AN INVESTIGATION OF GRADE 9 LEARNERS EDUCATIONAL CONCEPTIONS IN TWO SECONDARY SCHOOLS: A CASE STUDY.

BY

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DECLARATION

I declare that this document represents my own personal work, with the exclusion of references quoted and listed for the purposes of academic writing style adopted. I further declare that, to the best of my knowledge, a publication of exactly like this one has never been submitted for degree purposes to any university locally and international.

Signed: ______________________

T E Makhathini

December 2006

DEDICATION

I would like to dedicate this work to my mother Cabangile Alosia Makhathini and my late father, David Vela Basi (*peace be with him*).

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ABSTRACT

This research considers specific strategies that would enhance teaching and learning of fractional concepts in mathematics at a secondary school. The notion of the Zone of Proximal Development (ZPD) – Vygotskian view, is invoked as one of the fundamental frameworks for explaining fractional knowledge. This view is contested on the bases of that "*human thinking is inherently social in its origin*" (Goos, 2004: 259).

Another theory that bears testimony to mathematics education especially abstract concepts like fractions is that of *constructivism,* drawn from the works of, Lave (1996), Steffe (1990) and others. Learners' informal knowledge is investigated for the purposes of highlighting what learners *know* and can *do.* Therefore, the study examined the development of learners' understanding of fractions during instruction with respect to the ways their prior knowledge of whole numbers influenced the meanings and representations they construct for fractions as they build on their informal knowledge.

There were 30 participants (15 School **A** and 15 from School **B**) that were engaged in worksheets. Thereafter, 6 cases of the participants were carefully selected for *clinical interview* purposes. The overall methodology of this study is *participatory action research* (Kemmis & Mctaggart, 2000).

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CHAPTER ONE

OVERVIEW OF THE STUDY

1.1. MOTIVATION

Learning and teaching of fractions is one of the most challenging sections of school mathematics. The researcher noticed this challenge in his teaching of mathematics over the many years to learners of different backgrounds. Abstraction of concepts seemed to be a prevalent hindrance during the teacher-learner-classroom-interaction, which resulted in a lot of confusion and misconceptions. This is because, "*procedural knowledge, such as algorithms for operations, is often taught without context or concepts, implying that algorithms are an ungrounded code only mastered through memorization*" (Sharp, Garofalo and Adams, 2002:18). This idea will be noted in Chapter Two as more researchers are alluded to, and trying to investigate the problem of fractional knowledge.

1.2. THE RESEARCH PROBLEM

The challenges experienced by learners to learn and understand more abstract fractional forms where algebraic fractions are involved in higher classes is the actual problem that the researcher has noted. Students from both primary and secondary schools do not have a clear understanding of basic fractional concepts; hence the study undertakes to tackle this problem through research *focus questions* mentioned in section 2.2.

Another problem is that of overgeneralization of whole number concepts. Learners tend to perceive whole numbers operations as the same as fractions manipulations. For an example, the fact that multiplication makes big is translated into fractional numbers on the misconception of that the same would happen. To avoid misconceptions like these, a number of researchers have suggested pictorial representations during learning and teaching of fractions, which will be alluded in section 2.3.

1.3. RELEVANCE OF THE STUDY

The study would be of benefit to education practitioners and interested educationists, researchers and/or mathematics education policy makers. It will therefore be relevant to designers of in-service teacher development programmes, and teacher training institutions.

Material developers for learners' textbooks and teachers' guidelines would find this study very useful, especially during the transitional phase of curriculum changes in South Africa. It is also important to mention that, a majority of pre-service teacher-training programmes lays emphasis on *practical work* (discussed in details in section 2.4) as an important tool in the learning and teaching of complex/abstract concepts like fractions in mathematics education.

1.4. DEFINITION OF A CONCEPT

Novak (1997) cited in Mokapi, defines a concept as follows, "*a concept is an invention of man (sic) used to describe regularities in events or objects designated by some arbitrarily and culturally agreed upon sign or symbol" (p. 454).*

Therefore a concept may be concrete or abstract. To increase awareness and understanding, one must be more exposed to such a concept. That means, even if contexts were different, one would get familiarised with different meaning that the concept upholds. This is a very important view in this study, since the researcher investigates *fractional conceptions*.

1.5. SPONTANEOUS VERSUS SCIENTIFIC CONCEPTS

For Vygotsky (1962), the meaning of concepts learned in everyday life develops spontaneously from lived experiences. Mediation would therefore assist students to become conscious of both spontaneous and scientific concepts. This is what Vygotsky (ibid) calls the Zone of Proximal Development (ZPD). ZPD, according to Vygotsky (ibid), is the psychological distance between what one can do independently and what one can do with the help from others (more able or experienced). The psychological tools required are things like language, symbols and signs during this interaction, so as to make sense of new knowledge.

According to Piaget, drawing on the works of Constructivism, "*all knowledge is tied to action, and knowing an object or an event is to use it by assimilating it to an action scheme*" (von Glasersfeld, 1995: 56). In mathematics, what a learner can do is more important than knowing how to do it. Therefore, *knowing that* and *knowing why*, alluded to in section 2.3 concurs fully with the constructivists' epistemological views. These views (Vygotskian and Constructivist) draw a distinction between the *knowledge from community of practice* (Lave, 1996) and the *scholastic knowledge* (Irwin, 2001).

1.6. FRACTIONAL CONCEPTION

One part of the fractional concept is numerical (abstract) while the other part is quantitative (concrete) (Carraher, 1996). Dickson *et al* (1984) cited in Mokapi, suggests that the part-whole relation (concrete) can be discrete or continuous, (these concepts are alluded to in section 2.5). Examples of discrete relations are countable things representing three-quarters shaded like this:

Example of a continuous relation for still three-quarters shaded looks like this:

To the researcher, this is enough to get the learners completely confused about what exactly is the difference between these two diagrams. That is where the concept of number and fractional representation (concrete) comes in. Learners must be given an opportunity to understand why these two diagrams are both referring to the same fractional number. If concepts like this are not grounded very well, there is very little chance of success in learning and teaching algebraic fractional concepts.

Dickson *et al* (ibid), also argue that a fraction can be represented as division, ratio or a point on a number line (included in questionnaire items-Appendix A). The case of number line representation can be associated to natural numbers trying to fill in the gaps in between them. Dickson *et al* (ibid), (with regard to the number line) "*emphasises that the set of fractions form an extension of the set of natural numbers helping to fill the holes in between them*" (p. 282).

This therefore suggests a call for teachers, that it is important "*to show distinction between fraction as number and as part of a whole when teaching fractions*" (Fraser, 2001: 3).

1.7. TEACHING FRACTIONS

According to Carraher (1996), it is important to teach fractions in a way that relates concrete to abstract forms of fractions, "*the leaning of fractions entails becoming aware of special relations between numbers and quantities and learning to express these relations in diverse ways*" (p. 242).

Some teachers have a tendency to emphasise concrete (fractions as part of a whole) aspects, while others tend to emphasise formal symbolic algorithms (rules of operation). Paulsen (1994) cited in Mokapi (2002), indicates that students need to conceptualise fractions as quantities before they are introduced to the more conventional symbolic algorithms.

On the other hand, Newstead and Murray (1998) argue that the introduction of symbolic algorithms concerning operations with fractions need not be delayed. Otherwise, students tend to conceptualise fractions as single quantities and not as numbers that can be operated mathematically. This argument is discussed in details in section 2.5.2.

Overemphasis on symbolic algorithms causes students to fail to understand fractions from an elementary concrete perspective (Kerslake, 1991). On the other flip side of the coin, more emphasis put on diagrams, causes students to conceive fractions as parts of whole and not as numbers.

Another study done by Kerslake (ibid) in London, reveals that the part-whole model for learners limits them to fractions less than 1. That is students cannot visualize for an example $\frac{7}{5}$ diagrammatically. The researcher included a question item (no. 3) so as to check the applicability of this notion to the cases under study. Part-whole relations emphasis makes students find it difficult to accept that a part can be greater than the whole. Ndlovu (2003) in his paper presented at South African Association for Research in Mathematics, Science and Technology Education (SAARMSTE) conference found the same notion in his Swaziland study.

1.8. AIMS OF THE STUDY

The research questions mentioned in section 2.2 are driven by the following aims that are central to the study as a whole:

- To check if a significant number of learners in grade 9 have the necessary groundwork to continue with mathematics in higher classes.
- To improve mathematization processes to make conjectures.
- To provide educators (through literature review) with other strategies that can make learning and teaching of mathematics more meaningful in lower grades so as to increase learner confidence in the subject in higher levels.
- To determine, if learners are equipped with enough skills to deal with challenging algebraic fractions in higher classes.

1.9. CONCLUSION

This chapter was trying to lay a background based on the many years of the researcher's teaching of mathematics in several secondary schools. The chapter noted the difficulties experienced by learners in grasping the concept of fractions. The study is actually a long one that can take a lengthy period of time like the Mathematics Learning and Teaching Initiative (MALATI) project (explained in details in section 2.1 paragraph 2 and 3) to investigate exactly what makes learners not to understand fractions very well. This is noted through a variety of many research initiatives that were noted in the chapter that are both locally and internationally.

The issue of *practical work* seemed to play a critical role to most researchers and academics (e.g. Luthuli, 2003; Mokapi, 2002); hence the researcher included a section in the next chapter discussing *what, where, how*, practical work can be employed to derive maximum learner performance in fractional problem-solving. Not only is this study focussing on learners, but also educators are noted as they are included in the aims of this study and they also need to be skilled in thought provoking *problem-posing* (Hansraj, 2003, pers. comm.). This leads the researcher to the next chapter.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

According to Strydom (1983), fractions are equal parts of a whole. Learners can be shown these parts by cutting and pasting each of the fractions from a cut whole. Koomen (2001) also suggests a definition of fractions by saying, "*we sometimes need to divide a whole object into equal pieces, and these pieces are described by using numbers called fractions*" (p. 5). Therefore, use of colors and pasting is one strategy that can be employed when defining these pieces as fractions.

Notwithstanding the fact that, fractions have to be classified under continuous or discrete quantities (Smith III, 2002), there are three models suggested by Gearhart *et al* (1999). These models are *area model, fair sharing model and linear model.* These models are also noted in the Mathematics Learning and Teaching (MALATI, 2004) project conducted in the Western Cape, South Africa. This was a continuous research project taking the whole year for data collection in classrooms of grades 4 and 6. It was following the strategy of pre-test, teaching and post-test.

The MALATI project went further by including the *rational number sense*, as these were experienced researchers in the field of mathematics education; hence the researcher included their style in this study. There is an item in the questionnaire where learners had to make relations on non-integer division, since it is actually where fractional knowledge is needed in higher mathematics. The researcher would like to first mention the focus questions before Learning and Teaching of fractions is investigated.

2.2 FOCUS QUESTIONS

This study has investigated how mathematics education could improve by addressing the following research questions:

1. How can grade 9 learners' conceptions be investigated in practical problem-solving involving fractions?
2. What concepts do grade 9 learners hold in learning fractions?
3. What are the suggested literature views in learning and teaching of fractions?
4. To what extent can learners' informal knowledge be utilised in learning fractions?
5. What impact do the concepts in fractions at grade 9 have on later years?

2.3 LEARNING AND TEACHING OF FRACTIONS

Mathematics teaching and learning are viewed as social and communicative activities that require the formation of a *"classroom community of practice"* (Lave and Wenger, 1991 cited in Goos, 2004). This refers to fractions as well, since they are perceived to be one of the hardest sections of mathematics to teach learners. Wu (2001) argues the issue of the challenges faced by learners when dealing with fractions - it is not on computation but on conceptualization. Concurring with Wu is Hatch (2002: 133) accounting by saying, "*conceptual problems need to be met head on rather than avoided if real understanding is to be achieved*", and this is because teachers tend to focus on ready-made algorithms. Tirosh (2000) responded when I communicated with her, asking the step-by-step emphasis of teachers, about the article she wrote on fractions, she said, "*my experience, with many other student teachers, shows that these findings are general*" Tirosh (2003, personal communication: e-mail).

This suggests how detrimental the problem of learning and teaching fractions is. Tirosh's findings were based on the challenge for the prospective teachers if they are aware of discrepancies embedded in "*knowing that*" and "*knowing why*". Her findings called for immediate steps to be taken so as to challenge the rules or conventional algorithms entrenched in the mathematics curriculum in Israel. During her research, she asked prospective teachers to give incorrect responses that learners could come up with given the following division of fraction:

$\frac{1}{4} \div \frac{3}{5}$

Some of the participants wrote: $\frac{1 \div 3}{4 \div 5}$, as one of the incorrect responses. This is because, they have a specific rule of division and they think other mathematically justified methods like this one used, would be incorrect. This is how the method is justified:

$\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d} = \frac{\frac{a}{c}}{\frac{b}{d}} = \frac{\frac{a}{c}}{\frac{b}{d}} \times \frac{\frac{b}{d}}{\frac{b}{d}} = \frac{a}{c} \times \frac{d}{b}$, actually showing how the Invert and Multiply (IM) rule is mathematically justified.

I have included an example that would work easily by simply multiplying numerators and denominators in the paragraphs to come. Interviewees would be interrogated with regard to this method of division, so as to know if they are aware that it also works especially if the second fraction is actually a factor of the first one.

The *"invert and multiply,"* Siebert (2002) cited in Litweller and Bright (eds)(2002) yearbook cautions:

> *-unless we can actually point to where we invert and multiply in our pictures, children will still see the IM rule as an unexplainable and mysterious short cut to fraction division* (p. 247).

Hence, Wu (2001) is suggesting pictoral representation of the problem situation. He continues by saying that this approach *"solidifies learners existing knowledge and further develops generalized ideas about the operation"* (p. 175). Fennema and Franke (1992) cited in Meel (2002), found that teacher knowledge influences instruction since classroom interaction partially depends on teacher knowledge.

On the contrary, Sinicrope et al (2002) claims that the first step is to express both the divisor and dividend as fractions, concurring with Tirosh's (2000) method above, with like denominators, thus making denominators as units of measurement within the same quantity. For an example, $\frac{2}{3} \div \frac{4}{5} = \frac{10}{15} \div \frac{12}{15}$, thereafter just do $\frac{10}{12} = \frac{5}{6}$, dividing the numerators only. This is another approach seeking to "make sense" on fraction division. She then says, *"it is possible to relate procedural reasoning used to the invert and multiply algorithm"* (2002: 154). This concurs with Behr et al (1997) when they use division and multiplication of both denominator and numerator (respectively) by the reciprocal of the denominator fraction. Here is an example, $\frac{3}{5} \div \frac{2}{3} = \frac{\frac{3}{5}}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{9}{10}$, this method actually clarifies the *"invert and multiply"* rule. These views are sound in the sense that they can be mathematically justified following number analysis. But the big question is on practicability of each of the strategies. If learners cannot relate fractions to reality, the fractions become abstract and meaningless to their minds. According to Behr *et al* (1997), citing von Glasersfeld and Richards (1983):

> *There is a first act of abstraction that produces units from sensory-motor material, i.e. unitary things, corresponding to what Piaget calls 'simple' or 'empirical' abstraction; and there is a second act of abstraction that takes these units as the material for the construction of a unit that comprises them* (p. 48).

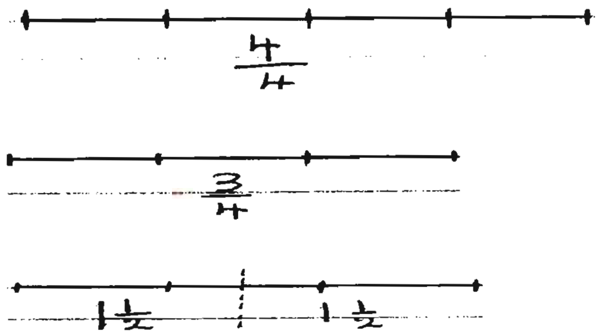
Wu (2001) addresses this problem through "*practical work*" which concurs with Luthuli (2003, personal communication). Luthuli uses a ruler to manipulate fractions. When he does division of fractions, for an example, $\frac{3}{4} \div \frac{1}{2}$, he takes a ruler (30 cm), mark off 20 cm, show three fourths through partitioning as 15 cm, take away one third (i.e. 5 cm) as **one unit** and then **half** one part of the other two parts, thereafter be left with $1\frac{1}{2}$ as the solution. According to Gal'perin and Geogiew (1969) cited in Behr et al (1997), "*all elementary mathematical concepts, regardless of the limitations of their content, assume the notion of unit*" (p. 1).

Note the following diagram representing practical approach with respect to a chosen unit for convenience:

STEP 1 — **20 cm** (for convenience)

STEP 2 — **15 cm**

STEP 3 — **7.5 cm** (measurement)



(Extracted from the researcher's own presentation to the masters students (Science and Mathematics) group in November 2003 to Prof C. Mitchel, as a requirement for field work)

The above diagrams show that fractions can be taught practically, contrary to just following normal routines of algorithms that do not make any sense to the learner's thinking. The method uses the linear model of fractional concepts. Simply take a convenient whole using a linear model (e.g. a 30 cm ruler). The dividend fraction must first be treated as a whole. For an example ¾ take 4/4, 4/5 take 5/5, and so on. The trick is to take a convenient number. Where the denominator is a 5^{th}, it is convenient to choose a number (in a ruler) that can easily be divided into fifths. The same principle works for 4^{th}, 6^{th} 7^{th}, and so on. From the chosen number then take the required dividend say ¾ then divide that dividend by the divisor (second fraction) then the solution can easily be read just like in the above example.

As an educator, the researcher was not aware of this approach until he got a lecture from Luthuli (2003) and doubt if there is a huge number of mathematics educators that are aware of these practical approaches to fractions. That is why the researcher concurs with Meel (2002) quoted in the following citation, "*division of fractions is rarely taught conceptually in school, most of the prospective teachers probably learnt to divide with fractions without necessarily thinking about what the problems want* (p. 141).

The problem does not only lie with the teachers, but also the materials and the textbooks used at schools emphasize the algorithms that are not concrete, but too abstract for learners' comprehension and thus doing more harm than good towards addressing fractional concepts. In the MALATI (2004) project, it is noted that the traditional way of Least Common Denominator (LCD) in addition and subtraction of fractions does not foster understanding of equivalent fractions. For an example,

$\frac{1}{6}+\frac{2}{3}$, could be done as $\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$. This is emphasizing the same unity of measurement (6 units) then different quantities can be added. Some learners are not equipped with these skills, which make it difficult for them (learners) to be able to deal with mixed fractions. According to Newstead and Murray (1998) traditional teaching of fractions results in misconceptions.

This brings my discussion to Boaler (2002) where he draws on the analysis, concurring with Siebert (2002) above, by saying,

> *traditional, textbook approach that emphasizes computations, rules and procedures, at the expense of depth of understanding, disadvantages students, primarily because it encourages learning that is inflexible, school-bound and of limited use* (p. 111).

From the South African point of view, I believe, if content prescription could involve as many educators as possible, some of the didactical problems could be addressed. What needs to be borne in mind is the fact that Outcomes- Based Education (OBE) is "*neither content free nor content based*" (Policy Document, 1997). This is a South African view on OBE, of which, according to the researcher, says a mouthful, but it is ignored on the assumption of that educators are skilled enough to deal with curriculum challenges during the transitional phase of a new curriculum.

In particular, Schifter (1997) cited in Meel (2002) asserting that:

- Teachers need to develop a richer understanding of the subject matter;
- Teachers need to gain more experience listening to students and sorting out the mathematical issues confronting those students; and
- Teachers need to learn to pose questions in order to gain additional insights into students thinking.

On the contrary, Crump (1995) cited in Meel (2002) claims that "*students will learn what they want to learn and will have difficulty learning material that does not interest them*" (p. 1).

There is however a need to let learners grasp important algorithms in mathematics, that makes the teachers job to unfold the learners' experiences so as to make sense of whatever learning and teaching interaction taking place in the classroom make sense. Thus, teachers of diverse classrooms must be aware of students' everyday knowledge and any misconceptions developed on the way to achieving scientific knowledge (Irwin, 2001).

There is a need for specifically designed problems that are of appropriate level of difficulty for students so that conflict of informal knowledge on fractional concepts leads to successful scientific or "*scholastic knowledge*", as some researchers like Wardekker cited in Irwin (2001) refer to this knowledge. Wardekker emphasizes the importance of reflection if students are to gain understanding so that this knowledge becomes knowledge-in-action and that this reflection usually happens through dialogue. Concrete models and the number line, incorporated in my research questionnaire items, can be used in many context so as to make fractions understood (Irwin, 2001).

According to Vygotsky (1987), cited in Irwin (2001), mathematical concepts that are not intuitive, such as fractions, fit within the definition of scientific concepts. Despite the optimism that exists, studies (e.g. Brown, 1981; Thipkong and Davis, 1991 cited in Irwin, 2001) involving both school students and adults reveal that the system of fractions is neither simple to learn nor general to understand. In Vygotsky's terms, cited in Mokapi (2002), students become conscious of both spontaneous and scientific concepts through mediation. Other authors (Jones, 1991; Lubienski, 2000) have shown that students from lower economic classes may resist a pedagogy that is based on integration of school and everyday knowledge, and that resistance may be due to the fact that specific problems or tasks that these students are asked to solve do not relate directly to their own personal milieu experiences.

Mack (1995) claims that, a substantive body of literature has suggested that many students perform operations on symbolic representations with little understanding of the meaning underlying the representations when citing Hiebert and Wearne (1988) and Kouba *et al* (1988). Consequently, a number of studies (cited in Mack, 1995) have been commissioned in this area of mathematical knowledge, looking at students' misconceptions related to symbolic representations for fractions are tied to knowledge of whole numbers (Behr, Lesh, Post and Silver, 1983; Behr, Wachsmith and Post, 1985; Kerslake, 1986; Kouba *et al*, 1988; Irwin, 2001).

Drawings on the works of Mack (1995) symbolic representations are introduced with respect to real-world problems the students could solve to encourage them to draw on their informal knowledge of fractions. According to Steffe (1990), cited in Lo and Wanatabe (1997), learning occurs when an individual adapts his or her schemes to cope with a problematic situation. Mack (1995) says "*ability of students to relate symbolic representation for fractions to their informal knowledge is influenced by their prior knowledge of symbolic representations of whole numbers*" (p. 436).

Hiebert (1988), and Hiebert and Carpenter (1992) cited in Mack (1995) claim that students may overgeneralize their prior knowledge of previously learnt mathematical symbol systems as they attempt to construct meaning for symbolic representations that are unfamiliar to them.

According to Goos (2004), *"-mental processes are mediated by tools and signs such as language, writing, systems for counting, algebraic symbol systems, diagrams, and so on"* (p. 260). This is line with the Vygotskian view of learning and teaching and Zone of Proiximal Development (ZPD) perspective where the scaffolding term is noted (Brunner, 1986; Rogoff and Wertsch, 1984 cited in Goos, 2004). The metaphor of scaffolding was introduced by Wood, Bruner and Ross (1976) to elaborate on the role of "*tutoring in enabling novices to solve problems beyond their unassisted efforts*" (cited in Goos, 2004: 260). This is actually the ZPD.

Noting the learners that were observed by Mack (1995), it became evident that these learners do not only overgeneralize prior mathematical knowledge, but also knowledge of new symbol systems is overgeneralized. This concurs with the MALATI (2004) project, where the researchers caution by saying that, "*half-heard or half-remembered rules*" can create problems. A sizable number of researchers (Behr *et al*, 1983; Kieren, 1988) have demonstrated that students overgeneralization in a variety of content domains are reflective of knowledge of whole numbers, where they (students) have a strong knowledge base for whole numbers and hence blindly mimic that knowledge to fractional symbols as the same knowledge base for just ordinary whole numbers written just *one on top of the other*.

On the other flip side of the coin, some researchers (cited in Mack, 1995) argue that students' overgeneralizations are influenced by their (students) prior knowledge of whole numbers, but by the proximity of the symbol system they have worked with most recently in lower classes (Davis, 1984; Kennedy, 1977). Therefore, understanding of symbols as shared communication plays a significant and critical role in the representations students use for different symbol systems (Kennedy, 1997; Pimm, 1987, cited in Mack, 1995). According to the Australian Education Council (1991) cited in Goos (2004), developing students communication and problem solving skills within which mathematical concepts are nurtured like conjectures, generalizations, proofs, refutations etc., should be the epistemological view of mathematics education.

It is evident that problems from prescribed school textbooks might not be appropriate for students designed for. This is a typical case even in South Africa, as learners from rural areas are subjected to school textbooks that have examples or activities like cricket scores or baseball scores of which learners have never played cricket or baseball and are not familiar with these sports at all. As a result, they cannot associate the activities at school with their everyday life experiences.

Knowledge appears to emanate primarily from the individual's real life experiences rather than from formal schooling instruction (Carpenter and Moser, 1983; Riley, Greeno and Heller, 1983 cited in Mack, 1995). Associating the instructional knowledge to personal experiences is of paramount importance. Wheatley (1991) cited in Lo and Wanatabe (1997) believes that knowledge originates in a learner's activity performed on mental constructs that are directly related to the action and experience of the learner.

Researchers have provided valuable insights into ways students may draw references to give meaning to mathematical symbols (Carpenter, Fennema, Peterson, Chiang and Loef, 1989; Mack, 1990; Streefland, 1991 cited in Mack, 1995). The following paragraphs discusses practical work in details with respect to literature as the researcher had touched on it (practical work) through demonstrating how possible practical illustrations can be used in fractional learning and teaching so as to maximize learner performance in fractional problem-solving.

2.4 PRACTICAL WORK

Two reasons that make the investment in time so as to let students explore on fractions behaviors, Smith III (2002: 9), are:

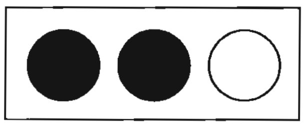
a) *To master order and equivalence, and*
b) *To be able to tackle the range of practical applications of ideas used in fractions in everyday life.*

According to Smith, learners are capable of constructing their own knowledge; the school mathematics teachers should have to incorporate that knowledge within the instructed knowledge for the benefit of the learners' mental capability and interrelatedness. If this rapport is neglected, schooling becomes senseless and misdirected (Sinicrope et al, 2002). On the very same note, according to the National Council of Teachers of Mathematics (NCTM) standards, Gearhart et al (1999: 286) view mathematics as a "*discipline of conceptual inquiry.*"

According to NCTM standards, mathematics is viewed as "*both conceptual and collaborative endeavor*", hence co-operative learning is nurtured if mathematics is approached practically and learners explore a variety of strategies towards problem solving. NCTM, 1989, 2000, cited in Goos (2004), emphasizes a shift in mathematics towards being more of problem solving, reasoning and communication.

Wu (2001: 175) also suggests the idea of that *"children's experiences with genuine problems"* which concurs with Gearhart et al (1999). Learners are equipped with skills of understanding the three relations, mentioned earlier on in this chapter, referred to by Gearhart et al (1999), which are *part-whole, part-part* and *equivalence relations*. However, Wu (2001) gives an account on that a whole itself can be a part (fraction) of a unit. Therefore, Sinicrope *et al* (2002), Gearhart *et al* (1999) and Wu (2001) are in agreement to the view of practical demonstration, not just *pen to paper* writing and claim to be mathematically viable, without any evidence of learner actions done practically. Dickson *et al* (1984) indicates that the part-whole aspect makes learners find it difficult to accept that a part can be greater than a whole. One example to illustrate how a fraction could be approached using pictures as Wu and Lamon are suggesting is given below:

The following example by Lamon (1999: 22) concurs with what Wu is suggesting:



Three circles are a whole, 1 unit, but taking one whole circle not shaded, implies $\frac{1}{3}$ unit. Though, one circle is a whole on its own, but when compared to three initial circles, it is a fraction. So a fraction in this scenario is in relation to number of circles not the partitioning of the circle itself like this one:

Where $\frac{1}{3}$ represents the not shaded part of the whole circle. According to Smith III (2002), a divided quantity is synonymous to a partitioned quantity. Hence, the fact that, "*proportional thinking involves projecting the same ratio to another situation (or situations)*" (Smith III, 2002: 4-5).

Here is another illustration given by Smith III (2002: 8), which emphasizes practical work in more details. Students should be able to explain in clear and convincing terms why not all the shaded quantities in the following figure represent $\frac{2}{3}$:

If students can be able to explain how they solve the above problem, included in my research questionnaire items (item 6.1), then that would call for them (students) to make their own generalizations, and that is one of the approaches suggested by Wu, concurring with Australian Education Council (1991) where generalizing is one of the important aspects that learners should be skilled with in mathematics education. This is therefore a unanimous agreement for these authors and the council. Since Wu (2001), Lamon (1999) and Smith III (2002) are all referring to partitioning from a variety of perspectives, the focus should be on exploring activities that are enhancing learners' concept of partitioning.

This brings me to the discussion on how exactly are fractions viewed in terms of cognitive functions (Lamon, 1999). Greeno (1983), cited in Behr et al (1997: 49), referred to a "*cognitive object that can be reasoned about directly, a cognitive object for which the system has procedures that take the object as the argument.*" This will be reviewed on literature based on the subject Proportional Reasoning, since it addresses the idea of proportionality based on fractions and ratios by Sinicrope *et al* (2002) above when she referred to procedural reasoning and invert and multiply algorithm and contrast ideas to those made by Lamon (1999) as well as the Chinese perspective.

Proportionality and Similarity are important concepts that are to be clearly understood early in the schooling years (as they come up in higher classes), otherwise learners would experience difficulty if these concepts are not grasped well in the earlier grades. Lo and Wanatabe (1997) claim that:

> *growing literature on elementary school teachers' difficulties with division and rational-number concepts* (Ball, 1990; Graeber, Tirosh and Glover, 1989; Simon, 1993), *it should not be surprising that these difficulties exist for middle school or high school students, thus interfering with their learning of ratio and proportion fractional concept* (p. 223).

The researcher will also consider the above sentiments during "*clinical interviews*" (Cohen and Manion, 1994) with the cases chosen for the study. Since a fractional study has to do with, but not limited to, proportional reasoning, the study will also focus on what is meant by this phrase in fractional terms considering different researchers working on this field on mathematics education. The researcher will now address the issue of proportional reasoning in more details in the next section.

2.5 THE DEVELOPMENT OF PROPORTIONAL REASONING

Lamon (1999) suggests that;

> *by providing children experiences with some of the critical components of proportional reasoning before proceeding to more abstract, formal presentations, we increase their chances of developing proportional reasoning* (p 3).

The question that might arise would be that of describing proportional reasoning *per se*. According to Lamon (1999), proportional reasoning is,

> *the ability to recognize, to explain, to think about, to make conjectures about, to graph, to transform, to compare, to make judgements about, to represent or to symbolize relationships of two simple types* (p. 8).

Very few teachers, in the researcher's opinion, are able to give learners such a great reasoning power. May be it is because they (teachers) themselves lack or perhaps are not equipped with such reasoning powers during their prospective teacher-training institutions, so they must first have such strategies themselves before being able to impart them to their learners.

According to Piaget and Inhelder (1975), cited in Lo and Watanabe (1997) proportional reasoning is a second-order relationship that involves an equivalent relationship between two ratios. The concept of ratio is also a complex one on its own, so the researcher cannot dwell much on it in this study. However, it is imperative to note that there is an interrelationship between fractions and ratios. From the Chinese perspective, ratios are introduced early in the years of mathematics education (Cai and Sun, 2002). This is actually a good idea as learners get progress to higher classes having grasped both critical aspects of fractions and ratios.

There are six areas that are to be noted when one has to provide activities that would assist to nurture proportional reasoning (Lamon, 1999):

1. Partitioning: - active "doing" to promote insight.
2. Ratio sense: - intuition acquired through experience in appropriate contexts.

3. Rational numbers: - connecting various meanings and operations in the world of numbers.
4. Quantitative reasoning: - the ability to interpret and operate with changing quantities.
5. Unitizing, and
6. Relative thinking: - both dealing with cognitive functions.

When dealing with fractional activities, it has to be kept in mind that proportionality is twofold, that is *direct and inverse proportionality*. This refers to the relationship between the two quantities that can either increase or decrease proportionally (direct), or one increases while the other decreases, and *vice versa* (inverse). This can be illustrated using a table or even *logarithmic functions* in higher classes. Further explanation on this could be another longitudinal study; hence this study is therefore beneficial to teachers from both lower and higher classes of mathematics.

In Chinese, students are taught to be able to decide on "*two forms of proportions*" (Cai and Sun, 2002: 201). They refer to students' discussions as important, since the problems presented can help students *"make coherent connections among ratio, proportion, fraction, and part-whole relations"* (2002: 204). This concurs with MALATI (2004) project where, "*posing challenging problems to be solved collaboratively"* (p. 5). Not getting into details on what is entailed in these activities, the researcher would like to highlight other concerns by looking at other literatures focusing on implications for learning and teaching fractions. The following two sub-sections seek to emphasize the concept of proportional thinking further considering the issue of terminology.

2.5.1 PROBLEM WITH TERMINOLOGY

The terms fraction and rational number are sometimes used interchangeably, *"but they are not identical concepts"* (Lamon, 1999: 28). Lamon also notes that a fraction might mean *"a piece of land", "a little bit", "a small part"*, etc and a fraction is used when rational numbers are intended and *vice versa* (1999: 26). On the same note Wu (2001) speaks about *part-part, part-whole and equivalence relations*, concurring with Gearthart *et al* (1999) referred to above. But formal symbolic interpretation of fractions is based on rational numbers, so one cannot emphasize either, while neglecting the other, as both are needed for formal mathematization of the whole process of fractional concepts.

According to Irwin (2001), selected ideas of equivalence can be related to both concrete and symbolic representations, as addition and subtraction of fractions represented symbolically requires the fact that there must be common denominators. However, there are contradicting factors when one uses fraction and rational number interchangeably, $\frac{\pi}{2}$ is a rational number not a fraction and $\frac{2}{3}$ is a fraction not a rational number (Lamon, 1999). For learners, this might draw a certain degree of confusion, until they do further mathematics at tertiary institutions.

Another distinction to note is by Smith III (2002), where he cautions on the miscommunication. He says fractions and ratios are both "*relational*" numbers, but they cannot be interchanged, as they are quite different in manipulation and meaning. This is because a fraction results to a *quotient* and a ratio gives a multiplicative sense of two quantities (this will be elaborated in the next sub-section by Lamon, 1999). Further is proportionality, which refers to reasoning with ratios according to Smith

III (2002). On the other hand, some authors argue "*-the development of ratio and proportion concepts is embedded within the development of multiplicative conceptual fields*" (Lo and Watanabe, 1997: 33). This concurs with Lamon (1999), Cai and Sun (2002) when they make reference to proportional reasoning and the curriculum coverage.

Constructivists view knowledge as the epistemological basis for examining children's mathematical thinking (Lo and Watanabe, 1997). In Chinese, percent literally means "*percent ratio*" (Cai and Sun 2002: 196). This is a very different view from that of South African mathematics education, since we view a percent as per 100. They even go further in Chinese referring to ratio as "*to compare or comparison*"; at least we (South African perspective) are on the same wavelength with that.

On the other perspective, Luthuli (2003, personal communication) gives an account on that a unit can be anything in relation to what one considers as the whole, as he uses a ruler and take any convenient number of centimeters as a whole to start of with. This concurs with Lamon (1999) when he says, a unit whole may consist of a single object or more than one objects. For an example, a unit may be a single object (1 box) of 10 dozens of eggs. If there are 5 boxes, and you take one of them that is $\frac{1}{5}$, but taking one dozen in one box that is $\frac{1}{50}$. Interestingly, is taking one egg out of 5 boxes is $\frac{1}{600}$. That makes the part-whole and part-part distinction quite prevalent and easy to figure out. In Chinese, this problem could be considered with a different approach, since Cai and Sun (2002) highlight that in Chinese students are equipped to see how $a:b$, $\frac{a}{b}$ or $a \div b$ are related. According to Cai and Sun (2002), division is used as a bridge to connect the concept of ratio and its representations.

2.5.2 RELATIVE AND ABSOLUTE THINKING

This is where two or more quantities are compared with respect to their change in size. Lamon (1999) gives an example of tree A and tree B. Tree A is 8 m tall and tree B is 10 m tall. Later on, after some days, these trees are measured: tree A is 14 m and tree B is 16 m. The question is which tree grew more? The researcher is not interested in the correct or wrong approach to the answer. To understand the mental phenomena, one needs to concentrate on the *"process of growth and change rather than the product of development"* (Goos, 2004: 260). But let's at the strategy towards arriving at the answer (product), which concurs with "*knowing that*" and "*knowing why*" approach by Tirosh (1999) referred to in the beginning of this chapter.

Lamon (1999) explicitly shows how significant is this part (*knowing why*) since if one notices that these two trees grew 6 m both of them, then that means the approach employed is that of *absolute thinking (i.e. additive).* But if one considers growth of the trees in relation to the original size that becomes different, it is now *relative thinking (i.e. multiplicative).* Focussing on the latter, since it is what is called *multiplicative thinking.* To elaborate further in multiplicative thinking, it is noted that tree A was multiplied by 1,75 to get 14 and tree B was multiplied by 1,6 to get the length of 16. This judiciously shows that tree A grew more than tree B as it has a higher rate of growth. This is a critical part of mathematization, actually introducing the concept of the *rate of change* in Calculus.

According to Irwin (2001), *"complete understanding requires multiplicative thinking, which is not natural, but requires a reconceptualization of the relationship of numbers from that required in additive relationships"* (p. 418).

Additive thinking is in fact adding the same quantity to both trees. Drawing on Vygotskian school of thought, cited in Goos (2004), human thinking is inherently social in its origin (Sfard, Forman and Kieran, 2001). One can figure out additive thinking easily, but multiplicative thinking is a skill that needs to be developed in mathematics education if teachers have to succeed in overcoming the difficulties experienced by learners' in grasping the knowledge of fractional concepts. Adding the same quantity would be pretty obvious to most learners, but finding the ratio at which the compared things are changing is a different approach altogether which demands *"multiplicative thinking"* as the authors (Lamon, 1999; Irwin, 2001; Goos, 2004) are suggesting. It is therefore incumbent upon the mathematics curriculum developers to see to it that they address whatever shortfalls that are limiting the proper learning and teaching of mathematics at school levels.

There are four suggested aspects to entice this view of reasoning about fractions supported by Smith III (2002: 11) focusing on:

1. *Divided quantities,*
2. *Numerical components,*
3. *Reference points, and*
4. *Numerical conversions.*

But at school we tend to quickly embark on numerical conversion without the students having the other three reasoning strategies encouraged. We even confuse learners by terms like "*numerator*" and "*denominator*" which can in fact be introduced at a later stage in schooling (Smith III, 2002). This is in accordance to Cai and Sun (2002), as in Chinese some terms or concepts are used differently to us, but they bear the same mathematical connotations. The researcher questionnaire items noted the above-mentioned aspects so as to validate learners' responses noted in Smith III (2002). The researcher has explained these items on chapter 3 entitled, Methodology and Research Design.

According to Lamon, children are faced with a "*big mathematical and psychological stumbling block*" (1999: 21). This is because, there is a huge "*conceptual jump*", thus contributing to children's difficulty in learning fractions. Cai and Sun (2002) gives a Chinese perspective by noting that, the use of student's knowledge to describe new situations is emphasized according to the Elementary Mathematics (1996a, 1996b). In the same document, the concept of proportion is built on understanding ratio. They also mention that the concept of ratio is clearly defined as the comparison of two quantities with *multiplicative relationships.*

The above is in accordance to what Lamon (2002), claims. According to Cai and Sun, 2002), "*examples used in Chinese help students to distinguish between multiplicative relationship and additive relationship*" (p. 197). Learners are given an opportunity to use scale drawings so as to link ratios and proportions (Cai and Sun, 2002). The ability to recognize structural similarity and the sense of co-variation and "*multiplicative comparison*" illustrated in such a reasoning process are at the core of algebra and more advanced mathematics (Confrey and Smith, 1995 cited in Lo and Watanabe, 1997). Concurring with these academics is Vergnaud

(1988) who used the term "*multiplicative conceptual field*" referring to "*all situations that can be analyzed as simple or multiple proportion problems*" (p. 141). He continues further to identify fractions in scalar and functional methods.

Lo and Wanatabe (1997) wondered "*if the multiple relationship might be performed more easily with quantities in continuous contexts than in discrete contexts*" (p. 221). To the question of delaying ratio and proportion to later grades, the findings of Lo and Wanatabe (1997) suggest a big "NO" (p. 223). They argue that, introducing ratio and proportion earlier could help students recognize the need for non-integer division. Concurring to the fact that "*intuitive knowledge of fractions that could be used to deal with a wide range of ratio and proportion tasks*", Lo and Wanatabe (1997: 234) is indispensable, arguing that the delaying of these concepts is not necessary. Greer (1994) cited in Lo and Wanatabe (1997) pointed out that the importance of paying special attention to the transition or shift from integers to fractions/decimals when extending the meaning of multiplication and division could not be overemphasized, as their study suggests the seriousness this transition demands. This view is in line with the issue of informal whole number knowledge noted by Behr et al (1983); Kieren (1988) and Mack (1995) earlier on.

It is clear that, we need more longitudinal studies to examine the development of the multiplicative conceptual field as a whole, hence my investigative study on fractional concepts in grade 9 in a South African school. The researcher would like to highlight the intertwined literature survey thereof by noting that, mathematical concepts (e.g. fractions) are of a unique nature and they need to be treated with careful consideration of factors underpinning them.

2.6 CONCLUSION

Considering comments and views about fractions, it has become clear that there is actually a lot of conceptualization that needs to be fully developed in lower grades of mathematics learning. Without the necessary groundwork, learners become too confused when it comes to fractions like $\frac{4}{8}$. They don't figure out how possible it is to share a small number into a big number *literally speaking*, hence they treat this as 2, ignoring that it is $4 \div 8$ not $8 \div 4$, since the whole number concept is just conveniently translated.

Investigating rules is a helpful activity as it also reinforces other mathematical principles and concepts, for an example, *division by zero*, can be explained clearly through demonstration, not just as a rule as it is normally done in textbooks by just saying it *meaningless or undefined.*

It is therefore a challenge to mathematics educators, to be aware of other methods that are actually *alternative conceptions* (Mokapi, 2002), like the one mentioned by Tirosh in the division of fractions. According to the researcher, in South Africa, there is a need to improve on the effectiveness of mathematics teacher associations. Here a platform for educators to meet and share ideas/strategies on several mathematical concepts so as to give learners as much options as possible to solve unfamiliar mathematical problems, would be much beneficial to the whole nation at large.

CHAPTER THREE

METHODOLOGY AND RESEARCH DESIGN

3.1 INTRODUCTION

The study is of a qualitative nature probing learners' conceptions about fractions. It is a case involving in-depth interviews with six grade 9 learners from two selected schools. These two schools are more or less of the same background with similar learner-teacher-support-material (LTSM). This study is going to be both descriptive and explanatory. It describes and explains explicitly the kind of conceptions that the six cases have concerning fractions, from actions observed and explanations provided during informal and formal interview proceedings.

3.2 METHODOLOGY

Participatory Action Research (PAR) would be the overall research methodology that the researcher will be used in gathering data which would be situated in a Case Study (Cohen and Manion, 1994). The study draws on the works of Kemmis and Wilkinson (1998); Cohen *et al* (2000); Kemmis and Mctaggart (1997); and Denzin and Lincoln (2000) form basis of the direction of the study for qualitative research purposes.

The researcher also used interviews of students, and qualitatively analysed the responses of six cases chosen in a strategy explained in section 3.3. Questionnaire addressing the focus questions stated in the previous chapter would be given to learners to complete in a controlled classroom environment, in the researcher's presence. This questionnaire was given to learners to attempt all problems based on fractions.

3.2.1 SELECTION OF CASES

The study used a *non-probabilistic* sampling method known as *purposive* sampling. In purposive sampling (Cohen and Manion, 1994),

> *The researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality. In this way, they build up a sample that is satisfactory to their specific needs* (p.89).

According to Bell (1993), gaining access to the place where a researcher wishes to conduct a study and cooperation between the interviewer and cases to be studied is crucial. For this study, the researcher needed a familiar school so that access to the school and cooperation of both educators and learners would not be a problem. Thus, as a researcher, I purposefully chose to conduct the study in my own school and the other school that I used to teach extra classes for mathematics during Saturdays for the I-Afrika Entsha Mathematics and Science Community project (2004).

The above project was initiated by a group of prominent people around Pietermaritzbug, who were a provincial cabinet minister, a pastor, a lawyer, and experienced educators who were teaching Mathematics and/or Physical Science in schools around the town. The researcher taught mathematics in a diagnostic way so as to find out what mathematical concepts were lacking and thus hindering learners' mathematical performance at a high school level. The project was taking place on Saturdays at the then Indumiso College of Education. This initiative gave the researcher a wide range of difficulties that were experienced by these learners in grasping fractional concepts in higher classes (grade 10, 11 and 12). The study was considered for trying to address the identified problem areas in grade 9 mathematics, since this is actually the exit level from elementary mathematics.

The learners I chose have already studied basic concepts of fractions. They have been taught different forms of fractions, namely fractions as part of a whole and fraction as number (Mokapi, 2002). Mokapi also notes that such learners to be investigated should be able to show the extent to which they can integrate concrete and the abstract forms of fractions. The researcher applied this notion in the questionnaire items used for the study which addressed the research questions mentioned in section 2.2. Grade 9 was the apt level for all my focus points and wherein ideal cases were to be located.

3.2.2 CHOICE AND SETUP OF CASES

Classroom observation in focus groups looking at how learners interact to socially constructed knowledge environments (Vygotstky, cited in Fraser, 2001), will also be incorporated within my PAR since I will try to validate my results by obtaining feedback from two selected schools, involved in using identical worksheets. Worksheets given to learners will be based on studies done in Lesotho by Mokapi (2000) and Fraser (2001) and modified to focus on my research questions. In both these research projects where my study will be based, learners work will be analysed and classified according to the method used to solve the problem as well as the manner in which the solution/answer was notated. I will also use the same problem-centred approach affording the learners an opportunity to construct their own ideas and to develop a deeper understanding of the concept.

A set of questions focussing on fractional concepts was given to 30 grade 9 learners. There were fifteen learners from each school. This questionnaire included items (which will be alluded to later) on equivalence problems, mixed fractions, fractional equivalence, part-/part-whole fraction, fraction as a number, linear models and contextualised word problems involving fractions. Learners had to write out their solution on the questionnaire and provide explanations where required so as to support their responses.

The tests took place in a controlled classroom situation without any resources like charts or displays that could assist or influence learners' answers. The researcher conducted and invigilated the tests, so as to ensure that even cheating or adult assistance could not take place, and thereafter collected the questionnaires himself. Initially, the researcher thought that he would select the 6 cases (for interview purposes) on merit. This initial plan could not work accurately when piloted. The researcher opted for another strategy explained hereunder.

Learners were classified according to their responses into three categories. This idea was derived from Mokapi (2002). The first category included learners with completely clear and correct responses and explanations. The second category included unclear responses and explanations. Lastly, the third category included incorrect responses and explanations. Two learners were chosen from each of the mentioned categories. This was done so as to have a typical range of responses based on learner responses rather than performance to the overall test.

Since the cases come from the 30 learners, it manifests itself that they were 6 cases altogether that would be considered for "*clinical interview*" (Truran and Truran, 1998:61). They explain a clinical interview as a set of questions, some prepared, some following from the subject's responses to previous questions. During the clinical interview, the interview is *"free to modify the sequence of questions, change the wording, and explain them"* (Cohen and Manion, 1994:271).

According to Posner and Gertzog (1982) and Ginsburg (1981), Piaget is a pioneer of clinical interview method (cited in Mokapi, 2002). It is noted that a number of researchers in mathematics education have adopted this method so as to probe learners' conceptions about mathematical knowledge (Ginsburg, 1981 cited in Mokapi, 2002). These are the justifications for my choice in adopting this method.

3.2.3 VALIDITY AND RELIABILITY OF RESULTS

Bell (1993) argues that it is possible that to a certain extent the researcher might exercise biasness during the interview proceedings, despite their carefulness.
He indicates that, *"researchers may be biased because as human beings, they are never neutral or explicit about their assumptions and orientations"* (cited in Mokapi, 2002: 58).

McMillan and Schumacher (1993:157) argue that *"bias is a form of systematic error"*, a factor that influences the results and undermines quality of the research, in particular credibility of the results. However, Bell (1993) indicates that, one can get reliable results provided s/he uses a valid instrument. This is because, though a test may prove to be highly reliable, it may at the same time be highly invalid, since reliability does not imply validity (Huysamen, 1993). Thus, the researcher was very careful not to overgeneralise the validity and the reliability of the results.

According to Halldórsson and Aastrup (2002) citing Erlandson *et al* (1993), there are three issues that are to be stressed when evaluating the research impact. These issues are:

- *Truth value*: referring to that, credibility must be guaranteed;
- *Application*: appropriate to intended audience; and
- *External judgement and neutrality of findings*: enabling cross checking of the findings to be possible.

The researcher would consider the above issues so as to make informed inferences about the results. This is because *"to validate means to check, to question and to theorise"* (Halldórsson and Aastrup, 2002: 329). Validity and reliability focus on many aspects that cannot be exhausted in this study, however, the researcher would like to mention that *"craftsmanship"* Halldórsson and Aastrup (ibid) is being explicitly aware of and maintaining a critical distance towards certain interpretations. This is done so as to establish the truthfulness of the study undertaken.

3.3 THE QUESTIONNAIRE

3.3.1 CATEGORIZATION OF ITEMS

The questionnaire has 25 items[1] altogether including sub-items divided into two sections, as **Section A-Concrete fractional concepts** and **Sections B-Abstract fractional concepts**. The questions were set in such a way to deal with the research questions given in my previous chapter.

[1] To have a look at all the 25 items and how they are phrased, refer to Appendix A.

These items were mentioned in section 3.2.2 above, looking at how each of the six the cases would perform on each item in comparison to other items. This would be more of criterion style of assessing learner performance to each item. Since the items are testing different fractional concepts, it became important for the researcher to note that were the learners performing the same in similar or related concepts, or there were diverse responses. This comparison would play a significant during the discussion of the findings.

3.3.2 PURPOSE OF EACH QUESTION

Questions 1.1 a), 1.1 b) and 1.2 address the issue of continuous and discrete fractions. The continuous model permits repeated and infinitely varied subdivision, while the discrete model permits dealing and counting as strategies with less obvious emphasis on the whole (Pitkenthly and Hunting, 1996). These items investigate if learners are equipped with skills of relating part-whole and part-part fractional concepts in equivalent relations. Items also challenges learners' capability of being able to relate fractional concepts referring to the same quantity used in different thought processes and contexts. Mokapi (2002) calls this typical idea a concept of equivalence represented in concrete models.

In answering question 2, learners had to check if the parts of a continuous whole were equal. Learners had to check this mathematical principle first and thereafter be able to give the answer of 3/5, otherwise 3 of the shaded unequal parts in a whole of 5 unequal parts does not constitute a fraction of 3/5, in one distractor used intentionally . According to Austin *et al* (1999) and Diskson *et al* (1984), learners tend to ignore the notion of equality of parts in continuously represented fractions. I wanted to check that this is also true to with my set cases.

Question 3 is concerned with the use of mixed fractions incorporated in the use of improper fractions with two continuous equal wholes. There are two rectangles each subdivided into five equal parts. In one rectangle all the parts are shaded, thus making the shades in the first one to be 5/5, that is equal to 1. While in the second one the shades make 3/5. Learners can also view this item as continuous whole of 8/5. Kerslake's (1991) study cited in Mokapi (2002) reveals that learners often treat mixed fractions represented continuously as proper fractions. This implies that, it is possible for learner to treat this fraction as 8/10.

In question 4, the focus is on the concept of half as applied in most cases in our daily lives. I used bread as a common practice in most "*tuck shops*" in townships and rural areas (focussing on my chosen cases). Hence, learners could associate with this item very well on what it is all about. Two unequal parts of bread are shown as Part A and Part B. Learners have to say whether the two parts are equal or not and why. From the school perspective concrete fractional concepts are not compared like this, but one or more parts are shaded so as to compare the shaded to the unshaded. So, this is unfamiliar to learners from the schooling perspective, but they are used to it in their everyday life. Austin *et al* (1999) argue that the manner in which half is used and talked about in everyday life practices and activities, encourages learners to perceive "*half as big and small*" (p. 39). This item could check if learners can be able to note that there is no half in these two parts, since pieces are not cut equally. During clinical interviews, I intend to shade one part and find out if there could be any changes in a case's response. This is because; at school learners are mostly given shaded parts to write as fractions.

Question 5.1 requires learners to find the whole given part length and its two part fractional formats. In the other part only the fractional format is given and learners are supposed to calculate the part length. Thereafter, the two part lengths added would give the height of the tree. There are many informal methods that could be employed to answer this question. According to Kerslake (1991:89) cited in Mokapi (2002), "*it is common for learners to find answer by using informal methods that are illogical and use imprecise language when answering problems of this kind*". This often leads to violating some mathematical rules and thus leading into incorrect answers. It would be interesting to note how the cases would respond to this question item.

Looking at question 5.2, I noted that usually learners are asked to interpret the already shaded in concrete models so as to give the numerical value of the shaded compared to unshaded plus shaded in total. I tried the opposite so as to see if learners are able to associate number representation to concrete fractional formats of objects. It is interesting to recognise that what they (learners) are supposed to shade is 9/24. This is related to questions 1.1 a) and 1.1 b) as they are all focussing on fractional equivalence. This idea came from noting that Austin *et al* (1999) and Dickson *et al* (1984) indicate that there is a tendency for learners to ignore the notion of equality of parts in continuously represented fractions. I would to see if they (learners) concur with these academics findings.

Question 5.3 is actually linear model of fractional representation (Gearhart *et al.*, 1999). This is where only the length is important not the area (usually shaded). This is a very important concept since it introduces learners to the "*discipline of conceptual inquiry*" (Natational Council of Teachers of Mathematics (1999) standards cited by Gearhart *et al.*, 1999). All what learners need to do is compare 3 over 15 and simplify as 1/3. It is possible for learners to look at the number of windows next to the car in comparison to the total windows of the building. That would be an incorrect assumption since there are no windows in between but the double arch door. The whole is not always referring to the object in its physical state. Wu (2001) gives an account of that a whole itself can be a part (fraction) of a unit.

In question 5.4, estimation is being introduced. A very important concept of mathematics in the senior phase in the Revised National Curriculum Statement grade 9 (RNCS), learning outcome #1 (DoE, 2002). This question addresses the notion of continuous fractional concept as learners are to compare part less that ¾ to the part left if it possible to be greater or less than ¼. Learners tend to ignore estimations and focus on exact numbers as the concept of estimation is too abstract for them. It is now emphasised in the new curriculum approach, Outcomes Based Education focussing Skills, Knowledge, Attitude and Values (National Curriculum Statement, 2003).

Question 6.1, adapted from, Smith III (2002: 8) focuses on fractions using the area model, Wu (2001) suggests the following relations: Part-Whole relations, Part-Part relations and Equivalence relations, which were also noted by Mokapi (2002). In this question, learners are to use the part-part relations. Learners have to recognize that not all shapes represent a fraction as parts must be equal to be compared to others in continuous fractions. Sinicrope *et al* (2002), Gearhart *et al* (1999) and Wu (2001) are all in agreement to the view of practical demonstration, not just *pen to paper* writing and claim to be mathematically viable, without any evidence of learner actions done practically. Luthuli, (2002, personal communication) also argues that *practical work*[2] can be very useful in mathematics education especially on learning and teaching of fractions. It is possible for learners to choose B., C. or D as they are all referring to 2/3, if one ignores the area model. This question ties with question 2, where if a learner sees 3/5 to be represented by b), then obviously, there is a misconception of that, equality of parts must be there so as to form a fractional part of a whole (Mokapi, 2002).

Questions 6.2 and 6.3 focus on the concepts of fraction equivalence in a different context from questions 1.1 and 5.2. I included these questions so as to validate the cases' responses of same concepts asked in different ways. As a researcher, it is important to consider validity and reliability constructs in instruments use in research so as to ensure that inferences drawn on data are valid (Fraenkel and Wallen, 1990). They argue that,

> *-reliability refers to the consistency of scores obtained from one administration of an instrument to another and from one set of items to another, and validity refers to the truthfulness of the results (p. 127).*

Objective interpretation as opposed to subjective interpretation is of paramount importance to the study (Maxwell, 1992). That is why I thought it would be of assistance to include more than one item referring to the same concept so as to draw

[2] In Chapter Two, Practical work is explained in details with illustrations.

informed inferences from a wide scope of the same concept being asked in different contexts.

In question 6.4, learners are engaged in the concepts of scaling. This is one of fundamental points when studying graphs so as to plot a fraction on a Cartesian plane. I have noticed that even in higher classes (higher than grade 9 that is under study in this research), learners struggle to plot mixed fractions on a number line, and thus they get incorrect estimations of the co-ordinates. Mokapi (2002) used the same concept and noted that some learners associate fractions to decimals. In the clinical interviews, I would ask if learners can be able to relate the decimals between 1 and 2 to the fractions in the same interval, and hence give point P as both fractional form and decimal form. This question is also in line with the number relations are introduced and estimation plays a significant role in decimals and fractions (DoE, 2002).

Question 7 ties very well with question 6.4 on a different conceptual notion. In this case learners are asked to compare if they can choose the smallest fraction out of the four given. Hart's (1989) study cited in Mokapi (2002) reveals that learners have a tendency to treat the components (numerator and denominator) of a fraction as separate entities. So in answering this question it possible for students to say ½ is the smallest, since 1 and 2 are both the smallest in numerator and denominator respectively. This is common problem as we see sometimes learners adding numerators and denominator to find the sum of two or more fractions (Behr, 1997). Luthuli (2002, personal communication) argues if this approach is not an "*alternative conception*" of manipulating fractions differently as teachers do it to find an aggregate result of scores for different subjects of a learner so as to write a school report. The next item would elaborate this notion fully as learners are now embarking on, a *"conceptual jump"* (Lamon, 1999: 21), to formal manipulation of fractions in addition and subtraction.

Question 8.1 focuses on the actual abstract mathematical manipulation of fractions in a schooling situation. Concrete fractional concepts knowledge should have been drilled very well before learners are engaged to these types of abstract fractional concepts. Cai and Sun (2001) argue that the algorithms and terms like "numerator" and "denominator" should be delayed to later years as that confuses learners. This notion concurs with Smith III (2002) but is contrary to Lo and Wanatabe (1997) where they argue that concepts like ratio and proportion should be introduced early to learners so as to make sure that they are aware of the mathematical fact of an integer divided by another integer. This item would therefore diagnose if learners are familiar and ready with algorithms and can also manipulate fractions following rules or informal knowledge which can be either be slips or alternative conceptions, if not misconceptions (Mokapi, 2002).

Question 9 tries to validate the cases' responses to question 1.1; 1.2; 5.2 and 6.3, as all these questions are based on fractional equivalence (Gearhart *et al*, 1999; Irwin, 2001). If participants could perform correctly to all these related items that would imply that they are aware of equivalence relations in fractions and hence the *"multiplicative thinking"* (Lamon, 1999; Irwin, 2001; Goos, 2004) as mentioned in the previous chapter, is nurtured. Since this question is, *"open but closed"*, it will play a significant role in translating quantities to different representations (e.g. ratios).

Question 10 involves division of fractions where the first fraction is carefully chosen so as to be exactly divisible by the second one. This question was chosen so that, during the clinical interviews, meaning: -*"free to modify the sequence of questions, change the wording, explain them or ask further questions"* (Cohen and Manion, 1994: 271), I will be able to check if learners are aware that there are actually other methods that could be employed to divide fractions alternatively. These strategies were noted in my previous chapter (Literature Review) in this thesis adapted from, Sinicrope *et al* (2002), Wu (2001) and Luthuli (2003, personal communication) where not only the *"invert and multiply"* (Litweller and Bright, 2002), rule is used but also other methods are appreciated as they actually *"make sense"* to the learners' minds. Litweller and Bright (2002) argue that:

> *Unless we can actually point to where we invert and multiply in our pictures, children will still see the IM rule as an unexplainable and mysterious short cut to fraction division* (p. 247).

Question 11 is straightforward terminology of fractional numbers with respect to its place value in the denary number system. I will not dwell much on what I mean by denary since the study of number systems is a complex one on its own and beyond the terms of reference of this study. However, it is important to say that, by denary number system I mean a *"number system of base 10"*, as there other bases that could be used to analyse number theory (e.g. binary number system used in computers). Cases are challenged to write the given word-fractions in number formats. Lo and Wanatabe (1997) claim that terminology should be introduced early to learners so as to avoid confusion when it comes non-integer division as Greer (1994) cited by these authors also concurs with these authors by claiming a smooth transition from whole numbers to fractions at an early stage. Cai and Sun (2002) are in agreement to the introduction of terms/concepts like "*ratio and proportionality*" terminology early in the schooling years. Hence, the researcher asked these number fractions to diagnose if learners are aware of them and their meaning would be interrogated during clinical interview sessions.

Although question 12 and 13 could result in a *language barrier* problem where code-switching (Mokapi, 2001) during clinical interviews could be used for the cases so as to find out whether it is a language or a mathematical problem that might hinder learner performance in these items. The researcher could not dwell much on "*language issues*" in learning and teaching mathematics as that is beyond the scope of this research and could in fact be a study on its own. The researcher decided to include these items for the purposes of giving learners an opportunity to analyze a given scenario and be able to apply mathematical "*symbol systems*" (Mack, 1995: 436). Hiebert (1988), and Hiebert and Carpenter (1992) cited in Mack (1995).

These questions could help the researcher to arrive at informed inferences on how learners construct their own knowledge (Carpenter and Moser, 1983; Riley, Greeno and Heller, 1983 cited in Mack, 1995) as these authors argue that knowledge appears to come from learners realistic experiences. The researcher would affirm or refute this assertion (considering the cases chosen) in the next chapter, where data analysis will be done so as to arrive at informed conclusions with regard to the researcher's cases.

3.4 CONCLUSION

The study will be analysed by coding the data source so as to make sense of participants' responses. Table 4.1 in the next chapter (Data analysis) shows how data was coded. After analysing data according to the set criterion of three categories (as mentioned earlier in this chapter), the researcher will choose two cases per category for *clinical interview* purposes. These cases will be interviewed so as to get some in-depth responses on how they perceived the items. During that period, the researcher would have a session with each interviewee in an unstructured interview. The following chapter elaborates participants' response and also what happens in each of the interview sessions.

CHAPTER FOUR

DATA ANALYSIS AND FINDINGS

4.1 INTRODUCTION

This chapter provides a descriptive and informed analysis of data sources gathered from the cases of two schools under study where thirty grade 9 learners were a sample and six of those learners took clinical interviews. The clinical interviews of the six cases' responses are attached in Appendix B. The researcher's analysis shows fractional conceptions displayed by these cases. The conceptions highlighted are in relation to part-whole concept, the concept of half, the concept of position of a mixed fraction in a number line and concepts observed in operations within fractions. The study classified these concepts into two sections, *viz.*: **Section A-Concrete fractional concepts** and **Section B-Abstract fractional concepts** (see Appendix A). In the latter section, there were also contextualised word problems.

4.2 STUDENTS' CONCEPTIONS

The study envisaged to seek answers to the research questions alluded to in chapter two. The researcher decided to first get a *Gestalt* view of both school A and school B learners' conceptions. Table 4.1 gives raw data with respect to the given column headings in line with the categorization of responses mentioned in chapter three. The study then analysed clinical interview responses. Each participant's response was taken into cognizance, so as to make sure that a list of common misconceptions is noted, thus answering research question number 2 (see section 2.2) mentioned in chapter two.

Table 4.1: Raw analysis of learners' responses per item (MALATI project strategy).

Items[1]	Correct Responses School A (n = 15)	School B (n = 15)	Most common learners' misconceptions (Percentage given in brackets respectively)
1.1 a)	15	14	Both fractions refer to the same thing. (3%).
1.1 b)	15	15	
1.2	10	9	¼ is bigger (37%).
2.	15	15	None
3.	8	6	Multiplying 5/5 by 3/5 (20%). Treating the blocks as separate entities (33%).
4.	12	10	Fractions associated to half (big) and quarter (small) (27%).
5.1	0	0	Just adding the two fractions, 3/5 + 2/5 = 5/10, resulting an *alternate conception* (60%)[2]
5.2	4	2	Just shading 3 parts (65%). Multiplying both top and bottom by 3 (15%).
5.3	12	8	Multiplying 3 by 15 = 45 (33%).
5.4	12	11	The part not shaded is equal to ¼ while the shaded part is ¾ (23%).
6.1	15	15	None
6.2	13	12	Confusing equivalence to 4/10 as the representation of 2/5 (17%).
6.3	14	13	Taking option B, which is actually 2/10 (10%).
6.4	12	11	Choosing ¼ since the point P is close to 1 and the numbers go up to 4 (23%).
7.	11	7	Taking ½ instead of 1/3 as the smallest by considering bigger denominator (40%).
8.1 a)	0	0	Just adding/subtracting the given numerators and then denominators (100%).
8.1 b)	0	0	
9.	8	7	Multiplying both numerator and denominator thereafter have 3/12 and 4/12 (50%).
10.	7	8	Dividing the numerators and denominators; this is actually an *alternate conception* (50%).
11.1	15	15	None
11.2	15	15	None
11.3	15	15	None
11.4	14	13	1/3 is three quarters??? (10%)
12.	0	0	No idea at all (100%)
13.	0	0	Only one case showed 240 ÷ 5/8 (3%).
	65%	**59%**	

[1] Refer to Appendix A for the details of each item.
[2] The other 40% did not even attempt the item at all.

4.2.1 DATA ANALYSIS OF THE TWO SCHOOLS

Generally, the overall performance of school A and school B was 65% and 59% respectively. This is actually an indication of that; there is generally an average understanding of fractions in schools A and B despite the contextual factors (like background and language issues towards the acquisition of knowledge). The worst performed (0%) items were 5.1, 8.1a), 8.1b), 12 and 13. If one can check these items (see Appendix A), they are actually demanding that a learner be familiar with problem solving where there are fractions involved and rules for mathematical operations (addition, subtraction, multiplication and division) in fractions, which are actually at the level of grade 9 content coverage.

Considering the scope and sample of the study, it is not possible for the researcher to generalize the results, but the overall impression noted could suggest that there were challenges that could hinder learners' progress towards mastering sections involving fractions at a secondary school level of mathematics. This assertion can be justified by looking at each item in the next paragraphs and looking at how the 6 cases that were carefully selected (looking at their significant uniqueness in responses) from 30 participants responded to the researcher's clinical interview sessions (see Appendix B).

In Table 4.1, items 2 and 6.1, all participants got them correct. So the cases have grasped the concepts of space and fractional comparison through using the shading approach. On the contrary to this notion is the fact that, in item 5.2 only six respondents got the items correct, which to the researcher was just the reverse thinking of what was done in items 2 and 6.1.

Further to the same concept of space and fractional representation is in items 5.3, 5.4 and 6.2. The performance of the participants (combined) was generally poor when compared to that they got items 2 and 6.1 correctly. These items had to do with the development of the rational number, together with a notational system for representing fractions (De Windt-King and Goldin, 2003). These are important concepts for later years in mathematics at school and beyond, so if they are not well developed and mastered, there could be negative consequences, or rather detrimental to solving complex mathematical problems in later years (research question number 5 checking the impact of fractional concepts).

As alluded to earlier, items 5.1, 8.1a), 8.1b), 12 and 13 were the worst performed. However, in item 13, one case tried to get $240 \div 5/8$ instead of $240 \times 5/8$ at least to start of with. The alternate conception was common (95%) where addition of numerators and then denominators was the order of operation these participants employed for the addition and subtraction of fractions in these items, which is questionable as it looses the actual meaning of fractions when compared to ratios.

It was interesting to note that in items 1.1a), 1.1b), 2, 6.1, 11.1, 11.2 and 11.3, there was an overall significant high performance (97%). A misconception that was noted is that of 1/3 being taken as three quarters. This item is noted to in Appendix B (Learner 6). The other item that is worth referring to, is item 10. In this item about half (50%) of the participants got the correct answer by chance (not actually always working). This is due to the fact they just divided numerators and then denominators, of which it happened coincidentally. If the second fraction could not divide the first one without leaving a remainder it would not have been so easily.

This item was deliberately put to check this *alternate conception* noted by Tirosh(2000). This notion is relevant in addressing research question number 4 looking at learners' informal knowledge.

It is also worth mentioning that the concept of equivalence (items 6.2 and 6.3) was well attempted with about 73% getting it correctly according to the items used. The concept of understanding fractional representation in a number line (item 6.4) was not well performed (with 23% getting it incorrect) at an expected level of grad 9 mathematics. Grade 9 learners are supposed to know these concepts as a number line is actually introduced from primary school mathematics. The researcher could not expand to each an every question. The items are actually related focusing on trying to find conclusions to the research questions. This leads to the clinical interviews analysis.

4.2.2 ANALYSIS OF CLINICAL INTERVIEW RESPONSES

The questions were chosen to elicit learners' conception regarding fractions as parts of wholes. The items used investigated practical and/or concrete operational stage of fractional thinking with regard to drawing differences and similarities of discrete and continuous fractional models. Further to these two models explanation of fractional knowledge is, De Windt-King and Goldin (2003: 29) who eloquently suggest that fractions provide a "*spatial extent*" or <u>continuous quantity model</u> and also on the other flip side of the coin, the "*kids in our classroom*" can be characterised as a "*set*" or <u>discrete quantity model</u>.

Items 1.1 and 1.2 were related in the sense that a learner had to figure out if there is an equivalence relationship of both discrete (1.1.a) and continuous 1.1 b) fractions. The following excerpt of Leaner 4 was noted as to answer research question number 4 (focussing on learner's informal knowledge) and the extent to which their (learners) informal knowledge could be utilised:

I : You said that 1/4 and 3/12 are all quarters of the shaded fractions. Why?

L4 : By taking 1 in 4 things repeatedly 3 times in 12 things makes the same fraction of parts taken at time.

I : What would have happened if I had 4/12? Would it still be equal to 1/4?

L4 : No, Sir.

I : Why not the equal to it?

L4 : It is because for it to be the same you will be taking equal things at a time to make the same fraction.

I : How?

L4 : Using groups 1, 2 and 3 with the same starting number of things, take the same number of things in each group like this one, 3 groups taking 1 thing in each group for 3 times you will end up with 3/12 if there were 4 things in each group.

I : Ok, that is correct.

L4 used a method that is not following routine mathematical rules of looking for the highest common factor in both the numerator and denominator and then divides by it so as to simplify the fraction. This method is actually using some informal knowledge alluded to in the literature review (Mack, 1995). L4 also gave an interesting response during the writing stages of the research as Ntenza (2006) suggests, "*It is that they are all the quarters of the fractions*" (L4, 2006: 1). During the interview, L4 explicitly gave a clear response to elaborate the statement, if not focussing on the English grammar used.

The other responses were very ambiguous and not to the point. For an example, L5 could not clearly explain question 5.1 that involved a tree. She just said that she did not understand or know the question even if asked during clinical interviews:

I : In question 5.1, we have this tree problem. How did you do this problem?

L5 : I saw that here (*showing the 8 m length*) in the stem, 2/5 + 3/5 = 5/10.

I : Is this how fractions are added?

L5 : I really don't know, but I was trying.

It is evident in her response in the question of writing the height of the tree as 5/10, which is clearly not showing any thinking process that took place. This performance concurs with the results from Table 4.1, where all participants could not get the solution to the tree problem. This question was addressing the very first focus question on problem solving using practical scenarios.

Item 2 comprised of three options, A, B and C. Options A and B tested students perceptions regarding the notion of equality of parts (Mokapi, 2002) as well as the naming of part-whole relations (Cai and Sun, 2002). In A, the rectangle had all the parts equally subdivided whilst B and C had ambiguous subdivisions. It is interesting to note that all the six cases got this item correct. That was showing some sense of understanding equality of parts relations which does not concur with the discoveries of Mokapi.

Another result that showed a significant milestone in these clinical interviews is that of item 11 where most (90%) participants got almost everything correct. Here is one excerpt from L5:

I : In question 11 you got all the correct answers. How did you do them?

L5 : I just looked at the names used and then wrote them as number fractions.

... ...

I : Thank you. You performed to your best level to most questions.

Although, this was also an informal knowledge, but it came with the desired outcomes. This suggests that sometimes informal knowledge work, but it needs to be carefully checked that it does not work by chance. Item 5.4 was not very clear to some (23%) of the participants, as a result they came up with some alternative conceptions. L4 is used as one of the examples to note these conceptions:

I : In question 5.4, you said that the other part is greater than 1/4. Why?

L4 : By looking at it and consider that 90^o would have been straight here (*showing on the diagram*) to form a part equal to 1/4. So as this line is after 90^o, it is therefore showing a part greater than 1/4. (*alternative conception*)

I : Is 90^o the same as 1/4?

L4 : If you take 90^o times 4 you get 360^o, which is a complete circle. So then 90^o could represent 1/4.

This was a very good relationship of relating a revolution angle to the fractions. This learner had an insight of what was required in a different perspective altogether. The researcher was very impressed with this idea.

Item 6.4 did not come up with a very good performance (only 23%). However, L3 managed to get it right during clinical interviews. The following is her excerpt:

I : In question 6.4 you had the correct answer as choice B. How did you know that other choices are not correct?

L3 : D is wrong, because it's not after 1, but before 1. But A should have been in between 1 and 2. Option C (*after taking a long pause*), I cannot explain it, but it's not a better estimation.

Here is another participant's (L5) response to the same item 6.4:

I : In question 6.4, which fraction is better close to 1 considering the point P?

L5 : I really didn't understand this one.

I : But, what's happening here is that you can see that P is after 1. Why did you choose A not B, C or D.

L5 : If P was at 1, I would have said 1/4 is the point P. (*misconception*)

Looking at the two responses, there are several questions that could be asked, but are beyond the scope of this research. With regard to the study on hand, focus question number two (*investigating concepts*), is addressed by this item and the performance speaks volumes about the learners' conceptions on a fraction represented on a number line in the correct fractional representation.

The above excerpts could also suggest the impact that the incorrect conceptions learners hold about fractions could result into great difficulties if not addressed at earlier stages. This comment is based on highlighting the researcher's focus question number 5 (impact of concepts to later years).

The next excerpt would be highlighted because the researcher found it also relevant to focus question number 1 (practical problem solving) from a quantity point of view. This idea is as a result of Luthuli (2002, personal communication) about practical work relevance to fractional problem solving. Here is the excerpt:

I : In question 7, which fraction is the smallest?

L5 : (*After a long silence with some signs of thinking*) Uh! I really don't know these things. I don't understand what's going on here.

I : Let's take this paper. In this paper (*to be folded into two equal parts*), how can I get half?

L5 : By taking the middle.

I : How?

L5 : (*silence*)

I : Folding it like this? (*showing the half folded paper*)

L5 : Yes.

I : What is represented by each part?

L5 : Half.

I : Now, let's divide the same paper into 3 tin

(folding the paper looking at L5 showing some sense of disbelieve)

I : What do you think has happened? Half was here (*showing the paper*) where is 1/3 of the paper?

L5 : Somewhere less than half.

I : So 1/3 is less than ½.

L5 : Yes.

I : Now, if you look at all the given options of fractions which one would be smaller?

L5 : I can say that A would be smaller.

Here is another discourse analysis again using just only the paper folding with L6.

I : In question 7. You chose ½. Why?

L6 : I saw that ½ is the smallest compared to others.

I : How? Is 1/3 not smaller than ½?

L6 : No. I think 1/3 is bigger than ½.

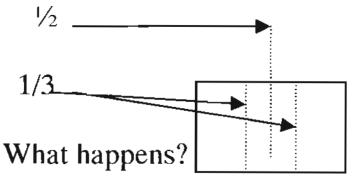
I : Do you think so?

L6 : Yes.

I : Let's use a paper folding. If I fold the paper (*showing it to him*), I get two parts both equal to ½ of the paper, Do you see?

L6 : Yes.

I : Now, let's divide the paper (*shown*) into three parts. What happens?



L6 : ½ is bigger than 1/3, Sir.

I : Which one is smaller now?

L6 : This one (*showing with his hand 1/3 part*).

I : So, the smallest is not ½.

L6 : Yes.

I : Now, if the paper is folded 6 times. What would be the smallest fraction, ½, 1/3 or 1/6?

L6 : The one of 1/6 would be smaller.

The above two excerpts *explicitly and judiciously* prove the value of practical work in early stages of mathematical concepts introduction. Practical work is in fact often time-consuming when employed to elaborate abstract concepts in question, but the benefits thereafter are worthwhile, as the time spent with learners to fully grasp concepts, gives better writing in mathematical concepts in later years (addressing the researcher's focus question number 5) of mathematics schooling, hence "*writing to learn*" (Ntenza, 2006: 321) in mathematics education.

In items 8.1a) and 8.1b), the researcher noticed that the participants performance was appalling, in fact a disaster (check Table 4.1) as there was no learner that actually got these items correct. This is really a course for concern. Let's look at the interview with L6:

I : Now, in question 8.1 (a), how did you do it?

L6 : I said 4 – 1 + 1 = 4 and then 5 – 2 + 15 = 18, then I got 4/18, Sir.

I : So you subtracted or added numerators first and then denominators and wrote the answers as numerator over a denominator?

L6 : Yes, Sir.

I : How did you do this one (*pointing at 8.1 (b)*).

L6 : I added the bigger numbers (*referring to 2 and 3*), got 5 and then numerators and denominators got 4/8.

I : But 4/8 can be simplified as what fraction?

L6 : It is the same as ½.

I : So the answer could have been what?

L6 : 5½, Sir.

I : Ok, let's go back to that one of 8.1 (a). You subtracted or added numerators and denominators.

L6 : Yes, Sir.

I : Is this how you add fractions, doing the tops and bottoms?

L6 : No, actually it was the first time that I got something like this.

I : You did not do these fractions at primary school?

L6 : No, Sir.

I : But you did other questions very well, that's good.

That the participant has never been taught (as she claims so) these fractions before, would be beyond the scope of this study to investigate the truthfulness of such a *shame* statement. The entire research noted with regard to research focus question number 2 (concepts learners hold in grade 9) is that, there were not at the acceptable level for progression to higher mathematics at a secondary school. If there are more learners like these in our schools that something has to be done with our primary school mathematics learning and teaching.

In items 12 and 13, there was also a 100% of either totally incorrect or not attempted at all. When the 6 cases were interviewed on why they could not get the solution to these problems, this is how some of them responded:

I : In question no. 12, you didn't even try, why?

L6 : It's because. I did not know and understand the question. But if I got an explanation, I would have tried it.

I : Ok, but you tried this one (*showing no. 13*).

L6 : It's because there was still time.

I : What is meant by 5/8 spent? If you spent 5/8 of the whole, how big of a whole is left in fractional form?

L6 : I think it is 2/8. (*misconception/slip*)

I : Thank you for allowing this interview to progress smoothly. Your responses are guaranteed anonymity.

… … ….

I : That was good. What about questions 12 and 13?

L5 : I really don't know what I was doing in these. I can't even explain.

I : Thank you. You performed to your best level to most questions.

… … …

I : You did not even try question 12 and 13. Were they difficult or you did not understand the questions?

L3 : I did not understand the questions. The English used was not familiar to me, that is why I did not even try it.

From the above excerpts, the researcher noted that there were a lot of contextual factors involved in attempting to solve these problems from the learners' perspectives. One of the factors is that of a *language barrier*. Since the participants were English second language speakers, it was not very clear whether that it could be generalized that they did not understand the questions (*language barrier*) or the mathematical concepts involved in answering these fractional problems (*mathematical concepts*).

The researcher noted that, common to most (4/6) cases is that they "*did not understand*" the question at all. To check if these are actually facts or just a limitation of their mathematics skills could be beyond the scope of this research to validate or draw inferences, since the language issue *per se* is a very broad one when it comes to the dynamics involved in the Language of Learning and Teaching (LoLT) which are very debatable from the South Africa perspective since there are 11 official languages.

In Table 4.1, only one participant (L2) tried to do the item like this 240 ÷ 5/8 during the worksheet stage. This was a good try, but only one concept was missing, just to use multiplication instead of division. The participant had a limited clue of these problems as she was not very clear on how to apply the algorithm. The participant that drew the researcher's attention in this item is noted in the following conversation:

I : In question 12 you were trying very hard, but you did not get the correct answers. You have 1/3 + 1/4. Can you explain your method?

L2 : I really could not understand the question. But now that you have explained it to me I can see that it is easy.

I : You tried question 13 very well. But it's like you did not understand it. Can you explain what you were thinking?

L2 : All I can say is that I had to divide the money left. For an example having R2, you use R1, you can say R2 – R1 = R1. (*Showing confusion*) We actually do not do these fractions. That's why I could not do well.

4.3 CONCLUSION

The performance of the cases (L1, L2, L3, L4, L5 and L6) was generally good but with some limitations to important mathematical concepts like addition/subtraction of fractions as they have some knowledge gaps associated to their low performance to most of the questions. That the majority used additions/subtraction of numerators and then denominators shows that, misconceptions are rife within these learners and they need to be identified and rooted our before it's already too late.

The researcher was very impressed with some of the informal knowledge that the participants possessed. For an example the idea of simplifying 4/12 to 1/3 was very well put. Another interesting point that is worth mentioning is that of using revolution angle to associate fraction ¼ to 90^{0}, this was a good approach. It shows that some learners are able to integrate geometry to algebra/arithmetic, which is actually a strategy of vital importance to understand mathematics in totality.

Lastly, on a low point, the researcher was very concerned with the fact that all participants could not do items 12 and 13. There is really a need to check if these learners had a language or mathematical conceptualization problem. This would be detailed further in Chapters 5 and 6 during discussions and implications for further research.

CHAPTER FIVE

DISCUSSION OF THE FINDINGS

5.1 INTRODUCTION

This chapter discusses the findings by relating to the research focus questions of the researcher that were also referred in chapter four as initially listed in chapter two. The researcher also compares the literature claims that were noted from chapter two with the findings that came up during the worksheets stage and clinical interviews.

Since the study was both of a quantitative and qualitative nature, it is imperative to highlight the results with regard to the methodology, i.e. Participatory Action Research, Kemmis & Mctaggart (2000) that was noted in chapter three. The learners' conceptions that were found by various researchers (Mokapi, 2002; Fraser, 2001; Irwin, 2001; Kennedy, 1997, Koomen, 2001, *et cetera*) are also discussed for the purposes of making sense in drawing conclusions and validating the outcomes of the current study from a different milieu and focus group.

5.2 LEARNERS' CONCEPTIONS

Drawing from the Vygotskian view, learners' conceptions with regard to fraction demand some *scientific knowledge.* This refers to the fact that for learners to grasp the concept of fractions, it could not be easily achieved through just quick fix strategies, but learners become conscious of both *spontaneous and scientific concepts through mediation* (Mokapi, 2002). Concurring with this claim is Irwin (2001) in suggesting that, *the system of fractions is neither simple to learn nor general to understand in both primary and secondary school students.*

The researcher noted these scientific concepts when analysing participants' responses using Table 4.1 (see chapter four). That no learner could do items 12 and 13 correctly. This is a direct implication of lack of integrating mathematical knowledge to practical problem solving. The first focus question was investigating these conceptions in problem-solving involving fractions. It became evident that, there was very little mathematics that these learners could use to solve these fractional problems.

There was a need to check if these learners hold any informal concepts (i.e. focus questions number 4). It came up that about 90% of these participants were adding/subtracting numerators and then do the same with denominators and hence write the sum as a fraction. The researcher was concerned with this notion as it is actually an *alternate conception* (Tirosh, 2000). It is in fact a direct *mimic of multiplication of fractions* (Wu, 2001). These participants could not relate fractions to number representation. Ndlovu (2003) concurs with this study's result that since multiplication of whole numbers makes the number bigger; learners tend to take the same method for addition/subtraction of fraction as well.

Interesting in this study was the *role of practical work* (Luthuli, 2003, pers. comm.) that was noticed in two clinical interview sessions (L5 and L6) from chapter four. The researcher used a *paper folding* method just to explain the fact that, the greater the denominator the smaller the fraction (Ndlovu, 2003). The researcher noted that the cases used in the study could not do the worksheet problems correctly, but during the clinical interviews when paper folding was used by the researcher (Participatory

Action Research); they managed to get the correct solutions (see excerpts of L5 and L6).

The value of practical work is also noted in the study conducted by Maharaj, *et al.* (2006), that the first step should be the use of familiar concrete experiences so as to *develop new abstract concepts and their symbolisation*. This assertion is in line with the focus question number 3 looking at the suggested literature views. The use of traditional normal teaching methods without any practical demonstrations is doing more harm than good. That is evident in the study as most of the learners could not do the reverse process of shading a fraction on a given grid (item 5.2), but they were able to do items 1.1a), 1.1b), 2 and 6.1. The study revealed that participants are only used at writing the part-regions as fractions not representing the fraction in using the given grid (item 5.2) where all but one case could not get the shading correct. This leads the discussion to the next section where mathematics education is related to and appreciates learners' informal knowledge.

5.3 OVEREMPHASIS OF MATHEMATICAL OPERATIONS

The researcher was very concerned with the overemphasis of algorithms in mathematics teaching and learning as it is like a *doctrine* of some kind. There are several logical methods that can enhance mathematics to learners without strictly relying on rules for operations of fractions, like the Highest Common Factor (HCF) method which is often confused with the Lowest Common Denominator (LCD). One case explained explicitly the simplification of 3/12 to get 1/4 equivalence by just using groups (check L4 excerpt). This is actually a direct testimony to the researcher's question number 4 (looking at informal knowledge), looking at the value prevalent to enhance mathematics education.

Another case of informal knowledge is that of item number 10, where learners just divided numerators and denominators to find the respective fractions. Tirosh (2000) claims that, there is nothing wrong with this method as it can be mathematically justified. So, learners' intuition with regard to division of fraction is actually correct and it is the same method that is used for multiplication, but when it comes to addition and subtraction, that intuitive thinking does not work towards giving a correct mathematical statement if the fractions are representing numbers *not ratios*.

The researcher was very impressed with one of the cases that used a revolution angle to represent that 1/4 is actually associated to 90° (see clinical interview with L4). This is another informal knowledge that is actually not taken care of in our algorithms of operating with fractions. The researcher is therefore of the opinion that there is a need for appropriately designed fractional problems that are of appropriate level of difficulty to learners so that their informal knowledge on fractional concepts leads to successful scientific or "*scholastic knowledge*", as Wardekker cited in Irwin (2001) refer to this knowledge.

5.4 LEARNER PREPAREDNESS

It was not clear, according to the researcher's perspective, whether the participants were all at the stage of being prepared very well to accomplish the Learning Outcomes of grade 9 - LO #1-*representing numbers and their relationships* (DoE, 2003). There were some participants that were vocal about the issue of that there were never taught some of the items before during their mathematics learning in the classroom *community of practice* (Lave, 1996), as a result they did not understand the exact questions (see L2, L3 and L6).

There can be a number of factors that can feature in the above assertion of whether they (participants) were taught or not. Maharaj, *et al.* (2006) study revealed *"half-measures and errors, or complete omissions of practical work from the lessons"* (p. 104). That, this observation is applicable to the participants of this study would be either a YES or NO if not both, depending on whether facts about their mathematics background could be established and investigated. This study is limited to just the participants, so the researcher could not draw any inferences on the general learner preparedness, but it was evident according to the results that there was not enough practice on fractional problem solving skills.

From a different perspective, still with learner preparedness, there can be a number of factors that could limit learner performance in items 12 and 13. One crucial point that is worth mentioning is the issue of *language barrier* (will be discussed in details in Chapter 6, section 6.4). All participants were English second language speakers and this might have an impact on their understanding of the questions and as a result failing to even attempt the items at all. Section 4.2.1 from the previous chapter alluded to this language issue and also the clinical interviews highlighted that there was a language barrier sometimes, as when explained clearly there were able to figure out the solution with the help of their vernacular language.

The other factor that can impact on learner preparedness is the familiarity of questions asked. If the style of questioning is not that of a *traditional textbook approach* (Siebert, 2002), there is very little chance of achieving the correct answers. In chapter two in the section 2.3, Schifter (1997) cited in Meel (2002) argued strongly that teachers must see to it that they do everything possible to gain learners insight and hence develop their subject matter so as to sort out informal mathematical issues confronted by learners in later years of schooling (focus question number 5). This is in line with research question number 5 which seeks to check the impact of later years in mathematics learning and teaching, including concepts like ratios in trigonometry and exponential/logarithmic functions; that if fractional concepts not clearly understood earlier, could result in dismay at tertiary mathematics education.

5.5 THEORETICAL RELEVANCE OF THE FINDINGS

Drawings on the works of Mack (1995) claiming that when symbolic representations are introduced with respect to real-world problems that students could solve to encourage them to draw on their informal knowledge of fractions. This is in line with the Piagetian view of *constructivism* – considering the fact that *knowledge constructed.* It is therefore of critical importance for educators to allow learners to construct knowledge that emanates from their immediate environments.

The other theoretical location that transpired in this study is that of the Vygotskian school of thought (Goos, 2004) which refers to *additive thinking* versus *multiplicative thinking*. This theory was noted in items 5.1, 8.1a) and 8.1b) of this study where participants just added numerators and denominators respectively.

According to Sfard *et al.* (2001), one can easily do *additive thinking*, but *multiplicative thinking* (as it was the case in 5.1, the tree problem, see Appendix A) is a skill that needs to be developed in mathematics education so that learners can succeed with mathematics in later years (focus question number 5).

Lamon (1999) concurs with Sfard *et al.* theory from the *cognitivist perspective* using proportional reasoning. This is a critical concept of introducing similarity of triangle in later years of mathematics education. When learners are not fully equipped with the skills of *multiplicative thinking* and *additive thinking*, as mentioned earlier in section 2.5.3, very little success could be achieved in later years of mathematics learning, that is often more abstract and complex in nature.

5.6 CONCLUSION

The researcher noted that in this chapter there were a number of common aspects that were suggested by other researchers from different studies (Mokapi, 2002; Ndlovu, 2001; Maharaj *et al.*, 2005). It is worth mentioning that although this study used only 30 learners from two schools covered in this study; the results that came up could be associated to similar studies that had been commissioned from local and international academic communities. This study had then contributed to the live debate that is rife and necessary for developing countries like South Africa.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 INTRODUCTION

This chapter tries to relate strategies that learners and teachers can adopt to improve learner performance with regard to fractional concepts. The researcher also refers to the learners' difficulties taken into account during the worksheet and the clinical interview analysis that was done in chapter four. Learner preparedness, teacher attitudes and language barriers are associated to present South African context. Further research projects are referred to so as to make some informed recommendations that teachers/researchers can implement to assist learners in improving the scientific scholastic knowledge of fractional concepts.

6.2 LEARNERS' DIFFICULTIES

Learners' difficulties and misconceptions that have been discussed in this study were actually evident in the errors or slips from the worksheets that participants attempted. It has been the researcher's assumption to expect to find conceptual errors among a group of learners who underwent clinical interviews after completing questionnaires. However, it was noted that not only are the learners having misconceptions, but there are exceptional cases where there was a significant improvement when the items were elaborated in either vernacular or simple terms during clinical interviews.

The study revealed in a sample (6 cases) of participants who were in the clinical interviews providing clear and correct response when the items were elaborated and with little hints and thought provoking questions. This actually indicates that, teachers should pay particular attention to all learners' responses, as some of them have informal knowledge that could be used to explain critical concepts in mathematics education.

6.3 TEACHERS AND THE SUGGECTED THEORIES

Unless teachers thoroughly check learners difficulties and/or misconceptions with fundamental concepts like fractions, with an aim to engage learners thinking skills, learners will continue to perceive mathematical concepts as too difficult to grasp and thus fail even to try their level best to put more oomph. This attitude would have a negative impact in later years of mathematics schooling where abstract complex concepts would be required to progress with mathematics even at tertiary education levels.

It is therefore recommended by the researcher that, there must be culture of investigating learners' conceptual understanding of mathematical concepts using dialogue on an ongoing basis. This was noted to be worthwhile when one of the cases managed to explain the concept of equivalence using *set theory* (see L4 excerpt in chapter 4). The researcher was very impressed with this idea as it showed that this learner could do well in set theory, as long as his thinking capabilities are channelled correctly.

It is important to note that, there is a room for improvement in mathematics performance if teachers could include dialogue on mathematical issues in their assessment programme in an integrated way. Some learners could be very good at debating and make investigations that can contribute to mathematics if given an opportunity.

6.4 LANGUAGE BARRIER

Another significant point to note in this study is that of a unique language that is used in mathematics concepts like fractions. Teachers should try to use technical jargon that learners able to relate their daily lives. This is because, *knowledge appears to emanate primarily from the individual's real life experiences rather than from formal schooling instruction* (Carpenter and Moser, 1983; Riley, Greeno and Heller, 1983 cited in Mack, 1995).

Therefore, relating instructional knowledge to personal experiences should be encouraged especially when abstract like fractional concepts are introduced. Wheatley (1991) cited in Lo and Wanatabe (1997) believes that knowledge originates in a learner's activity performed on mental constructs that are directly related to the action and experience of the learner. This is actually a constructivist view of knowledge as alluded to in chapter two.

The study revealed that some of the participants could answer the items when explained in their vernacular. The issue of Language of Learning and Teaching is a very critical one. To address it directly on the impact language has in grasping mathematical concepts should be another study on its own. The researcher can only comment on that, there was significant improvement on learners' responses when items were rephrased and *code-switching* used. Setati (1996) did a study on this issue of language and one could conduct the same study in a different milieu so as maximize generalisation of the findings in the role played by language in learning and teaching *scientific concepts*, as Mokapi (2002) and Irwin (2001) argue that there are two types of knowledge acquisition; *viz, spontaneous and scientific or scholastic.* The onus lies upon teachers to see to it that they take advantage of what the learner already knowledge by first identifying it and thereafter build new (scholastic) knowledge.

In actual facts, gaining learners ideas on the view on mathematical concepts is very important in the context of mathematics teaching and learning. This enables the teacher to actually get to the bottom of the actual problem or barrier to learning scientific concepts necessary for the progress of the learner. By doing diagnostic assessment, teachers could get the kind of concepts that learners can use independently and the once they the learners have *misconceptualised* (Mokapi, 2002). This in turn enables the teacher to decide on the contingency plans and remedial work that could be designed to assist learners' grasp the concepts that they have misunderstood. For an example, in this study it came up that most learners could not do addition and subtraction of fractions. It can be a starting point to rectify that problem before having complicated fractional problems without the proper background of basic concepts of simplifies fractions.

From the Vygotskian view, cited in Mokapi (2002), *to enhance understanding and formulation of true concepts, teachers may incorporate everyday concepts.* Over and above everything, learners should be encouraged to use their informal concepts so as to make sense of the formal concepts and thus being able to associate informal knowledge to formal/school knowledge in an integrated fashion. This assertion concurs with my focus question on the extent to which informal knowledge could be used to make sense of what the learner does at school and be able to relate if in the *community of practice* (Lave & Wenger, 1991 cited in Goos, 2004). This academics claim that mathematics teaching and learning is viewed as a social and communicative activity that requires the formation of inquiring minds. One of the strategies to be employed to achieve the goal of mathematical wizards could be to make sure that our learners are engaged in mathematical investigations/projects that are challenging the mathematics they do at school on its capabilities. The other approach would be to engage learner in critical thinking debates about the proofs of mathematical propositions that are possible to be challenged through the mathematics they possess in a particular grade.

6.5 SOUTH AFRICAN PERSPECTIVE

The National Curriculum Statement of Mathematics speaks of learners being able to make conjectures (DoE, 2003). It is therefore a challenge to all mathematics educators to check if all learners are equipped enough to deal with a classroom environment where mathematics is challenged and put in a test that teachers must prove its gigantic power and the divine truth possessed by the theorems, convention (agreed upon methods) and formulae. Such a culture would foster *esprit de corps* among mathematics learners and there would be more learners interested in taking mathematical related careers of which South Africa has a very little pool to draw from it.

It is however appreciated that there are number of initiative to improve mathematics educations in South Africa as there is an increased number of bursaries for Maths and Science prospective educators. Some of the initiative is to introduce Mathematical Literacy to the non-Mathematics learners so as to make South Africans a nation that is mathematically literate. All these government initiatives are appreciated and would probably have a positive impact on mathematics education in general only if *implemented, monitored and evaluated* from time to time to make mathematics a more viable subject in our country.

6.6 SUGGESTED FURTHER RESEARCH PROJECTS

Since this was a short study, there are a lot of questions that are still not having answers. For an example:

- That teachers know how to use practical work to teach fractional concepts,
- That learners are prepared enough to tackle abstract fractional problems,
- More needs to be investigated about the language issue,
- The methods that can make learners enjoy mathematical projects,
- How is Mathematical Literacy going to address the existing mathematics issues that have existed for years,
- When to introduce formal notation to maximize leaner performance in fractional concepts.

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APPENDIX A

☐ **Learner Worksheet**

Learner profile *(to be used for research purposes only)*
General information (Please tick or fill in)

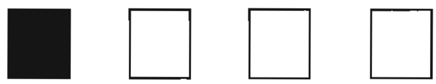
1. Boy ☐ Girl ☐
2. Age in years:——
3. Self rating on your grade 9 mathematics performance.
   Excellent >80% Very Good >70% Good >50% Average <40% Poor <30%
4. Rate yourself when it comes to **fractions** understanding in your class.
   Difficult ☐ Easy ☐
5. Do you think that you would have done better in mathematics if you understood fractions?
   YES ☐ NO ☐

Please answer the following questions neatly and in clear writing. Use the spaces provided to answer each sub-question. The space DOES NOT imply the length of the expected answer.

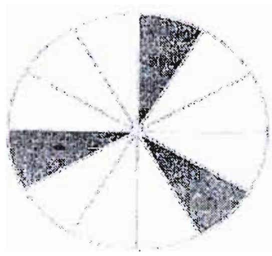
Section A: Concrete fractional concept

1. 1 In each of the following, write the shaded regions as a fraction.

a)



b)



a) Answer:	b) Answer:

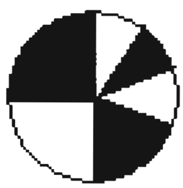
1.2 Compare the two answers above and state how they are related.

Answer:

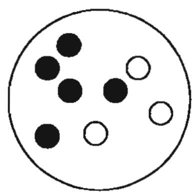
2. Which of the following shaded areas show the fraction $\frac{3}{5}$? Circle the letter e.g. (E) above the part representing the said fraction.

A.

B.



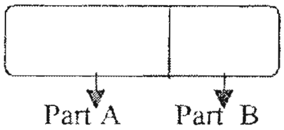
C.



3. What mixed fraction is shaded in the following two rectangles?

Answer:

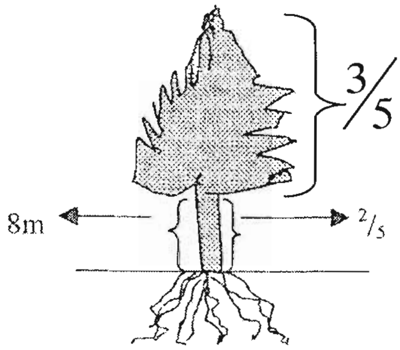
4. You work in a nearby home spaza shop on weekends. A child comes to buy one-half a loaf. You cut bread like this:



Are the two parts equal? Explain.

Answer:

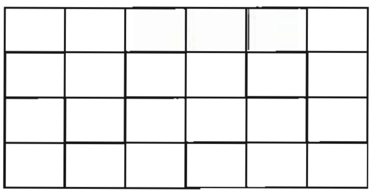
5.1 Using the information given on the diagram about the tree, find the height of the tree. Show all calculations



Answer:

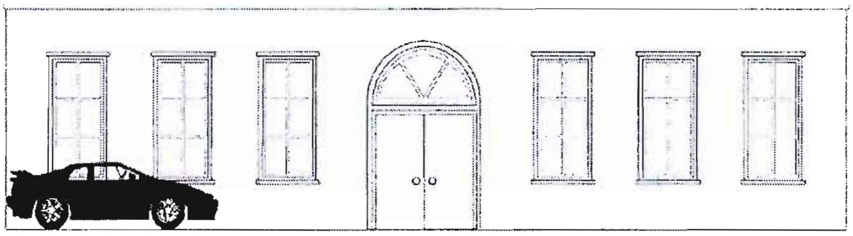
5.2 Shade in $\frac{3}{8}$ of the unit in the following grid:

N.B. DO NOT overlap shading in each block.



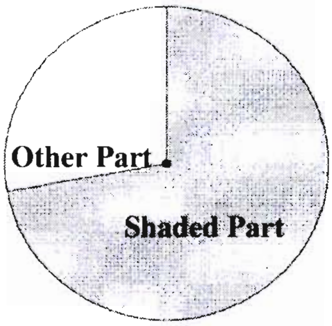
APPENDIX A

5.3 In the figure below, the car is 3 m long and the building is 15 m long. Write down a simplified fraction comparing the length of the car to that of the building.



Answer:

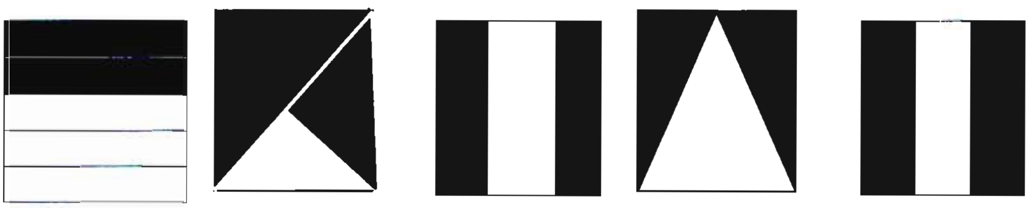
5.4 If the **Shaded Part** of the following circle represents a fraction less than $\frac{3}{4}$, estimate the **Other Part** as either less or greater than or equal to $\frac{1}{4}$. Explain your answer.



Answer:

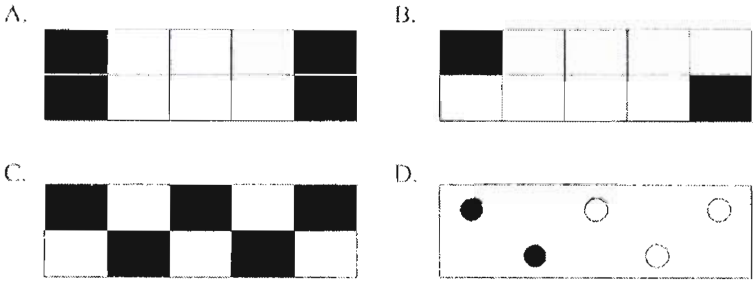
6.1 One of the following figures represents $\frac{2}{3}$ of the square shaded. Circle the letter e.g. (F.) above the part representing the said fraction.

A. B. C. D. E.



APPENDIX A

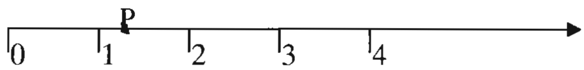
6.2 In the figures below, circle the letter showing the shaded part equals to $\frac{2}{5}$.



6.3 From the above diagrams, write the letter of the picture representing that $\frac{2}{5} = \frac{4}{10}$.

Answer: __________

6.4 Given the following number line,



Circle the better estimate for the point P.

A $1\frac{1}{5}$ B. $1\frac{1}{3}$ C. $1\frac{4}{5}$ D. $\frac{1}{4}$

Section B: Abstract fractional concept

7. Circle the smallest fraction in the following:

A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

8.1 Calculate the value of the following fractions by showing all workings; write the answer in its simplest form:

a) $\frac{4}{5} - \frac{1}{2} + \frac{1}{15}$ b) $2\frac{1}{4} + 3 + \frac{3}{4}$

Answer: **Answer:**

9. List TWO more fractions that are equivalent to $\frac{3}{4}$.

Answer:

10. Simplify: $\frac{6}{55} \div \frac{3}{5}$

Answer:

11. Write down the following terms as numbers in fractional form:

11. 1. two-thirds **Answer:** ___________

11.2. three-fifths **Answer:** ___________

11.3. one-half **Answer:** ___________

11.4. three-quarters **Answer:** ___________

12. Penny had a bag full of marbles. She gave *one-third* of the marbles to Rebecca, and then *one-fourth* of the remaining marbles to John. Penny then had 24 marbles left in the bag. How many marbles were originally in the bag? Show all your calculations.

Answer:

13. Jabu had R 240. She spent $\frac{5}{8}$ of her money. How much money did she have left? Show all your calculations.

Answer:

APPENDIX B

Interviewer (I) and Learner no. 1 (L1) dialogue.

I : In question 1.2 you said ¼ is bigger. Can you explain why?

L1 : I thought ¼ is bigger than 3/12. If you add the rest of these (*showing the shaded circle-referring to continuous quantities*), you get 3, but the others (*referring to discrete quantities*) will give 1 in all as there is only one part shaded.

I : In question 2 you chose A as the correct representation of 3/5. Why did you no choose B or C?

L1 : I did not actually understand the question. But I could see that the other two would not be correct as parts were not equal at B and at C we don't get the fraction of 3/5.

I : You had 5/5 and 3/5 as two separate answers. I you simplify 5/5 and add to 3/5, what fraction would you get as the answer?

L1 : 5/5 adding 3/5 we get 8/10. This is just adding the tops and bottoms here (*showing numerators and denominators*)

I : If I say the answer would be 1 and 3/5, would you accept it?

L1 : Yes as there is a whole and 3/5.

I : So 5/5 means a whole and what is a whole equal to?

L1 : Yes, a whole equals to 1.

I : In question 4, you said Part A is bigger than Part B. Can you explain what can be done to make the two parts both equal?

L1 : By putting the line at the centre.

I : Is it not at the centre?

L1 : No, Sir, it is not.

APPENDIX B

I : You did not even try question 5.1. Did you not understand the question or you don't know it at all? Can we try to do it together?

L1 : No, I did not understand it. May be if we try it together, I can see it. (*we did it and he was very pleased to see how easy it was*).

I : In question 5.2 you shaded 9 blocks in total. Can you explain why do you think that this is a representation of 3/8?

L1 : I multiplied 3 by 3 and got 9 and then 8 by 2 got 16. Thereafter I shaded 9 and be left with 16.

I : Is it 16 left?

L1 : (*A long pause*) No, I think I made a mistake.

I : You did not try question 5.3. Was it difficult or you did not understand the question?

L1 : I did not understand the question, but if explained, I can try it.

I : In question 5.4, you gave the correct answer. How did you know this?

L1 : It is because ¼ is bigger than ½.

I : Is that true?

L1 : (*Uh, after a long pause*) No.

I : In question 6.1, you gave E as the answer. Why are other choices like C incorrect?

L1 : It's because the pieces at C are not equal. But at E all the pieces were equal and only 2 were shaded out 3 equal pieces.

I : That's correct; I am very impressed with the way you explain.

I : In question 6.2 you had two choices A and D. Which one is actually correct?

L1 : Actually I changed my mind after writing, the right answer is A.

I : In question 6.3, you wrote unclear. What did you mean actually?

APPENDIX B

L1 : I meant A.

I : How did you know that the correct answer is B in question 6.4?

L1 : I cannot remember, but ½ is bigger.

I : In question 7, you got the correct answer as A. Why are other choices incorrect?

L1 : I did not fully understand this, but I thought 1/6 would be the answer.

I : It's like you were very confused in questions 8.1 a) and b). Did you understand the questions or not?

L1 : I was very confused as it was for the first time I get these questions at school. I have not been thought them before.

I : In question 9, it's not clear what the first fraction answer was. Can you write next to it? How did you get your fractions? What does equivalent mean?

L1 : It is 8/12.

I : Which is not correct?

L1 : Yes, I see.

I : In question 10, you wrote the answers as 11 and 2. What fraction are these numbers representing? You also came up with 11* 2 =22, What were you calculating there?

L1 : I divided the tops and the bottoms (*referring to numerators*), then got 11 and 2.

I : In question 11 you got all but 11.1 correct. What made you to say that three-quarters means 3/12?

L1 : I multiplied 3 by 4 got 12, and then I just wrote it as 3/12.

I : In questions 12, you had the answer as 38 but which is incorrect. Can you explain how?

APPENDIX B

L1 : I looked at the given numbers, and then got 38 out of them.

I : You did not even try question 13. Was it difficult or you did not understand it at all? Can we try it together?

L1 : I did not actually understand the question.

APPENDIX B

Interviewer (I) and Learner no. 2 (L2) dialogue.

I : You did not answer question 1.2. Can we try it again?

L2 : Yes, but I could not relate these (*showing the fractions*) fractions.

I : In question 4, you said the other part is bigger. How can you make the two parts equal?

L2 : (*Showing with her hands*) By moving this (*pointing at the dividing line*) closer to the side of Part A.

I : In question 5.1, you added 3/5 and 2/5 got 5/10. You then said that the height is 5/10. Can a height be this fraction? Please explain how you got the answer?

L2 : I did not understand the question. I just thought that we needed to add the given fractions and thereafter the sum would be the height of the tree.

I : In question 5.2 you shaded 4 blocks to represent 3/8. Can you explain how did you get that answer?

L2 : I was not very clear on what to do. I just shaded the first column of blocks with 4 blocks. I did not understand the question.

I : Question 5.3 is correct. Was it easy? Can you simplify it?

L2 : Yes, it was easy. I can simplify it by dividing by 3 on the numerator and the denominator then get 1/5 as the answer.

I : In question 5.4 you said part not shaded is equal to 1/4. Can you explain how you got that answer?

L2 : I did not notice that the line is not straight. I can now say that say that greater than.

I : In question 6.1 you got E as the answer. Why did you choose C?

APPENDIX B

L2 : Option C had got the middle part bigger. The others like A is actually 2/5 not 2/3.

I : Question 6.4, you got the answer as ¼. Can you explain how you got it?

L2 : I did not understand the question well. (*Interviewer marks off three equal parts between 1 and 2*). No I think I can say the correct answer must be $1^1/_3$.

I : In question 8.1 a) you got 4/19. It is like you were subtracting/adding numerators and then denominators. Can you explain how do you know that this to be a correct method?

L2 : This is the way I was taught to add fractions.

I : It's like in question 8.1 b) you did the same thing as you did in 8.1 a), can you explain?

L2 : I could not get 1/4 + 3/4 without following addition, that's how I was taught fractions.

I : You answered question 9 correct. Was it easy? What does equivalent mean?

L2 : No, it was not easy. I got 6/8 by multiplying by 2 both numerator and denominator. I also repeated multiplying by 2 and got another fraction as 12 /16.

I : Very good that in question 11, you got everything correct. Were they easy?

L6 : Yes, we did this in primary school.

I : In question 12 you were trying very hard, but you did not get the correct answers. You have 1/3 + 1/4. Can you explain your method?

L2 : I really could not understand the question. But now that you have explained it to me I can see that it is easy.

I : You tried question 13 very well. But it's like you did not understand it. Can you explain what were you thinking?

APPENDIX B

L2 : All I can say is that I had to divide the money left. For an example having R2, you use R1, you can say R2 – R1 = R1. (*Showing confusion*) We actually do not do these fractions. That's why I could not do well.

APPENDIX B

Interviewer (I) and Learner no. 3 (L3) dialogue.

I : In question 1.2 you said in a) the part that is shaded is one of the four parts and in b) the three parts shaded are three of the twelve parts. What did you mean?

L3 : If simplifying 3/12, you can get ¼.

I : In question 2 you chose A as the correct representation of 3/5. Why are the others B and C not correct?

L3 : In option C to be correct there must be 3 parts shaded and 5 in total.

I : You had 5/5 multiply by 3/5. Why did you decide to multiply?

L3 : A mixed fraction can only be found if multiplying.

I : You said Part A is a half and quarter in question 4 while part B is a half only. How do you know that the other part is half and the other one in half and quarter?

L3 : If it was whole bread, there should have been half-half.

I : In question 5.1 you multiplied 3/5, 2/5 and 8/1. Can you explain how you decided to did this?

L3 : I thought the height would be possible if I multiply the given numbers.

I : You shaded 4 blocks in question 5.2. What made you think that this represents 3/8?

L3 : I cannot remember why I chose 4 parts, but I can see that I was wrong.

I : In question 5.3 you multiplied 3/1 by 15/1. What made you to think this can help you answer the question?

L3 : I chose to multiply because I had to combine the two heights.

I : You did not answer 5.4. Is it because you don't know or you did not understand the question?

APPENDIX B

L3 : I did not understand the question. But if it can be explained, I can be able to do it.

I : You had two choices (C and E) in 6.1. Can you explain why do you think both are correct?

L3 : I wrote the correct one at the back (*turned the worksheet to show the interviewer*). E is the correct one. C is wrong as the middle part is bigger than other two parts.

I : In question 6.2 you had two choices (A and D). Can you explain why?

L3 : D is the correct answer. A is the same as D; as if you simplify 4/10 you get 2/5.

I : In question 6.4 you had the correct answer as choice B. How did you know that other choices are not correct?

L3 : D is wrong, because it's not after 1, but before 1. But A should have been in between 1 and 2. Option C (*after taking a long pause*), I cannot explain it, but it's not a better estimation.

I : In question 7 you said the smallest fraction is 2/3 compared to 1/6, 1/3 and 1/2. How do you know?

L3 : It's because if you simplify 2/3 you get 1 as 2 into 3 goes once and the remainder is 1.

I : In questions 8.1 a) and 8.1 b) you did not even try them. Is it because you don't know how to answer or you did not understand the question?

L3 : I did not understand the question, but if explained I think I can do it.

I : Very good that you got question 9 correct. Was it easy for you? What is meant by equivalent?

L3 : Yes, it was easy. Equivalent means "the same" thing.

APPENDIX B

I : You got question 10 as 2/11 the answer. How did you get it?

L3 : I divided 3 into 6 got 2 and divided 55 into 5 got 11. That is how I got 2/11 as the answer.

I : In question 11, you got 11.1, 11,2 and 11.3 correct but not 11.4 as you wrote 3/12 which is not correct. Why did you say three-quarters means 3/12?

L3 : It's because a quarter ¼, so if there are 3 quarters, it would be ¼ + ¼ + ¼ =3/12.

I : You did even try question 12 and 13. Were they difficult or you did not understand the questions?

L3 : I did not understand the questions. The English used was not familiar to me, that is why I did not even try it.

APPENDIX B

Interviewer (I) and Learner no. 4 (L4) dialogue.

I : You said that ¼ and 3/12 are all quarters of the shaded fractions. Why?

L4 : By taking 1 in 4 things repeatedly 3 times in 12 things makes the same fraction of parts taken at time.

I : What would have happened if I had 4/12? Would it still be equal to ¼?

L4 : No, Sir.

I : Why not the same?

L4 : It is because for it to be the same you will be taking equal things at a time to make the same fraction.

I : How?

L4 : Using groups 1, 2 and 3 with the same starting number of things, take the same number of things in each group like this one 3 groups taking 1 thing in each group for 3 times you will end up with 3/12 if there were 4 things in each group.

I : Ok, that is correct.

I : In the next question (*shown with a finger*), why did you choose A not B or C as the correct representation of 3/5?

L4 : It's because in A, all blocks are equal. So as 3 blocks are shaded there out of 5 blocks that would be a correct representation of 3/5. But in B and C not all blocks are equal. So you can't make a fraction shaded.

I : Ok, that is a very good explanation. In question 3, you have 5/5 and 3/5 as your answers. Could you show me how you got these fractions?

APPENDIX B

L4 : Here *(pointing at 5/5 shaded block)* all the blocks are shaded so that is 5/5. In this one (pointing at 3/5 shaded block) only 3/5 of the blocks are shaded.

I : Here (*showing question 4*), you have part A and part B. You said that they are not equal. A is bigger than B, you also said that you used measurement, how did you use measurement?

L4 : Take for instance a ruler and start from 1 cm to 10 cm. You can have half being at 5 cm. In this diagram (*referring to Part A and Part B*), it is clear that these parts are not equal at all.

I : Are you sure that, 1 cm to 5 cm is equal to 5 cm to 10 cm in a ruler? (slip)

L4 : No, you have to start from 0 cm.

I : Ok, that's correct.

I : In this one (*showing question 5.1*), you said that the height of a tree is 20 m. Can you explain how you got the answer as there are no calculations shown?

L4 : At 2/5 we have 8 m. This means that 2/5 means 8 m of the tree. Therefore, each 1/5 means 4 m of the tree. That means that 3/5 must be 12 m of the tree. Then 8 m plus 12 m is equal to 20 m. That is how I did it.

I : Very good. You are doing very well.

I : How did you do this (*referring to question 5.2*)?

L4 : I thought that since there are many squares, so I had to take 3 squares at a time.

I : Could you show me how you did that?

L4 : Actually in the whole block I noticed that, if I take 3 squares at a time, I end up with 8 squares of 3 blocks. Then I shaded 3 of the 3 squares as a 1 unit three times. I thought that represents 3/8th of the whole block

I : Very good method. I like you mathematical thinking.

I : In question 5.3 you got a simplified answer of 1/5, can you explain?

L4 : I wanted to take, uh! (*pause as he is reading the question again*). As the car is 3 m long and the house is 15 m long. I looked at how I can multiply 5 to make 15. I got that 3, 6, 9, 12, 15 are 5 times in total. So as there are five 3's that make 15 and one 3 that makes 3, then the simplest form of 3/15 would be 1/5.

I : That's correct.

I : In question 5.4, you said that the other part is greater than ¼. Why?

L4 : By looking at it and consider that 90^{o} would have been straight here (*showing on the diagram*) to form a part equal to ¼. So as this lines is after 90^{o}, it is therefore showing a part greater than ¼. {alternative conception}

I : Is 90^{o} the same as ¼?

L4 : If you take 90^{o} times 4 you get 360^{o}, which is a complete circle. So then 90^{o} could represent ¼.

I : Ok, in question 6.1 you chose E. What about C? Why not choose it?

L4 : If you look at C, the blocks there are not of equal sizes. So you could not compare the spaces shaded in fractional from.

I : Ok, in question 6.3, you chose A. How did you get that?

L4 : I took 2/5 and compared to 4/10. Then I treated A as a no line (*showing on the diagram the middle horizontal*). Then the shaded part was 2/5, but if I put back the line the shaded parts became 4/10.

I : You are doing so wonderful, that's excellent!

I : Now in this one (*referring to question 6.4*) you had a number line with point P. You chose A as better representation of P, why?

L4 : The point P is after 1, so I took A as it represents a fraction after 1.

I : What about the others B or C, they are also after 1. Why not choose them?

L4 : (*after a long pause took place*) This is actually confusing me as I really don't know these things.

APPENDIX B

Interviewer (I) and Learner no. 5 (L5) dialogue.

I : In question 1.1, you wrote ¼ and 3/12 for a) and b) as your answers. You also said that you think 4 can go into 12. How many fours are in 12?

L5 : That you can find into 12?

I : Yes.

L5 : There are 3.

I : Three, Ok. Now you said this (*looking at the question 2*) is 3/5. How do you know that this diagram represents 3/5?

L5 : I had to count the squares inside and found that there are 5 and then three are shaded. I know that the fraction is not the whole number. That is how I was taught.

I : Ok, what about the others (*pointing at the other options*)? Why are they not equal to 3/5?

L5 : In this one (*referring to B with his finger*), I had to count the parts and found that there were six.

I : So it couldn't be a correct option?

L5 : No.

I : In question no 3, how did you do it?

L5 : I really don't know this thing. I was just guessing.

I : You were guessing!

L5 : Yes, Sir.

I : These are two rectangles. Shaded in the first one are five parts out of five parts. Then in the other one 3 parts are shaded. You see?

L5 : Yes. But I really don't know these things.

APPENDIX B

I : Ok, let's move on. In this one (*referring to question*), you said the bread must be cut in the middle and the two parts would be equal. Could you explain further?

L5 : The teacher used to cut an object I for us in the middle and show us that the two parts that are equal represent ½. But in this one (*pointing at question 4*), the other side is bigger, so the two parts are not equal.

I : Ok, how can you make the two parts equal?

L5 : By moving more to part A than to part B.

I : In question 5.1, we have this tree problem. How did you do this problem?

L5 : I saw that here (*showing the 8 m length*) in the stem, 2/5 + 3/5 = 5/10.

I : Is this how fractions are added?

L5 : I really don't know, but I was trying.

I : I question 5.2, it seems as if you did not understand the question. Did you?

L5 : Yes, I did not understand the question.

I : But how did you make a decision to shade 3 parts?

L5 : I looked at the top row, but again I found that there were 6 squares. Then took 3 in the top row.

I : Did you notice that there are too many squares than to just shade 3?

L5 : I noticed that.

I : So, if you took 3 squares in 8 at a time repeatedly, how many would you end up with shaded?

L5 : Could you please repeat, Sir?

I : Taking 3 out of 8 squares at a time repeatedly, how many would you end up with shaded?

L5 : I think 9 would be shaded, Sir.

I : Ok, that's correct. Let's move on.

I : You wrote 3/15 as the answer in question 5.3. This fraction can be simplified as what fraction?

L5 : (*After a long silence taking place*) I do not get it, Sir. If you ask me now to get it, but I can calculate it if you can give me time.

I : Ok, let me give a few minutes to try it.

L5 : (*Pause*)

I : Do you get it now?

L5 : I get 1/5 using 3 into 3 and 3 into 15.

I : That's correct, so you know that 3/15 = 1/5?

L5 : Yes, Sir.

I : In question 5.4 you used 90^{o}. Why?

L5 : If I look at it, it's a circle. The teacher taught us that a quarter is 90^{o}. When we were doing "pie charts", so this is more than 90^{o} as it is not straight.

I : (*very impressed with this knowledge incorporated*). That means 90^{o} is the same as ¼.

L5 : Yes, Sir.

I : In question 6.1, you chose E. But I would have liked to choose C. Why did you not choose C?

L5 : I taught that E is the correct one.

I : But even A looks like 2/3. Why not choose it?

L5 : In A there are 5 blocks not 3. So you can see that this id not 2/3 but 2/5 by counting all the blocks.

I : What about C if you use the same method of counting, there are 3 blocks also here (pointing at C)?

L5 : C is not that wrong, but the blocks used are not equal (*awalingani*) as the part not shaded is bigger than the other 2 shaded. But in E all parts are equal (*ayalingana*) and only 2 are shaded that is why it represents 2/3.

I : Ok, that is correct.

I : I question 6.3 you chose A as a better representation of 2/5 = 4/10. How do you know that?

L5 : By lookin at the top row, there are 5 parts, and the bottom row has 55 parts that makes 10 parts in total. So I thought that, at the top you can get 2/5, but when using the whole block, you can get 4/10 shaded.

I : In question 6.4, which fraction is better close to 1 considering the point P?

L5 : I really didn't understand this one.

I : But, what's happening here is that you can see that P is after 1. Why did you choose A not B, C or D.

L5 : If P was at 1, I would have said ¼ is the point P. (misconception)

I : In question 7, which fraction is the smallest?

L5 : (*After a long silent with some signs of thinking*) Uh! I really don't know these things. I don't understand what's going on here.

I : Let's take this paper (*same method used to L6*). In this paper (*to be folded into two equal parts*), how can I get half?

APPENDIX B

L5 : By taking the middle.

I : How?

L5 : (*silence*)

I : Folding it like this? (*showing the half folded paper*)

L5 : Yes.

I : What is represented by each part?

L5 : Half.

I : Now, let's divide the same paper into 3 times.

(*folding the paper looking at L5 showing some sense of disbelieve*)

I : What do you think has happened? Half was here (*showing the paper*)where is 1/3 of the paper?

L5 : Somewhere less than half.

I : So 1/3 is less than ½.

L5 : Yes.

I : Now, if you look at all the given options of fractions which one would be smaller?

L5 : I can say that A would be smaller.

I : If A was 5/6, would it still be smaller?

L5 : I don't know really.

I : That means there would have been five papers cut into 6 equal parts. I you compare it with half, would 5/6 still be smaller or bigger than ½?

L5 : I think it 5/6 will be bigger than ½.

I : Ok, that's correct. Can you see that fractions are easy?

L5 : Yes, the way you do them makes tit easy to understand them.

APPENDIX B

I : In 8.1 a) and b) I can see that you added numerators and denominators. Is this how you normally do fractions?

L6 : Yes.

(The researcher did not want to dwell much on this one as it was alluded to in L6).

I : In question 9, how did you do it?

L5 : I really didn't know the term "equivalent."

I : Is it the term that was problem? But in question 10 you got the correct answer, did you notice?

L5 : Correct!

I : Yes, it's correct. How did you do it?

L5 : I said 3 into 6 and 5 into 55, than got 2/11 as fraction.

I : That was good, well done. Is this method always working when dividing fractions?

L5 : I don't know.

I : Let's take fractions 2/3 divided by 5/7. Could you do this one?

L5 : No, I can't.

I : You have not done these fractions before?

L5 : No.

I : In question 11 you got all the correct answers. How did you do them?

L5 : I just looked at the names used and then wrote them as number fractions.

I : That was good. What about questions 12 and 13?

L5 : I really don't know what I was doing in these. I can't even explain.

I : Thank you. You performed to your best level to most questions.

APPENDIX B

Interviewer (I) and Learner no. 6 (L6) dialogue.

I : You gave ¼ at 1.1 a) and 3/12 at 1.1 b).

L6 : Yes.

I : What did you mean if you say these numbers are related in question 1.2?

L6 : I meant that this (*showing ¼*) refers to a quarter, Sir.

I : Are they the same thing?

L6 : Yes, Sir.

I : Is 3/12 equals to ¼?

L6 : Actually, they are not equal.

I : Here you have 5/5. What did you mean? What is 5/5 equal to? Which number is equal to 5/5?

L6 : (*Very long pause*) 4/4 is the same as 5/5.

I : Yes 4/4 and 5/5 are the same, because they are both equal to which number?

L6 : (*Silent, but showing signs of thinking*)

I : Uhm? What are they equal to?

L6 : (*Still silent*)

I : If you divide these numbers (*showing one at the top and the other at the bottom*) What would be the answer?

L6 : It is equal to 1, Sir.

I : Then, this (*pointing at 5/5*) is the same as what number?

L6 : 1, (*showing a sense of being convinced with the analogy*).

I : Cut loaf (*Isinkwa esincane*) is shown. Is there any part that's half in this diagram?

L6 : No, Sir, they are not equal parts of the whole.

I : As you say that they are not equal. How can you make them equal? By moving Part A or Part B to get smaller?

L6 : Move Part B.

I : Here is a tree problem. How did you get 12 m?

L6 : This confused me, Sir.

I : But you got 12 m, how did you get it?

L6 : I added all these (*showing fractions on the worksheet*) Sir. No, I said 2/5 is 8 m and 3/5 is 12 m, because I saw 2/5 in 8 m and then looked at the other of 3/5 to be 12 m.

I : Very good, that is excellent, quite good performance. You mean, if you have 12 here (*showing on the tree part*) and 8 here (*showing on the tree part*). What would be the total height?

L6 : It is going to be 20 m, Sir.

I : That's good.

I : In the question (*pointing to 5.3*), you are asked to shade 3/8. You shaded parts only, why?

L6 : I had to count what can make 8 (*showing on the grid*) counting in 3's.

I : So you used 3's to count?

L6 : Yes, Sir. Then shade 3 parts.

I : If you look at the whole grid, how many parts are there?

L6 : I think there are 24 (*showing with his fingers that he calculated*).

I : Is there no way you can change this 8 to make it 24 and take 3 parts per each 8 in 24 parts?

L6 : I can see it that way now.

I : Then, how many would you have shaded?

L6 : 6 parts, no!!! 9 parts, Sir.

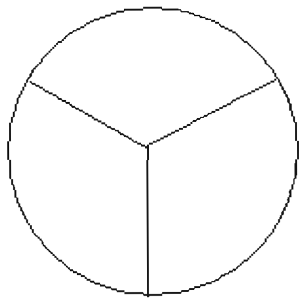
I : In question 5.3, how did you get your answer?

L6 : I read that 3 in 15, then simplified to 1/3.

I : OK that's correct. Right now in 5.4, you wrote measured, how did you measure?

L6 : I took a pen and placed it here (*showing the shade part to divide it into three equal parts*) then got 1/3.

I : So you looked at it like this (*drawn roughly on a piece of paper*).



L6 : Yes, Sir.

I : In question 6.1, why didn't you take C as the correct answer?

L6 : It is because the parts there (*pointing at C*) are not equal, especially the one in the middle (*showing with his finger*). So it cannot represent a fraction of 2/3.

I : When parts are not equal, is it not a fraction of 2/3?

L6 : No, I think parts must be equal, as if there are big and small parts a fraction cannot be formed.

I : What about the other options? Are they all wrong?

L6 : Yes, as they having unequal parts (*showing with his hand each option*).

APPENDIX B

I : In question 6.2, you ticked two answers (*pointing at them*). Which one is correct?

L6 : Actually, it is D the correct one (*showing with his hand*).

I : OK, it is D, why is the other one incorrect?

L6 : The other one (*referring to B*), can be correct if you look at 2/10 not 2/5.

I : You said that you answer is A in question 6.3, Why?

L6 : I noticed that 2/5 and 4/10 can be represented in this diagram. Looking at the top only it's 4/10.

I : That's right, in question 6.4, you did not answer, why?

L6 : I missed it, but I can try it now.

I : OK. Could you try it?

L6 : The point is A (*showing on the worksheet*).

I : In question 7. You chose ½. Why?

L6 : I saw that ½ is the smallest compared to others.

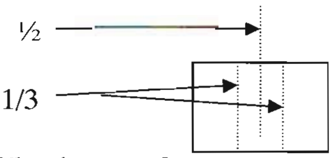
I : How? Is 1/3 not smaller than ½?

L6 : No. I think 1/3 is bigger than ½.

I : Do you think so?

L6 : Yes.

I : Let's use a paper folding. If I fold the paper (*showing it to him*), I get two parts both equal to ½ of the paper, Do you see?



L6 : Yes.

I : Now, let's divide the paper (*shown*) into three parts. What happens?

L6 : ½ is bigger than 1/3, Sir.

I : Which one is smaller now?

L6 : This one (*showing with his hand 1/3 part*).

APPENDIX B

I : So, the smallest is not ½.

L6 : Yes.

I : Now, if the paper is folded 6 times. What would be the smallest fraction, ½, 1/3 or 1/6?

L6 : The one of 1/6 would be smaller.

I : Now, in question 8.1 (a), how did you do it?

L6 : I said 4 – 1 + 1 = 4 and then 5 – 2 + 15 = 18, then I got 4/18, Sir.

I : So you subtracted or added numerators first and then denominators and wrote the answers as numerator over a denominator?

L6 : Yes, Sir.

I : How did you do this one (*pointing at 8.1 (b)*).

L6 : I added the bigger numbers (*referring to 2 and 3*), got 5 and then numerators and denominators got 4/8.

I : But 4/8 can be simplified as what fraction?

L6 : It is the same as ½.

I : So the answer could have been what?

L6 : 5½, Sir.

I : Ok, let's go back to that one of 8.1 (a). You subtracted or added numerators and denominators.

L6 : Yes, Sir.

I : Is this how you add fractions, doing the tops and bottoms?

L6 : No, actually it was the first time that I got something like this.

I : You did not do these fractions at primary school?

L6 : No, Sir.

I : But you did other questions very well, that's good.

I : Let's move on. Here you gave 4/12 and 8/24 as equivalent fractions to ¾. Can you show me how you got these fractions?

L6 : I had to count if there is ¾, and then there is 12. No (*showing some sense of regretting*), now I think I should have said 8/12.

I : How do you get 8/12 now, can you show me?

L6 : It's because, there is a one number difference in ¾ compared. If I count 4 in 8, I would be close to the number at the top and 12 being closed to the number at the bottom. This would be as if I used ¾ where the same thing applies.

I : Is 8/12 correct then?

L6 : I don't know. This is the way I think.

I : Ok, now let's look at like this; at the beginning we had a number like 3/12, we said it is the same as ¼. Why?

L6 : It's a quarter Sir.

I : How many quarters?

L6 : 3 quarters.

Taking a long pause took place to try and think actually what he meant

L6 : When simplifying, for an example 2/8, I say 2 into 2 get 1 and 2 into 8 gat 4, that's how I simplify fractions.

I : Ok, very good. Now if you want to increase both the numerator and the denominator to be having bigger numbers at the top and the bottom that are still equivalent to the original fraction, how do you do it?

L6 : Using the same numbers like these (showing those in the worksheet).

I : Yes.

L6 : To increase it?

I : Yes, to be bigger numbers.

L6 : You have to multiply now, Sir. (Multiplicative thinking)

I : Let's do it.

L6 : I can say 3 "*times*" 4 equals to 12, and then also use 4 at the bottom saying 4 "*times*" 4 equals to 16.

I : Why not use different numbers?

L6 : Numbers to multiply must be the same.

I : If there was 2 at the bottom and still 3 at the top. How would you have multiplied?

L6 : I would have to use the bigger number (*pointing at 3*) in this case. (alternative conception)

I : In question 10, you did not even try. Why?

L6 : It was the first time I see something like this.

I : You've never seen something like this before!

L6 : Not like this one.

I : (*I then referred to the item 8.1 (a) where the leaner followed the operation sign to manipulate fractions. This was showing no sense of knowing Lowest Common Denominator rules*). But you mange to do this one following the signs (*showing 8.1 (a)*). How can you do this one following the sign used?

APPENDIX B

L6 : I would have divided 6 by 3 and 55 by 5 and then put them in the order of fraction as 2/11.

I : You did not think about this during the time of writing the answers?

L6 : No, it just came up now.

I : What about these one? (*referring to question no. 11*).

L6 : I just took the first number "*name*" as the numerator and the second number "*name*" as the denominator.

I : Let's look at 11.4. You said *three-quarters* is 1/3. How did you get that?

L6 : A quarter is 1/3.

I : Is a quarter 1/3?

L6 : Yes, Sir. It was *three-quarters* so 1/3 is *three-quarters*. (Misconception)

I : In question no. 12, you didn't even try, why?

L6 : It's because. I did not know understand the question. But if I got an explanation, I would have tried it.

I : Ok, but you tried this one (*showing no. 13*).

L6 : It's because there was still time.

I : What is meant by 5/8 spent? If you spent 5/8 of the whole, how big of a whole is left in fractional form?

L6 : I think it is 2/8. (misconception/slip)

I : Thank you for allowing this interview to progress smoothly. Your responses are guaranteed anonymity.

APPENDIX C

Letter of Consent

To: Participant(s) and Parent/Guardian

Research Project: An investigation of fractional conceptions in "learning and teaching" mathematics at a secondary school in KwaZulu-Natal.

Year: 2006

Thamsanqa Makhathini (Final year Masters Student) is conducting a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** under the supervision of **Dr Deonarain Brijlall** contactable at 031-260 3491(work). Proposed research looks towards a better understanding of "learning and teaching" fractional concepts at a secondary school in KwaZulu-Natal: South Africa. In particular, this inquiry looks at the integration of informal and formal approaches to a course in algebraic fractions for grade 9 learners and investigating if learners grasp necessary skills before being introduced to abstract notions of fractions and mathematics concepts at a secondary school.

Learners and educators are requested to assist through participating in this research project as it would be of benefit to education practitioners and interested educationalists and/or mathematics teachers. However, participation is completely voluntary and has no impact or bearing on evaluation or assessment of the learner in any studies or course while at school. Participants may be asked to take part in the post-course surveys and open-ended interviews after the worksheets have been completed. These interviews will be recorded as the time progresses, but initially no recording will take place. All participants will be noted on transcripts and data collections by a *pseudonym* (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secure password protected server where authentication will be required to access such data.

Participants may revoke from the study at any time by advising the researcher of this intention. Participants may review and comment on any parts of the dissertation that represents this research before publication.

____________________ ______________

(Researcher's Signature) *(Date)*

DECLARATION

I,________________ *(Participant's NAME)*________________*(Signature)*

________________*(Parent's/ Guardian's NAME)*___________*(Signature)*

________________ *(Date)*

☐ Agree.

☐ Disagree.

N.B. Tick ONE

APPENDIX D

Isicelo sokusebenza nomntwana

Mfundi noMzali

Ucwaningo kwizibalo zamabanga aphezulu ezikoleni za-KwaZulu-Natal

Ngingu **Thamsanqa Makhathini** (owenza unyaka wokugcina kwizifundo eziphezulu mayelana nokufundiswa kwizibalo (Mathematics Education). Ngifunda e-University of KwaZulu-Natal – Edgewood Campus, sengifike esigabeni sokuba ngenze lolu cwaningo ngezingane. Kulolu cwaningo ngibhekwe u-Dr Deonarain Brijlall othalakala kulezinombolo- yasehovisini 031-260 3491.

Lolu cwaningo lubheka izingane ukuthi zinezinkinga kangakanani ekuqondeni izibalo zamaqhezu ebangeni lesishiyagalolunye (grade 9). Abafundi, othisha nabazali bayacelwa ukuba basize ekubambeni iqhaza kulolucwaningo oluzosiza kakhulu kubantu ababheke uhlelo lokufundisa kahle abafundi bezibalo ezikoleni zamabanga aphezulu, ngaphambi kokuba zenze izibalo ezithe thuthu ngokucabangisa.

Umfundi obamba iqhaza akaphoqiwe futhi akunamibandela noma imiphumela emayelana nokuphumelela kwakhe esikoleni. Abafundi bangacelwa ukuthi basize ekuqhubekeni nocwaningo ngokuhamba kwesikhathi ezingeni lokubuza imibuzo kube kuqoshwa amazwi. Ababamba iqhaza ngeke badalulwe amagama abo futhi bayocashiselwa abafunda lolucwaningo ndlela thizeni eyimfihlo.

Obamba iqhaza angacela ukuyeka noma nini ngokuvele azise umcwaningi ngolokho kuyeka kwakhe. Ekugcineni obambe iqhaza angabheka ukuthi akukho yini okumdalulayo.

________________________ ______________
(*Umcwaningi*) (*Usuku*)

ISIVUMELWANO

I,_____________________________ (*Umfundi*)_______________(*Sayina*)

________________________(*Umzali noma Umbheki*)________________(*Sayina*)

_________________ (*Usuku*)

☐ Ngiyavuma.

Khetha Okukodwa

☐ Ngiyaphika.

APPENDIX E

Application to do a Research in two Public Schools

To: Communications/Research officer: KwaZulu-Natal Department of Education

Circuit Manager: Lions River

Principals of Mpophomeni High School & Injoloba High School

Year: 2006

Research Project: An investigation of learners' fractional conceptions in "*learning and teaching*" mathematics at a secondary school in KwaZulu-Natal.

Thamsanqa Makhathini (Final year Masters Student) is conducting a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** under the supervision of **Dr Deonarain Brijlall** contactable at 031-260 3491(work). Proposed research looks towards a better understanding of "learning and teaching" of fractional concepts at a secondary school in KwaZulu-Natal: South Africa. In particular, this inquiry looks at the integration of informal and formal approaches to a course in algebraic fractions for grade 9 learners and investigating if learners grasp necessary skills before being introduced to abstract notions of fractions and mathematics concepts at a secondary school.

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Participants may revoke from the study at any time by advising the researcher of this intention. Participants may review and comment on any parts of the dissertation that represents this research before publication.

______________________ ______________

(Researcher's Signature) *(Date)*

DECLARATION

I, ______________________ ______________________(**NAME** and **SIGNATURE**)

principal/circuit manager on this day of ____ month______ 2006, hereby grant permission to the researcher to go ahead with the research in the above-mentioned schools following the terms of reference noted in this request letter.