CONCEPT DEVELOPMENT IN MATHEMATICS: TEACHING AND LEARNING OF QUADRATIC EQUATIONS, INEQUALITIES AND THEIR GRAPHS.

BY

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DECLARATION

I HEREBY DECLARE THAT THE RESEARCH IS THE RESULT OF MY OWN WORK.

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ABSTRACT

This was an evaluative study undertaken to unpack some of the factors which could explain Transkei matriculation students' apparent poor conceptual understanding of Mathematics and to throw some light on possible solutions to their problems. In addition the study attempted to examine how Mathematics as well as the learning and teaching of Mathematics, were viewed by Transkei teachers and students at the high school level.

The theory of quadratic equations, inequalities and their graphs constituted the mathematical content research area of this study. This topic was chosen because of the key role that it plays in the matriculation Mathematics syllabus. There were 8 research questions which led to 8 hypotheses.

The research sample comprised 311 matriculation students taking higher grade Mathematics and their 10 Mathematics teachers from 10 schools in the Umtata education circuit. Four researcher-designed instruments, namely: a diagnostic test (students'), a student interview schedule, a teachers' questionnaire, and a teacher interview schedule were used. The diagnostic test consisted of 38 items aimed at addressing the first 7 research questions. Students' mean scores for each group of items of the test addressing a particular research question were computed and compared against a criterion score of 60%, using the "Z" statistic. In addition, an analysis of students' scripts was carried out and clinical interviews on a sample of the subjects (students) were conducted to find out their conceptual difficulties/misconceptions.

The teachers' questionnaire and interview schedule were used to ascertain the teachers' disposition towards Mathematics teaching. Accordingly, teachers were divided into two groups A and B on the basis of their scores in relation to the median for the whole group. This enabled the testing of hypothesis 8. In this regard, means for the students taught by the two respective groups of teachers were compared by using "Z" statistic to establish if they were statistically different from each other. Teachers' reasons for their responses to some of the items in the questionnaire were analyzed and discussed with a view to finding out their favourite teaching styles and some of the difficulties they faced in order to be as effective as they wished to be.

Analysis of data for research questions 1-7 showed that students did not have sufficient pre-requisite knowledge, and did not display a satisfactory level of mastery in solving quadratic equations and inequalities, and interpretation of graphs for quadratic equations and inequalities. Students' difficulties identified from the findings of this study were classified into 7 categories, namely: mathematical terms, mathematical symbolic language, mathematical skills, form in mathematics, over generalisations, translation and conceptual difficulties. The "Z" test for hypothesis 8 showed that students taught by teachers whose teaching strategies were more student-centred performed better than those who were taught by teachers whose teaching was inclined towards teacher-centredness.

Finally, recommendations for teachers, curriculum planners, education authorities and other researchers are also made.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF CONCEPT MAPS

LIST OF SKETCHES

LIST OF FIGURES

APPENDICES

CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

This chapter includes a brief background of the problem, general statement of the problem, research questions, research hypotheses, a brief description of the research approach and methods, rationale and limitations of the study.

1.2 BACKGROUND

Education in Transkei was under the Department of Bantu Education of South Africa up to the year 1963, when the region was declared a self-governing territory. The Department of Education for the self-governing territory of Transkei came into being in 1964, and subsequently adopted the Cape European Education syllabus for primary schools in 1965 (Ngubentombi, 1989). Transkei was granted full independence on 26th October, 1976, and subsequently adopted the Syllabus of the white Education Department of the Cape province for secondary schools.

According to the education system of Transkei, Mathematics is compulsory for all students up to standard 7 (9th year of schooling). It is an optional subject in high schools (10th to 12th years of schooling). Consequently only those students who wish to continue with Mathematics in their high school and /or further studies take this subject. Students are free to opt for the **standard** or **higher** grade syllabus. The latter is much more demanding than the former.

The standard grade syllabus is aimed, amongst other things, at developing: (a) accuracy and (b) clarity of thought, so that mathematical techniques may be well understood (see appendix 1). On the other hand, the higher grade syllabus is aimed at developing: (a) accuracy and mathematical insight, and (b) clarity of thought and the ability to make logical deductions (see appendix 1). Hence, these syllabi differ both in their content and level of treatment thereof. In particular, the theory of quadratic equations and inequalities, the solutions of quadratic inequalities and the problems which lead to quadratic equations are all part of the higher grade syllabus but not of the standard grade syllabus (see appendix 1).

1.3 THE PROBLEM AND ITS CONTEXT

1.3.1 CONTEXT

One way of gauging students' achievement level in Mathematics is to look at their performance in their matriculation examination. Table 1.1 below (see next page) shows the matriculation Mathematics, Physical Science and Biology results of the Transkei Department of Education for a period of six years (1986 to 1991).

Table 1.1

Percent passes in Mathematics, Physical Science and Biology: in whole matriculation examinations (1986-1991) of the Transkei Department of Education:

KEY: Y = YEAR

M = MATHEMATICS PASS RATE (%)

P = PHYSICAL SCIENCE PASS RATE (%)

B = BIOLOGY PASS RATE (%)

HIGHER GRADE

Y = 1986	1987	1988	1989	1990	1991
M = 22.9	22.6	22.8	17.1	18.5	15.4
P = 27.0	22.3	23.7	23.6	21.2	20.5
B = 42.7	37.6	34.5	37.4	26.2	27.3

STANDARD GRADE

Y = 1986	1987	1988	1989	1990	1991
M = 37.0	40.6	41.1	38.3	37.3	36.0
P = 65.3	63.8	63.5	71.1	41.5	58.0
B = 78.3	48.3	54.5	60.4	46.2	43.8

Table 1.1 shows that the percentage pass of students who took standard grade Mathematics ranged from 36,0% to 41,1% and of those who took higher grade, ranged from 15,4% to 22,9% over the indicated period of time. These figures reveal that the students' achievement in Mathematics is very poor, and evidently the poor performance is recurring year after year and a worsening trend is evident.

In an attempt to unearth some of the factors contributing to these alarmingly low pass rates, some research studies have been conducted to find out the attitudes of students towards Mathematics in Transkei. The results of these studies differ. Matshayana (1985), for instance, observed that most of the students had some interest in the subject; that they enjoyed the learning of Mathematics and that the majority of them perceived Mathematics as relevant to their lives and to their anticipated occupations. In Matshayana's words (1985:167) :

> "It is quite possible that contrary to the view of some educationists, the high failure rate is due to some factors other than the unfavourable attitudes which were thought to be prevailing."

On the other hand, a research study conducted by Nyamende (1990) in Matatiele district of Transkei revealed that both standard seven pupils and their Mathematics teachers in that district had negative attitudes towards Mathematics; teachers did not use very effective teaching methods; students had poor background in Mathematics and most of the teachers did not qualify to teach the subject.

This researcher has been involved in Mathematics teaching as an active classroom teacher in high schools, as an in-service training course leader and currently as a Mathematics educationist offering academic support to needy students at the University of Transkei. Personal observations during the above mentioned interactions have tended to influence the researcher to arrive at the following tacit assumptions:

(a) that Transkeian high school students lack background knowledge in Mathematics;

(b) that they tend to be satisfied with memory-based problem solving technique rather than engaging in systematic problem solving techniques;

(c) that they are generally passive learners; and

(d) that they have poor conceptual understanding of Mathematics.

Research as a whole still remains inconclusive in terms of articulating clearly what factors may be responsible for the overall poor performance of students in Transkei. The foregoing assumptions have motivated the researcher to undertake this study in order to unearth some of the factors which may explain the students' apparent poor conceptual understanding of Mathematics and throw more light on the current situation. In doing so, the researcher chose the theory of quadratic equations, inequalities and their graphs as the research area. The rationale for this is presented below:

(a) This is the first topic students are taught in standard nine (pre matriculation year) Mathematics. It is, therefore, envisaged that adequate comprehension of this topic will accord the students the necessary background knowledge to enable them to cope with most of the subsequent topics in the matriculation syllabus. Stiff (1989) has reported on the importance of this approach. He observes (1989:239):

> "The findings indicate that teachers must be sensitive to the background knowledge and preparation of their students before selecting a teaching strategy."

(b) Thorough knowledge and understanding of this section is vital owing to the topic's wide and pervasive application in other areas of Mathematics, such as:

(i) solution of (quadratic) exponential equations - for instance, the equation $5^{2x} - 2.\ 5^{x} + 1 = 0$ is transformed into the form: $y^2 - 2y + 1 = 0$ where $5^x = y$ as a strategy for solving the equation for x;

(ii) solution of logarithmic equations - for instance, some logarithmic equations of the type: $\log_2(3x + 2) + \log_2 x=4$, or $\log_x(3x - 2) + \log_x(x - 1)=2$ transform into quadratic equations when written in the exponential form;

(iii) in sequences and series, where the problems related to the sum of arithmetic series and geometric series may reduce to quadratic equations or inequalities. For instance, the problem: 'find the minimum number (n) of terms of the series 3 +5 +7 + 9 ...which when added give a minimum sum of 40400, reduces to: $n^2 + 2n - 40400 \geq 0$;

(iv) calculus/sketching graphs of second and third degree polynomial functions. For instance, the problems such as: find those values of x for which the function $y = 2x^3 + 6x^2 + 6x + 2$ is (a) stationary, (b) increasing, and (c) decreasing, reduce to solving either quadratic equations or inequalities;

(v) solutions of some third degree equations - for instance, in the equation $2x^3 + 3x^2 - 8x + 3 = 0$ one root is found by using the factor theorem and the other two roots can be found by the methods of solving quadratic equations;

(vi) to find intersection sets of various functions: the problems related to the intersections of (a) a circle and a straight line, (b) a parabola and a straight line and (c) two parabolas reduce to the solution of quadratic equations;

(vii) solution of some trigonometric equations- for instance, some trigonometric equations such as: find the value(s) of 'A' such that, $\cos 2A + \cos A = 0$ and $0° \leq A \leq 360°$, reduce to a quadratic equation;

(viii) analytical geometry - the problems such as: find the values of 'x' such that the point(s) (x;2) are at the distance of 26 units from the point (1;3), reduce to quadratic equations.

(c) It is evident that students who experience difficulties in understanding this topic will be unable to meaningfully learn many other portions of the school Mathematics curriculum as a whole.

1.3.2 STATEMENT OF THE PROBLEM

In general, this study seeks to explore conceptual problems faced by high school students in the learning of quadratic equations, inequalities and their graphs. In particular, the study focuses on the effects of the following factors on students' conceptual development of quadratic theory:

(a) mathematical background knowledge,
(b) problem-solving techniques,
(c) students' mathematical language ability in solving direct and implied quadratic equations,
(d) teaching styles and their influence.

1.3.3 RESEARCH QUESTIONS

In addressing the afore-stated problem, the study seeks answers to the following specific questions:

1. Is there any difference in performance in theory of quadratic equations, inequalities and their graphs between students with poor pre-requisite knowledge and those with a good understanding of pre-requisites?
2. Do students possess the pre-requisite knowledge and skills required for learning quadratic equations, inequalities and their graphs?
3. Are students able to solve quadratic equations expressed in the standard form?
4. Are students able to identify and solve problems dealing with quadratic equations not expressed in the standard form?
5. Do students exhibit sufficient ability and understanding of the procedures of solving quadratic inequalities?
6. Are students able to interpret graphs of quadratic functions?
7. Are students able to apply the knowledge of quadratic equations, inequalities and their graphs to unfamiliar situations?

8. Is there any difference between the performance of students (on theory of quadratic equations, inequalities and their graphs) taught by "student-centred" and those taught by "teacher-centred" teachers?

1.3.4 RESEARCH HYPOTHESES

The above questions led to the following hypotheses:

Hypothesis 1. (Q 1): There is no significant difference in overall performance between students with poor pre-requisite knowledge and those with a good understanding of pre-requisites.

Hypothesis 2. (Q 2): Standard 10 Higher Grade Mathematics students of Transkeian high schools possess the required level of pre-requisite knowledge and skills required for learning the theory of quadratic equations, inequalities and their graphs.

Hypothesis 3. (Q 3): Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to solve quadratic equations and inequalities expressed in the standard form.

Hypothesis 4. (Q 4): Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to identify and solve problems dealing with quadratic equations not expressed in the standard form.

Hypothesis 5. (Q 5): Standard 10 Higher Grade Mathematics students of Transkeian high schools exhibit sufficient ability and understanding of procedures of solving quadratic inequalities.

Hypothesis 6. (Q 6): Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to interpret graphs of quadratic functions.

Hypothesis 7. (Q 7): Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to apply the knowledge of quadratic equations, inequalities and their graphs to unfamiliar situations.

Hypothesis 8. (Q 8): There is no difference between the performance of (Group B) students taught by (Group 2) teachers whose teaching style was more student-centred and those (Group A) students taught by (Group 1) teachers whose teaching style was less student-centred.

1.4 RESEARCH APPROACH AND METHODS

This section gives a brief introduction of the approach and methods used in this research study. The details are given in chapter 3.

1.4.1 RESEARCH DESIGN

This was an evaluative study involving matriculation (standard 10) students in Transkei. Firstly, a diagnostic test was developed and used to ascertain students' mathematical background, problem-solving ability, procedures and processes they used in relation to quadratic theory. Secondly, an interview schedule for students was developed and used as a follow up to the diagnostic test with a view to finding out their perceptions and conceptions of the various concepts and principles embodied in the theory of quadratic equations, inequalities, their graphs and the related background knowledge. Thirdly, a questionnaire (see appendix 2) was developed and used to ascertain the teachers' disposition towards Mathematics teaching. Fourthly, an interview schedule was developed and used to interview some of the participating teachers to verify their teaching approaches and disposition in the teaching of Mathematics. Data was analyzed by both computer and manually.

1.4.2 SAMPLE

The target population for this study were all high school students taking higher grade Mathematics in Transkei. The accessible population were all standard 10 high school students in the Umtata circuit taking higher grade Mathematics in 1992.

1.4.3 INSTRUMENTATION

Two instruments were constructed: (a) questionnaire for teachers (see appendix 2); and (b) a diagnostic test for students (see appendix 3). Two interview schedules for teachers and students were also used in the study.

(a) The teachers' questionnaire (see appendix 2) consisted of two sections, A and B. Section A consisted of 16 items ascertaining teachers' disposition towards Mathematics in general, whereas section B, consisting of 11 items, ascertained their teaching approaches for the teaching of theory of quadratic equations, inequalities and graphs. Furthermore, the teachers were requested to furnish reasons for choosing the response to each item.

(b) A diagnostic test (see appendix 3) consisting of 18 true and false, 19 multiple choice and 1 supply type items (where the candidate is expected to give a full solution), was constructed and administered to the students. Students were asked to furnish reasons for choosing a particular answer for each item.

(c) Clinical interviews of a sample of subjects (students) were also conducted. The questions asked during the clinical interviews were aimed at gaining more insight into their conceptual understanding of the theory of quadratic equations, inequalities and their graphs.

(d) Interviews of a sample of subjects (teachers) were also conducted. The questions asked were intended to supplement information gathered from teachers' questionnaire. The starting question was: Why do students not perform well in Mathematics? The rest of the questions were based on teachers' responses to each subsequent answer.

1.4.4 DATA ANALYSIS

Quantitative and qualitative analyses of the responses were done in order to test the hypotheses mentioned and also to ascertain the causes of some of the difficulties that students experienced in the learning of the theory of quadratic equations, inequalities and their graphs.

1.4.5 THE PILOT STUDY

A pilot study was conducted with a view to trying out the instrument and finding out students' cognitive growth patterns. Another aspect of the pilot study sought to investigate the students' conceptual development and the persistence of their errors over a period of one and a half years or more in solving quadratic equations and inequalities, and drawing or interpretation of their graphs. This analysis was based on examinations given by teachers during June and November (1990) and November (1991) to the 1992 standard 10 students in one of the schools in Qumbu education circuit.

Relevant sections of their scripts (of above mentioned examinations) were analyzed and the diagnostic test items for the main study were constructed keeping in mind the information revealed by the pilot study. For instance, the analysis of the scripts had revealed that:

(a) for some students confusion existed between a variable and a specific unknown given in an equation. The students substituted a given root (value of the variable) for the specific unknown in the equation;

(b) some students interpreted $-x^2$ as $(-x)^2$ whenever they substituted a numerical value for x in an expression or equation containing $-x^2$ as the first term;

(c) some students did not take into consideration '-' sign written before a specific unknown (e.g. before 'k' in $-x^2 - 8x - k = 0$) when they substituted a negative value for the specific unknown; and

(d) some students had both misconceptions mentioned in (b) and (c). In particular, the construction of item 33 (see appendix 3) in the main instrument was made in consideration of all three misconceptions. This item had -1 the coefficient of the leading term and a '-' sign before k, the specific unknown. The correct answer was option D. Options E, B, A and C were arrived at by following the lines of thought mentioned in (a), (b), (c) and (d), respectively.

1.5 RATIONALE

Booth (1988:31-32) remarks:

> " A continuing assessment of exactly what is involved in the learning of new mathematical topics, assisted by an analysis of the errors that students make and reasons for them, may provide us with extremely useful tools for deciding on ways to help children improve their understanding in mathematics."

The need to cultivate and promote the conceptual development of students in their learning of Mathematics cannot be overemphasised. As Stiff (1989:239) observes:

> "The point is not simply that relevant knowledge should be increased before a concept venture is undertaken but that certain type of strategies should be employed to achieve improvement in acquiring mathematical concepts."

The researcher believes that this study will throw some light on the specific reasons for the apparent conceptual difficulties experienced by students in learning quadratic equations, inequalities and their graphs. It is further hoped that recommendations made will prove valuable to students, teachers and researchers in Mathematics education.

1.6 LIMITATIONS OF STUDY

This study should be seen within the context of the following limitations:

(a) Many students' responses did not have reasons or working to support their answers. It was, therefore, difficult to find the roots of misunderstandings in such cases.

(b) The diagnostic test was not going to count towards their final year results, therefore, the students might not have taken genuine interest in answering it.

(c) The clinical interviews were conducted a few days after the students wrote the test. It was possible that they presented a changed version of their line of thought when they were interviewed.

(d) It was not possible for the researcher to conduct clinical interviews with all the selected subjects because of non-availability of those subjects for that purpose.

(e) The instrument (students' test) was perhaps too long in terms of the time required to answer all the questions. Even though the students were told, verbally, to take as long as they wished to answer the test many of them might have forced themselves to finish in two and a half hours because of the indicated time on the test paper. In such cases, some items might not have received due attention.

(f) Although the instruments (students' test and teachers' questionnaire) were validated by the experts it cannot be said that 100% success was attained in this aspect.

(g) Correction for guessing was not calculated.

CHAPTER 2

THEORETICAL CONSIDERATIONS

2.1 OVERVIEW

This chapter includes, different views of Mathematics and teaching and learning of Mathematics, definition of concept and conceptual development, role of language in concept development in Mathematics, sources of students' difficulties in learning Mathematics, and descriptions of different teaching styles. This chapter also looks at some learning theories and examines how these may elucidate the teaching and learning of Mathematics in general and, in particular, the teaching and learning of quadratic theory. These theories and their effects on the teaching and learning of Mathematics are also related to the research hypotheses of the study.

2.2 MATHEMATICS AND TEACHING/LEARNING OF MATHEMATICS

The process of teaching and learning of Mathematics is influenced by teachers' views of Mathematics and how they perceive the role of their students in the instructional process. The traditional view of Mathematics as content is fast giving way to the view of Mathematics as a process. Those who hold the traditional notion of Mathematics as content see it as a set of concepts, rules, theorems and structures. Others view Mathematics as a set of processes such as generalising, classifying, ordering, exploring patterns, etcetera. Moodley (1992(b):4) summarises the implications of these two opposing views about Mathematics for classroom practice in the teaching/learning of Mathematics as follows:

"the former view results in the learning of finished products of mathematical activity, emphasis on procedures and closed manipulation of techniques, limiting children to solving routine problems - characterised mainly by teacher showing and telling with students following and repeating;

- the other view results in the learning of Mathematics as a process (irrespective of the content material), emphasising meaningful development of concepts and generalisations, increasing the prospects of real problem solving, open enquiry and investigation - characterised mainly by teacher challenging, questioning and guiding with students doing, discovering and applying."

From the above quotation it is quite clear that those who hold the traditional view of Mathematics tend to view the learner as an empty vessel which can be filled with the required mathematical knowledge (Shuard, 1986). On the other hand, people with the latter view of Mathematics perceive the learner as an active processor of information (Moodley, 1992(b); Njisane, 1992). They believe that the learner interprets, organises, reorganises, structures and restructures new knowledge in order to assimilate and\ or accommodate so that ultimately s/he she learns it (Cobb and Steffe, 1983; Maher et al, 1988; Schifter et al, 1992).

This study attempted to examine among others how Mathematics and the learning and teaching of Mathematics in Transkei were viewed by teachers and students at the high school level.

2.3 CONCEPT AND CONCEPTUAL DEVELOPMENT

In order to understand what is involved in the teaching and learning of quadratic theory: equations, inequalities and their graphs, it is important at the outset to define the terms 'concept' and 'conceptual development'.

Information processing psychologists (e.g. Bruner, 1966) believe that learning takes place by way of analysing the information through the processes of observation and inferences. Through inferences, generalisations and predictions are made and events are explained. This results in the construction of knowledge called content that includes facts, concepts and generalisations (rules, principles, explanations for some phenomenons).

From the Information Processing point of view a learner finds relationships amongst concepts in the course of processing the information. Concepts are seen as 'form of data' obtained from categorizations of a number of observations (Eggen et al, 1979). Some relationships may result in generalisations, giving rise to principles and rules, whilst others may result in arranging these concepts into hierarchies, leading to superordinate, coordinate and subordinate concepts.

Heritage (1980(a):62) likens 'concept' to "the atom in elementary chemistry". Hence, heritage sees a concept as "the 'smallest', most primitive particle we need to think about and we can, for the most part, ignore its constituent parts. Further, Heritage observes that "higher concepts resemble molecules in that they consist of lower concepts, though with something added so that the whole is greater than the sum of its parts" (Heritage 1980(a):62).

Mathematical concepts may be classified into three categories depending on the basis of three factors. Firstly, concepts may either be concrete or abstract. The former are those which have direct concrete or physical referents, whereas the latter do not and can only be inferred from their effects. At the general level, examples of concrete concepts include tree, table, whereas abstract concepts may be represented by concepts such as justice, addition, covalent bond etc. In Mathematics concrete concepts include geometrical figures such as triangle whereas abstract concepts include operations such as division.

Secondly, mathematical concepts can be viewed in terms of their definitions. Definition of some concepts does not change from one context/stage to another context/stage. For example, the concept 'addition' remains the same in each context. Such concepts are said to have constant dimensions. On the other hand, there are concepts whose definitions change from one context to another. For example, the symbol '2' does not have the same meaning in the two numbers 2x and x^2. Such concepts have variable dimensions.

Thirdly, mathematical concepts differ in their complexity i.e. lower concepts and higher concepts. The number of attributes associated with a concept determines its complexity. Higher concepts also tend to be complex concepts (Heritage: 1980(a)). The attributes associated with the algebraic term 'x^2' make it more complex than the term 'x'. Thus complexity of quadratic expressions in the following example increases from left to right: $x^2 \rightarrow (x + 1)^2 \rightarrow x^2 + 2x + 1 \rightarrow x^2 + 2x + 4$. As such, although the teacher is expected to teach mathematical concepts in an understandable manner to his/her students, it is important to observe that most of these concepts tend to be abstract. This is where conceptual development comes in. Skemp (1971:32) observes the following two first principles of the learning of Mathematics:

> "(a) Concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples.
> (b) Since in mathematics these examples are almost invariably other concepts, it must first be ensured

that these are already formed in the mind of the learner."

It, therefore, becomes necessary that teachers and learners perform a concept analysis to find out the prior concepts embodied in the concept that is to be taught and/or learned. A concept map may be drawn by listing all the concepts which are required for the development of a new concept, followed by drawing of arrows in between them to indicate relationships between them. A full analysis of the concept indicates lines of thought to be followed by the learner (Heritage 1980(a)).

Concept analysis helps in planning teaching activities which result in an effective concept attainment process. Accordingly, the task of concept analysis includes (Eggen et al, 1979) concept name, definition, characteristics, exemplars, superordinate concepts, subordinate concepts and coordinate concepts. For instance, the concept analysis of the concept 'quadratic equation' may proceed as follows:

1 **Concept**: quadratic equation, i.e. $ax^2 + bx + c = 0$.

2 **Definition:** an equation, in single variable, with the leading term raised to the power 2.

3 **Characteristics:** an equation, single variable, leading term = a $(\text{variable})^2$, 'a' is a real number except 0.

4 **Exemplars:**

$$x^2 = 9;$$
$$2x^2 - 5 = 0;$$
$$3x^2 + 5x = 0;$$
$$5x^2 + 8x - 3 = 0.$$

5 **Superordinate concepts:** algebraic equations in one variable.

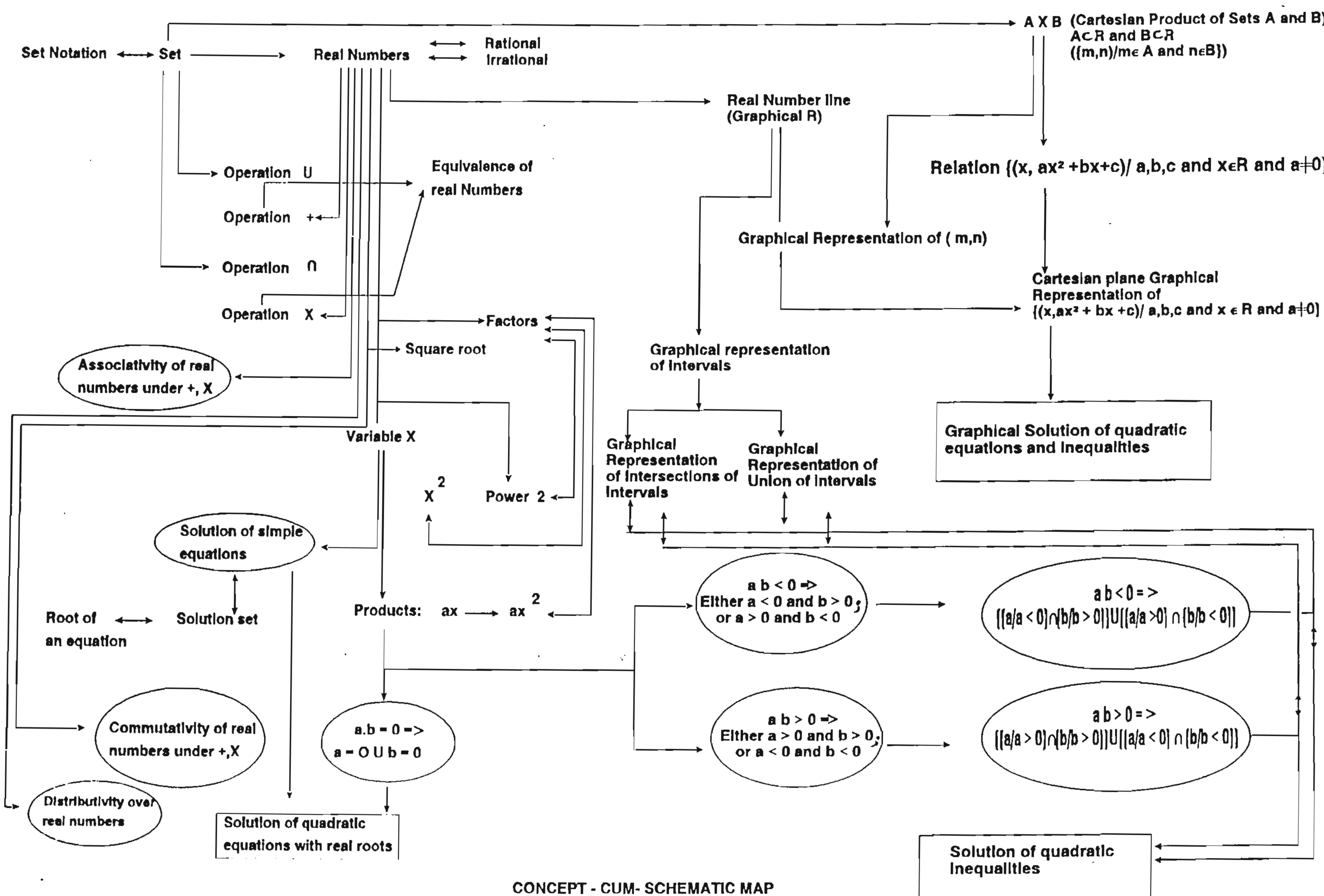
6 **Subordinate concept:** simple equations.

7 **Coordinate concept:** equations that can be reduced to quadratic equations.

Heritage (1980(b)) contends that the latter part of any Mathematics syllabus consists of results obtained from already existing concepts. These results are important in the teaching and learning of Mathematics, but have no place in concept maps. Therefore, Heritage (1980:b) supports schematic analysis maps. A schema is a collection of closely related concepts which have something in common and which together possess some measure of problem-solving capacity.

Not only should the individual meanings be understood by the student, but also the meanings of the interrelationships between and amongst them. This leads to the understanding of principles, or laws as well as theories. What this means, therefore, is that in terms of the quadratic theory, all the related subordinate concepts and principles (laws) need to be understood before students can be expected to grasp the meaning of the theory as a whole. A concept-cum-schematic analysis map of pre-requisite concepts and their relationships for the solution of quadratic equations and inequalities designed for this study is given as map 2.1

MAP 2.1



CONCEPT - CUM- SCHEMATIC MAP
Solutions of quadratic equations and inequalities

The map (page 20), for instance, illustrates that to be able to meaningfully solve quadratic equations with real solutions, the learner must be in possession of the knowledge of real numbers (rational and irrational); operations + and X on real numbers; associativity of real numbers under + and X; commutativity of real numbers under + and X; finding square roots of real numbers; concepts of variable and expression; factorisation of expressions; evaluating algebraic expressions; solution of simple equations; that $a . b = 0 \Rightarrow a = 0$ or $b = 0$; $a.b < 0 \Rightarrow$ either ($a < 0$ and $b > 0$) or ($a > 0$ and $b < 0$); $a.b > 0 \Rightarrow$ either ($a < 0$ and $b < 0$) or ($a > 0$ and $b > 0$); cartesian product of two sets; the concept of relation; the knowledge of real numbers in terms of the real number line; real number intervals; intersection of intervals; union of intervals; cartesian plane and graphical representation of a relation.

Accordingly, to learn meaningfully to solve a quadratic equation of the type $ax^2 = b$ and classify the solutions as rational or irrational a student must be able to:

(a) describe the terms: 'solve', 'roots', 'solutions', 'satisfy', 'real numbers', 'imaginary number', 'rational number' and 'irrational number';
(b) describe correctly the meaning of the equation $ax^2 = b$;
(c) identify different operations and order in which they must be carried out;
(d) determine square roots of both sides of the equation;
(e) distinguish between real and imaginary roots;
(f) distinguish between rational and irrational roots;
(g) check the validity of the solutions; and
(h) write the solution set in a correct mathematical symbolic notation (set notations).

The importance of a concept-cum-schematic map cannot, therefore, be overemphasised in its potential to direct and promote conceptual development in the learner.

2.4 SELECTED THEORIES OF LEARNING AND THEIR INFLUENCE ON THE INSTRUCTIONAL PROCESS

2.4.1 OPERANT CONDITIONING

Skinner (1953) was the father of the learning theory called operant conditioning. According to this theory some mental bonds are created whenever there is a set of stimuli and their responses. These bonds represent association between these stimuli and the responses. Reinforcement or repeated use makes these bonds stronger whereas punishment and lack of instances of use weaken the bonds. This theory of learning encourages the instructor to teach those things which 'go together'. In this regard, drill and practice are regarded as basic and key principles of teaching and learning. In the history of Mathematics teaching, the 'drill and practice' mode of instruction, which is a manifestation of the commitment to operant conditioning, was used in teaching Mathematics through the 1950's. This mode of instruction is still being used by some teachers and in some computer assisted instructional programmes. As Shuard (1986:179) observes:

> "mathematics in the U.K. is taught , at present, mostly by assuming that the students are 'empty vessels' and to fill these empty vessels with the knowledge about how calculations are performed by standard methods, is teachers' duty. The teacher also provides enough practice so that the students can do those calculations correctly."

This shows the extent to which this theoretical perspective has influenced classroom transactions in the teaching of Mathematics. The research questions of this study are bound to provide some idea of the situation in Transkei Schools.

2.4.2 GESTALTISM

Gestaltism recognises the complexity of the human mind and acknowledges that mental structures are more complex than construed by Skinner, the architect of operant conditioning. This complexity of mental structures must be taken into account in the process of teaching and learning. Accordingly, Wertheimer (1959) contends that the mastery of procedures acquired by the 'drill and practice' mode of instruction does not earn a student a long lasting, in-depth and flexible type of knowledge. The knowledge acquired in this manner cannot be used in alternative situations.

Gestaltist psychologists, therefore, believe that the learner perceives an object on the basis of the whole before s/he perceives its individual parts. They further contend that problem solving requires insight, and that for insight to occur the learner must have knowledge of basic elements. It is envisaged that knowledge acquired through insight is longlasting (Mwamwenda, 1989).

However, although Gestaltism believes in the development of mental structures it gives no suggestions about the mode of instruction that could be followed to achieve the development of such mental structures. It seems the interrelationships among the various elements of quadratic theory as per concept map 2.1 (page 20) become essential for the teaching and learning of it.

2.4.3 BEHAVIOURISM

The essence of learning, within the context of behaviourism, may best be discussed within the aegis of:
(a) Gagne's learning types;
(b) Diene's progressive stages of learning; and
(c) the notion of algorithmic learning as a construct of

behaviourism.

(a) Gagne's Learning Types and the Meaning of Learning

Gagne (cited by Bell, 1978:111) contends that there are eight types of learning. These are:

(i) signal learning
(ii) stimuli association
(iii) verbal association
(iv) discrimination learning
(v) chaining
(vi) concept learning
(vii) rule learning, and
(viii) problem solving.

Gagne believes that each type of learning occurs in the learner in four sequential phases, viz: the **apprehending, acquisition, storage and retrieval** phases.

Gagne (1977:3) sees learning as "a change in human disposition or capability which persists over a period of time and which is not simply ascribable to the process of growth."

According to Gagne, learning has taken place if a person is in a position to do something that s/he could not do earlier. Therefore, a student will be said to have learned a component of the theory of quadratic equations, inequalities and their graphs if s/he is in a position to answer the questions related to these constructs. For example, a student can be said to have learned what a quadratic equation is if s/he is able to identify all the quadratic equations from a given set of equations or, a student has learned a mathematical fact if s/he can state it and use it in various situations.

More specifically Gagne contends that a student has learned the fact that, $x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ in the equation $ax^2 + bx + c = 0$ if s/he can recall the formula when required to do so; s/he has learned the required mathematical skills if s/he can substitute the values of a, b, and c in the formula, $x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and compute the value correctly when using the formula in determining the solution set of a given quadratic equation, e.g. $4x^2 + 5x + 1 = 0$; a student has acquired the required conception of variable and constant if s/he identifies 4, k and 1 as constants and x as the variable in the quadratic equation $4x^2 + kx + 1 = 0$; or a student who can derive the quadratic formula, $x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ starting from '$ax^2 + bx + c = 0$ where $(a,b,c) \subset R$ and $a \neq 0$' by supplying reasons in each step has learned the principle: if $ax^2 + bx + c = 0$ then $x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(b) Diene's Progressive Stages of Learning Mathematics

On the other hand, Dienes (1973), who considers Mathematics as the study of concepts, advocates that there are six progressive stages in the learning of concepts. These stages are:

(i) free play
(ii) games
(iii) searching for commonalities
(iv) representation
(v) symbolisation
(vi) formalisation.

Free play is construed to be the simplest stage and **formalisation** the most complex. Dienes supports the 'concrete to abstract' mode of teaching and learning. Dienes also emphasises the transfer of old learning to new learning. This is only possible if the learner is equipped with all the related pre-requisite knowledge before s/he starts learning new material. A slow learner can be helped in the learning of mathematical concepts if the teacher presents the conceptual structure in as many different situations as possible. It is mainly through such diversity in the learning environment that transfer of learning is facilitated. In this respect, this study seeks to explore this matter and throw some light on the role pre-requisite knowledge can play in the learning of the theory of quadratic equations, inequalities and their graphs.

(c) Algorithmic Learning

One manifestation of behaviourism is algorithmic learning. Within this mode of learning many teachers and students of Mathematics take Mathematics merely as 'something that you do'. They do not take Mathematics as something that you think about. Gagne (1983) pioneered this outlook towards the learning of Mathematics by stressing the view that the right sequence of experiences, repeated with adequate frequency should generate the right learning. In Gagne's view, the following sequence of events should result in the attainment of intended educational objectives: divide the material to be learnt into small building blocks; let these building blocks be learned first. Then these building blocks can be combined to larger units of competency. Gagne's views (which to a large extent also reflect Diene's contention) are further articulated in Wenger's (1987:221) observation that:

(i) Students learn Mathematics primarily from examples and practice tasks - typically from text books. That is, they do not usually learn by understanding the explanations of procedures and using those explanations. Rather they figure out what the procedures are about by working through them.

(ii) An important force driving student behaviour is the need to make sense of things by creating simple, straight forward procedures that work. As students work problems, they invent rules for the procedures that seem to fit the expected answers. Often those rules are as simple as possible, and often they are incorrect.

(iii) The rules and procedures that students invent become part of their problem solving approaches. A student will continue to use one of these inferred procedures until (at least) encountering a task that reveals it being incorrect.

Gagne believes in 'learning hierarchies'. According to Gagne previously acquired capabilities (pre-requisite knowledge) and the capabilities to be acquired (material to be learned) constitute a learning hierarchy. The learner needs to use the previously acquired capabilities to learn the material at hand. Therefore, Gagne believes that the first step in instruction is to ensure that the students possess all the pre-requisite capabilities. However, behaviourists contend that errors or misconceptions that students may hold are not troublesome except that students need to be taught the correct versions of those concepts. In Gagne's (1983:15) words:

> "The effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules... This means that teachers would best ignore the incorrect performances and set about as directly as possible teaching the rules for correct ones."

In summary, behaviourism contends that mathematical knowledge can best be taught and learned by relating (individual's) old (pre-requisite) knowledge to the new knowledge, and by presentation of the new material as directly as possible through carefully articulated procedures - entailing sufficient repeated practice on the part of the learner. On the basis of this repeated application of procedures, the learner gains more insight into the operations and formulates his/her own rules and understanding of the concepts and principles involved. According to behaviourists therefore, direct instruction and feedback are essential components of the instructional process. Consequently, behaviourism further contends that mathematical facts and skills can best be learned through sequential and hierarchically well-planned step-by-step procedures. Memory, drill and practice play a great role in the learning of mathematical facts, concepts, skills and generalisations. However, it is doubtful that higher level conceptual learning (application onwards) can occur through such procedures of learning as articulated by behaviourism. Research questions of this study are directed towards gaining an insight into students' readiness in terms of possession of pre-requisite knowledge, for the learning of Mathematics in Transkeian high schools.

2.4.4 CONSTRUCTIVISM

The contemporary, most popular view of learning is constructivism (e.g. Piaget) where learning is construed as a two-directional flow of information between students and teachers (Bodner, 1986; Njisane,1992). Students are seen as active processors of information and do not merely absorb the knowledge that a teacher tries to pass on to them in class, as is. They interpret the teacher's instruction according to their understanding and experiences (Cobb and Steffe,1983; Maher et al,1988). This is accompanied by organisation and reorganisation; structuring and restructuring of this knowledge into larger sets of interrelated concepts (Schifter et al,1992). Through the complementary processes of accommodation and assimilation the new becomes a part of the existing 'schema' of the student (Olivier,1992). So, the constructivists view the students as interpreters of their experiences rather than as absorbers of knowledge.

Bodner (1986) views construction as a process in which knowledge is both built and continually tested. It is, nonetheless, important to observe that although individuals are free to construct any knowledge, their knowledge must be viable; it must 'work', not only for the individual but also in the wider context.

A number of studies have added to the increasing awareness that many students develop their own methods rather than depending on the taught methods in secondary schools. In this regard Booth (1981:176) observes that:

> "Many children are not using the "proper" mathematical methods taught them at school, but rather are relying upon naive intuitive strategies, and this is almost as true of students in their fourth year of secondary school education as it is of students in their first."

This means that teachers are not filling empty minds with knowledge. Students may have some perceptions, prior knowledge and misconceptions, about some of the subject matter they study. Such perceptions and misconceptions may hinder the students from making acceptable sense (of the mathematical world) from the instruction presented by the teacher. This may lead to another set of misconceptions which may add to the student's own mathematical world (Schoenfeld, 1987) and make subsequent learning even more difficult. Brodie (1991:17) observes that:

> "...it is important to develop the alternate perception that mathematical concepts are ideas that we can talk about, ask questions about, make sense of and come to understand."

Those students whose learning style is algorithmic are in the habit of applying algorithms and not looking back starting from their answers. They seemingly do not have the ability of finding out the suitability of their answer to the given question. In their study on Concepts and Strategies in Estimation, Sowder and Wheeler (1989:145) observed that:

> "Without the careful development of the foundational concepts of computational estimation, instruction on computational estimation may result in the learning of a set of discrete, rote algorithmic processes, distinct from a conceptual knowledge base."

Constructivism advocates that, in the process of learning Mathematics, one has to engage actively one's previous knowledge related to the concept being learned. As Heritage (1980(a):62) observes:

> "A concept cannot be given or received like a book or a brick or a bang on the head. What can be given is experience and guidance while the rest, the development of concept, is up to the learner. (S)\He needs to participate actively by paying attention and reflecting on the experiences (s)\he is having of the various embodiments of the concept. Where the concept is one which embodies other concepts, both teacher and learner must ensure that the learner is in possession of all these prior concepts before the process begins."

If students construct their own knowledge then their understanding must be limited by their perceptions. Many studies (e.g. Booth, 1988; Craig and Winter, 1990) have been conducted in different parts of the world relating to errors, misunderstandings, misconceptions or alternate frameworks of different mathematical concepts. In all these cases it has been observed that these misconceptions interfere with the learning of new concepts. It is, therefore, important for (the) teachers to know the errors and their causes. Knowledge of errors and their origins may help us in finding out different ways and means to help (the) learners. In this connection Booth (1988:31-32) remarks:

> " A continuing assessment of exactly what is involved in the learning of new mathematical topics, assisted by an analysis of the errors that students make and reasons for them, may provide us with extremely useful tools for deciding on ways to help children improve their understanding in mathematics."

Many studies have been conducted in Mathematics education which show that misconceived concepts create confusion in the learner and lead to misunderstandings of the related higher concepts. Schoenfield, et al (cited by Leinhardt et al, 1990) report a case of an advanced Mathematics student who had misunderstandings at various structural levels because of some basic confusion over coordinate system notation.

Failure to recognise the correct use of literals in Mathematics has been reported to be a source of misconceptions (Philipp, 1992). For instance, the literals a,b,c and x have been used in the equation $ax^2 + bx + c = 0$ where a, b, and c are any real numbers and x the variable. Some of the uses of literals are: literals as constants, unknowns, generalised numbers, varying quantities, parameters and abstract symbols.

Olivier (1992) advances the argument further in his observation that many misconceptions arise because of the wrong use of some 'sensible deep level guiding principles' that work well in some domains and do not work in other domains. Some of the principles that are erroneously transferred are:

(a) a notion that a figure (number) with more number of digits is larger than a figure (number) with a less number of digits interferes with the learning of comparison of decimal numbers. For example, 1531 (a number having four digits) is greater than 531 (a number with three digits). The same rule, if applied to compare decimal numbers 1,531 and 5,31, leads to a wrong conclusion, i.e. 1,531 > 5,31.

(b) an equation does not change if each term of the equation is multiplied by the same quantity, interferes with the learning of multiplication of an inequality with a negative number. For example, to solve equations $2x = 4$ and $-3x = 9$ one has to multiply them by 1/2 and -1/3 respectively, and solutions are given by the set $\{x: x = 2; -3\}$. The order relation (=) in the two equations does not change irrespective of the sign of the number(s) with which they are multiplied. On the other hand, to solve inequalities $2x < 4$ and $-3x < 9$ one has to multiply them by 1/2 and -1/3 respectively. The order relation (<) after the multiplication (by a positive number) in the former inequality remains the same whilst in the latter inequality the order relation on multiplication (by a negative number) changes from '<' to '>'. Therefore, the rule that equivalent equations can be obtained by multiplying an equation by any (positive or negative) number can lead to wrong conclusions when applied to inequalities.

Olivier further observes that some misconceptions are the result of overgeneralising over (a) numbers; and (b) operations. In the case of solution of quadratic equations, by factoring method it has been observed that students extend the 'zero product principle' i.e. if $(x - a)(x - b)=0$ then, either $x - a = 0$ or $x - b = 0$, to the statement, if $(x - a)(x - b) = c$ then, either $(x - a) = c$ or $(x - b) = c$. Matz (1980: cited by Olivier, 1992) reports the following errors which are, possibly caused by the overgeneralisation of "the distributive property" (Olivier, 1992).

$$\sqrt{a + b} = \sqrt{a} + \sqrt{b}$$

$$(a + b)^2 = a^2 + b^2$$
$$a(bc) = (ab)(ac)$$
$$\log(a + b) = \log a + \log b$$
$$\sin(a + b) = \sin a + \sin b$$

Further erroneous use of some deep level principles bringing about correct results have been reported (Lochhead, et al 1988, Olivier 1992). These kinds of errors or misconceptions are troublesome to overcome. In this regard Leonard and Sackur-Grisvard (cited by Olivier, 1992: 200) contend that:

> "Erroneous conceptions are so stable because they are not always incorrect. A conception that fails all the time cannot persist. It is because there is a local consistency and a local efficiency in a limited area, that those incorrect conceptions have stability."

Students' Beliefs and Success in Mathematics Learning

An extension of the constructivists' concerns about the role played by students' prior dispositions also include their belief systems. As such, certain student beliefs may also mediate between their chances of success or failure. Documented ones include the following (Garofalo, 1989):

(a) that the teacher is an authority on the subject and is always right;

(b) that the exercises in the text books can be solved only by following the procedures laid down in the text book or by the teacher;

(c) that only Mathematics expected in the examinations is important;

(d) that only creative people create Mathematics.

These beliefs contribute towards the emphasis on rote and passive learning. Schoenfeld (1987) contends that students who believe that mathematical understanding is simply beyond ordinary mortals like themselves become passive consumers of Mathematics, accepting and memorizing what is handed to them without attempting to make sense of it on their own. People who think that those who can learn Mathematics have mysterious insight and have minds to learn Mathematics, which they themselves do not have, develop: (a) fear about the subject and (b) negative attitude towards the subject (Schoenfeld, 1987).

It is, therefore, essential that the teaching of Mathematics is done in such a way as to dispel these notions and make the learner recognise his/her own abilities to learn Mathematics. This may be achieved through the use of concrete, relational (Skemp, 1971) modes of presentation closely linked with conceptual development and meaning.

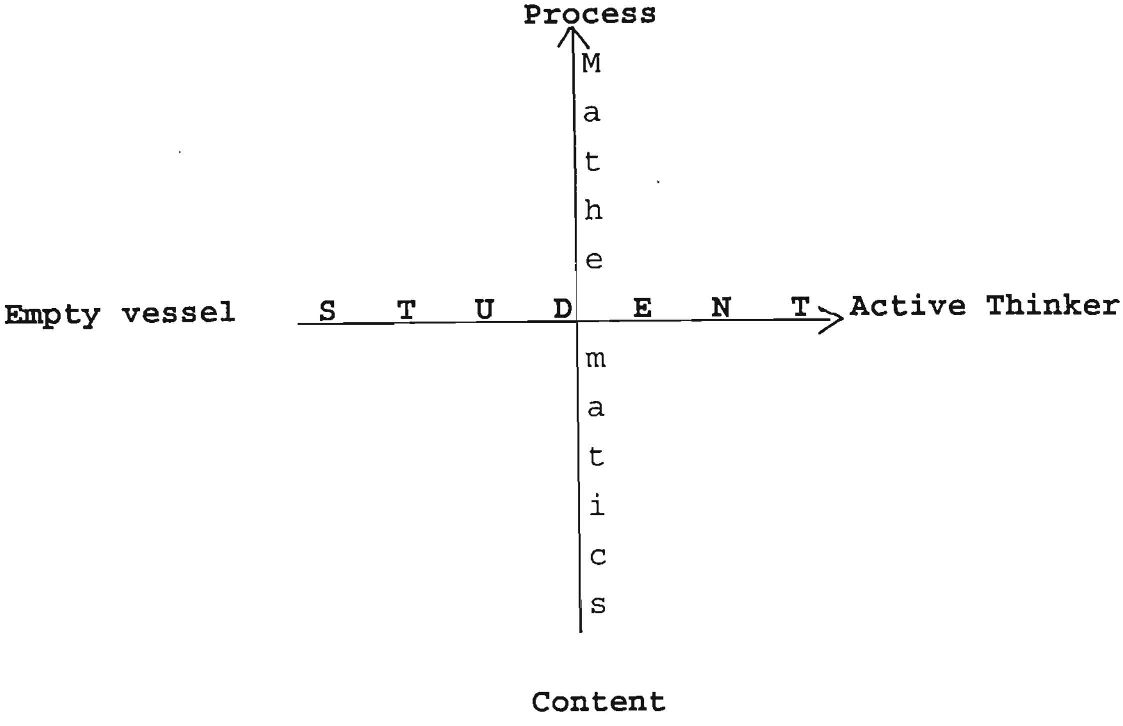
To encourage conceptual development and active learning, classroom activity can play a very important role. What happens in the Mathematics classroom has direct effect on creating beliefs about the subject. If the Mathematics teacher acts as a 'facilitator' and discussion leader, s/he can change students' perceptions about Mathematics and mathematical thinking. This will encourage students to get involved in more interesting and more representative aspects of Mathematics and mathematical thinking (Garofalo 1989:502-505).

In this section some learning theories have been presented.

Operant conditioning advocates 'drill and practice'. Gestaltism is in favour of the development of mental structures. Behaviourists believe in expository meaningful learning and the constructivists believe that the learner constructs his/her own knowledge. Information processing psychologists believe that the learner constructs his/her own knowledge by processing data through classifying and generalising.

There seems to be no single theory of learning that is sufficient to be followed on its own to plan the process of teaching and learning, as the various perspectives complement each other. For example, to teach/learn the concept of quadratic equation one looks at a number of exemplars in order to identify common characteristics and proceeds to define 'quadratic equation' (information processing). This is then followed by identification of quadratic equations from a set of equations (quadratic and non-quadratic) by comparing each of them with corresponding mental structures. This step may serve as 'drill and practice' for those who have already learnt the concept and this whole process invariably gives all the learners an opportunity to construct their own knowledge and understanding of the concept at hand. Therefore, whilst it is important to know the development of different views about learning, it is not possible to stick to only one of them and follow it rigidly in the teaching/learning situation. Whilst concepts and principles/rules may be developed through information processing, procedures may be developed by following expository meaningful learning together with questioning and discussions, and procedures and rules may be committed to long term memory (Gagne's storage and retrieval) by drill and practice.
Mathematics and the learner may be viewed on a continuum as represented in the sketch 2.1.

Mathematics and the learner continuums



SKETCH 2.1

This sketch places the learner on an 'empty vessel' - 'active thinker' continuum; and mathematical knowledge on 'content' - 'process' continuum. This means that a learner may be given direct instruction to start with (empty vessel concept) but soon s/he should be drawn into active participation by asking questions leading him from viewing Mathematics as merely comprising content to a view of the subject as a process, such as: discovering, conjecturing, generalizing, specialising, evaluating and applying. Consequently, this involves a paradigm shift for the teacher from the third quadrant to the first quadrant in the above sketch.

2.5 <u>LEARNING FOR EXAMINATIONS</u>

The majority of students prepare themselves and are prepared by their teachers to write examinations which test basically the facts, rules, formulae and procedures. This influences the students to believe that mathematical thinking essentially consists of these operations. As such, many students, as their teachers do, follow the textbook and learn various techniques in the various chapters of their textbooks. They can solve problems in the various exercises in individual chapters. Yet, these students have difficulty in solving problems in a test that covers a number of chapters at a time. Wenger (1987:218) observes that:

> "the students could use various techniques they had studied, but when problems were presented out of context and the students had to select the methods as well as use them, they had great difficulty."

2.6 LANGUAGE AND CONCEPT DEVELOPMENT IN MATHEMATICS

We communicate with each other through spoken or written language. A student interprets teachers' instructions according to his/her own understanding of the words that the teacher is using. There are chances that the students' interpreted meaning may not be what the teacher intended to mean. This may be due to the nature of words, the length of the sentences and the level of comprehension of language on the part of both the student and the teacher. Language and development of thought are linked. Amongst others, Bruner (1975) argues that language acts as a powerful tool for development of thoughts. Understanding of a concept gets deep rooted when the learner is in a position to verbalise it. Furthermore, mathematical English is different from ordinary English in that many mathematical terms have Greek or Latin roots. This may also create problems in the communication process between the teacher and the student (Brodie, 1991).

In addition to the English language, Mathematics has a language of its own. It has its own symbols (+, _, $\sqrt{}$, ≤, ∥) and principles (rules) ($y = mx + c$, $y = ax^2 + bx + c$) which should be used as intended to convey certain concepts and their relationships. The principles are presented in mathematical statements. An understanding of a rule or principle depends on the placement of symbols in its statement. Like spoken language Mathematics has its own equivalents of **naming** words (square, area, median), words denoting **properties** (two, congruent, similar) and **doing** words (add, differentiate, square).

Furthermore, certain words have more than one mode of usage; for example, a 'square' is a rectangle with all its sides equal. On the other hand, 'square' of 4 is equal to 16. So the students have to learn correct mathematical vocabulary and context for use. They also need to know correct usage of mathematical language (Burton, 1980). Some students may have difficulties in learning Mathematics because they have not understood the appropriate meanings of some of the symbols (Brodie, 1991).

2.7 SOURCES OF STUDENT DIFFICULTIES

The students' ideas about the focus of algebraic activity, the nature of answers, the use of notation & convention in algebra, the meaning of letters and variables have been identified by Booth (1988) as some of the root causes of students' difficulties in learning algebra.

Wagner (1981) observes that students do not view letters, in algebraic expressions, as representing numbers. They view algebraic expressions as incomplete statements (Collis, 1974 cited by Chalouh and Herscovis; 1988). The differences in the meaning of concatenation in algebra and in arithmetic have also been described as one of the difficulties in learning algebraic expressions (Matz, 1979: cited by Chalouh and Herscovis, 1988). The fact that expressions may stand for both the process and answer has also been reported to be a difficulty in the learning of algebraic expressions (Davis, 1975 cited by Chalouh and Herscovis, 1988). For example, the expression '$(x + 3)^2$' stands for: (a) instructions (processes)- 'add 3 to a number, and square the result'; and (b) represents the answer arising from carrying out the instructions (processes) mentioned in (a). Therefore, students who have misconceptions about different meanings of the algebraic expressions can have difficulties in solving, meaningfully, problems such as: 'I bought x number of pens at the rate of (x - 10) Rands per pen for 24 Rands. Find (a) the number of pens that I bought; and (b) the cost of each pen.'

An understanding of the concept of rational numbers is essential as a foundation for building on the understanding of elementary algebraic operations. This is a very important concept in the study of Mathematics. The student finds great difficulties both in learning and applying the rational number concept. This may be attributed to the fact that rational numbers can be seen as a set of six different sub-constructs and their inter-relationships, understanding of which requires various cognitive structures. These sub-constructs and their inter-relationships include: a part to whole, comparison, a decimal, a ratio, an indicated division, an operator and a measure of continuous or discontinuous quantities (Merlyn, et al 1983).

The direct use of the rational number concept comes up in the learning of the quadratic theory when, for instance, the students are required to discuss the nature of roots of a given quadratic equation. The roots of a quadratic equation are rational if the discriminant of the equation is a complete square. The students may be able to understand this if they know, in addition to the meaning and relationship of discriminant with quadratic formula, the difference between the rational and irrational numbers.

Markovits, et al (1988) found that students have some difficulties and misconceptions in relation to 'functions'. According to the author, there are different levels of difficulties. The pupils have problems with terms such as pre-image, image, pair, domain, range and image set. The next level of difficulties arises from the first level such as "... difficulties in locating pre-images and images on the axes in the graphical representations, identifying images,...and ignoring the domain and range of a function" (Markovits, et al, 1988:49). They further report that students have difficulties in locating pre-images and images on the axes in the graphical representation. Students do not appreciate that the domain and range of a function are represented by the x-axis and the y-axis in its graphical representation. They suggest that this difficulty may be because of the double role of points on the axes, namely:

(a) x-axis and y-axis can be taken as number lines. Any point on these axes represents a point on the number line. A point on x-axis represents pre-image and on y-axis represents an image;
(b) any point on these axes may represent a pre-image - image pair in the Cartesian plane, ie. $(x, 0)$ on the x-axis and $(0, y)$ on the y-axis.

This knowledge is essential for finding graphical solution of quadratic equations from the graphs of quadratic (parabolic) functions. The quadratic function defined by $\{(x, f(x)): f(x) = ax^2 + bx + c\}$ gives quadratic equation $ax^2 + bx + c = d$ when $f(x) = d$ and 'd' is a real number. On the graph of the above mentioned function the values of x-coordinates of all those points where $f(x) = d$ are the roots of the equation $ax^2 + bx + c = d$.

Therefore, students who have misconceptions mentioned by Markovits, et al (1988) will have difficulties in reading these graphs for, for instance, finding out:

(a) the roots of the equation $ax^2 + bx + c = 0$;

(b) the interval(s) which represents the solution set(s) of the inequalities of the type: $ax^2 + bx + c < \text{or} > 0$;

(c) the intersection set of two given sets; and

(d) the region that represents a given function of the form: $\{(x, f(x)) : f(x) \geq ax^2 + bx + c\}$, etc.

In addition these graphs can be used to find out if the equation $ax^2 + bx + c = 0$ has real or unreal; and equal or unequal roots.

There are some conflicting views about the usefulness of the number line in the teaching of some concepts of Mathematics. Whilst the use of number line diagrams in the teaching of whole number addition and subtraction is recommended by, among others, Reidesel and Burns(1977), Carpenter, et al (cited by Paul, 1985) found that in some cases the misconceived use of the number line becomes a source of error. Nonetheless, the use of number line diagrams is very important for understanding the intervals that form solution to various inequalities (linear as well as quadratic) in one variable (Paul, 1985).

Robert and Robinet (1989), (as cited by D'Ambrosio, et al, 1992) argue that knowledge or concept representations in one's mind are very stable. In order to change these representations the equilibrium must be disturbed by developing conflicts for students. This should be followed by resolution of the conflicts which should proceed to regain equilibrium.

In addition to the difficulties that students experience because of misconceptions, they also find difficulties in relating different concepts in solving mathematical problems. Some students can often perform required manipulations correctly, yet they fail to solve a problem because they do not know which approach to follow. Wenger (1987) contends that this "observation suggests that students' difficulties result not so much from the content of their mathematical knowledge base but from its organisation... The way in which that knowledge is organised, accessed, and used are equally important determinants of one's intellectual performance."

Olivier (1989) contends that a student may fail to solve a mathematical problem because:

(a) he may not have the required schema;
(b) he may have the required schema but fails to retrieve it;
(c) he may retrieve a 'flawed or incomplete' schema; and
(d) the retrieved schema may be 'inappropriate'.

The mode of presentation may also be another source of student learning problems. A problem may be stated in various forms of representation, for instance: spoken form, written in formal language, symbolic form, pictorial form - pictures, graphs, etc., and in the concrete order form. In the process of solving a problem students generally translate it from one representation to another in order to gain more understanding of the problem. They sometimes use more than one representation simultaneously for solving a problem. For example, solutions of quadratic equations and inequalities can be obtained both algebraically and graphically. Some students may find algebraic solutions easier than the graphical solutions, and vice versa. Consequently, where it is necessary to use both representations to solve a problem, these students would experience particular difficulties which will, almost invariably, lead to failure to finding the necessary solutions.

2.8 **TEACHING STYLES**

The "learning" of Mathematics, like its teaching, may take place through various ways. Some of these learning types depend heavily on rote learning whereas some may evolve around the understanding of the concepts in ways that embody meaning for the individual learner. This research, among others, explores whether much of Mathematics learnt in Transkeian schools is closely related to the teachers' teaching styles, as well as the teachers' persuasion in terms of their espoused theories of learning as briefly described above.

Mathematics classroom activities depend on, amongst other things, the teachers' views about the learner, the teachers' conception of Mathematics and how the student learns Mathematics. Thompson (1984:124-125) observes:

> "... teachers' beliefs, views, and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behaviour. In particular, the observed consistency between the teachers' professed conceptions of mathematics and the manner in which they typically presented the content strongly suggests that the teachers' views, beliefs and preferences about mathematics do influence instructional practice."

According to Elliot (1968) there are only two options that a teacher can adopt to pass on knowledge to the students. These options are: (a) tell them (lecture method, teacher-centred), and (b) ask them to tell you (enquiry method, student-centred). Elliot further observes that the 'tell them' option is quicker and faster. Consequently, on the basis of these two options, Elliot divides teaching strategies into three categories namely: teaching as **explaining**, teaching as **questioning** and teaching to **promote recall**.

Teaching as explaining. Elliot contends that explaining promotes understanding. In this strategy the teacher keeps in mind the level of students and the nature of the subject matter whilst preparing the explanation. The success of an explanation depends on the following three factors: (a) the problem to be explained; (b) the relevant characteristics of the explainee; and (c) the communication process. This strategy makes the students passive, and by and large, rote learners.

Teaching to promote recall. Whatever a student learns s/he must remember so that s/he can use this knowledge in future. In this strategy the teaching material is presented in a logical and sequential order; generalisations are made and main principles are stressed. A quick review of the lesson at the end of it and periodic reviews, later on, are essential.

Teaching as questioning. The use of questioning in teaching and learning arouses curiosity in students, and makes them think and discover things for themselves. This promotes creativity, dialogue and helps the students to become independent critical thinkers. In support of teaching by questioning Moodley (1992(a):141) makes the following observation:

> "Questioning is the key to promoting mathematical thinking in the classroom. In a questioning atmosphere assumptions are queried, questions are posed, conjectures are made, checked, modified, justified and accepted."

Cockcroft (1982, cited by Hoyles, 1985:205) supports dialogue in the teaching and learning of Mathematics. Cockroft observes:

> "Language plays an essential part in the formation and expression of mathematical ideas. School children should be encouraged to discuss and explain the mathematics which they are doing"

Through good questioning techniques and high levels of active student involvement in the learning process, attainment of higher order educational goals is possible. Students should be involved both in answering and asking questions (Moodley, 1992(a); Maher, et al, 1988). However, Moodley (1992(a)) discourages teachers from using questions which may promote "showing and telling" by the teacher and "seeing and following" by the pupil. On the other hand, questioning directed towards promoting confidence, positive attitude to Mathematics, mathematical thinking and problem solving is encouraged. According to Moodley (1992(a)) some of the questions that may succeed in developing mathematical thinking include the following:
Why is that so?
What if...?
Is there a pattern?
Is there a counter-example?
How can we make it easy?
Is there another way?
Can you prove it?
Moodley argues that questions of this kind promote the emphasis on the processes of Mathematics such as: discovering, specialising, conjecturing, generalizing, evaluating, applying, etc.

Garofalo (1989:504) especially favours Elliots' second option, i.e. 'ask them to tell you'.. Garofalo is of the view that Mathematics teaching should emphasise activities that encourage students to explore mathematical topics;

develop and refine their own ideas, strategies and methods, and reflect on and discuss mathematical concepts and procedures. Through class discussions the students get an opportunity to verbalise and clarify their own thoughts, develop a deeper understanding of mathematical ideas, and are encouraged to think mathematically. Furthermore, the teachers get a chance to listen to the students and become more aware of their difficulties and misconceptions (Brodie, 1991).

However, the idea that the lecture method of teaching necessarily promotes rote learning is not shared by Ausubel. Ausubel (1979) contends that expository teaching may lead to meaningful reception learning. Consequently, Ausubel argues that by putting more emphasis on the meaning of the concept, meaningful learning is possible by this method. Ausubel (1979:179) further contends that:

> "The very fact that the accumulated discoveries of millennia can be transmitted to each new generation,...is possible only because it is so much less time consuming for teachers to communicate and explain an idea meaningfully to pupils than to have them rediscover it by themselves."

Davis (1984) mentions another teaching strategy, namely: The Paradigm Teaching Strategy. The paradigm teaching strategy is based on the 'theory of representations'. In this strategy, the teacher makes use of the available representational devices to lead the student to create a representation of the problem situation and that of corresponding 'knowledge'. Davis gives an example of an activity called 'Pebbles-in-the-bag' which is used to explain that subtraction of a number from a smaller number results in a negative answer. The activity proceeds as follows:

Situation: There is a bag, partially filled with pebbles and there is a heap of pebbles on the table.

Actions:

(i) A student, say Sipho, claps his hands (starting time as reference point).

(ii) Another student is asked to take 4 pebbles from the heap and **put into** the bag.
This action is written on the board as:
+4

(iii) Then the student is asked to **remove** 10 pebbles from the bag.

This action (iii) is added to the writing on the board as follows: **+4 - 10**

This is followed by questioning.

Question 1 Are there more pebbles in the bag than there were when Sipho clapped his hands?

Answer (children answer): Less.

Question 2 How many less?

Answer (children answer): six (6).

This is then followed by a talk by the teacher as follows:

Talk:

We write $+4 - 10 = -6$ and read this as 'four minus ten is (gives) negative six.' The negative tells us that there were (now) fewer pebbles in the bag (than there were at the beginning).

'Assimilation paradigm' strategy may be used to introduce the translation of word sums into algebraic representations leading to quadratic equations.

Davis argues that the above mentioned activity is an example of 'assimilation paradigm'. In the example, the students' representations of **put in** (add), **remove from** (subtract), **more** and **fewer** are used for achieving the intended objective.

Davis contends that this is a successful strategy because the process helps the student to use his/her existing mental representations thereby making the process of assimilation and/or accommodation easier.

Freire (1970) classifies teaching strategies into two groups, namely: the problem-solving and the banking strategies. Freire defines banking strategies as those whereby the teacher follows more of a narrative teaching style and plays an active role in contrast to the passive role of the student. This implies that creativity and dialogue on the part of the students will largely be absent from such teaching and learning situations where banking strategies are used. On the other hand, the problem-solving strategies are based on more communication between the teacher and the taught through dialogue and activity. Freire contends that the banking strategies promote passiveness amongst the students and the problem-solving strategies help the students to become independent critical thinkers. Freire's view is supported by Lindgreen (1984) who contends that the students taught through problem-solving strategies evaluate, participate, weigh evidence, consider alternatives and arrive at conclusions through active participation in the teaching and learning process.

Schifter and Simon (1992:189) contend that:

> " Teaching mathematics is now to be understood as providing students with the opportunity, and the requisite stimulation, to construct powerful mathematical ideas for themselves, to guide them in that process, and to help them know their own power as mathematical thinkers."

Moodley (1992(b):12) recognises the need for both procedural and relational understanding in Mathematics teaching/learning and suggests that:

> "The instructional process should be characterised by an interactive model with a constant shift between procedural and relational understanding while spiralling to ever complex techniques and concepts/generalisations:"

Shuard (1986:179-180) sums up the comparison between the two types of teaching styles in the following words:

> "At present, substantial part of the way in which mathematics is taught - especially in work on numbers...are based on a conceptual model that children are 'empty vessels', and that it is (the) teachers' duty to fill those vessels with knowledge...In fact, recent work would suggest that another model of mathematics learning is in fact a better one; learners are conceptualised as active *mathematical thinkers*, who try to construct meaning and make sense for themselves of what they are doing, on the basis of their personal experience...and who are developing their ways of thinking as their experience broadens, always building on the knowledge which they have already constructed."

2.9 CONCLUSION

This chapter dealt with paradigm shift, concept, concept development in terms of information processing, selected theories of teaching and learning, a review of some literature that deals with the difficulties that students face in the learning of Mathematics, and different teaching styles.

What emerges from the review is that the process of teaching and learning involves a complex interaction among the teacher, the learner and the material to be taught and learned. Mathematics is seen as the collection of rules - i.e. instrumental understanding in Mathematics (Skemp, 1976) by some teachers as well as students. Others view Mathematics as the subject which deals with the development of processes such as abstraction, generalisation, etc. - relational understanding in Mathematics (Skemp, 1976). Teachers who believe in instrumental understanding in Mathematics promote instrumental learning in their students. An instrumental learner depends on memorized rules for each class of problems and often fails to use these rules correctly in a changed situation. On the other hand, teachers who believe in relational understanding in Mathematics promote relational understanding in their students. Not only do they learn which rule works to solve a problem, but also why it works. They learn the rules and the connections between them. Knowledge thus acquired can easily be remembered, recalled and put into use in a variety of related problems (Skemp, 1976).

In general, students may experience certain difficulties in the learning of Mathematics. These difficulties may either be due to their mathematical cognitive constructions, which may partly be attributed to the teaching, language, their learning set or their beliefs about Mathematics and/or the instructional process in general. Furthermore, the success of the teaching/ learning process depends on the readiness (in terms of previous knowledge) of the learner and the appropriate teaching strategies adopted by the teacher.

Most of the findings mentioned in this chapter emanate from research done in different parts of the world. The problems of students elsewhere may not, necessarily be the problems of the students of the Transkei. This researcher is interested in exploring the difficulties that students of Transkei are facing in the learning of Mathematics with special reference to theory of quadratic equations, inequalities and their graphs, and the teaching styles that are being used by their teachers.

The hypotheses of this study were presented in chapter one. The methods of data collection and analysis used for the study are discussed in the chapter that follows.

CHAPTER 3

METHODS OF STUDY

3.1 OVERVIEW

This chapter presents the design and methods of this study. A description of the instrumentation is given, together with a brief description of the validation procedures and computation of the reliability coefficient of the diagnostic test. The target and accessible populations are also described. The sampling and administration procedures are given, as well as the procedures for the collection and analysis of data.

3.2 DESIGN

This is an evaluative study concerning high school students' conceptual development of the concepts and principles embodied in the theory of quadratic equations, inequalities and their graphs. The study involved matric higher grade Mathematics students, in the Umtata educational circuit, Transkei, and their teachers. Both quantitative and qualitative research methodology were employed.

3.3 TARGET AND ACCESSIBLE POPULATION

The target population for this study was all high schools offering higher grade Mathematics in Transkei. The accessible population was all high schools in the Umtata circuit offering higher grade Mathematics. In all, 10 high schools (A, B, C, D, E, F, G, H, I, and J) were involved. Six of these schools (A, B, C, E, F, and H) are within Umtata town and the other four (D, G, I, and J) are in the range of 20 to 30 km from Umtata. The other three schools could not be reached for logistical reasons - mainly of distance and timetabling, as well as low enrolment levels of students in the higher grade stream.

Principals and standard 10 Mathematics teachers of the respective schools were approached in August, '92 with a view to discussing the purpose and methodology of the research and to prepare a timetable for administering the test. Administration dates ranged from the beginning of September to mid October, 1992.

3.4 CHARACTERISTICS OF THE TARGET POPULATION

Quadratic equations, inequalities and their graphs are taught in standard nine in Transkei schools. Students who constituted the target population were preparing themselves to write matriculation examination in October-November 1992. Obviously, all had passed standard nine promotion examinations held by respective schools or some were repeating. Whatever the case these students were expected to be conversant with the topic under investigation in this study.

3.4.1 THE SAMPLE

Students

In all, there were 311 (82.3% of the total number of students taking higher grade Mathematics in Umtata circuit) students who wrote this test. The sample did not include the 66 students who took this test for pilot study (see page 11). All students were blacks and had attended schools for blacks in their previous classes. Table 3.1 shows the gender distribution amongst the individual schools and the total sample. There were 140 male and 171 female students. The male to female ratio was, therefore, 1:1,22 in the total sample. Schools C and D had only male students whereas school F had only female students. Schools A, B, H and J had considerably higher proportion of male to female students. The number of male (M) students was less than that of the females (F) in schools with both sexes, except in school G where males were more than the females. The sample could be regarded as fairly representative of the total population.

Table- 3.1

Gender distribution: by ~~school~~ CLINIC

SAMPLE BHM CLINIC	**MALE (M)**	**FEMALE (F)**	**RATIO (M : F)**
SCHOOL A	7	17	1 : 2.43
SCHOOL B	4	6	1 : 1.5
SCHOOL C	21	0	1 : 0
SCHOOL D	4	0	1 : 0
SCHOOL E	69	89	1 : 1.29
SCHOOL F	0	6	0 : 1
SCHOOL G	8	7	1 : 0.875
SCHOOL H	17	33	1 : 1.94
SCHOOL I	9	11	1 : 1.22
SCHOOL J	1	2	1 : 2
TOTAL	140	171	1 : 1.22

Teachers

Teachers of the sampled students participated in answering the teachers' questionnaire (see appendix 2). These were 10 in number of whom 3 were female and 7 male. Every one had at least a university degree and 80% had a professional teacher's certificate.

3.5 <u>INSTRUMENTATION</u>

Two instruments constructed for this study were: (i) a questionnaire for the teachers; and (ii) a diagnostic test for the students.

3.5.1 TEACHERS' QUESTIONNAIRE

An instrument (see appendix 2) was constructed for finding out the teachers' styles of Mathematics teaching, generally, and their approach to teaching of the theory of quadratic equations, inequalities and graphs - in particular - to test research hypothesis 8 arising from research question 8. The instrument consisted of two sections. Section A consisted of 16 statements which were aimed at finding out teachers' dispositions (on a five point Likert type scale ranging from strongly agree to strongly disagree) towards students working in groups, examination oriented teaching, participation of students in the class and the development of students' problem-solving skills in general. There were 11 statements in section B of the instrument aimed at finding out how, more specifically, the teachers taught the theory of quadratic equations, inequalities and their graphs. In particular, these items were aimed at finding out whether the teachers:

(a) laid emphasis on derivation of the quadratic formula;

(b) laid emphasis on the correct use of the quadratic formula;

(c) supplemented current exercises with some reflection on pre-requisite knowledge;

(d) related solution of quadratic equations and inequalities to the graphical representation of the quadratic functions; and

(e) taught all aspects of quadratic equations and inequalities without any prejudice to any particular section.

The subjects were required to respond to statement of each item on a five-point (Likert) scale: strongly agree (SA), agree (A), neutral (N), disagree (D) and strongly disagree (SD). They were also provided with spaces to forward reasons for their responses to some items (1 to 16; 21, 25, 26 and 27) to gain further insight into their beliefs and notions about Mathematics teaching.

3.5.1.1 Validation

The instrument was validated by Prof. S.N.Imenda (Director of the Student Academic Support Centre; Acting Director of the Bureau for Academic Support Services and Head of the Staff Development Unit), Prof. M. Glencross (Prof and Head of Science and Mathematics Education) and Mr. Njisane (Lecturer in Mathematics Education) of the university of Transkei. Their suggestions were incorporated and the necessary changes were made. The final version was judged to be a valid instrument for ascertaining the respondents' teaching style in line with the constructs of student- versus teacher-centredness discussed in chapter two.

3.5.2 TEACHERS' INTERVIEW SCHEDULE

Interviews with the teachers started with the following question: Why do students not perform well in Mathematics? The rest of the interview questions were based on the answers given to the first and the subsequent questions. All these questions were directed towards supplementing information gathered from the teachers' questionnaire.

3.5.3 DIAGNOSTIC TEST (see appendix 3)

3.5.3.1 Item Construction

Objectives, Subject matter and Number of Items

The total number of items initially constructed for this instrument was 43. These items were designed to test the following objectives:

(a) Assumed prerequisite knowledge required for learning the theory of quadratic equations, inequalities and their graphs included:

(i) knowledge of the system of real numbers (rational and irrational number concept), the terms 'root', 'solution', 'satisfy' as used in the problems involving equations in general, and defining equations of x-axis and y-axis on the cartesian plane;

(ii) ability to multiply two binomials and factorise a quadratic trinomial;

iii) the skill to write a quadratic trinomial in the form: $a(x - p)^2 + q$;

(iv) the ability to recognise that the value of an algebraic expression, even if it can be written as a complete square will not necessarily be a rational number; and

(v) the ability to translate the mathematical information given in the verbal form into algebraic form.

(b) The ability to solve quadratic equations and inequalities by factorisation.

(c) The ability to solve equations by using a quadratic formula.

(d) The ability to transform quadratic equations into the standard form.

(e) The ability to solve problems related to the theory of quadratic equations in general, e.g. multiplication of an inequality by a negative number; finding of discriminant and discussion of the nature of roots of a quadratic equation; and determining the values of the unknowns in an equation when one or more root(s) is/are given.

(f) The ability to solve problems related to graphs of quadratic functions, e.g. relating the position of graph on cartesian plane with the possible equation of the graph; establishing the relationship between the graph of a quadratic function $y = ax^2 + bx + c$ and the equation $ax^2 + bx + c = 0$ and inequalities $ax^2 + bx + c \le$ or ≥ 0; relating a given point on the graph to the equation of the graph; and establishing the relevant equation at the points of intersection of two graphs.

(g) The ability to apply the knowledge of quadratic equations, inequalities and their graphs in unfamiliar situations, e.g. to check if a given quadratic function can have a zero value; use of a maximum or minimum value of a quadratic function to solve related problems; and any other problem that reduces to a quadratic equation or inequality.

The breakdown of items according to the above objectives was as follows (see table 3.2):

Table 3.2

Breakdown of items according to areas of concern

	Area of concern	**Number of Items**
(a)	Prerequisites	07
(b)	Inequalities by factorisation	02
(c)	Use of quadratic formula	02
(d)	Transformation into standard form	03
(e)	Quadratic theory	14
(f)	Graphs	12
(g)	Comprehension and Application of quadratic theory	03
	Total:	43

Initially, the instrument consisted of 21 true/ false, 21 multiple choice and 1 structured supply type items. Each multiple choice question had four distracters. A few items were at what Bloom calls knowledge/recall level but most of them demanded comprehension, application, analysis, synthesis and evaluation. An analysis of objectives as per scheme (Moodley, 1992(c)) tested by each item is presented below in table 3.3. In the table the symbol * represents the objective(s) tested in each item.

<u>Table 3.3</u>

Analysis of objectives tested by each item in diagnostic test

Key:
A = Knowledge;
B = Skills;
C = Comprehension;
D = Selection - Application;
E = Analysis - Synthesis.

Item No	A	B	C	D	E
1				*	
2			*		
3	*	*			
4X		*	*		
5	*				
6		*	*		
7	*				
8	*				
9			*	*	
10			*	*	
11			*		
12			*		

13X		*	*		
14	*		*		
15			*		
16X		*	*		
17		*			
18				*	*
19			*	*	*
20	*				
21	*				
22	*		*		
23	*			*	
24	*	*	*		
25	*	*			
26		*	*		
27		*	*	*	
28		*		*	
29		*	*		
30		*	*		
31	*	*			
32		*		*	
33		*		*	
34		*			
35X			*	*	
36X					*
37			*	*	*

38			*	*	
39			*	*	
40			*	*	
41			*	*	
42			*	*	
43			*	*	

(Items marked X were not included in final instrument)

It is evident from the table 3.3 that there was a fair spread of items over different objectives. This ensured an appropriate coverage of the subject in the diagnostic test in terms of the academic demands on the respondents. Furthermore, this spread of items ensured content validity.

The students were requested to show their working and/or write their reasons for their choice of answer in response to each item in the spaces provided within the instrument. A maximum of 2 hours 30 minutes was mentioned on the test. However, since this test was not a speed test students were allowed to take as long as they needed to answer all the items.

The breakdown of items in the final instrument according to the research questions was as given in table 3.4 (see next page):

Table 3.4

Breakdown of items according to the research questions

RQ	Content Area	Items Number
1	Pre-requisite knowledge vs. Theory of quadratic equations, inequalities and graphs.	(3, 6, 7, 8, 20, 27 and 29) vs. (The rest of the items in the test).
2	Pre-requisite knowledge.	3, 6, 7, 8, 20, 27 and 29.
3	Quadratic equations in standard form.	24 and 26.
4	Quadratic equations not in standard form.	1, 5, 22, 23, 25, 28, 30, 31, 32, 33 and 34.
5	Quadratic inequalities.	11, 14, 15 and 17.
6	Interpretation of graphs.	2, 9, 10, 18, 19, 21, 37, 38, 39, 40, 41, 42.
7	Application of quadratic equations.	12 and 43.

The low number (2) of items in the section testing research question 3 was due to the narrow scope (the use of quadratic formula) of the research question. Also the number (2) of items testing research question 7 (the ability to apply the knowledge of quadratic equations, inequalities and their graphs) was low because some items testing objectives of the same level were also included in other research questions. Thus inclusion of more items in this section was going to make the test more difficult and longer.

3.5.3.2 Validation

The instrument was validated by a panel of experts. These were: two practising and experienced teachers, the Mathematics examiner in the department of Education, Transkei; and Prof. Mishra of the Mathematics Department, University of Transkei. These experts made some suggestions and the revised paper was judged to be valid in terms of the Mathematics syllabus requirements and the objectives set by the researcher. Table 3.3 shows spread of items over a range of objectives. This fulfilled the validation requirements for face, content and construct validity.

3.5.3.3 Reliability

The reliability of the instrument was established by using a random sample of 66 students in a pilot study from three schools offering higher grade Mathematics outside the Umtata circuit. The split half (odd-even) Spearman's Rank correlation was used. A Spearman rank correlation coefficient of 0.57 was obtained for the test. This was significant at alpha = 0.01. Hence, statistically the instrument can be said to be reliable.

3.5.3.4 Item Analysis

Item analysis was used as a further check on the validity and reliability of the instrument. Specifically, the discrimination and difficulty indices of each item were calculated following the pilot study (Imenda 1991; Mammen, Imenda and Grewal, 1992; Stanley and Hopkins 1972). Item number 13 had a negative discrimination index, and items 4, 16, 35 and 36 were more difficult but had low discrimination indices. On the basis of these calculations, 5 items (No.4, 13, 16, 35 and 36) were subsequently deleted. The final instrument of 38 items was, therefore, used in the main study. All the items used in the final instrument had the discrimination indices falling in the range 5.9% to 49.4% and difficulty indices fell in the range 11.8% to 92.4%. This confirms that the items used in the diagnostic test were of a reasonable standard. Table 3.5 (see pages 65, 66 and 67) gives the discrimination and difficulty indices of each item for final test. The final instrument appears as appendix 3.

(See overleaf)

Table: 3.5

Correct scores distribution, item difficulty index and discrimination index: by item

(Rounded off to whole percentages)

Key: T = Total;

N = Number;

C = Correct options;

P = Percentage of correct options;

S = Item Difficulty (%);

D = Discrimination Index (%).

Q = QUESTION NUMBER
C = TOTAL CORRECT OPTIONS (OUT OF 311 STUDENTS)
P = PERCENTAGE OF CORRECT OPTIONS (100 X C/311)
$S=(N(C)_{top} + N(C)_{bottom}) \times 100/(N(T)_{top} + (N(T)_{bottom})$ FOR 85 STUDENTS
$D = (N(C)_{top} - N(C)_{bottom}) \times 100/N_{top\ or\ bottom}$ FOR 85 STUDENTS

Q	1	2	3	5	6	7	8
C	227	106	290	166	175	140	65
P	73	34	93	53	56	45	21
S	70	34.7	90.6	48.8	56.5	46.5	27.1
D	15.3	43.5	16.5	17.6	30.6	22.4	23.5

Q	9	10	11	12	14	15	17
C	46	155	139	99	161	158	154
P	15	50	45	32	52	51	50
S	18.8	50.6	42.4	34.1	52.9	48.2	48.2
D	25.9	35.3	16.5	23.5	30.6	28.2	20

Q	18	19	20	21	22	23	24
C	121	118	163	114	289	134	159
P	39	38	52	37	93	43	51
S	38.8	38.8	49.4	42.4	92.4	49.9	52.9
D	32.9	27.7	37.7	49.4	5.9	20	23.5

Q	25	26	27	28	29	30	31
C	195	120	91	89	94	159	76
P	63	39	29	29	30	51	54
S	60	38.8	30.6	35.9	35.3	51.8	30.6
D	18.8	30.6	37.7	29.4	25.9	47.1	35.3

Q	32	33	34	37	38	39	40
C	160	78	177	52	81	36	31
P	52	25	57	17	26	12	10
S	50	32.9	55.3	17.6	27.6	11.8	12.4
D	22.9	24.7	22.4	21.2	27.1	14.1	15.3

Q	41	42	43
C	136	60	66
P	44	19	21
S	45.9	17.1	24.7
D	32.9	17.1	23.5

3.5.4 STUDENTS' INTERVIEW

Questions put to the students during interviews varied from student to student. They were based on the student's (working) answers given for various items in the test. The questions were intended to seek explanations for students' working for those items which they had failed to answer correctly in the test. The main purpose of these interviews was to supplement data collected from the students' scripts. These interviews were audio taped.

3.6 ADMINISTRATION

The instrument was administered to standard 10 students taking higher grade Mathematics in 10 different schools in Umtata circuit of Transkei. A total of 311 students wrote this test. The researcher supervised all these test sessions except in two schools where students could not be accommodated in one classroom. In these two schools an assistant was employed to assist in invigilation. All the students were informed about the scope and purpose of the test at least a week before administering the test. So, they were all aware and presumably prepared for the test.

3.7 ANALYSIS

The teachers' questionnaire and students' test results were analyzed as described below.

3.7.1 STUDENTS' TEST

3.7.1.1 Marking

All the scripts were marked manually. A mark of 1 was awarded for each correct answer, while wrong answers or no answers were awarded 0 marks. Reasons forwarded in support of the answer were not considered in this exercise. The possible total score was 38. These scores constituted the raw data for statistical analysis. A comparison of the score distribution among the schools and the total sample is presented in Table 4.1. Score distribution of the total sample is also presented in figure 1. A comparison of the average scores and average score % of the individual schools and total sample is given in Table 4.2 and figure 2. Comparisons of sample's performance in items testing hypotheses 2 to 7 are also presented in figures 4 to 9.

3.7.1.2 Statistical Techniques for Data Analysis

The percentage mean scores on different sections of the instrument for the sample were computed. These percentage mean scores were used to test the hypotheses of the study. Sixty percent was used as a criterion for determining whether or not the subjects had the required level of pre-requisite knowledge for the learning of the theory of quadratic equations, inequalities and their graphs. Sixty percent was also used as a criterion for determining whether or not the subjects were successful in attaining the required level in the theory of quadratic equations, inequalities and their graphs. These criterion marks were arrived at arbitrarily and _a priori_, based on the premise that the learning of Mathematics is hierarchical. In particular, the literature (e.g. Heritage, 1980(a)) also suggests that for a complete understanding of the quadratic theory of equations, inequalities and their graphs mastery of subordinate concepts and principles is essential. These concepts/principles are tested in the seven items (3, 6, 7, 8, 20, 27 and 29) which illustrate the elementary nature of the pre-requisite knowledge particularly for standard 10 students. Although this criterion mark was arbitrarily set, it bears close resemblance to other studies with similar concerns (e.g. Thomas, 1975:152-153). In that study, Thomas fixed a score of 66% or more correct responses of the total number of responses as a criterion for concluding that a subject had attained the required level of mastery of the concept. Such a level of competency has, for the purpose of this study, therefore been set at 60%. Educationally, this could be regarded as a reasonable and acceptable level of mastery.

Pearson's Correlation Coefficient

A Pearson's correlation coefficient (Minium, 1970:148) between the subjects' performance in the pre-requisite knowledge section and their performance in the rest of the instrument was calculated. This was used to test the relationship between the subjects' pre-requisite knowledge and the learning of the theory of quadratic equations, inequalities and their graphs.

3.7.1.3 Script by Script Analysis

There was further analysis involving script by script examination of all the working procedures and reasons given in support of student answers. The analysis was conducted with a view to identifying the students' conceptual understanding of the concepts and principles in question. As such, students' conceptual strengths and weaknesses - together with students' own alternative strategies to intended answers - were examined. The analysis was conducted within the context of each of the research questions.

3.7.1.4 Students' Interviews Analysis

Further to the examination of the scripts, follow up clinical interviews were conducted with a randomly chosen sample of students from the highest, middle and lowest achieving students. Four students were interviewed from each of these categories. Students' audio taped interviews were transcribed on paper. These transcripts were then analyzed and apparent reasons for their poor performance were inferred from the students' responses. Relevant sections of the protocols were included in the discussions of the results of this study.

3.7.2 TEACHERS' QUESTIONNAIRE (see appendix 2)

3.7.2.1 Coding

The following five point Likert scale was used to code teachers' responses.

Response	Weighting
Strongly Agree	1
Agree	2
Neutral	3
Disagree	4
Strongly Disagree	5

These codes corresponded to a continuum of least favourable (1: strongly agree) to the most favourable (5: strongly disagree) statements, according to the contemporary educational theories favouring student-centred and activity-based teaching approaches as against teacher-centred and lecture method teaching approaches. Teachers' responses to items 1, 3, 13, 17 and 23 were reversely coded. A total score for each teacher was calculated. Participating teachers of schools A, B, C etc. were coded as teachers A, B, C etc., respectively. Their scores are presented in table 4.18 and in figure 10. The median score of the group was found. The teachers were grouped into two groups. Group 1 (n = 5) consisted of teachers scoring below the median score and the teachers who scored above the median score constituted Group 2 (n = 5). Further analysis of the teachers' questionnaire was carried out. The number of strongly agree and agree responses on each item were combined to give an affirmative score, and likewise, a dissenting score was obtained by combining the disagree and strongly disagree responses. These numbers were converted into percentages. A summary of these responses is presented in table 4.20.

3.7.2.2 Statistical Techniques for Data Analysis

Students' scripts were also divided into two Groups, A and B. The scripts of Group A (n = 194) and those of Group B (n = 117) consisted of those of students taught by Group 1 and 2 teachers, respectively.

The mean scores of the two Groups of students' scripts were calculated (presented in table 4.19 and figure 11 in chapter 4) and compared, by using z-test (Minium, 1970) to find out whether their difference was statistically significant. This established whether the performance of the students taught by teachers whose teaching style was more towards student-centred, was better than that of the students who were taught by teachers with a teaching style leaning towards a teacher-centred approach.

3.7.2.3 Analysis of Teachers' reasons for given responses and interviews

Teachers had given reasons for their responses to some of the items in the teachers' questionnaire (see appendix 2). These responses were analyzed with a view to finding out their favourite teaching styles and some of the difficulties they were facing in order to be as effective teachers as they wished to be.

3.8 CONCLUSION

The researcher is thankful to all those who assisted in the construction, validation and administration of students' diagnostic test and teachers' questionnaire. It was quite a cumbersome process because a number of sessions, some lasting as long as 2 hours had to be arranged with the experts mentioned in this chapter in order to discuss and re-discuss the items included in these instruments. Also, the co-operation of the department of education, principals, teachers and students in this project is acknowledged with thanks.

The researcher feels that there were some concepts which could not be tested in isolation. Therefore, some of the items tested more than one concept.

A few problems were, nonetheless, experienced during the administration phase of the instruments. For example, there were 12 secondary schools which were target schools, but the researcher could administer the instruments in only 10 schools. In one of the remaining schools the students had called a meeting to discuss some problems at their school, on the day they were supposed to write the diagnostic test. The second school could not be reached because it was raining on the time tabled day and the access road was impassable. Another date could not be arranged because of lack of time.

The researcher faced some problems in arranging a workable time table with some of the subject teachers and school principals. Some of them were, initially, reluctant to give this test to their students because of the timing of the test. The test was planned to be written in the month of September which was only a month before the students were to sit for their final matric examinations. However, through discussions it was made evident to them that the test was a part of a study which was intended to help both the teachers and the students in teaching/learning of Mathematics in general, and that the students would benefit from the test because they would have revised the theory of quadratic equations, inequalities and their graphs in preparation for writing the diagnostic test.

The researcher had also planned to find a Pearson's correlation coefficient between the subjects' performance (students) in the diagnostic test scores and their ranks in the final matric examination scores. However, it was not possible because the Transkei department of education would not release the students' scores for reasons of confidentiality.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 OVERVIEW

This chapter deals with the findings of this study in terms of data collected from both students and teachers, as described in the last chapter. Students' written responses were analyzed manually, script by script, while their scores were analyzed statistically by computer. The Pearson correlation coefficient was calculated to establish the relationship between the pre-requisite knowledge and the level of learning of the theory of quadratic equations, inequalities and their graphs, (i.e. Hypothesis 1). Furthermore, arithmetic means of the top and bottom 27% of the students (n = 85 for each) on previous knowledge and on the rest of the items were compared. Hypotheses 2 to 7 were tested by comparing the sample means with a criterion mean, using the "z" test statistic. The script by script analysis of students' written work, as well as interviews, provided supplementary information about some common errors/misconceptions. Hypothesis 8 was tested by comparing the means of two groups (Group A and Group B) of the sample (students) using the "z" test statistic.

4.2 STUDENT SCORE DATA

4.2.1 STUDENTS' SCORE DISTRIBUTION

Table 4.1 shows the score distribution by school and for the total sample (see next page).

Table: 4.1

Score distribution: by school and gender, and total sample

Key: S: Score;
A-J: Schools;
ST: Sample Total;
T: Grand Total

Entry of Frequency: (Top: Female; Bottom: Male)
(n = 311)

S	A	B	C	D	E	F	G	H	I	J	ST
3	-	-	0 2	-	-	-	-	-	-	-	0 2
4	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	1 0	-	-	1 0	-	-	2 0
6	-	-	-	-	0 4	-	-	1 0	1 0	-	2 4
7	-	-	-	-	1 1	-	-	0 1	-	-	1 2
8	-	-	0 2	-	1 2	-	-	-	-	-	1 4
9	-	-	0 2	-	6 1	-	-	1 1	1 1	-	8 5
10	-	-	-	-	5 1	-	1 0	2 0	2 2	-	10 3
11	-	-	0 3	-	4 0	-	0 1	2 0	-	0 1	6 5
12	-	-	0 3	-	6 5	-	0 1	3 0	1 1	-	10 10
13	-	1 0	0 2	-	12 5	-	0 2	2 1	2 1	-	17 11
14	2 0	-	-	-	11 10	-	2 0	3 2	1 1	1 0	20 13
15	1 1	-	0 1	-	6 10	-	0 1	8 2	1 0	-	16 15
16	1 0	-	0 1	0 2	9 8	2 0	2 1	3 2	-	-	17 14
17	2 0	-	0 2	-	8 5	-	2 1	2 1	1 2	-	15 11
18	1 0	1 0	-	-	9 3	-	0 1	0 2	0 1	1 0	12 7

19	-	0 1	0 2	-	4 2	1 0	-	1 0	-	-	6 5
20	-	-	-	0 1	-	1 0	-	2 2	1 0	-	4 3
21	2 0	-	-	0 1	2 4	-	-	1 0	-	-	5 5
22	0 1	1 1	-	-	0 3	-	-	0 1	-	-	1 6
23	1 2	-	-	-	2 2	1 0	-	-	-	-	4 4
24	1 1	0 1	-	-	1 1	-	-	0 1	-	-	2 4
25	1 1	1 0	-	-	1 0	-	-	1 0	-	-	4 1
26	1 0	1 0	-	-	0 1	-	-	-	-	-	2 1
27	0 1	-	0 1	-	0 1	-	-	-	-	-	0 3
28	0 0	1 1	-	-	-	1 0	-	-	-	-	2 1
29	0 0	-	-	-	-	-	-	0 1	-	-	0 1
30	1 0	-	-	-	-	-	-	-	-	-	1 0
31	1 0	-	-	-	-	-	-	-	-	-	1 0
32	2 0	-	-	-	-	-	-	-	-	-	2 0
TO	24	10	21	4	158	6	15	50	20	3	311

Figure 1 shows the score distribution for the whole sample.

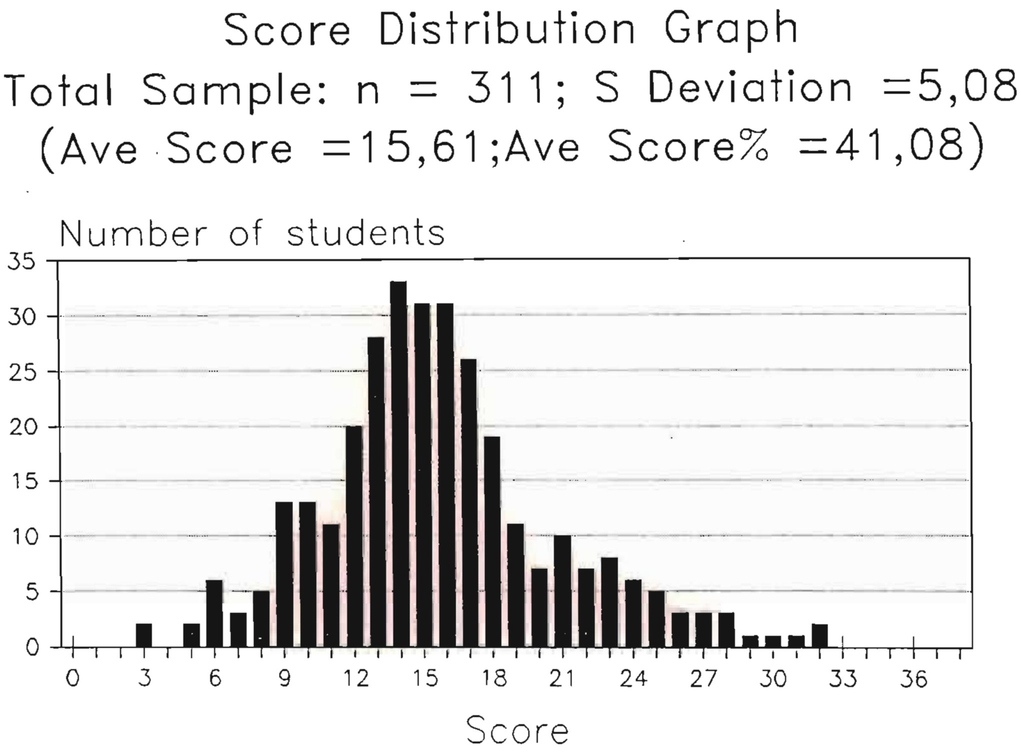


FIGURE 1

Out of a maximum of 38 scores, the highest score for any one student was 32 (84.2%) and only two students (0.643% of the total sample) could get this score. These were two female students. The weakest students were two male students. Each got 3 items correct. The graph is skewed to the right and shows that there were no students who scored more than 32 (84,2%). However, the sample gave a fair distribution of scores over a wide range i.e. from 3 (7,9%) to 32 (84,2%). The mode, average score and standard deviation of the sample were 14 (36,8%), 15,61 (41,1%) and 5,08, respectively. The average score was much lower than the criterion score i.e. 22,8 (60%) which indicated that the sample had not performed well in general.

4.2.2 AVERAGE SCORE (STUDENTS)

Table 4.2 and figure 2 show the comparison of average score and average score % of the individual schools and the total sample.

Table 4.2

Comparison of average score and average score %: by school

School	Average Score	Average Score %
A	22,29	58,66
B	22,5	59,21
C	12,43	33,21
D	18,25	48,03
E	14,9	39,21
F	20,33	53,51
G	14,6	38,42
H	15,2	40,0
I	12,95	34,08
J	14,33	37,72
Total sample	15,61	41,08

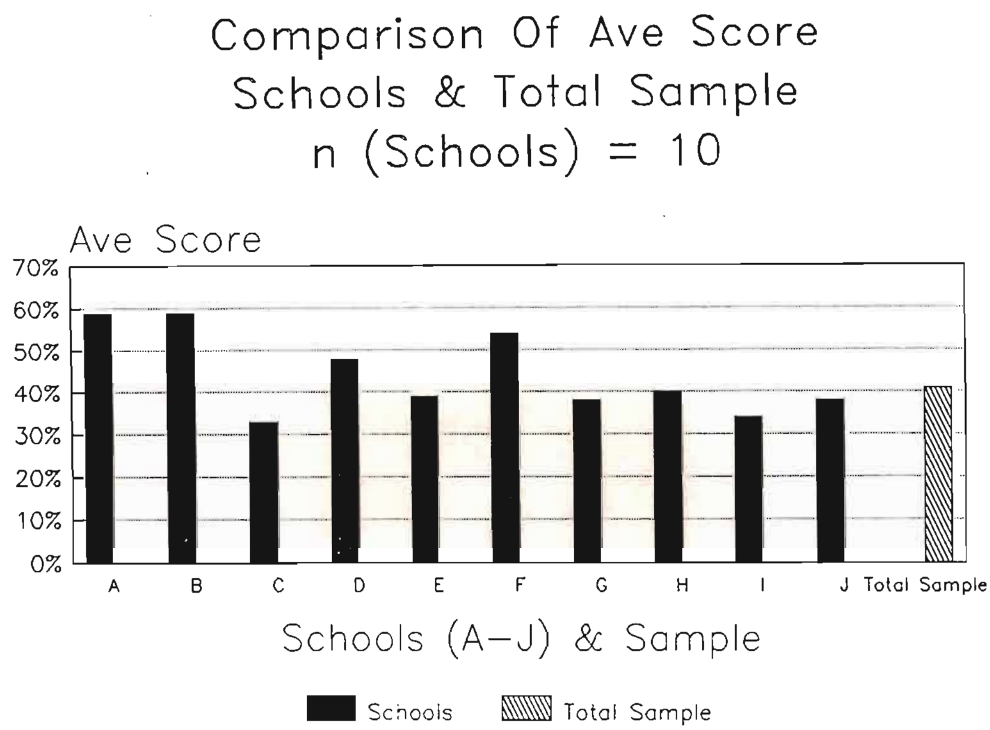


FIGURE 2

The average percentage score varied between 33,21% and 59,21%. The mean score of the sample was 15,61 (41,08%) out of a maximum of 38 scores. The performances of schools A, B, D and F were above the average score for the whole sample, while the rest fell below the overall average. In general, the low average percentage for most of the schools indicates that the understanding of concepts and principles was generally poor.

4.2.3 GENDER-SCORE & GENDER-SCORE PERCENTAGE DISTRIBUTION

Table 4.3 gives the comparison of performances of the sample by gender. Figure 3 gives the comparison of score distribution by gender for the whole sample.

Table 4.3

Average gender-score & average gender-score % distribution

	Arithmetic mean	Standard Deviation	$Z_{calculated}$	$Z_{critical}$
Male	15,6(41,13%)	8,1	-0,18	±1,96
Female	15,46(40.7%)	5,52		

Comparison:Females(n=171;Av Sc 15,46 (40,7%),SD = 5,52) vs Males (n=140;Av Sc 15,6(41,13%);SD=8,1)

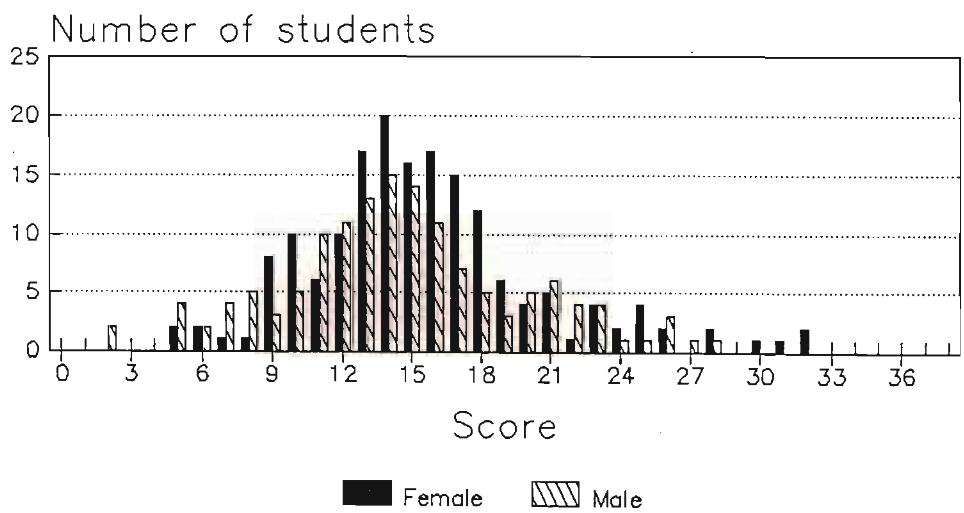


FIGURE 3

The figures suggest that there was no significant difference ("z"$_{calculated}$ (-0,18) > "z"$_{critical}$ (-1.96)) between the performance of male and female students. Some studies (e.g. Backman, 1972; Fennema & Carpenter, 1981) of sex differences in mathematical performance suggest that males' overall performance in Mathematics is better than that of females. On the other hand, other studies (e.g. Fennema & Sherman, 1977, 1978) reveal no difference between males' and females' performances. The observation, in this regard, made in this study was consistent with the latter.

4.3 TESTS OF HYPOTHESES AND DISCUSSION

The criterion mean (X_2) was set at 60% of the possible scores for research question 2 to 7. The "z" test was used to compare the observed means (X_1) against the criterion means (X_2), thereby testing the respective hypotheses.

4.3.1 PRE-REQUISITE KNOWLEDGE AND THEORY OF QUADRATIC EQUATIONS, INEQUALITIES AND THEIR GRAPHS

Hypothesis 1

There is no significant difference in their performance in theory of quadratic equations, inequalities and their graphs between students with poor pre-requisite knowledge and those with a good understanding of pre-requisites.
This was represented statistically as follows:

H_o: $r = 0$;

H_1: $r \neq 0$,

where r refers to the Pearson's Correlation coefficient.
The Pearson correlation coefficient (r) between prerequisite knowledge (items 3, 6, 7, 8, 20, 27 and 29) and scores on the remaining items (1, 2, 5, 9, 10, 11, 12, 14, 15, 17, 18, 19, 21, 22, 23, 25, 28, 30, 31, 32, 33, 34, and 37 to 43)(see table 3.4) of the test was computed yielding the value of 0,45164. This was statistically significant at alpha = 0,01. It can, therefore, be said that, statistically, the level of previous knowledge of students correlates with their performance in theory of quadratic equations, inequalities and their graphs. In general therefore, poor pre-requisite knowledge correlated with poor performance in theory of quadratic equations, inequalities and graphs, and good prerequisite knowledge

correlated with good performance in the remaining part of the test.

Table 4.4

Performance of the top and bottom 27% students' Previous Knowledge

	Arithmetic Mean	Standard Deviation	$Z_{calculated}$	$Z_{critical}$
Top	4,85	0,65	34,9	±1,987
Bottom	1,62	0,56		

Table 4.5

Performance of the above top and bottom 27% students' on remaining test items

	Arithmetic mean	Standard Deviation	$Z_{calculated}$	$Z_{critical}$
Top	15,89	4,24	9,6	±1,987
Bottom	9,85	3,96		

Furthermore, the arithmetic means of the top and bottom 27% of the students (n = 85 for each) on previous knowledge and on the rest of the items were computed. These are given in tables 4.4 and 4.5. The mean (4,85) for the top students on previous knowledge (see table 3.4) was significantly (statistically) different from the bottom students' mean (1,62) on the same items. The hypothesis that the students' poor previous knowledge contributed towards their poor performance in the rest of the test (see table 3.4) was supported by the fact that the same top students' arithmetic mean (15,89) in the remaining items of the test was also significantly (statistically) different from the bottom students' arithmetic mean (9,85) on the same items. In each case, the "z" test showed a statistically significant difference in the performance of the two groups (top and bottom) of students.

This is an important finding in that it reinforces the need for finding out if the students had learned the theory of quadratic equations, inequalities and their graphs which serve as important pre-requisites for the learning of other topics. As Heritage (1980(a):62) observes:

> "...both teacher and learner must ensure that the learner is in possession of all these prior concepts before the process (of teaching/learning) begins."

This, therefore, underlines the importance of the students' possession of the required pre-requisite knowledge and skills.

4.3.2 POSSESSION OF PRE-REQUISITE KNOWLEDGE

There were 7 items in the section of the test that tested the pre-requisite knowledge and skills required for learning the theory of quadratic equations, inequalities and their graphs. The performance of students on each item dealing with prerequisite knowledge was computed and this is shown in table 4.6 and figure 4 (see next page).

Table 4.6

Students' performance in items testing hypothesis -2

A: Item Number;

C: Number of Correct Responses (Percentages).

(n = 311)

A	3	6	7	8	20	27	29
C	93	56	45	21	52	29	30

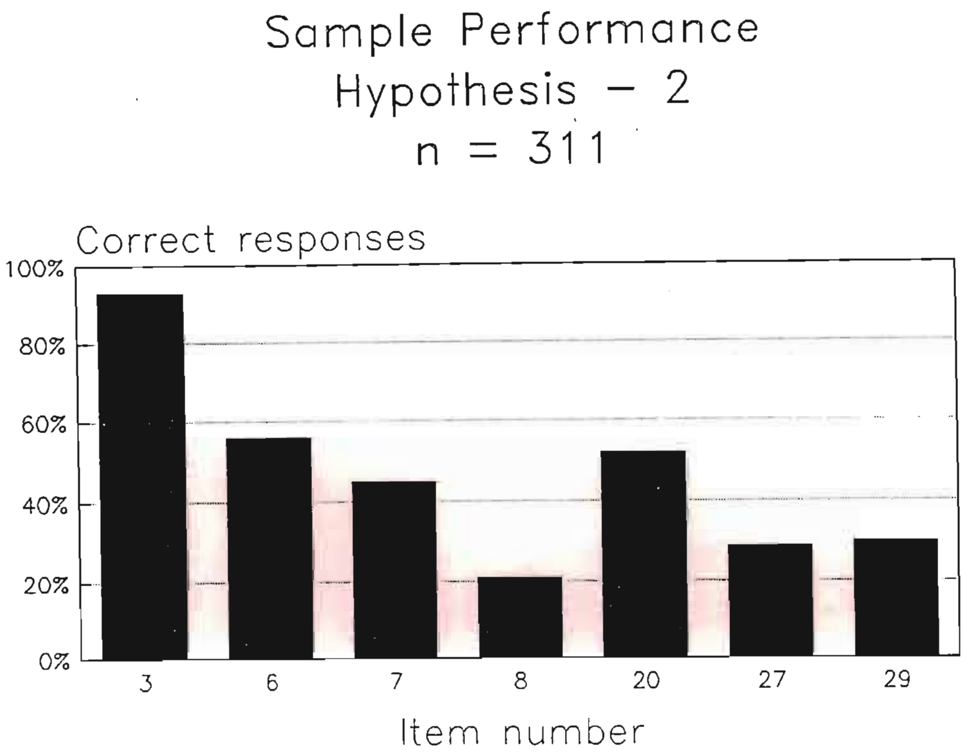


FIGURE 4

Statistical Test Results for Hypothesis 2

Hypothesis 2

Standard 10 Higher Grade Mathematics students of Transkeian high schools possess the required level of pre-requisite knowledge and skills required for learning the theory of quadratic equations, inequalities and their graphs.

This was represented statistically as follows:

$$H_o: X_1 - X_2 = 0,$$

$$H_1: X_1 - X_2 \neq 0;$$

where X_1 refers to the observed mean and X_2 refers to the criterion mean.

The criterion mean was set at 4,2 which was 60% of a total of a possible score of 7 scores (one score for each item). Statistics of items testing hypothesis 2 are presented in table 4.7.

Table 4.7

Statistical results of items testing hypothesis 2

n	ObservedMean	Criterion Mean	Standard Deviation
311	3,27	4,2	1,35

Contrast	z_o	z_c (alpha =0,05)	Observation
$X_1 - X_2 = 0$	-12,23	±1,96	Reject H_o

Since the calculated or observed "z" value ("z"$_o$ = -12,23) is lower than the $Z_{critical}$ (z_c = -1,96), the null hypothesis (H_o) is rejected at alpha = 0,05 in favour of the alternative hypothesis. The negative value of the calculated "z" indicates that the observed arithmetic mean (X_1 = 3,27) was (statistically) significantly lower than the criterion mean (X_2 = 4,2). This means that the students did not have sufficient pre-requisite knowledge needed for learning the theory of quadratic equations, inequalities and their graphs.

The picture that emerges from the performance of students in the above table is worrisome. The average score for this section was 46,6%. The items constituting this section were regarded by the researcher and the experts who validated the instrument to be central as pre-requisites for the understanding of the theory of quadratic equations, inequalities and their graphs, hence, the criterion mean of 60% was used for testing this hypothesis. It is clear from this table that a lot of pre-requisite material had not been properly understood by a significant proportion of the students. In particular, those concepts tested by items 8, 27 and 29 produced very poor results.

The analysis of students' written responses and interviews with some of the students revealed that they had some misunderstandings and misconceptions about some concepts that they had learned in the previous standards. The major areas of concern were as follows:

(a) Multiplication of algebraic expressions: Some students demonstrated lack of ability to multiply two algebraic expressions. Student number 7 (S7) multiplied (7a - 10) and (5a - 43) in item number 3 as follows:

$$(7a - 10)(5a - 43) = 35a^2 - 50a + 430.$$

This student failed to apply the distributive law correctly.

(b) Factorisation of trinomials: Students had difficulty in factorising trinomials. For instance, S3 factorised the expression $35a^2 - 50a + 430$ in item number 3 (see appendix 3) as follows:

$35a^2 - 50a + 430$

$(7a - 10)(5a - 43)$, and verified as follows:

$\therefore \quad 35a^2 - 50a + 430$

The student found wrong factors and also failed to multiply those factors correctly.

(c) Complete square: A number is a complete square if it can be factorised into two equal factors. Students were required to check in item number 6 (see appendix 3) if the expression $4x^2 - 20x + 25$ was a complete square. Some of the students did not know the meaning of 'complete square'. This was evident from some of their responses to item 6. For instance, S2, worked out this problem as follows:

$(2x - 5)(2x - 5) = 4x^2 - 20x + 25$

is not complete.

Another student, S14, in solving the same problem worked it out as follows: **$(2x - 5)(2x - 5)$**

$\therefore 2x/2 = 5/2$ or $2x/2 = 5/2$

In these cases the students factorised the given expression correctly but could not make out that the expression was a complete square since the two factors that they had found were equal. Such students would have problems in understanding the derivation of the quadratic formula. This was made more evident by a student during an interview (extract is given below) who admitted that s/he had confusion between squaring a number and checking it for a complete square.

(transcript of student's interview)

(Researcher: R): **Can you give me an example of a complete square?**

(Student: S): Two... complete square...is it not a number which when you square (it) gives you an exact number?

S: Silence

S: When you square two you get four.

R: When you square any number you get another number.

S: Yes.

R: Does it mean that all numbers are complete squares?

S: Well, I think so, if you square a number you double the number.

...

This student's conception of squaring a number was also faulty. Later on during the interview the student realised that squaring a number did not mean doubling the number, instead, it meant multiplying the number by itself.

(d) Square root of an expression: There were some students who understood the meaning of the term 'complete square' but applied the distributive property over the radical sign as follows:
(extract from student's, S16, script)

$$\sqrt{4x^2 - 20x + 25} = 2x\sqrt{-20x} + 5$$

This student did not know how to find the square root of an algebraic expression. A similar error has been reported by Matz (cited by Olivier, 1992).

(e) Rational and irrational numbers: In the discussion of the nature of roots of a quadratic equation a student cannot appreciate the rational or irrational nature of roots without being able to discriminate between rational and irrational numbers. The concept of rational and irrational was not clear to some of the respondents. The following transcripts of interviews with two students demonstrated the kind of misunderstandings some of them had.

Transcript 1

...

R: Give an example of a rational number.

S: Half.

R: Give an example of an irrational number.

S: Three.

R: Why do you say that 'half' is a rational and 'three' an irrational number?

S: My teacher said that rational numbers are fractions and irrational numbers are whole numbers.

...

This student did not have the concept or complete definition of a rational number, i.e. a number that can be written in the form p/q where p and q refer to integers, and q is not equal to zero. The teacher might have introduced a rational number as any number that can be written in the form of a fraction. The student had failed to realise that even whole numbers could be written as fractions with denominator equal to 1. Therefore, this student did not regard whole numbers as rational numbers and clearly did not understand that a set of whole numbers is a sub set of rationals.

Transcript 2

R: **Give me an example of a rational number.**

S: **A rational number?**

R: **Yes.**

S: **..half.**

R: **What about an example of an irrational number?**

S: **.....Five.**

.....

R: **How do you define a rational number?**

S: **A rational number is a number which is...which has perfect square.**

R: **Which has a perfect square?**

S: **Yah, for example 9, which has its root 3.**

There was no consistency in this student's understanding of the meaning of a rational number. The student's examples of rational and irrational numbers did not tally with his/her definition of a rational number.

It was clear that many students in the sample had difficulties with the rational number concept. Other researchers (e.g. Merlyn, et al, 1983) have also reported that students find great difficulties in learning and applying the rational number concept.

(f) Satisfy, Roots and Solution: When substituting a number for a letter, the number that makes the equation true is said to satisfy the equation. The number that satisfies an equation is called its solution or root. Some of the respondents had misunderstandings about the terms 'root' and 'solution'. The following transcript of one of the respondents' interview demonstrated this misunderstanding.

....

R: **What are you asked in question number 7?**

S: **They are asking if the roots of the equation are the same as solutions of the equation.**

R: **What is your response?**

S: **Roots of an equation are not the same as solution of the equation because the solutions are those numbers which you get on solving an equation. May be one of them is not correct and the roots are the numbers that are correct.**

R: **What do you mean by 'correct'?**

S: **Correct is that value of x which would make both sides of an equation equal.**

Clearly, this student did not know that the solution(s) of an equation is/are the same as the root(s) of the equation. This confusion could have been the result of lack of understanding of reasons for identifying which one(s) is/are the roots of the equation of the type $\sqrt{(2x-3)} = x - 4$. At one stage, in the process of solving this type of equation, it is squared. The resulting equation is a combination of two equations,

$$\sqrt{2x-3} = x - 4 \ldots\ldots\ldots \qquad (a)$$

and

$$\sqrt{2x-3} = -(x - 4) \ldots\ldots\ldots \qquad (b)$$

Therefore, the resulting solutions are the solutions of these two equations ((a) and (b)). These solutions are then tested by substituting in the original equation in order to identify its solution(s). In most cases, only one of these solutions satisfies the original equation, and is thus called the solution of the equation.

Another student explained that the roots of an equation do not satisfy the equation as follows:

> **When the roots are negative they are unreal and irrational and do not satisfy the equation.**

This student seemed to have confusion between 'root(s) of an equation' and 'discriminant' of a quadratic equation. According to the explanation the student had misconceptions of the terms roots, negative root, unreal and irrational.

(g) **Zero (0) not considered as a real root:** Some misconception/difficulty might have been caused by some students' conception of '0'. After solving a given equation and finding that x = 0, they then concluded that the equation had no real roots. This indicated that these students did not have a clear idea about 'root' as a number and '0' also as a member of the real number system. For instance, student S300 solved item 29 (see appendix 3) as follows and concluded that the equation had no real roots:

$(x + 2)^2 = (x - 2)^2$

...

...

$8x = 0$

$x = 0/8 = 0$

No real roots

(h) Square root of a negative number: Some of the students did not appreciate that the square root of a negative number is undefined under the real number system. This had an adverse effect on their performance in solving quadratic equations of the type $x^2 + 16 = 0$ (given in item number 30). One of the students (S15) solved the equation as follows:

$$x^2 + 16 = 0$$
$$x^2 = -16$$
$$x = \pm 4$$

(i) **Expression and an Equation:** The following transcript of an interview demonstrated that some of the students were not clear about the difference between an equation and an expression.

> R: **Can you give me an example of a quadratic equation?**
> S: **$2x^2 + 4x + 2$.**
> R: **Can you give me an example of a quadratic expression?**
> S: **Is there any difference between them?**
> R: **...between...**
> S: **O.K. Quadratic equation is $2x^2 + 4x + 2 = 0$ and expression ...is... $2x^2 + 4x + 2$.**

This kind of misunderstanding may lead the students to write equations and expressions interchangeably in their workings for various algebraic problems. This, in turn, could create difficulties in understanding the logical links between various steps in the process of working out problems.

It is evident from the last response that the student could not discriminate between an expression and an equation. However, the student was able to make out the difference between the two terms on the researcher's probing questions. This clearly illustrates and agrees with some educationists' view (e.g.Njisane, 1992; Garofalo, 1989; Moodley, 1992(b)) that a dialogue, discussion and an appropriate use of questioning techniques in the classroom could enhance students' participation and facilitate learning.

This section dealt with the students' performance in previous knowledge required for the learning of theory of quadratic equations, inequalities and their graphs. Statistical results (see table 4.7) for items in this section showed that the students did not have the required level of mastery of previous knowledge. These statistical results were supported by findings from the qualitative analysis of students' scripts as well as interviews. The qualitative analysis revealed that most students had difficulties with concepts such as rational numbers, perfect square, squaring a number, doubling a number, expression, equation, 'solution' of an equation, irrational numbers, square root of an expression, square root of a negative number and 'root' of an equation. Furthermore, they had difficulties in multiplication of algebraic expressions, factorising trinomials and the difference of two squares as well as checking whether a given algebraic expression could be written as a complete square.

4.3.3 ABILITY TO SOLVE QUADRATIC EQUATIONS

There were 2 items (24 and 26) that tested the students' ability to solve quadratic equations expressed in the standard form. The performance of students on each item dealing with this section was computed and this is shown in the table 4.8 and figure 5 (see next page).

<u>Table 4.8</u>

Students' performance in items testing hypothesis -3

(n = 311)

Item No	24	26
Correct responses (%)	51	39

The mean score of the sample in this section was 0,91

(44,86%).

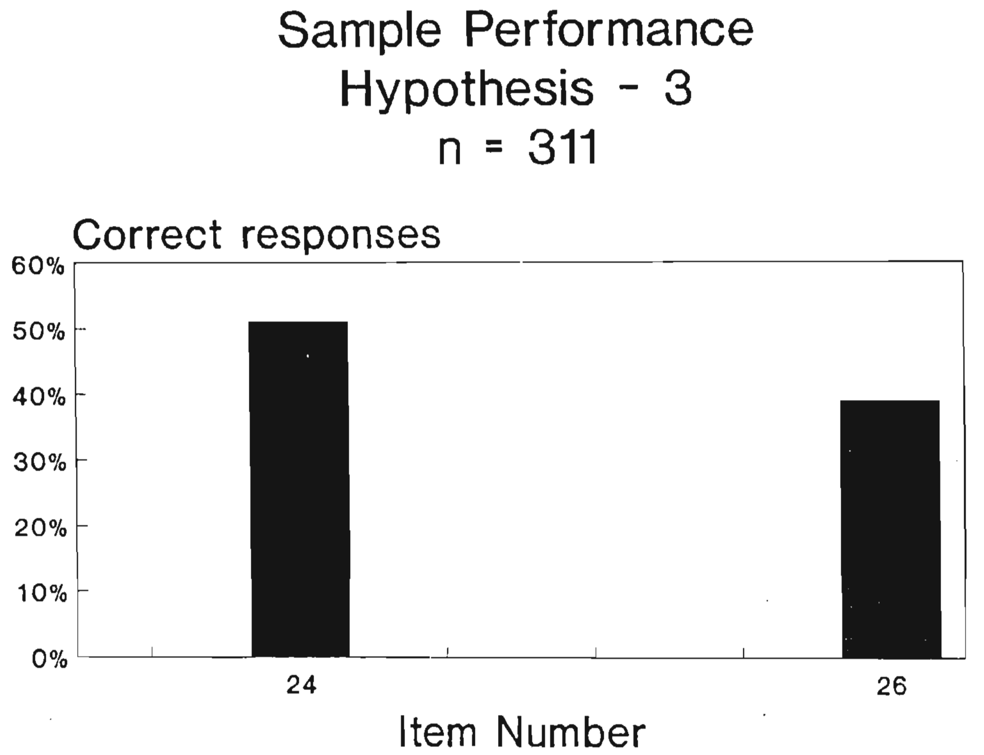


FIGURE 5

Statistical Test Results for Hypothesis 3

Hypothesis 3

Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to solve quadratic equations expressed in the standard form.

This was represented statistically as follows:

$$H_o: \ X_1 - X_2 = 0,$$
$$H_1: \ X_1 - X_2 \neq 0;$$

where X_1 refers to the observed mean and X_2 refers to the criterion mean.

The criterion was set at a mean score of 1,2 which was 60% of a total of 2 scores (one score for each item). Statistics of items testing hypothesis 3 are presented in table 4.9.

Table 4.9

Statistical results of items testing hypothesis 3

n	Observed Mean	Criterion Mean	Standard Deviation
311	0,91	1,2	0,74

Contrast	z_o	z_c (alpha =0,05)	Observation
$X_1 - X_2 = 0$	-6,9	±1,96	Reject H_o

Since the observed "z" value (z_o = -6,9) is lower than the $Z_{critical}$ (z_c = -1,96), the null hypothesis (H_o) is rejected at alpha = 0,05 in favour of the alternative hypothesis. The negative value of the observed "z" indicates that the observed arithmetic mean (X_1 = 0,91) is (statistically) significantly lower than the criterion mean (X_2 = 1,2). This means that the students did not have sufficient ability, in terms of the expected level of performance (i.e. 60%) to solve quadratic equations.

The analysis of students' scripts revealed that the main difficulties that the students appeared to have in this section were as follows:

(a) Over generalisation of 'the multiplication principle' in equations: According to the multiplication principle an equation equivalent to the original can be obtained by multiplying the equation by a non-zero number. Some students extended this principle to solve quadratic equations of the type $ax^2 + bx = 0$. They multiplied the equation by $1/x$, thus reducing the quadratic equation into a simple equation $ax + b = 0$. This led them to find one of the two roots of the equation $ax^2 + bx = 0$ as $-b/a$, thereby losing the other root '0'. S5 solved the equation $12x^2 = 6x$ in item 24 as follows:

$$\mathbf{12x^2 = 6x}$$

$$\frac{12x^2}{6x} = \frac{6x}{6x}$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

This problem could be redressed by sensitising students to special cases where a given principle may not be applied. This goal may be achieved, for instance, if students are asked to divide an equation of the type $ax = bx$ by x where a and b are constants and then discussing the correctness of the results. This type of discussion could lead students to realise the importance of the word 'non-zero' in the above mentioned 'multiplication principle'. This realisation may then be extended to cautioning the students against the division by 'x' in the solutions of quadratic equations of the type $ax^2 + bx = 0$.

(b) Difficulties in understanding the correct usage of the words 'and' and 'or': If the product of any two factors is equal to zero, then at least one of the two factors must equal zero.

If $a b = 0$ then
either $a = 0$, (1)
or $b = 0$, (2)
or both $a = 0$ and $b = 0$. (3)

All these three possibilities are included in the mathematical statement 'a = 0 or b = 0'. Therefore, the two mathematical statements, (i) 'a = 0 or b = 0', and (ii) 'a = 0 and b = 0' are not equivalent. The statement number (ii) is included in statement number (i). Thus the correct (all inclusive) interpretation of the statement (x - a) (x - b) = 0 is 'x = a or x = b' and **not** 'x = a and x = b'. Some students do not seem to know the distinction between the right and the wrong interpretations of ab = 0. For instance, S13 found roots of the equation $12x^2 = 6x$ by factorisation and gave a wrong interpretation of the statement 6x(2x - 1) = 0 as follows:

$$12x^2 = 6x$$
$$12x^2 - 6x = 0$$
$$6x(2x - 1) = 0$$
$$6x = 0 \quad \text{and} \quad (2x - 1) = 0$$
$$x = 0 \quad \text{and} \quad 2x - 1 = 0$$
$$2x = 1$$
$$x = (1/2).$$

This demonstrates the above mentioned misunderstanding which would appear to arise from the difference in the usage of the word 'and' in day-to-day speech and in mathematical sentences. Some students chose those wrong options perhaps because they appeared earlier than the correct option in the option list. Thus, it would appear that those students cared for only the numbers in the solutions and the words 'and' and 'or' did not make a difference to them. S304's response to item 24 (see appendix 3) illustrates the above mentioned view.

$$12x^2 = 6x$$
$$12x^2 - 6x = 0$$
$$2x\ (6x - 3) = 0$$
$$2x = 0 \qquad 6x - 3 = 0$$
$$x = 0 \qquad 6x = 3$$
$$x = 3/6$$
$$x = 1/2$$

This student neither used 'and' nor 'or' in the solution but chose the option which had the answer '0 and 1/2' (perhaps, because this option appeared before the correct option on the question paper).

The learning of mathematical language is a very important part of the teaching and learning of Mathematics. It is therefore necessary that all students learn the usage of the words 'and' and 'or' in the formation of compound sentences. There seems to be a gap in the current curriculum in that the concept of the 'truth value' of a compound sentence is not introduced to students as a formal topic, although it is being used in the teaching and learning of Mathematics at matriculation level. There is need to fill this gap in order to alleviate the difficulties that students face in the learning of the usage of compound sentences, conjunctions and disjunctions.

Possibly, the confusion between the use of **'and'** and **'or'** is compounded in the classroom, where on finding the solution of the equation such as $x^2 - 3x + 2 = 0$, it is often said '1 **and** 2 are the roots of this equation.' The students appear to have interpreted this statement as : The solution of the equation is, $x = 1$ **and** $x = 2$. This kind of mathematical language problem needs to be attended to in the class, as and when noted, and on a day-to-day basis.

(c) Difficulties with regard to the quadratic formula: The equation $ax^2 + bx + c = 0$ where $a \neq 0$, and a, b and c are real number constants, is the general form of quadratic equation. Its roots are given by the formula (called quadratic formula):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula can be used to find a solution set of any quadratic equation. Item number 26 (see appendix 3) was aimed at testing if the students could use the formula correctly. The correct roots of the equation $bx^2 + cx + a=0$ are as follows:

$$x = \frac{- c \pm \sqrt{c^2 - 4ab}}{2b}$$

39% of the respondents answered it correctly. The poor performance in this item indicated that the respondents had difficulties in solving the quadratic equations by using quadratic formula. There appeared to be two main noticeable reasons for the errors made by those students who did not get this item right.

Firstly, 18% of the respondents (i.e.14% who chose option A and 4% who chose option C) were not in a position to associate a, b and c in the formula with the coefficients of the terms x^2, x and the constant term respectively, of a quadratic equation. For instance, S182, item 26 (see appendix 3) wrote:

$$x = \frac{- b \pm \sqrt{b^2 - 4ac}}{2a}$$

but not:

$$x = - b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

The two formulae differ from one another in that in the former formula '-b' is also divided by '2a' whereas in the latter formula '-b' is not divided by '2a'. The correct values of roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the former and not by the latter. It is also possible that these students did not realise that the equation given in item number 26 (see appendix 3) was not the same as $ax^2 + bx + c = 0$ (the general form). Therefore, they gave the quadratic formula as it appears in their textbooks.

This difficulty is also reflected elsewhere in this test, such as with student S273 who worked out the answer to item 32 (see appendix 3) as follows:

$x^2 + px + s = p$
products 1X3 = c/a (probably product of roots) (1)
1X3 = s/1 (2)
...(continued).

From equations (1) and (2) in the above mentioned student's response, it is evident that s/he took s (in the given equation) as the constant term and 1 the coefficient of x^2. Thus this student failed to match this equation with the general form of a quadratic equation and, therefore, could not see (s-p) as the constant term. This has led the student to wrong conclusions. It is important that a student is able to determine whether or not a given equation is quadratic. S/He should also be able to identify each component of the equation. This may be enhanced by asking students to compare the given equation with $ax^2 + bx + c = 0$ and write down the values of a, b and c separately each time before they substitute them in the quadratic formula. This , it is hoped , will help in drawing students' attention to the need and importance of looking at the 'form' of the piece of Mathematics in general that the student is dealing with (Byers and Erlwanger, 1984).

Secondly, 20% of the respondents (i.e. those who chose option D) viz:

$$x = -c \pm \frac{\sqrt{c^2 - 4ab}}{2b}$$

appeared to have made this error because of a wrong/flawed schema of quadratic formula. For them the quadratic formula was

$$x = - b \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

S12 wrote the explanation for choosing

$$x = - c \pm \frac{\sqrt{c^2 - 4ab}}{2b},$$

for the same item as: **'The formula for solving quadratic equation'**.

Thus S12 identified

$$x = - b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

as the formula for solving a quadratic equation.

This error appears to arise from: (a) the ignorance of the difference between the two expressions, that is:

$$- b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

and

$$\frac{- b \pm \sqrt{b^2 - 4ac}}{2a}$$

which, in turn, perhaps arises from the lack of understanding of mathematical symbolism (Austin and Howson, 1979) or (b) the flaw in memory (Olivier, 1992) or in the retrieval of the formula from memory.

The occurrence of this error may be reduced by:

(i) introducing students to the following version of the quadratic formula:

If $ax^2 + bx + c = 0$ then:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a};$$

(ii) asking students to verbalise (Usiskin, 1988) the formula as follows: **'minus b', plus or minus square root of 'b squared minus four times 'a' times 'c' <u>all divided by</u> 'two times a'.** Symbol strings in the quadratic formula may further be interpreted and students' attention drawn to the common denominator '2a'. The following response of S17 to item 26, indicates that such an emphasis had been laid by some teachers and the students were sensitive to the phrase such as '<u>all divided by</u>':

(S17, item 26)

$$x = \frac{-c \pm \sqrt{c^2 - 4ab}}{2b}$$

Because the dividing line goes all the way to the -c on the outside of the square root sign.

This section dealt with the students' ability to solve quadratic equations. Statistical results (see table 4.9) for items (24 and 26) comprising this section ($z_o = -6{,}9$ and $z_c = -1{,}96$) indicate relatively poor performance. Some of the conceptual difficulties which could be the cause of this poor performance were revealed by qualitative analysis. Some areas of concern included:

(a) students' inability to find solutions of quadratic equations with one of the roots equal to zero;

(b) students' failure to make correct usage of 'and' and 'or' in mathematical sentences;

(c) application of quadratic formula to find roots of a quadratic equation in which a constant term and the coefficients of other terms are literals;

(d) students' inability to recall or learn correct quadratic formula;

(e) wrong interpretation of symbol strings used in the formula;

(f) lack of appreciation of the limitations of some generalisations, e.g. ignoring the importance of the word 'non-zero' in 'the principle of multiplication of equations by non-zero numbers'; and

(g) lack of appreciation for 'zero' as a real number.

4.3.4 QUADRATIC EQUATIONS NOT EXPRESSED IN STANDARD FORM

There were 11 items (Items 1, 5, 22, 23, 25, 28, 30, 31, 32, 33 and 34: see appendix 3) in this section. The section tested the students' ability to solve problems dealing with quadratic equations not expressed in the standard form. The performance of students on each item dealing with this research question was computed and this is shown in the table 4.10 and figure 6.

Table 4.10

Students' performance in items testing hypothesis -4

A: Item Number;

C: Number of Correct Responses (Percentages).

A	1	5	22	23	25	28	30	31	32	33	34
C	73	53	93	43	63	29	51	24	51	25	57

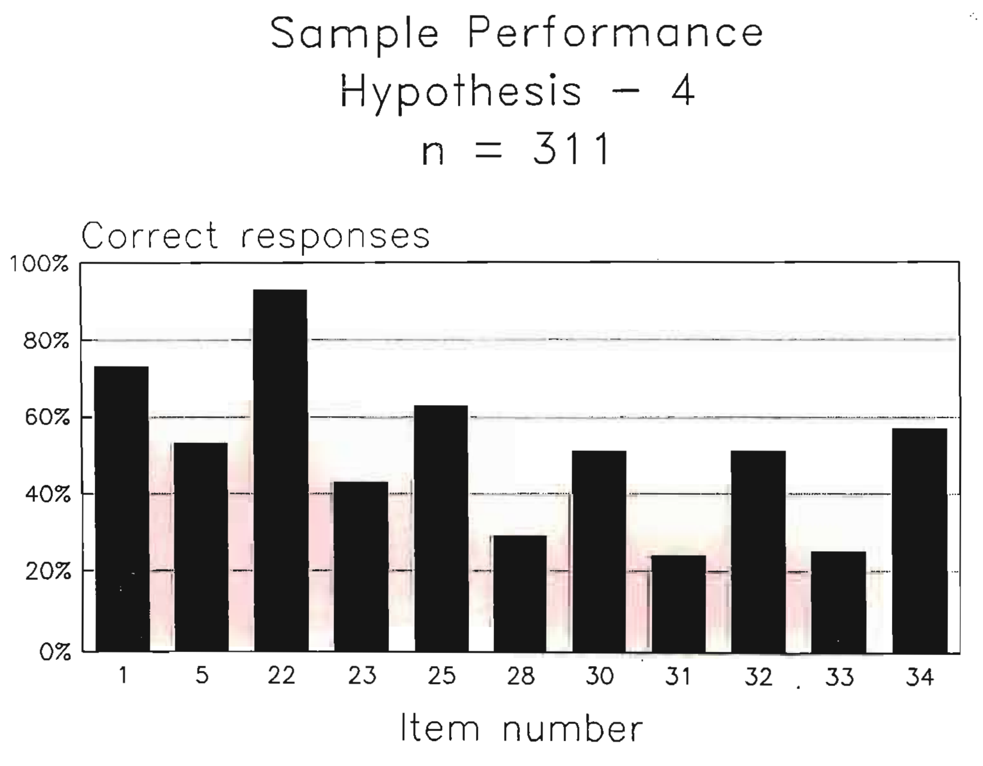


FIGURE 6

The average score of the sample in this section was 49.3%. The students performed very poorly in items 28, 31 and 33.

Statistical Test Results for Hypothesis 4

Hypothesis 4

Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to identify and solve problems dealing with quadratic equations not expressed in standard form.

This was represented statistically as follows:

$$H_o: \quad X_1 - X_2 = 0;$$

$$H_1: \quad X_1 - X_2 \neq 0.$$

X_1 refers to the observed mean and X_2 refers to the criterion mean.

The criterion mean was set at 6,6 which was 60% of a possible total of 11 scores (one score for each item). Statistical results of items testing hypothesis 4 are presented in table 4.11.

Table 4.11

Statistical results of items testing hypothesis 4

n	Observed Mean	Criterion Mean	Standard Deviation
311	5,55	6,6	2,13

Contrast	"z"$_o$	z_c (alpha =0.05)	Observation
$X_1 - X_2 = 0$	-8,66	±1,96	Reject H_o

Since the observed "z" value (z_o = -8,66) is lower than the $z_{critical}$ (z_c = -1,96), the null hypothesis (H_o) is rejected at alpha = 0,05 in favour of the alternative hypothesis. The negative value of the observed "z" indicates that the observed arithmetic mean (X_1 = 5,55) was (statistically) significantly lower than the criterion mean (X_2 = 6,6). This means that the students did not have sufficient ability to solve quadratic equations not expressed in standard form.

An analysis of students' scripts revealed that they seemed to have failed to answer some of these items correctly because of various difficulties which may be summed up in two groups:

(1) difficulties arising from background knowledge; and
(2) difficulties related to the conceptual development of the theory of quadratic equations, inequalities and their graphs.

(1) Difficulties related to the background knowledge

There were so many different errors students made in this section that it was not possible to find out the causes of all of them. Nonetheless, the following observations were made.

Students difficulties fell into three categories, viz: (i) difficulties relating to terminology; (ii) difficulties relating to mathematical symbolism; and (iii) difficulties relating to operations and procedures.

(i) Difficulties relating to the terminology

(a) Real and non-real numbers: The respondents appeared to have misunderstandings about the definitions of real and non-real numbers, as observed earlier. Some of them did not consider 'zero' and 'negative numbers' to be real numbers. For example , in item number 31 (see appendix 3) a student calculated the discriminant (Δ) of the equation ' $t^2 = 16$ ' to be '-64' and concluded that the discriminant (Δ) was imaginary, thereby demonstrating that the student considered negative numbers as non-real numbers. Another possible explanation for this error could be the confusion arising from the connection between the nature of roots of a quadratic equation and the nature (positive, zero or negative) of its discriminant. A quadratic equation has non-real roots if its discriminant is negative. It would appear that the students interpreted the phrase 'if the discriminant is negative then the roots of the equation are imaginary' as 'if the discriminant is negative then it is imaginary.' This demonstrates that rule learning without understanding is dangerous (Eisenberg and Dreyfus, 1988).

(b) 'Square root' and '$\sqrt{\ }$': Some difficulties in the correct usage of the term 'square root' and the symbol '$\sqrt{\ }$' were also observed. The following extract from one of the interviews illustrates this problem:

...

R: What is the square root of 16 ?

S: Four.

R: Find $\sqrt{16}$. (wrote on paper)

S: ±4. (student wrote on the paper)

...

This student had misunderstanding between the two instructions, i.e. (i) find $\sqrt{16}$ (answer is 4) and (ii) find square root of 16 (answer is ±4). The effect of this thinking was displayed in the students' responses to item numbers 25, 28 and 29 (see appendix 3). In these items they were required to find solution sets of equations $(x + 2)^2 = 9$, $(x + 6)^2 = 10^2$ and $(x + 2)^2 = (x - 2)^2$, respectively. Student S67 found the square root of both sides of these equations to arrive at the solution sets as follows:

Item 25:

$(x + 2)^2 = 9$

$(x + 2)^2 = 3^2$

$x + 2 = 3$

$x = 1$ **Answer: Option B, i.e. $x = 1$.**

Item 28:

$(x + 6)^2 = 10^2$

$x + 6 = 10$

$x = 10 - 6$

$x = 4$ **Answer: Option C.**

Item 29:

$(x + 2)^2 = (x - 2)^2$

$x + 2 = x - 2$

$0 = 0$ **Answer: Option A, i.e. No real root.**

In all these items the student lost one of the possible root(s) of each of these equations because of lack of knowledge of the outcome of the process of taking square root of these equations.

(c) Variable, root, unknown

Another problem lay in the students' inability to relate the variable of an equation to its roots. Some of them did not realize that the roots of an equation must satisfy the equation. For example, in item number 33 if '2' is one of the roots of equation ' $-x^2 - 8x - k = 0$' then '2' must satisfy the equation, i.e. $-(2)^2 - 8(2) - k = 0$. 32.8% of the sample wrote that if 2 is one of the roots of the equation $- x^2 - 8x - k = 0$ then $- x^2 -8x - 2 = 0$. These respondents substituted 2, the root of the quadratic equation $-x^2 -8x -k = 0$, for k which is not the variable in the equation.

This error appears to arise from the interchangeable use of the terms **variable** and **unknown** and that **the root of an equation is the value of the unknown in the equation**. In this particular example, the respondents appeared to have considered x as the variable in the given equation and k as the unknown. Therefore, they substituted 2 (the given root) for k. It would appear that this arises from the overgeneralisation of the commonly used statement in the classroom situation, viz: that 'the roots of an equation must satisfy the equation,' to the substitution of the root (value) for any unknown in the equation.

This error was also observed in item 32 (see appendix 3) in which the roots of the equation $x^2 + px + s = p$ were given to be 1 and 3. The students were asked to find the value of s. 51.5% of the respondents chose the correct option (B). However, a good number of these students arrived at this option by working out this problem as follows:

$$\mathbf{x^2 + px + s = p}$$

$$\mathbf{(1)^2 + 3(1) + s = 3}$$

$$\mathbf{\therefore s = 3-3-1}$$

$s = -1$, hence the option B.

This was another example that demonstrated students' misunderstanding of the term 'root' and its usage in relation to 'variable' and other 'unknowns' in an equation. It would appear that these students thought that x, p and s were three unknowns in this equation and that 1 and 3 were the values of the unknowns x and p, respectively. Therefore, they substituted 1 for x and 3 for p in the equation and found the value of s.

This kind of error may be minimised if students are made to realise that a quadratic equation can have more than one unknowns and the unknown which is raised to the power 2 is considered to be the <u>variable</u> in that equation. If there are more than one unknowns raised to the second power, then it must be indicated in the conditions which one is to be considered as the variable (Craig and Winter, 1990). An exercise of the following type may be helpful in bringing this idea home.

<u>Exercise</u>:

Each of the following is a quadratic equation. Name the variable and the other unknown(s).

(a) $m + 2m^2 = k$

(b) $3t^2 - (k + x) = 2x$

(c) $CR^2 + \mu = k$.

In addition to this, the words 'variable' and 'unknown' must be used carefully in the teaching and learning process, i.e. the word 'root' should always refer to the value of the <u>variable</u> in the equation.

The lack of recognition that an unknown in an equation/expression may stand for a number was observed in one of the interviews:

> ...
>
> R: **Is a + 1 a rational or an irrational number?**
>
> S: **Well, I don't think I can call it a rational or irrational number because a number is one term and a + 1 has two terms. I don't know.......**

The student in this case does not seem to appreciate that:

(a) 'a' in the expression represents a number (Wagner, 1981);

(b) if 'a' represents a number then 'a + 1' represents another number (closure);

(c) if 'a' is a rational number then 'a + 1' is also a rational number; and

(d) if 'a' stands for an irrational number then 'a + 1' is also an irrational number.

In summary, it is important to note that some students exhibited the following difficulties:

(a) They had incomplete concept of real numbers. They did not consider 'zero' and 'negative numbers' as part of the real number set;

(b) they did not have understanding of the use of discriminant in the discussion of nature of roots. The meaning of the phrase 'discuss the nature of roots of an equation' was not well understood by some students;

(c) a negative discriminant meant non-real discriminant;

(d) they did not appear to understand that the symbol '$\sqrt{}$' stood for the principal square root of a number, whilst the phrase 'square root of a number' stood for both positive (principal) and negative square roots of the number;

(e) literals used in expressions did not stand for number for some of them; and

(f) any literal used in an equation stood for a variable for some of the students.

(ii) Difficulties relating to mathematical symbolism

Students appeared to have difficulties in using mathematical symbols such as '=', '$\sqrt{\ }$','>' and brackets with understanding. Some students had difficulties in substituting a given negative root of a quadratic equation.

(a) <u>=, < and > signs</u>: The students in this sample also misused symbols such as =, < and >. For example, in item number 31, the students were required to find the discriminant of the equation $t^2 = 16$. One could have found the discriminant as follows:

$$\begin{aligned}\text{Discriminant } (\Delta) &= (0)^2 - 4\,(1)(-16)\\ &= 0 + 64\\ &= 64\end{aligned}$$

Some students started their working by writing '$b^2 - 4ac = 0$' or '$b^2 - 4ac > 0$'. For instance, student S15 found the discriminant of $t^2 = 16$ in item 31 as follows:

$$\begin{array}{ll} \mathbf{t^2 - 16 = 0} & \mathbf{b^2 - 4ac \le 0} \\ \mathbf{b^2 - 4ac = 0} & \mathbf{(16)^2 - 4.1(0) \le 0} \\ \mathbf{(16)^2 - 4\ (1)\ (0)} & \mathbf{16 - 0 \le 0;} \end{array}$$

and concluded that the discriminant was 16. This was a nonroutine type of item in the sense that most matriculation question papers contained questions on discussion of roots of a quadratic equation based on the value of the discriminant whilst in this item, the students were required only to find the discriminant. In addition to the students' inability to identify the values of coefficients of the leading, middle and constant terms correctly, the working indicated that some students misused '=', $\geq$ or $\leq$ signs without realising their mistake. Such students did not realise that they were asked in the problem to find out the value of the expression $b^2 - 4ac$ (the discriminant) whereas they had assigned it some value ($= 0$ or ≤ 0) in their working. This is clearly an example of difficulties that students faced in communicating through symbolic language.

Another possible reason for this kind of error may be lack of students' understanding of the difference between an expression and an equation. Some of them did not seem to understand that the value of an expression depended on the value(s) of the unknown(s) in the expression whilst an equation was a mathematical sentence which would be true only for certain value(s) of the variable(s).

Furthermore, some students might have considered the expression $b^2 - 4ac$ as an incomplete statement (Chalouh and Herscovics, 1988). Therefore, such students wrote every expression as equal to zero and then solved that equation even though such working was not required and not even applicable in the given situation.

Changing of equations into inequalities and vice versa during the process of solving a problem was also very common. For instance, in item number 36 (see appendix 3), a student wrote:

$$2x - \sqrt{2 - x} = 3$$

$$- \sqrt{2 - x} > 3 - 2x$$

This illustrated that either some of the students did not see any difference between equations and inequalities or they did not know the difference between = and > symbols.

(b) <u>Brackets</u>: Some students had problems in the proper use of brackets. For example, in item number 33, 2 was given as one of the two roots of equation $- x^2 -8x - k = 0$ and the students were required to find the value of k and substitute that value in the equation to obtain $-x^2 -8x + 20 = 0$. Some students substituted 2 in $- x^2 -8x - k = 0$ as follows: $(- 2)^2 - 8(2) - k = 0$. These students interpreted $-x^2$ as 'square of negative x'. This meant that they did not see any difference between the expressions $-x^2$ and $(-x)^2$.

The others proceeded correctly and found -20 as the value of k but made an error at the time of substituting this value in the equation. For instance, some of the students worked out this item as follows:

$$- x^2 - 8x - 20 = 0.$$

These students confused the negative sign before k (in the equation) with the negative sign of 20.

All the above mentioned errors appeared to arise from the lack of correct interpretation of the meanings of these symbols in a mathematical sentence. These observations are consistent with Brodie (1991) who contends that students' difficulties in learning Mathematics may be related to their (students') not understanding the appropriate meanings and use of some of the symbols.

Such errors may be minimised if students are given enough opportunity to translate word sums into expressions and/or equations and write stories about given expressions and/or equations. This exercise may prove to be useful if carried out every time when either a new mathematical symbol is used or a new meaning is given to a symbol already known to the students. In this regard Craig and Winter (1990) contend that the learner must attend to the detail of the problem, symbols +, -, X and ÷ in particular. These operational signs must be highlighted. The learner must also be made aware of the significance of brackets in mathematical "puzzles"; that is, brackets act as cues themselves - they delineate aspects or part of the whole, needing to be attended to (Craig and Winter:1990)

In summary, it is important to note that some students were not consistent in using symbols '=', '>' and '<' in the process of solving problems related to 'discriminant'. Many students did not appear to know the judicious use of brackets in evaluating algebraic expressions.

(iii) Difficulties relating to operations and procedures

The students' other difficulties included: (a) overgeneralisation of some principles; and (b) inability to factorise.

(a) <u>Over generalisation of $(x\ y)^m = x^m\ y^m$ to $(x + y)^m = x^m + y^m$:</u> In item 28, the students were required to find the solution set of equation $(x + 6)^2 = 10^2$. 23,8% of the sample solved this item as follows:
If $(x + 6)^2 = 10^2$, then $x^2 + 36 = 100$.
This appeared to arise from an over-generalisation of $(x\ y)^m = x^m\ y^m$ to $(x + y)^m = x^m + y^m$ (Matz, 1980: cited by Olivier, 1992). This, in turn, appeared to arise from the students' lack of understanding of the meaning of a power: namely, that a power is a brief way of writing a product of a repeated factor, e.g.
$2.2.2.2.2.2 = 2^6$,
$x.x.x.x.x.x.x.x.x.x = x^{10}$,
$x.y.x.y.x.y.x.y.x.y.x.y.x.y = x^7.y^7$
$(x + y)(x + y)(x + y)(x + y) = (x + y)^4$.

(b) <u>Difficulties in factorisation</u>: In item number 30, the respondents were required to find out the number of real roots of the equation $x^2 + 16 = 0$. Many students solved the equation by factorising the expression $x^2 + 16$. For instance, student S31 worked out this problem as follows:

$$\mathbf{x^2 + 16 = 0}$$

$$\mathbf{\therefore \quad (x + 4)(x - 4) = 0}$$

$$\mathbf{x = -4 \quad or \quad x = 4}$$

and concluded that the equation had **two real roots**.

Most students act out a paradigmatic sequence which had been imposed rather than reinvented and understood by students. Students seemed to have used the rule of factorising 'the difference of two squares'. According to this rule:
$(A)^2 - (B)^2 = (A + B)\ (A - B)$. They seemed to ignore the importance of the '-' sign, on the left hand side of the rule in the process of recognising the form of the algebraic expression at hand. For them as long as the two terms could be expressed in square terms, the rule of factorisation of 'the difference of two squares ' could be used. However, it is also possible that some students read the expression '$x^2 + 16$' as '$x^2 - 16$' and arrived at the above written sample solution.

This kind of error may be minimised if there is more emphasis on analysis and reasoning, rather than working out problems following procedures in the classroom (Moodley, 1992(b):4).

Fennema et al (1991) contend that teachers have theories and belief systems that influence their perceptions, plans and actions in the classroom. In order to bring about the shift from working out problems by following procedures to more emphasis on analysis and reasoning in the classroom, teachers' beliefs, views and preferences about Mathematics need to be modified from a common perception of Mathematics as content to the view of Mathematics as a process.

In summary, some of the students did not do well because of difficulties that they faced in carrying out some mathematical operations. Their difficulties included:

(a) over-generalization of the rule:
$(xy)^m = x^m.y^m$ to $(x + y)^m = x^m + y^m$; and

(b) factorisation of the difference of squares extended to the factorisation of the sum of two squares.

(2) Difficulties relating to the conceptual development of the theory of quadratic equations:

Generally, students had difficulties answering some of the items (1, 5, 22, 23, 25, 28, 30, 31, 32, 33 and 34) of this section correctly. The following observations were made:

(a) <u>The general form of a quadratic equation</u>: Some of the students had difficulties in identifying a given quadratic equation with its general form. This resulted in their inability to find the correct values of 'a', 'b' and 'c' for substitution in the quadratic formula and discriminant expression in items 30 and 31, respectively. 11% of the respondents made this error in item 27. For instance, S15 found the discriminant of $t^2 = 16$ in item 31 as follows:

$$\mathbf{t^2 - 16 = 0}$$
$$\mathbf{b^2 - 4ac = 0}$$
$$\mathbf{(16)^2 - 4\,(1)(0) = 0.}$$

Byers and Erlwanger (1984) contend that one of the main weaknesses in the teaching of school Mathematics is the failure to recognise the difference between the content of Mathematics and its form. In the present case, it would appear that this difficulty arises from lack of stress on the form of the equation and the characteristic features that make quadratic equations different from simple equations. This could have resulted in students not understanding the order in which the terms of the general quadratic equation are arranged and, therefore, lack of understanding of the meanings of a, b and c in the quadratic formula.

This difficulty may be overcome if the students are advised to write a quadratic equation in the standard form by filling in the missing terms by '0x' and '0' where required. This exercise would each time reinforce the notion that the coefficient of the term 'x' in a quadratic equation can be zero and at times the constant term may also be zero.

Students' attention should, therefore, be drawn to the fact that there are a maximum of three terms in a quadratic equation, i.e. the x^2 term, x term and a constant and that it may be written as follows:

$$Ax^2 + Bx + C = 0. \quad \text{--(1)}$$

In the equation above A, B and C stand for real numbers which may appear as digits or algebraic expressions, e.g.

$$2\ x^2 + 3x + 1 = 0;$$
$$x^2 + \sqrt{2}\ x + \sqrt{3} = 0;$$
$$k\ x^2 + (m + n)\ x + p = 0;$$
$$(k + 1)\ x^2 + (m + n)\ x + (p - q) = 0; \text{ and}$$

that A cannot be equal to zero so that (1) above remains a quadratic equation.

It may be mentioned here that students should also be made aware of what 'x' represents in equation (1) above. They may be introduced to the possibilities of 'x' in equation (1) above standing for an expression, a trigonometric ratio, a power and logarithm of an expression, etc. Given below are some possible examples:

$$A\ (k^2 + 1)^2 + B\ (k^2 + 1) + C = 0;$$
$$A\ (\sin m)^2 + B \sin m + C = 0;$$
$$A\ (\ 5^m\ )^2 + B\ \ 5^m\ + C = 0;$$
$$A\ (\log(x + 1))^2 + B \log(x + 1) + C = 0, \text{ etc.}$$

The above mentioned introduction to the meaning of the general quadratic equation $ax^2 + bx + c = 0$, will help the students recognise a quadratic equation even when the variable is not represented by a single letter. This will prove beneficial when applying the knowledge of solution of quadratic equations to the solutions of exponential, logarithmic and trigonometric equations. This will also help students to appreciate the beauty of recognition of form (Byers and Erlwanger, 1988) in Mathematics.

(b) Solving a quadratic equation by finding square root:
Quadratic equations of the form $(x + a)^2 = c$, can be solved by finding the square root of both sides as follows:

$$(x + a)^2 = c$$

$\therefore (x + a) = \pm \sqrt{c}$ {by finding square roots of both sides of the equation}

Some students committed the error of considering only one (positive square root) of the above mentioned two possibilities. Options B, C and A in items 25, 28 and 29 (see appendix 3) respectively were arrived at by considering only that possibility. 13% (item 25), 28% (item 28) and 42% (item 29) of the respondents chose these options as their answers. Some students made this error consistently. For instance, student S67 responded to these items as follows:

Item No 25.

$$(x + 2)^2 = 9$$
$$(x + 2) = 3$$
$$x = 1$$

Item No 28.

$$(x + 6)^2 = 10^2$$
$$x + 6 = 10$$
$$x = 4$$

Item No 29.

$$(x + 2)^2 = (x - 2)^2$$
$$(x + 2) = (x - 2)$$
$$x - x = 2 - 2$$

No real root.

Thus, s/he found one of the two possible solutions of these equations.

The students do not seem to appreciate that there are two possible square roots of a given positive number. Furthermore, there are some students who do not recognise that the expressions of the type x^2 and $(ax + b)^2$ also represent some positive numbers whose square roots are +x or -x, and (ax + b) or -(ax + b), respectively.

This misunderstanding may be minimised by drawing the students' attention to the fact that a quadratic equation could have a maximum of two real roots. Therefore, students should be urged to always start by looking at the possibility of finding two roots of a given quadratic equation. If a student finds only one real root of a quadratic equation then s/he should be able to explain why that is so (If the discriminant is equal to zero then the two roots are equal).

In addition, students' attention may also be drawn to the fact that when the principle of taking square roots of both sides of an equation is applied we get the following possibilities:

say $(x + a)^2 = c$ (1)

By taking square roots of both sides of the equation (1) we get:

$\pm (x + a) = \pm \sqrt{c}$ which means that:

Either $+ (x + a) = + \sqrt{c}$ (2)

Or $+ (x + a) = - \sqrt{c}$ (3)

Or $- (x + a) = + \sqrt{c}$ (4)

Or $- (x + a) = - \sqrt{c}$ (5)

On examination of (2) and (5); and (3) and (4) we find that equation (2) is equivalent to (5); and equation (3) is equivalent to equation (4). So finally we have:

Either $(x + a) = \sqrt{c}$

Or $(x + a) = - \sqrt{c}$; which is commonly written as:

$(x + a) = \pm \sqrt{c}$.

This explains why we add '±' only on one side of the equation when we take the square root of an equation.

(c) Over generalisation of the zero principle:

According to the zero principle if $(x - a)(x - b) = 0$ then either $(x - a) = 0$ or $(x - b) = 0$. This principle is used to solve those quadratic equations which can be written as, $(x - a)(x - b) = 0$.

34% and 12% of the respondents solved items 23 and 25 respectively, as follows:

Item No 23:

$$(x + 2)(x - 3) = -4$$

Either $(x + 2) = -4$ **Or** $(x - 3) = -4$

- - -

Item No 25:

$$(x + 2)^2 = 9$$

$$(x + 2)(x + 2) = 9$$

Either $(x + 2) = 9$ **Or** $(x + 2) = 9$

- - -

It would appear that these students overgeneralised the zero principle which indicates that they did not understand the importance of '0' in the application of the zero principle to solve the equation $(x - a)(x - b) = 0$. They have extended the zero principle to the statement, 'if $(x - a)(x - b) = c$ then, either $x - a = c$, or $x - b = c$. This was the same observation made by other researchers (e.g. Craig and Winter, 1990; Olivier, 1992).

This error may be minimised by giving students an exercise on solving quadratic equations written in the form: $(x - a)(x - b) = c$ by using the quadratic formula, and then the zero principle, followed by a comparison of the solutions obtained from these two methods. If the solution sets to a given equation thus obtained differ, then the students need to find out the reasons for such a difference and find out which one is the correct one. This will force them to think where they went wrong in the other one. This exercise may create situations of conflict in the students' minds and hopefully lead to the correct understanding of the zero principle (Lochhead and Mestre 1988). In addition, students should be discouraged from ending their equation solving exercise before they check if the roots that they have found do satisfy the equation (Bernard and Cohen, 1988).

In summary, some students had conceptual difficulties that contributed to their failing to answer some of the items in this section. These difficulties included:

(a) failure to relate quadratic equations of the form $ax^2 + b = 0$ with the general form of quadratic equations ($ax^2 + bx + c = 0$);

(b) ignoring the negative square root in the process of solving quadratic equations by finding the square roots of both sides of the equation; and

(c) extension of zero principle to : 'if $(x - a)(x - b)=c$ then, either $x -a = c$, or $x - b = c$.

Statistical results (see table 4.11) for items (1, 5, 22, 23, 25, 28, 30, 31, 32, 33 and 34) comprising this section (z_o = -8,66 and z_c = 1,96) indicate relatively poor performance. Some of the conceptual difficulties which could be the cause of this poor performance were revealed by qualitative analysis.

4.3.5 PROCEDURES FOR SOLVING QUADRATIC INEQUALITIES

There were 4 items (11, 14, 15 and 17: see appendix 3) which tested the students' ability to solve problems dealing with quadratic inequalities. The table 4.12 and figure 7 (see next page) give the performance of students on each item in this section.

Table 4.12

Students' performance in items testing hypothesis -5

A: Item number;

C: Number of Correct Responses (percentage).

(n = 311)

A	11	14	15	17
B	45	52	51	50

The mean score of the sample in this section was 49,2%.

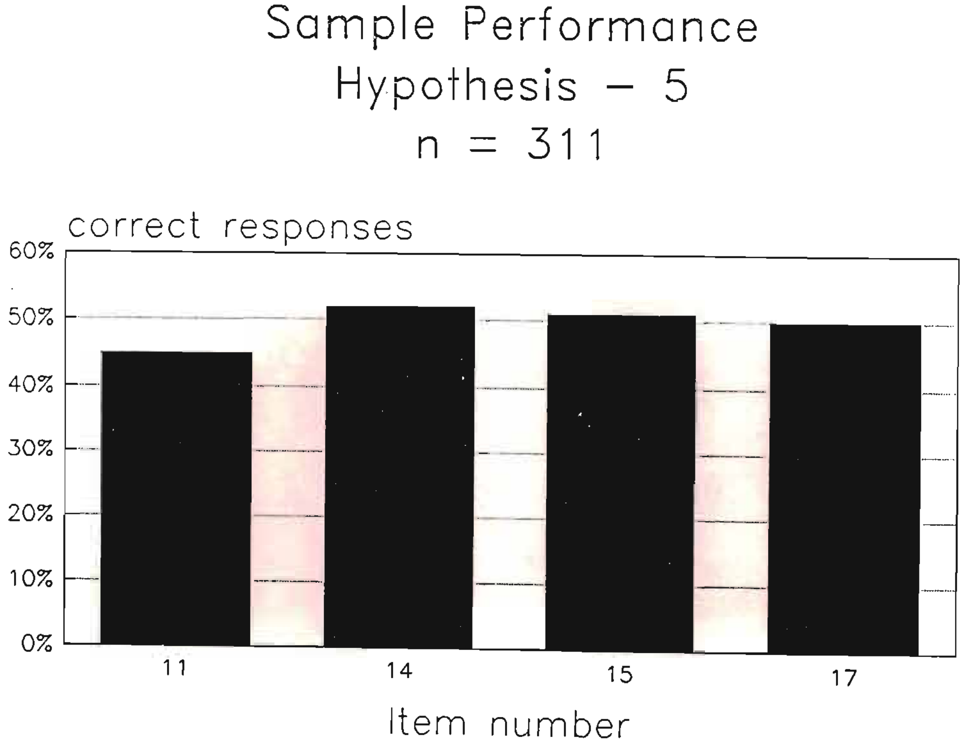


FIGURE 7

Statistical Test Results for Hypothesis 5

Hypothesis 5

Standard 10 Higher Grade Mathematics students of Transkeian high schools exhibit sufficient ability and understanding of the procedures of solving quadratic inequalities.

This hypothesis was represented statistically as follows:

$$H_o: \quad X_1 - X_2 = 0$$

$$H_1: \quad X_1 - X_2 \neq 0,$$

where X_1 refers to the observed mean and X_2 refers to the criterion mean.

The criterion was set at a mean score of 2,4 which was 60% of a total of 4 scores (one score for each item). Statistical results of items testing hypothesis 5 are presented in table 4.13.

Table 4.13

Statistical results of items testing hypothesis 5

n	ObservedMean	Criterion Mean	Standard Deviation
311	1,91	2,4	1,07

Contrast	z_o	z_c (alpha =0,05)	Observation
$X_1 - X_2 = 0$	-8,0	±1,96	Reject H_o

Since the calculated "z" value ($z_o = -8,0$) is lower than the $Z_{critical}$ ($z_c = -1,96$), the null hypothesis (H_o) is rejected at alpha = 0,05 in favour of the alternative hypothesis. The negative value of the calculated "z" indicates that the observed arithmetic mean ($X_1 = 1,91$) was (statistically) significantly lower than the criterion mean ($X_2 = 2,4$). This means that the students did not have sufficient ability and understanding to solve quadratic inequalities as measured by the criterion mark of 60% of the absolute score.

The analysis of students' scripts revealed that the main difficulties that the students appeared to have in this section were as follows:

(a) <u>The principle of product of two numbers positive or negative</u>: In order to solve a quadratic inequality, we write it in the form $(ax + b)(cx + d) < 0$. From the statement '$(ax + b)(cx + d) < 0$' we conclude that :

Either $ax + b > 0$ and $cx + d < 0$;
Or $ax + b < 0$ and $cx + d > 0$.

The students' ability to analyze the statement, '$(ax + b)(cx + d) < 0$' and make the required deductions was tested in item 11. 45% of the sample answered this item correctly. However, 39% of the sample had difficulties in using the words 'and' and 'or' correctly in the construction of compound mathematical sentences arising from the deductions made from the given statement. One of the typical examples was:

S 31.

$$\overset{-}{(2x + 1)}\overset{+}{(x + 2)} < 0$$

$$\begin{array}{ll} 2x + 1 < 0 & \quad 2x + 1 > 0 \\ x + 2 > 0 & \quad x + 2 < 0. \end{array}$$

The student has not used the words 'and' and 'or' in the solution. This is an example of students who think that :

(a) the product of two expressions can be negative only if one of the two expressions is positive and the other negative; and

(b) that the first expression is positive and the second negative, or the first expression is negative and the second positive are the two possibilities, but do not know how to express it in mathematical language.

The following extracts from the students' scripts strengthen this observation:

S296.

If any of them is negative the whole answer turns to be negative.

To obtain a product which is -ve either multiply - X + or + X -.

Incorrect understanding in terms of the recognition of the product of the two factors being negative was common. Some illustrative student responses were:

S42.

Because they can be negative .

S52.

Its depending on the signs.

S53.

All must have the given sign in < it must not change until you get the value of x.

S58.

When we conclude both they are greater than zero.

S50

2x + 1 > 0 or x + 2 > 0 and 2x + 1 < 0 or x + 2 < 0

S291.

This is impossible, it would be true only if there is equal sign but not inequality sign.

These responses indicate students' lack of appreciation that the product of two numbers is negative if one of the two is negative and the other positive. They also have difficulty in the correct use of symbolic language. Teachers can help students by leading them gradually from verbalisation to algebraic symbolism (Schoen, 1988). A class discussion on questions such as: (a) give me two numbers whose product is negative; (b) give me two numbers whose product is less than zero; (c) what could be two numbers whose product is less than zero, etc, may help students to establish in their minds that it is only when one of the two numbers is negative and the other positive, that the product will be negative. This may then be followed by writing it symbolically as follows:

$A.B < 0$ means that:

Either A is negative **and** B is positive, Or A is positive **and** B is negative. This is written as:

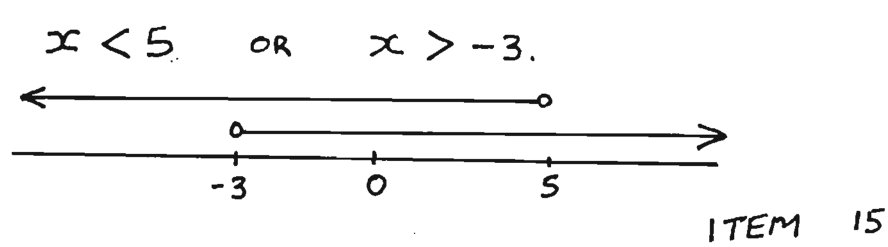
Either $A < 0$ **and** $B > 0$; Or $A > 0$ **and** $B < 0$.

52% and 51% of the respondents gave correct responses in items 14 and 15 respectively. This indicates that the rest of the students had difficulties with the meanings of the mathematical statements used in these two questions. The following two observations were made in this regard. Firstly, some of the students had difficulties in translating the algebraic representation of the statement into graphical representation and the others failed to correctly translate a graphical representation into its equivalent algebraic form (Markovits et al:1988). This also appeared to have occurred due to inconsistency regarding the meaning attached to 'and' and 'or' in terms of intersection and union of the two sets in the statements in items 14 and 15. The following two responses are typical examples of such difficulties.

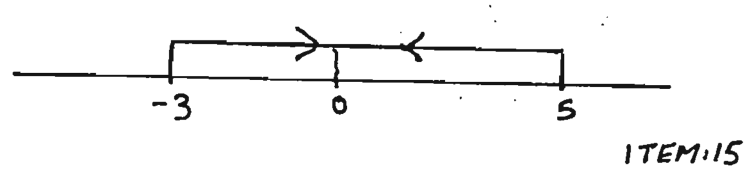
S289.

$$x < 5 \quad \text{or} \quad x > -3$$

$$\therefore -3 < x < 5$$



S293.



Another problem seems to stem from the emphasis on generalisation, 'choose the value smaller than the smallest/ greater than the greatest,' when looking for an equivalent statement to the statements of the type: $x > b$ and $x < c$. It was evident from the responses of some students given below that these generalisations were not applied correctly.

S308.

> **When we have two "greater thans"\ two "less thans" we take greater than the greatest or less than the least. From the above -4 is the least of the two numbers. Hence x < -4 is taken out.**

S287.

> **Because when there are two smaller than, we take smaller than the least.**

It would appear that the students applied this rule 'smaller than the smallest and greater than the greatest' without understanding that it is applicable only when we are looking for intersection of two sets of real numbers and not when we are looking for the union of the two sets. It is, therefore, necessary to draw students attention to the conditions under which a certain generalisation is applicable and to those where it is not applicable. It would be beneficial to the students if they are encouraged to make such generalisations themselves after going through the much needed reasoning and convincing processes (Moodley, 1992(b):4).

Some other responses such as: **'because they can be negative** (S42, item 11).'; **'Its depending on the signs +ve or -ve** (S52, item 11).'; **' All must have the given sign is < it must not change until you get the value of x** (S 53, item 11).'; **'They are both less than zero** (S 54, item, 11).' and **' When we conclude both they are greater than 0 > 0** (S 58, item 11).' etc., indicate that these students had confused ideas about the interpretation of the statement in item number 11.

(b) Treating the inequalities like equations: When an inequality is multiplied by a negative number the inequality changes. This concept was tested in item 17. 50% of the respondents gave correct response. A large number of students multiplied the inequality in their solutions by negative one (-1) but did not change the inequality. A typical example (solution of S221, item 17) of this is given below:

$$(-2x + 3) > x + 2$$

$$-2x + 3 > x + 2$$

$$\frac{-3x}{-3} > \frac{-1}{-3}$$

$$x > \frac{1}{3}$$

The above mentioned example shows that some students have extended the rules applicable to equations to inequalities as well. Other researchers (e.g. Craig and Winter: 1990) have also made similar observations.

In order to minimise students' difficulties in treating quadratic inequalities their attention should be drawn to the following points by giving simple examples.

(a) Multiplication by a negative number changes an inequality;
(b) taking the square root of an inequality is not permissible;
(c) multiplication of an inequality by an unknown number whose sign cannot be determined is not permissible; and
(d) a quadratic inequality could be solved by the factorising method as follows:

(i) If $(x - a)(x - b) > 0$ then solution:
$\{x:x - a > 0 \text{ and } x - b > 0\} \cup \{x:x - a < 0 \text{ and } x - b < 0\}$
(the meaning of 'and' and 'U' in the solution should be clearly understood).

(ii) If $(x - a)(x - b) < 0$ then solution:
$\{x:x - a < 0 \text{ and } x - b > 0\} \cup \{x:x - a > 0 \text{ and } x - b < 0\}$

(e) that the solutions of inequalities can be graphed on a number line.

This section dealt with the students' performance in solutions of quadratic inequalities. Statistical results (see table 4.13) for items (11, 14, 15 and 17) in this section (z_o =-8,0 and z_c=-1.96) show that the students did not have the required level of mastery in solving quadratic equations. These statistical results were supported by the findings from the qualitative analysis of students' scripts and interviews. The qualitative analysis revealed that some students had difficulties which included:

(a) lack of understanding of the principle, 'if $ab < 0$ then, either ($a > 0$ and $b < 0$) or ($a < 0$ and $b > 0$)';

(b) lack of ability to express the following expressions in mathematical language:
   (i) meaningful use of the words 'and' and 'or' in mathematical conjunctions and disjunctions;
   (ii) equivalence of 'and' and '$\cap$' in a conjunction;
   iii) equivalence of 'or' and 'U' in a disjunction; and

(c) extending some generalisations to domains where they do not apply, for example, applying the principle, 'that equations do not change when multiplied by non-zero numbers', to inequalities.

4.3.6 INTERPRETATION OF GRAPHS OF QUADRATIC FUNCTIONS

The test had 12 items (2, 9, 10, 18, 19, 21 and 37 to 42: see appendix 3) on the students' ability to interpret graphs of the equation $y = ax^2 + bx + c$ and inequalities y

$<$, $\leq$, $>$ or $\geq$ $ax^2 + bx + c$ in terms of translation of algebraic representations into graphical representations and vice versa. The table 4.14 and figure 8 give the performance of students on each item dealing with this section. The performance in items 2, 9, 37, 38, 39, 40 and 42 was very poor.

<u>Table 4.14</u>

Students' performance in items testing hypothesis 6

A: Item number;

C: Number of Correct Responses (Percentage).

(n = 311)

A	2	9	10	18	19	21	37	38	39	40	41	42
C	34	15	50	39	38	37	17	26	12	10	44	19

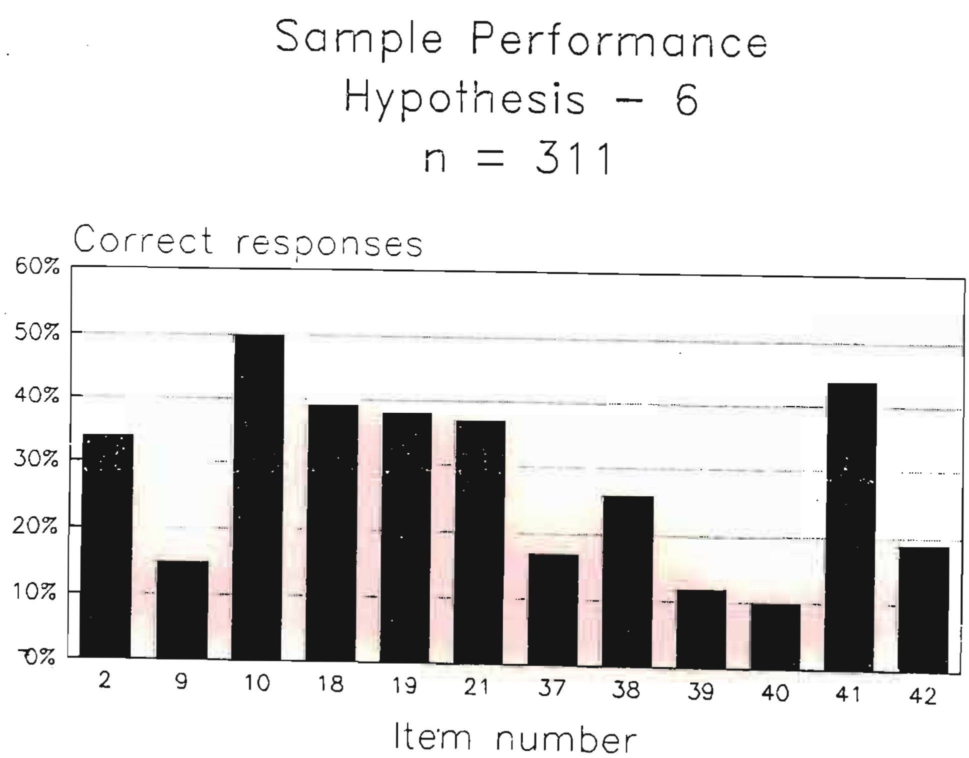


FIGURE 8

The average score of the sample in this section was 28.3%. Performance in this section was the second lowest as compared to that in the other sections of this test.

Statistical Test Results for Hypothesis 6

Hypothesis 6

Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to interpret graphs of quadratic functions.

This hypothesis was represented statistically as follows:

$$H_o: \quad X_1 - X_2 = 0$$

$$H_1: \quad X_1 - X_2 \neq 0;$$

where X_1 refers to the observed mean and X_2 refers to the criterion mean.

The criterion mean was set at 7,2 which was 60% of a total of 12 marks (one mark per item). Statistical results of items testing hypothesis 6 are presented in table 4.15.

Table 4.15

Statistical results of items testing hypothesis 6

n	ObservedMean	Criterion Mean	Standard Deviation
311	3,43	7,2	2,14

Contrast	z_o	z_c (alpha =0.05)	Observation
$X_1 - X_2 = 0$	-31,03	±1,96	Reject H_o

Since the calculated "z" value (z_o = -31,03) is lower than the $Z_{critical}$ (z_c = -1,96), the null hypothesis (H_o) is rejected at alpha = 0,05 in favour of the alternative hypothesis. The negative value of the calculated "z" indicates that the observed arithmetic mean (X_1 = 3,43) is (statistically) significantly lower than the criterion mean (X_2 = 7,2). This means that the students did not have sufficient ability to translate, relate and interpret algebraic and graphical representations of a given mathematical situation.

The graph of equation $y = ax^2 + bx + c$ has the following salient characteristics:

(i) the shape of the graph is called a parabola;

(ii) the graph opens upwards if 'a' is positive and downwards if 'a' is negative, and the width of the graph depends on the magnitude of 'a';

(iii) it is symmetrical about a line parallel to the y-axis and the position of the line is determined by the values of 'a' and 'b' in the equation;

(iv) the point where the graph and the line of symmetry intersect is called the turning point of the parabola;

(v) if the parabola opens upwards then the expression $ax^2 + bx + c$ has a minimum value, otherwise the expression has a maximum value. This minimum/maximum value is represented by the y-coordinate of the turning point of the graph;

(vi) the graph cuts the y-axis at one point whose co-ordinates are (0;c) where 'c' is the constant term in the given equation;

(vii) the graph may cut x-axis at 0, 1 or 2 points;

(viii) the x-intercepts of the graph are the real solutions of the equation $ax^2 + bx + c = 0$, which means that the equation has no, one or two real roots if the graph intersects x-axis at 0, 1 or 2 points, respectively;

(ix) the y-coordinate of any point on the curve of the graph is equal to the value of the expression $ax^2 + bx + c$ where 'x' is the value of the x-coordinate of the point;

(x) the y-coordinate of any point above the curve is greater than the value of the expression $ax^2 + bx + c$, where 'x' is the value of the x co-ordinate of the point;

(xi) the y co-ordinate of any point below the curve is less than the value of the expression $ax^2 + bx + c$, where 'x' is the value of the x co-ordinate of the point; and

(xii) the equation of a given sketch of a parabola must hold for the co-ordinates of the turning point and at least one more point on the graph, and/or for three points on the graph.

Analysis of the students' scripts revealed a number of difficulties that they experienced in answering the items in this section. The most common difficulties are discussed below:

(a) Equation $y = ax^2 + bx + c$ and the shape of its graph: Some students were totally unaware of the graphical representation of the equation $y = ax^2 + bx + c$. For instance, S202 on item 2 wrote:

The above graph is the parabola and the equation is for the hyperbola.

S133 on item 18 found the equation of the sketch of a parabola as follows:

$$y - y_1 = m(x - x_1)$$
$$y - 4 = m(x - 5)$$
$$\ldots$$
$$y = -2x + 10.$$

This also indicated that the student could not even classify or recognise the equation $y = -2x + 10$ as a linear equation whose graph should be a straight line, and not a parabola. Another student, S 182, in dealing with item 18 had similar difficulty which was quite evident from his/her answer:

Because the equation of the parabola is not a straight line ie $y = x^2 - 4x + 8$. Equation of parabola is $y = ax^2 + c$.

In the process of learning graphical representation of y =

$ax^2 + bx + c$, the first and foremost characteristic of the graph that a student should conceptualise is its shape. Some students did not have this knowledge.

(b) Lack of global analysis of the behaviour of the function: Some students failed to answer item 2 correctly because of their apparent inability to analyze the behaviour of the function $y = ax^2 + bx + c$ globally. Some students looked at the direction in which the given graph of a parabola opened and the nature of 'a', positive or negative, and concluded that the graph represented the given equation $y = ax^2 + bx + c$. For instance, S294 on item 2 wrote:

> **Because the coefficient of x^2 is positive $\therefore$ the graph should be a minimum graph not a max one.**

Some other similar reasons were given by other students. For instance, S161 on item 2 wrote:

> **$ax^2 + bx + c$ gives a parabola and when a is positive the arrow point upwards.**

This student also made a wrong conclusion that, **" 'a' in the given equation is positive"** (the given condition was, $a \neq 0$). Most probably, s/he made this observation from the (upward) direction of the parabola in the graph. Another student (S284, item 2) made a conclusion based only on the shape (parabola) of the graph and the form of the given equation. This student wrote:

> **$f(x) = ax^2 + bx + c$ is an equation for the graph of parabola. The graph is a graph of parabola.**

A number of students could not conceptualise that a parabola may or may not cut the x-axis but it must cut the y-axis. This was clear from S280's response on item 9 (see appendix 3):

> **This means that it cut x-axis twice, if it crosses y-axis.**

Another student, S279 (item 9) wrote that:
(an equation $ax^2 + bx + c = 0$ would not have two distinct real roots because) **the graph can cut the y-axis above the origin.**

This implied that if the graph of $y = ax^2 + bx + c$ cut the y-axis above the origin then the equation $ax^2 + bx + c = 0$ would not have '**two distinct real roots.**'

(c) <u>Y-intercept of the graph of a parabola and the equation $y = ax^2 + bx + c$</u>: There were instances of evidence that some students had not grasped the relationship between the y-intercept of the graph of a parabola and the constant term 'c' in its equation. The following transcript of an interview bears testimony to this:

Student- T

...

S: **I think they are asking me to find if this rough sketch is drawn in the correct way.**

R: **What do you mean by 'correct way'?**

S: **For example in $f(x) = ax^2 + bx + c$ as 'c' is positive here but they may put it as negative.**

R: **What about 'c' here? positive?..negative?**

S: **c is positive.**

R: **What is the value of 'c' in the given graph?**

S: **.....May be $c = 1$.**

R: **What tells you that c should be equal to 1?**

S: **Since it has been stated that $c \neq 0$.**

The student interpreted '$c \neq 0$' as 'c' to be positive. "c may be 1" indicates that the student had no knowledge of the relationship between the y-intercept, in the graph, and the value of 'c', in its algebraic representation.

Some students thought that the graph of $y = ax^2 + bx + c$ should cut the axis (**x-axis or y-axis**) at two points if $ax^2 + bx + c = 0$ had to have two distinct real roots. This was illustrated by student S287's responses to items 9 and 10. In item 9 the students were required to find out if the intersection of the graph of $y = ax^2 + bx + c$ with y-axis meant that the equation $ax^2 + bx + c = 0$ had two distinct real roots. S287's response was:

> **false, because if it has two real roots it suppose to intersects the y-axis at two points.**

In item 10 the students were required to find out if the intersection of the graph of $y = ax^2 + bx + c$ with x-axis at two points meant that the equation $ax^2 + bx + c = 0$ had two distinct real roots. The same student responded as follows:

> **Since it intersects x-axis at two points it suppose to have two distinct real roots.**

This student displayed not only an inability to find a graphical solution of the equation $ax^2 + bx + c = 0$ but also lack of appreciation for the fact that the graph would never cut y-axis at more than one point.

(d) Graphical solution of $ax^2 + bx + c = 0$ and the graph of $y = ax^2 + bx + c$: Item 10 tested the students ability to relate the number of real roots of the equation $ax^2 + bx + c = 0$ with the number of points of intersection of the graph of $y = ax^2 + bx + c$ with the axes. 50% of the respondents got this item correct. This was again not a satisfactory result. In item 41 the students were required to identify from the graph the distances that represented the solution set $\{(x, y): y = 0\} \cap \{(x, g(x)): g(x) = -x^2 + 8x + 5\}$. In other words they were required to identify those sections of the graph of $g(x) = -x^2 + 8x + 5$ which represented the solution set of the equation $-x^2 + 8x + 5 = 0$.

44% of the respondents got this item correct. This indicated that the other students had difficulties with the graphical solutions of the quadratic equations. For instance, S182 wrote on item 9 as follows:

> **Let the equation f(x) = $ax^2 + bx + c$ be $x^2 - 6x + 9$ and let f(x) = 0;** $\therefore x^2 - 6x + 9 = 0$
>
> $(x - 3)(x - 3) = 0$
>
> $x = 3$ or $x = 3$
>
> $\therefore$ **f(x) = $ax^2 + bx + c$ has no real roots.**

The student had difficulties with: (a) the graphical solutions of the equations; and (b) the description of the nature of roots of the equation $ax^2 + bx + c = 0$ from the position of the graph of $y = ax^2 + bx + c$ in terms of its intersection with x-axis. Another student, S298, item 9 wrote:

> **$b^2 - 4ac$ can be = 0 then there will be one point only if $b^2 - 4ac > 0$ that there are two real roots.**

The student probably thought that to discuss the nature of roots of a quadratic equation one must look at the value of the discriminant ($b^2 - 4ac$), not at the graph. That is, that the equation would have two real roots if the discriminant was a non-zero positive number, and one real root if the discriminant had a value of zero. Possibly, such students were not exposed to the graphical solutions of problems related to the expression $ax^2 + bx + c$.

(e) <u>A set of necessary conditions for an equation to be an algebraic representation of a given graph of a parabola</u>: There are an infinite number of parabolas that may have a common turning point. Therefore, an equation (of the type: $y = ax^2 + bx + c$) which may hold for a given point (turning point of graph of a parabola) may not necessarily be the algebraic representation of the graph. On the other hand, there is one, and only one, parabola (graph) that may have a given turning point (known coordinates) and pass through another given point. Therefore, an equation (of the type:

$y = ax^2 + bx + c$) which holds for both of these points (on the graph of a parabola) is necessarily the algebraic representation of such a graph.

Item 18 tested the students' ability to verify if a given graph of a parabola is a graphical representation of a given equation by checking if the given equation held for both the given points (coordinates are known) on the graph. 39% of the students got this item correct, suggesting that the majority (61%) of the respondents had difficulties in checking if a given equation was an algebraic representation of a given graph.

An algebraic equation, $y = ax^2 + bx + c$ and a graph of a parabola can only be called mathematically equivalent if:

(a) any three ordered pairs belonging to the relation $y = ax^2 + bx + c$ are also found on the graph of the parabola, and/or

(b) the turning point of the graph and at least another point on the graph also belong to the relation $y = ax^2 + bx + c$.

There were some students who displayed total inability to make this translation. Their reasoning was far from the acceptable techniques of checking the validity of statement of this item. For instance, student S36 wrote:

> **because the parabola has y-intercept & x intercept and also the T.P.**

Other students showed difficulties in identifying the relationship between the coordinates of points on the graph and the equation of that graph. For instance, S63, item 18 (see appendix 3), did not proceed after writing the coordinates of **C(4;5) and A(2;4)**.

Another student (S67) found (for item 18) the coordinates of the turning point of the parabola defined by $y = x^2 -4x$

+ 8 to be equal to those of the turning point of the given sketch and proceeded to conclude (wrongly) that the equation was the algebraic representation of the given sketch. The student did not appreciate the possibility of having an infinite number of parabolic sketches with a common turning point.

(f) Wrong use of the condition: if x_1 and x_2 are the x-intercepts of a parabola then its defining equation may be written as, $y = a(x - x_1)(x - x_2)$: Students had difficulties in identifying the correct links and schemas to answer item 18. Some made out that they could answer this item by finding and then comparing the equation of the given sketch with the given equation. However, they made mistakes in finding the equation. For instance, student S184, item 18 wrote:

$$\begin{aligned} y &= (x - x_1)(x - x_2) \\ 4 &= (x - 2)(x - 4) \\ &= x^2 - 2x - 4x + 8 \\ 4 &= x^2 - 6x + 8 \end{aligned}$$

Amongst other misconceptions, this student apparently also did not know that $y = a(x - x_1)(x - x_2)$ is the equation of a parabola whose x-intercepts are x_1 and x_2.

(g) Graphical solution of a system of two equations: In the graph of item 19, OC and OD were x-coordinates of points of the intersection set of $y = -x + 3$ and $y = 1/4\ x^2 + x + 5$. Therefore, the distances OC and OD represented the x-coordinates of the solutions of $-x + 3 = 1/4\ x^2 + x + 5$ which reduces to $x^2 + 8x + 8 = 0$. Students were asked to check if OC and OD represented the roots of the equation $x^2 + 8x + 8 = 0$. Only 38% of the students got this item correct. A great number of students could not make out what they were asked in this item, while others showed their inability to interpret these graphs correctly. Some associated the distances OC and OD with the equation y = -x

+ 3 only. The following examples illustrate this view: S152, item 19 wrote:

> **They show the roots of equation of $y = mx + c$. They show straight line graph;** whilst S182 wrote: **Because the equation of distances OC and OD is $y = -x + 3$.**

Other students associated those distances with the equation $y= 1/4\ x^2 + x + 5$ only. For instance, S301 wrote:

> **OC and OD represent the x-intercepts of $y = 1/4\ x^2 + x + 5$ which are the zeros of this equation that is they are the solution of x.**

This clearly shows that these students did not associate these distances with:

(a) the points of intersection of these graphs;
(b) the equations of the graphs; and therefore,
(c) could not conclude that the y values of the two graphs at the points of intersection were equal.

Other students could not associate the equation $x^2 + 8x + 8 = 0$ with the graphs. They gave different reasons in support of their answers. Some of those reasons are given here - all relate to item 19. S214 wrote:

> **These points are the distances and the $x^2 + 8x + 8$ is the equation for a parabola;**

S185 wrote:

> **They lie on the x-axis;**

and S306 wrote:

> **The roots of the equation are found where $y = 0$.**

The responses indicated that the respondents had not learned the graphical solutions of a system of equations.

(h) <u>Finding the turning point of a parabola when its equation is given in the form: $y = a(x - p)^2 + q$</u>: The point (p,q) is the turning point of a parabola whose defining equation is $y = a(x - p)^2 + q$. The students were required to check if $(-p;q)$ was the turning point of the parabola defined by $y = a(x - p)^2 + q$ in item 21. 37% of the respondents got this item correct while the majority of the respondents had difficulties in finding the co-ordinates of the turning point of a parabola from its equation written in the form: $y = a(x - p)^2 + q$.

It would appear that these students faced difficulties in conceptualising that: (a) at the turning point the expression, $a(x - p)^2 + q$ (the y co-ordinate of the turning point) had a minimum (if $a>0$) or maximum (if $a<0$) value; and (b) the minimum or the maximum value of the expression, $a(x - p)^2 + q$ was possible at that point where x was such that $a(x - p)^2$ acquired a zero value. This was possible only if $x = p$, since $a \neq 0$ in the given equation.

(i) <u>Identifying graphical representations of solution sets of quadratic inequalities</u>: Items 37 to 41 tested students' ability to identify, from a given figure, graphical representations of the solution sets of quadratic inequalities in one and two variables, as well as intersection sets of two functions given in set builder notation. Specifically, item 42 tested students' ability to recognise the graph of a quadratic function having an undefined minimum value.

The respondents performed very poorly in these items and very few gave reasons for their answers. However, some observations made from available responses to these items included the following:
The students were required to interpret the graph to find the graphical representation of the solution set of $-x^2 + 8x + 5 \geq 0$ in item 37. The set of x co-ordinates of all those points on the curve of g(x) whose y co-ordinates were positive or zero made the solution set of the inequality. There was no correct answer in the response options given in this item. 17% of the respondents chose the option 'c'

meaning that they did not know the difference between an open '()' and closed '[]' intervals. Some of the responses given by those who chose other options included the following: (a) **" because ≥ 0 means that it is positive therefore it lies on positives."** (S122); (b) **"where the y-intersect in the feasible region."** (S130); (c) **"the y-intercept is positive."** (S 151); (d) **"it is because it must be ≥ 0"** (S290); (e) **"and the arrows point upwards which means it is greater or equal to"** (S292). These responses indicated that the respondents probably had some vague idea that the sign '≥' in an inequality meant something above x-axis in the graph but had no idea what that something was. Others related the sign '≥' to the upward direction of arrows marked in the figure. Most probably, the respondents could not identify that (a) the solution set of $-x^2 + 8x + 5 \geq 0$ consisted of all those values of x for which the expression $-x^2 + 8x + 5$ had positive or zero values, and (b) the elements of the solution set were the x co-ordinates of all those points on the curve whose y co-ordinates were positive or zero.

These difficulties may be minimised if the teaching of algebraic solutions of inequalities is also related to their graphical solutions. In this particular case, the value of the y co-ordinate of any point on the curve is equal to the value of the expression $ax^2 + bx + c$. This means that the students should be encouraged to deduce that if at any point on the curve the y co-ordinate is positive then it follows that the value of the expression $ax^2 + bx + c$ is also positive. The students should also be made to realise that when solving for $ax^2 + bx + c \geq 0$ they are supposed to be looking for all those values of x for which $ax^2 + bx + c$ is positive or zero.

In general, the respondents seemed to have difficulties in reading the graphs for identifying the solution sets of:

(a) inequalities of the type: $-x^2 + 8x + 5 \geq 0$;

(b) intersection sets of two inequalities in two variables;

(c) intersection sets of two quadratic functions; and

(d) intersection sets of linear and quadratic functions.

Furthermore, the respondents had difficulties in identifying graphs of parabolic functions whose minimum values were undefined.

These difficulties appeared to arise from the respondents' lack of appreciation for the meanings attached to the graphical representation of a general point (x, y) in relation to the component of the graph of a function that represents (in the graph) its value for that value of 'x'. In other words, they did not seem to appreciate that:

(a) the y co-ordinate of any point on the graph of $y = ax^2 + bx + c$ represented the value of the expression $ax^2 + bx + c$; and

(b) the y co-ordinates of points not on the curve could not be equal to the value of the expression '$ax^2 + bx + c$'.

Items 38 and 40 (see appendix 3) tested the students' ability to identify the areas in the graph that represented the solution sets of a system of two inequalities in two variables. 26% and 10% of the respondents, respectively, got items 38 and 40 correct. These results indicated that the majority of the respondents had not grasped the concept of graphical representation of inequalities in two variables and the intersection thereof.

Item 39 tested students' ability to recognise that two functions are equal at the point(s) of intersection of their graphs. Only 12% of the respondents got this item correct. This indicated that students had difficulties in establishing the relevant equation at the point of intersection of two graphs.

In item 42, students were required to interpret the graph to identify the function which had an undefined minimum value. This is one of the items commonly used in examinations. Only 19% of the respondents got this item correct. 11% of the respondents thought that both functions f(x) and g(x) had undefined minimum values whereas 15% thought that f(x) had an undefined minimum value.

These misunderstandings may be redressed if the algebraic treatment of the equations and inequalities is done alongside the treatment of graphs of quadratic functions.

This section dealt with the students' performance in their ability to translate and interpret graphical representation of a quadratic function into its equivalent algebraic representation, and vice versa.

Statistical results (see table 4.15) for items (2, 9, 10, 18, 19, 21, 37, 38, 39, 40, 41 and 42) in this section ($Z_o = -31.0$ and $Z_c = 1.96$) show that the students did not have the required level of mastery of translation process. These statistical results were supported by findings from the qualitative analysis of students' scripts as well as interviews. The qualitative analysis revealed that most students had difficulties that included:

(a) inability to correlate graph of a parabola to the equation $y = ax^2 + bx + c$, and vice versa;

(b) failure to determine the value of the quadratic expression $ax^2 + bx + c$ from its graph;

(c) lack of appreciation of the characteristics of the graph of the equation $y = ax^2 + bx + c$ indicating whether the equation $ax^2 + bx + c = 0$ has two distinct real roots, two equal real roots or no real roots;

(d) lack of understanding about the meaning of the point(s) of intersection of graphs of two functions;

(e) lack of realization that a point can be the turning point of infinite parabolas; and

(f) lack of understanding of the relationship between the maximum/minimum value of the expression $a(x - p)^2 + q$ and coordinates of the vertex (turning point) of a graph of a function defined by the equation $y = a(x - p)^2 + q$.

It was necessary that the students exhibited a relational understanding (Skemp, 1976) in order to answer items in this section correctly. They had to translate, relate and interpret algebraic and graphical representations of mathematical situations. One can relate some of the problems experienced by the students to their lack of appreciation that the domain and range of a function are represented on the x and y-axes respectively in its graphical representation, as reported by Markovits, et al (1988). These problems were further compounded by the students' lack of appreciation for that from the algebraic representational point of view whilst the x values constituted the domain, the corresponding values of $ax^2 + bx + c$ constituted the range of the function defined by the equation $y = ax^2 + bx + c$.

4.3.7 APPLICATION OF KNOWLEDGE OF QUADRATIC EQUATIONS, INEQUALITIES AND THEIR GRAPHS

There were 2 items (12 and 43: see appendix 3) on the students' ability to apply the knowledge of the theory of quadratic equations, inequalities and their graphs to solve word problems. The table 4.16 and figure 9 give the performance of students on each of the two items in this section.

Table 4.16

Students' performance in items testing hypothesis -7

(n = 311)

Item No	12	43
Correct responses (%)	32	21

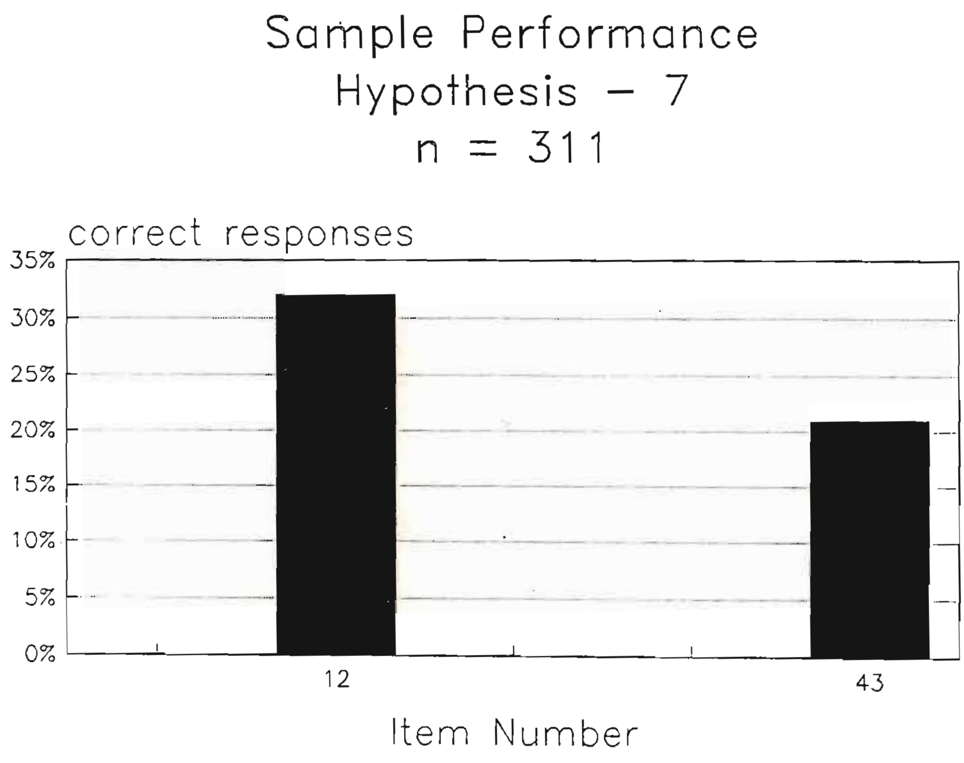


FIGURE 9

The average score of the sample in this section was 26,5% which was the lowest as compared to that in the other sections of the test. It was evident that a majority (73%) of the subjects had problems in answering items in this section.

Statistical Test Results for Hypothesis 7

Hypothesis 7

Standard 10 Higher Grade Mathematics students of Transkeian high schools are able to apply the knowledge of quadratic equations, inequalities and their graphs in unfamiliar situations.

This was represented statistically as follows:

$$H_o: \quad X_1 - X_2 = 0$$
$$H_1: \quad X_1 - X_2 \neq 0;$$

where X_1 refers to the observed mean and X_2 to the criterion mean.

The criterion mean was set at 1,2 which was 60% of a total of 2 marks (one mark per item).

Table 4.17

Statistical results of items testing hypothesis 7

n	Observed**M**ean	Criterion Mean	Standard Deviation
311	0,53	1,2	0,66

Contrast	z_o	z_c (alpha =0.05)	Observation
$X_1 - X_2 = 0$	-17,99	±1,96	Reject H_o

Statistical results of items testing hypothesis 7 are presented in table 4.17. Since the calculated "z" value (z_o = -17,99) is lower than the $z_{critical}$ (z_c = -1,96), the null hypothesis (H_o) is rejected at alpha = 0.05 in favour of the alternative hypothesis. The negative value of the calculated "z" indicates that the observed arithmetic mean (X_1 = 0,53) was (statistically) significantly lower than the criterion mean (X_2 = 1,2). This means that the students did not have sufficient ability to apply their knowledge of theory of quadratic equations, inequalities and their graphs in solving unfamiliar problems. Their difficulties included:

(a) Difficulties with the order of operations: Item 12 read as follows: 'The sum of a number and square of half the number is 48. If the number is x then: $1/2\ x^2 + x = 48$.' Students were required to check if the second part (algebraic form) of the statement was the correct translation of its first part (verbal form). Accordingly, there were two operations to be carried out (one after the other) on a number x. These operations were: (a) halve; and (b) square. There were two possible orders in which these operations could be carried out. These were:

(a) halve x then square (the result);or

(b) square x then halve (the result).

These two orders of operations when carried out on a number would yield two different results (except when the number is zero).

"Halve x, then square (the result) is written, algebraically as $(1/2\ x)^2$ which is arrived at as follows:

Number = x

Halve the number = $1/2\ x$ (First operation)

Then square (the result) = $(1/2\ x)^2$ (Second operation)

'Square, then halve (the result)' is written, algebraically as $1/2\ x^2$ and is arrived at as follows:

Number = x

Square the number = x^2 (First operation)

Then, halve (the result) = $1/2\ (x^2)$ (Second operation).

Item 12 required respondents to 'halve x and then square (the result)'. The following observations were made from the students scripts:

There was evidence that some students did not understand what was required of them. They solved the equation $1/2\ x^2 + x = 48$ for x.

There were others who tried to check the validity of the statement by substitution. Student S5 came up with 12 as the value of x and checked if it satisfied the equation $1/2\ x^2 + x = 48$ as follows:

$$1/2\ x^2 + x = 48$$
$$(6)^2 + 12 = 48$$
$$36 + 12 = 48$$

From the substitution it was clear that the student understood (wrongly) $1/2\ x^2$ as 'half x then square (the result)'. The student appeared to have difficulties in translating symbolic representation of a mathematical situation, especially the order of operations correctly into verbal form and vice versa.

(b) <u>Difficulties with the mathematical meaning of the words in the given statement</u>: Misconception that 'squaring a number means doubling the number' misled student S152. The student checked if $1/2x^2 + x = 48$ was valid for '$x = 24$' as follows:

$$\mathbf{1/2\ (24)^2 + 24 = 48}$$
$$\mathbf{1/2\ .\ 48 + 24 = 48}$$
$$\mathbf{24 + 24 = 48}$$
$$\mathbf{48 = 48}$$

The student carried out the required operations in the correct order but thought $(24)^2$ was the same as 24X2.

In order to apply the knowledge of the theory of quadratic equations, inequalities and their graphs in solving real world problems, it is essential that students are able to model a given situation into algebraic expression(s) and/or equation(s) correctly. It is only then that a student can apply this knowledge to arrive at correct results.

Item 43 read as follows: 'Find two possible numbers whose sum is 18 and whose product is the largest possible.' Only 21% of the sample got this item correct, all of whom but 4 found out their answer (9 and 9) by inspection as follows:

Numbers	**Product**
1 X 17	**17**
2 X 16	**32**
.....	
9 X 9	**81**

There was no special instruction given to the students to follow an algebraic way. Therefore, one would not conclude that these students did not know the algebraic solution to the problem. The four students mentioned above chose 'x' as one of the two numbers and therefore, '18 - x' the other. They found '$18x - x^2$' the product of 'x' and '18 - x'; rewrote '$18x - x^2$' as $-(x - 9)^2 + 81$ and concluded that the maximum value of the expression is 81 at $x = 9$. Therefore, the two numbers are 9 and 9, and the product 81.

A large proportion (79%) of the sample did not seem to know what to do in order to answer this item. The majority did not even attempt to answer this item.

This section dealt with the students' ability to apply the knowledge of quadratic equations, inequalities and their graphs in unfamiliar situations. Statistical results (see table 4.17) for items (12 and 43) in this section (z_o = -17,99, and z_c =- 1,96) show that the students did not have the required level of mastery to apply the knowledge of theory of quadratic equations and their graphs to solve related problems. These statistical results were supported by the findings from the qualitative analysis of students' scripts as well as interviews. The qualitative analysis revealed that most students had difficulties with:

(a) translating word problems into their equivalent mathematical models;

(b) mathematical meanings of the words used in the word problems; and

(c) relating the word problem to the relevant section of the theory of quadratic equations.

Other researchers (e.g.Lochhead and Mestre, 1988) have stressed the extent of seriousness of the difficulties that students face in solving word problems. Lochhead and Mestre (1988) suggest that the students should be provided with ample practice with the translation process itself in order for them to develop strategies to carry out these translations more efficiently and correctly.

4.3.8 TEACHERS' TEACHING STYLES AND STUDENTS' PERFORMANCE

10 respondents participated in answering the teachers' questionnaire (see appendix 2). Their scores ranged between 74 (lowest) and 115 (highest) out of a maximum of 135. The distribution of the teachers' score and their students' average percentage score is given in table 4.18 and figure 10. The central score of 96.5 was taken as the median score. Accordingly, the teachers were divided into two groups on the basis of this median score. Group 1 consisted of teachers E, F, G, I and J, and Group 2 consisted of teachers A, B, C, D, and H. Students of Group 1 teachers were grouped as Group A and those of Group 2 teachers as Group B.

Table 4.18

Teachers' scores and their students' average score %

Teacher	Teachers' Score	Students' Average Score(%)
A	103	22,29 (58,66)
B	97	22,5 (59,21)
C	103	12,43 (33,21)
D	115	18,25 (48,03)
* E	92	14,9 (39,21)
* F	96	20.33 (53,51)
* G	93	14,6 (38,42)
H	114	15,2 (40,0)
* I	90	12,95 (34,08)
* J	74	14,33 (37,72)
Total sample		15,61 (41,08)

(* indicate Group A students and Group 1 teachers)

Mean scores of Group B students of individual schools (Group 2 teachers' students) were generally higher than those of Group A students. However, there were some exceptions to this rule. The mean score of school C's (a Group 2 teacher) students was lower than those of Group A (schools marked *) students. The mean score of school F's (a Group 1 teacher) students was higher than those of Group B (individual schools) students. The mean scores of the two groups of students were calculated and they were 14,84 and 17,06, respectively for Group A and Group B. Figure 11 (see next page) represents the comparison of performance of Groups A and B students. A lower score suggested that the teachers' teaching style was inclined towards teacher-centredness (Group A students) and a higher score meant that the teaching style was leaning towards student-centredness (Group B students).

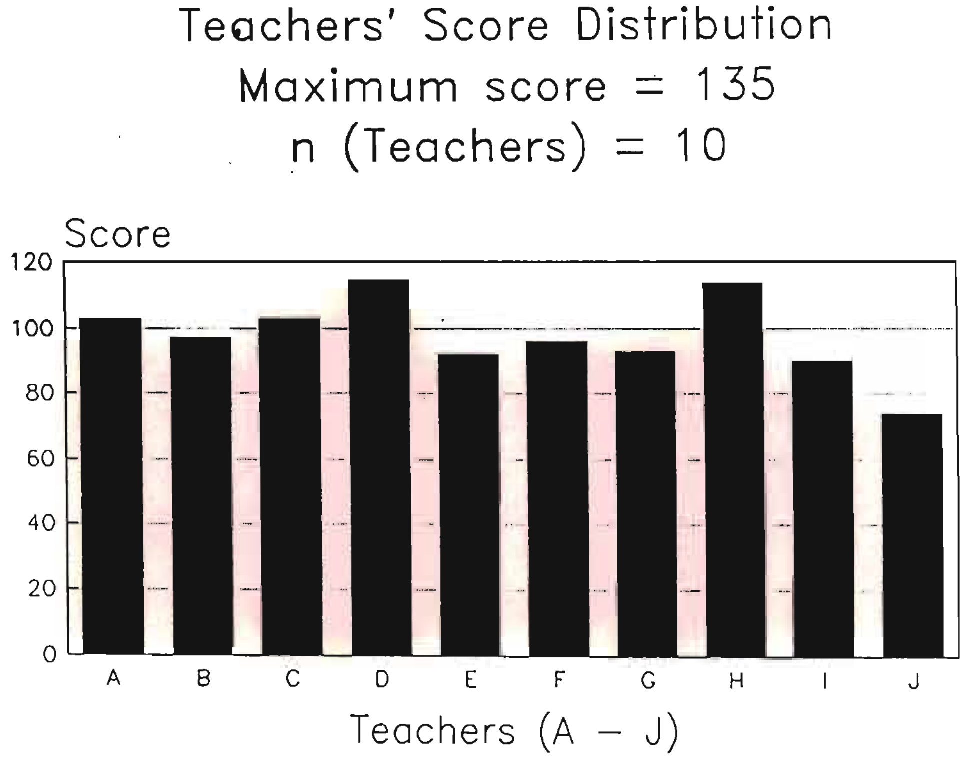


FIGURE 10

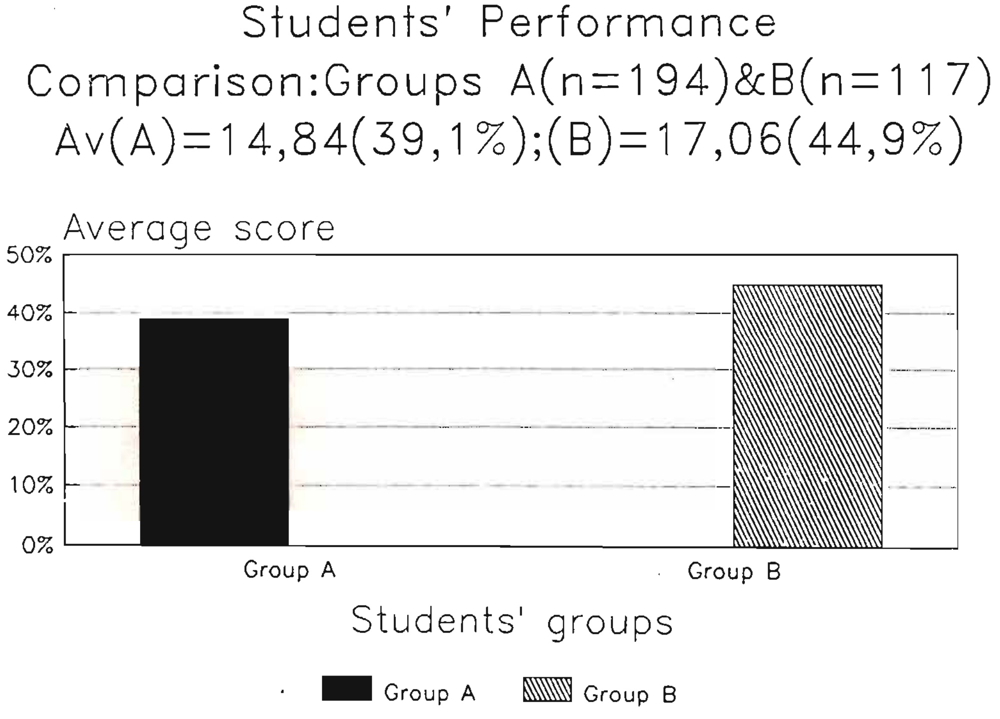


FIGURE 11

4.3.8.1 Testing Hypothesis - 8

Hypothesis 8 (Q 8).

There is no difference between the performance of (Group B) students taught by (Group 2) teachers whose teaching style was more student-centred and those (Group A) students taught by (Group 1) teachers whose teaching style was less student-centred.

This was represented statistically as follows:

$$H_o: \; X_1 - X2 = 0$$

$$H_1: \; X_1 - X_2 \neq 0;$$

where X_1 and X_2 refer to the arithmetic means of Group A and Group B students, respectively.

Table 4.19 gives the comparison of Groups A and B students' performance.

<u>Table 4.19</u>

Comparison of groups A and B students' performance

	Group 'A'	Group 'B'
Number Students (n)	194	117
Arithmetic Mean	14,84 (X_1)	17,06 (X_2)
Standard Deviation	1,01	6,19

Contrast	Z_{cal}	Zcrit (alpha=0.05)	Observation
$X_1 - X_2 = 0$	-3,71	±1,96	Reject H_o

Since the calculated "z" value (z_{cal} = -3,71) is lower than the $Z_{critical}$ (Z_{crit} = -1,96), the null hypothesis (H_o) is rejected at alpha = 0,05 in favour of the alternative hypothesis. The negative value of the calculated "z" indicates that the observed arithmetic mean (X_1 = 14,84) of Group A students was (statistically) significantly lower than the arithmetic mean (X_2 = 17,06) of Group B students. This means that the students taught by Group 2 teachers performed better than those taught by Group 1 teachers. This indicates that the students taught by teachers whose teaching styles were more student-centred performed better than those taught by teachers whose teaching styles were more teacher-centred. This is an important finding in that it reinforces the need for students' active involvement in the learning of Mathematics. Such a need is supported by other educationists (Brodie 1991; Garofalo, 1989; Moodley, 1992(a); Schifter and Simon, 1992; Shuard, 1986).

4.3.8.2 Analysis Of Teachers' Questionnaire And Interviews

This section deals with the analysis of the teachers' questionnaire (see appendix 2) and interviews. Teachers' response statistics by item are given in the table 4.20.

Table 4.20

Teachers' questionnaire response statistics (by item)

(see appendix 2)

A = Item number; C = Disagree (%);

B = Agree (%); D = Neutral (%).

(n = 10)

A	1*	2	3*	4	5	6	7
B	90	40	60	20	30	0	60
C	10	50	20	80	60	100	10
D	0	10	20	0	10	0	30

A	8	9	10	11	12	13*	14
B	40	20	50	50	50	70	70
C	50	80	20	40	50	0	30
D	10	0	30	10	0	30	0

A	15	16	17*	18	19	20	21
B	40	0	40	70	10	20	0
C	60	90	20	30	80	50	90
D	0	10	40	0	10	30	10

A	22	23*	24	25	26	27
B	10	90	0	20	30	10
C	90	10	100	80	60	90
D	0	0	0	0	10	0

Teachers' responses were evaluated on a continuum of least favourable (1: strongly agree) to the most favourable (5: strongly disagree) statements, according to the contemporary educational theories favouring student-centred and activity-based teaching approaches as against teacher-centred and lecture method teaching approaches. Teachers' responses to items 1, 3, 13, 17 and 23 (marked *) were reversely coded for scoring purposes.

Their views/beliefs related to the teaching/learning process are investigated and presented under various sub headings. Teachers' open ended responses to various items were also analyzed and their views about discovery method of teaching, group work, classroom discussions, problem solving sessions in classroom, questioning in Mathematics classes, attending to individual students' difficulties, testing students at the end of each topic, home work practices, place of explanations in classroom, importance of reasoning in learning Mathematics, and importance of attending to students' pre-requisite knowledge are presented below.

Discovery method of teaching: The majority (Item 1: 90%) of the teachers believed that Mathematics can be taught by discovery method and that:

(a) "it motivates the students";

(b) "concepts can be made understood by discovery method";

(c) "it is based on reality which is extrapolated to abstract and generalisations"; and

(d) "it is the most effective method."

However, the discovery method of teaching was not followed by most (50%) of the teachers because of various reasons which included:

(a) "there were no teaching facilities";

(b) "encouraging students to discover mathematical concepts will be a very slow procedure";

(c) "allocated time is short";

(d) "the students are not used to this approach of teaching from JSS (junior secondary schools), therefore, it will be time consuming to introduce this approach";

(e) "(this approach) requires that the students are good at the language used in class, which is not the case";

(f) "it (this approach) would be difficult at higher levels."

The others (20%) felt that the discovery method could be employed only for teaching:

(a) "certain topics"; and

(b) "responsible and above average students".

This indicates that discovery method of teaching Mathematics while appreciated by the teachers, was largely not implemented for the reasons mentioned above. One would, therefore, conclude that most teachers follow, **teaching as explaining** (Elliot, 1968) as their teaching style. The consequence of this would be that proper conceptual understanding of the material may not be facilitated.

However, one teacher contended that:

> "Every lesson is an opportunity for discovery. The size of the syllabus does not affect this (the choice of this method) too much...Isn't that what we always do - guide the students to discover mathematical facts."

Group work, discussions and problem-solving: 50% (item 2: see appendix 2) of the teachers in the sample were in favour of group work. These teachers believed that :

(a) "group work helps to share the ideas and it creates confidence (in students)";

(b) "working in groups helps below-average students, provided they all take part in the discussion";

(c) "there is a difference in levels of communication between teacher and pupils. Therefore, in groups weak pupils can benefit from group mates because their levels of communication are close"; and

(d) "group work allows them (students) to discuss problems among themselves."

Some of the teachers' responses indicated that they considered discussion as an important activity in a Mathematics class. These included the following:

(a) "Discussion is part and parcel of any teaching, especially for mathematics teaching. So it is recommended in the scheduled time...";

(b) "effective teaching is not done without discussions"; and

(c) "studying of Mathematics involves discussions in the class if the students should understand the topics."

However, it did not appear to be a regular activity in their classrooms. They used discussions occasionally, and for revision purposes mainly, and were discouraged from practising discussion methods for various reasons, which include:

(a) "periods are short";
(b) "...because this (discussion) creates noise and disturbs other classes"; and
(c) "...besides the syllabus has to be finished."

Those teachers who did not favour group work (item 2: 40%) and discussions (item 8: 40%) believed that:

(a) "dull students tend not to participate";
(b) "periods (time for Mathematics lessons) are short"; and (c) "because of messing up the work".

Others had mixed views about group work and class discussions. This was evident from their responses such as:

(a) "occasionally we have some discussions but not often because this creates noise and disturbs other classes. Besides, the syllabus has to be finished"; and
(b) "each student has his\her learning pace. It is a waste of time for the very good ones, although the slow learners do benefit."

80% (item 9: see appendix 2) of the respondents reported that they had time for problem-solving sessions in their Mathematics classes. They believed that:

(a) "It was a way of consolidating what had been taught"; and
(b) "problem solving helped the students to know their mistakes, misunderstanding of concepts and application of skills and theorems, etc."

The above remarks for those who are in support of group work are consistent with the literature. Indeed, small group discussions help in developing problem - solving processes and reflective thought. These discussions, if properly done, help in providing background information to those students who do not already possess it (Stoker, 1991). Knowledge of background information helps in the construction of meaning of new knowledge.

Questioning in Mathematics class: 80% (item 4: see appendix 2) of the respondents reported that they asked questions during their Mathematics lessons. They used questions for testing the students' previous knowledge and to assess if the students understood a lesson. One teacher reported that s/he asked questions in order to encourage students to verbalise what they thought. Another teacher reported that questioning brings an atmosphere of competition amongst the students in the class, suggesting that it encourages the students to participate in the learning process.

20% (item 4: see appendix 2) of the teachers appeared to have developed a negative attitude towards asking questions because of the apparent passive nature of their students. One of the teachers wrote:

> **"They won't respond to the questions asked except the few."**

Another commented:

> **"The majority tells itself that Mathematics is not meant for it (questioning)."**

It is evident from these responses that most of the students lack confidence and ability to talk about Mathematics. Reasons for this may be varied and may include the fear that their peers might ridicule them. The students come to the high schools at standard 8 level, and stay in the school for at least a period of 3 years. This is a long enough period to cultivate confidence in them.

This can be achieved if teachers encourage the culture of discussions and questioning as an integral part of classroom activity, starting from earlier on in the students' academic career - and across the entire school curriculum.

Attending to individual students' difficulties: 70% (item 14: see appendix 2) of the teachers did not have time to look into individual students' difficulties. They taught different batches of students of different classes (standards) which required more work from them. One teacher reported that all his/her students had similar difficulties. Therefore, most of the students' difficulties were attended to in class as a group. The others believed that it was important to look into individual students' difficulties, but they could do so only for "weak students" and only "sometimes" for lack of time.

It is necessary to identify and make students' aware of their difficulties in any given area (Booth, 1988). Students should then be provided with the necessary assistance to overcome such difficulties. Most of the teachers complained of overload, therefore, being unable to find time to attend to individual students. Teachers may be able to get closer to their students in the available time by dividing them into smaller groups. This will enable the students to identify each other's difficulties and subsequently arrange further classroom discussion sessions, under the guidance of the teacher, to address the identified problems. Discussions among pupils promote reflections, and lead to the clarification of errors and the prevention of misconceptions taking roots (Olivier, A; Murray, H. and Human, P: 1992).

Tests at the end of each topic: 60% (item 15: see appendix 2) of the teachers believed that testing at the end of each topic was possible and necessary in order to "ascertain their (students') degree of understanding of each topic." The other 40% (item 15: see appendix 2) of the teachers could not give tests at the end of each topic for a number of reasons, including:

(a) High student numbers: One teacher wrote: "**numbers are very high, so it is taxing to do so** " (to give tests at the end of each topic).

(b) Lack of time: One teacher wrote: "... not enough time to do whatever we want."

(c) Emphasis laid on completing the syllabus: A teacher wrote:
"we are always in a hurry to finish the syllabus."
Another teacher wrote: "syllabus is to be completed."

(d) Students not keen to write tests: One teacher wrote: "cooperation by the students is not there. They miss classes during tests. They...do not prepare for the test."

It is quite evident that some teachers do not give tests at the end of each topic. It was not clear, what methods these teachers employed in order to find out their successes and failures in teaching a particular topic. It is suggested that teachers give regular tests to find out their students' difficulties and take the necessary steps to help them. Literature (e.g.Johnson, 1990) also suggests that students who are subjected to short tests on regular basis perform better than those who are not given such tests.

Homework: All the respondents reported that they gave homework to their students. However, one teacher felt that copying of homework could be avoided if students were given simple exercises as homework. Some teachers (item 16: 20%) believed that even though students might be copying home work solutions from one another, nonetheless, "**they get familiar with questions and prepare for examinations.**"

Another teacher believed that some students "might seek help from others" which might be considered a part of the learning process. There was an element of dissatisfaction with regard to homework as indicated by one teacher that "(students did) not worry themselves by doing home work."

This indicates that most teaching was examination orientated. Accordingly, teachers gave homework to prepare their students for examinations. Holdan (1988) contends that homework exercises should combine both distributed and exploratory practice. In other words, homework should be aimed at not only providing students further practice and insights on the work that they have done, but also to lead them to the following lesson. In support of independent practice of concepts and skills, Holdan (1988) contends:

> "To ensure proper understanding of an algebraic skill or concept...at some point in the learning sequence the learner must be given the opportunity to engage in independent practice... There can be little doubt that homework is an integral part of instruction."

The importance of properly organised homework can, therefore, not be over-emphasised.

Explanations in classrooms: Some respondents (item 5: 30%) reported that they enjoyed writing on the board and explaining mathematical concepts to their students throughout Mathematics periods. They did so because they believed that "It teaches students about skills in problem solving", and "...Mathematics is nothing less than a language, so writing and explaining are necessary."

These comments indicate a teacher-centred approach. It is necessary for such teachers to see the value of the students' active involvement in the learning process and, consequently, favour teaching styles which encourage or give students an opportunity to meaningfully construct their own knowledge (Brodie, 1991; Garofalo, 1989; Maher, et al, 1988; Moodley, 1992; Olivier, 1992; Schifter and Simon, 1992).

A large number of the teachers (item 5: 70%) believed that whilst explaining and writing on the board was necessary, it was also important that the students be given an opportunity to participate through answering questions and working problems on their own in class. One teacher used teaching aids in Mathematics lessons and another encouraged students to discover facts for themselves.

Some teachers (item 13: 30%) believed that those students who had interest in the subject, and had good foundation followed their explanations. 40% of the teachers believed that most of their students followed their explanations. Some of their comments were as follows:

> "I am more open to them, they feel at ease in my presence",
>
> "I teach them in simple methods which they could easily understand"; and
>
> "they have interest in the subject and understand my explanations very clearly."

Some teachers (item 13: 20%) believed that at times their students did not follow explanations due to either language problems or lack of understanding of the basic concepts. Indeed, other researchers (e.g. Brodie, 1991 and Bruner,1975) have also found that some students find difficulties in understanding mathematical concepts because they fail to understand the meanings of mathematical terms. Seemingly, teachers had false belief that their students followed their explanations. If the students followed their teachers' explanations then their performance in the test would not have been unsatisfactory (see tests of hypotheses 2 to 7).

Mathematical reasoning: 100% (item 6: see appendix 2) of the respondents believed that understanding mathematical reasoning was an important aspect of Mathematics learning for their students. Some of their comments were as follows:

> "Mathematical reasoning is very important and useful to understand mathematical concepts";
> "maths reasoning forms basis for higher levels of maths learning i.e. application, synthesis and evaluations";
> "reasoning helps them to remember how to tackle the problems";
> "one cannot memorize Mathematics rather it needs understanding"; and
> "Mathematics is subject of logic and reasoning. We do not prescribe rules/laws just randomly. Each law and rule must have some reasoning."

These responses suggest that the teachers valued the importance of reasoning in Mathematics, but relied heavily on explanations in an attempt to develop their students' reasoning powers. This suggests that most of the teachers tended to be more teacher-centred in their teaching approach, believing that the more they explained concepts, the higher the likelihood of their students attaining the intended educational goals.

Teaching pre-requisite knowledge: 40% (item 11: see appendix 2) of the respondents reported that they taught lower level Mathematics to their students in the process of developing lessons; and/or whenever they felt the need to do so. This was evident from the comments made by some of the teachers, such as:

> "That is where I start my lessons - from previous classes then gradually integrating the current syllabus"; and

"lower level Mathematics is taught whenever we feel it is necessary to refer to it."

However, 50% (item 11: see appendix 2) of the teachers conceded that they did not teach lower level Mathematics because of lack of time. They argued that if they taught lower level Mathematics then they would not be able to finish the syllabus. One teacher stated it as follows:

"If I take time to do lower level maths, I will be left with no time for the relevant syllabus. Still, sometimes I try to do the lower-level topics for the sake of weak students."

The responses indicate that the teachers are aware of their students' difficulties in lower level Mathematics but they (teachers) do not have time to assist them in this respect. They seem to be worried about finishing the matriculation syllabus. It was not clear if the teachers were ignorant of the fact that in Mathematics a higher level concept can only be understood if the lower level related subordinate concepts are correctly understood. It is suggested that teachers incorporate the required lower level concepts into their lessons and make sure that the students understand them. As Skemp (1976) observes, the correct learning of higher concepts in Mathematics is based on the students' understanding of lower level related subordinate concepts. At the practical level, pre-requisite knowledge may be addressed through activities such as homework exercises if the teacher cannot find time to deal with the matter during regular class time.

Effects of the knowledge of linear graphing on the learning of graphs of quadratic functions: 20% (item 20: see appendix 2) of the teachers believed that the poor knowledge of graphical representation of linear functions did not pose any difficulties to the students in the learning of graphs of quadratic functions. Due emphasis on the teaching of graphs of quadratic functions was given by most (item 21: 90%) of the teachers. 90% (item 22) of the teachers believed that it was necessary for their students to know why 'q' was the y co-ordinate of the turning point of the graph of the function $\{(x;y): y = a\ (x - p)^2 + q\}$. They also believed that:

(a) (item 23: 90%) making the students draw a number of graphs of quadratic functions in different positions in a Cartesian plane would enhance their (students') understanding of (the features of) those graphs; and

(b) (item 24: 100%) it was necessary for the students to learn drawing quadratic function graphs irrespective of the examiners' preferences.

80% (item 25) taught graphical solutions of quadratic equations. 60% (item 26) believed that both solving and graphing of quadratic equations were equally important. 90% (item 27) believed that plotting of graphs of quadratic expressions was not a waste of time.

Those who did not find teaching graphical solutions of quadratic equations important commented as follows:

> "This does not appear in the examination";
> "it is easier/quicker to solve by factorisation";
> "students are asked to solve (equations) by factorisation in the exam(ination)."

These responses explain the students' very poor performance in the section on interpretation of graphs in the test. Although most teachers believe that graphing is important they appear not to have taught the solutions of quadratic equations and inequalities in an integrated manner vis-a-vis algebraic and graphical techniques.

In summary, it could be deduced that the majority of the teachers taught their students according to the requirements of the examiner. Their main emphasis seemed to be on the type of questions that were asked in the examinations. For example, some teachers did not teach plotting/sketching graphs satisfactorily because the students are never required to draw graphs in the examinations. Teachers appeared to have been giving in to an examination orientated teaching approach because of the expectations from parents, school principals and the department of education. Teachers who produce good results in matriculation examinations are labelled as good teachers and those who do not are construed as bad teachers in the eyes of the concerned parties.

Furthermore, it was clear that some teachers did not give an opportunity to their students to reflect on what they were learning by way of asking questions in their classrooms. This led to minimal or no student participation in the learning process whatsoever.

The notion that each individual has his/her own knowledge base on which he/she constructs new knowledge was not given importance. Teachers were aware that their students had difficulties emanating from earlier Mathematics classes. They, nonetheless, had no time to attend to these difficulties, yet they expected their students to learn new (higher level) concepts. To make the problem more serious, some teachers seemed to have no mechanism for finding out about the success of their lessons in terms of the gains that their students made. These teachers did not ask questions in class and also did not give frequent class tests.

The fact that some teachers did not pay attention to individual students' difficulties brought with them from lower level Mathematics, and that they did not consider the lower level Mathematics knowledge (e.g. knowledge of linear graphs for the learning of graphs of quadratic functions of one variable) essential for teaching the new concepts, points towards their conviction of instrumental understanding of Mathematics and a teacher-centred approach to teaching Mathematics. Hence, the teachers' convictions and teaching approach might have partially contributed towards students' poor performance in Mathematics, in general.

4.4 CONCLUSION

This chapter dealt with the presentation and analysis of results, as well as discussion thereof. It is clear that students had very poor background knowledge (46.6% correct responses) required for the learning of the theory of quadratic equations, inequalities and their graphs. This was one of the major reasons for students' poor performance in the test as a whole. Students' conceptions of concepts such as real numbers, non-real numbers, square root, root, real root and variable, etc., were by and large, faulty. The second area of students' difficulties was clearly their lack of ability to correctly use mathematical symbols. This posed problems for the students to communicate their mathematical thoughts correctly and understandably.

Furthermore, students' poor performance in graphs (28.3% average score) indicated that they had many misunderstandings, particularly in the interpretation of graphs. A more relational approach is required for answering the items in this section of the test. Poor performance (49.2% average score) in quadratic inequalities was linked to the students' difficulties in interpreting mathematical conjunctions and disjunctions, and graphical representation thereof.

In general, items requiring analysis, and application (higher level objectives) were answered poorly, giving an average score of 26,5% in the section dealing with the application of the theory of quadratic equations.

In general, the overall performance was poor. This indicated that the students in the sample had not learned the theory of quadratic equations, inequalities and their graphs to a satisfactory level of mastery.

Responses obtained from the teachers' questionnaire (appendix 2) indicated that most teachers relied on lecture methods and explanations as their teaching styles. The teachers could not monitor their students' written work, and could not assess their students difficulties on a regular basis. This promoted a teacher-centred approach in their classrooms.

The majority of the teachers taught Mathematics, mainly to enable their students to pass the matriculation examination. It was quite evident from their responses such as:

> "I have to confess that making the students pass the exam is first and foremost in my mind, but now and then I do give them some enrichment."

Matriculation examination results show that even though teachers' teaching was examination orientated, they did not seem to have achieved very good results.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 SUMMARY

This study was an evaluative study involving a total of 311 matriculation students taking higher grade Mathematics. In all, 10 high schools of Umtata Educational circuit and 10 Mathematics teachers took part in the study.

The purpose of the study was to determine some of the factors which may explain students' apparent poor conceptual understanding of Mathematics and to throw some light on possible solutions to their problems.

A diagnostic test (see appendix 3) on the theory of quadratic equations, inequalities and their graphs was constructed, validated and given to students. Items in the test were aimed at finding out answers to the following questions:

1. Is there any relationship between the students' overall performance and their pre-requisite knowledge?
2. Do the students possess the pre-requisite knowledge and skills required for learning quadratic equations, inequalities and their graphs?
3. Are the students able to solve quadratic equations expressed in the standard form?
4. Are students able to identify and solve problems dealing with quadratic equations but not expressed in the standard form?
5. Do the students display understanding in their problem solving procedures of quadratic equations and inequalities?
6. Are students able to interpret graphs of quadratic functions?

7. Are students able to apply the knowledge of quadratic equations, inequalities and their graphs to unfamiliar situations?

A total of 38 items made up the test. These items were grouped into 7 parts (see table 3.4); each part aimed at answering at least 1 of the 7 questions listed above. Students' scripts were marked out of a maximum of 38 scores (1 for each item). Students' raw scores were used to compute their mean score for each part of the test (see tables: 4.6, 4.8, 4.10, 4.12, 4.14 and 4.16).

Question 1

The pearson's correlation coefficient between pre-requisite knowledge and scores on the remaining items of the test was computed yielding the value of 0,45164 which is statistically significant at alpha = 0,01. This showed that those students who had the necessary pre-requisite knowledge performed well in the theory of quadratic equations, inequalities and their graphs (the rest of the items in the test) and those with poor pre-requisite knowledge performed poorly in the rest of the items.

Furthermore, arithmetic means of the top and bottom 27% of the students (n = 85 for each) on previous knowledge and on the rest of the items were computed. These arithmetic means are given in tables 4.4 and 4.5. The mean (4,85) for the top students on previous knowledge (see table 3.4) was significantly (statistically) different from the bottom students' mean (1,62) on the same items. The hypothesis that the students' poor previous knowledge (see table 3.4) contributed towards their poor performance in the rest of the test was supported by the fact that the same top students' arithmetic mean (15,78) in the remaining items of the test was also significantly (statistically) different from the bottom students' arithmetic mean (10,08) on the same items. In each case, the "z" test (see tables 4.4 and 4.5) showed a statistically significant difference in the performance of the two groups (top and bottom) of students.

Question 2

A "z" statistic (see table 4.7) was computed by comparing a criterion mean of 4,2 (60% of a maximum of 7) with the observed mean of 3,27 (46,6% of total of 7) yielding "z" = -12,23 which is less than z_c = -1,96. This indicated that the students did not have sufficient pre-requisite knowledge needed for learning the theory of quadratic equations, inequalities and their graphs.

Questions 3 - 7

The "z" statistic was computed by comparing a criterion mean of 60% with the observed means for hypotheses 3 to 7. In all cases, the null hypotheses were rejected in favour of the alternatives (see tables 4.7, 4.9, 4.11, 4.13, 4.15 and 4.17). Therefore, it was concluded that the students did not have sufficient ability, in terms of the expected level of performance:

(a) to solve quadratic equations and inequalities;
(b) to solve quadratic equations not expressed in the standard form;
(c) in demonstrating understanding to solve quadratic inequalities;
(d) to translate algebraic representations into graphical representations, and vice versa; and
(e) to apply knowledge of theory of quadratic equations, inequalities and their graphs in solving unfamiliar problems.

In addition, **an analysis of students' scripts** was carried out and clinical interviews on a sample of the subjects (students) were conducted to find out their difficulties/misconceptions. Students' difficulties/misconceptions discussed in chapter 4 have been summarised below under various categories, namely: mathematical terms, mathematical symbolic language, mathematical skills, importance of form in Mathematics, overgeneralisations, translation problems, and conceptual difficulties.

Mathematical Terms: The findings of this study suggest that the majority of the students had problems/difficulties with the mathematical terms used in the classroom. The observed specific areas of difficulty included the meanings of the following terms:

(a) a number which is a <u>complete square</u>;

(b) 'a number' is a <u>root</u> of an equation;

(c) <u>roots</u> of an equation <u>satisfy</u> the equation;

(d) set of these numbers is the <u>solution set</u> of the equation;

(e) the <u>difference</u> between an <u>equation</u> and an <u>expression</u> e.g. seeing $2x^2 + 4x + 1$ as an example of a quadratic equation;

(f) the <u>difference</u> between <u>roots</u> and <u>discriminant</u> of a quadratic equation;

(g) a <u>variable</u> in an equation;

(h) 'this is an <u>unknown constant</u> in the equation';

(i) the <u>difference</u> between <u>doubling</u> and <u>squaring</u>;

(j) the word '<u>reduce</u>' in the phrase 'the equation reduces to a quadratic equation'; and

(k) the difference between the instructions, '<u>find $\sqrt{16}$</u>', and '<u>find square root of 16</u>'.

Indeed, language acts as a powerful tool for development of thoughts (Bruner, 1975), and students may have difficulties in learning Mathematics if they have not understood the appropriate meanings of some of the terms used (Burton, 1980).

Mathematical Symbolic Language: In addition to their difficulties in mathematical English the students also had difficulties in mathematical symbolic language. These difficulties included the following:

(a) Students' inability to relate the words 'and' and 'or' to the symbols '$\cap$' and '$\cup$', respectively.

(b) The majority of the students did not appreciate that the literals used in expressions stood for numbers.

(c) Many students had difficulties in making sensible use of order symbols such as =, > and <. They changed from '=' to > or < during the course of solving a problem.

(d) A large number of students did not perceive any difference between $-x^2$ and $(-x)^2$, and $1/2\ x^2$ and $(1/2\ x)^2$ in items 33 and 12, respectively. Evidently, this arises from the students' lack of awareness of the significance of brackets in mathematical "puzzles". Consequently, the majority of students did not differentiate between the expressions:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and

$$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

These observations concur with literature wherein students' difficulties in learning Mathematics are seen as being related to their lack of understanding of the appropriate meanings and use of some of the symbols (e.g. Austin and Howson, 1979; Brodie, 1991).

Mathematical skills: The majority of students did not exhibit a functional level of mathematical skills embodied in the learning of the theory of quadratic equations and inequalities. It was evident, for instance, that they were unable to:

(a) multiply algebraic expressions, such as $(7a - 10)$ and $(5a - 43)$ in item 3;

(b) factorise the trinomials, such as $4x^2 - 20x + 25$ in item 6;

(c) find square roots of trinomials, such as $4x^2 - 20x + 25$ in item 6;

(d) expand powers with algebraic binomials as a base, such as $(x + 2)^2$ in item 29; and

(e) evaluate algebraic expressions.

Importance Of Form In Mathematics: Byers and Erlwanger (1984) contend that recognition of 'form' plays a very important role in the learning of Mathematics. In the case of equations and algebraic expressions, the operations (+, -, ÷ and x), the literals used, brackets and exponents determine the forms thereof (Craig & Winter, 1990). For instance, in the quadratic equation $ax^2 + bx + c = 0$, x stands for a variable, the leading term is ax^2, $a \neq o$, and a, b and c are real numbers. Similarly, $f(x) = ax^2 + bx + c$, where $a \neq 0$ is the equation of a quadratic function of x because the leading term is ax^2. In these cases the literals 'a' and 'b' are coefficients of terms x^2 and x, respectively, and c is the constant term. Many students appeared not to have paid much attention to this important aspect of Mathematics (i.e. form). This resulted in their inability to realise that:

(a) the algebraic expression '$a^2 - b^2$' represents a <u>difference</u> of two squares whereas the expression '$a^2 + b^2$' is the <u>sum</u> of two squares. Therefore $(a + b)$ and $(a - b)$ are factors of $a^2 - b^2$ and not of the expression $a^2 + b^2$;

(b) the equation $ax^2 + c = 0$ is in fact $ax^2 + bx + c= 0$, where $b = 0$. Therefore, the quadratic formula could be used to find roots of the equation $ax^2 + c = 0$; and

(c) the expression

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is different from

$$- b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Students also exhibited the misconceptions that:

(i) the equation $y = -2x + 10$ represented an equation of a parabola;

(ii) the equation $y = ax^2 + bx + c$ represented an equation of a straight line; and

(iii) the equation $y = ax^2 + bx + c$ represented an equation of a hyperbola.

In all the above cases, it is quite evident that students' difficulties emanate from the lack of recognition of the importance of 'form' in the learning/teaching of Mathematics.

Overgeneralisations: Students faced some difficulties in Mathematics because they did not pay attention to the domains within which certain generalisations were applicable. This resulted in students' overgeneralising concepts, principles and other relations to situations where they could not be used. Examples of this problem included:

(a) 'an equation remains unchanged on multiplying/dividing it by a **non-zero** number', **to** 'an equation remains unchanged on multiplying/dividing it by **any** number', e. g. dividing the equation $12x^2 = 6x$ by x (item 24);

(b) the law of exponents that '$(xy)^m = x^m \cdot y^m$' **to** '$(x + y)^m = x^m + y^m$', e.g. $(x + 6)^2 = x^2 + 6^2$ in item 28;

(c) 'if $(x - a)(x - b) = 0$ then either $\mathbf{x - a = 0}$ or $\mathbf{x - b = 0}$' <u>to</u> 'if $(x - a)(x - b) = c$, then either $\mathbf{x - a = c}$ or $\mathbf{x - b = c}$ ', e.g.
if $(x + 2)(x + 3) = -4$,
then either $\mathbf{x + 2 = -4}$ or $\mathbf{x + 3 = -4}$ (in item 23);

(d) 'if $x > b$ and $x > c$ then **x** is **greater** than **the greatest** of 'b' or 'c', **to** 'if $x > b$ and/or $x < c$ then x is greater than the greatest of 'b' and 'c' (in item 14);

(e) 'if $x < b$ and $x < c$ then **x** is **smaller** than **the smallest** of 'b' or 'c', **to** 'if $x < b$ and/or $x > c$ then **x** is smaller than the smallest of 'b' or 'c' (in item 15);

(f) 'an equation remains unchanged on dividing/multiplying it by a non-zero number' **to** 'an inequality remains unchanged on dividing/multiplying it by a non-zero number, e.g. if $-2x + 3 > x + 2$ then $x > 1/3$ in item 17.

Leonard and Sackur-Grisvard (cited by Olivier, 1992) contend that the type of misconceptions mentioned above are very stable in students' minds because of the efficiency and consistency that these concepts enjoy in their legal domains. Teachers need to build into their lessons deliberate and mindful attempts to draw their students' attention to the domains in which these generalisations work, and give examples of instances where they do not work.

Translation Problems: Bloom, Wood and others (cited by Moodley, 1992c) identify "translation" as a crucial aspect of comprehension in Mathematics. The majority of the students were very weak in translating verbal mathematical statements into their algebraic form. For example, only 32% of the students got item 12 correct in which they were required to translate the verbal statement into its algebraic form. They were also very poor in carrying out translations from the algebraic form to the graphical form, and vice-versa. Their difficulties lay in:

(a) their misconception that the defining equations of x and y-axes are $x = 0$ and $y = 0$, respectively;

(b) their inability to link the equation $f(x) = ax^2 + bx + c$ to the parabolic shape of its graph, in general;

(c) their misconception that $y = ax^2 + bx + c$ is the equation of a straight line;

(d) their lack of knowledge that the value of 'c' in the equation $y = ax^2 + bx + c$ represents the y-intercept of the parabola;

(e) their belief that for equation $ax^2 + bx + c = 0$ to have two real roots, the graph of $y = ax^2 + bx + c$ must intersect the y-axis at two points;

(f) lack of ability to correctly determine the nature of roots of the equation $ax^2 + bx + c = 0$ from the position of the graph of $ax^2 + bx + c = y$ in relation to intercepts on the x-axis;

(g) failure to recognise that if the co-ordinates of the turning point of a sketch of a parabola satisfied an equation of a quadratic (parabolic) function then the equation could not be said to be the algebraic representation of the graph **unless** another point on the curve satisfied the equation;

(h) the misconception that the defining equation of a parabola which intersects x-axis at x_1 and x_2 is $y = (x - x_1)(x - x_2)$ instead of $y = a(x - x_1)(x - x_2)$, where 'a' stands for a non-zero constant;

(i) the inability to identify the correct distances on the x-axis that represent a solution set of a system of equations whose graphical representations are given;

(j) mistaking (-p; q) for the turning point of the parabola whose defining equation is $y = a(x - p)^2 + q$;

(k) lack of clarity regarding what represents the graphical solution of quadratic inequalities (e.g. $ax^2 + bx + c > 0$) on the graph of $y = ax^2 + bx + c$;

(l) lack of knowledge regarding what represents:
   (i) an intersection set of two quadratic functions; and
   (ii) an intersection set of a linear and a quadratic function; and

(m) difficulties in finding out, from graphs which parabola had an undefined minimum/maximum values on the basis of the direction in which it "opened".

Conceptual Difficulties: It is quite possible that the difficulties identified under the above headings are essentially conceptual in nature, although the manifestation of this occurs in a manner that a casual observer may regard as "wrong" mathematical operations or calculations. In this regard, it was quite evident that students' conceptual difficulties included:

(a) lack of clarity of the meanings of 'doubling a number' versus 'squaring a number', e.g. square of 6 is 12 (in item 12);

(b) incomplete or "wrong" conception of rational numbers, e.g.

  (i) construing rational numbers as fractions and irrational numbers as whole numbers,

  (ii) holding the notion that a number which is a perfect square is a rational number;

(c) having no concept of non-real numbers, e.g. square root of -16 is -4;

(d) incomplete conception of the set of real numbers, e.g. a belief that zero and negative numbers are not real numbers;

(e) having no concepts of intersection and union of two sets represented in algebraic or graphical forms;

(f) an inability to arrive at different possibilities for the statement $(x - a)(x - b) < 0$ to be true, e.g. in item 11.

It was quite evident that misconceived concepts and principles created confusion in the learner's mind and led to misunderstandings of the related higher concepts.

Question 8

Furthermore, **a teachers' questionnaire** (see appendix 2) was developed, validated and given to the participating teachers. Items in the questionnaire were aimed at finding an answer to the following:

8. Is there any difference between the performance of students (in theory of quadratic equations, inequalities and their graphs) taught by student-centred versus more teacher-centred teachers?

An analysis of the teachers' questionnaire was carried out. The number of 'strongly agree' and 'agree' responses on each item were combined to give an affirmative score, and likewise, a dissenting score was obtained by combining the 'disagree' and 'strongly disagree' responses. These numbers were converted into percentages. A brief summary of these responses is presented below (also see table 4.20).

On the whole, the teachers believed that:

(a) Mathematics could be taught by discovery methods (1: 90%) - where "(1: 90%)" means that 90% respondents either agreed or strongly agreed with the statement presented in item 1 of the questionnaire;

(b) questioning in classroom was necessary (4: 80%):

(c) students tried to answer the questions that teachers asked in their classrooms (3: 60%);

(d) they enjoyed spending most of their classroom time writing on the board and explaining mathematical concepts to their students (5: 30%);

(e) students followed teachers' explanations (13: 70%);

(f) there was enough time for discussions in class (8: 50%);

(g) there was enough time for problem sessions (9: 80%);

(h) it was necessary for students to understand mathematical reasoning (6: 100%);

(i) it was possible to give students progress tests at the end of each topic (15: 60%); and

(j) they gave home work to their students (16: 90%).

Some of the reasons given by these teachers in support of the above response profiles included the beliefs that:

(a) the discovery method of teaching Mathematics was the most effective method;
(b) all students benefited from group work and class discussions;
(c) the importance of questioning in the Mathematics classroom could not be overemphasised;
(d) it was important to look into individual students' difficulties;
(e) whilst explaining and writing on the board was necessary, it was also important that the students be given the opportunity to participate through answering questions and working problems on their own;
(f) the importance of reasoning in Mathematics could not be overemphasised;
(g) it was important to see to it that the students understood pre-requisite knowledge; and
(h) the knowledge of linear graphs was important for the learning of quadratic function graphs.

However, a significant proportion of teachers also believed that:
(a) their teaching was examination oriented (item 7: 60%);
(b) they did not get any opportunity to teach Mathematics by following discovery methods (10: 50%):
(c) they felt that their students needed assistance in lower level Mathematics but had no time to offer any such assistance to them (11: 50%);
(d) their main goal in teaching their students was to help them pass the matric examination (12: 50%; 7: 60%); and
(e) they did not have time to look into individual students' difficulties (14: 70%).

In addition, the following apprehensions were also mentioned:
(a) the discovery method could not be used for teaching in Mathematics classes because the students were not used

to this way of learning and it was a time consuming method;

(b) discussions (i) created noise; (ii) took a lot of time to teach/ learn a concept; and (iii) were not a fruitful exercise in the class as periods were not long enough;

(c) questioning did not work in their Mathematics classes because the majority of students did not respond to questions;

(d) they did not have enough time to attend to individual student's difficulties;

(e) they could not give regular tests because of high student numbers, lack of time, students' non-co-operation, and the vastness of the syllabus in relation to the time allocated;

(f) home work exercises were given to provide facility for drill and practice;

(g) they had no time to attend to their students' difficulties in lower level Mathematics;

(h) graphical solutions of equations and inequalities were not important because they did not appear in the examinations; and

(i) the knowledge of linear graphs was not important for the teaching/learning of graphs of quadratic functions.

It was thus evident that the majority of the teachers followed teacher telling/explaining and students listening approaches to teaching/learning with the belief that their explanations and mathematical reasoning were followed by the majority of the students. This, they believed, would bring about success in achieving their main goal, thereby enabling students to pass the matriculation examination. Thus, these teachers' beliefs about the learner leaned more towards the left on the "empty vessel" to 'an active thinker' continuum given in sketch 2.1. Accordingly, it would appear that they believed mathematical knowledge to be 'content' which could be transferred to students mostly through explanations and expository learning. The low pass rate (see table 1.1) could, therefore, be attributed to the fact that the teachers:

(a) ignored the importance of their students' readiness (in terms of pre-requisite knowledge) in the teaching/learning process in that they did not help their students to upgrade their poor pre-requisite knowledge; and
(b) adopted mostly teacher telling/explaining (teacher-centred) approach to teaching of Mathematics which tended to direct the classroom instruction towards developing lower level objectives.

This indicated that on the average the teachers in the sample (in practice) were in the third quadrant of sketch 2.1 (see page 36). Within contemporary educational theory and practice, however, students' participation in the learning process is viewed as the central ingredient in all instructions. It is the student who should do things and talk more during learning sessions. The role of the teacher is mainly to point out the way and then supervise the students as they try things out - physically, mentally and verbally in discussions, laboratory activity or in supervised practice sessions (e.g. class tasks) so that they can achieve a whole range (knowledge, skills, comprehension, selection - application, and analysis - synthesis) of objectives.

With specific reference to the research questions of this study, teachers were divided into two Groups, 1 and 2, based on their teaching disposition (i.e. teacher-centred vs. student-centredness) as reflected in their scores in the questionnaire. Mean scores of their students' (Group A and Group B) performance were compared by using the "z" statistic. It was found that the arithmetic mean (14,84) of Group A students (taught by teachers with disposition towards teacher centredness) was significantly lower than the arithmetic mean (17,06) of Group B students (taught by more student-centred teachers). Hence, it could be said that the students' performance in the test was positively related to the degree of their teachers' student-centredness towards Mathematics teaching. This indicated, and one would conclude from it, that students taught by teachers whose teaching strategies are more student-centred perform better than those who are taught by teachers whose teaching is inclined towards teacher-centredness.

Muthukrishna (1994) reports a study in which two instructional approaches (one explicit, the other problem centred) were compared. The findings of the study indicated that the students taught following a problem solving approach (inclined to student-centredness) showed better conceptual understanding as compared to those who were taught following explicit instruction (teacher-centred). Muthukrishna's findings support those of the present study and, together, these studies suggest that students taught using predominantly student-centred approaches are intrinsically motivated and they deliberately and mindfully engage themselves in the learning process.

In summarising the findings of this study the following two points may be made: (a) the students in the sample had not learned the theory of quadratic equations, inequalities and their graphs to the satisfactory level of mastery; and (b) most teachers though appreciating the importance of classroom discussions and discovery methods of teaching Mathematics (student-centred approaches) employed the 'teacher telling' and 'students listening' approach to the teaching and learning of Mathematics for various reasons already mentioned. Consequently, their teaching was inclined towards teacher-centredness resulting in poor students' performance. However, the others were more student-centred and their results were significantly better.

5.2 CONCLUSION

This study has shown that for achieving desired objectives in teaching/learning of a mathematical concept(s)/procedure(s) the students' pre-requisite knowledge should be taken into consideration. In effect, the majority of students had serious problems in the work done at primary and junior secondary levels.

Furthermore, students learned Mathematics by rules and they faced difficulties in solving problems which required deductive reasoning and higher levels of understanding. Some of the hurdles that students faced arose out of their lack of understanding of mathematical language and symbolisms. As an extension of this problem, students did not appear to pay attention to the importance of 'form' in Mathematics and therefore failed to relate graphical representations to their equivalent algebraic representations. Furthermore, they did not exhibit the habit of checking the correctness of their answers.

New concepts/procedures need to be related to the students' existing cognitive structures for meaningful learning to take place. It was quite evident from the study that misconceptions about a mathematical concept in one area may create further misconceptions in the understanding of new concepts.

The teachers seemed to be aware of, at least, some of the problems their students had but the majority of them did not manage to help the students to overcome their difficulties owing to practical constraints, such as time, physical facilities, etc.

Finally, teachers had different views about the manner and extent to which a student should be involved in the teaching/learning process. Students taught by teachers whose teaching styles were more student-centred performed better than those whose teachers' teaching styles were teacher-centred.

In conclusion, it is important to mention that this study has succeeded in finding answers to some of the questions that the researcher set out to investigate. The findings are both important and significant in an attempt to enhance the quality of teaching and learning - not only in Transkei but the greater South Africa as well; and not only within the topic of 'theory of quadratic equations, inequalities and their graphs' but also other Mathematics topics. It is hoped that this study will make a significant contribution to the quality of teaching and learning of Mathematics generally - and, in particular, the teaching and learning of quadratic equations, inequalities and their graphs.

The study has, however, by implication raised a number of equally important questions, including the following:

1 Are the teachers teaching at junior secondary and primary schools qualified to teach Mathematics?
2 Do teachers at junior secondary schools teach all components of Mathematics without any bias?
3 Are students positively motivated to learn Mathematics?
4 Are Mathematics teachers using the teaching methods that may help their students in developing relational understanding of the subject?
5 Why are Mathematics teachers failing to attend to students' difficulties arising from their mathematical knowledge base that they should have acquired at lower levels?
6 Is there an adequate coordination among Mathematics teachers of standards 8, 9 and 10?

7 How can students be encouraged to participate actively in the learning process?

8 Are the teachers aware of their students' language related problems in the learning of Mathematics?

9 Are Mathematics teachers making special efforts to teach their students mathematical language?

10 Are pre-service and in-service teachers exposed (sensitised) to students' difficulties in learning Mathematics, particularly in relation to specific topics?

11 Can Mathematics teachers help each other, regionally and/or nationally, in developing efficient methods to teach different mathematical concepts?

12 Are in-service teachers kept informed of the outcomes of the research projects in Mathematics education both in and outside South Africa?

The findings of this study have led the researcher to make the following recommendations.

5.3 RECOMMENDATIONS

The recommendations which arise out of this study are presented in respect of teachers, education authorities, curriculum planners and researchers.

5.3.1 TO MATHEMATICS TEACHERS

(i) Emphasis needs to be laid on the meaning and correct usage of mathematical symbolism.

(ii) Meanings of verbs such as simplify, evaluate, transform, solve, satisfy, reduce, etc. need to be well understood by the students. Students may be encouraged to prepare a Mathematics dictionary in one part of their note books in which they should write the meaning of each and every new term that they come across.

(iii) Special attention needs to be paid to translation processes (i.e. from verbal to algebraic; algebraic to graphical representations, and vice versa).
(iv) Solutions of equations should be introduced to the students as a necessity to solve some problems.
(v) Steps should be taken to identify students' difficulties/misconceptions and discuss them in an effort to make the students aware of them.
(vi) An evaluation of students' relevant previous knowledge by means of diagnostic tests or other forms of monitoring student progress is necessary. "Students' IQ and other abilities may influence their success, but it is unlikely that average students cannot comprehend new mathematical concepts if presented with relevant subordinate concepts" (Craig & Winter:1990). Teachers must, therefore, consider readiness (in terms of previous knowledge) of their students for every lesson.
(vii) Teaching and learning of Mathematics should be made student-centred as much as possible.
(viii) Students should be made aware of the hierarchical nature of the subject and its implications in the learning of Mathematics.

5.3.2 TO EDUCATION AUTHORITIES

(i) Teaching of Mathematics requires a shift from a teacher-centred approach to a student-centred approach. Teachers may be encouraged to change their perceptions about teaching of Mathematics through organising discussions on how to involve students in the teaching/learning process in and outside the classroom during in-service courses.
(ii) Teachers must be encouraged to conduct research on how to improve teaching/learning in Mathematics under given conditions.
(iii) The information arising from the discussions

mentioned in (i) and (ii) may be shared through forums specifically created for such purposes (e.g. journals, symposia, etc.). Teachers should be kept informed of the findings of current research work going on in Mathematics education in and outside South Africa by being supplied with journals dealing with such materials. This could best be done through teachers' resource centres at educational circuit level.

(iv) A massive programme to innovate the teaching/learning of Mathematics at primary and junior secondary levels may be required through extensive in-service courses aiming at improving:

(a) the teachers' own understanding of concepts that they teach to their students; and

(b) teaching methods for teaching each concept through sharing of ideas amongst teachers on how to teach a given concept.

(v) It is very clear in this study that the majority of the students' poor performance in Mathematics is mainly linked to their poor background knowledge. There is need for this situation to be rectified by giving extra support to students with poor pre-requisite knowledge, otherwise these students have a very weak base to build on new concepts. From the findings of this study it is also clear that teachers, in general, do not have time to go back to previous work, despite the urgent need for remedial teaching in schools. Teachers should, therefore, be urged to consider remedial teaching as part of their <u>bona fide</u> duties. Their teaching loads should, where necessary, be adjusted to enable them to undertake this very important task.

5.3.3 TO THE CURRICULUM PLANNERS

(i) A separate section needs to be devoted in the curriculum to developing the ideas of conjunction and disjunction in mathematical statements.

(ii) This study revealed that the majority of students are very poor in interpreting graphs, in general, and graphs of quadratic equations in particular. Yet, interpretation of graphs is a very important part of Mathematics because of its application not only in other school subjects, but also to the understanding of many events occurring in students' day-to-day experiences. For example, many times data is represented graphically in the news media. Students are comparatively much weaker in this section, perhaps because not much importance is given to the interpretation of graphs in the classroom. Evidently, interpretation of graphs also receives less importance in lower level Mathematics examinations. Overall, this encourages teachers (most of whom teach for examinations) not to put much emphasis on graphs resulting in students also ignoring this section. This becomes a major cause of poor background for them when it comes to learning graphs in senior classes. It is, therefore, recommended that standards 7 and 8 examinations have a fair share of questions on drawing and interpretation of graphs. Curriculum developers could give the examiners guidelines as to what weighting each area of the syllabus should receive and what level(s) of questions and proportion there-of should comprise the question paper, at standards 7 and 8 levels.

(iii) Curriculum developers need to move away from just giving a list of topics for a given syllabus towards the development of:

(a) details of each component of the topic;
(b) the objectives to be achieved under each component of the topic; and
(c) guidelines as to how best these objectives may be achieved.

This will give clearer direction to teachers and students in terms of the expectations and demands of the curriculum. However, for relevance, appropriateness and effectiveness, the whole curriculum development exercise should be characterized by an active involvement of practising teachers. Such an active involvement of teachers will help generate concrete ideas and activities which could be carried out in order to achieve specified outcomes. Without such teacher input, the ownership of the curriculum becomes foreign to the teachers and, soon, problems emerge at the implementation stage.

5.3.4 TO OTHER RESEARCHERS

(i) Research studies similar to this one may be conducted on other topics playing key roles in the matriculation Mathematics syllabus.

(ii) There should be more classroom based research to investigate the actual effects of implementing the above mentioned recommendations. This will have the benefits of bringing career researchers and classroom teachers together for the benefit of the student.

(iii) More research on how to make homework exercises, class discussions and questioning effective in promoting students' conceptual development in Mathematics needs to be conducted.

(iv) This study revealed that students' poor performance in the theory of quadratic equations, inequalities and their graphs was partly linked to their poor pre-requisite knowledge. More efficient ways of incorporating previous knowledge in new lessons need to be developed and researched, within the constraints of third world Mathematics teaching (e.g. large classes; high teaching load; lack of availability of modern facilities such as video taped lessons, computer aided instruction etc.).

Given the crucial role of quadratic equations, inequalities and related graphs in high school Mathematics, it is important that the concerns raised by this study be addressed so that the quality of the learning and teaching of Mathematics in high schools is improved.

BIBLIOGRAPHY

Austin, J.L. & Howson, A.G. (1979). Language and Mathematics Education, Educational Studies in Mathematics, 10: 161-197..

Ausubel, D.P. (1977). The Facilitation of Meaningful Verbal Learning in the Classroom, Educational Psychologist, 12: 162-178.

Ausubel, D.P. (1979). Education For Rational Thinking: A critique.In Lawson A.E. (Ed.): The Psychology Of Teaching For Thinking And Creativity. 1980 AETS Yearbook, (pp 174-190), Columbus.

Backman, M.E. (1972). Patterns of mental abilities: Ethnic, socioeconomic, and sex differences, American Educational Research Journal, 9: 1-12.

Behr, M.J., Wachsmuth, I., Post, T.R., & Lesh,R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment, Journal for Research in Mathematics Education, 15(5): 323-341.

Behr, M.J., Wachsmuth, I., Post,T.R. (1985). Construct a sum: A measure of children's understanding of fraction size, Journal for Research in Mathematics Education, 16(2): 120-131.

Bell, F.H. (1978). Teaching and Learning Mathematics in Secondary Schools, William C Brown, Dubuque, Iowa.

Bell, A., Fischbein, E. and Greer, B. (1984). Choice of operations in verbal problems, Educational studies in Mathematics, 15 (2):129-147.

Bernard, J.E. and Cohen, M.P. (1988). An Integration of Equation-solving Methods into a Developmental Learning Sequence. In Coxford A.F & Shulte A.P., The Ideas of Algebra, K-12 (pp97-111). NCTM 1988 yearbook.

Bodner, G.M. (1986). Constructivism: A Theory of Knowledge, Journal of Chemical Education, 63 (10):873-877.

Booth, L.R. (1981). Child-methods in Secondary Mathematics, Educational Studies in Mathematics, 12 : 29-41.

Booth, L.R. (1988). Children's Difficulties in Beginning Algebra. In Coxford A.F & Shulte A.P., The Ideas of Algebra, K-12 (pp20-32). NCTM 1988 yearbook.

Brodie, K. (1991). Using language in the Mathematics Classroom: A Teachers' Approach, Pythagoras, 27:17-23.

Bruner, J.S. (1966). Toward a Theory of Instruction, Cambridge, Havard University Press.

Bruner, J.S. (1975). Language as an Instrument of Thought. In Alan Davies (ed); Problems of Language Learning, London, Heinemann.

Burton, L. (1980). The teaching of mathematics to young children using a problem solving approach, Educational studies in mathematics, 11 (1):43-58.

Burton, L (1984). Mathematical Thinking: The Struggle for Meaning, Journal for Research in Mathematics Education, 15 (1): 35-49.

Byers, V. and Erlwanger, S. (1984). Memory in Mathematical Understanding, Educational Studies in Mathematics, 15 (3): 259-276.

Carpenter, T.P., Coburn, T.G., Reys, R.E., Wilson, J.W. (1976). Addition and multiplication with fractions, Arithmetic Teacher, 23(2): 137-141.

Chalouh, L. & Herscovics, N. (1988). Teaching Algebraic Expressions in a Meaningful Way. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp33-42). NCTM 1988 yearbook.

Clements, M.A. (1980). Analysing Children's Errors on Written Mathematical Tasks, Education studies in mathematics, 11 (1):1-21.

Cobb, P. and Steffe, L.P. (1983). The Constructivist Researcher as Teacher and Model Builder, Journal for Research in Mathematics Education, 14 (2): 83-94.

Craig, A.P. & Winter, P.A. (1990). An Analysis of Learners' Engagement in Mathematical Tasks, South African Journal of Higher Education, 4 (1): 59-68.

D'Ambrosio, B and Campos, T.M.M., (1992). Pre-service Teachers' Representations of Childrens' Understanding of Mathematical Concepts: Conflicts and Conflict Resolution, Educational Studies in Mathematics, 23 (3):213-230.

Davis, R.B. (1984). Learning Mathematics, The Cognitive Science Approach to Mathematics Education. (pp.311-316) Ablex Publishing Corporation, New Jersey.

De Villiers, M. (1992). Inductive and deductive reasoning: logic and proof. In Moodley, M. et al (Ed.), Mathematics Education for in-service and pre-service teachers, (pp.45-59), Shuter and Shooter, Pietermaritzburg.

Dienes, Z.P. (1973). The Six Stages in the Process of Learning Mathematics, National Foundation for Education Research in England and Wales, Windsor, Berks, England.

Du Toit, et al, (1988). A Position Paper on a New Mathematics Curriculum for the Primary School (Grade 1/Sub A/Class 1 to Standard 4), The Mathematical Association Of South Africa, July 1988.

Eggen, P.D., Kauchak, P.D. and Harder, R.J. (1979): Strategies for Teachers - Information Processing Models in the Classroom. Prentice Hall.

Eisenberg, T. and Dreyfus, T. (1988). Polynomials in the School Curriculum. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp 112-118). NCTM, 1988 yearbook.

Elawar, M.C. (1992). Effects of Teaching Metacognitive Skills to Students with Low Mathematics Ability; Teaching and Teacher education, 8 (2):109-121.

Elliot.J. (1968). A Practical Guide to Teaching and Learning. (pp.31-48), Maskew Miller Longman (Pty) Ltd, Cape Town.

Fennema, E. & Sherman, J. (1977). Sex-related differences in mathematics achievement, Spatial visualization and affective factors, American Educational Research Journal, 14: 51-71.

Fennema, E. & Sherman, J. (1978). Sex-related differences in mathematics achievement and related factors: A further study, Journal for Research in Mathematics Education, 9: 189-203.

Fennema, E.& Carpenter, T.P. (1981). Sex-related differences in mathematics. In Carbitt, M.K.(Ed.), Results from the Second Mathematics Assessment of the National Assessment of Educational Progress. Reston, VA: NCTM, 1981.

Fennema, E., Carpenter, T.P. and Peterson, P. (1991). Teachers' decision making and Cognitively guided instruction: A New Paradigm for Curriculum Development, Pythagoras, 27: 27-35.

Freire, P. (1970). Pedagogy of the Oppressed. New York, Continuum.

Gagne, R.M. (1977). The Conditions of Learning. Holt, Rinehart and Winston, New York.

Gagne, R.M. (1983). Some Issues in the Psychology of Mathematics Instruction, Journal for Research in Mathematics Education, 14(1):7-18.

Garofalo, J. (1989). Beliefs and Their Influence on Mathematical Performance, Mathematics Teacher, 82(7):502-505.

Gonin, A.A., et al (1989). Modern Graded Mathematics Standard 9, For Higher and Standard Grade, New syllabus 1986. Nasou Limited.

Heritage, R.S. (1980(a)).Concept Analysis (What is a Mathematical concept?). In Wain, G.T. and Woodrow, D.(Ed); Mathematics Teachers Education Project Students' Material. (pp.61-62), Blackie and Son limited, London.

Heritage, R.S. (1980(b)). Schematic Analysis. In Wain, G.T. and Woodrow, D.(Ed); Mathematics Teachers Education Project Students' Material. (pp.62-66), Blackie and Son limited, London.

Holdan, G.(1988). Making Algebra Homework More Effective. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp 242-247). NCTM, 1988 yearbook.

Hoyles, C.(1985): What Is The Point Of Group Discussion In Mathematics?, Educational Studies In Mathematics, 16 (2):205-214.

Imenda, S.N.(1991). Assessment: Analysis of test scores. Staff Development Bulletin, 2(1); 13-20. University of Transkei.

Imenda, S.N. and Grewal, A.S. (In print): Application of Item Analysis in Student Academic Support Interventions: The Case of Special Mathematics Students at Unitra, International Journal of Mathematical Education in Science and Technology, accepted on 13th November, 1993.

Johnson, P.E. (1990). Effect of frequent testing on learning mathematics; International Journal of Mathematical Education in Science and Technology, 21 (5):733-737.

Karen, C.F. and Diane, J.B.,(1990). Using a base-ten block learning/teaching approach for First - and second - grade place-value and multidigit addition and subtraction, Journal for Research in Mathematics Education, 21 (3):180-206.

Leinhardt, G., Zaslavsky, O. and Stein M.K.(1990). Functions, Graphs, and Graphing: Tasks, Learning, and Teaching; Review of Educational Research, 60 (1):1-64.

Lindgreen, H.C. (1984). Discovery Learning, Encyclopedia of Psychology, 1, John Wiley and sons, New York.

Lochhead, J., Mestre, J.P. (1988). From Words to Algebra: Mending Misconceptions. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp 127-135). NCTM, 1988 yearbook.

Maher, C.A., Paca, J.P. & Pancari, J. (1988). Integrating Statistical Applications in the Learning of Algebra through Problem Solving. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12, (pp 223-228). NCTM, 1988 yearbook.

Mammen, K.J., Imenda, S.N. and Grewal, A.S. (1992): Application of item analysis in student academic support. Staff Development Bulletin, 3(1): University of Transkei.

Markovits, Z., Eylon, B.S., Bruckheimer, M. (1988). Difficulties Students Have with the Function Concept. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp 43-60). NCTM, 1988 yearbook.

Matshayana, V.Z., (1985). The assessment of the attitudes of standard 10 students of Engcobo Senior Secondary Schools towards mathematics, B.Ed. Thesis, University of Transkei, Transkei.

Merlyn, J.B.et al, (1983). Rational Number Concepts. In Richard Lesh, Marsha Landau (Ed), Acquisition of Mathematics Concepts and Processes,(pp.91-126). Academic Press, New York.

Minium, E.W. (1970). Statistical Reasoning in Psychology and Education , John Wiley & Sons, New York.

Moodley, M. (1990). Teaching/Learning Mathematics: What Counts?; Inaugural Address, Delivered at the University of Durban- Westville on 8 November 1990.

Moodley, M. (1992(a)): Questioning skills. In Moodley, M. et al (Ed.), Mathematics Education for inservice and pre-service teachers, (pp.135-143), Shuter and Shooter, Pietermaritzburg.

Moodley, M. (1992(b)): Teaching/Learning Mathematics: What Counts? In Moodley, M. et al (Ed.), Mathematics Education for inservice and pre-service teachers, (pp.1-16), Shuter and Shooter, Pietermaritzburg.

Moodley, M. (1992(c)). Objectives in the teaching/learning of mathematics. In Moodley, M. et al (Ed.), Mathematics Education for inservice and pre-service teachers, (pp.98-110), Shuter and Shooter, Pietermaritzburg.

Muthukrishna, N. (1994). Meatcognitive theory: A framework for mathematics instruction, Pythagoras, 33: 37-41.

Mwamwenda, T.S. (1989). <u>Educational Psychology, An African Perspective</u>; Butterworths, Durban.

Ngubentombi, S.V.S.(1989). <u>Education in the Republic of Transkei: Some Origins, Issues, Trends and Challenges</u>; Academica, Cape Town.

Njisane, R.A. (1992). Constructivism. In Moodley, M. <u>et al</u> (Ed.), <u>Mathematics Education for inservice and pre-service teachers</u>, (pp.26-37), Shuter and Shooter, Pietermaritzburg.

Nyamende, Z.H., (1990). The factors that contribute to the attitudes of standard seven pupils in mathematics in the Matatiele district, <u>B.ED.Thesis, University of Transkei</u>, Transkei.

O'Daffer, P.G., (1990). Inductive and deductive reasoning; <u>Mathematics Teacher</u>, <u>83</u>,(5):378-380.

Olivier, A.I. (1989). Handling pupils' misconceptions; <u>Pythagoras</u>, <u>21:</u> 19-21.

Olivier, A.I.,(1992). Handling pupils' misconceptions. In Moodley, M. <u>et al</u> (Ed.), <u>Mathematics Education for inservice and pre-service teachers</u>, (pp.193-209), Shuter and Shooter, Pietermaritzburg.

Olivier, A.; Murray, H. & Human, P. (1992). Problem-centred learning: The case of division, <u>Pythagoras</u>, <u>28</u>: 33-36.

Paul, E., (1985). The number line as a teaching aid, <u>Educational Studies in Mathematics</u>, <u>16</u> (4):411-424.

Philipp, R.A., (1992). The many uses of algebraic variables, <u>Mathematics Teacher</u>, <u>85</u> (7):557-561.

Polya, G., (1971). Mathematical Discovery. New York: John Wiley and sons.

Reidesel, C.A. and Burns, P.C. (1977). Handbook for Exploratory and Systematic Teaching of Elementary School Mathematics, Harper and Row, San Francisco.

Schifter, D. & Simon, A.M.,(1992). Assessing Teachers' development of a Constructivist View of Mathematics Learning, Teaching and Teacher Education, 8 (2):187-197.

Schoenfeld, A.H., (1987). What's All the Fuss about Metacognition?. In A.H. Schoenfeld (Ed.), Cognitive Science and Mathematics Education,(pp189-235). Lawrence Erlbaum Associates.

Schoen, H.L. (1988). Teaching Elementary Algebra With a Word Problem Focus. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp 119-126). NCTM, 1988 yearbook.

Shuard, H., (1986). Primary Mathematics towards 2000, The Mathematical Gazette, 70 (453):175-185.

Skemp, R.R., (1971). The Psychology of Learning Mathematics. London: (Penguin) Hazell Watson and Viney Ltd.

Skemp, R.R., (1976). Relational Understanding and Instrumental Understanding, Mathematics Teaching, December 1976:20-26.

Skinner, B.F., (1953). Science and Human Behaviour, Macmillan, New York.

Sowder, J.T. and Wheeler, M.M., (1989). The Development of Concepts and Strategies used in computational Estimation, Journal for Research in Mathematics Education, 20 (2):130-145.

Stanley, J.C. and Hopkins, K.D. (1972). Educational and psychology measurement and evaluation. London: Prentice Hall International.

Stiff, L.V., (1989). Effects of Teaching Strategy, Relevant Knowledge, and Strategy length on Learning a Contrived Mathematical Concept, Journal for Research in Mathematics Education, 20 (3):227-241.

Stoker, J. (1991). Constructing mathematical meaning through language: A way Forward in Primary Mathematics, Pythagoras, 25: 28-30.

Thompson, A.G., (1984). The Relationship of Teachers' Conceptions of Mathematics and Mathematics Teaching to Instructional Practice, Educational Studies in Mathematics, 15 (2):105-127.

Thomas, H.L. (1975). The Concept Of Function. In M.F. Rosskopf (Ed.), Children's Mathematical concepts, pp145-171. Teachers College, Columbia University.

Usiskin, Z., (1988). Conceptions of School Algebra and Uses of Variables. In Coxford, A.F & Shulte, A.P.(Ed.), The Ideas of Algebra, K-12,(pp 8-19). NCTM, 1988 yearbook.

Wagner, S., (1981). Conservation of Equation and Function under Transformation of Variable, Journal for Research in Mathematics Education, 12; 107-118.

Wenger, R.H., (1987). Cognitive Science and Algebra Learning. In A.H. Schoenfeld (Ed). Cognitive Science and Mathematics Education, University of California.

Wertheimer, M., (1959). Productive Thinking; Enlarged edition edited by Michael Wertheimer, New York, Harper.

Wollman, W., (1983). Determining the Sources of Error in a Translation from Sentence to Equation, Journal for Research in Mathematics Education, 14 (3):169-181.

APPENDIX - 1

Relevant sections of Senior Secondary Course: Syllabi for Mathematics Higher Grade, and Standard Grade. (Provincial Administration of the Cape of Good Hope: THE EDUCATION GAZETTE, PART LXXXIII No 7; 19 July 1984; pp 344-353)

2. **AIMS** (Higher Grade: p344)

2.3 To develop clarity of thought and the ability to make logical deductions.

2.4 To develop accuracy and mathematical insight.

5.1.3.1 Graphical representation of the function defined by:
(a) $y = ax^2 + bx + c$ $(a \neq 0)$

5.1.3.2 The deduction of the characteristics of the functions in 5.1.3.1 from their equations and graphical representation

5.1.3.3 Graphical representation of simultaneous equalities with respect to functions from 5.1.3.1 including their intersection with $ax + by + c = 0$

5.1.5 Quadratic equations and inequalities.

5.1.5.1 The roots of $ax^2 + bx + c = 0$ where a, b and c are rational.
(a) The solution of $ax^2 + bx + c = 0$;
(b) Conditions for which the equation is solvable on the set of real numbers;
(c) Equal and unequal roots; rational and irrational roots; real and non-real roots.

5.1.5.2 The solution of $ax^2 + bx + c < \text{or} > 0$.

5.1.5.3 Problems which lead to quadratic equations.

2. **AIMS** (Standard Grade: p350)

2.3 To develop clarity of thought, so that mathematical techniques may be well understood.

2.4 To develop accuracy.

5.1.2.1 Graphical representation of the function defined by:

$y = ax^2 + bx + c$ $(a \neq 0)$

5.1.2.2 The deduction of the characteristics of the functions in 5.1.2.1 from its equations and graphical representation

5.1.2.3 Graphical representation of simultaneous equalities with respect to functions from 5.1.2.1 and the function defined by $ax + by + c = 0$

5.1.3 Quadratic equations

5.1.5.1 The roots of $ax^2 + bx + c = 0$ where a, b and c are rational.

(a) The solution of $ax^2 + bx + c = 0$;

(b) Conditions for which the equation is solvable on the set of real numbers;

(c) Equal and unequal roots; rational and irrational roots; real and non-real roots.

APPENDIX - 2

TEACHERS' QUESTIONNAIRE:

PREAMBLE

Different teachers teach under different conditions or restrictions which may include, student characteristics, class size, teaching load, as well as other environmental factors. These factors may influence classroom transactions.

My aim is to obtain some information about some conditions that may be applicable in your teaching of Mathematics, in general, and in particular, the teaching of standard 10 students. There is no statement in this questionnaire which is either right or wrong. They are a matter of personal conviction or professional choice.

INSTRUCTIONS

Please, respond to each of the following statements as it applies to teaching and learning of Mathematics in your class, by ticking in the appropriate space corresponding to your view. The information supplied by you will be treated as strictly confidential. A space is left for your convenience to indicate the reason(s) for your answer. These reasons are just as important as the choice you are to indicate by your tick. If the space is insufficient, you may use the reverse side of the questionnaire indicating the section and item number to which your comments refer.

KEY

SA - Strongly Agree

AG - Agree

NT - Neutral

DA - Disagree

SD - Strongly Disagree

SECTION A. <u>GENERAL CLASSROOM TRANSACTIONS</u>

1. Mathematics can be taught by the discovery method.

   Reason(s) -------------------------

SA	AG	NT	DA	SD

2. I **do not** encourage students to work in groups for various reasons.

   Reason(s) -------------------------

SA	AG	NT	DA	SD

3. My students always try to answer questions in class.

   Reason(s) -------------------------

SA	AG	NT	DA	SD

4. I **do not find it necessary** to ask my students questions in class for various reasons.

SA	AG	NT	DA	SD

Reason(s) --------------------------

----

5. I enjoy writing on the board and explaining mathematical concepts to my students through out my Mathematics periods.

SA	AG	NT	DA	SD

Reason(s) --------------------------

----

6. I believe that my students **need not** worry about mathematical reasoning but should learn to use the procedures given to them.

SA	AG	NT	DA	SD

Reason(s) --------------------------

The Mathematics syllabus is so vast that (7 - 11):

7. I **can not avoid** teaching Mathematics based on what the examiners want from the students.

SA	AG	NT	DA	SD

Other reason(s) --------------------

----

8. I **do not** get enough time for discussions in class.

SA	AG	NT	DA	SD

Other reason(s)--------------------

----

9. there is **no time** for problem solving sessions.

SA	AG	NT	DA	SD

Other reason(s)--------------------

----

10. there is **no opportunity** to teach Mathematics by following discovery methods.

SA	AG	NT	DA	SD

Other reason(s)--------------------

----

11. I **do not get time** in class to teach my students lower level Mathematics even though I feel that they need that help.

SA	AG	NT	DA	SD

Other reason(s)--------------------

----

12. The department of education, school principal and the parents want their children to pass matric Mathematics examination, therefore I try to push my students through the matric examination.

Other reason(s) --------------------

SA	AG	NT	DA	SD

13. My students follow my explanations.

Reason(s) -------------------------

SA	AG	NT	DA	SD

I have to teach so many students that (14 - 15):

14. I **do not get time** to look into individual students' difficulties in Mathematics.

Other reason(s) -------------------

SA	AG	NT	DA	SD

15. it is **not possible** to give my students progress tests at the end of each topic.

Other reason(s) --------------------

SA	AG	NT	DA	SD

16. I **do not give** my students' home work because they copy from one another.
Other reason(s) -------------------

SA	AG	NT	DA	SD

SECTION B TEACHING OF QUADRATIC EQUATIONS INEQUALITIES AND THEIR GRAPHS.

17. I ask my students to memorize the quadratic formula by giving them a lot of practice problems.

SA	AG	NT	DA	SD

18. I often tell my students that simple inequalities can be solved the same way as we solve simple equations except that inequality changes when multiplied by a negative number.

SA	AG	NT	DA	SD

19. Students find quadratic inequalities difficult so I **do not spend** much time teaching this section.

SA	AG	NT	DA	SD

20. Although students have a very poor background in graphical representations of the linear functions, this **does not influence** their understanding of graphs of quadratic functions.

SA	AG	NT	DA	SD

21. Students find it difficult to interpret graphs of quadratic functions, therefore I **do not lay much emphasis** on this topic.

SA	AG	NT	DA	SD

Possible reason(s) ----------------

----

22. **I do not see the need** for the majority of students to know why 'q' is the y- coordinate of the turning point of the graph of the function $\{(x;y): y = a(x - p)^2 + q\}$ as long as they can write it out from the above mentioned equation.

SA	AG	NT	DA	SD

23. For proper understanding of graphs of quadratic functions students should be made to draw a number of graphs with different positions on a cartesian plane.

SA	AG	NT	DA	SD

24. The examiner **does not give much importance** to the sketching of the graphs of quadratic functions, so I **do not find it necessary** that the students should worry about drawing these graphs.

SA	AG	NT	DA	SD

25. I **do not teach** my students graphical solutions of quadratic equations.

SA	AG	NT	DA	SD

Reason(s) -------------------------

----

26. Of the two, graphing and solving a quadratic equation, solving is very important.

SA	AG	NT	DA	SD

Reason(s) -------------------------

----

27 . The plotting of graphs of quadratic expressions is a waste of time.

Reason(s) --------------------------

SA	AG	NT	DA	SD

Thank you very much.

APPENDIX - 3

TEST ON QUADRATIC THEORY: Equations, Inequalities and graphs.

PERSONAL PARTICULARS

(To be filled in by the candidate)

SURNAME: ....

NAMES: ....

NAME OF YOUR SCHOOL: ....

....

HOME/ CONTACT ADDRESS: ....

....

....

....

....

TELEPHONE NUMBER: ....

TIME ALLOWED: 2 HOURS 30 MINUTES.

Diagnostic Test on:

<u>Theory of Quadratic equations, inequalities and their graphs.</u>

NOTE: PLEASE ANSWER ALL QUESTIONS. <u>Time: 2 Hrs 30 minutes.</u>

<u>All answers should be written in the spaces provided. If you need more working space for any question you can use the extra sheet attached at the end.</u>

<u>SECTION 1.</u>

<u>Instructions:</u>

Please read the following statements carefully and categorise each of them under the following categories.

A. False

B. True, but not always

C. True

Write the corresponding letter in the space provided. Give reason(s) for your answer in the spaces provided. If your choice is '<u>true, but not always</u>' then give conditions when the statement is not true.

If your choice is '<u>false</u>' then write down your corrected statement.

---

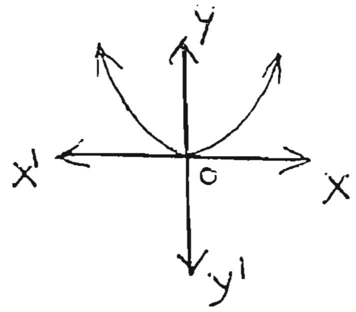
1. The expression $-x^2 + 2x - 5$ can not be equal to zero for real values of x.

Your answer:--------------.

Show your working and/or give reasons for your answer:

----

2. In $f(x) = ax^2 + bx + c$; a, b, c = 0.
The following graph is a rough sketch of f(x).



Your answer: ------------.

Show your working and/or give reasons for your answer:

----

3. $35a^2 - 50a + 430$ can be written as the product of $(7a - 10)$ and $(5a - 43)$.

Your answer: -------------

Show your working and/or give reasons for your answer:

----

5. The equation $(2x - 5)/(2 - 5x) = x;\ x \neq 2/5$ will reduce to a quadratic equation whose roots will satisfy it.

Your answer:-------------

Show your working and/or give reasons for your answer:

---

6. The expression $4x^2 - 20x + 25$ is not a complete square.

Your answer:------------

Show your working and/or give reasons for your answer:

---

7. The root(s) of an equation is (are) the same as solution(s) of the equation.

Your answer: -------------

Show your working and/or give reasons for your answer:

---

8. The root(s) of an equation do(es) not always satisfy the equation.

Your answer:--------------

Show your working and/or give reasons for your answer:

---

9. If the graph of $f(x) = ax^2 + bx + c$ intersects y-axis it means that $ax^2 + bx + c = 0$ has two distinct real roots.

Your answer:---------------

Show your working and/or give reasons for your answer:

---

10. If the graph of $y = ax^2 + bx + c$ intersects x-axis at two points, it means that $ax^2 + bx + c = 0$ does not have two distinct real roots.

Your answer:-------------

Show your working and/or give reasons for your answer:

---

11. If $(2x + 1)(x + 2) < 0$ then we can conclude that both $2x + 1 < 0$ or $x + 2 > 0$; And $2x + 1 > 0$ or $x + 2 < 0$ are possible.

Your answer:------------

Show your working and/or give reasons for your answer:

---

12. The sum of a number and square of half the number is 48. If the number is x then:

$$1/2\ x^2 + x = 48.$$

Your answer:------------

Show your working and/or give reasons for your answer:

----

14. The statement: [$x < -1$ and $x < -4$] is equivalent to: $x < -4$.

Your answer:---------------

Show your working and/or give reasons for your answer:

----

15. The statement: [$x < 5$ or $x > -3$] is equivalent to: $x > -3$.

Your answer:--------------

Show your working and/or give reasons for your answer:

----

17. The statement: (-2x+3) > (x+2) is equivalent to: $x > 1/3$.

Your answer:---------------

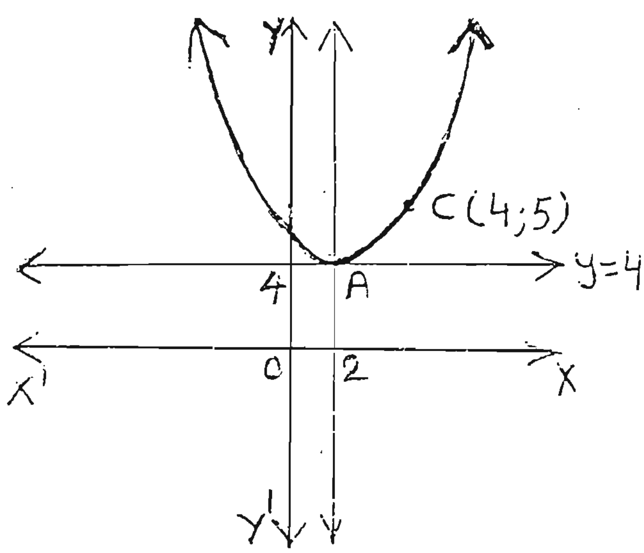
Show your working and/or give reasons for your answer:

----

18. Turning point A of the parabola shown in the graph lies on the intersection of lines y = 4 and x = 2. Point C (4 ; 5) lies on the parabola.

Therefore, the defining equation of the parabola is:

$y = x^2 - 4x + 8$.

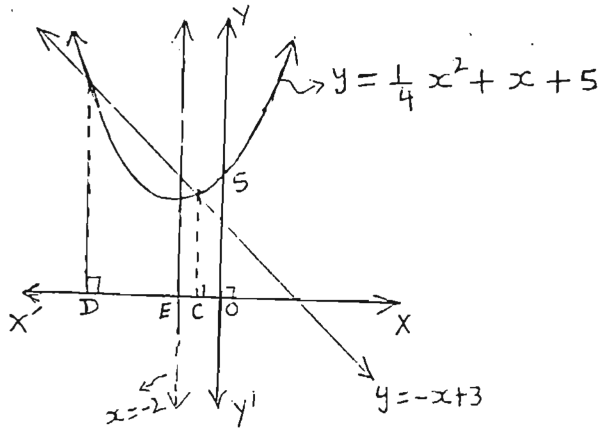


Your answer:--------------

Show your working and/or give reasons for your answer:

----

19. In the graph given below, the distances OC and OD represent the roots of equation $x^2 + 8x + 8 = 0$.



Your answer:-------------

Show your working and/or give reasons for your answer:

----

20. Defining equation of x-axis is, $x = 0$ and that of y-axis is, $y = 0$.

Your answer:------------

Show your working and/or give reasons for your answer:

----

21. To find turning point of the parabola defined by $\{ (x ; y): y = 2x^2 - x + 3 \}$ we write $y = 2x^2 - x + 3$ in $y = a(x-p)^2 + q$ form and from that we conclude that the turning point is: $(-p ; q)$.

Your answer:-------------

Show your working and/or give reasons for your answer:

SECTION 2.

Read the following questions carefully and choose the best answer; and write the corresponding letter in the spaces provided.

22. If $(x - 1)(y + 2) = 0$, then:

(A) $x = 1$

(B) $y = -2$

(C) $x = -1$ or $y = 2$

(D) $x = 1$ or $y = -2$

(E) $x = -1$ and $y = -2$

Your answer: -----------------

Show your working and/or give reasons for your answer:

----

23. If $(x + 2)(x - 3) = -4$, then:

(A) $x = -1$ only.

(B) $x = -6$ only.

(C) $x = -1$ or $x = -6$.

(D) $x = -1$ and $x = 2$.

(E) $x = -1$ or $x = 2$.

Your answer: -----------------

Show your working and/or give reasons for your answer:

----

24. If $12x^2 = 6x$, then:

(A) $x = 0.5$

(B) $x = 0.5$ and $x = 0$

(C) $x = 0.5$ or $x = 0$.

(D) $x = 2$.

(E) $x = 2$ or $x = 0$.

Your answer ---------

Show your working and/or give reason(s) for your answer:

----

25. If $(x + 2)^2 = 9$, then:

(A) $x = 7$

(B) $x = 1$

(C) $x = 1$ or $x = -5$.

(D) $x = 5$ or $x = -1$.

(E) $x = 5$ and $x = -1$. Your answer --------------

Show your working and/or give reasons for your answer:

----

26. If $bx^2 + cx + a = 0$, then:

(A) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(B) $x = \dfrac{-c \pm \sqrt{c^2 - 4ab}}{2b}$

(C) $x = -b \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$

(D) $x = -c \pm \dfrac{\sqrt{c^2 - 4ab}}{2b}$

(E) None of the above.

Your answer ------------------

Show your working and/or give reason(s) below :

----

27. If m is a root of $dx^2 - ef + x = 0$, then:

(*A*) $m = \dfrac{ef \pm \sqrt{e^2 f^2 - 4d}}{2d}$

(*B*) $dm^2 + m - ef = 0$

(*C*) $m = ef \pm \dfrac{\sqrt{e^2 f^2 - 4d}}{2d}$

(*D*) $m = \dfrac{-1 \pm \sqrt{1 - 4def}}{2d}$

(*E*) *None of the above.*

Your answer --------------

Show your working and/or give reasons for your answer:

----

28. If $(x + 6)^2 = 10^2$, then:

(A) $x^2 + 36 = 100$ and so $x = 8$.

(B) $x^2 + 36 = 100$ and so $x = 8$ or $x = -8$.

(C) $x + 6 = 10$ and so $x = 4$.

(D) $x + 6 = +10$ or -10 and so $x = 4$ or $x = -16$.

(E) $x + 6 = 10$ or -10 and so $x = 4$ and $x =-16$.

Your answer ---------

Show your working and/or give reasons below:

---

29. The equation $(x + 2)^2 = (x - 2)^2$ has:

(A) No real root.

(B) 1 real root.

(C) 2 real roots.

(D) 3 real roots.

(E) More than three real roots.

Your answer -----------

Show your working and/or give reason(s) below:

---

30. The equation $x^2 + 16 = 0$ has:

(A) No real roots.

(B) 1 real root.

(C) 2 real roots.

(D) 3 real roots.

(E) More than 3 real roots.

Your answer ------------

Show your working and/or give reason(s) below:

----

31. Discriminant of equation $t^2 = 16$ is:

(A) imaginary.

(B) 0.

(C) 16.

(D) 32.

(E) 64.

Your answer ------------

Show your working and/or give reason(s) below:

----

32. If 1 and 3 are roots of equation $x^2 + px + s = p$, then the value of s is :

(A) 3
(B) -1
(C) 7
(D) 2
(E) -4

Your answer -------------

Show your working and/or give reason(s) below:

---

33. If 2 is one of the roots of equation $-x^2 - 8x - k = 0$, then:

(A) $-x^2 - 8x - 12 = 0$.
(B) $-x^2 - 8x + 12 = 0$.
(C) $-x^2 - 8x - 20 = 0$.
(D) $-x^2 - 8x + 20 = 0$.
(E) $-x^2 - 8x - 2 = 0$.

Your answer -------------

Show your working and/or give reason(s) below:

---

34 $x = -2$ will satisfy the equation $x^2 = 8x - k$ only if k is equal to:

(A) 12

(B) -12

(C) -20

(D) 20

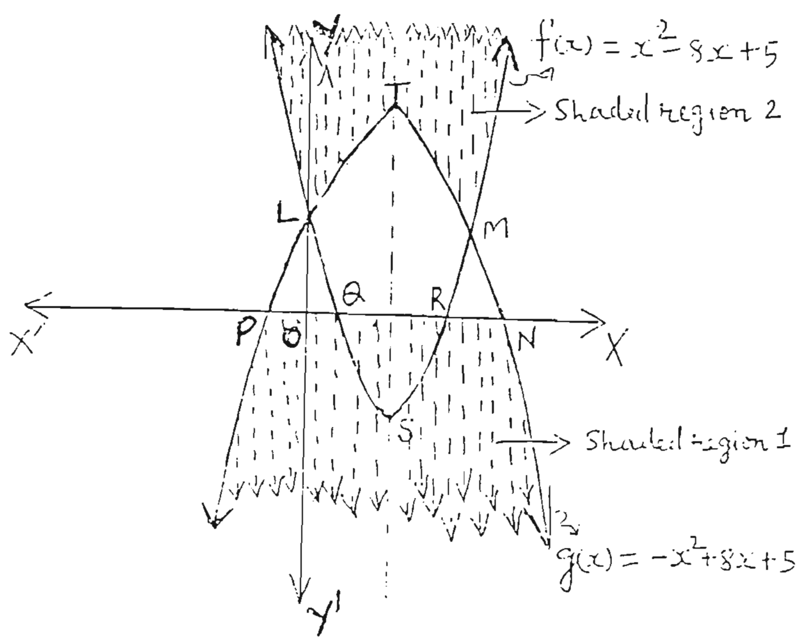
(E) 18

Your answer -------------

Show your working and/or give reason(s) below:

INSTRUCTIONS:

Refer to the following graph to answer questions number 37 to 42.



37. Solution set of $-x^2 + 8x + 5 \geq 0$ is represented by:

(A) the shaded region 1.

(B) the shaded region 2.

(C) the range of values of x represented by the interval (P,N).

(D) the range of values of x represented by the interval (Q,R).

(E) x co-ordinates of the points L and M.

Your answer:------------

Show your working and/or give reasons for your answer:

----

38. Solution set:

$\{(x;y): y \leq 0\} \cap \{ (x;g(x)): g(x) \leq -x^2 + 8x + 5 \}$ is represented by:

(A) x co-ordinates of the points L and M.

(B) the range of values of x represented by the interval (P,N).

(C) the range of values of x represented by the interval (Q,R).

(D) the shaded region 1.

(E) the shaded region 2.

Your answer:----------

Show your working and/or give reasons for your answer:

----

39. x co-ordinates of the points L and M are the solutions of the equation:

(A) $x^2 - 8x + 5 = 0$.

(B) $-x^2 + 8x + 5 = 0$.

(C) $x^2 - 8x = 0$.

(D) $-x^2 + 8x = 0$.

(E) none of the above.

Your answer:-----------

Show your working and/or give reasons for your answer:

----

40. The solution set:

$\{(x;g(x)):\ g(x) \geq -x^2+8x+5\} \cap \{(x;f(x)):\ f(x) \geq x^2-8x+5\}$

is represented by:

(A) the range of values of x represented by the interval (P,N).

(B) the range of values of x represented by the interval (Q,R).

(C) x co-ordinates of the points L and M.

(D) the shaded region 1.

(E) the shaded region 2.

Your answer:------------

Show your working and/or give reasons for your answer:

----

41. The solution set:

$\{(x;y): y = 0\} \cap \{(x; g(x)): g(x) = -x^2+8x+5\}$

is represented by:

(A) distances OQ and OR.

(B) distances OQ and ON.

(C) distances OP and ON.

(D) distances OR and ON.

(E) distances OP and OR.

Your answer:------------

Show your working and/or give reasons for your answer:

----

42. The function(s) that has/have undefined minimum value is/are:

(A) f(x) and g(x).

(B) f(x).

(C) g(x).

(D) h(x) = f(x) - g(x).

(E) h(x) = f(x) + g(x).

Your answer:------------

Show your working and/or give reasons for your answer:

Answer the following question in the space provided:

43. Find two possible numbers whose sum is 18 and whose product is the largest possible.