The Promotion of Mathematical Proficiency in Grade 6 Mathematics classes from the Umgungundlovu district in KwaZulu-Natal

by

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Abstract

The research conducted in this study is inextricably linked to a larger study of teacher quality and student performance in KwaZulu-Natal. The aim of the larger study was to explore and establish the relationship between teachers' mathematical content knowledge, teachers' practice and learner outcomes in grade 6 mathematics classrooms. This meant ascertaining teachers' mathematical content knowledge, teachers' pedagogical content knowledge and teachers' practice in mathematics classrooms. Videos of lessons were analysed for the following aspects: content coverage, mathematical proficiencies facilitated by the teacher, cognitive demand on learners and teachers' content knowledge. The analyses of all aspects were initiated at the same time, with different researchers/post-graduate students coding for separate aspects.

In this study, the notion of mathematical proficiency as originally developed by Kilpatrick and colleagues (Kilpatrick, Swafford, & Findell, 2001) was used to ascertain the promotion of the strands in the district of Umgungundlovu of KwaZulu-Natal. Essentially the larger study hoped to establish the prevalence and quality of these strands by viewing video recordings of lessons obtained from schools. This in turn would present a view on mathematics learning in the district. The larger study used random stratified sampling to identify schools after which the necessary ethical approval and clearance was obtained. Mathematics lessons of the identified schools were then video-taped and questionnaires and both teacher and learner tests were conducted. I have not included examples of test questions due to agreements about not reproducing these.

However, analysis of the recordings, in my view required the formulation of a construct that would interrogate the extent to which the strands of mathematical proficiency are promoted. This was necessary since the five strands in the original formulation represent 'goals of mathematical understanding. 'In order to achieve these goals, tangible evidence of teacher classroom practice must be observable. Using opportunities as a vehicle of identification of such practice, the notion was formulated. The analytical framework entrenches the notion of 'opportunity to develop mathematical proficiency' as a construct with its corresponding descriptor table and is the main feature of this study. This in turn informed the design of the instrument which reflected the notion introduced and allowed

ease of use. The research was not simply finding instances of what the instrument describes, but also trailing the applicability and strength of the instrument and the underlying notion of 'opportunities to develop mathematical proficiency'.

The findings reflect the current state of the promotion of mathematical proficiency. Not only is the quality of the promotion weak it is also irregular. An important off spin of the results is the alignment of these results to many studies including the recent 'Report on the Annual National Assessments 2011' issued by the Department of Basic Education.

The notion introduced in this study with its corresponding analytic scoring method indeed proved to be a useful key to unravelling the answers to the questions posed. The results and findings give a detailed description to the aspect of mathematical proficiencies facilitated by the teacher, one of the aspects the larger study aimed to explore and establish. In this respect, it also shows the applicability and relevance of the developed theoretical notion and the related instrument.

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Thoughts and thanks to my wife and children who have endured and stood by me throughout the duration of the research. This research is dedicated to my wife Lisa and children Tasneem and Zaheera.

Declaration

I, Noor Ally, declare that:

i. The research reported in this dissertation, except where otherwise indicated, is my own work.

ii. This dissertation has not been submitted previously for any degree or examination at any university or other higher education institution.

iii. This dissertation does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.

Signed:

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Chapter 1: Introduction

Reading through the extensive literature on mathematics education, an overwhelming sense of personal identification in a number of areas became evident. Specifically, curiosity centred on the mathematics knowledge and skills offered to learners during the course of a lesson as well as the depth of the mathematics imparted or discussed by teachers.

Teaching mathematics in a secondary school for many years and lecturing at a tertiary institution provided an extensive background in mathematics teaching to realise the complicated nature of the classroom environment. To create a norm in teaching mathematics was fraught with problems. Each mathematics class had its unique characteristics comprised of the collective personalities of the learners in the class. After negotiating that obstacle, which was achieved more readily with experience, the task of teaching mathematics for understanding was an even greater barrier. Every learner’s idea of any aspect of mathematics was shaped by their encounter with the subject at lower levels besides many other influences. Introducing each section of the mathematics syllabus at the secondary school level and now at a tertiary institution focussing on mathematics for engineering students presented obstacles simply because learners mathematical background knowledge was fundamentally flawed. The lessons were punctuated with episodes which sought to clarify basic mathematical concepts that learners are expected to possess. Ultimately, such learners progressed through a system which provided a ‘false’ sense of mathematical ability encountered at the primary school level. An opportunity to provide some answer to this presented itself when a large study with the project title, ‘A study of Teacher Quality and Student Performance in KwaZulu-Natal’ was to be conducted in grade 6 mathematics classes. This study was part of a much larger international study which aimed to investigate student academic performance along the South Africa and Botswana border and included grade 6 mathematics classes from North West province as well as the Western Cape and KwaZulu-Natal. An important phase of the larger study was teacher questionnaires and tests. Here information of teacher variables such as highest school qualification obtained or number of year’s mathematics teaching experience was noted. In addition a teachers test was administered. Learners also wrote a pre- and post- test to establish whether any learner gains, i.e. the difference between their first and second test scores, were made or not. Teacher questionnaires and test as well as learners' pre-and post test remained confidential at all times.

I was fortunate to be part of a group that would analyse data from the study conducted in KwaZulu-Natal. Other post-graduate students analysed the videos for the pedagogical content knowledge demonstrated by the teachers in the video recordings (Ramdhany, 2010); the opportunities to learn (Noubouth, forthcoming); the numeracy levels of the learners based on their performance on the tests (Maharaj, 2011); and the framing, pacing and classification of the lessons (Devcharran, forthcoming). I hoped that the participation would enlighten me and give me an idea of the reasons for poor mathematical understanding that students bring with them to university. This however, entailed viewing video recordings of the lessons and trying to ascertain the teaching quality. Mathematics in the public domain generally equates mathematical proficiency to 'skill in performing arithmetical calculations', an observation obtained from discussion with friends, parents, non-mathematics teachers and learners. Many encounters with artisans such as carpenters, bricklayers, boiler makers, electricians (many in my family) and fellow sportsmen in which on the job calculations are necessary, allude to the fact that mathematics in their job only involved arithmetical calculations. Repetitive procedure in these calculations served as a basis in their understanding of which calculations to use during particular situations which in turn equipped them with a degree of conceptual understanding allowing them to transfer that knowledge to unique situations. In other words, mathematical proficiency is more than just procedural work.

The term mathematical proficiency is aptly described by Kilpatrick et al (2001) in the book, 'Adding it up: Helping children Learn Mathematics.' Successful mathematics learning involves more than just procedural skills. Emphasis on mathematical procedures and skills has been the cornerstone of mathematics learning in classrooms for centuries. Learners who have mathematical proficiency should display attributes in mathematics that includes procedures, concept formulation, problem solving skills, reasoning ability and a positive disposition. The technological advances and job opportunities in the world today require a sound grasp of mathematics in more than just computational form. Kilpatrick and Swafford state, “Greater understanding of mathematics will be essential for today’s schoolchildren. Success in tomorrow’s job market will require more than computational competence. It will require the ability to apply mathematical knowledge to solve problems… [Students] need to have the mathematical sophistication that will enable them to take full advantage of the information and communication technologies that permeate our homes and workplaces. Students with a poor understanding of mathematics will have fewer opportunities to pursue higher levels of education and to compete for good jobs.” (Kilpatick and Swafford, 2002, p. 144).This term with its five strands provided the necessary theory that would allow me to analyse grade 6 mathematics lessons. A first encounter with coding created a personal dilemma. Here the presence of the strands of mathematical proficiency was acknowledged and recorded after discussion. At this stage I realised that viewing the lesson for the strands of proficiency appeared to be superficial and deeper engagement with this aspect was necessary. An approach was adopted that would mirror the mathematical experience of a learner in the lesson. An analogy to this could for instance be a principal ‘looking at’ one of the teachers in the school teaching mathematics as opposed to a learner ‘looking in’ and experiencing the mathematics taught by the teacher. This led to the development and formulation of a notion which is used principally in this study.

The theme of the research is thus looking for empirical evidence of the promotion of the five strands of mathematical proficiency. It is therefore framed within the larger study and will provide evidence of the quality of the mathematics discussed by adopting the theory of mathematical proficiency.

In Chapter 2: Literature Review, a historical perspective is adopted. Through the last century, an idea of how the term mathematical proficiency was regarded and how researchers change their views is explored. In this chapter the research questions are informed to a large extent by the bigger study. In Chapter 3: Theoretical Framework, the five strands are investigated in depth. Much of this chapter is informed by the conception of the term 'mathematical proficiency', what it entails and the realisation that it is composed of five strands that are inextricably linked. Identification of each strand is discussed and the manner in which each strand is seen in classroom practice is explained.

Initially coding videos using this coding procedure created an innate desire to find a coding description that suited and captured the essence of the research. This was formulated into a notion in which 'opportunities to develop mathematical proficiency' played a critical role. This notion, in the context of my study, allowed me to view the lesson through the opportunities afforded by the teacher. The theory proposed is explained in chapter 4, the Analytical Framework. The concept is explained and coding of a lesson is through a descriptor table in which description of each strand is detailed. The table incorporates the main features of the notion and provide a suitable and user friendly manner for coding lessons. Acronyms abound in this chapter and will be used prolifically throughout the remainder of the study.

In chapter 5, details of the design of the study, data collection procedures, classroom observation instruments used and method of coding will be highlighted. The appropriateness of video-taping for this study will be interrogated. Classroom observation using this method of collection is a rich source of information. The development of the instrument eventually used is recorded and explained. The final version shows an instrument that tries to capture as much of the mathematical content discussed in a lesson. A key component of the instrument is a section that describes episodes in the class that attempts to justify the coding of the strands of mathematical proficiency.

In order to synchronise the coding, chapter 6 contains extracts from the lessons. These extracts attempt to correlate the coding with the mathematical quality during classroom activity. The teaching presents opportunities which inevitably affect learners' understanding, though perhaps not in the desired or intended ways. Extracts relating these perceived opportunities are presented and discussed. The coding is informed by the descriptor table and these narratives provide the necessary information that synchronises classroom discourse with the descriptor table. The latter section of this chapter explains the manner in which class exercises, seated or group work was coded.

Results and analysis of the study is discussed over two chapters. Reasons for this include the theoretical and analytical frameworks as well as the research questions. The study investigates the promotion of mathematical proficiency in the district of Umgungundlovu of KwaZulu-Natal. Data from individual lessons are combined to give a total for the district which is then analysed as a whole. Hence, chapter 7 records the results of lessons across the entire district which is then analysed and mainly used to answer the first two research questions. Important in this analysis is the analytical scoring introduced and explained. This was crucial in the analysis in both the chapters. Data analysis is based on simple descriptive statistics. Tables and figures are used throughout the two chapters.

Individual lessons are analysed in chapter 8 followed by discussions. Comparisons are made and investigated further. The scoring, incorporating the presence of, as well as the degree that opportunities abound in a lesson, is used extensively here to provide a clearer picture of the quality of mathematics teaching in these lessons. The results from this chapter are predominantly used to answer research question 3 and 4. Throughout the results and analyses chapters, information is taken directly from the instrument used to code individual lessons. Thus any observation made in these chapters can be verified directly with the information recorded in the instrument.

In attempting to answer the research questions reference had to be made to both chapter7 and 8, the results and analyses. It therefore made sense to place this in the final chapter, viz. the conclusion. In this chapter the research questions are discussed informed by the previous two chapters. Research question 2 is answered before the first and reasons for this are given. Correlation analysis and comparisons were done to answer the remaining two research questions. The findings of the study are then briefly mentioned and discussed followed by a summary on the study as a whole and the notion advanced in this study.

Chapter 2: Literature Review

2.1. The notion of 'mathematical proficiency'

The focus and purpose of study alludes to a disquieting aspect of "Mathematical Proficiency", viz. the wide and varied use of the term in different contexts. The search for a comprehensive and all-inclusive definition and understanding has reached the highest levels of government in many countries around the world. In the United States the 'No Child Left Behind' Act of 2001(NCLB) was legislated so that all children should be proficient in reading and Mathematics by the end of the 2013-14 school year. By Federal law in the United States, each state has the power to define proficiency resulting in 50 different definitions as pointed out by Samuels and Hoff(2007). They clearly indicate that the lack of guidance in defining the term resulted in mixed interpretations and a variety of disconnected standards between the various states. This led to the development of one set of national indicators which were used as attainment of 'levels of proficiency' as observed by Parker-Burgard (2009, p. 42). The use of these national indicators was problematic due to the variety of interpretations of the term proficiency.

The use of the term 'Mathematical Proficiency' in many article titles further confirms the need to agree upon a universal definition. Wide acceptance of an agreed upon definition will see this term used more constructively. To the ordinary individual not familiar with the dynamics of mathematics, its multiplicity of inter, intra and extra connections is lost. The term is thus loosely used in society giving all and sundry a sense of 'knowing mathematics' when conferred with the term 'proficient'. The construct of Mathematical Proficiency however, has wider implications and broader meaning and Boaler (2002) warns that if scholars ignore this, then "we may be reduced to the dominant ideology that pervades public rhetoric, in which Mathematical Proficiency is equated with the reproduction of isolated mathematical procedures" (Boaler, 2002, p. 17). She further contends that the prevailing dogma about what it means to know and be proficient in mathematics is extremely narrow in most countries.

The search for 'successful mathematics learning' provided the impetus for developing a construct for Mathematical Proficiency. The search has a protracted history stretching back to the early 1900's. Kilpatrick et al (2001) gives a brief description of this transformation as a result of changes in society as well as schooling, and I summarise this briefly here.

Computational fluency dominated teaching in the first half of the century. Many teachers emphasized the need to perform arithmetical calculations effortlessly, flawlessly and rapidly. Others related better to the need of learning procedures with underlying meaning. The movement in the 1950's and 1960's shifted the emphasis primarily towards the understanding of the mathematics (conceptually) and its related ideas. Disputes during this era were of an internal nature mainly in the mathematics and mathematical committees. In the 1980's, documents published in the USA by the NCTM shaped the nature of mathematics and its delivery. The extensive use of calculators placed detractors of classical tradition in a dominant position. The late 1980's and 1990's resulted in proponents of progressive education advocating a student-centred, 'real-world' approach to the learning of mathematics. Thus, basic maths skills and principles were learnt by problem-solving of 'out of class' situations. Klein (2007) suggests that these changes were the result of two themes, viz. social justice and the needs of business and industry (Klein, 2007, p. 24). Against this background, which "reflected different goals for school mathematics by different groups of people at different times" states Kilpatrick et al. (2001, p.115), the need to have a comprehensive understanding of successful mathematics became a focal point of mathematics education stakeholders (Kilpatrick et al, 2001, p. 115). Further evidence of the need to capture the essence of why learners need to become proficient, were results from the Third International Mathematics and Science Study conducted in 1995 and repeated in 1999. This study showed that American students were on average similar to their counterparts in three dozen other countries. Strategic planning was necessary to improve students' mathematical learning.

In 1999, the National Research Council of the USA convened a group of experts to review and examine research on effective mathematics learning. One of its goals was to define ‘successful mathematics learning’. It needed to characterize such learning and eventually settled on the term “Mathematical Proficiency defining it in terms of five interwoven and interdependent strands to be developed in concert” (Kilpatrick et al., 2001, p. 106). The five strands, which will be unpacked in Chapter 3, are:

- *Conceptual understanding*
- *Procedural fluency*
- *Strategic competence*
- *Adaptive reasoning*
- *Productive disposition*

Prior to this definition, searches for frameworks for the construct of mathematical understanding were evident in the USA and elsewhere. In the USA, the National Assessment of Educational Progress (NAEP) featured three of the strands, viz. conceptual understanding, procedural fluency and problem solving (Kilpatrick et al, 2001, p. 117). The Singapore Mathematics Framework emphasized concepts, skills, metacognition, processes (i.e. reasoning) and attitudes (Stacey, 2002, p. 297). In this framework, skills were defined as manipulative skills that learners were expected to perform when solving problems. Processes meant strategic problem solving strategies while meta-cognition described the ability to reflect on one’s own thinking. Attitudes on the other hand included such things as “finding joy in doing mathematics, appreciating the beauty and power of mathematics, showing confidence in using mathematics and persevering in problem solving,” (Ginsburg et al, 2005). The Singapore framework has much in common with the five strands of mathematical proficiency mentioned above. In the USA, the NCTM’s framework identified five core mathematical processes. These include problem solving, communication of mathematical ideas, reasoning and proof which covered logical thinking skills, representation as the ability to move from abstract concepts to symbols and connections within mathematical ideas and in contexts outside of mathematics. In probing what it meant to teach for understanding, Goodell (2000) used the five forms of mental activity as proposed by Carpenter and Lehrer (1999). These five forms are: constructing relationships, extending and applying mathematics knowledge, reflecting about experiences, articulating what one knows, and making mathematical knowledge one's own. This American study of pre-service maths teachers' methods course revolved around designing activities that reinforced the five forms. Further investigation of the frameworks mentioned is necessary to establish the extent that they share commonalities with the five strands of mathematical proficiency.

The overarching differences and similarities are partly due to the emphasis placed on the mathematics. The NCTM and NAEP emphasised concepts via representations, connections and reasoning and proof. Singapore's framework focussed on problem solving processes including computational skills and heuristics for problem solving. Closer inspection of the frameworks may indeed produce additional similarities and differences between themselves and others. However, this would entail further investigation and analysis.

The definition of Mathematical Proficiency as expressed in the book, 'Adding it Up' (Kilpatrick et al, 2001), has indeed had an impact on mathematics researchers, mathematics educators, curriculum developers, mathematics teacher colleges, cognitive scientists and other stakeholders involved in mathematics education. Pape writes, in reference to the new goals for mathematics education, "these goals have become widely embraced by educators and scholars in the international community" (Pape, 2003, p. 180). The generic formulation of Mathematical Proficiency allows for implementation as a framework in many scenarios. For instance, Lang (2008) investigated the interdisciplinary approach to teaching maths, science and technology. This curriculum integration stems largely from instruction that promotes real-world problem solving. The instructional approach used was based on supporting the five strands of proficiency. They included the challenge, the learning cycle, making connections, concepts in context and problem solving using DAPIC [Define, assess, plan, implement, communicate] (Lang, 2008, p. 3). She further contends that only conceptual understanding could be attained through the traditional instructional approach – a statement which puzzled me, since my experience is that much teaching of mathematics focuses on the procedural aspect. Clearly, we may have different understanding of what each term means.

2.2. Studies of mathematical proficiency

There are a multitude of studies which have used the notion of mathematical proficiency in some way or other – a search in the MATHEDU database gave 86 returns, from around the globe. Here, just a few are summarised, in particular those from Africa.

The approach used by Suh (2007) seemed to be more encompassing and portrayed the construct of Mathematical proficiency as envisioned by Kilpatrick et al. She designed classroom practices for her American students that promoted the five strands of proficiency. Activities given to students throughout the year were structured to build the strands. This provided the student with the opportunity to develop mathematical proficiency. The practices included 'modelling maths meaningfully', 'math curse', 'math happening', 'convince me and 'poster proof'. (Suh, 2007, pp. 164-167).

She used a practice of 'Modelling mathematics meaningfully' to teach for and assess conceptual understanding. Students represented their mathematical understanding in five different ways, viz. manipulatives, pictures, real-life contexts, verbal symbols and written symbols. Students wrote problems with numbers, drew pictures, wrote a real life story and explained how they solved a problem through manipulatives. In this way she claims that she was able to gauge their conceptual understanding by assessing their representations.

The second practice was termed 'Maths Curse'. The focus was on a productive disposition towards mathematics. She implemented this by initially reading a book entitled 'Maths Curse'. This tale was about a boy who woke up one morning to view every situation in life as a mathematics problem. She then told her students that they were under the 'Maths Curse' and expected them to bring a mathematics problem that they encountered and discuss it in class.

'Mathematics Happening' was another practice utilised. She presented a personal encounter with a 'Mathematics Happening' in such a way that students became interested in helping her solve the problem. In this way she hoped that students became more familiar with the concept of problem solving.

The last practice was titled ‘Convince me’ and ‘Poster Proofs’. Here she states, ‘to develop strategic competence and adaptive reasoning students need opportunities to share and compare their solution strategies and explore alternative solution paths (Suh, 2007, p. 167). These classroom activities were created to provide opportunities to exercise reasoning and proof through verbal and written exercises.

Collectively she suggests that these activities provided opportunities to build conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and provide a productive disposition towards mathematics. Her most noticeable observation was the change in her students’ disposition towards mathematics. She notes that “my students became mathematicians ‘mathematizing,’ solving real-life problems, justifying and explaining interesting patterns and relationships” (Suh, 2007, p.168).

Samuelson (2010) used the five strands as a framework for assessing the impact of teaching approaches on Swedish learners’ progress in the first five years in school. He examined the effectiveness of the traditional and problem-solving approaches. The instrument used was designed based on the five strands. The learners of the two groups were presented with problems that tested each of the five strands. His results indicated that different teaching approaches do indeed affect the development of the learners.

Closer to home, Langa and Setati (2007) investigated the use of home language to support mathematics learning in South Africa. In this investigation they present debates that support the use of home language as a useful resource for maths learning. Others such as Howie (2001), who analysed the TIMSS data, believe that second language English learners who are taught mathematics in English should improve their level of proficiency in English. On the contrary, Langa and Setati (2007) used the five strands to analyze their data. This was achieved by presenting a task to students. A vignette is used to highlight how learners used their home languages to demonstrate four of the five strands. The interaction between learners promoted the following strands: adaptive reasoning, conceptual understanding, procedural fluency and strategic competence.

Moodley's 2008 research focussed on the development of mathematical proficiency in number skills of grade 10 Durban learners in both Mathematics and Mathematical Literacy. Analysis was structured according to the five strands. He was able to compare the mathematical proficiency between the two cohorts of students as well as monitor their mathematical proficiency progress over a four month period. In his dissertation, he outlined the criterion for placing each question in its respective strand. He suggests that instruction and learner support material must be of a nature that supports the development of mathematical proficiency.

The studies mentioned have a common thread, viz. the use of or promotion of mathematical proficiency. In some studies the strands were used to analyse data, others to design mathematical practices whilst some used it as criteria to investigate specific topics of mathematics instruction. In all cases the five strands were used as tools to highlight their results and findings. More empirical work within the South African context is needed to 'measure' the construct of Mathematical Proficiency as defined by Kilpatrick et al.

2.3. A South African perspective

One study of the mathematical proficiency encouraged in actual teaching done in South Africa is Pillay's 2006 small scale study located in the broad area of mathematical knowledge for teaching for mathematical proficiency. He investigated what teacher's knowledge and experience is used and how it is used as he confronted maths problems when teaching functions in a grade 10 class in the Witwatersrand area. In addition, he tried relating the resource pool of the teacher to the potential to promote mathematical proficiency in his learners. In his discussion, he attempts to link the teacher's knowledge and experiences, his resource pool, to having the potential to promote mathematical proficiency. This does not seem to be clear and lacked sufficient evidence to categorically state that learners indeed acquired mathematical proficiency. His findings seem to be limited but had the necessary core ideas to make significant contributions were the study conducted on a larger scale.

The design of the National Curriculum Statement (Department of Education 2003) embodies the essence of Mathematical Proficiency. It envisions a learner that can “identify and solve problems and make decisions using critical and creative thinking” (p. 2). The ‘Foundations for Learning Campaign’ announced in the Government Gazette on 14 March 2008, outlined the tasks expected of primary school teachers during a Mathematics lesson. Daily teaching activities in the grades 4-6 classes included 10 minutes of oral and mental work to develop learners’ mental skills. Questions such as ‘how many groups of 8 in 72’, is used as an example in this part of the lesson. This is followed by the teacher reviewing and correcting previous homework. A concept is then introduced and consolidated by further examples. Problem solving to challenge learners must be included, allowing learners to investigate different ways of solving problems. Group or pair work is recommended here. Thereafter, class discussion to share and explain thinking should assist with learners reflecting on their own efficacy to see mathematics as useful. Lastly homework tasks must be given with explanations by the teacher. A hint at the strand of conceptual understanding appears even though it is not as sophisticated as Kilpatrick et al. description. Strategic competence strand incorporates problem solving and reflection on their efficacy points to a productive disposition.

The South African Mathematics Curriculum Statement is central to the mathematics taught in schools. Sanni focused on the question ‘to what extent are the tenets of mathematical proficiency and mathematical practices promoted in the revised national curriculum statement for grade 7’ (Sanni, 2009, p. 1). In this investigation he classified the verbs that were used in describing assessment standards according to the five strands of mathematical proficiency. The demands of each assessment standard on the learner served as the criteria for placing in the categories. His findings indicate that 96.0% of the verbs used in the document could fit into one of the five strands of mathematical proficiency. However, the distribution is not uniform. The strands of conceptual understanding and adaptive reasoning are favoured with 37.3% and 23.1% of occurrences respectively. He also contends that some of the strands are not adequately provided for. These included productive disposition (0.0%), procedural fluency (8.6%) and strategic competence (13.5%).

Provision should be made for those strands inadequately represented in describing assessment standards to emphasise the relevance of their intertwined nature.

The literature outlined above suggests that Mathematical Proficiency as defined by Kilpatrick et al has had a significant impact on teaching and learning of mathematics at a macro and micro level. Curriculum designers seem to be noticing the relevance of the five strands, whilst teachers' promotion of the five strands needs to develop mathematically proficient learners. However, despite Pape (2003) believing that Kilpatrick et al 'goals' for mathematics education have been embraced, he notes that, "designing classroom environments and teaching pedagogies that effectively promote this vision, has proven more elusive" (Pape, 2003, p. 180).

Yet, we now have a clearer picture of what is deemed 'successful mathematics learning' that includes the proficient learner, proficient teacher and proficient classroom activities. The desirable future is aptly summarised by Barmore, who says "ahead of us is the mathematically proficient student" (Barmore, 2009, p.12). Herein possibly lies the struggle. We can adopt Kilpatrick et al (2001) vision of Mathematical Proficiency, but to create the proficient learner, the teacher not only has to be 'proficient', but has to promote the five strands in the classroom environment using available resources. Classroom teaching needs to be created, carefully planned and constructed so that each strand is adequately promoted over time. To what extent this happens, thereby feeding into more knowledge about teaching which facilitates learning is the focus on this study.

The literature review above suggests that the strands of mathematical proficiency have become a notion that can be adopted in different ways. Suh (2007) conducted mathematical activities in everyday teaching to promote mathematical proficiency while Samuelsson (2010) used the strands as criterion to assess the impact of teaching approaches in classes. Locally, Pillay (2006) attempted to relate the resource pool of a teacher to the potential to promote the strands of mathematical proficiency while Langa and Setati (2007) used the strands to test whether learners' home language had any impact in promoting mathematical proficiency. Sanni (2009) on the other hand identified verbs that appeared in the South African Curriculum statements that could fit in the five strands. My study hopes to use empirical evidence to ascertain the present position in mathematics lessons in relation to mathematical proficiency. The primary goal of the study is to describe and measure the extent to which teachers create opportunities to develop the five strands of mathematical proficiency in the classroom.

2.4. Research Questions

In the light of absence of studies on the extent South African teachers teach for mathematical proficiency, I want to focus specifically on this aspect. Secondary, though key to the larger study, is considering the correlation between the promotions of mathematical proficiency, teacher characteristics, and learning. This leads me to the following research questions:

Research Question 1:

Is the promotion of the strands of mathematical proficiency prevalent in the current practices of the grade 6 teachers' in the Umgungundlovu district of KwaZulu-Natal?

Research Question 2:

To what extent is each of the strands of mathematical proficiency promoted by grade 6 teachers' from the Umgungundlovu district of KwaZulu-Natal?

Research Question 3:

How does the promotion of mathematical proficiency vary, if at all, with the educational background of the teacher, the teacher content knowledge as reflected in the results from the teacher test and other background factors?

Research Question 4:

How does the teachers' promotion of mathematical proficiency correlate with the learning that took place during grade 6, according to the difference between the results on the two learner tests?

It should be noted that the focus in this study is on the teaching. It is about the mathematical proficiencies promoted by the teacher, even if these are not necessarily taken up by the learners. There are many variables that influence learners' mathematical learning. What the teacher knows and does in the classroom is probably the biggest influence which is emphasised by Hattie who says, "as such excellence in teaching is the single most powerful influence on achievement" (Hattie, 2003, p. 4).

Importantly, these questions serve the double purpose of interrogating the relevance of the developed instrument for the analysis of mathematics lessons.

Chapter 3: Theoretical Framework

The learner either actively engaged or passively involved in a mathematics classroom anticipates mathematics teaching that will ultimately lead to understanding. Whether the learner is acutely aware or totally unaware of the goals of mathematics learning, the interaction in the classroom influences understanding. Kilpatrick et al have envisaged the goals of mathematics learning in the term ‘mathematical proficiency’, composed of five interwoven strands. Accordingly, mathematics learning should develop all aspects of mathematical proficiency. In the classroom the learner is exposed to the instructional practice of the teacher. Mathematics lessons should include sufficient aspects that allow the learner to ultimately achieve these goals. The teacher needs to be able to help students with that development. In order to achieve these, tangible opportunities provided by the teacher within the classroom must exist. This is corroborated by Ball who states, “Students opportunities to develop mathematical proficiency are shaped within the classrooms through interaction with teachers and interaction with specific content” (Ball, 2003). The five strands provide the framework for the learning proficiencies of mathematics whilst the teacher in the classroom must promote mathematical proficiency by exposing learners to as many opportunities as possible. This chapter interrogates the five strands as goals of mathematical learning providing the necessary backdrop for expanding on opportunities to develop.

For a framework to fully and comprehensively reflect and define Mathematics Proficiency, current trends in all aspects of Mathematics as well as its history should be considered. It is essential that such a framework is all-encompassing but simultaneously lends itself to further improvement or adjustment. Lave and McDermott (2002) relates that narrow frameworks may not give us the perspective to question practices. Boaler (2002) believes that theoretical frameworks encourage researchers to pursue new ideas, and I would add that it offers new perspectives and understandings of practices. A framework for mathematical proficiency that encompasses the goals of mathematics learning, considers not only previous research and paradigms but also captures the dynamics of the interaction in the classroom, is the framework proposed by Kilpatrick et al.(2001) in the book 'Adding it Up', and it has gained widespread acceptance.

Here he suggests that there are five separate but intertwined strands that separately yet collectively combine to give us a conception of Mathematical Proficiency. As mentioned earlier, these are: Conceptual understanding, Procedural fluency, Strategic Competence, Adaptive Reasoning and Productive disposition.

This description is meant to encapsulate all aspects of mathematics learning. Yet, these categories need to be interrogated further. It is quite possible that additional categories might be necessary as researchers, teachers and educational officials dissect the strands – indeed, a sixth strand, namely the dimension of historical and cultural knowledge is under interrogation(Wilson et al., 2010). As they argue, historical knowledge of mathematics is likely to lead to deeper understanding and significance of mathematical conventions. However, in this study, this dimension has not been fore grounded. Firstly because it was not included in the international study of which my work is a part. Secondly, because it was not observed in any of the video recordings I watched while preparing the proposal for this study.

Boaler (2002) admits that the framework has the potential to offer something but will become clearer in time as researchers, teachers and education officials work with it. This mirrors my experience, as the analytical framework derived from it had to be revisited after the pilot analysis of some videos.

I will give a brief description of each strand as set out by Kilpatrick et al. (2001) trying to identify its appropriate classroom practice. It must be recognised that to some extent, this separation of the strands is against the way Kilpatrick et al thought about them, as they claim them to be intertwined (Kilpatrick, 2001, p.116).

3.1. Conceptual Understanding

This refers to a "grasp of mathematical ideas, its comprehension of mathematical concepts, operations and relations" (Kilpatrick et al, 2001). Learning with understanding, results in easier connection of facts and methods. The importance of mathematical ideas is understood. Conceptual knowledge would include the ideas of the nature of topics and is exemplified in the use, illustration or representation of concrete and semi-concrete models. It refers to the underlying structure of mathematics – the relationship, links and connections between and among mathematical ideas. It is sometimes described as the 'knowing why' of mathematics. Hiebert and Lefevre believe that conceptual knowledge is achieved in two ways: by "the construction of relationships between pieces of information" or by the "creation of new information that is just entering the system" (Hiebert and Lefevre, 1986, p. 46).

According to Kilpatrick et al, learners showing considerable conceptual knowledge attributes are likely to retain mathematical ideas and knowledge easier. Understanding and remembering methods as well as reconstructing the method, if forgotten, are easily and effectively accessed. Mathematical knowledge is organized as a coherent whole allowing monitoring of what is remembered, explaining and correcting methods themselves and eventually verbalizing their understanding. Deeper similarities between unrelated situations, ideas or representations are observed resulting in less to learn.

Applied to the current study, the interest is not in the cognitive aspect of conceptualizing, but rather whether, during the course of instruction, the teaching practice was such that a concept in whatever form was interrogated – so that we may with some fairness say that learner's conceptual understanding was promoted, even if to a limited extent. Such 'interrogation' could consist of explanations, linking the concept to previous concepts or processes, of engaging structural patterns of mathematics such as inverses or identities, or such structures and links evolving from learners engaging with tasks. Whether the mathematical definitions and representations were conceived either structurally as objects or as processes (cf. Sfard ,1991) is not the aim of this investigation. Stigler, Gallimore and Hiebert (2000), claim that the concept could be simply stated by the teacher or developed through examples, demonstrations and discussion.

In line with the above discussion this aspect of proficiency promoted by the teacher will be analysed as follows:

- Present or not present – the lesson contained instances where a concept was evident or not.

- Stated or developed – was the concept simply stated or developed? Stated would include descriptive or routinely algorithmic concepts with little mathematical justification. It is simply provided by the teacher or learners but not explained or derived. Development of the concept could be through the sub-categories: discussion, demonstration, linking multiple representations, or any other form in which concepts are mathematically motivated, supported and justified by the teacher or learners.

Diagrammatically:

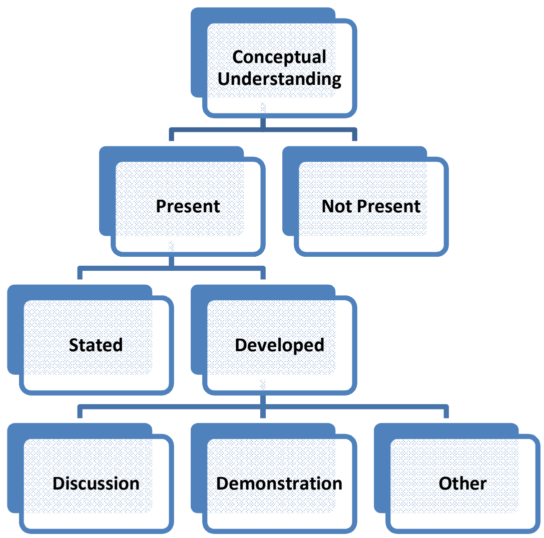

Figure 1: Conceptual Understanding mind map.

3.2. Procedural Fluency

Procedural knowledge is seen as the ability to solve mathematical problems using mathematical skills such as rules, algorithms and formulae. It can be seen as having two parts: (a) knowing the formal language and identifying the representations and (b) knowing the rules and the step by step procedure needed to complete mathematical tasks. Kilpatrick et al. (2001) coined the term procedural fluency as one of the strands of mathematical proficiency to refer to the "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them accurately, flexibly and efficiently" (Kilpatrick et al, 2001, p. 121).

In other words, it is not enough to be able to perform an operation or procedure, it is also crucial to be able to choose the correct one.

This strand tends to dominate mathematics learning in classrooms both locally and internationally. Surveys such as TIMMS support this view (Stigler et al. , 2000). Locally, Engelbrecht, Harding and Potgieter claim, "the general perception is that high school teaching of mathematics in South Africa tends to be fairly procedural" (Engelbrecht, Harding and Potgieter, 2005, p.1). In America a similar pattern is observed where procedural knowledge, specifically rote learning of rules and algorithms are emphasized. Eisenhart et al cite data from the 5th National Assessment of Educational Progress (Mullis, Dossey, Owen and Phillips, 1991) and an earlier 2nd International Mathematics Study (McKnight et al, 1987) that indicate dominance of rote learning and procedures in school mathematics in the United States of America.

Procedural fluency involves other skills. These include knowledge of ways to estimate the result of a procedure, apply procedures fluently and efficiently, accuracy in arriving at answers as well as using a variety of mental strategies to find solutions. Insufficient procedural fluency is likely to result in learners having difficulty understanding mathematics concepts or experiencing obstacles in the solution of problems. Procedural fluency and mathematical skill are synonymous with each other.

Based on Stigler et al.'s account of video analysis (2000), procedures could be demonstrated or developed by the teacher. Students are then expected to apply the procedure. This they referred to as 'task controlled'. On the other hand, students could be asked to develop procedures themselves, which they referred to as 'solver controlled' (Stigler, Gallimore, Hiebert, 2000, p.93). The analysis will follow a similar pattern as the first strand.

Diagrammatically,

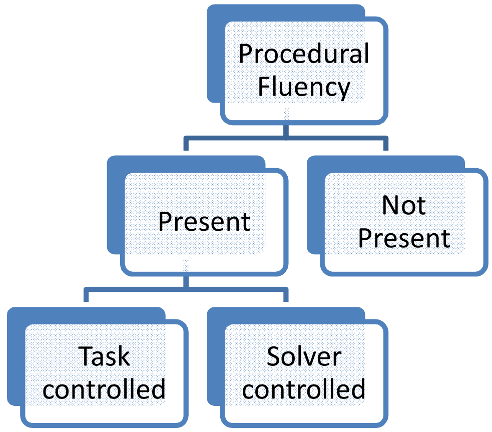

Figure 2: Procedural Fluency mind map

Procedural fluency can be thought of as part of the 'knowing how' of mathematical knowledge. The ability to quickly recall and accurately execute procedures assists in the solution. Aspects of procedural knowledge and fluency promoted by the teacher include:

- Recalling and using algorithms for completing tasks and procedures
- Sketching
- Using formulae
- Identify and using key words
- Knowing strategies for solving problems
- Knowing the formal language, or the 'symbol representation system'

At this juncture it must be noted that the distinction between conceptual and procedural knowledge is a problem. At times, they seem to function independently yet compensate each other. The debate about conceptual and procedural knowledge will continue. This is clearly identified by Kilpatrick et al. (2001) who emphasizes the interwoven and interdependent nature of the different strands of mathematical proficiency. This assertion is substantiated by Hiebert and Lefvre(1986) who suggest that procedural knowledge has meaning only when it is linked to conceptual understanding. A debate which then arises is one of a number of possibilities: should concepts be considered first followed by procedural knowledge or should procedures initiate concepts or possibly address one aspect only. Long (2005) contends that "different aspects of the debate apply differently to particular mathematical concepts, to different stages of mathematical development and to different learning styles". Wong and Evans, (2007) citing Rittle-Johnson et al (2001), state that "developing students' procedural knowledge had positive effects on their conceptual understanding, and conceptual understanding was a prerequisite for the students' ability to generate and select appropriate procedures." The debate between procedural and conceptual knowledge is likely to continue.

3.3. Strategic Competence

Strategic Competence is seen as "the ability to formulate, represent and solve mathematical problems. It is similar to what is generally called problem solving and problem formulation. Students need to encounter situations in which they need to formulate the problem so that they can use mathematics to solve it" (Kilpatrick et al, 2001, p.124). Thus it is necessary to engage students in problem formulation and problem solving throughout their schooling. Strategic competence is more than just procedural fluency. Students are required to generate problem-solving strategies when they encounter problems, evaluate the relative effectiveness of those strategies, and subsequently employ the chosen strategy to reach a solution. It requires procedural fluency as well as a certain level of conceptual understanding.

Problem solving has been researched in depth over the past decades with many models proposed to solve mathematics problems. Polya's celebrated four-phase model of the problem solving process in mathematics involved: (a) understanding the problem, (b) devising a plan, (c) carrying out the plan, and (d) reflection (Higgins, 1997). These steps form the basis of strands in mathematics problem solving. De Corte, Vershaffel and Masui's (2004) competent problem solving model underlying the learning environment included five steps viz. (a) build a mental representation of the model, (b) decide how to solve the problem, (c) execute the necessary calculations, (d) interpret the outcome and formulate an answer and (e) evaluate the solution.

In line with these models, strategic competence encompasses more than just the process. Students must first understand the problem in hand. The key elements of the problem are essential to identify. A mathematical representation of the problem that captures the core elements either numerically, graphically, symbolically or verbally must be formulated. Another way is to construct a mental model of the variables and relations. Kilpatrick et al (2001) suggest, "In becoming proficient problem solvers, students learn how to form mental representations of problems, detect mathematical relationships and devise solution methods when needed." Flexibility improves students' ability to solve non-routine problems. Routine problems are problems that the student has encountered before and which he knows how to solve. Non-routine problems require productive thinking, forcing a student to invent a way to understand and solve a problem. A student possessing strategic competence is able to have several approaches to the solution of a problem and then choose flexibly among them through reasoning and reflection on experience.

A theme characteristic in the above conceptions is the heuristics embedded in the problem-solving process. Researchers are divided as to the effectiveness of heuristics. Mixed opinions regarding the use of strategies have been recorded. The heuristics themselves appear to be a source of contention. Claims that problem solving strategies are themselves problematic and student specific have surfaced (Begle, 1979 cited in Higgins, 1997). Yet others claim that heuristics could be used as a means to enhance and improve problem solving ability. Higgins (1997) falls into this category when the effect of year-long instruction in mathematics problem solving was investigated. The research involved three classes of middle-school American students. These classes received one year of problem-solving instruction. They were compared with three classes of students who were taught mathematics in a more conventional setting. The results of the investigation showed that "the heuristic students' superior performance on the problems given at the end of the interviews suggest that problem solving instruction has a positive impact on students problem solving ability" (Higgins, 1997, p.16).

My study will seek to identify the strand of strategic competence by looking at the heuristics a teacher implements or employs as he/she encounters problem-solving situations. My interest is in the heuristics used and not the problem-solving model underlying the learning environment. Consequently the following heuristics will be identified in accordance with those used by De Corte, Verschaffel and Masui (2004) of Belgium when designing a framework for learning environments for thinking and problem-solving (De Corte, Verschaffel and Masui, 2004, p.372):

- Drawing pictures or figures
- Making lists, a scheme or a table
- Making a flowchart
- Guess and check or trial and error
- Looking for a pattern
- Formulate similar problems or modify a problem
- Other

Opportunities to engage these heuristics will be identified within the context of the lesson and includes any strategy that is used during mathematics problem solving encounters.

3.4. Adaptive Reasoning

Support for learners' mathematical thinking appears in the form of reasoning, explanations, justification and arguments amongst many other forms of mathematical practice. Explaining a procedure, justifying a mathematical idea or reasoning during computation underpins mathematical understanding and learning. Connecting concepts or representations requires logical thinking. Adaptive Reasoning refers to the "capacity for logical thought, reflection, explanation, and justification" (Kilpatrick et al, 2001). Mathematics is built on a foundation of reasoning. It is not just a collection of arbitrary rules. It is the strand that holds the others together. Kilpatrick et al emphasise the importance of this strand stating, "In mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning" (Kilpatrick et al, 2002, p.129).

Many constructs of mathematical reasoning have been around formal proof and deductive reasoning. Stylianides and Stylianides laments, "students' proficiency in proof can improve their mathematical proficiency more broadly" (Stylianides and Stylianides, 2008, p.104). They suggest that students' abrupt introduction to proof in high school and university as a possible explanation for the problems they face with proof and proving. He further proposes that students engage with proof throughout their schooling. His suggestion is supported by Powell, Francisco and Maher's 16 year longitudinal study on the development of mathematical ideas of a focus group of students. Educators viewing the video-recordings of classroom interactions from the project yielded "discourse that children so young could reason mathematically with such sophistication" (Powell, Francisco and Maher, 2003, p. 406). Stigler and Hiebert also mention that "deductive reasoning as a form of mathematical activity that is central in important mathematics" (Stigler & Hiebert, 1997).

Learners should not only possess mathematics knowledge but should be able to use and do mathematics. These mathematical practices are important in learning and doing mathematics. Horn contends that, "mathematical reasoning is supported by identifiable mathematical thinking practices" (Horn, 2005). Three core practices that identify mathematical reasoning and that have been substantially researched include the effective use of representations, the formulation of justifications, and the identification of patterns through generalisations. These activities support and become involved in establishing and contributing to mathematical knowledge. The first, representation, encompasses the use of symbolic notation – as well as graphs, tables, etc. Mathematics uses highly developed symbolic notation upon which work and thinking depend. This complex notation allows easier comprehension and manipulation and it is therefore necessary that it is fluently and flexibly implemented. Secondly, understanding mathematics depends crucially on justification. Mathematical ideas should not only be known but learners should also know why they are true. Justifying a solution ultimately leads to thinking about the problem resulting in greater understanding. The third area of practice is generalisation. Isolating patterns, structures and relationships in mathematical data within a class or across classes of situations helps in making important connections (we could have added others ... such as what-if thinking leading to extending concepts, ...).

Kilpatricks et al.'s notion of adaptive reasoning encompasses more than the brief review above. Learners should "think logically about the relationships among concepts and situations" (Kilpatrick et al 2001, p. 129). Structural comparison of mathematical systems is evident. Formal and informal reasoning is used and justification is commonplace. Adaptive reasoning not only involves formal proofs, deductive reasoning, informal explanation and justification but also includes intuitive and deductive reasoning based on patterns, analogy and metaphor. Analogical reasoning, metaphors and mental and physical representations are the ‘tools to think with’. Formal reasoning such as distinguishing between necessary and sufficient conditions, as well as informal reasoning such as reasoning from representation and creating and understanding appropriate analogies, is examples of adaptive reasoning ability.

In line with the above discussion this strand of mathematical proficiency will be identified by looking at the extent to which participants in classroom discourse engage in practices that encourages:

- reasoning
- informal explanation
- justification

Videos of learners in grade six are the subject of the study. As a result, identification of this strand takes into account the level of mathematical maturity they could display when reasoning and which they expressed in their own language, as well as the level of reasoning to which the teacher can fairly appeal.

Explanations of the thinking process learners undergo when arriving at solutions or when following a procedure must be forthcoming. Rationale used in arriving at answers or the reasoning used when comparing answers are indications of this strand. Reasons to someone else’s solution could also be part of reasoning during mathematical activity.

3.5. Productive Disposition

Learners who tend to see sense of when, how and where mathematics is used create a disposition towards their mathematics. Motivation shapes their attitude towards learning and understanding mathematics. Positive motivation would likely lead to interested and effective learners. Productive disposition refers to the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Kilpatrick et al, 2001, p. 131). According to Kilpatrick et al, this implies a perception that mathematics is useful and worthwhile. Learners' personal values and goals in mathematics are regarded as rewarding and they see the purpose and use of mathematics. Positive emotion toward the subject and enjoyment in its engagement is evident. There is a disposition toward mathematics that is personal and a willingness to engage in mathematical activity is forthcoming.

Practices that nurture positive motivation converge with learners' personal motivation. Learners present in classrooms in which greater emphasis on effort, learning, understanding and recognition is placed experience relatively more positive emotions. Learners do not always realise the importance of the effort they place in mathematics nor do they see the ultimate value in doing mathematics. Learners can be taught to realise their own effort leads to a belief that they can do mathematics and in so doing increase their motivation to work hard and to learn. Learners who see improvement through the effort and work they do will experience more success. In addition, recognising the accomplishment of worthwhile goals, increases motivation. Recognition of performance (e.g., getting answers correct, high grades) and encouragement of working autonomously are contributing factors of motivation, as are being able to solve challenging problems. A positive classroom atmosphere as well as the level of enthusiasm created and recognised fosters motivation. Persevering with a task should be encouraged. Learners displaying a productive disposition are likely to see a mathematics problem through to its conclusion and will persevere with a problem for long periods in order to make progress.

Students exhibiting productive disposition will notice mathematics in the world around them and apply mathematics principles to situations outside of the classroom. Links between the mathematics taught and the learner's out of school experiences needs to be made explicit. Such connections ultimately influence the learners' conceptions of mathematics understanding. Learners need to see that mathematics in the class and life in the real world are connected.

Promotion of productive disposition during classroom discourse could include:

- Encouraging perseverance with a task
- Encouraging confidence

- Relating out-of-class situations to mathematics

It must be noted that the above systematisation simply provides a first indication of how to recognise the strands in the observed lessons. It must be remembered that Kilpatrick et al. stress the highly intertwined nature of the strands, and thus it is plausible that several strands will operate at the same time.

This chapter provided the framework around which the study was centred. Whilst it provides the necessary theory that informs the study, how this translates to analysis of classroom observation still needs to be addressed. In the next chapter, I outline coding on a finer grain size and introduce the notion of 'Opportunity to develop mathematical proficiency' with its concomitant descriptors for each strand.

Chapter 4: Analytical Framework - Notion of OTDMP and Descriptor table

Previous chapters provided the framework around which the study was centred. This included the key themes and patterns, the framework and the rationale foregrounding the coding. In particular, the explanation of the coding seemed simple enough: view videotapes of lessons in segments of 5-minute intervals, grade for the various strands of proficiency encouraged by the teaching in each segment and, based on this observation code each strand of proficiency. Thus grading, in the case of procedural fluency, had unique characteristics that defined this strand which differed to that of conceptual understanding as well as to the other strands. Attempting to describe, define and explain explicitly the characteristics that uniquely differentiated between the sub-coding of the strands was indeed a thought, one which I pondered long and hard about. Ideally, this would mean, for instance, in the case of the strand of conceptual understanding, defining it in its entirety; its psychological, sociological and pedagogical characteristics. Doing this for each strand of mathematical proficiency would be a mammoth task and beyond the scope of this study.

The five strands of mathematical proficiency discussed in the theoretical framework chapter provide acceptable goals for mathematical understanding. Mathematical proficiency is described in terms of developing these five strands simultaneously due to their interwoven nature and connectedness. For learners to achieve these goals, exposure to the five strands is therefore crucial. Mathematics learning takes place largely in the classroom environment. Mathematics instruction directly impacts the learning of mathematics concepts, procedures, and skills as well as developing a disposition towards the subject. For this to materialise, learners must have had an opportunity to develop the understanding of mathematical ideas, practice mathematical procedures, solve non-routine problems and find the mathematics useful. It becomes clear that the promotion of the five strands of mathematical proficiency is manifested in the classroom by the opportunities to develop them. Whilst recognising the five strands as the goals of mathematical proficiency, identifying them in video recordings was problematic. A more tangible approach was necessary.

In the classroom, the opportunities presented during mathematical instruction develop mathematical understanding. The opportunities offered to learners by the teaching develop and enhance mathematical learning. Samuelsson (2010) citing Reynolds and Muijs (199) states, “a result of their review is that effective teaching is signified by a high number of opportunities to learn” (Samuelsson, 2010, p. 3). Opportunities play a key role in mathematics learning.

In this chapter I introduce the notion of ‘Opportunity to Develop Mathematical Proficiency’. Use of this notion and its acronym, OTDMP, as well as its components will be a feature of the study henceforth. The remainder of the chapter is devoted to the notion of OTDMP and the descriptor table showing characteristics of opportunities per strand of mathematical proficiency.

4.1. Opportunity to Develop Mathematical Proficiency-OTDMP

OTDMP Explained

Kilpatrick et al contend that the term ‘mathematical Proficiency’ was chosen to capture what we believe is necessary for anyone to learn mathematics successfully (Kilpatrick et al, 2001, p. 116). Proficiency in school mathematics was characterised in terms of the five strands. Accordingly, the expectation is that a successful mathematics learner is proficient in mathematics if he/she possesses the five component strands. These strands could be viewed as mathematical attributes of a mathematically proficient learner. These attributes are interwoven and interdependent and are a powerful combination that will reward a mathematically proficient learner handsomely. However, such attributes need to be developed and nurtured over time.

It is within this frame of reference that the notion of opportunity to develop mathematical proficiency has its roots. The opportunities to develop the strands must be present in a mathematics lesson. Hence the notion of “the existence of an opportunity to develop, promote or advance mathematical understanding via one or a combination of its component strands,” adequately describes the construct. These opportunities are fundamental in the attainment of the attributes needed by learners to become mathematically proficient. These traits develop provided the opportunities exist, occur regularly and with a corresponding degree to which the mathematics was encountered when the opportunity was promoted– of course influenced as well by the learners' personal attributes and circumstances. These opportunities impact the development of mathematics learning and understanding.

4.2. Characteristics of OTDMP

Engaging further with the notion of OTDMP may indeed provide additional characteristics. In this study, however, three important characteristics of OTDMP will feature. Firstly, the opportunity must be present. Development of any strand depends on the presence of the opportunity been perceived or felt during mathematics instruction. Naturally, absence of any opportunity has no developmental value and hence no promotion of mathematical proficiency.

Secondly, time spent in learning a concept, engaging in mathematics procedures, solving problems, reasoning or explaining mathematical ideas or acquiring a disposition to the subject is crucial to successful attainment of proficiency. It is expected that learners become increasingly proficient each year. Time span is a key property of acquisition of mathematical proficiency, both over the long term as well as over a shorter period. On a shorter time scale, e.g., the duration of a lesson, many examples should be done to illustrate how and why a procedure works or what a concept means. Failure to do this may result in failure to learn. Sustained periods of time should be spent doing mathematics. It is during a single lesson that OTDMP becomes prominent. They should not only be observable but sustainable over time within the lesson unit. Sustained periods of engagement with mathematical content create more opportunities to develop mathematical proficiency.The number of opportunities of mathematical proficiency occurring in lessons directly impacts the learners' propensity to develop the necessary attributes of a proficient learner. The number of opportunities created in lessons increases the learners' chances of gaining deeper understanding of the mathematics discussed.

Thirdly, the strength of the opportunity to positively develop a particular strand influences the learner’s propensity to learn mathematics successfully. Characterising the forcefulness or strength of the opportunity could vary and likely depend on the research in question. In my studies, a numerical rating was assigned which indicated the degree to which the opportunity was developed. Each strand was assigned ratings from 1, an opportunity that would hardly promote the development of a strand of proficiency, to a rating of 3 indicating a high degree of development of an opportunity. Perusal of the descriptor table, table 1 below, indicates the increase in the ratings of each opportunity with its corresponding descriptors. In this study the forcefulness or strength must be viewed in a mathematical sense. A more accurate term may indeed be ‘mathematical forcefulness’, indicating the relation between the opportunity and the manner in which it will impact mathematics learning and understanding. The higher rating indicates a greater promotion of the strand of opportunity. This in turn is more likely to lead to stronger development of the corresponding mathematical attribute of the learner.

4.3. Five Categories of OTDMP

The notion, OTDMP, refers to the existence of an opportunity to develop, promote or advance mathematical understanding via one or a combination of its component strands. Here, the term ‘opportunity to develop mathematical proficiency’ and ‘promotion of mathematical proficiency’ will be used interchangeably, having no discernible differences. Embedded within OTDMP is the potential for the development of the understanding of important mathematics. The notion of OTDMP is composed of five categories reflecting the five strands of mathematical proficiency which are matched on a one-to-one basis. Each strand has a corresponding opportunity to be developed. The use of these categories was crucial to establish whether the opportunities supported the advancement of mathematical proficiency. OTDMP is comprised of the following categories:

- Opportunity to Develop Conceptual Understanding – OTDCU
- Opportunity to Develop Procedural Fluency – OTDPF
- Opportunity to Develop Productive Disposition – OTDPD
- Opportunity to Develop Adaptive Reasoning – OTDAR

- Opportunity to Develop Strategic Competence – OTDSC

By its very nature, a mathematics lesson should show continuous instances of at least one aspect of OTDMP since one or a combination of the five strands should exist throughout the course of the lesson - assuming that mathematical proficiency indeed encompasses mathematical learning in its entirety.

Opportunities were analysed by attending to the forcefulness or strength of the opportunity to develop a particular strand. Analysis thus proceeded by characterising the forcefulness or strength of the opportunity and assigning a corresponding numerical rating which indicates the degree to which the opportunity is developed. The ratings 1, 2 and 3 were used for this purpose. These ratings were considered contextually, inclusive of teacher and learner mathematical knowledge, level of mathematical discussion, classroom demeanour, learner participation and many other characteristics of mathematical teaching and understanding that would promote that particular strand of proficiency. Amongst strands, clear differences in assigning these numerical ratings will emerge due to the unique character of each strand, discussed further below. Thus the reasons for assigning the value 2 for OTDPF will differ for the case of selecting a 2 for OTDAR and so on. In order to show these and to capture the features of the sub-coding, a table containing coding descriptors was used to characterize opportunities to develop or promote the strands of proficiency. The coding is summarized in the form of a rubric in Table 1. These descriptors were not pre-determined but rather co-evolved during the research, in particular after some preliminary coding of data. As alluded to in previous chapters, my coding for strands of proficiency presented more questions than answers and a personal conflict and unhappiness with the manner of coding. This resulted in striving for refinement which led to the design changes of the initial coding instrument. Further progression with the use of this instrument led to the development of the notion of OTDMP and the descriptor table for the numerical ratings. The table allows classification of each segment of the lesson using the description as the tools.

The table thus informs the instrument used to capture the strands of mathematical proficiency. In a sense, it is the bridge between the qualitative research and the quantitative analysis that follows. It is the concrete link that shows the degree to which an opportunity to develop the strands of proficiency and the overall analysis of the videos.

4.4. Table of Descriptors of OTDMP

The descriptors of each rating of the OTDMP strands to a large extent draw on the larger body of work in mathematics education, though implicitly. It should be noted, however, that a particular system of values could possibly be detected in the choice of descriptors – yet this is also true for the focus on mathematical proficiencies including adaptive reasoning, productive disposition and conceptual understanding in the first place. Furthermore, the descriptors are potentially more related to more teacher-centered or expository forms of teaching, and thus may need further development to be more inclusive. However, since teacher-centered methods interspersed with individual seat work mostly focused on practising taught work dominated all the videos, the current set of descriptors appeared adequate for the data collected in this study.

Table 1: Descriptors per opportunity to develop each strand per numerical rating.

Numerical rating→ Components of OTDMP↓	1	2	3
Opportunity to develop Conceptual Understanding – OTDCU	Few opportunities are provided to build understanding of concepts. Explanations or developments of concepts are not linked to other concepts in explicit ways (low discursive saturation of links). No attempt is made	Opportunities are provided which clearly clarify the concept with some explicit links made to other concepts, horizontally or vertically. Only mathematically key links are engaged – in other words, 'structural overload'	Clear explanations when stating or developing concepts, are provided. In addition, two of the following three opportunities to develop conceptual understanding are evident: (i) How and why specific concepts are used as well as

	to explain the relevance or significance of the concept. Representations are not linked to the concept in explicit ways or are irrelevant, in the sense that they do not capture essential aspects of the concept.	is avoided. Any representation is linked to the concept in explicit ways.	their significance is formulated and demonstrated. (ii) Connections to other concepts are indicated, as per rating . (iii) More than one representation are explicitly compared, discussed and connected, but only mathematically relevant representations are included, and not so many of these that the key characteristics become difficult to discern
Opportunity to develop Productive Disposition – OTDPD	Opportunities to encourage, persevere and instill confidence (eg. encouraging learners to persist, praising effort and performance, explain strategies, adhering to tasks) occurs at times, but is not consistent (as when learners are told to collaborate, then scolded for talking to each other; or the teacher has a negative disposition a lot of the time, e.g., displays anger, aloofness, sarcasm). Real world situations are described but opportunities to relate these to the mathematics are not made explicit to the learners.	Opportunities to reinforce effort, comment on learners' performance, encourage interest is developed, but only on occasion. Making sense of maths is identified and recognized but not fully exploited. Out of class situations are mentioned and used to motivate the mathematics, but the connections are only partially made explicit.	Opportunities to regularly reinforce effort as well as create enthusiasm in mathematics are developed and exploited. A positive approach to maths showing sensitivity, respect and interest in learners' responses and questions is evident. Learners are regularly encouraged to persevere -'keep trying'. Opportunities to create an environment conducive to fostering confidence is observable (eg. conveying that mistakes are okay, providing explanations to learners in difficulty, organizing content so that it is personally meaningful to learners). Opportunities to

			develop links between out-of-class situations and the mathematics are made explicit.
Opportunity to Develop Procedural Fluency – OTDPF	Development of procedures is inefficient and inappropriate. Opportunities to provide explanations of what and why a procedure is used are non-existent. Opportunities to develop reasons for doing a procedure in a particular way are not forthcoming and alternate procedures are not explored. Procedures may not be performed fluently.	Opportunities to perform procedures appropriately and fluently are developed. One procedure is used and an explanation of what procedure is used is communicated. Opportunities to provide reasons for doing the procedure are only partially made explicit. Alternate procedures are not explored.	Opportunities to develop appropriate, efficient and fluent procedures are maximised. What, when and how a procedure is applied is explicitly and competently communicated. Opportunities to break down a procedure into its components are developed. It is coherent, orderly and sequenced. Multiple procedures are used and opportunities to provide reasons for and differences of these are made explicit.
Opportunity to develop Adaptive Reasoning – OTDAR	Opportunities to develop reasons underlying explanations are incoherent and inconsistent. Formal or informal proof or justification is not observed. Justification may be simply with reference to authority (textbook, teacher, 'rule').	Reasoning is explicit and valid. Opportunity to develop informal proof or justification is sometimes used. Learners are not encouraged to neither justify nor prove their work.	Opportunities to develop explicit reasoning are validated. Explanations of procedures or concepts are immediately followed with informal proof, justification and/or deductive reasoning. Learners are encouraged to consistently justify their answers or claims.

Opportunity to Develop Strategic Competence – OTDSC	Opportunities to develop and use a heuristic (pictures, lists, flow chart, etc.) are inappropriately selected for the mathematical problem at hand.	The opportunity to develop a single heuristic (pictures, lists, flow chart, etc.) that is appropriate to the mathematical problem at hand is seized.	Opportunities to develop multiple heuristics to solve problems are evident. Opportunities to choose flexibly among these are explicitly linked to the mathematics at hand.

In this chapter I introduced the notion of OTDMP which will now permeate the remainder of the study becoming the mainstay of the research. It informs the final design of the instrument and will be an integral part of the results and analysis. The table is the core around which the theory of OTDMP was first formulated. The table went through multiple changes mainly in the belief that opportunities that develop mathematical content and quality play a key role in mathematical understanding. The overall quality of the lessons is seen in terms of "the opportunities that the lesson provided for students to construct important mathematical understandings" (Hiebert*et al*, 2003, p. 199). This vision, framed by the five strands of mathematical proficiency, eventually led to the ratings which merge the mathematical content with the quality. A key component of the formulation was the generic use of the table in all areas of mathematics learning viz. number concepts and operations, geometry, measurement and data handling. The table could be applied equally well in any of these domains.

The acronyms introduced in this chapter will be used substantially in the remainder of this study.

Chapter 5: Design and Methodology

The research is framed by the construct of mathematical proficiency defined in terms of its five strands. In previous chapters, a brief description of the literature that eventually led to the embodiment of mathematical proficiency composed of a number of strands was elucidated followed by the formulation and description of the notion of 'opportunity to develop mathematical proficiency'. In this chapter, details of the design of the study, data collection procedures, classroom observation instruments used and method of coding will be highlighted. It must be recognised that all of these except the method of coding were already given by the larger study of which this is part.

5.1. The appropriateness of a video survey

This study essentially involved a video survey, which was part of the design of the larger study (see chapter 1). A number of factors influenced the choice of and implementation of this method. Below is a brief indication of some reasons for the choice of a video survey as the chief source of data collection. The video survey was developed for the TIMMS video study to marry videotaping for qualitative analysis with national sampling (Stigler, J. W., Gallimore, R., &Hiebert, J. 2000).

The classroom interaction is a complex and dynamic situation. Stigler et al. (2000, p.90) says video analysis has crucial advantages over other analyses of classroom observations. To capture the essence of opportunities to develop mathematics proficiency as explained above, a video analysis was an obvious choice. A video of a lesson will provide the opportunity to analyse this complex interaction in the class from different points of view. Both aural and visual information is captured by a videotape. Subtle nuances in speech and non-verbal behaviour can be observed. Checklists, which refer to lists compiled by a researcher in order to analyse lessons, are restricted by observation since only a limited number of judgements can be made in an instant. Video can however, be paused, rewound, watched again, using any time interval selected by the researcher. They can be viewed multiple times so that salient features of the lesson can be identified. Video captures the moment by moment unfolding of a lesson. Most importantly, the data remains untouched, unanalyzed and raw (though it must be recognised that deciding where to focus the camera is in itself a selection). A teacher's style and classroom practice according to a particular framework can be identified, and the analysis subjected to inter-coder interrogation. In addition, surveying student response and behaviour provides an opportunity of observing student involvement in the lesson. Audio recordings are crucial in videotaping as this will provide additional opportunities to analyse lessons. Each statement, question, answer, intimation, etc. could be played repeatedly to establish the impact of the lesson at any stage. Teachers' voice patterns, volume, tone and other audio characteristics provide further detailed observations of the complex interaction in a classroom.

In agreement with this is Powell (2003) citing Bottork (1994) who notes density and permanence as two main potentials of video recordings as a resource for research. Density refers to the simultaneous details of the learning activity at a particular instant. Video recordings capture large amounts of both audio and visual data at particular instances. Any moment of the video can be dissected and viewed from multiple perspectives. Permanence refers to the shelf life of the data. Researchers can view recorded events as frequently as necessary, revisiting the learning scene when required. The technical features of video such as real time, slow motion and frame by frame allows greater flexibility of analysis. To capture the essence of opportunities to develop mathematical frequency (OTDMP) as explained in the Theoretical and Analytical framework chapters, a video analysis was an apt choice.

Video surveys also have their limitations. One is that the deep analysis it allows also takes substantial time, thus limiting the scope of a study. In the larger grade 6 study, only one lesson was videotaped in each classroom. It therefore offered only a partial picture of a teacher's style and classroom practice. Thus the information about the teacher and corresponding teaching practice is scant and may not provide a reliable picture. The camera effect is another potential problem. Teachers' and learners' behaviour during videotaping may be unrepresentative of normal practice. Despite these limitations, I still believe that to capture opportunities to develop mathematical proficiency, videotaping of mathematics lessons was the most appropriate method. Access to the videos from the larger study provided an opportunity to conduct my research.

5.2. Videotaping procedure

Videotaping in the larger project was conducted by handheld cam-video recorder with the operator positioned at the back of the classroom in order to be unobtrusive as possible. A tripod was used to stabilise the cam-video recorder. The general aim was to capture the mathematical interaction in the classroom. A major part of this interaction was classical with the teacher imparting the necessary mathematics using the chalkboard as a medium. As a result most videos focussed on the teacher and the corresponding mathematical examples which were written on the board. In some cases the operator held the video and walked around the classroom capturing images of learner's worksheets, moments in which learners were involved in group work or any other activity which was deemed as constituting mathematical learning. The camera was turned on at the beginning of the lesson and was only switched off once the lesson had ended.

Some problems did arise during coding as a result of camera operation. In one instance, the operator focussed on the teacher during instruction despite the teacher's reference to content written on the board. The written work was then videotaped after the instruction segment. In these cases the features of rewinding and fast forwarding to appropriate time intervals became extremely useful. The only drawback in coding such cases was the additional time needed. In another case, the initial video focussed entirely on the teacher in zoom view with poor audio quality. This continued for the first two segments of the lesson. These segments could not be coded. The operator managed to revert to the correct operation later in the lesson which was then coded as normal. Whilst the quality of the videotaping lacked a professional touch, it was generally adequate for the research under investigation.

The validity of data collection fell under the auspices of the larger study which ensured that the video tapes were relevant, reliable and comparable. Appropriate procedures were implemented to prepare team members for videotaping protocols.

A centrally managed system of entering, cleaning and storing data samples was implemented.

5.3. Sampling

In the sampling procedure of the larger study, forty primary schools were sampled from the Umgungundlovu Education district in KwaZulu-Natal, using stratified random sampling. Umgungundlovu is one of 12 education districts in the province. It comprised a total of 219807 learners in 2009, making it the seventh largest district in terms of number of learners. The district has 4 towns with services and only a few schools that are considered remote (60 kms or more from a town). The districts of Obonjeni, Umzinyathi and Empangeni have the highest number of remote schools. In terms of the matriculation pass rate in 2008, the Umgungundlovu district had a 63% pass rate, which is the second highest in the province. Thus we may expect that the results from the Grade 6 learners and teachers in this study are slightly better than the results in other districts.

All schools categorised by the Department of Education as Quintiles 1, 2 and 3 were re-coded into quintile 1 (paradoxically named) for this study, representing poorer schools, and schools usually categorised as quintiles 4 and 5 were re-coded into the new quintile 2, representing affluent schools. Approximately 76% of KZN grade 6 schools fall within the study’s quintile 1 (old quintiles 1, 2, 3) and 22% in the new quintile 2 (old 4 and 5), and 2% still need to be updated. In order to maintain the provincial representivity of schools, sampling was done within these strata. A random number of all sampled schools was generated and sorted in an ascending order. A rank was assigned to each school on the basis of this ascending order. 2 lists of schools were generated representing quintiles 1 and 2 respectively. The first 30 schools were selected from the list of quintile 1 schools and the first 10 from the list of quintile 2 schools. Thus the study sample was stratified to comprise 75% of less-resourced schools and 25% of better-resourced schools.

Approximately 78% grade 6 schools in KZN are rural, 18% urban and 4% not yet demarcated. The research team did not use the rural-urban field as a variable for sampling because they believed that there is a strong relationship between rural schools and schools in the lower socio-economic quintiles, and urban schools and schools within higher socio-economic quintiles. I find this assumption reasonable, and thus would expect the data to be fairly representative of the district and the province.

Although the intention was that 40 schools be sampled, four schools did not wish to participate and had to be replaced. The final number of schools in the larger study was 39. 30 of these 39 schools were used in this study due to availability and in some cases quality and damage of some videotapes.

5.4. Ethics

The ethical considerations in this study fall beneath the umbrella of the larger research. All participants in the larger study had to sign consent forms. This included the principals, teachers and parents/guardians of learners with the understanding that their names, the names of schools or any other form of identification will not be used to identify them. In addition guarantees that the videos will not be released to anyone and that an executive summary of the final report would be sent to all participating schools. Participants and schools were assured of anonymity at all times and that the video tapes/DVDs would be for the use of researchers only. Safe storage of these tapes and questionnaires was guaranteed in a secure location of the buildings of the university. Appendix 1 contains a copy of the ethical clearance application form.

5.5. Instrument design and Coding Process.

The instrument has its design roots in the larger study. The instrument from the larger study was initially designed to code for various aspects which included content coverage, mathematical proficiencies, level of cognitive demand and teachers content knowledge, pedagogical knowledge and pedagogical content knowledge. Columns were provided in the mathematical proficiency section to code for the five strands, viz. conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. This served as the core feature in the development of the instrument for this study. Initial changes were informed by the theoretical framework chapter. Sub-categories for each strand were introduced as understanding of mathematical proficiency increased resulting in changes in the instrument design. Ongoing subtle changes to the instrument occurred as the coding progressed and the theory of opportunity to develop mathematical proficiency (OTDMP) was formulated. This eventually culminated in an instrument which captured the essence of the study yet did not result in re-coding of the initial videos. The fundamental structure of the instrument remained intact which fore-grounded the theoretical and analytical framework. Changes included a section for recording observations in each segment. Here, the notes were descriptive and illustrated the material relevant to the research. These notes were later utilised to identify types of classroom interactions, for instance (see Chapter 6). Factual descriptions were used to describe the observations, steering clear of interpretation or inferences. Example of recordings would take the following tone: 'the teacher discusses an example of addition of mixed numbers, by initially converting to improper fractions.' Inferential remarks such as, 'he is trying to......' or 'an unsuitable diagram is chosen to....' were avoided. These descriptions allowed the coder to return directly to a specific segment of the lesson if required to do so. A description of the opportunities which were identified was recorded in the observation part of the instrument.

A segment of the lesson was defined as the time interval within which mathematical instruction occurred. A segment of 5-minute duration was the unit of analysis in this study. Thus each lesson was viewed in intervals of 5-minutes in line with the larger project. A maximum of 12 segments was inserted to accommodate for maximum lesson duration of 60 minutes. Initially a stop – pause – analyse and record procedure was implemented. As the process was repeated and the familiarity of the descriptor table increased, the procedure changed into viewing and recording without pausing save for exceptional circumstances. This was aided in part by the fact that the software used to view the videos clearly showed the time lapsed at any point in the viewing cycle.

As alluded to earlier in this chapter, the instrument used in the larger study was adapted to mirror the requirements of this project. Additional columns and split columns were inserted for each strand to record the finer grain size required for each opportunity. Columns were needed for OTDCU, OTDPF, OTDAR, OTDSC and OTDPD. Sub-categories were then used to provide further meaningful analysis for each strand of opportunity as discussed in the theoretical framework chapter.

Each lesson was then viewed in intervals of 5 minutes with each segment been assessed for the five strands of OTDMP. This coding instrument and rating was adapted from the larger study. This was necessary to analyse lessons using a finer grain size as opposed to that used in the larger study. A 2-step procedure of identification and rating was used.

Step 1: Analyse each interval and detect whether the opportunities were 'present' or 'not present'.

The following lettering indicated this step: P – present NP- not present

If 'NP', a '√' was then inserted in the appropriate cell of the video coding sheet.

Step 2: If 'present' then the grading system using the descriptor table explained in the analytical framework chapter was implemented. The values 1, 2 or 3 were inserted when the observed mathematical opportunity correlated with the ratings in the descriptor table. The list below indicates how each opportunity appears in the final instrument that was used.

OTDCU: S – stated D – developed

OTDCU		
NP	P	
	Stated	Developed

Figure 3: Instrument design for OTDCU

OTDPF: T – task controlled S – solver controlled

OTDPF		
NP	P	
	Task	Solver

Figure 4: Instrument design for OTDPF

OTDSC: Pictures, lists, flowcharts, trial and error, patterns, like problems and others.

OTDSC							
NP	P						
	Pictures	Lists	Flowchart	Trial and error	Patterns	Like problems	Other

Figure 5: Instrument design for OTDSC

OTDAR: Reasoning, justification and explanation.

OTDAR			
NP	P		
	Reasoning	Explanation	Justification

Figure 6: Instrument design for OTDAR

OTDPD: Perseverance, confidence and real world.

OTDPD			
NP	P		
	Perseverance	Confidence	Real World

Figure 7: Instrument design for OTDPD

An additional feature of the instrument included rows and columns used to collate the ratings for the respective strands. The insertion of rows was necessary during the analysis phase of the research. Results were collated by inserting formulae for the rows and columns that would allow comparative analysis. The aggregation of results within and across strands was used to answer the research questions.

The formulation of the notion of OTDMP followed by the descriptor table per opportunity incorporating the characteristics that inform the ratings contained sufficient information to assess the strands of mathematical proficiency for the lesson unit. Using the results of OTDMP, it would be possible to correlate the analysis of the videos with the results from the learner and teacher questionnaires and tests as well as with other variables. The notion of OTDMP will be the main focus and feature prominently during results and analysis. Appendix 2 shows the instrument used in the study.

5.6. Validity

Early stages of validation began during the preliminary stages of the larger project commencing in KwaZulu-Natal. Recorded lessons from other provinces were available and the coding process initiated when a workshop was held in Pretoria for this purpose. The primary aim of the workshop was to ensure consistency during the coding process in all geographical areas involved in the larger project. This workshop was attended by all co-ordinators from the various provinces as well as by students who would be involved in analysing videos. During the workshop the videos were coded in the areas of content discussed in the lesson, mathematical proficiency, level of cognitive demand and pedagogical content knowledge. A process of viewing five minute intervals followed by discussion in order to arrive at a consensus view was implemented. During this workshop initial exposure to coding and analysing videos was entrenched.

Further validation took place during supervision and analysis workshops which were organised to ensure coding assigned was consistent in the Umgungundlovu district of KwaZulu-Natal. This took place at UKZN, Pietermaritzburg, where videos were viewed and analysed by a group. If there was doubt in coding, the content was deliberated and discussed in order to arrive at a single view. It was during this process that the instrument developed was further refined.

These discussion sessions and viewing of videos provided the necessary forum for checking and consulting with others and obtaining a view on the ratings assigned by looking at instances where coding was in doubt. Such discussions ensured that coding the video lessons was consistent, as far as possible under the circumstances.

Many variables could have impacted on the study affecting its design, methodology, analysis and reporting of results. A brief description of these is given below.

The choice of five minute intervals was to a large extent driven by the larger study. Choosing varying time intervals was considered but not implemented. Ten, fifteen or possibly shorter three minute intervals could impact the reporting of the results. If the methodology applied is to record an opportunity in any chosen time interval, then the number of such recordings is likely to change proportionately. Longer time intervals would result in a lower number of recorded opportunities and vice versa. The study focused on the mathematical opportunities presented and developed during classroom interaction. The opportunity itself remains present irrespective of the observation methodology and approach used. Identifying, coding and rating these observed opportunities will vary depending on the aim of the researcher and his/her research goals. The mathematics and identified opportunities to develop mathematics proficiency is unlikely affected by the time frame of the coding intervals but rather to a limited extent by the methodology and coding process used by the researcher in reporting these opportunities.

Other aspect of the study to consider is that only a single lesson from each teacher was recorded. To what extent this differed from normal practice is debatable. Another is advantages and disadvantages of coding for opportunities within a time interval as opposed to identifying the time frame within which the opportunity occurred. These and other issues could be explored at a later stage.

Chapter 6: Coding

In this chapter extracts from the videos are related to illustrate the coding process. The extracts show the correlation between the discourse in the mathematics classes and the OTDMP as indicated in the descriptor table. The selection of the extracts was informed by the instrument which was designed to capture the opportunities to develop the strands of mathematical proficiency in each 5 minute segment of the lesson. Thus extracts that showed more than one strand in a single segment were chosen. This would reduce the number of extracts needed to explain the coding implementation as described in the descriptor table 1. However, for completeness, some extracts are included that show only one strand. In addition video segments that were difficult to code, those that bordered between sub-codes or that could not be differentiated between strands are also discussed. Transcripts of the segments were recorded by making use of headphones and using the stop, pause, rewind and play features of the video player.

6.1. Extract 1

This lesson began by the teacher writing the heading on the blackboard, viz. Multiplication by 10, 100, 1000 and 10 000. Six problems were written and learners were invited to write the answers on the blackboard. This was met with much enthusiasm and eagerness. No attempt at revising any concept or illustrating a previously taught procedure was made. The very first answer written was, '560'. This was correct, but what followed was an indication of the confusion to come. I relate the rest of the extract.

1 Teacher: Is it correct?

2 Class: Yes! (chorus)

3 Teacher: There is a space, ja?

This exchange suggests a specific manner in which the teacher expects the answers to be written. The first three answers are written on the blackboard corresponding to multiplication by 10, 100 and 1000. All of these are correct. A learner is then chosen from the multitude of hands to write the answer to the problem, 25 X

10 000. The learner writes 25, then counts the number of zeros' and subsequently writes the answer, 25 0000, with a space between the 5 and the zeros. In the background some learners shout, 'no, no, no'.

4 Teacher: Who said no? Is that correct?

5 Class: Yes (chorus by a few learners).

The teacher provides no explanation. Reasons for expecting the answer to be written in a particular way are not forthcoming. No attempt is made to explain neither what procedure is used nor why the procedure is used. Although the answer written was correct, the teacher's comment in line 4 and the class's chorus in line 2 appear to have created doubt amongst some of the learners. This is evident when the next attempt by a different learner yields the answer, 2500. The teacher then steps in trying to show the learner his error by making him read the question. Another learner is called. He begins writing his answer, viz. 200. The loud objections from the class, stops him. He tries again, writing down the answer, 20050. Once again, no explanation or attempt at indicating the nature of error is evident. Another learner then writes the answer, 250 000.

6 Teacher: Is this correct?

7 Class (chorus): Yes!

A learner is then invited to write the answer to the problem, 125 X 10.

8 Learner, writing on the board: 1250 [no spaces]

9 Teacher: Read your answer.

10 Learner: One thousand two hundred and fifty.

11 Teacher: How can you write 1250?

12 Learner: 125 0(space between 5 and 0)

13 Teacher: Is it correct? No.

Another learner writes the answer correctly, 1 250. This is followed by a different learner writing the answer to the last question, viz. 76 X 1 000 = 76 000. Both answers are applauded and worksheets are then circulated and learners proceed with class work for the remainder of the lesson.

Discussion

The only strand of proficiency recorded in this segment was OTDPF. The rating assigned was 1, corresponding to the descriptor in the table describing inefficiency in handling procedures and a lack of fluency when handling procedures. The extract suggests procedural inefficiency. Although the teacher may have a clear understanding of what the answer is and how it should be written, this is not conveyed to learners in an efficient manner, and there is no feedback indicating when the value of the answer is incorrect and when it is merely written in a format different to what the teacher wants. Feedback during mathematical lessons affects understanding tremendously. This is corroborated by Hattie who states that "Feedback is one of the most powerful influences on learning and achievement" (Hattie & Timberlie, 2007, p 81).The extract shows no positive feedback throughout this segment. There seems to be no clear direction or control. Alternate methods of writing numbers are not entertained during the course of the extract and this implies that any answer, although correct, not written in a specific way, is wrong. (Refer to line 8). However, no explanation for leaving or positioning spaces was given which could have avoided confusion. Writing numbers in a particular manner may be helpful when performing operations with numbers such as when multiplying and adding large numbers. In the comment below line 3 we observe a learner counting the number of zeros to arrive at a correct answer. This is one method that could be used to obtain the answer. Multiple methods arise in different domains of the curriculum and these should be explored by teachers. The rating 'NP- not present' was recorded for all other opportunities of proficiency as none were observed.

Extract 1: Strand Coding

NP – OTDCU, OTDSC, OTDAR, OTDPD

P - OTDPF (stated), rating 1

6.2. Extract 2

This slightly longer introductory lesson clearly shows a teacher-led approach to a new concept, that of symmetry. The concept is developed and I thought it prudent to delay the discussion until after the extract since more than one strand of proficiency is identified. The teacher greets the class and the lesson begins.

1 Teacher: Ok, today we are going to do some things, activities. These activities are going to mean something. Sometimes we do it alone, sometimes together. We have some things on the board. Let's begin with this one. What does it look like?

The teacher points to a hand drawn picture placed on the board.

2 Learners: A loaf of bread

3 Teacher: A loaf of bread. I am happy that you see it as a loaf of bread. We usually cut bread at home. Sometimes into slices, sometimes into quarters. Who can show me where can you cut this bread so that you have two equal halves?

A learner is selected from those that raised their hands. He draws a line through the centre of the picture.

4 Teacher: He will cut it like this. Ok. Thank you. You can cut it like here or like this. But he will cut it like this.

The edge of a blackboard set-square is used to indicate lines that would cut the bread in half. Teacher continues.

5 Teacher: Are the halves equal?

6 Class: Yes, sir

7 Teacher: How can we find out if there are two equal halves?

Without waiting for a response he removes the picture of the bread and proceeds with a demonstration.

8 Teacher: Ok, let us check now. We cannot do it to actual bread, but we can do it to this picture to check. Let me fold it along this line. Fold it here; but I want to be sure here. I am convinced that they are equal if both areas fall exactly onto each other. This corner on top of this corner; this line on top of this line; everything....every piece, everything falls exactly on top of the other. Ok, very good. Clap your hands. He has

cut the bread right into half. Ok, note the lines where the bread can be cut into two halves. We will talk about it later.

He points to the crease where the picture was folded in half.

9 Teacher: Okay, now I am the grandfather of two twins. (He points and touches the heads of two boys). ...two grandsons. I have a slab of chocolate and you or anyone is going to cut this chocolate in half; a funny looking slab of chocolate, a funny shape. I bring it and you are going to cut it into two equal halves and if you don't satisfy anyone of them they are going to cry. How can you cut the chocolate for them so that you get two equal halves? Is it possible? Look carefully. Look for the line where you can cut it and fold it. If you can we will fold it along that line to see if you get two equal halves like we did with my bread. If it is not possible just put your hand up and say, it is not possible. Can we cut my funny looking chocolate into two equal halves? ...look.....look....look. We did it with this one; can we do it here?

10 Learner: It's impossible

11 Teacher: He says it is impossible. Who else says it is impossible? Do you agree we can't cut this funny looking chocolate into two halves?

12 Class: Yes sir!

The teacher then demonstrates that the funny looking chocolate cannot be cut into two equal halves using the blackboard set-square and placing it along imaginary lines on the picture.

13 Teacher: Okay then. We have learnt something. That we can cut something's into two equal halves that are equal and identical. That is very important; they must be equal and identical. The one half looks exactly like the other half. Some things we can cut but other things we cannot cut. Okay let's look at the other objects

Teacher points to other pictures displayed on the board including the South African Flag, the letter 'F', a drawing of a person and some flowers. He checks that the class recognises the South African flag and then continues.

14 Teacher: Which things out of all of these can we cut like my bread?

15 Learner: South African Flag

16 Teacher: Can anyone come and show me where mustI cut the flag so that there are two equal and identical pieces. Can anyone come and show me?

A learner is chosen and proceeds to draw a line through the centre of the flag from left to right. Using the blackboard set-square he places it on the line the learner had drawn. The class unanimously agree that the line does divide the flag in two equal halves.

17 Teacher: Which other objects here can we cut like my bread?

18 Teacher: A human being

A learner is then asked once again to draw a line. Whilst the learner draws, the teacher relates a story from the bible how a dispute was settled by a king when confronted by two mothers, both claiming that a baby was theirs. The king suggests that the baby should be cut in half, in that way the two mothers can have an equal share of the baby. He then asks for other objects which cannot be cut into two equal halves.

19 Teacher: What we have been learning thus far, grade 6, is called ‘symmetry’. Usually it has a line where you can cut it. There is a line you can cut it so that you can get two equal and identical pieces. We call this line, ‘the line of symmetry’.

The teacher then draws a table dividing it into two columns, ‘symmetrical’ and ‘non-symmetrical,’ beneath which he writes ‘line of symmetry’ and ‘no line of symmetry’. With the help of the learners, he completes the table. The lesson then continues with learners divided into groups with each group finding two things in the class which are symmetrical and non-symmetrical. Class activities are then given using drawings on worksheets and folding of pages to consolidate the visual idea of symmetry.

Discussion

Two opportunities of Mathematical Proficiency seem to dominate this extract. The first is OTDCU. Symmetry and the associated line of symmetry is the core theme of the lesson. The teacher chose to develop the understanding of symmetry which extended over two of the 5-minute intervals. Development of the concept was through demonstration. Paragraph 1 of the extract indicates the teacher's intention of illustrating the concept by referring to activities which he intended doing either together or individually. This he did by drawing on the learners' real world experiences. Further evidence of development of the concept was the fact that the term, 'line of symmetry' was never mentioned throughout the extract. Only in line 20 of the extract do we see the term 'symmetry' associated with cutting into two equal pieces. Throughout this extract and the lesson the teacher shows confidence in the mathematics he is teaching and a clear and well prepared strategy. The preparation of drawings, albeit crude, adequately conveys the teacher's intention of using them as aids. Their flexibility allows the teacher to fold the bread along the line the learner had drawn. This visual aspect of the line of symmetry is fairly well covered. In paragraph 9 he emphasises the importance of areas falling exactly onto each other. Learners are in no doubt that the line has to be drawn so that it divides the object equally. This understanding allows the learners to easily answer the questions that follow. They appear to have no doubt, that edges should fall directly on the opposite edge once folded as explained in paragraph 9. The pace at which the teacher progresses also allows the learner to assimilate and internalise the concept. Besides the length of time spent in developing the concept, he allows learners an opportunity to think (paragraph 10 where he encourages learners to "look, look, look"). Towards the end of the extract he introduces the terms for the concept, viz. symmetry and line of symmetry. To summarise, this pictorial representation of the construct of symmetry was developed in a manner which would allow the learner to extend the concept to other representations at later stages of their mathematical education, for instance, the line of symmetry in graphs or theorems in geometry. Clarification of the concept is clear and links are made between the visual aspect of symmetry and the concept of line of symmetry. The actions of the teacher described below paragraph 16 and the teacher's summary in paragraph 19 explicitly connects the line of symmetry and the concept of symmetry itself. During the discourse the teacher shows how the line of symmetry is used. This was done by demonstrating with the use of the blackboard set-square. Consequently, a rating of 2 for conceptual understanding was assigned for both segments of the lesson in line with the descriptor. Although fairly conceptual for a grade 6 learner a rating of 3 was initially considered. However, no other representation is evident and explanations of why the representation is used as well as its significance are not clearly outlined. In addition in paragraph 9 he asks if a 'funny shape chocolate can be cut into two equal halves.' In the context of the lesson, correct mathematical terminology and usage is essential. The term 'halves' implies two areas or parts of equal value which contradicts his reference to the chocolate. Later he also uses a baby in his story as well as a human being to identify lines of symmetry which may not be true in all cases. The segment fell short in addressing and constructing pieces of mathematical information.

The strand of productive disposition is recognised throughout this extract. From the outset the teacher attempts to use the everyday experience of cutting bread to develop the concept since many learners would identify with this. He guides learners and uses their natural instincts to draw the necessary lines. Paragraph 14 and 15 indicates other objects that were used that learners could relate to. The objects were used and not simply referred to. However, the connection to the mathematics was not very clear and explicit. Sense of the mathematics is recognised but not fully exploited. The objects became learning and teaching aids, which did add real world value to the concept. The teacher also rewards effort (paragraph 8) where he encourages learners to clap their hands when a learner correctly completed a task. Although opportunities to comment on learners' performance or encourage interest existed, this was only done on occasion. Here, a rating of 2 for productive disposition was given for this segment of the coding. Once again, the above comments resonate with the description as it appears in Table 1.

The remaining segments of the lesson contained many more instances where this strand was promoted. The activity required groups of learners to identify two symmetrical and two non-symmetrical objects from within the classroom. Worksheets were distributed that had to be folded along lines of symmetry. These shapes included rectangles and types of triangles. Learners were actively engaged in the folding process and questioned when folding along certain lines. They persevered with the task and were encouraged by the teacher. Initially the opportunity, OTDPF was considered during the folding phase of symmetry. This was eventually shelved as it seemed more a question of dexterity than any form of mathematical activity.

Extract 2: Strand Coding

NP –OTDSC, OTDAR, OTDPF

P –OTDCU through demonstration (rating 2), OTDPD (rating2)

6.3. Extract 3

This extract occurs during the learner task phase of the lesson while learners were working individually with the mathematical task. The learners are actively engaged, or seem to be, with the task introduced and explained in the previous segments. The teacher moves between learners offering advice and identifying errors in learners work. Learners are involved in an application of addition and subtraction using real world situation, viz. purchasing items from a store. The teacher stops the learners, 10 minutes into this phase of the lesson. The lesson unfolds further.

1 Teacher: This is how much the items cost. R193. (Writes this value on the board.) How much Mrs X.takes out of her pocket to pay for the items?

2 Learner: R200

3 Teacher: That is your cash tendered. Now, she paid R200. She has to get change. How are we going to calculate her change? All these things are coming to R193. This is the total. She paid R200. How do I calculate her change? Come,....come,.....- one, two , three (referring to the number of learners with hands in the air. The teacher pauses).

Think. How do I calculate? If I buy something for R5 and I pay R10, how much is the change?

4 Learner: R5

5 Teacher: R5 is the change. It was R5, paid R10, R5 is the change. How did we get that? Now Mrs X. bought for R193. Paid R200, what is the change? What do I do? Come on, come on,.......Too few hands. Yes Tom

6 Learner: R7

7 Teacher: I don’t want the answer; I want you to tell me how do I get the R7. Yes Tom.

8 Learner (Tom): Minus

9 Teacher: Yes, we minus. What number minus what?

10 Learner: R193 – R200

11 Teacher: Hey, think carefully.

12 Learner: R200 – R193

13 Teacher: R200 – R193 (in agreement). That will give you your change.

The teacher then writes the problem, viz. R200.00 – R193.00 on the board and does the computation.

14 Teacher: 0 minus 0. This will become 1. This will become 9.

Whilst completing the computation, the teacher directs questions to the class as a whole, who responds in chorus. The class then continues with their class work.

Discussion

This particular segment contained discourse that incorporated a few strands, despite the shorter length of the extract. OTDPD appears in paragraph 3. The teacher encourages learners to provide an answer. The possibility exists that he is irritated that they are not giving him an answer. However, counting the number of raised hands could be an attempt at boosting learners’ confidence, not only in their thinking but in a belief that their answers are correct. This perseverance and encouragement is evident again in paragraph 5. Here he encourages learners to keep trying. In lines 9 – 12 he persists with a learner until the correct mathematical formulation of a subtraction problem is elicited, viz. larger subtract smaller. Writing the minuend and subtrahend correctly conveys deeper understanding of the concept of subtraction. Rewarding effort is present but not enthusiastic or explicit. In line 13 the teacher repeats the correct answer but without any subsequent personal comment to the learner. Answers from other learners are not considered. This traditional teacher led approach encourages learners to see if they are correct and to correct them if necessary though criteria are not always made explicit. There are references to everyday situations which link to the mathematical content. Using the descriptors from Table 1, for productive disposition the situation described was allocated a rating of 2.

The OTDAR appears in paragraph 3 when the teacher requires learners not to provide the answer, but rather explain how the answer is obtained. Here an attempt is made to coax learners to explain the procedure used to arrive at their answer. This is confirmed in line 7. Explicit explanation is required in an informal mathematical sense. It appears that the teacher recognised the problems learners were experiencing whilst completing the class work. The guidance offered by the teacher during this segment is an attempt to correct learners' errors and misunderstanding of a real world problem. A rating of 2 for OTDAR was penned for this extract. At the end of the extract the subtraction is performed fluently and appropriately and the answer obtained with the class's assistance. Alternate procedures are not explored. A rating of 2 for OTDPF was given.

During paragraph 5 of the extract the teacher uses a simpler example. This approach does not differ substantially from the original problem. No manipulative is used to convey deeper understanding of the mathematics involved. Reasoning underlying the procedure is not evident. However, this simpler version of the segment seemed to assist learners in arriving at the answer much quicker. Coding this part did initially present a problem, but by unpacking each strand of proficiency I concluded that this part merely reinforced both productive disposition and procedural fluency.

Extract 3: Strand Coding

NP – OTDSC, OTDCUP – OTDPF (stated), OTDPD, OTDAR – all rating 2

6.4. Coding Seated Work/Class work.

Coding this phase of the lessons presented a number of obstacles. The overarching thrust of the study is to establish the teachers' promotion of the strands of mathematical proficiency. A large proportion of mathematics lessons in South Africa are devoted to learners involved in activities of which class work dominates. Most lessons involved the following sequence of activities: Review previous material, show the procedure for the day on a set of problems, and then assign problems for learners to complete in the class. The instrument utilised inadequately allows for coding of the segments of the lesson that were used for class work or learner work time. This presented a major problem. The traditional approach to class work involved giving learners work to do and then walking around the class checking on what learners were doing, which could promote several strands of proficiency. Occasionally teachers would call learners to the table, correcting and marking work already completed, but otherwise it was difficult to judge what strands were being promoted during class work without access to the tasks they were working on and the feedback provided by the teachers.

A number of issues during coding of this part of the lesson surfaced. Below is list of some of the seated work that appeared often that adversely affected the coding.

i. The class work given could not be ascertained. The video recording did not show the actual work that learners had to complete. Video–recordings could not reveal the contents of learners' workbook. There were isolated attempts by the video crew to show worksheets or exercises in learners' books, but this was the exception and not the rule. It became difficult to correlate assigned class work to the actual mathematics discussed for the day.

ii. Conversations between individual learners and the teacher were not audible.

iii. Discussions during group work, amongst learners and between learners and teacher could not be verified.

iv. Many videos showed teachers using this time to correct and mark learners' books. Whether constructive engagement of errors and misunderstanding of concepts was identified could not be ascertained.

Before delving into the coding aspect of these cases, situations in which teachers identified shortcomings in learners' class work and corrected these with the entire class were coded as an opportunity and the strand of OTDMP observed was rated in line with the descriptor table. An example of this appears in extract 3, related above.

The study concerns the teacher's promotion of the strands of mathematical proficiency. During class work, the only observation that was meaningful was noting the actions of the teacher. Some teachers preferred walking around the class ensuring all learners are engaged in the activity. Other teachers checked individuals work in a predictable pattern, starting at one end of the class, finishing at the other and then repeating this until the end of the lesson. Yet other teachers visited each group, spending time depending on the number of learners in the group. Another batch of teachers preferred working from the table, calling individuals and marking their work. This was done either randomly or when learners completed the assigned tasks. There was very little doubt that such actions supported and favoured the fact that teachers provided opportunities to develop procedural fluency. Learners had to complete and practice exercises based on the concepts taught and the procedures that were used during the lesson. Other cases could include applying procedures in new situations or inventing procedures and analysing new situations. These cases were not discernible. Thus all segments of the lesson that conformed to this description were given a rating of 1 for procedural fluency and 'not present' for all the other strands. This was inaccurate but necessary since every segment of the lesson affected the coding.

Whilst tempted to provide additional extracts to further consolidate and correlate the coding with the description of OTDMP, those outlined above should provide sufficient evidence of the process of coding for the opportunities to develop the five strands of mathematical proficiency.

Chapter 7: Results and Analysis: The presence of OTDMP across Lessons

Results and analysis of the study is discussed over two chapters. The approach used in the chapters is influenced by the framework, both theoretical and analytical, as well as the research questions. Chapter separation is mainly due to analysis across lessons in the first chapter as opposed to individual lesson analysis in the second. This entails looking at the district in totality detailing results for the combined lessons followed by individual results of each lesson. The results of the study will be detailed using simple graphical interpretation of the data. The empirical material gathered in this study focuses on the observed opportunities that allow the development of the strands of mathematical proficiency. The results are framed by two key elements, viz. number of opportunities and scores of mathematical proficiency. In constructing the notion of OTDMP in chapter 4, repeated emphasis was placed on the observed opportunities and the degree to which it is developed. The results and analysis chapters embody these factors by considering the number of OTDMP recorded. Emphasis on the need to offer many opportunities during teaching to make it effective was placed in the analytic framework chapter. The chapter begins with the results aggregated from the entire sample followed by results and analysis strand by strand.

7.1. Consolidated Data Sheet

The focus in this study is the promotion of mathematical proficiency in a district of KwaZulu-Natal. Hence it was necessary to aggregate results in order to provide an analysis of the data obtained from all schools that participated thus providing a picture of mathematical proficiency in the district. Data obtained from the instrument used to code each lesson was consolidated on a single Excel spreadsheet. Simple statistical formulae were used to tally opportunities, ratings and scores (see chapter 7.3.below) for each strand as well as across strands. As analysis proceeded and additional information was required to inform the answers to the research questions, the consolidated spreadsheet provided the ideal tool to draw the necessary information. This information is then represented in terms of themes. As intimated earlier, number of opportunities and forcefulness characterises the nature of the construct of opportunity to develop mathematical proficiency in this study. The spreadsheet also contained the ratings per sub-category for each strand. The results and analysis are set out to show these features, the findings of which will be summarised to answer the research questions in the concluding chapter.

The results are set out to mirror the term 'mathematical proficiency' composed of its five strands. The synchronous nature of the strands of mathematical proficiency is emphasised repeatedly by Kilpatrick et al. In this research the analytical framework chapter details the characteristics of OTDMP and then sets out the descriptors for each strand of opportunity. The rating and recording of each opportunity promotes and strengthens that particular strand which in turn develops the promotion of mathematical proficiency as a whole. Thus the total number of opportunities afforded in lessons is crucial to the attainment of mathematical attributes of a learner. The first set of data thus shows aggregated totals obtained from the 30 recorded lessons coded in this study, followed by data from the separate strands. This sequence of analysis is not arbitrary but rather stems from the conception of the term, 'Mathematical Proficiency', as composed of its strands where the sum or whole is greater than its individual parts. Hence totals for all recorded lessons are dealt with first.

7.2. Total Results for Mathematical Proficiency

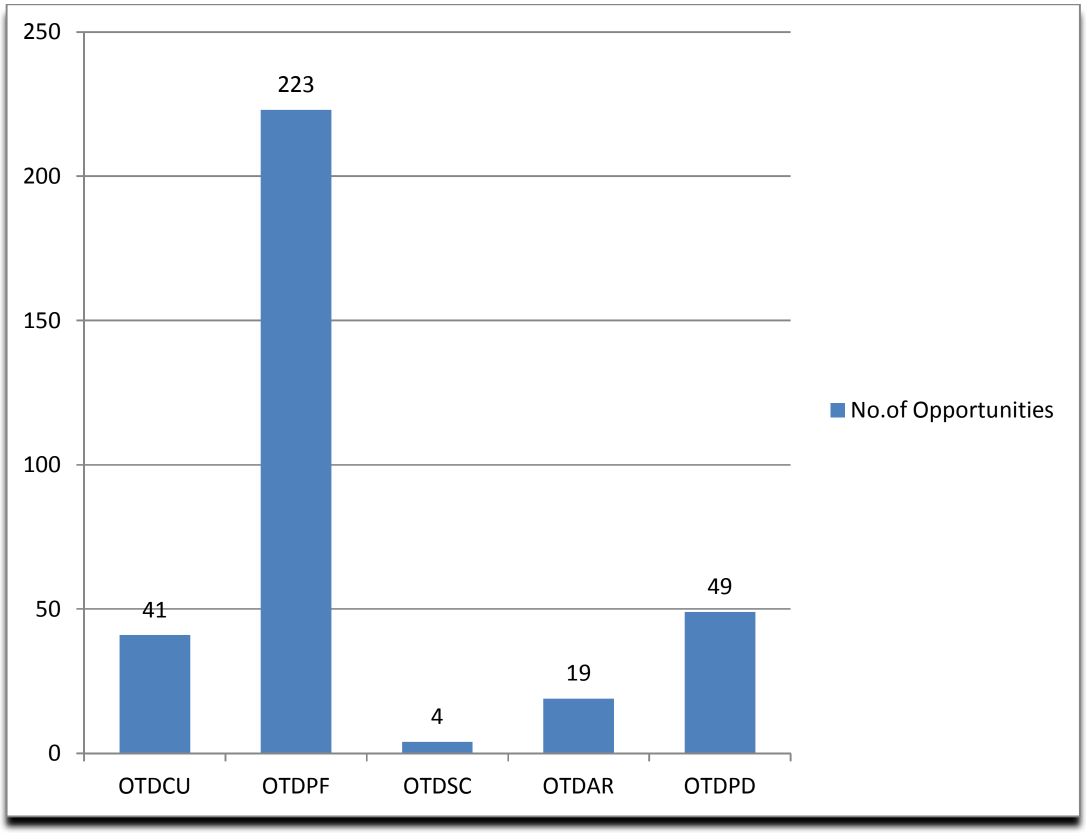

Figure 8: Number of Opportunities to Develop each Strand of Mathematical Proficiency (OTDMP) identified in 30 lessons.

Figure 8 shows the distribution of the opportunities recorded per strand for the entire sample of lessons in this study. A segment of the lesson was defined as the time interval within which mathematical instruction occurred. Recall that the unit of analysis is an opportunity afforded in a 5 minute interval. The figure therefore shows the total number of opportunities recorded from all recorded lessons for each strand. The opportunities to develop procedural fluency (OTDPF) dominate the result. A total of 223 OTDPF were observed in the 30 lessons. 49 opportunities to develop productive disposition (OTDPD) and 41 opportunities to develop conceptual understanding (OTDCU) were identified. This gives a fraction of just more than 1 opportunity per lesson in the strands of conceptual understanding and productive disposition that were developed per lesson. Only 19 opportunities to develop adaptive reasoning (OTDAR) were observed with a paltry 4 opportunities to develop strategic competence (OTDSC) recorded for all lessons. Combining all opportunities, a figure of 336 OTDMP occurred.

Viewed as a whole unit, opportunities to develop operations with numbers, calculate areas or perimeter of regular 2 dimensional figures, find averages in statistical data, draw bar graphs from given data and other procedural work in which an answer is obtained is strikingly dominant in lessons. A factor in the large number of OTDPF is the coding of seated work encompassing both individual and group work. Classroom exercises in which learners were actively engaged in completing activities or exercises were rated in the lowest category of OTDPF as explained in chapter 6.4.

Problem solving is virtually non-existent. In order to code for OTDSC, the heuristics employed in aiding the solving of a non-routine problem was coded. Different types of heuristics can be seen in the instrument. A specific column was inserted to accommodate for any other type of strategy used in solving the non-routine problem. Solving a non-routine problem was therefore identified in the opportunity to develop strategic competence. The 4 OTDSC identified is an indication of the lack of non-routine problem solving strategies in the analysed lessons.

OTDCU was recorded when development and knowledge of concepts was observed. Only 41 OTDCU in the sample of 30 lessons was noted. Translations between representations develop deeper understanding of mathematical ideas. It reduces the need to memorise allowing learners to engage with their mathematical background knowledge and link this to new concepts. Opportunities provided in this strand are clearly insufficient to develop concepts fully.

Opportunities that develop productive disposition included references to out of class situations. An observation which will be analysed in detail below is the relatively low degree in which situations in the real world are connected to the mathematics at hand. The number of OTDPD exceeds the number of OTDAR. This opportunity, which is supposed to bridge all the strands, was seldom recorded. Further engagement with this strand follows later in the analysis.

Crucial to the study is the ratings, ranging from low to high. Incorporating these into the results will be critical since a high rating indicates a high degree of an opportunity to develop a strand. Figure 9 gives the rating spread of all opportunities developed across all strands from all lessons.

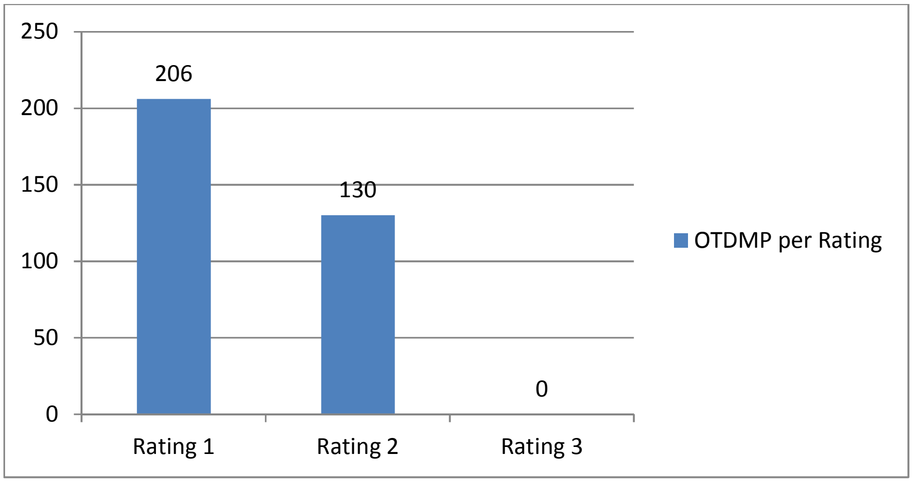

Figure 9: Rating Spread of OTDMP identified in 30 lessons.

206 of the total opportunities observed were classified in the lowest rating, viz. rating 1. The number of opportunities to develop mathematical proficiency falling in the low degree is far greater than the number recorded in the medium level. This is an indication that opportunities hardly promoted the development of mathematical proficiency as a whole. 130 opportunities fell into category 2 while none were observed in the highest category. Many variables could have played a role in this result. A contributing factor could be the descriptions of the highest rating in the descriptor table pitched at too high a level. Coding of the segments is another possibility. These and other factors will be interrogated further in publications after the study.

7.3. Scores of Mathematical Proficiency for the District.

Further analysis used 'analytical scoring' technique which incorporated the total number of segments noted in all recorded lessons with the ratings. Lesson duration ranged from a minimum of 3 segments to a maximum of 12. The data was again accessed and the total number of segments observed in the study was obtained. In total the 30 lessons observed recorded 242 five minute segments.

It was a challenge on the basis of this observation to say anything about the extent to which OTMP was prevalent in the observed lessons. In order to get a sense of this, a score for each strand of proficiency was then obtained by considering the highest possible rating that could be recorded, viz.3. This was merged with the total number of segments giving a maximum possible 'score' for each strand of proficiency that could be obtained from all lessons of 726. The 'scores of mathematical proficiency' is therefore a value that is viewed relative to the maximum that provides insight into the existence or prevalence and degree of opportunities to develop proficiency. It must be noted that this is seeing the observations against an entirely hypothetical situation of very high 'ratings' or OTDMP in all strands at all times, and thus is not referring to anything real; it is simply a different way of viewing the data.

Once again the data spreadsheet informs the table below showing the corresponding ratings obtained for each strand and total score calculated.. This information is then depicted graphically to allow a viewer ease of observation.

Table 2: Scores of Proficiency per strand

Opportunity	Rating	Number	Total score
OTDCU	1	24	58
	2	17	
	3	0	
OTDPF	1	142	304
	2	81	
	3	0	
OTDSC	1	3	5

	2	1	
	3	0	
OTDAR	1	10	28
	2	9	
	3	0	
OTDPD	1	27	71
	2	22	
	3	0	

In this table, a 'score of opportunity to develop conceptual understanding', for instance, is obtained by multiplying the number of opportunities with its corresponding rating. The score of each rating is then added to arrive at the total score for each opportunity. The maximum score does not in any way represent a perfect situation nor does the study attempt to find a best score. It does however give an indication by virtue of its remaining opportunity anti-score of the degree that a strand of opportunity is developed in the observed lessons.

Only the score for OTDPF at 41.9% shows any significant value. Scores for the remaining strands are 8.0% for OTDCU, 0.7% for OTDSC, 3.9% for OTDAR and 9.8% for OTDPD. This result will impact on the key research questions profoundly as it embodies the essence of OTDMP as constructed in this study.

Another possible approach is to consider the number of strands per recorded lesson. The connectedness and intertwined nature of this construct can also be observed by looking at the number of strands that appeared in lessons. This factor pervades the construct of OTDMP as it incorporates the need to develop opportunities simultaneously so that the combined and net effect is the total development of proficiency. Once again, information is gleaned from the data resulting in the following table.

Table 3: Distribution of Number of Strands

Number of strands	Number of observed lessons
1	5
2	5
3	16
4	2
5	2
Total	30

We see that the combined table shows that close to half of the lessons observed showed at least three strands prevalent whilst 16.7% of the lessons analysed only recorded a single strand of mathematical proficiency. An average of 3 strands of mathematical proficiency is prevalent in grade 6 KZN mathematics lessons. 2 lessons or 7% showed the promotion of all five strands of proficiency.

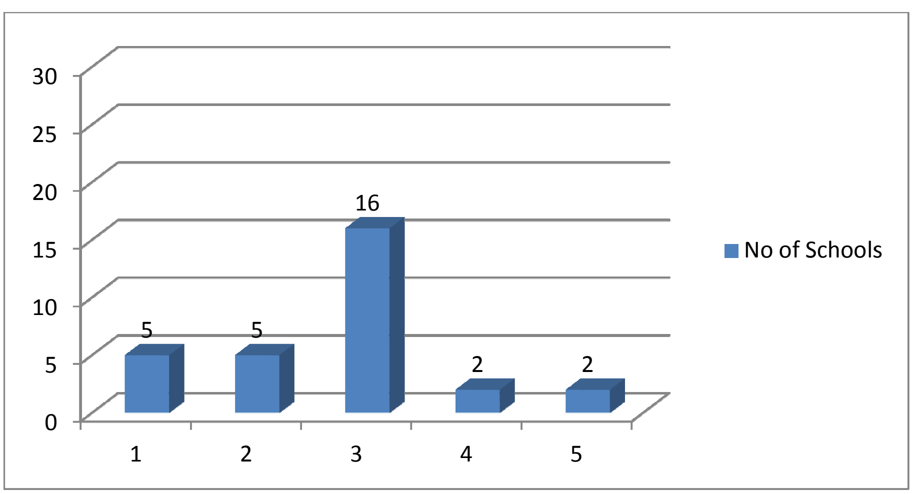

Figure 10: Comparison of distribution of strands.

In this comparison 2 observed lessons showed opportunities that promoted all strands in a lesson. Acknowledging the fact that various other factors may indeed impact the number of strands observed in a single lesson such as lesson type, mathematical content etc., the data indeed shows many lessons devoid of opportunities that develop different strands of mathematical proficiency. Another important consideration is the fact that 30 lessons from 30 different schools were viewed in this study and not a longitudinal study in which 30 lessons from one school was viewed over a period of time. The study provided a snapshot of 30 different teachers presenting different mathematical lessons over a period within which the lessons were recorded.

Closer interrogation of the results strand by strand is imperative so that different perspectives can inform the findings.

7.4. Results per Strand of Mathematical Proficiency

The distribution of these ratings for the five strands is depicted in figure 11, which will be used to analyse the results strand by strand. A deeper engagement and analysis of each strand follows showing some important observations of the results.

A distinguishing feature of the results and analysis henceforth is the number of opportunities as opposed to the score of mathematical proficiency. If the discussion focuses on number of opportunities then the word ‘opportunities’ will be prominent. However, when scores of opportunities are used in the analysis then it will be indicated as either just ‘score’ or ‘scores of proficiency.’

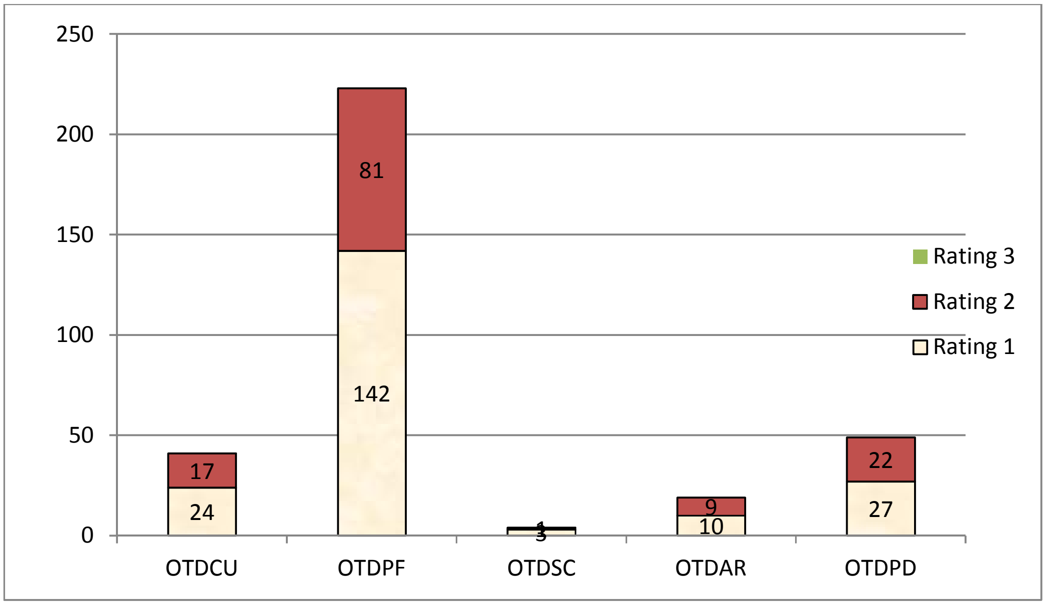

Figure 11: Ratings per strand of Mathematical Proficiency

OTDCU

OTDCU occurred in 16 of the 30 lessons viewed. 12% or a total of 41 of the 336 opportunities coded represented the strand of conceptual understanding. Of the 41 opportunities recorded, 24 fell into the lowest category and the remaining 17 was placed in rating 2. None were observed in the highest rating.

Opportunities in this strand were coded as either 'stated' or 'developed'. In this respect, 28 of these or 68% were stated, where the teacher simply stated or directed learners' attention to a concept. The remaining 22% were developed when a concept was formulated through discussion or demonstration and mathematically supported by teachers or learners. The 9 OTDCU that were developed were recorded in only 3 lessons leaving the remaining 32 conceptual understanding opportunities stated by the teacher, distributed amongst 13 lessons. Interestingly, all 9 instances of OTDCU that were categorised as 'developed' were recorded in rural schools.

14 lessons showed no opportunity to develop this strand. Lesson type could contribute to the minimal number of opportunities that were observed in this strand. However, just a single lesson was classified as revision, an indication that

lesson type played a minimal role in the low number of OTDCU. Representations and connections between concepts provide the foundation for successful mathematics understanding. Opportunities to develop these should abound during lessons. Learners' grasp of facts, ideas and methods are essential. Learning these with understanding supports retention. The key elements of identifying a rating of 1 for OTDCU occurred when few opportunities to build conceptual understanding were present. No attempt to explain the relevance of the concept and no link was made to any representations. Many OTDCU fell into this rating and compared to the size of the study too few were observed.

In one lesson, learners were introduced to the concept of decimal fractions. A simple counting procedure of consecutive values from 1.0 to 1.9 followed. Opportunities to build the idea of a decimal fraction and linking this to other representations were completely lost, and the counting procedure could reinforce the common misconception that real and rational numbers have successors the same way integers do (Roche, 2005). Learners seemed to provide consecutive values in a simple counting exercise. Too many observed OTDCU were of a similar nature.

A rating of 2 was implemented when opportunities were provided which clearly clarify the concept with some explicit links made to other concepts. A case in point was a lesson involving 2 and 3- dimensional figures. Learners were assigned an activity, either as an assignment or project, in which they constructed various 3-dimensional shapes such as a cube and pyramid. Use of these figures with explicit links was made to connect the objects and the corresponding terms defining this section including faces, edges and vertices. The concepts were stated over 6 segments covering a time of almost 30 minutes. 4 of the total 13 OTDCU with a rating of 2 appeared in this particular lesson.

Overall, it is fair to say that the extent of opportunities to develop conceptual understanding in the lessons observed appears limited.

OTDPF

Further meaning of these opportunities was given in relation to the manner in which the procedure was encountered. These were either 'task controlled' or 'solver controlled'. A staggering 99% of opportunities to develop procedural fluency were observed to be in the category of 'task controlled,' meaning that teachers either stated or developed the procedure. Only 1% of opportunities involved learners developing a procedure themselves. The opportunities in this strand dominate lessons. 142 were recorded in the lowest rating while 81 achieved a rating of 2 – partially due to the fact that learners' working on tasks was generally rated as OTDPF-1. Once again, no occurrences of this opportunity in rating 3 appeared. All of the observed lessons showed OTDPF.

The strand of procedural fluency dominated the mathematics lessons. 69% of observable opportunities noted were opportunities to develop procedural fluency. All, except one of these lessons were task controlled. In essence the procedure was either demonstrated or developed by the teacher. Generally, the lesson unfolded when examples were discussed during the acquisition phase of the lesson and then similar problems were assigned for the learners to complete during the application phase. This was done either individually or in groups. In most cases the teacher proceeded to either assist learners or correct work

Crucial in this result is the coding of class work. In section 6.4., I explained the rating assigned to this aspect of the mathematics lesson. A tally of 77 class work segments was coded of the total 223 OTDPF. This tally could be obtained since the instrument contained a section for the description of each segment. During coding of lessons expressions such as, 'learners involved in procedural textbook exercises', 'learners involved in completing the set task with teacher assisting individuals at their desks on an ad-hoc basis(OTDPF)' or 'classwork continues with teacher moving around class correcting and offering advice' allowed easy identification of such instances. The remaining 145 segments contained OTDPF through interaction with the teacher. All lessons showed instances of OTDPF. During mathematics lessons, the strand of procedural fluency is clearly prevalent in the grade 6 KZN teachers' current teaching practice. However, without OTDCU, it likely remains procedures without connections (Stein, Smith, Henningsen, & Silver, 2000), and thus its impact on the development of mathematical proficiency is supposedly limited.

Caution must be exercised when analysing these results. The unit of analysis was a five minute segment of the lesson. Thus when a particular opportunity was identified, it suggests that the segment viewed showed the opportunity appearing in that segment. The time interval that the opportunity occupied within the five minutes was not recorded. Whether the opportunity occupied the entire segment or just a portion of the interval was not considered since its presence was felt and the rating appropriately implemented in line with the characteristics of OTDMP discussed in the analytical framework chapter. When coding for class work, a rating of 1 was allocated. However, in this case class work occupied the entire five minute segment. Thus, if both class work and an opportunity was identified, the opportunity was rated in preference to the seated work and was not included in the tally for class work mentioned previously. Some segments thus contained elements of seated work and teaching.

Emphasis of this strand seems to continue unabated.

OTDSC

A total of only 4 of these opportunities were noted. Coding of this opportunity depended on the heuristics employed during problem solving. One each of the following strategies were recorded, viz. pictures, lists, trial and error, and similar or like type problems. No flowcharts, patterns or any other type of heuristic was observed. 3 of the 4 OTDSC moments fell into rating 1, whilst the other was rated 2. 4 OTDSC in 4 different lessons were observed. Earlier in this chapter, section 4.2., details of the coding for this opportunity was explained. It is directly linked to problem solving which is decidedly lacking in lessons. An example appeared in a lesson involving an analysis of a supermarket pamphlet indicating prices of various grocery items. Learners were required to calculate the total costs of items purchased from this store. Learners suggested drawing a list which the teacher then used to develop a neatly set out list showing all calculations. This strategy to arrive at a final purchase price also allowed learners to make informed decision about their budget constraints. Whether the lesson actually contained problem solving per se is debatable. Accordingly a rating of 2 for OTDSC was given as it closely correlated with that of the descriptor table viz., 'the opportunity to develop a single heuristic (pictures, lists, flow chart, etc.) that is appropriate to the mathematical problem at hand is seized.'

The lack of OTDSC in this district is an indication that examples of non-routine problems in mathematics lessons at the grade 6 level are seldom discussed. Problem solving in mathematics has been extensively researched over the years but has undergone radical changes in its meaning. Schoenfield (1992) states, "much of what passed under the name of problem solving during the 1980's has been superficial" suggesting different and sometimes opposing meanings existed at the time. This resonates with different models for problem solving highlighted in the Theoretical Framework chapter. However, to emphasise its importance Schoenfield (1992) cites Stanic and Kilpatrick (1989, p. 1) who state "the term problem solving has become a slogan encompassing different views of what education is, of what schooling is, of what mathematics is, and of why we should teach mathematics in general and problem solving in particular" (Schoenfield ,1992, p.9).

Despite guidance to mathematics lesson approach in the 'Foundations for Learning Campaign' emphasising problem solving in South African schools, the lack of OTDSC indicates major obstacles still exist. The wording itself suggest contradictory and possibly misguided views of the term problem solving. In the gazette it states, 'Problem solving: Interactive group or pair work should follow where learners engage with a problem or challenging investigation where they have to apply what they've learned in the earlier part of the lesson. Opportunities for learners to try out different ways to solve the problem should be encouraged, e.g. rounding off or adding on to subtract as two possible strategies for adding 3-and 4-digit numbers. The teacher should once again leave time for a short whole class or group review where different learners share and explain their thinking, methods and answers. Sufficient attention shall be given to questions requiring higher order thinking and the solving of word problems in particular.' (Foundations of Learning Campaign, Government gazette, March 2008). Recognising this discrepancy does not detract from the low number of lessons developing opportunities of strategic competence. This description relating to problem solving suggests different views exist regarding the term.

OTDAR

As mentioned earlier, 19 of these opportunities appeared. 13 of these involved reasoning, 4 involved explanations, while 2 used justification. In the adaptive reasoning strand, 9 received a rating of 1 and 7 a rating of 2. Only nine of the observed lessons exhibited moments of opportunities in which reasons, justification or proofs appeared. The importance of this strand is highlighted by Kilpatrick and others noting that it is "the glue that holds everything together" (Kilpatrick et al, 2001, p.129). Opportunities that allowed learners to justify the procedures they use or explanations that clarified the concepts and related the concepts to new situations were mostly absent. Few lessons viewed contained instances in which sustained and meaningful discussion in the mathematics occurred. A scan of the extracts related in the coding chapter suggests that learners participation in lessons is limited to affirmation, as in 'yes' or 'no', single words and numbers in answers to questions posed by teachers or chorus answers. These extracts indicate that such answers are common but not necessarily found in all lessons. Very limited classroom conversations between teacher and learner were noted and in most lessons viewed no follow through of the mathematics in reasoning, justifying or explaining procedures or concepts appeared. Few episodes contained questions that asked learners 'why' procedures worked or 'give a reason' for answers. Opportunities to develop adaptive reasoning were not developed since learners need to develop and express their own intuitive justification. Expressing their reasons and supplying explanations for the mathematics that they encounter entrenches and consolidates the concepts and procedures discussed. A possible connection is when relating this result to results for OTDPF of which 78% was stated and OTDCU in which only 1 lesson showed development of the concept, it is not surprising that OTDAR is largely absent in lessons. Strands are not independent but are interwoven. Conceptual understanding develops when connected pieces of ideas are merged by reasoning and justifying. In a review of the results of the TIMMS 1999 video study, Richland in providing an explanation for low achieving results for schools in the USA, contends that "American teachers introduced conceptually connected rich problems at rates similar to teachers from higher-achieving countries. However, they engaged students in connected reasoning and problem solving less often" (Richland, E. 2007).

The importance of this strand could have been embedded in the analytic scoring implemented. The strategy of weighting each strand equally in this study is based on the premise that 'the five strands are interwoven and interdependent in the development of proficiency in mathematics' (Kilpatrick, 2001, p.116). However, they fall short in categorically stating that emphasis on each of the strands should be equal, rather they "argue that helping children acquire mathematical proficiency calls for instructional programs that address all its strands" (Kilpatrick, 2001, p.116). A scenario to consider is attaching more weighting to OTDAR than the other opportunities since it is seen as holding the other strands together. This and other possibilities exists but will not be explored here.

OTDPD

41 opportunities to develop productive disposition occurred. The majority, viz. 31 or 76% of these considered real world examples. In 12, or 29%, both perseverance and confidence were observed. Opportunities in this strand were split almost equally, with 21 receiving a rating of 2 and 20 allocated a rating of 1.

Lessons seem to include many references to real world experiences half of which received the lowest rating. According to the descriptor table a rating of 1 is when 'Real world situations are described but opportunities to relate these to the mathematics, are not made explicit to the learners'. Recognising the need to link the mathematics to out of class situations, teachers explore these situations without connecting explicitly to the mathematics discussed. In a study of elementary school teachers, Garri and Okumu (2008, p.291) claim, "Their responses indicate that they do not recognize that mathematics plays any important role in technological and professional practices". Assuming this is so, attempting to use real life instances during mathematics classroom instruction without connecting to the mathematics, lessens the impact of the opportunity to develop productive disposition.

Encouraging learners to persevere, praising learners' efforts or instilling confidence in the mathematics that they are working with was seldom recorded. Many lessons showed episodes in which chorus answers were expected with teachers acknowledging correct or incorrect answers to the class group and not individually. Many cases were noted where teachers requested learners to 'clap your hands' in unison in response to a correct answer. Productive disposition includes self efficacy, where the learner feels confident in his own ability, to "see sense in the mathematics, to perceive it as both useful and worthwhile" (Kilpatrick et al, 2001). Lesson segments containing instances in which teachers coaxed learners to correct their own mistakes or praised a novel mathematical solution were non-existent. As indicated only 12% of the OTDPD recorded in all lessons were of this nature, i.e. perseverance or confidence.

Results for the district of Umgungundlovu in all strands have been interrogated in this chapter. Data from each lesson contributed to the results and analysis of the district. This chapter reflects the promotion of the strands of mathematical proficiency in the entire district. The next chapter seeks to view the results and analyse these lesson by lesson giving a view of the diversity of teachers' current practice in the promotion of mathematical proficiency.

Chapter 8: Results and Analysis: The range in quality of teachers' practice.

Chapter 7 contained analysis across lessons. However, it is important to consider how the presence of OTDMP relates to the other components of the study, in order to be able to get an idea what teacher and school variables correlate with the presence of OTDMP and to what extent OTDMP correlates with learner performance. Linking all information from individual schools provides an opportunity to engage these issues. As part of this, a short analysis of the highest and lowest scoring lessons will be given as well as other comparisons. Included in this discussion will be a latent evaluation of the analytic scoring method used as well as the suitability of the descriptor table.

8.1. Score of Mathematical Proficiency for a Lesson.

Calculation of the scores of mathematical proficiency per lesson follows. A similar approach is used to that in chapter 7.3. The number of strands for each lesson was used to calculate the highest score that each strand could achieve in a lesson by multiplying it by the highest rating of 3. Thus, if lesson duration consisted of 9 segments then that strand of opportunity would have a highest value of 27. A value was then obtained for each strand of opportunity by multiplying the number of segments that recorded an opportunity by its corresponding rating and finding a total. These were then converted to percentages to make necessary comparisons between lessons. The score of mathematical proficiency per lesson was then obtained by aggregating the value of each opportunity. Thus the total score of mathematical proficiency in this chapter has a maximum of 500.

By way of an example, consider a lesson with 9 segments. The maximum value is thus 27. If the number of segments recording OTDPF is 8 in which 3 had a rating of 2 and 5 were rated as 1, then the value obtained is 11 of the maximum of 27, or 40.1%, i.e. a score of OTDPF of 40.1 was assigned to this lesson. This score is then added to scores from the other four strands to arrive at a score of mathematical proficiency for a lesson. Analysis in this chapter proceeds using this analytic scoring method in order to compare lessons to each other, but *not* as a way of comparing the lessons against some unattainable 'goal'; it is a measure developed for analysis only. The maximum score of mathematical proficiency per lesson is thus 500.

8.2. Number of strands vs Total score

Results and analysis have centred on the number of strands of proficiency and the scores of mathematical proficiency. Combining these provide the scatter plot in Figure 12. It shows the number of strands observed in a lesson on the horizontal axis versus the total score of mathematical proficiency as construed earlier in this chapter on the vertical axis.

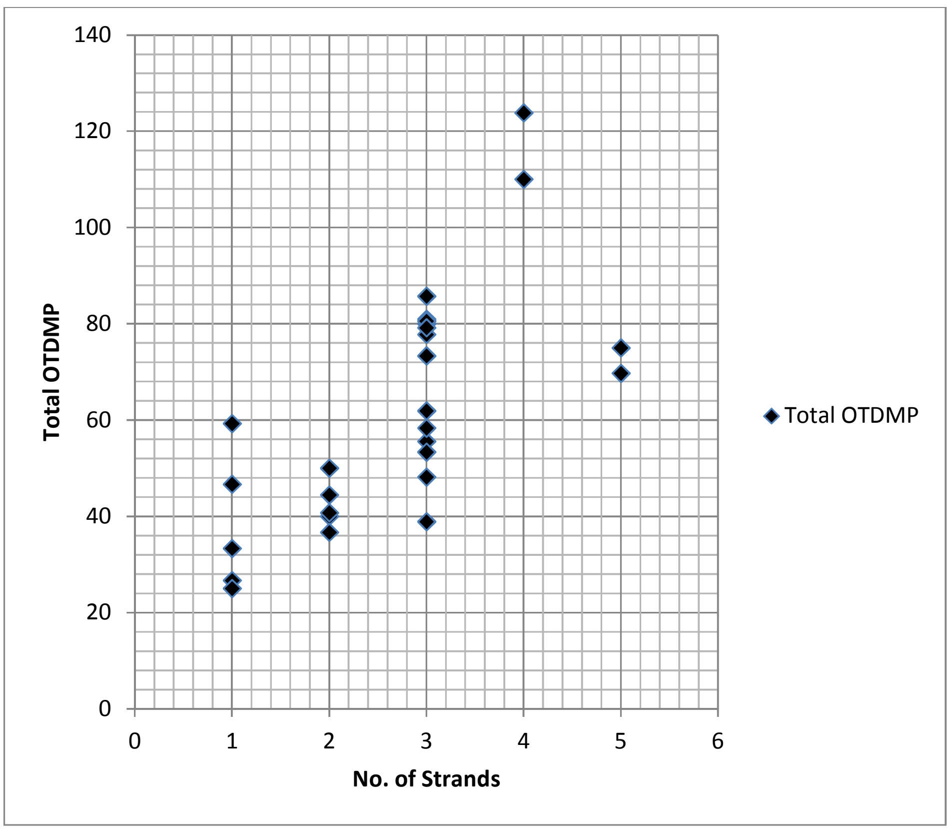

Figure 12: Comparison of Lesson Scores of Mathematical Proficiency with number of strands

Despite the small number of data, a correlation of 0.69 between number of strands and scores of proficiency is fairly high, suggesting some relationship between the variables. In other words, the total score does seem to be somewhat correlating with the number of strands recorded in a lesson. The ranges of scores in each strand are as follows:

Table 4: Lesson Score Range per Strand

No. Strands	Lowest Score	Highest Score
1	25	59
2	37	50
3	39	81
4	110	124
5	70	75

The high scoring lesson obtained for the group with 1 strand only is attributable to a lengthy procedurally driven lesson in which only two segments were recorded as seat work. Procedures in this lesson were clearly stated and displayed so that learners had many opportunities to develop their skill. Although only two problems were discussed, these were executed fluently with the teacher accepting chorus answers throughout the discussion. As mentioned in chapter 4.2., 'sustained periods of doing mathematics' are a teaching practice that is a component of the characteristics of OTDMP. This lesson promoted the strand of procedural fluency positively reflected in the high score of mathematical proficiency obtained.

A worthy observation is that the lowest value in the range of scores increases with each strand except for strand 4. This suggests that if more strands of mathematical proficiency in a lesson are promoted then there is a fair chance that the score of mathematical proficiency will increase thus enhancing the promotion of mathematical proficiency. However, the manner in which a score is compiled and the formulation of the descriptor table will to a degree offset the case to improve mathematical proficiency scores substantially if a teacher does not offer high scoring opportunities in the additional strand.

The information appearing in Figure 12, show the diverse nature of teaching mathematics in the Umgungundlovu district in KwaZulu-Natal. Comparisons of all lessons will be an onerous task but certainly possible. Discussion of all lesson comparisons will not be attempted here, however, particular interesting cases will be considered. The first case involves the comparison of the maximum and minimum scores recorded.

8.3. Maximum and Minimum score Comparison

The highest score of 124 appeared in a lesson during which fractions were discussed. A number of factors contributed to the high score. The lesson contained seven segments indicating lesson duration of 35 minutes. The only opportunity not promoted in this lesson was OTDSC. No problem solving occurred nor was any heuristics used. Oranges were used to merge real world with the mathematics at hand. A constant appeal to learners to 'think and reason' pervaded the lesson including persistence and patience with learners thus providing learners with OTDPD and OTDAR. In this lesson, only 1 segment contained 4 strands, 3 segments had 3 strands, 1 segment had 2 strands and 1 segment had 1 strand only. This observation is a feature of the results, viz. that the number of strands indicated in figure 12, is not an indication that they were observed in every segment of that lesson. Rather, it indicates the maximum number of strands that was recorded in any 5 minute segment of the lesson. As the lesson unfolded procedural work was explained in detail and concepts stated quite clearly. Only one segment was coded as seated work indicating a teacher that provided opportunities across strands during most of the lesson. In another high scoring lesson, the concept of line of symmetry was developed where learners were called upon to draw these lines. Learners were encouraged to persevere and were rewarded with a 'well done' or 'very good' remark. Real world situations were exploited and used to link the concepts whilst procedures were carried out fluently. The descriptions in the descriptor table, table 1 in chapter 4, were relevant and the rating awarded in this lesson were deemed appropriate and correlated closely with observation.

On the other side of the scale, the lowest score of 25 appeared in a lesson in which only one strand was noted. Only OTDPF was recorded over 4 segments with the lesson ending abruptly. Opportunities to develop procedural fluency were inappropriate and confusing. Attempting to explain measurement of an acute angle using a set of axes normally reserved for direction identification, left learners appearing bewildered. This brief yet important analysis of a high scoring and low scoring lesson appears to indicate that scores of mathematical proficiency by its connection to opportunities to develop mathematical proficiency and the rating may indeed provide a snapshot of the quality of teaching mathematics for understanding

The table below contrasts the spread of the highest and lowest scores of mathematical proficiency amongst strands.

Table 5: Score Comparison – Highest vs Lowest

Score of OTDMP	OTDCU	OTDPF	OTDSC	OTDAR	OTDPD	OTDMP
Highest	33	52	0	24	14	124
Lowest	0	25	0	0	0	25

N.B. Discrepancy in the Total OTDMP is a result of the rounding off.

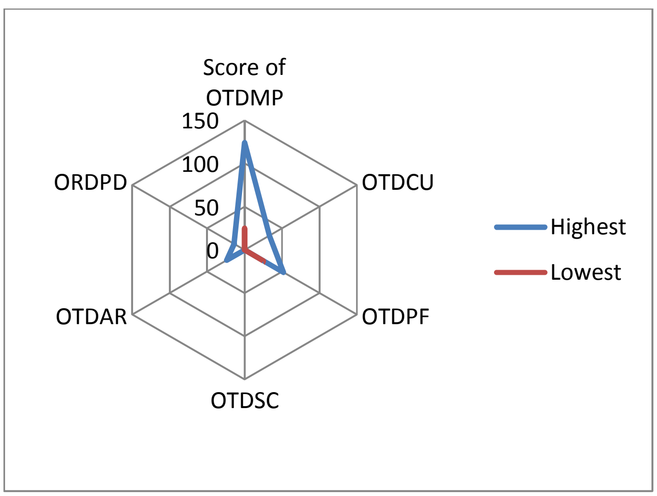

Figure 13: Score Analysis – Highest vs Lowest

Figure 13 contrasts the highest and lowest scoring lessons. The lowest scoring lesson has short tentacles along only two lines of the web whereas the maximum scoring lesson has a wider net with longer tentacles indicating higher scores along opportunities.

Yet another important factor contributing to either low or high scores was the practice of coding seated work as explained in chapter 6.4. This diminished a score of mathematical proficiency if much of the lesson was used for class exercises or activities. A case in point was a lesson that showed all strands of proficiency but recorded a score of 73. Closer inspection revealed that 6 of the 10 segments were coded as seated work and therefore only OTDPF was noted with the lowest rating of 1. In line with the characteristics of OTDMP discussed in chapter 4.2., the issue of time engaging in many examples and mathematical ideas, appears to negatively impact scores if less segments are utilised for teaching. Allocating a rating for class work could have impacted results favourably and boosted scores.

8.4. Highest scores comparison.

Table 6: Score Comparison – Highest Scores

	Score of OTDMP	OTDCU	OTDPF	OTDSC	OTDAR	OTDPD
Teacher A	124	33	52	0	24	14
Teacher B	110	13	53	0	7	37

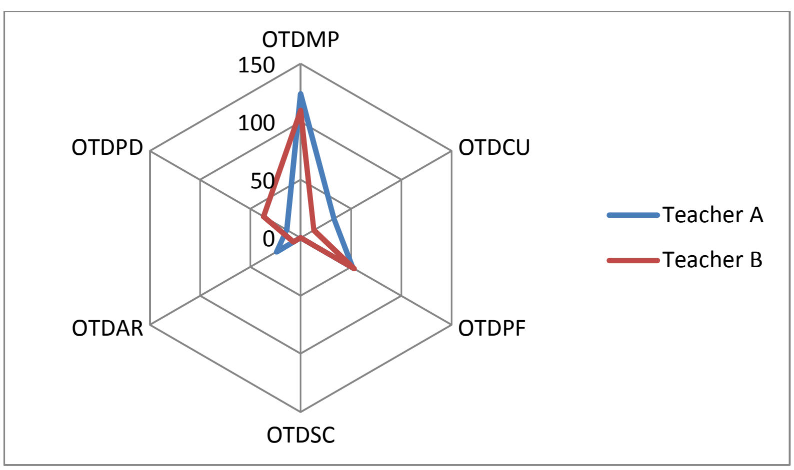

Figure 14: Score Analysis – Highest Scores

Figure 14 shows the scores of the two highest scoring lessons. Both lessons have a wider web indicating higher scores along opportunities. Analysis of scores show the different emphasis placed by teachers during lessons. Both teachers' procedural fluency score is similar. However, teacher 'A' places more emphasis on understanding the concepts taught as well as reasoning and explanation which shows in the scores of proficiency for OTDCU and OTDAR for the two teachers. The lesson contains more opportunities that weave the different strands together and combine to provide opportunities that develop and link ideas by thinking and reasoning mathematically. Teacher 'B' uses out of class situations considerably more but with less emphasis on OTDCU and OTDAR. Both teachers afford opportunities in 4 strands but their emphasis on different strands show in their final score, indicating that teacher 'A' promotes mathematical proficiency more than teacher 'B'. At this point it may be fair to say that the 'scores of OTDMP' as envisaged in this study reflect teaching mathematics for understanding. However, such a comment still needs to be interrogated further such as deciding which strands need to have less or greater weighting, for instance should OTDAR be weighted more than OTDPD. These and other factors may indeed be considered at a later stage.

The third case compares the two scores that showed instances of all five strands.

8.5. Maximum strands comparison

Table 7: Score Comparison – 5 Strands

	Score of OTDMP	OTDCU	OTDPF	OTDSC	OTDAR	OTDPD
Teacher C	75	8	46	4	8	8
Teacher D	70	12	45	3	3	6

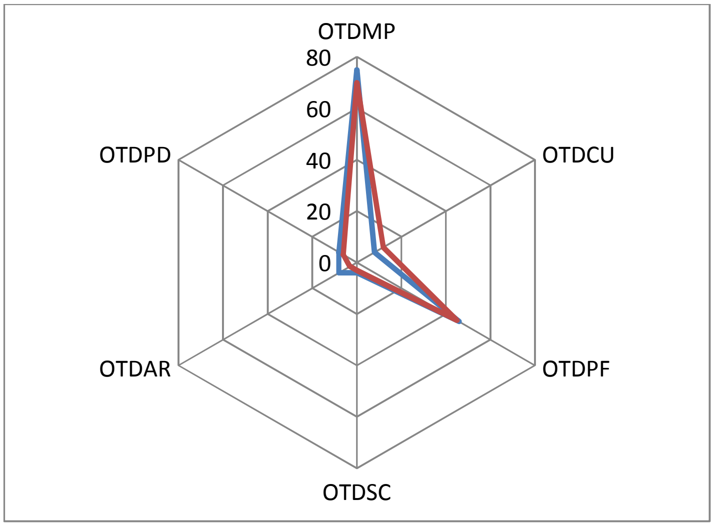

Figure 15: Score Analysis – Maximum number of Strands

The web for the two lessons is almost identical. In this analysis two lessons in which all five strands occur are compared. The scores for each strand are remarkably similar. OTDPF still dominate both lessons. OTDSC recorded in the one lesson used 'like problems' to connect the topic and was deemed appropriate and as such received a rating of 2 in OTDSC. In the other lesson 'trial and error' opportunity presented itself during the introductory phase of the lesson. Learners were requested to remove matchsticks to form a word other than the original. No connection to mathematics was observed and a rating of 1 was recorded for both OTDSC and OTDPD. In both lessons scores were diminished by the number of segments that recorded class work or seated work. In the case of teacher C, 5 of the 8 segments recorded learners working on worksheets while in the case of teacher D, 7 such class work segments were recorded of a total of 11. Once again the score for OTDMP seemed to reflect the quality of the lessons in terms of its ability to predict the 'the existence of an opportunity to develop, promote or advance mathematical understanding via one or a combination of its component strands,' as explained in chapter 4.

8.6. Correlations between scores and number of strands

Kilpatrick et al strongly emphasise the interwoven and connected nature of the strands of proficiency. Table 3 in chapter 7 only shows the distribution of strands, i.e. the highest number of strands observed in a lesson. If for instance a lesson contained the 3 strands OTDPF, OTDCU and OTDPD, further analysis to ascertain the connected nature of mathematical proficiency would be to establish which combination of strands did teachers' favour during lessons. In attempting to answer this, a table showing scores of OTDCU, OTDPF, OTDSC, OTDAR, OTDPD and number of strands for each lesson was compiled. A Spearman's rho 2-tailed test was performed to test the strength of correlations between strands. The results are indicated in table 8.

Table 8: Correlation between scores and number of strands.

Correlations

			OTDCU	OTDPF	OTDSC	OTDAR	OTDPD
Spearman's rho	OTDCU	Correlation Coefficient	1.000	.311	-.136	.024	.252
		Sig. (2-tailed)	.	.094	.473	.898	.180
		N	30	30	30	30	30
	OTDPF	Correlation Coefficient	.311	1.000	-.251	.549**	.397*
		Sig. (2-tailed)	.094	.	.180	.002	.030
		N	30	30	30	30	30
	OTDSC	Correlation Coefficient	-.136	-.251	1.000	.079	-.024
		Sig. (2-tailed)	.473	.180	.	.679	.901
		N	30	30	30	30	30
	OTDAR	Correlation Coefficient	.024	.549**	.079	1.000	.370*
		Sig. (2-tailed)	.898	.002	.679	.	.044
		N	30	30	30	30	30
	OTDPD	Correlation Coefficient	.252	.397*	-.024	.370*	1.000
		Sig. (2-tailed)	.180	.030	.901	.044	.
		N	30	30	30	30	30

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

The statistic of 0.549 is the highest between OTDAR and OTDPF at 1% level of significance whilst that between OTDAR and OTDPD was 0.370 at a 5% level of significance. Recall that OTDAR occurred in only 9 lessons. These correlation figures showed that the tendency of teachers in these lessons was to use reasoning, justification or explanations more often when procedures were taught. Promotion of productive disposition also occurred more often when procedures were discussed in the lesson. OTDPD and OTDPF were also strongly correlated at a 5% level of significance. This comparison indicates that opportunities are not developed simultaneously in the observed lessons, but also that teachers show a preference for developing pairs of strands when more than one strand is developed simultaneously.

The correlation is based on opportunities that were recorded during the course of the lesson.

Chapter 9: Conclusion

This final chapter attempts to synthesise the construct proposed in the study and the results and analysis found in the previous chapter. The purpose of the study does not hope to highlight best teaching practice or to identify model mathematical lessons. Rather, it sought to identify the promotion of the strands of proficiency in current teaching practice without interfering with the lessons themselves. However, in attempting to research this, some interference in lessons were unavoidable such as the presence of the cameraman in the classroom. Despite this intrusion it is hoped that this had a minimal effect on lessons. Even if teachers selected topics and teaching strategies in order to present their best sides on the videos, it is still likely to be indicative of the opportunities to develop mathematical proficiency the teachers will provide in her/his lessons.

The first part deals with the research questions followed by the findings and finally a discussion.

9.1. Research questions 1 and 2

The research questions were formulated prior to the formulation of the notion of OTDMP. Development of this notion materialised as viewing began and progressed leading to the realisation that a construct was necessary to inform the research questions. A bridge was needed to link the qualitative observations to a quantitative result which would then be used to answer the research questions. A consequence of the development of the notion with its characteristics was the order in which the research questions appeared. Prevalence and degree, a rather simple separation of the first two research questions, seemed to be intertwined, possibly reflecting the very construct of mathematical proficiency. Hence, in order to arrive at an answer for question one it would indeed be wise to rather pursue an answer for the second research question first.

Research question 2: To what extent are the five strands of mathematical proficiency promoted in KZN schools in the Umgungundlovu district?

In attempting to answer this question, the first results and analysis chapter serves as reference. The observation that 242 five minutes segments in the 30 lessons recorded contained 336 instances of opportunities to develop mathematical proficiency (OTDMP) is a starting point. There are two ways to add to this.

First, by considering that in the total number of 242 segments noted for the 30 lessons, a grand total of 1210 OTDMP would be possible. While it is unfair to expect all strands to be engaged at all times, it is still an indication of the extent of the OTDMP. Teachers in this district afforded learners only 27.7% of the total number of opportunities to develop the strands of mathematical proficiency theoretically possible.

Secondly, in the lessons analysed, OTDPF was the most prevalent, followed by OTDPD, OTDCU, OTDAR and, almost absent, OTDSC. The low score for OTDSC shows few lessons containing problem solving. The low frequency of OTDAR is of extreme concern. Despite this opportunity described as the “glue that holds everything together, the lodestar that guides learning” (Kilpatrick et al, 2002, p. 129), it is virtually non-existent in the mathematics lessons of the district. A result which is not too far from the results recorded in the TIMMS studies which showed that reasoning in mathematics lessons in the United States stood at 0% (Stigler, Hiebert, 200, p. 4). Even if a view of weighting this strand more is taken, it is unlikely that this result would have improved substantially due to the low number of opportunities offered.

The results for OTDPD is mostly due to the inclusion of real world or out of class situations that teachers tried to link and incorporate into their lesson, possibly hoping that the learners’ self efficacy will be positively impacted.

These figures and facts suggest that ‘the extent to which the five strands of mathematical proficiency is promoted in the district of Umgungundlovu in KwaZulu-Natal’ is far below expectation.

Research Question 1: Are the strands of mathematical proficiency prevalent in the current practices of the grade 6 teachers' in the Umgungundlovu district of KwaZulu-Natal?

The question attempts to establish whether the strands of mathematical proficiency are widely used, occurs commonly and is frequently practiced in classrooms. The answer to research question 2 above has a major influence here. A simple yet uninformed answer would be in the affirmative since lessons in the district showed the 5 strands. This view could possibly be substantiated by figure 11 which shows how many strands were promoted during lessons. All five strands were recorded amongst the 30 lessons viewed.

However, the question also relates to prevalence of strands simultaneously, a characteristic entrenched in the formulation of mathematical proficiency. Lessons must contain all strands that must be developed in unison since they are intertwined and work together to enhance and promote mathematical understanding. The instrument used in this study accommodates for this characteristic allowing simultaneous recording of opportunities during segments. During a discussion of the highest scoring lesson in section 8.3 in reference to how the number of strands was obtained in a lesson, it was stated that, 'the number of strands indicated in figure 13, indicates the maximum number of strands that was recorded in any segment of the lesson.' In that particular case only 1 segment recorded all four opportunities to develop strands whilst 4 segments showed 3 OTDMP. In fact the two lessons that recorded all five strands had no segments showing all the strands simultaneously. In both cases the maximum strands in a segment was indeed only three.

Across lessons the total number of segments noted was 242. 223 or 92.1% contained OTDPF, 41 or 16.9% showed OTDCU, 49 or 20.2% contained OTDPD, 19 or 7.9% had OTDAR and 4 or just 1.7 % contained OTDSC.

The above analysis seems to point towards a possibility that mathematical proficiency is not prevalent in the Umgungundlovu district of KwaZulu-Natal. The strands of mathematical proficiency as seen by the opportunities to develop them occur irregularly across the district and in moments of mathematical lessons.

9.2. Research Question 3 and 4

These two questions relate to the teacher questionnaire and the test scores. Both these questions can be approached in a quantitative manner. It must be mentioned initially that problems arose during statistical analysis when considering these questions. In some cases questionnaires given to teachers were not returned and in others they were incomplete. This led to complications during correlation analyses. Ideally it was expected that information from all 30 teachers who conducted the lessons was readily available. This was not the case. Nevertheless, statistical correlations continued but the sample size in each case would be indicated below, and indicate low validity of this part of the analysis, which remains tentative. Thus, the discussions that follow are only suggesting correlations and connections which would need further exploration.

Research Question 3: How does the promotion of mathematical proficiency vary, if at all, with the educational background of the teacher, the teacher content knowledge as reflected in the results from the teacher test and other background factors?

Data from 34 teachers in the larger study showed that eighteen of the teachers had passed Grade 12 without exemption/endorsement and 14 had passed with exemption. Two of the teachers had no teacher training, four had one year of teacher training, 5 had two years and 8 (25%) had three years of teacher training. Forty percent or thirteen teachers had more than three years of teacher training. This is representative of both the district of Umgungundlovu, where 39% of teachers are on REQV 14, and the province, where 40% of teachers have REQV 14. Eleven (37%) of the teachers had obtained their original teaching qualification between 1980 and 1987, thirteen (43%) obtained this between 1990 and 1999 and six teachers (11%) qualified as a teacher in the last decade (between 2000 – 2007). Four teachers did not respond to this question.

A table showing scores of OTDCU, OTDPF, OTDSC, OTDAR, OTDPD and number of strands for each lesson was compiled. This was then correlated to the following teacher variables: highest secondary school qualification[1] (Hssq), how many years of pre-service professional teacher training they had received (tt), how many years as a maths teacher (mty) and teacher score on the test from the larger study as a percentage (Mark%). A Shapiro-Wilks test of normality was first conducted on the data. It showed some variables not normally distributed at a significant level of 5%. A non-parametric approach to correlation was conducted using Spearman's correlation co-efficient based on ranks. The correlation result appears in table 9 below.

Table 9: Correlation table for OTDMP vs Teacher variables

		OTDCU	OTDPF	OTDSC	OTDAR	OTDPD	No. of strands	Total OTDMP
Q4 Hssq	Correlation Coeff.	.517*	.286	.207	.443	.171	.626**	.510*
	Sig(2 tail)	.028	.250	.409	.066	.499	.005	.031
	N	18	18	18	18	18	18	18
Q7_tt	Correlation Coeff.	.009	.125	.113	.154	.105	-.005	.137
	Sig(2 tail)	.973	.631	.667	.555	.687	.984	.600
	N	17	17	17	17	17	17	17
Q16_mty	Correlation Coeff.	.182	.170	-.153	.252	.151	.009	.212
	Sig(2 tail)	.469	.500	.544	.314	.550	.972	.397
	N	18	18	18	18	18	18	18
Mark_%	Correlation Coeff.	.004	.146	.075	.466	-.185	.257	.072
	Sig(2 tail)	.988	.563	.768	.051	.462	.303	.775
	N	18	18	18	18	18	18	18

The sample size in this correlation was reduced by factors mentioned to just 18. Nonetheless, there was a significant correlation (at the 5% level of significance) between Hssq and OTDCU, number of strands and total OTDMP. If we consider a 10% level of significance then the teachers' test score is correlated with OTDAR. The correlation result suggests that the number of years of pre-service professional training and the number of years of mathematics teaching has limited material

[1] It would perhaps be expected that all teachers would have a high school qualification providing access to further education. This will be discussed after the presentation of the correlations.

effect on the promotion of mathematical proficiency. The highest secondary school qualification of a teacher seems to have an impact during teaching. The quality of this qualification is reflected in the teachers' opportunity to develop conceptual understanding. The higher the secondary school qualification the greater the potential for the teacher to provide opportunities to develop the concept discussed. However, this variable is also strongly correlated to OTDMP overall. In chapter 8, a suggestion that the increase in the number of strands potentially increases OTDMP seems to also correlate with Hssq. The highest secondary school qualification of the teacher could impact on number and degree of opportunities that develop mathematical proficiency. Teachers possessing high secondary school qualification seem to be in a better position to provide opportunities to develop the strands of mathematical proficiency and therefore have a higher potential to develop the mathematical attributes of a mathematically proficient learner

The highest secondary school qualification can be traced to the history of education in the country. In the era during which education departments were divided along racial lines the highest certificate in a secondary school that could be achieved by a learner was the standard 10 matriculation certificate. Learners at the time attended racially segregated schools and colleges of education. Subjects were offered at higher grade and standard grade levels. Learners had to achieve a result of 40% or more for higher grade subjects and 33.33% or more for standard grade subjects to pass. Hssq in the teachers' questionnaire was a reference to the quality of the pass in subjects at the standard 10 level (currently grade 12). Teachers in this study went through the system of education just described and which continue to impact in their current teaching practice. A case in point is the highest score of mathematical proficiency in a lesson which was discussed in chapter 8.5. In this lesson the scores of OTDPF, OTDAR and OTDCU were significantly higher than other lessons. The teacher who taught this lesson had the highest level for Hssq in the teacher questionnaire. The strong correlation with number of strands and OTDMP suggest that teachers with higher secondary school qualifications are capable of developing more strands of proficiency at a higher degree. Teachers with lower Hssq showed the lowest scores of mathematical proficiency.

Correlation of the teachers' test score with OTDAR is an interesting result. The teacher test questionnaire included conceptually related mathematical problems that included for instance identifying learners' incorrect answers. The correlation suggests that teachers who scored high in these test which ultimately involved much reasoning and justification tend to use their adaptive reasoning ability during lessons.

Research Question 4: How does the teachers' promotion of mathematical proficiency correlate with the learning that took place during grade 6, according to the difference between the results on the two learner tests?

Learner test gain was calculated on the difference between the two tests that they wrote.

In order to link these gains to teachers, an average learner gain was calculated for each teacher. This average was then correlated with opportunities as in research question 3. A Spearman's Rho test was conducted and the correlation results indicated in the table below.

Table 10: Correlation between OTDMP and average learner gain.

		OTDCU	OTDPF	OTDSC	OTDAR	OTDPD	No. Of strands	Total OTDMP
	N	18	18	18	18	18	18	18
avg_l_gain	Correlation Coefficient	-.266	.168	-.104	.496	.153	.160	.172
	Sig. (2-tailed)	.320	.533	.703	.051	.571	.555	.524
	N	16	16	16	16	16	16	16

At the 10 % level of significance, OTDAR at a value of 0.051 is significantly correlated with the average learner gain. One possibility is that the generally low learning gains and the generally low OTDAR scores lead to a 'false' correlation. On the other hand, it is not impossible that these do link: reasoning skills, justification and explanation impacts mathematical learning. This strand holds the others together, an important aspect of mathematical proficiency that has been emphasised throughout the study. Teachers creating opportunities that develop adaptive reasoning positively impacts mathematics understanding and learner achievement. Learners exposed to opportunities that develop their adaptive reasoning which holds the other strands together, may enhance their understanding at a content level and at a deeper, more meaningful level.

9.3. Discussion

In this chapter, I reflected on the results and analysis and attempted to merge these with the notion of 'opportunity to develop mathematical proficiency' to answer the research questions. The characteristics of OTDMP were interrogated and further analysed so that a view in the formulation of the construct proposed in this study can be obtained. Discussion will be around a central theme, viz. the promotion of mathematical proficiency. Included in this chapter but interspersed within the discussion, salient features of this study showed connections to the larger project.

Time span is a key property of acquisition of mathematical proficiency, both over the long term as well as over a shorter period. This important characteristic affects the promotion of mathematical understanding. In this study the unit of analysis was a 5 minute segment during which recordings of OTDMP were captured using the instrument designed for this purpose. The length of time spent during lessons provides the opportunities to construct important mathematical understanding. Embedded in these opportunities is the potential for learners to develop and gain an understanding of mathematical ideas and concepts. More time spent on introducing new content and working with concepts that learners have not worked on before provides greater OTDMP. An observation to consider is the length that each opportunity spanned within each segment. Nevertheless, time available and time utilised for introducing new content and providing substantial examples to consolidate new concepts is crucial to assimilation of mathematics over time. Kilpatrick et al. emphasise the importance of proficiency developing over time. In this sense reference is engagement with the mathematics topic, illustration of the concept, attempting a multitude of examples on the concept, reasoning, building connections and developing understanding. Since learners acquire most of their mathematics knowledge in classrooms, it is reasonable to expect that many opportunities to develop the strand of mathematical proficiency should appear in a lesson. A substantial part of a lesson should be utilised in providing learners with opportunities thus promoting proficiency over time. The TIMMS study revealed that 75% of lesson time in the high achieving East Asian classrooms was spent dealing with new content (Leung, 2005, p 203). Although this study did not focus on time *per se*, it became a key feature of the notion of OTDMP but no definitive or conclusive observation regarding promotion of mathematical proficiency could be ascertained using time alone.

It necessitated interrogating other aspects of OTDMP. The importance of the presence of OTDMP, i.e. it must occur or be observed, is self explanatory. Hence, the other important aspect of OTDMP is the degree or forcefulness of the opportunity to develop the strand of mathematical proficiency. The emphasis in this aspect of OTDMP is the degree to which the opportunity promotes that strand in understanding the mathematics content. The focus is purely on the mathematics at hand and no social cultural contexts were considered when rating the OTDMP. The overwhelming number of ratings in the lowest category indicates that opportunities to develop deep mathematical understanding in lessons seldom appeared. In the strand of conceptual understanding, using multiple representations to develop mathematics understanding is critical. Creating opportunities that allow learners to connect complex representations during mathematics lessons assists in the promotion of mathematics understanding. OTDAR holds the other strands together. Learners' connect previous knowledge to new knowledge using their reasoning ability. These opportunities were seldom observed.

This study seems to confirm the position that mathematics in this country finds itself. Results from TIMMS are well documented. Howie and Plomp observe that, "Overall, South African pupils achieved 275 points out of 800 (standard error, 6.8) in the mathematics test, whilst the international average was 487. This result is significantly below the mean scores of all other participating countries" (Howie and Plomp, 2002). Recently conducted tests of primary school learners showed that the average learner performed well below the mathematical literacy level for learners in their age group. Results from learner tests conducted in the 'Annual National Assessments 2011' yielded an average score of 32% after re-marking of schools in KwaZulu-Natal (Department of Basic Education, 2011).

The answers to the research questions regarding quality of teaching resonate with other research. In a project motivated by persistence in poor mathematics results and the introduction of Outcomes Based Education curriculum in South Africa in 1998, the authors state, "The consequence has been that quality of outcome has varied wildly from school to school as the completeness and complexity of content to which learners are exposed came to depend on individual teachers" (Schollar, 2004). The focus of redress to such statements has been ever increasing budgetary allocations to education departments. The South African education budget is now one of the highest in the world today. In searching for the qualities of excellence in teaching in New Zealand schools Hattie states, "Interventions at the structural, home, policy or school level is like searching for your wallet which you lost in the bushes, under the lamppost because that is where there is light. The answer lies elsewhere-it lies in the person who gently closes the classroom door and performs the teaching act" (Hattie, 2003, p. 3). Inculcating and developing the necessary attributes in learners to become mathematically proficient occurs in the classroom where the teacher provides the opportunities to develop mathematical proficiency.

9.4. Findings

The findings of this research are indicated in point form below. All findings are applicable to the Umgungundlovu district in KwaZulu-Natal.

1. Mathematical proficiency in the district of KwaZulu-Natal.

The answers to the research questions indicate that the mathematical proficiency in the district of Umgungundlovu in KwaZulu-Natal is not strongly or consistently promoted. In fact, the promotion of mathematical proficiency seems to have a limited impact on teaching mathematics for understanding. The tendency in the district is to teach procedural fluency skills. Limited conceptual understanding moments appear in lessons and virtually no adaptive reasoning is present. Teachers use out of class situations but the connections to the mathematics at hand is lost. The vision of education departments and classroom practices as observed using the notion of OTDMP in this study suggest a chasm exists between them. Sanni analysed the verbs that appear in the assessments standards of the 'Revised National Curriculum Statement Grades R-9'. His investigation revealed that 60% of the assessment standard in the curriculum constitutes conceptual understanding and adaptive reasoning (Sanni, 2009, p. 28). In this study the strands of OTDCU and OTDAR were occasionally observed compared to the 60% envisaged by the department.

2. Mathematical Proficiency in lessons

Isolated lessons show promise in teaching for understanding. This is reflected in the number of strands recorded for the lesson and the overall score of mathematical proficiency. However, emphasis in these lessons still tends to be largely procedural using a single procedure which is stated effectively and fluently. This observation of isolated schools implementing best teacher practices is reflected in the statement in the report on the Annual National Assessments 2011 which states, "In all provinces there are schools within these quintiles which can be considered to be showing promise" (Department of Basic Education, 2011). The majority of lessons however, lack opportunities that promote the strands of proficiency thus hardly developing learners' mathematical understanding.

3. Teacher variables.

The teachers' highest secondary school qualification is strongly correlated to their teaching in providing opportunities to develop the conceptual understanding strand. This obviously applies to only those teachers where this strand was observed. This adaptive reasoning strand correlates to the teachers test mark indicating that teachers who apply this skill in their own mathematical encounters promote opportunities that develop adaptive reasoning. The strong correlation of highest secondary school qualification and score of mathematical proficiency suggest that teachers with a higher qualification have a tendency to promote more strands with a higher degree. Their teaching practice offers more opportunities across strands and with a better quality.

4. OTDMP

The above findings show similarities with many other studies as indicated previously. The major difference lies in the manner in which these findings were obtained. In this research the notion of OTDMP was formulated and tested. It was used to test the promotion of mathematical proficiency in 30 lessons. The descriptor table compiled to identify and rate each strand of opportunity proved to be the key behind the notion of OTDMP. The analytic scoring method used to find a score of mathematical proficiency proved to be useful in answering the research questions.

5. Connections to the larger study.

The larger study hoped to explore and establish the relationship between teachers' mathematical content knowledge, teachers' practice and learner outcomes in grade 6 mathematics classrooms. The study involved assessing teachers' practice in mathematics classrooms. The notion of OTDMP formulated provides 'the concrete link that shows the degree to which an opportunity to develop the strands of proficiency and the overall analysis of the videos' (Chapter 4, p. 34). Using this table of descriptors, viz. table 1, the promotion of the five strands of mathematical proficiency can be ascertained in teachers' classroom practice. The scores of proficiency could be an indicator of outcomes of learners' assessments and achievement. The findings detailed above provide the necessary information regarding teaching practice in the Umgungundlovu district of KwaZulu-Natal.

9.5. Reflections

In this study the notion of OTDMP was proposed. This notion together with its descriptor table is the main feature of the study. Throughout the study, I sought to entrench this position and care is taken not to deviate significantly from it. Early in the introduction emphasis is placed on the descriptor table as incorporating the main feature of the proposed notion. Here I suggest that it may provide a suitable user friendly manner for coding of mathematics lessons but did not delve deeply in this suggestion which will need substantial investigation. The analytical framework chapter again emphasises OTDMP and its components as a key feature in the research.

This study focussed on whether the notion of OTDMP and its descriptor table could be useful as an instrument in measuring the extent to which teachers provide opportunities for the learners to develop in the five strands of mathematical proficiency. In order to accomplish this, videotapes from the larger study were used. Recording of these lessons were completed before this research began. The lessons were from schools located in the Umgungundlovu district in Kwa-Zulu Natal. Reasons for choosing videotaped lessons in this study are clearly indicated in section 5.1. Although I was not present during these recording sessions the influence this might have had on the study remains unknown.

Coding of these lessons was completed over a period of time. The quality of videotapes was generally acceptable. Development and refinement of the instrument proceeded as more videotapes were viewed. As the researcher, all videotapes were coded by me with comments and suggestions made by my supervisor. At no time was another coder involved who may have added a different perspective and provided a greater sense of validity of the coding. This is one area that I intend exploring further with a group comprised of current grade 6 mathematics teachers, mathematics education lecturers and students pursuing doctoral studies.

The instrument attempted to bridge the qualitative research and the quantitative analysis that followed. Literature on data collection in qualitative inquiry abound (eg., Gough and Scott, 2000; Ziebland and Mcpherson, 2006). There was no attempt in this study of providing an overview neither of qualitative data collection procedures showing their strengths and weaknesses nor of quantitative analysis methods.

In the coding chapter extracts were noted and discussed in depth. This continued in the two results and analysis chapters in order to establish the legitimacy of the descriptor table as well as the analytic scoring introduced. During the course of the research and very specifically during the results and analysis phase, the formulation of the notion of OTDMP with its corresponding descriptor table proved to be invaluable. The scoring method that evolved as a result of this theory around which much of the analysis was based, proved to be an effective analytic scoring instrument.

During the results and analysis chapters I constantly attempted to indicate that the proposed notion of 'opportunity to develop mathematical proficiency' needs to be interrogated further. This is done on a number of occasions. Factors that could affect the scoring phase of the research for instance, are highlighted on occasions. Previously, I indicate that a 'contributing factor could be the descriptions of the highest rating in the descriptor table pitched at too high a level'. This is a direct reference to the descriptor table alluding to the fact that more investigation may be needed to fine-tune the instrument. I found that 'scores of mathematical proficiency' is therefore a value that is viewed relative to the maximum that provides insight into the existence or prevalence and degree of opportunities to develop proficiency' whilst I strongly indicated that 'the maximum score does not in any way represent a perfect situation nor does the study attempt to find a best score'. Tweaking this analytic scoring may indeed result in using it in other areas of mathematical instruction.

Earlier, I noted that 'closer interrogation of the results strand by strand is imperative so that different perspectives can inform the findings.' Included in this interrogation could be the consideration of in depth mathematical development as well as closer inspection of the mathematics discussed in lessons.

During the course of the analysis the theory of OTDMP, in my view, appeared to offer substantially more than this research. Armed with the skill developed during this research project, the broader mathematics education knowledge absorbed over the course of the research as well as the finer details needed to be an effective researcher, there is no doubt that pursuing the construct of 'Opportunity to Develop Mathematic Proficiency' in the future will become a personal goal.

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Appendices

Appendix 1: Ethical Clearance Certificate

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WESTVILLE CAMPUS
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UNIVERSITY OF KWAZULU-NATAL

10 DECEMBER 2009

PROF. V WEDEKIND (21106)
EDUCATION AND DEVELOPMENT

Dear Prof. Wedekind

PROTOCOL REFERENCE NUMBER: HSS/0738/09
PROJECT TITLE: "A STUDY OF TEACHER QUALITY AND STUDENT PERFORMANCE IN KWAZULU-NATAL"

FULL APPROVAL NOTIFICAITON – COMMITTEE REVIEWED PROTOCOL

This letter serves to notify you that your application in connection with the above has been reviewed by the Social Sciences & Humanities Research Ethics Committee on 26 November 2009. Your research protocol has bee granted full approval.

Any alterations to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study must be reviewed and approved through an amendment /modification prior to its implementation. Please quote the above reference number for all queries relating to this study.

PLEASE NOTE: **Research data should be securely stored in the school/department for a period of 5 years**

Best wishes for the successful completion of your research protocol

Yours faithfully

....

PROF. S COLLINGS (CHAIR)
HUMANITIES & SOCIAL SCIENCES ETHICS COMMITTEE

cc. Dr. W Hugo
cc. Dr. N Mthiyane

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

Ethical Clearance Certificate: Update.

Update of details for *A Study of Teacher Quality and Student Performance in KwaZulu-Natal* for the purpose of ethical clearance HSS/073/09

The project has changed slightly since the submission of the original ethical clearance application.

Firstly, the number of schools has been changed from 20 to 40. All sampling procedures and data collection instruments stay the same.

Secondly, the original list of investigators and students associated with the project has now expanded. The following must be added to the list:

NAME	TELEPHONE NO	EMAIL	DEPARTMENT/ INSTITUTION	QUALIFICATIONS
Noor Ally	076 432 5194	noora@dut.ac.za	DUT & UKZN (student)	Hons
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Professor Volker Wedekind

Appendix 2: Instrument Coding Grid

Videocodes: Mathematical Proficiency
School Name: ______________________
Grade 6:
Content: ....

	OTDCU			OTDPF			OTDSC								OTDAR				OTDPD			
	NP	P		NP	P		NP	P							NP	P			NP	P		
Clip		Stated	Developed		Task	Solver		Pictures	Lists	Flowchart	Trial & error	Pattern	Like problems	Other		reasoning	explanation	justification		Perseverance	Confidence	Real
0-<5																						
5-<10																						
10-<15																						
15-<20																						
20-<25																						
25-<30																						
30-<35																						
35-<40																						
40-<45																						
45-<50																						
50-<55																						
55-<60																						
Final																						
Total NP	0			0			0								0				0			
Total per rating	Rating 1	0	0		0	0		0	0	0	0	0	0	0		0	0	0		0	0	
	Rating 2	0	0		0	0		0	0	0	0	0	0	0		0	0	0		0	0	
	Rating 3	0	0		0	0		0	0	0	0	0	0	0		0	0	0		0	0	
	Total per strand	0	0		0	0		0	0	0	0	0	0	0		0	0	0		0	0	

Time Interval	Identified Opportunities to develop Mathematical Proficiency
0-<5	
5-<10	
10-<15	
15-<20	
20-<25	
25-<30	
30-<35	
35-<40	
40-<45	
45-<50	
50-<55	
55-<60	

N.B. Opportunities to develop: Conceptual Understanding (OTDCU); Procedural Fluency (OTDPF); Strategic Competence (OTDSC); Adaptive Reasoning (OTDAR); Productive Disposition (OTDPD)

Classroom Observations:

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