Subspace-based Channel Estimation for DS/CDMA Systems Exploiting Pulse-shaping Information

Mohammed Yunus Abdul Gaffar

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Signed: ____________________

Name: Prof. A.D. Broadhurst

Date: 15 October 2003

Abstract

Third generation wireless systems have adopted Direct-Sequence/Code-Division Multiple-Access (DS/CDMA) as the multiple access scheme of communication. This system would typically operate in a multipath fading channel. This dissertation only deals with the task of channel estimation at the base station where the multipath delays and attenuations for each user are estimated. This information is used to aid the recovery of data that was transmitted by each user.

Subspace-based algorithms are popularly used to perform the task of channel estimation because they have the desirable property of perfectly estimating the channel in a noise-free environment. In this dissertation a new subspace-based channel estimation algorithm for DS/CDMA systems is presented. The proposed algorithm is based on the Parametric Subspace algorithm by Perros-Meilhac *et al.* for single-user systems. The main focus of this dissertation is to convert the Parametric Subspace algorithm from a single-user system to a multi-user DS/CDMA system.

It has been shown in the literature that by using information of the pulse-shaping filter in the Channel Subspace algorithm, the variance of the channel estimates is decreased. However, this has only been applied to a single-user system. There are several subspace algorithms that have been proposed for DS/CDMA systems. Most of these algorithms sample the received signal at the chip rate, making it impossible to exploit knowledge of the pulse-shaping filter in the channel estimation algorithm. In this dissertation a new subspace-based channel estimation algorithm is derived for a DS/CDMA system with multiple receive antennas, where the output is oversampled with respect to the chip rate. By oversampling the received signal, knowledge of the pulse-shaping filter is used in the channel estimation algorithm. It is shown that the variance of the channel estimate for the proposed subspace algorithm is less than the Torlak/Xu subspace algorithm that does not exploit information of the pulse-shaping filter. A mathematical expression of the mean square error of estimation for the new algorithm is also derived. It was shown that the analytic expression provides a good approximation of the actual MSE for high SNR.

The Parametric Subspace Delay Estimation (PSDE) algorithm was developed by Perros-Meilhac *et al.* to estimate the multipath delays introduced by the communications channel.

The limitation of the PSDE algorithm is that the performance of the algorithm deteriorates as the power of the multipath signals decrease with increasing delay time. This dissertation proposes a modified version of the PSDE algorithm, called the Modified Parametric Subspace Delay Estimation (MPSDE) algorithm, which performs better than the PSDE algorithm in an environment where the power of the multipath signals varies.

The final part of this dissertation discusses the Torlak/Xu channel estimation algorithm and the Bensley/Aazhang delay estimation algorithm. In order to compare the performance of these two subspace algorithms, the Torlak/Xu algorithm is converted to a delay estimation algorithm that is called the Parametric TX algorithm. The performance of the Bensley/Aazhang delay estimation algorithm and the proposed Parametric TX algorithm are compared and it is shown that the Parametric TX algorithm offers the better performance.

Preface

The research work presented in this dissertation was performed by Mr Mohammed Yunus Abdul Gaffar, under the supervision of Prof. A. D. Broadhurst, in the Centre for Radio Access Technologies at the University of Natal's School of Electrical, Electronic and Computer Engineering. This work was partially sponsored by Telkom South Africa Ltd and Alcatel as part of the Centre of Excellence programme.

Parts of this dissertation have been presented at the SATNAC'2002 conference in Drakensberg, South Africa, the SATNAC'2003 conference in George, South Africa and submitted for publication to the IEE Proceedings Communications.

The entire dissertation, unless otherwise indicated, is the author's work and has not been submitted in part, or in whole, to any other university for degree purposes.

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Firstly, I would like to express my most heartfelt thanks to my creator Allah, the cherisher and sustainer of the worlds, for blessing me with a thirst for knowledge and showering me with many successes in my life.

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Words cannot begin to express the special thanks owed to my parents. They have taught me the value of working and playing hard throughout my life. Without their loving support and constant encouragement, this thesis would not be possible. My sincere appreciation also goes to my mathematically talented twin brother Yusuf and ingenious sister Tasneem for always being there for me. Their love has provided me the inspiration to overcome some of life's exciting challenges.

I wish to also thank Telkom SA Ltd and Alcatel for their much appreciated financial support and for providing the equipment necessary for the completion of my dissertation.

Lastly, thanks are extended to my postgraduate friends for our long, enjoyable and well-deserved tea breaks. Our Tuesday afternoon squash games, after tea ping-pong matches and the Thursday night bowling provided the perfect escape from the long hours spent at work.

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List of Acronyms

BPSK	Binary Phase Shift Keying
CR	Cross Relation
CRB	Cramér-Rao Bound
DECT	Digital Enhanced Cordless Telecommunication
DS/CDMA	Direct-Sequence Code Division Multiple Access
EDGE	Enhanced Data Rates for GSM Evolution
ETSI	European Telecommunications Standards Institute
EVD	Eigenvector Decomposition
FDD	Frequency Division Duplex
FIR	Finite Impulse Response
GSM	Group Special Mobile
IMT-2000	International Mobile Telecommunications-2000
IS-95	Interim Standard 95
ITU	International Telecommunications Union
JOSC	Joint Optimisation with Subspace Constraints
MCR	Maximal Combined Ratio
MCS	Modified Channel Subspace
MIMO	Multiple Input Multiple Output
MIP	Multipath Intensity Profile
MNS	Minimum Noise Subspace
MPSDE	Modified Parametric Subspace Delay Estimation
MSE	Mean Square Error
MUD	Multiuser Detection
PSDE	Parametric Subspace Delay Estimation
UMTS	Universal Mobile Telecommunications System
UTRAN	UMTS Terrestrial Radio Access Network
UWC	Universal Wireless Communications
SIMO	Single Input Multiple Output
SNR	Signal-to-Noise Ratio
SOS	Second-order Statistics
TDD	Time Division Duplex
TDMA	Time Division Multiple Access

TSML	Two Step Maximum Likelihood
TX	Torlak/Xu
TXK	Tong, Xu, Kailath
W-CDMA	Wideband CDMA

Notation and Common Parameters

Bold capital	Matrix
Bold lower case	Column vector
Lowercase	Scalar (complex or real valued)
$(.)^T$	Transpose
$(.)^H$	Hermitian transpose
c	Spreading code
d	Number of multipaths
g	Pulse-shaping filter
h	Impulse response of the channel
K	Smoothing factor
L	Channel length
L_c	Processing gain
P	Number of users
p	Oversampling factor
q	Number of receive antennas
s	Transmitted symbol
T	Symbol duration
T_c	Chip period
T_b	Bit duration

Chapter 1

Introduction

1.1 Mobile Wireless Communications

Since the introduction of wireline technology systems, people have been able to communicate almost instantaneously over large distances at a reasonable cost for the very first time. This advance in technology has certainly changed the way we are able to interact with the people in our life. The natural evolution of landline-based telephones is the "mobile phone" or "cell phone", which has shown tremendous commercial growth over the last decade. This growth is due to the affordable cost of mobile devices, which was made possible by the ingenious technological advancements of the wireless communications industry. The advantage of mobile phones over landline-based phones is that it is the preferred choice of technology for both the service provider and the consumer. Wireless technology takes away the restriction of the user only being able to communicate in a fixed location. On the other hand, the service provider enjoys the benefit of penetrating a larger area of consumers than landline-based systems that have inaccessible areas because they are too remote or they have intervening inhospitable terrains.

The South African cellular communications industry is an excellent example of the economic growth shown by service providers around the world. The South African cellular market is currently worth twenty-three billion Rands [Cellular Online03] and it is expected to grow to around forty-five billion Rands by 2004. The market size of mobile users in South Africa is currently over fourteen million and with approximately nine thousand users signing up per day, the number of mobile users is expected to increase to nineteen million in 2006.

With the large increase of mobile users that is expected in the near future and the need for high data rate services, current second generation (2G) systems like Group Special Mobile (GSM) are unable to satisfy these demands. Third generation (3G) systems were developed by the ITU (International Telecommunications Union) and the ETSI (European Telecommunications Standards Institute) to exceed the capabilities of current 2G systems by offering high data rate services such as video on demand, video conferencing, web browsing and e-mail retrieval and use advanced signal processing techniques to increase spectral efficiency. 3G technologies are seen as the next step in wireless communications that will enhance the multimedia capabilities of mobile phones.

1.2 3G Standards

The motivation for the development of third-generation wireless communication systems is to provide current 2G and 2.5G services with wideband services. With this aim in mind, two large standards bodies consisting of the ITU and the ETSI encouraged the evolutionary path towards 3G technologies. The ITU is a United Nations department responsible for co-ordinating global telecommunications standards and activities. The ETSI is a non-profit European body that is made up of representatives from a diverse communications background such as administrators, network operators, manufacturers, service providers, research bodies and users.

The ITU's global standard for the next-generation broadband mobile telecommunications systems is called IMT-2000 (International Mobile Telecommunications-2000) [Ojanperä98]. Their initiative is to define a method of accessing the global telecommunications infrastructure using satellite and terrestrial mobile systems. IMT-2000 is a flexible standard that allows operators around the world the freedom of choosing their preferred radio access method and core network so that they can openly implement and evolve their systems. Regulations and market requirements dictate how the operators are able to implement their systems.

There are five radio interface standards that fall under the IMT-2000 standard. Three of the radio air interface standards include CDMA and two of them encompass Time Division Multiple Access (TDMA):

1. IMT-MC: CDMA Multi-carrier (known as cdma2000 or IS-2000) [Hara97].
2. IMT-DS: CDMA Direct Spread (known as Wideband CDMA or W-CDMA-FDD) [Milstein00]. The Frequency Division Duplex (FDD) mode is used for applications

that require the same amount of radio resources in the uplink as in the downlink, which is also referred to as symmetrical applications.

3. IMT-TC: CDMA TDD (known a W-CDMA-TDD). Time Division Duplex (TDD) is optimised for symmetrical and asymmetrical applications with high data rates.
4. IMT-SC: TDMA Single Carrier (known as UWC-136 and EDGE). UWC-136 (Universal Wireless Communications) and EDGE (Enhanced Data Rates for GSM Evolution) provided higher data rate services with no changes to the channel structure, frequency or bandwidth.
5. IMT-FT: TDMA Multi-carrier (known as DECT, Digital Enhanced Cordless Telecommunication).

The standards that have CDMA [Prasad98] techniques have received the most amount of attention from the communications research community since it provides a logical evolutionary path for Interim Standard 95 (IS-95) based networks that were developed for 2G systems.

The ETSI's network solution for third-generation systems is driven by UMTS (Universal Mobile Telecommunications System) and it is called the UTRAN (UMTS Terrestrial Radio Access Network). The radio access technology is based on W-CDMA. A conceptual overview of W-CDMA is provided in [Milstein00]. The ETSI proposed UMTS because it is a logical upgrade path for 2G GSM networks.

A comparison between the air interface standards for IMT-2000 and UMTS is given in [Ojanperä98] and more detailed information on the UMTS and IMT-2000 standards based on W-CDMA can be found in [Dahlman98].

1.3 What is Channel Estimation?

This dissertation focuses on the task of channel estimation, thus it is important to first define what it meant by a channel. In the most general sense, a channel is the physical medium between the transmitter and the receiver through which the signal propagates. A few examples of physical mediums are free space, fibre, waveguides and copper wire. The characteristic feature of any channel is that the transmitted signal gets corrupted in frequency and phase before it reaches the receiver. This distortion is caused by the time dispersive behaviour of the channel and thermal noise.

Channel estimation is defined as the process of characterising the effect that the physical channel has on the input sequence. In most communications areas, the channel is assumed to be linear. In this case, the channel estimate corresponds to the estimate of the impulse response of the system. A channel estimation algorithm aims to estimate the mathematical representation of the channel. The channel estimation algorithm with the best performance is not the algorithm with the closest mean value to the actual channel but it is the algorithm that has the smallest estimation variance.

The purpose of channel estimation is to allow the receiver to estimate the impulse response of the channel that explains the behaviour of the channel. This knowledge of the channel's behaviour is well utilised in modern wireless communication systems. Multiuser detection (MUD) algorithms require knowledge of the channel to estimate the data that was transmitted by a desired user. Time diversity techniques such as the Maximal Combined Ratio (MCR) Rake receiver use channel estimates to implement a matched filter such that the receiver is optimally matched to the received signal instead of the transmitted one. Equalisers use the channel estimates to correct the distortion caused by the time dispersive behaviour of the channel.

1.4 Channel Estimation Algorithms

The channel estimation algorithms that have been proposed in the literature can be classified into three categories: training, semi-blind and blind methods. Traditional techniques for channel estimation use training data, which is a received sequence whose transmitted data is known. More recently, blind techniques have been researched intensively in the literature. Blind techniques don't use training data, but instead they use certain prior information that is inherent in the original strings of data symbols. Blind methods are preferred to training-based methods because they save on bandwidth used for training data and this gives blind techniques the potential to increase the data throughput in wireless systems. Semi-blind methods are a combination of existing training and blind methods that aim to estimate the channel using not only the known data in the transmitted signal and its corresponding observation, but also the observation of the unknown transmitted signal. Semi-blind channel estimation is motivated by the fact that in modern communication systems there are known pilot symbols that can be used to improve the performance of the channel estimation algorithm.

There are various different types of blind and training-based channel estimation algorithms that have been proposed in the literature. Some of these algorithms are listed in Table 1. The type of blind channel estimation algorithm that has received considerable research is the subspace method. Subspace algorithms use second order statistics of the received signal to estimate the channel. The desirable characteristic of subspace algorithms to other types of algorithms is that subspace algorithms are near-far resistant. Thus, subspace-based channel estimation algorithms have the ability to identify the channel of a user with a weak transmit power whose received signal at the base station is being drowned by other users with a stronger transmit power. A comprehensive overview of subspace algorithms can be found in [Tong98], [Giannakis01], [Abed-Meriam99], [Tugnait00].

Table 1 Channel estimation algorithms

Name of method	Algorithm type	More details can be found in:
Subspace techniques	Blind	[Tong98], [Giannakis01]
Higher Order Statistical Approaches	Blind	[Godard80], [Donoho81]
Expectation-Maximisation	Blind	[Weinstein90]
Maximum Likelihood	Training-based	[Bensley98], [Ström00]
Least Squares	Training-based	[Haykin84]
Neural Networks	Training-based	[Amari98]
Kalman Filter	Training-based	[Safaya97]

Semi-blind channel estimation has also attracted attention due to the need for fast and efficient channel estimation algorithms. A survey of semi-blind channel estimation algorithms is given in [de Carvalho00]. Semi-blind channel estimation assumes additional knowledge of the input sequence compared to blind algorithms. This gives semi-blind methods the ability to offer significant performance improvements to existing blind or training-based methods as shown in the evaluation of the Cramér-Rao lower bound in [de Carvalho97]. A popular approach of semi-blind methods is the formulation of an optimisation function that is developed by combining the optimisation function of an existing blind and training-based channel estimation algorithm. In most cases, a weighted linear combination of the cost function for the blind and training-based channel estimation algorithms are used to form the optimisation function of the semi-blind method [Gorokhov97].

1.5 Motivation for research

Third-generation systems are required to operate in a channel that is characterised by multiple reflections, diffractions and attenuation of the transmitted signal energy. These are caused by natural objects such as buildings and hills that are in the communication path between the base station and the mobile unit as shown in Figure 1-1.

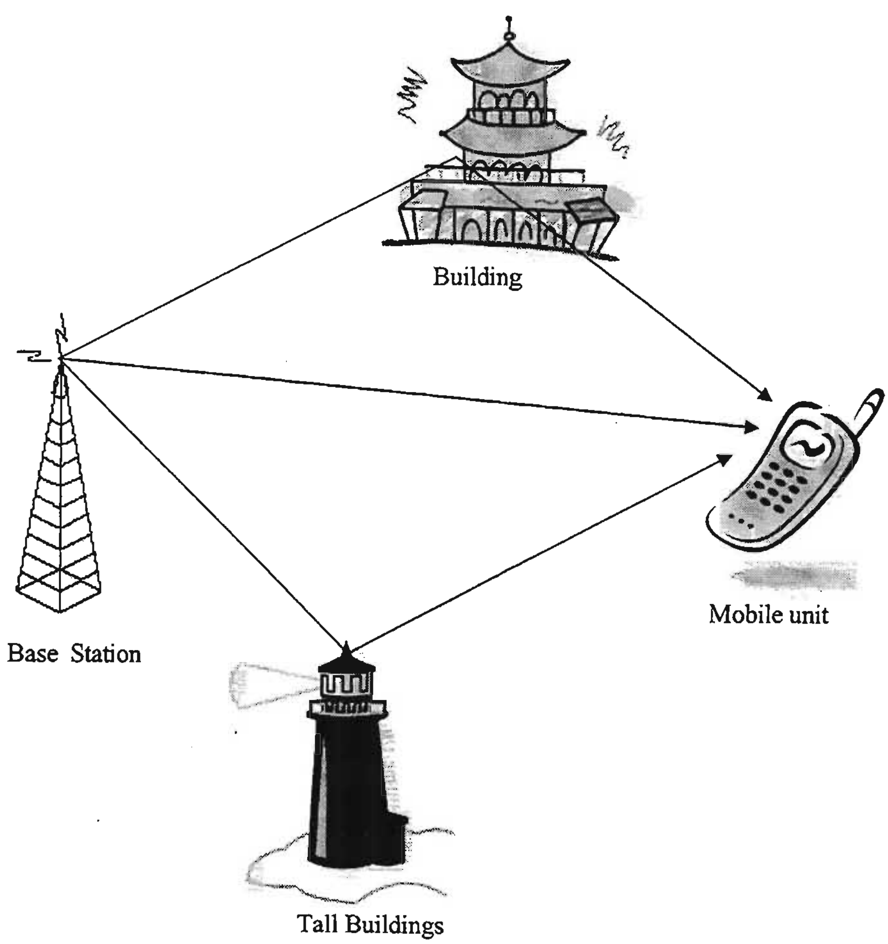

Figure 1-1 Illustration of a multipath channel in an urban environment

As a result, in the downlink, the mobile receives multiple copies of the same signal that are each scaled in amplitude and delayed in time. This leads to multipath propagation. Two effects resulting from multipath propagation are:

1. The signal that is transmitted from the base station arrives at the mobile at distinct time instants. This causes the arriving signal's transmitted energy to be distributed across time to form a multipath delay profile.
2. When there is an object positioned close to the mobile, the transmitted signal from the base station gets scattered into many paths that are received at virtually the same instant when compared to the duration of a single chip period. As a result, the scattered transmitted signal can add up constructively or destructively at the receiver causing fast fading.

The above two points describe how multipath propagation has the effect of distorting the signal that is transmitted by the base station. One of the countermeasures that have been proposed to combat the adverse affects of multipath propagation in 3G systems is the use of a Rake receiver [Holma02]. A Rake receiver is used to constructively combine the energy of the multiple signals that are received. However, the Rake receiver requires knowledge of the channel estimates. In 3G UMTS systems, pilot symbols are inserted to the start and the end of a packet to provide an estimate of the momentary channel state using a training-based channel estimation algorithm. In a time-varying channel, the channel state at the beginning of the received packet may be very different to the channel state at the end of the packet. Thus, a training-based approach is unable to identify a fast changing channel. Semi-blind methods have been proposed in [Lasaulce00] to solve the task of channel estimation for time-varying channels. The semi-blind algorithm in [Lasaulce00] uses the Torlak/Xu subspace algorithm [Torlak97] to blindly estimate the channel from the unknown data symbols and a Least Squares approach to identify the channel from the training symbols, as shown in Figure 1-2.

The channel is estimated using a weighted combination of the channel estimate from the subspace and the Least Squares algorithms. It is shown in [Lasaulce00] that from the training-based Least Squares algorithm, the blind subspace algorithm and the proposed semi-blind algorithm, the semi-blind method provides the best channel estimate. The emergence of semi-blind methods for 3G systems that use existing blind subspace algorithms has motivated the focus of this dissertation to investigate and improve the performance of blind subspace algorithms.

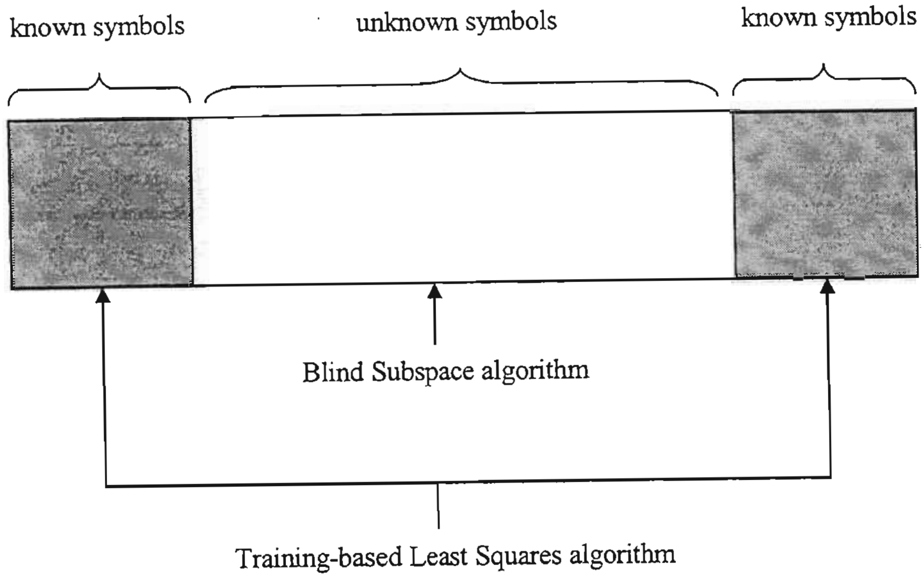

Figure 1-2 A 3G packet structure showing a semi-blind channel estimation approach

1.6 Outline of dissertation

In Chapter 2, blind subspace-based channel estimation algorithms that have been proposed in the literature for single-user systems are reviewed. The Channel Subspace algorithm is described together with the algorithms that have been proposed in the literature, which are related to the Channel Subspace algorithm. Some of the subspace-based channel estimation algorithms that have been developed for Direct-Sequence Code Division Multiple Access (DS/CDMA) systems are also discussed.

Chapter 3 describes the concepts, assumptions and the mathematical formulation of the Channel Subspace algorithm for single-user systems. The development of the Parametric Subspace algorithm is also discussed. The Parametric Subspace algorithm extends the Channel Subspace algorithm by incorporating pulse-shaping information into the channel estimation algorithm. Concepts of the Parametric Subspace algorithm are used to derive a new subspace-based channel estimation algorithm for DS/CDMA systems that incorporates pulse-shaping information. The performance of the proposed algorithm is evaluated via simulations. Lastly, a mathematical expression of the mean square error of estimation for the new algorithm is derived.

Chapter 4 explains the Parametric Subspace Delay Estimation (PSDE) algorithm and describes how it can be applied to multi-user DS/CDMA systems. A modified version of the PSDE algorithm, called the Modified Parametric Subspace Delay Estimation (MPSDE) algorithm, is proposed to improve the performance of the PSDE algorithm in an environment where the power of the multipath signals varies. Simulation results are presented to compare the performance of the proposed MPSDE algorithm to the PSDE algorithm. Chapter 4 concludes with simulation results that compare the performance of the new channel estimation algorithm in Chapter 3 that uses the MPSDE algorithm to estimate the multipath delays, to the new channel estimation algorithm that assumes all the multipath delays are known at the receiver.

In Chapter 5, the Torlak/Xu (TX) channel estimation algorithm [Torlak97] and the Bensley/Aazhang delay estimation algorithm [Bensley96] are discussed. In order to compare the performance of these two algorithms, the Torlak/Xu algorithm is converted to a delay estimation algorithm that is called the Parametric TX algorithm. Simulations results are presented at the end of Chapter 5 to compare the performance of the Bensley/Aazhang algorithm to the proposed Parametric TX algorithm.

A summary of the dissertation and concluding remarks are made in Chapter 6. Some future directions for further work are also discussed. In Appendix A, a numerical example of the Channel Subspace algorithm is given to enhance the reader's understanding of the Channel Subspace algorithm that is described in detail in Chapter 3.

1.7 Original contributions in this dissertation

The original contributions made in this dissertation include:

1. In Chapter 3, a new subspace-based channel estimation algorithm is derived for DS/CDMA systems. The proposed algorithm is based on the Parametric Subspace algorithm for single-user systems. Simulations are performed comparing the new algorithm that uses pulse-shaping information to the Torlak/Xu algorithm that does not use knowledge of the pulse-shaping filter. It is shown that the proposed algorithm performs better than the Torlak/Xu algorithm. A mathematical expression of the mean square error of estimation for the proposed algorithm is also derived. It

is shown that the analytic expression provides a good approximation of the actual MSE for high SNR.

2. In Chapter 4, a new delay estimation algorithm, called the Modified Parametric Subspace Delay Estimation Algorithm (MPSDE), is proposed. The MPSDE algorithm is based on the Parametric Subspace Delay Estimation (PSDE) algorithm in [Perros-Meilhac01]. It is shown by simulations that the MPSDE algorithm performs better than the PSDE algorithm in a communications channel where the power of the multipath signal varies.

3. In Chapter 5, the Torlak/Xu channel estimation algorithm is converted into a delay estimation algorithm that is called the Parametric TX algorithm. This was done to compare the performance of the Parametric TX algorithm to the Bensley/Aazhang delay estimation algorithm. Simulation results show that the proposed Parametric TX algorithm performs better than the Bensley/Aazhang delay estimation algorithm.

Parts of this research have been presented by the author at two local conferences and submitted for publication at an international journal:

- M. Y. Abdul Gaffar, A. D. Broadhurst, F. Takawira, "Performance analysis of a subspace-based channel estimation algorithm for CDMA systems", in *Proc. SATNAC 2002*, Champagne Sports, Drakensberg, South Africa, Sept. 2002.

- M. Y. Abdul Gaffar, A. D. Broadhurst, F. Takawira, "A comparison of parametric subspace delay estimation algorithms for DS/CDMA systems", in *Proc. SATNAC 2003*, Fancourt Hotel and Country Club Estate, George, South Africa, Sept. 2003.

- M. Y. Abdul Gaffar, A.D Broadhurst, F. Takawira, "An improved subspace-based channel estimation algorithm for DS/CDMA systems exploiting pulse-shaping information", *submitted to IEE Proceedings Communications.*

Chapter 2

Subspace-based Blind Channel Estimation Algorithms

2.1 Introduction

In recent years there has been considerable interest from both the signal processing and communications communities in the so-called "blind" problem. The motivation for the increased research activities in blind techniques is their potential applications in wireless communications, which have been experiencing explosive growth. The blind channel estimation problem involves a system model shown in Figure 2-1, where only the observation signal x is used to identify the channel h. This is in contrast to the classical channel estimation problem where both the input signal s and the observation signal x are used.

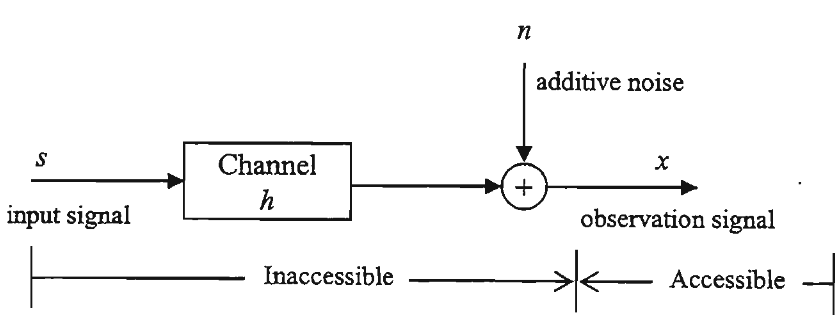

Figure 2-1 System model for blind channel estimation algorithms

In typical wireless communication systems, the distortion caused by multipath interference affects both transmission quality and efficiency in wireless communications. In order to mitigate such distortions, knowledge of the channel needs to be known. The current cellular communication system used in South Africa and Europe, which is known as GSM, uses "training" signals to perform the task of channel estimation. The transmission of training signals has the effect of decreasing the time available for transmitting information. For time-invariant channels, the loss in throughput is neglible because training is only done once. However, for time-varying channels, the loss of throughput becomes significant since training needs to be done periodically.

At first glance, the task of blind channel estimation illustrated in Figure 2-1 may not seem tractable. From the observation signal, how is it possible to distinguish the transmitted signal from the channel when neither is known? The main concepts used in blind channel estimation algorithms lies in the exploitation of the structure of the channel, when it is written in a matrix form, and the known properties of the input signal such as the probabilistic distributions and moments. Using these concepts, the problem of estimating the channel using the output statistics is related to the output's time series analysis. In communications systems, for example, the input signals may have a finite alphabet property that exhibits cyclostationarity. This property of the input signal was used in [Tong91], which was the first paper that showed the possibility of estimating a non-minimum phase channel using only the second-order statistics of the observation signal. The algorithm in [Tong91] led to the development of many blind subspace-based channel estimation algorithms.

Blind channel estimation algorithms can be classified into two categories: statistical and deterministic methods. These two categories are clearly explained in Section 2.2. It has been shown in the literature that deterministic methods offer better performance than statistical methods. For this reason, only blind deterministic methods are discussed in Section 2.3. The most extensively researched blind deterministic algorithm is the Channel Subspace algorithm [Moulines95]. For this reason, Section 2.4 describes the algorithms that have been proposed in the literature, which are related to the Channel Subspace algorithm. A brief comparison of three deterministic subspace-based algorithms: the Channel Subspace algorithm, the Cross Relation algorithm and the Two Step Maximum Likelihood algorithm can be found in Section 2.5. Lastly, Section 2.6 discusses some of the subspace-based channel estimation algorithms for DS/CDMA systems.

2.2 Classification of Blind Channel Estimation Algorithms

The blind channel estimation algorithms that have been proposed in the literature can be classified into statistical and deterministic methods.

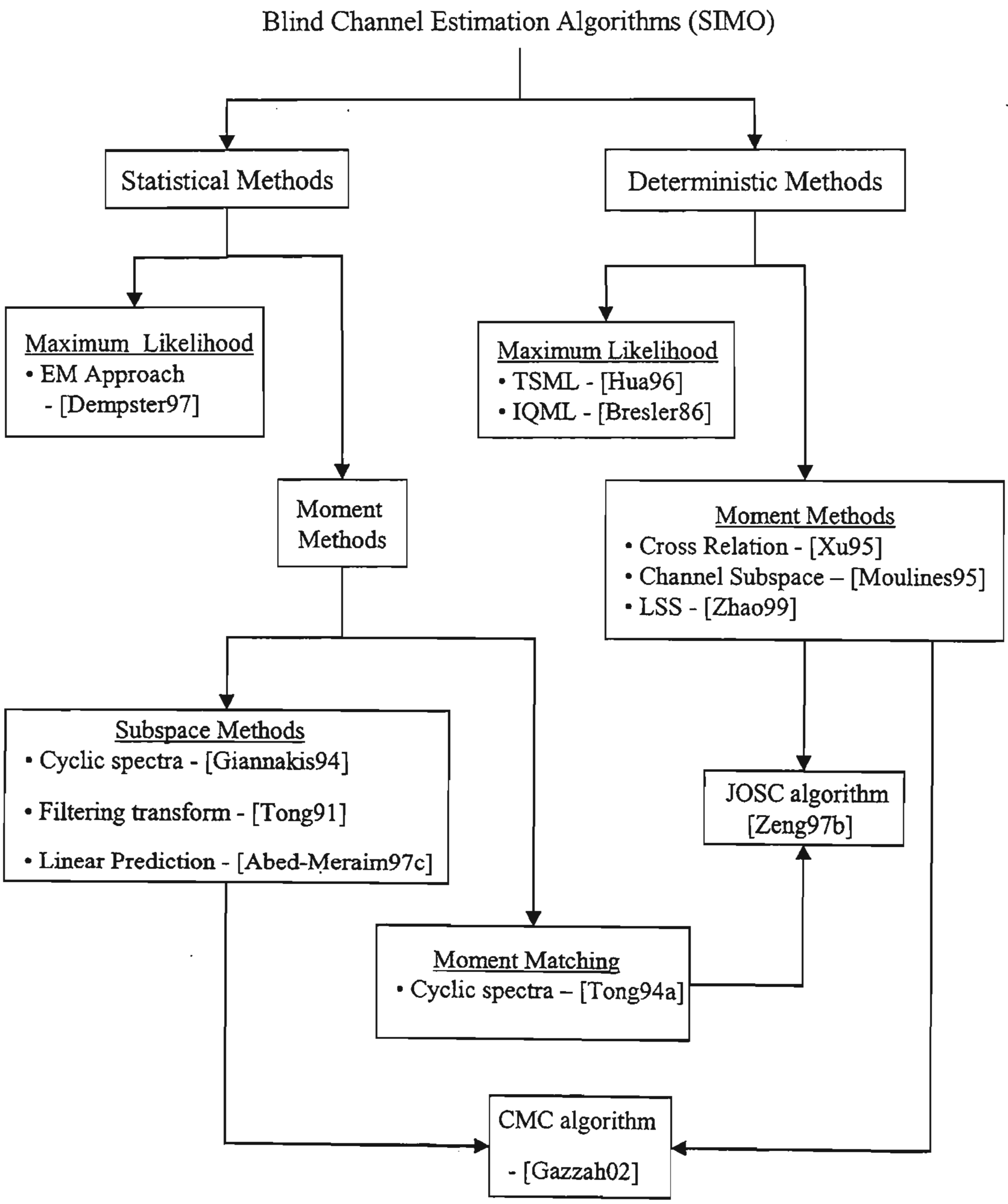

Figure 2-2 Classification of blind channel estimation algorithms

If the input s is assumed to be random with a known statistical distribution that is used by the blind channel estimation algorithm, the method is considered to be statistical. However, if the source does not have a statistical description, or although the source is random but the statistical properties of the input signal are not used by the blind channel estimation algorithm, the corresponding algorithm is classified as a deterministic method. Figure 2-2 shows a map of the different classes of subspace methods along with a few algorithms that have been proposed in each class.

Since Chapters 3, 4 and 5 are involved with improving the performance of deterministic subspace algorithms, the focus of this chapter is on deterministic methods that have been proposed in the literature.

2.3 Blind Deterministic Channel Estimation Algorithms

The algorithm proposed by Tong, Xu and Kailath [Tong91] was the first method to show that a channel could be identified using only the second order statistics of the received signal. This result was an important breakthrough since the performance of the algorithm in [Tong91] was better than previous higher-order statistical blind channel estimation algorithms [Godard80], [Donoho81] and [Benveniste80] that exhibited high variances. The key to this breakthrough was the consideration of a system model with multiple channels, corresponding to the user of an array of antennas possibly combined with fractional sampling reception techniques [Tong98]. Soon afterwards, similar algorithms that used only the second order statistics of the received signal, also referred to as subspace algorithms, were developed by Moulines *et al.* [Moulines95], Gurelli and Nikias [Gurelli95] and Slock [Slock94]. These algorithms showed that under mild assumptions, which can be found in [Hua96a], an unknown single input single output (SISO) polynomial transfer function representing the unknown channel can be identified using the eigenvector decomposition of the matrix obtained from the covariance of the received signal.

The Channel Subspace algorithm that was proposed by Moulines *et al.* in [Moulines95] was shown to be better than the TXK (Tong, Xu, Kailath) algorithm [Tong91] because it has a lower computational complexity and estimation variance. The Channel Subspace algorithm performs better than the TXK algorithm because the Channel Subspace algorithm works on the orthogonal property of the noise subspace and the column space of the channel matrix, whereas the TXK algorithm works on the matrix pencil property [Moon00]. The advantage of the orthogonal property is that in the presence of temporally correlated symbols, the orthogonal property is preserved, but the matrix property is lost.

Since the development of the Channel Subspace algorithm, there has been a lot of work done in improving the performance of this algorithm. The assumptions that are made for the Channel Subspace algorithm to give reliable channel estimates [Moulines95], [Hua96a], [Zeng97a] and [Abed-Meraim97d] are the following:

- The multiple channels arising from fractionally sampling the received signal or the use of an antenna array do not share common zeros.
- The order of the channel L needs to be known exactly.
- The input sequence should be complex enough to excite all the modes [Abed-Meraim99] of the channel.
- The number of samples from the received signal that is used by the channel estimation algorithm needs to be sufficient in order for the left nullspace of the channel matrix exists.

In [Abed-Meraim97a], the concept of minimum polynomial basis of a rational subspace is used to understand subspace methods. The rational subspace concept was also used in [Abed-Meraim97d] to show that the Channel Subspace algorithm leads to inconsistent channel estimates if the channel order is over-determined. It was not long before Liavas *et al.* proposed a channel order estimation algorithm for the Channel Subspace algorithm [Liavas99]. It was shown in [Liavas99] that the impulse response of real communication channels has terms that are "large" and a lot of terms that are "small". If the channel length is overestimated, the subspace algorithm estimates the "small" terms of the channel impulse response, which leads to a dramatically degraded quality of estimation. To avoid estimating the "small" terms, a channel length estimation algorithm is presented in [Liavas99] chooses the channel length from the ratio of two consecutively decreasing eigenvalues of the correlation matrix.

2.4 Algorithms Related to the Channel Subspace Algorithm

Subspace methods traditionally use the received signal to form the correlation matrix that is used to estimate the signal and noise subspaces. In a noiseless system, the received signal lies only in the signal subspace. The noise subspace is the subspace that is orthogonal to the signal subspace. The Channel Subspace algorithm uses the vectors that span the complete noise subspace to estimate the channel. The Minimum Noise Subspace (MNS) algorithm [Hua97] showed that in a noiseless system, only M-1 vectors from the complete noise subspace is sufficient to identify the channel to within a scalar factor, where M denotes the number of multi-channels. The advantage of the MNS algorithm is that it is computationally less expensive than the Channel Subspace algorithm but the estimation variance is greater. Hence, the MNS algorithm offers a trade off between complexity and performance. Soon afterwards, the MNS algorithm was extended to multiple input multiple output (MIMO) systems [Abed-Meraim97b] where it was also shown that the performance of the algorithm was not as superior in performance to the algorithm that uses the full noise subspace to perform the task of channel estimation.

Novel work by Zeng and Tong in [Zeng97a] lead to the development of analytical expressions for the performance of statistical and deterministic subspace methods. The following important limitations of these two subspace methods were found:

- For moment-based statistical methods, the condition number of the Jacobian matrix limits the performance of the channel estimation algorithms. The Jacobian matrix becomes ill-conditioned when any of the multi-channels share common conjugate reciprocal zeros. When the Jacobian matrix is ill-conditioned, the channel estimation algorithm does not always converge to the global minimum because the search space given to the algorithm is so large that it covers the local minimums as well.

- The condition number of the channel matrix limits the performance of deterministic subspace algorithms such as the Channel Subspace algorithm. The channel matrix becomes ill-conditioned when any of the multi-channels share common zeros. The exact value of the channel order is needed for the algorithm to give reliable channel estimates. If an overestimate of the channel order is used, the Channel Subspace algorithm provides channel estimates that are inconsistent.

Noting the limitations of the deterministic and statistical algorithms, the Joint Optimisation with Subspace Constraints (JOSC) algorithm [Zeng97b] was proposed to solve the limitations of the two types of subspace algorithms. The characteristics of the JOSC algorithm are as follows:

- An overestimate of the channel order need only be known.
- The algorithm is robust with respect to ill-conditioned channels that share common zeros.
- The final optimisation equation involves a search of parameters in a low dimensional space that uses techniques of statistical algorithms to search for the global minimum that identifies the channel.

The JOSC algorithm uses two deterministic subspace constraints that are derived from the principle component analysis and the noise subspace to reduce the search space of the final statistical-based optimisation equation. By reducing the search space, the convergence of the JOSC algorithm to the global minimum is improved.

Recognising that in many communication applications, the pulse-shaping information of the transmitted waveform is often known, Schell *et al.* proposed a subchannel response matching approach [Schell94]. Ding used the same idea as Schell and presented a Modified Channel Subspace (MCS) algorithm [Ding95] that incorporated knowledge of the pulse-shaping filter into the Channel Subspace algorithm. However, the performance of the MCS algorithm is dependent on the condition number of the matrix that contains pulse-shaping information. The Parametric Subspace algorithm [Perros-Meilhac01] solved the condition number limitation of the MCS algorithm by presenting a unique delay estimation algorithm that also uses pulse-shaping information.

Figure 2-3 shows a map that summarises the algorithms that are related to the Channel Subspace algorithm. In Chapter 3 the Parametric Subspace algorithm is discussed in detail and a new algorithm is proposed that converts the Parametric Subspace algorithm from single-user system to a to multi-user DS/CDMA system.

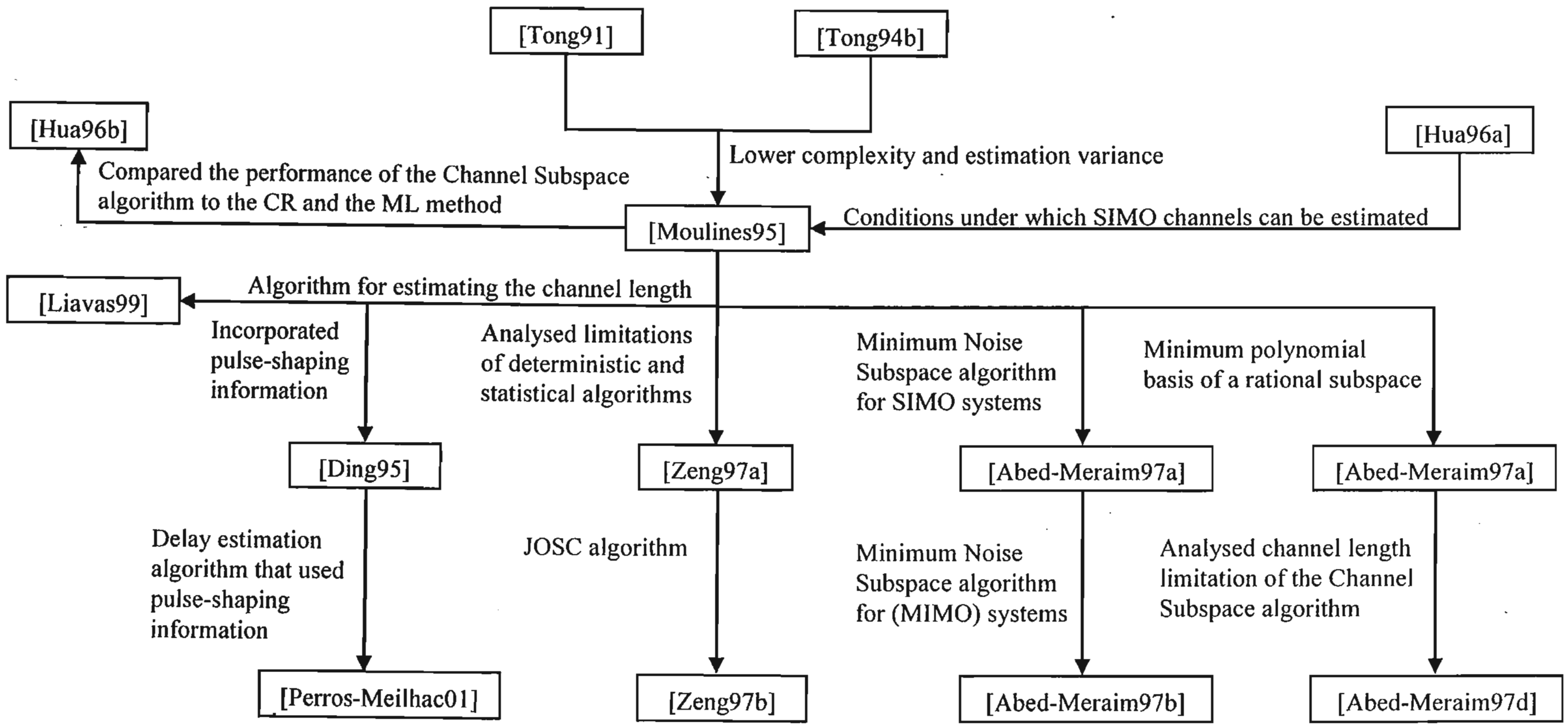

Figure 2-3 Map of channel estimation algorithms that are related to the Channel Subspace algorithm

2.5 Comparison of Deterministic Subspace-based Algorithms

A few other deterministic subspace-based algorithms that have been presented in the literature are the Cross Relation (CR) algorithm [Xu95] and the Two Step Maximum Likelihood (TSML) algorithm [Hua96b]. It was shown in [Zeng96] that the CR and the Channel Subspace algorithms are exactly the same when the number of multi-channels is equal to two. When the number of multi-channels is greater than two, the CR method only uses a portion of the complete noise subspace to estimate the unknown channel, whereas the Channel Subspace algorithm uses the complete noise subspace. The CR method was also shown to be the algorithm used in the first step of the TSML algorithm [Hua96b].

The performance of the TSML, CR and the Channel Subspace algorithms were compared in [Hua96b]. The following was found:

- When the channel matrix is well conditioned, the channel estimation variance of the TSML, CR and the Channel Subspace algorithms attains the Cramér-Rao Bound (CRB) at high signal-to-noise ratios (SNRs).
- For an ill-conditioned channel matrix, the TSML method outperforms the CR and the Channel Subspace methods. At a high SNR, the estimation variance of the TSML algorithm approaches the CRB.

2.6 Subspace-based Algorithms for DS/CDMA

Code-Division Multiple-Access (CDMA) is a technique whereby many users simultaneously access a communication channel. The users of the system are identified at the base station by their unique spreading code. The signal that is transmitted by any user consists of the user's data that modulates its spreading code, which in turn modulates a carrier. An example of such a modulation scheme is binary phase shift keying (BPSK). In a typical environment, the signal that is transmitted by a user is reflected by objects before it reaches the base station. Hence the receiver receives multiple copies of the same signal that are each delayed and attenuated by different amounts. The purpose of channel estimation is to find these unknown parameters to aid the recovery of the data transmitted by each user.

In the early practical implementation of DS/CDMA systems, such as IS-95, the computationally simple matched filter solved the task of estimating the multipath delays introduced by the channel. However, the simple matched filter was found to be non-resistant to multiple access interference [Simon85] and the optimal maximum likelihood solution [Verdú86] is too complex for practical purposes. Subspace-based methods are a desirable way of solving the delay and channel estimation problem because they offer near-far resistant algorithms that exhibit adequate performance with reasonable computational complexity.

Channel estimation algorithms are concerned with estimating the tap coefficients of the Finite Impulse Response (FIR) filter representing the channel. On the other hand, delay estimation algorithms obtain an estimate of the multipath delays introduced by the channel. Channel and delay estimation algorithms are also referred to as non-parametric and parametric methods respectively. The rest of this section discusses subspace-based delay and channel estimation algorithms that have been proposed in the literature.

The algorithm in [Ström96a] was the first subspace-based algorithm that estimated the multipath delays of a desired user in a DS/CDMA system. The algorithm was later modified for a multipath time-variant channel [Ström96b]. However, this algorithm for time-variant channels was not suitable for a channel that had several paths with time-variant path gains [Ström00]. A subspace method that solved this problem was proposed by Bensley and Aazhang in [Bensley96]. The limitation of the Bensley/Aazhang algorithm was the potentially complicated final optimisation equation, which was later solved to some extent by the same authors in [Bensley98].

A subspace channel estimation algorithm that extended the Channel Subspace algorithm from a single-user system to a multi-user DS/CDMA system can be found in the paper by Torlak and Xu [Torlak97]. One of the impractical assumptions made by the Torlak/Xu algorithm was the algorithm's knowledge of the exact channel length. A channel length estimation algorithm for the Torlak/Xu method was proposed in [Tugnait02]. Other subspace algorithms that have been developed to be computationally less expensive than the Torlak/Xu algorithm can be found in [Aktas00] and [Pi01].

2.7 Summary

Subspace techniques are a desirable way of solving the task of channel or delay estimation since they offer algorithms that are near-far resistant with sub optimal complexity. Subspace-based channel estimation algorithms can be classified into two categories, depending on whether or not they use statistical information. Deterministic methods that do not use statistical information were discussed in this chapter. From the literature it was seen that since the Channel Subspace algorithm was proposed by Moulines *et al.* in 1995, the research in deterministic subspace-based channel estimation algorithms was focused on improving the performance of the Channel Subspace algorithm. This chapter explained the development of many new improved channel estimation algorithms that were based on the Channel Subspace algorithm. Lastly, subspace-based channel estimation algorithms for DS/CDMA systems were discussed.

Chapter 3

A New Subspace-based Channel Estimation Algorithm for DS/CDMA Systems

3.1 Introduction

Ever since Tong demonstrated the feasibility of identifying non-minimum phase channels using second-order statistics (SOS) [Tong91], there has been considerable research in the area of blind identification of multiple FIR channels in SIMO systems. Over the last twelve years many blind SOS-SIMO channel estimation algorithms have been developed [Tong98], namely the Channel Subspace algorithm [Moulines95]. It has been shown that when the Channel Subspace algorithm takes into account knowledge of the pulse-shaping filter, it reduces the overall channel estimation variance [Ding95] and provides reliable channel estimates for a system with an overestimated channel length [Perros-Meilhac01], [Perros-Meilhac99].

In 1997, the Channel Subspace channel estimation algorithm was extended from a single-user system to a multi-user DS/CDMA system [Torlak97]. However, this multi-user method sampled the received signal at the chip rate, making it impossible to exploit knowledge of the pulse-shaping filter in the channel estimation algorithm. In this chapter a new subspace-based channel estimation algorithm is derived by extending the algorithm in [Perros-Meilhac01] from a single-user system to a multi-user DS/CDMA system. In the proposed system, the received signal is oversampled with respect to the chip rate and knowledge of the pulse-shaping filter is incorporated into the channel estimation algorithm with the aim of lowering the variance of the channel estimates compared to the algorithm that does not use information of the pulse-shaping filter.

The system model for a single-user system and the assumptions that are made about the system in [Perros-Meilhac01] are described in Section 3.2. Subspace algorithms use linear algebraic concepts to estimate the channel. These concepts are discussed in Section 3.3. The Channel Subspace algorithm and the way the algorithm is extended to incorporate pulse-shaping information to form the Parametric Subspace algorithm is described in Sections 3.4 and 3.5 respectively. A new subspace-based channel estimation algorithm for DS/CDMA systems that incorporates pulse-shaping information is derived and the performance of the new algorithm is analysed and confirmed via simulations in Section 3.6.

3.2 Single-user System Model and Assumptions

Consider a continuous-time single-user communication system with linear modulation over a linear time-invariant channel. The baseband representation of the continuous signal that is received on q antennas (Figure 3-1) may be expressed as:

$$\mathbf{x}(t) = \sum_{l=-\infty}^{\infty} \mathbf{h}(t - lT)s(l) + \mathbf{e}(t) \tag{3.1}$$

where $\{s(l)\}$ are the transmitted symbols and T is the symbol period. The observation vector $\mathbf{x}(t)$, the channel impulse response $\mathbf{h}(t)$ and the noise vector $\mathbf{e}(t)$ are all $q \times 1$ column vectors that are defined as :

$$\begin{aligned} \mathbf{x}(t) &= [x_1(t), \quad \cdots \quad , x_q(t)]^T \\ \mathbf{h}(t) &= [h_1(t), \quad \cdots \quad , h_q(t)]^T \\ \mathbf{e}(t) &= [e_1(t), \quad \cdots \quad , e_q(t)]^T \end{aligned}$$

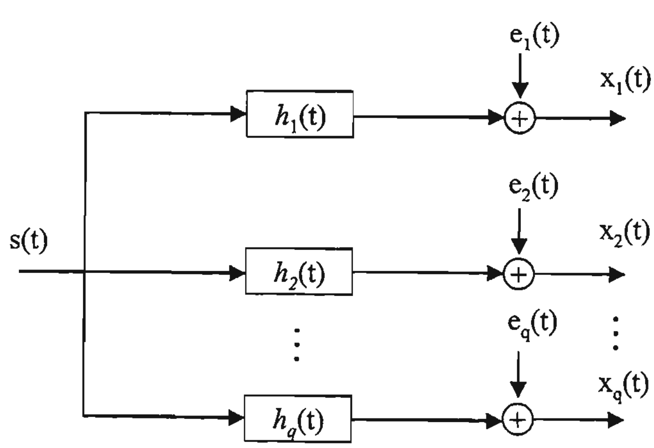

Figure 3-1 System model of a single-user system with one transmit and q receive antennas

The channel impulse response vector $\mathbf{h}(t)$ is a function of the pulse-shaping filter $g(t)$ and the propagation channel associated with each antenna. The channel is assumed to be the sum of a small number of paths. Each path is characterised by a delay and a spatial attenuation factor. The assumptions that are made on the multipath channel [Perros-Meilhac01] are as follows:

A1) The number of multipaths and the delay spread are known.

A2) The signal is narrowband with respect to the array aperture.

A3) The pulse-shaping filter $g(t)$ is known and has finite support $g(t)=0$ for $t \notin [0, L_g T]$, where L_g is the truncated length of the filter in symbol periods.

A4) Doppler shift and residual carriers are neglected. Hence the channel is assumed to be stationary over the observation period.

Under the above assumptions, the channel impulse response vector can be expressed as:

$$\mathbf{h}(t) = \sum_{k=1}^{d} \mathbf{a}_k g(t-\tau_k) \qquad \tau_1 < \cdots < \tau_d \tag{3.2}$$

where d is the number of multipaths,

τ_k is the delay of the k^{th} path,

$\mathbf{a}_k$ is a $q \times 1$ vector which is the spatial signature associated with the k^{th} path.

The spatial signature of the k^{th} path $\mathbf{a}_k$ is dependent on the array response $\mathbf{a}_k(\theta)$ to a point source for direction θ_k and the fading factor α_k:

$$\mathbf{a}_k = \alpha_k \mathbf{a}(\theta_k) \tag{3.3}$$

To mathematically formulate the observation vector $\mathbf{x}(t)$ that is sampled at the rate of T/p, where p is the oversampling factor, the following equations are defined for $1 \le i \le p$:

$$\mathbf{h}^{(i)}(n) = \mathbf{h}((i-1)T/p + nT)$$
$$\mathbf{x}^{(i)}(n) = \mathbf{x}((i-1)T/p + nT)$$

The spatial and temporal diversity can be combined to form a SIMO linear system with $r = pq$ outputs:

$$\mathbf{h}(n) = [\mathbf{h}^{(1)}(n)^T, \quad \cdots \quad , \mathbf{h}^{(p)}(n)^T]^T_{r \times 1}$$
$$\mathbf{x}(n) = [\mathbf{x}^{(1)}(n)^T, \quad \cdots \quad , \mathbf{x}^{(p)}(n)^T]^T_{r \times 1}$$
$$\mathbf{e}(n) = [\mathbf{e}^{(1)}(n)^T, \quad \cdots \quad , \mathbf{e}^{(p)}(n)^T]^T_{r \times 1}$$

Under the assumptions A1 and A3, the channel impulse response can be considered to be causal with duration LT, where $L = L_g + \lceil (\tau_d - \tau_1)/T \rceil$ [Perros-Meilhac01]. By defining

$$\mathbf{H} = [\mathbf{h}(0), \quad \cdots \quad , \mathbf{h}(L)]_{r \times (L+1)}$$

the oversampled observation vector from q antennas for one symbol duration may be expressed as :

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{e}(n) \tag{3.4}$$

where $\mathbf{s}(n) = [s(n), \quad \cdots \quad , s(n-L)]^T_{L+1 \times 1}$. By stacking K successive samples of the observation vector, the Toeplitz structure of the multi-channel system can be seen [Giannakis01]:

$$\underbrace{\begin{bmatrix} \mathbf{x}(\mathrm{n}) \\ \mathbf{x}(\mathrm{n-1}) \\ \vdots \\ \mathbf{x}(\mathrm{n-K+1}) \end{bmatrix}}_{\substack{\mathbf{x}_K(n) \\ Kr \times 1}} = \underbrace{\begin{bmatrix} \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(\mathrm{L}) & & & & \\ & \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(\mathrm{L}) & & & \\ & & & \ddots & & \ddots & & \\ & & & & \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(\mathrm{L}) \end{bmatrix}}_{\substack{\mathcal{H}_K \\ Kr \times K+L}} \underbrace{\begin{bmatrix} \mathrm{s}(\mathrm{n}) \\ \mathrm{s}(\mathrm{n}-1) \\ \vdots \\ \mathrm{s}(\mathrm{n-K-L+1}) \end{bmatrix}}_{\substack{\mathbf{s}_{K+L}(n) \\ K+L \times 1}} + \underbrace{\begin{bmatrix} \mathbf{e}(\mathrm{n}) \\ \mathbf{e}(\mathrm{n-1}) \\ \vdots \\ \mathbf{e}(\mathrm{n-K+1}) \end{bmatrix}}_{\substack{\mathbf{e}_K(n) \\ Kr \times 1}} \tag{3.5}$$

By writing the observation vector in terms of the Toeplitz structure of the channel matrix $\mathcal{H}_K$, an important structural relation that was developed in [Moulines95] can be used to formulate the simplified channel identification equation in the Channel Subspace algorithm. This is explained in detail in Section 3.4.

3.3 The Subspace Concept

Subspace-based algorithms [Moulines95], [Perros-Meilhac01] exploit the low rank of the Toeplitz matrix $\mathcal{H}_K$ to identify the unknown channel. The low rank structure of a matrix exists when the number of its rows is greater than the number of its columns. A matrix with low rank ensures that its left nullspace exists. This property is discussed below:

For a matrix **A** (m x n where $m > n$) with full column rank exhibiting a low rank structure:

- The rank equals to the number of its columns: rank = min(m,n) = n
- The dimension of the left nullspace equals to m – rank = $m - n > 0$

Since the dimension of the left nullspace is greater than zero it will exist.

It is the left nullspace of the Toeplitz channel matrix $\mathcal{H}_K$ that is used to identify the channel. This is described in great detail in Section 3.4. To explain the concept of how subspace-based algorithms work, the noiseless model of a stacked system model is studied. Ignoring the additive noise term in (3.5), it gives:

$$\mathbf{x}_K(n) = \mathcal{H}_K \mathbf{s}_{K+L}(n) \tag{3.6}$$

From (3.6) it can be seen that the received vector $\mathbf{x}_K(n)$ is an exact linear combination of the columns of $\mathcal{H}_K$. Thus, the received vector lies in the same subspace as the column space of $\mathcal{H}_K$. The column space of $\mathcal{H}_K$ is popularly called the signal subspace. The left nullspace of $\mathcal{H}_K$, that is orthogonal to the column space of $\mathcal{H}_K$, is popularly called the noise subspace. The Channel Subspace channel estimation algorithm uses the following algorithm to identify the channel:

1. The received vector $\mathbf{x}_K(n)$ is used to form a correlation matrix.
2. The correlation matrix is used to obtain an estimate of the noise subspace.
3. The noise subspace is used to form an optimisation equation that identifies the channel.

The algorithm given above describes the main concepts that are used in the Channel Subspace algorithm. The assumptions and equations that are developed to identify the channel are discussed in Section 3.4.

3.4 Identifying the Channel without Pulse-shaping Filter Information

The Channel Subspace algorithm was originally proposed without using any *a priori* knowledge of the pulse-shaping filter. This section describes the assumptions, concepts and equations that were derived to develop the Channel Subspace channel estimation algorithm.

3.4.1 Assumptions

Assuming the following:

- The data sequence is zero mean and $E\left\{\mathbf{s}_{K+L}(n)\mathbf{s}_{K+L}(n)^T\right\} = \mathbf{I}$.
- The noise is white with zero mean and $E\left\{\mathbf{e}_K(n)\mathbf{e}_K(n)^T\right\} = \sigma^2\mathbf{I}$.
- The noise and data sequences are statistically uncorrelated.

Then the correlation matrix of the observation vector for K bit durations can be written as:

$$\mathbf{R}_{xx}(0) = E\left\{\mathbf{x}_K(n)\mathbf{x}_K(n)^H\right\} = \mathcal{H}_K\mathcal{H}_K^H + \sigma^2\mathbf{I} \tag{3.7}$$

For the left nullspace of $\mathcal{H}_K$ to exist, the number of its rows must be greater than the number of columns:

$$\begin{aligned} Kr &> L + K \\ K &> \frac{L}{r-1} \end{aligned} \tag{3.8}$$

Thus, for the channel to be identifiable in the Channel Subspace algorithm, the stacking factor K needs to be chosen to satisfy (3.8) so that the left nullspace of $\mathcal{H}_K$ exists.

3.4.2 Eigenvectors of the Correlation Matrix

It has been shown that the eigenvectors of the correlation matrix that have eigenvalues equal to the noise variance correspond to the vectors that span the left nullspace of the Toeplitz channel matrix $\mathcal{H}_K$ [Giannakis01]. It is the vectors that span the nullspace of $\mathcal{H}_K$ that are used to form the channel identification equation [Moulines95].

The eigenvector decomposition (EVD) is a matrix factorisation tool that is used to compute all the eigenvectors and eigenvalues of a matrix. The EVD is performed on the correlation matrix to find all the eigenvectors that have eigenvalues equal to the noise variance. These eigenvectors correspond to the vectors that span the nullspace of $\mathcal{H}_K$. Performing the EVD operation on the correlation matrix:

$$EVD\{\mathbf{R}_{xx}(0)\} = [\mathbf{V}_s \quad \mathbf{V}_n]\begin{bmatrix} \Sigma_s & \\ & \Sigma_n \end{bmatrix}\begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \tag{3.9}$$

where $\mathbf{V}_s$ $(Kr \times K + L)$ is commonly called the signal subspace and it contains the eigenvectors that correspond to the $K + L$ largest eigenvalues of Σ_s.

$\mathbf{V}_n$ $(Kr \times Kr - K + L)$ is commonly called the noise subspace and it contains eigenvectors that correspond to the Kr - $K + L$ smallest eigenvalues of Σ_n.

The number of vectors that span the left nullspace of $\mathcal{H}_K$ gives the number of columns of $\mathbf{V}_n$. The vectors that span the estimated noise subspace are used by the Channel Subspace algorithm to identify the channel.

3.4.3 The Channel Subspace Algorithm

This section describes the mathematical formulation of the Channel Subspace algorithm. It is shown how the vectors that span the estimated noise subspace are used to identify the unknown channel.

Let $\mathbf{w}^{(k)}$ $(Kr+1)$ be the k^{th} vector of the nullspace of the $\mathcal{H}_K\mathcal{H}_K^H$ matrix. By multiplying both sides of (3.7) by $\mathbf{w}^{(k)}$ the following expressions can be formulated for a noiseless system:

$$\mathbf{R}_{xx}(0)\mathbf{w}^{(k)} = \mathcal{H}_K\mathcal{H}_K^H\mathbf{w}^{(k)} \tag{3.10}$$

$$\mathcal{H}_K\mathcal{H}_K^H\mathbf{w}^{(k)} = 0 \tag{3.11}$$

If the columns of $\mathcal{H}_K$ are linearly independent, (3.11) can be written as:

$$\mathcal{H}_K^H \mathbf{w}^{(k)} = 0 \tag{3.12}$$

$$\Rightarrow \left[\mathbf{w}^{(k)}\right]^H \mathcal{H}_K = 0 \tag{3.13}$$

In [Moulines95] an important structural relation is developed that is crucial in the formulation of the final channel identifying equation. Partitioning the noise subspace vector $\mathbf{w}^{(k)}$ into K subvectors each of length r, the following commutative identity holds:

$$\underbrace{\left[\left[\mathbf{w}_1^{(k)}\right]^H \left[\mathbf{w}_2^{(k)}\right]^H \cdots \left[\mathbf{w}_K^{(k)}\right]^H\right]}_{\left[\mathbf{w}^{(k)}\right]^H} \underbrace{\begin{bmatrix} \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(L) & & & \\ & \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(L) & & \\ & & \ddots & & & \ddots & \\ & & & \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(L) \end{bmatrix}}_{\mathcal{H}_K}$$

$$= \underbrace{\left[\mathbf{h}^H(1)\ \mathbf{h}^H(2)\ \cdots\ \mathbf{h}^H(L)\right]}_{\bar{\mathbf{h}}^H} \underbrace{\begin{bmatrix} \mathbf{w}_1^{(k)} & \mathbf{w}_2^{(k)} & \cdots & \mathbf{w}_K^{(k)} & & & \\ & \mathbf{w}_1^{(k)} & \mathbf{w}_2^{(k)} & \cdots & \mathbf{w}_K^{(k)} & & \\ & & \ddots & & & \ddots & \\ & & & \mathbf{w}_1^{(k)} & \mathbf{w}_2^{(k)} & \cdots & \mathbf{w}_K^{(k)} \end{bmatrix}}_{\mathcal{W}^{(k)}}. \tag{3.14}$$

But the dimension of the left nullspace of $\mathcal{H}_K$ or the noise subspace is equal to $Kr \times Kr - (K+L)$. Let the number of eigenvectors that span the noise subspace be z. Thus (3.14) can be written as:

$$\begin{bmatrix} \left[\mathbf{w}^{(1)}\right]^H \\ \left[\mathbf{w}^{(2)}\right]^H \\ \vdots \\ \left[\mathbf{w}^{(z)}\right]^H \end{bmatrix} \mathcal{H}_K = \bar{\mathbf{h}}^H \left[\mathcal{W}^{(1)} \quad \mathcal{W}^{(2)} \quad \cdots \quad \mathcal{W}^{(z)}\right] \tag{3.15}$$

The identity in Equation (3.15) is important to understanding the concept of the subspace channel identification algorithm. The expression on the right hand side of (3.15) states that the unknown channel $\bar{\mathbf{h}}$ lies in the left nullspace of the $\left[\mathcal{W}^{(1)} \quad \mathcal{W}^{(2)} \quad \cdots \quad \mathcal{W}^{(z)}\right]$ matrix. To state this in another way: if the eigenvectors that span the noise subspace are calculated and the matrix $\begin{bmatrix} \mathcal{W}^{(1)} & \mathcal{W}^{(2)} & \cdots & \mathcal{W}^{(z)} \end{bmatrix}$ formulated, the vector corresponding to the left nullspace of this matrix would yield a perfect estimate of the unknown channel $\vec{\mathbf{h}}$ in a noiseless system. However, in a practical system, noise is present and the channel is estimated by finding the eigenvector corresponding to the smallest normalised eigenvalue of $\mathbf{Q}$, under the constraint that $\|\vec{\mathbf{h}}\|=1$:

$$\hat{\vec{\mathbf{h}}} = \arg\min_{\|\mathbf{h}\|=1} \vec{\mathbf{h}}^H \underbrace{\sum_{k=1}^{z} \mathcal{W}^{(k)} \left[\mathcal{W}^{(k)}\right]^H}_{\mathbf{Q}} \vec{\mathbf{h}} \tag{3.16}$$

Since any scalar multiple of $\hat{\vec{\mathbf{h}}}$ would also lie in the noise subspace, any $\beta\hat{\vec{\mathbf{h}}}$ (β is a scalar) is also a solution. For this reason the unknown channel is only identifiable to within a scalar factor.

3.4.4 Summary of the Channel Subspace Algorithm

The Channel Subspace algorithm for a SIMO system that is described by the system model:

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{e}(n) \tag{3.4}$$

with q receive antennas and an oversampling factor of p, is summarised as follows:

1. Assume that the following information about the system is known:

    - The exact channel length (L)
    - The number of receive antennas (q)
    - The oversampling factor (p)

    Concatenate K successive observations of the received vector each of a bit period to form $\mathbf{x}_K(n)$:

$$\mathbf{x}_K(n) = \begin{bmatrix} \mathbf{x}(n) \\ \mathbf{x}(n-1) \\ \vdots \\ \mathbf{x}(n-K+1) \end{bmatrix} = \mathcal{H}_K \mathbf{s}_{K+L}(n) + \mathbf{e}_K(n) \tag{3.5}$$

Choose the stacking factor $K > \frac{L}{r-1}$ to ensure that the left nullspace of the Toeplitz channel matrix $\mathcal{H}_K$ exists.

2. Estimate the correlation matrix:

$$\hat{\mathbf{R}}_{xx}(0) = \frac{1}{N}\sum_{k=1}^{N} \mathbf{x}_K(k)\mathbf{x}_K^H(k) = [\mathbf{x}_K(1)\,\mathbf{x}_K(2)\ldots\mathbf{x}_K(N)]\begin{bmatrix}\mathbf{x}_K^H(1)\\ \mathbf{x}_K^H(2)\\ \vdots \\ \mathbf{x}_K^H(N)\end{bmatrix} \tag{3.17}$$

where N is the total bit duration of the received signal that is used to perform the task of channel estimation. The eigenvectors that span the noise subspace are obtained by:

- Performing the EVD on $\hat{\mathbf{R}}_{xx}$.
- Choosing the last $z = Kr - (L+K)$ eigenvectors, $\mathbf{w}^{(1)}\ \mathbf{w}^{(2)}\ \cdots\ \mathbf{w}^{(z)}$, that correspond to the smallest z eigenvalues of the correlation matrix.

3. Partition the k^{th} vector of the noise into K sub-vectors and form the Toeplitz matrix $\mathcal{W}^{(k)}$. Estimate the channel by finding the left nullspace of the matrix $\left[\mathcal{W}^{(1)}\ \ \mathcal{W}^{(2)}\ \ \cdots\ \ \mathcal{W}^{(z)}\right]$. Equivalently, the channel is estimated by finding the eigenvector corresponding to the smallest eigenvalue of $\mathbf{Q}$:

$$\hat{\bar{\mathbf{h}}} = \arg\min_{\|\bar{\mathbf{h}}\|=1} \bar{\mathbf{h}}^H \underbrace{\sum_{k=1}^{z} \mathcal{W}^{(k)}\left[\mathcal{W}^{(k)}\right]^H}_{\mathbf{Q}} \bar{\mathbf{h}} \tag{3.16}$$

A numerical example of the Channel Subspace algorithm is given in Appendix A.

3.5 Identifying the Channel using Pulse-shaping Information

The first algorithm that incorporated knowledge of the pulse-shaping filter in the Channel Subspace algorithm was presented in [Ding95]. The key to this breakthrough was noting that the unknown channel can be written in terms of the known pulse-shaping filter and the unknown spatial signature. The limitation of [Ding95] was that the algorithm assumed that the delays of a multipath channel are close together in time. When the multipath delays are separated far apart in time, the performance of the algorithm degrades drastically. This is because the condition number of the matrix that contains information of the pulse-shaping filter increases [Perros-Meilhac01], causing a loss in the number of reliable decimal digits of the estimated channel compared to when pulse-shaping information is not used.

The Parametric Subspace algorithm [Perros-Meilhac01] solved the increasing condition number limitation in [Ding95] by presenting a delay estimation algorithm. Information of the delay estimates are used to form a matrix that contains knowledge of the pulse-shaping filter in such a way, that the matrix's condition number does not increase as the separation of the multipath delay increases.

3.5.1 The Unknown Channel and the Known Pulse-shaping filter

That Parametric Subspace algorithm showed that by writing the oversampled channel as a function of the pulse-shaping filter, the Channel Subspace algorithm could easily be extended to include pulse-shaping information. This section describes how the vector form of the channel can be written in terms of the known pulse-shaping filter.

In [Perros-Meilhac01] the z-transform of the oversampled channel is given by:

$$\mathbf{h}_{T/p}(z) = \sum_{k=-\infty}^{\infty} \mathbf{h}(kT/p)z^{-k} \tag{3.18}$$

Defining $\mathbf{H}^{(i)}(z) = \sum_{k=-\infty}^{\infty} \mathbf{h}^{(i)}(k)z^{-k}$, (3.18) can be written as:

$$\mathbf{h}_{T/p}(z) = \sum_{i=1}^{p} \mathbf{H}^{(i)}(z^p)z^{-i} \tag{3.19}$$

From (3.2), $\mathbf{h}_{T/p}(z)$ can be expressed in terms of the known pulse-shaping filter as:

$$\mathbf{h}_{T/p}(z) = \sum_{k=1}^{d} \mathbf{a}_k g(\tau_k, z) \tag{3.20}$$

where $G(\tau,z) \triangleq \sum_{n=-\infty}^{\infty} g(nT/p-\tau)z^{-n}$ is the z-transform of the pulse-shaping filter. The pulse-shaping filter that is commonly used in communication systems is the raised cosine pulse. The time domain representation of the raised cosine pulse is given by:

$$g(t) = \frac{\cos(\beta\pi t/p)}{1-(2\beta t/p)^2}\frac{\sin(\pi t/p)}{(\pi t/p)} \tag{3.21}$$

where β is the roll-off factor. Equation (3.20) can be written in matrix form as follows:

$$\vec{\mathbf{h}} = \mathbf{G}_{L,d}(\boldsymbol{\tau})\mathbf{a} \tag{3.22}$$

where $\mathbf{a}_j = \begin{bmatrix} e^{-i\psi_1} & \cdots & e^{-i\psi_q} \end{bmatrix}_{q\times 1}^{H}$, $\psi_j = \pi(j-1)\sin(\theta_j)$, θ_j is the angle of arrival, for the j^{th} path of a linear uniform receive array and $\mathbf{a} = \begin{bmatrix} \mathbf{a}_1^T & \cdots & \mathbf{a}_d^T \end{bmatrix}_{qd\times 1}^{T}$, $\mathbf{G}_{L,d}(\boldsymbol{\tau}) = (\mathcal{G}(\boldsymbol{\tau}) \otimes \mathbf{I}_q)$, $\otimes$ is the Kronecker product, $\boldsymbol{\tau} = \begin{bmatrix} \tau_1 & \cdots & \tau_d \end{bmatrix}$ is the vector of the d multipath delays and $\mathcal{G}(\boldsymbol{\tau})$ is a $p(L+1)$ x d matrix that contains information of the pulse-shaping filter:

$$\mathcal{G}(\boldsymbol{\tau}) = \begin{pmatrix} g(0-\tau_1) & \cdots & g(0-\tau_d) \\ g\left(\frac{T}{p}-\tau_1\right) & \cdots & g\left(\frac{T}{p}-\tau_d\right) \\ \vdots & & \vdots \\ g\left((L+1)T-\frac{T}{p}-\tau_1\right) & \cdots & g\left((L+1)T-\frac{T}{p}-\tau_d\right) \end{pmatrix} \tag{3.23}$$

Equation (3.22) shows that the vector form of the unknown channel $\vec{\mathbf{h}}$ can be written in terms of the known pulse-shaping filter $\mathbf{G}_{L,d}(\boldsymbol{\tau})$ and the unknown spatial attenuation $\mathbf{a}$. This equation is used to develop the Parametric Subspace algorithm and the new channel estimation algorithm proposed in this chapter.

3.5.2 The Parametric Subspace Channel Estimation Algorithm

This section describes how the Parametric Subspace algorithm was developed by incorporating knowledge of the pulse-shaping filter in the Channel Subspace algorithm. The Parametric Subspace algorithm estimates the unknown part of the channel $\mathbf{a}$ that is given in (3.22). Recalling the channel identification equation of the Channel Subspace method:

$$\hat{\bar{\mathbf{h}}} = \arg \min_{\|\bar{\mathbf{h}}\|=1} \bar{\mathbf{h}}^H \underbrace{\sum_{k=1}^{z} \mathcal{W}^{(k)} \left[\mathcal{W}^{(k)}\right]^H}_{\mathbf{Q}} \bar{\mathbf{h}} \qquad (3.16)$$

Assuming that the exact channel order L and the number of multipaths d are known in the Parametric Subspace algorithm, the unknown part of the channel $\mathbf{a}$ is identifiable under the following conditions [Perros-Meilhac01]:

- The number of receive antennas are greater than or equal to the number of multipaths $(q \geq d)$.
- The angles of arrival for each path are distinct.

The Parametric Subspace method uses (3.22) and (3.16) to estimate the unknown part of the channel as follows

$$\hat{\mathbf{a}} = \arg \min_{\|\mathbf{a}\|=1} \mathbf{a}^H \mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau}) \mathbf{a} \qquad (3.24)$$

under the constraint that $\tau_i \neq \tau_j$ for $i \neq j$. The estimate of $\mathbf{a}$, for a fixed $\boldsymbol{\tau}$, is the normalised eigenvector corresponding to the minimum eigenvector of the matrix $\mathbf{a}^H \mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau}) \mathbf{a}$. The delay estimation algorithm for estimating all the multipath delays $\boldsymbol{\tau}$ in (3.24) is discussed in Chapter 4.

3.6 A New Subspace-based Channel Estimation Algorithm for Multi-user DS/CDMA Systems

There are several subspace-based channel estimation algorithms that have been developed for DS/CDMA systems, as discussed in Section 2.6. All these algorithms use the second order statistics of the received signal and the user's known spreading code to estimate the channel of the user. However, the limitation of all these subspace methods for DS/CDMA systems is that the received signal is sampled at the chip rate. This makes it impossible to exploit knowledge of the pulse-shaping filter in subspace-based channel estimation algorithms for DS/CDMA systems, which has been shown in single-user subspace-based channel estimation algorithms to decrease the overall estimation variance [Perros-Meilhac01], [Ding95].

In this section a new subspace-based channel estimation algorithm for DS/CDMA is derived. A system model for a DS/CDMA system is presented in Section 3.6.1 where the received signal is oversampled with respect to the chip rate. By oversampling the received signal, knowledge of the pulse-shaping filter is exploited in the proposed channel estimation algorithm. The proposed algorithm is an extension of the Parametric Subspace algorithm for single-user systems [Perros-Meilhac01] to multi-user DS/CDMA systems. The development of the new algorithm with respect to algorithms that are proposed in the literature is shown in Figure 3-2.

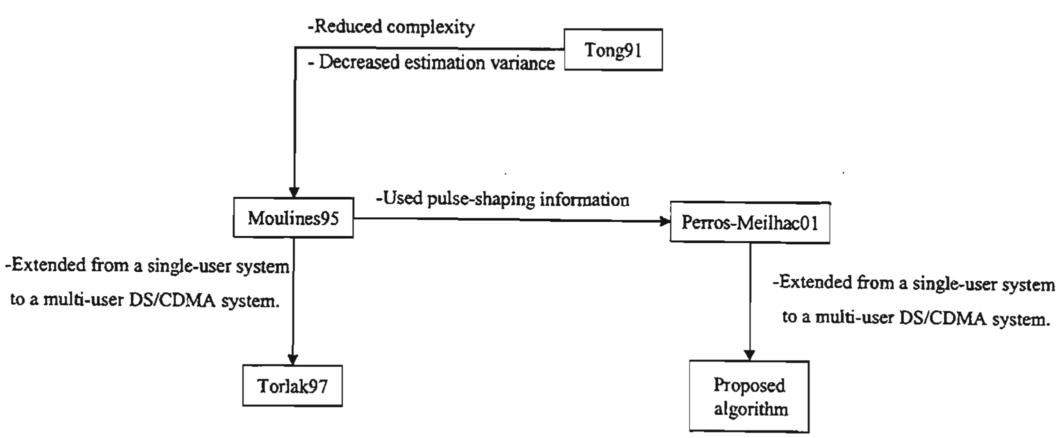

Figure 3-2 Development of the proposed channel estimation algorithm for DS/CDMA systems

3.6.1 Multi-user DS/CDMA System Model and Assumptions

In order to simplify notation, the contribution of noise to the received signal will initially be ignored. Noisy data is considered at the end of this section. The DS/CDMA system model that is presented in this section is similar to the multi-user data model in [Ghauri99]. Both models assume a system with a single transmit antenna, multiple receive antennas and oversampling of the received signal at each receive antenna. The only difference between the two system models is way the received signal is expressed. This difference is clearly explained after the complete system model is mathematically formulated.

The DS/CDMA system that is considered (Figure 3-3) assumes that P users transmit linearly modulated BPSK signals over a linear multipath channel with additive white Gaussian noise.

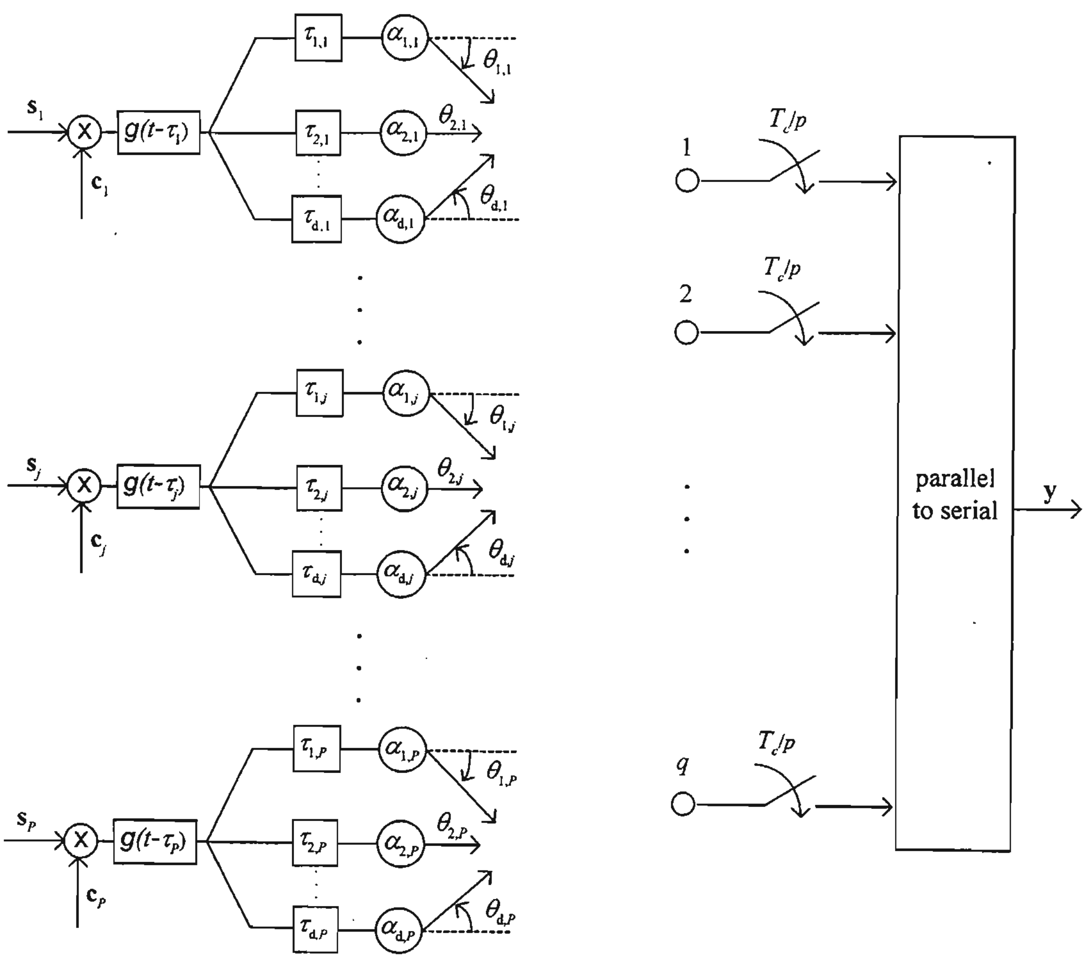

Figure 3-3 System model of the DS/CDMA system

The receiver is assumed to have q antennas in order to exploit the spatial diversity of the received signal. The noiseless continuous baseband signal that is received at the l^{th} antenna is given by:

$$y^l(t) = \sum_{j=1}^{P} \sum_{n} s_j(n) v_j^l(t - nT_s) \tag{3.25}$$

where $s_j(n)$ is the transmitted symbols from user j, T_s is the symbol period, $v_j^l(t)$ is the overall channel impulse response for the j^{th} user's signal at the l^{th} antenna which is given by the convolution of the spreading code $c_j(n)$ and actual channel $h_j^l(t)$ (assumed to be FIR) representing the multipath fading environment.

The overall channel impulse response for the j^{th} user can be expressed as:

$$v_j^l(t) = \sum_{n=0}^{L} h_j^l(nT_c) c_j(t - n - \tau_j) \tag{3.26}$$

where T_c is the chip duration, L is the channel length and τ_j is the delay of the j^{th} user's signal that arrives at the receiver. The symbol and chip duration are related by the processing gain L_c: $T_s = L_c T_c$. It is assumed that the delays of the j^{th} user's signal are an integer multiple of the chip period. Thus $\tau_j = k_j T_c$ where k_j is an integer.

In order to write (3.26) in vector form, it is assumed that the channel length is smaller than the processing gain $(L < L_c)$. By sampling the overall channel impulse response of the j^{th} user at a rate of T_c / p and concatenating the contribution from q receive antennas, the discrete vector of length $2pqL_c$ can be expressed as:

$$\mathbf{v}_j = [\mathbf{0}_{\text{I}} \quad v_j(0) \quad v_j(1) \quad \cdots \quad v_j\big((L_c + L - 1)pq - 1\big) \quad \mathbf{0}_{\text{II}}] \tag{3.27}$$

where $\mathbf{0}_{\text{I}}$ is a $1 \times k_j$ vector of zeros and $\mathbf{0}_{\text{II}}$ is a $1 \times (L_c - L)pq - k_j + 1$ vector of zeros. Equation (3.26) can be expressed in matrix form as:

$$\mathbf{v}_j = \underbrace{\left(\mathbf{c}_j \otimes \mathbf{I}_{pq}\right)}_{\mathbf{c}_{j,\mathbf{I}_{pq}}} \mathbf{h}_j \tag{3.28}$$

where $\mathbf{h}'_j(n) = \begin{bmatrix} h_j^1(n) & h_j^2(n) & \cdots & h_j^q(n) \end{bmatrix}_{q\times 1}^H$ is the contribution from all q receive antennas, $\mathbf{h}'_j = \begin{bmatrix} \left[\mathbf{h}'_j(0)\right]^H & \left[\mathbf{h}'_j(T_c/p)\right]^H & \cdots & \left[\mathbf{h}'_j\left((L+1)T_c - T_c/p\right)\right]^H \end{bmatrix}_{pq(L+1)}^H$ is the channel from q receive antennas that is sampled at the rate of T_c/p for a length of $(L+1)T_c$, $\mathbf{c}_j$ is a $2L_c \times L$ Toeplitz matrix whose first k_j rows are zero:

$$\mathbf{c}_j = \begin{pmatrix} 0 & 0 & & 0 \\ \vdots & 0 & & 0 \\ 0 & \vdots & & \\ c_j(0) & 0 & & \vdots \\ c_j(1) & c_j(0) & & \vdots \\ \vdots & c_j(1) & \ddots & 0 \\ c_j(L_c-1) & \vdots & \ddots & c_j(0) \\ 0 & c_j(L_c-1) & \ddots & c_j(1) \\ \vdots & 0 & \ddots & \vdots \\ & \vdots & & c_j(L_c-1) \\ & & & 0 \\ & & & \vdots \\ & & & 0 \end{pmatrix} \tag{3.29}$$

For the mathematical development of the system model, it is useful to break up the $\mathbf{v}_j$ vector into two vectors each of length pqL_c vectors as $\mathbf{v}_j = \begin{bmatrix} \mathbf{v}_j(1)^H & \mathbf{v}_j(2)^H \end{bmatrix}^H$. With this definition, the received vector due to the j^{th} user for a stacking factor of K bit durations can be written as:

$$\underbrace{\begin{bmatrix} \mathbf{y}_j(n) \\ \mathbf{y}_j(n-1) \\ \vdots \\ \mathbf{y}_j(n-K+1) \end{bmatrix}}_{\substack{\mathbf{y}_{j,K}(n) \\ pqKL_c \times 1}} = \underbrace{\begin{bmatrix} \mathbf{v}_j(2) & \mathbf{v}_j(1) & & & \\ & \mathbf{v}_j(2) & \mathbf{v}_j(1) & & \\ & & \ddots & \ddots & \\ & & & \mathbf{v}_j(2) & \mathbf{v}_j(1) \end{bmatrix}}_{\substack{\mathcal{V}_i \\ pqKL_c \times (K+1)}} \underbrace{\begin{bmatrix} s(n) \\ s(n-1) \\ \vdots \\ s(n-K) \end{bmatrix}}_{\substack{\mathbf{s}_i \\ (K+1)\times 1}} \tag{3.30}$$

where $\mathbf{y}'_j(n) = \begin{bmatrix} y^1(n) & \cdots & y^q(n) \end{bmatrix}^H_{q\times 1}$ is the received signal from all q receive antennas for the same time instant n,

$\mathbf{y}_j(n) = \begin{bmatrix} \left[\mathbf{y}'_j(n)\right]^H & \left[\mathbf{y}'_j(n-\frac{T_c}{p})\right]^H & \cdots & \left[\mathbf{y}'_j(n-\frac{(L_c-1)T_c}{p})\right]^H \end{bmatrix}^H_{pqL_c\times 1}$ is the received signal from all q receive antennas that is sampled at T_c/p for a duration of L_cT_c.

Partitioning $\mathbf{c}_{j,I_{pq}}$ $(2pqL_c \times pqL)$ (3.28) into two matrices each of length $pqL_c \times pqL$ as $\mathbf{c}_{j,I_{pq}} = \begin{bmatrix} \mathbf{c}_{j,I_{pq}}(1)^H & \mathbf{c}_{j,I_{pq}}(2)^H \end{bmatrix}^H$, the received vector due to the j^{th} user's signal can be expressed in terms of the spreading code, the channel matrix and the data vector of the j^{th} user:

$$\underbrace{\begin{bmatrix} \mathbf{y}_j(n) \\ \mathbf{y}_j(n-1) \\ \vdots \\ \mathbf{y}_j(n-K+1) \end{bmatrix}}_{\substack{\mathbf{y}_{j,K}(n) \\ pqKL_c\times 1}} = \underbrace{\begin{bmatrix} \mathbf{c}_{j,I_{pq}}(2) & \mathbf{c}_{j,I_{pq}}(1) & & & \\ & \mathbf{c}_{j,I_{pq}}(2) & \mathbf{c}_{j,I_{pq}}(1) & & \\ & & \ddots & \ddots & \\ & & & \mathbf{c}_{j,I_{pq}}(2) & \mathbf{c}_{j,I_{pq}}(1) \end{bmatrix}}_{\substack{C_j \\ pqKL_c \times pqL(K+1)}} \underbrace{\begin{bmatrix} \mathbf{h}_j & & & \\ & \mathbf{h}_j & & \\ & & \ddots & \\ & & & \mathbf{h}_j \end{bmatrix}}_{\substack{\mathcal{H}_j \\ pqL(K+1)\times(K+1)}} \underbrace{\begin{bmatrix} s_j(n) \\ s_j(n-1) \\ \vdots \\ s_j(n-K) \end{bmatrix}}_{\substack{\mathbf{s}_j(n) \\ (K+1)\times 1}} \tag{3.31}$$

Including the contribution of additive white Gaussian noise $\mathbf{E}(n)$ $(pqKL_c \times 1)$ and the signal from P users to the received signal, the observation vector is given by:

$$\mathbf{y}_K(n) = \underbrace{\begin{bmatrix} \mathcal{V}_1 & \cdots & \mathcal{V}_P \end{bmatrix}}_{\substack{\mathbf{V}_P \\ pqKL_c \times P(K+1)}} \begin{bmatrix} \mathbf{s}_1(n) \\ \vdots \\ \mathbf{s}_P(n) \end{bmatrix} + \mathbf{E}(n) \tag{3.32}$$

The difference between the system model presented in this section and the one in [Ghauri99] is the way the received signal is expressed. In [Ghauri99] the oversampled signal from each antenna is concatenated to form the received vector. In the system model that is presented in this section, the signal from all the receive antennas at the same time instant are concatenated to form an 'antenna vector'. The 'antenna vector' is then obtained for different time instances which are successively concatenated to form the receive vector.

3.6.2 Derivation of the New Channel Estimation Algorithm for DS/CDMA Systems

Subspace-based channel estimation algorithms [Moulines95], [Torlak97], [Perros-Meilhac01] use the concepts described in Section 3.3 to estimate the channel. The new subspace algorithm for DS/CDMA systems that is derived in this section is no different. The proposed algorithm uses the orthogonal property of the signal and noise subspace to estimate the channel. It also uses the commutative property [Moulines95] between a Toeplitz matrix and a vector to simplify the formula that identifies the unknown channel.

The derivation of the channel estimation algorithm that is presented in this section is similar to the algorithm in [Torlak97]. Both algorithms assume a DS/CDMA system with multiple receive antennas. The difference between the two system models is that the new algorithm considers a system where the received signal at each receive antenna is oversampled with respect to the chip rate. By oversampling the received signal, it gives the new algorithm the advantage of exploiting information of the pulse-shaping filter in the channel estimation algorithm, which has been shown in single-user systems to decrease the estimation variance [Ding95], [Perros-Meilhac01], [Schell94], [Cedervall97].

3.6.2.1 Assumptions

In the same way that assumptions were made about the single-user system model in Section 3.4.1, so that the Channel Subspace algorithm could be derived, similar assumptions are made about the DS/CDMA system model for the derivation of the proposed channel estimation algorithm. The following are assumed:

- The data sequence of the j^{th} user has zero mean and $E\left\{\mathbf{s}_j(n)\mathbf{s}_j(n)^T\right\} = \mathbf{I}$.
- The noise is white with zero mean and $E\left\{\mathbf{e}(n)\mathbf{e}(n)^T\right\} = \sigma^2\mathbf{I}$.
- The noise and data sequences are statistically uncorrelated.

Under the above assumptions, the correlation matrix of the received vector over K bit durations can be written as:

$$\mathbf{R}_{xx}(0) = E\left\{\mathbf{y}_K(n)\mathbf{y}_K(n)^H\right\} = \mathbf{V}_P\mathbf{V}_P^H + \sigma^2\mathbf{I} \tag{3.33}$$

The correlation matrix in (3.33) is used in the next section to derive the proposed channel estimation algorithm.

It was shown in (3.24) that the Parametric Subspace algorithm for single-user systems [Perros-Meilhac01] uses the vectors that span the estimated noise subspace to estimate the channel. For the noise subspace to exist, certain constraints are made on the stacking factor (3.8). The proposed channel estimation algorithm also requires the noise subspace to be estimated. The vectors that span the estimated noise subspace are used to identify the channel. The constraint that was made on the stacking factor for the noise subspace to exist in the DS/CDMA system model that is presented in Section 3.6.1 is explained below.

In the single-user system model that is described in Section 3.2, the received signal is given by:

$$\mathbf{x}_K(n) = \mathcal{H}_K \mathbf{s}_{K+L}(n) + \mathbf{e}_K(n) \tag{3.34}$$

For the noise subspace to exist, the number of rows of the channel matrix $\mathcal{H}_K$ needs to be greater than the number of columns. The number of vectors that span the noise subspace in a single-user system is given by the number of rows of $\mathcal{H}_K$ $(Kr \times K + L)$ subtracted by the number of columns: $Kr - K - L$. In the multi-user DS/CDMA system that was described in Section 3.6.1, the received signal is given by:

$$\mathbf{y}_K(n) = \mathbf{V}_P \begin{bmatrix} \mathbf{s}_1(n) \\ \vdots \\ \mathbf{s}_P(n) \end{bmatrix} + \mathbf{E}(n) \tag{3.35}$$

Similar to the single-user system, the number of vectors that span the noise subspace in a DS/CDMA system z_m is given by the number of rows of $\mathbf{V}_P$ $(pqKL_c \times P(K+1))$ subtracted by the number of its columns.

$$z_m = pqKL_c - P(K+1) \tag{3.36}$$

Hence, for the noise subspace to exist, the following constraint on the stacking factor K needs to be satisfied:

$$\begin{aligned} &pqKL_c > P(K+1) \\ &K > \frac{P}{pqL_c - P} \end{aligned} \tag{3.37}$$

3.6.2.2 Derivation of the Channel Identification Equation

Assuming that all the conditions in Section 3.6.2.1 are satisfied, the first step in the proposed channel estimation algorithm is to find an estimate of the signal and noise subspace. This is obtained by performing the EVD to the estimated correlation matrix $\hat{\mathbf{R}}_{yy} = \mathbf{y}_K(n)\mathbf{y}_K(n)^H$:

$$EVD(\hat{\mathbf{R}}_{yy}) = \begin{bmatrix} \hat{\mathbf{V}}_s & \hat{\mathbf{V}}_n \end{bmatrix} \begin{bmatrix} \Sigma_s & \\ & \Sigma_n \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_s^H \\ \hat{\mathbf{V}}_n^H \end{bmatrix} \tag{3.38}$$

where $\hat{\mathbf{V}}_s$ $\left(pqKL_c \times P(K+1)\right)$ is the estimated signal subspace. It contains the eigenvectors that correspond to the $P(K+1)$ largest eigenvalues of Σ_s.

$\hat{\mathbf{V}}_n$ $\left(pqKL_c - \left(pqKL_c - P(K+1)\right)\right)$ is the estimated noise subspace. It contains the eigenvectors that correspond to the $pqKL_c - P(K+1)$ smallest eigenvalues of Σ_n.

The proposed channel estimation algorithm that is derived follows the same concepts as the derivation of the Channel Subspace algorithm described in Section 3.4.3. Let $\mathbf{w}^{(k)}$ $\left(pqKL_c \times 1\right)$ be the k^{th} vector that spans the nullspace of the $\mathbf{V}_P\mathbf{V}_P^H$ matrix:

$$\mathbf{V}_P\mathbf{V}_P^H\mathbf{w}^{(k)} = 0 \tag{3.39}$$

Assuming that the columns of $\mathbf{V}_P$ are linearly independent, (3.39) can be written as:

$$\mathbf{V}_P^H\mathbf{w}^{(k)} = 0 \tag{3.40}$$

$$\Rightarrow \left[\mathbf{w}^{(k)}\right]^H \mathbf{V}_P = 0 \tag{3.41}$$

For the DS/CDMA system without noise, multiplying $\mathbf{w}^{(k)}$ to both sides of (3.33) gives:

$$\mathbf{R}_{yy}(0)\mathbf{w}^{(k)} = \mathbf{V}_P\mathbf{V}_P^H\mathbf{w}^{(k)} \tag{3.42}$$

$$\mathbf{R}_{yy}(0)\mathbf{w}^{(k)} = 0 \tag{3.43}$$

From (3.41) and (3.43) it can be seen that the $\mathbf{w}^{(k)}$ is also the vector that spans the left nullspace of $\mathbf{V}_P$ and the nullspace of the correlation matrix, which is also called the noise subspace. Hence $\mathbf{w}^{(k)}$ is estimated from selecting the k^{th} column vector of the estimated noise subspace matrix $\mathbf{V}_n$.

Equation (3.41) describes the relationship between the k^{th} vector from the noise subspace and the overall channel matrix from all the users. The overall channel matrix $\mathbf{V}_P$ does not exhibit a Toeplitz structure and the commutative identity involving a row vector and a Toeplitz matrix in [Moulines95] cannot directly be used to help identify the channel in a simplified way. The key to being able to use the commutative identity is noting that the matrices that make up the overall channel matrix $\mathbf{V}_P$ does exhibit a Toeplitz structure:

$$\mathbf{V}_P = \begin{bmatrix} \mathcal{V}_1 & \cdots & \mathcal{V}_P \end{bmatrix} \tag{3.44}$$

Since the row vector $\mathbf{w}^{(k)}$ is orthogonal to the columns of the overall channel matrix of all the users $\mathbf{V}_P$, it follows that $\mathbf{w}^{(k)}$ would also be orthogonal to the j^{th} user's overall channel matrix $\mathcal{V}_j$:

$$\left[\mathbf{w}^{(k)}\right]^H \mathcal{V}_j = 0 \tag{3.45}$$

$$\Rightarrow \quad \mathcal{V}_j^H \mathbf{w}^{(k)} = 0 \tag{3.46}$$

The advantage of (3.45) compared to (3.41) is that the channel matrix $\mathcal{V}_j$ does have a Toeplitz structure. Thus, the commutative identity in [Moulines95] can be used to simplify the channel identification equation for the j^{th} user. Partitioning $\mathbf{w}^{(k)}$ into K subvectors each of length pqL_c the following commutative identity holds:

$$\underbrace{\begin{bmatrix} \mathbf{v}_j^H(2) & & & \\ \mathbf{v}_j^H(1) & \mathbf{v}_j^H(2) & & \\ & \ddots & \ddots & \\ & & \mathbf{v}_j^H(1) & \mathbf{v}_j^H(2) \\ & & & \mathbf{v}_j^H(1) \end{bmatrix}}_{\mathcal{V}_j^H} \underbrace{\begin{bmatrix} \mathbf{w}_1^{(k)} \\ \mathbf{w}_2^{(k)} \\ \vdots \\ \mathbf{w}_K^{(k)} \end{bmatrix}}_{\mathbf{w}^{(k)}} = \underbrace{\begin{bmatrix} \left[\mathbf{w}_1^{(k)}\right]^H & & \\ \left[\mathbf{w}_2^{(k)}\right]^H & \left[\mathbf{w}_1^{(k)}\right]^H & \\ \vdots & \left[\mathbf{w}_2^{(k)}\right]^H & \ddots \\ \left[\mathbf{w}_K^{(k)}\right]^H & \vdots & \left[\mathbf{w}_1^{(k)}\right]^H \\ & \left[\mathbf{w}_K^{(k)}\right]^H & \left[\mathbf{w}_2^{(k)}\right]^H \\ & \ddots & \vdots \\ & & \left[\mathbf{w}_K^{(k)}\right]^H \end{bmatrix}}_{\left[\mathcal{W}^{(k)}\right]^H} \underbrace{\begin{bmatrix} \mathbf{v}_j(1) \\ \mathbf{v}_j(2) \end{bmatrix}}_{\mathbf{v}_j} \tag{3.47}$$

Combining (3.47) and (3.46) gives:

$$\left[\mathcal{W}^{(k)}\right]^H \mathbf{v}_j = 0 \tag{3.48}$$

For the DS/CDMA system model that is considered in this section, the number of vectors that span the noise subspace is given by Equation (3.36). Therefore (3.48) becomes:

$$\left[\mathcal{W}^{(1)} \quad \cdots \quad \mathcal{W}^{(z_m)}\right]^H \mathbf{v}_j = 0 \tag{3.49}$$

Performing the Hermitian operation of both sides of Equation (3.49):

$$\mathbf{v}_j^H \left[\mathcal{W}^{(1)} \quad \cdots \quad \mathcal{W}^{(z_m)}\right] = 0 \tag{3.50}$$

Substituting (3.28) into (3.50):

$$\mathbf{h}_j^H \underbrace{\left[\mathbf{c}_{j,I_{pq}}\right]^H \left[\mathcal{W}^{(1)} \quad \cdots \quad \mathcal{W}^{(z_m)}\right]}_{\mathbf{Q}_{tx,j}^H} = 0 \tag{3.51}$$

From (3.51) it can be clearly seen that in a noiseless system the channel of the j^{th} user lies in the left nullspace of the $\mathbf{Q}_{tx,j}^H$ matrix. For practical DS/CDMA systems additive white Gaussian noise is present and the channel of the j^{th} user is estimated by finding the normalised eigenvector corresponding to the smallest eigenvalue of $\mathbf{Q}_{tx,j}^H \mathbf{Q}_{tx,j}$

$$\tilde{\mathbf{h}}_j = \underset{\|\mathbf{h}_j = 1\|}{\arg\min}\ \mathbf{h}_j^H \left[\mathbf{c}_{j,I_{pq}}\right]^H \left[\sum_{k=1}^{z_m} \mathcal{W}^{(k)} \left[\mathcal{W}^{(k)}\right]^H\right] \mathbf{c}_{j,I_{pq}} \mathbf{h}_j \tag{3.52}$$

However, the channel that is estimated in (3.52) has a phase ambiguity that is present in all blind channel estimation algorithms. Equations (3.53) and (3.54) are used to correct the phase ambiguity of the estimated channel.

$$\theta_{j,h} = phase\left(\tilde{\mathbf{h}}_j^H \mathbf{h}_j\right) \tag{3.53}$$

$$\hat{\mathbf{h}}_j = e^{i\theta_{j,h}} \tilde{\mathbf{h}}_j \tag{3.54}$$

The channel identification equation in (3.52) does not use information of the pulse-shaping filter and it becomes the same optimisation equation given in [Torlak97] for an oversampling factor p of one. The channel estimation algorithm that does not use pulse-shaping information is easily extended to exploit knowledge of the pulse-shaping filter by using (3.55) that relates the unknown channel of the j^{th} user to the known pulse-shaping filter:

$$\mathbf{h}_j = \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\mathbf{a}_j \tag{3.55}$$

Therefore (3.52) and (3.55) can be combined to develop an algorithm that only estimates the unknown part of the channel for the j^{th} user by finding the eigenvector corresponding to the smallest eigenvalue of the matrix $\left[\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right]^H \mathbf{Q}_{tx,j}^H \mathbf{Q}_{tx,j} \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})$:

$$\tilde{\mathbf{a}}_j = \arg \min_{\|\mathbf{a}_j\|=1} \mathbf{a}_j^H \left[\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right]^H \underbrace{\left[\mathbf{c}_{j,\mathbf{1}_{pq}}\right]^H \left[\sum_{k=1}^{z_m} \mathcal{W}^{(k)}\left[\mathcal{W}^{(k)}\right]^H\right] \mathbf{c}_{j,\mathbf{1}_{pq}}}_{\mathbf{Q}_{tx,j}^H \mathbf{Q}_{tx,j}} \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\mathbf{a}_j \tag{3.56}$$

Equations (3.57) and (3.58) are used to correct the phase ambiguity present in (3.56).

$$\theta_{j,\mathbf{a}} = phase\left(\tilde{\mathbf{a}}_j^H \mathbf{a}_j\right) \tag{3.57}$$

$$\hat{\mathbf{a}}_j = e^{i\theta_{j,\mathbf{a}}}\tilde{\mathbf{a}}_j \tag{3.58}$$

Assuming that all users have the same number of multipaths and channel order. In typical urban channels, the number of multipaths d is less than the channel order L.

$$d < L \tag{3.59}$$

Under the above assumption, there is a complexity reduction advantage in estimating the unknown part of the channel $\mathbf{a}_j$ $(qd \times 1)$ compared to the actual channel $\mathbf{h}_j$ $(q(L+1)\times 1)$ because fewer parameters are estimated. The EVD is considered to be the most computationally complex operation to perform in subspace algorithms. The complexity of performing the EVD operation on a matrix is dependant on the size of the matrix. The new channel estimation algorithm offers a complexity reduction when performing the last EVD operation when the Torlak/Xu algorithm that does not used pulse-shaping information. This can be seen by comparing the dimensions of the final matrix of the Torlak/Xu algorithm that is used to estimate the actual channel $\mathbf{h}_j$ to the final matrix of the proposed algorithm that is used to estimate the spatial attenuation $\mathbf{a}_j$ of the j^{th} user:

- Estimating $\mathbf{h}_j$ (3.52): $\left[\mathbf{c}_{j,J_{pq}}\right]^H \left[\sum_{k=1}^{z_m} \mathcal{W}^{(k)} \left[\mathcal{W}^{(k)}\right]^H\right] \mathbf{c}_{j,J_{pq}} = q(L+1) \times q(L+1)$
- Estimating $\mathbf{a}_j$ (3.56): $\left[\mathbf{G}_{L,d}^j(\boldsymbol{\tau})\right]^H \mathbf{Q}_{tx,j}^H \mathbf{Q}_{tx,j} \mathbf{G}_{L,d}^j(\boldsymbol{\tau}) = qd \times qd$

Under the assumption made in (3.59), it is clearly seen that the EVD operation that is performed on the matrix that is used to estimate the actual channel $\mathbf{h}_j$ has a greater size than the matrix used to find the unknown part of the channel $\mathbf{a}_j$. Thus, when the unknown part of the channel is estimated, it results in the reduction of the computational complexity of the final EVD operation compared to when the actual channel is estimated.

3.6.3 Simulation Results

In this section simulation results are presented to compare the performance of the Torlak/Xu algorithm [Torlak97] that does not use knowledge of the pulse-shaping filter to the proposed channel estimation algorithm in Section 3.6 that does incorporate pulse-shaping information. The BPSK signal from each user is modulated by a raised-cosine waveform with a roll-off factor $\beta = 0.25$, truncated to a length of $L_g = 10$ symbol periods. One of the assumptions made in the Parametric Subspace algorithm for the unknown part of the channel to be identifiable (Section 3.5.2), is that the number of receive antennas q needs to be greater than or equal to the number of multipaths d. For the simulations $q = d$. The receive antennas are considered to be omnidirectional, spaced apart by half a wavelength and standard far-field propagation conditions are assumed. The oversampling factor is $p = 2$.

In all the results, the spreading codes were generated randomly in the same way as [Torlak97] and [Aktas00]. The elements of the spreading sequence are selected from $\left\{\pm 1/\sqrt{L_c}\right\}$ with equal probability. The length of the stacking factor is $K = 3$. The number of multipaths and the multipath delays for each user is assumed to be known. Delay estimation algorithms for estimating the multipath delays are presented in Chapter 4.

In the channel that is considered, it is assumed that the multipath delays change so slowly that they remain constant during the observation period. The channel is also assumed to have quasi-static fading: the attenuation of each path is fixed for the observation period. A linear multipath intensity profile channel similar to the channel used in [Perros-Meilhac01] is considered. The attenuation $\alpha_{k,j}$ of the k^{th} path for the j^{th} user is given by

$$\alpha_{k,j} = m_{k,j} p_{\tau}(\tau_{k,j}) \tag{3.60}$$

where $m_{k,j}$ is the complex fading variable that is generated using a normal random distribution with a mean of zero and a standard deviation of one,

$p_{\tau}(\tau_{k,j})$ is the power of the k^{th} path for the j^{th} user that varies linearly with the multipath delay in such a way that $p_{\tau}(0) = 1$ and $p_{\tau}(9T_c) = 0.1$.

The delays for the j^{th} user are chosen from a uniform distribution. Hence the delay of the k^{th} path $\tau_{k,j} \sim U[0, \quad T_c/2, \quad T_c, \quad \cdots \quad, 9T_c]$, where the multipath spread of the channel is set to $9T_c$. In the literature [Torlak97], [Aktas00], [Bensley96] the multipath delays are assumed to be an integer multiple of the chip period. In this simulation, the multipath delays are assumed to be an integer multiple of the oversampling rate T_c / p with the constraint of the minimum delay separation between two paths being greater than a chip period.

A channel with three paths is chosen. Therefore the number of receive antennas that is used is also set to three. The angles of arrival are set to $\boldsymbol{\theta} = (0, \quad 20°, \quad 40°)$. The number of Monte Carlo trials used in all the simulations is 100. Lastly, the asynchronous delay denoted as k_j for the j^{th} user is known. An asynchronous delay estimation algorithm is presented in [Torlak97]. In the simulation results presented the performance of new channel estimation is compared to the algorithm that does not use pulse-shaping information [Torlak97] by varying one of the following system parameters: SNR, length of observation window, number of users and power difference between five strong users and one weak user, and keeping the others constant. Table 2 gives the values of the system parameters when they are kept constant.

Table 2 Values of the simulation parameters used

Simulation Parameter	Value
Number of users	6
Length of observation window	200 bits
SNR	15dB
Power difference between the strongest and weakest user	0dB

The performance criteria that is used is the mean square error (MSE) which is the cumulative MSE for all the channel coefficients for all the users [Aktas00]

$$\mathrm{MSE} = \sum_{j=1}^{P} \left\| \mathbf{h}_j - \hat{\mathbf{h}}_j \right\| \qquad (3.61)$$

where $\hat{\mathbf{h}}_j = \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\hat{\mathbf{a}}_j$ for the new channel estimation algorithm.

The MSE as a function of the SNR is shown in Figure 3-4. The performance of the new channel estimation algorithm is clearly shown to be better than the algorithm that does not use pulse-shaping information [Torlak97].

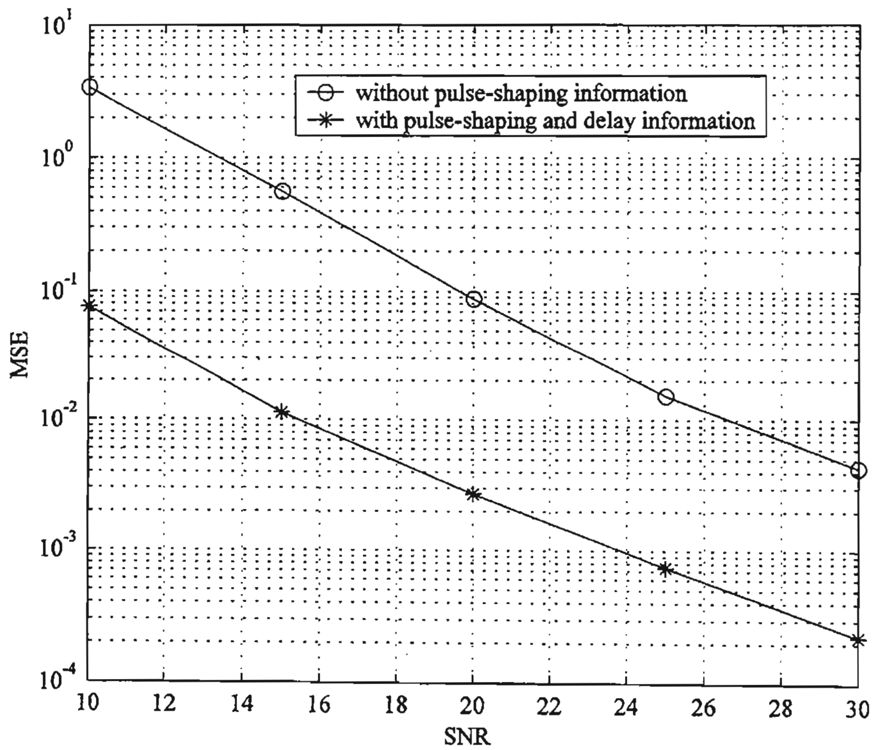

Figure 3-4 MSE as a function of SNR

It can be seen that performance gain achieved when knowledge of the pulse-shaping filter is used is approximately 11dB.

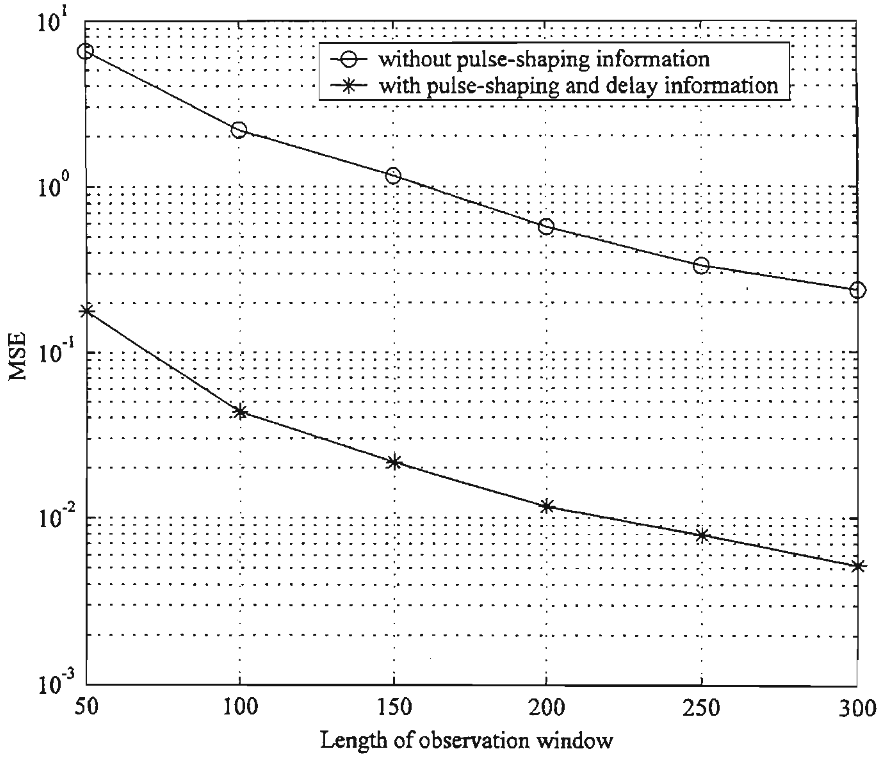

Figure 3-5 MSE as a function of the observation length

In Figure 3-5 the effect of the observation length as a function of the MSE is investigated. As the observation length increases, the channel estimates for both the algorithms become more accurate. The improved accuracy of the channel estimates is because a more accurate estimate of the correlation matrix is obtained as the observation length is increased. Since the noise subspace is obtained from the estimated correlation matrix (3.38), a more accurate estimate of the correlation matrix leads to a more accurate estimate of the vectors that span the noise subspace. Since the channel identification equations are a function of the vectors that span the noise subspace (3.56) and (3.52), a more accurate estimate of the noise subspace gives rise to a more accurate estimate of the channel.

The effect of the number of users as a function of MSE is shown in Figure 3-6. The MSE for both algorithms is expected to increase as the number of users in the system increase since the MSE expression is the cumulative MSE for all the channel coefficients for all the users. It is interesting to note that the new algorithm is able to accomodate many more users than the algorithm that does not use pulse-shaping information for both algorithms to offer the same performance.

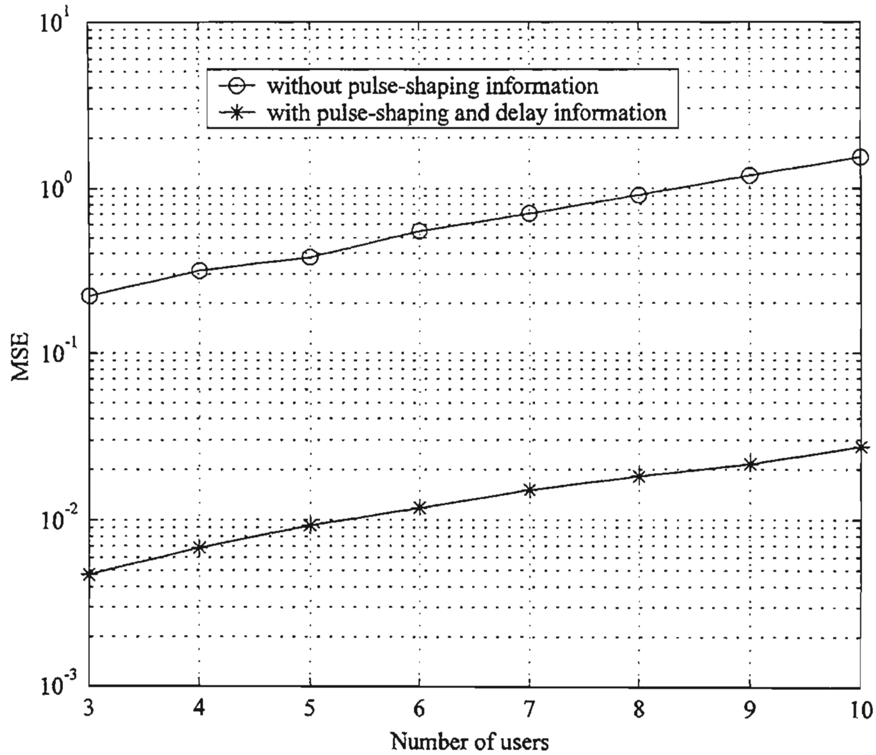

Figure 3-6 MSE as a function of the number of users

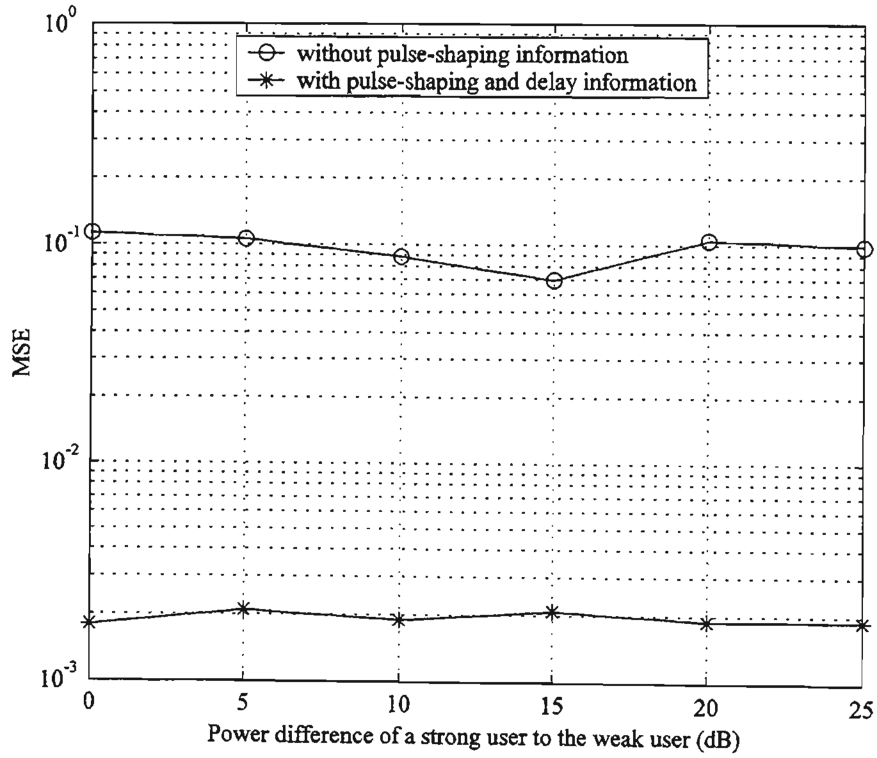

Figure 3-7 MSE of the weakest user as a function of the near-far ratio (dB)

In Figure 3-7 the channel estimation algorithms' ability to combat the near-far effect is investigated. A system with five users with equal strong power and one user with weak power is considered. The near-far ratio is defined as the ratio of the power of one of the strong users to the power of the weak user [Aktas00]. The SNR of the weak user is set to 15dB and the power of the strong users is varied. It is seen that the MSE of the weak user remains fairly constant as the near-far ratio is increased. Therefore it can be concluded that the channel estimates are resistant to the near-far effect. It should also be noted that the new algorithm that uses pulse-shaping information offers better performance.

3.6.4 Analysis of the Proposed Subspace Algorithm

Simulation results in the previous section show that the new subspace algorithm that uses pulse-shaping information has a better performance than the Torlak/Xu algorithm that does not use knowledge of the pulse-shaping filter. In this section, the performance of the proposed subspace algorithm is investigated analytically. A mathematical representation of the means square error of estimation is obtained using the results in [Aktas00].

An analytical expression for the MSE of the Torlak/Xu algorithm was presented in [Torlak97]. However, the MSE was for the estimation of the signature vector, which is the convolution of the channel with the spreading code of the desired user. Later, an analytical expression for the channel's MSE for the Torlak/Xu algorithm was derived [Aktas00]. It was shown that the MSE for the channel estimation of the j^{th} user can be expressed as:

$$\mathrm{MSE}\left(\hat{\mathbf{h}}_j\right) = \frac{\sigma_n^2}{(N-K)\sigma_s^2}\,\mathrm{tr}\left(\left(\left[\mathbf{c}_{j,J_{pq}}\right]^H \left[\sum_{k=1}^{z_m} \mathcal{W}^{(k)}\left[\mathcal{W}^{(k)}\right]^H\right]\mathbf{c}_{j,J_{pq}}\right)^{\dagger}\right) \tag{3.62}$$

where $\mathrm{tr}(\mathbf{A})$ denotes the trace of the matrix $\mathbf{A}$, $(.)^{\dagger}$ represents the pseudo-inverse operation and σ_s is the energy of the transmitted signal.

Recalling that the unknown channel can be written in terms of the known pulse-shaping filter and the unknown part of the channel :

$$\mathbf{h}_j = \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\mathbf{a}_j \tag{3.63}$$

the analytical expression for the MSE of the known part of the channel can be expressed as :

$$\mathrm{MSE}\left(\hat{\mathbf{a}}_{\mathrm{j}}\right)=\frac{\sigma_n^2}{(N-K)\sigma_s^2}\mathrm{tr}\left(\left(\left[\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right]^H\underbrace{\left[\mathbf{c}_{j,\mathbf{I}_{pq}}\right]^H\left[\sum_{k=1}^{z_m}\mathcal{W}^{(k)}\left[\mathcal{W}^{(k)}\right]^H\right]\mathbf{c}_{j,\mathbf{I}_{pq}}}_{\mathbf{Q}_m^H\mathbf{Q}_m}\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right)^{\dagger}\right) \tag{3.64}$$

Equation (3.64) shows that the MSE is inversely proportional to the signal-to-noise ratio and the length of the observation window. A detailed derivation of Equation (3.64), starting from the DS/CDMA system model in Section 3.6.1 can be found in Appendix B. Since the pulse-shaping filter is known exactly, the MSE of the estimated channel can be expressed as :

$$\mathrm{MSE}\left(\hat{\mathbf{h}}_{\mathrm{j}}\right)=\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\mathrm{MSE}\left(\hat{\mathbf{a}}_{\mathrm{j}}\right) \tag{3.65}$$

Figure 3-8 compares the channel's MSE of the proposed subspace algorithm that is derived via simulation in Section 3.6.2.2 and analysis in Equation (3.65) using simulation values given in Section 3.6.3.

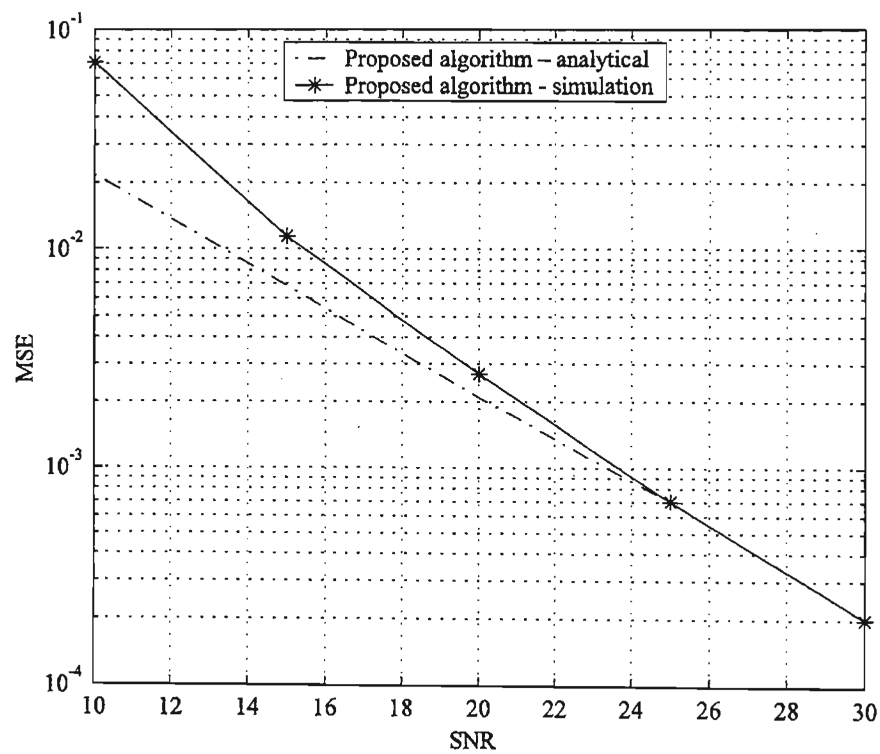

Figure 3-8 MSE obtained analytically and from simulation as a function of SNR

It is observed that the analytic expression that is derived provides a good approximation of the actual MSE for high SNR. This result is expected because the analytical expression is based on theory on the perturbed subspace approximation [Li93], which is only valid under the assumption of high SNR.

3.7 Summary

This chapter presented a new subspace-based channel estimation algorithm for DS/CDMA systems that used information of the pulse-shaping filter. A DS/CDMA system model with multiple receive antennas, where each antenna is oversampled with respect to the chip rate, was given. The new algorithm was based on the Parametric Subspace algorithm for single-user systems that also incorporated knowledge of the pulse-shaping filter. The DS/CDMA system model that was given was written in terms of the overall channel for each user that is in a Toeplitz matrix form so that the useful commutative property involving a Toeplitz matrix and a vector [Moulines95] could be used to simplify the channel identification equation.

The performance of the new algorithm was compared to a similar subspace-based channel estimation algorithm [Torlak97] that did not use pulse-shaping information, in a quasi-static linear multipath intensity profile channel. Simulation results showed that the new channel estimation algorithm offered better performance under all conditions (SNR, observation length, number of users and the near-far situation). It was observed that the new algorithm could operate at a SNR of 11dB lower than the Torlak/Xu algorithm for both algorithms to offer the same performance. Lastly, a mathematical expression of the mean square error of estimation for the new algorithm was derived. It was shown that the analytic expression provided a good approximation of the actual MSE for high SNR.

Chapter 4

A Modified Multipath Delay Estimation Algorithm

4.1 Introduction

In typical wireless communication systems, the digital data that is transmitted by a user gets reflected, scattered and diffracted by objects before it reaches the receiver. Therefore, the receiver obtains multiple copies of the same signal that are each scaled in amplitude and delayed in time. These multipath components distort the transmitted signal. The use of a Rake receiver has been proposed in next generation communication systems to combat the adverse effects caused by inter-symbol interference. The Rake receiver requires knowledge of the multipath delays to constructively combine the signal energy from multipath components. In communication systems, a delay estimation algorithm is performed at the receiver to estimate the multipath delays introduced by the channel.

In Chapter 3.6, a new channel estimation algorithm is proposed for DS/CDMA systems that estimates the unknown spatial attenuation that is introduced by the channel. However, an assumption is made in the proposed algorithm that all the multipath delays are known at the receiver. In this chapter a new delay estimation algorithm is presented to estimate the multipath delays for the new subspace algorithm in Chapter 3.6.

There have been many delay estimation algorithms that have been proposed in the literature, as discussed in Section 2.6. The only subspace algorithm, as far as the author is aware of, that incorporates knowledge of the pulse-shaping filter in the delay estimation procedure is the Parametric Subspace method [Perros-Meilhac01], which is referred to as the Parametric Subspace Delay Estimation (PSDE) algorithm. In [Perros-Meilhac01], the delay estimation variance of the PSDE algorithm is shown to be efficient enough to approach the CRB, which provides a theoretical bound of the minimum estimation variance for all estimation procedures based on second order moments [Abed-Meraim97]. For this reason, the PSDE algorithm is chosen as the method to estimate the multipath delays for each user in the DS/CDMA system proposed in Section 3.6.1.

The PSDE algorithm and its application to multi-user DS/CDMA systems are explained in Section 4.2. It is shown by simulations that the performance of the PSDE algorithm degrades when it operates in a DS/CDMA environment where the power of the multipath signals decrease with increasing delay. Thus, the Modified Parametric Subspace Delay Estimation algorithm is proposed in Section 4.3 to improve the performance of the PSDE algorithm in an environment where the power of the multipath signals varies. Lastly, simulation results are presented in Section 4.4 to compare the performance of the new channel estimation algorithm in Section 3.6 which uses the MPSDE algorithm to estimate the multipath delays, to the new channel estimation algorithm that assumes all the multipath delays are known at the receiver.

4.2 The PSDE Algorithm for Single-user Systems

In Section 3.5.2, the Parametric Subspace algorithm for single-user systems is discussed. The Parametric Subspace algorithm uses pulse-shaping information to estimate the unknown spatial attenuation of the channel. The equation that identifies the unknown part of the channel is given by:

$$\hat{\mathbf{a}} = \arg \min_{\|\mathbf{a}\|=1} \mathbf{a}^H \mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau}) \mathbf{a} \tag{3.23}$$

It should be noted that the unknown part of the channel is only identified by Equation (3.23) if the multipath delays $\boldsymbol{\tau}$ are known. This section describes the PSDE algorithm that is proposed in [Perros-Meilhac01] to estimate the multipath delays.

4.2.1 Concept of the PSDE Algorithm

Pseudocode for the PSDE algorithm is given in [Perros-Meilhac01]. However, no detailed explanation is given about how and why the algorithm works. This section explains the concepts of the PSDE algorithm, which gives understanding into the operation of the PSDE algorithm.

Stating Equation (3.23) in words, the unknown part of the channel is identified by the normalised eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau})$. In a noise-free environment, the smallest eigenvalue of the matrix $\mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau})$ would only correspond to the noise power of zero if the multipath delay vector $\boldsymbol{\tau}$ is correctly estimated. If the incorrect estimate of the multipath delays $\hat{\boldsymbol{\tau}}_e$ is used, the smallest eigenvalue of the matrix $\mathbf{G}_{L,d}(\hat{\boldsymbol{\tau}}_e)^H \mathbf{Q} \mathbf{G}_{L,d}(\hat{\boldsymbol{\tau}}_e)$ would correspond to a numerical value which is higher than the noise power of zero.

The PSDE algorithm uses the following algorithm to estimate the delay in a single path environment where noise is present:

- Each possible value of the delay $\tau_p \in [0, \ T_c / p, \ \cdots, \ T_m]$ is used to obtain the smallest eigenvalue of the matrix $\mathbf{G}_{L,d}(\tau_p)^H \mathbf{Q} \mathbf{G}_{L,d}(\tau_p)$, which is denoted by $J(\tau_p)$.
- The estimate of the delay $\hat{\tau}$ is obtained by minimising $J(\tau_p)$.

The next section explains how the PSDE algorithm estimates delays in a multipath environment.

4.2.2 The PSDE Algorithm

The following equation is defined so that the PSDE algorithm could be easily formulated:

$$J(\boldsymbol{\tau}) = \lambda_{\min}\left(\mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau})\right) \tag{4.1}$$

where $\lambda_{\min}(\mathbf{A})$ denotes the minimum eigenvalue of the matrix $\mathbf{A}$.

The flow chart of the PSDE algorithm for a two-path channel is shown in Figure 4-1. Assuming that the number of paths d is known, the PSDE algorithm uses the following iterative process to estimate the multipath delays assuming that $\tau_1 \neq \tau_2 \ \cdots \ \neq \tau_d$:

- Assume that $d=1$ and obtain $\hat{\tau}_{1,1}$, which is the first estimate of τ_1, by minimising $J(\tau_1)$.

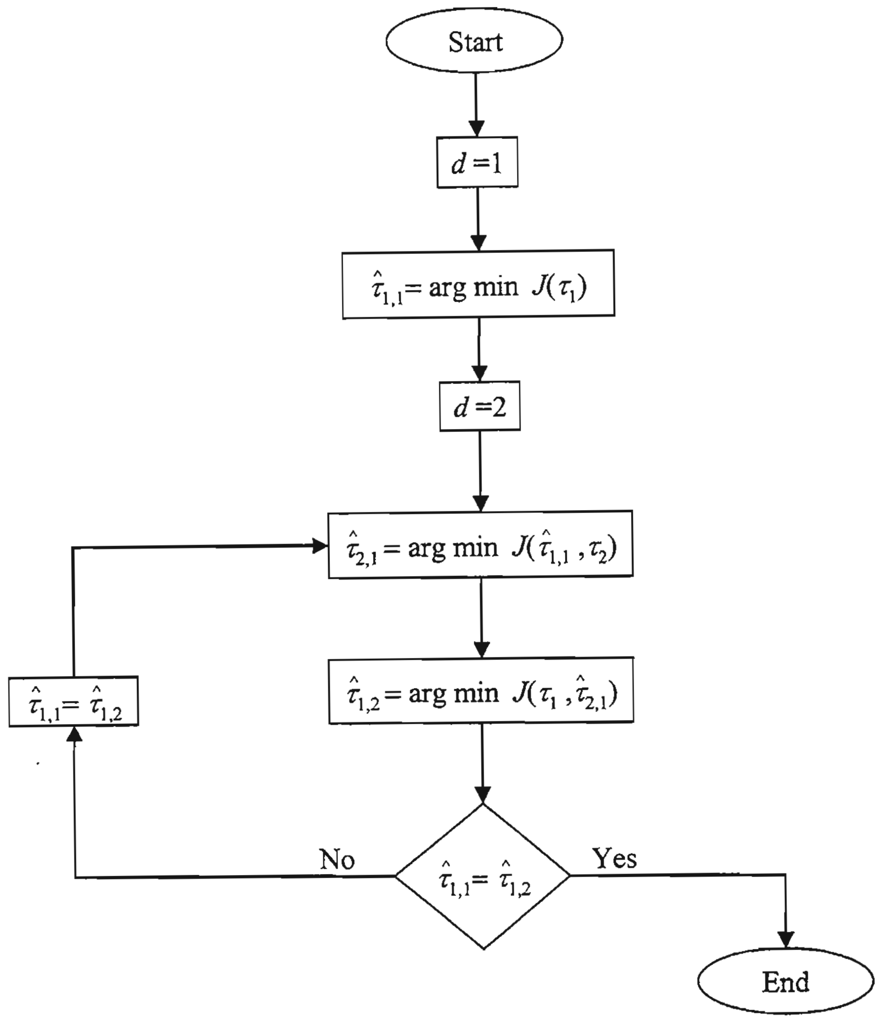

Figure 4-1 Flow chart of the PSDE algorithm for a two-path channel

- Assume that $d=2$ and obtain $\hat{\tau}_{2,1}$, which is the first estimate of τ_2, by minimising $J(\hat{\tau}_{1,1}, \tau_2)$, which is only a function of τ_2 with $\hat{\tau}_{1,1}$ fixed. Next obtain $\hat{\tau}_{1,2}$, which is the second estimate of τ_1, by minimising $J(\tau_1, \hat{\tau}_{2,1})$, which is only a function of τ_1 with $\hat{\tau}_{1,2}$ fixed.

If the first and second estimate of τ_1 are not equal, then $\hat{\tau}_{1,2}$ is made equal to $\hat{\tau}_{1,1}$ and $\hat{\tau}_{2,1}$ is replaced by the value that minimises $J(\hat{\tau}_{1,1}, \tau_2)$. This method is repeated until $\hat{\tau}_{1,1} = \hat{\tau}_{1,2}$.

- The process above is repeated until the number of paths d is reached.

4.3 The PSDE Algorithm Applied to DS/CDMA Systems

The PSDE algorithm was originally developed for a single-user system. In this section, the PSDE algorithm is extended to operate in DS/CDMA systems by modifying the minimisation equation $J(\boldsymbol{\tau})$. In a single-user system, the minimisation equation $J(\boldsymbol{\tau})$ is a function of the matrix $\mathbf{G}_{L,d}(\boldsymbol{\tau})^H \mathbf{Q} \mathbf{G}_{L,d}(\boldsymbol{\tau})$ that is also used to identify the unknown part of the channel in Equation 3.23. In the DS/CDMA system that is mathematically formulated in Section 3.6.1, the unknown part of the channel for the j^{th} user is identified by the matrix $\left[\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right]^H \mathbf{Q}_m^H \mathbf{Q}_m \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})$. Thus, for DS/CDMA systems the minimisation equation $J_j(\boldsymbol{\tau})$ of the PSDE algorithm is modified to:

$$J_j(\boldsymbol{\tau}) = \left[\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right]^H \mathbf{Q}_{tx,j}^H \mathbf{Q}_{tx,j} \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau}) \tag{4.2}$$

For a single-user system of two paths, it is shown in [Perros-Meilhac01] that the performance of the PSDE algorithm degrades when the power of the second path decreases with respect to the first path. Simulation results provided in this section show that the same performance degradation occurs when the PSDE algorithm is applied to DS/CDMA systems.

The simulation parameters that are used to obtain results are shown in Table 3. The system consists of six users, each having a two path quasi-static fading channel. The channel is assumed to change so slowly that the multipath delays are considered to remain the same within the observation period.

Table 3 Simulation parameters

Simulation Parameter	Value
Chip and data modulation	BPSK
Code sequence	Random code of length 32 where each element is chosen from $\{\pm 1/\sqrt{L_c}\}$ with equal probability.
Chip pulse shape	Raised cosine pulse with a roll-off factor of 0.25 and truncated to 10 symbol periods.
Channel	Two-path quasi-static fading
Oversampling factor	2
Number of receive antennas	2
Number of users	6
Observation window	200 symbols
Stacking factor	3
SNR ratio	15dB
Power difference between the strongest and weakest user	0dB
Number of Monte Carlo trials	100

The multipath delays of the first user is fixed to $\boldsymbol{\tau}_1 = [1.5T_c,\ 3T_c]$, whereas the delays for the other five users are chosen from a uniform distribution. For example, the delay of the k^{th} path for the j^{th} user is $\tau_{k,j} \sim U[0,\ T_c/2,\ T_c,\ \cdots,\ 9T_c]$. The performance criteria used is the probability of acquisition for the first user:

$$P_{ac,1} = \frac{\text{number of correctly estimated delays for user 1}}{\text{total number of delays for user 1}} \tag{4.3}$$

The multipath delays for the first user are estimated using the two-path PSDE algorithm described in Section 4.2.2, where the minimisation function is modified to Equation (4.2). In Figure 4-2, the power gradient between the two paths is varied. The power gradient is defined as $10\log_{10}(\beta_2/\beta_1)$, where β_2 and β_1 are the power of the first and second path respectively. It can be clearly seen that as the power gradient increases, the probability of acquisition for both paths decrease.

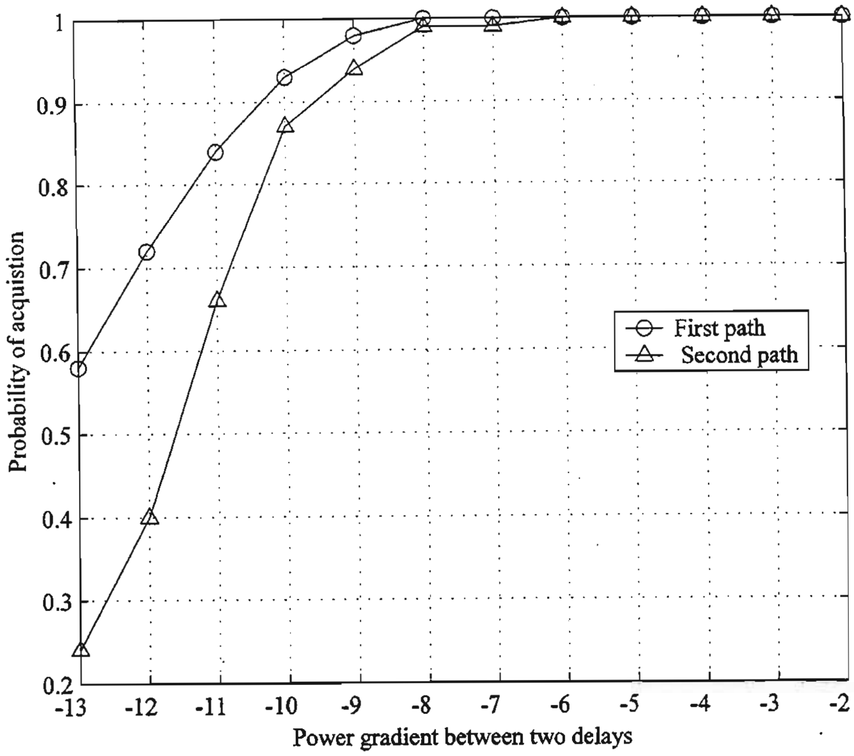

Figure 4-2 Probability of acquisition for the first and second paths as the power gradient increases

In order to understand why the performance of the PSDE algorithm deteriorates as the power gradient deceases, the graphical representation of the cost function $J_1(\boldsymbol{\tau})$ is investigated for power gradients of -3dB and -11dB. The power gradient is first set to -3dB. Following the flow chart in Figure 4-1, the function $J_1(\tau_1)$ is plotted for varying delays τ in Figure 4-3. It is clearly seen that when $J_1(\tau_1)$ is minimised, the first estimate of the first path's delay $\hat{\tau}_{1,1}$ is identified as $1.5T_c$.

Figure 4-4 shows that the delay for the second path $\hat{\tau}_{1,2}$ is correctly estimated as $3T_c$ by minimising the function $J_1(\hat{\tau}_{1,1}, \tau_2)$. Note that although the function $J_1(\hat{\tau}_{1,1}, \tau_2)$ has a global minimum at $1.5T_c$, this is not chosen as the estimate for the second path because it corresponds to the estimate of the first path's delay.

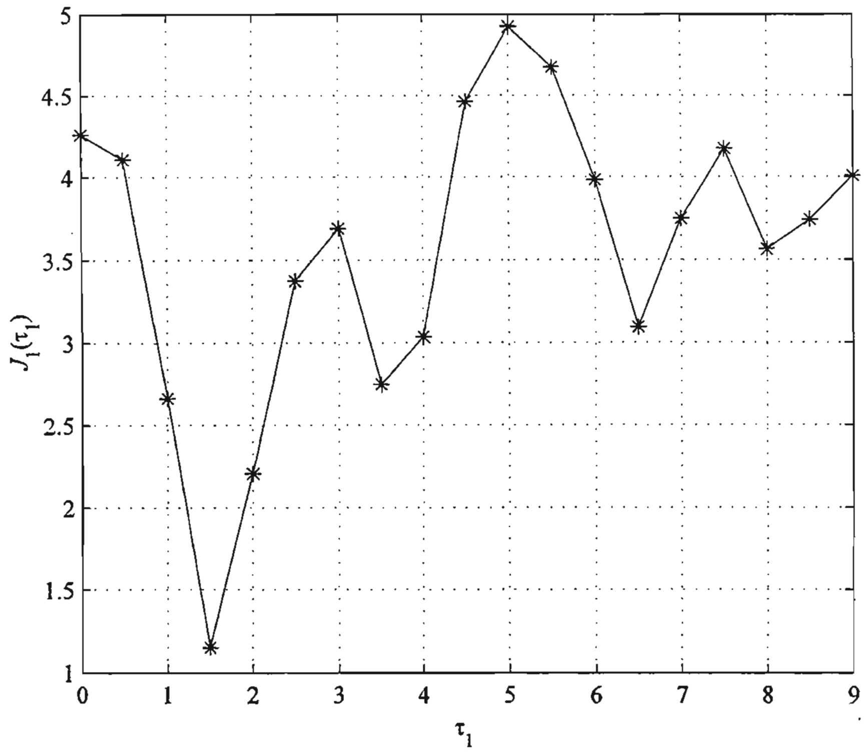

Figure 4-3 Graphical representation of $J_1(\tau_1)$ when the power gradient is -3dB

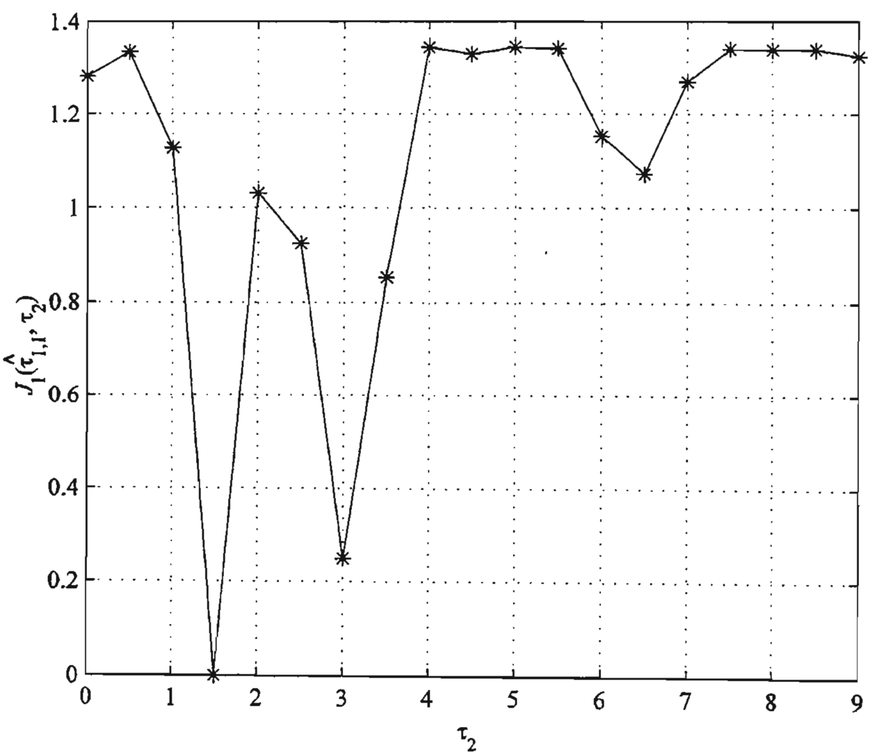

Figure 4-4 Graphical representation of $J_1(\hat{\tau}_{1,1}, \tau_2)$ when the power gradient is -3dB

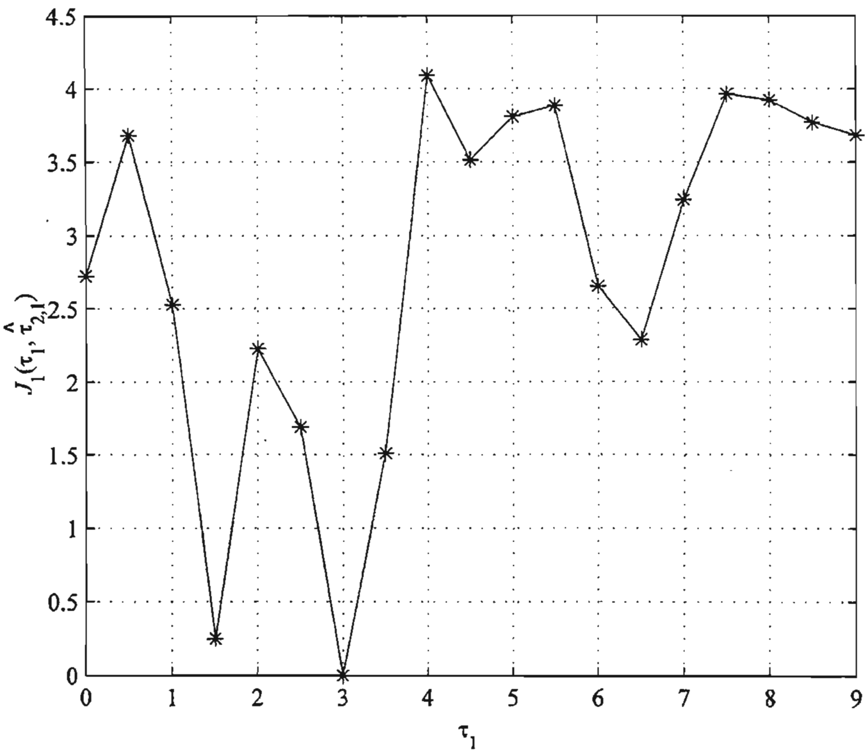

Figure 4-5 Graphical representation of $J_1(\tau_1,\hat{\tau}_{2,1})$ when the power gradient is -3dB

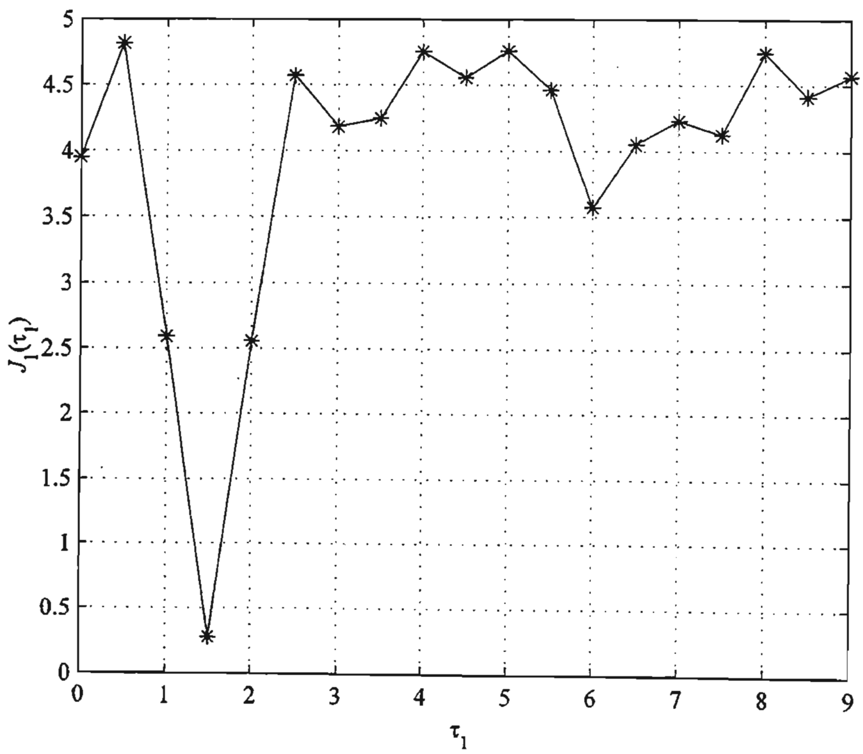

Figure 4-6 Graphical representation of $J_1(\tau_1)$ when the power gradient is -11dB

Finally, the first path's delay is correctly re-estimated as the first estimate value of $1.5T_c$ in Figure 4-5, thus the PSDE algorithm stops. Figure 4-6 illustrates the cost function $J_1(\tau_1)$ when the power gradient is set to -11dB. It can be seen that the first estimate of the first path's delay is correctly identified as $1.5T_c$.

The limitation of the PSDE algorithm is seen in Figure 4-7, where the function $J(\hat{\tau}_{1,1}, \tau_2)$ is used to obtain the first estimate of the second path's delay. Ignoring the minimum at $1.5T_c$ because it corresponds to the first path's delay estimate, the delay value $\tau_2 = 2T_c$ gives the minimum value of $J(\hat{\tau}_{1,1}, \tau_2)$. Thus, the second path's delay is incorrectly estimated by the PSDE algorithm. It can be seen from Figure 4-7 that the correct value for the second path's delay $\tau_2 = 3T_c$ can be found by choosing the second global minimum of $J(\hat{\tau}_{1,1}, \tau_2)$ if the smallest eigenvalue corresponding to the first delay estimate is ignored.

The function $J(\hat{\tau}_{1,1}, \tau_2)$ that is used to estimate the second path's delay gives a false global minimum that is positioned at a time of $T_c/2$ away from the first path's estimated delay when the power gradient is decreased to -11dB. From Figure 4-2, it can be seen that the probability of acquisition for the second path is non-zero. Thus, the likelihood of the PSDE algorithm locking onto the false global minimum has to depend on the phase difference between the two paths because it is the only parameter that changes in the simulation used to obtain Figure 4-2.

Using the same simulation parameters in Table 3, Figure 4-8 shows the probability of acquisition for the first user as the phase difference between the two paths in varied from zero to three hundred and sixty degrees. It can clearly be seen that the performance of the PSDE algorithm depends on the phase difference betweens the two paths and it performs the worst when the phase difference between the two paths is 180°.

The PSDE algorithm can be improved by decreasing the delay estimate resolution between two paths. This is done in the proposed Modified Parametric Subspace Delay Estimation algorithm that is discussed in the next section. The MPSDE algorithm makes the constraint that the minimum separation between any two paths must be greater than a chip period in order for the delays to be resolved by the MPSDE algorithm. Using this constraint, the PSDE algorithm overcomes the false global minimum problem that occurs in the PSDE algorithm.

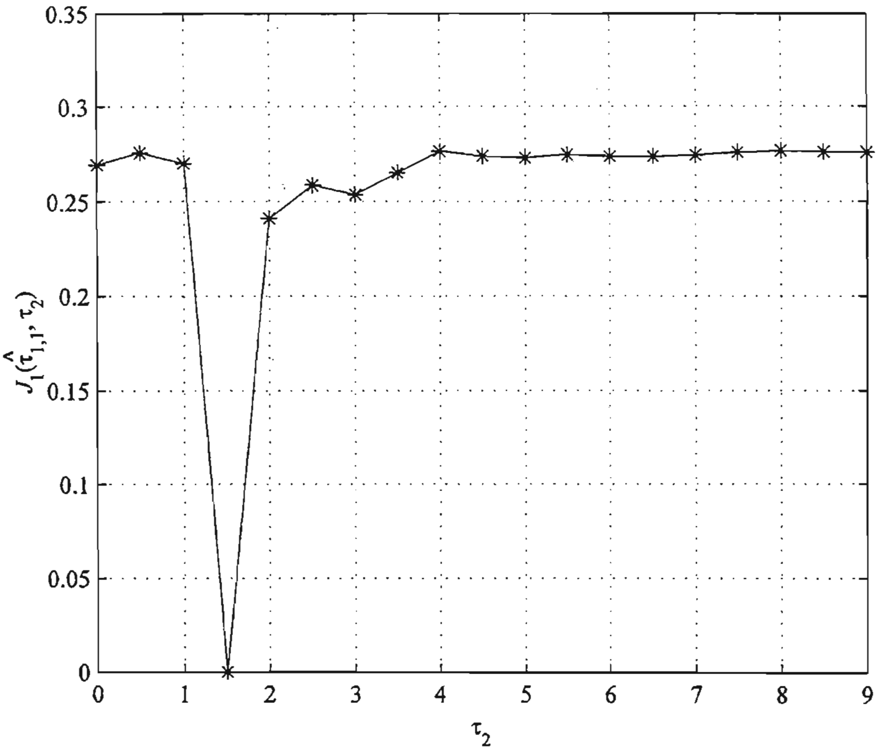

Figure 4-7 Graphical representation of $J_1(\hat{\tau}_{1,1}, \tau_2)$ when the power gradient is -11dB

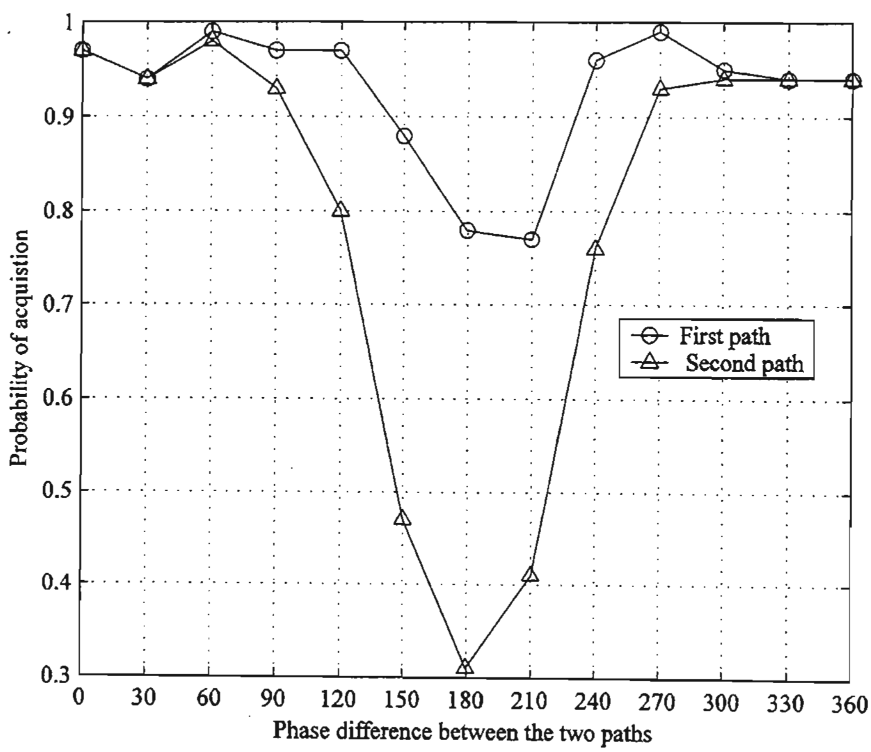

Figure 4-8 Probability of acquisition of the first user as the phase difference between the two paths is varied

4.4 The MPSDE Algorithm for DS/CDMA Systems

The MPSDE algorithm solves the limitation of the PSDE algorithm by making a trade off between the delay estimate resolution and the performance of the delay estimation algorithm.

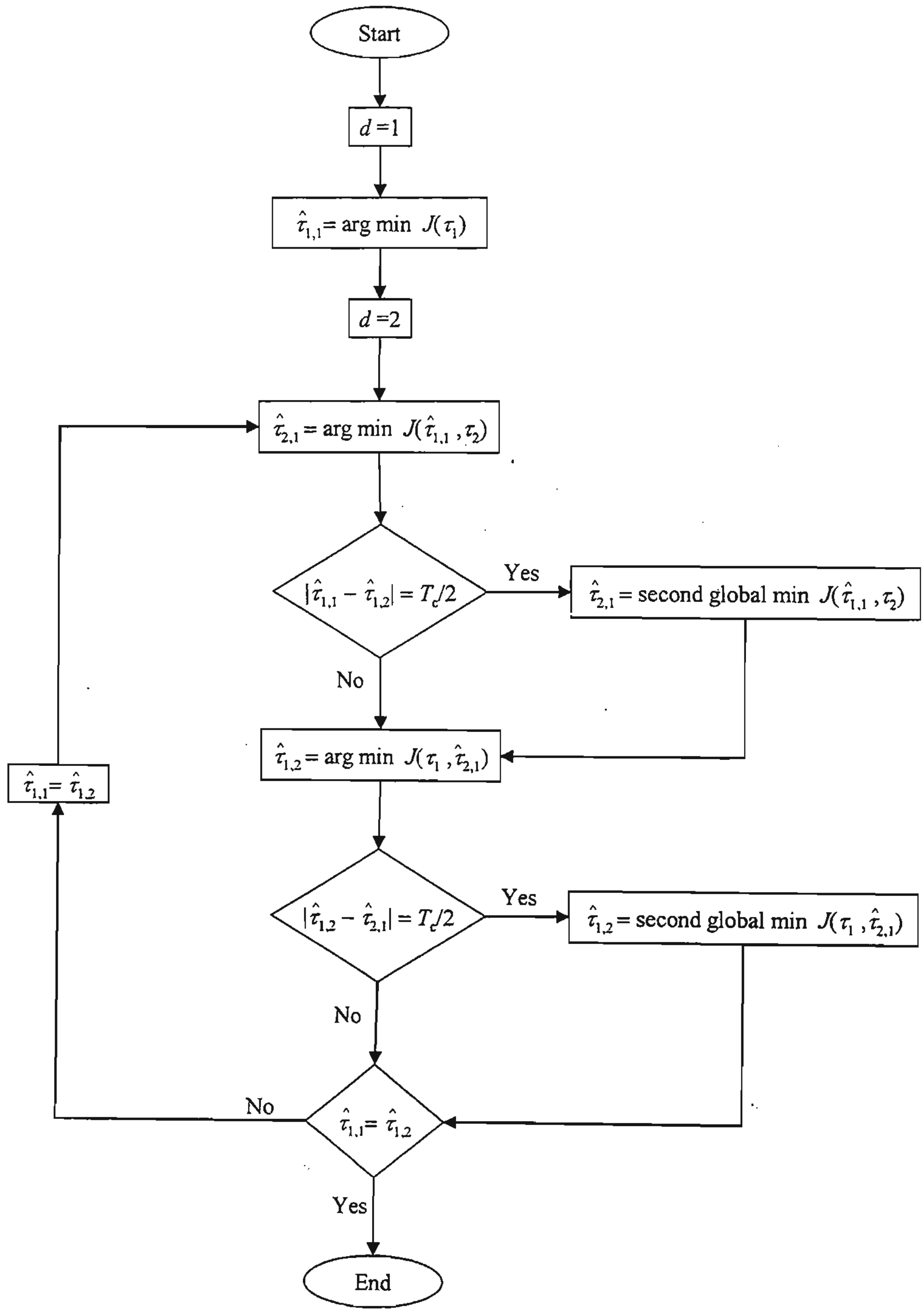

Figure 4-9 Flow chart of the MPSDE algorithm for a two-path channel

The MPSDE algorithm places greater emphasis on improving the performance of the PSDE algorithm when the power gradient between any two delays decreases to values less than -10dB. However, the delay estimation resolution between any two paths of the MPSDE algorithm is one chip period which is larger than the PSDE algorithm, which is half of a chip period.

The flow chart of the MPSDE algorithm for a two path channel that is shown in Figure 4-9, is very similar to the flow chart of the PSDE algorithm in Figure 4-1. In both algorithms, the same optimisation function that is given in Equation (4.2) is evaluated. The difference with the MPSDE algorithm is that if the delay estimate corresponding to the global minimum of the optimisation function is $T_c/2$ away from the other path's estimated delay, then the second global minimum of the optimisation function is chosen as the delay estimate. For example, if the MPSDE algorithm is estimating the second path's delay and it finds the global minimum of $J(\hat{\tau}_{1,1},\tau_2)$ to be $T_c/2$ away from the first path's estimated delay, then the second global minimum of $J(\hat{\tau}_{1,1},\tau_2)$ is chosen to be the second path's estimated delay.

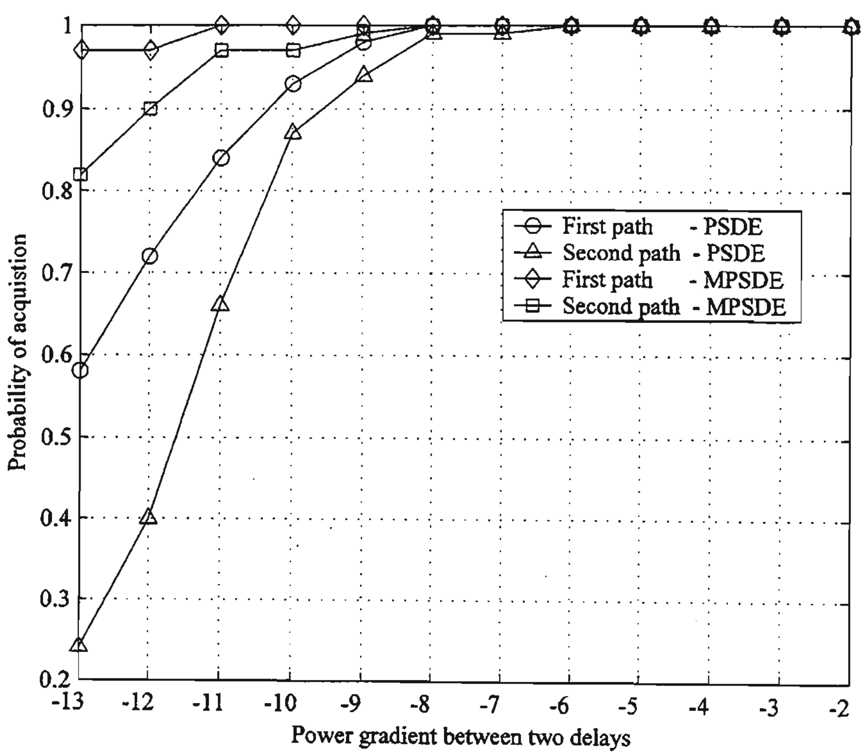

Figure 4-10 Probability of acquisition for the first and second paths of the PSDE and MPSDE algorithms as the power gradient decreases

The performance of the MPSDE and the PSDE algorithm is compared in Figure 4-10, using the simulation parameters in Table 3. The results show that the performance of the MPSDE is better than the PSDE algorithm when the power gradient between the two paths is lower than -8dB.

4.5 Results

In this section, simulation results are presented that compare the performance of the following blind subspace-based channel estimation algorithms:

- The Torlak/Xu algorithm [Torlak97] that does not use knowledge of the pulse-shaping filter.
- The proposed algorithm in Section 3.6 that incorporates knowledge of the pulse-shaping filter and assumes perfect delay estimation.
- The proposed algorithm in Section 3.6 that incorporates knowledge of the pulse-shaping filter and performs delay estimation using the MPSDE algorithm.

The simulation setup and parameter values that are used to obtain results are exactly the same as discussed in Section 3.6.3. The only difference is that in this section, results for a two path and a three path channel are presented. One of the performance criteria used in the results is the average probability of acquisition for all the users P_{ac}, which is defined as:

$$P_{ac} = \sum_{i=1}^{P} P_{ac,i} \tag{4.4}$$

where P is the number of users.

Figure 4-11 shows the performance of the three subspace algorithms for a two path channel, when the SNR is varied. It can be seen that the subspace algorithm that uses the MPSDE algorithm to estimate the two delays has exactly the same performance as the algorithm that assumes perfect delay for a SNR greater than 15dB. Therefore, it can be concluded that the MSPDE algorithm correctly estimates the two delays for all the users at a SNR greater than 15dB.

Figure 4-12 compares the performance of the three algorithms for a three path channel as the SNR varies. The graph shows that the performance of the proposed subspace algorithm that uses the MPSDE algorithm to estimate the three multipath delays is better than the Torlak/Xu algorithm, which does not use pulse-shaping information. Similar trends can be seen in Figures 4-14, 4-16 and 4-18. The average probability of acquisition is computed as one of the following simulation parameters is varied:

- Length of the observation window
- Number of users
- Power difference between the strongest and weakest user

while keeping the rest of the simulation parameters constant to the values given in Table 3. These are shown in Figures 4-13, 4-15, 4-17 and 4-19. It is shown that as the SNR or the length of the observation window increases, the MPSDE algorithm tends to correctly estimate all three delays because the average probability of acquisition approaches one. The MPSDE algorithm's ability to combat the near-far effect is shown in Figures 4-18 and 4-19. Since the probability of acquisition for the weakest user remains fairly constant as the power difference between the strongest and weakest user is increased, it can be concluded that the MPSDE algorithm is near-far resistant.

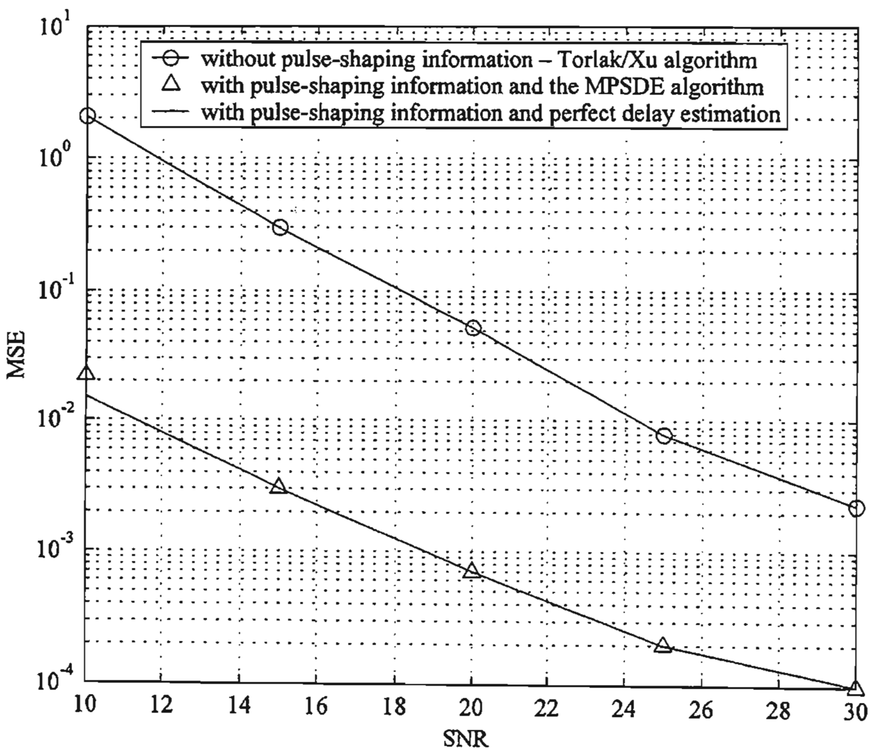

Figure 4-11 MSE versus SNR for a two path channel

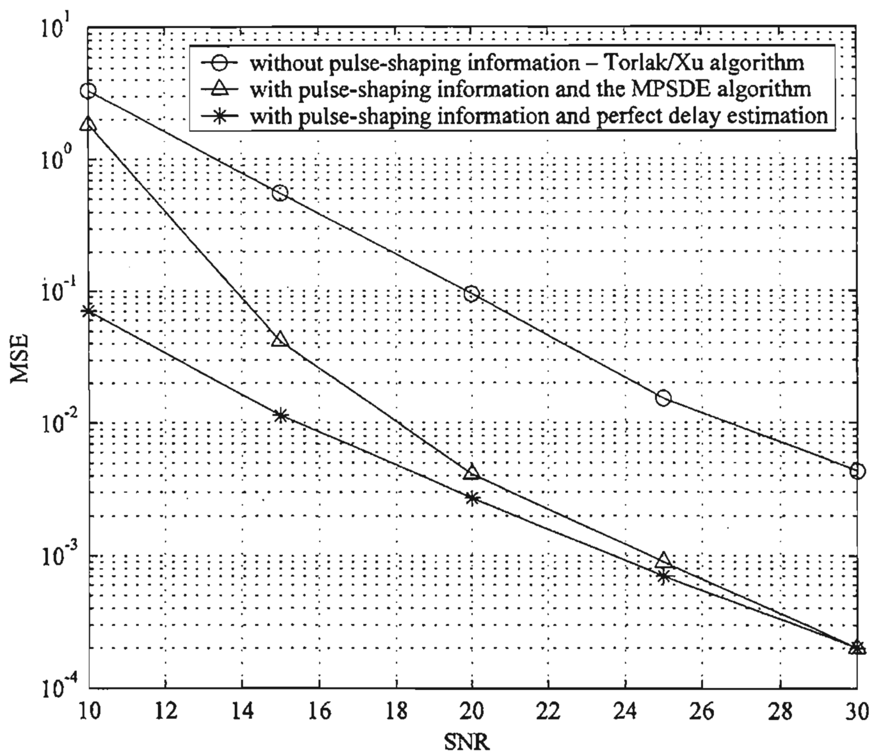

Figure 4-12 MSE versus SNR for a three path channel

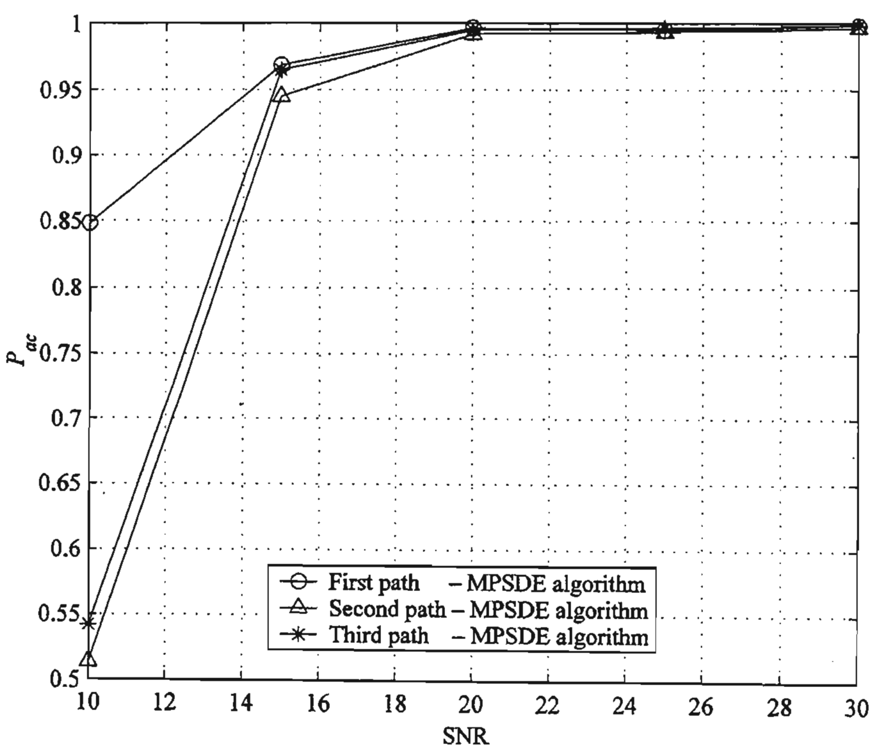

Figure 4-13 P_{ac} versus SNR for a three path channel

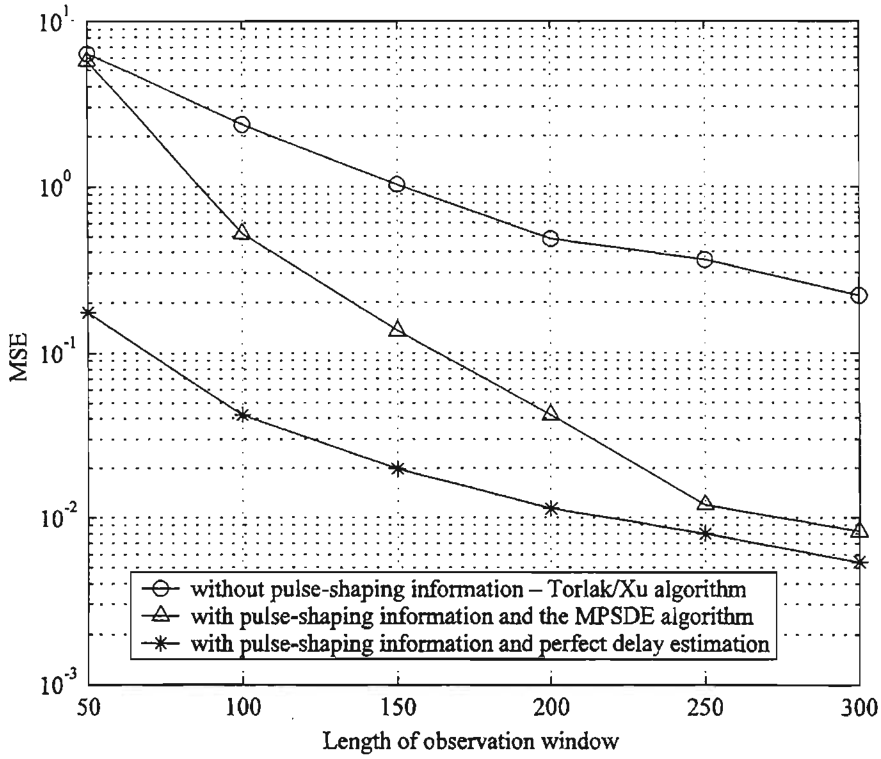

Figure 4-14 MSE versus length of observation window for a three path channel

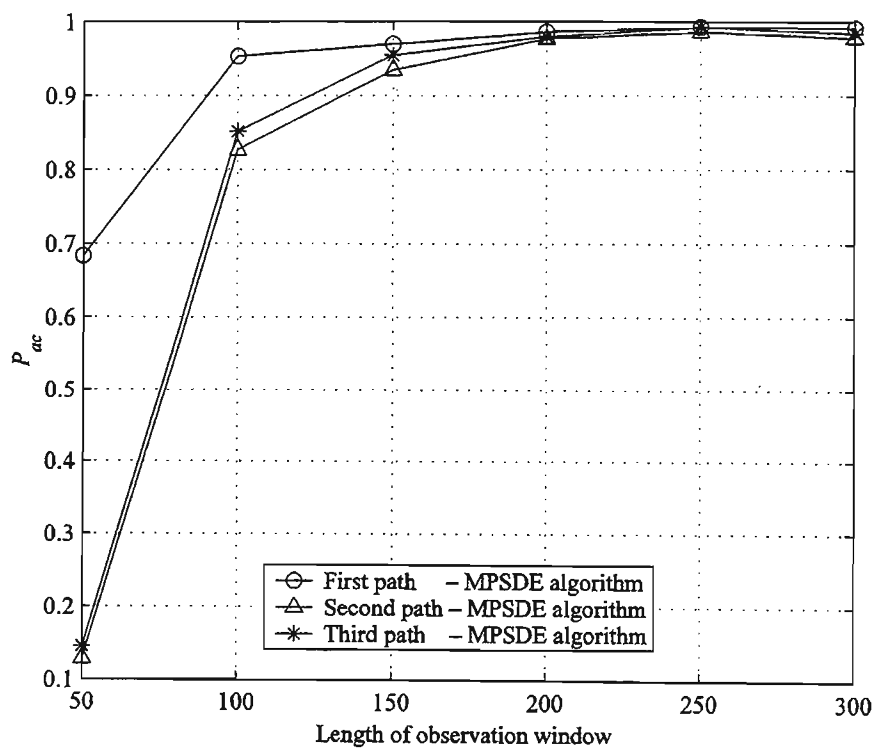

Figure 4-15 P_{ac} versus length of observation window

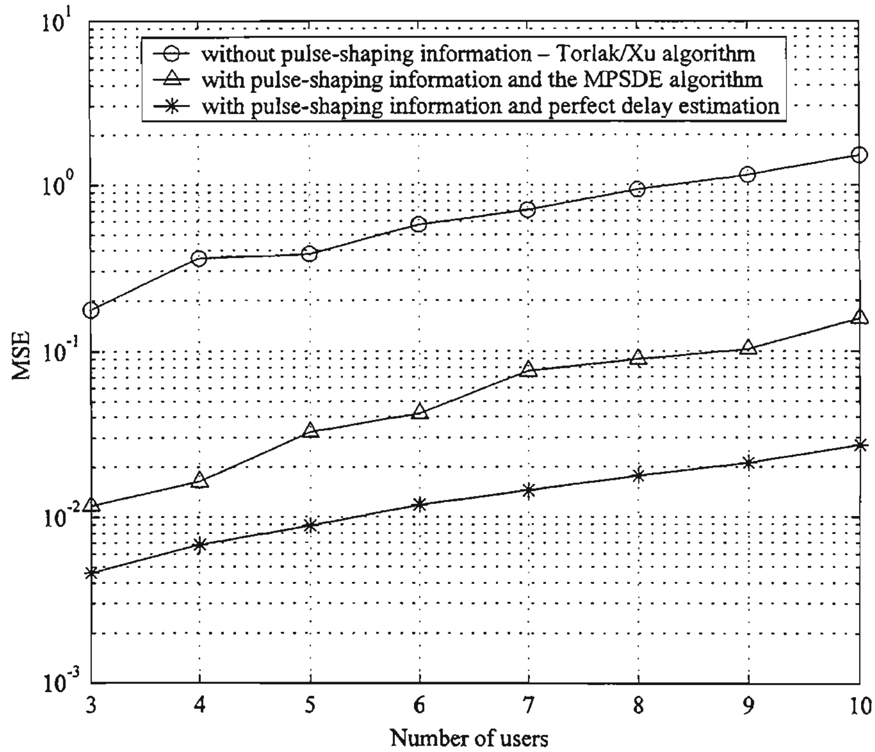

Figure 4-16 MSE versus number of users for a three path channel

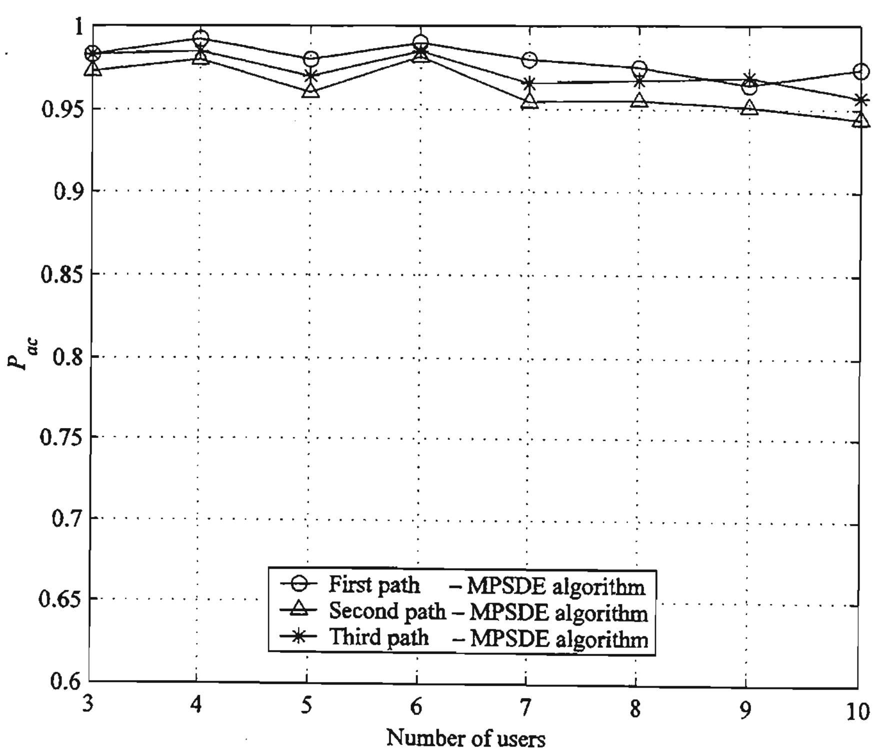

Figure 4-17 P_{ac} versus number of users for a three path channel

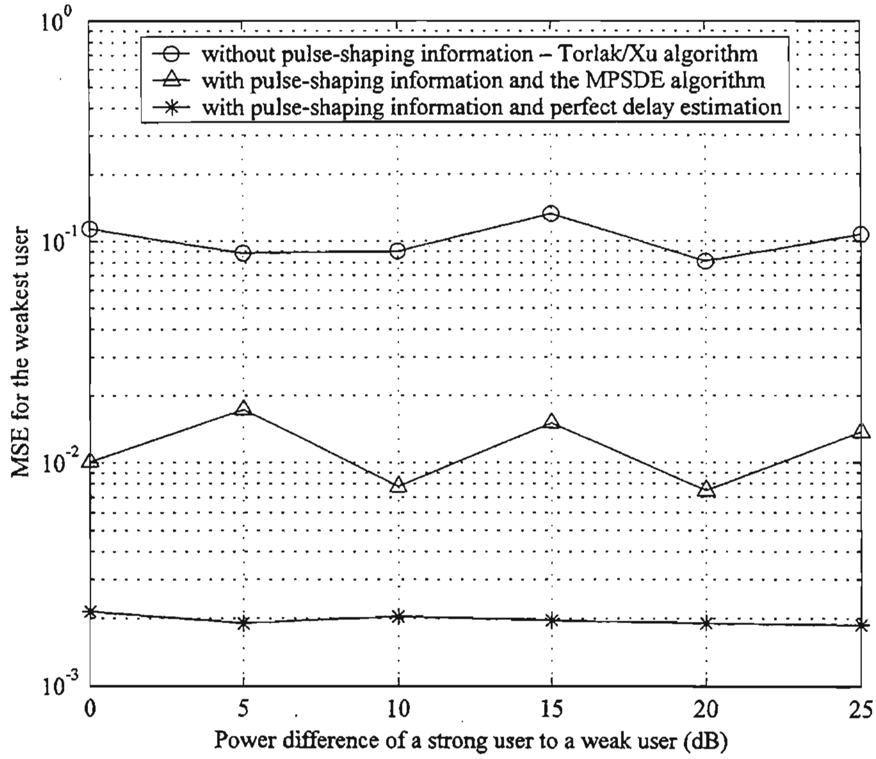

Figure 4-18 MSE for the weakest user as a function of the power difference of a strong user to a weak user for a three path channel

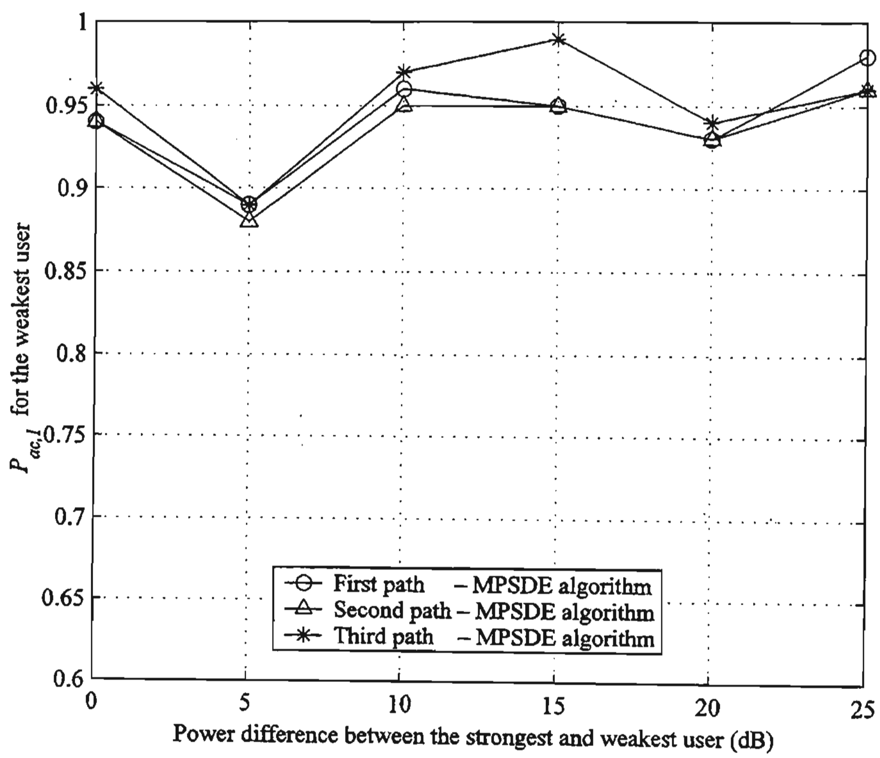

Figure 4-19 P_{ac} for the weakest user as a function of the power difference of a strong user to a weak user for a three path channel

4.6 Summary

This chapter presented a new delay estimation algorithm called the Modified Parametric Subspace Delay Estimation (MPSDE) algorithm. The MPSDE algorithm was based on the PSDE algorithm [Perros-Meilhac01] that uses pulse-shaping information to estimate the multipath delays introduced by the channel. It was shown by simulations that the performance of the PSDE algorithm degrades when it operates in a DS/CDMA environment where the power of the multipath signals decrease with increasing delay. This was seen as a limitation of the PSDE algorithm. The MPSDE algorithm was proposed to improve the performance of the PSDE algorithm in an environment where the powers of the multipath signals vary.

It was also shown by simulation that the performance of the proposed blind channel estimation algorithm in Chapter 3 that uses the MPSDE algorithm to estimate the multipath delays is better than the Torlak/Xu algorithm, which does not use pulse-shaping information. It was noted that for a high SNR that is greater than 30dB and a large observation window that is greater than 300 bits, the average probability of acquisition of the MPSDE algorithm approaches one. Lastly, the MPSE algorithm was seen to be near-far resistant, which is a traditional property of subspace algorithms.

Chapter 5

A Comparison of Subspace Delay Estimation Algorithms for DS/CDMA Systems

5.1 Introduction

In the early practical implementation of DS/CDMA systems, such as IS-95, the computationally simple matched filter solved the task of delay estimation. However, the simple matched filter was found to be non-resistant to multiple access interference [Simon85] and the optimal maximum likelihood solution [Verdú86] is too complex for practical purposes. Subspace-based methods are a desirable way of solving the delay estimation problem because they offer near-far resistant algorithms that exhibit adequate performance with reasonable computational complexity [Bensley96], [Ström96], [Luukkanen97]. There are many subspace-based delay and channel estimation algorithms that have been proposed for DS/CDMA systems some of which are discussed in Section 2.3.

Channel estimation algorithms typically model the multipath communication channel as a FIR filter and the objective of the algorithm is to estimate the tap coefficients of the filter representing the channel. For this reason channel estimation algorithms are called non-parametric methods. Delay estimation algorithms on the other hand are concerned with estimating the multipath delays introduced by the channel and thus they are referred to as parametric methods.

An example of a subspace-based channel estimation algorithm is the Torlak/Xu (TX) algorithm [Torlak97] that was proposed as an extension of the Channel Subspace algorithm [Moulines95] from a single-user system to multi-user DS/CDMA systems. The first subspace-based channel estimation algorithm was proposed by Bensley/Aazhang [Bensley96] and soon afterwards other subspace-based algorithms were developed [Ström96], [Luukkanen97]. The algorithm by Bensley/Aazhang is not a typical channel estimation algorithm that estimates the tap coefficients of the FIR filter representing the channel. However, the Bensley/Aazhang algorithm estimates the delay and attenuation of each path. Thus the algorithm can be called a parametric method since it is a delay estimation algorithm.

In communication systems, delay information is needed to achieve synchronisation and attenuation information is used in a maximal combining Rake receiver that is used to estimate the transmitted bits of the desired user. This chapter focuses on the task of synchronisation or delay estimation, therefore only the delay estimation portion of the Bensley/Aazhang algorithm is discussed. For this reason, the Bensley/Aazhang algorithm is referred to as a delay estimation algorithm.

Since the Torlak/Xu algorithm is a channel estimation algorithm and the Bensley/Aazhang algorithm is a delay estimation algorithm, the performances of the two algorithms have not been compared in the literature. The Torlak/Xu and the Bensley/Aazhang subspace algorithms are of special interest because they both exploit the orthogonal property of the signal and the noise subspace. However, the difference between these two algorithms is that the Torlak/Xu algorithm uses the Toeplitz commutative property developed in [Moulines95] to identify the channel whereas the Bensley/Aazhang algorithm does not. In this chapter the Torlak/Xu algorithm is converted from a non-parametric method to the proposed Parametric TX method. A quasi-static fading channel with a linear multipath intensity profile (MIP) is chosen to compare the performance of the Bensley/Aazhang delay estimation algorithm to the proposed TX algorithm. By comparing these two parametric subspace methods, the algorithm with the best performance is found.

5.2 DS/CDMA System Models

The delay estimation algorithm that was proposed by Bensley/Aazhang and the channel estimation algorithm developed by Torlak/Xu are discussed in this chapter. Both algorithms were developed for DS/CDMA systems. However, in [Bensley96] where the delay estimation algorithm is presented, the mathematical formulation of the DS/CDMA model is different to [Torlak97] where the TX channel estimation algorithm is discussed. The different mathematical descriptions of the system model are used to derive the channel estimation and delay estimation algorithms. For this reason the two system models in [Torlak97] and [Bensley96] are described in this section so that Bensley/Aazhang delay estimation algorithm and the Torlak/Xu channel estimation algorithm are able to be formulated in Section 5.4.

The model of the DS/CDMA system that is considered in this section is shown in Figure 5-1. In this system P users transmit data simultaneously over the same bandwidth. Each user employs BPSK modulation for data transmission, where a transmitted bit of duration T_b consists of L_c chips with duration $T_c = T_b / L_c$.

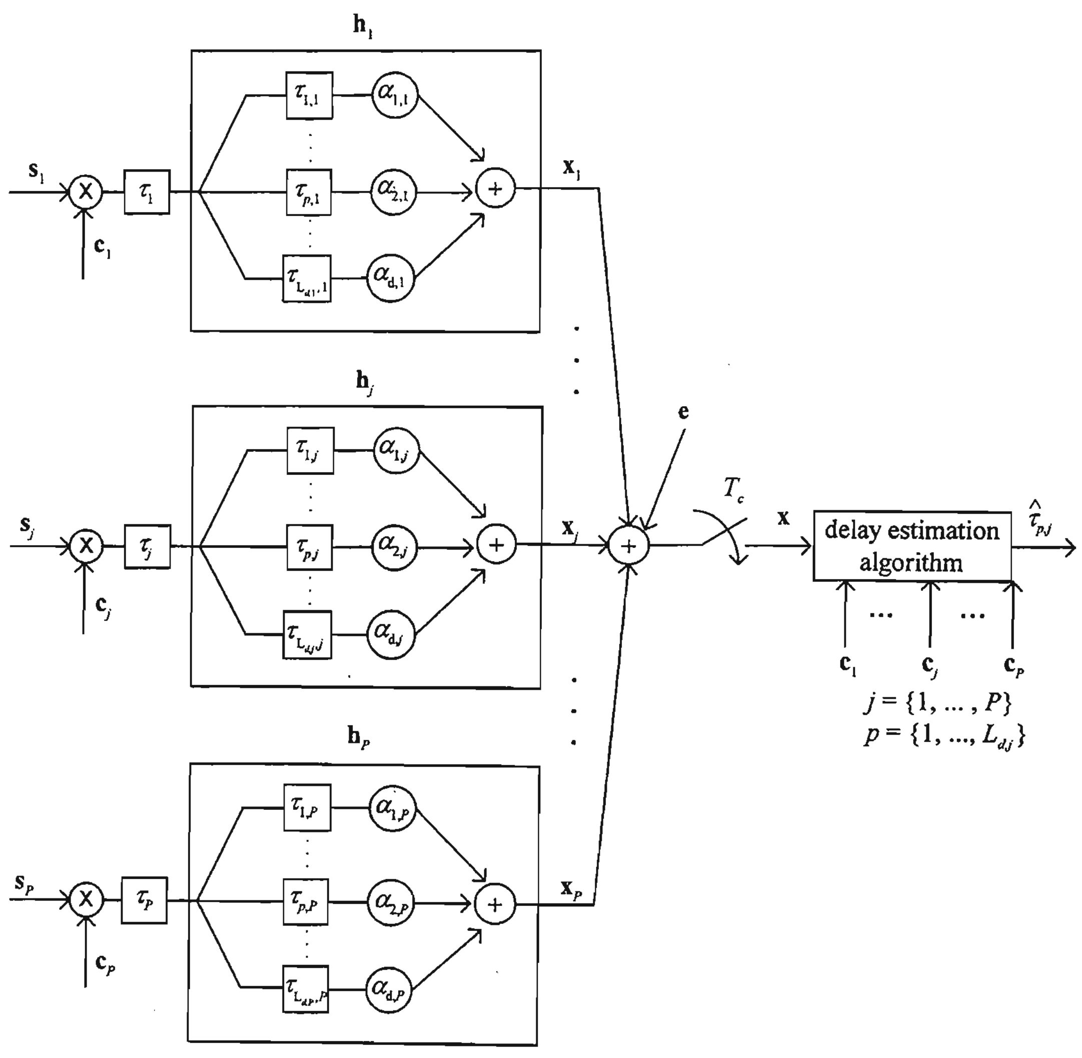

Figure 5-1 DS/CDMA system model

The following are defined for the j^{th} user:

$\mathbf{c}_j = \left[c_j(0) \quad \cdots \quad c_j\left((L_c-1)T_c\right)\right]$ is the discrete spreading code sequence,

s_j is the sequence of the transmitted BPSK symbols,

h_j is the channel that represents the multipath fading environment,

τ_j is the asynchronous delay that arrives at the receiver.

It is assumed that the asynchronous delay τ_j is an integer multiple of the chip period, thus $\tau_j = d_j T_c$ where d_j is an integer. In [Proakis95], it is shown that $h_j(t)$ can be written as:

$$h_j(t) = \sum_{q=1}^{L_{d,j}} \alpha_{q,j} u(t - \tau_{q,j}) \tag{5.1}$$

where $L_{d,j}$ is the total number of paths for the j^{th} user,

$\tau_{q,j}$ is the delay of the q^{th} path for the j^{th} user,

$\alpha_{q,j}$ is the attenuation of the q^{th} path for the j^{th} user,

$u(t)$ is a rectangular pulse of amplitude one and duration T_c.

5.2.1 The Torlak/Xu Mathematical System Model

In order to simplify notation, the contribution of noise to the received signal is initially ignored. Noisy data is considered at the end of this section. The continuous baseband signal that is received due to the j^{th} user is given by:

$$x_j(t) = \sum_{n=-\infty}^{\infty} s_j(n) \sum_{m=0}^{L_c-1} c_j(mT_c) h_j(t - \tau_j - mT_c - nT_b) \tag{5.2}$$

Sampling the continuous signal at the chip rate, we obtain the discrete-time observation of the received signal due to user j:

$$x_j(k) = \sum_{n=-\infty}^{\infty} s_j(n) g_j(k - nL_c - d_j) \tag{5.3}$$

where the global channel response is given by:

$$g_j(k) = \sum_{l=0}^{L-1} h_j(lT_c) c_j((k-l)T_c) \tag{5.4}$$

The constant L is defined as the channel length.

Adding the contribution due to noise and the received signals from all P users, the discrete time observation at the receiver is given by:

$$x(k) = \sum_{j=1}^{P} x_j(k) + e(k) \tag{5.5}$$

where $e(k)$ is a white Gaussian, random sequence with zero mean and variance σ_n^2.

To write (5.3) in vector form, we assume that the channel length is smaller than the spreading gain ($L < L_c$). In [Aktas00], the global response vector of length $2L_c$ is given by:

$$\mathbf{g}_i = \begin{bmatrix} \mathbf{0}_{\mathrm{I}} & g_i(0) & g_i(1) & \cdots & g_i(L_c + L - 2) & \mathbf{0}_{\mathrm{II}} \end{bmatrix}^T \tag{5.6}$$

where $\mathbf{0}_{\mathrm{I}}$ is a $1 \times d_j$ vector of zeros and $\mathbf{0}_{\mathrm{II}}$ is a $1 \times L_c - d_j - L + 1$ vector of zeros. Equation (5.4) can be written in matrix form as:

$$\mathbf{g}_j = \mathbf{C}_j \mathbf{h}_j \tag{5.7}$$

where $\mathbf{h}_j = \begin{bmatrix} h_j(0) & h_j(T_c) & \cdots & h_j((L-1)T_c) \end{bmatrix}^T$ and $\mathbf{C}_j$ is a $2L_c \times L$ Toeplitz matrix whose first column is given by:

$$\begin{bmatrix} \mathbf{0}_{\mathrm{I}} & c_i(0) & \cdots & c_i((L_c - 1)T_c) & \mathbf{0}_{\mathrm{III}} \end{bmatrix}^T \tag{5.8}$$

where $\mathbf{0}_{\mathrm{III}}$ is a $1 \times L_c - d_j$ vector of zeros. In [Torlak97], the $\mathbf{g}_j$ vector is broken up into two vectors each of length L_c as $\mathbf{g}_j = \begin{bmatrix} \mathbf{g}_j(1)^H & \mathbf{g}_j(2)^H \end{bmatrix}^H$. The observation vector due to user j can then be written as:

$$\mathbf{x}_j(n) = \underbrace{\begin{bmatrix} \mathbf{g}_j(2) & \mathbf{g}_j(1) & & & \\ & \mathbf{g}_j(2) & \mathbf{g}_j(1) & & \\ & & \ddots & \ddots & \\ & & & \mathbf{g}_j(2) & \mathbf{g}_j(1) \end{bmatrix}}_{\mathcal{G}_j,\, KL_c \times (K+1)} \underbrace{\begin{bmatrix} s_j(n) \\ s_j(n+1) \\ \vdots \\ s_j(n+K) \end{bmatrix}}_{\mathbf{s}_j(n),\, (K+1)\times 1} \tag{5.9}$$

The observation vector $\mathbf{x}_j(n)$ of dimension $KL_c \times 1$ corresponds to an observation window for K bit durations (bits from n to $n + K - 1$), sampled at the chip rate. The factor K is called the smoothing factor in [Torlak97]. When the smoothing factor is chosen to be greater than one, it aids the channel estimation algorithm to mitigate the effects of intersymbol interference [Aktas00].

Breaking up the $\mathbf{C}_j$ matrix into two $L_c \times L$ matrices as $\mathbf{C}_j = \left[\mathbf{C}_j(1)^H \; \mathbf{C}_j(2)^H\right]^H$, the global channel matrix of user j can be related to the spreading code matrix; $\mathcal{G}_j = C_j \mathcal{H}_j$

$$\mathcal{G}_j = \underbrace{\begin{bmatrix} \mathbf{C}_j(2) & \mathbf{C}_j(1) & & & \\ & \mathbf{C}_j(2) & \mathbf{C}_j(1) & & \\ & & \ddots & \ddots & \\ & & & \mathbf{C}_j(2) & \mathbf{C}_j(1) \end{bmatrix}}_{C_j,\, KL_c \times L(K+1)} \underbrace{\begin{bmatrix} \mathbf{h}_j & & & \\ & \mathbf{h}_j & & \\ & & \ddots & \\ & & & \mathbf{h}_j \end{bmatrix}}_{\mathcal{H}_j,\, L(K+1)\times(K+1)} \tag{5.10}$$

The matrix formulation of the observation vector is given by:

$$\mathbf{x}(n) = \underbrace{\begin{bmatrix} \mathcal{G}_1 & \mathcal{G}_2 & \cdots & \mathcal{G}_p \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{bmatrix} \mathbf{s}_1(n) \\ \mathbf{s}_2(n) \\ \vdots \\ \mathbf{s}_p(n) \end{bmatrix}}_{\mathbf{S}} + \mathbf{e}(n) \tag{5.11}$$

where $\mathbf{e}$ is a white Gaussian noise vector.

5.2.2 The Bensley/Aazhang Mathematical System Model

The signal that is transmitted by the j^{th} user is given by:

$$b_j(t) = \sqrt{2P_j}\, c_j(t) s_j(t) \cos(\omega_c t + \theta_j) \tag{5.12}$$

where P_j is the transmitted power, ω_c is the carrier frequency and θ_j is the carrier phase relative to the local oscillator at the receiver.

The spreading waveform and the transmitted symbols of the j^{th} user are given by:

$$c_k(t) = \sum_{n=0}^{(L_c-1)T_c} u_{T_c}(t-n) c_j(n) \tag{5.13}$$

$$s_j(t) = \sum_{n=0}^{(L_c-1)T_b} u_{T_b}(t-n) s_j(n) \tag{5.14}$$

where u_{T_c} and u_{T_b} are rectangular pulses of amplitude one with duration T_c and T_b respectively.

The signal that is transmitted by each user is reflected off objects in the channel before it reaches the receiver. As a result, the receiver obtains different versions of the same signal that are each delayed and attenuated by arbitrary amounts. In this section a channel with three multipaths is considered. The signal that is received at the receiver from the j^{th} user is given by:

$$\begin{aligned} x_j(t) &= \int_{-\infty}^{\infty} h_j(t-\tau) b_j(\tau) d\tau \\ &= \int_{-\infty}^{\infty} \left(\sum_{p=1}^{L_{d,j}} \alpha_{p,j} \delta(t-\tau_{p,j}) b_j(\tau) \right) d\tau \\ &= \sqrt{2P_j} \sum_{p=1}^{L_{d,j}} \alpha_{p,j} c_j(t-\tau_{p,j}) s_j(t-\tau_{p,j}) \cos(\omega_c t + \theta_j) \end{aligned} \tag{5.15}$$

where $h_j(t)$ is the impulse response of the channel and $\tau_{p,j}$ is the delay of the p^{th} path for the k^{th} user. When there are P active users transmitting at the same time, the total received signal at the receiver is given by:

$$x(t) = \sum_{j=1}^{P} x_j(t) + n(t) \tag{5.16}$$

where $n(t)$ is assumed to be white Gaussian noise with zero mean and a two-sided power spectral density of $N_0/2$.

Sampling the continuous received signal at the chip rate, the discrete-time signal x(k) is obtained. The discrete-time signal is buffered into blocks of length L_c to obtain the observation vector $\mathbf{x}(k)$, which is given by:

$$\mathbf{x}(k)=\begin{bmatrix}x(k) & x(k+1) & \cdots & x(N-1+k)\end{bmatrix} \tag{5.17}$$

The observation vector has a covariance function that has the desirable property of being invariant to a shift by L_c in both its arguments. Thus, it can be used in many traditional signal-processing techniques that are suited towards wide sense stationary processes. Although the length of observation vector corresponds to one symbol period, it was obtained without regard to the actual symbol intervals of the users. Thus, the observation vector would contain more than one symbol for each user. If the multipath delay spread of the channel is less than half the symbol period, the observation vector would contain the linear combination of $2P$ signal components plus noise.

The subspace-based delay estimation algorithm [Bensley96] is concerned with using the observation vector $\mathbf{x}(k)$ to estimate the channel impulse response $h_j(t)$. Let $\mathbf{u}_j^y$ be a function of the unknown channel impulse response and the known spreading code for the j^{th} user, where y denotes the y^{th} symbol that is contained in the observation vector. Let z_j^w be a function of the power and the w^{th} transmitted symbol of the j^{th} user. Since the observation vector will contain two symbols, let r denote the end of the previous symbol and m the beginning of the current symbol. The observation vector can be written in terms of the signal vector $\mathbf{u}_j^y$, which is estimated by the delay estimation algorithm and the unknown variable z_j^w that is not estimated by the algorithm, in the following way:

$$\begin{aligned}\mathbf{x}(k)&=\sum_{j=1}^{P}\left[z_j^{(w-1)}\mathbf{u}_j^r+z_j^{(w)}\mathbf{u}_j^m\right]+\mathbf{n}\\&=\mathbf{Az}+\mathbf{n}\end{aligned} \tag{5.18}$$

where z_j^w is a constant times $\sqrt{2P_j}s_j(t)$ and $\mathbf{n}=\begin{bmatrix}n(0) & \cdots & n((L_c-1)T_c)\end{bmatrix}$ is the additive noise vector whose elements are chosen from a Gaussian distribution that has a mean of zero and a variance of $\sigma^2=N_0/2T_c$. Matrices $\mathbf{A}$ and $\mathbf{z}$ are given by the following expressions:

$$\mathbf{A}=\begin{bmatrix}\mathbf{u}_1^r & \mathbf{u}_1^m & \cdots & \mathbf{u}_P^r & \mathbf{u}_P^m\end{bmatrix} \tag{5.19}$$

$$\mathbf{z} = \begin{bmatrix} z_1^{(w-1)} & z_1^{(w)} & \cdots & z_P^{(w-1)} & z_P^{(w)} \end{bmatrix}^T \tag{5.20}$$

The signal vectors $\mathbf{u}_j^r$ and $\mathbf{u}_j^m$ depend only on the j^{th} user's spreading waveforms for the r^{th} and m^{th} symbols respectively and the channel impulse response. Two symbols are contained in the observation vector because the symbol period that is used to make up the observation vector is selected without regard to the symbol period of the users.

The r^{th} symbol's spreading code that is contained in the observation vector corresponds to the end of the previous symbol and the m^{th} symbol's spreading code corresponds to the beginning of the current symbol. This is illustrated in Figure 5-2.

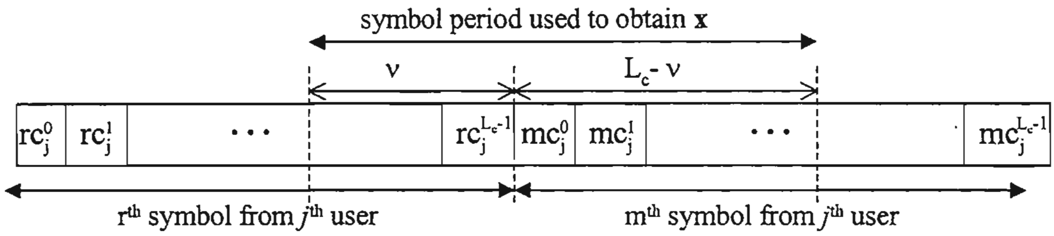

Figure 5-2 Diagram showing that the observation vector is made up of two signal vectors from the j^{th} user, where 'r' and 'm' represent the end of the previous symbol and the beginning of the current symbol respectively.

The symbol v represents the integer multiple of chip periods that are needed to achieve synchronisation between the start of the j^{th} user's symbol period and the symbol period used to obtain the observation vector.

The subspace-based delay estimation algorithm that is presented in [Bensley96] states that "it is possible to gain insight from a simple channel model where the vectors $\mathbf{u}_j^r$ and $\mathbf{u}_j^m$ are the right side of the j^{th} user's code vector followed by zeros, and zeros followed by the left side of the j^{th} user's code vector respectively". From Figure 5-2 it can be seen that the observation vector $\mathbf{x}(k)$ contains $\begin{bmatrix} rc_j^{L_c-1} & rc_j^{L_c-2} & \cdots & rc_j^{L_c-v} \end{bmatrix}$ from the r^{th} symbol and $\begin{bmatrix} mc_j^0 & mc_j^1 & \cdots & mc_j^{L_c-1} \end{bmatrix}$ from the m^{th} symbol.

For a simple channel model where each user's signal travels through a single path with no attenuation and a propagation delay νT_c, the signal vectors are defined in [Bensley96] as follows:

$$\begin{aligned}
\mathbf{u}_j^r &= \mathbf{c}_j^r(\nu) \\
&= \begin{bmatrix} c_j^{L_c-\nu} & \cdots & c_j^{L_c-1} & 0 & \cdots & 0 \end{bmatrix}^T \\
\mathbf{u}_j^m &= \mathbf{c}_j^m(\nu) \\
&= \begin{bmatrix} 0 & \cdots & 0 & c_j^0 & \cdots & c_j^{L_c-1-\nu} \end{bmatrix}^T
\end{aligned} \tag{5.21}$$

The propagation delay $\tau_{1,j}$ can be expressed as:

$$\tau_{1,j} = \nu T_c \tag{5.22}$$

where $\nu \in \{0, \ 1, \ \cdots \ T_m\}$ and T_m is the multipath spread of the channel.

In a multipath transmission channel, the delays are likely to be different. As a result, the signal vectors will be a weighted sum of the combinations corresponding to each path:

$$\begin{aligned}
\mathbf{u}_j^r &= \sum_{p=1}^{L_{d,j}} \alpha_{p,j} \mathbf{c}_j^r(\nu_{p,j}) \\
\mathbf{u}_j^m &= \sum_{p=1}^{L_{d,j}} \alpha_{p,j} \mathbf{c}_j^m(\nu_{p,j})
\end{aligned} \tag{5.23}$$

From the observation vector, the signal vectors are not known. However, the set of vectors that span the subspace of all the possible signal vectors is known. The signal vectors $\mathbf{u}_j^r$ and $\mathbf{u}_j^m$ lie in the column space of the matrices $\mathbf{V}_j^r$ and $\mathbf{V}_j^m$ respectively.

$$\mathbf{V}_j^r = \begin{bmatrix} 0 & c_j^{L_c-1} & c_j^{L_c-2} & & c_j^1 \\ \vdots & 0 & c_j^{L_c-1} & & c_j^2 \\ & \vdots & 0 & \ddots & \vdots \\ & & \vdots & & c_j^{L_c-1} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{5.24}$$

$$\mathbf{V}_j^m = \begin{bmatrix} c_j^0 & 0 & \cdots & 0 \\ c_j^1 & c_j^0 & \ddots & 0 \\ \vdots & c_j^1 & & \vdots \\ & \vdots & \ddots & 0 \\ c_j^{L_c-1} & c_j^{L_c-2} & & c_j^0 \end{bmatrix} \tag{5.25}$$

5.3 The Subspace Approach

Subspace-based algorithms use second order statistics to estimate the parameter of interest. Subspace-based delay and channel estimation algorithms are no different. They use the second order statistics of the received signal, called the correlation matrix. The correlation matrix is used to obtain an estimate of the signal and the noise subspaces. Finally, either the signal or noise subspace is used to estimate the channel or the multipath delays of the desired user.

In the Torlak/Xu channel estimation algorithm and the Bensley/Aazhang delay estimation algorithm different approaches are used to determine the dimension of the signal and noise subspaces that are derived from the mathematical description of the system model. Thus, both approaches are discussed in this section and it is shown later in the section that the dimension of the noise subspace in [Torlak97] and [Bensley96] are the same for a stacking factor of one.

5.3.1 Estimating the Signal and Noise Subspaces – Bensley/Aazhang

The observation vector $\mathbf{x}(k)$ that is given in (5.17) consists of $2P$ signal vectors. The column space of the signal matrix $\mathbf{A}$ given in (5.18) is the subspace for all signal vectors. Thus, in the absence of noise, the observation vector is a linear combination of the columns of the signal matrix $\mathbf{A}$. In practise the observation vectors are corrupted with noise. Thus, the observation space may be broken up into the signal subspace and the noise subspace. The Bensley/Aazhang delay estimation algorithm estimates the signal subspace and the noise subspace and then determines the impulse response of the channel by minimising the projection of the user's signal vectors into the estimated noise subspace. The algorithm has the desirable property that the impulse response of the channel for each user can be obtained independently of the others. This breaks down the channel estimation task into 1-D problems for each user, which drastically decreases complexity.

The column space of matrix **A** spans the true signal subspace. A matrix whose column space projects onto the signal subspace is the correlation matrix $\mathbf{R}_{xx}$. When the observation vectors are not corrupted by noise, the correlation matrix $\mathbf{R}_{xx}$ projects perfectly onto the true signal subspace. In the presence of noise, the correlation matrix $\mathbf{R}_{xx}$ projects onto the signal and noise subspace:

$$\mathbf{R}_{xx} = \mathbf{R}_{ss} + \mathbf{R}_{nn} \tag{5.26}$$

where $\mathbf{R}_{ss} = E\left[b(t)b(t)^H\right]$ is the signal correlation matrix and $\mathbf{R}_{nn} = E\left[n(t)n(t)^H\right]$ is the noise correlation matrix.

In this way the observation space can be divided into a signal subspace and a noise subspace. When there are P users in the system, the observation vector contains $2P$ signal components. Thus, the signal subspace has dimension $2P$. An estimate for the signal subspace is given by the span of the eigenvectors of the signal covariance matrix $\mathbf{R}_{ss}$. The estimate for the signal subspace is given by the eigenvectors corresponding to the $2P$ largest eigenvalues of the correlation matrix $\mathbf{R}_{xx}$. The correlation matrix is estimated using N observation vectors:

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N}\sum_{k=1}^{N} x(k)x(k)^T \tag{5.27}$$

The estimate for the signal and noise subspace can be obtained by performing the eigenvector decomposition on the estimated correlation matrix $\hat{\mathbf{R}}_{xx}$:

$$\mathrm{EVD}(\hat{\mathbf{R}}_{xx}) = \begin{bmatrix}\mathbf{U}_s & \mathbf{U}_n\end{bmatrix}\begin{bmatrix}\Sigma_s & \\ & \Sigma_o\end{bmatrix}\begin{bmatrix}\mathbf{V}_s^H \\ \mathbf{V}_o^H\end{bmatrix} \tag{5.28}$$

where $\mathbf{U}_s$ and $\mathbf{U}_n$ correspond to the estimated signal and noise subspaces respectively, Σ_s and Σ_n denote the eigenvalues corresponding to the eigenvectors of $\mathbf{U}_s$ and $\mathbf{U}_n$ respectively.

The estimated signal subspace $\mathbf{U}_s$ is obtained by choosing the eigenvectors corresponding to the largest $2P$ eigenvalues and the remaining $L_c - 2P$ eigenvectors span the estimated noise subspace $\mathbf{U}_n$.

5.3.2 Estimating the Signal and Noise Subspaces – Torlak/Xu

In the Torlak/Xu algorithm, the orthogonal property of the signal and the noise subspaces is used to form an optimisation formula to estimate the channel. The received vector is used to form a data matrix [Torlak97] by concatenating the observation vectors in the following way:

$$\mathbf{X} = \left[\mathbf{x}(n) \quad \mathbf{x}(n+1) \quad \cdots \quad \mathbf{x}(n+N-K-1)\right] \tag{5.29}$$

where N is the total number of received bits used to perform the task of channel estimation.

The estimates of the signal and noise subspaces are obtained by applying the EVD to the estimated correlation matrix $\hat{\mathbf{R}}_{xx} = \mathbf{X}\mathbf{X}^H$:

$$\text{EVD}(\hat{\mathbf{R}}_{xx}) = \left[\mathbf{U}_s \quad \mathbf{U}_n\right] \begin{bmatrix} \Sigma_s & \\ & \Sigma_o \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_o^H \end{bmatrix} \tag{5.30}$$

The columns of $\mathbf{U}_s$ and $\mathbf{U}_o$ correspond to the estimated signal subspace and noise subspace respectively, Σ_s and Σ_o correspond to the eigenvalues of $\mathbf{U}_s$ and $\mathbf{U}_o$. The dimension of $\mathbf{U}_o$ is $KL_c \times KL_c - P(K+1)$ and it is given by the left nullspace of the matrix $\mathbf{G}$ (5.11). Since the dimension of the estimated correlation matrix is $KL_c \times KL_c$, the remaining eigenvectors are used to form the estimate for the signal subspace $\mathbf{U}_s$ which has dimensions $KL_c \times P(K+1)$.

Lastly, the number of vectors that span the noise subspace for the Torlak/Xu and the Bensley/Aazhang system models are compared below:

Torlak/Xu system model: number of columns of $\mathbf{U}_o = KL_c - P(K+1)$ (5.31)

Bensley/Aazhang system model: number of columns of $\mathbf{U}_o = L_c - 2P$ (5.32)

It is clearly seen that for a stacking factor K of one, the number of vectors that span the noise subspace for both system models are the same.

5.4 Channel and Delay Estimation Algorithms

In this section the Torlak/Xu channel estimation algorithm and the Bensley/Aazhang delay estimation algorithm are discussed. The proposed parametric Torlak/Xu algorithm is also formulated so that the performance of the new algorithm can be compared to the Bensley/Aazhang algorithm.

5.4.1 The Bensley/Aazhang Delay Estimation Algorithm

The Bensley/Aazhang delay estimation algorithm determines the multipath delays for the j^{th} user by projecting the user's signal vectors (5.23) into the estimated noise subspace:

$$\begin{aligned} \mathbf{e}_k^r &= \left(\left(\mathbf{u}_k^r\right)^T \mathbf{U}_n\right)^T \\ \mathbf{e}_k^m &= \left(\left(\mathbf{u}_k^m\right)^T \mathbf{U}_n\right)^T \end{aligned} \tag{5.33}$$

When there is no noise present in the system, the signal vectors $\mathbf{u}_k^r$ and $\mathbf{u}_k^m$ lie within the signal subspace, thus $\mathbf{e}_k^r$ and $\mathbf{e}_k^m$ are zero. The channel parameter $\mathbf{h}_j$ is estimated using the property that the signal vectors lie in the nullspace of the noise subspace, which is the signal subspace.

However, the noise subspace $\mathbf{U}_n$ that is obtained from the eigenvector decomposition of the estimated correlation matrix defined in (5.28) is only an estimate of the true noise subspace. Thus, Bensley/Aazhang algorithm aims to determine the channel impulse response by minimising the Euclidean distance of the projection of the signal vectors into the estimated noise subspace:

$$\hat{\mathbf{h}}_j = \arg \min_{\|\mathbf{h}_j = 1\|} \left\|\left(\mathbf{u}_k^r\right)^T \mathbf{U}_n\right\|^2 + \left\|\left(\mathbf{u}_k^m\right)^T \mathbf{U}_n\right\|^2 \tag{5.34}$$

Substituting the matrices given in (5.24) and (5.25), whose columns span the subspace of the signal vectors, into (5.34) gives:

$$\hat{\mathbf{h}}_j = \arg \min_{\|\mathbf{h}_j=1\|} \mathbf{h}_j^T \underbrace{\left[\left(\mathbf{V}_j^r\right)^T \mathbf{U}_n \mathbf{U}_n^T \mathbf{V}_j^r + \left(\mathbf{V}_j^m\right)^T \mathbf{U}_n \mathbf{U}_n^T \mathbf{V}_j^m \right]}_{\mathbf{M}} \mathbf{h}_j \tag{5.35}$$

The minimisation equation given in (5.35) contains the channel's impulse response $\mathbf{h}_j$ that is unknown. By introducing the constraint $\|\mathbf{h}_j\| = 1$, the solution to (5.35) is found by choosing the normalised eigenvector corresponding to the smallest eigenvalue of $\mathbf{M}$. In this way $\hat{\mathbf{h}}_j$ is chosen so that that the projection of the user's signal vectors into the noise subspace is minimised.

The Bensley/Aazhang algorithm uses $\hat{\mathbf{h}}_j$ to determine the j^{th} user's delays for each path by performing a least-squares fit of a single path to each pair of adjacent coefficients. The solution to the least-squares problem is given in [Bensley96]:

$$\boldsymbol{\alpha}_j(\nu) = \hat{\mathbf{h}}_j(\nu) \tag{5.36}$$

where $\nu \in \{0, \ 1, \ \cdots \ , T_m\}$. The estimated delay for the strongest path is obtained by the index of the strongest value of $\boldsymbol{\alpha}_j$:

$$\hat{\nu} = \max \ \left|\boldsymbol{\alpha}_j(\nu)\right| \tag{5.37}$$

Hence the delay for the strongest path is given by:

$$\hat{\tau}_{1,j} = \hat{\nu} T_c \tag{5.38}$$

In order to estimate the second strongest path, the largest value of $\hat{\mathbf{h}}_j$ is set to zero and Equations (5.37) and (5.38) are repeated to find the next strongest path. This is done until an estimate for the given number of paths has been determined.

5.4.2 The Torlak/Xu Channel Estimation Algorithm

The Torlak/Xu algorithm uses the following properties to estimate the channel for the j^{th} user:

- The orthogonal property of the noise subspace and the global channel matrix $\mathcal{G}_j$.
- The commutative property [Moulines95] between a Toeplitz matrix and a vector.

In [Torlak97], it is noted that the columns of $\mathcal{G}_j$ are orthogonal to any vector in the noise subspace:

$$\mathbf{U}_o \perp \mathcal{G}_j \quad \Rightarrow \quad \mathbf{U}_o^H \mathcal{G}_j = 0 \qquad j = 1, \quad \cdots \quad ,P \tag{5.39}$$

$$\therefore \mathbf{U}_l^H \mathcal{G}_j = 0 \qquad l = 1, \quad \cdots \quad ,KL_c - P(K+1) \tag{5.40}$$

where l denotes the l^{th} column of the matrix $\mathbf{U}_0$.

Since $\mathcal{G}_j$ is a Toeplitz matrix, the commutative identity developed in [Moulines95] can be used to simply the optimisation formula that identifies the unknown channel $\mathbf{h}_j$. Therefore, the left hand side of Equation (5.40) can be written as:

$$\mathbf{U}_l^H \mathcal{G}_j = \mathbf{g}_j^H \mathcal{V}_l \tag{5.41}$$

where

$$\mathcal{V}_l = \begin{bmatrix} u_1^{(l)} & u_2^{(l)} & \cdots & u_K^{(l)} & \mathbf{0} \\ \mathbf{0} & u_1^{(l)} & u_2^{(l)} & \cdots & u_K^{(l)} \end{bmatrix}$$

$$\mathbf{U}_l = \begin{bmatrix} u_1^{(l)} \\ u_2^{(l)} \\ \vdots \\ u_K^{(l)} \end{bmatrix}, \qquad u_k^{(l)} \; is \; L_c \times 1, \quad k = 1, \ldots, K \tag{5.42}$$

Using (5.41) and (5.7), Equation (5.40) can be written as:

$$\begin{aligned} \mathbf{g}_j^H \mathcal{V}_l &= 0 \\ \mathbf{h}_j^H \mathbf{c}_j^H \mathcal{V}_l &= 0 \end{aligned} \tag{5.43}$$

It can be seen from Equation (5.43) that the channel is identified by the left nullspace of the matrix $\mathbf{c}_j^H \mathcal{V}_l$. Since there is a total of $KL_c - P(K+1)$ vectors that span the noise subspace, the minimisation equation used to identify the channel [Torlak97] is given by:

$$\tilde{\mathbf{h}}_j = \arg \min_{\|\mathbf{h}_j\|=1} \mathbf{h}_j^H \underbrace{\mathbf{c}_j^H \left[\sum_{l=1}^{KL_c - P(K+1)} \mathcal{V}_l \mathcal{V}_l^H \right] \mathbf{c}_j}_{\mathbf{M}} \mathbf{h}_j \tag{5.44}$$

$$\hat{\theta}_j = phase(\tilde{\mathbf{h}}_j^H \mathbf{h}_j) \tag{5.45}$$

$$\hat{\mathbf{h}}_j = e^{i\hat{\theta}} \tilde{\mathbf{h}}_j \tag{5.46}$$

The unknown channel of the j^{th} user is identified by the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{M}$. However, the estimated channel in Equation (5.44) has a phase ambiguity that is present in all blind channel estimation algorithms. Equations (5.45) and (5.46) is used to correct the phase ambiguity of the estimated channel.

The task of the non-parametric TX algorithm is to estimate the tap coefficients of the FIR filter representing the unknown multipath channel. In the next section it is shown how the non-parametric TX channel estimation algorithm is converted to the proposed Parametric TX algorithm that is concerned with the estimating the multipath delays of the desired user.

5.4.3 The Proposed Parametric Torlak/Xu Algorithm

The Parametric TX method estimates the delay of each path that is introduced by the channel. The multipath delays for the j^{th} user $\tau_{q,j}$ $(q = 1, \cdots, L_{d,j})$ are estimated by the proposed Parametric TX method by using the estimated channel $\hat{\mathbf{h}}_j$ obtained from the non-parametric TX method.

The delay of the q^{th} path for the j^{th} user is estimated using the following equations:

$$\hat{v}_{q,j} = \underset{v\in\{0,T_c,\cdots,T_m\}}{\arg\max} \left|\hat{\mathbf{h}}_j(v)\right| \qquad j\in\{1,\cdots,P\} \tag{5.47}$$

$$\hat{\tau}_{q,j} = \hat{v}_{q,j} \qquad q\in\{1,\cdots,L_{d,j}\} \tag{5.48}$$

where T_m is the multipath spread of the channel.

The delay of the path with the strongest energy is obtained using Equations (5.47) and setting q to one.

To find the next strongest path, the maximum value of estimated channel $\hat{\mathbf{h}}_j(\hat{\tau}_{l,j})$ is subtracted from $\hat{\mathbf{h}}_j$ and Equation (5.47) is evaluated, with q set to 2. This process is repeated until a delay estimate for each path has been obtained.

5.5 Simulation Results

Simulation results are presented in this section that compare the performance of the proposed Parametric TX algorithm to the Bensley/Aazhang delay estimation algorithm in a quasi-static fading channel. In all the results, each user has a Gold code with a spreading code length of 31. A linear MIP channel is chosen such that the power of the multipaths decrease linearly with the delay in such a way that $|p(\tau=0)|=1$ and $|p(\tau=15T)|=0.1$. The fading variable r is generated using a normal random distribution with a mean of zero and standard deviation of one. The attenuation of the q^{th} path is given by the expression

$$\alpha_{q,j} = r_{q,j}p(\tau_{q,j}) \tag{5.49}$$

The channel is assumed to change so slowly that the delays and the attenuations are fixed during the observation period. The Bensley/Aazhang delay estimation algorithm was formulated using a smoothing factor K of one. For this reason, the smoothing factor in the simulations is set to a value of one. In [Bensley96], the multipath spread of the channel is assumed to be less than half of a bit duration. Therefore, in the simulations the multipath spread T_m of the channel is set to $15T_c$. The channel of three paths is chosen, where the delays of the j^{th} user is chosen from a uniform distribution with $\tau_{q,j}\in U[0,\ T_c,\ \cdots\ ,15T_c]$ and $q\in\{1,\ 2,\ 3\}$.

The number of Monte Carlo trials used in all the simulations is 500. In both the parametric subspace algorithms the asynchronous delay d_j for the j^{th} user is known. A delay estimation algorithm for d_j is presented in [Aktas00]. In the simulation results shown in this section, the performance of the proposed parametric Torlak/Xu algorithm is compared to the Bensley/Aazhang algorithm by varying one of the system parameters: SNR, length of the observation window, number of users and the power difference between five strong users and one weak user, while keeping the other parameters constant. Table 4 gives the values of the system parameters that are kept constant in the simulations when it is not the parameter being varied.

Table 4 Values of simulation parameters used

Simulation Parameter	Value
SNR	10dB
Length of observation window	200 bits
Power difference between the strongest and weakest user	0dB
Number of users	6

The performance criteria used is the probability of acquisition, P_{ac}, which is defined as the average of all the users' probability of acquisition:

$$P_{ac,j} = \frac{\text{number of correctly identified delays for user } j}{\text{number of delays for user } j} \tag{5.50}$$

$$P_{ac} = \frac{1}{P}\sum_{j=1}^{P} P_{ac,j} \tag{5.51}$$

where the number of users P equals to six.

The probability of acquisition as a function of SNR is shown in Figure 5-3. The performance of the proposed Parametric TX method is clearly shown to be better than the Bensley/Aazhang delay estimation algorithm. It is interesting to note that despite fading, the probability of acquisition for both algorithms approach unity when thermal noise is reduced.

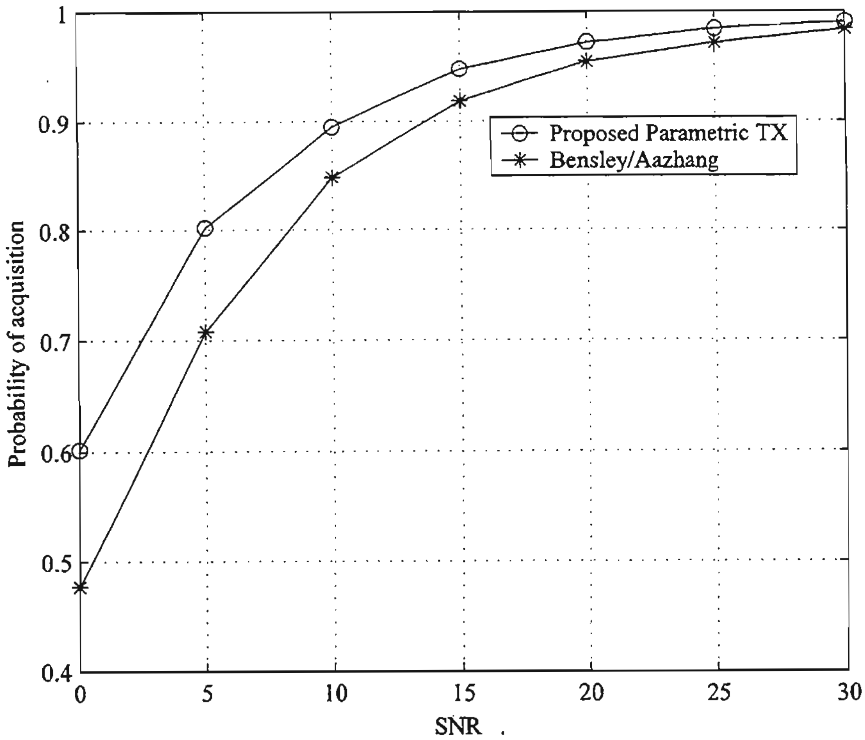

Figure 5-3 Probability of acquisition as a function of SNR

In Figure 5-4, the effect of the observation length is investigated. It is shown that the probability of acquisition of both methods increase as the length of the observation window increases. This result can be expected because as the length of the observation window is increased, the estimate of the correlation matrix $\hat{\mathbf{R}}_{xx}$ becomes more accurate. A more accurate estimate of the correlation matrix leads to a more accurate estimate of the noise subspace. Since the optimisation equations that identify the channel in (5.44) and (5.35) are a function of the vectors that span the estimated noise subspace, a more accurate representation of the noise subspace will result in a more accurate delay estimation algorithm.

In Figure 5-5, the influence of the number of users on the performance of both subspace algorithms is studied. As the number of users increase, the performance of the algorithms decreases. This can be explained by inspecting the dimension of the estimated noise subspace in Equations (5.31) and (5.32) as the number of users increase. It can be seen that as the number of users increase, the number of vectors that span the estimated noise subspace decreases. An inaccurate estimate of the noise subspace leads to a degraded delay estimation algorithm.

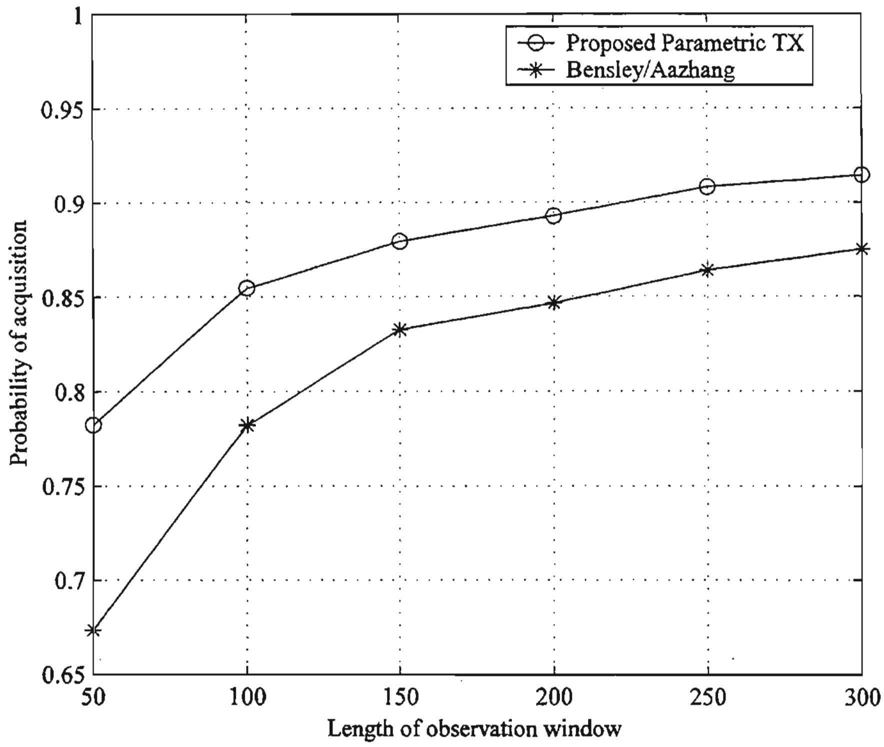

Figure 5-4 Probability of acquisition as a function of the observation window length

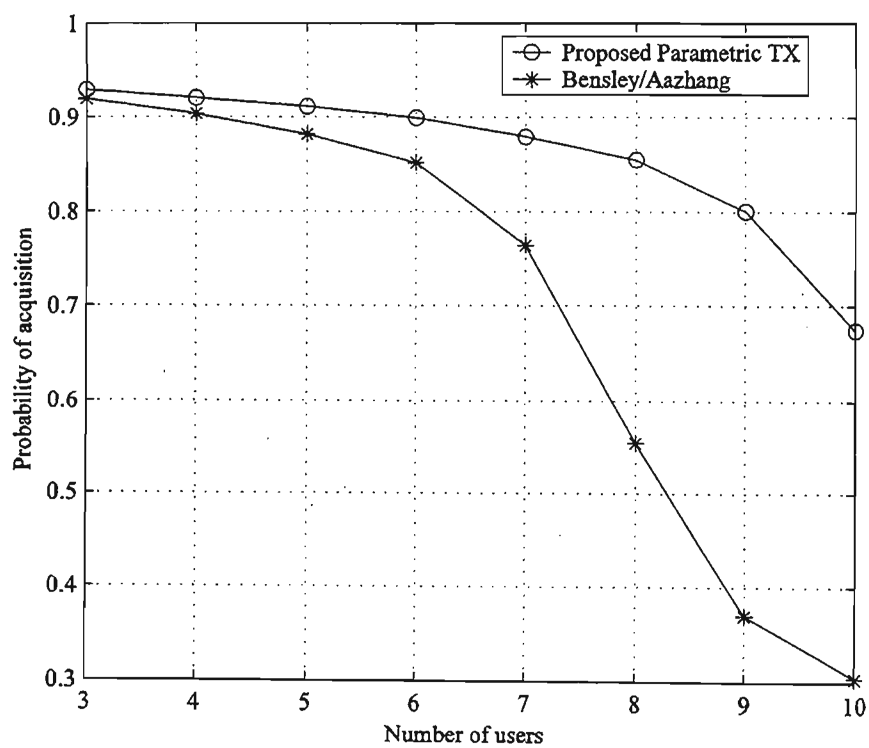

Figure 5-5 Probability of acquisition as a function of the number of users

In Figure 5-6, the near far-effect is investigated. A system with five strong users with equal power and one weak user is considered. The user with the weakest power is the first user whose power is denoted by P_1. The strong users' power is denoted by P_j, where $j \in \{2, \cdots ,5\}$. The probability of acquisition for the weak user is plotted against the power difference between the strong users and the weak user P_j / P_1. Since the probability of acquisition of the weak user remains fairly constant as the power of the strong users are increased, both subspace algorithms are resistant to the near-far effect. However, the proposed Parametric TX algorithm offers the better performance.

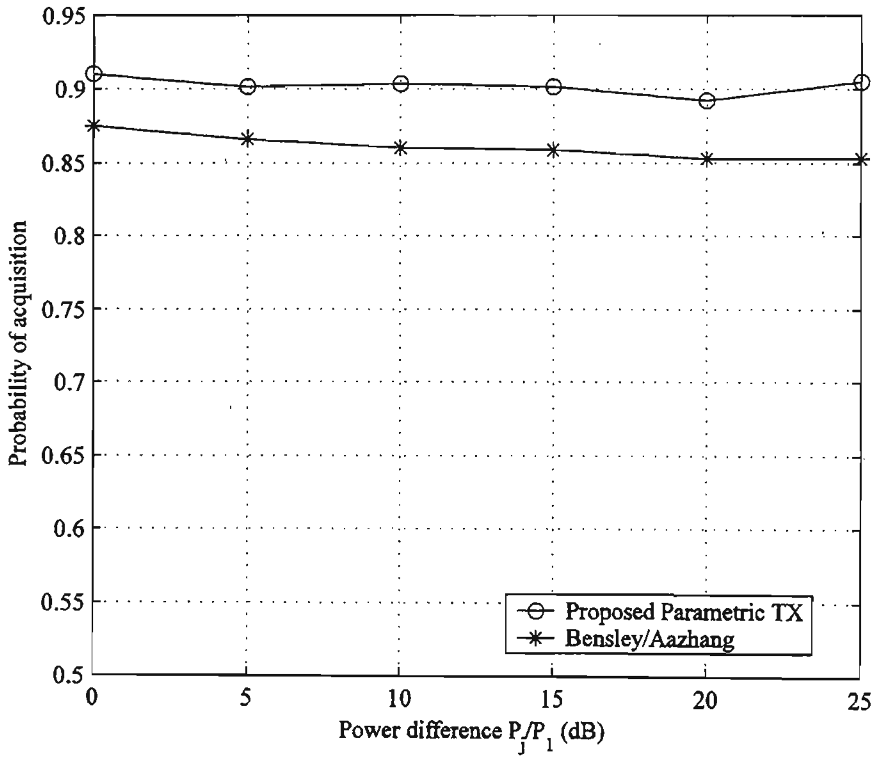

Figure 5-6 Probability of acquisition of the weakest user as a function of the power difference between the strongest and weakest user.

5.6 Summary

In this chapter the subspace-based channel estimation algorithm proposed by Torlak/Xu was converted from a non-parametric method to a parametric method. The performance of the proposed Parametric TX method was compared to the Bensley/Aazhang algorithm. This was done to find which subspace algorithm offered the better performance.

The performance of the two algorithms was compared by varying one of the system's parameters: SNR, number of users, length of the observation window and the power difference between the strongest and weakest user, while keeping the others constant. It was shown that the proposed Parametric TX algorithm offered the better performance under all conditions.

Chapter 6

Conclusion

Third-generation cellular communication systems have been proposed to increase the capabilities of 2G systems by providing high data rate services such as video on demand, video conferencing, web browsing and e-mail retrieval. These 3G systems are required to operate in a multipath channel that distorts the transmitted signal. One of the countermeasures that have been proposed to combat the adverse effects of multipath propagation in 3G systems is the use of a Rake receiver. However, the Rake receiver requires knowledge of the channel estimates.

Recently, semi-blind algorithms have been developed to estimate the channel in 3G systems using a combination of a blind subspace algorithm and a training-based algorithm. It is shown in the literature that the performance of the semi-blind method is better than the performance of the blind or the training-based algorithms alone. For this reason, the focus of this dissertation is to investigate and improve the performance of blind subspace algorithms. The new algorithms that are presented in this dissertation can be used in one of the WCDMA radio air interface standards for 3G systems.

6.1 Dissertation Summary

A broad overview of 3G systems and motivation for blind channel estimation algorithms was provided in Chapter 1.

In Chapter 2, a review of blind subspace-based channel estimation algorithms for single-user systems was presented. It was found from the literature that the subspace algorithm that attracted the most amount of research was the Channel Subspace algorithm. For this reason, Chapter 2 placed emphasis on the Channel Subspace algorithm together with the algorithms that were proposed to improve the performance of the Channel Subspace algorithm. Some of the subspace-based channel estimation algorithms that have been developed for DS/CDMA systems were also discussed.

Chapter 3 described the concepts, assumptions and the mathematical formulation of the Channel Subspace algorithm for single-user systems. It was shown how the Channel Subspace algorithm was made to incorporate pulse-shaping information to form the Parametric Subspace algorithm. Concepts of the Parametric Subspace algorithm were used to derive a new channel estimation algorithm for DS/CDMA systems that also incorporated pulse-shaping information. The new algorithm extended the Parametric Subspace algorithm from single-user systems to multi-user DS/CDMA systems. It was shown by simulations that the performance of the proposed channel estimation algorithm is better than the Torlak/Xu algorithm that does not use pulse-shaping information. It was also observed that the new algorithm could operate at a SNR of 11dB lower than the Torlak/Xu algorithm for both algorithms to offer the same performance. Lastly, a mathematical expression of the mean square error of estimation for the proposed algorithm was derived. It was shown that the analytic expression provided a good approximation of the actual MSE for high SNR.

In Chapter 4, a new delay estimation algorithm called the Modified Parametric Subspace Delay Estimation (MPSDE) algorithm was presented. The MPSDE algorithm was based on the PSDE algorithm that uses pulse-shaping information to estimate the multipath delays introduced by the channel. It was shown by simulations that the performance of the PSDE algorithm degrades when it operates in a DS/CDMA environment where the power of the multipath signals decrease with increasing delay. This was seen as a limitation of the PSDE algorithm. The MPSDE algorithm was proposed to improve the performance of the PSDE algorithm in an environment where the power of the multipath signals varies.

It was also shown by simulations that the performance of the proposed blind channel estimation algorithm in Chapter 3 that uses the MPSDE algorithm to estimate the multipath delays is better than the Torlak/Xu algorithm that does not use pulse-shaping information. It was noted that for high SNR that is greater than 30dB and a large observation window that is greater than 300 bits, the average probability of acquisition of the MPSDE algorithm approaches one. The MPSDE algorithm was also shown to be near-far resistant, which is a traditional property of subspace algorithms.

The Torlak/Xu channel estimation algorithm and the Bensley/Aazhang delay estimation algorithm are discussed in detail in Chapter 5. In order to compare the performance of the two subspace algorithms, the Torlak/Xu algorithm was converted to a delay estimation algorithm that is called the Parametric TX algorithm. The performance of the Bensly/Aazhang and the proposed Parametric TX algorithm was compared by varying one of the system's parameters: SNR, number of users, length of the observation window and the power difference between the strongest and weakest user, while keeping the others constant. It was shown that the proposed Parametric TX offered the better performance under all conditions.

6.2 Future Directions

There are some future research topics to build on the work presented in this dissertation:

- The new channel estimation algorithm proposed in Chapter 3 has a high computational complexity due to the final EVD operation used to identify the channel. One method that could be used to reduce the complexity of the new algorithm is to use a subspace-tracking algorithm [Attallah02] to adaptively estimate the time-changing noise subspace. A subspace-tracking algorithm would also be advantageous for the new channel estimation algorithm to easily track a fast changing channel.

- The semi-blind algorithm in [Lasaulce00] combines the blind Torlak/Xu subspace algorithm and the training-based Least Squares algorithm to estimate the channel. It would be interesting to propose a semi-blind algorithm that uses the new channel estimation algorithm proposed in Chapter 3 to blindly estimate the channel. The proposed semi-blind method is expected to have a better performance than the algorithm in [Lasaulce00] because in Chapter 3, the new blind channel estimation algorithm was shown to have a far superior performance to the Torlak/Xu algorithm.

Appendix A

The Channel Subspace Algorithm: A Numerical Example

In this section an example of a single-user SIMO system is presented and the channel is estimated using the Channel Subspace algorithm. This example is intended to enhance the reader's understanding of the Channel Subspace algorithm by putting numerical numbers into the equations that were formulated in Section 3.4.4.

1. A system model with the following properties is considered:
   - Channel length $L = 1$
   - Number of receive antennas $q = 1$
   - Oversampling factor $p = 2$

As a numerical example, the oversampled observation vector from q antennas for one symbol duration (3.4) is given by:

$$\mathbf{x}(\mathrm{n}) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{H}} \mathbf{s}(\mathrm{n}) + \mathbf{e}(\mathrm{n}) \tag{A.1}$$

Choosing a stacking factor K of two, the observation vector from two successive bit durations may be expressed as:

$$\mathbf{x}_2(\mathrm{n}) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{H}_3} \mathbf{s}_3(\mathrm{n}) + \mathbf{e}_2(\mathrm{n}) \tag{A.2}$$

Note that the value of K was chosen to satisfy the constraints in Equation (3.8) for the left nullspace of $\mathbf{H}_3$ to exist. The dimension of the left nullspace of $\mathbf{H}_3$ is given by the number of rows subtracted by the number of columns: $4-3=1$. Thus, the number of vectors that span the noise subspace is one.

2. The noise component in (A.2) is ignored to show that the Channel Subspace algorithm obtains perfect channel estimated in a noise-free environment. The correlation matrix is estimated by choosing data symbols $s(n)$ randomly from the set $\{-1,+1\}$. Choosing the total bit duration $N=200$ of the received signal to perform the task of channel estimation, the correlation matrix has numerical values:

$$\hat{\mathbf{R}}_{xx}(0)=\frac{1}{N}\sum_{k=1}^{N}\mathbf{x}_2(k)\mathbf{x}_2^H(k)=\begin{bmatrix}1.0000 & 0.0099 & 0.0099 & -0.1881\\ 0.0099 & 1.0000 & 1.0000 & -0.0297\\ 0.0099 & 1.0000 & 1.0000 & -0.0297\\ -0.1881 & -0.0297 & -0.0297 & 1.0000\end{bmatrix} \tag{A.3}$$

To find the eigenvectors that span the noise subspace, the eigenvector decomposition was performed on the estimated correlation matrix:

$$\begin{aligned}\mathrm{EVD}(\hat{\mathbf{R}}_{xx}\hat{\mathbf{R}}_{xx}^H)&=\begin{bmatrix}\mathrm{U}_1 & \mathrm{U}_2\end{bmatrix}\begin{bmatrix}\Sigma_1 & \\ & \Sigma_2\end{bmatrix}\\ &=\begin{bmatrix}-0.0413 & 0.7011 & -0.7118 & 0.0000\\ 0.7065 & 0.0145 & -0.0267 & 0.7071\\ 0.7065 & 0.0145 & -0.0267 & -0.7071\\ 0.0119 & 0.7128 & 0.7013 & 0.0000\end{bmatrix}\begin{bmatrix}4.0076 & 0 & 0 & 0\\ 0 & 1.1006 & 0 & 0\\ 0 & 0 & 0.9006 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\end{aligned} \tag{A.4}$$

From the EVD of the correlation matrix it can be seen that the vector that spans the one dimensional noise subspace is given by the eigenvector that corresponds to the smallest eigenvalue of zero: $\mathbf{w}^{(1)}=\begin{bmatrix}0 & 0.701 & -0.7071 & 0\end{bmatrix}^H$.

3. The vector that spans the noise subspace is partitioned into $K=2$ sub-vectors and the Toeplitz matrix $\mathcal{W}^{(1)}$ is formed:

$$\mathcal{W}^{(1)} = \begin{bmatrix} 0 & -0.7071 & 0 \\ 0.7071 & 0 & 0 \\ 0 & 0 & -0.7071 \\ 0 & 0.7071 & 0 \end{bmatrix} \tag{A.5}$$

The channel is identified by finding the left nullspace of $\mathcal{W}^{(1)}$. Equivalently, the channel is found by performing the EVD operation on $\mathcal{W}^{(1)}\left[\mathcal{W}^{(1)}\right]^H$:

$$\begin{aligned} \mathrm{EVD}\left(\mathcal{W}^{(1)}\left[\mathcal{W}^{(1)}\right]^{\mathrm{H}}\right) &= \begin{bmatrix} \mathrm{U}_1 & \mathrm{U}_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix} \\ &= \begin{bmatrix} -0.7071 & 0.0000 & -0.0000 & -0.7071 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ -0.0000 & 0.0000 & -1.0000 & 0.0000 \\ 0.7071 & 0.0000 & -0.000 & -0.7071 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{A.6}$$

and finding the eigenvector corresponding to the smallest eigenvalue. Thus the unknown channel estimated as:

$$\hat{\hat{\mathbf{h}}} = \begin{bmatrix} -0.7071 & 0 & 0 & -0.7071 \end{bmatrix}^H \tag{A.7}$$

4. Partitioning $\hat{\hat{\mathbf{h}}}$ into $K = 2$ sub-vectors, the channel impulse response matrix is identified as:

$$\begin{aligned} \hat{\mathbf{H}} &= \begin{bmatrix} -0.7071 & 0.0000 \\ 0.000 & -0.7071 \end{bmatrix} / \beta \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{A.8}$$

where $\beta = -1/0.7071$. Thus the Channel Subspace algorithm correctly estimates the channel up to a scalar multiple, since the estimated channel $\hat{\mathbf{H}}$ (A.8) and the actual channel $\mathbf{H}$ (A.1) are exactly the same for $\beta = -1/0.7071$.

Appendix B

Mathematical Analysis of the Mean Square Error

This section describes the mathematical derivation of the mean square error of the spatial attenuation estimation for the proposed algorithm in Chapter 3. The analysis that follows is based on the mathematical formulation of the mean error for the Torlak/Xu algorithm that can be found in [Aktas00] and [Aktas98]. It was shown in Chapter 3 that the spatial attenuation of the j^{th} user was found from the nullvector of $\mathbf{Q}_{ps,j}$, where:

$$\mathbf{Q}_{ps,j} = \left[\mathbf{G}^{j}_{L,d}(\boldsymbol{\tau})\right]^{H}\left[\mathbf{c}_{j,\mathbf{I}_{pq}}\right]^{H}\left[\mathcal{W}^{(1)} \quad \cdots \quad \mathcal{W}^{(z_m)}\right] \tag{B.1}$$

In the mathematical derivations that follow, the k^{th} vector that spans the true noise subspace, which is obtained from the observation without noise, is denoted by $\mathbf{w}^{(k)}$ $(pqKL_c \times 1)$ and the k^{th} vector that spans the estimated noise subspace, which is obtained from noise observations, is given by $\hat{\mathbf{w}}^{(k)}$. It is important to note that the spatial attenuation $\mathbf{a}_j$ for the j^{th} user is the null vector of $\mathbf{Q}_{ps,j}$, and the estimate $\hat{\mathbf{a}}_j$ is the null vector of $\hat{\mathbf{Q}}_{ps,j}$. The matrices and their estimates are related by:

$$\hat{\mathbf{w}}^{(k)} = \mathbf{w}^{(k)} + \Delta\mathbf{w}^{(k)} \tag{B.2}$$

$$\hat{\mathbf{Q}}_{ps,j} = \mathbf{Q}_{ps,j} + \Delta\mathbf{Q}_{ps,j} \tag{B.3}$$

$$\hat{\mathbf{a}}_j = \mathbf{a}_j + \Delta\mathbf{a}_j \tag{B.4}$$

The following matrices are defined to simply the derivations that follow:

$$\mathbf{V}_p = \left[\mathcal{V}_1 \quad \cdots \quad \mathcal{V}_P\right] \tag{B.5}$$

$$\mathbf{S} = \left[\mathbf{s}_1^H(n) \quad \cdots \quad \mathbf{s}_P^H(n)\right]^H \tag{B.6}$$

Thus $\mathbf{V}_p\mathbf{S} = \sum_{j=1}^{P} \mathcal{V}_j\mathbf{s}_j(n)$

The first order approximation of the perturbed subspace, which is given by Equation (11) in [Li93], is used to obtain the error in the spatial attenuation as:

$$\Delta\mathbf{a}_j = -\hat{\mathbf{Q}}^{\dagger}_{ps,j}\Delta\mathbf{Q}^{H}_{ps,j}\mathbf{a}_j \tag{B.7}$$

where † represents the pseudo inverse operation. The matrix $\Delta\mathbf{Q}^{H}_{ps,j}$ can be expressed in terms of $\Delta\mathbf{w}^{(k)}$ as follows:

$$\Delta\mathbf{Q}_{ps,j} = \left[\mathbf{G}^{j}_{L,d}(\boldsymbol{\tau})\right]^{H}\left[\mathbf{c}_{j,\mathrm{I}_{pq}}\right]^{H}\begin{bmatrix} \mathbf{0} & \Delta\mathbf{w}^{(k)}(1) & \Delta\mathbf{w}^{(k)}(2) & \cdots & \Delta\mathbf{w}^{(k)}(K) \\ \Delta\mathbf{w}^{(k)}(1) & \Delta\mathbf{w}^{(k)}(2) & \cdots & \Delta\mathbf{w}^{(k)}(K) & \mathbf{0} \end{bmatrix} \tag{B.8}$$

By defining $\mathbf{Z} = \mathbf{V}_p\mathbf{S}$, the first order perturbation equation can be applied once more to evaluate $\Delta\mathbf{w}^{(k)}$:

$$\Delta\mathbf{w}^{(k)} = -\mathbf{Z}^{\dagger}\mathbf{E}^{H}\mathbf{w}^{(k)} \tag{B.9}$$

Defining the following variables:

$$\mathbf{Z}^{\dagger} = \begin{bmatrix} \mathbf{Z}^{\dagger}(1) \\ \mathbf{Z}^{\dagger}(2) \\ \vdots \\ \mathbf{Z}^{\dagger}(K) \end{bmatrix} \tag{B.10}$$

$$\Delta\mathbf{w}^{(k)} = \begin{bmatrix} \Delta\mathbf{w}^{(k)}(1) \\ \Delta\mathbf{w}^{(k)}(2) \\ \vdots \\ \Delta\mathbf{w}^{(k)}(K) \end{bmatrix} \tag{B.11}$$

makes it possible to express

$$\Delta\mathbf{w}^{(k)}(m) = -\mathbf{Z}^{\dagger}(m)\mathbf{E}^{H}\mathbf{w}^{(k)} \tag{B.12}$$

where $\mathbf{Z}^{\dagger}(m)$ is the $\left(pqL_c \times N-K\right)$ submatrix of $\mathbf{Z}^{\dagger}$ $\left(pqKL_c \times N-K\right)$, and $\mathbf{w}^{(k)}(m)$ is the $\left(pqL_c \times 1\right)$ subvector of $\mathbf{w}^{(k)}$ $\left(pqKL_c \times 1\right)$.

Substituting (B.12) into (B.8) gives:

$$\Delta\mathbf{Q}_{ps,j} = -\left[\mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\right]^H \left[\mathbf{c}_{j,\mathbf{1}_{pq}}\right]^H \begin{bmatrix} \mathbf{0} & \mathbf{Z}^{\dagger}(1)\mathbf{E}^H\mathbf{w}^{(k)} & \cdots & \mathbf{Z}^{\dagger}(K)\mathbf{E}^H\mathbf{w}^{(k)} \\ \mathbf{Z}^{\dagger}(1)\mathbf{E}^H\mathbf{w}^{(k)} & \cdots & \mathbf{Z}^{\dagger}(K)\mathbf{E}^H\mathbf{w}^{(k)} & \mathbf{0} \end{bmatrix} \tag{B.13}$$

After substituting (B.13) into (B.7), the following expression for $\Delta\mathbf{a}_j$ is obtained:

$$\Delta\mathbf{a}_j = \hat{\mathbf{Q}}_{ps,j}^{\dagger} \underbrace{\begin{bmatrix} \mathbf{0} & \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(1) \\ \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(1) & \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(2) \\ \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(2) & \vdots \\ \vdots & \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K) \\ \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K) & \mathbf{0} \end{bmatrix} \mathbf{c}_{j,\mathbf{1}_{pq}} \mathbf{G}_{L,d}^{j}(\boldsymbol{\tau})\mathbf{a}_j}_{\Delta\mathbf{Q}_{ps,j}^H} \tag{B.14}$$

Using Equation (3.28), (B.14) can be simplified to

$$\Delta\mathbf{a}_j = \hat{\mathbf{Q}}_{ps,j}^{\dagger} \begin{bmatrix} \mathbf{0} & \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(1) \\ \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(1) & \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(2) \\ \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(2) & \vdots \\ \vdots & \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K) \\ \left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_j(1) \\ \mathbf{v}_j(2) \end{bmatrix} \tag{B.15}$$

By letting $\hat{\mathbf{Q}}_{ps,j}^{\dagger} = \left[\hat{\mathbf{Q}}_{ps,j}^{\dagger}(1) \quad \cdots \quad \hat{\mathbf{Q}}_{ps,j}^{\dagger}(K+1)\right]$, a compact form of $\Delta\mathbf{a}_j$ can be obtained by expanding out the brackets in (B.15):

$$\begin{aligned}
\Delta\mathbf{a}_j &= \hat{\mathbf{Q}}^{\dagger}_{ps,j}(1)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(1)\mathbf{v}_j(2) \\
&+ \hat{\mathbf{Q}}^{\dagger}_{ps,j}(2)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(1)\mathbf{v}_j(1) + \hat{\mathbf{Q}}^{\dagger}_{ps,j}(2)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(2)\mathbf{v}_j(2) \\
&+ \cdots \\
&+ \hat{\mathbf{Q}}^{\dagger}_{ps,j}(K)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K-1)\mathbf{v}_j(1) + \hat{\mathbf{Q}}^{\dagger}_{ps,j}(K)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K)\mathbf{v}_j(2) \\
&+ \hat{\mathbf{Q}}^{\dagger}_{ps,j}(K+1)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}(K)\mathbf{v}_j(1) \\
&= \sum_{i=1}^{K+1} \hat{\mathbf{Q}}^{\dagger}_{ps,j}(i)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)
\end{aligned} \tag{B.16}$$

In the above expression $\Delta\mathbf{a}_j$ is expressed in terms of the noise $\mathbf{E}$. In order to mathematically express the mean square error, the expectation operation is performed $E\left(\left\|\Delta\mathbf{a}_j\right\|^2\right)$ and evaluated. Let $\mathbf{e}_m$ be the m^{th} unit vector of length d. The mean square error for the m^{th} element of $\mathbf{a}_j$ is found as follows:

$$\begin{aligned}
E\left\{\left\|\Delta\mathbf{a}_j\right\|^2\right\} &= E\left\{\left\|\sum_{i=1}^{K+1} \mathbf{e}_m^H \hat{\mathbf{Q}}^{\dagger}_{ps,j}(i)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)\right\|^2\right\} \\
&= E\left\{\left(\sum_{i=1}^{K+1} \mathbf{e}_m^H \hat{\mathbf{Q}}^{\dagger}_{ps,j}(i)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)\right)\left(\sum_{i=1}^{K+1} \mathbf{e}_m \hat{\mathbf{Q}}^{\dagger}_{ps,j}(i)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)\right)^H\right\} \\
&= E\left\{\sum_{i=1}^{K+1}\sum_{l=1}^{K+1} \mathbf{e}_m^H \hat{\mathbf{Q}}^{\dagger}_{ps,j}(i)\left[\mathbf{w}^{(k)}\right]^H \mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)\mathcal{V}_j^H(l)\mathbf{Z}^{\dagger}\mathbf{E}^H\mathbf{w}^{(k)}\hat{\mathbf{Q}}^{\dagger^H}_{ps,j}(l)\mathbf{e}_m\right\} \\
&= \sum_{i=1}^{K+1}\sum_{l=1}^{K+1} \mathbf{e}_m^H \hat{\mathbf{Q}}^{\dagger}_{ps,j}(i)\left[\mathbf{w}^{(k)}\right]^H E\left\{\mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)\mathcal{V}_j^H(l)\mathbf{Z}^{\dagger}\mathbf{E}^H\right\}\mathbf{w}^{(k)}\hat{\mathbf{Q}}^{\dagger^H}_{ps,j}(l)\mathbf{e}_m
\end{aligned} \tag{B.17}$$

Assuming that the noise is white $E\left\{\mathbf{E}\mathbf{E}^H\right\} = \sigma_n^2\mathbf{I}$ and the data is uncorrelated with energy σ_s^2, it has been shown in [Aktas98] that the above expectation simplifies to:

$$E\left\{\mathbf{E}\mathbf{Z}^{\dagger^H}\mathcal{V}_j(i)\mathcal{V}_j^H(l)\mathbf{Z}^{\dagger}\mathbf{E}^H\right\} = \delta_{l,j}\frac{\sigma_n^2}{(N-K)\sigma_s^2} \tag{B.18}$$

Substituting (B.18) into (B.17) and using the fact that $\left[\mathbf{w}^{(k)}\right]^H \mathbf{w}^{(k)} = \mathbf{I}$:

$$
\begin{aligned}
E\left\{\left\|\Delta\mathbf{a}_j(m)\right\|^2\right\} &= \frac{\sigma_n^2}{(N-K)\sigma_s^2}\sum_{i=1}^{K+1}\left\|\mathbf{e}_m^H\hat{\mathbf{Q}}_{ps,j}^{\dagger}(i)\right\|^2 \\
&= \frac{\sigma_n^2}{(N-K)\sigma_s^2}\left\|\mathbf{e}_m^H\hat{\mathbf{Q}}_{ps,j}^{\dagger}\right\|^2
\end{aligned}
\tag{B.19}
$$

where $\|.\|^2$ denotes the Frebenious norm. Finally, a simple expression is obtained for the mean square error of the spatial attenuation estimation

$$
\begin{aligned}
E\left\{\left\|\Delta\mathbf{a}_j\right\|^2\right\} &= \sum_{m=1}^{d} E\left\{\left\|\Delta\mathbf{a}_j(m)\right\|^2\right\} \\
&= \frac{\sigma_n^2}{(N-K)\sigma_s^2}\left\|\hat{\mathbf{Q}}_{ps,j}^{\dagger}\right\|^2 \\
&= \frac{\sigma_n^2}{(N-K)\sigma_s^2} tr\left(\left(\hat{\mathbf{Q}}_{ps,j}\hat{\mathbf{Q}}_{ps,j}^H\right)^{\dagger}\right) \\
&= \frac{\sigma_n^2}{(N-K)\sigma_s^2} tr\left(\left(\left[\mathbf{G}_{L,d}^j(\boldsymbol{\tau})\right]^H\left[\mathbf{c}_{j,\mathbf{I}_{pq}}\right]^H\left[\sum_{k=1}^{z_m}\mathcal{W}^{(k)}\left[\mathcal{W}^{(k)}\right]^H\right]\mathbf{c}_{j,\mathbf{I}_{pq}}\mathbf{G}_{L,d}^j\right)^{\dagger}\right)
\end{aligned}
\tag{B.20}
$$

where $tr(\mathbf{A})$ denotes the trace of the matrix $\mathbf{A}$.

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