UNIVERSITY OF KWAZULU-NATAL

THE APPLICABILITY OF THE RISK-FREE RATE PROXY IN SOUTH AFRICA: A ZERO-BETA APPROACH

Ailie Charteris
204501681

This thesis is submitted in fulfilment of the requirements for the degree of **Master of Commerce** by research.

School of Economics and Finance
Faculty of Management Studies
Pietermaritzburg

Supervisor: Mr. Barry Strydom

2009

DECLARATION

I .... declare that

(i) The research reported in this dissertation/thesis, except where otherwise indicated, is my original research.

(ii) This thesis has not been submitted for any degree or examination at any other university.

(iii) This thesis does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.

(iv) This thesis does not contain other persons' writing, unless specifically acknowledged as being sourced from other researchers. Where other written sources have been quoted, then:

a) their words have been re-written but the general information attributed to them has been referenced;

b) where their exact words have been used, their writing has been placed inside quotation marks, and referenced.

(v) Where I have reproduced a publication of which I am author, co-author or editor, I have indicated in detail which part of the publication was actually written by myself alone and have fully referenced such publications.

(vi) This thesis does not contain text, graphics or tables copied and pasted from the Internet, unless specifically acknowledged, and the source being detailed in the thesis and in the References sections.

Signed: …………………..   Date:……………….

ACKNOWLEDGEMENTS

The encouragement, support and assistance that I have received in completing this thesis have been unbelievable, and I know that this will remain with me long after the content of this thesis has been forgotten.

To my supervisor Mr Barry Strydom, words cannot express my deep gratitude for your guidance and insight throughout this process. Our meetings have been filled with many hours of discussions, debate, tears (just mine) and laughter; and I am grateful not only for the contribution that you have made in assisting me with this thesis, but more importantly for the significant contribution that you have made to my career as an academic.

To my amazing parents, whom God has blessed me with, thank you for your enduring love, patience and support, especially throughout this period. Without you this thesis would never have been finished.

"And whatever you do, whether in word or deed, do it all in the name of the Lord Jesus, giving thanks to God the Father through him" (Colossians 3:17, *Holy Bible*, New International Version).

ABSTRACT

The Capital Asset Pricing Model (CAPM), despite criticism and debate regarding its validity, remains the most widely employed model to estimate the cost of equity for use in capital budgeting decisions, both in the U.S. and in South Africa. The risk-free rate specified in the model is generally estimated with the use of a government security, but there is some concern as to the appropriateness of this practice in the South African market. An alternative approach was derived by Black (1972), known as the minimum-variance zero-beta portfolio returns; but the suitability of this parameter in the South African market has not yet been examined.

The objective of this study therefore is to determine the best method to estimate the risk-free rate for applications of the CAPM in South Africa. A set of theoretical requirements that an asset must closely satisfy to be considered a suitable proxy for the risk-free rate are derived, with the most commonly employed proxies being compared to these criteria to ascertain their appropriateness. The zero-beta portfolio returns are computed, in conjunction with the rate that investors have historically viewed as the minimum required return, denoted by the intercept of the CAPM. Hypothesis tests of the equality of the two estimates of the risk-free rate and the minimum required return are conducted, as well as a comparison of the forecasting accuracy of the model using the different risk-free rate values.

The results of the analysis indicate that the South African proxies diverge substantially from the criteria, and are likely to overstate the true-risk-free rate. In complete contrast to this, the hypothesis tests reveal that the proxies understate the intercept estimate, whilst the zero-beta portfolio returns closely approximate this value. This finding that the zero-beta portfolio returns, which are larger than the proxy yields, are more suitable appears counter-intuitive given the goal to identify the minimum return from investing. This result can possibly be explained by the fact that the CAPM intercept represents the average of the riskless lending and borrowing rates, whilst the proxy only denotes the former. The borrowing rate is likely to be higher than the lending rate; thus giving reason for the average being greater. However, the possibility also remains that the results observed may be a consequence of the incorrect specification of the market portfolio, that the tests employed are inapt, or that the model itself is inappropriate.

The forecasting analysis confirms the greater accuracy associated with employing the zero-beta portfolio returns as the risk-free rate compared to the use of a proxy, but the improvement is small. Thus the choice for the practitioner is whether the increase in accuracy is justified by the difficulty and time involved with estimating the zero-beta portfolio returns.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1
THE SCOPE AND PURPOSE OF THIS STUDY

1.1 Background and Problem Definition

1.1.1 The Capital Budgeting Process: The Choice of Models and Parameters

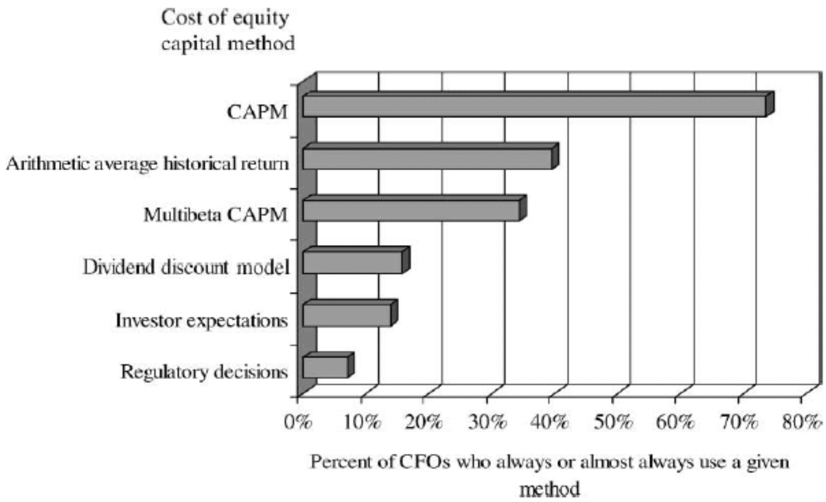
One of the most important lessons taught to commerce students at university is that financial and economic decisions should be made on the basis of the Net Present Value (NPV) rule, as this approach ensures that resources are allocated to the most productive uses in the economy. The results of a survey of the most common approaches to capital budgeting in the United States of America (U.S.), conducted by Graham and Harvey (2001: 197), provide evidence that what is taught in university is adhered to in industry, as 74.9 percent of chief financial officers surveyed always, or almost always, use the NPV rule. In a similar survey in South Africa, this trend was also identified, with 81.2 percent of the respondents always, or almost always, utilising this capital budgeting technique (Correia and Cramer, 2008: 36).

In order to implement the NPV methodology, estimates of the proposed investment's future cash inflows and costs are required, as well as an approximation of the firm's discount rate, so as to calculate the present value of the future cash flows of the project (Reilly and Brown, 2006: 686). When a proposed investment with a risk profile similar to that of the company is evaluated, the firm's weighted average cost of capital (WACC) is the most appropriate discount rate to use (Firer, 1993: 23). This discount rate comprises both the firm's cost of equity and after-tax cost of debt, weighted in proportion to the relative amounts of equity and debt in the firm's total capital structure (Brigham and Ehrhardt, 2005: 307). The cost of debt is easily discernible as it is a market-related value; however, it is difficult to determine the firm's cost of equity as this is a theoretical concept representing the relative risk of the firm, and hence is not directly observable (Firer, 1993: 24).

Various methods have been suggested in the financial literature for the estimation of a firm's cost of equity. Brigham (1985: 223) proposed the use of the CAPM, the Dividend Discount Model (DDM) and the Bond-Yield-Plus-Risk-Premium approach for this purpose. The CAPM is generally the favoured method, as the DDM has severe limitations because it only applies to firms that pay regular dividends, and it assumes that the growth rate in dividends is constant; whilst the Bond-Yield-Plus-Risk-Premium approach is dismissed because of the difficulties inherent in trying to obtain accurate estimates for the risk premium (Firer, 1993: 25). However, Brigham and Ehrhardt (2005: 311) argue that the three approaches are not mutually exclusive and are all subject to errors when implemented in practice; hence they advocate estimating a firm's cost of equity using all three techniques, with the practitioner choosing among the estimates based on their confidence in the reliability of the data for each specific case (Brigham and Ehrhardt, 2005: 311).

The results of the survey conducted by Graham and Harvey (2001: 201) of the methods employed to estimate a firm's cost of equity, summarised in Figure 1-1, show that whilst some practitioners do utilise more than one approach, the CAPM is by far the most extensively used method; 73.5 percent of the respondents surveyed always, or almost always, employ the CAPM.

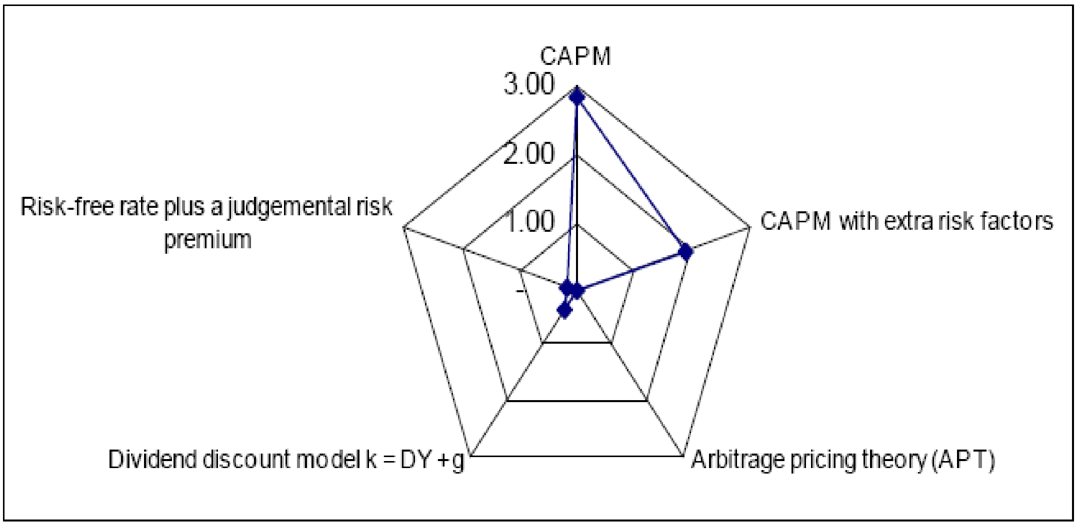
Figure 1-1: Cost of Equity Methods in the U.S.



(Source: Graham and Harvey, 2001: 203)

The findings of a survey conducted by Pocock, Correia and Wormald (1991) of the use of the CAPM in South Africa differed from those of the U.S. study, with more qualitative methods being preferred to estimate a firm's cost of equity rather than quantitative methods. However, more recent surveys conducted by PricewaterhouseCoopers (PWC) (quoted in Correia and Cramer, 2008: 41) and Correia and Cramer (2008: 41) indicate that a change in ideology has transpired, as the CAPM has become the most dominant method in South Africa to estimate the cost of equity for a firm. Correia and Cramer (2008: 41) observed that almost all practitioners surveyed compute the cost of equity using the CAPM or a variant thereof, such that the DDM, the Bond-Yield-Plus-Risk-Premium approach and the Arbitrage Pricing Theory (APT) are not used in practice. This is depicted in Figure 1-2 (on the following page), which illustrates the relative weightings attached to the various approaches based on the respondents of the survey, which are significantly weighted towards the CAPM and its variations.

Figure 1-2: Cost of Equity Methods in South Africa



(Source: Correia and Cramer, 2008: 41)

The CAPM depicts the equilibrium risk-return relationship, showing that the expected return on a risky security or portfolio of securities is the sum of the return on the risk-free asset and a premium for bearing risk (Pike and Neale, 2006: 249). The latter is a function of the security's non-diversifiable risk, as it is assumed that any firm-specific risk is eliminated by the investor through diversification (Sharpe, 1964: 438). The model can be depicted graphically by the Security Market Line (SML), the equation for which is given as:

$$E(R_k) = R_f + \beta_k [E(R_m) - R_f] \tag{1.1}$$

Where

- $E(R_k)$ is the expected return on the k^{th} security (the cost of equity of the firm)
- R_f is the risk-free rate of interest
- β_k is the measure of risk of the k^{th} security [equal to: covariance (R_k, R_m)/variance (R_m)]
- $E(R_m)$ is the expected return on the market portfolio

(Reilly and Brown, 2006: 240)

The issue for practitioners estimating the cost of equity of a firm is the choice of appropriate parameters to utilise in the CAPM; "…the determination of the parameters in the cost of capital calculations is fraught with difficulty…" (Correia and Uliana, 2004: 67). The reason for this is because the variables outlined in equation 1.1 are all theoretical values and thus proxies for these parameters must be identified (Firer, 1993: 25). The ramifications for financial decision-makers of using improperly specified proxies for these variables are substantial; cost of equity estimates which are too high can lead to the rejection of projects which would have created value, whilst cost of equity estimates which are too low can lead to the allocation of resources towards projects which may actually destroy value.

Bruner, Eades, Harris and Higgins (1998: 24) conducted an examination of the influence of differing proxies for the risk-free rate, the return on the market, and beta on the cost of equity estimates. The results showed that the cost of equity estimate for one of the firm's analysed varied between 12.1 and 20.25 percent, with the WACC estimates ranging between 9.3 and 12.8 percent, depending on the proxies chosen (Bruner *et al*, 1998: 24). The minimum present value of the perpetual stream of income of $10 million per year was thus $78 million and the maximum $118 million (Bruner *et al*, 1998: 23). Extending this analysis further, if the present value of the cost of the project was $100 million, then whether the computed NPV was positive or negative, and hence the decision of whether to proceed with the project or not would have depended directly on which proxies were selected. Accordingly the selection of the appropriate parameters for the CAPM is essential in order to guarantee the efficient and effective allocation of scarce resources in the economy.

Numerous studies have been conducted in the U.S. regarding the most suitable proxies to use for the market portfolio (Roll, 1977; Brigham and Shome, 1981; Amihud and Mendelson, 1991; Stambaugh, 1982; Vandell and Stevens, 1982; Carleton and Lakonishok, 1985; Kandel and Stambaugh, 1993; Roll and Ross, 1994; Levy and Roll, 2009) and the risk-free rate (Ibbotson and Sinquefield, 1979; Carleton and Lakonishok, 1985; Damodaran, 2001), as well as the most appropriate methodology to estimate beta (Jacob, 1971; Blume, 1971 and 1975; Breen and Lerner, 1972; Gonedes, 1973; Baesal, 1974; Rosenberg and Marathe, 1975; Alexander and Chervany, 1980; Eubank and Zumwalt, 1981; Bey, 1983; Hawawini, 1983; Carvel and Strebel, 1984). Within this literature, there is some degree of consensus about the best proxies to use for the various parameters; however, these conclusions may be inappropriate for applications of the CAPM in South Africa because of the dissimilarities between the two markets.

Research has consequently been conducted on the appropriate choice of proxy for the market portfolio in South Africa. Venter, Bradfield and Bowie (1992) (quoted in Ward, 1994: 102) and Bowie and Bradfield (1993) suggest that because of the segmentation of the South African market, appropriate major sector indices should be used as a proxy for the market rather than the overall index for Johannesburg Securities Exchange (JSE) shares. In contrast to this, Ward (1994) found that the perceived segmentation was not apparent in statistical testing and that using major sector indices as a proxy did not capture the significant dimensions of risk in these sectors; therefore he advocated the use of the All Share Index (ALSI). More recently, Correia and Uliana (2004) confirmed the suitability of the ALSI as a proxy, but they did acknowledge that its use may overestimate the beta values for industrial and financial shares.

The most suitable methodology to estimate beta has been analysed by Bradfield and Barr (1989), who highlighted the need to adjust beta estimates to account for the thin trading phenomenon on the JSE. Their results, in conjunction with those of Bowie and Bradfield (1997), support this assertion as the adjusted betas provide considerably more efficient forecasts than the unadjusted betas. A review of recent published research in South Africa revealed that whilst some scholars do follow the approach of Bradfield and Barr (1989) to adjust share and unit trust betas for thin trading, especially when the studies examined periods dating back to the 1970s and 1980s (Fraser and Page, 2000; Bradfield, 2002; Bhana, 2008); many do not (Oldfield and Page, 1997; Van Rensburg 2001; Akinjolirie and Smit (2003); Van Rensburg and Robertson 2003[1]; De Wet and Hall, 2006; Bhana, 2007; Samoulihan, 2007).

Very little research however has been conducted on the most suitable method to estimate the risk-free rate in South Africa; perhaps because of the perceived simplicity of the estimation of this parameter compared to beta and the market portfolio. Firer (1993: 37), in concluding his discussion on the matter stated "…at this point it should be clear that the issue of estimating the risk-free rate … is by no means resolved, and CAPM users have not been given any firm and theoretically sound guidelines on which to base their estimates". Yet, since Firer (1993) made this assessment of the state of knowledge regarding the estimation of the risk-free rate, no further research has been conducted. Given the importance of the CAPM in ensuring accurate capital budgeting decisions, it is imperative that the risk-free rate, as with the other parameters of the model, is correctly specified in South Africa.

1.1.2 The Estimation of the Risk-Free Rate

The risk-free rate is the rate of return on an asset that has no risk; that is, it has no variance or covariance with the returns on the market (Sharpe, 1964: 431; Harrington, 1987: 150), and it does not exhibit inflation, default, liquidity, currency or interest rate risk (Damodaran, 2001: 3; Blake, 2000: 86-87; Reilly and Brown, 2006: 22). Given these stringent conditions necessary for an asset to be considered riskless, it is not surprising that most scholars and authors are of the opinion that "…there is no such thing as a riskless asset" (Brigham and Ehrhardt, 2005: 312), and thus similarly to the other CAPM parameters, the risk-free rate has to be estimated.

[1] Van Rensburg (2001: 5) and Van Rensburg and Robertson (2003: 8) applied a thin-trading filter to remove all shares from the analysis that were not traded at least once during a month but did not adjust the betas for thin trading volumes.

The most common method for determining the risk-free rate is to use the yield on a government issued security as a proxy (Harrington, 1987: 150). According to Damodaran (2001: 5), in most developed economies, such as the U.S., where the government can essentially be viewed as default-free, the yield on a government security can validly be employed as an estimate of the return on the riskless asset. The findings of the survey conducted by Bruner *et al* (1998: 16) indicate that the choice of risk-free proxy in the U.S. is typically between the three month Treasury bill (T-Bill) and a long-term Treasury bond (T-Bond).

This method of estimating the risk-free rate is not, however, without criticism. Both U.S. T-Bills and T-Bonds exhibit variation and co-movement with the market over time (Carleton and Lakonishok, 1985: 40), and even the U.S. government has defaulted on payments to T-Bill holders (Nippani, Liu and Schulman, 2001: 263). An alternative approach to estimating the risk-free rate was derived by Black (1972), without reference to the existence of a single riskless asset. He developed a model that is almost identical to the traditional CAPM; the only difference being that the risk-free rate is substituted with the expected return on the minimum-variance zero-beta portfolio. This approach thus measures the risk-free rate as the return on a portfolio of assets, combined using both long and short positions, such that the portfolio returns do not move with those of the market; that is, the portfolio has a zero beta. Black (1972: 455) indicated that if a suitable proxy for the riskless asset does exist, then the rate of return on this security should be employed in the CAPM. However, if an appropriate surrogate risk-free asset does not exist, then the return on the minimum-variance zero-beta portfolio should be used.

Several empirical tests of the applicability of the traditional and zero-beta portfolio models in the U.S. have been conducted by Black, Jensen and Scholes (1972), Fama and MacBeth (1973), Stambaugh (1982) and Fama and French (2004), which follow an indirect approach to assessing whether the risk-free rate in the CAPM is more accurately measured by the returns on a government security or the zero-beta portfolio returns. That is, the historical minimum return required by investors (denoted by the intercept of the CAPM) was estimated via a number of different techniques and compared to the yields on government securities over the same period. If the risk-free rate proxy was statistically significantly different from the estimate of the minimum required return, then the conclusion drawn was that the zero-beta CAPM was more appropriate. In contrast, if the risk-free rate proxy was not statistically significantly different from the minimum required return, then the proxy was deemed a suitable means to estimate the risk-free rate. In general, the evidence presented in these studies supported the conclusion that estimating the risk-free rate in the CAPM using the zero-beta portfolio returns was more accurate than using a government security, as the proxy returns, surprisingly, were less than the estimated intercept values.

Despite these findings, the zero-beta portfolio approach is not implemented in industry or research in the U.S., partly because of the difficulty and complexity involved in computing this parameter (Copeland, Koller, and Murrin, 2000: 215)[2], but also because government securities are considered to be reasonable estimates of the risk-free rate (Damodaran, 2001: 4).

1.1.3 The Risk-Free Rate in South Africa: The Best Approach

Similarly to the U.S., the use of a proxy to estimate the risk-free rate is the most common approach in South Africa; with researchers favouring the three and twelve month T-Bill rates (Firer, 1993: 29), and practitioners the rate on long-term government bonds (Correia and Cramer, 2008: 42). However, there is evidence within the financial literature of scholars questioning the validity of this approach in South Africa, arguing that government securities do not satisfy the requirements for a risk-free asset as closely as in the U.S. (Graham and Uliana, 2001; Cloete, De Jonah, and De Wet, 2002; Akinjolirie and Smit, 2003; Correia and Uliana, 2004; Oldham and Kroeger, 2005; Msweli-Mbanga and Mkhize, 2007; Viviers, Bosch, Smit and Buijs, 2008). Consequently, these authors have preferred to use short-term private sector securities, such as Banker's Acceptances (BAs) and Negotiable Certificates of Deposit (NCDs) for this purpose. Damodaran (2001: 5) confirms this saying that in a developing country, where the government cannot be viewed as entirely risk-free, the use of government securities may not be entirely appropriate. However, as highlighted in Section 1.1.1, no research has directly focused on this issue in South Africa.

Further compounding the uncertainty regarding the best approach to use to estimate the risk-free rate is the fact that the zero-beta portfolio has not been reliably tested to ascertain its suitability in the South African context. Van Rhijn (1994), as a minor objective of his study, attempted to examine the relationship between a risk-free rate proxy and the minimum required return (based on the same methodology as established by Fama and MacBeth, 1973). He obtained an estimate of the intercept of the CAPM which was smaller than the proxy used for the risk-free rate (3 year government bonds) over the period 1984-1992, but was unable to test whether these returns were statistically significantly different because of data constraints; concluding simply that the difference between the two was small (Van Rhijn, 1994: 322). However, the inability of Van Rhijn (1994) to test the statistical equality of these values and the limited dataset used (only industrial shares were considered) brings these results into question.

[2] Black (1972) gave no suggestions as to how to estimate the minimum-variance zero-beta portfolio returns and despite considerable research (Black *et al,* 1972; Fama and MacBeth, 1973; Morgan, 1975; Alexander, 1977; Gibbons, 1982; and Stambaugh, 1982) there is no consensus in the literature as to the best approach to be used. Furthermore, there is also no clarity on the most suitable assets to include and how these assets should be formed into portfolios.

Moreover, Van Rhijn (1994: 322) did not explore the finding that the relationship between the minimum required return and the risk-free rate proxy was reversed compared to international studies and the consequences and the implications thereof.

It is thus clear that with regards to the estimation of the risk-free rate in South Africa there is uncertainty. Whilst the use of proxy securities is the only approach implemented, a variety of instruments are employed; and yet there are no theoretical guidelines to assist both practitioners and researchers in choosing the appropriate asset for this purpose. However, it is also possible that the zero-beta portfolio approach may be more suitable for estimating the risk-free rate in South Africa than the use of a public or private sector security, but there is little or no formal examination of this technique. Accordingly there is a need to assess both the validity of government and private-sector securities to estimate the risk-free rate in South Africa and the suitability of the zero-beta portfolio returns as an alternative means to estimate this parameter.

1.1.4 Research Problem

The research problem which is thus the focus of this study is summarised in the following question:

What is the best method for estimating the risk-free rate for use in applications of the CAPM in South Africa?

1.2 Research Objectives

The primary objective of this research is:

- To determine the appropriate parameter to use in the CAPM in South Africa for the risk-free rate, by estimating the historical minimum required return and the minimum-variance zero-beta portfolio returns and comparing these estimates to the returns on the most commonly used risk-free proxies.

The sub-objectives of this research include:

- To assess how closely the most commonly used proxies in South Africa adhere to the theoretical requirements for a risk-free asset and whether their divergence from the criteria is considerably greater than that associated with the most commonly used U.S. proxies.

- To answer the question of which of the various methods developed in the literature is the most efficient to employ in estimating the intercept of the CAPM.
- To ascertain whether allocating shares to portfolios on the basis of industry affiliations is more efficient than historical beta estimates in calculating the minimum required return and the zero-beta portfolio returns.
- To determine the optimal combination of assets that will result in the lowest variance intercept estimates and zero-beta portfolio returns, by expanding the analysis to include both bonds and preference shares in addition to ordinary shares.
- To identify the optimal time period over which the historical minimum required return and zero-beta portfolio returns should be estimated.
- To assess the impact on the forecasting accuracy of the CAPM of the use the zero-beta portfolio returns compared to a risk-free asset proxy.

The purpose of this study is to therefore establish whether the cost of equity calculated by the CAPM is robust to the choice between a risk-free rate proxy and the returns on the minimum-variance zero-beta portfolio in South Africa; and in so doing, expand the theoretical understanding of the risk-free rate of return, and provide practitioners with clear guidance on the best measure of the risk-free rate to employ in their financial decision-making.

1.3 The Scope and Method of this Study

1.3.1 The Scope of the Study

The identification of the correct parameters to employ in the CAPM is vital given the crucial role the model plays in capital budgeting decisions. This study, however, focuses only on the estimation of the risk-free rate and not the other variables, as a significant amount of research has been conducted on the most efficient methodology to estimate firm betas on the JSE and the most suitable surrogate to employ for the return on the market portfolio in the South African context, as explained in Section 1.1.1. Furthermore, it is assumed in this study that the CAPM relationships hold[3]. This supposition appears valid given the empirical results of Bradfield, Barr, and Affleck-Graves (1988), Bradfield and Barr (1989), Van Rhijn (1994), and Ward (1994), who found that "…the CAPM stood up reasonably well to empirical testing on the JSE" (Ward, 1994: 100). More recently, Van Rensburg and Slaney (1997), Van Rensburg (2001) and Van Rensburg and Robertson (2003) have found evidence disputing the validity of the model,

[3] There is a linear relationship between beta and return, returns are not dependent upon any other measures of risk, and that the market risk premium is positive (Fama and MacBeth, 1973: 613).

but for the purposes of this study, it is assumed that the CAPM holds in the South African context.

In order to estimate the minimum-variance zero-beta portfolio returns it is necessary to assume that investors are not restricted in short-selling risky assets. Although this condition may be unrealistic for institutional investors, it does not impinge on the critical focus of the study, of determining the appropriate procedure to estimate the risk-free rate for use in the CAPM in South Africa.

The relationships between the estimated intercept of the CAPM, the minimum-variance zero-beta portfolio returns and several risk-free rate proxies are examined from January 1993 to December 2008, with data for the period January 1990 to December 1992 being used as the out-of-sample data for the estimation of betas. The period over which this study is conducted was constrained by the availability of data as price and dividend information on all ordinary shares listed on the JSE in every year was required, but it was not possible to gather this information prior to 1990. Monthly prices and dividend yield information were obtained in accordance with the studies of Black *et al* (1972), Fama and MacBeth (1973), Stambaugh (1982) and Fama and French (2004) who all used monthly returns to estimate the average monthly minimum required return. This data was gathered from the JSE. However, shares listed on the development capital market, venture capital market and the alternative exchange (Alt-X) were not considered because of the impact of thin trading.

In addition to this, data was also collected from the JSE on preference shares to expand the assets included in the estimation of the minimum required return and the zero-beta portfolio returns. Bonds were also incorporated into this analysis; however due to the limited number of corporate and government listed bonds in South Africa, three of the Bond Exchange of South Africa (BESA) indices (short-term, medium-term and long-term) were selected to represent the bond market and data was collected for these from BESA. These indices were formulated in January 2001 and thus were only included in the study from this point in time.

With respect to the risk-free rate proxies utilised, U.S. three month T-Bills and ten-year government bonds were chosen as the proxies for the American risk-free rate, as these are the most commonly employed. With respect to South Africa, as discussed in Section 1.1.3, there has been a growing tendency of using three month BAs and NCDs within research as the proxy and thus these two assets were used in addition to the South African three month T-Bill and the R157 government bond (used as the equivalent T-Bond).

1.3.2 Research Methodology

This study consists of both a literature review and an empirical analysis. In the first part of the paper, a set of theoretical requirements for a risk-free asset are developed, with both the most commonly employed American and South African proxies being compared to these criteria to ascertain their suitability. This is followed by an explanation of the derivation of the minimum-variance zero-beta portfolio and an analysis of the empirical tests thereon, with particular focus on the U.S. market.

The second part of the study entails the estimation of the intercept of the CAPM and the returns on the minimum-variance zero-beta portfolio, and a comparison of these estimates to the risk-free rate proxies to determine the most suitable method to estimate the risk-free rate. The first step in the empirical analysis involves the calculation of the average monthly estimate of the minimum required return for each year, eight two year periods, three five-year periods (excluding 2008), two eight-year periods, and over the entire period 1993 to 2008. As there is no consensus in the literature as to the best approach for this purpose, the techniques of Black *et al* (1972), Fama and MacBeth (1973) and Stambaugh (1982) are used.

The ordinary shares are allocated to portfolios in accordance with the approaches adopted in all three studies; however, as mentioned in Section 1.1.2, there is no agreement as to whether beta-sorted or industry-sorted portfolios are the most efficient and consequently both methods are employed with two different samples being created. Two additional samples are also formed consisting of the ordinary share portfolios, combined with the preference shares (also allocated to portfolios) and bond indices. In this way, estimates of the minimum required return are computed for the three approaches across all four samples in order to ascertain the most efficient methodology, the most efficient asset allocation method and the most efficient combination of assets to include. The estimates are also compared based on the different time periods over which they are estimated to ascertain the optimal period over which the intercept of the CAPM should be computed. To statistically ascertain the most efficient estimates, the standard errors associated with the different estimates are compared based on an F-Test, as well as taking into consideration the extent to which the different approaches satisfy the CAPM relationships.

The majority of the methods suggested in the literature to estimate the minimum-variance zero-beta portfolio returns were formulated in the 1970s where simplifications were regularly made because of the lack of advanced computing power. The same is not true today, and therefore whilst the broad approach outlined by Morgan (1975) is followed, the specifics thereof are adjusted to make use of Microsoft Excel's solver function. The four samples formed for the estimation of the CAPM intercept are again used to estimate this variable with the results compared across the four samples in order to avoid assuming that the results with respect to the minimum required return will necessarily be appropriate for the zero-beta portfolio.

The most efficient set of estimates of the minimum required return and the zero-beta portfolio returns are used in the hypothesis tests to determine the equality between these values and the risk-free rate proxy in South Africa. To ensure that the results of these hypothesis tests are robust to the choice of risk-free rate, the two securities identified to most closely resemble the risk-free ideal (in Chapter 2) are chosen. The distribution of the test statistic used for the hypothesis test is a function of the results of the test of the most efficient methodology to estimate the intercept, as if the Stambaugh (1982) estimates are the most efficient, the normal distribution is appropriate; whilst for the estimates based on Black *et al* (1972) or Fama and MacBeth (1973), the Student t distribution is applicable.

The final component of this study is to compare the forecasting accuracy of the CAPM using both the minimum variance zero-beta portfolio estimates and the best risk-free rate proxies, to ascertain the impact on the reliability of the cost of equity calculated of using an inappropriate method to estimate the risk-free rate. Ten securities are randomly selected in each year and the expected return computed using the CAPM with the two estimates of the risk-free rate and compared to the actual returns. The mean-squared error, mean absolute percentage error and adjusted mean absolute percentage error are determined for this purpose, using the weighted average of the errors from the ten assets across all sixteen years. In this way, the results obtained in the hypothesis tests are assessed in terms of the actual influence on the CAPM.

1.4 Structure of the Study

This study consists of six chapters:

- **Chapter 1** provides a description of the background to the study, highlighting the lack of research in South Africa as to the most appropriate method to estimate the risk-free rate in the CAPM. This is followed by an explicit definition of the research problem and a comprehensive explanation of the research objectives. The scope and research methodology of the study are briefly discussed, with the chapter concluding with a concise outline of the structure that is adopted in the subsequent chapters.

- **Chapter 2** presents an overview of the securities that are used to estimate the risk-free rate in both research and practice in the U.S. and South Africa. A set of theoretical requirements that an asset must fulfil to be considered a suitable proxy for the riskless rate are derived from a number of sources. Thereafter, the extent to which both U.S. and South African proxies adhere to these requirements are assessed with reference to financial literature, current market-related information and a brief empirical analysis.

- **Chapter 3** provides an explanation of the derivation of the minimum-variance zero-beta portfolio in conjunction with a discussion of the theoretical suitability of this approach to estimate the risk-free rate. The empirical tests of the validity of this method compared to the use of a risk-free rate proxy in the U.S. are discussed, with reference to the more common indirect tests as well as the direct test of Morgan (1975). International evidence is also presented in this regard. The chapter concludes with an assessment of the veracity of the continued usage of proxies for the risk-free rate in the U.S. in light of the empirical findings and the inferences thereof for the estimation of the risk-free rate in South Africa.

- **Chapter 4** describes the methodology that is followed in this study with respect to the estimation of the intercept of the CAPM, the determination of the most efficient estimates thereof, the computation of the minimum-variance zero-beta portfolio returns, the hypothesis tests conducted, and the forecasting accuracy comparisons. The justifications for the methodologies adopted are also provided.

- **Chapter 5** presents the results from the various analyses described in the preceding chapter. The findings are analysed in conjunction with the theory discussed in Chapters 2 and 3 and the international empirical evidence.

- **Chapter 6** provides the conclusions and implications that can be drawn from the study, and makes recommendations for further research.

CHAPTER 2
THE SUITABILITY OF A RISK-FREE RATE PROXY IN SOUTH AFRICA

2.1 The Estimation of the Risk-Free Rate

2.1.1 The Role of the Risk-Free Asset in the Derivation of the CAPM

The CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966) is one of the cornerstones of financial theory and its development, in conjunction with modern portfolio theory (Markowitz, 1952 and 1959), revolutionised the field of finance; life without the CAPM "…would undoubtedly be miserable" (Lakonishok, 1993: 38). Markowitz's (1952 and 1959) modern portfolio theory was derived with reference to risky assets only, where risk is measured as the variability in returns around the mean value over time (Rees, 1995: 223). This theory illustrates that in a world of risky assets, a rational investor will choose to hold a portfolio of securities that is mean-variance efficient[4] and hence plots on the efficient frontier (Fama, 1976: 260).

Sharpe (1964: 433), in expanding upon the work of Markowitz (1952, 1959), introduced the assumption of the existence of a riskless asset, which by definition does not exhibit any variability in returns (the return is constant over the measurement period). The corollary to this assumption is that it is possible for investors to both borrow and lend at this rate (Reilly and Brown, 2006: 231). The inclusion of this instrument was critical in the development of the CAPM, as by introducing the riskless asset into the opportunity set of available securities, the mean-variance efficient investments, from which a rational investor will choose, provide higher returns for a given level of risk, or lower risk for a given level of return, than the investment options in a world of risky assets only; the new combinations are said to dominate the previous efficient portfolios (Van Horne, 2002: 58). Thus, an investor will select their optimal portfolio as a combination of risky assets (determined by the ray from the risk-free rate which is tangential to the efficient frontier) and the riskless asset, depending on the investor's aversion to risk (Sharpe, 1964: 432).

As a consequence of the assumption of the model that all investors have homogenous expectations regarding risk and return, every investor faces the same efficient frontier, and will select the identical optimal portfolio of risky assets to hold in conjunction with the risk-free

[4] Mean-variance efficient investments provide the highest possible return for a given level of risk, and the smallest possible risk for a given return of all attainable investments (Markowitz, 1959: 6).

asset (Reilly and Brown, 2006: 233-236). This optimal portfolio must represent the market portfolio consisting of all possible risky securities, as any asset not included in the portfolio will not be purchased (Brigham and Ehrhardt, 2005: 185).

Prior to the inclusion of a risk-free asset into the analysis, the risk of a portfolio was calculated as the sum of the weighted variance of each security and the covariance of each asset with every other asset in the portfolio; although Markowitz (1959: 114) proved that for a large portfolio the variance depends more on the covariance of each security with every other asset than an individual instrument's variance. Nevertheless, the computation of the variance of a portfolio was a burdensome task. However, with the introduction of a riskless asset, and the conclusion that all investors will hold the same portfolio of risky assets, the covariance component can be computed as the covariance of each security with the market portfolio instead of the covariance of the asset with every other asset in the portfolio; thereby dramatically simplifying the analysis. Thus, the crucial role of the risk-free asset in this model is that it identifies that all investors will hold some combination of this riskless security and the market portfolio of risky assets, which can thus be used as the basis for assessing the risk and return for individual securities, as is done in the CAPM.

2.1.2 The Role of the Risk-Free Asset in Applications of the CAPM

In the CAPM, the risk of an individual security is calculated on the supposition that the security is held as part of a well-diversified portfolio; that is, the unique or unsystematic risk of the security is assumed to have been diversified away (by holding it together with all other assets in the market portfolio) and hence securities are priced so that investors only receive compensation for market or systematic risk (Sharpe, 1964: 431). As proven by Markowitz (1959: 114), the risk of an individual security in a well-diversified portfolio can be computed by its covariance with the other assets; thus the systematic risk of an individual asset is determined as the covariance of the asset with the market, divided by the variance of the market[5], as denoted by beta (as shown in equation 1.1). The CAPM equation that Sharpe (1964: 440) therefore inferred consists of two components: compensation for the pure time value of money (the return on a riskless asset) and a risk premium. The risk premium is a function of the difference between the expected return on the market portfolio and the risk-free rate, scaled by the asset's unique beta (Pike and Neale, 2006: 249). This fundamental relationship between risk and return was shown in equation 1.1, and is reproduced on the following page.

[5] The covariance of the security with the market is expressed as a proportion of the total risk of the market portfolio, which can be measured as the covariance of the market with itself. The covariance of any security with itself is the variance (Bodie, Kane and Marcus, 2003: 197-198).

$$E(R_k) = R_f + \beta_k[E(R_m) - R_f] \tag{1.1}$$

In the CAPM equation, the risk-free rate appears twice. In the first case, it is the base or minimum return that investors require for investing, which is essentially the compensation that the investor requires to forgo current consumption and liquidity for a *certain* future amount (Harrington, 1987: 150). The second appearance of the risk-free rate is to determine the market risk-premium, which is calculated as the return on the market in excess of the minimum required rate of return.

As discussed in Section 1.1.1, in order to employ the CAPM to estimate the cost of equity for a firm, a value for the risk-free rate is needed. Sharpe (1964: 433) assumed the existence of a risk-free asset, but the question arises as to the validity of this assumption; that is, *does an asset actually exist which satisfies the definition of a riskless asset*? The answer to this question across a variety of sources is emphatic, and unequivocal: “No asset is totally risk-free” (Pike and Neale, 2006: 250), “No such asset exists” (Rees, 1995: 230) and “There is really no such thing as a truly riskless asset” (Brigham and Ehrhardt, 2005: 312). Thus, as with beta and the market portfolio return, the risk-free rate in the CAPM has to be estimated.

The risk-free rate is as important as the other two parameters of the model in obtaining accurate cost of equity estimates for a firm, but as Harrington (1987: 149) confirmed, it is the parameter to which the least attention has been paid in the U.S. literature. Moreover, as highlighted in Section 1.1.1, in South Africa almost no explicit research has been dedicated to the estimation of the risk-free rate.

The quality of output from any model, system, or process depends on the quality of inputs, and the same is true for the CAPM; incorrect parameter values will lead to erroneous cost of equity estimates and potentially wrong decisions regarding the allocation of scarce resources in the economy. It is for this reason that Harrington (1987: 3) emphasised the need for practitioners to be able to “…distinguish good input from useless garbage…” in estimating the parameters of the model. It is therefore necessary to assess the validity of the processes used to estimate the risk-free rate in South Africa to ensure that the values used for this variable are indeed an appropriate and accurate input. In light of the fact that the majority of research in this area is based on the U.S. market, it is of value to consider what practices are commonly implemented to estimate the risk-free rate in applications of the CAPM in America, before assessing the South African practices and their validity.

2.1.3 The Estimation of the Risk-Free Rate in the U.S.

The most common approach to estimating the risk-free rate for the CAPM in U.S. research for the past forty years (for example Black *et al*, 1972; Malkiel, 1995; Fama and French, 2004), and advocated in numerous texts (Harrington, 1987; Rees, 1995; Copeland *et al*, 2000; Brigham and Ehrhardt, 2005) is to use the yield on a government security as a proxy. The results of the survey of highly regarded U.S. companies, conducted by Bruner *et al* (1998: 16), confirmed that this method of estimating the risk-free rate is as prevalent in industry as academia. 85 percent of the corporations and 90 percent of the financial advisors surveyed used the yields on government securities as a surrogate for the risk-free rate (Bruner *et al*, 1998: 16).

The widespread usage of these instruments reflects not only the simplicity and ease with which data on the yields of government securities can be collected, but also the view that these assets closely resemble the risk-free ideal (Pike and Neale, 2006: 250). No government security is completely devoid of risk, as will be discussed further in Section 2.3; but according to Damodaran (2001: 5), in developed countries, such as the U.S., where governments can essentially be viewed as risk-free, these securities can validly be applied as surrogates for the riskless asset.

Various government securities are employed by practitioners to estimate the risk-free rate. An examination of research using the CAPM and of textbook descriptions of the model and its parameters revealed that the most commonly employed and advocated instruments are T-Bills and T-Bonds. T-Bills are zero-coupon obligations, meaning that they do not pay interest prior to maturity, but instead are sold at a deep discount to their face value so as to provide a positive yield to the holder, and they typically mature within one year or less; the most frequently traded being three and six month bills (Hahn, 1993: 105; Van Horne, 2002: 438). T-Bonds on the other hand, are usually semi-annual coupon bearing obligations that have considerably longer maturities than T-Bills, usually ranging between three and thirty years (Van Horne, 2002: 438)[6].

Historically, T-Bills were the most commonly used surrogate for the risk-free asset, and in fact, Harrington (1987: 149) argued that "…whether in academic research or in practical applications of the CAPM, the 90-day Treasury bill rate has been virtually the only proxy used for the risk-free asset". More recently, however, longer-term T-Bonds have become increasingly popular in industry and this changing practice is demonstrated in the results of the survey of Bruner *et al* (1998: 19), which found that almost all of the corporations and financial advisors surveyed (77

[6] U.S. T-Bonds however, are valued as floating rate zero-coupon bonds (Bodie, Kane and Marcus, 2002: 31).

and 80 percent respectively) favoured the use of securities with maturities greater than three years for the risk-free proxy. It is possible that the increased use of T-Bonds represents an attempt to reconcile practice with empirical evidence which has highlighted that the historical intercept of the SML is greater than the T-Bill return, as will be discussed in more detail in Section 3.3.1. This is not to say however, that there is complete agreement amongst scholars that a longer-term asset is better. Although some authors such as Copeland *et al* (2000: 162) and Brigham and Ehrhardt (2005: 312) favour this approach, a review of U.S. literature revealed there are very few occurrences of this security being employed in research as the means to estimate the risk-free rate.

It is therefore apparent that academics and practitioners in the U.S. appear to concur that a government security is the appropriate proxy to use, but that there is no agreement as to whether or not this proxy should be a short-term or long-term instrument. The choice between T-Bills and T-Bonds has important implications for the CAPM and the cost of equity estimates calculated, as in general, treasury yield curves are upward sloping implying that securities with longer maturities provide a greater return than short-term securities (Pike and Neale, 2006: 250). The choice of proxy for estimating the risk-free rate will thus have a material effect on the cost of equity estimated (Bruner *et al*, 1998: 12). For this reason it is imperative to consider which of these securities provides a better proxy, which will be considered in Section 2.3.

2.1.4 The Estimation of the Risk-Free Rate in South Africa

Firer (1993: 29) conducted a review of all research papers published in South Africa in which the author was required to estimate the risk-free rate as part of the methodology. The four articles examined, similarly to U.S. research publications, all employed a proxy as the means to estimate this rate (Firer, 1993: 29). Affleck-Graves, Burt and Cleasby (1988), and Page and Palmer (1991) both estimated the rate using the three month T-Bill yield; whilst De Villiers, Lowlings, Pettit and Affleck-Graves (1986) used the one year equivalent. In contrast, Bradfield *et al* (1988) diverted from the traditional approach and employed the twelve month fixed deposit yield.

In order to ascertain how practices have evolved in South Africa, a similar examination of published research was conducted over the period 2001 to 2008. The results of this investigation confirmed that American practices continue to be implemented in South Africa in terms of estimating the risk-free rate using an appropriate proxy; however, the instruments that have been employed for this purpose, whilst emulating the U.S., have also been adapted in an attempt to account for the unique characteristics of the local market. Friis and Smit (2004), Van

Rensburg (2001) and Samouilhan (2007) chose simply to estimate the risk-free rate in their respective studies as the three month T-Bill yield. In contrast, De Wet (2005), Moolman and Du Toit (2005), De Wet (2006), De Wet and Hall (2006), and De Wet and Du Toit (2007) all estimated the risk-free rate as the yield on government bonds. None of these authors provided any justification for their decision, but notably, shorter-term government bonds (maturities of less than five years) were favoured in all of these studies (represented by the R150 (expiry in 2006) and R153 (expiry in 2010) bonds), except that of Moolman and Du Toit (2005), who used the R157 bond (expiry in 2015) which represented a ten year instrument.

Several of the research papers examined questioned the validity of government securities as proxies for the risk-free rate in the South African market, and consequently chose to estimate the parameter using short-term (maturity of less than a year) private sector securities which they deemed to be more appropriate. Graham and Uliana (2001), Cloete *et al* (2002), Oldham and Kroeger (2005), and Msweli-Mbanga and Mkhize (2007) employed the rate on BAs, which are perceived to be the safest private sector short-term instrument, as the risk-free rate. The other private sector security that has become increasingly popular is the three month NCD, as this was used by Akinjolirie and Smit (2003), Correia and Uliana (2004) and Viviers *et al* (2008).

Corporations and financial advisors in South Africa, however, have not mirrored the growing practices in research of using private sector securities rather than government instruments to estimate the risk-free rate, as evinced in the results of several surveys in the past six years. The PWC Survey of 2003(referred to in Section 1.1.1) indicated that almost all of the firms surveyed employed the R153 government bond yield which, at that stage, had a maturity of approximately seven years (quoted in Correia and Cramer, 2008: 42). In the subsequent 2005 PWC survey, the results showed a split between the use of the R153 and the R157 (quoted in Correia and Cramer, 2008: 42); presumably because of the shorter maturity of the R153 some practitioners reverted to a longer maturity security.

Correia and Cramer (2008) analysed a more diverse spectrum of companies in their survey compared to the PWC surveys which tended to focus on financial institutions, and consequently observed slightly different trends. With respect to the instruments used to proxy the risk-free rate, they found that the majority (55 percent) employed the R153 government bond yield, with only fifteen percent using the R157 bond (Correia and Cramer, 2008: 42). The other respondents used the R186 (expiry in 2026), R196 (expiry in 2009), ALBI (the All Bond Index, consisting of the top 20 listed bonds, ranked by liquidity and market capitalisation), and R201 (expiry in 2014) as the selected rate (Correia and Cramer, 2008: 42).

It is certainly apparent that there is little or no clarity in South Africa as to the appropriate proxy to use for the risk-free rate in the CAPM and yet the choice can have a material impact on the cost of equity value obtained. Moreover, the questioning of some scholars (such as Firer and McLeod, 1999; Graham and Uliana, 2001; Correia and Uliana, 2004; Viviers *et al*, 2008) of the validity of T-Bills and T-Bonds to estimate the risk-free rate in South Africa raises the question of what the correct specification of the risk-free rate looks like and how closely both U.S. and South African surrogates adhere to this specification. Therefore, to address these issues, a set of requirements that an asset must satisfy so that the risk-free rate is correctly specified are formulated in Section 2.2. Thereafter, both the U.S. and South African proxies are compared to these criteria to assess their appropriateness.

2.2 Theoretical Requirements for the Correct Specification of the Risk-Free Rate

Obviously, by definition a risk-free asset should be devoid of risk. However, given the wide range of risks securities are exposed to, this requirement has a number of implications. In order to isolate these sources of risk, it is first necessary to consider what the risk-free rate should encompass, as is done in Sections 2.2.1 and 2.2.2, and thereafter identify those risk factors to which it should not be exposed (Sections 2.2.3-2.2.8).

2.2.1 The Pure Interest Rate

The first requirement that Sharpe (1964: 431) identified for a security to be classified as a risk-free instrument is that the return must be equal to the pure interest rate. Reilly and Brown (2006: 15) confirm this, suggesting that the real return should only depend on the time preference of individuals for the consumption of income and the investment opportunities available in the economy. This implies that the return earned should not be influenced by external factors, such as government policies or economic events (Harrington, 1987: 153). In this way, the relative independence of the yield on a risk-free security in comparison to other assets should assist in ensuring both that the returns are stable over time (Reilly and Brown, 2006: 15), and that the returns do not move in tandem with the returns on the market.

2.2.2 Inflation Risk

The presence of inflation erodes the return on an investment as the investor's purchasing power is reduced because of the increase in the prices of goods and services (Bodie *et al*, 2003: 146). Accordingly, investors require a return which includes an inflation premium to provide compensation for the expected rate of inflation over the investment horizon (Harrington, 1987: 40, 155; Reilly and Brown, 2006: 21). This return is known as the nominal return and Fischer (1930) showed that it can be calculated as the sum of the real return and the market's assessment of the expected inflation rate (quoted in Fama, 1975: 269).

One of the assumptions of the CAPM is that there are no market imperfections, which includes an absence of inflation (Reilly and Brown, 2006: 231). As with many of the other assumptions of the model, this is unrealistic, unless the effects of inflation are removed from the analysis; that is, only real returns are examined. Whilst this is a useful alternative, several problems still exist. These include trying to isolate the real returns by using various analysts' forecasts of the expected inflation rate, as well as the fact that the cost of equity estimate obtained from the CAPM will consequently not include inflation expectations, yet the other variables to which it may possibly be compared or analysed in conjunction with will be nominal returns. For these reasons, although several inflation-adjusted versions of the CAPM have been developed (Biger, 1975; Friend, Landskroner, and Losq, 1976), the most common approach is to use the nominal returns on all assets examined with the CAPM (Blake, 2000: 494).

A risk-free asset should thus provide a yield greater than the real return to compensate the holder for inflation (Firer, 1993: 28); that is, it comprises the pure interest rate plus a premium for inflation. However, a risk-free security must still satisfy the criterion that it is free of inflation risk, which arises when actual inflation differs from expectations and the premium included in the yield on the security does not accurately reflect the observed price level changes during the period of the investment (Blake, 2000: 86). In this way, if inflation is higher than expected, the real return earned by the investor will be eroded. Consequently, for an asset to be free of this risk, the expectations of, and the actual inflation level must be consistent. Inflation risk is thus partially a function of the volatility of inflation within an economy, as securities which trade in markets where inflation is more stable will have less exposure to this risk than those traded in markets with more volatile changes in price levels.

2.2.3 Variance in Returns

As discussed in Section 2.1.1, risk is measured in portfolio and capital market theory as the variation of the actual returns around the mean or expected value; measured by variance (Rees, 1995: 223). The formula used for this purpose is shown in equation 2.1 (on the following page). For an asset to be risk-free in an uncertain environment the variance must equal zero for the duration of the investment, implying that the actual returns earned over the period are always equal to the expected return (Sharpe, 1964: 431). The standard deviation of a security, the more commonly expressed measurement of risk, is calculated simply as the square root of the variance, and will thus also be zero for a riskless security.

$$\sigma_x^{\,2} = \sum_{i=1}^{n} P_i\,[\,R_{xi} - E\,(\,R_x)]^{\,2} \tag{2.1}$$

Where:

- σ_x^2 is the variance of asset x
- P_i is the probability of state i occurring
- R_{xi} is the return on asset x in state i

(Reilly and Brown, 2006: 204)

The requirement of zero variance has different implications for short-term and long-term securities. As explained in Chapter 1, the CAPM is the most commonly employed tool for estimating the cost of equity for capital budgeting decisions; the latter referring to the long-term investment decisions of the firm (Copeland *et al*, 2000: 215). Utilising a three month instrument, such as the T-Bill, as the risk-free rate in the CAPM analysis thus necessitates rolling this investment over for the duration of the project examined. Whilst the investor is guaranteed to receive the specified return by holding the asset in the initial period, if variation occurs in the returns over time, there is no assurance that the promised yield in the following period will be identical to that earned in the first period; hence short-term securities are exposed to reinvestment risk. Consequently, variation in the returns of short-term securities introduces an element of risk into the analysis, as there is uncertainty as to the future returns that will be earned; which violates the condition of zero variance for the risk-free rate.

In contrast, if a longer-term security is selected as the proxy, with a maturity that matches the proposed duration of the investment, fluctuations in the yields of these instruments will not affect the investor's wealth over the investment horizon. The reason for this is that the interest rate to be earned on the security is specified and fixed at the date of issue, and provided the security is held to maturity, the promised return (the coupon rate) will be earned; with the fluctuations in the market yield simply reflecting the rate at which the bond can be sold in the secondary market. As the risk-free proxy does not have to be rolled over or liquidated prior to maturity the investor will earn the return specified at purchase, and hence no risk is involved as a consequence of movements in the yield to maturity (Damodaran, 2001: 4). However, the bond valuation model implicitly assumes that the coupons earned on the investment can be reinvested at the same rate at which they are being earned (Blake, 2000: 86-87) and consequently fluctuations may have a nominal influence on the investors' wealth position in terms of the return earned on the reinvested coupons. Of far greater importance with regards to the variation of T-Bond yields is any potential maturity mismatch between the long-term proxy chosen and the project duration. Fluctuations in returns of longer-term securities will affect the value earned when the security is liquidated prior to maturity (price risk) or rolled over (reinvestment risk); the return in either of these situations is therefore not risk-free.

2.2.4 Interest Rate Risk

Interest rate risk comprises both reinvestment and price risk (Reilly and Brown, 2006: 775), which is a function of volatility in interest rates. The yield on the risk-free rate proxy should not include compensation for the risk of interest rates fluctuating unfavourably during the life of the security. However, if the returns of the proxy instrument vary significantly over time, then the possibility exists that the return may include a premium for interest rate risk. Thus, although as discussed above, variance in the returns of longer-term securities may not be of importance if the maturity matches that of the investment horizon, if a risk premium for volatility is included, the instrument will not satisfy the risk-free asset requirements.

2.2.5 Covariance with the Market

Covariance measures the relationship between two securities by the extent of co-movement of the returns of each instrument around their respective mean values (Reilly and Brown, 2006: 232), as illustrated in equation 2.2. For a riskless asset, whose returns are always equal to the expected return, this variation around the mean will be zero and thus it cannot be related to the movement of a risky asset's returns around its mean value. Hence the covariance of a risk-free security with any risky instrument must be zero (Sharpe, 1964: 431). In addition, the covariance of a riskless asset with the market portfolio, which includes all risky assets, should also be zero. Correlation, which is the standardised measure of the relationship between two securities, is calculated using the covariance value, as depicted in equation 2.3, and will also be zero for the risk-free asset and the market portfolio.

$$\sigma_{xy} = \sum_{i=1}^{n} P_i \left[R_{xi} - E(R_x) \right] \left[R_{yi} - E(R_y) \right] \tag{2.2}$$

$$\rho_{xy} = \sigma_{xy} / (\sigma_x \sigma_y) \tag{2.3}$$

Where:

- σ_{xy} is the covariance of assets x and y
- ρ_{xy} is the correlation of assets x and y
- σ_x is the standard deviation of asset x

(Bodie *et al*, 2003: 175)

As with the requirement of zero variance, the condition of zero covariance with the market has different implications for long-term and short-term securities. In the context of choosing between a long-term and short-term instrument, the presence of a significant relationship between the asset chosen and the market intimates that the fluctuations observed in the market will not only affect the security being analysed directly through the market returns but also via the risk-free asset returns. Consequently, this affect is likely to be pronounced for a short-term security being rolled over, whereas for a longer-term security matching the investment horizon, the correlation will, by definition, be zero.

2.2.6 Default Risk

The security employed as the proxy for the risk-free rate must be devoid of default risk meaning that there is no possibility that the issuer of the security will default on the repayment of the investment to the holder (Damodaran, 2001: 3). Government securities are therefore identified as suitable proxies as a government can, under most circumstances, print money or raise taxes to avoid default on its commitments (Damodaran, 2001: 4; Reilly and Brown, 2006: 19). The ability to obtain an asset that is risk-free is thus a function of the reliability and strength of the government within a particular country.

2.2.7 Liquidity Risk

Liquidity refers to the ease with which an asset can be sold without a significant reduction in value; liquidity risk thus arises from the difficulty in selling an asset in the secondary market with respect to both the time taken to sell the security and the price to be received (Reilly and Brown, 2006: 22). The risk-free rate in the CAPM should only provide investors with compensation for the temporary illiquidity of being without their funds for a single investment horizon (Firer, 1993: 28); and thus as a consequence of the fact that a risk-free asset must not exhibit any liquidity risk, no liquidity premium should be included in the yield.

2.2.8 Currency Risk

In applying the CAPM within a particular country, it is implicitly assumed that the cost of equity calculated is for a firm operating in that market. In fact, if the CAPM is being utilised to determine the cost of equity for a firm contemplating an investment in another country, a country risk premium, which includes compensation for currency risk, additional political risk, differential tax rates etc. is usually added to the domestic cost of equity estimate (Bruner, 2004:

361-362). Hence, the risk-free rate in the CAPM should not incorporate any compensation for the risk pertaining to fluctuations of the domestic currency as it is irrelevant for the firm. In selecting an asset for use as the risk-free proxy in the model the requirement is therefore not that the security must be free of currency risk, but rather that the yield must not include any compensation for this risk, if it exists.

2.3 The Suitability of Risk-Free Rate Proxies in the U.S.

Given these requirements that a security must satisfy to be considered a suitable proxy for the risk-free rate, it is of value to examine how closely U.S. T-Bills and T-Bonds satisfy these criteria before assessing the suitability of the most commonly used South African proxies.

2.3.1 The Pure Interest Rate

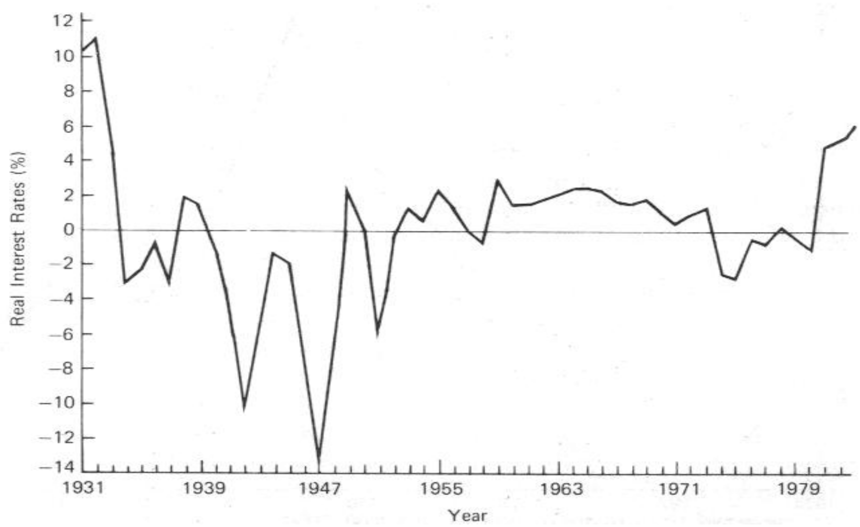
Interest rates are one of a government's major tools to implement monetary policy, as they are used as a tool to fight inflation, to stimulate/slow down the economy, and to adjust the exchange rate (Harrington, 1987: 153; Rees, 1995: 230). Whilst the actions of a government affect both short-term and long-term rates, short-term rates are impacted to a larger extent, because many of the government activities occur in the money market and thus influence money market rates (of which T-Bill yields are included) to a larger extent than long-term yields (Harrington, 1987: 153; Kelly, 1993: 123). Furthermore, economic events and various trends also tend to have a larger impact on short-term rates rather than long-term rates, as over longer periods of time short-term fluctuations offset each other, with only long-term, more stable trends affecting longer-term yields (Brigham and Ehrhardt, 2005: 312).

As Firer (1993: 27) alluded to, the rates on government securities reflect more than simply investors' compensation for illiquidity, but are priced to reflect other factors, which are independent of the risk of the instrument. It is therefore apparent that neither T-Bills nor T-Bonds completely reflect the pure interest rate; but in this regard, it appears as though longer-term T-Bonds more closely adhere to the theoretical condition than T-Bills.

2.3.2 Inflation Risk

Several studies in the U.S. have examined the inflation premia included in the yields of T-Bills and T-Bonds and the risk to which these instruments are exposed when these premia do not fully compensate the investor for the actual change in the price level over the investment horizon. Fama (1975: 277) found that real returns on T-Bills were reasonably constant over the period 1953-1971, with the variation in nominal yields caused primarily by changing inflation. An analysis of the real yield on three month T-Bills over a much longer period from 1931 to 1982 was conducted by Harrington (1987: 157) who, whilst confirming the observations of Fama (1975), found that during periods of substantial and volatile inflation the real returns were in fact negative. This result implies that the inflation premia did not accurately reflect the changes in prices; hence holders of these securities were exposed to substantial inflation risk in contravention of the risk-free asset requirement, as illustrated in Figure 2-1.

Figure 2-1: Real T-Bill Yields 1931-1983

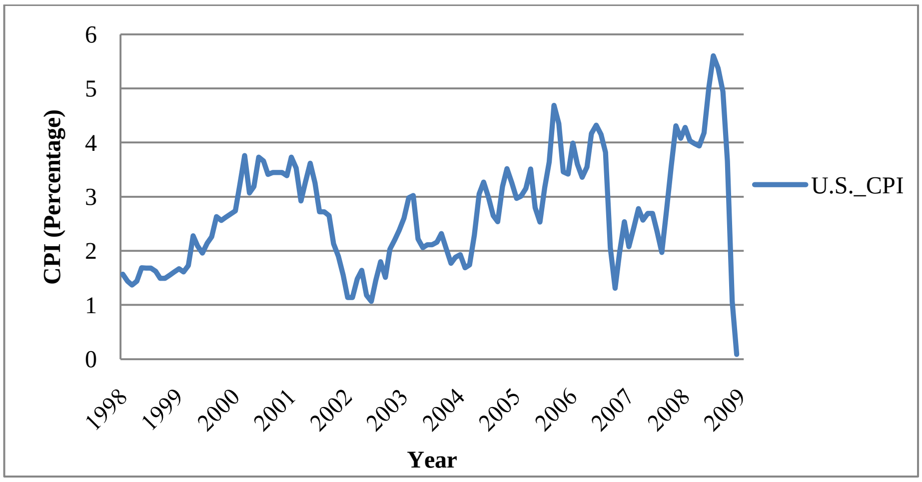


(Source: Harrington, 1987: 157)

T-Bonds, however, are also not exempt from the effects of inflation risk. Carleton and Lakonishok (1985: 41) showed that over the period 1976-1980, the returns on T-Bonds were particularly small compared to T-Bills (average of 2.1 percent versus 7.8 percent respectively), as higher unanticipated inflation contributed to greater interest rates and lower bond prices. Thus, the unexpected capital losses offset the coupons received resulting in lower realised returns (Carleton and Lakonishok, 1985: 41).

The periods over which the studies discussed have examined the effects of inflation on T-Bills and T-Bonds correspond with turbulent economic and political times in the U.S. that include the great depression, second world war, cold war and the oil crisis (Harrington, 1987: 157). It was therefore considered necessary to analyse the inflation risk of these proxies over more recent periods; however, a review of U.S. financial literature revealed that no studies have been conducted on this subject. Consequently, an examination of the monthly U.S. Consumer Price Index (CPI) over the period January 1998 to December 2008 was performed (using data collected from Econstats) in order to ascertain whether the volatility in inflation evinced in the U.S. from the 1930s through to the 1980s has continued in more recent periods. The results are depicted graphically in Figure 2-2 and illustrate that over the period, inflation only moved within a range of approximately 5 percent, reaching a high of 5.6 percent in July 2008 and a low of 0.9 percent in December 2008. The variance of this series was calculated (as per equation 2.1, but adjusted for the use of historical data, where the probability of the state occurring is replaced with 1/ (n-1) with n reflecting the sample size) as 0.01 percent squared.

Figure 2-2: Monthly Inflation Movements in the U.S. 1998 – 2008



(Source: Econstats)

This variance estimate of the U.S. CPI was tested to determine whether it was statistically significantly different from zero and thus whether the requirement was satisfied. Normally the variance of a sample can be tested against a hypothesised value using the chi-squared statistic (which follows the chi-squared distribution) (Keller and Warrack, 2000: 370-372), shown in equation 2.4.

$$\chi^2 \text{ statistic} = (n-1)\, \sigma^2 / \sigma_h^2 \qquad (2.4)$$

Where:

- χ^2 is the chi squared test statistic
- n is the sample size
- σ^2 is the variance of the asset
- σ_h^2 is the hypothesised value of the variance

(Keller and Warrack, 2000: 370)

The problem with this approach is that testing the null hypothesis that the variance is zero will result in a statistic that is undefined because the denominator will be zero. Thus, a value of 10^{-3} was selected as a suitable value against which to test rather than zero. A one-tailed chi-squared test was conducted, as depicted in equation 2.5, to determine whether the variance was equal to or greater than 10^{-3}.

$$H_0: \sigma_h^2 = 0.001$$
$$H_1: \sigma_h^2 > 0.001 \qquad (2.5)$$

The test statistic computed (136.11) was smaller than the critical value at the ten percent significance level based on 131 degrees of freedom (153.21) and hence the null hypothesis could not be rejected. The test conducted therefore confirmed the visual observation that the inflation risk to which U.S. investors have been exposed in the past eleven years has been relatively small and has decreased substantially as the economic and political environment has become more stable.

Equally important is the question of whether T-Bills or T-Bonds more closely resemble the theoretical condition of zero inflation risk. In this regard, there is consensus: T-Bills minimise the impact of changes in prices (Harrington, 1987: 156-157; Blake, 2000: 86; Reilly and Brown, 2006: 19; Pike and Neale, 2006: 250). The reason for this is that it is considerably harder for investors to anticipate long-term inflation movements with any accuracy and hence determine its likely impact on returns (Harrington, 1987: 155); consequently, yields on long-term bonds include a greater inflation risk premium to compensate investors for the possibility that the promised return will be eroded by unexpected changes in purchasing power (Pike and Neale, 2006: 250). For short-term investments inflation risk is less prevalent as the shorter duration enables more accurate expectations to be incorporated into the returns offered on the securities. With such assets, investors are also able to reinvest regularly and in so doing, adjust their decisions based on more reliable inflation expectations. Campbell and Viceira (2001: 106) confirmed these relationships for both stable and volatile periods in the U.S.; finding evidence of positive inflation risk premia associated with long-term compared to short-term government securities over the period 1952-1996, after adjusting for differing maturities.

It is therefore evident that government issued securities are exposed to inflation risk and that this is exacerbated during periods of volatile inflation. However, during more stable periods U.S. T-Bonds and T-Bills, in particular, do closely adhere to the requirement of zero inflation risk. Whilst there is no argument that T-Bills as opposed to T-Bonds minimise inflation risk, an alternative argument has been proposed that if a long-term investment involves greater inflation risk then using a proxy for the risk-free rate that incorporates an appropriate inflation premium may be suitable provided the maturity of the proxy matches the maturity of the investment horizon (Copeland *et al*, 2000: 217; Brigham and Ehrhardt, 2005: 312).

2.3.3 Variance in Returns

2.3.3.1 Original Studies

A riskless asset should, by definition, exhibit no variation in returns over time, and thus the extent to which both T-Bills and T-Bonds satisfy this condition can be assessed empirically by examining their variation over historical periods. Ibbotson and Sinquefield (1979) and Carleton and Lakonishok (1985) performed analyses of the returns and variance of these securities over the periods 1926-1976 and 1926-1980 respectively. Their results, which are shown in Tables 2-1 and 2-2, indicate that both securities exhibited variation in returns over time, but that this variation was considerably larger for T-Bonds than T-Bills.

Table 2-1: Return and Variance Analysis for U.S. Treasury Securities 1926-1976

	T-Bills	T-Bonds
Arithmetic Return (%)	2.5	3.20
Geometric Return (%)	2.55	3.40
Variance (%^{2})	5.15	32.17
Standard Deviation (%)	2.27	5.71
χ^2 Test of Variances	26.76	169.77

(Adapted from: Ibbotson and Sinquefield, 1979: 43)

Table 2-2: Return and Variance Analysis for U.S. Treasury Securities 1926-1980

	T-Bills	T-Bonds
Arithmetic Return (%)	2.8	3.2
Geometric Return (%)	2.8	3
Variance (%^{2})	7.29	32.49
Standard Deviation (%)	2.7	5.7
χ^2 Test of Variances	489.89	2183

(Adapted from: Carleton and Lakonishok, 1985: 40)

Similarly to the analysis of inflation risk, in order to accurately assess these results in light of the risk-free asset requirement, it was necessary to determine whether the variance estimates were statistically significantly different from zero. Neither of these authors presented results from such an analysis; however, using the information presented in the respective studies, it was possible to do so.

The chi-squared statistics, calculated based on the Ibbotson and Sinquefield (1979) data, are given in Table 2-1. The critical values from the chi-squared distribution based on 51 degrees of freedom at the one, five and ten percent significance levels are 77.39, 68.67 and 64.30 respectively. The statistic computed for T-Bills was less than the critical values and therefore it was concluded that the variance in returns over this period was not statistically significantly different from zero; whilst for T-Bonds, the test statistic was greater than the critical values revealing that this security did not satisfy the requirement of zero variance over the period 1926 to 1976. The statistics calculated from the Carleton and Lakonishok (1985: 40) data, shown in Table 2-2, result in the same conclusions being drawn regarding the statistical significance of the variance of T-Bills and T-Bonds; the value of the test statistics and critical values (760, 733 and 719 at the one, five and ten percent significance levels respectively) only differed because the Carleton and Lakonishok (1985) study employed monthly compared to annual data for the Ibbotson and Sinquefield (1979) study.

This significant volatility evinced in T-Bonds can partly be explained in terms of the dual influence of inflation and price risk on the yields of T-Bonds compared to T-Bills. As discussed, in the 1970s the U.S. market was characterised by high levels of inflation, which contributed to rising interest rates and falling bond prices, where the coupon income from these bonds was offset by the capital losses, resulting in lower realised returns, and in fact in many periods the returns were negative.

2.3.3.2 More Recent Evidence

Some texts referred to in Section 2.1.3, such as Harrington (1987) and Brigham and Ehrhardt (2005); advocate the use of T-Bonds as the more suitable proxy for the risk-free rate on the basis that these securities exhibit less variation in returns over time. This suggestion seems surprising in light of the results in the previous section that the variation in T-Bond yields was significant whereas this was not the case for the T-Bill. Consequently, it was deemed necessary to conduct a comparison of T-Bonds and T-Bills over a more recent period (January 1996 – December 2008 was chosen) to ascertain whether the volatility in T-Bill and T-Bond yields has changed over time, and whether T-Bonds more closely satisfy the zero-variance condition for a riskless asset. For this purpose, daily data was collected from Econstats on the yields of three month T-Bills and ten year government bonds.

The asset prices of the U.S. T-Bills were obtained as per equation 2.6 (on the following page), with the prices of U.S. T-Bonds calculated on a pure discount of the principal of the securities, illustrated in equation 2.7, as they are valued as zero-coupon bearing bonds with the yields determined at auction (Bodie *et al*, 2002: 31). The returns were computed as continuously compounded returns (equation 2.8) rather than simple returns, which guarantees that they are stationary (Brooks, 2006; 377-380); however, the high frequency of data means that the difference between the simple and compound returns is negligible. Thereafter the variance and standard deviations for the thirteen year period were calculated as per the methodology established in Section 2.2.3 (but taking into consideration the use of historical data).

U.S. Bill $P_t = 100 \times (1 - Y_t \times 91/365)$ (2.6)

U.S. Bond $P_t = F/(1+ Y_t)^m$ (2.7)

$R_t = \ln(P_t / P_{t-1}) \times 100$ (2.8)

Where:

- P_t is the closing price on day t
- Y_t is the stated yield on day t
- F is the face value of the bond
- m is the number of periods to maturity
- ln refers to the natural logarithm ($\log_e$)
- P_{t-1} is the closing price on day t-1

(Bodie *et al*, 2002: 30 and Brooks, 2006: 7)

The estimates and critical values are shown in Table 2-3 and support the findings of both Ibbotson and Sinquefield (1979) and Carleton and Lakonishok (1985) that the variance of T-Bills was not statistically significantly different from zero, whilst the variance of T-Bonds was significant, in contravention of the risk-free asset criterion. It is therefore clear that U.S. T-Bills satisfy the requirement of zero variation.

Table 2-3: Return and Variance Analysis for U.S. Treasury Securities 1996-2008[7]

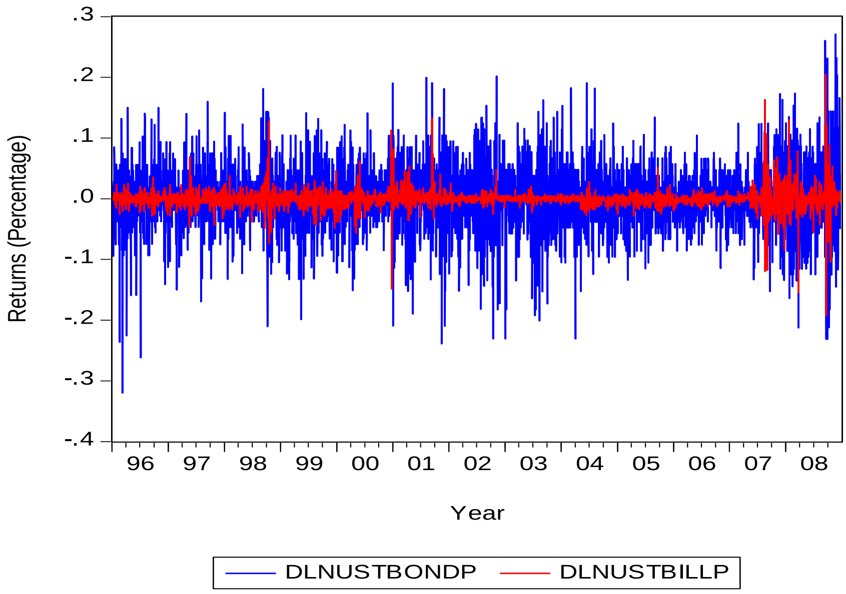
	T-Bills	T-Bonds
Arithmetic Return (%)	0.000383	0.00096
Geometric Return (%)	0.000381	0.00095
Variance (%^{2})	0.000254	0.003204
Standard Deviation (%)	0.0159	0.0566
Chi Squared Statistic	863	10870

* Chi Squared Critical Values based on infinite degrees of freedom are 1162, 1179 and 1213 at the ten, five and one percent significance levels respectively.

Figure 2-3 (on the following page) shows the stationary daily compounded returns for both T-Bills and T-Bonds over the period 1996 to 2008 and confirms the greater volatility of T-Bonds. This figure also indicates the increase in volatility of both instruments in 2007 and 2008, which coincides with the global financial crisis.

[7] The returns and variance values are not directly comparable to those from the studies of Ibbotson and Sinquefield (1979) and Carleton and Lakonishok (1985) in terms of magnitude because of the higher frequency of data used in this study and because of the use of holding period returns as opposed to quoted yields.

Figure 2-3: U.S. T-Bond and T-Bill Returns 1996-2008



The finding regarding the statistically significant variation in T-Bond returns contradicts various texts which propose the use of this security as a proxy for the risk-free rate because of its lower variation (Harrington, 1987; Brigham and Ehrhardt, 2005). However, in lieu of the discussion in Section 2.2.3, these suggestions perhaps refer to the fact that the variations in this instrument's yield are inconsequential if the security is held to maturity, as the coupon rate promised at issuance is earned and not the yield in the market, rather than its actual variation being lower.

2.3.4 Interest Rate Risk

The fact that U.S. T-Bonds exhibit variability over time is, as mentioned, not of considerable concern if the maturity of the proxy chosen matches the maturity of the investment being analysed. However, if this variability in returns means that investors are provided with a risk premium to compensate them for the volatility, then this contravenes the requirements for a risk-free asset. Strydom and Charteris (2009: 23-24) estimated a GARCH-M (1,1) model (Generalised Autoregressive Conditional Heteroscedasticity in Mean) for both U.S. T-Bills and T-Bonds over the period 1996 to 2008 to determine if a premium was included in the returns earned on these securities to compensate investors for volatility.

In the conditional mean equation for U.S. T-Bills, the risk premium was statistically insignificant as the null hypothesis that this value was equal to zero could only be rejected with ten percent confidence (p-value of 0.8970). The model estimated for U.S. T-Bonds also revealed a statistically insignificant coefficient for the risk premium (p value of 95.52 percent); therefore indicating that although these instruments do exhibit fluctuations in returns over time, no compensation for this movement is provided to investors holding the long-term instruments. Strydom and Charteris (2009: 20-25) thus proved that both U.S. T-Bills and T-Bonds adhere to the risk-free asset requirement of zero interest rate risk over the investment horizon.

2.3.5 Covariance with the Market

2.3.5.1 Original Studies

Similarly to the analysis of the extent to which both T-Bills and T-Bonds fulfil the constraint of zero variation over time, the degree to which these instruments satisfy the requirement of zero covariance with the market can be determined empirically. The covariance and correlation values computed in the Ibbotson and Sinquefield (1979) study are presented in Table 2-4. The covariance was not calculated by Carleton and Lakonishok (1985) in their study and insufficient information was presented in order to estimate it. The values shown indicate that T-Bonds do not move as closely with the market as T-Bills; but again, it is difficult to interpret these values in light of the risk-free asset criterion without tests of the statistical significance of the covariance or correlation estimates.

Table 2-4: Covariance Statistics for U.S. Treasury Securities 1926-1976

	T-Bills	T-Bonds
Covariance	-10.5	6.19
Correlation	-0.21	0.05
Z Test	-1.53	0.35

(Adapted from: Ibbotson and Sinquefield, 1979: 43)

Rather than testing the covariance, the correlation between the risk-free rate and market portfolio proxies was examined, following Freund and Perles (2003: 455-457), because of the difficulty in determining the distribution for the covariance measurement. The test statistic used for this purpose (which follows the normal distribution) and the hypothesis tested are depicted in equations 2.9 and 2.10 (on the following page). The statistics computed are also shown in Table 2-4, and provide evidence that the null hypothesis that the correlation coefficients were equal to zero could not be rejected at the ten percent significance level for both T-Bills and T-Bonds (critical value of 1.58); that is, both securities satisfy the theoretical requirement of zero correlation with the market.

$$Z \text{ statistic} = ((n-3)^{0.5} / 2) * \ln \{[(1+r)(1-\rho)] / [(1-r)(1+\rho)]\} \quad (2.9)$$

H_0: $\rho = 0$
H_1: $\rho \neq 0$ (2.10)

Where:
- r is the estimate of the sample correlation coefficient
- ρ is the hypothesised population correlation coefficient

(Freund and Perles, 2003: 456)

2.3.5.2 More Recent Empirical Evidence

In conjunction with the analysis of the variation in returns of T-Bills and T-Bonds for the period 1996 to 2008 in Section 2.3.3.2, the covariance and correlation of these securities with the returns on a market portfolio proxy were also calculated (as per equations 2.2 and 2.3 but adjusted for the use of historical data in the same way as the variance equation). For the purposes of this analysis, the Standard and Poor's top 500 share index (S&P500) was used to represent the market portfolio. However, monthly rather than daily data was preferred because of the way in which the test statistic is computed, where the number of observations has a substantial influence on the statistic calculated. The results of this analysis and the statistical tests thereon are tabulated in Table 2-5.

Table 2-5: Covariance Statistics for U.S. Treasury Securities 1996-2008

	T-Bills	T-Bonds
Covariance	-0.04	-0.91
Correlation	-0.14	-0.09
Z Statistic	-1.77	-1.1

* The critical values from the normal distribution are 1.64, 1.96 and 2.6 for the ten, five and one percent significance levels respectively.

The results follow the same patterns identified by Ibbotson and Sinquefield (1979), as the correlation of T-Bonds with the market was smaller than for T-Bills (although the sign of the relationship of T-Bonds with the market was opposite). However, in contrast to the analysis of earlier periods, the correlation of T-Bills with the market was statistically significant at the ten percent level (but not any lower); suggesting that there is some evidence that the returns do move in tandem with the returns on the market. These statistical tests therefore suggest that the co-movement of T-Bills with the share market is potentially a source of risk for the investor as a consequence of the need to reinvest over the life of the project. For T-Bonds, even if the security has to be liquidated prior to maturity or rolled over, the movement of this asset with the market will not represent a substantial source of risk for the holder.

2.3.6 Default Risk

As discussed in Section 2.2.6, theoretically, a risk-free asset should be free of default risk. Generally it is assumed that an asset backed by the government meets this requirement (Damodaran, 2001: 3) but there is some evidence to suggest that not even U.S. T-Bills, which have the highest possible credit rating, are entirely devoid of default risk. The default of the Treasury in 1979 and the threatened default in 1995/1996 have been analysed by Zivney and Marcus (1989) and Nippani *et al* (2001) respectively; providing insight as to whether default premia are included in the yields of these instruments, the relationship between the default premium and the time to maturity, and the impact of default on the yields of other securities.

In 1979, the U.S. Treasury was unable to fulfil its T-Bill obligations that matured on the 26th April, the 3rd and 10th of May to the combined value of $122 million, as a consequence of a number of factors (Zivney and Marcus, 1989: 475-476). Zivney and Marcus (1989: 476) estimated that the average delay of ten days in payment amounted to more than $325 000 in lost interest for the holders. Although the Treasury later compensated many of these investors for this loss so as to preserve their integrity as a ‘riskless’ institution, an analysis of interest rate movements around the time of the default revealed a substantial increase the day after the default announcement (Zivney and Marcus, 1989: 477). Thus, Zivney and Marcus (1989: 479) tested the hypothesis that the default led to a permanent increase in interest rates. The results of these tests revealed that the default was associated with an upward ratchet in both T-Bill yields and other interest rates, as the spread between T-Bills and other instruments remained at their pre-default level (Zivney and Marcus, 1989: 479-487). This increase in the general level of interest rates shows that government securities were perceived as more risky following the default and thus investors required a greater return.

Nippani *et al* (2001: 263) examined the validity of the use of T-Bills as a proxy for the risk-free asset over the period 1995-1996, when the U.S. Treasury threatened to default on their T-Bill commitments as a result of the inability of Congress and the President to agree to an increase in the Treasury’s debt limit. However, Congress eventually passed the Debt Extension Bill in March 1996, thereby avoiding default (Nippani *et al*, 2001: 264). Similarly to Zivney and Marcus (1989), Nippani *et al* (2001: 254-263) assessed whether the market perceived Treasury debt as more risky, and thus demanded a higher return as a consequence of the threatened default. They focused on examining the spread between commercial paper and T-Bills, as the spread represents the potential default risk of commercial paper; hence if the market perceives the default of T-Bills as more likely, the spread should decrease (Nippani *et al*, 2001: 254). A comparison of the pre-event spread and the spread at the time of the announcement showed that the yield on T-Bills increased causing a reduction in the spread (Nippani *et al*, 2001: 255-257). However, in determining whether the chain of events had a sustained effect on yields, the results were mixed (Nippani *et al*, 2001: 257-258). One of the tests found a persistent increase in the three month T-Bill yield, but not the six month yield; whilst the other tests supported the conclusion that there were no long-term effects (Nippani *et al*, 2001: 263).

There are several possible explanations for the varying results obtained by Zivney and Marcus (1989) and Nippani *et al* (2001); the foremost being that the threat of default in 1995/1996 did not have a sustained impact on the T-Bill yield as the events in 1979 did because the Treasury managed to avoid default over this period. Moreover, the differences in the results also illustrate that whilst the actual default led to the increase in the general level of all interest rates, thus leaving spreads unchanged; the threat of default impacted only on T-Bill yields. The results of these empirical examinations therefore indicate that in general, U.S. T-Bills are priced as if they are default-free and only under special circumstances do the yields include a default risk premium (Nippani *et al*, 2001: 263).

Another feasible explanation for the findings of Nippani *et al* (2001), especially that the threat of default appeared to have a sustained effect on the three month T-Bill yield and not the six month yield, is that the spread between these instruments may already include a default risk premium. This idea is supported by the results of Kamara’s (1988: 367) study in which he found that forward rates implied in secondary market T-Bill yields contain a premium for the risk that short-sellers will default. According to Kamara (1994: 1681), the implication of this is that the spreads between long and short-term T-Bill yields contain a premium for default. The logical extension of these findings is that other Treasury securities, such as T-Bonds, which have longer periods to maturity than T-Bills will thus include a more significant default premium. This argument that the shorter the maturity, the safer the investment, has been advocated since Guthmann (1929), Macaulay (1938) and Robinson (1960) (quoted in Johnson, 1967: 317). Robinson (1960) in fact confirmed this empirically by showing through an analysis of Aaa rated municipal bond yields that investors considered default risk an increasing function of time (quoted in Johnson, 1967: 318-319).

This idea that the longer the maturity of the asset the greater the default risk is based on the perception that there is greater uncertainty as to whether a government will be able to meet its commitments in ten years as opposed to in three months as a consequence of business cycles, government policies and various other uncertain factors (Pike and Neale, 2006: 250). In contrast, some authors have postulated that in fact long-term securities have lower default risk premia than short-term securities. Johnson (1967: 323-328) explained this viewpoint through the development of the maturity crisis hypothesis that default premia may be higher for debt of a short-term nature if investors believe that the borrower's inability to satisfy its obligations may be resolved in the future. In fact, this argument can be used to explain the results of Nippani *et al* (2001: 262) that the yield of the three-month T-Bill adjusted to the threat of default, but the six-month yield did not; thereby suggesting that investor's believed that the Treasury's inability to repay its debt obligations would be resolved in the future.

These studies thus highlight that, in general, U.S. government securities can be considered to satisfy the risk-free criterion of being devoid of default risk. However, there appears to be no conclusive evidence as to whether T-Bills, being short-term in nature, or T-Bonds, being longer-term instruments, more closely fulfil this condition.

2.3.7 Liquidity Risk

The effects of liquidity risk and the potential inclusion of a liquidity premium in the yield of a security are difficult to isolate. Short-term securities are generally more liquid than longer-term instruments, as they can be converted to cash more easily without the risk of losing capital value and hence investors generally prefer to lend for short periods in time (Blake, 2000: 83). Most borrowers however, prefer to issue long-term securities rather than having to roll over investments, and in so doing, exposing themselves to fluctuations in the interest rate (Blake, 2000:83). To entice investors to purchase securities of a longer-term nature, borrowers generally offer investors a liquidity premium to hold longer-term securities; thus the yields on long-term instruments tend to be greater than on short-term securities.

In order to ascertain the relationship between the yields on long-term and short-term securities, the yield curve is plotted, which graphs the return on a security against its time to maturity (Bodie *et al*, 2003: 322). Government securities are employed for this purpose as the risk level across the securities is reasonably constant (although default risk and inflation risk may vary depending on the maturity). For the theoretical reasons given, the yield curve should be upward sloping; any other shape is simply a temporary aberration (Pike and Neale, 2006: 250; Kelly, 1993: 125). Thus, an examination of the actual yield curve of the U.S. provides an indication as to whether T-Bills more closely resemble the risk-free criterion in this regard than T-Bonds.

Shiller and McCulloch (1987) and Campbell (1995) both showed that in general, the U.S. yield curve has conformed to the theoretical representation of being upward sloping, although there were periods where the yields on T-Bills exceeded the returns on long-term bonds. Ibbotson and Sinquefield (1979: 43) determined that over the period 1926 – 1976, the average maturity premium between T-Bonds and T-Bills was 0.7 percent based on geometric returns and 0.9 percent calculated using arithmetic returns. Campbell and Viceira (2001: 106) also confirmed empirically that positive liquidity risk premia existed on long-term government bonds compared to short-term bonds over the period 1952 to 1996.

In light of these results, T-Bills appear to more closely satisfy the theoretical checklist in terms of zero liquidity risk; this is reinforced by Reilly and Brown (2006: 22) who suggest that T-Bills exhibit "…almost no liquidity risk…". However, given that investors using the CAPM to make long-term investment decisions are likely to require compensation for the liquidity risk of tying up their funds for an extended period of time, it is arguable that incorporating an appropriate liquidity risk premium in the risk-free rate is justified. A further implication, however, is that it would then become essential to match the maturity of the risk-free proxy with that of the investment horizon in order to ensure the use of a suitable liquidity premium.

2.3.8 Currency Risk

Currency risk, as defined in Section 2.2.8, implies that no compensation is provided to investors, in the form of a premium, for fluctuations in the domestic currency when examining projects denominated in the local currency. It is widely considered across a number of sources that neither U.S. T-Bills nor T-Bonds include compensation for unexpected movements in the domestic currency and therefore conform to this risk-free asset criterion. Adler and Dumas (1984: 45-46) identified that U.S. T-Bills and T-Bonds are not influenced by fluctuations in the exchange rate, provided that investors perceive these fluctuations to be random (that is, not influenced by changes in purchasing power). The reason for this is because the yields of these securities are guaranteed over the life of the assets (Adler and Dumas, 1984: 46). The implication of this, in the context of identifying an asset to be used as a proxy for the risk-free rate in the CAPM, is that using a short-term security, which has to be rolled over for the duration of the project may expose the holder to currency risk caused primarily by changes in relative inflation rates (Adler and Dumas, 1984: 44); however, this can be avoided by holding T-Bonds that match the investment horizon

Domowitz, Glen and Madhavan (1998: 202) examined the currency risk associated with debt instruments of the Mexican government. They measured the currency risk premium in the yields of government securities by comparing peso-denominated bonds to U.S. dollar denominated securities also issued by the Mexican government, of identical maturity; as the yields on the U.S. dollar denominated securities do not include a premium for currency risk. In this way, all other characteristics of the bonds are identical expect for the currency in which they are denominated, allowing for a direct estimate of the currency premium to be computed (Domowitz *et al*, 1998: 193). This identical approach is adopted by Peter and Grandes (2005: 71) in examining the currency risk premium in South Africa, by comparing the yields on South African government dollar and rand denominated bonds (as will be discussed in Section 2.4.8).

It is therefore apparent that U.S. T-Bills and T-Bonds are considered to satisfy the theoretical requirement for a risk-free asset of zero currency risk, provided that the maturity of the instrument matches the investment horizon.

2.4 The Suitability of Risk-Free Proxies in South Africa

The examination of U.S. T-Bills and T-Bonds in the preceding sections revealed that these instruments provide a reasonable proxy for the risk-free rate; but that they are certainly not beyond reproach. In light of these observations of the apparent shortcomings of American proxies, it is of even greater necessity to consider the extent to which the most commonly employed South African equivalents (T-Bills, T-Bonds, BAs and NCDs) satisfy these requirements, and hence the validity of their continued usage in the CAPM.

2.4.1 The Pure Interest Rate

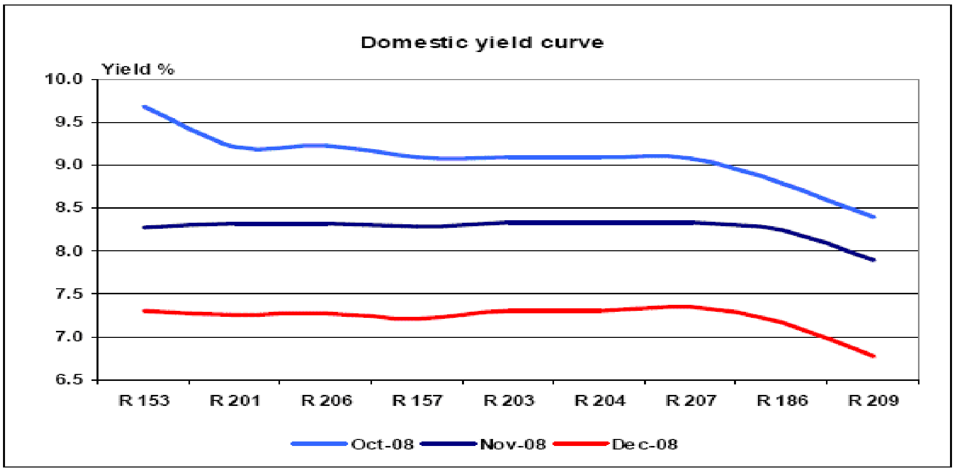
In Section 2.2.1, reference was made to Sharpe's (1964: 434) description of the risk-free yield as the "…pure interest rate". The U.S. proxies used for the risk-free rate, as was duly noted, do not satisfy this requirement, as the yields on these securities are influenced by government actions. In South Africa, the role of the government has historically been even more pronounced, especially in the T-Bill market. Morgenrood (1987, 1988a, 1988b), quoted in Firer and McLeod (1999: 6), in an analysis of T-Bills over the one hundred year period culminating in 1981, found that T-Bills could not be considered as a suitable proxy for the risk-free rate because of the government's "…enduring official reluctance to refrain from intervening in the financial markets…". In the latter part of this period, this problem was intensified by the introduction of the Banks Act in 1965 (Firer and McLeod, 1999: 6). This legislation required financial institutions to hold various types of assets that matched the liquidity of their liabilities (Firer and McLeod, 1999: 6). Both T-Bills and BAs were ranked as a liquid asset in terms of this legislation resulting in artificial demand for the instrument from banks; increased demand contributed to an increase in the price of these instruments and a subsequent decrease in their yields. The De Kock Commission of Enquiry (1985), described in Firer and McLeod (1999: 6), confirmed this, arguing that "…the South African Treasury Bill market was neither free nor competitive…" and therefore failed to produce realistic, market-related interest rates.

NCDs in contrast have only ever ranked as liquid assets or prescribed assets for pension funds and life assurance companies for short periods of time (Firer and McLeod, 1999: 6); hence they represent a more realistic market-determined rate. It is for this reason that Firer and McLeod (1999: 7) advocate this instrument as the most suitable proxy for the risk-free rate in South Africa. However, Firer and McLeod (1999: 6) indirectly suggest that it is the less liquid nature of the NCD which has prevented it from being listed as a liquid asset in terms of the required holdings of banks and other institutions. Yet, as has been discussed, a risk-free asset should be a liquid security and thus the use of the NCD as a proxy appears to contradict this requirement.

Whilst this historical analysis of the T-Bill, BA and NCD markets is not entirely appropriate for the estimation of the risk-free rate for future applications, it does provide some explanation as to why several scholars, such as those mentioned in Section 2.1.4, have favoured other instruments for this purpose when part of their historical analysis covered this period in time. More importantly, however, is the fact that T-Bills are still listed as a liquid asset in the Bank's Amendment Act of 1999 (SARB, 2003: 9); however, BAs have been removed from this list. This infers that there is still artificial demand for T-Bills, and that the yield on this security is not necessarily an accurate reflection of a pure interest rate. Although the removal of BAs from this list may be for other unrelated reasons, it may also infer that it is not perceived to be as liquid as the other securities on the list, which are all government or government institution securities; thus potentially highlighting the greater liquidity risk of this security.

The final source of influence on interest rates in South Africa is the government's monetary policy, in which their expressed aim is to maintain a stable inflation rate by adjusting interest rates (SARB, 2002: 2). This point is clearly illustrated in Figure 2-4 (on the following page) which shows the South African yield curve (based on long-term government bonds) for the last three months of 2008. The fall in the yield curve over this period is a direct consequence of government decisions to reduce the interest rate; thereby showing that the returns on these instruments do not only reflect the time preference of individuals for the consumption of income and the investment opportunities available in the economy; in contravention of the criterion of a pure interest rate.

Figure 2-4: South African Yield Curve October-December 2008

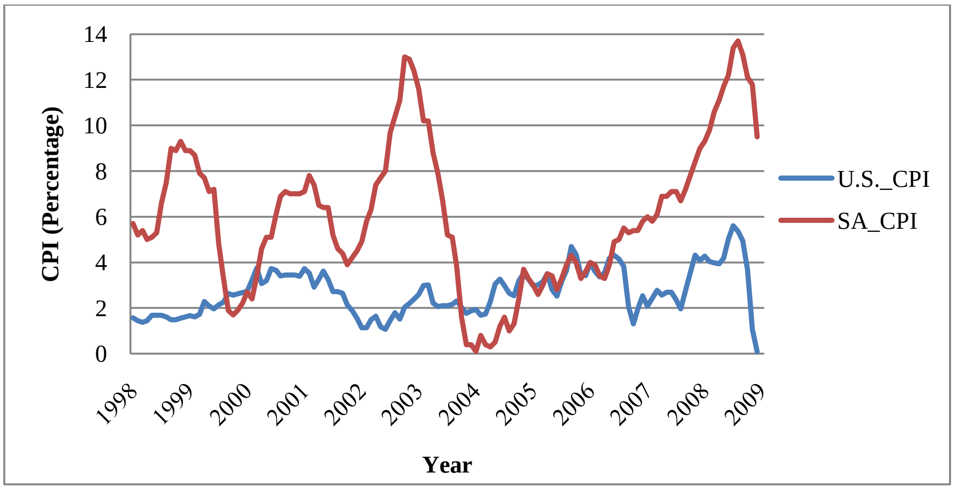


(Source: BESA Quarterly Report, 2009a: 4)

2.4.2 Inflation Risk

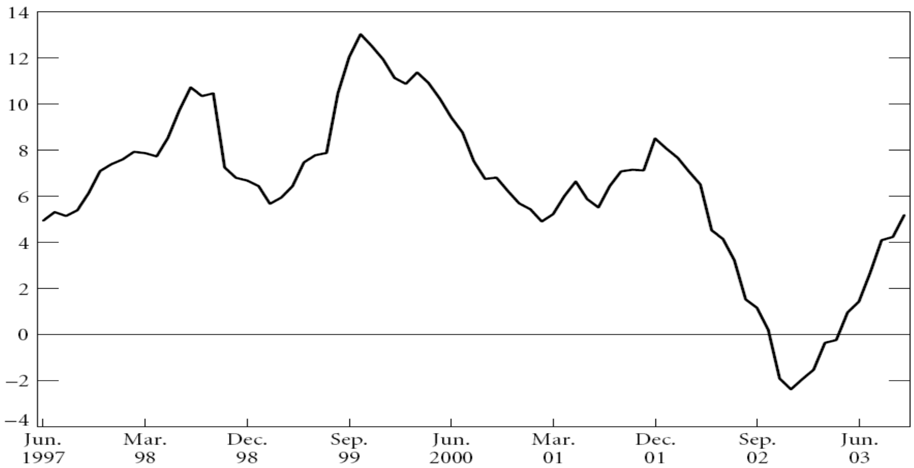
As discussed in Section 2.2.2, the risk-free rate should include a premium for expected inflation but should be free of inflation risk. The examination of the U.S. revealed that whilst there is some variability in inflation over time, over the past eleven years this movement has been relatively small. An identical analysis was conducted for South Africa using monthly CPI information gathered from the SARB for 1998 to 2008. As shown in Figure 2-5, inflation in South Africa has generally been higher than in the U.S., and considerably more volatile. To confirm this, a chi-squared test of the statistical significance of the variance of the South African CPI was conducted (using 10^{-3}); the results of which confirm that the variance was statistically significant as the chi-squared test statistic computed (1389) was greater than the critical value (172 at one percent).

Figure 2-5: U.S. and South African Consumer Price Index 1998-2008



It therefore follows that while inflation risk for U.S. securities is likely to be relatively low, South African securities are exposed to greater inflation risk, thereby reducing their suitability to act as a risk-free rate proxy. This uncertainty to which South African investors are exposed is confirmed by Ahmed and Ricci (2004: 2), who illustrate the extensive fluctuations in the real returns earned on long-term government bonds in South Africa over the period June 1997 to December 2003, as shown in Figure 2-6. The period of negative real returns corresponds to the period of high inflation revealing that the inflation premium did not fully compensate the investor for the actual changes in purchasing power experienced over the period.

Figure 2-6: South African Long-Term Real Interest Rates June 1997-December 2003



(Source: Ahmed and Ricci, 2004: 2)

2.4.3 Variance in Returns

The third requirement that the security chosen as the risk-free asset in South Africa must fulfil is that of zero variation over the investment period. An examination of South African finance literature revealed that no explicit research on T-Bills, T-Bonds, BAs or NCDs has been conducted in this regard. Consequently, an identical examination to the updated U.S. analysis was undertaken for the four most common South African proxies, with daily data collected from the SARB and Econstats[8]. The daily returns earned on T-Bills, BAs and NCDs were computed in the same way as for U.S. T-Bills (as per equation 2.6); the only difference being that the South Africa securities are priced based on 360 days compared to 365 days (Botha, 2006: 240). However, South African T-Bonds are not valued in the same way as the U.S.

[8] The R157 (coupon rate of 13.5 percent) was selected to represent the T-Bond, despite the evidence presented by Correia and Cramer (2008: 41), discussed in Section 2.1.4, that practitioners have generally favoured the R153 because of its slightly shorter maturity (it expires in 2010 compared to 2015). However, for the latter years of the period under analysis the maturity of the R153 would be shorter than that normally associated with a T-Bond (greater than three years), which is not the case for the R157; hence the reason it was chosen.

equivalents, as they are issued with a fixed semi-annual coupon as opposed to the zero-coupon rate associated with the U.S. T-Bonds. The bond valuation formula applicable for this purpose is shown in equation 2.11.

$$\text{SA Bond } P_t = \text{Coupon} \times [1-(1+ Y_t)^m]/Y_t + F/(1+ Y_t)^m \tag{2.11}$$

(Firer, Ross, Westerfield and Jordan, 2008: 192)

The returns and variation were computed for the period 1996-2008 and are shown in Table 2-6. A comparison of the variance estimates reveals that T-Bonds in South Africa exhibit by far the greatest daily fluctuations in returns. This is confirmed by the statistically significant chi-squared statistic which is much greater than the critical values based on infinite degrees of freedom. Similarly to the U.S., the variation of T-Bill returns was substantially lower than for T-Bonds; however, the chi-squared statistics show that this variation was still statistically significant in South Africa. Thus, neither T-Bills nor T-Bonds in South Africa adhere to the requirement of zero variation. This is of greatest concern for T-Bills as the need to roll-over this investment creates reinvestment risk that would not be associated with T-Bonds, assuming the maturity of the instrument matches the duration of the project being analysed with the CAPM.

Table 2-6: Return and Variance Analysis for South African Proxies 1996-2008

	T-Bills	**T-Bonds**	**BAs**	**NCDs**
Arithmetic Return (%)	0.000263	0.0136	0.00121	0.00125
Geometric Return (%)	0.00026	0.0107	0.00118	0.00118
Variance (%^{2})	0.00069	0.587	0.00739	0.0115
Standard Deviation (%)	0.026	0.766	0.0859	0.1074
Chi Squared Statistic	2344	1991449	25074	39140

* Chi Squared critical values at the ten, five and one percent significance levels are 1162, 1179 and 1213 respectively.

Interestingly, this analysis also reveals that neither BAs nor NCDs provide any better estimate of the risk-free rate in South Africa than T-Bills or T-Bonds, and in fact exhibit greater variation in returns than T-Bills. Given their short-term nature this greater variation will exacerbate the reinvestment risk if they are used as the proxy in the CAPM.

A graph depicting the stationary returns of the four South African securities is shown in Figure 2-7 (on the following page), which indicates the significantly greater volatility associated with the monthly T-Bond returns compared to T-Bills, BAs and NCDs. This finding that both U.S. and South African T-Bonds exhibit greater volatility in returns than the short-term securities is consistent with theory that the longer the time to maturity, the greater the influence of changes in interest rates on the price of the bond (and thus the returns earned) (Firer *et al*, 2008: 194). However, as highlighted in Section 2.2.3, this volatility is largely irrelevant for the holder of the

T-Bond if the instrument is held to maturity. In Figure 2-8 only the three short-term securities are illustrated and this confirms the results of the statistical tests that these three instruments do exhibit significant variation over time. Importantly this graph also reveals that the two private sector short-term instruments follow the movements of the T-Bill quite closely, although they do exhibit greater volatility than the T-Bill returns. It is also clear from Figure 2-8 that the volatility of these instruments has declined over time in accordance with the increased stability in South Africa. Moreover, the fact that neither Figure 2-7 nor Figure 2-8 reflects the same trend as the graph illustrated for the U.S., shown in Figure 2-3 (page 32), as the volatility has not increased in 2007 and 2008; confirms that the effects of the financial crisis have thus far been less pronounced in South Africa than in the U.S. It is important to note that the reason Figures 2-7 and 2-8 appear to show less volatility than Figure 2-3 is because of the different scales on the vertical axis, with the latter only ranging between positive and negative 0.4 percent, whereas for the South African graphs, the scale ranges between 8 and -12 percent.

Figure 2-7: South African Risk-Free Proxies 1996-2008

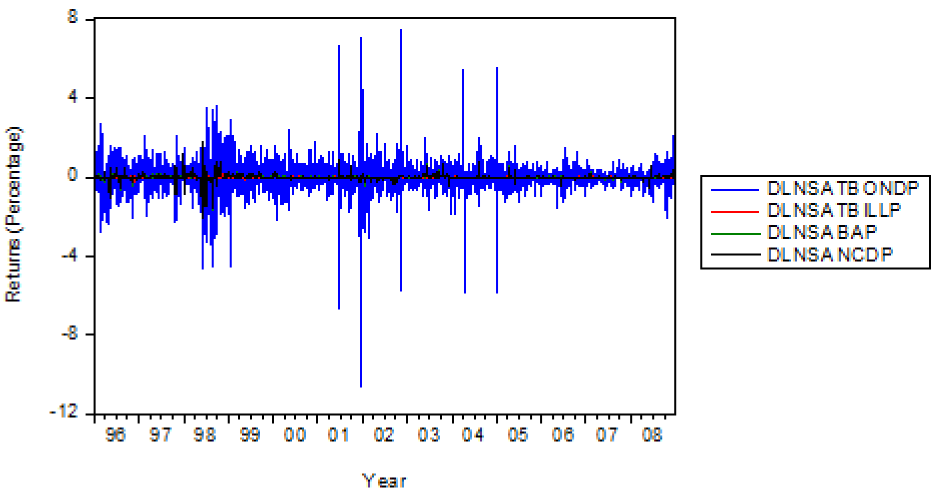
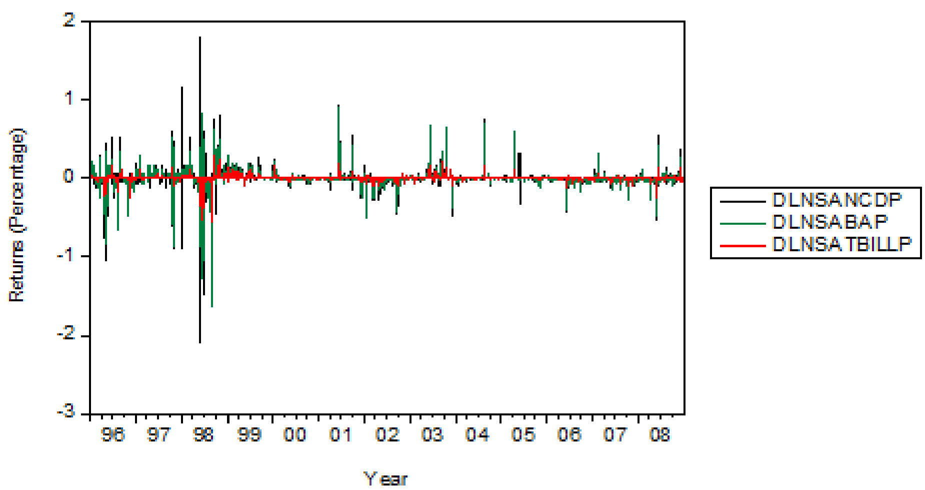


Figure 2-8: South African Short-Term Risk-Free Proxies 1996-2008



2.4.4 Interest Rate Risk

In light of this evidence that both short-term and long-term securities in South Africa exhibit statistically significant variation in returns, it is clear that investors are exposed to interest rate risk. The question that this therefore raises is whether the return offered on the security includes compensation for this volatility. Strydom and Charteris (2009: 24-26), referred to in Section 2.3.4, also examined the South African risk-free proxies using a GARCH-M model, to determine whether the returns earned on these securities include a volatility risk premium.

For T-Bills, the coefficient on the risk-premium term was highly statistically significant (p value equal to zero); thereby providing evidence that the returns offered on South African T-Bills include a premium to compensate investors for the risk inherent in these instruments caused by volatility over time (Strydom and Charteris, 2009: 24). The same conclusions were drawn for BAs and NCDs. For T-Bonds, however, the evidence with regards to the presence of a volatility premium was mixed, as the highest level at which the null hypothesis that the risk premium coefficient was zero could be rejected was 16.13 percent (Strydom and Charteris, 2009: 26). Thus, whilst this p value lies above the conventional significance levels, there was still some evidence to suggest that the inclusion of a volatility premium in the R157 bond yields could not be completely rejected.

The fact that the volatility premium is smaller in T-Bond than T-Bill returns in South Africa is somewhat at odds with the results obtained in the analysis of variance in the preceding section, which revealed the much greater volatility in the long-term instrument. A possible explanation for this is the different perspectives of investors purchasing securities of varying maturities. Investors purchasing longer-term T-Bonds generally have a long-term investment horizon and therefore short-term fluctuations in returns are not likely to have an impact on their wealth position. Consequently, because these investors are not necessarily concerned about this variability, they will not demand compensation for it in the returns earned on the instrument. Although the wealth position of short-term investors in T-Bills is guaranteed once the security is purchased, their ability to reinvest in the following period at the same rate depends on the movement in interest rates, and in a volatile market they are thus exposed to substantial risk from this source, for which they require compensation in the form of a volatility risk premium.

2.4.5 Covariance with the Market

In conjunction with the calculation of the returns and variance of the four proxies in South Africa, the correlation of each of these instrument's returns with the ALSI (used as a surrogate for the market portfolio) and the statistical significance thereof were also computed. The results are shown in Table 2-7. All four test statistics are greater than the critical values indicating that the movement of the returns of these instruments with the market are statistically significant. Thus these securities fail to satisfy the risk-free criterion of zero co-movement with the market, with particular concern for the three short-term assets where the statistically significant correlation will increase the reinvestment risk faced by an investor.

Table 2-7: Covariance Statistics for South Africa Proxies 1996-2008

	T-Bills	T-Bonds	BAs	NCDs
Covariance	0.21	4.73	1.08	1.31
Correlation	0.25	0.32	0.28	0.35
Z Statistic	3.2	4.09	3.62	4.51

* Normal distribution critical values at the ten, five and one percent significance levels for a two-tailed test are 1.64, 1.96 and 2.6 respectively.

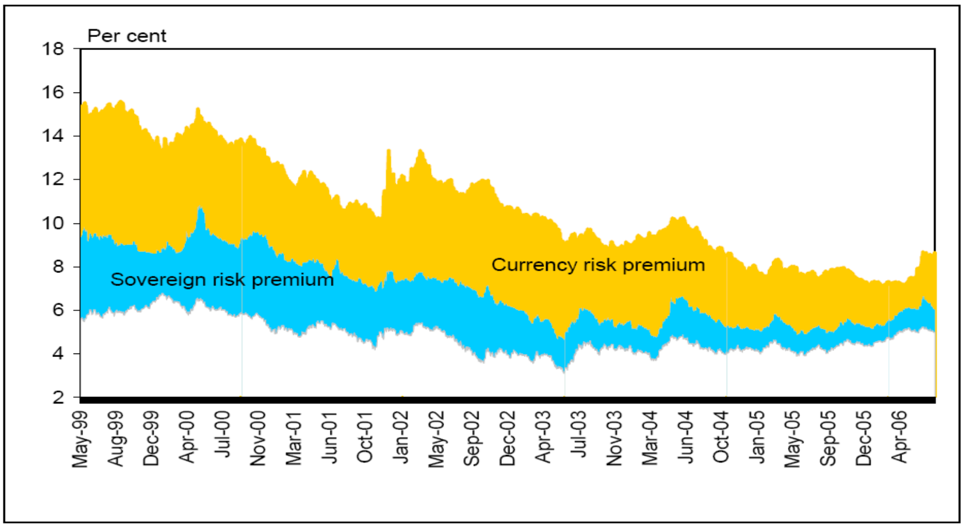
These findings are in complete contrast to the U.S., where the movement of T-Bonds with the market was found to be insignificant, whilst there was some evidence that T-Bills do move with the market. Moreover, the trend observed in the U.S. that T-Bonds exhibited the smaller correlation was not replicated in South Africa. The other notable difference was that where the correlation figures are positive for South African securities, they were negative for the U.S. securities. The reason for this may be that during downturns in the stock market, investors in the U.S. move out of equities and into treasury securities; whilst investors in the South African share market engage in disinvestment on a much wider front and favour securities of countries which are perceived to be of greater safety. The reversal of this simple relationship between equity and bonds highlights the dissimilarity between the two markets and in so doing, confirms the premise alluded to in Section 1.1.1 that practices appropriate in the U.S. are not necessarily appropriate in South Africa because of the differences between the countries.

2.4.6 Default Risk

2.4.6.1 Government Securities

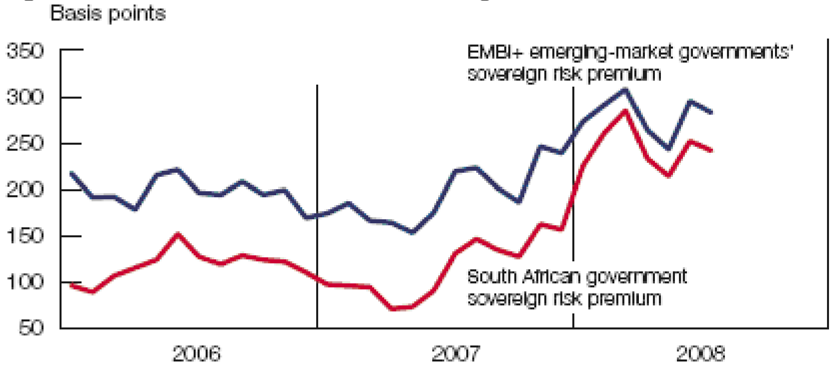
Government issued securities are generally considered to meet the requirement of zero default risk, but in the case of emerging markets a sizable default risk premium exists (Robinson; 2007: 5). Peter and Grandes (2005: 71) estimated the default risk premium (measured by the sovereign spread between South African and U.S. treasury yields) for South African government securities at approximately forty-five basis points for the period June 1997 to January 2005. Robinson (2007: 6) calculated a larger premium for the period 1999-2006, fluctuating from approximately four percent in 1999 to one percent in 2005 and 2006; this is shown in Figure 2-9. This reduction in the default premium signals an improvement in the perceived riskiness of the South African government (a consequence, according to Robinson (2007: 18), of effective and consistent fiscal policy). As depicted in Figure 2-10, the sovereign risk premium began to increase after 2006, from a low of 70 basis points in April 2007 to 285 basis points in March 2008 in response to declining yields in the U.S. and a ratings downgrade for South Africa from positive to stable (SARB, 2008: 60-61). In addition to the above findings, Peter and Grandes (2005: 50) reported that between October 1994 and May 2003, Moody's long-term credit rating on South African government bonds changed eight times (all positive).

Figure 2-9: Default and Currency Risk Premia in South Africa 1999-2006



(Source: Robinson, 2007: 6)

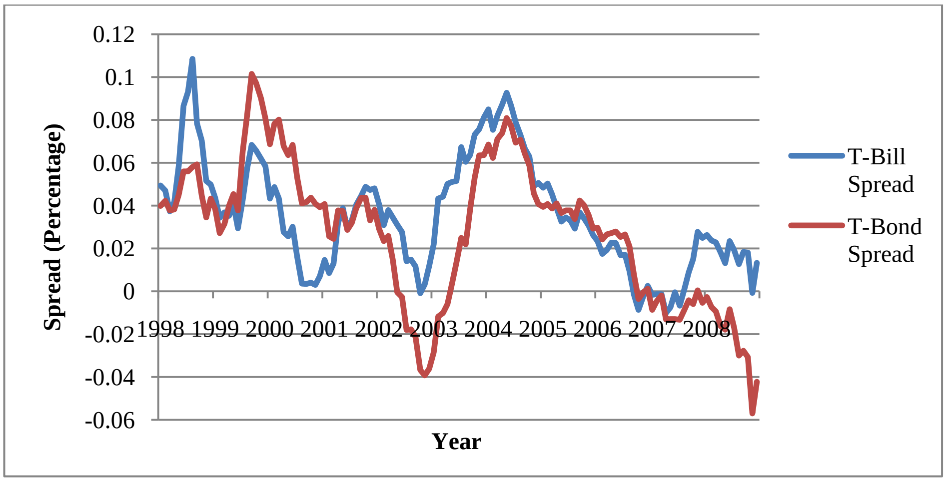
Figure 2-10: South Africa's Sovereign Risk Premium 2005-2008



(Source: SARB, 2008: 61)

In order to compare the sensitivity of T-Bills and T-Bonds to default risk the short-term sovereign risk premium was estimated by subtracting the real return on U.S. T-Bills from the real return on South African T-Bills, while the long-term premium was estimated by calculating the difference between the real return on U.S. Treasury Bonds and the R157 bond for the period 1998 to 2008. Figure 2.11 shows the result.

Figure 2-11: Sovereign Spread for T-Bills and T-Bonds 1998-2008



It can be seen that the sovereign spreads on T-Bills and T-Bonds have tended to move together, although short-run deviations did occur. It is also noticeable that in 2008 the long-term spread fell substantially below that of the short-term spread. A standard Z-test was conducted in order to test whether or not there was a significant difference in the means of these two spreads, as shown in equation 2.12. The null and alternative hypotheses are depicted in equation 2.13.

$$\text{Z-statistic} = [(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)] / (\sigma_1^2/n_1 + \sigma_2^2/n_2)^{0.5} \qquad (2.12)$$

$$H_0: \mu_1 = \mu_2 \text{ (which can be rewritten as } \mu_1 - \mu_2 = 0)$$
$$H_1: \mu_1 \neq \mu_2 \text{ (which can be rewritten as } \mu_1 - \mu_2 \neq 0) \qquad (2.13)$$

Where:

- $\bar{X}_1$ is the value of the T-Bill spread
- $\bar{X}_2$ is the value of the T-Bond spread
- μ_1 is the hypothesised value for the T-Bill spread
- μ_2 is the hypothesised value for the T-Bond spread
- n_1 is the number of observations in the T-Bill spread sample
- n_2 is the number of observations in the T-Bond spread sample

(Wegner, 1993: 230)

The statistic of 2.12 is less than the critical value at the one percent level of 2.58, but greater than the critical value at the five percent level of 1.96. Therefore at the five percent level the T-Bill spread is statistically significantly larger than the T-Bond spread; hence South African T-Bills appear to incorporate a more substantial sovereign risk premium compared to longer-term instruments. This finding that the shorter the time to maturity, the greater the default risk is consistent with the maturity crisis hypothesis developed by Johnson (1967), in which he argued that short-term securities may be exposed to greater default risk than longer-term securities because investors believe that the Treasury's inability to pay may be resolved in the future.

If the South African investment environment is inherently riskier than a market such as the U.S. then the inclusion of an appropriate risk premium in the risk-free rate for South African investments is arguably appropriate. However, the fluctuations in the premium are problematic as they imply that the appropriate return on investments are subject to change and that less reliance can be placed on CAPM valuations based on a proxy subject to such fluctuations.

2.4.6.2 Private Sector Securities

BAs and NCDs are issued by private sector banks and investors are thus, in theory, exposed to a greater probability of default than with government securities (Firer *et al*, 2008: 614). In South Africa, the institutions that issue these securities are large and reasonably stable companies and therefore are considered to be low in risk (Firer *et al*, 2008: 614). However, no security issued by any organisation can have a higher credit rating than the government, and thus the default premium included in these yields must be equal to or greater than the equivalent premium in government security yields. Peter and Grandes (2005: 37, 48) confirmed this in South Africa, as the long-term secured debt of major banks had a higher yield than government debt; not necessarily because of a greater possibility of default, but rather reflective of the default risk associated with the government, which places a cap on the credit rating of companies operating within the country.

Further to this, an important distinction between NCDs and BAs are that the former are unsecured short-term liabilities of the banks as opposed to the later which are guaranteed by a pledge of assets (Botha, 2006: 254-259). BAs are therefore safer instruments and this should be reflected by the yields offered on the respective securities. Based on the analysis conducted in Section 2.4.3, and the results thereof presented in Table 2-6 (page 43), over the past fourteen years, the returns on NCDs have exceeded the BA returns for securities with identical maturities, based on the arithmetic average.

Private sector securities in South Africa thus include a default premium of at least the same magnitude as government securities because no private institution can have a higher credit-rating than the government, even if they are perceived to be less risky. Accordingly, with respect to default risk there is no clear evidence that either BAs or NCDs provide a better proxy for the risk-free rate than T-Bills or T-Bonds in South Africa.

2.4.7 Liquidity Risk

U.S. securities are, in general, highly liquid instruments because of their perceived risk-free nature and are thus easily brought and sold in the secondary market at close to their true values (Fleming, 2003: 1). In so far as South African government securities are concerned, they do not exhibit the same levels of liquidity evinced in the U.S. market, as there are fewer market participants and the number of transactions are lower (Van Zyl, 2006: 302). If it is more difficult for an investor to sell either a T-Bill or T-Bond in South Africa compared to the U.S., there must be greater risk associated with both of these instruments compared to the American equivalents, as reflected by higher yields.

Compounding this is that historically T-Bonds in South Africa were reasonably illiquid instruments as a consequence of a lack of an active secondary market for these securities, as well as economic sanctions which resulted in little or no demand for the assets (Firer and McLeod, 1999: 4; Viviers *et al*, 2008: 46). With the demise of Apartheid and the consequent removal of trade sanctions, the bond market has become one of the most liquid in the emerging world (Van Zyl, 2006: 303); however, it certainly does not yet resemble the U.S. market. This is confirmed by comparing the average daily trading volumes on the bond markets in the U.S. and South Africa. The average daily trading volume in the U.S. for December 2008 was $704.8 billion (Securities Industry and Financial Markets Association, 2009); whilst in South Africa, the average daily trading volume over the same period was $5.94 billion[9]. The South African trading volume is less than one percent of the U.S. market on a daily basis; thus clearly confirming that in terms of liquidity the South African market is not comparable to the U.S. market. Therefore, South African securities are likely to include a liquidity premium to compensate investors for the greater difficulty associated with liquidating their position than in the U.S.

[9] The average monthly trading volume for South Africa in December 2008 was R1.3 trillion, including standard trades, repurchase agreements and offshore transactions (BESA, 2009a: 3-4). Based on 22 trading days per month, this equates to approximately R59 000 billion per day. The average Rand/Dollar exchange rate for the month of December was R9.95 (obtained from the SARB database) which means that the daily average trading amount was $5.9 billion for the South African bond market.

The issue of liquidity risk associated with BAs and NCDs was briefly discussed in Section 2.4.1, in examining the influence of government in the market for short-term securities. It was seen that NCDs have never been listed as a liquid security for banks whereas BAs have; suggesting that NCDs are less liquid. Firer *et al* (2008: 614) confirm this saying "…bankers' acceptances are regarded as the short-term securities with the lowest risk and highest liquidity issued by the private sector"; thus BAs more closely resemble the risk-free ideal than NCDs.

In terms of a comparison of T-Bills and BAs with respect to their exposure to liquidity risk, both securities are short-term in nature, traded frequently and earn similar returns which tend to trend together over time. Consequently, there does not appear to be any substantial evidence to suggest that one or other is more likely to include a larger liquidity premium.

2.4.8 Currency Risk

Peter and Grandes (2005: 71) and Robinson (2007: 5) examined several determinants of the yield spread between South African and U.S. government securities. The results of both of these studies indicate that the presence of a currency risk premium is one of the primary sources of the greater yields evinced in South Africa. Peter and Grandes (2005: 71) estimated the currency risk premium for South African securities over the period June 1997 to January 2005 as 784 basis points; accounting for more than 90 percent of the yield spread. Robinson's (2007: 5) analysis supports these findings, as the currency premium contributions to the spread ranged between 5 and 8 percent over that time period. This is illustrated in Figure 2-9 (page 47). Notably in the period following the study of Peter and Grandes (2005: 71) the currency risk premium decreased substantially to approximately 200 to 300 basis points in 2005 and 2006 (Robinson, 2007: 6).

This evidence thus proves that included in the yield used in the CAPM to represent the risk-free rate in South Africa is substantial compensation for a currency premium which is not relevant for South African investors. Thus, employing a proxy yield in the CAPM does not accurately represent the true risk-free return in South Africa.

2.4.9 The Suitability of Proxies in South Africa

This examination of risk-free proxies in South Africa has revealed that these securities, in general, do not adhere as closely to the theoretical requirements as their U.S. equivalents. Whether these deviations invalidate their use, however, is not clear. In light of these results it is necessary to consider an alternative approach to estimate the risk-free rate and compare whether these values or those of the proxies more closely approximate the rate which investors view as the minimum required return from investing.

2.5 Chapter Summary

An analysis of published South African financial literature and of surveys of industry practices was undertaken in this chapter and revealed that the U.S. method of estimating the risk-free rate using T-Bills and T-Bonds has largely been extrapolated and applied in South Africa; although there was some evidence to suggest that BAs and NCDs are also used by scholars for this purpose.

In order to assess the validity of using these public and private sector instruments to estimate the risk-free rate in South Africa, a set of theoretical requirements that an asset must satisfy to be considered a suitable proxy for this parameter were derived. The risk-free rate should represent a pure interest rate with zero inflation risk, variance, interest rate risk, covariance with the market, default risk, liquidity risk, and currency risk. An examination of the extent to which U.S. securities satisfy these requirements reflected digressions from the criteria; however, in comparison to the deviations identified for the South African proxies analysed, the deviations of the U.S. T-Bills and T-Bonds appeared small. The South African instruments were seen to include premia for currency, default, liquidity, inflation and interest rate risk, that the security yields exhibit considerable variability and co-movement with the market over time and are also influenced by government policies.

In light of these findings regarding the shortcomings of the proxy instruments in South Africa, it is necessary to consider an alternative approach to estimate this parameter. Black (1972) derived a different method to estimate the risk-free rate under the condition that a suitable proxy does not exist. The derivation of this parameter and its suitability will be discussed in Chapter 3.

CHAPTER 3
AN ALTERNATIVE APPROACH TO ESTIMATING THE RISK-FREE RATE

3.1 The Need for a Different Approach

The analysis in Section 2.4 revealed that the securities used to estimate the risk-free rate in South Africa digress significantly from the requirements established for the correct specification of the risk-free rate. But, highlighting the shortcomings of these instruments is of little value to either practitioners or scholars unless a viable alternative exists; the use of which must be justified both theoretically and empirically in the form of more reliable cost of equity estimates. This chapter will examine the alternative method derived by Black (1972) to estimate the risk-free rate, with particular emphasis on the empirical tests of the validity of this approach compared to the use of a risk-free rate proxy in the U.S.

3.2 The Derivation of the Zero-Beta CAPM

As a consequence of the results of several studies in the U.S. in the 1960s and early 1970s disputing the validity of the risk-free rate proxy approach to estimating the risk-free rate, and frequently the CAPM as well (such as Pratt, 1967 (quoted in Black, 1972: 445); Douglas, 1969; Friend and Blume, 1970), several scholars (Vasicek, 1971; Brennan, 1971; Black, 1972) reassessed the appropriateness of the assumption of the model that a risk-free asset exists, and that it is possible for investors to both borrow and lend at this rate. Whilst Vasicek (1971), Brennan (1971), and Black (1972) all derived models based on this broad subject area, the work of Black (1972) is primarily focused on in this study as his model provides a more generalised approach, which largely encapsulates the work of the other two scholars[10]. Black (1972) derived a model under identical conditions to the CAPM, but allowing only for investments in risky assets; with no assumption of the existence of a riskless asset, which was an essential component of the development of the CAPM as explained in Section 2.1.1. The assumption Black (1972: 446) did make, however, was that investors can take both long and short positions of any size in all risky assets. Black (1972: 446) acknowledged that the supposition of unrestricted short selling was not entirely realistic, but believed that restrictions thereon would not have a significant impact on the model derived.

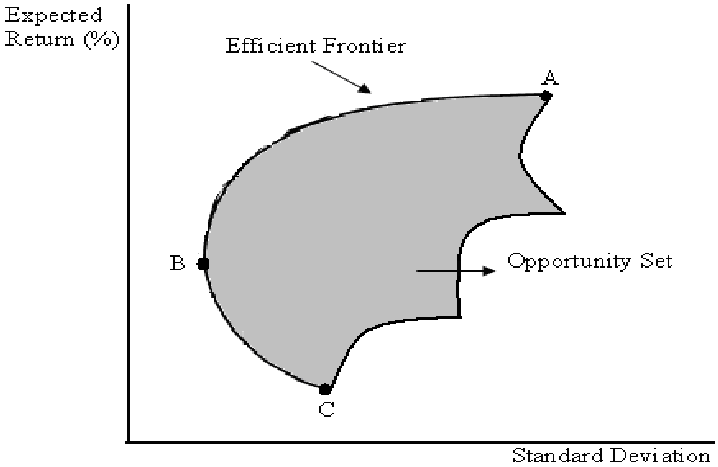
[10] Vasicek (1971) examined the appropriateness of the use of a risk-free asset when it is only possible to lend at this rate (which will be discussed in Section 3.2.5) and Brennan (1971) examined the influence of divergent borrowing and lending rates on the CAPM (which will be discussed in Section 3.3.4.6).

3.2.1 Determining the Minimum-Variance Zero-Beta Portfolio

3.2.1.1 Calculating Efficient Portfolios with Non-Unique Combinations

Similarly to the derivation of the CAPM, Black's (1972) model was based on Markowitz's (1952, 1959) portfolio theory. Markowitz (1952: 82) illustrated that all attainable investments can be plotted on a graph of the expected return of the investments against their risk, measured by standard deviation. An example of this is shown in Figure 3-1, with the shaded area representing the opportunity set of all attainable investments (Fama, 1976: 234). As explained in Section 2.1.1, Markowitz's (1952) theory shows that rational investors will only choose mean-variance efficient portfolios from the available possibilities; these portfolios plot along the boundary of the opportunity set from point A to point B, and the curve AB is thus known as the efficient frontier (Van Horne, 2002: 59).

Figure 3-1: The Efficient Frontier and Opportunity Set



(Source: Fama, 1976: 234)

Sharpe (1970: 59-69) illustrated that any mean-variance efficient portfolio can be described as the weighted combination of two basic portfolios, and by altering the weights of these two portfolios it is possible to derive the entire efficient frontier AB. The term basic means that the only constraint to the composition of the efficient portfolio is that the proportions invested must sum to one (Sharpe, 1970: 59). Black (1972: 447-448) reached the same conclusion as Sharpe (1970) regarding the composition of efficient portfolios via an empirical derivation, which provided the basis for the development of his adjusted CAPM. This is outlined below.

Investments that lie below B on the boundary of the opportunity set are not mean-variance efficient, as although they satisfy the minimum-variance constraint, they do not provide the maximum possible return for that level of risk; that is, there is another portfolio or investment on the efficient frontier which provides a higher return for the same level of risk. These portfolios however, in conjunction with efficient combinations, are referred to as minimum-variance portfolios[11] (Fama, 1976: 260). For an investor, k, to be able to obtain a minimum-variance efficient portfolio, the proportions invested in each of N assets (x_{ki}) must be selected so as to minimise the variance of the portfolio returns, subject to the constraints that the proportions invested in the individual assets must sum to one, and the expected return of the portfolio must equal the targeted return (Black, 1972: 447). Mathematically these conditions can be expressed as:

$$\text{Minimise: } \sigma^2_k = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ki}\, x_{kj}\, \sigma_{ij} \tag{3.1}$$

$$\text{Subject to: } E(R_k) = \sum_{j=1}^{N} x_{kj}\, E(R_j) \tag{3.2}$$

$$\sum_{j=1}^{N} x_{kj} = 1 \tag{3.3}$$

Where:

- $j = 1, 2, \ldots, N$
- σ^2_k is the variance of the returns of portfolio k
- x_{ki} and x_{kj} are the proportions invested in assets i and j respectively
- σ_{ij} is the covariance of assets i and j
- $E(R_k)$ is the expected return of portfolio k
- $E(R_j)$ is the expected return of asset j

(Black, 1972: 447)

Every point along the efficient frontier depicted in Figure 3-1 (page 54) is a minimum-variance portfolio and can thus be viewed as a solution to the equations 3.1-3.3, with each point corresponding to a unique expected return (Fama, 1976: 261-262). To minimise the variance subject to the constraints, Black (1972: 447) introduced Lagrangian multipliers (S_k and T_k) to include the constraints (3.2 and 3.3) into equation 3.1, as shown in equation 3.4 (on the following page). Thereafter, this equation was differentiated with respect to S_k, T_k and x_{kj}, and these partial derivatives were equated to zero (Black, 1972: 447). The first two derivatives reflect the imposed constraints (equations 3.2 and 3.3) and therefore are not shown, whilst the third (equation 3.5) is used to determine the proportions to be invested in each asset in an efficient portfolio, as can be seen in equation 3.6.

[11] An efficient portfolio must satisfy the minimum-variance constraint, but a minimum-variance portfolio is not necessarily efficient (Elton, Gruber, Brown and Goetzmann, 2007: 311).

$$\text{Minimise: } \sigma^2_k = \sum_{i=1}^{N}\sum_{j=1}^{N} x_{kj}x_{ki}\,\sigma_{ij} - 2S_k\left[\sum_{j=1}^{N} x_{kj}\,E(R_j) - E(R_k)\right] - 2T_k\left[\sum_{j=1}^{N} x_{kj} - 1\right] \quad (3.4)$$

$$\delta\sigma^2_k/\ \delta x_{kj} = 2\sum_{i=1}^{N} x_{ki}\,\sigma_{ij} - 2S_k\,E(R_j) - 2T_k$$

$$0 = 2\sum_{i=1}^{N} x_{ki}\,\sigma_{ij} - 2S_k\,E(R_j) - 2T_k \quad (3.5)$$

$$x_{ki} = S_k\sum_{i=1}^{N} D_{ij}\,E(R_j) + T_k\sum_{i=1}^{N} D_{ij} \quad (3.6)$$

Where:

- S_k and T_k are Lagrangian multipliers
- δ represents the derivative of the function
- D_{ij} is the inverse of the covariance matrix for assets i and j

(Black, 1972: 447-448)

According to Fama (1976: 280), by considering only those values of S_k and T_k that are consistent with the solutions to the problems stated in equations 3.1-3.3, it is possible, by varying S_k and T_k in equation 3.6, to ascertain the different values of the proportions of portfolios invested in all risky securities in minimum-variance portfolios. In this way, the weights invested in each security along the entire efficient frontier can be determined. To identify the different values of S_k and T_k it is necessary to consider their interpretation in equation 3.4. If the derivative of this equation is computed with respect to the portfolio's expected return, the solution is equal to the Lagrangian Multiplier, $S_{k,}$ as shown in equation 3.7. S_k thus represents the rate of change in the variance of the efficient portfolio with respect to a small change in the expected return (Fama, 1976: 265). The slope of the efficient frontier at point k is the change in the expected return of the portfolio with respect to a change in the minimum-value of the portfolio standard deviation; thus the Lagrangian Multiplier, S_k, is related to the slope of the efficient frontier as shown in equation 3.8.

$$\delta\sigma^2_k/\ \delta E(R_k) = 2S_k \quad (3.7)$$

$$S_k = \sigma_k/\ m_k \quad (3.8)$$

Where:

- m_k is the slope of the efficient frontier at point k

(Adapted from Fama, 1976: 265-266)

The slope of the efficient frontier decreases from B to A in Figure 3-1 (page 54) and thus ranges from positive infinity to a finite positive asymptote (Fama, 1976: 280-281). Furthermore, the standard deviation of the portfolio increases as the portfolio plots further to the right along the efficient frontier. Thus given the relationship between the slope of the efficient frontier, the standard deviation of the portfolio, and S_k; the latter can take on any positive number (Fama, 1976: 280). The set of minimum-variance portfolios on the efficient frontier can therefore be generated by varying S_k between zero and positive infinity and combining each value of S_k with an appropriate value of T_k. This can be determined as the value which ensures that the sum of the weightings of the individual assets equals one, given the estimate of S_k (Fama, 1976: 281).

In equation 3.6, the subscripts k, referring to the individual investor, appear only in the multipliers (the other values are not influenced by the choice of assets that the investor holds), and therefore it can be seen that each investor must hold a combination of two portfolios, with every efficient portfolio thus comprising these two portfolios (Black, 1972: 448). However, from equation 3.6, there is no guarantee that the sum of the weights of the individual assets in the two portfolios will be equal to one, as confirmed by Sharpe (1970: 69). To overcome this, Black (1972: 448) normalised the weights, meaning that both the terms on the right hand side of equation 3.6 were multiplied by one, in the form of the sum across all assets of the value of that weighting, divided by the same amount, as illustrated in equation 3.9. This was then simplified to obtain equation 3.10, with the two portfolios denoted p and q.

$$x_{ki} = S_k \sum_{j=1}^{N} D_{ij}\, E(R_j) \times \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}\, E(R_j)}{\sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}\, E(R_j)} + T_k \sum_{j=1}^{N} D_{ij} \times \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}}{\sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}} \tag{3.9}$$

$$x_{ki} = w_{kp}\, x_{pi} + w_{kq}\, x_{qi} \tag{3.10}$$

Where:

- w_{kp} is the weighting of p in portfolio k and is equal to $S_k \sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}\, E(R_j)$
- x_{pi} is equal to $\sum_{j=1}^{N} D_{ij}\, E(R_j) \,/\, \sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}\, E(R_j)$
- w_{kq} is the weighting of q in portfolio k and is equal to $T_k \sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}$
- x_{qi} is equal to $\sum_{j=1}^{N} D_{ij} \,/\, \sum_{i=1}^{N}\sum_{j=1}^{N} D_{ij}$

The weightings of asset i in portfolios p and q are the unique investor values multiplied by the sum of the original variables (the product of the expected return and the inverse of the covariance matrix between assets i and j for the weighting for portfolio p, and the inverse of the covariance matrix for the weighting of portfolio q) but across all assets. The proportions invested in portfolios p and q are denoted as the value of these variables for asset i as a proportion of the value of those same variables, but summed across all assets. In light of this specification, the sum of all the proportions of each asset in portfolios p and q respectively must sum to one as stipulated in equation 3.11.

$$\sum_{j=1}^{N} x_{pi} = \sum_{j=1}^{N} x_{qi} = 1 \tag{3.11}$$

(Black, 1972: 448)

Given the formula for w_{kp}, it is clear that this weighting can take on any value, as the double summation is a constant (either positive or negative) and S_k, as explained previously, can only be positive (Black, 1972: 448). Furthermore, Fama (1976: 282) highlighted that because the double summation in the formula for w_{kq} is also a constant, and thus independent of T_k; for any value of w_{kp}, the value of w_{kq} involves choosing T_k such that the proportions invested in each asset in the minimum-variance portfolio must sum to one, as can be seen in equation 3.12.

$$\sum_{i=1}^{N} x_{ki} = 1 \tag{3.12}$$

(Fama, 1976: 282)

By expressing the proportion of the asset held in portfolios p and q as a fraction of the value of the identical variables across all assets Black (1972: 448) ensured that that the proportions invested in each asset must sum to one. That is, by summing equation 3.10 across all values of j, the left-hand side of the equation is equal to equation 3.12, and the right-hand side $w_{kp} + w_{kq,}$ as a consequence of the relationships specified in equation 3.11. The efficient portfolio that investor k holds therefore includes investments in portfolios p and q, with weightings that sum to one, as illustrated in equation 3.13.

$$\sum_{j=1}^{N} x_{kj} = \sum_{j=1}^{N} (w_{kp}\, x_{pi} + w_{kq}\, x_{qi})$$

$$1 = w_{kp} + w_{kq} \tag{3.13}$$

(Fama, 1976: 282)

Equation 3.10 thus illustrates that the efficient portfolio held by any investor consists of a weighted combination of two basic portfolios p and q; confirming the findings of Sharpe (1970: 59-69). Effectively this formula implies that the proportion of asset i in investor k's efficient portfolio is equal to the proportion of asset i invested in portfolios p and q, adjusted by the weighting of each of these portfolios in the specific efficient portfolio chosen. This result can be applied to compute the return on an efficient portfolio, which is calculated as the weighted combination of the returns on portfolios p and q as shown in equations 3.14. Given that any two different minimum-variance portfolios have different expected returns, it is possible to generate all the points on the efficient frontier by varying the weights assigned to the two portfolios.

$$E(R_k) = \sum_{j=1}^{N} x_{kj} E(R_j) \qquad (3.2)$$

$$E(R_k) = \sum_{j=1}^{N} (w_{kp} x_{pj} + w_{kq} x_{qj}) E(R_j)$$

$$E(R_k) = w_{kp} \sum_{j=1}^{N} (x_{pj} E(R_j)) + w_{kq} \sum_{j=1}^{N} (x_{qj} E(R_j))$$

$$E(R_k) = w_{kp} E(R_p) + w_{kq} E(R_q) \qquad (3.14)$$

(Fama, 1976: 282)

3.2.1.2 Calculating Efficient Portfolios with Unique Combinations

Portfolios, p and q are not unique, meaning that it is not possible to identify any specific characteristics of these portfolios other than that they are minimum-variance (Black, 1972: 449). Other portfolios however can be created with specific attributes by appropriately weighting portfolios p and q, because as proven, all portfolios along the efficient frontier can be formed by these two basic portfolios. Black (1972: 449) transformed p and q into two different portfolios, denoted u and v, which must also be minimum-variance as they are the weighted combinations of two minimum-variance portfolios. This transformation is presented in equation 3.15, where the proportions of asset i invested in portfolios u and v are calculated as the weighted average of the proportions of asset i invested in portfolios p and q; the weightings representing the share of portfolios u and v invested in both p and q.

$$x_{ui} = w_{up} x_{pi} + w_{uq} x_{qi}$$
$$x_{vi} = w_{vp} x_{pi} + w_{vq} x_{qi} \qquad (3.15)$$

Where:

- w_{up} is the weighting of portfolio p in portfolio u
- w_{uq} is the weighting of portfolio q in portfolio u
- w_{vp} is the weighting of portfolio p in portfolio v
- w_{vq} is the weighting of portfolio q in portfolio v

(Black, 1972: 449)

These formulae can be rearranged to solve for the fraction of asset i in portfolios p and q respectively as a function of the proportions of asset i invested in portfolios u and v, where the coefficients represent the weightings of portfolios p and q in u and v (Black, 1972: 449). This is shown in equations 3.16.

$$x_{pi} = w_{pu}\, x_{ui} + w_{pv}\, x_{vi}$$
$$x_{qi} = w_{qu}\, x_{vi} + w_{qv}\, x_{ui} \qquad (3.16)$$

Where:

- w_{pu} is the weighting of portfolio u in portfolio p
- w_{pv} is the weighting of portfolio v in portfolio p
- w_{qu} is the weighting of portfolio u in portfolio q
- w_{qv} is the weighting of portfolio v in portfolio q

(Black, 1972: 449)

If the equations depicted in 3.16 are substituted into equation 3.10, the formula for the proportion of asset i in the k^{th} investor's efficient portfolio can be expressed solely in terms of the proportion of asset i invested in portfolios u and v, weighted according to their respective shares of the total efficient portfolio (Black, 1972: 449). That is:

$$x_{ki} = w_{kp}\,(w_{pu}\, x_{ui} + w_{pv}\, x_{vi}) + w_{kq}\,(w_{qu}\, x_{ui} + w_{qv}\, x_{vi})$$
$$x_{ki} = x_{ui}\,(w_{kp}\, w_{pu} + w_{kq}\, w_{qu}) + x_{vi}\,(w_{kp}\, w_{pv} + w_{kq}\, w_{qv})$$

Let:

- $w_{ku} = w_{kp}\, w_{pu} + w_{kq}\, w_{qu}$
- $w_{kv} = w_{kp}\, w_{pv} + w_{kq}\, w_{qv}$

$$x_{ki} = w_{ku}\, x_{ui} + w_{kv}\, x_{vi} \qquad (3.17)$$

Where:

- w_{ku} is the weighting of portfolio u in investor k's efficient portfolio
- w_{kv} is the weighting of portfolio v in investor k's efficient portfolio

(Black, 1972: 449)

Given the resultant equation expressed in 3.17, the return on the k^{th} efficient portfolio can thus be expressed as the weighted combination of the returns on portfolios u and v, as shown in equation 3.18, which is obtained in an identical derivation to that conducted in equations 3.14.

$$R_k = w_{ku}\, R_u + w_{kv}\, R_v \qquad (3.18)$$

(Fama, 1976: 282)

Portfolios u and v represent any two portfolios that can be formed with the original basic portfolios p and q; Black (1972: 449) therefore proved that every efficient portfolio can be expressed as a weighted combination of portfolios u and v, and that these portfolios do not have to be efficient, provided that they satisfy the minimum-variance constraint. In order to be able to generate every efficient portfolio as a weighted combination of p and q, these two portfolios must have different betas so that they have different expected returns so as to create the efficient frontier from all attainable investments. According to Black (1972: 449), the consequence of this is that the two new portfolios created can have arbitrary betas provided that the appropriate weights are selected; Black (1972: 449) selected the weightings of portfolios u and v such that the betas were 1 and 0 respectively.

The market portfolio, by definition, must have a beta equal to one, as it measures the covariance of the market returns with itself. Any portfolio with a beta of one has the same risk as the market. Black (1972: 449) illustrated that equation 3.10 can be adjusted to reflect the proportion of asset i held in the market portfolio, by multiplying this equation by the fraction investor k holds of total wealth, x_{mk}, and summing across all investors, as shown in equation 3.19. This equation indicates that the proportion of asset i in the market portfolio is a weighted combination of p and q and thus portfolio u, which is also a weighted combination of p and q, with a beta of one, must be the market portfolio.

$$\sum_{k=1}^{L} x_{mk}\, x_{ki} = \sum_{k=1}^{L} (x_{mk}\, w_{kp}\, x_{pi}) + \sum_{k=1}^{L} (x_{mk}\, w_{kq}\, x_{qi})$$

$$x_{mi} = (\sum_{k=1}^{L} x_{mk}\, w_{kp})\, x_{pi} + (\sum_{k=1}^{L} x_{mk}\, w_{kq})\, x_{qi} \qquad (3.19)$$

Where:

- x_{mi} is the proportion of asset i held in the market portfolio
- x_{mk} is the fraction of total wealth held by each investor

(Black, 1972: 449)

Portfolio v, assigned a beta of zero, meaning that its returns are completely uncorrelated with the market, is termed the zero-beta portfolio (Black, 1972: 449). Every portfolio with a beta equal to zero will have an identical expected return; however because the market and zero-beta portfolios are uncorrelated, and weighted combinations of these two must be efficient, the zero-beta portfolio used in the model must be the minimum-variance zero-beta portfolio (Black, 1972: 450; Blume and Friend, 1973: 20).

The major conclusions Black (1972: 449) therefore reached were that:

1. The entire set of efficient portfolios can be generated as a linear combination of any two minimum-variance portfolios.
2. It is possible to allocate different values to the betas of any two minimum-variance portfolios as these can be formed by appropriately weighting two other minimum-variance portfolios.
3. The entire set of efficient portfolios can be generated as a linear combination of the market and zero-beta portfolios, as shown in equation 3.20, and that the return on an efficient portfolio can be expressed as the weighted combination of the return on the market and zero-beta portfolios, depicted in equation 3.21.

$$x_{ki} = w_{km}\, x_{mi} + w_{kz}\, x_{zi} \tag{3.20}$$

$$\bar{R}_k = w_{km}\, \bar{R}_m + w_{kz}\, \bar{R}_z \tag{3.21}$$

Where:

- w_{km} is the weighting of the market portfolio in investor k's efficient portfolio
- w_{kz} is the weighting of the zero-beta portfolio in investor k's efficient portfolio
- x_{zi} is the fraction of portfolio z invested in security i

3.2.2 The Efficiency of the Market and Zero-Beta Portfolios

In deriving u and v, Black (1972: 449) highlighted that these portfolios did not necessarily have to be efficient for a portfolio consisting of investments in both to be efficient. Intuitively it is easy to understand that, as with the traditional CAPM, the market portfolio must be efficient. Given the assumption of the model that investors have homogenous expectations (as referred to in Section 2.1.1), all investors face the same efficient frontier, and all rational investors will choose portfolios that lie on this curve. The market portfolio is the weighted average of each investor's efficient portfolio where the weights are determined by the proportion each investor owns of all risky assets. Thus the market portfolio, as a combination of efficient portfolios, must itself be efficient (Fama, 1976: 279).

The zero-beta portfolio in contrast, despite being minimum-variance, is not efficient; this was proved mathematically by Fama (1976: 286) and Elton *et al* (2007: 311). They illustrated that in light of the relationships identified in equation 3.20, the variance of an efficient portfolio, k, can be expressed as shown in equation 3.22 (on the following page). There is no covariance term in the formula, because the covariance between the zero-beta and market portfolios is, by definition, zero.

$\sigma^2_k = w^2_{kz}\,\sigma^2_z + (1 - w_{kz})^2\,\sigma^2_m$ (3.22)

Where:

- σ^2_z is the variance of the zero-beta portfolio
- σ^2_m is the variance of the market portfolio

(Adapted from: Elton *et al*, 2007: 311)

To find the weights invested in the market and zero-beta portfolios in the portfolio that has the smallest possible variance, but is still efficient (as represented by B in Figure 3-1, page 54), Elton *et al* (2007: 311) calculated the derivative of equation 3.22 with respect to the fraction invested in the zero-beta portfolio, equated it to zero and solved for the weighting as follows:

$\sigma^2_k = w^2_{kz}\,\sigma^2_z + (1 - 2w_{kz} + w_{kz}^2)\,\sigma^2_m$

$\sigma^2_k = w^2_{kz}\,\sigma^2_z + \sigma^2_m - 2w_{kz}\sigma^2_m + w_{kz}^2\sigma^2_m$

$\delta\sigma^2_k / \delta w_{kz} = 2\,w_{kz}\,\sigma^2_z - 2\,\sigma^2_m + 2\,w_{kz}\,\sigma^2_m$

$0 = 2\,w_{kz}\,\sigma^2_z - 2\,\sigma^2_m + 2\,w_{kz}\,\sigma^2_m$

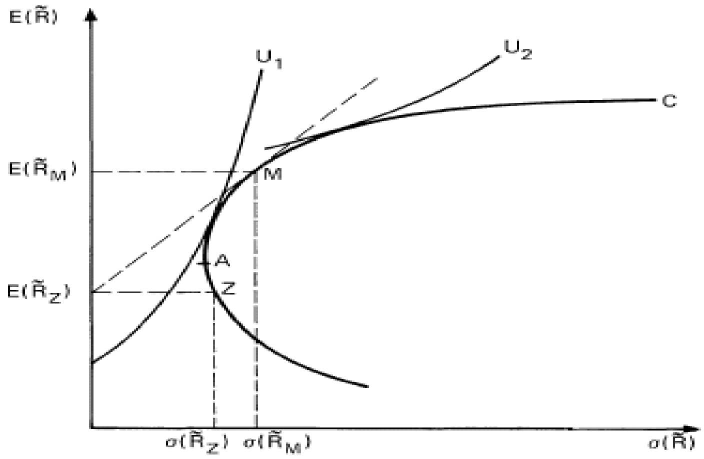
$w_{kz} = \sigma^2_m / (\sigma^2_m + \sigma^2_z)$ (3.23)

(Elton *et al*, 2007: 311)

Given that w_{kz} must be greater than zero and less than one and that the variances, being the standard deviation squared, must be positive, the portfolio with the smallest possible variance will have positive holdings of both portfolios (Elton *et al*, 2007: 318). For the market portfolio to be efficient and the classical risk aversion assumptions to hold, the return on the zero-beta portfolio must be smaller than the market portfolio return, and therefore portfolios with a combination of the two must have higher expected returns than the zero-beta portfolio (Elton *et al*, 2007: 319). Thus, as a result of the fact that the minimum-variance efficient portfolio has a higher return and a lower variance than the minimum-variance zero-beta portfolio, the latter cannot lie on the efficient frontier and hence is not efficient.

Jensen (1972: 376) graphically portrayed the efficient frontier with the market and zero-beta portfolios and the requisite relationships between the variables, as shown in Figure 3-2 (on the following page). The points Z and M represent the standard deviations and expected returns of the zero-beta and market portfolios respectively; hence all points on the curve ZAMC can be obtained (Jensen, 1972: 376). Z does not plot on the vertical (return) axis indicating that it does exhibit some variance (in the form of unsystematic risk). All portfolios between A and C are efficient in the Markowitz sense. Jensen (1972: 376) showed that as with modern portfolio theory, each investor will maximise their utility by purchasing that combination of Z and M at which their highest utility curve of the choice between risk and return is tangential to the efficient frontier, as depicted by u_1 and u_2 for investors 1 and 2 respectively. Investors in the aggregate hold the market portfolio, and thus the net holdings of the zero-beta portfolio (long minus short positions) must sum to zero (Elton *et al*, 2007: 311; Jensen, 1972: 377).

Figure 3-2: Equilibrium Portfolio Choices in the Absence of a Risk-Free Asset



(Source: Jensen, 1972: 376)

3.2.3 The Adjusted CAPM

As denoted in equation 3.21, the return on an efficient portfolio can be calculated as the weighted average of the returns on the market and zero-beta portfolios; however, it is necessary to transform this finding to a form that is directly comparable to the traditional CAPM, which is outlined below. Equation 3.5 in Section 3.2.1.1 is the derivative of the minimisation function (equation 3.4) with respect to x_{kj}, and because it holds for every security, it must also hold for a randomly selected security denoted b, as shown in equation 3.24. Since both equations 3.5 and 3.24 are equal to zero they can be equated as is done in equation 3.25.

$$0 = \sum_{i=1}^{N} x_{ki}\sigma_{ib} - S_k E(R_b) - T_k \qquad (3.24)$$

$$\sum_{i=1}^{N} x_{ki}\sigma_{ij} - S_k E(R_j) - T_k = \sum_{i=1}^{N} x_{ki}\sigma_{ib} - S_k E(R_b) - T_k \qquad (3.25)$$

(Adapted from Fama, 1976: 264)

Both sides of equation 3.25 are multiplied by the proportion of asset b held in portfolio k, x_{kb}, and the values summed across b as illustrated in equation 3.26 (on the following page) (adapted from Fama, 1976: 264-265). This equation can be simplified using the formulae specified in equations 3.2 and 3.3 for calculating the variance and expected return of a portfolio, adjusted to reflect security b, as shown below. In addition to this, the sum of the proportions invested in each asset in the efficient portfolio must sum to one as established in equation 3.12. Equation 3.27 displays equation 3.26 simplified using these three relationships and is rearranged further in equation 3.28.

$$\sum_{b=1}^{N} [\, x_{kb} \sum_{i=1}^{N} x_{ki}\, \sigma_{ib} - x_{kb}\, S_k\, E(R_b)] = \sum_{b=1}^{N} [\, x_{kb} \sum_{i=1}^{N} x_{ki}\, \sigma_{ij} - S_k\, x_{kb}\, E(R_j)] \qquad (3.26)$$

But from equations 3.2 and 3.3:

$$\sigma^2_k = \sum_{i=1}^{N} \sum_{b=1}^{N} x_{ki}\, x_{kb}\, \sigma_{ib}$$

$$E(R_k) = \sum_{b=1}^{N} x_{kb}\, E(R_b)$$

From equation 3.12:

$$\sum_{b=1}^{N} x_{kb} = 1$$

$$\sigma^2_k - S_k\, E(R_k) = [\sum_{i=1}^{N} x_{ki}\, \sigma_{ij} - S_k\, E(R_j)] \qquad (3.27)$$

$$S_k\, [E(R_j) - E(R_k)] = \sum_{i=1}^{N} x_{ki}\, \sigma_{ij} - \sigma^2_k \qquad (3.28)$$

(Adapted from Fama, 1976: 264)

As shown in equation 3.8 in Section 3.2.1.1, the Lagrangian multiplier S_k is inversely related to the slope of the efficient frontier at point k, and substituting this relationship into equation 3.28 and simplifying, yields equation 3.29. Furthermore, Fama (1976: 241-243) showed that it is actually more convenient to term the weighted average of the covariances between two securities as the covariance of the two return series, as shown in equation 3.30. Taking this into consideration, the equation presented in 3.31 is obtained.

$$S_k = \sigma_k / m_k \qquad (3.8)$$

$$[E(R_j) - E(R_k)] = m_k / \sigma_k \,(\sum_{i=1}^{N} x_{ki}\, \sigma_{ij}) - m_k\, \sigma_k \qquad (3.29)$$

$$\sum_{i=1}^{N} x_{ki}\, \sigma_{ij} = \mathrm{cov}(R_i, R_k) \qquad (3.30)$$

$$E(R_j) = E(R_k) - m_k\, \sigma_k + m_k / \sigma_k\, [\mathrm{cov}(R_i, R_k)] \qquad (3.31)$$

(Adapted from Fama, 1976: 243, 264)

As discussed and illustrated in Section 3.2.1, Black (1972: 447-448) proved that the return on an efficient portfolio can be computed using the returns on the market and zero-beta portfolios. Portfolio k can thus be viewed as the market portfolio, as it is assumed to be efficient. The slope of the efficient frontier at point m is computed based on a tangent to the frontier which touches at point m. The applicable formula for the calculation of the slope is given in equation 3.32. In equation 3.33, the slope of the efficient frontier is included in equation 3.31 and simplified.

$$m_k = [E(R_m) - E(R_z)] / \sigma_m \tag{3.32}$$

$$E(R_j) = E(R_m) - [[E(R_m) - E(R_z)] / \sigma_m]\sigma_m + [[E(R_m) - E(R_z)] / \sigma_m] / \sigma_m [cov(R_i, R_m)]$$
$$E(R_j) = E(R_z) + [E(R_m) - E(R_z)] [cov(R_i, R_m) / \sigma^2_m] \tag{3.33}$$

Beta, as identified in Section 2.1.1, is computed as the covariance between the security and the market portfolio divided by the variance of the market portfolio and substituting this into equation 3.33 results in equation 3.34. It is therefore clear from equation 3.34 that the only difference between the traditional form of the CAPM and the zero-beta CAPM, derived by Black (1972) is that the risk-free rate is replaced with the return on the minimum-variance zero-beta portfolio.

$$E(R_j) = E(R_z) + \beta_j [E(R_m) - E(R_z)] \tag{3.34}$$

(Black, 1972: 450)

In summary, what Black (1972: 446-452) found was that the absence of riskless borrowing and lending does not have a significant influence on the model. The relationship between risk and return is still linear; the important observation being that if a suitable surrogate for the riskless asset does not exist, the minimum-variance zero-beta portfolio should be calculated to estimate the risk-free rate.

3.2.4 The Zero-Beta Portfolio as the Means to Estimate the Risk-Free Rate

Effectively the minimum-variance zero-beta portfolio offers an alternative way of measuring the risk-free rate; that is, instead of trying to identify a single security which is devoid of default, liquidity, inflation, currency and interest rate risk, whose returns have no variance or covariance with the market, and whose yield reflects the pure interest rate, the zero-beta portfolio is a collection of risky assets combined in such a way that the returns do not move with the market (i.e. a zero beta) but does still exhibit some unsystematic risk. The formation of this portfolio rests on the concept of short selling, where an investor essentially ‘borrows’ securities from a broker and sells them in the market in anticipation of being able to replace them later at a lower price (Harrington, 1987: 36). A short-asset will thus have a beta with the opposite sign to that normally associated with the asset. Due to the fact that the beta of a portfolio is simply the weighted average of the security betas (Harrington, 1987: 18), a zero-beta portfolio can be obtained by balancing long and short asset positions; creating a riskless portfolio in any environment. Thus, as Harrington (1987: 36) suggested, Black (1972) effectively used short-selling as a proxy for the riskless asset.

There are certainly several theoretical advantages to employing this zero-beta portfolio approach to estimating the risk-free rate in the CAPM. The foremost of these being that the use of the zero-beta portfolio returns is theoretically justified in a market with no suitable risk-free surrogate (Black, 1972: 455). Closely associated with this is the fact that the zero-beta portfolio returns do not, by definition, vary with the market portfolio proxy employed; thereby more closely approximating the risk-free condition, and avoiding the myriad of statistical problems that arise when implementing and testing the CAPM when there is correlation between the risk-free and market portfolio proxies (Harrington, 1987: 36).

In addition to the above two reasons in favour of the zero-beta portfolio returns, the considerable theoretical discrepancies associated with both long-and short-term government issued securities highlighted in Sections 2.3 and 2.4, such as default risk, inflation risk, liquidity risk, and interest rate risk are avoided with the theoretical alternative of the zero-beta portfolio returns.

That is not to say, however, that the employment of the zero-beta portfolio as the means to estimate the risk-free proxy in the CAPM is beyond reproach. One of the major criticisms levelled against its use is the reliance on the assumption of unlimited short-sales. Large professionally managed portfolios may be limited in the quantity and exposure of short-sales, and thus the applicability of using the returns on such a portfolio as the minimum required rate of return is certainly questionable (Harrington, 1987: 36). Moreover, market regulators may restrict the number of short-sales and the sector in which they are permitted, such as occurred in the U.S., Canada, England, Australia, Hong Kong, Japan and Pakistan during the recent financial crisis (Hulbert, 2008: 1; Nelson, 2008: 1; Imeson, 2009: 4).

For these reasons, it is evident that the assumption of unlimited short-selling is not entirely practical or realistic. However, Black (1972: 446) argued that restrictions on short-selling would not bring the model into disrepute; as long as it remained possible for some assets to be sold short, it would be possible to form a portfolio with a zero-beta. Further to this, Blume and

Friend (1973: 30) argued that the model would be robust to the violation of this assumption if no investor actually held this position. That is, the net holdings of individual investor's investments in the zero-beta portfolio must sum to zero, and thus it is possible that in combination with the market portfolio no actual short-sale positions need to have occurred; hence rendering this assumption less restrictive than the risk-free asset assumption (Blume and Friend, 1973: 30).

An additional criticism of the zero-beta portfolio approach is the complexity of calculating these returns (Copeland *et al*, 2000: 169); the reason for this being that Black (1972), in deriving the zero-beta portfolio, gave no indication as to how to estimate these returns. Black *et al* (1972), Fama and MacBeth (1973), Gibbons (1982), and Stambaugh (1982) all developed different approaches for this purpose in testing the zero-beta CAPM, whilst Morgan (1975) and Alexander (1977) focused specifically on this issue in their respective papers. Despite this considerable attention there appears to be no conclusive evidence in the literature as to which approach is best or the easiest to implement. Compounding this is the fact that there is also no clarity on the types of assets to include in the portfolio (common and preference shares or bonds), or the number thereof. Overall, however, in a market where no suitable proxy for the risk-free asset exists, the theoretically appropriate alternative is to estimate the returns on the minimum-variance zero-beta portfolio rather than use a government security.

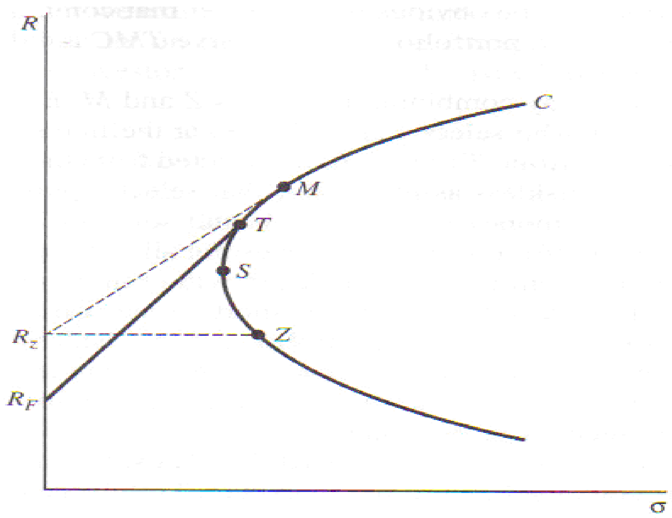
3.2.5 The Zero-Beta Portfolio Returns and the Risk-Free Rate

Given that the zero-beta portfolio approach provides an alternative estimate of the risk-free rate for use in the CAPM it is important to consider the relationship that is likely to be observed between the two parameters if a suitable risk-free asset exists.

To determine the relationship between these two variables, Black (1972: 452) relaxed his assumption regarding the absence of a riskless asset, assuming instead that if such an asset does exist, that it is only possible for investors to lend at this rate (the same scenario examined by Vasicek, 1971). This is certainly feasible, as if government securities are considered reliable estimates of the risk-free rate investors can only lend their funds at this rate and not borrow. By allowing this, the range of efficient portfolios from which investors can choose increases, consisting of weighted combinations of the market and zero-beta portfolios, and weighted combinations of the riskless asset proxy with a single portfolio of risky assets, denoted portfolio T (Black, 1972: 452). Portfolio T is determined, as in the original CAPM derivation, as the tangent point between a ray from the risk-free rate and the efficient frontier (Fama, 1976: 289).

All investors who wish to invest in the riskless asset and a portfolio of risky assets will choose portfolio T and thus lie on the line R_fT, as illustrated in Figure 3-3 (Elton *et al*, 2007: 313). In light of Black's (1972: 449) finding that every efficient portfolio can be formed as a combination of the market and zero-beta portfolios, under the conditions of restricted borrowing at the risk-free rate, the investor who holds the riskless asset will also hold a combination of the market and zero-beta portfolios combined in specific proportions in portfolio T.

Figure 3-3: Market Equilibrium with Restricted Borrowing



(Source: Elton *et al*, 2007: 313)

Unlike the classical case of riskless lending and borrowing, when restrictions are imposed on borrowing, the tangential point between the ray from the risk-free rate and the efficient frontier (portfolio T) does not represent the market portfolio (Fama, 1976: 290). In fact, the market portfolio must lie to the right of portfolio T on the efficient frontier, otherwise an investor who does not hold the riskless asset would be worse off as they would not lie on the efficient frontier, but rather on the curve below T (Elton *et al*, 2007: 313). The efficient frontier with restricted borrowing is thus given as the straight line segment R_fT and the curve TM in Figure 3-3 (Elton *et al*, 2007: 313). Given that portfolio T, the combination of the market and zero-beta portfolios must lie to the left of the market portfolio, it is clear that the zero-beta portfolio return must be greater than the risk-free return from lending. Black (1972: 453) confirmed this relationship between the zero-beta portfolio return and the risk-free yield mathematically.

3.3 The Validity of the Zero-Beta Portfolio Approach

The most logical way to assess the validity of the zero-beta portfolio as a means to estimate the risk-free rate would be to form the minimum-variance zero-beta portfolio, calculating the returns earned over a specific period of time, and then comparing these returns to the rate which investors' viewed as the minimum required return for investing over the period (determined by the intercept of the SML), and the chosen risk-free proxy yields. In this way, it would be possible to ascertain which approach provides a better estimate of the minimum required return in the CAPM, and the relationship between these two estimates.

The majority of the tests of the validity of the zero-beta portfolio approach have not followed this methodology; possibly as a result of the difficulty inherent in estimating the zero-beta portfolio directly, as well as the fact that these examinations were conducted as part of more comprehensive tests of the CAPM as a whole and thus this issue was one of numerous concerns and not the primary focus. Instead the methodology followed in these studies was referred to by Morgan (1975: 363) as an indirect approach, as opposed to the more direct approach suggested above, which was followed, to some extent, by Morgan (1975).

3.3.1 Indirect Empirical Tests

The initial research that Morgan (1975: 363) criticised for implementing an indirect approach to testing the validity of the risk-free proxy and the zero-beta portfolio was the work of Black *et al* (1972) and Fama and MacBeth (1973); however, the methodology followed by these authors has been replicated by many others. In general, these authors proposed that if the other CAPM relationships held over the period, then the intercept must be equal to either the risk-free rate or the zero-beta portfolio return (Black *et al*, 1972; Fama and MacBeth, 1973; Blume and Friend, 1973; Gibbons, 1982; Stambaugh, 1982). Thus, if the risk-free proxy was statistically significantly different from the intercept values obtained, the conclusion drawn was that the zero-beta portfolio return provided a better estimate of investor's minimum required return, and hence Black's (1972) CAPM was deemed to be more appropriate than the traditional form of the model. These tests can therefore be viewed as indirect examinations of the most appropriate technique, as the minimum-variance zero-beta portfolio returns were not explicitly estimated and compared to the value of the intercept and the risk-free rate proxy.

3.3.1.1 Black *et al* (1972)

Black *et al* (1972) sought to conduct in-depth tests of the CAPM. To improve the efficiency of the parameter estimates compared to earlier studies on the model (such as Douglas, 1969), Black *et al* (1972: 10) combined the shares in the sample (all shares listed on the New York Stock Exchange (NYSE) over the period 1931-1966) into portfolios rather than using the shares individually. The portfolios were formed, each year, using historical estimates of beta based on out-of-sample data, as ranking stocks based on current values of beta would not have been efficient (Black *et al*, 1972: 12). To estimate the beta values for the shares, the traditional CAPM formula was rearranged as shown in equation 3.35, so that the variable on the left-hand side represents the excess return (the returns earned in excess of the risk-free rate), which is a function of the security's beta and the market risk premium. This relationship, however, assumes that all securities plot on the SML, which Jensen (1968) showed is not a realistic prediction; they may plot above or below the line as the securities may have outperformed or underperformed the market relative to their risk level. Consequently, in estimating the model using ordinary least squares (OLS), it is necessary to allow for an intercept term to reflect these differences. The intercept of any regression model can be interpreted as the value that the dependent variable will assume when the independent variables are all simultaneously zero (Brooks, 2006: 51). With reference to equation 3.36 which was used to estimate the betas by Black *et al* (1972: 10), the intercept can be interpreted as the excess return earned when the excess market portfolio returns are zero. However, in the context of employing equation 3.36 to estimate betas to be used in additional tests regressions, the value of α_j was not deemed to be of importance (Black *et al*, 1972: 10).

$$R_{jt} - R_f = \overline{\beta}_j [R_m - R_f] \tag{3.35}$$

$$\overline{R}_{jt} = \alpha_j + \overline{\beta}_j \overline{R}_m + e_{jt} \tag{3.36}$$

Where:

- R_{jt} are the returns of security j over time
- R_f is the risk-free rate
- $\overline{\beta}_j$ is the estimated coefficient
- R_m are the market portfolio returns over time
- $\overline{R}_{jt}$ are the mean excess returns over time for a set of securities ($R_{jt} - R_f$)
- α_j is the estimated intercept of the model
- $\overline{R}_m$ are the mean excess returns on a market portfolio ($R_m - R_f$)
- e_{jt} are the error terms of the time-series regression

(Source: Black *et al*, 1972: 2-3; 10)

The portfolio betas were utilised, as the independent variable, to estimate the test equation derived by Black *et al* (1972: 12), shown in equation 3.37 (on the following page). Effectively this regression entails plotting the SML based on the returns and beta estimates for each portfolio (that is, a cross-sectional as opposed to a time series sample) so as to infer the values for the market portfolio and intercept.

$\overline{R}_p = \lambda_{p0} + \lambda_{p1}\hat{\beta}_p + u_p$ (3.37)

Where:

- $\overline{R}_p$ are the excess portfolio returns
- λ_{p0} is the estimated intercept of the model
- λ_{p1} is the estimated coefficient of beta
- $\hat{\beta}_p$ is the estimated systematic risk of the p^{th} portfolio
- u_p is the error term of the cross-section regression

(Source: Black *et al*, 1972: 2-3; 18)

The coefficient on the beta term represents the market risk premium as it shows by how much the excess returns (the dependant variable) change when beta changes. In contrast to the interpretation of the intercept in equation 3.36, in equation 3.37, the intercept represents the excess return when beta is zero; that is, the return earned over and above the risk-free rate. OLS methodology estimates the coefficients of the model so as to minimise the error variance and thus the intercept, by definition, should represent the return earned on a portfolio which has no risk (i.e. a zero-beta) with the lowest possible variance based on the returns given (but not necessarily *the* minimum-variance zero-beta portfolio return identified by Black (1972)). It therefore follows as to why the estimate of the intercept can be interpreted as the return on an asset or portfolio with a zero-beta.

The results show that for the entire period and across all four sub-periods the intercepts of the SMLs differed from zero and the slopes differed from the theoretical predictions (Black *et al*, 1972: 21-25). Moreover, the tests of the statistical significance of the difference between the risk-free proxy returns and the minimum required rate of return, as shown in Table 3-1, revealed highly significant statistics, implying that the two values were statistically significantly different (Black *et al*, 1972: 23).

Table 3-1: Intercept Estimates and Test Statistics from Black *et al* (1972)

Period	λ_{j0}	$t(\lambda_{j0})$
January/1931-December/1965	0.00359	6.52***
January/1931-September/1939	-0.008	-4.45***
October/1939-June/1948	0.00439	3.2***
July/1948-March/1957	0.00777	7.4***
April/1957-December/1965	0.0102	18.89***

*** Statistically significant at the 1% level

(Source: Black *et al*, 1972: 21-24)

Black *et al* (1972: 22-25) acknowledged that these t-statistics may be overstated because of misspecification, as although the time series and cross-sectional regressions both followed the simple CAPM form as suggested by Miller and Scholes (1972) (quoted in Elton and Gruber, 1995: 346); the fact that the beta estimate was based on a potentially erroneous proxy for the risk-free rate means that its value may have been wrong and the form of the model tested incorrectly specified. These results indicate that the minimum required return determined by the model was statistically greater than the risk-free rate proxy returns; evidence, according to Black *et al* (1972: 25) that the zero-beta portfolio return is a more appropriate method to estimate the risk-free rate than the use of a proxy. However, this appears counterintuitive in light of the fact that if the proxy is inappropriate, it would be expected that the alternative rate used would be lower rather than higher, as it is meant to reflect the minimum return that investors require from investing as discussed in Section 3.2.5. But, based on the results shown in Table 3-1, it appears as though the yield on the one month T-Bill (used as a proxy for the risk-free rate (Black *et al*, 1972: 20)) understates the minimum required return.

Black *et al* (1972) also tried to obtain more explicit estimates of the minimum required return rather than basing them on a regression analysis, although they were still reliant on estimates of the other CAPM parameters. For this purpose, Black *et al* (1972: 35) estimated the excess returns by adjusting equations 3.35 and 3.37 to take into consideration the effects of the excess minimum required return on the beta estimate obtained, as illustrated in equation 3.38. This equation was in turn rearranged in order to isolate the excess minimum required return (equation 3.39), where the estimate thereof was obtained from the pth portfolio.

$$\overline{R}_p = \overline{\gamma}_0 + \overline{\beta}_p(\overline{R}_m - \overline{\gamma}_0) \qquad (3.38)$$

$$\overline{\gamma}_{0p} = 1/\,(1- \hat{\beta}_p) * [\overline{R}_p - \hat{\beta}_p \overline{R}_{mt}] \qquad (3.39)$$

Where:

- $\overline{\gamma}_0$ is the excess minimum required return
- $\overline{\gamma}_{0p}$ is the excess minimum required return obtained from the pth portfolio

(Black *et al*, 1972: 36)

In order to compute a more accurate value, Black *et al* (1972: 35) combined the estimates from each of the ten portfolios according to equation 3.40 (on the following page). These estimates from the ten portfolios had to be combined to minimise the error variance associated with the individual portfolio estimates of the parameter, subject to the constraint that the sum of the weightings of each estimate, h_p, must equal one (Black *et al*, 1972: 36). As with the constrained optimisation problem identified in Section 3.2.1.1, Black *et al* (1972: 36) used Lagrangian Multipliers to solve this problem, with the resulting formula for the individual weightings replicated in equation 3.41. The values of beta used in equations 3.39 and 3.41 were obtained from the times series regressions of the excess return CAPM, as shown in equation 3.36.

$$\bar{\gamma}_{0t}^{*} = \Sigma\, h_p \bar{\gamma}_{0p} \tag{3.40}$$

$$h_p = (1- \hat{\beta}_p)^2 / \Sigma_p (1- \hat{\beta}_p)^2 \tag{3.41}$$

Where:

- $\bar{\gamma}_{0t}^{*}$ is the average excess minimum required return from all portfolios
- h_p is the weighting of each estimate of the excess minimum required return

(Black *et al*, 1972: 36)

The estimates of the excess minimum returns obtained by Black *et al* (1972: 38) are depicted in Table 3-2. For the entire period 1931 to 1965, the excess return was estimated at 0.338 percent per month. The t-statistic of 1.62 falls slightly below the ten percent significance level critical value (1.65), but it is high enough to suggest that the minimum return and risk-free proxy returns differed over the period. In the first sub-period (1931-1939) the t-statistic was negative meaning that the proxy returns were larger than the minimum required return (although statistically insignificant); thereafter inversing and increasing substantially across the remaining periods, such that in the third and fourth sub-periods the t-values of 4.03 and 4.49 comprehensively reject the equality of the two values at the one percent significance level.

Table 3-2: Explicit Intercept Estimates and Test Statistics from Black *et al* (1972)

Period	$\bar{\gamma}_{0t}^{*}$	**t(**$\bar{\gamma}_{0t}^{*}$**)**
January/1931-December/1965	0.00338	1.62
January/1931-September/1939	-0.0085	-1.35
October/1939-June/1948	0.0042	0.946
July/1948-March/1957	0.00782	4.03***
April/1957-December/1965	0.00997	4.49***

* Statistically significant at the 10% level
** Statistically significant at the 5% level
*** Statistically significant at the 1% level

(Source: Black *et al*, 1972: 38-40)

The findings of Black *et al* (1972: 38) across the two analyses thus, in general, provide support that the use of a proxy to estimate the risk-free rate is inappropriate.

3.3.1.2 Fama and MacBeth (1973)

Fama and MacBeth (1973) intended to calculate the actual risk premium and the predicted intercept of the CAPM, and determine whether any other risk factors or more complex relationships than those posited by the model existed. They adopted a similar approach to Black *et al* (1972) by forming portfolios based on out-of-sample beta estimates, but favoured the use of the market model, as depicted in equation 3.42, to estimate the betas of the shares and portfolios (Fama and MacBeth, 1973: 615-616). This ensured that they maintained consistency (as per Miller and Scholes, 1972 (quoted in Elton and Gruber, 1995: 346)) but avoided the possible misspecification associated with the Black *et al* (1972) results caused by the use of a potentially inappropriate risk-free rate proxy. In contrast to the formula employed by Black *et al* (1972) for this purpose, the risk-free rate was not included in this specification. The intercept of this model is interpreted as the return earned when the market portfolio returns are zero, and the coefficient on the market portfolio reflects the change in the returns earned for a given change in the market portfolio returns; that is, the major difference to the model in equation 3.36 is that the returns examined are not excess returns.

$$R_{it} = \psi_j + \beta_i R_m + e_{jt} \tag{3.42}$$

Where

- ψ_j is the estimated intercept of the model
- β_I is the estimated coefficient

(Fama and MacBeth, 1973: 616)

Using these time series estimates of beta, Fama and MacBeth (1973: 619) estimated the regression shown in equation 3.43 for the period 1935 to June 1968 and over a variety of sub-periods. For this model, the intercept provides a direct estimate of the minimum return required by investors over the periods examined (that is, when all independent variables are assumed to be zero) and can be compared to a risk-free rate proxy yield to ascertain the similarity between this instrument and the rate which investors view as the appropriate base rate for investing.

$$R_{pt} = \gamma_{p0} + \gamma_{p1}\widehat{\beta}_{pt} + \gamma_{p2}\widehat{\beta}^2_{pt} + \gamma_{p3}S_{ei} + \eta_{pt} \tag{3.43}$$

Where

- γ_{p0} is the estimate of the intercept
- γ_{p1} is the coefficient estimate of the market risk premium
- γ_{p2} is the coefficient estimate indicating any non-linear relationships between risk and return
- γ_{p3} is the coefficient estimate of the residual risk term
- $\widehat{\beta}_{pt}$ is the beta of portfolio p
- $\widehat{\beta}^2_{pt}$ is the squared value of the portfolio beta
- S_{ei} is the standard deviation of the error terms for the securities in portfolio p
- η_{pt} is the error term of the regression

(Fama and MacBeth, 1973: 616)

Fama and Macbeth's (1973: 620) results proved that residual risk was not an important factor in determining returns (γ_{p3} was statistically insignificant), and no non-linear relationship between risk and return was detected either ($\gamma_{p2}= 0$). Given these findings, equation 3.43 was re-estimated, without the two insignificant variables, so as to obtain more accurate estimates of the intercept and the risk premium (Fama and MacBeth, 1973: 632), as shown in equation 3.44.

$$R_{pt} = \gamma_{p0} + \gamma_{p1} \widehat{\beta}_{pt} + \eta_{pt} \qquad (3.44)$$

The results, which are reproduced in Table 3-3, are similar to those of Black *et al* (1972). The statistics obtained for the test of the equality of the relationship between the estimated intercept and the risk-free proxy were, in general, smaller than those tabulated in the results of Black *et al* (1972: 23) because the potential misspecification had been accounted for. For six of the nine sub-periods, the statistics indicated that the difference between the minimum required return and the risk-free rate was not statistically significant; however, across the entire period and the other three sub-periods, the evidence strongly favoured the observation that the minimum return required by investors was not equal to the T-Bill rate (the test statistics were significant at the five and one percent significance levels respectively). Moreover, the trend highlighted in the previous study that the intercept was actually greater than the risk-free rate proxy was evident in all but one of the periods (1961-June 1968). Thus these tests, as with Black *et al* (1972), purport the potential inappropriateness of the use of a proxy to estimate the risk-free rate in the CAPM in the U.S. But rather than overstating the intercept of the SML, as was expected based on the analysis in Section 2.3, the findings indicate that the risk-free proxy returns in fact understate the estimated minimum required return in the U.S.

Table 3-3: Intercept Estimates and Test Statistics from Fama and MacBeth (1973)

Period	γ_0	γ_{0-Rf}	$t(\gamma_0 - Rf)$
1935-June/1968	0.0061	0.0048	2.55**
1935-1945	0.0039	0.0037	0.82
1946-1955	0.0087	0.0078	3.31***
1956-1968	0.006	0.0034	1.39
1935-1940	0.0024	0.0023	0.31
1941-1945	0.0056	0.0054	1.22
1946-1950	0.005	0.0044	1.1
1951-1955	0.0123	0.0111	4.56***
1956-1960	0.0148	0.0128	4.89***
1961-June/1968	0.0001	-0.0029	-0.8

* Statistically significant at the 10% level
** Statistically significant at the 5% level
*** Statistically significant at the 1% level

(Source: Fama and MacBeth, 1973: 622)

3.3.1.3 Gibbons (1982)

Gibbons (1982) and Stambaugh (1982) expanded this area of research by developing alternative approaches to determine the validity of the CAPM as a whole, including the risk-free proxy/zero-beta portfolio debate. They both used a multivariate regression model (MVRM) comprising both the market model and the traditional CAPM (Gibbons, 1982: 4-6). If both models hold simultaneously, Gibbons (1982: 6) showed that the CAPM places the constraint, depicted in equation 3.45, on the intercept of the market model. Gibbons (1982: 7) used this constraint as the basis for his test of the validity of the CAPM, as if this relationship (tested as the null hypothesis) holds, then the model must be consistent with the data; whilst if it is rejected in favour of the alternative hypothesis, $\psi_j \neq \gamma_0 (1-\beta_i)$, it can be concluded that neither form of the model is consistent with the data over the period examined.

$$\psi_j = \gamma_0 (1-\beta_i) \tag{3.45}$$

Where:

- ψ_j is the intercept of the market model (as per equation 3.42)
- γ_0 is the zero-beta portfolio return or the risk-free rate proxy yield
- $\gamma_0 (1-\beta_i)$ is the intercept of the CAPM

(Gibbons, 1982: 5-6)

The problem that Gibbons (1982: 8) encountered was that whilst the coefficients of the market model estimated using OLS under the alternative hypothesis would be efficient, this was not true for estimation under the null hypothesis, as the reduction in parameter space meant that the error terms would not be normally distributed. Consequently, Gibbons (1982: 8) proposed the use of Maximum Likelihood (ML) to estimate the coefficients; however, he employed a one-step Gauss-Newton procedure, where the coefficient estimates still had the same desirable properties of consistency and asymptotic efficiency associated with the ML estimators, but the approach was much simpler (Gibbons, 1982: 10).

The one-step Gauss-Newton method linearises the imposed restriction (equation 3.45) using a Taylor series expansion around consistent estimators of the intercept and the coefficient of the market risk premium (Gibbons, 1982: 10), as shown in equation 3.46 (on the following page).

$$(R_{it} - \widehat{\gamma}_0 \widehat{\beta}_i) = \gamma_0 (1- \widehat{\beta}_i) + \beta_i (R_m - \widehat{\gamma}_0) + \eta_{it} \quad (3.46)$$

Where:

- $\widehat{\gamma}_0$ is the estimate of the intercept of the SML
- γ_0 is the coefficient value for the intercept of the SML
- $\widehat{\beta}_i$ is the estimate of the security beta
- β_i is the coefficient value for the security beta

(Gibbons, 1982: 10)

Estimates of the minimum return were calculated using the explicit methodology of Black *et al* (1972), and the values of beta were obtained from ranked portfolios of shares, formed using the same period estimates of beta[12] (Gibbons, 1982: 10). Gibbons (1982: 12-13) estimated equation 3.46 (the restricted regression) and the market model depicted in equation 3.44 (the unrestricted regression) and compared the statistical similarity between these two regressions using a likelihood ratio test (LRT).

The results provided overwhelming evidence against the null hypothesis (p values equal to zero); that is, the CAPM did not adequately explain the data over the period 1926 to 1975. As a consequence of these findings, Gibbons (1982) did not examine the relationship between the risk-free rate proxy returns and the intercept; instead only highlighting the greater efficiency of the one-step Gauss-Newton estimates of the minimum required return compared to those obtained using the approach of Black *et al* (1972).

3.3.1.4 Stambaugh (1982)

Stambaugh (1982: 246), whose primary focus was to probe how the choice of market portfolio influenced the statistical inferences of the CAPM, also tested for linearity between risk and return, the size and sign of the market risk-premium, and the equality of the intercept of the SML and risk-free proxy returns. The equation he tested was that of the market model, depicted in equation 3.44. As with earlier studies, Stambaugh (1982: 251) tested this equation with portfolios of shares, rather than individual securities. He expanded the previous methodologies of forming portfolios based on estimates of beta, by instead compiling industry based portfolios, including not only ordinary shares, but also preference shares and bonds (Stambaugh, 1982: 251). However, in a MVRM framework, Stambaugh (1982: 253) faced the same problem as

[12] Given the different form of the test statistic conducted by Gibbons (1982: 12) it was not necessary to rank shares based on out of sample beta estimates.

Gibbons (1982) that his coefficient estimates using OLS would be inefficient and he thus estimated the equation using ML.

To test the relationship between risk and return, Stambaugh (1982: 250) employed a Lagrangian Multiplier (LM) test, rather than the LRT of Gibbons (1982), arguing that in a MVRM, the latter test tends to reject the null hypothesis too frequently as the number of portfolios on the left-hand side of the equation increases. Accordingly, his results provide a complete contrast to those of Gibbons (1982), as the p-values associated with the LM test were, on average, greater than 70 percent across all four market portfolio variations; therefore indicating that the fundamental CAPM relationships did hold over the period examined (Stambaugh, 1982: 254). Given these positive findings, Stambaugh (1982: 255) proceeded to examine the coefficient estimates, including a statistical test of the equality of the intercept of the model and the risk-free proxy (one-month T-Bills). The results of this test, depicted in Table 3-4, show that the null hypothesis that these two values were equal from 1953 to 1976 was rejected across all four market portfolios, as all p-values were less than one percent. Furthermore, Stambaugh (1982: 256) highlighted that these ML estimates of the intercept had lower standard errors (greater efficiency) than any of the previous studies, as a result of the inclusion of bonds and preferred shares, which led to a greater dispersion in the values of beta.

Table 3-4: Intercept Estimates and Test Statistics from Stambaugh (1982)[13]

	Estimates of γ_0				
Market Index	2/53-3/59	4/59-5/65	6/65-7/71	8/71-12/76	Test statistic and p-value
1	0.118 (0.026)	0.192 (0.022)	0.172 (0.036)	-0.007 (0.054)	4.82 0.00%
2	0.11 (0.025)	0.19 (0.022)	0.172 (0.036)	-0.01 (0.055)	4.59 0.00%
3	0.099 (0.027)	0.187 (0.027)	0.172 (0.036)	-0.011 (0.055)	4.23 0.00%
4	0.094 (0.026)	0.172 (0.024)	0.177 (0.036)	-0.01 (0.055)	3.83 0.20%

Note: Values in parentheses indicate standard errors

(Source: Stambaugh, 1982: 255)

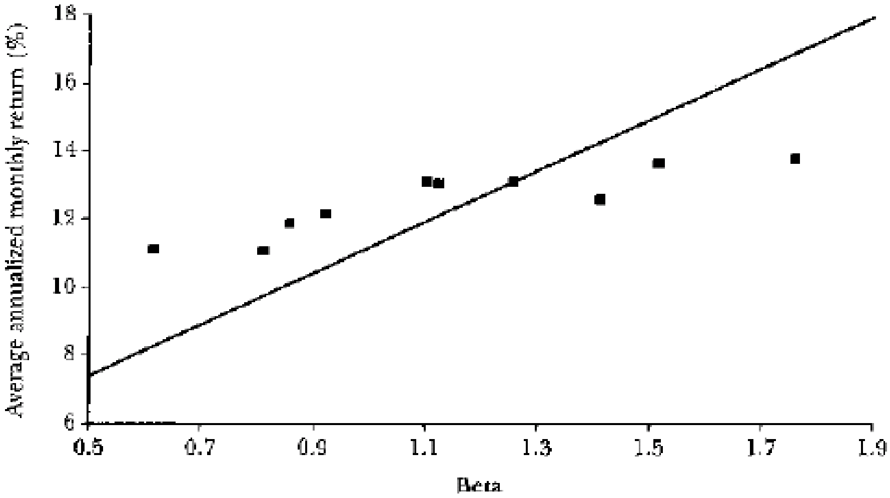
[13] The main purpose of Stambaugh's (1982) study was to show that the inferences made by the CAPM were robust to composition of the market portfolio. Portfolio 1 included all ordinary shares listed on the NYSE; portfolio 2 added corporate bonds, government bonds and T-Bills to portfolio 1 in proportion to their respective market values; portfolio 3 included house-furnishings, cars and real estate in addition to the assets included in portfolio 2; and portfolio 4 represented the same composition as portfolio 3 but with imposed weights for the respective asset classes, rather than weights determined by market value (Stambaugh, 1982: 244).

3.3.1.5 More Recent Empirical Tests

Following these initial studies of the relationship between the risk-free rate proxy and the estimated intercept (and indirectly, the zero-beta portfolio returns), very little empirical work has been dedicated to this particular issue in the U.S., instead focusing on the development of more sophisticated tests of the CAPM. Shanken (1985 and 1986), Gibbons, Ross and Shanken (1989) and Chou (2000) derived more advanced methods of testing the validity of the CAPM compared to the LRT and LM statistics used by Gibbons (1982) and Stambaugh (1982). In these tests, they assume that the intercept of the model is unknown and thus, as with beta, must be estimated. That is, they assume that the intercept is best explained by the zero-beta portfolio returns and hence is unknown, as opposed to the risk-free rate proxy which is a defined variable. In the tests of the CAPM that employ these approaches, the risk-free rate proxy and zero-beta portfolio relationship is thus not discussed (Gibbons *et al*, 1989 and Chou, 2000). The very nature of this research therefore appears to indicate that scholars believe that the zero-beta portfolio provides a more accurate representation of the risk-free asset for the CAPM than T-Bills or T-Bonds. Fama and French (2004: 35) confirmed this stating "…the success of the Black version of the CAPM in early tests produced a consensus that the model is a good description of expected returns."

Fama and French (2004: 32-33), in a paper reviewing the CAPM and the results of numerous empirical tests thereon, did however, conduct an updated version of Fama and MacBeth's (1973) research, using all stocks listed on the NYSE for the period 1928-2003, AMEX (the American Stock Exchange) for 1963-2003 and NASDAQ (The National Association of Securities Dealers Automated Quotations) for 1972-2003. They ranked betas based on out-of-sample estimates and used these rankings to form ten portfolios; the returns of which were then calculated for the next twelve months (Fama and French, 2004: 32). This process was repeated for each year from 1928 to 2003 (Fama and French, 2004: 32). Using the 912 monthly returns on each beta-sorted portfolio, and a value weighted portfolio of all stocks as a proxy for the market portfolio, Fama and French (2004: 32) estimated equation 3.44 for each portfolio. The results are presented graphically in Figure 3-4 (on the following page), in conjunction with the theoretical SML based on the one month T-Bill rate and the average excess market return.

Figure 3-4: Theoretical SML versus Actual SML 1928 – 2003



(Source: Fama and French, 2004: 33)

As with the earlier studies, the actual CAPM relationship (shown by the dotted points) was characterised by a higher intercept and a flatter slope than the theoretical line (shown by the solid line). The intercept value based on the entire period was approximately 10.9 percent, whilst the T-Bill average return was 7.5 percent (Fama and French, 2004: 32). Although this average T-Bill return calculated by Fama and French (2004: 32) appears high compared to the averages calculated by authors for earlier periods and more recent yields, the T-Bond return was, at most, only 50-100 basis points greater than the T-Bill rate. Thus, although the use of a ten-year T-Bond return would more closely resemble the minimum required rate of return, the evidence seems to suggest that the use of any government security is an unsuitable approach for estimating the risk-free rate.

3.3.2 A Direct Empirical Test - Morgan (1975)

Morgan (1975: 363) criticised Black *et al* (1972) and Fama and MacBeth (1975) for not using the available knowledge to estimate the zero-beta portfolio returns directly, as Long (1972) derived two techniques for this purpose (quoted in Morgan, 1975: 363). For his study, Morgan (1975: 365) chose to employ one of these methods to determine a value for the zero-beta portfolio (R_z). The approach derived by Long (1972), as explained by Morgan (1975: 363-364), initially resembles the first part of the derivation of the zero-beta portfolio explained in Section 3.2.1.1; however, the expected return constraint is replaced with the requirement that the covariance between the returns on the portfolio and the market portfolio must equal zero. The formula obtained for the zero-beta portfolio is shown in equation 3.47 (on the following page).

$z = \lambda_2 D1 - \lambda_1 m$ (3.47)

Where

- z is the minimum-variance zero-beta portfolio vector
- λ_1 and λ_2 are Lagrangian Multipliers
- D is the inverse of the covariance matrix
- 1 is a vector of ones
- m is the market portfolio vector

(Morgan, 1975: 363-364)

The difficulty with employing this equation for the purpose of estimating the zero-beta portfolio returns is the inversion of the covariance matrix; however, Morgan (1975: 364) overcame this problem by deriving an additional portfolio (called the global minimum-variance portfolio) in an identical manner but without the covariance constraint, as shown in equation 3.48. Moreover, the constraint that the portfolio weights must sum to one was used to isolate λ_2 in terms of λ_1 (equation 3.49). The final formula used by Morgan (1975: 365) to determine the zero-beta portfolio is shown in equation 3.50.

$q = D1/1^t D1$ (3.48)

$\lambda_2 = (1 + \lambda_1) /1^t D1$ (3.49)

$z = [(1 + \lambda_1) /1^t D1] (q \, x1^t D1) - \lambda_1 m$

$z = (1 + \lambda_1) q - \lambda_1 m$ (3.50)

Where:

- q is the global minimum-variance portfolio vector
- t denotes the transpose of the vector

(Morgan, 1975: 363-364)

The estimate of the intercept of the SML was determined using the methodology of Fama and MacBeth (1973) (Morgan, 1975: 362). Rather then testing the statistical significance of the estimates, R_z and γ_0, to determine their equivalence, Morgan (1975: 366) compared the predictive power of the two models with the different parameters included. With the 275 shares in the sample that had not been used to form the portfolios from which the two parameters were computed, Morgan (1975: 366) estimated the betas for each share based on the market model (equation 3.44). Equations 3.51 and 3.52 (on the following page) were then estimated for each share using both the explicit zero-beta portfolio return and observed market portfolio return (the weighted average of the all the shares in the sample) in the former, and the estimated intercept and market risk premium from the Fama and MacBeth (1973) tests in the latter. The squared error terms from these equations were combined according to equations 3.53 and 3.54 respectively (on the following page). These terms formed the basis of the Wilcoxon (1945)

Matched Pair Signed Rank Test used to evaluate the predictive power of the two models (as per Morgan, 1975: 366).

$$\hat{e}_{jt} = R_{jt} - \hat{R}_z - \widehat{\beta}_j (R_m - \hat{R}_z) \tag{3.51}$$

$$\hat{\eta}_{jt} = R_{jt} - \hat{\gamma}_0 - \hat{\gamma}_1 \widehat{\beta}_j \tag{3.52}$$

$$\phi(e_t) = \sum_{j=1}^{J} \hat{e}^2_{jt} \tag{3.53}$$

$$\phi(\eta_t) = \sum_{j=1}^{J} \hat{\eta}^2_{jt} \tag{3.54}$$

Where:

- $\phi(e_t)$ is the Wilcoxon test statistic

(Morgan, 1975: 366)

The null hypothesis for this test was that the two models predict equally well, whilst the one-sided alternative was that the direct estimates of the zero-beta portfolio return provided a better prediction than the model with the estimated intercept (Morgan, 1975: 366). The results of the hypothesis test showed that the difference in the predictive power of the two models was small (Morgan, 1975: 366-367). This has two important implications; firstly, that the estimated intercept of the SML can validly be interpreted as the zero-beta portfolio return, and secondly, estimating the zero-beta portfolio via an indirect methodology, which is considerably easier than a direct technique appears to provide a good estimate of the zero-beta portfolio.

Morgan (1975: 367-368) also examined the variance of the two parameters using a Box Tiao (1973) test as opposed to the more common F-test of the equivalence of variances due to the effect that large kurtosis may have on the results obtained. In this regard, the results strongly suggest that R_z had a lower variance than γ_0 and thus whilst the estimated intercept closely resembles the minimum-variance zero-beta portfolio returns, this value exhibits greater variation over time than associated with the estimate of the minimum required return.

3.3.3 International Empirical Evidence

In general, the evidence presented in both Sections 3.3.1 and 3.3.2 favours the zero-beta portfolio approach to estimating the risk-free rate for the CAPM in the US; however, it is of value to consider similar studies conducted in the United Kingdom (U.K.), Australia, and in the global marketplace to assess whether the use of a government security as a proxy for the risk-free rate in other markets has more empirical support.

Fraser, Hamelink, Hoesli and MacGregor (2000) tested the CAPM relationships on the London Stock Exchange using the approach of Fama and MacBeth (1973) and three other, more advanced techniques. In this study, the excess return CAPM was subject to investigation, such that the intercept values provided an estimate of the difference between the minimum required return and the risk-free proxy (Fraser *et al*, 2000: 13). Similarly to the Black *et al* (1972) study described in Section 3.3.1.1, the t-statistics associated with the test of the statistical significance of the intercepts therefore represented the test of the equality of the minimum required return and the risk-free proxy returns. Whilst the estimates of the intercept varied considerably across the four estimation procedures, ranging from a high of 1.35 percent to a low of 0.08 percent per annum, all of the t-statistics were below one (Fraser *et al,* 2000: 14); therefore providing support that over the period 1975-1996, the difference between the intercept of the SML and the risk-free proxy in the U.K. was not statistically significant. Thus, the risk-free proxy provides a good means to estimate the risk-free asset in the U.K.

Faff (2001) tested the empirical veracity of the zero-beta CAPM in the Australian market using the LRT statistic suggested by Gibbons (1982), but adjusted for the small sample bias; the coefficients of the CAPM regression, however, were estimated using ML (Faff, 2001: 160). Across twenty-four portfolios comprised of both industrial and resource stocks, the average annualised minimum return for the period 1974-1995 was 14 percent (Faff, 2001: 162-163). Whilst Faff (2001: 162) did not test the statistical difference between this estimate and the average equivalent return on thirteen-week Australian Treasury notes used as the proxy for the risk-free asset, he acknowledged that the estimated parameter provided "…a plausible estimate for the expected return on the zero-beta portfolio" as it was greater than the risk-free proxy yields in keeping with the U.S. evidence. Thus this evidence conforms to the U.S. results that the estimate of the intercept exceeded the risk-free rate proxy returns in Australia.

Chou and Lin (2002: 873), rather than assessing the validity of the zero-beta portfolio approach to estimate the risk-free rate in a specific country, aimed to test the CAPM and this relationship in particular, in an international setting, where the possibility of finding a universal risk-free rate proxy is not "…a well justified assumption". The various tests (based on Gibbons *et al,* 1989 and Chou, 2000) were applied to the returns of seventeen national markets (where each country was treated as a separate asset) from January 1980 to December 1997 (Chou and Lin, 2002: 878). The ML estimate of the monthly minimum return for the entire period and across all countries was 1.63 percent (Chou and Lin, 2002: 882). The authors did not statistically compare this return to the most suitable proxy for an international risk-free rate (the London Interbank Offered Rate); concluding simply that the equivalent monthly yield of the proxy was smaller. Thus, in an international context, the use of a proxy for the risk-free rate in the CAPM is also likely to understate the true risk-free rate.

3.3.4 Interpretation of the Collective Findings

Overall the findings of Black *et al* (1972), Fama and MacBeth (1973), Stambaugh (1982) and Fama and French (2004) indicate that in the U.S. the risk-free proxy differs significantly from the estimates of the intercepts obtained. Furthermore, Morgan (1975)'s direct test of the validity of the zero-beta portfolio approach revealed that there was no observable difference in the predictive power of the CAPM using either the estimated intercept or the directly computed minimum-variance zero-beta portfolio returns. These results appear, as discussed in Section 3.3.1, counterintuitive as the rate that investors require exceeds that of the T-Bill yield, used as a proxy for the risk-free rate in these studies, when the analysis in Section 2.3 indicated that it is more than likely that this security overstates the true risk-free rate. Thus, if these proxies are unsuitable it would be expected that the alternative method for estimating the risk-free rate would provide a lower value than these returns, rather than higher. Several explanations have been proposed in the literature to account for this empirical irregularity (Harrington, 1987: 157), and these are discussed in the following sections in order to obtain a greater understanding of the estimation of the risk-free rate in the CAPM.

3.3.4.1 The Tests of the CAPM

Harrington (1987: 152) indicates that some scholars have argued that the surprising results obtained in the tests of the CAPM can be explained by the fact that the tests use historical data (such as those described in Section 3.3.1) instead of expectations, on which the model is based. Accordingly, to correctly assess the CAPM and in particular the relationship between the risk-free rate proxy and the intercept of the SML, forecasts of the share and market returns should be employed rather than historical values. However, Vandell and Stevens (1982) conducted a test of the CAPM using a reputable bank's forecasts for these values and found that their results mirrored those of the historical tests, as the estimated intercept of the SML was found to be greater than the risk-free proxy returns, whilst the other CAPM relationships were as predicted by theory.

The other criticism related to the tests of the CAPM is that of the methodology employed. The approaches adopted by Black *et al* (1972), Fama and MacBeth (1973), Stambaugh (1982) and Fama and French (2004), whilst differing slightly in implementation, follow the general technique of estimating the intercept of the model by extrapolating the observed data points back to the vertical axis. The problem with this approach is that if the observations are clustered far away from the axis the estimated intercept may not be that accurate (Brooks, 2006: 51-52). Brooks (2006: 52) states that where the observations lie far from the axis, it cannot realistically be expected that the estimate of the intercept will be robust. From Figure 3-4 (page 81), the smallest beta value was in excess of 0.5 and thus the values computed were quite far from the axis. As discussed in Section 3.1.1.4, Stambaugh (1982) expanded his dataset to include preference shares and bonds in order to widen the range of beta values (as these two asset classes tend to have lower betas than associated with ordinary shares). His results provide more efficient estimates of the intercept of the CAPM and yet the same observation is true with respect to this value and the risk-free rate proxy. Thus, whilst this explanation of the empirical anomaly certainly highlights the weaknesses of this approach to testing the model, it does not provide sufficient explanation for the findings identified.

3.3.4.2 The Model

Many scholars have also pointed to the results of several empirical tests of the CAPM (such as Basu, 1977 and 1983; Banz, 1981; Gibbons, 1982; Keim, 1983; Rosenberg, Reid and Lanstein, 1985; Bhandari, 1988; Fama and French, 1992), arguing that the theoretical relationship between risk and return postulated by the model does not hold in practice and that the observed relationship between the estimated minimum return and the risk-free rate proxy is further proof of this (Fama and French, 2004: 38; Reilly and Brown, 2006: 252). Additional research has been conducted on the CAPM, but these tests have focused on more complex versions of the model rather than the traditional and zero-beta forms to which this study has been limited. For example, Breeden, Gibbons and Litzenberger (1989) examined the consumption CAPM; Kothari, Shanken and Sloan (1995) analysed the traditional CAPM but including a risk variable for size; Petengill, Dundaram and Matthur (1995) tested a dual-beta CAPM; and Jagannath and Wang (1996) considered the conditional CAPM.

The findings of these studies regarding the validity of the theory underlying the CAPM is positive; that is, the results indicate that the relationships postulated by the model hold and that by expanding upon the traditional CAPM, the measure of how well the model explains the data increases significantly compared to the traditional CAPM. With respect to the findings of these studies regarding the relationship between the estimated intercept and the risk-free rate, however, the evidence is mixed. Kothari *et al* (1995), Petengill *et al* (1995), and Jagannath and Wang (1996) still find that the estimated intercept is greater than the risk-free rate proxy used (a short-term T-Bill). Jagannath and Wang (1996: 25) confirm this stating "… the estimated value of the average zero-beta rate is rather high when compared to the average T-Bill rate and the average risk premium of stocks. Hence there is cause for concern even though our CAPM specification does substantially better than the static CAPM in explaining the cross-section of average returns on stocks". In contrast, Breeden *et al* (1989) did not find evidence of this empirical irregularity in their estimated model of the consumption CAPM, as the intercept of the SML was not statistically significantly different from the risk-free rate proxy returns used (short-term T-Bill).

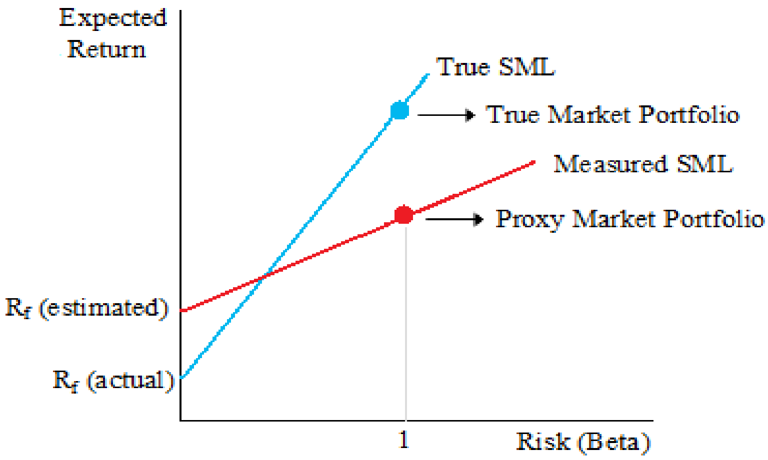
3.3.4.3 The Influence of Negative Beta Assets

More recently, Cloninger, Waller, Bendeck and Revere (2004) proposed an alternative explanation to account for this empirical irregularity that the CAPM incorrectly specifies the returns earned on negative beta assets. The SML is assumed to be positively sloped for all assets meaning that negative beta assets provide a negative risk premium and hence a lower return than the risk-free rate (Cloninger *et al,* 2004: 398). However, Cloninger *et al* (2004: 398) point out that negative beta assets vary to the same degree as an asset with a beta identical in magnitude, but opposite in sign. Shorting a high positive beta asset produces the same size negative beta asset, and investors will therefore not accept any lower return for holding the short position rather than the long position (Cloninger *et al*, 2004: 398). Cloninger *et al* (2004: 398) thus argue that the SML should be "v-shaped" or reflexive meaning that it is negatively sloped for negative beta assets and positively sloped for positive beta assets. Accordingly, the previous tests of the CAPM discussed may include a positive bias in the intercept and a downward bias in the slope estimate. Whilst this explanation for the empirical anomaly is theoretically appealing, the results of tests conducted by Cloninger *et al* (2004: 400-401) do not provide convincing evidence in favour of a reflexive SML.

3.3.4.4 Inappropriate Specification of the Market Portfolio

One explanation which has also been proposed is not that this anomaly is caused by the incorrect specification of the model, but rather that it is a consequence of the incorrect specification of the market portfolio proxy. Roll (1977) argued that the market portfolio can never be correctly specified meaning that it does not satisfy the condition of mean-variance efficiency, because it is impossible to create a portfolio that accurately weights all possible assets that an investor could hold. He inferred that the use of an incorrect market portfolio proxy could result in erroneous beta estimates and a biased intercept value. This can be seen in Figure 3-5 which shows that employing an inappropriate market proxy is a feasible explanation for why the estimated intercept was greater than the risk-free proxy in the empirical tests.

Figure 3-5: The Implications of the Use of an Incorrect Market Portfolio Proxy



(Adapted from: Reilly and Brown, 2006: 258)

Stambaugh (1982), as discussed in Section 3.3.1.4, proved empirically that the inferences of the CAPM were robust to the choice of the market portfolio proxy, and therefore concluded that although the portfolio may not be mean-variance efficient, the relationships postulated by the model still hold. In contrast to this, Reilly and Akhtar (1995) and Reilly and Wright (2004) examined the influence of the choice of market proxy and found that employing a broader market portfolio proxy that includes international shares and bonds (which Stambaugh (1982) did not consider) does result in more accurate parameter estimates. More importantly Reilly and Wright (2004: 72) showed that when the security betas and the SML were estimated using the Brinson GSMI (an index that includes U.S. and international shares and bonds), the estimated intercept was not statistically significantly different from the risk-free rate proxy (six month T-Bills) (Reilly and Wright, 2004: 72). Therefore, the selection of the appropriate market portfolio proxy does have a material effect on the SML estimated and could possibly account for the results obtained.

3.3.4.5 Liquidity and Inflation Premia

The explanations for the empirical irregularity that have been discussed thus far have focused on weaknesses of the model or the tests thereof; however, the possibility remains that the finding is a consequence of the incorrect specification of the risk-free rate proxy, "… perhaps they should have, instead, questioned their choice of proxy for Rf" (Harrington, 1987: 152).

The proxy used in the studies was a short-term T-Bill of either one or three month maturity. For an investor with a longer-term investment horizon, the liquidity and inflation premia included in these instruments may not be sufficient[14]. As discussed in Section 2.3.7, U.S. T-Bills have almost no liquidity risk and therefore do not provide compensation to the investor for the inconvenience of being without their funds for a substantial period in time. Furthermore, in Section 2.3.2, the inflation premium included in the yields of short-term securities was shown to be small in comparison to longer-term securities as investors holding instruments of a longer-term nature are exposed to greater uncertainty with regards to changes in purchasing power. Thus, for an investor with a long-term horizon, a T-Bond with a maturity that matches the investment horizon may be more appropriate than a T-Bill as the inflation and liquidity premia included reflect the greater risk involved with investing for a longer period in time. Consequently, the greater return observed in the empirical tests may reflect greater inflation and liquidity premia that are not incorporated into the yields of T-Bills.

The results of the survey of Bruner *et al* (1998) discussed in Chapter 2 indicated that amongst practitioners T-Bonds are more commonly used to estimate the risk-free rate, which represents an attempt to reconcile practice with empirical evidence. An analysis of the values of both T-Bills and T-Bonds presented in Ibbotson and Sinquefield (1979) and Carleton and Lakonishok (1985), over similar periods to those examined by Black *et al* (1972), Fama and MacBeth (1973) and Stambaugh (1982), revealed that the realised returns on T-Bonds were approximately 20 to 85 basis points higher than for T-Bills. T-Bonds thus more closely resemble the estimate of the intercept of the SML in the U.S; however, Harrington (1987: 151) inferred that even the yields of these instruments were statistically significantly different from the estimated intercepts of the SML across the majority of periods examined in these studies.

3.3.4.6 Divergent Borrowing and Lending Rates

As discussed in Section 2.1.1, the assumption that Sharpe (1964: 433) made in expanding upon the work of Markowitz (1952 and 1959) was that a risk-free asset exists and that it is possible for investors to both borrow and lend at this rate. Thus far in this study, the focus has been on ascertaining the validity of the assumption of the existence of a risk-free asset, with the corollary of being able to both borrow and lend at this rate not considered. However, the

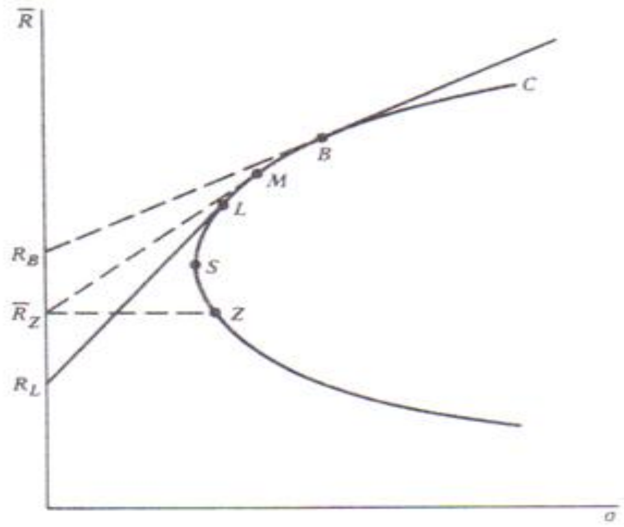
---

[14] Obviously, this is not applicable to speculators who have a very short investment horizon of a few hours to a few days.

violation of this assumption may explain the empirical findings (Reilly and Brown, 2006: 255). T-Bills and T-Bonds are debt instruments issued by the government that investors can hold and consequently the yield on these ‘riskless’ securities represents a close proxy to the lending risk-free rate, but investors certainly cannot borrow at this rate. Individuals are generally more risky than the government as they cannot simply raise taxes or print money to repay debt obligations. Consequently, individuals are likely to borrow at a premium to this rate; the value of the premium being dependent upon their risk profiles. Thus, the proxies used do not take into consideration borrowing costs.

Brennan (1971) examined the issue of divergent borrowing and lending rates on the basis that whilst individuals may be able to both borrow and lend at a riskless rate, these returns are not necessarily equal, with the borrowing rate, denoted R_B, usually greater than the lending rate, R_L (Brennan, 1971: 1199). Brennan (1971: 1201) showed that all net lenders will hold the same portfolio of risky assets, and all investors who are net borrowers will hold an identical risky portfolio. That is, unlike the traditional CAPM where all investors hold the same combination of risky assets, the market portfolio; with divergent borrowing and lending rates, net borrowers and net lenders will hold different, but efficient, portfolios of risky assets. This is clearly evident in Figure 3-6, which shows that those investors holding the riskless asset and portfolio L will lie along the line R_LL, while net borrowers will hold portfolio B and will plot along the line R_BB, but from point B onwards (Fama, 1976: 293).

Figure 3-6: Market Equilibrium with Divergent Borrowing and Lending Rates



(Source: Fama, 1976: 294)

For individuals with no investment in the riskless asset (short or long), they will hold a portfolio somewhere between L and B, as if they were to invest in a portfolio below L or above B on the curve they would be worse off as they could obtain a higher return for a given level of risk, or lower risk for a given return if they held a short or long position in the riskless asset. Therefore, the straight-line segment R_lL, the curve LB, and the straight-line from B onwards represent the efficient frontier with divergent borrowing and lending rates (Elton *et al*, 2007: 322). As depicted in Figure 3-6, the market portfolio must lie between portfolios L and B (Fama, 1976: 294). The reason for this is reasonably intuitive as the market portfolio represents the weighted average of all individual investor's investments in risky assets. Thus, where investors hold either portfolio B or L or an intermediate portfolio; the market portfolio, as the average of these, must lie between the two extreme points. The specific location of M will ultimately depend on the number of net borrowers and lenders. Turnbull (1977: 331) specified the equation for the SML with divergent borrowing and lending rates that Brennan (1971: 1202) alluded to as:

$$E(R_k) = (R_L T_L + R_B T_B) + \beta_k [E(R_m) - (R_L T_L + R_B T_B)] \quad (3.55)$$

Where:

- T_L is the weighted average of the proportion of the investor's funds lent at the risk-free rate
- T_B is the weighted average of the proportion of the investor's funds borrowed at the risk-free rate

(Turnbull, 1977: 331)

Brennan (1971: 1204) referred to the intercept of the SML as the "… market's equivalent risk-free rate …", as it represents the return on a security which has zero-covariance with the return on the market portfolio, and is a weighted combination of the individual's lending and borrowing rates; thus it is constrained to lie between R_L and R_B. The risk-free rate that is used in the CAPM, therefore, should reflect the average of the borrowing and lending rates that are applicable to investors in the market, weighted according to the number of market participants engaging in each activity. The fact that the rate currently used as the proxy only represents the lending rate infers that it is likely to understate the true risk-free rate average as it does not take into consideration the additional return that those investors who borrow funds to invest would require to cover their borrowing costs; thus explaining the irregularity observed.

Obviously the problem with this approach is that in order to implement it in practice it is necessary to have some idea of the relative proportions of investors who borrow and lend, and to be able to identify appropriate riskless borrowing and lending rates. Brennan (1971: 1204) suggested that the minimum-variance zero-beta portfolio return derived by Black (1972)[15] would be a good approximation of this value. The reasoning for this is that it represents a return

[15] Brennan (1971) refers to Black's working paper on the topic, with the citation as a forthcoming article in the Journal of Business.

which has zero covariance with the market and must, as discussed in Section 3.2.5, be greater than the lending risk-free rate. The difference between this model therefore and that discussed in Section 3.2.5 is that in this case, it is assumed that investors can borrow at a riskless rate but that this rate is higher than the lending rate. On the basis that there are at least a few investors borrowing at the riskless rate in the market, the equivalent risk-free rate will be greater than the lending risk-free rate and consequently should closely resemble the zero-beta portfolio rate.

3.4 The Choice Between a Proxy and the Zero-Beta Portfolio

3.4.1 The Choice in the U.S.

Both the direct and indirect tests of the suitability of the use of a proxy to estimate the risk-free rate in the CAPM compared to the zero-beta portfolio favour the use of the zero-beta portfolio approach in the U.S. This is despite the fact that the risk-free rate proxies actually yield a lower return than the zero-beta portfolio. However, it was clear from the analysis in Section 2.1.3 that in both practice and research in the U.S. government securities continue to be employed to estimate the risk-free rate. The question therefore is why this practice persists even though it appears inappropriate.

As is clear from Section 3.3.4, the possibility exists that practitioners and scholars do not employ the zero-beta portfolio approach to estimate the risk-free rate because the empirical results observed may be a consequence of a variety of potential factors, and not necessarily that the risk-free rate is incorrectly specified. However, the views expressed in the literature do not posit this argument as justification for why the zero-beta portfolio approach is not used, but rather that it is because of the difficulty inherent in estimating the correct values for this parameter. That is, the choice between the two approaches is viewed as something of a trade-off. Although the zero-beta portfolio may result in more accurate cost of equity estimates, the difficulty in estimating this value is not seen to be justified by the improvements in accuracy, as T-Bills and T-Bonds are seen as reasonably good proxies. "In theory, the best estimate of the risk-free rate would be the return on a zero-beta portfolio, constructed of long and short positions in equities in a way that produces the minimum-variance zero-beta portfolio. Because of the cost and complexity of constructing minimum-variance zero-beta portfolios, they are not practical for estimating the risk-free rate" (Copeland *et al*, 2000: 215). However, in light of the findings, it is clear that the use of a T-Bond compared to a T-Bill appears to not only more closely satisfy the theoretical criteria established in Chapter 2 for the correct specification of the risk-free rate, but also provides a higher yield which is in keeping with the historical minimum return required by investors.

3.4.2 The Choice Internationally

The international evidence presented in Section 3.3.3 revealed that in Australia as well as in the global market, the same phenomenon was observed as in the U.S. that the proxies understate the intercept of the SML. Thus the choice for practitioners employing the CAPM in these markets is whether or not the increased accuracy of the cost of equity estimates caused by the use of the zero-beta portfolio returns compared to the use of a T-Bill or T-Bond justifies the cost and complexity of using the zero-beta method. As with the U.S., if the alternative approach does not provide statistically significantly more accurate estimates of the cost of equity in the CAPM, the use of a longer-term proxy compared to a short-term proxy is likely to be preferable. However, given the difficulty associated with identifying an appropriate proxy for the risk-free rate in an international setting (Chou and Lin, 2000: 873), it may be harder to justify such practices.

The results of Fraser *et al* (2000) for the U.K. contradict those for other countries as the estimated intercept was not found to be statistically significantly different from the risk-free rate proxy. These findings may be a consequence of several factors which certainly necessitates additional research in order to be able to make a reliable choice between a proxy or the zero-beta portfolio returns to estimate the risk-free rate. These factors may include that the number of net borrowers compared to net lenders is small, that there is not a large difference between riskless lending and borrowing rates, that the risk-free rate proxy used may be high as it includes substantial premia that mean that it lies quite close to the zero-beta portfolio rate or any combination of these factors.

3.4.3 The Choice in South Africa

From the discussions in Chapter 2, it is clear that U.S. practices regarding the estimation of the risk-free rate in the CAPM have generally been extrapolated and applied in South Africa. But, whilst the deviations of the U.S. proxy securities from the theoretical requirements are considered small enough to be ignored, the same is not necessarily valid in South Africa, as the deviations of the proxy securities from the criteria are more pronounced.

Following a thorough investigation it appears as though only one study, Van Rhijn (1994), has attempted in any form to address this issue in South Africa, and similarly to the U.S. based research, this approach followed the indirect methodology. However, as explained in Section 1.1.3, because of data constraints, Van Rhijn (1994) was unable to conduct any statistical tests to determine the difference between the intercept and the risk-free yield, simply concluding that the difference between the two values was small. This result seems to suggest that South Africa mirrors the pattern observed in the U.K. that the proxy appears appropriate, but because of the small sample on which van Rhijn (1994) based his tests it is not possible to obtain meaningful conclusions. It therefore remains to determine the correct approach to estimate the risk-free rate for applications of the CAPM in South Africa, so as to ensure that the values used for this purpose are accurate and hence the cost of equity estimates are valid.

3.5 Chapter Summary

In this chapter, an alternative approach to estimating the risk-free rate was derived based on the model developed by Black (1972), known as the minimum-variance zero-beta portfolio. Black (1972) postulated that using this parameter as the risk-free rate would be applicable in a market where no suitable proxy for the risk-free asset exists. Several of the indirect tests that have been conducted on the applicability of this approach compared to the use of government securities were examined, with the results indicating that the intercept of the SML exceeded the risk-free proxy returns. Many of these scholars consequently proposed the zero-beta portfolio approach as a more suitable technique for estimating the risk-free rate; this conclusion was confirmed in the direct study of Morgan (1975). The explanations presented for the surprising result that intercept was higher than the proxy yields were analysed and whilst these findings may be a consequence of the CAPM not adequately describing reality or the market portfolio being incorrectly specified, a feasible explanation may also be that the risk-free rate proxies used only represent the riskless lending rate and not the average of the borrowing and lending rates.

It was therefore shown that in the U.S., despite empirical evidence in support of the zero-beta portfolio approach, government securities are still used to estimate the risk-free rate; the reason being that the complexity of estimating the zero-beta portfolio returns does not validate the increased accuracy of the cost of equity estimates. However, simply extrapolating the U.S. practices to South Africa is not necessarily appropriate because of the differences between the two countries. In South Africa no comprehensive study has been conducted on the relationship between the estimated intercept of the CAPM, the risk-free rate proxy and the minimum-variance zero-beta portfolio returns and thus it is not possible to conclude which approach to estimating the risk-free rate is best. The methodology that will be followed in order to perform such an analysis in South Africa will be described in Chapter 4.

CHAPTER 4
METHODOLOGY

4.1 Introduction

The research question posed in Chapter 1 of this study was to determine the most suitable approach to estimate the risk-free rate for use in the CAPM in South Africa. In light of the analyses conducted in Chapters 2 and 3, in order to fully address this question it was deemed necessary to examine empirically whether government securities or the minimum-variance zero-beta portfolio provide the most accurate estimate of this parameter in South Africa. The methodology that was followed in this regard is described in this Chapter and entails three distinct empirical analyses. Firstly, the techniques followed to estimate the historical minimum return required by investors (as reflected by the intercept of the SML) and the zero-beta portfolio returns are described. Thereafter, the hypotheses to be tested are outlined, in conjunction with the methods employed to test the equality of the zero-beta portfolio returns, the risk-free rate proxy and the minimum required return. Finally, the procedures employed to assess the forecasting accuracy of the CAPM, using both the zero-beta portfolio returns and the risk-free proxy returns are examined.

As discussed in Section 3.3.4.1, the CAPM is based on expectations and thus to correctly estimate the parameters in this study expected values should be used. The unavailability of large-scale systematic data on expectations has been circumvented in the U.S. studies by employing historical data. There are three primary reasons postulated to justify this adaption:

1. If expectations are, on average, correct, then over a significant period of time, actual events can be taken as a surrogate for expectations (Elton and Gruber, 1995: 342).
2. Studies that have employed expectational data (such as Vandell and Stevens, 1982) have identified the same trends and relationships as those using historical data (Harrington, 1987: 152).
3. If both the market model and CAPM hold simultaneously, then the CAPM can be seen to be both an expectational model and a historical model. Elton and Gruber (1995: 342-343) proved this mathematically, as outlined below.

The market model specified in equation 3.42:

$$R_{it} = \psi_j + \beta_i R_{mt} + e_{it} \qquad (3.42)$$

Introducing expectations[16]:

$E(R_{it}) = \psi_j + \beta_i E(R_{mt})$

$E(R_{it}) - \psi_j - \beta_i E(R_{mt}) = 0$

Adding this to the right-hand side of equation 3.42 and simplifying,

$R_{it} = \psi_j + \beta_i R_{mt} + e_{it} + \{E(R_{it}) - \psi_j - \beta_i E(R_{mt})]\}$

$$R_{it} = E(R_{it}) + \beta_i [R_m - E(R_{mt})] + e_{it} \quad (4.1)$$

But, according to the traditional CAPM specified in equation 1.1,

$$E(R_{it}) = R_f + \beta_i [E(R_{mt}) - R_f] \quad (1.1)$$

Substituting equation 1.1 into 4.1:

$R_{it} = \{R_f + \beta_i [E(R_{mt}) - R_f]\} + \beta_i [R_m - E(R_{mt})] + e_{it}$

$$R_{it} = R_f + \beta_i [R_{mt} - R_f] + e_{it} \quad (4.2)$$

If Black's CAPM (equation 3.34) holds instead of the traditional CAPM:

$$E(R_j) = E(R_z) + \beta_j [E(R_m) - E(R_z)] \quad (3.34)$$

$$R_{it} = R_{zt} + \beta_i [R_{mt} - R_{zt}] + e_{it} \quad (4.3)$$

Equations 4.2 and 4.3 thus confirm that the CAPM can be viewed as a historical model as well as an expectational model. Therefore in light of these three justifications, in this study the research question was addressed by examining historical data rather than forecasted values for the parameters.

4.2 The Dataset

4.2.1 Time Period and Frequency of Data

The period chosen for this analysis was constrained by the availability of reliable data. Share prices and dividend yields (the justification for the inclusion of dividends is discussed in Section 4.2.3) for all shares listed on the JSE in every year were collected from the JSE statistics and records department from 1990 to 2008[17]. None of the other databases accessed (Sharenet, McGregor BFA Station and I-Net Bridge) provided consistent values for all shares

[16] $E(\psi_j) = \psi_j$ as the expected value of a constant is the constant, whilst $E(e_{it}) = 0$, as per the definition of the error term (Brooks, 2006: 146).

[17] Over the years 1990 to 1993 there were some missing observations in the dataset, but these were obtained from historical Financial Mail records to provide a complete sample.

any further back than this date and thus the nineteen-year time period, January 1990 through to December 2008, was chosen for this study. However, as will be explained in Section 4.3.1, the first three years (1990-1992) were allocated for the initial estimation period and consequently the period over which the parameters were computed and the hypothesis tests conducted was January 1993 to December 2008 (sixteen years).

The frequency of the data to be used was determined based on similar international studies, which predominantly employed monthly observations for the share prices (Black *et al*, 1972; Fama and MacBeth; 1973; Stambaugh; 1982; Faff, 2001; and Fama and French, 2004). Monthly closing share prices and monthly dividend yields were thus gathered from the JSE.

4.2.2 Assets Included

U.S. based research on the appropriateness of the risk-free proxy and zero-beta portfolio utilised all shares listed on the NYSE for every period of the study (Black *et al*, 1972; Fama and MacBeth, 1973; and Stambaugh, 1982), and more recently Fama and French (2004) employed not only all shares listed on the NYSE, but also AMEX and NASDAQ. In comparison, the study of Van Rhijn (1994) used shares listed only on the industrial sector of the JSE; thereby substantially limiting the scope of the analysis. Thus this study utilised all possible shares listed on the main board of the JSE, at the beginning of each year, over the period 1990 to 2008 inclusive. Adjusting the number of shares used per year ensured that the sample was not biased towards low-beta shares; as high-risk shares tend to disappear from the market more quickly, some of which would be excluded if the condition that only shares that remained listed during the entire period had been implemented (Morgan, 1975: 365; Van Rensburg and Robertson, 2003: 8; Mutooni and Muller, 2007: 17). Shares that were delisted within a particular year were excluded from the analysis from the end of the previous year provided a full calendar year of trading history was available. Similarly, for new shares listed during a year, they were only included in the analysis from the beginning of the following calendar year for which a full year of trading history was available. Information on the dates of new listings and de-listings was also obtained from the JSE statistics and records department, which facilitated this process.

Shares listed on the development and venture capital boards were not considered in this study, as they tend to be very small and highly illiquid (Mutooni and Muller, 2007: 17). In addition to this, very little research has been conducted on these companies and according to Mutooni and Muller (2007: 17) they may exhibit substantial unsystematic risk. Consequently, the shares listed on Alt-X, which was introduced in 2003 to replace these two largely defunct boards, were also not considered in order to maintain consistency.

The study of Stambaugh (1982), referred to in Section 3.3.1.4, differed from previous research on the estimation of the intercept of the SML, by including preference shares and bonds as part of the dataset. It was this expansion beyond ordinary shares that Stambaugh (1982: 256) believed contributed to the greater efficiency of his parameter estimates compared to previous studies. For this reason, these two asset classes were also included in this analysis. Price and dividend information on all preference shares listed on the JSE over the period 1990 to 2008 was gathered from the same source as the ordinary share price data. Due to the limited number of traded bonds in South Africa, rather than using individual instruments combined into portfolios, three BESA bond indices were selected; the 1-3 year, 7-12 year, and 12-30 year maturities. Information on the indices was obtained from BESA, in conjunction with McGregor BFA Station. These bond indices however, were only formulated in January 2001 and thus these assets were not included in the analysis prior to this date.

In Section 2.4, the three month T-Bill, R157 government bond, three month BA and three month NCD were used as proxies for the risk-free rate. For this part of the study, however, only the T-Bill and R157 bond were used, as the results of the analysis revealed that the yields of the two short-term private sector instruments closely follow those of the T-Bill. Therefore in the context of the hypothesis tests to be conducted, they were unlikely to provide any different results than those obtained with the T-Bill. T-Bills and T-bonds, due to their varying maturities, do exhibit different yields, and respond differently to changes in the market and investors perceptions; and consequently may result in contradictory conclusions in the hypothesis tests. The yields of these instruments were only required from the beginning of 1993 and not for the initial estimation period (1990-1993), and consequently, there were no concerns with respect to the availability of data on the R157 bond yield, as this instrument was issued by the South African government on the 18th of January 1991 (BESA 2009b).

In order to conduct the analysis it was also necessary to identify a suitable proxy for the market portfolio. Theoretically, the market portfolio should consist of all risky assets in the universe; that is, not only ordinary shares, but also bonds, unit trusts, real estate, coins, stamps etc. (Van Rhijn, 1994: 79). Compiling a portfolio consisting of the returns on all of these assets would be an impossible task and for this reason a comprehensive market index is normally used as a proxy. As discussed in Section 1.1.1, several studies in South Africa, such as Venter *et al* (1992) (quoted in Ward, 1994: 102) and Bowie and Bradfield (1993) have suggested the use of appropriate major sector indices as proxies because of the segmentation of the South African share market. Ward (1994) found that this segmentation was not apparent in statistical testing and therefore suggested the ALSI for this purpose. More recently, Correia and Uliana (2004) confirmed the suitability of the ALSI as a proxy, although they did acknowledge that its use may overestimate the beta values for industrial and financial shares.

In contrast, the U.S. studies referred to previously on this topic have typically made use of an equally weighted index of all ordinary shares as the proxy for the market portfolio. None of the studies provide any justification for this reasoning, but perhaps the unavailability of a broad-based index, especially over early periods of their studies (such as the 1920s and 1930s), or the desire to ensure a more significant relationship between each asset and the market portfolio (i.e. a higher beta) were contributing factors. To implement a similar approach in this study has certain weaknesses; most notably the fact that it does not take cognisance of the vast disparity in sizes of shares listed on the JSE and consequently its accuracy would certainly be questionable. It was thus considered most appropriate to employ the ALSI as the market portfolio in this analysis. The monthly index values (adjusted to include dividends) for the period 1990 to 2008 were also obtained from the JSE.

4.2.3 The Calculation of Returns

Before the returns were computed, several adjustments had to be made to the data. Over the entire period outlined for the study (1990 to 2008) a variety of corporate activities occurred which affected the returns earned on the shares. Such activities included share splits, swaps, mergers and acquisitions, and name changes. The data collected from the JSE had been back-dated to reflect share splits and swaps; however, appropriate adjustments had to be made for name changes and mergers and acquisitions. The double entry of companies resulting from pure name changes was eliminated based on the Who Owns Whom record of name changes over the period. Adjusting for mergers and acquisitions however, proved to be a more difficult task.

Given the way the shares are combined into portfolios based on the beta-sorting procedure (to be discussed in Section 4.3.1), it was ideal to have shares listed for as long a period as possible; thus to treat both the acquiring and target companies as delisted as of the date the acquisition occurred and the consolidated company as newly listed on the same date impeded upon this[18]. However, combining the share prices of the acquiring and the newly formed company in some instances resulted in share price movements in excess of 40 percent per month which had a substantial impact on the period beta estimates computed (as acknowledged by Mutooni and

[18] As explained in the preceding section, delisted shares were removed from the end of the previous calendar year and newly listed shares only introduced from the beginning of the following year.

Muller, 2007: 18). Consequently, dealing with mergers and acquisitions was viewed as a trade-off between obtaining shares listed for longer periods and beta estimates not significantly impacted by outlier observations. Based on the Who Owns Whom records historical prices of the acquiring company were combined with the merged entity's prices where substantial similarities were apparent between the acquiring company and the merged entity. If this was not the case, then both the target and acquiring company were recorded as delisted from the date of the merger and the merged entity as a newly listed firm from that same date.

The question of whether dividends should be included in the calculation of total returns earned by holding shares is not clear, as an analysis of the financial literature both internationally (Sharpe and Cooper, 1972; Fama and MacBeth, 1973; Soe and Dash, 2008) and in South Africa (Van Rensburg 2001; Van Rensburg and Robertson, 2003; Samoulihan, 2007; Samoulihan and Shannon, 2008) revealed studies where they have been both included and excluded. Sharpe and Cooper (1972: 50) proved in their study that the inclusion of dividends did not significantly improve the estimates of beta obtained and therefore concluded that given the difficulty in obtaining data on dividends and verifying this information, they can be excluded from the calculation of total returns. The majority of the studies discussed in Chapter 3 (Black *et al*, 1972; Fama and MacBeth, 1973; Gibbons, 1982; Stambaugh, 1982 and Fama and French, 2004) do not discuss the issue of dividends explicitly stating simply that the information on share returns was gathered from the Centre for Research in Security Prices. Black *et al* (1972: 10) and Fama and MacBeth (1973: 614) however note that this information is adjusted for dividends. Thus it appears that these authors do not follow the conclusion reached by Sharpe and Cooper (1972). More recently, the importance of dividends in an investor's portfolio has come to the fore because of their role in protecting portfolio returns during market downturns. Soe and Dash (2008: 2) show that in the U.S. the proportion of total returns accounted for by dividends has increased by 30 percent in the past decade. It therefore appears that excluding dividends from the calculation of total returns may understate the true value and potentially hide important relationships between those shares that pay substantial dividends and those that do not.

An analysis of recent South African literature confirmed the inclusion of dividends in the computation of total returns by several authors, such as Van Rensburg (2001: 5), Van Rensburg and Robertson (2003: 8) and Mutooni and Muller (2007: 18). In lieu of this evidence, both internationally and within South Africa, dividend yields were included in the computation of total returns in this study.

The common approach to determine the total returns on both ordinary and preference shares is to compute the natural log of the end of period share price and dividend less the natural log of the beginning period share price, as illustrated in equation 4.4. This closely resembles equation 2.8 used to compute the T-Bill and T-Bond returns and yields the continuously compounded return from holding the share. The monthly dividend information obtained from the JSE was expressed as an annual percentage and thus it was not possible to determine the exact amounts and dates dividends were paid, as they had been smoothed across each month. Consequently, the continuously compounded returns could not be calculated without the actual dividend amounts and thus holding period returns were determined for the ordinary and preference shares, as per equation 4.5, which indicates the actual return earned by holding the asset over the period[19]. The dividend yield was obtained by dividing the equivalent annual yield by twelve, in accordance with the approach adopted by Mutooni and Muller (2007: 18), as given the nature of the data gathered it was not necessary to compute the monthly dividend based on the assumption that the annual yields were compounded. The same procedure was employed for the calculation of total returns for the preference shares.

$$\text{Returns} = [\ln (P_1 + D_1) - \ln (P_0)] * 100 \quad (4.4)$$

$$\text{Holding Period Returns} = [(P_1 - P_0)/P_0 + D_1/P_0] * 100 \quad (4.5)$$

Where:

- P_1 is the price at the end of the month
- P_0 is the price at the beginning of the month
- D_1 is the rand value of the dividend

(Brooks, 2006: 7-8)

The three bond indices chosen are total return indices meaning that the index value represents both the bond price and accrued interest earnings, and thus no adjustments had to be made for interest. To ensure consistency with the share returns, the holding period returns were also calculated for the bonds, as the difference in the index values over the period divided by the original index value, as can be seen in equation 4.6. Finally for the market portfolio returns, the identical approach to the bond indices was employed, as the index values obtained were, as specified in Section 4.2.2, adjusted for dividends paid by the firms comprising the index.

$$\text{Holding Period Returns} = (P_1 - P_0)/P_0 * 100 \quad (4.6)$$

4.3 The Estimation of the Minimum Required Return

In Section 3.3.1, several empirical tests of the relationship between the risk-free rate proxies and the intercept of the CAPM were discussed, from which it was evident that there is no

[19] To calculate the returns for January 1990, the share prices were obtained as at the 31st December 1989 from the Financial Mail.

agreement in the finance literature as to the best approach to estimate the minimum required return. As specified in Chapter 1, one of the sub-objectives of this study was to determine which of these techniques is the most efficient, and thus three approaches were implemented. The methods that were followed are based on the work of Black *et al* (1972), Fama and MacBeth (1973), and Stambaugh (1982), as it is clear from Table 4-1 that these three techniques have been employed by other scholars to estimate the CAPM parameters, and in particular the intercept. The most efficient of these estimates were then utilised in the tests of equality with the risk-free rate proxy and the zero-beta portfolio returns (See Section 4.6). That is not to say however, that these approaches were replicated in their entirety in this empirical investigation, but instead were adjusted to account for more recent developments, as well as for the unique characteristics of the South African market.

Table 4-1: Approaches Employed in Empirical Tests of the CAPM

Approach of Black *et al* (1972)	**Approach of Fama and MacBeth (1973)**	**Approach of Stambaugh (1982)**
Gibbons (1982)	Morgan (1975)	Faff (2001)
Jagannath and Wang (1996)	Breeden *et al* (1989)	Chou and Lin (2002)
	Fama and French (1992)	
	Kothari *et al* (1995)	
	Pettengill *et al* (1995)	
	Jagannath and Wang (1996)	
	Fraser *et al* (2000)	
	Fama and French (2004)	

4.3.1 Ordinary Share Portfolio Formation

Black *et al* (1972: 9) were the first scholars to suggest estimating the CAPM parameters based on portfolios of ordinary shares consisting of all securities in the sample, rather than based on the individual assets. This procedure grew rapidly in popularity as it results in more efficient and precise values for the parameters (Fama and French, 2004: 31). Estimates of beta based on individual securities are usually characterised by the error in the variables problem which is reduced substantially in a portfolio because the estimates tend to offset each other provided that the errors in the individual beta estimates are not perfectly positively correlated. The downside to this approach is that it may mask certain risk-return relationships as the grouping procedure contracts the range of betas and decreases statistical power (Fama and French, 2004: 31). But, in light of the widespread usage of this approach (such as Blume and Friend, 1973; Black *et al*, 1972; Fama and MacBeth, 1973; Morgan, 1975; Stambaugh, 1982; Fama and French, 1992, Kothari *et al*, 1995; Pettengill *et al*, 1995; Jagannathan and Wang, 1996; Fama and French, 2004) it is apparent that the benefits of forming portfolios outweigh the disadvantages and hence this methodology was adopted.

According to Fama and French (2004: 31), the most widely accepted method for forming portfolios, is the beta-sorting procedure originally suggested by Black *et al* (1972: 9). As briefly described in Section 3.3.1.1, this process entails classifying shares based on their betas, with the first portfolio containing those shares with the highest betas up to the last portfolio consisting of the assets with the lowest betas, so as to obtain the maximum possible dispersion (Black *et al*, 1972: 9). However, these portfolios cannot be formed based on in-sample beta values, as this introduces a selection bias in that the high beta securities will tend to be biased upwards above their true values and low beta securities biased downwards (Black *et al*, 1972: 9; Fama and MacBeth, 1973: 615). To overcome this problem, Black *et al* (1972: 9) suggested using historical estimates of beta to rank the securities as these values are likely to be highly correlated with the required period beta, but observed independently. Thus, in their study, Black *et al* (1972: 11) used the immediately preceding five years worth of data (1926-1930) to estimate betas for all securities listed on the NYSE as of January 1931. Using the beta estimates, the shares were then allocated to portfolios, with the ten percent of shares with the highest betas being combined equally in portfolio 1, up to portfolio 10, with the ten percent of shares with the lowest beta values (Black *et al*, 1972: 11).

To avoid assuming that the beta estimates remained stationary over the entire period of the study, the portfolios were reformed each year based on beta estimates using the immediately preceding five years worth of data (Black *et al*, 1972: 11). Share price information had to be available for at least 24 months for the beta values to be estimated; otherwise Black *et al* (1972: 11) did not include the shares in the analysis until such information was obtainable. Fama and MacBeth (1973) and many more recent scholars (such as Kothari *et al*, 1995; Pettengill *et al*, 1995; Fama and French, 2004) employed an identical approach, tweaking only the length of the periods used for testing and portfolio formation.

This approach however has been criticised with respect to the validity of the instrument used; that is, how reliable is the assumption that historical estimates of beta provide a good approximation of future values (Stambaugh, 1982: 251). To avoid relying upon this assumption, Stambaugh (1982: 251) adopted a more straight-forward approach of classifying shares into portfolios based on industry affiliations, in a similar way to Morgan (1975). This approach satisfies the requirement of maximum possible dispersion of beta values as risk varies across industries, but is reasonably constant within an industry as reflected by similar beta values for the shares. Stambaugh's (1982) results were consistent with those of previous studies; thus positing industry affiliations as a reliable alternative to beta-sorting as the basis for portfolio formation. However, Fama and French (1992: 432) vindicated the use of the beta-sorting procedure by illustrating that historical estimates of beta based on a minimum of thirty-six months and a maximum of sixty months previous data provide a statistically valid approximation for the true beta values.

Van Rensburg and Robertson (2003: 8) employed a beta-sorting procedure for allocating shares to portfolios in their tests of the CAPM in South Africa; however, their procedure differed slightly to the methods described as they used only five portfolios, the composition of the portfolios was changed each month as opposed to each year, and they used between twelve and 30 months on a rolling basis to estimate the average monthly betas. However, the results obtained, in contrast to earlier research on the reliability of the CAPM in South Africa (Bradfield *et al*, 1988; Bradfield and Barr, 1989; Van Rhijn, 1994; and Ward, 1994), indicated that risk and return were not positively related (Van Rensburg, 2003: 10). Whilst these results may reflect that the CAPM is an inappropriate model for the South African market, it is also possible that the beta-sorting procedure is not suitable in South Africa as it may hide certain risk-return relationships.

In addition to this potential evidence against the beta-sorting procedure in South Africa, there are other problems associated with applying this methodology in this particular study. Firstly, to implement the beta-sorting procedure would ideally require at least 36 months (based on the results of Fama and French, 1992) of data to obtain beta values for all the shares listed on the JSE at the beginning of the period (1990); however, as explained in Section 4.2.1, trying to obtain data for the period 1987-1989 in South Africa is exceptionally difficult. Given this constraint, the dataset used for the entire analysis would have to be contracted by three years (1993-2008) to allow the observations for 1990-1992 to be used to estimate the beta values to allocate all the shares listed on the exchange as of the beginning of 1993 to the appropriate portfolios. This contraction of the dataset may be largely inconsequential in a study spanning 50 years (such as those U.S. studies mentioned), but is less desirable in a study such as this.

Further to this, implementing the beta-sorting procedure will effectively remove all shares listed on the JSE post 2006 from the analysis as there will be insufficient information to estimate beta values for these shares. However, of the total number of shares listed as of the 31st December 2008, 20.6 percent were listed from 2006 onwards[20] (JSE Annual Report, 2008: 5). To exclude such a large percentage of shares from the analysis may hamper the accuracy of the parameter estimates obtained.

[20] 61 companies were listed on the main board of the JSE from January 2006 to December 2008 and there were 294 companies listed on this board at the end of 2008.

Forming portfolios based on industry affiliations, whilst overcoming some of the abovementioned limitations, brings with it its own shortcomings. There is no one standardised classification system, as it depends to large extent on the degree of specialisation which is inferred. The Financial Mail for example, usually classifies the JSE shares into ten major industries and 19 sub-industries, with Sharenet also identifying ten major sectors, but 26 sub-industries, and I-Net Bridge 38 (as of 31st December 2008). Too wide a focus will result in portfolios consisting of one or two shares, whilst too few may result in shares being grouped together that exhibit little similarity in terms of beta values and variations. In this respect, the beta-sorting procedure, despite using up valuable data, does provide a statistically based sorting procedure, as opposed to industry classifications which rely more on subjectivity.

In light of the fact that neither of these two techniques appears to provide a full-proof approach in the South Africa market, from a theoretical perspective at least, both procedures were used; that is, two separate datasets of portfolios were created using both the beta-sorting and industry-sorting procedures. These two sets of portfolios were employed for the Black *et al* (1972), Fama and MacBeth (1973), and Stambaugh (1982) techniques to estimate the intercept, which were then compared to ascertain which set of values was more efficient. Given the decision to implement the beta-sorting procedure and thereby use the first three years of data for the initial estimation period, the period over which all parameters were estimated and hypothesis tests conducted was constrained to January 1993 to December 2008. In this way, it was also possible to ascertain whether the results of Van Rensburg and Robertson (2003) were a function of the beta-sorting procedure or whether the CAPM is not entirely valid in South Africa.

For both grouping procedures, it was necessary to determine the optimal number of portfolios to form each year. The literature provides very little guidance with regards to the optimal number of portfolios, or number of shares to include therein. Black *et al* (1972: 11) and Fama and French (2004: 32) constrained the number of portfolios to ten, whilst Stambaugh (1982: 251) employed 19 ordinary share portfolios, and Fama and MacBeth (1973: 615) 20. The number of shares listed on the JSE over the period of study fluctuated dramatically; however, the number of shares is considerably smaller than listed on the NYSE. Therefore, irrespective of the number of portfolios chosen, it was probable that the number of assets included in each portfolio would be smaller than those formed in the U.S. studies. The quantity of portfolios selected has a direct impact on the reliability of the cross-sectional regression coefficient estimates as they represent the sample to be used; consequently, selecting too few portfolios will adversely affect the accuracy of the results. Although one of the sub-objectives of this study is to conduct a direct comparison of the efficiency of the two portfolio grouping procedures, it was not considered imperative that both procedures have the same number of cross-sectional units provided that in both cases the number to be employed was sufficient to ensure reliable parameter estimates.

For the industry-based portfolios, it was not possible to simply select one of the current industry classification systems because of the vast disparities with the types and number of shares listed on the JSE at the beginning of 1993 compared to the beginning of 2008. Consequently, based on the industry classifications as of each year in the analysis a list of 20 was derived, as reflected in Table 4-2. The composition of the portfolios was adjusted each year for any newly listed or delisted firms.

Table 4-2: Industry Classifications

Insurance
Banks and Financial Services
Property
Chemicals, Oil and Gas
Health Care
Media
Automobiles and Parts
General Mining
Platinum, Diamond, Coal and Precious Metals
Gold Mining
Basic Resources
Building and Construction
Industrial Engineering
Industrial Transportation
Other Industrial
Technology and Electronics
Food and Beverage
Travel and Leisure
Personal and Household Goods
Retail

For the beta-sorted portfolios, ten equally-sized portfolios were formed (compared to five by Van Rensburg and Robertson, 2003: 8), with the composition thereof changing each year to reflect changes in the sensitivity of the shares to movements in the market portfolio and any newly listed or delisted shares, using the immediately preceding 36 months worth of data to estimate the beta values. In line with the work of Black *et al* (1972), the shares had to be listed for at least 24 months in order to be included in the analysis. The beta-sorting procedure implemented in this study thus differed from that of Van Rensburg and Robertson (2003: 8) not only with regards to the number of portfolios formed, but also with respect to the regularity of changes to the portfolio compositions and the minimum historical sample size for beta estimation. The composition of the portfolios was only changed annually as opposed to monthly by Van Rensburg and Robertson (2003: 8), because, according to Graham and Uliana (2001:

16) there is no clear evidence of superior estimates when portfolios are formed less frequently than a year. Accordingly, because the portfolios were not reformed on a monthly basis it sufficed to only roll the samples over annually in this study. In addition to this, the periods used to estimate the betas were longer in this study, because at least 24 months of returns had to exist in contrast to twelve with Van Rensburg and Robertson (2003: 8), with the maximum being 36 months compared to 30.

To calculate the historical betas in order to allocate shares to portfolios, Black *et al* (1972:11) used the traditional CAPM, as shown in equation 3.36 (reproduced below). However, as explained in Section 3.3.1.1, the problem with this approach was that it employed an estimate for the risk-free rate which may have been incorrect and hence biased the parameter values and standard errors. Consequently, this formula was not used, but rather the approach adopted by Fama and MacBeth (1973: 632) in which they estimated the beta values using the market model, with no reference to the existence of a risk-free asset. This formula (equation 3.42) was estimated in accordance with OLS methodology in EViews 6.

$$\overline{R}_{jt} = \alpha_j + \overline{\beta}_j \overline{R}_m + \eta_{jt} \tag{3.36}$$

$$R_{it} = \psi_j + \beta_i R_m + e_{jt} \tag{3.42}$$

Bradfield and Barr (1989), Bowie and Bradfield (1997) and Bradfield (2002) advocated adjusting beta values in South Africa to account for thin trading. As discussed in Section 1.1.1, there is evidence of this approach being implemented in published studies, especially when the study covered early periods such as the 1970s and 1980s (Fraser and Page, 2000; Bhana, 2008), but there are also several authors who have not made any adjustments (Oldfield and Page, 1997; Akinjolirie and Smit (2003); de Wet and Hall, 2006; Bhana, 2007; and Samoulihan, 2007). Van Rensburg (2001: 5) and Van Rensburg and Robertson (2003: 8) did not adjust the beta estimates in accordance with Bradfield and Barr (1989), but instead applied a thin-trading filter to remove those shares that were not traded at least once during the month. An examination of the dataset used in this study revealed that there were very few shares that were not traded at least once during any month and almost all of these occurrences were during the years 1990 to 1993. Given the fact that betas were estimated over at least 24 months but mostly 36 months, sufficient different observations existed over this period such that it was not deemed necessary to adjust or remove any shares that were thinly traded.

4.3.2 Expanding the Analysis Beyond Ordinary Shares

The generally accepted approach to estimating the CAPM parameters, including the minimum required return, is based solely on ordinary shares. However, as mentioned in Sections 3.3.1.4 and 4.2.2 Stambaugh (1982: 251) expanded the analysis to also include portfolios of preference shares and bonds. However, Stambaugh (1982: 251) was only able to form four preferred share portfolios and five bond portfolios in combination with the 19 ordinary share portfolios. He acknowledged that these nine portfolios certainly did not comprehensively represent the two additional asset classes, but his goal was to "...extend the range of asset types and parameter values beyond those typically encountered in tests of the CAPM" (Stambaugh, 1982: 251). In so doing, Stambaugh (1982) expanded the range of beta values in the analysis; thus overcoming one of the problems associated with the grouping procedure in that it contracts the range of beta values, thereby decreasing statistical power.

The parameter estimates obtained by Stambaugh (1982: 256) were more efficient, as reflected by lower standard error estimates, than those obtained by Black *et al* (1972) or Fama and MacBeth (1973). This increase in efficiency could have resulted from not only the inclusion of two other asset classes, but also the different estimation procedure employed by Stambaugh (1982). Thus, despite Stambaugh's (1982: 256) claims that the increased efficiency was attributable to the wider spread of assets; no other studies have considered this issue to verify the findings. Therefore the direct effects of expanding the assets used beyond ordinary shares were examined by estimating the parameter with and without the additional asset portfolios.

For the preference shares, Stambaugh (1982: 253) selected four firms, with data available for the full period of the study, with each representing a unique portfolio. Whilst this approach was not ideal, as the beta estimates may have been characterised by the error in the variables problem, Stambaugh (1982) faced data constraints preventing him from forming portfolios of preference shares based on identical or similar industry classifications to those used for the ordinary share portfolios. In this study, for the beta-sorted portfolios, the preference shares were sorted into two portfolios based on their historical beta values (in exactly the same way as with the ordinary shares) but these shares were not combined in the ordinary share portfolios as the impact of the different beta values of the preference shares may be minimal in a large portfolio dominated by ordinary shares (as per Stambaugh, 1982). The JSE listing of preference shares is considerably smaller than the main board listed ordinary shares; with a high of 79 in 1993 and a low of 25 in 2003 and 2004 (See Figure 5-1, page 138). Two portfolios were thus considered sufficient for this purpose, as any more would have resulted in too few shares in each portfolio, in conjunction with the preference share-based portfolios having too big a weight in the number of cross-sectional units in relation to the number of common share portfolios.

It was not feasible to allocate the preference shares into industry-based portfolios as the affiliations of the listed preference securities are as broad as for the ordinary shares. Whilst the number of preference share portfolios could have been increased in lieu of the greater number of common share portfolios in the industry-based analysis compared to with the beta-sorted sample (twenty compared to 10), the impact of a single share on the results obtained when too few shares are combined in a portfolio, did not justify altering the number of portfolios. Consequently, the same two beta-sorted preference share portfolios were retained.

Finally, with respect to the formation of bond portfolios, the methodology of Stambaugh (1982), where he combined both corporate and government securities with differing maturities, was not easily replicable in the South African market because of the limited debt traded, especially corporate debt. To attain the dispersion in beta values that would be expected across debt of varying maturities, the BESA indices for short-term (one to three years), medium-term (seven to twelve years), and long-term (twelve to thirty years) bonds were used, as explained in Section 4.2.2, to represent three bond portfolios. The compositions of these portfolios thus did not have to be adjusted annually to reflect any newly listed or de-listed securities, as was the case for the ordinary and preference share portfolios. Importantly, because the bond portfolios were only formed from 2001 onwards, the analyses conducted with these expanded samples only included two additional portfolios (the preference share portfolios) for the period 1993 to 2000 and thereafter five additional portfolios from 2001 to 2008.

To estimate the betas of the preference shares and bonds, the ALSI was used for the market portfolio; that is, the market portfolio was not adjusted to include these two asset classes. The justification for this is seen in the results of the market-portfolio proxy analysis conducted by Stambaugh (1982), which clearly show that the inclusion of these additional assets in the market portfolio has little or no impact on the returns because the weightings of these assets are small in comparison to the share of ordinary shares. Their impact on variability is also inconsequential, as this is overwhelmingly dominated by the variation in ordinary share returns. The same appears to be true for South Africa, as the bond market is only approximately fifteen percent of the size of the share market (BESA, 2009a: 4), and preference shares contribute a very small fraction of the total equity value.

The estimation of the intercept of the CAPM based on the work of Black *et al* (1972), Fama and MacBeth (1973) and Stambaugh (1982), was thus repeated for four different samples – the beta-sorted ordinary share portfolios (sample 1), the industry-sorted ordinary share portfolios (sample 2), the beta-sorted ordinary share, preference share and bond portfolios (sample 3), and the industry-sorted ordinary share, preference share and bond portfolios (sample 4).

4.3.3 Methodology Based on Black *et al* (1972)

The study conducted by Black *et al* (1972) is one of the most frequently cited with reference to empirical tests of the CAPM relationships (such as Morgan, 1975; Gibbons, 1982; Shanken, 1985; Faff, 2001). As explained in Section 3.3.1.1, Black *et al* (1972) initially estimated the minimum required return using a cross-section regression; however, the authors also derived what they believed to be a more explicit approach for estimating this parameter that was not directly reliant on a regression analysis and it is this technique which was implemented in this study. The first step of this approach is to determine average monthly beta estimates for each of the portfolios formed for each year from 1993 to 2008. As with the estimates of the individual share betas, the beta values for the portfolios were not estimated according to the traditional CAPM equation as per Black *et al* (1972: 10), but rather using the market model to avoid explicit denotation of a value for the risk-free rate. Thus equation 3.42 specified previously was used for this purpose.

In accordance with the approach specified by Black *et al* (1972: 35), these beta estimates were combined with the market portfolio proxy returns, as per equation 4.7[21], to determine an estimate of the minimum required return based on each portfolio in each year. The portfolio specific estimates were then combined according to the formula derived by Black *et al* (1972: 35), shown in equation 4.8.

$$\hat{\gamma}_{0p} = 1/(1 - \widehat{\beta}_p) * [R_p - \widehat{\beta}_p R_{mt}] \tag{4.7}$$

$$\hat{\gamma}^*_{0t} = \sum_{p=1}^{J} h_p \hat{\gamma}_{0p} \tag{4.8}$$

$$h_p = (1 - \widehat{\beta}_p)^2 / \Sigma_p (1 - \widehat{\beta}_p)^2 \tag{4.9}$$

Where:

- $\hat{\gamma}_{0p}$ is the estimate of the minimum required return from the p^th^ portfolio
- $\hat{\gamma}^*_{0t}$ is the combined estimate of the minimum required return
- h_p is the weighting of the estimate of the minimum required return from the p^th^ portfolio

(Adapted from Black *et al*, 1972: 36)

---

[21] Equations 4.7-4.9 closely resemble those outlined in equations 3.39 to 3.41 but have been adjusted in order to reflect total returns rather than excess returns.

The key component of this formula is the weighting given to each of the individual portfolio estimates of the parameter, h_p, which was calculated in order to minimise the error variance of the portfolio estimates (Black *et al*, 1972: 36). As can be seen from equation 4.9 the weightings are proportional to the value of the portfolio's beta; where the parameter estimates of those portfolios with betas significantly lower or higher than the market average are afforded greater weight in determining the overall intercept value. The final estimate is calculated simply as the weighted average of the individual portfolio values.

Due to the fact that these parameter estimates were not computed from a regression, the associated standard errors were not directly observable. Therefore, these values were calculated by measuring the deviation of each of the portfolio estimates of the minimum required return from the weighted average value, according to the formulae depicted in equations 4.10 and 4.11.

$$\sigma^2 (\hat{\gamma}^*_{0t}) = \sum_{p=1}^{J} [(\hat{\gamma}^*_{0t} - \hat{\gamma}_{0p})^2 * h_p] \tag{4.10}$$

$$\sigma (\hat{\gamma}^*_{0t}) = \sqrt{\sigma^2(\hat{\gamma}^*_{0t})} \tag{4.11}$$

In addition to the annual monthly average estimate of the minimum required return, this process was repeated to obtain estimates of the average intercept value for longer periods (eight two year sub-periods, three five-year sub-periods (excluding 2008), two eight-year sub periods, and over the entire sixteen-year period). However, for those estimates based on the expanded asset samples (samples 3 and 4), it was not possible to compute estimates of the parameter based on the entire period because of the differing number of cross-sectional units; whilst for the five-year period estimates, the first two (1993-1997 and 1998-2002) were computed using only the ordinary and preference share portfolios, whilst for the period 2003-2007 the datasets including bonds were employed.

4.3.3.1 Estimation Considerations

Ordinarily in utilising OLS methodology to estimate linear regressions it is imperative to ensure that the sample and model adhere to the assumptions of the classical linear regression model (CLRM), as any deviations from these assumptions may result in biased and inefficient parameter estimates and inappropriate distributions being used for hypothesis tests (Brooks, 2006: 145).

In light of the fact that the beta values for all the shares and portfolios were estimated using returns over a particular period in time (that is, using time series data); the most likely violations of the CLRM are those of autocorrelation, heteroscedasticity and non-normality of the residual distribution. Autocorrelation occurs when the error terms (e_{jt}) are correlated across time meaning that the assumption that the covariance between error terms is zero is violated (Gujarati, 2003: 442). In the presence of autocorrelation, the beta estimates remain unbiased, such that they provide a reliable estimate of the true parameter; but these estimates are not efficient, indicating that the standard errors are not minimum-variance (Brooks, 2006: 166). However, because the standard errors of the beta estimates were not needed for the remainder of the analysis, and the beta values estimated still provided an unbiased estimate of the true parameter, it was not considered necessary to test for autocorrelation in the estimation of the minimum required return based on Black *et al* (1972: 7).

The CLRM also assumes that the error terms of the regression are homoscedastic, which means that their variance is constant; when the error terms vary with the explanatory variables in the regression, it is known as heteroscedasticity (Wooldridge, 2003: 266). In such circumstances the parameter estimates would still have been unbiased but not minimum-variance; and thus, similarly to the problem of autocorrelation, in light of the uses of these parameter estimates in this study, it was not considered necessary to test for heteroscedasticity. The same is true for the violation of the assumption of normality as the estimators remain unbiased and thus the impact of the violation of this assumption is also only reflected in the standard error estimates (Brooks, 2006: 163). Accordingly, no tests were conducted for this purpose either.

4.3.4 Methodology Based on Fama and MacBeth (1973)

The Fama and MacBeth (1973) approach to estimating the CAPM has been implemented by numerous authors, as indicated in Table 4-1 (page 102). This approach is analogous to that of Black *et al* (1972) and should result in similar estimates for the intercept of the CAPM. This technique was again used to estimate the minimum required return based on the four different samples and over the differing estimation horizons. The beta estimates for each portfolio obtained in the first step of the approach of Black *et al* (1972) were utilised. The only difference therefore in this study between the Black *et al* (1972) and Fama and MacBeth (1973) procedures implemented is that instead of using derived formulae to calculate the intercept of the CAPM, a regression was estimated. The returns and betas of each of the portfolios represented a cross-sectional unit in the regression, as indicated in equation 3.44 (reproduced below)[22]. The standard errors of these estimates were directly observable, as the regression analysis in E-Views computes the standard errors automatically.

$$R_{pt} = \gamma_0 + \gamma_1 \widehat{\beta}_{pt} + \eta_{pt} \quad (3.44)$$

Where:

- γ_0 is the minimum required return
- γ_1 is the market risk premium

(Fama and MacBeth, 1973: 632)

4.3.4.1 Estimation Considerations

In order to apply the approach described above to test the CAPM relationships, Fama and Macbeth (1973: 611-612) made the assumption that the share (and therefore the portfolio) returns followed a normal distribution such that the coefficients and residuals estimated in the regression model must follow a multivariate normal distribution. This requirement is not specific to the context of testing the CAPM; but pertains more generally to any model estimated based on OLS in order to conduct single or joint hypotheses about the model parameters (Cont, 2001: 225). Fama and MacBeth (1973: 612) do not provide any justification for the suitability of assuming that share returns are normally distributed; however, Gibbons (1982: 5) and Stambaugh (1982: 245), who infer the identical assumption, refer to the work of Blume (1970), Officer (1971) (quoted in Stambaugh, 1982: 245), Blattberg and Gonedes (1974) and Fama (1976) in support of this supposition. In general, this research indicates that for *monthly* ordinary share returns, the normal distribution is a good approximation; but for weekly data the returns more closely resemble the Student t-distribution.

More recently, however, the results of other research have brought the validity of this assumption into question. The findings of Shanken (1985), Bollerslev, Engel and Nelson (1994), Pagan (1996) and Cont (2001) indicate that share returns do not always follow a normal distribution, but in fact more closely resemble a leptokurtic distribution, which has fatter tails than a normal distribution. This means that there are more extreme values in the sample that lie away from the mean than associated with a normal distribution (Cont, 2001: 225). In South Africa, Page (1993) found evidence of non-normality among the 244 shares examined, where 45 of these were considered to be frequently traded. More recently, and in a significantly more

---

[22] In their original papers the differences between the approaches of Black *et al* (1972) and Fama and MacBeth (1973) were not only the formulae used, but also the specification of the equations as Back *et al* (1972) employed excess returns and Fama and MacBeth (1973) total returns. However, as indicated in Section 4.3.1, total returns were used for all analyses in this study to avoid the specification of an incorrect value for the risk-free rate.

comprehensive examination of the distributional properties of shares on the JSE, Mangani (2007: 64) confirmed the observations of Page (1993) finding "…unequivocal evidence of leptokursis on the JSE…that renders no support for the assumption of normality in the distribution of security returns".

If the residuals of an OLS regression violate the assumption of normality and no correction is made for this disparity, then the results of any hypothesis tests involving the parameter estimates may be invalid. In light of the importance of hypothesis tests in this study in order to compare the estimates of the intercept of the CAPM, the zero-beta portfolio returns and the risk-free rate proxy yields, the residuals of each of the OLS regressions in the Fama and MacBeth (1973) based approach were tested to determine whether the assumption of normality was violated, using the Bera-Jarque (BJ) statistic, and if this assumption was found to be violated corrections were made.

The BJ statistic is the most commonly applied test for normality, and is based on the premise that a normal distribution is symmetric (i.e. not skewed) around the mean value, and mesokurtic, where kurtosis measures how fat the tails of the distribution are (Brooks, 2006: 179). The BJ test thus jointly tests the hypothesis that the coefficients of skewness and excess kurtosis are zero. The null and alternative hypotheses tested are shown in equation 4.12. The test statistic, depicted in equation 4.13, can be computed using the residuals and variance of the regression and follows the Chi-Squared distribution with two degrees of freedom (Brooks, 2006: 180). If the BJ statistic was greater than the critical value at the chosen significance level, then the null hypothesis of normally distributed residuals was rejected in favour of the alternative hypothesis of non-normality and vice-versa.

$$H_0: \eta_{pt} \sim N(0, \sigma^2)$$
$$H_1: \eta_{pt} \neq N(0, \sigma^2) \qquad (4.12)$$

$$BJ = n\,[b_1^2/6 + (b_2 - 3)^2/24] \qquad (4.13)$$

Where:

- $b_1 = E(\hat{\eta}^3)/(\sigma^2)^{3/2}$
- $b_2 = E(\hat{\eta}^4)/(\sigma^2)^2$

(Brooks, 2006: 180)

As referred to in the statistical analyses conducted in Chapter 2, the most common significance levels used for the selection of the critical values are one, five and ten percent and thus any values lower than a cut-off of ten percent would suggest non-normality is a problem in the sample. In conducting hypothesis tests, there are two possible errors that can be made: *rejecting* the null hypothesis when it is really *true* (known as a type I error) or *not rejecting* the null hypothesis when it is in fact *false* (known as a type II error) (Brooks, 2006: 79). Using a lower significance level (such as one percent compared to five percent) provides a more stringent requirement for rejection with the result of reducing the possibility of a type I error but increasing the possibility of a type II error. The opposite is true when a higher significance level is employed. With respect to the hypothesis test at hand, type II errors were considered more serious as this means that the non-normality of the variables may be ignored and consequently, the results of any hypothesis tests conducted with these values may be incorrect. In contrast, type I errors will result in the regression being corrected for non-normality, when this is not necessarily a problem, which was not deemed a serious setback. Therefore, the significance level used for this hypothesis test was increased above the conventional levels to 20 percent to guarantee efficient and accurate standard error estimates.

The finding of non-normality, however, was not easy to resolve. Increasing the dataset, which in the case of this study would mean increasing the number of portfolios, was one possible solution, as the violation of the normality assumption is almost inconsequential in a large sample (Brooks, 2006: 183). Across the four samples used, the number of observations increased (from ten to 25) and yet in the samples with the larger number of observations, there were in fact, more occurrences of the violation of the assumption of non-normality. This result can be interpreted in that despite the increase in the number of cross-sectional units; the samples are still comparatively small.

The other suggestion Brooks (2006: 183) advocates is to examine the dataset for any outliers (extreme observations that might contribute to the non-normality of the distribution) and use dummy variables to model the extreme values. This approach to addressing the assumption of non-normality of the residuals is often criticised because the inclusion of a dummy variable that takes on the value of one for only a single observation effectively removes the observation from the analysis (Cont, 2001: 227). Cont (2001: 227) argues that "…large market movements, far from being discardable as simple outliers, focus the attention of market participants since their magnitude may be such that they compose an important fraction of the return aggregated over a long period". Brooks (2006: 185) supports this assertion, suggesting that dummy variables should only be used if it is based on theory or at the researcher's discretion in the event of a financial crisis or some other event which is unlikely to be repeated. Many of the regressions which were observed to have extreme values did coincide with periods of financial turmoil in the South African markets (1998, 2000 and 2008); however, there were also some periods where the introduction of the dummy variable was harder to justify.

The solution to the problem of non-normality in this study was thus viewed as a trade-off in that in order to satisfy the precondition of normally distributed parameters for an OLS regression, variables had to be adjusted which were not necessarily completely justified by theory. However, in lieu of the primary objective, it was considered more important to satisfy the normality constraint at the expense of slight disparity with theory.

Graphs of the residuals from each regression, where the assumption of normality was rejected, were examined to identify any outlier observations. A new variable was created with a value of one for the extreme observation and zero for the rest of the sample. The regression was re-estimated, as per equation 4.14, and the residuals were again tested to ensure that they satisfied the normality constraint. In only three of the regressions, across all four samples, was more than one observation considered a notable outlier. In these cases, introducing a dummy variable for only that observation did not provide a sufficiently high statistical probability of normality and thus a second dummy variable was included. These regressions were estimated using samples 2 and 4, which had at least twenty observations; and therefore the effect of introducing two dummy variables was not considered to have a significant negative impact on the sample size.

$$R_{pt} = \gamma_0 + \gamma_1 \beta_{pt} + \gamma_2 D_{pt} + \eta_{pt} \tag{4.14}$$

Where:

- D_{pt} is the dummy variable

(Adapted from Brooks, 2006: 183-184)

In addition to the testing and correction for the violation of the normality assumption, it was also necessary to test for heteroscedasticity because the methodology of Fama and MacBeth (1973), in contrast to Black *et al* (1972), is based upon a second-pass regression. This was a concern in terms of the reliability of the standard errors of the coefficients (including the intercept), as these values would be used both to compare with the estimates obtained via the other approaches and potentially to be used in the hypothesis tests. Autocorrelation, however, was not considered as it tends to be mainly observed in time series as opposed to cross-sectional samples (Gujarati, 2003: 442).

Each regression was thus subjected to a statistical test to ascertain whether the variance of the error terms was constant (i.e. homoscedastic) or not. The null and alternative hypotheses tested are shown in equation 4.15 (on the following page). Given that each regression was based on between ten and 25 observations, the Breusch-Godfrey-Koenker (BGK) test was used rather than the White test, as it does not consume degrees of freedom and thus has greater power in a small sample (Wooldridge, 2003: 267-270). The BGK test entails the estimation of an auxiliary regression, using the squared error terms from the original regression as the dependent variable and the identical independent variables, as illustrated in equation 4.16. Significant coefficients in this regression indicate a relationship between the error term and the explanatory variables, and the violation of the assumption of homoscedasticity. The test statistic, depicted in equation 4.17, follows a Chi-Squared distribution with m degrees of freedom, where m refers to the number of independent variables in the auxiliary regression[23]. If the test statistic was greater than the critical value at the 20 percent significance level[24], the null hypothesis was rejected in favour of the alternative hypothesis of heteroscedasticity; whilst if the test statistic was smaller than the critical value the null hypothesis of homoscedasticity was not rejected.

H_0: variance $(\eta_{pt}) = \sigma^2 < \infty$

H_1: variance $(\eta_{pt}) \neq \sigma^2$ (4.15)

$$\hat{\eta}_p^{\,2} = \alpha_1 + \alpha_2 \beta_p + v_p \quad (4.16)$$

$$NR^2 \sim X_m^{\,2} \quad (4.17)$$

Where:

- α_1 and α_2 are the estimated coefficients
- v_p are the error terms of the regression

(Brooks, 2006: 150)

The problem of heteroscedasticity was easily remedied using the White (1980) standard error transformations in EViews.

4.3.5 Methodology Based on Stambaugh (1982)

As discussed in Section 3.3.1.3 and 3.3.1.4, Gibbons (1982) and Stambaugh (1982), both expanded upon certain elements of the original approaches to estimating the CAPM parameters in order to improve the accuracy of these estimates and hence the reliability of the conclusions drawn. The major advancement was the extension of the methodology from a univariate to a MVRM; that is, they were effectively testing whether both the market model (equation 3.42) and the CAPM (equation 3.36) hold simultaneously (Stambaugh, 1982: 247) and hence considering the two models concurrently in a multi-variable environment. This fact has implicitly been assumed in the estimation of the minimum required rate of return based on both

[23] In light of the model being tested, the degree of freedom was one.

[24] As with the test of normality, the possibility of type II errors was deemed to be of greater concern than type I errors (especially given the ease with which the problem of heteroscedasticity can be resolved) and thus the significance level chosen was increased to 20 percent.

the Black *et al* (1972) and Fama and MacBeth (1973) approaches. The consequence of this assumption is that the intercept of the market model must be equal to the minimum required rate of return multiplied by one minus the beta estimate, as shown in equation 3.45.

The problem, according to both Gibbons (1982: 8) and Stambaugh (1982: 249), is that if both of these models do hold simultaneously, then the OLS estimates of the parameters (i.e. the coefficients estimated according to the methodology of Fama and MacBeth, 1973) are inefficient compared to a non-linear MVRM estimation procedure because of the reduction in the parameter space. Gibbons (1982: 10) therefore proceeded to employ a one-step Gauss-Newton methodology and Stambaugh (1982: 253) ML estimation to overcome this problem. As discussed in Section 3.3.1.4; the latter approach is preferable to the one-step Gauss-Newton approach and consequently this was the technique that was implemented.

The identical equation to the Fama and MacBeth (1973) approach (equation 3.44) was used to estimate the CAPM parameters based on the method implemented by Stambaugh (1982). The key difference between OLS and ML estimation is that where the OLS approach estimates the coefficients so as to minimise the error terms between each observation and the predicted value, ML estimation involves finding the most likely values of the parameters given the actual data (Brooks, 2006: 49, 455). To employ ML in E-Views it was necessary to specify the log likelihood functions (LLF) for each regression; the LLF for each estimate, based on Gujarati (2003: 115) is expressed as shown in equation 4.18. The method of ML therefore entails maximising the LLFs for each time period. The OLS estimates of the parameters were used as the starting values for this process. As with OLS estimation, once the LLFs were specified, E-Views automatically generated the standard errors of the coefficient estimates.

$$LLF_t\ (\gamma_0, \gamma_1, \sigma^2) = -n/2 \ln \sigma^2 - n/2 \ln (2\pi) - 1/2\ [(R_{jt} - \gamma_0 - \gamma_1 \widehat{\beta}_j)^2 / \sigma^2] \qquad (4.18)$$

(Gujarati, 2003: 115)

In order to model the regression function, estimates of the asset betas were required. The same asset portfolios formed for the purposes of the methodologies of Black *et al* and Fama and MacBeth (1973) were used, which conforms to the approach specified by Stambaugh (1982: 248) with respect to using portfolios rather than individual assets. Further to this, the same beta estimates for each portfolio were employed, as estimating these values using OLS still provided efficient and consistent estimates (Stambaugh, 1982: 251).

Parameter estimates obtained using ML have the desirable properties of sufficiency, consistency, and efficiency; none of which, according to Myung (2003: 90) can be said about

OLS estimates. Myung (2003: 90) argues that most statisticians would not even consider OLS as a general method for parameter estimation, but rather an approach used for linear regression models, and they would certainly not rely on OLS as the basis for constructing confidence intervals and hypothesis testing. Due to the fact that one of the primary purposes of estimating the implied minimum required return was to test the equality of this value with the zero-beta portfolio and risk-free rate proxy returns, it is imperative that these values are accurate; thus justifying the expansion of this study to obtain estimates of the intercept using ML. In addition, more recent international studies such as Faff (2001) and Chou and Lin (2002) have used the ML approach to estimate the minimum required return.

4.3.5.1 Estimation Considerations

As suggested in Section 4.3.4.1, the fact that both Gibbons (1982: 5) and Stambaugh (1982: 245) also made the assumption that the coefficient estimates follow a multivariate normal distribution indicates that it is also a necessary assumption for the application of ML (Gujarati, 2003: 114; Myung, 2003: 92). Consequently, the same discussion regarding the pros and cons of adjusting the sample for non-normality is applicable in this context. However, in light of the desire to be able to accurately compare the estimates of the CAPM (and the associated standard errors) obtained across the three approaches the identical dummy variables were introduced into the ML regressions where the BJ test statistics were found to be statistically significant in the equivalent OLS regressions.

Homoscedasticity is also a necessary condition for the ML estimation procedure; however, testing and correcting for heteroscedasticity with ML estimation output is substantially more difficult than with OLS (Marazzi and Yohai, 2000: 174). Marazzi and Yohai (2000: 175) suggest employing a weighting procedure to remove the effects of heteroscedasticity; but acknowledge the complexity associated with this task as it requires some knowledge regarding the form of the heteroscedasticity. The form of the heteroscedasticity was not known in those regressions where this problem was identified and therefore the weighting alternative was not a feasible solution. Instead, for those OLS regressions where heteroscedasticity was identified and the coefficient standard error estimates adjusted based on White's (1980) transformations; the variance from these modified regressions were used as the value of the variance in the log-likelihood functions as depicted in equation 4.18. In this way, the parameter values used were reasonably consistent across the Fama and MacBeth (1973) and Stambaugh (1982) approaches and thus allowed for more accurate comparisons across the two sets of estimates.

4.3.6 The Most Efficient Method to Estimate the Intercept of the CAPM

In determining which estimates of the minimum required return were the most efficient, as reflected by the lowest standard errors, four of the sub-objectives of this study were addressed:

- which method of estimating the intercept was the most efficient,
- whether the portfolio formation technique used had a statistically significant impact on the efficiency of the estimates of the parameter,
- whether the inclusion of two additional asset classes increased the efficiency of the intercept values, and
- the optimum period over which the minimum required rate of return should be estimated

In this section, the method followed in determining the most efficient method to estimate the intercept of the CAPM is described, with the techniques employed to assess the other three sub-objectives discussed in the following section.

The standard errors of the estimates of the minimum required return should be used as the basis for assessing the first sub-objective, as the standard errors provide a measure of the degree of uncertainty associated with the coefficients estimated (Brooks, 2006: 58-59). That is, the smaller the standard error, the more precise the estimate and the more confidence that the estimates of the parameter will not vary considerably from one sample to another sample. Moreover, an estimate of a parameter is said to be efficient if no other estimator has a smaller standard error (Brooks, 2006: 58).

The three tests proposed in econometric literature for the statistical comparison of standard error values are the F-Test, Bartlett's test and Levene's test (Conover, Johnson and Johnson, 1981: 353). The F-test is used to determine if the variances of two populations or samples are equal; whilst Bartlett's and Levene's tests determine whether samples have homogeneous variances, with Levene's test being preferable if departures from normality are observed (Conover *et al*, 1981: 353-354). For this analysis, the point was not to test the variance of the regression functions within each sample, but rather to compare the variance of one parameter estimate based on one approach to an estimate based on another, and therefore it was more appropriate to employ the F-Test. Morgan (1975: 367) chose to use the Box-Tiao (1973) test for this purpose rather than the F-test because of the unreliability of the latter in the presence of significant kurtosis. However, kurtosis was not a concern in this regard as the OLS and ML regressions had been adjusted to ensure that the residuals were normally distributed (the coefficient of excess kurtosis was zero) and therefore the F-test was deemed appropriate[25].

The F-test statistic is computed as the larger sample variance divided by the smaller sample variance, as shown in equation 4.19, with the null and alternative hypotheses tested depicted in equation 4.20. The F-statistic follows the F-distribution with n_1 and n_2-1 degrees of freedom, where n_1 refers to the sample size of the numerator and n_2 the sample size of the denominator (Brooks, 2006: 102). If the test statistic was greater than the critical value at the chosen significance level from the F distribution, then the null hypothesis that the two variances were equal was rejected; concluding that the variance from the numerator sample was statistically significantly larger than the other variance estimate.

$$F = \sigma_1^2 / \sigma_2^2 \qquad (4.19)$$

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 \neq \sigma_2^2 \qquad (4.20)$$

Where:

- σ_1^2 is the larger sample variance
- σ_2^2 is the smaller sample variance

(Brooks, 2006: 102)

The difficulty with implementing this test was the selection of the numerator and denominator, as across the different time periods, one set of standard errors were not always larger than the other. To correctly apply this statistic would entail considering each time period individually. The problem with doing this was that in analysing the test statistics it would have been more difficult to identify in which cases sample 1 was larger and in which cases sample 2 was larger. Therefore to aid in the appropriate analysis, the sample where the majority of standard errors appeared larger was used as the numerator throughout. When the relationship was reversed (reflected by an F-statistic less than one), the statistics were compared to the inverse of the critical values, but with the numerator and denominator degrees of freedom reversed.

For these hypothesis tests, the three most common significance levels (one, five and ten percent) were chosen rather than 20 percent employed for the normality and homoscedasticity tests. The reason for this is because in a hypothesis test of this nature type I and II errors are equally problematic as opposed to those discussed in the previous sections. Moreover, using all

[25] The assumption of normality was not a necessary requirement for the implementation of the technique based on Black *et al* (1972) and thus the influence of kurtosis on the tests of equality of the variance estimates of this approach compared to the other two techniques was not of concern.

three of the common statistics enabled easy identification of those statistics computed that were sensitive to the choice of critical values within these high confidence levels. The efficiency of the three techniques was compared across the four samples to ensure that the results obtained were robust to the selection of the best portfolio formation technique, asset composition and time period of estimation.

It was also considered of value to compare the actual intercept estimates across the different approaches to assess how the methodology affects the magnitude of the estimate of the minimum required rate of return. The test statistic and hypotheses established in Section 2.4.6 (equations 2.12 and 2.13), were employed for this purpose and are reproduced below, the only difference being that the test statistic is specified as a t- rather than a z-test.

$$\text{t-statistic} = [(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)] / (\sigma_1^2/n_1 + \sigma_2^2/n_2)^{0.5} \qquad (2.12)$$

H_0: $\mu_1 = \mu_2$ (which can be rewritten as $\mu_1 - \mu_2 = 0$)
H_1: $\mu_1 \neq \mu_2$ (which can be rewritten as $\mu_1 - \mu_2 \neq 0$) (2.13)

In testing the estimates across the three approaches the selection of the appropriate distribution of the statistic specified in equation 2.12 is important. The estimates of the intercept based on the approaches of Fama and MacBeth (1973) and Stambaugh (1982) follow the Student t-distribution and normal distributions respectively, as a consequence of the underlying theory of the estimation procedures. For the explicit estimates, Black *et al* (1972) applied a t-distribution, which is also assumed to be applicable for the purposes of this study. For the comparisons of the Black *et al* (1972) and the Fama and MacBeth (1973) based estimates the t-distribution was used as both estimates follow this distribution. Similarly, despite the differences in distributional characteristics between the Stambaugh (1982) based estimates and the other two approaches, the t-distribution was also employed on the suggestion of Keller and Warrack (2000: 409), as the t-distribution exhibits fatter tails thus allowing for greater deviation from the mean value.

The t-statistics were compared to the critical values (at the one, five and ten percent significance levels)[26] to determine whether the values were statistically significantly different across the varying estimation techniques adopted. The estimation technique identified from Black *et al* (1972), Fama and MacBeth (1973) and Stambaugh (1982) that was the most efficient was then used to ascertain the best portfolio formation technique, the most efficient asset classes to include and the optimal time period over which to estimate the intercept.

[26] The reasoning behind the choice of significance levels is identical to that explained for the variance tests.

4.3.7 The Choice of Portfolio Formation, Asset Inclusion and Time Period

As highlighted in Section 1.3.1, in order to employ the methodology of either Black *et al* (1972), Fama and MacBeth (1973) or Stambaugh (1982) to estimate the intercept of the CAPM, it is necessary to assume that the other CAPM relationships hold; that is, that there is a linear relationship between risk and return, that the risk premium is positive and that returns are not dependent upon any other measures of risk (Fama and MacBeth, 1973; 613). The original tests conducted on the CAPM in South Africa (Bradfield *et al*, 1988, Bradfield and Barr; 1989, Van Rhijn; 1994, and Ward; 1994) found that "…the CAPM stood up reasonably well to empirical testing on the JSE" (Ward, 1994: 100). In contrast, Van Rensburg and Robertson (2003) found over the ten-year period July 1990 to June 2000 that the relationship between risk and return was negative. However, as specified in Section 1.3.1, for the purposes of this study, the CAPM is assumed to be valid for examining the risk-return relationship of shares on the JSE.

Whilst the objective of this study is *not* to examine the validity of the CAPM, if there are any periods where the relationships postulated by the model did not hold, this may incorrectly influence the estimate of the intercept obtained, and for this reason the conclusions drawn regarding the most efficient methods to estimate this parameter. Of particular concern in this regard are the assumptions that beta is an important determinant of returns[27] and that the market risk premium is positive. The violation of these assumptions should not affect the choice of the most efficient method of estimating the intercept because these approaches are estimated using the same samples, but it is possible that the extent to which these assumptions hold may be a function of the choice of sample. The problem therefore with assessing the standard errors across the different samples as was done with the choice of methods is that if the CAPM relationship did not hold over a particular period of time based on one sample but did based on another sample, then comparing the estimates of the standard error of the intercept values is not necessarily a good indicator of the validity of the technique.

The reason for this is that the efficiency of coefficient estimates in a linear regression (as per the methodology of Fama and MacBeth (1973) and Stambaugh (1982))[28] is a function of the dispersion of the data points vertically and thus the intercept estimates based on one sample may be clustered together thus reflecting a low standard error for the intercept value despite the fact that the slope of the line estimated is negative, when in reality the market portfolio returns were positive and considerably larger than the risk-free rate. Therefore examining the estimates

[27] If beta is an important determinant of returns then this means that there is a significant linear relationship between risk and return, irrespective of whether there are also more complicated relationships, or whether other factors play an important role in determining returns (Stambaugh, 1982: 246)
[28] The same is true for the approach of Black *et al* (1972).

of the standard errors of the intercept without first considering if the CAPM does not consistently hold in any of the samples may result in inappropriate conclusions being drawn regarding these sub-objectives.

To examine the extent to which beta was an important determinant of risk was reasonably straight-forward if the intercept values based on either the methodology of Fama and MacBeth (1973) or Stambaugh (1982) were found to be the most efficient. As described, both of these approaches involved estimating regression functions, which included not only estimates of the intercept of the CAPM, but also the market risk premium which is computed as the coefficient on the beta term. If this coefficient was statistically significantly different from zero then it satisfied the condition that beta was an important determinant of returns. The hypothesis tested in this regard and the test statistic (which follows the t- distribution or the normal distribution depending on whether the Fama and MacBeth (1973) or Stambaugh (1982) estimates were found to be the most efficient) are shown in equations 4.21 and 4.22 respectively.

H_0: $\gamma_1 = 0$
H_1: $\gamma_1 \neq 0$ (4.21)

t-statistic/z-statistic = $\gamma_1 / \sigma(\gamma_1)$ (4.22)

Where:

- γ_1 is the estimate of the risk premium
- $\sigma(\gamma_1)$ is the standard deviation of the risk premium estimate

(Adapted from Brooks, 2006: 67)

For the reasons discussed in Section 4.3.6, the one, five and ten percent critical values from the relevant distribution were used. If the absolute value of the test statistic exceeded the critical value at these significance levels, it was concluded that the risk premium was statistically significantly different from zero; that is, that beta is an important determinant of returns. The converse was true for those statistics that were smaller in absolute value than the critical values. The problem however, was that if the Black *et al* (1972) based estimates of the intercept were found to be the most efficient, the market risk premia estimates were not easily obtainable. However, given the similarity in this approach to that outlined by Fama and MacBeth (1973), the only difference being that the Black *et al* (1972) did not rely on the regression technique, it was concluded that the best approach would be to employ the Fama and MacBeth (1973) estimates as approximations for the market risk premia.

Examining the extent to which the second assumption that the market risk premium is positive held was considerably more difficult; the primary reason being that the validity of this assumption is itself questionable. The CAPM theory implies that the SML, which shows the relationship between risk and return, is upward sloping (Reilly and Brown, 2006: 240-241). But, as explained in Section 3.2.3, the underlying condition for this relationship to hold is that the returns on the market portfolio must be positive and greater than the returns on the risk-free asset (the market risk premium is positive). However, intuitively it is clear that if the market return was not greater than the return from a riskless asset over time, investors would only hold the riskless asset and would not purchase shares.

But, the returns on the market portfolio proxies employed do violate this condition over time, and in fact, if the returns on the market were always greater than the risk-free rate, then no investor would ever hold the riskless asset (Pettengill 1995: 102). Therefore the possibility must be taken into consideration that the returns on the market portfolio can be smaller than the risk-free asset returns or negative in the CAPM formulation. If it is assumed that the CAPM holds under both of these conditions, then the SML should be downward sloping; that is, the lower the risk, the higher the return (or the smaller the negative return) earned as these securities are effected less by the downturn in the general market movements than securities of greater risk (Pettengill *et al*, 1995: 104).

In order to take into account the possibility that over some periods examined the estimated risk premium may be negative, but still in accordance with the theory underlying the CAPM, a simple analysis was conducted based on the work of Pettengill *et al* (1995: 105). They computed the actual market returns for each period examined and the market risk premium (using the three month T-Bill as a proxy) to gauge those periods where the actual risk premia were negative (Pettengill *et al*, 1995: 105). In the same way, in this study, the actual returns earned on the ALSI and the market risk premia, using both T-Bills and T-Bonds, were computed for each period coinciding with the periods over which the CAPM was estimated. These computed values were compared to the estimated market risk premia (from the regressions) in order to assess whether the CAPM values based on any of the samples consistently estimated values for the risk premium which were negative, even in periods where the market returns and market risk premia were seen to be substantially positive. In this case, a sample where the risk premia was frequently negative signalled a violation of the theory underlying the CAPM that as risk increases returns increase, rather than reflecting that the actual market returns were negative or smaller than the risk-free proxy yields. This examination, whilst not beyond reproach, provided an indication of the extent to which the different samples complied with the underlying theory of the CAPM, where the asset allocation procedures used differed or additional asset classes were included. In those cases where the risk premia computed using either the T-Bill or T-Bond were close to zero, allowances were made in comparing the estimated risk premia for opposite signs as if the proxies overstate or understate the true risk-free rate this may be reflected in differing signs.

Following these examinations, the efficiency of the parameter estimates across the asset allocation procedures, asset compositions and time periods were assessed. The equality of the mean values across the samples was also tested as per the methodology established in the previous section; in so doing determining whether the procedure used to classify assets or the inclusion of preference shares and bonds has a significant impact on the results obtained and the impact of this on future applications of the model. Based on the results of the analysis of the risk premia estimates, the tests of the equality of the standard errors and the mean values, the most efficient estimates of the minimum required return were selected.

4.4 The Estimation of the Zero-Beta Portfolio Returns

4.4.1 Methodology of Morgan (1975)

The previously discussed methods for estimating the intercept of the CAPM are frequently cited as a means to estimate the zero-beta portfolio returns (albeit indirectly) based on the argument that if the intercept is not equal to the risk-free rate proxy then it must represent the minimum-variance zero-beta portfolio returns. Thus, the argument that is postulated is that if the other CAPM relationships are not violated, then either the traditional or zero-beta CAPM must hold.

Morgan (1975: 362) however, criticised the research of Black *et al* (1972) and Fama and MacBeth (1973) for not utilising the available knowledge to estimate the zero-beta portfolio value directly so as to more accurately ascertain which version of the CAPM is more appropriate. Consequently, as highlighted in Section 3.3.2, Morgan (1975: 363-364) derived a direct approach to obtain the desired estimate of the zero-beta portfolio returns. This method closely resembles the first steps of Black's (1972) derivation of the zero-beta CAPM, discussed in Section 3.2.1.1, but is specified in terms of vectors (in this section), in accordance with the description by Morgan (1975: 363-364). To determine the proportions invested in each asset in any minimum-variance portfolio it is necessary to solve equation 4.23 subject to imposed constraints (on the following page). Morgan (1975: 363) imposed that the sum of the proportions invested in each asset must equal one (equation 4.24) and that the covariance between the returns on this particular portfolio and the market portfolio must equal zero (equation 4.25). Using Lagrangian multipliers to incorporate the imposed constraints, the appropriate equation to be minimised is as illustrated in equation 4.26. Moreover, to minimise this function, it was differentiated with respect to x, the partial derivative equated to zero and the solution for x obtained as in equation 4.27. This arbitrary portfolio has the minimum possible variance and has zero covariance with the market portfolio, and can thus be viewed as the zero-beta portfolio (as per equation 3.47 reproduced below).

Minimise: ½ variance ($x^t R$) (4.23)

$\Sigma x_i = 1$ (4.24)

Covariance ($x^t R$, $R^t m$) = 0 (4.25)

Minimise: ½ Var ($x^t R$) + λ_1 Cov ($x^t R$, $R^t m$) + λ_2 (1 – Σx_i) (4.26)

$x = \lambda_2 D1 - \lambda_1 m$ (4.27)

$Z = \lambda_2 D1 - \lambda_1 m$ (3.47)

Where:

- x^t is an arbitrary portfolio vector
- R^t is a vector of returns on the assets in the market
- m^t is the market portfolio vector
- x_i is the proportion invested in each asset in portfolio x

(Morgan, 1975: 363-364)

The difficulty with employing equation 3.47 for the purpose of estimating the zero-beta portfolio returns is inverting the covariance matrix and for this reason Morgan (1975: 364) expanded his analysis to resolve this issue. Firstly, as shown in Section 3.3.2, he introduced an additional parameter, termed the global minimum-variance portfolio and showed that, in the same way as the zero-beta portfolio had been estimated, as in equations 4.23 through 4.25, the solution for the global minimum-variance portfolio could be determined by simply excluding the covariance constraint of equation 4.25 (Morgan, 1975: 364); the result of which is given in 3.48 (reproduced below). Secondly, in the derivation of the formula for estimating the zero-beta portfolio returns, the condition that the portfolio weights must sum to one enabled Morgan (1975: 364) to express λ_2 in terms of λ_1, as shown in equation 3.49. Incorporating these relationships yields the function depicted in equation 3.50, which does not include the covariance matrices or inverses thereof; thus simplifying the analysis.

$q = D1/1^t D1$ (3.48)

$\lambda_2 = (1 + \lambda_1) / 1^t D1$ (3.49)

$Z = (1 + \lambda_1) q - \lambda_1 m$ (3.50)

(Morgan, 1975: 364)

To use equation 3.50 as the basis for estimating the zero-beta portfolio returns, values for λ_1, the market portfolio and the global minimum-variance portfolio are required. Whilst the values for the market portfolio are known from the previous methods, the other values are not. λ_1 can be obtained by combining equation 3.49 and the conclusion reached in Chapter 3 (equation 3.20) that every minimum-variance portfolio can be expressed as a weighted combination of the zero-beta and market portfolios as evidenced in the formulae below. The final equation for calculating λ_1, given in 4.28 indicates that λ_1 is proportional to the estimated beta of portfolio q. The fact that this portfolio would have to be estimated based on the market model used to estimate the betas of the asset portfolios in the previous methodologies suggests that this ‘direct’ approach to determining the zero-beta portfolio returns is still reliant on some on the same methodology as the indirect approaches; it is the second stage of the estimation procedure, using the derived formulae that distinguishes this technique.

From equation 3.20:

$$q = (1-\beta_q) Z + \beta_q m$$

$$Z = (1/1-\beta_q) [q - \beta_q m]$$

Therefore:

$$\lambda_1 = \beta_q /(1-\beta_q) \tag{4.28}$$

Obtaining values for a global minimum-variance portfolio is substantially more difficult than λ_1, and in fact, Alexander (1977: 232) used this difficulty as justification for abandoning this direct approach to estimating the zero-beta portfolio; especially given that Morgan (1975: 364) gave no indication as to what values should be used for this value. Alexander (1977: 332) instead advocated an approach to estimating the zero-beta portfolio returns based on Lemke’s (1965) algorithm. This approach, similarly to Morgan’s (1975), entails trying to find a simpler method of determining the zero-beta portfolio returns because of the lack of computational power. However, in light of the availability of advanced computer-based programmes, estimating the minimum-variance zero-beta portfolio is no longer as complicated a task as both Morgan (1975) and Alexander (1977) suggested; and thus the approach that was used for this purpose, based on the derivations above, is described in the following section.

4.4.2 Derived Methodology

Microsoft Excel’s Solver function was used to estimate the zero-beta portfolio returns; making use of the derivation process of Morgan (1975) to implement the appropriate methodology. This function allows a particular value to be minimised or maximised by altering other values subject to any constraints imposed. Therefore, with reference to the original derivation of Morgan (1975), the Solver function was employed to minimise the portfolio variance by altering the weights of all the assets to be included, subject to the constraints that the sum of the weights invested in the assets must be equal to one, and that the sum of the weighted individual asset's covariance with the market must be equal to zero[29]. However, in order to simplify the analysis, the last constraint was replaced by the portfolio betas computed in the previous analyses.

The key part of estimating the zero-beta portfolio was the assets which were included. Morgan (1975) utilised the same portfolios created for estimating the intercept, via the methodology of Fama and MacBeth (1973), as the basis for determining the zero-beta portfolio returns. Furthermore, as discussed in Section 3.2.3 and illustrated in Figure 3.2 (page 64), the zero-beta portfolio is shown in the graphical plane of all assets and thus it is logical to employ as many assets as possible in creating this portfolio. Thus, all possible assets were used in the same four samples derived to estimate the zero-beta portfolio returns, which were then compared to determine the most efficient values. The implication of using the same portfolios however is that the zero-beta portfolio returns are likely to be similar to the intercept values from the regressions based on the Fama and MacBeth (1973) and Stambaugh (1982) estimates as these two approaches calculated the intercept by minimising the error variance of the estimates.

The zero-beta portfolio returns were computed as the weighted average returns of the individual asset portfolio returns, as shown in equation 4.29. To calculate the portfolio variance, the variance/covariance matrix of the returns from the asset portfolios was required. The co-movement of any portfolio with itself is its variance and therefore the diagonal line of the matrix is represented by the portfolio variances. However, for the purposes of this analysis the variance values are replaced with ones because only the covariance values between the shares are of interest (Ruppert, 2004: 153). Thus the formula for calculating the portfolio variance that was used is the same as equation 3.1. The constraint that the individual weights must sum to one is identical to equation 3.3, with the second constraint that the weighted sum of the asset portfolio betas must equal zero illustrated in equation 4.30 (which mirrors equation 4.25), shown on the following page.

$$R_z = \sum_{p=1}^{N} x_{zp} R_p \tag{4.29}$$

$$\sigma^2_z = \sum_{p=1}^{N} \sum_{q=1}^{N} x_{zp} x_{zq} \sigma_{pq} \tag{3.1}$$

$$\sum_{p=1}^{N} x_{zp} = 1 \tag{3.3}$$

[29] This is proof of the argument proposed by Fama (1976: 243) discussed in Section 3.2.3 that it is easier to examine the covariance as the weighted average of the individual covariances.

$$\sum_{p=1}^{N} x_{zp}\,\beta_p = 0 \tag{4.30}$$

Using this information, the average monthly minimum-variance zero-beta portfolio returns were estimated for each year, as well as over two year, five year and eight-year periods, and over the entire sixteen year period from 1993 to 2008 (where appropriate). The estimates of the zero-beta portfolio returns were then compared across the four samples in an identical fashion to the minimum required return estimates described in Section 4.3.6 in terms of the statistical comparison of the mean values across the samples and the variance values. The issue of the extent to which the different asset allocation procedures and different asset classes satisfy the other CAPM relationships was not considered relevant with respect to identifying the best estimate of the minimum-variance zero-beta portfolio returns, as the zero-beta portfolio should consist of all assets combined so as to minimise the variance, irrespective of the extent to which the component portfolios individually and/or collectively adhere to the risk-return relationship. The focus therefore in this analysis was solely on the differences in the mean and variance of the estimates of the zero-beta portfolio returns.

4.5 Estimation of the Risk-Free Rate Proxy

As mentioned in Section 4.2.2., the yield on the T-Bill and T-Bond securities gathered for the analysis conducted in Chapter 2 were used in this part of the study as well. However, the returns computed based on the yields were not appropriate for the comparison with the intercept and the zero-beta portfolio returns in the hypothesis tests, as it was necessary to provide a comparable value of the amount earned by holding either of the instruments for a month.

The R157 government bond has a coupon rate of 13.5 percent meaning that the owner of the bond earns 13.5 percent per annum by holding this instrument to maturity from the date of issue. The yield which the bond is trading at in the market reflects the rate which an identical bond would earn if it was issued at that particular point in time and therefore reflects changing market conditions and changing perceptions of investors over time. Consequently, this market yield reflects the return that investors expect to earn on this instrument at that date. The appropriate T-Bond yield was therefore computed as the average monthly yield to maturity of the R157 bond, assuming the bond was held to maturity. The R157 daily bond yield data gathered from the SARB was expressed as effective annual yields and thus, the annual percentage rate assuming monthly compounding (m was equal to twelve) was computed as shown in equation 4.31, with the monthly rate calculated by dividing the annual percentage rate by twelve as shown in equation 4.32.

$$APR = [(EAR + 1)^{1/m} - 1] * m \quad (4.31)$$

$$\text{Periodic Rate (T-Bond)} = APR/12 \quad (4.32)$$

Where:

- APR is the annual percentage rate
- EAR is the effective annual rate
- m is the number of periods

(Adapted from Firer *et al*, 2008: 163)

The computation of the T-Bill yield to be used in the hypothesis tests was easier than that of the R157 yield, as T-Bills are priced assuming simple interest as opposed to compound interest (Botha, 2006: 240). For this reason, the average monthly return earned by holding the asset was determined by dividing the equivalent annual rate by twelve, as per equation 4.33.

$$\text{Periodic Rate (T-Bill)} = EAR/12 \quad (4.33)$$

Given the evidence presented in Section 2.4.3 that South African T-Bills and T-Bonds exhibit statistically significant variation over time, it was considered necessary to compute the variation associated with the average monthly yield estimates to be taken into consideration when testing the hypotheses of the equality of the risk-free proxies with the minimum-variance zero-beta portfolio returns and the estimated intercept. The reason for this is that it cannot reliably be assumed that the returns on these instruments do not exhibit statistically significant volatility over time as is assumed in theory. These variance estimates were computed as explained in Section 2.3.3.2.

4.6 Hypothesis Tests

The hypotheses tested in order to address the primary research objective are quantified in this section with the methodology followed for this purpose described thereafter.

4.6.1 The Hypotheses

According to the theory discussed in Chapter 3, if government and other private sector securities are suitable proxies for the risk-free rate in South Africa, then these values should be equal to the intercept of the CAPM, which represents the rate which investors have viewed as the minimum required rate of return over the period analysed. This means that the returns earned on these instruments (R_f) provide a reasonable estimate of the minimum return that investors require from investing (γ_0). This is presented as the null hypothesis in equation 4.34, with the alternative hypothesis that the minimum required rate of return was not equal to the risk-free rate proxy returns over the different periods examined.

Hypothesis 1:

H_0: $\gamma_0 = R_f$

H_1: $\gamma_0 \neq R_f$ (4.34)

A two-tailed test is specified despite international evidence which suggests that only a one-tailed test is necessary (with the alternative hypothesis being that the minimum required return is greater than the risk-free rate proxy return), as the results of Van Rhijn (1994), discussed in Chapters 1 and 3, suggest that this relationship may actually be reversed in South Africa. Using a two-tailed test means that the percentage of significance is allocated equally to both ends of the distribution and hence whatever significance level is chosen is effectively halved. The disadvantage of using a two-tailed test therefore is that the reduction in the significance level results in a higher chance of a type II error (that is, not rejecting the null hypothesis when it is false), as highlighted in Section 4.3.4.1. In order to overcome this problem 20 percent was also selected as a significance level, in conjunction with the conventional one, five and ten percent values, as this equates to ten percent if a one-tailed test had been conducted.

The second hypothesis, specified in equation 4.35, tests the relationship between the zero-beta portfolio returns (R_z) and the intercept of the CAPM to determine whether this parameter provides a better estimate of the value that investors use to estimate the base return that they require from investing. Similarly to the first hypothesis, a two-tailed alternative was examined.

Hypothesis 2:

H_0: $\gamma_0 = R_z$

H_1: $\gamma_0 \neq R_z$ (4.35)

The final hypothesis examined the relationship between the risk-free rate proxy and the zero-beta portfolio returns. In Section 3.2.5.2, it was shown that if a suitable proxy for the lending risk-free rate then the return on this asset should be less than the zero-beta portfolio returns. In light of the substantial digressions of South African T-Bills and T-Bonds from the theoretical requirements that must be closely satisfied for an asset to be considered a suitable proxy, it is possible that the risk-free proxy return will be greater than the zero-beta portfolio returns if the proxies are inappropriate. Consequently, it was considered of value to test the relationship between these two variables as set out in equation 4.36. A two-tailed test was performed because of the uncertainty regarding whether or not the risk-free rate proxy returns would be greater or smaller than the minimum-variance zero-beta portfolio returns.

Hypothesis 3:

H_0: $R_z = R_f$

H_1: $R_z \neq R_f$ (4.36)

4.6.2 Testing the Hypotheses

The most efficient estimate of the intercept was used to test both hypotheses one and two. To test an estimated parameter against a hypothesised value, a standard t- or z-test of the forms shown in equation 4.37, which follows either the Student t- or normal distribution, can be employed (Brooks, 2006: 67). The choice between the two distributions was made based on the estimates of the intercept that were most efficient, as the z-statistic was appropriate for the estimates based on Stambaugh (1982); whilst the t-statistic was applicable for the Black *et al* (1972) and Fama and MacBeth (1973) estimates.

$$\text{t-statistic/ z-statistic} = (\gamma_0 - R_f) / \sigma(\gamma_0) \qquad (4.37)$$

Where:

- $\sigma(\gamma_0)$ is the standard deviation of the minimum required return estimate

(Adapted from Brooks, 2006: 67)

Employing these statistics to test the first two hypotheses necessitates assuming that the hypothesised value is constant over the period under review, but as mentioned previously, the analyses in Chapters 2 and 3 revealed that this assumption is not appropriate for the risk-free rate proxy or for the zero-beta portfolio. A more appropriate test statistic must therefore be computed which takes into account the variation of both the estimated coefficient and the hypothesised value. The t-test and z-tests for assessing the difference between two coefficient estimates taking into consideration their respective standard errors were discussed in Sections 2.4.6 and 4.3.6. The appropriate equation is reproduced on the following page (equation 4.38) but adjusted to reflect the parameters to be used in testing hypothesis one. These statistics were thus employed as they ensured that the variability in all the parameters being assessed was fully accounted for with all three hypotheses. If the test statistics computed were greater (in absolute terms) than the critical values from the appropriate distribution at the one, five, ten and 20 percent significance levels, then the null hypothesis of equality was rejected in favour of the alternative hypothesis that the two values differed over the period estimated. If the test statistic was smaller in absolute terms than the critical value, the null hypothesis of equality was not rejected.

t-statistic/ z-statistic = $[(\gamma_0 - R_f) - (\mu_1 - \mu_2)] / (\sigma^2(\gamma_0)/n_1 + \sigma^2(R_f)/n_2)^{0.5}$ (4.38)

Where:

- μ_1 and μ_2 are the hypothesised values for γ_0 and R_f, where the difference between the two is assumed to be zero
- $\sigma^2(R_f)$ is the variance of the risk-free proxy

As mentioned in Section 4.3.6, these hypothesis tests were conducted for all the different time periods over which the parameters were estimated to ensure that the major trends are identified and are not incorrectly influenced by deviations from the trends (as observed by Black *et al* (1972) and Fama and MacBeth (1973)) across varying time periods.

4.7 Assessing the Forecasting Accuracy of the Model

As discussed in Chapter 1, the CAPM is commonly employed to estimate the cost of equity for a firm making capital budgeting decisions. Consequently, it is imperative that the cost of equity calculated is accurate to ensure that correct capital budgeting decisions are made and that resources are allocated to the most productive sources. It is therefore of value to consider the impact on the cost of equity estimates of using either a risk-free rate proxy (the T-Bill or T-Bond) or the zero-beta portfolio returns. To do this, the forecasting accuracy of the model using the three different parameters was compared (both risk-free proxies were used).

The assessment of the forecasting accuracy of a model entails computing a forecasted value and comparing this to the actual value (which is known); the difference is known as the forecasting error (Pindyck and Rubinfeld, 1991: 516). The key component therefore is to accurately estimate the forecasted value. With time-series models, this typically involves distinguishing an ‘in-sample’ and an ‘out-of-sample’ period, with the model only being estimated with the ‘in-sample’ data (Brooks, 2006: 279). The ‘out-of-sample’ observations are then used in the model to determine a forecasted value for the dependent variable for each observation (Pindyck and Rubinfeld, 1991: 518). Excluding the ‘out-of-sample’ data points ensures that the relationships between the parameter estimates and the selected data points are not unrealistically significant. This approach was not, however, entirely applicable given the differing nature of the sample in this study; but similar principles were applied to ensure the forecasting comparisons conducted were accurate. Ten assets (ordinary or preference shares or bonds) were randomly selected[30] in each year using Microsoft Excel's random number function. Although ten appears small in the context of the total number of shares in the sample in each year (see Figure 5-1, page 138), by employing too many shares, the discrepancies between the actual and forecasted values would be averaged out, whereas the purpose is to assess how accurate the zero-beta portfolio returns are compared to the T-Bill or T-Bond yields on a individual share basis. Using ten thus ensures that the impact of any extreme values are limited but without hiding any important findings.

The ten assets chosen in each year were not excluded from the estimation of the zero-beta portfolio returns, as the influence of a single asset relative to the total number included in the analysis in each year (and that portfolios were formed thereby reducing the impact of one share even further), was not considered sufficient to favour the zero-beta portfolio estimates rather than the risk-free proxy. The betas of the shares from the asset allocation procedure were employed as this entailed using the prior period returns (either 24 or 36 months) to obtain values for beta; thus ensuring that the beta values were not determined by the market portfolio returns used in the calculation of the cost of equity and thereby exhibiting unrealistically significant correlation. The values for the risk-free rate proxies, zero-beta portfolio and the market portfolio returns in each year were employed rather than estimates based on the same periods as the beta values, as this would result in the forecasted value being equal to the average value of the dependent variable (Brooks, 2006:282-283). Using the actual period values for the market portfolio and risk-free asset instruments rather than expectations of these values may have understated the error associated with the model, but the focus is not on the absolute values of the accuracy of the model, but the relative accuracy across the models.

Using the risk-free rate measurement, the market portfolio returns and the estimated betas, the expected returns of the shares were computed in accordance with the CAPM equation (equation 1.1 and 3.34) for each year 1993 to 2008. Thereafter, the forecasted values for each share based on the three models were compared to the actual average monthly return over the year by computing the forecast error. Rather than subjectively assessing the size of the forecast errors across the models, several statistics, which aggregate the forecasting errors, were computed. In line with the work of Samoulihan and Shannon (2008: 25), non-utility based measures were employed for this purpose as they do not require any knowledge of the shape or properties of the relevant utility functions as per West, Cho and Edison (1993).

[30] These assets were chosen from the beta-sorted sample as this ensured that there was at least two years (preferably three) of data to be used to estimate the betas for the computations of the forecasted returns.

The most widely used of these is the Mean Squared Error (MSE), which is calculated as shown in equation 4.39. The MSE from one model can be directly compared with that of the other model, provided the same data and forecast period are used, with the model with the lowest value of the error measure being the most accurate. Both Brooks (2006: 287) and Samoulihan and Shannon (2008: 25) highlight that one of the criticisms of this approach is that it affords a larger weighting to larger forecast errors. However, in light of the importance of accurate cost of equity estimates, the greater weight afforded to the bigger forecast errors is not necessarily inappropriate in the context of the models being examined.

$$MSE = 1/ [T - (T_1 - 1)] \sum_{t=T1}^{T} (y_{t+s} - f_{t+s})^2 \quad (4.39)$$

Where:

- T is the total sample size (in-sample and out-of-sample)
- T_1 is the out-of-sample size
- y_{t+s} is the actual value for s periods from time t
- f_{t+s} is the forecasted value for s periods from time t

(Brooks, 2006: 288)

An alternative measure to the MSE is the Mean Absolute Percentage Error (MAPE), which has the advantage over the MSE that the answer can be expressed as a percentage which allows for easier interpretation (Makridas, 1993: 528), as illustrated in equation 4.40. The Adjusted Mean Absolute Percentage Error (AMAPE) provides the same benefit as the MAPE in that it yields an easily interpretable value, but also overcomes the problem of asymmetry associated with both the MSE and MAPE (Makridas, 1993: 528). Makridas (1993: 528) in fact argues that it is the measure that incorporates the best characteristics of the various criteria. The formula for calculating the AMAPE is provided in equation 4.41.

$$MAPE = (100/ [T - (T_1 - 1)]) \sum_{t=T1}^{T} | (y_{t+s} - f_{t+s})/ y_{t+s} | \quad (4.40)$$

$$AMAPE = (100/ [T - (T_1 - 1)]) \sum_{t=T1}^{T} | (y_{t+s} - f_{t+s})/ (y_{t+s} + f_{t+s}) | \quad (4.41)$$

(Brooks, 2006: 288)

These three criteria were used to assess the forecasting accuracy of the CAPM with both risk-free rate proxies and the zero-beta portfolio returns. In this way, the impact of the choice of parameter for the risk-free rate was fully conceptualised and appropriate recommendations made on the most suitable value to use in future applications of the model.

4.8 Chapter Summary

In this chapter the methodology that was followed to determine the appropriate parameter to use to estimate the risk-free rate in the CAPM in South Africa was discussed in detail, including the justifications for the techniques chosen. It was clear from an analysis of the related literature that the CAPM parameter estimates are more efficient if they are estimated using portfolios of shares rather than the individual securities. However, it was not apparent as to which of the methods of allocating shares to portfolios is the most efficient, what assets to include in these portfolios, the optimal period over which the variables should be estimated, nor in fact what general technique is best to estimate the minimum required return. Consequently, it was proposed to examine both the beta-sorting and industry-based allocation methods, the influence of including bonds and preference shares, several estimation periods and the techniques of Black *et al* (1972), Fama and MacBeth (1973) and Stambaugh (1982) to calculate the intercept.

In examining the literature regarding the techniques adopted to estimate the zero-beta portfolios returns, it was clear that simplifying adjustments had been made due to the lack of advanced computing capacity. In light of the fact that this is no longer a problem, a method was derived for this purpose based on Morgan (1975), where Microsoft Excel's Solver function will be used.

Three hypotheses were specified for testing. The first was that the risk-free rate proxy is equal to the minimum required return; the second that the zero-beta portfolio returns are equal to the minimum required return; whilst the third examines the relationship between the proxy and the minimum-variance zero-beta portfolio returns. The final component of the methodology that was discussed was the comparison of the forecasting accuracy of the CAPM which will be conducted, using T-Bills, T-Bonds and the zero-beta portfolio returns.

In Chapter 5, the results of these procedures and hypothesis tests are analysed based on the theory discussed and where applicable, with the results of similar international studies.

CHAPTER 5
DATA ANALYSIS AND RESULTS

5.1 Introduction

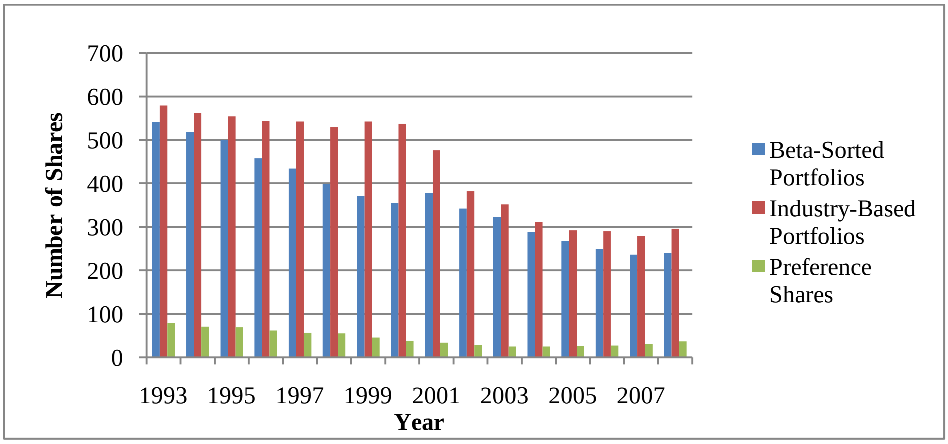
In this chapter, the results of the procedures discussed in the previous chapter for estimating the intercept of the CAPM and the zero-beta portfolio returns are examined. The hypotheses are tested and the findings are analysed with reference to the theory set out in the preceding chapters, as well as the results of comparable international studies. Furthermore, the tests of the forecasting accuracy of the CAPM, with the different measurements of the risk-free rate, are conducted and the results are examined in conjunction with the findings of the hypothesis tests.

5.2 Preliminary Data Analysis

5.2.1 The Number of Shares Included in the Samples

The numbers of ordinary shares included in the beta-sorted and industry-sorted portfolios, in each year in the analysis, are shown in Figure 5-1 by the blue and maroon columns respectively. As is clearly evident, the portfolios formed based on the industry-classification system included larger numbers of shares, as the requirements for inclusion were less stringent than for the beta-sorted portfolios. The general decrease in the number of shares in both samples from 1993 to 2006 reflects the decline in the number of listed companies on the JSE over this period. The increase in the number thereafter is not however reflected in the blue columns, because, as discussed in Section 4.3.1, the companies listed after the beginning of 2006 did not have sufficient trading history to be included in the beta-sorted portfolios.

Figure 5-1: The Number of Shares Included in Each Sample per Year



The number of preference shares that satisfied the beta-sorted conditions in each year are also depicted in Figure 5-1 (the green columns), which are substantially smaller than the number of ordinary shares. The listing pattern of these securities closely mirrors that of the ordinary shares, as the number gradually declined to a low of 25 in 2003 and 2004 before increasing marginally thereafter.

5.2.2 Formation of Portfolios

The ordinary shares, preference shares and bonds were allocated to portfolios in accordance with the methodology described in Sections 4.3.1 and 4.3.2. Table 5-1 shows the beta values for the ordinary share, preference share and bond portfolios for the years 1993, 2000 and 2008 based on the beta-sorting procedure, with Table 5-2 (on the following page) providing the beta values for the ordinary share portfolios based on the industry-sorting method for the same periods. These three years were selected in order to provide an overview of the relationships identified from the beginning of the period under review, the middle and the end.

Table 5-1: Selected Portfolio Betas from the Beta-Sorted Portfolios

	Betas 1993	Betas 2000	Betas 2008
Portfolio 1 (Largest)	1.0601	1.2412	0.6716
Portfolio 2	0.7927	0.8912	0.4987
Portfolio 3	0.8725	1.0963	0.7127
Portfolio 4	0.5452	0.9235	0.6338
Portfolio 5	0.5972	0.8621	0.3987
Portfolio 6	0.6307	0.6376	0.4529
Portfolio 7	0.6038	0.5998	0.2749
Portfolio 8	0.4981	0.4521	0.3491
Portfolio 9	0.585	0.6634	0.0979
Portfolio 10 (Smallest)	0.5043	0.0663	0.1325
Portfolio 11 (Preference shares largest)	0.1916	0.1883	0.0579
Portfolio 12 (Preference shares smallest)	0.1021	0.1401	0.0869
Portfolio 13 (Bonds short-term)			-0.0073
Portfolio 14 (Bonds medium-term)			0.0035
Portfolio 15 (Bonds long-term)			-0.0495

As the results in Table 5-1 suggest, the estimation period ranking in the beta-sorting method was typically a good indicator of the actual period risk measures, as the highest beta portfolios in the estimation period exhibited the largest beta values in the test period and the smallest beta portfolios in the estimation period the smallest beta values for the test period. However, there were several discrepancies in this regard, such as the third-largest ordinary share portfolio in 2008 providing the highest beta and the eighth portfolio in 1993 being the smallest. Moreover, what is also clear is that the beta values tended to be clustered together despite the goal of trying to achieve the maximum possible dispersion in beta values. This clustering of the actual period values of the portfolio betas based on the beta-sorting procedure is similar to that observed in the study of Van Rensburg and Robertson (2003), and is discussed further in the analysis of the validity of this approach in Section 5.5.1.

It is also clear from Table 5-1 that the betas of the preference share portfolios were generally lower than for the ordinary shares meaning that the response of these shares to market movements is smaller, on average, than ordinary shares. Two of the bond portfolios (portfolio 13 and portfolio 15) exhibited negative betas for the year 2008, implying that factors which caused a positive response in the market as a whole resulted in a negative response in the prices of bonds and vice versa for negative movements in the market. Therefore by including these additional asset classes, the range of values examined is expanded.

Table 5-2: Selected Portfolio Betas from the Industry-Sorted Portfolios

	Betas 1993	Betas 2000	Betas 2008
Portfolio 1 (Insurance)	0.5922	1.0864	0.9538
Portfolio 2 (Banks and Financial Services)	0.3463	0.7217	0.3863
Portfolio 3 (Property)	0.4464	0.8622	0.2701
Portfolio 4 (Chemicals, Oil and Gas)	0.3904	0.4198	0.1335
Portfolio 5 (Health Care)	0.3907	0.1346	0.3174
Portfolio 6 (Media)	0.3256	0.7911	0.2924
Portfolio 7 (Automobiles and Parts)	1.6991	0.5975	0.7512
Portfolio 8 (General Mining)	1.1251	1.1446	1.1771
Portfolio 9 (Platinum, Diamond, Coal and Precious Metals)	0.6586	0.4791	1.4928
Portfolio 10 (Gold Mining)	1.4034	0.8825	1.1017
Portfolio 11 (Basic Resources)	0.7230	0.4187	0.5014
Portfolio 12 (Building and Construction)	0.3467	0.9750	0.4564
Portfolio 13 (Industrial Engineering)	0.8546	1.3297	0.3445
Portfolio 14 (Industrial Transportation)	0.3414	1.1574	0.5887
Portfolio 15 (Other Industrial)	0.4914	0.6484	0.2754
Portfolio 16 (Technology and Electronics)	0.5054	1.8666	0.4946
Portfolio 17 (Food and Beverage)	0.1618	0.5490	0.1695
Portfolio 18 (Travel and Leisure)	0.4167	0.6818	0.3449
Portfolio 19 (Personal and Household Goods)	0.4225	0.9552	0.0565
Portfolio 20 (Retail)	0.5012	0.6073	0.1873

For the industry-sorted portfolios, there was no prior expectation regarding specific portfolios having the highest betas or the lowest betas provided that sufficient range in the estimates was obtained. The most notable observation from Table 5-2 is that the betas of the respective industries varied quite considerably across the three years presented, which reflects changes in the nature of business over time. A pertinent example of this is portfolio 16, consisting of all technology and electronics companies. The beta of this portfolio was much higher in the year 2000 compared to the other two periods, as this period corresponded with the dramatic volatility evinced in this industry at that time which included both the boom and the beginnings of the collapse. The three portfolios related to the mining industry (general mining; gold mining; platinum, diamond, coal and precious metals) exhibited high betas over the three years, with the betas of industries such as health care, travel and leisure, and retail remaining low over the periods examined. Comparing the beta values across Tables 5-1 and 5-2 it is apparent that the industry-sorted portfolios generally exhibited greater range in the beta estimates than associated with the beta-sorted portfolios.

5.3 The Estimates of the Minimum Required Return

5.3.1 Black *et al* (1972)

The methodology described in Section 4.3.3 for estimating the intercept of the CAPM based on the approach originally derived by Black *et al* (1972), but adjusted for more recent developments, was implemented and the results obtained are presented in Table 5-3 (on the following page). As discussed, it was not necessary to conduct any tests of the adherence of the samples to the assumptions of the CLRM. The relationships between the estimates of the intercept and the associated standard errors across the four samples will be analysed in conjunction with the estimates based on the other two techniques in Sections 5.4 and 5.5.

5.3.2 Fama and MacBeth (1973)

Using the portfolios consisting of ordinary shares, preference shares and bonds the methodology established in Section 4.3.4, based on Fama and MacBeth (1973), was employed to obtain estimates of the intercept of the CAPM over the various investment horizons. However, in estimating a regression according to the CLRM model several assumptions must hold; the most important identified in the context of this study being that the parameters are normally distributed and that the variance of the residuals is homoscedastic.

The residuals of each of these regressions were tested, using the BJ test, to determine whether the assumption of multivariate normal distributions was violated, as explained in Section 4.3.4.1. The test statistics and p-values for each regression are shown in Table 5-4 (page 143). EViews automatically calculates the p-value associated with the test statistic, where the p-value represents the lowest significance level at which the null hypothesis can be rejected (Gujarati, 2003: 137). Accordingly, those regressions where the p-values were below the chosen significance level of twenty percent (as highlighted in blue in Table 5-4) were observed to violate the assumption of normally distributed residuals.

Table 5-3: Estimates of the Intercept of the CAPM in South Africa (1993-2008) Based on the Approach of Black *et al* (1972)

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**Intercept**	**Standard Error**	**Intercept**	**Standard Error**	**Intercept**	**Standard Error**	**Intercept**	**Standard Error**
1993	6.7756	3.3674	2.6147	2.5907	4.4246	3.3257	2.5288	2.3482
1994	5.2226	1.7379	4.8319	2.5237	4.5454	2.0114	4.583	2.5337
1995	2.0838	1.6623	2.5001	3.2507	1.9802	1.384	2.3732	2.9718
1996	1.7957	4.8251	1.4354	4.8391	1.9726	1.8475	1.648	3.837
1997	0.2702	3.0964	0.3917	266.135	0.9826	2.4334	0.8009	266.5312
1998	0.7797	2.388	0.1401	6.1422	0.8789	1.8472	0.4103	5.1159
1999	2.83	4.2331	0.1818	9.1272	2.237	2.9637	0.7565	7.2512
2000	1.449	1.1416	-0.7775	0.8715	1.587	1.3188	-0.0564	0.2627
2001	2.4622	1.1416	3.3228	2.792	1.9942	1.3986	2.6281	2.3454
2002	3.4345	1.8032	1.8223	5.3717	2.5197	1.8521	1.6199	4.6467
2003	2.843	2.9302	2.8762	3.8906	2.1135	1.9567	2.3228	2.9686
2004	3.7856	3.6413	4.61	2.1461	2.8496	3.1737	3.8281	2.3819
2005	1.6396	2.7637	3.5683	2.4711	1.4639	1.9423	2.644	2.2414
2006	2.3944	3.0196	2.5618	1.9658	1.0624	1.71	1.6356	1.7429
2007	1.076	1.4212	0.6852	3.2423	0.759	1.1763	0.534	2.57
2008	-2.7321	1.2589	-2.4525	2.9483	-0.7542	1.9582	-1.1921	2.8045
1993/1994	6.8212	2.2708	3.8231	3.7505	4.9414	2.8987	3.5585	3.5004
1995/1996	2.3106	0.874	1.8945	2.9924	2.0741	0.6075	1.8969	2.4611
1997/1998	0.6971	1.7292	-0.0636	3.4712	1.0796	1.3569	0.4591	2.938
1999/2000	1.7646	3.2782	1.875	5.4997	1.7077	2.2849	1.8483	5.1569
2001/2002	3.2967	0.2282	3.933	7.043	2.3258	3.7557	2.9327	9.2948
2003/2004	5.9235	3.3229	4.9589	4.1486	3.3204	3.0198	3.5468	3.5928
2005/2006	2.3832	2.5263	2.7137	2.1014	1.4835	1.7136	1.8389	1.7618
2007/2008	-0.481	0.4609	-0.8817	0.8559	0.2667	0.2183	-0.1756	0.2087
1993/1997	3.5426	1.9352	2.8332	2.3066	2.6532	1.4523	2.5926	1.9649
1998/2002	2.4372	1.3378	3.7003	3.9244	2.0125	1.169	3.0302	3.4766
2003/2007	3.8145	2.1226	3.4056	1.6996	2.0078	1.8323	2.3013	1.7235
1993/2000	2.5825	3.1494	3.6736	1.7028	2.1403	2.0402	2.0849	1.8202
2001/2008	3.2613	0.963	2.9203	1.4982	1.9795	1.2267	2.0333	1.4454
1993/2008	3.2103	0.8253	2.9213	1.4986	NA		NA	

Table 5-4: BJ Statistics and P-Values for the Test of Normally Distributed Residuals[31]

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**BJ Statistic**	**P-Value**	**BJ Statistic**	**P-Value**	**BJ Statistic**	**P-Value**	**BJ Statistic**	**P-Value**
1993	1.0609	0.5883	0.0042	0.9979	1.2877	0.5253	0.05	0.9753
1994	0.2995	0.8609	1.3972	0.4973	0.1355	0.9345	1.8884	0.389
1995	0.1721	0.9175	0.0473	0.9766	0.0341	0.9831	0.3764	0.8284
1996	3.1514	0.2669	1.9169	0.3835	7.0113	0.03	1.787	0.4092
1997	0.3283	0.8486	1.2248	0.5421	0.4227	0.8095	0.8125	0.6661
1998	0.6107	0.7369	1.6473	0.4388	0.513	0.7738	1.6535	0.4375
1999	0.4581	0.7953	1.8645	0.3937	0.187	0.9108	3.6524	0.1611
2000	0.6186	0.7339	0.4521	0.7977	0.7427	0.6898	0.3809	0.8266
2001	0.3681	0.8319	0.861	0.6502	0.4438	0.8001	0.2578	0.8791
2002	0.0919	0.9551	21.6437	0	0.7282	0.6948	37.6475	0
2003	3.9506	0.1387	0.562	0.755	0.5155	0.7728	0.1067	0.9481
2004	1.0081	0.6041	1.4386	0.4871	3.0147	0.2015	1.5082	0.4704
2005	1.0829	0.5819	3.3658	0.1858	1.378	0.5021	17.5011	0.0002
2006	0.6902	0.7082	2.2329	0.312	0.7066	0.7024	2.9696	0.1966
2007	1.6778	0.4322	1.8385	0.3988	4.2963	0.1167	2.1906	0.3344
2008	0.5552	0.7576	1.1525	0.562	5.2672	0.0718	1.132	0.5678
1993/1994	0.6336	0.7285	1.8202	0.4025	0.8318	0.6598	1.8913	0.3884
1995/1996	1.4323	0.4886	5.0353	0.0806	0.9322	0.6275	3.1174	0.2004
1997/1998	0.3024	0.8596	1.9602	0.3753	0.328	0.8487	2.0493	0.3589
1999/2000	0.0977	0.9523	66.5746	0	0.0004	0.99998	91.6612	0
2001/2002	1.8376	0.399	5.3452	0.0691	1.2123	0.5454	11.9445	0.0025
2003/2004	1.9373	0.3796	1.3218	0.5164	18.0525	0.0001	1.9746	0.3726
2005/2006	1.1657	0.5583	4.9461	0.08433	1.7887	0.4089	10.3664	0.0056
2007/2008	0.5713	0.7515	2.9845	0.2049	10.9722	0.0041	12.7201	0.0017
1993/1997	2.8758	0.2074	0.8102	0.6669	0.5857	0.7461	1.1804	0.5542
1998/2002	0.4479	0.7994	1.5763	0.4547	0.8984	0.6381	6.4422	0.03991
2003/2007	1.0677	0.5863	0.0975	0.9524	7.7359	0.0209	0.399	0.8191
1993/2000	2.7869	0.2482	10.9844	0.0041	3.8037	0.1493	24.1413	0
2001/2008	0.8872	0.6417	8.028	0.0181	2.2717	0.3212	3.2723	0.1947
1993/2008	0.6957	0.7062	0.3961	0.8204	NA	NA	NA	NA

[31] A p-value below twenty percent was used to identify those regressions where the assumption of normality was violated (highlighted in blue). As can be seen, where the p-value was only slightly above 20 percent (sample 1 1993/1997; sample 2 2007/2008; sample 3 2004; and sample 4 1995/1996), the null hypothesis of normally distributed residuals was also rejected in favour of the alternative hypothesis that the error terms were non-normally distributed.

The assumption of normality was rejected in 30 of the 118 regressions (25.42 percent) and therefore to have assumed normally distributed parameters (as Fama and MacBeth (1973), Gibbons (1982) and Stambaugh (1982) did) may have resulted in erroneous conclusions being drawn from the hypothesis tests. Moreover, this finding that the returns earned on South African shares diverge from the normal distribution is in keeping with the results of Mangani (2007) discussed in Section 4.3.4.1. It is certainly clear from Table 5-4 that the occurrence of non-normality is more prevalent in samples 2 (industry-sorted ordinary shares), 3 (beta-sorted all assets) and 4 (industry-sorted all assets) than in sample 1 (beta-sorted ordinary shares). In light of this, it was expected that the inclusion of preference shares and bonds in samples 3 and 4 may represent outlier observations if they differed substantially in terms of the relationship between risk and return than associated with ordinary shares. However, the examination of the residual graphs in these periods indicated that this was not the case.

For those regressions where the assumption was violated, adjustments were made through the introduction of a relevant dummy variable as explained in Section 4.3.4.1. To assess the extent to which the dummy variables resulted in the assumption of normality being satisfied, the regressions were again tested, with the relevant BJ statistics and p-values shown in Table 5-5 (on the following page). In all cases except for three, the use of one dummy variable improved the distribution of the residuals sufficiently so as to satisfy the requirement that the p-value associated with the BJ statistic computed exceeded 20 percent. However, for sample 2 2001/2008, sample 4 2001/2002 and sample 4 2001/2008 the p-values were still smaller than 20 percent and thus an additional dummy variable was included for the second most extreme observation. As depicted in Table 5-6 (on the following page), the removal of the second observation was sufficient to ensure the normality assumption was upheld.

After the adjustments were made for non-normally distributed residuals all the regressions were also tested for heteroscedasticity using the BGK test. The statistics from these tests and the p-values are shown in Table 5-7 (page 146), with the p-values below 20 percent highlighted in blue. Heteroscedasticity was identified in 38.14 percent (45 of 118) of the regressions. The standard errors associated with the estimates of the intercept in those regressions where the null hypothesis was rejected are tabulated in Table 5-8 (page 147), with the adjusted standard errors, after applying White's (1980) transformation, shown alongside.

One of the consequences of the presence of heteroscedasticity is that the standard errors are not necessarily efficient. Accordingly, it was anticipated that following the adjustment based on White's (1980) transformation, the standard errors of the intercept estimates would decline.

Table 5-5: BJ Statistics and P-Values after the Introduction of a Dummy Variable[32]

	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**BJ Statistic**	**P-Value**	**BJ Statistic**	**P-Value**	**BJ Statistic**	**P-Value**	**BJ Statistic**	**P-Value**
1996					1.0803	0.5826		
1999							0.6731	0.7142
2002			1.2857	0.5258			0.5267	0.7685
2003	0.6891	0.7085						
2004					0.6612	0.7185		
2005			0.6041	0.7393			1.9799	0.3716
2006							0.5682	0.6388
2007					0.2622	0.8771		
2008					0.4101	0.8146		
1995/1996			1.3185	0.5172				
1999/2000			0.6055	0.7388			0.4762	0.7881
2001/2002			2.9353	0.2305			28.4523	0
2003/2004					1.9836	0.3709		
2005/2006			0.7123	0.7004			0.857	0.6515
2007/2008			0.4533	0.7972	3.9821	0.3589	0.1349	0.9348
1993/1997	1.0357	0.5958						
1998/2002							1.5582	0.4588
2003/2007					2.0475	0.3592		
1993/2000			1.426	0.4902	1.1953	0.5501	1.0386	0.5949
2001/2008			13.6428	0.001			3.2671	0.1952

Table 5-6: BJ Statistics and P-Values after the Introduction of a Second Dummy Variable[33]

	2: Industry-Sorted Ordinary Shares		4: Industry-Sorted All Assets	
	BJ Statistic	**P-Value**	**BJ Statistic**	**P-Value**
2001/2002			1.8653	0.3935
2001/2008	0.8434	0.6559	1.9199	0.3829

[32] A p-value below 20 percent was used to identify those regressions where the assumption of normality was violated (highlighted in blue).
[33] A p-value below 20 percent was used to identify those regressions where the assumption of normality was violated (highlighted in blue).

Table 5-7: BGK Statistics and P-Values for the Test of Homoscedasticity[34]

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	BGK Statistic	P-Value	BGK Statistic	P-Value	BGK Statistic	P-Value	BGK Statistic	P-Value
1993	2.8163	0.0933	0.1072	0.7434	0.0333	0.8551	0.4147	0.5196
1994	1.2664	0.2604	3.6161	0.0572	2.8683	0.0903	2.4059	0.1209
1995	1.772	0.1831	1.2762	0.2586	1.8643	0.1721	0.4636	0.4959
1996	3.169	0.075	0.033	0.8559	0.4434	0.8011	0.5201	0.4708
1997	1.0253	0.3113	0.3637	0.5465	1.6741	0.1957	0.4445	0.5049
1998	0.7099	0.3995	0.0003	0.9864	2.0049	0.1568	0.1791	0.6722
1999	0.5175	0.4719	3.2841	0.07	1.7098	0.191	0.4916	0.7821
2000	0.507	0.4765	1.0466	0.3063	0.0048	0.9448	2.3327	0.1267
2001	2.03	0.1542	1.0232	0.3118	1.6929	0.1932	0.0258	0.8725
2002	0.0002	0.9889	4.3515	0.1135	0	0.9944	4.9283	0.0851
2003	3.1824	0.2037	2.2051	0.1376	6.947	0.031	4.0961	0.043
2004	0.3609	0.548	2.4	0.1213	2.0017	0.3676	5.4154	0.02
2005	1.3402	0.247	2.5059	0.1134	2.0323	0.154	0.729	0.6946
2006	0.3002	0.5838	0.2846	0.5937	1.2651	0.2607	0.137	0.7113
2007	0.0791	0.7786	3.3446	0.0674	6.7967	0.0334	0.8099	0.3682
2008	2.5166	0.1127	0.0283	0.8664	1.0671	0.5865	0.0809	0.7761
1993/1994	2.2067	0.1374	0.6498	0.4202	1.142	0.2852	0.4525	0.5012
1995/1996	0.0664	0.7967	2.3095	0.3151	1.5105	0.2191	0.07	0.7914
1997/1998	0.3588	0.5492	0.6956	0.4043	0.8849	0.3469	0.7813	0.3767
1999/2000	2.3484	0.1254	3.0351	0.2193	3.7474	0.0529	3.8702	0.1444
2001/2002	0.323	0.5698	0.3407	0.8434	4.0687	0.0437	1.2783	0.7343
2003/2004	2.1848	0.1394	5.2035	0.0225	0.861	0.6502	5.0589	0.0245
2005/2006	0.0253	0.8735	1.2161	0.5444	0.0116	0.9142	1.6442	0.4395
2007/2008	3.2601	0.071	0.7864	0.6749	2.3009	0.3165	1.7771	0.4112
1993/1997	0.8102	0.6669	2.5389	0.1111	0.5772	0.4474	0.7828	0.3763
1998/2002	1.9507	0.1625	0.387	0.5339	0.4866	0.4855	0.9682	0.6163
2003/2007	1.7729	0.183	0.0116	0.9143	1.0881	0.5804	0.8287	0.3627
1993/2000	4.4623	0.1074	1.4764	0.478	4.4623	0.1074	1.5324	0.4648
2001/2008	2.2376	0.1347	8.0309	0.0454	0.0215	0.8835	4.0133	0.26
1993/2008	0.8498	0.3566	0.1415	0.7068	NA	NA	NA	NA

[34] A p-value below 20 percent was used to identify those regressions where the assumption of homoscedasticity was violated (highlighted in blue).

Table 5-8: Intercept Standard Errors before and after the Adjustment for Heteroscedasticity[35]

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	Before	After	Before	After	Before	After	Before	After
1993	1.4902	1.2993						
1994			0.5076	0.4222	0.5273	0.6366	0.5022	0.4461
1995	0.5373	0.4165			0.4144	0.3452		
1996	1.4875	2.0881						
1997					0.4693	0.3563		
1998					0.6376	0.4972		
1999			2.0765	2.0255	0.7435	0.5774		
2000							1.1462	0.9153
2001	0.4179	0.2989			0.3484	0.4408		
2002			0.4553	0.3962			0.3723	0.3403
2003	0.5869	0.6707	0.9078	0.9113	0.2795	0.2538	0.5784	0.4611
2004			0.5894	0.7846			0.5815	0.711
2005			0.5919	0.8962	0.36	0.4458		
2007			0.7379	1.3025	0.3547	0.2359		
2008	0.5583	0.7914						
1993/1994	0.6669	0.7837						
1995/1996					0.2193	0.1386		
1999/2000	0.9211	0.7687	0.681	0.3779	0.5741	0.3841	0.6051	0.3402
2001/2002					0.4191	0.5324		
2003/2004	0.8691	1.1558					0.5405	0.3935
2007/2008	0.727	0.8814	0.4971	0.4697				
1993/1997			0.4402	0.4734				
1998/2002	0.4941	0.326						
2003/2007	1.0324	1.1728						
1993/2000	1.3128	1.8679			1.3128	1.8679		
2001/2008	0.5906	0.5971	0.2489	0.2585				

[35] The yellow highlighted values show the regressions where the standard errors increased after the application of White's (1980) transformation.

As is clear from the results presented in Table 5-8, this was only true in half of the regressions (53.33 percent), with the standard errors that increased highlighted in yellow. However, according to Brooks (2006: 152) this is to be expected, as the adjustment to the standard errors reflects the sign of the relationship between the residuals and the explanatory variable that is causing the violation of the assumption of homoscedasticity. That is, if the variance of the residuals is positively related to beta (the explanatory variable), then the standard errors will increase after the adjustment. The implication of this is that more evidence would be required against a null hypothesis being tested before it is rejected (Brooks, 2006: 152). The opposite is therefore true with respect to a negative relationship being identified between the variance of the residuals and beta. Therefore, it can be concluded that in approximately half of the regressions identified to violate the assumption of homoscedasticity the relationship between the variance of the residuals and beta was positive and in approximately half the relationship was negative.

Taking into consideration the necessary adjustments made for heteroscedasticity and the non-normality of the residuals, the estimates of the minimum required return and the associated standard errors based on the Fama and MacBeth regressions are shown in Table 5-9 (on the following page).

5.3.3 Stambaugh (1982)

Lastly, the results obtained for the intercept and associated standard errors based on the approach of Stambaugh (1982), adjusted for heteroscedasticity and non-normally distributed residuals, are given in Table 5-10 (page 150). The relationships between these estimates of the intercept and those based on the methodology of Black *et al* (1972) and Fama and MacBeth (1973) are examined in the following section.

Table 5-9: Estimates of the Intercept of the CAPM in South Africa (1993-2008) Based on the Approach of Fama and MacBeth (1973)

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**Intercept**	**Standard Error**	**Intercept**	**Standard Error**	**Intercept**	**Standard Error**	**Intercept**	**Standard Error**
1993	4.747	1.2993	2.5682	0.6276	2.7465	0.9193	2.4863	0.5401
1994	4.732	0.4319	5.0838	0.4222	4.0691	0.6366	4.7803	0.4461
1995	2.9973	0.4165	2.6356	1.2195	2.4741	0.3452	2.45	0.747
1996	2.2021	2.0881	1.8177	1.1258	1.8358	0.2974	1.913	0.8379
1997	0.4626	0.844	0.3977	0.9636	1.1248	0.3563	0.8617	0.7043
1998	2.1132	1.0597	-0.3908	1.7683	1.5121	0.4972	0.1546	1.3289
1999	3.7394	1.124	0.5001	2.0255	2.7044	0.5774	2.0456	0.9714
2000	2.025	0.8283	-0.7707	1.3906	1.9969	0.5482	-0.0463	0.9153
2001	3.4847	0.2989	3.8323	0.6981	2.1777	0.4408	2.7442	0.5073
2002	3.4236	0.6118	2.1362	0.3962	2.5211	0.5025	1.8686	0.3403
2003	2.0797	0.6707	2.1208	0.9113	1.4205	0.2538	1.8733	0.4611
2004	4.2309	0.7849	4.4974	0.7846	1.8127	0.5164	3.6641	0.711
2005	1.1162	0.5803	3.4753	0.8962	1.2157	0.4458	1.9358	0.3132
2006	0.7961	0.9135	2.615	0.5057	0.581	0.3327	1.4761	0.3205
2007	1.3678	0.5874	0.6945	1.3025	-0.0103	0.2359	0.4992	0.5196
2008	-2.1605	0.7914	-1.9421	0.5706	0.4006	0.2941	1.5006	0.6761
1993/1994	4.7826	0.7837	2.9503	0.593	2.9988	0.663	2.7393	0.5301
1995/1996	2.6261	0.4567	0.1945	0.6542	2.0683	0.1386	1.5709	0.7037
1997/1998	1.7825	0.7465	-0.9918	0.9886	1.6727	0.4156	0.0941	0.763
1999/2000	1.7156	0.7687	0.3368	0.3779	1.7422	0.3841	0.533	0.6051
2001/2002	4.074	0.3846	3.0962	0.7621	2.3678	0.5324	1.9173	0.3646
2003/2004	4.2076	1.1558	2.6184	0.7598	1.7223	0.313	1.9463	0.3935
2005/2006	1.1577	0.7005	1.6119	0.4409	1.0012	0.3472	1.2067	0.274
2007/2008	-0.2875	0.8814	-0.283	0.4697	0.6915	0.0698	0.1893	0.3246
1993/1997	1.4126	0.2037	1.6583	0.4734	1.9104	0.1758	1.78	0.3321
1998/2002	3.5387	0.326	4.0301	1.1789	2.3172	0.4523	2.6027	0.8851
2003/2007	2.5111	1.1728	2.7085	0.5226	1.0147	0.2819	1.613	0.3436
1993/2000	1.4214	1.8679	0.9587	0.8166	1.6383	1.8679	1.4766	0.4889
2001/2008	3.9479	0.5971	1.8196	0.2585	1.5612	0.3422	1.355	0.2166
1993/2008	3.9021	0.4226	2.0619	0.6924	NA	NA	NA	NA

Table 5-10: Estimates of the Intercept of the CAPM in South Africa (1993-2008) Based on the Approach of Stambaugh (1982)

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	Intercept	Standard Error	Intercept	Standard Error	Intercept	Standard Error	Intercept	Standard Error
1993	4.7477	2.2915	2.5682	0.6206	2.7465	0.9185	2.4863	0.5572
1994	4.732	0.3766	5.0837	0.6655	4.0691	0.4047	4.7804	0.5737
1995	2.9974	0.7093	2.6356	0.6457	2.4741	0.4824	2.45	0.6197
1996	2.2022	1.4735	1.8173	1.014	1.8223	0.4855	1.913	0.8884
1997	0.4626	0.8091	0.3977	0.7919	1.1248	0.6016	0.8616	0.7168
1998	2.1132	1.1238	-0.3908	2.3232	1.5121	0.777	0.1546	1.9258
1999	3.7394	1.2344	0.5001	2.9753	2.7044	1.1613	1.7329	1.5271
2000	2.025	0.803	-0.7706	1.6432	1.9968	0.6762	-0.0463	1.5823
2001	3.4847	0.5336	3.8323	0.7066	2.1778	0.3024	2.7442	0.515
2002	3.4237	0.7648	1.9569	0.4818	2.5211	0.5408	1.6317	0.3471
2003	1.7469	1.6608	2.1207	0.9137	1.4271	0.3044	1.8733	0.7504
2004	4.231	0.7729	4.4974	0.5219	1.8414	0.7994	3.6642	0.457
2005	1.1163	0.4789	3.3415	0.7781	1.2158	0.3148	1.9531	0.4222
2006	0.7961	1.0104	2.615	0.6937	0.581	0.3902	1.581	0.4349
2007	1.3679	0.8195	0.6944	0.5011	-0.0662	0.5707	0.4992	0.3803
2008	-2.1605	0.3802	-1.9421	0.5435	0.3418	0.2985	-0.8183	0.5096
1993/1994	4.7827	0.9	2.9502	0.7742	2.9988	0.4976	2.7392	0.623
1995/1996	2.6256	0.5387	0.8456	0.8683	2.0683	0.3324	1.3087	0.6108
1997/1998	1.7825	0.9219	-0.9918	1.1723	1.6727	0.5481	0.0943	0.8076
1999/2000	1.7156	1.1225	1.8547	2.0835	1.7422	0.932	1.8295	1.8952
2001/2002	4.074	0.4418	3.0312	1.9194	2.3679	0.3957	2.244	0.9552
2003/2004	4.2078	1.4503	2.6184	1.0748	1.745	0.2944	1.9463	0.7744
2005/2006	1.1576	0.8625	1.6152	0.4043	1.0012	0.3817	1.212	0.2652
2007/2008	-0.2876	0.7906	-0.2806	0.4911	0.6692	0.1844	0.2981	0.5087
1993/1997	1.4058	0.1532	1.6583	0.4242	1.9104	0.1827	1.78	0.4132
1998/2002	3.5387	1.1944	5.3587	1.425	2.3172	0.3727	2.6232	1.0279
2003/2007	2.5111	0.9947	2.7085	0.5005	1.0305	0.28	1.613	0.3657
1993/2000	1.4213	1.5368	0.9583	0.9423	1.6382	1.1712	1.4766	0.7185
2001/2008	3.9478	0.8178	1.8193	0.5558	1.5612	0.5294	1.355	0.265
1993/2008	3.9022	0.3153	1.6135	0.693	NA	NA	NA	NA

5.4 The Most Efficient Methodology to Estimate the Intercept

Mean and variance tests were conducted across all four samples and across all time periods in order to ascertain the differences in the estimates of the intercept computed based on the different approaches. As specified in Section 4.3.7, it was not considered necessary to examine the extent to which the CAPM relationships held for this purpose as all three methods used the same samples to estimate the values and consequently the extent to which these relationships held should not impact on the results obtained. The mean and variance tests for the comparison of the Black *et al* (1972) estimates to those based on the Fama and MacBeth (1973) and Stambaugh (1982) approaches are shown in Tables 5-11 (page 152) and 5-12 (page 153) respectively, with the statistical comparisons between the Fama and MacBeth (1973) and Stambaugh (1982) values shown in Table 5-13 (page 157).

The tests of the equality of the average monthly minimum return estimates of the Black *et al* (1972) and Fama and MacBeth (1973) approaches across the four samples revealed that the number of estimates where the differences were statistically significantly different was small; only 21 of the 118 (17.80 percent) test statistics across the four samples were statistically significant at the ten percent level or lower. For the comparison to the Stambaugh (1982) based estimates 18 (15.25 percent) were statistically significantly different. Thus, whilst the estimates of the intercept via the approach of Black *et al* (1972) do differ, this difference is not consistently significant. Moreover, there is no conclusive evidence as to whether the Black *et al* (1972) based estimates are consistently larger or smaller than the other values; in fact, across all four samples, the divide between the number of observations that were larger based on the Black *et al* (1972) technique compared to the other two approaches is almost equal (54.24 and 53.38 percent are positive across Tables 5-11 and 5-12 respectively, which reflects larger values for the Black *et al* (1972) based estimates).

The fact that the Black *et al* (1972) based estimates were not adjusted for the violation of the assumption of non-normality may have contributed towards the differences observed in the mean returns, as the Black *et al* (1972) values included observations that were not present in computing the Fama and MacBeth (1973) or Stambaugh (1982) values. A comparison of the years where dummy variables were included to the results presented in Tables 5-11 and 5.12, indicates that several of the statistically significant differences did occur in these years (52.38 percent and 46.06 percent respectively of the total) but that these adjustments did not fully account for all the statistically significant differences observed. The modifications for heteroscedasticity may have also impacted on the statistics through the standard error values.

Table 5-11: Test Statistics for the Comparison of the Black *et al* (1972) and Fama and MacBeth (1973) Based Estimates[36]

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**Mean Test[37]**	**Variance Test[38]**	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**
1993	1.7773	6.7166***	0.0779	17.0384***	1.6847	13.0878***	0.0827	18.9051***
1994	0.8663	16.1953***	-0.4404	35.7230***	0.7821	9.9827***	-0.3598	32.2549***
1995	-1.6858	15.9268***	-0.1745	7.1052***	-1.1995	16.0778***	-0.1176	15.8262***
1996	-0.2444	5.3394***	-0.3441	18.4746***	0.2533	38.5807***	-0.3165	20.9716***
1997	-0.1896	13.4606***	-0.0001	76287.24***	-0.2002	46.6463***	-0.0011	143211.60***
1998	-1.614	5.0783***	0.3714	12.0647***	-1.1468	13.8044***	0.2269	14.8195***
1999	-0.6566	14.1835***	-0.1523	20.3053***	-0.5362	26.3502***	-0.8264	55.7198***
2000	-1.2915	1.8998	-0.0188	0.3927*	-0.9943	5.7866***	-0.0499	0.0824
2001	-2.7399**	14.5856***	-0.7918	15.9962***	-0.4847	10.0660***	-0.242	21.3754***
2002	0.018	8.6871***	-0.2604	139.22***	-0.0028	13.5865***	-0.2668	186.5014***
2003	0.8164	55.6418***	0.8455	18.2252***	1.3579	49.0047***	0.7481	41.4425***
2004	-0.378	21.5214***	0.2205	7.4827***	1.2489	37.7632***	0.3298	11.2234***
2005	0.5861	22.6832***	0.1581	7.6026***	0.4824	18.9794***	1.5646	51.2244***
2006	1.6021	10.9254***	-0.1174	15.1124***	1.07	26.4124***	0.4501	29.5734***
2007	-0.6001	5.8535***	-0.0118	6.1962***	2.4833*	24.8552***	0.0664	24.4642***
2008	-1.2157	2.5302*	-0.76	26.6943***	-2.2586*	44.3288***	-4.6671***	17.2049***
1993/1994	2.6835**	8.3961***	1.0279	39.9966***	2.2631*	19.1145***	1.0853	43.6098***
1995/1996	-1.0117	3.6625**	2.4821**	20.9244***	0.0322	19.2154***	0.5975	12.2314***
1997/1998	-1.8225	5.3661***	1.1501	12.3285***	-1.448	10.6578***	0.5639	14.8252***
1999/2000	0.0461	18.1872***	1.2414	65.2195***	-0.0516	35.3871***	1.1882	72.6217***
2001/2002	-5.4958***	0.3522 ˣ	0.5283	85.4071***	-0.0429	49.7643***	0.5458	650.0459***
2003/2004	1.5423	8.2654***	2.4818	29.8118***	2.0387	93.0662***	2.2141*	83.3468***
2005/2006	1.4783	13.0074***	2.2950*	22.7167***	1.0685	24.3523***	1.773	41.3546***
2007/2008	-0.6152	0.2734**	-2.7052**	2.9648***	-7.2410***	9.7735***	-4.7270***	0.4134 ˣˣˣ
1993/1997	3.4614*	90.2305***	2.2313*	23.7383***	1.759	68.2621***	1.9126	35.0034***
1998/2002	-2.5296*	16.8444***	-0.36	11.0815***	-0.8422	6.6800***	0.5589	15.4273***
2003/2007	1.6996	3.2756**	1.7534	10.5755***	2.0749	42.2499***	1.9582	25.1549***
1993/2000	1.105	8.8503***	6.4291***	4.3481***	0.9191	13.3269***	1.6139	13.8627***
2001/2008	-1.9219	2.6589*	3.1775***	14.4435***	1.2721	12.8491***	2.3206*	44.5186***
1993/2008	-2.3593*	3.8146**	2.3283*	4.6843***	NA	NA	NA	NA

* (ˣ) Statistically significant at the 10% level
**(ˣˣ) Statistically significant at the 5% level
***(ˣˣˣ) Statistically significant at the 1% level
The ˣ denote where the inverse of the F statistics are statistically significant.

---

[36] The Black *et al* (1972) estimates are used as the first value and the numerator in the mean and variance tests respectively.

[37] The t-critical values were used as both sets of estimates follow this distribution. The degrees of freedom for Samples 1 and 2 were nine and 19 respectively; whilst for Samples 3 and 4, eleven and 21 were used for periods before 2001, and fourteen and 24 for periods thereafter.

[38] The F-critical values differed across the four samples because of the different numerator (n_1) and denominator (n_2-1) degrees of freedom.

Table 5-12: Test Statistics for the Comparison of the Black *et al* (1972) and Stambaugh (1973) Based Estimates[39]

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**Mean Test**[40]	**Variance Test**[41]	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**
1993	1.5744	2.1594	0.078	17.4288***	1.6848	13.1090***	0.0826	17.7574***
1994	0.8724	21.3014***	-0.4316	14.3809***	0.8042	24.7051***	-0.3564	19.5050***
1995	-1.5986	5.4916***	-0.1829	25.3468***	-1.1672	8.2311***	-0.1187	22.9969***
1996	-0.2549	10.7228***	-0.3454	22.7732***	0.2726	14.4828***	-0.3156	18.6547***
1997	-0.1901	14.6468***	-0.0001	112936.63***	-0.1964	16.3607***	-0.0011	138258.80***
1998	-1.5977	4.5156**	0.3615	6.9903***	-1.0946	5.6511***	0.2194	7.0568***
1999	-0.6522	11.7602***	-0.1483	9.4103***	-0.5086	6.5131***	-0.618	22.5475***
2000	-1.3051	2.0215	-0.0166	0.2813 xxx	-0.9579	3.8031**	-0.0297	0.0276 xxx
2001	-2.5658*	4.5766**	-0.7911	15.6123***	-0.4969	21.3875***	-0.2418	20.7427***
2002	0.0174	5.5598***	-0.1116	124.3268***	-0.0028	11.7308***	-0.0127	179.2624***
2003	1.0291	3.1127**	0.8454	18.1318***	1.3424	41.3275***	0.734	15.6493***
2004	-0.3783	22.1974***	0.2281	16.9092***	1.1931	15.7612***	0.3379	27.1616***
2005	0.59	33.3010***	0.3914	10.0848***	0.4882	38.0685***	1.5146	28.1836***
2006	1.5873	8.9305***	-0.1143	8.0305***	1.0628	19.2023***	0.1519	16.0599***
2007	-0.5627	3.0076**	-0.0125	41.8701***	2.4443*	4.2484**	0.067	45.6734***
2008	-1.3746	10.9623***	-0.7613	29.4248***	-2.1430*	43.0446***	-0.6558	30.2848***
1993/1994	2.6391*	6.3661***	1.0193	23.4689***	2.2881*	33.9321***	1.0808	31.5699***
1995/1996	-0.9702	2.6324*	1.5055	11.8769***	0.029	3.3409**	1.088	16.2365***
1997/1998	-1.7516	3.5181**	1.1329	8.7678***	-1.4042	6.1287***	0.5615	13.2340***
1999/2000	0.0448	8.5297***	0.0154	6.9674***	-0.0485	6.0097***	0.016	7.4039***
2001/2002	-4.9427***	0.2669 xx	0.5525	13.4640***	-0.0431	90.0609***	0.3685	94.6929***
2003/2004	1.4965	5.2496***	2.4424*	14.8974***	2.011	105.2464***	2.1773*	21.5250***
2005/2006	1.4518	8.5798***	2.2957*	27.0174***	1.0641	20.1538***	1.7594	44.1307***
2007/2008	-0.6685	0.3399 xx	-2.7240**	3.0376***	-5.4565***	1.4016	-0.7546	0.1007 xxx
1993/1997	3.4807**	159.5919***	2.2403*	29.5613***	1.758	63.2077***	1.8982	22.6089***
1998/2002	-1.9422	1.2546	-1.7764	7.5839***	-0.8602	9.8392***	-0.34	7.7397***
2003/2007	1.7583	4.5541**	1.7597	11.5318***	2.0422	42.8293***	1.9533	22.2056***
1993/2000	1.0478	4.1998***	6.2396***	3.2654**	0.8265	3.0344**	1.5544	6.4175***
2001/2008	-1.7183	1.3868	3.0814**	7.2668***	1.2124	5.3692***	2.3080*	29.7409***
1993/2008	-2.4762*	6.8520***	3.5425**	4.6756***	NA	NA	NA	NA

* Statistically significant at the 10% level
**(xx) Statistically significant at the 5% level
***(xxx) Statistically significant at the 1% level
The x denote where the inverse of the F statistics are statistically significant.

[39] The Black *et al* (1972) estimates are used as the first value and the numerator in the mean and variance tests respectively.
[40] The t-critical values were used as explained in Section 4.3.6, with the same critical values as for Table 5-11.
[41] The F-critical values differed across the four samples because of the different numerator (n_1) and denominator (n_2-1) degrees of freedom.

Nine of the 21 statistically significant test statistics in Table 5-11 and six of the 18 in Table 5-12 coincided with periods where White's standard errors were employed and more significantly, only three (in both tables) of these corresponded to the same periods as the adjustments made for non-normality. That is, between these two procedures, 80.95 and 82.35 percent respectively of the statistically significant statistics were accounted for; thereby suggesting that the results observed are, to a large extent explained by changes in the sample rather than actual differences in the estimation techniques employed. This appears to suggest therefore that because it is assumed that the Black *et al* (1972) based estimates follow a t-distribution it would have been more accurate to adjust for non-normality and heteroscedasticity. However, adjusting for the former would only have been possible based on the results of the BJ statistics from the Fama and MacBeth (1973) regressions; with no apparent solution to the problem of heteroscedasticity. This highlights one of the weaknesses of this approach, as it is not feasible to make the necessary adjustments to obtain reliable parameter and standard error estimates.

There does not appear to be any notable trends with respect to the equality of the estimates across these two procedures over time; however, it is of value to note that the estimates of the intercept from the approaches based on Fama and MacBeth (1973) and Stambaugh (1982) across both samples 1 and 2 differ significantly from the Black *et al* (1972) values for the entire period 1993 to 2008 (which have not been adjusted for violations of either of the assumptions). This finding therefore intimates that over long periods of time, the estimates of the intercept based on the methodology of Black *et al* (1972) may differ significantly from those obtained if either of the other two approaches was implemented and hence may impact on the results of any hypothesis tests or analyses conducted.

Whilst the differences between the Black *et al* (1972) estimates of the intercept and the Fama and MacBeth (1973) and Stambaugh (1982) values may be small enough to regard as inconsequential the differences in the standard errors are substantial. As can be seen in Tables 5-11 and 5-12, across all four samples the F-test statistics are statistically significant at the ten percent level and lower; almost all of the test statistics were in fact larger than the critical value at the one percent level (91.5 and 80.5 percent in Tables 5-11 and 5-12 respectively). Moreover, the statistics are greater than one meaning that it is the variance of the estimates of the intercept based on the approach of Black *et al* (1972) that are statistically significantly less efficient than the other two approaches. The impact of the adjustments for non-normality and heteroscedasticity on the results obtained were assessed, and revealed that these adjustments did not influence the findings. The F-test statistics that were below one indicated that in the particular periods in question, the standard error of either the Stambaugh (1982) or Fama and MacBeth (1973) estimates of the intercept were larger than that of the Black *et al* (1972) based estimates. Therefore, as explained in Section 4.3.6, these test statistics should be computed as the inverse of this value; however, for ease of analysis these values were retained and were compared to the inverse of the critical values (but adjusted for the degrees of freedom) to ascertain whether they were also statistically significant. Only six were significant across both sets of test statistics.

The findings of this analysis thus provides evidence that, on average, the methodology of Black *et al* (1972), in computing the intercept in this study, provided less efficient estimates than the other two methods. It can therefore be inferred that the estimates of the intercept, which did differ, especially over longer periods, from those using the other two methods are likely to be less reliable and thus are not employed in any further analyses in this study.

This finding regarding the efficiency of the Black *et al* (1972) based estimates is not entirely unexpected as it does not exhibit the theoretical fundamentals upon which the other two approaches are founded. OLS estimation, the basis of the Fama and MacBeth (1973) approach, effectively determines the standard error of the estimates as the difference in the actual observation to the predicted value, with the predicted values computed so as to minimise this difference (Brooks, 2006: 48). Thus, the impact of any outlier observations on the results obtained should be reasonably small if the other data points are distributed closely together. Similarly, for ML estimation (the basis of the Stambaugh (1982) approach), the regression coefficients are computed as the most likely values and if the majority of the observations are congested together the coefficient estimates will be based on these values rather than the extreme observations which are perceived as less likely to occur (Gujarati, 2003: 115). With the Black *et al* (1972) approach the impact of outlier observations on the results obtained is likely to be more substantial if the extreme observation is associated with a very large or very small beta value for the portfolio. That is, the average estimate of the intercept and the associated standard error based on this approach is not a function of how closely distributed the various values are, but rather is a weighted average of the various individual portfolio estimates, where the weightings are determined by the betas of the portfolios.

There is a clear example of this in the results obtained, which can be seen by examining Tables 5-11 and 5-12. The F-test statistics in the year 1997 for samples 2 and 4 (the industry-sorted portfolios) are *very* large (between 76 000 and 145 000) compared to the other statistics which are also statistically significant at the one percent level. This was caused by one portfolio where the relationship observed between risk and return was in contradiction to the theory underlying the CAPM. That is, the portfolio had a large beta estimate (1.011) and given that the market portfolio returns were negative (-0.5681), the portfolio returns should have been similar in size and sign to the market returns. The returns earned however, were substantially positive (2.79 percent); therefore, the estimate of the minimum required return based on this portfolio was very large and negative. Therefore, because this portfolio had the second largest beta in comparison to other portfolios in that year, this estimate contributed a large proportion to the overall value. In contrast, the estimates of the intercept and the variance based on the approaches of Fama and MacBeth (1973) and Stambaugh (1982) did not differ notably from other years and therefore the impact of this outlier observation was small (as observed in Table 5-13, page 157). Moreover, this outlier observation did not result in the normality assumption being violated for these samples in that year, meaning that it was included in determining the intercept values based on both these methods.

For the tests of the equality of the mean values across the Fama and MacBeth (1973) and Stambaugh (1982) based estimates, the t-test statistics were, on average, statistically insignificant. This finding in not surprising given that the ML estimation employed in this study utilised the OLS coefficient estimates as the starting values for these estimates. As indicated in Table 5-13, within samples 1 and 3, there were no exceptions to this conclusion; whilst in samples 2 and 4 there were four statistically significant test statistics in each. A more thorough examination of the periods in which these apparent contradictions occurred revealed that six of these eight observations across both samples occurred in periods where adjustments were made for non-normally distributed residuals (all six cases) and heteroscedasticity (two of the same periods). The findings regarding the influence of heteroscedasticity are not surprising given that the adjustment made with the ML estimates was not comprehensive, but the finding with respect to the adjustments for non-normality is surprising given the inclusion of the identical dummy variables.

With respect to the values of the standard errors of the Fama and MacBeth (1973) and Stambaugh (1982) estimates, the results of the test statistics are not clear-cut. Across the four samples, only 36.64 percent (43 of the 118) of the F-test statistics indicated that the efficiency of the two sets of estimates differed significantly based on the chosen critical values. Of the test statistics which were significant, 29.66 percent revealed that the Fama and MacBeth (1973) estimates were more efficient, with only 6.78 percent favouring the Stambaugh (1982) estimates as the most efficient. However, in comparing the periods in which the assumption of non-normality was identified to the results in Table 5-13, it was evident that there is some correlation (33.33 percent) between the introduction of a dummy variable and the statistical significance of the standard errors.

Table 5-13: Test Statistics for the Comparison of the Fama and MacBeth (1973) and Stambaugh (1982) Based Estimates[42]

Sample	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Assets		4: Industry-Sorted All Assets	
Year	**Mean Test[43]**	**Variance Test[44]**	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**
1993	-0.0009	0.3215 ^{xx}	0	1.0229	0	1.0016	0	0.9393
1994	0	1.3153	0.0007	0.4026 ^{xx}	-0.0001	2.4748	-0.0004	0.6047
1995	-0.0002	0.3448 ^{x}	0	3.5674**	0.0004	0.512	0	1.4531
1996	-0.0002	2.0083	0.001	1.2327	0.0821	0.3754 ^{xx}	0.0002	0.8895
1997	0	1.0881	-0.0001	1.4804	0.0001	0.3507 ^{xx}	0.0003	0.9654
1998	0	0.8892	0	0.5794	0.0002	0.4094 ^{xx}	0	0.4762 ^{xx}
1999	0	0.8292	0.0001	0.4634 ^{x}	0	0.2472 ^{xx}	0.8104	0.4047 ^{xx}
2000	0	1.0641	-0.0001	0.7162	0.0004	0.6572	0	0.3347 ^{xxx}
2001	-0.0001	0.3138 ^{xx}	0.0001	0.976	-0.0004	2.1247*	0.0002	0.9704
2002	-0.0001	0.64	1.2099	0.893	0	0.8634	2.4365**	0.9612
2003	0.6167	0.0559 ^{xxx}	0.0001	0.9949	-0.0623	0.8433	-0.0001	0.3776 ^{xxx}
2004	-0.0002	1.0314	-0.0001	2.2598*	-0.1167	0.4174 ^{xx}	-0.0004	2.4201**
2005	-0.0003	1.4681	0.5043	1.3265	-0.0012	2.0058	-0.1648	0.5502 ^{x}
2006	0.0001	0.8174	0	0.5314 ^{x}	0	0.727	-0.971	0.5431 ^{x}
2007	-0.0003	0.5138	0.0002	6.7574***	0.3507	0.1709 ^{xxx}	0.0001	1.8670*
2008	0	4.3325**	-0.0003	1.1023	0.5431	0.971	13.6943***	1.7602*
1993/1994	0	0.7582	0.0005	0.5868	0	1.7752	0.0004	0.7239
1995/1996	0.0022	0.7187	-2.6785**	0.5676	0	0.1739 ^{xxx}	1.3194	1.3274
1997/1998	0.0001	0.6556	-0.0002	0.7112	0	0.575	-0.0008	0.8927
1999/2000	-0.0001	0.469	-3.0969**	0.1068 ^{xxx}	0	0.1698 ^{xxx}	-3.0567**	0.1020 ^{xxx}
2001/2002	0	0.7578	0.1407	0.1576 ^{xxx}	-0.0002	1.8097	-1.5977	0.1457 ^{xxx}
2003/2004	-0.0002	0.6351	0	0.4997 ^{x}	-0.2045	1.1309	-0.0002	0.2583 ^{xxx}
2005/2006	0.0001	0.6596	-0.025	1.1893	-0.0001	0.8276	-0.0694	1.0671
2007/2008	0.0001	1.2432	-0.0149	1.0246	0.5092	0.1434 ^{xxx}	1.7773	0.2436 ^{xxx}
1993/1997	0.0841	1.7687	0.0002	1.2453	0.0001	0.926	0	0.6459
1998/2002	-0.0001	0.0745 ^{xxx}	-3.2126***	0.6844	0.0004	1.4729	-2.1296*	0.5017 ^{x}
2003/2007	0	1.3903	0	1.0904	-0.1543	1.0137	0.0003	0.8828
1993/2000	0.0002	0.4745	0.0014	0.751	0.0001	0.2277 ^{xxx}	-0.0001	0.4629 ^{xx}
2001/2008	0.0003	0.5216	0.0018	0.5031 ^{x}	-0.0003	0.4179 ^{xx}	-0.0002	0.6681
1993/2008	-0.0003	1.7963	2.0471	0.9981	NA	NA	NA	NA

* Statistically significant at the 10% level
**(^{xx}) Statistically significant at the 5% level
***(^{xxx}) Statistically significant at the 1% level
The ^{x} denote where the inverse of the F statistics are statistically significant.

[42] The Fama and MacBeth (1973) estimates are used as the first value and the numerator in the mean and variance tests respectively.

[43] The t-critical values were used as explained in Section 4.3.6, with the degrees of freedom identical to those used in Tables 5-11 and 5-12. .

[44] The F-critical values differed across the four samples because of the different numerator (n_1) and denominator (n_2-1) degrees of freedom.

Of greater importance however, is the correlation between those periods where heteroscedasticity was identified as the statistically significant variance tests occurred in 57.1 percent of the periods. Combining the non-normality and heteroscedasticity results (taking into consideration double counting) revealed that 76. 19 percent (32 of the 42 estimates) were in these periods. These results therefore confirm the expectation that the adjustments made for heteroscedasticity in the ML estimates were potentially inadequate, and more importantly that there does not appear to be a notable difference in the efficiency of the estimates based on the Fama and MacBeth (1973) and Stambaugh (1982) procedures.

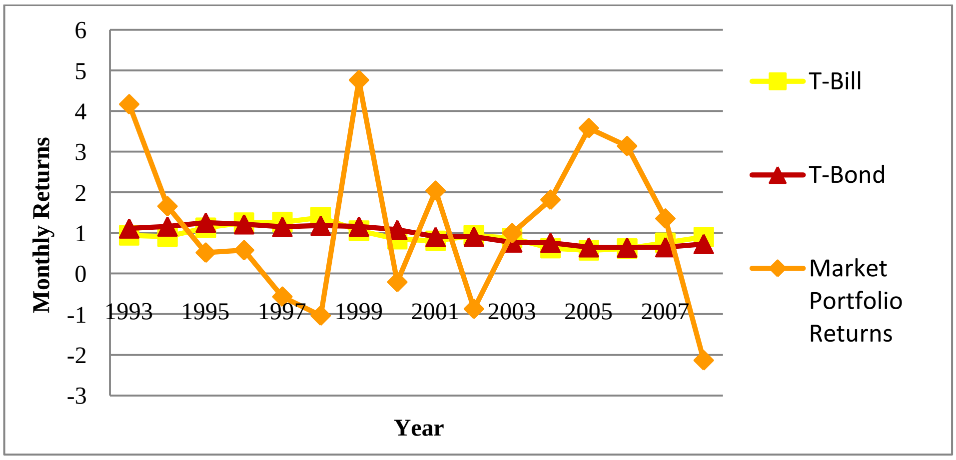
If a model adheres to a linear form then it is to be expected that the OLS estimates are the most efficient as they are the 'best' estimates (Gujarati, 2003: 115). According to Gibbons (1982) and Stambaugh (1982) however, because of the assumption imposed that both the market model and CAPM hold simultaneously, OLS estimates will be inefficient. If the majority of the test statistics had indicated that the OLS estimates were the most efficient then this finding might suggest that in fact this assumption does not hold in South Africa (in accordance with Gibbons, 1982 and Stambaugh, 1982). However, in this study, the majority of the test statistics infer that there are no significant differences between the efficiency of the two approaches and thus, there is no reason to suggest that the assumption that both the market model and the CAPM hold simultaneously is inappropriate in South Africa. Table 4.1 (page 102) in Section 4.3 presented the approaches that various authors have employed to both estimate and test the traditional CAPM and extended versions of the model. It is clear from this table, that a substantial number of scholars have employed the Fama and MacBeth (1973) approach with the assumption that both the market model and CAPM hold simultaneously post both Gibbons (1982) and Stambaugh's (1982) research. Therefore these studies may suggest that these authors do not believe that employing ML is likely to provide any more reliable estimates than OLS.

In light of the similarity between the estimates of the intercept and the standard errors based on the approaches of Fama and MacBeth (1973) and Stambaugh (1982), employing either of these two sets of estimates should result in identical conclusions in the remaining analyses in this study. Any differences will be in those periods where heteroscedasticity was not appropriately corrected for in the ML regressions as was done in the OLS regressions. Accordingly, the OLS regression estimates of the intercept, based on the methodology established by Fama and MacBeth (1973), were selected as the best estimates of the intercept of the CAPM in this study.

5.5 Optimal Asset Allocation, Asset Inclusion and Time Period

In examining the optimal asset allocation method, asset classes to include, and the period over which the CAPM parameters should ideally be estimated the extent to which the CAPM relationships, that beta is an important determinant of risk and that the market risk premium has the correct sign, were assessed. Figure 5-2 shows the average monthly market returns, T-Bill and T-Bond returns[45] for each year from 1993 to 2008. The ALSI clearly exhibited substantial volatility over the period and consequently, there are numerous years in which the returns were negative or smaller than the T-Bill or T-Bond yields. Examining the monthly market returns over the longer estimation horizons (five-year, eight-year and the entire sixteen year period) contained in column 5 of Table 5-14 (on the following page), it is clear that, except for the period 1998 to 2002 (which includes a significant market downturn), the remainder indicate substantial positive returns earned by holding the market portfolio that are higher than those associated with either the T-Bill or T-Bond (as shown in column 6 by the computed market risk premia). Over the entire period for example, the average monthly return was 2.8595 percent[46], compared to the T-Bill and T-Bond returns of only 0.9301 and 0.9616 percent.

Figure 5-2: Average Monthly Returns for the ALSI, T-Bills and T-Bonds 1993-2008



The values for the risk-premia estimated using the Fama and MacBeth (1973) equation, as specified in equation 3.44, where γ_1 represents the market risk premium, are also displayed in Table 5-14 (columns 1-4), with the t-test statistics of their significance shown below.

---

[45] The T-Bill and T-Bond returns were computed as specified in Section 4.5 and were the same values used in the hypothesis tests and forecasting comparisons.

[46] Although this value appears high, the ALSI grew from 3268.58 points (31 December 1992) to 21587.89 points (31 December 2008), which equates to a simple monthly return of 2.8595 percent. As mentioned in Section 4.2.3, simple returns were computed throughout this study because of the dividend data constraints.

Table 5-14: Estimates of the Market Risk Premium[47]

Period	(1) Sample 1 Beta-Sorted Ordinary Shares	(2) Sample 2: Industry-Sorted Ordinary Shares	(3) Sample 3: Beta-Sorted All Asset Classes	(4) Sample 4: Industry-Sorted All Asset Classes	(5) Market Return	(6) Market Risk Premium (T-Bill and T-Bond)
1993	0.5787 1.6559*	1.4266 1.6274*	3.3177 2.2876**	1.5095 1.9125*	4.1667	3.2231 3.0524
1994	-1.2565 -1.6693*	-1.1409 -1.6726*	0.6504 1.7495*	0.9861 -1.1829	1.6567	0.7465 0.4941
1995	-4.0909 -3.3188***	-2.2044 -0.8658	-3.16 -2.8644**	-1.8936 -1.6452*	0.5146	-0.6137 -0.7430
1996	-0.8876 -0.4799	-0.3423 -0.3088	-0.7114 -2.2462**	-0.429 -0.496	0.5752	-0.6818 -0.6415
1997	-1.0001 -0.8275	-0.718 -0.5231	-1.902 -2.4864**	-1.3352 -1.7707*	-0.5681	-1.8340 -1.7215
1998	-3.2854 -2.0208**	-0.901 -2.3623**	-2.449 -2.4234**	0.1797 2.0919**	-1.0336	-2.4180 -2.2164
1999	-1.9435 -2.6882***	2.6338 1.9240*	-0.2073 -2.2045*	0.1056 2.0886**	4.7621	3.7077 3.6002
2000	-3.0185 -2.9489***	-0.7384 -1.6768*	-2.9862 -4.0405***	0.0373 2.0273**	-0.2073	-1.0507 -1.2860
2001	-4.4729 -4.2667***	-2.9401 -2.0994**	-1.6463 -1.6859*	-1.5467 -1.9259*	2.0388	1.2340 1.1337
2002	-2.7816 -1.7908*	2.2017 2.8096*	-1.9492 -1.2765	2.3455 1.9260*	-0.8681	-1.8119 -1.7796
2003	2.5022 2.5570**	0.9081 1.7376*	1.8135 3.1207***	1.2397 1.7003*	0.9947	0.1244 0.2266
2004	0.684 1.6680*	2.0921 1.0671	3.1035 7.8062***	1.0168 2.0732**	1.8166	1.1905 1.0570
2005	4.4818 5.0021***	0.2086 2.1678**	4.2441 5.6879***	2.1102 6.6061***	3.5793	3.0057 2.9309
2006	3.4953 2.8300***	0.25 2.3492**	3.7607 6.8492***	1.5298 2.5283**	3.1382	2.5202 2.4963
2007	0.4254 1.8851*	1.3254 1.8691*	2.367 3.6715***	1.5006 2.2194**	1.3548	0.5963 0.7037
2008	-0.8277 -0.5239	-1.7946 -2.0088**	-5.6021 -7.2465***	-3.2039 -3.7891***	-2.1341	-3.0347 -2.8644
1993/ 1994	0.2031 2.1913**	2.9746 3.0615***	2.9302 2.4314***	3.2131 3.5365***	3.3258	2.3989 2.1874
1995/ 1996	-2.0513 -3.0024***	1.9984 1.9842**	-1.2411 -4.1538***	0.55 2.5767**	0.5626	-0.6300 -0.6745
1997/ 1998	-2.941 -2.5759***	1.6005 1.7377*	-2.7982 -4.0346***	0.1479 2.1301**	-0.7656	-2.0907 -1.9337
1999/ 2000	-1.3654 -1.7667*	1.3352 1.7832*	-0.3912 -0.5363	1.1992 1.7162*	2.2181	1.2692 1.0978
2001/ 2002	-6.3968 -5.2417***	-1.0919 -0.5607	-1.8757 -1.2133	1.1477 2.1195**	0.4791	-0.3951 -0.4291
2003/ 2004	-1.1954 -1.7027*	1.2988 2.7049***	2.4348 3.7784***	2.3673 1.9511*	1.5141	0.7658 0.7503
2005/ 2006	3.7386 3.4224***	2.3888 3.5420***	3.886 5.8993***	2.9276 6.2375***	4.0327	3.4369 3.3876
2007/ 2008	-0.1329 -0.0736	-0.1982 -0.2887	-2.3425 -8.5581***	-0.7633 -1.616	-0.5631	-1.3927 -1.2539
1993/ 1997	1.2811 4.5894***	1.1619 1.9145*	0.6862 2.5468**	0.9943 1.9010*	1.5031	0.4021 0.3221

[47] T-statistics are shown below except for column 7, where the risk premia with the T-Bill is at the top and the T-Bond below. Degrees of freedom were based on the number of portfolios comprising each sample less one.

Table 5-14 Continued

Period	(1) **Sample 1 Beta-Sorted Ordinary Shares**	(2) **Sample 2: Industry-Sorted Ordinary Shares**	(3) **Sample 3: Beta-Sorted All Asset Classes**	(4) **Sample 4: Industry-Sorted All Asset Classes**	(5) **Market Return**	(6) **Market Risk Premium (T-Bill and T-Bond)**
1998/ 2002	-3.5621 (-5.1528)***	-3.8047 (-1.9645)*	-1.6344 (-1.9315)*	-1.598 (-1.0628)	0.8277	-0.1784 -0.2210
2003/ 2007	1.1154 (1.6443)*	0.3673 (2.4106)**	3.2655 (5.6181)***	2.087 (3.1801)***	3.5319	2.8425 2.8379
1993/ 2000	1.414 (2.8962)***	1.3391 (1.9928)**	1.414 (1.8792)*	0.5107 (1.6704)*	2.3515	1.2535 1.4514
2001/ 2008	-3.5673 (-3.4198)***	0.217 (1.8726)*	-1.3614 (-1.7984)*	1.3105 (3.4585)***	3.5858	0.8913 0.9007
1993/ 2008	-3.13 (-4.4644)***	0.825 (1.9076)*	NA	NA	2.8595	1.9294 1.8979

* Statistically significant at the 10% level
** Statistically significant at the 5% level
*** Statistically significant at the 1% level

The yellow highlighted values in Table 5-14 reflect those regressions where the estimated risk premia (depicted in columns 1-4) and the computed market risk premia (column 6) were both negative, whereas the blue highlighted values indicate where the risk premia estimated were negative but the computed risk premia was positive. In light of the discussions in Section 4.3.7, the yellow highlighted values were not considered to be in complete contrast with the theory as over short periods in time it is conceivable that the market portfolio returns may be negative and hence the SML downward sloping (Pettengill *et al*, 1995: 105.). Of greater concern however, were the blue highlighted values which show those periods, where based on the different samples, the slope of the SML was estimated to be negative (columns 1-4), when it should have been positive (as per column 6). This is of particular concern over the longer estimation horizons, such as the five-year, eight-year and the entire sixteen-year period, where the market portfolio returns were large and positive and the T-Bill and T-Bond returns considerably smaller. Therefore the estimation of a significant negative risk premium infers that the relationship between risk and return, predicted by the CAPM did not hold over the period. Only one of the estimated risk premia highlighted in blue was statistically insignificant (Sample 3, 1999/2000) confirming that there were periods where the CAPM relationship was violated.

There were also four periods (based on the one-year or two-year estimates) where the market returns were negative (three cases) or smaller than the T-Bill or T-Bond returns (one case) but the estimated risk premia were positive. However, all of these observations (as denoted by the green highlight), except for one, were from sample 4. This sample comprises preference shares and bonds, which are not included in the market portfolio, and therefore may have contributed to the estimated positive risk premia. The relationship between the occurrences of the violations and the nature of the samples employed to estimate the various regressions and the time periods over which they are estimated are discussed within the analysis of each of the three sub-objectives in the following sections.

As mentioned, the statistical significance of the risk premia estimates are also indicated in Table 5-14 and the results suggest that on average, the risk premia were statistically significantly different from zero; confirming that beta is an important determinant of returns. In fact, the estimates of the risk premia were only statistically insignificant in 13.56 percent (sixteen of 118) of the regressions, with all of these except for one occurring over the shorter estimation horizons. The divide between the four samples in terms of the insignificance of the beta estimates is four, six, two and four for samples 1 to 4 respectively. This appears to indicate that the absence of a statistically significant relationship between beta and returns is more prevalent in the industry-sorted portfolios (samples 2 and 4) than in the beta-sorted portfolios (samples 1 and 3), but this difference was not considered sufficient to warrant further investigation. Moreover, the inclusion of the additional asset classes had a positive impact on the importance of beta as the number of regressions where the relationship was not significant was reduced in samples 3 and 4 compared to samples 1 and 2 respectively. The conclusion that is therefore evident is that the composition of the samples does not have a notable effect on the importance of beta in determining returns and is thus not considered in the following sections.

5.5.1 Asset Allocation

In order to assess the optimal asset allocation procedure samples 1 (the beta-sorted ordinary share portfolios) and 2 (the industry-sorted ordinary share portfolios) were compared, as the only difference between these samples and samples 3 (beta-sorted with all asset classes) and 4 (industry-sorted with all asset classes) respectively was the inclusion of the preference shares and bonds. Thus employing the latter two samples to determine the optimal method may reflect the inclusion of the preference shares and bonds rather than the asset allocation technique only.

23.33 percent (seven of 30) of the estimates of the market risk-premium based on sample 1 were negative when the difference between the market portfolio returns and the risk-free rate proxy was positive with all indicating statistically significant negative market risk premium estimates. Moreover, one of these disparities was observed during the period 2001-2008 where the average monthly return was 1.6533 percent and the other over the entire period 1993 to 2008 in which it was shown that the average monthly return was 2.8595 percent. These results certainly do not appear to correspond with the theory underlying the CAPM and point towards the inadequacy of the model and/or the beta-sorting procedure in the South African market.

In examining the risk premia associated with the tests of the industry based portfolios, however, the results conformed to a large extent to the findings of Bradfield *et al* (1988), Bradfield and Barr (1989), Van Rhijn (1994), and Ward (1994) that the CAPM is a reasonable model to employ in South Africa as 93.33 percent of the risk premia were statistically significant and corresponded closely to the predicted signs of the market risk premia. Moreover, those that failed to satisfy these criteria, as can be seen in Table 5-14 (page 160-161), occurred when the estimates were based on only one year's worth of observations. This is not unexpected as the returns earned on a monthly basis in a year are likely to have a more substantial impact on both returns and the beta values estimated than when a longer period of time is considered as dramatic fluctuations are averaged across the longer time horizon. This will be discussed further in Section 5.5.3.

It is surprising however that the industry-sorted portfolios appear to provide more reliable estimates of the CAPM parameters given that the allocation of shares to these portfolios was not based on any statistical measurements, but on a more subjective approach. The fact that the beta-sorting procedure in this study reflects a negative relationship between risk and return in periods where the market portfolio returns were large and positive in almost a quarter of the regressions, may reflect that the beta-sorting procedure is not adequately distributing the shares such that when they are combined the average movements of the portfolio do not adhere to the principle that the greater the risk, the greater the return.

As clearly specified in Section 4.3.1, the purpose of forming portfolios based on historical estimates of beta is to obtain a wide dispersion of beta values which are frequently lost when portfolios are formed rather than using the individual assets. The problem that is evident in this study with the beta-sorted portfolios is that to a large extent, the dispersion in beta values is not achieved; however, with the industry-sorted portfolios reasonable dispersion in betas values is attained (as noted in Section 5.2.2). This conclusion is apparent by examining graphs of the data points over the longer estimation horizons, comparing the distribution across samples 1 and 2. In Figures 5-3, 5-4 a and b (page 164), and 5-5 a, b and c (page 165), the beta-sorted and industry-sorted data points are plotted for the entire period, the two eight-year periods and the three five-year periods respectively. The blue points represent the ten beta-sorted portfolios with the twenty maroon points depicting the industry-formed portfolios.

Figure 5-3: Comparison of Beta- and Industry-Sorted Share Portfolios 1993-2008

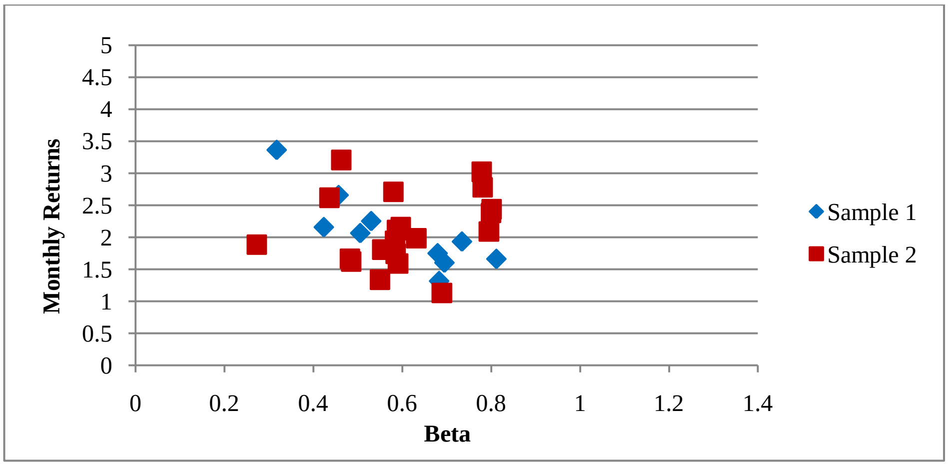
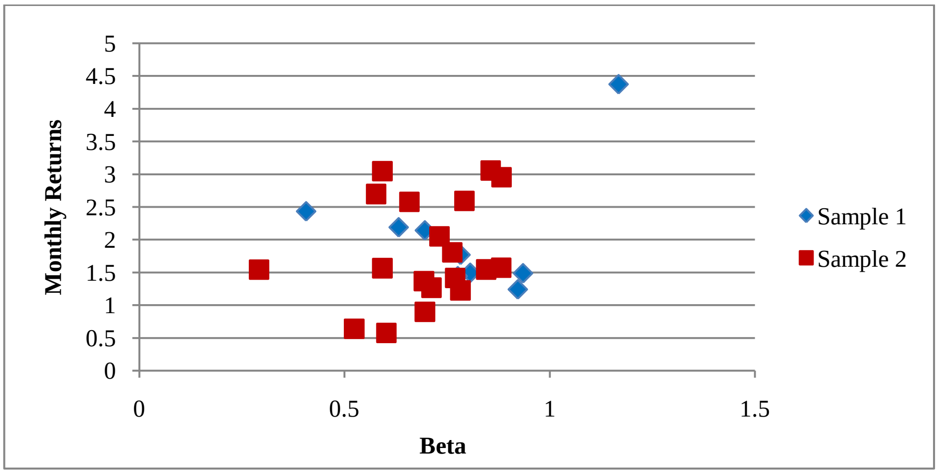


Figure 5-4: Comparison of Beta- and Industry-Sorted Ordinary Share Portfolios

Panel a: 1993-2000



Panel b: 2001-2008

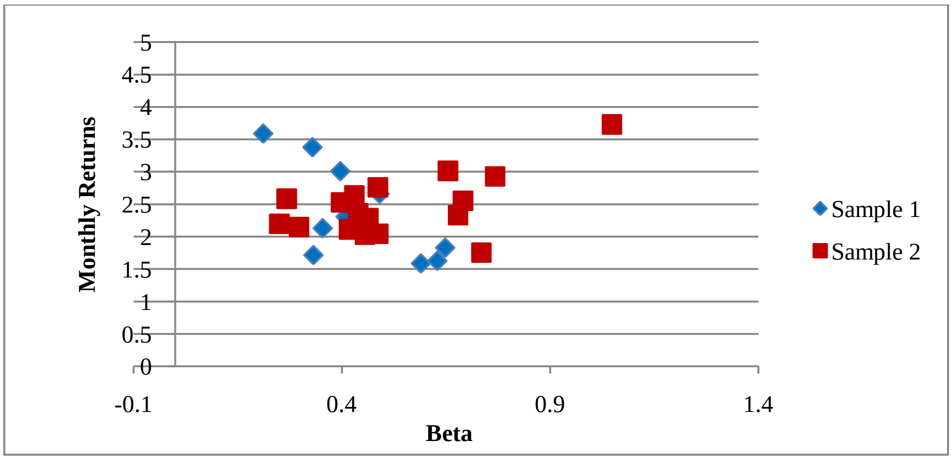
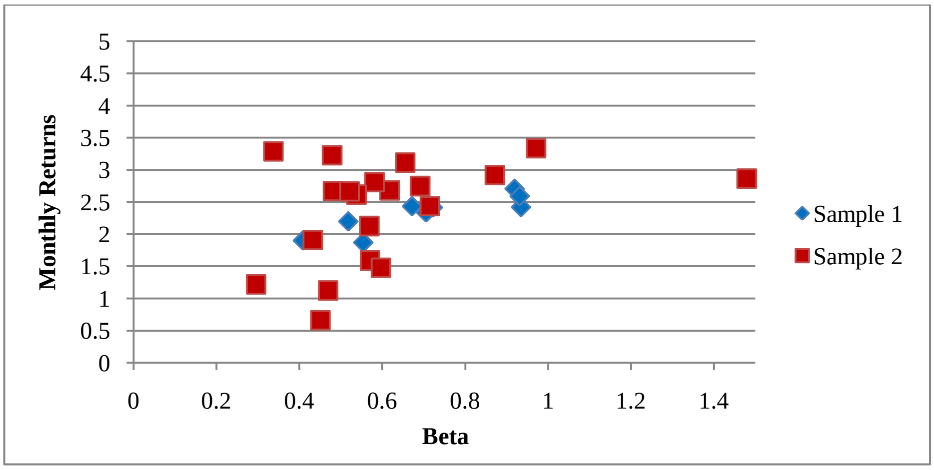
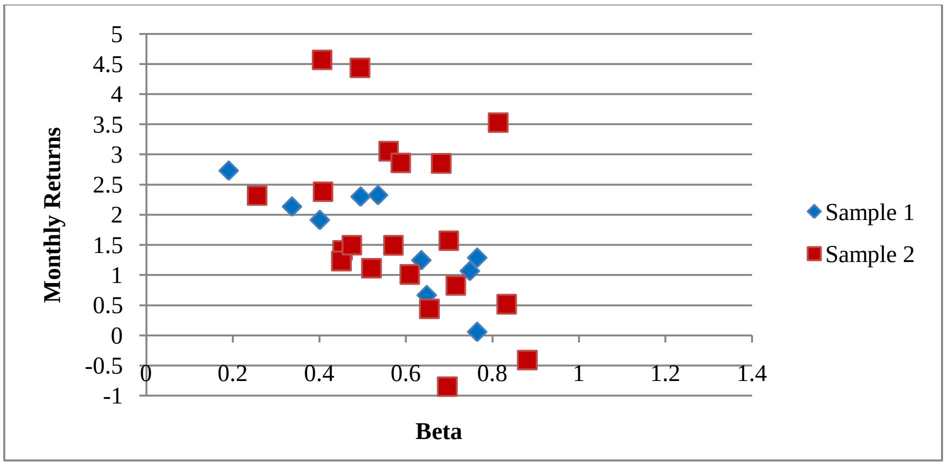


Figure 5-5: Comparison of Beta- and Industry-Sorted Ordinary Share Portfolios

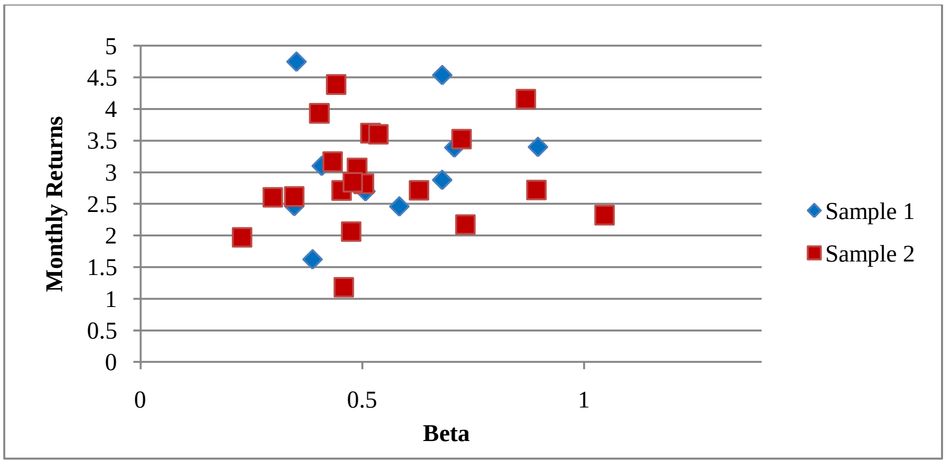
Panel a: 1993-1997



Panel b: 1998-2002



Panel c: 2003-2007



Across all the graphs, it is clear that the beta values do not vary across a large range, but generally the range is larger for the industry-based rather than beta-sorted portfolios. This is particularly apparent in panel b of Figure 5-4 (page 164) where the betas range from 0.25 to 1.04 for the industry-sorted portfolios and only from 0.2 to 0.65 for the beta-sorted portfolios. A possible reason for the lack of dispersion of the beta-sorted portfolios is that the historical estimates of beta do not necessarily provide a good indication of the following period betas. Hence the betas of the shares in a particular portfolio do not exhibit substantial similarity in the period used for estimating the regression, such that in averaging the returns across all shares unique risk-return relationships are clouded. Consequently, because the beta-sorted portfolios plot closely together with regards to the measurements of both risk and return, in some periods the relationship that is estimated is negative and is thus opposite to that predicted by the CAPM when the market portfolios returns are positive and exceed the risk-free proxy yields.

In Figure 3-4 (page 81), the data points from the analysis conducted by Fama and French (2004: 38) were shown. It is clear from this graphical representation that the beta values were quite widely dispersed, ranging from approximately 0.65 to 1.75. Although this range is larger than that achieved in this study using only ordinary shares (the range, as will be shown in Section 5.5.2, increases when preference shares and bonds are included), Fama and French (2004: 38) included ordinary shares listed on NASDAQ and AMEX in addition to those on the NYSE. Shares on these two junior exchanges are likely to exhibit greater risk (as measured by a higher beta) because they tend to contain newer firms in sometimes riskier, high-growth industries. Consequently, it might be expected that if Alt-X shares were included in this analysis a greater distribution may be achieved. A further reason for the lower beta estimates compared to the U.S. study and in particular, that the lowest beta values from the portfolios estimated in South Africa are generally smaller than with Fama and French (2004), may be a consequence of the effects of thin-trading, which "…results in the smaller and, most likely, less traded securities having downwardly biased-beta estimates" (as quoted in Van Rensburg and Robertson, 2003: 9, but in conjunction with the conclusions drawn by Scholes and Williams, 1977; Dimson, 1979; Bradfield, 1990 and Bradfield, 2002). The flipside of this however, is that the intercept estimates are likely to be more robust because, as discussed in Section 3.3.4.1, the values lie closer to the vertical axis.

As highlighted in Section 4.3.1, Van Rensburg and Robertson (2003: 8) implemented a similar beta-sorting procedure to allocate shares to portfolios as was adopted in this study; the only differences being the number of portfolios formed, the frequency of reforming the portfolios and the length of the estimation periods. However, the differences in the implementation of this approach do not appear to have had any impact on the results obtained, as Van Rensburg and Robertson (2003: 11) also found that the CAPM relationships did not hold over the ten-year period of their study (July 1990 to June 2000). They concluded, "This study finds, if anything, an *inverse* relationship between risk and returns!" (Van Rensburg and Robertson, 2003: 11).

These authors use this result in conjunction with their tests of the extent to which price-earnings ratios and size influence return to conclude that beta is not an appropriate determinant of returns in South Africa. However, from this study, where industry-based portfolios are used as the sample from which the CAPM coefficients are estimated, beta was, on average, an important determinant of returns and as beta increased, returns increased when the market portfolio returns were positive. In light of the results obtained in this study, in conjunction with the findings of Van Rensburg and Robertson (2003), it appears as though employing the beta-sorting procedure to allocate shares to portfolios in South Africa is flawed. This suggests that perhaps unsystematic risk continues to play an important role in the returns earned on securities in South Africa, such that securities with similar risk exposures in the estimation period may react differently to movements in the market in the period following the estimation of the security's beta.

Despite these results which favour the use of the estimates of the intercept based on the industry-sorted portfolios for the remainder of the statistical analyses in this study, it was considered of value to also examine the differences in efficiency across the two approaches where both samples estimated intercepts of the same sign. Essentially, the efficiency of the parameter estimates (as measured by the standard error) in regressions (where the method applied is identical) is a function of this vertical dispersion as the closer the points lie to the predicted line, the lower the standard error (Brooks, 2006: 62). As can be seen from the graphs on pages 164 and 165, the dispersion of the data points vertically were not equal across the two approaches. Panel a of Figure 5-4 (page 164) shows greater vertical dispersion of the data points for the beta-sorted portfolios; in contrast, Panels a and b of Figure 5-5 (page 165) reflect greater vertical dispersion for the industry-sorted portfolios. In Panel b of Figure 5-4 (page 164) and panel c of Figure 5-5 (page 165), there is no noticeable difference in terms of the vertical dispersion of the data points from the two samples. Therefore the results of the graphical analysis suggest that whilst differences in efficiency may occur across the two samples in different periods, on average neither is consistently more efficient.

This conclusion is confirmed in the results of the statistical tests of the equality of the standard errors of the intercepts of those regressions where both risk premia estimated were of the same sign as the calculated risk premium. Of the 23 estimates in Table 5-15 (on the following page), sixteen of the F-statistics were statistically significant; with ten indicating that the beta-sorted portfolios were more efficient and six that the industry-sorted portfolios were more efficient.

Table 5-15: Asset Allocation Mean and Variance Tests (Sample 1 versus Sample 2)[48]

	Mean Test[49]	Variance Test[50]
1993	5.0182***	4.2856***
1994	NA	NA
1995	1.1945	0.1167 ^{xxx}
1996	0.544	3.4400***
1997	0.1894	0.7672
1998	4.8310***	0.3591 ^{xx}
1999	NA	NA
2000	6.8763***	0.3547 ^{xx}
2001	NA	NA
2002	5.8889***	1.806
2003	-4.2484***	0.4147 ^{xx}
2004	-0.8767	1.0009
2005	-8.6823***	0.4192 ^{xx}
2006	-5.8631***	3.2639**
2007	1.9492	0.2034 ^{xxx}
2008	-0.7774	1.9234
1993/1994	6.5192***	1.7463
1995/1996	11.8295***	0.4874 ^{x}
1997/1998	8.5787***	0.5701
1999/2000	NA	NA
2001/2002	4.6704***	0.2547 ^{xxx}
2003/2004	NA	NA
2005/2006	-1.8734	2.5241**
2007/2008	-0.0153	3.1446**
1993/1997	-1.9826	0.1852 ^{xxx}
1998/2002	-1.7363	0.0765 ^{xxx}
2003/2007	-0.5077	5.0360***
1993/2000	1.2135	1.6806
2001/2008	NA	NA
1993/2008	NA	NA

*(^{x}) Statistically significant at the 10% level
**(^{xx}) Statistically significant at the 5% level
***(^{xxx}) Statistically significant at the 1% level
The ^{x} denote where the inverse of the F statistics are statistically significant

As explained in Section 4.3.7, tests of the equality of the mean values were performed and in this respect the results were also not conclusive, as shown in Table 5-15. Eleven of the test statistics were statistically significant with eight positive (beta-sorted portfolio intercept was larger) and three negative (industry-sorted portfolio intercept was larger). However, 52 percent of the test statistics were insignificant, especially over the longer investment horizons (five-year and eight-year periods). The results therefore imply that over shorter periods it is likely that the values of the intercept may differ across the two procedures with the estimate from the beta-sorted portfolios being smaller, but over longer periods the difference is likely to be statistically insignificant.

[48] The estimates from Sample 1 were used as the first value and the numerator in the mean and variance tests respectively.
[49] The t-critical values were used with 14 degrees of freedom (average of the two samples).
[50] The F-critical values were used with 10 and 19 numerator and denominator degrees of freedom respectively.

The expectation that the beta-sorted and industry-sorted procedures are close substitutes was thus not found to be valid is this study, as the beta-sorted portfolios did not adhere to the CAPM relationships as consistently as the industry-sorted portfolios; suggesting that this portfolio formation technique does not necessarily provide the best results in South Africa. This finding confirms the results of Van Rensburg and Robertson (2003). Furthermore, in those periods where the CAPM estimated based on both samples satisfied the underlying theory of the model, the results of the statistical tests provided further support that the two approaches were not identical both in terms of the value of the estimates and the standard errors. However, on average, the differences in value were not statistically significant and neither approach consistently provided more efficient estimates than the other.

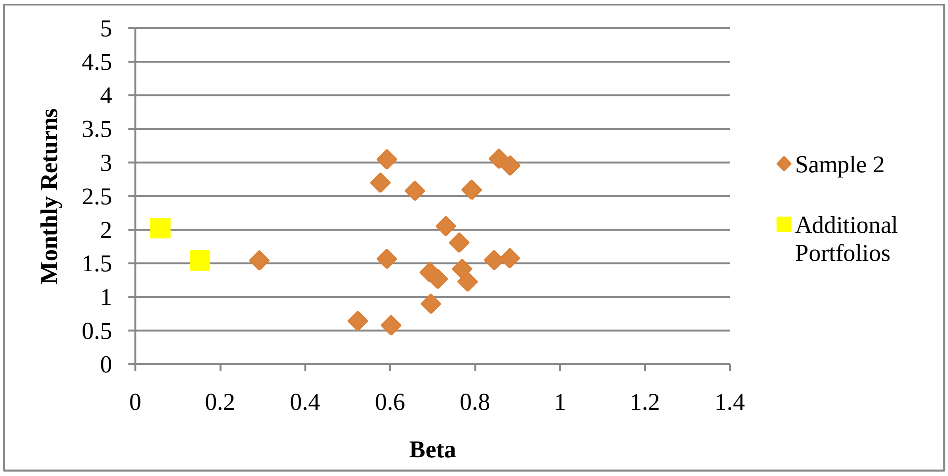
In light of these results, in the remaining analyses conducted in this study, the industry-sorted sample was used. The next question therefore is whether or not including preference shares and bond portfolios improves both the extent to which the CAPM relationships hold in practice and the efficiency of the intercept estimates.

5.5.2 Asset Inclusion

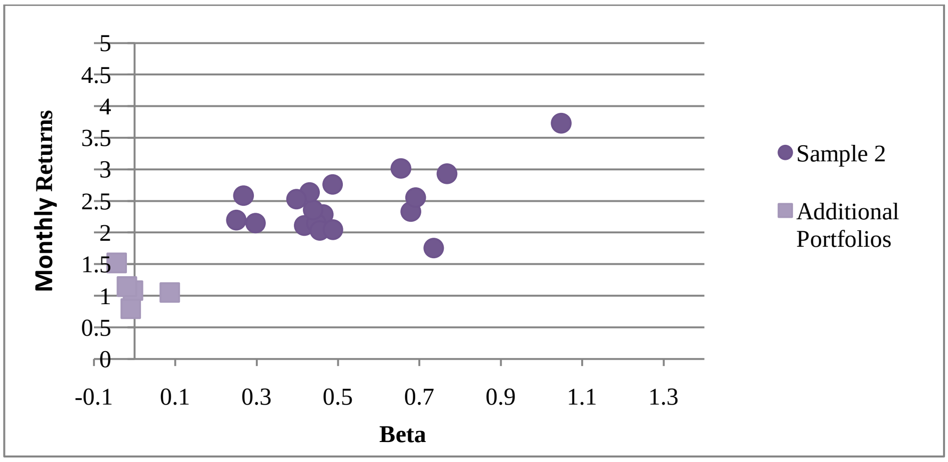
In Table 5-14 (page 160-161), the risk premia and test statistics of the significance of these values for the expanded asset samples (with preference shares and bonds) are also shown as reflected by samples 3 and 4. Specifically analysing the difference in the industry-sorted portfolios (samples 2 and 4), the addition of preference shares and bonds did not appear to have a notable impact as the number of regressions that did not satisfy the condition that the estimated risk premium must have the same sign as the computed risk premium decreased from two to one (as indicated by the blue highlighted values). Figure 5-6 (on the following page) shows the industry-sorted portfolios with only ordinary shares and the additional preference share and bond portfolios for the two sub-periods, 1993 to 2000 and 2001 to 2008 (it was not possible to estimate the intercept for the whole period using the preference share and bond portfolios because bonds were only introduced into the analysis from 2001 onwards). As can be seen in panel a of Figure 5-6, there are only two additional portfolios which represent the two preference share portfolios, whilst in panel b there are five additional portfolios – two preference share portfolios and three bond portfolios.

Figure 5-6: Comparison of Industry-Sorted Portfolios with Additional Asset Classes

Panel a: 1993-2000



Panel b: 2001-2008



From both graphs it is clear that the preference share and bond portfolios lie quite far from the ordinary share portfolios with the beta values close to zero and the returns also relatively low. Thus, the horizontal dispersion desired from forming portfolios according to either industry classifications or historical beta-estimates is further enhanced with the inclusion of these two asset classes. Furthermore, the returns on both of these asset classes also appear, on average, to satisfy the CAPM theory as they only provide low returns to compensate investors for the low risk, and this is particularly clear in panel b of Figure 5-6. In light of these graphical representations, it is not surprising that the results of the statistical tests of the risk premia estimated across the different samples reflected greater adherence to the CAPM relationships for those estimated based on the samples including preference shares and bonds, and the difference was more noticeable where both asset classes were employed (post 2000). As discussed in the preceding section, according to Van Rensburg and Robertson (2003: 8), the low beta values for the ordinary share portfolios are possibly attributable to the effects of thin trading on the JSE. Examining the daily trading volumes of shares on the JSE, it is clear that preference shares are not as frequently traded as ordinary shares, hence reducing their betas.

The other advantage of the fact that these additional asset classes plot close to the vertical axis is that the intercept value estimated is likely to be significantly less sensitive to the choice of sample than with the portfolios comprising only ordinary shares, which plot further away from the vertical axis reflecting their greater risk. In fact, the bond portfolio betas are very close to zero over the period 2001 to 2008 (especially for the medium-term, 7-12 year bonds) and thus, in effect, provide an estimate of the returns on a zero-beta portfolio, although this is not guaranteed to be the minimum-variance zero-beta portfolio returns.

From the two graphs presented in Figure 5-6, especially for the period 2001 to 2008, it is evident that the preference share and bond portfolios were clustered quite close together vertically and thus this should result in lower standard errors for the intercept estimates as there is greater certainty as to the value of this parameter. The results of the variance tests, shown in Table 5-16 (on the following page), confirm this conclusion, as the standard errors of the estimates of the intercept based only on the portfolios of ordinary shares were statistically significantly larger than those where the preference shares and bonds were included. Of the 26 estimates which satisfied the condition that both risk-premia were the same sign as the computed risk premium, thirteen of the statistics were significant at the ten percent level or lower indicating the smaller standard errors associated with the expanded asset sample. In addition to this, only two of the estimates in total showed that the standard error was larger for sample 4 and neither were statistically significant at the conventional significance levels. It therefore appears clear that the sample including ordinary shares, preference shares and bonds provides the most consistently reliable estimates of the CAPM relationship and the most efficient estimates of the intercept. This finding is consistent with the conclusion Stambaugh (1982: 263) reached regarding the inclusion of these assets.

With respect to the equality of the estimates of the intercept across the two samples, the evidence is mixed. It was expected, in light of the graphical representations shown above, that the inclusion of the additional asset classes would most likely reduce the intercept value because these portfolios generally offer lower returns, but the extent of the influence of these on the intercept depends on whether the returns were substantially lower given the lower level of risk. The results of the tests were inconclusive in this regard; fifteen of the 26 estimates were statistically significant, with five showing a negative sign (ordinary share portfolio estimates smaller) and ten a positive sign (ordinary share portfolio estimates larger). Therefore although the preference share and bond portfolios generally offer lower returns due to the lower risk associated with these securities, this risk-return relationship does not appear to be sufficiently smaller in all cases than the average risk-return relationship for shares; thus the effect on the intercept estimated is not consistently negative.

Table 5-16: Asset Inclusion Mean and Variance Tests (Sample 2 versus Sample 4)[51]

	Mean Test[52]	Variance Test[53]
1993	0.4511	1.3506
1994	NA	NA
1995	0.5875	2.6652**
1996	-0.309	1.8055
1997	-1.7669	1.8717
1998	-1.1213	1.7706
1999	NA	NA
2000	-1.973	2.3082*
2001	NA	NA
2002	2.1859*	1.7902
2003	1.1065	3.9058***
2004	3.6897**	1.2177
2005	7.3326****	8.1897***
2006	8.7625***	2.4893**
2007	0.6314	6.2840***
2008	-18.5173***	0.7123
1993/1994	1.211	1.2517
1995/1996	-6.5687***	0.8641
1997/1998	-3.9567**	1.6786
1999/2000	-0.9829	1.2664
2001/2002	6.3602***	4.3701***
2003/2004	3.5893**	3.7277***
2005/2006	3.5925**	2.5899**
2007/2008	-3.6690**	2.3442*
1993/1997	-0.9559	2.0322
1998/2002	4.4030***	1.7739
2003/2007	8.0804***	2.3131*
1993/2000	-2.5005*	2.7905**
2001/2008	4.7304***	3.3117***

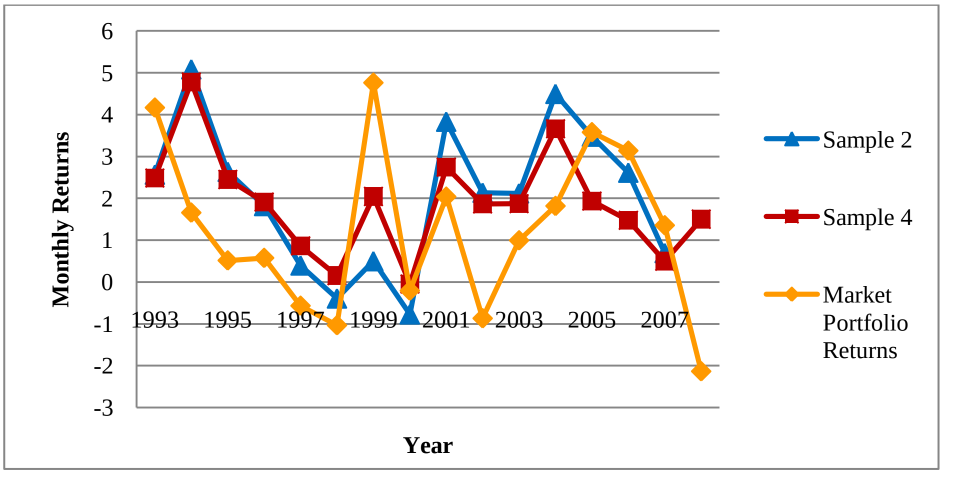
*(^{x}) Statistically significant at the 10% level
**(^{xx}) Statistically significant at the 5% level
***(^{xxx}) Statistically significant at the 1% level
The ^{x} denote where the inverse of the F statistics are statistically significant

This uncertainty regarding the relationship between the estimates of the intercept consisting of only ordinary shares and ordinary shares plus preference shares and bonds is confirmed in Figure 5-7 (on the following page) which shows the one-year estimates of the intercept based on sample 2 (the blue line) and sample 4 (the maroon line)[54]. To a large extent Figure 5-7

[51] The estimates from Sample 2 were used as the first value and the numerator in the mean and variance tests respectively.
[52] The t-critical values were used with 21 degrees of freedom pre 2001 and 22 post 2001.
[53] 20 (20) and 24 (21) numerator and denominator degrees of freedom were used for the period pre (post) 2001 from the F-distribution.
[54] The values for 1994, 1999 and 2001 are not considered in examining the patterns identified in this graph as between the two sets of estimates these were the years in which the computed market risk premia did not match the sign of the estimated values.

illustrates that the changes in the values of the intercept over time track the changes in the market portfolio, represented by the orange line. In general, the minimum required return falls as the market portfolio returns decrease and increases as the market portfolio returns rise, but not as substantially. The differences in the estimates of the intercept based on the two samples can thus be seen to reflect the differing behaviour of preference shares and bonds in response to changes in the market.

Figure 5-7: The Intercept and the Market Portfolio Returns 1993-2008



For example, in 2008 when the market fell markedly as a consequence of the global financial crisis, the intercept value based only on ordinary shares was negative. However, with the inclusion of preference shares and bonds the intercept value remained high and actually increased from the 2007 value, reflecting the good performance of these asset classes, in particular bonds, over this period. In 2004, when the market was performing better, the estimates of the intercept based only on ordinary shares exceeded those based on the expanded dataset, showing that preference shares and bonds did not perform as well in this year as ordinary shares; thus resulting in the discrepancies observed. As is clear from both Figure 5-7 and Table 5-16, over the period 1993 to 1997 the difference in the intercept values across the two samples was statistically insignificant, with the difference only becoming more prominent post 2001. This indicates that the inclusion of preference shares in the sample did not necessarily have a notable impact, whereas the inclusion of bonds from 2001 did appear to have a more substantial effect. Whether this is a consequence of the behaviour of bonds only, or the combined effect of the larger number of portfolios including these different asset classes is however, not clear.

In light of this analysis, it is clear that the inclusion of preference shares and bonds into the sample increases the extent to which the CAPM relationships hold and more significantly, increases the efficiency of the intercept values computed. Consequently, for further analyses using the minimum-required return estimate, the industry-sorted portfolios including preference shares and bonds (as denoted by sample 4) were employed.

5.5.3 Estimation Horizon

In order to assess the impact of the estimation horizon on the extent to which the CAPM holds and the efficiency of the intercept estimates, both the risk premia across the various periods were examined and the standard errors of the intercept values. As already mentioned in Section 5.5.1, and as is clearly evident in Table 5-14 (page 160-161), the number of regressions which did not satisfy the condition of matching the computed market risk premium decreased as the estimation horizon increased. In particular, with respect to the most efficient sample, the only observation (2001) which did not fulfil this condition was based on the one-year time horizon. This is not surprising given that when average portfolio returns (and hence betas) are computed over a longer period of time, the influence of an outlier value is considerably smaller than when this value is computed over a much shorter period. An analysis of research conducted on the CAPM in both the U.S. and South Africa discussed in Chapter 3 did not reveal any studies where the model parameters were estimated over such short periods in time – the closest being Black *et al* (1972) with two-year estimates. The justification for using such short time periods in this study was to obtain more regular estimates of the intercept to compare to the annual average risk-free rate proxies. The results indicate that whilst over the shorter periods the intercept estimates were more prone to deviations from the CAPM relationships, the frequency was not sufficient so as to completely invalidate estimating the model over short horizons.

To examine the effect of the additional time periods on the efficiency of the estimates obtained, an analysis of the standard errors was conducted based on the one-year, two-year, five-year and eight-year estimates. To accurately assess these measures of efficiency, the average standard errors were estimated across the four time periods[55] and the statistical tests conducted, as established in Section 4.3.7, using these values. The results are shown in Table 5-17 (on the following page) and confirm the expectation that generally, the longer the time period over which the intercept was estimated, the lower the standard error (although the average is slightly higher over five-year periods compared to two-year periods). This relationship however, was only statistically significant when comparing the one-year, two-year and five-year period estimates to the average eight-year estimate, and not for the intermediate periods. Therefore the

[55] The average value for 2001 was excluded from the calculation of the one-year average standard error as this was the one regression which did not satisfy the imposed constraint from the industry-sorted portfolios with preference shares and bonds, as identified in Table 5-14 (page 160-161).

results from this study infer that when choosing between time periods employing an eight-year time horizon is likely to result in more efficient estimates than any shorter periods. Due to the differing number of portfolios from 2001 onwards in the analysis it was not possible to ascertain whether estimating the intercept value over any longer periods would have been more efficient and therefore it would certainly be of value in future studies to determine the time period from which the most efficient estimates can be obtained.

Table 5-17: Time Period Mean and Variance Tests[56]

Period	Average Intercept	Average Standard Error
One Year	1.8309	0.6463
Two Years	1.2746	0.4948
Five Years	1.9986	0.5203
Eight Years	1.4158	0.3527

	Mean Test[57]	Variance Test[58]
One Year versus Two Years	3.4234***	1.7057
One Year versus Five Years	-1.0124	1.5428
One Year versus Eight Years	2.8254***	3.3565***
Two Years versus Five Years	-5.0415***	0.9045
Two Years versus Eight Years	-0.9608	1.9679*
Five Years versus Eight Years	4.6359***	2.1756**

*(^{x}) Statistically significant at the 10% level
**(^{xx}) Statistically significant at the 5% level
***(^{xxx}) Statistically significant at the 1% level
The ^{x} denote where the inverse of the F statistics are statistically significant

These findings with respect to the optimum time period over which to estimate the minimum required return are similar to those of Black *et al* (1972: 41) in which their estimates of this parameter over the entire period (1931 to 1965) and four eight-year periods had lower standard errors than the bi-annual estimates. However, the standard errors associated with the intercept values based on Fama and MacBeth (1973: 622) did not differ across the varying time periods over which the parameter was estimated; the entire thirty-three years, three ten-year periods and six five-year periods. These results appear to suggest that there may be an optimal time period

[56] The estimates from the shorter-periods were used as the first value and the numerator in the mean and variance tests respectively.
[57] The t-critical values were selected based on 24 degrees of freedom (adjustments were not made for the smaller number of portfolios prior to 2001).
[58] F-critical values were based on 25 and 24 numerator and denominator degrees of freedom.

over which to estimate the CAPM and its parameters and beyond this time horizon any longer periods are not justified by a substantial increase in efficiency.

The mean intercept values, as the averages of the one-year, two-year, five-year and eight-year estimates were also tested, as in theory these averages should be similar. The results, shown in Table 5-17, dispute this as the mean values differed across the estimation periods, thereby inferring that the selection of the estimation period is important in determining the most efficient (and hence the most accurate) estimates. Given the volatility evinced in the one-year average monthly estimates of the intercept values shown in Figure 5-7 (page 173), it was also considered of value to examine how the minimum required return changed over the longer estimation periods given that these estimates are more efficient. For the purposes of this analysis only the five-year period and eight-year period estimates were considered. The test of equality of the eight year periods resulted in a t-statistic of 1.1373. This statistic is smaller than the critical value at the ten percent significance level (1.6449 for a two-tailed test) and thus it was concluded that the estimates of the intercept did not change over the eight-year horizons, despite the inclusion of bonds in the second sample which were not included in the first. Comparing the periods 1993-1997 and 2003-2007 the test statistic (1.5452) also pointed to the same conclusion that the values did not change significantly over time; however, when compared to the estimate of the minimum required return across the period 1998-2002 the test statistics (4.0816 and -4.8889 respectively) did indicate a statistically significant difference in the estimates of this value. It is therefore evident that, although changes in the value of the intercept appear to be driven by changes in the market portfolio returns over short periods in time (Figure 5-7), over longer, more efficient, estimation horizons, the estimates of the intercept are reasonably stable across periods.

In testing the hypotheses, the estimates of the intercept based on the industry-sorted portfolios including all asset classes were conducted over all periods so as to ascertain whether the use of shorter time periods compared to longer periods has any impact on the results obtained.

5.6 The Zero-Beta Portfolio Returns

As discussed in Section 4.4.2, the zero-beta portfolio returns were estimated using all four samples to avoid assuming that the conclusions drawn with respect to the estimation of the intercept were necessarily applicable with respect to the estimation of the minimum-variance zero-beta portfolio; the results of which are shown in Table 5-18 (on the following page). Prior to examining the results obtained, it was expected that similar findings to the intercept would be found in terms of there being little notable distinction between the efficiency of the beta-sorted portfolios compared to the industry-sorted portfolios and that a wider range of asset classes would be favoured to portfolios of ordinary shares only. However, the findings for the zero-beta portfolio returns with respect to efficiency were not entirely consistent with the previous results, but were reasonably similar for the differences in the mean returns across samples. The zero-beta portfolio returns are assumed to follow a t-distribution, which, according to Morgan (1975: 370) is a reasonable assumption because of the nature of the methodology employed, where the individual portfolio values are weighted to minimise the variance subject to the imposed constraints.

Table 5-18: Estimates of the Zero-Beta Portfolio Returns 1993-2008

	1: Beta-Sorted Ordinary Shares		2: Industry-Sorted Ordinary Shares		3: Beta-Sorted All Asset Classes		4: Industry-Sorted All Asset Classes	
	Zero-Beta Portfolio Returns	Standard Error	Zero-Beta Portfolio Returns	Standard Error	Zero-Beta Portfolio Returns	Standard Error	Zero-Beta Portfolio Returns	Standard Error
1993	4.7472	1.2623	2.5724	0.4235	2.7491	0.7279	2.4898	0.3796
1994	4.7317	0.4311	5.0791	0.2481	4.0671	0.3818	4.7726	0.2356
1995	2.9977	0.6433	2.6294	0.3820	2.4744	0.4844	2.4445	0.3389
1996	2.2029	1.5750	1.8207	0.5146	2.0699	0.5976	1.9153	0.4035
1997	0.4628	0.8561	0.3894	0.5287	1.1248	0.4976	0.8567	0.4005
1998	2.1161	1.1242	-0.3830	0.6805	1.5130	0.6920	0.1574	0.5330
1999	3.7413	0.8712	0.4378	0.6121	2.7056	0.5880	0.9595	0.4836
2000	2.0257	0.7998	-0.8093	0.5368	1.9948	0.5887	-0.0675	0.4531
2001	3.4849	0.5229	3.8378	0.3801	2.1790	0.3112	2.7496	0.2736
2002	3.4247	0.3480	2.1991	0.2628	2.5192	0.2736	1.9193	0.2261
2003	1.7470	0.6671	2.1150	0.4477	1.6563	0.3769	1.8698	0.3194
2004	4.2322	0.4165	4.4926	0.3480	2.9154	0.3268	3.6553	0.3070
2005	1.1165	0.5156	3.4770	0.3945	1.2159	0.3442	2.5676	0.3001
2006	0.7956	1.0718	2.6169	0.5120	0.5810	0.4510	1.5819	0.3563
2007	1.3675	0.5430	0.6960	0.4228	0.7973	0.3716	0.5451	0.3320
2008	-2.1597	0.7282	-1.9416	0.3805	-0.0116	0.3902	-0.8105	0.2931
1993/1994	4.7834	0.9872	2.9481	0.3693	3.0002	0.6660	2.7368	0.3393
1995/1996	2.6257	1.2537	0.8198	0.6089	2.0682	0.5987	1.3090	0.4540
1997/1998	1.7827	1.1346	-0.9934	0.7222	1.6729	0.6746	0.0930	0.5438
1999/2000	1.7176	0.8303	1.9185	0.3101	1.7469	0.5794	1.8869	0.2883
2001/2002	4.0737	0.5047	3.3237	0.4795	2.3683	0.3117	2.2439	0.3063
2003/2004	4.2079	0.7137	2.6131	0.5109	2.2451	0.4026	1.9429	0.3532
2005/2006	1.1582	0.7605	2.0354	0.6068	1.0014	0.4062	1.3761	0.3768
2007/2008	-0.2880	1.1006	-0.7576	0.4517	0.5214	0.4332	-0.0623	0.3256
1993/1997	1.7007	1.2685	1.6570	0.5900	1.9105	0.6467	1.7795	0.4644
1998/2002	3.5385	0.9866	5.3657	0.8687	2.3160	0.6516	3.3045	0.6117
2003/2007	2.5107	1.0483	2.7082	0.6432	1.2219	0.4497	1.6130	0.3928
1993/2000	1.4217	1.1329	1.6366	0.6594	2.6440	1.0563	2.1033	0.6514
2001/2008	3.9481	1.0535	1.5615	0.4116	1.5113	0.6590	1.2499	0.3704
1993/2008	3.9021	1.2630	1.6133	1.0054	NA	NA	NA	NA

With respect to the mean tests for comparing the asset allocation methods, the test statistics shown in Table 5-19 (page 179) indicate that, on average, the values obtained differed significantly across the two approaches (60 percent were statistically significant); however, neither of the sets of estimates were found to be significantly larger or smaller than the other. For the comparison of the impact of the inclusion of preference shares and bonds on the results, the test statistics were generally greater than the critical values at the ten percent significance level (55.17 percent for the beta-sorted portfolios and 72.4 percent for the industry-sorted portfolios). Although the majority of estimates were statistically significant, they were not all of the same sign; with the divide between positive (estimates based on ordinary shares were greater) and negative (estimates based on the expanded sample were greater) test statistics reasonably similar. There was thus insufficient evidence to reliably conclude that the portfolios including ordinary and preference shares and bonds consistently resulted in estimates of the minimum-variance zero-beta portfolio returns which were lower. In examining the mean statistics across both the asset allocation and asset inclusion tests, there were no notable differences in the trends identified when the parameter was estimated over longer periods rather than the shorter one-year or two-year periods.

For the tests of the comparison of the variance estimates of the zero-beta portfolio returns across samples 1 and 2 which consist only of ordinary shares, 70 percent of the test statistics (21 of 30) were significant at the ten percent significance level or lower, where the test statistic was computed with the numerator as the standard error from the beta-sorted portfolio and the denominator as the standard error from the industry-sorted portfolio. Therefore the test statistics indicate that the variance of the zero-beta portfolio estimates was statistically significantly larger for the beta-sorted portfolios than for the industry-sorted portfolios. In fact, all of the test statistics were greater than one confirming that across all the regressions the variance was smaller based on the industry-sorted portfolios and consequently, these values are likely to more closely resemble the minimum-variance zero-beta portfolio returns.

Consequently, to assess the impact of the inclusion of additional asset classes on the efficiency of the zero-beta portfolio returns, the industry-sorted portfolio statistics were specifically examined, although the tests across both samples 1 and 3, and 2 and 4 are shown in Table 5-19 (on the following page). For the variance test across samples 2 and 4 (the industry-sorted portfolios), 44.83 percent of the test statistics were significant at the ten percent level or lower which showed that the standard errors of the estimates including the preference share and bond portfolios were lower than those portfolios consisting only of ordinary shares. The remaining test statistics, whilst not significant, all pointed to the greater efficiency of the estimates including the additional asset classes. These conclusions regarding the efficiency of the estimates are consistent across both the shorter estimation horizons (one-year and two-years) and the longer time horizons (five-years and eight-years).

Table 5-19: Test Statistics for the Comparison of the Zero-Beta Portfolio Estimates Across the Four Samples[59]

	Test of Asset Allocation: Sample 1 versus Sample 2		Test of Asset Inclusion: Sample 1 versus Sample 3		Test of Asset Inclusion: Sample 2 versus Sample 4	
Period	**Mean Test** [60]	**Variance Test** [61]	**Mean Test**	**Variance Test**	**Mean Test**	**Variance Test**
1993	5.301***	8.8857***	4.4293***	3.0076**	0.6628	1.2448
1994	-2.3607*	3.0187**	3.7907***	1.2747	4.0952***	1.1087
1995	1.6696	2.8361**	2.1201	1.7638	1.6521	1.2703
1996	0.7477	9.3678***	0.2524	6.9463***	-0.6583	1.6265
1997	0.2485	2.6217**	-2.1601	2.9592**	-3.2042***	1.7428*
1998	6.4624***	2.7292**	1.4789	2.6391**	-2.8454**	1.6303
1999	10.7384***	2.0259*	3.2006**	2.1953*	-3.044**	1.6019
2000	10.1271***	2.2199*	0.1014	1.8457	-4.8142***	1.4032
2001	-1.8982	1.8923	7.1034***	2.8226**	10.7662***	1.9306*
2002	9.8252***	1.753	6.9253***	1.6179	3.7736***	1.3513
2003	-1.5761	2.2203*	0.3907	3.1331**	2.0655	1.9649*
2004	-1.7027	1.4321	8.4187***	1.6245	8.447***	1.2849
2005	-12.7337***	1.7082	-0.5349	2.2431*	8.5235***	1.7277*
2006	-5.0909***	4.3820***	0.5987	5.6490***	7.6752***	2.0655**
2007	3.4259***	1.6495	2.8989**	2.1355*	1.3067	1.6215
2008	-0.8884	3.6635***	-8.5461***	3.4835**	-10.9479***	1.685
1993/1994	5.6836***	7.1473***	4.8638***	2.1971*	1.9257	1.1845
1995/1996	4.3081***	4.2387***	1.2889	4.3849***	-2.9281**	1.7990*
1997/1998	7.0552***	2.4684**	0.2689	2.8287**	-5.4647***	1.7637*
1999/2000	-0.74	7.1712***	-0.0942	2.0533	0.3415	1.1568
2001/2002	3.9014***	1.1078	9.5418***	2.6218**	8.7442***	2.4502**
2003/2004	6.3053***	1.9514*	7.8999***	3.1422**	4.9898***	2.0917**
2005/2006	-3.1765**	1.5709	0.5976	3.5064**	4.2474***	2.5932**
2007/2008	1.296	5.9378***	-2.214*	6.4552***	-5.7862***	1.9238*
1993/1997	0.1034	4.6220***	-0.4741	3.8481***	-0.7422	1.6143
1998/2002	-4.9716***	1.29	3.3558**	2.2925*	8.8101***	2.0169**
2003/2007	-0.5465	2.6562**	3.669***	5.4346***	6.6823***	2.6812**
1993/2000	-0.5546	2.9516**	-2.7146**	1.1503	-2.3718*	1.0247
2001/2008	6.9054***	6.5510***	6.5143***	2.5557**	2.6371**	1.2346
1993/2008	4.9938***	1.5779	NA	NA	NA	NA

* Statistically significant at the 10% level
** Statistically significant at the 5% level
*** Statistically significant at the 1% level

[59] The sample 1 estimates are used as the first value and the numerator in the mean and variance tests respectively for the asset allocation tests. Samples 1 and 2 are used as the first value and the numerator in the mean and variance tests for the two asset inclusion tests respectively.
[60] The t distribution was used (based on the methodology of Morgan (1975)) and the critical values mirrored those employed for the respective tests based on the intercept estimates.
[61] The F critical values were identical to those used for the respective tests based on the intercept estimates.

It is therefore clear that the industry-sorted portfolios comprising ordinary shares, preference shares and bonds result in the estimates that most closely resemble the theoretical ideal of the minimum-variance zero-beta portfolio returns.

The point that must be acknowledged, however, is that the differing number of portfolios included in the four samples is likely to have had an impact on the standard error estimates obtained in these computations because of the methodology adopted. That is, based on the simple premise of diversification, the more securities that are included in a portfolio, the smaller the variance of the returns as the movement in the returns of the different shares offset each other as long as the correlation between the securities is less than positive one (Bodie *et al*, 2003: 190). In light of this, it is not surprising that the finding that the sample (industry-sorted portfolios consisting of all three asset classes) with the largest number of securities (as reflected by the number of portfolios) provided the most efficient estimates. Empirical tests of the effects of diversification have generally found that including approximately twenty shares in a portfolio will achieve close to the optimal benefits from diversification (Evans and Archer, 1968; Tole, 1982; Statman, 1987). However, Neu-Ner and Firer (1997) showed that in South Africa, it is necessary to include up to thirty shares in a portfolio to achieve close to maximum benefits from diversification, without incurring excessive transaction costs, which will reduce this benefit. Therefore it is not surprising that the results reflect a notable difference in the measurement of variance across the four samples because, except for sample 4, they do not closely adhere to the requirements of 30 shares (portfolios).

It is thus apparent that the results from this analysis of the most efficient method to estimate the zero-beta portfolio returns may be skewed because of the differences in the number of portfolios used to estimate the parameter across the different samples. For the purposes of this study, however, the most efficient estimates (from the industry sorted portfolios including preference shares and bonds) were selected, but it is certainly necessary for additional research to be conducted on the optimal number and composition of portfolios in order to obtain the most reliable estimates of the minimum-variance zero-beta portfolio returns.

In addition to these analyses of the efficiency of the zero-beta portfolio returns across the four samples created, tests of the efficiency were also conducted across time using the industry-sorted portfolios with preference shares and bonds included. As with the tests of the minimum-required return over time, averages were computed across the one-, two-, five- and eight-year horizons of the standard errors and the F-test statistics computed based on these values. The results are shown in Table 5-20 (on the following page).

Table 5-20: Time Period Mean and Variance Tests for the Zero-Beta Portfolio[62]

	Average Zero-Beta Portfolio Returns	Average Standard Error
One-Year	1.7254	0.3522
Two-Years	1.4408	0.3734
Five-Years	2.2323	0.4896
Eight-Years	1.6766	0.5109

	Mean Test[63]	Variance Test
One-Year versus Two-Years	2.7723***	1.124
One-Year versus Five-Years	-4.2024***	1.9326*
One-Year versus Eight-Years	0.3931	2.1042**
Two-Years versus Five-Years	-6.4272***	1.7193*
Two-Years versus Eight-Years	-1.8632*	1.8720*
Five-Years versus Eight-Years	3.9265***	1.0888

*(^{x}) Statistically significant at the 10% level
**(^{xx}) Statistically significant at the 5% level
***(^{xxx}) Statistically significant at the 1% level
The ^{x} denote where the inverse of the F statistics are statistically significant.

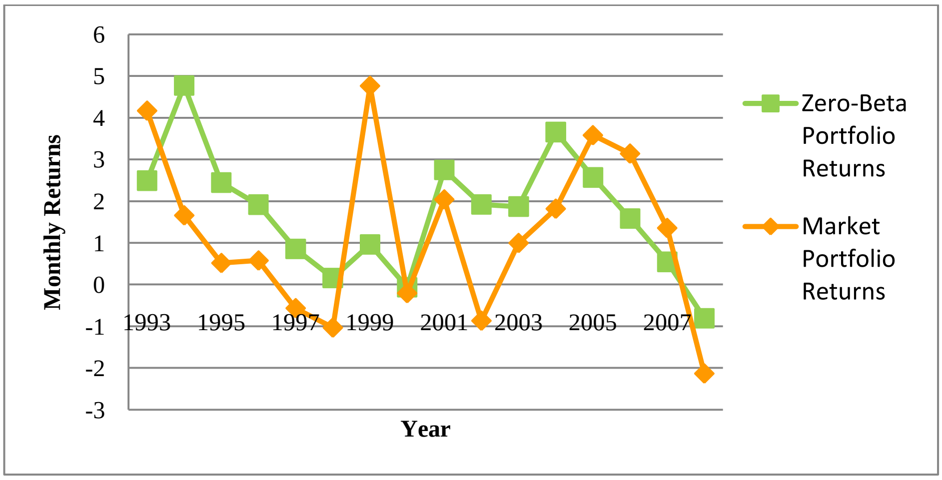
As can be seen in the top half of the table, the average standard errors increased as the estimation horizon increased, which was in contrast to the identical examination conducted for the intercept values, where the standard errors decreased as the estimation horizon increased. Consequently, in computing the F statistics, the longer period standard error was employed as the numerator and the shorter period as the denominator. The difference in efficiency between the estimates based on one-year and two-years is insignificant and the same is true for the difference between the five-year and eight-year estimates. However, the one- and two-year estimates are statistically significantly more efficient (at the five and ten percent significance levels) than either the five-year or eight-year estimates. The finding that the estimates of the minimum-variance zero-beta portfolio returns are more efficient over shorter periods is surprising, and it is not immediately clear what the possible causes of this may be. Accordingly, there is a need for further investigation in this regard.

With regards to the mean estimates of the zero-beta portfolio returns across the various time horizons, five of the test statistics, as depicted in Table 5-20, are statistically significant at the ten percent level or higher but they do not show any consistency in terms of sign. It is therefore

[62] In contrast to the previous time period based tests, the longer-period estimates are used as the first value and the numerator in the mean and variance tests respectively.
[63] The t-critical and F critical values used were identical to those employed for the intercept time period tests.

clear that using differing time horizons results in different estimates of the zero-beta portfolio returns and consequently, the selection of the most efficient time period is of importance in ensuring the results are reliable. Moreover, examining how the zero-beta portfolio returns change over time (as was done for the intercept values), revealed that over the two eight-year periods the return on the zero-beta portfolio was statistically significantly different (t test statistic of 5.6939) with the returns being larger over the period 1993 to 2000 than over the period 2001-2008. With respect to the five year periods, the findings mirrored those for the intercept values as over the periods 1993 to 1997, and 2003 to 2007 the zero-beta portfolio returns were not statistically significantly different (t test statistic of 1.3686), whereas the values in both these periods differed significantly from the estimate for the period 1998-2002, which was much larger (t test statistics of -9.9288 and 11.6346 respectively). As can be seen in Figure 5-8 these movements, similarly to the intercept value, track changes in the market returns. It is thus clear that the zero-beta portfolio returns do fluctuate over time (as observed by Morgan, 1975: 372), which has important implications for the use of this parameter in the CAPM as the risk-free rate. That is, if the zero-beta portfolio returns change over time, practitioners employing this value need to ensure that their forecasts for this parameter match the time period of the project being analysed with the model.

Figure 5-8: Comparison of the Zero-Beta and Market Portfolio Returns 1993-2008



5.7 Hypothesis Tests

The hypothesis tests using the intercept estimates, based on the Fama and MacBeth (1973) approach and employing the industry-sorted portfolios comprising all asset classes, were conducted for all periods excluding 2001 (where the sign of the estimated market risk premium was incorrect). Both T-Bonds and T-Bills were used as the risk-free rate proxy instruments in the hypothesis tests so as to ensure that the results of these tests were not sensitive to the choice of proxy. As alluded to in Section 1.3.2, because the estimates of the intercept based on the technique of Fama and MacBeth (1973) were chosen, the Student t distribution as opposed to the normal distribution was used to obtain the critical values for the tests of hypotheses one and two, with the t-distribution also appropriate for the tests of hypothesis three.

5.7.1 Hypothesis One

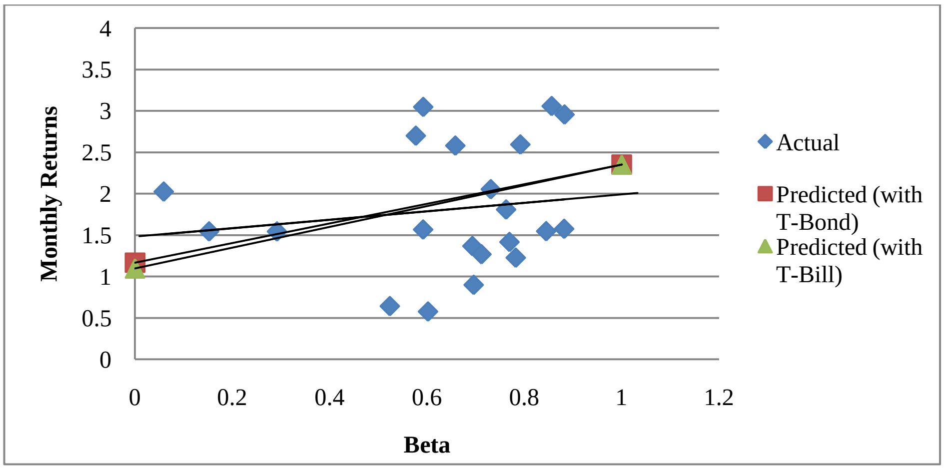
As specified in Section 4.6.1, the first hypothesis tested is given as:

$$H_0: \ \gamma_0 = R_f$$
$$H_1: \ \gamma_0 \neq R_f \qquad (4.34)$$

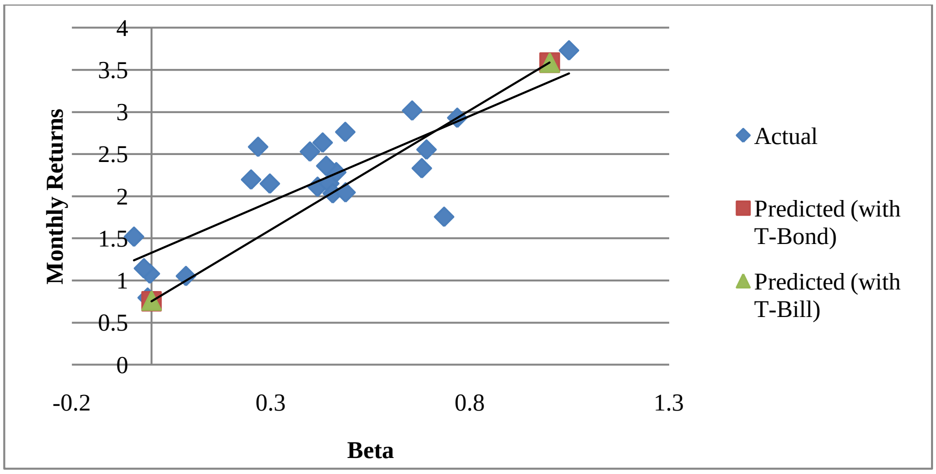
Panels a and b of Figure 5-9 graphically depict the actual relationship between risk and return over the periods 1993-2000 and 2001-2008 respectively, with the predicted relationships based on T-Bill and T-Bond yields and the average market portfolio returns also shown. From these figures it is evident that the difference between the monthly returns earned on the three month T-Bill and the R157 government bond were small, especially over the period 2001-2008 where it is impossible to distinguish between the two values. Moreover, the actual intercept of the CAPM in both of these periods exceeded the predicted value; with the difference being more pronounced over the latter period. As a consequence of the fact that the intercept exceeded the risk-free proxy return, it is not surprising that the actual SML's were flatter than predicted, because the slope, determined by the market risk premium, was smaller as a result of the higher intercept value.

Figure 5-9: Actual and Predicted (with Proxies) CAPM Relationships

Panel a: 1993-2000



Panel b: 2001-2008



The test statistics shown in Table 5-21 (on the following page) for the test of hypothesis one confirm this graphical conclusion, as the test statistics were significant at the 20 percent level or lower for all periods, with the T-Bill as a proxy, and all periods except three with the T-Bond. The extension of the significance level to 20 percent in order to take into consideration the effect of employing a two-tailed hypothesis test rather than a one-tailed test did not have a significant impact on the results, as only three statistics were significant at this level which would otherwise not have been considered significant. In fact, this highlights the important observation that for both T-Bills and T-Bonds, the majority of the test statistics were significant at the one percent level (78.57 percent for both proxies); thereby confirming that the intercept values do differ significantly from the risk-free rate proxy returns.

Of equal importance as the statistical significance of the test statistics are the signs of these statistics. 75 percent (21 of the 28 estimates) were positive indicating that the returns of the intercept were greater than the risk-free rate proxy returns using both T-Bills and T-Bonds. The seven estimates which were negative all occurred over the one-year or two-year estimation horizons, which as confirmed in Section 5.5.3, are likely to be less reliable estimates of the minimum required return. The fact that in some of the periods the risk-free rate proxy returns exceeded the intercept is in keeping with the results of Black *et al* (1972) and Fama and MacBeth (1973) discussed in Section 3.3.1.2, in that in both of these studies certain periods were observed where the relationship was inversed. It is also of value to note that whilst the use of the T-Bond as a proxy for the risk-free rate in the CAPM does more closely approximate the intercept value and more closely satisfies the theoretical requirements for the correct specification of the risk-free rate, the impact on the statistical tests is largely inconsequential.

It is therefore clear that despite the shortcomings of the proxy instruments in South Africa, highlighted in Section 2.4, which seemed to suggest that these instruments are likely to overstate the true risk-free rate that investors require from investing, the intercept value exceeded the proxy returns in 75 percent of the regressions estimated.

Table 5-21: Hypothesis Test Results[64]

	Hypothesis Test One		Hypothesis Test Two	Hypothesis Test Three	
	T-Bills	T-Bonds		T-Bills	T-Bonds
1993	13.3042***	11.6916***	-0.0250	18.8382***	16.3678***
1994	39.8833***	34.5520***	0.0720	71.8035***	54.1743***
1995	8.2752***	7.4055***	0.0314	17.9620***	15.6051***
1996	3.6535***	3.8715***	-0.0113	7.4845***	7.8875***
1997	-2.6818**	-1.9327	0.0290	-4.7388***	-3.4218***
1998	-4.2074***	-3.5913***	-0.0091	-9.1295***	-8.4847***
1999	4.6767***	4.2599***	4.6947***	-0.8441	-1.9493*
2000	-4.5573***	-5.7534***	0.0975	-9.4148***	-11.7688***
2001	NOT COMPUTED				
2002	12.7272***	13.7141***	-0.6203	18.7959***	21.0860***
2003	9.4718***	11.9324***	0.0313	12.1321***	17.0955***
2004	21.3386***	20.3291***	0.0570	49.0166***	45.9951***
2005	21.6852***	20.4667***	-7.2830***	33.1145***	31.8258***
2006	12.9160***	12.7547***	-1.1032	13.1375***	12.9771***
2007	-2.4595**	-1.4564	-0.3718	-3.1048***	-1.5829*
2008	4.4232***	5.6470***	15.6808***	-28.7094***	-25.1400***
1993/1994	15.9469***	13.8369***	0.0189	24.6795***	20.9035***
1995/1996	2.5048**	2.2114*	1.4668	1.1837	0.7320
1997/1998	-7.3711***	-6.5437***	0.0057	-10.1056***	-9.1147***
1999/2000	-3.1334***	-4.5303***	-9.4742***	13.6067***	12.2042***
2001/2002	13.7788***	13.6736***	-3.4295***	21.2167***	21.4349***
2003/2004	13.8326***	14.9458***	0.0324	15.0540***	16.5824***
2005/2006	10.9671***	10.1648***	-1.8188*	10.2641***	9.6576***
2007/2008	-9.5030***	-7.5682***	2.7359**	-13.2000***	-11.3321***
1993/1997	9.2044***	8.3022***	0.0046	6.7074***	5.9859***
1998/2002	8.3402***	8.1890***	-3.0595**	17.1122***	17.0992***
2003/2007	12.9937***	13.2532***	0.0005	11.4538***	11.6167***
1993/2000	3.5539***	2.9628**	-0.2423	2.9906***	2.5308**
2001/2008	12.8680***	13.3592***	1.2239	6.4431***	6.6197***

* Statistically significant at the 20% level
* Statistically significant at the 10% level
** Statistically significant at the 5% level
*** Statistically significant at the 1% level

The findings in this study provide contrasting results to Van Rhijn (1994), who observed that the difference between the estimated intercept and risk-free rate proxy returns was small in South Africa. There are a number of possible reasons as to why the findings differ from that of Van Rhijn (1994). Van Rhijn (1994) only considered industrial shares listed on the JSE which was a considerably smaller sample than was employed in this analysis such that his number of observations was insufficient to conduct statistical tests. Furthermore, Van Rhijn (1994)

[64] The test statistics are computed with the intercept estimates as the first values for hypotheses one and two, and the zero-beta portfolio returns first for hypothesis three. The critical values from the t-distribution were based on 21 degrees of freedom for periods pre 2001 and 24 thereafter.

employed a beta-sorting procedure to allocate shares to portfolios and, as was shown in this study and that of Van Rensburg and Robertson (2003), this approach does not appear to produce the best results in the South African context.

The results obtained in this study that the intercept exceeded the proxy returns over the majority of the periods, despite the presence of risk premia in the yields of the proxy securities as compensation for additional risk, at first appear counter intuitive. They closely mirror, however, the results of studies conducted in the U.S, Australia and in the international marketplace. These studies were discussed in Sections 3.3.1 and 3.3.3, and a table summarising the results from the whole periods examined (and not the various sub-periods) is presented in Table 5-22 for ease of comparison. Although none of these international studies examined the identical time period employed in this study, comparing the results enables some general conclusions to be drawn.

Table 5-22: Summary of International Studies

Study	Country	Period	Monthly Intercept Estimate (%)	T/Z Statistics ($\gamma_0 = R_f$)	Conclusion
Black *et al* (1972)	U.S.	1931-1966	0.36	6.52	Statistically significant
Fama and MacBeth (1973)	U.S.	1935-June 1968	0.61	2.55	Statistically significant
Stambaugh (1982)	U.S.	February 1953-December 1976	0.12	4.82	Statistically significant
Fraser *et al* (2000) [65]	U.K.	1975-1996	0.86	0.43	Insignificant
Faff (2001)	Australia	1974-1995	1.1	Not given	Statistically significant
Chou and Lin (2002)	International	1980-1997	1.63	Not given	Statistically significant
Fama and French (2004)	U.S.	1928-2003	0.87	Not given	Statistically significant
This study (2009)	South Africa	1993-2000 2001-2008	1.48 1.36	2.9628 13.3592	Statistically significant

The only exception to the finding that the risk-free rate proxy returns generally understate the observed minimum required return is in the U.K. where the estimated intercept (reflecting the minimum required return less the proxy returns) was statistically insignificant, although the difference of 0.86 percent appears substantial. Comparing the magnitude of the estimate of the intercept across the U.S. studies certainly indicates that the value has increased as more recent time periods are included in the analysis. The exception to this is Stambaugh (1982), but given

[65] This study estimated the intercept as the difference between the minimum required return and the risk-free rate proxy and therefore the monthly intercept represents the difference in the two values. The t-statistic shown can be interpreted in the same manner as for the other studies, as it reflects a test of whether the intercept was statistically significantly different from zero.

that his estimates included preference shares and bonds this may account for the considerably lower values for the intercept (although they were still statistically significantly different from the proxy yields). The estimates of the monthly average intercept obtained in this study for the periods 1993-2000 and 2001-2008 were 1.4766 and 1.3550 respectively, as shown in Table 5-22. These values more closely approximate the values estimated by Faff (2001) for Australia, and Chou and Lin (2002) for the international market than the study of the U.S. by Fama and French (2004). This is not entirely unexpected given the less advanced nature of the South African economy compared to that of the U.S., and in fact, the South African economy is seen to closely resemble the Australian market given the dependence of both economies on mining and financial services (Faff, 2001: 158). These results therefore appear to suggest that in countries which may be perceived to be of greater risk, investors require a larger base return from investing; in light of the discussions presented in Chapter 2 this conclusion conforms to expectations.

The surprising fact therefore is that despite the high interest rates observed in South Africa, the yields on government securities still understate the intercept of the CAPM, and in many instances the differences are even more substantial than in the U.S., Australia and the international marketplace as reflected by the high t statistics computed (as shown in Table 5-21, page 185). For the period 1993-2000 the test statistic (using a T-Bond) was 2.9628 which was significant at the five percent significance level and resembles the magnitude of the t-statistics of Fama and MacBeth (1973). This period coincides with relatively high interest rates in South Africa. In contrast, for the period 2001-2008 the t-statistic was 13.3592, as the average interest rate levels were substantially lower than for the previous period. These statistics are higher than even the Black *et al* (1972) values computed which were likely to be overstated because of the misspecification of the risk-free rate. Therefore, in some periods examined it appears as though the difference between the proxy yields and the estimated intercept of the CAPM may be more substantial in South Africa than in other countries.

Although not reflected in Table 5-22 (page 186), but discussed in Section 3.3.1, the other similarity in the results obtained in this study of South Africa compared to these international studies (specifically Black *et al*, 1972; Fama and MacBeth, 1973; and Stambaugh, 1982) is that the minimum required return does fluctuate over time and is not constant as the risk-free rate proxy should be over the investment horizon.

In light of the deviations of the South African risk-free proxies from the theoretical criteria, it appears strange that the minimum-required rate of return was larger than the proxy yields over the majority of periods examined. Several alternative explanations have been proposed to account for this empirical irregularity in the U.S. literature and therefore it is necessary to assess the validity of these explanations in the South African context.

As was discussed in Section 3.3.4.1, one possible explanation for these results may be the inadequacy of the estimation and testing procedures employed, in particular that historical data is used to test an expectational model. However, in the U.S. tests have been conducted (such as Vandell and Stevens, 1982) using forecasts of the market portfolio, share returns and risk-free rate, with the results mirroring those using historical data. A review of South African literature revealed that no similar tests have been conducted and thus although this explanation does not account for the relationships observed in the U.S., it is not possible to make such deductions in South Africa without further research on the subject. The second argument postulated is that the CAPM in its entirety, or in its simplest form, is inappropriate. As referred to throughout this study, there is evidence to support the use of the CAPM in South Africa (Bradfield *et al*, 1988; Bradfield and Barr, 1989; Van Rhijn, 1994; and Ward, 1994), but there have also been more recent studies which provide evidence for the opposite standpoint (Van Rensburg and Slaney, 1997; Van Rensburg, 2001; Van Rensburg and Robertson, 2003). As intimated in Section 3.3.4.2, there is some support that this empirical irregularity is not observed in the U.S. with more advanced forms of the CAPM (such as the consumption CAPM), but as with the forecast-based tests of the model, no research on more advanced forms of the CAPM was identified in South African financial literature to verify or dispute this explanation. More recently, Cloninger *et al* (2004) entered the debate regarding the relationship between the risk-free rate proxy and the intercept, suggesting that the SML is incorrectly presumed to be positively sloped for all beta values, when it should be negatively sloped for negative beta assets (when the market portfolio returns are positive); thus potentially accounting for the relationship observed. These authors did not present sufficient evidence to support this proposition in the U.S. market and in light of the results obtained in this study; it is certainly a necessary area for further research in South Africa as well.

One possible explanation for this result that certainly necessitates attention in the South African context is the potential incorrect specification of the market portfolio proxy. The ALSI was used in this study in lieu of the results of several examinations of the validity of this instrument (Ward, 1994; and Correia and Uliana, 2004), but its use is certainly not beyond reproach given the segmented nature of the South African market and the dominance of the mining and financial sectors in the ALSI (Van Rensburg and Slaney, 1997). Thus, as explained in Figure 3-5 (page 88), the incorrect specification of the market portfolio may account for the relationships observed.

The possibility also remains that the results observed may be a consequence of the use of an incorrect proxy for the risk-free rate. In the U.S. the use of a T-Bond which incorporates relevant inflation and liquidity premia certainly explains some of the difference observed between the T-Bill and intercept values and to a large extent, as was observed in Table 5-21 (page 185), the same is true for South Africa. However, the difference in the yields of the two instruments representing the liquidity and inflation premia in South Africa is not sufficient to fully account for the observed patterns. In addition to this, one of the most logical suggestions proposed to account for this anomaly is that the risk-free rate proxy only represents the lending risk-free rate and does not consider the costs associated with borrowing; that is, investors who borrow will require more as their base return as compensation for the costs associated with borrowing. The "…market equivalent risk-free rate…" (Brennan, 1971: 1204) should therefore represent the weighted average of the riskless borrowing and lending rates. This certainly has some validity as in all of the markets examined in Table 5-22 (page 186), excluding the U.K., the same relationship between the proxy and the intercept has been observed which suggests that this anomaly is not necessarily a flaw unique to a particular country but perhaps a flaw with the model or the specification of the parameters. Due to the difficulty in determining an optimal borrowing rate and the weighted proportions of each, Brennan (1971: 1205) suggested that the zero-beta portfolio returns should provide a good estimate of this value.

The results of the test of hypothesis one therefore indicate that the intercept of the CAPM exceeds the risk-free rate proxy returns for both T-Bills and T-Bonds in South Africa over the period 1993 to 2008. The possible causes of this finding however are not easily identifiable, but in order to fully analyse the best estimate of the risk-free rate in the CAPM it is necessary to consider the results of the tests of hypotheses two and three.

5.7.2 Hypothesis Two

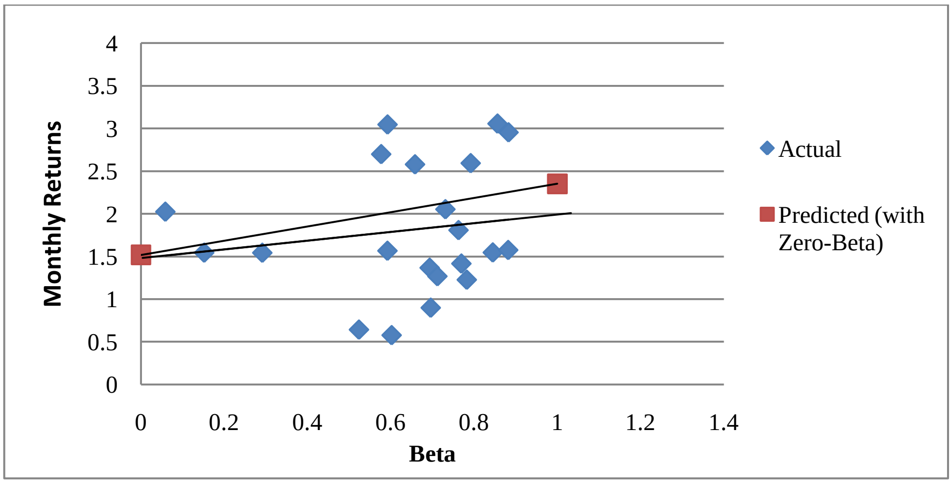
The second hypothesis tested in order to ascertain the most suitable method to estimate the risk-free rate in South Africa was specified in equation 4.35, and is reproduced below.

$$H_0: \gamma_0 = R_z$$
$$H_1: \gamma_0 \neq R_z \qquad (4.35)$$

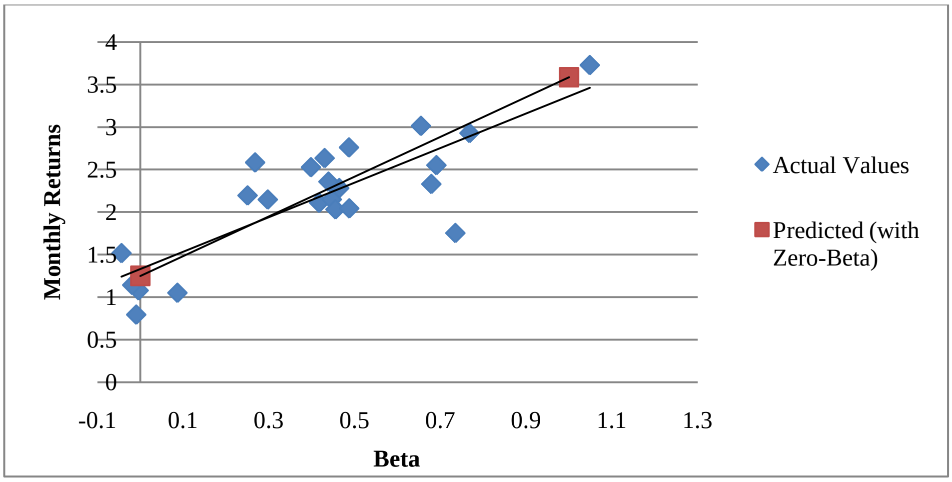
The identical graphs to Figure 5-9 (page 183) are depicted in panels a and b of Figure 5-10 (on the following page) but with the predicted SML being based on the zero-beta portfolio returns rather than the T-Bill or T-Bond values. For both periods, the graphs clearly indicate that it is not possible to distinguish between the estimated intercept and the zero-beta portfolio returns; that is, the zero-beta portfolio returns provide a good estimate of the minimum return required by investors. However, it must be acknowledged that this is largely to be expected given the identical datasets used to estimate both parameters and the similarity in the estimation techniques used to determine the two parameters. From both graphs it is also noticeable that even with the use of the zero-beta portfolio returns, the market risk premia are not identical, but the differences are small; confirmed by statistical tests of their equality (0.4229 and 1.3460 respectively which are both smaller than the t-statistic at the ten percent critical value of 2.0595). In both cases the predicted values overstate the actual risk premia.

Figure 5-10: Actual and Predicted (with the Zero-Beta Portfolio) CAPM Relationships

Panel a: 1993-2000



Panel b: 2001-2008



The statistical tests of the equality of the zero-beta portfolio returns and the estimated intercept values (shown in Table 5-21 on page 185) confirm the conclusions drawn from examining the graphs that the zero-beta portfolio returns and the intercept values were not consistently statistically significantly different. Of the 28 estimates, only eight were statistically significant and seven of these coincided with periods where adjustments were made for non-normality in estimating the intercepts that were not made to the zero-beta portfolio returns. It can therefore be concluded that the zero-beta portfolio returns closely approximate the estimated intercept values of the CAPM, the only differences observed in this study being a consequence of the introduction of dummy variables in the estimation of the intercept terms. This result that the zero-beta portfolio return closely approximates the estimated intercept of the CAPM mirrors that of Morgan (1975) in the U.S. discussed in Section 3.3.2.

The results of the tests of hypotheses one and two thus indicate that the zero-beta portfolio returns more closely resemble the intercept of the CAPM than the risk-free rate proxy; however, this finding may be a consequence of a number of factors and does not necessarily indicate that the zero-beta portfolio approach *is* a more suitable method to estimate the risk-free rate in South Africa. The results obtained could be a function of the incorrect market portfolio specification, the incorrect specification of the form of the model or whether the zero-beta portfolio returns provide an approximation of "…the market equivalent risk-free rate…" which is the weighted average of the riskless borrowing and lending rates (Brennan, 1971: 1204).

A corollary to the tests of the equality of the zero-beta portfolio returns and the estimated intercepts over the various periods is the question of the similarity of the associated variance of the two measures based on the two procedures. Morgan (1975) also examined this issue in his study and found that the direct measurements of the zero-beta portfolio were more efficient than the indirect method of estimating this parameter. Consequently, similar results were expected in comparing the standard errors of the two sets of values in this study. As can be seen in Table 5-23 (on the following page), the results in this regard generally adhere to the findings of Morgan (1975) as 64.29 percent of the estimates were statistically significant, where the intercept term represented the numerator in the test statistic and the zero-beta portfolio returns the denominator. Of concern however, is that over the longer periods (the five-and eight-year estimation horizons), the statistics were frequently less than one meaning that the zero-beta portfolio standard errors estimates were higher and in three cases these statistics were significant at the ten percent level or lower. These different patterns identified across the shorter and longer estimation periods represent the unique trends identified with respect to both sets of estimates over time. That is, the efficiency of the intercept estimates were seen to increase as the time horizon increased whereas the opposite was observed with respect to the zero-beta portfolio returns. Therefore, although the majority of the test statistics point towards the greater efficiency of the direct zero-beta portfolio estimates, this is largely a function of the greater number of short-time period estimates and consequently it is not possible to draw a definitive conclusion with respect to the efficiency of these two sets of estimates.

Table 5-23: Variance Tests of the Intercept and Zero-Beta Portfolio Returns

Period	Variance Test[66]
1993	2.0246**
1994	3.5843***
1995	4.8578***
1996	4.3122***
1997	3.0925***
1998	6.2173***
1999	4.0347***
2000	4.0802***
2001	3.4389***
2002	2.2649**
2003	2.0849**
2004	5.3629***
2005	1.0888
2006	0.8092
2007	2.4492**
2008	5.3214***
1993/1994	2.4408**
1995/1996	2.4027**
1997/1998	1.9689**
1999/2000	4.4066***
2001/2002	1.4164
2003/2004	1.2412
2005/2006	0.5286^{x}
2007/2008	0.9939
1993/1997	0.5114^{xx}
1998/2002	2.0941**
2003/2007	0.7652
1993/2000	0.5632^{x}
2001/2008	0.3420^{xxx}

*(^{x}) Statistically significant at the 10% level
**(^{xx}) Statistically significant at the 5% level
***(^{xxx}) Statistically significant at the 1% level
The ^{x} denote where the inverse of the F statistics are statistically significant

5.7.3 Hypothesis Three

The final hypothesis tested was the relationship between the risk-free rate proxy returns and the zero-beta portfolio returns, as denoted in equation 4.36 and reproduced on the following page.

$$H_0: \; R_z = R_f$$
$$H_1: \; R_z \neq R_f \qquad (4.36)$$

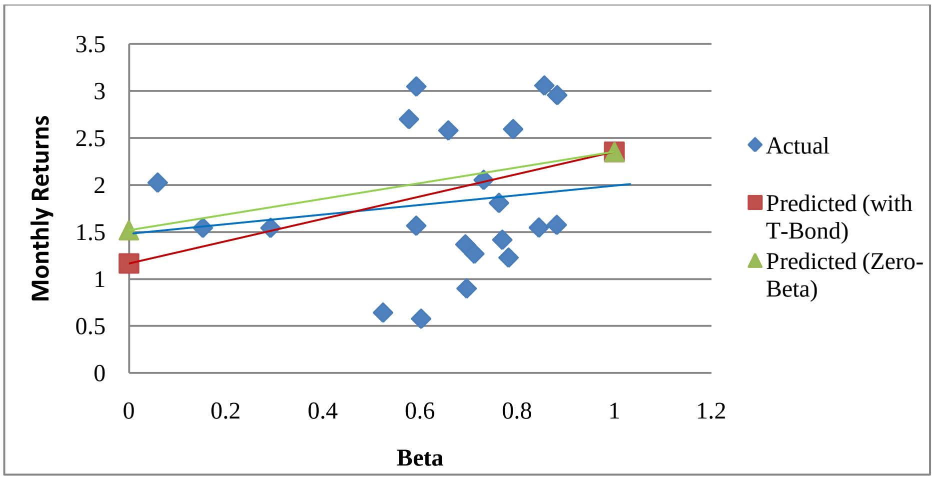
In light of the results of hypothesis two that the zero-beta portfolio returns were not, on average, statistically significantly different from the intercept, and hypothesis one that the intercept

[66] The intercept estimates were used as the numerator in the F-statistics. Both the numerator and denominator degrees of freedom were adjusted to reflect the changes in the sample size from 2001 onwards.

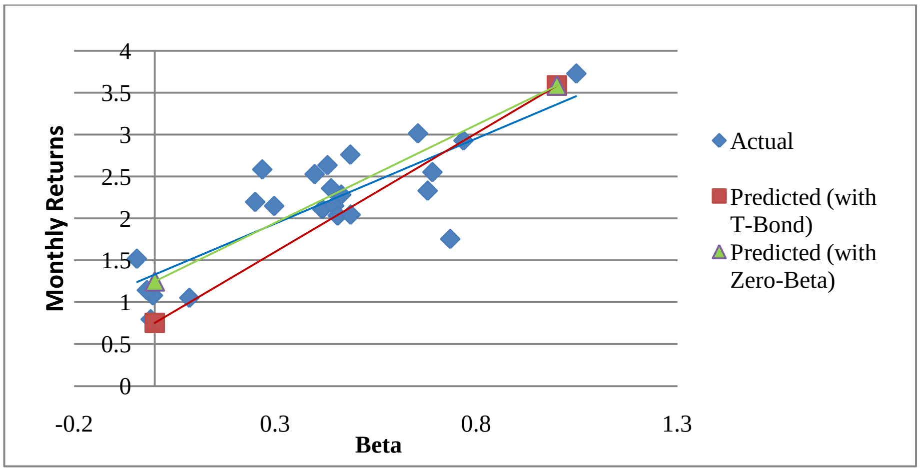
values differed significantly from the risk-free rate proxy returns, it was expected that these tests would result in the rejection of the null hypothesis that the zero-beta portfolio returns and the proxy yields were equal. The graphs presented in Figures 5-9 (page 183) and 5-10 (page 190) were combined, as shown in panels a and b of Figure 5-11 so as to compare the difference in the estimated SMLs employing both risk-free rate measurements. As was anticipated, the zero-beta portfolio returns lie substantially above the risk-free rate proxy returns, and consequently the SMLs have a flatter slope than with the risk-free rate proxy.

Figure 5-11: Actual and Predicted CAPM Relationships

Panel a: 1993-2000



Panel b: 2001-2008



The results of the statistical tests presented in Table 5-21 (page 185) confirm the conclusion regarding the equality of the zero-beta portfolio and the risk-free rate proxy returns as 96.43 percent were significant at the 20 percent level or lower, where the T-Bond was employed as the proxy, and 92.86 percent when the T-Bill was used. This provides evidence therefore that the null hypothesis can be rejected in favour of the alternative hypothesis that the two values were not equal over the various periods examined.

5.7.4 Conclusions

The hypothesis tests therefore indicate that the risk-free rate proxy returns were lower than the intercept of the estimated CAPM over the period 1993 to 2008 in South Africa, whilst the zero-beta portfolios returns closely approximate this value. At face value, these findings certainly intimate that the zero-beta portfolio approach to estimating the risk-free rate in South Africa is more suitable than the use of a proxy; however, as was discussed, this may not necessarily be the direct consequence of the zero-beta portfolio approach being more appropriate, but perhaps because of the incorrect specification of the model, the market portfolio or the risk-free rate in terms of ignoring the costs of borrowing. Notwithstanding this uncertainty, it is necessary to ascertain whether the choice between these two measurements of the risk-free rate is economically significant; that is, does the use of the zero-beta portfolio return provide consistently more accurate estimates of the cost of capital from the CAPM. The forecasting tests conducted for this purpose are discussed in the following section.

5.8 Forecasting Comparison

For the comparison of the forecasting accuracy of the traditional CAPM and the zero-beta CAPM, the T-Bond and T-Bill returns and the direct estimates of the zero-beta portfolio utilised in the hypothesis tests were employed for the respective models. It was not considered necessary to analyse the forecasting accuracy of the model using the intercept due to the similarity between this value and the zero-beta portfolio returns as revealed in the findings of hypothesis three, as was shown by Morgan (1975), discussed in Section 3.3.2.

The MSE, MAPE and AMAPE criteria were computed for each of the three models tested, in accordance with the methodology established in Section 4.7. The results are presented in Table 5-24 (on the following page) for each year over the period 1993-2008 and are also averaged across the sixteen years. Although to appropriately assess the forecasting accuracy of the CAPM, in general, the results obtained should be compared to an alternative model such as the APT, the computed values for the MSE, MAPE and AMAPE do indicate that the CAPM appears to be a reasonably reliable model as the forecasting errors were quite small. As mentioned in Section 4.7, interpreting the values computed with the MSE calculation is difficult in absolute terms and hence it is of more use in comparing the relative forecasting accuracy of the different models. In contrast, the MAPE and AMAPE criteria can be interpreted as percentages and thus the results suggest that on average, the forecasting error was between approximately 2.3533 and 2.4397 percent based on the MAPE and between 2.4376 and 3.9599 percent for the AMAPE. These results are however likely to be biased downwards as the actual market portfolio proxy and the risk-free instrument returns in the period where the returns were being forecasted were used to represent the expectations of these values. In practice however, these values will also be forecasts and thus their suitability will impact on the reliability of the estimates of the CAPM.

Table 5-24: Forecasting Accuracy of the CAPM

	MSE			MAPE			AMAPE		
	T-Bill	T-Bond	Zero-Beta	T-Bill	T-Bond	Zero-Beta	T-Bill	T-Bond	Zero-Beta
1993	0.3230	0.3173	0.2782	0.9135	0.8973	1.0248	0.4883	0.4739	0.4864
1994	0.1265	0.1240	0.1343	1.6726	1.6245	1.9088	2.5797	4.1635	1.1066
1995	0.2304	0.2265	0.1991	1.3127	1.2642	1.1283	1.2112	1.1649	0.9571
1996	0.1614	0.1607	0.1743	1.8218	1.8085	2.0588	4.0208	4.4648	3.6751
1997	0.2788	0.2777	0.2752	1.8834	1.8758	1.8558	2.9073	2.8172	2.6197
1998	0.1969	0.1982	0.1963	1.6750	1.6555	1.6874	1.5638	1.5708	1.5604
1999	0.1506	0.1470	0.1538	2.9400	2.9234	2.9547	0.5746	0.5629	0.5852
2000	0.1524	0.1586	0.1313	2.3643	2.4443	2.0550	22.3255	6.2328	9.0767
2001	0.6322	0.6241	0.4876	1.6412	1.6309	1.5331	1.6761	1.6683	1.5914
2002	0.2448	0.2458	0.2192	2.5829	2.5929	2.6054	4.7623	4.8438	3.3683
2003	0.4167	0.4213	0.3807	1.9468	1.9410	2.0042	1.6586	1.6696	1.5970
2004	0.2810	0.2709	0.1344	2.4179	2.3524	1.4680	2.7607	2.6804	1.6742
2005	0.0534	0.0513	0.0394	2.0681	2.0330	1.8130	0.8723	0.8396	0.5260
2006	0.1033	0.1023	0.0728	2.1578	2.1511	2.1733	0.9786	0.9720	0.8657
2007	0.1277	0.1294	0.1314	5.2173	5.1782	5.1519	5.1620	5.7778	7.2000
2008	0.3335	0.3199	0.2261	6.4192	6.4005	6.2312	9.8172	5.3040	2.1116
Average	**0.2383**	**0.2359**	**0.2021**	**2.4397**	**2.4233**	**2.3533**	**3.9599**	**2.8254**	**2.4376**

The MSE, MAPE and AMAPE estimates for the CAPM using the T-Bond as a proxy were more accurate (i.e. had a lower forecasting error) than the model using the three month T-Bill value. Whilst the differences between the two models were small based on the MSE and MAPE criterion (0.0024 and 0.0164 percent), the difference was more substantial as per the AMAPE (1.5223 percent) calculations. As discussed in Section 4.7, the AMAPE is widely considered in both texts and applications to be the most reliable of the criteria computed, as it is calculated such that all errors are given equal weighting rather than those which lie further away from the actual value being more heavily weighted as is the case for MSE and MAPE. Thus, it can be concluded that the forecast error associated with the CAPM using a T-Bond is certainly smaller than if a T-Bill was employed. It is however, not possible to ascertain the statistical significance of this difference as the distribution that these statistics follow is not easily determined (Brooks, 2006: 290). These results confirm the theoretical deductions in Section 2.4 that T-Bonds more closely satisfy the requirements for a risk-free asset than T-Bills.

With respect to the CAPM estimated using the minimum-variance zero-beta portfolio returns, all three criteria indicated that this model was more accurate than that using either T-Bills or T-Bonds. As with the comparison of T-Bills and T-Bonds, the difference in the forecasting accuracy measurements between the model with the T-Bond and the zero-beta portfolio returns were smaller than anticipated (0.0338 for the MSE, 0.07 percent for the MAPE and 0.3878 percent for the AMAPE). Therefore these results reflect that the CAPM determined cost of equity values are likely to be more reliable using the minimum-variance zero-beta portfolio returns, but that the improvement in accuracy is not necessarily as substantial as previously thought. Examining the annual estimates it is clear that there were some periods where the use of a T-Bill or T-Bond would have resulted in more accurate results but, on average, the criterion revealed the zero-beta portfolio CAPM as the most accurate model.

The greater forecasting errors associated with the CAPM using either a T-Bill or T-Bond compared to the minimum-variance zero-beta portfolio returns is consistent with the results of the hypothesis tests, as the results of hypotheses one and two indicated that the historical minimum return required from investing exceeded the returns on the risk-free rate proxies (both short-term and long-term) but that the zero-beta portfolio returns closely approximated these values. Although a number of different reasons were proposed to account for the results obtained in this regard, it is clear that for the form of the model that is currently employed in South Africa and the value used for the market portfolio proxy, that the use of the minimum-variance zero-beta portfolio returns is likely to result in more accurate cost of equity estimates.

In light of the results of the comparison of the forecasting accuracy of the CAPM using T-Bills, T-Bonds or the zero-beta portfolio, it appears as though the choice of the appropriate parameter to estimate the risk-free rate in South Africa is not necessarily unambiguous but rather something of a trade-off. That is, employing the zero-beta portfolio returns is likely to result in more accurate estimates of the cost of equity for a firm, but this increase in accuracy, which is small, has to be weighed up against the increased cost and complexity associated with estimating the returns on such a portfolio compared to the ease associated with employing a T-Bond or T-Bill. This situation closely mirrors the comment by Copeland *et al* (2000: 215) discussed in Section 3.4.1, with reference to the U.S., stating that whilst the minimum-variance zero-beta portfolio is the best way to measure the risk-free rate, its use is not justified in the U.S. by the cost and difficulty in determining the correct value to employ.

5.9 Chapter Summary

In this Chapter, the results of the procedures described in the preceding chapter for the estimation of the various parameters were discussed in conjunction with the findings of the hypothesis tests and forecasting analysis. The tests of the comparison of the three estimation techniques revealed that the Fama and MacBeth (1973) and Stambaugh (1982) estimates were more efficient than the Black *et al* (1972) method, with the former being favoured in this study due to the more accurate adjustment for heteroscedasticity. With regards to the choice of asset allocation procedure, the industry-sorted portfolios were selected as the most suitable, as these samples adhered to the theory underlying the CAPM to a much larger extent that the beta-sorted portfolios. The inclusion of preference shares and bonds did not have a substantial impact on the extent to which the CAPM relationships held, but were seen to significantly increase the efficiency of the intercept estimates and thus were chosen for use in the hypothesis tests.

The results of the tests of the minimum-variance zero-beta portfolio returns indicated that the industry-sorted portfolios including the additional asset classes were, similarly to the estimation of the intercept, found to be the most efficient. However, the fact that the number of portfolios in each of the four samples differed had a direct impact on the results obtained, as because of the influence of diversification, the sample with the largest number of portfolios would have the lowest variation in returns (industry-sorted portfolios with all asset classes).

The results of the hypothesis tests conducted were clear; the risk-free rate proxies understate the estimated intercept of the CAPM (similarly to international studies), the direct estimates of the minimum-variance zero-beta portfolio returns closely approximate the intercept values and the risk-free rate proxy yields differ significantly from the returns of the zero-beta portfolio. These results, at first appearance, certainly favour the zero-beta portfolio approach; however, the results obtained may also be a function of the incorrect specification of the model, the market portfolio or the use of an inappropriate proxy as the T-Bill and T-Bond only represent the lending rate without taking into consideration the cost of borrowing. The forecasting comparisons confirmed the results of the hypothesis tests, as the CAPM using the zero-beta portfolio returns rather than a proxy was identified to be the most accurate. However, the increase in accuracy associated with using this value was small, and given the difficulty associated with estimating the zero-beta portfolio returns, it was not clear whether this increase in accuracy was justified.

In Chapter 6, the results and conclusions presented in this study are summarised with recommendations for further research also provided.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Review of Research Objectives

The CAPM, despite considerable criticism and debate regarding its validity, remains the most widely employed model to estimate the cost of equity for use in capital budgeting decisions, both in the U.S. and in South Africa. The difficulty for both practitioners and scholars employing the model is the computation of the appropriate values to use for the variables, as they are all theoretical constructs that have to be estimated. As was discussed in Section 1.1.1, considerable research has been conducted in the U.S. regarding the appropriate methods to determine beta, the market portfolio and the risk-free rate, but there is no guarantee that the techniques deemed to be suitable in the U.S. are necessarily appropriate in South Africa due to the dissimilarities between the two markets. This is confirmed in the South African research addressing the estimation of beta and the market portfolio as different techniques and adjustments have been identified in order to accurately implement the CAPM in South Africa, primarily as a consequence of thin-trading and the segmented nature of the market.

Despite this, very little research has been conducted on the estimation of the risk-free rate in South Africa, with U.S. practices largely extrapolated and applied. However, there is no guarantee that these practices are necessarily optimal and given the importance of the CAPM in ensuring that scarce resources are allocated to their most efficient uses, it is imperative that all of the model's parameters are correctly estimated so that the cost of equity calculated is accurate. Consequently, the primary motivation underlying this study was to ascertain the most suitable method to estimate the risk-free rate in South Africa. In analysing this issue, a number of sub-objectives were also considered including:

- To assess how closely the most commonly used proxies in South Africa adhere to the theoretical requirements for a risk-free asset and whether this divergence is considerably greater than that associated with the most commonly used U.S. proxies.
- To answer the question of which of the various methods developed in the literature is the most efficient to employ in estimating the intercept of the CAPM.
- To ascertain whether allocating shares to portfolios on the basis of industry classifications is more efficient than historical beta estimates in calculating the intercept value and zero-beta portfolio returns.

- To determine the optimal combination of assets that will result in the lowest variance intercept estimates and zero-beta portfolio returns, by expanding the analysis to include both bonds and preference shares in addition to ordinary shares.
- To identify the optimal time period over which the intercept of the CAPM and zero-beta portfolio returns should be estimated.
- To assess the impact on the forecasting accuracy of the CAPM of using the zero-beta portfolio returns compared to a risk-free asset proxy.

The extent to which each of the objectives was addressed is best gauged by considering a summary of the specific findings. The results of the analyses of the sub-objectives (excluding the forecasting analysis) are discussed prior to the main objective, as the results of these sub-objectives directly impacted upon the values employed in conducting the hypothesis tests, which formed the basis of the assessment of the primary objective. The results of the forecasting analysis are examined in conjunction with the findings of the hypothesis tests given their combined importance for the overall conclusions of this study.

6.2 Summary of Study Findings

6.2.1 The Risk-Free Rate Proxy in South Africa

The most common approach to estimating the risk-free rate in the U.S. is the use of a government security, such as a T-Bill or T-Bond. In South Africa, these instruments are also commonly employed for this purpose but primary sector securities, such as BAs and NCDs, have also become popular due to the perception amongst scholars that government securities do not necessarily provide a good proxy for the risk-free rate. In light of this concern regarding the appropriateness of government securities to estimate the risk-free rate, a set of theoretical requirements that an asset must closely satisfy to be considered a suitable proxy were compiled to be able to compare the South African securities against. These include that the return on a suitable risk-free asset candidate must represent the pure interest rate and include a premium which accurately incorporates inflation expectations. In addition to these two criteria, the risk-free asset must not exhibit any variation in returns over time or co-movement with the returns on the market and must be devoid of interest rate risk, default risk, liquidity risk and currency risk. U.S. T-Bills and T-Bonds were initially examined to assess how closely these securities meet the requirements so as to provide a basis for comparison of the suitability of using the equivalent instruments in South Africa. The results of this examination indicated that whilst these assets are by no means beyond reproach, provided that a security is chosen whose maturity matches that of the investment horizon being analysed, the impact of the deviations of these securities from the requirements is considered small.

From both the qualitative and quantitative analysis conducted, it was clear that the same cannot necessarily be said about the most commonly employed proxies for the risk-free rate in South Africa. These securities were observed to be subject to considerable inflation risk, exhibit greater default, liquidity, interest rate and currency risk, and also displayed more pronounced variation in returns over time and correlation with the market than the U.S. equivalents. Although some of these problems are reduced through the selection of an asset with an appropriate maturity; there was evidence presented in Chapter 2 to conclude that the South African T-Bills, T-Bonds, BAs and NCDs do not closely satisfy the requirements established for the correct specification of the risk-free rate in the CAPM.

6.2.2 The Estimation of the Minimum Required Return

In order to address the primary research objective of determining the best approach to estimate the risk-free rate in South Africa, the historical minimum rate which investors have required from investing (not taking into consideration the effects of risk) was computed. However, in analysing the literature on this procedure it was apparent that there were a number of issues which were not resolved; specifically, the choice of the best estimation procedure for this purpose, the methodology of combining shares to portfolios to use as the sample from which the CAPM is estimated, the choice of asset classes to include in the sample, and the optimal time period over which to estimate this value. Consequently, these four areas were assessed in this study and the results obtained in this regard are summarised below.

6.2.2.1 The Most Efficient Methodology

Three unique approaches were identified to have been used for the purpose of estimating the CAPM in the literature based on the original work of Black *et al* (1972), Fama and MacBeth (1973) and Stambaugh (1982). However, there was no consensus as to which of these approaches produces the most efficient estimates of the intercept. Therefore the methods derived in all of these studies were implemented.

The tests of the equality of the standard errors showed clearly that the estimates based on the approach of Black *et al* (1972) were less efficient than those based either on the methodology of Fama and MacBeth (1973) or Stambaugh (1982). Very little difference was found between the efficiency of the estimates based on the techniques of the latter two studies. What differences were observed did not visibly favour one approach; and in fact, these differences were largely seen to be a consequence of the insufficient adjustments for heteroscedasticity in the estimates based on the procedure of Stambaugh (1982). In light of this, the Fama and MacBeth (1973) based estimates were selected as the most efficient in this study and employed in the consequent analyses.

6.2.2.2 Asset Allocation Procedure

Early tests of the CAPM (such as Douglas, 1969) used individual shares as the sample for estimating the model; however, Black *et al* (1972: 8-9) proposed allocating shares to portfolios when estimating the CAPM parameters rather than basing the estimates on individual shares, as the latter approach can lead to the error-in-the-variables problem, as referred to in Section 4.3.1. The best system of allocating the shares to portfolios however is not clear, and consequently, an investigation was conducted to address this issue by forming portfolios based on both procedures (historical beta estimates and industry affiliations) and comparing the results.

In this respect, the findings of the analysis conducted showed that the beta-sorting procedure does not provide the most accurate estimates of the CAPM parameters in South Africa. It was seen that over periods where the market portfolio returns were large and positive, the estimates of the risk premium were significantly negative; thereby signalling that the relationship between risk and return over the period was negative, which is in violation of the theory underlying the model. These results regarding the suitability of the beta-sorted procedure are consistent with the findings of Van Rensburg and Robertson (2003). The industry-sorted portfolios in contrast, were largely seen to satisfy the CAPM relationships, and provided equally efficient estimates of the intercept as the beta-sorted sample, when the latter were consistent with the CAPM theory. Accordingly, the industry-sorted samples were used for further analyses in this study.

6.2.2.3 Asset Classes

As was discussed in Sections 3.3.1.4 and 4.3.5, Stambaugh (1982) expanded upon the traditional approach (of Black *et al*, 1972 and Fama and MacBeth, 1973) to estimating the intercept of the CAPM by including not only ordinary shares, but also preference shares and bonds in the sample. He claimed that his estimates of this value were more efficient than in previous studies; however, because he also adopted an alternative approach to estimating this value, the direct influence of the inclusion of additional asset classes was not clear. As highlighted, no further research was identified to have been conducted in this area confirming or disputing the observations of Stambaugh (1982) and consequently, the effect of the inclusion of additional asset classes on the reliability of the intercept values was also addressed.

The inclusion of preference shares and bonds into the industry-sorted portfolios had little impact on the extent to which the CAPM relationships held over the various time periods examined, but they had a substantial effect on the efficiency of the intercept estimates. The expansion of the dataset to include the additional asset classes significantly reduced the standard errors of the estimates; thus increasing their efficiency. Thus the estimates based on the industry-sorted portfolios using all three asset classes were employed in the hypothesis tests conducted.

6.2.2.4 Time Period

Similarly to the issues of the most suitable methodology, asset allocation procedure and asset classes, the question of the optimal time period over which to estimate the intercept was not clearly evident from previous research, and hence was examined in this study. In keeping with expectations, the findings of this analysis revealed that the longer the time horizon, the more efficient the parameter estimates. However, the differences in the standard error estimates were only statistically significant when the one-, two- and five-year estimates were compared to the eight-year values and not between the shorter periods. Despite these results that the estimates of the intercept computed over longer estimation horizons were more efficient, all period estimates were employed in the hypothesis tests to ensure that the conclusions drawn were robust to the choice of time period.

6.2.3 Zero-Beta Portfolio Returns

As identified in Section 3.2, Black (1972) derived an alternative approach to estimate the risk-free rate in the CAPM known as the minimum-variance zero-beta portfolio. Due to the nature of the estimation procedures adopted for the computation of the intercept of the CAPM, this value can be interpreted as the returns on a zero-beta portfolio. However, this is considered to be an indirect approach to estimating the minimum-variance zero-beta portfolio returns and hence, in accordance with the suggestions of Morgan (1975), the value for this parameter was estimated directly based on his technique but adapted for use in Microsoft Excel's Solver function. However, in estimating this parameter the identical issues highlighted with respect to the estimation of the minimum required return were identified; that is, the optimal asset allocation procedure, assets to include in the samples and the most efficient time period over which to compute the values. Rather than assuming that the results obtained with respect to these issues in estimating the intercept would be identical for the zero-beta portfolio returns, the same analyses were conducted for this set of estimates.

With regards to the optimal asset allocation procedure and the asset classes to be included in the estimation of the zero-beta portfolio returns, the results of the F-tests conducted were identical to the findings from the estimation of the intercept, as the industry-sorted portfolios including preference shares and bonds were found to be the most efficient. The problem however was that because of the way in which the direct estimates of the zero-beta portfolio were computed it was inevitable that the portfolio with the largest number of portfolios would be the most efficient, because the effects of diversification were taken into consideration. Consequently, in this regard whilst the industry-sorted portfolios comprising all asset classes had the lowest variance in this study, the questions of the optimal asset allocation procedure and assets to be included in determining the zero-beta portfolio returns were not resolved.

For the tests of the zero-beta portfolio estimates across time, it was found, contrary to expectation, that the shorter the time horizon over which the parameter was estimated, the lower the variance; the reason for this finding however, was not clear. Notwithstanding this, as with the intercept values, all time period estimates were used for the hypothesis tests,

6.2.4 The Best Approach to Estimate the Risk-Free Rate in South Africa

To determine the most suitable method to estimate the risk-free rate in the CAPM, the risk-free rate proxy returns and the zero-beta portfolio returns were compared to the intercept of the CAPM across the various periods over which the parameters were estimated. The first hypothesis tested examined the relationship between the estimates of the intercept of the CAPM and the yields of both T-Bills and T-Bonds; the results of which indicated that the risk-free rate proxies were statistically significantly smaller than the minimum required return in the majority of the periods. These results contradict the findings of Van Rhijn (1994) who observed that the difference between the risk-free rate proxy yields and the estimated intercept of the CAPM was inconsequential in South Africa. However, they closely mirror the findings for the U.S., Australia and the international market, suggesting that this relationship between the risk-free rate proxies and the intercept of the CAPM is not unique to the South African market. For the second hypothesis tested, the relationship between the direct estimates of the minimum-variance zero-beta portfolio returns and the estimates of the intercept were compared, with 73.43 of the test statistics computed signalling that there was no significant difference between these two sets of values over the various time horizons examined; that is, that the minimum-variance zero-beta portfolios returns closely approximated the base return required by investors. Finally, the results of the third hypothesis test confirmed that the risk-free rate proxy returns were not statistically similar to the minimum-variance zero-beta portfolio returns.

At face value, the results of the three hypothesis tests infer that the zero-beta portfolio approach to estimating the risk-free rate appears more appropriate in South Africa than employing a proxy because the returns on this security closely match the base return required by investors over the various periods examined, whereas the risk-free rate proxy returns understate this value. However, as acknowledged in Section 5.7.1, it seems counter-intuitive to advocate a value for the risk-free rate which exceeds the proxy yields when these instruments appear to overstate the true riskless rate. A number of possible reasons for the relationships identified were explored including the incorrect specification of the model, the use of an inappropriate market portfolio proxy and the possibility that the risk-free rate proxy yields do not take into consideration the costs associated with borrowing which would increase the base value that investors require from investing in order to cover these costs.

It was also considered of value to examine whether the cost of equity estimates calculated using the CAPM are likely to be more accurate when employing the zero-beta portfolio returns compared to a T-Bond or T-Bill. The results of this analysis revealed that using T-Bonds as the risk-free asset results in a lower forecasting error than associated with using T-Bills, but that the lowest errors occur when the minimum-variance zero-beta portfolio returns are employed. However, the difference in accuracy between the CAPM based on the T-Bond yield and the minimum-variance zero-beta portfolio returns were smaller than expected. In light of these results, it appears that the choice of the appropriate parameter to estimate the risk-free rate in South Africa is a trade-off between increased efficiency and the difficulty and costs involved with estimating the zero-beta portfolio returns. What is clear is that if a proxy is to be used, the T-Bond is more favourable than the T-Bill.

6.3 Opportunities for Further Research

This study, in addressing the question of the most appropriate approach to estimate the risk-free rate for applications of the CAPM in South Africa, has contributed to the knowledge and understanding of the estimation of this parameter. However, the research objective posed in Chapter 1 of this study has not been fully addressed due to the inability to disregard other possible explanations of the results obtained, seemingly unrelated to the question of the appropriate choice of estimation procedure for the risk-free rate, because of an absence of research on these topics. Therefore, in order to be able to completely answer the question of the appropriate technique to estimate the risk-free rate, further research must be conducted regarding the issues which may assist in explaining the relationships identified and the consequences thereof for estimating the risk-free rate. In addition to this, some suggestions for further research with respect to the sub-objectives examined are also made.

6.3.1 The Reliability of the CAPM

The first possible explanation proposed for the relationship identified between the risk-free rate proxy returns and the estimated intercept of the CAPM is that the model is incorrect (Fama and French, 2004: 38). This could imply that more complex versions of the CAPM are more appropriate or that another model altogether is suitable, such as the APT. There is some evidence to support the use of a multi-factor model such as the APT in South Africa (Van Rensburg and Slaney, 1997; Van Rensburg, 2001), but the fact that the tests of the CAPM do not completely invalidate the relationship between risk, as measured by beta, and return does not necessarily indicate than another model is more appropriate, but perhaps that the form of the CAPM applied is incorrect. That is, the consumption-based CAPM, conditional CAPM, dual-beta CAPM or the Cloninger *et al* (2004)-form of the model may be better suited to the South African market, but no research has been conducted on the validity of these more advanced versions of the model and whether the empirical irregularity that the risk-free rate proxies understate the true minimum required return from investing is still apparent when estimated.

6.3.2 Tests of the CAPM

The possibility also remains that the observed relationship between the risk-free rate proxy returns and the CAPM intercept may be a consequence of inappropriate tests of the model. As discussed in Section 4.3.4.1, one of the necessary assumptions to be able to employ either OLS or ML is that the dependent and independent variables are normally distributed such that the residuals of the regression also satisfy this distributional property. As was clearly illustrated in both the international and South African literature, this assumption is not valid; however, as was shown in this study, this problem is not easily resolved with either OLS or ML estimation. Generalised Method of Moments (GMM) is an alternative estimation procedure, which does not require the specification of any distributional properties, and therefore can validly be employed to estimate the CAPM (Hall, 2005: 1-2). A review of the more recent research in the finance literature revealed that this approach has become popular for precisely this reason (Jagannathan, Skoulakis and Wang; 2002); however, there does not appear to be any work in this area with respect to the traditional form of the CAPM in the U.S. or with regards to any form of the CAPM in South Africa. Thus, using this technique to estimate the model is certainly likely to improve the reliability of the estimates of the intercept and contribute further to the knowledge of not only the appropriate proxy for the risk-free rate and other model parameters, but also the suitability of the CAPM in South Africa.

Another possible explanation to account for the intercept value exceeding the risk-free proxy returns is that the tests of the model are incorrect as they rely on historical data but should be based on expectations. However, in the U.S., tests using expectations have identified the identical relationship. Whether the same is true for South Africa has yet to be proven. Thus, it is imperative to conduct tests on the CAPM using forecasted values for the market portfolio returns, share returns and risk-free rate proxies to be able to ascertain whether the relationship observed between the risk-free proxies and the intercept in this study is a function of inappropriate tests of the model, or that the zero-beta portfolio approach to estimating the risk-free rate is more suitable.

6.3.3 The Specification of the Market Portfolio

One of the most widely postulated reasons for the results obtained in the U.S. regarding the relationship between the intercept and the risk-free rate is not that the risk-free rate is incorrectly specified, but that the market portfolio is incorrect. In the South African context, the correct specification of the market portfolio comprises two separate issues. Firstly, the question arises as to whether the ALSI (used in this study) accurately represents the market or whether, because it is too heavily weighted towards resource and mining shares, the beta estimates for financial and industrial shares are inaccurate. The research conducted on this topic in South Africa (such as Venter *et al*, 1992; Ward, 1994; Correia and Uliana, 2004) is mixed with respect to the choice of using the ALSI or appropriate major sector indices. In light of these results it is suggested that the analysis conducted in this study be repeated but employing major sector indices for the market portfolio rather than the ALSI in estimating the portfolio betas and the results compared. The second issue is whether any of these proxies in South Africa actually satisfy the mean-variance condition, as this has not been considered in the research discussed. More specifically, the newer procedures for examining this issue should be employed (as per MacKinlay and Richardson; 1991, Faff and Lau, 1997; Britten-Jones, 1999; and Levy and Roll, 2009), where these techniques also entail the use of GMM. In this way it will be possible to assess whether the incorrect specification of the market portfolio accounts for the disparity between the risk-free rate proxy returns and the estimated intercept of the CAPM.

6.3.4 The Specification of the Risk-Free Rate

Whilst the issues discussed above may explain the results observed regarding the relationship between the risk-free rate proxy returns and the estimated intercept of the CAPM, the possibility remains that this relationship is simply a function of the incorrect specification of the risk-free rate proxy. That is, these returns may be less than the actual intercept value because they do not take into account the additional costs for those investors who are net borrowers; that is, the proxy is only a proxy for the riskless lending rate. This argument is based on the ideas proposed by Brennan (1971) that the intercept of the CAPM represents the weighted average of the riskless borrowing and lending rates, weighted in accordance with the number of investors who are net borrowers and lenders in the economy. From a theoretical perspective, this argument is intuitively appealing, but based on a review of the financial literature; it does not appear to have been examined empirically to ascertain its validity. Therefore it would certainly be of value to try and test this model of divergent borrowing and lending rates so as to determine whether this is a feasible explanation for the results obtained and if so, whether this is unique to South Africa or whether it explains the identical relationships observed in the U.S., Australia and the international marketplace.

6.3.5 The Estimation of the Minimum-Variance Zero-Beta Portfolio Returns

If the minimum-variance zero-beta portfolio returns are to be used in practice as the means to estimate the risk-free rate, there are several issues which remain to be resolved following this study. As highlighted, one of the limitations of this research was that it was not possible to ascertain the optimal asset allocation method and assets to include in estimating the minimum-variance zero-beta portfolio returns, because of the different number of portfolios in each sample. Consequently, the most efficient estimates were those from the sample with the most portfolios because of the effects of diversification. Therefore to fully address this issue, it is necessary to create the same number of portfolios for all four samples to accurately determine whether the results in this regard will replicate the findings for the estimation of the intercept.

As a corollary to this, it is also essential to determine the optimal number of portfolios from which to calculate the zero-beta portfolio returns. As discussed in Section 5.6, research conducted on the benefits of diversification in the U.S. indicates that including approximately twenty shares is sufficient to achieve close to the maximum benefits from diversification (Evans and Archer, 1968; Tole, 1982; Statman, 1987); whilst in South Africa it appears necessary to include up to thirty shares (Neu-Ner and Firer, 1997). These studies thus provide a starting point from which to work from in this regard but the possibility also remains that shares rather than portfolios should be employed for this purpose.

As mentioned in Section 5.6, the variation in the standard error estimates of the zero-beta portfolio returns over time could not be adequately explained. Therefore it will certainly be of value to determine whether the zero-beta portfolio returns based on only one year of data are consistently more efficient that those estimates based on five or eight years of data.

6.4 Conclusion

The primary objective of this study was to determine the best method to estimate the risk-free rate in South Africa by considering both the use of proxy securities and the zero-beta portfolio returns. The analysis revealed that the proxies used digress substantially from the theoretical requirements that an asset should satisfy to be considered a suitable proxy; thereby suggesting that the returns on these instruments are likely to overstate the true risk-free rate. In complete contrast to this, the results of the tests of equality of these proxies with the intercept of the CAPM revealed that the returns on these instruments were statistically significantly smaller than the minimum required rate of return. The zero-beta portfolio returns however, closely approximated the estimated intercept value. Several possible explanations to account for the anomaly were discussed but due to a lack of research on these issues in South Africa, it was impossible to ascertain whether the results obtained truly indicate that the zero-beta portfolio approach is a more suitable means to estimate the risk-free rate, or whether the model tested was incorrectly specified, the tests were inappropriate, or the market proxy was erroneous.

BIBLIOGRAPHY

Adler, M., and Dumas, B. (1984). Exposure to currency risk: Definition and measurement. *Financial Management*, 13(2): 41-50.

Ahmed, F., and Ricci, L.A. (2004). Exploring determinants of long-term interest rates. Unpublished Working Paper, International Monetary Fund: Washington D.C.

Affleck-Graves, J.F., Burt, G.H., and Cleasby, S.J.M. (1988). The premium on acquisition in South African mergers. *South African Journal of Business Management*, 19(4): 155-160.

Akinjolirie, A., and Smit, E. (2003). South African unit trust performance and strategy in a changing climate (1989 – 2002). *Investment Analysts Journal*, 58: 41-50.

Alexander, G.J. (1977). An algorithmic approach to deriving the minimum-variance zero-beta portfolio. *Journal of Financial Economics*, 4(2): 231-236.

Alexander, G.J., and Chervany, N.L. (1980). On the estimation and stability of beta. *Journal of Financial and Qualitative Analysis*, 15(1): 123-138.

Amihud, Y., and Mendelson, H. (1991). Liquidity, maturity and the yields on U.S. Treasury securities. *Journal of Finance*, 46(4): 1411-1425.

Baesel, J. (1974). On the assessment of risk: Some further considerations. *Journal of Finance*, 29(5): 1491-1494.

Banz, R.W. (1981). The relationship between return and the market value of common stocks. *Journal of Financial Economics*, 9(1): 3-18.

Basu, S. (1977). Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. *Journal of Finance*, 32(3): 663-682.

Basu, S. (1983). The relationship between earnings yield, market value and return for NYSE common stocks: Further evidence. *Journal of Financial Economics*, 12: 129-156.

Bey, R. (1983). Market model stationarity of individual public utilities. *Journal of Finance and Qualitative Research*, 18(1): 67-85.

Bhana, N. (2007). The market reaction to open market share repurchases announcements: The South African experience. *Investment Analysts Journal*, 65: 25-36.

Bhana, N. (2008). The market reaction to capital expenditure announcements. *Investment Analysts Journal*, 68: 57-68.

Bhandari, L. (1988). Debt/equity ratio and expected common stock returns: Empirical evidence. *Journal of Finance*, 43(2): 507-528.

Biger, N. (1975). The assessment of inflation and portfolio selection. *Journal of Finance*, 30(2): 451-467.

Black, F. (1972). Capital market equilibrium with restricted borrowing. *Journal of Business*, 45(3): 444-455.

Black, F., Jensen, M., and Scholes, M. (1972). The capital asset pricing model: Some empirical tests. Social Science Research Network, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=908569 [Accessed 28th February 2008].

Blake, D. (2000). *Financial Market Analysis*, 2nd Edition. New Jersey: John Wiley & Sons.

Blattberg, R., and Gonedes, N. (1974). A comparison of the stable and student distributions as statistical models for stock prices. *Journal of Business*, 47: 244-280.

Blume, M. (1970). Portfolio theory: A step towards its practical application. *Journal of Business*, 43(2): 152-173.

Blume, M. (1971). On the assessment of risk. *Journal of Finance*, 26(1): 1-10.

Blume, M. (1975). Betas and their regression tendencies. *Journal of Finance*, 30(3): 785-796.

Blume, M., and Friend, I. (1973). A new look at the capital asset pricing model. *Journal of Finance*, 28(1): 19-33.

Bodie, Z., Kane, A., and Marcus, A. (2002). *Investments*. New York: McGraw-Hill.

Bodie, Z., Kane, A., and Marcus, A. (2003). *Essentials of Investments*, International Edition. New York: McGraw-Hill.

Bollerslev, T., Engel, R., and Nelson, D. (1994). ARCH models. In Engel, R. and McFadden, D. (Eds.). *Handbook of Econometrics Vol. IV*. Amsterdam: North Holland: 2959-3038.

Bond Exchange of South Africa (BESA) (2009a). BESA Quarterly Update Q1/2009. http://www.bondexchange.co.za/besa/action/media/downloadFile?media_fileid=11254 [Accessed 25th October 2009].

Bond Exchange of South Africa (BESA) (2009b). Listed bonds: All bonds. http://www.bondexchange.co.za/besa/view/besa/en/page30630 [Accessed 10th January 2009].

Botha, Z. (2006). The money market. In: Van Zyl, C.; Botha, Z. and Skerrit, P. (Eds.) (2006). *Understanding South African Financial Markets*, 2nd Edition. Pretoria: Van Schaik: 229-283.

Bowie, D., and Bradfield, D. (1993). A review of systematic risk estimation on the JSE. *South African Journal of Accounting Research (De Ratione)*, 7(1): 6-22.

Bowie, D., and Bradfield, D. (1997). Some evidence on the stability of beta coefficients on the JSE. *South African Journal of Accounting Research*, 11 (2): 1-20.

Bradfield, D. (1990). A note on the estimation problems caused by thin trading on the Johannesburg Stock Exchange. *South African Journal of Accounting Research (De Ratione),* 3: 22-25.

Bradfield, D. (2002). Investment Basics XLVI: On estimating the beta coefficient. *Investment Analysts Journal*, 57: 47-53.

Bradfield, D., and Barr, G. (1989). Risk-estimation in the thinly traded JSE environment. *South African Journal of Business Management*, 20(3): 169-173

Bradfield, D., Barr, G., and Affleck-Graves, J. (1988). Asset pricing in small markets – the South African case. *South African Journal of Business Management,* 19(1): 11-21.

Breeden, D., Gibbons, M., and Litzenberger, R. (1989). Empirical test of the consumption-oriented CAPM. *Journal of Finance*, 44(2): 231-262.

Breen, W., and Lerner, E. (1972). On the use of beta in regulatory proceedings. *Bell Journal of Economics and Management Science,* 3(2): 612-621.

Brennan, M. (1971). Capital market equilibrium with divergent borrowing and lending rates. *Journal of Financial and Quantitative Analysis*, 6(5): 1197-1205.

Brigham, E. (1985). *Financial Management: Theory and Practice*, 4th Edition. New York: Dryden Press.

Brigham, E. and Ehrhardt, M. (2005). *Financial Management: Theory and Practice,* 12th Edition. Ohio: Thomson South-Western.

Brigham, E., and Shome, D. (1981). Risk premium approach to estimating the cost of common equity capital, in Proceedings of the Iowa State University Regulatory Conference, Ames, Iowa.

Britten-Jones, M. (1999). The sampling error in estimates of mean-variance efficient portfolio weights. *Journal of Finance,* 54(2): 655-671.

Brooks, C. (2006). *Introductory Econometrics for Finance*. United Kingdom: Cambridge University Press.

Bruner, R. (2004). *Applied Mergers and Acquisitions*. New Jersey: John Wiley & Sons.

Bruner, R., Eades, K., Harris, R., and Higgins, R. (1998). Best practices in estimating the cost of capital: Survey and synthesis. *Financial Practice and Education*, 8: 13-28.

Campbell, J. (1995). Some lessons from the yield curve. *Journal of Economic Perspectives*, 9(3): 129-152.

Campbell, J., and Viceira, L. (2001). Who should buy long-term bonds? *American Economic Review*, 91(1): 99-127.

Carleton, W., and Lakonishok, J. (1985). Risk and return on equity: The use and misuse of historical estimates. *Financial Analysts Journal*, 41(1): 38-47.

Carvell, S., and Strebel, P. (1984). A new beta incorporating analysts' forecasts. *Journal of Portfolio Management*, 11(1): 81-85.

Chou, P. (2000). Alternative tests of the zero-beta CAPM. *Journal of Financial Research*, 23(4): 469-493.

Chou, P., and Lin, M. (2002). Tests of the international asset pricing model with and without a riskless asset. *Applied Financial Economics*, 12(12): 873-883.

Cloete, G., de Jonah, P., and de Wet, T. (2002). Combining Vasicek and robust estimators for forecasting systematic risk. *Investment Analysts Journal*, 55(3): 37-44.

Cloninger, D., Waller, E., Bendeck, Y., and Revere, L. (2004). Returns on negative beta securities: Implications for the empirical SML. *Applied Financial Economics*, 14(6): 397-402.

Conover, W., Johnson, M., and Johnson, M. (1981). A comparative study of tests for homogeneity of variance with applications to the outer continental shelf bidding data. *Technometrics*, 23: 351-361.

Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, 1: 223-236.

Copeland, T., Koller, T., and Murrin, J. (2000). *Valuation – Measuring and Managing the Value of Companies*, 3rd Edition. New York: John Wiley & Sons.

Correia, C., and Uliana, E. (2004). Market segmentation and the cost of equity of companies listed on the Johannesburg Stock Exchange. *South African Journal of Accounting Research*, 18(1): 65-81.

Correia, C. and Cramer, P. (2008). An analysis of cost of capital, capital structure and capital budgeting practices: A survey of South African listed companies. *Meditari: Research Journal of the School of Accounting Science*, 16(2): 31-52.

Damodaran, A. (2001). Estimating risk-free rates. Stern School of Business, http://www.bonato.com.ar/Archivos/riskfree.pdf [Accessed 4th May 2008].

De Villiers, P., Lowlings, A., Pettit, T., and Affleck-Graves, J. (1986). An investigation into the small firm effect on the JSE. *South African Journal of Business Management*, 17(4): 191-195.

De Wet, J. (2005). EVA versus traditional accounting measures of performance as drivers of shareholders value – a comparative analysis. *Meditari: Research Journal of the School of Accounting Sciences*, 13(2): 1-16.

De Wet, J. (2006). Determining the optimal capital structure: A practical contemporary approach. *Meditari: Research Journal of the School of Accounting Sciences*, 14(2): 1-16.

De Wet, J., and du Toit, E. (2007). Return on equity: A popular, but flawed measure of corporate financial performance. *South African Journal of Business Management*, 38(1): 59-69.

De Wet, J., and Hall, J. (2006). An analysis of strategic performance measures of companies listed on the JSE securities exchange. *South African Journal of Economic and Management Sciences*, 9(1): 57-71.

Dimson, E. (1979). Risk measures when shares are subject to infrequent trading. *Journal of Financial Economics*, 7: 197-226.

Domowitz, I., Glen, J., and Madhavan, A. (1998). Country and currency risk premia in an emerging market. *Journal of Financial and Quantitative Analysis*, 33(2): 189-216.

Douglas, G. (1969). Risk in the equity markets: An empirical appraisal of market efficiency. *Yale Economic Essays*, 9: 3-45.

Elton, E. and ,Gruber, M. (1995). *Modern Portfolio Theory and Investment Analysis*, 5th Edition. New Jersey: John Wiley & Sons.

Elton, E., Gruber, M., Brown, S., and Goetzmann, W. (2007). *Modern Portfolio Theory and Investment Analysis*, 7th Edition. New Jersey: John Wiley & Sons.

Evans, J., and Archer, S. (1968). Diversification and the reduction of dispersion. *Journal of Finance*, 23(5): 61-67.

Eubank, A., and Zumwalt, J. (1981). Impact of alternative length estimation and prediction periods on the stability of security and portfolio betas. *Journal of Business Research*, 9(4): 321-325.

Faff, R. (2001). A multivariate test of a dual-beta CAPM: Australian evidence. *Financial Review*, 36(2): 157-174.

Faff, R., and Lau, S. (1997). A generalised method of moments test of mean variance efficiency in the Australian stock market. *Pacific Accounting Review*, 9(1): 2-16.

Fama, E. (1975). Short-term interest rates as predictors of inflation. *American Economic Review*, 65(3): 269-282.

Fama, E. (1976). *Foundations of Finance: Portfolio Decisions and Securities Prices*. New York: Basic Books.

Fama, E., and French, K. (1992). The cross section of expected stock returns. *Journal of Finance*, 52(1): 427-465.

Fama, E., and French, K. (2004). The capital asset pricing model: Theory and evidence, *Journal of Economic Perspectives*, 18(3): 25-46.

Fama, E., and MacBeth, J.D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 81(3): 607-636.

Firer, C. (1993). Estimating the return parameters of the capital asset pricing model. *South African Journal of Accounting Research (De Ratione)*, 7(1): 23-39.

Firer, C., and McLeod, H. (1999). Equities, bonds, cash and inflation: Historical performance in South Africa 1925 to 1998. *Investment Analysts Journal*, 50: 7-28.

Firer, C., Ross, S., Westerfield, R., and Jordan, B. (2008). *Fundamentals of Corporate Finance*, 4th South African Edition. Berkshire: McGraw-Hill.

Fleming, M. (2003). Measuring treasury market liquidity. *Economic Policy Review*, 9(3): 1-26.

Fraser, P., Hamelink, F., Hoesli, M., and MacGregor, B. (2000). Time-varying betas and cross-section return-risk relation: Evidence from the UK. *Working Paper Series*, Social Science Research Network, http://ssrn.com/abstract=216929. [Accessed 20th February 2009].

Fraser, E., and Page, M. (2000). Value and momentum strategies: Evidence from the Johannesburg Stock Exchange. *Investment Analysts Journal*, 51(2): 1-9.

Freund, J., and Perles, B. (2003). *Statistics: A First Course*, 8th Edition. United States of America: Prentice-Hall.

Friend, I., and Blume, M. (1970). Measurement of portfolio performance under uncertainty. *American Economic Review*, 60(4): 561-575.

Friend, I., Landskroner, Y. and Losq, E. (1976). The demand for risky assets under uncertain inflation. *Journal of Finance*, 31(5): 1287-1297.

Friis, L., and Smit, E. (2004). Are some fund managers better than others? Manager characteristics and fund performance. *South African Journal of Business Management*, 35(3): 31-40.

Gibbons, M. (1982). Multivariate tests of financial models: A new approach. *Journal of Financial Economics*, 10: 3-27.

Gibbons, M., Ross, S., and Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica*, 57(5): 1121-1152.

Gonedes, N. (1973). Evidence on the information content of accounting numbers: Accounting-based and market-based estimates of systematic risk. *Journal of Financial and Quantitative Analysis*, 8(3): 407-443.

Graham, J., and Harvey, C. (2001). The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics*, 60: 187-243.

Graham, M,. and Uliana, E. (2001). Evidence of a value-growth phenomenon on the Johannesburg Stock Exchange. *Investment Analysts Journal*, 53(1): 7-18.

Gujarati, D. (2003). *Basic Econometrics*, 4th Edition. New York: McGraw-Hill.

Hahn, T. (1993). Treasury Bills. In: Cook, T., and Laroche, R. (Eds.). *Instruments of the Money Market*, 7th Edition. Richmond: Federal Reserve Bank of Richmond: 75-88.

Hall, A. (2005). *Generalized Method of Moments: Advanced texts in Econometrics*. Oxford University Press: New York.

Harrington, D. (1987). *Modern Portfolio Theory, the Capital Asset Pricing Model, and Arbitrage Pricing Theory: A User's Guide*, 2nd Edition. New Jersey: Prentice Hall.

Hawawini, G. (1983). Why beta shifts as the return interval changes. *Financial Analysts Journal*, 39(3): 73-77.

Hulbert, M. (2008). Maybe short-selling isn't so bad after all, *New York Times* 28th September, http://www.nytimes.com/2008/09/28/your-money/28iht-28stra.16527650.html?_r=1 [Accessed 15th May 2009].

Ibbotson, R., and Sinquefield, R. (1979). Stocks, bonds, bills and inflation: Updates. *Financial Analysts Journal*, 35(4): 40-44.

Imeson, M. (2009). Short-selling bans anger fund managers, The Banker 2nd February, http://www.thebanker.com/news/fullstory.php/aid/6445/Short-selling_bans_anger_fund_managers.html [Accessed 15th May 2009].

Jacob, N. (1971). The measurement of systematic risk for securities and portfolios: Some empirical evidence. *Journal of Financial and Quantitative Analysis*, 6(2): 815-834.

Jagannathan, R., Skoulakis, G. and Wang, Z. (2002). Generalized method of moments: Applications in finance. *Journal of Business and Economic Statistics*, 20(4): 470-481.

Jagannathan, R., and Wang, Z. (1996). The conditional CAPM and the cross section of expected returns. *Journal of Finance*, 51(1): 3-53.

Jensen, M. (1968). The performance of mutual funds in the period 1945-1964. *Journal of Finance*, 23 (2): 389-416.

Jensen, M. (1972). Capital markets: Theory and evidence. *Bell Journal of Economics and Management Science*, 3(2): 357-398.

JSE Limited (2008). Annual Report 2008. Johannesburg: JSE.

Johnson, R. (1967). Term structure of corporate bond yields as a function of risk of default. *Journal of Finance*, 22 (2): 313-351.

Kamara, A. (1988). Market trading structures and asset pricing: Evidence from Treasury bill markets. *Review of Financial Studies*, 1(4): 357-375.

Kamara, A. (1994). Liquidity, taxes and short-term treasury yields. *Journal of Financial and Quantitative Analysis*, 29(3): 403-417.

Kandel, S., and Stambaugh, R. (1993). Portfolio inefficiency and the cross-section of expected returns. *Journal of Finance*, 50(1); 157-184.

Keim, D. (1983). Size related anomolies and stock return seasonality. *Journal of Financial Economics*, 12(1): 13-32.

Keller, G. and Warrack, B. (2000). *Statistics for Management and Economics*, 5th Edition. United States of America: Duxbury Thomson Learning.

Kelly, M. (1993). *Financial Institutions in South Africa: Financial Investment and Risk Management.* Pretoria: Juta Academic.

Kothari, S., Shanken, J., and Sloan, R. (1995). Another look at the cross section of expected stock returns. *Journal of Finance*, 50(2): 185-224.

Lakonishok, J. (1993). Is beta dead or alive? In Harrington, D., and Korajczyk, R. (Eds.) *The CAPM Controversy: Policy and Strategy Implications for Investment Management*. New York: AIMR: 38-42.

Levy, M., and Roll, R. (2009). The market portfolio may be mean-variance efficient after all. *Working Paper*, Jerusalem School of Business Hebrew University, http://econ.haifa.ac.il/~zimmerman/Moshe_Levy_PAPER.pdf [Accessed 5th June 2009].

Lintner, J. (1965). The valuation of risky assets and the selection of stock portfolios and capital budgets. *Review of Economics and Statistics*, 47(1): 13-37.

MacKinlay, A., and Richardson, M. (1991). Using generalized method of moments to test mean-variance efficiency. *Journal of Finance*, 46(2): 511-527.

Makridas, S. (1993). Accuracy measures: Theoretical and practical concerns. *International Journal of Forecasting*, 9: 527-529.

Malkiel, B. (1995). Returns from investing in equity mutual funds 1971 to 1991. *Journal of Finance*, 50(2): 549-572.

Mangani, R. (2007). Distributional properties of JSE prices and returns. *Investment Analysts Journal*, 66: 57-72.

Marazzi, A., and Yohai, V. (2000). Robust Box-Cox transformations for simple regression. In: Hubert, M., Pison, G., Struyf, A., and van Aeslt, S. (Eds.). *Theory and Applications of Recent Robust Methods (Statistics for Industry and Technology)*. Germany: Birkhauser, 173-190.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7(1): 77-91.

Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*, 2nd Edition. United States of America: Blackwell Publishing.

Moolman, E., and du Toit, C. (2005). An econometric model of the South African stock market. *South African Journal of Economic and Management Sciences*, 8(1): 77-91.

Morgan, I. (1975). Prediction of return with the minimum-variance zero-beta portfolio. *Journal of Financial Economics*, 2: 361-376.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, 34(4): 768-783.

Msweli-Mbanga, P., and Mkhize, H. (2007). The risk-adjusted performance of companies with female directors: A South African case. *South African Journal of Economic and Management Sciences*, 10(2): 207-213.

Mutooni, R. and Muller, C. (2007). Equity style timing. *Investment Analysts Journal*, 65: 15-24.

Myung, I. (2003). Tutorial on maximum likelihood estimation. *Journal of Mathematical Psychology*, 47: 90-100.

Nelson, S. (2008). Financial crisis leads to restrictions on short sales, *Traders Magazine Online News* 24th July, http://www.tradersmagazine.com/news/101682-1.html?zkPrintable=true [Accessed 15th May 2009].

Neu-Ner, M., and Firer, C. (1997). The benefits of diversification on the JSE. *Investment Analysts Journal*, 46: 1-12.

Nippani, S., Liu, P. and Schulman, C. (2001). Are Treasury securities free of default? *Journal of Financial and Quantitative Analysis*, 36 (2): 251-265.

Oldfield, C., and Page, M. (1997). Assessing portfolio performance: The case of South African unit trusts. *Investment Analysts Journal*, 44(2): 1-13.

Oldham, G. and Kroeger, J. (2005). Performance, persistence and benchmarks of selected South African unit trusts for the period 1998-2002. *South African Journal of Business Management*, 36(4): 81-90.

Pagan, A. (1996). The econometrics of financial markets. *Journal of Empirical Finance*, 3: 15-102.

Page, M. (1993). The arbitrage pricing theory: An assessment of the robustness of empirical techniques employed under conditions of thin-trading and in the presence of non-normalities. Unpublished PhD Thesis, University of Cape Town.

Page, M. and Palmer, F. (1991). The relationship between excess returns, size and earnings to price ratio on the JSE. *South African Journal of Business Management*, 22(3): 63-72.

Peter, M. and Grandes, M. (2005). How important is sovereign risk in determining corporate default premia? The case of South Africa. *Working Paper Issue 2217,* International Monetary Fund.

Pettengill, G., Dundaram, S., and Matthur, I. (1995). The conditional relation between beta and returns. *Journal of Financial and Quantitative Analysis,* 30(3): 101-115.

Pike, R., and Neale, B. (2006). *Corporate Finance and Investment*, 5th Edition. Essex: Prentice Hall.

Pindyck, R., and Rubinfield, D. (1991). *Econometric Models and Economic Forecasts*. New York: McGraw-Hill.

Pocock, A., Correia, C., and Wormald, M. (1991). An analysis of the approaches used by industrial companies listed on the JSE to identify their cost of capital. *South African Journal of Accounting Research (De Ratione),* 5(1): 27-40.

Rees, B. (1995). *Financial Analysis,* 2nd Edition. Hertfordshire: Prentice-Hall.

Reilly, F., and Brown, K. (2006). *Investment Analysis and Portfolio Management*, 8th Edition. Ohio: Thomson South-Western.

Reilly, F., and Akhtar, R. (1995). The benchmark error problem with global capital markets. *Journal of Portfolio Management*, 22(1): 33-52.

Reilly, F., and Wright, D. (2004). Analysis of risk-adjusted performance for global market assets. *Journal of Portfolio Management*, 30(3): 63-77.

Robinson, Z. (2007). The sovereign risk premium in South Africa: An empirical approach [Online],https://editorialexpress.com/cgi-bin/conference/download.cgi?db_name=IIPF63andpaper_id=187 [Accessed 8th April 2009].

Roll, R. (1977). A critique of the asset pricing theory's tests. *Journal of Financial Economics*, 4(4): 129-176.

Roll, R., and Ross, S. (1994). On the cross-sectional relation between expected returns and betas. *Journal of Finance*, 49(1): 101-121.

Rosenberg, B., and Marathe, V. (1975). Tests of the capital asset pricing hypothesis. Working Paper No. 32 of the Research Program in Finance, Berkley: Graduate School of Business and Public Administration, University of California.

Rosenberg, B., Reid, K., and Lanstein, R. (1985). Persuasive evidence of market inefficiency. *Journal of Portfolio Management*, 11(3): 91-97.

Ruppert, D. (2004). *Statistics and Finance: An Introduction*. New York: Springer.

Samoulihan, N. (2007). The price of risk in the South African equity market. *South African Journal of Economics*, 75(3): 442-458.

Samouilhan, N., and Shannon, G. (2008). Forecasting volatility on the JSE. *Investment Analysts Journal*, 67: 19-28.

Scholes, M., and Williams, J. (1977). Estimation betas from non-synchronous data. *Journal of Financial Economics*, 5: 309-327.

Securities Industry and Financial Markets Association (SIFMA). Average daily trading volume in the U.S. bond markets, http://www.sifma.org/research/research.aspx?ID=10806 [Accessed 25th October 2009].

Shanken, J. (1985). Multivariate tests of the zero-beta CAPM. *Journal of Financial Economics*, 14(3): 327-348.

Shanken, J. (1986). Testing portfolio efficiency when the zero-beta rate is unknown: A note. *Journal of Finance,* 41(1): 269-276.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3): 425-442.

Sharpe, W. (1970). *Portfolio Theory and Capital Markets*. New York: McGraw-Hill.

Sharpe, W., and Cooper. G. (1972). Risk-return classes of New York Stock Exchange common stocks, 1931-1967. *Financial Analysts Journal*, 28(2): 46, 48-54, 81.

Shiller. R., and McCulloch, J. (1987). The term structure of interest rates. NBER Working Paper Series No. 2341, National Bureau of Economic Research: Cambridge.

Soe, A., and Dash, S. (2008). Dividend Investing. Standard and Poor's and McGraw-Hill, http://www2.standardandpoors.com/spf/pdf/index/Dividend_Investing.pdf [Accessed 10th January 2009].

South African Reserve Bank (SARB) (2002). The objectives and importance of inflation targeting, 13th November, *Business Day*: Johannesburg.

South African Reserve Bank (SARB) (2003). Bank's Amendment Act of 1999 (Revised 2003), Pretoria: SARB.

South African Reserve Bank (SARB) (2008). Annual Report 2007/2008: March. Pretoria: SARB.

Stambaugh, R. (1982). On the exclusion of assets from tests of the two parameter model. *Journal of Financial Economics*, 10: 237-268.

Statman, M. (1987). How many stocks make a diversified portfolio? *Journal of Financial and Quantitative Analysis*, 22(3): 353-363.

Strydom, B., and Charteris, A. (2009). A theoretical and empirical analysis of the suitability of South African risk-free rate proxies. Conference Paper, Cape Town: African Finance Journal Annual Conference July 2009.

Tole, T. (1982). You can't diversify without diversifying. *Journal of Portfolio Management*, 8 (2): 5-11.

Turnbull, S. (1977). Market imperfections and the capital asset pricing model. *Journal of Business Finance and Accounting*, 4(3): 327-337.

Van Horne, J. (2002). *Financial Management and Policy*, 12th Edition, New Jersey: Prentice Hall.

Van Rensburg, P. (2001). A decomposition of style-based risk on the JSE. *Investment Analysts Journal*, 54(4): 1-12.

Van Rensburg, P., and Slaney, K. (1997). Market segmentation on the Johannesburg Stock Exchange. *Journal of Studies in Economics and Econometrics*, 23(3): 1-23.

Van Rensburg, P., and Robertson, M. (2003). Size, price-to-earnings, and beta on the JSE Securities Exchange. *Investment Analysts Journal*, 58: 7-16.

Van Rhijn, H. (1994). The capital asset pricing model for financial decision-making under South African conditions. DBA Thesis, University of Pretoria.

Van Zyl, C. (2006). The bond market. In: Van Zyl, C.; Botha, Z. and Skerrit, P. (Eds.). *Understanding South African Financial Markets*, 2nd Edition. Pretoria: Van Schaik: 285-328.

Vandell, R., and Stevens, J. (1982). Personal taxes and equity security pricing. *Journal of Financial Management*, 11(1): 31-40.

Vasicek, O. (1971). Capital market pricing model with no riskless borrowing. Unpublished Manuscript, Wells Fargo Bank.

Viviers, S., Bosch, J., Smit, E., and Buijs, A. (2008). The risk-adjusted performance of responsible investment funds in South Africa. *Investment Analysts Journal*, 68: 39-55.

Ward, M. (1994). Risk on the Johannesburg Stock Exchange. *South African Journal of Accounting Research (De Ratione)*, 8(2): 99-114.

Wegner, T. (1993). *Applied Business Statistics: Methods and Applications*. Cape Town: Juta & Co Ltd.

West, K., Cho, D., and Edison, H. (1993). A utility based comparison of some methods of exchange rate volatility. *Journal of International Economics*, 35: 23-45.

White, H. (1980). A heteroscedasticity consistent covariance matrix estimator and a direct test of heteroscedasticity. *Econometrica*, 48: 817-818.

Wooldridge, J. (2003). *Introductory Econometrics: A Modern Approach*, 2nd Edition, Ohio: Thomson South-Western.

Zivney, T. and Marcus, R. (1989). The day the United States defaulted on Treasury bills. *Financial Review*, 24(3): 475-489.