Exploring teachers' use of visualisation tools when teaching Grade 9 problem-solving in mathematics. A case of Umlazi District Dinaledi Schools in South Africa.

By

Makhosazana Faith Shoba

215080713

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Supervisor: Prof. Jayaluxmi Naidoo

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DECLARATION

I, Makhosazana Faith Shoba, declare that

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DEDICATION

This thesis is first dedicated to God Almighty who was my guide, strength, pillar, and comforter through it all. It is through his guidance that I made it up to this far. It is also dedicated to my wonderful parents for the sacrifices they made to raise me, develop me spiritually and give me a tremendous gift of education. Words cannot explain my sincere gratitude for their love and support from my childhood. Not forgetting, my husband, Patrick, who has been a pillar of strength throughout this journey; and my adorable kids for their kind support and understanding especially when I could not spend quality time with them.

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ABSTRACT

This study focused on exploring Grade 9 mathematics teachers' use of visualisation tools when teaching problem-solving in their classrooms. This issue has been a challenge in South Africa, particularly in matric and grade 9 mathematics Annual National Assessment. The use of visualisation when teaching problem-solving in the mathematics classroom has been viewed as critical to learner's performance, in response to the abstract nature of mathematics. However, problem-solving and the importance of the use of visualisation is emphasised in the Curriculum Assessment Policy Statement for the Senior Phase. Moreover, it is also included in every topic of the learners' Grade 9 mathematics workbook for everyday classroom activities.

Therefore, this study aimed to answer the questions of *what* visualisation tools teachers use and *how* they use these when teaching problem-solving. Lastly, *why* do they use them during their lesson in their classroom? Polya's 4-step problem-solving and Activity theory was used as a theoretical framework for this study. A qualitative case study of two Dinaledi Comprehensive Technical High School in Umlazi District was conducted to explore the use of visual tools by five grade 9 mathematics teachers during their teaching of problem solving. Teacher's questionnaire, classroom observations, and semi-structured interviews for teachers were used to generate data.

The findings revealed that mathematics teachers do teach problem solving in their Grade 9 classrooms as stated by the policy document. However, the use of visualisation tools in the mathematics classroom seems to be infrequent. Therefore, the teachers highlighted the lack of resources and understanding of what problem-solving is, as a challenge to their use of visualisation tools. However, the study suggested that the department of Kwa-Zulu Natal education should provide in-service training for Grade 9 teachers on the effective use of visualisation tools when teaching problem-solving. It was also suggested that schools should provide resources that can enhance problem solving, and mathematically related resources for their mathematics lessons. It was further suggested t schools to have a mathematics classroom, which will provide a mathematics atmosphere with relevant mathematics resources for effective and efficient teaching and learning of mathematics.

ACRONYMS

ABET	Adult Basic Education and Training
ACE	Advanced Certificate in Education
ANA	Annual National Assessment
BEd	Bachelor of Education
AMESA	Association of Mathematics Teachers in South Africa
BSc	Bachelor of Science
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DoE	Department of Education
FET	Further Education and Training
FETC	Further Education and Training Certificate
GET	General Education and Training
KZN	Kwa-Zulu Natal Province
IBE	Independent Examination Board
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
OBE	Outcomes-Based Education
PCK	Pedagogical Content Knowledge
PGCE	Post Graduate Certificate in Education
SAARMSTE	South African Association of Research in Mathematics, Science and Technology Education
SP	Senior Phase
STD	Secondary Teachers Diploma
TCK	Teacher's Content Knowledge
TIMSS	Trends in International Mathematics and Science Study

FIGURES USED IN THE STUDY

TABLES USED IN THE STUDY

LIST OF APPENDICES

CODING USED IN THE STUDY

Code	School
School A	1st Comprehensive Technical High School in Umlazi
School B	2nd Comprehensive Technical High school in Umlazi

Code	Pseudo names	Teachers
Teacher A1	Portia	First observed teacher in School A
Teacher A2	Patrick	Second observed teacher in School A
Teacher A3	Noxolo	Third observed teacher in School A
Teacher B1	Sisanda	First observed teacher in School B
Teacher B2	Sibusiso	Second observed teacher in School B

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CHAPTER 1 INTRODUCTION

1.1 Introduction

This chapter presents an overview of the study and its critical questions, rationale, and objectives. It further discusses a brief background of teachers' use of visualisation tools when teaching Grade 9 problem-solving in mathematics classrooms.

1.2 Overview of the study

This study aimed at exploring grade 9 mathematics teachers use of visualisation tools when teaching problem solving in their classrooms. The major problem in South African education was the high failure rate in Grade 12 mathematics (DBE 2016). This is due to the poor teaching of mathematics in South Africa (Du Plooy, 2015) and, since 2010, the highest percentage pass in mathematics in South Africa has been 59.1% (DBE, 2016). Moreover, when Viser, et al. (2015) note that mathematics has been and still is a major challenge for learners, teachers, schools, and the Department of Basic Education at large, it raised the attention of this study to explore how mathematics is taught in the classroom especially in grade 9. This study concentrated on the teaching of problem solving and the use of visualisation tools in mathematics.

Therefore, it was critical to explore grade 9 mathematics teachers us of visualisation tools as it forms the foundation of teaching and learning mathematics during problem solving. It was also important to observe how the mathematics teachers use visualisation tools during their problem-solving mathematics lesson. This was done in this study through the lens of Polya's 4-step problem solving and activity theory. There teachers were therefore, observed and interview on their use of visualisation tool during problem solving to explore the visual tools that they use, how they use them and why they use them.

1.3 Background of the study

In South Africa, mathematics teaching and learning have amounted to a significant challenge to learners, teachers, schools, and the Department of Education (Nkhwalume, 2013). In a similar vein, as a long-serving mathematics teacher, I noted and discovered that mathematics teaching and learning are challenging to learners and teachers. Analysing mathematics pass rates for

Grade 12 in South Africa confirms this observation. It shows the poor performance of learners, especially in mathematics. Spaull (2013) notes that South Africa is underperforming in education, especially in mathematics, pointing out the poor quality of mathematics teaching evident in teachers' failure to answer questions on their curriculum. Reddy, Visser, Winnaar, Arends, Juan, Prinsloo, and Isdale, (2017) echo the same sentiments about mathematics performance in Grade 9 which is low compared to other TIMSS participating countries. Moreover, Table 1.1 shows the pass percentages of Grade 12 learners in mathematics for Comprehensive Technical Schools in the Umlazi District within the KwaZulu-Natal province, South Africa. This represents information from the inception of the Curriculum Assessment Policy Statement (CAPS) curriculum in 2014.

Table 1.1: Grade 12 Mathematics performance in KwaZulu-Natal Comprehensive Dinaledi Technical Schools

School Name	2014		2015		2016		2017		2018	
	No. wrote	**Achieved at 30%**	**No. wrote**	**Achieved at 30%**	**No. wrote**	**Achieved at 30%**	**No. wrote**	**Achieved at 30%**	**No. wrote**	**Achieved at 30%**
Esizibeni Sivananda Comp. Tech. H.S.	90	28%	80	10%	85	19%		31%	82	59%
KwaMakhutha Comp. Tech H.S.	60	32%	103	20%	124	31%		32%	180	24%
Ogwini Comp. Tech. H.S.	489	15%	433	24%	500	24%	534	19%	528	15%
Umlazi Comp. Tech. H.S.	461	36%	425	26%	345	29%	289	38%	306	34%
Sibusisiwe Comp. Tech. H.S.	181	56%	228	50%	200	70%		67%	230	77%

Adapted from Department of Basic Education [DBE] (2018, p. 245-249) and DBE (2014; p.80-89)

These results show a serious challenge where mathematics performance is concerned. Several strategies were implemented while trying to improve the Grade 12 results. Despite several strategies to improve the learner's mathematics performance including extra classes and programs to capacitate teachers on how they can effectively teach mathematics, learners are still performing poorly. However, Table 1.1 shows that in the past four years of teaching the CAPS curriculum in the four comprehensive high schools at Umlazi district, only one school achieved above a 60% pass rate in their mathematics Grade 12 results. If comprehensive technical schools with adequate support from the Department of Education and Businesses vested in Mathematics education still do not perform well, then it follows that there is a need for further intervention, especially where mathematics performance is concerned.

The focus of attention was previously on Grade 12 mathematics results, whilst the problem at stake does not necessarily begin in Grade 12. However, Du Plooy, (2015) pointed out that in most cases national testing might be misleading as it does not represent the major gaps in lower grade levels. This became evident in the previous Annual National Assessment (ANA) results. It was discovered that the overall performance of mathematics in ANA 2014 was at the 'not achieved' level, that is, ten percent (DBE, 2014). Those results proved that learners were struggling with mathematics even in Grade 9, which might result in poor performance at Grade 12 level. However, Caprioara (2015) states that mathematics is problem-solving by nature and problem solving can be easily done through visualisation.

Moreover, one major way of doing mathematics is solving problems rather than just a goal of learning mathematics (Kojo, Laine, & Näveri, 2018). Therefore, in the mathematics Grade 9 CAPS document, problem-solving is emphasised as an outcome where it states that the learners should be able to identify, solve problems and make critical and creative decisions (DBE, 2011a). Furthermore, problem-solving can be considered as a cornerstone of effective mathematics teaching and learning as is done in countries such as America, Singapore, Australia, Indonesia, and Japan where they value the use of problem-solving in mathematics education. Additionally, Caprioara (2015) defines a problem situation in mathematics as a learning situation in which the teacher creates an environment for reflection and analysis around a problem or question to be solved.

Mathematics teachers refer to problem-solving, ‘as a process that requires learners to develop a rich network of ideas that they can use when faced with difficult situations’ (Tripathi, 2009, p. 67). This means that a mathematician must be able to solve problems of numbers, algebra, geometry, trigonometry, patterns, shapes, and diagrams. Nevertheless, a problem solver needs the skill of visualisation to effectively solve mathematics problems as supported by Krawec (2014) who emphasises that effective problem-solvers use visualisation to comprehend the problem. This is evident when analysing the Grade 9 mathematics curriculum, whereby problem-solving is found in all Grade 9 mathematics topics and is stated in the curriculum statement. Moreover, this shows the importance of problem-solving in understanding mathematics concepts. In a similar vein, in the CAPS document for mathematics, the importance of making use of visualisation and learners being problem solvers is emphasised (DBE, 2011b). As result, this indicates that problem-solving and the use of visualisation is crucial in the teaching and learning of mathematics in South African schools, mostly in Grade 9.

However, in this study the problem was whether teachers are using visualisation tools and, if they do, how and why they use them during mathematics problem-solving in their Grade 9 lessons. Makhubela and Luneta (2014) advocate the use of visualisation tools to enhance the effective mastering of mathematics concepts during teaching and learning. On this basis, the present study explores effective ways of using visualisation skills in Grade 9 classes when they solve mathematical problems. If learners achieve mathematical problem-solving at the Grade 9 level, it implies that their results in Grade 12 will be good. This is parallel with McCarthy and Oliphant’s (2013) who discussed an outcry that besides the analysis of the ANA results it has been noted that Further Education and Training (FET) (Grades 10, 11 &12) teachers in the National Curriculum Statement (NCS) are expected to make up for large deficits from lower classes such as Grade 9. Hence, this study saw the need to attend to the learning gaps seen in the lower classes such as Grade 9 compared to Grade 12 for developing an effective mathematical foundation for Grade 12.

1.3 Problem Statement

The major problem in the South African education sector is the high failure rate in mathematics in Grade 12. According to the DBE (2016), since 2010 the highest percentage pass in mathematics in South Africa has been 59.1%. This has resulted in wasting money; since 19.7% of government money in 2015 and 2016 was spent on education (World Bank, 2015). This funding was also earmarked for providing resources to improve learners' mathematical understanding. The underperformance in mathematics has not only been witnessed in Grade 12 but also in the Grade 9 ANA results. In a similar vein, Spaull (2011) states that 76% of Grade 9 learners in 2011 showed no understanding of basic mathematics problems.

It is worth noting that solid performances in Grade 9 mathematics should improve matric results. This is because most of FET mathematics topics and concepts are based on Grade 9 work. However, Govender (2015) indicates that there is a possibility that the absence of visual strategies might be the main issue that obstructs learners from effectively solving problems. Therefore, in this study, the major problem to be discussed is if the use of visualisation tools during mathematics problem-solving has a link to learners' poor performance. Considering that Makhubela and Luneta (2014) state that the poor performance in Grade 9 mathematics in the ANA is a symptom of the state of mathematics teaching and learning in the Senior Phase (Grades 7-9) Intermediate and Foundation, Phase in South Africa.

This study explores whether teachers' use of visual tools in teaching problem-solving assists learners in mastering mathematics concepts which may lead to an improvement in learners' mathematics performance. If the teachers can use visualisation and help learners form visual images to solve mathematical problems, it would be easy for the learners to solve any type of problem they come across at any level of their learning. Tseng, Chang, Lou, and Hsu, (2013) confirm visualisation to be a powerful cognitive tool in problem-solving. Consequently, the CAPS documents outcome which states that learners should be able to communicate using written, oral, visual, and graphic and other forms of communication (DBE, 2011b, p.27) can be effectively met.

1.4 Significance of the study

This study will be important for mathematics education in South Africa specifically since there was an outcry of poor educational performance of learners in the country caused by the poor teaching of mathematics (Du Plooy, 2015). Therefore, it was important for this study to be carried out. Remarkably, the CAPS curriculum advocated for the teaching of problem-solving and the use of visualisation when teaching mathematics (DBE, 2011b). As a result, problem-solving, and visualisation appear several times in the CAPS document.

Moreover, the high failure rate of mathematics in schools is a major concern especially to the department of education (DBE, 2016). Furthermore, it is regretful for a country such as South Africa to be placed in the third last position in TIMSS compared to other performing countries (Reddy, et al. 2017), not to mention, underperforming in the National Senior Certificate in matric (DBE, 2014) which acts as a mirror of the country's education. Relevant stakeholders: for example, the Department of Education, mathematics specialists and researchers, curriculum developers, ANA personnel, TIMSS department, and the mathematics examination council have been concerned about the performance of mathematics in school (Du Plooy, 2015). Consequently, this study will be important when it comes to demonstrating how the use of visualisation tools during problem-solving might enhance the performance of learners in mathematics.

This study is also important to mathematics teachers, as it explores how teachers can effectively teach problem-solving using visualisation for the improvement of their learners' performance in mathematics. Additionally, the use of visualisation tools helps learners' understanding of mathematics and makes it easier for them to master mathematics problem-solving skills. If the learners do well in their Grade 9 mathematics, then there is a high possibility of them doing well in Grade 12 (Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2014). This implies that FET teachers will no longer be expected to make up for the deficits of lower classes (McCarthy & Oliphats, 2013). If learners' Grade 12 mathematics results are good, it follows that the school results will also be improved. This is because, when comparing the provisional subject results in Grade 12 from 2013 to 2016, mathematics was the worst-performing subject (DBE, 2018b).

Moreover, improved past rates will help the country to save on finances to employ people to facilitate in improving mathematics results. This will also result in reduced programs to help improve mathematics performance. As stated in the budget 2017/18, 48 % was allocated for education in the provincial allocation (ages 5-17 years) and therefore, this money will result in a surplus (Malope, Gaetsewe, Khanie, Molefhi, & MacKenzie, 2018). Consequently, the country will be producing mathematically literate, local, and internationally mathematically competent citizens. An increase in the mathematics pass rate will address the shortage of highly needed mathematics skills (Nel, 2015). Moreover, the country's economy will improve because there will be a competent mathematical workforce and South Africa will not need to outsource expertise.

1.5 Research Questions

The research questions that framed this study's inquiry are as follows:

1. What visualisation tools do teachers use when teaching problem-solving in Grade 9 mathematics classrooms?
2. How do teachers use visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?
3. Why do teachers use these visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

1.6 Definitions of terms

Key terminology used in this study are defined as follows:

i.Problem-solving

Problem-solving in this study refers to a mathematics statement that requires a solution using one or more operations. Problem-solving is a mathematical concept that presents knowledge at three levels: concrete (for example, manipulative), pictorial (visual), and abstract. It is mainly viewed according to Polya's four-step problem-solving process. This process involves: understanding the problem, devising a strategy, carrying out the strategy, and looking back.

ii.Visualisation

Visualisation is the ability to form and transfer a mental image necessary for problem-solving in mathematics (Naidoo 2011). Visualisation is the use of visual representations to inspire ideas and allow a deeper understanding of the relationships involved in problem-solving (Abdullah, N., Halim, L., & Zakaria, E.,2014). It mainly involves the use of diagrams, pictures, manipulatives, highlighting on the board, the use of technology, and the use of gestures.

iii.Dinaledi Schools

Dinaledi schools were established in 2001 to improve the number of learners passing physical science and mathematics (DBE, 2010). These schools obtain resources and financial support from the Department of Basic Education and other stakeholders invested in the improvement of mathematics and science at schools.

iv.Comprehensive Technical High Schools

A comprehensive technical school is one that includes technical or technology subjects (electrical, chemical, civil, and mechanical engineering, science, and mathematics technology) on a comprehensive level. These schools are reminiscent of TVET colleges.

1.7 Chapter layout

The thesis consists of seven chapters organised as follows:

Chapter 1: Introduction

This chapter serves as an introduction of the whole research, laying the foundation and orientation of the study. It consists of the introduction, background of the study, objectives, research questions, problem statement, and discussion of the significance of the study and the structure of the thesis.

Chapter 2: Literature Review

In this chapter, the relevant literature about the topic is discussed. The chapter focuses mainly on the existing literature that is significant in carrying out this study. The literature review surveys what other authors have done and discovered concerning the current study's research objectives. The gap in the previous literature in terms of the study is analysed and attended to.

Chapter 3: Theoretical Framework

This chapter deals with the framework or lens in which the visual tools were viewed as to how teachers use them and why they use them in the way they did. In this study, the theoretical framework was a merging of two frameworks. The study was framed by activity theory and Polya's 4-steps problem-solving process.

Chapter 4: Research Methodology

This chapter deals with the research paradigm, research approach, and research method. It is where the research philosophy is discussed compared to other philosophies. Furthermore, research instruments that have been developed and utilised in pursuit of the study are introduced and discussed in detail. Additionally, this chapter brings attention to the population and the sample as well as the sampling strategies.

Chapter 5 and 6: Results, Discussions, and interpretation of findings

In this chapter, the findings, analysis of results, and discussion of findings are presented. The thematic analysis of results using different themes formed from the objectives is discussed and interpreted.

Chapter 7: Conclusion and Recommendations

This chapter discusses what has been concluded and recommended based on the findings of this study.

1.8 Conclusion

A brief discussion of the background of the study and statement of the problem including insights from the Department of Basic Education documents were shared in this chapter. An overview of the connection between problem solving and visualisation has been presented. The research question, objectives, definition of terms, and layout of the study were outlined.

The next chapter discusses the literature which was reviewed based on the focus of this study.

CHAPTER 2 REVIEW OF LITERATURE

2.1 Introduction

This study explores the use of visualisation tools when teaching Grade 9 problem-solving in mathematics classrooms. Related literature is discussed under the following sections: a) mathematics teachers and teaching, b) teaching Grade 9 mathematics, c) problem-solving skills in the mathematics classroom and d) the use of visualisation when teaching problem-solving. Figure 2.1 portrays the structure of the chapter.

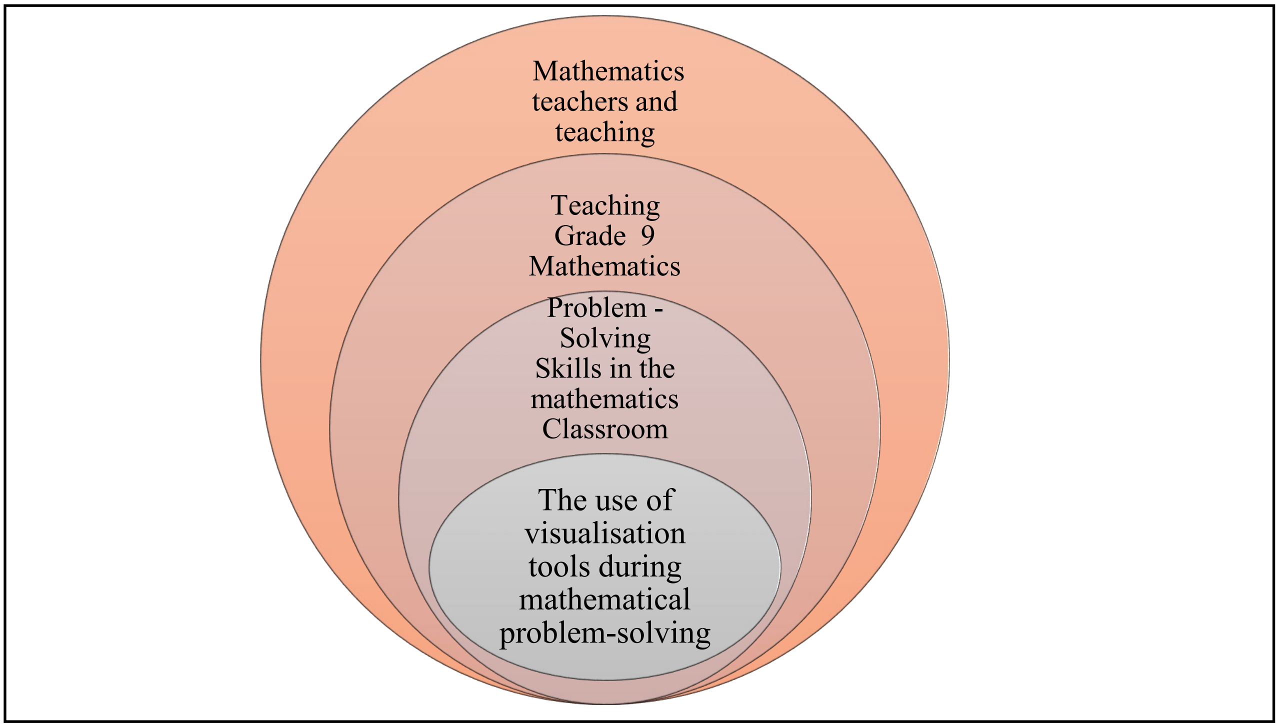

Figure 2.1: Layout of the topics to be discussed in the literature review

This study explored the teaching of mathematics problem-solving using visualisation tools in South Africa, specifically within Grade 9 mathematics classrooms in the Umlazi District of KwaZulu-Natal, South Africa. It is based on the current situation in schools parallel to curriculum expectations from the teachers regarding the current mathematics policy document (CAPS). According to the policy document, learners are expected to be able to use visualisation encompassing creative and critical thinking during problem-solving (DBE, 2011a). Moreover, the importance of this study was focussed on exploring the teaching of Grade 9 mathematics problem-solving using visualisation to be able to interrogate if this type of teaching enhanced learners' performance in mathematics.

Further discussions in this study are based on the use of visualisation as an effective and pivotal tool in teaching mathematics problem-solving for effective learner performance in mathematics. Problem-solving is discussed based on Polya's 4-step problem-solving process. It unpacks what happens in each step as far as mathematical problem-solving in the classroom is concerned and is aligned with each mathematics teacher's praxis as they teach problem-solving. Another important aspect discussed in this study is visualisation, especially the different visualisation tools used in a mathematics classroom. The use of visualisation is displayed in this study as a tool that plays a pivotal role in effective problem-solving which is needed when teaching Grade 9 mathematics. Consequently, the use of visualisation during problem-solving helps in encompassing the mathematics CAPS document requirements which emphasise the importance of making use of visualisation to foster learners who are problem-solvers (DBE, 2011b). The chapter begins by discussing mathematics teachers and their teaching.

2.2 Mathematics teachers and teaching

Teaching mathematics according to Alfaro-Carvajal and Fonseca- Castro (2018) involves an approach of teaching algorithmic techniques and procedures. Additionally, Ambrus (2014) points out that teachers view mathematics from three different perspectives, namely the Platonist perspective which states that mathematics exists in an abstract plane; the formalist perspective which views mathematics as a game where one manipulates symbols and the intuitionist perspective which is the construction of human minds. Furthermore, the perspective the teacher adopts will influence the presentation of mathematics in the classroom (Sousa, 2008). Therefore, this study critically explores Grade 9 mathematics teachers' use of visualisation tools when teaching problem-solving.

The role of mathematics is emphasised by Alfaro-Carvajal and Fonseca-Castro (2018) as a human activity related to the need to solve problems. Moreover, within the South African curriculum, according to CAPS around the Senior Phase, mathematics is described as follows:

*"Mathematics is a language that makes use of symbols and notations for describing numerical, geometric, and graphical relationships. It is a human activity that involves observing, representing, and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to developmental processes that enhance logical and critical thinking, accuracy, and problem-solving that will contribute in decision-making," (*DBE, 2013, p.8).

The main aim of teaching and learning mathematics is therefore to enhance problem-solving skills. These skills may be enhanced if teachers understand their roles and what they are supposed to do in the mathematics classroom (Buthelezi, 2016).

2.2.1 A mathematics teacher

Adler (2017) brings to our attention that the issue of whom to teach mathematics and how, is a matter of concern, since there is a connection between appropriate knowledge of mathematics and pedagogical knowledge as well as improved teaching and learning. A mathematics teacher is any person who is qualified or has the necessary experience to impart mathematical knowledge to learners in their different levels of learning. In European countries, they put in place several requirements for the teaching of mathematics in their schools, for example, that a mathematics teacher must have a master's degree in mathematics to teach mathematics (Kilpatrick, 2020). However, in South Africa, a teachers' minimum qualification is a Bachelor of Education Degree (Buthelezi, 2016) and some even have an Education Diploma.

In a research study conducted in 2012, it was found that 89% of mathematics teachers had either mathematics or mathematics education included in their training (Mullis, Martin, Foy & Arora, 2012). However, Buthelezi (2016) noted that South Africa is facing a challenge in the quality of mathematics teachers in schools. All these factors have an impact on how mathematics is effectively taught within the South African mathematics classrooms.

Furthermore, Nel, and Luneta (2017) state that South African teachers are exposed to numerous professional development initiatives but there is still a poor performance of learners which is said to be due to teachers' poor performance in the classroom. Buthelezi (2016) confirms this notion whereby in a research study conducted with North West Province Grade 6 teachers, it was found out that in a Grade 6 mathematics test, teachers scored an average of 40%. Additionally, Carnoy, Chisholm, and Chilisa (2012) note that only 32% of Grade 6 learners in South Africa possess the appropriate content knowledge in mathematics. Therefore, it is worth exploring the use of visualisation when teaching problem-solving in the classroom. Since 2010, the highest percentage pass in mathematics in South Africa has been 59.1% (DBE, 2016). These dismal results in a developing country such as South Africa can be linked to Teacher's Content Knowledge (TCK) (Mogari, Kriek, Atagana, & Ochonogor, 2016). As a result, this study explored the use of visualisation tools when teaching Grade 9 problem-solving in mathematics.

2.2.2 Expectations of the curriculum from mathematics teachers

Mosotshwane (2012) views teachers as managers of the curriculum who are concerned at seeing the success of the curriculum, correctly and adequately done. In this study, the teacher's duty is described as the ability to manage the instilling of problem-solving in learners' minds hence improving the learner's performance in mathematics. Subsequently, it is also concerned with the exploration of how mathematics teachers are managing their curriculum and making sure that problem-solving is used for effective teaching of mathematics. Furthermore, Nkhwalume (2013) suggests that nowadays mathematics concepts should be used for unfamiliar and familiar real-world problems as compared to before when they were used for the enjoyment of mathematics.

A teacher in today's classroom serves many roles and so, too mathematics teachers. According to Tylor (2016), norms and standards for a South African teacher are as follows: a teacher is a learning mediator between the curriculum and attainment of its objectives, interpreter of the curriculum to learners, and designer of learning programs and materials. To be a learner mediator simply entails unpacking the curriculum to the learners effectively and efficiently that will enhance the improvement of their learning. What is stated in the curriculum needs to be displayed in teaching and hence seen in the product produced by the learners. South African teachers are expected to unpack the curriculum for learners and help them attain the learning outcomes of the curriculum. Moreover, teachers are also expected to be able to interpret the curriculum to the learners. Consequently, for this study, it was important to explore the teachers' use of visualisation tools when teaching problem-solving as they mediate the curriculum for learners and interpret it clearly for them as they unpack the concepts. Teachers are supposed to make learners able to learn whilst enjoying and understanding mathematics concepts.

Since a teacher is a leader, he/she must be able to lead or model how learners will be able to solve routine and non-routine problems. The greatest service that teachers are accountable for, is being a subject specialist who will be modelled dynamically and will allow the learners to produce a certain character that can be seen in their real-life or when solving real-life situations. This kind of character can also be displayed in their mathematics performance in the examination and hence their results. Hence, the teaching of mathematics becomes pivotal during the use of visualisation when teaching problem-solving.

2.2.3 Teaching of mathematics

Mathematics teachers' Content Knowledge (TCK) is seen as a factor that contributes to the effective teaching of mathematics (Pournera, Hodgen, Adler & Pillay, 2015). On the same note, Venkat and Spaull (2015) found that teachers in Limpopo who lack subject knowledge tend to use 'safe talk' supported by notes on board, constant referral to books, and repetition of examples and exercises. Additionally, Pournera et al. (2015) argue that the best teaching of mathematics can be identified by a more thoughtful selection of examples and tasks and by explanations that focus clearly on the concept or procedure that those teachers purpose learners to learn. This implies less time for creativity and innovation in such a classroom during the teaching of mathematics while problem-solving in mathematics require innovation and creativity

Despite this, an increase in the Grade 12 mathematics pass rate is needed to address the shortage of highly needed mathematics skills (Nel, 2015) which will help reduce the gap of outsourcing mathematics literate individuals from other countries. Mabizela and Green-Thompson (2019) note that the national benchmark tests at South African Universities reveal 67% of matriculants are not prepared to cope with the academic demands of university. Therefore, an increase in the pass rate can be attained by teaching the correct mathematics content to learners as early as Grade 9 and lower classes.

Buthelezi (2016) notes that the Independent Examination Board (IEB), which comprises a few private and independent schools in South Africa, had proved to produce internationally competent learners at Grade 12 level especially in mathematics. Hence, mathematically competent teachers may be able to teach mathematics which will improve the Grade 12 learners' results and produce competent learners such as those from IEB. Since IEB is also done in South Africa and the learners are taught by the same South African teachers, prepared at the same universities and colleges who in most cases can use problem-solving, this implies that it is also possible with the current curriculum to obtain competent results in South Africa. Nevertheless, appropriate mathematics can be best instilled by the correct modelling of problem-solving skills using visualisation in the classroom by mathematics teachers.

2.2.4 Mathematics teachers and teaching of mathematics in secondary schools

Across the world, there are attempts to improve teachers' mathematical content knowledge to raise learner's performance (Pournara et al., 2015). According to the National Council of Teachers of Mathematics NCTM, it was emphasised in 1989 that problem-solving should be the central focus of mathematics in the United States. Subsequently, in 1990 problem-solving in mathematics was included as a major goal in Australia. Additionally, in Japan, there has also been an open-ended approach that centres on problem-solving (Munroe, 2015). Furthermore, Indonesia adopted a competency-based curriculum where problem-solving is a key component (Har, 2009). Similarly, since 1992 in Singapore, where a problem-solving curriculum was adopted, where their learners did well in international comparative studies in mathematics whilst all learners were expected to be exposed to challenging mathematics (Toh, Chan, Tay, Leong, Quek, Toh, & Dong, 2019). Moreover, it is worth noting that an increase in problem-solving and visualisation are included in the Hungarian mathematics curriculum as compared to European, American, and South African curriculum.

Finland, which is ranked average on the list of best education systems in the world, requires a teacher to have a master's degree as the minimum qualification as opposed to South Africa where a Bachelor of Education degree is their minimum (Malinen, Väisänen, & Savolainen, 2012). Therefore, it is important to consider the influence of the CAPS curriculum which started in 2012 in South Africa, focusing on problem-solving and visualisation as core to mathematics education.

Notably, South Africa's current curriculum now follows in the footsteps of well-performing countries (Spaull, 2013) such as Finland. Therefore, this study is of importance concerning exploring the use of visualisation during problem-solving in mathematics classrooms in the Umlazi district within KwaZulu-Natal, South Africa.

When comparing South Africa's educational system with other Southern African Development Community (SADC) neighbouring countries it was found that they have almost the same historical background, but their educational performance is not the same (Buthelezi, 2016). Therefore, the difference in the performance of learners in mathematics in neighbouring countries gives rise to the question as to what they are doing well to improve their mathematics learner's performance which South Africa might not be aware of. Similarly, it was noted that in Zimbabwe 8% of government expenditure goes to education while South Africa spent 19.7% of its budget on education in 2015 (World Bank, 2015). However, Zimbabwe has changed its curriculum once after independence while South Africa has changed several times causing destabilisation of the system (Graven, 2014). However, in South Africa, a TCK is arguable due to what happened during the Apartheid era, and this still has an impact on the current education curriculum as it makes it insufficient and irrelevant (Mogari, et al. 2016). By the Apartheid era we mean the period of segregation to maintain white domination while extending racial separation (Nordling, 2018). Moreover, this resulted in inappropriate and ineffective instructional methods (Mogari et al., 2016). Therefore, one of the teaching methods which the teachers can use is visualisation pedagogy, especially when teaching problem-solving.

Carnoy, Chisholm and Chilisa (2012) note that in a content knowledge test for teachers in South Africa, only 32% possessed mathematics content knowledge compared to their poorer neighbouring countries such as Kenya (90%), Zimbabwe (76%), and Swaziland (55%). Moreover, it was noted that teachers could not impart knowledge to learners which they have not acquired themselves (Spaull, 2013).

2.3 Teaching Grade 9 mathematics Curriculum

Previous assessment in South Africa was only done in Grade 12 (DBE, 2013) except for TIMSS which requires even lower grades to be assessed to compare them with national standards.

However, there is now a shift nationally, by using the Annual National Assessment (ANA), which is based on the lower grades including Grade 9. According to Spaull (2013), the Grade 9 ANA was first written in 2012 with a national average of 12.7%, then in 2013 with an average of 13.9%, and in 2014 the national average dropped to 10.8%. These results show that mathematics in Grade 9 needs just as much attention as in Grade 12. However, this study aimed at exploring how mathematics was taught in the Grade 9 classroom using visualisation tools during problem-solving.

Spaull (2013) discloses in his report that 76% of Grade 9 learners in 2011 had not acquired a basic understanding of whole numbers, decimals, operations, or basic graphs. The Grade 9 ANA document (DBE, 2011) distinguishes three levels: knowledge of basic concepts, application of concepts, and non-routine problem-solving. In response to that, DBE (2013) states that Grade 9 learners demonstrate a limited range of necessary basic skills and knowledge when they are in a critical transition grade to the Further Education and Training (FET). Subsequently, this study offers solutions to how Grade 9 mathematics can be effectively taught for learners to be able to understand mathematical concepts in preparation for the FET. However, it was worth questioning whether Grade 9 teachers were aware of what problem-solving is or how to effectively teach problem-solving in a mathematics classroom.

The poor performance of Grade 9 mathematics in ANA is therefore an approximate state of mathematics teaching in the senior phase (Grades 7-9) or even in the Junior Phase in South Africa (Makhubela & Luneta, 2014). Moreover, this brings awareness of how problem-solving is delivered in the classroom by the teachers. Spaull (2013) brings to our attention that South African mathematics teachers were lacking in content knowledge. This situation may affect the learners' performance as a teacher cannot deliver what they did not acquire. Moreover, teaching problem-solving can be a greater challenge since it occurs on a higher level concerning the cognitive levels. Subsequently, Makhubela and Luneta (2014) note that the Grade 9 teachers tend to concentrate on knowledge type questions which from only 20 percent of the four cognitive levels. The present study, therefore, explores how teachers teach problem-solving using visualisation tools.

Mathematics classrooms are supposed to have mathematics ‘feel and touch’ that assists learners to enjoy doing mathematics and be willing to study mathematics. The lack of resources, for example, models, drawings, graphics, calculators, and charts tend to be a factor contributing to learners’ lack of interest in mathematics (Makgato, 2007). Hence, Makhubela and Luneta (2014) allude to the fact that telling learners without exposing them to concrete materials in the mathematics classroom does not enhance learning. The importance of visualisation in a mathematics classroom for learners to be able to effectively solve problems is therefore clear.

2.4 Problem solving skills in mathematics classrooms

In this study problem-solving is discussed based on Polya’s steps. Youngchim, Pasiphol, and Sijiva (2014) bring to our attention Polya’s steps to problem-solving, which are understanding the problem; devising a plan; carrying out the plan and looking for feedback. This is discussed concerning what problem-solving is and what ought to take place in the mathematics classroom.

2.4.1 Meaning of Problem Solving in mathematics

Polya (1945) views problem-solving as an art in which the learner learns through imitating what the teacher does in the classroom and practices what he or she sees the teacher doing.

“Mathematical Problem-Solving is a practical art like swimming or playing the piano: you can learn it only by imitation and practice…. If you wish to learn to swim you have to go in the water, and if you wish to become a problem-solver you must solve problems,” (Polya, 1945, p. 49).

Donaldson (2015) dovetails with this when he says that the three perspectives to problem-solving are problem-solving as context, as skill, and as art. Problem-solving as a skill is viewed as valuable curriculum outcomes that deserve special attention. Problem solving as art is whereby it is reduced to problem-solving as a skill. Therefore, the perspective of Donaldson (2015) intersects mainly on art and skill when attempts are made to implement Polya’s (1945) ideas. However, in this study, the way the teacher model’s problem-solving for learners to be able to use it in their classroom will be viewed as pivotal, especially as an art.

Furthermore, Polya (1962, p. 56) defines problem-solving as ‘the capability to solve non-routine problems that are a difficulty that needs a degree of independence, judgment, originality, and creativity as one solves the mathematical challenge'. Moreover, Donaldson (2015) views problem-solving as a process that involves solving non-routine exercises that combine creativity and intuition. Furthermore, this study focused on problem-solving of non-routine problems in Grade 9 classrooms rather than routine problems. However, Govender (2015) defines problem-solving in South Africa and other countries as one distinct attribute that makes mathematics important.

2.4.1 Problem-solving skills in the mathematics classroom

"Proposed intervention was that routine and non-routine word problems should feature in almost all mathematics lessons. Learners should be taught the techniques to solve word problems; they should be able to read the problem, underline the numbers in the problem and the questions, and thereafter work out the solution to the word problem by drawing, writing the number sentences, or using a number line to work out the answer. Problem-solving should be an oral, practical, and written activity," (DBE, 2013, p.56).

This intervention of the Department of Basic Education as a response to the ANA results shows that the Department of Basic Education considered problem-solving for mathematics. However, it is not only about problem-solving, learners must be able to visualise or see the mathematics problems that they are solving. Subsequently, according to DBE (2014), the main aim of studying mathematics is to solve problems. Without it the usefulness and effect of mathematical ideas knowledge and skills are limited.

Therefore, in a mathematics classroom learners are expected to be taught the techniques to solve word problems through underlining the numbers in the problem and questions and thereafter work out the solution in the word problem by drawing a diagram, writing the number sentence or using a number line to work it out (DBE, 2013). In a similar vein, as the learners proceed, Abdullah, Halim, and Zakaria, (2014) propose that attention and focus should be placed on increasing learners' conceptual and strategic knowledge to ensure awareness and control of the thinking process that occurs in their minds. Therefore, the teacher becomes the key person in making sure that the learners master how to go about solving problems in a mathematics classroom. It is for that reason this study focused on teachers' use of visualisation during problem-solving in the Grade 9 classroom. Moreover, this can also be done through teachers strengthening their learners' problem-solving skills through using open-ended problems that encourage people to learn their mathematics in a meaningful way (Intaros, Insprasitha & Srisawadi, 2014). Furthermore, the way teachers teach problem-solving is pivotal and how they model solving word problems is key to this study.

Mathematical understanding serves as a primary goal of instruction in a mathematics classroom (Donaldson, 2015). Additionally, in a mathematics classroom, a problem is a question or difficult situation given to learners, which needs to be solved (Caprioara, 2015). This means that problem-solving in a mathematics classroom is a problem situation in which the teacher brings about to create a space for a learning situation. However, Liljedahl, Santos-Trigo, Malaspina, and Bruder (2016) further view mathematical problem-solving as a process whereby previously acquired knowledge is used in a new and unknown difficult situation to come up with a solution. Contrary to this, Donaldson (2015) concludes that there is no clear version from the teachers of what problem-solving is and how to teach it. However, these debates were taken care of in this study when observing how the teachers use problem-solving in the classroom.

Learners are expected to use previously known concepts or procedures to solve unfamiliar situations or problems in mathematics after being exposed to them by their teachers in the classroom. Subsequently, Mogari, et al. (2016) state that drilling and remembering facts and procedures when teaching mathematics do not encourage thinking and problem-solving. Therefore, it was important for this study to explore how mathematics problem-solving is taught. However, drilling and making learners remember facts and procedures is not necessarily when tackling problem-solving. Moreover, a task is said to be a problem if its solution requires the teacher to combine previously known information in a way that is new especially to the learners (Pehkonen, 2014).

Polya (1965) views developing learners' problem-solving skills as one of the main aims of high school mathematics. Subsequently, Polya views it in line with the principle and standards for school mathematics, which states that problem-solving is a pivotal part of school mathematics (NCTM, 2000). Consequently, it is greatly supported by mathematics education researchers with the same understanding that problem-solving is a cornerstone not only in doing mathematics but also in the teaching and learning of mathematics (Lester & Charles, 2003; NCTM, 1980, 1989, 2000, Schoen, 2003 & Donaldson, 2015). Mathematics teachers are in other words expected to teach through problem-solving. Therefore, this study explores if mathematics teachers do teach through problem-solving and how they use Polya's 4-steps of problem solving.

Problems given to learners in the classroom need to engage the learners to think on their own rather than to recall or recite what the teacher said in the classroom. Similarly, Mogari, et al. (2016) proposed that approaches and strategies are problem-solving should not be just doing the activity but rather comprehensive and involve the activity of thinking. Moreover, a teacher is expected to use strategies that involve the activity of independent thinking of the learners. Sadly, the current way of teaching mathematics nowadays is on drilling and registration of facts and procedures (Abdullah, et al., 2014). In a similar vein, this study aimed to discourage the drilling and registering of facts and procedures in the mathematics classroom.

The current way of teaching mathematics in schools has resulted in Makhubela and Luneta (2014) concluding that poor performance in Grade 9 mathematics is a result of mathematical teachers' lack of pedagogical content knowledge, especially where problem-solving is concerned. Additionally, Siswono, Kohar, Kurrniasara, and Astuti (2015) point out that teachers theoretically understand pedagogical problem-solving knowledge and problem-solving and implementation of problem-solving but, when teaching, show less understanding of problem strategies and the meaning of problem-solving when doing it in the classroom. Nevertheless, this study looked closely at how teachers teach problem-solving in the classroom including content and pedagogical knowledge. This study considers Polya's 4-step problem-solving model when learning mathematics.

2.4.2 Polya's 4-step problem solving process

Polya (1945, p. xxxvi-xxxvii), explains the problem-solving process as illustrated in Figure 2.2 that follows:

Understanding the problem → Devising the plan → Carrying out the Plan → Looking back

First. UNDERSTANDING THE PROBLEM.

- You must understand the problem.
- What is the problem?
- What are the data?
- What is the condition?

Second. DEVISING THE PLAN

- Find the connection between the data and the unknown.
- You may be obliged to consider auxiliary problems if an immediate connection cannot be found.
- You should obtain eventually a plan of the solution.

Third. CARRYING OUT THE PLAN.

- Carrying out the plan of your solution, check each step.

Fourth. LOOKING BACK.

- Examine the solution you obtained.

Figure 2.2: Summary of Polya's 4- step problem solving process
Adapted from Donaldson (2015)

Schoenfeld (1992) further identifies the following as crucial episodes in problem-solving as:

- The problem
- Making a plan
- Carrying it out
- Checking the answers against the question asked.

Kilpatrick (1969) views a mathematical problem as a prior index of processes used to arrive at the solution. However, Donaldson (2015) argues that there is a challenge around a clear picture of what problem-solving is and how to teach it and alluded that it is meaning and procedures of teaching it is complex. Therefore, Kilpatrick (1985)'s description of problem-solving as 'word problems' is purposed to allow learners to apply what they have learned in their classroom. However, this study also viewed problem-solving considering non-routine mathematical problems which emphasise the complexity and mathematical interest as opposed to routine problems as a focus of other mathematics researchers (Donaldson, 2015).

Moreover, the DBE (2015), in response to the ANA results, shows that the Department of Basic Education views mathematics with an emphasis on problem-solving. Furthermore, it must not just be problem-solving, but the learners must be able to visualise or conceptualise the mathematics problems that they are solving. According to the DBE (2014), the main aim for studying mathematics is to solve problems, and without the ability to solve problems the usefulness and effect of mathematical ideas, knowledge, and skills are limited. This shows that problem solving is the major goal of teaching mathematics. As a result, the objectives of the CAPS curriculum can be met if the problem-solving feature is considered in all teaching and learning of Grade 9 mathematics.

2.4.4 Problem Solving in South African mathematics classrooms following the National Curriculum Statement (NCS-CAPS)

The major general aim of the South African curriculum as stated in the curriculum and assessment policy (CAPS) document is as follows, 'the national curriculum statement from grade R-12 aims to produce learners that can identify and solve problems and make decisions using critical and creative thinking' (DBE, 2011a, p.4). Moreover, the CAPS document identifies two of its six mathematical skills essential for the Senior Phase (Grades 7-9) learners as follows: to develop learners who are able ' to pose and solve problems and build awareness of the important role that mathematics places in real life situation including the personal development of a learner' (DBE, 2011b, p.58). From the document for South African mathematics teachers, the importance of problem-solving in a mathematics classroom is viewed as highly important. As a result, it is important to note that failure to adhere to the policy might result in teachers underperforming in their duties towards the Department of Basic Education. Since problem-solving is not just an aim of learning mathematics but also an important way of doing it (Pehkonen, 2014), it becomes the foundation for effective teaching of mathematics.

Additionally, Govender (2015) advises schools about how mathematics Olympiads act as tools to promote problem-solving among learners. In a similar vein, Buthelezi (2016) notes that the IEB curriculum in South Africa has proved to produce internationally competitive learners, especially in mathematics. Additionally, Govender (2015) views mathematic Olympiads and competitions to have more influence on emphasising problem-solving skills. Mathematics Olympiads and competitions are seen to promote excellence in mathematics teaching and tend to enrich the learner's mathematical knowledge. This can be one strategy to improve learner's problem-solving skills but only if put in place in most schools.

2.5 Use of visual tools when teaching Grade 9 mathematics problem-solving

2.5.1 Meaning of visualisation

"Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools to deplete and communication information, thinking about and developing previously known ideas and advancing the understanding," (Arcavi, 2003, p. 217).

Duval (1995) defines visualisation as a process that refers to the use of representations such as figures, images, diagrams, symbols for illustrating or verification of different concepts. This is done using visualisation when teaching. It can be performed either by using pictures, images, diagrams, manipulatives, or technological tools such as computers and interactive whiteboards. Moreover, Naidoo (2011) views visualisation as the ability to form and transfer a mental image necessary for problem-solving in mathematics. Mental images can be transferred in the form of diagrams pictures, images which make solving mathematics problems simpler and more possible. The use of visual representation aims to inspire ideas and allow a deeper understanding of the relationships involved in problem-solving. It follows that for learners to better understand how to solve a problem, the use of visualisation can be helpful.

Some researchers have pointed out that visualisation has been an important part of a mathematician's ways of thinking (Hadamand, 1945; Presmeg, 2006 & Zarzycki, 2004). In support of this, Zarzycki (2004) concludes that mathematical concepts cannot be introduced without illustrating them through pictures, drawings, graphs, and manipulatives. Therefore, the use of visual tools in a mathematics classroom seems to play a major role. Furthermore, Naidoo (2011) lists visual tools used in a mathematics classroom as gestures, graphs, shapes lines, and diagrams. Hence, the visualisation tools used by the different scholars in this study are those used and expected to be used by Grade 9 mathematics teachers when teaching problem-solving.

Maries and Sing (2013) argue that using pictures, images or diagrams are effective in solving higher order mathematics problems. On the same note, using visual tools in an adjustable and corresponding manner has the potential to help learners understand mathematical concepts and relationships as they solve the problems (Bautister, Canadas, Brizuel & Schliemann; 2015). Barmby, Bolden, and Thompson (2014) further argue that the use of a variety of visual tools in mathematical problem-solving helps in explaining mathematical ideas while solving a given problem.

Additionally, Taylor and Reddi (2013) point out that the lack of resources such as models, drawings, graphics, calculators, and charts is a contributing factor to learners' negative attitudes towards mathematics. Moreover, Makhubela and Luneta (2014) state that mere telling without exposing the learners to concrete materials do not enhance learning. In the same vein, Spaull (2013) recommends that models, pictures, drawings, graphics, manipulatives, and charts should be used in explaining mathematical concepts and relations. Therefore, the mathematics classroom should allow learners to feel and touch mathematics (Taylor & Reddi, 2013), which implies that the surroundings in a mathematics classroom are supposed to reflect a mathematical space and inspire learners to want to learn more mathematics. It can be characterised by having mathematical charts, manipulatives, and 3D shapes placed around the room. Dovetailing with Taylor and Reddi (2013), Makhubela and Luneta (2014) suggest that principals must ensure that their schools are mathematically well resourced, while the resources will help to form concepts in learners' minds. Therefore, a classroom of that nature would be of value in the learners' visualisation process when solving their mathematical problems.

2.5.2 Types of Visualisation Tools used when teaching problem-solving

2.5.2.1 Diagrams

Van Garden (2006) argues that the use of diagrams during problem-solving increases learners' ability to achieve this. However, Maries and Sign (2013) argue that it is not just the use of diagrams, but the effective use of diagrams that leads to successful problem-solving while failure to draw the diagram correctly may result in an inability to solve the problem. Additionally, diagrams act as an important step in organising and simplifying given information (Van Garden, 2006). Therefore, once information has been simplified into a diagram it becomes easy to solve a problem. Affirming this, Maries and Sign (2013) aver that they see diagrams as practical problem-solving heuristics. Similarly, Bautista, et al. (2015) emphasise that teachers with strong mathematical backgrounds normally use diagrams to explain concepts. The use of diagrams was seen to be evident in the mathematics classroom since the participating teachers used them as visual tools.

2.5.2.2 Pictures/ graphs/ figures / gestures

Graphs or pictures allow learners to visualise a mathematical problem and may work in the same manner as using diagrams and images (Bautista, et al., and 2015). When using graphs teachers can use verbal statements, number lines, data tables, or symbolic notation. According to Duval (1995), a figure gives a figural representation of a geometric mathematical situation that is shorter and easier to be understood than giving a verbal explanation. However, gestures are body movements produced in a silent, non-communicative, problem-solving situation (Chu & Kita, 2011). Extant research shows that people perform visualisation tasks better when they are instructed to move their hands in a way congruent with what they need to visualise (Wexler, Kosslyn, & Berthoz, 1998; Smith, & Pereira, 2009). This was evident from teachers using gestures as a visualisation tool during the lesson on problem-solving.

2.5.2.4 Manipulatives

Manipulatives are physical objects designed to represent explicit and concretely abstract mathematical ideas (Furner & Worrell, 2017). However, Ball (1991) questions the effectiveness of using manipulatives to reach correct mathematical concepts but stresses the context in which manipulatives are used to create meaning. Moreover, Baroody (1989) supports Ball (1991) by stating that the use of manipulatives is not guaranteed in solving mathematical problems but supports its use because he believed it is a useful tool.

Furthermore, teachers believe that manipulatives are not necessary for solving mathematics problems and usually advise their learners to solve problems mathematically (Furner& Worrell, 2017). On the contrary, Golafshani (2013) in his study, examined the 9^{th}-grade mathematics teachers' use of manipulatives and discovered that teachers prefer using manipulatives but lack time to use manipulatives as a resource and knowledge of how to use them to explain mathematical concepts. Therefore, Furner and Worrell (2017) emphasise the use of manipulatives as a useful tool for mathematical problem-solving. This does support the use of manipulatives in a mathematics classroom to explain concepts and to solve mathematical problems.

2.5.2.5 Videos, computers, and other technology equipment

The National Council of Teachers of Mathematics (NCTM) in the USA considers technology as one of its six principles: 'technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning' (NCTM, 2000, p. 24). Along similar lines, Alfaro-Carvagal and Fonseia- Casto (2018) view the use of computers or technology as tools that directly influence interactions with a problem and conceptualisation and as a result, contribute to the development of learners' competencies and how these can be explained. Consequently, this brings to our attention the importance of the use of computers and technology in mathematics teaching and learning.

Moreover, computer-aided algebra resources whose core comprises symbolic manipulators can be used as a visualisation tool (Chiu & Churchill, 2015). Furthermore, to support the use of technology, Lois and Milevicich (2008) state that they can be used as mediation strategies that allow teachers to mentor, motivate, and stimulate learning while assisting learners to reason and search for knowledge. Additionally, Sutch (2010) argues that technology strengthens the relationship between the teacher and learners as it brings them closer to conceptual co-learning. Therefore, technology was seen in this study as one of the important visualisation tools which may help in problem-solving.

Furthermore, the use of digital tools allows for the introduction and reflection of new mathematical knowledge in line with the development of a learner's competency and as a result, brings about rethinking. Additionally, Caglayan (2014) suggested the use of materials that help visualise algebraic expressions or numbers to construct mathematical formulas meaningfully.

Chiu and Churchill (2015) suggested three principles to maximise learning with written words and images: deleting irrelevant words and graphics; highlighting important words and graphics; and presenting words next to corresponding graphics simultaneously. This can also be expressed using whiteboards. Additionally, Lois and Milevicich (2008) argue that whiteboard and video equipment can also be used as ways of solving problems while facilitating the concepts to be learned and how they can be carried out.

2.5.2.6 Calculators

Karadeniz and Thompson (2018) argue that the use of a calculator in a mathematics classroom changes the teacher's role to that of a facilitator. Moreover, using calculators in a mathematics classroom helps promote learner achievement and attitude (Tan, 2012). Additionally, Spinato (2011) notes that calculators promote learners' reasoning skills. It is for these reasons that this study explores the teachers' use of visualisation tools when teaching problem-solving in their classrooms. Furthermore, Karadeniz and Thompson (2018) suggest that calculators allow for common starting points for teachers and their learners. Moreover, Lazakidou, and Retalis (2010) states that calculators save time, increases learners' efficiency, and their outcomes in mathematics.

2.6 Conclusion

A gap in research on the use of visual tools when teaching problem-solving in Grade 9 has been noted. There have been studies done on visualisation in different topics especially in Grades 10-12. More studies done on visualisation in South Africa focussed on the Foundation Phase. In other countries, problem-solving has been noted to be more prevalent while in South Africa it is only found effectively in the Olympiads, AMESA competitions, and in IEB schools. It was thus important to consider this study concerning exploring teachers' use visualisation tools when teaching problem solving in their Grade 9 mathematics classroom

CHAPTER 3 THEORETICAL FRAMEWORK

3.1 Introduction

The previous chapter discussed related literature concerning a) mathematics teachers and teaching, b) teaching Grade 9 mathematics in South African schools, c) problem-solving skills in the mathematics classroom, d) the use of visualisation when teaching problem-solving and e) how teachers may use visualisation tools to allow learners to effectively solve problems in the mathematics classroom. This chapter focuses on the theoretical framework that was used in the study, that is, Activity Theory and Polya's 4-step problem-solving process.

Grant and Osanloo (2014, p. 56) describe a theoretical framework as 'a form of research in totality based on its philosophy, epistemology, methodology, and analytical approached. It is where the epistemological perspective of the study is put across concerning the problem, purpose, significance, research question, literature, and the analysis of the study'. The first-generation activity theory and Polya's 4-step problem-solving process were used as a theoretical framework for this study based on their dovetailing with the study.

A theoretical framework is based on existing and prior proven theories (Kumar, et al., 2011). Therefore, activity theory is a prior-proven and existing theory. It was used by different scholars, especially in mathematics education (Engeström, 1987; FitzSimons, 2003; Yamazumi, 2008; Rasmussen & Ludvigsen, 2009; Naidoo, 2011). In this study, it was linked to the use of visualisation tools when teaching problem-solving. It was used to connect the tools used by the teachers when teaching problem- solving. However, Polya's 4-step problem-solving process is also an existing and prior-proven theory in mathematics education (Maluleka, 2013; Shuilleabhaim, 2014; & Malesala, 2016). Its relevance in this study was to examine the steps used by teachers during problem-solving. Grant and Osanloo (2014) argue that there is no one best theory that best fits with an inquiry. Therefore, this study used both theories for an effective explanation of the problem under study.

Groves and Dale (2004) propose that activity plays an important role in mathematics learning and development. If activity places a major role in the teaching of mathematics, then the problem-solving process plays a major role in how mathematical problems are solved in the classroom. Activity theory focuses mainly on the human activity of a teacher, on the use of visual tools in Grade 9 mathematics classroom while Polya's 4-step problem-solving model emphasises how teachers teach problem-solving in a mathematics classroom.

3.2 Activity theory

Activity theory is founded on the works of a Russian psychologist Vygotsky and his students such as Leontieve (Hasan & Kazlauskas, 2014). According to Engestrom (1999), activity theory has its threefold origin in classical German philosophy (from Kant to Hagel) in the writing of Marx and Engels, and in the Soviet Russian cultural psychology of Vygotsky, Leont'eve and Luria. Therefore, activity theory is about who is doing what (Hasan & Kazlauskas, 2014). In this study, we are focusing on Grade 9 mathematics teachers who are teaching problem solving using visual tools. The theme of activity is formed by the relationship between mathematics teachers who are subjects or doers and the use of visual tools when teaching problem solving which becomes the object within the activity system under focus. This is illustrated in Figure 3.1 which shows the core of the activity.

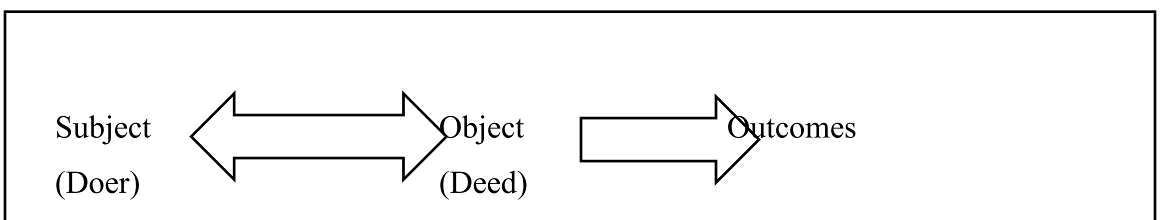

Figure 3.1: The core of an activity

Adapted from Hasan and Kazlauskas (2014, p.9)

In this study, Activity Theory was used to explain how an individual activity of mathematics teachers interacts with the subject and tools in their environment. Naidoo (2011) states that Activity Theory encourages interaction and social cohesion among participants. Additionally, Hasan and Ekelauskas (2014, p. 46) concur and state that activity theory is a human activity driven by human needs to achieve a certain purpose. This means that the central part in Activity Theory is that a human doer (the teacher) who is driven by a motive to do an activity in different operations. Furthermore, Naidoo (2011) emphasises that an effective mathematics environment is one that constitutes actions, group activity, and operations as illustrated by Figure 3.2.

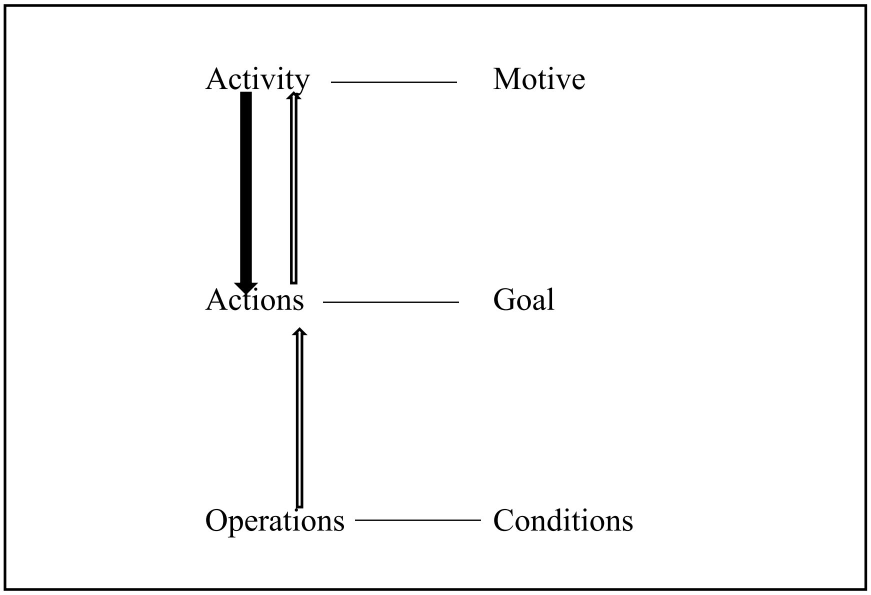

Figure 3.2: Three levels of an activity

Adapted from Kuutti (1996, p. 2)

Therefore, Activity Theory aims at reflecting the richness in alternatives of content, form, and mobility (Hasan & Ekelauskas, 2014). Subsequently, this study uses Activity Theory to explain the richness in each teacher's use of visual tools during problem solving in the classroom. Engestrom (1999) outlines three generations of Activity Theory but this study was based on the first generation of Activity Theory as shown in Figure 3.3.

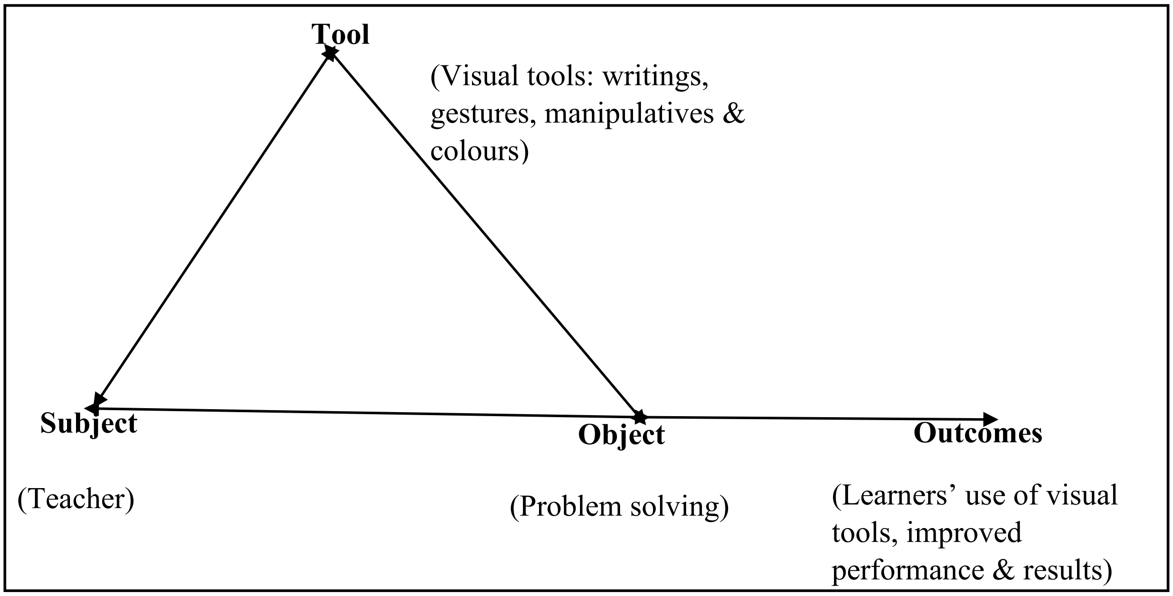

Figure 3.3: First generation Activity theory model
Adapted from Hardman (2005, p.1)

Figure 3.3 shows an Activity Theory approach which is drawn from Vygotsky's concept of mediation and further included the concepts of this study. Therefore, the first-generation Activity Theory was used because this study was only interested in the interaction of the teacher with visual tools during mathematical problem-solving. However, this study does not place emphasis on the community and the rules. It focused on the human activity of the subject, which in this study is a Grade 9 mathematics teacher. The object or deed is mathematical problem-solving, and the tools are the visual tools used such as drawings, gestures, maps, manipulative, pictures, colors, and computers. The outcome was the ability of Grade 9 mathematics teachers to use visual tools when teaching problem-solving in a mathematics classroom which produced learners who can use visual tools and hence improve their performance and results.

3.3 Polya's 4-step Problem-Solving Process

According to Van Hanegem (2017), doing mathematics is not about reproducing readymade mathematics, but about promoting mathematical thinking and understanding and developing problem-solving abilities and research skills. In mathematics, problem-solving can be explained following Polyas'4-steps problem-solving process. It involves being presented with a written problem in which the learner has to interpret the problem, devise a method to solve it, follow mathematical procedures to achieve the results, and then analyse the results to determine any acceptable solution to the current problem (Polya, 1945). Therefore, in this study, it refers to a classroom situation whereby a teacher is presented with a written word problem to be taught to the learners through interpreting it, devising a plan, applying the plan through mathematical procedures, and finally analysing the results.

However, it is important to note that there is no single procedure that works all the time in problem-solving because each problem might be slightly different from the other (Caglavan, 2014). This means that there are many ways of solving problems mathematically. Therefore, a teacher may use different and several procedures for teaching learners to solve problems. Nevertheless, Polya's 4-step problem-solving process as shown in Figure 3.4 was used as a guide for this study on how teachers teach problem-solving.

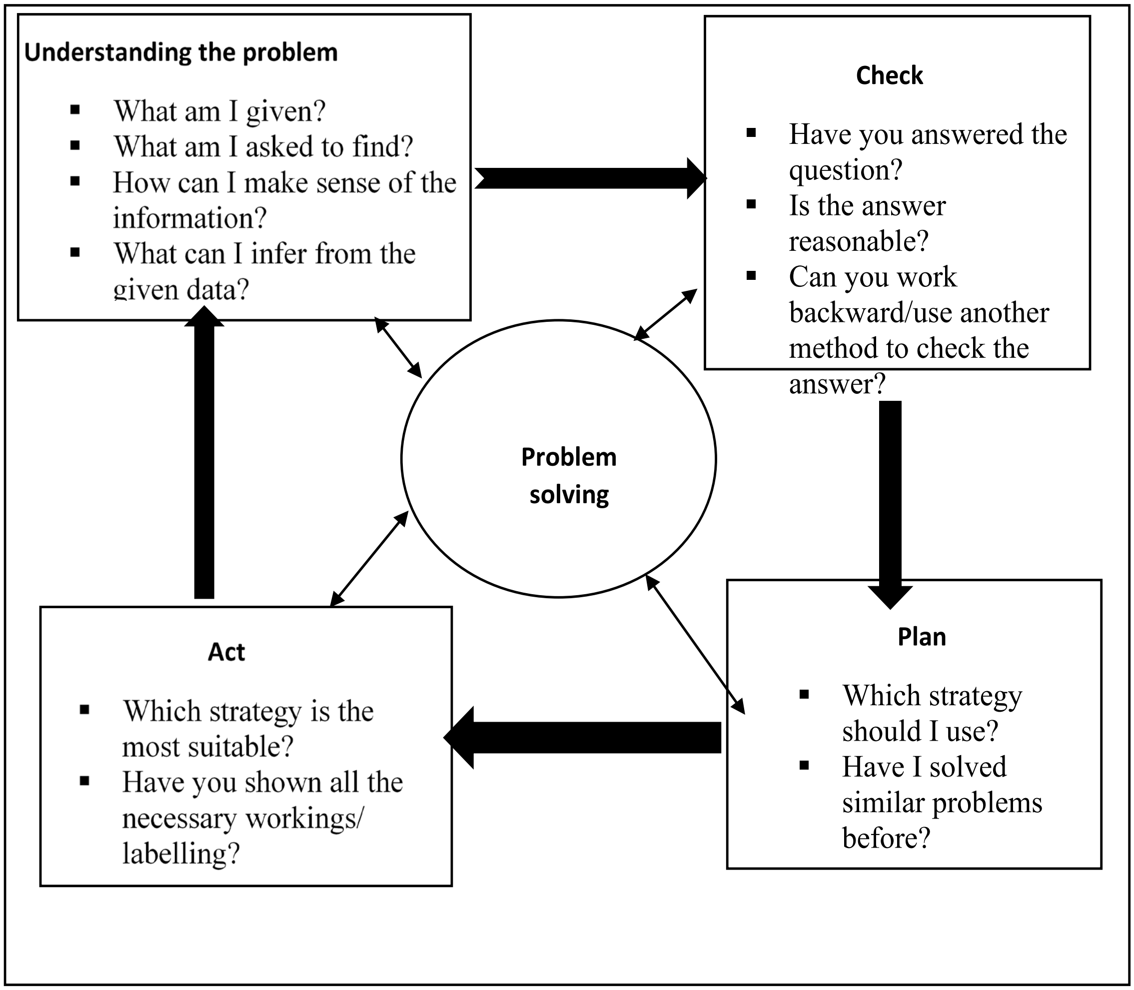

Figure 3.4: Polya's 4-step problem-solving process
Adapted from Caglavan (2014, p.15).

The first step of Polya's 4-step problem-solving was understanding the problem, which constitutes reading the problem carefully, listing, or underlining keywords involved in the problem. Therefore, Step 1 shows if one has understood the problem. Then teachers who used this step of problem-solving tended to read the problem aloud, underline keywords or list them with the learners. Step 2 was about devising a plan, which involved a teacher devising a strategy of solving the problem or setting up an equation to solve the problem. However, learners could be asked to suggest ways they think could be used to solve the problem. Then, Step 3 was about carrying out the plan. If an equation was created, this was where the equation was solved or where the plan was carried out. Finally, Step 4 looked back on what had been done to see if all information given was used correctly and helped to detect if the answer made sense. In this study, the two theories were incorporated together because they were both needed to include all the concepts of this study.

3.4 Activity theory and problem-solving process as a framework

The use of Activity Theory and Polya's 4-step problem-solving process was purposefully done to meet the objectives of this study. The Activity Theory framework was used to find out the kind of visualisation tools that were used by the teachers and how they were used. Hardman (2005) presents the diagram of first-generation Activity Theory and Fernandez, Hadaway, and Wilson (1994) present the model for problem-solving which were jointly used in this study. Bringing together the two diagrams assisted in presenting a single theoretical framework for this study as shown in Figure 3.5.

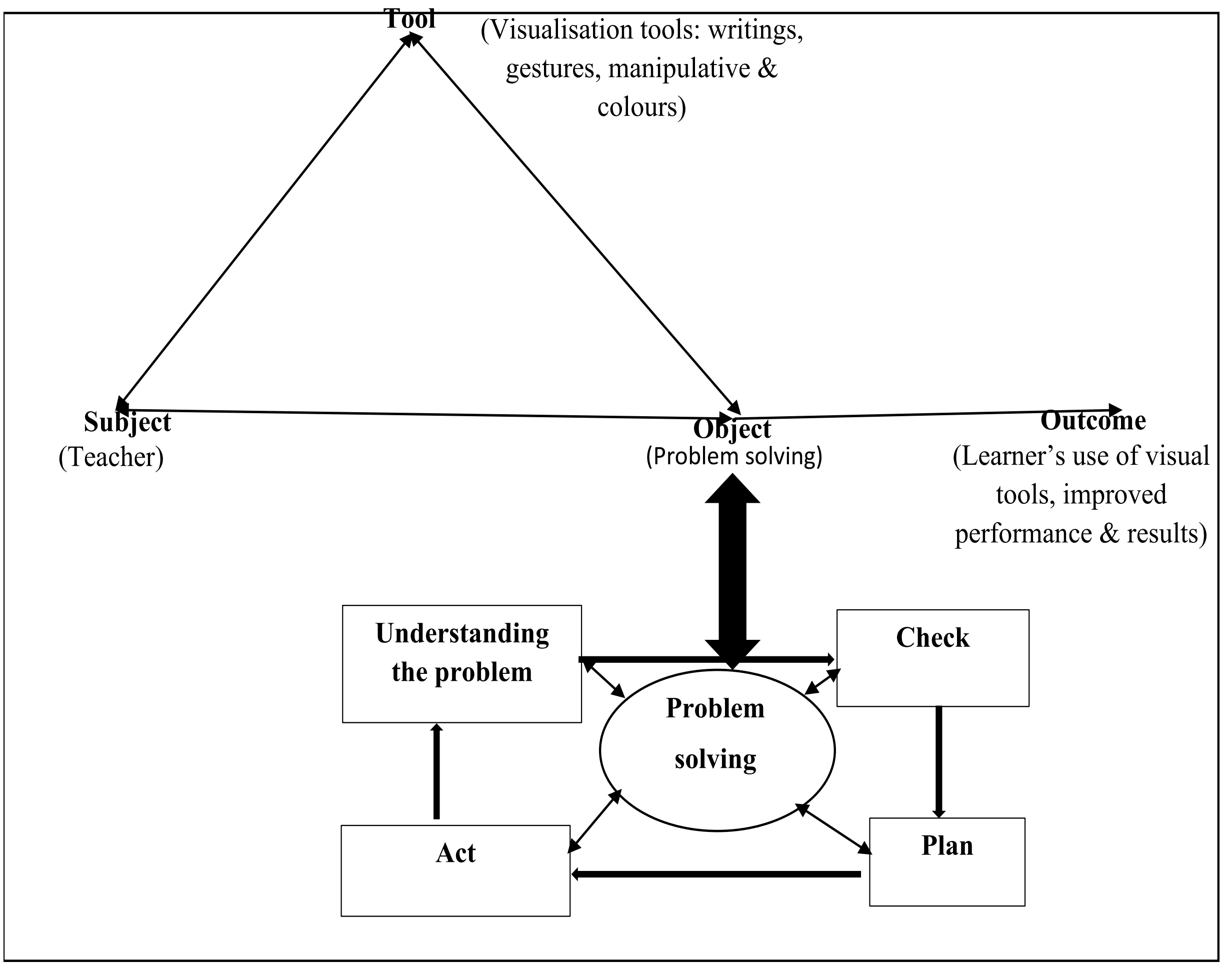

Figure 3.5: Activity theory and Polya's 4-step problem-solving process
Adapted from Hardman (2005).

This framework was important for this study since, after finding out what visual tools (such as graphs, gestures, drawings, manipulatives, and computers) were used by Grade 9 teachers, there was a need to examine how and why they used them during problem-solving, by using the 4-step model and activity theory. Moreover, it was also important to design a framework that would bring together visualisation and problem-solving processes as a model since the Grade 9 curriculum involves problem-solving and the use of visualisation throughout the CAPS document. Furthermore, it helped explore if teachers used visual tools when teaching problem-solving in their mathematics classroom. Hardman (2005) sees the main aim of Activity Theory as one that reflects the richness in a variety of content, form, and mobility. However, Bautista et al. (2015) argue that using multiple representations in mathematical problem-solving play an important role in the explanation of mathematical ideas. These ideas thus bring about the richness of using both theories.

3.5 Integrating the theoretical framework to the study

This study adopted Activity Theory and 4-step problem-solving process which was solid and reliable frameworks on which to build and inform the use of visual tools when teaching Grade 9 problem-solving. Grant and Osanloo (2014) highlight that a theoretical framework is integral in developing the key components of a study that include the problem, purpose, significance, research question, literature review, research method, and analysing of research results.

3.5.1 Integrating the theoretical framework with the problem

The study found that the major problem in South African education is the high failure rate in Grade 12 mathematics (DBE 2016). This is due to the poor teaching of mathematics in South Africa (Du Plooy, 2015) and, since 2010, the highest percentage pass in mathematics in South Africa has been 59.1% (DBE, 2016). Moreover, when Visser, et al. (2015) note that mathematics has been and still is a major challenge for learners, teachers, schools, and the Department of Basic Education at large, it raised the attention of this study to explore how mathematics is taught in the classroom. This study explored the use of visual tools when teaching problem in grade 9 mathematics classroom. Activity theory was used to show that visual tools such as manipulatives, graphs, different colors, computers, diagrams, and pictures may be used as mediating artifacts in a mathematics classroom. The teacher was referred to as the subject and problem-solving was regarded as the object as shown in Figure 3.5. Problems in a mathematics classroom may be solved using Polya's 4-step problem-solving process which entails understanding the problem, devising a plan, carrying out the plan, and looking back (Grant & Osanloo, 2014).

This study aimed at exploring how the teachers teach problem-solving using visualisation tools which in turn improve learners' problem-solving skills. The theoretical framework of this study may help teachers understand that teaching problem solving can be effectively achieved by using the 4-step problem-solving model. It is also important to view teaching as a human activity where the teacher performs an activity to solve the problem using visual tools. This then helps to respond to the importance of the use of visualisation and problem-solving as key concepts in the teaching of mathematics (DBE, 2014).

Grant and Osanloo (2014) state that research questions and the theoretical framework are complementary. This is because the research question has a direct influence on the theoretical framework. Green (2014) states that the research question for a study acts as a link between the existing knowledge and the problem one wants to solve. In mathematics teaching, it is referred to as a human activity and in solving problems, there is a certain way of going about the process. The use of visual tools during mathematics teaching is an activity that links the subject and the mediated artifacts and hence answers the question of what visual tools are used in the classroom. Figure 3.6 illustrates how the research questions were integrated into the theoretical framework of the study.

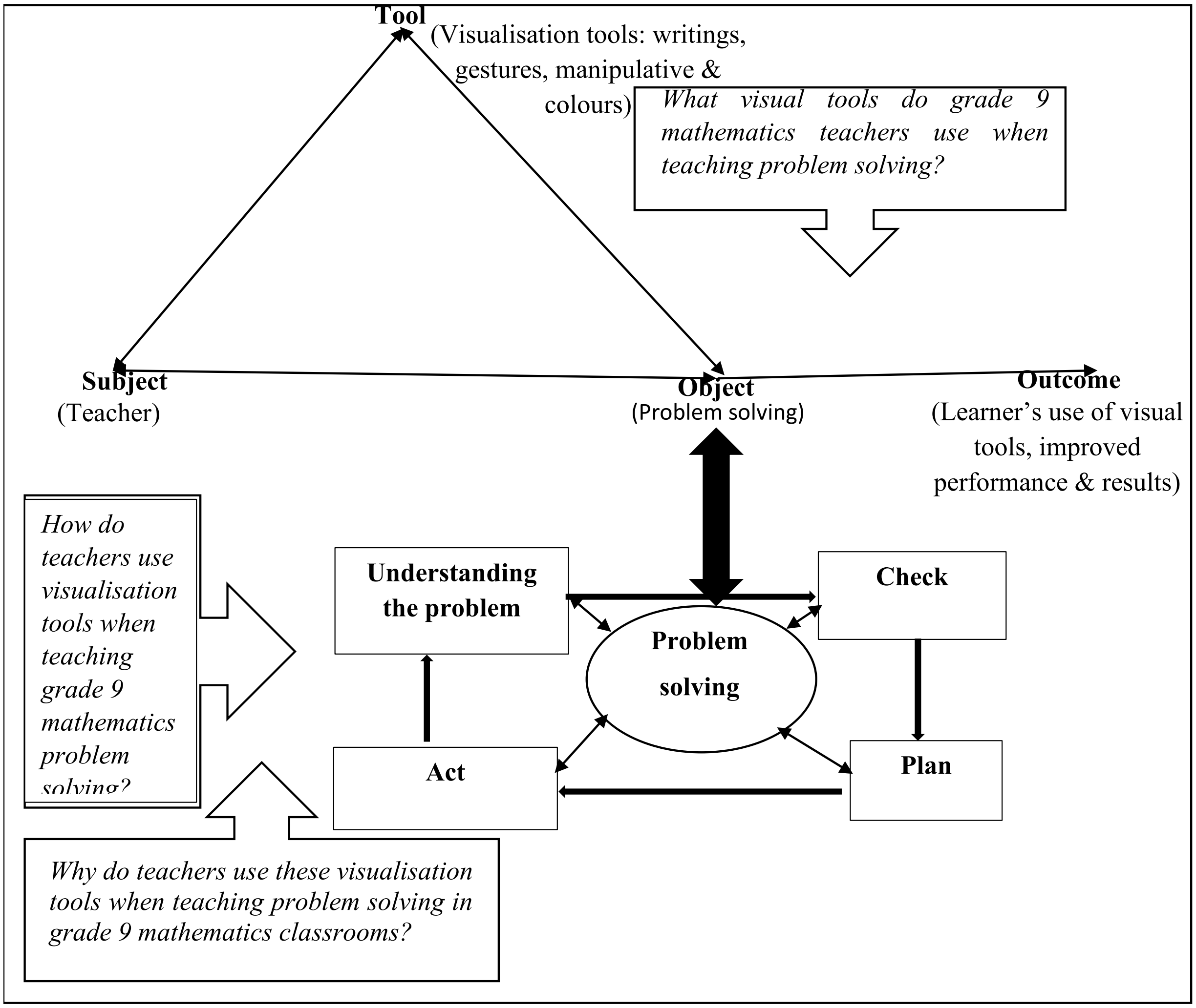

Figure 3.6: Theoretical framework integrated with the research question

Researcher's diagram

Figure 3.6 shows how the research question was included into the theoretical framework of the study. The first question of what visual tools were used by mathematics teachers was well fitted in the tools part of the activity theory which also link it to the object which is problem solving. Then, question two of how the teachers us the visualisation tools linked the subject (teacher) of the activity theory to Polya's 4-step problem solving which then also connects to the third question, why the teachers use those tools. Therefore, question one and two was linked more to the activity theory whilst two was also linked to Polya's 4-step problem solving together with the third question.

3.5.2 Integrating the theoretical framework with the literature review

This study argues that there is a process mathematics teacher may follow to solve problems during their activity of teaching mathematics in a classroom. Therefore, the literature review was defined as a discussion of one's knowledge about the topic under study and how that topic was elucidated by extant research (Grant & Osanloo, 2014). This implies that the literature review shows its benefit to the whole population and provides a theoretical basis of the work under study. In chapter 2 of this study, gaps were identified in specific areas whereby the proposed theoretical framework was used to try and fill up those gaps. Grant and Osanloo (2014) suggest that there is a need to develop concepts from existing literature that may or may not fit in with the theoretical framework. Therefore, in the case of this study, concepts from Activity Theory and Polya's 4-step problem-solving were used.

Moreover, in chapter 2, mathematics teaching in South Africa was discussed. It was noted that the CAPS curriculum emphasised the use of problem-solving and visualisation when teaching mathematics in the classroom (DBE, 2014). Since the CAPS document emphasises the teaching of problem-solving and the use of visualisation, the theoretical framework which combines problem-solving and visualisation as a framework in a mathematics classroom was relevant to this study. Furthermore, considering the content of the mathematics Grade 9 workbook, it was noted that problem-solving is included in every topic as a daily activity for learners. Therefore, the activity of using visual tools when teaching problem-solving becomes a daily activity that mathematics teachers ought to undertake in their classrooms.

Additionally, Motshwane (2012) states that mathematics teachers are managers of the curriculum's success; meaning that they need strategies to manage it successfully. In this study, the success of the curriculum is perceived to be based on what the teachers can effectively use as a framework or model for their mathematics teaching. Moreover, Naidoo (2011) completed her study with master teachers on the use of visual tools when teaching mathematics, demonstrating that the use of visualisation and problem-solving has a certain link to a certain level of educational qualification or experience. This study did not focus on any qualification or experience level but did take the teachers' qualifications and experience into consideration. Subsequently, it brought an awareness of a model or framework that could be used by mathematics teachers for effective teaching. Furthermore, Bryan (2011) found that Limpopo teachers who lacked subject knowledge tended to use self-talk and class-based notes from the book during their teaching. Notably, it can be said that content knowledge can skew teachers' use of other creative teaching methods such as visualisation.

Moreover, Nkhwalume (2013) argues that the state of current mathematics education no longer compares with previous years where it was done for enjoyment but rather it is taught to help learners to understand and use mathematics concepts to solve real-word problems, familiar as well as unfamiliar. There has been a shift from previous styles of teaching mathematics to a current and effective framework that can fit the present generation (Buthelezi, 2016). The framework should be able to help teachers to produce mathematically competent and skilful (DBE, 2013) learners. Competent and skilful learners can meet the national benchmark tests (NBT) required by South African universities. This is a response to Cliff's observation that 67% of matriculants are not prepared to cope with the academic demands of the university (2015). South African learners can then cope with university standards.

Since the DBE (2014) stated that the main aim of studying mathematics is to solve problems, without the ability to solve problems the usefulness and effect of mathematics ideas, knowledge and skills are critically limited. Subsequently, Mogari, et al. (2016) states that drilling, facts, and procedures when teaching mathematics do not encourage thinking and problem-solving. Noting the need for an effective way of teaching problem-solving, the theoretical framework of using activity theory as a human activity together with the problem-solving steps come into play. It serves to address the problem of drilling facts and procedures and aims to help learners effectively solve mathematics problems using visualisation.

Furthermore, Polya (1949) views problem-solving as an art in which learners imitate what the teacher does in the classroom and practice what they see the teacher doing. Understanding the activity of mathematics teachers when teaching problem-solving is important to influence how the learners solve mathematics problems. Additionally, the DBE (2013) emphasises the need for teachers to teach learners the techniques to solve problems through underlining numbers and questions, drawing diagrams, and using number lines to work out the answers. Hence, Polya's

4-step problem solving process is of importance. Subsequently, Mogari et al. (2016) emphasised that approaches and strategies used during problem-solving are comprehensive and involve the activity of thinking. Therefore, these approaches and strategies may employ the use of Activity Theory and Polya's 4-step problem-solving approach of this study.

3.5.3 Integrating the theoretical framework with the methodology, analysis, and findings

The theoretical framework of Polya's 4–step problem-solving process and Activity Theory was identified to support and build the methodological plan of this study. The methodology can be understood as a bridge between theory and data (Grant & Osanloo, 2014). Additionally, Engestrom (2016) states that it goes beyond the collection of specific methods and techniques and puts forward and implements a theory-driven set of principles, upon which the choice of specific methods is based, starting from data collection to examples of methodologically informed choice of methods. Since this study was qualitative, the use of visual tools was observed, and subsequently, teachers were interviewed on how and why they used visual tools when teaching problem-solving using the lens of this framework.

Since Activity Theory focuses on human activity such as where the subject is the teacher, the teacher was asked questions if he or she used visual tools as mediating artifacts when teaching problem-solving in mathematics. Consequently, there was a need to observe how the teacher used the tool, which was visual, as well as observing the problem-solving steps used in the activity. Therefore, the data collection methods used were gleaned from Engestrom (2015) when stating the importance of using the triangular model of activity theory as a graphical model for interpretive data analysis. Furthermore, Lochmiller and Lester (2015) argue that there must be a framework that will make sense of that data. Subsequently, data were analysed using a content mapping analysis of related literature, the theoretical framework, and the data gathered for the study. Therefore, the framework knitted together the data collection resulting in an effective model for teaching problem-solving using visualisation tools.

Research findings can be used to support, extend, or modify (Grant & Osanloo, 2014) a borrowed theory presented and applied in a study. Therefore, the findings of this study helped to endorse the use of Activity Theory and 4-step problem-solving as a theory that can be used for effective teaching of problem-solving using visualisation tools. They also brought an understanding of the scenario that had to take place in mathematics classrooms when teachers used visual tools when teaching problem-solving.

3.6 Other studies which used Polya's 4-step problem solving process and Activity Theory as a framework

Engeström, (1999) views Activity Theory as an advancing model which can also be manageable and founded. Activity Theory is said to further describe learning activities and learners' differences in processes and outcomes in problem-solving (Perels, Gürtler, & Schmitz, 2005). Moreover, studies rooted in Activity Theory relate to acquiring substantial learning (Engestrom, Sannino, & Virkkunen, 2014; Sanninno, Engestrom, & Lerios, 2016). Furthermore, Gedera and Williams (2015) provide examples of Activity Theory used in their classroom curriculum. However, this study focused on how teaching problem-solving in Grade 9 mathematics was related to an Activity Model.

Anthony (2011) used Activity Theory with the integration of technology planning systems through a longitudinal case study while this study was on mathematics education and not the use of technology. Moreover, Clark and Fournillier (2012) analyse teacher's experiences in mathematics and explore using Activity Theory to examine how those experiences impact their teaching. Huang and Lin (2013) focus on Activity Theory as a theoretical framework to analyse the influence of cultural factors on Taiwanese mathematics learners. Furthermore, Treffert-Thomas (2015) use Leontiev's (1981) Activity Theory framework to sort the teaching of linear algebra on three levels: activity-motive, actions-goals, and operations-conditions. However, in this study, the focus is on the activity of Grade 9 mathematics teachers when teaching problem-solving.

Researchers who use Polya's 4-step problem-solving process include Pehkonen (2014) whose aim was to find possibilities of advancing mathematical problem-solving while focusing on different importance of the problem-solving steps. Additionally, Polya (1949) carried out heuristics in problem-solving. Furthermore, Schoenfeld (1979) assumed that teaching and learning would enhance learners in problem-solving strategies and improve their problem-solving abilities. Moreover, Serwerin (1979) successfully directed program strategies on problem-solving. Notably, most of these studies did not focus mainly on mathematics education.

In the 1980's more attention was given to requests for promotional opportunities in daily teaching. However, according to NCTM (1980), the learning of problem-solving in mathematics at schools was compulsory. Some researchers focussed on specific aspects such as attitudes, emotion, and self-regulated behaviour (Kretschmer, 1983; Schoenfeld, 1985, 1987, 1992) where problem-solving is concerned. Therefore, the framework in this study takes a certain angle while immersing itself in mathematics teaching.

3.7 Conclusion

The idea of bringing together Activity Theory and Polya's 4-step problem-solving process was useful in responding to the three questions of this study. It also assisted in summing up the whole research process. Activity theory and Polya's 4–step problem-solving process were used in the study to construct a model that would help mathematics teachers to effectively use visualisation when solving mathematics problems in their classroom. It was found suitable for this study since it was able to respond to the research question and achieve the main aim of the study. The next chapter focuses on the research design, methodology, and philosophy of this study.

CHAPTER 4 RESEARCH METHODOLOGY AND PROCEDURE

4.1 Introduction

The previous chapter provided an overview of Activity Theory and Polya's 4-step problem solving process as a theoretical framework guiding this study. In this chapter, the underlying philosophical assumptions of the research, approach, strategy, research design, and methodology are discussed. This chapter further discusses the population, selection of the sample, research instruments and outlines the issues related to their management such as piloting and administration of the research instruments. Furthermore, it outlines the limitations governing the study and then concludes with a brief discussion of the ethical considerations for the study.

Research is often initiated when there is a need to find a solution or a better solution than the one that exists to a problem or to contribute new knowledge or inventions. This was discussed around the research questions which influenced the methodology of this study which read: what visual tools do teachers use when teaching problem-solving in Grade 9 mathematics classrooms? How do teachers use visual tools when teaching problem-solving in Grade 9 mathematics classrooms? And why do teachers use these visual tools when teaching problem-solving in Grade 9 mathematics classrooms?

4.2 The research design

A research design is a structure that integrates the research and permits the researcher to address the research questions in a suitable, efficient, and constructive way. McMillan and Schumacher (2010) state that the research design relies on the nature of the study and the purpose of controlling the study. The purpose of this study was to explore visualisation tools used by mathematics teachers when teaching problem-solving. According to Flick (2014), a qualitative research design is a proposal for collecting and analysing verification that will make it possible for an explorer to answer study questions. Correspondingly, Brundrett and Rhodes (2013) view a qualitative approach as one that focuses on understanding an occurrence from a closer viewpoint. Therefore, this study did not concentrate on many respondents (quantitative) but rather a few purposefully chosen participants who used visual tools when teaching problem-solving in their Grade 9 mathematics classrooms. In a similar vein, it uncovered trends in thoughts and opinions of teacher's use of visual tools during mathematics problem-solving through in-depth searching into the problem (Wyse, 2011). Quantitative research did not qualify for this study as it did not quantify a problem by generating numerical information or data that could be generated into useable statistics and generalises results from a large population (Brundrett, & Rhodes, 2013). Therefore, this study was qualitative, since the sample population was seen to contain in-depth information of the context because it was mainly focussed on only those who used visual tools when teaching problem-solving in mathematics.

Moreover, qualitative research designs have the following types: exploratory design, descriptive design, casual-comparative design, and correlational design (McMillan & Schumacher, 2010). However, qualitative research mostly follows exploratory research which is used to acquire an understanding of underlying reasons, opinions, and motivations for a phenomenon (Wyse, 2011). Additionally, Creswell and Poth (2016) describe qualitative research as one that addresses an exploration research problem about which little is known. However, descriptive research was more suitable for this study since it is more structured than exploratory research (Van Wyk, 2012). It followed a more structured exploration of an understanding of underlying reasons, opinions, and motivation for the use of visual tools in a Grade 9 mathematics classroom. Moreover, this study was targeted at providing a valid representation of factors or variables that pertain to the research questions, which could be accurately explored through a descriptive research design.

Therefore, a qualitative descriptive approach involves identifying the characteristics of the phenomena or exploring possible correlations among two or more variables. Notably, descriptive research helps in acquiring the value of human experiences in research. Furthermore, Anderson (2012) emphasises that it helps in uncovering universal knowledge as opposed to multiple and different perspectives about the world.

An advantage of using qualitative research in this study was that it provided an in-depth understanding of the materials (Ito, Gutiérrez, Livingstone, Penuel, Rhodes, Salen, ... & Watkin, 2013). The researcher was able to obtain a deep understanding of how and why mathematics teachers used visual tools when teaching problem-solving. Moreover, Flick (2014) states another advantage of qualitative research as the fact that it is an appropriate method and theory because its data requires in-depth probing through semi-structured interviews. The processes followed in this study were natural using real-life settings which allowed the researcher to develop a more accurate understanding of the context Ito, et.al (2013).

Moreover, the data obtained was based on human experiences of the teacher's setting which had power and was more compelling (Anderson, 2012). Creswell (2014) states that it was quantitative research because it sought for explanations rather than measuring number data. Furthermore, McMillan and Schumacher (2010) point out that it also produces findings arrived at from real-world settings where the phenomenon of interest unfolds naturally. Therefore, analysing numerical data using statistical measures was contrary to this study because it used context and themes to analyse data.

4.3 Research philosophy

According to Rhodes (2013), a research philosophy involves a discussion of the research paradigms which incorporate the fundamental philosophical concepts and values about the nature of reality and the scientific pursuit of knowledge Therefore, since this study was qualitative, it followed an interpretive research paradigm based on a phenomenological school of thought. A research paradigm or approach 'is an opinion held by a community of researchers that is founded on a set of shared assumptions, concepts, values and practices' (Johnson, 2010 in Christensen, 2014, p. 31). However, the participants of this study were viewed as active with feelings and reacted following certain behaviours that could be affected by the researcher.

Denzin and Lincoln (2005) define a paradigm as the researcher's basket that holds ontological, epistemological and methodological beliefs, and Adebesin, Kotzé, and Gelderblom (2011) adapt and summarise four philosophical assumptions as described by Vaishnavi, Kuechler, and Petter (2013), as shown in Table 4.1.

Table 4.1: Philosophical assumptions of the four research paradigms

ASSUMPTIONS	PHILOSOPHICAL ASSUMPTIONS			
Research Paradigms	**Ontology**	**Epistemology**	**Methodology**	**Axiology**
Positivist	- Single, stable reality - Law-like	- Objective -Detached observer	- Experimental - Quantitative -Hypothesis testing	-Truth (objective) -Prediction
Interpretive	*-Multiple realities* *-Socially constructed*	*- Empathetic* *-Observer subjectivity*	*- Interactional* *- Interpretation* *- Qualitative*	*-Contextual understanding*
Critical/ Constructionist	-Socially constructed reality -Discourse - Power	- Suspicious -Political -Observer constructing Version	- Deconstruction - Textual analysis -Discourse analysis	-Inquiry is value bound -Contextual understanding - Researcher's values affect the study
Design	-Multiple, contextually situated Realities	-Knowing through making -Context-based construction	- Developmental - Impact analysis of artifact on the composite system	- Control - Creation - Understanding

Adapted from (Adebesin et al., 2011, p.310; Vaishnavi and Kuencher, 2013, p.235)

The four philosophical assumptions shown in Table 4.1 are ontology, epistemology, methodology, and axiology. Since this study follows the interpretive paradigm it is conceptualised as 'having relativist ontology with a subjectivist epistemology and is always aligned with postmodern thought' (Levers, 2013, p.1). The researcher believes that relativist ontology is a finite subject experience (Denzil & Lincoln, 2005) whereby the participants or teachers are of critical importance where the nature of reality is concerned. Therefore, the researcher encouraged dealing with the participants' knowledge of the use of visual tools and whatever they did in the classroom during the teaching of problem-solving. Moreover, subjective epistemology is the belief that knowledge is always filtered using language, gender, social class, race, and ethnicity (Denzil & Lincoln, 2005). Therefore, in this study knowledge was filtered using English as a medium of communication while gender was not of much importance but rather the teaching of mathematics problem-solving using visualisation tools by Grade 9 teachers.

Since ontology raises questions about the nature of reality, the researcher reasoned with participants on their use of visual tools through an interview. However, a phenomenological stance of ontology within the interpretive paradigm is distinguished by a concept of describing the real world rather than revealing the mental processes of observers (Antwi, & Hamza, 2015) and emphasises the social construction of knowledge from the participants together with the researcher's stand around knowledge on their use of visual tools. Subsequently, epistemology, or the study of knowledge, is 'a way of understanding and explaining how I know what I know' (Crotty, 1998, p. 3). According to Vaishnavi and Kuencher (2015), epistemology contemplates the relationship between an inquirer and the object of inquiry, that is, how a researcher reflects about the nature of knowledge or reports the truth about information gained. This paradigm puts emphasis on the importance of the subjective human creation of meaning but can also be influenced by the objectivity of the researcher.

The methodology entails developing or formulating an exploration of the impact of a development or construct in its context of use (Adebesin et al., 2011). Therefore, the methodology used in the study was interrelated with the questions or objectives by examining the visual tools used and how and why the teachers used them in their classrooms. It relates to the literature review as it tried to mend gaps through understanding the link between mathematics teaching and the way teachers use visual tools during problem-solving. The activity theory and Polya's 4-steps problem-solving theoretical frameworks assisted in the interpretation of the use of visual tools in the classroom and forming a theory that would influence the use of visuals tools in a mathematics classroom.

Moreover, axiology reflects the values of a researcher concerning the environment of research (Adebesin et al., 2011, & Vaishnavi et al., 2013). Axiology within the interpretive paradigm brings about contextual understanding enabling the researcher to come out with themes of different concepts about the use of visual tools. The conceptual understanding was based on the examples of typical qualitative methods of research employed in an interpretive research paradigm as case studies, interviews, and observations where the focus was on understanding context (Adebesin et al., 2011; Bhattacherjee, 2012; 2010; Oates, 2005; & Hevner, March, Park, & Ram, 2004). However, Vaishnavi et al., (2013) emphasise that interpretive researchers are not neutral, and researchers are expected to acknowledge their involvement and influence.

Furthermore, it was understood that this study followed the interpretive approach because the researcher tried to explore and understand a particular social phenomenon as it allowed the researcher to ask open-ended questions, observe and live with the participants in their natural social context (Crotty, 1998 & Pring, 2000). In a similar vein, interpret beliefs are guided by the researcher's set of beliefs and feelings about the world and how it should be understood and studied (Denzin & Lincoln, 2005). Therefore, within the interpretive paradigm, the focus is primarily on identifying and narrating the meaning of human occurrences and actions (Fossey, Harvey, McDermott, & Davidson, 2002) as was done through interviews and observation in this study.

4.4 Research strategies

According to Oates (2005), different research strategies can be linked to philosophical paradigms. Since this research involves a qualitative or phenomenological approach and an interpretive research paradigm, the most common research strategy available was a case study. This is because it involves a single site or few sites studied over a certain period concerning the in-depth study of a phenomenon (Yin, 2012). This study explores a case of five Grade 9 mathematics teachers from two comprehensive technical high schools within the Umlazi district. Moreover, a case study design was considered because it answered the 'how' and 'why' questions of the study. Considering the nature of a case study, that it requires a significant amount of time and resources to be spent on studying one program, this study it was only concerned about the use of visual tools during problem-solving in Grade 9 mathematics classrooms. In a similar vein, it helped in acquiring insight into the broader issue of Grade 9 mathematics teacher's use of visual tools during problem-solving (Springer, 2010). In support of that, Rule and John (2011) and Stake (2013) emphasise that a case study is more relevant in descriptive research. However, this study follows a descriptive case study that encompasses a thorough description of a phenomenon within the classroom setting (Yin, 2012 & Flick, 2014). This was because it explored and interpreted the experiences of teachers' use of visual tools. Bertram and Christiansen (2014) argue that descriptive case studies intend to describe the condition of a state in detail.

Vaishnavi et al., (2013) state that an advantage of case studies is that they are widely used because they offer insights that might not be achieved with other approaches, such as design or survey research. Therefore, in this study in-depth information on how and why the teachers in the two comprehensive technical schools used visual tools was explored. Moreover, they also allow for the identification of a group of individuals with diverse experiences. This was done by first giving teachers' questionnaires where they stated if they used visual tools when teaching problem solving. Secondly, they were purposefully selected for observation on how they used the visual tools during their problem-solving. Finally, they were interviewed with an open-ended type of semi-structured interview to get a broader perspective on their use of visual tools in their problem-solving lessons.

4.5 Study setting: comprehensive secondary schools within the Umlazi district

The case study was carried out in two Dinaledi comprehensive technical secondary schools within the province of KwaZulu-Natal (KZN) in South Africa. KZN has a population of about 10 919 100 and is in the South-Eastern part of the country. According to Ndlovu, and Demlie (2018) neighbouring provinces for KZN are the Eastern Cape, Free State, Mpumalanga, and Gauteng. Durban city is situated close to the Indian Ocean. There are 12 districts in KZN including Umlazi where the study was conducted. Umlazi district is the largest with 16 circuits having approximately 537 schools. The approximate percentage of the languages used in these schools are 77, 8%, IsiZulu; 13, 1%, English and 1, 5%, Afrikaans (Lehohla, 2012). The high schools used for the study were in a township area with mainly African learners and teachers. Most learners from these schools came from poor and disadvantaged backgrounds as many of them received social grants and lived in Rural Development Project (RDP) houses as mentioned by different participants. The school enrolment in the district ranged between 700-3000 learners. However, in the two-school chosen, they had a high enrolment of more than 2000 learners each as compared to others, but what was common in all schools was that they were under-resourced yet striving to obtain excellent results (Yan-Di Chang, 2007).

These schools were located within a low socio-economic area since most parents depended on government grants. These were both comprehensive technical Dinaledi schools. A comprehensive technical school is one that includes technical or technology subjects (electrical and mechanical engineering, automotive, electrical, and graphic design, civil engineering, welding, and Technical Science and Technical Mathematics). Dinaledi schools were established in 2001 to improve the number of learners passing physical science and mathematics (DBE, 2010).

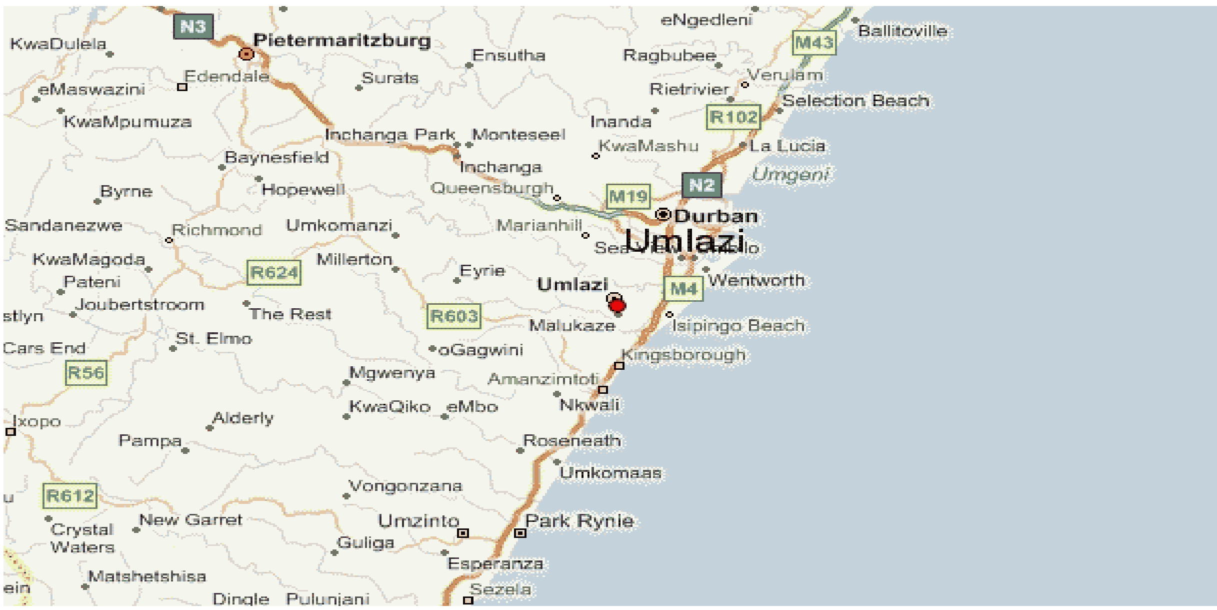

Figure 4. 1: Map of Umlazi

Adopted from (Ndlovu, & Demlie, 2018)

Figure 4.2: Umlazi Location

Adopted from (Yan-Di Chang, 2007, p.23)

4.6 Target population

According to McMillan and Schumacher (2010), a population is described as a group of cases in which we intend to generalise the results of the study from a criterion. The population for this study was made up of Grade 9 mathematics teachers from five comprehensive technical high schools within the Umlazi district. There were nineteen (19) Grade 9 mathematics teachers from the five schools. Weiss (1998) stresses that although the sample for qualitative evaluation may be a small subset of the population it could generate a large amount of data about each participant considered.

4.7 Sampling strategy

There are two types of sampling: probability sampling and non-probability sampling. In probability sampling, the sample is selected such that the probability of a subject being selected is known. Therefore, all members of the population have equal chances of being selected and hence bias is greatly minimised. Moreover, subjects in probability sampling are representative of the whole population, hence the findings from such a sample can be generalised to the whole population (Creswell, & Poth, 2016). However, non-probability sampling is not about selecting participants randomly, but they are rather selected based on specific characteristics, accessibility, and relevance to the study.

Moreover, non-probability sampling may be categorised into convenience sampling, purposeful sampling, and quota sampling (McMillan & Schumacher, 2010) and is mostly suitable for qualitative research. Therefore, the purposeful non-probability sampling method was chosen for this study. The sampling method around Grade 9 mathematics teachers also allowed for the selection of participants based on characteristics, that is, those who were currently Grade 9 mathematics teachers; relevance, that is, they used visual tools when teaching problem-solving and accessibility when needed, for generating data for the study. Furthermore, purposive sampling helps one to be precise and clear about the targeted sample (Creswell, & Poth, 2016). Therefore, the five teachers in the two schools were chosen on condition that they used visual tools during their teaching of problem-solving in their Grade 9 classrooms.

A questionnaire was administered to twelve (12) Grade 9 mathematics teachers in the four Comprehensive Dinaledi technical high schools within the Umlazi district. Therefore, the administering of questionnaires helped in selecting teachers who used visual tools during problem-solving in their classroom. It was a form of a selection tool for the observation and the interview.

Five teachers, that is, three from school A and two from school B, who were then using visual tools in their classroom, were observed during their mathematics lessons. Concept sampling by the case was used in selecting the three teachers who were said to be using visual tools when teaching problem-solving in their Grade 9 mathematics classrooms (Yin**,** 2017). Concept sampling helped in focussing only on teachers who used visual tools to be studied in the research rather than selecting those who were not familiar with or did not use the concept.

Table 4.2: Profile of Participants

Pseudo names of the participant	Age group	Gender	Qualifications	Subject Major(s) Grade 9	Teaching Experience	Teaching of problem	Use of visual tools
Patrick- Teacher A2	20-30	male	Master's in applied mathematics	Mathematics	4	yes	yes
Sisanda- Teacher B1	41-50	Female	B.Ed. (Hons.) & masters in curriculum studies	Mathematics& physical sciences	8	yes	yes
Noxolo- Teacher A3	51-65	Female	STD, B.Ed. (Hons.)	Mathematics	16	yes	Yes
Nothando	41-50	Female	B.Ed.	Mathematics	10	yes	Not often
Sibusiso- Teacher B2	41-50	Male	B.Ed. (Hons)	Mathematics	17	yes	yes
Portia- Teacher A1	31-40	Female	B.Ed.	Mathematics and natural sciences	1	yes	yes
Hendry	41-50	Male	B.Ed.	Mathematics	15	yes	no
John	41-50	Male	B.Sc. Chem Eng. SPTD, B. Com	Mathematics & Physical Science	20	yes	yes
Thando	40	female	STD; FETC Math's, science	Biology & Math's	20	yes	Not always

4.8 Pilot Study

Zikmund, Carr, and Griffin (2013) advocate the importance of conducting a pilot study before embarking on a larger study. Therefore, McMillan and Schumacher, (2010) define a pilot study as one which is about testing the research instruments with the participants and is important in ensuring that the research instrument contains items that are clear to the respondent and are related to the focus of the study. Hence, a similar school as that for the study and a teacher who used visual tools when teaching problem solving in his/her Grade 9 mathematics teacher, was selected. Moreover, Zikmund et al. (2013) state that pilot studies are critical in refining questions and reducing the risk of distorting the study. Piloting was vital to this study because it gave insight on other visual tools that can be used by teachers and it also helped to discover the need for word problem solving question templates to be used as guidelines during the observation (See appendix I). These helped to avoid diverse and unrelated concepts during the observation. Furthermore, the pilot study helped to examine each research instrument's clarity around questions. Therefore, the teacher who participated in the pilot study was given the questionnaire to answer and was subsequently observed on how he/ she used visual tools when teaching mathematics problem-solving. Then a semi-structured interview was administered for probing and clarifying reasons for using those visual tools.

The pilot study was found to be vital in checking if the instrument was able to measure what it was designed to measure and whether the questions were clear to the respondent. It was discovered that the instrument questions were able to answer the research question. Moreover, it assisted in discovering a link between the literature review and the responses from the teachers. However, a loophole was discovered in that the questions in the interview schedule needed some additional probes for obtaining clarity of responses. Furthermore, it was discovered that pictures were not clear when sitting at the back. Therefore, the researcher positioned herself closer for clear pictures during the observations in the main study.

4.9 Data generation

Data generation is an integral part of any research effort, especially in qualitative research. It can be defined as a methodology used by researchers to produce data from participants who responded to the research questions (Ritchie, Lewis, Nicholls, & Ormston (2013). Table 4.3 shows the data generation plan for this study.

Table 4.3: Data generation plan

Research question	Data generation Method	Sampling process and participants	Data source	Anticipated analysis
What visual tools do teachers use when teaching problem-solving in Grade 9 mathematics classrooms?	Questionnaire and Observation	Purposive Sampling; Grade Nine teachers	Twelve teachers (from four schools)	Thematic analysis
How do teachers use visual tools when teaching problem-solving in Grade 9 mathematics classrooms?	Observation	Purposive sampling: Grade Nine teachers who use visual tools in their lesson	Five Teachers	Thematic analysis
Why do teachers use these visual tools when teaching problem-solving in Grade 9 mathematics classrooms?	Semi-structured interviews	Purposive sampling: Grade Nine teachers who were observed	Five Teachers	Thematic analysis

4.9.1 Questionnaires

According to Blandford, (2013) questionnaires are a well-established technique for collecting demographic data and users' opinions. Therefore, questions in questionnaires should be unambiguous and set out to collect facts or information from human respondents as honestly as possible. However, short, easy-to-answer questionnaires have a better chance of being answered.

Another advantage of questionnaires is that they reach large numbers of respondents simultaneously and are more economical than interviews and observation.

On the other hand, the disadvantages of using questionnaires are that researchers are not in total control of the data-exchange process and are not able to probe certain responses (Rule & John, 2011). Therefore, the questionnaires were personally hand-delivered by the researcher to the participants and collected after they were completed. This helped to avoid delays in responses, avoiding the loss of questionnaires and timeous completion of the questionnaires. However, the inter-personal experience of face-to-face interaction and collection of data regarding feelings and emotions were not as easily captured in questionnaires as compared to interviews.

4.9.2 Lesson observations

Observations generate practical and convenient data for a case study (Rule & John, 2011) as it was practically generated on how the participants used visual tools. Additionally, Cohen, Manion, and Morrison (2013) describe observations as these provide the researcher with an opportunity to generate real data from naturally occurring public situations. Moreover, Yin (2017) adds that observations grasp and contribute to the liveliness of the situation within its real-life context. Therefore, each lesson from each participant was observed as it naturally unfolded. The lesson observation engendered the following observations: types of visual tools that the teacher used, how the visual tools were used to solve the problem, and the steps used in solving the problem including gestures and the use of manipulatives. During each observation, all-important occurrences were noted, and pictures were taken, also around video recording of each observation.

Ritchie, at.al, (2013) state that observations are useful in that the researcher becomes alert to intuitive interaction and qualities which could not be clarified in interviews. The observation on-site occurring during the teaching of Grade 9 problem-solving in mathematics and enabled the researcher to get first-hand information. The observation schedule was used simultaneously with the video recording of each observed lesson. After the observation, the participants were then invited for an interview based on what transpired in the classroom. However, constraint to the observation was the ‘Hawthorne effect’ which is the presence of the interviewer influencing participant behaviour. Therefore, this affected how the participants taught and seemed to be teaching in a way that they would without the presence of the researcher.

4.9.3 Semi-structured interviews

Rule and John (2011) concurs with Yin (2012) that an interview is the most common and familiar technique of generating data in qualitative research and often used in case studies. Moreover, interviews help to gather suitable data considering participants' feelings, beliefs, emotions, and experiences (Oates, 2005). This study, therefore, chose interviews as one of the instruments to collect data. Furthermore, Blandford (2013) lists four main types of interviews: open-ended or unstructured, semi-structured, structured, and group interviews. This study used open-ended semi-structured interviews for the participants to share information on why they used the visual tools that they used in their problem-solving lessons. McMillan and Schumacher (2010) define semi-structured interviews as ones in which the researcher prepares open-ended questions and allows participants to share information, feelings, experiences, and emotions in conversation.

Moreover, semi-structured interviews are suitable for exploring the views, attitudes, and beliefs of an individual concerning a subject (Van Teijlingen, 2014). Furthermore, Blandford, (2013) explains that exploratory data is gathered in semi-structured interviews through pre-determined, identical but open questions for every participant, and the goals of the interview session should be clear and well understood by participants. It was appropriate for the researcher to use this type of interview because it had open-ended questions which allowed for further probing to obtain a broader description of the experiences of the participants (Kvale & Brinkmann, 2009; Van Teijlingen, 2014).

This semi-structured interview was audio-recorded and then transcribed. The researcher focused on the participant's view as he or she explained the reasons and gathered more information than just words using communication skills such as listening, being creative, sensitive prompting, and encouraging and directing the conversation. The researcher was also flexible as she explored what was said, listened carefully, improvised, and was open to acknowledging differences in participants such as shyness or fatigue. All information was kept in a safe and secure place. The researcher also ensured that facts were verified with participants and maintained ethical commitments to participants and organisations.

4.10 Data analysis

Jansen and Vithal (2010) state that the analysis of data involves procedures that a researcher undergoes to prepare the raw data for analysis. Additionally, Cohen et al. (2013) state that qualitative data includes organising, accounting for, and clarifying the data; that is, making sense of the data in terms of the participants' descriptions of the situation, remarking patterns, themes, groupings, and consistencies. Therefore, data were analysed qualitatively using thematic analysis. Vaismoradi, Turunen, and Bondas (2013) note that thematic analysis is an independent and consistent method of analysing data, which is descriptive and provides basic skills to researchers for conducting many other forms of analysis in qualitative research. This allowed the researcher to discuss and analyse the data in terms of different themes used during the teaching of mathematics. The themes were based on the observation, prescription from interviews, and questionnaires linking it with the theoretical framework and what transpired from related literature discussion.

Moreover, the level at which thematic analysis was used can be associated with the essentialist, constructionist, and interpretive paradigm (Msiza, 2016). Rohleder and Lyons (2014) suggest that thematic analysis provides more flexibility compared to other forms of qualitative analysis as it is not tied to a specific theoretical, epistemological, or ontological position. The thematic analysis mainly includes both deductive and inductive approaches whereby the former is mostly driven by the interest of the researcher and the latter by the data (Vaismoradi et al., 2013; Rohleder & Lyons, 2014). Therefore, data that was collected using an audio recorder from the interviews were personally transcribed into written form by the researcher.

Furthermore, the transcribed data from both interviews and the observation schedule was analysed by the researcher using inductive analysis. According to McMillan and Schumacher (2010), inductive analysis is the process of making meaning from the raw data by synthesizing it, starting with specific data, and ending with categories and patterns. This process involves coding, categorising, and interpreting the data to provide explanations for the fundamental phenomenon of the study. However, the researcher used the data from the transcriptions of the audiotaped responses and observations to synthesise and infer meaning related to the research phenomenon without the assistance of any statistical software. After data was analysed and findings emerged, each participant was invited to an interactive workshop to discuss the findings.

4.12 Limitations of the study

The major limitation of the study was that since the study followed the interpretive paradigm, the small size of the sample limited the scope of generalising the research findings to other mathematics teachers in other schools, districts, and provinces. This therefore confine the study into teachers in Umlazi district in Durban where one cannot be 100% sure if it can be the same case with teachers in other parts of South Africa. Since this was a National issue in mathematics education, a need to sample even a wider scope was therefore necessary. Additionally, this was also limited by the fact that the researcher was doing her studies part time whilst working and thus limiting the scope and time of the study.

4.13 Trustworthiness

According to Msiza (2016), validity and reliability in qualitative research are based on honesty, depth, richness, trustworthiness, and quality of the study. However, within qualitative research, validity and reliability of a study cannot be tackled in a similar way as in quantitative research; instead, the concept of trustworthiness is used (Golafshani, 2003; Shenton, 2004). Since this study is qualitative, it aimed at trustworthiness as opposed to validity and reliability, which includes four criteria: 'credibility, dependability, confirmability and transferability,' (Shenton, 2004, p.29).

Credibility measures the consistency between the researcher's findings and the participants' responses (Shenton, 2004). Additionally, Schwandt, Lincoln, and Guba, (2007) point out that credibility in qualitative studies is one of the key measures used to ensure the trustworthiness of a phenomenon being studied. Thus, to enhance credibility in this study, audiotape recordings were used during the interviews and field notes were acquired during observations (Creswell, & Poth, (2016). Similarly, participants were told that they had the right to read the transcribed data if they wanted to check whether the transcriptions portrayed their experiences in the ways they meant it. Additionally, triangulation was used to increase credibility. According to Stake (2013) and Bertram and Christiansen (2014) triangulation refers to various resources of data gathering and gives a detailed assessment of what is being said. Therefore, interviews, observations, and questionnaires were used as a way of triangulation. The participants were encouraged to be free and truthful as it was not about wrong or right responses (Shenton, 2004).

Consistency and stability of data gathering are defined in terms of dependability in qualitative research (Schwandt, et. al, 2007). The researcher noted everything observed and obtained from the interviews during the field study to ensure dependability (Shenton, 2004). Moreover, data gathered was compared with different sources to verify if the findings were consistent.

By confirmability, Houghton, Casey, Shaw, and Murphy (2010) refer to objectivity and truthfulness of the data. Therefore, confirmability was maintained using triangulation to minimize the influence of bias (Shenton, 2004). Similarly, during data generation, the researcher tried not to be biased. The findings were only based on the experiences and ideas of the participants as stated in their interviews (Shenton, 2004). Vaismoradi et al. (2013) indeed indicate that interpretive researchers need to be careful not to impose their expectations as their expectations may influence what they hear and what they observe.

Finally, transferability refers to the extent to which the findings of a study can be generalised to other settings and populations (Shenton, 2004; Cohen et al., 2013). However, the findings of a case study cannot be generalised, since a case study focusses on a single instance in a specific environment and individuals (Stake, 2013). Therefore, this study intended not to generalise the findings, but to understand and interpret the phenomenon under research. However, the findings could apply to other similar contexts.

4.14 Ethical considerations

4.14.1 Ensuring informed consent

Informed consent was meant for participants' awareness of the purpose of the study and they had a choice to take part in the research. It meant that participants knew that their participation was voluntary. Additionally, they were free to withdraw at any time. A copy of the informed consent form signed by participants before they participated in the research is attached in Appendix E.

4.14.2 Ensuring that no harm comes to participants

This includes information that may lead to negative consequences (McMillan & Schumacher, 2010) especially to the participants. Since harm to participants may include physical, mental, and emotional harm observation and interviews were conducted in a place where the participants felt safe and free to talk. Moreover. The video recordings targeted only work done on the board and not the learners or the teacher. However, if any participants appeared in pictures, their faces were blocked so that they will not be recognised.

4.14.3 Ensuring confidentiality and anonymity of participants

McMillan and Schumacher (2010) state that respondents should be assured of confidentiality and protection of their privacy. This was ensured by making sure that they signed the concerned form which stated that their information would be kept confidential and would not be used in a way that would harm them (see Appendix E). Moreover, they were also informed that they were not obliged to answer all questions if they felt uncomfortable answering them. They were also informed that the information would be kept in a safe place with the supervisor and then burnt and destroyed after a certain period.

4.14.4 Ensuring permission is obtained

To ensure that there was no harm to the participants, an application of permission to conduct the study was done via the University of KwaZulu-Natal (UKZN) ethical clearance department. The ethical approval letter is found in Appendix A. In a similar vein, permission to conduct a study was obtained from the UKZN department of education (see Appendix D) and letters were written to principals of the schools for permission to conduct the study (see Appendix C). Moreover, pseudonyms were used to protect participants' identity and the schools used for the study.

4.15 Conclusion

This chapter began with a discussion of the qualitative research design and methodology used in this study. Then the target population and the sample selection procedures used were discussed. Similarly, research instruments used to generate data for this study were discussed. Moreover, a discussion of how the pilot study was conducted to test the instruments for reliability and validity was unpacked. Furthermore, a detailed discussion of how the data was generated, collected, presented, and analysed to address the research phenomenon and limitations associated of the study were acknowledged. Finally, it discussed the considerations of ethical compliance. The next chapter focuses on the discussions and interpretation of findings concerning the first two research questions.

CHAPTER 5: DISCUSSION AND INTERPRETATION OF FINDINGS CONCERNING RESEARCH QUESTION ONE AND RESEARCH QUESTION TWO

5.1 Introduction

The previous chapter provided an overview of the research methodology. This included the research design, research philosophy, and the research strategy including a discussion of the population, selection of the sample, research instruments, and piloting. Moreover, it outlined the limitations governing the study and then concluded with a brief discussion of ethical considerations. This chapter discusses the results and interprets the findings. Following the following research questions:

1. What visualisation tools do teachers use when teaching problem-solving in Grade 9 mathematics classrooms?
2. How do teachers use visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

These were analysed and discussed based on two Dinaledi Comprehensive Technical High Schools in the Umlazi district. Table 5.1 provides an outline of this chapter.

Table 5.1: Outline of chapter 5

Schools	**Themes**			
	Teachers	**Problem--solving steps (Object)**	**Visual tools used**	**How teachers use visual tools**
School A	Teacher A1; Teacher A2 & Teacher A3	How the teachers in School A used Polya's 4-steps process of problem-solving.	Chalkboard, chalk and gestures, and fingers.	Pointing, underlining, body language, and gestures.
School B	Teacher B1 & Teacher B2	How the teachers in school B used Polya's 4-steps process of problem-solving.	Chalkboard, chalk, whiteboard, markers, fingers, and gestures	Pointing, underlining, body language, use of colors, and gestures
Conclusion	All the Grade 9 (School A & B) mathematics teachers who participated in this study	How the teachers teach problem solving using Polya's 4-step process of problem-solving.	The most observed visual tools were used by the participants.	Pointing, underlining, body language, use of colors, and gestures.

Table 5.1 shows how the chapter was divided into cases of two schools. Firstly, this chapter discusses and analyses the use of Polya's 4-step problem solving by the three participating teachers in School A and two participating teachers in School B. Moreover, it discusses and analyses the first research question which was the type of visualisation tools used by participating teachers in both schools. Lastly, it discusses and analyses how the participant teachers in School A and B used the visualisation tools during problem solving in their classroom. However, it is important first to describe Grade 9 mathematics classrooms in Umlazi schools.

5.2 A description of teaching and learning in Grade 9 mathematics classrooms at Umlazi Dinaledi Comprehensive Schools

This study focused on five participant teachers from two participating schools where the participants agreed that they used visualisation when teaching problem-solving. School A included three Grade 9 mathematics teachers from a school in a location with an enrolment of about two thousand learners. The school had chalkboards but not whiteboards or projectors. The learners possessed mathematics learner's books and calculators. As a Dinaledi and technical school, the school obtained support from the Department of Education and companies such as Toyota. These supporters provided the school with resources and teachers with workshops that might have been useful in helping to improve the school's mathematics results. Three teachers were interviewed in School A after an observation on how they used Polya's 4-step problem-solving process and how they used visualisation tools when teaching problem-solving in their classroom.

Figure 5.1 shows the type of mathematics classroom in School A. However, the mathematics classroom in this school was just a room with a chalkboard, chalk, a teacher, desk, and learners as shown in Figure 5.1.

Figure 5.1: A kind of one of Grade 9 mathematics classroom in School A

However, in school B, two teachers were observed around teaching using Polya's 4-step problem-solving process and visualisation tools when teaching problem-solving. This school had an enrolment of more than three thousand learners. The school was also supported by the Department of Education and companies such as Toyota, Transnet, and others who provided them with needed resources and workshops that might have helped them improve their learners' mathematics results. Notably, school B was better resourced than school A. They had chalkboards, whiteboards, projectors and mathematics DVD's, charts and computers, and a printer in the classroom. However, as in the case of school A, their classroom was seen to be a common classroom because it did not show any pictures of mathematically related objects except for a poster of mathematics work schedules. Figure 5.2 shows a picture of a Grade 9 mathematics classroom in School B.

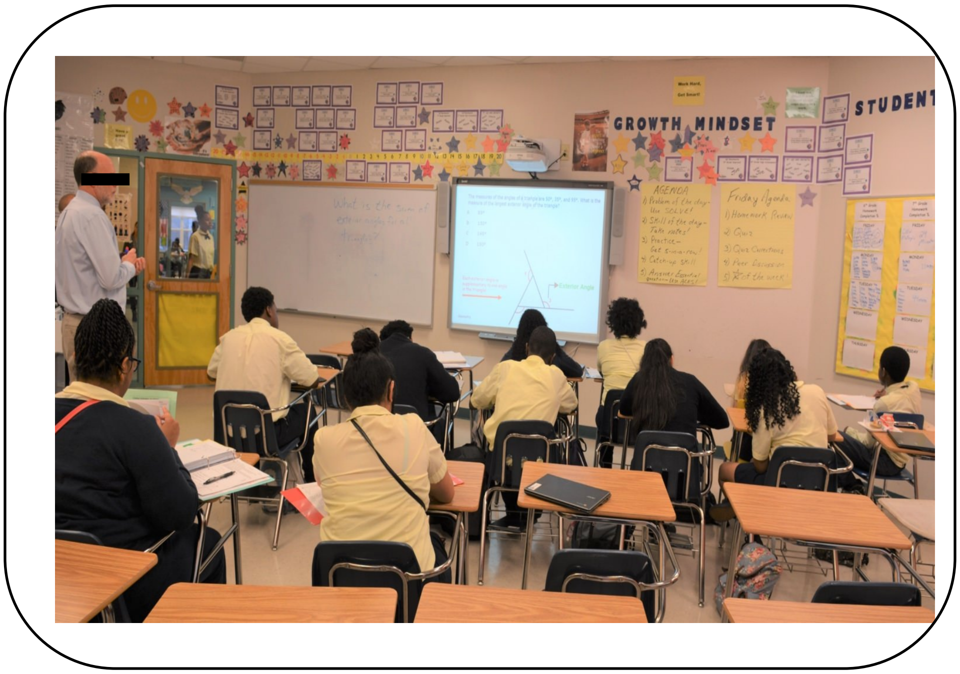

Figure 5.2: A Grade 9 mathematics classroom in School B

Moreover, the observations of each class activity lasted between 30-45 minutes succeeded by a semi-structured interview which took between 15-20 minutes. However, the academic profile of the participants was also of interest to this research.

5.2.1 Academic profile of the participants

The teachers were given pseudonyms: A stands for School A and B for School B while Teacher A1 is the first observed and interviewed teacher in School A. Two females and one male teacher participated in School A. The qualifications of participants in this study were important in understanding the level of the teachers' use of visualisation tools when teaching problem solving. Therefore, the profile of the participants showed that Teacher A1 from School A had a master's degree in Applied Mathematics, while Teacher A2 had an honors degree in mathematics.

However, teacher A3 was a novice teacher who possessed a Bachelor of Education Degree in mathematics. The study concurs with a statement made by Buthelezi (2016) that the minimum qualification of mathematics teachers in South Africa should be a Bachelor of Education Degree. Teacher A3 also had one year of teaching experience while the other two teachers (A1 and A2) had more than five years of experience teaching mathematics in School A. It was also revealed in this study that two out of three of the participants qualified for mathematics education. However, one out of three of the participants did not have any mathematics education modules in their qualification. This was supported by Mullis, et al. (2012) who noted that eighty-nine percent of mathematics teachers had either mathematics or mathematics education in their training. This study revealed that Teacher A1 did not have mathematics education in his qualification but rather a master's degree, which was also in Applied Mathematics.

In-School B, two Grade 9 mathematics teachers participated in the study. They were referred to as Teacher B1 and Teacher B2. They indicated that they used visualisation tools when teaching problem-solving in their mathematics classroom. They were a male teacher and an experienced female teacher who had more than five years teaching mathematics at School B. Moreover, one of them Teacher B1 had a master's degree in curriculum studies (physical sciences), and the other, Teacher B2, a bachelor's degree in mathematics education but over twenty years' experience as a mathematics teacher in his current school. Therefore, the case of School B does not dovetail with Azzarello, et al. (2013) who state that a master's degree in mathematics education is a prerequisite to teach mathematics in European countries. Moreover, data from School B shows that the participants had a hundred percent mathematics education in their qualifications. Furthermore, participants in School B were more experienced in teaching mathematics than those in School A. Although Teacher B2 did not have a master's degree he was observed to be more knowledgeable in teaching mathematics problem solving using visualisation tools. Nevertheless, Teacher B1's master's degree was not in mathematics education.

5.3 Teaching of problem-solving by Grade 9 mathematics teachers

All the participants in this study indicated that they did teach problem-solving in their Grade 9 mathematics classroom. Therefore, they were observed on how they used Polya's 4-step problem-solving process when teaching their learners how to solve word problems in algebra. The classroom activity was to solve three Grade 9 mathematics word problems (see Appendix I). Subsequently, they were observed using any two of the questions regarding how they taught the learners how to solve problems using Polya's 4-step process.

5.3.1 Problem solving steps followed by Grade 9 teachers when solving algebra word problems

The main objective during the observation was to determine how teachers used Polya's 4-step problem-solving process when teaching problem-solving. The study thus first discussed how the participants used Polya's 4-step problem-solving process. However, the participants in School A and B followed Polya's 4-step problem-solving very well. Youngchim, et al, (2014: p. 38) categorise Polya's 4-step problem-solving process as consisting of these steps:

- Understanding the problem,
- Devising the plan,
- Carrying out the plan, and
- Looking back

5.3.1.1 Understanding the problem

Understanding the problem entails teaching the learners how to work through the problem to obtain the desired solution. The participants at this stage taught learners to point out the critical parts/ clues of the problem; the problem / unknown or the data and the conditions attached to the problem. The first step of Polya's 4-step problem-solving process may be marked under the following conditions as indicated by (Maluleke, 2013, p.8):

- What is the problem/ unknown?
- What are the data?
- What is the condition?

The analysis of School A's participant teachers' response at this stage was that two participants (Teachers A2 and A3) asked their learners to read the problem on the board and to identify the data and conditions. This meant that two out of three of the participants were able to follow the conditions of the first step of Polya's 4-step problem-solving process. However, Teacher A1 simply picked the conditions for the learners without making them understand what the problem was and did not write the problem on the board. At School A, two of the participants asked the learners to state the unknown and then went on to underline the unknowns on the board. The participant (Teacher A3) who simply stated the conditions did not write the question on the board. The learners were not able to see the question or able to state the conditions because the question was only read to them. Identification of unknowns is shown by Figure 5.3 around the data and conditions from case study A3 teacher who was able to make learners pick the conditions for question 3 and then she underlined them on the board.

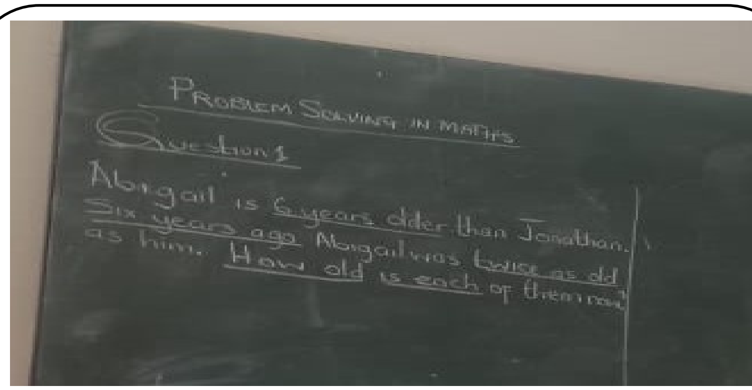

Figure 5.3: Identification of unknowns from Q.3 by Teacher A3

However, at School B, the teachers showed this stage by underlining keywords on the board. Both participants at School B asked learners to read the question with understanding so that they were able to pick clues, data, or conditions of the questions. Therefore, at School B, participants were able to follow the first step very well. Teacher B2 started by demonstrating to the learners how they ought to identify the clues or conditions to solve word problems, as shown in Figure 5.4.

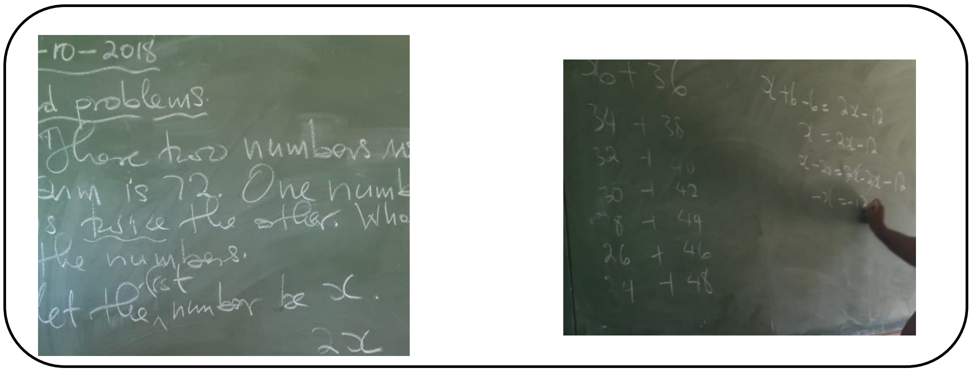

Figure 5.4: Teacher B2 using an example to represent how to identify unknowns from a given word problem

As presented in figure 5.4, Teacher B2 started by first demonstrating to learners how they could follow the first step of Polya’s 4-step problem-solving. Moreover, he reminded them about how to identify the unknowns and conditions when given an algebra word problem to solve. This dovetails with the argument of Pournera et.al (2015) argument that the best teaching of mathematics can be recognised by having a teacher who carefully and thoughtfully selects and explains tasks and examples which enhance learning.

It was noted that four out of five participants in this study were observed to follow step one of Polya’s 4-step problem-solving adequately. This is because at both schools it was discovered that four of the participants were able to teach the learners to identify the unknowns in the problem. Moreover, they were also able to underline the unknowns on the board for the learners, given the data, and given conditions to solve the problem. Therefore, the participants were seen to understand the first step of Polya’s 4-step problem-solving process. Subsequently, the teachers were able to move to the second step of Polya’s 4-step problem-solving which is devising a plan.

5.3.1.2 Devising a plan

Devising a plan entails dividing each part of the problem or difficulties into as many parts as possible and in a way that it will give direction towards obtaining the desired solution (Maluleka, 2013, p.38). According to Polya (1945, p. 8), ‘We can only say there is a plan after knowing the problem, which data/calculations, ways or solving / computations, or ways/constructions we have to perform to obtain the unknown'. Therefore, this step entails:

- Finding the connection between the data and the unknown,
- Considering auxiliary problems if an immediate connection cannot be obtained and
- Eventually obtaining a plan of the solution.

All participants as observed at School A and School B at this stage formulated equations in terms of x to represent the unknown. This was one way in which they devised a plan to solve the problem. They wrote the equations on the board as shown in Figure 5.5.

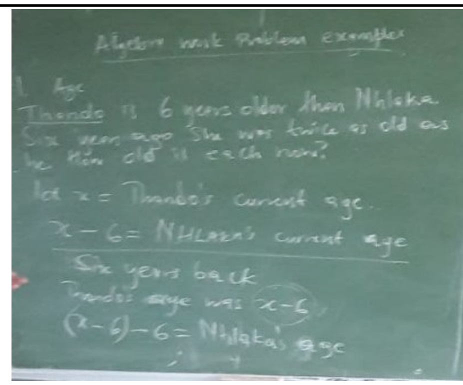

Figure 5.5: Teacher A3 devising a plan to solve the problem in Q.1

In both schools, all the participants were able to devise a plan by writing the unknown in terms of x to assist in solving the equations. Therefore, the x was used to create a condition that needed to be solved. However, in the case of Teacher A3, it was observed that even if you did not underline the unknowns/ data and conditions but if you know them you could be able to devise a plan to solve them. Similarly, an argument arises as to whether the learners were able to learn how to carry out the first step in this stage since they could not visualise it on the board. This is related to Bryan (2011) who notes that teachers who lack subject knowledge tend to use 'safe talk' supported by notes on board, constantly using books, and duplication of examples and exercises. Similarly, in this case, the teacher was observed to be reading from a book and talking rather than illustrating the work on the board and allowing learners to participate. Nonetheless, the next and third step of Polya's 4-step problem-solving process is carrying out the plan.

5.3.1.3 Carry out the plan

The third stage of Polya's 4-step problem-solving process is to carry over reflections from step 2 in due order. Participants completed this step using the plan they devised in step 2. At both schools, all the participants planned to solve the problem that was presented in terms of x. This showed that participants were able to teach learners to carry out their plan of solving a linear equation formulated from a word equation. Figure 5.6 shows how Teacher A2 carried out the plan to solve the problem for Q.1. This was done regarding how the participant had taught the learners to solve the unknown variable x.

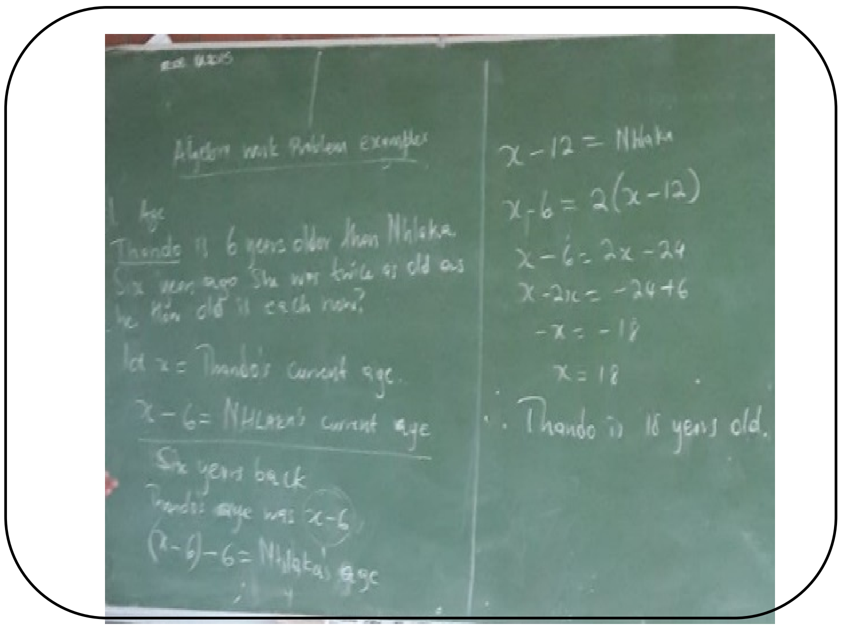

Figure 5.6: Teacher A2 carrying out the plan for solving Q.1 problem

The participant illustrated on the board how to carry out a plan of solving algebraic equations formulated from word problems. In this part, every step of the plan was clearly shown. In a similar vein, the learners were seen to be following this step very well as they solved for the unknown x from the data and conditions that they have underlined. Therefore, this step just entailed continuing with the plan that was chosen (Brijlall, 2015) in step 2. However, Figure 5.7 shows how teacher A2 was observed carrying out a plan to solve Q.3.

Figure 5.7: Teacher A2 carrying out a plan to solve the Q.3 problem

Question 3 was interesting since it involved integrating information from Natural Sciences (NS). Therefore, Teacher A2 began by drawing on the board the situation illustrated by the problem in the question. Then they used formulae from NS to devise a plan and carry out the plan. Moreover, those drawings tended to arouse interest and enhance understanding as the learners were heard saying, ‘Oh woo’ when they saw those formulae. However, Figure 5.8 shows pictures of how Teacher B2 taught learners how they could use the trial and error method as a plan to solve Q.2.

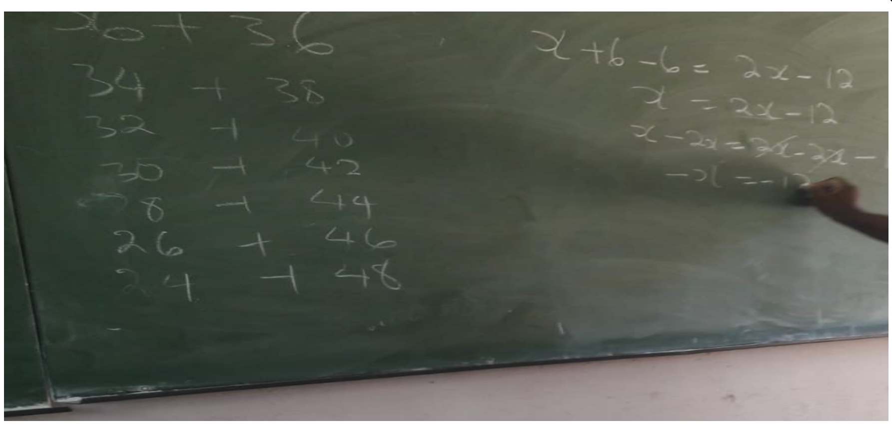

Figure 5.8: Teacher B2 guiding learners on using a trial and error method to answer Q.2

Similarly, Teacher B2 started by making an example to learners on how they could solve the problem of question 2 using trial and error dovetailing with the argument of Pournera et.al (2015) that the best teaching of mathematics can be recognised by having a teacher who carefully and thoughtfully selects and explains tasks and examples that enhance the learning. As a result, that example made learners discover why it was not advisable to use this trial and error method to solve such questions even though it ended up giving the correct answer. Therefore, he was able to explain to them the importance of first identifying the unknowns and conditions, then devising a plan and carrying out the plan before arriving at the answer.

Based on the data generated in this study, it was evident that participants from both schools were able to teach their learners how to carry out step three of Polya's 4-step problem-solving. It shows that a hundred percent of the participants could teach learners how to solve linear equations that were formulated from word problem-solving equations. However, since they were solving for the unknown x in this step, the next step required participants to teach their learners to look back at the problem. This leads to the final step of Polya's 4-steps problem-solving process.

5.3.1.4 Looking back

Looking back is the final stage of Polya's 4-step problem-solving process whereby the research entails looking back at what the question required or to find out what the problem was. All the participants were able to teach their learners that after obtaining the value of x, which was an unknown variable, they were supposed to look back to the question and check what was required to be solved or what the problem was. Figure 5.9 shows how Teacher A2 taught the learners to look back on what Q 1.

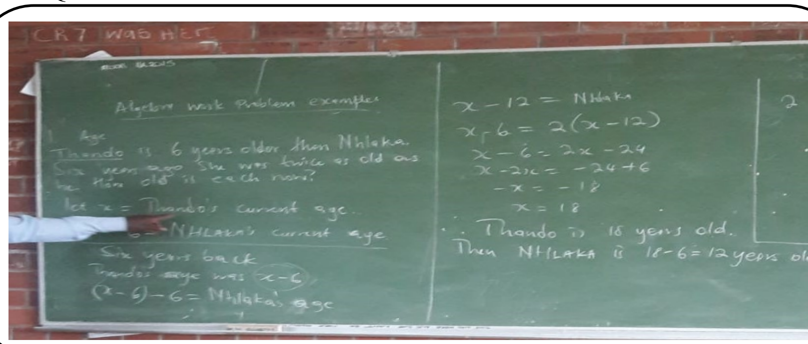

Figure 5.9: Teacher A2 guiding learners on how to look back when solving word problems

Teacher A2 is seen pointing to learners what the unknown was and reminding them what they did when devising the strategy, that is, the unknown was written in terms of x . Therefore, in this stage, they had to look back at what the question required them to solve. Then the answer was written in terms of what was originally asked in the question. Similarly, in this stage, they looked back to see whether the solution made sense considering the given data and conditions. Moreover, the same procedure transpired when Teacher B1 was doing question 2.Therefore, checking was found to be a way of looking back into the question to see if the solution made sense (Caglavan, 2014) and it was seen to play a vital role in this stage.

At both schools, all participants were observed to be able to teach problem-solving to their learners. They were able to teach their learners to solve algebra word problems following Polya's 4 -step problem solving process. This showed that a hundred percent of the Grade 9 participants in this study knew how to use Polya's 4-step problem-solving process to solve a given word problem. It was discovered that a hundred percent of the participants in this study were seen creating an environment for the learners to reflect and analyse around the problem (Capriora, 2015) as they went through their problem-solving steps. However, apart from using Polya's 4-step problem solving process to solve word problems, it was important for this study to explore visualisation tools used by the participants when solving those word problems.

5.4 Research question 1: What visualisation tools were used by Grade 9 mathematics teachers when teaching problem-solving in their classroom?

When the participants were observed on their use of visualisation tools when teaching problem-solving, it was discovered that the most evident visual tools used were the use of gestures and chalk. Table 5.2 shows the visualisation tools used by participants at both schools during the observations.

Table 5.2 Visualisation tools used by Dinaledi Umlazi Grade 9 mathematics teachers when teaching problem solving

Cases	Teachers	Visual tools used
School A	Teacher A1;	Chalkboard, white chalk, gestures & fingers.
	Teacher A2	Chalkboard, white chalk, underlining & noting some important things, gestures & fingers to point at what he was talking about.
	Teacher A3	Chalkboard, white chalk, underlining& circling important things, gestures, and fingers.
School B	Teacher B1	Whiteboard, colored markers, gestures, & fingers to point at what she was talking about.
	Teacher B2	Chalkboard, white & colored chalk, use of fingers and gestures, use some relevant figures on the board.
Conclusion	All teachers	Board, chalk, gestures, use of fingers, underlining, and facial expressions.

5.4.1 Gestures

Gestures are body movements produced in silent, non-communicative, problem-solving situations (Chu & Kita, 2011). They can be seen in the classroom when teachers make movements to elaborate or explain something to their learners. In this study, this was observed in both schools where participants used their hands to illustrate or emphasise something about the problem that they were solving. It showed that all the participants made use of this type of visualisation tool. Therefore, at both schools all the participants used gestures; for example, pointing to what they were talking about, using their fingers to illustrate something and facial expressions, and pausing while talking. Extant research shows that people perform visualisation tasks better when they are instructed to move their hands in a way congruent with what they need to visualise (Wesler, et al. 1998; Wohschlager & Wohlschlager, 1998). Other than gestures, pictures/graphs were also used in this study.

5.4.2 Pictures or graphs

Graphs or pictures allow learners to visualise a mathematical problem and they serve the same purpose as using diagrams and images (Bautista, et al., 2015). When using graphs teachers may use verbal statements, number lines, data tables, and symbolic notation; this was confirmed by Teacher B2 when he talked about question 2.

Teacher B2**:** **'***It's because mam, you see, when you say to a learner for example let the first number be x and the second is twice the other, it becomes complicated to them. Once you talk about x and y and equations to them it becomes complicated to them, but visualisation and understanding, like when you have something that they can see on the board, it becomes easier for them. Therefore, they will be able to solve the equation on their own without bringing the concept of equations.'*

Similarly, in figure 5.10, Teacher A3 was observed using diagrams and pictures on the board to illustrate question 3. Moreover, during the observation, all participants from School A used diagrams to clearly show question 3.

Figure 5.10: Diagrams drawn by Teacher A3 to explain Q.3

Teacher A3 as seen in Figure 5.10 illustrated how to use diagrams to create an image in the learner's minds of what was happening. It would be very difficult to attempt this problem without the use of a diagram. Therefore, he had to illustrate the movement of the trains, and then they can think of possible ways to solve it. Bautista, et al. (2015) emphasise that teachers with strong mathematical backgrounds normally use diagrams to explain concepts. However, Maries and Sign (2013) emphasise that failure to draw the diagram can result in an inability to solve the problem. Moreover, bringing the speed, distance, time triangle from Natural Sciences (NS) helped familiarised them with the problem, and thus they gained the confidence to solve such a problem. Therefore, it can be concluded that pictures familiarise a problem and make it routine.

It was observed that teachers might use some previously known information to help learners remember how to solve unfamiliar problems. Additionally, it could help them visualise how they ought to solve those word problems. Teacher B2 used variables to represent the unknown, for example, a plus (+) sign to represent positive. This allowed learners to familiarise the problem with concepts that they previously learned. Moreover, diagrams act as an important step in organising and simplifying given information (Van Garden, 2007). At both schools, four of the participants directly worked on the given problem rather than allowing learners to first visualise the task using previous information on what they were going to do in the lesson showing that using pictures or graphs is not widely used in Grade 9 mathematics classroom since four out of five teachers avoided the use of pictures or graphs to illustrate the problem. However, another visualisation tool was the use of a chalkboard or whiteboard.

5.4.3 Chalkboard or whiteboard

At School A, all the participants used the chalkboard to provide their learners with the opportunity to visualise the problem and assisted them to go about solving them. At school B, one of the participants used chalkboards and the other one used the whiteboard. Moreover, it was also noted that even at School B, where the participants had access to projectors and whiteboards, they did not opt to use visuals projected on the board but rather preferred writing on the board. Figure 5.11 shows Teacher A3 using a chalkboard and chalk to help learners visualise the problem on the board. However, Figure 5.12 also shows Teacher A3 using a whiteboard and colored pens to help learners visualise the problem on the board.

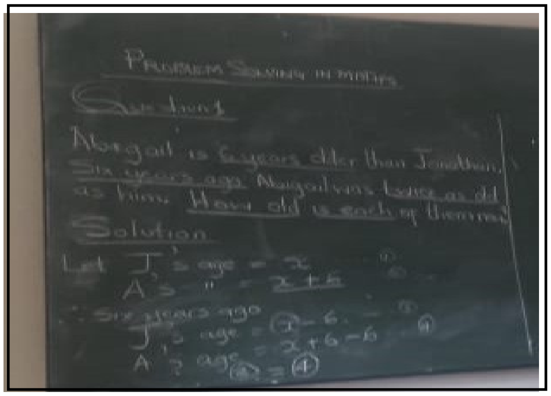

Figure 5.11: Teacher A3 using a chalkboard and chalk as visual tools

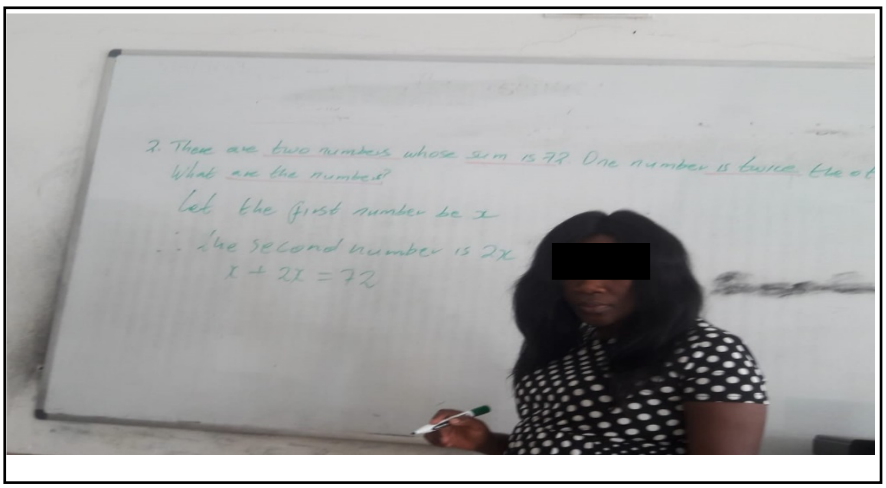

Figure 5.12: Teacher B1 using a whiteboard and colored pens as visual tools

It was noted at both schools that the most available visual tool in schools was the chalkboard and chalk. Therefore, all the participants in this study drew or sketched the concepts on the board which was time-consuming while their teaching period was between 45 minutes and 60 minutes. Chiu and Churchill (2015) indeed suggest three principles to maximise learning with written words and images through using the chalkboard or whiteboard: deleting irrelevant words and graphics, highlighting important words and graphics, and presenting words next to corresponding graphics simultaneously. However, participants' teaching aids seemed to be insufficient, a sentiment echoed by teacher A2 during the interview, stating that the question of two trains would have been understood better if learners had access to toy trains rather than presenting it on the board.

Teacher A2: '*There are other times when one needs the learners to have a picture of what you are talking about. You can demonstrate the concept of speed, time, and distance well if you use visuals such as toys and show them how they move.'*

5.4.4 Coloured chalk or pens

The participants at both schools were seen underlining words or procedures on the board. Moreover, some of them even used colored chalk and pointing at important aspects of the problem on the board. At both schools, a hundred percent of the participants used chalk, colored chalk, and colored pens to underline important clues for the learners to visualise the concepts they were talking about. The figures show the use of chalk and colored pens. However, Teacher B1 was observed underlining the unknowns and conditions using colored markers. This was seen to arouse interest and attention to the learners. In a similar vein, it made learners to pay attention to the underlined and colored data and conditions. Underlining the conditions and data made learners to be able to identify the problem.

It was noted in this study that the teachers tried to use some visualisation tools in line with what was stated by Krawec (2014) that an effective way that problem-solvers use visualisation is for understanding the problem. However, it was noted that only a few visualisation tools were used for the learners to visualise and understand the problem. Nevertheless, the participants did acknowledge the lack of some visual tools that could have enhanced the effective teaching of problem-solving. It was noted that participants used only available resources as their visual tools. Teacher A3: *'Mathematics laboratory is very good. Projectors are also good though learners tend to concentrate on the projector and forget the concept being demonstrated. I like the lab more than the projector.'*

Similarly, this showed agreement to Spaull (2013) who argues that a mathematics classroom needs to have mathematics feel and touch. Moreover, it was also noted that in both participating schools, there was no mathematics classroom with figures, drawings, manipulatives, and any diagrams on the walls that showed some of the concepts the teachers were talking about. These were just normal classrooms without anything related to mathematics. At both schools, all the participants had to write something on the board to assist learners to remember how they solved algebra word problems.

Therefore, it shows a lack of consideration of the Department of Education's norms and standards by almost all the Grade 9 teachers in both case studies. DBE (2015) of South Africa views a teacher as a designer of teaching and learning programs and materials. Therefore, it is a goal in education for these teachers to constantly and consistently design materials to help learners visualise what they were learning. However, this goal of norms and standards seemed to be less considered by the participants when teaching their Grade 9 mathematics problem-solving. Notably, instead, they used readily available resources as visual tools that were common and seemed not to arouse interest to the learners. Taylor and Reddi (2013) note that the lack of such resources as models, graphics, drawings, calculators, and charts are contributing factors to learners' lack of interest in mathematics. This lack of interest was evident as there were very few visualisation tools used in the classroom.

It was discovered that the visualisation tools used by the participants tended to be limited and monotonous as teaching methods. However, they did enhance the teaching of mathematics to a certain extent. Spaull (2013) recommends that models, pictures, drawings, graphics, manipulatives, and charts should be used when explaining mathematical concepts. It was noted that the teachers only made use of gestures, fingers, chalk, or whiteboards to enhance learners' visualisation of the mathematical concepts. During the interview they stated that other visual tools could be used to clearly explain the concepts; however, time to prepare them was their major constraint to using them. Moreover, this study further explored how the participants used those visualisation tools when teaching problem-solving in their classrooms.

5.5 Research question 2: How teachers used visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

As per this second research question of this study, several strategies were observed in the classes during data collection. When discussing how the participants used visualisation tools in this study, we consider what the teachers did to help learners visualise the problem. This includes using gestures, pointing, underlining, pictures, manipulatives, projectors, or some drawing on the board for learners to understand the problem to be solved. The first strategy of the teachers' use of visual tools was using gestures.

5.5.1 Gestures and body language

The participants in this study in both cases were observed using mostly gestures including using their hands to show learners what they were talking about as discussed in response to research question 1. The participants were observed to frown, use hands for emphasis or show what was happening. Figure 5.13 shows Teacher A1 using gestures to assist learners to visualise question1.

Figure 5.13: Picture of Teacher A1 using gestures to assist learners to visualise Q. 1

She was trying to compare the two scenarios of the children's ages as she is seen using her hands to show the numbers. Mudaly and Naidoo (2015) state that gestures can be used as a strategy to emphasise materials, echoed by Teacher A2 during the interview.

Teacher A2**: '***Most of the message may not be necessarily spoken but visualised. Also, when you are using gestures such as using your hands to point, even those learners who were not concentrating tend to refocus and follow.'*

5.5.2 Pointing for attention

The participants pointed at the board for attention to learners so that they could visualise what was being discussed. Figure 5.14 shows how teacher A3 used her finger to point to what learners needed to pay attention to.

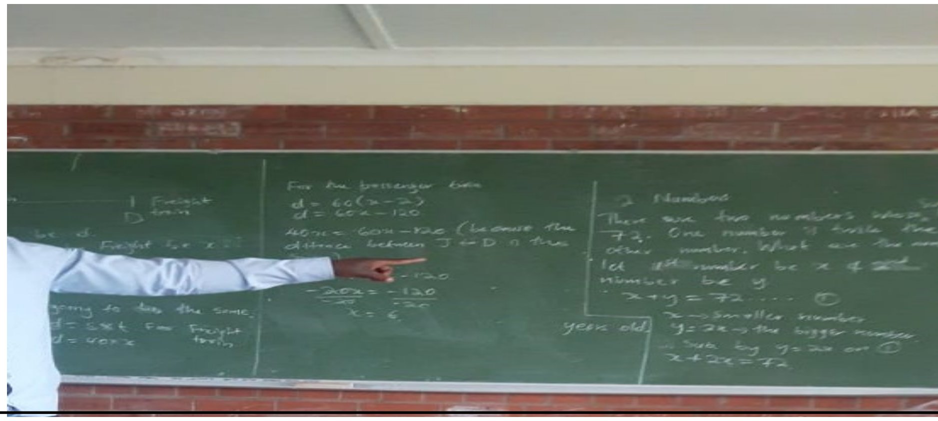

Figure 5.14: Illustrates how Teacher A3 use his fingers as a visualisation tool during solving Q.1 problem

5.5.3 Constant interaction with learners

As the teachers were teaching, it was observed that their constant interaction with the learners was another strategy around the use of visual tools. Figure 5.14 shows Teacher A3 showing his learners how to solve the question, explaining the materials to a few individuals in their notebook.

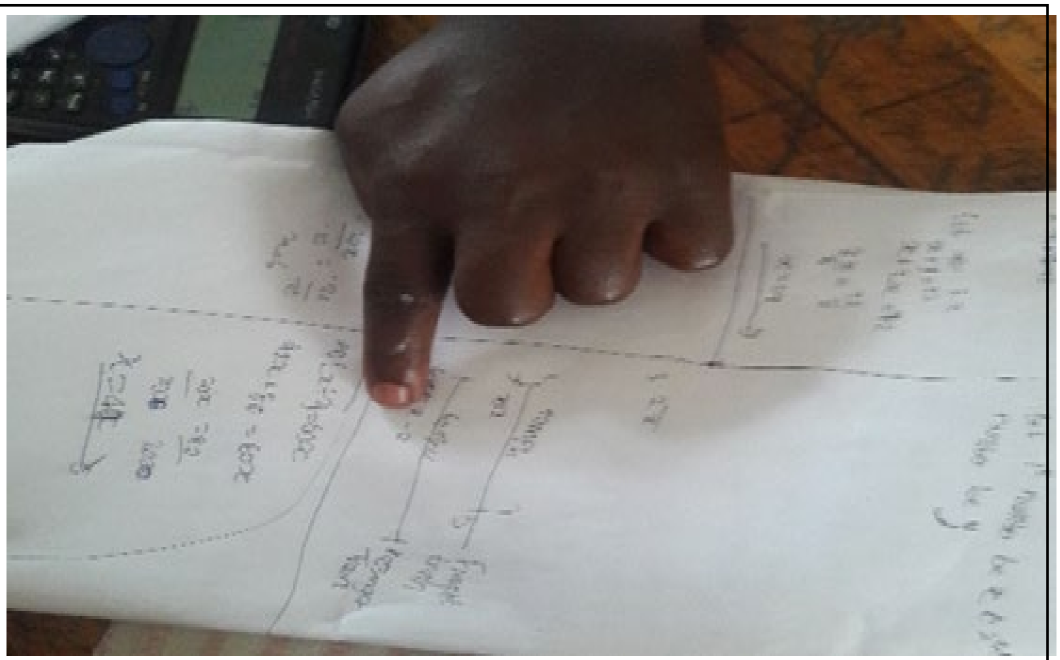

Figure 5.15: Teacher A3 explaining Q.3 concepts to learners

Figure 5.14 shows that the learners were able to visualise the problem and as a result wrote it down and discussed it with their teacher during their constant interaction.

5.5.4 Underlining and circling important concepts

The participants pointed at the board and underlined or circled important concepts as a strategy to use visualisation tools during problem-solving. Several participants in this study were observed underlining and circling important concepts as a way for them to code that information in their minds and hence visualise the problem. Figure 5.15 shows Teacher B2 underlining and circling important concepts to help learners visualise the problem

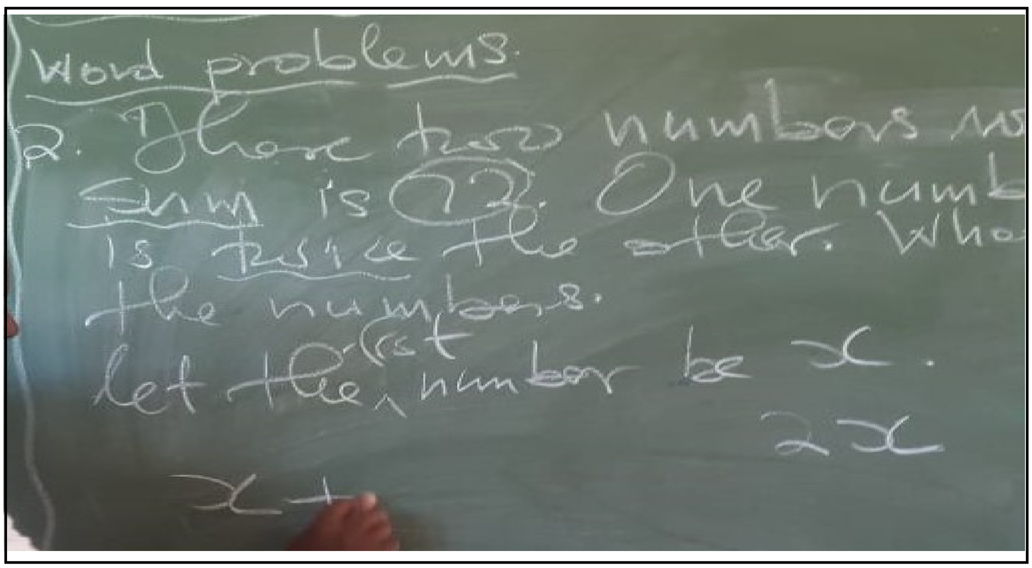

Figure 5.16: Teacher B2 underlining and circling important concepts

5.5.5 Pictorial representation and diagrams

Some participants represented the information in a pictorial form while others used diagrams for the learners to visualise the problem. When the learners saw the pictures or diagrams, they seem to relate and visualise how they could solve the problem. Therefore, Mudaly and Naidoo (2015) support the use of diagrams and pictures because they see it as a tool that can be used to remove misconceptions compared to when explaining the concepts verbally.

5.5.6 Writing on the board with colored chalk

Writing on the board, especially using different colors, is one way of making learners visualise the mathematics concept or problem under discussion. All participants display the information on the board and two out of the five participants even use different colors for learners to concentrate on what was on the board. Moreover, Teacher A1 supported the use of writings on the board as a strategy to make her learners follow the solving of the problem. This is supported by Makhubela and Lunete (2014) when they started that 'self-talk' and reciting of the textbook does not enhance the learning of mathematics.

Teacher A1**:***'If we refer to the chalkboard, solving a problem verbally without using writing the solution on the board can make the learners not to be able to follow. When learners see the problem or solution, they learn well than when they listen.'*

5.5.7 Mathematical symbols and words

According to Mudaly and Naidoo (2015), the use of words and mathematical symbols helps learners understand the problem and results in the correct solution. In this study, the participants were observed using symbols and even tried to explain what they meant.

Moreover, Naidoo (2011) argues that in her study with master teachers, she observed that they used visuals and language rather than using the language for code-switching. Code-switching is defined as the concurrent use of two or more languages in a conversation.

5.6 Conclusion

This chapter discussed results and interpreted the findings on the participants' use of Polya's 4-step problem solving process, the visual tools that they used when teaching problem-solving and how those visual tools were used. It was discovered that the participants did teach problem-solving using Polya's 4-step problem-solving process.

It was also revealed that there were only a few visual tools that they used when teaching problem-solving. These visual tools were discovered to be the only readily available tools for example gestures, fingers, and the chalkboard. The participants stated time constraints as an issue of concern. They said that they needed more time to prepare for other materials to be used as visuals in their classrooms. Furthermore, the lack of resources such as a mathematics laboratory was a matter of concern where using visualisation tools was a concern. The next chapter discusses results and findings concerning research question 3 which focussed on the reasons why the participating teachers used those visualisation tools.

CHAPTER 6: DISCUSSION AND INTERPRETATION OF FINDINGS CONCERNING RESEARCH QUESTION THREE

6.1 Introduction

The previous chapter discussed, and interpreted findings based on research questions 1 and 2 of this study. It discussed findings around the participants' use of Polya's 4-step problem-solving, the visual tools that were used when teaching problem-solving and how participants used those visual tools. This chapter will discuss and interpret the results and findings concerning research question 3:

3. Why do teachers use these visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

Research question 1 was used to identify visualisation tools used by the participants when teaching problem-solving in Grade 9 mathematics classroom. Secondly, research question 2 was employed to establish how the participants used these visualisation tools when teaching problem-solving in the Grade 9 mathematics classroom. Lastly, research question 3, discussed in this chapter, examines the reasons why the participants used those visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms.

6.2 Teacher's philosophy on their use of visualisation tools when teaching problem-solving

All five participants were interviewed following the semi-structured interview questions as a guide. The interview with Teacher A1 who was a novice teacher participant in School A with one year of the experience lasted 14 minutes. The participant stated that her philosophy of teaching using visualisation tools was that it should be the foundation for every mathematics lesson.

Teacher A1**:** '*I think visuals should be the foundation of every lesson. Before developing any lesson, one must ensure that all the learners have the same understanding and have the same picture of what the lesson is about.'*

This participant emphasised learner's formation of visual images of a concept in mathematics which echoed Makhubela and Luneta (2014) when arguing that visualisation enhanced forming a mental picture necessary for problem-solving.

However, Teacher A2 had been in School A for almost 5 years of teaching mathematics. He also confirmed the use visualisation tools when teaching mathematics problem-solving in his Grade 9 classroom. He was observed teaching using Polya's 4-step Problem Solving Process and some visualisation tools such as gestures, fingers, chalkboards, and calculators. The interview with Teacher A2 lasted for 15 minutes whereby he stated that his philosophy for using visualisation tools centered on the flexibility of a teacher when using problem-solving. The participant believed that any teacher using problem-solving in his classroom needs to be flexible.

Teacher A2**:**

'*I think I don't have a specific philosophy as such, but it is about being flexible to the problems as they come because they are unique and trying different options until you get it right. The teacher needs to be flexible during problem-solving.'*

This participant's statement dovetails with an argument by Pournera et.al (2015) that the best teaching of mathematics can be recognised by having a teacher who carefully and thoughtfully selects and explains tasks and examples which enhance learning. The flexibility comes with the careful and thoughtful selection of appropriate tasks and examples that enhance visualisation during problem-solving.

Teacher A3 was an experienced teacher who had been teaching mathematics in school A for more than 10 years. She agreed that she used visualisation tools when teaching mathematics problem-solving. The interview with Teacher A3 lasted for 16 minutes. This participant stated that she believed in the perfect use of visualisation tools which resulted in learners' mastery of mathematics concepts.

Teacher A3**:**

'*I can say perfect use of good visuals leads to a good understanding of mathematics concepts.'*

Teacher A3's statement can be aligned with how Naidoo (2011) defines visualisation as the ability to form and transfer a mental image necessary for problem-solving in mathematics. The participants believed in the importance of learners' good understanding of mathematics using good visuals. This use of good visuals could then be transferred to mental images which could help them in problem-solving.

Teacher B1 was an experienced mathematics teacher who had more than 10 years' experience in school B. She also confirmed that she used visualisation tools in her Grade 9 mathematics classroom when teaching problem-solving. She was the only participant who used a whiteboard and colored whiteboard markers. Teacher B1's interview on why she used those visual tools lasted for 15 minutes. She stated that her philosophy for teaching using visualisation tools was that visualisation was the key to teaching mathematics and very important when doing mathematics.

Teacher B1**:**

'*Without visualisation, you cannot do mathematics and science. With science and mathematics visualisation is an important thing.'*

This statement from Teacher B1 dovetails with an argument by Krawec (2014) that it is known that effective problem solvers use visualisation to comprehend the problem. This can be one of the reasons the participant valued visualisation during teaching and learning of mathematics. The participant's philosophy of using visualisation tools was also in line with the mathematics CAPS Document which emphasises the importance of making use of visualisation and the importance of the fact that learners should be problem solvers (DBE, 2011b). This shows that this participant understood the CAPS document and taught in line with it.

Teacher B2 was the most experienced participant in this study where mathematics teaching is concerned. He had been teaching mathematics in school B for more than 20 years. He indicated that he used visualisation tools in his Grade 9 classroom when teaching problem solving. Regardless of having a fully resourced classroom with whiteboards and projectors, he used the chalkboard during the observation. The participant used the chalkboard, chalk, gestures, and body language as his visualisation tools during the observation. He was also observed using some diagrams and writings on the board to explain some concepts to the learners. The interview with teacher B2 took 13 minutes where he stated his philosophy of using visualisation tools when teaching problem-solving as one that makes it easier for learners to remember some concepts.

Teacher B2**:**

'*If a learner has seen something it will be easier for him to remember it.'*

This shows that the participant believed that children remembered more of what they had seen than what they heard. It can be concluded that visualisation needs to be encouraged in our mathematics classrooms. His statement also meshes with Mogari, et.al (2016) when they state that drilling and remembering of facts and procedures when teaching mathematics do not encourage thinking and problem-solving whereas visualisation does.

6.3 Reasons for using visual tools when teaching mathematical problem solving

All five participants from both schools agreed that it was important to use visualisation tools when teaching mathematical problem-solving. This study showed that the participants fully agreed about the use of visual tools when teaching mathematics problem solving. In both School A and B, the participants were able to state their reasons for using visualisation tools, though they these differed from one participant to the other. This included among others that visuals were used as a foundation for every lesson, to help learners to relate concepts to real life, and to make the information stay in the minds of the learners. This was further elaborated by Teacher A1 when she stated her thoughts about the importance of visual tools:

Teacher A1**:**

'*Yes. I think visuals should be the foundation of every lesson, even if it is not problem-solving.'*

With this statement the teacher echoed what is stated in the GET Mathematics CAPS Document, DBE (2013) that teachers should use visualisation when teaching mathematics referring not only to teaching problem-solving but all mathematics concepts. Teacher A2 however emphasised the importance of the age of learners when using visuals.

Teacher A2**:**

'*It is very important, but it depends on the type of problem you are to solve. In some problems like the last one that we did, it is impossible to come up with a visual for it. It also depends on the age of the learners you are teaching. The younger ones learn better with visuals as compared to the older ones. For the younger ones, seeing is better than listening. When you tell them of the half, they understand better if you show them something visible.'*

This participant's statement showed a limited scope of the meaning of visualisation and what the CAPS Document emphasises about visualisation. In this case, he seemed to be referring to manipulatives other than visualisation tools. This was also contrary to how Naidoo (2011) defines visualisation when she states that it is the ability to form and transfer mental images necessary to problem-solving. However, Arcavi (2003, p.217) defines visualisation as

'the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools to deflect and communicate information, thinking about and developing previously known ideas and advancing understanding.'

This shows that 'visualisation must go beyond the use of manipulatives or gestures but involve other visualisation tools using pictures, images, diagrams, manipulatives and technological tools like computers and interactive white boards' as stated by Naidoo (2011, p.23).

Since the participants agreed that their use of visualisation tools was important to teaching mathematics problem solving in their classroom, this study then discussed what motivated them to use those visuals. It was discovered in both schools that forty percent of the participants suggested that visualisation helped learners to follow what was being discussed in the classroom. The other forty percent stated that it brought about focus among learners on what was being done. Twenty percent of the participants stated that visualisation catered for different capabilities and another twenty percent stated that it caused learners to easily remember materials and made teaching easier, as found in Teacher A1's response on what motivated her to use visualisation tools.

Teacher A1**:**

'*I think it does help because it gets learners to understand what you are talking about and to be able to relate mathematics to reality so that they do not think mathematics is just theoretical.'*

Teacher B1 gave similar reasons on what motivated her to use the visualisation tool.

Teacher B1:

'Yes, Hamm, I think the use of visual is very important because once you visualise something it not easy to forget. It has not to like when somebody is just saying something. Once something is shown/projected, you see clearly, and it stays in your mind for longer. So, I think the use of visuals is very important.'

This indicates that forty percent of the participants agreed that the use of visualisation tools helped learners understand mathematics concepts that stayed in their minds.

The third theme of this study was to establish the reason why the teachers chose those visual tools that they used in their mathematics classroom. The first reason, as discussed, was that using visualisation tools in a mathematics classroom was important not only for problem solving but to the whole of mathematics and to all mathematics learners. It was thus discovered that a hundred percent of the participants in both schools in the study agreed that the use of visualisation tools was important. What was the reason? Those reasons were stated as follows: it helped learners understand mathematical concepts, helped explain mathematical ideas, brought about positive attitudes of learners towards mathematics, helped learners relate mathematical concepts to real life, improved mathematical results, and was a requirement in a Grade 9 Caps document.

6.3.1 Helps in solving mathematics problem

In both schools in this study, a hundred percent of the participants agreed that the use of visualisation tools enhanced solving mathematical problems. This affirms Makhubela and

Luneta (2014) who state that mere telling without exposing learners to concrete materials do not enhance learning as echoed by Teacher B1 when stating why visualisation helped in mathematics problem-solving.

Teacher B1***:*** *'Yes, it does because you will be pointing out and highlighting what is important. You will be pointing at what the learners must take care of. I believe it does help learners solve mathematical problems.'*

This showed that most of the participants in this study believed that visualisation tools enhanced teaching problem-solving in mathematics. It was discovered that sixty percent of them agreed that it helped to highlight and pointed out important points to the learners during problem solving. Teacher A3 stated that using key points or clues were very important for learners to understand a problem. Since mathematics is viewed as an abstract subject, especially around problem solving, forty percent of the participants in both schools agreed that the use of visualisation tools enhanced learner's ability in solving mathematics problems and helped them relate mathematics concepts to real life, thereby making them less difficult. Since the use of visualisation tools was important to help solve mathematics problems, it was discovered that a hundred percent of the participants in this study also agreed that it helped learners understand mathematical problem-solving concepts.

6.3.2 Help learners understand mathematical problem-solving concepts

It was discovered from this study that the use of visualisation tools helped learners understand mathematical problem-solving. This echoes Makhubela and Luneta (2014) who state that visualisation enhances with forming a mental image necessary for problem-solving. Therefore, Teacher A1 stated that her learners tend to follow a lesson when she uses visualisation tools.

Teacher A1**:** '*By using visuals, I am sure that my learner would follow my lesson and they will not be left behind. Not using visual may be deceiving in that you may think that the learners understand what you are talking about as a teacher when they are not.'*

This participant practically modelled this scenario in her classroom during the observation. She did not write the question on the board for learners to visualise it. During that time, they seemed to be lost and no longer interested in the lesson. However, when she started showing some work on the board and using her body language to illustrate the problem learners were seen to be actively participating and interested in the lesson. It can be concluded that she realised that the class was lost and immediately regained their attention by helping them visualise the concept under discussion. Moreover, Teacher A3 believed that using visualisation tools led to a good understanding of mathematics concepts.

Teacher A3**:** '*I can say perfect use of good visuals leads to a good understanding of mathematics concepts.'*

This statement shows that the participant believed that using visualisation tools when teaching mathematical problem-solving enhanced learners' ability to understand the concepts. However, the participants gave different positive reasons concerning how using visualisation tools enhanced learners' understanding of the problem-solving concepts. It was also revealed in this study that forty percent of the teachers believed it enhanced understanding of problem-solving by bringing the mathematical concepts to reality. To further support that, forty percent of the participants in both schools stated that it made the mathematical problem solving less complicated and thus the learners could relate to it. Furthermore, forty percent of the participants stated that it catered for different learner's capabilities. This meant that even a single learner who seemed to be struggling when using visualisation tools might end up finding it less complicated. This would make the learner end up understanding the concepts considering that the teachers believed that it did enhance understanding of the mathematics concepts.

Teacher A3 confirmed this by stating that the use of visualisation tools helped her to cater for all learners in the lesson. She emphasised the use of visualisation tools enabling her to reach all learners' capabilities.

Teacher A3**:** '*The level of understanding of the learners is not the same. I am therefore motivated to use visuals to reach all the learners, considering that they have different capabilities.'*

The participants were able to state how their use of visual tools enhanced their learners understanding of mathematics problem-solving concepts. It made mathematics problem solving less complicated. They also stated that using visualisation tools assisted in explaining mathematical problem-solving ideas. These statements dovetail with Makhubela and Luneta (2014) who said that mere telling without exposing the learners to concrete materials do not enhance learning. They emphasise that learners should be allowed to visualise their mathematics concepts using concrete material.

6.4.3 Help teachers explain mathematical problem-solving ideas

Several researchers view the use of visualisation as a crucial skill in teaching mathematics (Tilmann, 2012; Avgerinou, 2009; Bleed, 2005; Bamford, 2003). Tilmann (2012) proposes that using visualisation tools play a supportive role when new concepts are introduced. The participants in both schools also agreed that using visualisation tools was crucial as it helped them explain mathematical problem-solving concepts and ideas to the learners. Some of the participants had different ways of indicating how the use of visual tools assisted them to explain the mathematics concepts to their learners. Teacher A1 emphasised on how the chalkboard helped him explain the mathematical problem-solving ideas.

Teacher A1**:** '*If we refer to the chalkboard, solving a problem verbally without writing the solution on the board can make the learners not to able to follow. When learners see the problem or solution, they learn well than when they listen.'*

This participant emphasised that the learners should see the work on the board for them to understand the concept and be able to solve it. This means that according to this participant mathematics cannot be taught as if you are telling a story but must involve the active participation of learners using the chalk and chalkboard. Moreover, Teacher A2 emphasised catering for all learners' capabilities, that is, the visual, kinaesthetic, and the auditory.

Teacher A2**:** '*Learners have different learning capabilities. Some may not understand when you only explain verbally or may even be bored. If the is something visual for such learners, the learner may concentrate.'*

This shows that all the participants agreed that visualisation tools assisted them in their classrooms when teaching problem-solving. This is also true for any teacher who has been using these tools for several years in the classroom. Teaching with visualisation tools yielded more results than teaching without them.

6.4.4 Promote a positive attitude among learners towards mathematical problem-solving

Hussain, Mutalib, and Zainol (2014, p.3) argue that 'learners are visual learners.' This is to say that using visualisation may promote a positive attitude and interest among learners towards mathematics problem-solving. However, Taylor and Reddi (2013) state that the lack of resources, like models, drawings, graphics, calculators, and charts contribute to learners' negative attitudes towards mathematics. The participants from both schools stated that the use of visualisation tools aroused interest among learners and made them pay attention when asked how visual tools benefited their teaching.

Teacher B1: 'L*earners become more interested because you know learners want to see more than hearing they then seem interested as you show every step. If they are watching seeing what and how you are doing it. It motivates them as well because they will do exactly what you showed them and exactly how you did it'.*

This participant emphasised that when learners visualised the concept, they would be able to follow the procedure and would also be able to use them to reason during tests. This dovetails with Hussain, et. al (2014, p.3) who argue that 'learners are visual learners.' I also think that learners understand more when they can visualise the concepts rather than when they are taught verbally.

6.4.5 Help learners relate mathematical problem-solving concepts to real life

Spaull (2013) recommends that models, pictures, drawings, graphics, manipulatives, and charts should be used in explaining mathematical concepts. Interviews showed that participants especially in School A felt that using visual tools helped learners relate mathematical problem-solving to real life. Teacher B2 clearly stated that mathematics was difficult but could be made easier using visualisation tools when asked about the importance of using visualisation in her mathematics classroom. Additionally, when Teacher B2 was asked if the use of visualisation tools made any difference to his learners his statement was as follows:

Teacher B2**:** '*Of course, it does, learners say mathematics is difficult and if he can create something that he can see and refer to will make it understandable and relate it to their everyday life.'*

This shows that he believed that it helped them relate to real-life when solving problems in the classroom. This brings to our attention that mathematics is what happens in our everyday life and if the learners can be taught that, they will no longer view it as a difficult subject. It also shows that there is a need for teachers to relate mathematics to real life as much as possible and make use of materials and resources that learners can relate to when studying mathematics.

Question 1 of the three-algebra Grade 9-word mathematics problem-solving, reads:

1. Age:

Abigail is 6 years older than Jonathan. Six years ago, she was twice as old as he. How old is Abigail and Jonathan now?

This question involves an everyday life problem that happens in our lives at home. However, in this study, sixty percent of the participants realised that when you used visualisation during problem solving you were helping them relate the problem to their real-life scenarios. Moreover, Teacher A2, stated that in the questions concerning the passengers' train and freight train, he would have brought some working toy trains to demonstrate the problem and let learners solve it after seeing the scenario.

Teacher A2**:** '*Yes, it does. There are other times when one needs the learner to have a picture of what you are talking about. You can demonstrate the concept of speed, time, and distance well if you use visuals such as toys. In the question of the trains, it was going to be easily explained with two moving train toys. The learners would have seen that at a certain time they will meet each other and the other one then passes the other.'*

This dovetails with Krawec (2014) who states that effective problem solvers use visualisation to comprehend the problem. This is supposed to mean that it would be very difficult for a learner to comprehend mathematics concepts without visualising it.

6.4.6 Helps improve mathematics problem-solving and overall mathematics results

In this study, a hundred percent of the participants in both schools agreed that using visualisation tools in the classroom improved learners' problem-solving skills and hence their mathematics results. They elaborated on how they thought it improved their learners' problem-solving skills and their mathematics results.

Teacher A3**:** '*From experience, I have tested the benefits of using visuals against not using them and realised that visuals are very helpful. The learners perform better when taught using visuals.'*

This participant highlighted that she has proved that when using visualisation tools, the learners performed better. This means that if the teacher can use visualisation tools most frequently in their daily teaching, mathematics results can improve. Moreover, Teacher B1 stated that they were able to benefit when all working, and calculations were exposed to them. The participant emphasised that the use of visualisation tools helped the learner to show all the steps and since mathematics problem-solving is more about the working and not just the solution, the learners, in this case, would be able to show all the working on any given task, which was confirmed by Teacher B1 when she emphasised that if learners could show all the working, it meant that would achieve better marks and hence improve their mathematics results.

Teacher B1***:*** *'Yes, it does benefit them. It makes them remember what the teacher did in class or showed them in class. It makes them remember. When they are writing tasks or assessments, even when they get stacked, it helped them remember.it also enhances them to show and display everything. When you are marking, they tend to show everything and as a result, their working gives them marks.'*

This shows that a hundred percent of the participants agreed that using visualisation tools enhanced learners' performance. Therefore, if the learners' performance increased, their mathematics results would increase. Teacher B2 emphasised that it also helped them to solve unfamiliar and non-routine problems.

Teacher B2*: 'Obvious mem, for understanding it does help. In the way when the learner understands the problem, the marks will increase. Because he will do much better. You can give the child any problem, if he knows to analyses it in a certain way he will understand and that will improve the marks.'*

This confirms the importance of certain aspects of Realistic Mathematics Education and in South Africa IBE performance due to the use of visualisation and problem-solving. These improve learners' problem-solving skills and the results of their learners are good. Buthelezi (2016) noted that the IEB curriculum in South Africa has proved to produce internationally competitive learners in contrast to matric, especially in mathematics. This is because they mostly used visualisation in those schools. Therefore, it can be concluded that the use of visualisation in the mathematics classroom is imperative to the performance of learners and hence mathematics results.

6.4.7 As a requirement for Grade 9 mathematics CAPS Document

In the Mathematics Grade, 9 Curriculum and Assessment Policy Statement (CAPS) document, problem-solving is emphasised in its outcomes where it states that learners should be able to identify and solve problems and make decisions using critical and creative thinking (DBE, 2011a). The CAPS documents for mathematics further state that mathematics emphasises the importance of making use of visualisation and the importance of learners acting as problem-solvers (DBE, 2011b).

In this study, the participants were not asked if using visualisation tools did help them meet the Department of Education CAPS Document policy requirements and standards. However, it was expected that they used it as a guide to their teaching in any event, since we believe that they understood their CAPS document and frequently made use of it. They were expected to reflect on this as their major reason to teach problem-solving using visualisation. It transpired that some participants lacked knowledge or awareness of what the Department of Education Policy or CAPS Document required from them. They could not refer to the CAPS Document as a driving force in their use of visualisation in their mathematics classroom. Nonetheless, it was clear to all that it was important for learners to make use of visualisation and become problem-solvers.

6.5 Grade 9 teachers' concept of the use of visual tools when solving problems in a mathematics classroom

The study found that participants from both schools believed in the use of visualisation when teaching problem-solving in their Grade 9 mathematics classroom. They even stated their philosophies on the use of visualisation when teaching problem-solving. They stated that visualisation catered for every learner in the classroom that it had to be the foundation of every mathematics lesson, that teachers had to use visuals perfectly, and that it was the key to doing mathematics and would make learning mathematics easier.

The participants also believed in the use of a mathematics classroom that would have a mathematics feel and atmosphere confirming the work of Taylor and Reddi (2013). This implies that the surroundings in a mathematics classroom are supposed to reflect a mathematical space and inspire learners towards wanting to learn more about mathematics. Such classrooms can be characterised as having mathematical charts, manipulative and 3D shapes placed around the room. In both schools, a hundred percent of the teachers believed in having such a mathematics classroom which is more of a mathematics laboratory. As a result, there was a single case where the participant brought to my attention during the interview that the class that we did the observation in was not a mathematics classroom and did not show any sign of mathematics to learners.

Teacher B2*: 'It is very important and helps learners to see what they are learning. This class that we are having is not a mathematics class since it is silent about mathematics. The class must speak mathematics to the learners even when the teacher is not there. A learner can have all resources but if the learner does not know how to use them, then it means nothing.'*

This participant also believed that a mathematics classroom had to communicate mathematics to the learners. It had to show that it was there for the use of mathematics. He even eluded to the fact that a mathematics classroom had to communicate mathematics even in the absence of the teacher. Moreover, Teacher A1 emphasised a mathematics classroom which displayed material and concepts that made them understand mathematics easier. She questioned a classroom that does not communicate any mathematics to the learners.

Teacher A1**:** *'I think it is very important. It is something that we need to have because it is challenging to teach a learner who does not understand what you are talking about. If they see the visuals, they will believe and accept what they are being taught.'*

Taylor and Reddi (2013) suggest that principals must ensure that their schools are mathematically well resourced, and Makhubela and Luneta (2014) state that those resources will help in the formation of mathematical concepts in the learners' minds. All the participants in this study accentuated the importance of having a mathematics classroom in their school and how it could help their teaching and improve their learners' mathematics performance. However, they did not further state how they felt about their principal making sure that they resourced their mathematics classroom.

Since we are now at the dawn of the fourth industrial era, it was considered important to understand how the participants viewed the use of projectors and interactive whiteboards in their mathematics classroom. This helped to understand if teachers were aware that they were supposed to be flexible and accommodate the technological era. It was worth noting that a hundred percent of the participants in both schools agreed that projectors could be used and could help in saving time.

Teacher A2: '*Projectors are very helpful because if one must draw a graph on the board for learners to see the concept of the gradient, it may take time, yet a projector can be helpful.'*

This emphasised the importance of projectors in saving time and that they would also help explain the mathematical concepts effortlessly. This is related to working smartly rather than working hard as a mathematics teacher. Teacher B1 further emphasised the effectiveness and time-saving effect of using projectors in a mathematics classroom.

Teacher B1**:** *'The use of projectors does help especially because we have limited time. Projecting makes it easier and faster. As you are talking, the learners can see, and you will be able to cover more work in a short space of time.'*

Though all the participants viewed using projectors to enhance learners' visualisation of mathematical concepts, Teacher B2 slightly differed by saying that it did not matter what you used if it allowed learners to visualise the concept. However, he did emphasise that since we were now living in a technological era, it was necessary for our learners.

Teacher B2: *'The most important thing is that the learner sees what you are doing whether on board, smartboard or project but the most important thing is to see something that you can see, is easier to remember. But then since we are living in a technological era, modern technology is recommended to make learning easier.'*

It was also revealed at both schools that the participants believed in training on the use of the visual tools to effectively employ them in their classrooms. However, sixty percent of them stated that they learned to use visualisation tools through self-experience. It was noticed that those who said they learned in this way were the most experienced teachers in both schools.
Teacher B2**:** '*No, through experience and interacting with other colleagues and workshops.'*

In as much as they said they were experienced; they still emphasised the need for training on the use of visualisation tools. Since we are now living in a technological era, it follows that it is important for teachers to get training on how they can effectively use the visualisation tools provided by current technology for effective teaching of mathematics.

Teacher A3**:** '*No. Not as such. Yes, partly because sometimes we are always encouraged to use different strategies. Self-training helps you to be able to see which problems need visualisation and which one can be solved without visuals.'*

However, the least experienced, that is, forty percent of the teachers, revealed that there was a need for them to be trained on how to use visualisation tools in their classroom.
Teacher B1: *'... It's very important to be trained because most of us are challenged in those.'*

This shows that not all the participants viewed the use of visualisation tools from the same angle, probably due to their experience, as indicated by some of the participants. However, some claimed to have trained themselves around how to use the visualisation tools as they taught with the help of experience. Experience cannot be ignored in any field because it influences how a person handles any given task. This can be supported by Naidoo's (2011) study on master teacher's use of visual tools.

6.6 Using the theoretical framework when employing visualisation during problem-solving by Grade 9 mathematics teachers

It was notable in this study that the Activity Theory when teaching problem-solving was apparent among the participants as informed by the researcher's perspective of course. The participants were discovered to be subjects in activity theory. The visualisation tools used were mostly gestures, body language, fingers, chalkboard, chalk, calculators, whiteboard, and drawings. These were tools within the framework and the object was problem-solving. Polya's 4-steps of the Problem-Solving Process concerning what the participants did in class was extensively discussed in the previous chapter. The reason for using those visualisation tools is that it produces the department of education's desired outcome, which is to improved mathematics performance in Grade 9 and hence improving mathematics matric results in the country as shown in Figure 6.1.

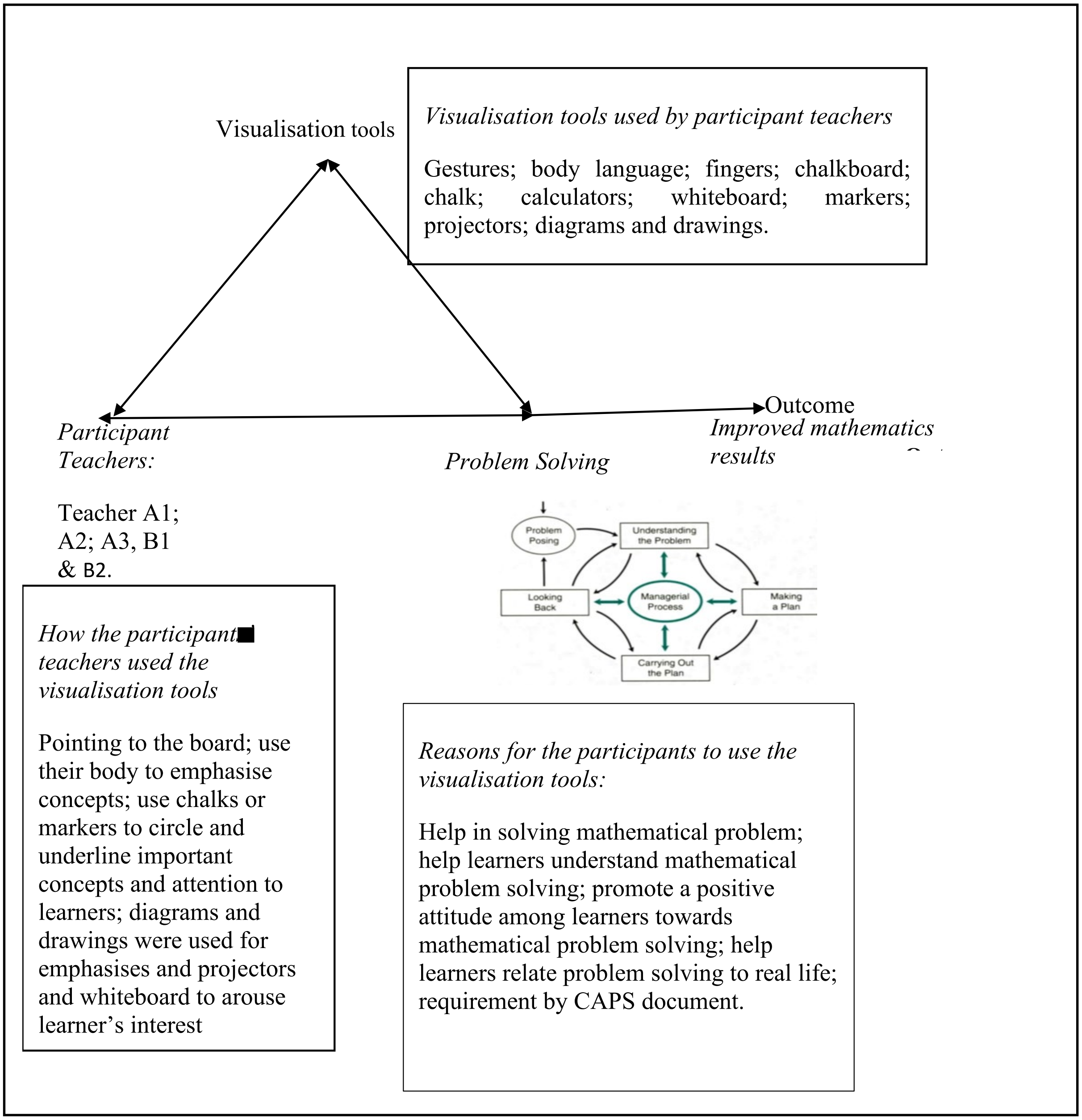

Figure 6.1: Activity theory and Polya's 4-step problem-solving process

Adapted from Hardman (2005, p.1) and

Fernandez et al. (1994, p.21). This has also been modified by the researcher

Figure 6.1 summaries the activities that took place in the Grade 9 mathematics classrooms as the participants were using visualisation tools when teaching algebra word problems. It was discovered that they answered the three research questions of this study.

6.7 Conclusion

This chapter discussed findings on why the participants from the two Dinaledi comprehensive high school in Umlazi used visualisation tools when teaching problem-solving in their Grade 9 mathematics classrooms. These findings were engendered in response to theme three of this study which was to find the reason why Grade 9 mathematics teachers used visualisation tools when teaching their problem-solving in the classroom. It was also discovered that the opinions of participants in this study dovetailed with scholars where the importance of using visualisation tools in the classroom is concerned. On a concluding note, it was discovered that this study fitted in well with the theoretical framework of activity theory and Polya's 4-step problem-solving process. The next chapter will discuss conclusions and recommendations.

CHAPTER 7: SUMMARY, RECOMMENDATION, AND CONCLUSION

7.1 Introduction

Chapters 5 and 6 discussed the findings of the study based on the data collected from the interviews and observations conducted with the participants. This chapter summaries the findings based on the purpose of the study, the research questions, the theoretical frameworks, and the methodology used. Additionally, this chapter discusses the implications and conclusions of the study.

7.2 Summary

7.2.1 On the purpose of the study

The major problem in this study was a high failure rate in mathematics Grade 12 results in South Africa (DBE, 2016). Moreover, this failure rate was not only seen in matric but also in the Grade 9 results as stated by Reddi and Juan (2014). Thus, regardless of several strategies towards developing mathematics programs in South Africa after the effects of apartheid schooling and teacher education, there seems to be little progress especially concerning learners' achievement. More so, there are summative national and international assessments which show those effects such as Trends in International Mathematics and Science Study (TIMSS), Program for International Student Assessment (PISA), Southern Africa Consortium for Monitoring Education Quality (SACMEQ), the Annual National Assessments (ANA), and the National Senior Certificate (NSC) exams. The overall performance of mathematics in the ANA 2014 was at the 'not achieved' level which was at 10% (DBE, 2014). TIMSS also confirms that South Africa is underperforming in its Grade 9 mathematics results. Juan and Feza (2015) discovered that in Grade 9 TIMSS South Africa sits at position fifty-seven out of sixty-three participating countries.

However, the Department of Basic Education introduced the Curriculum Assessment Policy Statement (CAPS) targeted minimising the failure rate in schools. The CAPS document in Grade 9 mathematics states that Grade 9 learners should be able to recognise and solve mathematics problems using visualisation (DBE, 2011a). Furthermore, in the DBE (2011b) the Grade 9 CAPS document highlights the importance of making use of visualisation and the importance of learners being problem-solvers.

This study was aimed at exploring the use of visualisation tools when teaching problem-solving in Grade 9. The findings of this study exhibit that teachers do indeed teach problem-solving well and use visualisation strategies when teaching problem-solving in their Grade 9 mathematics classrooms. However, it was revealed that the visualisation tools used at Dinaledi schools were limited. The teachers were seen to be using visualisation tools that were easily accessible to them like the chalkboard, gestures, and body language. These visualisation tools were seen by the teachers to enhance the results and learners' better understanding of mathematics. Teachers stated in the interviews that their use of these visualisation tools when teaching problem-solving was aimed at improving the mathematics results in their schools. Nevertheless, the minimum use of visualisation tools appeared to be a loophole in the effective teaching of mathematics for the improvement of the results. This was evident when they stated that they needed more time to prepare for their use of projectors to enhance learner's visualisation of the problem during the lesson.

It was also discovered in this study that the teachers would like to have a mathematics classroom that will create a mathematics atmosphere during learning. All the teachers suggested that their school must have a kind of mathematics laboratory which would enhance the learners' visualising mathematics concepts. They even stated that the kind of a classroom in which the learners studied their mathematics needed to communicate mathematics to the learners and make work easy for the teacher as he or she can easily refer to things around them.

The teachers agreed that the use of various visualisation tools in the Grade 9 mathematics classroom especially during problem solving could increase their learner's mathematics results and hence Grade 12 mathematics results. It was found in this study that using relevant and good visualisation tools for learners to see and understand what the teacher is teaching them, may enhance their mathematics performance.

7.2.2 On the research question

The main research question was to explore Grade 9 mathematics teachers' use of visualisation tools when teaching problem-solving in their mathematics classrooms. The main question of the study was further divided into three research questions:

1. What visualisation tools do teachers use when teaching problem-solving in Grade 9 mathematics classrooms?
2. How do teachers use visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?
3. Why do teachers use these visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

These research questions were initially aimed at answering the main research question which formed the main aim of the study. The findings were discussed and summarised in chapter 5 and 6 of this study. Chapter 5 discussed and summarised the findings of the first two questions and chapter 6 discussed and summarised the findings of the third question. The three research questions in this study for their reflection will be discussed in the sections that follow.

7.2.2.1What visualisation tools do teachers use when teaching problem-solving in Grade 9 mathematics classrooms?

In this study, all five teachers used readily available visualisation tools found in their classrooms. The most prevalent visualisation tools used in this study were the chalkboard/ whiteboard, chalk/ markers, colored chalks/colored markers, hands, gestures, body language, or drawings on the board. They believed that those visualisation tools could enhance learners' understanding of mathematical problem-solving hence improving their mathematics performance which could then result in improving mathematics results. This is in line with the argument of Fardin, Alamolhodaei, and Radmehr (2011) who indicate that the lack of school resources is a major factor that results in the increased failure rate.

It was worth noting that the teachers' understanding of visualisation was not very clear. Remarkably, in the literature (Naidoo, 2011, Tillmann, 2012, Botha et al., 2019) define visualisation tools as resources that include the ability of a teacher to use and interpret visual material regarding a specific topic like problem-solving in algebra. However, the visual tools in this study were seen to be used as a general aspect relevant to any topic and any subject as compared to being specific to problem-solving when solving algebra word problems.

However, they stated that time to prepare for each lesson was a major constraint for the effective use of visualisation tools. They suggested the use of projectors for their mathematics lesson and having a mathematics classroom that had all the resources necessary for them to clearly explain the concept to learners for better understanding. This dovetailed with Spaull (2013) who suggests that the mathematics classroom is supposed to have a mathematics feeling and touch that helped learners enjoy doing and always be willing to study mathematics. Moreover, Taylor and Reddi (2013) note that lack of resources like models, drawings, graphics, calculators, and charts tends to be a contributory factor to learners' lack of interest in mathematics. It was worth noting that teachers in these schools could not use all the resources as recognised by Taylor and Reddi (2013) except for the learners to use their calculators. It then follows that the teachers recognised that they needed to use different and relevant visualisation tools. On contrary, they mentioned that there was a need for more time to prepare the visualisation tools and that there was a need for them to be available at school.

7.2.2.2 How do teachers use visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

It was revealed from this study that the teachers used their visualisation tools by pointing on the important points on the board or underlining the important points on the board. They used white or colored chalks or markers to underline the points to catch the learner's attention. They sometimes used gestures and body language to emphasise important points. Only twenty percent of the teachers used diagrams on the board to further emphasise important points and clarify important concepts to the learners.

It was, however, noted that as much as some schools do have projectors and mathematics videotapes, but the teachers rather preferred not to use them. Moreover, it can be worth noting that using the same visualisation tools for every lesson can result in monotonous lessons and make learners end up losing interest in the subject. This can be linked to Buthelezi (2016), who noted that South African teachers lacks content knowledge. Therefore, the lack of content knowledge and pedagogical knowledge of teachers can result in a lack of teacher's innovation in the mathematics classroom.

This can be viewed parallel to what is done in International Baccarat Education (IBE) schools and some former Model C schools where they value visualisation of mathematics concepts by their learners. Many of those kinds of schools have mathematics classrooms that have pictures and objects that help learners visualise their mathematics concepts. They also use projectors, videos, and games for learners to clear understand mathematics concepts.

7.2.2.3 Why do teachers use these visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms?

Makhubela and Luneta (2014) argued that the importance of using a visualisation tool is to enhance with forming a mental image necessary for problem-solving. Moreover, the teachers in this study valued that as the importance of using visualisation tools especially when teaching problem solving. It was noted that the use of the visualisation tool is a requirement of the Grade 9 mathematics CAPS curriculum statement. However, the teachers did not state that during the interview. It seemed that they were not familiar with their CAPS curriculum statement.

They all agreed that the use of visualisation tools enhanced learners' understanding of mathematics concepts, which creates a mental image in their minds. Moreover, they further stated that the learner's understanding of mathematics concepts helped in improving their performance in mathematics and hence their mathematics results.

7.2.3 On the theoretical framework

The theoretical framework of this study fitted well with this study and was a perfect tool for the study. This was discussed in detail in the previous chapter. It was discovered that the activity theory and Polya's 4-step problem-solving connect very well with the research questions of this study. It produces a solid framework which helped in answering all the research questions in this study.

7.2.4 On the methodology

It was revealed in this study that the questionnaires were able to help in acquiring the demographic data of the participants. This was also the research instrument in which the participants stated that they were using visualisation tools when teaching problem solving. Within their demographical data, the participants also listed their work experience and qualifications. This assisted the researcher to compare each participant's manner of using visualisation tools with their qualifications and their experience in the field. Through interviewing the teachers, the researcher was able to conclude that novice teachers needed more workshops about how to teach problem solving and the use of visualisation tools during their mathematics lessons compared to the more experienced teachers. However, it can also be concluded that even experienced teachers needed to be workshopped to help improve their use of visualisation when teaching problem-solving.

The observation was very relevant to this study. It assisted the researcher to acquire primary data on what exactly happened in the Grade 9 mathematics classroom concerning the use of visualisation tools when teaching problem-solving. It helped the researcher to be able to link what the teachers said about their use of visualisation tools and the real action that happens in the classroom. It was found that the teachers used fewer visualisation tools which seemed not to arouse interest to the learners and these visualisation tools did not seem specific to problem-solving.

Finally, the interviews which were used to acquire an in-depth understanding of the reasons why the participants used visualisation tools were also very relevant for the study. It was used to find what they said was their importance of using visualisation tools during problem-solving in a mathematics classroom. It was one tool that helped the researcher to discover that teachers believe that using visualisation helped learners understand the mathematics concepts and master mathematics well. They also emphasised that it helped to improve learner's performance in mathematics and hence their mathematics results.

7.3 Implications of this study

Since the teachers were aware of the use of visualisation tools and the importance of using them especially when teaching mathematical problem-solving, it was also of importance to see the use of visualisation tools in mathematics classroom improving. This can be done through Grade 9 teachers using the relevant visualisation and those that suit the content under discussion especially problem-solving. It also implies that there is a need for the Department of Basic Education to organise workshops which will equip teachers on the effective use of visualisation tool which can enhance learners understanding of mathematics and hence improve their mathematics performance and results.

7.4 Findings and recommendations for further research

It was revealed in this study that mathematics teachers used minimal, readily available, and routine visualisation tools when teaching problem-solving in their mathematics classrooms. It was also found that the visualisation tools used in the Grade 9 mathematics classrooms when teaching problem-solving were in no way different to any basic or normal routine visualisation tools that can be used for any mathematics topic or rather any school subject. Notably, there was also no specific association of the visualisation tools to mathematics problem-solving for easier remembrance or understanding of the concept by the learners. It also makes the lesson boring to the learners and can result in a lack of interest from the learners.

Though the mathematics teachers stated time constraint as their major challenge towards their use of diverse and various visualisation tools, there was no solution reached on how the time constrain can be sorted. However, they stated the need to have technological resources, mathematics classrooms, and projectors in their classrooms. They also highly recommend the Department of Education to workshop them on the use of visualisation tools in their mathematics classrooms specifically both novice and experienced teachers.

The South African Department of Education and other Education authorities should take into consideration measures to improve the teaching of mathematics in South African secondary schools:

1. Emphasise the use of relevant and diverse visualisation tools for effective teaching of problem-solving in Grade 9 mathematics classrooms.

2. Workshop mathematics teachers on how to use relevant and appetising visualisation tools effectively and continually in their mathematics classrooms especially when teaching problem-solving.
3. Encourage principals and school management to supply relevant resources that will enhance the use of visualisation tools in mathematics lessons.
4. Motivate mathematics teachers to use visualisation tools especially during their teaching of problem-solving in their mathematics classrooms as per the GET Mathematics CAPS Curriculum Statement.

After a thorough study on the use of visualisation tools when teaching problem-solving in Grade 9 mathematics classrooms, it was discovered that there was a need for further research on this topic. One aspect was noted on how learners use visualisation tools during their problem-solving lessons. Another one was that since this study was limited to comprehensive Dinaledi schools at Umlazi, there seems to be a need to expand the research to diverse secondary schools in South Africa. However, this study was of more interest to mathematics education and mathematics curriculum studies as it seemed to align the South African education system to other countries such as Australia, Japan, Indonesia, the United States of America, Singapore, and Hungary. These countries emphasise problem solving and visualisation within their curriculum and seem to be performing well in their mathematics (Kilpatrick, 2020).

7.5 Conclusion

It can be concluded that Grade 9 mathematics teachers in this study were aware of the value of using visualisation tools when teaching problem-solving in their mathematics classrooms. They also valued the use of projectors and mathematics classrooms which would have objects and pictures of mathematics concepts to enhance visualisation of mathematical problem solving during their teaching. Having a mathematics classroom was also of major importance to the participants as they believed that it was going to help communicate mathematics to their learners. The major conclusion of this study was that using the relevant visualisation tools for concepts like problem-solving in mathematics classrooms might enhance learners' performance in mathematics. This would enhance in addressing learners' underperformance in mathematics. This might entail that the country's mathematics results could improve and be on par with at least their neighbouring countries such as Zimbabwe and Botswana that are said to be performing better in mathematics (Spaull, 2013). Moreover, it could also meet the standard of some European countries which have a strong mathematics culture because of their use of visualisation and problem-solving in their curriculum.

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APPENDICES

APPENDIX A: Ethical Clearance Letter

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

17 August 2017

Mrs Makhosazana Faith Maseko (215080713)
School of Education
Edgewood Campus

Dear Mrs Maseko,

Protocol reference number: HSS/1004/017M
Project title: Exploring teachers' use of visualisation tools when teaching Grade 9 problem solving in Mathemathics. A case of Umlazi District Dinaledi Schools in South Africa

Approval Notification – Expedited Application

In response to your application received on 29 June 2017, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Prcject, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

[redacted]

Dr Shamila Naidoo (Deputy Chair)

Cc Supervisor: Dr Jayaluxmi Naidoo
Cc Academic Leader Research: Dr SB Khoza
Cc School Administrator: Ms Tyzer Khumalo

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za
Website: www.ukzn.ac.za

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100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

APPENDIX B: Letter to the Department

To: HOD: KwaZulu Natal Department of Education

From: Makhosazana Faith Shoba

Date: 28 August 2018

Subject: Application for Permission to Conduct Research in KwaZulu Natal Department of Education Institutions

Dear sir/ madam

1. The above subject bears reference.
2. Please find attached my completed new form for application to conduct research in the KZN department of education schools.
3. I hope my request will be gladly accepted.

Thanking you in advance

Yours faithfully

[redacted]

Faith Shoba

APPENDIX C: Letter to the Principals

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

COLLEGE OF HUMANITIES:

To: The Principal

From: Faith Shoba (Master's Student at UKZN)

Date: 20 October 2018

Subject: Request for permission to conduct a research

I am currently registered as a master's student in University of KwaZulu-Natal (UKZN) in the school of Education, Edgewood campus. I am doing a master's program in mathematics education.

The aim of my study is to explore the use of visualisation tools when teaching grade 9 mathematics classrooms.

I request for the permission to administer questionnaires to your grade 9 mathematics teachers, do a classroom observation in one of the lessons during a mathematics lesson and an interview with the teachers after the observation. Teachers' participation is strictly voluntarily and he or she can withdraw from the project any time he or she wishes to. There will also be no financial gain in participation in this research.

Thanking you in advance.

Yours in education

[redacted]

Shoba M.F.

APPENDIX D: Permission Letter from the Department of Basic Education (KZN)

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Enquiries: Phindile Duma   Tel: 033 392 1063   Ref.:2/4/8/1627

Mrs MF Shoba
University of KwaZulu-Natal
Durban
4000

Dear Mrs Shoba

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"EXPLORING TEACHER'S USE OF VISUALISATION TOOLS WHEN TEACHING GRADE 9 PROBLEM SOLVING IN MATHEMATICS CLASSROOM. A CASE OF UMLAZI DISTRICT DINALEDI SCHOOLS IN SOUTH AFRICA"**, in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 11 September 2018 to 01 March 2021.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers below.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

[redacted]

Dr. EV Nzama
Head of Department: Education
Date: 13 September 2018

..Championing Quality Education - Creating and Securing a Brighter Future

KWAZULU-NATAL DEPARTMENT OF EDUCATION
Postal Address: Private Bag X9137 • Pietermaritzburg • 3200 • Republic of South Africa
Physical Address: 247 Burger Street • Anton Lembede Building • Pietermaritzburg • 3201
Tel.: +27 33 392 1063 • **Fax.:** +27 033 392 1203• **Email:**Phindile.Duma@kzndoe.gov.za •**Web:**www.kzneducation.gov.za
Facebook: KZNDOE....Twitter: @DBE_KZN....Instagram: kzn_education....Youtube:kzndoe

APPENDIX E: Informed Consent Letter

Shoba Makhosazana Faith
School of Education, College of Humanities, University of KwaZulu-Natal,
Edgewood Campus,
Dear Participant

INFORMED CONSENT LETTER

My name is Makhosazana Faith Shoba I am a master's candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in the use of visualisation tools when teaching Grade 9 mathematics. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person but reported only as a population member opinion.
- The interview may last for about 45 minutes to 1 hour.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalized for taking such an action.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether you are willing to allow the interview to be recorded by the following equipment.

Equipment	**Willing**	**Not willin**
Audio equipment		
Photographic equipment		
Video equipment		

I can be contacted at:
Email: makhfaith@gmail.com
Cell: 078 514 2006 /071 516 7401

My supervisor is Dr. Jayaluxmi Naidoo who is located at the School of Education, Edgewood campus of the University of KwaZulu-Natal.
Contact details: email: naidooj2@ukzn.ac.za
Phone number: +27312601127.

You may also contact the Research Office through:
Ms P Ximba (HSSREC Research Office)
Tel: 031 260 3587
Email: ximbap@ukzn.ac.za)
Thank you for your contribution to this research.

DECLARATION

I………………………………………………………………………… (Full names of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT DATE

……………………………………… …………………

SIGNATURE OF PARENT (If participant is a minor) DATE

………………………………………… …………………..

APPENDIX F: Letter from Editors

cumlaude language practitioners

Director: CME Terblanche - BA (Pol Sc), BA Hons (Eng), MA (Eng), TEFL
22 Strydom Street
Baillie Park, 2531
Tel 082 821 3083
cumlaudelanguage@gmail.com

DECLARATION OF LANGUAGE EDITING

I, Christina Maria Etrecia Terblanche, hereby declare that I edited the research study titled:

Exploring teachers' use of visualisation tools when teaching Grade 9 problem solving in mathematics. A case of Umlazi District Dinaledi Schools in South Africa.

for **Makhosazana Faith Shoba** for the purpose of submission as a postgraduate research study. Changes were indicated in track changes and implementation was left to the author.

Regards,

[redacted]

CME Terblanche

Cum Laude Language Practitioners (CC)

South African Translators Institute accr nr: 1001066

Full member of the Professional Editors Guild

1

APPENDIX G: Turn it in Certificate

Exploring teachers' use of visualisation tools when teaching Grade 9 problem solving in mathematics. A case of Umlazi District Dinaledi Schools in South Africa.

ORIGINALITY REPORT

17%	13%	4%	5%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	repository.nwu.ac.za Internet Source	1%
2	ulspace.ul.ac.za Internet Source	1%
3	uir.unisa.ac.za Internet Source	1%
4	my.unisa.ac.za Internet Source	1%
5	researchspace.ukzn.ac.za Internet Source	1%
6	dctgdansk.pl Internet Source	1%
7	Submitted to University College London Student Paper	1%
8	www.saarmste.org Internet Source	1%

APPENDIX H: Teacher's Questionnaires

TEACHER QUESTIONNAIRE

a) School Profile

1. School name: ____

2. School Address: ____

3. Number of teachers on staff: ________________________________

4. Learner enrolment: ____

5. Learner- Teacher ratio: ______________________________________

6. Number of Grade 9 mathematics Teachers: _______________________

7. Number of learners in the classroom: Girls: _______ Boys: _______________

b) School Infrastructure

1. Number of classrooms:

____

2. Does all the classroom have furniture?

____

3. Does the school have electricity?

____

4. Does the school have a copier?

____

5. Do learners have textbooks?

____

6. Does the school have an interactive whiteboard?

____

c) **Use of visualisation when teaching mathematics problem-solving**

1. Is there any different way that you teach problem-based mathematics questions in the classroom?

____

2. What do you use when teaching problem based / solving mathematics problems in the classroom?

____

d) Teachers Profile

1. Surname:

____

2. Title: ____
3. First names (in full): ____
4. Gender: ____
5. Age group: (tick) 20-30 31-40 41-50 51-65
6. Qualifications:

____

7. Subject teaching:

____

8. Number of years teaching:

____

APPENDIX I: Observation Schedule

OBSERVATION SCHEDULE

School: ____

Teachers Name: ______________________________________

Class: ____

Date: ____________________Time: ______________________

Visuals used	Always	Sometimes	Rarely	Not at all
Manipulative				
Gestures				
Diagrams				
Pictures				
Videos				
Colours (pencil, pens or chalk) in the board				
Other visuals				

How do teachers use the visualisation tools?

____

____

____

____

____

GRADE 9 PROBLEM-SOLVING QUESTIONS

Algebra word problem examples

1. Age:

Abigail is 6 years older than Jonathan. Six years ago, she was twice as old as he. How old is each now?

2. Numbers:

There are two numbers whose sum is 72. One number is twice the other. What are the numbers?

3. Rate, Time, and Distance

A freight train starts from Johannesburg and heads for Durban at 40mph. two hours later a passenger train leaves the same station for Durban traveling at 60mph. how long before the passenger train overtakes the freight train.

4. Mixtures

A mixture containing 6% boric acid is to be mixed with 2 quarts of a mixture which is 15% boric acid in order to obtain a solution which is 12% boric acid. How much of the 6% solution must be used?

5. Coins

Michael has some coins in his pocket consisting of dimes, nickels, pennies. He has two more nickels, and three times as many pennies as nickels. How many of each kind of coins does he have if the total value is 52 cents?

APPENDIX J: Interview Schedule

SEMI-STRUCTURED INTERVIEW SCHEDULE

How and why do teachers use visualisation tools?

1. Do you value the use of visuals when teaching problem-solving?

____

____

2. What motivates you to use visualisation tools when teaching problem-solving?

____

____

____

____

____

3. Does the use of visuals help you cover the key mathematics concepts during problem-solving in your mathematics classroom? If so, elaborate how?

____

____

____

____

____

4. Does using visuals make any difference when solving mathematical problem? How?

____

____

____

____

____--

5. Has the use of visuals benefited your learners? If so, please explain.

6. Did you need training to use visuals in the classroom?

7. What is your philosophy of teaching using visuals?