in mathematics (Kazima, 2008). This holds true in cases where English is the medium of instruction as is the case of the majority of secondary schools in South Africa. This may create a problem for the majority of learners

in South Africa because the language of learning and teaching (LOLT) is not their main language. With the high probability of discord between the LOLT and the learner's main language, many setbacks in the internalisation process may emerge.

When an individual experiences a problem with the internalisation process, externalisation is often necessary. To clarify, one cannot comprehend internal activities if they are divorced from external activities. External and internal activities are seen as interrelated, complementary acts, because they transform into each other. Sometimes external influences alter some components of activities causing contradictions (Uden, 2007). Activity theory uses the term contradictions to signify a breakdown or a clash between activities or elements within the activity system. This is seen as a source of development (Kuutti, 1996), given that activities are always in the process of working through contradictions that consequently support transformation (Uden, 2007). Breakdowns related to the function of a process occur when work is interrupted. One possibility for this occurring is that the artefacts within the activity system perform differently than expected (Bodker, 1996). Whilst contradictions cause disturbances, they also provide the possibility of allowing for the identification of different ground-breaking strategies for altering an activity. Contradictions assist in transforming activity systems.


In order to elaborate, I provide an example. During the lesson observations at one of the schools, the Master teacher had prepared to use a diagram as a visual tool (artefact) to explain a concept in trigonometry; however, during the course of the lesson she realised that the visual was not adequate. If she continued teaching with this tool, this would have led to a breakdown in the activity system. Learners would not have been able to reach the outcome of the lesson. Fortunately, due to the Master teacher's experience, and good pedagogic and mathematics content knowledge, she managed to modify the tool. This allowed for the successful mediation of the modified tool to achieve the intended outcome. This example demonstrated that the Master teacher engaged in reflection-in-action to be successful within the classroom. By being reflective the teacher was able to alleviate the

contradiction or disturbance within the activity system. This assisted in creating a smooth transition from the internalisation to the externalisation process.

3.2.2.4. Mediation

All human activities contain and are mediated by culturally created signs or tools (Kuutti, 1996). The teaching of mathematics is an activity and a mathematics teacher uses tools such as language, books, pictures and the calculator to teach his/her learners mathematics. Tools can be anything from a sign or language to machines and computers (Hashim & Jones, 2007). Vygotsky claimed that signs were the products of the internalisation processes and referred to these signs as “psychological tools” (Maschietto & Bartolini Bussi, 2009, p. 145). These tools are available when we start an activity, but can also be a product of the activity. Due to their dynamic nature, tools can constantly change as a result of the activity. For example in this study in the first observed lesson, Karyn started her discussion on the calculation of perimeter with an uncomplicated diagram of an isosceles triangle. An isosceles triangle is a triangle with (at least) two sides equal. As she taught the lesson, she realised that she needed to modify the diagram to allow her learners access to the mathematics being taught. This was done through a discussion and negotiation within the learning community. At the end of this discussion, the diagram was adjusted to make the content being taught more accessible to the learners. Once this was achieved (based on the observations of the rest of the lesson), the outcome of the lesson was successfully accomplished.

According to Peirce, “we think only in signs” (Chandler, 2009, p. 1). Signs take the form of words, images, sounds, acts or objects, but such things have no intrinsic meaning and become signs only when we assign them with meaning. This implies that nothing is a sign unless it is interpreted as a sign. There are three kinds of signs, icons, indications and symbols. Icons identify aspects of material they represent by emulating them, indications communicate meaning of things they represent by being connected with them and lastly, symbols convey meaning of things they represent through usage (Peirce, 1894).

Saussure's work as discussed by Chandler (2009) offered a two-part model of the sign. A sign was defined as being composed of a 'signifier', the form which the sign takes; and the 'signified', the concept the sign represents. For example in this study Penny used the following image.  This is the signifier, or the form the sign takes. The signified which is the concept that it represents is the shape of the parabola as depicted in the image that follows.



What was evident here was that the visual tools may be used as both cognitive tools and social tools for mediation. Tools delineate how individuals communicate with the world. When used as a cognitive tool, learners process and manage their own meaning making. When used as a social tool, learners use the tool to communicate (Swain, et al., 2002) with the rest of the learning community. In the study, the Master teachers used visuals as tools both cognitively and socially within the mathematics classroom. These tools included gestures, diagrams, pictures, technology, manipulatives, shapes, signs and lines. Similarly, Waite (2005) claimed that activity theory exposes the relationship between human knowledge and physical artefacts. In this study, human knowledge and artefacts were worked on and propelled through a continual process of engagement and assessment. This implied that an artefact may be used as a descriptive tool for an activity system rather than a predictive one (Nardi, 1996). The activity system was represented in various ways during its conception. These different ways are referred to as the different generations of activity theory. The different generations will be discussed in Chapter 3.3 of the study.

3.2.2.5. Development

Activity theory supports the notion that development is not just the object of analysis but also a general research methodology. This is so because activities are not inert, but they are transforming and developing. The basic research methods that support activity theory are those that advance active participation. These methods involve the monitoring of the

development that occurs with the participants. Although activities are based on social, cultural and historical aspects, each activity has a history of its own. This history is conveyed to the objects and tools.

For example, when each Master teacher engaged with their learners whilst they were working on different mathematical tasks, in this context the interjection by the Master teachers represented the cultural and social factors. The use of different visual tools to scaffold the problem solving process incorporated a history of its own. This history could relate to the Master teachers tacit knowledge, experience or pedagogic knowledge. The use of the visual tools was designed to initiate and encourage mathematical development within each learner.

3.3. The three generations of activity theory

There exists three generations of activity theory. The first generation centred on Vygotsky's work. His model created the idea of mediation (Engeström, 2001). The initial model used to illustrate the basic framework developed by Vygotsky resembled Figure 5.

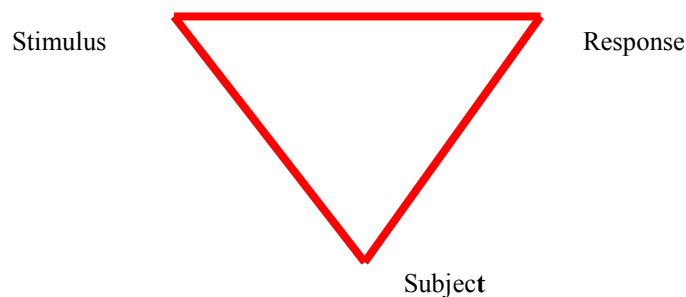


Figure 5: Vygotsky's model of a mediated act
Adapted from Engeström (2001, p. 134)

Initially it was not clear whether the subject was incorporated within this activity system or not. Some literature represents this activity system such that the subject does not appear as one of the vertices of the activity system triangle. Instead of the subject an arbitrary symbol X appears. Due to the lack of transparency and the existence of many different

interpretations whilst comprehending this model, Vygotsky's model was reformulated as shown in Figure 6 that follows.

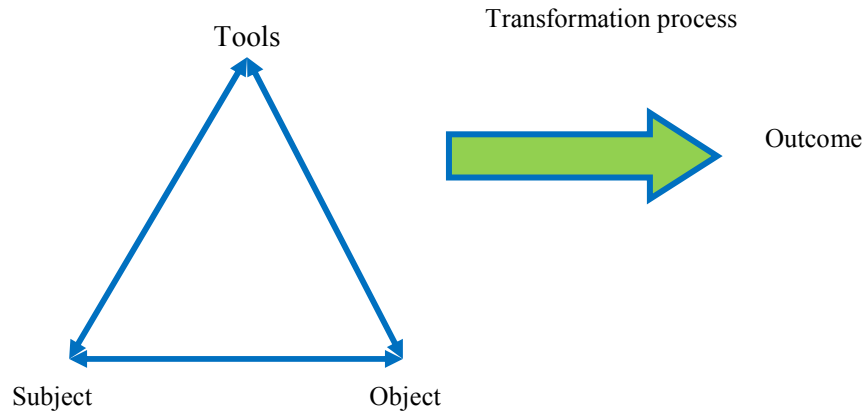


Figure 6: Vygotsky's model reformulated - Mediated relationship at the individual level
Adapted from Kuutti (1996, p. 28)

An interpretation of Figure 6 implies that an activity necessitates a subject and object, both of which are required to be mediated by a tool. Uden (2007) claimed that this is done in an attempt at attaining the outcome of the activity. Rajkumar (2005) regarded the motive of the activity as the 'problem space' at which the activity is focussed in order to bring about a change or the desired outcome. The object can be material (mathematics curriculum statements), or less concrete (a plan) or even elusive (a common idea). All this can be possible, provided the object can be managed and transformed by the subject/s of the activity (Kuutti, 1996). The subject is the individual/or group of individuals who carry out the activity (Uden, 2007).

However, Engeström (2001) argued that Figure 6 did not capture the absolute relationship between the subject and the environment in an activity. This is so because the unit of analysis is still individually focussed. Since most human activities are shared ones taking place in rich social milieus, the earlier models were inadequate. The earlier models failed to demonstrate the complete associations between an individual and his or her environment in an activity. This shortfall of the first generation model necessitated the need for the second

generation activity theory. The second generation activity theory centres on Leontiev's research. In his example of the "primeval collective hunt" (Leontiev, 1981, pp. 210 - 213), he used the example of the "bush beater" to explain the relationship between an individual action and a collective action.

Within a tribe, a group of hunters hunting for food and clothing (skin of the animal); each had a specific role to play during the hunt. The beater's role was to scare the animal/s towards the rest of the hunters so that they could capture/kill the animal. The beaters' activity was the hunt and his action was the frightening of the animal/s. This was the basis of his making the distinction between an activity, action and operation.

The upper level of activity (often collective) is driven by an object-related motive; the middle level of action (often individual) is driven by a conscious goal; and the lowest level of automatic operations is driven by the environment and tools of the action at hand. However, the shortcoming of Leontiev's research was that he did not use a graphical representation to extend the first generation activity theory. Whilst Leontiev himself never graphically expanded Vygotsky's original model into a model of a collective activity system, Engeström (1987) did so.

This model was more proficient at illuminating shared activities and collaborative work. Engeström expanded the original triangular design in order to advance the development of activity theory (Daniels & Cole, 2002). Figure 7 was based on Engeström's 1987 conceptualisation. This model is referred to as the second generation activity system.

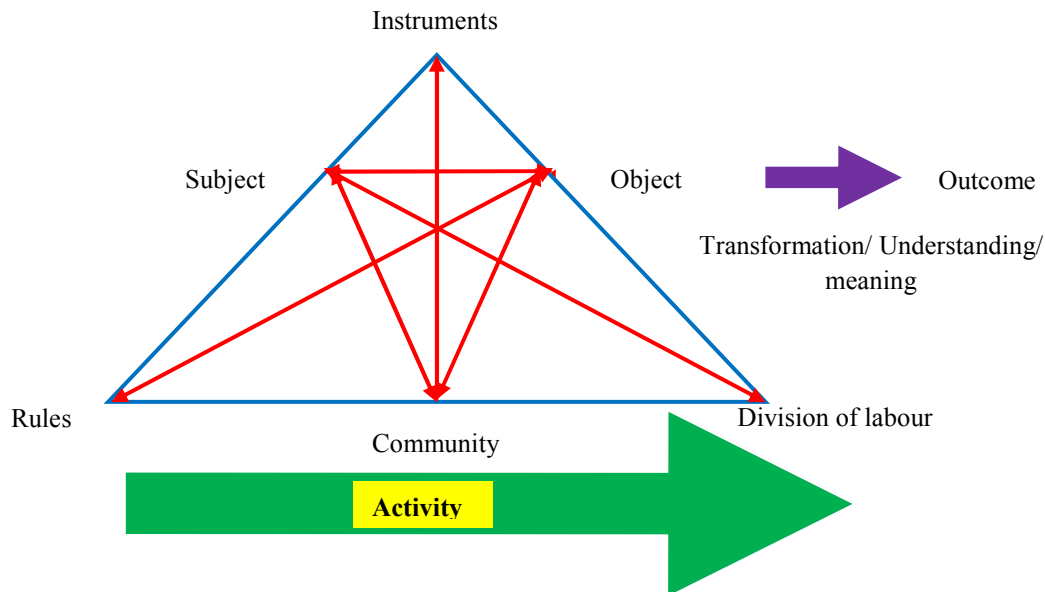


Figure 7: Conceptual model of the human activity system, the second generation activity system
Adapted from Engeström (1987, p. 78)

Thus with this improved model, two new relationships were formed, subject-community and community-object, both of which were mediated. The tools, (Christiansen, 1996; Kuutti, 1996; Rivers, et al., 2009; Slay, 2002; Uden, 2007) mediated the subject-object relationship. This suggested that in order for the subject to work on achieving the object of the activity system, mediating tools were required. The subject-community was mediated by rules, indicating that in order for a relationship to exist between the subject and community of the activity system, rules of the activity system needed to be in place. Finally the division of labour (Kuutti, 1996; Roth & Lee, 2004) mediated the community-object relationship. Each member within the community was required to have a responsibility within the activity system. To reiterate, one or more individuals who share the same object as the subject, constitute the community. The unit of analysis within the activity system is the actual activity (Uden, 2007). Whilst human activities are advanced by need, individuals wish to attain a specific outcome.

To demonstrate these notions, let us take the hypothetical case of planning a dinner party as discussed in Table 5. The activity is planning a dinner party. The actions are the building blocks of the activity. The actions in this example would be planning the menu, making place cards, ordering the food and inviting guests. Operations are the methods used to carry out the actions for example, using a computer programme to make the place cards, using a pen to write out the menu, using a telephone to invite guests and changing gears when driving the car to pick up the food.

To explain further, let us use another hypothetical example within the scope of mathematics education. The mathematics curriculum developers (subject advisors, policy makers, Master teachers and the minister of education) are working on designing a new mathematics curriculum. The activity (designing a new mathematics curriculum) consists of goal-directed hierarchy of actions (Figure 3) that are used to complete the object. It consists of a chain of conscious actions such as working on the needs assessment, drawing diagrams and setting up meetings.

Each action consists of a chain of operations, for example, note taking, spreadsheet entries and telephone calls. All operations are actions at first because they require a conscious effort to complete. With practice and internalisation, these activities are collapsed into actions that in turn collapse into operations as they become more automatic and require less thought (Jonassen & Rohrer-Murphy, 1999). The object in this example is the existing curriculum that needs to be transformed into the updated new curriculum (outcome). The curriculum developers are the community sharing the object. There is a certain division of labour: amongst the minister of education and policy makers, material developer, subject advisors and Master teachers. Team members have agreed on roles determined by skills, preferences or availability (Jonassen & Rohrer-Murphy, 1999). There is a set of rules covering what it means to be a part of the team of curriculum developers. Some may be explicit, set by the Department of Education, or by parent organisations or even the team manager.

Some rules will be implicit as part of general work culture or these rules may be developed as the team of curriculum developers' work together. Some rules may even be constructed solely for this project, for example, those learning outcomes and assessment standards from the old curriculum that will be included in the new one. Essentially different communities agree on different conventions and customs (Jonassen & Rohrer-Murphy, 1999). In each stage of the transformation process whilst developing this new mathematics curriculum, different tools are being used – for example, computers, computer programmes, analysis methods, curriculum material from other countries, and so on. These tools have a history, they are the result of a process of development and rejection at all levels within the community. Any additions or deletions may occur during this project. Primarily whatever the curriculum developers accomplish during this project is fashioned by the milieu of the activity.

Whilst this activity is taking place, other activities may also be taking place simultaneously. Partaking in an activity implies implementing activities that one is conscious of, as well as activities that have an instant unmistakable purpose (Kuutti, 1996). A possibility exists that some of the team members are competing against each other for an available position higher up on the 'vocation ladder'. Whilst working on the curriculum development project, the minister of education or other subject advisors could be weighing the capabilities of Master teachers to take on new positions. Again, there exists a division of labour, rules and tools, and, since activities are open systems, the presence of inner contradictions become the principal force that advances change within an activity system. There could be a third or fourth activity taking place simultaneously with the two discussed activities.

When the activity system under the microscope adopts a new element external to the activity system, the new element creates a variance or tension. The conflict invokes the third generation activity system. The third generation activity system addresses situations

where there are more than one activity system influencing the one under interrogation. Figure 8 represents a model of the third generation activity system.

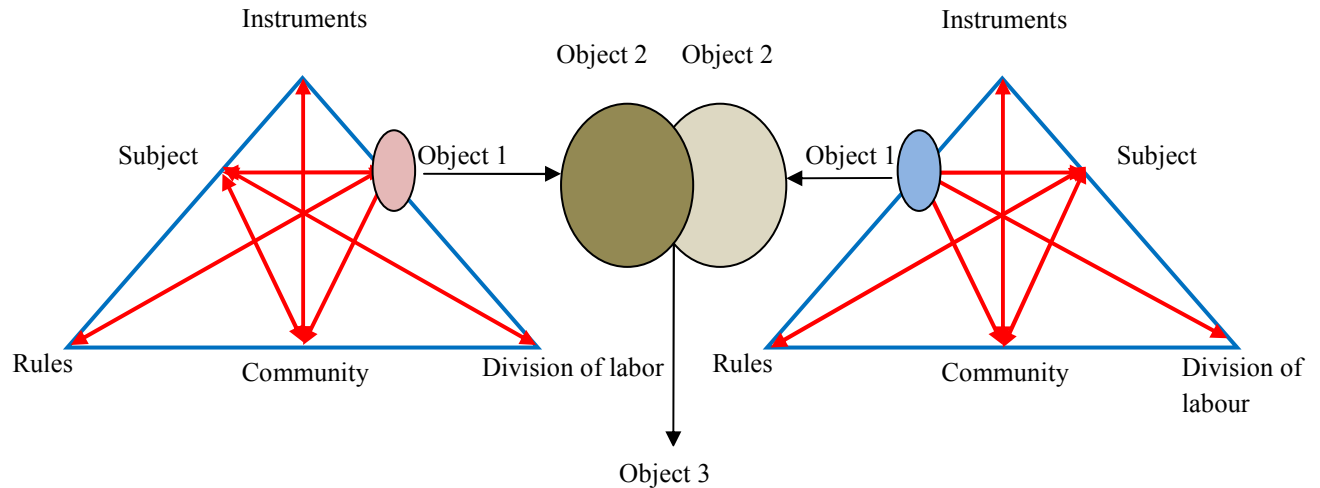


Figure 8: Two interacting activity systems as a model for the third generation activity system

Adapted from Engeström (2001, p. 136)

The third generation activity system is aimed at developing conceptual tools to comprehend discourse, multiple perspectives and networks of interrelated systems (Daniels & Cole, 2002; Engeström, 2001). In case scenarios of this nature, knowledge that is learned is learned simultaneously with development. It is in this manner, through transforming or expanding the object within the activity system, that new mediating tools have the power to change activity systems. The ongoing production of problem solving tools affords an opportunity for expansive learning and enhanced professional practice (Daniels, 2008). Expansive learning encompasses the creation of new knowledge for a newly emerging activity. A new mediating tool, rules or technology may “trigger” this transformation (ibid. p. 127). Learning can be categorised into different levels. The table that follows, Table 6 depicts the three levels of learning as summarised by Bateson.

Level	Description	Example
1	Conditioning through the acquisition of responses deemed correct within a given context.	Learning the correct answers and behaviours in a classroom.
2	Acquisition of the deep – seated rules and patterns of behaviour characteristic to the context itself.	Learning the ‘hidden’ curriculum of what it means to be a student.
3	Radical questioning of the sense and meaning of the context and the construction of a wider alternative context.	Learning leading to change in organisational practices.

Table 6: Bateson’s levels of learning
Adapted from Daniels (2008, p. 126)

Whilst all three levels of learning may take place within expansive learning, the levels of learning acquire an altered meaning as part of the expansive process. For example, learning at level three is instrumental in transforming the activity system and changing the objects of the activity, it involves the conception of new tools and the reconstruction of problems. Thus, expansive learning focuses on overwhelming existing contradictions. It draws on the strengths of shared analysis and fosters the tangible transformation of current practice (Engeström, 2005). It entails collaborative learning and involves action, reflection and advancing praxis.

3.4. Activity theory as a theoretical framework in other research

Much research (Beyer & Holtzblatt, 1998; Heiskanen & Niva, 1996; Lindell, 1991; Miettinen & Hasu, 2002; Vicario & Troilo, 1998; Wilson, 2006) has been done using activity theory. Most of this has been located in the computer-orientated sectors and human computer interaction (HCI) (Kaptelinin & Nardi, 1997; Kuutti, 1996; Nardi, 1996). The field of HCI presented a theoretical challenge to researchers trying to establish this field as an incorporated area of study (Kaptelinin, 1996). One of the reasons for this difficulty is the multidimensional role of the computer in an activity system. The role of a computer within an activity system may be threefold. Computers are tools in certain activity systems, while in another activity system, it may be an object. Lastly, computers could also be considered as an outcome in a new activity system.

Research incorporating activity theory as a framework has also been done in the field of education (Engeström, 1987, 1999), communities of practice (Engeström, 1993), information systems (Bodker, 1991), library science (Spasser, 2002), cultural psychology (Ratner, 2006) and information science (Hjorland, 1997; Wilson, 2006). Whilst the above mentioned researchers focus on different aspects of activity theory for different areas of study, they provide advice to researchers who desire to use activity theory in future studies. Research propagates the idea that activity theory does not offer readymade techniques, but is a conceptual tool. The characteristics of any artefact can only be understood within the perspective of human activity (Kaptelinin, 1996). Activity theory thus, allows us to explore the association between the development of the individual and the social milieu within which the individual exists (Bodker, 1996).

Nardi (1996, pp. 94 - 95) describes four methodological considerations for activity theory.

1. Allocate enough time to understand the user's objects.
2. Focus on broad patterns of activity.
3. Use varied sets of data collection techniques; don't rely on only one method.
4. Use a user-centred inquiry process.

Engeström (2001) formulated five principles to assist researchers when working with activity theory. The first principle reminds the researcher that the unit of analysis in activity theory is the activity system. To ensure that the researcher is on the right track, one ought to interrogate the activity system under scrutiny. Secondly, the notion of "multi-voicedness" needs to be considered. This refers to the multiple viewpoints and interest that would be prevalent in an activity system. This prevalence may cause conflict within an activity system. Thirdly the principle of historicity implies that the history of an activity system may assist in identifying both the negative and positive aspects that may influence the system. Researchers who are aware of this aspect may gain added insight whilst working with this theory. Fourthly, contradictions whilst causing pressure within an activity system may also assist in modifying the activity system. Researchers, who are aware of this notion, will be less likely to abandon the theory when faced with the first tension in an activity system. Finally, the notion of expansive learning is discussed. Whilst undergoing transformations, it is possible that wider learning is encouraged and initiated.

This wider learning may include the conception of new knowledge for a newly emerging activity.

Nevertheless, whilst activity theory has been widely used, there has also been critique of both the theory and Engeström's well-known triangle. Garrison (2001) contemplated that activity theory partitioned operational processes. Garrison (2001) in discussing these partitioned processes utilised Dewey and Bentley's (1989) research on three forms of action: self-action, inter-action and trans-action. Self-action refers to actions that are independent, inter-action refers to elements that are balanced against another element in casual intersection and lastly, trans-action occurs:

“... where systems of description and naming are used to deal with aspects and phases of action, without attribution to ‘elements’ or ... independent ‘entities’, ‘essences’, or ‘realities’, and without isolation of presumptively detachable ‘relations’ from such detachable ‘elements’.” (ibid, p. 101- 102)

Garrison (2001) voiced his concern with the use of the word action. The point that he was making is that when one is using the word action, to which action is one referring to? Are we referring to a self-action, inter-action or a trans-action? Rajkumar (2005) highlighted another drawback of activity theory; he argued that the language was too intricate to understand and the use of certain delineations proved to be confusing. Taking both the advantages and disadvantages into consideration, I employed the use activity theory but I have added on an ‘Operational definition of key concepts’ section so that activity theory used in this study can be more accessible to a wider audience. I also define what I consider to be action within my study.

3.5. Activity theory and this study

In this study, the activity system under the microscope is the act of teaching in mathematics classrooms. The community in this study are the learners within the mathematics classroom, the Master teachers, the staff and learners at each school and the parents within the community. The tasks are what the Master teacher expects their learners to complete.

The tasks that I am referring to within my study are the examples, visual tools and group work that each teacher works with in order to assist in accomplishing the goal of teaching for understanding. Moreover the relationship between the task and larger motivating activity is made clear, as discussed by Waite (2005). Thus, activity theory provided a descriptive framework allowing one to better understand and classify the processes involved in performing a task.

Whilst the teacher was working with these activities, artefacts were used. These artefacts were not only physical as traditionally understood. They were sign systems, instruments, procedures, machines, methods and laws (Rajkumar, 2005). One of the notions of activity is that while subjects engage and interact within their environment (the actual activity), instruments/ artefacts /tools are produced and may transform an activity (Bellamy, 1996). As can be construed, the tool is a dependent entity made by humans. Tools are culturally specific and shape the way people think and act as suggested by Jonassen and Rohrer-Murphy (1999). Therefore, learning is not an isolated act. Learning occurs as individuals interact with each other, and these interactions are mediated by tools. The action that we are focusing on, are those that require cooperation and not independence.

For activity theory, the basic unit of analysis is the activity. Activities take place within an explicit situation within a precise context (Uden, 2007); they arise through social transactions (Bellamy, 1996). Likewise, Barab, Schatz and Scheckler (2004) proposed that activity theory emphasised the mutual nature of learning and doing, of tool use and community and of content and context. To clarify, as the learning community within each activity system in the study works, plays, and solves problems together, they develop a new set of values and notions. The activity system refers to a group of individuals who share a common objective. Values and notions that are developed in each activity system are significant within the specific context. For example, when Penny uses smiley and sad faces, Penny together with her learners negotiated a meaning for these visual tools within their context. These visual tools would not necessarily have the same meaning within another

activity system. This visual tool is discussed in further detail in Chapters six, seven and eight of this study.

I use Engeström's (1987, 1993, 1994, 2001), second generation activity theory model in my study. Whilst using this model I acknowledge that there were tensions between external activity systems and each activity system within each classroom in my study. The tensions that were most apparent relate to that of language, parental involvement, historical privilege and the history of poor resourcing. The impact of these external activity systems on the individual activity systems of each classroom are explored in Chapter five, six, seven and eight of this study.

The model that follows (Figure 9) emerged from this study. This model explains the overarching notion of the use of visuals as mediating tools, graphically. A detailed exploration of each activity system within each Master teacher's classroom is represented in Chapter five of this study.

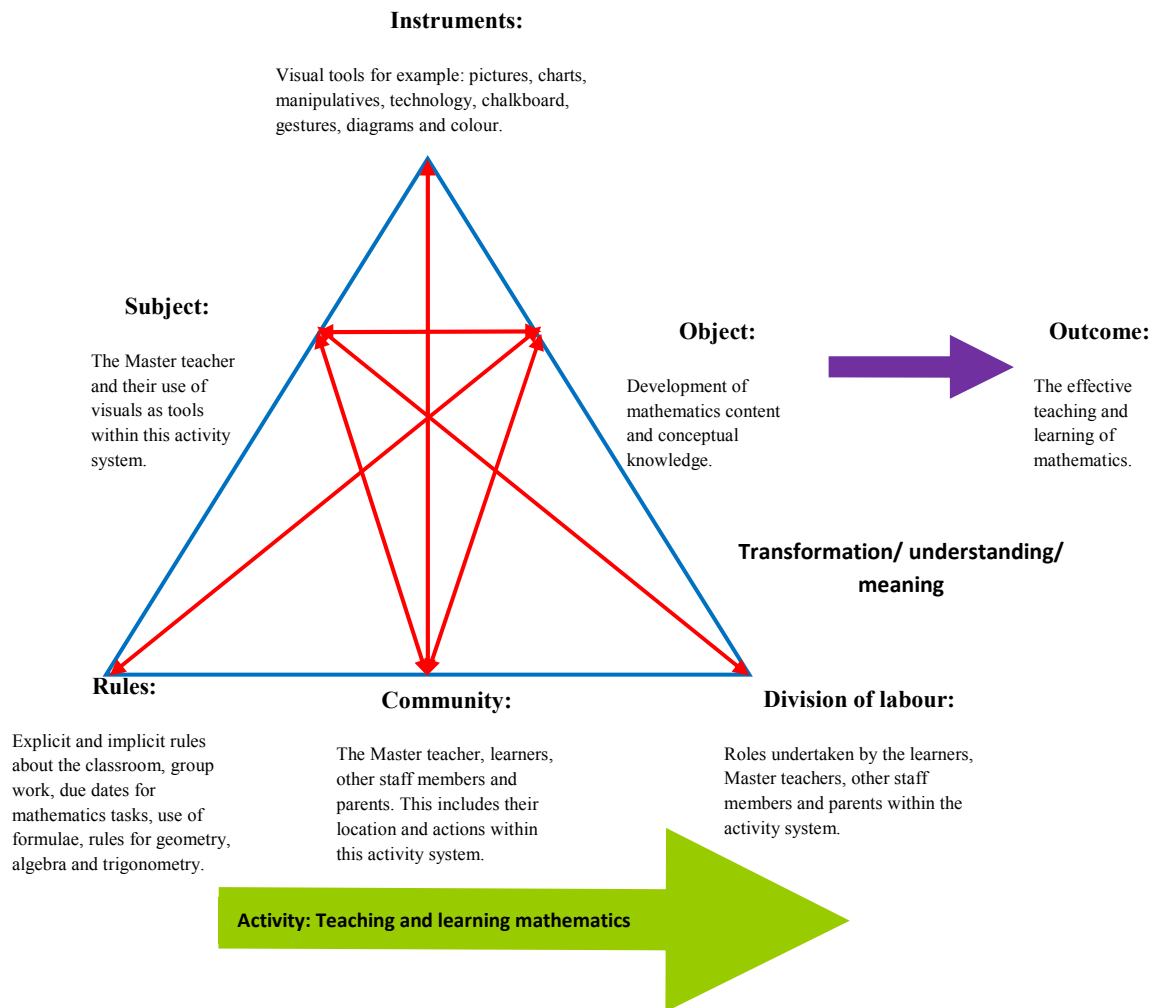


Figure 9: Conceptual model of the human activity system within this study
Adapted from Engeström (1987, p. 78)

To clarify the notions highlighted in Figure 9, we ought to relate the Engeström's conceptual model to the current study. The comprehension of the link between activity theory and the study is further justified by the operational definition of key concepts. These definitions are discussed in the following section.

3.5.1. Operational definition of key concepts used in this study

Activity in this study refers to the teaching and learning of mathematics. The activity becomes the starting point for all interpretation. All instruments used mediate each activity. Instruments or artefacts signal the tools, signs and various other types of representations. A ‘tool’ can be anything that is used in the transformation process. It may be material or intangible. Tools could include ‘tools’ for thinking, for example culture and language as well as material ‘tools’, for example a computer, smart board, a coloured marker and so on.

In this study, ‘tools’ referred to visuals used during the observed lessons. For example, the gestures, OHT¹⁵, diagrams, equations, pictures, graphs, books and facial movements were used during the observed lessons. Both software and hardware are included in the definition of ‘tools’ within this study. Vygotsky pointed out that one uses artefacts to attain achievements that would otherwise have remained out of reach. With respect to mental activities, they are supported and developed by means of signs (Maschietto & Bartolini Bussi, 2009).

The ‘subject’ generally refers to the individual whose point of view is adopted in the analysis. The ‘subject’ may also be a group of individuals. In the study, the ‘subject’ refers to each Master teacher. Learners learn through active engagement initiated by the teacher. With respect to obtaining information, this would include the Master teacher questionnaire, the classroom observations, the Master teacher interview, and the focus group interview with the learners.

The ‘object’ refers to the objective, task or problem area to which the activity is directed. In this study, the ‘object’ signifies the mathematics content or mathematics conceptual understanding. This refers to the attainment of new mathematics knowledge or improving existing mathematics knowledge. This would include data obtained from the focus group interviews with the learners and the classroom observation schedules. Whilst learning

¹⁵ OHT - Overhead Transparency

mathematics at school would be the object, at university level it becomes the tool. The purpose at university level would be attaining a university qualification.

In terms of the word ‘action’, when learners work independently, I refer to their ‘actions’ as ‘self action’. When learners collaborate with other members within the learning community, I refer to the ‘action’ as ‘interaction’. The ‘outcome’ indicates the decided or preferred consequence of the activity. ‘Outcomes’ may be both negative (unintended) and positive (intended). In the current study it refers to the comprehension of the content or concept at hand, for example knowledge of trigonometry ratios achieved. Fundamentally achieving success in mathematics would be the desired and intended outcome.

‘Rules’ refers to all constraints whether explicit or implicit that govern the each action. This can also include written and unwritten ‘rules’. In the study, this refers to all the common practices pertaining to the specific content areas, and includes all the mathematics rules and proofs. This also incorporated the lesson structure in terms of what was acceptable and what was not within the classroom and school. To determine the rules within each activity system in this study, I needed to take cognisance of the collaboration between the learners and each Master teacher

‘Community’ refers to all participants within the activity system who share the same object. In this study, this refers to all the learners, teachers, other staff members and parents within each of the sample schools. ‘Division of labour’ indicates the distribution of tasks with the activity system. This refers to the explicit or implicit structuring of the community with respect to the transformation process of the object into the outcome. This included the roles undertaken by each member of the community.

3.6. Conclusion

This chapter explored the theoretical framework within which this study was located. The chapter commenced with a comprehensive discussion of activity theory including the principles of activity theory; this discussion was followed by a discussion on the three generations of activity theory and activity theory in other research. Subsequently activity theory and the link with this study was interrogated. Activity theory offers a complete and context based method of discovery and therefore supported the qualitative and interpretive research methodology. What is evident in this study was that activities could not be comprehended or analysed outside the social milieu within which they takes place (Daniels & Cole, 2002; Jonassen & Rohrer-Murphy, 1999).

Furthermore, as part of the study a large amount of data was collected and analysed. So whilst I use activity theory as a theoretical framework for my study, at the same time I wish to demonstrate the usefulness of activity theory as an analytical lens. This will be done in more detail in Chapter five of this study. The next chapter discusses the research design and methodology for this study.

Chapter Four: Research Design and Methodology

“True education is not for a mere living, but for a fuller and meaningful life.”
Bhagavan Sri Sathya Sai Baba

4.1. Introduction

Chapter three provided an argument for the use of activity theory as the theoretical framework for this study. As discussed in the previous chapter, activity theory provides an ideal framework for discussing the practice of teaching. The theory is relevant because it describes an activity (teaching and learning) within the context of individualised classrooms or activity systems. With this in mind the research questions and research instruments were constructed.

This chapter reintroduces the critical research questions and explicates the limitations governing the research. It then goes on to discuss the methodology of the research by describing the paradigm within which the study was located, the research design adopted and the methods employed to conduct the study. Additionally the chapter concludes with a brief summary of the methodology.

4.2. The critical research questions

The study proposed to explore Master mathematics teachers’ use of visuals as tools within mathematics classrooms. It sought to identify the visuals used and projected to explore the reasons why Master teachers use visual tools in the mathematics classroom. I hoped that by exploring these reasons, the study would be able to make a positive impact on the teaching and learning of mathematics. The critical questions posed were:

1. What visuals do Master teachers use as tools in mathematics classrooms?
2. How do Master teachers use visuals as tools in mathematics classrooms?
3. Why do Master teachers use visuals as tools in mathematics classrooms?

4.3. The interpretive research paradigm

Traditional interpretive research focussed on the “subjective understanding or interpretation (Verstehen) of human action” (Babbie, Mouton, Vorster, & Prozesky, 2010, p. 30). Wilhelm Dilthey, a German scholar believed that the intention of human sciences was to understand rather than to explain. He referred to this approach of research as hermeneutics. Thus, hermeneutics was founded in humanities and the focus of this interpretation was a written text. Hermeneutics is the science of text interpretation and was originally derived from studying biblical texts (Silverman, 2005). Contemporary or modern hermeneutics methodology encompasses not just issues involving the written text, but everything within the interpretative process. This includes verbal and nonverbal forms of communication.

This approach was ideal for my study, since my focus was on written, verbal and nonverbal forms of communication. Furthermore, the interpretive research paradigm incorporates research approaches that emphasise the meaningful nature of individual’s participation in social and cultural life. Since I needed to analyse the meanings the Master teachers’ and learners’ within my study were making when confronted with visual tools, I felt that the interpretive research paradigm would be an appropriate paradigm within which to work. In this study, meanings were considered in each activity system with reference to the actions of individuals within each learning community.

Most of the data was collected within the classroom setting. The classroom involves various individuals engaged at different levels of interactions. When examining individual’s behaviour, discourse, actions or thoughts we focus on the lives of individuals over time. These individuals come from different social, cultural and political contexts. I was therefore mindful of the fact that the context often impacts on how teachers do their work, in addition to how their work is interpreted, as noted by Loef Franke, Kazemi and Battey (2007).

According to Cohen, Manion and Morrison (2000), in the interpretive research paradigm, individuals are studied with their many characteristics, behaviour, opinions, and attitudes. The interpretive paradigm assists the researcher in obtaining information by examining the world in diverse ways. Therefore, its advantage is in finding a meaningful observation of objects. This paradigm provided me with an opportunity to seek understanding and make sense of others' perspectives. Through this paradigm, I was able to gain a broader understanding of meanings, reasons, and actions. However this bias may result in the inability to analyse and interpret information objectively. Nevertheless, it is generally acknowledged that it is more difficult to be objective in human research than in scientific settings. Therefore, subjectivity is an integral aspect of such research.

An interpretive researcher studies a text, such as a conversation, to draw out elusive verbal communications in order to discover embedded meanings (Pillay, 2004). I needed to study the Master teacher's interview transcripts to interpret the meanings that they constructed in order to draw out the embedded implications. Through the interpretive paradigm, I was able to observe situations with different approaches to solving problems. A multiple number of possible solutions as well as interpretations emanated. The solutions and interpretations will be discussed further in Chapters six, seven and eight of this study.

4.3.1. Ontological and epistemological assumptions in the interpretive research paradigm

The interpretive research paradigm assumes that the world is inter-subjective. It encompasses the social construction of reality where the key focus is the search for patterns of meaning. The goal of this research paradigm in this study is to describe meanings, and recognise and comprehend the Master teachers' definitions of situations. In addition, since interpretive research seeks to understand the importance of social events, thus allowing one to obtain a deep understanding of human activities and occurrences, I examined how objective truths were produced. The meanings that were placed under the lens were based on situations occurring within the normal classroom context.

Interpretive inquiry aspires to characterise how individuals experience the world, the way in which they collaborate, and the settings in which these interactions occur. This research paradigm assumes that people employ interpretive schemes that necessitate understanding, and that the nature of the local context ought to be articulated. This paradigm locates subjects and objects within inter-subjective social fields that structure and constrain activity. Subjects are actively involved in the reproduction of these fields. The underlying assumption of the interpretive paradigm is that the whole needs to be examined in order to understand a phenomenon. To reiterate these thoughts within the framework of my study, I needed to examine the entire activity systems in order to comprehend the use of the visual tools in each lesson. The interpretive paradigm proposed that there are multiple realities of phenomena, and that these realities can differ across time and place. I needed to test this out with numerous observations within different contexts.

Consequently, the function of epistemology in an interpretive paradigm is to acquire knowledge by investigating the phenomena in many ways; additionally, the interpretive paradigm accentuates the notion that the world has different meanings. A single factor influences the change in social context. As a result, different researchers may reach different conclusions for the same observation.

4.3.2. Methodological assumptions in the interpretive research paradigm

Cohen, Manion and Morrison (2000) suggested that due to the demand to understand the subjective world of human experience, efforts are made to get inside the participant to understand from within. Similarly, in the interpretive paradigm the researcher embarks with the participant and begins to understand the participant's interpretation of the world. Theory ought to be based on data generated by the actual research; thus the research needs to occur first in order for the theory to allow the researcher to understand and interpret the participant's behaviour.

The interpretive paradigm assumes that research requires the description of persons and communities, through narrative articulation and interpretation. To assist the reader, I have compiled a detailed description of each Master teacher and each school in Chapter five of this study.

4.4. Research design and methodology

The research methodology describes the selected design and sampling method used in this study. It is evident from the critical questions that this study focuses on the use of visual tools. My reason for focusing on visual tools is that in today's world we are continuously bombarded with visual tools wherever we go, whether we are walking down the street or visiting a restaurant. Moreover, due to the increasing sophistication of computer graphics, visualisation is undergoing a rapid resurgence (O' Halloran, 2005). Due to technological globalisation, there are a vast number of software packages readily available to schools. If teachers are made aware of the value of using visual tools in the teaching of mathematics, given the opportunity, they would most probably resort to this type of teaching methodology.

To reiterate, visual tools in mathematics are specialised types of visual representations, in the form of graphs, signs, symbols and diagrams. The visual tools I focused on in this study were diagrams, lines drawn on the chalkboard, squiggles, shapes and symbols on the board, pictures, transparencies, the use of colour, mathematics manipulatives, gestures, body movements and facial movements. Essentially, I focused on any visual tool that could be considered as a trigger or catalyst in prompting the understanding of mathematical concepts.

This research represents a qualitative interpretation of Master teachers' use of visual tools within mathematics classrooms. My intent is to focus on the Master teacher and to understand, interpret and explore the Master teacher's use of visual tools in the mathematics classroom. The Master teachers in this research study were probed with

respect to their actions within the classroom as well as their responses on the questionnaire, to assist in gaining an in-depth understanding about their use of visual tools.

In essence, the methodology encompassed the following structure.

Structural components of interpretation

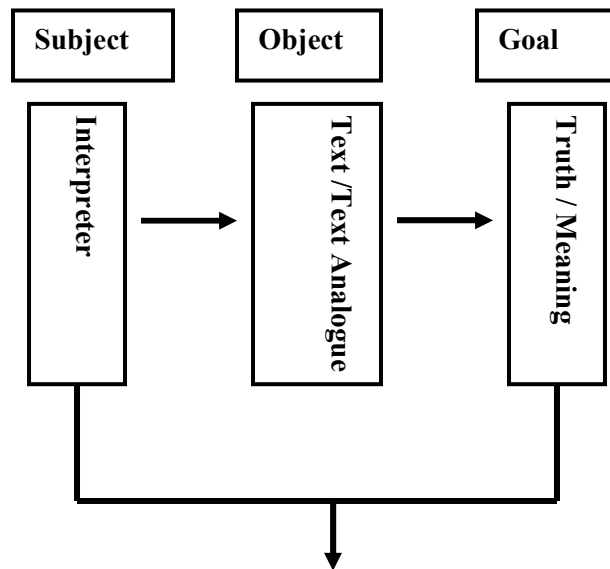


Figure 10: Systems of interpretation
Adapted from Demeterio III (2001, p. 1)

Demeterio's system of interpretation has been reworked to exhibit the nuances of this study. The figure that follows (Figure 11) represents the systems of interpretation within this study.

Structural components of interpretation

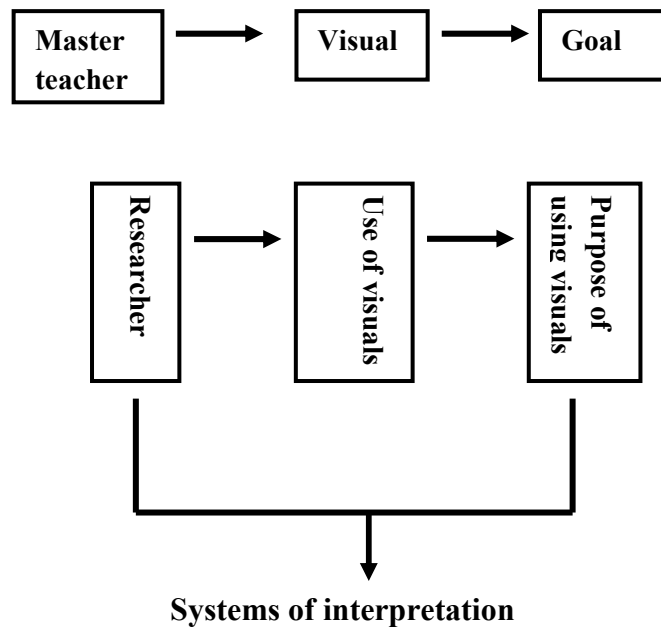


Figure 11: Reworded structural components of systems of interpretation based on this study

Based on an interpretation of the above flow diagram, I concur with Alsina (2008) who purported that the interpretive methodology seeks to discover the meanings of social actions, their practices and discourses. In this study, it is hoped that the rationale behind each Master teacher’s use of visual tools in the mathematics classroom will be comprehended. It is considered that using an interpretive approach would assist in suggesting “alternate moral points” from which the problem can be interpreted and assessed (Denzin, 2001, p. 2).

4.5. Research methods

A description of, and justification for, the research method used is discussed. Gee (1999) claimed that a research method is made up of various tools of inquiry and strategies for applying them. Along similar lines Denzin and Lincoln (2005) maintained that a solitary method is not proficient enough to unpack the subtle nuances and disparities in ongoing

social interactions. Thus since within this study, the focus was on situated meanings, cultural models and activity based learning it was necessary to make accurate descriptions and explanations of all interactions. To assist in ensuring that all experiences and interactions that were studied in this study were interrogated to obtain the best possible explanation, multiple tools of inquiry were used. These tools incorporated the use of interview schedules, observations schedules and a questionnaire. These tools of inquiry were designed to describe and explain what was taken to be of importance in this study. Example of these tools may be found in Appendix B.

Although ‘method’ triggers in our minds a step by step guide or set of rules, it also entails a set of thinking devices with which one may investigate questions with due regard to how others have investigated such questions (Gee, 1999). In addition, following what Titscher, Meyer, Wodak and Vetter (2000, p. 31) proposed, those who wish to carry out empirical research must pose four questions to themselves:

1. What research question am I trying to answer?
2. What analysis will provide a useful response to the question?
3. What data do I need and from whom?
4. What are the practical steps to obtain and record this data?

I posed most of these questions to myself in order to clarify and justify why I conducted the study in the way that I chose to conduct it. I used the notions of interpretivist methods to conduct this study. The study involved the methods of lesson observations, video recordings, a questionnaire and interviews as a means of acquiring information. For practicality purposes, I also used interviews with learners to assist in deriving more information about the use of visual tools in the mathematics classrooms.

4.5.1. Data Collection

To reiterate this study was a qualitative study. When one conducts qualitative research, the research encompasses the use of a multiplicity of empirical methods (Denzin & Lincoln, 2005). These methods were the source of the data that was collected. The data was collected based on the following plan:

Critical Research Questions	Participant/s under focus	Method	Timeframe
1. What visuals do Master teachers use as tools in mathematics classrooms?	Master teacher	Classroom observations (3/4). Field notes. Video recordings of mathematics lessons.	Mar - Jun 09 Mar - Jun 09 Mar - Jun 09
2. How do Master teachers use visuals as tools in mathematics classrooms?	Master teacher	Classroom observations (3/4). Field Notes. Video recordings of mathematics lessons.	Apr - Jun 09 Apr - Jun 09 Apr - Jun 09
3. Why do Master teachers use visuals as tools in mathematics classrooms?	Master teacher Learners	Perusal of video recordings by Master teachers. Interviews with Master mathematics teachers (1). Focus group interviews with learners (1).	Jun - Sept 09

Table 7: Data collection plan

4.5.2. Gaining access and inviting participation

The purpose of the study was to explore Master mathematics teachers' use of visuals as tools. For this study I needed to conduct research at schools. Firstly, I needed to identify the schools that I wanted to work with and then I needed to gain access into the selected schools. To gain access to schools in KZN, permission needs to be obtained from the research office of the Department of Education.

To select the teachers for the study, I employed a purposive sampling technique. I needed my sample group of teachers to meet specific requirements (Cohen, et al., 2000). I wanted

to work with experienced Master teachers. After speaking to fellow researchers and colleagues, I learned that there were two cohorts of Master mathematics teachers.

The first cohort of teachers for this study was selected from all schools in KZN. The first cohort proved problematic as the criteria for this selection was not transparent or readily available from the Department of Education. People in the field (other teachers, managers of schools, department officials and union members) did not value this selection by the Department of Education as a true reflection of Master mathematics teachers in all respects. This selection led to many deliberations by teachers, school based managers, departmental officials and unions. The selection of a second cohort for this study was initiated because of these challenges.

The second cohort of Master teachers for this study was selected from Dinaledi schools. This cohort comprised of teachers that I had worked with previously. These teachers were kept abreast of changes in the curriculum; they were engaged in professional development workshops focussing on pedagogy and content on a regular basis (at least four times a year). The Dinaledi teachers were given a platform to engage with other educators on a regular basis with respect to their challenges and victories in the classroom. Teachers from the Dinaledi schools were given an opportunity to try out new techniques and unconventional teaching strategies in the classrooms. They were exposed to and assisted with professional development by subject specialists on a regular basis.

Once I decided on which cohort to focus my research, I selected 20 schools out of the 88 Dinaledi schools. These schools were selected based on geographic position as well as convenience for me the researcher. I contacted the Research Officer at the Department of Education via email and posted my proposal for perusal. Once permission was granted by the Research Office, I contacted each principal telephonically. Out of the twenty schools, eleven responded positively. I arranged to work with five schools for the pilot study.

4.5.3. The pilot study

A pilot study has several purposes, most essentially to increase the validity and reliability of my research instruments and procedures. The pilot study was conducted at five secondary schools with a similar background to the schools that were to be used in the main study. The participants for the pilot study were five Master mathematics teachers of different cultures and races. The sample was reasonably balanced with regard to gender. The data collected during the pilot study helped me to think through my concerns about the study. A better focus of the study was developed and the best possible data collecting procedures were determined.

The existing instruments were further refined; this assisted me in maintaining reliability and validity of my research instruments. After improving my research instruments, based on the feedback from my pilot study, I conducted the main study at the remaining six schools. The list of schools and research participants has been provided in Chapter one. I realised early on that when conducting research a myriad of factors might affect the success of the study. These factors could include gender, race, rejection and withdrawal.

4.5.4. Gender

Whilst gender was not being researched in this study, I decided to have an equal number of male and female teachers. I chose to select an equal number because I did not want any lurking variables like the gender of the Master teacher affecting the study. Thus, the six schools I chose for the main study were selected based on the gender of the Master mathematics teachers. I worked with three male and three female Master teachers.

4.5.5. Race

We claim that we are in a democratic South Africa and equality is prevalent with respect to race, colour and creed. To test this claim out in the real world, I selected Master teachers

that were of different race groups teaching in classrooms that were diverse. The schools were diverse with respect to the teachers' and learners' race and culture.

4.5.6. Informed consent

Conducting research is a sensitive issue, I had to take into consideration the following factors; informed consent, rights to withdraw, confidentiality, methodological rigor and fairness. Before I proceeded with both my pilot and main study, I provided each teacher and learner that participated in the study an introductory letter. This letter discussed and defined informed consent, the right to withdraw and confidentiality. The letter provided each participant with the reasons and purpose of my study. Each participant was required to provide their written consent. In the case of the learners, their parents/ legal guardians were required to provide written consent. I also explained the procedures that would be followed during the research process, I provided timeframes and relevant contact details of personnel at the University as well as the DoE. A copy of this letter may be found in Appendix A.

4.6. Stages of data collection

The structure of classroom discourse is very similar in different parts of the world; this similarity indicates that schooling is both dependent on society and autonomous from society. Schooling is dependent on society in that as a transmitter of culture it obtains its specific cultural content from the society of which it is a part (Meehan, 1985).

People tend to do and say things in some circumstances that they would not do in other circumstances, for this reason the use of multiple lenses on the same phenomenon is essential, thus observations, questionnaires and interviews were all used in order to challenge, confirm and expand information gathered from each other (Schoenfeld, 2007). Triangulation or the use of multiple methods may overcome the limitations that flow from the use of only one method. Triangulation is considered to be one of the best ways to enhance validity and reliability in qualitative research (Babbie, et al., 2010; Miles &

Huberman, 1994). The process for collecting data involved three stages, six methods, and five research instruments.

4.6.1. Stage 1: The Master teacher questionnaire

Hiebert and Grouws (2007) proposed that having good teachers made a difference, thus in the first stage a sample population was chosen from practicing Master FET¹⁶ mathematics teachers as identified by the DoE. One Master mathematics teacher from each of the six sample Dinaledi schools was selected; these teachers were asked to complete a Master teacher questionnaire (See Appendix B).

Whilst questionnaires are generally used with a large sample size, I felt that it would be easier for the teachers in my study to answer the questionnaire in the comfort of their own homes. I considered that they would answer the questions with the least amount of disruptions if I gave them more time to think about each question at their leisure. I also did not want the teachers to feel inconvenienced or pressured for a response. I allowed each Master teacher to take the questionnaire home and in order for them to give considerable thought when answering the questions (Wragg, 1999), I arranged to pick up the questionnaires one week later.

The questionnaire comprised of three sections that included the school profile, the school infrastructure, and the Master teacher profile. Through discussions with each Master teacher, I found that each Master teacher spent an average of about half an hour to answer the Master teacher questionnaire.

¹⁶ Formal education in South Africa is categorised according to three levels:

- General Education and Training (GET) –Grade 1-3 make up the foundation phase, grades 4 – 6 make up the intermediate- phase and grades 7 – 9 make up the senior phase.
- Further Education and Training Band (FET) - encompasses grades 10, 11 and 12.
- Higher Education and Training Band – Tertiary Education includes education for undergraduate degrees, postgraduate degrees, diplomas and certificates.

4.6.1.1. School profile

Schools play an important role in any society, as emphasised by Donald, Lazarus and Lolwana (2007) and since the process of learning occurs within a particular social milieu, I needed to know as much as possible about the social context within which each school was located. I needed to know more about the school with respect to enrolment, location and demographics.

I felt that I needed information regarding the teacher-learner ratio, the ratio of boys and girls in the secondary phase of the school, and the textbook being used by the mathematics learners. Although this information forms part of the Master teacher profile, by locating these questions in this part of the questionnaire allowed me the opportunity to make comparisons across the schools easily.

Donald et al. (2007) claimed that access to teaching materials such as textbooks have an influence on the process of teaching and learning, I felt that there was a need for this information. Stein, Remillard and Smith (2007) concurred with these sentiments and added that the teaching context is an important factor in a teachers' use of curriculum material. Because effective teachers recognise the particular strengths and weaknesses of text books and material they are using, many effective teachers have an array of materials they use when teaching mathematics (Sowder, 2007). This was evident in this study in that the Master teachers did not only use one textbook but rather used a variety of resources.

Furthermore, much like the mass media, educational discourse derives its power from its enormous scope and, unlike most other types of texts; textbooks are obligatory reading for many people. Moreover Van Dijk (2008) claimed that many studies have shown that most textbooks reproduce a nationalistic, ethnocentric or racist view of the world. The choice of text books at the different schools also assisted me in finding out more about the values the Master teacher and their schools embraced. This section of the Master teacher

questionnaire provided me with a means of doing comparisons with respect to the use of textbooks at the sample schools.

Since the social context plays an important role in school development (Donald, et al., 2007), in order to move forward with my study I needed to know and understand the context within which the teaching and learning was occurring. Once I was aware of the different contexts, I could now begin to theorise with respect to similarities and differences that I found at the schools.

Based on my previous experience in doing school based research, I have found that gatekeepers are crucial in terms of gaining access. I felt that it was important for my study to have good communication between all relevant parties; so I included a section in the questionnaire that requested contact details of school officials as well as officials from the DoE. These details were used to communicate with all necessary gatekeepers when planning visits to each school.

4.6.1.2. School infrastructure

Many factors inside and outside of school influence what and how well learners learn (Hiebert & Grouws, 2007). These factors embrace the social milieu within which the school is located, the learning environment as well as the learning community. While some South African schools have excellent infrastructure, others lack essential services such as water and sanitation (Gibberd, 2007). Regardless of the differing infrastructure, learners are required to write the identical Grade 12 examinations and compete for the same limited positions at tertiary institutions across South Africa. Inequalities in resource allocation, differences in infrastructures among rural schools and urban schools call for a far more radical interventionist strategy in education (Vambe, 2005).

"School infrastructure" refers to the site, buildings, furniture and equipment that contribute to a learning environment. It is widely recognised that learner performance is strongly

affected by the design and suitability of those facilities in which it takes place (DEECD, 2009, p. 1). The structures (for example school committees, formal staff and learner groupings) of a school reflect its subsystems. The procedures (for example the schools rules, regulations and other various methods of communication) show how they relate to one another (Donald, et al., 2007). Research suggests that learners prefer infrastructure that can be controlled easily, is safe and that invite exploring (Dutta Roy, 2008).

The school infrastructure section in the Master teacher questionnaire assisted me in gaining an idea of the facilities that were made available to the learners at each school. It also helped me in finding out about the safety of learners at school. A safe school has a positive climate where people are trusted, respected, and involved, and the physical school environment affects learner behaviour (Bucher & Manning, 2003). I did not want any peripheral issues like the lack of safety at a school to influence my data; I wanted the schools in my study to be similar to each other in most respects.

4.6.1.3. The Master teacher profile

Teaching plays a major role in shaping learning opportunities for learners (Hiebert & Grouws, 2007). There are many views of what criteria teachers ought to satisfy in order to be labelled a good teacher. Some of the characteristics of being a good teacher were explored in Chapter two. What constitutes good teaching is controversial and will always be controversial (Loef Franke, et al., 2007). The Master teacher profile, part of the questionnaire, was useful in that I had knowledge of each teacher with respect to his or her background and level of qualification.

This section also provided me with useful information about each Master teacher's professional experience. Since being a scholar and lifelong learner is an essential part of being a teacher (Donald, et al., 2007), I was also able to ascertain the teachers' interest in their own professional development. I acquired information regarding the professional

organisations they belonged to and their level of interest in professional development. In this section, teachers also shared their ideas and thoughts about good practice.

4.6.2. Stage 2: Classroom observations

Although observation is an interesting hobby it is not always a noteworthy way to achieve understanding. Watkins (2005) maintained that observation of classrooms are challenging since their complexity makes distinguishing the most meaningful aspects problematic. Disregarding all the complications, I felt that classroom observation would have many valid and important educational purposes. In this study, it allowed me a ringside seat as I observed first hand a part of the teachers' practice. These observations in a teacher's class afforded me vast opportunities to interrogate skills related to the use of visual techniques.

Teaching consists of classroom interactions between teachers and learners. The interactions focus around initiating learners' attainment of goals (Hiebert & Grouws, 2007). I was interested in how learners constructed meaning of mathematics content and concepts. I approached these observations as a means of looking at the specific visual methodologies teachers used, in order to assist the learners to construct mathematical meaning. These observations provided the lens that noted all visual techniques used. I made a note of both the implicit and explicit techniques. In some instances, teachers had no option but to use a visual tool, for example, when drawing graphs. However, in other instances, teachers' hand gestures and facial movements were not planned. The gestures and movements occurred in the midst of teaching, and were intuitively used by the teacher. Although the teacher automatically used these gestures and movements, they came at critical moments in the lesson. These gestures assisted in triggering responses to key concepts and ideas in the mathematics classroom. They appeared to be significantly useful for the untrained learner.

As an essential part of my research, classroom observations followed soon after the Master teacher questionnaire was completed. I set up an observation timetable with each Master teacher within my sample. This timetable was developed in consultation with each Master

teacher. The timetable afforded each teacher adequate time to prepare if they needed to, I did not want the teachers to feel pressurised by the study in any way.

When someone new comes into a classroom, their presence may influence what happens within the classroom (Wragg, 1999). Keeping this in mind, I took full cognisance of my 'outsider' presence in the classroom. I tried to be discreet (Loef Franke, et al., 2007). Learners became accustomed to me, and I blended into the classroom routine by visiting the class a few times before I began the actual lesson observations. Similarly, Babbie et al. (2010) maintained that the researcher ought to get as close as possible to participants in order to gain credibility and trustworthiness. All lessons were videotaped and an observation schedule completed. Firstly, the observation schedule included a checklist of common visual tools and the frequency of use within each lesson. A space to add other visual tools that were used by each Master teacher was also included. Secondly, part of the observation schedule afforded the researcher the opportunity to describe how each visual tool was used within the lesson. Finally, a space was provided where additional researcher notes, reflections and comments could be added. A copy of the observation schedule may be found in Appendix B.

Trying to capture multiple features through pure observation was difficult. I used video recordings because I realised that a teaching method consists of multiple features that interact with one another in countless ways (Hiebert & Grouws, 2007). In order to take cognisance of each teacher's body movements, gestures, actions, interactions, symbols, tools, technologies, values, attitudes, beliefs and emotions I recorded each observed lesson. By viewing the video recording of these observations, I was able to capture even the subtle nuances of the classroom environment such as the posture of the teacher, gestures, facial expressions and eye contact. All of these formed a part of the data.

The observation schedule was extensive with critical details noted in a field diary. I used iconic symbols to assist in noting where learners sat or stood in relationship with others

(Wragg, 1999). Where possible I observed one lesson in geometry, one lesson in algebra and one lesson in trigonometry. A total of eighteen lessons were observed.

I mapped a socio-gram for each lesson observation in order to represent communication and interactions within the classroom. The socio-grams assisted in multiple ways. I used them in the data analysis stage as well as during the interviews with the teachers. The socio-gram also assisted me with selecting learners for the focus group interviews. I had a good idea of those learners who were active and non-active within the classroom environment. The focus groups were set up to include participants from both groups.

Each socio-gram represented the teacher and learner communication as they interacted within the classroom. ‘Communication’ is understood in many ways; in this study, I define communication as the process through which individuals as well as institutions exchange information within a specific social milieu. Individuals are required to live and work with others who differ from themselves within the social milieu (Tracy, 2001). Figure 12 represents an example of a socio-gram based on one of my observations in the field.

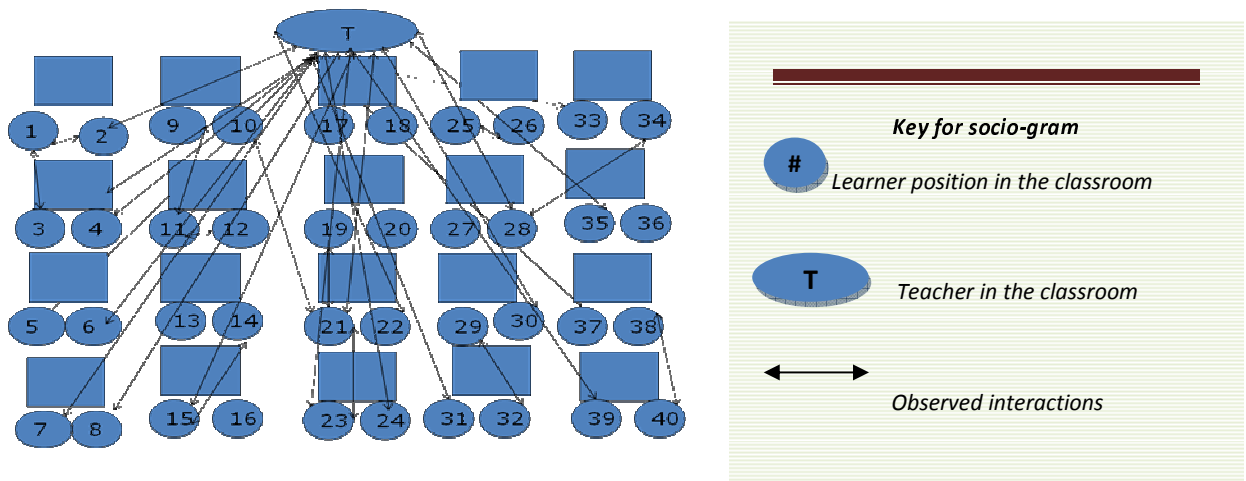


Figure 12: Socio-gram of observation 2 at Orchid Secondary School

4.6.3. Stage 3: The interviews

Interviewing has become a sensitive and powerful method for investigating individual's private and public lives, as claimed by Kvale (2006). In qualitative interviews social scientists investigate varieties of human experience. They attempt to understand the world from the individual's point of view. It is for this reason that interviews have been regarded as a beneficial method for social research.

Similarly, it makes sense that if you want to find out something you ought to ask people about their experience of the phenomenon (Seale, 2004). Whilst we use an interview to obtain this information, there are a number of different types of interviews. The interview type depends on what the purpose of the interview is. Hitchcock and Hughes (1989, p. 79) lists the following types: structured interview, survey interview, counselling interview, diary interview, life history interview, ethnographic interview, informal/unstructured interview, and conversations. Similarly, Cohen, Manion and Morrison (2000) group and discuss four main kinds of interviews, the structured interviews, the unstructured interview, the non-directive interview and the focused interview. I used semi-structured interviews in this study. The reason for the use of semi-structured interviews was that each activity system that was under interrogation was individualised. Thus, I constructed the instrument so that it was specific to the activity systems 'under the microscope'. I had a few basic questions that I asked each Master teacher to start off the interview and the rest of the interview was based on how the Master teacher responded based on his/her activities with the classroom. A copy of the semi-structured interview may be found in Appendix B. A similar strategy was employed for the focus group interviews. The interviews are discussed in more detail in the sections that follow.

Interviews are an important part of research as they provide the opportunity for the researcher to probe and delve deeper, to solve problems and to gather data which could not have been obtained in other ways (Cunningham, 1993). My interviews took a specific form, in the first part I asked the teacher and learner questions about their lives, homes,

communities, interest and schools. This is called the ‘life part’ of the interview (Gee, 1999, p. 119). In the second part, the teachers and learners were asked more ‘academic like’ explanations and opinions about clips from the video tapes. I call this the ‘society part’ of the interviews (ibid. 119).

These semi-structured interviews offered a versatile way of collecting data (Welman & Kruger, 2001) as they raised key questions and allowed me to have some natural conversation with the teachers and learners about critical incidents (Wragg, 1999). These conversations assisted me in establishing a good rapport with both the teachers and learners. Ordinary conversation is the most basic form of talk and the main way in which people come together, exchange information, and maintain social relations (Paltridge, 2000). After observing the Master teachers in the classroom, I conducted semi-structured interviews with each Master teacher. The purpose of the interview was to seek clarification, probe further and to explore how and why the different Master teachers used visual tools in their mathematics classrooms.

4.6.3.1. The Master teacher interviews

The Master teacher interviews were flexible enough to range over issues that the teachers brought up. I made the interview the focus of a meeting in which I also tried to develop an understanding about where they lived, how they worked, the relationships with others and their interests.

These conversations were also applied to place the Master teachers at ease. It was imperative that they understood that the focus of the interview was not to scrutinise or inspect their mathematical knowledge or teaching strategies. They needed to understand that the focus of the interview was to understand their practice. The interviews were based on the Master teacher questionnaire as well as the lesson observations.

The interview itself was comprised of five aspects. Each aspect was noted as a section. Section A focused on the value the teachers placed on the use of visual tools. Section B focused on each Master teacher's thoughts on the actual visual tools used whilst teaching. Section C focused on the support the teachers needed for using visual tools. Section D focused on the actual lesson observations, and lastly Section E focused on each Master teacher's philosophy of using visual tools. To peruse the interview schedule please refer to Appendix B.

Thus the main themes covered during the interviews embraced the following aspects:

- The Master teacher profile.
- The value of using visual tools when teaching.
- The use of visual tools within different contexts.
- Reasons for using visual tools at specific points during the lesson.
- Support needed or necessary to assist teachers when using visual tools.

I presented a selection of video clips of their observed lessons to each Master teacher. I used these clippings as a means of probing responses and generating discussion. Since the Master teachers varied in their ability to articulate their thoughts and ideas, I used various questioning techniques to probe responses. I wanted to deepen their responses in order to increase the richness of the data. I also used the video clips as a technique to remind the Master teacher of visual tools that had been used in the class whilst teaching.

Follow-up questions were asked to clarify the Master teachers' responses. Throughout the interviews I tried to elucidate and broaden the meanings of each teacher's statements to avoid misinterpretations and misunderstandings. Whilst, the time for each interview varied, on average each interview lasted about an hour. Each interview was recorded on tape and subsequently transcribed. After analysing the Master teacher interviews, the field notes and video tapes, I conducted a semi-structured focus group interview with the learners. I wanted to know how the learners felt about the use of visuals as tools in the mathematics

classroom. These focus group interviews were used as a means of triangulating the data collected. All sections of the interviews, the classroom observations and questionnaire assisted in answering the three critical research questions of this study.

4.6.3.2. The focus group interview with learners

Focus group interviews are also referred to as in-depth interviews (Welman, Kruger, & Mitchell, 2005). Many researchers use focus group interviews because they are less cumbersome than other research methods (Vaughn, Schumm, & Sinagub, 1996) and there is a quick turnaround from implementation to findings. Kreuger (1988, p. 18) defined a focus group as a "carefully planned discussion designed to obtain perceptions in a defined area of interest within a permissive, non-threatening environment". Focus groups are group discussions organised to explore a specific set of issues, the group is focussed in the sense that it involves a collective activity such as viewing a video clip (Kitzinger, 2004).

Focus group interviews offer researchers the opportunity to conduct qualitative research and gather qualitative data. Qualitative research focuses on words and observations to convey reality. The fundamental component in this type of research is the engagement of individuals where their revelations are encouraged in a nurturing environment. Qualitative research taps into human tendencies where attitudes and perceptions are formulated through interaction with other individuals. In most classrooms there is continuous interaction between the learners and teachers. The teacher usually controls communicative events and dispenses speaking turns. Since teachers have control over the educational discourse, learners talk in classrooms only when talked to and invited to speak (Van Dijk, 2008). Additionally, in general learners often talk when they are not meant to. I wanted to create an environment where the learners in the study articulated their thoughts freely and felt unconstrained whilst talking to me. For this purpose I chose to use focus group interviews. An advantage of focus group interviews is their 'loosening effect'. In a calm and inviting atmosphere where participants acknowledge that their opinions and

experiences are valued, participants are more likely to articulate their opinions and perceptions freely.

When arranging a suitable venue my main goal was to promote comfort of the participants, hence I chose a venue on the school property for the convenience of my participants. I used a round table with comfortable chairs placed close together to create a relaxed atmosphere, this ensured that everyone participated. Refreshments, snacks and beverages are integral in establishing an atmosphere for interaction. I informed my participants in advance that light snacks would be served before the focus group interview. In this environment, participants hear and interact with each other and the interviewer, which affords different data than if individuals were interviewed separately. During a group discussion, individuals may change their opinions due to the influence of other comments. Alternatively, views may be held with certainty.

Similarly, Beck, Trombetta and Share (1986, p. 73) described the focus group as an “informal discussion among selected individuals about specific topics relevant to the situation at hand.” Glesne and Peshkin (1992) suggested that interviewing more than one person at a time is often useful; some young people need company to be encouraged to talk, and some topics are better discussed by a small group of people who know each other. When we honour a plethora of stories and interpretations and when we attempt through discussion to find a place within our stories for some of these perspectives, we can rebuild and reinvent commonalities (Brookfield & Preskill, 1999). When using focus group interviews the items that are most relevant from the participant’s perspective are less likely to be overlooked. This type of interview assists in explaining the relationship between a stimulus and an effect. In my research the stimulus was the Master teacher’s use of visual tools and the effect was the impact these visual tools had on the learners’ understanding of mathematics.

Focus groups are structured small groupings of individuals; they should have no fewer than 6 participants in order to get good group discussion and no more than 10 participants so that less assertive participants will still contribute to the conversation. My focus groups ranged from between 6 learners to 10 learners. My participants were selected through purposive sampling. Purposive sampling is a procedure by which researchers select participants based on “predetermined criteria” about the extent to which the selected subjects could contribute to the research study (Vaughn, et al., 1996, p. 58). As mentioned earlier, I developed and used a screening process based on the classroom observations. I used each socio-gram as a scaffold in the selection process. Another criterion that I used in the selection process was gender. The composition of the group by gender was of interest to me because the interaction styles of males and females are influenced by whether same sex or cross sex grouping occurs.

I used these interviews as a means of triangulating the data collected by allowing the learners to focus on specific clips of each observed lesson. Miles and Huberman (1994) describe this type of triangulation as triangulation by data source. In most cases these interviews were a means of verifying what I had observed as well verifying to a certain extent what teachers were saying. This type of interview resembled a chat, during which the respondents sometimes forgot that they were being interviewed. Most of the questions asked flowed from the classroom observations. With good questioning techniques, I was able to facilitate the learners’ accounts in order to obtain quality data from them. In line with Patton (1987) who proposed that if interviewers ask double barreled or ambiguous questions they are prone to have problems with their questioning techniques, I asked clear questions. I used words that made sense to the learner; my questions were short and devoid of jargon. This eliminated any unnecessary burden of interpretation for the learners.

Interpretation takes many forms and applies to many different objects (Lamarque, 2000). Interpretation is often regarded as a matter of opinion (Gracia, 2000), and the clarification of meaning. When I asked a question, I ensured that the learners interpreted the question in

the manner in which it was intended. I confirmed whether the question made sense to them. I verified that they understood what I was asking. Essentially to interpret something is to make sense of something (Lamarque, 2000; Mantzavinos, 2005), and understanding is the process of interpreting and comprehending the meaning that is expressed (Denzin, 2001). Understanding in this sense implies calling up both knowledge which is presupposed by language and knowledge that is implied in discourse (Lederer, 2003). To facilitate understanding, I took great care in sequencing my questions moving from general to specific, from broad to narrow. I allowed the learners to explore as they liked, but I kept a rough checklist of ideas or areas I required to be explored. The use of the checklist assisted in maintaining control of the interviews.

Since understanding is also related to lived experiences (Mantzavinos, 2005), I used semi-structured interviews because I wanted to gather information about the learners' experiences. More specifically I wanted to know their views on the Master teacher's use of visual tools in the mathematics classroom. These interviews were flexible enough to allow learners to provide other input as they saw fit. The interviews encouraged understanding and interaction among participants during discussions (Vaughn, et al., 1996). Learners were continuously reminded about the purpose of the study and were reassured that they did not have to answer any questions if they were uncomfortable about answering them. They were also reminded about their right to withdraw from the study at any point.

I established good rapport with my interviewees by respecting their opinions, supporting their feelings, and recognising their responses. This was supported by my tone of voice and facial expressions. In addition, Kvale (1996) suggested that good contact is established by listening, exhibiting interest, understanding, and respecting the interviewees' opinions. Balance is one of the keys to a good discussion; when one or two people dominate a discussion, the benefits for the entire group are diminished (Brookfield & Preskill, 1999). I allowed the interviewees to finish what they are saying, and allowed them to proceed at

their own rate of thinking and speaking. I ensured that everyone was heard and no one dominated the discussions.

To encourage success, I did this by probing and redirecting responses and questions. The discussions during the focus group interviews focused on the visual tools used during the mathematics lessons and how learners responded to them. I wanted to know if the visual tools did indeed have the desired effect during the lessons. Learners were shown selected clips of the video recordings; these were used to assist learners in recapping what had occurred during the observation. These clips also assisted in the probing of responses and generation of discussion.

The learners were guided into the questioning; I started with general questions first. I initiated and encouraged interactive discussions during the interviews. Discussions assist learners in exploring a diversity of perspectives and develop the capacity for a transparent communication of ideas (Brookfield & Preskill, 1999). Learners voiced their views and compared their views with each other. Learners were inhibited at first but after a few minutes of chatting and discussing their views and opinions they articulated their thoughts freely. Very often they took over the discussions and mentioned other instances (including lessons that were not video recorded for the study) of visual tool use that were beneficial.

As the learners started to share ideas, I encouraged and ensured that each learner had a chance to be heard. This encouraged respectful and attentive listening and allowed the learners to become connected to the topic (Brookfield & Preskill, 1999). Notes and member checks during the interview were recorded. All verbal and nonverbal responses of participants to key issues were also recorded. If participants made a statement with an emotional message (for example: sarcasm, enthusiasm, anger or frustration), I noted this down during the interview, I intended to use this information to assist me whilst analysing the data (Vaughn, et al., 1996).

Before the end of each focus group interview, I summarised the main themes and ideas that emerged from the group on a flip chart. This allowed the focus group the opportunity to support or refute what had been interpreted. This also allowed participants to express their final thoughts on the focus group in general. Each focus group interview lasted between one to one and a half hours. All interviews were subsequently turned into a textual form by transcription (Lee & Fielding, 2004).

4.7. Conclusion

This chapter thus serves as an overview of how this study was conducted with respect to methods and procedures. The chapter started with a list of the critical research questions and a discussion of the interpretive research paradigm ensued. This discussion was followed by a discussion of the research design, methodology and methods adopted. As can be comprehended the research methodology serves as a guideline and point of reference for the study with respect to data collection and procedures followed. The data collection process together with the research instruments were discussed at length in this chapter. Once all the data was collected and interviews transcribed, the data analysis process commenced. The analysis of the data is discussed in detail in the chapters that follow.

Chapter Five: Introducing each Master teacher

“You, who deal with children and their upbringing and upliftment, have to be aware of this preciousness and of the need to express it in action.”

Bhagavan Sri Sathya Sai Baba

5.1. Introduction

In the previous chapter, the research design and methodology was discussed. A detailed description of each tool of inquiry and data sources were provided. Additionally, the data analysis process was revealed. In this chapter, the data that was collected will be disclosed by means of master teachers’ stories. Furthermore, this chapter aims to explore and respond to the first critical question of this study.

This chapter is organised in the following manner. I start with the story of each Master teacher. Information for this section was obtained from the Master teacher questionnaire, the Master teacher lesson observations and the Master teacher interview. I then progress to the research questions and the discussion surrounding the data collected in order to answer these questions.

Each classroom, with its many complex relationships and associations, was considered to be an individual activity system. All findings from observations and video recording for each Master teacher were collated and a single activity system emerged. The information for each activity system was supplemented by data collected via the Master teacher questionnaire, Master teacher interview and focus group interviews with the learners.

In this chapter and throughout the thesis I have italicised where I have used the Master teachers’ or learners’ actual words. The rest of the text is my paraphrasing of their words and a filling out and connecting of data in other instances. This was done to create an articulate discussion around fundamental issues.

5.2. Presenting the Master teacher's stories

5.2.1. Alan

Alan¹⁷ teaches at Orchid Secondary School¹⁷ in a predominantly Indian suburb. Due to the political and social changes that have occurred over more than a decade of democracy, the school attracts learners from all races in the neighbouring areas. This is due to the good Grade 12 pass rates in mathematics (83% in 2008 and 80% in 2009), reasonable school fees and an average allocation of resources. The learners at the school have access to a library as well as functional science and computer laboratories. The learners also have access to the internet and email system. There are fifty six teachers employed at the school; fifty are DoE paid teachers and six are employed by the SGB¹⁸. The teachers are responsible for 1560 learners and the learner - teacher ratio is approximately 34: 1. There are eight mathematics teachers at the school.

Alan has been teaching mathematics at the FET phase for the past 13 years. He has an undergraduate degree with majors in Mathematics and Computer Education, and he is currently completing his second mathematics degree. Alan belongs to the Association for Mathematics Education of South Africa (AMESA). He plays an active role in the organisation, attends AMESA conferences and uses AMESA journal articles and workshop material to assist in preparing for his lessons.

Alan believes in teaching in a manner that encourages learners to visualise what he was teaching. He uses both technology and the traditional chalkboard in the classroom. Alan made decisions about which would be the best method for teaching different sections in

¹⁷ All the names of the Master teachers and schools are pseudonyms to protect their identity for ethical purpose.

¹⁸ A School Governing Body or SGB is the group of people that are chosen by election to assist with the running of a school. The main objective of the SGB is to ensure that the school is successful in providing the learners with the best possible education. This is done by assisting the principal in making the school efficient and effective. Each school governing body ought to comprise of the principal; elected members (parents of learners at the school, educators at the school, and members of staff who are not educators) and the learners themselves (Grade 8 and higher).

mathematics, for example he used the board to teach basic algorithms, but when he was teaching transformation geometry he used concrete manipulatives and computer generated diagrams to teach rotations and reflections. Research has shown that Information Communication Technologies (ICTs) have enhanced mathematical thinking and learning (Zevenbergen, 2004). The use of technology also provides the teacher with the opportunity to perform dynamic transformations (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000) thus allowing learners to visualise these (once) abstract concepts in mathematics.

Additionally Garofalo et al. (2000) proposed that in order to use technology appropriately in the teaching of mathematics, the following guidelines need to be adhered to:

- Introduce technology in context
- Address worthwhile mathematics with appropriate technology
- Take advantage of technology
- Connect mathematics topics and
- Incorporate multiple representations

(Garofalo, et al., 2000, p. 67)

Alan used a similar guideline when he was teaching using technology in that he made the mathematics relevant to the learners and context within which they were located. I observed this practice during each lesson observation. He preferred expanding his learners' knowledge and not just restricting them to syllabus requirements. He believed that they needed to be exposed to more advanced mathematics. His sentiments are reflected in the statement below.

Alan IE1¹⁹: “... *you can't move away from the chalkboard ... at the same time technology must be part of your lesson. ... if they (the learners) can see what's happening ... if they can see the uses ... it makes the understanding easier and the work easier ... I strongly*

¹⁹ In excerpts from transcripts, pauses and hesitations are represented by dashes -. Longer pauses are represented by multiple dashes. Where a portion of the original transcript is not included, I represent this with Where I use my own words within the excerpts, I represent this with normal font placed within brackets. For the complete original transcripts, please see Appendix C and D.

believe that you (the teacher) should know more than what the syllabus requires, don't restrict the child ... you will pick up the mathematics results. If you give them a chance they will manage, you just need to steer them in the right direction ... I would like to see technology being brought in because it is needed...

By learners being exposed to richer contexts and situations learners pick up both implicit and explicit knowledge, for example they may pick up theories of learning without being taught what learning is about (Hung & Chen, 2001). Additionally, Alan used multiple representations whilst teaching. For example, he used manipulatives, the OHP²⁰ and gestures to teach the concept 'rotation' (a concept taught within the transformation geometry section). His use of visuals made the mathematics he was teaching easier to comprehend. This sentiment is reflected in the learners' comments as indicated below:

L4 OFG1: *"He (Alan) makes it easier to understand ..."*

L8 OFG2: *"... it (the mathematics concepts in trigonometry) becomes easier to understand ... it (the visual tool) does make the lesson more easier to understand ... the understanding becomes simpler through his (Alan) use of diagrams..."*

L7 OFG6: *"Yes, it (the visual tool) made me understand it better ..."*

L9 OFG6: *"... that (the diagram of the Cartesian Plane) helped us in the understanding..."*

L3 OFG6: *"It (the mathematics concepts in trigonometry) becomes much easier to grasp, the diagram was like helpful."*

The use of multiple representations in mathematics engages all learners because it caters for the different learning styles. Using multiple representations efficiently comes with experience, where teachers often find what works best through trial and error. According to Brown and Coles (2009), a useful skill to possess when using trial and error methods in mathematics is the skill of being organised.

²⁰ OHP - Overhead Projector.

Alan was organised; he knew when he would be using different representations in his classroom and prepared accordingly. He was also aware of the latest developments in the mathematics field, and he preferred to see some of the latest developments within his own classroom. Alan was also willing to try new interventions and approaches, and he was not afraid to ask for help or to learn from other teachers. His willingness is captured in his response below.

Alan IC: “...*the more you are exposed to ... you pick up ideas, things that work, things that don't work and different ways to approach something ... the different charts or picture to show something because ... you always learn...*”

Whilst it is commendable and important for teachers to seek new approaches and move away from the traditional ‘talk and chalk’ method, interventions and approaches that are selected must support and strengthen the didactic aspirations of the learners based on their age, needs and abilities (Steadly, Dragoo, Arafah, & Luke, 2008). The needs of the learner are a complex matter, and teachers ought to consider this when planning for each lesson. A guide to meeting the needs of a learner was initially documented in 1970 by A. H. Maslow. This has been discussed earlier on in Chapter 2.4. Based on an interpretation of Figure 1 in Chapter 2.4, an effective school is one in which the learning community feels safe and are loved and respected. The experiences gained in each activity system within the school, ought to enable the learning community to move towards self-actualisation (Pollard & Triggs, 1997).

Throughout this study, attention was focussed on specific activity systems. Whilst the study focuses on the second-generation activity theory as justified in Chapter three, there is evidence of the possibility of external activity systems affecting the activity system under the microscope. As discussed earlier on in Chapter three, when an activity system assumes a new element external to the activity system, the new element creates tension or internal conflict. This tension brings into play the third generation activity system.

The diagram that follows illustrates the activity system within Alan’s classroom.

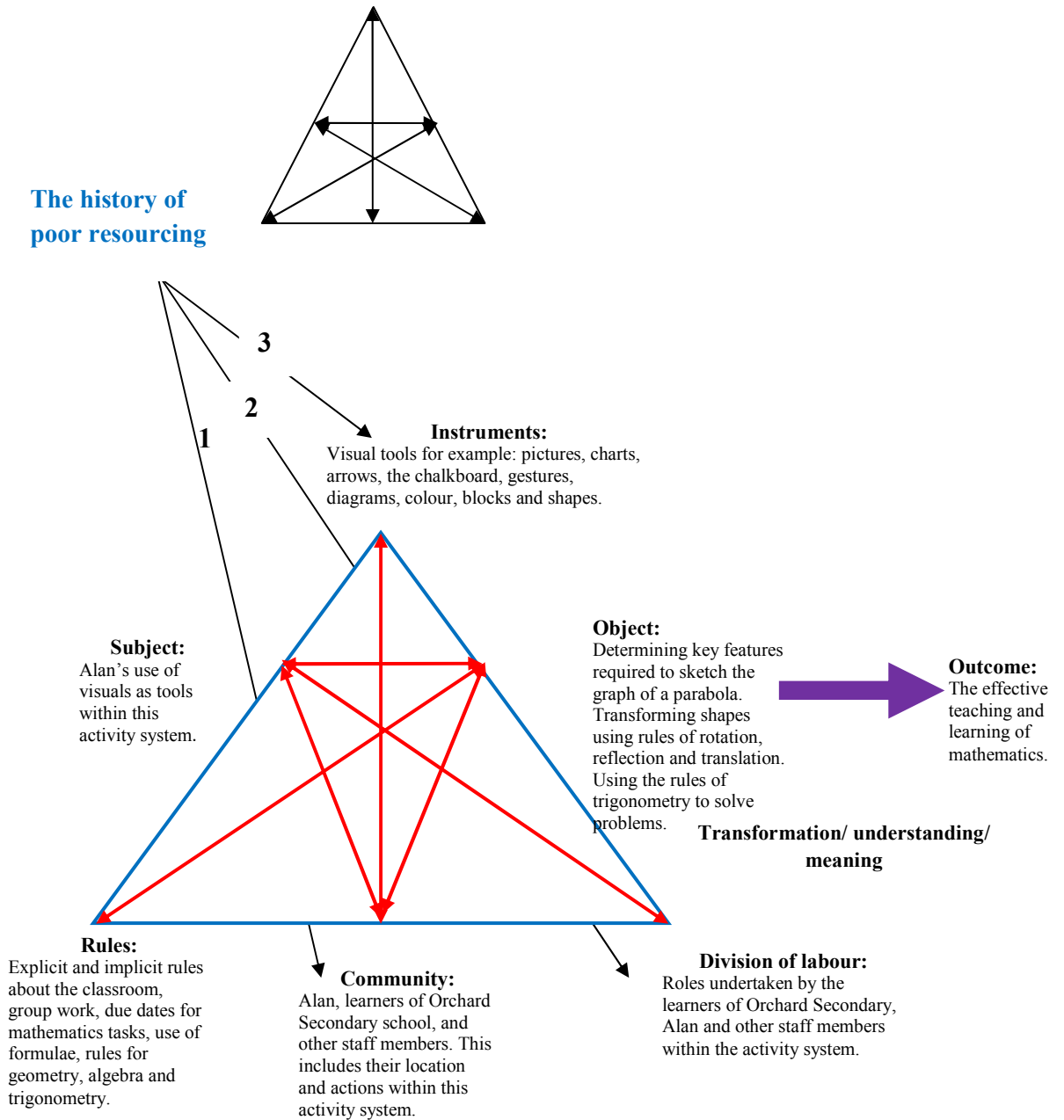


Figure 13: Alan’s activity system at Orchard Secondary School

As mentioned earlier on in Chapter two, for decades the history of poor resourcing created disadvantages across schools in South Africa. Whilst the DoE are currently addressing

these disadvantages, at Orchid Secondary teachers are still sharing their limited resources. Sharing of resources causes conflicts with the planning and time management of lessons. For effective teaching and learning to occur it is necessary to make maximum use of teaching time (Pollard & Triggs, 1997). Apart from the sharing of material resources, human resources are also limited. This implied that teachers at this school spend a great deal of their NTPs²¹ serving relief for teachers that were not present at the school. This lack of an adequate support structure, limits the teacher's time for effective planning and preparation for future lessons. As depicted in Figure 13, the external activity system caused tension within the main activity system in three ways.

Arrows 1 and 2 signifies the tension that arises because of limited human resources. Limited human resources impact negatively on both the community and division of labour within the activity system. Members of the teaching staff within the learning community were burdened by the added pressure of providing adequate support to all learners within the learning community. Regardless of the number of teachers at the school, it is still necessary for teaching and learning to go on. Teachers are compelled to take on more responsibility and a greater workload. This external activity system leads to teachers being overworked, worn-out and stressed. This could lead to more teachers being absent than at other advantaged schools.

Arrow 3 demonstrates the conflict that arises when resources are being shared. This sharing influences what is readily available for the teacher to use in each lesson. As a result of the limited resources, the teacher is required to be more resourceful than colleagues at other more advantaged schools, if Level 5 of Maslow's hierarchy of needs is going to be attained. In many cases, the teacher will need to reflect on previous experiences and use a combination of tacit and explicit knowledge in order to be effective in the classroom.

²¹ Non teaching period generally set aside for planning and preparation

Doubtless, there are other external activity systems influencing Alan’s activity system; whilst this was not the focus of my study, it is worth mentioning the above external activity systems that were in ‘play’. In the next section, the next Master teacher, Karyn is introduced.

5.2.2. Karyn

Karyn is a teacher at Rose Secondary School. Rose Secondary is located in a predominantly White suburb. Despite this, the learners at her school are of different races and come from all parts of KZN. Although the school fees are above average (but below that of a private school), the school has a long waiting list for entrance to the different grades.

In South Africa a good state-aided school, offering small class sizes (around 20), may cost between R6 000 to R15 000 per year compared with a private school costing from R10 000 to R55 000 ²² per year (excluding boarding, which could cost an extra R20 000 a year). Private or independent schools are schools that receive no funding from the government; they are funded entirely by fees paid in by the parents and guardians of learners. The following tables provide the reader with additional information regarding schools in KwaZulu-Natal.

Number of Public (State funded) schools	Number of Private (Independent) Schools	Total number of schools in KZN
5937	223	6160

Table 8: The number of schools in KwaZulu-Natal
Adapted from Department of Education (2010)

²² There are a few private schools that have low standards and are cheaper than this.

Number of primary schools	Number of secondary schools	Number of combined schools	Total number of schools in KZN
3902	1575	683	6160

Table 9: The number of schools in KwaZulu-Natal arranged according to the different levels of schooling

Adapted from Department of Education (2010)

State funding is organised on a quintile system, in which schools are divided into five categories according to the poverty levels in the areas they serve. Schools are categorised according to five quintiles: Q1, Q2, Q3, Q4 and Q5. Q1 represents the poorest schools and Q5 represents wealthier schools. More money is allocated to the poorest schools and lesser funds are allocated to the wealthier schools. The criteria used to determine the quintile that schools belong to are based on the national census data for school catchment areas (Mbuli, 2010). Three factors are predominant within this criteria: the income level, the unemployment rate and the level of education (Kanjee & Chudgar, 2009).

The following table provides information with respect to the number of schools and learners within the quintile system in KwaZulu-Natal.

Quintile	Number of schools (Primary, Secondary and Combined)	Number of learners
Q1	1 912	569 909
Q2	1 687	618 047
Q3	1 158	562 050
Q4	696	478 875
Q5	484	326 424
Total	5 937	2 555 305

Table 10: The number of schools in KwaZulu-Natal arranged according to the quintile system

Adapted from Mbuli (2010, p. 1)

This information is also depicted in the pie graph that follows.

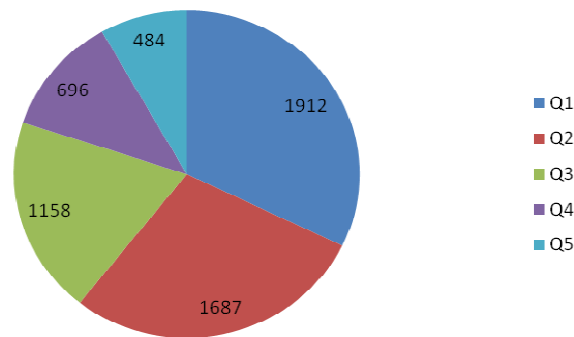


Figure 14: Pie graph representing the number of schools in KwaZulu-Natal arranged according to the quintile system

Poorer schools are given larger state subsidies, and so have lower school fees, while wealthier schools are given smaller subsidies, and have higher fees. In the poorest areas of all, parents are completely exempt from paying school fees (Kanjee & Chudgar, 2009). These schools are referred to as ‘no fees schools’, none of the schools in this study were ‘no fees schools’.

Rose Secondary, although not private, is one of the wealthier schools and commands a good reputation with respect to discipline. Rose Secondary belongs in the Quintile 5 category. The school has small class sizes and excellent Grade 12 pass rates (100% pass rate in mathematics in 2008 and 2009). Rose Secondary School has excellent resources, well-equipped science and computer laboratories, excellent sporting facilities (including an operational gym and swimming pool) and an up to date library. Additionally, learners at the school have access to the internet, email and interactive mathematics software. There are seventy teachers on staff; forty are state paid (DoE) and thirty are employed by the SGB. There are ten mathematics teachers on staff. Currently there are 1230 learners enrolled at the school and the learner-teacher ratio is 18: 1.

Karyn is a well-qualified mathematics teacher; she has a teaching diploma, an undergraduate degree and a postgraduate degree (B.Ed Honours in Mathematics

Education). Karyn has been teaching mathematics in the FET phase for 15 years. She believes in updating her pedagogic, content and professional knowledge by attending workshops coordinated by the KZN Department of Education. She uses many resources (textbooks, mathematics manipulatives, transparencies and charts) to enhance her teaching including the smart board. Karyn is of the notion that the use of visual tools in mathematics is very important. She believes that learners need to see the mathematics in order for them to understand the mathematics being taught. Her beliefs are captured in the statement that follows.

Karyn IE1: *“... important for the learners to see ...if you don't have all these facilities it can be very difficult to explain some of the mathematics concepts ...the more they see the better...”*

Her beliefs were echoed in her learners' views as indicated in the statements that follow.

L6 RFG1: *“I think it kind of clarifies what's what, if someone had to tell you this is the hypotenuse if you had not heard it before you would not know what it is but if you like show it on the actual triangle it is the longest side; it's easier to understand so you kind of link everything together.”*

L3 RFG4: *“Well she (Karyn)is trying to makes sure that everyone is understanding what's happening so she is not leaving anything out, so if she shows okay that's A that's B and that's C ... you will begin to understand ... so that you can see where you went wrong...”*

L7 RFG6: *“Sometimes if you see it like especially if you are using colour ... you have a visual and you will always remember ...”*

Karyn exhibits a high level of planning and preparation and uses various websites and textbooks to supplement her lessons. Whilst she uses the smart board effectively and efficiently in the classroom, she aspires for her pedagogic knowledge to be developed further in this respect. These sentiments were apparent during her interview, and are reflected in her response that follows.

Karyn IC: *“...we do use each other as teachers, I can inform you that we would like to have more (more workshops)... if we could go on a course to use the smart board properly. ...it is all about technology, smart board, websites, and availability of software...”*

Thus, the use of technology has been encouraged in Karyn’s classroom because it increased learner engagement in learning (Murphy, 2001). Karyn also demonstrates evidence of being a lifelong learner, since she is willing to learn more in an attempt to be better equipped to teach her learners. In the spirit of the vision for the new education and training system in South Africa, lifelong learning is an essential objective (Walters, 2006). Learning is a complex process for all parties involved. There are many factors influencing or affecting this process. On interrogation of Karyn’s activity system, I believe that there are external influences that would influence what transpires in her classroom.

Karyn’s activity system is depicted in Figure 15 that follows.

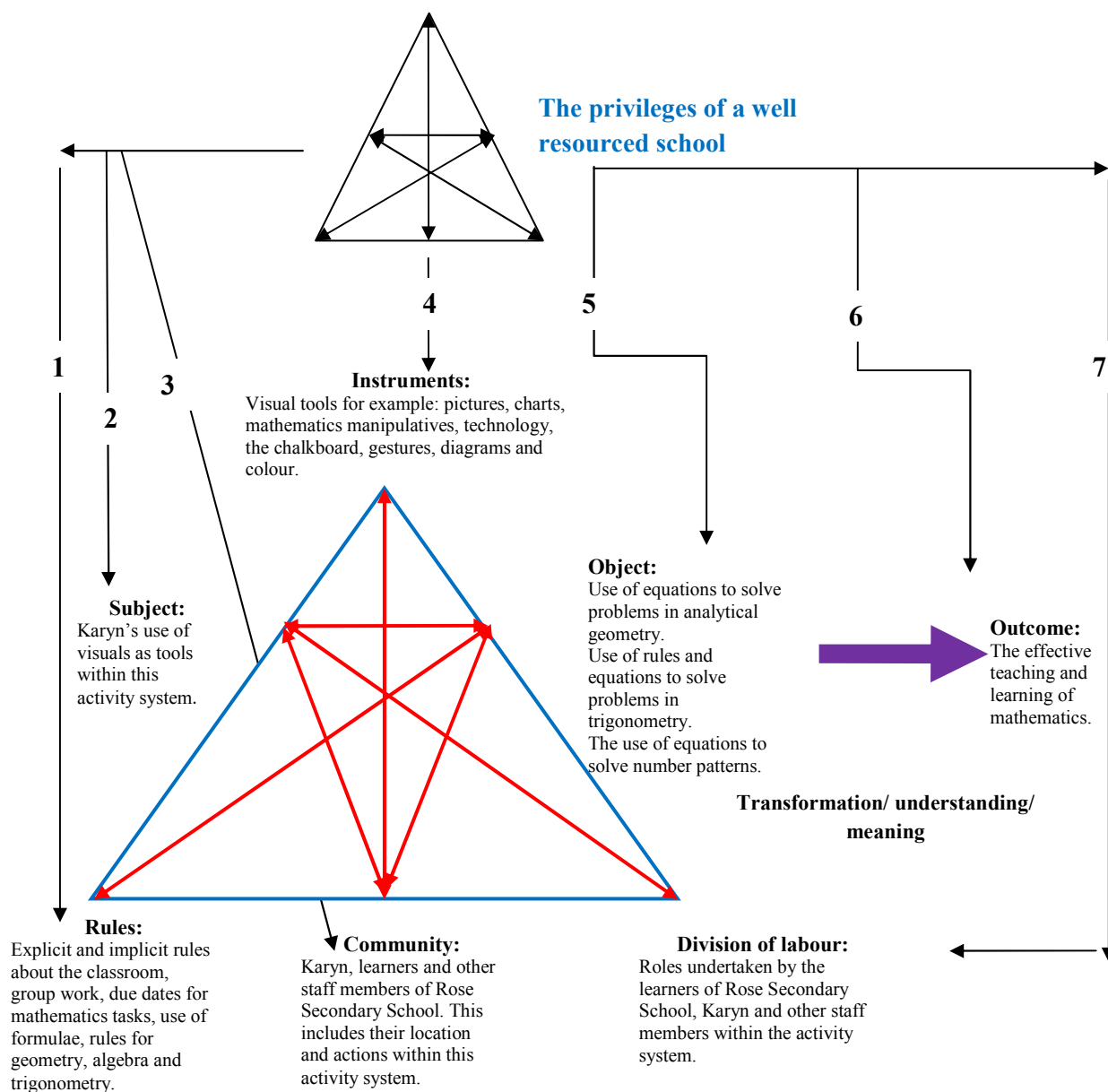


Figure 15: Karyn's activity system at Rose Secondary School

The most noticeable external system that influenced this activity system is one of privilege. This activity system was privileged in all respects when compared to the other activity systems in this study, because it was one that had better material and human resources. Teachers at the school were in abundance, and they were well qualified. This external

activity system influenced all aspects of this activity system. The first arrow (arrow 1) indicates the effect on the rules within this activity system. Most learners at the school came from privileged backgrounds and other advantaged primary schools. They were accustomed to the rules of this type of learning environment. The minority of the learners who did not come from privileged backgrounds conformed to what was socially acceptable within the learning environment. They developed a shared understanding with other members of the learning community, this suggested that these learners assumed the dominant practices of this activity system (Naidoo, 2006).

The second arrow indicates the influence of the external activity system on the subject in this learning environment. The Master teacher at this school came from a privileged background and could identify with the learners in this activity system. The teacher had a range of tools from which to choose and she used her tacit knowledge to make tool use (arrow 4) in this activity system beneficial. This dynamic use of tools led to a varied approach to the teaching and learning of mathematics. This is discussed in detail in Chapters six, seven and eight of this study.

With respect to issues of community and division of labour, most of the learners came from better socio-economic backgrounds and hence had the parental support ensuring success at schools. Most parents could afford to provide their children with extra resources to assist with effective learning. These resources included, for example, study guides, computer access, internet access and extra tuition. This extra parental support influenced the roles of the learning community (arrow 3) and the division of labour (arrow 7) within the classroom. Learners were able to take more responsibility for their own learning. Additionally, being a well resourced school, there was more funds available for employing additional staff members (SGB appointed) as the need arose. This relieved the burden of relief teaching and support. Responsibilities could be shared with other staff members and hence more time could be spent on planning and preparing for lessons.

What is evident from this discussion is that the external activity system of privilege positively influenced the object (arrow 5) and eventual outcome (arrow 6) of Karyn's activity system. Almost certainly, other external activity systems might have influenced this activity system, but the identification of all external activity systems was not the focus of this study. The study explored each Master teacher's use of visual tools in the mathematics classroom. The external activity system that was observed has been highlighted. It has been acknowledged that this external activity system may well have influenced the activity system under scrutiny. In the next section Dean's activity system is discussed.

5.2.3. Dean

Dean was the most experienced of all the Master teachers in the study. Dean has been teaching mathematics in the FET band for the past 26 years. He is also the Head of Department (mathematics) at Daisy Secondary School. Daisy Secondary School is located in a predominantly Indian suburb, but serves a range of learners diverse with respect to ability level, race and culture.

Deans' school is similar to Alan's school with respect to the allocation of resources (both material and human). Whilst the school has a well-resourced library, functional science and computer laboratories, the school does not have any internet access or email. With respect to human resources, the school has well qualified teachers. There are thirty four DoE teachers and five SGB teachers at the school. There are only eight mathematics teachers at the school. The Grade 12 mathematics pass rate at the school in 2008 was 91% and 87% in 2009. The learner enrolment at the school is 897, and the learner – teacher ratio is approximately 35:1. Dean is well qualified; he has two degrees, the first majoring in Mathematics Education and the second majoring in mathematics and science. Dean is currently enrolled for his Honours degree majoring in Mathematics Education. Dean focused his lesson preparations on various textbooks, past exam papers and his personal teaching experience. He is of the belief that the use of visual tools in teaching mathematics

is extremely important because learners need to see in order to understand. He believed that if learners saw something concrete; it would make the abstract mathematics more accessible. His philosophy is that by making the mathematical rules and procedures visible, the learners' comprehension of these rules and procedures will be improved (Karadag & McDougall, 2009). His view is exemplified in the following statement.

Dean IE: *"...I would say that we definitely need it (visual tools) when teaching mathematics ... pupils need to see things ..., things must be concrete to them (the learners). If they (the learners) can see things and understand concrete concepts, from there it would be easy to move on to more abstract stuff..."*

Dean's views were supported by his learners. This was apparent during the focus group interviews. Some of his learners' comments are reflected below.

L1 DFG1: *"It (the visual tool) helps us to remember stuff, so in the exam we can picture it."*

L3 DFG1: *"We can understand the work easier."*

L5 DFG9: *"Yes because if he (Dean) hadn't used the stick and just explained, then we would have been confused, we would not have known which direction we were moving in."*

Dean valued the input provided by colleagues and believed that teachers can learn a lot from other veteran teachers because experience plays an important role in developing techniques and strategies that are beneficial. This would also influence the teaching and learning of mathematics because learners learn more from experienced teachers as suggested by Gardner (2005). The following response from Dean reflects this view.

Dean IC: *"... the training comes with teaching practice and also advice from other senior persons..."*

Dean was cognisant of the fact that technology is important in the teaching and learning of mathematics. He realised that the use of technology in the classroom enhanced mathematical teaching and learning with the technology itself (Garofalo, et al., 2000). He is willing to learn more about how he could implement technology in his classroom; this view is captured in Dean's response that follows.

Dean IC: *"...we are now moving towards things like the smart board ... although we (the school through the Dinaledi incentives) have acquired a smart board recently; we need to get practice and guidance...how to use this because I think that is one of the ways to go..."*

Although Dean is an experienced teacher, he is comfortable to admit that he does need to learn new techniques and approaches to the teaching and learning of mathematics. He is willing to learn more to improve his practice with the aim of being a 'more knowledgeable other'²³ to his learners. The 'more knowledgeable other' refers to someone who has a better understanding of a particular task, process, or concept than the learner, this could include teachers and more experienced teachers (Putnam & Borko, 2000). This feeling is common with teachers at schools that are disadvantaged with respect to human and material resources. Whilst Dean does have a well resourced library, functional science and computer laboratories, the school is not as advantaged as Rose and Lily Secondary. As mentioned earlier Dean's school is similar to Alan's school, and hence had a similar external activity system influencing the activity system under scrutiny.

Dean's activity system is illustrated in Figure 16 that follows.

²³ The more knowledgeable other (MKO) is normally thought of as being a teacher or older adult, but the MKO could also be a peer, a younger person, or even computers.

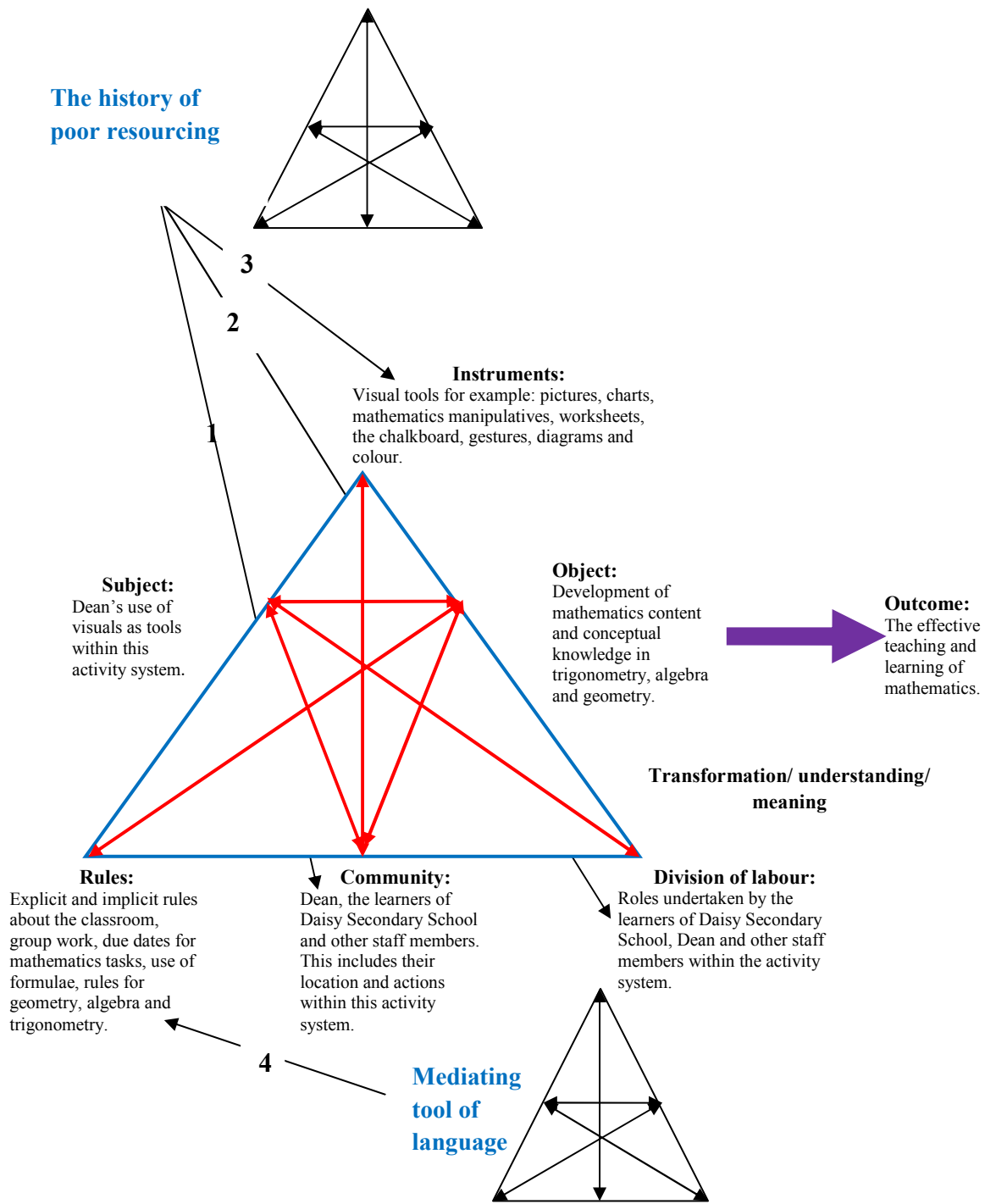


Figure 16: Dean's activity system at Daisy Secondary School

As with Alan's activity system, the history of poor resourcing had a similar impact on Dean's activity system. The external activity system affected the use of tools (arrow 3), the role of the community (arrow 1) and the division of labour (arrow 3) within Dean's activity system. Similar notions with respect to the lack of adequate support structures created demanding situations for teachers at the school. Teachers had limited time for planning and preparing lessons at this school.

There was evidence of another external activity system that appeared to influence Dean's activity system. The mediating tool of language affected how the rules and instructions were understood within the learning environment (arrow 4). Learners at Dean's school came from different language backgrounds and at times these differences created issues of miscommunication within the classroom. The majority of learners at the school spoke English as a second or even third language. Due to differences between the dominant language at home and the language of instruction at school, certain mathematics concepts and rules had to be revisited. This revisiting of mathematics concepts added pressure on the time allocated for syllabus coverage. Regardless of these external conflicts, the syllabus had to be completed in time for the national tests and examinations. To overcome these conflicts, Dean had to be reflective and resourceful in the classroom. This is explored further in Chapters six and eight.

As with the other activity systems in this study, I acknowledge that it is possible that there are other external activity systems influencing Dean's activity system. I focus on the ones that I believed were of greater importance. In the next section the next Master teacher, Penny is introduced.

5.2.4. Penny

Penny has been teaching mathematics for the past 24 years; she teaches at Tulip Secondary School. This school is located along the borders of a triad of suburbs that were once termed Black, Indian and Coloured suburbs. The school accommodates diverse learners from

multicultural backgrounds. The school fees are equivalent to that of other state schools, but here again learners flock to the school because of the high Grade 12 pass rates as compared to the other schools in the vicinity (70% in mathematics in 2008 and 2009).

The school has an enrolment of 1629 learners; the learner- teacher ratio is approximately 26:1. There are sixty three teachers on the staff; fifty seven are DoE teachers whilst six are employed by the SGB. This school is not as well resourced as the other schools discussed thus far. Whilst the learners have science and computer laboratories, these are not as well resourced. The school does not have a functional library but the learners do have access to the internet and email system.

Penny has a teaching diploma, a further diploma in education and she is working on her Bachelor of Science degree. Penny is the Head of Department (mathematics) at the school and teaches both Mathematics and Mathematics Literacy. Based on her experience, Penny was of the notion that the use of visual tools in mathematics is imperative to teaching and learning. She believed that learners need to make some internal connection in order for the mathematics that they saw to make sense.

She stated “...*I believe that children need to see things to remember it ... things that stick in our minds are things that we have seen in life and those things actually get stored in our memory ... I can't see mathematics being taught without visuals...it's something they (the learners) have to see to be able to internalise...*” (Penny IE).

Penny used textbooks, pictures, charts and other resources to prepare for her lessons. She often used information picked up at workshops to assist in her teaching. Penny is also an EHV (Education in Human Values) facilitator. Education in Human Values is a value based teaching programme whereby the core human values are discussed. The diagram that follows represents the core components of the EHV programme.



Figure 17: Diagrammatic representation of the key themes in Education in Human Values
Adapted from <http://sathyaibaba.files.wordpress.com>

Values of truth, love, peace, right conduct and non-violence are encouraged during each lesson. EHV facilitators teach through experiential lesson plans based on stories about life, relationships, music, poems and activities (BISSE, 2010)²⁴. Lessons are based on socially relevant real world contexts. Penny often used her tacit knowledge and knowledge acquired as an EHV facilitator to assist with the teaching of mathematics. This strategy created a different approach to mathematics teaching, one that the learners enjoyed. Although Penny has many years of teaching experience, Penny believed that she could learn more. She was keen to attend workshops on the use of technology in the mathematics classroom.

What is evident here is that Penny like the other Master teachers in the sample was aware of the fast changing technology currently available. She realised that through the use of technology there would be innumerable creative learning possibilities available to her learners. A similar notion has been proposed by Alsina (2002) in that technology provides a greater number of alternatives and strategies for learners to interact with. In Penny's case, hers was an innate need to improve her teaching of mathematics by incorporating technology within her lesson. Penny was focussed on her professional development.

²⁴ The British Institute of Sathya Sai Education.

This was apparent in the following response.

Penny IC: “...I have attended many workshops ... in preparation for OBE ... in terms of using a laptop and projector and using the Cartesian plane and moving of graphs... the use of the computer and mathematics ... I have not mastered that...”

Professional development in education is part of lifelong learning that includes assuming personal responsibility for learning about teaching (Carroll, 2005). In this study this was done with the intention of enhancing the learning of mathematics (Chval, Abell, Pareja, Musikul, & Ritzka, 2008). Here again this Master teacher exhibited indications of being a lifelong learner and demonstrated her commitment to her learners. Professional development is an intensive, ongoing and systemic process that aims to improve teaching, learning and school environments by developing new attitudes, mindsets and insight (Carroll, 2005; Chval, et al., 2008). Penny was willing to forsake her personal time to go for planned professional development workshops. Her intention was to make mathematics more accessible and comprehensible for her learners.

This noble intention was compromised in many ways by the various external activity systems impacting on Penny’s activity system, regardless of this, though; Penny was triumphant in her classroom.

Penny’s activity system is depicted in Figure 18 that follows.

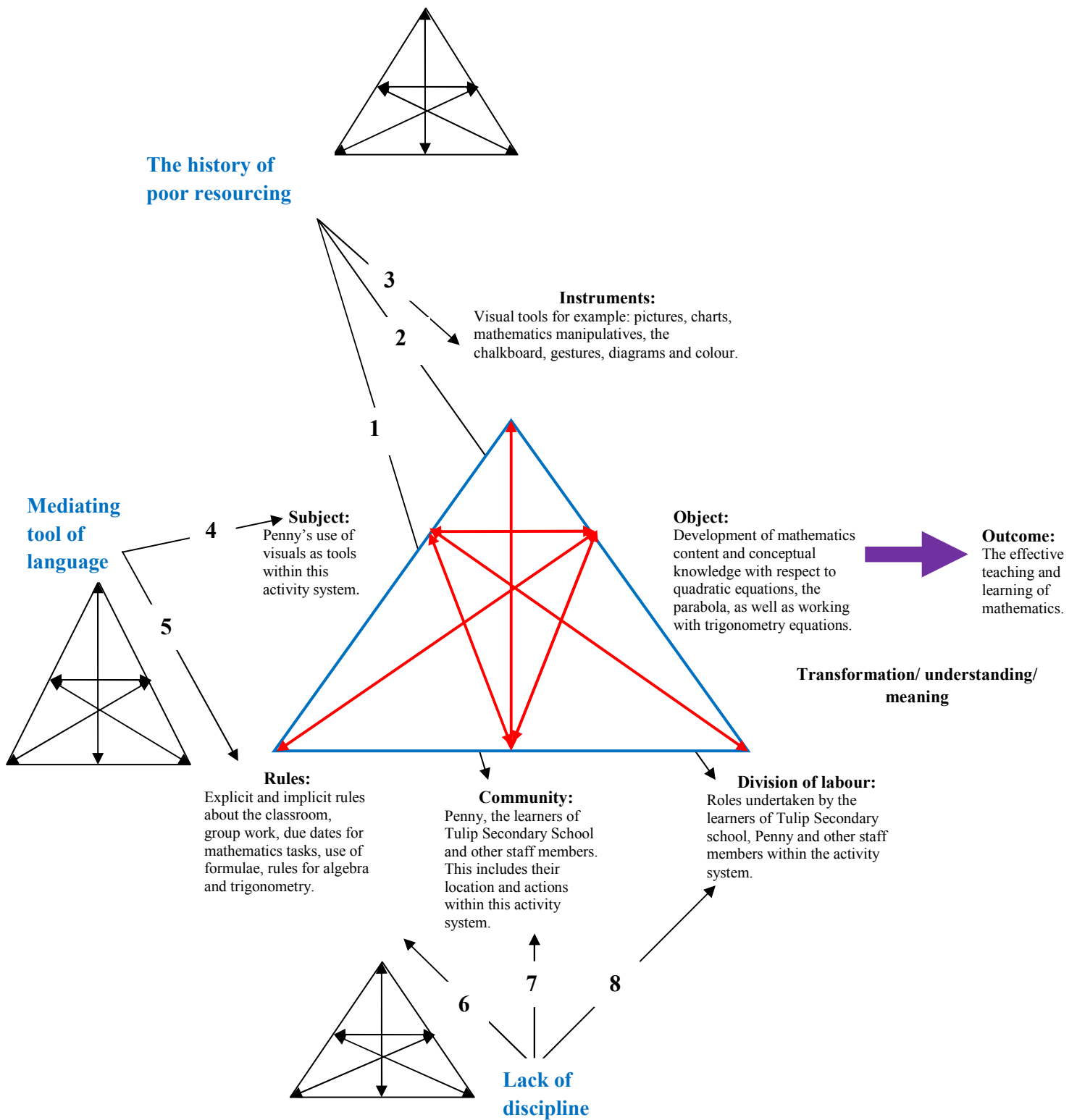


Figure 18: Penny's activity system at Tulip Secondary School

As in Dean's and Alan's activity system, the external activity system of the history of poor resourcing had a similar impact on the division of labour (arrow 2), community (arrow 1) and instruments used (arrow 3) within this activity system. Apart from this external conflict, two other activity systems influenced Penny's activity system. They were the mediating tool of language and the lack of discipline. Penny used various strategies to ensure that the learners' behavioural problems that were evident in other classrooms did not manifest itself in her classroom. This has been discussed earlier on in this chapter. Whilst Penny used her knowledge gained from being an EHV educator to make her classroom more conducive to teaching and learning for her learners, she would not necessarily have used these strategies and methodologies if this external activity did not begin to manifest itself at her school.

With respect to the external activity system of language, language influenced how the teacher taught the lessons as well as how rules and instructions were interpreted within the classroom context. Language is a cultural matter; it is a way of communicating meanings and of coding events. Generally, children attain a basic mastery of their mother tongue before they start school. As mentioned earlier, learners at this school came from three differing contexts and cultures. Since learners find it difficult to follow instructions in a language that is not their mother tongue, this may account for their poor academic performance (Mwamwenda, 2004). This caused additional conflict and tension within Penny's activity system. Likewise Zevenbergen (2001a) proposed that the language that learners learn from their homes is prone to locate them more or less favourably at school. This notion is dependent on the correlation between the home and school languages. This suggests that the home and language are connected, with the home being very significant to how the learner communicates within the classroom (Naidoo, 2006).

The links between the school and the home tend to be stronger with families from advantaged backgrounds than with families from disadvantaged backgrounds. Learners from advantaged backgrounds use an "elaborated code" whilst learners from disadvantaged

backgrounds use a “restricted code” with respect to language (Bernstein, 1971, p. 76). The issue of language has been discussed earlier on in Chapter two. Learners from advantaged backgrounds have access to both codes and this locates them as having a more dependable voice within the community. Learners from disadvantaged backgrounds may not have access to the elaborated code and this partly explains their underachievement at school (Boaler, 2002).

The personification of characteristics of tastes, disposition and language can be seen to be the structure of habitus. “Habitus” according to Bourdieu is the embodiment of culture and presents the lens through which the world is construed (Zevenbergen, 2001b, p. 202). Each activity system in this study exhibited their own ‘habitus’ whereby each learning community constructed the learning of mathematics diversely. Each learning community in this study used their own symbols and visual tools to make the mathematics more accessible to members within the learning community. In this activity system, Penny used many differing strategies to compensate for the differing backgrounds of her learners. Penny also used a combination of group work and individual during her lessons. This is explored in Chapter six.

The third external activity system of note was the lack of discipline. This affected the rules of the classroom (arrow 6), the learning community (arrow 7) and the division of labour (arrow 8) within Penny’s activity system. In every classroom there are rules to be followed. This assists in attaining Levels 2 to 4 of Maslow’s hierarchy of basic needs (Figure 1). Due to the lack of discipline at some schools, rules are compromised and more responsibility is shifted to the learning community which in turn leads to an increase in the division of labour. The reason Penny did not have a discipline problem in her classroom, as experienced by the other teachers at the school, was that she used diverse teaching strategies and methodologies in her classroom. This included the use of scaffolding, stories, pictures and colour. These are explored in Chapter six, seven and eight of this study. In the next section, the next Master teacher, Sam is introduced.

5.2.5. Sam

Sam teaches in a similar school as Penny with respect to resource availability. However, his school has the least amount of resources compared to all the other schools in the study. Sam teaches at Carnation Secondary School, which is located in a predominantly Black suburb. The school has a learner enrolment of 1305 and the learner–teacher ratio is approximately 40:1. There are forty three teachers on staff; the DoE employs forty one and the SGB employs two. There are six mathematics teachers based at Carnation Secondary. Although the school has functional science and computer laboratories and a moderately resourced library, the learners do not have access to the internet or email system. However, learners still desire to attend this school because of the good discipline and good Grade 12 pass rates (70% in mathematics in 2008 and 71 % in mathematics in 2008) as compared to other schools in the vicinity.

Sam is the Head of Department (mathematics) at the school and has been teaching mathematics in the FET phase for the past 25 years. He is a well-qualified mathematics teacher. He has a teaching diploma (mathematics major), a further diploma in education and a Bachelor of Arts (B.A.) degree. When Sam is not teaching at school or workshopping teachers within his mathematics department, he facilitates mathematics workshops at district level for the DoE. In South Africa each province is divided into districts. There are fifty two districts in South Africa, eleven of which can be found in KZN.

Sam believes that teachers ought not to take anything for granted, more so with respect to learners' foundation knowledge that is required when starting a new section. Sam believes that teachers ought to access the learners' prior knowledge first before starting with a new section. This is so because the comprehension of new information can only be understood in relation to prior knowledge (Myhill & Brackley, 2004). If the teacher ignored or disregarded prior knowledge this would result in the learner learning information in conflict with the teacher's intention.

Moreover, Sam believed that all the learners ought to have a common grounding before starting with new work. This makes the mathematics classroom an impartial space and everyone has the ability to do well because all the learners have the same opportunities and equal access to knowledge. This creates a stimulating environment and atmosphere for learners to work and participate in, unconditionally. Sam's beliefs are captured in the following statement.

Sam IE: *"... don't take things for granted ... if you are introducing a lesson ... you have to go back previously ... in mathematics ... the children have a tendency of saying that mathematics is difficult ... you have to create an atmosphere where they can see that ... nothing is impossible ... they can do it..."*

Sam also believes that when teaching mathematics the use of a diagram is important. He also values the use of technology in the mathematics classroom, provided the teacher has access to this technology. Sam stated that *"...for math teachers you have to use the diagram ... I think that a computer has precedence ... teachers ... may get laptops and I am hoping ... they start with math teachers..."* (Sam IC).

As can be seen, Sam displayed enthusiasm and passion for mathematics. Based on the lesson observations and interview with Sam, he was committed to helping his learners in making a difference in their lives. He wanted to provide access to a better future for his learners; he wanted to help them change their circumstances. Sam used mathematics as the vehicle for driving this transformation. His actions involved reflection, which is the medium for becoming an adaptive mediator. Schön's (1983) notion of reflecting-on-action and reflecting-in-action characterise the kind of reflective thinking that leads to an adaptive teacher. The adaptive teacher focuses on the context within which the teaching and learning occurs. As with the other activity systems in the study, many external tensions influenced the activity systems within Sam's classroom. The activity system is illustrated in the Figure 19 that follows.

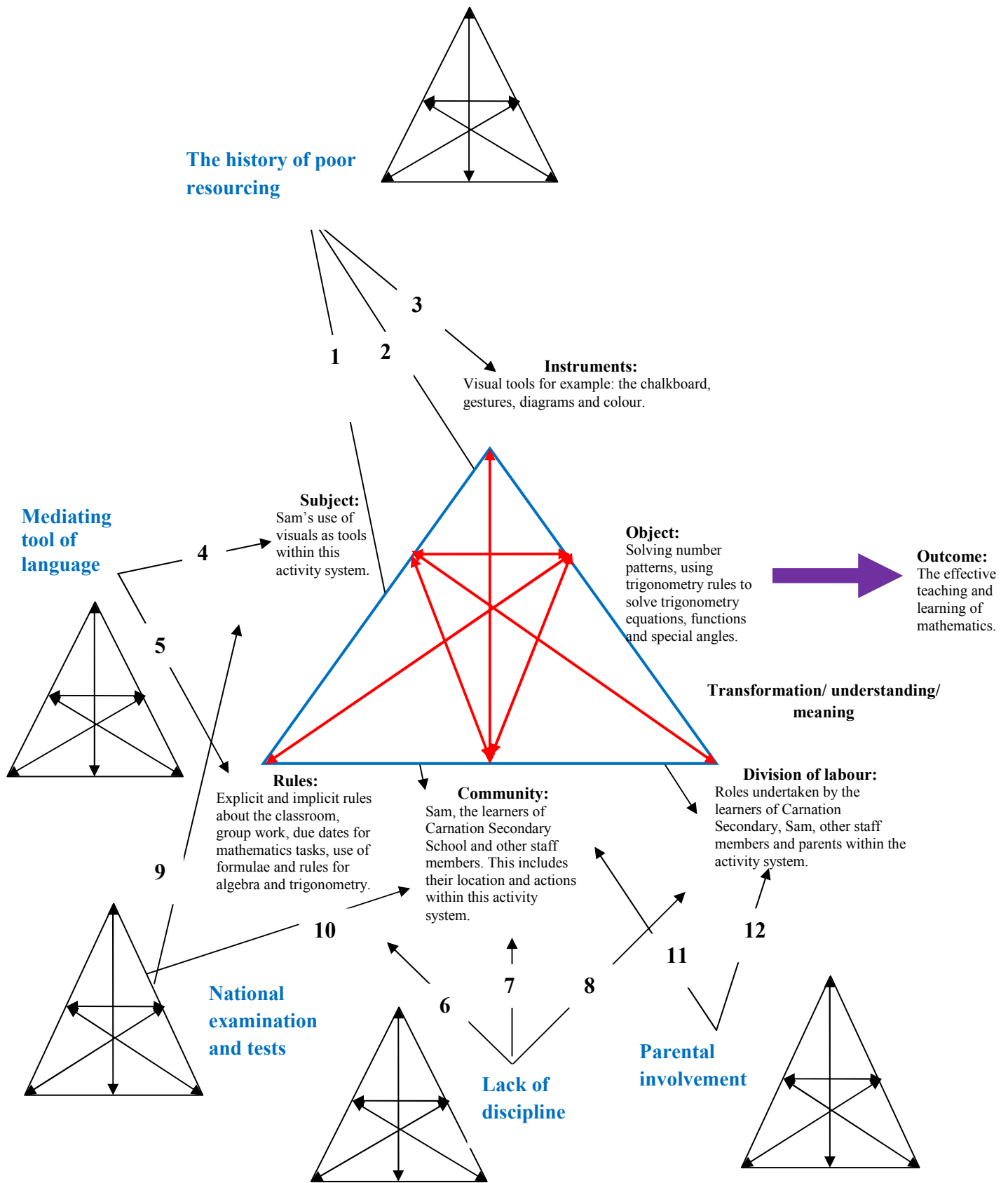


Figure 19: Sam's activity system at Carnation Secondary School

The first external activity system that impacted on the activity system within Sam's classroom was the history of poor resourcing. Schools across South Africa are unequally resourced in terms of material and human resources. With policy changes, the DoE are addressing this issue, but until then, teachers in poorly resourced schools have an added dilemma when it comes to the teaching and learning of mathematics. Within this activity system, this lack of resources caused tension between the visual mediating tools and the Master teacher (arrow 3). Sam reflected on his practice and used manipulatives, and visual tools that were easily available to learners within the classroom environment. He used bricks, desks, coloured chalk and mental images to assist his learners. The use of these visual tools influenced the use of instruction time in the classroom. More time spent in the drawing of these diagrams and mediating tools than in the interaction and engagement within the classroom. As in the case of the other Master teachers (Alan, Dean and Penny), the lack of resources affected the community (arrow 1) and the division of labour (arrow 2) within Sam's activity system.

The second external activity system that affected Sam's activity system was the mediating tool of language. As discussed in Chapter two, due to the changes in curriculum, many learners learn mathematics in a language that is not their first. The mediating tool of language in the home environment caused conflict with the mediating tool of the language of instruction. This conflict posed problems for the Master teacher (arrow 4) when trying to engage learners with the mathematics being taught. If one does not have exposure or experience of certain words and concepts, one would not be able to communicate effectively within a classroom context. One would be functioning within Bernstein's "restricted code" of language (Bernstein, 1971, p. 76).

Rules related to the content taught and concepts learnt will most likely be misunderstood (arrow 5). Thus, apart from the use of visual images to assist in making the language of mathematics more comprehensible and accessible, the teacher encountered an added dilemma. Sam needed to ensure that the visual tools used would be understandable to all

his learners. He needed to ensure a levelling of the mathematics 'playing field' before any teaching and learning could occur.

The third external activity system that influenced the activity system within Sam's classroom was that of the national examinations and tests. Regardless of the context of the school, all schools write national examinations and tests. When teaching time is spent on drawing diagrams and levelling out the mathematics 'playing field', this has an impact on syllabus coverage. The added pressure on the teacher (arrow 9) of completing the syllabus in a shorter period affects the community (arrow 10) within the activity system. The community becomes overworked and the division of labour is unfairly distributed.

The fourth external activity system that I observed was the lack of discipline within the learning environment. Whilst this did not hamper Sam in his teaching this was significant. This suggested that Sam could control how the learners behaved within his activity system. He did this by making his classroom milieu more accessible, inviting and conducive to the learning of mathematics. He used pictures, diagrams, concrete manipulatives and gestures to attain this atmosphere. As mentioned earlier in Penny's activity system, the lack of discipline could have an impact on the rules (arrow 6), the community (arrow 7) and division of labour (arrow 8) within an activity system.

The fifth external activity system that I believed was of significance was that of parental involvement. The learners who attended Carnation Secondary come from less privileged backgrounds. Whilst the parents wanted their children to succeed in school and obtain the best possible education, the home environments caused conflict with the community (arrow 11) and the division of labour (arrow 12) within this activity system. Not all parents could provide educational support for their children. Most parents could not afford additional materials and tuition to assist in improving their children's educational abilities. This added pressure on the community within the school to provide an environment that supported these needs. This in turn added extra responsibilities on the already overextended teachers

at the school. This caused an unfair disadvantage when compared to other schools within the more affluent suburbs of KZN. The division of labour amongst the learning community was unfairly distributed.

Doubtless, there are other external activity systems that might have influenced this activity system, but this was not the focus of the study. Those external activity systems that have been observed to have influenced the activity system under scrutiny have been highlighted. In the next section, the last Master teacher in the sample, Maggie, is introduced.

5.2.6. Maggie

Maggie teaches at a similar school to Karyn's school. She teaches at Lily Secondary School. Lily Secondary is situated in what was once called a White suburb, but here again because of the reputation of good discipline, small class sizes and good Grade 12 pass rates (98% in mathematics in 2008 and 96% in mathematics in 2009), the school often has a long waiting list for admittance to the different grades.

Lily Secondary School has excellent material and human resources. There are fully functional computer and science laboratories, a well-stocked library and excellent sporting facilities (including an operational gym and swimming pool). The learners have access to the internet and the email system. With respect to human resources, there are sixty six teachers at the school, the DoE employs fifty teachers and sixteen teachers are employed by the SGB. There are ten mathematics teachers based at the school. The learner enrolment at Lily Secondary is 1100 and the learner-teacher ratio is approximately 30: 1.

Maggie is the Head of Department (mathematics) at the school and she has been teaching mathematics in the FET phase for the past 18 years. Maggie is a well-qualified mathematics teacher; she has a teaching diploma (Mathematics Education major), a Bachelor of Science degree and an Honours degree (BSc honours). Maggie's philosophy of teaching mathematics includes preparing her learners for the future. Her passion and love

for the subject came through as she spoke about her teaching strategies. Maggie was of the notion that if the teacher valued and enjoyed mathematics, the learners would feel the same way. Yara (2009) maintained that a teacher's personality and methods of mathematics teaching impact on a learner's attitude towards mathematics. With this in mind, a teacher ought to exhibit a positive attitude and passion toward mathematics if the teacher expects their learners to value and enjoy mathematics. Maggie appeared to portray all of these traits.

Maggie also believes in the value of visual stimulation in the mathematics classroom. These beliefs are captured in the following response "*...I teach them (the learners) to be able to answer questions ... I tend to go with the method ... they can apply ... if they understand the method it just makes it that much easier ... I would not be able to teach without visuals and I don't think maths teachers can actually teach maths without ... you got to have visual stimulation...maths for me is really interesting, I love teaching maths ... you have to enjoy it as well ...*" (Maggie IE).

Maggie valued the use of technology and recognised the importance of using this technology in the classroom. Her sentiments were echoed by her learners. This was apparent during the focus group interviews. Some of the learners' comments are as follows.

L3 LFG2: "*It (the visual tool) helps you understand better, it helps you understand what she (Maggie) is talking about, if she just wrote text, honestly I would not understand what is going on compared to if she draws a diagram ...*"

L5 LFG4: "*It helps us to actually understand what she (Maggie) is teaching ... because you get to see her demonstrating on the board and showing us what diagrams and how to apply it and then by using things like gestures and pictures she like emphasises what she is teaching us and shows us certain key points.*"

Similar to the other Master teachers in the study, Maggie would like further training and development in the use of technology in the mathematics classroom. This was apparent in the following statement.

Maggie IC: "... yes I did go for some training ... it would always be nice to have further training ... I have been for an intensive course on the use of Geometer's Sketchpad so I know how to actually use it, but as I said it is always good to learn..."

What was also evident in Maggie's lessons was that the use of the smart board made it easier for Maggie and her learners to communicate with respect to diagrams and formulas (Smith & Ferguson, 2004). Maggie's activity system is depicted in Figure 20 that follows.

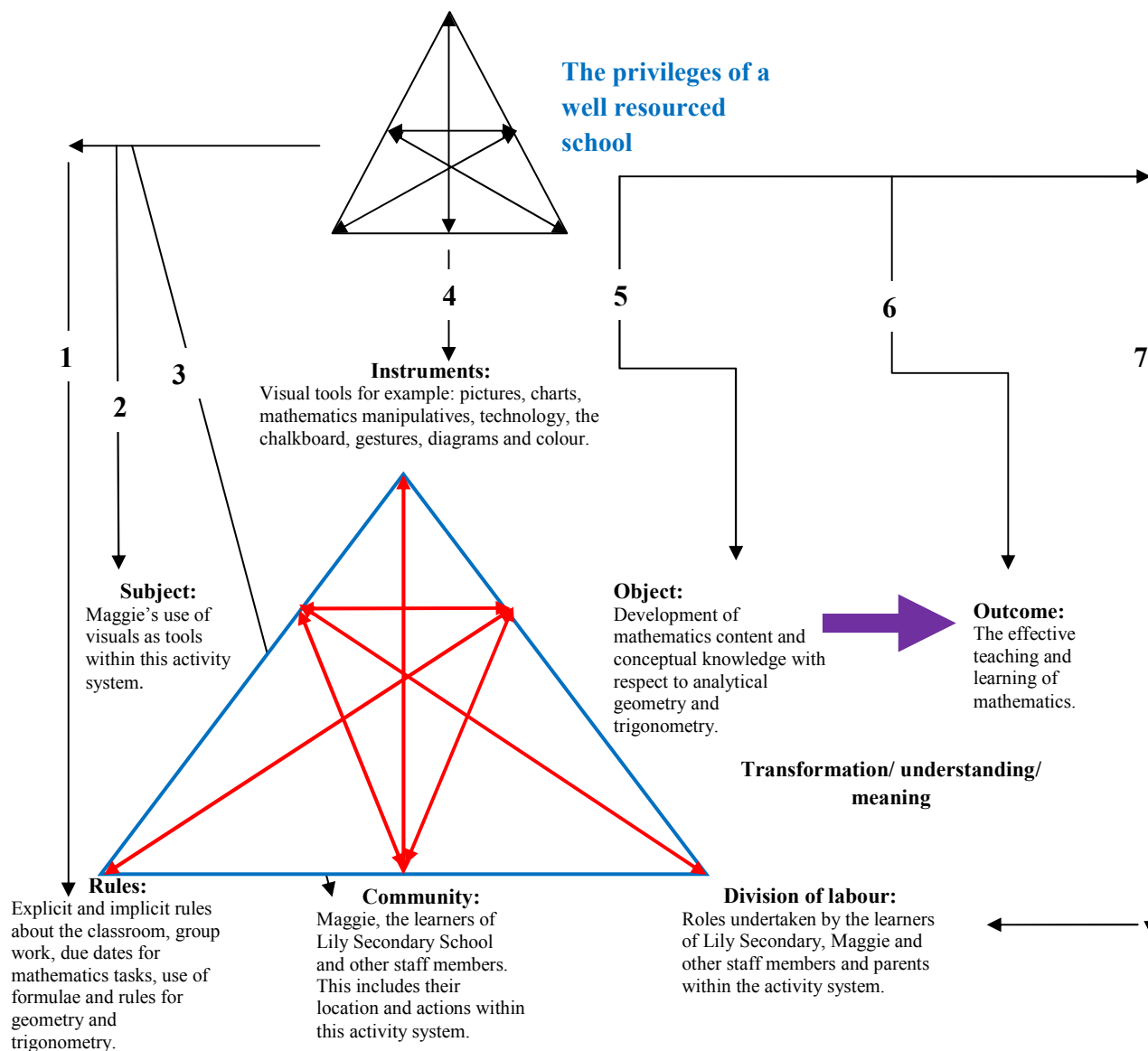


Figure 20: Maggie's activity system at Lily Secondary School

As with Karyn's school, Maggie's activity system shared a similar external influence. Her activity system was positively influenced by the privileges of teaching at a well-resourced school. Every aspect of Maggie's activity system was influenced by the privilege of resources. This has been discussed in detail whilst interrogating Karyn's activity system. Once again, whilst there may have been other external activity systems that could have influenced Maggie's activity system, the activity systems that was of most significance have been acknowledged and discussed.

5.2.7. A broad overview

Based on all the Master teachers' views three common beliefs have been identified. Firstly all the Master teachers believed that the use of technology was important in the mathematics classroom. Secondly, the Master teachers believed that it was essential for mathematics to be visual in order to make mathematics more accessible comprehensible for the learners. Thirdly, the Master teachers were of the notion that teacher training workshops were important for teachers regardless of how experienced or inexperienced they were.

What came out strongly in the interviews and observations was that whilst the Master teachers had their own ways of teaching, their methods were both grounded in tradition and modified as they taught; each Master teacher was willing to share their practice. It was through this practice that they could comment on the usefulness and appropriateness of their tools for specific sections in mathematics. This resonates strongly with the notions of activity theory, whereby human beings mediate their activities using tools.

In the next section, the first research question is discussed. The data collected (from the lesson observations, the field notes, the video recordings, the Master teacher interview and the focus group interview) in the study was employed in an effort to answer the three research questions.

5.3. The visual tools used by each Master teacher

The Master teachers in the study preferred to use a variety of visuals as tools in the mathematics classroom. When interviewed about these tools, the Master teachers mentioned the chalkboard, over head projectors (OHP), over head transparencies (OHT), gestures, the smart board, diagrams, charts, graphs, text books, pictures, mathematics manipulative materials, highlighters and coloured chalk.

Some preferred diagrams and gestures but tried not to use pictures, like Alan who said that he would use gestures and diagrams but will not necessarily use pictures. The following statement captures Alan’s partiality.

Alan IA1 “...*pictures not necessarily, gestures and diagrams yes ...*”

Based on the classroom observation, it is without doubt that these tools form an integral part of their teaching (Anghileri, 2006; Christiansen, 1996). All visual tools that were used as tools during the observed lessons were recorded both on an observation schedule as well as in a field diary. This information was later tabulated for easy reference. Table 11 that follows illustrates the visual tools used during the observed lessons.

Research Question	Alan	Karyn	Dean	Maggie	Penny	Sam
What visuals do Master teachers use as tools within mathematics classrooms?	Gestures Colour Diagrams Signs, shapes, symbols and lines Blackboard Mathematics manipulatives OHP	Gestures Colour Diagrams Signs, shapes, symbols and lines White board Smart board OHP	Gestures Colour Diagrams Signs, shapes, symbols and lines Blackboard Mathematics manipulatives Highlighters	Gestures Colour Diagrams Signs, shapes, symbols and lines White board Smart board OHP	Gestures Colour Diagrams Signs, shapes, symbols and lines Blackboard Mathematics manipulatives Pictures Charts	Gestures Colour Diagrams Signs, shapes, symbols and lines Blackboard Mathematics manipulatives

Table 11: Visual tools used by each Master teacher in the study.

For the purpose of discussion, I have coded the visual tools used under the following categories: Colour and highlighters, diagrams, gestures, manipulatives, technology, shapes,

symbols and lines. These categories will be discussed in detail in Chapter eight of the study.

5.4. Conclusion

This chapter commenced with the Master teacher's stories. Each Master teacher was discussed with respect to the schools they taught at, their professional qualifications and their notions about teaching mathematics. Based on lesson observations, the focus group and the Master teacher interview, activity systems for each Master teacher were developed. These activity system diagrams supplemented each Master teacher's story and were thus unique to the Master teacher. The chapter concluded with a summary of the visual tools that were used by each teacher. A table representing the visual tools that were used by each master teacher was compiled so as to make it easier for the reader to identify the visual tools used in each classroom.

Whilst this chapter has introduced each Master teacher and has provided a list of the visual tools used during the lessons, the next chapter deals with in-depth issues regarding **how** each Master teacher used visual tools in the mathematics classrooms.

Chapter Six: Scaffolding in the mathematics classroom

“Wherever students are, silence, serenity and security should prevail, for such an atmosphere promotes study.”

Bhagavan Sri Sathya Sai Baba

6.1. Introduction

The Master teachers in the study used various visuals as tools whilst teaching mathematics. These visual tools have been identified and discussed in Chapter five. This chapter of the study deals with the manner in which each Master teacher used their selected visual tools.

Based on my observations, the Master teacher interviews and the focus group interviews, I have found that rather than using direct teaching strategies or the traditional approach to teaching mathematics, all the Master teachers incorporated scaffolding techniques to support their learners’ development in mathematics.

Scaffolding, which was introduced as an educational concept by Wood, Bruner and Ross in 1976, describes the support given by a more expert individual during interactions (Sherin, Reiser, & Edelson, 2004). This support can be thought about as a bridge in mathematics. For example, in the real world, a bridge above a river allows a person to move from one side of the river to the other side. If a person learns how to swim and navigate the river, s/he would be able to swim across the river. The person would then not necessarily need the bridge. Similarly, a bridge in mathematics enables the learner to move from one level of mathematical understanding to another level of mathematical understanding. Once the learner is able to navigate the mathematics content independently, the teacher can remove the bridge or scaffold.

Scaffolds are thus seen as tools and strategies which assist in allowing learners to attain a higher level of understanding by encouraging divergent and creative thinking (Brush & Saye, 2001; Mccosker & Diezmann, 2009). In some instances the interactions amongst the learners enhance their experiences and they scaffold each other’s thinking, on occasion the thinking of the group is challenged by the teacher. It is in this way that learners are

supported to think at a metacognitive level. This higher level of thought processing would not necessarily have been achieved if the learners worked independently or used another means of acquiring knowledge and comprehension. Consequently, learners are able to actively construct meaning as they engage significantly within established mathematical practices (Anghileri, 2006). Scaffolding techniques provide learners with opportunities to carry out tasks that they would not be able to do on their own.

The use of scaffolding techniques necessitates that teachers supply their learners with tools that are necessary for learning. Jacobs (2001) suggested that as learners are able to do more and more independently, teachers should then slowly withdraw this assistance. In the mathematics classroom, these tools could include diagrams, pictures, technology, and mathematics formula. Tools also embrace the use of colour, mathematics manipulatives, gestures, and hints for an effective systematic solution process.

6.2. The different levels of scaffolding

According to Anghileri (2007), there are three levels of scaffolding. These are discussed in the subsections that follow.

6.2.1. Level 1: Examining the learning environment

Level 1 incorporates environment provision. Before working with learners, teachers scaffold learning by means of the environment and atmosphere they construct within the classroom. Environmental provision includes the prompts and incentives that exist within the classroom context (Siemon & Virgona, 2003). Level 1 scaffolding relates to the manner in which the teacher organises the mathematics classroom. This organisation includes the use of artefacts. Artefacts embrace pictures and charts used in the form of wall displays. The use of wall displays is a characteristic of a reflective teacher, whereby the teacher uses the wall space to advance active learning (Preen, 2007).

To achieve the same end, the teacher may also use seating arrangements and diverse groupings of learners as a means of making the most of peer collaboration. Peer collaboration may be encouraged by allowing learners to work on engaging mathematical tasks. This collaboration, when combined with the effective sequencing and pacing of the lesson, is beneficial in the teaching and learning of mathematics. Sequencing and pacing refers to the manner in which the teacher moves from one concept to the next within a mathematics topic or strand. Ideally, the mathematics teacher paces his or her teaching in the lesson to ensure the maximum use of instruction time. This technique is often connected with the experience and expertise of the teacher. The more experience a teacher has at teaching a specific concept or topic, the more opportunities are available for cultivating good praxis.

Hand in hand with good praxis is the ability to promote emotive feedback. Emotive feedback refers to the encouragement and affirmation the teacher provides to learners whilst learners are engaged with working on activities. Positive and encouraging feedback creates an appropriate milieu for effective teaching and learning within activity systems. The classroom environment is also positively influenced when learners are provided with a range of different types of activities. This could include free play, structured tasks and self-correcting tasks. Free play refers to the opportunities the teacher affords the learner to experiment with mathematical manipulatives in the classroom. This includes instances for example when learners are allowed to ‘play’ with blocks or tangrams. This play is at first undirected, later on the instruction becomes more formalised and guided. Through the initiation of free play, the teacher provides a valuable opportunity for learners to engage and interact with concrete mathematics manipulatives. This interaction lays the foundation for later development. It is also in this manner that the learner acquires mathematical knowledge through feedback from the more knowledgeable other. Feedback occurs whilst the teacher starts guiding the learner through a more formalised approach to ‘playing’ with concrete manipulatives.

Structured tasks serve a similar purpose. The tasks may take the form of worksheets or directed activities, whereby the learner is provided with instructions and teacher support through the lesson or activity. The other type of task that assists the teacher in providing Level 1 scaffolding is the self-correcting task. Self-correcting tasks may include looking up the answer to a calculation or by using the reverse operation to verify an answer. For example, when the learner is working with problems involving division, he or she may check their solutions by working with multiplication. To clarify in the following example $24 \div 6 = 4$, to check if this is correct, the learner can perform the following computation $6 \times 4 = 24$.

Primarily, the diagram that follows, captures the crux of Anghileri's (2006) notions about Level 1 scaffolding:

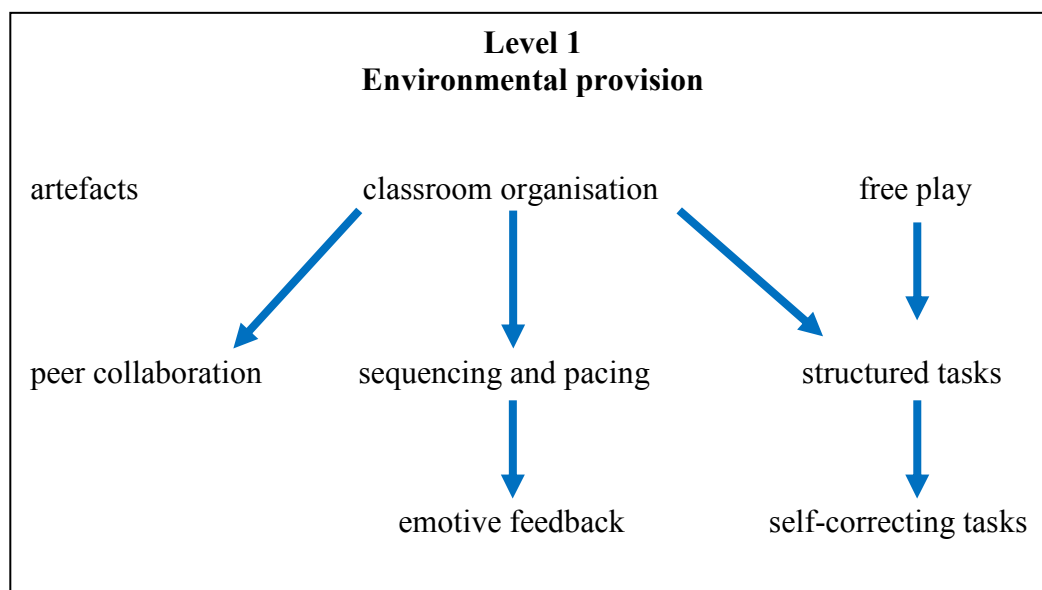


Figure 21: Teacher strategies for scaffolding learning at Level 1
Adapted from Anghileri (2006, p. 39)

On interrogating the data collected in the study, a deduction was made that the majority of the Master teachers (4 out of 6) in this research study utilised pictures, mathematics diagrams and charts as wall displays. The displays depicted mathematics theorems and

proofs and were displayed prominently on the walls of the classroom (See Appendix F for photographs of the classroom walls). The tools that were readily available in all of the classrooms were calculators and coloured chalk.

Through environmental provision, the Master teachers created an engaging and supportive atmosphere within the classroom. The classroom environment in each activity system signalled that these teachers were already supporting the learners as they walked through the classroom doors. Learners were constantly reminded of important mathematics information throughout their time in the classroom. Knowledge of mathematics became a fundamental component of what was expected of them.

6.2.2. Level 2: Exploring the teacher-learner interaction

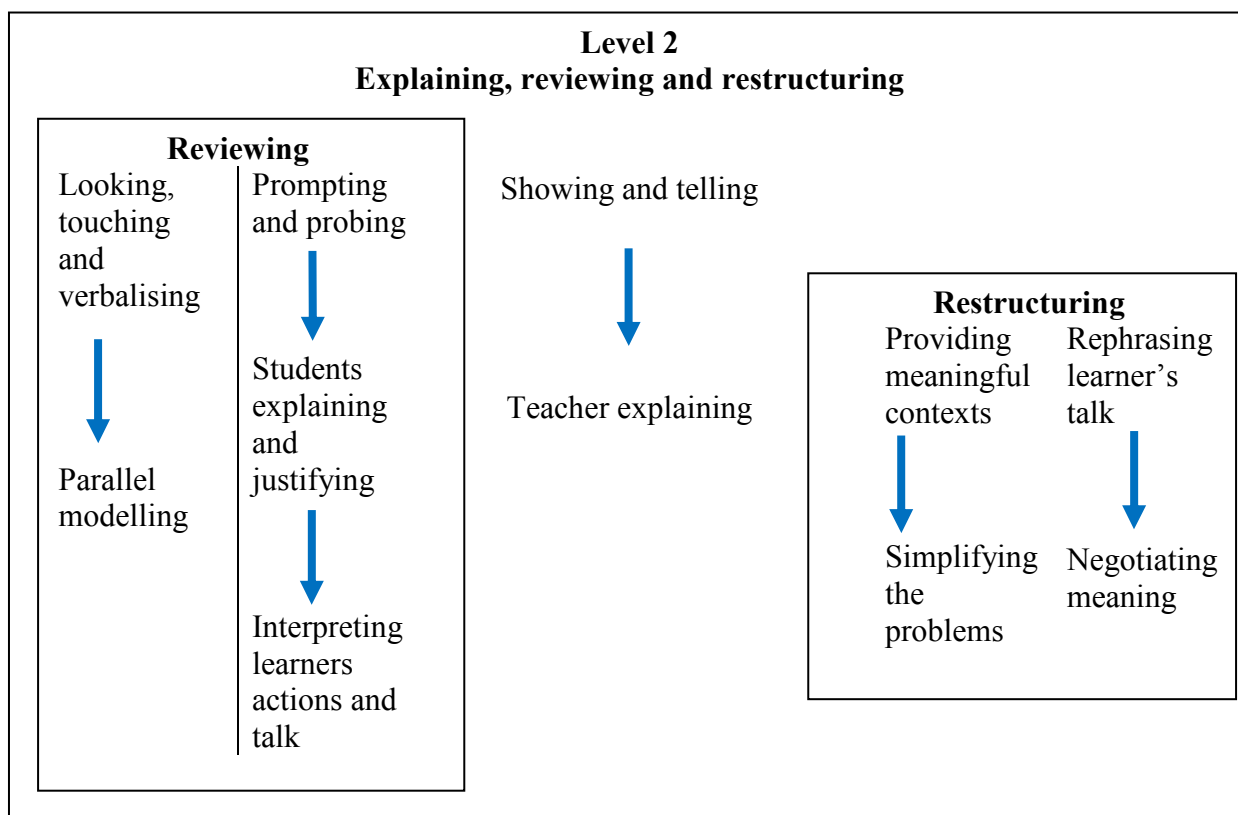


Figure 22: Teacher strategies for scaffolding learning at Level 2
Adapted from Anghileri (2006, p. 39)

When teachers demonstrate, discuss and explain concepts to their learners, this can constrain learners. Learners and teachers need to interact with each other, and there needs to be collaboration between all members of the learning community. Scaffolding at Level 2 includes different levels of teacher-learner interaction. This type of interaction relies on teachers' reviewing and restructuring what is happening within the classroom.

During the reviewing process, learners ought to be encouraged to verbalise what they see and think. The learners need to be motivated to explain and justify their actions and comments. Through interpretation of learner comments, prompting and asking probing questions, teachers have a higher probability of identifying misconceptions and misunderstandings in mathematics thinking and learning. This leads to parallel modelling whereby, based on the teacher's identification of learner misconceptions and learner misunderstandings; the teacher creates and collaboratively solves tasks that share similar characteristics to the learners' problem.

In the restructuring of tasks, the teacher simplifies the problem or rephrases the learners' comments with the aim of negotiating meanings and taking the understanding forward. Meaningful contexts are created so that abstract situations become more accessible to the learner. All the Master teachers in the study incorporated Level 2 scaffolding to varying degrees. The interactions between the Master teachers and learners were focused on specific tasks. Instead of compelling learners to work with mathematics exercises independently, the Master teachers demonstrated and discussed different problem solving strategies and evoked discussion from their learners. Learner comments were probed and learners were pressed to provide meaningful explanations (Mccosker & Diezmann, 2009). These strategies were characteristic of Level 2 scaffolding.

Therefore, whilst the teacher was in control, as is common with traditional approaches to teaching, the Master teachers also involved their learners in the discussion. They reviewed and restructured tasks to accommodate their learners' needs. Showing, telling and

collaborative meaning making (Siemon & Virgona, 2003) ensued. This outcome is similar to Brown's (2008) views that mathematics is created within the classroom whilst being constructed collectively by the learners and teachers.

6.2.3. Level 3: The use of representational tools

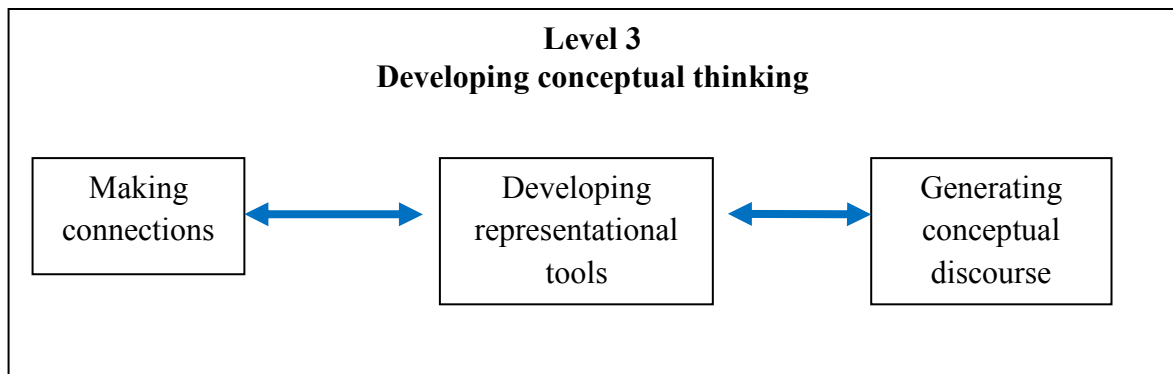


Figure 23: Teacher strategies for scaffolding learning at Level 2
Adapted from Anghileri (2006, p. 39)

Level 3 scaffolding refers to the use of representational tools with the aim of generating conceptual discourse within the learner (Verenikina & Chinnappan, 2006). All the Master teachers used scaffolding to make connections between their learners' prior knowledge and with new information that needed to be assimilated. They assisted learners by using symbolic tools in order to initiate abstract discussions (Siemon & Virgona, 2003). This type of scaffolding conforms to Level 3.

The teachers wanted to use concrete materials to make abstract mathematics more accessible to their learners. When mathematics becomes less abstract to learners, there is a higher level of comprehension of concepts. The Master teachers' concrete materials included pictures, diagrams, graphs, coloured markers, paper, sticks, rubber bands, gestures and body movements. The concrete material that is discussed in this study does not only conform to sensory concrete (physical objects), but also includes integrated concrete

knowledge. Integrated concrete knowledge refers to conceptual knowledge that is connected to other ideas and is built as learners learn (Clements, 1999).

Making connections is imperative in supporting mathematics learning. For example, in the following problem (50% of 80, $\frac{1}{2}$ of 80, 0.5×80), different versions of the same calculation may provide learners with the opportunity to make connections between fractions, decimals and percentages (Anghileri, 2007). Thus whilst scaffolding has become useful for teachers (Verenikina & Chinnappan, 2006), the purpose of scaffolding is to provide learners with a teacher-supported transition. This implies that from seeing and hearing the teacher illustrate a particular mathematics concept, learners are now required to perform the skill independently.

However, whilst the use of scaffolding in mathematics is necessary, scaffolding is useless on its own. It is necessary that scaffolding be complemented by mathematical understanding, together with the ability to think, perceive and analyse mathematically (Lewis, 2010). Based on my observations, there seemed to be an instinctive understanding by the Master teachers in this study that scaffolding was useless without conceptual understanding. Dictated by individual preferences and contexts, each Master teacher used his/her own approach to scaffold the mathematics concepts being taught.

6.3. The Master teachers' use of scaffolding

Read-Griffin and Carter, (2004) claimed that technology has the ability to change immeasurable aspects of classroom instruction. This could influence the amount of resource material or practice examples the learners are exposed to. This could even influence the amount of time spent on specific topics or the amount of teacher talk. Whatever the case may be, one must concede that using technology to support the teaching of a topic in mathematics requires some form of transformation (Ferrara, Pratt, & Robutti, 2006).

For example, in Karyn's class, the transformation was evident by her pedagogic perspective. Instead of using a traditional approach of 'chalk and talk', Karyn embraced technology in her classroom. This notion pervaded her lessons. She used the smart board in her lessons much to the delight of her learners. Her learners seemed to welcome the use of technology and compared her mathematics classroom favourably with that of other teachers at the school. One learner stated that "*...we haven't had smart boards forever, in our other classes we don't have smart boards... it's just ... better now that we do have one ...*" (L4 RFG9). The use of technology within Karyn's activity system made her activity system unique from the other activity systems within the school.

In general the learners' response to her use of the smart board was positive. This was epitomised in the following statement.

L6 RFG9: "*... with the smart board it makes everything much more interactive... you get to do so much more ... it makes it (learning mathematics) so much more fun and it encourages all the other students ... it gives them (the other students) the interest and opportunity to actually know what is happening and then they (the other students) can also be interested in the subject and obviously if you are more interested then you want to learn*"

From the lesson observations and focus group interview with Karyn's learners, the learners appeared to be interested, active and involved in her lessons. Researchers (Moyer, 2001; Tobias, 1994) concur with the notion that learners become engaged in lessons and find mathematics fun because they enjoy tasks which is of interest to them.

Based on the lesson observations, the teacher and focus group interviews, it appeared as if Karyn used the smart board in all her mathematics lessons. She used the smart board as a scaffold to support her learners. In doing so, this became her approach of making the mathematics more understandable for her learners. For example, Karyn taught her class analytical geometry in the first observed lesson. The object of the lesson was to

successfully solve problems using key formulae in analytical geometry (See Figure 15, Karyn's activity system). At the start of her lesson, she provided a worksheet for each learner (See Appendix E for worksheet). A collaborative discussion was encouraged before learners were asked to work independently on the worksheet. This discussion was aimed at supporting the learners' understanding by using the learners' ideas as a starting point for discussion.

One of the questions on the worksheet necessitated that the learners work out the perimeter of the triangle. Whilst repeating the question, Karyn traced out the outline of the triangle. By using this deictic gesture Karyn provided the learners with the first step in the solution process, she provided them with a scaffold that they needed to solve this question. It is in this way that Karyn used her gestures to elucidate and rectify misconceptions (Goldin-Meadow, 2004). Based on her experience as a mathematics teacher, Karyn knew that learners often have problems with area and perimeter. Area is a complex topic in mathematics and learners develop area concepts over time (Clements & Sarama, 2009). She believed that they often confuse one for the other and she used her gestures as a means to avoid this confusion.

On other occasions, Karyn drew shapes (rings and blocks) around specific steps in a solution process or around a particular formula. She used these symbols as signifiers within the lesson. For example, during the lesson on number patterns and analytical geometry, Karyn used coloured blocks and rings to signify the key formulas learners ought to use whilst engaged in problem solving (See Appendix F). Key terminology was also highlighted using these (rings and blocks) symbols. These symbols mediated the learning for the learners in this teaching and learning community. In mathematics, signs mediate two processes, an internal one whereby understanding of the mathematics concept associated with the sign is cognitively contrived (occurs within the learner), and an external one, whereby understanding of the mathematics concept associated with the sign is socially contrived (Berger, 2005). When the teacher intentionally uses artefacts as tools of semiotic

mediation (Bartolini Bussi & Mariotti, 2008) they monitor the transformation of the signs produced by the pupils into mathematical texts (Maschietto & Bartolini Bussi, 2009).

In Karyn's classroom, the learner's mathematical knowledge was both cognitively and socially constituted. Through Karyn's scaffolding technique of offering hints in the solution process, learners realised that these markings signified key elements in a solution process or important information worthy of remembering. The learners used inductive reasoning. They drew inferences from observations made during Karyn's teaching to generalise reasons for why she had made markings. She thus provided the learners with the starting point in either a problem solving process or an initial bit of information required to spur on their thinking process. This information was used as a framework for the current lesson as well as subsequent lessons; Karyn used these markings as a scaffold for existing knowledge.

However, she did not leave the learners with just this scaffold. She provided them with strategies to use this scaffold to the maximum by using other examples to explain to the learners how these highlighted or ringed expressions ought to be used. Her response to a question about why she used deictic gestures to teach was "... *I like to explain with examples...*" (Karyn O9). She created an opportunity for learners to enhance their thinking ability by applying their newly acquired knowledge and skills to other examples and problems.

Karyn used parallel modelling (Anghileri, 2007) when she used examples similar to the ones learners have a problem with. She did this when she reviewed, restructured and worked on the solution until the problem solving process made sense to her learners. She shaped problems and examples stemming from her learners' comments. She supported her learners' understanding of tasks by operating from their ideas. Modelling of correct actions assists the learner in comprehending how good problem solvers use different strategies appropriately (Steadly, et al., 2008). Teacher modelling also motivates learners to achieve

positive outcomes at school (Wentzel, 2002). Once Karyn was confident that her learners could work on their own, she allowed them to work independently. Good teachers do not tell learners too much directly (Tanner & Jones, 2000), rather their commitment plays a key role in learner performance (Howie, 2003). Although Karyn allowed her learners to work independently, she still walked around monitoring her learners' progress and providing support if needed. This strategy is called soft scaffolding (Brush & Saye, 2002) and can be justified using Vygotsky's Zone of Proximal Development (ZPD²⁵). The ZPD concerns each learner's potential to make sense of information or content.

In Vygotsky's theory, the learner works with someone more knowledgeable within the ZPD (Swain, et al., 2002; Swan, 2001) in order to acquire knowledge that is just beyond what they can learn independently (González, Andrade, Civil, & Luis, 2001; Verenikina, 2008). If support given to learners is suitable and significant, a learner's understanding may be extended beyond what they would have understood on their own. There is therefore an inverse association between learner independence and teacher support (Siemon & Virgona, 2003). This suggests that a learner may be able to achieve more when supported by a 'more knowledgeable other' in the presence of mediating tools (Read-Griffin & Carter, 2004) since cognitive development begins with mediation (Adhami, 2006).

Thus, the smart board served as Karyn's mediating tool (See Figure 15, Karyn's activity system) and canvas on which she depicted graphs and diagrams, illustrated the motion of salient points and emphasised important formulae. This type of teaching allowed the learner to see the mathematics in action whilst simultaneously allowing the teacher to stop at certain aspects of the lesson to discuss the relevant processes and procedures (Starkings & Krause, 2007). It was in this manner that Karyn followed a logical method of teaching, moving from the known to the unknown. She frequently drew little sketches or lines to make the mathematics more meaningful. For example, when Karen was teaching analytical

²⁵ Zone of proximal development is referred to as the difference between what a learner can do without help and what he or she can do with help.

geometry during the first video-recorded observation, she started the section by reminding her learners about their previous work on lines and gradients of lines. She was tapping into their prior knowledge base because she was of the notion that learning ought to begin with ample prior knowledge in order to achieve understanding (Myhill & Brackley, 2004).

According to McDaniel (2009), prior knowledge is very important because this is how learners make sense of their world. Roschelle (1995) claimed that it is impossible to learn without prior knowledge because this assists in developing the learning and understanding of new information (Myhill & Brackley, 2004). This is so because learners try to take new information and assimilate with existing knowledge in order to create meaningful information. Thus, the information becomes more accessible because it makes sense and is thus more memorable. This suggests that accessing prior knowledge is a fundamental stage in the learning process (Christen & Murphy, 1991).

Karyn was revisiting the idea about gradients of lines and the direction in which the lines faced. Since the drawing of straight lines by using the gradient is important within analytical geometry, she proceeded to write symbols on the board and drew lines through them as depicted below:



Figure 24: Karyn's use of symbols to represent the slope of a line

Essentially, she used this visual tool as a means of making the concept of gradients and lines easier to remember. Her discussion revolved around how symbols denoting the greater than, less than or equal to signs facilitated the learners' understanding or knowledge with respect to the slope of the line. When examining the first symbol; it is discernible that the direction of the bottom part of the 'less than' sign is similar to the downward slope of a

line. In the second symbol, the bottom part of the ‘equal to’ sign is similar to a horizontal line. With the last symbol the bottom part of the ‘greater than’ sign looks identical to the upward slope of a line.

When asked about her reasons for using this approach to teach the direction of the slope of lines, Karyn believes that by using this visual manner of teaching, it would assist in making gradients and directions of slopes easier to remember. The following statement captures this notion.

Karyn O5: “...it makes it easier for them (the learner) to remember, can you see every time when I draw a line I use the bottom line and when it is like that I use the other side and when it is horizontal I use the straight line. It’s just a way for the learners to remember...”

Karyn supplemented this visual tool by using iconic gestures as shown below. Iconic gestures are gestures that look like their concrete representation (Edwards, 2009). These gestures depicted the direction of the slope of each line.



Figure 25: Karyn’s hand gestures used to represent the slope of lines

The use of visual tools during teacher learner communication effectively promoted knowledge gain (O'Donnell & Dansereau, 2000); to expand, when Karyn used her symbols and her gestures, this increased the chance of her learners adding on to their knowledge base. Learners who were high achievers used a more varied number of strategies to make meaning of mathematics than other students (Harries, Barrington, & Hamilton, 2006). By

Karen's use of visual tools and language she supported the teaching and learning in her classroom. Her visual tools served as a scaffold in the classroom.

Karyn was not alone in using visual tools as a scaffolding technique. In a similar manner, Maggie used the overhead projector, the smart board, coloured markers, diagrams and gestures as a means of making mathematics more accessible to the learners. Maggie used her visual tools as her instruments within her activity system (See Figure 20, Maggie's activity system). Maggie used her pictures as icebreakers and as a means of making her lessons fun. When asked about the reason for using a cartoon character on a worksheet displayed on the smart board, she stated "...*that was just for fun...*" (Maggie O2). Apart from being fun, pictures also play an important role in communicating mathematical ideas (Elia & Philippou, 2004; Harries, et al., 2006).

To elaborate, when a teacher wants to explain an abstract mathematical concept, by using pictures this concept can be more easily accessible to the learner. The picture helps in making the mathematics more concrete. For example, when teaching the concepts area and perimeter, displaying this idea pictorially may make more sense than just verbalising the concepts. By shading in what the area means on a diagram and by marking off on the diagram what it means to calculate perimeter, this will assist in alleviating any confusion that a learner may experience. The concepts of area and perimeter are often confused by learners (Van de Walle, Karp, & Bay-Williams, 2010). The use of a representation assists in communication by alleviating misconceptions and aiding individual understanding.

Apart from using pictures, Maggie also used her interactive smart board as one of her visual tools (See Figure 20, Maggie's activity system). She used her smart board to introduce mathematics manipulatives. For example, she used diagrams of triangles during the observed lesson on analytical geometry. She manipulated the size of these diagrams whilst discussing the concept of area. Based on experience, it is faster, more accurate and easier to demonstrate changes in height and angle measurements using virtual

manipulatives (manipulatives using the computer) than it is to teach using the traditional ‘chalk and talk’ method. The time saved can be spent more profitably on probing learner comments and initiating interactive discussions. It is in this manner that the use of virtual manipulatives advances scaffolding for problem solving (Clements & McMillen, 1996).

Maggie allowed her learners to guide and support her decisions regarding her use of the smart board and the OHP. She urged her learners to provide meaningful explanations of problem solving strategies by asking them to verbalise their thoughts and actions. The use of technology here allowed the learners and the teacher to focus on the screen and discuss the various transformations that only technology allows (Garofalo, et al., 2000).

For example, in the first observed lesson Maggie taught her learners a section in algebra dealing with ‘finding equations of graphs’. This was the object of her first observed lesson (See Figure 20, Maggie’s activity system). Maggie had placed a prepared transparency on the OHP (See Appendix F for photograph of transparency). She had used four different colours (green, red, blue and black) on the OHT. Each colour signified something different. The question was written in green, the graph was drawn in blue, and the axis was drawn in green. The key strategy and clues that Maggie wanted the learners to use whilst solving this problem were written in red. The actual solution was written in black.

Throughout the lesson, Maggie continued to demonstrate and explain key points and strategies to her learners, often going back to draw on their prior knowledge (questions regarding equations of lines, finding gradients and shape of graphs were asked), acknowledging that prior knowledge is the strongest basis for learning (Myhill & Brackley, 2004). Brush and Saye (2002) referred to this strategy as ‘hard scaffolding’. Hard scaffolding refers to support that has been prepared in advance. In this case, the OHT was prepared before the lesson. It was well thought out and designed for that specific lesson. Maggie used hard scaffolding predominantly in the lesson until the learners were confident

of the different types of questions one could be asked in this section, the different approaches to use to answer a question, and the value of given information.

It was through this intervention that learners were guided and supported during the problem solving process; Maggie used her knowledge of colours to scaffold her learners' learning. Maggie's learners valued her approach. This was apparent during the focus group interviews. Her learners' comments are reflected below.

L3 LFG5: *"... you have to use different colours because if you look at something and it's all in one colour its quiet hard to pick out certain things..."*

L6 LFG5: *"... it's going to very difficult for me to differentiate between diagrams and different formulae whereas if is in different colours ... I find it helps me."*

By using different colours Maggie believed that learning would improve since the learners' attention would be focused only on relevant content and there would be no distractions (Rakes, Rakes, & Smith, 1995). This approach to teaching mathematics may be categorised as Level 2 scaffolding, whereby the Master teacher, explained, reviewed and restructured her questions. She did this to attain the aim of the lesson; the different colours helped to direct attention to particular aspects as and when required. The use of colours promoted inductive reasoning within this community of learning. The learners reasoned that when Maggie used different colours their focus was directed onto what she was highlighting on the board. Their reasoning was based on what they had observed during her previous lessons.

In another observed mathematics lesson, Maggie taught a section in analytic geometry. She had displayed an example together with a diagram using the smart board, and learners were asked to spend a few minutes reading the question. Maggie then redrew the diagram and initiated a discussion. She used a systematic method to redraw the diagram, directing her

learners' attention to specific aspects of the diagram by using gestures. As she spoke about the problem, and used deictic gestures to illustrate key points on the diagram, the learners visibly engaged more interactively in the lesson. They began to verbalise their thoughts using fluent mathematical language. Maggie continued supporting their thinking and commenced writing down their thoughts in different coloured markers. Encouraging complex thinking is imperative in concept formation as it allows learners the opportunity to think coherently (Berger, 2005).

Maggie also encouraged her learners to verbalise what they were seeing and how they were thinking; she prompted learners and probed their responses. Encouraging learners to use the language of mathematics discourse promotes better learning and understanding of mathematics (Mercer & Sams, 2006). Maggie was cognisant of the fact that too little support may lead to frustration and too much may lead to learners being removed from the task (Read-Griffin & Carter, 2004), thus when it was apparent that the learners could continue independently with the problem, Maggie encouraged this independence with the problem solving process. She also afforded the learners an opportunity to try out their newfound confidence by providing a second problem for exploration. It was in this manner that her scaffolding tools served as mediators for learning (Read-Griffin & Carter, 2004). As can be deduced, encouraging learner confidence while preserving high expectations is significant in promoting effective classroom practice (Ollerton, 2006).

In the same lesson, Maggie used another approach to achieve similar results. She used her mathematical language to serve as a scaffolding tool. Maggie started talking about the area of triangles and asked her learners to think about calculating the area of triangles (this is generally taught in the grade 9 mathematics classroom). She spoke about perpendicular lines, the base of triangles, heights of triangles and the vertex of the triangle. As Maggie mentioned prior terms, it was evident that the learners understood. It seemed as if they were creating pictures in their minds as they acknowledged the fact that they knew the meanings of these terms. She then asked her learners to verbalise what they were thinking,

she started interpreting her learner's comments and used iconic gestures to prompt others to respond or comment. Through this probing and discussing, using the language of mathematics, the learners reasoned that the areas of the triangles being discussed would be the same if all the triangles had the same height and base. This reasoning followed a deductive approach.

It was apparent here that Maggie used the smart board not only for demonstration but as a stimulus for discussion as well (Ainley, 2001). Maggie wanted to draw on her learners' understanding. She stated "*...if I just write that on the board it could mean anything, so I have to indicate to them exactly what it is ... I have got to show them what it is ... I want them to understand ... the method so if they understand the method behind what we are doing ... then they will be able to answer future questions...*" (Maggie O10).

This is an example of how scaffolding has been used by Maggie to support her learners until they could work independently. After providing sufficient support, she then gradually removed the scaffolding. Once Maggie could see that her learners could go on with the problem solving independently, she stopped illustrating on the board. Maggie also stopped rephrasing student's comments, and she stopped negotiating meanings. She allowed her learners to continue on their own. This led to an enhanced ability to transfer the acquired rules and knowledge to new activities. This indicated that the learners were now learning at a metacognitive level.

On the other hand, Penny used diverse methods to teach her learners. She believed that if her learners felt respected and valued they would participate enthusiastically in the lesson. As mentioned earlier in Chapter 5.2.4., Penny used lessons focusing on human values as the foundation for her lessons. This was apparent in her statement that follows.

Penny O5: "*... I am an Education in Human Values facilitator ... we facilitate the values aspect ... we work with tools that will assist pupils ... one of the main tools is breathing ...*

we encourage them to do deep breathing in order to feed their brain with some oxygen during the lesson but it also calms them down ... it focuses their attention...”

She preferred to focus her learners’ attention so that she could help them explore their understandings. Ollerton (2006) claimed that a learner’s achievement is linked to a teacher’s expectations. In this classroom, Penny expected her learners to participate in her lessons; she expected them to engage with the mathematics being taught, and she expected them to collaborate with each other in her classroom. These were the unspoken rules underpinning Penny’s activity system (See Figure 18, Penny’s activity system). These positive notions manifested itself in her teaching. Her learners were actively encouraged by her attitude to engage with individuals within the learning community in this activity system.

Penny attempted to create a culture of learning in her mathematics classroom that encouraged flexible ways of thinking and working (Harries, et al., 2006). As discussed earlier (Chapter 5.2.4.) she used diverse methods to teach her lessons. She discussed her choice and reasons for using diverse methods in the following statement.

Penny O9: “... *I tend to use stories and I wanted to relate it to something in the lesson ... I want to bring the children into the classroom and I intend to bring out values into the lesson in every lesson ... the interest that children show is amazing ... it ... focuses their attention but I also draw them into the mathematics with it ... children enjoy it because it helps them to remember and also it prepares them for life itself...*”

From the statement above, it was apparent that Penny chose to use stories to focus her learners’ attention and pique their interest. Penny was cognisant of the fact that being interested contributes to learning by invoking deeper types of comprehension processes. This in turn may stimulate more connections and associations with prior knowledge (Tobias, 1994).

Whilst Penny used unusual methods to start of her lessons, based on her exam results (discussed earlier in Chapter five), she had great success with these methods. The reasons behind Penny's use of symbols and pictures were in harmony with those of the other Master teachers in the study; the difference was in **HOW** she used these symbols and pictures. For example, she likened the happy face and positive parabola to that of a happy child. Penny believed that when the learners were faced with parabolas they instantly remembered how she had taught them the aspect of parabolas. She taught them this topic by using pictures and symbols. She used these symbols as her visual tools (See Figure 18, Penny's activity system) to concretise an abstract mathematics concept, the sketching of the graph of a parabola. She stated that "*... the happy face and a positive parabola ... bring in the values of a child being positive ... children remember it ... being positive being happy that is how they do remember it, when they see the positive a, they know, they smile to tell you that's the way the graph is supposed to go...*" (Penny O1).

Whilst one must concede that in mathematics positive and negative parabolas do not exist, within Penny's activity system it was understood that the positive parabola referred to parabolas with the equation $y = ax^2 + bx + c$, and with $a > 0$. This meaning of the 'happy face' was socially constructed within this classroom (See Figure 18, in Chapter five). What can also be observed here is the shift from a visual way of thinking (happy face, symbols) to the analytical way of thinking. This shift following along the lines of the visualiser/analyser model as discussed earlier on in Chapter 3.2.2.2, made the concept of graph sketching more meaningful and accessible to the learners within Penny's activity system.

Moreover, Clements (1999) proposed that when educators speak about making concepts in mathematics more concrete, this does not always imply the use of physical objects, learners at higher grades are expected to have a concrete understanding that goes beyond

manipulatives. This was evident in Penny's classroom when she used the following symbols to concretise the parabola concept.



Figure 26: Symbols used by Penny to make the abstract parabola more concrete

Penny also used pictures and diagrams on the board as a means of making the mathematics more concrete. Whilst it is expected that the use of pictures in a mathematics class ought to be related to mathematics, however, Penny used many kinds of unusual pictures for a mathematics classroom, such as those of dolphins. She did this to assist in changing the mindset of the learner. She wanted them to feel free and uninhibited in the mathematics classroom, as the dolphins do in the wild. Penny essentially used a non-mathematical picture to make learners focus and pay attention to the mathematics being taught. Language and pictures are therefore two distinct kinds of representations which complement each other (Elia & Philippou, 2004).

In Penny's second observed lesson, she taught a section in trigonometry. She drew a Cartesian plane on the board to remind learners of the different positions and signs of angles. This allowed learners to see the 'actual' position of each angle, in this way this section of trigonometry became more accessible. Penny made the mathematics more accessible by supporting her learners' thinking by means of a diagram with which the learners were familiar with from Grade 10 mathematics (Hazzan & Zazkis, 2005). Learners could identify with what was being taught. Penny went on to use deictic gestures to demonstrate movement and orientation of the angles. As Penny articulated the motion, she gestured as well. Therefore, whilst language was a tool for communication her gestures were the action for this communication (Sfard, 2009). The diagram illustrating the Cartesian plane, Penny drew on the board is represented in Figure 27 that follows.

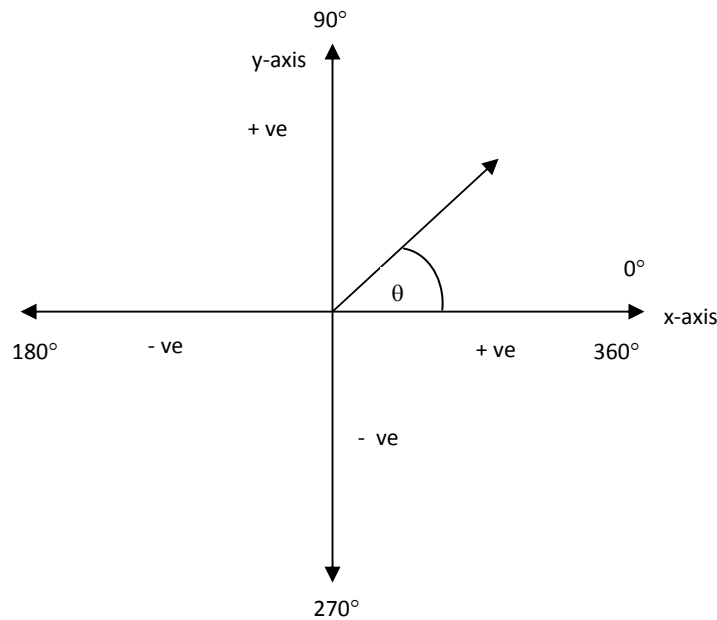


Figure 27: Diagram of the Cartesian plane that Penny used in her lesson

As discussed earlier, Penny used visuals as a tool to concretise mathematics concepts. This was used as a means of scaffolding mathematical thinking in her lessons. As an example, Penny used the diagram of the Cartesian plane to make the mathematics more concrete. The diagram was also used as a way of supporting and guiding learners through the next part of the lesson. For the second part of the lesson, learners were expected to answer questions on a worksheet (See Appendix E VII for Worksheet). The Cartesian plane formed the framework of this activity. Penny, whilst using the diagram, guided her learners during the problem solving process. She interacted with her learners in their ZPD, scaffolding the learning with the goal of assisting them in reaching their higher level of performance (Jacobs, 2001).

As Penny noticed that her learners were becoming more competent and confident in the problem solving process, she gradually allowed the learners to continue with solving the problems independently, thereby creating an atmosphere of shared responsibility between the learners and herself (Read-Griffin & Carter, 2004), until learners gradually took

ownership for their own learning. Penny exhibited all three levels of scaffolding during her lessons with her Grade 11 mathematics learners.

In a similar manner as the other Master teachers in the sample, Alan also used diagrams, gestures and coloured markers as visual tools in his class. Alan believes that learners learn best in an interactive environment. He created an interactive environment that constructed an opportunity for ‘hands on experience’ (Jacobs, 2001, p. 127). As a concrete visual tool, Alan used a sheet of paper (See Figure 13, Alan’s activity system). He folded the paper along different lines of reflection (See Figure 28 that follows).

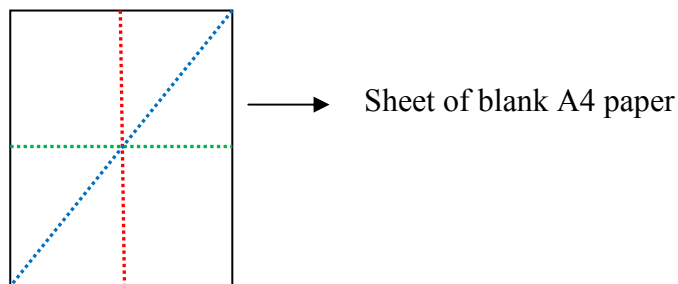


Figure 28: Paper folding used by Alan to demonstrate different lines of reflection

Key:

- Blue dashed line represents $y = x$ (line of reflection)
- Red dashed line represents y axis (line of reflection)
- Green dashed line represents x axis (line of reflection)

He then used deictic gestures to indicate positions of different points as they went through various transformations. The different coordinates were discussed and learners started developing their own rules for transformations about different lines of reflections. It was in this way that this piece of paper was used as a mathematics manipulative. The ideas of rotation, reflection and translation, as part of transformation geometry was reinforced and strengthened. The learners were thus able to grasp the abstract concept of reflection, rotation and translation with the help of this concrete manipulative. This is evidence of how good manipulatives assist learners in shaping, reinforcing and linking different representations of mathematical ideas (Clements, 1999).

Good teachers have the ability to reflect on their own teaching, with the aim of reflecting on practice and making improvements. They recognise the different learning styles of their learners and create instructional approaches that will support these styles (Montgomery, 2001). Gardner (2005) proposes that, owing to their commitment and flexibility in promoting effective teaching and learning, we must identify and keep good teachers. Having taught mathematics for more than 10 years, Alan has had many opportunities to reflect on his teaching with the aim of improving his methods. He realised that learners find it difficult to grasp abstract mathematics concepts, for example, angles on a Cartesian plane. Instead of drawing another diagram to explain what 370° looks like in terms of the different quadrants on the Cartesian plane, Alan chose to use body movements.

He made learners take note of his initial position in the classroom, he then did a full rotation of 360° with his body, he explained what he was doing all the time he was moving. Once learners were comfortable with the concept of 360° , he moved another 10° ; fundamentally, he used his body to represent 370° .

By Alan moving his body in this manner, he taught the learners something abstract by using something concrete and something visual. His reasons for using body movements whilst teaching, is captured in the statement that follows.

Alan IA2: *“...because certain movements or ... gestures bring out more understanding for the child because it can bring up more association than working in a vacuum. You can show the child more than 360 does not mean much, ... as 10 degrees with regards to your position then you can see exactly what is happening and the problem of seeing something foreign is being removed, because the child now understands something foreign is the same as something basic...”*

What was evident here was that body orientations symbolised important resources for establishing consistency; this allowed Alan's learners to appropriately connect his body movements with the concept, and in turn with the words (Pozzer-Ardenghi & Roth, 2005). Alan also used diagrams as a scaffolding tool whilst he was teaching mathematics, he believes that too many rules and formulae may lead to confusion, and if learners fail to remember them then it could become a catastrophe. His notion that since learners used a diagram, which was visual, they would remember it. This was something he accepted as important. The diagram provided the support necessary for learners to work with problem solving in trigonometry. The diagram formed the foundation knowledge that the learners required in order to move further along in the activity. Alan realised that the diagram would only support learners' understanding if it made sense to them, and this was an important consideration.

Alan and the learners socially constructed meaning from the diagram and that social construction allowed learners to complete the task. The diagram was drawn on the board and Alan asked questions regarding the different axes, and queried what would happen if the co-ordinates of points were on different parts of the diagram. He prompted learner comments and encouraged the use of mathematical discourse. He wanted the learners to understand what the signs of each coordinate of arbitrary points would be when positioned in different quadrants of the Cartesian plane. It was evident here that mathematical meaning is not integrated with prior knowledge in a ready-made form. Mathematical meaning undergoes considerable development for the learner, as the learner uses this knowledge in communication with the teacher and other learners. The construction of mathematical knowledge becomes socially regulated (Berger, 2005).

Alan used every opportunity to get learners to construct their own diagrams. Such mathematics teaching and learning is informed by social constructivism, whereby learners actively construct meaning as they interact and participate in mathematical practices (Anghileri, 2006). The environment, the collaboration within the learning community and

the involvement of the more knowledgeable other increases the effectiveness of the learning process. The nuances of Alan's diagram will be discussed in more detail in the Chapter eight, when I discuss **WHY** Alan used this diagram at that specific point in the lesson.

Like all the other Master teachers in the study, Sam used his many years of teaching experience to dictate how he would conduct lessons in mathematics. Sam taught in a school that was not well resourced, he did not have the luxury of an interactive smart board, the overhead projector, white board and expensive manipulative material. What he did have was a black board, the classroom and his body. He used all three as best as he could to make mathematics more meaningful and accessible to his learners. He used all three as his instruments within his activity system (See Figure 19, Sam's activity system).

Based on my observations and the interviews with Sam as well as those with his learners (during the focus group interview), it seems that Sam used visual tools in his classroom as a scaffold for teaching and learning of mathematics. Sam's learners came from disadvantaged backgrounds with limited resources. This had an effect on how Sam taught his lessons. The effects of a disadvantaged background persist throughout schooling for learners coming from disadvantaged backgrounds (Weir, 2001). Mathematics is seen as the key to help them change their lives and change their status, mathematics acts as a social filter (Zevenbergen, 2001a).

Sam realised this need at his school, but he also realised that mathematics needs to make sense to the learners in order for them to succeed. Sam was cognisant of the fact that learners coming into his school had different backgrounds and came from a variety of cultures as suggested by Jacobs (2001). He therefore used visual tools that were easily accessible; he used, for example, bricks as concrete scaffolding tools. Whilst he was teaching number patterns, to make the abstract concept of number patterns more concrete and accessible, he spoke about the bricks on the classroom walls. Sam stated that "... *I*

start from the concrete, something they can see ... they can see the pattern in bricks right in front of them ... I think it makes things easier if you are talking about something you can see ...” (Sam O1).

Scaffolding is a process that supports learners with tasks that the learners may not have been able to accomplish independently (Sherin, et al., 2004). Apart from using concrete tools for scaffolding, Sam also used diagrams as a scaffold for mathematical thinking. For example, when he taught an introductory lesson to trigonometry he used diagrams. By the end of the lesson, learners were expected to calculate trigonometric ratios and identities. He drew different diagrams to serve as support mechanisms for the lesson. The first diagram was used to remind learners of prior knowledge which would be required to solve problems in the current lesson. Sam focused on using a diagram that his learners were familiar with. This diagram focused on all aspects of the circle. His reason for using the diagram is revealed in the following response.

Sam O6: “... *they (the learners) must know when you are talking about a circle ... before I can come up with that system of axes there (referring to the Cartesian plane) ... in the Cartesian plane I am trying now to show them that ... use x and y and r , where does this r come from... they don't understand where we are coming with the radius...it's easy now when I am drawing the radius, they can see now, they can compare ...*”

Sam's diagram is redrawn below:

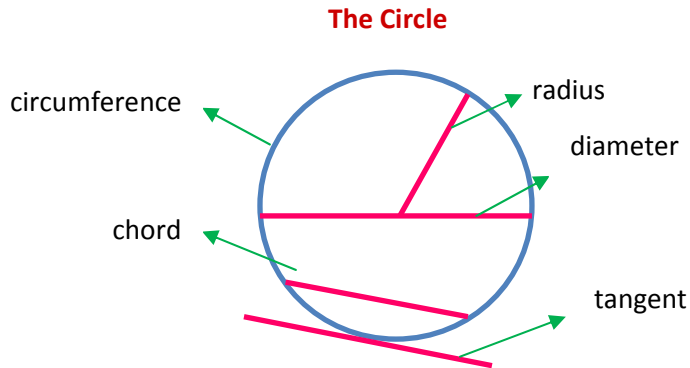


Figure 29: Sam's diagram representing key features of a circle

The diagram became an effective tool to dispense with misconceptions as opposed to verbally explaining to learners. To elaborate, it was easier to point out on a diagram the differences between, for example, the chord, diameter and the radius, than it is to verbalise the differences.

The next diagrams Sam used were two versions of the CAST diagram. This is typically used in Grade 11 to teach learners the positions and signs of trigonometry ratios in the different quadrants on the Cartesian plane. The first diagram displays the positioning of the four basic trigonometry ratios. The second diagram displays the quadrants within which the ratios and their co-ratios are positive. The diagrams are redrawn in Figure 30 that follows.

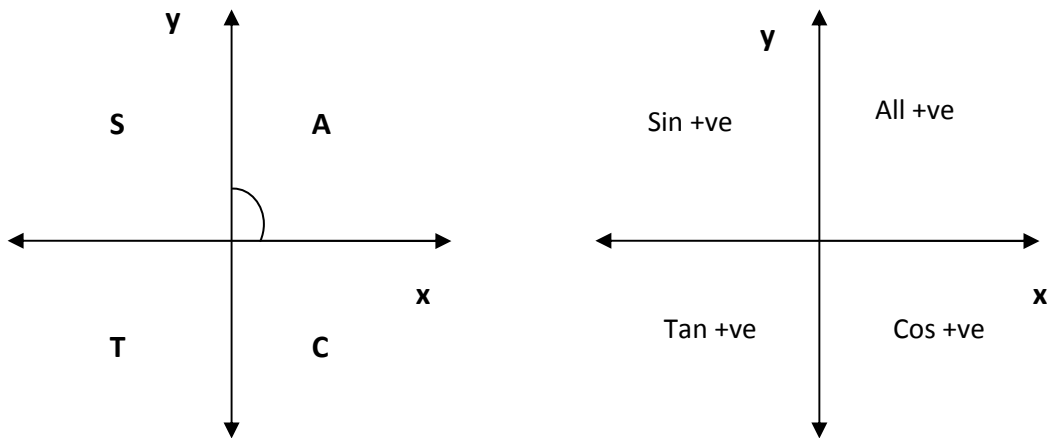


Figure 30: A representation of Sam's CAST diagrams

Sam used the diagrams as a scaffold to support and guide learners during the solution process that followed. He used the different levels of scaffolding. These levels have been discussed in detail in Chapter 6.2. Sam initiated interaction with his learners when he drew his diagrams. Through collaboration with community members within his activity system (See Figure 19), the learners were able to grasp key concepts and understand the mathematics being taught. Sam used level three scaffolding when he used his representational tools to initiate and generate mathematical discussions. He also used the three diagrams as a way of subtly ‘levelling out the playing field’ within his mathematics classroom. Many of his students came from other mathematics classrooms as well as other schools, so he used these diagrams to ensure that they all would have the necessary prior knowledge before moving further with the section. Sam was cognisant of the fact that regardless of how learners were grouped, there is always a difference in the depths of knowledge (Ollerton, 2006).

When learners come into a classroom, they do so with a range of experiences and they come from different cultures. Learners have varying ideas, knowledge and conceptual understandings, some of which may be incorrect. It is the task of the teacher to build or change this existing knowledge in order to increase knowledge retention (McDaniel, 2009). Sam was aware of this need within his own classroom. He stated that “...*we have got different learners ... many learners coming from this informal settlements, ... you cannot leave your learners behind ... I am repeatedly going back and showing them where this is coming from ... so everybody will understand the lesson...*” (Sam O9).

Researchers (McDaniel, 2009; Roschelle, 1995) maintained that tapping into prior knowledge is imperative for the effective teaching and learning because it serves as a sense making tool, whereby learners use prior knowledge to make sense of new knowledge with the aim of merging the two. When learners lack the necessary prior knowledge, they are unable to achieve the aims of the lesson (O'Donnell & Dansereau, 2000). Effectively, Sam

wanted all the learners in the class to be able to solve problems based on the Cartesian plane, he wanted all his learners to participate in his classroom unconditionally.

Hence, Sam's practice tells us that it is the teacher's primary responsibility to scaffold learners' understanding (Read-Griffin & Carter, 2004). Dean had a similar belief; he also used his visual tools as sense making, scaffolding tools. He constructed a stick with coloured rubber bands and used this visual tool within his activity system (See Figure 16, Dean's activity system) to demonstrate reflections about the x and y axis.

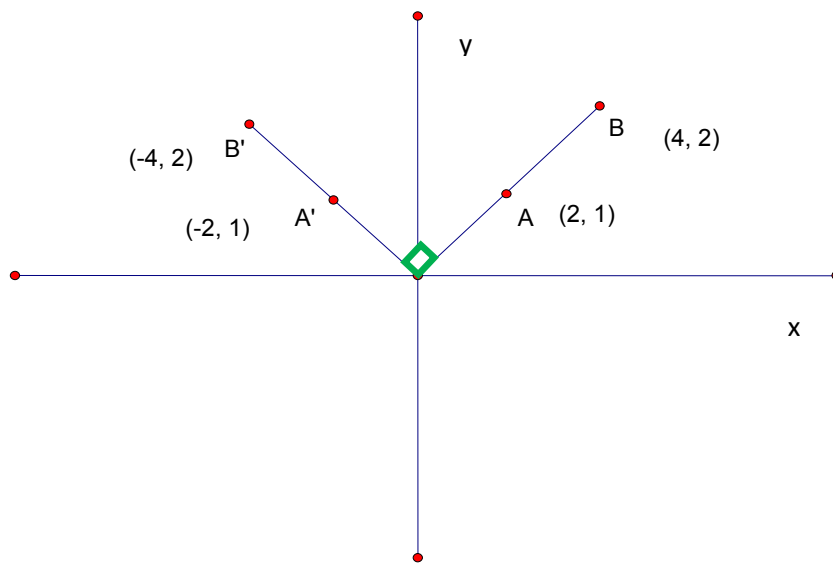


Figure 31: Dean's use of concrete sense making tools

Dean could have chosen to use the traditional approach of teaching by talking his way through the reflection exercise. But using the experience gained from over 20 years of teaching mathematics, Dean realised that learners needed to see something concrete to make the abstract mathematics meaningful. Dean believes that using concrete manipulatives, makes the mathematics concepts and ideas easier to see. He stated that “...by showing them (the learners) the rotation using the stick ... can see exactly how the position changes and I wanted them to see that initially...” (Dean O8).

Dean also used parallel modelling (Anghileri, 2007) to teach this section of rotation. He created and demonstrated the problem solving process; the problem he was working on had similar characteristics to the problems the learners in his class could not work with on their own. They could not rotate points 90 degrees about the y axis. Once he had completed his demonstration, Dean prompted his learners to try to solve the initial task. This was done with great success. Thus with his method of reviewing and restructuring the problem together with his method of parallel modelling (Anghileri, 2007), Dean was able to effectively scaffold the teaching and learning of transformation geometry within his classroom. He made the mathematical rule of rotation visible to his learners so as to improve his learners' understanding (Karadag & McDougall, 2009) of this concept. His learners could now apply their new knowledge and skills to solve tasks that required a higher level of cognition.

6.4. Conclusion

In this chapter, we have discussed **how** the visual tools were used by the Master mathematics teachers. Master teachers used visual tools and concrete tools as a means of scaffolding mathematics thinking and learning. Scaffolding has become popular because it provides an alternative to traditional teaching that has its foundations entrenched in direct instruction (Verenikina & Chinnappan, 2006). The different levels of scaffolding were introduced in this chapter. This was followed by a detailed description and discussion of how mathematics was scaffolded within each activity system.

In the next chapter, aspects pertaining to Sections A, B and C of the Master teacher interview will be discussed. These aspects of the interview schedule dealt with factors leading to the use of visual tools, the preparation required in order for teachers to use visual tools whilst teaching and finally the support and training that was required or necessary for a teacher to use visuals as tools in the mathematics classroom.

Chapter Seven: Motivation, adaptation, preparation and support

*“Students must develop extensive interests. They must visualise wide horizons.”
Bhagavan Sri Sathya Sai Baba*

7.1. Introduction

In Chapter six, scaffolding in mathematics classrooms was discussed. The levels of scaffolding were interrogated and the examples of use in specific classrooms were highlighted. In this chapter, parts of the Master teachers’ interviews are interrogated. Apart from the three critical research questions, the Master teachers were asked various questions pertaining to their use of visual tools in the mathematics classrooms. It was necessary to ascertain their personal views on the use of visual tools.

7.2. Section A of the Master teacher interview

In Section A of the interview, the Master teachers were asked about the use of visual tools when teaching. The discussion centred around whether or not using visual tools advanced learning, the benefits afforded to the learners by using visual tools in the classroom as well as whether or not the teacher themselves benefited from teaching with visual tools in the mathematics classroom. Whilst these issues are developed further in Chapter eight of the thesis, in this section I focus on the Master teacher’s motivation for using visual tools in the classroom.

7.2.1. Motivation for using visual tools whilst teaching

Karyn indicated that she used visual tools mainly to gain her learners attention. She used her visual tools instrumentally. By getting learners to obtain clarity about a concept, she induced interest in a lesson by using visual tools (See Figure 15, Karyn’s activity system). Observations of her teaching showed that she used visual tools on a regular basis, from hand, body and facial gestures to diagrams, technology and different coloured chalk/markers on the board. Much of the notes that learners have received is in a visual form, for

example worksheets and diagrams (See Appendix E for examples of worksheets used). The visual tools used included mathematical equations, worksheets, charts, mathematical symbols, different coloured texts, diagrams; a squiggle on the board, pictures, transparencies, graphs, and manipulatives. Essentially a visual tool incorporated any trigger or catalyst that prompted the understanding of a mathematics concept or situation.

Whilst Penny also used her visual tools (See Figure 18, Penny's activity system) instrumentally to attain her learners' attention, Penny and Sam were of the opinion that visual tools were absorbed into cognition better than words were. Penny also believed that when visual tools are used in the classroom the information would be retained better because the learner was more interested in the lesson when visual tools are used. On the other hand, Sam believed that diagrams supported his verbal message.

Diagrams and gestures were used to facilitate understanding of concepts Sam was talking about in the classroom. For example in the second lesson that I observed, Sam taught a section in trigonometry called Reduction Formulae (See Figure 19, Sam's activity system). He drew the 'CAST'²⁶ diagram to explain to the class the different quadrants in which different angles could be found. He drew the diagram because he believes that diagrams assist learners in seeing and remembering the different quadrants angle could be found in. This was evident from his response that follows.

Sam O8: "*... if you are talking about a 210 degree angle they can see that ... 210 it's in the third quadrant, its 180 +, so if you say 180 +... then it makes it easy to remember...*"

Whilst $180^\circ +$ is a rule, by the teacher verbalising and using deictic gestures to indicate where the quadrant lies, this action now becomes a visual tool that supports the understanding of the rule. This comment is further justified when Sam also intimated that mathematics was more than just about talking; teachers were required to demonstrate what

²⁶ The CAST diagram was presented in Chapter six with a more detailed description.

they are talking about. He stated that “... *in mathematics you cannot just be talking and talking, I need to have a diagram to refer to what I am saying to whatever I am doing...*” (Sam IA2).

Whilst mathematics teachers need to verbalise everything including what was written on the chalkboard and on the OHT, visual representations²⁷ show the most promise for promoting effective mathematics instruction (Steedly, et al., 2008). Sam’s words appear to be linking a verbal dialogue with a diagram. Based on the lesson observation it appeared that learners found it easier to relate to questions of this nature when they had a diagram to work with. By referring to diagrams, Sam was able to illustrate important concepts that he was teaching. Furthermore, Sam felt that he could tell by his learners’ expressions when looking at the diagram, whether they understood a particular concept or not. He stated that “...*I can see in their faces that they didn’t get what I was doing, and then sometimes my emphasis is on the gestures as you can see...*” (Sam IA2).

When learners were confused or did not follow what was happening during the lesson, Sam would respond to this challenge by adapting his teaching strategy to include gestures. On some occasions he would attempt to change the level of his explanation. This reflection in action exhibited that Sam was aware that if instruction was at a different level to a learner’s thinking learning would not occur. This thinking was synonymous with the van Hiele levels of geometric reasoning. The work of two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof prompted the classification of five levels of geometric understanding i.e. visualisation, analysis, informal deduction, formal deduction and rigor.

The first three levels acknowledged thinking and learning that occurred during mathematics lessons at primary school, whilst the next two levels entailed thinking that was required and celebrated during secondary school and university mathematics courses

²⁷ Manipulatives, pictures, number lines, charts, diagrams, graphs and colour.

(Naidoo, 2006). All levels are sequential and hierarchical and are commonly used when teaching space and shape in mathematics.

Table 12 that follows illustrates a synthesised adaptation of the van Hiele theory.

Level	Description of level
Level 1 : Visualisation	The object is seen as a whole, individual properties are therefore not distinguished.
Level 2 : Analysis	The object can be identified by the properties; each property is seen in isolation. Hence there is no comparison.
Level 3: Informal Deduction	The objects are still determined by their properties; however the relationships between properties and figures evolve.
Level 4: Formal Deduction	At this level learners can construct theories and deductive proofs.
Level 5: Rigor	Proofs that contradict intuition may be accepted provided that the deductive argument is valid. The learner is now able to interrogate and work with geometric statements such as axioms, definitions as well as theorems.

Table 12: A synthesis of the van Hiele model
Adapted from Naidoo (2006, p. 71)

Alan similarly saw the use of diagrams as a means to increase understanding. His belief centred on visual tools promoting understanding. He explained that diagrams were useful “... *mainly from the point of view of understanding* ...” (Alan IA2).

Alan’s views were echoed by his learners. This was evident during the learner focus group interviews. Some of the learner comments are reflected below.

L3 OFG6: *“It (the mathematics concept) becomes much easier to grasp, the diagram was like helpful...”*

L8 OFG8: *“... when you draw it is more simplified ... so by actually drawing this diagram it puts everyone in the same level of understanding.”*

Alan also added that when one used a visual tool that related to something the learner already knew this was beneficial. This may indeed be a significant statement because Alan intimated that visual tools helped to link prior knowledge to new concepts (See Figure 13, Alan’s activity system). Knowing and seeing something learners can relate to, is significant, according to him. Dean and Maggie held similar opinions in that they believed that visual tools were a source of stimulation. This visual stimulation initiated and sustained learner involvement in the lesson. This involvement thus assisted the learner in *“... understanding concepts better...”* (Dean IA2).

What came out in this part of the interview was that half of the Master teachers (Alan, Maggie and Sam) stated that visual tools appeared to evoke a relationship between old and new knowledge/information. The visual tools served as a scaffold that supported the teaching and learning of mathematics. The Master teachers incorporated the use of different levels of scaffolding to support their teaching. This has been discussed in detail in Chapter six of the study.

Overall, the consensus was that visual tools made the lesson more interesting. This perception was captured by Maggie’s statement that follows.

Maggie IA2: *“... I think it (using visual tools) makes the lesson much more interesting so that the learners don’t get bored and so they (the learners) don’t fall asleep or stop listening and get inattentive. I think that they (the learners) need a bit of visual stimulation ...”*

7.3. Section B of the Master teacher interview

In Section B of the interview, the Master teachers in the study were asked various questions pertaining to how their visual tools were used in different classrooms and different contexts. This part of the interview focused on whether or not the Master teachers had used their visual tools in the manner in which they initially designed their visual tools to be used. The preparation the Master teachers needed to carry out in order to teach using visual tools in the classroom was also discussed. In addition to these questions, the Master teachers were asked whether they believed that using visual tools in the classroom improved their teaching.

7.3.1. The intentional use of visual tools

From the classroom observations and interviews it was apparent that all the Master teachers used visual tools that they had developed, however each Master teacher used their visual tools with some modification. For instance, Karyn, whilst teaching the concept of perimeter had initially intended using a diagram of triangle. However after asking the class to calculate the perimeter of this shape, and on getting no response, she started tracing the distance around the triangle. She evidenced the characteristics of a reflective teacher; she reflected in and on her action. Tracing the distance around the triangle meant she was finding the perimeter and hence answering her own question. Schön's (1983, 1987) research defined reflection-in-action as the teacher's ability to reflect during a lesson rather than after the lesson.

When asked during the interview why she had used this gesture she replied “... *to make sure that they (the learners) know what is perimeter, definitely ... if you use a word, make sure that they (the learners) understand what you are talking about...*” (Karyn 018). Whilst this may appear to be a trivial arbitrary response, there is something significantly implicit in her words. Karyn has been teaching mathematics for 15 years and has been fortunate enough to have the experience of teaching in different types of schools (ex Model C, private and religiously orientated). What she has encountered is that regardless of the

context in which learners are found, learners often have similar misconceptions. For example, the perimeter of a shape and the area of a shape are often confused. It was apparent that Karyn saw visual tools as an avenue to reinforce concepts and definitions. In addition, she saw her tools as being useful in removing or even preventing misconceptions.

Using her experience and tacit knowledge of learner misconceptions, Karyn traced the outline of the shape. This made the concept she was teaching obvious to the learners. It made the fact that she was talking about the distance around the shape and not the surface area covered by the shape very clear. She had intended using only the diagram of the triangle, but supplemented this with another visual tool in the form of a gesture in order to promote understanding. Karyn resorted to single loop learning. Single loop learning refers to the procedures that occur when the teacher realises that the method or approach s/he is using is not achieving the goal of the lesson.

Other Master teachers in the study also exhibited similar notions. Penny explained that she often adapted her visual tools to achieve the maximum response from her learners. Similarly, Alan explained that his primary objective in the class was to get the mathematics across to the learner. He explained that if this was not happening in the class he would “...*adapt and change*...” (Alan IB1). This adaptation and change would be based purely on what his learners required.

Dean encouraged the use of teacher-formulated diagrams. He believed that the teacher ought to work on the diagram first to see whether it was suitable for the learners within the class. This needed to be done because not all classes or learners were the same or have the same ability levels. Sam and Maggie had similar ideas, but included the need to prepare using lesson plans; CDs and DVDs. Maggie also sensed that it was necessary to change existing software to accommodate the teacher’s individual teaching style and methodologies. This was apparent when she stated that “... *CDs and DVD ROMs but I don’t actually like the way they are presented so I tend to change them so that it is more to*

the way that I teach. My way of teaching..." (Maggie IB1). She believed that very often the notion of 'one size fits all' learning material that are readily available, do not necessary provide as much support within the classroom. The responsibility thus rests with the teacher to 'tailor make' the learning material to suit the needs of the learners within the classroom.

Throughout this part of the interview, it seemed apparent that the dedication and commitment of the Master teachers to their learners was of a high level. The Master teachers admitted to having had experience with success and failure in the mathematics classroom. They were not inhibited by or afraid to discuss their successes and failures. The Master teachers were willing to provide advice on best practice; they were willing to disclose what worked and what did not work in the mathematics classroom.

The Master teachers were comfortable with changing or adapting their visual tools and teaching strategies in order to benefit their learners. They were not afraid to try new ideas if it benefited their learners. This demonstrated their confidence in both their content knowledge and pedagogic knowledge. All the Master teachers displayed evidence of using their tacit knowledge to reflect on their praxis. They displayed evidence of reflecting in and on action.

7.3.2. The preparation required for using visual tools in the mathematics classroom

All the lessons observed during the data collection process appeared to be skilfully thought out, expertly structured and time sensitive. It seemed as though, whilst preparing for the lesson, apart from all the necessary detail to the assessment standard requirements, the teacher also took into consideration the time learners would require for discussion, group work, and individual activities. This strategy allowed the learners the freedom to explore new ideas and discuss alternative strategies without rushing through the content. Moreover, most of the lessons were predominantly learner centred.

All the Master teachers agreed that it did take a long time for preparing effective lessons. This was due largely to the fact that all the Master teachers did a lot of research in order to plan effective lessons. As Dean went on to say that “... *it takes a lot of preparation because you have to use diagrams that are suitable for the class... you can't use ... diagram you used ... in the previous year ... pupils have changed, you have to work for the pupils that you have...*” (Dean IB2). Dean did not believe in using existing material from previous years, but rather he believed in improving and adding on to the materials in order to make it relevant to his current learners. His choice of methods and strategies were influenced by his learners and not content orientated as is the case in most traditional mathematics classrooms. He believed in catering for the needs of his current learners within his activity system (See Figure 16, Dean's activity system).

Alan believed in having knowledge beyond what he is required to teach in the classroom. He proposed that to be effective and successful in the classroom one ought to continuously develop one's self and be prepared for any question. He was also of the opinion that the mathematics needed to be relevant to the learner. This ensured that the learner was more aware of why s/he was learning specific sections and topics in mathematics. This knowledge was a motivating factor in encouraging learners to take an active interest in what is happening within the class. According to Alan, this teaching strategy “... *comes with knowledge beyond the school syllabus...*” (AlanIB2).

In the same vein, most of the Master teachers did not rely on just prescribed textbooks, but also used the internet, computer software, charts and so on. Karyn expressed the idea that “... *you have to play around on the smart board to know all the functions ... so that takes quite a lot of time. Then you page through old exam papers for exam type questions especially for seniors and to page through text books and ... you also have Autograph and different websites ...*”(Karyn IB2).

She believed that it was important to practice all the different motions of the lesson first before going to the classroom. Apart from the technology (smart board) she also consulted past exam papers and other textbooks. This allowed the learners more opportunities to work with different resources and have a wide range of experiences with different types of questions and techniques for problem solving.

Sam felt that although he prepared well in advance by consulting with many resources, on many occasions he found himself resorting to using a visual tool intuitively. He did so without thinking about it. The gesture came almost naturally during the course of the lesson. This notion is reflected in the following statement.

Sam O3: “... *sometimes you know, when you are talking ... use hands or all those things. I think that it is natural ...*”

Gestures were inevitably and intimately connected with the larger phenomena of thinking, learning, acting, and speaking (Nemirovsky & Ferrara, 2009) in Sam’s classroom. Moreover, even when the most basic resource like electricity was not available, this did not deter the Master teacher. Time and effort were still put in to ensure a learner centred lesson. Where some learners would have been told to read the text book or notes to gain understanding, Penny used chalk, diagrams and charts to stimulate understanding in this bleak situation. This was evident when Penny stated “... *I don’t have electricity in my room so we would love to use a laptop and projector ... but unfortunately it is not available ... so I use the charts and chalk and the board...*” (Penny IB2).

Admittedly whilst the Master teachers did talk about the amount of hours used for preparation, they did concede that it was worth the effort. Karyn reflected her thoughts on the issue by responding that “...*it is very time consuming but if you see how much kids enjoy this effort, it’s definitely worthwhile...*” (Karyn IB2). As can be seen the Master teachers considered the effort worth every bit of the time put into preparing for their

learners, due to the learners' positive response to the use of visual tools. This once again highlighted each Master teacher's commitment to their learners.

7.3.3. The ways in which the use of visual tools facilitate and improve teaching

When asked this question, all the Master teachers believed that using visual tools in the classroom did improve their own teaching. Karyn and Sam saw the use of visual tools as stimulating and beneficial from a teacher perspective. Karyn felt that by using visual tools in the classroom, the teachers' teaching could improve. Using visual tools based on her understanding also meant doing more research, gaining more information and improving teachers' own understanding. This was evident when Karyn stated that *"...it can only improve teachers' teaching. The diagrams and pictures, again you put in an effort to get some information ... the teacher also learns..."* (Karyn IB3). This to her implied that the teachers' content, theoretical and pedagogic knowledge base would increase and improve.

Similarly, Sam believed that when teachers use visual tools this in turn leads to a more efficient and effective lesson. Sam stated that visual tools are *"...helping and if you look at these NCS, ... in question papers you will find the pictures, ... a picture that shows about the compound increase ... it's useful to use that pictures as well as diagrams that can enable you to make your lesson more efficient."* (Sam IB3). The other Master teachers saw the use of visual tools as a key factor in improving a teacher's teaching from a learner's perspective. They believed that if the learner benefited in the classroom this is directly linked to an improvement in the teachers' teaching. Alan and Dean felt that the teacher's use of visual tools in the classroom was an excellent way to associate the mathematics with relevant realistic issues. Dean explained that diagrams allow learners to *"... see more..."* which enabled the learners to grasp concepts and *"... make connections with previous knowledge..."* (Dean IB3).

During the interview with Alan, he discussed how the use of visual tools was “... *the best way...*” to make any association (Alan IB3). He perceived that this would be of benefit to both the learners and the teacher. Learners benefit because they will be able to retain more information since they may want to pay more attention in the mathematics classroom. This is so because according to Alan, using visual tools can “... *make it (the lesson) a bit enjoyable...*” (Alan IB3). Similarly, Penny explained that when she used visual tools in her classroom, “...*it improves learner participation...*” (Penny IB3). She explained that her learners were more enthusiastic to work in the class when she used different, interesting teaching strategies, this was apparent when she stated that “...*it motivates them to learn...*” (Penny IB3). Maggie also expressed the same view when she said that “...*using visual stimulation would benefit a teacher’s teaching...*” (Maggie IB3).

Essentially all the Master teachers appeared to be saying the same thing in different ways. They believed that using visual tools facilitated the teaching and learning that went on within the classroom. They believed that visual stimulation improved the quality of teaching and learning within the mathematics classroom.

7.4. Section C of the Master teacher interview

In Section C of the interview schedule, the Master teachers were asked questions pertaining to the support and training required when using visual tools in the mathematics classroom. The majority (5 out of 6) of the Master teachers agreed that they had some training in order to use visual tools effectively in the classroom.

7.4.1. Training required to use visual tools in the classroom

Karyn explained that she had support from other teachers also using the same visual aid – the smart board. They used each other as a support network. Penny and Dean both agreed that they had received training and support from the DoE via workshops and guidance from subject advisors. Dean also believed that “...*the training comes with teaching practice...*” (Dean IC1). In other words he was of the opinion that the more practice you have with

using visual tools in the class, the more effective you become in teaching of the content. He strongly believed in ‘practice makes perfect’.

Maggie expressed her interest in the use of *Geometer’s Sketchpad*. *Geometer’s Sketchpad* is a dynamic software program used to teach concepts and application in geometry. She attended a workshop at the University of KwaZulu-Natal that focused on using this software program in the classroom. Manufacturers of teaching aids for the smart board arranged the other workshops that she attended. The workshops were conducted at her school. The teachers at her school were ‘work shopped’ on how to obtain teaching aids for the smart board. Those that attended these workshops were also introduced to the various websites and internet links that could assist whilst using the smart board.

Overall, the majority of the Master teachers felt that the training and support for using visual tools were necessary, especially when one wanted to use technology and computer software. However they believed that once you were shown the basics of using the visual tools and new technology in the classroom, it would not be too difficult to continue on your own.

7.4.2. Support needed to use visual tools whilst teaching

The teachers unanimously agreed that they needed support to use certain visual tools in the mathematics classroom. Although both Karyn and Maggie have been using the smart board quite effectively in their classrooms, they still felt that they needed further training using the software. Maggie expressed the idea that “... *it would always be nice to have further training on the stuff that I have not been using in my lessons and others out there have been using...*” (Maggie IC2).

Karyn explained that although she was using the smart board she would like to “...*go on a course to use the smart board properly...*” (Karyn IC2). It appeared as though both Maggie and Karyn aspired to be at the forefront of using the smart board efficiently and effectively.

This was indicative of Master teachers who are committed and dedicated to developing themselves professionally, with the ultimate goal of improving teaching and learning at schools.

Alan expressed his eagerness to learn from his peers. He would like to meet with other mathematics teachers so that they could collectively discuss effective teaching strategies. He believed that the more opportunities one has to discuss what works and what doesn't, the better the chances of you picking identifying "... *things that work, things that don't work and different ways to approach something and different diagrams to illustrate a point for something, ... the different charts or picture to show something...*"(Alan IC2). Dean echoed the same sentiments, in that he believed that there was a need for collaboration between mathematics teachers in order to promote effective teaching and learning of mathematics. This was evident when he stated that "... *we need to get practice and guidance on as to how to use this because I think that is one of the ways to go...*" (Dean IC2).

Sam and Penny come from schools that are not as well resourced as the other schools in this study. However, whilst they used visual tools in the classroom, they would like to use digital technology as well. For this, they would prefer support and training. Sam explained that although he does use pictures, diagrams, charts and graphs, he felt that by using a computer it might make teaching a bit easier. He said that "...*using ... a computer, sometimes it can make things easier ... having those diagrams ... in front of them (the learner) so I think that a computer has a precedence ... with math teachers...*"(Sam IC2).

Sam felt that when certain sections were being explained in mathematics, learners would benefit more if they saw what you were discussing. For instance, if you were talking about the parabola in algebra and you wanted to try to explain aspects of the shape of the graph in terms of the value of a , it would be most effective to use a software program to demonstrate this. Penny echoed the same sentiments as Sam and went on to explain that

she would like to master the use of the laptop and data projector, she explained that “...in terms of using a laptop and projector and using the Cartesian plane and moving of graphs etc. I have not mastered those concepts ...” (Penny IC2).

7.5. Conclusion

To sum up, all the Master teachers in the study felt it was necessary to receive some sort of training or support to use visuals as tools in the classroom. They needed support on different levels, some to refresh how they used certain software and computer programs and others required support to learn new strategies in teaching and learning.

Whilst this chapter dealt with issues surrounding perceptions, motivation, preparation, support and training, a more focused discussion on the reasons behind **why** the Master mathematics teachers used visual tools will be discussed in the next chapter.

Chapter Eight: Exhibiting principal themes on the use of visual tools

“Carry out your duties, as teachers in the spirit of dedication, love and service, and stand forth as bright examples to the country and the world.”
Bhagavan Sri Sathya Sai Baba

8.1. Introduction

In Chapter seven, the motivation, adaptation, preparation and support required for using visuals as tools in the mathematics classrooms has been focussed on. In this chapter reasons provided by each Master teacher regarding their use of visuals as tools is illuminated. Whilst observing the Master teachers in the classroom, I noticed that they went about accomplishing their aims during each lesson in a sequential manner. It appeared as if they were following a familiar ‘recipe’. I use the word familiar because it seemed as though they were well acquainted with the ratio of the ‘different ingredients’. It would seem that due to their experience (an average of 20 years); they knew what needed to be done and how to approach different situations. When asked what preparation he required to use visual tools in the classroom, Alan stated that “... *again it comes with experience...*” (Alan IB2).

Montgomery (2001) associated these types of teaching strategies to good teachers and further elaborated that good teachers develop instructional approaches that will accommodate the different learning styles of learners. Good teachers cater for the needs of individuals as well as the groups by continuously modifying their approaches in response to the feedback they are receiving from their learners (Tanner & Jones, 2000). All the Master teachers in this study used different teaching strategies to assist with achieving success within their individual activity systems (See individual activity systems of all the Master teachers in Chapter five). Each Master teacher used visual tools as their instruments to engage learners within the classroom. Both explicit and implicit rules were adhered to during this engagement. Rules may be seen as a part of the culture or they may be developed as the (mathematics) community within the activity system work collectively.

Mathematical learning is viewed as a process of acculturation (Barnes, 2005). The use of culturally modified tools allows meanings to be made, distributed, and transformed within the learning community²⁸ (Cooney, et al., 1998). In this study, the Master teachers achieved the outcome of each lesson via mediation between the visual tools and the object (teaching and learning of a mathematical concept).

Teacher-learner and learner-learner interaction was encouraged in all the observed lessons. These interactions provoked further questions and discussion (Mercer & Sams, 2006). The learning community within each activity system had a common understanding regarding the visual tools used and how these visual tools were used in the different circumstances; this understanding was negotiated within each context. This action by the learning community is consistent with what Kuutti (1996) proclaimed to be an activity that is mediated by culturally created tools. The teachers had different notions with respect to why they used visual tools. I used thematic analysis to capture the dominant notions present in the interview transcripts (Franzosi, 2004). The preponderance of emerging ideas and notions resulted in the need to categorise these ideas into themes. These notions and ideas are discussed in the themes that follow.

8.2. Theme 1: To make mathematics more concrete, accessible and comprehensible

8.2.1. Preamble

With the increasing need to encourage and motivate learners to become mathematically literate, the use of visual images has become popular (Silverman, 2001). The use of visual tools assists the teacher in making mathematics more concrete and accessible to learners. With the concretisation of abstract mathematics concepts, mathematics becomes more comprehensible. Whilst instruction ought to develop concretely preferably with the use of visual tools, this does not guarantee meaningful learning (Sarama & Clements, 2009).

²⁸ In this study the learning community encompassed the Master teacher and learners.

Learners may need concrete objects to build meaning initially, but thereafter they are required to reflect on their use of the concrete objects. This reflection assists the learner in developing a meaningful understanding of concepts and the ability to think at a higher level. Thinking at a metacognitive level is targeted and encouraged by teachers.

Meaningful learning also depends on the teacher creating a culture of mathematical learning within the classroom that supports the social construction of meaning. Learners ought to be encouraged to work in collaboration with other members of the learning community to construct shared meaning and understanding socially. Likewise, Davis (1999) maintained that it is important to remember that learners learn best when they are actively involved. Meaningful learning demands that new knowledge and concepts are compatible with the learners' existing knowledge banks. This meaning making, understanding and collaboration may differ from one activity system to the next. To assist the learner in assimilating new knowledge, the teachers in the study used different visual tools (See individual activity systems in Chapter five for the list of visual tools) as concrete tools to make mathematics more accessible to the learners.

8.2.2. Focussing on concrete knowledge

Whilst the term 'concrete' brings to mind physical objects that learners can hold in their hands, the term in this study incorporates two types of concrete knowledge (as discussed earlier in Chapter 6.2.3), sensory concrete knowledge and integrated concrete knowledge. Sensory concrete knowledge necessitates the use of sensory tools (tools that we can touch and feel). For example, in this study Dean and Sam used concrete visual tools in the form of bricks, sticks and rubber bands. Sam used examples like the bricks on the wall to demonstrate what a number pattern was. He believed that if they (the learners) see the patterns on the walls, they could then associate it with the mathematical concept.

Therefore, whilst abstract concepts are fundamental to mathematics, they make mathematics difficult to grasp. Sam's need was to concretise the abstract concept of

number patterns in order to make this concept more comprehensible. Whilst, these manipulatives have an important place in the teaching and learning of mathematics, their physicality is not essential in supporting meaningful learning. Realising this notion, the Master teachers also used gestures, diagrams and symbols as visual tools in their classrooms. These types of visual tools stimulate a learner's integrated concrete knowledge. Integrated concrete knowledge refers to knowledge that is connected in different ways (Sarama & Clements, 2009). A mental structure interconnects existing knowledge, the physical object, its manipulation based on different rules, procedures, and symbolic representations with new knowledge. This is how learning occurs.

8.2.3. The use of different colours and highlighters

The teachers used colour as a useful tool to assist in increasing learner understanding of concepts because colour stimulation improves performances (Imhof, 2004; Kercood & Grskovic, 2009). The majority of the Master teachers in the study (83%) used colour to make mathematics more accessible to the learners. By using colour as a visual tool, the teachers felt that they could make mathematics easier to work with and more comprehensible. To find meaning, the learner could now mentally manipulate the aspects that colour represented, on the board. This manipulation supported the teaching and understanding of the mathematics. Colour presented a focus for the learners. Their attention converged to sections of equations or diagrams on which the teacher used different colours. The use of colour also alleviated confusion and acted as a scaffold in the teaching and learning process. This was evident in the following statement made by Penny.

Penny O8: “... *it does not confuse them as to where the focus is...*”

In instances of colour use on diagrams, the colour helped learners in diagram segmentation. Different sections of the diagram were identified by using different colours. For example, whilst teaching Maggie indicated parts of the diagram that were equal in red, and sections of the diagram that were parallel in blue. Her use of colour assisted learners in making

sense of the diagram. The facility to make sense of a diagram is exceptionally important in developing mathematical thinking. Whilst a diagram provides the learner with a physical representation of a mental image (Mudaly, 2010a), using different colours makes it easier for learners to see aspects within a mathematics diagram. In Sam's classroom, he used different coloured chalk to indicate the concept of special angles in his diagrams. Sam maintained that "... *I am using coloured chalk and it is helping them (the learners) a lot because they can see ...*" (Sam O12). He also used red chalk to indicate the process of rationalising the denominator whilst working with trigonometry equations.

In the study, different colours were used for specific reasons and did not necessarily signal aesthetic value. The learner was required to decipher what the reason was. The learners reasoned inductively based on their observations guided by the use of colour. The reasons could be that the angles within the shape are equal or the section that is shaded or coloured in needs to be calculated. The responses that follow indicate a selection of Master teachers' responses regarding the use of colour in their lessons.

Sam O12: "... *I marked, they are equal by putting coloured chalk on the board, I am just showing them...*"

Karyn O19: "... *I think ... colour it in ...*" (referring to the diagram of the triangle, when asked how she would illustrate that the area of the triangle needs to be calculated).

Whatever the reason, the learning community was required to mediate the use of this visual tool. Sometimes it was necessary for the teacher to negotiate reasons for use with the learners within each activity system (See Master teacher's activity systems in Chapter five). This mediation of actions through the use of visual tools were imperative for cognitive development (González, et al., 2001). Through this type of mediation, learners were then able to construct their own understanding of mathematical concepts and ideas.

I found that in this study, this type of mediation was prevalent in geometry more so than in any other aspect of mathematics. Concepts of size, shape, position, and properties of figures in space are synonymous with geometry. Since this strand is generally wordy and lengthy, learners have problems understanding or following geometric arguments. This sentiment was echoed by Maggie when she stated that “... *geometry ... is quite wordy almost like writing an essay...*” (Maggie O7). Learners were required to communicate their understanding using words in a logical order. This implied that structuring and restructuring of available information needed to occur in order to achieve mathematical success.

Additionally, the use of colour attracted the learner and acquired the learners’ attention. These beliefs are captured by the following statements.

Penny O8: “... *it attracts children ...*”

Karyn O10: “... *to get the learners attention ...*”

Karyn O11: “... *when you have colour on the screen you tend to just get their (the learners) attention ...*”

Karyn’s responses captured above indicate that she strongly believed that in order to achieve understanding, the first step was to attain and maintain focus and attention. Her response was in harmony with what Back, Brooksbank and Faux (2007) proposed in that learners learn best when they enjoy their learning in a relaxed, positive and focussed environment.

The Master teachers also used the idea of highlighting with different colours to illustrate important formula or key words. The following comments substantiate this claim.

Karyn O14: “...*highlight or take out your keywords... which are the most important words, highlight them and from there you choose your formula and then do your calculations...*”

Dean O6: “... *want to stress something that is important...*”

Based on the focus group interviews, the learners were of the same view as their teachers with respect to the use of highlighters and colour. This is reflected in the comments below:

L5 DFG14: “*He uses it (colour) for the important stuff.*”

L6 DFG14: “*It (colour) makes important stuff stand out.*”

L7 RFG3: “... *she (Karyn) uses the colour to separate the pieces that go together ...*”

Maggie used different coloured markers to illustrate important aspects of diagrams, for example, perpendicular lines, equal angles, equal sides and 90-degree angles. Maggie also used colours to illustrate the formula, the question, as well as what required solving. Whilst Maggie did not reveal the reasons for the use of the different colours to the learners, the community within this activity system mediated and successfully negotiated this tool.

A similar mediation occurred in Dean’s classroom. When Dean highlighted using coloured markers/chalk, his learners knew that the highlighted words, rules, and formula were important to the section. This was evident based on what Dean had said during the interview. Dean stated that “... *I think that it is beneficial ... I highlighted only certain parts ... so they (the learners) know exactly which parts are undergoing some transformation ... instead of giving them just a set of notes ... they can see exactly which part we are manipulating...*” (Dean O5).

A common thread in the reasoning of the Master teachers was that using colours was a mechanism to alleviate the tedium in a mathematics classroom. Thus they introduced the idea of using different colours to highlight differences, similarities and important information. To elaborate, to assist the learners and mediate understanding, these Master teachers used different colours to differentiate one concept from another. This is affirmed by Penny’s statement that follows.

Penny O8: “... it also separates concepts...”

Colour was also used to indicate or signal the different steps involved within the mathematical process. Maggie stated that “... it’s my way of distinguishing either one step from the other or a question from the explanation... I tend to use different colours **just** to highlight the differences...” (Maggie O7).

Whilst Maggie used the word “*just*” casually as if to signify that it was not anything spectacular that she was doing, she did indeed use colour very strategically, which warranted more credit.

Maggie used coloured markers to represent different steps that needed completion in order to achieve a solution. She also used colour to indicate something new, this is affirmed in the following statement. Maggie O1: “... using the red as a new set of axis...”

It would prove difficult for learners to distinguish between the different steps without the use of different colours; differences and similarities would be concealed. For example, when working with numerous diagrams sketched on the same axis as is commonly done in algebra, geometry and trigonometry, the diagrams drawn become difficult to interpret and identify if they are all sketched in one colour. When using the different colour it would be unproblematic to identify the midpoint of the blue line segment, or the tangent to the red circle.

It was evident that in this study when the learners saw colour in the mathematics lesson, it signalled an important aspect. The learners responded by focusing their attention on the coloured object. This finding was in harmony with what Kercood and Grskovic (2009) proposed that adding stimulation (e.g. in the form of colour) to mathematics problems could assist learners in focusing their attention to relevant details within that task. Likewise, Imhof (2004) maintained that colour stimulation improved performance in mathematics.

8.2.4. Using diagrams whilst teaching

The Master teachers in this study used diagrams as visual tools, but they used this visual tool diversely. Sam used diagrams to concretise concepts and mathematical ideas. He believed that teaching ought to expose learners to something that is substantive rather than just abstract. Learners ought to relate to something before moving on to something abstract or new. Sam stated that “...*I start from the concrete, something they (the learners) can see...*” (Sam O1). The ‘seeing’ played an important part in Sam’s philosophy of teaching. Sam believed that his learners were required to know what he was teaching and talking about. This notion was evident in the following extract.

Sam O6: “... *they must know when you are talking about a circle what is involved... that in a Cartesian plane you can use x and y and r , where does this r come from, ... so now after we have done that (introducing the circle with the key aspects labelled)... it’s easier now too refer to ... Cartesian plane ...*”

Using diagrams when solving problems actively engages learners in meaning making (Mudaly, 2010a). What is important here is the need for learners to be diagram literate in order to unpack the nuances of a diagram.

Alan used diagrams to assist the learners with completing mathematics exercises. His task originated from a section in transformation geometry. He wanted his learners to find an arbitrary point that was positioned along the line $y = x$. Based on his experience, he believed that if he did not illustrate what is required, learners would have a problem comprehending what was meant to be done. Alan stated that “... *experience tells me that these kids won’t be able to do it ... if they draw the line skew then it is wrong according to the equation...*” (Alan O1). Alan used the following diagram to illustrate how learners could go about finding any point positioned along the line $y = x$.

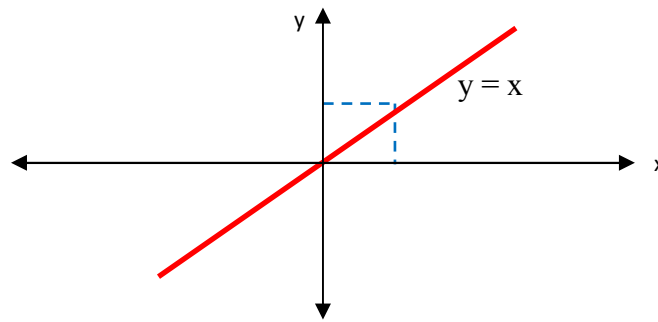


Figure 32: A representation of Alan's graph of $y = x$

The learners could solve questions of a similar nature through the use of this diagram as an example. Essentially, Alan used this diagram as a method to enhance the learners' ability to apply their new knowledge to other examples and situations. Learners were now functioning at a higher level of comprehension because of this visual tool.

Other researchers (Stokes, 2000; Thomas, et al., 2008) also claimed that using visual elements in teaching yields positive results and enhanced teaching. Karyn believed that using pictures and diagrams assisted in improving comprehension in mathematics. She guided her learners by using systematic instructions. Karyn stated that "... *I always say if they get an exam paper or test and if there is no picture with it, you draw a picture or a diagram ...*" (Karyn O20).

The drawing of self generated diagrams makes it possible for learners to alter their drawings whilst exploring possible scenarios within the problem situation (Nunokawa, 2006). Karyn believed that drawing a diagram would assist her learners in the comprehension of the problem, this notion is in accordance with what Mudaly (2010a) asserted. Karyn used an example of a defined instructional sequence to assist her learners in comprehending the task. The drawing of a diagram was identified as the first step to follow when solving a mathematics problem. The next step was identified as the linking of new ideas and concepts with old ones. This strategy of systematic and explicit instruction promotes the effective teaching and learning of mathematics (Steadly, et al., 2008).

Maggie focused on the need to illustrate all possibilities in the classroom. When she was teaching she wanted to ensure that all interpretations of a problem were considered. The following statement is indicative of this notion.

Maggie O12: “... *to indicate to them how it actually looked in the different types of triangles. They actually need to be able to visualise and see ...*”

Learners are required to see relationships in mathematics, so that they are aware of how to link ideas and concepts (Rakes, et al., 1995). This linking of ideas and concepts was necessary for attaining understanding. Maggie also believed that using visuals as tools assisted her in the teaching of mathematics, she stated that “... *at some point we need to draw it so that they see it visually...*” (Maggie O12). Similarly, Rakes, et al., (1995) claimed that visual tools play an important part in reinforcing and supplementing content. Moreover, Modally (2010a) argued that it is not simply the act of drawing a diagram that establishes better comprehension, but rather, the diagram ought to be constructed as a self explanatory tool in order to sustain enhanced solutions.

8.2.5. The use of gestures

Apart from using colour and diagrams, the Master teachers used gestures to make the mathematics they were teaching more concrete, accessible, and comprehensible to their learners. The Master teachers in the study used different types of gestures diversely. Alan claimed that he used gestures to assist the learner in comprehending particular concepts. He believed that in the mathematics classroom, language becomes an issue, because in most cases the language of learning and teaching (LOLT) is not the learner’s first language. When his learners saw his body movements, they were able to connect the movements and actions with the words. Likewise, Setati, Molefe and Langa (2008) suggested that this phenomenon was evident in the majority of the classrooms in South Africa, because English is not a main, home or second language of the learners and teachers.

Within Alan's activity system, words eventually started replacing gestures (Radford, 2003). Once learners became more familiar with the mathematics register, they became more confident with its use. The mathematics register encompasses words from the English language that has a specialised meaning in mathematics, for example 'power', 'difference' and 'similar'. This register also includes words that have been borrowed from other languages, for example 'quadrilateral' and 'isosceles' (Orton, 1992; Pimm, 1987). The more competent a learner is in English, the more competent s/he is in mathematical reasoning and comprehension of word problems. Language is thus important for conceptualisation and reasoning in mathematics (Vorster, 2008).

Similarly, Mercer and Sams (2006) claimed that improving the learner's use of language for reasoning improves a learner's understanding of mathematics. This kind of reasoning could apply globally, for example when travelling in a foreign country. You may not know the language but if you wave, the foreign national would understand that you are greeting them. Even a smile is generally regarded as acknowledgement or friendliness; when one smiles this discloses pleasure (Goldin-Meadow, 2004).

However, the smile meant something different in Penny's class. The community within this activity system had mediated and negotiated their own understanding of the smiley and sad faces. They had this 'shared understanding' so that they did not find the need to draw or write down this information anymore. Penny confirmed this notion by stating that: "... when they see the positive a , they know, they smile to tell you that's the way the graph is supposed to go..." (Penny O1). If another teacher from a different context came in to teach the class, the new teacher would not necessarily have made the connection between the learner's smile and the 'positive a '. They would have most probably seen this (the learner's smile) as learners being pleasant, happy or embarrassed because they did not know the answers. This is about students learning from their experiences. The transformation of experiences creates knowledge (Kolb & Kolb, 2005).

As can be seen the meaning of the gestures in different contexts may be different because the contextual composition of situations are different. This suggests that the same gesture may have a different meaning in different situations (Williams, 2009). Similarly, Alan was aware that at certain times when you talk in the class and explain mathematical concepts, learners are thinking about something very different. To alleviate this confusion, Alan used a variety of gestures in the classroom. He believes that when learners see they understand.

This is substantiated by the following statement.

Alan O10: *“...it is a question of visualisation, the learner can see exactly what you are doing ... sometimes you are saying certain things and the kid is thinking of something totally different ... if he thinks of something else his entire trig would suffer...”*

Sometimes Alan needed to represent the objects and sometimes he needed to show the location of objects (Clark & Van der Wege, 2001). He used deictic, iconic, and metaphoric gestures to assist learners in understanding concepts that may otherwise be misunderstood.

8.2.5.1. Using deictic gestures

Alan used deictic gestures when he explained the effects of reflection on the x and y values. His purpose was to emphasise what needed to be done; he believed that if the learners hear and see, it would make a bigger impact on the learners' understanding. Essentially his actions reinforced what he was saying. This purpose is reflected in the following statement.

Alan: O6: *“...the moment you point ... focus is more towards that concept that you are trying to highlight ... there are times I will say it and I will go and point that's what you need to do...”*

Dean also felt the same way about his own gestures, he stated that *“...if you have a look at the gestures ... they correspond to what I am saying... I am basically acting all that I am*

saying...” (Dean O10). Thus, in these classrooms words and gestures were symbiotic and acted as a prop for each other (Sfard, 2009).

Karen also had a tendency to use deictic gestures whilst teaching. She often pointed to the board or at terms or expressions during the lesson. She believed that her learners ought to visualise something in order to grasp the concept and remain focused. She was also intent on not allowing her learners to be bored during the lesson. These views are captured in the statements that follow.

Karyn O9: “... *here again they need to visualise it, to see it ... if you just talk to them they might fall asleep...*”

Karyn O26: “...*it might be boring ... I think if you point you can just get their attention ... this is what I am talking about...*”

According to Tobias (1994) when teachers talk about boredom in their classroom, this usually means that the learners are unconcerned about the curriculum. Karyn did not want this to be the position in her classroom. She wanted her learners to value and take hold of the importance of mathematics. She wanted mathematics to empower her learners to be the best that they could be. Her philosophy of teaching mathematics to achieve these ends came out strongly in her interview.

Penny used deictic gestures to show position and shape of graphs. Whilst working with graph sketching, Penny believed that, the only way to teach changes in the standard equation for the parabola was by means of hand gestures. This belief is encapsulated in the following response.

Penny O2: “...*initially we had taught them the aspect of q , which moved up and down and when it was positive we taught them... the positive part of the y axis is moving up and down ... using my hands was the only way I could do ... it's basically coming down to their level. I honestly believe they understand my movement and visuals...*”

Whilst one must concede that in mathematics, the movement as described by Penny is not on q and lines cannot move ‘up and down’ because it will coincide with itself, but rather the line can move ‘up or down’. Furthermore one must realise that the movement occurs on the graph as q changes with the Cartesian Plane fixed or when the Cartesian plane is moved vertically as the graph is maintained fixed. However the latter descriptions would have shortcomings if more than one graph is drawn on the same Cartesian Plane. It was evident in Penny’s classroom that the learners shared an understanding with the teacher with respect to what this gesture meant. The community within this activity system socially constructed meanings for the Master teachers discussion and visuals. Good visual tools assist learners in constructing, strengthening and establishing connections with various mathematical ideas (Sarama & Clements, 2009).

Moreover, gestures that learners use often indicate to teachers what learners know and don’t know about tasks (Goldin-Meadow, 2004). For example when Karyn asked her learners about the calculation of the perimeter, she noticed that learners were sinking down into their seats; this indicated to her that they were not confident about their knowledge of perimeter. Karyn continued with the lesson but introduced deictic gestures to indicate what perimeter meant. Her reason for introducing these gestures is described in the following response.

Karyn O18: “... *to make sure that they know ... perimeter ... because ... some of them...are just too shy to ask ... make sure that they understand what you are talking about ...they ... get confused between the perimeter and area...*”

Essentially gestures effectively assisted the learners to interpret and understand the mathematics being taught (Sfard, 2009).

8.2.5.2. Using iconic gestures

Teachers produce gestures that influence what their learners acquire from the lesson (Goldin-Meadow, 2004). When Dean and Penny were teaching direction and movement of angles they used similar iconic gestures to indicate clockwise and anticlockwise movement. These two Master teachers taught at different schools, to different learners within varying contexts, yet their gestures were similar. Their reasons for choosing to use gestures are depicted below.

Dean O9: *“... to show the initial position of the points ... to show the turning position ... we take things for granted that pupils know what anticlockwise is and what clockwise is, so I wanted them to actually see it on the diagram...”*

Penny O7: *“...the concept of a clock is lost now with the digital time machines ... children are not ... aware of clockwise and anticlockwise ... I can't take that chance so I use the gesture to show it...”*

This observation affirms that teachers' ways of discussing specific mathematical content with learners may be very similar despite diversities in pedagogic style (Chronaki, 2000).

Alan used iconic gestures to simulate the movement of a line. He stated that *“...once again it is a question of visualisation , the learner can see exactly what you are doing so I used my hand to show how it is moving then he can see what you are talking about the line itself is moving the angles are moving...”* (Alan O10). This type of visual tool relies on the learner's integrated concrete knowledge in order to make the mathematics more accessible and meaningful. Learners come into the classroom with preconceived notions, ideas, and knowledge. The learner ought to make a connection with the visual tool, the concept in focus, and prior knowledge. Likewise (Radford, 2003) maintained that gesturing while speaking emphasises and corroborates the information that individuals wished to communicate.

Alan also used iconic gestures when he tried to explain the term ‘blowing up’, or ‘enlargement’. Alan demonstrated the viewpoint that the learner made associations between hand movements and the words uttered. Alan believed all this was possible because the learner visualised the concept of enlargement. Visualisation as discussed earlier in Chapter two, refers to the ability to represent, transform, generalise, communicate and reflect on visual information (Gal & Linchevski, 2010). This notion is captured in the following statement.

Alan O8: “... *I am always using gestures ... if you don’t know the meaning of a certain word but you can see the action then you can make the association...*”

Karyn used gestures frequently in her classroom. Whilst Karyn was teaching gradients of lines she used an efficient iconic hand gesture to show the slopes of lines that were positive or negative (See visual in Chapter 7.3). She used something visual to complement her discussion. This is encapsulated in her statement that follows.

Karyn O3: “... *with the straight line work in mathematics it’s ... in three different sections you work with straight lines ... so straight line is really a concept in mathematics that they (the learners) need to understand properly. They need to know if the slope is positive it will be in that direction and if it is negative it will be in the other direction (demonstrating hand motions to the left and right) ...*”

When asked about why she used gestures, she explained that “...*I think it’s just to make sure that they listen to you ... it’s just my way of teaching...*” (Karyn O17). Karyn believed that learners tend to listen if you used your hands to formulate gestures during the lesson. Karyn wanted to ensure that her learners achieved the most they could whilst in her classroom. She was concerned about her learners and wanted to make sure that her learners knew exactly what she was talking about. Her response is captured below.

Karyn O25: “... to make sure that they know what you are talking about, after a few years of teaching you have habits that you are not aware off...”

Language plays an important role in the teaching and learning of mathematics. It assists with comprehension of word problems and provides access to the mathematics content. It is important to encourage the learning community to communicate competently using the language of the mathematics register. Gaining control over the mathematics register is crucial to the learning of mathematics (Vorster, 2008). This aspect was evident in Karyn’s classroom. When Karyn used her gestures, she used them simultaneously with mathematics discourse. She wanted to share and negotiate ideas so that the learning community within the classroom came up with joint decisions and shared understanding (Mercer & Sams, 2006) about the concepts under discussion. The use of these gestures depicted her knowledge and experience; she knew what would work in different situations. Goldin-Meadow (2004) concurred with this notion that gestures materialise in classrooms of experienced teachers.

8.2.5.3. The use of other gestures

Maggie used a combination of iconic and deictic gestures to represent the action of cross multiplication. She used the gesture as a way of making the concept of cross multiplication more accessible. Maggie explained that “... *I have to indicate to them exactly what it is they are cross-multiplying... I want them to understand, mostly when I teach these things on the board I want them to understand the method... to answer future questions...*” (Maggie O10). Whilst the gestures were made in the air, and the numbers that were to be cross multiplied were only imagined, not physically present, this was a moment of practical realisation (Sfard, 2009).

To achieve similar results, Alan sometimes used metaphoric gestures. For example in the first observed lesson on transformation geometry, he wanted to explain the change in position of the x and y coordinates when the graph undergoes a reflection or rotation. Alan

demonstrated a simple process using his fingers to represent something complicated. He discussed this gesture in the following statement.

Alan O7: “...*fingers interchange and as a result the coordinates interchange and the fact that the fingers have interchanged it hasn't really changed the finger it's just the position ... the coordinates have not changed but x becomes y and y becomes x ...*”

By using this gesture, he made the concepts of rotation and reflections more accessible to his learners. The learners mentally manipulated the x and y coordinates to achieve the result of rotation and reflection. Using this metaphoric gesture, Alan evoked a higher level of thought processing within his learners.

Whilst there were many other cases indicative of teachers' use of gestures to emphasise some concept within the lesson, I have highlighted a few. The intention was to allow the reader to grasp an understanding of what it meant by stating: teacher's gestures influence what their learners leave their classrooms with. It seems, based on empirical evidence, that gestures were important for effective mathematical communication.

8.2.6. The significance of mathematics manipulatives

The use of mathematics manipulatives in this study played an important role in mathematical meaning making and communication. Dean used a stick with coloured rubber bands; Alan used a piece of A4 paper whilst Karyn used the smart board as her manipulative. She believed that the use of technology was more effective than any other tool that she could use to show changes in graphs. She maintained that “... *this is so much more effective than anything else ...*” (Karyn O15), thus, in this classroom, technology influenced the way algebra was understood. By using the smart board, Karyn was able to manipulate her graphs to illustrate to her learners the effect of changes in the equation on the graph of a quadratic function.

Throughout her 15 years of experience, Karyn has found nothing else that compares with her smart board. When using paper and pencil or chalk on a board to draw graphs, it becomes time consuming and tedious especially when explaining different examples. The graph is generally the end product of various actions on an equation. No further action can be carried out unless you redraw. However when technology (the smart board for example) is used, modifications in algebra are done with ease because technology allows the teacher to directly adjust graphs of functions (Ferrara, et al., 2006). This saves time and affords the learner the opportunity to obtain more practice with a greater number of examples within the same timeframe. Her response is captured below.

Karyn O15: “...but in the old days when I did not have it (the smart board) ... what you had to do is...just draw lots of lines...all the possibilities...no it is definitely not as effective...”

Technology-infused classrooms support Vygotsky’s emphasis on the importance of social interaction for learning (Wepner & Tao, 2002). Learners are more actively involved in the lesson as opposed to the traditional ‘talk and chalk’ approaches. When using the traditional approach, lessons are teacher dominated. Technology-infused classrooms encourage teacher collaboration with learners in order to facilitate the shared construction of meaning. Learning therefore becomes a mutual practice for all within the learning community. Technology adds more value as compared to the traditional approaches to teaching and learning mathematics (Ferrara, et al., 2006) and is essential in teaching and learning mathematics (NCTM, 2000)²⁹. However, in South Africa, one ought to concede that these technology-infused classrooms are a minority. Many South African classes do not have electricity, let alone, a smart board. Hence, teachers need to find other alternatives to demonstrate shifts in graphs.

Nevertheless, whilst most classrooms did not have access to the smart board, all the learners in this study had access to another form of technology – the scientific calculator. All the schools in the study are Dinaledi schools. As part of the incentive aimed at

²⁹ National Council of Teachers of Mathematics

improving Grade 12 mathematics results, each learner in the FET phase was provided with a scientific calculator. These calculators were efficiently used during the mathematics lessons and assisted learners in calculating, simplifying and verifying activities (Ferrara, et al., 2006). Good scientific calculators are highly developed. With the proper training on the use of a calculator, learners do not need to worry about wasting time doing tedious calculations. The calculator does this for them. This allows the learner to concentrate his/her energy and time on more substantial growth in their understanding of mathematical concepts. In these classrooms, it seemed that the scientific calculator contributed to the teaching, learning and understanding of mathematics.

8.2.7. The use of different shapes, symbols and lines

Some Master teachers used underlining techniques as visual tools to emphasise important concepts. These tools were used in different ways to enable the learner's transformation or formation of new knowledge. Important words were underlined so that learners would be able to distinguish what the focus of the lesson was. Maggie's views are denoted below.

Maggie O3: *"...the question is quite wordy being a geometry question, just to distinguish from the question and where the question actually ends and the proof part actually starts I underlined ..."*

In Karyn's classroom she underlined the key words so that learners could focus on them with a view of ensuring that they understood what these words meant. This was captured in her response that follows.

Karyn O13: *"...a huge problem now is language ... you make sure they understand what this means ... focus on the key words..."*

The use of language in the mathematics classroom includes ordinary English as well as the mathematical register. This could be confusing to learners especially if English is not their main or second language. Great effort ought to be taken by the teacher to ensure that

learners comprehend the concepts being taught. For this purpose underlining was used. Teachers underlined so that learners could ‘see’ and comprehend what was under ‘the microscope’. This was apparent in Alan’s interview. His response is documented below.

Alan O5: “... *once again it is a question of them (the learners) hearing something and seeing it ... the fact that you are underlining it, is also emphasising...what you are talking about ... draw attention to something ... I think that is critical ...*”

The teachers used different tools based on their learners and the knowledge they had about these learners. The context determined the use of the different tools in the classroom. Hence the use of visual tools in this study was context dependent. For example, at Carnation Secondary School, Sam used the block to accentuate that there were more steps to be completed in the mathematical process; Penny at Tulip Secondary School used the blocks to illustrate the intention of the lesson. She stated that “...*the block draws attention to the whole purpose of the lesson...*” (Penny O4). Blocks in this study refers to the following type of visual tool.

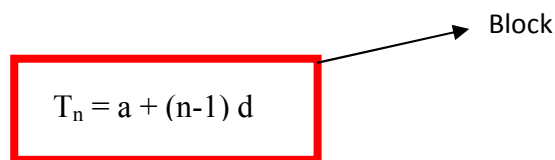

$$T_n = a + (n-1)d$$

Figure 33: An example of blocks used as a visual tool during the observed lessons

Penny also felt it necessary to allow learner the opportunities to recognise the final stages of the mathematics problem. She believed that teachers ought to “... *make them aware that that is the final stage of the answer...*” (Penny O4). Since mathematics is seen as a complicated science involving a number of steps to each problem, teachers need to provide support to learners so that they become more confident at working within a mathematical framework. To assist in achieving this, teachers need to create an environment that is conducive to teaching and learning. This was apparent in Sam’s response as depicted below.

Sam IE: “... don't forget that the children have a tendency of saying that mathematics is difficult. You have to create an atmosphere where they can see that this thing, no there is nothing impossible here, this thing is easy, and you can do it...”

This notion reinforces the idea that learning ought to be seen as a process, whereby the outcome is not necessarily the solution of the problem but rather the understanding (Kolb & Kolb, 2005).

Karyn at Rose Secondary School used the blocks to highlight formulae. She regarded the formula as being important to the solution process, she maintained that she used blocks because “... it's just again to highlight important information they need to pay attention to...” (Karyn O27). Karyn felt that learners needed to remember them and pay attention to them. On reflection, this would have served the same purpose as highlighting the formulae or using different colours as was done in other contexts in this research study. What we see here is the mediating of visual tools to gain a common, shared understanding.

Therefore, whilst the learning community mediated these tools, one must concede that there are implicit rules when using certain visual tools. For example, arrows in mathematics were used to indicate first and second difference, arrows were also used to indicate one to one mapping when working with graphs, Venn diagrams and coordinate systems. These reasons were evident based on the following comments made by some Master teachers.

Sam O5: “...it's...an arrow showing...the first difference...the linear is the first difference...”

Sam O5: “...showing... the one to one mapping ...”

Alan O5: “... I am using it (the arrow) as a symbol of mapping, the coordinates of one set is being mapped onto a completely different set ...”

Essentially, since the learner starts communicating with peers and teachers using signs of mathematics objects before the learner fully comprehends the meaning of this sign, it is through this communication with others that initial access to this new mathematical object is granted (Berger, 2005). The connections between mathematics and language reinforce the idea that mathematics is a social and cultural product (Gorgorio' & Planas, 2001).

As argued by activity theory, the subject under analysis (the Master teacher) has a relationship with the object (mathematics content) and the visual tool (diagrams, gestures, manipulatives, blocks, colours, and highlighters). The visual tool mediates the object. Mediation of tools may result in a common understanding regardless of the context. By this I mean some visual tools used by the Master teachers served as a representative tool. For example, whilst Maggie used a set of symbols to represent angles, Karyn used another representation to show time, and Dean used arrows to indicate movement and position. The Master teachers provided the following reasons for using visual tools as a representative tool:

Maggie O8: *“... to show two angles that are equal, you don't have to use a tick you could use two circles, two dots or two crosses, but in a triangle you get three corners so I generally use a tick and a dot and maybe a cross to indicate that one set is similar to the other...”*

Karyn O23: *“... broken lines ... will represent 5 minutes...”*

Dean O5: *“... my intention was to show the movement ... from one side of the equal sign to the other ...the correct position of where the number is supposed to go...”*

It is evident here that signs bring together an image or word and a concept (Silverman, 2001) and the use of signs is an essential part of concept formation (Berger, 2005) and meaning making in mathematics (Berger, 2004).

8.3. Theme 2: To make mathematics more interesting and fun

The conceptions that learners have about mathematics is that mathematics is dull and uninteresting. To alleviate the notion that mathematics is dull, boring and uninteresting, researchers (Fenton, 2002; Heiede, 1996) suggested that the history of mathematics ought to be taught during mathematics lessons. In this respect, mathematics is seen as a vibrant entity with a story and substance rather than as a dead subject. Watson (2008) suggests that to advance attempts at piquing learners interest in mathematics, learners ought to be exposed to exciting ideas in mathematics. According to Alan's learners, his use of different and less formal teaching strategies made mathematics "... *more interesting...*" (L6 OFG1).

Alan made his lessons, interesting and more meaningful. To encourage meaningful learning, Alan used paper folding to teach transformation geometry. He stated that "...*I don't like teaching kids to use formulae and rules ... if you forget it (rules and formula), it becomes a hassle ...*" (Alan O2). He believed that it was important for learners to see what was happening, he did not want to rely on just rules and formulae. He believed that "... *by folding it, they can visually see it ...*" (Alan O2). Alan intimated that by 'seeing' the mathematics, learners would not forget the mathematics. He believed that seeing a rule was more reliable than learning a rule by rote.

Thus, in Alan's classroom, it was not acceptable practice to learn by rote: he did not endorse rote learning. He supported meaningful learning because meaningful learning required that content be processed effectively (Rakes, et al., 1995). Similarly, Goos (2004) claimed that all classrooms are communities of practice but differ depending on what is construed as acceptable learning practices within that learning community. By Alan's demonstration, not only did he focus his learners' attention, he also piqued their interest by using a fun method to teach the concepts of rotation and reflection.

When learners cannot see the relevance of mathematics concepts or if they cannot construct meaning, they have no place to fit this concept into and learn by rote for the sake of

learning the concept. This concept does not generally fit into the existing schemas and does not connect to prior knowledge that the learner possesses. Alan realised that he needed to be the expert guide and offer both challenges and encouragement to his learners whilst assisting them in the process of constructing new meanings and knowledge.

Sam and Karyn concurred that their use of colour assisted in making their lessons more interesting. Karyn was prepared to go to great lengths to make her lessons more interesting, she was fortunate that she could do this because she had access to the smart board. Their responses are captured below.

Sam O12: “... *to make the lesson more interesting...* ”

Karyn O11: “... *it is more interesting ...* ”

Karyn O12: “... *anything to make the lesson more interesting...* ”

All Karyn’s observed lessons centred on the use of her smart board. She used different ‘fun’ symbols and bright colours (red, yellow, blue and green). Her borders around worksheets and diagrams were made out of stars and smiley faces (See Appendix E). Karyn’s smart board provided the ideal tool to focus attention and promote learners’ interest in the teaching and learning of mathematics (Becta, 2005).

Penny did not have access to a smart board but was aware that whilst learners were physically in the classroom, they may not be there mentally. Penny used stories as a means of developing learners’ interest and involvement in the lesson. This has been discussed earlier on in Chapter six and Chapter seven.

Other visual tools like smiley faces, stars and circling of terms were used by the Master teachers within their individual activity systems to make lessons fun and interesting. Fenton (2002) suggested that another strategy a teacher could use to encourage interest in mathematics is by practising and doing the mathematics. For example, when teachers are teaching probability in mathematics, instead of talking about how probabilities are

calculated and worked out, rather than using the ‘chalk and talk’ method, they should allow learners to do the experiments. This is because learners’ understanding about chance and probability ought to develop from experiments and experience (Van de Walle, et al., 2010). Allowing learners to work in groups encourages interaction with each other and the manipulatives (dice and coins). This interaction promotes the effective investigation of theoretical and experimental probability. Likewise, the use of mathematical play³⁰ improves learning and comprehension in mathematics (Holton, et al., 2001).

8.4. Theme 3: To establish an alternative teaching strategy

In South Africa the lack of financial resources poses serious problems in mathematics classrooms (Nieuwoudt, Nieuwoudt, & Monteith, 2007). Schools are under resourced and oversubscribed as discussed earlier on in Chapter two. It becomes the responsibility of the teacher to actively engage learners with these limited or no resources. Teachers are expected to create environments that are conducive to learning. Learners are expected to be enthusiastic, involved and willing to participate effectively in the classroom. This is not an easy task for many teachers. The desire to try different approaches to teaching is crucial in creating environments that are beneficial to learning mathematics. The Master teachers in this study resorted to using visual tools as alternative teaching strategies within their activity systems.

An important way to teach learners strategies to assist in learning mathematics is for teachers to model that strategy. For example, when the Master teachers used colour or highlighters to show the movement from one step to the next in the problem solving process, they modelled thinking processes that were required to analyse and solve problems. They modelled an unconventional strategy to the traditional ‘chalk and talk’ method.

³⁰ Mathematical play is the process that involves both experimentation and creativity to generate ideas in mathematics.

Sam taught in a school that had a high enrolment of students who came from disadvantaged backgrounds. By disadvantaged backgrounds, I mean that these learners came from households where there was evidence of unemployment and poverty. According to Weir and Milis (2001) the disadvantages associated with poverty are aggravated when there are many learners from poor backgrounds within a school. Sam used gestures to level out the 'playing field'. By being perceptive to the fact that his learners came from different backgrounds, he acknowledged that they had different needs. He was also aware that he had mixed ability learners within his class, and due to his experience, he recognised that in order to teach successfully he needed to try different teaching strategies.

Sam was of the notion that every learner was capable of achieving, and it was his responsibility to provide a supportive environment to make this attainable (Ollerton, 2006). He approached new concepts and content by building them from the foundation level upwards. He did not take his learners' prior knowledge for granted. Sam was aware of the fact that prior knowledge also refers to knowledge that is developed outside the classroom and school setting (Myhill & Brackley, 2004). Sam stated that teachers ought to "... *make sure that they (the learners) understand the grass root concepts that they (the learners) need to know...*" (Sam O9).

Sam exhibited many characteristics of an experienced teacher. He was committed and dedicated to his learners. Experienced teachers use a perceptive approach to prevent or cope with their learner's difficulties (Gal & Linchevski, 2010). When Sam taught trigonometry, he preferred to use a diagram rather than an alternative mathematics manipulative or tool. His preference is revealed in the following statement.

Sam O10: "... *although I know that by using the calculator you can come out with the same answer but ... by means of the circle there they can find the special angles...*"

He used the unit circle method (Steer, et al., 2009) because he wanted his learners to be aware that there is more than one strategy to use when solving problems in trigonometry.

Likewise Mudaly (2010a) suggested that diagrams are excellent tools for sense making and ought to be used wisely when presenting tasks to learners.

By Sam introducing his learners to the diagram of the unit circle, he offered his learners a choice to select an appropriate strategy when they needed to. This draws our attention to the idea that there are many different ways to solving mathematics problems, so instead of using only one method, the teacher can use as many as possible in order to accommodate learners with different learning styles within the classroom (Montgomery, 2001). This allows for a greater chance of all learners participating unconditionally in the classroom.

For this to surface in classrooms, the teacher ought to be agreeable to trying out new ideas and methods. Sam explained that a teacher “... *must be flexible when you are talking about math you cannot just rely on one and the same thing...*” (Sam O7). Apart from making mathematics more accessible to the learner, this also alleviates boredom and piques interest. Sam went on to explain alternative diagrams teachers could use when teaching trigonometric ratios. This is captured in the following statement.

Sam O7: “... *without any Cartesian plane ... I draw ... right angled triangle... I think it is very important to have a triangle there ...*”

Sam has essentially provided three methods with three different diagrams of teaching trigonometric ratios. Likewise, Diezmann (2000) claimed that it could be more beneficial if more than one diagram is produced to assist with the teaching of concepts in the classroom.

The methods that Sam used are redrawn in Figure 34 below.

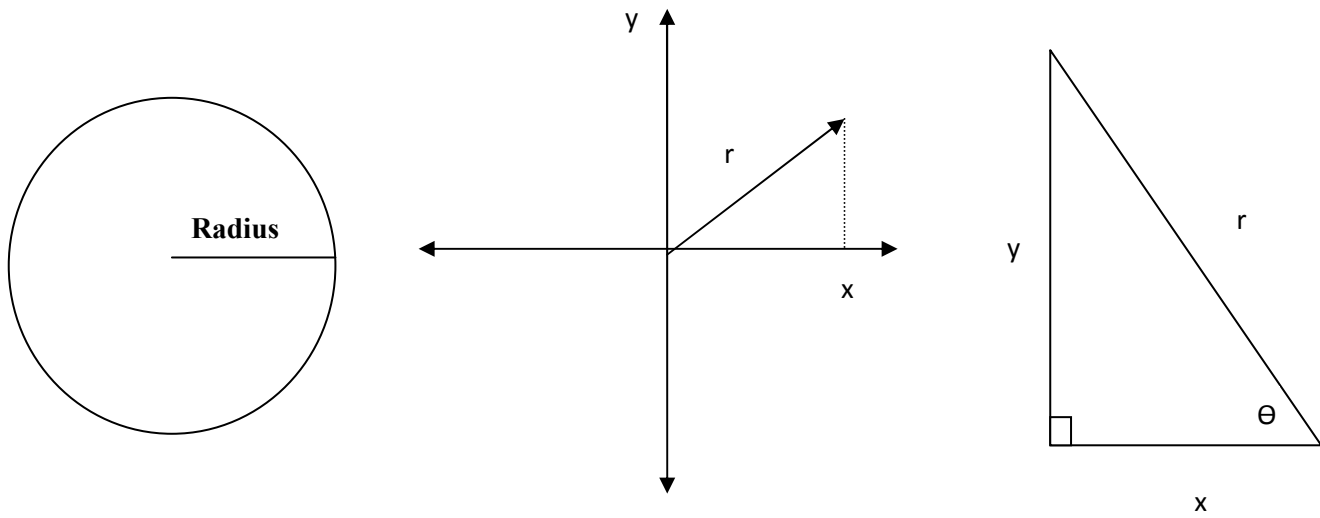


Figure 34: A diagrammatic representation of graphs Sam used to teach trigonometric ratios

Sam displayed evidence of being the ‘more knowledgeable other’ within his activity system. Sam possessed more content knowledge than his learners in trigonometry. He was the expert in this activity system. With his guidance, his learners learnt more about trigonometry. Sam supported them in this learning process until they could work independently within this content area. Likewise Swain, Brooks and Tocalli-Beller (2002) maintained that the ‘more knowledgeable other’ plays an essential role when learners are constructing knowledge. Thus, teachers can be important participants within the knowledge construction process, provided they have a strong knowledge of their content in addition to well-established pedagogic practices. Good pedagogic content knowledge helps learners fill in the gaps. This strategy enhanced the teaching and learning of mathematics in Sam’s classroom.

Penny also drew diagrams on the board and used deictic gestures to indicate angles on the Cartesian plane. She had a similar view as Sam in that she wanted to ensure that learners knew their foundation knowledge before moving on to more complex information. This foundation knowledge ought to be linked to the Cartesian plane. Penny discusses this notion in the following statement.

Penny O6: *“...these are tools that I expect children to make use of in their classrooms ... to focus on that aspect of the diagram ... when I point I am directing their attention to the concept involved at that point...”*

What was evident in these classrooms was that representations were important in the teaching and learning of mathematics (Font, Godino, & D'Amore, 2007).

Penny used her experience as a mathematics teacher to make decisions about what she wanted to achieve during her lesson. Her aim was to obtain the maximum attention of her learners. Thus teachers' sense making of their classroom is central to how they function (Ben-Peretz & Halkes, 1987) and influences their teaching strategies. If teachers become more aware of how learners understand mathematics within their own classrooms, teachers will be more likely to learn from their learners' reactions (Margolinas, Coulange, & Bessot, 2005) with the probability of having a positive impact in the classroom.

Penny used group work effectively when she asked her learners to present their attempts on chart paper. Group activities offer significant opportunities for learners to create solutions for themselves through talk, which would not be accessible during whole class instruction (Mercer & Sams, 2006). The use of group work assisted learners in Penny's class by focusing attention, eliciting interest and responses. This was noticeable from the classroom observations and the following statements.

Penny O12: *“... the aspect of group work helps those that are not picking up the concepts easily ... the enthusiasm is so much greater and everybody wants to be a part of it. It helps to encourage peer learning. ... I use that technique where I get groups to do a few and they are presenting it ... by the end we have 10 examples done in the classroom where we could have just had 2 done by the whole class ... presented their work on charts which is visual as well. So they get to see it...”*

Here, two important issues confront us. Firstly, Penny used charts as a manipulative that encouraged learners to work collaboratively in a visual environment. Secondly, she recognised that by separating the learners into groups, she had the facility to expose them to more visual examples. The use of learner centred approaches to learning promotes the development of higher order skills such as critical thinking and problem solving (Brush & Saye, 2002). Learners were also encouraged to work collaboratively with each other and discuss their ideas in front of the class. This strategy reinforces the importance of peer learning because it provides learners with the confidence to talk about mathematics. This strategy also allows learners the opportunity to become a part of the group whereby they feel supported and validated (Dodge & Kendall, 2004).

In this classroom, discussion and co-action were valued (Goos, 2004), and, regardless of the wide range of contributions (Ollerton, 2006), learners felt respected and important. Additionally, calling learners out to the front of the classroom promoted individual accountability (Hancock, 2004); this assisted and encouraged learners in becoming critical and independent learners. Teachers could also use cooperative learning activities to assist with this. The teaching of mathematics whilst using activities engaged learners and allowed for the implementation of their own ideas (Kazima, et al., 2008). Learners would now be able to bridge the gaps between the concrete and abstract level of instruction through the interaction with other learners. Gaps in knowledge would surface and be filled with the help of peers.

Likewise, when Alan used paper folding, his learners developed a conceptual understanding of reflecting along the x axis because he demonstrated this reflection by using multiple ways of representation (Shaw, 2002). Alan used a sketch on the board in conjunction with the paper folding. The learners had prior knowledge about the Cartesian plane and the rules associated with movement in the four quadrants (x and y are positive in the first quadrant; x and y are negative in the third quadrant; in the second quadrant x is negative and y is positive; in the fourth quadrant, x is positive and y is negative). He

focussed on the links between his learners' prior knowledge and new learning (Myhill & Brackley, 2004). The reflection along the x-axis of points and shapes and their effects on the different coordinates required integration with existing knowledge. Since the class work exercise was successfully completed, this evidenced the effective assimilation of knowledge.

Dean conceded that in his experience there were too many rules and theorems in mathematics; he had developed different techniques of teaching mathematics for maximum recall by his learners. He was fond of using mnemonics to teach aspects of mathematics. Dean went on to explain that *"... from ... the experiences that I have had, I have noticed that pupils when they are given the triangles to learn ... the special angles ... they do have problems ... I found as I have been teaching that they find it very easy to remember this (the mnemonic) which is ... beneficial to them ..."* (Dean O8). Apart from equipping learners with mathematics content knowledge, it is also necessary to equip learners with strategies (Stylianou, 2002) and skills to cope with the pressures of mathematics.

Similarly, when Karyn used diagrams and pictures, she proposed that different sections in mathematics require different strategies and methods of teaching. She believed that *"... it's the section that lends itself ... analytical geometry is all about pictures ..."* (Karyn O20). This suggested that due to her experience, she had developed different techniques for teaching different sections in mathematics. Montgomery (2001) also emphasised the need for teachers to develop different instructional strategies. Not only does this accommodate different sections in mathematics but this also assists in accommodating learners with different learning styles. In this respect, everyone within the individual activity system is completely included in the lesson.

8.5. Theme 4: To assist in remembering

Memory can be measured in a number of ways including recall, recognition and preparation (Knowlton, 1998). For many the words recall and remember may signify

different ideas, but for the purpose of this study recall is defined as ‘to remember something’ and remember as ‘to be able to recall’. These two words are used synonymously in this study.

Rules, processes, theorems, and proofs are key components of mathematics. In algebra, a strong memory assists in successfully manipulating and completing mathematical operations, which are generally procedural in nature. Learning in geometry, however, entails understanding how theorems work and how they could be used to find new knowledge. Geometry in mathematics requires learners to create and discover new conjectures (Kulp-Brach, 2004). To achieve this goal, a learner ought to be able to understand concepts and move from one concept to the next with ease. This demands that the learner is adequately prepared and possesses the foundation knowledge necessary to progress further.

Learners are not only required to remember what they have already learnt, but they are also required to call on that knowledge in multiple situations (Kulp-Brach, 2004). For this to be possible, a learner ought to be able to recall key rules, formulae, and proofs with ease. The skill to remember also expects that learners establish links and understand relationships between concepts. Given the importance of both the procedural and explicit memories for mathematics, it is important for the teacher to assist in making these connections. This can be done by channelling information from one part of the learner’s brain to the next as suggested by Davis, Hill and Smith (2000). It is important for learners to acquire this skill, experience these relationships and learn how to link these ideas for the effective teaching and learning of mathematics (Rakes, et al., 1995).

All the Master teachers in this study had various notions about the value of remembering in the mathematics classroom. Penny’s ideas and notions epitomises most of the Masters teachers’ views. Penny believed that seeing promotes understanding and stimulates

memory. She tied her notions in with important statements that she had taught her learners. This is revealed in her comments below.

Penny O3: “... if you look at my children’s’ books you will see at the back of their books 3 statements written there which says: I hear and I forget, I see and I remember, I do and I understand...”

Penny acknowledged that whilst a teacher may stress all of the above, the one matter that teachers do not have control over is compelling learners ‘to do’. She maintained that “...as much as we stress the doing part it does not always happen ... that part we cannot make happen... as a teacher the visual part is very important...” (Penny O3). Whilst realising the limitations of being a teacher, Penny is determined to make a difference, she emphasised that using visual tools assisted in learners recalling information (Graber, 1996).

Penny exhibited her enthusiasm and interest for mathematics, by using concrete objects to make something abstract more relevant to the learners. Whilst teaching the parabola she used smiley faces and sad faces. This is supported by Elia and Philippou (2004), who claimed that visual tools play an important role in communicating mathematical ideas and supporting the process of reflection. Penny believed that when learners were exposed to these symbols in future lessons, on reflection, they would remember what the symbols signified. She explained that “... children remember it ... I use it for that purpose expecting them to remember it better and that is how they do remember it ...” (Penny O1). Remembering requires the learner to consciously recollect seeing the concept prior to being exposed to the concept in the class (Knowlton, 1998).

Within the learning community, the teacher (Penny) used previous knowledge to trigger responses to current work. The recall and application of older skills are crucial as new concepts and symbols are learned (Steadly, et al., 2008). Similarly, Maggie used symbols

that her learners were exposed to in previous lessons or grades. She explained her reasons for using the symbol that represented a 90 degree angle as follows.

Maggie O6: “... *standard symbol that we use in geometry...it is something that is standard and you learn from grade 8...*”

Whilst this concurs with the notion that one learns from experience (Kolb & Kolb, 2005), the teacher must be mindful of taking things for granted. So although, mathematical signs and symbols have a cultural meaning which is derived from its established usage in mathematical discourse (Berger, 2004) meanings are also made within the present learning community.

In Alan’s lesson on transformation geometry, when the learners saw what Alan was doing with the paper folding and the sketch, it reinforced their notion of reflection. They understood and there was a strong possibility they will remember (Graber, 1990). Thus, the idea behind Alan’s use of his manipulative was to make the mathematical rule of reflection visible to his learners so as to improve his learners’ understanding (Karadag & McDougall, 2009). With this enhanced understanding due to the visible representation, it was more likely that the learners would recall this rule for future lessons and activities. Graber (1990) advocated that seeing is remembering, and that this capacity is further enhanced when pictures are combined with words (Graber, 1996). This implies that teachers need to talk even when using transparencies, diagrams, and graphs (Steadly, et al., 2008). Teacher talk is imperative because it is through this engagement that learners can gain exposure and experience with the language of mathematics.

8.6. Theme 5: To use both deliberately and intuitively

In Chapter six of this study, the unique manner in which each Master teacher employed the use of visuals as tools in the mathematics classroom was discussed. Detailed descriptions of how and why the Master teachers in this study went about using these visual tools were provided. The preparation teachers required to use these visual tools were also discussed in Chapter seven. What we need to consider now is that in some instances at critical incidents in the study the Master teachers resorted to using visual tools intuitively. Critical incidents occurred in two modes in this study, firstly whilst the teacher was teaching and secondly, when learners were working on a task and experienced an ‘aha’ or ‘light bulb’ moment. Thus, the critical element may be something that is observed or experienced as well as, it could be a misconception or a misunderstanding of concepts or instructions. Whatever the case, it is not necessarily the result of something planned by the teacher; it may just happen within the lesson. It was the manner in which the Master teacher dealt with these issues that was of importance.

Dean believed that mathematics was not a verbal subject, it was much more. He used diagrams, mathematics manipulatives and gestures because he believed that apart from interacting with the mathematics discourse, learners needed to see the mathematics. For example when he designed the manipulative to explain rotation in transformation geometry, this was an example of a deliberate use of visual tools. He prepared the visual tool in advance because he considered that it was important for learners to see what they were being taught. He was of the notion that since there were many rules in mathematics; explaining or listing these rules (especially for transformation geometry) may create difficulty in comprehension. These difficulties would prevail if learners fail to recall these rules correctly.

From Dean’s point of view, seeing the rule of rotation of 90 degrees about the y-axis revealed to the learners how points on the Cartesian plane transform. He believed that seeing assisted the learner to learn. This is encapsulated in the following statement.

Dean O8: “... *if you just show rotation and if you just tell them (the learners) or give them (the learners) the rule... they (the learners) are not going to see anything, they (the learners) are not going to learn anything they (the learners) are just going to go by the rule and then apply...if they (the learners) can remember the rule...by showing them (the learners) the rotation ... see ... how the position changes...*”

Dean used a stick with a red rubber band and a blue rubber band to assist his learners in visualising the concept of rotation (See Figure 29 in Chapter six). In this case, the visualisation enabled the learner to use the concrete object (the stick with coloured rubber bands) to grapple with the abstract concept (rotation of points A and B, 90 degrees about the y axis) as discussed by McLoughlin (1997).

Apart from the deliberate use of visual tools in the mathematics classroom, Dean also used visual tools intuitively. For example, in the first observed lesson Dean taught quadratic equations (completing the square) (See Figure 16, Dean’s activity system). Whilst walking around the classroom he realised that learners were having a problem with keying in information on the calculator. He asked the learners to stop what they were doing and to look at him. He used this technique to focus learners’ attention on him. He uttered the sequence of steps that were required to be followed. He used systematic and explicit instruction to promote the teaching and learning of mathematics in his classroom (Steedly, et al., 2008).

He went one-step further, whilst providing the sequence of keys that needed to be punched on the calculator, he used gestures to support and demonstrate what each step meant. He used these gestures intuitively. Firstly, he used iconic gestures to represent the ‘opening and closing’ of brackets. This was a critical incident in the lesson; previously the learners were told what needed to be done but they could not complete the process successfully. When Dean added gestures to his sequenced process, the act of seeing reinforced the doing. The learners successfully completed the problem.

Secondly, together with the use of gestures to illustrate the ‘opening and closing’ of brackets, Dean used a gesture to represent balancing the equation. He started his discussion about balancing equations by talking about a familiar playground toy, the seesaw. A seesaw is a long, narrow board pivoted in the middle so that, as one end goes up, the other goes down. He went further than just describing the association between a seesaw and a balanced equation. He used iconic gestures to represent how a balanced seesaw would appear and how it would move to one side if it were not balanced. To represent a seesaw he used outstretched arms and moved them diagonally to the left and then to the right. This was a ‘pivotal’ moment in the teaching of the lesson. When Dean was asked about the use of these gestures his response was “...it’s the force of habit, I am not aware that I am doing this ...” (Dean O3). His gestures became a part of his thinking process (Sfard, 2009). This intuitive use of gestures assisted the learners in successfully completing the tasks set out for them.

Alan admitted that some of the gestures that he used were intuitive; he stated that “... it’s subconscious for me...” “...some gestures are subconscious...” (Alan O3/4). When Alan used the paper folding whilst teaching transformation geometry, this was an example of a deliberate visual tool. He wanted to prepare an activity that would support his learners’ learning. Similarly, Montgomery (2001) proposed that teachers ought to plan instruction that serves as a scaffolding technique. However, not all of Alan’s visual tools were planned and structured in advance. Some of the visual tools he used in his classroom were intuitive. For example, whilst teaching the reflection of images using the paper folding technique, he pointed out what he wanted his learners to ‘see’. He wanted learners to see the reflected image about the line $y = x$. To stress his instruction he used his palm. When his open palm faced him, he called this the ‘new image’, when he demonstrated the ‘reflected image’ he used his palm again, but the open palm now faced the learners. He essentially swivelled his palm to create the impression of a reflected image. So whilst he used his language as a tool for communication, his gestures became the action for this communication (Sfard, 2009).

Similarly, Sam also used gestures intuitively. He explained that “... *sometimes it is natural... it's natural ... some people ... they use hands ... when I am teaching I use hands...*” (Sam O3). In the first observed lesson, Sam taught number patterns. Whilst he was teaching first and second differences he started speaking about the term constant, whilst talking about the first difference being constant, he used a metaphorical gesture. His gesture depicted a straight line, what he signalled here was that if something (the first difference) was constant, it followed the path of a straight line. Learners could link this gesture and what it signalled; for example, with a car travelling at constant speed, there is no acceleration. This gesture was not planned; it came naturally, as Sam taught. This was a critical incident in the lesson, in that learners could now associate this word ‘constant’ from the mathematics register with a visual image. Therefore, whilst he did not reveal to his learners that this is about a car travelling at constant speed, the learners interpreted this intuitively.

Likewise, Church, Ayman-Nolley and Mahootian (2004) proposed that gesturing is predominantly intuitive and since gestures are the bonds that associate different forms of information (Goldin-Meadow, 2004), listeners also acknowledge and interpret gestures intuitively. Since mathematics is viewed as an interpretive activity as purported by Brown (2008), the use of this intuitive gesture assisted in including learners in tasks that were beyond their reach. This reinforced the idea that knowledge cannot be transmitted from one person to another but rather that each person constructs his or her own knowledge. This knowledge has to be meaningful; it has to make sense to the learner. It is seen as a product of individual cognitive acts (von Glasersfeld, 1990). Along similar lines, researchers (Hein, 2002; Jaworski, 1996) emphasised that knowledge is not passively received but rather, human beings manufacture the world they know. What is evident here is that knowledge is acquired both as a result of social activity as well as a result of individual cognitive acts.

8.7. Conclusion

Essentially throughout the study it became apparent that in order to be successful in the mathematics classroom, teachers needed to be highly motivated, creative and flexible. Teachers needed to prepare in advance of their lessons. All the Master teachers agreed that visual tools improved the teaching and learning of mathematics. They believed that some training and support would be necessary to use visual tools successfully in the mathematics classroom.

This chapter served to inform the reader the reasons behind why the Master teachers used visual tools in the mathematics classroom. To recap the following notions were discussed in great depth in this chapter. Master teachers used visual tools in the mathematics classroom:

- to make mathematics more concrete, accessible and comprehensible
- to make mathematics more interesting and fun
- as an alternative teaching strategy
- to assist in remembering and recall
- both deliberately and intuitively

The next chapter concludes the study by discussing the researcher thoughts, findings, recommendations and limitations of the study.

Chapter Nine: Thoughts, summary, recommendations and limitations

“If teachers play their role properly, the nation can be transformed.”
Bhagavan Sri Sathya Sai Baba

9.1. Preface

In Chapter eight the themes uncovered in this study was discussed. These themes represented each Master teachers reasons and ideas. In this chapter researcher thoughts, a summary of the study, recommendations and limitations are presented. This study began with the exploration of Master teachers’ use of visuals as tools in mathematics classrooms. In undertaking this project Master teachers from Dinaledi schools were selected as participants. Three male Master teachers and three female Master teachers were observed, video recorded and interviewed over a six month period.

The study explored what visual tools Master teachers used and how and why they used visuals as tools in their mathematics classrooms. To achieve these ends, five measuring instruments were employed in this study. The first was a Master teacher questionnaire. This questionnaire was used to assist in becoming acquainted with both the Master teacher and the context within which s/he was teaching.

The second measuring instrument was an observation schedule. This instrument was employed to investigate the visual tools implemented by the Master teachers during the teaching process. The third measuring instrument used in this study was a Master teacher interview schedule. Individual in-depth interviews were conducted using a semi-structured technique with a set of interview questions to direct and guide the inquiry. This instrument assisted the research study in many ways. The interview schedule assisted in probing each teacher’s thought process about teaching within the classroom. By probing each teacher’s responses, it was possible to establish greater insight into the use of visual tools.

Probing was important because during the lesson observations, it was apparent that at certain instances in the lesson, the Master teachers used visual tools intuitively. When questioned about the use of these visual tools, many of the Master teachers indicated that they used these visual tools to clear up misconceptions and misunderstandings as well as to scaffold their explanations. The intuitive use of visual tools played an important role in mediating the learners' understanding of concepts, rules and processes in mathematics. The deliberate and intuitive use of visual tools has been discussed earlier in Chapter eight.

The fourth measuring instrument was a field diary. This assisted in the documentation of ideas and thoughts as each lesson was observed. Details regarding the atmosphere of the learning environment and the interactions that occurred within each classroom were noted. The teacher-learner and learner-learner interactions were depicted on a socio-gram. Each socio-gram assisted in the selection process of participants for each focus group.

The fifth and final research instrument was the focus group interview schedule. This was a semi-structured interview schedule for learners. The instrument was used to explore how the learners experienced the use of visual tools in the mathematics classroom. The data collected during the focus group interviews was used as a means of triangulating the empirical evidence. This was important when probing learners' views about both the deliberate and intuitive use of visual tools in the classroom.

To evaluate the validity and reliability of each measuring instrument, a pilot study was conducted prior to the main study. The participants in both the pilot as well as the main study were Master teachers and learners from Dinaledi schools in KwaZulu-Natal, South Africa. The participants were representative with respect to race, culture and gender.

The qualitative methodology used in the study allowed for the emergence of rich details. Personal interviews with the research participants exposed a thick description of the factors that affected the Master teacher's decisions to employ visual tools in their mathematics

classrooms. This rich data created a large body of information for the analysis of common themes. Descriptive information was derived from the video recorded lessons and audio taped sessions. A thematic coding system was developed in order to analyse the data collected. This information was inductively coded into common categories. Major themes emerged from the coded transcriptions. The categories and themes are included for perusal in Appendix B. These themes were systematically analysed through the process of data analysis. Key findings, recommendations and limitations were uncovered. This chapter provides a synthesis of the analysis and arguments developed thus far. The chapter also highlights recommendations for future research and documents the limitations of the study.

9.2. Researcher thoughts

Mathematics is regarded by many as a complicated subject. The extent of this complexity envelops both the learning and teaching of mathematics. This complication presents a stumbling block in society since success in South Africa is generally measured by the amount of mathematics you know. Thus, it is important to be well equipped with the knowledge of mathematics. To substantiate this point, in order to gain access to higher paying occupations, learners are required to attain a high pass rate in their Grade 12 mathematics examination.

Whilst the Department of Education invests copious amounts of money and resources in developing the mathematical proficiency of both teachers and learners, based on researcher observations, many teachers are failing to teach important mathematical concepts (for example, concepts that are examined in Mathematics Paper 3³¹). This is the result of many factors. One of which could be linked to a teacher's own inadequacies as a result of the manner in which were taught. Many teachers are scarred by the affects of being educated

³¹ The current Grade 12 mathematics examinations is divided into two compulsory papers (Paper 1 – Algebra: equations, exponents, surds, financial mathematics, number patterns, sequences, calculus and linear programming and Paper 2 – Analytical geometry, transformation geometry, trigonometry and data handling) and one optional paper (Paper 3 – Euclidean geometry, recursive sequences, descriptive statistics, interpretation and bias, probability, bivariate data and data handling). Whilst paper 3 is optional in terms of writing the examination, the concepts covered by the paper ought to be taught in all classrooms.

during the apartheid era. However in the midst of all the politics and bureaucracy some teachers are successful in making a difference in the mathematics classrooms. This success in some instances is difficult to explain and warrants exploration.

It was as a result of these success stories that this study emerged. What was prevalent among these success stories was the use of visual tools. Visual tools were used within the mathematics classroom to allow all learners access to the mathematics being taught. In some cases the Master teacher in this study used visual tools to assist in the communication of mathematical ideas when the language of instruction was not the learners first. In many instances it served the same purpose as code-switching³². However instead of only using language to code-switch the Master teachers used visual tools and language. In other cases, the Master teacher used the visual to clarify concepts being taught when language was not the issue.

Each master teacher used their mathematics lessons to make a difference in their learners' lives. They were motivated and determined to make a difference. Master teachers were more interested in the solution process rather than focussing on attaining the correct answers. Visual tools were also used to make the abstractness of mathematics more accessible.

In light of the research done in this study, the contribution that this study makes is the knowledge that these techniques and strategies (for example when Dean used a piece of wood to demonstrate the rotation of a point in transformation geometry) may be used in any classroom within any social milieu. Apart from poorly resourced schools these techniques may also be used in schools where the behaviour of learners proves to be the biggest obstacle to learning. These tools may also be used in classrooms where learners are not streamed into ability levels as is the case of the majority of schools in South Africa.

³² Code-switching is the concurrent use of more than one language. It means switching back and forth between two or more languages in the course of a conversation.

These tools also proved to be highly effective in large classrooms. Additionally the Master teachers demonstrated the usefulness of their visual strategies in large classrooms with limited resources. The Master teachers in this study demonstrated that anything is possible provided that the teacher is determined and committed to making a difference in mathematics education. The ongoing professional development of the Master teachers also proved to impact positively on how each Master teacher taught in the classroom.

Essentially what was apparent in all the classrooms was that the Master teacher was not the distributor of knowledge, but rather each teacher acted as a guide for the educational experience of their learners. The role of the Master teacher in these classrooms was to help learners identify associations between their unprompted, everyday concepts and the formal concepts of the mathematics discipline. In this way learners played a more active role in their own learning and this led to an intrinsic motivation. The learners' enjoyment, fulfilment and interests were emphasised. It was through these observations that the classrooms in this study were seen as progressive learning spaces rather than traditional teaching spaces. Whilst these researcher thoughts are based on what was observed, each Master teacher also provided reasons for their use of each visual. This may be perused in the section that follows.

9.3. Summary

This study proved to be a challenging though significant undertaking in the field of mathematics. Observations and findings were notable with respect to the realities of the mathematics classroom within different social milieus. It was evident from the data collected and discussed thus far, that the use of visual tools had a positive impact on the teaching and learning of mathematics. Within the limits of the present study, the following conclusions with respect to the critical questions of the research study could be drawn.

9.3.1. What visuals were used as tools in mathematics classrooms?

The first question dealt with identifying all the visual tools that each Master teacher used in their mathematics classrooms. The Master teachers in this study used a variety of visuals as tools in their classrooms. Each teacher used visual tools that were easily accessible to him/her within the context of their teaching. The choice of visual became their personal preference. Some of the Master teachers used technology and cutting-edge visual tools. Other Master teachers used their tacit knowledge to implement the use of innovative handmade visual tools to achieve success in the classroom. This has been discussed in detail in Chapter five. Pictures of these visual tools may be found in Appendix F.

Whilst examining each visual used, activity theory was implemented as the framework to discuss the interactions within each classroom. Each classroom was considered to be an individual activity system. The evidence for each interaction within the different activity systems were compiled using data collected via the classroom observations, video recordings, Master teacher questionnaires, Master teacher interviews and focus group interviews with the learners.

Whilst interrogating each activity system, the feasibility of the different generations of activity theory was examined. It was decided not to work with the first generation activity theory as it would be inadequate for this study. This was so because the first generation did not allow for social interactions to be discussed, the unit of analysis was individually focussed (Engeström, 2001). Since this study explored the Master teachers' use of visual tools within different mathematics classrooms, the empirical data was located within mathematics classrooms in different contexts. It was therefore important to interrogate interactions within each social milieu. The second generation activity theory allowed for this type of discussion and had to be explored.

As opposed to the first generation activity system, the unit of analysis in the second generation activity system was the activity itself (Uden, 2007). The activity being

examined in this study was the teaching and learning of mathematics. Each Master teacher's activity system was examined using the second generation activity theory as the lens. Second generation activity theory is directed at single activity systems only. However, whilst exploring the activity systems using the second generation activity theory, other external activity systems surfaced. These external activity systems appeared to influence the activity system being interrogated.

The external activity systems demonstrated evidence of creating tension and conflict within the activity systems that were under scrutiny. The third generation activity theory concentrated on networks of interacting activity systems. This generation acknowledged that single activity systems may be influenced by other activity systems (Daniels & Cole, 2002; Engeström, 2001). With the manifestation of conflict and tension, it became necessary to explore the third generation activity system.

Whilst several external activity systems surfaced, six seemed to be worthy of exploration. The first was the history of poor resourcing. As discussed earlier in Chapter two, the apartheid era affected many schools. Some schools were affected positively and others were affected negatively. The former Model C schools were well resourced in all aspects, whereas the former Black, Indian and Coloured schools were poorly resourced. This was noticeable at four of the six sample schools. Presently these disadvantages are being addressed. Until they are resolved, schools are operational within these unfair environments. Thus, despite not having many resources, the Master teachers were innovative in creating their own manipulatives and mathematics equipment using easily accessible materials. These resources have been discussed in Chapter six and eight of the study. Pictures of these resources may be found in Appendix F.

Apart from material resources, human resources were also scarce at these disadvantaged schools. Researchers (Adler, 2001; Reddy, 2006) claimed that this dilemma is not uncommon at South African schools. South African schools are riddled with injustices and

inadequacies. This has been discussed earlier on in Chapter two of this study. Time became a mammoth deciding factor at the schools. Master teachers did not have enough time to prepare or do research for their lessons. Most of their non-teaching periods were either spent doing administrative tasks or taking care of learners in lieu of teachers who were either absent or not appointed as yet. This however was not evident at the two well resourced schools. Based on researcher observations and experience, teachers here had ample time allocated within school hours to prepare for their lessons using the latest technological tools. There were numerous administrative assistants to take care of administration. There were SGB appointed and paid for relief teachers. Teachers at the well resourced schools were not distracted from their job of teaching mathematics.

Despite the immense pressure to fulfil many roles and duties in the absence of human and material resources, the Master teachers at the disadvantaged schools were successful in encouraging and motivating learners to obtain good mathematics results. They exhibited the characteristics of good teachers by being determined and committed to making a difference at their schools. The characteristics of good teachers were discussed earlier in Chapter two. These characteristics included the need for teachers to have effective communication skills, be creative, possess both pedagogic and content knowledge as well as have the ability to model concepts in their content area (Polk, 2006). Based on evidence obtained during data collection, the Master teachers used these skills to produce mathematically literate learners, capable of becoming critical citizens in a “democratic and open society” (Department of Education, 2003b, p. 1).

Master teachers at the four disadvantaged schools used their creativity and tacit knowledge to construct innovative manipulatives which they used as tools to scaffold the teaching and learning of mathematics concepts. What was of note here was the tenacity of the Master teachers not to give up in the bleakest of situations. Even without electricity Penny used coloured chalk, gestures, diagrams and pictures to make the mathematics ‘speak’ to her learners. Penny used her visual tools to communicate that mathematics was the key to a

better future. In order to escape their depressing and frustrating circumstances, learners were encouraged to embrace the various strategies Penny used. These strategies provided the scaffold they needed to gain access to the mathematics being taught. The use of these innovative visual tools in mathematics has been discussed in Chapter six of the study.

The second external activity system that surfaced was the benefits of teaching at a well resourced school. This was apparent at two of the six schools in the study. The privileges of a well resourced school influenced each component of the activity systems positively. The Master teachers at these schools focussed on the teaching and learning of mathematics. They did not have to concern themselves with learners not coming prepared into their classrooms. To assist with this aspect, parents played an important role at the school and ensured that learners completed homework at home. Parents also ensured that most learners came to the classroom with mathematics equipment. This included mathematics sets, rulers, coloured pencils and highlighters. This assisted the Master teachers tremendously.

It was as a result of learners being well equipped that the teachers could use visual tools effectively in the mathematics classroom. For example, it became easier for learners to see the relationships between angles on a diagram if they used coloured markers to mark them or highlight the angles under scrutiny. An accurately constructed diagram also made much easier the task of identifying the sides and angles that were equal, corresponding and alternate. Apart from the correct mathematics equipment and support from parents, most learners also had the advantage of attending private tuition in mathematics. Private tuition may be viewed as a method through which learners broaden their learning in order to grasp a better understanding of content and key concepts. Bray (1999) asserted that private tuition benefits both the learner and the society of which s/he is a part. In a sense learners came more prepared into these classrooms. This provided the teacher with more time to use dynamic visual methods and strategies to reinforce the understanding of mathematics concepts.

The third external activity system that surfaced was the mediating role of language. This was evident at two of the six schools. Learners at these schools spoke English as a second or even third language. This created an added tension to the already disadvantaged activity system. Teachers used visual tools in an effort to strengthen their explanations and discussions of key mathematics concepts. Despite the language barrier, the Master teachers at Tulip Secondary and Carnation Secondary Schools were successful in achieving the desired outcome of their activity systems. Despite all the adversities with which they were faced, they taught mathematics effectively at each school. Much of this success could be attributed to the conscious and intuitive use of visual tools. The planned visual activities mitigated the absence of useful and necessary resources.

The fourth external activity system was the lack of discipline. This was noticeable at two of the six sample schools. The majority of the learners who attended these schools came from poverty stricken backgrounds. Most of these learners were struggling to achieve the lowest levels of Maslow's hierarchy of needs. Maslow's hierarchy of needs has been discussed in Chapter two. Learners were focused on achieving their physiological and security needs (Pollard & Triggs, 1997). They were concentrating on meeting their need for food, shelter and safety. Without satisfying these basic needs it would be very difficult to concentrate on a mathematics lesson.

Regardless of this, the Master teachers at these two schools persevered until they earned the trust and respect of their learners. They implemented their innovative visual tools to focus their learners and level out the 'mathematics playing field'. These Master teachers worked tirelessly to use a combination of colour, diagrams, gestures, pictures and stories to provide a safe environment for learners. At Tulip Secondary, the class that was observed had a school wide reputation of being boisterous and troublesome. Penny used her experience of teaching EHV³³, to calm and placate these learners. Education in Human Values is a value based teaching programme. In these programmes the core human values

³³ Education in Human Values.

of truth, love, peace, right conduct and non-violence were encouraged during each lesson. Penny respected and cared about her learners, the learners reciprocated her actions. She used pictures to compose her learners and guide their attention. Her use of visual tools in this manner served as a scaffold in the teaching and learning of mathematics within her activity system. This aspect has been discussed in detail in Chapter five and chapter six.

The fifth external activity system that surfaced was the impact of the national examinations and testing policies. The Department of Education has implemented policies to ensure that all learners are provided with the appropriate level of education. To ensure that all teachers follow the same curriculum and complete the syllabus in good time, learners write common tests and national examinations. Whilst this system has many merits, the drawback is that often learners at disadvantaged schools and rural areas bear the brunt of an unequal education system that seems to favour the more advantaged schools. Learners are compelled to write the common tests and national examinations even though the language of teaching and learning (LOLT) is not their first. Most often, due to the disadvantages associated with being in an under resourced school, the syllabus is not completed. These drawbacks further entrench the desperation and vulnerability felt by many learners within the South African education system. It was apparent that the Master teachers based at disadvantaged schools used their visual tools to bridge the gap between what was known and what was required to be known.

The sixth external activity system that surfaced was the lack of parental involvement. Parents in low socio-economic areas struggle to provide for their families. Some parents often work long hours and have more than one job. As a result these parents do not have the time to be involved in their children's education. This in turn adds pressure on the already oversubscribed teacher. The teacher is now responsible for both the work done at school as well as the work that ought to have been done at home. Teachers when faced with this situation have two choices, either s/he re-teaches the section and treats the homework

as a class work activity. The other choice is that the teacher ignores the fact that home work is not done, and moves on with the syllabus.

With the latter choice, learners will not have the proper foundation to move forward with content. The Master teachers in this study realised this and pursued different versions of the former choice. For example Penny often taught sections and allowed learners to work in groups when solving problems. This allowed for active learner-learner and learner-teacher interactions. Once the learners had a chance to actively engage in the problem solving process, each group used chart paper to model the solutions. These ‘posters’ were then displayed around the classroom for others to see and interrogate. The charts became visual tools that covered more problems in one lesson than would have been covered using the traditional ‘chalk and talk strategy’. It was in this way that homework was lightened or converted.

In other instances, for example, to lighten the teachers work load, learners were initially cajoled into working with abstract mathematics concepts by using concrete visual tools. This was apparent for example when Alan used a piece of A4 paper to explain key concepts in transformation geometry. This was also apparent when Dean used a stick with coloured rubber bands. This assisted by reducing the amount of time the Master teacher spent explaining the abstract concepts. This created more time and opportunity for the Master teacher to provide individual support and encouragement whilst walking around the classroom. Often whilst the teacher walked around the classroom, they used many gestures intuitively whilst supporting their learners thinking. It appeared as though the act of seeing, for example, what a limit of a function looked like would trigger the solution process. Sometimes whilst the teacher walked around encouraging the learner, they would draw diagrams in learner’s books to stimulate the solution process or illustrate diverse possibilities.

In other instances, the use of concrete visual tools made the abstract mathematics more accessible. This accessibility provided the learner with the confidence to attempt other more abstract mathematics concepts. The confidence motivated the learner to work independently both in class and at home. Learners were motivated by their teachers to become independent critical thinkers. According to Brush and Saye (2002), this type of teacher-learner engagement may be called soft scaffolding. This refers to the monitoring, support and encouragement provided by the teacher whilst allowing the learner to work independently. This has been discussed in Chapter six of the study.

Whilst the various external activity systems have been briefly addressed both here and in Chapter five, the exploration using the third generation activity system was not the focus of the study. This could be the starting point for future research. Researchers could use this study as the basis of exploring and interrogating similar activity systems using the philosophy of the third generation activity theory.

9.3.2. How do Master teachers use visuals as tools in mathematics classrooms?

The second research question focussed on how each Master teacher used visuals as tools in the mathematics classroom. In order to answer this question the data collected from all the research instruments were interrogated and analysed. Master teachers used visual and concrete tools as a means of scaffolding mathematics thinking and learning. Scaffolding provided an alternative approach to the traditional ‘chalk and talk’ approach to teaching. Each Master teacher used different levels of scaffolding to ensure success in the classroom.

Level 1 scaffolding revolved around the classroom environment. Each Master teacher created a classroom atmosphere that was conducive to teaching and learning. Master teachers organised their classrooms in specific ways. Learners had specific seating arrangements to promote group work for certain lessons, whilst for some lessons they sat in a manner that supported individual work. This organisation also included the use of

pictures and charts during lessons. Some Master teachers used innovative wall displays to motivate and encourage learners during their lessons. Pictures of these wall displays may be found in Appendix F.

Scaffolding at Level 2; included different levels of teacher-learner and learner-learner interactions. These type of interactions relied on the reviewing and restructuring of all activities within the classroom. The reviewing and restructuring was initiated by each Master teacher. During the reviewing process each Master teacher encouraged their learners to verbalise both what they saw and what they were thinking. Learners were encouraged to explain and justify each action and comment. Master teachers also restructured tasks that were posing a problem in class. They did this in two ways; firstly, the Master teacher simplified the problem. Secondly, on some occasions the Master teacher rephrased the learners' comments with the aim of negotiating and probing meanings. This was apparent in for example Penny's classroom when she taught the sketching of parabolas and also for example in Karyn's classroom when she was teaching analytical geometry. This strategy also assisted in shifting the understanding forward. Master teachers also used relevant real life contexts or situations so that problems involving abstract concepts became more accessible to the learner. Creating this real world, relevant and meaningful conditions led to Level 3 scaffolding.

Level 3 scaffolding refers to the use of tools with the aim of developing conceptual understanding. Each Master teacher used concrete materials and mathematics manipulatives with the intention of making the abstract concepts in mathematics more accessible. This has been discussed in detail in Chapter eight of the study. Scaffolds that were used became tools and strategies which assisted learners in achieving a deeper level of conceptual understanding. Scaffolding encouraged divergent and creative thinking in the mathematics classroom. This was apparent during the lesson observations and some evidence of creative thinking may be found in Appendix F. The Master teachers were able to assist more learners using their visual tools in this manner.

9.3.3. Why do Master teachers use visuals as tools in mathematics classrooms?

The third question sought to find out why Master teachers used visuals as tools in their mathematics classrooms. This section deals with what each Master teacher revealed with respect to why they used visual tools in their classrooms. Descriptive information was derived from the observations, video recorded lessons, focus group interviews and Master teacher interviews. This information was coded into common categories. On analysing the data, major themes emerged from the coded transcriptions. These major themes were organised through the process of data analysis. It emerged in the study that Master teachers used visual tools in the mathematics classroom for five major purposes.

Firstly, each Master teacher wanted to make mathematics more concrete, accessible and comprehensible for their learners. Secondly, they wanted to make mathematics more interesting and fun. Thirdly they used visual tools as an alternative strategy to the traditional and tedious ‘chalk and talk’ method. Fourthly, the Master teachers used visual tools to assist their learners in remembering and recalling important concepts and procedures. Finally, the Master teachers used visual tools both deliberately and intuitively whilst teaching. Each of these objectives was discussed in detail in Chapter eight.

9.4. Recommendations

Six Master teachers from six different Dinaledi schools within KZN were participants in this study. This implies that the data that has been used in the analysis has been based purely on Master teacher accounts from the Master teacher questionnaire, observation schedule, video recordings and Master teacher interview schedule. In addition data collected from the learner focus group interviews was used to triangulate research findings. Whilst this has been adequate in exploring Master teachers use of visuals as tools, for future research, it is recommended that a broader research that includes more Master teachers within the province be conducted. Alternatively, this study may be extended to any mathematics teacher. All teachers ought to exhibit most characteristics of a good

teacher to make a difference in the mathematics classrooms. This implies that teachers need to be able to communicate effectively, be creative and have good pedagogic content knowledge. Thus including any mathematics in a similar study would provide valuable data.

It has been found that the Master teachers in this study believed that the use of technology was important in the mathematics classroom. This was so because the Master teachers are of the notion that mathematics needs to be visual in order to make mathematics more accessible and comprehensible for the learners. Since the use of visual tools in mathematics classrooms has proved to be beneficial, it is recommended that the Department of Education and teacher training institutions provide ongoing support in this area. Teacher training workshops are important for teachers regardless of how experienced or inexperienced they are.

Networks involving teachers within the same social milieu ought to be encouraged so that peer support is provided. This would be highly beneficial for teachers that struggle in the classroom. Master teachers could be asked to assist in coordinating and facilitating endeavours of this nature. This would assist in improving teacher morale and the praxis of teaching across the South African landscape. Being appreciated for work well done would serve as an incentive, thus providing the platform for encouraging more teachers to work harder at attaining success.

Whilst the use of second generation activity theory was adequate in this study, however, it is recommended that for future research; researchers explore the possibilities of the third generation activity theory. This has been discussed in detail in Chapter three. As mentioned earlier on in this Chapter, researchers could use this as a starting point for future research into the classroom as an activity system. Researchers could employ the use of the third generation activity theory to interrogate and expand on the relationships between the external activity systems on each activity system under examination.

9.5. Limitations

Firstly, this study was a relatively small-scale study. Only aspects directly related to the sample Master teachers and their visual tools that were used to assist in their mathematics classrooms were investigated. Doubtless, there are various other issues that could have been investigated, however these were not within the scope of this doctoral study.

Secondly, since this study has been conducted in secondary schools that were purposively chosen because of accessibility and co-operation from all gate keepers and the willingness of the Master teachers and learners to be observed, the situations in other schools may differ. This may be the case because variables differ from school to school. Whilst it is not the intention to claim that the findings of this study conducted in six secondary schools in KZN South Africa may be generalised to all secondary schools, the findings of this study are worthy of attention.

Thirdly, in order to gain access to schools and gain consent from each participant, the Master teachers were provided with a letter outlining details of the research study and the processes that would be followed. This knowledge may have influenced how the Master teachers taught each lesson. Lessons could have been structured according to what each Master teacher thought I was looking for. However, I have addressed these concerns by using focus group interviews with the learners. As mentioned earlier, the purpose of the focus group interviews was to triangulate the data collected.

Fourthly, the Master teachers that were based at the Dinaledi schools were appointed by the Department of Education. When the school allowed me access to conduct research, I had no choice but to work with the teacher at the school.

Fifthly, the timeframes for each lesson differed across the different schools and different days. This meant that on some days I observed a 40 minute lesson at one school and other days I observed a 60 minute lesson at another school. This also meant that on for example a

Monday at Tulip Secondary I would observe a 55 minute lesson and on a Friday at the same school, each lesson was 35 minute. These timeframes impacted on the way in which the teacher engaged with the topic and the learners. Timeframes also influenced which visual would be used as a tool in the classroom.

Finally, on interrogating each video recording, it may have been beneficial to have used two video cameras, one directed at the Master teacher and one focused on the learners. This would have captured the learners' responses and expressions especially in situations that were considered critical moments in each lesson. This would have also captured relevant learner responses when the Master teacher resorted to using intuitive gestures in the classroom.

9.6. Conclusion

This chapter commenced with researcher thoughts based on observations made throughout this study. A summary of the research study followed. Within this summary key aspects directly related to each critical research question was discussed. This study focussed on exploring Master teachers' use of visuals as tools in mathematics classrooms. Based on data collected in this study, visuals proved to be beneficial to learners regardless of the context within which they were located. Through the use of visual tools, Master teachers made the mathematics being taught more accessible and comprehensible to the learners within their own activity systems. Additionally during the teaching and learning process community members within each activity system used their own socially constructed meanings of visual tools. Master teachers used different levels of scaffolding within their classrooms to assist in the teaching and learning process. This chapter concludes with researcher recommendations and limitations that were identified.

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Appendix

Appendix A:

Mr Sibusiso Alwar
The Research Officer
RESEARCH, STRATEGY, POLICY DEVELOPMENT AND ECMIS DIRECTORATE
PRIVATE BAG X9137
PIETERMARITZBURG
3200

25 November 2008

Mrs J. Naidoo
10 Clover Crescent
Sarnia
Pinetown
3610

Re: Application to do research in KZN schools.

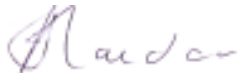
Dear Sir

I am a currently PhD student at Edgewood Campus, UKZN. My specialisation is in mathematics education and I am researching master mathematics teachers in KZN. I wish to explore master teachers' use of visuals as explanatory tools within socially integrated mathematics classrooms. I hereby wish to apply for permission to conduct research in schools in KZN.

I have attached a copy of my research proposal, ethical clearance form, research instruments and my sample list of schools. I agree to provide the Department of Education with full details of all findings, copies of all articles, papers, thesis, etc., once the research is completed.

If there is anything you wish to clarify, please do not hesitate to contact me. I look forward to hearing from you.

Yours in education
Mrs Jayaluxmi Naidoo (Student Number 205524804)





RESEARCH OFFICE (GOVAN MBEKI CENTRE)
WESTVILLE CAMPUS
TELEPHONE NO.: 031 – 2603587
EMAIL : ximbap@ukzn.ac.za

07 August 2009

MRS. J NAIDOO (205524804)
MATHEMATICS, SCIENCE, COMPUTER AND TECHNOLOGY EDUC

Dear Mrs. Naidoo

ETHICAL CLEARANCE APPROVAL NUMBER: HSS/0054/09D

I wish to confirm that ethical clearance has been approved for the following project:

"Exploring master teachers use of visuals as explanatory tools in socially integrated mathematics classrooms"

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years

Yours faithfully

MS. PHUMELELE XIMBA
ADMINISTRATOR
HUMANITIES AND SOCIAL SCIENCES ETHICS COMMITTEE

cc. Supervisor (Dr. V Mudaly)
cc. Mr. D Buchler



PROVINCE OF KWAZULU-NATAL
ISIFUNDAZWE SAKWAZULU-NATALI
DEPARTMENT OF EDUCATION
UMNYANGO WEMPUNDO

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INHLOKHOHOVISI

PIETERMARITZBURG

HEAD OFFICE

Imibuzo:
Enquiries: Sibusiso Alwar

Reference:
Inkomba: 001/2009

Date:
Usuku: 26 January 2009

Mrs J Naidoo
10 Clover Crescent
Sarnia
Pinetown
3610

RESEARCH PROPOSAL: EXPLORING TEACHER'S USE OF VISUALS AS EXPLANATORY TOOLS WITHIN SOCIAL INTEGRATED MATHAMETICS CLASSROOM

Your application to conduct the above-mentioned research in schools in the attached list has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educator programmes are not to be interrupted.
5. The investigation is to be conducted from 02 February 2009 to 02 February 2010.
6. Should you wish to extend the period of your survey at the school(s) please contact Mr Sibusiso Alwar at the contact numbers above.
7. A photocopy of this letter is submitted to the principal of the school where the intended research is to be conducted.
8. Your research will be limited to the schools submitted.
9. A brief summary of the content, findings and recommendations is provided to the Director: Resource Planning.

RESOURCES PLANNING DIRECTORATE: RESEARCH UNIT
Office No. G25, 188 Pietermaritz Street, PIETERMARITZBURG, 3201

The Principal
_____ School

02 March 2009

Mrs J. Naidoo
10 Clover Crescent
Sarnia
Pinetown
3610

Re: Application to do research at _____ Secondary School.

Dear Sir/Madam

I am currently a PhD student at Edgewood Campus, UKZN. My specialisation is in mathematics education, I am researching master mathematics teachers in KZN. I wish to explore master teachers' use of visuals as explanatory tools within socially integrated mathematics classrooms. I hereby wish to apply for permission to conduct research in your school.

I have attached a copy of my ethical clearance, and informed consent form. The consent form describes the research process in detail. I agree to provide the school with full details of all findings once the research is completed.

If there is anything you wish to clarify, please do not hesitate to contact me. I look forward to hearing from you.

Yours in education
Mrs Jayaluxmi Naidoo (Student Number 205524804)
031 260 1639 (W)
074 475 2938 (Cell)
naidooj2@ukzn.ac.za





Dear Teacher / Learner

My name is Jayaluxmi Naidoo. I am a postgraduate student at the University of KwaZulu Natal in the Science, Mathematics and Technology Education Department at Edgewood Campus. I am currently conducting research in mathematics education under the supervision of Dr. V. Mudaly. The purpose of this research is to assist me in exploring master mathematics teachers' use of visuals as explanatory tools in socially integrated mathematics classrooms. The mathematics teacher based at your school has been identified as a master mathematics teacher.

If you agree to participate in this research study, the following will occur:

1. Three grade 11 mathematics lessons will be video recorded, the recording will last the duration of each lesson.
2. An observation schedule will be used to assist in the observations.
3. Master mathematics teachers will be asked to complete a questionnaire as well as participate in an individual interview; the interview will be between 30 min – 45 min. Selected learners within the class will be asked to participate in focus group interviews.
4. A semi-structured interview schedule will be used. All interviews will be audio taped.
5. If you agree to participate in this research study, an audiotape of the interview and videotapes of the lessons will be made for research purposes.

CONFIDENTIALITY:

The records from this study will be kept as confidential as possible. No individual identities will be used in any reports or publications resulting from the study. All videotapes, audiotapes, transcripts and summaries will be given codes and stored separately from any names or other direct identification of participants. Research information will be kept in locked files at all times. After the study is completed and all data has been transcribed from the tapes, the videotapes and audiotapes will be held for five years and then destroyed. You will receive a copy of the final transcript, so that you have the opportunity to suggest changes to the researcher, if necessary. Participation in this study is voluntary and you are free to withdraw your participation from this research study at any point. You will be given a copy of this consent form for perusal.

QUESTIONS

If you have any further questions or queries about the study, please contact:

Mrs. J. Naidoo: (031) 260 1639 (W) / 0744752938 (C) / naidoj2@ukzn.ac.za (email)

2. Dr Vimolan Mudaly: (031) 260 3682 or mudalyv@ukzn.ac.za (email)

Mrs Krishnie Naidoo: (033) 341 8500 or Krishnie.Naidoo@kzndoe.gov.za (email)

CONSENT FORM FOR PARTICIPANTS

I agree to take part in the study on exploring master mathematics teachers' use of visuals as explanatory tools in socially integrated mathematics classrooms. I am aware that the mathematics lessons are going to be videotaped. I am prepared to be videotaped. I am aware that the researcher is going to conduct interviews. I am willing to take part in focus group interviews as well as individual interviews.

I am aware that each interview will be audio taped. I have read and understood the accompanying letter. I know what the study is about and the part I will be involved in. I know that I do not have to answer all of the questions and that I can decide not to continue with this research at any time.

I prefer my face to be blurred/obscured in the videotape?

No

Yes

Name _____

Signature _____ Date _____

Participant

Signature _____ Date _____

Parent/Guardian if participant is a minor

Signature _____ Date _____

Researcher

Crispin Hemson
15 Morris Place
Glenwood
Durban
South Africa 4001

hemson@ukzn.ac.za

C: 082 926 5333

H: 031 206 1738

TO WHOM IT MAY CONCERN

This is to record that I have edited the dissertation by Jayaluxmi Naidoo, entitled
EXPLORING MASTER TEACHERS' USE OF VISUALS AS TOOLS IN
MATHEMATICS CLASSROOMS.

.

A handwritten signature in cursive script that reads "Crispin Hemson". The signature is written in dark ink on a light-colored background.

Crispin Hemson

25th September 2010

Appendix B:

Coding

Pseudonym	Code	Interviews	Observations
Alan	A	AI	AO
Sam	S	SI	SO
Dean	D	DI	DO
Karyn	K	KI	KO
Penny	P	PI	PO
Maggie	M	MI	MO

Learner Number	School	Code
1 - 6	Daisy Secondary	L 1- 6 DFG
1 - 10	Tulip Secondary	L 1- 6 TFG
1 - 8	Rose Secondary	L 1- 6 RFG
1 - 6	Lily Secondary	L 1- 6 LFG
1 - 9	Orchid Secondary	L 1- 6 OFG
1 - 7	Carnation secondary	L 1- 6 CFG

Key

The coding is illustrated below; it can be interpreted as follow:

For example

SO2 – Sam Observation 2

AIC2- Alan Interview Section C, response to question 2

L6 RFG 7 – Learner number 6, Rose Secondary Focus Group Interview, response to question 7

L 1-3 OFG 10 – Learner number 1, 2 and 3. Orchid Secondary Focus Group Interview, response to question number 10.

Master Teacher	Section A	Section B	Section C	Section D	Section E
Alan	AIA1 AIA2 AIA3 AIA4 AIA5 AIA6 AIA7	AIB1 AIB2 AIB3	AIC1 AIC2	AO1 AO2 AO3	AIE1
Sam	SA1 SA2 SA3 SA4 SA5 SA6 SA7	SIB1 SIB2 SIB3	SIC1 SIC2	SO1 SO2 SO3	SIE1
Dean	DA1 DA2 DA3 DA4 DA5 DA6 DA7	DIB1 DIB2 DIB3	DIC1 DIC2	DO1 DO2 DO3	DIE1
Karyn	KA1 KA2 KA3 KA4 KA5 KA6 KA7	KIB1 KIB2 KIB3	KIC1 KIC2	KO1 KO2 KO3	KIE1
Penny	PA1 PA2 PA3 PA4 PA5 PA6 PA7	PIB1 PIB2 PIB3	PIC1 PIC2	PO1 PO2 PO3	PIE1
Maggie	MA1 MA2 MA3 MA4 MA5 MA6 MA7	MA1 MA2 MA3	MIC1 MIC2	MO1 MO2 MO3	MIE1

Code:

Master Teacher Questionnaire
A. School Profile

1. School Name	
2. School Address	
3. District	
4. Circuit	
5. Number of teachers on staff	
6. Number of DoE teachers	
7. Number of Governing Body teachers	
8. Number of mathematics teachers	
9. Learner Enrolment	
10. Learner – Teacher Ratio	
11. Number of mathematics learners in Gr. 10 - 2009	Girls _____ Boys _____
12. Number of mathematics learners in Gr. 11 - 2009	Girls _____ Boys _____
13. Number of mathematics learners in Gr. 12 - 2009	Girls _____ Boys _____
14. What textbooks do you use for Gr. 10 mathematics?	
15. What textbooks do you use for Gr. 11 mathematics?	
16. What textbooks do you use for Gr. 12 mathematics?	
B. School Infrastructure	
1. Number of classrooms	
2. Do all the classrooms have furniture?	
3. Do all the classrooms have windows?	
4. Does the school have running water?	
5. Does the school have electricity?	
6. Does the school have ablution facilities?	
7. Does the school have a security fence?	
8. Does the school have a security guard?	
9. Does the school have laboratories?	
10. Does the school have laboratory equipment?	
11. Does the school have a library?	
12. Does the school have a copier?	
13. Does the school have internet/email access? (Provide email address)	
14. Does the school have computers?	
15. Does the school have a computer room?	
16. Does the school have a staff room?	

C. Master Teacher Profile:

1. Surname _____

2. Title (Mr/Mrs/Ms/Dr/Prof) _____

3. First Names (In full)

4. Gender _____

5. Age Group (tick) 20 – 30 31 – 40 41 – 50 51 – 65

Other (Please specify) _____

6. Qualifications

7. Subject/s Teaching

8. Grade/s teaching

9. Number of years teaching mathematics _____

10. Highest grade level in mathematics that you have taught _____

11. The number of years you have taught the above grade _____

12. Total number of years teaching _____

13. Do you use textbooks when preparing for lessons? _____

14. If so, name them.

15. What other sources have you used when preparing your lessons?

16. Name some of the journals, websites, etc. you have consulted. (Question will be phrased depending on what /if teacher consults other resources).

17. Do you engage in any professional development activity? _____

18. If so, please elaborate.

19. Home Address

20. Home Telephone _____

21. Cell Number

22. Email Address (If available)

Code:

Semi – structured interview schedule *:
Schedule for follow up interview with master mathematics teachers who have used visuals as tools in the classroom.
School _____

Teachers name _____

Date _____ **Time** _____

Indicator:

Master mathematics teachers use visuals as tools when teaching.

Critical question 2 & 3:

How and why does the master mathematics teacher use visuals as tools within mathematics classrooms?

A. Do teachers value the use of visuals (pictures/gestures/diagrams)? *

1. Do you always use visuals when teaching?
2. What motivates you to use visuals when teaching mathematics?
3. Did the use of the visuals help you cover the key mathematics concepts in the selected topic?
4. What can you say about your learners response to the visuals used? ***

Checklist/ Indicators	Notes
Enjoyment	
Excitement	
Boredom	
Understanding	
Confusion	
Indifference	
Participation	
Inspiration	
Motivation	
Other (specify)	

5. Do you think that using visuals in the mathematics classroom can help children learn mathematics?
6. Why do you say so?
7. Has the use of visuals benefited your learners?

8. Have you learnt anything new from using visuals to teach your lesson?

B. How are visuals used within different contexts?

1. Do you feel that you have used your visuals in the way that you intended?
2. What preparation did you have to do in order to use these visuals in your class?
3. In what ways can the use of visuals help teachers improve their teaching?

C. What support do teachers need to use visuals?

1. Did you need training to use visuals in the classroom?
2. Do you need any further support to use visuals? If so, what support do you need?

D. Interview questions based on observations of mathematics lessons.

E. What is your philosophy of teaching using visuals?

Reflections/ Comments:

* All videotapes will be perused first before interviewing the master mathematics teacher.

** The question will be phrased depending on what visual the particular teacher used.

*** The checklist/ indicators will assist in the recording responses to Section A Question 4.

Code:

Semi – structured focus group interview schedule for learners. *

School _____

Focus Group _____

Date _____

Time _____

Indicator:

Master mathematics teachers use visuals as tools when teaching.

Critical question 2 & 3:

How and why does the master mathematics teacher use visuals as tools in mathematics classrooms?

Do learners value their teacher's use of visuals? **

3. Your teacher uses visuals to teach mathematics, what is your view about him/her teaching in this manner?

Checklist/ Indicator	Notes
Enjoyment	
Excitement	
Boredom	
Understanding	
Confusion	
Indifference	
Participation	
Inspiration	
Motivation	
Other (specify)	

4. Did your teacher's use of the visuals help you understand the key mathematics concepts in the selected topic?
5. What did you learn in this lesson?
6. Can you explain how you learnt this?

Appendix C

Section A: Do teachers value the use of visuals

1. R: Do you use visuals when teaching?

Karyn: I would say it all depends on how much time I have to prepare but yes I really do try to prepare properly and to use some interesting things and just to make things more clear. So I would say 80% of the time yes.

Penny: Yes, I think visuals help children to remember better

Alan: Pictures not necessarily, gestures and diagrams yes. Obviously it depends on the section involved, ... because sometimes the kids will see something better if they can associate it with some other link, ... but it depends on the lesson, sometimes in maths you can't design the lesson in a certain way, but where it can be done I try and use it.

Sam: Oh, yes, that's the way I do it. So that to emphasis some of the things.

Dean: Yes.

Maggie: Yes. One of either a gesture or pictures. I try to use a diagram because sometimes it helps in the understanding but sometimes we try to use diagrams because its more mathematical ... generally gestures, yes.

2. R: What motivates you to use visuals when teaching mathematics?

Karyn: It's just to get the learners attention; I think that's the most important reason.

Penny: The children's interest in it, they obviously show more interest in the lesson itself if they have got something to look at and work with.

Alan: I suppose mainly from the point of view of understanding, ..., because certain movements or a gesture brings out more understanding for the child because it can bring up more association than working in a vacuum. Association with that gesture or action or picture with what is being done, so if you can look at the one time when it was more than 360

degrees (one of the observed lessons), it is the same as ..., 370 degrees is the same. You can show the child more than 360 does not mean much, ... as 10 degrees with regards to your position then you can see exactly what is happening and the problem of seeing something foreign is being remove, because the child now understands something foreign is the same as something basic.

Sam: More especially in mathematics you cannot just be talking and talking. I need to have a diagram to refer to what I am saying to whatever I am doing. The learners will have to refer to the diagram and if I can see that they have a problem I can see in their faces that they didn't get what I was doing, and then sometimes my emphasis is on the gestures as you can see.

Dean: Eh, pupils, eh..., they seem to get involved when they see pictures or diagrams, they like eh..., understand certain concepts better.

Maggie: I think it makes the lesson much more interesting so that the learners don't get bored and so they don't fall asleep or stop listening and get inattentive. I think that they need a bit of visual stimulation.

3. R: Did the use of the visuals help you cover the key mathematics concepts in the selected topic?

Karyn: Oh, definitely yes, ja it's just so much easier when the learners visualise what you try to explain, I would say it's a definite yes.

Penny: Yes very easily.

Alan: It might not necessarily help me cover the key concept but it might assist in the understanding. Because the concepts thereafter, ... , the diagram it might,....., depending on the diagram it might assist in the covering of the concept but more especially I think it should assist in the explaining of it because we can understand what is being done and what is being taught and then syllabus coverage becomes easier.

Sam: Oh, yes, without those diagrams I don't think that I can reach my purpose. If there are no diagrams... because you cannot take it for granted that the learners have done the same grades, you need to go back each and every time and refer to the diagram. You cannot just simply start with that lesson, at that time you need to go check previously that they recall what they have done.

Dean: Yes.

Maggie: Yes. Well it depends on the actual section that you are doing, for e.g. if you are doing a section in geometry you have to have a diagram to teach that particular section. You go to the visual in front of you as well as a hard copy in front of them as well (the learners) so that really helps. I tend to do a bit of writing on the board as well because I find that even though I put stuff up on the smart board they like me to go step by step (the learners) and write on the white board so as I am writing and explaining they pick it up. So certain aspects we need to use visual stimulation.

4. R: What can you say about your learners response to the visuals used?

Karyn: They just enjoy it so much more, you can see that you have their attention and they are just more involved. They definitely enjoy it more.

Penny: They are more enthusiastic to the lesson itself because we often bring them to the board and get them to work on the diagrams etc and they enjoy that participation as well.

Alan: It depends, sometimes when they just don't get it which means you need to change the approach. You need to use another method or another diagram, but experience has taught me that obviously the silly and ridiculous work better than the extremely abstract. So if something does not work to get the point across you just have to improvise and do something else.

Sam: I think with them they have, I don't see that they have any problem, I think that they do understand, they do understand what I was teaching them. More especially now and again when referring to the diagram, I think they are getting so easily.

Dean: Uh, I would say that there is a positive response, because they seem to get involved in the lesson via the use of diagrams.

Maggie: Once again it depends on the section, if the section is just mostly equations. Yes, in a sense that we got to have the equation up on the board, we cannot teach maths lesson verbally, and you got to have some sort of visual aid to teach a maths lesson.

5. R: Do you think that using visuals in the mathematics classroom can help children learn mathematics? Why do you say so?

Karyn: Oh definitely yes. The more visually they can see the better.

Penny: Definitely. Well I understand the brain and the way it functions and the bottom line is that visuals enable children to remember things better. If you turn even a story into something that they can visualise they remember even the story better as well. You know anything you want to teach a child even words; the simple thing like a chair will mean nothing to a child until they see a chair. I believe that is how children internalise concepts by seeing it. So diagrams and visuals do that for children.

Alan: Ja, without a doubt. Because of the points I made, because if they can see that shifting a graph that they can see through a computer programme or data projector and they can see what is happening it makes drawing the graph $y = \sin x + 10$ it is easier. Like $\sin x$, because they can see the change in the formula, and that they are not really doing something difficult it is something basic. So other than that I mean, back in the old days they had to work it out on their own.

Sam: Ja they can, perhaps a lot. Perhaps because I don't really believe you can just do, just solving the problem without, it depends on what topic you

are doing. My topic was on more especially on the trigonometry so you have to go back to the diagram to remember all these ratios sin, cos, tangent using the diagrams. So you cannot just simply say sin is just a ratio which is y over r, you need to refer to the diagram so that they can understand and also recall what they have done in grade 10.

Dean: Yes most definitely. Eh, because without diagrams or without pictures it's eh, the mathematics becomes like abstract to them. With the use of diagrams they are able to see things and make connections.

Maggie: Definitely. It helps them, I know some boys think their thinking is different to others so when they see the diagram they get a better understanding of what is expected of them especially when it comes to sections like geometry and graphs and functions where you got to actually show them the diagram and explain to them the concept of a graph so it is absolutely crucial.

6. R: Has the use of visuals benefited your learners? Why do you say so?

Karyn: Well you don't know what the results would be without using those diagrams and pictures ..., but I think they definitely benefit. They (the learners) ask perhaps more questions and as I said they are more involved in the lesson, and they can visualise what you have explained to them.

Penny: Absolutely. I have often asked them to make charts etc and you would see a beautiful response to things like that. Yet if you just give them a problem, a normal algebra problem to work, without this kind of support, they are not as enthusiastic.

Alan: It would have probably benefitted with regards to understanding the concept ... but I generally do it when I try to illustrate a basic concept to them. I can't see how it can be done with more concrete content based things. It can be used to introduce a lesson, and in that introduction part they may have an idea of what the section is all about ... but for the maths

behind it they have to know that diagram. If they don't know the maths behind it or what the section is all about it does not really help them at all.

Sam: Ja I think so, I think it has benefited them because even, let me take for instance an example, for instance let's say it's a reduction formulae, so in the reduction formulae you have to give them a diagram, that CAST diagram. So that CAST diagram and you also have to have another diagram which is showing them whether in the first quadrant is tetha, second 180 minus tetha, 180 plus, 360 minus, so with that they can easily remember everything that we have done. Using the diagrams helps them to understand them.

Dean: I would say yes, eh..., because they seem to 'see' more things in a diagram or in the pictures.

Maggie: Yes, the use of technology has benefited them because when we draw on the board it is not accurate its sort of estimation. If you want to draw something accurately it's going to take you quiet a long time. Then we went on to using the OHP which is quiet accurate, ..., but then when they see it up on the smart board you can actually see animation as well, depending on what you are looking at. I think the use of new computer generated software they tend to respond more to that kind of stimulation.

7. R: Have you learnt anything new from using visuals to teach your lesson?

Karyn: Oh yes I learnt how to use the web, I also learnt that learners are really enjoying a lesson when you put in an effort to prepare properly and to make it interesting.

Penny: I think it's basically new in terms of my teaching and getting across to the children that has improved. Yes in terms of teaching methods and strategies.

Alan: I learnt what works and what doesn't that's the best way that I can say it because if something works, I got two matrices and if something doesn't work in the first class I will try something else, then in the second class I will not try what did not work so from that way you learn.

Sam: What I can say there, ja I have learnt that teaching you need each and every time to have something like a teaching aid, you need to have something that you can refer them to so I think, from my diagram I think they are, on my side it makes it easy to teach them.

Dean: Yes, firstly when you are looking at a diagram in mathematics for example like a triangle we look at just one orientation of the angle in, for example the area of a triangle, but by using different types of diagrams or different orientations pupils like eh,..., seem to get, they seem to understand more about the topic.

Maggie: Yes, because of the new technology instead of preparing lessons on an OHP we actually now prepare our lessons on the computer. So power point presentations are something I actually had to learn more about. So that actually helped me a lot, also using the net... the internet for various sources that helps a lot, as well in lesson planning and things like that.

Section B: Use of visuals

1. R: Do you feel that you have used your visuals in the way that you intended?

Karyn: No, it's always a surprise, lessons are always a surprise, it all depends, and the learners come up with different questions than you might be prepared for. Ja you always make some changes on some errors sometimes, ja its quiet an experience especially in the beginning.

Penny: Yes, though sometimes you have to adapt it to the way the children respond to it as well. As I have said sometimes I could use a diagram just to teach directly from the board but at times I have got to bring the child to the board to do the work and that works. That is adapting it to them, you know it does not always work the way that we intend.

Alan: I think so, because I think the point comes across, if it doesn't as I said, I change it because you are using it to get the point across and if that is not achieving its objective you have got to adapt and change.

Sam: Oh yes, I think ja, you see sometimes you can find that sometimes you can prepare lessons maybe more especially as I have said in that one. That one specific one that I was dealing with it needs a diagram so you have to refer them to a diagram even if I am doing my lesson plan. Although in my lesson plan I will just be looking at the outcomes all those things but I have to go further than that having my diagrams.

Dean: Eh, I would say yes, in the sense in most of the cases the diagrams that I use are diagrams that I make up myself and I use this to see how I understand and maybe pass it on to the pupils.

Maggie: Most times when I plan if I use that kind of presentation that's what I would actually use it for, I tend to try and use it in the way I planned to use it a lot of programmes we can get on the net and even buy CD's and DVD ROMs. But I don't actually like the way they are presented so I tend to change them so that it is more to the way that I teach. My way of teaching.

2. What preparation did you have to do in order to use these visuals in your class?

Karyn: You have to play around on the smart board to know all the functions, to know all the things that are available. So that takes quite a lot of time. Then you page through old exam papers for exam type questions especially for seniors and to page through text books and you page you play

with more programs than just the smart board facility you also have autograph and different websites. So it is very time consuming but if you see how much kids enjoy this effort, it's definitely worthwhile.

Penny: Okay well most of the time I use just the talk and chalk method because of the lack of electricity. I don't have electricity in my room so we would love to use a lap top and projector etc but unfortunately it is not available to us. So I use the charts and chalk and board.

Alan: Once again it comes with experience, it comes with knowledge beyond the school syllabus you ought to know for example functionality of a parabola, because just drawing the graph you can't show them how it is used in abridge for example or how it is used to construct head lamps or the focal lengths.

Sam: I look at my lesson, whether my lesson would need those diagrams then I have to prepare the lesson. Then I have to think whether I have to include my diagrams, if it is necessary I have to include them but just that sometimes you know when you are talking somehow with other people they use hands or all those things. I think that it is natural.

Dean: It takes a lot of preparation because you have to use diagrams that are suitable for the class. You can't use for example a diagram you used last, in the previous year because the pupils have changed, you have to work for the pupils that you have.

Maggie: Firstly you got to get whatever section you are teaching, whatever topic it is, you got to get information, examples and things like that. You can get from books, I do get it from books, you can get it from the net, now a lot of the hard copy is available on the net and then you got to put those examples onto slides, so you got to be able to put those onto a slide, if you just want to put it onto a slide and put it up, that's fine to. But if you want a presentation where you want the slide to run as you are explaining then you got to put it onto power point.

3. In what ways can the use of visuals help teachers improve their teaching?

Karyn: I think sometimes after you've used it you might be the one that understands the work better even as a teacher. It can only improve teachers teaching. The diagrams and pictures, again you put in an effort to get some information and from that the teachers also learns.

Penny: As I said, at best, it improves learner participation in the lesson. I believe that is, the enthusiasm of the learners' is much better, in terms of, when you make use of visuals. You would rather have the child enthusiastic to learn mathematics than just sit there and think that they are just wasting their energy being with you. You want them to be a part of your class and part of your lesson and you know most of our children believe that they are totally useless in mathematics but at least they enjoy the lesson and having visuals etc gives them that bit of satisfaction. They are not falling off to sleep. It motivates them to learn.

Alan: I would say visualisation is the best way to make any type of association but the problem with maths is there aren't that many sections that lead to the use of that. That is the handicap with that if you are teaching factorisation you can't really show them something visual or nothing I can think of hand. So that is the problem with maths so they got to take sections where they can make it a bit enjoyable because kids find maths a bit boring if they are not good at it and maths becomes a hassle so those sections like the parabola, general solutions things like that where they can associate it with a graph besides all the numbers, make it more associative. Something they can learn from, associate with something they are familiar with. In those sections they should do because when the kids see the instructor just writing out numbers and x 's all the time there is some method in that madness and with some sections there is some logic in it, show them how calculus works what is the derivative why is the limit and how to do differentiation and everything else it makes life easier.

Sam: I think that pictures and diagrams can help, it is helping and if you look at these NCS, this new curriculum, you will find even in question

papers you will find the pictures, there will be pictures of, those pictures you will have to look at the pictures and then more especially I will just refer to a mathematics of finance. Math of finance sometimes they can come out with something, a picture that shows about the compound increase, all those things percentage, the rate, with pictures. So I think it's useful to use that pictures as well as diagrams that can enable you to make your lesson more efficient.

Dean: As I said earlier, in mathematics especially, pupils see more in a diagram than in just calculations so by using diagrams they are able to make or grasp the topic and make connections to previous knowledge and such.

Maggie: I think using visual stimulation would benefit a teachers' teaching, we have access to a smart board and computers when you plan a lesson maybe on power point or you can actually print that lesson out and other teachers can actually benefit from it, so you can print out a worksheet that you presented to your class and you can make copies and run it out and you can copy it on disk for them so that they could actually use it, either on a smart board or on whatever other visual aid that they use.

Section C: Support

1. Did you need training to use visuals in the classroom?

Karyn: We do have teachers that are more computerised than others, we do use each other as teachers but there is not really workshops and stuff like that. I can inform you that we would like to have.

Penny: Not really, well I have attended many workshops etc, I suppose it happened in the normal education practices that we have been exposed to in terms of workshops in preparation for OBE etc, so nothing else.

Alan: No.

Sam: It depends on a person, I don't think you can just go and get a, I don't think it depends to an individual whether you need it or not, but for math teachers you have to use the diagram.

Dean: The training comes with teaching practice and also advice from other senior persons.

Maggie: Yes I did go for some training I wouldn't say a lot of training. We had a workshop where we had a little bit of training on, not on the use of the smart board so much but on aids. How we can get aids for the smart board like the websites and things like that and we had training at our school, but you know it is not difficult to use because if you use a computer its exactly the same. So if you are familiar with the computer then the smart board it is just a touch screen computer and power point presentation , it was just me playing around with the computer.

2. Do you need any further support to use visuals? If so, what support do you need?

Karyn: It would be wonderful if we could go on a course to use the smart board properly I know that there is one during the July vacation especially teachers teaching teachers and it is all about technology smart board, websites, and availability of software.

Penny: I think so in terms of using a lap top and projector and using the Cartesian plane and moving of graphs etc. I have not mastered those concepts, the use of the computer and mathematics basically I have not mastered that.

Alan: I do because the more you are exposed to. I mean I could use gestures from one to 10 and someone else is using 20 gestures and the more you are exposed to it, you pick up ideas things that work, things that don't work and different ways to approach something and different diagrams to illustrate a point for something, the different charts or picture to show something because it is when you are exposed to something or even with people in the same field or even with people not in the same field tell you something that you have never thought of, so you always learn.

Sam: Oh ja, I think that as you know that now everything technologies using for instance you have got a computer. Sometimes it can make things easier not even having those diagrams because you are talking about something that is in front of them. So I think that a computer has a precedence say that teachers they may get laptops and I am hoping that we will, more especially they start with math teachers.

Dean: Ja, I would say yes, because we are now moving towards things like the smart board and we need to, although we have acquired a smart board recently, we need to get practice and guidance on as to how to use this because I think that is one of the ways to go.

Maggie: It would always be nice to have further training on the stuff that I have not been using in my lessons and others out there have been using. Yes definitely, I would like to be involved. I know we do try and keep in touch with schools that do have smart boards and use this kind of presentation and

we want to buy, we don't have that much technology we are looking at buying things like Geometry sketchpad which is good for graphs and things like that so it helps all of us draw diagrams even if it is on a worksheet. I have been for an intensive course on the use of Geometry sketchpad so I know how to actually use it, but as I said it is always good to learn.

Section D: Interview based on lesson observations

Karyn - Observation1

4. R: When you were explaining distance between two points, you drew a line, what was the purpose of drawing this line? (1:23)

K: It's to explain to them the distance between two points, so they can see a point and a point and what the distance between these points is.

5. R: Do you think that if you had drawn it on a Cartesian plane it would have made a difference?

K: It will be much neater and ja, it would be wonderful if I could do that, I could do that yes, it is a very nice idea to perhaps include in that page, that smart board page a little Cartesian plane and show them perhaps points and do that example over there. I can't remember if I did an example in the next page.

6. R: You used hand gestures / movements to show the slope of a line, what was the aim of using these gestures / movements? (4:21)

K: With the straight line work in mathematics it's about in three different sections you work with straight lines. Functions, analytical geometry and at the end in Calculus. So straight line is really a concept in mathematics that they need to understand properly and they need to know if the slope is positive it will be in that direction and if it is negative it will be in the other direction.

7. R: So you felt that there was a need to use gestures to show this?

K: Oh, definitely there is always a need to know, yes if the value of the gradient is positive yes the line will be like that (uses hand gestures during interview) and it is always very useful for them to know that because if they sometimes have to determine the gradient and if the line, the slope is in the positive direction that if the answer is negative they need to notice stop, something is wrong here, I need to relook at my calculations.

8. R: What was the purpose of drawing a line through the \langle , \rangle and the $=$ sign? (4:29)

K: Okay, the purpose of that is just that it makes it easier for them to remember, can you see every time when I draw a line I use the bottom line and when it is like that I use the other side and when it is horizontal I use the straight line. It's just a way for the learners to remember.

9. R: Why do you think that they would remember, what are you doing for them to remember it?

K: I don't know, they can see it, they can visualise it.

10. R: If you had to show for a vertical line, how would you do this?

K: Okay, no vertical line does not have a gradient. So that is why it is not part of that. It's undefined.

11. R: Why did you choose to point at the x and y coordinates when talking about finding the midpoint? (6:04)

K: Okay the little formula says here you have to use your x; your two x coordinates on the edges of your line to find the midpoint.

12. R: If you had just told them that, would it have had the same impact?

K: Here again they need to visualise it, to see it, ja and I think that they just remember it easier. I like to explain with examples. If you just talk to them they might fall asleep.

13. R: You tend to use a lot of different colours when teaching, what is the purpose of using different colours? (6:16 – coloured boarders, coloured diagrams, 22:10 – different coloured markers, Observation2 – Clip 1 – coloured markers, Observation 3 – coloured boarders, coloured backgrounds, 11:13).

K: It is again just to get the learners attention; I think that is my one and only reason.

14. R: So how would it be different if you did not have colour?

K: If you don't have colour it is all so dull, and they might just think of something else, and when you have colour on the screen you tend to just get their attention. I don't know if I can answer that question more better. It's just not so dull.

15. R: I have noticed that you use various visuals on the board, what is the aim of these visuals? (Stars – Observation 1, Clip 2 – 20:07, smiley faces – Observation 1 Clip 3 - 00:16, Observation 2 Clip 1 – 14:38)

K: That's me, ja again I think you noticed when I did the teaching when they see something new or different they say "Oh mam there's a smiley face !" and what do you have, you have their attention and that's what you want, so ja anything to make the lesson more interesting. Sometimes it's hard to think of something.

16. R: Why did you underline (equidistant from p)? (21:20)

K: Okay, I think the reason there was, lots of, a huge problem now is language and I think that why I now when you have equidistant you make sure they understand what does this mean. These questions can become very difficult with equidistant, is it equidistant from this point to that point or is it twice the distance from here to there so it is very important for them to focus on the key words and the key word there is equidistance.

17. R: When you teach you tend to highlight quite frequently, what is the purpose of highlighting? (Observation 1 – Last clip – 1:47, Observation 3 – 6:14, 6:19, 6:30, 6:43, 7:33)

K: Highlighting again is to get the idea of what is important and when I teach most of the times I always say read your question you highlight or take out your keywords and then usually you find you select your formula you substitute and solve. So it's for me very important that when they read questions they need to get used to fact which are the most important words highlight them and from there you choose your formula and then do your calculations.

18. R: It is very interesting what you did here, you used your software and moved the line, do you think that if you did not have your smart board you would have demonstrated this a effectively or was it in fact effective?

K: This is so much more effective than anything else but in the old days when I did not have it what you had to do is then you just draw lots of lines all the possibilities so it will not be as neat and you will have about twenty lines there to explain to them that point can be at any point on that line. So you will draw from P about twenty lines to that dotted line there (refers to the diagram on the DVD). So no it is definitely not as effective.

19. R: If you had used a compass do you think you would have had the same effect?

K: No not really, because you are moving the point on a straight line, with a protractor you won't be able to do that because the protractor it will be from a point and it will go like that (making an arc with her hands on the desk) and my point to show them was this (showing a straight line using hand movements on the desk). On a straight line and the protractor will form a circle. (Karyn meant compass when she referred to a protractor). No it won't work.

20. R: What about your hand gestures, why do you move your hands when you teach?

K: I am an active person, I am busy, I don't know a reason for that but I think it's just to make sure that they listen to you. Hard to say, it's just my way of teaching.

21. R: When you were teaching, you asked the class to find the perimeter; you traced the triangle to ask them to find the perimeter, why did you trace the triangle?

K: To make sure that they know what is a perimeter, definitely. Because they could be, some of them, sometimes they are just too shy to ask as well and again its language you need to make sure that they understand if you use a word make sure that they understand what you are talking about. They always get confused between the perimeter and area.

22. R: If you were teaching area, what would you have done?

K: I think I would colour it in (referring to the diagram of the triangle). No we will shade it, definitely.

23. R: Throughout these lessons you continuously used diagrams, why did you do so?

K: I think it's the section that lends itself to it, it is not all the sections, I mean if you do algebra you will not use many diagrams but analytical geometry is all about pictures. I always say if they get an exam paper or test and if there is no picture with it you draw a picture or a diagram. So I think it is more the section than really my way of teaching all the time.

Observation 2:

24. R: Why do you use arrows when teaching? (19:24, 21:39, 27:57, 30:47, Observation 3 Clip 1 – 16:24, 28:26)

K: For calculation purposes, That is what they need to type into their calculator, I think I needed more space there, just to take them and say this is what I am doing there, how do you do this question on your calculator, I think it's just an issue of space. I did not have enough space there to put that in. I used the same formula there, in another example. So in the first one I substituted t with 1 and then I used the same formula and I substituted t with 2. I think that is all. For space, I think I did not have enough space to write out 60 degrees so I just take it out. I like my arrows, I like my blocks.

25. R: If you had used something else instead of an arrow, would it have had the same effect?

K: You don't have to use anything there actually, you can just carry on, and I don't think the arrow is necessary.

26. R: You drew broken lines on the diagram as you were teaching, what was the aim of drawing these lines? (Clip 2 – 5:48).

K: I think it was a word problem and they have to select which one of the lines will represent 5 minutes and I think I just made a dotted line to highlight it again.

Observation 3:

27. R: When teaching differences, what was the purpose of drawing converging lines? (6:04)

K: That is to see a common difference, the difference between 5 and 8 is 3 and the difference between 8 and 11 is 3, that's the way the textbooks do it and that's the way we usually explain it, it's an easy way for them to see, if it is common or not, if you have the same difference between all the terms or not.

28. R: I have noticed that you point to the board whilst teaching, why do you do so? (12:13, 19:38, 25:09)

K: To make sure that they know what you are talking about, after a few years of teaching you have habits that you are not aware of.

29. R: What if you had to make the statement without pointing?

K: It might be boring, I don't know, you can say it without pointing I am sure but I think if you point you can just get their attention back and say listen this is what I am talking about.

30. R: What is the purpose of using rings / borders / blocks around certain expressions / numbers / terms? (Clip 1 3:53, 18:16, 22:28, 28:24)

K: I think it's again the more important things that you need to try to highlight to them. It's the formula, it's the little formula. I think it's just again to highlight important information they need to pay attention to.

31. R: When you are preparing to teach do you find yourself consciously thinking of a visual way to teach?

K: I think we are all dreaming of having well prepared lessons for every lesson and I think the best is to go and find a nice idea how to approach the section you are going to teach and ja that's my dream really but we have so much happening and other stuff that you are running around for so you do not really get the time and if you do put in an effort to plan a proper lesson, you can see the difference in the learners definitely. They enjoy it and as a teacher you enjoy it more. It's wonderful to be prepared for a nice lesson especially if you have all these facilities.

32. R: You constantly say that if you did not do something it would be boring or dull are you saying that visuals make it easier for learners to understand or are you saying that it really does not make a difference?

K: It definitely makes a difference in understanding because they visualise it, they see it and when you put in colours and the smiley faces and stars they just enjoy looking at it, you have their attention. It definitely makes a huge difference in their learning.

33. R: Have you tried teaching without the visuals?

K: Oh well we always did, I think with the visuals you do save time definitely teaching time is saved because for example that line I used, if I had to draw 20 lines it is going to take me longer and they are not going to understand the real concept. It is wonderful and so useful. We did teach in the past without all these equipment and I think the learners still passed, we were learners in schools with only the blackboards, it's possible but I think that it is not that much fun and it is time consuming and so forth.

Penny: Observation 1:

1. R: You used a drawing of a happy and sad face to explain which way a graph turns, why did you choose this visual to explain this? (00:00 – 00:18)

P: The happy face and a positive parabola basically and we bring in the values of a child being positive and children remember it because we often talk about being positive being happy and basically I use it for that purpose expecting them to remember it better and that is how they do remember it, when they see the positive a, they know, they smile to tell you that's the way the graph is supposed to go.

2. R: What was the purpose of using hand gestures/ movements to show movement to the left and right? (31: 00 – 31:12)

P: I think that was about movement of the graph to the left and the right, initially we had taught them the aspect of q, which moved up and down and when it was positive we taught them, they understand the positive part of the y axis is moving up and down and with horizontal movement it's the opposite direction so it was important to stress the difference in that concepts. Using my hands was the only way I could do it. I don't even know if they all understand the movements left and

right, we have to emphasis it and ensure that every child is with us. It's basically coming down to their level. I honestly believe they understand my movement and visuals.

3. R: You drew a graph to ask the class where the graph was originally situated, why you chose to draw the graph, why not just ask the question. (31:20)

P: As I said visuals is the only way the child understands what I am talking about and making them see it will help to remember it, if you look at my children's' books you will see at the back of their books 3 statements written there which says: I hear and I forget, I see and I remember, I do and I understand. So I stress, I believe that they have to see something to be able to remember it as much as we stress the doing part it does not always happen. Unfortunately that part we cannot make happen. As a teacher the visual part is very important.

4. R: When you say the visual part is important, did it take time for you to realise it?

P: I actually picked this up from a course that I taught at one time, I learnt how to remember a 100 numbers within 5 minutes by using visuals. I would have a list of numbers 1 to 20 and I could ask you to put numbers next to each one and I would then take each line and create a picture from it. Once I have the picture in my mind for number 1, I can tell you every number in that line. I used to actually facilitate such a course.

Observation 2:

5. R: What was your aim in using breathing exercises to start off the lesson?

P: I am an Education in Human Values facilitator so in that aspect we facilitate the values aspect to all teachers and in it we work with tools that will assist pupils in the classroom. One of the main tools is breathing because the brain actually receives only 30% of the oxygen it needs in normal breathing so we encourage them to do deep breathing in order to feed their brain with some oxygen during the lesson but it also calms them down and brings them into the classroom. It focuses their attention.

6. R: You drew a Cartesian plane on the board and used hand gestures / movements to show angles on the Cartesian plane, what was your purpose in drawing this Cartesian plane and what was your purpose in using the hand movements / gestures? (9:38) →

S: These are tools that I expect children to make use of in their classrooms we need to know exactly when we are making use of it and what thought processes are happening when you need to focus on that aspect of the diagram or the mathematics itself that is the idea. To make sure that they are focused on what we are looking at and what we are working with. When I point I am directing their attention to the concept involved at that point.

7. R: I have noticed that you use many hand gestures / movements when you teach, what is your purpose for using these movements / gestures? (15: 30, to show movement of angles, 15:37, to show anticlockwise direction, 30:19, to show where angles lie, 35:30, to show which position angles lie between, observation 3, 19:32 on the x and y value of the graph)

P: I had to show them because the concept of a clock is also so lost now with the digital time machines so the children are not so aware of clockwise and anticlockwise as much as they may know it I have to ensure that they do know what I am talking about. I definitely cannot assume that every child is aware. I can't take that chance so I use the gesture to show it.

8. R: I have also noticed that you use different pieces of coloured chalk whilst teaching, what is the purpose of using different coloured chalk? (43:00 , 12:43 – Observation 3, 13:38 – Observation 3)

P: It attracts children definitely and it does not confuse them as to where the focus is and it also separates concepts on the board.

Observation 3:

9. R: Why did you choose to start the lesson off with a story?

P: Well generally I tend to use stories and I wanted to relate it to something in the lesson. Basically what I do with stories is I want to bring the children into the classroom and I intend to bring out values into the lesson in every lesson. Some values in the child, so the stories help me to do that and the interest that children

show is amazing so it basically focuses their attention but I also draw them into the mathematics with it and if I am not mistaken in this lesson I never actually got to the aspect where I brought the story back and explained how it had impact on the lesson but that is what I generally do. There is a concept in the lesson that I would relate the story as well, and children enjoy it because it helps them to remember and also it prepares them for life itself. It draws their attention to me and to the lesson itself, they must want to hear my voice that's the bottom line. We cannot just kill them when they enter with 'shut your mouth' or 'be quiet' so once I start a story I can get silence like that , they are all 'ears' with even the worst of classes I experience that.

10. R: You drew graphs on the board to remind learners of previous work, why not mention the name of each graph to the learners?

P: Once again I think it goes back to the aspect of visuals and when they see it they remember what we had done, just to recall or bring back the memories that we had put into their brains last year and just telling them or relating the words is not enough for children, they need the visuals to take them back there to those memory banks they have. Also there are sometimes children in our classes who may have not been there the year before, etc so we have to recall everything that is necessary in this lesson before you take them into the new concept as well.

11. R: When you write answers down, you draw a block around it, what is the aim of drawing a block around the answer? (11:51)

P: This was the aspect of finding the equation of the graph and I once again make them aware that that is the final stage of the answer where they need to realise exactly what the equation is and not thinking that basically to make them realise that that is the focus of the lesson. The block draws attention to the whole purpose of the lesson.

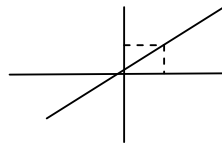
12. R: You gave learners chart paper to work with, why did you choose to do this instead of letting learners use their books? (38:09)

P: The aspect of group work helps those that are not picking up the concepts easily or may not have remembered things from previously, so they help each other and when you ask them to present that chart on the board then the enthusiasm is so

much greater and everybody wants to be a part of it. So it does facilitate learning, peer learning. It helps to encourage peer learning. I try to do this as much as possible especially when we are not pushed for time in terms of the syllabi but it does not often happen that we can do that, it depends on the lesson as well and when time permits it I enjoy doing it and sometimes when you want to cover many examples then I use that technique where I get groups to do a few and they are presenting it and by the end we have 10 examples done in the classroom where we could have just had 2 done by the whole class. I use that quite often especially in my literacy classes and it works beautifully. They have more answers to work with answer banks. As much as a child has worked with two problems but they have been exposed to 10 by the end of the lesson, the other have presented theirs on charts which is visual as well. So they get to see it.

Alan: Observation 1

9. R: You drew a diagram, what were you trying to achieve by doing this? (3:06 – 4:27)



A: On that particular worksheet I remember they were looking at a point symmetrical along the line $y = x$ so I needed for them to get that line drawn that is the only reason why, on the worksheet because it was symmetrical and we had one point and we needed the line of symmetry, experience tells me that these kids won't be able to do it because if you tell them to draw the line even at grade 12 level they are used to having a x intercept and the gradient. Even when they use the normal the dual intercept method they substitute 0 there is a problem there is only one point and then they will draw the line. Even if they draw the line skew then it is wrong according to the equation. So that is the only reason why.

10. R: Why did you choose to use the paper folding for the exercise on reflection?

A: Because I don't like teaching kids to use formulae and rules like that because if you forget it becomes a hassle so if they can see it, if I am reflecting along the x

axis if I fold it I am seeing the exact same thing, so there is no need for me to break my head to find that if I am folding it along the x axis that the x axis stays the same and the y axis the sign changes. But by folding it they can visually see it, even if I don't know which sign is changing and which side stays the same, by folding it I am getting my results, because even sometimes people use things like SOHCAHTOA, kids can't even spell it and they don't know which ratios are what. So personally I don't like to use too many formulae, you must learn how to do without that is my strategy.

11. R: You used hand gestures / movements to represent the reflected image, what was the purpose of doing this? (9:35 – 9:38)

A: Its subconscious for me it would have worked you are reflecting it, it's something you can see even if it is not happening, it is not working on your piece of paper you can see what you got and what you should be getting. Once again it comes back to the first part if a child tries to visually see his result before he obtains it if he has an idea of whether he is right or wrong then when he is doing the work then he should know if something is wrong because it should be in a certain way but it is not which means I need to go back and check.

12. R: What were you trying to achieve when you used hand movements to represent “marking the point”? (11-20)

A: Sometimes some gestures are subconscious like that one.

13. R: What was the purpose for drawing the arrow? $\{(x,y) \longrightarrow ()\}$ (14:57)

A: I am using it as a symbol of mapping, the coordinates of one set is being mapped onto a completely different set or the kids will look at it as from one point moving onto the other so changing from x, y moving to x +3 or y – 1 or whatever the case is.

14. R: What did you hope to achieve by pointing to the x and y values? $\{(x,y) \longrightarrow (y,x)\} / R(3,0) \longrightarrow R'(0,3)$ (17-08)

A: Because that way the moment you point or say look here then besides the child listening to you he pays more attention because he could be doing something when

you are speaking but the moment you stop and point to something on the board although you are saying the same thing his focus is more towards that concept that you are trying to highlight. Because if you don't, you are saying it and he's hearing you but it is not registering and if it does not register then it is a problem and he can't get the task done or he is lost in the task. So there are times I will say it and I will go and point that what you need to do.

15. R: Why did you choose to use finger movements / gestures to represent the x and y coordinates being swapped around? (28-51)

A: The finger was here, now it is there, fingers interchange and as a result the coordinates interchange and the fact that the fingers have interchanged it hasn't really changed the finger it's just the position as you viewing it as changed but your forefinger is still your forefinger so the same thing is true of the coordinates. The coordinates have not changed but x becomes y and y becomes x but the sign and value has not changed.

Observation 1 second clip

16. R: To represent 'blowing up' or 'enlargement' you used hand gestures, why did you choose to do so? (4: 22)

A: I am always using gestures; it's just something that I do because once again if you can see what's happening, even if you don't know the meaning of a certain word but you can see the action then you can make the association. So if something is contracted you will squeeze your hand together if he does not know the word contracted but he can see the gesture then he will understand the word and associate it with what is happening the same thing is true.

Observation 2

17. R: I noticed that you tend to underline expressions / terms whilst you are teaching, why do you do this? (23:00)

A: Once again it is a question of them hearing something and seeing it because if you tell them you take -3 -1 and they hear you and they know what you are saying but it does not register sometimes but if you highlight it then they focus on it. I use it mainly to draw focus to something like I did it with the equation and I showed them which is a constant. They know which is a constant but the fact that you are

underlining it is also emphasising the fact of what you are talking about. So that is the only reason why, because underlining will emphasis certain aspects of the formulae or certain word, draw attention to something I think that is critical trivial as it might be but it is critical in the sum for some particular reason trivial in a sense that it is a constant .

Observation 3

18. R: You used hand gestures to simulate a line moving, what was the purpose of doing this? (11:19).

A: Once again it is a question of visualisation , the learner can see exactly what you are doing so I used my hand to show how it is moving then he can see what you are talking about the line itself is moving the angles are moving. The sign has not changed in anyway except for position, it hasn't become a curve all of a sudden so it has kept its basic characteristics because in a way sometimes you are saying certain things and the kid is thinking of something totally different, and, if you don't show them what is happening you are making the assumption that they know what is happening but if he things of something else his entire trig would suffer.

Sam - Observation 1:

1. R: Why did you choose to use windows, bricks, etc, whilst teaching the lesson? (3:48 – clip 1)

S: I think that, I start from the concrete, something they can see, something they can, it's just in front of them when I am talking about the bricks they can see the pattern in bricks right in front of them without even going outside, that is why I refer to the pictures, I use bricks for that lesson. I think it makes things easier if you are talking about something you can see.

2. R: You drew converging lines on the board whilst teaching, what was the purpose of using these lines? (5:44)

S: Oh, yes, these converging lines they help just to see the difference between the first term and the second term and they can even because now with those lines they can see that I am talking about the difference between 1 and 4 as you can see there 4 and 9. So they can even go further then and see that there is still another

difference between 3 and 5 that is why I think it helps to see from the beginning that I am talking about 1st term, 2nd term, etc.

3. R: Whilst teaching you use various hand movements / gestures. What did you hope to achieve by using these movements / gestures? (Observation 1 – 7:21, Observation 2 – 41:40, Observation 3 – 2:06, 22: 54)

S: Sometimes it is natural, it's natural. Some people cannot just talk straight without doing anything. They use hands even you can see other interviews, politicians they use hands so even myself it's something with me, when I am teaching sometimes when I am teaching I use hands.

4. R: I noticed that you drew a block around the general form, what was the purpose in drawing the block? (Observation 1 – Clip 1 – 10:32)

S: Oh yes, that block you know with the learners sometimes they forget what you want to achieve , what you want to get in that second question, for instance in that one you can see you want to find the formula, the equation so we have got that formula. So to emphasise, that block that I put there is just to emphasise that you are not yet finished if you did not achieve what you want to you have to come out with the equation at the end, that's why I, it's just an emphasis on that one. You see sometimes students they can just leave, c, as you can see $c = -1$, and they leave it like that. They forget that what are you looking for, what is the question behind that. I used it to emphasise that don't forget what we are looking for; we are looking for the formula for this number pattern.

5. R: When you are teaching, you tend to draw arrows quite frequently, what is your purpose for drawing these arrows? (Observation 1 – Clip 1- 7:45, 29:20, Observation 3 – 6:10)

S: Okay, no as I mentioned before that the way that I started with using the block and the arrow that it's just come to an end there, so my problem there that's where it ends, but with the first one that's showing, I was referring that the linear, the linear equation must have the first difference I know sometimes this arrows, but the one that is implied is not the same as the one that is implied it's just an arrow showing that where we are now, we are in the first difference that's why I am showing them the linear is the first difference. So the arrow there it shows that, I

am showing them that the linear is the first difference. It's like when you showing them the one to one mapping, in one to one mapping we use that this element it corresponds to that one, so the arrow there it is the same thing as I am doing here. I am just doing that the linear here, it's for the first difference. I could have gone further, that the second difference is having the quadratic pattern. It must have this second difference so it's just like that there is no other meaning for that.

Observation 2:

6. R: What was your purpose for drawing the circle at the beginning of the lesson?

S: This circle, if I have to draw a circle the first thing that I come up to my mind it's not for the sake of drawing a circle, they must know when you are talking about a circle what is involved. In the circle there is a centre of a circle, there is diameter, that is terminology that they must use each and every time there is a chord there is tangent and for those that are doing math paper 3 when you come to that just remember, recall what we have done with the circle. They know about the circle, so in other words I don't think that when you draw a circle it's just a matter of talking about only that particular lesson that we are doing. They must know about a circle before I can come up with that system of axis there. As I mentioned that to draw a circle you need to know everything concerning that circle. They must know everything concerning the circle, so now I am coming to the Cartesian plane, in the Cartesian plane I am trying now to show them that in a Cartesian plane you can use x and y and r , where does this r come from if I didn't talk about it in the first place there about the circle that there is a radius there, they don't understand where we are coming with the radius. So now after we have done that its easier now to refer to come back to as I am doing now this Cartesian plane. It's easy now when I am drawing the radius, they can see now, they can compare now, oh! We have been talking about the radius in that circle.

7. R: You drew a diagram to show the signs of ratios, what was the purpose for doing this?

S: It's a right angle triangle; you must be flexible when you are talking about math you cannot just rely on one and the same thing that they will only use r , the y and x those ratios. The problem can come out without the y , x and r ratios so they must

know that if it is a right angled triangle, what is involved in the right angled triangle, so those are the basic things without any Cartesian plane. That's why I draw that right angled triangle, they must see that opposite side depends on which angle you are using as you can see there repeatedly or I am emphasising that this angle theta there is opposite this side and they must know about the theorem of Pythagoras. But because this is related to what I have been doing there in the Cartesian plane there is $x^2 + y^2 = r^2$, if I can come out with that right angled triangle its showing them the theorem of Pythagoras. I think it is very important to have a triangle there without a Cartesian plane to have a right angled triangle, they must know how to this trig ratios.

Observation 3:

8. R: Why did you draw a diagram to show the cast rule and α , $180^\circ - \alpha$, $180^\circ + \alpha$, etc?

S: I believe that it would make things easier for the learners more especially if you say without using a calculator because they must get this without using a calculator so that diagram it has everything even if the child is writing the exam they can write this down in a rough paper so it means that it shows us that particular child knows what she is doing so now easily you can remember that so if you are talking about a 210 degree angle they can see that, okay it is not in the first quadrant, it is not in the second quadrant, 210 it's in the third quadrant, its $180 +$, so if you say $180 +$ what to get 210, then it makes it easy to remember.

9. R: Whilst you are teaching you tend to point to the board, what is the aim of doing this? (9:04, 9:20)

S: You know, now you see we have got different learners you cannot for instance in this model C schools and with us having many learners coming from this informal settlements, it is not that easy just telling them. You must think about those slow learners there, you must go with them, you cannot leave your learners behind and then you go all by yourself. By pointing out, you see in my lesson I am repeatedly going back and showing them where this is coming from so that when by the time I give them the work so everybody will understand the lesson so that's the way of making them understand, because as I have said in our school we take every Dick, Tom and Harry in our school. We try and teach them to make them to understand,

so for slow learners you have to think about that, not only about the bright ones because if you keep on trying they can surprise you coming with a test. You find that most of them fail, so that to get a number of passes you make sure that they understand the grass root concepts that they need to know.

10. R: Why did you draw a diagram of the circle to show the learners the radius?
(15:50)

S: Okay if you can look at what you got there, you got the method of getting special angles without having a calculator, special angles where are they really coming from not simply telling them that when you are talking about special angles its 0, 30, 45, 60, 90 no, but they can see now from the diagram that they drew why is it 60, why not 50, so that's why we came out with an equilateral triangle, we are going back to the basics of the properties of an equilateral triangle so in other words everything is coming from that telling me that all sides are equal that all angles are equal, each angle is 60 degrees that's why we came out with those special angles 60 degrees. In the equilateral triangle they can see that all sides are equal so I put there the length of each side and then I came out with what's 60 degrees, 30 degrees as you can see it bisects that angle, it bisects the vertex there then you come out with 30 degrees. So that is part of special angles. If now we know that we are talking about the angle that's less than 90 degrees but what about more than 90 degrees like 180, 270, 360? That's why I came out with this diagram, the circle as you can see, so the circle includes everything, it includes 0 up till 360 degrees. Although I know that by using the calculator you can come out with the same answer but I want them to know if we are still using because this is the continuation where we have started with the ratios even there we can also use the ratios to find the special angles, by means of the circle there they can find the special angles.

11. R: Why did you circle the terms when you were simplifying the expression?
(21:57)

S: You know our children, these are the mistakes that always occur you see any other child can start by subtracting half minus 1, by putting that circle there I was showing them that something that they know from lower grades. They are talking about BODMAS, so my emphasis there is that you cannot start with the subtraction, you must start with multiplication first and then you can come to the subtraction.

So it's built on what they know from the previous classes. So that was the emphasis because these are common mistakes that are happening with most of the students even in grade 12 you can find a child that are not multiplying they are starting by saying half minus 1. Even in marking centres I can find one and the same thing so that's why I was trying to emphasise that.

12. R: Why did you use different coloured chalk when you were teaching? (24:43).

S: If you look there I put that special so that they can even remember what they have done because this we are rationalising the denominator, it is something we have done early this year, even in grade 10 I think so. The coloured chalk is to show them guys don't leave any number in the denominator with a square root sign you have to rationalise make it a whole number it must be a rational number that is why I put that colour chalk. Its showing them that it is kind of a warning, don't leave the answer like that it is not complete, you have to use the skill the one that you have done in grade 10 rationalising the denominator. I usually use colour chalk it helps me more especially children they can see now with that colour chalk it means something they can see. For instance let me take for example if you are a geometrical rider and you are getting maybe a parallelogram there inside or a cyclic quad once the thing is marked they are equal by putting coloured chalk on the board. I am just showing them don't forget in mathematics they don't put anything without using it. By putting it there, it is marked that it's equal because we are going to make use of it so it is to show them that there is nothing. They are not trying to make a beautiful diagram, it means something. I am using coloured chalk and it is helping them a lot because they can see what is important.

Dean: Observation 1:

1. R: Whilst teaching the lesson, you used an arrow, $\{x + 3\} \Rightarrow \sqrt{5}$, what did you hope to achieve by doing this? (11:35)

D: My intention was to show the movement so called movement from one side of the equal sign to the other and the possible position or the correct position of where the number is supposed to go.

2. R: On reflection do you think you achieved this goal?

D: It took some time I would say but with the use of the calculator the pupils, most of the pupils or some of the pupils' did get it right. Then this was the first lesson and the kids were a bit scared (The first video recorded lesson).

3. R: Whilst teaching you used hand gestures / movements to represent steps to be followed when using the calculator, why did you choose to do so? (14:57).

D: It's the force of habit, I am not aware that I am doing this in the sense, how do I answer this question. It somehow works when you do it like that because when I did this the sign language of a bracket then they can see it on the calculator that's how it is on the calculator so they know which key to press.

4. R: I noticed that whilst explaining a step to a learner one on one, or whilst teaching the entire class you chose to use an arrow, what was your aim in using the arrow? (27:11, 30:31, 3:03 (2nd clip))

D: So that the kids can identify certain parts of the number, the arrow is pointing to a specific part so the pupils will be able to identify that, if you just tell them that this is the surd, and then there is no visual there. It does not stick long in their memory. At that moment in time they can identify the surd, they know exactly what you are talking about when you say this is the surd.

Observation 1, 2nd clip:

5. R: You ask learners to use highlighters in their books, what is the purpose of doing this? (12:11, 13:13)

D: I think that it is beneficial because when you use a highlighter and I highlighted only certain parts here, the parts that they are working with and the parts that are undergoing a transformation so they know exactly which parts are undergoing some transformation and when they read through it instead of giving them just a set of notes 'take this to the left' or whatever here they can see exactly which part we are manipulating.

Observation 2:

6. R: Whilst teaching you sometimes use different coloured chalk, what were you hoping to achieve by using different coloured chalk? (9:26, 15:12 (Observation 3))

D: Normally you use a different colour maybe when you want to stress something that is important or you want the kids to focus on this. If there is a different colour obviously they will, because of different colours they will be focusing and if you tell them that this is important they are going to be focusing on that. I want them to focus on something that was important at that stage.

7. R: Whilst talking about the sign of x , you used hand gestures to show the movement of x , why did you choose to use hand gestures? (10:00)

D: I am sure that this would be beneficial to the kid because, one answer is that it is a force of habit because of the way the y axis is, if it is positive you go up, I am even doing it right now, maybe because from experience. I find that pupils find it difficult plotting points on the Cartesian plane and maybe by showing them movements up and down with my hand it makes it easy for them at that time.

8. R: I noticed that you used a stick with coloured rubber bands whilst teaching the lesson, what was the aim of using this manipulative? (13:13)

D: If you just show rotation and if you just tell them or give them the rule, in my opinion they are not going to see anything, they are not going to learn anything they are just going to go by the rule and then apply it if they can remember the rule. But by showing them the rotation using the stick, we can see exactly how the position changes and I wanted them to see that initially.

9. R: You explained counter clockwise direction to the learners using hand gestures, why did you choose to do so? (20:58)

D: I would say that there are two reasons for this; one is to show the initial position of the points and by the use of the arrow to show the turning position. The second we take things for granted that pupils know what anticlockwise is and what clockwise is so I wanted them to actually see it on the diagram.

10. R: I noticed that you tend to use many hand gestures/ movements whilst teaching, why to choose to do so? (26:11, 35:40, 36:28, clip 2, 00:08)

D: Although I said it was unconscious, if you have a look at the gestures they act, they correspond to what I am saying in most cases whether I am moving to the left or the right or up and down. I think I am basically acting all that I am saying.

Which in my opinion is beneficial to the kid because you don't only listen, you are also seeing what's happening and that plays a beneficial part in developing new knowledge?

Observation 3:

11. R: You used a mnemonic to teach special angles, why did you choose to do so?

D: From my past experiences or the experiences that I have had I have noticed that pupils when they are given the triangles to learn to get to the special angles, that they do have problems and in most of these questions where they say without the use of a calculator and if they got the triangle wrong they get the entire thing wrong. I also found as I have been teaching that they find it very easy to remember this which is then beneficial to them.

Maggie:

1. R: What was the purpose of using different colours on the OHT? (00:04)

M: The red lines indicates the shift of the actual graph from the basic graph it indicates the actual vertical and horizontal shift so you have to draw in the shift the lines that indicate the asymptotes and then you got to move the graph according to the red lines. So the red lines actually now become sort of your new set of axes. To highlight that we are now moving away from the original axis and we actually using the red as a new set of axis. If I draw them in the same colour and I try to explain it to them it might be a bit difficult for some of them to realise that we are actually moving the original graph.

2. R: Why did you choose to use a picture of a fish? (00:02)

M: That was just for fun.

3. R: Why did you choose to underline what was given in the question? (00:04)

M: Because that is what we had to prove. That is what they actually have to prove in the rider. Because the question is quiet wordy being a geometry question just to distinguish from the question and where the question actually ends and the proof

part actually starts it just makes it that much easier, I underlined in that kind of question.

4. R: What was the purpose of drawing the diagram again? (11:00)

M: One reason is that my smart board does not, my pen facility is not clear so I am unable to draw on the smart board, which definitely should happen if I was able to draw and add to that then there would be no need for me to actually redraw the diagram, but I could not do that at the time.

5. R: What was the aim of pointing to what was given in the question? (12:20)

M: Just to draw their attention to particular aspects that I am actually trying to focus on. Because once again it is a whole lot of writing and the pupils tend to get lost sometimes if you don't keep emphasising certain things to them. It just becomes monotonous I think.

6. R: Why did you use symbols to represent 90 degrees on the diagram? (12:27)

M: That is the standard symbol that we use in geometry when you draw a diagram that is the symbol that we use to indicate 90 degrees. It is something that is standard and you learn from grade 8.

7. R: I noticed that you tend to use different colours on the board, what is the purpose of using different colours? (12:45)

M: I think it's my way of distinguishing either one step from the other or a question from the explanation or a proof because geometry once again is quiet wordy almost like writing an essay in maths. Because every proof is different so it tends to get complicated there is no one way of writing. So I tend to use different colours just to highlight the differences.

8. R: What was the aim of using the 'tick' symbol on your diagram? (14:03)

M: I think when I was trying to indicate to the learners that two sets of angles were equal angular that is just the way I tend to show two angles that are equal, you don't have to use a tick you could use two circles, two dots or two crosses, but in a triangle you get three corners so I generally use a tick and a dot and maybe a cross to indicate that one set is similar to the other.

9. R: What was the purpose of nodding your head? (14:06)

M: I think someone was asking me a question and I was indicating my approval saying that yes, it was true.

10. R: What was the purpose of use the gesture/hand movement to represent ‘cross multiplying’? (15:49)

M: You have to because, not all of them would get cross multiplying, If I just write that on the board it could mean anything, so I have to indicate to them exactly what it is they are cross multiplying. The only way, if it is already there I have got to show them what it is I want. I want them to cross multiply. I want them to understand, mostly when I teach these things on the board I want them to understand the method so if they understand the method behind what we are doing and even if we don't get to the final answer and then they can understand the method then they will be able to answer future questions.

11. R: Why did you draw broken lines in a different way on the diagram? (30:58)

M: I was trying to show them how to draw perpendicular lines from the vertex of one to the base so to indicate the difference between the actual triangle and the perpendicular line I indicated it as a dotted line, I think I used the same colour so I indicated in a different form of line. So if they saw it as two different types of lines they would be able to see that that was the perpendicular line. Because if you look at it and I put just a solid line then there would be two triangles, I was trying to emphasis the fact that it was a perpendicular line drawn from the vertex.

12. R: What was the purpose for drawing the diagrams on the board? (32:02)

M: I think the triangles that I was showing them were different, one was acute, one was obtuse, and each triangle had the vertex. The perpendicular or the bisector was different so I had to indicate to them how it actually looked in the different types of triangles. They actually need to be able to visualise and see, if I had to say to them vertex to the base I think it would mean absolutely very little to them. Oh, they would listen to it, they would probably understand for its words but they would not get it. At some point we need to draw it so that they see it visually because this was a visual section, geometry.

Section E: Philosophy of teaching

1. R: If you had to sum up your philosophy of teaching mathematics using visuals what would it be?

Karyn: I think it is unbelievably important for the learners to see and we are really spoilt with the smart board. If you don't have all these facilities it can be very difficult to explain some of the mathematics concepts, so ja the more they see the better.

Penny: I don't know whether I am going to be saying the same thing again, but at the end of the day I believe that children need to see things to remember it and I think most of the things that stick in our minds are things that we have seen in life. Those things actually get stored in our memory and we can use it again. So I can't see mathematics being taught without visuals it is not, you are not telling a story, it's something they have to see to be able to internalise.

Alan: I generally feel that you can't move away from the chalkboard and the chalk but at the same time technology must be part of your lesson because I have emphasised throughout this interview and I always emphasised this, and my HOD will tell you that for this kids if they can see what's happening then it isn't a hassle so if the kids can see anything. If they can see whatever, if they can see the uses, the first thing a kid will tell you is why do I need to do this I am not going to use this in my life but if he can see the use and that sin graph and a sound wave are very similar and he is using this kind of graph all the time and the parabolic graph is used in construction all the time. The focus in the parabola is where your satellite dish is something they can see, they can all these things. It makes the understanding easier and the work easier. And also especially with all the tedious calculation he can see, he has to do the calculation but we have programmes that they can use. But there also a few values is the calculation

and they can see exactly how they do the calculation like financial mathematics he can see that his loan will decrease the amount if he pays at the end. You normally do about 20 examples and you see the figures changing all the time so it expedites matters. So you are doing something that is needed in the syllabus with regards to different loan payments but you not taking 50 examples to establish the point in his head you can see it with 50 examples which you are working out with technology. So I strongly believe that you should know more than what the syllabus requires, don't restrict the child if you push him he will learn and you will pick up the mathematics results. If you give them a chance they will manage, you just need to steer them in the right direction. So that is what I would like to see happen, I would like to see technology being brought in because it is needed.

Sam: Don't take things for granted, don't take that a child is in grade 11 it means that they know everything that we have done. So by having that coloured chalk we are trying to get through to them that this we have done before so they must make good use of it. So if you take things for granted I don't think you can have a good pass rate. Even if you are introducing a lesson introducing a lesson does not mean that you start from that lesson I think that you have to go back previously. Those are the things that are helpful. Now and again to have to go back in mathematics, don't forget that the children have a tendency of saying that mathematics is difficult. You have to create an atmosphere where they can see that this thing, no there is nothing impossible here, this thing is easy, and you can do it.

Dean: I would say that we definitely need it (visuals) when teaching mathematics. I would say in all grades because pupils need to see things and to see things, things must be concrete to them. If they can see things and understand concrete concepts from there it would be easy to move on to more abstract stuff.

Maggie: Basically I think when I teach mathematics I teach them to be able to answer questions to be able to do this in the future. I don't just teach them something to answer that particular question. I tend to go the method where I show them the method and if they understand the method behind how to get to the answer it is very important because they can apply that to almost any question they get. In this way they are able to answer questions that might even not be the same as the ones that we get in class, but the ones that you get in an exam. But if they understand the method it just makes it that much easier. I think that is a very good aid in the mathematics classroom now that we all have it is necessary now. I would not be able to teach without visuals and I don't think maths teachers can actually teach maths without visuals if you have got worksheets once again you have to have diagrams on the worksheet you have to be able to explain to them (the learner) what it looks like so we have to even equations you can't actually just stand and read out of the page you got to have visual stimulation. Maths for me is really interesting, I love teaching maths, I think if I didn't like teaching maths it would be very difficult for me to teach to a lot of boys who don't actually like the subject and you have to love it, you have to be able to, you have to enjoy it as well.

Appendix D:

Code: DFG

Semi – structured interview schedule for learners. *

Focus Group: Daisy Sec School

Date: September 2009

Time: 1 pm

Indicator:

Master mathematics teachers use visuals as tools when teaching.

Do learners value their teacher's use of visuals? **

19. Your teacher uses visuals to teach mathematics, what is your view about him/her teaching in this manner?

- *L1: It helps us to remember stuff, so in the exam we can picture it.*
- *L3: We can understand the work easier.*
- *L4: It is a unique teaching method, no other teacher uses it, and it helps you to learn more.*
- *L1: It also makes maths more fun, it makes your learning experience more fun, and people are more interested to participate in the lesson.*

20. Did your teacher's use of the visuals help you understand the key mathematics concepts in the selected topic?

- *L1, L2, L4: Yes (emphatic YES)*

21. Why do you say so?

- *L4: Because it is more understandable, because we can understand it more better.*
- *L1: We just like the way in which he explains.*
- *L5: Also because he draws things on the boards so that we can understand it better.*
- *L6: We like want to go for maths we want to learn more.*

22. In one of the lessons your teacher used a stick with coloured rubber bands.

What did you learn in this lesson?

- *L2: Yes rotation, we like actually saw the movement, there was reflections of angles of 90 degrees anticlockwise.*

- ***L6: Keep the thing on the spot and just rotate it, the rubber band will go onto the angle.***
23. Do you think it was worth it him using that manipulative?
- ***L1 – L6: Yes.***
24. If he did not use it, how would it have affected the lesson?
- ***L6: It would be dull.***
25. But would you have still learnt the maths if he did not use the manipulative?
- ***L6: Not as well and clearly as we did, because we could see it.***
26. Can you explain how you learnt this?
- ***L6: By actually seeing the thing move and understanding how it moved.***
 - ***L4: We had a visual of how it moved.***
27. Do you need that visual?
- ***L5: Yes because if he hadn't used the stick and just explained, then we would have been confused, we would not have known which direction we were moving in.***
28. What were you thinking when your teacher used these gestures/diagrams/pictures?
- ***L4: It's exciting.***
 - ***L3: It's a fun way of learning.***
 - ***L1: Its helps us to learn.***
29. Did you understand it better in this way?
- ***L5: Yes much better, compared to the other subjects.***
30. I also noticed that your teacher tends to point at the board; this is a gesture, what does this mean to you in the class when he points to something?
- ***L4: He is getting our attention.***

31. You say that he is very different from other people and the way that he teaches, tell me a bit about this.

- ***L3: Other teachers just tell us to write stuff, they are really not explaining.***
- ***L2: If we don't understand...he helps us.***
- ***L4: He catches our attention.***
- ***L1: He uses stuff that we know.***
- ***L6: He uses stuff that we can relate to.***

32. What do you think about the use of diagrams/gestures/pictures or colours in mathematics?

- ***L5: He uses it for the important stuff.***
- ***L4: To show us the differences.***
- ***L3: It does make a difference.***
- ***L2: It's better.***
- ***L6: It makes important stuff stand out.***

33. How and in what way would you like your teacher to teach mathematics?

- ***L5: We would not change anything.***

34. Why do you say so?

- ***L4: We had many teachers and no one teaches as well as he does and finally we got this teacher and he just teaches us so well.***
- ***L5: I enjoy his teaching.***

Code: RFG

Semi – structured interview schedule for learners. *

Focus Group: Rose Sec School

Date: September 2009

Time: 1 pm

Indicator:

Master mathematics teachers use visuals as tools when teaching.

Do learners value their teacher's use of visuals?

1. R: Your teacher uses visuals to teach mathematics, what is your view about him/her teaching in this manner?

- *L1: Well I think it's good with the diagrams and stuff because it gives us a visual of what she is actually talking about so especially with like geometry and stuff like that so that we can have an idea of how the picture looks so it is easier for us to do the actual calculations and it is easier for us to understand what she is trying to teach us and for me it makes it easier to remember what she is teaching us and what we are doing.*
- *L2: And especially the lessons that you observed, we were doing trig and geometry and stuff so you need diagrams when you are doing it, eh... those sections and it just made it easier.*
- *L7: And we can also like when we work out like when we have trig graphs and we get a solution we can compare that to the actual sin graphs so it actually makes sense as to what we are solving for.*
- *L5: Ja when we go back to the notes when we are learning it will be easier for us.*
- *L3: We need them to understand, well it makes it better for us to understand because if we just have a bunch of numbers like in trigonometry if they just give us y and r or certain degree angles it is not easy for us to see what they are actually talking about and which sides work out but with diagrams we are able to see easily and we can once we have it on a diagram we can see what we need to find and how we can go about to find out.*

- ***L4: It's easier to summarise mathematical information because sometimes they give you the information in paragraphs and you don't know what you are looking for until everything is on the diagram because then you can see what is missing.***
- ***L6: I think it kind of clarifies what's what, if someone had to tell you this is the hypotenuse if you had not heard it before you would not know what it is but if you like show it on the actual triangle it is the longest side; it's easier to understand so you kind of link everything together.***

2. R: Did your teacher's use of the visuals help you understand the key mathematics concepts in the selected topic?

- ***L8: Yes definitely.***

3. R: When your teacher taught the lesson she used lots of colour and gestures, I remember she used gestures to show you the slope of a line, and she was talking about the gradient, did those gestures help you understand what she was talking about with respect to gradients? What went through your mind when she was doing this?

- ***L6: It just helps you remember like, because you remember it instead of just a diagram or something on a piece of paper you remember her like doing this with her body and or you remember the colours and stuff and well it's easier to remember stuff like that.***
- ***L3: As well as like maths is a very straight forward subject and with the like gestures, expression and the use of colour it makes it more interesting and you actually look forward to the lesson because it is not just her reading a book and telling you to do an exercise, it's actually like watching a story play out because she does the action and then there is the use of colour so she makes it very interesting.***
- ***L7: And also the colour sometimes pieces of information go together but they don't go with other pieces of information so she uses the colour to separate the pieces that go together and also her gestures and stuff, if you like thinking about it yourself, you can use your own body and make it like the other stuff.***

4. R: I also noticed that when your teacher was teaching she talked about a triangle ABC. She actually pointed to the board. What do you think about this pointing?

- *L3: Well she is trying to makes sure that everyone is understanding what's happening so she is not leaving anything out, so if she shows okay that's A that's B and that's C and you carry on and listen to her and watch her do everything you will begin to understand even if you are not having a problem, and if you get confused so say if you get something wrong then she does it on the board and she explains from the beginning so that you can see where you went wrong and you can correct your answer.*
- *L5: And it's just putting the visual and the audio together because some people can listen without looking on the board know what they are doing but some people need to see what you are actually talking about. So to get a full understanding or grasp the actual topic she does this.*

5. R: When your teacher underlines something, what does this mean to you?

- *L3: It's important.*
- *L7: She is reinforcing something.*
- *L1: She is emphasising that point is important.*

6. R: So why can't your teacher tell you it's important?

- *L7: Sometimes if you see it like especially if you are using colour and everything that she does if you see something and it is underlined you have a visual and you will always remember it compared to actually it is easier to remember a visual compared to for me if she just says it is important and if she underlines it you can do it as well in your book so that when you are learning you can see that is important.*
- *L5: And also like from the time we have been small whenever something is being underlined its important so even if you weren't paying attention and you were doing your own old sum there as soon*

as you look at the board you can see that is underlined and you know okay I should make a note of that or I should remember it.

7. R: Did you understand it better in this way?

- *L7: Yes it does help.*
- *L1: I think she uses it to emphasis because she does say that it is important but I think like it is better to have two ways than just say something is important just write it, if she does it both ways, it's just double reinforces the idea that whatever she is underlining or highlighting you need to what it is.*
- *L4: And also for the individual you can always refer back to the important point if she just says that it is important you are going to forget about it.*
- *L3: I also think that because most of us copy that important thing down it also helps us to learn because we say this is important because we highlighted it.*

8. R: But do you think you would have done this in your own way if your teacher had not underlined it? Would you have highlighted it in your books if your teacher did not do this?

- *L6: Some people do but I don't.*
- *L2: No.*
- *L4: I don't really take notes but whenever mam does underline something, I make it a point to just write it down so even though I don't usually take notes down if she is highlighting and I see it is important then that stuff I do take down so I thinks that's important that she does underline and highlight.*

9. R: What do you think about the use of diagrams/gestures/pictures or colours in mathematics?

- *L2: Highly effective.*
- *L1: Ja.*

- *L5: It's more interesting, you want to go for maths.*
- *L3: Its brings maths to life with colours and gestures.*
- *L6: With the smart board it makes everything much more interactive, I mean you get to do so much more and more of the features like stars, smiley faces and things it makes it so much more fun and it encourages all the other students to also if they don't; like the subject and they have to do it then it gives them the interest and opportunity to actually know what is happening and then they can also be interested in the subject and obviously if you are more interested then you want to learn.*
- *L1: You will also want to work hard at the subject.*
- *L7: And we are not only using our imagination, it makes it easier for us actually we are seeing we are seeing visually what she is talking about so it makes it much easier to learn and grasp concepts.*
- *L8: I think also it doesn't have to be a smart board like it could just be a diagram on the board and you can use coloured chalk or something like that it does not have to be high tech or anything.*
- *L4: We haven't had smart boards like forever, in our other classes we don't have smart boards even then they use diagrams on the board so it's also effective. It's just a little better now that we do have one.*
- *L6: And I think that if a teacher just uses colour it will be easier for the children not to forget, especially in maths diagram allot of information in one colour it is difficult to see where everything fits and what you are supposed to find but if things are separated with different colours it would be much easier.*
- *L4: And another thing about the smart board is that it saves lots of time and in our case time is education so she does not have to actually physically draw it she just presses a button and bam there's your triangle so it saves lots of time.*

10. R: How and in what way would you like your teacher to teach mathematics?

- *L7: I think we are kind of happy with the way she teaches.*
- *L4: She must not be boring she must be fun and want us to learn.*

11. R: Why do you say so?

- *L7: If your teacher is boring and she has a negative attitude or she is not very enthusiastic about maths it reflects onto you and you become all down and you don't really want to learn mathematics or do maths, but if your teacher is enthusiastic you will become enthusiastic about mathematics because I think that's important.*
- *L4: It's also how they look at the subject and their emotions and so if our teacher was like all boring and though the subject was just maths I think we would be feeling that way to.*
- *L5: I think any teachers needs to adapt to the student she has because even if you are not in a fortunate enough school and if you don't have all the facilities you need to find ways and methods to make it interesting for your pupils so that they can want to learn.*
- *L1: Even if a teacher takes the effort to like when you bring physical models to the class to show to the children.*
- *L1: It is not a good enough excuse to say you have no resources.*
- *L3: There are lots of teachers who have taught well without the smart board.*
- *L6: I don't think that having a smart board makes you a better teacher it just makes things easier.*
- *L6: And also like practical ways like when we were in primary school a very effective way was that we drew a triangle and everybody drew their own size triangle and then we had to cut the corners and we had to take it and paste it on a straight line and they all made a 180 degree angle and I like never forgotten that and we actually did like we coloured it in and we pasted it and it was exciting so even without a smart board you can have like little practical things.*

- *L5: And in our other subjects we don't have smart boards, so our teachers make use of things like the white board, sometimes even by just talking to us and teaching us you know with their own way that they think would have an impact on us and it works.*
- *L8: Ja the way they talk to us like with expression.*
- *L2: Because as well we do physics and we don't have a smart board for that but the way our physic teachers as well because she doesn't have a smart board but she uses colour and the white board and it is also very effective.*
- *L7: She uses like reality itself.*
- *L8: It also helps for the teacher to link up certain concepts to real life situations.*
- *L4: Like the other day we are doing trigonometry in maths and we did an example with an aeroplane so even in maths we are learning how to apply why they use trig and why we need trig. In triangles and stuff so it relates to subjects and it peaks your interest.*
- *L8: I like it when she uses things like reality stuff then it makes me understand more better even with maths when you use reality, real life examples it makes it easier to understand.*

12. R: Is there anything else you would like to add?

- *L3: Colour and stuff also helps you concentrate and stuff.*
- *L7: One more thing just like in terms of teaching methods especially with maths we normally like get our class work and our homework and like the nice thing about our teacher is the next day when she begins the lesson she always starts off with the homework and what we did not understand and the she will go over it in class so that they are not like gaps in your mind as to I could not do that question I don't know what's going to happen so if it comes out in the exam I am stuck.*
- *L8: Even if it's one person that does not understand, she will go through it with the whole class or she will come to that person and work with them individually. Even during her lesson if you don't*

understand something, she will come to you, even when she is doing class work she will ask does anyone need help, like constantly.

- *L4: And I think she is very great since our class has very different people and we work at different paces so like some will be way ahead and don't need help and some people need help but mam manages to bring a balance like when we need help she will be there. There is always something to do, either you are doing listening or some work.*
- *L6: And she does that by asking how many people need help, if like three people say that they don't she will work to the whole class and then she will move to the 3 individual people alone so that not everybody has to be held back and they can carry on but those 3 people can learn with the help that they need.*

Code: LFG

Semi – structured interview schedule for learners.

Focus Group: Lily Sec School

Date: 19/08/09

Time: 1:30 pm

Indicator:

Master mathematics teachers use visuals as tools when teaching.

Do learners value their teacher's use of visuals?

1. R: Your teacher uses visuals to teach mathematics, what is your view about him/her teaching in this manner?
 - *L3: Excellent.*

2. R: Why do you say so?
 - *L3: It helps you understand better, it helps you understand what she is talking about, if she just wrote text, honestly I would not understand what is going on compared to if she draws a diagram. I may be more visual and I may be able to understand a diagram to see exactly what she means.*

3. R: You say that you are a visual learner, what do you mean by that?
 - *L3: I can learn something better or understand something if I see it in practice.*
 - *L1: I think we apply it better if we see it in use instead of learning theory we can actually practically apply our knowledge which we get in class.*
 - *L4: Mind you, I think that a person learns better with a presentation plan because it is visual, we also are looking at something visual and I think a person who is standing up and explain what the work is very cool, it adds to it a sort of personal touch, I think the understanding becomes much better the concepts become part of who you are.*

4. R: Did your teacher's use of the visuals help you understand the key mathematics concepts in the selected topic?

- *L5: It helps us to actually understand what she is teaching us more, because you get to see her demonstrating on the board and showing us what diagrams and how to apply it and then by using things like gestures and pictures she like emphasises what she is teaching us and shows us certain key points.*
- *L2: She also like uses colour and all sorts of symbols.*

5. R: You picked on the use of different colour; can you tell me anything more about using different colours?

- *L3: I mean you have to use different colours because if you look at something and it's all in one colour its quiet hard to pick out certain things that you need to look at and differentiate between different concepts but if use say like red, green and blue like what we usually do you can do like one part in blue another part in green and you can see like the differences and what you were supposed to do.*
- *L6: Well myself being I have a photographic memory, to remember something and it is in colour its easier than if it is one tone, like if I have to learn everything in two shades black and white on a well black on a white board it's going to very difficult for me to differentiate between diagrams and different formulae whereas if is in different colours then you can remember the situation really where you can remember a different formulae if you can remember how it was coloured, I find it helps me.*

6. R: you say that you have a photographic memory, what do you mean by this?

- *L6: I can recall certain things by; I can see it in front of me. I can see certain things in front of me when, like certain diagram I can see in front of me that I remembered in the past so it makes an impact on me I remember certain situations I can see basically the whiteboard, I can the teacher and everything, I take random photos of my surrounding, it works like that. I can see it in my mind. So if you are*

stuck you think about when did we learn this, like I mean you can picture yourself even back in the situation and you can recall what the teacher said or what the teacher was drawing and probably you can use the information that you remembered in your mind to apply it to the situation.

- *L4: Mind you everyone is talking about colour about how colour aids and how you understand and everything I think colour is used to organises everything, like for me, if certain things are written out in blue and the working is done out in red, it sort of makes it easier to understand what's really going on it separate things because of the different colours and I suppose it makes things easier to memorise also.*
- *L6: Use a certain colour for certain things but don't overdo it.*

7. R: When you talk about colour and the teacher using it, do you all use colour in your books?

- *L3: No.*
- *L1: Never.*
- *L5: No, I don't.*

8. R: Do you think now that you are talking about it, it would make a bit of difference?

- *L4: Mam, we also learnt that colours stimulate different parts of our brain, so the teachers' that are using colour are actually helping us in a way.*
- *L2: Sometimes the diagrams that you get are ambiguous diagrams there is more than one thing in that diagram and mam separates those things using different colours so it makes it easier to work out the different angles.*

9. R: What did you learn in this lesson (picture of a fish on the OHP)?

- *L1: We learnt about the proportionate sides and the theorem, we were doing analytical geometry as well.*

- ***L3: Basically a different form of geometry that is more theoretical.***

10. R: Can you explain how you learnt this?

- ***L4: In every lesson we learn, probably the combination of actually looking at the smart board and seeing what it is and actually looking at mam doing it herself to further emphasise it we were able to understand it more clearly.***

11. R: What were you thinking when your teacher used these gestures/diagram/pictures?

- ***L1: What's going on?***

12. R: What do you mean by that?

- ***L1: Well just a simple observation as she is going along she will draw the triangle and she will say like point A and then she would redo point A and point B and point C and then as she goes along explaining what she does she redraws the triangle basically with either the same colour or even with a different colour, she is actually redoing line AB just to emphasise that this is the line that you will use and this is your measurements.***

13. R: So if she just said, without emphasising without drawing anything, okay line AB is perpendicular, would that make a difference to you, would you understand that?

- ***L1: No.***
- ***L3: Absolutely not.***
- ***L4: To be visually shown it you would be actually be able to differentiate between what it is what is perpendicular, what is parallel, etc. So you will never get confused in an exam with terminology in that way.***

14. R: Did you understand it better in this way?

- ***L1: Yes.***

- *L6: We all know what perpendicular is when drawing it on the board the perpendicular line speaks for itself so it actually helps us.*

15. R: What do you think about the use of diagrams/gestures/pictures or colours in mathematics?

- *L5: I think let's take for example trig, trigonometry, that kind of what we are doing now that certainly involves triangles and geometry measures sides, angles etc. In a situation like that obviously I think it's impossible to learn without some drawing. But say for example you are doing something like compound interest and simple interest I find it hard to sort of say where a diagram will be used there unless it's some kind of bar graph or a number line or something so I think certain areas of mathematics definitely require the use of your visual aids but I think it also brightens up the day.*
- *L1: If it's something different you remember it.*
- *L3: I won't lie maths sometimes gets tedious but by using thing like colours, and pictures it actually sparks interest in it so you actually like sit up, it helps to keep students attention. When you sue pictures it helps keep the interest it actually helps. You learn more.*
- *L5: By using diagrams you can look at one diagram and it will be like almost equivalent to what reading through the whole paragraph and it is much easier to understand because you don't have to actually read it, you can get it at one glance and you can understand.*
- *L4: Pictures convey a thousand words.*

16. R: How and in what way would you like your teacher to teach mathematics?

- *L3: To be honest, she teaches quiet well.*
- *L5: We can all probably agree that she teaches well, but I think that the only thing is like homework like we cut down a bit.*
- *L4: She is probably one of the best maths teachers that I had.*
- *L6: Sometimes I feel that she is bit disorganised. She is like a steam train, she take some time to start.*

17. R: Why do you say so?

- *L4: If you do not understand she takes the time to explain, how she got to that point, that's wonderful because some mathematics teachers that I had in the past, they will try but they want to move on. So they just give you a quick crash course and tell you where they went wrong and move on, but mam will stop the whole lesson and tell you where you went wrong which is great because somebody else may have the same problem they can also just learn by telling you.*
- *L6: She needs to let you catch up when you start a new section, I know she wants us to like practice and all, but we need to know what's happening first.*
- *L5: What we do in physics is to first before we start a new section is to have a first lesson or so or at least half a lesson of theory on it to just give us some background on the new section to tell us how it is applied what it is used for, before we start the section.*
- *L2: Maybe to have something to show you how it works, not to just give you a formula and say okay here is a triangle now solve. So that 10 years down the line when somebody asks you about something you can remember.*
- *L1: Ja her teaching methods are very good.*

Code: OFG

Semi – structured interview schedule for learners. *

Focus Group: Orchid Sec School

Date: September 2009

Time: 1 pm

Indicator:

Master mathematics teachers use visuals as tools when teaching.

Do learners value their teacher's use of visuals?

1. R: Your teacher uses visuals to teach mathematics, what is your view about him/her teaching in this manner?
 - *L4: He makes it easier to understand, because if you got a teacher that just comes in the front and just sits down and doesn't show any interest, but sir always explains stuff.*
 - *L2: It also creates a more interactive feel between the teacher and the learners as such, he doesn't become like monotonous in a sense, he is there and we know by the way carries himself and off course the gestures that he uses, of the interaction with the class, you can see that he is there.*
 - *L6: It also makes the lesson more interesting.*

2. R: Did your teacher's use of the visuals help you understand the key mathematics concepts in the selected topic?
 - *L8: Absolutely because it becomes easier to understand, I think that once it becomes comfortable with the teacher and off course with the interaction, we have become more comfortable with him, it does make the lesson more easier to understand and through his diagrams and stuff like that we are able to see like firsthand like as he is doing it on the board instead of just looking at a already drawn out picture in the text book, as he is doing it on the board he is explaining it. As he is doing stuff we are able to see it first hand and the understanding becomes simpler through his use of diagrams.*

- *L4: Just say you are looking at a square, on the board you can just explain but he shows us you can see it in 3D so it is much easier to know what actually happens.*
 - *L2: Especially when you are doing areas and volumes it becomes better instead of just looking at it in 2D on a chalkboard.*
3. R: What did you learn in this lesson?
- *L1: Yes, I think by folding the page we got to see for ourselves that it was actually a mirror image that was created on the other side instead of the teacher just sort of telling you okay the picture that is going to be formed is going to be a mirror image of the first one, by having us fold the paper and doing it on our own we are actually seeing for ourselves no this is actually true. It becomes again easier to understand because you are doing it for yourself and actually seeing the proof in front of you, instead of just picking it up from somewhere or just listening to someone say you know what, what is formed is a mirror image. So in that sense it is better, we have learnt through that.*
4. R: Can you explain how you learnt this?
- *L1: We do believe that it is going to be a mirror image but by actually doing it for yourself and actually folding the page and finding out that it is an actual fact that the point we predicted is going to come out it gives a more truthful, it becomes more true in a sense.*
5. R: What were you thinking when your teacher used these gestures/diagram/pictures?
- *L7: Confused. It seemed strange. But the way he was doing it and how to get it.*
6. R: Did you understand it better in this way?

- *L7: Yes, it made me understand it better, with those rules like you can apply it and it is easier to do it that way and which part of the Cartesian plane it goes.*
- *L9: It was easier to understand because you know why in a certain quadrant the ratio becomes negative or why it is positive, you can actually see it on the Cartesian plane because he is showing you here you y is negative, here your x is positive and that is why you know your ratio is going to be such instead of like you know, because what we knew from prior knowledge is that all was positive so by showing it to us, and the reason why it becomes negative, that helped us in the understanding instead of just saying that you know it becomes negative.*
- *L3: It becomes much easier to grasp, the diagram was like helpful.*
- *L5: And the pneumatic helps. CAST, All Smoke Texan Cigarettes.*

7. R: I also noticed that your teacher tends to point to the board, what does this do for you?

- *L4: It helps us to concentrate better, it helps us to focus.*
- *L1: As you speak you may just get lost in what we are saying, but his pointing helps us focus our concentration.*
- *L3: Well what exactly is the question asking; it goes back to your question. It shows us like what to do.*

8. R: What do you think about the use of diagrams/gestures/pictures or colours in mathematics?

- *L8: When you say verbal it is so hard to picture it in your mind. It's a mouthful to actually picture. When you draw it is more simplified. Not all children have the same level of understanding stuff, so by actually drawing this diagram it puts everyone in the same level of understanding.*

9. R: You said you need to picture it, why do you say so?

- *L6: In the exams it is easier to remember stuff and to study. Sometimes the rules are so close that one can easily misinterpret it for the next rule and by having a diagram we can actually pinpoint which is the exact rule.*

10. R: I noticed that your teacher tends to use different coloured chalk, what do you think about this? What do you think about using different colours on the board?

- *L3: It really prevents confusion, when there are different steps involved.*
- *L6: I think when there are graphs involved and let say both equal to negative 90 and we use different coloured chalk we won't be confused, that's what I think.*
- *L7: It's boring if you just use white chalk, colour makes it attractive.*
- *L8: It draws your attention to it and I think sir also uses coloured chalk at points where he wants your attention to be focused on so like when you looking at it although all the steps may be important the one where he feels that children are most likely to have difficulty with, he will highlight it even if it's not by colour chalk he will put an asterix there or something to say, you know what as you are going through be careful and at this point more especially.*

11. R: How and in what way would you like your teacher to teach mathematics?

- *L1: I think if he makes it more practical so that we can see what happening rather than just seeing it on the board especially with area.*
- *L5: We are trying to say not just on a projector or something but you actually want to see the cube or something.*
- *L3: He sometimes takes a page and folds it and shows us the top or bottom and tells us what it is.*
- *L7: Even though we don't have the actual models he like takes a pencil case or something to demonstrate what he is trying to say so I*

think that makes it easier but all in all I don't think that I will change anything because I think I am very lucky.

Appendix E:

NUMBER PATTERNS

1)

1.1 Study the following patterns made up of sticks.



Pattern 1
8 sticks



Pattern 2
15 sticks



Pattern 3
22 sticks

1.1.1 How many sticks will the fourth (4^{th}) pattern have?

1.1.2 Write down a conjecture that describes the relationship between the pattern number and the number of sticks required for the pattern.

1.1.3 Write down an algebraic formula for the number of sticks in the n^{th} pattern.

2)

Dots are arranged to form a sequence of patterns as shown below:



Pattern 1
(1 dot)



Pattern 2
(5 dots)



Pattern 3
(13 dots)



Pattern 4
(25 dots)

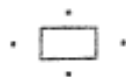
2.1 Write down the number of dots in pattern 5.

2.2 Determine a formula for the number of dots in the n^{th} pattern.

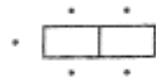
2.3 Which pattern number has 2 113 dots in it?

3)

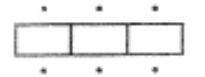
3.1 A restaurant owner sets his tables in a pattern to cater for his customers to be seated as follows:



Pattern 1
4 Customers



Pattern 2
6 Customers



Pattern 3
8 Customers

3.1.1 Write down the number of customers that will be seated in Pattern 5.

3.1.2 Write down a formula in terms of n for the number of customers that will be seated in the n^{th} pattern.

3.1.3 Can the owner continue to set the tables in the pattern above? Give a reason for your answer.

3.2 Given the sequence: 18; 28; 44; 66; ...

3.2.1 Write down the next term in the sequence if the pattern continues.

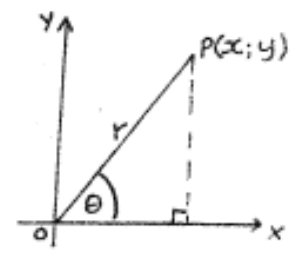
TRIG.

1. DEFINITIONS

$\sin \theta = \underline{\hspace{2cm}}$

$\cos \theta = \underline{\hspace{2cm}}$

$\tan \theta = \underline{\hspace{2cm}}$

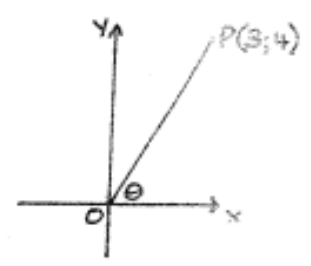


To find x ; y and r use :

Application

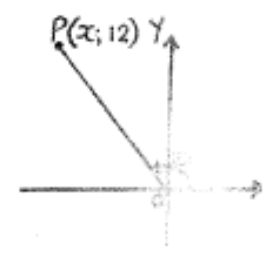
1. $P(3; 4)$ is a point in the Cartesian plane and $\widehat{POX} = \theta$

- 1.1 State the length of OP
- 1.2 State the 3 trig. ratios of θ .
- 1.3 Prove that: $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- 1.4 Calculate the size of θ correct to 2 decimal digits.

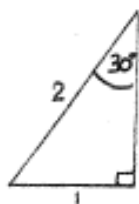


2. $P(x; 12)$ is a point in a Cartesian plane and $\widehat{POX} = \theta$. $OP = 13$ units

- 2.1 Find the value of x .
- 2.2 Evaluate: $13 \sin \theta + 10 \tan \theta$



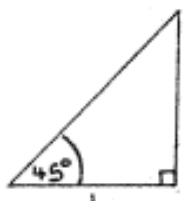
- 2.3 Evaluate: $\sin^2 \theta + \cos^2 \theta$
- 2.4 Determine the value of θ (1 decimal digit)

SPECIAL ANGLES1. 30°

$\sin 30^\circ = \dots$

$\cos 30^\circ = \dots$

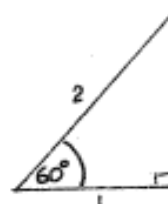
$\tan 30^\circ = \dots$

2. 45°

$\sin 45^\circ = \dots$

$\cos 45^\circ = \dots$

$\tan 45^\circ = \dots$

3. 60°

$\sin 60^\circ = \dots$

$\cos 60^\circ = \dots$

$\tan 60^\circ = \dots$

EXAMPLESSimplify without a calculator

1. $\sin 30^\circ \times \cos 30^\circ$

$= \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

2. $\tan 45^\circ - \cos 60^\circ$

$= \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

Application :Evaluate without using a calculator

1. $\sin 60^\circ \cos 30^\circ$

2. $4 \sin 30^\circ - 2 \cos 60^\circ$

3. $\tan 60^\circ \cos 30^\circ$

4. $\sqrt{2} \cos 45^\circ + \sqrt{3} \sin 60^\circ$

5. $\cos 60^\circ \sin 30^\circ - \tan 45^\circ$

6. $\sin^2(60^\circ)$

7. $\sqrt{\frac{\tan 60^\circ \cos 30^\circ}{\sin 30^\circ} + \tan(45^\circ)}$

GRAPHS SUMMARY

	LINEAR FUNCTION Straight Line	QUADRATIC FUNCTION Parabola	HYPERBOLA	EXPONENTIAL
Equation	$y = ax + q$	$y = a(x-p)^2 + q$	$y = \frac{k}{x+p} + q$	$y = a \cdot b^{x+p} + q$
Shape	$m > 0$ / $m < 0$ \ $m = 0$ — $y = c$ horizontal line $x = c$ vertical line (m is u.d.)	$a > 0$ Minimum $a < 0$ maximum	$k > 0$ $k < 0$	increasing $a > 0, 0 < b < 1$
Vertical shift	$y = q$	$y = q$ (mini/ maxi)	asymptote : $y = q$	asymptote : $y = q$
Horizontal shift	none	A.O.S. $x = -p$	asymptote : $x = -p$	asymptote : $x = -p$
Intercepts	y -int : $(0; q)$ x -int : let $y = 0$ and solve	y -int : let $x = 0$ and solve x -int : let $y = 0$ and solve	only if there are shifts	y -int : $(0; a)$ x -int : only if shifts
Significant points	gradient $(m) = a$ gradient = $\frac{y_2 - y_1}{x_2 - x_1}$ pt of intersection with inverse	T.P. $(-p; q)$ Finding equation Translations	NB : Translations	translations inverse is the Log graph
domain & range	domain $x \in \mathbb{R}$ range $y \in \mathbb{R}$	domain $x \in \mathbb{R}$ must be restricted for inverse to be a function range $a > 0, y \geq q$ $a < 0, y \leq q$	domain $x \in \mathbb{R}, x \neq -p$ range $y \in \mathbb{R}, y \neq q$	domain $x \in \mathbb{R}, x \neq q$ range $y > q$

GRADE 12

GRAPHS REVISION WORKSHEET

1. Sketch the following graphs each, on separate axes. Show all intercepts and significant points.

1.1 $f(x) = 4 - (x + 1)^2$

1.2 $g(x) = \frac{-x}{2} + 3$

1.3 $k(x) = 2x$

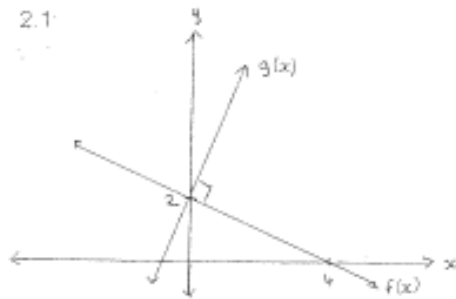
1.4 $p(x) = -2 \cdot 4^x + 4$

1.5 $q(x) = x^2 + 4x - 5$

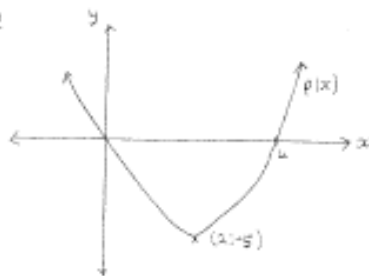
1.6 $h(x) = \frac{6}{x+1} + 2$

2. Determine the equation of each of the following graphs:

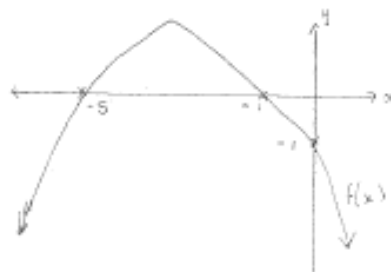
2.1



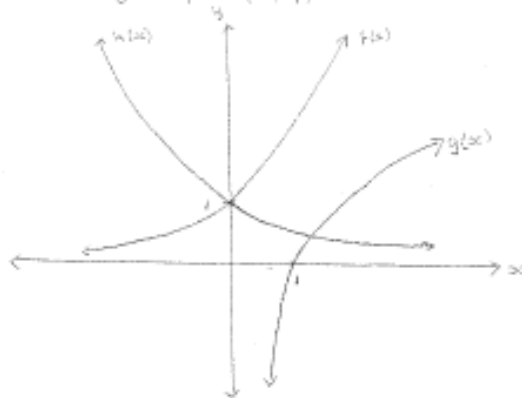
2.2



2.3

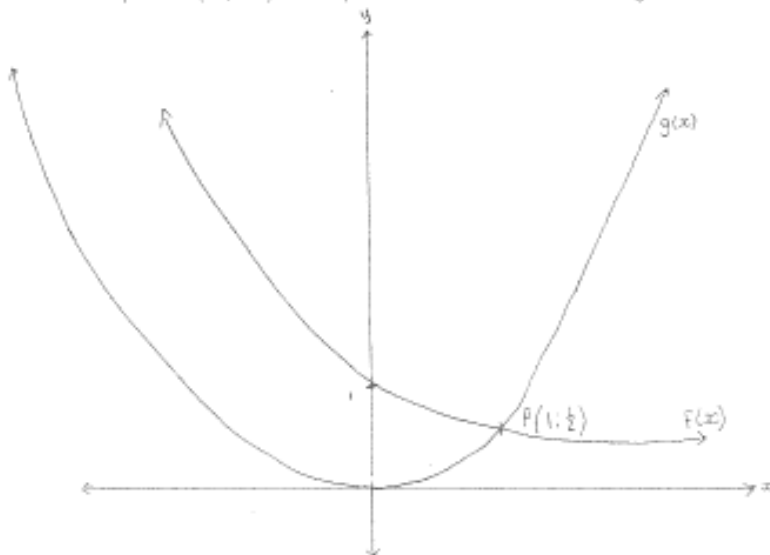


5. The diagram below shows the graph of $f(x) = a^x$. It is further given that the graph passes through the point $(2; \frac{16}{9})$



- 5.1 Calculate the value of a
- 5.2 Write down the equation of $g(x)$, if $g(x) = f^{-1}(x)$
- 5.3 Write down the equation of $h(x)$, if h is symmetrical to f about the y -axis
- 5.4 Determine the equation of $s(x)$, if s is the reflection of h in the x -axis
- 5.5 Determine with the aid of the above sketch, the x -values for which:
 - 5.5.1 $f(x) - h(x) = 0$
 - 5.5.2 $\log_{\frac{1}{3}} x \leq 0$

6. The diagram below shows the graphs of $f(x) = a^x$ and $g(x) = bx^2$. The point $P(1; \frac{1}{2})$ is the point of intersection of f and g .



GRADE 11 TRIGONOMETRY - INTRODUCTION

ANGLES ON THE CARTESIAN PLANE

1. SHOW BY SKETCHING, IN WHICH QUADRANT WILL OP BE IF $\hat{P}OX$ IS:-

1.1 50°



1.2 -30°



1.3 120°



1.4 460°



1.5 -200°



1.6 650°



2. WRITE DOWN 3 OTHER ANGLES THAT WOULD HAVE THE SAME FINAL POSITION AS 30°

3. STATE THE INTERVALS FOR θ , $-360^\circ < \theta < 360^\circ$ IN THE FOUR QUADRANTS

SECOND QUAD	FIRST QUAD ($0^\circ < \theta < 90^\circ$)
THIRD QUAD	FOURTH QUAD

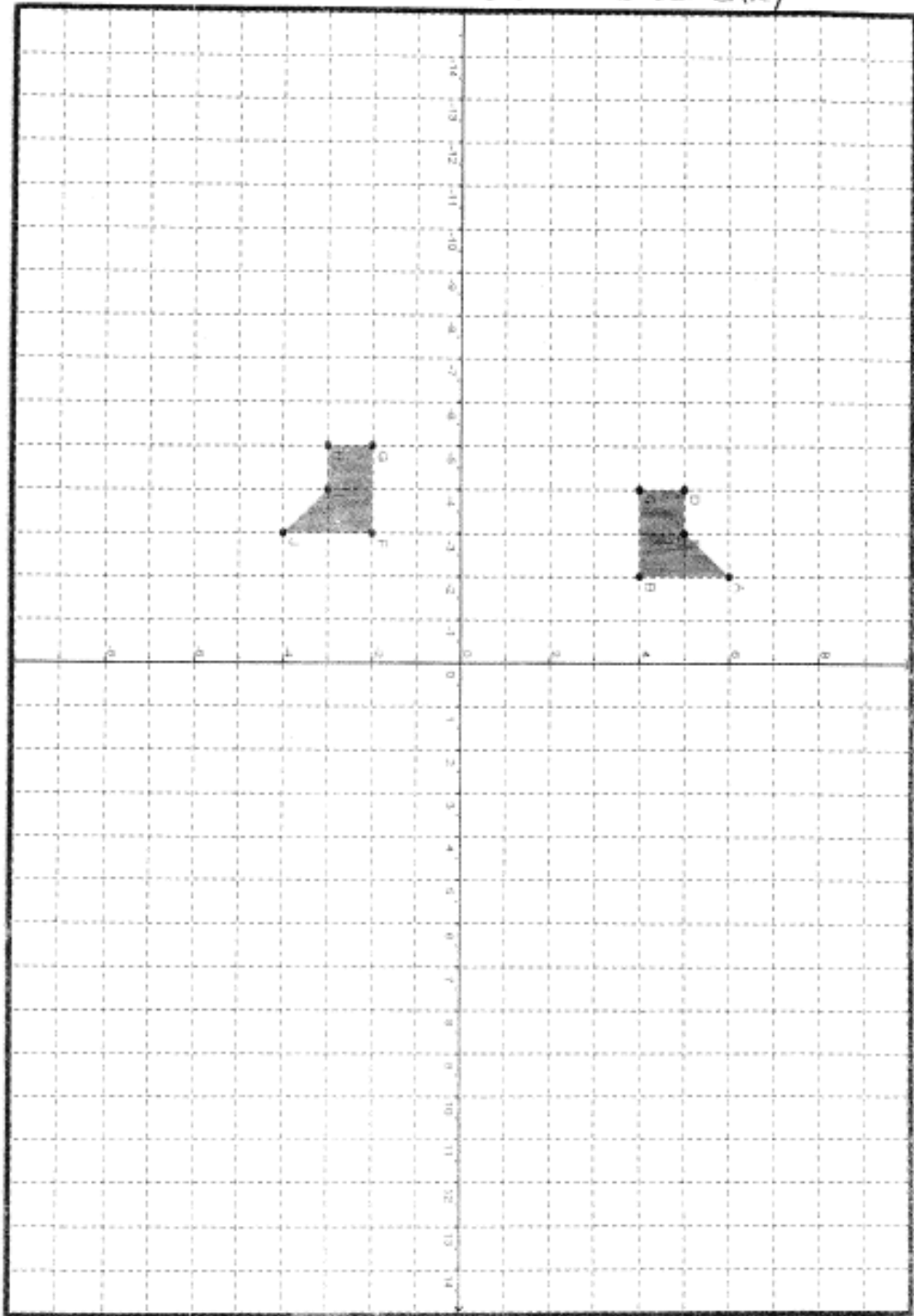
4. STATE THE SIGNS OF x AND y IN THE FOUR QUADRANTS AND THEREAFTER RATIOS THAT ARE +VE IN EACH QUADRANT

SECOND QUAD X: -VE Y: +VE	FIRST QUAD
THIRD QUAD	FOURTH QUAD

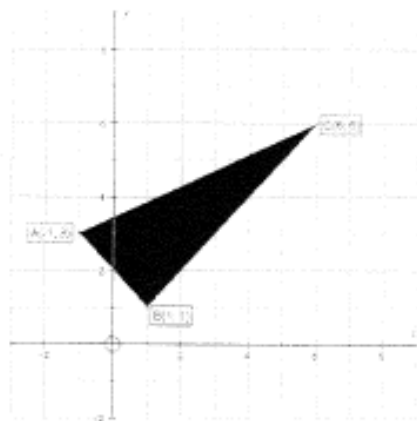
5. DECIDE WHETHER THE FOLLOWING FUNCTIONS WILL BE +VE OR -VE:

- | | | | |
|----------------------|-------|-----------------------|-------|
| 5.1 $\sin 135^\circ$ | _____ | 5.6 $\cos 257^\circ$ | _____ |
| 5.2 $\cos 169^\circ$ | _____ | 5.7 $\tan 312^\circ$ | _____ |
| 5.3 $\tan 121^\circ$ | _____ | 5.8 $\sin 300^\circ$ | _____ |
| 5.4 $\sin 197^\circ$ | _____ | 5.9 $\cos 75^\circ$ | _____ |
| 5.5 $\tan 225^\circ$ | _____ | 5.10 $\cos 312^\circ$ | _____ |

ORCHID SECONDARY



Analytical Geometry Gr.11
Worksheet 1



1. Determine the perimeter of triangle ABC
2. Determine the gradient of AB
3. Determine the gradient of BC
4. What do you notice about AB and BC. why?
5. Find the midpoint of line AC.

NUMBER PATTERNS
GRADE 11

A mathematician invents a row of numbers so that whatever n stands for,
the n th number is $3n + 2$

This means that:

the 1 st number is	$3 \times 1 + 2$
the 2 nd number is	$3 \times 2 + 2$
the 3 rd number is	$3 \times 3 + 2$
the 9 th number is	$3 \times 9 + 2$

i.e.

Term	1	2	3	9	n
Number	5	8	11	29	$3n + 2$

EXERCISE 1:

1. In a certain row of numbers the n th number is $3n + 1$. Find:
- | | |
|-------------------------|------------------------------------|
| 1.1 the first 3 numbers | 1.2 the 100 th number |
| 1.3 which number is 601 | 1.4 $(n + 1)$ th number |

EXERCISE 2:

If the following numbers go on in the same way, give the next number, the 100th number and the rule in terms of n (i.e. the n th number)

- | | |
|----------------|---------------|
| 1.1 1, 2, 3 | 1.2 3, 6, 9 |
| 1.3 3, 4, 5 | 1.4 1, 3, 5 |
| 1.5 10, 20, 30 | 1.6 9, 19, 29 |
| 1.7 5, 9, 13 | 1.8 4, 9, 14 |

Interesting discoveries can be made about number sequences by looking at the differences between the successive terms of a sequence. Example:

	2	5	10	17	26
1 st difference	3	5	7	9	
2 nd difference		2	2	2	

Example

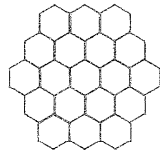
Allan is using regular hexagonal carpet tiles to make floor rugs. He has arranged the tiles in the following patterns to make rugs of different sizes:

RUGS:

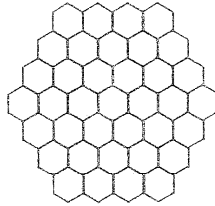
Pattern 2
(7 tiles)



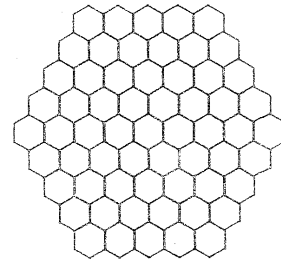
Pattern 3
(19 tiles)



Pattern 4
(37 tiles)



Pattern 5
(61 tiles)



1. How many tiles will Allan need to make pattern 6 in this sequence?

2. Make a conjecture that describes the relationship between the pattern number and the number of tiles needed for the pattern.

3. Use variables to write an algebraic statement to generalize the relationship between the pattern number and the number of tiles.

Exercise 5

Do exercise 1.13 on page 15 nos 3, 4 a

We display the information in a diagram similar to this.

Given: $T_n = an^2 + bn + c$

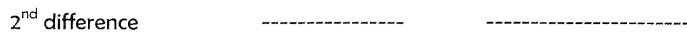
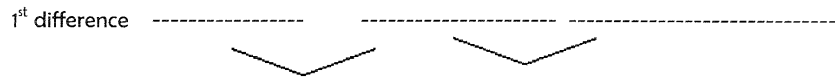
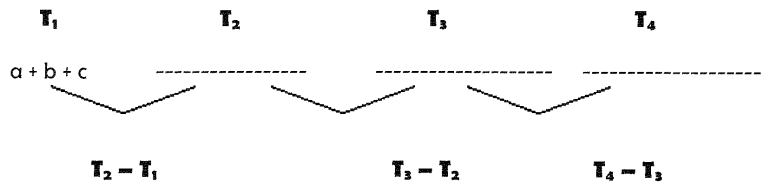
$$T_1 = a(1)^2 + b(1) + c = a + b + c$$

Similarly write T_2, T_3 and T_4 in terms of a, b and c .

$T_2 =$

$T_3 =$

$T_4 =$



What do you notice about the second difference? _____

Is this sequence quadratic? Give a reason for your answer.

There are 3 unknown values we need, a, b , and c . Therefore we need 3 equations. We identify these three equations in terms of a, b and c .

Appendix F:

Appendix F: What visuals did the Master teacher use as tools in the mathematics classroom?

Orchid Secondary

$$\begin{aligned} (1) \quad y &= 2x^2 + 4x \\ &= 2(x^2 + 2x) \\ &= 2(x^2 + 2x + 1 + 0 - 1) \\ &= 2[(x+1)^2 - 1] \\ y &= 2(x+1)^2 - 2 \end{aligned}$$

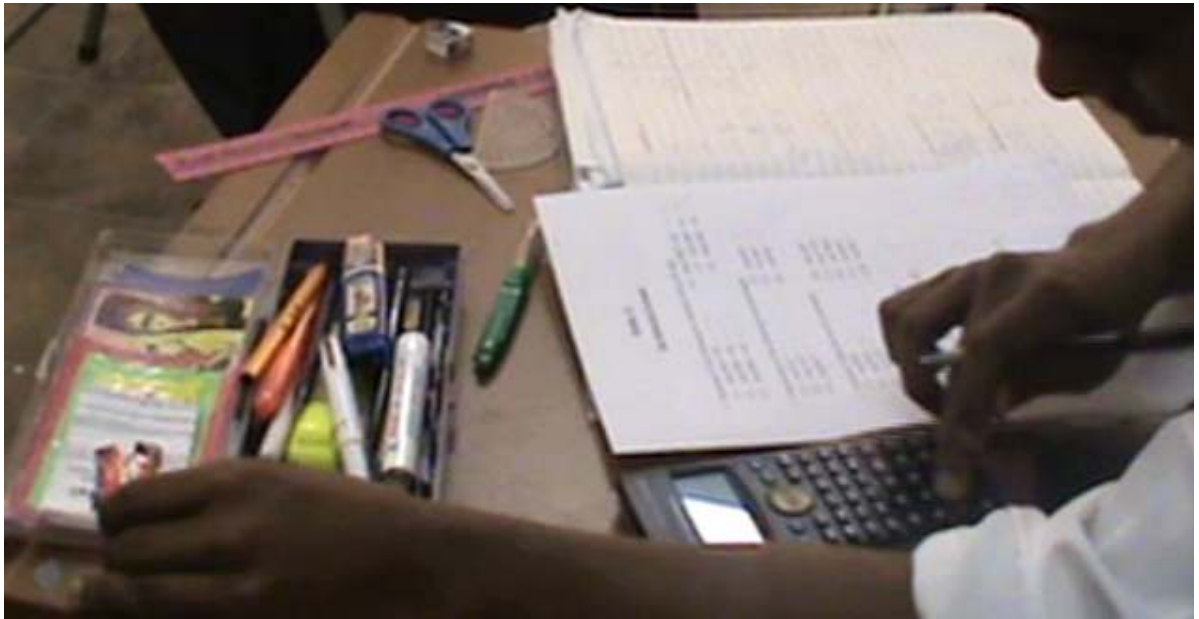
Use of arrows at Orchid Secondary

Rewriting FROM $y = ax^2 + bx + c$
INTO $y = a(x-p)^2 + z$

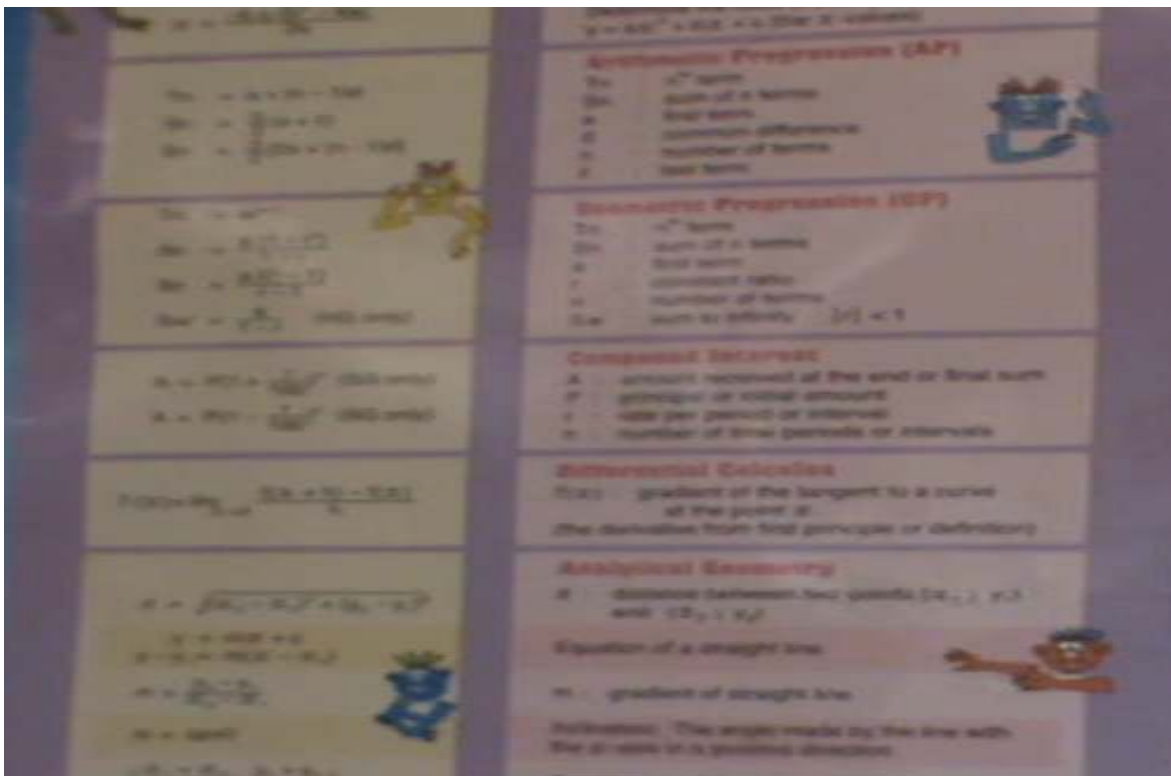
$$\begin{aligned} (1) \quad y &= x^2 - 2x - 3 \\ &= [x^2 - 2x + 1] - 3 - 1 \\ &= (x-1)^2 - 4 \end{aligned}$$

(1) x -int ✓
(2) y -int ✓
(3) $A=1$, $x=p$
(4) TP $(p; z)$

Use of blocks, shapes and symbols at Orchid Secondary

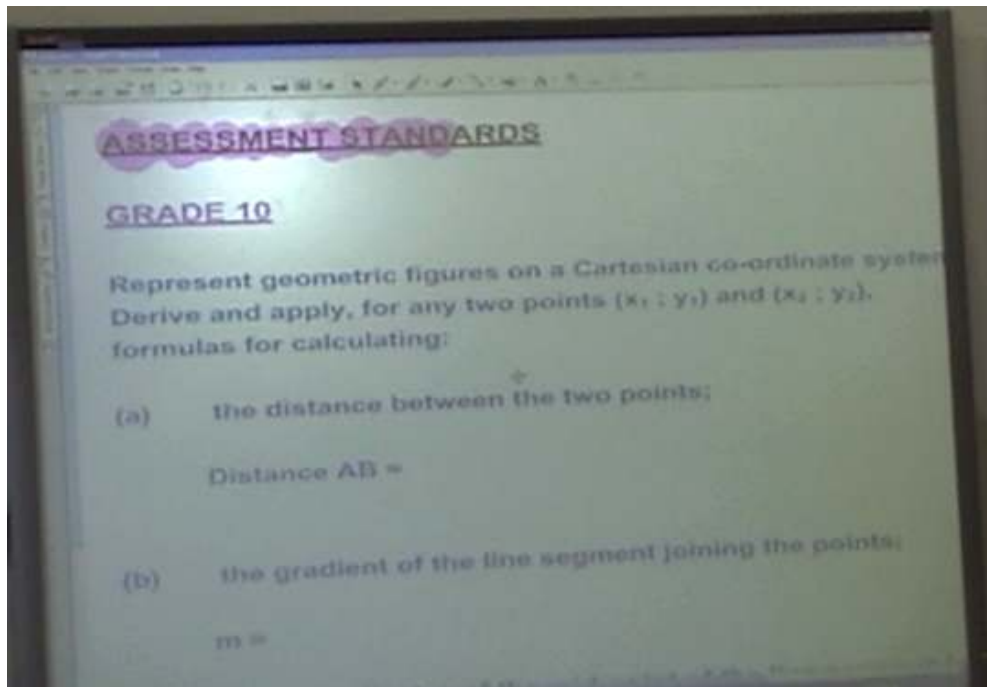


Use of calculators and other mathematics equipment at Orchid Secondary

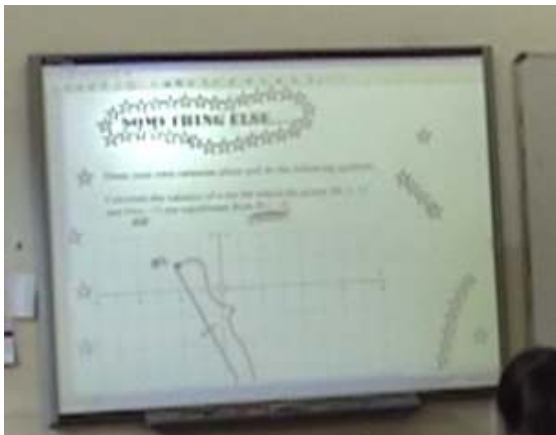


Example of Level 1 scaffolding at Orchid Secondary

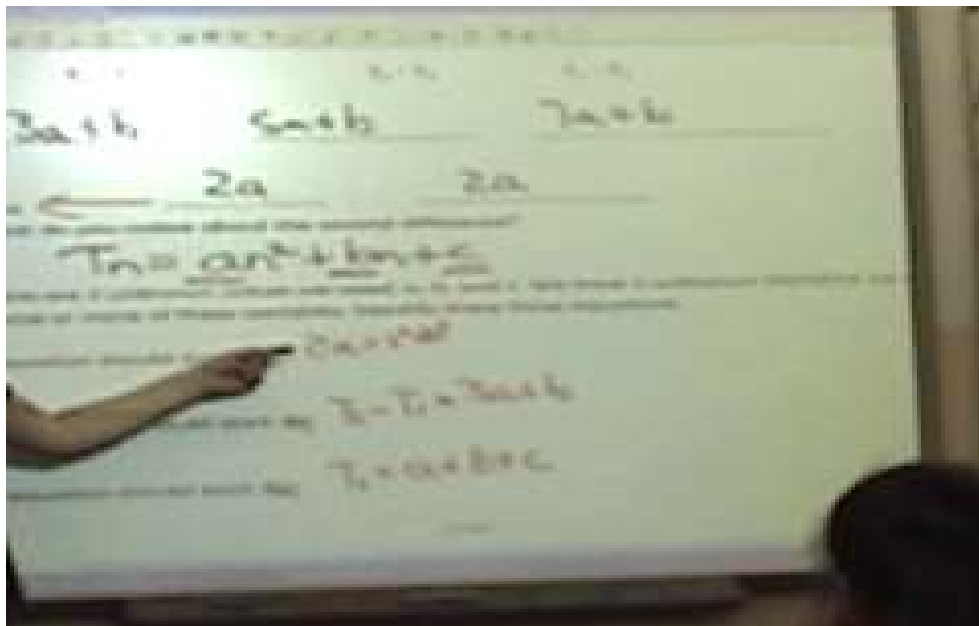
Rose Secondary



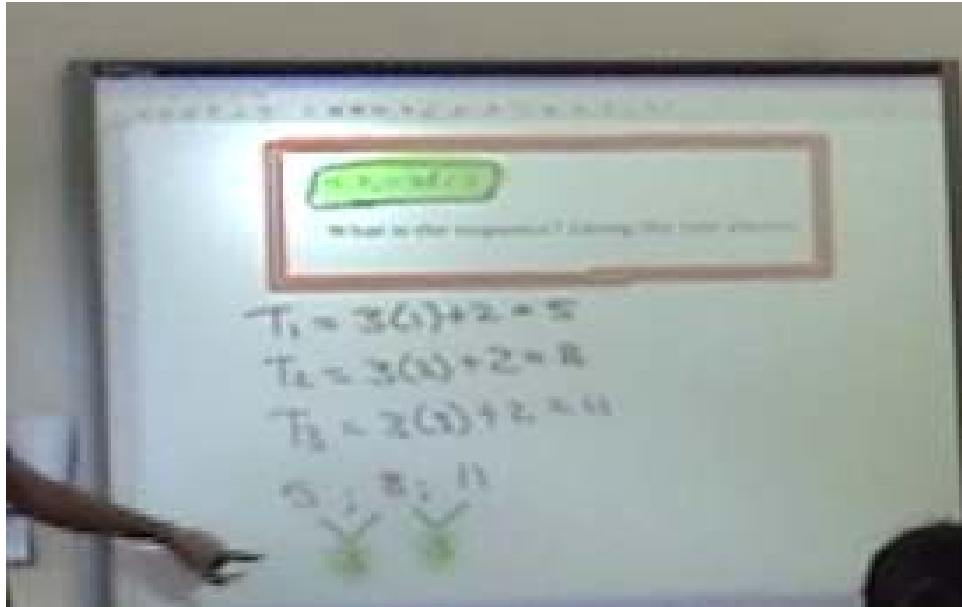
Use of colour at Rose Secondary



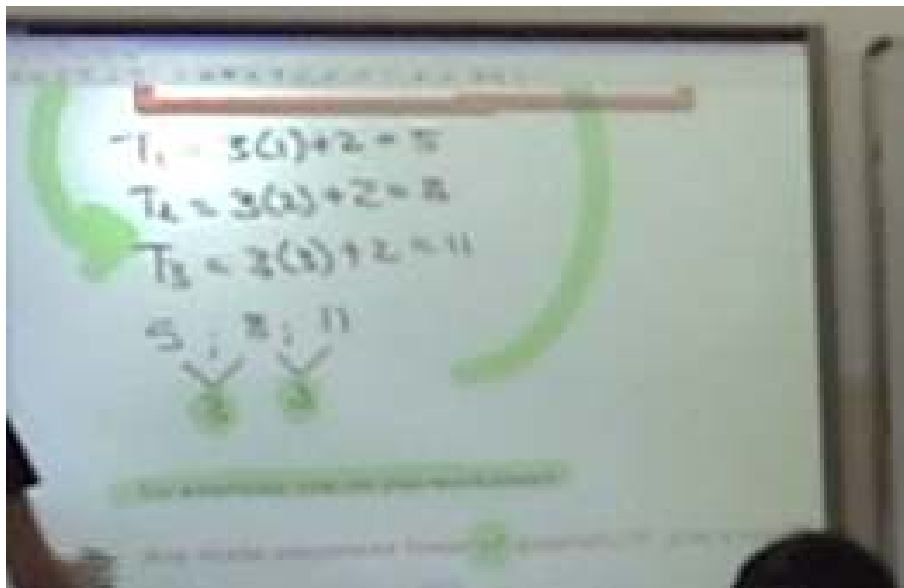
Use of coloured borders at Rose Secondary



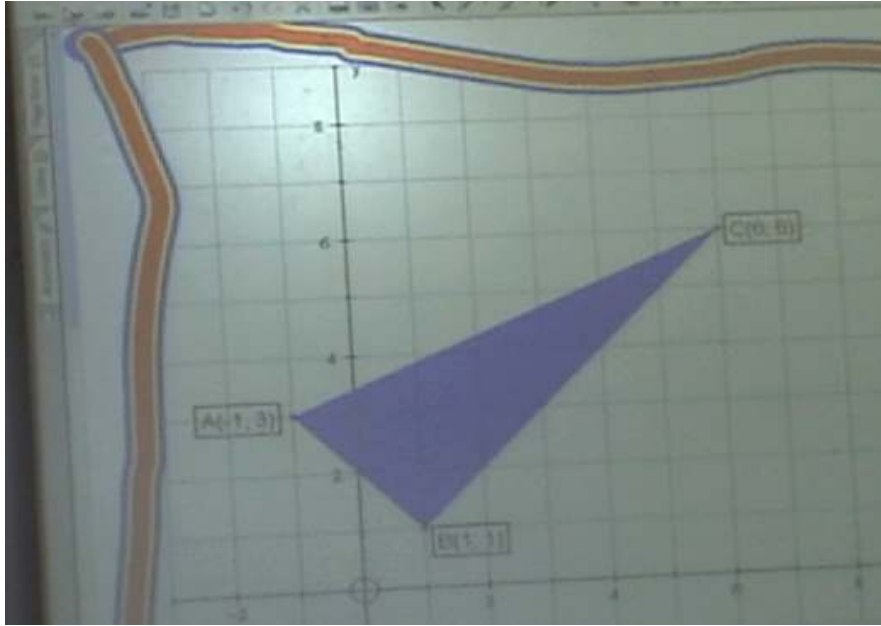
Use of deictic gestures, underlining and colour on the board at Rose Secondary



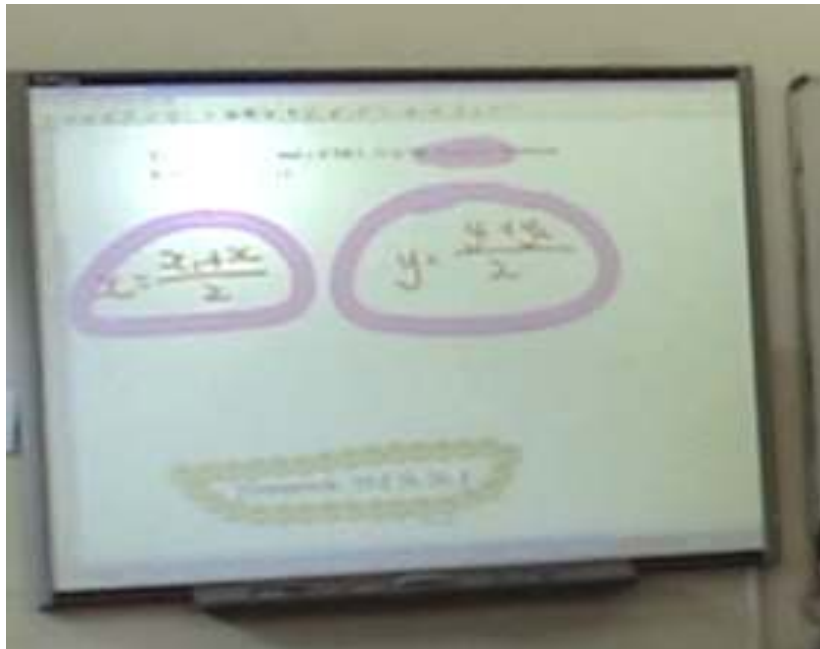
Use of deictic gestures, highlighters, blocks and colour at Rose Secondary



Use of highlighters, symbols and colour at Rose Secondary



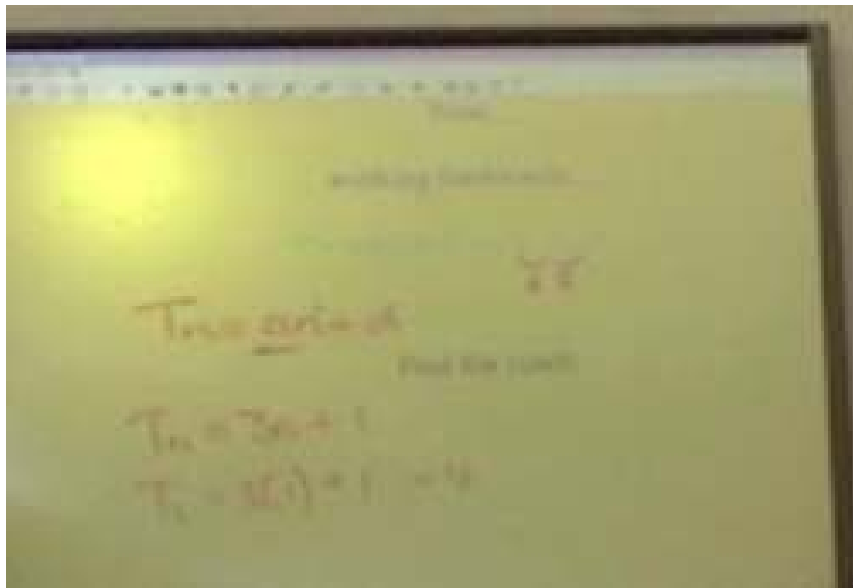
Use of colour at Rose Secondary



Use of symbols and colour at Rose Secondary



Use of iconic gestures (representing the gradient of a line) at Rose Secondary



Use of colour at Rose Secondary

A photograph of a whiteboard with handwritten mathematical formulas. The first line shows the general formula for the nth term of an arithmetic progression: $T_n = an + d$. The second line shows a specific formula, $T_n = 4n + 1$, which is enclosed in a hand-drawn rectangular box. The third line shows the calculation of the first term: $T_1 = 4(1) + 1 = 5$.

Use of symbols, blocks and colour at Rose Secondary

Daisy Secondary



Example of Level 1 scaffolding at Daisy Secondary



Example of Level 1 scaffolding at Daisy Secondary



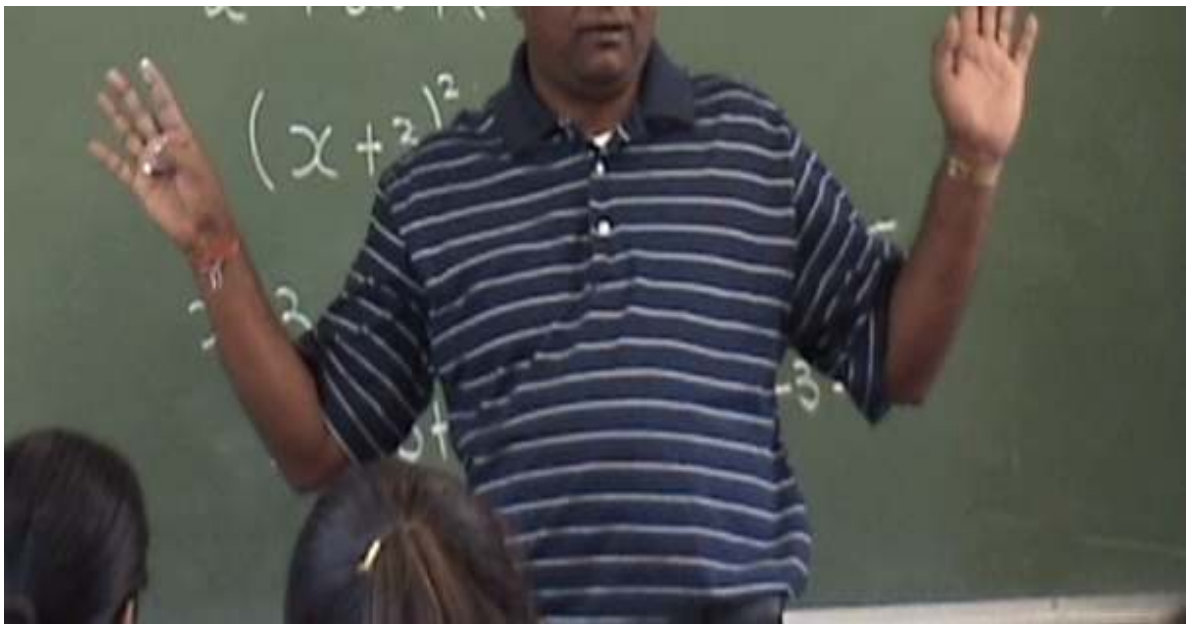
Concrete manipulative used at Daisy Secondary

Handwritten mathematical work on a chalkboard. The first line shows the equation $(x+3)^2 = 11$. The second line shows $x+3 = +\sqrt{11}$, with an arrow pointing from the word "sqrt" to the square root symbol. To the right of this line is the text "or $x+3$ ". The third line shows $x = -3 + \sqrt{11}$, with "or x " written to its right. In the bottom left corner, there is a small handwritten number "5" inside a square root symbol.

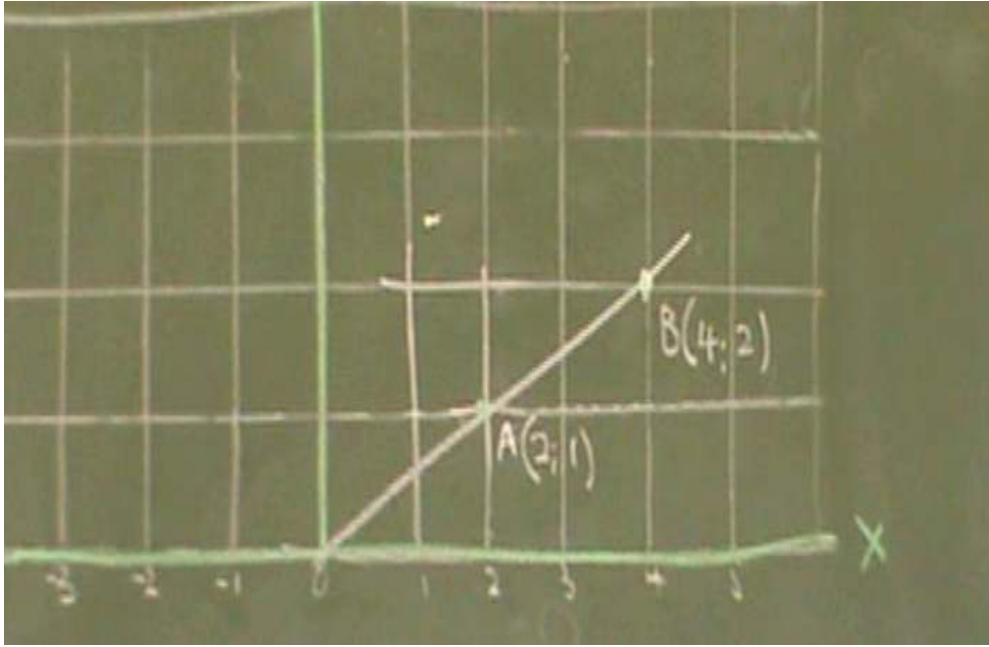
Use of arrows at Daisy Secondary



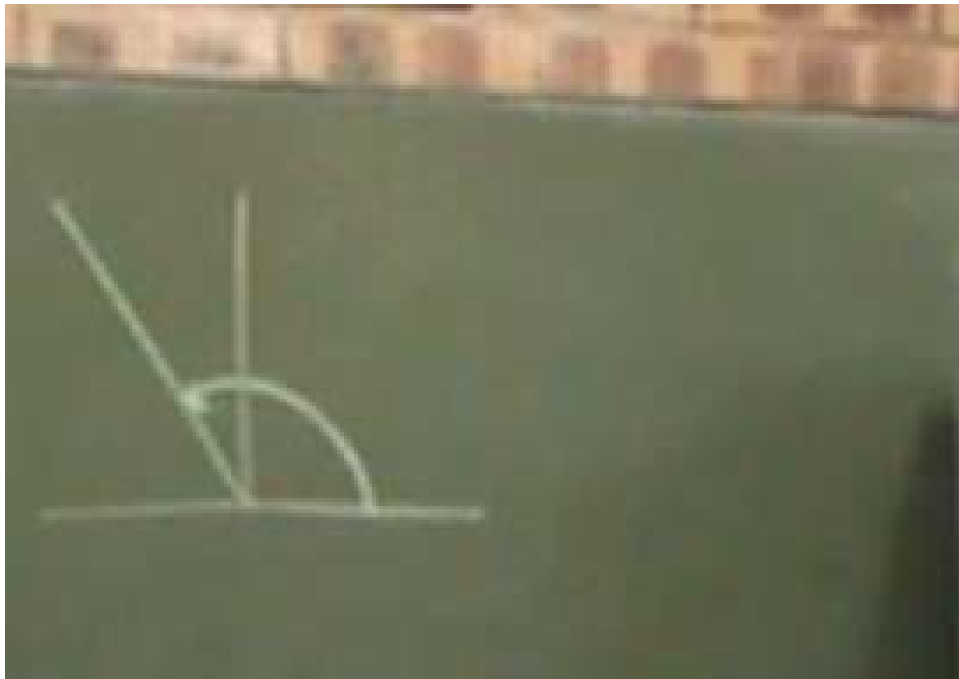
Use of iconic gestures (using a calculator) at Daisy Secondary



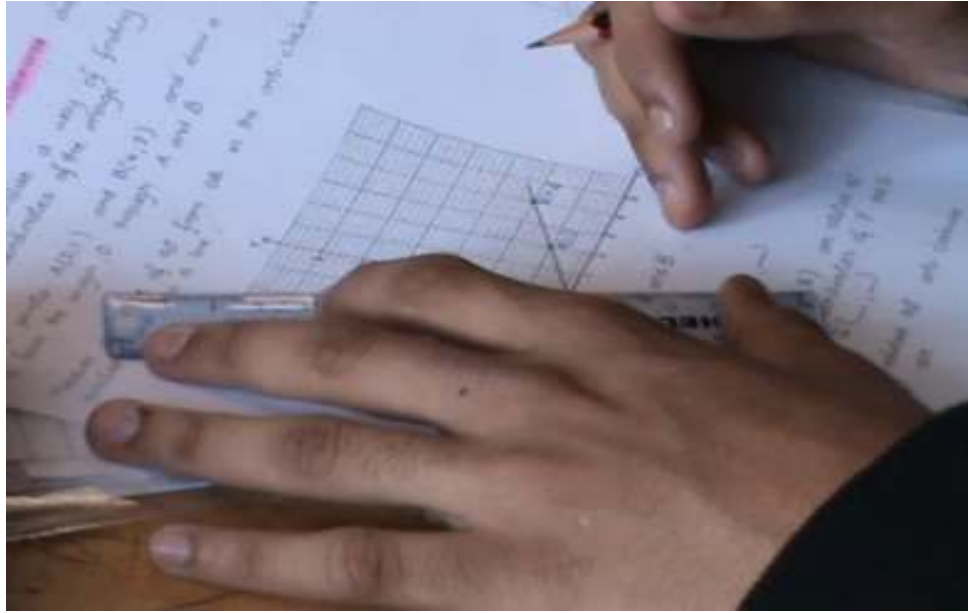
Use of gestures (asking learners to stop and look at the board) at Daisy Secondary



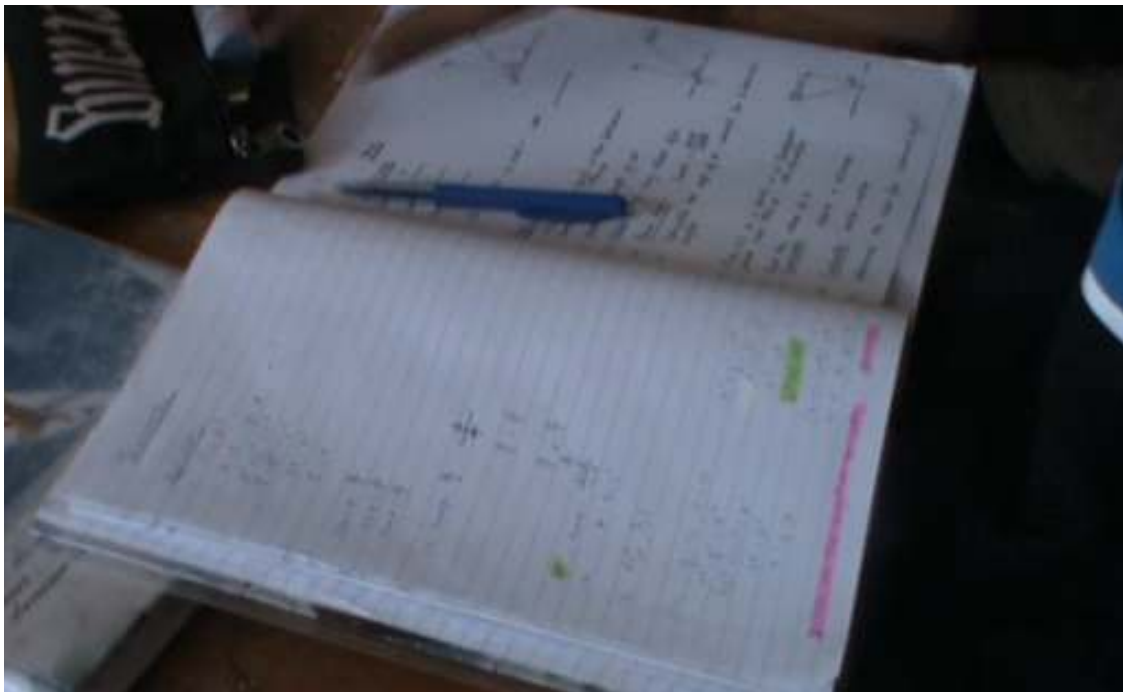
Use of colour on the board at Daisy Secondary



Use of colour on the board at Daisy Secondary



Learner's use of correct mathematics equipment (ruler and pencil to sketch graphs) at Daisy Secondary



Learner's use of highlighters at Daisy Secondary

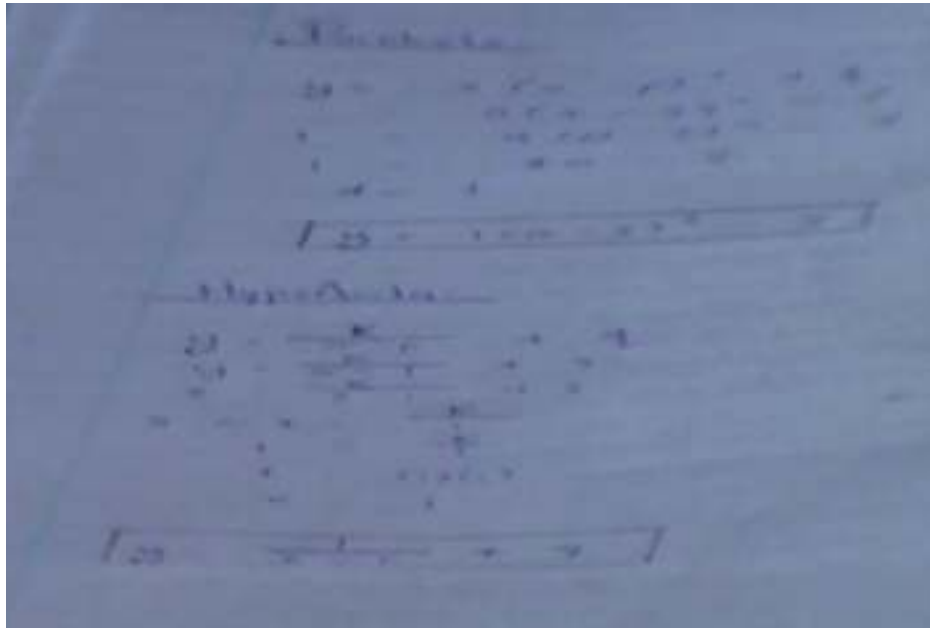
Tulip Secondary



Example of Level 1 scaffolding at Tulip Secondary



An example of a picture used at Tulip Secondary

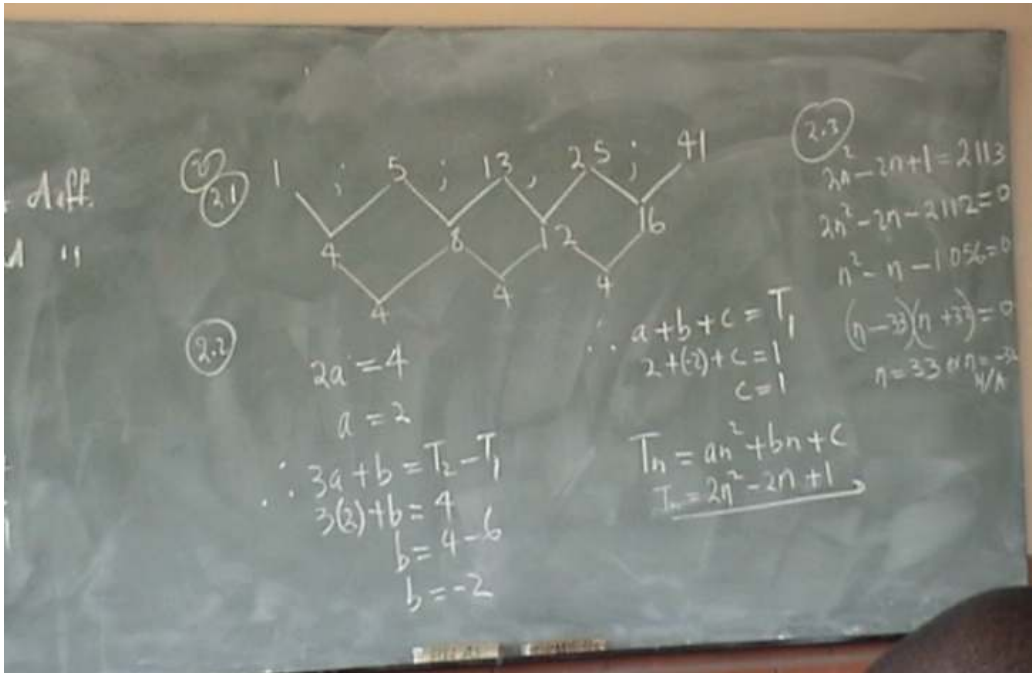


Learner's use of blocks at Tulip Secondary

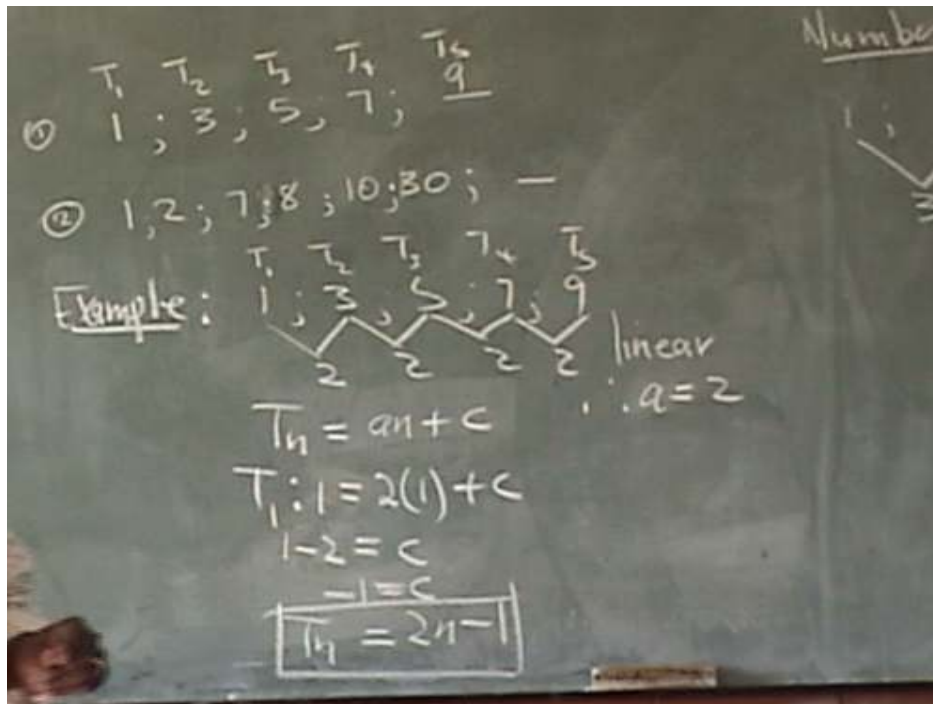


Learner's use of the calculator at Tulip Secondary

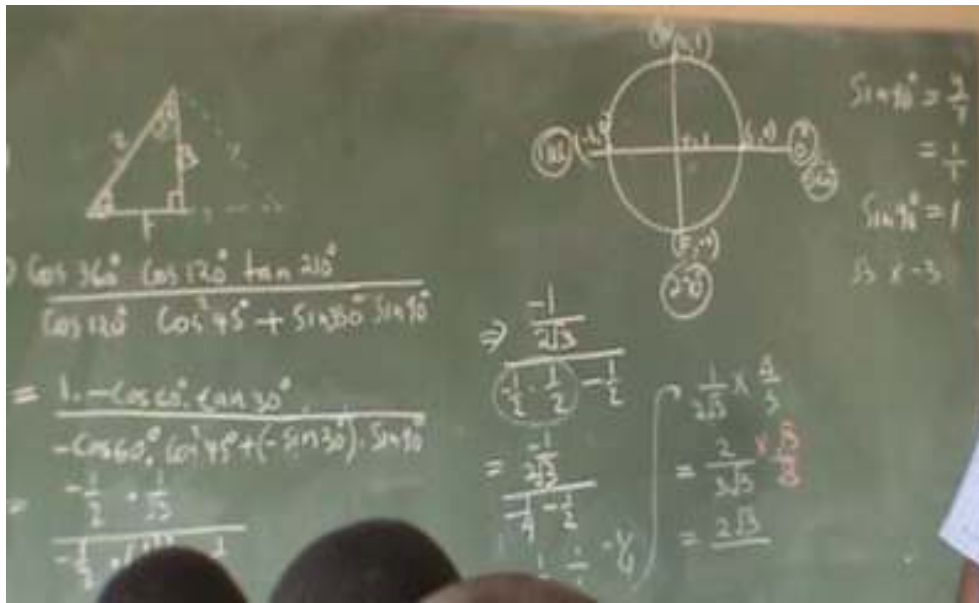
Carnation Secondary



Use of arrows and different symbols at Carnation Secondary



Use of blocks at Carnation Secondary

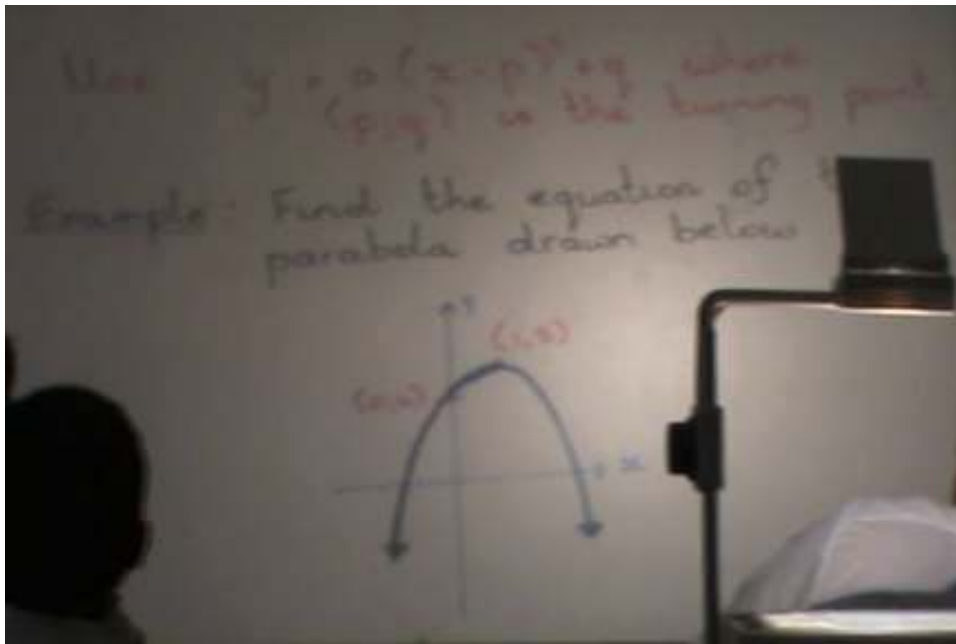


Use of symbols and different colours at Carnation Secondary

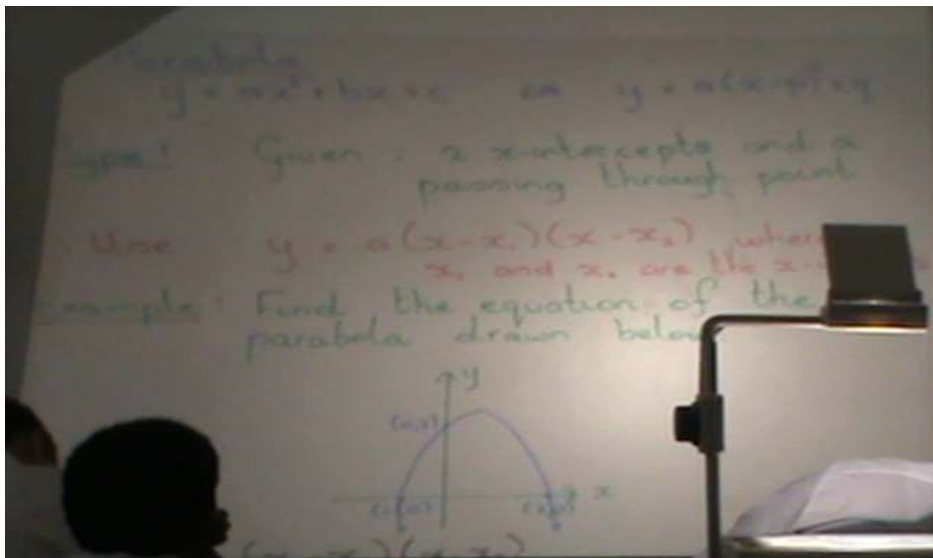


Use of deictic gestures at Carnation Secondary

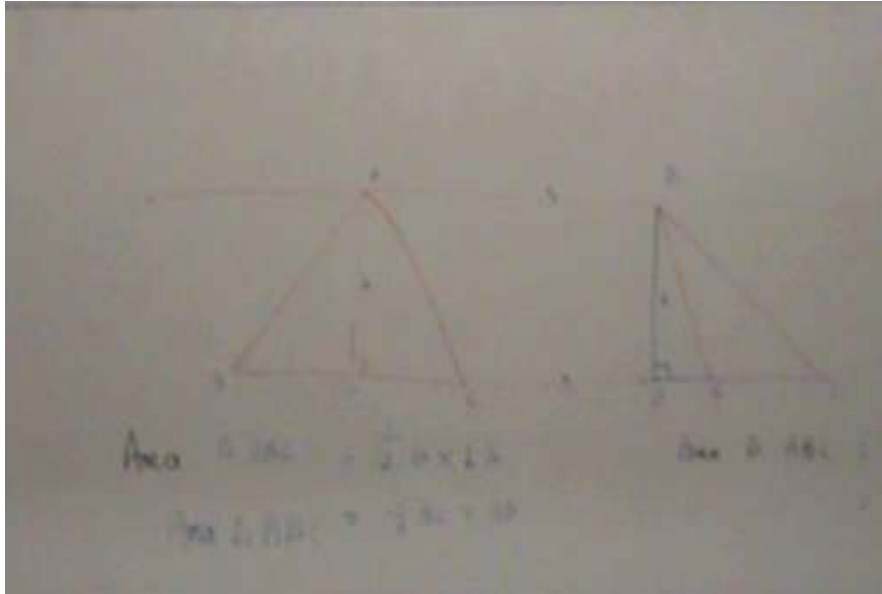
Lily Secondary



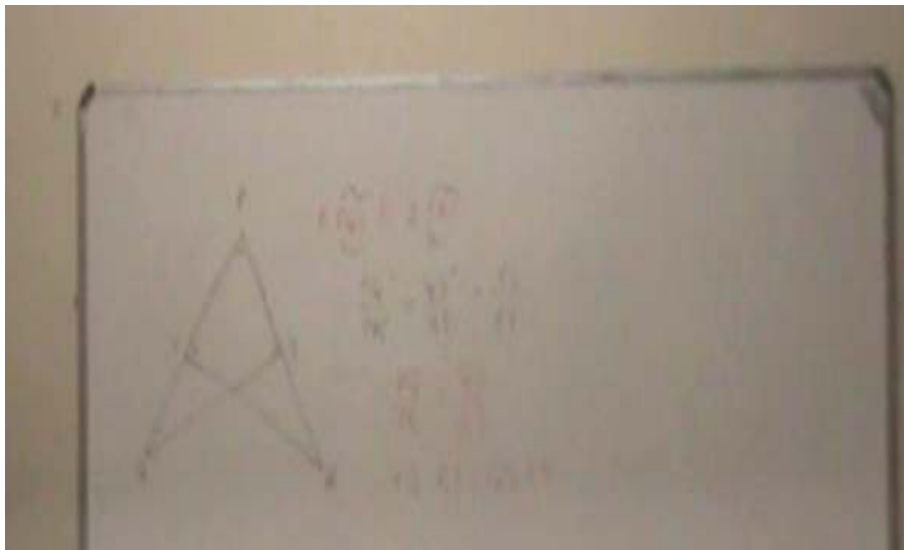
Use of different colours on the OHT at Lily Secondary



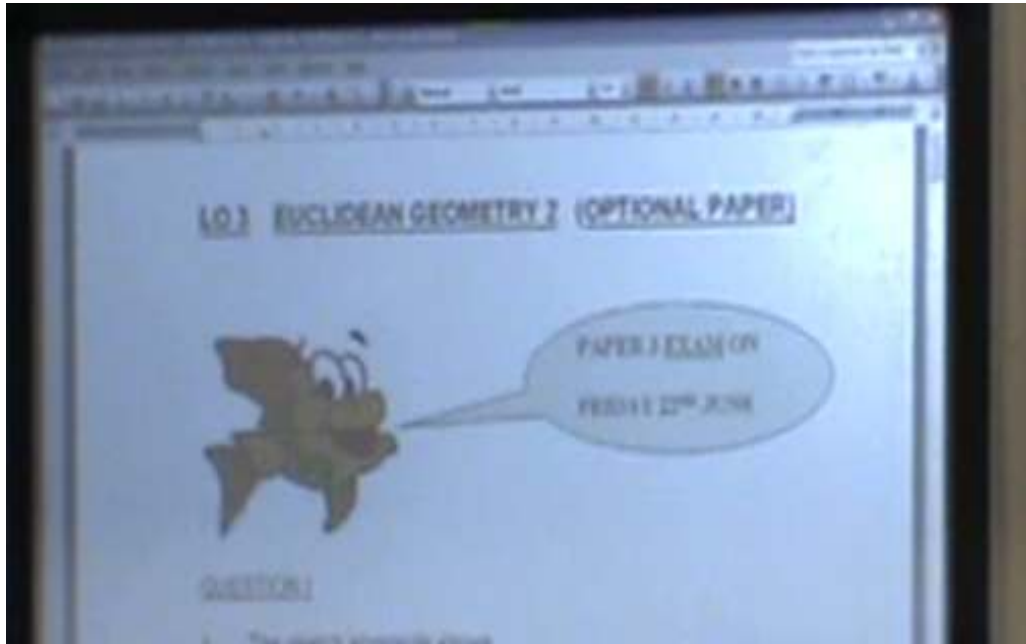
Use of different colours on the OHT at Lily Secondary



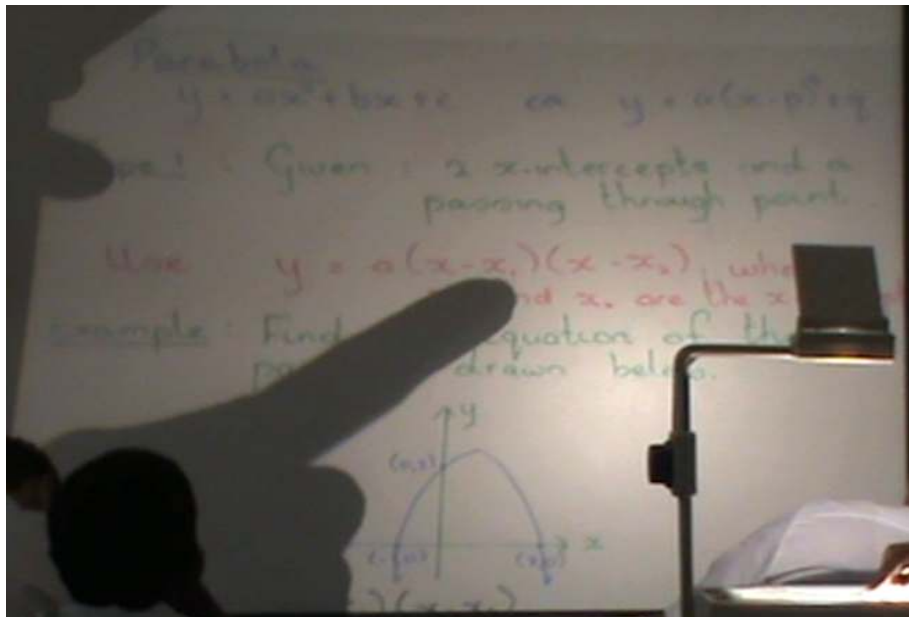
Use of different colours on the white board at Lily Secondary



Use of different colours on the white board at Lily Secondary



Use of cartoon characters to make mathematics more interesting at Lily Secondary



Use of deictic gestures at Lily Secondary

