AN EXPLORATION OF PRESERVICE TEACHERS' MATHEMATICS KNOWLEDGE FOR TEACHING IN TRIGONOMETRY AT A HIGHER EDUCATION INSTITUTION

BY

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DECLARATION

I, Benjamin Tatira hereby do declare that this dissertation is an original work. It does not contain copied data, tables, graphs or writings of other authors. Sourced information from other persons were referenced accordingly.

16 March 2020

Candidate: Benjamin Tatira   Date

This dissertation has been duly submitted with my approval.

Supervisor: Prof V Mudaly   Date

DEDICATION

Dedication for this dissertation goes to the Almighty God who made it possible for me to accomplish this noble feat. The words of this song always ring in my mind:

Got any rivers you think are uncrossable?

Got any mountain you can’t tunnel through?

God specialises in things thought impossible,

And He will do what no other person can do.

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ABSTRACT

This study investigates the extent of final-year preservice teachers' understanding and development of the mathematics knowledge for teaching in trigonometry. Teachers' lack of adequate mathematics knowledge to teach mathematics effectively is one of the major source of low mathematics attainment in South Africa. On this basis, the readiness of prospective teachers to teach mathematics must be established at the point of exit. The purpose of the present research study is to explore preservice teachers' understanding and development of content knowledge and pedagogical content knowledge in teaching trigonometry. The review of literature revealed that many preservice teachers lack the conceptual understanding of school mathematics. Thus, preservice teachers exit teacher education and enter the world of teaching with limited skills and abilities of teaching mathematics. The content test, task-based interview, lesson planning and lesson observations were used to gather data on preservice teachers' understanding of content knowledge in trigonometry in response to three research questions. The sample of the study was composed of fifteen mathematics final-year preservice teachers who were registered for a Bachelor of Education degree programme at a rural-based institution of higher learning in South Africa. The sample was selected purposively. The mathematics knowledge for teaching conceptual framework by Ball, Thames and Phelps was used to structure the present study and provided lens for data analyses. The analysis of the content test results revealed that preservice teachers' mastery of content knowledge in trigonometry was inadequate. The results from the task-based interview, lesson plan and lesson observation analyses indicated that the preservice teachers' mastery of pedagogical content knowledge in trigonometry was limited. Moreover, the extent of preservice teachers' development of mathematical knowledge for teaching based on results from classroom practices was sub-standard. The traditional teaching methods and learner-misconceptions never left preservice teachers all through the four years of teacher education. Therefore, more needs to be done by the higher education institution to accelerate growth of content knowledge and pedagogical content knowledge through the provisions of methodology, content and teaching practices courses. The interplay of the three, methodology courses, content courses and teaching practice form the basis of an ideal preservice teacher.

I-ABSTRACT

Lolu cwaningo luphenya ubungako bokuqonda kothisha wokugcina umsebenzi wokugcina kanye nokuthuthukisa ulwazi lwezibalo lokufundisa nge-trigonometry. Ukuntuleka kothisha kolwazi lwezibalo okwanele ukufundisa izibalo ngempumelelo kungenye yomthombo omkhulu wokutholwa kwezibalo okuphansi eNingizimu Afrika. Ngalesi sisekelo, ukulungela kothisha abazoba abafundisi bezibalo kumele kusungulwe lapho bezophuma khona. Inhloso yocwaningo olukhona manje ukuhlola ukuqonda kwabafundisi abasebenza ngokuthile kanye nokuthuthuka kolwazi lokuqukethwe kanye nolwazi lokuqukethwe okuzenzakalelayo ekufundiseni i-trigonometry. Ukubuyekezwa kwezincwadi kuveze ukuthi othisha abaningi abasebenza ngezinsizakusebenza abanakho ukuqonda okuqondakalayo kwemathematics esikole. Ngakho-ke, othisha abazisebenzelayo baphuma emfundweni yothisha futhi bangene ezweni lokufundisa ngamakhono alinganiselwe namakhono wokufundisa wezibalo. Ukuhlolwa kokuqukethwe, inhlolokhono esekwe emisebenzini, ukuhlelwa kwezifundo nokubukwa kwezifundo kusetshenziselwe ukuqoqa imininingwane ekuqondeni kothisha okuhlinzeka ngemininingwane yokuqukethwe kwe-trigonometry ukuphendula imibuzo emithathu yocwaningo. Isampula yalolu cwaningo belakhiwa othisha abasebenza iminyaka eyishumi nantathu ababhaliselwe uhlelo lweBachelor of Education esikhungweni esisebenza emaphandleni semfundo ephakeme eNingizimu Afrika. Isampula lakhethwa ngamabomu. Ulwazi lwezibalo lokufundisa uhlaka lwangempela lweBall, iThames nePhelps lwalusetshenziselwa ukwakha lolu cwaningo lwamanje futhi lwahlinzeka ngelensi yokuhlaziya idatha. Ukuhlaziywa kwemiphumela yokuhlolwa kokuqukethwe kuveze ukuthi ukuphathwa kahle kothisha okuhlinzeka ngemininingwane yokuqukethwe kwe-trigonometry bekunganele. Imiphumela evela kwinhlolokhono esekwe emisebenzini, uhlelo lwezifundo kanye nokuhlaziywa kokubuka izifundo, iveze ukuthi ukuphatha othisha abalungiselela kakhulu ulwazi lokuqukethwe okuphathelene ne-trigonometry kukhawulelwe. Ngaphezu kwalokho, ubukhulu bokuthuthukiswa kothisha abalungiselela ulwazi lwezibalo ngokufundisa okusekelwe kwimiphumelo evela ekilasini okwakwenziwa kungaphansi. Izindlela zokufundisa zendabuko kanye nemibono eyiphutha yabafundi ayikaze ibashiye othisha abafundile kuyo yonke le minyaka emine yokufundisa kothisha. Ngakho-ke, kuningi okudingeka kwenziwe yisikhungo semfundo ephakeme ukusheshisa

ukukhula kolwazi lokuqukethwe kanye nolwazi lokuqukethwe okufundwayo ngezinhlinzeko zendlela yokufundisa, okuqukethwe nokufundisa ngezifundo. Ukudidiyelwa kwezifundo ezintathu, izindlela zokufundisa, izifundo zokuqukethwe kanye nokuzijwayeza ukufundisa kuyisisekelo somfundisi ofanelekile wokuphakelwa.

Key words: mathematics knowledge for teaching, pedagogical content knowledge, initial teacher education, trigonometry, lesson plan analysis, video-teaching episodes, task-based interview, content test.

DEFINITION OF TERMS

Preservice teacher: denotes a student studying towards a teaching qualification at a teacher-training institution that makes him/her eligible to join the teaching profession. It is synonymous with prospective teacher.

Higher education institution: is a place where people of all ages acquire post-school education in a designated field of study, which is usually a university or college in most countries.

Pedagogical content knowledge: it represents a special integration of skills of teaching and subject-matter expertise that teachers need to teach different topics in a curriculum.

Content knowledge: refers to the concepts and facts that learnt and taught in a given field of study.

Mathematics knowledge for teaching: represents a combination of content and pedagogical knowledge skills which mathematics teacher need to teach mathematics concepts in ways best understood by learners.

Teacher education: is the formal instruction of procedures and procedures designed to equip prospective teachers with necessary skills required to become qualified teachers. It is synonymous with teacher training.

Initial teacher education: it is a post-school training offered to candidates who are working towards a first qualification in teaching.

Task-based interview: is defined as a scenario where the interviewee talks during or immediately after answering a question on paper, whereupon the interviewer can probe to seek further clarifications if necessary.

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CHAPTER 1: INTRODUCTION

1.1. Background of the study

Mathematics is a high-status subject in the South African curriculum which is done by all learners except at the Further Education and Training phase where they have an option between mathematics and mathematical literacy. Mathematics is a gateway subject for professions in health sciences, science, engineering and technology. To be relevant in the modern technologically- and scientifically-oriented world, children need to be taught and learn mathematics concepts competently. The success of learning mathematics is reflected in the pass-rate of learners who are doing the subject. As in any other educational undertaking, many players are involved in achieving that success. The following will be considered in this background section; the crucial role played teachers and teacher knowledge in classroom practice, the performance of South African learners in mathematics and the teaching and learning of trigonometry. The state of preservice teachers' current mathematical knowledge for teaching and the description of the initial teacher education brings the background of the study to an end.

1.1.1. Key role of teachers in the classroom

As classroom practitioners, the role of teachers in the classroom is irreplaceable. Teachers have the noble role to mould and organise the classroom learning context, which places them at the centre stage of what must be taught and learnt in the classroom (Cross, 2009). Teachers are the catalysts to teaching and learning upon whom the education system and reform depends. The ability of a teacher to breakdown content knowledge in meaningful chunks and engage it in the classroom in such a way to enhance learner understanding of mathematics has been a central theme in mathematics education (Kulm, 2008).

It was noted that teachers play an important role of helping learners to overcome problems in learning mathematics, and this forms the starting point for current reform efforts in the South African education system (Dooren, Verschaffel & Onghena, 2002; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008). However, with the full knowledge of supposed key teacher-role, the teaching and learning of subjects such as science and mathematics is still of poor quality and below par (Sayed, 2002). The learning of mathematics in particular is in such a critical state (Jansen, 2011) that memorisation of facts and other procedural teaching strategies dominate the classroom teachings (Jansen & Christie, 1999). Sometimes mathematics teachers lack confidence in some aspects of subject-matter, which lead them to skip certain topics completely which they feel are too challenging.

1.1.2. Teacher knowledge

The art of teaching is an intricate activity that is built on the diverse kinds of knowledge, which are, knowledge of school contexts, pedagogical content knowledge, curriculum knowledge, content knowledge and knowledge of learners' misconceptions. The uncertainty in the right kind of knowledge needed for teaching has given rise to much research on teacher knowledge since the 1800s. In the 1870s, the knowledge of pedagogy was essentially ignored to the extent that teachers were assessed for their teaching competence based only on content knowledge. There was a turnaround in the 1980s wherein concerns of teacher competency were judged through pedagogical knowledge assessment only. Shulman, together with his colleagues started the "Knowledge Growth in Teaching" project 1986, in which they queried the basis of teacher knowledge by posing questions such as, "Where do teachers explanations came from? How do teachers decide what to teach? What are the sources of knowledge?" (Shulman, 1986, p.8). The result of their project was to open a debate as to what constitutes the vital elements of teachers' knowledge and how teachers' knowledge is organised (Gess-Newsome, 1999a; Fennema & Franke, 1992).

Up to now there is no theory on teacher knowledge, but a plethora of models of teacher knowledge were born. Teacher knowledge is a broad system which fundamentally renders its components difficult to study in isolation (Fennema & Franke, 1992). After considerations about the diverse kinds of knowledge that inhabit teachers' minds, Shulman (1986) came up with three key categories of teachers' knowledge, which are, content, curricular and pedagogical content knowledge. Content knowledge was the basic knowledge of facts of a content area that teachers are required to teach, as well as the knowledge of the underlying structures of those facts. Curricular knowledge was defined as the various specific teaching and learning resources made available by the teacher to teach the given content. Curriculum knowledge also encompasses the knowledge of where the current content fits into the broad scope of an educational program.

What is striking in Shulman's (1986) work was the novel knowledge type which he termed the pedagogical content knowledge, which is the overlap of the pedagogical and content knowledge domains. Shulman (1986) perceived pedagogical content knowledge as the "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (Shulman, 1987, p.8). Pedagogical content knowledge then became the start of a new way of thinking about knowledge for teaching which took the scope of teacher knowledge a step beyond both content and pedagogical knowledge that constitute it (Chick & Harris, 2007). Ball, Phelps and Thames (2008) refined Shulman's teacher knowledge model and came up with what is termed the mathematics knowledge for teaching framework. Of all the diverse teacher knowledge types, two of these, the pedagogical content knowledge and content knowledge have direct bearing on learner achievement. These two constitute what is termed the mathematics knowledge for teaching in this study. It is expected that the pedagogical content knowledge and content knowledge of qualified teachers be adequate for effective teaching to take place.

Ever since the times of Shulman (1986) and Ball, Thames and Phelps (2008), teacher knowledge research has undergone enormous growth. The strategies and methods for studying the teachers' mathematics knowledge for teaching have broadened and developed, to include lesson plan studies, lesson observations, content tests, interviews and video-teaching analysis (Borko & Livingston, 1989; Kieran, 2007; Wilson, Floden & Ferrini-Mundy, 2002). This study used a content test to assess preservice teachers' content knowledge. Pedagogical content knowledge was assessed through task-based interviews, lesson plan analyses and video-teaching analyses. From this section, the researcher concludes that Shulman (1986) and Ball (1991) broadly categorised teachers' knowledge into pedagogical content knowledge, content knowledge and curriculum knowledge. These three teacher knowledge types underscore all efforts to create a fruitful classroom instruction and learning situation. The present study explored pedagogical content knowledge and content knowledge with a specific focus on preservice teachers, as the chief determinants of effective mathematics teaching. Curriculum knowledge was dropped in this study since in the South African context, detailed work schedules are unilaterally supplied to all schools and all teachers which embodies all the curricular matters for each subject in all the grades.

It is true some aspects of teacher knowledge are assumed to be developed as teachers gain more experience (Borko & Putnam, 1996). However, the undergraduate modules and teaching practice that preservice teachers undertake at university should equip them with rudiments of mathematics knowledge for teaching to be ready for their first year of teaching (Kilić, 2007). In other words, the initial teacher education is the formal and rational source of teacher knowledge. It can be seen that teachers need to acquire deep understanding of mathematics knowledge for teaching during their training because teacher knowledge is key to learner-achievement. Currently, products of the South African teacher education institutions in mathematics and science do not meet the standards of the Department of Basic Education. Newly qualified teachers are obliged to have adequate content knowledge and the necessary instructional skills to teach that content.

1.1.3. Performance of learners in mathematics

There is growing evidence that the low learner-performance in mathematics in both primary and secondary schools is attributed to limited mathematics knowledge for teaching amongst South African teachers (Howie, 2002). In an investigation of the persistent low-level of mathematics performance among South African primary school learners, it was discovered that good quality teaching is favourable to good learner-achievement (Carnoy, Chisholm & Chilisa, 2008). Also, poor teaching is the main cause of low learner-achievement in mathematics and science in South Africa and so many current teachers are not teaching mathematics well (Centre for Development and Enterprise, 2011). According to Ball, Lubienski and Mewborn (2001), primary and secondary experienced teachers and preservice teachers have general weaknesses in understanding the basic concepts in mathematics and South African mathematics teachers were found to have inadequate pedagogical content knowledge (Brijlall &

Maharaj, 2015; Bansilal, Brijlal & Mkhwanazi, 2014). Given the key role played by teachers in the classroom, the future of mathematics teaching in South Africa is at stake.

Studies to this effect partly ascribe this scenario to inadequate training of teachers, especially in the previous political era of white minority rule (Fiske & Ladd, 2004; Howie & Plomp, 2002). Other factors of learners' poor mathematics attainment have been inappropriate teaching methods, poor infrastructure in some schools and lack of adequately trained mathematics teachers (Spreen & Vally, 2006; Stols, Kriek & Ogbonnaya, 2008). Thus, the mathematics knowledge for teaching is fundamental to their ability to teach effectively (Ball, Hill & Bass, 2005; Kreber, 2002). Hence, learners' poor performance in mathematics is mostly linked to teachers' inadequate knowledge to help the learners learn meaningfully. It is of interest to investigate preservice teachers to determine if teachers' challenges to teaching originate in teacher education (Mudaly, 2016).

During high school mathematics teaching, teachers often do not put emphasis on the conceptual skills. Routine application of algorithmic procedures is favoured, thus, when the prospective teachers enter teacher education, their mathematics background is often not well-formed (Nicol, 2002). Pournara (2005) asserts that the most mathematics preservice teachers might not have been accepted into a science degree programme mainly because their matriculation mathematics results would have been quite low. The implication of this is that many prospective teachers enrol for teacher education studies with meagre conceptual understanding of school mathematics. This, according to the Department of Education (2001), continuously produces teachers who perpetuate the cycle of mediocrity as their future learners would also enter teacher education upon the same basis. As a result of the poor quality of current teachers, the South African education system is under pressure, particularly in terms of science and mathematics attainment in the National Senior Certificate examinations (Centre for Development and Enterprise, 2011). The South African government's expenditure on education at all levels is one of the highest amongst less developed countries. However, the education output in terms of learners' performance results does not match the expenditure committed, as compare to less developed countries. Thus, there is need to conduct studies of this fashion to check if the education woes in South Africa commence in teacher-training, one of the instrumental sources of teacher knowledge development and production.

1.1.4. The state of trigonometry knowledge in South Africa

South Africa needs teachers with strong pedagogical content knowledge and content knowledge in all the mathematics topics, especially in the concept of trigonometry. Trigonometry forms an integral part of Grade 10 to 12 mathematics which make use of algebraic, graphical and geometric reasoning when solving problems involving trigonometric expressions, triangles and real-life application of trigonometry (Nabie, Akayuure, Ibrahim-Bariham & Sofo, 2018). Full understanding of trigonometry at high school forms a foundation for conceptual learning of many other concepts at higher education institutions. Trigonometry is all over in differential and integral calculus, linear algebra, real analysis, geometry and differential equations. Nevertheless, it was observed that find learners grapple with trigonometry, whereby they find trigonometry predominantly abstract, hence challenging relative to other mathematics concepts (Gür, 2009; Weber, 2005).

A study of mathematics topics which South African learners find most challenging to learn revealed that even though many of them find many topics difficult to learn, the largest group (46 percent of the learners in the sample) reported that they find learning the topic of trigonometry quite difficult to grasp (Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya & Makwakwa, 2009). As some of these learners progress to teacher education, their ill-formed ideas of trigonometry cause them to fare less in their understanding of undergraduate and school trigonometry. Hence, literature reports that say preservice teachers lack adequate knowledge of trigonometry (Fi, 2003). Dündar and Yaman (2015) observed that preservice teachers' procedural trigonometry knowledge was high at the expense of conceptual knowledge. Frequently, preservice teachers discuss the concept of trigonometry from its algorithmic standpoint but lack the strong mastery of the meaning of those concepts. Conceptual knowledge is required when transforming a problem situation into mathematical notation.

1.1.5. Preservice teachers’ mastery of mathematics knowledge for teaching

The current state of mathematics teachers is that they lack adequate mathematics knowledge for teaching, as was explained in section 1.1.2. An investigation of preservice teachers of mathematics might inform the policy makers of the future state of mathematics teachers. Once prospective teachers get their mathematics knowledge for teaching in order whilst in teacher education, then they would be able to play their expected classroom roles without difficulty. Preservice teachers must acquire knowledge in school mathematics and advanced undergraduate mathematics in the initial teacher education programmes. Advanced mathematics is good, but it is not directly connected to classroom teaching and learning, hence the mathematical knowledge for teaching focusses exclusively on school mathematics. In South Africa, most prospective teachers possess inadequate school mathematics content knowledge (Biyela, 2012).

Studies have even demonstrated that preservice teachers possess inadequate conceptual understanding of the mathematics content they would be expected to teach upon qualification (Southwell & Penglase, 2005; She, Matteson, Siwatu & Wilhelm, 2014). Prospective teachers enrol in teacher education programmes to gain teaching skills by means of content, pedagogy and methodology modules, and teaching practice. Nevertheless, preservice teachers often leave these programmes with more or less the same knowledge base as when they first entered with regard to school mathematics (Benken & Brown, 2008; Seaman, Szydlik, Szydelik & Beam, 2006). Thus, it implies that not much is done by teacher training institutions to advance preservice teachers’ knowledge of school mathematics. Though it is difficult to measure the extent of change brought about by the exposure of preservice teachers to teacher training programmes, research results reveal that some initial teacher programmes are hardly able to equip preservice teachers with essential mathematics knowledge skills to become fully-fledged teaching professionals (Barnes, 2009).

As such, it is of necessity to conduct research studies to assess the level of preservice teachers’ understanding of mathematics knowledge for teaching as they exit higher education institutions. Instead of taking mathematics knowledge for teaching as supposed, there is need to carefully measure it and map it. Such attempts have been made in this study by means of multiple tools, which are content tests, pre- and post-observation interviews, lesson plan analyses and classroom observation. The shortage of studies that evaluate the preservice teachers' level of mathematics knowledge at the point of exit and the alleged failure of some programmes to equip preservice teachers with adequate mathematics knowledge necessitated the present study. Hence in this study the researcher sought to investigate the preservice teachers' perceived understanding of mathematics knowledge for teaching in response to pleas of repeated production of teachers who seem to be inadequately trained to teach school mathematics.

1.1.6. Initial teacher education in South Africa

The misconstrued interpretation of the findings from the National Teacher Education Audit of 1995 resulted in the improper perception that they were too many teachers in South Africa. This duly led to the closure of all teacher education colleges, but then the initial teacher education deteriorated. There was now a shortage of teachers, and more pronounced in the gateway subjects such as mathematics, science and accounting. Higher education institutions solely took over all initial teacher education concerns, which guaranteed that all new teachers graduate with a Bachelor of Education or a Post-Graduate Certificate qualification upon completion of teacher training which was initiated on a large scale. University graduates seemingly are considered better qualified than the diploma-holder teachers who used to be produced by the old system of colleges of education. Still preservice teachers did not perform well in the mastery of teacher knowledge and other teaching skills. It remains the responsibility of universities to change the mediocre mathematics teaching and learning standards by providing the preservice teachers with strong mathematical content and pedagogical content knowledge (Sam, 2005).

In South Africa there has been some criticism on the role played by the current teacher education programmes in reforming mathematics education (Biyela, 2009). However, the critics are oblivious to the fact that teacher education institutions have challenges too, for example the difficulties they encounter in recruiting mathematically competent learners into teacher education and inclement financial budgets. Operational challenges of hosting practising teachers and the financial constraints limit the teacher education institutions' capacity to train in-service teachers. Teachers' content and pedagogical content knowledge is also acquired through workshops and seminars, and from on-the-job experience. But this route has financial repercussions for the service providers and sometimes it takes time. An imbalance in financial resources at most education departments in the country's universities makes it difficult to prepare confident and competent prospective teachers. Thus most higher education institutions are heavily funded by the state so that they can offer quality initial teacher education to prospective teachers.

Teacher education has the potential to influence teacher knowledge growth in preservice teachers (Ijeh, 2012). Preservice teachers occupy the link between teacher education and teacher knowledge. Teacher education imparts pedagogical content knowledge and content knowledge through undergraduate instruction and school practice teaching as part of the initial teacher education training. This is the feasible way to produce effective future teachers. Attempts to redress the inadequacies of teacher knowledge for qualified teachers is not without challenges.

Now that teachers' lack mathematics knowledge for teaching, it is best to address this issue in teacher education (Aslan-Tutak & Adams, 2015), the fundamental and formal source of teacher knowledge. Mastery of the mathematics knowledge for teaching is a function of the quality of initial teacher education received, thus the the notion of teacher knowledge was brought up in this study on preservice teachers. In-service training could be used too, to advance teachers' pedagogical content and content knowledge, but, according to Carnoy, Chisholm and Chibisa (2012), that does not lead to much improvement in the overall quality of teachers' knowledge. Practising teachers usually develop a teaching philosophy which naturally resists any attempt to change. That being the case, the focus of teacher quality improvements turns out to be the initial training of preservice teachers. Therefore, it lies upon the heart of teacher training institutions to thoroughly prepare preservice teachers in terms of their knowledge in subject-matter and the skills of how to teach it. The investigation of mathematics knowledge for teaching centred on the final-year preservice teachers' who represents the end product or climax of teacher education.

In mathematics teaching, besides knowing the content, it is critical for a teacher to know how to impart content knowledge to learners. There are some studies indicating that pedagogical content knowledge is also related to the quality of the preservice teacher education received (Auliffe, 2013). It is the right time to start asking ourselves why our teacher education courses are not having the desired improved effect on the mathematics performance of our learners. According to the model by Fennema and Frenke (1992) on teacher knowledge, effective teaching of mathematics occurs when the content knowledge, pedagogy knowledge of teachers and context-specific knowledge form an integral part of the teacher-preparation programmes. Thus, these components of Fennema and Franke's (1992) model of teachers' knowledge need to be advocated when the universities implement teacher training programmes (Mohr, 2006). In normal classroom situations, teachers have the duty to decide on the teaching strategy and what to teach. The syllabus and other policy documents act as guides within which teachers plan their lesson to achieve learning goals. Teacher's pedagogy and content knowledge skills thus play a major role, which guides teachers' decisions about the correctness or falsity of learners' responses. Consequently, this study reckons that it is frequently the unsuitable and insufficient preparation and training of teachers which leads to learners failing to understand mathematical concepts. Teachers normally resort to the easy route of direct transmission of knowledge to learners through teacher-centred teaching methods if they are faced with limited teacher knowledge.

1.2. Statement of the problem

Teachers are the key players of classroom learning and their knowledge on teaching and learning is the most reliable predictor of learner achievement. Darling-Hammond (2000) posited that the missing link to learners' academic success in the modern-day classroom is teacher knowledge. According to the model of teacher knowledge by Ball, Thames and Phelps (2008), teachers need to be in possession of two knowledge domains in order to be successful and contribute to the improvement of mathematical thinking of learners. These two are pedagogical content and content knowledge domains. The National Policy Framework for Teacher Education and Development envisage equipping South African teachers with all the necessary skills and knowledge for them to face the current demands for better education (Department of Education, 2006). Furthermore, the Department of

Higher Education and Training developed an instrument called the Minimum Requirements for Teacher Education Qualifications policy. This instrument advocates that teacher candidates and beginning teachers should be in possession of robust content knowledge and to know how to teach that content (Department of Higher Education and Training, 2011b). They should also be able to select, sequence and allot topics in tune to the demands of the syllabus of each of the subjects they will be teaching. Thus, the initial teacher education ought to instil competency in preservice teachers for a successful school teaching journey upon completing their studies (Gierdien, 2012). Preservice teachers ought to have an in-depth knowledge of the mathematics content they will soon teach for them to be competent future mathematics teachers (Ball, Thames & Phelps, 2008). This includes topics like trigonometry, which improves the reasoning capacity of learners and whose properties constitute many aspects of calculus, including curve-sketching, limits, continuity, differentiation and integration.

In South Africa, some higher education institutions tend to comply with the government's policy requirements on the initial teacher education superficially (Department of Higher Education and Training, 2010). This compromises production of new teachers who are competent to teach. As a result, there is a serious challenge facing education whereby teachers have limited conceptual understanding of content they are expected to teach (Department of Higher Education and Training, 2011b; Brodie, 2004). These are some of the teachers who passed through the same teacher education system Ihave today. The prospective teachers who are still under training did not fare any better. Their pedagogical content knowledge is not robust; it was revealed that novice teachers and preservice teachers often resorted to factual and closed questions in their teaching and also administer lower-order tasks which mainly focus on procedures and rote memorisation (Lloyd, 2006; Crespo, 2003). Moreover, preservice teachers were observed to lack the depth and breadth in the mathematics content knowledge they are required to teach (Wilburne & Long, 2010) and they have insufficient knowledge of learners' mathematics conceptions (Ball, 1990; Kilič, 2011). As for trigonometry, preservice teachers' understanding was found to be at insufficient levels (Fi, 2003). It was also shown that there is ineffective vetting of applicants prior to admission in teacher education due to weak passes of those who decide to train as teachers (Department of Higher Education and Training, 2010). Hence, preservice teachers enter and exit teacher education with meagre conception of mathematics.

If mathematics educators wish to improve the mathematics knowledge for teaching of preservice teachers, then the first step is to explore their current knowledge levels. Only after an assessment of the extent of their current level of knowledge can plans for modifications to the initial teacher education be hatched. Hence, the present study purposed to investigate final-year preservice teachers' mathematics knowledge for teaching in trigonometry at a South African higher education institution, to see if the current production of mathematics teachers meets the expectations of the Department of Higher Education and Training, the Department of Basic Education and other interested parties. This in turn hopefully would enlighten our quest for optimum training of mathematics preservice teachers and help to curtail the alarming low performance of mathematics amongst South African school learners.

1.3. Research questions

The present study attempted to answer the following research questions:

1.3.1. What is the level of preservice teachers' understanding of trigonometric concepts?

1.3.2. What pedagogical content knowledge do preservice teachers possess in trigonometry?

1.3.3. To what extent do preservice teachers develop the mathematics knowledge for teaching of trigonometry in initial teacher education?

1.4. The purpose of the study

The intention of the present research study was to explore preservice teachers' understanding of pedagogical content knowledge and content knowledge in teaching trigonometry and to determine the extent of their development in teacher-training. This was accomplished by assessing the depth of preservice teachers' mathematics knowledge for teaching in trigonometry as they exit teacher training.

1.5. Delimitations of the study

The selection of participants was confined to rural preservice teachers studying at a rural-based higher education institution in South Africa. The institution consists of a wide range of students from diverse ethnic and impoverished backgrounds. For some consecutive weeks in the last two years of their four-year study, preservice teachers go for home-based teaching practice. The rural areas where preservice teachers hail from were limited to Limpopo and Mpumalanga provinces. The exploration of preservice teachers' knowledge in this study was limited to trigonometry.

1.6. Significance of the study

Even though based on a cohort of final-year preservice teachers at one higher education institution, the findings of the present research study have far-reaching contributions, potential benefits and prize to the mathematics education community. This study has shown that the training and preparation of preservice teachers is two-faced. One facet is that preservice teachers receive training that make them fully competent by the time they graduate from teacher education institutions. This is the stance taken by policy makers who sponsor and expect teacher candidates to possess knowledge of content and knowledge of how to teach that content by the time they complete their studies (Department of Higher Education and Training, 2011b). Upon that basis, the exploration of preservice teachers' mastery of mathematics knowledge for teaching was conducted, such knowledge which they would need to put into practice when they are beginner-teachers. On the other hand, some argue that preservice teachers acquire knowledge of teaching through actual teaching practice experience. Teacher education therefore only provides a framework and structure, to which novice teachers complement by their individual experiences. Kilić (2009) concurs by stating that preservice teachers should possess the basic pedagogical content knowledge at teacher-training, which they would later improve as they gain actual field experience. Thus, both preservice teachers and teacher educators should strike a balance on the two facets teacher-training.

Curriculum planners of teacher education also stand to benefit from the findings of this study. As those responsible for drafting university programmes realise that teacher candidates exit the system with scant knowledge of content knowledge they are expected to teach, adjustments can be done to their curricula in order to optimise mastery of content knowledge in teacher education programmes of study. Sometimes it does not make much sense for preservice teachers to do advanced undergraduate mathematics content in applied mathematics, statistics, calculus, analysis and linear algebra, yet lack knowledge in school mathematics. According to Kilpatrick, Swartford and Findell (2011), it is a problem if preservice teachers learn highly abstract concepts in university-level mathematics yet they are unable to unpack the same concepts in ways understandable to learners. Preservice teacher education should strive at its best to prepare prospective teachers for school teaching (Gierdien, 2012). Teacher education should bridge the divide between university-level mathematics and school mathematics to empower preservice teachers with skills and expertise to teach mathematics when they begin their teaching career.

Most higher education institutions, including the research site for this study, design all the first-year mathematics modules to be a re-cap of selected Grade 12 mathematics concepts and topics and for a long time trigonometry has been one of them. In addition to content knowledge modules, methodology modules at all levels of study impart pedagogical content knowledge skills to preservice teachers by referring to appropriate secondary school mathematics concepts. Teaching practice at all levels of study is at hand to apply in the real classroom situation all the theory of teacher knowledge which preservice teachers have been taught in teacher education. These measures ensure prospective teachers' mastery of the mathematics knowledge for teaching is well-grounded in the minds as part of their training.

This study is important in the sense that it reveals the need to give due support to novice and beginning teachers. Many new teachers to the profession are given full responsibility to teach and left to figure out their own way into effective teaching ways because they are holders of a university teaching qualification. To most of the preservice teachers, real support to teaching ends with supervised teaching practice, both from the university supervisors and school-based mentors. New teachers still need teacher-mentors for support in their early years of teaching. Weak mathematics knowledge for teaching for both in-service and qualified teachers is a clear testimony that continued support is a necessity.

The knowledge which preservice teachers get from teacher education is not enough to face the reality of school teaching, but it has the potential to shape their future teaching if given necessary support from universities, schools and Departments of Higher Education and Training and Basic Education (Bailey, 2014). Many inexperienced teachers of mathematics do not know how to develop, adapt and employ pedagogical content knowledge in their teaching (Halim & Meerah, 2002). Consequently, they may start to dislike and skip teaching certain topics. This is something that is avoidable if necessary support is rendered to teachers' weaknesses in content and pedagogical content knowledge. Moreover, the chief factor attributed to the South African learners' under-performance in mathematics is the lack of teachers' knowledge and experience (Howie, 2002; Department of Basic Education, 2012). The findings of this aspect of the research will help generate valuable input for novice teachers to be given continual professional development, this time as in-service teachers through continuous professional development.

This study contributes by highlighting many issues and perspectives related to challenges faced by preservice teachers as they undergo training to teach mathematics. Preservice teachers face serious challenges of understanding some topics of mathematics which they would teach. Trigonometric functions are one such area of school mathematics that learners often find specifically difficult to comprehend (Akkoć, 2008; Tuna, 2013; Weber, 2005). Having been learners themselves, preservice teachers enrol at teacher education institutions with the same difficulties in understanding trigonometry. In this respect, it was noted that preservice teachers experienced problems in conceptualising and teaching trigonometry and many of the issues centred on preservice teachers' own inadequacies. One reason for the lack of comprehension is that basic concepts making up trigonometry call upon learners' reasoning skills, especially in the aspect of shifting from algebra to geometry (Tuna, 2013). Instead, this study revealed that most preservice teachers exhibited strong elements of procedural instruction of trigonometry such as the SOHCAHTOA acronym, CAST diagram and memorization of ratio of sides of special angles (Berenson, van Der Valk, Oldham, Runesson, Moreira, Berenson & Laurie, 2005). This obviously counteracts attempts of getting learners to develop reasoning skills in trigonometry. More so, there has been little research on final-year preservice teachers' understanding of trigonometry (Akkoć, 2008).

The other significance of this study rests on the premise that it will shed light on the shortcomings of mathematics education programmes in the teacher-training institutions. From the onset of democratic rule in 1994, the South African national departments of education and higher education have invested heavily in training and re-training of teachers. These measures were taken in order to improve this country's education system and the training of teachers (van der Sandt & Nieuwoudt, 2005). However, it is surprising that despite these efforts, the status of education in South Africa has not yet shown remarkable improvements (Brodie, 2004), notably in science and mathematics. Therefore, it is of necessity at this point to find out the current status quo of teacher candidates who are on the verge of becoming novice teachers whether they have a good command of mathematics knowledge for teaching.

The level of teacher knowledge informs classroom instructional strategies, which sequentially leads to improved learner-achievement (Cunningham, 2005). Teacher knowledge is acquired mostly in preservice and in-service teacher education, hence the national government willingness to fund it. This study and others reveal that preservice teachers leave teacher education with inadequate knowledge of content and how to teach it (Fi, 2003; Mudaly, 2016; Carnoy, Chisholm & Chilisa, 2008). If that is the case, then there are some shortcomings in the current teacher education system and value for the money invested into it is not realised. It is true some aspects of teacher knowledge are assumed to be developed as teachers gain more experience (Borko & Putnam, 1996). However, the undergraduate modules and teaching practice that preservice teachers undertake at university should equip them with rudiments of mathematics knowledge for teaching to be ready for their first year of teaching (Kilić, 2007).

Lastly, investigation of preservice teachers' content knowledge of mathematics is relatively easy. Most studies highlighted in the literature review chapter, as well as this study relied on using a content test instrument. The writing of the test is always supervised and then scored objectively. On the contrary, it is a mammoth task to measure preservice teachers' pedagogical content knowledge, which is by nature unique, specialised and develops in cycles rooted in classroom practice (Miller, 2006). In brief, pedagogical content knowledge varies among individuals (Jong, Van Driel & Verloop, 2005), hence not comparable across the breadth of participants. To alleviate that, three instruments were used to gain insight into preservice teachers' pedagogical content knowledge, which were lesson plan analyses, classroom observations and task-based interviews. The contribution of this study was to show that pedagogical content knowledge is nebulous and difficult to isolate from other domains of teacher knowledge. Notwithstanding, gauging preservice teachers' pedagogical content knowledge is necessary and provides a good starting point in gaining insight into the dynamics of teacher knowledge. Such knowledge is vital for in-service teachers too and is a headway to discovering new and improved methods of teaching problematic topics in mathematics. This helps to bridge the gap between teachers' classroom practices and teacher knowledge domains.

1.7. Justification of the study

1.7.1. Personal

The rationale for this study was multi-faceted, spanning personal interests, limited teacher knowledge, learners' under-achievement in mathematics and need for improvements in the training of new teachers of mathematics. The experience of the researcher in training and working with preservice teachers revealed their lack of confidence to explain trigonometric concepts. The core of teacher education programmes is to empower preservice teachers with requisite knowledge needed for classroom instruction by the end of their teacher education studies. Though it is not easy to measure the degree of transformation brought about by the exposure of preservice teachers to the teacher-training system, some programmes leave preservice teachers ill-equipped with essential teaching skills. Consequently, preservice teachers were found to possess inadequate mathematical content knowledge and pedagogical content knowledge (Fi, 2003; Mudaly, 2015). Thus, I have come to realise the need to add accountability to the production of new teachers coming through higher education institutions. One way of doing that was to conduct studies of this nature so that researchers can have a glimpse of the state of teacher knowledge in the products of higher education institutions. This would give a glimpse to the shortcomings of the current initial teacher education programmes and also preservice teachers would perform in their initial years of school teaching. All these personal dispositions have been made possible by the existence of a research gap in literature on preservice teachers' mathematics knowledge for teaching in trigonometry in poor ethnic students at a rural-based higher education institution.

1.7.2 Teacher knowledge

There is necessity to delve into the depths of the domains of teacher knowledge, since it is key to classroom instruction and possible improvements in learner-achievement. By means of well-mastered mathematics knowledge for teaching, teachers would be able to choose the best possible manner of delivery of content to their classes. This places the mathematics knowledge for teaching at the fore of what takes place in the classroom. According to Ball (1990), effective teaching of mathematics rests squarely on the teachers' mastery of content and pedagogical content knowledge. It also influences both lesson planning and other classroom practices. Also learner performance has been observed to be favourably high when a teacher has robust disciplinary knowledge they are supposed to teach (Hill, Rowan & Ball, 2005). Frequently, learners fail to grasp mathematics concepts due to inefficient preparation and training of teachers. For example, once a teacher dislikes or lacks confidence in teaching a particular topic, he/she may skip it or be selective in what he/she teaches in that topic (Furner & Robison, 2004). In a way, teachers pass down their own weaknesses in certain topics along to their learners. One such area of study of mathematics which teachers are not comfortable with is trigonometry, as explained in the next section. The discussion above justifies a careful consideration of knowledge domains of teachers as they apply to teachers and teacher-candidates. The mere possession of a qualification in mathematics teaching does not transform to effective teaching. Some of the results of effective teaching reflect in learner-performance results, which are currently below expectations in South Africa.

1.7.3. Trigonometry

Trigonometry is an important concept in school mathematics which has the potential to improve learners' reasoning capabilities (Dündar & Yaman, 2015). It is also a topic which many learners dislike and grapple with (Gür, 2009). One of the reasons cited for the perceived difficulties in trigonometry is that most trigonometric concepts are usually taught from the algorithmic approach, without necessary emphasis on the conceptual understanding (de Villiers & Jugmohan, 2012). In addition to that, teachers' knowledge of trigonometry has not received enough attention among mathematics education researchers (Akkoč, 2008) in a way to identify their weaknesses and possible channels of assisting them. Preservice teachers too were found to lack knowledge of trigonometry (Fi, 2003; Čižmešija & Milin Šipuš, 2013). Thus, there is need to tap into the preservice teachers' understanding of mathematics knowledge for teaching in trigonometry to check their readiness to teach it when they complete training. Having such knowledge would go a long way in enabling teacher education institutions to better equip future teachers of mathematics. In conclusion, a mixture of my personal experiences as a mathematics education lecturer, the knowledge domains of teacher knowledge and the poor teacher and learner-understanding of trigonometry is the justification of the present study.

1.8. Overview of chapters

In order to situate this study in the broader perspective of existing research, an in-depth literature study was done and reported in Chapter 2. Following on that, Chapter 3 expounds the underpinning conceptual framework of this study, which is the mathematics knowledge for teaching by Ball, Thames and Phelps (2008). Chapter 4 addresses the manner wherein this research study was conducted, that is, its design as well as other methodological issues to collect and analyse data. Chapter 5 inter-mixes the presentation and analysis of research data from the four data collection instruments. Chapter 6 presents the discussion of findings of the study based on the analysis of data. Finally, Chapter 7 presents the summary, conclusion and recommendations for future research, as well as the limitations of the study.

1.9. Chapter conclusion

The chapter started by indicating that the background to this study was rooted in the concern about the South African learners' under-achievement in mathematics and the need for effective teaching to redress this situation. The inadequate preservice teachers' understanding of the mathematics knowledge for teaching and the prominent role played by the initial teacher education in the training of new teachers also formed part of the background. The problem statement highlighted the ideal situation where preservice teachers are expected to possess competence of teaching school mathematics as they graduate from training institutions. However, literature reports that preservice teachers lack the depth and breadth of the knowledge of school mathematics. Hence, the goal of this research study was to explore preservice teachers' understanding of the mathematics knowledge for teaching and the extent of the development of the same in classroom teaching in trigonometry, in the sample of participants chosen from a rural-based higher education institution in South Africa.

The significance of the study was that it can inform teacher education programmes for improved preservice and in-service teacher training on how best their programmes can be structured to engender competent mathematics teachers. The study also will bring to the fore the South African state of teacher education, in tune with the efforts by the Department of Basic Education and the Department of Higher Education and Training to produce competent future teachers. The rationale of the study was the quest for quality mathematics teaching, especially in topics like trigonometry, which pose challenges to both learners and teachers.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

The various aspects of literature that informed this study are explained in this chapter. The next section (2.2) of this chapter deals with the constructs of teacher knowledge by highlighting their meaning and importance, as well as the critical role played by the mathematics knowledge for teaching in classroom practice. The discussion on teacher knowledge is centred on the ground-breaking work by Shulman (1986, 1987) and other studies which later built on Shulman's work. Section 2.3 discusses qualified teachers' mastery of pedagogical content knowledge in selected topics. This was done as a reference point for an investigation into preservice teachers' pedagogical content knowledge. Practising teachers experience the growth of pedagogical content knowledge as they gain experience of teaching, whereas preservice teachers lack the exposure of such growth of pedagogical content knowledge.

The section 2.4 places the significance of appropriate teacher knowledge for the success of classroom teaching practices. It will be shown that if teachers keep their role in the classroom through the knowledge which they have, learners' performance is bound to improve. Section 2.4 takes further the notion of teacher knowledge by narrowing it down to preservice teacher knowledge in the South African context, followed by other contexts from all over the world. Section 2.5 highlights the qualification route for new teachers in South Africa, as well as explaining the guidelines to be met by successful prospective teachers. The Minimum Requirements for Teacher Education Qualifications policy embodies mathematics knowledge for teaching by specifying that new teacher graduates ought to have rigorous subject knowledge and the know-how of teaching it. Thereafter, section 2.6 deals with the topic of trigonometry, by tracing its origin and providing the rationale for its inclusion in the secondary school mathematics curriculum, especially in South Africa. Coming after this is an explanation of the challenges of teaching and learning of trigonometry in schools.

Section 2.7 highlights literature of the extent to which preservice teachers' content knowledge on selected school mathematics topics fare, in a way to explore their preparedness as mathematics teachers. Teachers can only teach meaningfully what they know. Section 2.8 focusses on literature on explaining preservice teachers' understanding of pedagogical content knowledge. Prospective teachers' perceptions on teaching specific topics, explanations and critical thinking skills will be discussed. The subsequent section (2.9) gives an overview of preservice teachers' mathematics knowledge for teaching, which they acquire in teacher training institutions. This section lays background for the upcoming two sections, which highlight literature on preservice teachers' mastery of content knowledge, followed by one on preservice teachers' pedagogical content knowledge. Finally, the chapter conclusion is presented in section 2.10, whereby it will be shown the importance of teachers' successful mastery of the different types of teacher knowledge in order to be effective teachers.

2.2. Pedagogical content knowledge in mathematics teaching

Mastery of pedagogical content knowledge by teachers of mathematics is key to the teaching and learning process that it was observed that teachers with inadequate mathematics knowledge find it difficult to appropriately sequence or connect topics (Hristovitch & Mitcheltree, 2004). This study has shown that pedagogical content knowledge is nurtured in accordance to growth of job experience, hence some studies focussed on exploring qualified teachers' potentially shifting knowledge. While the present study focuses on prospective teachers, this study benefits from the insight drawn from the studies conducted on qualified teachers' mathematics knowledge for teaching.

The study by Hristovitch and Mitcheltree (2004) reported of a professional development programme designed to improve teachers' pedagogical skills and content knowledge. The first phase of the project focused on identifying the areas in which teachers' knowledge needed improvement and the second phase focused on the issues of the content that should be made aware to teachers so that learners' achievements in mathematics can be realised. Three teachers participated in the study where they were observed teaching fractions and decimals in primary schools. It was evident from this study that while teachers tried to use innovative approaches like problem-solving in their teaching strategies, the instructional activities employed did not lead to learner-conceptual understanding and the expected connections of ideas. These teachers failed in organising and sequencing mathematics concepts in a way that would present mathematical ideas coherently. The consequence of such teachers' failure to organise and coherently sequence mathematics concepts was to teach them as isolated bits of information. The topics were also taught devoid of real-world application which is known to aid learners to conceptualise mathematics concepts (Lott & Souhrada, 2000). Sufficient mastery of mathematical knowledge for teaching goes a long way to help teachers address teaching and learning issues highlighted above.

In cases where teachers had challenges with the mathematical knowledge for teaching, teaching was characterised by rigid teaching methods, which predominantly relied on textbooks as the sole source of instructional knowledge (Lott & Souhrada, 2000; Turnuklu & Yesildere, 2007). The then Department of Education perceived mathematics as a process- and concept-driven subject that requires learners to authentically connect and apply its concepts in the real-world context (Department of Education, 2005)**.** This requirement cannot be fully achieved if teachers do not possess adequate pedagogical knowledge and content knowledge. Furthermore, the study by Hristovitch and Mitcheltree (2004) indicated that teachers had problems explaining mathematical ideas. For instance, some teachers had difficulties in relating fractions to division of whole numbers and to explicitly show the transition from fractions to decimal fractions.

A study by Yusof and Zakaria (2010) sought to explore and describe the extent of pedagogical content knowledge held by three mathematics teachers when teaching functions at secondary school level. That study endeavoured to explore the components of pedagogical content knowledge and to determine the three teachers' mastery of pedagogical content knowledge in the concept of functions. The research was qualitative in nature and conducted as a case study design. Data collection instruments used were interviews, classroom observations and document analyses. The pedagogical content knowledge components which they found were the use of analogies, symbolic representations and instructional strategies. These were found to have been used by the three teachers to stimulate the learning process. The three teachers' lessons were found to lack accuracy and instructional clarity for the learners due to lack of adequate conceptual knowledge of functions.

In a study by Chick, Baker, Pham and Cheng (2006), the theme of the study was the exploration of the primary school teachers' pedagogical content knowledge as they taught subtraction in the lower grades. The research design was qualitative and the case study methodology was employed in that study. Data were collected by interviews, lesson observations and questionnaires. Chick, Baker, Pham and Cheng (2006) created three distinct projections in their study. Firstly, if teachers had sufficient subject matter knowledge, this would have been evident through teachers' exhibition of thorough and deep conceptual understanding of concept taught. Secondly, if teachers had adequate knowledge of instructional methods, it would have been evident in their use of appropriate activities during the instruction phase. Lastly, if teachers had ample knowledge of learners' conceptions, evidence would have been obvious if they displayed interest in the learners' prior knowledge and dealt with learners' difficulties and misconceptions on the concept taught. The findings were that teachers in the study lacked firm content knowledge on subtraction. The teachers were also observed to rely on a sole teaching method, and that method did not connect to the learners' known environment. The teachers had ostensibly good lesson presentations, but they lacked knowledge of how to identify and correct learners' misconceptions.

In all, the literature presented above indicates that qualified teachers had inadequate mastery of pedagogical content knowledge, for example, their sequencing of topics was lacking, prior knowledge was not squarely placed where it belonged and insufficient attention was paid to addressing learners' difficulties and misconceptions. Teachers' difficulties with content knowledge were observed in the teaching of subtractions, functions and sequences topics. Teachers' application of instructional strategies was mediocre, which could have been higher since the focus was on qualified teachers with many years of teaching experience. Resorting to sole teaching strategies is a sign of weak mastery of pedagogical content knowledge. Thus, for these teachers, their mathematics knowledge for teaching was inadequate.

2.3. The role of teacher knowledge in the classroom

Though it is commonly agreed that the teacher factor has a direct influence on learners' educational attainment (National Council for Teacher of Mathematics, 2000), the extent of that influence and the kind of knowledge structure that produces effective teaching has been in the spotlight of researchers. Teacher knowledge is pivotal in teachers' execution of their duties and this critical role of teachers' knowledge was acknowledged from time immemorial. Shulman (1986) concurred by saying that successful teachers require a highly organised and extensive body of knowledge of their respective disciplines. The National Council for Teacher of Mathematics also asserted the significant role of teacher knowledge when they stated that effective teaching requires understanding and knowing mathematics and commensurate pedagogical strategies to teach that content (National Council for Teacher of Mathematics, 2000). Nothing can surpass the role played by teacher knowledge in the classroom. Challenges to teaching and learning are encountered if the role of teacher knowledge is compromised, say, by lack of adequate mathematical knowledge for teaching. Hence, this study advocates assessing current preservice teachers' state of teacher knowledge so that they are not found embroiled in perpetuating the cycle of learner mathematics underperformance in South Africa when they eventually join the teaching service.

Globally, there is an outcry about low learner achievement in science and mathematics subjects (Moloi & Strauss, 2005; Howie, 2003; van der Walt & Maree, 2007). While the cause of poor achievements in mathematics is multi-faceted, many researchers attributed it to lack of mathematics knowledge among teachers, teaching of mathematics by unqualified or under-qualified teachers and outdated teaching practices (Pournara, 2005; Mji & Makgato, 2006; van der Walt & Maree, 2007). This implies that despite the existence of other teaching and learning considerations, the teacher factor remains the fundamental determinant of effective learning in the classroom. Mudaly (2016) posited that if teachers' levels of knowledge are poor, then it may be argued that teacher education programmes are to blame. In the South African landscape, preservice teacher-training was designed and developed in accordance with the guidelines set out in the Minimum Requirements for Teacher Education Qualifications policy document, to which all higher education institutions must comply with (Department of Higher Education and Training, 2011c). The guidelines spelled out in the Minimum Requirements for Teacher Education Qualifications outlines the kind of teacher that South Africa ought to have. Notwithstanding, the quality management approach of preservice teachers cannot guarantee that the end product is ready and capable to teach, as is the case with assembly-line products. These are human beings who have different temperaments and behave in a unpredictable ways. The challenge facing South Africa is how to ascertain that preservice teachers are fully equipped for the world of teaching at the point of exit from teacher-training institutions.

The Department of Higher Education and Training has acknowledged that poor content and conceptual knowledge is rife among practising teachers (Department of Higher Education and Training, 2011b). Thus, something needs to be done to identify and possibly rectify these teacher deficiencies in the grassroots. Teacher education is one such place of doing that. A good foundation in mathematics knowledge for teaching is a springboard for a successful career in mathematics teaching. According to McAuliffe (2013), there are some knowledge domains connected to the acquisition and application of mathematical knowledge for teaching purposes, namely: disciplinary, pedagogical and practical. Discipline knowledge is the study of specialised content that is related to an academic discipline sustaining the teaching of a learning area. Pedagogical knowledge incorporates general methodologies of teaching, such as knowledge of understanding the curriculum, the learners, the learning process, instruction and assessment. Practical knowledge is the application of theories of teaching and learning identified in pedagogical knowledge, thus creating a platform for future improved practice. These domains resonate to the teacher knowledge domains by Shulman (1986), which are content knowledge, pedagogical content knowledge and curriculum knowledge. Each type of the previously knowledge domains is pertinent to the development of teachers' mathematics knowledge for teaching to produce capable teachers who are geared towards teaching mathematics.

This is in line with the Minimum Requirements for Teacher Education Qualifications policy, which states that new teacher graduates ought to have rigorous subject knowledge and the knowhow of teaching it (Department of Higher Education and Training, 2011b). Selecting, sequencing and pacing content to suit the needs of the learners and requirements of a discipline are some of the expected outcomes of recently qualified teachers. While there are other fundamental competences that prospective and beginner teachers might need, Shulman's knowledge domains highlighted above link seamlessly to the focus of this research study, that of an exploration of prospective teachers' mathematics knowledge for teaching in trigonometry. Although the preservice teacher is not yet a beginner teacher, the undergraduate courses that they take at higher education institutions and the school teaching practicals are intended to build their competencies towards becoming a fully-fledged teacher. A recent South African government plan to address the challenges in teacher education has been the Integrated Strategic Planning for Teacher Education and Development for 2011 to 2025 period (Department of Higher Education and Training, 2011a). It is a long term and ambitious set of recommendations to turn around the South African education system. The plan was compiled with inputs from a range of stakeholders from across teacher education and provides a detailed set of strategies outcomes and outputs.

From above, it is evident that teachers need to acquire deep understanding of mathematics knowledge for teaching during their training because teachers are key to learner achievement. Currently, products of the South African teacher education institutions in mathematics and science do not meet the standards of the Department of Basic Education. Newly qualified teachers are obliged to have adequate content knowledge and the necessary instructional skills to teach that content. Teacher education institutions refuse to take the blame by citing operational challenges due to stringent budget provisions from the fiscus.

2.4. Initial teacher education in South Africa and other countries

With universities honoured with the sole responsibility of bringing forth future teachers in South Africa, one wonders if all higher education institutions are adequately equipped to achieve that. Not all institutions, especially those which were historically disadvantaged, could offer the same levels of training, engagement or support to prospective teachers. Lecturers too at these institutions often must tolerate lack of support in teaching and resources. Preservice teachers complained of large class sizes, reduced teaching practice supervision by lecturers and lack of facilities as some of the factors that inhibit possible growth of mathematics knowledge for teaching at the higher education institution where the data was collected. It is one of the historically disadvantaged institutions in

South Africa, which is still rural-based in terms of its location, source of its enrolment and deployment of its graduates.

In high school teaching, teachers often strive for scores and grades as the current matriculation standards are examination-oriented. Hence, learners who would have passed high school mathematics having no conceptual understanding of mathematics when they get to tertiary education. Faced with learners who enter teacher education with mediocre knowledge of mathematics, higher education institutions must find the best common ground. Consequently, some South African higher education institutions end up adjusting their academic standards down accordingly, which is an unpleasant way to accommodate the rising tide of incoming ill-prepared prospective teachers (Jansen, 2018). If higher education institutions do not compensate for the weak incoming students, they might be faced with high failure rates rising out of the matriculants who passed well. Because government subsidies and funding to higher education institutions are based on comfortable pass-rates and enrolments, in most cases, institutions trade-off quality education for government funding. They must deliver tertiary education and break-even at the end of the financial year.

Currently, are two possible routes of qualification to becoming a teacher in South Africa, which is graduating with an appropriate undergraduate degree with teaching modules coupled with a Post-Graduate Certificate in Education or an undergraduate four-year Bachelor of Education degree. These qualification routes came to be known as the Initial Professional Education of Teachers which indeed makes preservice teachers become qualified teachers (Department of Education, 2006). Ideally, by attaining a qualification in education, one becomes sufficiently qualified as a teacher. These newly qualified teachers are naturally expected to become good teachers through their post-qualification experience. However, a study by Mudaly (2016) revealed that the South African initial teacher education was on a downward trend, as evidenced by the production of teachers who are not adequately trained for school teaching. In fact, the Department of Basic Education consequently recommended that current approaches to teacher education must be reviewed and bolstered. To make matters worse, state-funded South African teacher education institutions operate under a stringent budget allocation, which adversely affects provision of human and economic resources. Without sufficient resources at hand to work, these higher education institutions grapple to give meaningful education to the preservice teachers. This leads to inadequate training of new teachers in initial teacher education.

If preservice teachers are expected to make a difference in the mathematics classroom, then they must show a deep understanding of school mathematics concepts they will be required to teach. Normally, this deep understanding is the result of their schooling experience and university undergraduate degree tuition. Notwithstanding, instances abound where preservice teachers who have just completed their teaching degree possess meagre understanding of basic mathematical concepts needed at secondary school (Taylor, 2011). Both the Department of Basic Education and the Department of Higher Education and Training corroborated that frequently higher education institutions deliver the initial teacher education programmes that do not meet required standards (Council of Higher Education, 2010). The undergraduate degree courses which preservice teachers take in the initial teacher education programmes unfortunately fails to fill the conceptual gaps that the prospective teachers take along to teacher education. Teacher-trainers oftentimes assume that prospective teachers possess the basic understanding of the school mathematics concepts, when in fact they may not.

The study conducted by Kayhan and Argun (2009) compared the performance of preservice teachers on the two assessments conducted during the first and tenth weeks of their final year of study. The scores obtained in the tenth week by preservice teachers were better than the scores obtained during the first week. The preservice teachers showed improvement in the performance of content knowledge, the teaching process and communication of ideas. It is also the same period where preservice teachers embark on supervised teaching practice. The study conducted by Crespo (2003) tracked changes in the way thirty-four preservice teachers posed questions to primary school learners in a period of eleven weeks. It was noted that preservice teachers' later problem-posing practices were significantly better than the earlier ones. Instead of posing traditional single-step and computational problems, they ventured into posing problems that were open-ended and exploratory. Hereby is highlighted the importance of teaching practice experience in developing the pedagogical content knowledge of preservice teachers. The testimony here is that preservice teachers' understanding of knowledge for teaching develops as they experience teacher training. It may be a matter of the magnitude of the development that may vary across individuals.

Consequently, undergraduate modules on content and methodology, as well as teaching practice are universally agreed as key to initial teacher education (Ashby, Hobson, Tracey, Malderez, Tomlinson, Roper, Chambers & Healy, 2008; Department of Basic Education, 2013). And one of the best place for preservice teachers to grow in teacher knowledge is during their supervised teaching practice. It is during this time that preservice teachers grow a repertoire of pedagogical content knowledge to teach mathematics appropriately (Makonye, 2017). During teaching practice, they also learn first-hand how to teach and apply what they have learnt in theory at teacher education in a real classroom situation, which enhances the pedagogical content knowledge.

2.5. The scope of trigonometry in the South African curriculum

Trigonometry is a branch of mathematics that focus on the ratio of angles and sides of different kinds of triangles. The development of trigonometry as the study of triangles commenced with the early Egyptians, but Leonard Euler is credited as the founding father of the current form of trigonometry. Studying trigonometry happens to be useful as a foundation for future mathematics knowledge to other topics and in professions like engineering and architecture (Beyers, 2010). Being foundation to other mathematical knowledge domains, trigonometry has connections to several other mathematical concepts and structures, for example algebra of functions, proving formulae and identities, solving equations, periodicity, inverse functions and the Cartesian plane. Trigonometry normally commences at middle secondary school level of education in most countries. In South Africa, trigonometry's maiden appearance in the mathematics curriculum is at Grade 10. At that level of education, children are generally able to think rationally according to Piaget's formal stage of cognitive development. The content of the South African Grade 10 to 12 curriculum was reconfigured in the recent Curriculum and Assessment Policy Statements document and the main theme was now placed on the mathematical content aspect (Department of Education, 2011), a move meant to transform the new curriculum to match international standards (Adler, 2011). Shown in Table 2.1 are the key sections of trigonometry that are currently covered in the South African curriculum.

Table 2.1. The concepts under trigonometry to be covered in Grade 10 – 12 mathematics.

Grade	Aspect of trigonometry
10	(a) Define trigonometric ratios $\cos\theta$, $\sin\theta$ and $\tan\theta$ in right-angled triangles. (b) Extend the definitions of $\cos\theta, \sin\theta$ and $\tan\theta$ to $0^0 < \theta < 360^0$. (c) Derive and use values of the trigonometric ratios (without using a calculator for the special angles $\theta \in \{0^0;\ 30^0;\ 45^0;\ 60^0;\ 90^0\}$) (d) Define the reciprocals of trigonometric ratios. (e) Solve two-dimensional problems.
11	(a) Derive and use the identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$. (b) Derive and use the reduction formulae. (c) Determine the general solution and/or specific solutions of trigonometric equations. (d) Establish the cosine, sine and area rules. (e) Solve two-dimensional problems.
12	(a) Proof and use of the compound angle and double angle identities. (b) Solve two-and three-dimensional problems.

As can be seen from Table 2.1, most of the ground-work on trigonometry is covered in Grades 10 and 11, while Grade 12 aspects mainly focus on compound and double angles formulae, as well as the three-dimensional application of trigonometry. Furthermore, current key exit-level outcomes for South African secondary school trigonometry topic are knowing:

- Definitions of cosine, tangent and sine, as well as their corresponding inverses, for all angles in the domain $-360^0 \leq x \leq 360^0$.
- How to solve any triangle if given necessary sides and angles?
- The standard trigonometry identities.

- How to simplify complicated expressions in trigonometry involving arbitrary positive or negative angles, double angle and compound angles formulae by using the standard trigonometry identities?
- How to sketch and interpret graphs of trigonometry graphs?
- How to solve trigonometric equations?
- How to compute height and length of mountains, buildings, and so on, by using horizontal and vertical distances, and angles of depressions and elevations?

The outcomes above compare well with the expectations of other countries also at secondary school level. As a study of the ratio of sides and angles, trigonometry is known to improve learners' cognitive understanding by effecting a transition from algebra to geometry (Tuna, 2013). The introduction of trigonometry at Grade 10 has engendered mixed feelings; for the first time, learners come across new types of functions which are represented by a name, rather than in symbols, as in polynomials. This poses a challenge to learners as they must, for the first time, establish a relationship of given triangles to numbers and then manipulate the symbols in the form of trigonometric ratios. These challenges in the underlying principles of trigonometry, denote that South African learners unilaterally resorted to memorisation and procedural learning (de Villiers & Jugmohan, 2012). By promoting memorisation of formula of trigonometric ratios in a bid to inculcate learner-understandings, teachers exacerbate learners' woes in trigonometry. They frequently resort to mnemonics such as the CAST diagram and SOHCAHTOA in a bid to help learners to remember signs of each trigonometric ratio in different quadrants and formula for trigonometric ratios respectively (Brown, 2005). This detaches learners from the reality of trigonometry as there seemingly appears to be an easy and simple rule to apply in order to conceptualise trigonometric concepts (Cavanagh, 2008; Wongapiwatkul, Laosinchai & Panijpan, 2011).

Learners unfortunately find trigonometry baffling as they consider it abstract rather than practical. The theory of trigonometrical ratios, their reciprocals and their inverses are unfortunately not part of learners' and teachers' everyday lives per se. To some extent, the application of trigonometry, like the 2-dimensional is rather superficial because no person will ever get to calculate the altitude of a plane as observed by an observer on horizontal ground at a given angle of elevation. Satellite imaging and other machines in construction use the concept of trigonometry, but none is used in learners' everyday life. Teachers are uneasy teaching trigonometry too, having passed through the same education system their learners are. Thus, procedural approach dominates the teaching of trigonometry at both secondary and tertiary levels. This creates a revolving door effect, where prospective teachers with abstract understanding of trigonometry pass it to their future learners.

2.6. A chronicle of teacher education in South Africa

This study has shown that teacher knowledge is instrumental in children's learning and this section places teacher training as a necessary foundation for competent practice in teaching. The importance of training competent teachers in South Africa has been in cognisance since the crafting of the Norms and Standards for Educators of 2001. However, the Norms and Standards for Educators initiative was not a success in the production of well-trained teachers because it lacked clear goals of the process of producing competent teachers. Initially, the Norms and Standards for Educators intended to achieve competence in teacher-training through an integration of fundamental, practical and reflexive competence, but that was inadequate to prepare new teachers (Sibaya & Sibaya, 2008). It turned out that it was unfruitful on beginning teachers because the focus of teacher-training during those days was mainly on the teaching methodology and the nature of the school curriculum. This was at the expense of the underlying content and conceptual knowledge needed for teaching (Brodie, 2004). Designers of teacher education programmes were oblivious to the fact that prospective teachers were not taking enough mathematics at high school. Consequently, Lloyd (2006) went on to say that preservice teachers ended up having weak mathematics knowledge and a narrow view of mathematics. This would set a wrong footing for their teaching career.

In South Africa, preservice teacher education and continuous professional development for practising teachers are substantially funded by the state at all levels of education. But the outputs of such teacher education efforts are not commensurate with the capital investment pumped into it (National Planning Commission, 2010). In-service training, workshops and seminars are conducted regularly which focus on mathematics content, mathematics curriculum, improvement of critical thinking, instruction and assessment. Comparatively, workshop attendance is more pronounced in South Africa than in other African countries (Reddy, 2006). These measures are taken to improve the level of education and training. However, it is surprising that despite these efforts, the status of mathematics education in South Africa has not yet shown conspicuous improvement (Brodie, 2004). This has conscientised the Department of Basic Education to start investing in teacher education in a way to improve future teachers of mathematics and science, having seen the weaknesses of training practising teachers. They have been doing that through student bursaries and research support into teacher education. However, in the face of all these efforts, preservice teachers' knowledge of teaching is still weak.

The seriousness of teachers' deficiency in mathematical knowledge for teaching is well pronounced among preservice and novice teachers. Usually, novice teachers lacking the skills to solve classroom mathematical problems in more than one way (Leikin & Levav-Waynberg, 2007). This is the result of the historical trend in mathematics education where the methods of teaching had preference over mathematics content knowledge (Onwu & Mogari, 2004; Mji & Makgato, 2006; Taylor, 2009). Despite of the pedagogy and the content modules taken by preservice teachers during their training, they frequently leave teacher-training institutions with more or less the same knowledge base as when they first entered (Benken & Brown, 2008). This assertion is supported by the findings of Sibaya and Sibaya (2008), who said that novice teachers who are the product of the bygone Norms and Standards for Educators did not master the content of the subjects they were expected to teach upon completion of their training. Consequently, these novice teachers felt less competent regarding their command of the subject matter, thus lacked the confidence to teach effectively. This meant teachers lacked adequate content knowledge in mathematics, as well as the confidence to teach proficiently (Even & Ball, 2009; Taylor, 2008; Crespo, 2003). Moreover, both the Departments of Higher Education and Training and Basic Education have recognised that many programmes of the initial teacher education of most South African higher education institutions are of poor quality (Council on Higher Education, 2010). This has then led to new minimum requirements being initiated for all initial teacher education programmes offered at higher education institutions (Centre for Development Enterprise, 2014).

Kilpatrick, Swartford and Findell (2011) went on to say that for the successful learning of mathematics, mathematical proficiency is highly needed. Subject knowledge makes it possible for teachers to be proficient in teaching mathematics, hence preservice teachers must train and specialise in mathematics (Benken & Brown 2008; Hill & Ball, 2004). Proficiency of preservice teachers in school content knowledge is acquired through teaching practice experience, content and methodology modules (Kayhan & Argun, 2009). The mathematics proficiency of preservice teachers has a direct effect on the quality of mathematics education in their learners (Kayhan & Argun, 2009). Preservice teachers' proficiency also reflects the effectiveness of the teacher education programme that they have experienced (Tatto, Schwille, Senk, Ingvarson, Rowley, Peck, Bankov, Rodriguez & Reckase, 2012).

2.7. Preservice teachers' mathematics knowledge of teaching

Oftentimes, that preservice teachers take many rigorous mathematics modules in teacher education, yet they struggle with understanding school mathematics content. This is the content they will be teaching during teaching practice and later when they qualify. Rather, mathematics undergraduate modules being studied by preservice teachers must at least align to the school mathematics curriculum. Many preservice teachers find themselves unable to study school mathematics in depth at teacher training institutions. This has undesired effects to preservice teachers' efforts to teach with meaning, during teaching practice and upon qualification. Consequently, many preservice teachers still have content gaps in knowing how to teach and apply the mathematics encountered in the school mathematics curriculum. If preservice teachers' content knowledge in mathematics is weak, they will not be able to explain and connect mathematical concepts necessary for learners to understand the content.

Having been learners themselves at some stage, there is a tendency for preservice teachers to teach the same way they have been taught by their teachers. To break that tradition, teacher education ought to challenge and extend preservice teachers' content knowledge and pedagogical knowledge of school mathematics. Many preservice teachers enrol at teacher training institutions with inadequate grasp of school mathematics (Conference Board of Mathematical Sciences, 2001). Additionally, they receive little mathematics knowledge for teaching and upon completion of their studies, they find themselves unprepared to teach mathematics to the next generation of learners. Later, some of these learners enter teacher-training institutions, thus perpetuating the vicious cycle of school mathematics inadequacy. In respect of this, there is a call for reforms in teacher education programmes so that they at least close the school-university divide. A balance of the two kinds of mathematics prepares prospective teachers well in the content aspect of mathematics teaching.

According to the Conference Board of Mathematical Sciences (2001), the pertinent challenge for teacher-trainers is to foster preservice teachers' knowledge of basic concepts in school mathematics. Even experienced teachers themselves do admit they never got to really understand the school mathematics they are supposed to teach during training, until they have gained experience through teaching it later. Hence, preservice teachers need rigorous post-school study of school mathematics in order to have confidence and competence to teach it. Preservice teachers were complacent to improving their school mathematics knowledge and teaching to break away from the traditional teaching of mathematics into meaningful content-rich teaching strategies.

One such study on school mathematics was conducted by Even (1993), who investigated teacher candidates' content knowledge of functions and its relationship to their pedagogical content knowledge. Data for that study were collected by means of questionnaires which were completed by 152 preservice teachers. Interviews were also conducted with ten of the initial 152 participants. Even discovered that preservice teachers tended to rely mainly on their previous learning about functions instead of mixing their prior knowledge of functions with the new knowledge they have learnt in teacher education. As a result, preservice teachers could not master the modern test of functions, but they relied solely on the "vertical-line test" to tell if given relations were functions or not. The modern definition of functions would have enabled them to recognise that some relations are still valid functions even though they fail the "vertical-line test". Thus, when preservice teachers teach functions, they would likely mislead learners about what a function is, especially in cases where the “vertical-line test” fails.

Avalos, Telez and Navarro (2010) sought to find the mathematical knowledge base possessed by preservice teachers in six Chilean teacher-training institutions. A questionnaire was administered, which contained both content and pedagogy. The results of that study indicated that, for the final year preservice teachers, 50 percent correctly answered the mathematics content questions and only thirty-two of them managed to correctly answer the questions on the pedagogy of mathematics. These results indicated that the preservice teachers might encounter difficulties of effectively teaching mathematics when they finally enter the world of teaching (Avalos, Telez & Navarro, 2010).

Another study indicated that preservice teachers had difficulties unpacking mathematical ideas, which in turn influenced their abilities to teach those ideas meaningfully (Kinach, 2002). She noted that the preservice teachers were successful at teaching execution of rules and formula but could not explain in clear terms why those rules work. Preservice teachers viewed teaching as giving rules, demonstrating to learners how to apply them in examples and then giving an activity to perfect the skill. By limiting teaching to identifying and applying algorithms, preservice teachers deprive learners the skill of decoding and proving conjectures.

Furthermore, Ball (1990) investigated elementary preservice teachers’ knowledge of division of fractions, on the basis of the data collected for the *Teacher Education and Learning to Teach* study. She realised that preservice teachers in that study had some difficulties to explain the skill of division of fractions to learners, even though they could easily perform the procedures of division of fractions. Preservice teachers often are limited to the traditional and procedural teaching of school mathematics, at the cost of conceptual teaching. Ball’s (1990) study illuminated conceptual and instructional challenges that many preservice teachers have with the aspect of division of fractions. Preservice teachers’ knowledge of mathematics was procedural; hence they could not explain reasons behind their calculations. The limitations of procedural fluency of mathematics are that it cannot be transferred to similar situations. According to Ball

(1990), preservice teachers' beliefs and preconceptions led them to perceive mathematics as consisting of rules and facts, and doing mathematics is following procedures intending to arrive at an answer. Teaching mathematics is to successfully teach learners how to repeat algorithms, rather than teach them about the underlying reasoning that makes such algorithms work the way they do. The following are recent studies on preservice teachers' understanding of the mathematics knowledge for teaching.

2.7.1. Preservice teachers' knowledge transformation geometry

Exploration of preservice teachers' mathematics knowledge for teaching has been under investigation for a long time, spanning diverse topics and countries. Noto, Priatna and Dahlan (2019) undertook an investigation into the Indonesian preservice teachers' mastery of content knowledge only in proofs under the topic of transformation geometry. The research purposed to identify learning obstacles that preservice teachers' encounter in executing mathematical transformation geometry proofs. The study was qualitative in nature and the design was a case study on a group of nine purposively chosen mathematics preservice teachers at an Indonesian state university. Just like the current study, data collection were done in the form of a content test and an interview task-sheet. A total of four problems on proving transformational geometry concepts constituted items in both instruments for that study. The analysis of data was done, and the findings revealed a plethora of obstacles that inhibit preservice teachers from effective mastery of the concepts which have been duly taught to them at teacher education. Two outstanding categories were identified, that is, understanding the concept and understanding the problem. Under obstacles of understanding the problem, the following were identified, namely, not knowing how to start the proof, not knowing how to use the definition to construct the proof, inability to state a definition, and not knowing the use of technical language and mathematical notation. Under obstacles of understanding the concept, they identified inability to visualise the geometrical object and not knowing how to determine the required principle. As a result of these observed obstacles, the content knowledge of those preservice teachers was thin and shaky (Ball, 1991).

2.7.2. Preservice teachers' perceptions and knowledge of trigonometry

In a quest to determine what causes learners' fears and failures in trigonometry in Ghana, a study by Nabie, Akayuure, Ibrahim-Bariham and Sofo (2018) explored preservice teachers' perceptions of trigonometry. The way teachers think determines their classroom instructional actions, which in turn influences the way learners understand concepts being taught. Coupled with exploring preservice teachers' perceptions on trigonometry, Nabie, Akayuure, Ibrahim-Bariham and Sofo (2018) also sought preservice teachers' conceptual knowledge of trigonometry in order to get a clear picture of how they view teaching trigonometry. Their quest was to yield results of what preservice teachers require for them to become competent future mathematics teachers. A convenient sampling technique came up with 119 second-year college of education students who completed a questionnaire on their perceptions of trigonometry teaching. To explore conceptual knowledge, a one-hour long assessment test on trigonometry was administered to the sample of preservice teachers.

A thematic analysis of data was performed on the nature of errors committed by preservice teachers in the assessment test, while the Statistical Package for Social Sciences was used to analyse the responses from the Likert-questions on the questionnaire. The result of that study unearthed that the preservice teachers in that study had serious difficulties in almost all school trigonometric concepts. In the example given, it was revealed that none of the 119 participants could honestly derive that the square identity, $\sin^2 x + \cos^2 x = 1$. The poor performance was in part a result of the negative perception towards trigonometry registered by the participants.

Though the participants admitted that learning trigonometry increases analytical and reasoning skills, they perceived that it was tedious and too abstract for them to understand. Their relational skills of trigonometric concepts were not good either, as they grappled with application of basic trigonometric concepts. They lacked the crucial link between basic concepts and their application. For instance, most prospective teachers had clear realisation of the connection between the sine and cosine ratios, however, they could not tell why $\tan x$ is positive in the third quadrant. Thus, despite the initial teacher education that prospective teachers receive, they still have inadequate conceptualisation of relational knowledge of trigonometry, coupled with negative perceptions of learning trigonometry. The same would be passed down to their future learners when they start teaching many years later. This result in learners failing and fearing trigonometry too.

2.7.3. Preservice teachers' level of content knowledge of radians

An examination of preservice teachers' understanding of measures of angle under trigonometry, with a particular interest on the radian measure was the object of Tuna's (2013) study. Tuna (2013) was interested in preservice teachers' levels of understanding on the radian measure, in conjunction with the degree measure amongst third- and fourth-year teacher candidates. Akkoç (2008) also investigated in-service and preservice teachers' content knowledge of the radian in Turkey. Both researchers made use of a content knowledge test, followed by a written one-to-one task-based interview as data collection instruments. Both instruments were administered under the researchers' supervision so a specific amount of time was spent on them. From those who sat for the content test, a few were sampled for the interview based on their initial performance in the test. For the content test, data analysis was performed by means of descriptive data analysis where correctness of the responses was key. This was followed by content analysis to unpack error patterns and sources in participants' responses in the task-based interviews.

The prominent result from the two studies was that preservice teachers' understanding of the radian measure was not robust. Their understanding of radians was rather dwarfed by the understanding of the degree measure. All angle measures were treated as degree measures, even if the domain of that trigonometric ratio was given as real numbers. For example given the mapping, $f: \mathbb{R} \to \mathbb{R}$ and $f(x) = x \sin x$, results show that $f(x)$ was erroneously stated as $30^0 \times \sin 30^0$. The misunderstanding was further exacerbated by the fact that 30^0 is not a real number, thus it is not possible to find the product of it with $\sin 30^0$, which is real. The concept image of degree measure dominated the preservice teachers' understanding, where degree measures were treated as real numbers. Radians, being the proportion of two lengths, do not carry a unit measure. They are just real numbers in the true sense. Very few of the preservice teachers could figure this out. Asked to define radians, none could give a correct and full definition thereof. Their attempts to define radians evolved around the use of π, which they treated to be different from the normal π, a real number. Fi (2003) discovered that preservice teachers were versatile with converting radians to degrees and vice-versa but failed to precisely define radians as a proportion of two lengths. Therefore, if the preservice teachers harbour negative perceptions about trigonometry (Nabie, Akayuure, Ibrahim-Bariham & Sofo, 2018) and possess inadequate content knowledge of the radian measure, their future learners will have difficulties too.

2.7.4. Preservice teachers' understanding of diagonals of quadrilaterals

Several investigations of preservice teachers' level of knowledge in this study have been on concepts covered in the secondary school curriculum. This was the case since some topics like trigonometry and calculus are a domain of the middle to senior secondary mathematics curriculum only. That aroused mathematics education researchers' interest to explore also prospective primary school teachers' level of knowledge in the content they would be teaching too. Naturally serious challenges of mastery of primary school mathematics concepts are not expected since the concepts are at the foundation level. However, this was not the case as it was reported that primary school teachers find some topics difficult to teach (Salman, 2009) and that primary school teachers lack the necessary depth of primary content knowledge (Long & Dunne, 2014). Considering this, Ayvaz, Gündüz and Bozkuş (2017) sought to determine the level of understanding amongst primary preservice teachers on the concept of diagonals of common quadrilaterals covered in the primary school curriculum.

In that study, the research methodology was a case study. Criterion sampling was used to select seven second year preservice teachers on the basis that they have successfully completed a course in geometry. They sat a diagnostic test which consisted of questions on the concept of diagonals and their associated properties. After initial analysis of test results, a further four preservice teachers were selected from the initial seven for a 25-minute one-to-one interview. The interview prompted further explanations from the participants, in line with the answers they provided in the diagnostic test. Thematic data analysis was used in the final data analysis where associated concepts based on participants' responses to each question were used to formulate categories. The data obtained from the interviews were analysed under these categories.

The findings revealed that preservice teachers in that study had basic knowledge about what a diagonal is, but they could not define it succinctly. The conceptual knowledge of preservice teachers regarding the diagonals of rectangles, rhombuses, squares, parallelograms and kites was inadequate. In other cases, preservice teachers could state claims about diagonals well, however, they could not prove those claims (Almeida, 2000). This meant that they had poor reasoning skills, which later affected their ability to prove statements. Similarly, Bukova-Güzel (2010) investigated preservice teachers' pedagogical content knowledge as they taught solid objects. Lesson plan analyses and video-recordings of instructional implementations, as well as semi-structured interviews were the data collection instruments used. The pre-determined pedagogical content knowledge components that were used in data analysis of that study were knowledge of teaching strategies, knowledge of learners and curriculum knowledge. The findings of that study were that the scope of participating teachers' teaching strategies was narrow and they did not pay particular attention to possible learner difficulties and misconceptions.

2.7.5. Preservice teachers' understanding of problem-solving techniques

The study by Avcu and Avcu (2010) explored the performance of preservice teachers on problem solving. Preservice teachers were given five problems to solve using different problem-solving strategies. Some of the strategies inferred were drawing a diagram, creating a model, visualising the problem, identifying key elements and working backwards. The findings of that study were that prospective teachers could follow a single routine of problem solving meaningfully, while the use of different problem-solving techniques was limited. Avcu and Avcu (2010) further concluded that the limited understanding of preservice teachers in problem-solving techniques was attributed to the lack of pedagogical knowledge.

Among the studies in preservice teachers' understanding of content knowledge, Peressin, Borko, Romagnano, Knuth and Willis (2004) conducted a research study that traced the development of teacher knowledge of preservice teachers from two reform based teacher education programmes all the way into their early teaching careers. From their study they concluded that preservice teachers enter and exit teacher education programmes with insufficient content knowledge needed to provoke reflective teaching. Similarly, Schoenfeld's (1987) study which concluded that despite numerous mathematics content modules undertaken by preservice teachers, they still have rule-bound knowledge of mathematical concepts they are expected to teach. Thus, according to Peressin, Borko, Romagnano, Knuth and Willis (2004), teacher-training institutions have the responsibility to design teacher preparation and teacher development programmes that model good mathematics teaching, which enable preservice teachers to develop mastery of both content and pedagogical skills.

2.7.6. Instructional explanation

Preservice teachers manifest pedagogical content knowledge in different measurable traits. In this study, teacher explanation is one of the pedagogical content knowledge components that was evident under both the task-based interviews and video-lesson analyses. Thus, this section is about literature on the preservice teachers' explanation skills during both the school teaching practice and in the interview. Teachers' abilities to explain an idea to a class is a prized possession. A study by Murtafiah, Sa'dijah, Chandra, Susiswo and As'ari (2018) sought to explore the different types of instructional explanations instituted by preservice teachers on a learning activity during the time they were doing school practice teaching. The learning activity involved in the explanation was on computing limits and derivatives of given functions. The research sample was five mathematics education undergraduate students drawn from two universities in Indonesia. The sampling technique was purposive and the five participants were carefully chosen to include those who were good in content knowledge of limits. Data were collected through observation, semi-structured interviews and video-recorded lesson analysis.

The collected data were analysed by means of deductive data condensation to concurrently give meaning to data and allow drawing of conclusions. Information from the three instruments of data collection were processed to fit into one of the categories. Among others, three types of instructional explanations were pre-determined, which became the focal point of data analysis based on data from the three instruments used. Of the three types of explanations, namely, descriptive, interpretive and reason-giving, it was noted that descriptive explanations were the most prevalent of the three. These featured mainly in the description of mathematical processes and procedures as a response to the “how” questions. Already the results show that the preservice teachers predominantly taught for mastery of rules and procedures, with little regard to deriving the procedures, and why the procedures work. They leaned more towards teacher-centred instruction which does little to promote learners’ construction of knowledge. This happens to be an obvious weakness of novice and prospective teachers. Reason-giving explanations were chiefly used to elucidate reasons based on mathematical principles in response to the “why” questions, whilst the interpretative explanations were used to expound concepts and facts of mathematics based on the “what” question types. Nevertheless, it was noted that instructional explanation is part and parcel of pedagogical content knowledge, which, according to the present study, is one of the predetermined pedagogical content knowledge component.

2.7.7. Preservice teachers’ critical thinking skills

It was agreed that teachers’ content and pedagogical knowledge are instrumental to learners’ academic success in the modern-day classroom (Darling-Hammond, 2000). In South Africa, learners study secondary school mathematics for the development of mental processes which help to augment critical thinking skills, computational accuracy and problem-solving skills. These in turn build individual learners’ decision-making capabilities (Department of Basic Education, 2011). Furthermore, the Grades R-12 National Curriculum Statement sought to produce learners who can make decisions using critical and creative thinking (Department of Basic Education, 2011). Now for teachers to assist learners develop these skills, teachers need to possess them first before they think of imparting them to learners. That being the case, a qualitative case study by As’ari, Mahmudi and Nuerlaelah (2017) sought to explore the level and readiness of critical thinking proficiencies of preservice teachers specialising in mathematics. The participants were third year undergraduate and some postgraduate preservice teachers at the Universitas Negeri in Indonesia. Each of these groups of participants were given a non-routine general mathematical problem to solve. Data were collected by observations as participants worked through the problem, and also by means of one-to-one interviews. The interviews were to probe the extent of the participants’ critical skills potential.

The research findings for the study by As'ari, Mahmudi and Nuerlaelah (2017) revealed that preservice teachers did not have exposure to hone their critical skills because the undergraduate curriculum places emphasis on routine problems. By modifying Paul and Elder's (2008) six stages of critical thinking development, these authors created a four-stage classification of critical thinking. The categories are non-critical, emergent critical, developing critical and mastering critical thinkers. All the participants' responses were categorised into one of the four stages, making the analysis of data deductive in nature. The research findings were that in all the cases that were considered, the critical thinking expectations of preservice teachers was at the basic stage. Whether these preservice teachers would be able to help learners to surpass the basic level of critical thinking when they hardly possess any themselves is anyone's guess. Thus, learners end up not having a role model to guide their critical thinking skills. As a result, they too operate at the non-critical thinker level stage (Dam & Volman, 2004). It is the responsibility of teacher education institutions to improve preservice teachers' critical thinking skills through instructional practices that prioritise non-routine problems in preservice teachers training.

2.7.8. Preservice teachers' strategies in solving limits

In calculus, calculating limits of a function is one of the fundamental concepts. The teaching and learning of the concept of limits has attracted the attention of mathematics education researchers. According to Kim, Kang and Lee (2015) and Row (2007), learners find it problematic to prove and apply the concept of intuitive definition of limits. What often happens is that learners end up memorising the algorithmic processes in proving the formal definition of limits, without full understanding of the procedures. In advanced calculus at universities, preservice teachers are taught skills of solving and proving limits using the formal definition of limits. In that regard, Oktaviyanthi, Herman and Dahlan (2018) sought to investigate preservice teachers' strategies when solving limits using the intuitive method and also explore their common mistakes when constructing proofs of limits.

The purposive sampling technique was used to come up with twenty preservice teachers who sat a one-hour mathematics content test on evaluating limits using the formal definition. This was done under the supervision of the researcher. Based on the initial analysis of test results, a further nine preservice teachers were selected for the one-to-one task-based interview. The interview task items were the same as the written test so that the participants could provide more explanation into their earlier responses as the researcher poses probing questions. Inductive analysis of data was carried out, whereby preservice teachers' answering processes and mistakes were classified into respective categories. The findings were that teacher candidates could use various strategies to solving limits by the formal method. Most of the preservice teachers used the strategies of preparation of proof and the algebraic techniques in the actual proving and these formed a framework for correct solving of limits. Nonetheless, these correct strategies were fraught with numerous errors so that the preservice teachers could not arrive at the accurate final solution. Their common mistakes in proving limits by the formal definition were many, which impacted their ability to sequence steps of proving. The other common mistake was the inaccuracies in simplifying algebraic expressions, thus obstructing the proving process. The last common mistake was the limited experience in proving limits by the formal definition method. Hence, knowing the strategy of solving a problem must be accompanied by accuracies in handling the key skills of key stages in the solution process.

2.7.9. Implications of literature to the present study

The focus of the present study was to investigate preservice teachers' understanding of content knowledge and pedagogical content knowledge in trigonometry. In the literature given above, some studies were investigating pedagogical content knowledge, some on content knowledge, whilst others focussed on both. For example, exploring instructional explanation, examining critical thinking skills and determining teaching strategies and common preservice teachers' mistakes constituted pedagogical content knowledge. Content knowledge was reported in literature on preservice teachers' knowledge of diagonals and transformation geometry. Some studies looked at both content and pedagogical content knowledge, like preservice teachers' perceptions and content knowledge on trigonometry, and preservice teachers' perceptions and knowledge of radians. This study investigated both types of teacher knowledge on trigonometry and attempted to draw a common conclusion.

Two instruments of data collection were generally used; a content test to assess mastery of content and interview or questionnaire to capture pedagogical content knowledge. This study followed a similar plan but went further and assessed pedagogical content knowledge by means of lesson plans and video-teaching analyses. Under interviews, task-based semi-structured interviews were widely used, as in the present study. The order of implementation of instruments was that a timed content test was administered first under the supervision of the researcher. After an initial analysis of test results, fewer participants were then selected to go for the interview based on the test results. Usually the participants for the test were selected based on the purposive or criterion sampling techniques. Those selected for the subsequent tools of data collection were based on performance in the initial tool(s), for instance, a balance of least- and best-performers. In other words, no new participants were considered.

Data analysis fell into three broad categories; deductive, inductive and descriptive. Deductive analysis was common for qualitative data in pedagogical content knowledge data analysis where analysis criteria was pre-determined from literature, and all participants' performances fitted into one of the criteria. Inductive analysis was common again for qualitative data under pedagogical content knowledge, where common categories start to emerge as data is analysed. These categories led to themes in the analysis, on which the discussion of data and conclusion of the study was based on. Descriptive analysis was widely used in quantitative data analyses whereby data was described, sometimes with the aid of statistical analysis in order to arrive at justifiable findings of the study. The present study made use of all these types of data analyses, each appropriately applied to the type of data under consideration. Form the presented literature and other studies (Ball, 1991; Fi, 2003), the discussion of results led to the conclusion that preservice teachers' mastery of content knowledge was inadequate, as evidenced in knowledge of radian measure, diagonals and transformation geometry. The preservice teachers' understanding of pedagogical content knowledge was limited, dogged by many common mistakes, lack of critical thinking, negative perceptions of trigonometry and radians, and teacher-centred instructional explanations.

2.8. Conclusion

From the above deliberations, it can be concluded that the performance of preservice teachers in mathematics knowledge for teaching depends on the quality of teacher-training they received and the type of knowledge they possess, which can be traditional, pedagogical and reflective. It was also shown that proficiency of preservice teachers in mathematical knowledge for teaching grows as they experience school teaching practices. This was supported by the evidence where preservice teachers' later problems posing strategies were better previously.

The literature review addressed three important aspects within this study, namely, teacher knowledge, teacher education and related research on preservice teachers. The first aspect of this review starts with the ground-breaking work by Shulman's (1986) types of teacher knowledge. Other researchers linked this concept of teacher knowledge to subsequent research within the field of mathematics education (Ball, Thames & Phelps, 2008; Davis & Renert, 2009; Rowland, Thwaites, Huckstep & Turner, 2009; Watson & Barton, 2011). It was highlighted that teacher knowledge is pivotal in teachers' practice and continuous professional development. However, it was noted in literature that both preservice and qualified teachers lacked conceptual knowledge to teach specific topics in mathematics, hence the pivotal role of teacher knowledge loses its appeal. In the South African context, it was observed that initial teacher education is not really producing the teachers that are required in schools (Department of Basic Education, 2013), thus not helping the challenge of teachers' lack of conceptual knowledge. Hence, topics like trigonometry are in limbo, with learners finding them difficult to learn and teachers find them difficult to teach. Finally, South African learners are not achieving in subjects like mathematics and science, in part due to the limited ways wherein teachers conduct classes.

CHAPTER 3: THE CONCEPTUAL FRAMEWORK

3.1. Chapter introduction

This chapter is a description of the conceptual framework of teacher knowledge, based on Shulman's (1986) work. The mathematics knowledge for teaching conceptual framework was used to anchor discussions on the types and relationships between different knowledge types for mathematics teacher education (Ball, Thames and Phelps, 2008). Other models of teacher knowledge were briefly explained to give an enhanced perception of the categories and nature of teacher knowledge, though they were not to be used in the analysis of data and discussion of findings. This chapter presents identified frameworks of teacher knowledge in order to provide a conceptual base for exploring and establishing the existence of the teacher knowledge domains of the preservice teachers in this study.

This study starts by presenting an overview and justification of using a conceptual framework of teacher knowledge under section 3.2. Though there are several relevant teacher knowledge models, only two have been considered in this study. Thus section 3.3 elucidates the models of teacher knowledge which underlie this study, starting with Shulman (1986) who was the first to investigate the modern view of types of teacher knowledge. Shulman's model was generic to all content areas, as he did not satisfactorily expound the variables that may affect teachers' knowledge in specific content areas. As a result, this has led to some modifications to Shulman's model of teacher knowledge to accommodate different content areas. Ball, Thames and Phelps (2008) proposed the mathematics knowledge for teaching framework as a modification to Shulman (1986), which entirely focussed on mathematics teaching amongst primary school preservice teachers. An overview of the models of teaching has been presented in section 3.4, which includes ideas from other models of teacher knowledge other than the stated two. The chapter ends with a chapter conclusion in section 3.5 by citing the purpose of a conceptual framework in a study. It also highlighted the chosen model of teacher knowledge which will be used as lenses in data analysis in this study.

3.2. The teacher knowledge conceptual framework

Teacher knowledge has been under study by academics for as long as there was teacher education, but there has been no theory developed on it up to now. The absence of a theory of teacher knowledge has engendered multiple conceptual frameworks which attempt to give structure to the study of teacher knowledge. Researchers contemplated various concepts and created relationships with them in order to find answers to research problems. A framework in research is a well-structured resource that portrays and identifies key research concepts and how the concepts are connected to each other. Such a structure would be used to develop data analysis procedures and draw conclusions.

3.2.1. Conceptual framework definition

A conceptual framework consists of a collection of logical and interrelated concepts that guides one's research study, determines the premise one will measure and the kind of relationships one will look for. The purpose of a conceptual framework is to describe and support the concept of teacher knowledge by explaining phenomena as well as providing a context for analysing data. In some cases, where there is an existing theory, then a theoretical framework is the one to be used to direct the conduct of the study, effectively leading to generalisable results. However, if the proposed theory does not fully address the research problem, then it may be adjusted, giving rise to a conceptual framework.

The teacher knowledge conceptual framework is comprised of several models that explain it. Models are defined as simplified and schematic forms of symbolic representations of phenomena which assists researchers to express concepts that may be abstract in nature. Models are also used to determine the possible interrelationships of concepts. In this study, Shulman’s model was considered as the default model, being the starting point of the modern view of teacher knowledge constructs. However, Shulman’s model was not used in this study for the following reasons: his contributions were broad-based to fields of psychology, medicine and science, and the pedagogical knowledge domain encompasses generic teaching knowledge that is not focussed on content needed in mathematics teaching.

3.2.2. The mathematics knowledge for teaching framework

The mathematics knowledge for teaching framework is the most appropriate framework for informing and analysing teacher preparation programmes for preservice and in-service teachers (Gess-Newsome & Lederman, 1999). It is a framework that has been widely used to categorise knowledge domains needed for mathematics teaching, as well as to determine their existence in data analysis. It is a practice-based approach, which means it focusses on teaching mathematics teaching. It is based on the premise that teachers are required to possess content knowledge of the mathematics concepts and knowledge of how engaging these concepts in teaching mathematics. The mathematics knowledge for teaching framework also expertly expounds the processes by which content knowledge and pedagogical content knowledge are established and situated in a normal learning environment. Oftentimes, prospective teachers of mathematics might indeed know subject matter, however, transforming that knowledge into meaningful instructional knowledge understandable to learners is something different.

Earlier, it was shown that a conceptual framework may consists of models which are appropriately used to facilitate understanding of concepts and variables in a particular study. Secondly, because a conceptual framework is specific to a unique research study, it is something that may not be readily found in literature. Modifications to existing models may have to be done to create a fitting conceptual framework. The model by Ball, Thames and Phelps (2008) was used as the prominent conceptual framework in this thesis after careful examination of my thesis title and research problem, as well as identifying key variables in the concept of teacher knowledge. These variables are content and pedagogical content knowledge, both of which play a vital role in moulding competent preservice teachers for classroom teaching. A well-managed conceptual framework goes a long way to strengthen the structure of a given research study in terms of data collection aspects and the scope of data analysis. This is achieved by connecting the researcher to an existing body of knowledge. A focus on specific concepts properly defines the scope of data, which reduces the possibility of generalisation of results. The next section gives a detailed account of the models of teacher knowledge, as well as the illustrations of models by Shulman (1986) and Ball, Thames and Phelps (2008).

3.3. Models of teacher knowledge

The conceptual framework of teacher knowledge has been explained through many established models, and all these elaborate on Shulman's (1986) ground-breaking work (Fennema & Frank, 1992; Gess-Newsome, 1999a; Grossman, 1990; Banks, Leach & Moon, 1999; Ball, Thames & Phelps, 2008). It is noteworthy that the models by Fennema and Frank (1992), Gess-Newsome (1999a), Banks, Leach and Moon (1999) and Grossman (1990) had one thing in common: pedagogical content knowledge is the culmination of the three knowledge domains of content knowledge, knowledge of context and pedagogical knowledge. The way these three were intermixed leading to pedagogical content knowledge depends on what Gess-Newsome's (1999a) explained as the dual taxonomies: integrative and transformative models. The integrative taxonomy places pedagogical content knowledge at the intersection of the three knowledge types. A good teacher in this category is one who possesses well-organised individual knowledge to which he/she can effortlessly tap into during the lesson. In the transformative taxonomy, the knowledge of pedagogy, context and content are infused into pedagogical content knowledge, which then becomes the new and sole knowledge type to be used in instruction. Under the transformative taxonomy, a competent teacher should have well-developed pedagogical content knowledge for every concept that they must taught, without explicit reference to the original constituents of teacher knowledge.

Neagoy (1995) had a different perception of pedagogical content knowledge from others. Indeed, he identified fairly the usual three teacher knowledge types of pedagogical, content and learners (akin to context), but he treated pedagogical content knowledge itself to be on par with teacher knowledge. She stated that, "pedagogical content knowledge is not one among these sets of knowledge, but rather, contains them all" (Neagoy, 1995, p.19). This study does not dispute Neagoy's (1995) standpoint about the central and absolute role of pedagogical content knowledge, but there are still other knowledge types which exist independent of pedagogical content knowledge. These are knowledge of students, knowledge of curriculum and knowledge of school contexts, among others.

The models by Shulman (1986) and Ball, Thames and Phelps (2008) are slightly different and do not fit into the preceding descriptions as they excluded general pedagogical knowledge as one of the knowledge types. Moreover, even though pedagogical content knowledge results from the intercourse of pedagogy and content, it is still treated as one of the types of teacher knowledge just like content knowledge. This fact is unique to Shulman (1986) and Ball, Thames and Phelps (2008) models only. In this study, pedagogical content knowledge does not occupy the central part of all knowledge types. Also, general pedagogical knowledge was excluded since teaching is rooted in content-specific pedagogy, which is pedagogical content knowledge. The purpose of presenting these models in this section was to create a firm foundational base to explore the mathematics knowledge for teaching of preservice teachers in this study. By addressing the two models of Shulman and Ball, Thames and Phelps, the intention was to draw the similarities between them so that the researcher can understand the conceptual base of this study as a basis for analysis of data. The next sub-section is a presentation of Shulman's (1986) model.

3.3.1. Shulman's (1986) teacher knowledge model

The skills and knowledge that teachers use in the classroom is a major contributor to effective teaching and learning. Thus, in 1986, Shulman claimed that the emphasis on mastery of pedagogical and content knowledge was addressed unconnectedly. He was of the idea that the thrust of teacher education should be to approach pedagogical and content knowledge jointly. Out of this quest, pedagogical content knowledge was born, which represented a special mixture of pedagogy and content. His initial description of teacher knowledge included seven types: knowledge of learners; knowledge of educational contexts; knowledge of educational ends, values and purposes; pedagogical knowledge; pedagogical content knowledge; curriculum knowledge; and content knowledge. As explained, the first four knowledge categories under Shulman were concerned with general forms of knowledge of teachers that dominated the 1980s teacher education programmes. Shulman dropped them in his later work because they were generally applicable to a wide educational setting. He put emphasis on the last three knowledge types, which had something to do with content and referred to them as the pre-determined domains in teacher education study, "a blind spot with respect to content that characterises most research on teaching" (Shulman, 1986, p.7). It is these three types of knowledge that Shulman incorporated into his teacher knowledge model, illustrated in Figure 3.1.

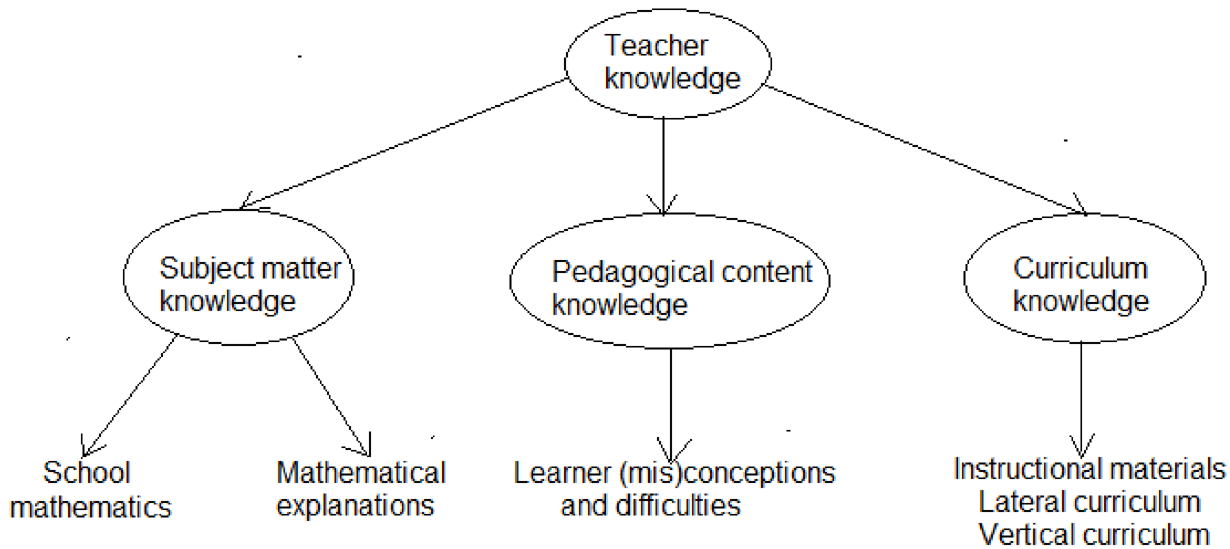


Figure 3.1. Three types of teacher knowledge according to Shulman's (1986) model.

Below each type of knowledge in Figure 3.1 are some (and not all) of the sub-types of teacher knowledge, though it is not exhaustive. A full description of each of these three types and their sub-types are alluded to in the next three sections.

Content knowledge

At higher education institutions, prospective teachers are taught advanced mathematics, some of which have direct connections to what they will teach after qualification, and some do not. The former is mathematics content knowledge for teachers, while the latter is content knowledge for mathematicians. Shulman also classified the underlying structures of mathematics in its entirety by saying content knowledge for teachers transcends mere mastery of the basic facts and concepts in any given topic (Shulman, 1986, 1987; Grossman, Wilson & Shulman, 1989). Shulman (1986) further posited that knowledge of the subject content knowledge supersedes knowing facts and concepts. Teachers need to figure out the principles and salient underlying structures of each topic. Knowing mathematics topics should be accompanied with an understanding of why these concepts are so. Ideally, to show understanding of content knowledge, teachers are expected to justify inclusion or exclusion of certain topics in a given discipline. Hence, it can be inferred that the classification of content knowledge is composed of teachers' knowledge of what to teach. In literature, it has been shown that content knowledge is necessary for effective teaching as it affects lesson planning and assessment and feedback thereof (Baumert, Kunter, Blum, Brunner, Voss & Jordan, 2010; Shulman, 1987).

To preservice teachers, content knowledge is useful for the selection of relevant learning materials and textbooks (McNamara, 1991). Furthermore, McNamara posited that teachers with low content knowledge may sometimes avoid teaching topics they perceive difficult. This is what Shulman (1987) defined as content knowledge. Content knowledge instruction gets emphasis in teacher-training, on the basis that teachers cannot teach what they do not know in reality. A preservice teacher who apparently lacks subject matter knowledge finds it hard to cope with other knowledge types during teacher-training (Brown & Borko, 1992). In some countries, new teachers are required by law to sit a qualifying examination in content knowledge as a pre-requisite for getting a position to teach. The mastery of content knowledge, or lack of it, is often acute and prevalent among novice and preservice teachers. This comes about when teacher education focuses on the pedagogy and methods of teaching relative to content knowledge, especially in mathematics education (Onwu & Mogari, 2004; Benken & Brown, 2008; Mji & Makgato, 2006).

Preservice teachers rely mostly on the content knowledge they acquire in the undergraduate mathematics content modules and their own schooling mathematics experience. According to Sam (2005), if teacher candidates exit teacher education with inadequate mathematics content knowledge, it will not be easy to gain it when they start practicing. Pedagogical content knowledge no doubt can be acquired as teaching experience increases. Nevertheless, there is some truth that teaching experience indeed brings about positive effects on content knowledge (Banks, Leach & Moon, 1999). Hence, a balanced view is that novice teachers build on the fundamental content knowledge which they acquire in teacher education to enrich this content knowledge as years of teaching goes by. Thus, beginning teachers ought to commit themselves to keep current with changes in content knowledge in their respective disciplines.

The performance of teachers in mathematics especially, depends on degree of mastery of subject knowledge in their possession. As for preservice teachers, their fluency in content knowledge reflects the teacher training which they underwent (Tatto, Schwille, Senk, Ingvarson, Rowley, Peck, Bankov, Rodriguez & Reckase, 2012). When school principals seek to engage newly qualified mathematics teachers for instance, they look for those who have specialised in the subject and are able to use content knowledge to become proficient in teaching school mathematics (Hill & Ball, 2004; Benken & Brown, 2008).

Pedagogical Content Knowledge

A unique and personal way of mixing pedagogical and content knowledge gave rise to a new knowledge type which Shulman (1986) termed pedagogical content knowledge. It is defined as the teachers' specialised knowledge concerned with how to teach specific topics in a way that is understandable to learners being taught that topic. Teachers can facilitate children's learning of a concept by using appropriately selected instructional strategies and vivid explanations that counteract learners' challenges to learning.

> Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. (Shulman, 1987, p.4).

Pedagogical content knowledge is more than an integration of pedagogy and content but is a unique type of knowledge quite different from its constituents. Pedagogical content knowledge is contextualised to each topics and specific to individual teachers and represents teachers' smart ways of expressing concepts in comprehensible way to the ones taught. On the other hand, good content knowledge makes knowledge comprehensible to teachers themselves. In reality, the pre-requisite to a well-developed pedagogical content knowledge is good content knowledge. Pedagogical content knowledge understanding hangs on a deep understanding of content knowledge (Nathan & Petrosino, 2002; Hill & Ball, 2004; Piccolo, 2008). Nevertheless, content knowledge on its own is insufficient for good teaching. For instance, teacher candidates who have already earned mainstream Bachelor of Science degrees were observed to have difficulties connecting their rich content knowledge to classroom practice (Nicol, 2002).

Teachers ought to be skilled at interpreting concepts to their learners and that skill is rooted in the knowledge of how topics are connected and how they anticipate learners' challenges to learning (Ball & Bass, 2000 cited in Davis & Simmit, 2006). Moreover, pedagogical content knowledge also encompasses teachers' understanding as to what renders the teaching and learning of certain topics to become difficult or easy, and the anticipation of misconceptions that learners bring to class in the teaching and learning of those topics. By being a metamorphoses of content and pedagogy, pedagogical content knowledge encompasses knowledge of instructional strategies, knowledge of teacher explanations, knowledge of assessment, knowledge of learners' difficulties and misconceptions and knowledge of subject matter.

One of the debates in literature pertains to how and under what circumstances teachers acquire pedagogical content knowledge. Borko and Putman (1996) posited that pedagogical content knowledge is experiential knowledge, which is directly related to the practice of teaching. Contrary to this, is the theoretical view, which claims that pedagogical content knowledge is acquired through structured in-service and preservice teacher training. By the time preservice teachers exit teacher education, they would have amassed meaningful pedagogical content knowledge through school teaching practicals, methodology modules, general pedagogy modules and several years of their own school learning (Bailey, 2014). This study acknowledges the theoretical development of pedagogical content knowledge thus; this study has attempted to explore the pedagogical content knowledge of preservice teachers at the point of exit from teacher training. This is corroborated by Carnoy, Chisholm and Chilisa (2012) who said emphasis should be placed on preservice teacher-training so that they develop pedagogical content knowledge before they qualify.

Curricular Knowledge

Knowledge of the curriculum is the last sub-domain of teacher knowledge under pedagogical content knowledge, which Shulman (1986) defined as follows:

> … represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (p.10).

The content knowledge in any discipline must find a place in the broader perspective of a program of study. In brief, curriculum knowledge is what teachers ought to teach at a certain level of study, which requires a good realisation of learners' espoused potential and the national policy expectations. Teachers teach based on the curriculum of a particular grade and program of study which they interpret and contextualise. Curriculum knowledge also includes the various teaching and learning resources which teachers make available to teach specific mathematics content. Shulman's (1986) description of curriculum knowledge also encompasses identifying relevant materials and textbooks specific to teaching given content areas. There exist two categories of curriculum knowledge that Shulman considered important for teaching, which are vertical and lateral curriculum. Lateral curriculum entails content knowledge of the curriculum in one situation relating to other topics or subjects being taught to learners (Shulman, 1986). Vertical curriculum infers to familiarity with what learners are taught in one topic or course which should connect their knowledge to grades and topics already been taught or are yet to be taught in later years in the same content area (Shulman, 1986).

Through the passage of time, what contributes to curriculum knowledge may change as new curricula are implemented by the policymakers. In the South African education landscape since independence in 1994; there was the Outcome Based Outcomes philosophy from 1994 up to 2005. Thereafter, the Curriculum 2005 was introduced, followed by the National Curriculum Statement in 2007. At present South Africa has the Curriculum Assessment Policy Statement which began in 2012 which went beyond defining what is to be taught by also setting out how teachers should plan, teach and assess.

Overview of the types of teacher knowledge by Shulman (1986)

Shulman (1986), just like Ball, Thames and Phelps (2008), focussed the basis of their quest for teacher knowledge domains on the preservice teachers. Both were involved in teacher education at their respective universities in psychology and mathematics respectively. Thus, there are many references were made to preservice teachers' knowledge in their models. Shulman commented that preservice teachers lack the know-how of using curriculum material and lack the necessary skills in interpreting and engaging curriculum demands. Inadequate curriculum knowledge implies limited pedagogical content knowledge in mathematics, as the two are closely related (Beyer & Davis, 2012; Roselle & Wilson, 2012). Analysing curricula material for planning lessons is instrumental in teaching, which is initially learnt in methodology modules at higher education institutions. Knowledge of curricula is needful in the South Africa context where curriculum-planners occasionally change, and at short notice. This often leads to some mathematics topics coming and going with the curriculum changes, for example, mathematical induction gave way to calculus at high school with the introduction of the National Curriculum Statement of 2007. Already we perceive a close connection between curriculum knowledge and content knowledge. New topics may be added to the current curriculum which teachers must be able to teach. Either teachers receive training, or they do self-adjustments to accommodate the new topics.

The three categories of knowledge are tightly interwoven, such that the development of one supports or depends on mastery of another. Pedagogical content knowledge is known to support both curriculum and content knowledge development. Knowing how to teach certain concepts leads to better interpretation of the curriculum and content included in that curriculum. Also, pedagogical content knowledge is based upon having a broad and deep understanding of mathematics content and the curriculum (Plotz, Froneman & Nieuwoudt, 2012). Lastly, once teachers can fit the current topics into the broader spectrum of an educational program, teachers close in on those concepts, thereby leading to improved practices in content and how to teach that content.

In this sub-section, it was shown that curriculum knowledge requires teachers to be skilled in identifying resources necessary for teaching specific mathematics content and where that content fits into the broader scope of learners' program of study. With curriculum policies changing with such rapidity, teachers need to keep pace and remain current with the expectations of the policy-planners. Teacher-trainers need to make necessary adjustments in teaching prospective teachers, by addressing the current curriculum practices in the teaching of methodology modules. According to a study by Beyer and Davis (2012), preservice teachers lack experience in using and implementing curriculum materials when they begin their teaching career, as a result of inadequate training. Thus, under Shulman's model, successful school teaching commences with the teacher recognising what is to be learnt in a particular program of study, how the identified content is to be taught and the manner in which teaching with meaning has to take place.

3.3.2. The model for mathematics knowledge for teaching

From the inception, many researchers have taken Shulman's (1987) model of teacher knowledge to describe specific content knowledge of teachers in diverse disciplines. One such model was developed by Ball, Thames and Phelps (2008) which deals with teaching school mathematics. The model was developed by investigating the actual work of teaching mathematics in primary schools and each component in their framework has been empirically tested in classroom practice. This model came to be known as the mathematics knowledge for teaching, which helped in identifying the elements that were fundamental to mathematics teaching. Ball, Thames and Phelps (2008) studied the knowledge of teaching mathematics in classroom practice instead of studying it in theory, in order to analyse the mathematical knowledge demands of teaching. Figure 3.2 illustrates the agreed domains in this model (Ball, Thames & Phelps, 2008; Hill, Rowan & Ball, 2005). The mathematics knowledge for teaching model features only two knowledge domains in its structure, which are the subject matter knowledge and pedagogical content knowledge.

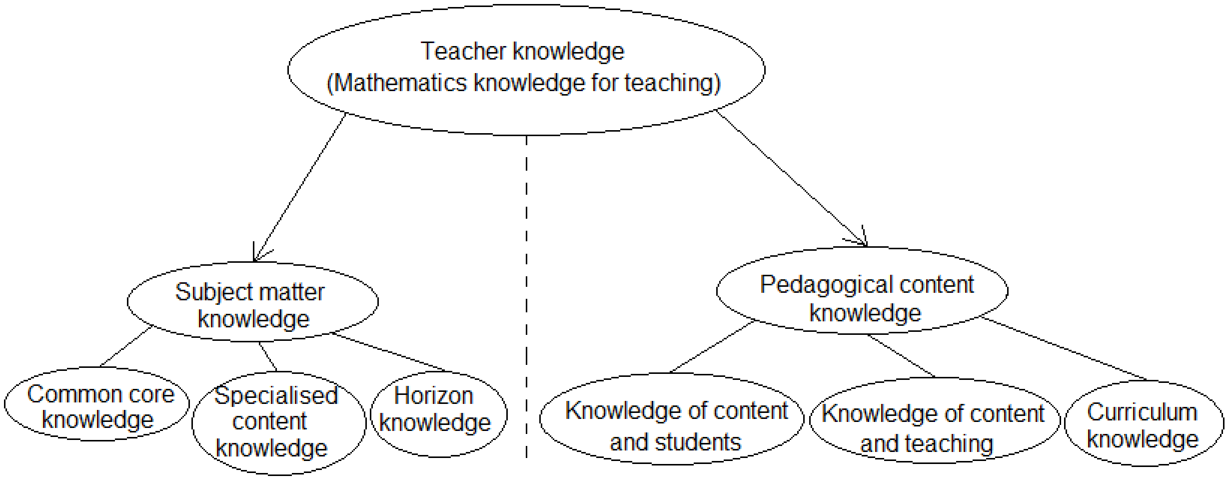


Figure 3.2. Model of teacher knowledge proposed by Ball, Thames and Phelps (2008).

The common content knowledge, the specialised content knowledge and the horizon knowledge are the three sub-domains identified in this model under the category of subject matter knowledge. Common content knowledge refers to the mathematics skills and knowledge pertinent to all mathematicians in mathematics-related professions. Mathematics teachers possess it too, but then this knowledge type is not confined to teaching only. It is commonly used by other mathematicians, for example, solving a mathematics problem using a particular algorithm and defining a mathematics concept. The specialised content knowledge is denoted by mathematical skills and knowledge that only mathematics teachers use for teaching and learning purposes. Teachers portray specialised content knowledge skills in instances where they must identify learners' error patterns or assess the logic of learners' non-routine solutions. Only teachers possess this type of knowledge, hence it plays a key role in teachers' overall mastery of content knowledge required for teaching. Specialised content knowledge is the most significant of the three, thus Ball, Thames and Phelps (2008) allocated a very large area to this domain in their diagrammatic representation of teacher knowledge categories. The third sub-domain, horizontal knowledge, is the teachers' awareness of the salient relations among mathematics topics within a grade. The connection of topics across grades in a program of study also forms part of horizontal knowledge (Ball, Phelps & Thames, 2008). Horizontal knowledge is about teachers' realisation of what mathematics knowledge they are currently teaching looks like in grades above or below.

Together with subject matter knowledge, pedagogical content knowledge is the other type of teacher knowledge according to the model by Ball, Thames and Phelps (2008). Pedagogical content knowledge has three sub-domains, which are knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. The knowledge of content and students is a conglomeration of teachers' knowledge of subject matter and of learners. Mainly this know-how equips teachers' to be familiar with how children learn and think for any given topic, as well as a sound anticipation of learners' challenges to learning (misconceptions, possible errors and difficulties). Knowledge of content and teaching treads on the grounds of teachers' knowhow of appropriate teaching strategies are commensurate with the topic under consideration and learners' calibre. Teachers obviously evaluate instructional strategies used to teach specific topics to be able to identify appropriate methods and procedures (Ball, Thames & Phelps, 2008). These two previously mentioned sub-domains square up to Shulman's (1986) precepts of pedagogical content knowledge: "the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" as well as, "the ways of representing and formulating the subject that make it comprehensible to others" (p.9).

Finally, knowledge of content and curriculum is about teachers' knowhow of mathematics as it is taught in the broader context of the national curriculum. Included also is knowledge of how different teaching and learning support materials could be employed to teach different aspects of the curriculum (Mudaly, 2016). The curriculum knowledge category of Shulman (1986) is subsumed into the knowledge of content and curriculum in the mathematics knowledge for teaching model (Ball, Thames & Phelps, 2008). It is worth noting that in South Africa, the provincial governments supply schools with annotated work schedules and pacesetters for each grade level. These provide in detail the topics to be covered, their duration and sequencing. What would remain then is for teachers to plan daily lessons within the ordered topics in a coherent and conceptual manner. The idea behind studying these knowledge domains was to establish the basis for effective mathematics teaching and teachers ought to be well-versed in all the knowledge types under the model by Ball, Thames and Phelps (2008).

As much as these sub-divisions of teacher knowledge exists on paper, in reality, it may be difficult at times to isolate them from each other. More so, the distinction between the knowledge of content and students, and the specialised content knowledge is rather arbitrary. Both involve teachers having a unique mathematical interpretation to assist learners to understand the concepts being taught to clear any difficulties of learning if they exist. An example is best suited to highlight what one is trying to portray. In teaching the concept of special angles in trigonometry to learners, a smooth transition happens across all knowledge types, except for horizontal and curriculum knowledge. Stating the values of the special angles calls upon teachers' command of the common content knowledge, whilst recognizing learners' errors as they relate to the special triangles requires specialised content knowledge. Teachers' recognition about which initial special angles would expose learners' challenges to derive further special angles entails good mastery of knowledge of content and students. Finally, teachers' decisions about what to do in order to alleviate or rectify the identified learners' errors and misconceptions to learning special angles involve the use of knowledge of content and teaching.

A salient distinction is observable too between horizontal knowledge and the knowledge of content and curriculum. Both speak about the need for teachers to understand the span of particular mathematics content in the context of the given curriculum in a program of study. However, unlike horizontal knowledge, knowledge of content and the curriculum spells out teachers' knowledge of teaching materials that teachers make available when teaching given topics contained in the current curriculum. In this study, the focus was on specialised content knowledge, common common knowledge, knowledge of content and students, and knowledge of content and teaching in the teaching of trigonometry. Knowledge of content and curriculum and horizon knowledge were omitted as they do not reflect individual teacher efforts since they are unilaterally supplied by the Department of Basic Education.

3.4. Overview of teacher knowledge models

In this chapter considering Shulman's (1986) model of teacher knowledge was an obvious choice as it is the foundation upon which other models were built. Based on Shulman's (1986) model, Ball, Thames and Phelps (2008) developed the mathematics knowledge for teaching model, in order to improve researchers' understanding of teacher knowledge at that time. To situate data collection and analysis and the discussion of findings, the present study used the conceptual framework by Ball, Thames and Phelps (2008). As it was in the original consideration of the model by Ball, Thames and Phelps (2008), this study has also investigated university preservice teachers' understanding of the mathematics knowledge for teaching in a specific topic in mathematics.

Nevertheless, the two models discussed in this chapter have not been without perceived weaknesses. These frameworks exclusively focussed on studying individual teachers without accounting for the roles played by school teaching contexts and culture in teacher knowledge study (Ellis, 2007). Also, teachers' thinking, learners' temperaments in the learning process and culture have not received satisfactory exploration. All these contextual factors have the potential to influence teacher's knowledge from time to time, and if not given due consideration, the perceived teacher knowledge is superficial. Furthermore, teacher knowledge was handled as if it is real and static, when yet it is known to be fluid and dynamic (Hashweh, 2005). Teacher knowledge changes constantly through a teacher's career, but Shulman viewed it as externally determined and fixed. Banks, Leach and Moon (1999) differed from Shulman in this regard by acknowledging that teacher knowledge is dynamic. Also, the model of mathematics knowledge for teaching was highly contextualised and culturally specific to America, hence the proponents of that model did not guarantee results to other unknown cultural groups. School contexts and teachers' beliefs still do not have a spot in the mathematics knowledge for teaching model up to now, although, as suggested by other researchers, teacher beliefs do influence the way teachers make decisions in teaching (Fennema & Franke, 1992; Grossman, 1990).

In response to the previously mentioned shortcomings of the Shulman (1986) and Ball, Thames and Phelps (2008) models of teacher knowledge, other researchers effected slight changes to their models, but were still based on Shulman's model. Two of these by Grossman (1990) and Fennema and Frank (1992) included aspects of teacher beliefs as being instrumental to pedagogical content knowledge. But beliefs as a teacher knowledge type was not directly connected to the centralised pedagogical content knowledge, hence the link was not robust. School context knowledge was one of the three types of teacher knowledge introduced in the models by Fennema and Franke (1992) and Grossman (1990), together with other models by Banks, Leach and Moon (1999), Gess-Newsome (1999a) and Neagoy (1995). Context knowledge influences teachers' instructional practices through teachers' experience in teaching, school settings and learners' thinking and learning processes. For Shulman (1986) and Ball, Thames and Phelps (2008), their closest to context knowledge is teachers' knowledge of the curriculum.

The common defining feature for the five previously mentioned models was the fact that pedagogical content knowledge was central, being a culmination of pedagogical, context and content knowledge types. Pedagogical content knowledge represents teachers' unified conceptualisation of distinct and diverse teacher knowledge categories. These categories are only useful to instruction when they are transformed to pedagogical content knowledge. Banks, Leach and Moon (1999) used the term teachers' personal constructs to refer to the same pedagogical content knowledge, which underpins teachers' unique and dynamic professional knowledge. By so doing, they differed immensely from Shulman (1986) and Ball, Thames and Phelps (2008) in that the latter never considered pedagogical content knowledge to be the ultimate, but just as one of the types of teacher knowledge. Furthermore, general pedagogical knowledge does not feature as a stand-alone type of teacher knowledge but is embodied in pedagogical content knowledge. In the present study, the conceptual framework chosen is the model of Ball, Thames and Phelps (2008), with minor variations. This model portrays pedagogical content knowledge as a stand-alone entity, on par with content knowledge. Knowledge of the curriculum and other contextual factors are loosely classified under the sub-domains of subject matter knowledge and pedagogical content knowledge. Knowledge of school contexts and of learners were excluded from this conceptual framework by virtue of preservice teachers having no consistent and full-time school teaching experiences.

However, researchers feel Shulman (1986) and Ball, Thames and Phelps (2008), were justified in their consideration of teacher knowledge types, as they were based on the context of preservice teachers. The other mentioned researchers' constituents of pedagogical content knowledge clearly display that they focussed on practising teachers, thus their mention of culture, school contexts and knowledge of learners.

3.5. Conclusion

This chapter attempted to highlight what a conceptual framework is, its composition, its difference with theoretical framework and how it strengthens a research study. If there exists an adequate theory covering a study, then researchers make use of a theoretical framework. If not, theory exists, researchers rely on concepts to create a relationship with them, leading to the use of a conceptual framework. This study saw that a conceptual framework consists of models used for a particular study which facilitates the understanding of concepts and variables in that study. Then, since a conceptual framework is specific to a particular research study, it is something that is not readily found in literature. The conceptual framework is engendered through a literature review. Hence this chapter was preceded by a chapter on literature review and the scope of the thesis study. This conceptual framework has been developed after careful examination of my research title and research problem, as well as key variables in the concept of teacher knowledge. A conceptual framework is made up of one or more models which attempt to describe it in a symbolic way.

After careful consideration of Shulman's (1986) framework, this study settled for the model by Ball, Thames and Phelps (2008), which is termed the mathematics knowledge for teaching. This study has focussed our investigation of preservice teachers' knowledge in trigonometry on the two knowledge bases of subject matter knowledge and pedagogical content knowledge, which is based on the model by Ball, Thames and Phelps (2008). These are known to be instrumental to preservice teachers who are still undergoing teacher-training. General pedagogy, knowledge of learners, curriculum knowledge, school contextual factors and teachers' beliefs were excluded from this study as they have not been part of frameworks by Shulman (1986) and Ball, Thames and Phelps (2008) on preservice teachers too. These are the domains of practicing teachers too, hence falls outside the scope of this study. For preservice teachers, only school practical teaching, content modules, methodology modules and their content knowledge as learners themselves are the major contributing factors to their knowledge of teaching development. An overview of the models of teacher knowledge was presented, from which some striking differences and similarities among them. The placing of pedagogical content knowledge as being central or not is one such difference. Teachers' beliefs, experience, knowledge of learners and other school contextual factors were included in some and excluded in others. Shulman (1986) and Ball, Thames and Phelps (2008) had a similar approach to teacher knowledge where general pedagogical knowledge was not part of the final framework. Other proponents of teacher knowledge had similar conceptions of teacher knowledge types, whereby the climax of all teacher knowledge comprehensions was pedagogical content knowledge (Neagoy, 1995; Banks, Leach & Moon, 1999; Gess-Newsome, 1999a; Grossman, 1990; Fennema & Franke, 1992). This pedagogical content knowledge was a product of general pedagogical and content knowledge domains. However, even though other researchers developed their own models which differed from those of Shulman (1986), they all used his work as a springboard for their studies.

CHAPTER 4: THE RESEARCH DESIGN AND METHODOLOGY

4.1. Introduction

This chapter presents an outline of the research plan that was applied to realise the purpose of this study. This chapter begins with an explanation of the research paradigms, which informs the three research designs that exist in educational research. Then the qualitative research design was identified as the one which sufficiently describes the present study. A description of the research design chosen for this study follows, which highlights the unit of analysis, sources of data, the pilot study undertaken and the rest of the general research procedures. These research procedures included sampling techniques, administration of data collection instruments and the role of the researcher in order to minimise bias. An explanation of the research methodology is given next, which specifies the context within which the research procedures are conducted. This is followed by measures of quality which make the study trustworthy to the reader and the ethical considerations that were put in place to safeguard the participants against abuse of their personal rights.

4.2. The research paradigm

A paradigm is a pattern or a set of practices and beliefs that guide a field of study (Morgan, 2007). The term paradigm originated from the Greek word “paradeigma” which means pattern. Specifically, research paradigms are the various approaches to conducting a research that have been verified and practiced by researchers for many years. The paradigm in which the researcher operates in, whether consciously or subconsciously, set the expectations of a research study. All researches make use of at least one research paradigm as a rigorous guideline to conduct the research and to take on the research venture in a manner that is most valid and appropriate. Attempt to categorise researches into types of paradigms led to three major paradigms that clearly stand out: interpretivism, positive and pragmatism. Each of these can be categorised further by examining their: ontology, epistemology and methodology. Positivism and interpretivism are like two extrema on the paradigm continuum to which many research topics broadly fall into. The occasional need by seasoned researchers to amend their philosophical assumptions over time leads to pragmatism, which lies on the middle of the positivism-interpretative continuum.

Interpretivism is paradigm that seeks to understand the ever-changing social order. The interpretive paradigm construe that each individual constructs his/her own view of the world based on experiences and perceptions. Researchers tend to "rely more upon the participants' views of the situation being studied and recognises the impact on the research of their own background and experiences" (Creswell, 2003, p8). Krauss (2005) refers to the interpretivist researcher as most likely to rely on the use of qualitative methodologies and analyses, and in some cases, the mixed-methods inquiry. Positivism dominates in the natural and physical sciences by assuming that phenomena consists of independent facts about a single perceivable reality. Positivists perceive knowledge acquisition to describe the phenomena where observation and measurement are central scientific endeavour (Krauss, 2005). To the researcher, observable reality out there is orderly and predictable, waiting to be discovered.

Pragmatist researchers are process- and semantic-driven in their pursuit of answers to the research questions by focussing on the 'how' and 'what' of a research problem (Creswell, 2003). The pragmatic researcher takes the lead in research by deciding what he/she wants to research on, guided by his/her personal value systems. Appropriate research methods are those that help to answer the research question at hand and which leads to action being effected. Pragmatists differ with the positivist position in that truth about the real world can be investigated solely by the scientific method. To interpretivism, pragmatism differs by limiting the nature of subjectivity of this approach and the overmuch research bias thereof. In most cases, pragmatists may combine both the positivist and interpretivist paradigms within the scope of a single research study in response to the research questions.

This study was framed within the interpretivist paradigm, as it assists in our understanding of the contemporary social world of preservice teachers' mathematics knowledge for teaching. Interpretivist paradigm acknowledges the subjective nature of human action and understanding which is required to explore preservice teachers' knowledge mastery. This study taps on benefit of interpretivism that allows the use of researchers' experiences to construct and interpret phenomena. This paradigm enables researchers to view reality through the experiences and perceptions of the participants. In seeking the answers to the research questions, the researcher who follows interpretive paradigm uses those experiences to construct and interpret his understanding from gathered data. Specifically, the interpretivist paradigm support researchers to explore the world by interpreting the understanding of individuals. This study sought to explore preservice teachers' understanding and development of the mathematics knowledge for teaching in trigonometry at a rural-based university in South Africa.

4.3. The research design

A research design describes the proposed plan of action for conducting a research study. It encompasses procedures that provide details of the planned activities of the various stages within a research study (Creswell, 2009; Yin, 2009). The research design guides the researcher with the preliminary research, collection of data, the sampling techniques and the analysis of results. The three categories of research methodologies are quantitative, qualitative and mixed methods. Though these three are treated as disjoint in their approach to guiding a research study, they should not be perceived as polar opposites. They are best viewed as the extreme terminals on a continuous line. Some studies tend to be more quantitative relative to qualitative in varying degrees and vice-versa. It is rare to find studies that are purely quantitative or qualitative. This gives rise to mixed methods research design, which integrates both aspects of quantitative and qualitative approaches and straddles the median of the said continuous line. The quantitative research design concentrates on theory testing through cross-checking the behaviour of variables that explain substances or social behaviour. Social context is seen as objective, quantifiable and external to the social phenomena. This makes it possible to manipulate variables in order to achieve desired research goals. According to Creswell (2009, p.4), a qualitative design seeks "exploring and understanding the meaning individuals or groups ascribe to a social or human problem". The variables in each and every research study are best examine in their natural settings as much as possible. The world is not 'out there' and separate from those involved in its construction. People are creators of their "social world rather than passive objects" under the qualitative research design (Bryman, 2008, p.34).

This study is an exploratory research geared at investigating final year preservice teachers' mathematics knowledge for teaching the topic of trigonometry, which fits well with qualitative research design. This study is a qualitative research design which uses both quantitative and qualitative data analyses to reach the conclusion of the extent of preservice teachers' understanding of the mathematics knowledge for teaching trigonometry for selected final-year preservice teachers. Quantitative data emanated from participants' scores from the content test, which were to "yield specific numbers that can be statistically analysed" (Creswell, 2005, p.510). Qualitative data were obtained from task-based interviews, lesson plan analyses and classroom observations. In brief, the design for this study took this form: develop data collection instruments; do a pilot test of the instruments; administer the instruments in the natural setting of events; ascertain the quality of data collected; perform data analysis; discuss the findings; and draw the conclusion of the research problem. Figure 4.1 gives an illustration of these procedures.

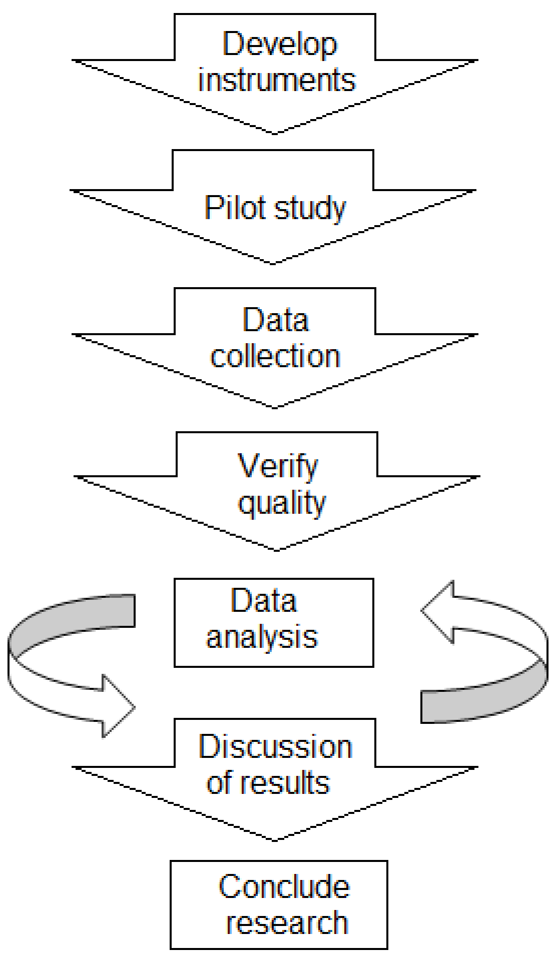


Figure 4.1. The steps in the research design for conducting this research study.

The sketch in Figure 4.1 is a simplified framework of the qualitative research adopted for this study. It is simplified in the sense that in reality the stages may not happen in a linear fashion as shown. Qualitative research designs are highly flexible as initial plans at each stage are likely to change as the study progresses. It is also notable that there is an iteration between analysing data and the discussion of findings. After initial analysis of data, new insights often crop up whenever another analysis of data is performed in qualitative research designs, which in turn effects a corresponding change in the discussion of results. This obviously hinges on the type of the analysis of data as well as the researchers' subjective viewpoint in the analysis of data. That makes data analysis a process.

4.4. Research methodology

Qualitative research designs that were explained in section 4.2 offer diverse strategies of inquiry towards a research problem. These approaches to inquiry are called research methodologies, such as narrative research, ethnography, case studies, grounded theory and others. The case study research methodology was employed to achieve the research purpose of this study. A case study research methodology is defined as an in-depth exploration of a well-defined phenomenon in a population within a space of time. This is made possible by collecting detailed information of a setting using multiple data collection procedures in a confined setting (Creswell, 2009). Thus, case studies make it possible for a researcher to focus on specific phenomena, which was, in this case, the mastery of mathematics knowledge for teaching in trigonometry in a well-defined population of preservice teachers at a higher education institution in South Africa. Moreover, case study methodologies can be used in studying many situations (Merriam, 2001) and "virtually any phenomenon can be studied by means of the case study methodology" (Gall, Borg & Gall, 1996, p.544). The unit of analysis is the major entity that frames what is being studied of analysed. It is the subject of study upon which a researcher may generalise. In this study, my unit of analysis are the preservice teachers from which I will obtain their level of understanding of the mathematics knowledge for teaching. Preservice teachers are the main focus of this study and the conclusion of this study shall say something about preservice teachers' degree of mastery the mathematics knowledge for teaching. The unit of observation is preservice teachers' understanding of mathematics knowledge for teaching. The research questions played a major role in determining the unit of analysis for this study, while the unit of observation is determined by the data collection methods.

Case studies come in different styles and types. Yin (2003) provides a classification of case studies based on their purpose. He identified three case study categories, which are, descriptive, exploratory and explanatory. The descriptive case study gives a narration of phenomena in a research study, as well as the contextual factors in which the phenomena occurred (Yin, 2003). The exploratory case study seeks to reveal situations in which the case being evaluated has not been investigated before and could possibly lead to further research. The exploratory case study was applied to this study, as the researcher sought to investigate the preservice teachers' understanding of mathematics knowledge for teaching in trigonometry at a particular point in time. According to de Vos (2000), case studies are useful tools of exploratory studies, hence researchers still use exploratory case studies in well-planned empirical and real-life situations (Soy, 2006). An explanatory case study seeks to give details of what happens in a particular case, why it happens and often involves verifying existing theory. Furthermore, case study methodologies can also be classified according to the number of cases involved. Yin (2003) differentiates between single case studies where only one phenomenal setting is considered and multiple case studies, in which many variables in different settings are under consideration at the same time in order to address a research problem. This study was a single case study as only the preservice teachers' mathematics knowledge for teaching at one research site was used to find answers to the research problem.

A case study methodology has many positive implications to the researcher and contextual settings. It promotes collaboration between the participants and the researcher, which leads to a better realisation of the actions and perceptions of participants (Baxter & Jack, 2008). Cordial working relations were established between the researcher and the participants as I was both a researcher and lecturer at the research site. This allowed for comprehensive investigation of phenomena within natural settings, as well as to obtain in-depth data from multiple sources. The study took place in 2016 and 2017, a sufficiently long period of time during which I was able to collect data from different sources, that is, content tests, task-based interviews, lesson planning analyses and video-lesson analyses. These provided rich and in-depth information of the context and phenomenon. The long period was able to cultivate trust and build good working relations between participants and the researcher.

Critics of the case study are sceptical that a single case study may not adequately offer a platform for establishing reliability and generality of research conclusions (Yin, 2009). Also, case studies cannot be replicated, therefore research results are not easy to corroborate. And because of that, it implies that data and results remain valid for only that time-bounded phenomenon. Case studies provide slim basis for generalisation since the study is only focussing on a sole case, which is bound by locality and time. Again, case study reports are frequently lengthy and detailed to sufficiently read and decode data. Case studies can be time consuming due to intensive data collection procedures and subsequent data analysis. While Yin (2009) acknowledges that using case studies is associated with possible challenges, he argues that each of these weaknesses can be overcome if researchers plan their work well. He suggests that researchers using case study methodology should report fairly and without bias, and case studies are not formulated for generalisations across populations. Rather, case studies seek to understand a particular behaviour and establish the value of that case to literature.

4.5. Data collection instruments

In any research study, data provides the researcher with the necessary primary evidence to effectively provide the credible findings to a research study, as opposed to speculating what the findings could be. According to Altricher, Feldman, Posch and Somekh (2008), data are immutable, thus the researcher can safely interpret what has been transcribed onto a paper. Usually, three processes in qualitative research are involved in data collection. Firstly, the researcher observes research participants under natural settings. Then he communicates with the participants to find meaning into why they act and behave in a certain way. Thereafter, the researcher attempts to ascribe meaning to the data which were collected. Contrary to quantitative methodologies where data are collected in controlled settings, the researcher has no control over the actions and behaviours of the participants under a qualitative research methodology (Yin, 2009). It is therefore imperative for the researcher to connect the study of participants' everyday lives to the demands of the overall proposed plan of data collection.

As seen earlier, the use of multiple data collection techniques is a defining property of case studies. For instance, Li and Smith (2007) used survey questionnaires and mathematics content tests to investigate a group of prospective teachers' knowledge of mathematics and pedagogy acquired during the time they spent in a teacher education programme. The study conducted by Benken and Brown (2008) used survey questionnaires, content examination and interviews to investigate the effects of integrating mathematics content, general pedagogy and methodology courses on preservice teachers' mastery of pedagogical content knowledge. It has been common practice to assess teachers' competence in content knowledge using paper-and-pencil content tests. Pedagogical content knowledge at its best has been commonly investigated through lesson delivery in a real classroom. Thus, in addition to content tests, the present study used task-based interviews, lesson plan analyses and classroom observations to investigate preservice teachers' mastery of pedagogical content knowledge in trigonometry. The reason for incorporating lesson planning and presentation was that the researcher wanted to get data based on what preservice teachers do during teaching practice in order to illuminate their existing knowledge of content and pedagogical content knowledge. It is fitting to explore pedagogical content knowledge from diverse stand-points due to the complexity of assessing it. Above all, pedagogical content knowledge is unique to each teacher and implicitly expressed. Hence, surety of capturing its presence or absence is by means of dissimilar instruments. Each of the four instruments for this study are presented in the next sub-sections.

4.5.1. The content test

A test on trigonometric knowledge was developed to investigate preservice teachers' understanding and knowledge of selected concepts in high school trigonometry. Paper-and-pencil tests can tap into preservice teachers' mathematics content knowledge much better than other instruments. The conceptual understanding of trigonometry was in line with the perceived conception of teaching and learning of mathematics geared for optimising learner understanding. The content test was intended to elicit the information on the level of subject matter knowledge which preservice teachers have acquired by the time that they exit their training. Content tests are the most profound instrument used to examine teachers' mastery of subject matter knowledge in literature. Furthermore, experts in mathematics education evaluated the test and suggested necessary improvements of the items. The final form of the test is attached in the Appendices section (Appendix E).

Aspects of trigonometry knowledge that were assessed in the content test were: special angles diagrams; deriving and proving identities; solving trigonometric equations; applying the reduction formula; sketching trigonometric functions; and identifying solution sets of inequalities. The content test was formulated by adapting problems from textbooks and past examination papers, as well as some identified school trigonometry problems from literature. In that regard, most of the problems from textbooks and past examination papers comprised the items for this test. Our desire was for participants to handle problems in trigonometry that would be analogous to those they would encounter when they would be teaching the topic in schools.

4.5.2. The task-based interview

Ideally, task-based interviews are defined as a scenario where the interviewee talks during or immediately after answering a question on paper, whereupon the interviewer can probe to seek further clarifications if necessary. In a task-based interview, the interviewer and the participant interact on a given task at hand. Pioneered by Piaget in the 1970s, task-based interviews have evolved by taking the form of structured think-aloud protocols and open-ended prompting (Clement, 2000). Evidence exists testifying that task-based interviews have the potential to reveal the state of participants' ways of thinking and understanding as they solve given tasks in mathematics (Schoenfeld, 1985). That gives rise to a unique interviewer and participant interaction over the given mathematics problem. This type of data collection relies on the participants' capabilities to think aloud and verbalise as they respond to given questions which requires minimal intervention from the researcher, and therefore prevents any of the interviewer-imposed bias. The researcher should also try to minimise interfering with the participants' solution process. It is advisable to wait until the respondent has finished responding before the interviewer intervenes for clarity-seeking. Task-based interviews have for long gained popularity among mathematics education researchers (Boris & Guershon, 2007).

In the preparation of the tasks for the interview, care was taken to avoid compound questions that tend to confound respondents. If that happens, faulty data is generated (Merriam, 2001). Other question-types avoided were leading questions that obviously pre-empt participants' responses. Yes-or-no question types were avoided too since they do not generate much meaningful information, but guesses and thoughtless responding. As such, the interview was composed of eight problems ranging from proving identities, identifying learners' errors, providing expected explanations and anticipating learners' challenges to learning certain concepts. Some of the items in the interview were hypothetical scenarios of learners' experiences in the learning concepts of trigonometry, which participants were then required to construe. This was felt necessary for an instrument that was designed to measure participants' pedagogical content knowledge skills. Task-based interviews may open the door to obtain information on what preservice teachers practically do to give insight into their perceived knowledge of pedagogical content knowledge. Appendix F gives the task-based interview instrument used in this study.

4.5.3. Preparation and analyses of lessons

Lesson plans are defined as teacher's step-by-step intended plan of teaching and learning activities for a particular lesson. As a research instrument, lesson plans have been necessitated by the fact that a lesson plan is an important component of teaching and learning. The design and development of lesson plans have been a feature of teacher education for a long time and have been key in assessing preservice teachers' understanding of knowledge of teaching (Murphy, 2009). The preparation and analyses of lesson plans have been used to investigate the extent of pedagogical content knowledge development amongst preservice teachers since the days of Shulman (1986, 1987). Lesson plans can be an effective indication of the preservice teacher's perceptions of teaching (Murphy, 2009). Preservice teachers should engage in cycles of lesson planning, lesson implementation and lesson reflections in order to improve their knowledge of instruction beyond the current lesson under consideration (Fernandez, 2005). The importance of lesson planning is that it gives focus to a lesson by ensuring that teachers teach the right content in the appropriate ways in the given time period.

Six preservice teachers in their final year of study were tasked with planning a 30-minute lesson during the time they were doing school practice teaching. The participants were at liberty to use any of the resources they usually use to assist them do the lesson-planning, which includes the research site's School of Education lesson plan template (see Appendix H). This was an attempt to make the act of lesson planning as natural as it could be. The drafted lesson plans were analysed for components of pedagogical content knowledge and that information constituted data for the present study. The School of Education's lesson evaluation form was used, with which all participants were familiar (see Appendix I).

4.5.4. Classroom observations

In addition to task-based interviews and lesson plan analyses, one of the best instruments to assess teachers' pedagogical content knowledge is video-lesson observation and analyses (Kapyla, Heikkinen & Asunta, 2009; Ijeh & Onwu, 2013; Rollnick, Bennett, Rhemtula, Dharsey & Ndlovu, 2008; McConnell, Parker & Eberhardt, 2013). Video-recorded lessons have proven to be a valuable tool for capturing teaching episodes and many benefits of using video reviews in teacher education researches have been reported (Coffey, 2014). Video-recordings as data collection tools are reliable because they have the potential and capacity to capture rich and complex events of what happens in the classroom. Investigating pedagogical content knowledge by video-recorded lesson analyses enables the researcher to delineate elements of pedagogical content knowledge that are difficult to document by other means (Janik, Najvar, Slaviik & Trna, 2009). In addition to capturing the complexities of teaching on a permanent electronic record, video-recordings can store the original voice and non-verbal cues of a teaching scenario for a long time without deterioration of quality. Researchers can at any time replay the video-recordings as often as they wish in the analysis of data process. They can also rewind, fast-forward and annotate the video to observe events and elements that might otherwise have eluded them in the initial views.

Based on their planned lessons mentioned in section 4.4.3, the six preservice teachers were tasked to implement the lesson to a real class of learners under video recording. The researcher also took observation notes in each class to complement the video-recordings. These field notes and video recording analyses were the fourth and last data collection instruments for this study. The lesson evaluation schedule for the School of Education was used and is attached as Appendix I.

4.6. The Pilot Study

4.6.1. Definition

In qualitative research, a pilot study is miniature research conducted before the full-scale research project in order to evaluate the feasibility of such a study in terms of cost, time and other things like ethics. It can also be a means to pre-test the structure and administration of instruments of data collection before the main study. It is common practice that a pilot study be conducted only after the researcher has tabulated all the necessary procedures for a research study. These procedures are the research problem, research questions, the research methodology and instrumentation which are to be applied to a research study. The instruments which were trialled for this study were the task-based interview and content test. The other two, lesson plan analysis and classroom observations could not be piloted, as it was not possible to take the chosen participants to the real classrooms where the main study would take place. A thought came to me about making use of peer-group teaching to pilot the two instruments, but I realised that it would be superficial since peers will never behave in any way close to school learners. As an advantage, the official School of Education templates for lesson planning and classroom evaluation tool were to be used to collect data during the formal teaching practice session. Data collection was intended to be done in the participants' natural setting of teaching practice.

4.6.2. Value and purpose of pilot study

The pilot study in this study was conducted in order to shape two of the data collection instruments. Major precepts that were trialled were the time estimates, number of items to be included, the level of difficulties of the items, a balance in the types of questions and bringing clarity to the terminologies used in the two instruments. There was need to identify potential defects in the measurement procedures of the instruments and to detect ambiguous questions in the items of both instruments. Other important issues like ethical consideration were highlighted in the pilot study so that necessary steps could be taken to minimise discomfort or embarrassment that may be experienced as a result of the contents of the instruments. In brief, the pilot study served the purpose of assessing the practicality as well as the appropriateness of the research instruments before the main study was administered.

Pilot studies are valuable in this regard: “You may think that you know well enough what you are doing, but the value of pilot research cannot be overestimated. Things never work quite the way you envisage, even if you have done them many times before, and they have a nasty habit of turning out very differently than you expected” (Blaxter, Hughes & Tight, 1996, p.122). Even though similar research may exist in literature, each study is unique under qualitative research designs, making a pilot study to be a priority. A pilot study bring harmony to the ideal scenario on paper to the practical research environment which exists in reality by bringing to the fore probable challenges that could lead to the demise of the main research. It is a fact that pilot research may not in earnest guarantee the success in main study, however, it has the potential to increase the probability of success. This was the main reason for conducting the pilot study prior to the main study. The pilot study setting, procedure and outcomes are outlined in the next sections.

4.6.3. The setting

Four preservice teachers were selected to take part in the pilot study based on their willingness to participate. However, to avoid possible contamination of data in the potential pool of participants, participants in the pilot study were not the same as those who will be selected for the main research study. They were selected from the then fourth-year students, who, by virtue of pressure of work to complete their studies, were not going to be able to volunteer for participation in the main study.

4.6.4. Procedure

After the participants were briefed of the activity which they were about to undertake, they signed the consent forms. Then the four were gathered in a quiet environment to respond to the content test questions under the supervision of the researcher. Supervision of the administration of the content test was to ensure that only what they have mastered in trigonometry goes on paper, instead of what their colleagues, textbooks, YouTube or Google has to say. Thus, they were not supposed to discuss, consult their notes or share ideas. Calculators were not used in some items, in line with typical questions in school trigonometry where calculators are not to be used in certain concepts. The completed question papers were collected and marked objectively using a marking guide. Based on their performance in the content test, the best- and least-performing participants were called in for the task-based interviews on a one-to-one basis on the following day. The aim was to attain maximum variation in the participants for the interviews. The two instruments were intentionally not timed so that the participants responded until they finished, thus providing a clue to the approximate time to be used for the actual study. A brief analysis of data was carried out after the two interviews to check for elements of pedagogical content knowledge. As explained before, the other two instruments were not piloted as the tools from School of Education teaching practice unit were going to be used.

4.6.5. Results of the pilot study

This section presents the analyses and findings that were obtained from both instruments of the pilot study. Information gained from the content test and how it brought about changes to the instrument were presented first, followed by the interview information and the changes thereof to that instrument.

Content knowledge test

Several adjustments were effected to the content test emanating from the pilot study findings. A challenge was discovered with the duration of the test. The longest participant took over two hours to complete. As a result, the items in this instrument were curtailed so that it could be completed in about one hour. Hence, a total of five items which were duplicating ideas that were already assessed were removed. Most of the items were unaltered, signifying the wording and the structure was above board. The angles in the items of solving trigonometric equations which were initially stated in radian measures were changed to degrees, in accordance with the requirements of mathematics covered at high school in South Africa. Participants had difficulty with applying the reduction formula and expansion of compound angles, however, no adjustments were made to these items because the questions were deemed relevant. The sketching of graphs and solving equations were underperformed as well, but the items were again maintained as such. Two-dimensional and three-dimensional applications of trigonometry were removed outright to reduce the test load. Lastly, space to write down responses to the questions was provided just below the item on the question paper after realising that separate answer sheets were difficult to account for.

Task-based interview

The duration for both interviews was about 40 minutes for both participants. Though this was considered adequate, the duration was changed to 45 minutes for the main study. The number of items was adjusted up from eight to nine. The ninth item was on the definition of a negative angle, which I felt ought to be included in the interview to increase items on common content knowledge. It was also observed that there was no space for participants' written explanations to each item, hence that space was created just below each item. Under questions 5 and 8, the participants had difficulty with vocabulary, thus the following words were altered: plausible to possible; support and refute changed to agree and disagree respectively. In both cases the theme of the question remained the same after the words were changed. Again, question 8 initially had four parts, but these were reduced to two in order to minimise unnecessary duplication of the transformation of function for $y = \sin bx$ for $0 < b < 1$ and $-1 < b < 0$. The coefficients $b < -1$ and $b > 1$ were omitted. The closing comment part was broad and general, thus, to minimise confusion to participants, the comment specified that the comments should relate to the interview and trigonometry. The pilot research for this present research study was necessary to guide the refining of the two instruments and to increase the likelihood of success of the main study.

4.7. Research procedure

This section presents the research procedure for the main research study, which is also called the research process. It is defined as an orderly and systematic way of conducting a research study. The researcher commences with an overview of issues related to the research site, the population, the sample and the sampling techniques. The explanation of the data collection processes and the description of the data analysis plan follows next. The identification of the role of the researcher to minimise research bias, the quality criteria measures and ethical considerations takes the rear. Having explained and put in perspective the research problem, the review of relevant literature and the conceptual framework in the first three chapters of this study, what needs to be explained now are the specifics of where to start our research and how to go about doing it.

The first step in embarking on this project was seeking permission from the research site to collect data. This was duly obtained in 2016, and the ethics clearance certificate is attached as Appendix A. However, the research study started in earnest with the approval of research proposal by the Human and Social Science Research ethics committee at the University of KwaZulu-Natal in June 2016 (see Appendix B). The pilot research was actioned in August of 2016 after obtaining the consent of the participants. The intention of the research was explained to the potential participants for the main study during the same month of August 2016. The briefing of information was effectively done both in writing and verbally. The following were explained to the potential participants: purpose of the study, type of instruments to be used, procedures to be utilised, expected duration of participation, any risks of the study and potential benefits of the study. The prospective participants were given two copies of a consent document, one from the university where the researcher is registered for his studies and the other one from the ethical clearance committee at the research site. Both copies of the information letter and the consent document may be found in Appendices C and D.

4.7.1. The research site

The research site was a higher education institution in South Africa which came into being in 1959 as a result of the extension of the then University Education Act of 1959. The Act made provisions for the establishment of rural-based ethnic black population universities for the majority black South Africans. Hence, from its humble beginnings, this institution remained historically disadvantaged and situated in the densely populated areas that used to be called homelands. Even now, one of the institution's chief aims is to be a leading rural-based university catering for the disadvantaged and ethnic communities. The Faculty of Humanities with a student enrolment of about two thousand is one of the four faculties at the site. The School of Education is the largest of the three schools under the Humanities faculty, where all students enrol for a teaching undergraduate degree in languages, commerce, humanities, and science and mathematics. The Department of Mathematics, Science and Technology Education is the popular and pride of the School of Education, having a huge responsibility of churning out mathematics and science teachers, who are still in short supply in South Africa (Zezekwa, Mudau & Nkopodi, 2013). Preservice teachers in the Department of Mathematics, Science and Technology Education have a common curriculum for the first two years of their Bachelor of Education Senior Phase and Further Education and Training degree. At third-year, they chose two majors amongst Physical Science, Mathematics, Life Sciences and Technology. The researcher was a member of the teaching staff in the Department of Mathematics, Science and Technology Education at that time. The next section elaborates on the participants for this study.

4.7.2. Participants

The subjects of this study were the third-year preservice teachers who had elected to take mathematics as one of their majors of specialisation. About 250 were third year students who were registered for a four-year teaching qualification at the institution of higher learning on full-time basis. The participants were aged between 21 and 28 years, and they all came from the provinces of Limpopo and Mpumalanga, except one who hailed from Gauteng. They all spoke an African home language and English was a second language. The majority of students at this higher education institution come from rural and impoverished communities, and would most probably return and teach in those same communities upon qualification.

The preservice teachers in this study were a cohort, meaning that during their participation in the program they all took the same courses at each particular point of the program, which ensured that they had at least comparable academic experiences. These participants had also completed teaching practice in schools from the time they were in first year. Teaching practice equips preservice teachers to the basics of instructional strategies, the learners' challenges to learning in certain concepts and to the reflection on mathematics teaching skills. Hence, preservice teachers had developed the essentials of pedagogical content knowledge through real classroom teaching experiences, which exposes them to the three phases of teaching, which are planning the lesson, implementing the lesson and reflecting on the lesson. In all, the participants possessed typical experiences and attributes of preservice teachers in their late years of teacher education just prior to beginning school teaching as qualified teachers.

In particular, the participants for this study had opportunities to develop an understanding of the Grade 10 – 12 mathematics Curriculum and Assessment Policy Statement syllabus through interactions with curricula materials in methods modules and teaching practice. The participants had also intricate notions about mathematics first as learners during their schooling years and as students in the first- and second-year undergraduate content modules. Such pedagogical, content and curriculum capabilities of mathematics would have an impact on their mastery of knowledge of mathematical teaching (Fennema & Franke, 1992).

4.7.3. Sampling

A sample is a group of subjects that are taken from a larger population for purposes of measuring certain attributes of a research study. Sampling is the procedure of choosing a small group of participants from the larger population. The population was all the 250 third-year student teachers registered for a mathematics major degree at the higher education institution of learning. Researchers chose to study a sample rather than the entire population because it is sometimes impractical and not important to consider all the population. Kumar (2005) emphasises that the accuracy of the results in any given research study depends to a large extent upon the sampling technique and not on the quantity of participants. In case studies, a relatively small number of units are selected to achieve in-depth study of a phenomenon. The small sample in this case provided a close representation of the population which is being studied.

The researcher therefore chose to employ non-random sampling techniques for this research study since the study was not working towards representativeness or generalisability (Kumar, 2005). Cohen, Manion and Morrison (2005) point out that in a non-probability sampling, subjects of the population do not have the same likelihood of becoming part of the sample. The researcher used judgemental sampling where the chief concern was the judgement of the researcher on the selection of subjects from whom rich data can be measured. To select a sample, the researcher therefore goes to those members of the population who, according to his/her perception, are likely to yield the expected information and be able to communicate it (Kumar, 2005). Some of those approached by the researcher were unwilling to participate due, in part, to the recording format of some of the data collection procedures. Thus, a combination of judgemental and willingness to participate were eventually used to select fifteen preservice teachers for the content test, which was phase one. The selection of four males and two females chosen for phase two data collection, that is, the task-based interviews, lesson preparation and video-teaching episodes was based on their performance in the initial analyses of the content test data. There were supposed to have been two low performers, another two average performers and the last two were good performers. However, after the best performed participant showed no interest for the phase two data collection, the researcher was left with one high-performer, three medium-level performers and two low-content performers. This was done to bring to effect the maximum variation type of sampling (Creswell, 2007), which brings a balance in the range of capabilities of participants.

4.7.4. Administration of data collection instruments

This study was an exploratory case study intended to investigate the preservice teachers' command of the mathematics knowledge for teaching in trigonometry. It is known that case studies make use of many data collection methods carefully selected to give a detailed exploration of the case under consideration. The conceptual framework of the research informed the choice of data collection instruments. The subject matter knowledge was assessed objectively using a content test, which yielded quantitative data. The pedagogical content knowledge, which is not easy to measure, was assessed by three data collection instruments, and these gave rise to qualitative data. These three were task-based interviews, lesson plan analyses and lesson analyses using video recordings. This research study uses quantitative data from the content test that were statistically analysed in conjunction with qualitative data in the form of interviews, lesson plan and lesson observation analyses (Creswell, 2005). Data collection began in August

2016 and ended in May 2017. Each of these four instruments is explained in detail in the coming sub-sections.

The content test

After the consent forms were read, understood and signed, the participants were gathered in a lecture hall where they sat for the content test at one sitting under the supervision of the researcher. Different terminologies have been used for this instrument, for example task-sheet (Mudaly, 2016), questionnaire (Akkoč, 2008) and test of trigonometric knowledge (Fi, 2003). But the goal is the same; to explore preservice teachers' common content knowledge in a given content area. Marks were not allocated to the questions, but a marking rubric was used since the interest was not just scores. There were seven questions (some with part questions) which were supposed to be completed in one hour. Most of the questions on special angles and the reduction formula do not require the use of a calculator, thus their use was not envisaged. The participants' identities on the content test were needed since selection for phase two of data collection rested on the preliminary analyses of data from the content test. A copy of this instrument is attached in the appendices section (see Appendix E).

Task-based interviews

After the initial analysis of data from the content test, a further six participants were selected for the one-to-one task-based interview. A quiet place devoid of interruptions was selected for the interviews. The interviews were captured on audio-recordings, as well as written responses which were done in the spaces provided below each question on the paper. These responses were collected to support what was captured on audio-recordings. Each interview was about 40 minutes long and they were conducted sequentially, until all the participants had their chance. An attempt was made to transcribe interview audio-recordings at the earliest possible time.

The interviews attempted to uncover possible interrelationships between the content knowledge and knowledge of teaching of trigonometry. As participants tackled the written problems contained in the instrument one at a time, the researcher came in with probing questions to bring out the intricacies of mathematics knowledge for teaching. That made the interviews to be semi-structured. The participants faced problems which required some explanations, proofs of identities, sketching of graphs and definitions of key concepts in trigonometry. Some tasks in the interviews were made up of constructed hypothetical scenarios based on learners' thinking and calculations. Participants were then required to refute or validated those scenarios, some of which were rife with common misconceptions and errors. Appendix F illustrates a copy of the task-based interview.

Preparation and analysis of lesson plans

All the six participants who participated in the interviews proceeded to the next stage of data collection, lesson planning and lesson analyses. These were tasked to prepare a 30-minute lesson plan on any concept in trigonometry using the institution's School of Education lesson plan template (see Appendix H). Trigonometry starts at Grade 10, so the participants planned for Grade 10 or 11 classes. As student teachers, school principals do not often trust them with Grade 12 classes. Usually, by the month of May, Grade 12 learners would be in high gear for preparation of the national examinations, hence they need minimum interruptions. Lesson plans are effective indicators of preservice teacher's mastery of the mathematics knowledge for teaching. Lesson planning was done as part of the preservice teachers' roles as they engage in normal school teaching practice, thus they could use any material they deemed necessary from the schools where they were practicing. The copies of lesson plans were submitted to the researcher for evaluation and analyses on the day of the school visit by the researcher. A copy of the instructions to plan a lesson and teach it under observation is given in Appendix G.

Video-lesson analyses

Following the lesson plans which they developed in the preceding section, the six preservice teachers went on to deliver those lessons to the classes for which they had planned. The length of each lesson was 30 minutes, which was in line with most school timetable durations. The whole teaching episodes were captured on video-recordings. This exercise was successfully conducted in April and May 2017 during the time when the fourth-year students went out to schools for practicals for a period of six weeks. When the preservice teachers were teaching their planned lessons in schools, the researcher was present as an observer and only wrote down observation notes while the lessons were video recorded. A brief post-observation discussion followed the implementation of the lesson plans to give reasons behind some of the actions that would have been observed during the lesson. The video-recordings were transcribed soon after the lesson presentation with the help of the School of Education teaching practice evaluation form (see Appendix I).

Of significance was the need to find means whereby prospective teachers' subject matter knowledge and pedagogical content knowledge interplayed in the real classroom setting (Rowland, Thwaites, Huckstep & Turner, 2009). Mathematics knowledge for teaching is an intricate combination of pedagogical content knowledge and subject matter knowledge as they intermix in the course of teaching of mathematics. Teachers require knowledge in several different domains, but this research supports the belief that mathematics knowledge for teaching is not only located in the mind but also through the practice of teaching.

4.7.5. Role of the researcher

As is common in most qualitative studies, researchers are the key players for all the research processes, for example, collection of data, analysing and discussing the findings (Merriam, 1998). The researcher for this study had to oversee the implementation of each stage of the research design for this study. Hence, it is important to state the researcher's position in the study to minimise possible bias that could negatively affect the research results. My role in the study was both practitioner and researcher. As a lecturer at the higher education institution, I taught entry-level mathematics content modules, thus I was not directly responsible for teaching final-year students. However, I was an active participant in the teaching practice evaluation for both Bachelor of Education and Post-Graduate Certificate of Education students. The subjectivity of the researcher is an inevitable part of case study research methodologies and I did not take it as a weakness. Rather, it gave me first-hand detailed and rich knowledge of what was happens in the classrooms while the preservice teachers delivered their lessons (Simons, 2009). This was done in a friendly and cooperative environment since the participants were familiar with the researcher. Hence, instead of perceiving researchers' contributions to the entire research process as detrimental, it could be made positive and useful.

4.7.6. Minimisation of research bias

Research bias stands for any interfering factors that provides a distortion of truth or accuracy of information in any part of the processes of research (Polit & Beck, 2014). The risk of bias exists in all components of qualitative research, but normally arises in the data instruments, the respondents and the researcher. Researchers in qualitative designs are an integral part of the methodology and final product, and any attempts to separate the two may not be possible nor desirous. The concern instead should be whether the researcher has been transparent about the processes by which data have been collected, presented and analysed. Thus, the goal of reducing bias is not to have a flawless research report, but to make sure that respondents do reveal their true feelings without distortions and researchers analyse data without prejudice. To address potential bias, the researcher piloted the research instruments and part of the methodology to see if the planned data collection and research procedures reflect the research problem. Then, the researcher attempted to triangulate data collection for the pedagogical content knowledge to reduce the risk of the limitations and biases of a single data source (Cohen, Manion, & Morrison, 2005).

In designing the instruments, the researcher was careful to avoid leading questions in the content test and task-based interview, as well as to minimise probing and follow-up questions as these are prone to bias. Some critics say a researcher probing data is like he is mining for data that will affirm his own preconceptions. The wording of questions was carefully chosen to avoid confusion and ambiguity. There was not much which the researcher could do to minimise bias emanating from participants, except to create a conducive environment where they feel free to express their ideas without fear or favour. There was not much risk of acquiescence or social desirability bias because the researcher made it clear that there was no material gain in participating in the study. The researcher-participant relationship was not social but professional.

4.8. The data analysis plan

This section presents the proposed data analysis plan for this study. The full-scale data analysis appears in Chapter 5, where this analysis plan stated here was applied to the collected data. Qualitative analysis is a well-structured spectrum of processes and procedures which leads to some form of interpretation or explanation of the phenomena under investigation, based on the raw data that have been collected. The procedure of data analysis transforms information collected during research into findings. The analysis of data for phase two was done through the lens of the identified conceptual framework in order to confirm presence or absence of established elements of pedagogical content knowledge established. The content analysis of the content test responses sought to establish themes in the data. A theme is a pattern that is considered significant to describe a phenomenon which transcends all data sets. Thematic analysis is a common form of data analysis in qualitative research which attempts to pinpoint, examine and record notable patterns within a data set.

Like most research methodologies, the data analysis can be inductive or deductive. Thematic analysis of data is an induction approach, where patterns identified are strongly data-driven and the researcher knows little about the outcome of the phenomenon. It is data-driven by identifying emerging themes in the data in order to develop explanations for social behaviour. Induction is not based on a structured on pre-existing frameworks. Deductive approaches are based on a structure predetermined by the guidance of the theoretical framework. Predetermined frames render deductive analyses less descriptive. These one or two specific aspects of data that were determined prior to data analyses then became the focus of analyses. The research design governs the choice between the inductive and deductive approaches. In this study both were applied to data analysis; deduction based on the predetermined categories of teacher knowledge as informed by the conceptual framework. Induction was used to identify emerging patterns within each predetermined strand of teacher knowledge and from the content analyses of the content test responses. For a single case only, deductive analysis can be pertinently used in a case study methodology and facilitates confirming certain traits in data without necessarily leading to generating new generalisations. Figure 4.2 shows the overview of the data analysis plan.

The data analysis plan was descriptive statistics for the phase one quantitative data from the paper-and-pencil content test since it “yields specific numbers that can be statistically analyzed” (Creswell, 2005, p.510). The content test data was subjected to content analyses too so that the researcher can delve into participants' nature of understanding of trigonometry. The content analysis was done in an inductive perspective by means of thematic analyses of data. This study is qualitative in design, which used quantitative data as part of collected data. Quantitative analyses of data were followed by qualitative analyses to obtain detailed and specific information the results of statistical tests. Deductive analysis was used on phase two qualitative data, which was based on the predetermined elements of pedagogical content knowledge components. Deductive analyses of data checks for established generalisations in the phenomena and efforts were done to identify the generalisation in the collected data. The quantitative analysis is explained first, followed by qualitative analyses in the following sub-sections.

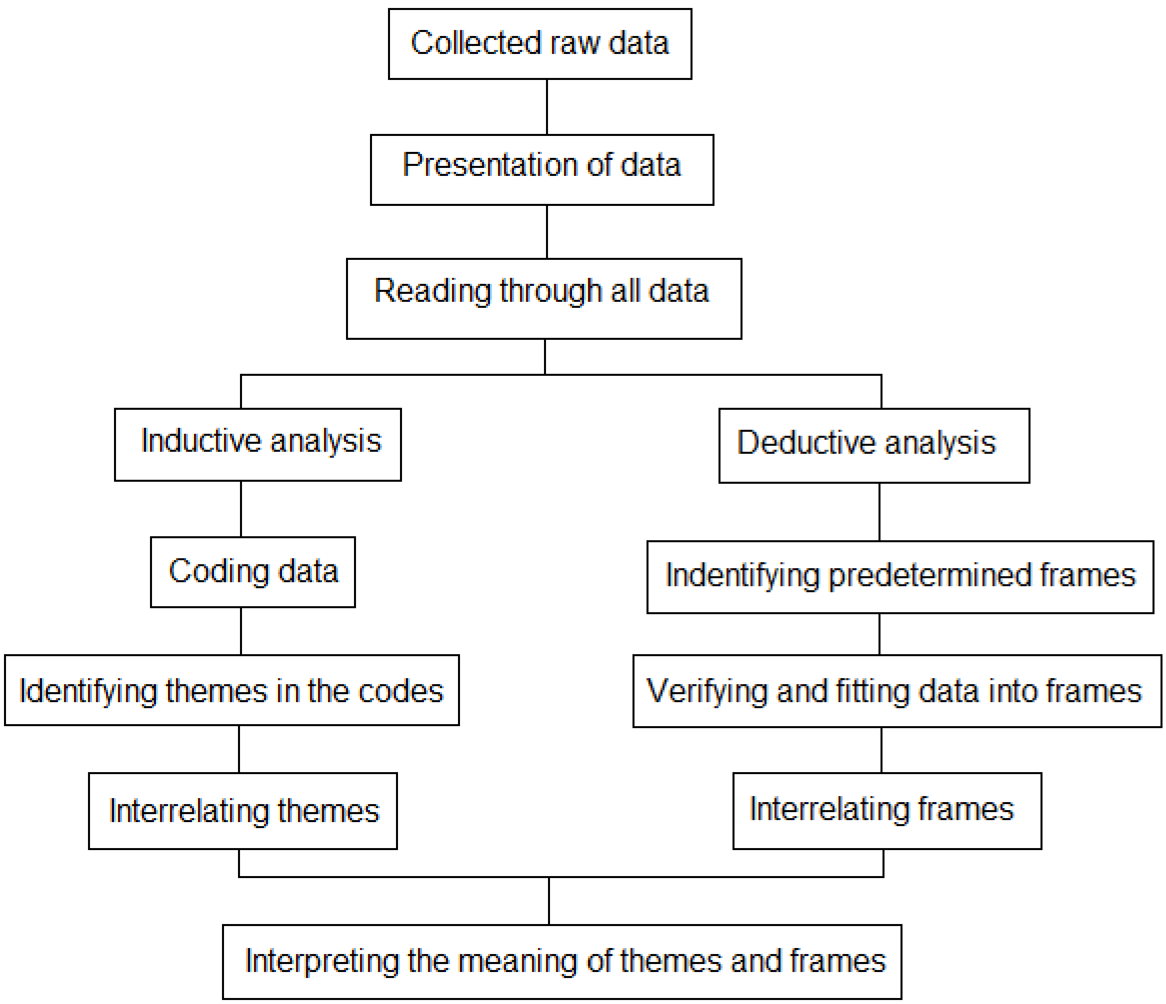


Figure 4.2. Stages in the data analysis process, modified from a model by Creswell (2009).

4.8.1. Quantitative analysis

In the first phase of data collection, both qualitative and quantitative data were gathered and analysed to respond to research question one, “What is the level of preservice teachers’ understanding of trigonometric concepts?” Fifteen participants took part in this phase. Primarily, descriptive statistics shed light on the extent to which preservice teachers understand the concepts of trigonometry based on the content test data. Thus, the content knowledge of preservice teachers was determined through their scores in the content test. A preservice teachers’ understanding to an item was regarded as robust if the response was correct, otherwise it was considered weak. In addition to quantitative data, item analysis of content test data was explained and supplemented the initial quantitative results. The completed content tests were collected and each question was scored using a rubric with scores ranging from 0 to 3, since no marks were allocated to the questions. The total score for each preservice teacher was recorded. However, due to the nature and requirements of each question, the general rubric at times was modified to suit individual questions. Thus, for all the 13 questions, each participant was rated on a scale of zero to a possible 39 marks. The percentages of participants who reached a certain level of performance were calculated. To summarize the overall performance, average scores per participant were reported. Furthermore, the percentage of participants who demonstrated mastery of a given level of performance on all the items were reported. Further qualitative analysis in the form of item analysis were conducted to highlight any discernible patterns of understanding in the participants’ responses from the data.

4.8.2. Qualitative analyses of data

Qualitative analysis was done to address the second research question, “What pedagogical content knowledge do preservice teachers possess in trigonometry?” Pedagogical content knowledge was explored through three data instruments and all the six participants took part in the data collection activities. The task-based interview consisted of nine open-ended questions and the responses were analysed in order to understand the nature and structure of preservice teachers’ responses to trigonometry teaching. Data were described on preservice teachers’ understanding of pedagogical content knowledge under three elements, namely, knowledge of explanation, knowledge of learners’ possible misconceptions and knowledge of content. The interviews were audio-taped and the analysis began by listening to the audio-taped interviews and transcribing them. Each interview transcription was analysed item by item and by the category of element of pedagogical content knowledge. Transcribed notes were summarized to avoid many pages of transcription if each interview were to be transcribed word for word for each of the six participants. The verbal transcripts were merged with the participants' written responses for each question.

The lesson plan analysis sought the preservice teachers' skills in pedagogical content knowledge. Five elements of pedagogical content knowledge that were predetermined under lesson plan analyses were content knowledge, knowledge of learners' misconceptions, knowledge of instructional strategies, knowledge of assessment and knowledge of lesson plan structure. Lesson planning was done by the six participants using a lesson plan template (see Appendix H). The completed lesson plans were completed and evaluated using the lesson evaluation sheet (see Appendix G). The extent of preservice teachers' understanding under each element were recorded as part of analyses.

Lesson observations were done in class by the researcher, coupled with the analyses of video-recordings of the six preservice teachers done using the lesson evaluation form (see Appendix H). The six predetermined elements for video-teaching episodes were: knowledge of teaching strategies, of content, of learners' misconceptions, of assessment, of lesson structure and questioning techniques. A brief post-observation discussion was conducted with each participant after the lesson delivery to substantiate the participants' intentions for both lesson planning and video-recorded lesson presentation.

The third research question, "To what extent do preservice teachers' develop the mathematics knowledge for teaching of trigonometry in the initial teacher education?" was responded to through the qualitative analyses of the four data sets. The tracking of each of the six preservice teachers' understanding across the four data sets were also done... However, even though predetermined themes were used in analysis, a door was left open for possible themes that may arise based on participants' responses. The content knowledge element featured in all the three of pedagogical content knowledge instruments because pedagogical content knowledge is all about teachers' art of teaching specific topics, and this hinges on the level of mastery of teachers' content knowledge.

4.9. Measures of quality criteria

The term quality criteria refers to the efforts and procedures that researchers put in place to ensure the integrity and accuracy of data collected, following a chosen research methodology. Attention has been aimed at the quality of qualitative research designs in recent years. Lincoln and Guba (1985) consequently proposed four measures of quality which feature prominently in the criteria used to assess integrity of both the qualitative and quantitative research. These are objectivity, reliability, external and internal validity. However, these criteria for measures of quality are not readily applicable to qualitative research, hence alternative qualitatively oriented criteria were crafted. These are credibility, transferability, dependability and confirmability. These four reflect better the tenets of qualitative research designs. A comparison is illustrated in Table 4.3 to relate the quantitative and qualitative measures of quality.

Table 4.1. The analogous measures of quality criteria for quantitative and qualitative research designs.

Quantitative researches	**Qualitative researches**
Internal validity	Credibility
External validity	Transferability
Reliability	Dependability
Objectivity	Confirmability

Trustworthiness is a term referring to establishing all the four measure of criteria in qualitative data. Credibility, a parallel to internal validity under quantitative design, is concerned with the researcher checking and verifying the accuracy and appropriateness of the results of a qualitative research study (Creswell, 2009). It is the most important aspect in establishing trustworthiness as it links the findings with reality. Triangulation of methods of collection of data was one of the techniques of establishing credibility that was employed in this study. Four methods were used to collect information on preservice teachers' understanding of mathematics knowledge for teaching to ensure that the findings are robust and well-developed (Bryman, 2008). Secondly, member-checking to the six preservice teachers was used also to establish credibility of the data collected. Findings emerging from the data were shared with the participants which gave them an opportunity to clarify what their intentions really were and provide additional information in some instances. Also, the interviews and the lessons that were observed were recorded electronically so that the researcher could re-visit them at any time with ease to ensure that the reality that the researcher had recorded was not a fabrication. However, credibility should not be seen as absolute in qualitative research due to the subjectivity and attitudes of respondents which would be difficult to guarantee in this case. Should the instruments be re-administered to the same participants, it would be human nature to try and answer the questions differently if the participants felt that their initial response was incorrect. Thus, the honesty, depth of the data collected, and the objectivity of the researcher can only establish credibility of results of a study.

Transferability, the alternative to external validity, is the extent to which results from a research study can be applied to a research in another context. Transferability rests in the hands of the interested parties; readers can then decide on their own whether the results of the study could be applicable to their own research contexts or not. However, the researcher enhanced transferability by giving thick description of the setting with sufficient details of the findings which make the results richer and more realistic (Creswell, 2009). As this was a qualitative study, no substantive generalisations are expected. Nevertheless, Yin (2003) posited that for case study research, there is room for generalisation to other settings in the same broader theory. As this study focusses on teacher knowledge theory of preservice teachers, other applicable settings could be on novice or experienced teachers.

Dependability, a parallel of reliability in quantitative research, pertains to the degree of consistency of results and the possible replicability in measuring a construct. The researcher’s approach to creating findings should be consistent across different settings and different researchers (Gibbs, 2007). Care must be exercised to avoid measuring the same construct twice and expecting to get uniform results. Research subjects tend to give different responses to the same question as they feel their initial response was not correct.

Also, measuring a construct the second time in human sciences may not necessarily mean the scenarios are the same. If one cannot guarantee perfectly matching settings, one is bound to get different results. To enhance the dependability of the data collection instruments for this study, the content test and task-based interview were piloted to the participants. The instruments which were not piloted were standard institutional tools for teaching practice, and their use was an effort by the researcher to situate this study in preservice teachers' natural setting of operation. Multiple revisions were carried out based on the pilot study feedback, leading to the final versions of the content test and task-based interview. Secondly, triangulation of methods enhances dependability; in this study task-based interviews, lesson plan analyses and video lesson analyses were used to try to figure out preservice teachers' understanding of pedagogical content knowledge. The results obtained from the three instruments were similar, an affirmation that triangulation serves its purpose. Furthermore, validation of the content test and task-based interview instruments of data collection were done by experts in mathematics education at the research site.

Lastly, confirmability refers to the degree of confidence that the research results are a true reflection of the respondents' perceptions, rather than shaped by the researcher's opinion. If not careful, researchers can have bias, which has the potential to mar the confirmability of research results of that study. Confirmability can be established by good data management, so that results can be traced back to the data. In this study, confirmability was enhanced by keeping safe the collected data that was used for interpretation safely, so that any interested parties can access the same data for inspection and confirmation. Also, in what is called the audit-trail, the researcher detailed the processes of data collection, data analysis and discussion of findings. Confirmability criterion was one of the simplest to establish, as it was just a matter of explaining in detail the decisions that were made in the research methodology. The details from the four quality criteria alluded to previously, were worthy steps taken to improve the trustworthiness of this study by providing valuable insight for readers to understand how the research results emerged from the data.

4.10. Ethical issues

Ethics is a philosophy branch which involves the dynamics of societal values and norms that are regarded as wrong and right. Human science is to some extent governed by laid down societal values and a set of written and unwritten rules (Fouka & Mantzorou, 2011). Research ethical committees govern how research is performed at most research institutions such as universities and how findings are to be published in a way that protects the dignity of human and animal participants and the environment. According to Creswell (2009), ethical issues transcend all the major stages of conducting a research study. Thus, researchers talk of ethical issues in the problem statement, in the purpose of study, in the collection of data, in data interpretation, in discussion of results and in the dissemination of research results. Research ethical considerations are important for the following reasons: they hold researchers accountable for their actions; they uphold community moral values and standards; they build public trust research-work; and they give meaning to collaborative work through mutual respect and fairness.

A request to collect data from the higher education institution's School of Education was submitted to the Ethics Research committee through the offices of the Director of School and the Dean of Faculty of Humanities. Approval to collect data was duly granted (see Appendix A). The Ethics Committee at the University of KwaZulu -Natal's Humanities and Social Sciences Research also approved the application for ethical clearance (see Appendix B). When the preservice teachers were recruited, two informed consent letters were provided to the participating volunteers (see Appendices C and D). Consent to audio-tape interviews and video-record the lesson presentations was duly obtained from the participants. Issues of voluntarily participating in the study were explained in the consent forms, as well matters of harm, adverse effects and personal gain. For lesson plan analyses and observations in schools, participants were assured that their participation was not to influence their performance in teaching practice evaluations by the same researcher. I had the privilege of visiting all the participants during their April/May 2016 teaching practice sessions in the schools from where they were practising.

There was no ethical clearance from any of the learners or schools for the classes that were taught under video recording because they were not the focus of the study. The transcribed information from the video recordings was mute on information that could identify the participants, the learners or the schools where preservice teachers were practising. In addition, the name of the institution where the participants attended was not disclosed, but was referred to as a higher education institution.

All the participants in the study were assigned pseudonyms for confidentiality. Protecting the identity of participants is what researchers call confidentiality. On the other hand, anonymity grants absolute protection of participants' identities from all interested parties, including the researcher himself/herself. Hence anonymity supersedes confidentiality in all respects. However, participants were not anonymous in this study because the researcher had to further select six participants for the task-based interviews based on their performances in the content test. Also respective participants took part in the four instruments of data collection, thus there was need to identify each one of them in order to trace their performance. The only way that could be possible was by using some form of identification which was known to the researcher alone. Cohen, Manion and Morrison (2005) argue that a respondent consenting to a face-to-face interview can be assured of confidentiality, but cannot expect anonymity. Even though the researchers can identify participants from the given information, or may know very well who has provided particular data, they will not disclose the information publicly.

The limits of confidentiality were also made clear to the participants before data collection commenced. By this, it was clarified that their data would be used for the purposes of this research only. The data collection materials, it was explained again, would be kept in a secure place with the researcher and the copies thereof with the supervisor for five years. These include the answers to the content test, audio-recordings of interviews, lesson plans and the analyses, and video-recordings of lesson presentations and evaluation sheets. Thereafter, they would be destroyed completely. All these deliberations on ethical issues bring to the fore the idea that researchers are unilaterally accountable for the moral integrity of the entire research process.

4.11. Conclusion

The qualitative research design as it was used in this thesis was outlined in this chapter. To address the research problem and provide answers to the research questions, an exploratory qualitative design was employed. A case study methodology was enacted to explore preservice teachers' mathematics knowledge for teaching. A preliminary study was conducted which paved way for improvements in data collection instruments. The research procedure for the main study was given after the pilot study, which highlighted the research site, participants, sampling techniques, collection and analysis of data. Judgemental sampling was used to select fifteen final-year preservice teachers for this study and the concept of maximum variation sampling was used to select six participants from the initial fifteen for the second phase of data collection. A content test was constructed and used to assess preservice teachers' understanding of content knowledge in trigonometry. The task-based interviews, lesson plan analyses and video lesson analyses were used to explore preservice teachers' mastery of pedagogical content knowledge in trigonometry. The description of the data analysis procedures came next, which was carried out to answer the research questions through the lens of the conceptual framework. Both the inductive and deductive approaches to the analysis of data were explained.

The researcher's role in this study was given, in line with the efforts made to eliminate research bias. Quality control issues in qualitative studies of trustworthiness of this study were elaborated and explained, and how to enhance each one of the four measures of quality criteria. The chapter ends with a consideration and expounding of research ethical issues pertaining to social science research. These included participants' voluntary participation, informed consent and the confidentiality of responses of participants and of the institution the preservice teachers attended. The identity of learners and the schools they attended were also kept confidential. The interpretation of data and discussion of findings will be explained and unpacked in the next two chapters respectively.

CHAPTER 5: INTERPRETATION AND ANALYSIS OF RESULTS

5.1. Introduction

This chapter endeavours to organise and arrange the results from the four data collection instruments. A detailed analysis of the fifteen participants' understanding of trigonometry in the content test is presented first under phase one data analysis in section 5.2. Quantitative data analysis was performed on the content test scores followed by a content analysis on the same data set. Statistical presentations were performed in the quantitative analysis whilst the content analysis was done thematically since it was qualitative in nature. Thereafter, section 5.3 elaborates on phase two qualitative data analysis of six participants who respectively participated in the task-based interviews, lesson plan analyses and video-recorded lesson presentations. The qualitative analysis was both inductive and deductive. Section 5.4 gives a traverse of the performance of each of the six participants across the four data sets. The process of data analysis rested on four key activities: data reduction, data presentation, discussion of findings and drawing of conclusions. Data reduction is akin to transcription of data and was done continuously and in most cases soon after the administration of the data collection instruments. Statistical diagrams, tables and screenshots of participants' work are part of the presentation of data. The discussion of findings will be done in Chapter 6 and drawing of conclusions in Chapter 7. The chapter conclusion, section 5.5 culminates this chapter on the analysis of data.

5.2. Phase one results

The content test was the first instrument to be administered, which sought to explore preservice teachers' understanding of content knowledge in various aspects of the topic of trigonometry. The results for this study have been organized into demographics data, quantitative data from participants' scores and qualitative data from content analyses.

5.2.1. Biographical information

Demographic data were collected on a variety of variables including gender, location of high school attended, province of origin and subject majors. These data were presented to give an indication of the nature of the participants for this study. Concerning the gender of the participants, five were females and ten were males. For the participants' age groups, most belonged to the 22 to 24-year age group and only one was above 28 years, as shown in Table 5.1. The information in Table 5.1 represents typical prospective teachers in South Africa, who normally commence tertiary education at an average age of 19 years. All of them hailed from the rural provinces of Limpopo and Mpumalanga, while only one had a suburban background from Gauteng province. The participants last attended high schools were all rural based, except one who attended a suburban school in Gauteng. This was in line with the vision of the higher education institution where data was collected, which was to be a leading rural African university epitomising global competitiveness and academic excellence in addressing the needs of rural communities through innovation. Concerning the subject majors, mathematics was done by all and the numbers registered for the second major subject were distributed as follows: technology were two, physical sciences were eight and life sciences were five. The male participants dominated in physical sciences and technology, while females were more populous in life sciences.

Table 5.1. The distribution of participants' ages.

Age (years)	19 – 21	22 – 24	25 – 27	Above 27
Frequency	2	10	2	1

5.2.2. Quantitative analysis of content test data

The section of the content test, which was composed of objective items yielded quantitative data, which were the percentage scores of the participants. A rubric was used to mark the questions and each item had a maximum score of 3, with partial credit awarded as shown in Table 5.2.

Table 5.2. A generalised rubric for scoring items on the content test.

Score	Criteria
0	Did not understand the problem, no answer provided, or inappropriate solution was provided.
1	Serious and major errors in the solution process but shows an understanding of the question.
2	Slight errors in solving the problem which leads to a partial answer.
3	The solution and the procedure are precise and appropriate.

Modifications to the holistic rubric were done where necessary in order to reflect the specific and individual structures of the items. For example, the rubric for item 1 on sketching and labelling the two special triangles commonly used in trigonometry is shown in Table 5.3.

Table 5.3. Modified rubric for item 1 on drawing and labelling special triangles.

Score	Criteria
0	Incorrect triangles drawn or no answer given.
1	Angles correct but with wrong ratio of sides on both diagrams.
2	Only one diagram is correct.
3	Both diagrams are correct for both angles and ratio of sides.

The fifteen participants' percentage scores were recorded and displayed in Table 5.4, as well as the participants' scores for each item. The total score is the sum of the individual thirteen items in the content test. The names of the participants have been changed to hide their identities. Remember the highest possible score was 39 marks since the highest possible per item was 3.

Table 5.4. Scores per item of each participants' performance.

Participants	**Sex**	**Items**													**Total**	**%**
		1	**2**	**3a**	**3b**	**4**	**5.1**	**5.2**	**6.1**	**6.2**	**6.3**	**7.1**	**7.2**	**7.3**		
Koka	F	3	3	3	3	0	3	2	3	3	1	3	3	2	**32**	**82**

Shab	M	3	3	2	2	3	2	3	3	2	3	2	3	0	**31**	**79**
Mahl	M	3	3	1	1	3	3	3	3	2	0	2	0	1	**25**	**64**
Mhla	F	3	1	1	1	3	3	0	1	3	3	2	3	0	**24**	**62**
Sell	F	3	3	1	1	0	1	0	3	3	0	1	3	3	**22**	**56**
Deli	F	0	3	1	1	0	1	3	0	2	2	2	3	0	**18**	**46**
Tume	M	2	3	0	1	0	0	2	0	0	1	1	3	1	**14**	**36**
Mupa	M	2	3	0	1	0	1	0	0	0	2	1	3	0	**13**	**33**
Rach	M	1	0	0	0	0	1	0	1	1	0	1	3	3	**11**	**28**
Malu	M	1	0	1	1	0	1	0	1	1	0	1	2	0	**9**	**23**
Mohu	M	1	0	0	0	0	1	2	0	1	0	2	2	0	**9**	**23**
Modi	M	1	0	0	0	0	1	2	0	0	0	1	3	0	**8**	**21**
Moga	M	1	0	0	1	0	1	0	3	0	0	1	1	0	**8**	**21**
Leng	M	0	0	2	0	0	2	0	0	0	0	0	3	0	**7**	**18**
Maka	F	0	0	0	0	2	1	0	1	1	0	1	0	0	**6**	**15**
Average																**41**

As can be seen from Table 5.4, the highest mark recorded was 82 percent which came from a female participant Koka and the lowest was 15 percent, which came from a male participant Maka. The average performance per participant was 41 percent. The same participants' scores have been displayed in the grouped bar graph shown in Figure 5.1. The modal class as can be seen is the 20-29 percent category.

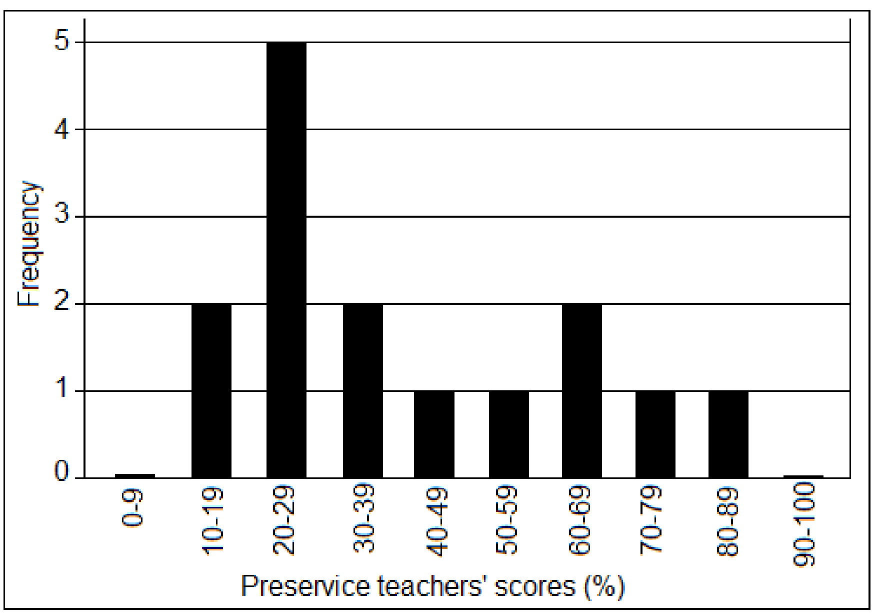


Figure 5.1. Bar graph showing participants' percentage scores in the content test.

The median score, which gives a better measure in this particular case due to the presence of outlier scores of 82 percent and 79 percent, was 33 percent. Concerning the spread of the scores, the range was 67 percent and the standard deviation was 24 percent, portraying a large spread of participants' percentage scores. Also, the data were skewed to the right, as can be seen in the five-number summary in Figure 5.2.

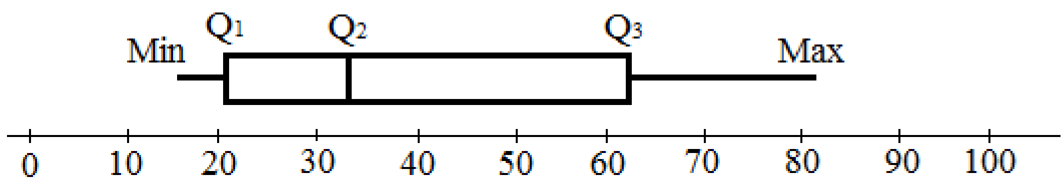


Figure 5.2. Box-and-Whisker diagram showing participants' percentage scores.

With the lower quartile of 21 percent and the upper quartile of 62 percent, the inter-quartile range was 41 percent, which again portrayed a large spread of scores. This shows lack of consistency in preservice teachers' level of understanding of trigonometry.

5.2.3. Qualitative analysis of content test data

The construction of the content test was in line with the model of teacher knowledge adopted in this study. The test was measuring preservice teachers' subject matter knowledge, which is known to have the subdomains of specialised content knowledge, common content knowledge and horizon knowledge. It is noteworthy that all teachers need to know specialised content knowledge (Mudaly, 2015), hence a total of seven items in the content test were devoted to it. These seven assessed preservice teachers' abilities on giving justifications, explaining and general problem-solving skills. The specialised content knowledge type is specifically related to teachers' everyday mathematics instruction (Mudaly & Moore-Russo, 2011; Ball, Thames & Phelps, 2008). Specialised content knowledge was followed by six items on the common content knowledge, since it is less specific to teach but still important. These items focussed on preservice teachers' skills on drawing diagrams and sketches, proving identities and computing exact values of trigonometric ratios. No items were made available for horizon knowledge, which is less applicable to the South African educational landscape by virtue of policy documents to which teachers must adhere. The sequencing of topics and concepts, recommended textbooks and formal assessment tasks are unilaterally provided by the Department of Basic Education. Table 5.5 illustrates the distribution of items in the content test.

Table 5.5. Illustration of the distribution of subject matter knowledge subdomains.

Content knowledge type	**Specialised content knowledge**	**Common content knowledge**	**Horizon knowledge**	**Total**
Items	2, 3(a), 3(b), 4, 6.1, 6.2 and 6.3	1, 5.1, 5.2, 7.1, 7.2 and 7.3	None	13

The qualitative data analysis under the content test was done to complement the quantitative data analysis done in section 5.3.2. Content analysis was the strategy used to qualitatively analyse data from the items in the content test. Under content analysis, attempts were made to make valid inferences from data to the context in which it is situated in, to provide new insights and a representation of facts and figures. The content analysis commenced with the tabulation of participants' performance per item according to the sub-type of subject matter knowledge have been tabulated in Table 5.6.

Table 5.6. Participants' performance per item in each sub-category of subject matter knowledge.

Type of subject matter knowledge	**Item number**	**Performance per item (%)**	**Average score per knowledge type (%)**
Specialised content knowledge	2	49	34
	3(a), 3(b)	28 (average)	
	4	24	
	6.1, 6.2, 6.3	35 (average)	
Common content knowledge	1	53	48
	5.1, 5.2	44 (average)	
	7.1, 7.2, 7.3	49 (average)	
Horizon knowledge	-	-	-

From Table 5.6, common content knowledge recorded the greatest percentage score, measuring 48 percent. Specialised content knowledge came next with an average percentage score of 34 percent.

The question-by-question analysis revealed that participants' performances were not all the same. For example, question 7.2 which was on curve sketching saw ten out of fifteen participants getting it all correct (67 percent). In contrast, question 7.3, which was on interpreting the solution set of an inequality statement was only managed by two of the fifteen participants (13 percent). The results of participants who scored all the marks, partial marks and no marks on each item are displayed below in Table 5.7.

Table 5.7. Distribution of frequencies of participants' performance per item.

Item	**1**	**2**	**3a**	**3b**	**4**	**5.1**	**5.2**	**6.1**	**6.2**	**6.3`**	**7.1**	**7.2**	**7.3**	**Average**
Frequency of score 3	5	7	1	1	3	3	3	5	3	2	1	10	2	
Percentages	33	47	6	7	20	20	20	33	20	13	6	67	13	23
Frequency of score 1 or 2	7	1	7	9	1	11	4	4	7	4	13	3	3	
Percentage	47	6	47	60	7	73	27	27	47	27	87	20	20	28
Frequency of score 0	3	7	7	5	11	1	8	6	5	9	1	2	10	
Percentage	20	47	47	33	73	7	53	40	33	60	7	13	67	39

From Table 5.7, on average, there was a chance of 23 percent for the participants to get all the thirteen items perfectly correct. The likelihood of obtaining partial marks, that is, a 1 or a 2 of the items was 28 percent. The probability of failing to get a single mark in any of the items in the content test was 39 percent. Participants had challenges to solve connected problems. For example, ten participants got the correct sketches of $f(x)$ and $g(x)$ but of those, only two managed to get the correct solution set for the regions where $\mathrm{f(x)} < \mathrm{g(x)}$, in item 7.3. As a result of failing to read-off the solution set from the graph, item 7.3 was the least performed, with ten participants getting no mark or skipping it completely. It was not difficult to simply read-off values of the required regions from a perfectly correct sketch, but many participants failed to do that, as shown in one participant’s attempts in Figure 5.3.

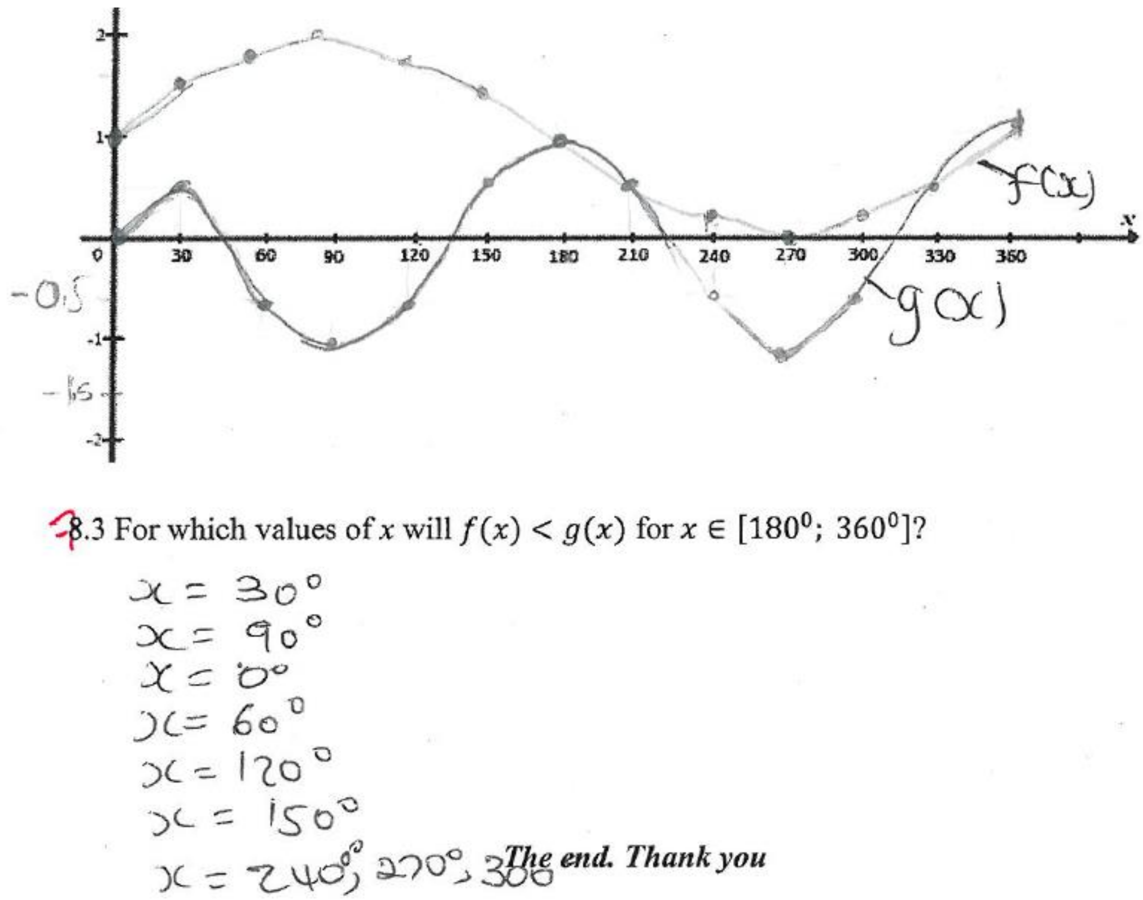


Figure 5.3. Correct sketches of $f(x)$ and $g(x)$ but with incorrect solution set for $f(x) < g(x)$.

Also poorly performed was item 4 where eleven participants had serious challenges to the extent that they left the item completely unanswered, an indication that they could not carry out the required proof. One of their challenges was that a sketch was not provided for this question, which could have made their life much easier if it had been. The two participants who got this correct had to produce their own sketches first before proceeding to prove. The concept of area rule which was under the spotlight here eluded many participants. Exactly one participant saw the need to use the area rule, however, she failed to get to the required result as she abruptly stopped, as shown in Figure 5.4. Only one stage of re-arranging the last equation was left to show the expected result.

$$\sin A = \frac{2S}{bc}$$

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$S = \frac{1}{2}ab\sin C$$

Figure 5.4. A correct attempt at applying the area rule but left incomplete.

Similarly, the other two proving items (2 and 5.1) were not satisfactorily done due to lack of understanding of the necessary underlying concepts. These were the square identity and the reduction formula respectively. Those who performed well, like Koka and Shab easily got the proof in 5.1 correct, but it was a heavy struggle for the rest. Also, the solution of equations registered low performance (items 3(a), 3(b), 5.2 and 7.1). Interestingly, though the procedures were known to participants, they could not go all the way to the final answer as expected. Most erroneously thought that supplying only the reference angle was sufficient, as illustrated in Figure 5.5.

a) $\tan x = \tan 30^0$ $\tan x = 0,58$ $x = \tan^{-1}(0,58)$ $\therefore x = 30,11^\circ$	b) $\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$ $\tan x = 1$ $x = \tan^{-1} 1$ $\therefore x = 45^\circ$

Figure 5.5. Partially correct solutions to the trigonometric equations.

As for item 5.2, all participants managed to realise that undefined values for the identity easily come from the expression appearing on the right-hand side of the identity ($-4\tan 2x$), rather than the complicated expression on the left-hand side. Deli got both solutions correct by dividing the undefined values of $\tan x$ in the given domain (90^0 and 270^0) by 2 to get correct values of 45^0 and 135^0 (as shown in Figure 5.6). This attempt was successful even though item 5.1 of proving the identity was poorly done. Deli was a mediocre performer.

5.2. For which values of x in the interval $0^0 < x < 180^0$ will the identity be undefined?

$= -4 \tan 2(45^\circ)$

$= -4 \tan (90^\circ)$

at $x = 45^\circ$

$x = 135^\circ$.

Figure 5.6. Correct solutions of the undefined values of $f(x) = -4\, tan\, 2x$.

Koka, a high performer in the content test, tried and missed the solution due to the misunderstanding of transformation of trigonometric functions. Koka's solution is illustrated in Figure 5.7 which was in the backdrop of a correct proof in item 5.1.

5.2. For which values of x in the interval $0^0 < x < 180^0$ will the identity be undefined?

tan x is undefined at $x = 90^0$

$-4\tan 2x$ is a multiple of -4 of $\tan 2x$ which is also undefined at $x = 90^0$

$x = 90^0$

Figure 5.7. Correct approach to undefined values of $f(x) = -4\, tan\, 2x$ marred by misunderstanding of transformation of functions.

The problem of rounding-off is well-pronounced in Figure 5.5 (a) where accuracy was lost by rounding-off $\tan 30^0$ to two decimal places. Rounding-off marred an otherwise correct solution. As for item 7.1, 13 participants knew very well they must equate the two equations in order to find the intersection points, but some could not successfully simplify and solve the resulting equation, as can be seen in Figure 5.8.

$F(x) = g(x)$

$1 + \sin x = \cos 2x$

$1 = \cos 2x - \sin x$

Figure 5.8. A correct attempt at solving an equation but aborted due to conceptual challenges.

On a different note, it was notable that some participants displayed unwarranted errors in simplifying trigonometric expressions using knowledge of quadrants and the Pythagoras theorem. They repeated the error of saying that if it is given that $\sin \alpha = \frac{4}{5}$, then $\alpha = \frac{4}{5}$, as is evident in Figure 5.9.

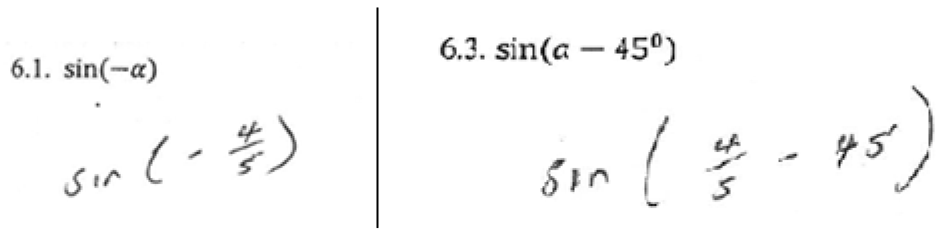


Figure 5.9. An error of misrepresenting $sin\, \alpha = \frac{4}{5}$.

The downside of preservice teachers harbouring such errors is that they may pass them to their learners when they begin teaching or during school teaching practicals (Haciömeroglu, 2009). From the deliberations above, it can be seen that the participants may have forgotten much concerning trigonometry which they last encountered many years back during high school days or sometimes in undergraduate classes, or else it was just poor understanding of those concepts, which they were obviously taught at some point.

5.2.4 Overview of the content test qualitative and quantitative analysis

The quantitative scoring of the instrument was done using a rubric with scores from 0 to 3. Of that, 24 percent of the participants managed to score a three, that is, 24 percent of the participants obtained certain items perfectly correct. Next, 13 percent had minor errors in some of the items (scored a 2 in the rubric). Then, 25 percent of the participants performed major errors in certain items (obtained a score of 1). Finally, 38 percent of the participants left some items undone at some stage or had no outright idea of the solution process as is illustrated in Table 5.8.

Table 5.8. The distribution of rubric participants' scores based on the rubric criteria.

Score	3	2	1	0
Percentage	24	13	25	38

The concepts which were poorly performed in the content test were proving statements, the solution set to inequalities and solving trigonometric equations. On the other hand, labelling special triangles, sketching graphs and proving identities were well-performed. Minor errors were registered in finding intersection points of equations. Major errors were identified in applying the reduction formula and simplifying trigonometric expression using knowledge of quadrants.

Concerning the individual participants' performance, Leng had serious difficulties to the extent that ten out of the thirteen items were not attempted at all. He only managed to get full marks in sketching of functions and minor errors were recorded in applying the reduction formula and solving the equation involving tangent ratio. Two more participants, Madi and Malu each left as many as eight items unattempted. These two had full marks in sketching graphs and computing intersection points of two functions. A female participant, Koka staged the best performance by registering nine out of thirteen perfect solutions. Her only serious problems were in proving a given statement based on the area rule. She had minor problem in simplifying the statement $\sin(\alpha - 45)$ given the value of $\sin(\alpha)$ and the solution set of inequalities.

5.3. Phase two results

Maximum variation sampling technique was use to select a further six participants as a sub-sample for the case study (van Putten, 2011) from the initial 15 for the second phase of data collection. This was chosen in order to address the diversity of preservice teachers' performances in the content test (SükrüBellibas, Özaslan, Gümüs, & Gümüs, 2016). A wide range of extremes need to be captured so that by collecting data from a very different selection, their aggregate answers can approximate the entire population's. The sample of 15 participants was not big enough to warrant use of other sampling techniques, for instance, random sampling. The characteristic which differed in the initial analysis was scores in the content test. To achieve a maximum variation type of sampling according to Creswell (2007), two low-scoring and one high-scoring preservice teachers were selected from the initial group of 15, as well as three participants that scored near the mean. These are referred to as the medium in this study (Brown, 2011). The sub-sample is shown in Table 5.9.

Table 5.9. Distribution of participants' performance selected for the second phase of data collection.

Position	Participant	Gender	Score (%)	Category
1	Mahl	M	62	Medium
2	Mhla	F	62	Medium
3	Leng	M	18	Low
4	Shab	M	77	High
5	Sell	F	56	Medium
6	Malu	M	23	Low

Leng and Malu were classified as low content-knowledge performers. Mahl, Mhla and Sell were regarded as medium performers and only Shab was regarded as a high performer. Some of the participants who did very well in the content test such as Koka chose not to participate in phase two since it was within the participants' discretion to participate. And because of similar reasons, only two female participants were willing to participate further than the content test, Mhla and Sell. The selected participants all took part in the three categories of data collection under the second phase, namely, task-based interview, lesson planning and video-recorded lesson presentation. The three instruments gathered data on preservice teachers' competence in pedagogical content knowledge and all the data generated were qualitative in nature.

5.3.1. Task-based interviews

This section presents overall results for the six interviewees. Data for the interview were made up of the transcribed audio-recordings coupled with participants' written responses to the tasks during the interview process. The transcriptions of the audio-recordings and written responses of the task-based interviews were coded and analysed in line with the multiple predetermined dimensions of pedagogical content knowledge. The three dimensions that were applicable to this instrument were knowledge of learners' difficulties and misconceptions, knowledge of content of trigonometry and teacher explanation. This was done to adhere to the precepts of deductive analysis of data, which is often used when the researcher wishes to test an earlier model in a new context (Marshall & Rossman, 1995). Being based on an earlier model, a deductive approach to analysis of data in this study was inevitable. The general model of teacher knowledge was a transition to get to specific results for this study (Burns & Grove, 2005). The general conceptual framework of teacher knowledge by Ball, Thames and Phelps (2008) from the conceptual orientation discussed in Chapter 3 was the lens used to focus data analysis in this chapter and the next one on discussion of findings.

Task-based interview analyses per question

In this section, the researcher checked how the participants' performed in each of the three elements of pedagogical content knowledge identified above.

Item 1

All the participants except one managed to give a good explanation of expansion of compound angles under sine and were precise at pinpointing learners' difficulties in compound expansion of sine. The one participant who could not explain the proof that $\sin(a+b) \neq \sin a + \sin b$ used particular values to verify (illustrated in Figure 5.10).

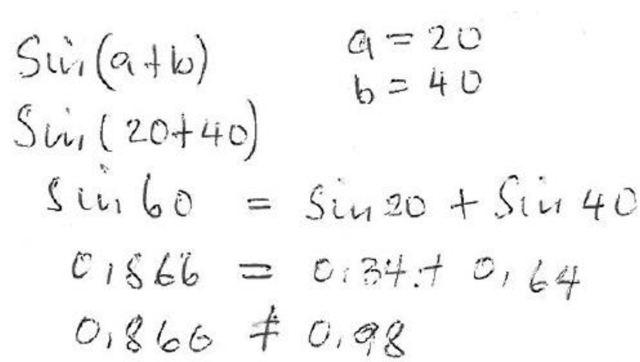


Figure 5.10. Using specific values to prove a general statement.

Mastery of content knowledge was the least performed in this item, as a result of participants failing to perform the correct expansion of $\sin(a+b)$. Overall participants' performance on this item was satisfactory.

Item 2

All participants easily identified possible learners' misconceptions correctly, as well as explaining the correct way of doing the proof of that fashion. However, mastery of content knowledge for the same participants was mediocre, as three out of six of the participants could not perform the correct proof themselves. They could not tell that the best way to prove $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$ was by rationalising the denominator, as what was correctly done by one of the participants in Figure 5.11.

$$LHS = \frac{\cos\theta\,(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta\,(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta\,(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta} = RHS$$

Figure 5.11. Correct solution to proving an identity by rationalisation.

Item 3

This item specifically required participants' knowledge in content and explanation. Knowledge of learners' difficulties and misconceptions was inapplicable. All except one used the graphical approach to explain that at 90^0, the tangent function is undefined. In the explanation, mention was made to the existence of an asymptote at 90^0. Sell was the only participant who could not get this task correct. He was of the idea of using the calculator to show the fact that $\tan 90^0$ is undefined, shown in the following dialogue.

Participant: [Reads the question aloud] [after some silence] Uhhh, how can I explain it that $\tan 90$ is undefined?

Researcher: Ok. Let me hear from you. But first of all do you agree that $\tan 90$ is undefined?

Participant: Yeah!

Researcher: Verify with a calculator [handing it to him].

Participant: [silence] Yeah, its undefined. So, the way to explain this...

Researcher: Uh huh.

Participant: Without the use of a calculator...

Researcher: Eeh. Yeah, obvious without using a calculator.

Participant: [silence]

Researcher: [interjecting] So would they understand that $\tan 90$ is undefined?

Participant: [more silence]

Researcher: [interjecting] How can you explain it to them? Yes, they have calculators, but now the calculator won't help with the explanation.

Participant: [silence]

Researcher: Or should we go to the next?

Participant: Yeah, let's move to the next. This one I don't know how to explain it.

Researcher: Ok [*Dialogue duration: 1 minute 23 seconds*]

As can be seen, the participant had no clue of where to start, other than using the calculator.

Item 4

Four participants managed to draw a diagram from which they established the trigonometric ratios of sine and cosine. In possession of that, a connection was established between the trigonometric ratios to the Pythagoras theorem, which completes the proof. All except one had good explanation of how to derive the trigonometric identity $\sin^2 x + \cos^2 x = 1$. The odd participant, Leng, just knew that the identity is correct but he could not explain its derivation. The following dialogue gives a highlight of what took place:

Participant: [reads question aloud] First of all I do agree that this identity is true.

Researcher: Uh huh.

Participant: Sin squared x plus cos squared x is equal to one.

Researcher: [silence]

Participant: Yeah, my question is ... I mean, the question is how do I explain this?

Researcher: Yes.

Participant: [silence]

Researcher: Where do you start so that you convince your learners that this identity is always true?

Participant: [silence]

Researcher: We know it’s true, but, uh huh, learners: they want an explanation why is it true?

Participant: [silence]

Researcher: Then you have to explain as educators to those learners.

Participant: [silence]

Researcher: Jump to the next.

Participant: This one I know also that it is true, but I do not know how to explain it.

Researcher: That’s why I am suggesting we jump question 5. [*Dialogue duration: 1 minute 33 seconds*]

The participant here just claimed to know the identity is true without being able to explain it. No attempt was done at all to show some clues of his purported understanding. The researcher wonders if he really knew what he claimed to know.

Item 5

Four of the participants could not identify the possible error whereby learners divide both sides of the equation by the function $\cos(x)$. Doing so does not only lead to a loss of set of solutions but violates a known mathematics rule of division by zero, for those instances where function is zero. These participants were trapped into the misconception which they were supposed to identify. Only two managed to recognise that the correct method to solve the equation $\cos^2(x) = \cos(x)$ is by the factorisation method. Thus, for this item, participants' identification of learners' misconceptions was limited, as well as their content knowledge of solving these kind of equations. Their explanations were not satisfactory either, which centred on solving for $\cos x = 1$ after dividing by $\cos x$ on both sides. One participant suggested that learners can make a mistake of saying $\cos^2(x) - \cos(x) = \cos(x)$. This kind of error amongst learners is uncommon since they know well that $\cos^2 x$ and $\cos x$ are not like times. The following dialogue depicts the situation where participant, Sell, repeats learners' misconception of dividing both sides by a function.

Participant: [Reads aloud the question]

Researcher: Yes.

Participant: [re-reads portion of the question] Possible errors...

Researcher: Or let do it this way! Can you solve that equation, perhaps in the process of solving you will be able to see those possible errors?

Participant: That cos squared $theta$ is equal to $cos\ theta$.

Researcher: Yes.

Participant: Here the question is actually ... we are solving for what?

Researcher: We are looking for $theta$.

Participant: Oh, we are looking for $theta$.

Researcher: Eeeh.

Participant: Ok.

Researcher: That's the unknown.

Participant: Alright. First of all, I would divide both sides by the cos of $theta$.

Researcher: Uh huh

Participant: And then here at the left-hand side the cos squared is going to divide cos and I am left with cos is equal to the right-hand side cos divide by cos is 1. And then $theta$ is going to be the $arccos$ of one. And then ... [using calculator] of which is, eeh, is zero meaning the angle here is 0.

Researcher: Uh huh. Alright.

Participant: [silence]

Researcher: So, the question I think is still; learners can make errors in the process of doing that. Where can they go wrong?

Participant: [silence]

Researcher: It could be a misconception? It could be a misunderstanding?

Participant: Aaah. Let me...

Researcher: Uh huh

Participant: Ok, I think ... I think if they can use this ... take cos to the other side

Researcher: Then.

Participant: They might have a challenge or might commit errors whereby they will say cos squared of $theta$ minus $cos\ theta$ and they are saying $cos\ theta$ is equal to 0 because the other side is zero. And then from there they will find the $arccos$ of 0.

Researcher: But wait a minute? Where is this 0 coming from?

Participant: Here, the question, the the... The original question was cos squared $theta$ is equal to $cos\ theta$. So now you jump… take $cos\ theta$ to the other side. So, meaning you are going be left with cos squared $theta$ minus $cos\ theta$ is equal to one.

Researcher: One. Yeah, we are together. No, there is no 1. It's zero.

Participant: Is equal to zero. It's the equation. So now cos squared $theta$ minus $cos\ theta$ they might say its $cos\ theta$. [Continues]

The participants' performance on this item was below average and many insightful observations were noted.

Item 6

In this item the learner-misconception was pre-identified. Participants were only required to explain using the correct content knowledge that the amplitude of all trigonometric functions is not always ± 1. Half of the participants remembered that in all trigonometric functions, amplitude is determined by the value of a as in $y = a \sin x$ and $a \in \mathbb{R}$. And these three managed to give an accurate explanation of the concept of amplitude of any trigonometric function, which is half of the range. One participant, in addition to highlighting the effect of the coefficient a, also brought in the idea of shifting of the function up or down, as shown in Figure 5.12. The linear analogy was also given to further explain the concept of change in amplitude.

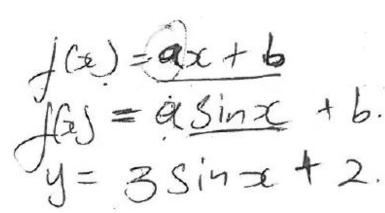
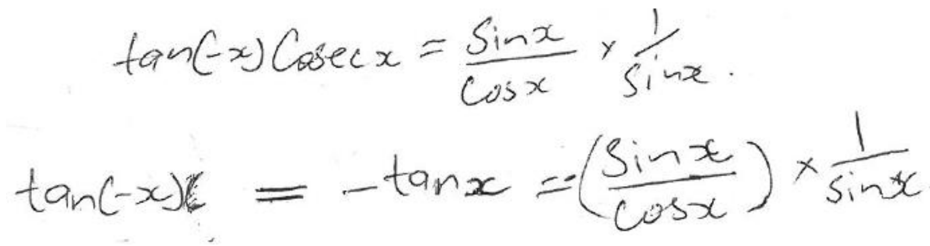


Figure 5.12. The effects of a and b in changing the amplitude of a trigonometric function.

From the explanation, it was clear that it is not always the case that the range is -1 to +1. Counter-examples were provided to further clarify the misconception, for example the function $f(x) = \tan x$ whose amplitude goes up to infinity at certain values of x. The other three, after some attempts, could not give meaningful explanation and subsequently gave up.

Item 7

All participants managed to pinpoint the learner difficulty of omitting the negative sign in simplifying the odd function $\tan(-x)$. Even those who had changed $\tan(-x)$ to $\frac{\sin(-x)}{\cos(-x)}$, they still managed to pinpoint the missing negative sign emanating from $\sin(-x)$, as shown in Figure 5.13.



$$\tan(-x) \operatorname{Cosec} x = \frac{\operatorname{Sin} x}{\operatorname{Cos} x} \times \frac{1}{\operatorname{Sin} x}.$$

$$\tan(-x) = -\tan x = -\left(\frac{\operatorname{Sin} x}{\operatorname{Cos} x}\right) \times \frac{1}{\operatorname{Sin} x}.$$

Figure 5.13. Correct handling of the negative sign in odd trigonometric ratios.

However, only four participants managed to explain using the appropriate content knowledge for odd functions, $-\tan(x) = \tan(-x)$. Two participants made errors themselves in simplifying the odd functions tangent and sine so that they could not get the correct statement on the right -hand side. Consequently, the explanation of those two was not satisfactory to justify the omitted negative sign on the right -hand side.

Item 8

Only one participant got both parts (a) and (b) correct, where the positive fractional coefficient of x in $0 < b < 1$ has the effect of expanding the graph whilst the negative coefficient of x in $-1 < b < 0$ causes the graph to shrink in the opposite direction. The following dialogue highlights how he did it:

Participant: [Reads aloud the question]

Researcher: Yes.

Participant: Can I draw the graph?

Researcher: Yes. The space is yours. Write as much as you want.

Participant: I am trying the graph of $y = \sin x$. [Drawing] Then from there, let's check the argument of the learner. From 0 going to 1.

Researcher: Uh uuh that one is for (b). They are saying b lies between 0 and 1.

Participant: (Interjecting) Oh b is the coefficient of x!

Researcher: And remember b is the coefficient of x. Inside the bracket.

Participant: Inside the brackets. OK, I get the point.

Researcher: In other words, we are factoring bx.

Participant: We have to factor out this one. But then, eeh, first thing b has to give us the period of the graph if I am not mistaken. [Reading question aloud] There is a horizontal shift because bx is smaller that x.

Researcher: In other words, can you see b is going to a fraction here.

Participant: Yah.

Researcher: A number between 0 and 1 is a fraction

Participant: is a fraction [interrupting]. Meaning it's going to be smaller.

Researcher: Yah. That's what the learning is arguing.

Participant: Oh

Researcher: In other words, the bracket is less than x. Remember the standard is here [pointing to $y = \sin x$].

Participant: Oooh ok

Researcher: So, the learner is arguing that that bx is going to be smaller, because we are multiplying by a fraction. And bx is now smaller, he is saying the graph is now going to shrink horizontally. Not vertically.

Participant: Yah, it's going to shrink horizontally because the the …

Researcher: Or sometimes it expands.

Participant: Ok if the value of

Researcher: It depends on the value of the period.

Participant: Yah, it depends on the value of the period. If the value of b is smaller, then the period is going to be bigger.

Researcher: Ok.

Participant: Meaning the graph … If the value of b is smaller, then the graph is going to-to stretch. Then if the value of b is greater is going to shrink.

Researcher: So now let's come to the answer. Remember you are choosing.

Participant: Eh I have to agree. I think I have to disagree with the shrink because it's supposed to stretch. Because the value of b is smaller.

Researcher: Ok fine. Next question.

The rest of the participants could not identify the misconception that for fractional coefficients of x, the graph does not shrink. They explained that for $0 < b < 1$, the period of the standard function is divided by the value of b, causing the graph to shrink. One participant in particular said that, for, $y = \sin\left(\frac{x}{2}\right)$, every point of x is halved, therefore the graph shrinks. For example, what was $\sin 45^0$ now becomes $\sin 22.5^0$, and the latter is smaller in value that the former. However, the shrinkage of the graph is in terms of the period, not the amplitude. The only consolation was that for $-1 < b < 0$, they concurred that the graph is opposite to that of $0 < b < 1$. Thus, all participants except one displayed complete lack of understanding of transformation of functions. All participants except one had flawed explanation and content knowledge, as they fell for the misconception that they were supposed to have identified.

Item 9

Exactly three of the participants displayed good understanding of what is meant by a negative angle. They explained well, some supporting with diagrams. These three's explanations managed to clarify what learners normally find difficult in understanding angle measures. The other three participants did not realise that a negative angle denotes measurement in the clockwise direction, in contrast to the normal angles which are measured in an anticlockwise direction. Two of these participants attempted to explain the definition of a negative angle but eventually gave up. One had a correct idea about negative angles but then lacked confidence to express himself fully. Herein, mastery of content knowledge of negative angles was successfully used by half of the participants in teacher explanations, while the other half could not.

Analysis of interview results per participant

Descriptions are given for each of the six participants concerning their performance in the three pre-determined elements of pedagogical content knowledge.

Participant 1 Mahl

This participant had low mastery of subject matter knowledge in the task-based interview, which was contrasted with his mediocre performance in the content test. He attempted to explain without success that the amplitude of trigonometric functions which are bigger than 1, proving identities and defining the negative angle measure. He was rather weak in teacher explanations, though he was good at using examples in his explanations. He used the concept of distributive law $x(a+b) = xa + xb$ to contrast $\sin(a+b) = \sin a + \sin b$. Identifying learner difficulties and/or misconceptions was weak, as he found he committed common learner-errors he was supposed to pinpoint. He commented that questions on trigonometry were difficult for him in general because he lacks sufficient knowledge on it. His overall performance in the tasks for the interview was mediocre.

Participant 2 Mhla

She was weak in subject matter knowledge, as she could not give the correct compound expansion of $\sin(a+b)$. Her performance of content knowledge was medium. Her explanations were good as she could follow correct procedures to prove identities and solve equations. She was weak in identifying learner difficulties and misconceptions, as she unknowingly fell into the learner-misconceptions herself unknowingly. Thus, she mentioned the importance of practice of trigonometry in order to get a better performance.

Participant 3 Leng

This participant's overall performance in the interview was above average when it came to explaining concepts in trigonometry. However, his content knowledge was just below average, which tallies with his low performance in the content test. He suffered greatly in solving equations, transformation of graphs and definition of negative angles, identification of learner difficulties and misconceptions was doubtful as he could not identify them in context but exposed his own misconceptions.

Participant 4 Shab

His overall performance in the interview was brilliant. All the content knowledge, knowledge of learner difficulties and misconceptions, and explanations were perfect. He was a high performer in the content knowledge test. He was the only one who could accurately solve trigonometric equations and describe the transformation of trigonometric functions. This tallies well with the good performance which he registered in the content test. The only flaw in his explanation and content knowledge was in suggesting the use of a calculator to enter specific values of a and b in $\sin(a+b)$ as proof of $\sin(a+b) \neq \sin a + \sin b$. Unfortunately, this approach may be true for the specific values entered but may not necessarily be true for any other angle.

Participant 5 Sell

She managed to achieve a mediocre overall performance in the interview, but her content knowledge was low. In the content test, her content knowledge was medium. She only performed well in the definition of a negative angle and dealing with odd functions $\sin x$ and $\tan x$. Her explanations were not good as she grappled with the square identity, asymptotes at 90 degrees for the tangent function and amplitude of a trigonometric functions. Sometimes she could manage to explain a procedure but due to lack of sufficient content knowledge, she could not execute the procedure (as in proving identities). She also seriously got trapped in the learner-misconceptions. Thus, she commented that trigonometry is a challenging topic which teachers needs to approach with caution as they teach it to learners.

Participant 6 Malu

Good overall performance by this participant as all three elements of pedagogical content knowledge were performed equally well. This was contrasted to the low performance in the content test. All proofs of identities were explained and performed well. He had some problems though with explanations and the mastery of content knowledge when it came to solving equations and understanding the transformation of graphs. This led him to commit common learner-misconceptions himself in both concepts. He made use of examples and diagrams to aid his explanations. He commented that lack of practice is the main reason participants do not fare well in trigonometry.

Overview of elements of pedagogical content knowledge under interviews

Knowledge of learner difficulties and misconceptions

All participants managed to identify possible learner difficulties for the compound expansion of $\sin(a+b)$. All the participants managed to identify the missing negative sign emanating from the concept of odd functions, which says that $\tan(-x) = -\tan x$. It is common practice for learners to miss the negative sign in odd trigonometric functions. On the contrary, all participants except one fell for the learner misconception that for $y = \sin bx$, for $0 < b < 1$, the period decreases by a factor of b in comparison to that of the standard function $y = \sin x$. A further four did not know the effect of the negative sign to the coefficient of x in $y = \sin(bx)$ for $-1 < b < 0$. Four participants fell for the learner misconception that if given $\cos^2 x = \cos x$, then divide both sides of the equation by $\cos x$ to yield $\cos x = 1$ (illustrated in Figure 5.14).

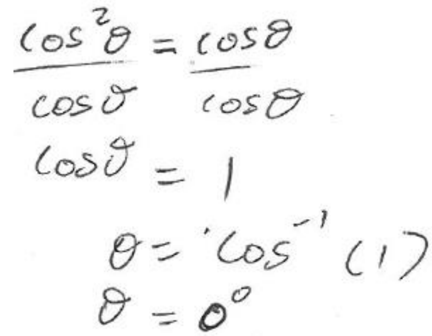


Figure 5.14. A common learner-misconception in solving quadratic trigonometric equations.

It can be deduced that preservice teachers' ability to anticipate learner misconceptions was inadequate, as most of them ended up trapped in learners' misconceptions themselves. That implies preservice teachers will end-up perpetuating misconceptions to the learners. As for learner difficulties, participants' explanations managed to show that they were somehow knowledgeable about them.

Subject matter knowledge

As in the content test, the participants had some serious challenges with the mastery of content knowledge. Only one out of six of the participants managed to identify that for $0 < b < 1$, the sketch of the function $y = \sin(bx)$ expands relative to the standard function $y = \sin x$. The rest thought that the transformed graph shrinks as if every point in the domain is multiplied by the fractional coefficient of x. Four participants realised that a negative sign in the coefficient of x in $y = \sin(-bx)$ has the effect of reversing the graph in the x-axis plane. Also, two participants used factorisation to solve the quadratic equation $\cos^2 x = \cos x$, while the rest divided both sides by $\cos x$. Exactly half of the participants could correctly expand $\sin(\mathrm{a} + \mathrm{b})$. In proving the identity $\frac{(1-\cos x)}{\cos x} = \frac{\cos x}{(1+\sin x)}$, only three participants managed to do so. Even though all the participants knew that the identity $\sin^2 x + \cos^2 x = 1$ is always true, only half of them knew how to derive it.

Moving on to the next question, half of the participants got the correct interpretation of amplitude of trigonometric functions, some by using the equation like $y = a \sin x$ where a is the amplitude and others using a sketch. Three participants knew the correct understanding of what is meant by a negative angle. Four participants managed to express $-\tan x = \tan(-x)$ in order to account for the missing negative sign on the right-hand side of $\tan(-x) \operatorname{cosec} x = \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$. Five out of six participants used graphical explanation to show that $\tan 90^0$ is undefined. None of the participants got everything correct in the tasks covered under the interview regarding content knowledge mastery. In some cases, participants resorted to the use of a calculator to prove certain statements.

Knowledge of explanations

All the participants could explain the correct way of proving identities, that is, the left-hand and right-hand sides should be equal. All managed to recognise the property of odd functions that $-\tan x = \tan(-x)$. Five out of six participants got the right explanation of the compound angle $\sin(a + b)$. The only exception was when a calculator was used to verify the compound angle using only specific values. Five out of six of the participants gave the correct explanation of the square identity $sin^2x + cos^2x = 1$. The idea of an asymptote at $\tan 90^0$ was well explained by all participants except one. The exceptional participant preferred to use a calculator to show that $\tan 90^0$ is undefined. Four participants explained well the misconception that all trigonometric function have an amplitude of ± 1 by citing counter examples where the amplitude is something other than 1 or -1. Sketches were used too to add clarity to the explanations. Also, four participants provided the correct explanation of the effect of the negative sign on b in the function $y = \sin(-bx)$, by saying it is opposite that of $y = \sin(bx)$. But only one got the correct explanation that the period of a trigonometric function $y = \sin(bx)$ increases for $0 < b < 1$ by means of a formula $\frac{360}{b}$. The rest believed the period shrinks by a factor b. Half of the participants got the accurate explanation of a negative angle by correctly stating that it is measured in the clockwise direction. Half managed to give the correct explanation of the solution to $\cos^2 \theta = \cos \theta$.

5.3.2. Lesson plan analyses

A total of six participants were tasked with the job to prepare a detailed lesson plan using the familiar School of Education template at a time when they were doing school-based teaching practice. Again, the School of Education lesson evaluation form was used to assess the six lesson plans. Five elements of pedagogical content knowledge were assessed, which are knowledge of content, teaching strategies, learner misconceptions, assessment and lesson plan structure. The lesson plan analyses constituted the third data collection instrument in this study. The analyses of these five elements per each participant are detailed below.

Participant 1 Mahl

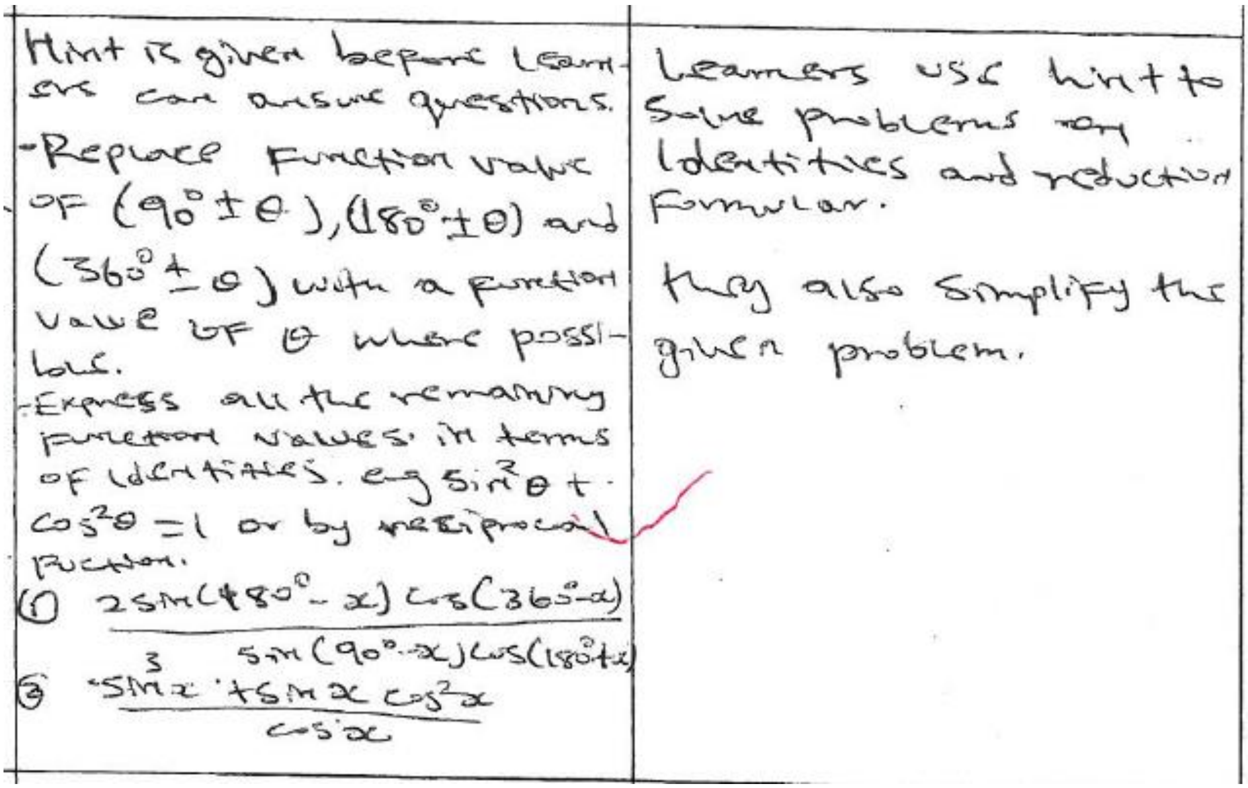
This participant planned for the concept of reduction formula at Grade 11 level. The goal of the lesson was stated, however, the challenge was that the participant seemingly planned too much for the lesson. He tried to encompass all the formulas for each of the quadrants in one lesson, as shown in Figure 5.15. Examples of problems similar to planned assessment were not provided.

Lesson Presentation	15mins	Teacher must! Demonstrate on how identities and reciprocals are derived. Explanation of how and when do we use or apply the following reduction formulae to simplify expressions such as : $\sin(90° \pm \theta)$ and $\sin(180° \pm \theta)$, $\cos(180° \pm \theta)$ and $\tan(180° \pm \theta)$ $\sin(-360° \pm \theta)$; $\cos(360° \pm \theta)$ and $\tan(360° \pm \theta)$ + $\sin(-\theta)$; $\cos(-\theta)$ and $\tan(-\theta)$	learners use reduction formulae to simplify problems. give explanation of why do they choose a Reduction Formulae of $\sin(90° \pm \theta)$ · or $\cos(90° \pm \theta)$ · · $\tan(180° \pm \theta)$

Figure 5.15. An overloaded lesson plan on the reduction formula.

In the prior knowledge section, there was mention of revising the square and quotient identities, but, the link between this and the current concept was weak, if not non-existent. The teacher explanation method was planned for and it fitted well with the content being taught. The teacher had so many explanations to do in this overloaded lesson. Other methods were also stated, which were question-and-answer and discussion. No anticipation of possible learner difficulties and misconceptions were stated. Assessment was planned, however, no similar examples were in place to make it easy for the learners to do the activity. Moreover, question two of the assessment item had nothing to do with application of the reduction formula, as shown in Figure 5.16. It is on the concept of simplifying trigonometric expressions.

It appears this participant intended to teach the square and reciprocal identity in this particular lesson as well, and besides overloading the lesson plan, these are unrelated to the lesson topic. No resources were planned for, save the usual chalkboard and sometimes textbooks. The lesson topic was stated as just “Trigonometry (Reduction formula)”, which show that the participant lacked skills in coming up with a proper lesson topic. The lesson plan was concluded well, though learners were required to take down notes in the conclusion. No new knowledge is to be introduced in the conclusion section.

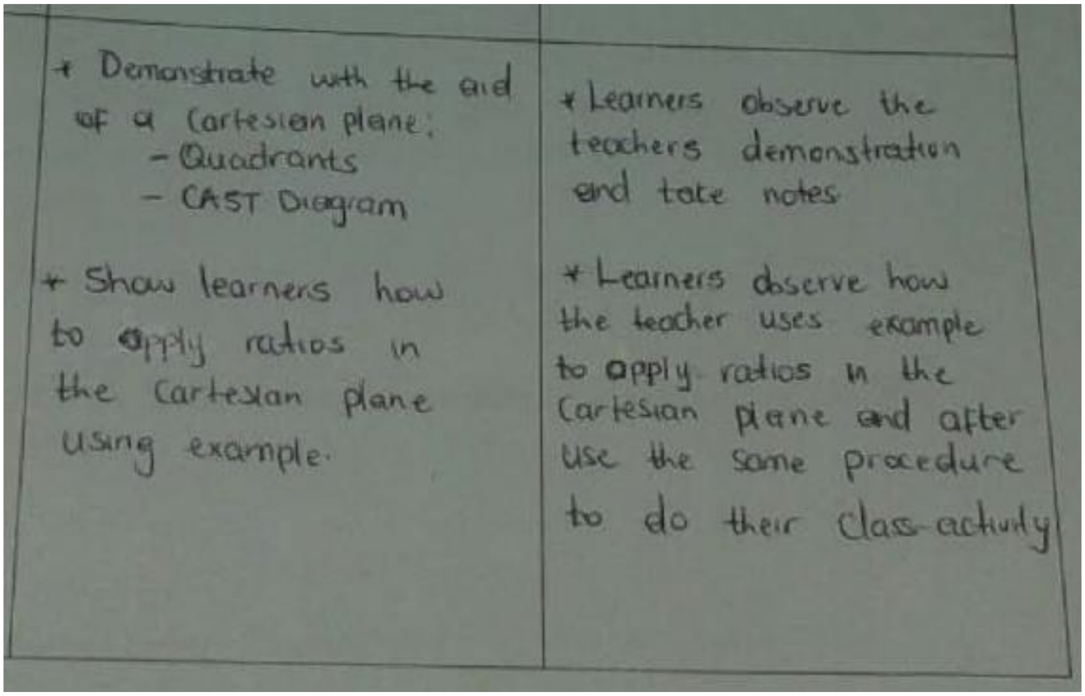


Hint is given before learners can answer questions. -Replace function value of $(90^\circ \pm \theta)$, $(180^\circ \pm \theta)$ and $(360^\circ \pm \theta)$ with a function value of θ where possible. -Express all the remaining function values in terms of identities. eg $\sin^2\theta + \cos^2\theta = 1$ or by reciprocal function. (1) $\dfrac{2\sin(480^\circ - x)\cos(365^\circ - x)}{\sin(90^\circ - x)\cos(180^\circ + x)}$ (3) $\dfrac{\sin^3 x + \sin x \cos^2 x}{\cos x}$	Learners use hint to solve problems on identities and reduction formular. They also simplify the given problem.

Figure 5.16. Assessment question unrelated to the application of the reduction formula.

Participant 2 Mhla

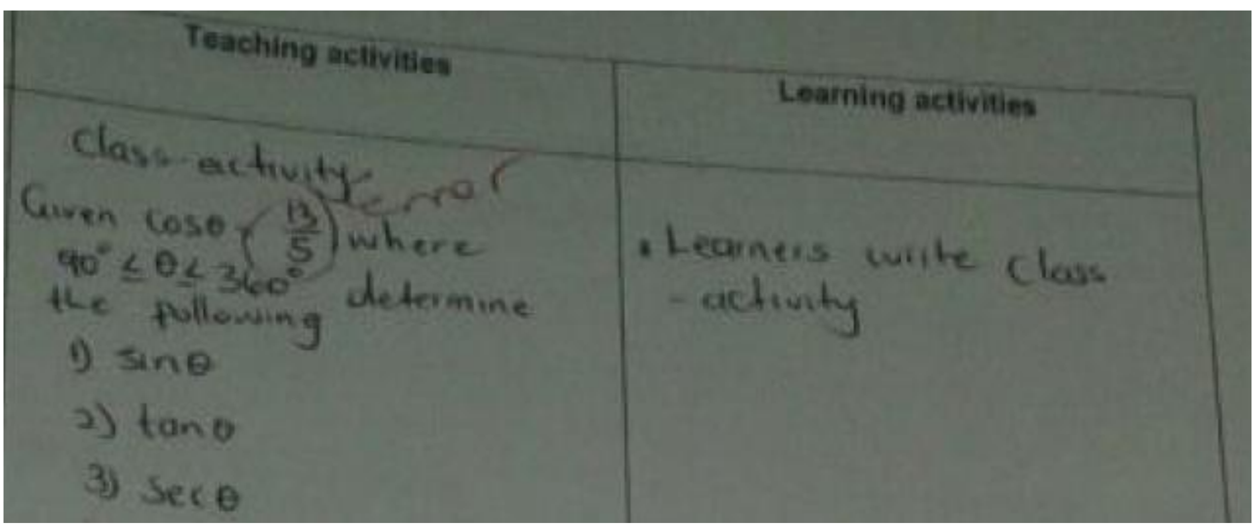
This participant planned a lesson on the application of trigonometric ratios at Grade 10 level for 30 minutes. Prior knowledge was presented in the introduction, but the concept of Pythagoras Theorem was not addressed, which was key to achievement of the lesson objective. The lesson plan clearly lacks examples of the key concept taught as none appears in the lesson development stage as can be seen in Figure 5.17. The participant went on to administer an activity without referencing relevant examples, which puts cognitive pressure on learners.



* Demonstrate with the aid of a Cartesian plane: - Quadrants - CAST Diagram	* Learners observe the teachers demonstration and tate notes
* Show learners how to apply ratios in the Cartesian plane using example.	* Learners observe how the teacher uses example to apply ratios in the Cartesian plane and after use the same procedure to do their class-activity

Figure 5.17. Lesson plan lacking sufficient details in the lesson development stage.

Moreover, the activity planned contained a flaw which led to the square root of a negative number after applying the Pythagoras theorem, as shown in Figure 5.18. It is assumed the participant meant to say $\cos\theta = \frac{5}{13}$. The screenshot in Figure 5.18 immediately followed the section shown in Figure 5.17.



Teaching activities	Learning activities
Class-activity error Given $\cos\theta = \left(\frac{13}{5}\right)$ where $90° \leq \theta \leq 360°$ determine the following 1) $\sin\theta$ 2) $\tan\theta$ 3) $\sec\theta$	• Learners write Class-activity

Figure 5.18. An assessment activity with an error.

The teacher demonstration method was dominant in the lesson plan; though other methods were planned, like teacher explanation and question-and-answer. Thus, the lesson plan appeared more teacher-centred as can be seen in Figure 5.16 where the teacher is at the helm of the lesson.

Potential learner difficulties and misconceptions for this lesson were not planned for as anticipated. The teachers' explanations do not feature in the lesson plan so that it is not known if they addressed succinctly potential learner-difficulties. The lesson plan was unfortunately written in structure form; no details of what to be covered in the lesson is conspicuous. No resources were planned for this lesson, save the chalkboard. There was a mix-up in the lesson plan as some aspects of assessment featured in the conclusion, like marking of class-activity and the corrections thereof. The lesson plan was concluded on a high note with real-life examples of the application of trigonometric ratios, as in navigation of ships and aeroplanes to determine their location.

Participant 3 Leng

This participant planned a lesson on deriving the reduction formulae and their application to simplifying trigonometric expressions to a Grade 11 class. The lesson topic was stated "Trigonometry" only and coupled with a broad objective, this lesson was overloaded for a 30-minute presentation. All the formula for quadrants 2, 3 and 4 for the three trigonometric ratios were planned in one lesson. The reduction formula was not addressed in the lesson introduction; only identities were deliberated on. There was no indication of how the teacher was going to demonstrate and explain the derivation of all the stated reduction formulae. The question-and-answer and explanation methods were stated, and they seem fitting to the details of the lesson plan. There was no identification of possible learners' conceptualisation challenges, hence no remediation plans were put in place.

The planned assessment activity covered the major facets of the reduction formula as expected. However, similar problems to what is assessed do not feature in the lesson development stage. No background was given for application problems similar to the last one in the class-activity: $\frac{\cos 10^0 \cdot \cos 120^0}{\cos 80^0 \cdot \sin 150^0}$. The lesson was concluded with a problem as a wrap-up, thereafter a homework activity was to be given. The homework problems were not stated, so it is not evident how they would fit into the lesson conclusion as can be seen in Figure 5.19.

Lesson Conclusion	5 minutes	A problem that includes both identities and reduction formulae is given:. × Simplify. $\dfrac{1-\sin^2(90°-\theta)}{\sin(90°-\theta)\cdot\cos(180°-\theta)}$ thereafter a homework is given	learners are answering questions given to see if they all understood.

Figure 5.19. A unique way of concluding a lesson of giving a problem to summarise.

No teaching resources were planned, other than the usual chalkboard and textbook. The lesson lacked details of what would take place in class but is given in skeleton format.

Participant 4 Shab

The participant prepared a lesson plan for Grade 11 for a duration of 30 minutes. The lesson objective addressed the transformation of graphs in the form $y = \sin(kx)$ and $y = \sin(x + p)$, and extended to other trigonometric ratios. It appeared the lesson plan was overloaded for all the concepts to be covered in one lesson, regardless of the lesson duration. It was stated that prior knowledge would be discussed but the full details were missing. The learners' and teachers' activities were detailed but the teaching strategy to bind these together was omitted. The sequencing of the concepts was fine, with some evidence of progressing from simple to complex, as can be seen in Figure 5.20.

The possible learner-misconception on the period of transformation of trigonometric functions was dealt with by means of a formula, as shown in Figure 5.21. Learners commonly think the graph stretches for $y = \sin(kx)$ for $k > 1$. Another misconception amongst learners is that they assume the graph shifts to the right if it is transformed by the function $y = \sin(x + p)$, relative to the standard function $y = \sin(x)$.

2.2 Main Body (Lesson presentation)

The teacher will give learners graphs to sketch on the set of axes

Learners should do them one at a time with the teacher correcting and discussing each graph after an allotted time.

The teacher must discuss the concepts of: period, amplitude, domain, range and asymptotes.

Suggested Graphs:

(1) y = sin (2x)

(2) y = sin (½ x)

(3) y = cos (2x)

(4) y = sin (x+30°)

(5) y = sin (x – 30°)

(6) y= tan (x + 60°)

Figure 5.20. Lesson development portraying progression from simple to complex.

The Sin and Cos graphs have periods of 360°, while Tan has a period of 180 °.

For the graphs of y = sin (kx), y = cos (kx) and y = tan (kx), multiplying x by k is a horizontal stretch by a factor of $\frac{1}{k}$.

In other words, k changes the period.

New period = $\frac{OriginalPeriod}{k}$

For the graphs of y = sin (x+p), y= cos (x + p) and y = tan (x + p),

the p value causes a horizontal shift left and right.

If p > 0, the graph shifts left by p units.

If p < 0, the graph shifts right by p units.

Figure 5.21. Teacher's activities addressing two anticipated learners' misconceptions.

No assessment activity was visible in the lesson plan. The conclusion section does not wrap-up the whole lesson, but only administers a homework task, as illustrated in Figure 5.22. The fact that the conclusion has been allocated ten minutes denotes that it was to be covered during lesson time. However, it defies logic to administer a homework activity during class time.

2.3 <u>**Conclusion**</u> The learners must sketch the following graphs for homework: (1) $y = \text{Tan}\ (2x)$ for $x \in [-180^\circ ; 270^\circ]$ (2) $y = \text{Cos}\ (x - 60^\circ)$ for $x \in [-360^\circ ; 360^\circ]$	**2.3** Learners must sketch the given graphs for homework.	**2.3** 10 min

Figure 5.22. Lesson conclusion planned as answering homework problems.

The participant did not plan for resources to use in the lesson other than textbooks and the usual chalkboard. The lesson topic just appears as “Trigonometry”, which is a chapter topic on its own.

Participant 5 Sell

This participant planned to teach transformation of graphs to a Grade 11 class. The lesson objective was stated but it focussed entirely on learners’ prior knowledge, that is, learners’ ability to sketch the function $y = \cos x$, which was learnt in Grade 10 (see Figure 5.21). The checking of pre-knowledge during introduction confirmed the same, from which the participant linked with current focus, $y = a \cos x + q$. Two teaching methods were stated; discussion and question-and-answer. However, the discussion method was not used in the entire lesson. The nature of the lesson may have been better addressed by other methods, for example, teacher demonstration. There was no prior plan for anticipated learner difficulties and misconceptions, though so many conceptual problems abound for this concept. A quick glance at the lesson topic revealed that the participant did not understand how to write a lesson topic. The lesson topic was just stated as “Trigonometric Functions” as shown in Figure 5.23.

Lesson Topic	Trigonometric Functions
Specific objectives(s)	Be able to draw the graph of $y = \cos x$
Teaching and Learning resources	-Chalk and duster -Textbooks (mind action series and via AFRICA) -Ruler -Calculator
Teaching methods	Discussion and question and answer methods

Figure 5.23. Lesson topic header showing weakness in lesson topic and objectives.

A single problem was planned for the assessment activity, which unfortunately did not capture the effect of q on the standard function. The usual teaching and learning resources of chalkboard and textbooks were the only ones stated. The introduction section was entirely devoted to prior knowledge; no mention of the current concept was done (see Figure 5.24).

Activities	Time allocation	Teaching activities	Learning activities
Lesson Introduction	05 min	Teacher must: -Ask learners to draw a graph of $y = \cos x$ they learnt from the previous grade (10)	Learners must: -be able to draw the graph of that Function.
Lesson Presentation			

Figure 5.24. Lesson introduction focussing entirely on prior knowledge.

Participant 6 Malu

This participant planned a Grade 11 lesson and the key concept was clearly stated, which was investigating the effects of a and q in the function $y = a\sin x + q$. The effects of q and a were investigated in a learner-centred approach. Prior knowledge was blended well in the introduction, which took the form of the basic definition of graph terminology and sketches of standard functions. As for the teaching strategies, many were stated and the main one, discovery, had been effectively used throughout the lesson plan. Several questions to be asked were written down, confirming the stated question and answer method. The snapshot in Figure 5.25 captures it all.

Lesson Topic	Trigonometric functions.
T and L resources	graph paper, Calculator and Textbook.
Specific objectives(s)	Participants should be able to draw sin x grap and able to know and understand the effect of a and q on the graph
Teaching methods	Discovery, observation, discussion and Q & A

Figure 5.25. Lesson plan header showing lesson topic, objectives, teaching resources and teaching methods.

The lesson was well-detailed with sufficient information on the teachers' and learners' activities. This had the effect of addressing possible learner difficulties. The assessment task was planned as an investigation exercise, in line with the discovery teaching strategy planned. As for the lesson plan structure, a good attempt was made in the lesson plan. Graph papers were planned for this lesson to maximise learner involvement when sketching the trigonometric graphs. The objectives were well stated, but the lesson topic was misunderstood; a chapter topic appears instead of the current lesson topic. Finally, the conclusion of the lesson was provided as a wrap-up of the whole lesson, alongside expanded opportunities for learners arising from this lesson.

Overview of the lesson plan analysis

The accuracy of the concepts planned for was good since they were self-chosen in line with Department of Basic Education work schedules to plan their lessons. Generally, procedures for lesson development were given in detail and the sequencing of concepts were performed well. The normal starting point for most of the lessons was a re-cap of prior knowledge. Nevertheless, a notable challenge concerning checking prior knowledge was that most of the time allocation for the lesson introduction was spent on prior knowledge, as shown in the sample of lesson plan in Figure 5.26.

Activities	Time allocation	Teaching activities	Learning activities
Lesson Introduction	05 min	Teacher must: -Ask learners to draw a graph of $y=\cos x$ they learnt from the Previous grade (10)	Learners must: -be able to draw the graph of that Function.

Figure 5.26. The introduction was made up of prior knowledge only.

In other words, due to lengthy pre-knowledge checking, the introduction of the current concept did not receive enough attention. Lesson plans were developed from simple to complex for some participants. Others brought in unrelated concepts in the same lesson plan. Assessment activities were present in all the lessons, but one participant planned an assessment activity with an error which would lead to undefined results. The level of assessment was fair except for some participants whose assessment tasks contained questions of concepts not planned. Learners should be prepared for the assessment tasks through similar examples in the lesson development stage. Another skill lacking in the participants was that of giving learners insufficient examples which match planned assessment problems. This has the effect of demotivating the learners as they struggle and take longer to accomplish the activities. All the participants had serious problems with drafting a lesson topic; since they were planning to teach trigonometry, to them "Trigonometry" was their favoured lesson topic. It turns out that they would keep on writing the topic "Trigonometry" every day until they finish the topic on trigonometry, which takes a total of at least six weeks at Grade 11.

Concerning planned teaching strategies, multiple strategies were identified by each participant and the evidence of their application was present. The dominant strategy was teacher explanation for the concept of reduction formula and sketching of functions, which was used by four out of six participants. The drawback was that this led to a teacher-centred learning environment. Only one did very well by using the discovery method to teach sketching graphs. Lesson plans were concluded in most cases, and two were exceptional in that they incorporated real-life examples as part of their conclusion; navigation for trigonometric rations and life-support machines for sinusoids.

Concerning addressing learners' difficulties and misconceptions, none of the participants explicitly noted and anticipated them, thus, no intervention or remediation were put in place to address such. However, two of the participants addressed this by giving well-detailed teachers and learners' activities which would somehow counteract possible learner difficulties. It was unfortunate preservice teachers did not pay attention to the need to gather, devise or improvise teaching and learning resources. Only the chalkboard and textbooks were the easy pickings. The Grade 10s and 11s learners are still at a stage where they greatly benefit from seeing and interacting with teaching and learning resource materials. Finally, participants had no problem with situating the planned concepts into the broader curriculum. All the concepts planned were well suited to the relevant grades taught, mainly because the education system in South Africa provides schools with the work schedules to be adhered to for the whole year. All the content and the sequencing wherein teachers are to cover the content identified in the teaching plan are provided by the Department of Basic Education. The content indicated within the teaching plan for each quarter is the minimum content that must be covered in that particular term, thus ambitious teachers, if they so wish, have some flexibility.

5.3.3. Video-recorded lesson analyses

The same six participants took part in the process of delivering the lesson plans which they had prepared earlier. Classroom observation notes and subsequent video analyses of the lessons constituted the fourth data set for the present study. Using video recordings made it possible to re-analyse the data repeatedly. Classroom teaching was considered the best form of exploring preservice teachers' pedagogical content knowledge in trigonometry. The School of Education lesson evaluation form was also used. The following six pedagogical content knowledge criteria were considered in video-lesson analyses: knowledge of subject matter, instructional strategies, learners' difficulties and misconceptions, assessment, lesson management and questioning techniques. Knowledge of learners and school contextual factors were excluded based on the premise that the preservice teachers were in schools for only a couple of weeks, thus they were not well positioned to understand the learners or the school context. Some knowledge types are best suited for investigating practising teachers, for example, knowledge of teachers' beliefs, culture, learners and school contexts. These do not apply to study of preservice teachers, as such Shulman (1986) and Ball, Thames and Phelps (2008) did not include them in their models. Each participant's results from the video-lesson analyses is presented below.

Results of video-recorded lesson presentations

This summary of analysis is that of participant 1 Mahl, a male preservice teacher who was teaching the introduction of reduction to a group of Grade 11 learners at a rural school (illustrated in Table 5.10).

Table 5.10. Analysis of participant 1's summary of video lesson analysis.

Elements of pedagogical content knowledge	**Details**
Knowledge of subject matter	The participant's mastery content knowledge was fine but it was tainted by errors of formulae on the chalkboard, for example he wrote on the chalkboard, $\cos(90 \pm \theta) = \sin(\theta)$ and $\sin(90 \pm \theta) = -\cos(\theta)$. Poor sequencing of concepts was observed when preservice teachers stated some identities not needed for this lesson, like the quotient and square identities. There was still over reliance on memorisation of formula in the form of SOHCAHTOA and the CAST diagram. The linking of prior knowledge to the current lesson was done well.
Knowledge of teaching strategies	The question-and-answer and explanation methods used were fitting for the reduction formula. The stated discussion strategy was not used. No learner participation during lesson development was observed, except for occasional chorus answers.
Knowledge of learners' conceptions	No difficulties or misconceptions were identified for this lesson.
Knowledge of the assessment	There was no feedback to the assessment activity done, as the assessment came late and assessed skills not taught in this lesson.
Knowledge of lesson management	Teacher facilitation was lightly done during the assessment activity. No instructional resources were used, save the chalk-and-talk. The participant took too long to get to the reduction formula, until expressions

	like $180 + \theta$, $360 - \theta$ were eventually not taught, though they were planned. The reduction formula did not get enough time thus left hanging. Also, the class-activity took extra-long, hence the lesson was not concluded, and no feedback given to learners.
Questioning techniques	Good presentation and communication of ideas were observed. The questioning technique was poor, characterised by a series of incomplete statements which learners finished in a chorus row.

As in task-based interviews and lesson plan analyses, participant 2 Mhla was a lady who taught a 30-minute lesson on the application of trigonometric ratios to a Grade 10 class at a rural secondary school. The summary of results is shown in Table 5.11.

Table 5.11. Analysis of participant 2's classroom observation findings.

Element of pedagogical content knowledge	**Details**
Knowledge of content	There was good mastery of subject matter knowledge since the participant chose her own lesson topic. However, a serious error in the assessment spoiled the remainder of the lesson. The teacher did not recognise that if $\cos\theta = \frac{13}{5}$, it is impossible to evaluate $\sin\theta$. Prior knowledge was well-linked at the right moment. Strong elements of memorisation of formula was evident, with heavy reliance on SOHCAHTOA and the CAST diagram.
Knowledge of teaching strategies	A fitting strategy of teacher demonstration was chosen, which was then solely used throughout the lesson. Consequently, the lesson was more teacher-centred, with little class discussion or learner-engagement taking place.
Knowledge of learners' conceptions	The participant was aware of possible difficulties, for example, the solution to $x^2 = 16$ was correctly given as $x = \pm 4$. She then led learners to eliminate the inappropriate solution where it was inapplicable.
Knowledge of the assessment	A single problem was administered, but it was fraught with errors which confounded learners.

Knowledge of lesson management	No teaching and learning resources were used. Facilitation was done but it was not thorough since an error was not detected in the process of assessment. No real-life examples were given to support learner understanding. Pacing was not good to the extent that the class-activity took too long, hence the lesson was not concluded nor was feedback given to learners.
Knowledge of questioning techniques.	The questions asked were never directed at individuals, thus they elicited chorus answers, for example, “You all understand?” Hence no reinforcement, probing nor scaffolding were made possible.

The participant 3 Leng’s summary of analysis was that of a male preservice teacher who was teaching the introduction of reduction formula to a class of Grade 11 learners at a rural school (illustrated in Table 5.12).

Table 5.12. Analysis of participant 3’s classroom observation details.

Elements of pedagogical content knowledge	**Details**
Knowledge of subject matter	The sequencing of concepts was not good, evidenced by lengthy explanation of identities and reciprocal functions, which had no coherence with the rest of the lesson. The participant went to a greater extent to derive the square identity and most of the trigonometric ratios per each quadrant. There was no good linkage because of the emphasis on reciprocal ratios which were not needed in reduction formula. Prior knowledge was merged well with the current lesson.
Knowledge of teaching strategies	A single teaching strategy was used throughout the lesson, which was teacher explanation. However, that strategy led to serious teacher domination of the lesson proceedings. Learners were reduced to spectators. Not much class discussion took place in the classroom.
Knowledge of learners’ conceptions	The participant was careful that the solution to $x^2 + 5^2 = 13^2$ was ±12. He explained that the third side of triangle was ±12, of which the negative value was the one used for the second quadrant calculation. However,

	no other difficulties or misconceptions were identified or addressed in class.
Knowledge of assessment	No assessment was administered in class at all, though it was planned. The supposed classwork was withdrawn and a quick look portrayed that it was going to be difficult for learners since no similar problems were addressed in class.
Knowledge of lesson structure	No instructional resources were used, save the chalk-and-talk. Teacher facilitation was almost nil because no assessment was given, and no meaningful teacher-learner interaction took place as a result.
Knowledge of questioning techniques	Low-order questions were asked which only elicited factual information.

The participant 4 Shab's summary of results was for a male preservice teacher who was teaching the sketching of transformation of trigonometric graphs in Grade 11 and the details are given in Table 5.13.

Table 5.13. Analysis of participant 4's video lesson analysis.

Elements of pedagogical content knowledge	**Details**
Knowledge of subject Matter	The concepts were well-taught, and procedures were provided, though the teacher did everything for the learners. The sequencing of concepts was good, and those concepts were well-linked. The participant knew well what was to be taught and he presented facts accurately. He had good mastery of subject matter knowledge, like he expertly distinguished amplitude from range when there was a query of such from the class.
Knowledge of teaching strategies	The discussion method was mentioned, but mainly teacher explanation strategy dominated the lesson. It was fitting but the drawback was the lesson ended up being teacher-centred. The participant lacked strong teacher-learner interaction due to dominant teacher-talk. A single teaching strategy used throughout the lesson.

Knowledge of learners' conceptions	No learner difficulties nor misconceptions were anticipated, thus it was no surprise that one of the learners interjected; "That period of 180^0 for the tangent function, where does it come from?" The misconception of the behaviour of the two functions $y = \sin(\frac{1}{2}x)$ and $y = \sin(2x)$ was planned and clarified, where the former doubles and the latter halves the period.
Knowledge of assessment	No assessment task was given in class even though it was planned. Learners were told to sketch $y = \sin(2x)$ without scaffolding during lesson development, hence they took extra-long to complete the task, squeezing out time for assessment.
Knowledge of lesson management	There were no instructional resources used, save the usual chalk-and-talk. Pacing was fine, though the practice activities took longer, choking the lesson conclusion. The function $y = \sin(x + p)$ was planned but was not addressed also due to time constraints.
Knowledge of questioning techniques	Poor questioning techniques were executed, for example, "Anyone with a different idea?", "Who is going to tell …" There was no reinforcement of learners' answers, and no probing was done to clear learner difficulties.

Participant 5, Sell, was a female who taught Grade 11s transformation of trigonometric graphs and their sketching at a rural school (illustrated in Table 5.14).

Table 5.14. Summary of analysis of participant 5's classroom observations.

Elements of pedagogical content knowledge	**Details**
Knowledge of subject matter	A narrow concept was considered for this lesson which excluded the sine and tangent functions. Good mastery of content knowledge was displayed, but the procedures were rather imposed to learners instead of being derived. The checking of prior knowledge took a bit longer than necessary.

Knowledge of teaching strategies	The teacher demonstration method was used fittingly, but it was overused. Discussion and question-and-answer were also planned but were never put into practice. A single teaching strategy was used throughout the lesson.
Knowledge of learners' conceptions	No misconception or difficulties were identified thus no intervention was suggested. She failed to connect with what was covered previously about effect of q (shifting up or down). The effect of a was imposed on learners as it not derived and done inductively.
Knowledge of the assessment	An assessment task with one problem was administered. However, the teacher gave feedback of the entire solution to the learners on the chalkboard without involving learners.
Knowledge of lesson management	No teaching and learning resources were used, save the chalk-and-talk and board ruler. The participant did not facilitate during the class-activity but instead went on to grant learners the solutions on the board. Lesson pacing suffered, as the activity took too long, hence the lesson was not concluded.
Knowledge of questioning techniques	Some leading questions were posed which elicited chorus answers from learners in most cases, for example, "Do you all understand?" There was no varying of order of questions types asked.

The participant 6, Malu's summary of analysis was that of a male preservice teacher who taught sketching of transformation of trigonometric graphs to a Grade 11 class (illustrated in Table 5.15).

Table 5.15. The summary of the classroom observation analysis of participant 6.

Elements of pedagogical content knowledge	**Details**
Knowledge of subject Matter	The key concepts were well-taught, namely, domain, range, amplitude and period. The lesson procedure was sketching of graphs using table of values. He started with standard trigonometric functions and built onto

	the transformation $y = a\sin x + q$. There was good mastery of content knowledge.
Knowledge of teaching strategies	The discovery method was used to determine the effects of a and q which helped learners to master the effects by themselves, rather than the teacher imposing information. The question-and-answer method was also used effectively so that the lesson was more learner-centred. Group-work at some stage was introduced, even though it was not used effectively since individual work was still promoted. There was good learner involvement and lively class discussions. The participant managed to draw learners' attention and addressed them individually by name.
Knowledge of learners' conceptions	No misconceptions or difficulties were identified by the participant; thus, no interventions were put in place beforehand. However, he handled learners' difficulties well, for example, learners had different opinions to the value of $\sin 0^0$ at some stage.
Knowledge of assessment	Part of the assessment was testing prior knowledge, for example, the sketch of $y = \sin x$. Multiple problems were posed covering the scope of the lesson.
Knowledge of lesson management	Graph papers were used for accurate sketching by the learners, but no other resources were used. The lesson was completed and concluded, spiced by real world examples of trigonometric graphs in the lesson wrap-up. Pacing was fine, though checking prior knowledge took too long.
Knowledge of questioning techniques	The participant did not follow-up learners' answers, for example, a learner responded to the question, "What is an angle?" by saying, "An angle is a measure where two lines meet." This was not followed up to qualify it as expected. Higher-order questions were asked, and learners gave sensible answers to these.

Overview of video-recorded lesson analyses

All the concepts which were taught were well-fitting to the relevant grades mainly because the education system in South Africa provides schools with the proposed work schedule for the whole year. One of the strengths of the Curriculum and Assessment Policy Statement is the clarity within which it indicates the content and the sequencing wherein teachers are supposed to cover the content identified in the teaching plan. This is achieved by means of a teaching plan per term for each grade. The content indicated within the teaching plan for the term is the minimum content that must be covered in that particular term, thus giving ambitious teachers some flexibility to cover more if they so wish. The participants just fitted into the system without much personal thought to the knowledge of the curriculum. Hence, knowledge of content and curriculum was not emphasised under the sub-categories of pedagogical content knowledge in this study. The researcher also find that the participants possessed good communication and presentation of ideas to their classes. All the lessons were taught in English and code-switching was only introduced when addressing individual learners at a personal level. The researcher now focus on each of the six components of pedagogical content knowledge in the following sections as part of the pedagogical content knowledge overview. The researcher commences with preservice teachers' understanding of content knowledge.

Mastery of content knowledge

The participants had the liberty to choose their own concepts to teach, hence all of them had good mastery of content knowledge in the aspects they addressed. Incidentally, most chose to teach aspects with which they were comfortable. However, for two of the participants, this good mastery was marred by some errors in the assessment activity and lesson development, both of which went undetected to the eye of the concerned participants. In one, $\cos x = \frac{13}{5}$ was planned in one of the class-activities by a participant. Another participant made a slip in the cofunction's formulae. It is not entirely true that $\cos(90 \pm \theta) = \sin(\theta)$ and also $\sin(90 \pm \theta) = -\cos(\theta)$ as was engraved on the chalkboard, as shown in Figure 5.27.

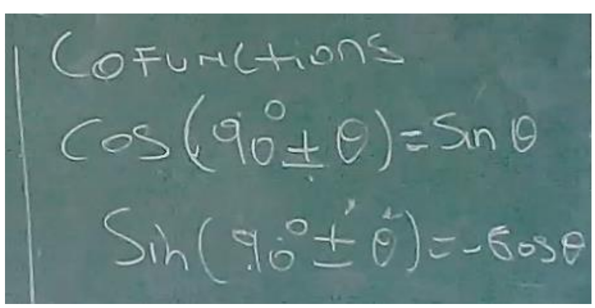


Figure 5.27. Conceptual errors in the cofunction's formulae by a participant.

There were some elements of promotion of memorisation of facts and formulae, as were in the use of mnemonics like SOHCAHTOA and the CAST diagram, shown in Figure 5.28.

Figure 5.28. Memorisation of facts and formula using the mnemonics taken from two different classes.

If teachers are not careful, the nature of most concepts in trigonometry lean towards memorisation of facts and it would appear as if learners have mastered those concepts, when they might not have. The sequencing of concepts was mixed-up by two participants, evidenced by the unrelated concepts being bundled in the same lesson. The derivation of the square identity was taught in a lesson for the reduction formula. The results show that preservice teachers planned much more than what a single lesson can handle, regardless of the duration of the lesson period. As a result, the lesson would end up being rushed through, and assessment and the lesson conclusion suffer the most. All participants blended well the prior knowledge to the current, adding coherence to their lessons.

Knowledge teaching strategies

Many eye-opening experiences were observed under teaching strategies. Five out six of the participants used a sole strategy throughout the lesson, and that strategy was a blend of teacher explanation and demonstration. Consequently, the preservice teachers dominated the lesson, leaving little room for learner involvement. The lessons observed lacked substantial teacher-learner interaction due to the dominant teacher talk. Moreover, during feedback to class activities, learners hardly went to the chalkboard to present an answer, even in cases where they could contribute meaningfully. Only one participant managed to include multiple strategies, namely discovery, question-and-answer and group-work, which rendered the lesson fully learner-centred.

Learners' difficulties and misconceptions

In planning a lesson, teachers need to take into consideration what the learners can do with or without difficulty, as well as identify possible misconceptions. Two of the participants handled the common error in solving quadratic equations where only the positive root is considered, as in $x^2 + 5^2 = 13^2$ and $x^2 - 16 = 0$, and subsequently helped the learners to select the appropriate solution where applicable. Another participant handled well the misconception amongst learners concerning the change in period of a trigonometric function $y = \sin(bx)$ where $0 \leq b \leq 1$. By making use of the formula $\frac{360}{b}$, he managed to clarify the misconception that the period of the function $y = \sin\left(\frac{x}{2}\right)$ doubles relative to the standard function. In the class of one of the participants, a learner raised a hand and inquired about the origins of 180^0 as the period the function of $y = \tan x$. The participant had not expected this and obviously he had not explained that idea well in class. The preservice teacher did not respond to this question well either, but imposed that fact to the class (See the dialogue).

Teacher: Our amplitude for this graph is going to be infinity. Now let's check the period of the tan graph. Period of the tan graph?

Learners: 180.

Teacher: Someone says the period of tan graph is always 180. 180 degrees, Neh. Ok, can we move on. Is there anyone who is left behind?

Learners: Eeh.

Teacher: Yeah, question?

Learner W: Period of 180, where does it come from?

Teacher: Right. 270+90?

Learners: 360.

Teacher: Sorry.

Learners: 360.

Teacher: Eeeeeh. Aaaaah. Say 270+180. We need to check from here up to there [pointing]. Check the other asymptote. Say tan 270. Tan 270?

Learner X: [short silence] It's undefined

Teacher: It's undefined. Right. So, it's going to be undefined. Say tan 360.

Learners: [silence]

Teacher: I need to show you how we are going to develop the next asymptote.

Learners: Its zero.

Teacher: Its zero right. Say 360+90. It's going to give you 450 right. Tan 450?

Learners: [silence]

Teacher: tan 450?

Learners: Undefined.

Teacher: Undefined neh. 450 is going to be undefined [drawing on the axes]. I need to answer this question of why are saying our period is 180. Right.

Learners: [silence]

Teacher: Then now we are going to introduce our, … another graph here. [Drawing]. This function is going to be on this fashion. So now we have our full graph here. Right. And our full graph here. So, if we have to take … From here going to that side it's going to be just 180. You know why we are adding this half side here? If we are going to continue we are going to have our … another asymptote being minus 90. Right. It's going to be in this fashion. The graph is going to behave in this fashion. Are you covered? So, a tan graph must be involve this 90. Are you covered?

Learners: Yes.

Teacher: tan graph is from, eeeh… Just check the distance [mumbled] to cover 145. These are the tan graphs. It has to cover 180.

In addition, many concepts in trigonometry were imposed upon the minds of learners, without any derivation performed. For example, no derivation was done for the effect of a in the function $y = a\sin x + q$. No other misconceptions in trigonometry were evident and observed to which the participants could have pinpointed and addressed accordingly.

Knowledge of assessment

Insufficient examples were given during lesson development stage to thoroughly prepare learners for the coming assessment activities to follow. As a result, most of the problems were quite difficult for the learners, hence learners struggled and took extra-long to complete the tasks. Four of the participants gave out a classwork activity to assess lesson objectives, while two did not do any form of assessment. Other forms of assessment were not attempted by the participants. Some find it easy to sacrifice assessment if they are under the pressure of time. In one extreme case, an error in the assessment activity (shown in Figure 5.29) went undetected during planning and administration of the assessment activity.

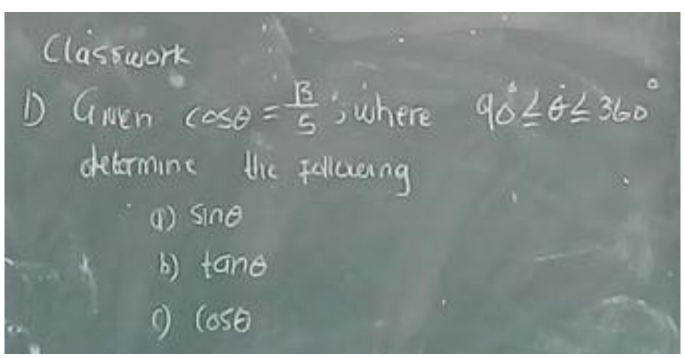


Figure 5.29. An undetected error in an assessment activity.

The participant meant to say $\cos\theta = \frac{5}{13}$ since the hypotenuse is the longest side in a triangle. Also, part (c) of the question was extraneous, because it was given in the opening statement. In all the four cases where an activity was given, two did not give feedback to the learners. One of the two who did give the activity and the feedback, went straight to provide solutions to the activity without any learner involvement. Figure 5.30 illustrates a situation where the teacher did all the stages of the corrections by herself; drawing and completing the table of values, plotting points and sketching the graph.

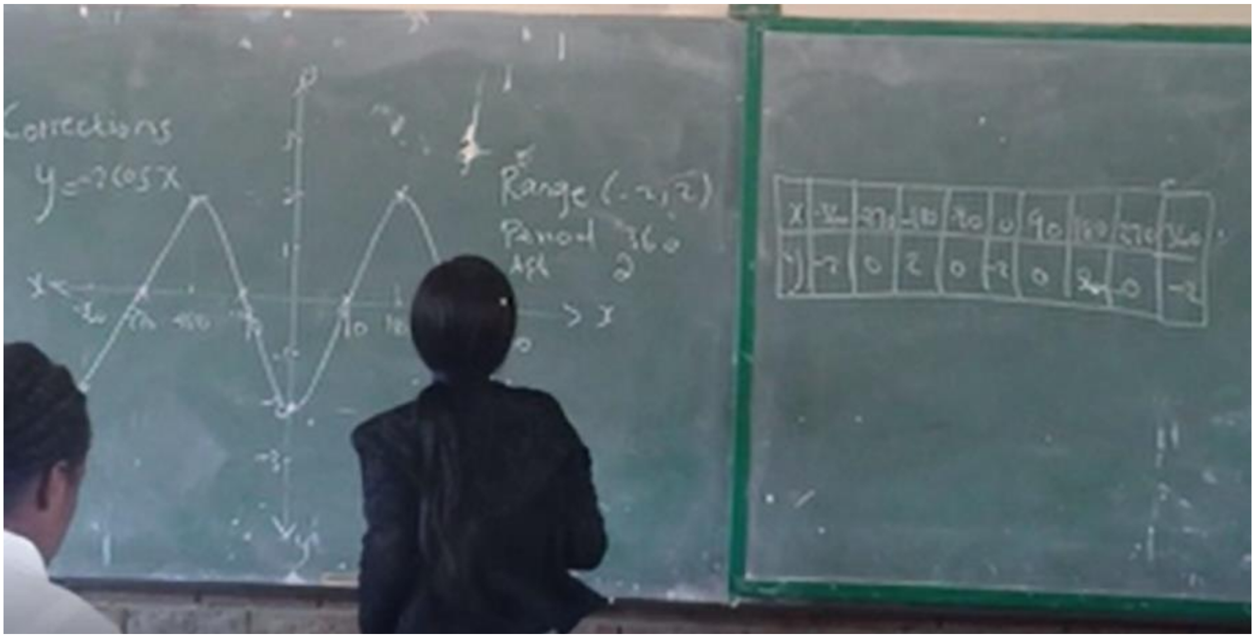


Figure 5.30. The preservice teacher granting the entire feedback to learners.

Knowledge of lesson management

Not all the participants made use of any instructional resources during their lesson presentations but resorted to the usual chalkboard and sometimes textbooks. There were situations where participants could have devised or improvised resources, yet they did not. For example, a chart could have been used to summarise key points in the reduction formula. Teacher facilitation during answering of practice examples and in the assessment task was absent in three participants, and it was not thoroughly done by those who did some facilitation. Learners grappled with tasks when the preservice teacher could have made it easy for them if facilitation was thorough. Also, mistakes in assessment tasks could have been easily identified timeously before learners got so confused in attempting to solve a problem with an error. Two participants did not administer an assessment activity at all, which necessarily meant facilitation was reduced to minimal levels.

Lesson pacing was a challenge for many participants. Half of the participants administered class activities which took too long to complete, until there was no time left for feedback or lesson conclusion. The two who never administered class activities still hurried through the lesson, suffocating the lesson conclusion. Only one participant completed the lesson on time, gave feedback to the learners and concluded the lesson well by incorporating some real-world examples. All participants had a tough time creating a lesson topic. Most just wrote on the chalkboard “Trigonometry” or “Trigonometric functions” as the lesson topic, as shown in Figure 5.31. They failed to realise the distinction between a lesson topic and a chapter topic. A lesson topic is specific to the current lesson of 30 or so minutes, whilst the chapter topic of trigonometry is taught in a span of four weeks apiece for all Grades 10 to 12 (Department of Basic Education, 2011).

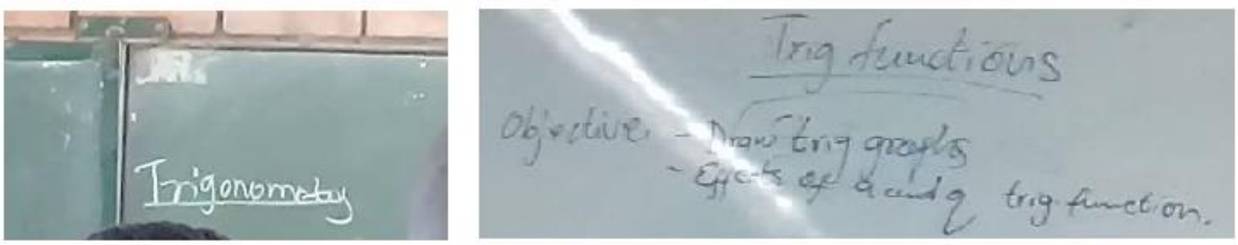


Figure 5. 31.A lesson topic misrepresented from two instances.

Figure 5.32 depicts a situation where the participant tried to narrow down the lesson topic from “Trigonometry” to “Reduction formula”. But the narrowed down topic was still too broad and generalised; there is no way everything under the reduction formula can be covered in one lesson.

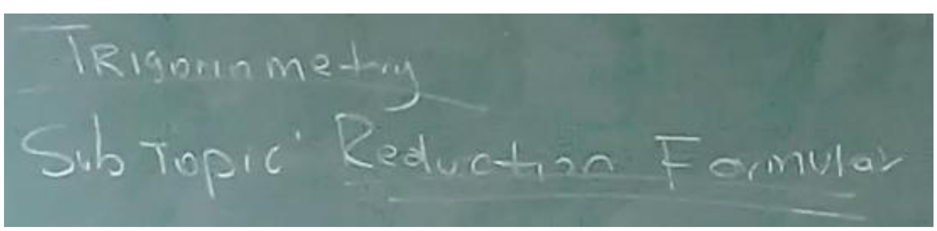


Figure 5.32. A general lesson topic on the reduction formula.

The same general lesson topics were also written the same way in the respective lesson plans, to show that it was not just an error. This led to overloaded lesson, because the lesson topic was not specific. The lesson pacing, was not planned sufficiently to cover up for the extra content matter to be taught in one lesson. Uncompleted lessons were a common feature. Some participants could argue and say time allocated was not enough for them. Preservice teachers should develop skills to plan effective lessons for any given duration, great or small. When they complete training they would encounter different school environments which may call for such skills.

Knowledge of questioning techniques

Lower-order question types were the order of the day in all the classrooms; consequently, there was no probing or scaffolding of learners' responses to clear any learning difficulties. Open questions (not directed at specific individuals) and non-specific group questions answers were quite common as well, which elicited group answers. Only one out of the six had the ability to pose both lower- and higher-order questions. That one also was in the habit of calling learners by name as they responded to the teacher's questions. It was observed that whenever participants mix English and the vernacular language in instruction, it oftentimes invoked leading questions. The Sesotho term *akere* (which loosely mean *isn't it?)* was the norm, which was responded to by a huge *yes* by the class. Some poor questioning techniques were common, for instance, a series of incomplete statements which learners finish in a chorus note. For example, "The formula for sine of is …". Participants lacked the capacity to create dialogue, discussion and reinforcement of learners' answers. Learners, on the other hand, did not pose clarification questions to the teacher either. This was captured in the dialogue shown here.

[The teacher completing the sides of the triangle given $\sin\theta = \frac{3}{5}$ on the sketch]

Teacher: But we are not given the value of x. To be able to answer the question we have to first determine the value of x. To find the value of x, what rule are we supposed to use?

Learners: [Chorus] Theorem of Pythagoras.

Teacher: Theorem of Pythagoras. Which states?

Learners: r squared is equals to x squared plus y squared. [chorus]

Teacher: Then this is a matter of just substituting what we are given. We know that r is …

Learners: Five.

Teacher: And not the forgetting the square. And we are not given x so we leave it as it is. Plus y is …

Learners: Three.

Teacher: Squared. What is five squared?

Learners: Twenty-five.

Teacher: And three squared?

Learners: Nine.

Teacher: If we want to find the value of x squared, … If we want to find the value of x squared we transpose the value of nine to the …

Learners: Left-hand side.

Teacher: Then we are gonna have twenty-five minus nine, which is equal to …

Learners: Sixteen.

Teacher: We are not looking for x squared but x, then what do we do?

Learners: We square both sides.

Teacher: We square both sides. And attention on this one. x will be equal to plus or minus …

Learners: Four.

Teacher: The reason x equals plus or minus four is because if we multiply positive four by positive four we get positive 16. And we if multiply negative four by negative four we still get a positive sixteen.

Learners: [Together with the teacher] A positive sixteen.

Teacher: Then you make a choice. You make a choice. You make a choice with respect to what? The horizontal axis is in the quadrant that you are working on. Here we are working on the first quadrant. The horizontal axis in this quadrant is …

Learners: [Chorus but not all learners] Positive.

Teacher: Positive right.

Learners: Yah.

Teacher: This means here when you make a choice you look at the quadrant you are working on and the horizontal and vertical axes. This means you are gonna pick a positive value. Which is …

Learners: Four.

Teacher: Then, now we answer the question. The question says, with the aid of a sketch, find the values of cos theta and tan theta. Now here we have a sketch that has all the sides. Now let's answer the question. Find cos theta with respect to theta. What is this cos theta?

Learners: [Again not all learners] Four over five.

Teacher: Cos theta we are saying it is the adjacent over …

Learners: The hypotenuse.

Teacher: Which is what on the triangle?

Learners: Four over five.

Teacher: And the tan of theta?

Learners: Opposite over the adjacent.

Teacher: Which is what on the triangle?

Learners: Three over four.

Teacher: Simple right.

Learners: Yes

Teacher: This is just a matter of system of angles and if you know how to do the … You remember the CAST which I told you use all All Students Take Coffee. If you remember the CAST rule and you know the trig ratios and you are able to define quadrants and the horizontal and vertical axes and the signs. You won't be able to face problems when answering the questions. Right.

Learners: Yes.

Teacher: Now here is a class-activity. Oh any questions before I continue? Anyone with a question?

Learners: No.

Teacher: Did you all understand?

Learner: Yes.

Teacher: Here is a class-activity.

5.4. Tracking of participants' performance in the four data instruments

This section is a record of the tracking of the six participants' performance in all the four data collection procedures that were conducted for this study. The intention was to reveal the implication and growth of the mathematical knowledge for teaching to classroom practice, in response to research question three.

Participant 1 Mahl

This participant's performance was 64 percent for the content test and was selected for the second phase of data collection under the category of medium performing. He managed to score full marks in six out of the thirteen items and obtained zero marks in two of the items, which were in simplifying the compound angle $\sin(\alpha - 45)$ and curve-sketching. His best performance was in the concepts of special triangles, proving the modified square identity, application of the area rule and the reduction formula. Major errors were identified in solutions of trigonometric equations and inequalities. Some minor errors were noted in simplifying $\cos\alpha$ given the value of $\sin\alpha$ and computing the values of x where $f(x) = g(x)$.

Concerning the interview results, this participant got trapped in the two misconceptions; that of dividing both sides of $\cos^2\theta = \cos\theta$ by the function $\cos\theta$. The same misconception was highlighted in the 2013 Diagnostic report, “Learners should be made aware that it is not possible to divide an equation by a trigonometric ratio, as there is a possibility that one is dividing by zero. Division by zero yields a meaningless or undefined result.” (Department of Basic Education, 2014, p.148). The other was on the transformed function of $y = \sin\left(\frac{1}{2}x\right)$, which stretches the sketch, relative to that of the standard function $y = \sin x$. The 2015 Diagnostic report on mathematics pointed out that high school learners are not aware of transformations of trigonometric functions and the resulting impact they have on these functions (Department of Basic Education, 2016). Thus, this preservice teacher’s knowledge of anticipation of learners’ misconceptions and difficulties was at low levels as he thought and made errors as learners do. Concerning the mastery knowledge of content knowledge, the participant had problems with negative angles, which he could not define and explain well. However, he managed to identify the important property of odd functions; $\sin(-x) = -\sin x$. Finally, his explanations were quite good, which he oftentimes supported with examples, for example, he used the analogy of the algebraic expansion of $a(b+c) = ab + ac$ to contrast the compound angle expansion of $\sin(a+b) \neq \sin a + \sin b$.

For the lesson planning, this participant apparently planned too much for a 30-minute lesson so that other parts of that lesson plan were not consummated. No feedback to the class-activity was given to the class and the lesson was not concluded as a result. The teacher demonstration strategy was well fitting to the lesson taught; however, it was overly and solely used to the extent that the lesson was teacher-centred. The concept of the reduction which was planned and taught was characterised by gross memorisation and facts and formulae, with the SOHCAHTOA and CAST acronyms heavily relied on. The mix-up in this lesson took another dimension when ideas which were not planned and taught under the reduction formula, suddenly featured in the class-activity. These included the concepts of $180 + \theta$ and $360 - \theta$. Consequently, learners struggled a lot with the activity and the teacher's facilitation, which was insufficient, did not help the learners much. This was a good sign that the participant did not anticipate learners' difficulties, to which he would have planned and implemented intervention strategies to rescue the situation. Finally, the participant's questioning techniques were poor, as only lower-order questions were posed.

There was no change in content knowledge from the content test to the task-based interview and classroom practice. Presumably, he entered teacher education with average performance in mathematics, which remained at that level all through teacher education. Misconception from school mathematics remained firm in him. His teaching style was teacher-centred which relied greatly on teacher explanation of ideas. This inevitably promoted memorisation of facts and formulae. The teacher-learner involvement was low, epitomised by lack of meaningful facilitation of learning during the lesson. Finally, he resorted to teaching the reduction formula to which he was particularly good at in the content test, thereby shunning concepts with which he was not conversant. Teacher education, by all intents does not promote such teaching etiquettes, which means preservice teachers cling to old habits of teaching from past experiences as learners.

Participant 2 Mhla

This participant obtained a score of 62 percent in the content test and was selected for the second phase of data collection as a medium performing participant. Out of the thirteen items in the content test, she scored full marks in six items. These items included labelling special triangles, the application of the area rule and the reduction formula, simplifying trigonometric ratios and curve sketching. Minor errors were identified in solving equation and proving identities. Lastly, the participant was quite weak in finding solution sets to trigonometric inequalities.

In the interview, her explanation and identification of learner difficulties were fine, but her content knowledge was shaky. She could not expand $\sin(a + b)$ correctly nor could she attempt to explain that the range of trigonometric functions is not always +1 or -1. The participant unknowingly chose to divide both sides of the equation $\cos^2 \theta = \cos \theta$ by $\cos x$

in the process of solving the equation. Furthermore, she erroneously deduced that the sketch function $y = \sin\left(\frac{x}{2}\right)$ shrinks compared to the standard function $y = \sin x$, because every point of x is divided by two. Her understanding of negative angles was partial, in that she managed to explain well that the effect of the negative sign on the function $y = \sin\left(-\frac{x}{2}\right)$, but could not define a negative angle. Moreover, she could not justify the idea that for odd functions, $\tan(-x) = -\tan x$, so as to account for the missing negative sign on the right-hand side of $\tan(-x)\,\mathrm{cosec}\,x = \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$. Lastly, the participant obtained the correct proof of the square identity $sin^2x + cos^2x = 1$ only after probing by the interviewer.

The teacher demonstration strategy was planned and used in the lesson development; however, it was over-used so that there was no notable class discussion and learner engagement. A glance at the lesson plan of this participant revealed that the lesson development stage was devoid of examples. During the administration of the class activity, facilitation was done but was not thorough since a flaw in the activity was not picked up. Real-life examples were stated in the lesson plan but were not enacted in the lesson delivery. This was in part due to poor lesson pacing, which led to insufficient time being devoted for completing the class-activity and concluding the lesson. There was evidence in lesson delivery of memorisation of formula, marked by heavy reliance the SOHCAHTOA and CAST acronyms. The participant made efforts to anticipate possible learner difficulties, for instance, she highlighted that for $x^2 = 16$, it implies that $x = \pm 4$. The importance of this aspect was that the true answer was $x = -4$ since the angle was in the second quadrant. No teaching and learning aids were planned nor implemented in class, save the usual chalkboard and textbook. There was good clarity in the participant’s explanation of concepts to the class. Finally, the questioning technique was mainly composed of low-order questions which at times invoked chorus answers. Thus, there was no evidence of probing, scaffolding and reinforcement as a follow-up to learners’ responses.

From the content test to the task-based interviews, the mastery of content knowledge deteriorated. She faced interview tasks that were more classroom-based than the stand-alone one in the content test leading to a reduced performance. She taught what she had relatively done better in the content test, which does not show knowledge growth. At the end of the day she will be faced with teaching all the topics in her class. Traces of traditional teaching feature often, as observed in teacher-domination, at the expense of learner-involvement and facilitated learning. She failed to adopt modern teaching styles which promotes learner-involvement in the creation of their own knowledge.

Participant 3 Leng

With a performance of 18 percent in the content test, this participant went on to the second phase of data collection under the low-performing category. Indeed, in the content test, his content knowledge was basic, evidenced by the fact that ten of the thirteen items were not attempted, and in cases when they were attempted, the score obtained was zero. These zero-score items encompassed identifying special triangles, proving the square identity, solving inequalities and simplifying trigonometric ratios. He managed to score full marks in only one item, which was on sketching trigonometric functions. Some minor errors were made in two items, namely, solving trigonometric equations and application of the reduction formula. In the task-based interview, the participant had difficulties with correct handling of misconceptions, as he made committed both misconceptions; dividing by a functions when solving equations and that the transformation of functions for standard functions shrink for $0 < b < 1$ in $y = \sin\left(\frac{x}{2}\right)$. The participant could explain most concepts covered in the interview, but his proofs were not accurate. For instance, he could explain the correct procedure of proving identities, but could not give the correct proof of the identity $\frac{(1-\cos x)}{\cos x} = \frac{\cos x}{(1+\sin x)}$. He also explained well that $\tan 90^0$ is undefined and that the range for trigonometric functions is not necessarily ± 1. These required the participant to explain only, without involving proofs.

Concerning the lesson planning and presentation, this participant inadvertently planned more than enough for a 30-minute lesson on the reduction formula concept. The checking of pre-knowledge took longer than necessary so that in the end, the lesson was hurried through. The assessment task which was duly planned for was not actioned and the conclusion for the lesson was not carried out. A sole teaching strategy, teacher explanation, was used throughout the lesson, though others had been planned for in the lesson plan. That teaching strategy, though suitable for reduction formula at Grade 11 level, rendered learners to spectators. The lesson was devoid of learner involvement and any form of discussion. The participant had some anticipation of possible learner misconception when solving quadratic equations, thus he went to some extent to highlight that if $x^2 + 5^2 = 13^2$, then the solutions are $x = \pm 12$, instead of just $x = 12$. Some formulae were derived but those that were derived had no linkage to the lesson topic. Finally, the questioning technique centred on low-order questions which then lacked reinforcement or probing aspects.

Some improvements to content knowledge from the content test to classroom teaching practice were observed. He did not just follow the routine of teaching their best-performed concepts, but he, having scored high marks in sketching graphs, chose to teach the reduction formula. However, there was no development in skills of identifying and dealing with misconceptions as he progressed through teacher education. Again, he felt that teaching rests on good explanation of concepts and procedures. Unfortunately, some procedures lacked substance as he fails to execute those procedures in full, as in proving identities. This he did through a sole teaching strategy of teacher explanation.

Participant 4 Shab

This participant's performance in the content knowledge test was one of the best in the group, with the score of 79 percent. He scooped full marks for over half of the thirteen items. His only major weakness was on finding the solution set of the inequality $f > g$. Minor errors were identified in solving trigonometric equation equations and the application of the reduction formula. In the interview, the participant could not explain algebraically the expression $\sin(a + b)$; rather he chose to use specific values of a and b to verify that the $\sin(a + b) \neq \sin a + \sin b$. Other than that, his proofs of identities were above-board, coupled with clear explanations. He also expertly identified learners' misconceptions and gave the correct explanations thereof, frequently supported by sketches.

As for lesson planning and presentation, the lesson objectives were stated clearly. No prior knowledge was stated in the lesson plan nor actioned in class. Discussion teaching method was stated in the lesson plan, but teacher demonstration and explanation dominated the lesson presentation. The result was a teacher-centred lesson, with had little or nil learner involvement. No teaching and learning resources were planned for the lesson nor used in lesson delivery. A remarkable observation was that the participant did not anticipate learner difficulties, thus, it came as shock to him when a learner asked from nowhere why the period of the tangent function is 180 degrees. The possible learner-misconception as to which transformation stretches the sketch of the standard function $y = \sin(x)$ between the two functions $y = \sin(2x)$ and $y = \sin\left(\frac{x}{2}\right)$, was handled well by this participant. An assessment task was planned but was not administered, partly due to time constraints as the lesson was rushed through towards the end. Finally, the participant's questioning was composed of low-order questions which did not bare any probing or reinforcement element.

This participant had good understanding of content in all the data collection instruments. However, the delivery of mathematics concepts was done in a teacher-centred environment. All learner-to-learner, learner-to-teacher and learner-to-resources forms of interactions were absent. Surprisingly, he consciously planned discussion as a teaching strategy in the lesson plan but did not enact it in the classroom practice. Therefore, he needs skills in linking lesson plans to lesson implementation on some of the expected teaching practices.

Participant 5 Sell

With a score of 56 percent in the content test, this participant was classified as medium performing. She registered impeccable performance in six out of the thirteen items and her strengths were in identifying special triangles, handling the square identity, curve sketching, simplifying trigonometric ratios and the solution set of the inequality $f > g$. She encountered major errors in solving equations and applying the reduction formula. Finally, she did not completely understand three of the 13 items, which were the application of the area rule proof, the solution set of a function where it is undefined and simplifying the compound angle $sin(\alpha - 45)$.

In the interview outcomes, she had a good explanation of concepts but lacked the necessary content to substantiate it. For example, when asked to explain the $\sin(a+b) \neq \sin(a) + \sin(b)$, she started well by contrasting the algebraic expansion $a(b+c) = ab + ac$ to that of $\sin(a+b)$. However, she could not expand the compound function $\sin(a+b)$ correctly. Furthermore, she could identify well the correct way of proving identities, that is, equating the right-hand side and the left-hand side, but could not prove the identity given as $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$. At one stage she resorted to the use of a calculator to verify the identity above, which signifies the lack of cognitive understanding. As for the misconceptions in trigonometry, she also made the two learners' possible misconceptions; division by a function $\cos\theta$ and the transformation of standard functions. The participant managed to handle the effect of the negative sign on trigonometric graphs well, as depicted in $\tan(-x) = -\tan(x)$ and $y = \sin\left(-\frac{x}{2}\right)$, as well as the general definition of a negative angle.

The lesson planned by the participant was of a narrow focus whereby she looked at the sketching of the function $y = a\cos x + q$ only. There was no transfer learning to other functions like tangent and sine. The teacher demonstration method was solely used throughout the lesson to the detriment of class discussion and the question-and-answer methods that were stated in the lesson plan. The checking of baseline knowledge took a bit longer, reducing the rest of lesson in terms of pacing. As a result, learners were not involved in feedback to the activity and the lesson was not concluded. There was no clarity in the lesson plan whether q in $y = a\cos x + q$ was classified as prior knowledge or part of the current lesson. Thus, the effect of q on the standard function of $\cos x$ was imposed on the learners, rather than derived. Only one problem was posed for the class-activity which was fitting for a 30-minute lesson. There was no evidence of anticipation of learner difficulty and misconceptions. There was good presentation and communication of ideas. Leading questions featured quite often and there was no varying of the order of question types.

The scale of this participant's development of knowledge of teaching was hampered by lack of adequate content knowledge. Her preferred teaching method of teacher explanation lacked substance, as some of the explanations required good mastery of content knowledge to give them substance. This was seen in processes of solving equations and proving of identities. Thus, she resorted to teaching only the concepts she felt she knew- the content. Even her own misconceptions were common as a result of low content knowledge mastery.

Participant 6 Malu

With a score of twenty-three in the content test, this participant was selected for the second phase of data collection as a low-content performer. No full marks were scored in any of the thirteen items under consideration. He made minor errors in curve-sketching, which was his best, as shown in Figure 5.33. The function $g(x)$ was done well, but some confusion hit him when it came to $f(x)$. Major errors were made in solving equations and inequalities, labelling special triangles, computing intersection points of two functions and the application of the reduction formula. Nevertheless, the interview performance was quite good for this participant. He could easily identify learner difficulties and provide possible solutions himself. The only serious concern was on learner-misconceptions; he got trapped in the misconception he was supposed to identify. He effortlessly divided both sides by a function $\cos\theta$ and suggested that the sketch of the transformation of $y = \sin\left(\frac{x}{2}\right)$ shrinks compared to that of the standard function $y = \sin(x)$. He portrayed good understanding of the meaning of negative angles and their application to transformation of functions as in $y = \sin\left(-\frac{x}{2}\right)$.

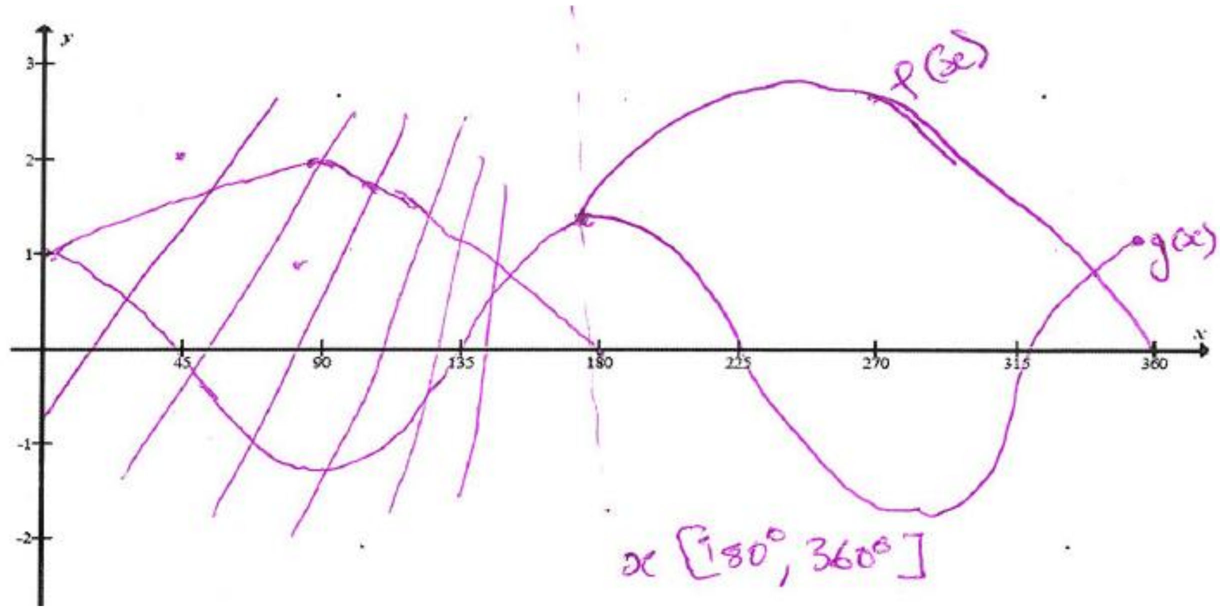


Figure 5.33. Minor errors in sketching of the functions $f(x) = 1 + sin(x)$ and $g(x) = cos(2x)$.

His lesson plan was well-written and detailed. All the planned teaching strategies were well-implemented in the lesson presentation – group-work was used for maximum learner involvement and the discovery method was used to teach transformation of graphs, starting from the simple to complex problems. The question-and-answer method was effectively used to grip the attention of the learners. The participant's mastery of content knowledge was good. Variation of question types was clear though no follow-ups were done to the learners' responses. He seemed to lack the skill to build-up on learners' partial answers. He completed the lesson within time and managed to provide feedback to the class-activity. The conclusion was spiced with real-life examples of common occurrence of graphs. Graphs papers were sought and used in class for accurate sketching, which was done as a group activity.

This participant defied all odds of low-content performance to stage a good performance in classroom practice. His lesson planning and implementation were informed by modern teaching propositions whereby the interests of the learners were put first. He made content knowledge so simple to learners that they were highly engaged throughout the lesson. It was only his level of mastery of misconceptions that took a knock from low-content mastery, as he repeated most of the learner-misconceptions himself.

5.5. Conclusion

This chapter was a presentation and interpretation of data from the four data collection instruments, namely, the content test, task-based interview, lesson plan analysis and video recorded lesson analysis. From the content test, quantitative data in the form of participants' percentage scores were obtained, as well as qualitative data which arose in the content analysis. In all, fifteen participants took part on the content test and the average performance per participant was 41 percent. After the initial analysis of content test scores, one high-performer, three mediocre-performers and two low-performers were selected to take part in the second phase of qualitative data collection, which was the task-based interview, lesson planning and video-recorded lesson presentation. In the interview results, participants could easily explain the concepts under consideration but could not conduct accurate proof of identities in the same concepts. Regarding misconceptions, preservice teachers' thinking was at the level of learners as results depict them committing errors reported in the Diagnostic reports for Grade 12 national examinations.

In the lesson plan analyses, it was observed that the participants could not frame a lesson title appropriately, planned too much for a thirty-minute lesson and planned insufficient examples for their lessons. The analyses of video classroom observations revealed that participants employed a sole teaching strategy and some of them were trapped in the learners' misconceptions they were supposed to identify. Thus, the content and pedagogy that the preservice teachers were able to remember and use in dealing with trigonometric concepts was limited. A trace of the six participants' performance in the four data instruments was done, which portrayed stagnated growth in content and pedagogical content knowledge. The discussion of the research findings is reported next in Chapter 6 of this study whilst Chapter 7 deals specifically with the drawing of conclusions and the generation of new knowledge.

CHAPTER 6: DISCUSSION OF RESULTS

6.1. Introduction

This chapter attempted to discuss the findings of this study in line with the literature review and the conceptual framework. The researcher starts with the discussion in section 6.2 with a review of research question one, which hinges on the findings from the content test. Section 6.3 is a discussion of the research question two, whereby the pedagogical content knowledge elements are explained in the light of findings from the task-based interviews, lesson plan analyses and video-teaching observations. The subsequent section focusses on the discussion of research question three, the extent of development of the mathematical knowledge for teaching based on classroom practice, in the light of the results from all the data collection instruments. All the discussions were done in cognisance of the conceptual framework that guided the study. Section 6.5 presents an overview of the discussion of findings considering all the data sources. The chapter conclusion (section 6.6) consummates the chapter by a wrap-up of the key ideas in the findings.

6.2. Subject matter knowledge discussion

Subject matter knowledge was investigated using the content test. The content test analysis, both qualitative and quantitative, formed the basis for the discussion that follows. This discussion addressed the first research question, "What is the level of preservice teachers' understanding of trigonometric concepts?" The participants were typical South African preservice teachers with a rural background who were studying for a mathematics and science teaching degree. In South Africa, several preservice teachers are dogged by weak mathematics content knowledge background (Biyela, 2012), and most of these preservice teachers come from under-resourced rural schools. On this basis, teacher education institutions may still produce teachers of mathematics enshrined with mathematics content incompetence (Brodie, 2004). The quality of teaching is rooted in teacher education, thus, to improve standards of mathematics teaching, there is need to investigate current teacher preparation programmes.

The Minimum Requirements for Teacher Education Qualification implores recent teacher graduates to possess robust subject matter knowledge in order to have a lasting impact on children's learning (Department of Higher Education and Training, 2011b). Mathematics and numeracy performance of South African learners has been under the spotlight for a long time, especially among the disadvantaged and rural populace (Jansen, 2011; Howie, 2002; Reddy, 2006). Thus, preservice teachers enter teacher education with meagre understanding of school mathematics. If preservice teachers do not happen to receive sufficient undergraduate instruction in school mathematics they will teach, they qualify and enter the classroom ill-prepared to teach mathematics to the next generation of learners. This perpetuates a vicious cycle of poor school mathematics mastery (Department of Education, 2001; Conference Board of Mathematical Sciences, 2001). Both Shulman (1986) and Ball, Thames and Phelps (2008) investigated the types of teacher knowledge based on preservice teachers.

The quantitative findings revealed small measures of central tendencies but high measures of spread. This was an indication of low mastery in content knowledge by the participating preservice teachers. From the data, only five of the fifteen participants obtained an overall mark of at least fifty percent. Combined with the other descriptive statistics mentioned above, it suggests that the participating preservice teachers possessed insufficient understanding of trigonometry. This was tantamount to probable weak performance in the preservice teachers' future early years of teaching (She, Matteson, Siwatu & Wilhelm, 2014).

The study by Ogbonnaya and Mogari (2014) did show that teachers' subject matter knowledge on trigonometry directly influences learners' achievements at Grade 11 level. Therefore, it implies that if there must be an improvement in performance by learners in mathematics, a good foundation is to strengthen the teachers' content knowledge base. If preservice teachers are to be productive in the mathematics classroom, they must show good mastery of the school mathematics concepts they are going to teach upon qualifying as mathematics teachers. This mastery is perfected by their schooling years, as well as the undergraduate degree studies. Howald (1998) discovered that trigonometry was one of the least understood topics by the prospective teachers and experienced mathematics teachers. Also, Lloyd (2006) went on to say that preservice teachers often have weak mathematics knowledge and a narrow view of mathematics they are required to teach.

Furthermore, Dündar and Yaman (2015) revealed teacher candidates similarly displayed serious lack of knowledge of trigonometry. In all, available research findings on in-service and preservice teachers give an indication that their mastery of content knowledge of school mathematics is not connected and robust in accordance to the earnest expectation of mathematics education community (Ball, Lubienski, & Mewbom, 2001).

Finer details on preservice teachers' performance in content knowledge become more evident when the researcher considers each of the two types of content knowledge according to the model by Ball, Thames and Phelps (2008). The third category, horizon knowledge was dropped in this study. The specialised content knowledge type which should be possessed by all mathematics teachers was allocated seven out of the thirteen items. However, preservice teachers' success rate in the items on specialised content knowledge was 34 percent, denoting limited mastery of specialised content knowledge for many preservice teachers (Mudaly, 2015). Teachers use specialised content knowledge in the everyday tasks of teaching when they give explanations, chose relevant examples to support conceptualisation, handle learners' questions appropriately and responses and pose questions to probe and scaffold learning. Thus, if the preservice teachers' unique knowledge to teaching show weak conceptualisation, it implies that their teaching capabilities of these concepts is under jeopardy.

The items were devoted to exploring the preservice teachers' common content knowledge in the content test. Preservice teachers' performance on these stood at an average score of 48 percent. This denotes that preservice teachers had a mediocre understanding on the content knowledge needed to solve and compute general mathematical problems. The preservice teachers did not appear to have much difficulty with common content knowledge: solving problems, recognising incorrect answers and using mathematical terms and notation (Ball, Thames and Phelps, 2008). Preservice teachers thus were relatively better in mathematics knowledge where explanations, justifications or reasoning were not necessary. Similarly, Mudaly and Moore-Russo (2011) investigated in-service teachers' common content knowledge of gradient of a straight line and discovered that it was weak for some and strong for others. Mathematics teacher educators must contend with teacher candidates who enter teacher education institutions with inadequate or non-existent common content knowledge. Common content knowledge ought to have been performed better since it is a springboard upon which specialised content knowledge and horizon knowledge develops (Mudaly & Moore-Russo, 2011).

This study also uncovered the fact that many participating preservice teachers could not fully apply known algorithms or given conditions flexibly into the various contexts given due to their lack of content knowledge in trigonometry. This conclusion was drawn based on the preservice teachers' poor performance in both items 3 on solving trigonometric equations and item 6 on proving identities. They knew well the procedures to solve and prove, but they could not precisely execute them all the way to the required end. This brings to the fore elements of procedural knowledge being put into practice but with underlying conceptual gaps or complications. In most cases South African learners tend to resort to applying rules and procedures whenever they do not fully understand the underlying principles in topics like trigonometry (de Villiers & Jugmohan, 2012). Having been learners themselves in the same education system, preservice teachers still harbour the same over-reliance of procedural learning of concepts, upon which they now base their teaching (Baturo & Nason, 1996). Ball (1990) posited that preservice teachers are frequently limited to algorithmic understanding of school mathematics, in particular trigonometry. Trigonometry is mostly taught in schools from a procedural perspective, which in turn causes its conceptual aspects to be neglected (Dündar & Yaman, 2015; Li & Smith, 2007).

Generally, preservice teachers' mastery of content knowledge was below 50 percent for most of the concepts addressed in the content test, except for sketching trigonometric functions. Interestingly, one of the low-performing participants who scored no marks in eleven of the thirteen items scored full marks in curve-sketching. Thus, curve sketching was independent on the performances in other items. Item analysis indicated that there were more zero marks scored than any other; zero scores appeared 38 percent in the participants' responses relative to 24 percent who obtained full marks in their responses. Most of the participants chose to leave items unanswered when they had severe challenges.

Similar studies involving other knowledge domains by Mudaly (2015), Brijlall and Maharaj (2015), and Fi (2003) also arrived at the same conclusion, that preservice teachers' content knowledge is inadequate. The Department of Higher Education and Training (2011b) indeed acknowledged that teachers' content and conceptual knowledge are poor. Measures to address this challenge have been put in place, starting with preservice and in-service teacher education. What is evident in this study is a remarkable dearth of knowledge of trigonometry by participating preservice teachers. A conjecture for the weak conceptual understanding of trigonometry was a result of information-loss over time. Preservice teachers last encountered trigonometry at high school or sometimes in entry-level modules at university (Fi, 2003), which transforms to be at least two years prior. Upon further inquiry, preservice teachers lamented on the necessity to delve deeper into the fundamental ideas of trigonometry in their undergraduate mathematics content modules, which they did not adequately learn or understand when they covered these at high school. It is of notable concern that teacher educators need to take into consideration. There is need for a re-examination of school mathematics content from an advanced platform in higher education institutions (Usiskin, Peressini, Marchisotto & Stanley, 2003). Teacher education programmes should acknowledge the necessity to re-refresh preservice teachers to the foundational mathematics content concepts which they will happen to teach upon completion of their studies. Hence, Ball (1990) suggesting that content knowledge should be the principal focus of teacher education instruction so that novice teachers may be able to teach mathematics effectively.

In conclusion, the results emanating from this study highlight the fact that preservice teachers generally lack school mathematics content knowledge. They notably have insufficient knowledge of mathematics up to the point of exit from teacher-training institutions (Even & Ball, 2009). This finding affirms the previous results of similar studies and existing theories that preservice teachers' understanding of mathematics they have acquired from teacher education is inadequate for school mathematics teaching (Mji & Makgato, 2006). As for trigonometry, most concepts encountered in teacher education modules are well covered in school mathematics. Thereby, preservice teachers have an opportunity to consolidate what they have learnt in school and gain additional understanding of the trigonometry concepts connected to what they have already covered. This was supposed to boost preservice teachers' chances to excel in trigonometry, however, preservice teachers still experience some difficulties in grasping trigonometry concepts. As for them being able to teach effectively, the answer depends on preservice teachers' mastery of the pedagogical content knowledge, which is the subject under discussion in the next section.

6.3. Pedagogical content knowledge discussion

This discussion on exploring preservice teachers' pedagogical knowledge was in response to the second research question, "What pedagogical content knowledge do preservice teachers possess in trigonometry?" Pedagogical content knowledge, unlike content knowledge, is elusive to researchers on the premise that it is a unique and isolated teacher construct, which makes it difficult to assess. While having no standard instrument for measuring teachers' pedagogical content knowledge, researchers and other interested parties could well learn about the nature and extent of pedagogical content knowledge using different instruments such as interviews, preparation and analysis of lesson plans, and video-recorded classroom observations (An, Wu & Kahn, 2004; Ijeh, 2012). Notwithstanding, evidence exists to the effect that pedagogical content knowledge is a function of the quality of initial teacher education which preservice teachers receive during training (McAuliffe, 2013).

In this study, task-based interviews with hypothetical instances of learners' work, lesson plan analyses and classroom observations were used to evaluate preservice teachers' pedagogical content knowledge. The best acknowledged means to evaluate pedagogical content knowledge is through observing classroom practices because pedagogical content knowledge has its deep roots in classroom practice (Toerien, 2011; Lee & Luft, 2008; Gess-Newsome & Lederman, 2001; Bütün, 2005). The researcher has divided the discussion of pedagogical content knowledge into two parts; one dealing with types of pedagogical content knowledge according to the theoretical framework and the other based on the pre-determined components of pedagogical content knowledge.

6.3.1. Types of pedagogical content knowledge

The discussion below is based on the results from the task-based interviews, lesson plan analyses and lesson observations, as the results of this study are discussed according to Ball, Thames and Phelps's (2008) categories of pedagogical content knowledge.

Knowledge of content and students

In the interviews, preservice teachers had some insight into the reasons for learners' mistakes in the given hypothetical situations. Thus, even with low mastery of content knowledge, good identification of learner difficulties was possible. In the lesson planning and presentation, preservice teachers did not make an effort to anticipate learners' potential difficulties and possible sources of learners' errors. In simple terms, there was no prediction of learners' thinking, though some difficulties were fairly handled as they cropped up in the lesson presentations. Also, the lesson plans and presentations lacked enough practice examples designed to make it easy for learners to understand the concept at hand.

On the other hand, identification of learners' misconceptions was poorly performed by almost all the preservice teachers in the task-based interviews. The fact that preservice teachers committed the same learners' misconceptions was a clear testimony to this (Haciömeroglu, 2009). The implication is that preservice teachers' thinking was at the level of learners, and as a result, preservice teachers might unknowingly exacerbate learners' misconceptions. Inadequate mastery of content knowledge was one of the reasons for preservice teachers to be incognisant of pertinent learners' misconceptions. Similarly, incorrect teacher explanation was also associated with poor identification of learner-misconceptions. Hence, the preservice teachers' mastery of knowledge of content and students was below expectation.

Knowledge of content and teaching

The pedagogical content knowledge sub-domain of knowledge of content and teaching, which identifies what teachers do about learners' errors and misconceptions, was typified by teachers' explanations in the interviews. Even though preservice teachers had insight into learners' challenges to learning in trigonometry, they could not give clear-cut explanations as to how best they should help the learners who had such challenges. The preservice teachers only managed to identify and address learners' errors and misconceptions of the content with which they were familiar (Moru & Qhobela, 2013). For example, identification of misconceptions in solving trigonometric equations and transformation of functions was at its worst because the preservice teachers' content knowledge was bad in these concepts. Furthermore, preservice teachers' explanations based on learners' errors were done relatively well, in contrast to the flawed explanations emanating from learners' misconceptions. Thus, preservice teachers lack understanding in identifying and rectifying learners' misconceptions to some concepts in school mathematics.

In the lesson planning and lesson presentation, teacher explanation was the teaching strategy of choice to most of the preservice teachers. Teacher explanation strategy, as the oldest and most commonly used teaching strategy (Odora, 2014), was solely used by most of the preservice teachers. As a result, teacher-centred strategies were promoted, to the detriment of other inquiry-based learning strategies. Consequently, memorisation of facts and formulae was prevalent in the lesson delivery, epitomised by the SOHCAHTOA and CAST diagram repertoires. Ordinarily, mathematics teachers' perception of concepts influences the teaching strategy which they are to employ in classroom teaching of a particular idea. Preservice teachers felt that being able to explain well makes them effective teachers, as most of them were taught through teacher explanation during school days. Frequently, preservice teachers resort to direct instruction whenever they are not sure of an appropriate strategy to use at a particular time (Ozden, Usak, Ulker & Sorgo, 2013). Hence, preservice teachers lacked knowledge of instructional strategies in both lesson planning and implementation stages.

In the lesson plans, preservice teachers planned concepts were planned from the known to unknown by means of commencing with baseline knowledge. This was in line with the constructivists' view of teaching and learning asserts that children start the learning process by refining and converting that which they already know into advanced knowledge. Thus, preservice teachers realised the main source of new knowledge development is prior knowledge. As for the teaching and learning resources, none was planned for and used in actual lesson delivery by all the participating preservice teachers. During teaching practice, all preservice teachers taught either Grades 10 or 11, who still operate at the concrete level of Piaget's stages of cognitive development. Teacher-centred instruction which advocated for teacher explanation which was favoured by preservice teachers does not advocate for the use of teaching and learning support materials in instruction. The support materials help to create interaction between learners and the materials, learner-to-learner and learner-to-teacher. Learner-interaction was absent in most of the classrooms observed. Use of learning aids is highly recommended at that stage in order to achieve effective teaching and learning.

In addition to absence of teacher-learner interaction, facilitation of learning during the assessment and lesson development stages was not thoroughly done. Hence, fewer examples were given in class. Also, some errors in the class-activity went undetected by the teacher. Moreover, two preservice teachers did not even administer their planned assessment activities to their classes, hence no facilitation was possible. Preservice teachers are challenged with pacing of the lesson so much that class-activities feedback and lesson conclusions became victims of time. Finally, teachers use varied and effective questions to analyse the depth of learners' ideas and to evaluate the learning process during lesson delivery (Tanisli & Kose, 2013). Unfortunately, only one of the preservice teachers was able to engage varied instructional type questions, while the rest resorted to lower-order question types only. Thus, preservice teachers' command of knowledge of content and teaching was weak.

6.3.2. Components of pedagogical content knowledge

Pre-determined components of pedagogical content knowledge, which spanned the three instruments, were analysed individually to evaluate preservice teachers' understanding of pedagogical content knowledge in trigonometry. These were knowledge of content, instructional strategies, learners' misconceptions, assessment, questioning techniques, lesson management, lesson plan structure and explanation. Each of these pedagogical content knowledge components are explained in the next sub-sections.

Content knowledge

In section 6.2 the results of a quantitative assessment of preservice teachers' subject matter knowledge in trigonometry were discussed. Though this was enough to give an understanding of preservice teachers' academic knowledge, it does not give an assessment of the quality of preservice teachers' knowledge. This can only be determined through analysis of data from classroom practice (Ball, Thames & Phelps, 2008; Capraro, Capraro, Parker, Kulm & Raulerson, 2005). In reality, "teaching necessarily begins with a teacher's understanding of what is to be learned and how it is to be taught" (Shulman, 1987, p.7). Thus, the discussion which follows is on preservice teachers' understanding of mathematics concepts in the context of classroom practice. Preservice teachers' content knowledge understanding is part of pedagogical content knowledge in that without having a deep knowledge of content, it may not possible to teach mathematics effectively (Turnuklu & Yesildere, 2007).

The discussion was based on results from task-based interviews, lesson planning analysis and classroom observations. Generally, the component of content knowledge in the interview was not done well, as key constructs like transformation of functions, solving quadratic trigonometric equations and proving identities were badly performed. All these constructs are part of school mathematics syllabus, hence it's a sign that the preservice teachers did not understand or had forgotten knowledge of trigonometry since their school days and undergraduate courses (Fi, 2003). Factual knowledge like the definition of negative angles were performed well by some preservice teachers. Also, preservice teachers could tell that a given statement was incorrect, but could not supply the corrected one, as in the expansion of $\sin(a+b)$. The findings indicate that despite the existing commonality in trigonometric concepts covered by teacher education and school mathematics, preservice teachers still find it hard to grasp some concepts in trigonometry (Biyela, 2012).

The analysis of content knowledge in the lesson plan analyses revealed that most preservice teachers were aware of good sequencing of lesson content and addressed relevant prior knowledge appropriately in the introduction of the lesson. All preservice teachers found it easy to present concepts with accuracy, because they had the liberty to choose a concept of their choice for the lesson plan. It is human nature for one to go for what one is comfortable with, if given the chance. Hence, problematic concepts like transformation of functions, solving equations and inequalities and proving identities were never planned for this classroom observations.

Notably, one lesson plan was marked by a flaw in the assessment activity, whereby it was stated that, “Given $\cos\theta = \frac{13}{5}$, determine the $\sin\theta$.” The flaw obviously led to undefined solutions, to the amazement of the learners. The reasons for this error may include negligence, a lack of knowledge of the mathematical concepts or the preservice teachers not comprehending what is expected in designing a mathematics task (Swan, 2001). This is reflected in Brodie’s (2014) research, in which teachers are held responsible for learners’ subsequent errors. As most of the learners struggled with the activity, some prudent learners when faced with this error had a shrewd way of dealing with it to get the expected answer of $\sin\theta = \frac{12}{13}$. In the post-observation discussion, the participant admitted that she saw the error when it was too late, so she chose to let it continue.

In the video-teaching episodes, most of the key concepts for the lesson were precise and well-taught by all participants, mainly because these concepts were self-chosen by the participants. Two of the participants’ committed errors which went undetected; one was an error in the class-activity emanating from the lesson plan and the other was done on the chalkboard, where the participant hastily indicated that $\cos(90 \pm x) = \sin x$. Unlike some errors in mathematics which are part of the learning process (Rach, Ufer & Heinze, 2013), these unintended teacher-made errors were disastrous to learners in that they added more confusion to the learning process.

Just like their limited content knowledge in the content test, the quality of preservice teachers’ content was moderate in the classroom practice. The participants had the benefit of choosing their own concepts to plan and teach, and so they naturally avoided concepts they were weak at. The future is bleak for these preservice teachers because without sufficient subject matter knowledge, many of pedagogical processes are thwarted (Southwell & Penglase, 2005). Content knowledge is instrumental to teaching and learning and limited content knowledge hinders the development of the processes of lesson planning, instruction, assessment and learner’ conceptualisation (Shulman, 1987; Brown & Borko, 1992). In South Africa, initial teacher education programmes are inclined towards pedagogy and methodology rather than content knowledge (Brodie, 2004), when content knowledge should be the focal point for all teacher education endeavours. Hence, South African preservice teachers have a weak content knowledge background in mathematics (Biyela, 2012).

Ostensibly, trigonometry has always been a problematic area in high school mathematics (Gür, 2009; Breidenbach, Dubinsky, Hawks & Nichols, 1992), denoting that current preservice teachers also had the same challenges when they were still at high school. Trigonometry should be one of the priority mathematics sections for preservice teachers as one would expect them to compensate for the gaps left by their own high school experience (Biyela, 2012).

Instructional strategies

The second component of pedagogical content knowledge in this study was instructional strategies (Grossman, 1990). Results from lesson plan analyses and lesson observations form the basis for this discussion. Preservice teachers must consider appropriate teaching strategies for the trigonometry concepts they have chosen to teach based on available resources and specific learners. The pedagogical content knowledge component of teaching strategies was fairly done by all participants, with at least one strategy being mentioned in the lesson plans. Not only were the teaching strategies stated, but there was evidence of their use in the rest of the lesson plan. Two dominant strategies emerged from those listed by the participants, namely teacher demonstration and teacher explanation. Now, these two alone lead to teacher domination of the lesson. Furthermore, it appears that most of the preservice teachers used teacher-centred instructional strategies despite widespread promotion of learner-centred strategies taught to them in methodology and pedagogy modules. Teacher explanation represents direct teacher instruction of knowledge and preservice teachers resorted to it whenever they were unsure of an instructional strategy to use.

In the lesson plans, multiple teaching strategies were stated, however, during lesson implementation, most of the participants displayed a sole strategy throughout. The combination of teacher explanation and teacher demonstration was the dominant strategy, used by five out of six participants. That meant that almost all the lessons taught leaned more towards a teacher-centred environment, which is known to inhibit learners' educational growth (Ahmed, 2013). Only one participant explicitly used learner-centred strategies in the form of discovery, which relied heavily on hands-on activities and small groupwork. Thus, for most of the participants, special learning techniques like class discussion or inquiry-based learning were hardly used throughout the lessons.

From the deliberations above, it is evident that preservice teachers' application of the knowledge of instructional practices was limited. A study by Ozden, Usak, Ulker and Sorgo (2013) reached the same conclusion that preservice teachers lacked appropriate pedagogical content knowledge of teaching strategies that are related to a given topic. One can argue that knowledge of instructional strategies is tied to classroom experiences, to which preservice teachers do not happen to have any. However, they do receive instruction on these in methodology modules, which they then enact in actual classroom teaching environments during teaching practice sessions. From both the lesson plans analyses and lesson observations, findings show that most preservice teachers leaned more towards teacher-centred instructional strategies, which is consistent with findings from other related studies (Ball, 1990, 1991; Fi, 2003). Constructivist teaching strategies like cooperative learning, discovery learning and learner-demonstration were conspicuous by their absence. Neither was there varied use of instructional strategies in a single lesson, which is known to assist learners to anchor mathematical knowledge while avoiding pertinent challenges to learning (Gersten & Benjamin, 2012).

Inadequate and sometimes unsuitable preparation and training of teachers have future classroom repercussions whereby learners end up not grasping the concepts they are taught. This happens as the teachers take comfort in straight transmission of knowledge, by which they may have been taught themselves when they were still learners. They live the adage, "teach the way you were taught" (Grossman, 1990, p.170). Many concepts in trigonometry are frequently taught from their procedural aspects, which may lead to the conceptual dimension being abandoned (Dündar & Yaman, 2015). Ball (1990) alluded that preservice teachers are confined to a limited understanding of school mathematics, which obviously leads them to put emphasis on rules and how to execute procedures without a clear explanation of why the algorithms work (Kilić, 2009). Incidentally, the procedural success coming from repeating algorithms and applying formulae masks underlying conceptual difficulties or gaps (de Villiers & Jugmohan, 2012). Findings show heavy use of acronyms for trigonometric ratios and quadrants in the lesson presentations. In summary, preservice teachers were inadequate in their pedagogical content knowledge and more so, they appeared to possess insufficient knowledge about instructional strategies (Shulman, 1986; Ozden, Usak, Ulker & Sorgo, 2013).

Learners' misconceptions

An understanding of common learners' misconceptions and difficulties to learning and the subsequent strategies put in place by teachers to combat such misconceptions are examples of pedagogical content knowledge. Hence, the knowledge of anticipating and dealing with learners' misconceptions was included herein as a component of pedagogical content knowledge. Discussion of learners' misconceptions are based on the findings from the task-based interviews, lesson plan analyses and lesson observations. Moreover, teachers need to be able to determine the source of learners' misconceptions and errors in order to correct them meaningfully (Kiliģ, 2011). Teachers should be able to eliminate learning difficulties and misconceptions by using appropriate tasks or asking probing questions.

An important finding of this study was that two-thirds of the preservice teachers themselves had misconceptions about some of the key concepts in the interviews, for example, solution of trigonometric equations and transformation of trigonometric functions. Like the findings by Stephen (2006), the weak preservice teachers' content knowledge is the likely cause for the difficulty in identifying and anticipating learners' misconceptions in trigonometry. According to Turnuklu and Yesidere (2007), if teachers have inadequate mathematics content knowledge, they may not see the relationship between the simple concepts such as solving equations and transformation of sinusoids. On the other hand, five out of six preservice teachers managed to precisely identify learners' difficulties and errors in the given hypothetical cases in the interview. This was evident when they identified correctly learners' erroneous way of proving identities and manipulating the negative sign in odd trigonometric functions. Preservice teachers were relatively good in questions which involved learner-errors, but their performance was low in questions involving learner-misconceptions. Preservice teachers have misconceptions themselves, hence they tend to repeat them, instead of identifying and challenging them. The unfortunate part is their misconceptions are bound to end up as learners' misconceptions in future, completing a vicious cycle of misconceptions (Tanisli & Kose, 2013).

One of the findings from the lesson plan analysis was that prospective teachers had weak insight of learners' misconceptions in the topic of trigonometry. No anticipation for learners' misconceptions were put in place in the lesson plan, which according to Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball (2008), should be detected at the beginning to lead to simple understanding of the concept being taught. This in turn adversely affected the way in which they chose to address the important concepts. Learners with difficulties fared better as results show them addressing prior knowledge and getting the correct sequencing of concepts in the lesson plan analyses. However, in studies by Shulman (1986, 1987) and Usak (2009), preservice teachers were not cognisant of learners' conceptual difficulties, especially when they had problems in mastery of content. Lesson plans not only demonstrate the preservice teachers' intended plan of action for the day, but can be an indication of the preservice teachers' perceptions of the topics which they plan to teach (Murphy, 2009). If they are not careful, preservice teachers might transmit to learners their own misconceptions whenever they do not possess adequate content knowledge (Even, 1993; Özden, 2008). Thus, it is essential to commit preservice teachers to the repertoire of lesson planning, lesson implementation and lesson reflections to improve their knowledge of learners' challenges to learning a particular concept (Haciömeroglu, 2009).

As for the anticipation of learners' misconceptions in learning the selected trigonometric concepts, none was explicitly evident from the data transcribed from the video-lesson analyses. No misconceptions were observed in the lesson delivery on the part of learners or teachers. Nevertheless, some difficulties featured in the lesson, arose from the way the lessons were taught (Penso, 2002). For example, learners struggled to solve an impossible situation of finding $\sin\theta$ given $\cos\theta = \frac{13}{5}$. Because they had understood that well-taught concept, learners had a shrewd way of getting the required answer of $\frac{12}{13}$. They had difficulties not from the concept being taught, but from an unnoticed error in the assessment task.

Assessment

Preservice teachers' knowledge of assessment featured in this study as pedagogical content knowledge component. The discussion of assessment is based on the results from the lesson plan analyses and video-lesson observations. Preparation of assessment activities was fairly done by most of the participants as five participants planned fitting class-activities. This contrasted with a study by Bukova-Güzel (2010), which revealed that preservice teachers had some difficulties in creating appropriate assessment activities. Items were appropriately kept to the minimum due to tally with available time of the planned lessons. One participant had a mishap of planning for a homework within the lesson timeframe. This defies logic in the sense that homework is supposedly done after class, thus it should appear as an extra activity after the lesson has been concluded. Furthermore, one participant prepared a class-activity for Grade 10 learners, but the activity contained an error which rendered the problem unsolvable. At that level, learners cannot easily pick up the teacher's error, thus in attempting to solve the flawed problem, learners exacerbate their own errors.

Mixed performances were observed in assessment among preservice teachers' lesson presentations. Firstly, two participants did not give assessment activities at all, even though they happened to be present in their lesson plans. By this, it meant preservice teachers were unaware that learning also occurs during assessment (Kornell & Bjork, 2007). Also, assessment can fruitfully promote and inform subsequent action by teachers and learners (Clarke, 1996). Some preservice teachers administered class-activities which were too long, so that learners had difficulty completing them on time. As a result, no assessment feedback was given to the class, which is equally significant for successful teaching. Preservice did not vary the modes of assessments in both the lesson planning and lesson implementation, which is profitable in an environment of mixed-ability learners (Bukova-Güzel, 2010). Only class-activities were administered. One class-activity had a flaw which went undetected in class, thus the solutions to that problem were undefined.

This is corroborated by Reyneke, Meyer and Nel (2010) who commented that teachers appear to be lacking in designing quality assessment items.

Questioning techniques

The art of posing instructional questions in a lesson is important so that teachers can gauge the depth of learners' ideas and direct their thinking towards achieving learning goals (Moyer & Milewicz, 2002). As such, questioning techniques found their way into pedagogical content knowledge studies. Analyses results from the lesson observations formed the base for the discussion that follows. Firstly, even though question-and-answer as an instructional strategy featured in some lesson plans, no participant used it during lesson delivery. Less confident teachers dread the task of devising the necessary and appropriate questions which are to drive the learning process (Haciomeroglu, 2009).

In this study, the participants had weaknesses in questioning skills regarding types and levels of questions required to build conceptual understanding of trigonometric functions (Auliffe, 2013). Lower-order question types were the order of the day in all the classrooms; it was mainly the *what*-questions that were posed to the learners (Lloyd, 2006). For example, "What is the amplitude?" "What is the range?" There was no posing of higher-order question types involving the *why* or *how*. Higher-order questions create room for teacher reinforcement of what learners already know and scaffolding of learners' ideas from the particular to the general. Also, the tasks preservice teachers administered to the learners in examples and assessment were of low cognitive demand, which mostly focussed on procedures and memorisation of facts and formula (Crespo, 2003). It was observed that leading questions were invoked whenever participants mixed English and the vernacular language. For example, in teaching the transformation of graphs, one participant had to code-switch and asked, "Do you see *gore o tšhentšitše* only the amplitude, *akere?*" (Do you see that what has changed is only the amplitude, isn't it?" Thus, teacher questioning was insufficient as an instruction cue to convey concepts to learners. Questioning was supposed to unravel the directions of what learners are to do at a particular time and also how they are to do it.

Lesson management and control

Lesson control and management reflects preservice teachers' grasp of how to teach, hence, it has been included here as a component of pedagogical content knowledge. Results from the video classroom observations were the source of data for discussion herein. None of the participants used teaching and learning aids, except, in some cases, save the usual chalkboard and textbooks. Preservice teachers sometimes teach junior classes which are still at a stage where they stand to benefit from use of such resources. As a result, the lesson was steered away from learner-centred to more of teacher explanation. Another observation was that participants took longer to get to the concept under consideration, as a result of lengthy prior knowledge checking. This had the effect of choking the lesson, thereby leaving insufficient time towards the end of the lesson for feedback to assessment and conclusion. Another important aspect of lesson structure was teacher facilitation. Three participants did not facilitate classroom learning, especially during assessment activities. One did not administer an assessment activity at all, thus there was not much to facilitate anyway, while the other two simply stood idle as learners sweated out on the assessment task. According to Ball and Bass (2000), one of the principal things that make teachers better positioned to teach is the extent to which they facilitate the learning process. To those who facilitated, it was seemingly not thorough enough, to the extent that one participant did not detect a technical error in the assessment items in the process.

Lesson plan structure

Shulman (1986) was led to introduce the concept of pedagogical content knowledge having seen that the then current research on teacher knowledge was ignoring issues dealing with the contents of a lesson. Contents of the lesson are visible in the lesson plan structure thus, lesson plan structure as a component of pedagogical content knowledge is included. This discussion is based on the results from the lesson plan analyses. All participants addressed the issue of prior knowledge well as a foundation to concepts in the introduction. Four of the participants provided lesson conclusions, some even spiced with real-life applications of the concepts taught. However, two participants had no conclusions at all, and because a template was used to draft the lesson plan, a homework activity was planned in the space of conclusion by one. In addition to that, four participants did not know how to frame a specific lesson topic; they simply stated the lesson topic in its broad sense as “Trigonometric functions”. The lesson plan template which they used required a lesson topic for that particular lesson only, not the broad textbook chapter. None of the participants planned for teaching and learning resources, save the usual chalkboard. Prospective teachers lacked appropriate pedagogical content knowledge of using materials related to the lesson concept under consideration (Ozden, Usak, Ulker & Sorgo, 2013). Preservice teachers should appropriately use learning materials and instructional designs in their lessons to assist learners with realising the goals of the lesson (Bukova-Güzel, 2009; Department of Basic Education, 2014). Finally, five participants did not include some examples to build the lesson towards the assessment activity. Learners may not be able to obtain correct answers to mathematical problem due to lack of sufficient knowledge of the concept(s) under consideration (Gardee & Brodie, 2015).

Explanation

The last component of pedagogical content knowledge in this section is teacher explanation. The task-based interviews and classroom observations results form the basis for the discussion hereunder. The capacity to explain well is quite significant as it leads to good teaching (Havita, 2000). Teachers' explanations are reflective of what they know and, hence, of what their learners will eventually learn (Ball, 1990). Having relied on teacher- centred teaching techniques in lesson planning and implementation, preservice teachers banked on good explanation skills to deliver content and this they did quite well. Over reliance on the teacher explanation method became counter-productive in this study. The participants all possessed good expression of ideas and a good level of communication skills of content material to learners. This was made easy by the fact that four participants planned and used teacher explanation as their main method of instruction.

Four participants gave correct explanations wherein they were asked how they would react to some hypothetical teaching situations. In all, good explanations came from concepts where preservice teachers had good content knowledge, which confirmed that teacher explanation is directly connected to level of content knowledge. However, some participants managed to explain concepts for which they did not have sufficient content knowledge as all participants were asked to explain the correct way of proving identities but were unable to do the actual proof the correct way. Moreover, teacher explanation was above board where there was involvement of identifying learner errors, otherwise, there was a plethora of incorrect explanations where learners' misconceptions were involved. Thus, ability to explain eloquently will not necessarily make preservice teachers good teachers. This affirms the notion that content knowledge on its own is not a prerequisite to effective teaching (Barker, 2007; Kahan, Cooper & Bethea, 2003).

6.4. The development of mathematics knowledge for teaching

This section expounds the intricacies of the development of mathematics knowledge for teaching, especially in the light of classroom practices, in response to the third research question, which asks, "To what extent do preservice teachers develop the mathematics knowledge for teaching of trigonometry in the initial teacher education?" The basis for this discussion is the results from all the data collection procedures in this study. Attempts were made to unravel the application, implications, impact and extent of preservice teachers' development of mathematics knowledge for teaching. The Curriculum and Assessment Policy Statement emphasises the content and instructional processes involved in teaching must be entrenched in appropriate contexts that are bound to enhance the application of mathematics (Department of Education, 2011). Thus problem solving contexts cannot exist in isolation.

6.4.1. Participant track in all data collection instruments

This section tracks the performance of individual participants as they experienced the four data collection instruments. This would portray a wholesome picture of participants' performance, and growth in knowledge of teaching, by examining the preservice teachers' content knowledge and pedagogical content knowledge.

Participant 1 Mhla

This participant was a medium performer in phase one of data collection. She was strong in identifying special triangles, applying the area rule, simplifying trigonometric ratios and sketching trigonometric functions. Her serious weaknesses were in solving inequalities and equations in the form $f > g$ and $f = g$ respectively. In the interview and lesson presentation, her explanations of concepts were adequate, but her command of content knowledge in the same concepts was medium, in line with the content test outcomes. For instance, she could explain well that $\sin(a+b) \neq \sin a + \sin b$, but could not give the expansion of the expression $\sin(a+b)$ precisely. Also, her content knowledge was dampened by an undetected flaw in one of the assessment items for the lesson plan and presentation.

The limited content knowledge resulted in her making misconceptions that learners often make in the change in period of transformation of trigonometric functions. Thus, this preservice teacher would invariably pass on this misconception to her learners (Haciömeroglu, 2009). Because she was good at explanations, the teacher explanation method, which is the oldest method of teaching (Odora, 2014), was the chosen teaching strategy for both the lesson plan and presentation. For the lesson plan, this participant chose to teach on the simplification of trigonometric ratios, to which she was particularly good at from the content test results. Her questioning techniques were generalised in that they were never directed at individual learners thus it was difficult for her to make follow-ups to learners' responses.

Participant 2 Mahl

This participant was chosen for phase two data collection as a medium performing preservice teacher in the content test. His best performances in the content test were in identifying special triangles, proving the square identities and the application of the reduction formula. His weaknesses were in solving equations, simplifying trigonometric expressions and curve sketching. In the interview, he also displayed mediocre mastery of content knowledge, which led him to commit two misconceptions he was supposed to identify, in solving equations and transformation of functions. Based on his strengths and weaknesses in content knowledge, he chose to plan and teach the reduction formula, at which he was particularly good. Teacher explanation, the most commonly used teaching method (Odora, 2014), was solely planned and used in the lesson presentation. This was detrimental in that the lesson ended up being predominantly teacher- centred. Furthermore, there was over-reliance on memorisation of formula rather than of active learner-involvement in knowledge building, a bad tendency noticeable in many teachers of mathematics. For a 30-minute lesson, too much on the reduction formula was planned, so that the lesson was left incomplete.

Participant 3 Leng

With quite a low performance in the content test, this participant struggled a lot on content knowledge both in the content test and the task-based interview. He performed poorly in all the concepts covered in the content test save the sketching of graphs of functions. In the interview, though his explanations were good, they were marred by lack of competence in content knowledge. This participant was not good at identifying misconceptions and difficulties thus he fell for the all the learner misconceptions posed to him. He chose to plan and teach the concept of reduction formula, which he had performed relatively well in the content test. However, due to the nature of the reduction formula in different quadrants and on different trigonometric ratios, this participant ended up planning too much for the 30-minute lesson. As a result, the lesson was not concluded nor was the planned assessment activity administered due to time constraints. Another effect of low mastery of content knowledge was that this participant spent some time on reciprocal identities which had no bearing on the current lesson topic. In line with that, only lower-order questions were posed to the class, which failed to challenge learners. He did pose expected higher-order questions due to his weak content knowledge (Haciömeroglu, 2009).

Participant 4 Shab

This participant registered fairly good scores in the content test on content knowledge. He only had some difficulties with the solution of the inequality in the form $f > g$. His explanations were above board, as well as his identification of learners' difficulties and misconceptions. Thus, good mastery of content gave meaning to teacher explanations. But his questioning techniques lacked specificity and were never directed at individual learners. Insufficient and insignificant questions were posed, so the preservice teacher could not analyse learners' thoughts and the learning process effectively. His pacing was not good to the extent that the lesson was not concluded nor was the planned assessment task administered. A sole teaching strategy was used in the lesson presentation, which was teacher demonstration. He did not plan to use teaching and learning resources for the lesson.

Participant 5 Sell

This participant's performance in the content knowledge test was mediocre, but she was the only participant to obtain the correct solution set of the inequality $f > g$. She managed to solve the inequality graphically, for she had an advantage in curve-sketching in the content knowledge test, in addition to identifying special triangles and simplifying trigonometric expressions. Thus, she chose to teach a lesson on sketching trigonometric functions, at which she was good. She had weaknesses in solving equations, application of the reduction formula and proof of the area rule.

In the interview, her explanations were good, though she lacked sufficient content knowledge to substantiate it. For instance, she could tell the correct procedure of proving identities, but she failed to prove a given identity in one of the tasks. She portrayed that she was not good at identifying learners' misconceptions, since she committed two of the learners' misconception herself in the interview tasks. The stated teacher demonstration method was solely used during the lesson which led to the participant providing solutions to the assessment activity herself, with no apparent involvement of learners at all. Lower-order and leading questions were common during the lesson presentation, which did provoke learners' reasoning capacity well. In addition, no resources were used in the lesson presentation, save the chalkboard ruler.

Participant 6 Malu

This participant was a low performer in the content knowledge assessment in the content test. His only remarkable strength was in curve-sketching, while he was weak in solving equations and inequalities, labelling special triangles, application of the reduction formula and computing the intersection points of two functions. He was seemingly good at establishing learner difficulties but had challenges with identifying learner misconceptions. In marked contrast to the low content test performance, his explanations and content knowledge mastery in the interview was superb. He was the only participant who had the know-how to use multiple learner-centred teaching strategies, which were question-and-answer, class discussion and discovery methods. The sitting arrangement of the learners was in small groups of about five for a greater part of the lesson, and the learners were actively involved in investigating graphs of functions hands-on. The questioning technique was varied throughout the lesson, which also included higher-order question types. However, the lesson was devoid of follow-up to learners' responses in order to qualify them when it was necessary to do so.

Across all the tracking of participants, there was no obvious cause-effect relationship observed between performance in content knowledge and preservice teachers' instructional capabilities. The majority indeed portrayed a scenario where poor content knowledge adversely affected subsequent pedagogical processes like explanations, instructional strategies, recognising learners' misconceptions and assessment in both the lesson plan analyses and lesson observations. However, they were exceptions to this. Findings show that Shab was fairly good in mastery of content knowledge in all the data sources, but his classroom practice skills were unimpressive. His teaching methods and questioning techniques were inadequate, and he did not even administer the planned assessment activity as pacing of the lesson failed him. On the other hand, Malu indeed grappled with content knowledge in the content test and interview but could not allow himself to be handicapped by this. He managed to exercise excellent classroom practice skills in instructional skills which intermixed question-and-answer and discovery learning performed in small groups. Learner-involvement was high in the lesson and feedback to assessment activities. This confirms that content knowledge is necessary to becoming a teacher but is not sufficient for pedagogical content knowledge processes (Ball & Bass, 2000; Borko & Putnam, 1996).

6.4.2. Implication of mathematics knowledge for teaching

The discussion of the implications commences with the development of mathematics knowledge for teaching. Such understanding of how mathematics knowledge for teaching is developed in practice would facilitate planning of workable teacher education programmes for preservice teachers. To describe how the mathematics knowledge for teaching develops, this study starts by identifying possible sources that add value to mathematics knowledge for teaching development (Haston & Leon-Guerrero, 2008; Evens, Elen & Depaepe, 2015; Kind, 2009). Afterwards, this study focuses our attention on the application of content knowledge and pedagogical content knowledge to classroom practice as preservice teachers taught real classes during their school teaching practice.

Development

During their time as learners, prospective teachers garner perceptions of what is mathematics and how to teach it. "Long before they enrol in their first education course or math methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988, p.13). It is upon these interconnected ideas that each preservice teacher meticulously constructs his or her mathematics teaching skills (Uusimaki & Nason, 2004). Thus, prospective teachers enter teacher education with rudimentary mathematics knowledge for teaching. Oftentimes teachers have been observed to base their mathematics knowledge for teaching skills upon memories of experiences in teaching and learning (Haston & Leon-Guerrero, 2008). These form a platform for the development of the mathematics knowledge for teaching. Unfortunately, these memories may also limit teachers who become rigid on their reliance on preconceptions. What prospective teachers bring into teacher education is a good starting point to isolate subsequent mathematics knowledge for teaching development. To be admitted in teacher education, prospective teachers must have a compulsory pass in mathematics with 50 percent or better at matriculation. However, the objective test results did not give a promising picture. With a 41 percent and a 33 percent average score and median score in school mathematics respectively, their content knowledge growth was somehow adverse. These content knowledge results came about after four years of entry-level and advanced mathematics courses.

Now the major driving force of mathematics knowledge for teaching for preservice teachers in teacher education programmes, is the graded modules in methodology and content they are formally taught from year one to four. Grossman (1990) investigated the bearing of formal teacher training on the growth of knowledge for teaching and discovered that it contributes to the knowledge growth of teachers. As this study was conducted towards the end of preservice teachers' programme of study, it duly gives us a better picture of growth of knowledge as a result of exposure to teacher education.

Concerning pedagogical content knowledge, so much was noted which did not portray remarkable development. Procedural practices in some aspects of trigonometry took the place of the expected learners' conceptual understanding of those concepts. This was portrayed in the lesson plan analyses and classroom observations. An important observation is that participants had some good ideas but could not make necessary connections to complete the solution process to the final stage. This was echoed by Tatto and Senk (2011) who remarked that teachers may be well-versed with procedures and facts on some mathematics content they are expected to teach, but they would be weak in conceptual understanding of the same matter. A deep conceptual understanding is essential for the mathematics which preservice teachers will teach when they graduate (Conference Board of Mathematical Sciences, 2001). For instance, the teaching of reduction formula was highly procedural. Coupled with procedural teaching was the dominant use of teacher-centred strategies. Most preservice teachers were taught by their teachers that way and not much has changed despite learner-centred approaches which are promoted in teacher education.

Large measures of spread of data indicated that preservice teachers' content knowledge of trigonometry was uneven and that several fundamental ideas of trigonometry were poorly understood (Fi, 2006). Combined with the other descriptive statistics, it suggests that the participating preservice teachers possessed insufficient understanding of trigonometry. Thus, after four years of teacher training, preservice teachers' mathematics knowledge for teaching was not fully developed and more needs to be done to redress this scenario in teacher education.

Moreover, the preservice teachers' need for improvement in their content knowledge hampered them from effectively identifying learner thinking processes and misconceptions (Tanisli & Kose, 2013). The unfortunate thing was that whenever preservice teachers have limited content knowledge on certain concepts or maybe misconceptions, learners are bound to inherit such misconceptions (Haciomeroglu, 2009). It is the responsibility of teacher education instruction to break this perpetuity of misconceptions before they get to the classroom. If it gets there, it creates a vicious cycle of misconceptions in trigonometry. Lesson objectives would be difficult to achieve if learners' difficulties and preconceptions are not addressed by the teacher during planning and implementation stages. This leads to poor delivery of subject matter in classroom practice (Penso, 2002; Carzola, 2006; Westwood, 2004). Teachers can obtain possible learners' misconceptions from the learners' responses from the learners' solutions of exercises and during oral probing. It is an essential aspect of teaching for teachers to expertly recognise and then address learners' misconceptions. The way in which teachers plan their lessons reflects those misconceptions that would be addressed during the lesson. The lesson plan analyses, classroom observations and task-based interviews corroborated that preservice teachers in this study knew little about learners' misconceptions and difficulties in trigonometry. Hence, the preservice teachers' development of mathematics knowledge for teaching was not robust in this regard, as most of the learner misconceptions appeared in secondary school. Preservice teachers, after four years, cannot think higher than the learners they are supposed to teach.

Another source of mathematics knowledge for teaching is teaching practice experience. Any research into teachers' understanding of mathematics knowledge for teaching that excludes lesson observations with real learners might not fully convey the required information (Ball, Thames & Phelps, 2008). For final-year preservice teachers, yearly teaching practice sessions provide the much-needed teaching experience for significant mathematics knowledge for teaching development. Data for this study on pedagogical content knowledge was collected in actual school settings when the preservice teachers were doing their teaching practice. All the yearly teaching practice sessions did not lead to much realisation of the mathematics knowledge for teaching as it was seen that the teachers' questioning techniques, quality of assessment tasks and teaching methods were not done at their best. It appears that preservice teachers need more time in field experiences to master the art of classroom teaching.

Furthermore, the development in framing a lesson topic was problematic to the preservice teachers. The result was that more than necessary was planned, owing to the broad lesson topics. The use of teaching and learning aids in class was limited. Preservice teachers failed to break away from the traditional methods of teaching to which they themselves experienced. Not a single example was provided by all the participants in the lesson development stages in lesson plans and lesson presentations. Preservice teachers are expected to develop their knowledge of instruction beyond the particular lesson and content as they engage in lesson plan preparation (Fernandez, 2005). This confirms the notion that indeed formal tuition at teacher education and teaching practices has the potential to contribute to the development of preservice teachers' pedagogical content knowledge (Kilić, 2011). Preservice teachers unfortunately, did not gain much from these which explains why some preservice teachers had under-developed knowledge of teaching.

Implications

Herein is discussed the implication of results of this study to classroom teaching practice in the South African mathematics education landscape. The results that came from obtained from the task-based interview informed us about the misnomer of dividing an equation by a term containing a function. More than half of the preservice teachers in the study effortlessly resorted to division by a function $\cos x$, a common fallacy amongst learners. The Diagnostic Report of the 2013 national examination in mathematics emphatically states that learners should be made aware to avoid dividing both sides of an equation by a trigonometric function (Department of Basic Education, 2014) or any other function in general. If that function is not confirmed to be non-zero at every point in the domain, there is a possibility that one is dividing by zero at one or more points. This is known to be undefined, according to the rules of mathematics. Many of the preservice teachers often harbour these misconceptions from their high school days when they were still learners. If that is the case, then the four years of teacher education could not untangle this idea. And they would pass this on to the next generation of learners during teaching practice and early years of teaching. This unfortunately creates a revolving door effect, which is self-sustaining to doom.

Preservice teachers in this study demonstrated that the instructional strategy which they find comfortable with is the lecture method, another term for teacher explanation. This traditional method of teaching features a teacher directing children to learn via recitation and memorisation of facts and formulae techniques. This deprives learners the skills to learn independently and develop critical thinking capabilities. This violates one of the tenets of the Curriculum and Assessment Policy Statement, which advocates "encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths" (Department of Basic Education, 2011, p.4). The traditional approach to teaching was a dominant feature of classroom instruction until the 1980s educational reforms. Preservice teachers ditched the diverse modern teaching strategies which they are taught in undergraduate studies. The understanding of facts suffered in favour of memorisation of principles, hence preservice teachers could well explain how to prove identities but were unsuccessful in application of such knowledge to a given problem.

To give relevance and meaning to the concepts being taught, teachers ought to give examples at different stages of the lesson are that are drawn from learners' familiar world. Real-world contexts render mathematics non-abstract and more connected to the learners' world (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008). Only two out of the six participants coined some real -world applications of the trigonometrical concepts they were addressing at that time. The Curriculum and Assessment Policy Statement categorically states that mathematical concepts do not exist in isolation somewhere out there but should lead to the required acquisition and application of knowledge in ways that are meaningful to learners' everyday way of life (Department of Education, 2011).

Five preservice teachers never gave examples of problems similar to what learners would encounter in the assessment activity. Class-activities must be administered in every lesson to consolidate what has been learnt in that lesson. Learning by following examples works best in mathematics. Consequently, learners were observed to struggle with class-activities tasks upon which they never had seen how similar problems have been solved by the teacher. In support of that, the 2014 Diagnostic Report for Mathematics advised teachers to give learners sufficient examples to practice upon to empower them to develop competence in the manipulation and simplification of trigonometric expressions (Department of Basic Education, 2015). If preservice teachers had a good command of specialised content knowledge, they would have no challenges of choosing examples and representations that support effective teaching.

The utilisation of learning and teaching support materials also revealed that preservice teachers could not relate their knowledge of pedagogical content knowledge to classroom practice. According to Bukova-Güzel (2010), preservice teachers should devise and utilise instructional designs and materials for each topic in normal classroom contexts. In this study, none of the preservice attempted to use ready-made or commercial resources to facilitate teaching and learning, which happens to be a relic of the past traditional teaching methods, where the lecture method was the obvious choice among teachers. The Diagnostic Report of 2014 on mathematics advocated for improvements in the utilisation of learning and teaching support materials to help learners understand the concepts they are taught (Department of Basic Education, 2015). In some cases, learners had some difficulties with certain concepts simply because there were no materials used in the lesson for them to visualise the ideas presented (Bukova-Güzel, 2010). The preservice teachers lacked appropriate pedagogical content knowledge skills of using technology or other resources. Only the chalk-and-talk and textbooks were used. Preservice teachers should see and harness the world outside the traditional classroom from a mathematical viewpoint. Resources, if utilised in a group-work environment provides collaborative learning, interactive engagement and sustains learners' interest in learning mathematics.

Learners' prior knowledge to specific concepts must be considered during lesson delivery. This effectively forearms the teacher to be wary of challenges to learning a particular topic that could crop up in the learning process (Penso, 2002). The process of teaching and learning is compromised if the pre-knowledge of learners' is not identified at the onset of the lesson. This accords the teacher the opportunity to effectively address challenges to learning that learners might encounter during teaching (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008). In this study, most preservice teachers were aware of the need to address learners' pre-knowledge first before getting into the content for the day. Upon the basis of full knowledge of baseline knowledge, preservice teachers safely proceeded with their lessons. The obvious drawback to this procedure was that too much time was then spent on checking prior knowledge to the detriment of other stages of the lesson. Some lessons ended abruptly as shortage of time squeezed out feedback to assessment and conclusion. The pacing skills of the lesson need to be inculcated in the preservice teachers, especially in terms of curtailing prior knowledge in the introduction. Though Gür (2009) found out that preservice teachers did not cite the Pythagoras theorem before teaching trigonometry. In this study, some preservice teacher aptly derived the square identity as a precursor to trigonometry teaching. Thus, prospective teachers made some attempts to relate their pedagogical content knowledge to classroom practice.

6.5. Overall discussion of findings

The knowledge domains that have been well-recognised in the works of Shulman (1986, 1987) and Ball, Thames, and Phelps (2008) are subject matter knowledge and pedagogical content knowledge. The two subject matter knowledge sub-types that were considered in this study were specialised content knowledge and common content knowledge. A very low performance in specialised content knowledge meant that preservice did not possess adequate knowledge that is uniquely needed for teaching, which includes identifying and interpreting the causes of learners' errors. An unsatisfactory performance in common content knowledge showed that preservice teachers were to some extent capable of knowing how to define concepts, perform calculations and identify patterns. Preservice teachers had notable challenges in both types, indicating that their acquisition of content knowledge in trigonometry was inadequate. Similarly, mastery of content knowledge in the interview was shaky. Thus, at the point of exit from teacher education, preservice teachers had forgotten so much of what they did on trigonometry from their high school and teacher education experiences. If left unattended, such low performances in content knowledge would have an adverse effect on their future years of classroom practice.

However, in the lesson plan and lesson presentation, preservice teachers displayed good content knowledge skills through good sequencing of concepts, good use of prior knowledge in the lesson foundation and precision in computations. This could be attributed to the fact that preservice teachers had the liberty to choose their own concepts in trigonometry to plan and teach. As such, preservice teachers chose to plan and teach concepts in trigonometry with which they had no content challenges to in the content test and the task-based interview. Curve sketching was the best-performed concept in the content test and results showed that three preservice teachers chose to plan and teach on it as a result. Trigonometric equations and inequalities were the least performed and no preservice teacher chose to plan and teach them. Unrelated concepts brought in by the preservice teachers, and errors in the assessment activities and in teacher explanations dampened some preservice teachers' content knowledge mastery. Frequently, teacher errors are the source of subsequent learner errors (Moru, Qhobela, Poka & Nchejane, 2014). Moreover, even though preservice teachers could explain the correct procedure of answering some mathematical questions like proving identities, they could not give the correct prove of the given identities themselves. This typifies a scenario where the preservice teachers' content knowledge on trigonometry was inadequate.

Without a firm grasp of content knowledge, many pedagogical processes are negatively affected. Thus, in a way content knowledge is necessary but is not enough in teaching (Moru, Qhobela, Poka & Nchejane, 2014). Shulman (1986) suggested that content knowledge symbolised a person who is training to be a mathematician, whereas pedagogical content knowledge represents a person who is training to be a mathematics teacher. Knowledge and anticipation of learners' misconceptions and difficulties is the driving in of the knowledge of content and students. Preservice teachers' knowledge on misconceptions was lacking as findings showed that they repeated learners' supposed misconceptions, especially in the task-based interviews. Their own misconceptions would prevent them from knowledge of learners' thinking, errors and misconceptions (Tanisli & Kose, 2013).

The dominant feature under the knowledge of content and teaching is the teachers' mastery of instructional strategies. In this study we found that preservice teachers did not display sufficient knowledge in this regard. They failed to engage multiple teaching methods in a single lesson. As if that was not enough, their favoured teaching method, teacher explanation, was not appropriate in efforts to create an enabling learning environment to which learners endeavour constructing their own knowledge. Direct instruction and procedural teaching were observed in the classroom discourses. Consequently, preservice teachers could not help learners overcome learning challenges which they encountered. In the interviews, preservice teachers managed to identify learner-difficulties, but they could not meaningfully address those them.

No meaningful contribution was noted under the knowledge of content and curriculum; the preservice teachers simply adhered to the provincial government's provided work schedules and pacesetters. These contain detailed sequencing of topics and their durations for each grade and each learning area. Teachers often are bound to these settings because common end of term tests would be based on the specified topics that are assumed to have been covered up to the time of the test based on the work schedules. And when preservice go to schools for teaching practice, they are encouraged to blend in with what their teacher mentors would be dealing with at that particular time.

The tracking of each participant's performance in the four data sets was aimed at presenting the progression of teacher candidates' mathematical knowledge for teaching in trigonometry teaching. The progression of preservice teachers' pedagogical content knowledge emanates from strong content knowledge; they chose to plan and teach concepts in trigonometry to which they were obviously good at if given the choice. When teachers are well-versed with the subject matter they are supposed to teach, they appear confident teaching the content, thereby design effective lessons (Wilburne & Long, 2010). Otherwise, they end up being selective in what they teach, which may entail eliminating a certain concept completely (Furner & Robison, 2004).

Preservice teachers could explain concepts well whereto they had good content knowledge. Hence, teacher explanation was the dominant teaching strategy. In cases where their content knowledge was poor, incorrect explanations were common and learners' misconceptions were not identified. In the task-based interviews, the three components of pedagogical content knowledge; content knowledge, knowledge of instructional explanation and knowledge of learners' misconceptions were interwoven. Instructional explanation and the identification misconceptions hinged on good command of content knowledge. Good explanation led to good identification of learners' challenges to learning. Then, if a preservice teacher could explain and understand learners' challenges to learning mathematics, this led to good mastery of content knowledge.

6.6. Conclusion

Chapter 6 was a narration of discussion on the mathematical knowledge for teaching of preservice teachers based on the results from all the four data instruments of this study. Firstly, a discussion of results on preservice teachers' mastery of content knowledge based on quantitative analyses of the content test data was presented. That which the preservice teachers were able to recall on trigonometric concepts was limited. They displayed serious problems in solving trigonometry equations and proving identities which calls for need to stress understanding of school mathematics in high schools and in undergraduate education courses.

The discussion on pedagogical content knowledge came next, and the main divisions of this were the components and types of pedagogical content knowledge. The performance on instructional strategies and questioning techniques, which fall in the category of knowledge of content and teaching was quite weak. Preservice teachers' performance on the knowledge of content and students, epitomised by skills in anticipation of learners' misconceptions and assessment, was inadequate. Knowledge of content and curriculum was indecisive in this study because all teachers are encouraged to comply with the government provided annual work-plans for each level of study, with the effect of masking teachers' true skills in curriculum interpretation.

The response to research question three was drawn from findings from all the data sources to assess the development of the mathematics knowledge for teaching in preservice teachers' classroom practice. This illustrated that deep conceptual understanding of mathematics content is necessary for effective classroom teaching. In this study, just because preservice teachers' content knowledge was shallow, other elements of teaching such as instructional methods, resolving learners' misconceptions and questioning techniques were ill-achieved. The teacher explanation approach to teaching implied that teaching of concepts was procedural, which did not transform into conceptually inclined classroom teaching practices.

CHAPTER 7: SUMMARY, CONCLUSION, RECOMMENDATIONS AND IMPLICATIONS

7.1. Introduction

This chapter concludes this study in seven sub-divisions. The first section introduces the chapter by presenting the outline of the chapter. The abridged summary of the entire study is given in the second section of this chapter. Of interest was the omission of horizon knowledge and the knowledge of content and curriculum since they do not have direct teacher efforts in the South African context. The third section gives the conclusion of the study in response to the study research questions and highlights the new knowledge generated in this study. The fourth section narrates the recommendation for future studies. Several limitations of the study in the way the study was conducted were noted and presented in the fifth section. Implications of the study in the classroom, on research and teacher training make up section six. Finally, the chapter conclusion is given in the last section.

7.2. Summary of the study

Preservice teachers acquire mathematics knowledge for teaching in teacher education through an interplay of teaching practice, methodology and content modules. If teacher knowledge is key to the success of learning and teaching of mathematics, then it is paramount that preservice teachers master and continue developing the mathematics knowledge necessary for teaching in specific content areas. The present study purposed to explore the preservice teachers' understanding of content knowledge and pedagogical content knowledge of final-year preservice teachers in trigonometry. Trigonometry was chosen because its concepts improve preservice teachers' reasoning ability and occupies a unique transitional position from algebra to geometry (Tuna, 2013; Dündar & Yaman, 2015).

Prospective teachers bring to teacher education rudiments of knowledge of teaching of school mathematics which higher education institutions should develop through the initial teacher education programmes. Owing to low and varied quality of the initial teacher education programmes in South Africa, it cannot be taken for granted that preservice teachers have all the skills to teach school mathematics upon completion of their undergraduate degree programmes. Ever since colleges of education were shut down after the controversial 1995 National Teacher Education Audit, there was a marked reduction in enrolments at teacher-training institutions (Centre for Development and Enterprise, 2015). Consequently, the supply of teachers could not match the demand. In the course of attempting to meet the growing demand of teachers, higher education institutions went into mass production of teachers, leading to reduced compliance with government policy requirements by some higher education institutions. Thus, the initial teacher education is to some extent compromised, which has led to situations where teachers have limited conceptual understanding of the subject matter knowledge they are required to teach (Department of Higher Education and Training, 2011b).

There were three research questions that underlined this study, which were: “What is the level of preservice teachers’ understanding of trigonometric concepts?”, “What pedagogical content knowledge do preservice teachers possess in trigonometry?” and “To what extent do preservice teachers develop the mathematics knowledge for teaching of trigonometry in initial teacher education?” Participants of this study were confined to rural prospective teachers who were approaching the end of their teacher education studies at a rural-based institution of higher learning. This study is significant in that it reveals the need to render necessary support to beginning teachers, since from anecdotal knowledge, holding a teaching qualification is not a passport to good teaching.

The literature review of this study centred on three key constructs, which were teacher knowledge, teacher education and preservice teachers’ mathematics knowledge for teaching in particular school mathematics topics. The work by Shulman (1986) formed the basis of the modern understanding of teacher knowledge. Other researchers further developed the concept of teacher knowledge by refining Shulman’s seminal work. The concept by Ball, Thames and Phelps (2008) is one such development which was instrumental in shaping this study. Teacher education as a source of teacher knowledge and cradle of new teachers was highlighted mainly in the South African context. It is acclaimed that new teachers need to acquire a deep understanding of mathematics knowledge for teaching during their training because teachers are key to learner achievement. Currently, products of South African teacher education institutions in the category of mathematics and science fail to meet the expectations of the Department of Basic Education. By analysing teacher education, researchers get to know if challenges of inadequate teacher knowledge emanate from teacher-training. Trigonometry was chosen for this study as one of the school mathematics topics which learners encounter for the first time at Grade 10 level and has the potential to develop learners' rational skills. That late start in the curriculum may be the reason learners find trigonometry challenging which is coupled with teachers who find it difficult to teach trigonometry. Related research on preservice teachers' understanding of both the pedagogical content and subject matter knowledge of school mathematics concluded the review of literature.

The review of literature revealed that many preservice teachers lack the conceptual understanding of school mathematics (Ball, 1990; Vaiyavutjamai, Ellerton & Clements, 2005) and the processes needed to teach it (Ball, 1988a; Ball, Thames & Phelps, 2008; Masingila, Olanoff & Kwaka, 2012). Thus, preservice teachers exit teacher education and enter the world of teaching with limited skills and abilities in teaching school mathematics. Although it is well known that teachers acquire and perfect pedagogical content knowledge as they gain classroom experience, it is felt that teacher candidates ought to have the basics of pedagogical content knowledge developed in the initial teacher education programmes. Preservice teachers should try to grasp pedagogical content knowledge in their undergraduate content, methodology and through practical teaching engagements to prepare for their initial years of school teaching experiences (Borko & Putnam, 1996).

After the literature review, the conceptual framework was presented, which gave this study a sturdy foundation. The framework also served as a window through which the analysis of data and discussion of results were done. Shulman's (1986) conception of teacher knowledge, later refined by Ball, Thames and Phelps (2008), was the chosen conceptual framework for this study. The two types of teacher knowledge according to the model proposed by Ball, Thames and Phelps (2008) are pedagogical content knowledge and subject matter knowledge. The analysis of data was to investigate the depth of these two types of knowledge in a different context as to that of the proponents of teacher knowledge framework. The zenith of this study, therefore, was to explore preservice teachers' competence in these two key types of knowledge, as well as their implication to classroom practice.

The research paradigm and how the study was conducted were elaborated in the research design. The qualitative research design was applied to this study, whereby preservice teachers' mathematics knowledge for teaching was explored at best in the normal classroom situation. A single case of preservice teachers at a higher education institution was studied in-depth complemented the qualitative research design. The case study research methodology was appropriate for this study as a well-defined population of preservice teachers and a specific topic of school mathematics were explored.

Data collection for this study comprised four instruments. The first to be administered was the content test, which was designed to assess preservice teachers' content knowledge of trigonometry, in response to research question one. The next three instruments to be administered were the task-based interview, lesson plan analyses and classroom observations, all addressing research question two. The third research question, which was on preservice teachers' development of pedagogical content knowledge, was addressed by findings from all the four data collection instruments. Even though it is known that pedagogical content knowledge of teachers is not easy to isolate and study from other teacher knowledge bases (Miller, 2006), in this study, attempts were made to map it in the context of classroom practice. This in part provides a starting point for collecting and analysing data regarding other aspects of teacher knowledge (Miller, 2007).

The sample of the study was composed of fifteen mathematics final year preservice teachers doing a Bachelor of Education degree programme at an institution of higher learning in South Africa. The sample was selected based on the purposive sampling technique. Before data were collected, ethical clearance was obtained from both the research site and the institution where the study is registered. Moreover, informed consent to audio- and video-record the task-based interviews and classroom teaching episodes respectively was obtained from the fifteen participants who participated in the study.

A preliminary study was conducted with fourth-year preservice teachers who were not going to be part of the sample for the actual study. Only the content test and the interviews were trialled, which was conducted on campus. Adjustments to the duration of the two instruments were duly done, as well as some changes to the items. Adjustments were done to strike a balance to the specialised content and common content knowledge items in the content test. Re-phrasing of some terms in the interview was performed to minimise ambiguities emanating from grammar or vocabulary. The pilot study was in August 2016 and it paved the way for the real study research procedures to begin. For the other two instruments, templates from the School of Education at the institution were to be used so that preservice teachers' experiences during teaching practice were as natural as possible. Thus, there was no need to pilot the lesson plan template, lesson plan evaluation sheet and the classroom observation schedule as all participants were already used to them by that time.

After the pilot study was concluded data collection for the actual study commenced in earnest. The content test and interviews were conducted on campus towards the end of 2016 and the classroom practice instruments implemented during the April/May 2017 teaching practice sessions. The same subjects took part in all the data collection stages. The administration of the content test had an obvious advantage of being applied to a large sample compared to interviews or lesson presentations (Pino-Fan, Godino, Font & Castro, 2012). Thus, more participants in the content test and fewer in the other modes of data collection.

The role of the researcher at the institution as a lecturer and supervisor of teaching practice was made clear. However, instead of witnessing unwanted research bias, the study benefitted from the cordial working relations between the researcher and the participants. Participants' cooperation was noble as a result. In any case, research bias in qualitative research is never an issue since researchers are an integral part of the research design and the final research study. Researchers therefore strive for transparency of the research processes so that participants may reveal their true feelings and data is to be analysed without prejudice, to lead to credible research results. Conducting a pilot study and triangulating data collection of pedagogical content knowledge were attempts to reduce research bias. Still on the integrity and accuracy of research data, four measures of data quality were explained and applied to enhance quality. For qualitative research studies, these are credibility, transferability, dependability and confirmability, and these four measures are collectively called trustworthiness. The final concern intended to give trust to this research study was ethical considerations. To all the key processes of this study, that is, data collection, analysis, interpretation and reporting, the dignity of human subjects was respected, and the researcher assumed accountability for all his actions. This was accomplished by initially getting permission to conduct the study from responsible authorities. Even though the participants were known to the researcher, their consent to involve them in the study was formally sought and confidentiality in terms of their identity and responses was actioned.

Due to the volume of data, the analysis of data was spread over three chapters. Chapter 5 took care of the interpretation and presentation of what emerged from the data from all the sources only. Chapter 6 presented the discussion of the findings from the data interpretation in the context of the broader literature. In Chapter 7 section 7.3, the conclusion of the study was detailed, in response to the research questions of this study. The three stages of analysis were in a linear fashion; data presentation informed the discussion of results, and discussion led to the research conclusion. In all, the conceptual framework was the lens through which data was analysed. The content test was presented in three parts; biographical, brief statistical analysis and the content analysis under qualitative analyses. The content test was poorly performed, with an average score of 41 percent. There were four items devoted to specialised content knowledge and three on common content knowledge. No items were assessed on preservice teachers' horizon knowledge. The preservice teachers' overall performance in subject matter knowledge was not robust.

The second domain of teacher knowledge, pedagogical content knowledge, was interpreted and analysed deductively. Some key components of pedagogical content knowledge were pre-determined based on literature. The presence or absence and the degree of mastery of each of these components constituted the analysis of data. This was done corresponding to the pedagogical content knowledge sub-categories as spelled out in the conceptual framework. These were knowledge of content and students and knowledge of content and teaching. The third one, knowledge of content and curriculum did not render meaningful contribution to the study, hence, it was omitted. The results from the three data sources for pedagogical content knowledge all contributed to the analysis, especially lesson planning and class observations which were practice-based. The mastery of preservice teachers' pedagogical content knowledge in trigonometry was limited.

The third research question was addressed by a consideration of the results from the four data sources. Preservice teachers' high school experiences on what to teach and how to teach did not build much into what they were to learn from teacher education. The traditional ways of teaching and learning never left them up to the point of exit from teacher education, militating possibilities of positive development of mathematics knowledge for teaching. The implication for classroom practice was that teacher-centred teaching strategies were dominant, and no resources were engaged in lesson delivery.

7.3 Conclusion of the study

To bring this study to an end, the researcher summarised the discussion of the findings in the perspective of the conceptual framework. In the process, responses to the study research questions were made available, since the conceptual framework was in synchronisation with the research questions. The subject matter knowledge correlates to the research question number and pedagogical content knowledge connects with research question two. The implications of mathematics knowledge for teaching on preservice teachers' classroom practice relates to research question three. The idea of preservice teachers' mathematical knowledge has been well-studied starting with the ground-breaking work by Shulman (1986), hence, the original contribution of this study emerged from small gaps within that saturated area of study. The established models of teacher knowledge were taken and applied to a novel context. This study confirmed or refuted what is already known about preservice teachers' mathematics knowledge for teaching in school mathematics in a specific population at a higher education institution in South Africa. This created an extra pane in a window through which reserchers can explore what preservice teachers know and how they teach it (Fi, 2003).

7.3.1. Preservice teachers' subject matter knowledge

The findings from the analysis of the content test results revealed that preservice teachers' content knowledge in school mathematics was inadequate. This was in line with the findings from other studies in South Africa and worldwide (Ball, 1990, 1991; Fi, 2003; Mudaly, 2015; Biyela, 2012). These findings held true for several school mathematics topics, including trigonometry, which was the focus for this study. Preservice teachers have a serious lack of knowledge of trigonometry (Dündar & Yaman, 2015). In all, available research findings on in-service and preservice teachers give an indication that their content knowledge of school mathematics is not connected and robust in accordance with the expectations of the mathematics education community (Ball, Lubienski, & Mewbom, 2001). With such limited content knowledge, preservice teachers may not be able to adequately answer the questions found in the subjects they are responsible to teach (Centre for Development and Enterprise, 2013). Even many experienced teachers in a report by Spaull (2013) cited in Mudaly (2016) were unable to respond to questions intended for the learners who they are teaching.

A closer look at the results of the two sub-domains of subject matter knowledge revealed varying participants' performance per each. As was explained in Chapter 3, two sub-domains of subject matter knowledge were applied, that is, specialised content knowledge and common content knowledge. Horizon knowledge was later dropped because the proponents of the model of mathematical knowledge for teaching left it hanging, promising to return and explore it theoretically and empirically. No mention was made concerning horizon knowledge, especially in its application to teacher-training studies. Knowledge of the awareness of how topics or concepts are related over the span of mathematics prior or future to what is being taught is subsumed in specialised content knowledge (Ball, Thames & Phelps, 2008).

Items 1, 5 and 7 focussed on drawing sketches, proving identities and computing values were classified under common content knowledge. From Section 5.2.3, result showed that the preservice teachers' average performance in these items was 47 percent. Items 2, 3, 4 and 6 featured on giving justifications and explanations, and solving problems intuitively. These were categorised under specialised content knowledge. The average preservice teachers' performance on these items stood at 34 percent, according to the explanation in section 5.2.3. These two statistics testify that all was not well with preservice teachers' performance in subject matter knowledge. Findings also showed that performance was relatively better in common content knowledge where prospective teachers answer problems without necessarily providing any justifications or using representations. But anyone who knows mathematics can do the same. The mathematical knowledge unique to teaching, which involved interpretations in diverse ways, flexible thinking and drawing generalisations based on valid justifications, was less performed. Most of the teachers' everyday tasks rest on their competence in specialised content knowledge (Ball, Thames & Phelps, 2008). With the content test items drawn from Grade 12 past examination papers and textbooks, prospective teachers were unable to answer with meaning questions in the curriculum they are training to teach (Centre for Development and Enterprise, 2013). Frequently, poor learner performance in mathematics in most schools is largely due to the poor subject knowledge of teachers.

The exploration of preservice teachers' content knowledge crossover into the second data collection instrument were the task-based interviews. Task-based interviews were conducted on a one-to-one basis, which turned out to be a good tool to also capture preservice teachers' performance in content. Of interest were items that dealt with identifying learners' errors and defining terms. Exactly three out of six preservice teachers were unable to give the correct definition of a negative angle measure. Also, on being asked to perform certain mathematical calculations, some preservice teachers expressly failed to do so. This was true for proving identities, solving quadratic equations in trigonometry and conceptualising horizontal transformations of trigonometric functions.

This study has shown that teachers' skill of identifying learners' errors in a mathematical computation or explanation is part of common content knowledge. In this study, most preservice teachers were able to pinpoint with accuracy some of the errors in the hypothetical situations presented in the task-based interviews. However, the same errors that they successfully identified, the way in which preservice teachers explained, corrected and justified those errors was not matching. If a teacher manages to identify an error but fails to render the correct solution to the same problem, then his or her mastery of common content knowledge is questionable. How preservice teachers handled the errors in terms of explanations, correcting and justifications are part of knowledge of content and students, to be covered in the next section. This study disagrees with Ball, Thames and Phelps (2008, p.400) who placed looking for patterns in learners' errors and "sizing up the nature of an error" as a type of specialised content knowledge. This study has classified knowing the nature of learners' errors as a skill under specialised content knowledge, but it does not have much to do with interpreting the errors in the light of content material. Thus, the poor mastery of common content knowledge and specialised content knowledge by the preservice teachers in this study denotes that their mathematics content knowledge was weak. In South Africa, many teachers have a poor grasp of the knowledge they are required to teach (Deacon, 2012). Thus, it could be that these teachers were inadequately equipped in content knowledge in various aspects of school mathematics during their training (Centre for Development and Enterprise, 2014).

7.3.2. Preservice teachers' pedagogical content knowledge

The climax of our exploration of preservice teachers' pedagogical content knowledge was attained during the classroom-based practices. This was done in real classrooms during formal teaching practice sessions. Lesson observations were conducted at a time when preservice teachers went out for their routine teaching practice. From this study, the knowledge of content and curriculum did not contribute much in our exploration of preservice teachers' mathematics knowledge for teaching. The provincial Departments of Basic Education have taken upon themselves to annually supply all schools with the list of recommended textbooks, assessment plans and work schedules. Work schedules encompass the ideal, which encompasses ideal sequencing and duration for all the topics for each grade and subject. The best teachers can do faced with this is to simply adhere and comply with the policy recommendations for teaching and learning. Non-compliance to work schedules may have negative implications on learners' readiness for common assessment tasks which are drawn in accordance with the work schedule. Hence, there was no chance of variability in preservice teachers' knowledge of curriculum who were using the same working documents. Some of the aspects of curricular knowledge, such as crafting appropriate teaching methods to match the goals of the curriculum (Ijeh, 2012) have been categorised under knowledge of content and teaching, which is the subject of discussion in the upcoming paragraph.

Under the exploration of pedagogical content knowledge, the premise was that what teachers know is central to teaching, thus hereunder, focus is placed on what teachers can do with what they know. With common content knowledge, teachers can define, solve mathematical problems and identify learners' possible errors. Building on common content knowledge, specialised content knowledge takes teachers' knowledge a step further by enabling them to solve mathematical problems by using different representations, understanding unfamiliar solution processes to problems and explaining with justifications their chosen approaches to mathematics phenomena.

Then teachers are expected to recognise content material that likely cause learners great difficulties under the category of knowledge of content and students. The manner wherein teachers prepare and deliver their lessons must reflect possible learners' difficulties and misconceptions. These then should be addressed during the lesson to successfully achieve learning goals. Possible learners' difficulties can be obtained from the learners' responses during oral probing and from the learners' solutions of exercises that are carefully presented. Sometimes, if difficulties and misconceptions concerning a specific concept are not apparent, teachers may have to anticipate them as they prepare to teach a lesson. This is one thing which was not consummated by preservice teachers in this study. No difficulties or misconceptions were explicitly addressed and anticipated, even in cases where they existed. In fact, as preservice teachers worked through the hypothetical cases of learners' work laden with probable errors and/or misconceptions in the task-based interviews, they instead revealed their own misconceptions (Fi, 2003). In more serious cases, there is a high likelihood of teachers passing down their own misconceptions and difficulties to their learners. Misconceptions emanating from high school when preservice teachers were still learners inherently follow them all through teacher education. This was the case in division by a non-zero function and in the horizontal transformations of trigonometric functions. Though preservice teachers had no serious problems in identifying learners' errors in computations, they had challenges of handling and anticipating those errors in task-based interviews and actual classroom practice.

Knowledge of content and teaching is concerned with how to address learners' misconceptions and difficulties in order to achieve effective teaching and learning. Knowledge of content and teaching is concerned with planning to teach and implementing intricate teaching strategies so that teachers can unpack mathematical knowledge to help learners to understand. Preservice teachers in this study unfortunately took great comfort in the traditional teaching approaches in the lesson plans and in video-teaching episodes. The traditional method of teaching is when a teacher directs learners to learn through explanation and memorisation techniques. This in effect deprives the learners of much-needed development in decision-making and critical thinking skills. The choice of preservice teachers' methods of teaching in the lesson plans and lesson implementation bore testimony to this. Five out of six of the preservice teachers chose to use the teacher explanation method in their lessons. Explanation as a teaching method is akin to lecturing, which obviously is not the best way of teaching. Modern teaching methods are proponents of "working with the learner" and learners working in small groups to create an enabling environment for learner participation and interaction (Somayajulu, 2012). These two facets are essential for fostering good learning.

The discussion on the elements of pedagogical content knowledge revealed that at the end of their undergraduate studies, preservice teachers still had challenges with necessary skills needed for classroom practice. No teaching and learning resources were planned for or used in the lesson delivery, which confirms that the hands-on approaches just mentioned in the preceding paragraph were not part of the lessons. This is reminiscent of past teaching practices which dominated classrooms until the 1980s until educational reforms were introduced. Thus, the traditional mode of teaching never left the preservice teachers despite four years of undergraduate studies which they undertook, whereby emphasis was placed on the constructivist learning paradigm.

The level and order of questions that were planned and executed in class were of lower-order, to the extent that they invoked factual and definitions of terms. Higher-order question types were not actioned at all. Preservice teachers grappled with framing a lesson topic for the lesson plan and lesson implementation. Lesson topics for more than half of them were just textbook chapter topics, though the lesson objectives were truly specific. Regarding assessment, two preservice teachers saw no need to assess the work they taught. It could have been due to time constraints because both had something on class-activities planned. Indeed, with few worked examples given during lesson presentation, some of the planned assessment activities took up so much lesson time as learners struggled with them. Also, during lesson observations, teacher facilitation to learning during assessments was below expectation as half of the preservice teachers stood aloof as if they were administering a test, when in fact it was class-activity time.

Again, from the discussion, it was shown that given a chance, preservice teachers would plan and teach concepts with which they are familiar. This confirms that content knowledge is necessary but not sufficient for pedagogical content knowledge. The concept of trigonometric graphs was one of the best performed in the content test, hence three out of the six preservice teachers chose to plan and teach sketching of trigonometric functions for the purpose of this study. On the other hand, none opted to teach concepts such as proving of identities or solving trigonometric equations, which were poorly done in the task-based interviews. Preservice teachers, even though they follow work schedules and pacesetters in their daily practice teaching, still had a choice on what aspect of trigonometry to address on the day of the class observation visit by the researcher-supervisor. Thus, some topics and concepts are omitted or taught in a haphazard way whenever teachers feel they do not fully understand the content. Only when teachers work as teams can they inter-teach classes as per their capabilities or else teachers pass down their own weaknesses to their learners.

7.3.3. The extent of development of mathematics knowledge for teaching

Prospective teachers do not go to teacher education empty-handed but have some ill-formed ideas of mathematics and how to teach. Deacon (2012) posited that a very strong influence on prospective teachers' preconceptions of mathematics for knowledge teaching is the way in which they themselves were taught when they were still learners. In any case, learners' long years of observations during their schooling years may have a greater influence on them than their subsequent formal preparation from teacher education. However, the ill-formed ideas of what to teach and how to teach it may militate future attempts by teacher education to train preservice teachers in other ways. Teacher-centred approaches to learning, non-usage of learning and teaching support materials and the dearth of real-world application of the concepts taught were all hard-wired relics of preservice teachers' schooling days.

Furthermore, regarding responding to content knowledge items in the task-based interviews and the content test, many preservice teachers relied on their shaky high school mathematics knowledge to guide them on how to answer and to explain misconceptions. Ideally, they were supposed to draw on the recent experiences in undergraduate mathematics courses in responding to subject matter-related questions in the content test and during the task-based interviews. In cases where some school mathematics topics were not part of the teacher-training curriculum, preservice teachers resorted to high school conceptualisations without properly filtering faulty dispositions as expected. As a result, certain aspects like misconceptions and the traditional teaching methods followed them from high school through to consummation of teacher education. The misnomer of dividing both sides of an equation by a function is a classic example of a misconception which the 2013 national Diagnostic Report alluded to (Department of Basic Education, 2014). It could also be the fact that preservice teachers may not have co-opted teaching practice experiences and the methodology modules as tools for mathematics knowledge for teaching (Gess-Newsome, 1999b). Some preservice teachers may still need more exposure to teaching practice in order to transfer what they have learnt in teacher education to classroom teaching. Having relied a lot on high school mathematics knowledge on trigonometry, there was no meaningful development of mathematics knowledge for teaching as a result of initial teacher education exposures.

Attracting increasing numbers of high-achieving matriculants into teacher education programmes has always been tricky for education departments in higher education institutions (Centre for Development and Enterprise, 2015). Most high-performing learners in the science stream opt for other high-status programmes such as health sciences, engineering and technology. At the higher education institution where the data was collected, a minimum of 50 percent is needed to secure a place of study for the

Bachelor of Education degree programme while medical science degree programme had a minimum of 60 percent for mathematics. Preservice teachers enrolled with basic passes in mathematics, and that performance plummeted as they progressed through teacher education. In the content test, participating preservice teachers scored an average of 41 percent in trigonometry. Hence, there is no evidence of marked development of preservice teachers' mathematics knowledge for teaching from commencement of teacher education all the way to the exit of the same. Lowrie and Jorgensen (2016) concurred and further said that preservice teachers' school mathematics content knowledge might remain unchanged throughout the duration of their teacher training period, even though the pedagogical content knowledge may develop during the same period. This is a result of the exposure to teaching practice experiences and methodology modules. However, the development of pedagogical content knowledge again was mediocre, as preservice teachers demonstrated traditional teaching approaches and displayed their own misconceptions in understanding some trigonometry concepts.

Ideally, content knowledge alone is not sufficient for pedagogical processes to take place. The tracking of each preservice teacher' performance in the four data sources yielded no commonalities of facts. It was seen that the only teacher candidate who planned and employed good learner-centred teaching strategies had performed very badly in the content test. To the contrary the teacher candidate who performed very well in the content test and content components of the task-based interview later settled for mediocrity in the lesson plan and video-teaching episode. Based on the findings of this research, it is important to note that preservice teachers commence in earnest their teaching careers with inadequate mathematics knowledge needed for teaching mathematics. Most of the preservice teachers seemingly appear to be at the same level as high school learners in terms of mathematics knowledge. Consequently, one cannot expect meaningful improvement in South African learners' results for mathematics (Biyela, 2012).

7.3.4. Resolving the conundrum

The preceding subsections have shown that based on the results of this study, mathematics preservice teachers at the higher education institution investigated portrayed lack of mathematics knowledge for teaching in trigonometry. The rationalisation from the four data sources also revealed that even the growth and connectedness of mathematics knowledge for teaching in school mathematics is not robust amongst final-year preservice teachers. This has happened in disregard of concerted efforts by teacher education programmes at the higher education institution, which are enshrined in the noble role of inculcating and refining preservice teachers' mathematics knowledge for teaching. This undesired status quo has the likelihood of negatively impacting preservice teachers' future classroom practice when they complete training and enter the world of work.

Teacher education programmes duly render much general pedagogy, content, teaching practice and content-specific methods modules geared to support the growth and development of professional knowledge for teaching. In the quest to develop knowledge to teach a particular content area, general pedagogy modules get relegated in the development of an ideal mathematics teacher. Therefore, the interplay of content, methodology and teaching practice modules contributes to the development of mathematics knowledge for teaching. This happens at the intersection of the three sources of preservice teachers' knowledge, as illustrated in Figure 7.1.

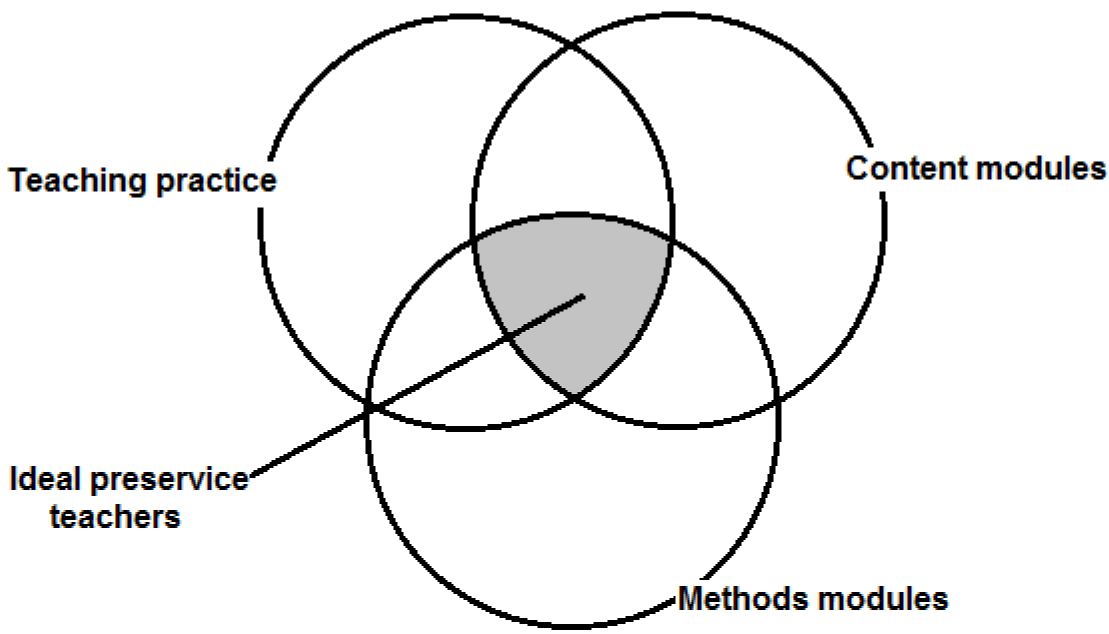


Figure 7.1. The triadic model showing the place of an ideal preservice teacher.

Hence, I believe at the higher education institution and other similar institutions in South Africa should strive to develop initial teacher education programmes that look at teaching practice, methodology and content knowledge as part of a triadic model where the ideal teacher is situated at the intersection of all three.

Content knowledge is important in that teachers by nature teach what they know successfully and knowledge of content forms the basis of many pedagogical processes such as lesson planning, questioning techniques, correcting learners' misconceptions and errors and assessment. Again, preservice teachers need good grounding in content knowledge as they interact with learners in the real classroom situation during teaching practice. Of importance is the need for discovering pertinent ways to bridge the university-school mathematics divide. Advanced undergraduate mathematics is acceptable in teacher education, but too much of it is not suitable as a base for mathematics (Department of Higher Education and Training, 2010). The content test attempted to uncover the extent to which preservice teachers in the study knew about the mathematics content that is informed by the mathematical knowledge for teaching, which is specific to teaching school mathematics (Morrow, 2007).

Methods modules came into prominence when criticisms were levelled against teacher education programmes in South Africa when they then construed teaching and learning as generic activities, "with scant reference to the content of what is being taught or learned" (Gierdien, 2012, p.140). In mathematics methods modules, preservice teachers are instructed how to teach mathematics that is embedded in content-specific contexts instead of generic teaching methods. For instance, preservice teachers are presented with carefully selected samples of learners' misconception riddled work and required to analyse the learners' thinking and possibly generate ways of eradicating such misconceptions (Kiliç, 2011). These skills are put to test and internalized in real classrooms when the preservice teachers go to schools for formal teaching practice. The task-based interview in this study addressed the aspects of methods modules in investigating preservice teachers' mathematics knowledge for teaching.

Finally, teaching practice accords preservice teachers the opportunity to learn first-hand how to teach and apply knowledge and understanding learnt in theory during university-based undergraduate classes to challenges encountered in real classrooms. However, there are disparities in the length and quality of school teaching practice of teacher candidates in the various teacher education institutions. On the duration of teaching practice, the Minimum Requirements for Teacher Education Quality emphatically states that preservice teachers should spend not less than sixteen weeks and not more than twenty-four weeks on supervised school- based practice over the four years of their degree programmes. Supervisor visits to schools should be sufficient, which normally is a financial hiccup to historically disadvantaged institutions as the one in this study. There should be common understanding among teacher education institutions, schools and teacher-mentors in terms of their mutual role in the development of prospective teachers. School teaching practice by virtue of its significance should become central and integral to the teacher education curriculum, instead of adjunct to the curriculum as in some cases. The lesson planning and video-teaching episodes which were practice-based in this study took care of the teaching practice experiences of preservice teachers.

7.4 Future research

There are many possible recommendations for future research and extensions based on this study. More in-depth explorations of the growth of preservice teachers' mathematics knowledge for teaching in trigonometry can be done. This can be in the form of a longitudinal study where a snapshot of preservice teachers' knowledge of trigonometry is done at the beginning. Possible interventions are conducted in between in methodology modules, content modules and teaching practice to increase preservice teachers' knowledge throughout the preservice years. Another snapshot is taken at the end of their studies and these two are compared for possible growth of mathematics knowledge for teaching. Also, the present study focused on the mathematics knowledge for teaching of trigonometry only. Similar studies could investigate mathematics knowledge for teaching other significant topics of school mathematics, which would benefit our understanding of the extent of prospective teachers' knowledge in mathematics.

There is a need to investigate the reason preservice teachers still experience weak performances in school mathematics, after having been in the higher education institutions for four years. Are these institutions doing justice when teaching mathematics content to the preservice teachers? Or do they assume that preservice teachers know the mathematics they will be teaching upon completion of their training? Alternatively, one can explore the mathematics knowledge for teaching for qualified teachers in specific topics of school mathematics. Other future studies could compare relative understandings of knowledge of teaching trigonometry by preservice teachers and qualified teachers. The mathematics knowledge for teaching of the in-service teachers could be investigated and then contrasted with the preservice teachers' mathematics knowledge for teaching. Moreover, future researchers can study and focus on pedagogical content knowledge and content knowledge separately. Finally, because this study was not representative of all the higher education institutions in South Africa, more research could be conducted, which focuses at other South African higher education institutions to broaden the scope of studies like these.

7.5. Limitations of the study

Analysing preservice teachers' classroom behaviours and practices, and then categorising them into pre-determined components of pedagogical content knowledge is not without challenges. Pedagogical content knowledge is by nature unique, specialised and develops in cycles rooted in classroom practice (Miller, 2006). Time allotted for the classroom observations was seemingly insufficient since the lessons were timed. The normal running of the school timetable and of the teaching practice programme had to be factored in. However, that limitation was accounted for in the task-based interviews and lesson planning, where the participants directed the pace and time required. Moreover, this study was conducted with final year preservice teachers at a rural-based university. However, the exploration was restricted to one particular year group and it is not known what the understanding would be for other year groups doing the same programme at the same higher education institution and other institutions of higher learning in South Africa (Dos Reis, 2012).

The act of teaching practice does not provide irrefutable evidence of the preservice teacher's mathematics knowledge for teaching because the preservice teacher would be someone's classroom, the school mentor. The classes they teach belong to the mentor-teacher, and they teach under the mentor's expectations and work with learners who are used to the teaching ways of their "teacher". However, the teacher-mentor scenario is the way it is done in all teacher education programmes and preservice teachers benefit from that mentorship relationship. Mentors do have formal roles assigned to them such as lesson evaluation and writing reports at the end of the teaching practice session. Preservice teachers do not assume full responsibility of a class during their practicals. Also, only one lesson was taught under observation by the teacher candidates when the researcher visited them at their practicing schools, and it is highly likely that under these circumstances, impressions may be given which are not true in everyday teaching experiences. Secondly, watching a preservice teacher teaching for just a single lesson does not allow for the full spectrum of classroom behaviours to be observed. Some classroom etiquette is known to occur over a period of time in the mathematics classroom, with which the teacher has to cope appropriately as time passes (van Putten, 2011).

7.6 Implications of the study

Preservice teachers commented that they were not taught trigonometry conceptually at school, which to some extent effected the weaknesses that were revealed. The undergraduate content modules which preservice teachers take at teacher education institutions still leave the content gaps un-filled in some school mathematics topics. This argument is a worthy criticism that the mathematics education community needs to take seriously. Key school mathematics topics should be well detailed at higher education institutions before advanced undergraduate mathematics take the stage. In some countries, training of mathematics teachers has already started to move towards providing content modules that re-capitulate school mathematics at a higher level (Conference Board of the Mathematical Sciences, 2001). Having seen that developing the mathematics knowledge for teaching of preservice teachers is central to all initial teacher education programmes, checks and balances should be put in place before preservice teachers begin full-time teaching. There was a call for all teacher-graduates to sit an assessment-of-competency examination as a requirement for employment in public schools (Centre for Development and Enterprise, 2014).

7.7 Chapter conclusion

This chapter provided a synopsis of the entire research study and answers to the research questions. The findings of this study shed some light on the extent and development of preservice teachers’ understanding of school mathematics. Preservice teachers' content knowledge was inadequate for trigonometry because they were not properly taught when they were learners themselves and teacher education programmes scarcely address high school mathematics concepts. Their pedagogical content knowledge competency was also limited mainly due to weak content knowledge and over-reliance on the traditional way of instruction. There was no smooth development of mathematics knowledge for teaching based on what preservice teachers brought to teacher education from high school; they were still inclined to teach the way they were taught. The undergraduate studies and teaching practice experiences were necessary but not enough to effectively empower teacher candidates with skills for their future teaching job. Some facets of their subject matter and pedagogical content knowledge were still inadequate, even after many years of undergraduate studies.

The limitation of this study was that the content test and task-based interviews data sources were not based on classroom practice and mapping preservice teachers’ pedagogical content knowledge was not easy since it is a teacher’s personal construct. The study has implications on teacher education by advocating that school mathematics needs to get more coverage at teacher training in order to equip preservice teachers with necessary skills and knowledge to effectively teach high school mathematics. Exploring content knowledge separate from pedagogical content knowledge and comparing preservice and in-service teachers’ performance in mathematics knowledge for teaching were some of the recommendations of this study.

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APPENDICES

APPENDIX A. Ethical clearance certificate from the research site.

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 2212, Fax: (015) 268 2306, Email:noko.monene@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE CLEARANCE CERTIFICATE

MEETING: 05 July 2016

PROJECT NUMBER: TREC/62/2016: IR

PROJECT:

Title:	An Exploration of pre-service secondary school mathematics teachers' knowledge of trigonometry: A case of a higher education of learning institute
Researcher:	Mr B Tatira
Supervisor:	Prof M de Villiers
Institution:	University of KwaZulu-Natal
Degree:	Independent Research

PROF TAB MASHEGO
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:

i) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee.**

ii) **The budget for the research will be considered separately from the protocol.**
PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.

APPENDIX B. The University of KwaZulu-Natal ethics clearance certificate.

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

21 June 2016

Mr Benjamin Tatira 214585806
School of Education
Edgewood Campus

Dear Mr Tatira

Protocol reference number: HSS/0522/016D
Project Title: An exploration of preservice secondary Mathematics Teachers' knowledge of trigonometry at a higher education institute.

Full Approval – Expedited Application

In response to your application received 09 May 2016, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment /modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

Dr Shamila Naidoo (Deputy Chair)
Humanities & Social Sciences Research Ethics Committee

/pm

Cc Supervisor: Dr V Mudaly
Cc Academic Leader Research: Dr SB Khoza
Cc School Administrator: Ms Tyzer Khumalo & Ms B Bhengu

APPENDIX C. Informed consent form for the research site.

TURFLOOP RESEARCH ETHICS COMMITTEE

PART III

INFORMATION FOR PARTICIPANTS

PROJECT TITLE: *An exploration of preservice secondary Mathematics Teachers' knowledge of trigonometry at a higher education institute.*

PROJECT LEADER/SUPERVISOR: *Dr Vimolan Mudaly*

1. You are invited to participate in the following research project:

   An exploration of preservice secondary Mathematics Teachers' knowledge of trigonometry at a higher education institute.

2. Participation in the project is completely voluntary and you are free to withdraw from the project (without providing any reasons) at any time.

3. It is possible that you might not personally experience any advantages during the project, although the knowledge that may be accumulated through the project might prove advantageous to others.

4. You are encouraged to ask any questions that you might have in connection with this project at any stage. The project leader and her/his staff will gladly answer your question. They will also discuss the project in detail with you.

5. This research project shall involve determining preservice mathematics teachers' content and pedagogical knowledge in the topic of trigonometry. Data shall be collected by means of a questionnaire, interview and lesson evaluation. There are no known alleged risk-factors nor side-effects emanating from participating in this research study. There are no extraneous advantages associated with participating in this study, except for contributing to the advancement of mathematics teacher education.

6. Should you at any stage feel unhappy, uncomfortable or is concerned about the research, please contact **Ms Noko Shai-Ragoboya at the University of Limpopo, Private Bag X1106, Sovenga, 0727, tel: 015 268 2401.**

PART IV

CONSENT FORM

PROJECT TITLE: *An exploration of preservice secondary Mathematics Teachers' knowledge of trigonometry at a higher education institute.*

PROJECT LEADER/SUPERVISOR: *Dr Vimolan Mudaly*

I, ____ hereby voluntarily consent to participate in the following project:

____

____

I realise that:

1. The study deals with *the exploration of preservice teachers' content and pedagogical content knowledge in trigonometry.*

2. The procedure or treatment envisaged may hold some risk for me that cannot be foreseen at this stage.

3. The Ethics Committee has approved that individuals may be approached to participate in the study.

4. The research project, ie. the extent, aims and methods of the research, has been explained to me.

5. The project sets out the risks that can be reasonably expected as well as possible discomfort for persons participating in the research, an explanation of the anticipated advantages for myself or others that are reasonably expected from the research and alternative procedures that may be to my advantage.

6. I will be informed of any new information that may become available during the research that may influence my willingness to continue my participation.

7. Access to the records that pertain to my participation in the study will be restricted to persons directly involved in the research.

8. Any questions that I may have regarding the research, or related matters, will be answered by the researcher/s.

9. If I have any questions about, or problems regarding the study, or experience any undesirable effects, I may contact a member of the research team or **Ms Noko Shai-Ragoboya.**

10. Participation in this research is voluntary and I can withdraw my participation at any stage.

11. If any medical problem is identified at any stage during the research, or when I am vetted for participation, such condition will be discussed with me in confidence by a qualified person and/or I will be referred to my doctor.

12. I indemnify the University of Limpopo and all persons involved with the above project from any liability that may arise from my participation in the above project or that may be related to it, for whatever reasons, including negligence on the part of the mentioned persons.

____
SIGNATURE OF RESEARCHED PERSON

____
SIGNATURE OF WITNESS

____
SIGNATURE OF PERSON THAT INFORMED THE RESEARCHED PERSON

Signed at ____________________ this ____ day of _______________ 2016

APPENDIX D. University of KwaZulu-Natal information and consent form.

UNIVERSITY OF
KWAZULU-NATAL

INYUVESI
YAKWAZULU-NATALI

COLLEGE OF HUMANITIES:
School of Education
Edgewood Campus

16 May 2016

Dear valued Participant

INFORMED CONSENT LETTER

My name is ***Benjamin Tatira*** I am a Mathematics Education PhD candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in determining the content and pedagogical content knowledge of final year undergraduate mathematics education students on the topic of trigonometry among. I am studying a case from the University of Limpopo, Turfloop Campus. Your community is one of my case studies. To gather the information, I am interested in asking you some questions in the form of a questionnaire and interview. You are also prepare a 40 – 45 minutes lesson plan on any concept under the topic of trigonometry, which shall be audio-taped and evaluated. The lesson shall be taught to your peer group of nineteen.

Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person, but reported only as a population member knowledge.
- The questionnaire may last about one hour to be done individually.
- The task-based interview may last for about 1 hour, which involves both written and verbal responses.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data on audio-tapes and written responses will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalized for taking such an action.
- The research aims at exploring mathematics preservice teachers' content and pedagogical content knowledge on trigonometry and how the two are related.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.

1

- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

	Willing	Not willing
Audio equipment		
Video equipment		

I can be contacted at:
Email: Benjamin.Tatira@ul.ac.za
Telephone: 015 268 3886
Cell: 083 505 1054

My supervisor is Dr. Vimolan Mudaly who is located at the Department of Mathematics, Science and Technology Education, Edgewood campus of the University of KwaZulu-Natal. Contact details: email: mudalyv@ukzn.ac.za and telephone number: 031 260 3682.

You may also contact the Research Office through:
Prem Mohun (Mr)
University of KwaZulu-Natal
Research Office: Ethics
Govan Mbeki Centre
Tel: 031 260 4557
Fax: 031260 4609
E-mail mohunp@ukzn.ac.za

Thank you for your contribution to this research.

Kind regards

Benjamin Tatira [Mr]

DECLARATION

I .... (full names of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

....

Signature of participant Date

3

APPENDIX E. The content test data collection instrument.

Questionnaire on trigonometric knowledge

SECTION A: PERSONAL INFORMATION

Please complete the following questions by ticking the appropriate response.

Gender: ⬭ Male ⬭ Female

Age range: ⬭ 19 – 21 ⬭ 22 – 24 ⬭ 25 – 27 ⬭ over 28

Location of high school attended: ⬭ Rural
⬭ Urban - Township
⬭ Urban - Suburban

Province you attended:
⬭ Limpopo ⬭ Mpumalanga ⬭ Gauteng ⬭ North-west ⬭ Other

Undergraduate major:
⬭ Physical Science ⬭ Mathematics ⬭ Life Sciences ⬭ Technology

---

SECTION B: KNOWLEDGE OF TRIGONOMETRICAL CONCEPTS

This is a questionnaire on secondary school trigonometric knowledge, as such, it contains items that are basic, intermediate and advanced. Answer all questions to the best of your ability in the spaces provided. If an explanation or justification is required, give an algebraic and/or geometric justification. Graphs can also be used to help you crystallise your reasoning and explanations. State all properties and laws that you call upon to reach the desired goal where necessary. **No calculator is allowed.**

1. Draw two special triangles commonly used in trigonometry to calculate exact solutions. Label all angles with their measures.

2. State whether $\sec^2 x + 1 = \tan^2 x$ is *True* or *False*. Give reasons.

1

3. Solve the following trigonometric equations for $0^0 < x < 360^0$.
a) $\tan x = \tan 30^0$

b) $\sin x = \cos x$

4. Show that for any triangle, $\sin \alpha = \frac{2S}{bc}$, where S is the area of the triangle, and b and c are the sides that include α.

5. Consider the identity:

$$\frac{8 \sin(180 - x) \cos(360 - x)}{\sin^2 x - \sin^2(90 + x)} = -4 \tan 2x$$

5.1. Prove the identity

5.2. For which values of x in the interval $0^0 < x < 180^0$ will the identity be undefined?

6. Given that $\sin\alpha = \frac{4}{5}$ and $90° < \alpha < 270°$, determine the value of each of the following in its simplest form:

6.1. $\sin(-\alpha)$

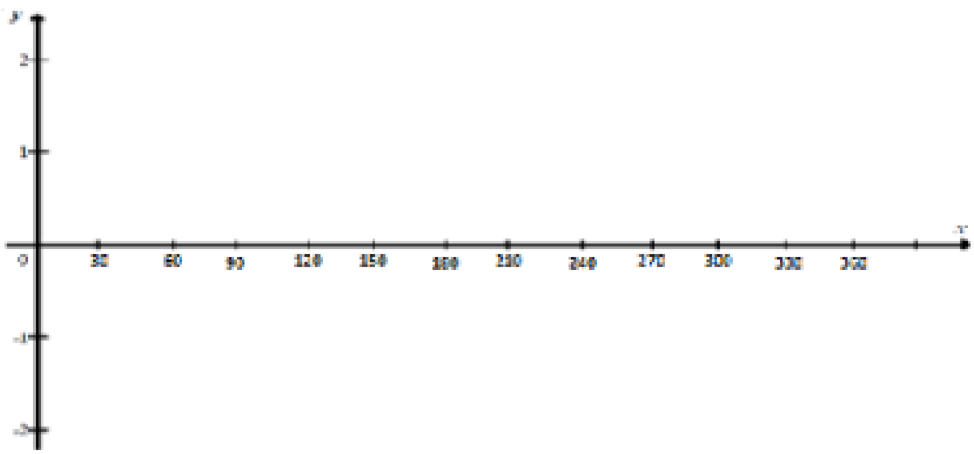
6.2. $\cos\alpha$

6.3. $\sin(\alpha - 45^0)$

7. Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

7.1 Calculate ALL the points of intersection of the graphs f and g for $x \in [180^0;\ 360^0]$.

7.2 Draw sketch graphs of f and g for $x \in [180^0;\ 360^0]$ on the same system of axes provided on the DIAGRAM SHEET below.



7.3 For which values of x will $f(x) < g(x)$ for $x \in [180^0;\ 360^0]$?

The end. Thank you

APPENDIX F. Task-based interview for qualitative data.

TASK-BASED INTERVIEW

The main purpose of this interview is to revisit your work with respect to trigonometry. You were selected for this interview based on your responses to the questionnaire on the trigonometric knowledge administered earlier. In **40 minutes** I will require you to explain and respond to the problems on trigonometry shown below, in a way to further learners' understanding of the topic of trigonometry. The interview will be recorded on audiotapes to serve as a future reference to your responses and my notes. You can use the space provided below the question for your working.

1. How will you teach your learners that $\sin(a + b) \neq \sin a + \sin b$?

2. You ask your class to answer the following question:

   Prove that $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$.

   Learner X's answer:

   $$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$$
   $$(\cos\theta)(\cos\theta) = (1+\sin\theta)(1-\sin\theta)$$
   $$\cos^2\theta = 1 - \sin^2\theta$$
   $$\cos^2\theta = \cos^2\theta$$
   $$\therefore \text{LHS} = \text{RHS}$$

   Comment on Learner X's answer and explain/show the correct method for proving this identity.

1

3. How would you explain to your learners that $\tan 90^0$ is undefined?

4. How you would explain to your learners that the identity $sin^2x + cos^2x = 1$ is true?

5. Identify any possible error(s) your learners could do in the process of solving the equation: $\cos^2\theta = \cos\theta$. Then suggest possible reasons for those errors.

6. Learners will come to your class with various misconceptions. How would you explain this misconception to a learner who thinks that all trigonometric functions have a range of -1 to 1?

7. Analyse the working shown below which contains an error. Identify the error and explain how you would help the learner who made that mistake to realise his/her error.

$$\tan(-x)\,\text{cosec}\,x = \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

8. *Choose to agree or disagree to the following work by a learner. Provide reasons for your decision.*
   Asked to explain the effect of the parameter b in $y = \sin(bx)$ on the standard function $y = \sin x$, a learner argued as follows:
   (a) If $0 < b < 1$ then there is a horizontal shrink because bx is smaller than x.

   (b) If b is negative and $b > -1$ then there is a horizontal shrink opposite that obtained in *(b)* above.

9. What does a negative angle measure represent? Assume that the angle is in standard position.

Closure

Is there anything else that you would want to share related to this interview and the topic of trigonometry?

Thank you for contributing to the advancement of mathematics education research.

4

APPENDIX G. Lesson planning and video lesson observation.

Lesson planning and lesson observation

You are required to plan a lesson of 30 minutes based on a concept of your choice covered in the topic of trigonometry as is expected at Grade 10 – 12. You will then be required to deliver that lesson plan in a real class to one of your class during teaching practice. The lesson delivery will be captured on video to serve as evidence of your participation. You are free to use the School of Education lesson plan template. At the same time the School of Education lesson evaluation form will be used to assess your work.

APPENDIX H. Lesson plan template.

School of Education

TEACHING PRACTICE LESSON PLAN 2: [Week 2-4] University supervisor

Grade		Subject		Phase	

Date		Duration	

Lesson Topic	
Specific objectives(s)	
Teaching and Learning resources	
Teaching methods	

Activities	Time allocation	Teaching and activities	Learning activities
Lesson introduction			
Lesson Presentation			
Lesson Conclusion			
Expanded opportunities			
Self-reflection after the lesson is taught: Briefly discuss the strong and weak points of your lesson and how you plan to improve on the weak points in the next lesson:			

APPENDIX I. Lesson evaluation form for the lesson plans and lesson observations.

School of Education

[redacted]

TEACHING PRACTICE EVALUATION FORM (LECTURERS)

STUDENT NAME..STUDENT NO.............................

NAME OF SCHOOL...

LESSON TOPIC...

DATE...

Score: 1=Poor; 2=Need attention; 3=Average; 4=Good; 5=Outstanding		1	2	3	4	5
	LESSON PLANNING					
1	Learning objectives(s) clearly stated					
2	Appropriate choice of teaching methods					
3	Appropriate choice of teaching aids					
4	Clearly written statement of Teacher and Learner's activities					
5	Well thought out time allocation of each step of the lesson					
6	Appropriate and adequate method of assessment and evaluation					
	Suggestions	MARK OBTAINED				
		MARK			**30**	

Score: 1=Poor; 2=Need attention; 3=Average; 4=Good; 5=Outstanding		1	2	3	4	5
	LESSON PRESENTATION					
1	Linking new topic to pre-knowledge					
2	Introduction of new topic					
3	Step by-step approach to lesson presentation					
4	Evidence of varied teaching strategies (Explanation, Discussion, Demonstration, etc)					
5	Appropriate use of questioning techniques					
6	Well organized and well balanced teacher and learner's activities					
7	Appropriate use of teaching media (e.g. chalkboard, textbook, visual aid)					
8	Mastery of subject content					
9	Teacher's communication skills					
10	Level of assessment during the lesson					
11	Management of development stages of the lesson					
12	Well-presented lesson conclusion					
13	Time for each step of the lesson was well managed					
14	Teacher as a facilitator					
	Suggestions	MARK OBTAINED				
		MARK			**70**	

Evaluator's signature:_______________________________ Student signature:____________________