AN EXPLORATION OF PRE-SERVICE TEACHERS USE OF VISUALIZATION WHEN TEACHING AND SOLVING PROBLEMS IN THE MATHEMATICS CLASSROOM

BY

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ABSTRACT

One of the objectives of mathematics instruction, according to the Department of Education (DoE) in South Africa, and elsewhere globally, is to prepare learners to become proficient in mathematics problem solving. There are many factors that contribute to learners becoming proficient in problem solving. The literature and many studies mentioned within this research present the many arguments for the field of problem solving and visualization.

Extant literature related to the range of problem solving is plentiful but there is insufficient or limited studies in the neglected field of visualization especially in how pre-service teachers use visualization and problem solving strategies in the classrooms. This study examines the use of visualization to support the teaching of problem solving strategies by pre-service teachers. The literature survey within this study intimates that a relationship is forged between solving problems and visualization. The available literature suggests that visualization assists learners to develop problem solving skills as it allows them to interpret the problem and show an understanding of the mathematical concepts. Literatures also indicate that when problem solving strategies are used in conjunction with the visual skills, the learners become more proficient in solving problem in the mathematics classroom. Thus, this research looks in a fine grained manner at how visualization and problem solving strategies are used by pre-service mathematics teachers.

Data was collected in phases from the pre-service teachers using a questionnaire, lesson observations, semi-structured interviews, evaluation worksheets and learner’s books. The pre-service teacher’s verbal and written responses were examined and their classroom practices were observed in conjunction with learner’s material.

The results from the data analysis have shown that some of the pre-service teachers have limited knowledge in the use of visualization and mathematical strategies when solving problems. It was also noted that they need to improve their mathematical content knowledge and how to use mathematical problem solving strategies together with visualization when teaching problem solving. These aspects need to be urgently addressed during their teacher training programmes.

DEDICATION TO:

1. The Divine Mother Saraswathie, the giver of knowledge, I offer my obeisance. By Your will and grace I begin this process of learning. Let my efforts be crowned with success.

2. My parents, mum Vindamuthee Durgapersad and late dad Budram Durgapersad, for instilling in me the value of education. Your personal sacrifices made for me to succeed will always be remembered and cherished.

I, RAJESH BUDRAM, declare that the research involved in my dissertation submitted in the fulfilment of the Doctor of Philosophy degree in Mathematics Education, entitled An Exploration of Pre-Service Teachers use of Visualization when teaching and solving problems in the mathematics classroom represents my own and original work.

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RAJESH BUDRAM

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DATE

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PROFESSOR VIMOLAN MOODLEY

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ABBREVIATIONS

AMESA	Association for Mathematics Educators of South Africa
ANA	Annual National Assessment
CAPS	Curriculum and Policy Statements
CK	Content Knowledge
CCK	Common content knowledge
DoE	Department of Education
EMS	Economic and Management Sciences
FAL	First additional language
HCK	Horizon Content Knowledge
KCS	Knowledge of content and students
KCT	Knowledge of content and teaching
LCT	Learning, Curriculum Content, Teaching Model
LOLT	Language of learning and teaching
MKT	Mathematical knowledge for teaching
NCLB	No Child Left Behind
NCS	National Curriculum Statement
NECT	National Education Collaboration Trust
NLSMA	National Longitudinal Study of Mathematics Abilities
OBE	Outcomes Based Education
PCK	Pedagogical content knowledge

PISA Programme for International Student Assessment

PK Pedagogical knowledge

RNCS Revised National Curriculum Statement

SACMEQ Southern Africa and Eastern African Consortium for Monitoring Educational Quality

SCK Specialized content knowledge

SLT Structural Learning Theory

TEMAG Teacher Education Ministerial Advisory Group

TIMMS Trends in Mathematics and Mathematics Sciences

TK Technology Knowledge

WHOT We Help our Teachers

ZPD Zone of Proximal Development

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CHAPTER ONE

1.1 INTRODUCTION TO THE STUDY

The educational authorities in South Africa have made many changes to the mathematics curriculum in the post-apartheid era. South Africa saw the curriculum evolve from one based on Christian principles to Outcomes Based Education and currently Curriculum and Assessment Policy Statement (CAPS). The inequalities pre-apartheid, changes in the demographics of our society and anxiety in respect of learner attainment have impelled such changes. The equalities and concerns still linger long after the demise of apartheid twenty five years later. The quality of mathematics results in South Africa is questioned annually due to the performance of the learners in the Senior Certificate examinations and previously in the Annual National Assessments (ANA). According to the Department of Basic Education (2018:11) teaching and learning in Mathematics is not yielding the relevant outcomes as expected and this is revealed by "*the low learner achievement levels*" in ANA and other educational studies. Reports such as the Trends in International Mathematics Study (TIMMS) (2008) and Southern and Eastern Consortium for Monitoring Education Quality (SACMEQ) (Spaull, 2011) indicated that South African teachers experience great difficulty in the field of mathematics when compared globally and the Department of Education has acknowledged this in their CAPS amendments document

(Department of Education, 2019). It was the learners and teachers poor results that forced the South Africa's Minister of Education to halt any participation in future TIMMS and other comparative studies. More recently the national Minister of Education in South Africa took the decision to 'discontinue' the ANA in South African schools until further dialogue has taken place between teacher unions and representatives in the educational arena. In my opinion, the decision to discontinue the participation in TIMMS and the non-implementation of ANA in schools and the constant reviewing of CAPS indicates the parlous state of mathematics in South Africa.

Mathematics teachers are not born gifted. In order to become a successful teacher one has to go through the learning process of acquiring the knowledge of mathematics. There are many factors that affect the quality of teaching mathematics and one of them is the content knowledge. Learner's marginalization of mathematical knowledge, evident in the ANA and TIMMS evaluation, will be reliant on current teacher knowledge for improvement. There is an urgent need for the acquisition of mathematical content knowledge because the pre-service teachers will soon find themselves as fully fledged teachers in front of a class of eager faces. The question one needs to ask is: how can mathematics knowledge be taught effectively?

Since the beginning of civilization, visuality (visual awareness) has been a key component to communication and understanding. I have examined the use of visualization in problem solving, more especially its central role in problem solving when used in conjunction with any problem solving strategy. The learners are reluctant to use visualization in mathematics due to not understanding the importance of it in mathematics and teachers accord little significance to visual aspects of mathematics in the classroom viewing it as time wasting. I am of the opinion that visualization skills are essential for a clearer perception and interaction with those around us in the classroom and this is given credence by Ahmad, Tarmizi and Nawani (2010:357) who found that the use of visual representations in mathematics word problems are exceptionally valuable. In order to use visualization as a tool, pre-service teachers should be aware of what it entails.

Visual skills and mathematical representation in any form has become a significant component of communicating mathematics in everyday life and is essential for extracting information, obtaining knowledge and building successful educational outcomes (Bamford, 2003). Further technological advancements at schools have made it possible for increased investigation, enhanced representations and communication of ideas thus allowing learners "*to solve problems in ways that are often impossible without these tools*" (van de Walle, Karp and Bay-Williams, 2014:3).

Teaching mathematics is a very complex activity and fewer teachers are willing to teach this subject due to its challenging content material. The ever political demands to change the curriculum in South Africa to cater for the masses meant there was a necessity to examine what mathematics content was to be taught in the different grades, teaching methods to be used by teachers in putting across the content and more so the tools and strategies that were needed to support these methodologies. In the current education climate in South Africa there is an extreme difference in the quality of teaching amongst teachers. The vast majority of teachers still function at the level of traditional teaching at the expense of using visualization and technology in the classroom. Although it can be argued that not all aspects of the mathematics curriculum can be taught using visualization, it must be upgraded to that of been supportive to both teachers and learners in the mathematics classroom.

In South Africa our school population has become more diverse making teaching and learning more culturally and linguistically challenging. Teaching occurs in direct face to face interaction with the learners and in divergent educational settings. The TIMMS report (Spaull, 2011) delivered an in-depth argument of the education scenario in South Africa and provided comparative results on the curricula and teaching practices in South Africa with other countries. The results placed South Africa in a precarious situation when compared internationally. It has been suggested that the universities and the faculties within them that are responsible for training teachers of mathematics should look at ways to improve the quality of education in order to achieve better results (Pavlekovic, Kolar-Begovic and Kolar-Super, 2013). The way pre-service teachers are trained to handle these changes will eventually have a knock on influence on the results that schools produce.

A lot of emphasis has been placed on problem solving in South Africa and internationally (National Council of Teachers of Mathematics, 2000). In many countries problem solving is seen as a critical component within the mathematics curriculum (Duru, Peker, Bozkurt, Akgun and Bayrakdar, 2011) thus problem solving is seen as the essence of mathematics and central to teaching mathematics. I have found that problem solving is fundamental to teaching and learning and this view is shared by Wilson, Fernandez and Hadaway (1993:66) who stated that "*the art of problem solving is the heart of mathematics*".

Whilst problem solving is now been given more prominence as the cornerstone of school mathematics, George Polya, considered to be the pioneer in problem solving, found that the aim of the mathematics curriculum is to develop the learners aptitude to solving problems (Polya, 1965:100). Learning how to solve mathematical problems, for various reasons, has been a long standing difficulty faced by learners in South Africa. The poor literacy level has been cited as one of the reasons for the poor performance. Furthermore, mathematical problem solving does not simply entail simple computational tasks but are mathematical problems which require appropriate choice of strategies and decisions that will lead to logical solutions (Ahmad et al, 2010:356). The problems are more unique and challenging than ordinary mathematics task. Hence the learners must use knowledge and skills in a flexible way not only for solving problems but to also use it as a foundation for learning new problem skills and knowledge (Department of Basic Education, 2018:15).

The primary purpose of this study was to identify these strategies (problem solving strategies and visualization) used by pre-service teachers when teaching problem solving. I found that teachers neglected the teaching of problem solving and problem solving strategies hence me focussing on the pre-service knowledge to teach problem solving using visualization. Anthony and Nalshaw (2009) stated that to assume that all teachers (I will also include pre-service teachers) to be experts is both unreasonable and unnecessary but they need to have a firm grip and expertise to solve problems.

1.2 MOTIVATION FOR THE STUDY

As the study of education in mathematics advances and the complexities of teaching problem solving is recognised, there is still a necessity to further investigate teaching problem solving in a bid to prepare future teachers for the challenges that lie ahead in teaching mathematics. According to Stein, Boaler and Silver (2003) the crucial research question for the next decade is: what happens inside mathematics classrooms in which problem solving approaches are used?

I have been a mathematics teacher for the last 33 years and have witnessed drastic changes to the mathematics curriculum. The recent standardization of the curriculum has made it more accessible to the learners but complicated for the teachers because the challenges have not been met in how teachers teach problem solving. Although problem solving is a prescribed component of the mathematics lesson (Department of Basic Education, 2011), I have observed that the teachers are reluctant to incorporate the teaching of problem solving into their daily mathematics lesson. This observation is based on my supervision programme of educators at school level and the moderation of teacher portfolios at ward level. Stein et al (2003) stated that teachers are resistant to problem solving even though the curriculum specify it be taught. Noticeably this lack of interest or negativity has had a rippling effect in that schools are reluctant in participating in problem solving competitions with the teachers citing difficulty as the main reason.

The rationale for choosing problem solving and visualization was the lack of interest on the part of classroom based teachers and their attitude towards this crucial aspect in the mathematics curriculum hence this angle of study with the pre-service teachers. Duru et al (2011) stated that the pre-service teachers will become prospective educational leaders and they will become responsible to educate the learners in problem solving in the future. Many pre-service teachers are enrolled in teacher education programmes at private and public higher education institutions in South Africa. The area of concern to me is the standard of training given to these pre-service teachers in the field of mathematics at the various tertiary institutions (considering that South Africa does not have any fully fledged teacher training institutions).

Any future problem solving research must pay closer attention to the subject mathematical content knowledge and pedagogical knowledge proficiencies a pre-service teacher should possess. Research must look at mathematics in such a manner that it takes a "*new pedagogic-content outlook*" in order to transform Mathematics teaching in South Africa (Department of Basic Education, 2018:12). Duru et al (2011:3463) stated that the application and adaptation of a variety of appropriate strategies to solve problems is necessary because teacher's content specific knowledge, beliefs and attitudes influence students learning outcomes. Therefore the prospective teachers need to be well trained and supported in the classroom. In general, pre-service teachers need to be given specialist training in the field of mathematics on how to use mathematical strategies effectively because it requires time and motivation.

The purpose of this study was to investigate pre-service teacher's problem solving skills and preference of using visualization in problem solving.

These factors motivated the present study.

1.3 KEY RESEARCH QUESTIONS

This study was guided by the following questions to investigate:

1. What strategies do pre-service teachers use when solving problems?
2. How pre-service teachers teach problem solving in the classrooms?
3. How do visual strategies affect the teaching and learning of problem solving?

1.4 STRUCTURE OF THE STUDY

This study comprises six chapters, bibliography and appendices. The chapters in this study are as follows:

Chapter 1 introduces the background to this study. The key research questions are also stated together with the background of why this study was undertaken.

Chapter 2 presents the relevant literature on the areas under investigation namely problem solving strategies and visualization.

Chapter 3 presents the theoretical framework. The relevant theories utilized in this study are discussed herein.

Chapter 4 presents the research design, the research methodology and processes undertaken to complete this study. It also discusses the research instruments used to conduct this study.

Chapter 5 details with the findings and analysis of the data obtained from the questionnaire, classroom observations, semi-structured interviews, evaluation worksheet and examination of learner's books.

Chapter 6 is the final chapter, which presents the limitations of the study; the conclusion and recommendations are made to be considered by the pre-service teachers, teachers, curriculum planners in the Department of Education and those in the greater education fraternity.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

We are living and teaching in a globally challenging and technologically evolving period. The rapid transformation and complexity of today's world has presented fresh challenges and demands on our education system. The changes to the mathematics curriculum have also led to many instructional and pedagogical challenges in the classroom. Added to this the mathematics curriculum and assessment content is forever changing internationally (National Council of Teachers of Mathematics, 2000) and in South Africa (Department of Education, 2019). These changes to the curriculum have been made to accommodate the needs of South African teachers and learners. The learners are now being exposed to a higher level of content that need to be taught in each grade and the teachers need to adapt to teaching the new curriculum. Santos and Domingos (2013:3237) stated that the changes made to the mathematics curriculum were to cater for new methods of teaching and learning mathematics. The new methods are needed to help improve the declining academic results in mathematics in South African schools.

Curriculum reformation in South Africa post-apartheid made it nigh on impossible for teachers to perform at their optimum level. These changes to the curriculum (Outcomes Based Education) made teachers struggle to adapt to the content before it was modified again. This meant that teachers needed continuous comprehensive re-training in order to undo the old (NCS/RNCS) and implement the new curriculum (CAPS) in order to meet the educational needs of the masses. This meant that South African teachers needed to be trained to accommodate the introduction of the content driven CAPS. This affected teaching and learning as it was now a great jump from what was previously taught. Despite these changes to the curriculum and the attempt at re-training teachers, there has been strong resistance to implement these changes by teachers. This is very noticeable at Department of Education organised refresher courses where teachers complained about the challenges they faced in the classrooms in implementing these changes (content and assessments). Although the current curriculum was approved as National Policy in 2011, it is only now that the Department of Education (2019:3) has acknowledged that it "*has considered the concerns and has agreed to undertake a holistic review of the CAPS documents*". This acknowledgement comes after a number of concerns were received from teachers and education stakeholders "*about the challenges of implementing of the Curriculum and Assessment Policy Statements*" (Department of Education, 2019:3). It is essentially factors like "*curriculum overload*", "*poor curriculum coverage*", "*lack of guidance*" and "*the need to create more time for teaching*" (Department of Education, 2019:3) that will affect how the pre-service teachers will be trained to teach, how they will build their curriculum knowledge during their training and how they will cope with these changes over time. All of more these aforementioned factors and more must be considered during curriculum reformation otherwise in the years to come CAPS will endure other amendments to meet other curriculum challenges. According to Zikre and Eu (2016) both teachers and learners will need 21st century skills to succeed in today's mathematical challenging world. The teachers need to examine the techniques that they will use to teach and also determine how they will engage with the mathematical subject matter. Hence the demands on the pre-service teachers will be many as several 21st century teachers are now realizing that teaching strategies and skills (visual) are crucial in order to exist in a vastly complex mathematics world (Tufte, 2008:1). In South Africa, the Department of Basic Education believes that CAPS has the potential to equip the learners "*with the skills for the 21st Century*" and also "*prepare them adequately for the demands of the 4th Industrial Revolution*" (Department of Basic Education, 2018:12). Why then are we as a country under performing in the field of mathematics? The shift of mathematics from the 20th century to the 21st century will prove challenging to South Africans.

The literature on problem solving and visualization that is presented as part of this review outlines my motivation for this study. It also guides the discussion around the research questions. The literature review provides an overview of problem solving and visualization. It also provides the sub aspects that are interrelated to the broad traits of problem solving and visualization. This review will also provide background knowledge and explanations on some key issues involving problem solving and visualization.

2.2 PROBLEM SOLVING

Due to the importance of its theoretical knowledge that is needed in practical form in all works of life, mathematics has long been hailed as a subject that opens doors to many professions and occupations. Not having the expected theoretical knowledge places people at a total disadvantage and the innumeracy deprives people of the opportunity of functioning in everyday tasks. Be it a local trader on the street or an actuarial scientist, the acquisition of theoretical knowledge in a practical form to face situational issues cannot be ignored. All professions and occupations are sometime or the other confronted by one aspect, namely, solving problems. Problem solving, albeit a complex process is the best approach for learners to experience and learn mathematics. It has a central importance in a subject like mathematics and achievement in this aspect is highly dependent on acquiring problem solving strategies. It does not only involve converting words into numerical operations but requires higher order thinking skills and strategies. Schoenfeld (1987) stated that mega cognitive and cognitive skills will assist learners create a thinking plan that will entail strategies and mathematical skills to solve problems.

Many studies have been conducted in problem solving by Polya (1945), Schoenfeld (1983), Lubienski (2000), NCTM (2000), Montague (2005) and Peker (2009). Its importance in mathematics has been a focal point in the United States of America (USA) (National Council of Teachers of Mathematics, 2000), England and Wales (Cockcroft Report, 1982) and an area of concern internationally in countries like New Zealand, Singapore, China, Malaysia and South Africa. The results of these studies have become the catalysts for focussing more on problem solving in mathematics.

Mathematics as a subject includes aspects of acquiring knowledge, understanding concepts and using the acquired knowledge in complex situations. In problem solving situations ideas are formed in the mind when learners are confronted with mathematical concepts. These concepts become an idea which "*enables learners to make sense of mathematics*" and "*make connections between ideas*" (Department of Basic Education, 2018:15). These ideas are actual thoughts created in the mind and is communicated by means of signs, symbols and schematically onto paper. These signs and symbols are representations which portrays ones understanding. These ideas are written or represented as a solution in a manner that is comprehensible to others. Montague (2006) stated that problem representation and execution is important in the problem solving process. The representation of the problem indicates that the learner has comprehended the problem and this serves as a guide to solving the problem.

As an experienced classroom mathematics teacher for the past 33 years, including been an independent contracted lecturer at a private higher education institution, examiner of the AMESA mathematics problem solving competitions, examiner and moderator of the Mathemagica Plus problem solving competition in the Mafukuzela-Gandhi Circuit, examiner and moderator of mathematics common paper in the Phoenix North Ward and the management and supervision of mathematics teachers, I have found that there is a distinct lack of focus to incorporate the teaching of problem solving strategies together with visualization into their teaching although it is prescribed component as stated in CAPS (Department of Basic Education, 2011).

The current curriculum changes in mathematics have affected the teaching and learning process in South Africa. In a modern day scenarios in schools especially from grades 4 to 12, a lot of 'learning' takes place through procedural application and memorization. Mathematics is often taught as a series of steps to memorize and reproduce what was taught. Mathematics teachers, who are seen as the only source of mathematical knowledge in the classroom, provide exercises to learners to complete repeatedly in order to 'master' the problem solving process through arithmetical means. Unfortunately "*it is no longer sufficient for learners only to learn how to reproduce the steps in the calculations that they are shown by teachers*" as it is "*insufficient for progression in mathematics*" (Department of Basic Education, 2018:15). Schoenfeld (2013) stated that there is a huge focus on conceptual understanding and mastery of skills and procedures in schools as a result there is a negligible amount of problem solving. The teachers need to cultivate the learner's interest in mathematics problem solving such that they become fascinated with finding and understanding solutions to the problem which in turn rouse their curiosity in engaging more effectively with mathematical situations. According to the Department of Basic Education (2018:15) "*learners need teachers to thoughtfully and strategically push them to progress*". Therefore, teachers need to be seen as those who can assist the learners reason with mathematics so that they can connect with what is been taught. A central feature of the modern day teacher is to rely on conventional methods to teach mathematics as a result learners are denied the essence to learn and enjoy the beauty of mathematics. The learners need to engage with the content in mathematics. Anthony and Walshaw (2009:150) stated that "*simply inviting students to contribute a response to a mathematical problem may not achieve anything more than co-operation from students*". A simple 'yes' or 'no' is not sufficient in the classroom. The teacher needs to communicate and engage the learners in the subject matter to create a better understanding of mathematical concepts. Following a conventional style of teaching will affect the learning process in the classroom. It can lead to high level of retrogression in the subject resulting in a high failure rate as the learners do not fully understand the mathematics they are being taught.

Throughout the last three decades numerous attempts have been made in numerous countries to make problem solving the focal point of school mathematics rather than teach it in isolation. Due to its powerful characteristics problem solving has been accorded great significance in the mathematics curriculum as a skill to be taught, as an objective for mental developmental and as an approach for teaching (Fadlemula and Cakiro, 2011:2). Although the curriculum guides, conference reports and textbooks maintain problem solving become fundamental to mathematics, mathematics teachers do not have sufficient knowledge about what mathematics problem solving is and what problem solving refers to (Lester, 1985).

The phrase 'problem solving' may have different interpretations depending on the context it is used. Some may view it as a teaching method, some as an exercise whilst some may view it as dealing with problematic situations. Expecting learners to complete a classroom activity on word problems from the textbook to test their knowledge on the taught algorithms is not problem solving. Reading and recalling an algorithm make it an exercise. These kinds of problems are solved by applying a simple algorithm. I refer to this as a mechanical regurgitation of the lesson because the learners reproduce work in the classroom because 'my teacher told me to do it like this'. To put problem solving in its perspective it will be important to examine the ensuing definitions of problem solving:

Polya (1962:v) described problem solving "*as finding a way out of a difficulty, a way around an obstacle attaining an aim which was not immediately attainable*".

Hyde (2006:8) stated that a problem is "*a task for which the person confronting it wants to find a solution, but for which there is not a readily accessible procedure that guarantees or completely determines the solution*".

National Council of Teachers of Mathematics (2000) mentioned that problem solving means engaging in a task for which the solution method is not recognizable in advance.

Mayer (1992) stated that a problem occurs when you are confronted with a given situation (let's call that the given state) and you want another situation (let's call that the goal state) but there is no obvious way of accomplishing your goal. Thus one can mention that problem solving refers to the process of moving from the given state to the goal state of a problem

The Cockcroft Committee (Backhouse et al, 1992) described problem solving as the ability to apply mathematics to a variety of circumstances in reality.

To summarise the above definitions, problem solving can be regarded as a process that involves using mathematical knowledge, higher order reasoning and decision making skills and strategies to solve a problem when a solution is not readily attainable. It is something that learners grapple with since a solution is not readily available and it requires them to fashion a solution using multiple strategies and representations. It is like crossing a swollen river during a storm not knowing where the banks are.

Two types of word problem are normally used in the classrooms, namely routine and non-routine problems. Routine problems are mainly used by teachers to test learners understanding of concepts taught in a lesson. These kinds of problems can be easily solved as the learners have been previously exposed to or have the necessary acumen to solve such problems.

An example of a routine mathematical problem:

Peter had 15 marbles. He gave Vani 7. How many marbles does Peter have left?

In routine problems the solution is easily identifiable. In the above example the teacher is testing the learner's knowledge on subtraction. In a normal classroom scenario the teacher demonstrates the learners how to use the appropriate operational sign or alternatively use visual representations to communicate their answer. Learners are taught in grade R to use objects, fingers or representations of each of the given numbers in the problem. They subtract and then count what is left over to arrive at their answer. I disprove of this kind of teaching as the learners are being incorrectly taught to solve problems by simply applying algorithms. This kind of teaching will lead learners to guessing the operation or the answer without demonstrating any understanding of why they are they using the operation or how they arrived at the answer. The Department of Education want teachers to move away from this kind of teaching because "*if children learn procedures without understanding, their knowledge may be limited to meaningless routines*" (Department of Basic Education, 2018:16).

A non-routine problem, where the solution is not readily available, requires a higher level of thinking. Solving such kinds of problems also requires reading for understanding and conceptual understanding. The learners need to be proficient in reading and have good comprehensions skills in order to make connections with the concepts in the mathematical problem. The learners understanding the mathematical concepts (conceptual understanding) "*leads into the procedures that a learner will use*" when solving problems (Department of Basic Education, 2018:16). This understanding will enable them, the learners, to choose an appropriate problem solving strategy or try to formulate their own when confronted with new problems. Accordingly the Department of Basic Education (2018:17) has indicated that "*learners should be able to make sensible decisions on what strategies to employ or to devise their own strategies to solve certain problems*". Since there is more than one way to solve a problem and depending on the degree of difficulty of the problem, the learners should apply the chosen mathematical strategy or their strategies to solve problems.

An example of a non-routine problem: The distance from town A to B is 15km. The distance from town C to D is 35km and the distance from town B to D is 70km. Virginia cycled one quarter of this journey, hitch-hiked two fifths of the journey and used public transport to complete the remainder of the journey. Determine the distance travelled by public transport. In the above problem one needs to comprehend it and then decide how to tackle the problem.

In mathematics there is no specific way to solve problems but literature over the years have directed teachers in using specific models or strategies as a guide to teach problem solving. One such model was designed by George Polya. George Polya's work in mathematics (Polya, 1945), more importantly his 4 step model (Figure 1), has been widely accepted as the norm to solve problems and this model continues to be the cornerstone on how problem solving is taught in schools.

In his book How to Solve It (Polya, 1945) Polya addressed the many issues in problem solving. He treated problem solving as experimental when using his 4 step method. Polya's four steps (Figure 1) involved understanding the problem, devising a plan, carrying out the plan and looking back to examine the logic of the solution. Referring to understanding the problem, Polya (1957:6) stated that "*it is foolish to answer a question that you do not understand*". He mentioned that teachers should ask general questions which the learners can apply to other problems as well. He stated that teachers need to commence with a general question and progress to a more specific question until a response is elicited in the learners mind (Polya, 1957). The problem must be such that it must motivate the learner to find a solution. Devising a plan is the crux of the problem solving process and Polya supports its importance when he mentioned that "*the main achievement in the solution of a problem is to conceive the idea of the plan*" (Polya, 1957:8). When the learner arrives at a solution he needs to look back or reflect on what was achieved. According to Polya (1957, 1962) a good teacher impresses on his learners that no problem solution is completely exhausted. The learner needs to check the solution, examine other possible solutions or make connections to other problems. By looking back at the completed solution and re-examining the result could consolidate their knowledge and develop their ability to solve problems (Polya, 1957).

Polya's model (Figure 1) to problem solve is linked to one of the earliest attempts by Wallas in 1926 in The Art of Thought (Reynolds and Flagg, 1983:232). It can be deduced that Polya's and Wallas's steps (Figure 2) are interwoven. The first step understanding the problem corresponds with preparation; devising a plan and carrying out the plan is similar to incubation and illumination and looking back in order to check the results corresponds with the verification stage. Taking into consideration the steps set out by both Polya and Wallas, one gets the impression that problem solving is straight forward as following a recipe. It is not the case as solving problems requires various strategies and techniques in order to arrive at the solution. I dislike presenting the entire Polya's plan to my learners. I prefer emphasizing step one and step four and let them find their own way in between. The learners will be forced to engage with the problem by moving between the stages in order to find the solution.

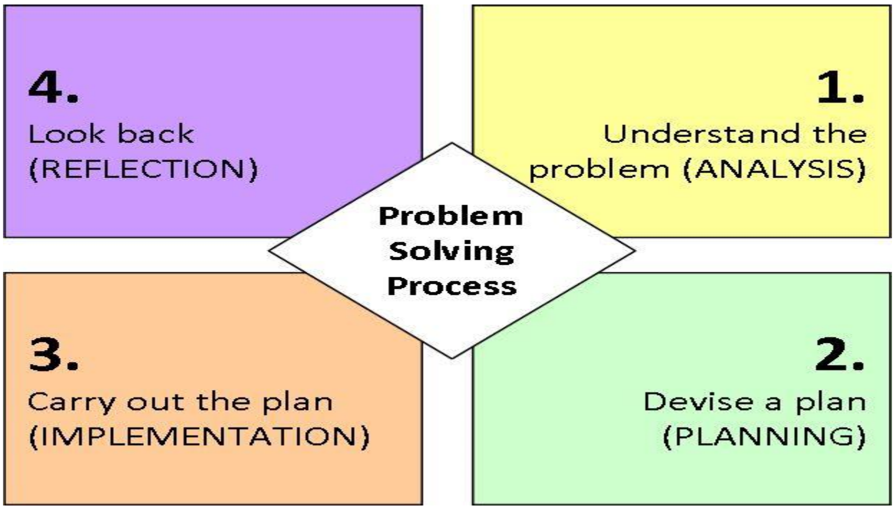


Figure 1 George Polya's Problem Solving Steps.

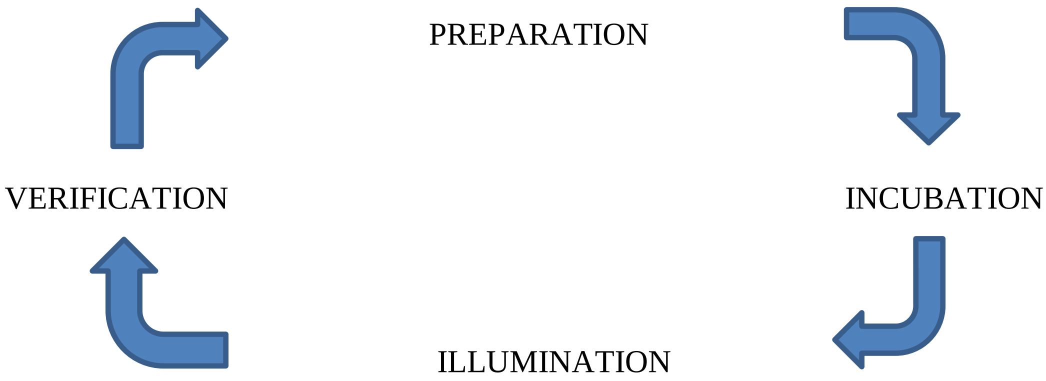


Figure 2 Wallas's Problem Solving Steps (Reynold's and Flagg, 1983)

According to Verschaffel, Greer and de Corte (2000) solving problems involves many phases. The first phase concerns reading the problem, understanding the problem and making a representation of the problem. The next phase involves mathematizing, that is, translating the problem into a mathematical form, identifying the numerical and linguistics elements contained in the problem and identifying the relation between these elements. The last three phases in the problem solving process are executing the mathematical operations within the problem, interpreting the outcome and formulating a solution and evaluating the solution.

Problem solving is clearly seen as a teaching method with links to experiential learning (discussed in chapter 3) and this is supported by the research and writing of Vygotsky on childhood development and learning. Learner's individuality and their personal experience allow them to also believe that the curriculum cannot be compartmentalized because problem solving in mathematics plays an important role in maintaining the interconnectedness. Problem solving should not be an isolated part of the mathematics curriculum. Problems cannot be taught in isolation because one of the four cognitive levels in CAPS is problem solving (Department of Education, 2011) and is linked to the others which are knowledge, procedures and complex procedures. In order to make this link understandable, mathematics as a subject and language requires practice and understanding. According to Foshay (2003:1) problem solving can serve as a core curriculum strand because it joins together the various disciplines, rules, concepts, strategies and skills in mathematics. The understanding of the various links between the innumerable rules, concepts, strategies and skills in mathematics results in the creation of mathematical knowledge. This mathematical knowledge is constructed by individuals or groups. In the problem solving process learners construct their knowledge through experience which is acquired from their real world.

According to Mayer and Wittrock (2006) and Kandemir and Gur (2009) problem solving share an incredible relationship with thinking, reasoning, decision making, critical thinking and creative thinking. All of the aforementioned requires higher order understanding on the path of the learners. Thinking refers to the cognitive processing of the problem solver, that is, directed thinking (problem solver) and undirected thinking (daydreaming) (Mayer and Wittrock, 2006). According to Carson (2007) thinking is more important in problem solving because teachers use problem solving to teach learners how to think abstractly. Attaining a relative higher level of thinking skills enables the learners to break up the problem into component parts. Reasoning involves drawing conclusion by using logical rules based on induction or deduction. Decision making is when one has to choose from several alternates based on some criteria. The problem solver has to resolve on the soundness of the strategy to find the solution. Creative thinking entails looking at the problem in various ways, generating alternatives or using resources in unique ways to arrive at the solution (Mayer and Wittrock, 2006; Church, n/d). Learners need to be encouraged to think and respond on how they have solved the problem without them been told that they are incorrect. Critical thinking is also acknowledged as logical or sequential thinking. It is the capacity to mentally break down the problem into segments (Figures 1 and

2) by classifying, comparing for similarities and differences, analysing them and then testing these alternatives in order to arrive at a solution (Mayer and Wittrock, 2006; Church, n/d).

Teachers find it difficult to determine the difference between giving a task and asking their learners to do a problem solving exercise in mathematics. Teachers believed that mathematic problems were exercises to be solved at the conclusion of lesson in order to rehearse or consolidate what was taught in the lesson. They also believed that these problems were merely a translation of a standard problem into a number sentence and had only one specific answer. However, Schoenfeld (1988) distinguished between problems and exercises. He stated that both are vital in the mathematics classrooms. The learners engage mainly in completing exercises and are seldom challenged to solve problems. A problem is not immediately obvious and it takes time to unravel. For a task to be a problem it must contain different situations and must also contain an intensity of challenge (Fadleluma and Cakiro, 2011).

Learning is not a spectator sport. The learners do not learn just by sitting in the classroom listening to the teacher. A conducive and supportive classroom atmosphere is necessary to build the confidence of learners in the teaching and learning of the problem solving. Schoenfeld (2013) mentioned that a rich classroom atmosphere and environment should embrace learner's involvement in the framing of the questions, explaining themselves through disciplined arguments and making resources to support their claims. This is relative to constructivism and the work of Piaget, Vygotsky and Dewey. Piaget noted that children in particular construct knowledge out of their actions with the environment because that environment gives specific content (Harlow and Cobb, 2014). The learners must be able to talk about what they are learning, write about it and relate to it from their past experiences. They need to be given the opportunity to orally engage and explore the problem with the diverse group of their peers in the classroom where they challenge each other in a convincing manner and question to demonstrate their understanding.

Learning about mathematics and concepts therein creates conceptual knowledge in the mind of an individual. Understanding the mathematical concepts is a mental activity that learners try to connect in their heads thus learning becomes an active process. It is through an interactive learning environment that learners are provided with ample opportunities for learning. They question and debate with each other in a cordial manner over solutions provided by their peers before accepting the solution. Thus, orally or written, they communicate their ideas and results. This rich interaction in the classroom raises the chances of productive learning. Vygotsky's theory of social constructivism comes to the fore in this milieu. This connection between learners and the environment through communication (oral and written) creates a social order.

In this way they become responsible for the creation of their own knowledge as they make what they learn part of themselves. Dewey reflects the beliefs of Piaget that learners need to explore and experience the classroom atmosphere in a bid to learn and if teachers want to understand the cognitive development of the learners then they must be thoroughly aware of the environment in which the learners learn.

Schoenfeld (2006) stated that with the right kinds of instruction, learners could develop into more effective problem solvers. The same can be said about the pre-service teachers as well. They need to be guided through the problem solving process with proper instructions. They need to have a good conceptual understanding, relative comprehension of what is required and the necessary strategic competence in order to solve the problem. To gain such proficiency the pre-service teachers need to garner ways to teach with varied instructions to solve problems. They need to develop proficiency and gain fluency which according to the Department of Basic Education (2018:16) is "*developed through much repetition and practice*". It is through the scaffolding component that pre-service teachers of can be trained to solve problems through strategic competence. I believe that by using Polya's steps (Figure 1) in conjunction with the process devised by Marjorie Montague (Figure 3) it is possible to provide scaffolding (guided instruction) to the pre-service teachers.

1	READ (for understanding)
2	PARAPHRASE (your own words)
3	VISUALIZE (create a picture or drawing)
4	HYPOTHESIS (the plan to solve the problem)
5	ESTIMATE (predict the answer)
6	CHECK (make sure all the steps were completed correctly)

Figure 3 Montague Problem Solving Process (Montague, 2005)

In 1 (Figure 3), the pre-service teachers reads the problem to recognise what the problem is about, that is, getting to know the problem at hand. They read and reread the problem as they advance through the problem. Thus, they learn much of the problem by identifying and comprehending the key concepts within the problem. The pre-service teachers need to be aware that critical comprehension strategies are needed to "*translate the linguistic and numerical information in the problem into mathematical notations*" (Montague, 2005:3).

In 2 (Figure 3), the pre-service teachers need to be encouraged to break the problem down to state it in their own words. They need to "*ask themselves what the question is and what they are looking for*" (Montague, 2005:3). By breaking down the problem it will give the pre-service teachers a better understanding of which key information is to be selected so that they will know where to start.

In 3 (Figure 3), the pre-service teachers need to be creative and visualize the problem. They must engage with the information and represent it in another way. This engagement allows the pre-service teachers to familiarise themselves with the concepts in the problem. They examine the concepts from a mathematical perspective and visualize it creating a mathematical representation. This makes it easier to choose a strategy to solve the problem. The pre-service teachers need to recognize that they have many strategies to choose from to solve the problem and by brainstorming they are likely to come up with as many solutions as possible. The schematic illustrations (pictures or drawings) of the problem allow the pre-service teachers an opportunity to examine the problem from varying angles. The importance of visualization is evident in this process as it reveals the association between the significant parts of the problem (Montague, 2005:3). The diagrammatic representations allow the pre-service teachers to map out their mental imaging to build an understanding of the problem which leads them to see the solution clearly. Mental imaging is an enjoyable and interesting way to learn as they "*are now ready to develop a solution path*" (Montague, 2005:3).

In 4 (Figure 4), the pre-service teachers hypothesize. In their hypothesizing they weigh up the solutions, the operations involved and thereafter devise a plan to solve the problem. The pre-service teacher may use many representations and other problem solving strategies to solve a problem (Montague, 2005:3). In solving a problem both conceptual and procedural knowledge is used. The combination of both allows them to develop their thinking ability, their flexibility to choose the strategy and creativity in their representations.

In 5 (Figure 3), the pre-service teachers make estimations through guess and check in an attempt to predict the answer. The solutions are evaluated to determine the best possible plan (Montague, 2005). Luneta (2013:82) referred to this stage as specialising which is seen as an integral part of problem solving. The pre-service teachers try out further examples to achieve a sense of what they are trying to do to unravel the problem. They review the steps to the solutions and decide how to continue.

In 6 (Figure 3), the pre-service teachers need to ensure that the necessary steps were completed and they met all the criteria before implementing the solution. They need to ensure that they have used the correct steps to check if the answer is acceptable and it works (Montague, 2005:3).

In order to support reading, paraphrase and visualize (Figure 3), I discovered that the Frayer Vocabulary Model (Florida Department of Education, 2010) as an important to assist the pre-service teachers to understand the concepts in a mathematical problem. Whilst scholars such as Levenson, Tirosh and Tsamir (2004) stated that younger learners may well experience difficulties in understanding mathematical concepts and explanations or verbal explanations may be beyond their comprehension, the same can be applicable to the pre-service teachers. The challenges pre-service teachers face in respect of explaining mathematical concepts are discussed in chapter 4. Using the Frayer Vocabulary Model one can commence with a definition and thereafter use visual assistance to reinforce a concept to support their understanding.

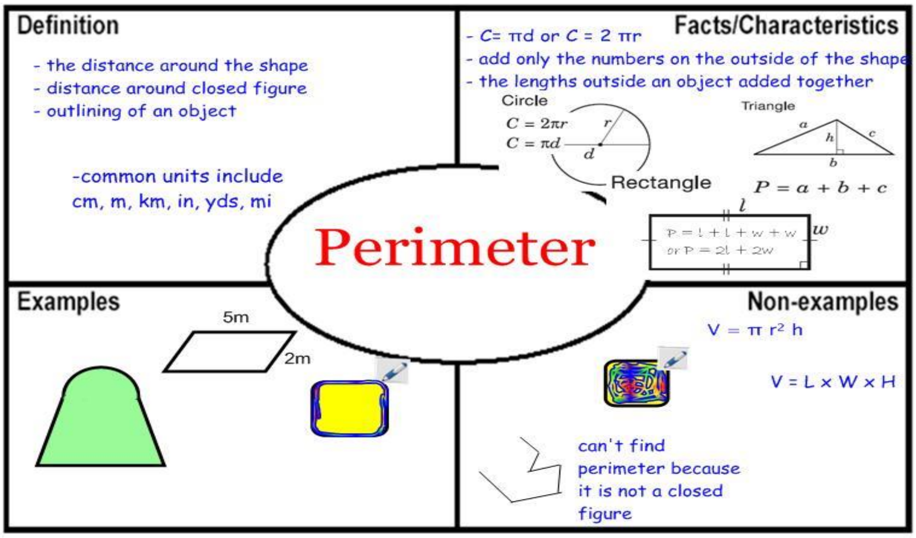


Figure 4 The Frayer Vocabulary Model

Mathematics content material is often the most difficult to interpret. Comprehending and understanding mathematics is attributed to its vocabulary. I believe that the Frayer Vocabulary Model (Figure 4) will suit the South African classroom situation because many of the mathematical concepts used in the lessons have diverse meanings due to the large cultural divide and linguistic differences that exists in South Africa. According to the Department of Basic Education (2003:32) the "*part of what makes learning mathematics in non-mother instruction difficult is that there are poorly developed lexicons for most of South Africa's indigenous languages*". The Frayer Vocabulary Model, as a concept map, makes it possible for pre-service teachers to make relational connections with the used vocabulary and by doing so understand concepts.

The pre-service teachers can use the Frayer Vocabulary Model advantageously in their teaching and learning classroom. They list the mathematical word, example, Octagon (Figure 4). They ask the learners to provide definitions of the word in their own words. Once this word is associated to mathematics it becomes a mathematical concept. To reinforce this concept the learners list the main characteristics and draw examples and non-examples related to this concept (Florida Department of Education, 201:9). I believe this an important phase in the problem solving process as it reflects insight and understanding on the part of the learners as they are able to translate verbal to visual. This can be likened to killing two birds with one stone in the classroom. The learners interact with the vocabulary by linking what is read and comprehended to create their visual mathematical concept. Using the Frayer Vocabulary Model constructively in the classroom will give the pre-service teachers an opportunity to gauge any forms of misconceptions since the words used in mathematics to represent mathematical concepts may differ depending on the learner's locale knowledge. Due to the diverse nature of the modern day classrooms, meanings will differ. As a measure of caution the mathematics teachers must discuss and reinforce meanings with the learners to avoid any forms of misinterpretation. I use the concept of revolution as an example. Revolution as a mathematical concept is rotation of 360˚ whilst in the historical context it refers to the forceful removal of a government or leader. Any misconceptions that are detected must be dealt with immediately and in an effective manner. Also by asking the learners to elucidate new concepts in their own words can assist them in assimilating it by forcing them to re-express their innovative ideas in their existing vocabulary. They draw inferences to the real things in their world thus activating prior knowledge. This is beneficial because by visualizing the concept in the problem the learners get a better understanding and by relating it to the real world they learn new mathematics with greater understanding.

Understanding the problem relies on many factors like mathematical terminology, comprehension of the problem, the ability to visualize the problem and make connections with existing ideas. Comprehension in the problem solving process is imperative in order to find the pathway to the problem situation. The comprehension of the problem can be divided into three groups, literal comprehension, integral comprehension and fine comprehension (Barake, El-Rouadi and Musharrafeh, 2004). In the literal comprehension, the learner understands the key words and ideas in the text; in the integral comprehension the learner comprehends the problem fully. It builds on the literal comprehension and adds to the representation of the text which allows the learner to deduce the salient information; in the fine comprehension the learner has to have a deep understanding of the problem in order to obtain implicit information by looking for clues and other information (Barake, El-Rouadi and Musharrafeh, 2004). A similarity is very noticeable with Polya's problem solving process when one closely examines the comprehension aspect as described by Barake, El-Rouadi and Musharrafeh (2004). Within this comprehension sphere emerging ideas created literally and through representation become integrated with existing ones thus improving their mathematical power of understanding and reasoning.

When mathematics progression is compared internationally, it has been found that the Asian nations like Shanghai, Singapore, Hong Kong, Taiwan, Korea, Macao and Japan are ranked in the first seven places in the world (Programme for International Student Assessment, 2012). These are the countries that underwent curriculum reforms. With its well trained teachers and excellent pedagogical content training in the curriculum there has been an improvement in their results. The curriculum and pedagogical approach was clear and distinct in what constituted school mathematical knowledge. Norton and Zhang (2015) examined the success of the Chinese learners in mathematics and Takahashi (2008) described the Japanese learner's growth in problem solving. In China the most important role of school mathematics is to establish a strong foundation for learners to acquire strong basic skills and knowledge so that the learners can problem solve in advanced mathematics. According to Norton and Zhang (2015) the Chinese approach to problem solving is that when basic knowledge and skills have been acquired, problem solving can be accomplished. In adopting the Chinese approach, the Chinese learners follow the four steps in their learning approach, namely, the learners commit the primary knowledge to memory; they comprehend the meaning of the material; they try and relate the understanding to situations that call for such knowledge and then finally they enter a higher level of enquiring and adapting the original material (Norton and Zhang, 2015). The Japanese teachers give their learners the problem without showing them how to do it and they need to construct their knowledge by solving the problem (Takahashi, 2008). There is an enormous amount of pressure on these learners where they have to have a high memory recall, precision and speed in mathematics. This kind of engagement inculcates a higher level of thinking. This interaction with the material improves teaching and learning as it enhances the learner's ability to see mathematics as meaningful to their lives. As the world changes there is a lot of emphasis to learn through authentic tasks in the classroom. This is invaluable to learners as they are able to learn within a school environment and relate their acquired curriculum knowledge to out of school situations. This is referred to as reality mathematics.

When a learner is confronted with a mathematical problem outside the school environment, he has to think, analyse the data and concepts in the problem, and decide on the strategies to be used before working towards a solution. The learners need to know how to use their mathematical knowledge, prior knowledge and mathematical skills to solve problems. Although assisting learners how to utilise strategies and develop problem solving skills lies in the hands of the teacher, a teacher must allow them the freedom to explore alone. Whenever a class is working on developing problem solving skills the teacher must allow them to work without guidance. He needs to step back and allow them to develop their problem solving and decisions making skills. In the South African classrooms this style of teaching is sadly lacking. The teachers 'spoon feed' the learners or give them examples repeatedly to do to test their understanding of procedural knowledge. This is not an ideal teaching and learning situation. The learners need to be left with a challenge when problem solving. By doing so learners develop their own approach in solving the problem thus enabling them to utilise their own knowledge rather than relying on the teacher. This will allow the learners to demonstrate their confidence and independence when faced with a problem in reality.

Besides teaching learners the elementary skills on how to add, subtract, multiply and divide in mathematics, there is much debate on how problem solving ought to be taught. The question then, how is problem solving taught? In today's mathematics environment the jury is out on whether "*teaching for problem*", "*teaching about problem solving*" and "*teaching through problem solving*" (van de Walle and Williams, 2014:54) is appropriate. These three approaches are highly accepted by the international mathematics community and it is through these approaches that the foundation is laid for future problem solving exercises.

Teaching for problem solving involves assisting the "*learners to obtain the knowledge, understanding and skills*" (Killen, 2013:258). Through teaching for problem solving, learners are taught mathematical ideas which they learn through understanding rather than through rote. A distinction needs to be made between learning by rote and learning by understanding. Rote method of instruction leads to good performance on retention tests but poor performance on transfer tests whilst learning for understanding leads to superior retention and excellent transfer performance (Mayer and Wittrock, 2009). Rote instruction creates reproductive thinking – applying already learnt procedures to a problem – whereas meaningful instruction leads to productive thinking – adapting what was learnt to new kinds of problems (Mayer and Wittrock, 2009). Foshay (2003:5) stated that problem solving is a situational and context bound process, is dependent on knowledge and experience. He further declares that when teaching problem solving, authentic problems in realistic contexts are critical.

Teaching about problem solving involves teaching learners the processes on how to solve problems (Killen, 2013:258). Problem solvers follow Polya's four step plan, where he proposed understanding the problem, devising a plan, carrying out the plan and looking back. According to the Ministry of Education (2007:34) teaching about problem solving focuses on having learners discover and develop problem solving strategies and processes. Teaching about problem solving allows learners and teachers to work collaboratively to construct strategies and to confer (formally or informally) during the problem solving stages details on how to arrive at the solution.

Teaching through problem solving involves using problem solving as a means to gain knowledge of other things (Killen, 2013:258). Teaching through problem solving is about using problem solving as the means for teaching mathematical content. It is to "*explore, develop and apply a conceptual understanding of a mathematical concept*" (Ministry of Education, 2007:6). It starts with the problem whereby learners discover for themselves and develop their own strategies and problem solving methodology in order to do it.

The underlying philosophy here is that the learners must do the mathematics themselves. When learners experience problems in solving problems, teachers should provide guidance as long as the problem remains problematic. Learners need to engage in problem solving so that it motivates them to investigate the concepts, expand and apply their individual understanding of these concepts. One can therefore conclude that mathematics concepts and procedures can be taught through problem solving.

In teaching through problem solving, the teacher believes that the learners can solve problems using their own strategies thus he remains in the background as a facilitator 'watching' learners as they engage themselves actively in the problem solving process by using representations (diagrams, pictures) to achieve a deeper understanding of mathematics. Whilst learners are encouraged to work autonomously, a lot of collaboration still occurs as learners interact with their peers sharing ideas in order to reach the solution especially when they become stuck with the problem. One of Kilpatrick five categories of how to teach problem solving is co-operation (Kilpatrick et al, 2001:9). According to Kilpatrick learners must work together to solve problems. The learners must be given opportunities to share their strategies and also consider other strategies given by their colleagues thus enriching their understanding of solving problems. The teacher guides the discussion by questioning thus ensuring that misconceptions are rectified when they occur and at the same time examines the accuracy and the procedures used to arrive at the solution. Misconceptions and errors in problem solving must not be seen as carelessness or guessing. It must be dealt with immediately and effectively. The probable cause must be investigated by the teacher to ensure there is no repetition of errors.

2.3 THE TEACHER AND PROBLEM SOLVING

Teaching mathematics in an interesting and challenging way has always brought anxiety to mathematics teachers. More of a concern to teachers was the changes to the mathematics curriculum content for the various grades and also how to teach it. The changes made to the mathematics curriculum content between the various grades have result resulted in many learners being ‘held back’ due to them performing poorly as they were not able to adjust to master and understand the content taught at that grade level. To ease the anxiety of these changes the focus was placed on mathematics problem solving.

One of our goals as teachers is to assist learners turn into better problem solvers, and to reflect on problem solving as a common, even exciting and engaging process. It was envisaged that using problem solving as a foundation in mathematics will provide learners with a deeper understanding of mathematical concepts and how solutions can be reached. Another associated objective is to recognise and discuss with learners the problem solving strategies that they will utilize in their problem solving process. By applying problem solving techniques allows the learners to build their understanding of the mathematical concepts while increasing their level of their confidence. The teachers are expected to be the agents of change in this mathematics curriculum transformation.

Duru et al (2011:3464) stated that the teacher plays an important role in how learners solve problems. Improving learning is dependent on the teaching abilities of the teacher. Since one of the most important objectives in teaching mathematics is to expand learner’s mathematical problem solving skills, mathematics teachers must make sure that they are provided with opportunities to struggle with mathematics. Understandably not all teachers know how to teach in an efficient and effective manner. To change this scenario in schools, the pre-service teachers, as future teachers, should be introduced to a diverse assortment of strategies with a focus on the development of creativity and collaborative problem solving skills. By being creative will allow the teacher to teach the content material practically thus allowing the learners the freedom of discovery. Whereas previously the focus was on memorizing formulas and methods, an understanding of the problem solving strategies will make it possible for the teachers to assist make the problem clearer, simpler and more manageable. The pre-service teachers should learn more about problem solving strategies and be able to expose their learners to these mathematical skills.

Effective teachers are those who stimulate classroom relationships that permit learners to deliberate for themselves, to ask questions and to make rational risks when solving problems (Anthony and Walshaw, 2009). Problem solving skills do not develop within a few weeks of schooling but it is a slow and progressive way to becoming a skilled problem solver as the learner progresses through the grades. The teacher as a facilitator in the realm of problem solving must assist the learners by providing them with challenging age appropriate problems; encouraging and accepting learners own strategies; supporting and extending learners learning abilities; using questioning techniques and prompting learners in the correct direction; observing and assessing learners during the problem solving process; identifying learners who encounter conceptual blocks and assisting them to recognize and rectify these misconceptions.(Ministry of Education, 2007:26).

In order to assist the learners the pre-service teachers themselves need to be coached to implement the mathematical strategies and techniques in the classroom. They need not be experts but rather make an attempt to make it part of their practice by integrating the relevant problem solving strategies to maximise teaching and learning in the classroom. This must be part of their teacher training modules at their tertiary institutions. In their theory learning they need to have access to the various teaching and learning tools which will enable them to train learners to improve their attitude towards problem solving. In practice they need to engage the learners with the mathematical problems to develop their critical thinking.

Besides learning how to use problem strategies to promote effective learning the pre-service teachers need to learn how to implement the teaching techniques in the classrooms. According to the Ministry of Education (2007:32) questions and prompts are critical when providing guidance to learners. Draper (2002:527) stated that teachers can provide learners with metacognitive prompts while they read and learn mathematics. The idea will be for teachers to provide most of the prompts first and then wean learners from relying on them. The type of questions and prompts that are used can assist the teacher to scaffold support for the learners. According to a teacher, from the Lorantffy Zsuzsanna Reformed Church School in Oradea, who uses the Varga method to teach mathematics, "*if it is needed we help them with prompts/questions that may lead them to the solution*" (Debrenti, 2013:90). Providing too much of information during the prompting phase or asking lead questions can lead directly to the solution to the problem and defeat the purpose of the task at hand. The teachers must also know when to prompt or ask questions as to not derail the learners thought processes. When asking a question or providing a prompt, a teacher needs to give the learners a reasonable period of time to allow them to comprehend and reason further.

A teacher together with good questioning techniques and prompts can also use probing. According to Killen (2013:153-154) "*probing is the process of seeking clarification or more information when a learner attempts to answer a question*". Probing can be successfully used during the discussion phase of the lesson especially when solving a problem. This is when learners discuss and justify their strategies on how they arrived at their solutions. The learners are asked to elaborate on the methods they used and demonstrate their solutions in order to provide a better understanding to other learners in the classroom. Thus probing can be used as a teaching and learning tool by the teacher in the classroom in order to get learners attempt to clarify an answer. This is to compel learners to raise their thinking levels and to also gain a clearer picture of learners understanding (Killen, 2013). Probing through redirection in the classroom can lead to an intense discussion when a teacher seeks further information from the learners. I have used probing in my mathematics lessons and often ask questions that lead learners to 'doubt' their solutions. This forces them to revisit the problem and check the validity of their solutions. When the learners are confident they will justify their answers or they will back to verify their answers or collaborate with others in the classroom. Whilst probing has its benefits, Killen (2013) cautions its use in the classroom.

Teachers may use probing and prompting in order to push learners towards a solution but Killen (2013:154) stated that they must be wary as not to embarrass the learners because they cannot read and comprehend the initial problem. In such circumstances, especially in the lower grades, teachers are known to read the problems to the learners. The teachers read in a manner such that they draw the learners' attention to search for key words (clue words) in the problem so that it keeps their concentration on the said problem and exclude unwarranted information that distracts them. Whilst many will agree that giving learner's guidance in finding and using the magic words (clue words) in a problem is beneficial, I out of experience, oppose such a strategy. It is a known fact that mathematics as a language is difficult to understand and many a word, due to having dual meaning, can leave learners stranded especially when deciding what operation to use to find the solution to the problem. van de Walle, Karp and Bay-Williams (2014) cautioned against using key words or clue words as they can be misleading. Many problems may have no clue words but the learners will look for words as any easy way of solving the problem.

I identify a few distractors such as 'altogether', 'in all', 'difference', 'give/gave', 'larger than and greater than', 'less and more than', 'share and divide' that can be problematic.

Let me put forth the following examples:

Tony had 250 marbles. He gave Anthony 120. How many did he have left altogether?

Whilst as a learner I remember the teacher stressing that 'when you see the word altogether you must add' and 'when you see the word difference, less or give you must subtract'. If this style of teaching continues in the modern day classrooms then learners will be found wanting in their choosing of operations in given problems.

What number is 247 greater than 550?

In the above example ‘greater than’ is used in the problem. This problem created two misconceptions. Firstly, I found that my learners had used the symbols ‹ and › in their answer due to them learning in the previous grades that greater than and less than can be represented by a symbol. Secondly, the learners wrote out their solution as 247 – 550. This problem actually required learners to add both the number to find the solution, namely, 797.

I have witnessed situations where the teacher gave the learners the solution out of sheer frustration or compassion when they discovered that the learners were not making progress towards a solution. According to National Research Council (2001:335) the learners may start pressing the teacher to reduce the challenge by specifying the procedures for them to perform. In certain circumstances the learners often ask the teacher on how to solve the problem when they themselves cannot solve the problem. The teacher leads the learners to an answer by “*telling them what to do*” (National Research Council, 2001:335). . Bauersfeld (1988) referred to this as funnelling. Funnelling is when the teacher assists the learners by asking simple questions thus pushing them towards the answer and when the expected answers are not exactly to the teacher’s answer he provides the answer (Bauersfeld, 1988). In this situation the learners copy the teacher’s solution without giving the learners an opportunity to use their own solution strategies. This type of ‘learning’ (giving and asking for the answers) continues in a vicious cycle in ‘teaching’ grade after grade. This prevents the learners from developing their skills and strategies to becoming independent problem solvers. When learners realise that the teacher will provide them with the answer they will have no motivation to work through the problem on their own accord. The teachers should therefore resist the impulse to give the learners the answers. Therefore the learners ought to be given a chance to decide for themselves what the problem is about and how to solve them.

Mathematics teaching and learning problem solving is entwined. Therefore it is important for pre-service teachers to become acquainted with the problem solving approach and it may be necessary to provide them with knowledge of the problem solving approach that will support their teaching and learning of mathematics. The problem solving approach should allow pre-service teachers to make the connection between the strategies and problem solving. Bahtiyar and Can (2016:2109) conferred that “*through this way, pre-service teachers can be trained for considering the teaching of problem solving skills as one of the most effective ways*”.

Figure 5 Pre-service teachers explaining their solution/problem solving strategy

To expose the current cohort of pre-service teachers to the problem solving approach, I introduced them to problem solving during their lectures. They participated in a gallery walk as a novel way of learning problem solving strategies. Initially they worked in groups of five to solve the given problems. I encouraged them to show all the steps involved in finding the solutions to the problems. When they completed the task I displayed the charts in the lecture room. They all walked around the room checking the solutions and making notes of the different strategies used by their peers. In this manner they learn by seeing. Once the walk was completed they moved into a discussion session with their colleagues. The pre-service teachers were given an opportunity to use their chart and the white board to display their effort (Figures 5 and 6). They made constructive input, in some cases, offering alternate solutions or indicated how solutions could be improved on what they saw.

Figure 6 A pre-service teacher using the whiteboard to explain her solution/strategy

A crucial area of concern of the mathematics lesson is the conclusion which in my opinion is sadly and badly neglected. Most often at the end of the lessons the teachers put up the answers on the board and request the learners to do their corrective work. No discussion occurs. In getting the pre-service teachers to discuss their solutions and strategies, I needed to make a statement. I used the pre-service teacher's discussion to demonstrate to them how important the conclusion aspect is in a mathematics lesson. I stressed to them that it is imperative that the teachers engage their learners to show how they found their solutions. It is only in this manner that the teacher can identify misconceptions and errors made by the learners. He must be able to focus, reinforce and summarize the explanation of concepts that were misunderstood. This to me is sound educational practice as it prevents the 'carrying on' of misconceptions in the learner's scholastic career.

2.4 PROBLEM SOLVING STRATEGIES

Mathematical knowledge alone is not adequate to make learners good problem solvers and teachers good at teaching problem solving. Problem strategies are necessary to aid teachers develop into efficient problem solvers. They are also necessary in order to assist problem solvers use their resources and knowledge efficiently and effectively. Reflecting on the past, mathematics was strictly taught from the textbook according to certain prescribed formulas. Gradual changes to the mathematical curriculum resulted in teachers using the textbook as a resource to teach some sort of word problem. By the early 1980's the idea of strategies found its way into the school curriculum and textbooks were printed with a few problem solving strategies (Hyde, 2006:8). Today the learners are exposed to some form of problem solving strategies in the various textbooks utilised in South African schools and the workbooks provided by the Department of Basic Education.

Polya (1957) mentioned that if the problem solver does not have a firm understanding of the problem then arriving at the solution is well-nigh on impossible. The learners need to read, understand and apply skills and strategies to the required mathematical problem. Equally important is the learner's procedural knowledge of mathematics. Procedural knowledge plays an important part in both learning and doing mathematics. The learners need to have a sound understanding in procedures so that they can skilfully apply the algorithms when solving the problems. Ozdemir and Reis (2013:86) stated that if understanding and mathematical knowledge is combined with a problem solving strategy, finding a solution is possible. When learners construct meaning of the concepts in whatever form or medium it shows their understanding and when they encounter different approaches to the problem it stimulates their minds to develop creative thinking (Debrenti, 2013) and also their problem solving ability in true life situations. As teachers of mathematics we must link the teaching of concepts, strategies and procedures so that learners can apply them in unison when working with the problems. By having these sound mathematical skills, it builds learner's mathematical ability to make sense of everyday situations in their lives by utilising the acquired strategies.

There is much debate on whether problem solving strategies need to be taught to learners or not. Schoenfeld (1987:290) explained that when teaching problem solving "*there is evidence that when students get coaching in problem solving that includes attention to such things …… - their problem solving performance can improve dramatically*" and Bahtiyar and Can (2016:2108) suggested that teachers should not only give learners assistance on how to solve problems but should also help them in assimilating problem solving skills. Bahtiyar and Can (2016:2109) further stated that assisting learners to develop their problem solving skills is one of the most key focus points for education and training at all levels. I support this view that problem solving strategies must be taught to learners and they should be assisted to develop their problem solving skills. Likewise, I believe the same kind of support must be given to the pre-service teachers in their lectures such that a statement like "*their problem solving performance can improve dramatically*" Schoenfeld (1987:290) can also be made. There should be a form of dialogue between the pre-service teacher and the learner such that the pre-service teacher guides the learner's thinking towards choosing the strategy that will lead to the solution. Pre-service teachers need to be wary that they must not be seen as the 'giver' of knowledge and learners as the 'receiver' of knowledge. In the traditional way of teaching a mathematics lesson, knowledge is seen to be in the teacher's head and this knowledge is transferred to the learners in a parrot like fashion. Thus, there is a need to shift away from the traditional way of teaching mathematics to a more innovative one.

The learners love to imitate their teachers. Kilpatrick et al (2001) stated that imitation is one of the five categories on how to teach problem solving. He mentioned that teachers should model problem solving for their learners. Polya (1962) also supported this method and mentioned that the learners must be given an opportunity to imitate their teachers solving problems as imitation and practice is seen to be vital to problem solving. When a teacher directs a lesson a lesson the learners pay attention and imitate the manner and procedures used by the teacher. By following the discussion and the teacher's guidance the learners will learn to choose the strategies to successfully solve problems. Although mathematical problem solvers are flexible and confident thinkers, the learners need to be taught the strategies and how to replicate these systematically to problem solve in other situations. This is very important for education as learners will be able to apply such strategies in their real life situations. In this manner they become skilled in choosing and applying the appropriate strategy when confronted with any real life problem.

According to Boonen, Reed, Scoonenboom and Jolles (2016:58) word problems, especially the non-routine type "*cannot be represented in a prescribed way*" and visual representations, pictorial and arithmetical representation, may seem appropriate to solve them. Pictorial representations denote a detailed image of some aspect of the problem whilst arithmetical representations support the calculation method needed to compute the answer. When using representations with problem strategies it is important that pre-service teachers know when and how to use them so that they can teach learners how to use them in diverse ways (Boonen et al, 2016:60).

Learning problem solving strategies will make it possible for learners to deal more efficiently with the majority types of mathematical problems as they become confident in the application of prior knowledge and processes. According to Debrenti (2013:87) when learners used acquired knowledge it encourages them to make connections and in addition raises their logical, critical and divergent thinking. When this occur learners gain confidence to problem solve and this heightens their reasoning skills which fosters understanding. Since mathematics is all about conceptualization and procedural knowledge, concepts and procedures to solve a problem must be taught. When learning how to solve problems one needs to identify certain procedures, strategies and how to utilize it appropriately. The how, when and what type of questions need to be demonstrated to learners when using a problem.

Problems are generally dreaded and to manipulate words to numbers and symbols can be like a torture (Alexander, 2015:1) therefore learners need to be taught how rephrase the question in their own words. In this manner the learners follow the teacher and get some practice on how to use the various strategies. This kind of teaching is within the Zone of Proximal Development (ZPD) whereby learning occurs under adult supervision and guidance. Using the ZPD in the mathematics classroom the learners can be taught how to solve problems visually. They choose a strategy and manipulative to transform the problem into some visual form. In this way the problem is made easier to understand and becomes solvable. They will eventually follow the teacher's guidance to reason, make connections and apply appropriate mathematical strategies when faced with new problem situations.

Some of the problem solving strategies (Ministry of Education, 2007) is discussed below and the examples have been adapted from the Mathemagica Plus Competitions.

2.4.1 **Draw a diagram**: the problem solver translates the problem by drawing pictures or some form of representations to solve the problem. By drawing a picture to illustrate the data make the problem real for learners.

Example: Ten light poles are placed 10 metres apart on the road. What is the total distance from the first pole to the last pole?

2.4.2 **Make a table**: this strategy enables learners to arrange the data from the problem and to see the relationship as well as visually consider their alternatives to unravel the problem.

Example: the class teacher asked Peter to count in sevens, Tammy to count in fives and Themba in tens. What number will be common for all three of them?

2.4.3 **Guess and check**: also referred to as guess and improve or trial and error. The problem solver makes guesses or estimations and then checks the answer by using a process of elimination of the 'wrong guesses until they find the correct solution. These guesses are based on a learner's prior knowledge or experiences.

Example: a farmer has chickens and cows on his farm. He counted a total of 80 heads and 212 legs. How many chickens did have on his farm?

2.4.4 **Use a model**: a representative model is used to find the correct solution.

Example: 4 cubes are stacked upon each other. There is a red, brown, orange and white cube. In how many different ways can you stack the cubes one upon the other?

2.4.5 **Find a pattern**: the problem solver looks for patterns and relationships in order to solve the problem. The use of geometric or number patterns together with colour coding plays a significant role in this strategy.

Example: John builds rectangles with match sticks. When the length of the rectangle is 3 there are 8 match sticks; when the length of the rectangle is 7 there are 16 match sticks. How many match sticks does he need to make a rectangle with a length of 20?

2.4.6 **Start at the end**: also known as working backwards. The problem solver commences with the answer. By working inversely and using the various algorithms arrives at the solution.

Example: Mala had a secret number. When the number was double and 15 was subtracted from it the result was 25. What was Mala's secret number?

2.4.7 **Making a list**: a list is made using the given data to arrive at the required solution. This involves listing all the possible outcomes systematically until all the possible outcomes are accounted for.

Example: Ashlek made a list of all the whole numbers between 1 and 100. How many times did he write the number five?

2.4.8 **Use a formula**: a formula is discovered to find the solution to the problem.

Example: the length of a rectangular garden is one and half times its breadth. If the perimeter is 150 metres, find the breadth in metres.

2.4.9 **Logical thinking**: the problem solver uses logical thinking as they investigate the numerous types of problems. The learners are encouraged to evaluate the information, use clues and reason logically to arrive at the solution.

Example: Jane is older than Kim; Kim is older than Shaun; Shaun is younger than Jane and Rachel is older Jane. Who is the youngest?

2.4.10 **Act it out**: in this strategy the learners act out the problem situation in order to find the solution. Acting out the problem in reality extends their understanding of the situation at hand.

Example: the Principal and six teachers met on the first day of school. They shook hands with each other once. How many handshakes were there altogether?

The mentioned strategies and skills (teaching methods and strategies) are acquired over a period of time and with experience, therefore to teach the subject teachers need to discover appropriate teaching methods and strategies in their formative years to make the content understandable.

2.5 PROBLEM SOLVING AND METACOGNITIVE KNOWLEDGE

Problem solving is one of the most important reasons for studying mathematics as it is regarded as a dynamic thinking and imaginative invention (Luneta, 2013). Mathematics problem solving has moved away from the drill and practice method as a result the demand of the mathematics curriculum on learners metacognitive and cognitive ability has increased tenfold. Metacognition warrants special attention due to its role it plays in problem solving. Sharma (2016:1) stated that "*problem solving in any setting is a complex cognitive activity*" therefore learners need greater metacognitive knowledge to investigate complex problems and make the connections between mathematical ideas. Kribbs and Rogowsky (2016:65) stated that the combination of the following metacognitive skills (comprehension, mental representation, solution construction and solution execution) when used in a strategic manner allows the learners to engage in solving the most complex problems.

One of the earliest goals of problem solving was the development of metacognitive skills. Metacognition warrants special consideration as it involves thinking, choosing the appropriate strategy to solve the problem and evaluating the chosen strategy to see if the solution made sense. Metacognition is often studied and related to problem solving as it involves the ability to think, read and write. According to Gurat and Medula (2016) metacognition was developed between the 1970s through the 1990s and it is during these years that metacognition became a dominant tool involving the thinking process. Gurat and Medula (2016:6) studied the use of metacognitive strategy knowledge involving mathematical problem solving amongst pre-service teachers and they stated that "*metacognition refers to one's knowledge concerning one's own cognitive processes…..or anything related to them*".

Studies have shown that there is a strong link between metacognition and problem solving. Many researchers have identified that metacognition is a key aspect in the problem solving process. According to Gurat and Medula (2016:4) there are three types of metacognition, namely, metacognitive skills, metacognitive experience and metacognitive knowledge. Posamentier and Jaye (2006) indicated that as learners develop their metacognitive skills become more successful in problem solving. Learners needed to ask themselves, "*What technique did I use to solve a similar problem in the past*?"; "*How do I find the derivative*?"; "*Is there anything I don't understand*?"; "*Am I headed in the right direction*?"; "*Have I made any careless mistakes*?" (Posamentier and Jaye, 2006:80). In metacognitive knowledge there are three broad types which are of importance, namely, strategy knowledge – this refers to the learner's knowledge of general strategies for learning, thinking and problem solving; task knowledge – this refers to understanding of cognitive tasks as well as the when and the why to use these strategies; person knowledge – this refers to familiarity about the person (self), cognitive issues and the motivation to perform. This is all indicated in Figure 7.

Metacognitive strategy knowledge involves issues of when and where to use cognitive and metacognitive strategies and "*also involves the skills needed to solve a problem such as prediction/orientation, planning, monitoring and evaluation*" (Gurat and Medula, 2016:2).

The metacognitive strategy knowledge of Isagani is discussed in Gurat and Medula (2016:12-15). In the Isagani process (Figure 7) the problem needs to be presented in written form and then orally. The problem is first read and reread to bring about understanding. When analysing the problem the learner visualizes the problem by creating drawings or detailing the needed details. Certain details not related to the problem are discarded. According to Alexander

(2015) when drawings are used learners are less likely to be bogged down as the excess language is removed from the problem. The problem is broken down into pieces so that it can be examined individually and a relation is created to real life situations.

Alexander (2015:2) stated that the teachers must always try and relate the problem to reality as much as possible and to events that are current in the learner's life. In this way learners can make a connection to real life situations. Various illustrations are created using the details provided in the problem and the strategy to be used is chosen. The strategy to be used will depend if the problem is recognizable or was encountered previously. If the problem is recognizable then the learner uses the known formula or considers the several strategies and subsequently chooses the strategy that suits the problem. In the event of the problem not been familiar it is read repeatedly and the trial and error method. If this fails, the learner then requests assistance from others to verify their understanding against their own. When the problem is understood and the learner is positive of using the correct strategy, he uses the steps methodically to arrive at the solution. On arriving at the solution he reverts to the problem to determine if the answer is apparent and there is no need to go through the entire process again. If time permits he reflects on the problem by rereading the problem, analysing it thoroughly, devising a plan and carrying it out.

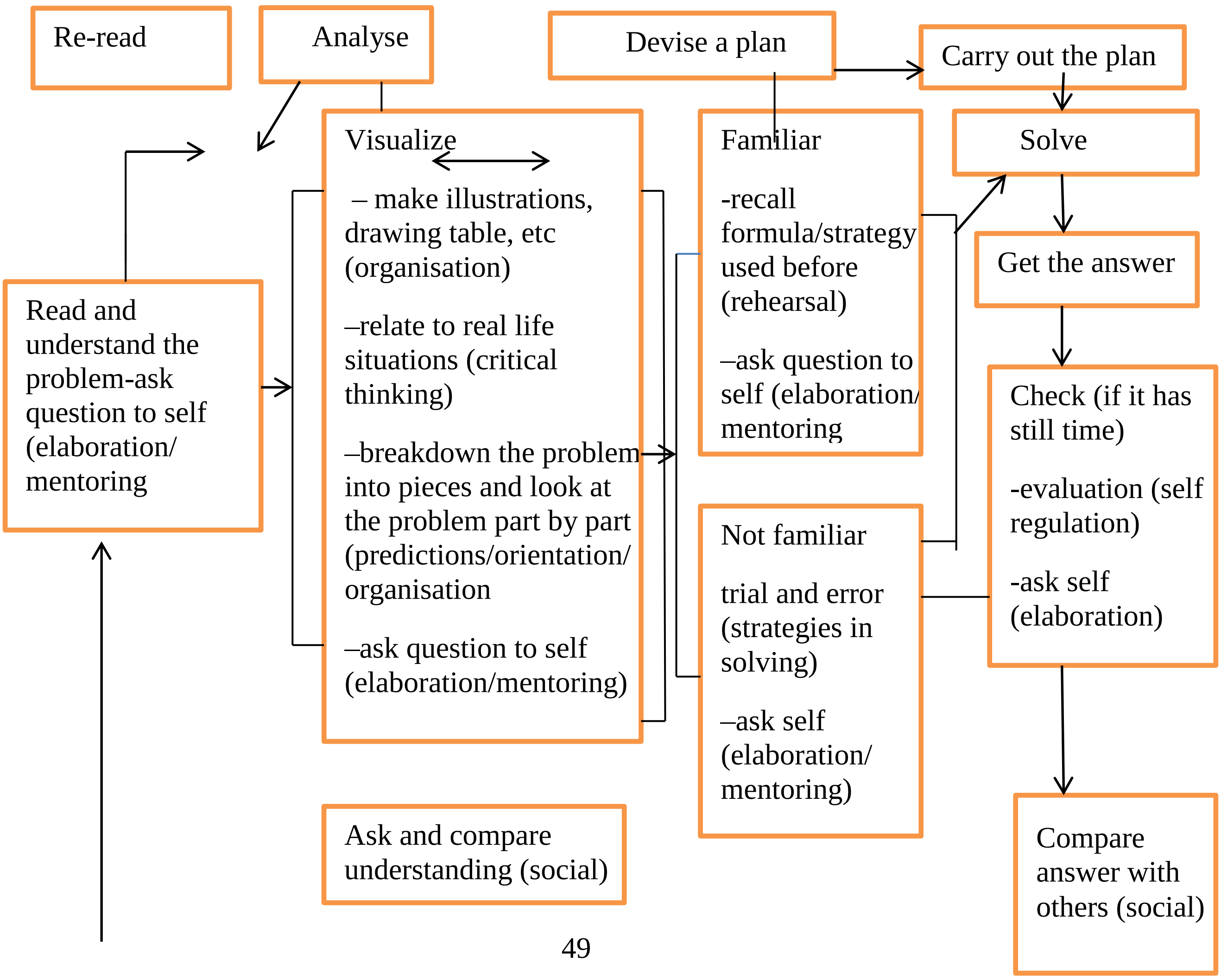


Figure 7 Metacognitive Strategy Knowledge of Isagani (Gurat and Medula, 2016:13)

According to the Ministry of Education (2007:4-5) when the learners participate in problem solving, they will engage in an extensive range of cognitive experiences that will be of assistance to them and prepare them for the many problem solving situations they will encounter in their lives. They will be trained to discover and learn mathematical concepts with understanding and perform skills in context; reflect on the nature of inquiry in the mathematics world; develop strategies that can be useful to new circumstances; connect the mathematics they study at school with its relevance in their daily lives; make associations between the concepts in mathematics; represent mathematical ideas and replicate (model) situations using concrete materials, pictures, diagrams, graphs, tables, numbers and symbols; move from one representation to another and recognize the connections linking them to other representations; through collaboration communicate their explanations and take note of the explanations of their peers and persist in tackling fresh challenges (Ministry of Education, 2007). Taking the aforementioned into consideration one can deduce that the focus is on the learner. According to Debrenti (2013:88) it is important to make the learner part of the problem as it will "*involve him in the solving procedure, offer him the possibility of self-expression or manifestation, help him to experience success and for him not to be afraid of failure, make him understand that mistake is allowed and lead him towards the pleasures of solving a problem*".

Kuzle (2013:258) sums it up nicely by stating that "*mathematics teacher education programs should allow pre-service teachers with opportunities to learn about a variety of pedagogical and learning issues, and means for implementing problem solving within the lessons, as well as to also experience them with respect to (meta)cognitive and non-cognitive aspects of problem solving*".

2.6 PRE-SERVICE TEACHER EDUCATION

The system of teacher training inherited in 1994 by the first democratic government of South Africa was part of the apartheid dispensation. The newly elected government went to great extents to make drastic changes to transform the apartheid structured education. Caught up in this transformation was teacher training education which then was racially segregated. Since then our nation's higher education institutions have gone through various transitions in their roles and responsibilities in educating the masses. One such transition was pressure on teacher training. The teacher training programmes in South Africa faced existential challenges. During post-apartheid and pre-democracy South Africa had dedicated teacher training colleges albeit for the different race groups. Professor Kader Asmal, the first Minister of Education in the newly elected democratic government, through the Higher Education Act incorporated the colleges of education and placed teacher training under the jurisdiction of the higher education institutions which eventually saw the closure of all teacher training colleges of education. Teacher training became the responsibility of the universities.

There have been both strong print and electronic media comments by the public and educational leaders that South African educational institutions have dropped their standards in teacher training when compared internationally. I believe that this is due to these institutions been forced to take in students from quintile one and two schools to the detriment of other students who have performed better from the higher ranked quintile schools in South Africa. In 2013 the Australian government provided a directive that in order to be accepted into a teaching degree the potential student had to achieve a mathematics mark in the top 30 percent (Lowrie and Jorgensen, 2015:2). I am of the opinion that this is the route that should be followed by the South African educational authorities to ensure that the South African schools get the best students to be trained as teachers to lead education into the future otherwise we will be scraping the bottom of the barrel to put competent people in the classrooms. One of the recommendations listed under "Address teachers and teaching issues" in the report by the Ministerial Task Team on Mathematics, Science and Technology was that "*there is a critical need to intervene in pre-service teacher production in order to ensure the HEIs produce competent and credible new teachers of sufficient quality and in sufficient quantities to service the MST needs of the school system*" (Department of Basic Education, 2013:53). This recommendation has been sadly overlooked to the detriment of education in South Africa. South Africa needs to produce mathematics of substance (competent and confident) otherwise we face the prospect of becoming the pariah nation in the world of education.

Pre-service teachers become skilled in many ways. Apart from the training the pre-service teachers receive at their campuses, an essential part is to get their training for teaching at schools. Training under a professional teacher is a critical attribute for future teachers. Centuries ago teachers needed only needed to be familiar with and understand the subject content determined by the grade that they taught and they were seen as competent teachers. During a period in South Africa, learners from rural schools who had completed grade 12 with 'good marks' in mathematics were taken on as mathematics teachers due to the shortage of qualified mathematics teachers in the rural areas. Whilst some rural schools still face this kind of challenge the Department of Education has moved away from this practice. Having some content knowledge in mathematics does make a teacher thus teacher training needs to focus much more than on the subject level at which the pre-service teacher is going to specialise. The pre-service teachers need to be given opportunities to learn mathematics. They need to become skilled at how their learners learn, think and do the mathematics in the classroom. To become a fully endowed teacher they need to know the structure of the curriculum, how to use instructional materials, assess in the correct manner and more important know the subject content. Exposure to CAPS and other relevant educational policies is a must during their teaching experience as it is within this space that the pre-service teachers can learn in context.

The implementation of the new content in the various grades will leave teachers exposed with inadequate knowledge and skills. Teacher training needs to give much attention to pedagogy and methodology that will eventually influence mathematics as a subject. Boonen et al (2016:60) stated that from a theoretical perspective, it is important that teachers possess adequate mathematical content knowledge (MKT) (discussed in chapter 3). This is needed to support mathematical learning and reaffirms the proposal that those who are approved to turn into teachers must undergo intensive professional teacher training and I believe that microteaching should be part of a module to develop pre-service teachers.

Much of the literature on improving the quality of teacher education focuses on the closer alignment between teacher training institutions and field experience. Microteaching bridges the divide between training and teaching. Microteaching is a teacher training technique when pre-service teachers are introduced to the complexity of teaching practice. According to Basturk (2016:239) "*pre-service teachers can find an opportunity to transform their subject matter knowledge and pedagogical content knowledge into practice*". Microteaching has always been part of the teacher training module at teacher training colleges pre-apartheid. Currently there is no literature available to indicate that there is a microteaching module in any teacher qualification offered in South Africa.

Since learning to teach is an active constructive process, it allows pre-service teachers to connect with field experience (classroom practices). Pre-service teachers are given an opportunity to develop and present a lesson to demonstrate their pedagogical content knowledge in a real teaching situation. Microteaching provided the pre-service teachers with an opportunity to make mistakes and a chance to improve their confidence to face a classroom (Higgins and Nicholl, 2003). Through microteaching enactment pre-service teachers are provided a platform to practise teaching and thereafter are given constructive feedback by peers and mentors on their performance allowing them to improve. Basturk (2016:239) stated that teaching can only be done by doing and the pre-service teachers get an opportunity to improve their weaknesses of teaching skills. In this manner pre-service teacher's pedagogical knowledge (how to teach the subject) can be strengthened.

The purview of teacher's mathematical knowledge may be influenced in teacher training institutions. Teacher training education lays the basis for future teachers and teacher's performances in the classrooms will be the product of what happens at these teacher training institutions. The various higher education institutions in South Africa have differing or specific modules in respect of obtaining a teaching degree. The pre-service teacher's education programs vary considerably at all higher education institutions in South Africa. The variation occurs in the content of the modules, pedagogy and the duration of the teaching practice. The modules and the content offered for the teaching qualification at the various higher education institutes and at private colleges are not really relevant or aligned to the school curriculum. Ball, Thames and Phelps (2008:404) stated that the overwhelming courses in the teacher training programs are more academically inclined, immaterial and remote from the realities of classroom teaching. According to National Research Council (2001: 375), the specialized knowledge that the pre-service teachers need is different from the mathematical content contained in most colleges mathematics courses. These modules do not do justice to prepare the pre-service teachers for teaching the subject per se as many of these modules do not have any relevance to actually teaching in the classroom. Mudaly (2016:67) stated that "*schools of education are offering more generic courses than subject specific courses*" as a result pre-service teachers are been subjected to a more generalised education than specialization. They need to know the subject content they teach thus these modules have to be designed in such a manner that they have a better understanding of the mathematics curriculum and the interrelationship between the various areas in the subject. Therefore there is a need for the pre-service teachers to be exposed to more mathematics modules that will build their conceptual and pedagogical knowledge in mathematics. Lowrie and Jorgensen (2015:2) claimed that the shift is towards mathematics lecturers been forced to "*teach mathematics content courses …… to undergraduate education students*". The teacher training modules needs to focus on the subject level at which they are going to specialise and they must know the subject matter specifically to mathematics (Rosas and West, 2011; Mudaly, 2016). Debrenti (2013:55) stated that in their training programme great emphasis needs to be placed on word problems, their interpretation, understanding the steps in problem solving and representations. Thus the onus is now on the educational institutions to prepare the pre-service teachers for meeting the challenges of teaching mathematics problem solving.

According to Killen (2013:132) "*good teachers are made, not born: and the making of a teacher is a complex process".* Teacher certification in a form of obtaining a teacher degree does not ensure that they have sufficient knowledge to teach the content in the subject. Simply put obtaining a qualification does not make pre-service teachers qualified teachers. It is imperative that those that are fostering the next generation of teachers and developing their specialized knowledge, skills and competencies need to keep abreast of the changes and challenges in teacher education. Mudaly (2016:155) stated categorically that besides been trained in subject knowledge and pedagogic knowledge, the pre-service teachers must also have a sound knowledge of schooling and learning. The question one needs to ask: How do the teachers develop their mathematical knowledge and use it for teaching? The teachers need to develop their own mathematical knowledge through intensive and extensive professional development. Continuous participation in professional development courses increases their understanding of teacher knowledge, especially familiarity with the subject matter and pedagogical skills. This has a cyclic effect as it leads towards an improvement in effective teacher training and learner development.

Professional training is critical for the pre-service teachers. Pre-service teacher training is mainly done in schools where the pre-service teachers do their teaching practice for approximately three to four weeks annually over a four year period depending on the institution they attend. They normally go to schools for a week of observation early in their academic year and go to schools later in the year for their teaching practice. They are normally attached to a mentor teacher for the duration of the teaching practice. These school visits are primarily envisioned as an opportunity for the pre-service teachers to learn as much as possible from the mentor teacher. The idea herein being that the mentorship will assist in building the capacity of the pre-service teachers in teaching their respective subjects. In-between observing their mentor teacher they are also required to teach a mandatory number of lessons in certain prescribed subjects as determined by the institutions they attend. It must be mentioned that the prescribed subjects change over the four years thus not giving the pre-service teachers sufficient exposure to the content knowledge or practice in teaching the subject. What I deem necessary during the teaching practice sessions is that the pre-service teachers observe a problem solving lesson or teach a mathematics lesson on how to solve problems. This kind of observation-teaching exercise is actually an extremely valuable learning experience for them. The experience of observing and teaching these lessons encourages them to develop their foundation of content and pedagogical knowledge. The observation-teaching experience will enable them to reflect on the mathematics lessons and examine ways on how to improve on them. This observation-teaching experience will allow the pre-service teachers to adapt the content knowledge and pedagogy to eventually teach in their own unique way when in the classroom. According to Grootenboer (2006) observing a problem solving lesson taught by an experienced teacher will aspire the pre-service teachers to teach in such a manner taking into consideration learner participation and content knowledge of the lesson.

My focus on the pre-service teachers is to determine if they have sufficient content knowledge on teaching problem solving and training to use visualization in how to solve problems. Lowrie and Jorgensen (2015:30) stated that "*mathematics content knowledge is critical for effective teaching of mathematics*". The pre-service teachers involved in this study have been exposed to a wide repertoire of theory (teaching styles and strategies) in their mathematics modules and teaching practice (pedagogical) but without sufficient training they will not be able to incorporate these ideas in the classrooms. I have observed that the newly qualified mathematics teachers have little training in teaching the subject let alone teaching problem solving or using visualization in their teaching. The problem stems from during the teaching practice. The problem is compounded as the pre-service teachers are not given sufficient guidance by their mentor teachers, for whatever reasons, during this period. As the area of problem solving was and still is neglected in schools, it has now become a critical area of focus in the South African mathematics curriculum. Due to the extreme neglect, the Department of Basic Education has now placed an enormous emphasis on problem solving because it has come to realise that problem solving is the heart of mathematics in the curriculum (Department of Basic Education, 2011).

Many previous training colleges of education in South Africa offered modules which focussed on the subject content knowledge and also the pedagogic aspects (how to teach the subject) and the interrelatedness proved to be very effective. The pre-service teachers were well trained to enter the classrooms confidently and prepared to use the curriculum to teach effectively. This was evident. In the modern teacher training era there is an increasing advocacy that the prospective mathematics teachers should have a balance between "*didactics and pedagogical training and the knowledge of mathematical content*" (Santos and Domingos, 2013:3239). I believe that the teaching modules should be aligned to the mathematics curriculum and structured in such a manner over the four years that the pre-service teachers will be able to confidently walk into the classrooms and do justice to teaching mathematics. For this to happen I propose that there should be strong co-operation between universities and schools. It is advisable that the schools and the institutions that are offering teacher training modules come together and design a teacher training model that will be beneficial to education development in South Africa. In the largest study of its kind carried during the 1960s, the director of the National Longitudinal Study of Mathematics Abilities (NLSMA) mentioned that "*the more a teacher knows about his subject matter, the more effective he will be as a teacher*" (National Research Council, 2001:374).

As the study of learning and problem solving in mathematics advances and the complexity of the problem solving processes is acknowledged, there is still a want for further research in order to support teaching practices. Although problem solving has been investigated from various perspectives (Polya, 1945; Ballew and Cunningham, 1982; Schoenfeld, 1983; Malloy and Jones, 1998; Holton et al, 1999; Lubienski, 2000; Pantziara et al, 2004; Pape, 2004; Kotze and Strauss, 2007; Ozdogan et, 2011; Institute of Educations Sciences, 2012; Gurat and Medula, 2016) there is relative silence on the use of visual literacy as a skill and visualization as an alternate method in the teaching and learning of problem solving by pre-service teachers. Whilst the importance of visualization in mathematics is well documented in the literature (Boaler et al, 2016; Boonen et al, 2016; De Guzman, 23002; Kadunz and Yerushalmy, 2015) I discovered that there has hardly been any research on how pre-service teachers use visualization in the teaching of problem solving. Despite its importance (Juersvich et al, 2009), there does not appear to be much research on how teachers and pre-service teachers use visualization for teaching mathematics problem solving. In Finland attempts are being made to develop visualization training using pre-service teachers (Malaty, n/d) and it was found that it will take comprehensive training to prepare them for the classroom. In fact, in early studies it was found that it takes between three to five years before teachers become competent and feel confident enough to teach. It will take longer for pre-service teachers to develop sufficiently to use visualization confidently in the classrooms.

As teachers play such a vital role in developing learner's mathematical knowledge, they need to be properly trained so that they can guide learners effectively and appropriately. According to National Research Council (2001:373) "*to develop prospective teacher's understanding of the mathematics they will teach, careful attention must be given to identifying the mathematics that teachers need in order to teach effectively*". If training programmes are to be effective then it is vital that pre-service teacher's knowledge be interrogated frequently to ensure that educational institutions are producing knowledgeable practicing teachers. As the mathematics lecturer responsible for the current cohort of pre-service teachers at an independent higher education institution, I made recommendations to the course developers in 2016 that the mathematics modules incorporate pedagogical aspects so that the pre-service teachers are trained in both aspects to enter the schooling system with confidence. The Department of Basic Education (2018:82) is recommending that "*the curricula planned for their courses*" should include at least one module related to its "*methodology courses, become part of assessment, used in planning for the practicum*". The National Minister of Education has stated through Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding (Department of Basic Education, 2018:82) that the pre-service teachers need to be prepared to teach the mathematics curriculum in the classroom. The pre-service teacher's module guides and portfolio of evidence tasks have been revised and designed in such a manner that they will now have more exposure to both content and pedagogical content knowledge.

According to Ozdogan et al (2011:2283) the pre-service teachers should "*improve their problem solving skills at university years*". According to Peker (2009) understanding the problem solving process is the first step in learning how to teach it and using problem solving strategies gives them an opportunity to learn how to teach it. The current cohorts of pre-service teachers are exposed to problem solving activities at the commencement of every lecture or are asked to prepare a problem solving lesson together with resources of their choice to support their lesson. They are given non-routine problems to solve. I encourage them to try work independently or work within groups. Working within groups allows the pre-service teachers to work collaboratively. When faced with any challenges they are allowed to discuss their solutions with their peers. The pre-service teachers either volunteer or are randomly selected to discuss their solutions on the whiteboard. I have found this to be an interesting feature because their peers offer suggestions if there are any shortcomings (I don't use the word error in this context) in finding the solution or they offer alternative solutions to the problems. Whilst engaging them in their presentation of their problem solving lesson, I have seen them become innovative in using their visual resources to present the strategies. They also receive constructive criticism and reflection is encouraged. Besides the peers giving each other confidence during their lesson presentation during lectures, they also offer support which is largely missing during their teaching practice sessions (Harlow and Cobb, 2014; McMaster and Cavanagh, 2016).

The pressure of teaching mathematics is great. Lowrie and Jorgensen (2015:3) asked "*what do teachers need to know to be good teachers of mathematics*". As there is no perfect way of teaching mathematics, the pre-service teacher's professional training needs to focus directly on the development of instructional practice so that it supports the actual work to be taught in the classroom (Ball and Forzani, 2011:19). Educating teachers in how to utilize visual representation is a complex process. It requires continuous rigorous professional development and support (Ozmantar et al, 2010). The pre-service teachers must remember that using visualization for them is a new undertaking and they should master the theory and practice. In order for them to realize the role of visualization in mathematics, the pre-service teachers need to be conscious of the role visualization plays in the teaching and learning of mathematics. According to Boaler et al (2016) when teachers are given visual experiences they gain understandings into mathematical concepts and ideas they had never experienced before. According to Kadunz and Yerushalmy (2015:463) a "*mathematicians success owes a considerable amount to visualization skills*" and future mathematics teachers must be competent enough to teach confidently using visual skills. The pre-service teachers should endeavour to make the role of visualization during problem solving clear to learners if they are to take maximum advantage of it during problem solving.

In my first interaction with the current cohort of pre-service teachers, I innocently asked how many of them will be specialising in mathematics. The reaction to this question was incredulous. The responses to this question were:

"*Not me*";

"*I barely made it in mathematics*";

"*It is a difficult subject to teach*";

"*I don't understand primary school maths*".

These responses are not new to education. The above responses by the pre-service teachers indicated that they do not feel self-assured in their capacity to teach the subject. In the current educational set up experienced educators who are not mathematics oriented, especially in the intermediate and senior phases are refusing to teach mathematics citing that they are not trained to teach the subject. Burdening them to teach the subject causes anxiety amongst them. Why then do these seasoned teachers reject teaching mathematics? The trend may well continue with the many pre-service teachers as well! Since mathematics is a must pass subject or otherwise known as a compulsory subject to meet the pass requirements in the foundation, intermediate and senior phases, the expectant high failure rate (for whatever reasons), is seen as a put off. It is seen as a failure and stigma and is one of the reasons that lead to further anxiety. I firmly believe that a solid foundation in mathematics must be laid in the years that the pre-service teachers spend at the higher education institutions.

Mathematics anxiety in pre-service teachers is well researched (Gresham, 2008; 2009; 2010) and achievements in the classroom will depend on the training of the pre-service teachers. A pre-service teacher that is devoid of adequate knowledge of mathematics is unlikely to be competent to convey sound mathematics teaching to the learners. To be knowledgeable in the subject matter, it is important for pre-service to have an understanding of the mathematics curriculum and knowledge of non-traditional methods to facilitate mathematics efficiently (Gresham, 2010). To overcome this, the mathematics curriculum should be become the focal point so that the pre-service teachers can be prepared to master mathematics skills, understand and solve mathematics problems.

The mathematics curriculum in countries like Turkey and South Africa emphasizes the importance of problem solving and problem solving strategies (Duru et al, 2011). Singapore, one of the better performing Asian countries in mathematics, has adopted the problem solving approach. Duru et al (2011) stated that the importance aims of the mathematics curriculum are to enable learners to develop their own abilities in mathematical problem solving. This can only be done if the curriculum "*have a specific pedagogic-content outlook that informs how teaching and learning of the subject should be approached*" (Department of Basic Education, 2018:11).

Any reformation of the curriculum and its success or failure eventually gets translated through the teacher in the classroom. The National Department of Education in South Africa develops the changes for the classroom but in reality it is the teachers who are responsible for the implementation of these changes. They, the teachers, are considered as 'agents of change'. When Outcomes Based Education (OBE) was introduced in South Africa, I was vociferous in opposing its implementation during the one week of in-service training for teachers. I pointed out to the leader of the facilitators that besides OBE failing overseas, the content and teaching methods will fail the South African learners. My argument was based on with amalgamation of the various education departments in South Africa, including the Education Departments from the former independent states (created during the period of apartheid), support and guidance to all teachers to handle these changes will be challenging and will not be forthcoming from the Department of Education or subject advisors in the foreseeable future. Furthermore, the learners will be exposed to more than one language of instructions at schools due to migration between the newly established provinces. The biggest challenge was that it will prove difficult for teachers (old and new) to implement due to inadequate training and unfamiliarity with the content and teaching methods involved in OBE. It is now widely acknowledged that the teachers eventually had to fend for themselves and many 'did their own thing in the classroom' during the tenure of OBE in schools. Many will remember OBE to have failed the masses in South Africa destroying more than a decade of learner's lives. OBE was replaced with Curriculum 2005. This in itself was prescriptive in how mathematics ought to be taught and time frames were set within which the teachers had to complete 'teaching' the prescribed content. Thus teachers focussed on completing the syllabus to meet the assessment requirements rather than actually teaching the content. The damage that Curriculum 2005 did to education in South Africa is well documented and evident in the results produced at the senior certificate examination level.

The Revised National Curriculum Statement (RNCS) became the National Curriculum Statement (NCS). The challenges that the mathematics teachers faced in implementing the content was immense. There was a shift in content from the higher grades to the lower grades, example, certain sections were taken out of the grade 8 scope of work and included in the grade

7 scope of work. This introduced new content across all the grades. The teachers in the primary schools were not in a position to teach content from the 'high school'. Along came the Curriculum, Assessment and Policy Statement (CAPS). Implementing CAPS had its own challenges. The teachers were constrained by many factors to teach effectively. The learners themselves were exposed to numerous stipulated types of assessments in all subjects. With CAPS, like the Curriculum 2005 and NCS, the teachers are forced to adhere to deadlines to teach the stipulated content for the term so that the learners can complete their assessments within a limited time frame. This is what I call 'forget teaching, get the job done'! The Department of Basic Education (2018:11) has confirmed the failure of mathematics from its own research and has admitted that "*too many students struggle with passing the subject*" and has stated that "*the teaching and learning of mathematics in South African schools is not yielding the intended outcomes of South Africa's education policies and curricula*".

Organisations with links to education, in this instance the National Education Collaboration Trust (NECT), have jumped onto bandwagon under the guise of assisting teachers in the classroom. Two districts, Pinetown and Uthungu, which fall under the jurisdiction of the KwaZulu Natal Department of Education, piloted the Jika Imfundo document over two years. Jika Imfundo is collaborated by the NECT. The Jika Imfundo programme was rolled out to all schools in all districts in. The Jika Imfundo pilot programme caused a lot of anxiety amongst teachers. Many teachers attending the training sessions complained that the school curriculum is so expansive at the moment that it was virtually impossible to do justice by covering it due to the limited instructional time they have. The teachers have been subjected to intense scrutiny to implement this programme at their schools alongside CAPS. Besides feeling pressurised to complete the prescribed amount of assessment tasks in the curriculum, the teachers now have to also contend with using the Jika Imfundo document in conjunction with the CAPS document, the prescribed textbooks together with completing tasks from the departmental supplied workbooks. I feel that due to these challenges faced by the current teachers, it is imperative the pre-service teachers to given sufficient support in their initial years of training to manage the mathematics curriculum with the intention to help them rise above the challenges they are likely to be confronted as newly qualified teachers. Fadlelmula and Cakiro (2011:10) found in their studies that the pre-service teachers need to scrutinize their course programs both related to mathematics content and teaching in order to be directly aligned to the mathematics curriculum.

According to Santos and Domingos (2013:3239) differing views are advocated in respect of how the pre-service teachers should be trained. One thought is that the pre-service teachers should have a solid foundation of didactics and pedagogical training with mathematical content knowledge to be acquired through experience. The other thought is that the pre-service teachers should be given sound mathematical training with the pedagogical issues been acquired with experience. I am of the opinion that the pre-service teachers must be given adequate grounding in acquiring both mathematical content knowledge and pedagogical training. Due to changes to the mathematics curriculum in South Africa, schools are facing difficulties to place adequately trained teachers to teach the subject due to them not having both sufficient content knowledge and pedagogical training to teach the mathematical content. Therefore it is imperative that the teacher training modules be so designed at South African educational institutions to meet the challenges of preparing the pre-service teachers to enter the classrooms. In Portugal the mathematics curriculum at higher education institutions have been designed such that their pre-service teachers receive a firm mathematical foundation, complete training for teaching mathematics and didactics and pedagogical training on how to teach mathematics (Santos and Domingos, 2013:3238). The National Minister of Education is recommending that the same happens in South Africa (Department of Basic Education, 2018:11).

The current mathematics curriculum has become more aimed at the acquisition of skills required for life. The pre-service teachers in South Africa are not conversant with CAPS per se and how it is supposed to be taught. It is more important that the pre-service teachers know the guidelines of modern mathematics training and changes recommended for mathematics teaching and learning in the South African schools. Knowing how to implement them will hold the pre-service teachers in good stead as a new breed of knowledgeable modern mathematics professionals influencing the teaching and learning process to improve learner performance and achieve good results in mathematics.

2.7 VISUALIZATION

The history of visualization dates back to the 1880s with the first studies conducted in the early 1960s. Visualization as a technique and strategy had been explored for many decades and was widely used a strategy to teach reading and was generally used as a technique by creative mathematicians. Today it is seen as a necessity in teaching and learning of mathematics. Visualization plays an important role in mathematical activities because it is seen as useful for building the understanding of mathematics concepts and its benefits have been consequential. Visualization provides information allowing one to develop deeper and richer concepts. Therefore, the proponents of visualization seemed to favour the utilization of visual approaches in teaching and learning because they have found visual representations effectual in teaching mathematics more especially in problem solving (Zimmermann and Cunningham, 1991; Presmeg, 1986). More recently Mudaly and Budaloo (2016:45) have argued that visualization contributes to the successful teaching of mathematics and their study revealed positive findings in terms of the role in successful mathematics teaching.

In this literature review chapter many specific terms relating to visualization are discussed and their inter relationship is described.

Let us consider some of the definitions of visualization:

According to Arcavi (2003:215) "*visualization, is both the product and process of creation, interpretation and reflection upon pictures and images, is gaining increased visibility in mathematics and mathematics* education".

Zimmermann and Cunningham (1991) described visualization as the constructions and creation of internal images (the creation of mental images) or external images (images created as illustrations with the aid of pencil and paper) and then using the images effectively for mathematical discovery and understanding.

The statement made by Arcavi (2003:217) is both just and concise when describing visualization. Visualization is described as "*the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding".*

From the above definitions it can be noted that visuals can be created and used in many ways. The manner that visualization is used can only enhance the learner's understanding and attainment in mathematics.

At the inception of this literature review it is important to examine the various forms of visualization and its significance in mathematics.

De Guzman (2002:4) made reference to the different types of visualization, namely, isomorphic visualization, homeomorphic visualization, analogical visualization and diagrammatic visualization. The method in which visual manipulations of objects can be transformed into abstract mathematical relationships is called isomorphic visualization. De Guzman (2002) proclaimed that a great part of our visualization in mathematics is of the isomorphic kind due to the objects having an exact connection with the representations. In homeomorphic visualization the elements have mutual relations that replicate the relationships between abstract objects so that they can provide support to guide our imagination in mathematical processes like proving and conjecturing (de Guzman, 2002:5). In analogical visualization, used by Archimedes and Descartes, we mentally replace or substitute the objects with which we are working with another because it is easier to work with (de Guzman, 2002:6). The learners work has shown them to use pictorial or schematic expressions when working with representations in problem solving to obtain a better understanding. Debrenti (2013:57) stated that they should be conscious of visual representation as it facilitates understanding the problem and memorization of the problem. The use of diagrammatic visualization (using a diagram–pictorial visualization) is a helpful medium to assist our thinking processes (de Guzman, 2002:7). The use of symbolizations and diagrams in representing the data and the relations between them will assist in the solving the problem. Since pictorial representations play an important role in the learning process, the ability to visualize the data and communicate their interrelations contributes to mathematics problem solving very easily (Debrenti, 2015:21).

It would seem that most researchers in the field of visualization are unanimous that it plays an integral role and has the potential to be the heart of mathematics. As early as 2007 the British Columbian Government, recognising the importance of visualization, revolutionised its mathematics curriculum and introduced it as a teaching component. There are still several issues concerning visualization in South African school mathematics which require significant reflection. Thus, in my opinion, the use of visualization in school mathematics should be re-evaluated.

According to Luneta (2013:93) "*mathematics is a unique form of communication*" and it is "*important to understand the way the world can be viewed and interpreted*". Mathematics enjoys the luxury of having together visualization as a communication tool and also as a language as both can be interpreted differently by individuals. Visualization is used to convey mathematical information, namely, translation of the written problem to make information visible (external representations). Mathematics as a language allows the learners to use symbols, figures and objects to arrive at the solution in the problem (Debrenti, 2013). These visual representations align learners thinking to communicate their mathematical ideas.

Since problem solving is difficult for many learners and it involves any complex situation that needs to be resolved, visualization can often provide powerful results. It allows the learners to understand mathematical concepts in a simpler manner because when a teacher explain concepts the learners try to form a mental image of what is been said. Visualization assists the learners to connect with the content and draw on their prior knowledge enabling new ideas to be created. Visualization, irrespective of diversity as a barrier to learning and varying learning styles in the classroom, can assist the learners in problem solving by making associations with concepts this improving the potential to improve mathematical reasoning and also allowing the learners to elevate their thinking to another level.

The Van Hiele pedagogical theory identified the different levels of understanding through which a learner passes when learning (Naude and Meier, 2015). According to the van Hielies, learning and understanding is hierarchical in that a learner cannot operate with understanding on one level without having gone through the previous levels. It is impossible for learners to bypass or skip a level in this hierarchy of understanding. This is supported in the Van Hiele's levels of geometric development. Visualization is the first level making reference to recognition and classification (Naude and Meier, 2015). This alone explains the importance of visualization in mathematics. When learners use visualization they become more informed and knowledgeable about mathematical concepts. Visualization will allow learners to process information by classifying and interpreting it visually. They form a deeper understanding when they see something shown to them. Thereafter they externalize their thoughts by communicating them through any written means (diagrams). Since learners do not learn in a vacuum but in an interactive location (the classroom), the provided thoughts (representations) is tested through collaboration with other learners' thoughts as well. Thus one can say that new layers of understanding occur when these visual thoughts are recognised and improved upon. The relevance of Arcavi's definition stated above is noted here.

According to Ahmad et al (2010:357) difficulties arise in solving word problems when translating the word representations into mathematical representations. Since representations produced do not originate automatically and logically from external presentations, many factors are required to be considered, namely, identifying and placing the concepts involved in the problem in context, examining the solution processes and applying previous knowledge to communicate their understanding. In order to overcome the translation difficulty, the teachers should present the learners with opportunities to develop powerful visualization and visual representations when engaging in problem solving (Lowrie, 2001:360). One such an opportunity is allowing the learners to be creative. According to Kilpatrick et al (2001) the ability to solve problems is not only a skill but it is an activity that incorporates creativity. Creativity allows for live and rich mathematical learning to take place in the classroom. Creativity is a mental activity and thinking instrument as the learners use their minds to manipulate the data (Ayllon, Gomez and Ballesta-Claver, 2016). As a thinking instrument (cognitive activity) problem solving engages the learner's higher order thinking ability allowing them to use their prior knowledge to create original representations. By allowing them to create their ideas and solutions in novel ways allows for better understanding. It is within the realm of representations that learner's creativity comes to the fore. The learners may use brightness of colours, patterns and objects to work towards a solution. In this manner the learners produce their own authentic ideas of conceptual understanding when they translate the written data their representations.

Problem solving in a mathematics lesson can be described as phase of ingenuity on the part of the learner. It brings out the creative manner of the learners. Ayllon et al (2016:203) stated that as a mathematical activity "*creativity is a way to solve problems*" and "*problem solving is an efficient way to develop creativity*". As much as the teachers use innovative means to teach mathematics, creativity allows the learners the freedom to display their inventiveness in learning mathematics. Therefore it is important for mathematics teachers to see creativity as a construction making process of ideas within the mathematics problem solving process. The teachers must encourage the learners to use their imagination and externalise the many representations they conjure up during the problem solving situations. They should be able to systematically direct their learners thinking and allow them to communicate their mathematical ideas and understanding of concepts independently to bring forth their creativity. By visualizing and drawing their mathematical ideas it assists the learners in interpreting the problem and formulating these ideas in an innovative manner. Thus by applying these ideas effectively to mathematics it demonstrates their vivid understanding of the problem.

The learners need to make the connection between concrete and abstract therefore in the process of problem solving the learners should be able to translate the concrete to the abstract and the abstract to the concrete in order to gain a better understanding of mathematical concepts. According to Yenilmez and Kakmici (2015:190) it is difficult to comprehend and describe abstract concepts therefore concrete words must be used to make it comprehensible. This is called concretization (Yenilmez and Kakmici, 2015:190). According to Luneta (2013) to learn mathematical concepts the learners first need concrete materials to indicate their mastery of the concepts. Their mastery is shown when the learners are able to convert the concrete to abstract and vice versa. Visual means assist the learners to present the abstract ideas into concrete form.

To overcome the difficulty to translate these abstracts, the teachers must allow the learners to use creativity. This will allow the learners to create real or close to real representations thus giving meaning to these abstracts. It is not only representing concepts on paper that makes it concrete. The teachers can use nursery rhymes or use the learners in role playing situations in the classroom. This will give the learners additional means to make mathematics enjoyable as they will be able to understand the mathematical ideas in play form. This will build the learner's confidence to remember the associations. This will assist them to internalise and externalise concepts imaginatively from abstract to concrete with confidence. According to Piaget (Luneta, 2013) a learner between the ages of 7 to 14 starts to use his imagination and this is the stage of concrete operations. Due to the development of their imagination the learners can through the process of visualization express mathematical ideas in concrete and abstract forms.

According to Dryden and Vos (2012:323) "*visualising is a powerful learning tool*" and at any given time we work with images in our brains. The optical processes of our eyes and the vision we see in our brain is a very complex process. According to de Guzman (2002) this vision or visualization does not only appear naturally giving birth to a mathematical thought but also forms new relationships with mathematical objects and the communication processes involved in the mathematical activity. According to Bezemer and Kress (2008:169) this can be defined as "*semiotic material [that] is moved across modes*". All of these differences hinges on how the learners react to diagrams and visuals. The manner in which learners interpret the information as imagery from these diagrams and visuals is called transduction. What the teacher sees is not what the learners see and vice versa because what is created in the mind cannot simply be told to the learner by the teacher. The pre-service teachers must therefore be aware of the need to promote the use of imagery during visualization in mathematics. The various representations used solving problems serve as different ways through which the learners understand the problems and the solutions.

Using visualization is an effective way to assist the learners to solve mathematical problems. According to Lowrie (2001:360) the teachers should provide the learners with opportunities to develop powerful visualization when engaged in problem solving. This notion is supported by Kashefi, Othman, Alias, Kahar, Buhari and Zakaria (2015:803) who revealed that "*visualization among student must be enhanced in order to boost the knowledge insight in mathematics*".

2.8 VISUALIZATION AND REPRESENTATIONS IN PROBLEM SOLVING

Much of the information we assimilate in our brain is through visual means (eyes) thus one conclude that we rely a lot on visual representations for learning to occur.

At the onset I will like to discuss the sameness and differences of visualization and representations which are closely linked to problem solving. Tufte (2008) initially viewed visualization as external representation of information in the form of pictures, diagrams and the like. Later the view on visualization shifted to indicate that visualization included diagrams. Visualization is not only when learners rely on the construction mental models of their internal representations but it also involves the way the learners represent the diagram to gain an understanding. Gilbert (2005) went further to state that visualization has to do with a formation of an internal representation after been exposed to an external representation. People normally say that representation stands for something but according to McKendree, Small, Stenning and Conlon (2002:59) representation is seen as "*a structure that stands for something else; a word for an object, a sentence for a state of affairs, a diagram for an arrangement of things, a picture for a scene*". According to Gilbert (2005) representation is the portrayal of anything within the confines of external representation and internal representations whilst Schneider (1995) stated that representation is a tool which is used to illustrate concepts verbally, numerically and algebraically. The external representation is anything created in a visual symbolic form whilst internal representation is constructed mentally by an individual. Using a model (modelling process) as an example, it is an external representation (can be seen) of a mental model (internal representation). In the modelling process an idea develops in our brain to explain something (possible problem or description of an object). From the description provided it is not difficult to see the closeness of representation and visualization. Both representation and visualization is a strategic tool in solving problems and this is discussed interchangeably within this chapter.

There is debate around the role of visualization in the teaching and learning of mathematics problem solving. Kadunz and Yerushalmy (2015:463) stated that in the history of mathematics visualization has been avoided to a certain extent. It is only in the 19^{th} and 20^{th} century that mathematicians used the idea of visualization to gain better and new views when confronted with a problem (Kadunz and Yerushalmy, 2015). Some great minds have contributed substantially to our civilization like Einstein and Descartes amongst others have described their methods to problem solving as being highly visual. Einstein mentioned that he never though in terms of symbols but he though in terms of images. When this occurred the pictures came first and the descriptions later (internal representation and then external representation). Researchers like Goldin (1998) and Hiebert and Carpenter (1992) identified both internal and external representations as important. Internal representations are cognitive mathematical thoughts developed using mental models by making relationships with previous experience and external representations is the expression of a person's thoughts using visual objects to define a concept.

According to Kashefi et al (2015) the learning process is divided into three parts, namely, enactive, iconic and symbolic. Enactive is the crucial level of visualization which performs the association between the levels of understanding or acts as the mediator of communication (Deliyianni, Monoyiou, Elia, Georgiou and Zannettou, 2009). It does not only help the learners to ascertain the connection of the mathematical imagery but it is also an effectual manner in solving any problem from a syntactically, semantically and pragmatically perspective (Schnotz, 2002).

According to Carney and Levin (2002) there are five functions of pictures, namely, transformational, organizational, decorative, interpretational and representational. Transformational pictures is when pictures are used to improve the learners recall memory of information especially the text; organisational pictures are a structural framework that is valuable for the text content; decorative pictures relates to the text content; representational pictures are an illustration of a component or of the whole problem and interpretational pictures comprehend the understanding of the question (Carney and Levin, 2002). Using these types of pictures will enhance the learners understanding of the problem. This will allow them to commit information better to memory when the text and picture is used frequently. By tapping into their memory when using images to visualize the problem the learners will eventually get to see the bigger picture and this can guide the learners towards the solution.

Today teachers have access to information on understanding the influence of visual imagery and representations in the problem solving process (Hoffman, 2016). They have the benefit of understanding the learner's cognitive process involved in internal and external representations and are able to guide the learning process in the mathematics classroom. Boonen et al (2016:60) stated that it is essential that the teachers focus on the construction process, namely, making the representation and teaching their learners how to construct visual representations.

Studies by Skemp (1982) and Presmeg (1986) indicated that visualization in particular came to be recognized as critical to how mathematical and non-mathematical concepts are taught and understood. In a recent study Chandra (2015:3) found that from an educational point of view visualization is a "*powerful method*" for "*understanding concepts*". Visualization enables the learner to understand mathematical concepts as it prompts mental development to create picture like representations of the problem. According to Debrenti (2013:56) "*numerous psychological studies confirm that visuals in teaching helps a deeper understanding of concepts*" because individuals remember the visual aspects better. According to the Singapore Ministry of Education (2007) visualization is seen an indispensable skill that is critical in the learning and application of mathematics and this is supported by Presmeg (2006). According to Presmeg (1985:2006) the learners have a great need for visual methods of solving problems as visualization is gaining increased visibility in mathematics and mathematics education. There should be a greater focus on conceptual knowledge and understanding of the problem. The learners should subject the problem to mental, cognitive and visual processes thus eliminating unwanted information. For this to occur the learners need to visually represent their thoughts and the pre-service teachers need to understand how to encourage their learners to do this. By representing the problem the visual process becomes apparent and the interrelations to the problem become evident.

According to Sajadi, Amiripour and Rostamy-Malkhalifeh (2013) since problem solving is such a complex process and challenging task, the learners need to be taught efficient skills and one of these skill is representation. The importance of representation is of vital importance in mathematics (NCTM, 2000) because the use of diagrams, tables and graphics is seen as principal in expressing mathematical thoughts (Bal, 2014) as a representation is an actual situation related in another way. Previously in mathematics and in some current cases obtaining an answer is seen as the most essential outcome with visual aspects such as representations seen as a transient step to acquiring mathematics (Thornton, 2000:251). In the modern day classroom the focus is now on how the answer is obtained therefore it is of paramount importance for the teachers to teach representations. This is to ensure that the learners show an understanding of the changes within the initial problem and using the different representations through visualizing deliver a solution to the problem. That is why understanding and using representations together with visual skills in mathematics problem ought to develop into a dynamic component of the mathematics problem solving process.

Representations play an important role in the development of mathematical thinking and conceptual understanding. In order to reinforce or build the learners conceptual understanding more than one representation may be used. The use of more than one or different representations is known as multiple representations. One type of representation may not be sufficient to solve the problem motivating the learners to use multiple representations and choosing the best representation to arrive at the solution. The usage of multiple representations promotes a better mental understanding of mathematical concepts. Multiple representations are vital in conceptual expansion in learners as it was found to stimulate learners learning. It is beneficial in supporting the learners in problem solving as they can transfer one representation to another to build multiple representations in order to solve problems.

Representations in problem solving can take many forms. According to Scheiter, Gerjets and Catrambone (2007) visualizing solutions to examples can be done by representing static pictures, animations or mental imagery. Static pictures (pictorial representations) are acknowledged to encourage retention of the text. Representation of the problem in picture form will assist to understand the key features of the problem and their interrelatedness which will lead to a solution. The diagram or picture that the learners employ or use to enhance their understanding will automatically generate a big picture in their mind to discover the solution to the problem (Deliyianni et al, 2009). Additionally visualizing the solution steps and comparing the illustrations to these steps will assist the learners when applying a solution to the problem. These frames show the changes from the initial step to the goal step (solution). Imagery is when the learners imagine the problem. The problem is presented in written form without any pictorial illustration and the learner pictures may learn in a more dynamic manner because they have to make multiple visualization comparisons to understand the solution procedures as the information is visible. Using mental imagery has a possible advantage in the latter in that these images are self-generated images and can be adapted taking into consideration the learner's prior knowledge.

Bal (2014:2350) stated that the pathway to how the learners solve the problem, the stage where representations are used and how the representations influence the solution must be highlighted. It is further emphasized that the pathway of solving the problem must be given importance as representation, single or multiple, will generate individual thinking thus giving better scope for understanding (Bal, 2014:2350). According to Montague (2005) problem solving progresses through two main stages, namely, problem representation and problem execution and both are equally necessary for successful problem solving.

Successful problem solving is not possible without initially representing the problem. In problem representation (can be an amalgamation of Polya's and Wallas steps one and two) the learners identify the issues to be solved and create representations of it using existing knowledge. The learners need to use apposite representation to show their understanding of the problem. They are able to translate the key words (concepts) in the problem into representations using paper and pencil or computer graphics. These representations are their internal representations formulated in their imagery (visualization) and shown as external representations (diagrams or symbols). The learner's external representations can be verbal, graphical, algebraic or numeric representations (Bal, 2014:2353). These representation images often provide the learners with visual alternate to words and to also guide their thinking. When the learners use apt representations it provides evidence that they have perceived the problem and this eventually assists in guiding them towards the solution (Sajadi, Amiripour and Rostamy-Malkhalifeh, 2013). Montague (2005:2) stated that visualization is one of the most powerful problem representation strategies. It allows the learners to utilise their mental images to create verbal representations (learners express the solution verbally), graphical representation (using diagrams in explaining the problem), algebraic or symbolic representation (use of mathematical symbols or arithmetic calculations) and numeric representation (using a format to explain the problem) (Bal, 2014, 2353-2354). During their years at university the pre-service teachers should be subjected continuously to such measures described above as it through this kind of training and exposure that they will influence how learners solve problems.

Problem execution is when the learners design a possible solution and executes it. They use their representations such as pictures, graphs, symbols or other forms of displays considering technological advancements to attempt and solve the problem. The visual representation of the problem assists in providing a clearer picture of the learner's conceptual understanding and also allows the learners to seek the many alternatives to finding the solution. Debrenti (2015:23) in her studies concluded that "*there is a strong need for the use of concrete and visual representations in the teaching of mathematics in schools*" as representation confirms the learners reading and comprehension which will indicate the correct solution (Sajadi et al, 2013). Seifi et al (2012) in their study found that the learners had a problem in representation due to them not been able to understand the problem as a result they used incorrect strategies which affects the problem execution. This was also evident amongst pre-service teachers when asked to solve problems (discussed in chapter 4). Sajadi et al (2013) stated that if teachers did not teach efficient representation then the learners will not be able to comprehend the problem which will affect them arriving at the correct solution. Pre-service teachers need to understand the importance of teaching their learners how to use representation as a visual means to solving mathematical problems. According to Sajadi et al (2013:4) most researchers and teachers agree that representation is linked to understanding and communicating mathematical concepts thus visual representations in problem solving is necessary as it seems to ease the problem solving process.

Pre-service teachers are fresh out of their matriculation year where much focus was on algebraic and geometric aspects with little or no focus on problem solving. Peker (2009) stated that the pre-service teachers understanding the problem solving process is the first step in them learning how to teach it. Duru (2011) investigated the pre-service teacher's problem solving preferences and it was discovered that some of their preferred problem solving strategies were using a model, guess and check and algebraic strategies. Taking Duru and Peker statements into consideration it is important that the pre-service teachers are taught how to use and reinforce representation and visualization as strategies in their mathematics modules at higher educational institutions because for future teachers these will become "*creative and fantastic methods for problem solving*" (Sajadi et al, 2013:8) and beneficial (Bal, 2014).

2.9 VISUALIZATION AND TECHNOLOGY

Technology is the future of our educational system and it can play a major role in the teaching and learning process. In the South African context, the curriculum reformation in CAPS is preparing the teachers to use technology effectively in teaching and learning as it can immensely enhance the classroom environment (Department of Basic Education, 2014). Much of the modern generation are very much knowledgeable technologically having taught themselves digital literacy through using the internet and social media platforms. They consider technology to be part of their lives and there is no sense depriving them of using technology at school. The result of the advancement of technology is that it has now impacted greatly as a means of communication for teaching and learning in schools and higher education institutions as it bridges the gap between the classroom and the outside world.

The strategic use of technology is important in teaching and learning. The focus on technology and its association with mathematics has great ramifications for teaching and learning. Kuzle (2012:8) stated that "*taking into consideration the influence of an increasingly global and technological society on teaching practices, teachers need to become aware of the pedagogical and implications of technology and be able to take advantage of technology as a powerful and engaging teaching tool*". Teaching problem solving methodology has become a central focus of instructional activity via technology education. It is presumed that technology permits topics to be studied in greater intensity through more collaborative ways through the use of simulations and descriptions. According to the White Paper on e-Education (Department of Education, 2004) teaching using technology will enhance the quality of learning. It allows mathematical situations and concepts to be brought to life. Since it stimulates the learner's interest, the pre-service teachers should maximise the use of technology to create visual means to teach problem strategies and skills in the classroom. Technology, besides being a powerful tool to hold the learners attention span, has the capacity of opening learners to new frontiers of learning by allowing them to acquire skills that can be used when solving problems. Technology in the mathematics classroom supports learning as it allows the learners an opportunity to adapt visualization by working collaboratively with the material. Engaging interactively with concepts and technology simplifies many mathematical aspects thus making learning fun and easy as the learners have a visual medium to effectively support their understanding. This will help create an authentic learning experience for the learners. Such a learning experience allows them to use their creativity, analytical skills to develop and construct their own understanding. In this way they can determine what is needed before making an informed decision on the strategy to be used to solve the problem. This kind of learning in the mathematics classroom is supported by the Department of Education where the learners need to use their order thinking skills to deliver higher order performance.

Investigation in teaching and learning and developments in technology have prompted significant changes in how mathematics is taught. The role of technology via the use of visual resources has shown that visualization in mathematics has grown immensely during the last decade and the teachers and learners are now using technology and visualization more regularly. The immense changes in the technological field and the manner in which information is communicated to the masses has seen an increase to using 2D, 3D, animations and representative models as visual material without difficulty in the classroom. The teachers are now being encouraged to focus on important mathematics through the use of visualization, appropriate use of representations supported by communication technologies (Luneta, 2013:14).

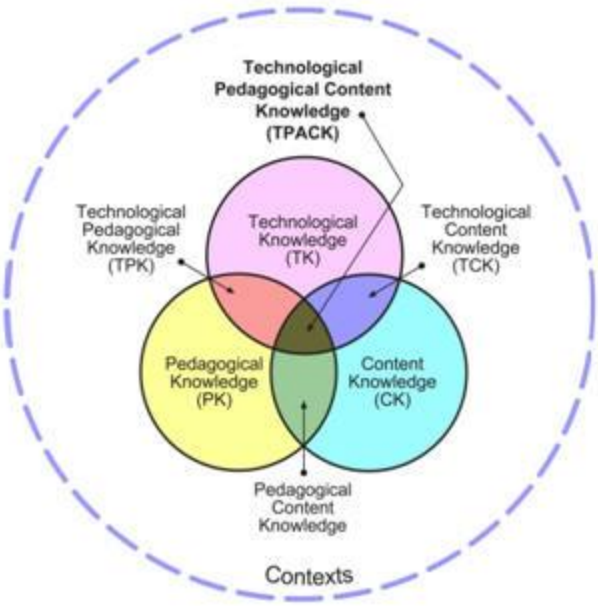


Figure 8 Framework for Technological Pedagogical Content Knowledge

The above model (Figure 8) was developed by Mishra and Koehler. The letters CK denotes Content, PK denotes Pedagogical and TK denotes Technology. The circles overlap creating new categories of knowledge. Why create another knowledge category when the pre-service teachers need common content knowledge (CCK), specialized content knowledge (SCK); knowledge of content and students (KCS) and knowledge of content and teaching (KCT) to teach the subject efficiently. Several issues linked to pedagogical content knowledge necessitate the need to add technology knowledge. This model as a new category is justified by Mishra and Koehler on the basis that the present changes in technology impacts on both content and pedagogy. They are of the view that the intertwining of the three sources of knowledge, namely, content, pedagogy and technology "*is the basis of all good teaching*" (Hyde and Edwards, 2011:85). An interesting area to explore is how the pre-service teachers are able to make didactic uses of technology to teach content knowledge. It is necessary for them to know how to manage technological advancements into their teaching and also assist their learners to use technology effectively in their learning.

In Figure 9 the diagram indicates the relationship of the student (learners), teachers and the teaching facility (classroom). The central route to learning is the learner.

In Figure 10 the learner is still the focus of teaching and learning. The pre-service teachers now need to focus on how information technology can make their lessons more dynamic. Hence they need to become proficient in using this in the classroom in order to put across content material more informatively as it lends itself to experiential learning and it brings real life situations into the classroom. This kind of approach will allow learners to learn more because when they visualize they retain more of the knowledge as it allows for concepts to be vividly explained.

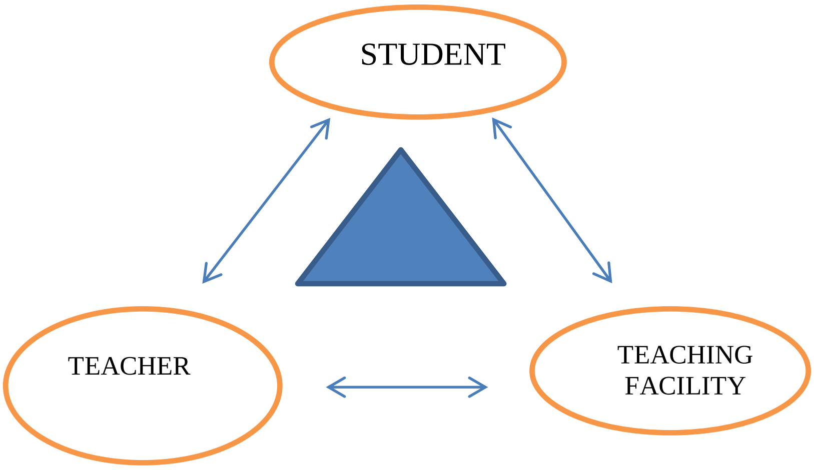


Figure 9 Didactic Triangle (Pavlekovic, Kolar-Begovic and Kolar-Super, 2013:132)

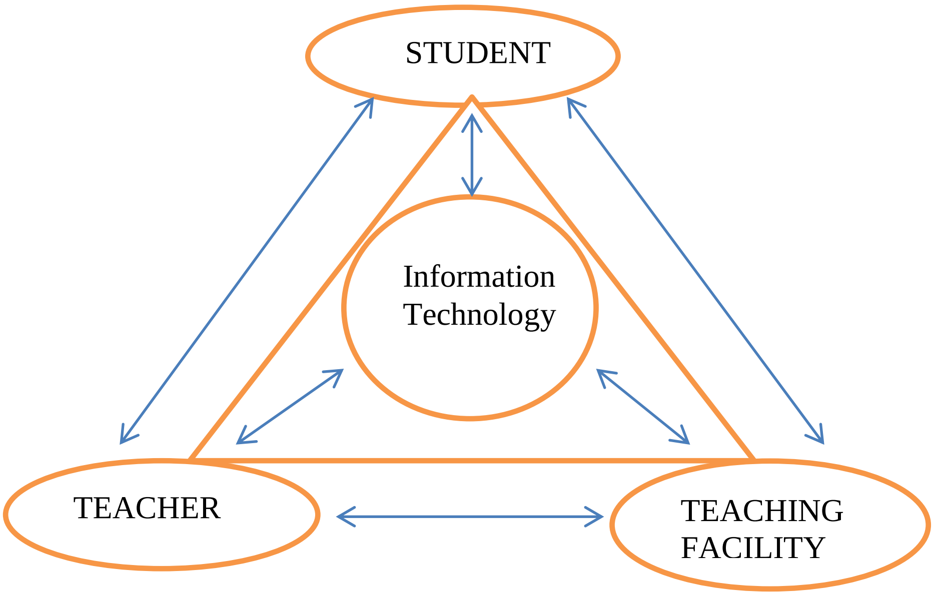


Figure 10 Didactic Triangle (Pavlekovic, Kolar-Begovic and Kolar-Super, 2013:133)

Teachers and the pre-service teachers, whilst exposed to the advancements in technology, do not have the necessary capacity to use it successfully in the classroom a s teaching resources or are not prepared to foster its implementation to improve pedagogical practices. Hyde and Edwards (2011) found in their studies that the pre-service teachers found it difficult to transfer technological skills to the classroom. According to Abbitt and Klett (2007) it has become the responsibility of the teacher training programs at higher education institutions to train the pre-service teachers effectively to use technology in the practice of teaching. With the advent of technology explosion internationally, schools will expect technologically literate teachers to model teaching differently to cater for the differing learning style of a new generation of learners. Schools and institutions, as a cost cutting strategy, have resorted to putting all teaching and learning material on educational programs accessible via tablets, word processors and smart phones. By using technologies, recognizable by learners, the teachers will be able to stimulate their lessons in an experiential and authentic way. By doing so the teachers will be able to engage the learners as active learners and also boost creative thinking and learning. Worldwide there has been a rapid development in information technology. This is influencing and reshaping the learning styles of learners in schools. By training the pre-service teachers to engage with technology correctly in conjunction with the relevant content material, higher education institutions will strengthen the use of technology in teaching and learning. Technology is currently a predestined component of the 21st century and if used to support visualization in a variety of contexts, it can assist learners in becoming prolific problem solvers. Therefore, it crucial that the higher education institutions take cognizance of modern technology and the impact it has on mathematics teaching and learning and teach the pre-service teachers pedagogical content simultaneously with technological pedagogical content knowledge in a way that encourages and optimizes learning.

2.10 SPATIAL VISUALIZATION

The following are definitions of spatial visualization:

According to Lowrie, Logan and Ramful (2016:408) spatial visualization is the ability to "*manipulate or transform the image of spatial patterns into visual arrangements*".

Augustynaik, Murphy and Philips (2004) described spatial visualization as a vital skill for understanding and developing crucial mathematical skills and is an opportunity to better problem solving.

Lohman (2000) defined spatial visualization as an adeptness to comprehend imaginary movement or the aptitude to manipulate objects in the mind.

To place spatial visualization into the perspective of visualization, it can be described as mentally manipulating a pictorial stimulant to understand the visual information. Idris (1998) found in his study that spatial visualization is related to mathematics achievement and goes further to state that visualization does not only influence mathematics success but also improves the learners overall academic accomplishment on the whole. Rabab'h and Veloo (2015) stated that the learner's low achievement in mathematics is of concern to the teachers. By having high spatial visualization ability motivates them to enhance their academic success.

Given the role played by visual spatial skills in visualization and the development of mathematics, it is significant to build on it early on in the learner's academic life as it will support mathematical development in later stages (Meyer et al, 2010). According to Kim and Cameron (2016:11) visual skills is basically overlooked in school settings and given its magnitude in school readiness, the teachers need to focus on developing these skills. A suggestion is made by Diamond and Lee (2001) that visual skill programs should be made part of the school curriculum such that the learners are able to engage in real world tasks.

Ozdemir and Yildiz (2015) examined the pre-service teacher's spatial skills through the SOLO model to raise awareness for their own visual skills. The SOLO taxonomy evaluated the pre-service teacher's mathematical understanding of concepts and their thinking skills. It consists of five levels, namely, prestructural, unistructural, multistructural, relational and extended abstract levels (Ozdemir et al, 2015:219). In the prestructural level finding the solution is not adequate. There are aspects in the problem which are a distraction. In the unistructral level the focus is on the problem. A part of the information is used and as a consequence the data in the problem cannot be related with previous situations thus resulting in an incoherent answer. In the multistructural level the multiple data is used to arrive at the answer but there are still inconsistencies in the answer. Within the relational level all the data in the problem is utilized to arrive at the solution and the association is seen with other data in the problem. In the extended level when a solution is arrived at, generalizations can be made thus creating new thinking styles (Ozdemir et al, 2015:219). In their study it was discovered that the pre-service teachers were within the multistructural and relational level – working with the problem and making associations with known data.

The Institute of Education Sciences (2012:26) made a strong recommendation that visualization be used in mathematics because it was found in their studies that "*students with learning disabilities performed better when taught to use visual representations*". Visual spatial learners have a different brain structure and they learn differently from other learners. They learn visually thus visual representations assist these learners in organizing the data which is then analysed leading to a solution. As a result of these learners learning styles they need more than one representation to solve a problem. Due to its importance in mathematics, representations (included herein is multiple representations) "*helps learners by employing their own thinking and learning habits*" (Ozdemir and Reis, 2013:86). Multiple representations provide visual material for problem solving. It is helpful to these learners in the problem solving process in that it can make their solutions visible (Ozdemir and Reis, 2013:86). Spatial visualization and visual representations assists in organizing the given problem for better understanding thus paving the way for the solution.

Visuospatial skills are a vital foundation for the learners to learn mathematics (Uttal et al, 2013; Mix and Cheng, 2012). Even though visualization demands spatial skills, it can be used to improve spatial skills as visual spatial skills contribute to the development of the learner's mental representations as they use strategies to solve problems.

Visualization research has shown that the learner's spatial skills and prior knowledge are related to the use of external and internal representation. External visualization can refer to the use of diagrams, graphics and models in learning whilst internal visualization is used to portray mental construction of ideas. In order for the learners to externalize their thoughts prior knowledge is needed. This prior knowledge is a means of association of previously acquired knowledge. This is stored in the brain as visual images thus allowing the learners to create new visual pictures in order to learn. The learners who have dominant visual-spatial intelligence learn best through visualizing entities, events or by studying with images, drawings and colours (Yenilmez and Kakmaci, 2015). According to Dryden and Vos (2012:323) the learners should be encouraged to "*visualize precisely*". They must first see the bigger picture and grasp the concepts for learning to occur. In this manner the learners will be able to apply their visuals to the problem and reinforce their learning.

Language is an important component to visual spatial learning. Language originates within an individual thus the learners need to verbalise it in order to acquire certain types of mathematical knowledge. As in English, the learners sound out the letters of the alphabet in phonic form to learn words. These words have to be constructed and translated into making meaning of mathematical concepts. The learners verbalize these words to develop their conceptual knowledge to communicate, example, learning to count using their fingers. Seeing their fingers as representation of numbers, an association is made between a word and the number of fingers seen visually. This type of counting skill will assist the learners to represent the numbers they are counting. Visual spatial learners will by association reproduce this imagery diagrammatically for a better conceptual understanding.

According to Lowrie et al (2016) diagrams are critical to success in mathematics. The learners think critically, decode their information and use the diagrams to represent their thoughts. In their imagining the learners think vividly and they organise ideas from within their world of experience. Thus one can state there is a link between spatial thinking and mathematical thinking.

Jitendra, Star, Dupuis and Rodriguez (2013) found in their study on Schema Based Instruction (SBI) that it has links with spatial visualization and problem solving skills. This was a four step strategy, namely, priming the mathematical structure of the problem. According to Jitendra et al (2013) this involved schema training in unravelling the relevant from the irrelevant information; using visual spatial representations; using pictorial and schematic illustrations which are an indication of an individual's idea and instructing through problem heuristics. Jitendra et al (2013:115-117) stated that the learners draw their representations or use strategies to represent and analyse the solutions to the problem. The teacher aids in modifying details in a step by step teaching process.

2.11 VISUALIZATION, COGNITIVE AND METACOGNITIVE SKILLS

Luneta (2013) made reference to two types of representations, namely, cognitive representation and instruction representation. Visualization, seen as the development of mental images, falls under the broad area of cognitive representation. These are the representations that the learners create individually in order to comprehend and understand the mathematical concepts and content.

Just as learners develop physically due to maturation, changes also alter the structure of the brain cognitively. The teachers generally begin to teach mathematics to the learners from a young age through visual means. The learner's cognitive and metacognitive skills develop in their formative years and it is during this phase that the learners are exposed to the process of problem solving in mathematics.

As problem solving is an intellectual activity based on constructivist theories of teaching and learning (NCTM, 2000) it will develop learner's cognitive and metacognitive skills. Cognitive can be described as the internal mental processes of an individual doing and knowing. According to Kirk (2006:142) metacognition is described as "*the ability to think about one's own thinking*". It is the process of thinking about you think. The learners are often unaware of how to think and engage with the problem. Thus the teacher assists the learners to learn independently by explicitly guiding them to plan and evaluate their learning strategies. This type of guidance consolidates the learner's decision making skills when dealing with problem solving.

Problem solving is a complex cognitive activity. Figure 11 indicates the interrelationship involving metacognition and cognition during problem solving. The interaction between metacognitive and cognitive activities is highly important as it impinges on the learner's problem solving performance. According to Kuzle (2013) cognitive problem solving actions not accompanied by appropriate metacognitive actions will lead to unproductive efforts in mathematics. One needs to determine how language (in the given problem) is turned into knowledge. The teacher teaches metacognitive skills by asking the learners to explain what they are thinking and what strategies they will use to solve the problem. Explaining their thoughts should take the form of 'thinking aloud'. The learners should be given the opportunity to read the problem and state their thoughts verbally (thinking aloud) thus bringing alive their mental images. By engaging with the learners in this manner allows the teacher the possibility to better prepare them for their lives and learning in the future.

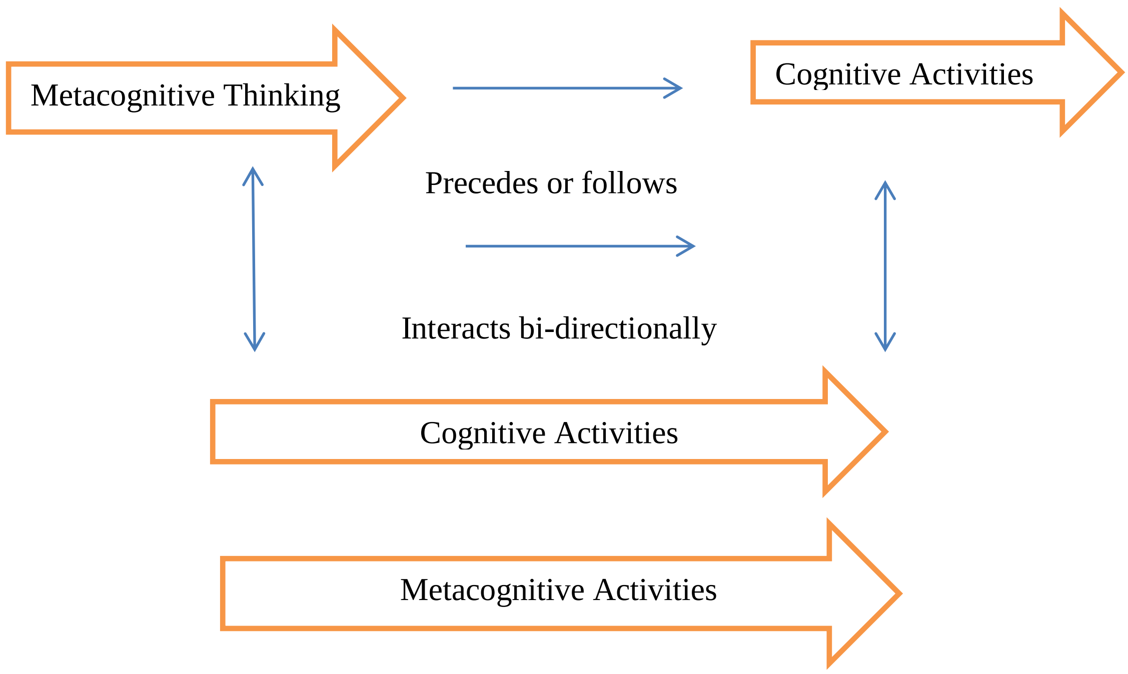


Figure 11 Relationships between Metacognitive and Cognitive (Sagirli, 2016:642)

Piaget (Luneta, 2013) described four distinctive stages of an individual's cognitive development, namely, sensorimotor, preoperational, concrete operational and formal operational. In the sensorimotor stage, whilst there is cognitive growth, there is no scientific research to support cognitive growth in relation to representation and differentiation. In the preoperational stage (2-7 years) a child-learner is able to "*use symbolic representations (e.g. drawing and graphics) and to develop the ability to imagine events*". This is a critical learning period in a learner's life as it is during this phase that the learner's communication, social and motor skills develop. It is beyond comprehension then why these cognitive skills are enhanced by the teachers early in the formative years of the learner's scholastic career. During the concrete operational stage (7-11 years) a learner's cognition increases as he is able to imagine, classify and reclassify physical objects and draws on previous experience. In the formal operational stage (11 years to becoming an adult) the learner has the capability to hypothesise reason and construct ideas in multiple aspects of the problem (Luneta, 2013:29). The learners write down their hypotheses. When this is done they are able to compare with the other learners in the classroom. Using a step by step process they arrive at a conclusion and this becomes their solution. What is very important is the manner the teacher uses the different questioning techniques, prompts and probes to cater for learners with differentiated cognitive abilities. Luneta (2013:29) sums it up adequately by stating that "*it is valuable for teachers to be aware of Piaget's stages*". There is sufficient evidence that these stages are crucial as they lay the foundation to problem solving from an early age. Referring to the age groups mentioned by Piaget (Luneta 2013), Deliyianni et al (2009) stated that using visualization is very much age dependent and this is supported by studies made by Boaler (2016).

According to Boaler (2016) as the learner's age they develop their ventral visual pathway which is responsible for visual brain activity. During the stages as described by Piaget (Luneta , 2013) and studies undertaken by Boaler (2016), visualization can be used from an early stage to support the learner's understanding of abstract concepts. During their formative years of learning the learners move from using concrete examples to abstract thinking and since visualization is an internal schematic representation of their understanding of concepts this allows the teacher to engage with the learners to support this understanding.

According to Boaler (2015) the greatest learning is achieved when the two areas of the brain are communicating. One division of the brain is used when we work by means of numbers and the other when we work with the visual information. Thus one can conclude that the brain can deduce both cognitive and visual information.

Considerable research indicates that the learners understanding can be enhanced by visuals. Richard Meyer (2001) who formulated the Cognitive Theory of Multimedia Learning stated that integrating content can activate the senses (visual, auditory, pictures) of the learners to visualize further. Pascal Wallisch, a neuroscientist, described visuals as more appealing to the human brain than words (Hoffman, 2016). Thus one can determine that the brain is instrumental in assisting with visual learning. According to Brown (2002) visual learning is the utilization of images to facilitate and enhance learning at all levels. New research in the area of learning has shown that 90% of information communicated to the brain is visual (Hoffman, 2016). This has created a better understanding of how the learners process and remember new information. Since the brain processes all visual functioning at one tenth of a second (Hoffman, 2016; Brown, 2002) it has the ability to apply visualization to a problem such that a learner is able to respond with a solution through simple illustrations to the given information rather than trying to solve it laboriously. We are unbelievable good at remembering pictures (Hoffman, 2016) thus when using illustrations the learners must learn to think visually and communicate their ideas using various thinking and problem solving skills so that they can expand their understanding of mathematical concepts.

The future of learning depends on our visual awareness on our visual awareness as technology is also playing a vital role in the developing of brain function as they enhance the cognitive powers of individuals during thinking, problem solving and learning. Considerable research has revealed that learning is enhanced by well communicated images as visual imagery attracts and maintains the learner's concentration on the lesson. According to Paivio (1990) imagery and written language have different cognitive representations as a result the memory systems uses verbal memory and image memory for comprehending the various types of information it is exposed. There is an interchange of information in memory, namely, when the brain acquires visual information it moves from the sensory memory to the visual processors when verbal information is received by the sensory memory it moves to the verbal processors. According to Sharma (2016) when one is solving a problem the attention is on the information provided in the question in order to understand it; the relevant information (concepts) is received, comprehended and then held in the working memory; the long term memory is scanned for find aspects (definitions, concepts and procedures) that it can relate to; the information is reformulated with that found in the long term memory and existing memory; the pertinent information is then selected to satisfy the requirements of the question and is then executed as a solution. All of this occurs within minutes. Mareno and Mayer (2000) suggested that active learning occurs when a learner engages in three types of cognitive processes, namely, selection, organisation and integration. According to Mareno and Mayer (2000) in selection the learner selects the key words and visual for verbal processing and visual processing respectively; in organisation the learners organises the words into verbal models and images into visual models; in integration the verbal and visual processes are integrated and the visual information is interlinked as they assist each other to create meaning.

Taking these into consideration Sharma (2016) examined the influence of the working memory on problem solving which plays an integral part in problem solving. The working memory stores information in our mind for a short period of time and this is used in many facets of our life, example, reading, comprehension and mental planning. According to Sharma (2016) many lower and higher order cognitive thinking skills, language processing, visuo-spatial and reasoning skills are involved in the memory in order for a learner to see and represent the problem. Problem solving involves a unique form of brain information processing because it uses techniques and information from our prior experience that is stored in our memory to support learning (Reynolds and Flagg, 1983:250). Greeno, one of the foremost who explained the connection between memory and the processes involved in problem solving, proposed a memory model for problem solving. This became known as the Greeno's memory model (Figure 12).

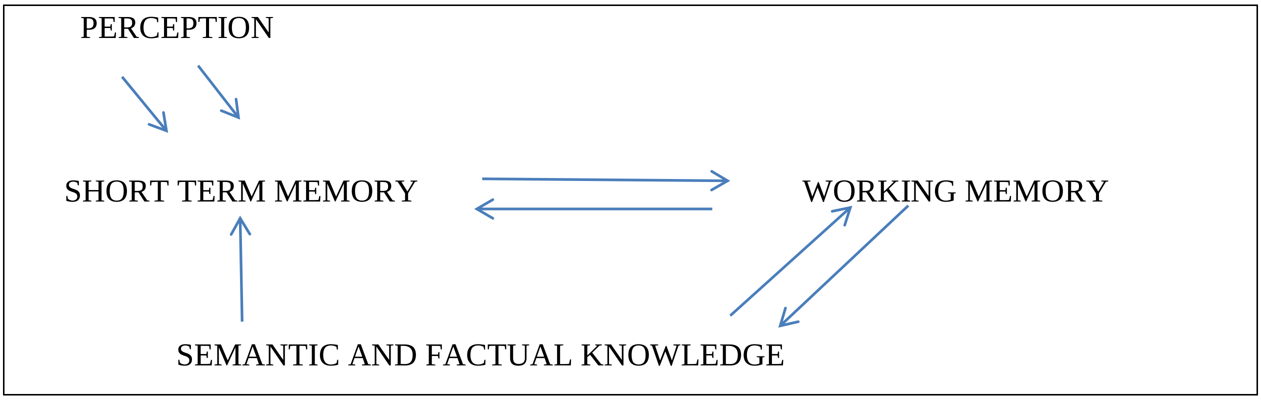


Figure 12 Greeno's Memory Model (Reynolds and Flagg, 1983:250)

In the short term memory, as the name suggest, only a limited amount of information can be stored here. It also stores information that is easily retrievable. The external description of the problem is input. In the long term memory past experiences such as facts, heuristics and related problems are stored. The long term memory is able to store large amounts of information. The working memory stores information in our minds for a short duration. It contains information that is actively in use and this information is processed in the working memory. The information from the short term memory and long term memory interact and a solution to the problem is generated. The problem solver is able to draw old information from the past and manipulate it in the working memory. Thus an internal representation of the problem is formed in the working memory. During the problem solving process certain features from the problem may activate the stored knowledge in the working memory which leads to a form of representation. The arrows (Figure 12) indicate the flow of information from perception (recognition) to the working memory and how it is consolidated as factual knowledge. The semantic and factual knowledge stores factual knowledge (long term memory plan). Using the stages of Greeno's problem solving model where the relations are established, problem representation occurs in the working memory. It is in the working memory that the information from the short term memory and the long term memory interact to generate a solution. It is in the working memory that conceptualization and mental processing of information (interaction) occurs (Sharma, 2016). The information is incorporated with the internal knowledge to show understanding.

Cubing (Bornman and Rose, 2010) is a versatile strategy that embraces all of lower and higher order cognitive thinking skills, language processing, visuo-spatial and reasoning skills (Sharma, 2016). This is a versatile differentiation strategy that can be utilised to challenge the learner's problem solving skills. As a cube comprises six sides, this approach uses six aspects such as describing, comparing, associating, analysing, applying and arguing. It also reflects similar levels to Blooms Taxonomy of learning. In step one, the lesson involves the learners recognising and recalling facts from memory (rote); thereafter the learners create an understanding by comparing the facts (step two); in step three the learner applies the facts to the given situation; he analyses the facts by breaking them into smaller parts (step four); in step five the learners evaluate the information by arguing the solution and in step six the learner creates new knowledge by applying the facts to new or similar situations (Bornman and Rose, 2010:78-80). When using the cubing theory the teacher's questioning technique should contain memory skills, comparing skills, application skills and analysing skills.

Psychologists constantly refer to the left and right brain when discussing how people think. In school there are many learners who learn differently due to the role of the brain in seeing and processing information. The left brain is responsible for the seeing and processing of stimuli from within the environment. This develops the conversion of processing to language formation in the left brain. As the child grows his ability to use language to express, describe and attach meaning to things indicates the development of his cognitive and metacognitive skills.

According to Luneta (2013) when the teacher ask metacognitive questions it allows the learners to reflect on their thoughts and their responses indicate to the teacher their understanding of a specific concept. Since a literal meaning to visualize means to see or think about something such that a mental picture is created, subsequently the central to the thinking processes in which memory is employed is visualization. As memory determines metacognitive competence, Gilbert (2005:15) asks, "*why should metacognition in respect of visualization exist?"* Visualization is an act of bringing into focus an image into the mind (Kotsopoulous and Cordy, 2009) and the mind creates a visual imagery which becomes the representation of the object visualized. It is the internal images that facilitate cognitive function in a meaningful manner which leads to vivid external representations. A mental picture constructed by an individual is a personal image which is then communicated to demonstrate the learner's level of understanding. According to Debrenti (2013:57) the way a learner represents his knowledge externally indicates the manner he represents the information internally.

According to Ball (Ball, Thames and Phelps, 2008:392) some representations are especially powerful whilst others although supposedly correct do not expose the learners to create meaningful ideas. Brating and Peljare (2008:354) stated that "*visuals can be interpreted in a variety of ways depending on what is being said or asked*". What a human being constructs in his brain is his understanding of concepts and the learner's external representations can contain errors and this is due to misinterpretation. Thereafter it is important that the pre-service teachers be trained "*to see through a child's eye*" (Debrenti, 2013:57) to understand how the learners visualize to create different representations and show their relationships between visualization and the eventual solution.

2.12 VISUAL LITERACY AND VISUALIZATION

Visual literacy was championed by Lida Cochran (Visual Literacy, 2007). Her work reminds us that visual literacy is a fundamental part of making meaning.

Many definitions of visual literacy have surfaced over the years, namely:

Bamford (2003:1) stated that visual literacy "*is what is seen with the eye and what is seen in the mind*".

Stokes (2002:3) defined visual literacy "*as the ability to interpret images as well as to generate images for communicating ideas and concepts".*

Psychologists view that seeing comes before words therefore visual literacy is related to our field of vision which we experience in our daily life. Visualization is a related construct as it allows the learners to read, interpret and understand the relevant information presented in pictorial form. As we see we construct a mental picture of what is seen. It assists the visualizer to transform the image and communicate it verbally or externalise the imagery mechanically. Mudaly (2008) made a distinction between visual literacy and visualization by stating that visual literacy is "*visualization combined with logical thought".* Visual literacy provides the learners with an opportunity to be innovative and imaginative during the problem solving process. The learners need to learn the visual language to communicate and the teachers need to learn how to teach visually (Stokes, 2002:12). According to de Guzman (2012) visual language can be a powerful way to transmit information. The learners create mental images of the mathematical concepts. When they do this through the use of visual literacy they are able to comprehend, understand and express mathematical concepts more easily thus making them reliant on the use of words alone in the problem.

The teachers should be encouraged to promote the use of visual literacy in the mathematics classroom as it assists in decoding the ideas that the learners perceive and interpret (Visual Literacy, 2007). The importance of visual literacy in mathematics is acknowledged by researchers as it allows the learners to view their thought process as they present their ideas visually. This leads to them using visualization.

Visualization is described in different ways by many researchers and the definitions are indicated elsewhere in this chapter. Simply put it is a mental image that is created in the mind that is eventually externalized in a representation. Kadunz and Straber (2004:247) stated that "*visualization is understood as linking images and diagrams….*" thus the learners create their images, represent them and then evaluate them in a critical manner.

Within the domain of visual literacy and visualization, I make reference to the distinction between the following kinds of imagery, namely, concrete/pictorial imagery, pattern imagery, memory images of symbolic notation, dynamic imagery and mental operations or transformations of images (De Windt-King and Goldin, 2003). According to De Windt-King and Goldin, 2003) these characteristics of visual imagery are regarded as internal constructs of external behaviour. A brief examination of these reveals that concrete/pictorial imagery is an internal which is represented as an external object or image; pattern imagery leads to the transformation of patterns which ultimately leads to the process of generalization; memory images refers to the visualization of the symbolic notation and reference is made to the visualization of formal mathematics of the formal expressions; in dynamic imagery the image undergoes a change; mental operations or transformations refers to the act by the imager to transform the image (De Windt-King and Goldin, 2003:5).

I believe that the non-acquisition of visual literacy skills and non-enhancement of visualization as part of teaching and learning in the mathematics classroom is one of the barriers to the learners performing poorly in mathematics.

2.13 VISUAL THINKING AND MATHEMATICAL LEARNING

The manner in which our learners utilise their thinking skills in the mathematics classroom can be viewed with trepidation. Sometimes, as mathematics teachers there is caution that is non-existent. The reliance on the teacher, textbooks and workbooks in the primary school has resulted in information overload leading the learners to resort to memorization of mathematical procedures rather than using their thinking abilities. This can be dangerous to a subject like mathematics as the learners will lack the basic thinking and learning skills to understand and apply the content imparted to them. Therefore we as teachers need to encourage our learners as well as expose them to utilise their thinking ability. To support their higher level thinking ability teachers can alternatively encourage their learners to start thinking visually as we all have the innate ability for visualization.

Using visualization as a starting point, the learners will be in a better position to demonstrate their understanding to learn visually and also make inferences when confronted with mathematical information. This will indirectly contribute to their mathematical thinking.

Visual thinking is the ability to turn all types of information into different forms of visuals that aids the communication of the information. According to Martins (2014) thinking visually is powerful and highly efficient and does not require much effort. This allows the learners to develop their conceptual skill of insight using their visual insights to unlock knowledge (Martins, 2014). All learners have different ways to interpret and learn visually based on their intellectual experience, example, seeing a policeman. When the learners see a policeman different thoughts conjure up in their mind. Based on their experience or prior knowledge, some may associate him with trouble and others may associate him as someone who can be approached when one is in trouble.

The sight of things when the learners are exposed to different teaching mediums in the classroom example, a diagram will elicit a response in their minds. Focussing visually on the image (diagram) may result in them thinking and making their own associations. Inadvertently a mental outcome is produced in the learner's mind. I make reference to an example of an incident I observed in a teacher's classroom. A geography teacher wanting to teach his learners the cardinal points drew a diagram (Figure 13) to denote the four cardinal points. He asked his learners to name the four cardinal points referring to them as directions. One response from a learner was *top*, *down*, *left* and *right*. Obviously this was not the answer the teacher was expecting but it was what was perceived in the learner's mind.

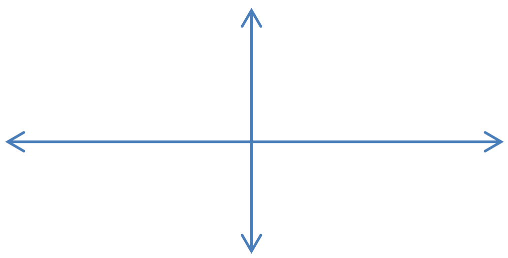


Figure 13 Teacher representation of the cardinal points

According to Puphaiboon and Woodcock (2005) understanding a diagram (whether it is drawn on paper or manipulated in some other manner) is part of the thinking process. Through mathematical thinking visual images are drawn on paper or created using visual tools of concepts. Boaler (2016) suggested that the teachers ask the learners how they see mathematical ideas and illustrate what they perceive. It is in this ways that ideas are germinated in the brain. When the learners learn through these visual means mathematics changes for as they require a deeper and better understanding of the concepts in the problem. These conceptual images consolidate their understanding and capture the most important aspects of their mathematical thinking which leads to mathematical learning (Figure 14).

Mathematical thinking, according to Kilpatrick et al (2001), must be seen in the same vein as learners having proficiency in mathematics. Proficient learners are those who are skilful in the five strands, namely, conceptual understanding where the learners are able to comprehend mathematical concepts, understand the purpose and their relations; procedural fluency when the learners are able to carry out procedures accurately, efficiently and in appropriate context; strategic competence where the learners are able to formulate, represent and provide solutions to mathematical problems; adaptive reasoning when the learners are able to think logically and reflect, explain and justify their answers; productive disposition when the learners are able to see mathematics as sensible, useful and worthwhile in their lives coupled with one's efficacy (Kilpatrick et al, 2001:116).

According to Figure 14 mathematical thinking occurs during the processing of concepts. Learning takes place from linking procedures and concepts. This occurs in amalgamation with compression. Compression is a thinking procedure used to elucidate the development of concepts. This mental process can be considered to be the means of a disciplined problem solving procedure from which concepts are developed (Sangpom, Suthisung, Kongthip and Inprasitha, 2016:74). Aptly described visual thinking (mathematical thinking) leads to visual learning (mathematical learning). This can be translated to talking, reading and writing their thoughts. This allows the learners to organise and consolidate their mathematical thinking. They gain insight of the meaning of the concepts making a solution to the problem possible. This is also evident in the statement made by Yilmaz et al (2009) who asserted that through the process of visualization a mental transformation takes place that eventually leads to successful understanding of concepts.

Learning is not merely talking or absorbing content knowledge. It is a complex process in which conceptual knowledge is created, recreated and understood. The learning process depends on the learner's personal perception, previous knowledge and skills which they use to create new knowledge. As teachers we must be aware that cognitive structure of all learners and their individual learning styles they have will vary. We also need to recognize that learners due to their backgrounds will attribute dissimilar meanings to the imparted content knowledge and concepts. The learners who do not comprehend the vocabulary or words in the problem or have difficulties to understand the language of instruction can fall back on their visual ability to learn. Learners observe directly. The visual will trigger a response thus making an association possible. Also through observation they will learn from the visual cues of the other learners and teachers in the classroom. Therefore the pedagogical value of using illustrations and demonstrations in teaching and learning cannot be overemphasised.

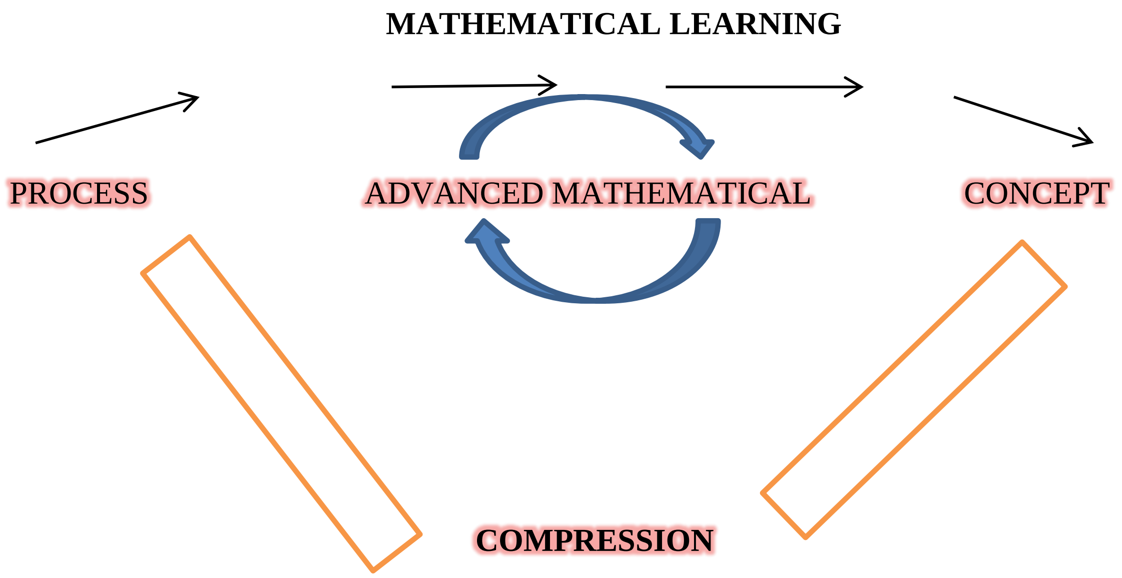


Figure 14 Mathematical Thinking and Mathematical Learning (Tall. 1991)

The understanding of mathematical concepts and the processes involved during problem solving occurs at the same time with visualization. The Visualizer/Analyser (VA) model (Figure 15) is based on Piaget's analysis of the interdependence of intelligence and perception using the visualization and analysis techniques (Stylianou, 2002) or views visual and analytic reasoning as complements. Zazkis, Dubinsky and Dautermann (1996) acknowledged the importance of both these forms of representations in solving problems in mathematics. Rather than attempting to solve problems purely through analytical or visual methods, an attempt should be made to integrate the two.

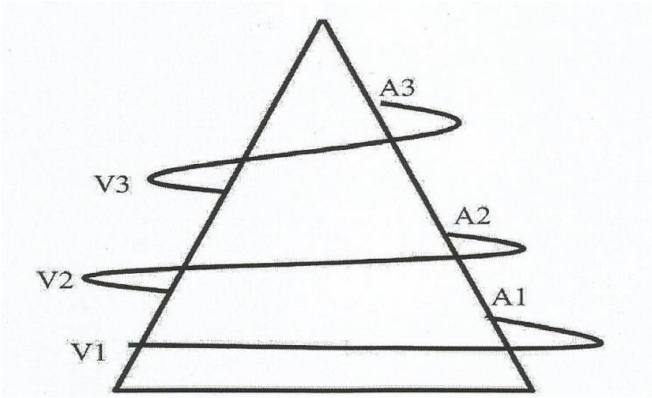


Figure 15 The Enhanced Visualizer/Analyser Model (Halpen et al, 2016)

The model as described in Figure 15 shows how the process of thinking unfolds. It starts with the concept of visualization in V1. This can be any visual representation, either in the form of a drawing or any image or a mental image. This image is then analysed during the process in A1. During this process an analysis of the visualized material takes place. This is followed by a second visualization step in V2 which is enriched as a result of A1. The process is repeated and hence there is a constant iteration between the visual and analytical processes (Stylianou, 2002). The iteration ends as the problem solver comes to a better understanding of the problem he/she was solving.

2.14 LANGUAGE, COMMUNICATION AND VISUALIZATION

Unknown to many outside the education sphere, mathematics as a language is learnt by many from an early age albeit at different levels and varying experiences. Mathematics as a language used in many forms to communicate and it is important for the pre-service teachers to understand what the learners have learnt before they enter the schooling environment. To understand the mathematics language better during the communication process, visualization steps in as a provider of clarity of the spoken word. Thus a symbolic relationship is created between language, communication and visualization. The language that is used to communicate the visual has to be apt and within the level of the learners. Visualization in written or oral form and communication in visual form is essential for the learners to construct meaning. Communicating through visualization makes it a powerful learning strategy as it becomes easier for the teacher to teach challenging mathematical concepts. It becomes easier for the learners to understand the concepts as they will be able to organise their thinking by seeing the relationship between the concepts and the visual mean.

The pre-service teachers need to understand that success in mathematics is dependent on the quality of language communication. The Department of Basic Education (2018:83) acknowledges that "*language plays an important and critical role in the teaching, learning and understanding of mathematics*" and that the learners need assistance to build their "*mathematical language so that it is easier for them to explain their mathematical thinking*" (Department of Basic Education, 2018:47). Figure 16 indicates the ten principles of mathematics pedagogy, the role of the teacher and learner in classroom teaching (Anthony and Walshaw, 2009) with mathematical language and mathematical communication prioritised as necessary for effective learning.

These principles will inter discussed within the language, communication and visualization context. Lerman (2002:107) defined learning mathematics as "*learning to speak mathematically*" and if the learners are to make sense of mathematical ideas then they require to understand the mathematical language used in the classroom including terms and expressions (Anthony and Walshaw, 2009:153). The language of mathematics is often a barrier to understanding the concepts within problem solving. In the context of South Africa, which has nine official languages and other dialects, mathematics teachers are sometimes forced to code switch in order to clarify a concept. The Department of Basic Education (2018:7) refers to code switching as the "*conscious switching from one language to another language during teaching and learning*" and makes mention of "*translanguaging*" which refers to "*a flexible use of language seen as an internal strategy by which speakers use all of their linguistic resources to communicate*". This can be problematic in that certain words in a certain language can have a different meaning compared to the words (concepts) used in mathematics. It is in this context that visualization that can be used to provide learners with a sound understanding of words used in the mathematical language. The difficulties of relating culturally learnt words to mathematical concepts can be alleviated by using visual representations to clarify concepts which will assist the learner's mathematical understanding. According to Presmeg (2006) when the medium of instruction is in a language that is not the home language of the learner then having visual elements as part of the lesson can assist in the comprehension of the material been taught. Whilst problem solving in the classroom is dependent on the teacher's mathematical knowledge to teach concepts, it is equally important that teacher's knowledge of visual representations is necessary to assist the learners understand the concepts in problem solving (Boonen et al, 2016).

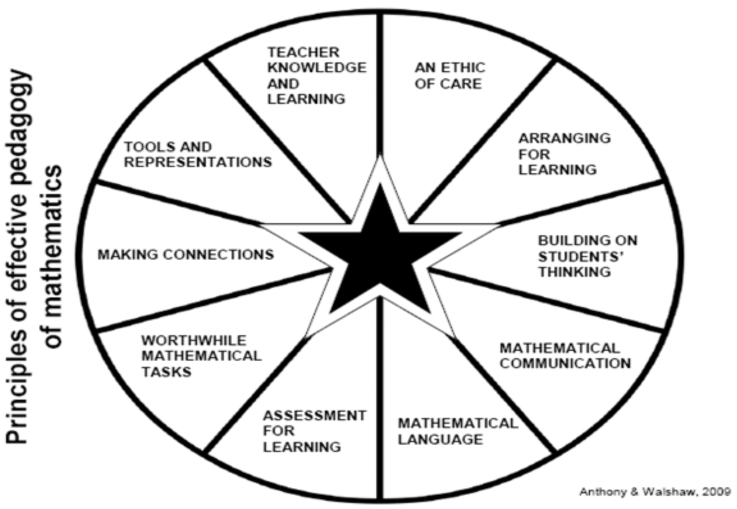


Figure 16 Principles of effective pedagogy of mathematics (Anthony and Walshaw, 2009:148)

One of the goals of mathematical problem solving is to develop oral and written communication. Lampert and Cobb (2003:237) described communication and language as "*as a primary means by which mathematics is taught and learned*". Communication in the mathematics classroom provides a teacher with proof of what the learners know and also remediating their weaknesses (misconceptions). Hyde (2006:7) stated that for the learners to comprehend mathematical concepts they need to utilize language. It is therefore important that the correct mathematical language is used effectively in the classroom for communication so that the learners can understand the language of mathematics. Therefore the teachers and the learners need to use and understand language to enhance a variety of communication skills.

Hyde (2006) stated that people deny the importance of language in mathematics as a means of communication. Moyer (2000) emphasises the strong link between the use of language and learning mathematics. The learners can negotiate the meaning of concepts, expressions and procedural knowledge with their peers and teachers as well as also add their new ideas (Anthony and Walshaw, 2009). Since language is versatile, it can be rearranged and combined limitlessly in order to communicate further (Jalongo, 2000:50). It also plays a vital role in mediating and negotiating learning therefore it is essential to engage the learners in various forms of oral and written communication as it will develop a permanent record of the development of their knowledge (Luneta, 2013).

According to the Department of Basic Education (2018:79) "*the use of language should not interfere with the learner's ability to speak about what they are doing*" and "*the spoken language needs to be used in such a way that learners are able to express their thoughts as clearly as possible*". Through oral communication the learners develop a better understanding and it allows the teacher to gauge the learners understanding. Oral communication in the form of storytelling with images offers explanations to concepts. The learners listen to the story and absorb what is being said and understand the concepts that they were not familiar with previously. The use of visual representations of concepts in the story can be an effective method to greatly influence the learners understanding. It is important that the teachers utilise appropriate visuals to match the concepts used in the story because the learners put a label to the concept when displayed visually, example, a circle is associated with something round or a revolution. The teachers need to ensure that the visuals provided are as close to reality as these visuals by association are stored in either their short or long term memory. Later in their learning when confronted with a similar concept they will be able to recall and utilise it to share a variety of solutions (verbally and visually) and also re-evaluate their ideas (Anthony and Walsh, 2009:152).

Discussion through questioning between learners and teachers create a better understanding of concepts. By teachers asking questions based on the conceptual images (key words), the learners develop their own creative train of thought. When asked by the teacher how they arrived at the answer, the learners by making their connections with prior knowledge, are able to justify their thought process leading to a solution. The teachers can use this advantageously during discussion by allowing the learners to grow their ideas off each other. They, the teachers, can intervene intermittently by guiding the discussion by asking pertinent questions. This will create open dialogue in the classroom and allow them to learn at their own pace therefore the teachers need to allow the learners to about how they arrived at the answer. Through open discussion and constant dialogue within the classroom validates their conceptual understanding. These types of proactive lessons assist in shaping the learners thought process leading to successful knowledge construction as they use their ideas to build their thinking with the new ideas obtained from their peers. Discussion as a form of dialogue between the learners indicates to the teacher their understanding or misconceptions. In the event of misconceptions, discussion in the social construct of the classroom allows for its undoing. The learners have the power to support each other in their knowledge construction. By giving individual responses, a consensus can be reached on the understanding of mathematical concepts thus overcoming any misconceptions. The learners can also voice their problems that they are experiencing, allowing the teacher to address them instantaneously.

Lerman (2002) stated that it is not only oral communication that is important in mathematics but written mathematical communication is its equal. Luneta (2013) stated that written communication entails more reflection than oral explanation and should not be underestimated. Written communication stimulates a learner's imagination and creativity (Luneta, 2013) and should be encouraged in the classroom. Written communication, as an essential collaboration of both oral and written (text and visual) is critical as it lays the foundation for the development of mathematical skills. Written mathematical work allows the learners to be expressive. It is a concrete manner in articulating what they have learnt in a creative manner. The learners create meaning from the text and through visualization one can see their thoughts in writing. How and what they communicate will depend on their prior knowledge of the given information or their knowledge of the concepts. The written work provides an overview of the learners understanding as they write and represent key mathematical concepts. As teachers, we must acknowledge that no two learners will interpret concepts the same way due to their environmental experience and prior knowledge. Any work in any type of written form (text or visual representations) allows the teacher to evaluate the learners understanding and progress.

According to a mathematics teacher (in Silver, 2017) "*Writing in mathematics gives me a window into my student's thoughts that I don't normally get when they just compute problems. It shows me their roadblocks, and it gives me, as a teacher a road map*". As an educator I have observed that writing helps improve the learners understanding in problem solving. The written aspect aids their reasoning skills as they are able to see the greater picture in front of them.

They are able to focus their attention on the various steps allowing them to examine and re-examine their solutions. On assessing the learner's tasks, the teachers read the learners written work. They are able to see how the learners justify how they arrived at their answer. The teacher is then in a position to provide constructive feedback or describe strategies for improvement (Anthony and Walshaw, 2009:154).

The mathematics classroom is moving away from being more text oriented. Modern mathematics teaching is no more reliant only on the textbook. The teachers are exposing their learners to a multitude of learning material and workbooks with illustrations in mathematics lessons. They are now using technology to support their teaching. Many learners in today's classrooms are having difficulty with language and the visual mediums assist to fill the vacuum. The teachers have recognised the importance of the visual component in mathematics and it can be regarded as a mathematical language on its own. Visualization as a language allows the teachers to utilise a vast range of models, visual mediums and representations to support their lessons in the mathematics classroom as these are valuable tools to provide concrete explanations when language fails. Visualization allows for visual representations to overcome the language barrier in the classrooms. These visual mediums as visualization tools, together with speaking, allow the learners to externalise their thinking. According to Novick (2004:307) diagrams are among the oldest preserved examples of written mathematics. It can be used constructively by the learners to communicate effectively when they cannot find words to communicate their thoughts. Hence using visual representations can be an effective way to support problem solving. This allows the learners to convert words from the problem into pictorial or schematic representations to communicate their thinking. Thus using the combination of oral and written communication together with visualization allows the learners to incorporate their mathematics literacy strategies by using the varied opportunities in the classroom to link their language and ideas. In this manner the teachers are able to see the learners understanding of the problem before they solve it.

The pre-service teachers need to realise that the mathematical language they use to communicate to their learners must be correct. The mathematical language they use will hinge on their mathematical knowledge (discussed in chapter 3). Mathematical knowledge is also indicated as one of the principles of effective pedagogy (Figure 17). It is therefore imperative that they have a good grasp of the mathematical concepts and pedagogical content knowledge to increase the learner's mathematics knowledge using sound communication and visualization skills.

2.15 VISUALIZATION AND VISUAL REASONING

The term visualization refers to the multifaceted nature of visual imagery. It plays an eternal role in all meaning, understanding and all reasoning on the part of the learner. The use of visual reasoning can be traced back to the early days of mathematics in Mesopotamia and Greece (Stylianou, 2002) where diagrams seemed to be an indispensable part of mathematics in the 18th century. The acceptance of visual reasoning however lost its place when it was found that it was misleading in several cases. However, in modern mathematics classroom visual reasoning is the new trend in teaching and learning of mathematics. Mudaly and Budaloo (2016:45) proposed that visual reasoning become the foundation of an innovative model for successful mathematics teaching because their studies have revealed that visual reasoning is an indispensable tool in the teaching of mathematics.

Mathematics teachers have found that visual reasoning plays a far more important role in today's mathematics than it generally acknowledged. Wheatley (1997:285) stated that using visual reasoning during the visual process allows the learners to reflect on what they have externalised. This gives the learners an opportunity to revisit the representation by redrafting and making changes before working towards a solution.

Yilmaz et al (2009:131) are of the opinion that one probable way of trying to understand visual reasoning is to perceive it in milieu that "*mathematics is a subject that is concerned with objectification and representing abstractions from reality and many of these representations appear to be visual, having roots in visually sensed experiences*". As mathematics teachers we have to understand that visual representations are not always easier to understand. The learners see and relate what they know because the representation that is created is exactly what the mind is seeing in reality (image). An image is a mental construction of the given problem and when represented in whatever form it creates a visual medium for the learners to understand the problem better. The mind makes an internal deduction of what is relevant as it eliminates unwanted data as the problem unravels and externalises the learners understanding in visual form. Therefore one can state that there is a strong relationship between internal and external representations.

2.16 VISUALIZATION AND READING

Reading is such an integral part of learning that if not nurtured correctly in mathematics can be detrimental to the learning and understanding process. In the 1990s the Programme for International Student Assessment (PISA) investigated learners reading knowledge and readiness as part of an essential skill to be part of normal society (Programme for International Student

Assessment, 2012). The Progress in International Reading Literacy Study (PIRLS) assesses reading and monitors trends in reading literacy every five years with the recent study in 2016. For me as a mathematics teacher, the result of the PISA investigation and The Progress in International Reading Literacy Study (PIRLS) is startling. The first startling fact is that there was no change in the reading abilities of learners in South Africa between PIRLS (2011) and PIRLS (2016). In 2016 PIRLS report, statistics showed that South Africa is the lowest performing country out of 50 countries and using the PIRLS scale of 40 points equalling 1 year of schooling, South Africa may be 6 years behind the top performing countries.

According to Debrenti (2013:1) reading, learning and understanding are the fundamental aspects to all kinds of learning "*even more important in the case of learning mathematics*". Much research has shown that reading is a historic barrier to learning in South Africa and I don't want to subject this study in that direction. Suffice to state that it has been noted in classrooms in South Africa that when the learners cannot read the teacher reads the problem to them so that they are able to comprehend the problem. If this does not happen then the learners have to navigate themselves through the lesson like a sailing boat in the open ocean on a windless day. It is expected that all teachers should practice the idea of initially reading to their learners. The importance of this is reiterated in Draper (2002). As long as mathematicians pose and examine problems in order to unravel solutions, the teachers will have to support the learners make meaning of the literal text (Draper, 2002). In this manner mathematics can become achievable for each and every learner.

Hite (2009:7) reiterated that "*to be a good math student, one needed to have solid reading skills*". Over the years I have discovered that reading gives the learners confidence to overcome the mathematics-reading disabilities. From my initial years of teaching to current, I have witnessed the deterioration of reading in schools. There are sufficient studies available (SACMEQ I, II, III and PIRLS) to lend credence to my observation. Factors lending itself within the South African education landscape are many. Over the years I have experimented and implemented many aspects relating to reading to assist me in my mathematics lessons. This was not limited to individual or choral reading, testing the spelling of mathematical concepts or teacher reading. Those learners who were chronically poor readers, I encouraged one to one peer reading. This assisted the learners to identify the key words within the problem and to comprehend the problem at hand. The 'difficult' words, or concepts, went into their mathematics dictionary together with the meaning and a representation of the concept. Since visualization has aided reading as a teaching strategy over the years I discovered this to be an invaluable benefit to the subject. According to Hyde (2006:67), a strong proponent of using visualization, stated that "*there are two ways students use visualization in mathematics that* *should come as no surprise: creating mental images as they read and creating representations of their mental images*". Training the learners to use visual skills in reading will cement the foundation for its use in mathematics problem solving. I read to my learners and ask them to illustrate what comes into their mind when I make reference to concepts in the problem. These illustrations reflect their mental understanding of the problem. As the learners progressed through the grade, I observed that by remembering the visual illustrations they were able to apply the concepts in non-routine problems or other mathematical situations. This plan of reading and illustration has brought much success in my years of teaching.

Reading develops both thinking and understanding. As reading is a development of constructing meaning from written language, the learners will need to think to comprehend and understand. They should therefore be encouraged to think, reflect and imagine. Since all learners have varying cognitive structures they will gather and assimilate information differently. By evaluating the visual representations (visualization will assist the learners to externalize their solutions) they will re-represent them in a logical manner in order to acquire a better understanding.

Much has been written on how learners must learn mathematics with understanding but I have discovered that many learners have extensive difficulties with reading with understanding. This in turn impacts on them determining the mathematical operations involved in the problem. This I have put down as poor reading accuracy and comprehension. This is further substantiated in The Annual National Assessment Report of 2014 (Department of Education, 2014). The difficulties arise when learners cannot comprehend a problem they revert to copying the solution of a previously given problem. The misunderstanding herein is that the learners cannot assimilate and relate to the given problem. One can therefore safely conclude that due to poor reading, thinking and comprehension skills that this tendency seems to be a barrier to successful problem solving.

Debrenti (2015) stated that it is imperative that we place immense importance on problem solving as it plays a significant role in developing comprehension. In the same vein I must stress that it is imperative that English and Mathematics teachers, albeit teaching two different subjects, join forces to cement the comprehension and mathematical vocabulary. Foshay (2003:2) stated we as teachers must continue placing importance on basic literacy skills with the learners in schools because problem solving skill depends on mastery of basic literacy skills. Since reading skills are often seen as an obstacle to developing comprehension, pre-service teachers should be aware of the magnitude of placing emphasis on making reading part of the mathematics class every day. They should be taught how to utilise mathematical dictionaries, journals, word lists, post problems and use technology to support both their learners and their teaching.

Studies have shown that it is not only the learners who are battling to read. Ozdogan et al (2011:2283) made a disquieting statement in their studies. They found that the "*pre-service teachers do not understand what they read*" and "*pre-service teachers need to read more books in their university years*" (Ozdogan et al, 2011:2283). This to me does not bode well for the future of the mathematics classroom and should be investigated further. Whilst it is an accepted norm universally that lecturers talk a lot during their lectures, I have lessened the 'talk time' during lectures and have set the pre-service teachers more tasks which includes reading. These tasks are set to interrogate their thinking and are translated to discussion forums for developing their mathematical knowledge. They are 'forced' to read the prescribed chapters or extracts from their reference textbooks to glean the material for discussion. They have to prepare a summary as notes. This assists them in using the acquired content knowledge to answer questions based for discussion. In this manner I engage them constructively in the discussion and at the same time ensuring that reading occurs concurrently.

Another avenue that needs to be explored is the pre-service teachers having reading included in one of their modules for their degree. If one considers the findings of Ozdogan et al (2011) then the pre-service teachers are least likely to teach efficiently as teaching involves reading in the classroom as well.

CHAPTER THREE

THEORITICAL FRAMEWORK

3.1 INTRODUCTION

Since learning is all about lasting change, learning theories provide a pedagogical base for all teaching and learning and how knowledge is constructed through understanding. It is a set of principles that guides learning in the classroom. There are numerous theories regarding how learning occurs and these embrace varying implications on how the teachers can support the learners learn by using various resources and activities. The theory used should justify the learning that occurs in the classroom.

The theories discussed in this chapter are important theoretical foundations and have strong links to creating knowledge and understanding in the classroom. I have chosen Kolb's Experiential Learning Theory Teacher Knowledge, Structural Learning Theory and Theory of Understanding because I found it relevant and present in modern day classroom. Their relevance and relation to problem solving, pre-service teacher's knowledge and visualization will be discussed in this chapter.

3.2 KOLB'S EXPERIENTIAL LEARNING THEORY

Kolb's Experiential Learning Theory draws on the work of prominent twentieth century scholars Dewey, Jean Piaget, Kurt Lewin and others who had as their aim of developing a dynamic and holistic model of learning through experience. Research based on experiential learning theory has occurred throughout the world and it was found that the holistic nature of the learning process occurs at all levels of human society (Kolb and Kolb, 2011). Experiential learning is viewed as a continuous progression where the learners expressively articulate their own knowledge and prior experience to learning in the classroom (Fry, Ketteridge and Marshall, 2015). The experiential learning theory is a learner centred approach which involves experience and knowledge, reviewing, interacting and reflecting in the learning process and applying what is learnt on the subject matter. Its participatory methods allow for active learner participation in the learning process and it promotes communication and group work. This experiential and participatory approach enhances valuable skill transfer, in order to assist conceptual and attitudinal growth and to bring about changes in learner's behaviour.

Experiential Learning Theory (ELP) is defined as "*the process whereby knowledge is created through the transformation of experience*" (Kolb, Boyatzis and Mainemelis, 2000:2). Many researchers have stated that for learning to occur it has to be connected to the learner's lives.

Kolb (1984) emphasised that there should be a link between what is done in the classroom and also for which the classroom is preparing the learners, namely, real life experience. By doing this they become creators of knowledge and become independent thinkers (Beaudin and Quick, 1995:3). According to Beaudin and Quick (1995) learning tasks requires active participation and action is the core to experiential learning. This is similar to what Piaget referred to as discovery learning whereby the learners learn best through doing. According to Rossman (1993) experiential learning centres around the learning strategies on problem solving and learner analysis of the problem. Thus problem solving in the classroom allows the learners to think independently, relate to the problem and apply their life experience to the problem.

Kolb (1984) describes experiential learning as a continuous four stage process (Figure 17) from concrete experience (the learners engage themselves in new experiences); reflective observation (the learners reflect on their experiences); abstract conceptualization (the learners integrate and conceptualize the experience) and active experimentation (the learners test their knowledge and understanding incorporating it with previous experience) and leading to a new situation.

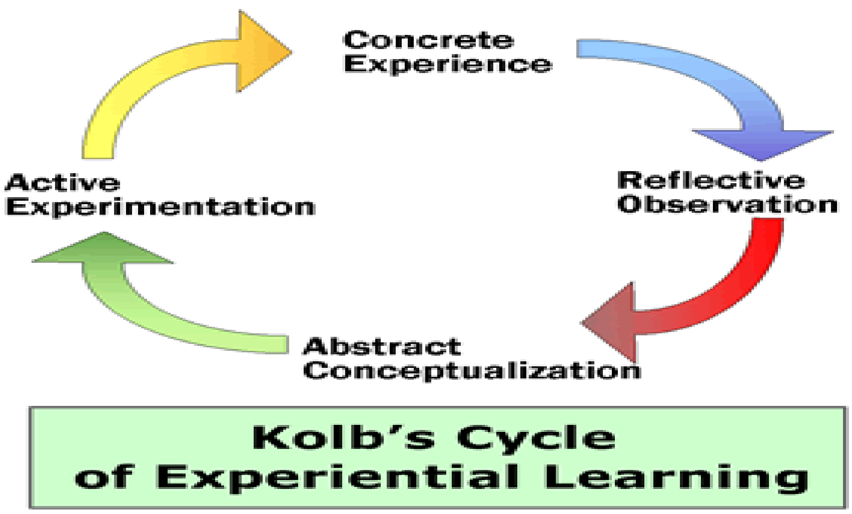


Figure 17 David Kolb's Experiential Learning Model (Kolb, 1984)

According to Kolb (1984), Sharlanova (2004) and Cherry (2016) in this cyclical process of Kolb's Experiential Learning Model (Figure 17) the concrete experience is the feeling phase. The learner physically experiences the here and now situation by doing the task. The reflection stage is the observing stage. The learners gather their thoughts that were previously experienced and deduces what is working or failing, consequently allowing for adjustments to be made. The abstract conceptualization stage is the thinking phase. According to Kolb (1994) this phase involves more logic and understanding. The learners use analytical skills to comprehend, interpret and understand the connections between the ideas. Generalizations and deductions are made and then the learners determine the means and ways to improve on what was reflected on. The active experimentation phase is the doing phase. The learner must be able to use the problem solving skills to actively test and generate new ideas that was gained from prior experience.

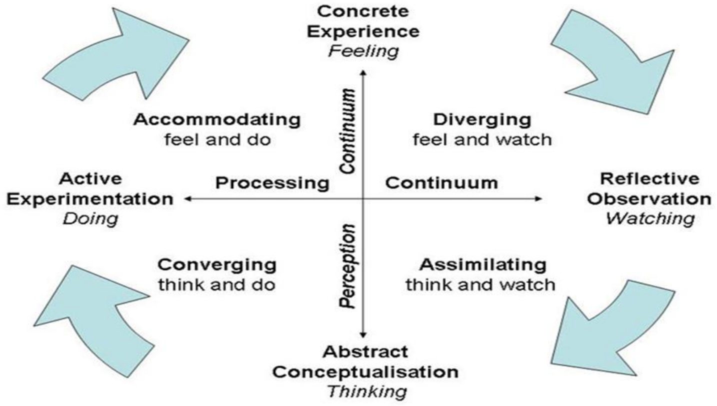


Figure 18 Kolb's Learning Styles (McLeod, 2013)

Four learning styles are identified in Kolb's Learning Styles (Figure 18) and it relates to the different stages of the learning cycle. The learners who learn best by reflecting on the concrete experience are known as divergent learners. These learners are superior at viewing (watching) concrete situations from an array of perspectives. They are excellent at brainstorming and generating ideas and are also good imaginatively. The learners who learn best by developing abstract theories and models of observation are known as assimilating learners. These learners can deal with an extensive array of information and organize it in a concise and logical manner. They are more interested in ideas. The learners who are fond of putting theories into practice are known as convergent learners. These learners find realistic uses for ideas and discover through experimentation. They are superior at problem solving. The learners who like to experiment to plan new concrete experiences are known as accommodating learners. These learners favour hands on action oriented learning and engaging in challenging experiences (Fry, Ketteridge and Marshall, 2015:75-77).

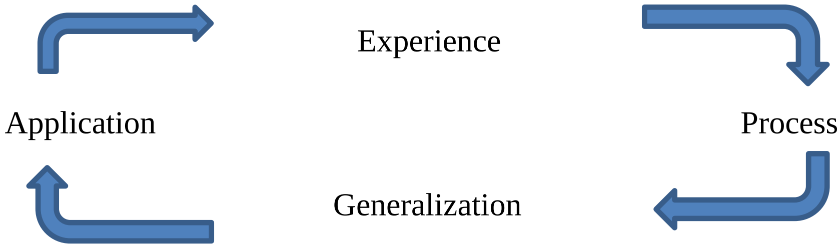


Figure 19 Experiential Learning Cycle (McCaffey, 1986)

MaCaffey (1986:3-4) explained the experiential learning cycle (Figure 19) as a cyclical process where the experience stage is described as data producing component and the doing period. The learners participate in a range of activities which may include reading, writing and listening activities, participating in role playing, completing a class exercise, practicing a skill or listening to a class lesson; the process stage reflects on the said activity embarked on during the experience stage. The learners interact with others in the group either individually or as group participants. They are encouraged to think critically about their experience and also articulate their feelings and insight by the teacher; the generalization stage is best characterized by asking: "*What did you earn from all this*?" and "*What more general meaning does this have for you*?" In this stage the learners draw conclusions based on any generalizations derived from the first two stages. The teacher needs to assist the learners to draw conclusions relating to their lives; in the application stage, once the learners have formed their generalizations, they must be guided by the teacher into the application stage. Drawing on their insights and conclusions attained during the generalization stage and previous stages, the learners thereafter incorporate what they have learnt into their lives which will effectively bring about a behavioural change in the future. Some of the techniques which can be used in this stage are action plans; reviewing peer's action plans; sharing action plans with the whole group and discovering further learning needs. According to McCaffey (1980) evaluation is an integral part of the experiential learning approach. This takes the form of feedback where the learners "*identify specific applications of the lessons learned".*

According to Kolb and Kolb (2011:45) the concept of learning style portrays individual differences in learning and is based on the learner's inclination for employing the different phases of the learning cycle. ELT posits that all learning is the major determinant of human development and how individuals learn, to shape the path of their individual growth.

3.3 STRUCTURAL LEARNING THEORY

The Structural Learning Theory (SLT) is a prescriptive theory conceived and developed by Joseph Scandura. This theory has been expansively applied to mathematics and its primary focal point as a theory is problem solving instruction (Scandura, 1977). According to Ikegulu (1996:6) the structural learning theory is an instructional theory that concerns itself to what occurs within the learner's brain during teaching (instruction) and the learning process. It is assumed that the learners organise their learning in the form of rules and they adjust and apply it to modify their existing knowledge. Educational psychologists Jean Piaget and William Perry who developed the cognitive approach also paid attention to what went on inside a learner's head hence they focussed on the mental processes involved.

In the SLT what is learned are rules of domain, range and procedure (Scandura, 1977). According to Scandura (1977) and Ikegulu (1996:10-11) domain is a set of inputs or internal cognitive structures that is relevant to a learning situation; range is outputs that the rules are expected to produce and procedure is the sequence of operations or the progression of unfolding events that is essential to generate the desired outputs.

SLT prescribes teaching the simplest solution path for a problem and then teaching more complex paths until the entire rule has been mastered (Ikegulu, 1996:8). When the higher order rules are facilitated then these rules generate new rules. This in turn makes it possible to solve complex problems by making it possible to learn new rules. This theory also suggests a strategy for teaching only the rules which the learner has not mastered. Content should be taught in the form of rules. This is a logical sequence by which the solution to the problem is derived. The lower order rules need to be taught, namely, teaching the basic concepts so that the learners are able to apply them when confronted with other problem situations. When applying the lower order rules, this brings about conceptual understanding to the problem. Through this, the higher order rules are derived which enables the learners to develop their background knowledge and solve mathematical problems in various forms (Ikegulu, 1996; Scandura, 1997). When the higher order rules are used, namely, rules that generate new rules, then problem solving may be facilitated whereby the learners are capable of solving problems that they have not confronted before (Ghazali, 2011; Scandura, 1977).

According to Scandura (1977) SLT is a methodology for identifying the rules to be learnt for a given topic and then breaking them down into components. SLT identifies the components for solving problems and this procedure is known as structural analysis. According to Ghazali (2011) and Scandura (1977) the major steps in structural analysis are to select a representative sample of problems; identify the rules for unravelling the rules for finding a solution to the problem. One needs to take into account the capabilities of the learners. The various operations involved in solving the problem must be taught according to the capabilities of the learners. The learners should therefore be taught procedural knowledge (step by step calculation procedures) that will enable them solve the problem; convert each solution rule into a higher order problem thereby eliminating lower order rules; identify a higher order solution rule for solving the new problems thus showing their understanding of the problems; assess and improve the rules by eliminating the redundant solution rules.

An importance aspect of this theory is the importance of learner's prior knowledge that will aid in creating new ideas (Scandura, 1977). According to Ikegulu (1996:19) having a well-structured knowledge connection allows for easier retrieval of prior knowledge and the facilitation of new knowledge.

SLT is a cognitively oriented model combining learning theories, instructional theories and instructional development theories (Ikegulu, 1996:7). As a cognitively model it is important to review the learner's previous level of understanding to establish their skills and knowledge for the lesson (Ikegulu, 1996:16-17). This will allow the teacher to present new material content in little steps using unambiguous language and positive examples. The teachers provide feedback when the answers are correct or reteaching occurs when conceptual understanding or answers are incorrect. The teachers spend more time on evaluating and teaching until the learners have an understanding and confidence in their techniques to solve the problems.

According to Ikegulu (1996:21-22) the SLT as an instructional theory support the learners creativeness and reduces or limits the learners rate of forgetting and recollection. This is attained through graphic organizers. This takes the form of diagrammatic expressions (using some sort of representation). The teacher provides the learner with a wide variety of diagrammatic representations and at the same time the learners are taught and encouraged to develop their own graphic representations to assist them problem solve. Together with this the learners should be provided with guided practice on solving problems on the chalkboard using examples. The learners should be engaged in discussing the steps to solving the problem (individually, group work or as a class). This practice is consistent with the cognitive model because it "*paves the way for learners to construct their own mental model, adjust their existing schema, and reconstruct a "NEW" schemata"* supported by the new knowledge obtained from the teacher (Ikegulu, 1996:25).

3.4 MATHEMATICAL KNOWLEDGE

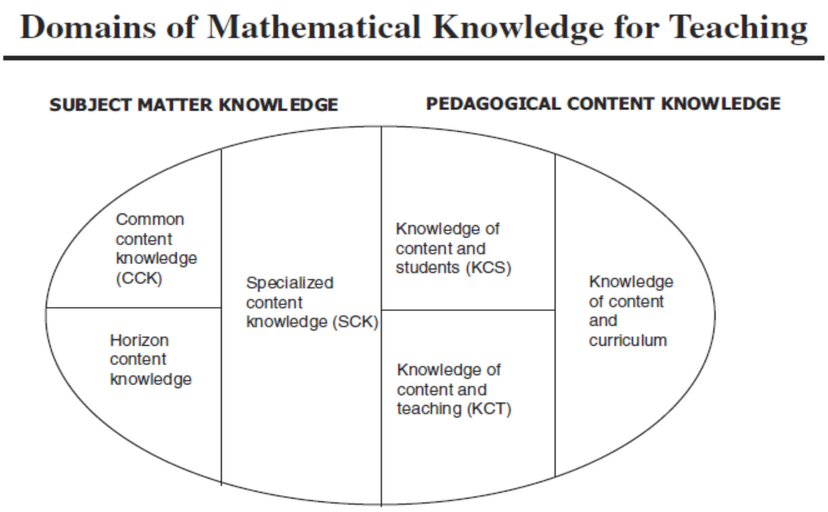


Figure 20 Domains of Mathematical Knowledge

According to Hill and Ball (2009:69) "*it would be foolish to say that mathematical knowledge is not important to teaching mathematics*". They found that mathematical knowledge for teaching (MKT) contained a combination of knowledge of content (subject knowledge) and pedagogical content knowledge. Preparing to become a pre-service teacher of mathematics poses great challenges to education as "*many teachers enter the classroom without comprehensive* *understanding of mathematics*" (Rosas and West, 2011:4). The institutions of higher education that provide teacher training are faced with a major challenge to ensure that the pre-service teachers are well prepared to teach mathematics. In an effort to improve the academic success of learners the No Child Left Behind Act of 2001 (NCLB 2001) was passed in America which required all teachers to be highly qualified in the content area they teach. The mandate of this requirement was that the pre-service teachers must "*have the subject content knowledge to instruct all children*" (Rosas and West, 2011:4) and it is imperative that the higher education teacher training institutions take cognisance of this well. In Australia The Teacher Education Ministerial Advisory Group (TEMAG, 2014) recently made mention on how the teacher training programs can be improved to prepare the pre-service teachers practical skills and as specialists in their subjects so that they will be confident to enter the classrooms as teachers and this was readily accepted by the Australian Government.

Comparatively the situation in South Africa the situation is totally different. Currently there is a mass production of teachers. Those that are exiting the higher education are not fully prepared to meet the demands of teaching subjects in schools. Many of them exit these institutions with subjects like Life Orientation and Economic Management Sciences (EMS). In a school situation they are allocated subjects like Social Sciences, English, Natural Sciences and Mathematics to teach. It is not they cannot teach these subjects. A firm understanding of the subject content knowledge and specialist pedagogical content knowledge is needed to transmit the content to the learners. The newly qualified teachers cannot manage teaching the 'difficult' as they lack the pertinent content knowledge and contributes to rocking the foundation of the education system in South Africa.

Teachers need to have a definite mathematical knowledge as well as knowledge of their learners and how they learn. There is a need to explore the different types of knowledge that the pre-service teachers need to have before entering the classroom. Ball, Thames and Phelps (2008) made reference to the following domains of mathematical knowledge for teaching (MKT). The broad heading of subject matter knowledge encompasses common content knowledge, horizon content knowledge and specialised content knowledge; pedagogical content knowledge comprises knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum. Content knowledge is common in all of the afore mentioned and it is vital that the pre-service teachers know their subject content matter that is been taught as it is a "*critical component of the goals and activities that constitute professional curriculum*" (Forzani, 2014:359). A description of the different teacher knowledge is discussed.

According to Ball, Thames and Phelps (2008:6) Common Content Knowledge (CCK) is closely related to content of the curriculum and it is "*mathematical knowledge that we would expect a well-educated adult to know".* In the first domain, CCK, 'common' knowledge is something that is relevant to one's daily life and not only exclusive to teaching. CCK refers to the knowledge and skills of mathematics that individuals need to possess in contexts other than in teaching (Mudaly and Singh, 2016). It involves calculating a simple answer or calculating a solution for a mathematics problem. The teachers need to be acquainted with the content they teach; determine when the solutions provided by the learners are correct or incorrect and use terminology and notation correctly (Ball, Thames and Phelps, 2008; Ball and Forzani, 2011). Recognising an incorrect answer is CCK. It is therefore crucial that all the pre-service teachers have a good understanding of CCK because "*it is the knowledge teachers need in order to be able to do the work"* (Ball, Thames and Phelps, 2008:6).

Example: It is knowing the algorithm on how to multiply together two numbers, namely, 2 x 12.

Another sort of MKT is Horizon Content Knowledge (HCK). According to Ball, Thames and Phelps (2008) (HCK) is "*[A]n awareness of how mathematical topics are related over the span of mathematics included in the curriculum*". In the third domain a vision is needed in the field of teaching mathematics – a view is required of the larger mathematical landscape that teaching requires. HCK is an understanding of how the mathematical subject matter (content - mathematics topics) is interconnected over the duration of teaching mathematics in the curriculum. The interaction between the teachers in the various grades is important in this regard because they need to know the content knowledge and how it unfolds across the spectrum of the school curriculum. The teachers need to know at what optimum level their learners should be performing before they progress through to another grade. In South Africa the Curriculum Assessment and Policy Statement (CAPS) prescribes what content needs to be taught in each grade. It very much shows the progression and development that is expected for each grade.

Example: knowing the algorithm to multiply together two numbers is related to multiplying together polynomials.

Ball, Thames and Phelps (2008:261) stated that Specialized Content Knowledge (SCK) is the mathematical knowledge and skill unique to teaching but "*not yet requiring knowledge of students or knowledge of teaching*". SCK refers to the information that the teacher has that is specifically related to the subject being taught and also how the teacher will organise the content sequentially (Mudaly and Singh, 2016). According to Lowrie and Jorgensen (2015), Carpenter, Fennema, Peterson and Carey (1988) content knowledge plays a critical role in the study of teaching and is very critical for the teaching of mathematics. What must teachers know and be able to do with mathematics content and how to get it across to the learners? (Ball, 2011).

Example: knowing how the algorithm to multiply together two numbers connects to place value and the distributive property.

Shulman and his colleagues constructed the Pedagogical Content Knowledge (PCK) framework (Ball, Thames and Phelps, 2008). According to Ball, Thames and Phelps (2008:390) PCK has been used to refer "*to a wide range of aspects of subject matter knowledge and the teaching of subject matter*" and the notion was that PCK "*bridged the gap between content knowledge and practice of teaching*". Pedagogical content knowledge (PCK) is where the teacher is expected to be acquainted with slightly more than the pure content knowledge of the subject matter. It is a blend of mathematical knowledge (content) and pedagogical knowledge and includes knowledge of the conceptual and procedural that the teachers bring to the learning of a topic, the misunderstandings about the topic that they have taught and the phases of understanding that they are likely to pass through in moving from a situation of having little understanding of the topic to the mastery of the subject matter (Carpenter et al, 1988; Luneta, 2013). It also includes procedures of assessing the learners existing knowledge and understanding, diagnosing their misconceptions and knowledge of instructional strategies that allows the learners to connect to knowledge that they already have.

According to Ball, Thames and Phelps (2008:261) Knowledge of Content and Students (KCS) is "*a type of pedagogical content knowledge that combines knowing about students and knowing about mathematics.* In the fourth domain, KCS, is combined knowledge involving knowing the learners and the subject content in mathematics. A teacher's familiarity with the learner's errors and determining which are likely to occur is KCS. The teachers must expect what learners are likely to think and what they will find unclear.

Example: knowing that when multiplying together two numbers the learners create the error of not appropriately 'shifting' the terms to be added.

Knowledge of Content and Teaching (KCT) is "*is knowledge that combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require mathematical knowledge that interacts with the design of instruction*" (Ball Thames and Phelps, 2008:261). In the fifth domain, KCT, is an amalgamation *of* knowing about teaching and knowing about mathematics (Ball, Thames and Phelps, 2008:401). This domain involves having a mathematical understanding of the design of the instruction, which entails preparing the content for the lesson. Appropriate examples and instructional tools need to be used to unpack the content in a logical manner using appropriate teaching methodology and methodologies. During the lesson discussion, the teacher needs to be aware of the direction the lesson is going and also of the learner's contribution to the lesson (their responses and asking questions for clarity).

Example: knowing what teaching strategies to utilize so that the learners when multiplying two numbers discover how and why to appropriately 'shift' the terms to be added.

The sixth domain, Knowledge of Content and Curriculum, is described by Ball, Thames and Phelps (2011) as been "*[R]epresented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to these programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances*".

Example: knowing what instructional resources are accessible for teaching and learning multiplication of two numbers and what approach these materials take.

The pre-service teacher education needs to be examined in light of the findings of Spaull (2011:5) who declared that "*many South African mathematics teachers have below-basic levels of content knowledge, with high proportions of teachers being unable to answer questions aimed at their pupils*". This finding by Spaull (2011) is not a completely new discovery but this is a damning but true statement. As an examiner of the Annual National Assessments and examiner and moderator of the common papers for grades four to seven, it was not unusual to see certain schools produce a common answer that is wrong.

I provide an example where reading and the teacher's content knowledge was questionable:

1. Give the **place value** of the 5 in 5 344.	2. Give the **value** of the 5 in 4 567
a. **5 000**	a. 500
b. Thousands	b. **Hundreds**
c. Thousandths	c. Hundredths
d. 5	d. 5

By examining the concepts (place value and value) we can see that in both the questions the answer is incorrect. In number 1 above, the question asked for the place value of the number but the common answer amongst the majority of the learners was 5 000 which was incorrect.

Similarly in number 2 above, the question asked for the value of the number and majority of the learners provided the answer hundreds which was incorrect. By examining the concepts of place value and value one can see that in both the questions the learners provided incorrect answers. It was noticeable from the learner's answers that they were taught these concepts incorrectly. The teachers from the schools with these types of common errors and those schools that produced results less than the expected national benchmark were identified for professional development, in other words, the teachers were taken for in-service training where they taught how to answer questions that were included in the ANA and improve their content and procedural knowledge. The Department of Basic Education (2018:79) has identified these shortcomings and has stated that intensive and systematic teacher development programmes are needed for every teacher from grade R to 12 and has recommended that teacher development should include structured systematic activities workshops for both in-service and pre-service teachers. It is imperative that the teachers have a sound knowledge of mathematical concepts as it impacts negatively on the learners. If the teacher's level of content knowledge is limited or construed, then it may be argued that the teacher training programmes are not proving to be adequate. Another possible reason is that the teachers who are teaching mathematics are not suitably qualified or lack the mathematical acumen to teach it. It is important that the higher education institutions take cognizance of what is happening in schools so that they can better prepare their teacher trainees with adequate subject knowledge. Failing which, these pre-service teachers will continue making the same mistake in their teaching resulting in a vicious cycle of misconceptions.

Shulman (Ball, Thames and Phelps, 2008) made a key contribution drawing attention that teaching a subject requires a great deal more than simply knowledge of the subject (Figure 21). He created two distinct fields in what teachers needed to know in respect of becoming proficient in teaching, namely, content knowledge (what to teach) and pedagogy (how to teach).

The first four categories (Figure 21) address a general dimension of teacher knowledge and Ball, Thames and Phelps (2008:391) stated that they were the foundation of teacher programs at that period of time.

The remaining three categories (Figure 21) define content specific dimensions to which Shulman (Ball, Thames and Phelps, 2008) described it as the missing paradigm in research on teaching. The first category is referred to as content knowledge. This includes knowledge of the subject and its organizing structures. The second category is referred to as curricular knowledge. This refers to the complete array of programs and the variety of instructional that is planned for teaching subjects and the appropriate topics at that plane. Shulman (Ball, Thomas and Phelps, 2008) described two other dimensions of curricular knowledge that he considered significant for teaching, namely, lateral curriculum and vertical curriculum knowledge. Lateral curriculum knowledge relates to knowledge of the curriculum being taught to the learners in other subjects and vertical curriculum knowledge refers to the subject matter (topics) that will be taught in the subject in the different grades preceding the current grade.

General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter Knowledge of learners and their characteristics Knowledge of educational contexts, ranging from workings of the group classroom, the governance and financing of school districts, to the character of communities and cultures Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds Content knowledge Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Figure 21 Shulman's Major Categories of Teacher Knowledge (Ball, Thames and Phelps, 2008)

The third category, arguably the most significant of the three categories, is pedagogical knowledge.

Lowery (cited in Ball, Thames and Phelps, 2008:394) describes pedagogical content knowledge as "*that domain of teacher's knowledge that combines subject matter knowledge and knowledge of pedagogy*".

Magnusson, Krajcik and Borko (cited in Ball, Thames and Phelps, 2008:394) stated that "*pedagogical content knowledge is a teacher's understanding of how to help students understand specific subject matter topics, problems, and issues can be organised, represented and adapted to the diverse interest and abilities of learners, and then presented for instruction….*"

Interrogating the above definitions one can conclude that both subject and pedagogical knowledge are both necessities in teaching and neither one can be divorced from each other. Thus the cross correlation of both subject and teaching is noted. Teaching is all aspects that a teacher is concerned with in supporting the knowledge of their learners. Besides the lesson presentation in the classroom, it also includes preparation of the lesson, evaluating the learner's work, assessing the learner's tasks (assessments and tests), parent interaction when reporting on the learner's progress and management issues at school (Ball, Thames and Phelps, 2008:395). Besides having sound proficiency of mathematical ideas and skills, the teachers also need to have a good grasp of concepts used in teaching and explaining procedures. The teacher needs to understand the procedures involved in calculating the algorithms. In this instance the teachers actually need to know more about mathematics.

Let us consider this example: 327 – 103

This algorithm involves subtraction of three digit numbers. The concepts involved here are subtraction, difference, subtrahend and borrowing. In the foundation phase and in grade 4 this is taught as a breakdown sum. When using the breakdown method and the teacher is unlikely to explain all of the mentioned concepts due to the way the sum is set (horizontally).

327 – 103

300 + 20 + 7 – 100 + 0 + 3

300 – 100 + 20 – 0 + 7 – 3

Answer: ?

Procedural knowledge is guiding the learners "to do" mathematics. The working in subsequent steps involves using the subtraction and addition signs and grouping of the hundreds, tens and units. The reason why addition and grouping is used in subtraction needs to be explained to the learners. It must be remembered that the teachers use the BODMAS rule when teaching mathematics. In examining the above example, the use of the addition sign will influence the learner's calculation and lead to misconceptions. The teacher normally uses an example to 'show' the procedures to calculate this algorithm. This 'showing' allow the learners to 'watch' the teacher 'demonstrate' the method to 'calculate' the sum but if learners "*learn procedures without understanding, their knowledge may be limited to meaningless routines*" (Department of Basic Education, 2018:16).

As the learner progresses to the intermediate phase they are introduced to the column method (setting the sum vertically). Whilst one may conclude that the computational task involved in calculating the following algorithms is easy but it can be difficult for the learners.

327	327	327
- 103	- 163	- 103
204	244	4210

As a subject teacher I can frankly state that the learners are in a dilemma on how subtract when changing from the horizontal to vertical setting.

In the first example, the error is in the tens column. A common area of misconception amongst the learners is that 0 minus any number will be 0 hence 0 – 2 = 0.

In the second example, the error is in the tens column. The concept of ‘borrowing’ has not been taught (I do not like using the concept ‘borrowing’ as the value that is borrowed is not returned!). Not understanding the concept of borrowing, the learner as a means of ‘convenience’ has subtracted the smaller number from the larger number. Another error, which is not indicated above but it is a common error amongst learners, is when the learners start the subtraction process from the hundreds column instead from the units column. This can be attributed to the fact that in the breakdown method the learners subtract the grouping of the hundreds first, followed by the tens and then the units. This is inherently incorrect. Thus learners should not be taught in manner that will result in them using the incorrect procedure (Department of Basic Education, 2018:17).

In the third example, the learner has used the inverse operation of addition instead of subtracting. The hundreds, tens and units columns has been added. This is a serious misgiving amongst teachers.

The foundational knowledge and skills in mathematics is laid in the learner’s formative years in school, namely, from grade R to grade 3. According to the Department of Basic Education (2018:18) if the learners are not “*enabled to develop appropriate knowledge, skills and understanding of mathematics in the early grades*” it may result in “*lost opportunity and wasted potential*”. Although the vertical method is recommended for the Foundation Phase it is important that the teachers discuss with learners “*why the vertical algorithm works with addition and subtraction*” (Department of Basic Education, 2018:18). Learners need to be taught to see the interconnectedness between the breakdown method and vertical method. This interconnectedness will aid them in not struggling with using the vertical method when transitioning from the Foundation Phase to the Intermediate Phase.

In the mathematics classroom a teacher will encounter numerous ideas and solutions provided by the learners. The learners have their own ideas and these must be shared with the others in the classroom. The learners themselves must respect the ideas of the others in the classrooms. The teacher need not be judgemental. He needs to be able to determine if the solutions are mathematically correct and if not, identify and rectify misconceptions, errors and mistakes as soon as they occur. Procrastination on the part of the teacher can lead to instruction time being lost and it also creates doubt in a learners mind that the teacher does not know the mathematics.

Velsoso (cited in Santos and Domingos, 2013:3238) stated that "*Issues related to mathematical preparation of future teachers have been investigated in view of training and teaching on education*" and it was discovered that "*mathematical knowledge is not present in many teachers*". The pre-service teachers spend approximately four weeks in a year at schools and the amount of time spent on 'teaching modules' related to the curriculum is minimal. I believe this is insufficient time to teach these pre-service teachers the various facets of becoming a quality mathematics teacher. This includes content knowledge and how to teach it (pedagogics). It becomes equally frustrating trying to teach the pre-service teachers aspects from the mathematics curriculum when they show an indifferent attitude towards the subject. With this kind of attitude their subject content knowledge stagnates and the prospective teachers' mathematics content knowledge remains unchanged throughout their teaching degree (Lowrie and Jorgensen, 2015). Baumert et al (2010) mentioned that is imperative that the pre-service teachers have a command of the mathematics content knowledge for the effectual teaching of mathematics. Lowrie and Jorgensen (2015:4) further reiterates that "*it is increasingly important for teachers to have strong content knowledge in order to be better teachers of mathematics".* I strongly recommend that both the issue of content knowledge and pedagogy be strongly addressed in the teaching modules preceding the pre-service teachers entering schools and mathematics lecturers are given the "*scope and flexibility to reinforce and build this content knowledge*" (Lowrie and Jorgensen, 2015:12).

According to Killen (2013:30) in order for the teachers to teach effectively, they need to be knowledgeable in the subject, have an excellent content knowledge and sound pedagogical knowledge (Figure 22). They need to have an understanding of the basics in the subject that includes concepts and principles; how to teach and guide and learning in the classroom and how to teach effectively.

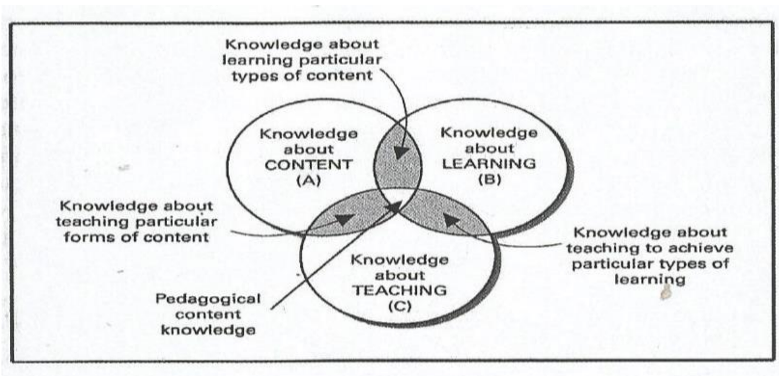


Figure 22 A model of teacher knowledge (Killen, 2013)

In Figure 22 the overlapping circles A and B indicate that the teachers need a deep understanding of the content and the learning theories to understand how the learners learn the content. The teachers need to know concepts that they want the learners to learn. The overlapping circles A and C demonstrate how the teachers need to teach those concepts. The overlapping of circles B and C indicates that the teachers need to understand the pedagogical implications of the learning theories involved in the subject. In the centre of the figure there is an intersection of A, B and C which shows the interaction between content, learning and teaching knowledge. Without a sound knowledge of concepts, pedagogical knowledge and learning traits, successful teaching and learning cannot occur in the classroom. If the teachers show a deficiency in mathematics knowledge, then the teacher training and continuing professional training need to be examined. These activities will not only improve their own classroom practice immensely but also enable them to make a contribution to the profession.

3.5 THEORY OF THE GROWTH OF MATHEMATICAL UNDERSTANDING

How often in our classrooms have we heard the learners mention, "I don't understand' to 'Please explain that again'. This is usually the first sign that they do not understand. Defining understanding is not easy therefore it is important for teachers to first understand the meaning of understanding.

According to the Department of Education (2008) understanding refers to the learner's ability to interpret, summarise and classify information. Mathematical understanding, which is crucial to all learning, is meaningful learning of what was taught to the learners in their formative years. When something is understood it is remembered for a great deal longer and it can be built upon to generate further understanding which eventually creates creativity. Therefore it is imperative that the teachers teach mathematics in such a manner that it promotes understanding of the concepts and procedures.

The definition of understanding relates closely to the constructivist learning theory. As with construction work, the workers will require a plan, materials, the know how to build and determination to finish the work within a prescribed or agreed time frame. Similarly in mathematics for understanding to occur in the mind of the learner (Luneta, 2013) the learners need a plan of action, an existing plan or knowledge and ideas. Working within a classroom and implementing the latter mentioned, they are able to tackle problem solving. According to constructivism the learners need to develop their own understanding from their previous experience by selecting, absorbing and adjusting what they experience in the real world because the more connections they make the better the new idea is understood. According to Haylock (2006) the more connected the experience the greater the understandings as the learners build connections between the new experiences and previous learning. I found the Growth of Mathematical Understanding (Figure 23) appropriate. It's within the realm of their connection between their experience and reality that learners understand better. Meyer (2001) mentioned that in order for the learners to gain an understanding of mathematics it must be real and experience related so that the learners learn with understanding. By relating to previous knowledge and experience, the learners are able to use that understanding to transfer the acquired knowledge to new situations. According to Meyer (2001:239) "*mathematical understanding is structured and interconnected"* and the learning of mathematics takes place by "*the progress through the levels of understanding*".

Pirie and Kieren (1989) developed the theory of the Growth of Mathematical Understanding. This theory has eight levels, namely, primitive knowing, image making, image having, property noticing, formalising, observing, structuring and inventising (Cobb, 1994; Pirie and Martin, 2000). As the learners progress the various levels they make connections through the use of drawings, diagrams, symbolic notations and models (Meyer, 2001:238).

According to Pirie and Martin (2000) and Cobb (1994), level one is primitive knowing. The term primitive refers to something that is important or having previous experience. This level is the starting point of mathematical understanding. It is when the learner draws from previous constructed knowledge (experience) and brings it to the lesson. It is important that the appropriate knowledge is selected and used as the foundation for growth of understanding. Level two is image making where a mental picture is created of the concepts or of the given problem. If the learner cannot create a schema of what is seen then the other levels of understanding cannot be understood. Level three is image having. This involves seeing. The learner constructs mental representations of what is asked in the problem. He attaches meaning to demonstrate an understanding. Level four is property noticing. At this level understanding occurs. The learner draws from previous experience and combines with the existing images. Teacher intervention occurs at this level to check if the learner has an understanding of concepts. Level five is formalising where the learner creates a mathematical definition. It is a process of seeing and communicating his thoughts for all to see and understand. Level six is structuring which involves the formal application of the theory. Level eight is the last level where the learner asks questions, tests the solutions and looks for alternate solutions to get a better understanding.

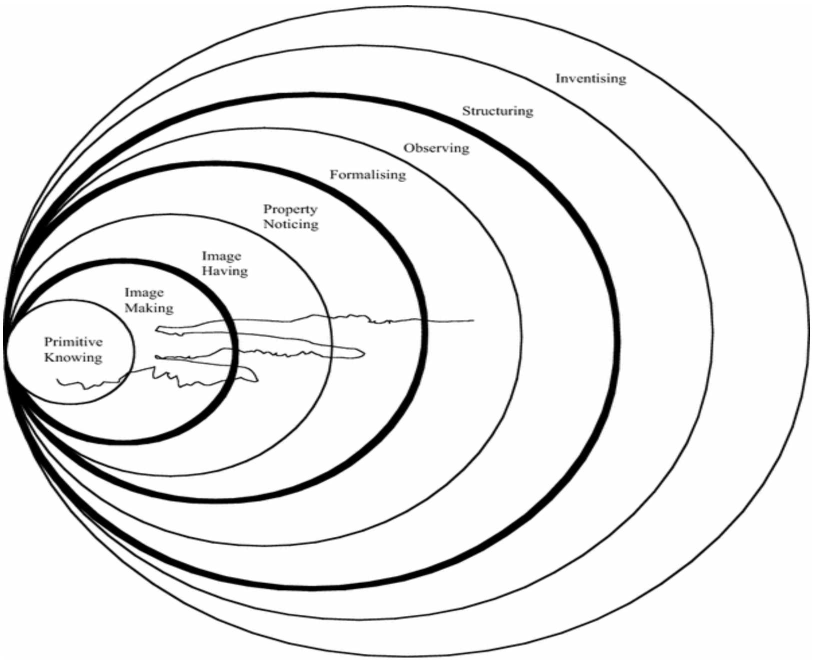


Figure 23 Model of Understanding (Susan Pirie and Thomas Kieren, 2000)

The learners must be given an opportunity to reflect on what they have learned as they progress through the levels of understanding. By being reflective learners, they engage themselves in such a manner that it forces them to search for ideas to find solutions. When a learner is not able to solve the problem then he returns to the inner layers of understanding to find another part to the solution. According to Pirie and Martin (2000) for growth of understanding to occur, the learners move forth and back through the layers reflecting and reconstructing their current knowledge since knowledge constructed is based on personal experiences.

David Asubel, a pioneer in the field of understanding as a component of the learning process, stated that two things are essential for understanding to transpire. Firstly, the learning content must be relevant and meaningful to the learner since individuals interpret and construct knowledge differently and secondly the learner must be able to relate this content to his previous knowledge since new information is linked to prior knowledge. According to van de Walle, Karp and Bay-Williams (2014:24) understanding occurs when there is a connection between the new idea and prior knowledge. The greater the connection is with the new idea the better the understanding. According to Hiebert and Carpenter (1992) and van de Walle, Karp and Bay-Williams (2014) understanding exists along a continuum (Figure 24). The two ends, relational understanding and instrumental understanding, were named by educational psychologist Richard Skemp. At one end a meaningful network of concepts and procedures (relational understanding) exists and on the other end ideas or rules are completely isolated (instrumental understanding). Relational understanding contains a rich web of ideas which leads to understanding whilst instrumental understanding contains information of isolated ideas which learners easily forget (Hiebert and Carpenter, 1992). Relational understanding implies knowing the reasoning behind the rules and this understanding can be gained if the learner has reflected through and can produce the rules for himself. This learning tends to be deeper, more lasting and more easily recalled to memory. The more mathematics available at automatic recall the more mathematics we learn to understand and the less the load on our working memory.

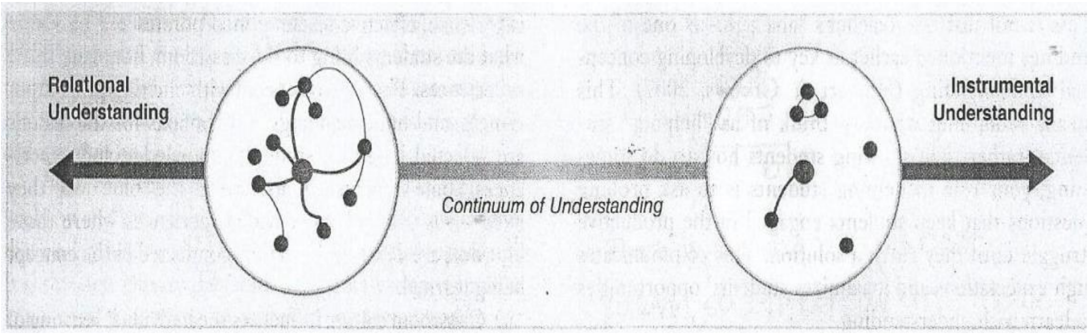


Figure 24 Cotinuum of Understanding (van de Walle, Karp and Bay-Williams, 2014)

In the problem process, the learner is expected to relate to the problem and understand the concepts within the problem. The teacher, as a facilitator, in the problem solving process has to ensure that the learners have mastered the understanding of concepts and the procedures related to and are able to correctly use it in simultaneously in mathematics problem solving. Skemp (1976) referred to this as instrumental understanding. The learner is able to use the concepts and procedures to complete an exercise. The failure to understand the concepts, namely, linking it with previous knowledge, experience and able to relate it to their real life situations, can result in obstacles in understanding.

DISCUSSION

Many higher education institutions do not model their training according to classroom practices and it makes it difficult to determine to what extent the pre-service teachers are exposed to the curriculum. This is further exacerbated by the fact that they are not adequately prepared in their teaching practices. Thus the pre-service teachers are left exposed if their training does not expose them to the above theories. By having a good grounding of the above theories they will be able to fend for themselves as the younger teachers are more likely to incorporate educational changes into their practice.

The above theories are closely linked to constructivism and I use this as my base in offering a brief discussion to show the relationship between the theories and their relevance to teaching and learning. According to constructivism the learners construct their own knowledge and the theories discussed above all have a common feature in that previous experience and prior knowledge is needed to create knowledge. Within constructivism we have endogenous constructivism which focuses on an individual's construction of knowledge. This view is derived from the Piagetian theory where individuals must negotiate the meaning of their experiences. The notion is that the learners are thought to be the creator of their own knowledge and this is accomplished through individual or socially constructed activities. Problem solving is such that experience, previous knowledge and understanding are necessary to use visualization as a concrete strategy to create knowledge. Kolb's Experiential Theory (discussed in 3.1) is a very much a learner centred approach where the learners create their own knowledge from utilising previous life experience, through active social participation and interaction in the classroom. The teacher is the facilitator who guides the learners through the problem solving process. Through using acquired skills, the teacher allows the learners to progress individually in understanding concepts and ensuring the application of procedures in solving the problem is correct. Since problem solving is a mental process, the learners rely on their memory to recall and utilise previous experience to show their understanding. Mathematical understanding emerges from mental activity, reflecting about their experiences and articulating what they know is important. What they the learners know shows their understanding. According to Pierie and Martin (2000) understanding occurs at various levels in the learners mind. Since creating understanding is a mental activity where one's mental capacity is expanded to replicate the world, the brain cells processes, creates and elaborates information based external realism. The more ways the learners think about the mathematical concepts the better they will understand. Therefore previous knowledge obtained from the real world (realism) or insightful knowledge is imperative to know and show an understanding of concepts. Mayer (1995) made reference to insightfulness (mental activity) in problem solving. Gestalt psychologists offered several ways of conceptualizing what happens during insight within the brain. As a mental activity insight involves building a diagram in which all the parts fit together; it involves reorganising the visual information so it fits collectively to solve the problem; it involves restating a problems objective in an innovative manner that makes the problem look easier to unravel; it involves removing mental blocks and finding a problem analog, namely, a similar problem that the problem solver already knows how to solve. I believe that good conceptualization in mathematics determines the learners application of procedural knowledge as indicated in the Structured Learning Theory (discussed in 3.2).

The SLT supports the learner's creativeness. Since problem solving activities encourages the learners to write, draw and converse as they work, the learners devise ways of representing the problem that will enable them to make connections and arrive at a solution. The learners create relationships, extend and relate (show their understanding in similar or related situations) their mathematical knowledge (Draper, 2002). In these instances teacher knowledge (discussed in 3.3) is vital. The teacher needs to be knowledgeable as their understanding of concepts and knowledge (domain knowledge) is vital together with pedagogical content knowledge. The teacher through acquired content knowledge (subject knowledge) is in a position to ensure the concepts are not misconceptualized and procedures (rules) are implemented in order. A teacher's common content knowledge informs whether a learner's definition of the concept, procedure and answer is correct and specialized knowledge is also necessary to demonstrate on how to teach using various representations.

The above four theories can be summarized from a constructivist platform. According to research there is an agreement amongst constructivists that learning is influenced when the learners create their own individual knowledge. In order to construct knowledge the learners must make sense of their new understanding and must be able to relate it to the topic been discussed. The creation of new knowledge will depend on the learners existing state of mind. Understanding is created through intense dialogue which occurs through social interaction. The learner's understanding is further assisted through constantly asking questions and offering explanations through feedback. The shared commitment and kindred spirit of learning communities creates an ideal vehicle to understand concepts which is the significant role of social interaction. Co-operative or collaborative learning is the key in social learning. It allows for two way learning. It is through spirited dialogue that the learners will learn more from each other. In this regard one can make reference to Vygotsky's concept of ZPD. By working with their peers learner who experiences difficulties can learn from another learner in that social set up who has a better intelligence acumen. Thus a new zone is created for learning. The learners need to be exposed to real life situations in order to build on their knowledge. Real or authentic tasks need to be generated from their environment to enhance their intellectual skills. The learners negotiate and refine what they have to learn. This increases their knowledge base.

CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 INTRODUCTION

The methods of research are tools by which we study the problem. Methodology consists of the systematic procedures by which we travel from the preliminary identification of the problem to the conclusion. The function of the research methodology is to complete the study in a valid manner. Research methodology involves identifying of the research problem, formulating hypotheses, review of the literature, designing of methodology, identification and designing of research instruments, sampling procedures, data collection, data analysis, drawing conclusions and making recommendations and the preparation of the concluding report.

In this chapter I explain precisely how the research study was conducted with reference to the methodological approach, research design, the research instruments used, data collection, access, sampling and ethical considerations.

4.2 METHODOLOGICAL APPROACH

According to Creswell (2014:5), the broad research approach is the "*plan or proposal to conduct research*". The purpose of the research aids in deciding the methodology and the design of the research and the methods are the range of approaches used to gather data (Cohen, Mannion and Marrison, 2011). As a researcher it is very important for me to design the research method. The aim of a research methodology is to assist me to develop a research strategy, understand the process of data collection and to be able to analyse the data.

All aspects of this research study are interrelated as it determines why the qualitative, quantitative or mixed approaches were used. According to Creswell (2014:4) qualitative research is used to explore and comprehend the meaning that individuals ascribe to a human problem and quantitative research tests the "*objective theories by relationships variables*". In the qualitative research the researcher is the primary tool to conduct the research. Qualitative research allows for the phenomena under investigation to be explored and is designed to support and understand people in the environment they live or work in. It presents data of a descriptive nature, in this research observing the pre-service teachers teaching mathematical problem solving in the classroom. In summation, the methodological approach is reference to the broad research methodologies or paradigms that guide the study.

The data was collected more or less simultaneously and this is known as convergent parallel mixed methods. Convergent parallel mixed methods are a "*form of mixed methods*" (Creswell, 2014:15) as it allows the researcher an opportunity to amalgamate both qualitative and quantitative data in order to present a "*comprehensive analysis*" of the research problem (Creswell, 2014:15).

I engaged in multiple methods to acquire evidence and knowledge on the parallel phenomenon using different research measures. Multiple methods involve mixed methods, i.e., when two or more methods are used to collect data, in this instance the qualitative and quantitative methods (Cohen et al, 2011). Since this study will follow a mixed methodological approach by conducting a qualitative and quantitative study, it will result in the amalgamation of data and this is called triangulation. Triangulation is when qualitative and quantitative data (methodological triangulation) is obtained from several sources to strengthen the research design (Kothari, 2014) and is used to evaluate and to ascertain if it corroborates and validate the research findings (Creswell, 2014). If one method shows weakness during the study, there is an opportunity through triangulation to verify the facts and findings (Kothari, 2014; Denizen and Lincoln, 2011). Triangulation improves trustworthiness and uncovers biasness.

Denizen and Lincoln (2011) described triangulation as crystallization which is more than the concept of triangulation. In this process a story is told through data gathered from a range of sources. The crystallization process involved the temporary suspension of examining data in order to reflect on the analysis experience and an attempt to identify and articulate patterns and themes during the immersion process. The immersion is the process when the researcher immerses himself in the data he has collected by reading and examining the data in detail and the dual processes continue until all the data have been examined and substantiated in detail (Denizen and Lincoln, 2011; Cohen et al, 2011).

Cohen et al (2011) and Lincoln, Lynham and Guba ( )*stated that qualitative research is interested how individuals understand themselves and construct meanings of their lives. The qualitative research is designed to enlighten the researcher (how) and the why (meanings) things ensue in order to get an in-depth understanding of the situation. Cohen et al (2011), Denizen and Lincoln (2011) and Lincoln, Lynham, and Guba ( )* described that the quantitative research method as being able to generate data that would be justifiable and unbiased, as they produce knowledge which de-personalizes the obtained data. The use of both quantitative and qualitative methods is therefore necessitated by the need to verify and understand the perspectives of the participants.

4.3 RESEARCH DESIGN

According to Creswell (2014) and Denizen and Lincoln (2011) a research design comprises an investigation with qualitative and quantitative and mixed method approach. This study will follow a mixed methodological approach by conducting both a quantitative and qualitative study. For the purpose of this study I used a combination of research instruments. The instruments used were a questionnaire for pre-service teachers (Annexure 1), observation of pre-service teachers in the classrooms (Annexure 3), semi –structured interviews with pre-service teachers (Annexure 2), evaluation worksheets for both pre-service teachers (Annexures 4 and 5) and learners (Annexures 6,7,8 and 9) and examination of learner's books to garner evidence. I used these instruments as it would assist me in garnering the necessary data for this study.

These instruments are discussed below.

4.4 QUESTIONNAIRE

A questionnaire is a paper and pen method of collecting information from participants. It is a popular and advantageous way to gather data from a wide range of issues as it is through this medium that data is collected. As a data collecting instrument, a questionnaire is often used in a wide range of settings to gather a variety of information. The questionnaire for this study (Annexure 1) comprehensively covered the relevant areas that I prosed to investigate. The said questionnaire was divided into three sections. The first section focused on the pre-service teacher's knowledge of the Curriculum Statement and Assessment Policy (CAPS) and their knowledge of problem solving; the second section examined their classroom observation and teaching experience knowledge relating to strategies used to teach problem solving and the third part focused on the pre-service teacher's knowledge on visualization and their ability to use this to teach and solve problems in the classrooms. The questionnaire comprised multiple choice questions, dichotomous questions, open and closed ended questions and Likert scale ratings was used to obtain an in-depth understanding on the pre-service teacher's knowledge and utilization of visualization skills in teaching and solving mathematical problems.

A questionnaire is also practical in that some will find it easier to write than to talk and much more information can be garnered from participants via a questionnaire than they will contribute to through an interview (Cohen et al, 2011). The questionnaires were handed out to the pre-service teachers during their lectures and they were requested to return it within the agreed time frame, that being a week. I allowed them this period as they had lectures and assignments to contend with. It also gave them an opportunity to complete it on their terms and in an independent environment. The stress free environment allowed the pre-service teachers to think over their responses before responding to the questions. Removing myself from the equation (environment) as their lecturer alleviated the possibility of biasness as they did not have to provide responses to satisfy me. Furthermore, the neutral environment also assured their confidentiality and anonymity.

In order to obtain the appropriate data the questions should be relevant and carefully constructed. Wherever the alternatives were being given the 'answers' should not be obvious to the respondent. Explanatory type of questions must be free from ambiguity. If this exists then the respondents will have limited opportunities for elaboration. Furthermore they can leave questions unanswered or provide superficial responses. In order to collect authentic data, the set questions must give all respondents an opportunity to provide an answer to all the questions. The respondents must be competent to provide the responses. All of these factors were considered when designing the questionnaire. As this study focused on the responses of the pre-service teachers it qualified their responses to the study.

When conducting a research study using a questionnaire the following factors, namely, informed consent, confidentiality, and the right to withdraw without prejudice have to be considered. A covering letter (Annexure 12) outlining the rationale of the study and afore mentioned factors were explained and attached to the questionnaire. Another letter (informed consent) from the institution these pre-service teachers attend was attached as well. These letters gave an overview of my intention as the researcher and purpose of the study. I allowed the pre-service teachers an opportunity to peruse both the attachments. I drew their attention to their rights to confidentiality and the right to withdraw from the study. I placed a lot of emphasis on both these aspects as I didn't want the pre-service teachers to feel threatened and prejudice the study by not responding objectively.

Whilst the questionnaire can be considered as the prime method of collecting data in a research study, it has its limitations. The response rate is low from individuals; there is a refusal to co-operate; providing brief explanations or ambiguous and incomplete answers to questions (Kothari, 2014; Cohen et al, 2011).

4.5 OBSERVATION

Observations require a researcher to develop an observation schedule that will direct what needs to be observed by the researcher. According to Cohen et al (2011) observation is the gathering of information as they occur and at the site it occurs as it allows for obtaining 'live' data from 'live' situations. The lesson observations allowed for the direct collection of data from an authentic source. The goal of the lesson observations was to understand the approaches and specific methods used by the pre-service teachers in the classroom. According to Lester (1985) classroom observation is the most appropriate medium to obtain first-hand information. Information can be gathered from extensive observation of actual teaching of authentic mathematics in the classroom. For the purpose of this study, the information is sought by the researcher in his own direct surveillance as it is currently happening.

Lester (1985) advocated qualitative methods for conducting research in problem solving. He stated that "*adopting a holistic view of problem solving and problem solving instructions necessitates the use of naturalistic (inquiry) rather than traditional scientific research paradigms*" (Lester, 1985:52). Naturalistic inquiry also known as constructivist inquiry (Lincoln and Guba, 1985) is the reference to qualitative research been done in a natural setting to examine a social or human issue. In this study the natural setting is the classroom and the area of investigation is how pre-service teachers teach problem solving. The pre-service teachers were observed teaching mathematics during their teaching practice at their allocated schools. I needed to get a sense of their teaching practices, procedural knowledge and how they taught problem solving. Observing the pre-service teachers in different geographical settings allowed for the testing of various teaching practices in the classroom.

I used a pre-determined observation checklist (Annexure 3) when observing the lessons in the classrooms. The reason, using reliability as a factor, was to ensure that all pre-service teachers were observed accordingly so that the data collected can be considered valid and free from bias.

4.6 SEMI-STRUCTURED INTERVIEW

Interviews are a valuable source of information. As interviews are face to face interviews it can be classified as a two directional conversation (Cohen et al, 2011; Maree, 2007) as the interviewees are able to express themselves by providing their own explanations and interpretation of the questions. Semi-structured interviews allow for leading questions to be asked of all participants and also allows for the possibility of asking unplanned questions based on the responses of the participants. The responses to the questions are direct and given without assistance or prompts. The interview, in my opinion, provides more reliable data than a questionnaire or the evaluation worksheet. The answers to the questionnaire can be obtained from the internet or copied from colleagues whilst in the interview process you have to give your own true response to the questions asked.

A semi-structured interview was used. I intended discovering first-hand the thoughts of the participants. A list of standardised open ended questions was asked (Annexure 2) and the responses were recorded. The standardized framing of the questions allows the interviewer to ask more than one interviewee the same question. This reduces researcher bias and allows for an increased intensity of comparability (Cohen et al, 2011).

The questions were based on visualization skills and problem solving strategies and how it was used in the classroom. Open ended questions also formed part of this interview. The use of open ended questions in these face to face interviews reveals the thoughts of the interviewees. This secures a more spontaneous and factual response. In an interview the interviewees reveal their true thoughts and feelings without inhibitions. These kinds of interviews allow for additional questions to be asked in the event of obtaining further clarity to a response. As the interviewer I am in a position to ask the interviewee additional probing questions in order to delve further or to ask them to elaborate their responses (Cohen et al, 2011; du Plooy-Cilliers et al, 2014). The asking of the additional questions also avoids misinterpretations of questions. The interview allowed me an opportunity to get a sense of their beliefs on problem solving and the teaching strategies or practices they employed. I was very much interested on how problem solving was taught using visualization.

According to Maree (2007:87) a semi structured interview can be used to "*to corroborate data emerging from other sources*". The data obtained from the interview was used to corroborate data obtained from the questionnaire, lesson observation, evaluation worksheet and learner's books.

Scheduling the interviews with the pre-service teachers was problematic. The participants in the interview process all live in different areas from their institution and logistically it was difficult to get them together for the interview process. This was further compounded as they attended lectures at different times. As time was a determining factor, all the interviews were scheduled for approximately thirty minutes. This allowed me access to the pre-service teachers during their non-lecture periods. To ensure reliability I formulated the questions as clearly as possible for all pre-service teachers. The interviews were audio recorded, transcribed, coded and categorised to suit the research questions.

The interview as a data collecting process has its disadvantages. It is a very time consuming and an arduous task to transcribe and compile the data. It also brings the problem of the interviewee to be punctual for the interview. With the interviewer been part of the interview it can lead to the contamination of data. The possibility may arise that the interviewer will ask additional questions to obtain data to suit the research study.

4.7 EVALUATION WORKSHEETS

Two evaluation worksheets (Annexures 4 and 5) comprising mathematical terminology and non-routine mathematical questions were given to the pre-service teachers. The reasoning behind using mathematical terminology and concepts in the evaluation worksheet was to gauge their pedagogical knowledge. The non-routine mathematical questions were taken from previous Association for Mathematics Educators of South Africa (AMESA) and Mathemagica Plus mathematics problem solving competitions. The purpose of administering this evaluation worksheet was to determine the methods or strategies used by the pre-service teachers when problem solving. This evaluation worksheet was given to the pre-service teachers as an activity during a routine lecture period and collected thereafter.

An evaluation worksheet comprising five non routine problems (with/without diagrams) was given to learners in grades four to seven (Annexures 6, 7, 8 and 9). These evaluation worksheets were given to the pre-service teachers to administer to their classes during their teaching practice sessions. The purpose of the worksheet was to give the learners an opportunity to show how they solved these problems using mathematical strategies and visual skills. The evaluation worksheet was within the ability levels of these learners and was sourced from AMESA and Mathemagica Plus problem solving competitions. Every endeavour was made to ensure that the learners had no come across these questions.

4.8 EXAMINATION OF LEARNER'S BOOKS

A casual examination of learner's books was done whilst I was in the classroom observing the pre-service teachers. I wanted to determine what work was done in respect of problem solving and what strategies and visual skills (if any) were being used by learners to solve problems during their normal mathematics lessons with a qualified mathematics teacher. It was also to determine if learners were being taught problem solving strategies.

4.9 DATA ANALYSIS

I aligned the analysis to the key research questions. The qualitative and quantitative data from the questionnaire, observation, evaluation worksheets and learner's books was identified separately for later analysis. It was thereafter coded into dominant themes, clustered into categories and a detail summary made thereof.

4.10 ACCESS

It is vitally important to seek authorization early from stakeholders to carry out a research study. I am an independent lecturer contracted to the institution which the pre-service teachers attend.

To secure permission I formally requested permission (Annexure 11) from the academic head for research. Permission was granted from the controlling body of this institution to use their students for the purpose of this study.

In this study the schools visited falls under the jurisdiction of the KwaZulu Natal Department of Basic Education. Permission was sort from the KwaZulu Department of Basic Education via the University of KwaZulu-Natal to pursue this study. Access to collect data for this study from schools under its jurisdiction was granted by the Department of Basic Education. Although the school visits for the lesson observations was facilitated by the institution which the pre-service teachers attend, I took the liberty to write to the Principals of the said schools (Annexure 12) informing them of my studies. The teaching practice coordinator corresponded with the various schools to place the pre-service teachers for their teaching practice. Permission was granted by the respective school Principals to the institutions for the pre-service teachers to do their teaching practice at these schools.

I was given a schedule of visiting schools in the North Durban towns of Phoenix and Verulam. This was based on convenience for travelling. These schools are attended by Indian and Black learners from within the vicinity of the school, from neighbouring townships, low cost housing complexes and informal settlements.

Many of these schools rely purely on state funding and are way under resourced. They also have a high learner-teacher ratio and this has an impact on teaching and learning in the classrooms. All these schools have English as the language of learning and teaching (LOLT) and IsiZulu or Afrikaans as their first additional language (FAL). Whilst the Indian learners attending these schools have English as their mother tongue many of the Black learners attending these schools have IsiZulu or Xhosa as their mother tongue making English their first additional language. None of these schools cater for Xhosa as an additional language. This puts them at a distinct disadvantage. This has a negative impact on teaching and learning in the classroom as many of the teachers at these schools are not trained to converse in IsiZulu, Xhosa and Afrikaans. Taking into consideration all of these factors (judging from the composite results submitted at district level) these schools were still able to produce results ranging from good to excellent.

4.11 SAMPLING

Apart from having research methodology and instrumentations, research studies also rely on the quality of the sample. The following factors need to be considered in respect of sampling, namely, the sample size, access to the sample, sample strategy and its representatives (Cohen et al, 2011: Killen, 2015). Sampling is a process by which you reduce the total research population to a number which is practical. Random sampling (also known as probability sampling) occurs when every member of the population (in this case the pre-service teachers) has an equal and autonomous chance of being selected. I chose the participants that were registered for the Bachelor of Education (B.Ed.) degree. These participants (pre-service teachers), both males and females, are representatives of the race groups in South Africa.

Sampling frame is the populace that you will interact with for the study (Cohen et al, 2011). Simple random sampling was used to select participants for the study. Simple random sampling is when every participant is given an opportunity to be part of the study (Maree, 2007; Cohen et al, 2011; Davis and Bezuidenhout; du Plooy-Cilliers et al, 2014). The pre-service teachers were categorized geographically (for travelling convenience). Thereafter they were randomly selected by the institution from their data base and allocated to me. If the selection is done correctly then a high degree of probability is a true reflection of the sampling process. The sample size should be neither too large nor too small to guarantee consistency and representativeness (Killen 2015). For the purpose of this study the sample size chosen was adequate.

4.12 ETHICAL ISSUES

Ethical issues are a vital part of a research (Maree, 2007; Cohen et al, 2011). It is a complex yet very significant component of the study. According to Bak (2004); Maree (2007); Cohen et al (2011); Killen (2015) the following ethical guidelines must be considered by the researcher whose overall responsibility is to:

- abide by national and provincial law and policy with professional principles governing how to conduct a research;
- design, conduct and report the research in accordance with recognised principles of scientific aptitude and ethical research;
- seek clarity from the university ethics committee about any unclear ethical issues;
- obtain appropriate permission from the individual subjects (participants);
- guard the human identities and security of those involved;
- guarantee the confidentiality of the information given to the researcher;
- Minimise the possibility that the result will be misleading.

Permission was obtained from the controlling body of the education institution attended by the pre-service teachers involved in this study. Permission was sort from the KwaZulu Department of Basic Education via the offices of the University of KwaZulu Natal to pursue this study. I attached a letter to the questionnaire (Annexure 12) which allowed me to introduce myself to the pre-service teachers and the topic under review. A further letter was given to the pre-service teachers. This letter was from the institution that the pre-service teachers attend. This letter confirmed that permission was granted for me to conduct my study. More importantly it outlined their rights of participation in this study and they had to sign off their consent to participate in this study. Whilst these two letters had the necessary information in respect of my study, I had a face to face discussion with the pre-service teachers. I impressed on them that this study was being forced on them and they would be able to withdraw at any stage of the study without any ramifications. Since no names where required on any data collecting instruments, they were assured of confidentiality and anonymity.

In this study, I had to consider the following ethical principles, namely, autonomy, nonmaleficence and beneficence (Maree, 2007; Cohen et al, 2011). In the first principle, autonomy had to be considered. Informed and written consent was obtained from the pre-service teachers. According to Maree (2007); Cohen et al (2011) informed consent is when individuals decide whether to participate in an investigation after being informed of the details that would be likely to influence their decisions. A letter was given to the participants. In this letter, I introduced myself; gave a brief outline of this study; indicated to the participants that he/she was under obligation to participate in the study and if they did participate then they had the right to withdraw without any ramifications; ensuring their confidentiality, anonymity and non-traceability in the case of publications (Maree, 2007; du Plooy-Cilliers, 2014; Cohen et al, 2011). Regarding the second principle of nonmaleficence (Cohen et al, 2011), I had to ensure that the participants did not endure physical, emotional and social harm or exposure to any forms of endangerment. The third principle of beneficence (Cohen et al, 2011), I had to ensure that this research benefitted the participants and education at large. Problem solving and visualization are two fields of critical interest in mathematics and the results obtained from this study will address areas of concern to educators, aid learners and ensure that pre-service teachers become adept in problem solving.

4.13 CONCLUSION

This chapter serves as an overview of how this study was conducted with respect to procedures and methodology. The research methodology discussed herein served as a guideline to me on aspects of procedures and data collection at various stages of this study. It allowed me to provide a detailed explanation and discussion of the research design taking into consideration the key questions related to this study.

The next chapter focuses on the findings and interpretation of the study and will be discussed in detail.

CHAPTER FIVE

DATA ANALYSIS

5.1 INTRODUCTION

This chapter will report on the results of the research involving the participants (pre-service teachers) who are part of the Bachelor Education degree cohort and grade 4-7 learners.

In this chapter I report on the results of the statistical analysis. I have analysed the data collected from the questionnaires, classroom observations, interviews, evaluation worksheets and the learner's books.

This research has the potential to assist bring awareness to pre-service teachers who intend specialising in teaching mathematics, mathematics teachers and educationists who have an interest to improve their mathematics skills.

5.2 QUESTIONNAIRE

5.2.1 Introduction

A questionnaire (Annexure 1) was administered so that the pre-service teachers could write their responses on their own and provide possible explanations. The questionnaire comprised both open ended and closed questions which were directly focussed on key aspects related to this study. The pre-service teachers' responses to the questionnaires were analysed in each category, the quality of their responses were graded and the findings reported on in this chapter.

5.2.2 Analysis of the Questionnaire

The CAPS document lays the foundation for understanding the curriculum and has relevance to the everyday life of a teacher hence this question on the CAPS document. When asked whether they read the CAPS, only fifty eight percent of the respondents indicated that they did read the CAPS and thirty three percent did not provide a response. The forty two percent, made up of those that did not read the CAPS document and also those that did not provide a response, is disquieting in the sense that these pre-service teachers had already been to schools for their teaching practice and their mathematics lessons were supposed to have being designed around the CAPS document. CAPS was developed for South African teachers so that it could guide their teaching in mathematics in such a manner that it improves the learner performance. As future teachers it is imperative that the pre-service teachers become au fait with CAPS document because it provides guidance to all teachers in the classrooms. It also outlines the conceptual knowledge to be taught to the learners as they progress through the grades. CAPS also prepare the pre-service teachers to build on their subject content knowledge.

When asked if they knew the concept of problem solving sixty-seven percent of the pre-service teachers indicated that they understood problem solving. Thirty-three percent of them did not provide a response. Within the sixty-seven percent who indicated that they knew what problem solving was, only sixty-two percent provided an explanation.

Some of the pre-service teacher's responses indicated that they understood the concept of problem solving as a means of facilitating problem solving, developing learners' problem solving skills, following procedures and formulating rules and explaining mathematical concepts. Their responses included:

- *Sifting through information to find a solution or an answer;*
- *When a solution to a problem isn't readily available;*
- *Solving problems by looking at factors and coming up with a plan of action;*
- *There are specific strategies and methods used;*
- *Working with something to solve an issue in order to gain a solution;*
- *Problem solving refers to analysing and interpreting a specific 'problem' or question and formulating solutions according to the details given in the problem;*
- *Finding solutions to solve a problem using various methods and explanations;*
- *Solving problems by looking at factors and coming up with a plan of action.*

The following extracts indicated that besides using strategies and different methods, the problem given must be of a scenario which the learner can relate to or has prior knowledge of. In doing so they will be able to understand and apply their acquired knowledge to the problem situation. Their responses included:

- *Using skills and acquired knowledge to solve problems by finding suitable solutions*
- *Problem solving to be an instance where a scenario is provided to the student. The student must be able to extract the necessary data and correctly apply their knowledge to acquire the answer;*
- *Problem solving in the intermediate phase consists of everyday situations as well as the operations learnt in maths combined so learners can solve problems.*

The non-response from the respondents indicated that they did not possess knowledge of what problem solving entailed. This is significant because these pre-service teachers do not possess adequate subject content knowledge (discussed in chapter 3) to function as mathematics teachers. Literature discussed in this study indicates that pre-service teachers do not possess sufficient or efficient knowledge and skills in schools today.

When asked about non-routine problem, only forty-five percent of the pre-service teachers indicated that they knew about non-routine problems and thirty-three percent of them did not provide a response. Only ninety percent of those who said yes provided an actual explanation. However, these responses were not clear enough and would have needed follow up questions to seek further clarification.

Some of the pre-service teacher's responses below indicated that they understood the concept of non-routine problems as been mathematics questions not usually used in their normal classroom teaching or mathematics questions that required a higher level of thinking.

Some of the responses included:

- *Problems that don't follow a pattern and need extra thought;*
- *Complex mathematical problems where various methods are needed to find a solution;*
- *Problems which can be solved in multiple ways, as there is no single solution or method;*
- *A complex problem that requires creativity and originality to solve;*
- *It requires you to be creative and there is no 'set' method to solve the problem;*
- *These problems require you to think. These are challenging and difficult questions;*
- *Problems that don't follow the normal routine;*
- *Problems that aren't really taught but require a brain storm to solve an issue.*

One response *'never heard of it'* indicated the pre-service teacher's lack of exposure to problem solving situations and knowledge in mathematical conceptual understanding. The thirty three percent of pre-service teachers not knowing or failing to provide a response in respect of this concept indicated that they lacked experience in mathematical knowledge on problem solving.

When asked to provide a response on their knowledge of routine problems only fifty one percent of the pre-service teachers indicated that they knew about routine problems and thirty three percent of them did not provide a response. Of the fifty one percent, only forty-nine percent of the pre-service teachers provided an explanation.

Some of the pre-service teacher's responses, amongst others, indicated that they understood the concept of routine problems as giving the learners the opportunity to practise certain algorithmic skills in the classroom.

Some of their responses:

- *Straight forward maths problem;*
- *Problems that require less thinking;*
- *Routine problems only had one way of solving them;*
- *Ability to do something using methods and with particular understanding;*
- *Using at least one operation to solve a problem.*

The responses below indicated the relevance of the structural learning theory in mathematics. According to SLT mathematics is taught to learners as rules and procedures. These rules are so often implemented by learners without them understanding why these steps or formulas are been used.

- *Problems that are taught to us based on a curriculum and we are trained to solve through parrot form;*
- *Involves using one of the four operations to solve problems;*
- *They follow routine and there is only one way of solving the problem;*
- *Problems that have a specific strategy or solution;*
- *These are problems that are solved using formulas and symbols that help get to an answer;*
- *They follow a step by step process;*
- *These problems involve particular steps in order to solve a problem. It might make use of an equation or algorithm. There is a sequence of actions which have to be followed to reach a solution.*

The thirty three percent of non-responses together with fifty one percent of the pre-service teachers not providing an explanation for their understanding of what routine problems are indicated that the pre-service teachers lacked an understanding of mathematical conceptual knowledge. Understanding these concepts is of paramount importance as they link key mathematical concepts and underpin the content material in problem solving. To manage the teaching and learning within a spiralling curriculum it is significant that the pre-service teachers understand these mathematical concepts so that they have the mathematical expertise to teach the learners in a constructive manner.

When asked if their learners understood non-routine and routine problems, only twenty-four percent indicated the learners understood non-routine problems and forty percent indicated that the learners understood routine problems. Thirty eight percent did not provide a response for non-routine problems and thirty three percent did not provide a response for routine problems respectively.

When the pre-service teachers were asked which of the problem types they used in their teaching, only four percent indicated non-routine problems and only sixteen percent mentioned routine problems. Only twenty two percent indicated that they used both types of problems in their teaching. When making a comparison with whether their learners understood non-routine and routine problems with the type of questions that they used in their teaching, it can recognized that the learners not understanding was due to them not using these questions in their teaching. One response indicated that routine problems were used as they *are easier to keep track of as a class.* Another response indicated that he used *both types, however, we tend to use routine problems move often to keep the class on the same level and to mark.* One non-respondent stated that *there hasn't been a need to classify routine and non-routine problems.* It is important that the pre-service teachers make the distinction between these two types of problem posing as both have distinct purposes in the mathematics curriculum.

The pre-service teachers were asked to respond, based on their classroom observation and teaching experience, whether problem solving is neglected in the mathematics curriculum. Only twenty percent of the pre-service educators agreed that problem solving is neglected in schools. Forty seven percent disagreed. Thirty three percent did not provide a response. Although the small group of respondents that agreed that problem solving is neglected might seem minimal, it is still an indication that problem solving is neglected in the school mathematics curriculum.

Mathematics is allocated six hours in the Intermediate Phase and five hours in the Senior Phase per week respectively. It is expected that mathematics teachers engage in problem solving every day as specified in the Foundations for Learning Document (DoE, 2008). When asked how often do you use problem solving during your teaching experience at schools, only twenty percent of the respondents indicated that problem solving was part of their lesson on a daily basis, thirteen percent indicated that used once a week, eighteen percent indicated twice a week and thirteen percent indicated other.

The following responses were provided for 'other':

- *Problem solving is only done in those areas of CAPS where the syllabus involves it.*
- *It depends on the content/section being taught in the classroom;*
- *It may be given only for certain sections and not frequently;*

- *Problem solving has been allocated a time frame in a particular term. It is however included in revision exercises.*

The above responses indicated that the majority of the respondents held an underdeveloped understanding of the mathematics curriculum. The above responses indicated that these pre-service teachers have a shallow knowledge of the contents of CAPS. If they did then they would have deduced that problem solving is an everyday activity. Hence there is no relationship between their responses when fifty-eight percent indicated that they have read the CAPS document but only twenty three percent are aware that problem solving must be an everyday component of mathematics lesson. If the pre-service teachers had read CAPS then they would have discovered that problem solving is not only handled in certain sections but all five content areas in Mathematics involve problem solving. They would also have known that one of the general aims of the South African Curriculum as stated in the National Curriculum Statement Grades R-12 is that learners *"identify and solve problems and make decisions using critical and creative thinking"* (DoE, 2011:5).

The disconcerting factor here is the thirty three percent of the respondents who did not give a response. The possibility exists, as pre-service teachers, they have no knowledge about when and how problem solving is taught. If this is the case then more time is needed to be focussed on teaching them how to use CAPS during their teaching experience lectures.

Mathematics lessons in the primary schools are normally 60 minutes. The pre-service teachers were asked to respond to how much of the lesson time is spent of teaching problem solving. Thirty-one percent spent approximately fifteen minutes of the lesson teaching problem solving; twenty seven percent used approximately half an hour and only seven percent used more than half an hour. This translates to less than the norm of time in a day spent in teaching problem solving. Thirty five percent did not provide a response. The use of approximately fifteen minutes is limited time for teaching problem solving considering that teachers must spend at least ten minutes daily fostering learning and practicing of oral computational procedures through drill and practice (DoE, 2011).

The response to 'Do you think that this sufficient time to teach problem solving', thirty-one percent responded that it was not enough and thirty-two percent responded that the time was enough. Those who indicated that the time was insufficient did not make an attempt to spend more time on teaching problem strategies. I should have probed the time factor further with the pre-service teachers to determine their reasons for their responses. It is possible that the anxiety of completing the curriculum to meet the assessment demands or other contextual factors prompted such responses. The content within the mathematics curriculum is not taught in isolation and cannot be time restricted. Furthermore, the mathematical content continues from one term to another on a developmental basis and this leaves no room on the part of pre-service teacher to disregard spending sufficient time teaching problem solving.

The pre-service teachers asked to respond on how often they teach problem solving strategies in your mathematics lesson. Only twenty-four percent of the pre-service teachers indicated that they teach problem solving strategies every day, eleven percent once a week, sixteen percent twice a week and sixteen percent said other whilst thirty three percent did not provide a response. Problem solving is not to be considered as a separate teaching activity and strategies are not to be taught in isolation. The mathematical content lends itself to be taught via any possible means with understanding as literature supports the need for teachers to demonstrate to the learners the skills and strategies needed in mathematics. It can be argued that as the lesson progresses problem solving can be introduced and the problem strategies should follow suit. In this approach the teacher forces his learners to raise their level of thinking to strategize and apply the strategies when problem solving. The thirty-three percent of non-responses indicated not knowing when to teach problem solving strategies. If this is too believed then it can lead to problem strategies been totally ignored as part of mathematics lessons.

Considering the pre-service responses to questions based on the time spent teaching problem solving and how often they teach it, the actual time spent on problem solving can be a determining factor depending on the ability of the learners. The fashion in which learners learn can be attributed to various reasons. Learners learning styles, ability levels and level of understanding need to be considered as some learners take longer than others to assimilate and solve the problem. This can linked to Kolb's experiential learning theory. This theory is based on the four stage learning cycle. Kolb's model offers a description of the different learning styles which is applicable to all of us. Much will depend on the teaching style of the teacher as well on how, where and when problem strategies are taught in the lesson. Today's teachers are curriculum and assessment driven therefore they are forced to focus on the completion of the syllabus in order to meet the demands of the prescriptive assessment programme.

The responses given for the 'Other' include:

- *Only done in revision or is a specified term;*
- *Usually once a month to once in two months, maybe more longer;*
- *It depends on the section being taught.*

The above responses indicated there is a grave misgiving that problem strategies are taught at such lengthy intervals or considered to be part of the revision programme and is dependent on the content sections being. Teaching of problem solving strategies and *'revision'* are two different components within mathematics teaching. Teaching occurs when the teacher exposes the learners to the mathematical content and revision is a refreshing and consolidation process to assess how much the learners remember from the teaching programme. Furthermore, revision shows the level of the learners understanding. The teaching of mathematical problem solving strategies is not content dependent. The mathematics curriculum is structured in such a manner that all five content strands lend themselves to problem solving.

The pre-service teachers were asked to respond to naming the problem solving strategies that they used in their teaching or observed the class teacher using in the mathematics lesson.

Some of their responses were that they used:

- *Mathematics illustrations;*
- *Trial and error – first extract all the important information that is provided then look at what needs to be found then work from there;*
- *Logical thinking;*
- *Look for a pattern;*
- *Guess and check;*
- *Draw a diagram and tables;*
- *Explaining what data will be needed to get a specific answer and discussing various methods on achieving it;*
- *Graphs and visuals images as a source of concrete material.*

The above responses indicated that the pre-service teachers were familiar with some of the frequently used problem solving strategies and have used them in their teaching.

The responses for naming the problem strategies that the teacher used included:

- *Tactile models;*
- *Diagrams;*
- *Trial and error;*
- *Use charts, boards and graphs*

The observation, by the pre-service teachers, of what strategies were used by teachers indicated that the teachers were not constantly exposing their learners to the many problem solving strategies that can be utilised in solving problems. A general conclusion that can be arrived at is that the mathematics teachers have limited or no knowledge about problem solving strategies.

The following response '*give learners problems, put them in groups and to work it out*' indicated a shallow understanding of a problem strategy as placing learners in groups is actually a teaching method and not a problem solving strategy. Furthermore '*use of algorithms on the chalkboard*' and '*key words are identified in story problems*' are not a teaching problem solving strategy but rather the way a teacher demonstrates to learners how to do their calculation. It is important that the pre-service teachers differentiate between a problem strategy and teaching method.

The following response '*usually students ask for help as they are unable to extract the correct data'* indicated the pre-service teacher's fallacies about problem strategies. It indicates what occurs in a mathematics lesson when a learner does not understand the problem as they cannot read, comprehend and understand the problem.

The following response, '*pose a problem, look at the work covered in the lesson, find a solution*', is the testing of the learner's application on a problem based on what was taught in the lesson. This creates an impression that a problem strategy can be or is a mere transmission and application of the passive knowledge received from the teacher during the lesson.

The response '*the teacher recaps previous lesson to clarify errors – helps children to relate to other examples*' is something that that is related to how a teacher commences a lesson. The teacher consolidates the learners understanding of the previous lesson by discussing their acquired knowledge, clarifying misconceptions and relating it to further examples before commencing with a new topic.

The responses '*circle numbers, underline questions*' are similar to giving instructions to learners by the teachers on how they the learners are expected to answer the question in a test or examination paper. The following responses '*the cubes method, use box numbers*' are indicative of the type of method learners are expected to use to do their calculations. These responses indicated that the pre-service teachers are unaware of basic mathematical problem solving strategies.

Referring to the response, "*using a worksheet with multiple sums'* is a massive misconception about problem solving strategies. Teaching in this manner of asking learners to complete numerous sums does not provide exposure to problem strategies or the consolidation of mathematics concepts. It is a mere use of sums to test their use of the four algorithms. In the event of no have conceptual and procedural knowledge the learner is bound to answer the sums incorrectly.

The '*use of visuals and diagrams; charts, boards and graphs*' is not a problem strategy but rather the resources one can use to support teaching and learning in the classroom. These become the support in presenting the lesson.

Whilst the responses by the pre-service teachers indicated that they have knowledge and have used mathematics problem solving strategies in lessons, the observation of the limited use of the problem solving strategies by the more experienced mathematics teachers may create an impression on the pre-service teachers that it is not important to teach problem solving strategies to learners. Furthermore the fifty-one percent of non-responses to this question created an impression that the pre-service teachers lack the necessary mathematical knowledge to teach problem solving strategies.

The pre-service teachers were asked to list the sources from where they obtained their questions. The responses included '*internet, past papers, grade textbooks*'. This indicated they are exposed to obtaining material from other sources. The following responses '*from my mind, my general understanding of problem solving*' indicated that the pre-service teachers lack the imagination to pursue other sources to obtain material for their teaching of problem solving. Stating using the '*CAPS document*' as a source to obtain questions is more damning because if their actually read the CAPS document they would have established that it dealt more with the curriculum content material, assessments and policy plus how CAPS should be implemented in the various grades rather than providing questions relating to problem solving.

When asked if the learners enjoyed problem solving only forty-two percent indicated that their learners enjoyed problem solving and eighteen percent stated otherwise. Forty percent of them did not provide a response. The considerable difference between 'YES' and the other responses indicated that there is a definite dislike for problem solving in mathematics.

The pre-service teachers were required to provide a reason if their response was NO. One response was '*they find it difficult*' can be attributed to many reasons. The learners are confronted with a myriad of information in a problem, they experience reading barriers to learning, and they are affected by contextual factors in the classroom or not having sufficient exposure to solving problems or strategies to solve problems. In order to overcome the difficulty in problem solving necessitates learners be given problems that are appropriate to their maturity so that they can see it as relevant and relate to it. If learners are not assisted in overcoming their difficulties then educators will be the contributing factor in learners been poor problem solvers. The above response can also lead to speculation as the pre-service teachers were not required to explain their responses.

Another response to 'NO' was '*they might find it challenging and they might prefer straight question-answer rather than question-think-search-evaluate-answer*'. The reference to *'straight question-answer'* within the response is a possible reference to learners solving simple routine problems in the classroom. These kinds of problem exercises have only one answer. Learners are quite often exposed by the teachers to these question-answer types of questions which require the translation of the operational sign and the numbers within the problem to arrive at the answer. This is not an effective way to learn as learners learn very little other than the sequence of steps (Killen, 2015:261). According to Killen (2015:259) these kinds of exercises are suitable to reinforce understanding but they do not fit the description of problem solving. Using these types of problems can influence future teaching and learning. Learners may constantly look for 'quick fix' method to solve problems. When exposed to problems where the learners are expected to *'question-think-search-evaluate-answer'*, possible reference to non-routine type of problems, learners are not likely to attempt the problem to find a solution due to the level of difficulty. Innovation is needed on the path of the teacher to ensure that learners move away from the mindset of solving basic routine problems to a higher level of thinking to show their understanding of mathematics.

There can be more plausible reasons for the following responses:

Some students struggle with the extraction of data;

It's complex and complicated and it takes time to figure out;

They sometimes find it difficult to sift through the information;

Some find it hard to filter irrelevant data.

Many learners have difficulty in the extraction of data. In the modern school environment in South Africa learners are faced with a language barrier as many are second language learners. They have trouble in reading and communicating their responses due to '*difficulty with languages as many find it hard to understand*'. Reading and poor comprehension skills on the path of learners are a constant problem confronted by teachers in a mathematics classroom. Those learners who have a language problem will lack the necessary confidence in solving problems as they will be confronted by words and concepts which will hamper their comprehension and have a direct impact on their problem ability. The use of visual skills and mathematical representation plays a significant role in communicating mathematics and extracting information (Diezmann, 1995; Bamford, 2003). The represented visuals make understanding the concepts easier as they represent the key ideas in the problem.

The following responses '*some students are fixated on only a few problem solving methods*' and '*some pupils have a fixed mental set which makes their approach inflexible and more difficult to solve problem*' has relevance. Having a fixed mentality that problems can be only solved in certain ways may result in learners been reluctant to give up their initial ways of solving problems. Furthermore they want the teacher to provide the steps that will eventually lead to the answer as '*they are lazy to think of the problem solving process as they want something to do with little effort*' when working with problem solving. I will discuss this aspect further in what I observed in learner's books and in the analysis of the evaluation worksheets in this chapter. With this restricted belief of the different problem solving strategies learners lose confidence and this dents their confidence.

How do you make problem lessons enjoyable?

Use a variety of different mediums to pose problems;

Using tangible items to explain it – find a solution;

Use visuals were possible;

Introduce fun activities and things that are visual so that they are able to see clearly and think creatively and critically.

The above responses indicated that pre-service teachers are aware that visual mediums, visual representations and the use of concrete modelling can make the lesson enjoyable. Pre-service teachers and learners are exposed to using modern gadgets. The pre-service teachers can build on using technology and visual resources advantageously to enhance learner's interest. Using brightly coloured visuals attracts the learner's attention and they must also be encouraged to use similar strategies in their work books. Teachers can '*show pictures as well*' and use concrete models to engage learners. The pictures allow learners '*to see clearly*' thus a teacher may attach a problem to the picture. The picture can be used as a point of stimulation to engage the learners with the problem. By doing so the learners would be able to relate the problem to the picture thus getting a better grasp of the concepts and strategies involved. The use of concrete models and other manipulatives allow the learners to focus their attention on the problem. In this way learners traverse from been passive learners to active participants.

The following responses '*relate it to them; make/use relevant links to life*'*;* '*using scenarios that learners can relate to*'*;* '*make or use relevant links to everyday life*'*;* '*provide a variety of examples*'*;* '*by using examples that learners can relate to as well as making the lesson a visual one*'*;* '*using every day or relatable topics to explain it*' indicated that the pre-service teachers are aware that by relating the problem to learner's real life it will create a better understanding. By relating the problem to the learners reality brings about an association with the learners prior experience. Kolb's experiential theory holds relevance here because it describes how learning occurs. Kolb's theory sees learning as the process where knowledge is created through experience. When the association is made between the problem and the learners past experience learning occurs and knowledge is created. Piaget and Vygotsky's theories also emphasized that learners construct new knowledge from what they already know and their understanding is based on their prior experience. Linking information from within the problem to the learners past experience builds foundational knowledge and it makes possible for the learner to deal with situations of a similar nature in the real world.

The following responses indicate that learning does not occur in isolation:

I ask learners to show the rest of the class how they had solved a problem;

Group work – where learners work together to reach a common goal. They enjoy it as it can be challenging;

Let learners be part of the lesson.

Group activities lend itself to collaboration amongst learners. Collaborative learning lends itself to problems solving thus activities in the form group work must be encouraged. The use of group activities makes learners active participants as they are given opportunities to express their ideas. It provides a basis for encouragement and future motivation in classroom participation. Learners are active knowledge seekers thus a teacher must allow the learners to become active participants in the lesson. They like to show off their work therefore teachers should encourage open discussion in the classroom so that '*all learners feel engaged in class when doing activities*'. Therefore one needs to have the constructivist, social constructivist and mathematical understanding theories in mind when working with learners within a classroom. Learners construct their own knowledge. Through social interaction with other learners the construction of ideas becomes cemented when co-mingled with other ideas. The cementing of ideas indicates the learners understanding of concepts. For this to occur the data and concepts presented within the problem must be within the level of understanding of the learners. This helps to make the problem as realistic as possible to the learners so that they can draw on their prior knowledge to create new knowledge. When attempting a solution to the problem they will be able to successfully apply their gained cognitive knowledge.

This part of the questionnaire sought information on the pre-service knowledge on visualization and its role it played in the teaching and learning.

What is your understanding of visualization?

The ability to see something in your head without actually seeing it or having seen it once and trying to remember it;

Ability to see something without it being in front of you;

Ability to use mental processes to see the visual;

Imagining the invisible data into something visible. An image must be produced. It needs to be readable and recognizable. We need to identify the main components then make sense of it;

Thinking in your mind;

Picturing ideas and concepts in your mind;

The process of being able to form a picture in your mind;

See;

Representation of data or information in the form of an image;

Think about things visually;

The use of pictures, charts, videos and worksheets to aid in the understanding by students;

Visualization consists of us visualizing aspects in maths that better our understanding. .

What is your understanding of visual literacy?

Being able to interpret, grasp knowledge and understanding of information presented to a learner in the form of a visual;.

To understand a topic;

Learning by seeing through practical visual examples/methods;

Visual literacy refers to the ability to read, write and create visual images. It's also about language, communication and interaction;

Using an image and analysing, understanding and interpreting the data given by it. One tries to make meaning from the image;

The ability to interpret what is placed before you and to create a link with existing knowledge or to accept it as new knowledge;

Visual literacy is the ability to interpret, negotiate and make meaning of information presented in the form of an image;

Interpretation of visuals.

The pre-service teacher's responses indicated that they understand the concepts visualization and visual literacy. To summarise their understanding of both these concepts: learners describe what they see in their minds; they use words and different mediums to describe their internal thoughts; they make associations by using illustrations to represent words in the problems; analysis is made through interpretations and mental connections are made; by making the link between the mental constructs and the visuals, the learners consolidate their understanding of the problem. The theoretical frameworks in this study seem to be able to explain that experience and understanding is important to consolidate visualization in mathematics problem solving.

When asked, do you use any visualization techniques in your lesson only fifty-six percent of the pre-service teachers indicated that they did. Taking into consideration the pre-service teachers responses to understanding the concept visualization the responses below indicated that they recognize the value of visualization in mathematics.

The following reasons were provided for the YES response:

Power point to explain the example;

Powtoon to explain concepts;

I use objects and colours at the beginning;

Narrate problems and ask learners to picture it;

Posters, aid, maps;

Slide shows, images, cylinders and cubes;

Modelling objects to represent a problem;

Visual thinking;

Thinking about situations and what answers could come out;

Get the students to visualize the problem and try and see the outcome;

Story telling;

Look at pictures or listen to sounds then think;

Map analysis, photo analysis, cartoon analysis, poster analysis.

The response for '*it is time consuming*' probably indicated that the pre-service teacher has no designs of using visualization in mathematics lesson especially in teaching problem solving.

Only fifty-six percent indicated that their learners understood mathematics better with visual stimuli when asked if learners understand the lesson better with visual stimuli or without stimuli. Eleven percent indicated that the learners learnt better without stimuli whilst thirty-three percent did provide a response. The evidence obtained from this question is overwhelming as it supports the basis that visualization is a foundation on which mathematical understanding (discussed in chapter 4) can be built.

It is often said that one retains knowledge through sight and sound. The following responses indicated that the learners have a better understanding when visual stimuli are used. In this manner they can relate or make connections to aspects in their environment or their lives.

Children prefer visual stimuli as they are more attracted to pictures with colours and movement;

Children learn better when they can 'see' a problem. It makes it easier for them to think of solutions;

Learners when they are able to see something physically are much more capable of grasping a topic quicker and easier;

They can see things which make them understand better;

Images and visuals stimulate their interests and this keeps them involved in the lesson;

They prefer to see things.

The following response, 'c*hildren find the use of modelling objects interesting, exciting and fun*' indicated that modelling a situation or object can lead to a fun filled activity. Mathematics is not about memorization of formulas. The learners must be given an opportunity to model situations, namely, role playing, so that what they see can be remembered.

The following responses indicated that the pre-service teachers associated using visual stimuli to cater for the different learning styles in the classrooms. Not all learners are receptive to the talk and chalk method of teaching. Therefore an alternate manner to support the learners is to use a wide variety of visual aids to cater for their varying learning styles, namely, auditory and visual.

Meets all types of learning needs;

There are different learners – many prefer to see because they learn better;

Children are visual learners and need visual stimulation to learn;

Visual learners will learn better;

Visual stimuli provides a basis for learners who are unable to link ideas mentally;

Visual helps the learners relate to better with the work set before them;

They enjoy looking at colour.

The following responses that visual stimuli '*helps learners create links*' as '*visuals helps the learners relate better with the work set before them*' thus '*there is something that they can see and work with without forgetting aspects*' indicate that learners can create a better understanding when they are able to link and relate with what they see with their prior knowledge. This helps to improve their mathematical understanding and retention levels in their long term memory.

Only fifty-six percent indicated that visual skills can be used in teaching of problem solving when asked: do you think mathematics problem solving can be taught using visual skills. Forty-forty percent did not provide a response. The large percentage of respondents who agreed indicated that visual skills are a necessity when teaching problem solving.

The pre-service teachers were asked: what are some of the visual skills that an educator can use when teaching problem solving? The following response '*linking the problem with the pictures efficiently*' indicated the pre-service understood the value of linking the picture with the problem. The picture can be a starting point to engage the learners in answering the question. '*By drawing images and pictures on the board*' the learners develop their thoughts '*creatively*' by developing further '*illustrations*' of flow charts and diagrams that will aid in finding the solution to the problem.

Time is priceless when teaching a subject like mathematics. Time constraints is massive in mathematics as the mathematics curriculum is quite comprehensive thus leaving little or no room for the teacher to deviate from their normal teaching. Using technology in the modern day classroom has become the norm for the modern teacher. The pre-service teachers can now be placed in the category of the modern teacher as they are quite advanced in the field of technology. They are able to '*use more visual approaches such as the use of videos, pictures, charts, drawing, diagrams, photographs*' to enhance their lessons. This kind of teaching will allow the pre-service teacher to bring real life examples into the classroom thus '*model the problem solving process*' through visualization. These responses indicate that the pre-service teachers believe that the use of visual skills will improve teaching problem solving.

When asked if the learners provide visual solutions in their classwork books only thirty-six percent responded that learners used visual solutions in their books. Forty percent did not provide a response. Comparatively, the obtained data above are not in agreement. Although pre-service teachers have indicated that visual stimuli aided the learners in mathematics and that problem solving can be taught using visual skill, the twenty four percent for NO indicated no real attempt had been made to encourage learners to use the visual mediums and skills that are taught in the mathematics lesson. Whist claims are made that visualization and visual skills play an integral role in mathematics, the statistical evidence obtained above is evidence enough how the use of visual skills is neglected in the mathematics classroom. The pre-service teachers are at the start of the learning curve and they must be motivated enough to utilise visual skills at all levels in their teaching of mathematics.

The pre-service teachers were asked to respond to: what are some of the barriers to problem solving?' The pre-service teacher's acknowledged that learners have barriers to learning. They need to take cognisance of these barriers to ensure that they assist their learners overcome these barriers at an early stage to avoid them getting stagnated in their learning.

Learners with language barriers are well documented in South Africa. The following pre-service teacher's responses, '*language barriers*'; '*reading*'; '*learners have problems with their English and their reading is one of their greatest barriers*' and *'linguistic understanding and comprehension*' amongst others confirm this. It is important to note that all knowledge is created through the medium of a language. Reading and problems with the English language is prevalent in South African schools. It must be noted that South Africa has nine official languages and majority of the learners attending school do not have English as a home language. Such is the position in South Africa that the migration of the learners within provinces further compounds the language problem. The learners are taught in their mother tongue in schools during their formative years. When they enter the intermediate phase they are confronted with a first additional language and sometimes a second additional language. The confrontation with the new language of instruction makes it almost impossible to understand. It must also be remembered that the mathematics language is a language on its own and has its own vocabulary. Not been able to follow direct instructions or follow conceptual aspects within the mathematics lesson will become a factor to learners performing poorly in the subject. If learners lack the basic understanding of concepts and the mathematical vocabulary, then acquiring and comprehending mathematical information will nigh become impossible.

Mathematics must not be downgraded as a subject using language as a determining factor but rather English should become the language of instruction for all learners from their inception year in South African schools. If all the learners in the South African schools had access to a common language from their inception year then that will allow them to express their understanding and improve their learning. Thus communicating mathematically will not become a regular barrier to learning. Pre-service teachers as future mathematics teachers, must create a social interactive classroom environment so that learners are encouraged to talk and learners become part of collaborative learning. By talking they will be able to verbally express themselves thus ensuring that any mathematical misconceptions are be rectified instantaneously.

The scope of work or content knowledge in the mathematics curriculum taught for each grade ought to be re-considered. The Department of Education, in order to shorten the grade twelve mathematics syllabus, has reworked the mathematics curriculum in such a manner that there has been a rippling effect of sections from grade twelve to the grades below. Learners in the Foundation Phase are battling to cope with the content knowledge as it is not age related taking into account the learner's entry into school in South Africa. This impedes the learning process which contributes to the high failure rate in schools. These '*learners need extra support*' and teachers are not professionally trained or in a position to provide the support as they themselves have to meet targets as set by their schools and the Department of Education.

The responses, '*previous knowledge not fully grasped*'; '*some learners might not grasp the concepts*' is something that teachers must not ignore. It is important that teachers do not gloss over the mathematical vocabulary but should rather engage learners so they understand the concepts fully before progressing to other aspects in the curriculum. Teaching and learning styles in all classrooms need to be considered as each classroom is unique. There are learners of various abilities and in order to improve learner competence, the teacher must have all learners within their focus so that no child is left floundering behind otherwise teachers will find that the learners '*cannot apply concepts to be able to solve problems*'. It is important that the learners have a full grasp of concepts so that they can apply this knowledge to other problems.

Taking into consideration the following responses, '*can't apply methods to a problem* and '*not knowing how to apply data to arrive at the answer*' it is important that learners are trained to solve problems using the basic skills and strategies as mentioned in the mathematics curriculum. The learners need to be encouraged and advised to use their visual skills and make connections to the data in any given problem.

The teachers need to encourage learners to express themselves without consciously making them aware of their errors. In this manner learners will attempt to apply whatever means they have at their disposable to work towards a solution thus overcoming their 'lack of confidence' and '*anxious attitude towards problem solving*'.

The responses, '*teacher cannot explain properly, learners unable to fully understand the problem to solve it*' and '*not understanding the problem leads to confusion and failure especially if the teacher moves on without explaining thoroughly*' indicates it's either a communication problem or the lack of pedagogical knowledge on the part of the pre-service teacher. It is imperative that pre-service teachers, since they are in learning realm, should be able to communicate concepts clearly and adequately. In order to do that they should bolster their necessary pedagogical and mathematical content knowledge to ensure that the learners '*grasp the concepts'* and overcome the barrier of '*not making connections to the problems they are trying to solve*'. Literature on problem solving within the theoretical framework (chapter 4) stated that learner's ability to problem solve is reliant on teachers specific knowledge. The lack of teacher knowledge has a direct bearing on learner's mathematical understanding. In the event of teachers explaining concepts incorrectly will result in learners having misconceptions. As teachers we must not make assumptions that learners understand everything that is said to them or that they understand everything that they read. It is important that the teacher explain concepts in a manner that does not cause confusion in the child's mind. Instructions and explanations need to be explicit using whatever resources available to concretise data thus making it relevant to the learner. The learner, whether orally or written, will be able to apply this type of guidance to solve problems.

It must be noted that the responses listed within the analysis of the questionnaire is not exhaustive. There are wider issues outside the mathematics classroom that have a detrimental impact on learner's performance within the classroom. Mathematics has always been viewed as difficult and learners possibly out of sheer apathy do not apply themselves fully to the subject. Issues such as '*learners have a negative attitude*'; '*children are often fixated on particular methods*'*;* '*lack of confidence*'*;* '*learners forget what they looking for*'; *'don't know their time tables' and* '*cannot do basic addition, subtraction, multiplication and division*' further compounds the problem. These issues can be negated by the pre-service teachers if they themselves know how to overcome them early in their teaching career. Concretisation of bonds and tables in the formative years will assist the learners in applying these skills in becoming adept in calculating algorithms. Learners need not be pushed into answering problem solving questions to suit the needs of the teacher. There is a huge shift towards a more learner centred teaching and learning environment than that of a teacher centred dominant environment. Kolb's Experiential Theory supports this kind of learning. By giving the learners the opportunity to experiment and create their knowledge will boost their self-confidence leading to them overcoming their anxiety to learn mathematics.

The mathematics curriculum has undergone a drastic change and these changes have had a negative impact on learner's performance in the classroom. Learners make the assumptions that mathematics is difficult. Their confidence needs to be built therefore one of the underlying principles of mathematics teaching should be to remove the negativity to learning mathematics. If one has to consider Piaget's developmental stage (age factor), our learners are not age ready to tackle the aspects within the curriculum due to these changes. As teachers we need to get learners to enjoy mathematics by allowing them to express themselves mathematically in whatever way they feel confident. Teachers must value learner's problem efforts (Killen, 2015:271). By allowing learners to use their acquired skills and imagination can assist in building learners confidence to engage more mathematically. It is also important that teachers use intrinsic motivation to get the best out of their learners as a means to remove anxiety and other possible constraints. To remove the anxiousness teachers must have a sound open dialogue relationship with their learners. This allows learners the freedom to express themselves and take ownership of their learning. Open dialogue and constant encouragement can remove the anxiety that creeps into the learners mind about mathematics as a difficult subject.

Teachers function under constraints but these constraints should not be placed on learners. Using prescriptiveness to force learners to follow the teacher's method as the only set method that works in the classroom must be reviewed. Understandably the scope of work needs to be covered to meet the demands of the Department of Education but the teachers should not teach the curriculum in order to prepare learners just to obtain a pass at the end of the year but rather educate the learners to apply their learning to the world they encounter.

Fifty three percent were in agreement that visualization can overcome barriers to problem solving when asked if they think visualization can overcome these barriers. Eleven percent stated NO and thirty six percent of the pre-service teachers provided no responses.

The following responses, amongst others, indicated that visualization is of importance to the teaching and learning of mathematics.

If visual representations are linked to the problem, example, if apples are measured in the problem; a visual of apples can be used to add, subtract, multiply or divide by adding even more or by removing the visual.

If learners have difficulty to read then looking at the pictures would help them understand;

They won't need a specific language if they understand what they see. They can figure out what they see;

They can learn better when they have something that represents the problem;

If they have visualization they would be able to see and understand through seeing and experiencing.

The visual technique of 'seeing is believing' is important when dealing with problem solving especially in the formative years of schooling. It is here that the foundation is laid to future learning. Associations are made using concrete items linked to concepts in the problem. Learners make meaning by means of associations using previous experiences. This can lead to a better understanding. They can see and initiate the use of pictures to schematically represent the concepts and solve the problem. The use of visual representations to understand the problem is significant as mathematical language is translated to visual representations indicating learners understanding of what they comprehended.

The responses below indicate that the pre-service teachers believe that using visualization can be the beginning to an end to solving a problem.

Visualization enables them to step out the box and see the problem from different angles to be able to solve the problem creatively and cleverly.

Learners may visualize the problem and create a starting point to work with. They will feel more confident and reassured if they understand and can 'see' the problem. It will encourage creative thinking rather than fixated solutions.

To create an understanding of the problem a learner can visualize and create representations. These representations enable the learners to overcome the language barrier and examine the problem in illustrative form. These representations can be revised allowing the learner to seek more than one solution for the problem. Learners always visualize the problem and create a starting point to work with. They will feel more confident and reassured if they understand and can see the problem. It will encourage creative thinking rather than fixated solutions.

The following response, '*to an extent – it has to be taught to learners so that learners can be taught this skill and will help learners solve the issues*' indicate that learners should be taught this skill to assist in problem solving. This is in keeping with the literature in chapter 2 that learners be taught problem skills and assisted with problem solving. One can argue in support of this statement. When learners develop sound skills in problem solving then it becomes practically possible for them to apply it to other similar situations when confronted. Learners need to be exposed to problem solving skills and strategies from an early age. They should not be reliant on following mathematical procedures but rather be encouraged to be creative in their mechanics when attempting to solve the problem.

The following comments, '*visualization won't always make them understand*' and '*problem solving is hard and need to be constantly practiced and learners to have constant support*' might seem inconsequential but it indicates that the pre-service teachers doubts the power of using visualization in problem solving. Learners will not pick up everything from a problem when confronted for the first time. They need to be guided through the problem solving process and asked to relate to the concepts or the data from their environment or prior knowledge. The teacher, as the gatekeeper to the acquisition of knowledge, must ensure that the visual mediums are used constantly and learners must be encouraged to use their visualization skills when attempting problems. When learners are confronted with visual mediums they should be related to the problem. The teacher should draw their attention to important information in the visuals used and ask them to apply it to the problem. With proper guidance and continuous practice the learners will eventually learn to convert text into visual representations. In this manner they become trained to use problem solving strategies and visualization to understand better.

5.2.3 CONCLUSION

The importance of doing, learning and teaching mathematics problem solving has long being emphasised of as been crucial to help learners gain a deeper mathematical understanding (NCTM, 2000). The questionnaire was used to collect the data. The primary goal of this data collection was to determine pre-service teacher's knowledge on problem solving strategies and their use of visualization to support the teaching of problem solving. The questionnaire contains the thoughts and descriptions of the pre-service teacher's knowledge on problem solving strategies and visualization. There were commonalities as well difference in their responses to the questions on the questionnaire. The analysis of the questionnaire showed that the pre-service teacher's knowledge is limited in respect of problem solving strategies. This is shown in their responses, or the lack of it, in their knowledge of mathematical teaching concepts and mathematics terminology used in teaching and learning. If they are to develop learners confidence and as prolific problem solvers then they must be able to gain proficiency in problem solving strategies. Whilst acknowledging the importance of problem solving strategies and visualization, it is not known whether the pre-service teacher's non responses and short responses represented their lack of pedagogical or content knowledge or they were hesitant to fully respond to the questions. The responses should encourage the pre-service teachers to take note of them and ensure they attempt to build on their teacher knowledge as this will play an active role at all levels of their future teaching in choosing visualization as a teaching method to teach learners the skills in solving problems.

Visualization which combines skill interpretation of the problem and diagrams has a potential to increase the learning potential of learners but the pre-service teachers lack the pedagogical knowledge in teaching learners how to use visualization in teaching of mathematical problem solving. Reading the problem is a challenge to learners. Visualization can be used as an effective tool to ensure all learners have the ability to access and comprehend the problem. It allows learners to familiarize themselves with the problem and to be creative to represent a strategy which contributes to an improvement of comprehension amongst learners.

Problem solving strategies and visualization need to be an area of focus for the pre-service teachers.

5.3 CLASSROOM LESSON OBSERVATIONS

5.3.1 Introduction

Basturk (2016) stated that one of the best ways to appraise the (pre-service) teachers is to observe them during teaching. All of the participants in this study were observed presenting their lessons during lectures and some during their teaching practice.

In their lectures the pre-service teachers were required to prepare and deliver a lesson. These lessons were classified as teach back lessons. For the purpose of this study, I have considered the observation of one of the groups, namely, Mars.

During their school visits the pre-service teachers were all expected to teach a prescribed number of lessons under the supervision of their mentor teacher. For the purpose of their evaluation they had to teach a lesson as per the scope of work of the chosen grade.

The observation process in this research process had a dual purpose. I observed the pre-service teachers during these teaching sessions to collect data for this study. While observing the pre-service teachers I used two sets of evaluation documents, one provided by the institution to evaluate the lessons for their academic purposes and my own independent document for this research (Annexure 3). The evaluation instrument provided by the institution had clearly set out teaching and learning criteria together with a rubric that the pre-service teachers had to achieve. During the lesson observations, whilst using both the evaluation instruments, I also made field notes.

The pre-service teachers, who were observed within this study, all voluntarily chose to prepare and teach a mathematics lesson. During the observation their teaching was evaluated. My main focus, for the purpose of this study, was on the use of both the visualization and problem solving aspects used during the lesson. I focussed on the skills and strategies used by the pre-service teachers during their teaching and those of the learners when solving problems. I also focussed on the pre-service teacher's mathematical and pedagogical knowledge in the classroom.

The mathematics lesson was built around a three tier lesson plan, namely, an introductory activity, the teaching of the lesson content itself and the conclusion. The introductory phase required the pre-service teachers to determine learner's prior knowledge so to familiarise them on what was going to be taught during the lesson. The content phase required them to clearly set out the aspects to be taught in line with the lesson objectives and during the closure phase or conclusion the pre-service teachers needed to indicate how they will summarise and end the lesson. The pre-service teachers had to also state the objectives of the lesson, the teaching strategies to be used, resources to be used during the lesson and also produce a learner activity.

Pseudonyms have been used to protect the identity of the schools and that of the pre-service teachers.

5.3.2 Analysis of the lesson observations

The pre-service teachers were randomly placed in groups of six. As a group they had to prepare a lesson and present the lesson to their colleagues. These lessons were classified as teach back lessons or co-teaching. I also saw this as didactic teaching and learning. In the absence of microteaching in their training I used this opportunity to expose the pre-service teachers to the intricacies of teaching mathematics and to learn mathematics. During their co-teaching they were exposed to mathematical knowledge for teaching (MKT) and pedagogical content knowledge (PCK) used by their colleagues. This was to gauge their content knowledge and how they developed their teaching using visualization and problem solving strategies. This is supported by Boonen et al (2016) who argue that besides professional teacher training the pre-service teachers need to be exposed to pedagogy and methodology.

The pre-service teachers were randomly placed in groups. In their groups they had to choose a topic from the mathematics curriculum. Although all groups presented their lessons I chose the first group for the purpose of this study.

Mars

This group, Mars, presented a lesson on Data Handling. Two pre-service teachers working in tandem introduced the lessons to their colleagues. Using a Powtoon presentation combined with a video clip they presented a clip on transportation in South Africa. As the video clip was presented the pre-service teachers had to indicate the concepts alongside key aspects on the worksheet to give meaning. What was very interesting during this introductory phase was the manner in which they discussed the concepts. They pointed out to their colleagues that certain concepts repeated themselves and using Powtoon the pair reinforced the concepts in a fun filled manner (figure 25). The use of the video clip and Powtoon brought the concepts alive creating a better understanding.

Figure 25 Pre-service teacher presentation of a teach back lesson

The next pair was responsible for the teaching phase. As the concepts such as tally, bar graph, mean mode and median were already discussed in the introductory phase, they presented a case study relating to travel in South Africa. The pre-service teachers were placed into groups of six. Using the learner centred approach they had to gather data from amongst themselves. According to experiential learning, learners need to experience learning as learning takes place through action (Beaudien and Quick, 1995) and I expected the same from the pre-service teachers. The pre-service teachers were required to organise the collected data in a table, then into a tally table and then a graph. Once this was completed questions were presented to them. The questions were both open ended and closed. They were quite ingenious in producing their answers or solutions to these questions. The use of problem solving strategies as using a diagram and working backwards was evident. Tables, sketches and diagrams were used to represent their understanding of the questions. According to Bal (2014:2) representation is a formation of a mathematical concept and it is a way to show an actual situation from a different view. This group had to choose a group leader to present their solutions. The group leader, by using a chart, presented the solutions in a very logical manner. The pair responsible for the teaching phase posed additional questions to the group leader on how certain solutions were arrived at. The pre-service teachers that were observing the lesson was asked to comment or request further explanations on how solutions were reached. The group leader, obtaining assistance from her group, used the white board to explain the manner on how they arrived at the solutions. I deduced from this presentation that the representations used to find the solution followed the structure of the question. The pre-service teachers translated the problem algebraically in a sequential order. Furthermore the idea of collaborative engagement during the discussion opened up avenues for innovative learning among the pre-service teachers. They were in a position to support each other learn mathematics.

The pre-service teachers displayed adequate MKT and PCK during their teach back lessons. This indicated that they had the confidence to teach mathematics.

Mala

The school where Mala was observed is situated in a suburb on the North Coast. It has large population of predominantly Indian learners from within the suburb and a small population of Black learners who travel from the nearby townships. This school receives a proportionate funding based on its roll but majority of the funds are received via school fees levied on the learners. It is a well-resourced school, has a team of well experienced teachers and often produces excellent results.

Mala was observed teaching a grade 4 class. The mathematics lesson began orally with a quick mix of bonds and tables. As prescribed in the CAPS document and the Jika Imfundo tracker 'mental maths' must be done daily as it is forms an important pre-requisite in all mathematics lesson. This provides a vital foundation in mathematics as the learners are provided with opportunities to use their mental counting strategies to develop their proficiency with numbers.

The lesson was based on the measurement of time. Mala introduced a problem related to the times of television programming schedules. She fostered the idea on learners that they must try and relate all problems to themselves and their environment. She discussed a problem solving strategy on the chalkboard as a means to guide her learners to solve the problem. Using Polya's steps of problem solving (discussed in the literature review) she initiated the discussion. She read the problem to her learners and asked them to respond to what the problem required. In this manner she provided a start to the problem. She used appropriate questions (Do you think you are on the correct path?) and prompts (try an alternate step) to initiate responses from her learners. She led the learners through the steps in a logical manner using simple rules working from the known to the unknown. The Structural Learning Theory (discussed in chapter 3) supports this method as learners need to be taught the simple steps and rules in order to proceed to more complex steps.

Mala mentioned a mathematical concept from within the problem and asked her learners to draw what they thought it meant or represented. In this manner she was determining if her learners displayed mathematical understanding (discussed in chapter 3) of the concepts. This is an important part of problem solving as learners construct their own meaning of the concept.

The learners must be given opportunities in the mathematics classroom to represent their understanding of mathematical concepts in a variety of ways. Mala had an understanding of Polya's problem model and briefly mentioned to her learners what would have occurred in steps one and step two of the problem model. It must be noted that she never mentioned the model by name or diagrammatically showed the learners the steps involved. The learners responded positively to the teacher. They were eager to show off to the other learners in the classroom what they had drawn. She randomly chose volunteers and asked them to explain their diagrams to the class. The discussion although noisy was encouraging. The learners used their basic drawings to explain and justify their steps of the problem. When she was satisfied that her learners were coping with the basic parts of the problem she encouraged them to work towards a solution. At first the learners worked individually. On completing their work the Mala asked her learners to discuss their efforts with their peers. When learners assist each other, they will feel more successful, empowered, and confident about their learning. I found the discussion to determine if their friend's solution was any different to theirs was very constructive. During this activity Mala walked around the classroom and the learners openly engaged her to give her decision if their answer was the correct one. She tactfully cajoled them to find out for themselves. Once the entire class was done with this activity the she asked for volunteers to put up their solutions on the chalkboard. These solutions were discussed with the learners. They were given an opportunity to determine which of the solutions on the chalkboard were correct. Mala consolidated the learner's efforts by discussing the concepts within the problem and also showed them how each step was related to each other.

For the written activity of the lesson the learners were given a problem solving worksheet. They were instructed to use any method that they were comfortable with as long as they showed their working on the worksheet. Although encouraged to work individually learners were observed talking to their peers discussing certain problems on the worksheet. Instead of curtailing this, the pre-service teacher allowed this to continue. I thought that this was a useful change this as this is a constructive approach to collaborative learning. This collaboration allowed the learners to progress with the problems especially in the stages they felt they were facing a challenge.

On the completion of the written activity Mala asked for a volunteer to present the first solution to the class. Thereafter she randomly asked the learners for any alternative answers. Intermittently during the classroom discussion she engaged the learners by asking questions or making general statements, example, '*Do you think your answer is right?*' or '*I think something is missing*' and also putting an incorrect solution on the board. I found this to be an interesting ploy on her part as this created doubt and some of the learners were forced to go back and check their solutions. Some of the learners were confidently stating that their solutions were correct and the other learners were very convinced the second time around that their answers were correct.

Some of solutions were presented numerically, others schematically or diagrammatically and a few had a mixture of numerical and diagrammatic solutions. The solutions that were represented arithmetically showed that the learners merely used the numbers that were given in the problem to calculate the solution. They did not comprehend the association between the numbers in the problem and the concepts within the problem. The other arithmetical solutions were long and the learners themselves were confused while trying to articulate the answers to their peers. The learners who represented their solutions schematically, diagrammatically or involving the mixture of arithmetical and diagrams, showed that they understood problem. The representations showed their visualization skills as they illustrated their understanding of the concepts from the problem. What is important to note here is that not all learners constructed the same visual images due to their own prior knowledge and experience. I observed that these representations indicated the learners had heeded their teacher's advice to use their prior knowledge and experience when solving the problems. It was indicative that it was their mathematical understanding drawn from using their own previous knowledge now been presented as their ideas in the solutions. This is relevant to Kolb's Experiential Learning Theory and Kolb's Experiential Learning Cycle. It is expected that the learners utilise their previous experience, process the concepts and apply it to the problem. By using this learning cycle learners were able to apply their knowledge and understanding to significantly provide their solutions.

Mala possessed the relevant knowledge of content and teaching. Her readiness for the lesson was evident as she had come prepared with her charts as a teaching resource. The creative use of the charts enhanced the lesson as the majority of the learners were able to engage in the lesson. The use of concrete representation on the chart allowed the learners to make direct connections to concepts and relate to them. This was very evident when she asked her learners to explain where they had come across what was shown on the chart. A notable feature in this lesson was the confidence in which Mala used her pedagogical content knowledge, common content knowledge and specialized content knowledge to explain the mathematical content and explain the concepts to the learners. The pre-service teachers need to acquire the necessary types of knowledge (discussed in Chapter 3) before they enter the classrooms. According to Killen (2015:30) teachers need knowledge of their subject and must understand the concepts to engage the learners. In this way it will allow the pre-service teachers to feel secure about their knowledge, understanding and skills and their capability to assist learners learn (Killen, 2015:33). They will be able to teach in a logical manner and make the subject understandable to the learners (Killen, 2015:30).

Tina

Tina's school is located on the periphery of a suburb, surrounded by a low cost development area and informal settlement. The school population is predominantly Black with majority of the learners from within the area and some travel from the nearby township. This is a quintile two school making it a no fees paying school. The school receives all funding from the Department of Basic Education and is well resourced and has the basic amenities. The school has an experienced mixed teaching staff.

I observed a mathematics lesson based on 2D shapes and their respective properties. Tina went straight into the lesson without delving into the learner's prior knowledge. According to Structural Learning Theory, a theory used within this study and discussed in Chapter 3, prior knowledge is essential. Determining the learner's prior knowledge lends itself to developing the learners existing knowledge and building understanding. According to Ikegulu (1996) making a well-structured knowledge association allows for easier retrieval of prior knowledge and the facilitation of new knowledge.

In the observed lesson Tina presented certain concepts on the chalkboard as the lesson progressed. She explained these concepts verbally. I saw this as a mere superficial explanation of the concepts. According to Killen (2015:50) teachers need to build their lessons around the primary mathematical concepts in the classroom. The learners were placed at a disadvantage during this lesson as some of the concepts mentioned were not within the grasp of the learners. The CAPS document sets out specific content and concepts that needs to be taught in each strand in each grade. This is to ensure conceptual progression through the grades in the various phases. Tina had chosen concepts that were not within this grade. If she had engaged with the CAPS document in her planning, where the order and progression of topics are carefully stated, she would have discovered this important aspect. Furthermore her lesson plan indicated scant content material. To be truly effective as a teacher and to ensure active learner engagement the lessons must be thoughtfully pre-planned and then presented. All the pre-service teachers are supposed to engage with their mentor teachers when planning their lessons. If Tina had done so then she would have prevented any short comings in her planning and presentation of the lesson.

Furthermore she missed an ideal opportunity to utilise concrete models or visual means to show the learners the connection between the mentioned concepts and their properties. By using the concrete models or allowing learners to visualize and represent their thoughts would have allowed the learners to perceive the relationship between the representations and the mathematical concepts. This would have developed their conceptual understanding and this knowledge would have boosted their mathematical thinking and understanding.

According to Kolb's Learning Theory it is important that that the learners have proficiency and understanding of the subject matter in order to progress further. Reference to abstract conceptualization is made in Kolb's Learning theory. This is the thinking phase and understanding phase. During this phase assimilation occurs. The learners make use of existing ideas to new understand new ideas. The learners learn to make the connections with what mathematical aspect is been in taught with what they know. This overlaps with the structural learning theory. According to the structural learning what is inside the learners head (prior knowledge) is important. This in turn is linked to the mathematical understanding as learners use this prior knowledge to make connections to understand what is been taught.

Very early in the observation I determined that Tina was using the teacher centred approach as she was taking centre stage in this entire learning process. Very little opportunities were been afforded to the learners to become active participants. Mathematics teachers need to realise that if learning is to take place then the lessons are not to be teacher oriented. They need to move away from being a dispenser of knowledge to becoming a facilitator of knowledge as mathematics is less about the teacher and more of what the learners are doing. When this type of methodology is applied in the classroom then the learners will become the producer or constructors of their own knowledge.

During the lesson Tina merely stated the properties of the 2D shapes without making any direct reference to the shapes in a visual form. The relevance of these properties was lost on the learners.

She should have directed the lesson enabling the learners to arrive at an understanding of what was been taught. According to the theory of Mathematical Understanding, learners cannot develop a level of understanding if they do not engage actively in the lesson. According to Killen (2015:66) learners should not be merely given information but the teacher should rather guide their learning to bring about a better understanding. In this lesson the use of visual or representation means would have satisfied the learning objectives of this lesson. According to the Curriculum and Policy Statement (2011) the use of representations in whatever form is an essential learning tool and it lends itself to the development of important mathematical skills.

In this lesson the learning activity entailed the learners copying a summary from the chalkboard. The teacher summarised the properties and the learners were asked to copy the summary from the chalkboard. No mathematical skills were taught or reinforced during this lesson. This is a dangerous manner to teach a difficult subject like mathematics. Failure to teach this lesson effectively was her lack of knowledge of teaching methodology. Teaching and learning suffers in the classroom if there is a mismatch between the learners' learning styles, teacher's teaching styles and teaching methodologies. With a more skilful teaching approach and using a variety of ideas together with effective teaching resources she would have created a better mathematical understanding environment for her learners. In this regard I do not think that Tina was aware of the mathematical ability and proficiency of her learners.

A homework task was given to the learners and they were asked to commence with it once they had completed copying the summary from the chalkboard. This exercise was given from the workbooks supplied by the Department of Basic Education. This book had colourfully illustrated 2 D shapes. Partial drawings of the 2D shapes were provided and the learners were expected to complete the drawing by applying their knowledge of what they taught the shape was. The illustrations provided in the workbook caught the attention of the learners and I could gauge they were enthusiastic to engage with this exercise. Unfortunately since it was a homework task I was not in a position to follow up on their performance and understanding of the given task.

The conclusion I arrived at the end of the lesson was that Tina lacked teacher pedagogical knowledge and specialized content knowledge. She did not know how to use appropriate teaching strategies to influence her lesson. Furthermore she lacked knowledge of content and teaching as did not know how to use visualization in a fruitful way to make the lesson more interesting for her learners indicating that she was not aware of the power of visualization.

Pat

Pat and Tina were observed at the same school.

Pat began the lesson with a game. She used this game to build her learner's conceptual knowledge and to test their understanding. The learners were placed randomly in groups of twos, threes and fours. She distributed to each group a set of twelve cards. On her instruction they had to share the cards equally between themselves. Each group was asked to indicate how many each one of them had. In this manner Pat reinforced the concept of half, thirds and quarters. She went on further to draw a set of seven fruits on the board and asked the learners a basic division problem of sharing it with two friends. The learners were asked to draw a diagram to show their answer. Many of the learners realised that one was left over and wanted to know from the teacher what to do with it. The teacher remarked *"share it"*. The learners were randomly selected to come to the chalkboard and show off their diagrammatic solutions to their peers. The realisation soon materialised that they will each receive a half each.

Pat used elicitation, essential in Kolb's experiential learning theory and the theory of understanding, in this introductory activity. According to Killen (2015:56) elicitation is when a teacher engages the learners in a learning activity that requires them to reveal their prior knowledge and understanding that he will use to facilitate further learning. The introductory activity, beginning the lesson with an experiential game, is a component of Kolb's learning styles. It is when learners experiment with their learning experience to gather knowledge.

Having consolidated learner's prior conceptual knowledge she proceeded to give the learners a worksheet. They were given a host of diagrams, with whole and fractional parts, and they had to determine equivalent fractions. Pat, whilst circulating her class, observed that the learners were making associations by looking at the diagrams and the number of pieces the shape was divided into irrespective of the type. They associated a square divided into four parts with a triangle divided into four parts as equivalent to each other. This indicated that learners lacked conceptual understanding. She stopped the learner activity and gathered the class attention. She proceeded to use a fractional chart to present learners with a clearer description of determining equivalent fraction. The chalkboard was used creatively and effectively with bright colours to show the equivalence between the fractions. The use of the bright colours was to enhance the clarity of the concepts. Teachers will always be confronted with all types of learner's solutions thus they need to understand the procedures, concepts, have the ability to identify right from wrong and fix the learners misconceptions (Ball, Thames and Phelp, 2008:8)

According to the structural learning theory learners formulate rules to guide their learning and produce solutions that reflect their thinking. The manner in which these learners were making the association of equivalence with the various shapes indicated what they were thinking. They made associations by looking at the number of parts the shapes were divided into. Conceptually it was incorrect. By stopping the lesson Pat was able to put into practice an underlying feature of the structural learning theory. This theory allows for the teacher to review their learners understanding and reteach concepts to improve their understanding. Pat was therefore in a prime position to prevent any further misconceptions in respect of learners understanding equivalent fractions.

Pat drew a few a diagrams on the chalkboard and randomly called the learners to the chalkboard to match and shade in the equivalent part of the fraction she had drawn on the board. The results were dramatic as the learners were now able to see and get a deeper understanding of the concept equivalent. Getting the learners to work on the board to show their understanding is linked to Kolb's experience based learning. Using experiential based learning, the learners learn by doing it through experience. By using pictorial representation as a visual medium allowed the learners to observe (reflective experience) and draw on their prior knowledge to make the connection with various illustrations on the chalkboard. By retrieving information from their memory allowed them to connect between their folders of mental knowledge. This showed Pat that the learners understood the concepts equivalent, half, thirds, quarters, fifths and tenths.

Pat provided a fractional problem. She simplified the problem diagrammatically and explained the mathematical concepts in the problem together with the mathematical procedures to be followed to solve the problem. Thus Pat was able to simplify the link between the concepts and the procedures. This kind of teaching indicated her level of teaching within the Zones of Proximal Development (ZPD). She continued her teaching within the ZPD dealing with the learners individually and guiding their learning.

The learners were given a practice example to complete in their books. They worked individually. The work alone time allowed the learners to work uninterrupted. This was an ideal way to check learners understanding. As they worked individually Pat was in a position to provide scaffolding support to those learners who became 'stuck' whilst seeking a solution. After providing the support the she withdrew and allowed the learners to proceed with the problem. Interacting with the learners whilst they were attempting the problem allowed her to observe the areas the learners were experiencing difficulty. It is important that teachers walk around the classroom interacting with the learners as they are in a position to check the learner's level of understanding and share in their cognitive ability. This interaction with the learners resulted in them receiving constant feedback from her. During her movement around the class Pat was particularly sensitive to those learners who were having difficulty with the problem and she provided scaffolding and encouraged them to find a solution.

The chalkboard was divided and the learners were invited to share their solutions with other learners in the classroom. Two learners produced the solution arithmetically and the third had a mixture of a diagram and an explanation. Pat did not indicate whether the solutions put on the chalkboard where correct or incorrect. This is important when learners are involved with problem solving. Teachers must desist from openly evaluating learner's answers as been correct or incorrect, as it will prevent learners from trying and it can also curtail mathematics discussion.

Once the learners were finished writing the solutions, she asked if any of other learners had a different answer to what was on the board. With no further answers forthcoming she discussed each of the solutions with the learners. As she proceeded she indicated to the learners the areas they had made errors. The first learner produced his answer arithmetically and he had used the incorrect operation. The second learner had managed to arrive at the answer but had difficulty explaining to the class how she eventually arrived at the answer. The third learner's diagram was only partially correct but the arithmetical solution was correct. When the learner was asked to explain how she arrived at her answer, she was able to use the diagram to support the written answer.

Based on the theories within this study it is important to listen to the learners carefully. By listening to what they are saying gives an indication of what they are thinking (abstract conceptualization in Kolb's experiential learning theory). Their explanation of ideas is their thinking which indicates their understanding which is drawn from their existing schemas. Pat's manner of evaluation of the solutions (active experimentation in Kolb's experiential learning theory) was a means to consolidate the learner's efforts by showing them the areas where they could have used alternate methods. In this manner she was able to build a foundation of understanding using the learners existing knowledge with the newly acquired knowledge.

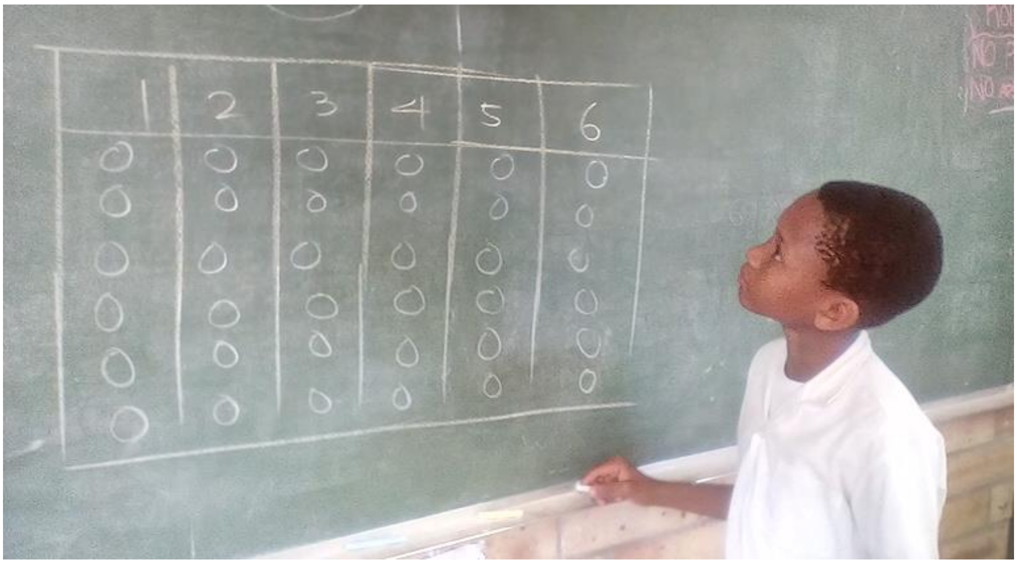


Figure 26 Learner explaining a solution on the board

The learners were given a worksheet as a class activity. The worksheet was divided into two parts. In the first part they had to identify, name and shade the fractions. A picture of a fraction wall was provided and the learners had to use the mathematical signs indicating greater than, less than and is equal to complete a few arithmetical problems. This assisted them in answering the second part of the worksheet. Pat had set two graded problem solving questions.

She was flexible in the sense that the learners were encouraged to use whatever methods they wanted, to work towards a solution. Pat became an observer during this activity without much interaction with the learners.

On completion of the task she discussed the problems on the board and provided the learners with the solution. She should have given the learners an opportunity to provide the solutions on the chalkboard (this indicated the lack of application of the knowledge of content and teaching) to gain an insight to their mathematical thinking. The learners had produced incorrect answers and the opportunity was lost to show them their areas of weakness. If Pat had possessed common content knowledge and knowledge of content and teaching, it would have been possible to diagnose the learner's area of weakness if they had solved the problem on the chalkboard. During their classroom activity I randomly checked their work (this will be discussed later under the analysis of the class work books) and I found that many of the learners had attempted to problem solve using procedural strategies (using steps and rules to solve it). Those learners who managed to find the correct answer were able to apply the acquired knowledge to the problem. Their constant use of diagrams, illustrations and the fraction chart assisted them to visually understand their solutions and they were able to explain to their peers when asked to do so. The learners had used steps and rules to find the solutions where within the ambit of the structural learning theory and mathematical understanding theory. The structural learning theory dictates the formulation of rules to assist learners solve problems. When these rules are constructed learners are in a position to use their experience to apply these rules to solve problems.

The third part of the worksheet was a homework activity. This part involved the solving of basic fraction calculations. Here too I had no knowledge of the learner's performance because I was not able to follow up on the next day.

Bheki

Bheki was observed at a well-established school in the Mafukuzela Gandhi Circuit. The school is situated within an affluent area and has a mixed learner population. The average teacher-learner ratio at this school is 1:35. This school is a fee paying school and it also receives a state subsidy based on the learner population. The school is well resourced in respect of teaching and learning. The learners have access to a well-resourced library and the classes have sufficient subject related charts. The school has a full staff complement of qualified teachers employed by the Department of Education and the Governing Body. The language of learning and teaching (LOLT) at this school is English and the first additional language (FAL) is Isizulu.

I observed Bheki teaching fractions in a grade four class. During the introductory phase of the lesson Bheki experienced difficulties in explaining the concepts and algorithm to the learners due to his lack of knowledge on fractions. He based his teaching of fractions on his knowledge. He gave the learners following example to complete:

$$\frac{3}{8} + \frac{1}{8}$$

The example above involved the addition of fraction with like denominators. The learners were required to add the fractions. When the learners completed this sum Bheki provided the solution.

Solution provided by Bheki: $\frac{3}{8} + \frac{1}{8} =$ **4/16**

As teachers we must always remember that "*what is learned depends on what is taught*" (National Research Council, 2001:334). The solution **4/16** is incorrect and it indicated that he had a weak conceptual understanding of the addition of fractions. The manner in how he arrived at the solution was incorrect. The denominator is never added or subtracted when adding or subtracting fractions with like denominators. The numerators are only added. The learners were been taught incorrectly. According to National Research Council (2001:378), in some instances just because teachers had inadequate conceptual knowledge, it resulted in them presenting incorrect procedures. Ball, Thames and Phelps (2008) stated that learner's achievement in the subject is dependent on the teacher's content knowledge and Bheki's lack of content knowledge resulted in learners having difficulty in grasping these concepts. According to the literature on teacher mathematical knowledge it is critical that they be able to perform calculations that will be assigned to their learners thus they need to have common content knowledge (Ball, Thames and Phelps, 2008:6). The teacher's explanations were directed verbally at the learners and they seemed restless consequently not taking much interest in the lesson. The learners were obviously confronted with unfamiliar concepts (numerator, denominator and eights) and the possible reason was that he did not start with the algorithm. It is important that discussions occur in the classroom when new concepts are been taught. In this situation there should have been "*discussions around the role of the numerator and denominator*" followed by "*conceptual demonstration*" to "*enable learners to understand the idea that we can add fractions of the 'same kind'*" (Department of Basic Education, 2018:37).

When such a situation presents itself in the classroom the learners are bound to become overwhelmed. I subtly intervened by giving the learners a brief task and spoke to Bheki about the incorrect solution on the chalk board. I asked him to use a fractional strip or play dough to discuss the example. Bheki went back and continued teaching. He allowed the learners to make the dough into a shape of a ball. They were then asked to divide the play dough into fractional parts as he indicated on the board, example, half, thirds (Figure 27).

Figure 27 Learners creating fractional parts with play dough

He then introduced the fractional strip **(Figure 28)**. By doing so Bheki was able to transform the representation of fractions as given parts a whole. The learners were asked to shade the required parts on it. The strips were placed side by side. The learners had to count the shaded parts and write down their answer. During this phase of his teaching he stressed to his learners that the ‘denominator must never ever be added’.

Solution: $\frac{3}{8}$ + $\frac{3}{8}$ = 6/8

Figure 28 Fractional strip

The lesson changed when Bheki introduced the resource charts that he had prepared for the lesson. The learners were now able to relate to what Bheki was explaining. By linking the concepts with pictures brought about specific meaning thus making it easier for the learner to understand. By directing their attention to the charts the teacher made them understand the mathematical concepts easier.

Although Bheki used the charts to introduce the lesson he did not build on the momentum to move the lesson forward by using other visual approaches. He fell into the routine of using the traditional teaching methods of showing the learners the basic procedures how to calculate. It was evident that whenever he made a reference to the fraction diagram on the chart **(Figure 29)** the learners showed signs of understanding. The use of the visual aspect during teaching indicated that the learners were able to follow his teaching.

The learners were given a worksheet to complete during the learner activity. This worksheet had questions based on fractional representations provided on the worksheet and a pie graph. The questions merely tested their knowledge on what they were seeing in the diagrams, namely, if the shape was divided into two parts the learners had to state the name of the fraction. The shaded diagrams were indicative of the answers required. The learners were able to complete this part of the worksheet timeously. On completion of the exercise Bheki asked the learners for the answers. There was a lot of chorusing on the part of the learners as they shouted out of the answers in unison. It was not possible to really determine who understood the work or not.

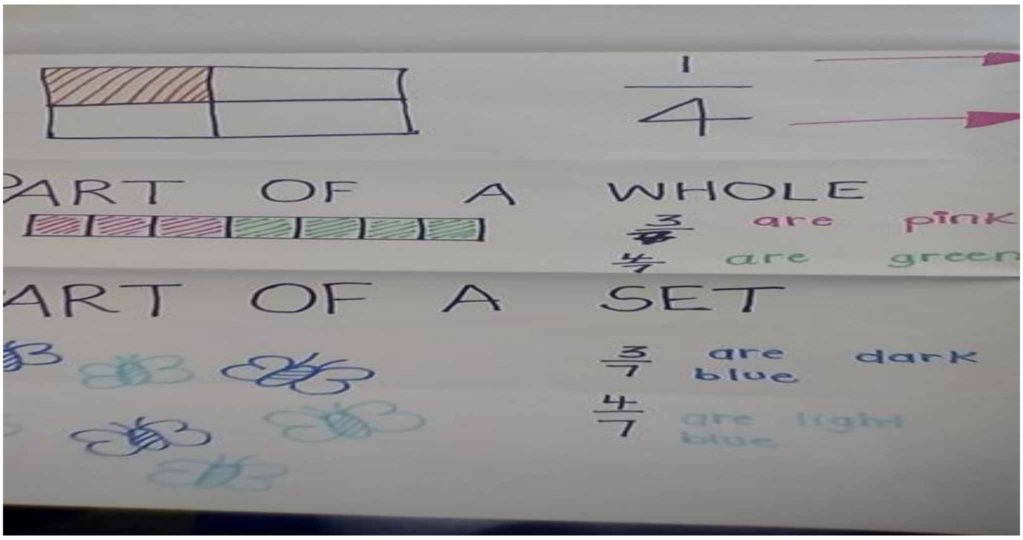


Figure 29 Bheki's Teaching resource on fractions

The second activity required the learners to answer the questions based on the pie chart which was divided into eights. The learners needed to deduce from the pie chart the fraction of learners who liked pizza, fish, burgers and chicken using the key listed next to it. The next question needed them to use these fractional answers and determine the number of learners (they were given 48 learners) who liked pizza, fish, burgers and chicken. The majority of the learners experienced real difficulty in applying procedural knowledge to calculate the answers. They gave responses to the questions which they saw appropriate and very noticeable was that there was a lot of guessing. This was envisaged when I randomly examined this activity in a walkabout during the lesson. Although they were experiencing difficulties, shown by them constantly asking each other what to do, the learners very occasionally asked Bheki for assistance. Other than answering the odd enquiry, interaction with his learners was absent. With no interaction and by not providing assistance to the learners meant that there was no eliciting of solution strategies and misconceptions by him. He walked around the class and was rather isolated from the learners during this activity. Bheki should have used this part of activity to show learners how they could have easily divided the pie chart fractionally. The learners would have used this as a starting point to work towards the answers.

Whilst it is essential that the pre-service teachers have mathematical common content knowledge, pedagogically knowledge and specialized content knowledge it is very vital that they have knowledge of content and teaching. Although the various aspects of teacher knowledge is particular and relevant to the teaching of mathematics, knowledge of content and teaching sets in motion the direction the lesson will take. It actually guides the manner the mathematical content will be taught during the lesson. The CAPS document has in a guided manner set out the mathematics content knowledge as "content focus" for each grade and it guides the teacher how to use this in the lesson. It is imperative that the pre-service teachers regularly engage with this document to upgrade their teacher knowledge and use it appropriately.

Bheki concluded the lesson by providing the answers. There were a few learners who asked him if their answers were correct even though they had used other methods to find the answer. One learner volunteered her answer. She wanted to tell the class how she got the answer. Her discussion was long winded and Bheki halted her explanation and asked her to write her answer on the board. She provided a diagrammatic illustration of the steps she used to find the answer. She indicated that she first divided the pie graph into halves, then the halves into quarters. According to Killen (2015) understanding is linked to the way when something makes sense to them and the manner in which the learner divided the pie graph indicated her level of understanding. This kind of thinking and learning is linked to Kolb's experiential learning theory and the Theory of Understanding. It is important that teachers listen to their learners ideas because it is in the brain that the assimilation and accommodation of these ideas occur which eventually leads learners to construct their own solution strategies. The learner was in a position to utilise her knowledge and transfer it into practice (Kolb's experiential learning theory). In this manner she showed she had utilised her acquired knowledge on fractions by dividing the pie into fractional parts to indicate her mathematical understanding. A learner who has a sound conceptual understanding is "*more able to transfer this knowledge to new situations and apply it to new contexts*" (Department of basic Education, 2018:38). The learner was able to show her level of sense making of what she had deduced for herself. Bheki failed to capitalise on the learner who provided an alternate explanation. He should have used the same example or a different one and built on this explanation to show his learners that besides relying on mathematical rules it was possible find an answer if one used representation. Bheki wrote down the answers to the other questions and the learners were asked to correct their work.

This was to an extent a passive lesson with not much interactive learning taking place other than the single learner engaging with the class. This goes against the attributes of an experiential learning activity where learners are required to be in action and not sit passively (Burnard, 1989). No attempt was made to teach any problem strategies. Bheki would have created a better understanding in the learner's minds if he had switched from using a theoretical strategy to a visual strategy or used both simultaneously in his normal teaching. Bheki did not have the knowledge on how to elicit learner's misconceptions thus the learner's difficulties were not reviewed or discussed with the entire class. The possibilities are that these learners will repeat these misconceptions and errors in future mathematical activities.

Bheki lacked the necessary specialized content knowledge and knowledge of content and teaching. Been competent by having these types knowledge would have allowed Bheki to be more confident in his teaching. He would have taken his learners from knowing (concrete) and applying their understanding (abstract) of the fractional concepts. He used his pedagogical and common content knowledge on fractions advantageously in his teaching.

Thobela

Thobela's school is located in a suburb next to a low cost development on the North Coast. The school has a mixed population of Indian and Black learners but has a predominantly Indian majority. The Indian learners attending this school are from within the catchment area and the Black learners travel from the nearby areas. This is a quintile three school and is a fee paying school. The school receives partial funding from the Department of Basic Education to address some of its needs. The School Governing Body plays an integral role in the functioning of this school. This school is well resourced and is well maintained. The school has an experienced mixed teaching staff and has consistently maintained a high degree of academic excellence.

The lesson commenced with Thobela doing the corrective work for the previous day's homework. The majority of the learners did not do their homework and getting them to do the corrective work impacted on the days teaching. This aspect proved to be time consuming. The learners had to copy the corrections from the chalkboard into their books and this was proving to be frustrating for Thobela as valuable teaching time was been lost.

Due to time constraints Thobela overlooked the introduction of the lesson. She simply stated to the learners what section they were going to do and discussed a few examples with them. She worked the few examples on the chalkboard and the learners were expected to follow the examples. Thobela relied on procedural knowledge to teach the fraction concepts and did not see the need to assist learners with using concrete objects or any form of representations and pictures. According to the Association for Experiential Education (2017) mathematics is more than the use of algorithms and procedural knowledge. She had not recognised the use of using visualization or visual resources to represent concepts. Although she used procedural knowledge she did not recognise the need to ascertain the learner's prior conceptual knowledge. In mathematics having both procedural proficiency and conceptual understanding is equally important. If she had given her learners an opportunity to use their ideas and engage in critical thinking, she would have developed both these aspects thus reinforcing the learners understanding of what was taught in the lesson. Using pictorial resources would have effectively supported the learners to understand concepts. Experiential learning theory dictates that the learners engage with the content so that learning becomes authentic and mathematical understanding occurs (Association for Experiential Education, 2017). Teachers must realise that the human world and experience is a source learning experience (Burnard, 1989). Thus when preparing material for the lesson pre-service teachers must consider the background of the learners, the learning environment, the concepts to be taught and how these concepts will be taught (Association for Experiential Education, 2017).

The learners were given an exercise as a learning activity and they completed the exercise in a short space of time. On conclusion of this activity Thobela put the answers on the board as corrective work and learners were asked to mark their work and forward the books for marking.

In this lesson learners were not given an opportunity to directly or actively participate in the lesson. There was hardly any discussion or explanation on how these answers were obtained. No discussion was conducted at the end of the lesson to consolidate the learners understanding of what was taught. I gathered that Thobela was making completing the lesson timeously her priority without focusing on the task at hand. In today's educational environment there has to be shift from teachers pursuing to complete the curriculum without focussing on the learner's acquisition of knowledge. This kind of teaching attitude and failure on the part of teachers to teach competently to build a solid foundation will have a detrimental impact on learner's learning lives as they proceed through their schooling career. There has to be a shift from teaching via algorithmic procedures to using a more flexible open model whereby learners and teachers can function in tandem to construct knowledge to sustain lifelong learning. The lack of understanding and the learning of the rules and procedures (structural learning theory) to calculate algorithms in a single grade will leave learners with major difficulties in other grades. The CAPS document is designed to show mathematical content development and progression in the different grade levels and taking the route of 'not teaching' due to time constraints will definitely show the shortcomings of the teaching and learning process. It is therefore important that pre-service teachers focus on the teaching aspect rather than time.

The classrooms have changed to become an interesting and stimulating arena of knowledge. There has to be a shift from teaching to test learner's knowledge to teaching mathematical concepts for understanding. To do so they need to have sound teacher knowledge and content knowledge. They must be able to use the teacher knowledge at the appropriate level to put across the content as stated in the CAPS document. I totally agree with Hughes (2016:320) who found that pre-service teachers understanding of mathematics subject knowledge (content knowledge) and pedagogical knowledge is reflected in their teaching practice. Thobela's teaching methods were sometimes skewed. Mathematics allows for both collaboration amongst learners and independent thinking, but Thobela never allowed her learners the space to develop these aspects or apply their own ideas. Although the learners may face difficulties in expressing their thinking, they should be encouraged to talk about their understandings of the problem in order to build their confidence (Department of Basic Education, 2018)

Teaching in a procedural manner does not allow for ideas to be solicited from the learners especially when a teacher is fosters his procedural knowledge 'I want you to do it like this' on his learners. This contradicts the theories within this study, namely, Kolb's experiential theory of learning, mathematical understanding and structural learning theory. Learners must be given opportunities to observe, think and make connections with what they know. This is called assimilation. When this happens learners are able to use their prior knowledge (old ideas) and synchronise them with new ones. This eventually leads to the expansion of their mental abilities as they acquire new knowledge.

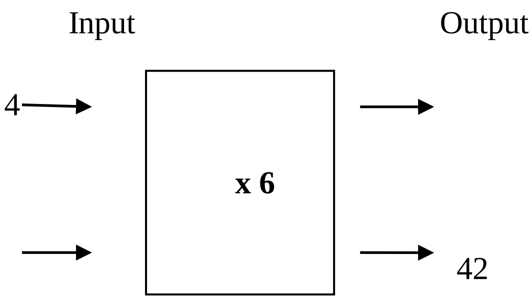
Robin

Thobela and Robin were observed at the same school.

During the introductory part of the lesson Robin took the learners through a 'two minute' mental activity. It involved the rapid asking of bonds and tables. She asked questions like, *"what is 2 plus 3?* or *"what is seven minus 5 ?"* to which learners responded in unison. It was a simple teaching exercise were learners mental capabilities were been tested. This was a smart exercise as it primed the learners for the next part of teaching and learning. This activity reinforced learning by reviewing the necessary algorithmic prerequisite skills and knowledge needed for the completion of flow diagrams.

At the start of the lesson the learners had to fill in the 'input' and 'output' on a simple flow diagram. Robin wrote the example on the board and the learners were required to work out this flow diagram in their books. This was an exercise to determine the learner's conceptual understanding of input and output. The concepts, input and output, was new to learners. Robin discussed the concept of input and output by inserting the 'answers' where they were missing.

The majority of the learners found that when the input was known then finding the output to complete the flow diagram unproblematic. They experienced difficulty when the output was given and the learners had to determine the input. Further difficulties arose when the flow diagram was provided without the rule. When the learners had to determine the rule in the flow diagram it proved difficult. According to Killen (2015:40) learners cannot learn new things and understand new concepts devoid of the necessary background knowledge. One of the primary aims of teaching for mathematical understanding is to assist the learners to develop a relational understanding of the mathematical ideas. When this happens learners are able to see the connection and understand.



Robin used this opportunity to alleviate their difficulty by teaching them a problem solving strategy on how to work backwards and also how to use the inverse operation. She commenced firstly by starting with the output. She wrote out the algorithm **4 x 6 =** □ stating that is an open number sentence since no answer is given. Robin highlighted the various numbers and the operational sign in different coloured chalk. This type of visual illustration was to focus the learner's attention on what she was trying to emphasise in her teaching. Robin explained this algebraic solution by showing her learners that one should calculate from left to right, namely, 4 x 6 = 24. She mentioned to her learners this algorithm was similar to them learning their four time tables. It is important that learners develop a strong number sense as this will encourage them to develop their own calculation strategies (Department of Basic Education, 2018). Robin asked the class to recite their four times table after which showed them the link between the input and output. As they recited the four times table, they were able 'generate' the next number. She thereafter went on to explain to the learners the concept of inverse operation.

42 ⟶ was the output

x 6 ⟶ the inverse operation to multiplication is division

Robin explained to her learners that when solving for the missing input they have to work from right to left and substitute the multiplication sign with division. By using the highlighted numbers and mathematical operation the learners were able to identify what was required. Robin gave her learners a few more examples to consolidate their understanding and the learner's responses showed that they understood the necessary concepts and operations. Once she had consolidated this she continued to extend her learners to determine a single rule or two sets of rules in the flow diagram by providing the input and output. A lot of guessing took place in the classroom and she allowed it as learners themselves were disputing whose answer was correct. When the learners gave an answer Robin asked them to explain their response. When the learners had difficulty explaining the operation she asked them to work out the flow diagram on the chalkboard. The learners themselves were able to point out to their peers where they erred and helped them to correct the errors. The learners should be encouraged to observe each other's solutions in order to clarify or make improvements to their solutions. In this manner they make knowledge construction their own. This type of learner participation in the lesson is in keeping with the attributes of experiential learning (Burnard, 1989), as much emphasis is placed on utilising learners' experiences to create knowledge. According to Burnard (1989) learners must not sit passively and accept what is mentioned but should rather be active participants. The teachers must realise that learners are a rich source of experience and by been active participants along with sharing ideas leads to new experience which lays the foundation for future teaching and learning (Burnard, 1989).

The learners were given a worksheet to complete as a learning activity. This worksheet was an exercise consolidating what was taught in the day's lesson. Whilst she had taught her learners a problem solving strategy, working backwards, the learners still tried to calculate for the missing input algebraically.

Robin had a sound foundation of mathematical knowledge. She used it to accommodate her teaching and her learners learning styles. Her use of manipulates to focus on the concepts and getting her learners actively involved in the lesson indicated she was also aware of the learning theories used in teaching and learning.

Pinky

The lesson commenced where Pinky asked the learners to draw a diagram representing the listed 2 D shapes, example, square and triangle (the names of the shapes were written on the board). Once completed volunteers was sought to draw their shape on the chalkboard. Further volunteers were selected to write the properties of the shape next to it. As the lesson progressed Pinky asked the learners to indicate if these shapes had other properties. When the learners completed this activity Pinky discussed the properties of each shape with the learners. Pinky used this moment to reinforce the learner's prior knowledge and the concretising of concepts.

Pinky carried this momentum of the lesson further and mentioned that the square now resembled a table and asked *"How many people can sit around this table?"* A learner was randomly selected and she answered *'four'*. Pinky asked her how she got the answer. She mentioned that she identified the spaces around the table. The learner's ideas are important in problem solving as they construct their knowledge in any possible way. Pinky congratulated the learner as a means of encouragement.

She then posed a real life problem: For Megan's birthday her mum had eight square tables set next to each other in a straight line. How many of Megan's friend could she seat altogether? Many of the learners responded *'eight'*; some gave the answer *'thirty two'* whilst some of the learners mentioned it was too hard. Pinky used prompts to assist them, *"how should you start?"* in order to direct the learners towards a solution. She accepted the learner's responses and then mentioned *"if I joined two tables how many people will be able to sit around it?"* She asked them to test their answer by drawing the two tables. Some of the learners, who did not draw, still gave *'eight'* as an answer whilst the others who attempted the diagram gave the answer as *'six'*. Pinky asked for a volunteer from both the 'six' and 'eight' camps to explain and display their answer on the board. This is an important feature of problem solving. The learners must be given an opportunity to solve the problem in any possible way and share their solutions with the class as it shows their thought processes.

When the answers were displayed the Pinky asked the learners if they agreed with the diagrams. There was still a difference of opinion amongst the learners. To demonstrate this aspect she asked for eight volunteers. She asked them to join two learner's desks. She then asked the eight learners to find a seat for themselves. Two learners found themselves without a seat. Pinky asked the learners to compare the seating arrangements and the diagrams on the chalkboard and determine the similarities or the differences between both the diagrams. She asked her learners to note that when the tables were joined nobody could be seated between them. Using the visual technique of the diagrams and the role playing of seating the learners enabled her to explain 'joined' and 'in a straight line'. The learners reached consensus the answer of eight was incorrect.

The best way to learn mathematics is to do the mathematics and Pinky asked the learners to attempt the problem again. By following Pinky's strategy of drawing a diagram (extending the diagram to represent eight joined tables) and role playing many of the learners arrived at answer of eighteen.

The learners were now in a position to use representations to reason through problem. The change from using a theoretical strategy to a visual strategy enabled the learners to get a clearer understanding of the solution. Pinky's use of a real life problem enabled them to relate to the problem. Furthermore by asking her learners to draw their solutions and providing them with a role playing scenario (seating demonstration) empowered her learners with instructional opportunities to problem solve in a way that made sense to them. Through the process of externalization of their internal thought process (drawing of the tables) the learners were able to comprehend the problem and show their understanding in their solutions.

The learners were given a worksheet to complete as a classroom activity. They learners had to complete the exercise on 2D shapes by naming the shapes, grouping them by the number of sides they had and indicate if the shapes had curved, straight or both curved and straight lines. The last activity required them to measure the sides of the shapes and indicate the measurement. The lesson was supposed to be based on 2 D shapes and their properties. Besides getting the learners to name the shapes and discussing their properties Pinky went into a problem solving strategy without focussing on the type of lines these shapes had and how the learners had to measure in both millimetres and centimetres.

The learner activity, to a large extent, became disruptive as the learners wanted to know from Pinky how can lines be curved. Some wanted to know what they needed to do when the shapes had both straight and curved lines. Pinky had not discussed the concepts of straight and curved lines. The learner's questions suggested a great deal of confusion. Pinky tried handling the learner's questions and provided explanations on an individual basis. She should have stopped the lesson and explained the concept of curved and straight lines and also reinforced that when the lines are curved it is still a line. Research has shown that the lack of conceptual understanding often relates to the success in mathematics as learners progress through the grades. Failure to clarify how a curved line is still a line can result in learners thinking that when a line is straight then only it is a line. The lesson became more chaotic when the learners attempted to measure the sides of the shapes. The learner's responses indicated that they were inefficient in using a ruler properly. Pinky stopped the learners from continuing with this activity mentioning that they will continue with it the next day. Unfortunately Pinky closed the lesson at this juncture and asked learners to do their corrective work. According to Killen (2015:151) it is very important that the pre-service teachers keep in mind the structure of their lesson or it will lead to learners unlikely engaging appropriately and constructively in learning.

There when planning to teach mathematics, pre-service teachers must be able to identify the different ways to put across material to the learners.

According to Hughes (2016:319) teaching mathematics places a huge demand on the teacher's knowledge. Pinky had pedagogical content knowledge and common content knowledge but lacked the knowledge of content and teaching. Her teaching strategy showed a deficiency. Structural learning theory makes mention of teaching learners until they show understanding. They should be guided in their learning. By assuming that the learners know the basics of using a ruler in the foundation phase or they are competent enough by the time they reach the intermediate phase can be disastrous for any teacher. She should have stopped the learner activity and demonstrated to the learners on how to use the ruler. According to the Kolb's experiential learning theory the teacher must engage the learners to use their reflective experience (observation how the teacher demonstrated using the ruler) and abstract conceptualization (show their understanding of how to use the ruler and also make the connection between the different units of measurement – mm/cm). It is important that the pre-service teachers gain specialized content knowledge whilst at higher education institutions. This will allow them to understand how the subject content is distributed in the mathematics curriculum in various grades.

5.3.3 CONCLUSION

The pre-service teachers showed proficiency in the subject to a certain extent as they were able to utilise the textbook knowledge to teach. They understood the basic classroom mathematics and some appeared to be familiar with the strategies in problem solving but they lacked the appropriate knowledge and skills regarding putting it into practice. Their deficiency of content knowledge during teaching practice raised questions about how well they would be able to teach without the required knowledge and skills. This is thought-provoking in terms of the quality and effectiveness of teacher education in respect of problem solving and using visualization in the field of mathematics.

The majority of the pre-service teachers taught their lessons in a 'show and tell' manner and expected their learners to follow what they demonstrated in the lesson. In this context, in light of the lesson observations, highlights the need for more practice and effective pedagogical and content knowledge regarding teaching solving through the means of visualization. Explicit knowledge and skills, which is vital, is needed in applying teacher knowledge and technological pedagogical knowledge in using visualization in the mathematics classroom. In order to equip teacher candidates with the required skills and information, more emphasis should be placed on te.aching problem solving strategies in their pedagogical courses, so as to provide opportunities for them to use and practice. Furthermore, in light of the findings of this observation, it is also possible to give suggestions for the direction of how the mathematics education modules should be drawn up to meet the challenges of the classrooms. According to Ball, Thames and Phelps (2008:12) this kind of knowledge and skills are not typically taught in the course of their formal mathematical preparation.

Understanding visualization and using it is difficult for novice teachers thus it requires substantial improvement taking into consideration the rapidly changing nature of today's educational climate. The pre-service teachers must be exposed to new methodologies, problem solving abilities and new learning tools to make the mathematics lesson interesting for learners. Some of the pre-service teachers did not see the importance of encouraging the use of visualization to arouse student's interest and to keep them fixated on the lesson in the classroom. Studies are needed about the content and teaching-learning process of pedagogical courses to establish reasons for their shortcomings in preparing the pre-service teachers to teach. In the foreseeable future, a pedagogical course need be designed to include visualization as a teacher training module in order to skill teachers to tackle teaching.

Teacher candidates' perceptions could be investigated regarding other teaching approaches and methods, and their skills in implementing these. Another research topic could concern to what extent teacher candidates use PBL in their lessons after graduation; the results of this could then be compared to their perceptions before graduation. In this way, reasons for teacher candidates using or not using may be revealed.

The lesson plan used by the pre-service teachers is divided into three main parts but is loaded with other aspects which I found to be irrelevant to the teaching and learning process.

5.4 SEMI-STRUCTURED INTERVIEWS

5.4.1 INTRODUCTION

An interview is necessary if we cannot observe people. The aim of the semi-structured interview was to determine the pre-service teacher's thoughts on their planning, problem solving and visualization. All the participants in this interview process were asked the same questions (Appendix 2) and their responses were recorded, transcribed verbatim and reported.

The core questions, amongst others, are highlighted and the pre-service teacher's responses are presented in italics and the analysis thereof follows.

5.4.2 ANALYSIS OF SEMI-STRUCTURED INTERVIEWS

Planning is an integral part of teaching and learning. Besides understanding the concept of planning, it is essential that the pre-service teachers know and understand the steps involved in planning a mathematics lesson. Planning is an ideal starting point to prepare the mathematical content and adapt teaching methodologies to engage the learners in the classroom. When asked are you offay with the steps in planning a mathematics lesson all of the respondents indicated they knew the steps involved in planning a lesson.

According to du Toit (du Toit, Louw and Jacobs, 2016:140) the following questions are imperative during the planning process, namely, "*what do I teach?; who are my learners?, why am I teaching this?, how can I teach this?*" and "*how successfully do I teach?*" The pre-service teachers were asked: Name some aspects that that you will consider when planning a lesson. Some of their responses were:

Learner diversity;

Prior knowledge;

Language;

Misconceptions;

Integration;

Content – what you will do with the lesson;

Aspect of time;

Various levels of understanding.

These responses provided the relevant responses to the questions posed by du Toit. The classrooms in South Africa are a cauldron of diversity. In addition to the language challenges facing learners there are other academic impediments teachers have to take into consideration. Some of the issues are mentioned above have to be by the pre-service teachers to meet the educational needs of the learners. The pre-service teachers, when drawing up a lesson plan, have to consider all of these aspects. All of learner diversity, prior knowledge, language and misconceptions can be classified together. Learners bring their own understanding of concepts due to the language they speak and the environment they live in. This forms their prior knowledge that they bring into the classroom. Therefore the pre-service teachers need to build from what the learners share as common knowledge. The correct use of language when explaining mathematical concepts can clear any misconceptions that the learners may have and the pre-service teachers need to be aware of these factors when they go out for their practice teaching so that the learners are catered for the in the different activities.

The pre-service teachers had to explain how they used some of the aspects used in planning. They are expected to use the lesson plan provided by the higher education institution they attend in all their teaching preparation. Provision is made for routine information, namely, topic, *time;* objectives for the lesson and resources to be used to support the lesson. The majority of the interviewees indicated that they follow the given headings to complete the aspects on the lesson plan.

In the lesson preparation there is an interrelationship between the introductory, teaching and conclusion phases. This interrelationship allows for the free flow of the content material during teaching. Determining learner's prior knowledge, catering for diversity in the classroom, managing misconceptions, content to be taught and language of learning and teaching must be considered in the planning stage. All of these are of the utmost importance to meet the needs of the learners. In the introductory phase the lesson is introduced in such a manner that the teacher is able to determine the learners' prior knowledge so that the teacher will '*know how to go about teaching the rest of the lesson*'. The teacher, by having an open discussion with the learners, will also be in a position to manage misconceptions by clearly explaining the vocabulary specific to the language of mathematics. By selecting the correct grade, the pre-service teachers would be able to determine the lesson objectives and '*curriculum content*' to be taught. In order to do so they need to understand curriculum progression material for the relevant grades in the CAPS policy document. The CAPS document lists the key elements and content that needs to be considered when preparing a lesson therefore the pre-service teachers need to be trained to use this document. The pre-service teachers must take the initiative and plan the objectives, select the '*content*' material to suite the grade, time needed for the lesson, determine which teaching strategy or strategies will be appropriate to accommodate the learner's learning style. To successfully engage in all aspects mentioned above the pre-service teachers must have pedagogical content knowledge (how to make the subject matter accessible to all learners irrespective of their learning challenges).

I am very critical of the lesson plan used by the pre-service teachers during their teaching practice. The lesson plan used still has resemblances to the one used during Outcomes Based Education (OBE). It involved a lot of writing out of information not relevant to the planning and teaching process. The Department of Education has made changes, since the introduction of CAPS, to the lesson plans in all subjects and has provided templates of the lesson plans to all schools. These lesson plans are currently in use. The pre-service teachers are expected to be mentored by subject teachers when they are out at schools doing their teaching practice and this includes been exposed to using the lesson plan used in the schools. Proper mentoring does not take place because teachers are not willing to check the long lesson plans used during teaching practice. The teachers consider it a laborious and tedious process. Therefore it is imperative that the institution implement changes to the existing prescribed lesson plan. These pre-service teachers need to be exposed to the current trends in lesson planning hence they should use what is already in practice at schools in order to equip them with the proper skills needed for teaching and learning .

The pre-service teachers must also factor contextual factors, namely, '*language levels of the learners; learner's ability and reading*' into their planning and preparation which might impact on their teaching. The Department of Education admission requirements states that the learners cannot be denied admission based on their understanding of a language. Learners in South African schools are expected to be taught in their mother tongue in their formative years at schools. The migration of learners between provinces, especially the rural areas, forces the parents to seek out schools that have English as their LOLT. The general consensus is that obtaining schooling in an English medium school bolsters the learner's chances of achieving academic success to gain entry into tertiary institutions and inevitably in the job market. Thus schools are forced to accommodate learners who do not have the schools LOLT as their mother tongue. These learners inability to communicate effectively in the classroom will impact on teaching and learning. Reading will become a barrier to these learners. This will inevitably have an impact on the learner's ability to learn to read and solve problems during the teaching and learning process.

It is expected that the pre-service teachers have the expertise in preparing teaching and learning activities. If the pre-service teachers answered yes to using certain aspects in planning then they had to explain how they used them.

Pre-service teacher: *I look at the learner's text book for material for my lesson.*

This response indicated the pre-service teacher's reliance on the textbook as the primary source of material for teaching. Low level of readiness to source sufficient teaching and learning material on the part of the pre-service teacher can impact on the learner's learning experience. It is expected that the teacher content knowledge is higher than that of the learner knowledge.

Pre-service teacher: *I check for the resources at the school to prepare for my lesson because I don't really know how to teach the subject without it.*

This indicated that the lack of reference resources at schools will impede the creation of knowledge for teaching. The pre-service teacher will be in a predicament if the he lacked the basic pedagogical content knowledge to teach the subject.

Mathematics teaching has become technologically revolutionised to such an extent that being a highly qualified teacher means being able to use technology effectively to support teaching learning in the classroom. According to National Council of Teachers of Mathematics (2000) technology is an absolute necessity in supporting teaching and learning as it influences the manner how mathematics is taught. It also supports how the learners learn the subject effectively as it engages the varying learning styles of the learners. It is therefore important, never mind the location of the school, to utilize whatever resources is available. The pre-service teachers were asked: do you use any of the following in your lessons: worksheets/charts/OHP/laptop/other creative media. The majority of them indicated that they used '*charts, laptop and worksheets*' to support their teaching. These are readily available at most schools. Majority of the South African schools have the relevant technological equipment to support teaching and learning. Certain technological companies in South Africa have formed partnerships with the Department of Basic Education and have provided schools with tablets loaded with mathematics learning material. Teachers have not made any real effort to utilise them in their teaching as they lack the basic technological pedagogical knowledge to utilise technology as part of their teaching programme. The pre-service teachers are in an advantageous learning situation since they are exposed to Information Communication Technology (ICT) as one of their modules at the university. This is proof enough that they are techno-savvy and knowledgeable in the field of technology. Whilst studying ICT as a learning module they need to acquire technological pedagogical knowledge to be able to use technology efficiently and effectively in their teaching. The effectiveness of using technology as a support to visualization will rely heavily on their readiness in implementing the visual aspect in teaching the subject. They need to demonstrate how this technological pedagogical knowledge can be used to show how innovative learning environments can be created in the classrooms.

One of the pre-service teachers indicated that they will also use games. Games, especially board games, are inherently visual and support the learning of mathematics. Integrating the lesson using games can drive active engagement amongst the learners. As they strive to win they are also learning valuable techniques from each other. This valuable technique can help learners develop their language and problem solving skills.

The pre-service teachers had to provide reasons why they used these technological resources to support their teaching.

Pre-service teacher: *it serves as a visual guide which the learners can always refer to.*

To make the language of mathematics more understandable, providing the learners with visual support develops their understanding of mathematical concepts. Problem solving strategies produced diagrammatically can prove a valuable guide for future problem solving activities.

Pre-service teacher: *these aspects will make the learners remember and gain a better understanding of the content.*

In order for learners to understand better, the learning activities need to be visually aesthetic and relevant to learners. The visual medium will engage their level of thinking allowing them to learn and understand in a fun way. This kind of learning improves their understanding of mathematical content.

Pre-service teacher: *accommodate inclusive education all types of learners, learning needs and styles.*

Various aspects affect the learner's ability to grasp the steps involved in solving problems. The pre-service teachers need to be aware that South African classrooms have learners of mixed academic ability and amongst them are learners who learn visually. These learners learn best by drawing diagrams. To further accommodate the learners, they should be exposed to diverse opportunities for speaking, explaining, reading and writing to construct meaning of the mathematical problem.

Pre-service teacher: *it keeps the learners focussed in the lesson and it is more engaging as well.*

Knowledge creation has a starting point when learners are exposed to working with concrete material, seeing or listening to things in the immediate environment. The pre-service teachers were asked: Which visuals are used when solving mathematical problems? Majority of the respondents mentioned that they mostly used mostly charts. These visual learning resources are readily available at most schools as schools buy subject related and content related charts from their learning and teaching supplementary material (LTSM) allocation. Teachers are also expected to supplement their teaching with additional resources and the most common of them all is the use of charts.

Pre-service teacher: *definitely writing on the board.*

This is one of the oldest of all educational media but still the most valuable medium to teach. Chalkboards are a permanent fixture in all classrooms and can be used creatively by the teacher to teach and consolidate mathematical concepts. Leaners thoughts, as mentioned by them, can be written down as the lesson develops. Whatever is written or illustrated on the board can be seen by all learners. The teacher can create a visual medium at an appropriate time to highlight significant points of the lesson and an explanation can be provided alongside to draw learner's attention to what is being studied.

Pre-service teacher: *Videos, provided that the videos must be appropriate to the content you are teaching.*

Teachers and schools have become technologically advanced and have the necessary equipment to source material to teach mathematics. The availability of videos to enhance teaching has enabled teachers to engage in a wide variety of content related material as a visual means to solving mathematical problems. Teachers, pre-service teachers to be included here, must ensure that the video material sourced on how to teach learners to solve problems must not lead them or their learners to become dependent on watching the material to solve problems.

Twenty first century education demands that we provide learners with a sound academic education together with skills that will prepare them for a full and productive life. There must be a shift away from the traditional teaching to ensuring our learners can read, write and become critical thinkers. Authentic learning needs to occur in the classroom and visualization gives the learners an opportunity to organise their knowledge acquisition by creating and their communicating intellectual though. Teachers at all levels, sometime or the other have used visualization in their lessons. The pre-service teachers were asked: what is your understanding about visualization?

Pre-service teacher: *visualization is something that the learners are able to see and create a picture of their own to understand.*

Visualization creates a situation for the learners to see the process in their minds when they read the problem. They are able to structure the mathematical process mentally and then produce a written structure of it. This written structure is a representation of steps used during the problem solving strategies.

To visualise to create a picture stretches the learners thinking abilities as they use visual, physical and oral aspects to better show or interpret a problem which leads to knowledge building.

Pre-service teacher: *Visualization is when the learners conceptualize in their mind the problem.*

Learners develop their conceptual and content knowledge by bringing their prior knowledge into the classroom. Visualization anchors these concepts in their minds for use when confronted with mathematical issues in their lives.

Pre-service teacher: *Visualization allows them to build their knowledge on it*.

According to Kolb and Kolb (2011:61) people construct new knowledge and understanding from what they already know. This is based on their previous experience. The learners come to school from various cultural backgrounds and socio-economic conditions. They bring with them to school their own ideas of certain mathematical concepts and construct their own meaning of them. Their understanding of the concepts in the problem will show in their working of the problem thus visualization becomes the medium to consolidate their existing knowledge and creation of new knowledge.

The curriculum demands on teachers are such that they have got into the routine of 'chalk and talk.' Visualization offers an alternate change away from the traditional teaching and learning where the teacher taught and the learners solved problems using a set method. The use of visualization in solving problems is a shift away for learners from learning through memorization and swotting rules to understanding concepts better. Memorization of rules does a disservice to mathematics because learner's risks forgetting a sequential step in the problem solving process thus getting the problem wrong. Conceptualizing the problem in their minds and using prior knowledge to support understanding leads to further concretizing of concepts. This brings forth their understanding and creativity to solve the problem.

In order to become adept in their teaching in the classroom the pre-service teachers need to adopt an on-going attitude to using visualization in problem solving. Instead of teaching learners mathematics in the traditional manner, pre-service teachers need to learn on how to encourage learners to solve the problem in a manner that makes sense to them. The use of diagrams and pictures was at one stage a dominant feature in mathematics lessons and pre-service teachers themselves have learnt the importance of using such visual mediums during their teaching practice.

Pre-service teacher: *Visualization is when one will use illustrations to present to the learners so that they can see how certain mathematical concepts are explained*.

This indicated that visualization plays an important role in mathematics. Visualization is used to explain concepts in an illustrative manner thus making it easier for learners to understand them. As learners are able to visualize concepts differently they are in a position to use their own methods to show how the maths problems are solved. When using visualization, the learners actively engage with the concepts to make sense of the problem and this intensifies their understanding of mathematics. They are able to represent the concepts, explain the methods and communicate all of this simultaneously as they are able to picture the mathematical process. This is authentic learning in practice as they are in a space to express their solutions differently. When learners convey their ideas through means of diagrams and pictures it indicates their cognitive responses to the problem as visualization assists in the depiction to convey ideas, concepts or methods.

The responses provided above by the pre-service teachers indicated that they understood the concept visualization.

Using visualization allows the learners to proficiently transfer their acquired knowledge to new situations. When asked, do you think your learners perform better when the lesson is supported by the above (visualization), the respondents were all in agreement that learner's performance improve with visualization.

Pre-service teacher: *yes, they will perform better. Visualization will help them put down what they are thinking and as a teacher you can gauge whether they are on the right track and what they are thinking about, how they are thinking about a problem.*

When learners translate their thinking onto paper the teacher is in a position to assist the learners rectify any misconceptions. Through scaffolding the teacher is a position to engage with the learners to create a better understanding of the concepts and steps used in the problem solving process.

Pre-service teacher: *yes, because I teach in the Intermediate Phase and these learners mainly learn and grasp certain aspects through the use of visuals.*

Researcher: Why do you mention through the use of visuals?

Pre-service teacher: *Learners who do not have English as their home language will be able to relate to pictures and diagrams.*

It is well documented that learners in South Africa have language issues when it comes to teaching and learning. The teaching and learning situation in schools is further compounded because most learning materials, textbooks and work books, are written in the English language. This inhibits the learning process. Using visualization as a teaching medium puts information into a better perspective. The visual information used by the teacher becomes easier for learners to understand allowing them to have a clear picture of the problem. The concepts in the problem are learned best when they are presented through a visual medium as it will make them understand and grasp the concepts quicker and better. It will enable the learners to make meaning and understand mathematical concept better because there is a visual to guide their thinking. Understanding the concepts correctly will enable the learners to select an appropriate problem strategy.

Pre-service teacher: *Yes, it will attract the learner's attention therefore they will engage in learning*.

Learners are able to work visually with mathematical problems making it more interesting and engaging. The visuals keep the learners motivated and engaged in the learning process. They are able to engage in the learning by creating illustrations as they progress. By allowing the learners to engage with visual representations, they are given an opportunity to use their creative thinking thus making the learning process an active one. With the lesson being interactive, there is a free flow of knowledge from the learner to the teacher and not vice versa.

According to Boaler (1997) success in traditional mathematics had rested on learning and using rules but that has all changed because mathematics has now become explorative and thought provoking. Teachers need to realise that visualization is necessary in mathematics because it presents a clearer understanding rather than just listening to the teacher talk about it.

Pre-service teacher: *Yes, they would because they will be able to see the content and it's more concrete for them*.

Having in-depth content knowledge, which is the fundamental component of teaching knowledge, is indispensable to teaching. A teacher needs to have this essential knowledge so that the learners can understand what you are talking about. By using and representing appropriate examples, learners have a visual of the content or concepts you are teaching. The relationship of seeing and understanding the visual they create a mental model. This will impact on learner achievement in the classroom as the association of the represented concept with the visual, the concept becomes embedded in the learner's memory as concrete knowledge and they will remember it. When teaching mathematics, teachers need to consider their teaching methodology and their learners learning styles. Learners learn differently, namely, some learn visually and others through auditory means. When choosing their teaching methods teachers need to understand that learners are exposed to a wide variety of visual material on a daily basis and what learners see they internalize it. Visual learners are interested in pictures and diagrams. They need to see the information presented to them because the pictures assist the learners to see the link with the mathematical concept and consolidate their understanding. This in turn concretizes the mathematical concepts and improves their content knowledge.

Pre-service teacher: *Yes, whatever types of visualization is used they become interested hence they become more curious and it will assist them in their understanding*.

In Polya's model of problem solving there is a stage related to understanding the problem. Visualization is associated with images and creation of diagrams that supports the understanding of ideas in problem solving. Curiosity and inquisitiveness makes learners want to know more. This increases their brainstorming ability as ideas are created during this phase. This helps learners visualize what is happening. They create mind maps or thinking maps. The creation of such maps leads to the visual synthesis of mathematical ideas thus making it easier for them to understand and learn. Through the visualization process the mathematical ideas are created as mental pictures in their minds. These mental images make understanding easier as learners make connections to what is happening and generate strategies to obtain their solution. The learners are in a position to apply simple problem skills or strategies to problems so that they become more accomplished at solving problems skills in later stages. To facilitate the application of this embedded knowledge, the learners will be able to engage their memory when recalling the association of these concepts for future learning experience.

Problem solving must not be seen as learning to drive a car from point A to point B and to point C. When learning to drive the pre-service teachers will come to realise, like solving problems is not straight forward, that there will be twists, turns and potholes on the roads. According to the Structural Learning Theory when learning to solve problems learners need to be taught the basic skills to assist in finding the simplest solutions. The learners learn the lower order rules which assist in conceptual understanding. Once the learners have mastered the basic skills and lower order rules in problem solving they will be in a position to solve problems requiring higher order thinking. Pre-service teachers, like driving instructors, have to use their knowledge, in this instance their pedagogical and content knowledge, to guide the learners in the learning process. One of the key twenty first century skills that learners need to acquire is to solve problems. The pre-service teachers were asked: do you consider problem solving an important aspect in mathematics? All of the respondents agreed that problem is important in mathematics.

Pre-service teacher: *It allows for critical thinking.*

Engaging learners to think critically is an important aspect to becoming a successful problem solver. It moves the learners away to thinking abstractly. It develops their cognitive ability and heightens their mathematical reasoning power.

Pre-service teacher: *Yes, builds not only critical thinking but also builds cognitive skills and it is used in everyday life. As well as it will help learners understand and build concepts.*

Today learners are exposed to a myriad of real world problem situations in the classroom and they are expected to think critically, make decisions and communicate their thoughts as a solution. What they learn as learners will point them into adulthood as they will be able reason with understanding.

Pre-service teacher: *Yes, problem solving teaches learners to think out of the box. Problem solving helps to apply what they have learnt to a scenario and this shows the teacher whether the learner has understood certain content of that section*.

Teachers when providing a question show the learners the steps involved to solve the problem. It is in this way that learners make connections when provided with similar problem at a later stage in their lives. According to experiential learning the teacher guides the learning process. They must design and plan the mathematical lesson such that it assists the learners to gain knowledge and skills from within the content been taught. The acquired content and skills will enable the learners to apply it to various other mathematical scenarios.

Pre-service teacher: *Yes, without problem solving no maths sums will be able to be solved.*

Traditional mathematics revolved around talk and record. The teacher spoke and the learners recorded the endless use of formulas which became redundant in learner's lives once they finished their schooling career. The learners needed to master the rules to learn the content material being taught and this made mathematics as a subject unattractive. However, modern mathematics has placed demands on the learners to acquire the necessary problem solving skills to function in this fast changing world. Learning to problem solve builds their understanding. Understanding motivates the learners to expand their skills which lead to its application in problem solving situations. In a nutshell the acquisition of mathematical skills provides a pathway to new learning.

Pre-service teacher: *Learners will be encouraged to pursue different ways to solve the problem.*

In traditional teaching learners used their algorithmic knowledge procedural knowledge to solve problems. There has been a shift in the modern classrooms where learners have to use both algorithmic knowledge and procedural knowledge to explain their solution with drawings and diagrammatically. In order to improve learner's problem solving ability the pre-service teachers need to bring innovation to teaching the subject. This involves teaching mathematical strategies, methods and using visual resources to build learner's knowledge and skills for beyond the classroom. This method of teaching will enable the learners to use their cognitive ability to collect the information from the problem and think abstractly.

The teachers become facilitators in the teaching and learning process. The pre-service teachers should acquire teacher knowledge on how to assist learners who are experiencing difficulties to solve problems. They must have the ability to lead the learners through the question in sequential order.

Pre-service teacher: *it will assist learners reading ability as they will read mathematics on their own.*

This can be debateable. I agree that learners need to be challenged to work independently when solving problems but not having the necessary language proficiency can be a major contributing factor in problem solving. It is common knowledge that South Africa has a high illiteracy rate and many of the learners who are still in school will further contribute to this number. It is a proven fact that in South African schools, learners who exhibit reading difficulties will also have language difficulty. Those learners who have difficulties with reading are likely to encounter difficulties with problem solving. Reading of the question leads them to comprehend the problem. Visualizing the problem is reliant on successful comprehension. When reading to the learners, the pre-service teacher must break up the question focussing on the key words which are the concepts. The Frayer Vocabulary Model can be used constructively as the key word recognition strategy increases the learner's comprehension of the mathematical terms and concepts. This is essential so that the learners are able to recognize and interpret the words correctly, analyse the mathematical processes or operations in order to construct mathematical representations of the problems. Through the use of visualization learners are able to demonstrate their comprehension ability of the problem and provide a solution. It is in this manner that they construct their own knowledge as visualization enables them to see the problem in totality. It is in this context that the pre-service teachers need to acquire subject content knowledge. Understanding the content and the inter connectiveness of reading, comprehension and visualisation the pre-service teacher will be in a position to strengthen learners understanding of the problem.

Mathematics problem solving, due to it being a pariah in the mathematics curriculum, will continue to be an insurmountable challenge to both the learners and pre-service teachers. The quicker the challenges are met the better for the field of mathematics. In this regard, the pre-service teachers were asked: how often do you TEACH problem solving in the class**?**

Pre-service teacher: *To be honest sometimes.*

Pre-service teacher: *Not always.*

The learners need to engage in problem solving on a daily basis. According to the CAPS problem solving is part of the everyday lesson. It is mooted in CAPS that problem solving is be taught daily. When this occurs the learners will be directly exposed to problem solving as a daily cognitive learning activity.

Pre-service teacher: *I would say that in any lesson one should have a problem sum because you know problem solving is a necessity in the aspects of mathematics.*

Since problem solving lends itself to every strand within the mathematics curriculum pre-service teachers can take the initiative to learn to integrate, teach and nurture problem solving. They need to inculcate a good habit of teaching problem solving as a means to develop learner's skills and increase their knowledge. In this way learners will learn from the teaching process and put into practice the knowledge and skills they have learned. According to experiential learning learners will become motivated to learn when teachers pose problems and involve them actively in the learning process.

Pre-service teacher: *I think problem solving, well is done in every activity after the content is taught.*

There is no set problem solving lesson nor is there a problem solving curriculum. It is possible that the pre-service teacher has an incorrect notion that one has to teach the content before engaging learners in problem solving. The mathematics curriculum is structured in such a manner that it allows the teacher to engage with the learners at any time. The given mathematical problem can force learners to engage with in many ways. They need to display the ability to solve the problem.

In order to teach pre-service teachers should possess subject content knowledge and knowledge of instructional strategies as these are vital to learner's academic achievement. With this in mind the pre-service teachers were asked: do you teach problem solving strategies? The majority of the pre-service teachers indicated that they did. They were given an option of stating *yes* or *no* and if they stated yes they were asked: name them and state why use them. I needed to know if they knew any strategies thus I asked them to name the problem strategies they used. It was here that I discovered there were grave misconceptions of what they considered were problem strategies.

Pre-service teacher: *The most common strategy I would use is the cube strategy where the learners will have to follow the system of underlining. So I will use cube.*

Pre-service teacher: *for example the concept of money. That's a problem solving strategy.*

Pre-service teacher: *Teaching them money as they will be using it in everyday life like going shopping and probability as well.*

The above responses indicated that they lacked the general content knowledge on problem solving strategies. The '*cube strategy*' or the '*concept of money*' is not problem solving strategies. Not stating any strategies indicated that the pre-service teacher's lacked pedagogical mathematical knowledge. I question the pre-service teachers ability to equip learners with the necessary skills to problem solve if they themselves do not know any problem solving strategies or do have the pre-requisite knowledge to teach the strategies. Where do they acquire such mathematical problem solving strategies to build their knowledge? As the course modules in their teacher training programme do not teach them these strategies I raise concerns about the quality of teachers we will be putting into the classrooms. In-depth mathematical knowledge and competence to problem solve is necessary to teach the subject. Not knowing how to teach strategies will put the learners at a distinct disadvantage. It is therefore crucial that these pre-service teachers be trained on how to teach problem solving strategies to obtain success in mathematics

Pre-service teacher: *yes, they can draw a visual like a diagram to help them actually gain a better understanding of the problem and then solve it.*

Drawing of diagrams is a common feature in mathematics. This technique allows the learner to comprehend diagrammatically. Using a drawing strategy (visual representation), allows the learner to see the relationships between the problem and the drawing. The diagram can be used to break down the question as a basis to understand the problem before the solution becomes known. It assists the learners to enhance their understanding by creating a bigger picture in their minds as they create logical steps to solve the problem according to what was drawn.

Modern mathematics no longer involves routine processes but rather an amalgamation or integration of curriculum material including technology. The pre-service teachers were asked: how do you use visualization techniques when working with problem solving. Teachers are good at talking as a result the mathematics lesson tends to become chalk and talk, hence this response:

Pre-service teacher: *I will use charts to basically represent to show them the step by step how things need to be done.*

When using the teacher centred approach, effective teachers spend more teaching time demonstrating to their learners how to solve problems. It is a form of guided instruction which serves as a framework for learners to follow on how to solve the problem using a logical method. The learners follow the explanation of what to do and how to do when solving problems. This type of teaching is not conducive to engaging learners in problem solving. The inadequate instructional knowledge on how to use a visual in their teaching compounds the teaching and learning process.

Pre-service teacher: *I will use charts and the chalkboard. I will write down the steps for the learner's to understand better.*

The use of teaching the steps to solve problems is related to following of logical steps in Polya's model of solving problems. This is discussed elsewhere in this study. Whilst some researchers see the need for learners to emulate teachers on how to problem solve, this does not give them the opportunity to work independently to conceptualize concepts. Although the idea of using charts as a visual aid is beneficial, the need to borrow ideas from the teacher to solve the problem impedes their level of thinking.

Pre-service teacher: *I use them in the form of charts to show them the steps or strategies.*

The above responses indicated that the pre-service teachers intended to show the learners the various steps involved in solving problems. Many learners who struggle to read the question would be able to follow the steps shown in the chart to process the information. To solve the given problem the learners must essentially identify what is needed in the problem, follow sequential steps and then write out the solution. This type of learning is to support the learners. According to Vygotsky it enables the teacher to work in the learner's zone of proximal development which is leading them from the unknown to the known. At times the complexity of problem will necessitate teacher support and will also dictate the need for collaboration amongst the learners to demonstrate their expertise but to become adept problem solvers learners need to work independently. This type of independence develops learner cognitive abilities as they are left to their own devices to come up with a solution.

Pre-service teacher: *Visualization techniques will be probably using a piece of paper to breakdown the problem and more or less write down notes or draw pictures whatever the learner feels like doing which represents the problem in the way they see.*

It is not enough to simply tell the learners to solve a problem without checking what they are doing. This can only be done when learners externalize their solutions to show their own methods. This kind of learning, externalization of the solutions, takes place best when learners are given opportunities to express their ideas. The above response indicated that the pre-service teacher encouraged the learners to use any picture and also make notes next to the picture. This gives an overview of the learner's thoughts in pictorial form and the notes alongside it indicate how they developed the answer. The teacher is in a situation to see what visualization techniques were used and also understand the way learners are thinking. The use of visualization will enable the teacher to see what the learners are doing and if the learners are approaching the problem incorrectly the teacher will be in a position to support the learners. Before providing any support the learners must be asked to explain why they are doing. This is to rectify any misconceptions. For those learners who become stuck at a certain stage must be prompted to continue. This will help their self-confidence grow as they experience the success in learning by showing their expertise when providing the solution.

Presentation and communication skill is vital in mathematics. Teaching and learning should be such that all knowledge and skills learnt in the classroom are done in a meaningful and functional way. Learner participation in the lesson is central to all learning as this is the only manner the teacher acquires learner knowledge. Opportunities must be created for learners to join in the learning situation so that the teacher is in a position to determine learner's conceptual and procedural knowledge. The pre-service teachers were asked: do you allow learners to discuss their solutions with each other in classroom?

Pre-service teacher: *Yes.*

Researcher: *Do you think this is beneficial to them?*

Pre-service teacher: Yes.

Researcher: *Explain.*

Pre-service teacher: *By sharing ideas they get different insights and different methods of working out.*

The discussion of the solutions allows the learners to discuss their ideas with each other thus showing their conceptual understanding. According to Kolb and Kolb (2011:43) "*all learning is re-learning*." The experiential learning theory states that learning occurs when learners are able to test their ideas and assimilate it with enriched ideas (Kolb and Kolb, 2011:43). This is important as learners are exposed to a variety of strategies as well as the idea that there may be more than one way to reach a solution. The oral communication of sharing of ideas enables a classroom climate of engagement with their peers. Collating other learner's ideas reinforces their conceptual understanding and this can used at a later stage. The acquired problem solving methods (strategies) and ideas are stored in the short term memory. These can be recalled to assist them in arriving at new knowledge when confronted with a similar or different problem.

Pre-service teacher: *Yes.*

Researcher: *Do you think this is beneficial to them?*

Pre-service teacher: *Yes.*

Researcher: *Explain.*

Pre-service teacher: *Learner's will be able to get different viewpoint from one another so it is like peer helping each other as well as they will be able to build on prior knowledge and listen to their peers talking and they gain their own different perception on the topic.*

Active learning through discussion allows the learners to experience the mathematical process. Active learner participation is akin to using the learner centred approach in the classroom. In the learner centred approach the learners are fully and actively involved in the learning process because they are responsible for their own learning. Through collaboration with other learners they generate their own understanding of the content using their prior knowledge This kind of learning activates a lot of ideas with '*peer helping*.' Peer interaction is important as learners are able to support each through cooperative problem solving. The learners bring their own prior knowledge into the classroom. Participating in conversation with their peers, learning and discussing of new words can develop a learner's vocabulary. This kind of collaboration through learning around the sharing of knowledge will result in better understanding of concepts. This results in differentiated thinking. This sharing of information will improve their mathematical understanding and motivate them to participate actively in the lesson. Their confidence is boosted resulting in them in answering questions and working independently later on.

The lesson must be prepared in the context of the learner's socio-cultural background and intellectual capabilities to ascertain the level of the learner's knowledge. The selected content material taught using the learner centred approach will allow for a better interchange of information between all learners. This will result in better peer understanding as learners will be able to apply their knowledge and transfer it to the content being taught. This kind of support and collaborative attitude is crucial in the classroom as it assists the learners because they are in a position to support each other to rectify any difficulties they are experiencing. Pre-service teachers will also benefit as they too will be able acquire knowledge and skills relating to using the learner centred approach in the classrooms.

If they stated **no** they had to offer an explanation on why the learners were not given an opportunity to do so.

Pre-service teacher: *No. I do not make use of that as all learners will decide they want to come to the board so we will have the whole class fighting to see who wants to come to the board and write their solution.*

This type of situation can be used advantageously. The experiential theory, according to Kolb and Kolb (2011:43), encourages "*conflict, differences and disagreement*" because that is *"what drives the learning process*." The teacher should encourage competition amongst the learners. As a guided activity they should be called to the chalkboard to show off their solutions. The learners can be randomly selected or the different groups can nominate a learner from within their group to show their solution on the board. Learners must be told that it is learning situation and not a fault finding activity.

Pre-service teacher: *Not really. I have tried it in the past and it creates a lot of chaos. When the learners get the problem wrong they could get teased.*

In the problem solving process learners must be allowed to take the wrong turn without having the feeling of failure. The mocking of learners for providing incorrect solutions must not be accepted but learners should rather be encouraged to see how the problem can be fixed. If teasing others for the failure to provide a correct solution is allowed to happen then the learners will lose their confidence to solve problems. As teachers of modern day mathematics, we must realise that learning mathematics creates anxiety amongst learners. Therefore teachers must not show the lack of confidence in their learner's ability to solve the problem. The learners will always have this fear of failure and approach the field of mathematics with trepidation. With social interaction in the classroom the teacher must make the learners aware that sometimes they will make errors and it must be accepted as a learning curve. Encouragement from both the teacher and learners must be forthcoming at all times.

If they stated ***yes*** then the pre-service teachers had to explain if this had any advantage.

Researcher: *Are learners given an opportunity to work out their solutions on the board?*

Pre-service teacher: *Yes. They should be given opportunities to work on their solutions on the board. It allows them to be confident, they can explain themselves in many ways that one wouldn't believe they can and they also motivate other learners in the classroom.*

By allowing a learner to display the solution on the board allows other learners to identify and relate their thinking and answers. Once the learners have solved the problems they must be encouraged to share their strategies with their peers. They must co-operate with each other as they get the opportunity to reflect and share ideas. In this manner it allows them to take ownership of the construction of mathematical knowledge for themselves. The learners must be taught it is acceptable to either agree or disagree with their peers given solution. When the learners are given an opportunity to talk about their answers it allows the teacher to see what they understood to get the specific answer. It also gives them a chance to justify their line of thinking. Alternately they forge a solution by understanding their peers reasoning for their solutions.

Researcher: *Are learners given an opportunity to work out their solutions on the board*?

Pre-service teacher: *Yes. It allows the teacher to see the common mistakes that the learners have.*

There are positives to allowing the learners to work their solutions on the board as it allows the teacher to pick up any misconceptions learners have. As previously mentioned learners socio-economic and cultural backgrounds affect understanding of mathematical concepts. In an open forum discussion the pre-service teachers are in an ideal position to determine any misconception. They must be able to listen to what the learners say to make sure that they are not guessing the answer. The learners must be able to show an acceptable mathematical method and the teacher must also check if they have understood what they are doing.

The manner that the learners calculate their answer must show that they have understood the mathematical content material taught in the lesson. The pre-service teachers were asked: do learners use visual techniques/representations in their books? All of the respondents indicated that the learners used representations in their written work.

Pre-service teacher: *Yes they do. They make their own diagrams and graphs and maps and whatever else they need to solve the problem.*

Relying on algorithmic calculations does not give the teacher a true reflection if learners have understood the problem or concepts within it. When the learners read they look for contextual clues in the problem. The recognition of key words or known words in the problem relate to something they identify with. These words are translated into ideas and eventually a picture is created in their minds. This idea is represented as their understanding. The representation provides the teacher with a better understanding of the learner's solution for the problem. It also gives the teacher a wider scope to understand the learner's thinking.

The best way to learn mathematics is to be able to understand the concepts through visualization. When examining the learner's books for the purpose of this study, there was evidence that the learners were using visual techniques in their books. The array of representations used were not necessarily problem solving related. The teachers used representation, namely, basic graphs to show learners how to represent data or used fraction diagrams to show their understanding of addition and subtraction of fractions in routine problems. In cases where problem solving was attempted the learners used inappropriate methods and the diagrams indicated misconceptions or had no relevance to the given problem.

According to Boaler (2016) when we don't ask learners to imagine visually, teachers miss an incredible chance to boost their understanding. The learners need to explore their ideas. They must be allowed to use their knowledge and utilise their cognitive skills to work towards a solution. The pre-service teachers were asked: do you encourage them to use visualization when problem solving? They had to respond either NO or YES and then provide a response to support their response. The majority of them indicated that they encouraged the use of visualization.

Pre-service teacher: *We are going to be teaching it. Students will constantly make use of diagrams, make use of visuals so if they have the visuals they will be able to break down the problem and also be able to solve it. That's why I will encourage them.*

Mathematics problem solving is a complex task and no one has actually figured a prescriptive method to solve problems. Although many use Polya's four step method as a guide to problem solving or Kolb's experiential learning style, many have adopted these strategies or have designed their own problem solving strategies and included as part of these strategies is the use of visualization in problem solving. The learner's breakdown the problem using the problem strategies together with using visuals to create a better a understanding.

Pre-service teacher: *I think with problem solving because of the complexity of it you cannot use your fingers so you know you will need to draw. The learners will have to find ways best to suit them to represent the problem so they can draw the problem. So they write out a story as long as it is a representation of the problem in their own way that makes sense to them and they arrive at the correct method of solving the problem and answer.*

Researcher: Why do you ask them to draw?

Pre-service teacher: *If it makes it easier for them to understand then I will be happy with the drawings. As this is one of the best ways to express their thoughts.*

When they choose the words or concepts from the problem the learners should be encouraged to write some sort of story or provide a brief explanation of their understanding. Alternately they must be encouraged to draw what comes into their mind and then they should be asked them to use the ideas from the meanings to work out their solution.

The teaching of mathematics problem solving through traditional methods of drill and practice poses many challenges for teachers. The pre-service teachers were asked: do you think problem solving strategies is important in teaching problem solving? They were all in agreement that problem solving strategies are important when teaching problem solving.

Pre-service teacher: *Strategies are step by step processes to get your answer. So by understanding the step you will be able to apply them at any time or to other situations.*

According to the Cockcroft report (Cockcroft, 1982) problem solving is considered to be the heart of mathematic and the learner's must be given ample opportunities to apply their acquired knowledge to a range of situations. Within the mathematics classroom teachers are expected to use the content to engage their learners in problem solving activities in such a manner that they acquire a deeper mathematical understanding. The learners frequently solve problems in ingenious ways therefore they must be able to articulate their ingenious strategies. The manner they use the strategies show in their mathematical solutions which most often than not reflect their thinking. When learners are given the opportunities to solve problems they will use their own means that make sense to only them. This reflects their higher order cognitive skills and understanding. They are able to craft solutions and communicate them using an array of visuals and explanations. The pre-service teachers must recognize that simply learning mathematics content in order to teach is not sufficient. They need to develop learner's cognitive abilities by exposing them to using problem strategies when teaching problem solving. This will enable learners to put across the content in ways that they understand.

When examining the learner's books the superficiality of using problem solving strategies was noted. What was also noted was that the teachers did not do corrective work for the given problems. The conclusion that I arrived at was that the teachers are ill prepared or inadequately trained to teach problem solving strategies. It is therefore imperative that the pre-service teachers learn the mathematical problem solving strategies to avoid falling in the same quagmire as the teachers. Failing which learners development to problem solve will be negatively affected.

The pre-service teachers were asked: do you think visual strategies are used effectively in the teaching of problem solving? Their responses indicated a mixed reaction.

Pre-service teacher: *Yes, it can be effective.*

Researcher: Explain how.

Pre-service teacher: *Visual strategies are used effectively. It can be used effectively in the teaching of problem solving. Just provided they are on track with what the problem entails and it's not misleading the learners.*

When making a drawing it becomes a visual representation. The learner sees the relationship of concepts in the drawing and then attempts to solve the problem according to the diagram. The use of a problem solving strategy together with a visual guides the solution process. In order to ensure that visual strategies are used effectively in the teaching of problem solving teacher engagement is important. The learners need to know 'the what' and 'the why' of using visual strategies. This is to ensure that the representations that the learners create do not mislead them. Guided instruction on the part of the teacher is needed to ensure learners relate the problem to their visual representation culminating in a reasonable solution.

Pre-service teacher: *Yes*

Researcher: So if I was using visual strategies will I be able to demonstrate the steps that are involved in problem solving.

Pre-service teacher: *Yes. You will be able to.*

Researcher: Do you think that will be advantageous to the learner?

Pre-service teacher: *It will but at the same time you cannot deviate from the fact they may also have their own problem solving steps*.

Learners are very impressionable and they are normally drawn to aspects that hold their attention. Visual representations keep learners interested and focused on the lesson. The use of technological equipment, diagrams, charts, concrete items, games and demonstrations can hold their attention. The use of colour and sound also lends itself to visual attraction as the learners are drawn to presentations that are brightly presented. The learners at primary school level are normally stimulated visually when the teachers engage them in the lesson by using teaching resources to make it more concrete and to create a better understanding. This visual stimulation is to create an interest in a task. Therefore teachers should direct the learners in combining the use of colour to represent their visual ideas. Furthermore they should be encouraged to use their creativity and present their work using the various means of writing material at their disposal when communicating the mathematical ideas.

The following responses indicated that the pre-service teachers felt that the visual strategies were not being used effectively in mathematics classroom.

Pre-service teacher: *No. The teachers that I have observed so far they don't know how to teach themselves so basically they just come and put sums on the board and they explain the content. They using direct instruction. They are not using visual strategies in the class. They do not know how to work problem solving themselves. So no.*

The statement, that teachers themselves don't know how to problem solving, can be categorised as damning. The scenario in many schools is that untrained mathematics teachers are allowed to instil the subject knowledge to learners. This raises the concern whether we have teachers of quality in the profession. These teachers lack the necessary pedagogical skills for teaching the subject which inadvertently negatively affects the teaching and learning process in the classroom. To avert a further crisis, teacher training modules must link the pedagogical knowledge to classroom practices.

Due to the age cohort, as mentioned by Piaget, learners develop understanding at different levels. These factors are totally ignored as the teachers do not see the advantage of using visual means to teach problem solving as they consider learners slow or it is considered as additional work. Pre-service teachers need to realise that visual means engage learners in meaningful learning. Visual strategies and tools can be used in any educational environment, discussed in any social context and also used as scaffolding techniques to allow for better conceptual understanding.

Pre-service teacher: *I will say sometimes.*

Researcher: Explain.

Pre-service teacher: *It depends on how the teacher plans the lesson and their method of teaching.*

It is important that the pre-service teachers have the relevant curricula knowledge to aid their planning of the lesson. This includes the knowledge of using curriculum resources to support the teaching and learning process in the classroom. The manner in which a lesson is taught is dependent on pre-service teacher's pedagogical content knowledge (**discussed in chapter 3**). Pedagogical content knowledge (PCK) can be described as an incorporation of understanding of teaching materials (content knowledge) and understanding the way of educating (pedagogical knowledge). Pre-service teachers need to make appropriate instructional decisions on how to use a teaching method effectively to explain the content to the learner. This knowledge needs to be owned by the pre-service teacher.

5.4.3 CONCLUSION

According to Kolb and Kolb (2011:63) the experiential learning theory has been widely accepted as a framework for life-long learning. The key concepts from experiential learning theory, namely, the learning cycle and learning style, can be used to determine the learning process at an individual or team level (Kolb and Kolb, 2011: 63). Teaching will always be a multifaceted occupation and one needs to be learned and continually improve one's self to meet the challenges of the mathematics classroom. The analysis shows a critical area of concern in respect of using problem solving strategies and visualization. Problem solving is a critical area in mathematics. The pre-service teachers need to engage with problem solving strategies with more practice. In this way they will develop and learn much of the new mathematics in the curriculum for themselves. A relationship exists where visualization supports learners to problem solve. Since in problem solving there is no clear cut solution it is possible to use visualization to put learners in another thinking level. Visualization provides additional information to the problem as it allows learners to think visually. This also builds learners understanding. The pre-service teachers are in a position to learn from their modules and ensure they are able to address the possible challenges faced by learners in the classroom.

5.5 ANALYSIS OF EVALUATION WORKSHEETS

5.5.1 Analysis of the learner worksheets

The evaluation worksheet was a problem solving activity consisting of non-routine problems. The problems were appropriate for testing the learner's proficiency to solve problems. These worksheets were designed for the learners in grade four (Annexure 6), grade five (Annexure 7), grade six (Annexure 8) and grade seven (Annexure 9). There is a lot of focus on these grades at school level and I needed to see the learner's level of understanding and use of visualization and problem strategies as they progressed through the grades. I chose these grades based on literature and my experience as a teacher. The grade four learners have progressed from the Foundation Phase to the Intermediate Phase. In the Foundation Phase a lot of the mathematics teaching takes place through visual means. Charts and the workbooks are used on a daily basis to support teaching and learning. The learners are given the opportunity to demonstrate their understanding, knowledge and skills by producing diagrammatic solutions to word problems in their books or using the diagrams in their workbooks to calculate the answers. I needed to discover if the learners continued using this kind of learning into grade four. The Department of

Basic Education had previously targeted the grade six for evaluation through the Annual National Assessment (ANA). The grade six is at the end of the Intermediate Phase. The Department of Basic Education found that the learners *"still experience challenges in providing responses to questions that require higher order cognitive skills"* and it was also found that *"learner performance in Mathematics tends to decline progressively from the Intermediate to the Senior Phase"* (Department of Basic Education, 2015:4). I chose the grade seven to determine if there was basis to this statement.

All the questions had relevance to the scope of work from the mathematics curriculum. The objective of using this worksheet was to ascertain what methods, skills and strategies the learners utilised to provide solutions to the particular problems. These questions, whilst challenging, required careful reading and understanding but were within the grade ability of the learners. These evaluation worksheets were handed to the pre-service teachers in advance to give to their learners to complete as a class activity whilst they were at schools during their practice teaching sessions. They were requested to give all the learners an opportunity to participate in the completion of this worksheet. Every precaution was taken to ensure that the learners had not been confronted with the same problems before.

When evaluating the questions on these worksheets, I examined the correctness of the solution, whether any visual techniques were used and if any the learner had used any problem solving strategy. These results are displayed accordingly under the various grades.

Table 1 shows a question-by-question statistical analysis for all four grades. I analysed the answers from the worksheets to determine its correctness.

Using the data in Table 2 and 3 together with the scanned solutions extracted from the various grades I provide a detailed evidence of learner knowledge, use of problem solving strategies and visualization when solving problems. I have also included an analysis of common misunderstandings, wherever possible, that occurred during their problem solving process.

The data in Tables 2 and 3 respectively indicated that the learners were able to use visual techniques and problem solving strategies when solving problems.

To keep this discussion in perspective I have grouped the discussion according to the grades.

Table 1 Learner performances in the various grades

	SOLUTIONS CORRECT (%)			
QUESTION	**Grade 4**	**Grade 5**	**Grade 6**	**Grade 7**
1	17	69	22	-
2	58	35	29	50
3	83	31	44	5
4	50	31	78	10
5	42	-	50	55

Table 2 Analysis of visual techniques

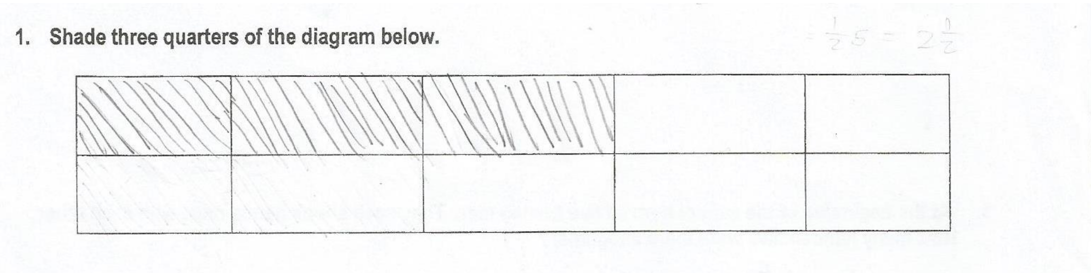
QUESTION	**Used Visual Techniques (%)**			
	Grade 4	**Grade 5**	**Grade 6**	**Grade 7**
1	100	85	28	-
2	-	46	50	50
3	67	50	100	5
4	17	23	50	30
5	42	50	55	45

When analysing the worksheet from the grade learners, four problem solving strategies emerged (Table 3). The learners were able to draw a diagram, draw a table, follow a pattern and use algebraic means when solving the problems.

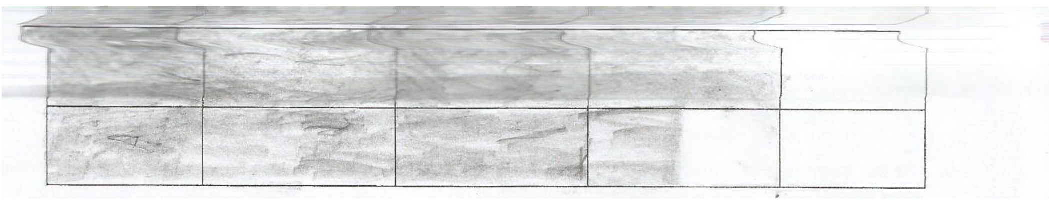
Table 3 Problem solving strategies used by the learners

Strategy	**Draw a diagram**				**Draw a table**				**Follow a pattern**				**Algebraic**			
Grade	**4**	**5**	**6**	**7**	**4**	**5**	**6**	**7**	**4**	**5**	**6**	**7**	**4**	**5**	**6**	**7**
Q1	100	73	28	-	-	-	-	10	-	-	-	20	-	27	72	70
Q2	92	46	50	-	-	-	-	10	-	-	-	15	8	56	50	75
Q3	17	83	100	5	34	-	-	-	34	-	-	-	15	17	-	95
Q4	16	-	-	15	-	39	6	10	-	33	39	5	8	28	55	70
Q5	34	27	39	45	-	19	-	-	-	27	44	-	66	27	17	45

In question one (Figure 30) the diagram was given and the learners were required to shade in three quarters.



Solution1



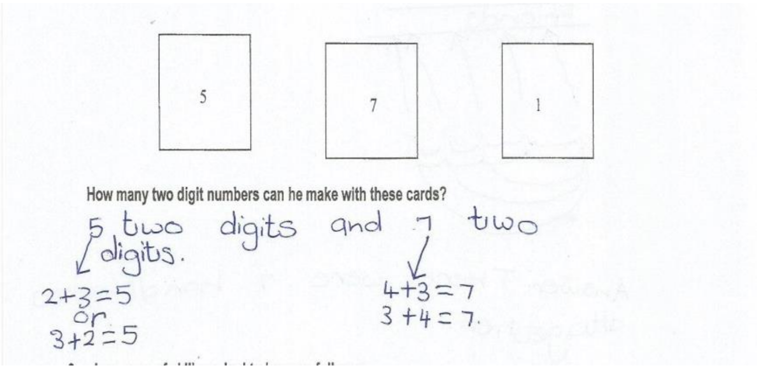
Solution 2

Figure 30 Grade 4 – learner's solution question 1

The learners had to solve this problem by making meaning of the drawing and identify the parts of the whole. With the diagram given, learners had to demonstrate their understanding of the concept three quarters. Seventeen percent of the learners shaded it in correctly (Figure 30 - solution 2). I had expected them to first recognise half of this diagram on the first row and then determine another one quarter on the second row. Majority of the learners shaded the diagram incorrectly with many of them shading three blocks on the diagram to indicate three quarters

(Figure 30 – solution 1). This indicated that the learners did not have the conceptual understanding of three quarters.

Learners are expected to demonstrate an understanding of two and three digit numbers at grade four. In question 2 the learners were given three cards as a visual medium (Figure 31) and they had to use them to create two digit numbers, namely, using the 5 and 7 will result in 57 or 75.



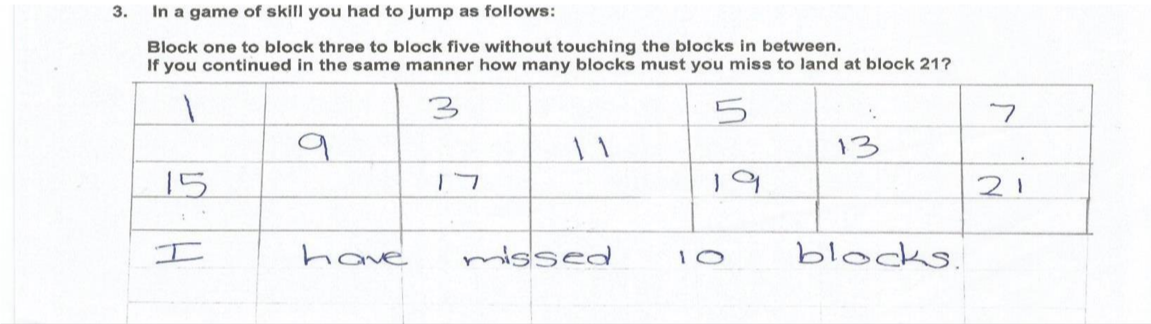
Solution 1

Figure 31 Grade 4 – learner's solution question 2

For this problem (figure 31) fifty eight percent of the learners were able to use these numbers and they provided the correct solutions. Figure 31 shows an incorrect solution to the problem. The learner attempted to devise a set of rules resulting in an inappropriate solution. The structural learning theory makes reference to learning mathematics through rules but when the rules are misunderstood and applied incorrectly it can be problematic as the learner progresses through the content in the grade. Teachers, in such situation, need to be mindful that if learners follow rules and sequence to solve problems they may learn very little other than apply it to a particular type of problem (Killen, 2015:261). Killen (2015) goes further to state that it is not an effectual way to help learners learn.

At grade four level learners are taught a section on whole numbers. Within this strand learners are exposed to the number system, namely, counting forward and backwards within a number sequence or using consecutive numbers, identifying odd and even numbers and rounding off to the nearest ten, hundred and thousand. Using their understanding of these aspects they had to apply it to answer question three (Figure 32).

Eighty three percent of the learners provided a correct solution. Sixty seven percent of those who attempted this question used a visual technique. The learners were able to solve this problem using a range of problem solving strategies (Figure 32 – solution 1, 2 and 3). The learners were able to find the solution by sequencing the numbers using natural numbers and odd numbers. The strategies as used in their solutions, drawing a diagram (Figure 32), indicated the learners proficiency in using visual means and their prior knowledge of the number system to solve the problem. The learner (Figure 32 – solution 1) drew a table and showed her understanding of ‘missing a block’ whilst the learner (Figure 32 – solution 3) showed her understanding of ‘jump’ from one number to the another missing the umber inbetween. Teachers who allow their learners to use their own strategies when solving problems are those who do not adhere to the traditional way of teaching. These solutions (Figure 32) showed the learners varied level of thinking and their ability to create solutions mentally and how to externalise them.

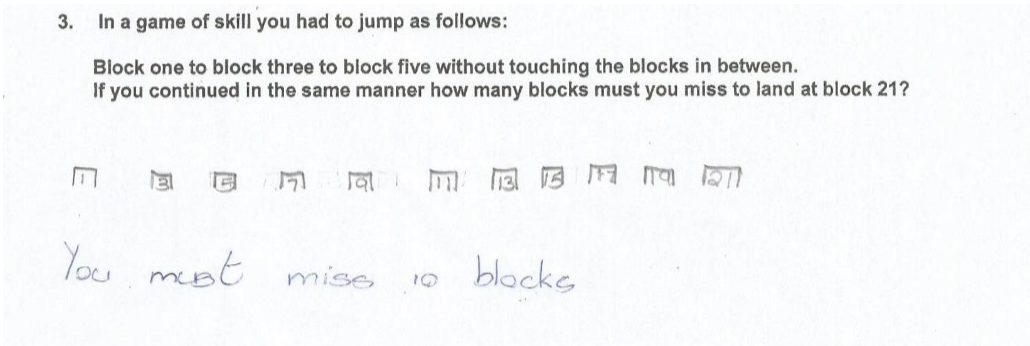

3. In a game of skill you had to jump as follows:

Block one to block three to block five without touching the blocks in between.
If you continued in the same manner how many blocks must you miss to land at block 21?

1		3		5		7
	9		11		13	
15		17		19		21

I have missed 10 blocks.

Solution 1

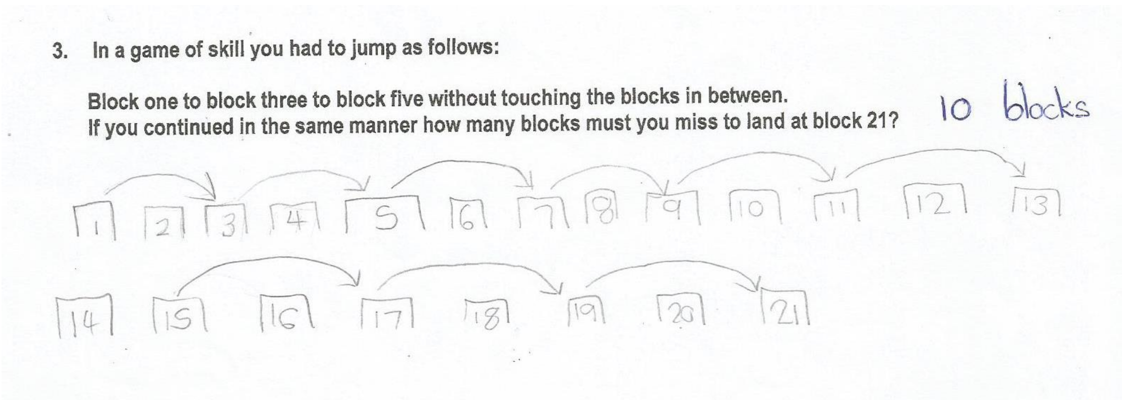

3. In a game of skill you had to jump as follows:

Block one to block three to block five without touching the blocks in between.
If you continued in the same manner how many blocks must you miss to land at block 21?

1 3 5 7 9 11 13 15 17 19 21

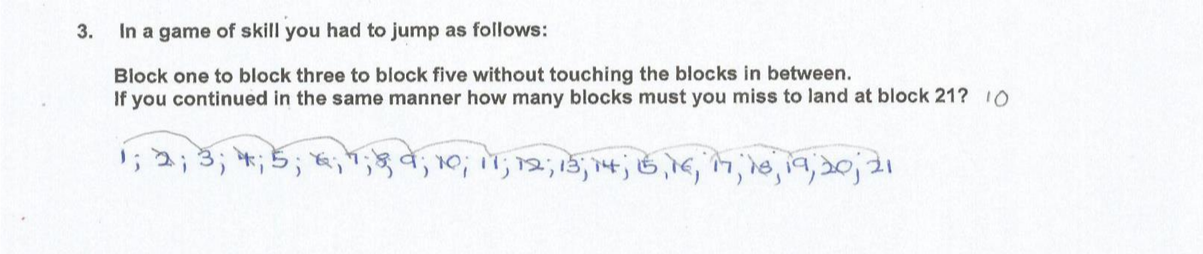
You must miss 10 blocks

Solution 2


3. In a game of skill you had to jump as follows:

Block one to block three to block five without touching the blocks in between.
If you continued in the same manner how many blocks must you miss to land at block 21? 10 blocks

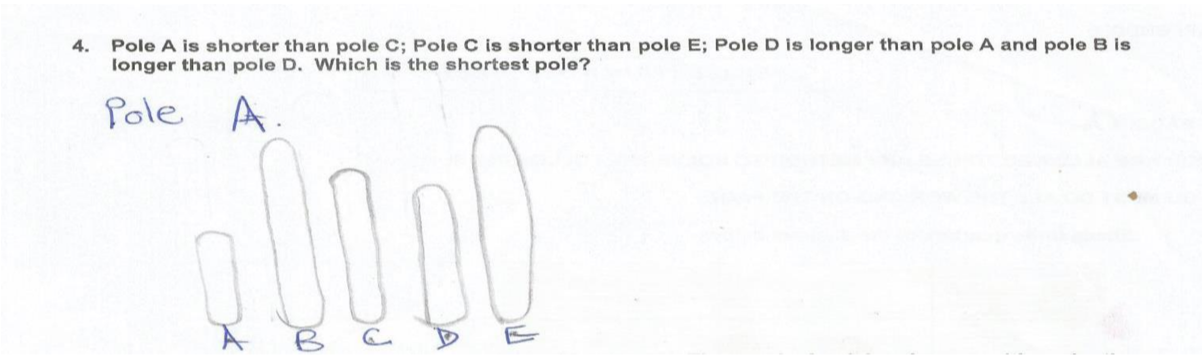
1 2 3 4 5 6 7 8 9 10 11 12 13
14 15 16 17 18 19 20 21



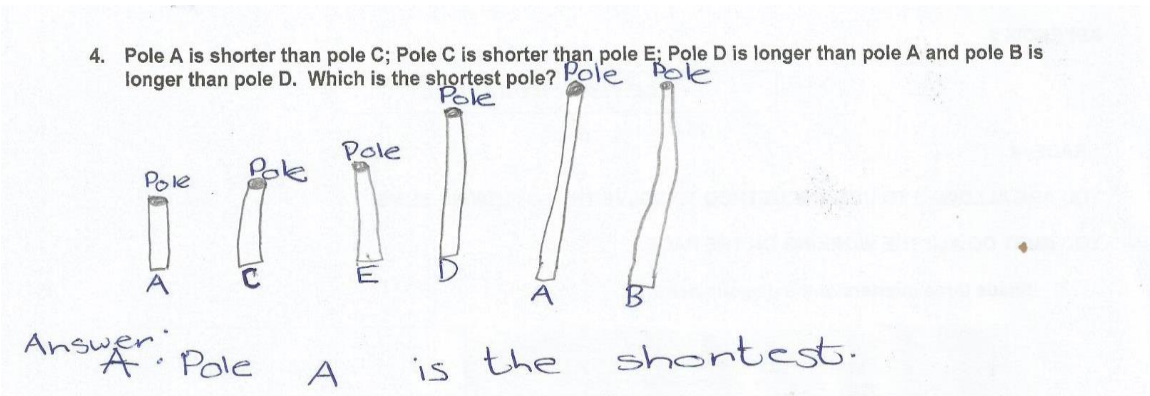
Solution 3

Figure 32 Grade 4 – learner's solution Question 3

Problem solving can help guide learners thinking in a systematic and schematic manner. Question 4 (Figure 33) needed learners to determine the shortest pole given a range of alternate heights of the other poles. Systematic thinking and a schematic representation of the poles would have provided the learners with the correct solution. Learners are made to stand in height order during assembly and I expected them to use their prior knowledge of their heights and transfer it to this question. Fifty percent of the learners provided a correct solution and seventeen percent of the learners used some sort of visual technique to illustrate their solution. Sixteen percent of the learners attempted this question by using a diagram to provide their solutions (Figure 33 –solution 1 and 2).



Solution 1



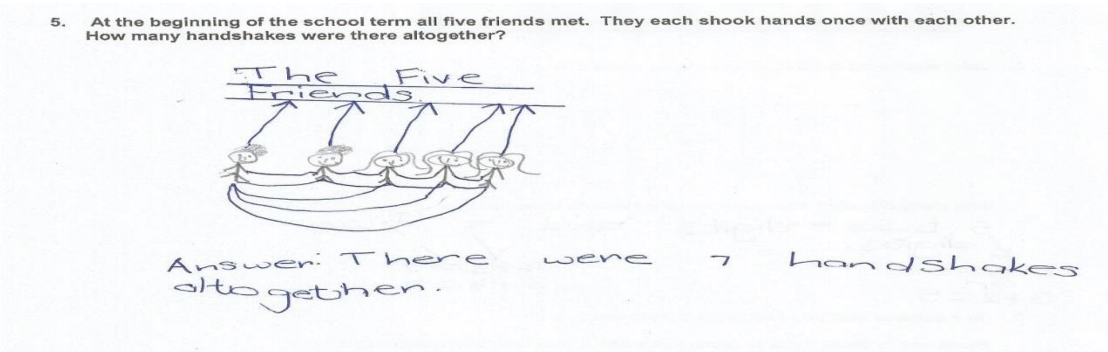
Solution 2

Figure 33 Grade 4 – learner's solution Question 4

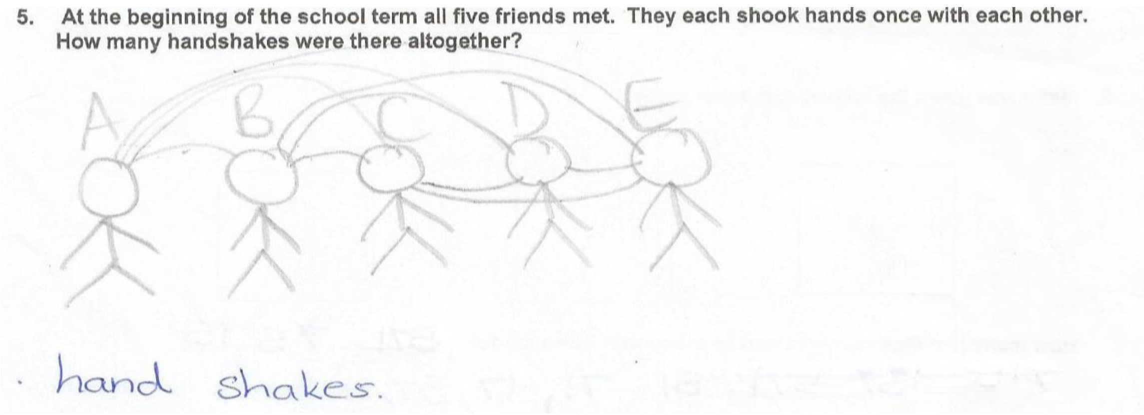
The learner (Figure 33 – solution 1) arranged the letters alphabetically but still indicated the correct answer. The poles, represented by the letter of the alphabet, were schematically arranged according to their required height. When one looks at Figure 33 – solution 2 one may find the diagram strange. The learner has represented pole A twice and still managed to state that A is the shortest pole. The learner commenced with A as mentioned at the beginning of the question and continued creating a schematic representation of the poles as the question unfolded. Pole A, mentioned again in the latter part of the question, is represented again inbetween D and B to show the height difference between them.

Killen (2015) mentioned that it is worth giving learners problems that they can relate to in the real world. The problem (Figure 34) is something that the learners can relate to in a school situation. This question can be translated into reality and the learners can figure out the answer by ‘shaking hands’ with their friends.

Forty two percent of the learners produced a correct solution and forty two percent attempted to answer this question using a visual technique. Not all the solutions with the visual techniques were correct. The learner used a visual diagram (Figure 34 – solution 1) but the solution is incomplete. The learner only indicated the first person shaking hands with the rest of the group whilst the question stated *‘shook hands once with each other’*. The inability to comprehend the problem and drawing an incomplete diagram raises the question if diagrams actually aid problem solving. The visual representation of matching the friends with the handshakes (Figure 34 – solution 2) indicated the correct interpretation of the question which led to the correct solution.



Solution 1

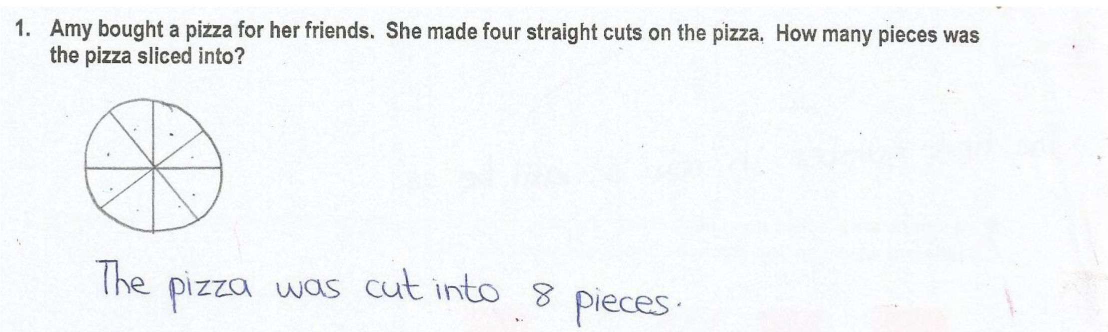


Solution 2

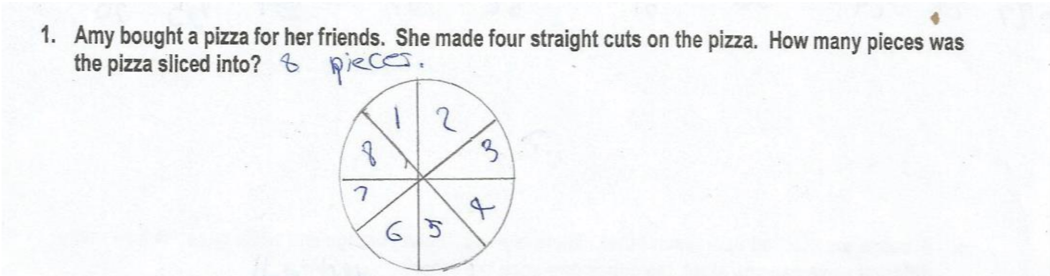
Figure 34 Grade 4 – learner's solution Question 5

The following solutions were extracted from the **grade five** learner's worksheets.

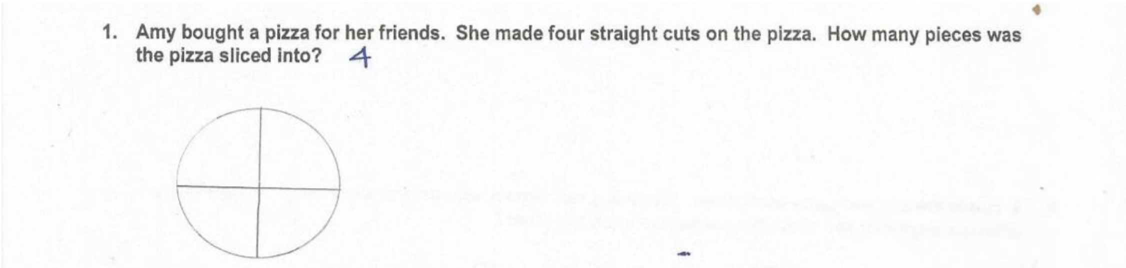
When open ended questions are posed to learners, teachers must expect a multitude of answers. It becomes more interesting when these kinds of problems are linked to 'real world problems' thus enabling learners to generate many different ways to find the solutions. This question did not state how many friends were going to share the pizza nor did it state it was going to be shared equally. This question paved the way for divergent thinking. This resulted in the learners interpreting the question differently. Question 1 (Figure 35 – solutions 1, 2, 3 and 4) produced differing strategies. Sixty nine percent of the learners produced correct solutions. Eighty five percent of the learners used a visual technique. Seventy three percent of the learners attempted this question using a diagram. We see here how the understanding of mathematical ideas developed when the learners used their real world knowledge of how a pizza is cut and served from an outlet to create their solutions (Figure 35 – solution 1 and 2). They produced a representation of a pizza to associate the image to the problem thus one can see the association of previous experience to the problem.



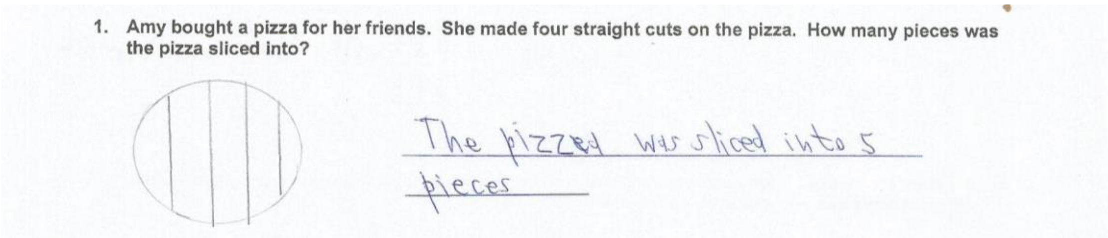
Solution 1



Solution 2



Solution 3



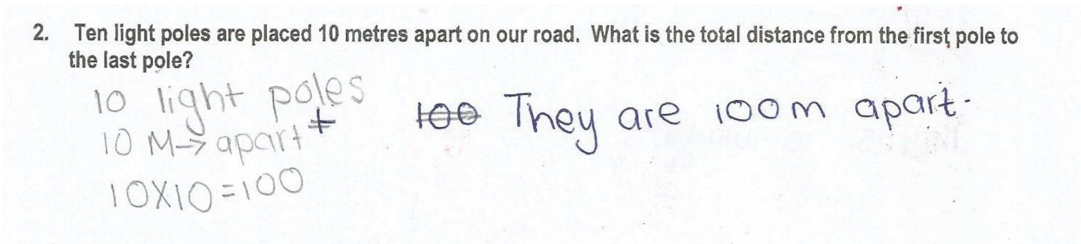
Solution 4

Figure 35 Grade 5 – learner's solution Question 1

The solution (Figure 35 – solution 3) showed the learner's understanding of the question in respect of the concept 'straight cuts'. The learner sliced the pizza into quarters. For this to occur, it meant that only two straight cuts were made. To make a point of the learner's diagram, it is possible that the learner made four cuts starting from the centre and cutting outwards thus dividing the pizza into quarters. The solution (Figure 35 – solution 4) indicated the learner's mental interpretation of the problem and the eventual solution. It is possible that the learner had prior knowledge of how a pizza is sliced but did not apply that knowledge to this question. The inclusion of the words 'straight cuts' resulted in the learner interpreting the problem differently hence illustrating such an answer.

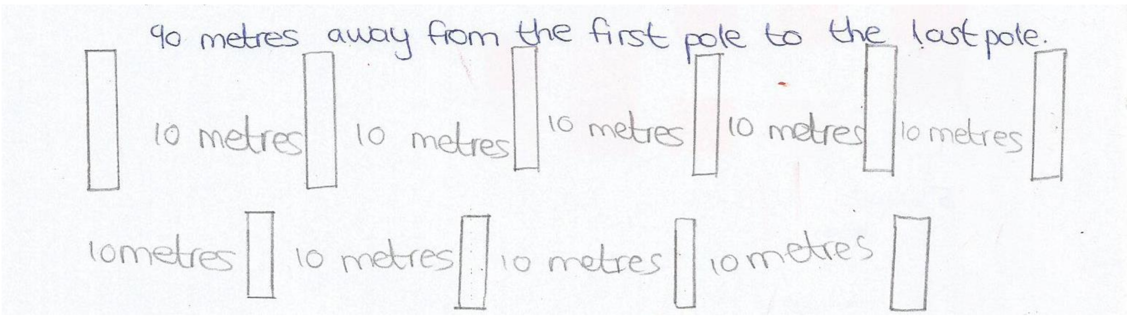
According to Killen (2015) learners can produce different solutions to real world problems only if they can bring it into realism. In Figure 36 – solution 1 the learner literally interpreted the question. The response indicated that the learner did not give this question much thought. The numbers in the problem was translated and using the multiplication sign the learner worked out the answer 100 which was incorrect.

The array of representations for the question two (Figure 36) indicated the different thinking and visualization levels of the learners. The learners were able to translate the written language into symbolic and schematic representation. Thirty five percent of the learners obtained the correct solution for question two from the learner's worksheet. Forty six percent of the learners used visual means to work towards a solution and forty six of them used a diagram to attempt a solution. Fifty six percent of the learners attempted to solve the problem through algebraic means and many produced solutions similar to those in Figure 36 – solution 1.

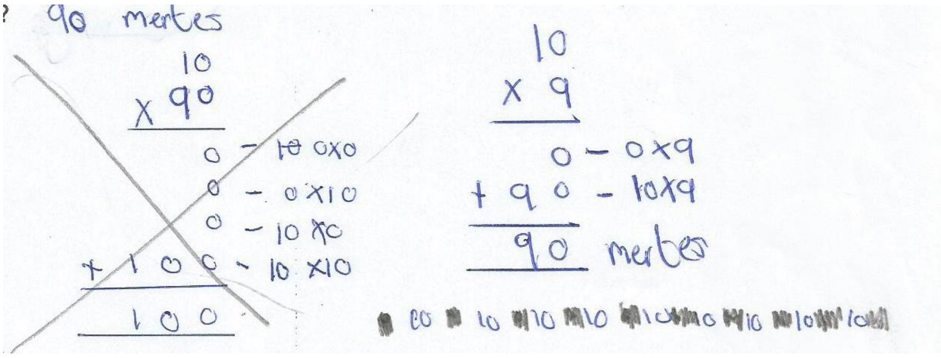


Solution1

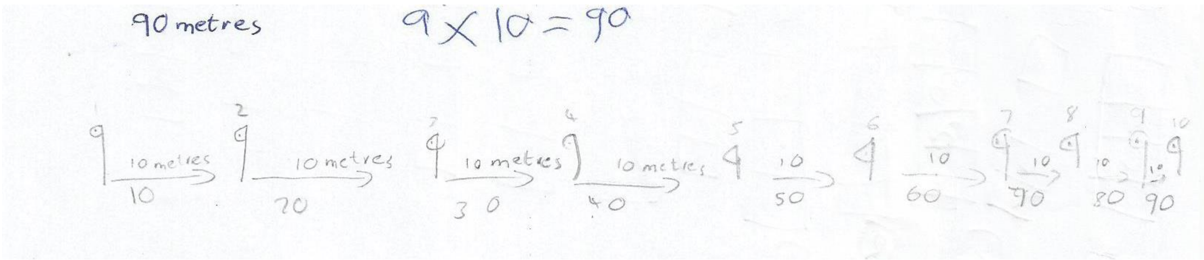
Solution 2



Solution3



Solution 4



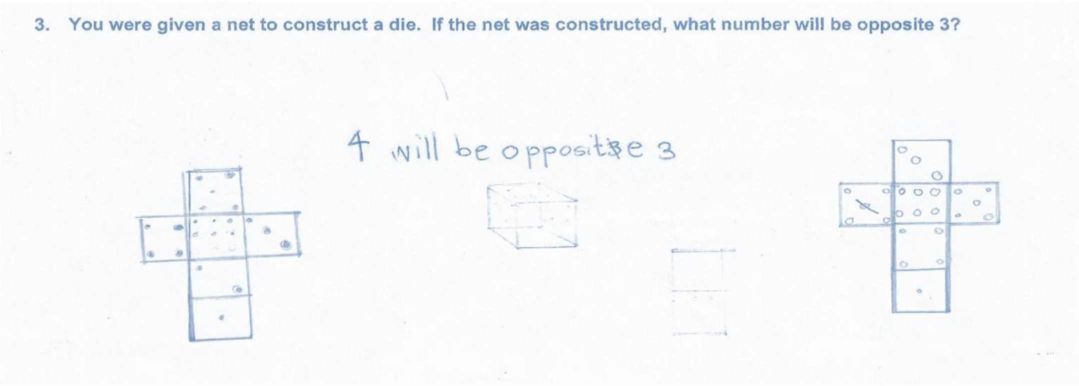
Solution 5

Figure 36 Grade 5 – learner's solution Question 2

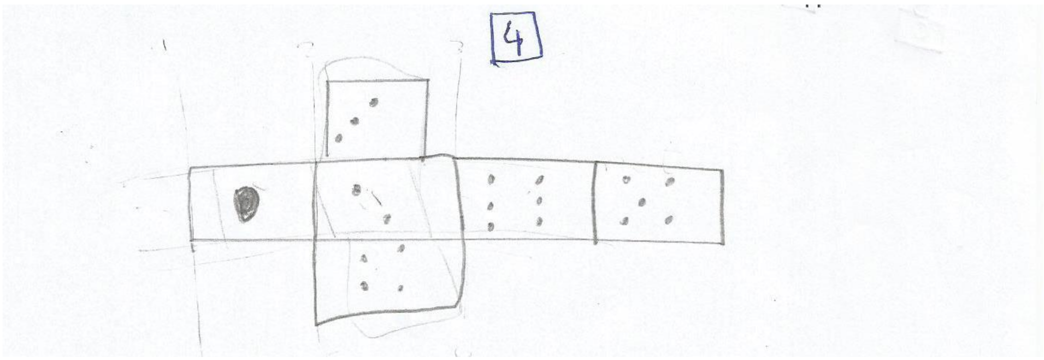
By restructuring the text with a schema and applying their knowledge of calculating distance the correct solution was arrived. All the diagrammatic representations (Figure 36 – solutions 2, 3, 4 and 5) showed the different illustrative styles employed by the learners to arrive at the answer. In Figure 36 – solutions 2, 3 and 5 the learners represented the ten poles and then counted up the spaces in-between. In Figure 36 – solution 4 the learner used the multiplication algorithm and a diagram to verify the correctness of his answer.

The solutions (Figure 37) extracted from the learners worksheet indicated that some of the learners had prior knowledge of a die. The learners were expected to show an understanding that the opposite sides add up to seven. Thirty one percent of the learners provided a correct solution to this problem. Eighty three percent of the learners used a diagram to attempt a solution.

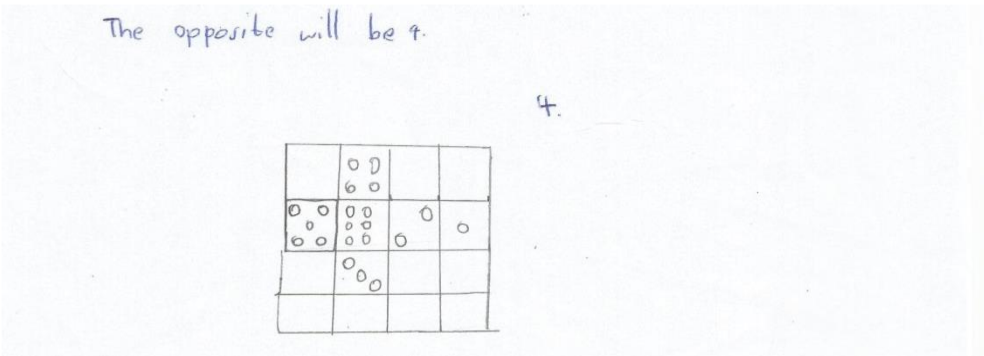
The learners have constructed 3D objects (cube and rectangular prism) when working through the content in the Space and Shape strand. The die is represented as a cube and the learners were able to use their prior knowledge of open nets (Figure 38 –solutions 1, 2 and 3) to represent the numbers. They were able to place the numbers in the correct places on the net (Figure 38 –solutions 1, 2 and 3) and when the learner folded the net it represented a model of a die (Figure 38 – solution 1).



Solution 1



Solution 2



Solution 3

Figure 37 Grade 5 – learner's solution Question 3

The learners were expected to complete the table by continuing the counting process whilst adding one additional block on either side of the next rows (Figure 38).

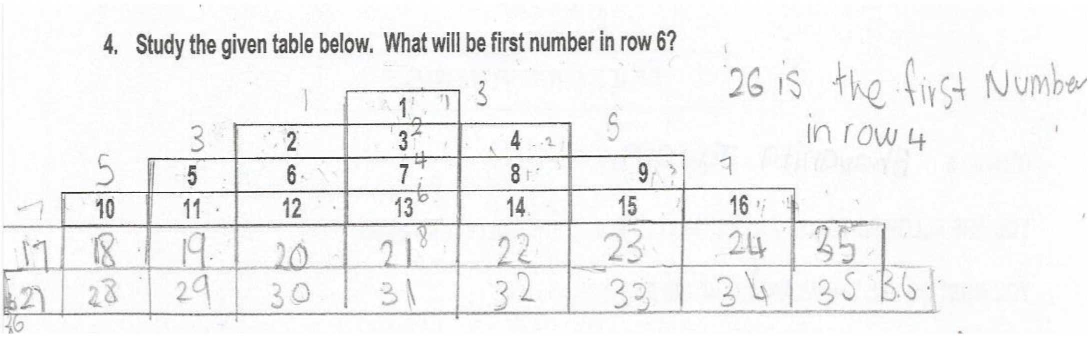
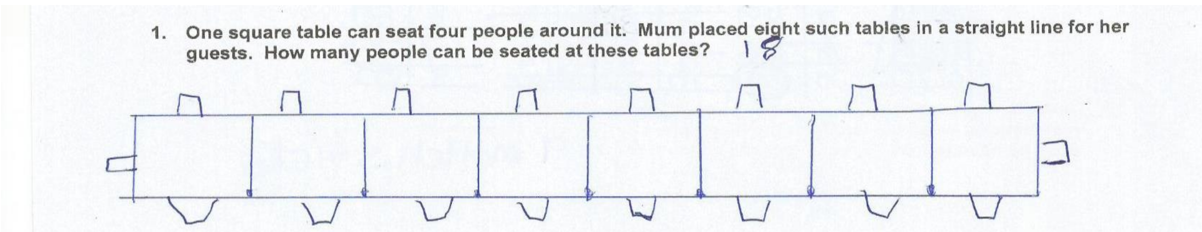


Figure 38 Grade 5 – learner's solution Question 4

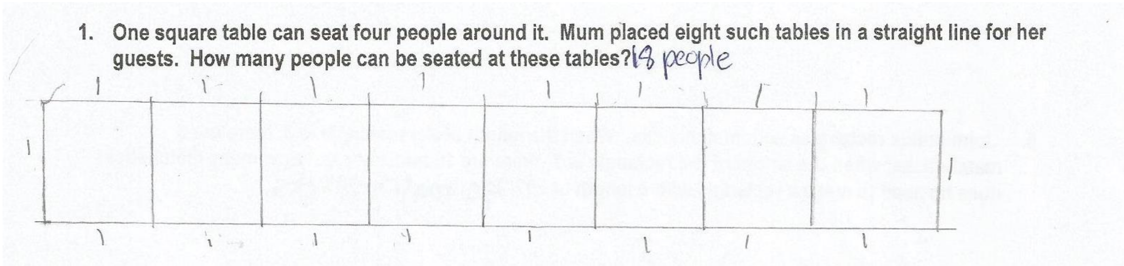
Majority of the learners were not able to complete the number pattern from left to right of the given table as a result only thirty one percent of the learners provided the correct solution. Twenty three percent of the learners used visual techniques to find a solution. Thirty nine percent of the learners drew a table whilst thirty three percent of them followed a pattern as a problem solving strategy to find the solution. A common error made by the learners was not to add on the additional block on either side of the next two rows. Some learners misinterpreted the question and only calculated the first number for row 5. If the learners examined the table carefully they would have discovered that they would have found the solution by looking for a pattern vertically and diagonally.

The following solutions were extracted from the **grade six** learner's worksheets.

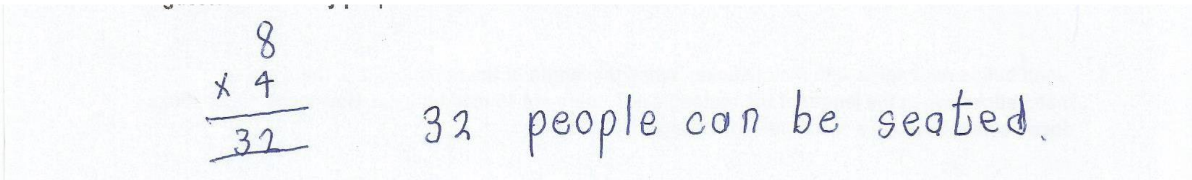
Only twenty-two percent of the learners provided a correct solution to the problem (Figure 39). Twenty eight percent of the learners attempted this problem through using visual techniques. The solutions (Figure 39 – solution 1 and 2) indicated the learner's strategic competence. They were able to generate and create an appropriate diagram to solve the problem correctly. The learners demonstrated their understanding of the question and the concepts therein to work out the solution.



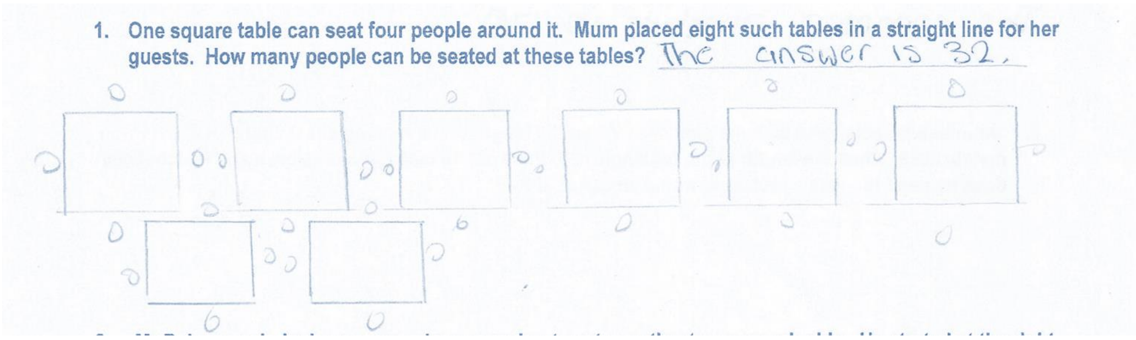
Solution 1



Solution 2



Solution 3



Solution 4

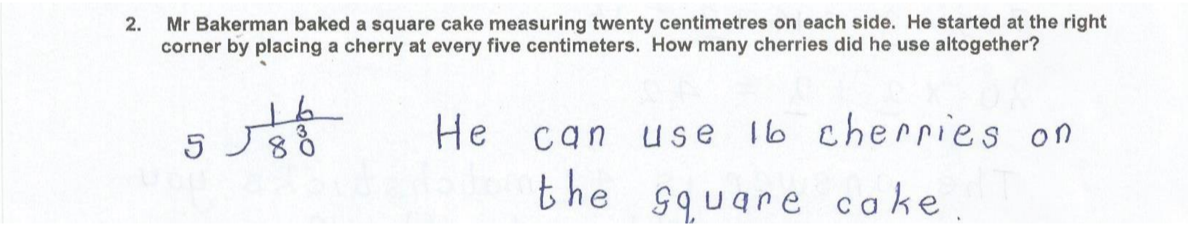
Figure 39 Grade 6– learner's solution Question 1

Seventy eight percent of the learners attempted this problem using algebraic means and twenty eight percent of them used a diagram as a strategy. The analysis of the solutions (Figure 39- solution 4) indicated a clearly defined solution was missing. In (Figure 39 - solution 3) the learner opted to translate the numbers within the problem and provide an algebraic solution. In (Figure 39 - solution 4) the learner whilst using an illustration to assist in solving the problem did not fully comprehend the question thus creating an incorrect diagram. The question asked for the tables to placed in a 'straight line'which was not done (Figure 39 - solution 4). It must be noted here the diagram denoted the learner's interpretation of the question. This question if placed in real life context of seating arrangements at a party, the learners would have arrived at a solution.

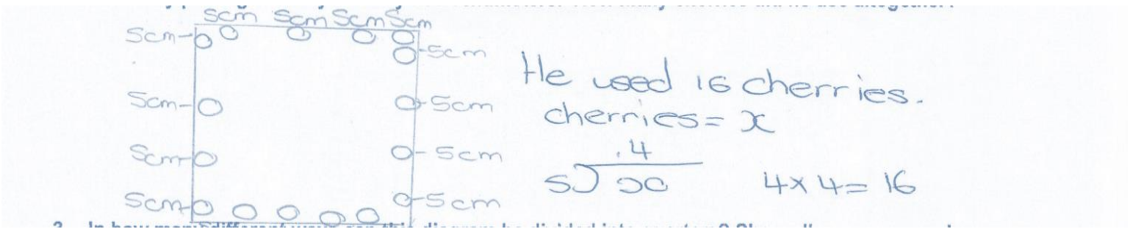
A similar problem was used by a pre-service teacher (see analysis of lesson observation) and learners there too provided a similar kind of solution. The pre-service teacher, in order to overcome this misunderstanding of a straight line, used concrete means (desks) to show the learners the idea of setting the desks in a straight line.

According to the Department of Basic Education (2015) one of the strands where learners experience difficulties was Measurement. In answering this question, I needed the learners to demonstrate their knowledge of knowing and drawing a square, identifying and using the unit of measurement as required in the question and applying their knowledge of counting in fives or using their prior knowledge of sharing. In the given question (Figure 40 – solutions 1, 2 and 3) the learners understood the problem and correctly represented the number cherries.

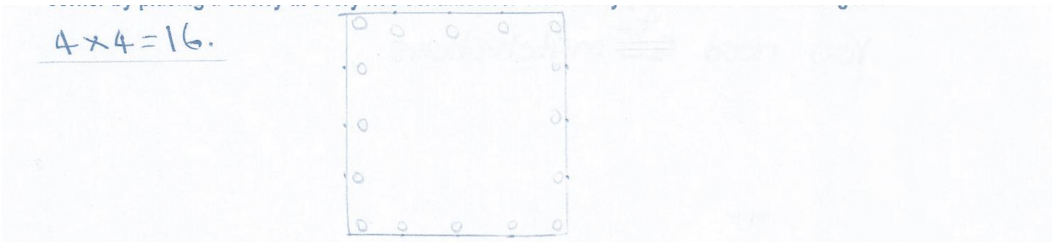
Twenty-nine percent of the learners provided a correct solution to this problem indicated in Figure 40. Fifty percent of the learners used some sort of visual techniques to represent the concept in the question whilst others used a combination of both an illustration and algebraic means (Figure 40 – solutions 2 and 3). These learners demonstrated their understanding of the question and the concepts thus applying their algebraic knowledge to work out the solution.



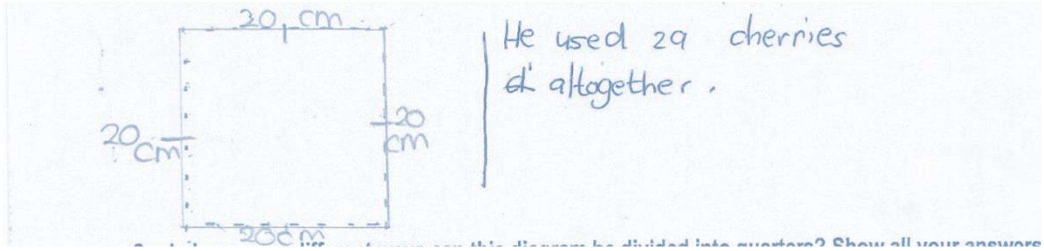
Solution 1



Solution 2



Solution 3



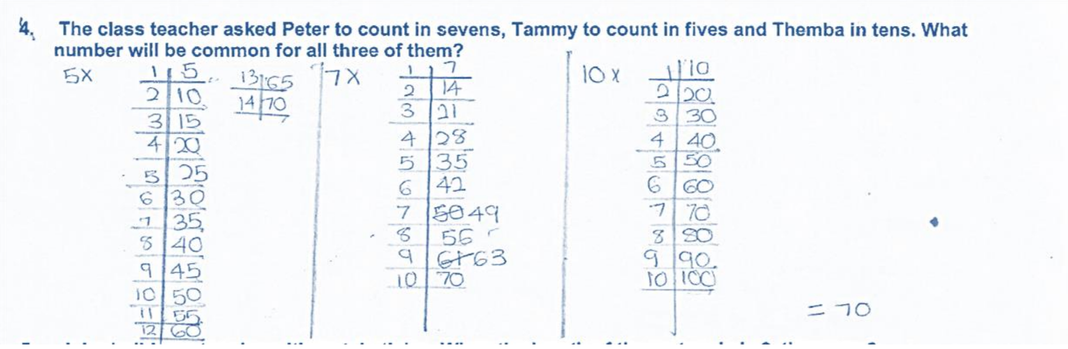
Solution 4

Figure 40 Grade 6 – learner’s solution Question 2

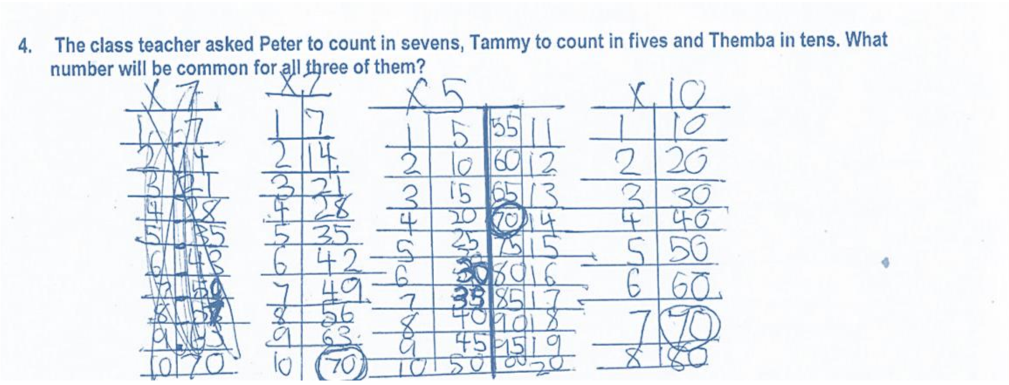
The learner (Figure 40 – solution 1) preferred to use her algebraic skills to process the solution. She used her sound knowledge of sharing and division when calculating this answer. To determine the correctness of the answer the learner used both a diagram and an algebraic calculation. The substituting of algebraic means with an illustration guided the learner in achieving the correct solution. From the above solutions (Figure 40 – solutions 2 and 3) it can be noted that the learners used both algebraic and diagrammatic means to provide their solutions. In Figure 40 (solutions 2 and 3) the learner drew a square and distributed the cherries equi-distance apart. The diagrams supported their calculations to indicate that they had arrived at the correct solution. The learner used a diagram to the illustrate the solution **(**Figure 40 – solutions 4). Whilst showing the correct measurement on all four sides the learner was not able to comprehend the problem correctly and failed to place the cherries equi-distance apart.

In the Foundation Phas, from grade one to grade three, learners are expected to count forward and backwards in muliples and apply this knowledge to complete number patterns (Department of Basic Education, 2015). Seventy-eight percent of the learners produced a correct solution similar to the one in Figure 41. Fifty five percent of the learners attempted to solve this problem through algebraic means whilst the other learners used a problem solving strategy (draw a table, following a pattern) to arrive at the answer. The learners used their knowledge of writng out their multiplication tables (Figure 41 – solutions 1 and 2) to determine their answers.

The common error made by the learners were in their counting techniques especially when counting in sevens. They lacked the knowledge of counting in mutliples of seven and they did not demonstrate their ability to go beyond 21 when counting in sevens. This can be attributed to their lack of knowing their time tables or poor number sense.



Solution 1



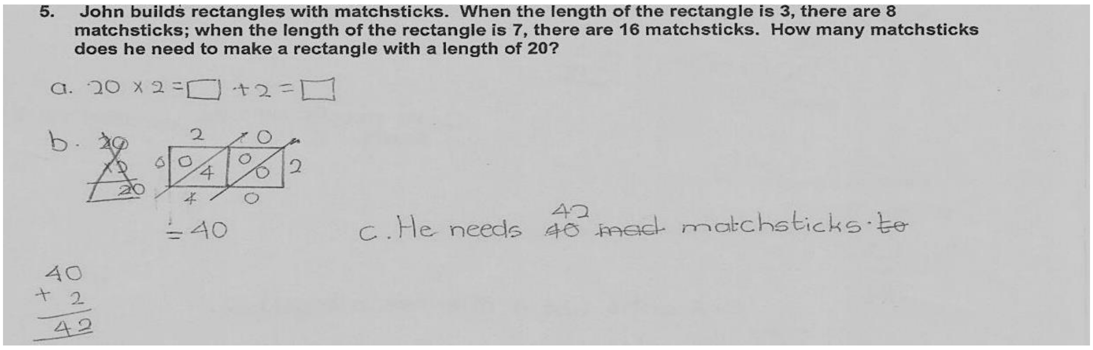
Solution 2

Figure 41 Grade 6– learner's solution Question 4

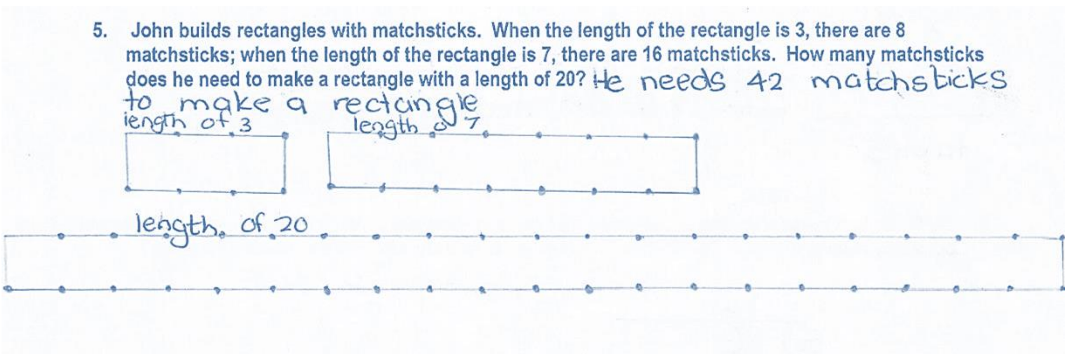
When solving problems, learners develop their own understanding of the problem that makes sense to them. It is their mental representation that shows their construction of new learning. Learners identify with situations through personal interpretations and within their own cognitive abilities Thus by allowing the learners the opportunity to solve problems using their own methods increases the quality of the mathematics. The solutions produced in Figure 42 indicated the varying problem solving abilities of the learners. Fifty percent of the learners provided the correct solution to the problem and fifty five percent of the learners used visual techniques to arrive at a solution. The learners drew a diagram as one of their problem solving strategies for this problem.

Constructivism calls for learners to be be empowered to create their own knowledge and solutions that they can understand. This implies that for knowledge to be meaninful to the learners and lasting it has to make sense to the learners. An interesting feature of the Figure 42 – solution 1 was the learner's use of vedic mathematics when calculating the answer. This kind of calculation is not prescribed in the CAPS document as part of the content to be taught in grade six. The learner demonstrated a higher level of cognitive understanding and I will classify this kind of thinking as mathematical maturity. It is this kind of learning process that must be nurtured in schools.

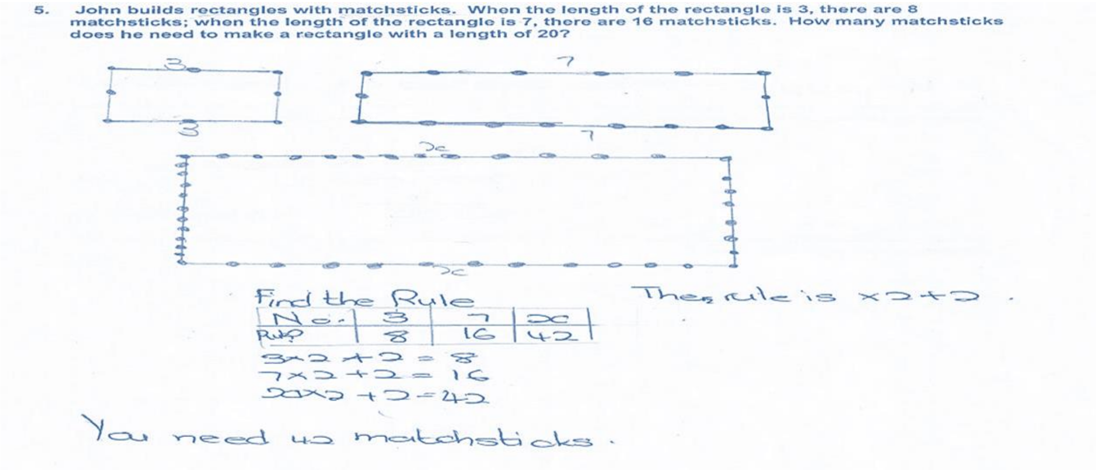
The extracted learner solution (Figure 42 – solution 2) showed that she was in a position to use her prior knowledge of perimeter when solving the problem. The visual representation contributed largely to the learner developing an understanding of the problem. By determining the required shape was a rectangle, the learner placed the 'matchsticks' opposite each other in an equitable manner. The integration of both the text and visual elements enabled the learner to represent her solution. The learner (Figure 42 – solution 3) drew a diagram and then re-drew it to get a better understanding. She then drew a table to determine a rule (Figure 42 – solution 3). By using the rule she was able to arrive at a reasonable solution. As mathematics teacher we must expect learners to engage with the problem in various ways. In solution 4 and 5 (Figure 42) the learners looked for a rule and pattern to determine their answers. The learner (Figure 42 – solution 5) displayed adaptive reasoning. She was able to use and justify the use of the diagram in order to solve this problem. If no explanation is provided on how they arrived at the answer then the learners must be allowed the opportunity to write out their rules or an explanation to the solution similar to Figure 42 – solution 3.



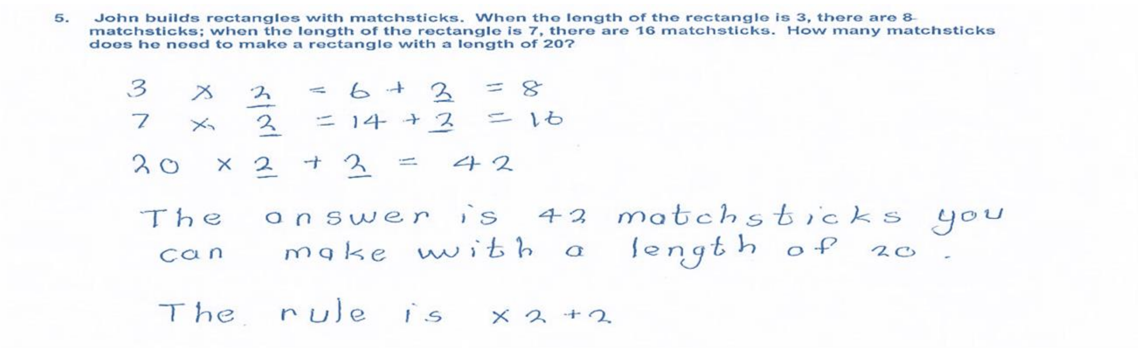
Solution 1



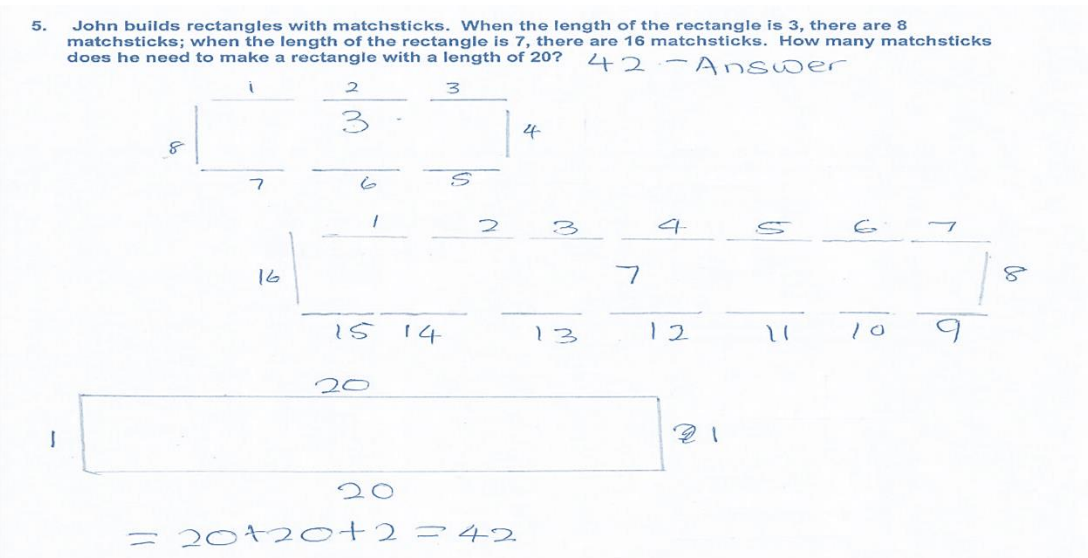
Solution 2



Solution 3



Solution 4



Solution 5

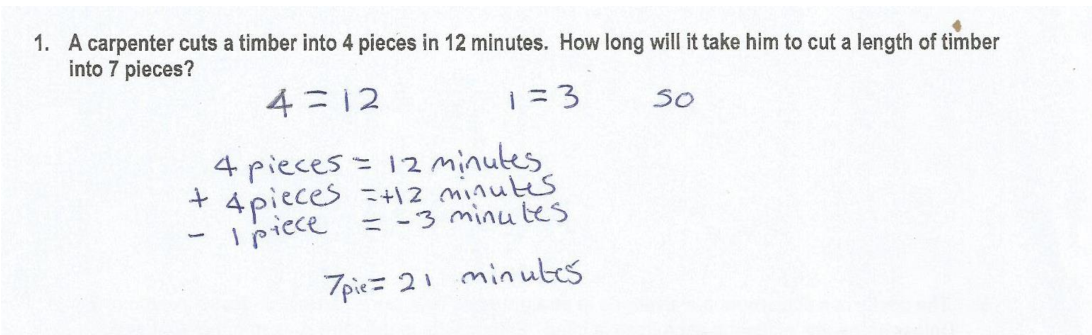
Figure 42 Grade 6– learner's solution Question 5

The following learner's solutions were extracted from the **grade seven** worksheets.

Learners who find it difficult to read and comprehend mathematical problems will continuously face learning difficulties as their progress through the schooling system. The manner in which they read, comprehend and visualize the problem will impact on the solutions they produce. The analysis to the question (Figure 43) showed that one hundred percent of the learners provided an incorrect answer to this question. It was noted that the learners displayed very poor comprehension skills and looked at the question in a literal way. They attempted to solve this problem algebraically (Figure 43 – solutions 1, 2 and 3). They merely translated the words in the problem into numbers to attempt a solution. The learners were firstly supposed to determine how many minutes it will take to cut one piece. In Figure 43 (solutions 1 and 2) the learners divided the minutes by the number of required pieces. If they had represented the timber and tried 'cutting' it the learners would have realised that they need to cut the timber three times to get the four pieces, namely, 12 minutes divided by the 3 cuts = 4 minutes a cut. By applying this same procedure to get seven pieces it would have required six cuts with the solution been 24 minutes.

A possible contributing factor to them getting this question (Figure 43) incorrect was their inclination not to use any form of visual representation. The findings from the analysis of the learner's solutions to the question showed that none of them attempted to use any form of visualization and not a single learner attempted to use any form of visual techniques.

According to Arcavi (2003) the process of solving a problem is through using visualization as it helps to grasp the definitions and understanding of the concepts in the problem.

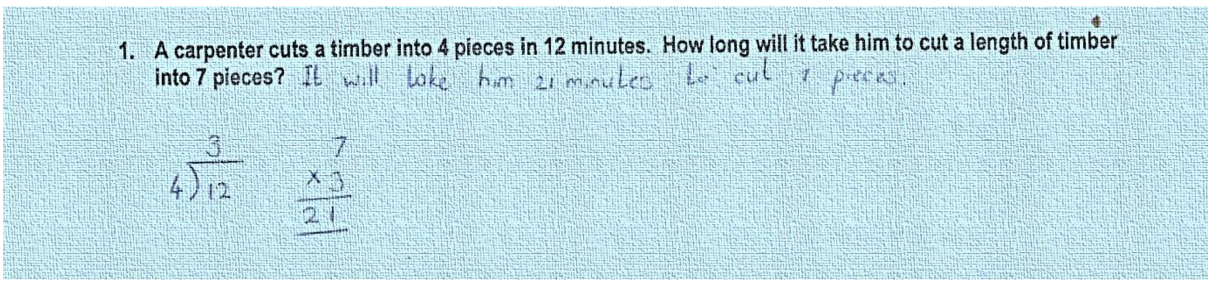

1. A carpenter cuts a timber into 4 pieces in 12 minutes. How long will it take him to cut a length of timber into 7 pieces?

4 = 12 1 = 3 so

4 pieces = 12 minutes
+ 4pieces = +12 minutes
− 1 piece = −3 minutes

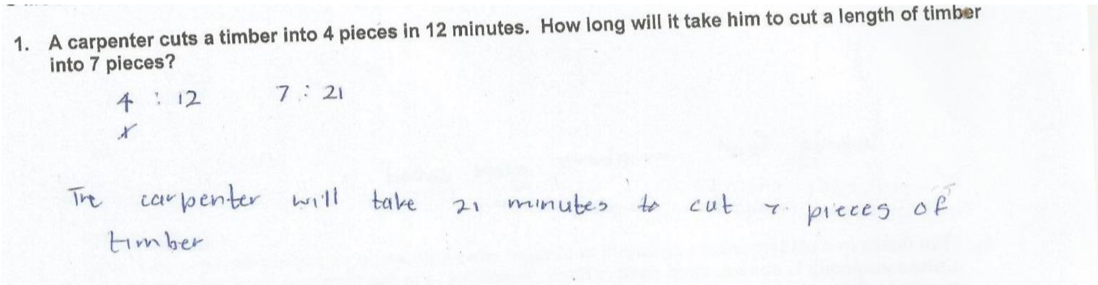
7pie = 21 minutes

Solution 1


1. A carpenter cuts a timber into 4 pieces in 12 minutes. How long will it take him to cut a length of timber into 7 pieces? It will take him 21 minutes to cut 7 pieces.

12 ÷ 4 = 3 7 × 3 = 21

Solution 2


1. A carpenter cuts a timber into 4 pieces in 12 minutes. How long will it take him to cut a length of timber into 7 pieces?

4 : 12 7 : 21
x

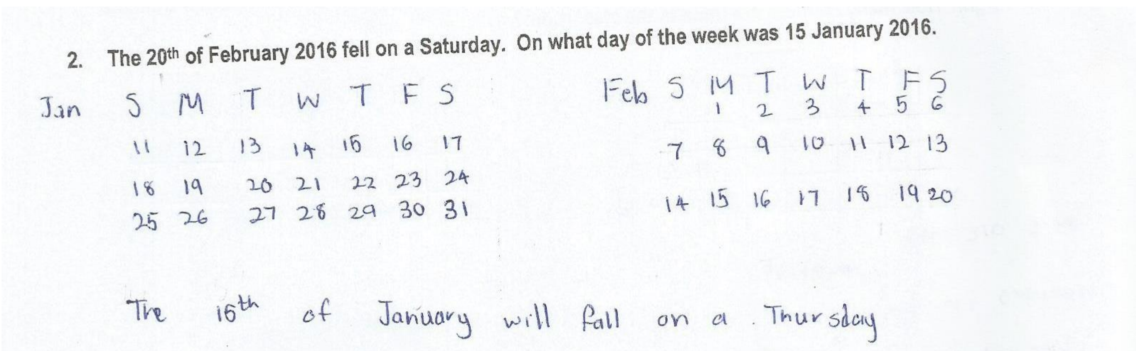
The carpenter will take 21 minutes to cut 7 pieces of timber

Solution 3

Figure 43 Grade 7– learner's solution Question 1

The learners solutions indicated that they had no understanding of the physical activity involved in this problem or could not relate it to a real world situation. Supporting the Theory of Growth of Mathematical Understanding, Meyer (2001) stated that if learners are to understand then the problem must be real and experience related for them. According to Walle, Karp and Bay-Williams (2014) understanding can only occur if there is a connection between the problem and the real world. The danger exist that if no remedial measures are undertaken to improve the learners understanding and problem solving abilities then the learners will continue applying this meaningless algebraic rules to other problem situations.

It is possible that not all learners are able to reason logically especially when placed in real life situations. Learners are exposed to writing the date daily in the school books. The question (Figure 44) required them to use the given dates and calculate the day on which it fell in the previous month. I needed to determine if the learners were able to use their daily knowledge in finding the correct day. Fifty percent of the learners provided a correct solution to the problem. As a problem solving strategy they drew a table format representing the dates on a calendar (Figure 44) to aid them find them answer. By establishing a table and representing the days of the week the learners proceeded to determine the correct day.

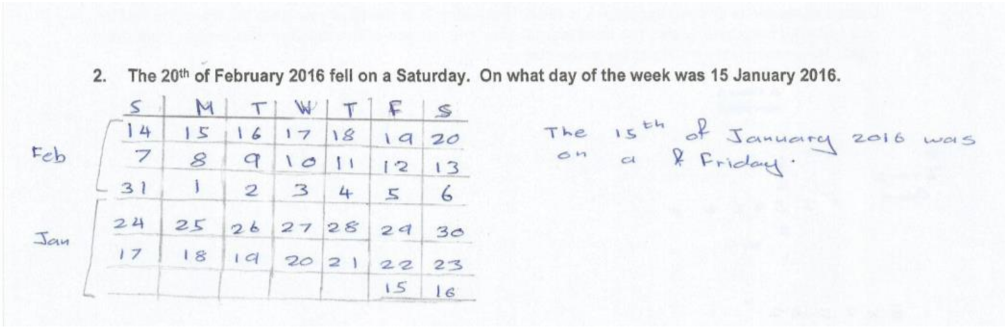

2. The 20th of February 2016 fell on a Saturday. On what day of the week was 15 January 2016.

Jan	S	M	T	W	T	F	S
	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	31

Feb	S	M	T	W	T	F	S
		1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20

The 16th of January will fall on a Thursday

Solution 1



2. The 20th of February 2016 fell on a Saturday. On what day of the week was 15 January 2016.

	S	M	T	W	T	F	S
Feb	14	15	16	17	18	19	20
	7	8	9	10	11	12	13
	31	1	2	3	4	5	6
Jan	24	25	26	27	28	29	30
	17	18	19	20	21	22	23
						15	16

The 15th of January 2016 was on a Friday.

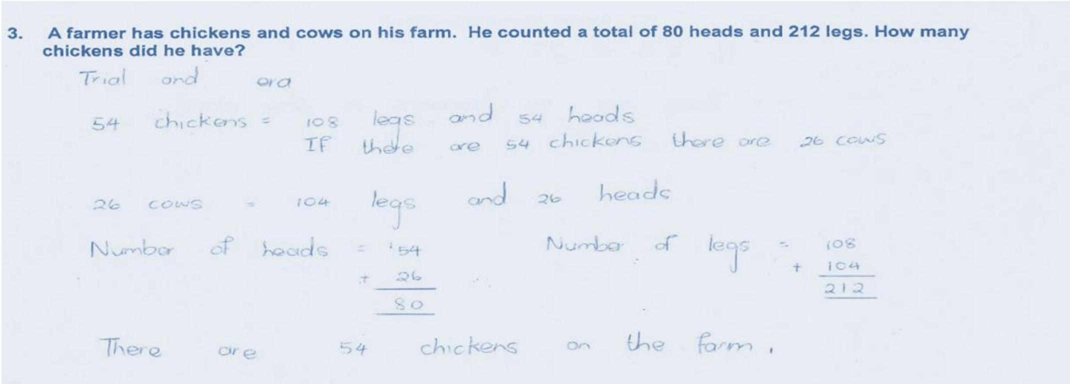
Solution 2

Figure 44 Grade 7 – learner's solution Question 2

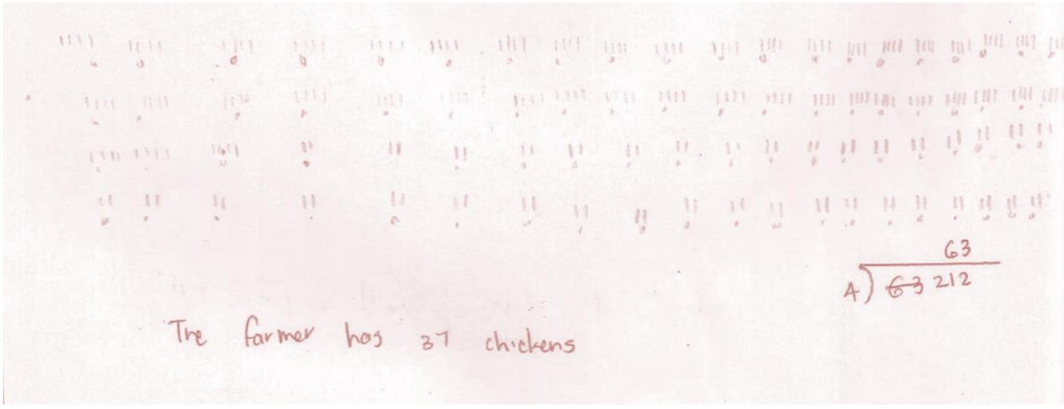
The learners (Figure 44 – solution 1 and 2) used counting backwards as a problem solving strategy. The learner (Figure 44 – solution 1) erred in her calculations. She commenced correctly in determining the dates for February which began on Monday, 1 February. Instead of placing the

31 January in the Sunday column she proceeded to place 31 January in the Saturday column. In reality if one month ends on a Saturday the next month will commence on a Sunday. Whilst using visible means of a calendar format the learner's sequencing rules did not provide the correct answer. The learner (Figure 44 – solution 2) showed her competence in using the working backwards strategy. She commenced her counting and progressively worked her way towards the correct solution.

Learners differ in the way they reason and they can, logically or illogically, represent their solutions in many ways. The mathematical solution, when put on paper, indicates the transformation from the written form to their interpretation of the mathematical idea. A mental transformation occurs and the solution is displayed in written form again. Only five percent of the learners arrived at a correct solution for the question indicated in Figure 45. Many of the learners resorted to guessing of the answers. Although drawing a diagram (Figure 45 – solution 2), the learner resorted to using her algebraic knowledge. In solution 2 (Figure 45) the calculation is incorrect, 212 divided by 4 is 53 and not 63. The learner used the trial and error (Figure 45 – solution 1) to determine the answer. The learners comprehension of the questions indicated her mathematical reasoning ability. The learner used this problem solving strategy and provided a well defined written explanation to quantify the answer (Figure 45 – solution 1). In order to find the solution, the learner was able to apply this strategy together with the algorithms.



Solution 1



Solution 2

Figure 45 Grade 7– learner's solution Question 3

Learners who lack reading skills may battle to read deductively hence their inability to select key and relevant words within the problem. Such inability was noted in the analysis of the question indicated in Figure 46. Only ten percent of the learners figured out the solution for the this problem. In Figure 46 the learner used a tree diagram to solve this problem. Noticeably this is something different. According to Arcavi (2003) visualization has a powerful role in promoting understanding through illustrating the solutions in a symbolic manner By using this diagram and a set of rules the learner was able to understand and solve this of the problem. Structural learning theory supports the use of creating rules to solve problems.

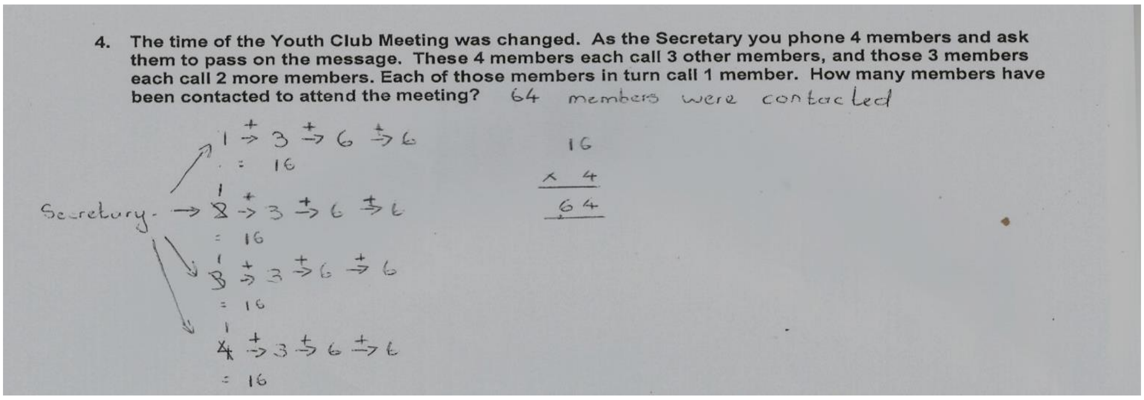
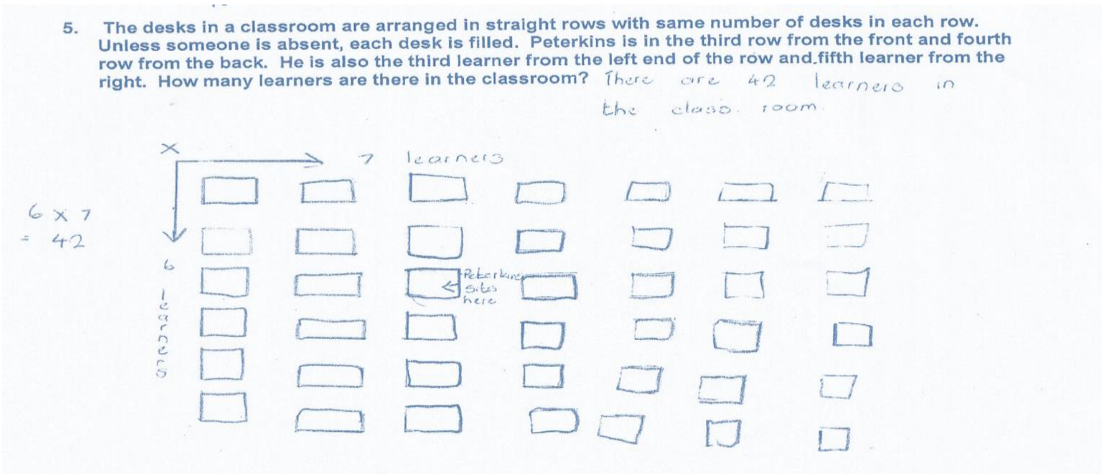


Figure 46 Grade 7– learner's solution Question 4

Reading a complex question and constructing an image or diagram requires a higher level of cognitive skills. The question indicated in Figure 47 expected the learners to read and describe the position of where the said learner was seated. Fifty-five percent of the learners provided a correct answer to this problem (Figure 47) and fifty percent of the learners attempted to find a solution using visual techniques. The extracts show that both the learners (Figure 47 – solutions 1 and 2) represented the rows of desks schematically.



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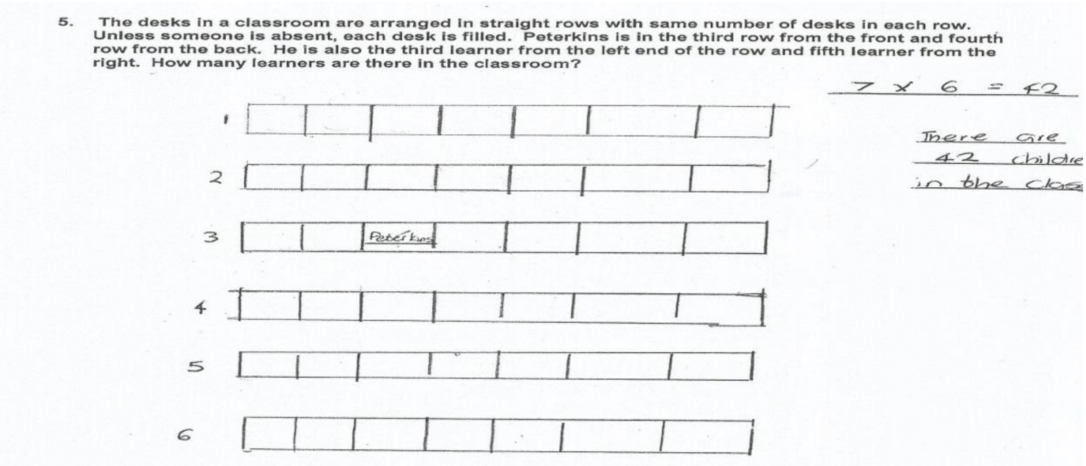
Solution 1



Solution 2

Figure 47 Grade 7– learner's solution Question 5

5.5.2 CONCLUSION

Whilst the questions were non-routine problems and were curriculum related the analysis of the evaluation worksheets indicated that the learners lack problem solving abilities. From the analysis above it showed that the learners lacked the ability to read and interpret the question and apply the correct operations. Some of the learners showed an understanding of the mathematical concepts but majority of them were not in a position define the concepts and transfer their knowledge to the problem solving situations. Only a small percentage of learners showed an inclination towards solving problems in an innovative and creative manner. They were also in position to use their prior knowledge to new situations or related the problem as part of their daily life. A small percentage of the learners also used both a visual diagram and algebraic means when solving the problem.

There is evidence from the analysis of the worksheets that the learners used graphical representation (using shapes and diagrams to explain the solution), numeric representations (table) and algebraic and symbolic representation (using mathematical and arithmetic symbols) to explain the problem. Within the Theory of Growth of Mathematical Understanding, level two involves the creating of a mental picture of the concepts and level three is the construction of the mental representation. The creation of the visual representations enabled the learners to make the connection between their own acquired experience and the mathematical concepts given in the problem. This indicated that visualization is a critical to mathematical thinking and it showed the learners understanding of the problem through their representations.

It was evident that the majority of these learners were not exposed to these kind of questions. It must be stated that these questions were based on curriculum content knowledge which the learners should have acquired as they progressed through the grades. The results showed that the learners did not have the ability to solve the problems due limited experience in and with the problems. It is important that the pre-service teachers have knowledge on their learner's thinking and decide how they will expose their learners to solving problems of this nature.

The majority of the learners experienced difficulties mainly due to the mathematical terminology used in the problem. The words in the given problem resulted in them not comprehending and understanding the problem. It is the teacher's responsibility to ensure that they have the expected content knowledge in the subject to ensure the concepts are not misconceptualized. Furthermore contextual factors were also noticed at certain schools. The learners experienced language barriers (the language of instruction was not their mother tongue). These learners attempted the problem without applying any form representations to find the solution. They tried to arrive at the answer but were not able to solve it. They struggled to comprehend thus showing insufficient understanding resulting in them providing incorrect operations or guessing the answer.

A distinct feature in the analysis of the worksheet was the performance of the grade seven learners. They performed poorly in the problem solving process. The learners made limited use of visual techniques and problem solving strategies when working towards a solution. Teaching problem solving and the use of problem solving strategies is not been given the due recognition it deserves in the day to day teaching in grade seven otherwise the learners would have mastered the problem solving techniques before departing for secondary school. The grade seven's performance within this study gives credence to the findings of the Department of Basic Education (2015) where it was found that there was a decline in the results from the intermediate phase to the senior phase.

To assist in the understanding of the problem learners need to be exposed to visualization skills to assist in identifying the meaning of the concepts. Since representation of mathematical knowledge shows interconnectedness to the learners thoughts the use of visual skills will assist in enhancing their understanding of the problem by transforming the internal representations into mental models. By enhancing their visual skills learners can use it as alternative resource to understand and solve the mathematics problems in the classroom.

5.6 PRE-SERVICE TEACHER'S EVALUATION WORKSHEETS

5.6.1 INTRODUCTION

The literature review in Chapter 2 and the discussion of the learning theories within the Theoretical Framework in Chapter 4 emphasises the need for problem solving and how the importance thereof respectively. The pre-service teachers were given two worksheets, one at the beginning of the semester and the other during the semester. The first worksheet (Annexure 4) consisted of non-routine questions which were taken from the past years Mathemagica Plus Problem Solving Competitions. The worksheet was given to the pre-service teachers to complete as a lecture activity. All the pre-service teachers were encouraged to attempt the problem and provide a solution to the problem using any logical manner. I never mentioned the concepts of problem solving strategy or visualization but I did mention that I wanted to see all their working on the worksheet. I needed to see for the purpose of this study how they arrived their solutions.

The second worksheet required the pre-service teachers to indicate their understanding of mathematical terms and do basic algorithmic calculations based on the curriculum. The pre-service teachers were exposed to the given terminology and algorithms during their classroom observation and teaching experience. I needed to know their level of understanding of the mathematical concepts and also gauge their mathematical curricular knowledge.

5.6.2 WORKSHEET ONE ANALYSIS

A statistical analysis is provided on the pre-service teachers attempt to answer the non-routine problems. This is followed with extracts of their solutions from the worksheets and a summarised discussion follows thereafter. My focus was not so much on the correctness of the answer, although important, but I needed to discover for the purpose of this study how these problems were solved. The analysis of the worksheet is based on the use problem solving strategies and visualization.

The data provided in Table 4 is a question by question based on the mathematical correctness of the solution provided by the pre-service teachers.

Table 4 Analysis of non-routine questions

	Q1	**Q2**	**Q3**	**Q4**	**Q5.1**	**Q5.2**	**Q6**	**Q7**	**Q8**
Correct %	6	72	28	56	33	28	28	56	50

Since I wanted to determine if the pre-service teachers used problem strategies or visualization skills when problem solving, an analysis of the questions (Table 5) is provided where the pre-service teachers used visual techniques during problem solving.

Table 5 Visual Techniques used in solving problems

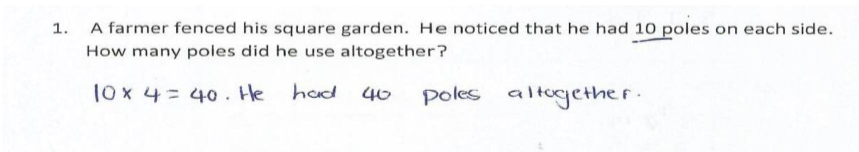
QUESTION	VISUAL TECHNIQUES USED (%)
1	28
2	84
3	22
4	-
5.1	11
5.2	-
6	-
7	39
8	61

The pre-service teachers were given the worksheet to complete as an activity in the first lecture. There were a few pre-service teachers who were reluctant to answer the questions deeming the questions to be “stupid”. Notwithstanding this attitude, I encouraged (rather cajoled) them to attempt the questions. In my interaction with many of them (whilst they worked through the activity) they mentioned that they ‘never across these types of questions’.

The solutions yielded some interesting data regarding the pre-service teacher’s problem solving abilities.

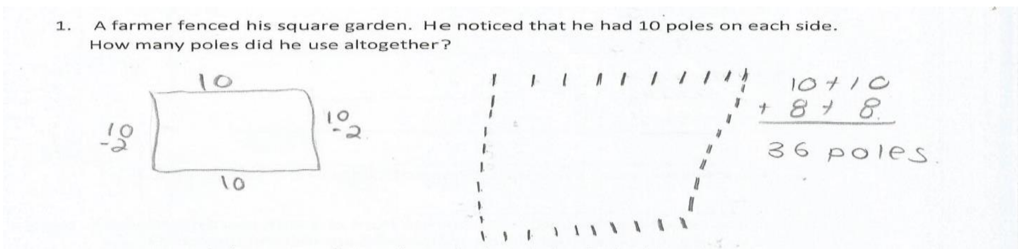
Many of them showed an inclination to use arithmetic-algebraic means to solve the problems. (Figure 48 - solution 1 and 2).

Question 1. A farmer fenced his square garden. He noticed that he had 10 poles on each side. How many poles did he use altogether?

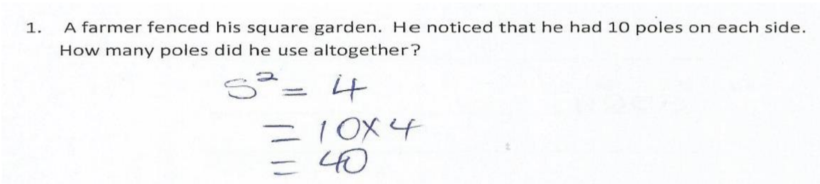

1. A farmer fenced his square garden. He noticed that he had 10 poles on each side. How many poles did he use altogether?

10 x 4 = 40. He had 40 poles altogether.

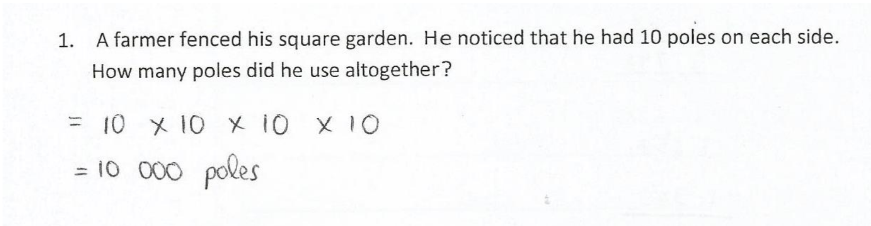
Solution 1


1. A farmer fenced his square garden. He noticed that he had 10 poles on each side. How many poles did he use altogether?

Solution 2



Solution 3



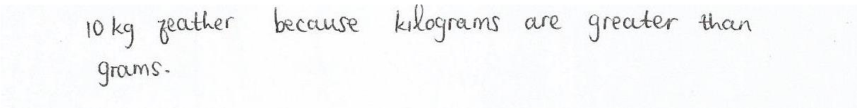
Solution 4

Figure 48 – pre-service teacher's solutions

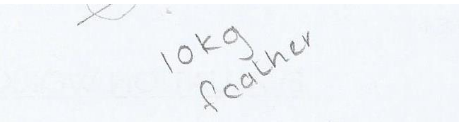
In attempting question one only six percent managed to get it correct (Table 4). This indicated that the pre-service teachers had difficulty in determining a solution for question 1 (Figure 48). It was very noticeable in their calculations that their arithmetic-algebraic method of calculating the problem was yielding incorrect solutions (Figure 48 - solutions 1, 3 and 4). The majority of the pre-service teachers took the question literally or read it without giving it much thought. This was noticeable in their calculations (Figure 48 – solutions 1 and 4) and interpretation of the question (Figure 48 – solution 3). They misinterpreted the mathematical problem and produced algebraic solutions of 4 x 10 = 40 (Figure 48 - solution 1) and 10 x 10 x 10 x 10 = 10 000 poles (Figure – solution 4). The solution 4 (Figure 48) indicated that the pre-service teacher did not read the question properly as the question mentioned '10 poles on each side'. Some of them were looking for words, namely, square and altogether. They translated square, according to their knowledge, to mean a shape with all sides equal hence solution in Figure 48 (solution 3) where the pre-service teacher used the symbol S^2 which is normally used in area. I asked the pre-service teachers to read the question very closely and check if they could attempt the problems in other ways as well. Only twenty percent of the pre-service teachers attempted to use a visual strategy to solve the problem (Table 5). The solution (Figure 48 – solution 2) indicated that some of the respondents lacked basic mental mathematical knowledge. I can surmise the lack of mathematical knowledge so early in the semester and poor comprehension skills were some of the contributing factors to producing incorrect solutions. One can hypothesise that a contributing factor to their poor problem solving skills could be that whilst they were at school, the mathematical curriculum that they were acquainted with (OBE) and the one they need to acquaint themselves (CAPS) are totally different. I surmised from their responses that many of them were not given the opportunity to work with non-routine problems; did not have the relevant mathematical content knowledge or the majority of them did not have any knowledge about how to visual skills when solving problems.

Question 4. Which do you think will be heavier, 10 000g of lead or 10kg of feathers?

Solution 1



Solution 2

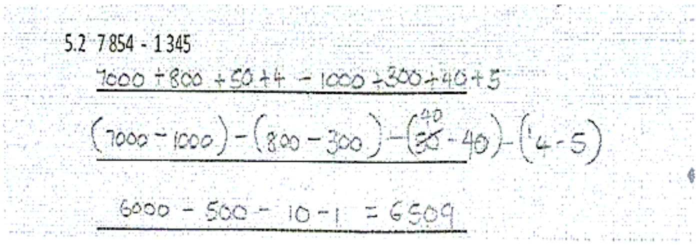


Solution 3

Figure 49 – pre-service teacher's solution

The current cohorts of pre-service teachers are all accomplished matriculants having obtained a pass in mathematics. The units of measurement are taught from the foundation phase all the way through to grade 12. The expectation was that they would have remembered the units used in mass. The pre-service teachers inherently failed to comprehend question 4 or did not have the basic mathematical knowledge associated with mass. This is indicated by the responses in Figure 49. CAPS indicate measurement as a large strand. The attributes within measurement includes length, time, mass, temperature and capacity. The various units within measurement are used continuously throughout the grades, namely, grade one to twelve, and are considered basic mathematical knowledge. Only fifty six percent of the respondents obtained a correct answer. They did not know the quantity value of the unit of measurement used in the problem. Figure 49 (solution 1 and 3), indicated that the pre-service teachers lacked the knowledge of making a comparison and to convert the units of measurement involving mass. Those that failed to provide the correct answer chose 10kg of feathers (Figure 49 – solution 2) identifying it as possibly the larger of the two units used in mass. The measurements are the equivalent due to the fact when 10kg is converted to grams it will be 10 000g.

I analysed the pre-service teachers solutions from worksheet one and worksheet two. At the beginning of this chapter I did indicate the first worksheet was given to them early in the first semester and the second worksheet towards the latter part of the semester. The solutions to in Figure 50 were taken from worksheet one and the solutions in Figure 51 are from worksheet two.

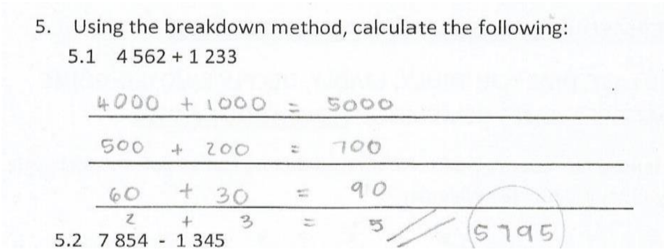

5.2 7 854 - 1 345

7000 + 800 + 50 + 4 - 1000 + 300 + 40 + 5

(7000 - 1000) - (800 - 300) - (50 - 40) - (4 - 5)

6000 - 500 - 10 - 1 = 6509

Solution 1


5. Using the breakdown method, calculate the following:

5.1 4 562 + 1 233

4000 + 1000 = 5000

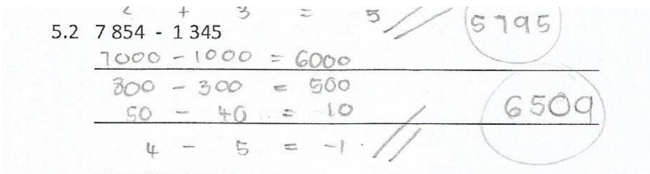
500 + 200 = 700

60 + 30 = 90

2 + 3 = 5 // 5795

5.2 7 854 - 1 345

Solution 2


5.2 7 854 - 1 345

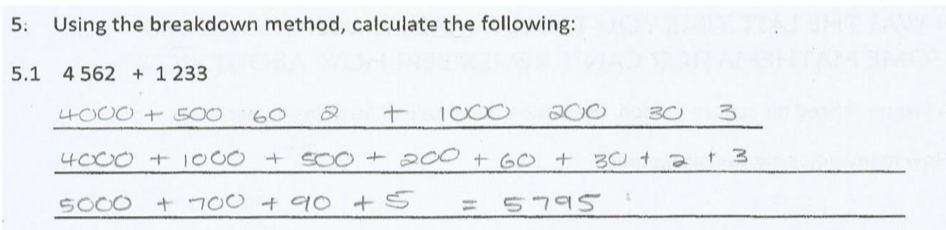
7000 - 1000 = 6000

800 - 300 = 500

50 - 40 = 10

4 - 5 = -1 // 6509

Solution 3


5: Using the breakdown method, calculate the following:

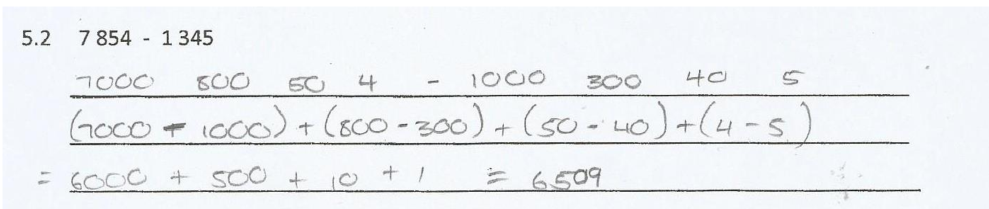
5.1 4 562 + 1 233

4000 + 500 60 2 + 1000 200 30 3

4000 + 1000 + 500 + 200 + 60 + 30 + 2 + 3

5000 + 700 + 90 + 5 = 5795

Solution 4

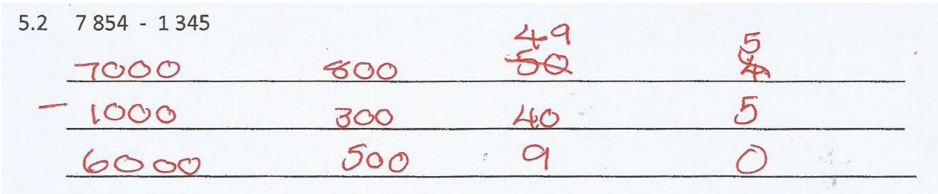

5.2 7 854 - 1 345

7000 800 50 4 - 1000 300 40 5

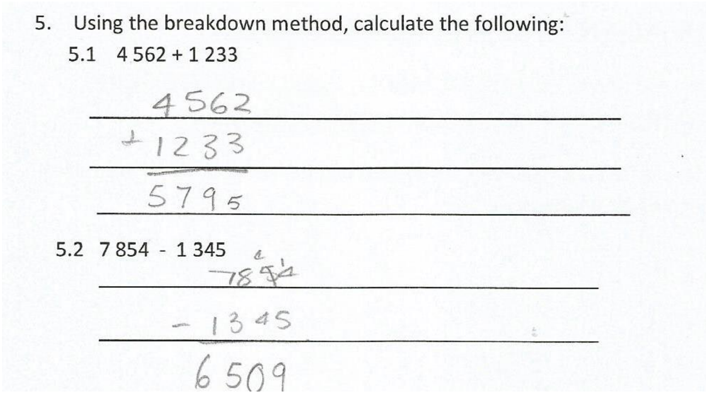
(7000 - 1000) + (800 - 300) + (50 - 40) + (4 - 5)

= 6000 + 500 + 10 + 1 = 6509

Solution 5



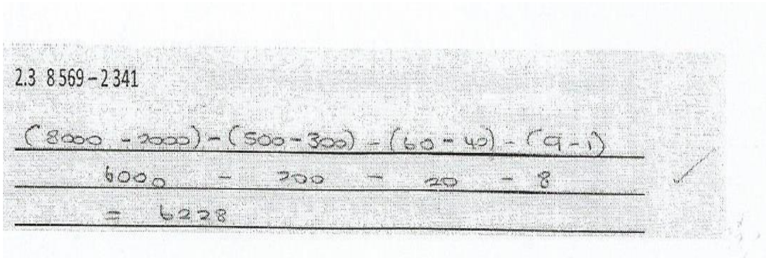
Solution 6



Solution 7

Figure 50 pre-service teacher’s solutions - the breakdown method

In worksheet one (Annexure 4) only thirty three percent of the pre-service teachers (Table 4) provided a correct solution for 4 652 + 1 233 and only twenty eight percent of them (table 4) provided a correct solution for 7 854 – 1 345. In worksheet 2 (Annexure 5) fifty seven percent of the pre-service teachers provided a correct solution to 2 452 + 3 125 and thirty nine percent provided a correct solution to 8 569 – 2 341. Comparatively a slight increase is noted in the pre-service teacher’s mathematical content knowledge when the responses from worksheet one was compared to the responses in worksheet 2. This marginal increase is still not sufficient to indicate that the pre-service teachers have acquired sufficient knowledge or are competent to teach the breakdown method to the learners. According to the Department of Basic Education (2018:9) it is expected that “*learners need to perform mathematical procedures accurately and efficiently*” and if the pre-service teachers lacked procedural fluency then it will be hard to envisage how they will be able to teach the learners the breakdown method. The solutions extracted from worksheet one indicated that the pre-service teachers lacked conceptual knowledge, subject content knowledge and procedural fluency. These three aspects are vital for the teaching of mathematics.

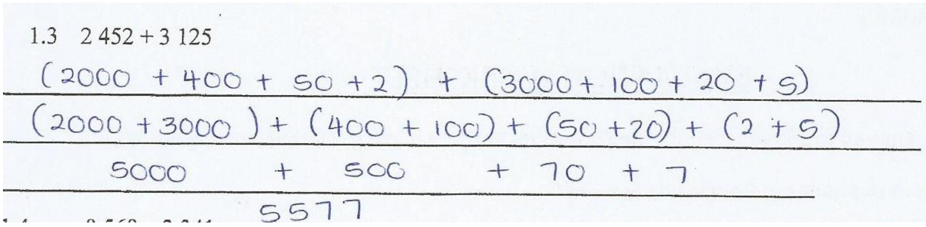

2.3 8 569 – 2 341

(8000 – 2000) – (500 – 300) – (60 – 40) – (9 – 1)

6000 – 200 – 20 – 8

= 6228

Solution 1


1.3 2 452 + 3 125

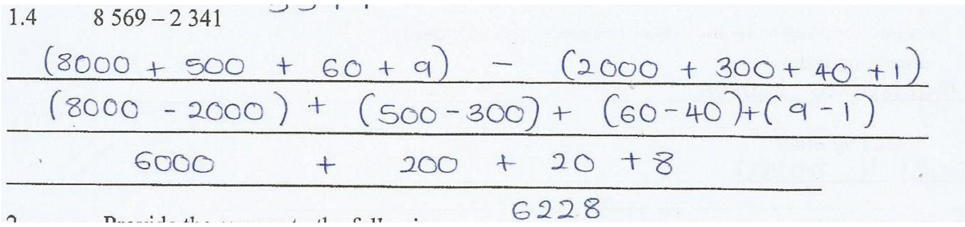
(2000 + 400 + 50 + 2) + (3000 + 100 + 20 + 5)

(2000 + 3000) + (400 + 100) + (50 + 20) + (2 + 5)

5000 + 500 + 70 + 7

5577

Solution 2


1.4 8 569 – 2 341

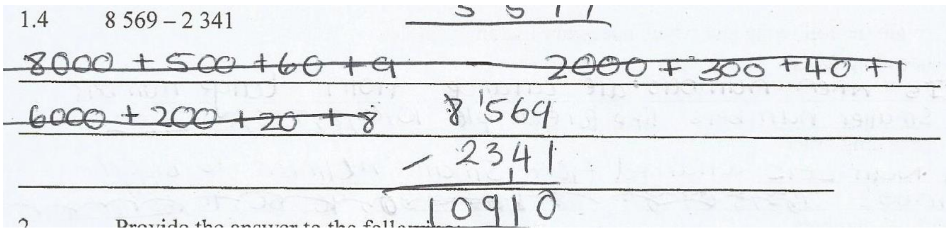
(8000 + 500 + 60 + 9) – (2000 + 300 + 40 + 1)

(8000 – 2000) + (500 – 300) + (60 – 40) + (9 – 1)

6000 + 200 + 20 + 8

6228

Solution 3


1.4 8 569 – 2 341

~~8000 + 500 + 60 + 9 – 2000 + 300 + 40 + 1~~

~~6000 + 200 + 20 + 8~~ 8 569

– 2341

10910

Solution 4

Figure 51 Worsksheet 2 – pre-service teacher's solutions - breakdown method

According to CAPS the breakdown method is introduced in the foundation phase and the learners continue using this method of calculation up until grade four. If the pre-service teachers had read CAPS then they will have discovered that is an important content knowledge. In Figure 50 (solution 4) and Figure 51 (solutions 2 and 3) are correct. This indicated that the pre-service teachers had the necessary conceptual knowledge of the breakdown method and were able to apply the correct operational sign to calculate the said algorithm. In Figure 50 (solution 1) the answer is correct but the operational sign used between the brackets is incorrect, namely, subtraction instead of addition. This makes it procedurally incorrect. If the algorithm was calculated as indicated in the solution, then the answer would be 5 489 instead 6 509. Although the pre-service teachers had the opportunity to learn about this method during their teaching experience at school, similar errors was still noted when the second worksheet was given (Figure 51 – solution 1). Through my observation as a mathematics teacher, the errors made by the pre-service teachers (Figure 50 – solution 1 and Figure 51 - solution 1) are very noticeably amongst the learners. It is possible that the learner's error evolve from the manner they are taught. The learners are taught expanded or extended notation, example, 4 562 = 4 000+ 500 + 60 + 2, early in their schooling career. This is indirectly the breakdown method. Once the learners have grasped the concept of extended notation, then they are taught grouping of numbers according to their value, example, hundreds with hundreds, tens with tens and units with units. It must be emphasised here that during their teaching the teachers do not use the subtraction symbol when breaking down the number.

The answers produced in Figure 50 (solutions 2 and 3) indicated that the pre-service teachers used their own interpretation of the breakdown method in their calculations. The pre-service teacher's calculations, albeit differently as required by CAPS, indicated that they had some knowledge of breaking down the given numbers.

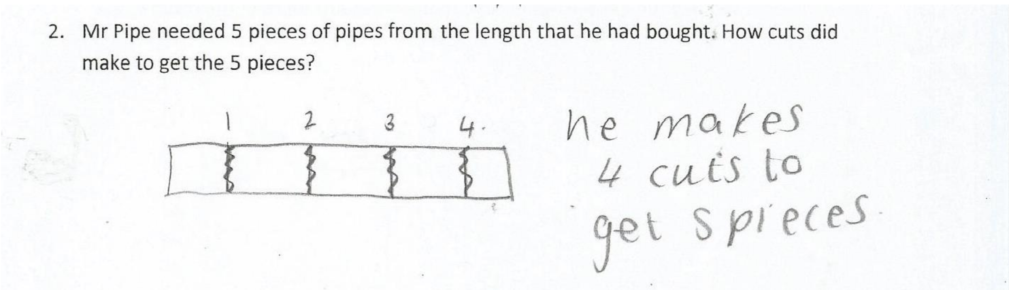
In Figure 51 (solution 3) the pre-service teacher's solution indicated the indecision of how to calculate. The pre-service teacher commenced calculating using the breakdown method but then struck it out and used the column or vertical method. Instead of subtracting, the pre-service teacher added the given numbers.

In Figure 50 (solution 3 and 5) is procedurally flawed. In the grouping of the units, 4 – 5 will provide a negative answer of -1 which cannot be written as the difference in the unit column. In Figure 50 (solution 6) the concept of borrowing is procedurally incorrect used. In normal teaching the teachers use the concept of borrowing incorrectly. The learners are told to 'borrow 1' from the next column when they cannot subtract the larger number from a smaller number (Figure 50 – solution 3). This is exactly what the pre-service teacher did. Instead of borrowing a ten from the tens column, he borrowed 1 from the tens column. This indicated the pre-service teacher's lack of understanding of the 'rules' that are used when teaching grouping of numbers. In this case the pre-service teacher did not understand the concept of place value. The principle of place value is based on the Hindu-Arabic number system and teaches the place value of numbers in the correct columns, namely, units, tens, hundreds, thousands. The manner that the learners come to understand mathematics is dependent on the teacher's understanding of concepts and procedures. According to National Research Council (2001:377), "*teachers are unlikely to be able to provide an adequate explanation of concepts they do not understand*". It is important that the teachers of mathematics teach mathematics so that the learners clearly understand the processes involved when working with algorithms. According to the structural learning theory certain mathematical aspects are taught and learnt through following rules and procedures (steps) but the manner the rule of application is applied here is incorrect. Whilst mathematics as a subject is a broad discipline, the pre-service teachers are expected to at least have the subject content knowledge of the rules of addition and subtraction. According to French (2005:3) it is important that elementary mathematics beyond the school level be reinforced so that the pre-service teachers have the ability to make the connection with the mathematics in the school curriculum.

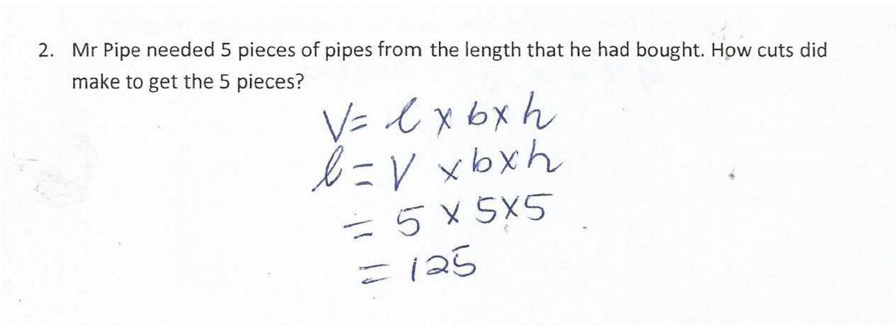
The majority of the pre-service teachers were unable to choose an appropriate visual strategy to provide a solution (Table 5). I had interacted with the pre-service teachers whilst they worked through both the worksheets, although at different time intervals. Many of them lacked the ability to solve the problems citing that they lacked familiarity in solving problems as given in the worksheet. When asked to provide an explanation on how they arrived at their answer, the majority of them mentioned that they guessed the answer. Guessing an answer is not an option in mathematics. If the pre-service teachers display a 'guessing technique' then it will be challenging for them to teach mathematics problem solving. If they lacked knowledge of the strategies used in problem solving or procedural knowledge, it can be hypothesised that they would struggle to teach it these at school. According to Ball, Thames and Phelps (2008) it is important that the teachers have sound content knowledge to teach the subject effectively. Within Shulman's categories of knowledge (Shulman, 1987) content based knowledge is highlighted as necessary professional mathematical knowledge for teaching. According to Ball, Thames and Phelps (2008) mathematical knowledge is referred to as that the teachers need to know to carry out their jobs as teachers of mathematics. This point is reiterated by the Department of Education (2018:82) which stated that pre-service training must take into account content knowledge across the curriculum.

As mathematics teachers we should not be satisfied to in only accepting arithmetical answers from the learners. It is possible that the learners are capable of working out the answers mentally but in their formative years of schooling they should be taught to produce a visual element on how they arrived at their answers to show their conceptual and procedural understanding. An analysis of their worksheets indicated the majority of the pre-service teachers did not use any visual techniques when trying to provide a solution to the problem (Table 5). Whilst in their years of studying they need to practice displaying their solutions in a practical manner when problem solving. In this manner they will build a foundation for themselves on how they will teach their learners to understand mathematical concepts and implement mathematical strategies.

The question (Figure 52), posed to the pre-service teachers had some similarities to one posed to the grade 7 learners (Figure 43).



Solution 1



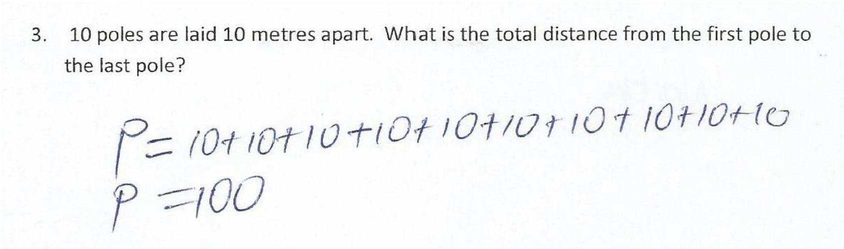
Solution 2

Figure 52 – pre-teacher's solutions

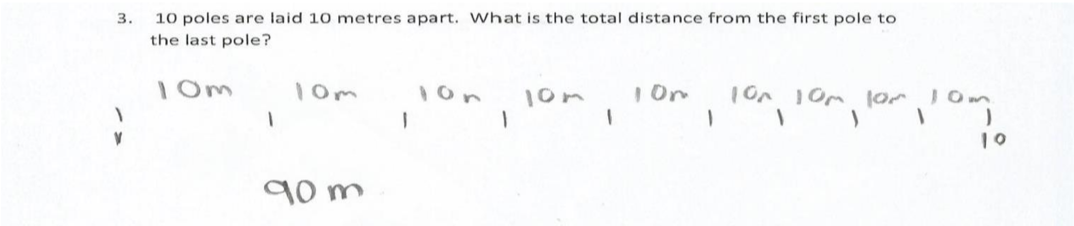
Eighty four percent of the pre-service teachers used varying visual techniques to find a solution to the question (Figure 52) of which seventy-two percent of them provided the correct solution. By indicating the 'cuts' on the diagram they were able to 'see' the number of cuts needed to obtain the five pieces.

In Figure 52 - solution 2 the pre-service teacher used the formula for volume and that to it is applied incorrectly when step 2 is examined. This indicated that the pre-service teacher had no mathematical knowledge of how to apply the formula. One hypothesise that the pre-service teacher had a reading problem or did not comprehend the question.

Both the grade 5 learners and the pre-service teachers were given a similar problem (Figure 52). Comparatively, the grade 5 learners fared better with thirty five percent of them obtaining the correct solution and twenty eight percent of the pre-service teachers getting it correct.



Solution 1



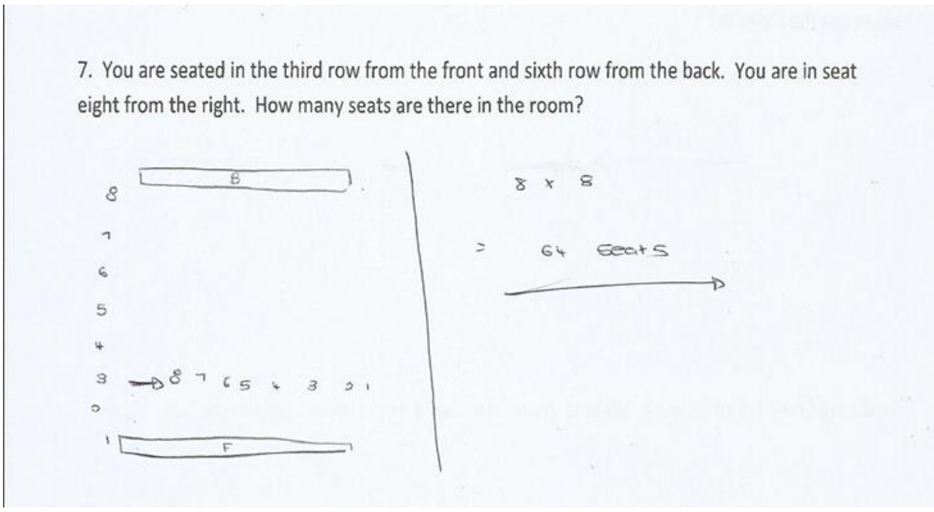
Solution 2

Figure 53 - pre-service teacher's solution

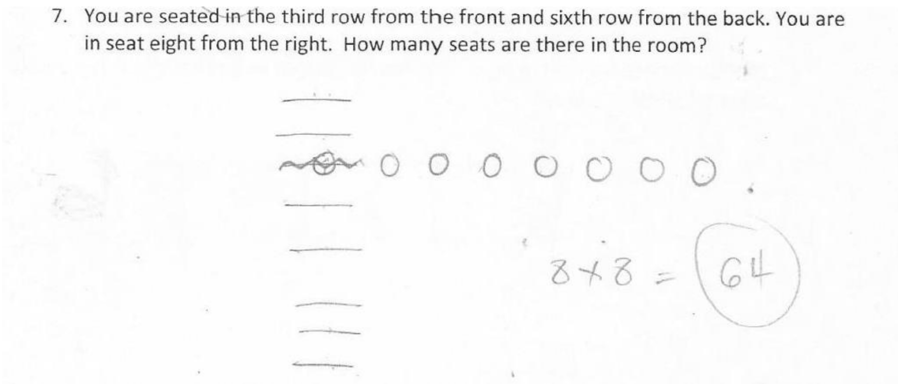
The pre-service teachers like the learners tried to solve the problem using their basic mathematical knowledge. They multiplied or added. They, the pre-service teachers and the learners, presumed that since there were 10 poles and the distance apart in between the poles was 10 metres the answer will be 100 metres (Figure 53 - solution 1). For those who managed to obtain the correct answer the visual representation aided them to get a clearer perspective of what was asked for. The schematic representation (Figure 51 – solution 2) provided by the pre-service teachers indicated that there were only nine spaces in between the 10 poles thus 9 x 10 = 90 metres.

The grade 5 learners and the pre-service teachers were given a similar problem (Figure 53) and (Figure 36) respectively. Only thirty five percent of the grade 5 learners and fifty six percent of the pre-service teachers were able to provide the correct solution with them using different strategies in their calculations.

Solving a problem like the one in Figure 54 through visual means enabled the pre-service teachers to translate the written language into an illustration. The drawing of the floor plan for this problem enhanced their comprehension as it gave them an avenue to explore other ideas and techniques to better their solutions. The pre-service teacher's representation of the problem diagrammatically indicated that they were able to generate the idea in their minds and translate this onto paper thus communicating their mathematical ideas creatively.



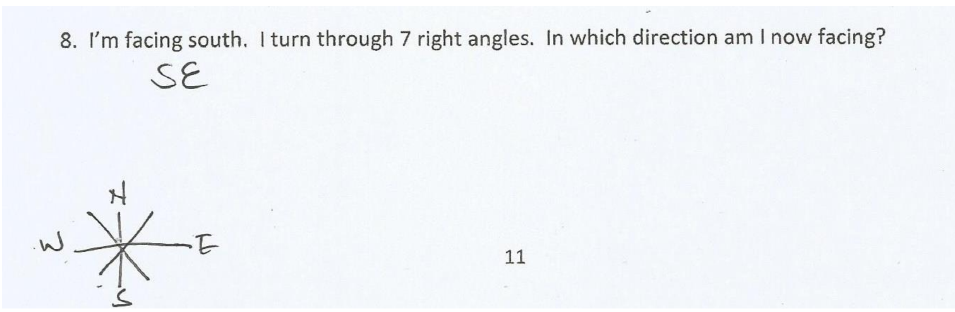
Solution 1



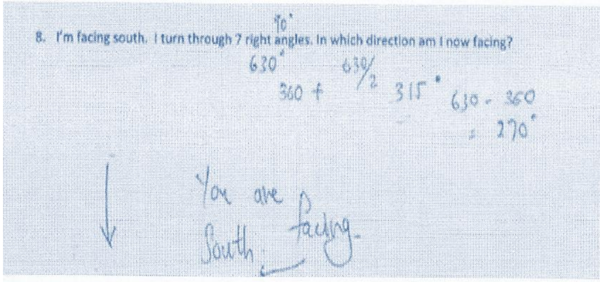
Solution 2

Figure 54 pre-service teacher's solutions

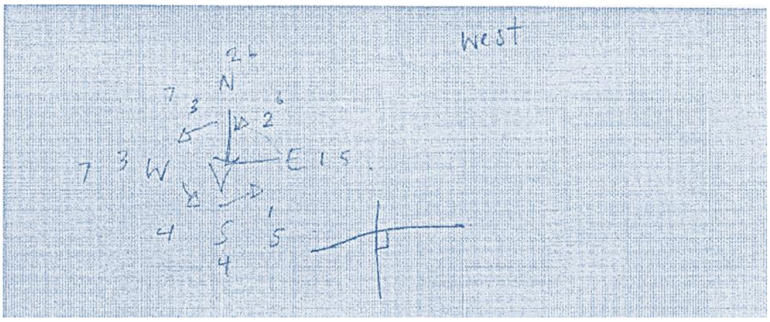
Sixty one percent of the pre-service teachers used a visual strategy to answer the question (Figure 54). Only fifty percent of those who used a diagram provided the correct solution.



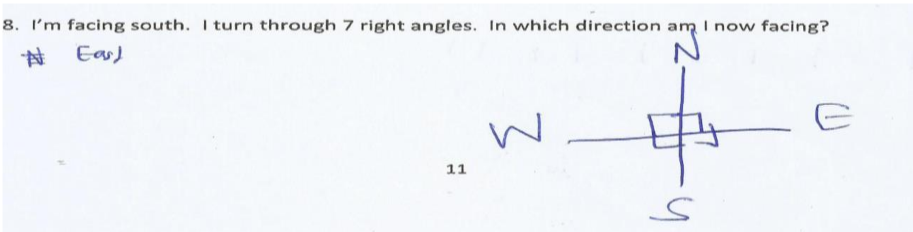
Solution 1



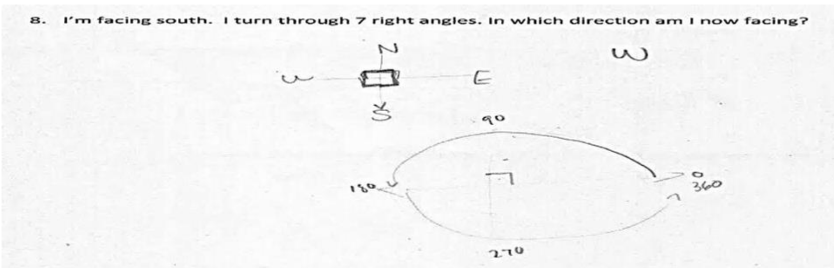
Solution 2



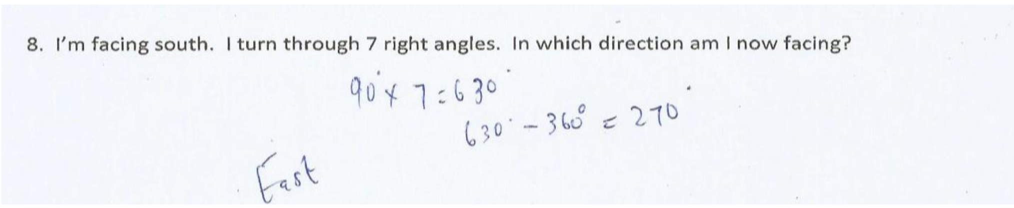
Solution 3



Solution 4



Solution 5



Solution 6

Figure 55 pre-service teacher's solutions

In order to answer the question above the pre-service teachers had to understand the concept rotation and realise that the rotation had to follow through seven right angles. In Figure 55, solution 1, provided by the pre-service teacher indicated that he did not have any conceptual knowledge about right angles. If he had, then he would have known that a right angle measures 90º. The pre-service teacher only turned through 45º hence his answer of South East which was incorrect. In Figure 55 (solution 2) the pre-service teacher provided a solution through algebraic means. 90º written above the words 'right angles' in the question indicated that the pre-service teacher had conceptual knowledge of right angles measuring 90º. He multiplied the 90º by the 7 right angles obtaining 630º. He thereafter divided 630º by 2 getting an answer of 315º. 630º - 360º = 270º. A similar calculation is indicated in Figure 55 (solution 6). Both these calculations do not show relevance on how he arrived at the answer South or East respectively. In Figure 55 (solution 4) the pre-service teacher answered the question by using a diagram. An illustration was drawn to show a right angle. The arrows were used to show the direction of the turn. In Figure 55 (solution 3) the pre-service teacher turned in anti-clockwise direction and obtained the answer West. The pre-service teachers are considered more academically advanced than the learners. I mention this because the question did not state whether one had to turn clockwise or anti-clockwise. It is debateable whether the pre-service teacher who provided the solution in Figure 55 (solution 3) had prior knowledge that the earth rotates clockwise on it axis and if such knowledge was used in answering the question. If this type of misconception of rotation and direction is ignored then the concepts are bound to be incorrectly embedded in their minds. Strong problem solving abilities is not sufficient to be a competent mathematics teacher. Teachers will have to focus on aspects such as clarifying the question and it will also be the learner's responsibility to ask for clarification especially when the question seems ambiguous.

A significant observable feature in the analysis of worksheet one was the lack of use of visual strategies by the pre-service teachers. Also noticeable, was the attempt by the pre-service teachers to solve the problems using traditional and algebraic methods.

5.6.3 ANALYSIS OF WORKSHEET TWO

Mathematical knowledge is a unique form of communication and teacher knowledge is extremely important in order to communicate in the field of mathematics. The teachers cannot expect the learners to understand their teaching if what they are teaching confuses them. A teacher who does not understand the mathematical concepts and teaches the subject content incorrectly will result in the learners learning incorrectly. If this is allowed to occur over a period of time, it will result in the learners carrying this misconception baggage forever in the schooling career. Using concepts repeatedly reinforces mathematical understanding hence me continuously 'testing' the pre-service teachers understanding of the basic concepts used in the mathematics classroom. According to Killen (2015) it is important that the teachers help the learners master the language of the subject they are teaching in order to understand its content.

In worksheet two the pre-service teachers were asked to explain the concepts (Table 6) without using any search engines or their reference textbooks. They had to provide an example to show that they understood the said terminology. I separated the explanations into two categories, namely, acceptable (correct) and unacceptable (incorrect) and added a third column to indicate the percentage of non-responses (Table 6).

Table 6 - Definitions of mathematical concepts

Concept	Acceptable (%)	Unacceptable (%)	Non-response (%)
Descending order	94	6	-
Ascending order	83	17	-
Cardinal numbers	6	17	77
Ordinal numbers	18	22	60
Place value	44	44	12
Value of a number	22	34	44
Objectives	83	-	17
Remedial work	22	56	22
Assessments	94	6	-
Planning	100	-	-

The following mathematical concepts, descending order, ascending order, cardinal numbers, ordinal numbers, place value and value of a number are highly misconceptualized by learners because teachers themselves don't understand them. In the literature review within this study I have discussed some of these concepts that are misunderstood by teachers. I wanted to determine if the pre-service teachers had the necessary mathematical knowledge or were they too on the same plain as the teachers.

The following responses were considered acceptable.

Descending order is described as "*when objects or numbers are placed in a specific order being biggest to smallest or highest to lowest*"; " *you start counting from biggest to smallest, example, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1*". Ascending order is " *a sequential whereby numbers are placed from the smallest to biggest, example, 1, 2, 3, 4, 5*"; cardinal numbers are a "*specific* *number name given for how many in a given collection of objects*"; ordinal numbers are " *the number word used to indicate the position, example, first, second, etc.*"; the place value of a number "*is in thousands, hundreds, tens or units*"; the value of a number "*is the actual amount of the number*".

Each subject has its own terminology and concepts which becomes its subject language. This is translated to teacher knowledge and is included in the curriculum. It is this knowledge that shapes their planning and implementation of pedagogical teacher knowledge in the classroom. In order to work within the confines of teaching and learning, the pre-service teachers need to have a sound knowledge of aspects that will assist them. Thus the pre-service teachers need to have a solid foundation in understanding the concepts. Objectives, remedial work, assessments and planning are terms which are continuously used during lessons preparation. Objectives are described as a statement of intent, namely, what does the teacher hope to achieve with the learners at the end of the lesson. The pre-service teacher indicated that it "*is something that you want to achieve, the aim or purpose of doing a particular activity*". During lesson preparation objectives is listed as something measureable and observable with the intention of teaching learners a skill that can be achieved at the end of the lesson. It is described as "*specific goals which one is set to be reached at the end of what they are doing*". According to du Toit (du Toit, Louw and Jacobs, 2016) the learners cognitive abilities need to be taken into consideration when writing out the objectives for the lesson. The objectives should contain an action verb (it must be measurable) to test the learner's competencies such that they are able to gain information to make informed choices (du Toit, Louw and Jacobs, 2016:145).

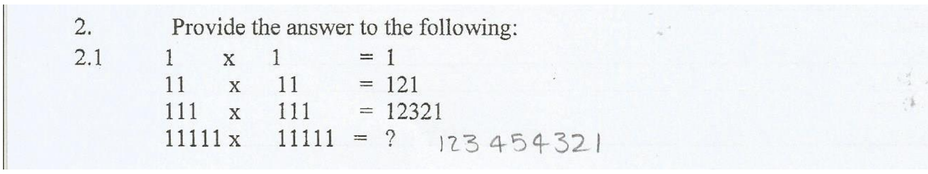
Whilst the majority of the pre-service teacher's responses indicated they understood the meaning of objectives, many of them could not write them out correctly during the planning. One response was that "*the learner must participate in the lesson if they want to learn*". It is important that the pre-service teachers formulate the lesson objectives in such a manner that it will have a desired effect on the learner's attitudes and grow their content knowledge and mathematical understanding (du Toit, Louw and Jacobs, 2016). Remedial work is undertaken when the teacher undertakes a diagnostic analysis after the learners have written an assessment or identifies an area of weakness amongst the learners resulting from a weakness in his teaching. Pre-service teachers indicated that remedial work is "*often done after a test/exam*"; "*to assist those learners who are struggling and need help and "reinforcement of work done*". In CAPS assessments are classified as formal or informal otherwise also known as summative and formative assessments respectively. The pre-service teachers indicated that "*informal and formal assessments test the learner's knowledge and understanding acquired from a lesson*"; formal assessments are administered to "*to get feedback on learner's performance*" for reporting purposes; informal assessments are normally given at the end of a section or topic as "*an activity used to determine a learner's level of understanding or knowledge of specific content*". Planning was described as "*necessary because you will be organised and it will be easy for you to teach*". It was also described as "*a process whereby goals, content and teaching strategies are chosen before a lesson so that lessons are purposeful and structured*". The pre-service teachers need to plan accordingly to meet the needs of the learners as described in Kolb's Experiential Theory. According to Shulman's description on teacher knowledge, a teacher needs to have sound subject and pedagogical knowledge to plan effectively.

In the context of mathematics, the following responses were classified as unacceptable:

Descending order is described as "*factors placed on a value which cause it to escalate*"; ascending order is "*when numbers or objects are placed from highest to lowest*"; cardinal numbers are "*numbers that have a remainder*" and "*the main numbers on a protractor/compass*". The latter description is a misconception between cardinal numbers and cardinal points. Ordinal numbers are "*the equal numbers that don't have a remainder*"; "*1; 3; 5; 7; 9; 11 odd numbers*". A misconception is the association of ordinal numbers with odd numbers. Place value of number is indicated as "*numbers that can divide that particular number*"; "*the value of a specific number within a series of numbers, example, 2$\underline{6}$3 → 6 represents 60*"; "*the place of the number underlined, example, 6$\underline{3}$7, place = 30*". The descriptions provided for place value is similar to the misconception of the teachers in the ANA. This is discussed in my literature review. Remedial work and assessments is an important component of teaching and learning. Remedial work is described as "*revising over previous work in order to get them (learners) ready for tests and exams*"; "*a way of revising work what was done, example, doing revision before a test*". Assessment is indicated as "*homework given to learners which teachers assess to see if the learners understand*". There is a blatant misconception between revision and remedial work and assessment and homework. These definitions can be described as unacceptable as not knowing these concepts will result in learner's poor grasping of the concepts. As these concepts are regularly used in mathematical teaching, it is important that the pre-service teachers acquire an in-depth knowledge of the concepts to teach efficiently. The constant use of CAPS, the learner's textbooks and the teacher guide will broaden their base of mathematical knowledge.

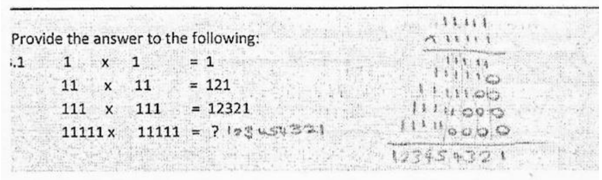
As previously mentioned, the second evaluation worksheet was given to the pre-service teachers in the middle of the semester. This evaluation worksheet entailed the pre-service teachers to calculate algorithms used in the intermediate phase and also included non-routine problems.

Question 2.1 The pre-service teachers had to calculate the product.


2. Provide the answer to the following:

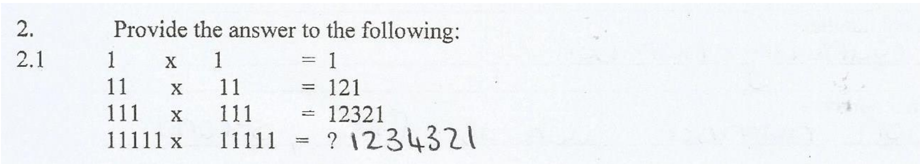
2.1 1 x 1 = 1
11 x 11 = 121
111 x 111 = 12321
11111 x 11111 = ? 123454321

Solution 1


Provide the answer to the following:

.1 1 x 1 = 1
11 x 11 = 121
111 x 111 = 12321
11111 x 11111 = ? 123454321

Solution 2


2. Provide the answer to the following:

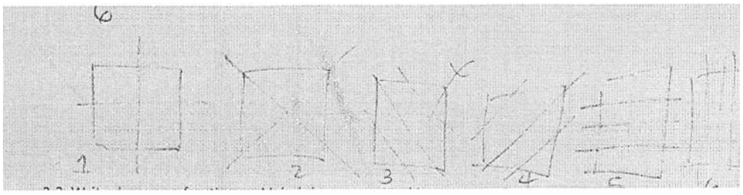
2.1 1 x 1 = 1
11 x 11 = 121
111 x 111 = 12321
11111 x 11111 = ? 1234321

Solution 3

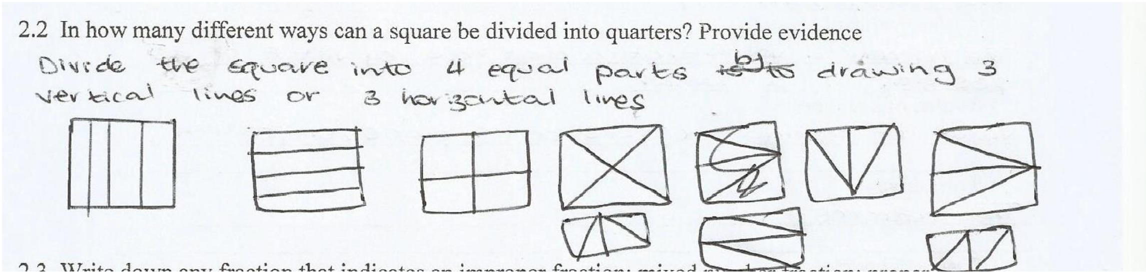
Figure 56 – pre-service teacher's solutions

The numbers represented in question (Figure 56) are represented in a pattern. Only thirteen percent of the respondents were able to deduce the pattern and showed a correct solution. By following the product of the multiplicative algorithm carefully from the first line to the second line and the second line to the third line the pattern was visible. 1 x 1 = 1 and when 11 was multiplied with 11 the product was 121. If 1 was added to 1 (digits taken from 11) the pattern unfolded as the middle number in the second line is 2. Likewise 111 multiplied by 111 the product was 12321. By adding 1 + 1 + 1 (the digits taken from 111) the middle number is 3. The solution indicated by the pre-service teacher in Figure 56 (solution 2) found the pattern but still multiplied 11111 by 11111 to verify the answer. In Figure 56 (solution 3) the pre-service teacher provided the subsequent step which would be correct if the question read 1111 x 1111. This kind of mistake is also made by learners.

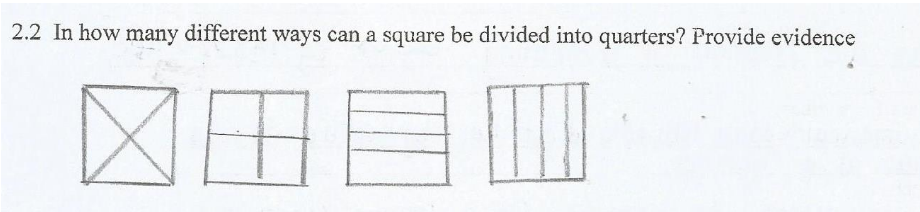
Fractions are commonly described as a part of a whole where the whole is divided into the required number of equal parts. They are also written differently from a whole number in that it has a numerator and a denominator, example, ⅗. Therefore it is essential for the pre-service teachers to understand the proportional representation of a whole into fractional form to teach concepts associated with fractions effectively.



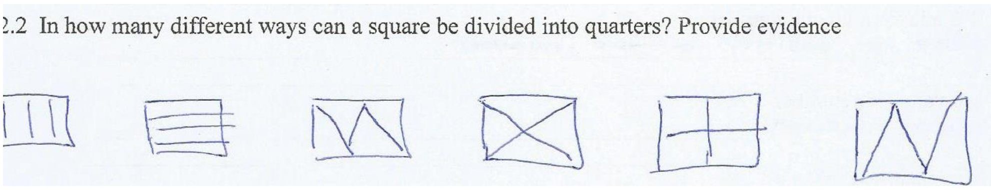
Solution 1



Solution 2



Solution 3

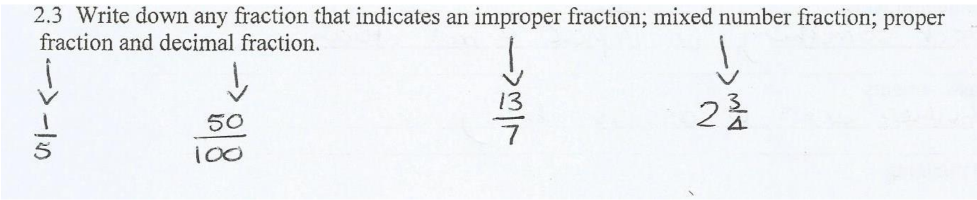


Solution 4

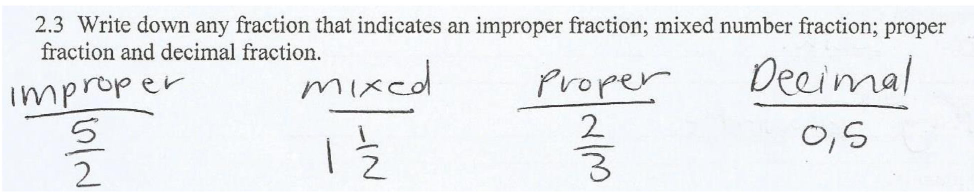
Figure 57 pre-service teacher's solutions

Only forty three of the pre-service teachers produced a correct solution to the question (Figure 57). Misconceptions when dealing with fractions abound when the learner's basic conceptually knowledge of fraction has not been concretized. If this is the case then writing and representing a whole as a fraction, would create all sorts of problems for the learners as they progress through the grades. The third and fourth illustration in solution 1 (Figure 57) showed the pre-service teachers erroneous understanding of the concept of quarters. Whilst their representations indicated a partial correct answer, the incorrect representation was indicative of their thinking. These representations (Figure 57 - solution 1- illustration 3 and 4) indicated that dividing a shape into four parts does not necessitate it is divided into quarters. To be a quarter a shape has to be divided into four equal parts so that it can have 4 as a denominator (⁄4). French (2005:3) stated that the pre-service teachers have a limited understanding and technique view of the subject therefore it is important that we extend the subject knowledge of our students or they will go on to reinforce their view in their teaching. The pre-service teachers need subject knowledge so that they become aware of the different mathematical ideas and of the common misconceptions associated with them so they can use it for effective teaching (French, 2005:2). The illustrations of quarters in Figure 57 (solutions 2, 3 and 4) indicated that the pre-service teachers have mathematical knowledge of representation

A fraction is written as one number over another with a bar line separating them, example, ⅛. The function of the denominator is to denote the number of equal parts a whole has been divided into and the numerator indicates the number of parts one is working with, example, in ⅛ the 8 represents the denominator (eights) and the numerator is indicated by the 1. Fractions are taught from the Foundation Phase. According to CAPS it is in this phase that learners learn to represent shapes or classify items into halves, thirds, quarters and fifths. As they progress into the intermediate phase they learn about sixths and upwards. It is within this phase that they learn about identifying and writing fractions as proper fractions, improper fractions, mixed number fractions and decimal fractions. The pre-service teachers were asked to write down any an improper fraction, mixed number fraction, proper fraction and decimal fraction.



Solution 1



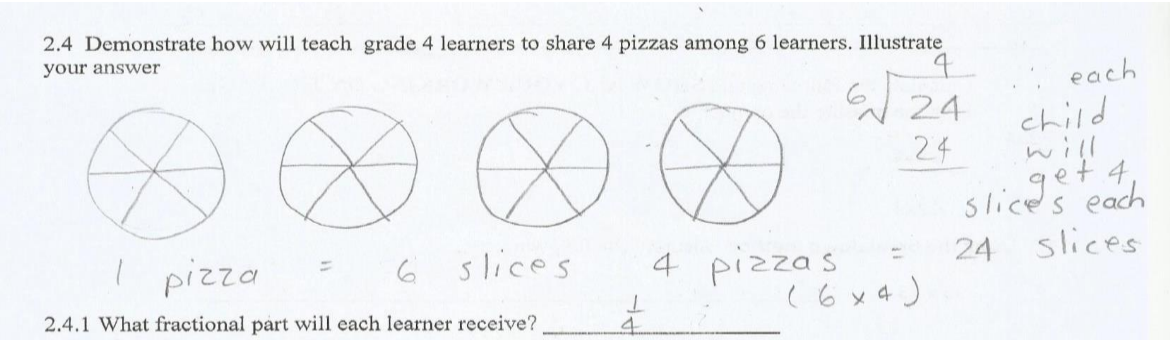
Solution 2

Figure 58

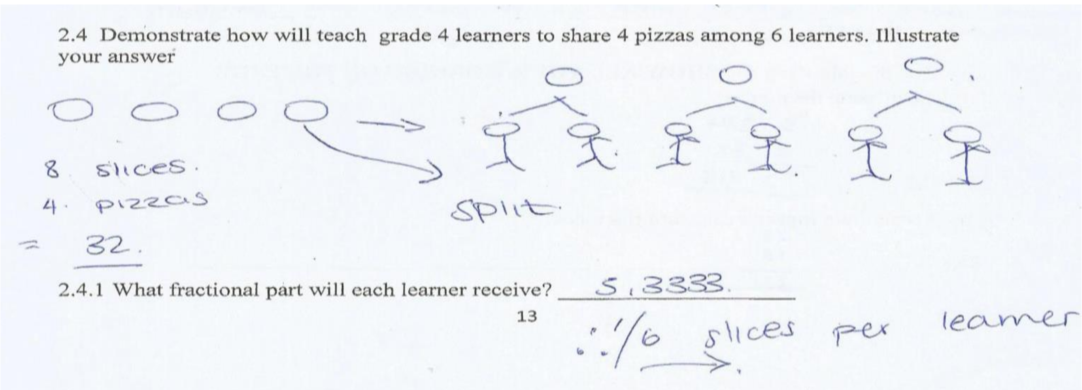
Majority of the respondents wrote out the fractions as requested (Figure 58 - solution 2). In Figure 58 (solution 1) the decimal fraction is indicated incorrectly. I can hypothesise that the respondent did not have the necessary mathematical knowledge about representing a decimal fractions correctly.

Representations are a fundamental way to teaching and learning of mathematical problem solving. It provides insightful information on ones thinking ability. When communicating with the learners through the use of representations, it is important that the pre-service teachers understand how to use teacher content knowledge in representations. Any visual misconception on the part of the pre-service teacher will affect the learner's mathematical achievement as the learner will be taught incorrectly.

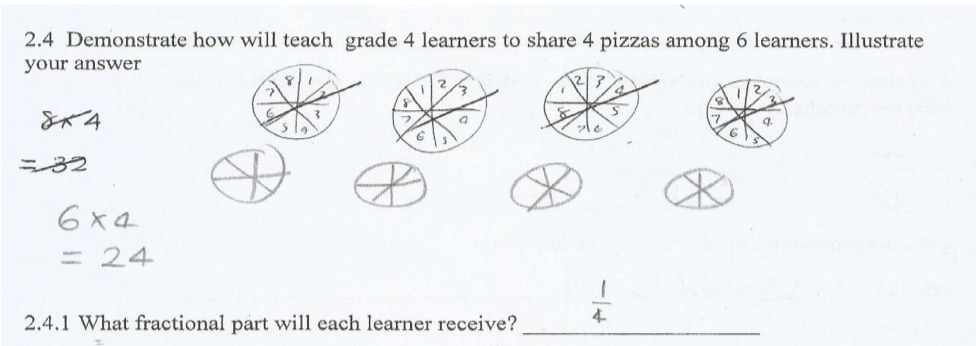
In explaining mathematical concepts to learners, the pre-service teachers must be able to use text and picture to explain the concepts to them. In answering question (Figure 59) the pre-service teachers had to provide an explanation to support their illustrations.



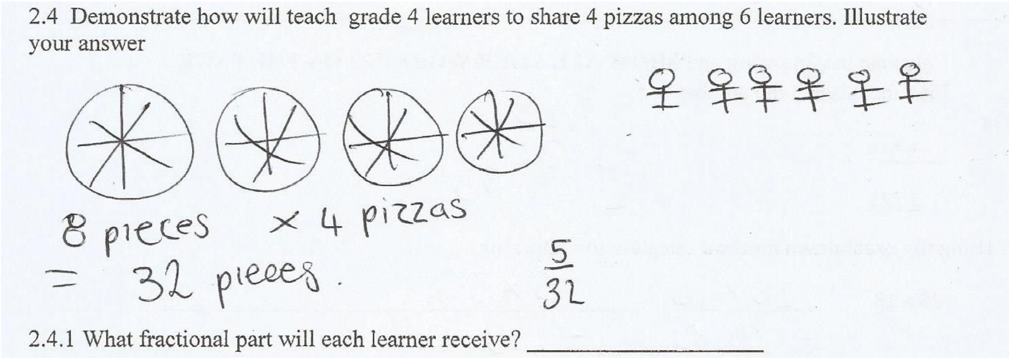
Solution 1



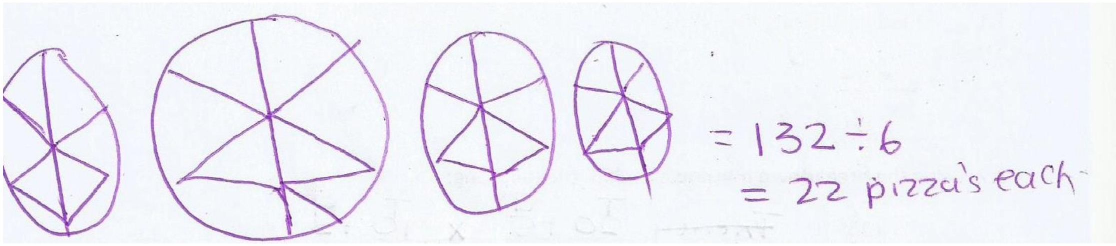
Solution 2



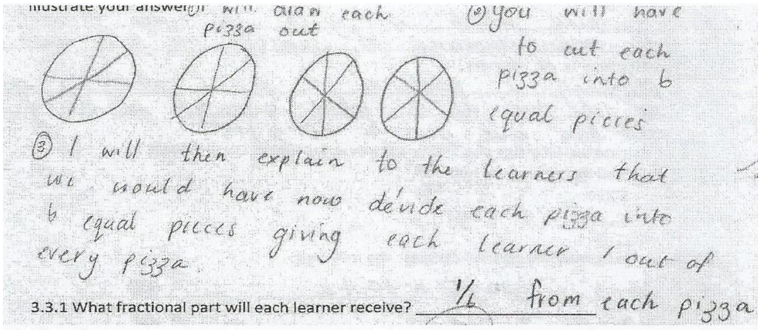
Solution 3



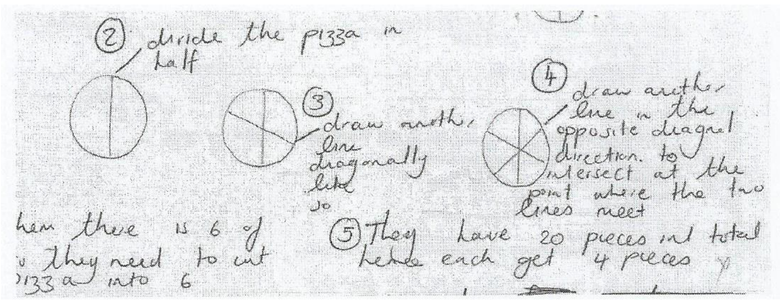
Solution 4



Solution 5



Solution 6



Solution 7

Figure 59 pre-service teacher's solutions

The illustrations in Figure 59 are a representation of the pre-service teacher's conceptual understanding of fractions. Only four percent of the pre-service teachers provided a correct solution and it is difficult to hypothesise if they were asked to teach this question whether it would be done logically. In Kolb's Theory of Learning having prior knowledge is an essential for all learning. When a person reads he creates a relational model connecting his internalized knowledge with the external representations, namely, associating old with new. They commenced answering the question by drawing circles thus making a diagrammatic representation of a pizza. In Figure 59 (solution 1) the pre-service teacher drew four circles. Each circle was divided it into 6 pieces to indicate 1 share per learner. The representation of the four circles divided into 6 pieces provided a total of 24 pieces. The pre-service teacher then divided the 24 pieces by the 6 learners to obtain an answer of 4. In the calculation the incorrect symbol is used for division, namely, the square root sign ($\sqrt{}$) was used instead of the symbol used for division. The answer indicated was $\frac{1}{4}$.

In Figure 59 (solutions 3 and 4) the pre-service teachers used their prior knowledge that a pizza is normally sliced into eights (8 pieces) thus they divided their circles representing the pizza into eight pieces. In Figure 59 (solution 2) although drawing four circles the pre-service teacher only shared three circles between the six learners indicating an answer of $\frac{1}{6}$ and 5,3333.

In Figure 59 (solution 4) although the illustrations and explanation indicated 32 pieces it is difficult to hypothesise how the pre-service teacher arrived at the solution of $\frac{5}{32}$.

In Figure 59 (solution 5) I found the representation and explanation illogical and it would have been very difficult for the learners to understand such an intricate diagram and explanation. While there are relevant elements present in the diagram to the question, there was no logical link from illustration to the other. Mathematics is a language and a diagram must be explained in a logical manner. In this instance the diagram is incomprehensible. It is important that clear and concise language is used and the illustration must be as realistic as possible for the learner to understand.

In Figure 59 (solution 6) the pre-service provided an explanation to accompany her illustration. In this manner the diagram assisted to reinforce the explanation in a step by step manner which definitely is an asset when teaching mathematics. This is normally called self-explanatory. Likewise in Figure 59 (solution 7) the pre-service teacher drew the diagram and provided an explanation for each of the diagram. The error is noted in step 5 where the explanation indicated 20 pieces whilst the explanation in step 4 stated that the pizza needed to be cut into pieces.

5.7 CONCLUSION

Teaching mathematics involves engaging the learner's prior knowledge, linking it to the teacher's content knowledge and extending the mathematical ideas to the real world. Whilst the majority of the pre-service teachers appeared to understand the mathematical terminology or concepts, there were those who misconceptualized the mathematical concepts. These misconceptions materialise out of the pre-service teacher's own experiences or having little or no knowledge of the meanings of the terminologies used in mathematics. Learning and understanding of mathematical concepts and terminology used in mathematics is important and this statement is supported by National Research Council (2001:371) who stated that teachers need to understand concepts correctly and perform procedures accurately. As future teachers it is important that they have a clear understanding of all concepts as it lays the foundation of teacher knowledge. Whilst having the necessary conceptual knowledge it is not the same as knowing how to teach the subject.

Overall, the majority of the pre-service teachers who participated in this study performed poorly in answering the mathematical problems. Many incorrect solutions where produced. This resulted from them lacking the necessary mathematical knowledge, weak conceptual understanding, poor comprehension levels and not making use of visual skills.

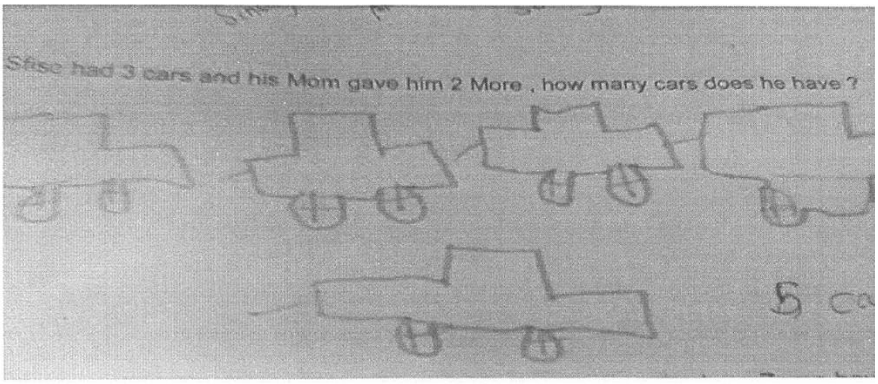
5.8 ANALYSIS OF EXAMINATION OF THE LEARNERS' BOOKS

I provide a brief summation of how the learners in the primary schools are progressing in respect of problem solving and using visualization. The grades four, five, six and seven learners have been engaging with word problems since the foundation phase. The teachers have used diagrammatic representations to represent the value of numbers when teaching them how to calculate algorithms. In turn the learners have imitated their teachers to do likewise in their classwork books.

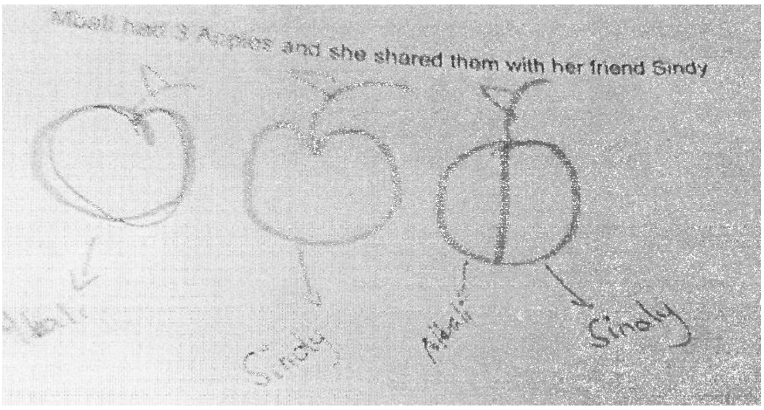
The learners have an exercise classwork book for their daily written work and they also use the departmental supplied workbooks as additional support material. The workbooks have many visuals, many of which represent the concepts to be learnt. Most of the exercises on word problems in the textbook are at the end of the chapter. Prior to the exercise on word problems, most of the exercises are based on learning algebraic skills which the learners then use to answer the word problems. The exercises in the workbook have very limited problem solving opportunities as the tasks are set to learn or reinforce the learner's computational skills.

For learners to become prolific problem solvers, they should encounter unfamiliar mathematical problems. The CAPS states that problem solving must be an everyday occurrence in the classroom. The learners are expected to be exposed to solving problems not necessarily based on the content taught during a mathematics lesson.

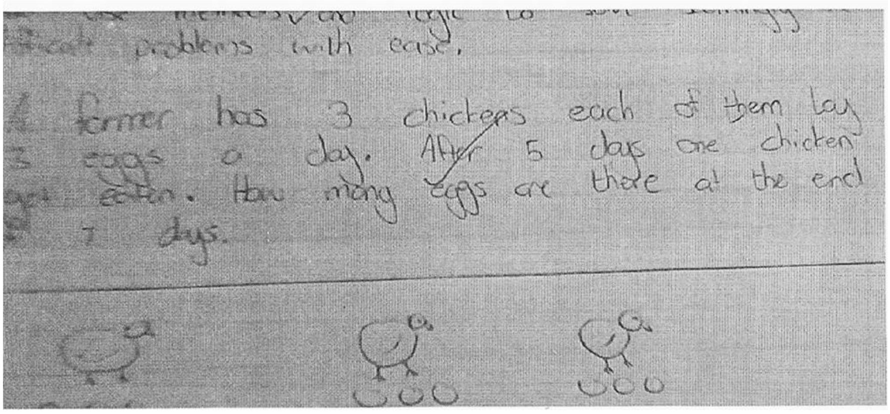
The grade four learners were mainly exposed to solving problems based on reinforcing computational skills. The questions set were mainly of the routine nature (Figure 60 - solutions 1 and 2). The questions did not offer any challenges nor did it assess the learner's reasoning skills. This resulted in the learners providing a single step diagrammatic solution to the problems which was actually a translation of the word problem.



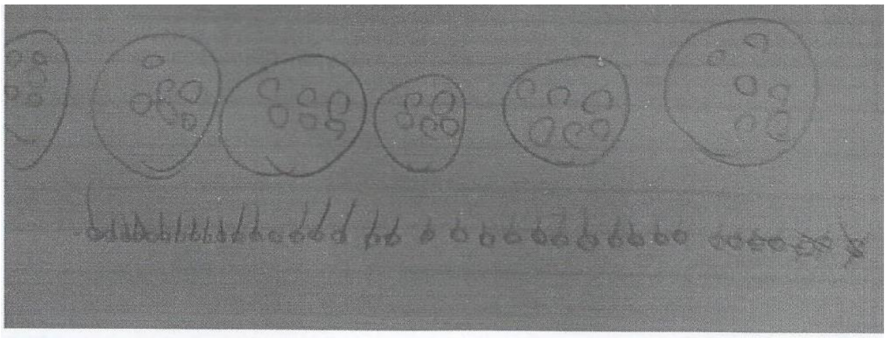
Solution 1



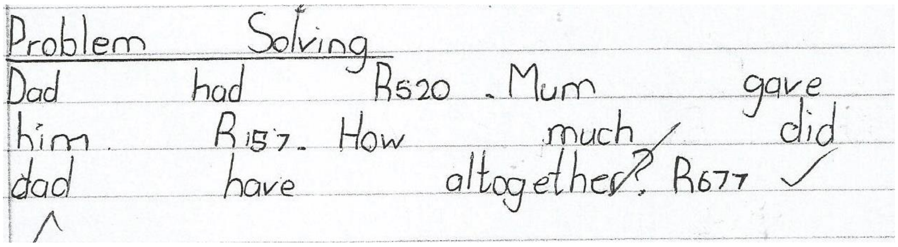
Solution 2



Solution 3



Solution 4



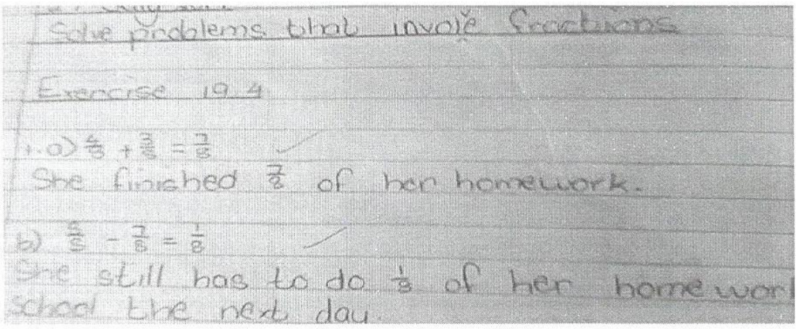
Solution 5

Figure 60 – solutions from grade 4 learner's books

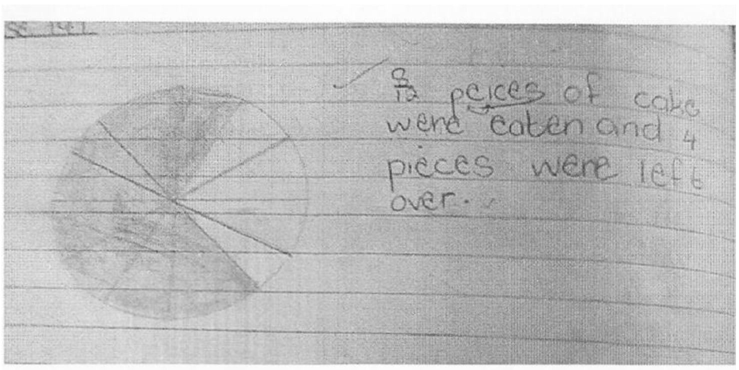
The word problems in Figure 60 (solutions 1 and 2 required the learners to use their relevant algebraic skills and convert the given word problem into a solution. When working with word problems, the learners invent informal modelling or counting strategies for their calculations (Figure 60 – solution 4). In Figure 60 (solution 1) the learner represented 3 cars and then added 2 more cars to obtain an answer of 5. The solution is readily visible as one reads the problem. In Figure 60 (solution 2) the learner needed to share 3 apples with her friends. The diagram indicated the learner understood the concept of sharing which is the foundation of learning division. In Figure 60 (solution 3) the question is not straight forward. The learner had to analyse the problem. The illustration indicated that the learner understood the initial part of problem and represented it as stated and the teacher marked the answer as correct which actually is incorrect.

Besides the type of problems indicated in Figure 60, the learner's books showed no evidence that they were exposed to non-routine problems nor taught problem solving strategies. Their classwork books showed that the learners mainly completed exercises relevant to the content as per the scope of work for the grade four. Likewise the learners did repetitive exercises from their workbooks when the teacher finished teaching the concepts. These exercises were given to determine if the learners understood what was taught to them in the lesson.

An analysis of the grade five learner's books (Figure 61) indicated that many of them were exposed to solving routine problems. There was evidence that the learners attempted the problems by using diagrams. The visual representations were used to represent the solutions. As mathematics teachers, we are preparing our learners for a life time of learning. We need to encourage them to make use of their combined visual and mental faculties to attempt, revise and rethink solutions in all situations including circumstances outside the classroom.



Solution 1

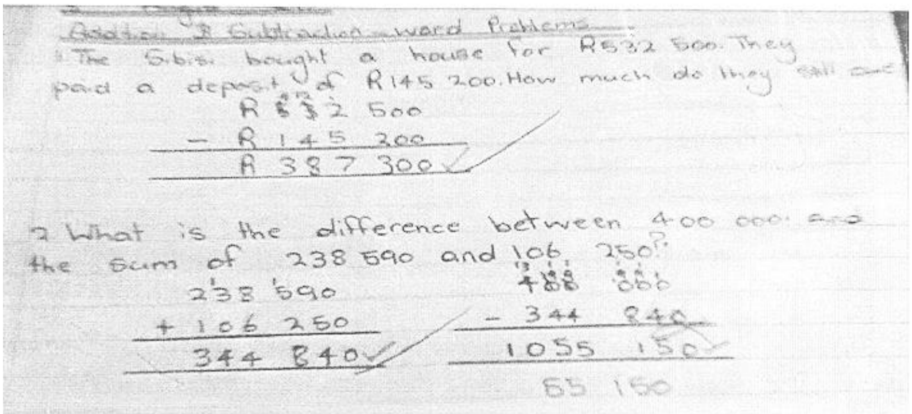


Solution 2

Figure 61 – solutions from grade 5 learner's books

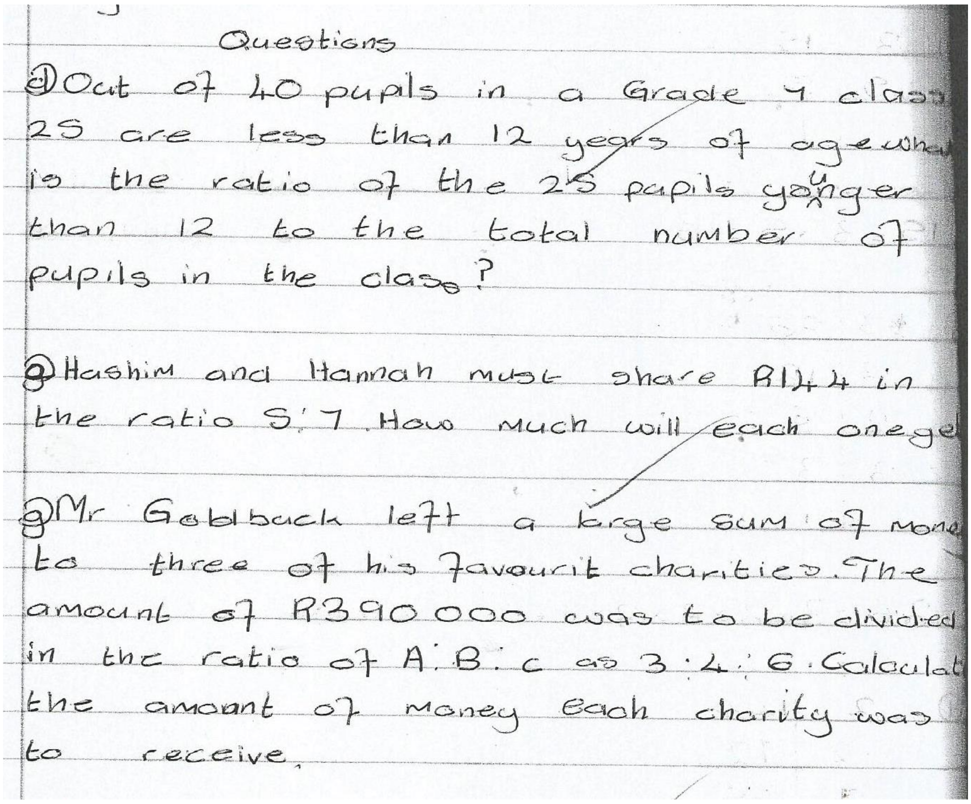
Many of the grade 5 learner's books revealed that that they were required to copy the mathematical problems from the chalkboard. I am of the opinion that this works favourably for the learners. It gives the learners an opportunity, whilst writing, to feel the problem. Teaching the learners to adapt from a transcription to an illustration strategy will allow the learners to develop their ideas and indicate their conceptual understanding through diagrams. Whilst transcribing the problem they read, comprehend, visualize and plan what needs to be written. They look for patterns, make connections and eventually communicate their thinking.

The study of the grade 6 and 7 learner's books indicated that the teachers were not focussing on problem solving. If they did, then the focus was on calculating routine problems from the textbook or workbook (Figure – solution 1). As this was a general analysis of the work done in the learner's book, it was difficult to determine if the learners were reluctant to use visualization in solving problems or was it apathy on the part of the teacher to teach the learners how to use visualization when solving problems. The grade 6 and 7 learners rarely used problem strategies in their written work. The mathematics teachers need to scaffold learners on how to use visualization. If a teacher rarely uses visualization in their teaching then the learners are bound not to use it in mathematical problem solving.



Solution 1 – solution from a grade 6 learner's book

Figure 62 – solutions from grade 6 learner's book



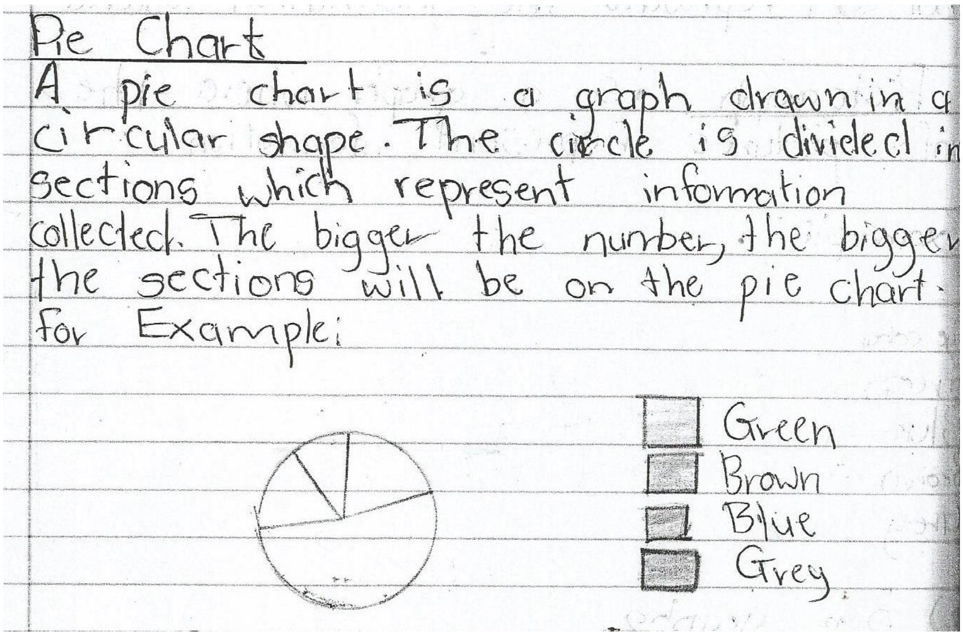
Questions

① Out of 40 pupils in a Grade 7 class 25 are less than 12 years of age what is the ratio of the 25 pupils younger than 12 to the total number of pupils in the class?

② Hashim and Hannah must share R144 in the ratio 5:7. How much will each one get

③ Mr Goldback left a large sum of money to three of his favourit charities. The amount of R390 000 was to be divided in the ratio of A:B:c as 3:4:6. Calculate the amount of money Each charity was to receive.

Solution 1



Pie Chart

A pie chart is a graph drawn in a circular shape. The circle is divided in sections which represent information collected. The bigger the number, the bigger the sections will be on the pie chart. For Example:

Solution 2

Figure 63 learner's work

The grade 6 and 7 learner's written work indicated that their work is highly prescriptive. They mostly followed the step by step method as indicated in their textbooks or taught by their teachers. The danger of using the step by step method can result in the learner's applying the learnt skills incorrectly (Figure 62 - solution 1). The learners are taught that when they cannot subtract a number (subtrahend) from the minuend in a subtraction algorithm, then they need to borrow from the next column. In Figure 62 (solution 1), in the second step the learner borrowed from the tens column although in the units column the 0 needed to be subtracted from 0. The manner how learners obtain their answer is dependent on how the teachers teach the learners. The corrective work provided by the teacher is also incorrect. In the literature review (chapter 2) I made mention of these types of errors committed by both teachers and learners. In this chapter, when discussing the pre-service teacher's evaluation worksheet, such errors were observed.

A teacher who is expansive in his though will move away from following this type of methodology and engage the learners in a variety of ways to do their calculations. The manner in which a teacher organises his teaching will impact on teaching and learning in the classroom. It is obvious from the examination of the learner's books that the teachers were using prescriptive methods hence the learners were copying and producing work of such a nature. The learners need to be encouraged to use their own discovery methods. This will evidently allow them to explore the different ways to solve problems and also give them an opportunity to try and use them in real life situations.

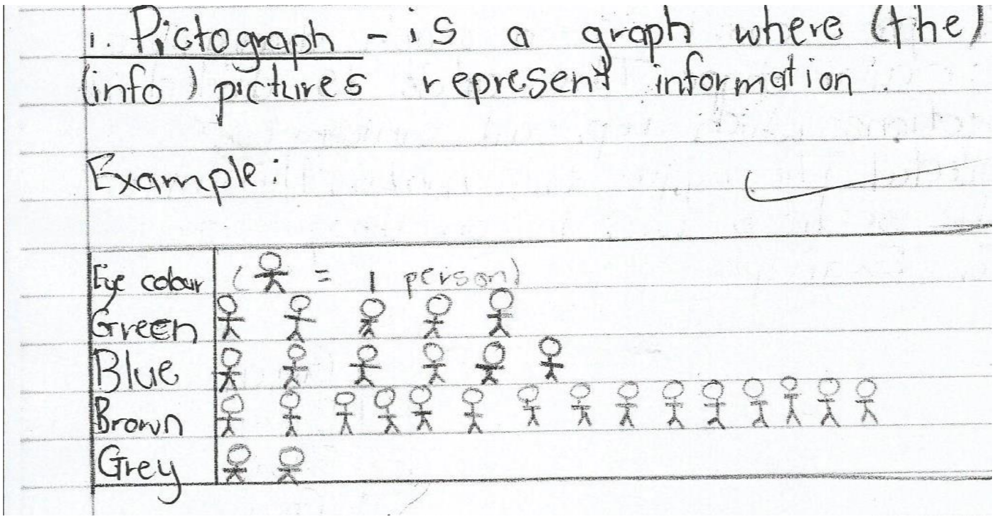


Figure 64 learner's work

It must be stated that although not using visualization in teaching learners how to solve problems or teaching them problem solving strategies, there is evident that the teachers are using visualization in teaching mathematics (Figure 63 solution 2, Figure 64 and Figure 65).

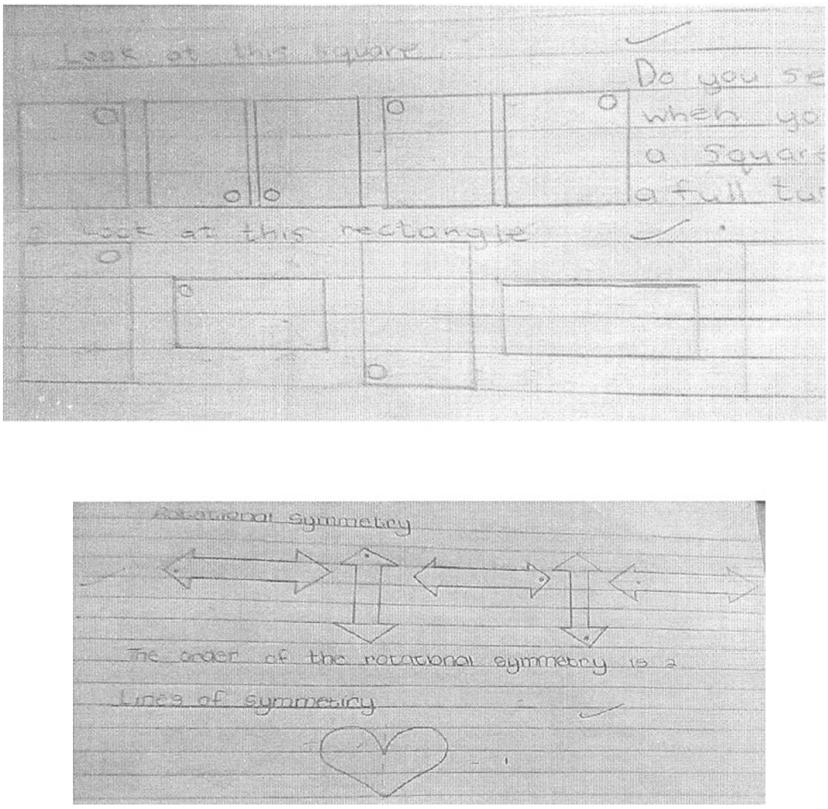


Figure 65 learner's work

5.9 CONCLUSION

From my observation of the learner's books, there is a definite need for the teachers to move away from slavishly using the textbook methods in their teaching. Understandably they have a set curriculum to complete but they need to pay attention to teaching learners to problem solve and engage with problem solving strategies. More attention needs to be paid to using non-routine problems in their teaching. The mathematical content in each grade allow for teachers to use their specialist content knowledge, pedagogical knowledge and technological knowledge to engage their learners in solving problems. They need to move out of their comfort zone of teaching from the textbook and motivate their learners to adapt to learn mathematics problem solving strategies through visualization. This type of engagement allows the teacher to move away from the conventional chalk and talk method to getting the learners to be adaptive with their mathematics in all situations. The complementarity use of both visualization and problem solving strategies will surely assist the learners overcome the mathematical challenges that may present itself in their academic career or their personal lives in the future.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

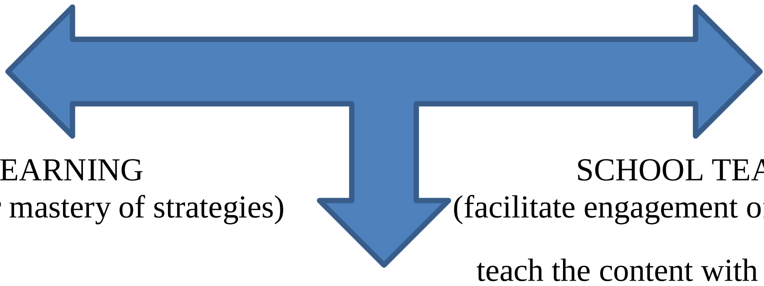
The goal of this study was to examine pre-service teachers' use of visualization when teaching and solving problems in the mathematics classroom. To address the study, three research questions were posed: firstly, what strategies do pre-service teachers use when solving problems?, secondly, how do pre-service teachers teach problem solving in classrooms? And thirdly, how do visual strategies affect the teaching and learning of problem solving? In this chapter I discuss the findings of this study and make recommendations that may contribute to the effective use of visualization when solving problems.

6.2 CONCLUSION

Acquiring problem solving strategies is one of the most crucial focal point to develop pre-teacher's ability in problem solving. The pre-service teachers need to educate themselves in new teaching strategies which will lead to better taught learners. The first question in this study sought to determine what type of strategies the pre-service teachers used when solving problems. Most of my lectures start with an engaging activity. In my very first lecture the set activity required them to solve a few mathematical problems which I had taken from the Mathemagica Problem Solving Competition. The pre-service teachers were hesitant to answer the questions. A few of them questioned me how they were learning mathematics by solving the problems with one pre-service teacher stating '*why are we been asked stupid questions*?' This took me by surprise but I managed to cajole them to complete the said task. The responses, whilst not used in this study, showed that the pre-service teachers lacked problem solving skills and were also not familiar with problem solving strategies. During my walk about during this activity it became evident that majority of them were using algorithmic procedures to find the solutions and many of them were not answering the questions. Those that answered the questions literally translated the given problem into numbers and symbols and some even tried using equations to solve the problems. On completion of the task, a discussion followed and I gave the pre-service teachers an opportunity to share their solutions with their colleagues. It was quite noticeable that the pre-service teachers were facing challenges in finding solutions to the mathematical problems. I delved further to ascertain why they were battling to solve the given problems. Some of the responses indicated that they were not exposed to problem solving strategies whilst at school as the focus was more on solving algebraic, trigonometry and calculus type of questions. Mathematical problem solving and use of problem solving strategies was not part of their teaching and learning norm. That in turn would mean that their previous mathematical knowledge on problem solving was weak.

In the 21st education system set up at schools teaching and learning mathematics is anything but simple. With this study I have discussed teacher knowledge (discussed in chapter 3). I have made a modification to this representation to indicate school content knowledge. The teachers in the secondary schools are curriculum, subject content and assessment driven. They focus on preparing their students to write the Senior Certificate Examinations (SCE) such that the students are given typical questions that will most likely appear in the examinations. These responses, "*the type of questions I give them is similar to that they will receive in an exam or an assessment*", "*because you know what you're going to be assessing them and because of time constraints, you can teach the content that's in the assessment*. *I'm afraid that that's the sort of thing that has crept in*" and "*the task of a secondary school is to follow the curriculum*" obtained in a study by Hong, Kerr, Klymchuk, McHardy, Murphy, Spenser, Thomas, and Watson (2009:244) is similar to the predicament that the South African teachers find themselves. The South African Government sets store by the results obtained in the SCE as it is one of the dynamics that gauge the success of the South African education system. Whilst the focus is on the school curriculum, the lack of basic knowledge like problem solving skills affects the secondary school students who want to pursue a career as a mathematics teacher. Guzman, Hodgson, Robert and Villani (1998:748) stated that "…*the secondary-tertiary transition can be seen as a major stumbling block in the teaching of mathematics*".

I have discussed teacher knowledge in chapter 3 but I have modified this model to include school content knowledge. Whilst the model of teacher of knowledge (Killen, 2015:31) focuses on the acquisition of knowledge about teaching and learning, I have added school content knowledge (Figure 67: A) due to the challenges experienced by the pre-service teachers. According to Kazazi, Al-Rashdi and Al-Azri (2016:211) there is great need for students to have a proper grounding and for the teachers to ensure that the school content knowledge is "*properly and adequately fulfilled thoroughly*". I have created a diagrammatic representation of a Learning, Curriculum and Teaching model (LCT) (Figure 66) to support the modification to the teacher knowledge model (Figure 67). Figure 66 is indicative of how teaching occurs in the South African context. In normal school circumstances, school learning is reliant on how teaching occurs in the classrooms. Teaching in turn depends on the type of content prescribed in the school curriculum. Whilst the content in the South African context is prescribed, it's the teacher ingenuity that is required to ensure that the students are challenged intellectually to show that understanding and learning has occurred. There must be a shift from teaching mathematics theoretically in schools. The focus should be such that teaching and learning revolves around the teacher acquiring knowledge (learning) of the various problem solving strategies, examining the probability of selecting the strategies and structuring them from the school curriculum content (Figure 66) and using it practically in the teaching process in order to prepare the learners for tertiary studies especially those that intend pursuing a career in the teaching profession (Figure 67-A). Learning requires a demand for the mastery of mathematical strategies such that the teachers can facilitate the engagement of these strategies from the school curriculum in their teaching so that they can teach the curriculum content with accuracy (Figure 66). If this does not materialize then the students are going to struggle a lot to survive during their studies as they try to acquire the new knowledge (Kazazi, Al-Rashdi and Al-Azri, 2016:211). A student, in a study conducted by Thomas, Klymchuk, Hong, Kerr, McHardy, Murphy, Spencer and Watson, (2010:28), responded that "*I think that's because the school syllabus doesn't go into enough depth, and if someone's going to do math in uni anyway, then they should learn things properly in high school, I think that's quite important*". What is needed during the school learning phase of the LCT model and knowledge of teaching (Figure 67-C) is a strong focus on the mastery (learning) of the various problem solving strategies (discussed in chapter 2).



SCHOOL LEARNING
(demand for mastery of strategies)

SCHOOL TEACHING
(facilitate engagement of strategies so to teach the content with accuracy)

SCHOOL CURRICULUM CONTENT
(selecting strategies and structuring them to accommodate the mathematical content)

Figure 66 Learning, Curriculum Content, Teaching Model

Both the first critical question, namely, what strategies do pre-service teachers use when solving problems and the second critical question, namely, how pre-service teachers teach problem solving in classrooms must be seen in context. The responses obtained from the questionnaires were relevant to the first question, namely, what strategies do pre-service teachers use when solving problems? The pre-service teachers listed guess and check, draw a diagram and trial and error as problem solving strategies. Although some of the pre-service teachers listed these strategies in the questionnaires, the same strategies were not replicated when solving problems. This was evident from the data collected from the pre-service teacher's evaluation worksheet (discussed in chapter 5).

As future teachers, the responsibilities of the pre-service teachers have become more conspicuous to accommodate learner’s skill development, cognitive thinking, creativity, decision making and reasoning ability. The mastering of the teaching mathematical content (knowledge of teaching) is not the only challenge that the pre-service teachers face. They need to develop a system of learning (Figure 67-C) whereby they learn varied practices beyond the context of learning procedural techniques (knowledge of content and knowledge of teaching). This is supported by Darling-Hammond, Hyler, and Gardner (2017:v) who stated that teacher learning is one of the many complex skills that needs to be acquired in preparation for further education and work in the 21st century. Although the pre-service teacher’s mathematics module had a pacer, I allowed myself to reorganise the module material and I started to teach them the basic steps involved in problem solving and how to incorporate the various problem strategies in their lessons. This kind of teaching is synonymous with the structural learning theory. The structural learning theory advocates the teaching of the simplest solution path for a problem and then teaching the more complex paths until the entire rule has been grasped. This shift brought about an increase change on how they solved problems. This change is supported by the data obtained during their teaching experience. The data from the lesson observation revealed that some of pre-service teachers attempted using the problem solving strategies in their teaching. I can assume that this was due to them having more exposure to the various strategies during lectures before they went for their teaching experience which normally happens in the third term. This was supported by the qualitative data obtained from the questionnaire and lesson observations (discussed in chapter 5). From my initial observation to the conclusion of this study, the data shows that there was an improvement from pre-service teachers not having prior knowledge on problem solving strategies to them showing adequate knowledge on how to engage with problem solving strategies. Thus one will note the importance of knowledge of content, knowledge of learning and knowledge of teaching (Figure 67- B and C).

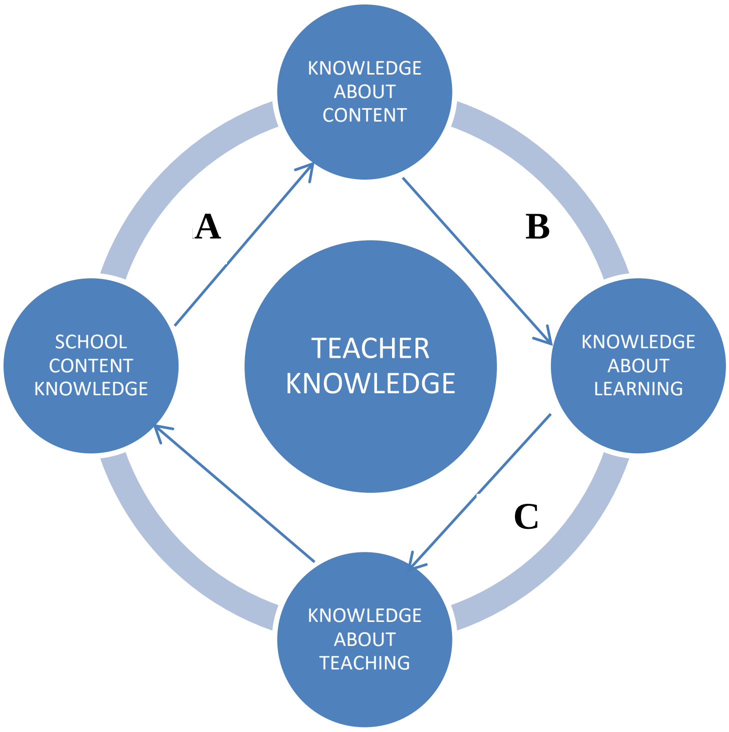


Figure 67 Teacher Knowledge

Many of the higher education mathematics modules are an outline of the school's mathematics curriculum. Most of the curriculum related content material in these modules lack focussing on problem solving strategies. In order for the pre-service teachers to acquire content and pedagogical knowledge (Figure 67-C), the higher education modules need to move away from focusing on the general content. In discussing the teacher knowledge model (Lecture 67), the mathematics modules studied at the higher education institutions need to include content material on problem solving strategies for each of the five content areas covered in the mathematics curriculum allowing the pre-service teachers an opportunity to engage with the content and develop their own possible strategies within it. This can only be done during their years of study as the training would involve the pre-service teachers engaging with the probable examples of the problem strategies against the content material. According to Polya (1945) problem solving is a practical skill and it requires a lot of training and experimentation thus during the learning phase the pre-service teachers will need explicit training to master mathematical problem solving strategies so that they can use them to bring new mathematical information to mind which will invigorate their thinking levels. In light of the obtained data it is of extreme importance that the pre-service teachers have more exposure to develop mathematical strategies. The mastery of these strategies will allow them the opportunity to experiment and use them in the many contexts. According to Ball and Forzani (2009:499) allowing the pre-service teachers opportunities to practice the work will allow them to adequately hone their problem solving skills. Thus the pre-service teachers need explicit training to master and consolidate the strategies for solving problems so that they can tackle mathematical problems with enthusiasm.

The mathematics curriculum content has been designed such that it shows the progression of content to be taught from grade to grade. When engaging with content (Figure 67-B) the pre-service teachers can select strategies and structure them to accommodate the mathematical content taught in schools. When doing so they create their own problem solving strategies guide. According to Ball and Forzani (2009:505) "*the work of designing instructional activities for teaching and learning is a vital part of developing a curriculum*". Designing their own problem solving guide will place them at a distinct advantage as this can used to complement the mathematics curriculum. In doing so the pre-service teachers will play a pivotal role in transforming each of the five content areas, namely, numbers, operations and relationships; patterns, functions and algebra; space and shape; measurement; data handling. Instead of only focussing on the algebraic development, they will now be in a position to factor in the problem solving strategies to create problem solving situations when working with the various aspects of the content. The first step is for the pre-service teachers to learn the skills and then prepare themselves to incorporate the strategies for each content area so that it can develop their skills (Figure 67-B and C). This will assist to demonstrate their understanding of mathematics and the strategies can be aligned in such a manner that it shows interconnectedness within the five content areas allowing the learners to see the link between the different sections of content. I will discuss the section of measurement as an example to show the possible interconnectedness of the mathematical content. Measurement includes the sections of mass, length and capacity. The content in these sections can be taught alongside each other. Although these sections have different units of measurement, the idea of conversion from one unit to the other is the same. I use one example, namely, 1 000 millimetres = 1 metre; 1 000 milligrams = 1 kilogram and 1000 millilitres = 1 litre and vice versa. The pattern of similarities is noticeable and since mathematics is all about sense making, there is no need to teach the sections in isolation as the learners will be able to fathom out the pattern in order to make a reasonable mathematical choice for their answers. Furthermore, there is no need to teach these sections separately as the teaching methodology and the applicable procedural knowledge is the same. Rather than teach sections in isolation, this type of teaching will allow the teacher an opportunity to also merge other sections in their lessons. Working with multiple maths content areas within one activity might influence teachers to cover more maths material in one lesson. Once the learners see the interconnectedness of the content areas they are more likely to cultivate their own individual conceptual ideas and make useful connections.

Whilst the first question focussed on the problem solving strategies, the second question focused on how the pre-service teachers teach problem solving to their learners in the mathematics classrooms. The significance of this study lies in its attempt to reveal pre-service teacher's abilities to teach problem solving and to what extent they have abilities and skills to do so.

This study revealed some important results on pre-service teacher's training. Poor performance in mathematics is a global phenomenon and this can be attributed to what people learn, what they are taught and how they are taught. This statement is supported by Ball and Bass (2000) who stated that the acquisition of mathematical knowledge significantly influences how and what teachers teach and how and what their students learn. The pre-service teachers prior to their teaching practice went on an observation period to schools. I had asked them to make notes on how lessons were taught in schools. On they return, they reported that almost all mathematical lessons began with the teacher checking homework. The teacher then presented a few sample problems and demonstrated to the learners on how to solve them. The conclusion phase of the lesson involved the teacher asking learners questions based on the procedures shown to them and checking some of the answers of the given problems and then assigning similar problems for homework. The data collected in respect of the second question indicated that many of the pre-service teachers preferred using this kind of practice in their teaching, i.e., using the traditional method of working procedurally when solving problems. Also, my observation during lectures showed that the pre-service teachers lacked problem solving skills. They preferred to use the routine traditional method to teach the mathematical content without much focus on problem solving. Although the pre-service teachers had different planes of teacher knowledge (discussed in chapter 3), it was observed that they failed to put these into practice. According to National Research Council (2001:371) "*knowing mathematics for teaching also entails more than knowing mathematics for oneself*". Thus the pre-service teachers should have the expertise and proficiency to interpret the curriculum and mathematical content to prepare to teach effectively. In order for that to happen, teacher training and development should become an area of important focus. For that reason I advocate that the pre-service teachers be trained according to a model I named, We Help Our Teachers (WHOT). According to Darling-Hammond et al (2017:1) mastery of challenging content and complex problem solving requires sophisticated forms of teaching skills in order to develop 21st century learners. It is at higher education institutions that we help pre-service teachers learn to teach. The importance of the WHOT can be seen when positioned within the teacher knowledge model (Figure 67). With the WHOT model (positioned as B and C in Figure 67), pre-service teachers need to be given opportunities to learn so that they can teach effectively and the teacher knowledge model focuses on areas that the pre-service teachers need to deliberate on to become effective transmitters of knowledge. In this manner they can refine the pedagogies required to teach these problem skills (Darling-Hammond, Hyler and Gardner, 2007:1). Ball and Bass (2000:94) claim that not providing teachers with concrete mathematical knowledge (this should also include pedagogical knowledge) undermines efforts to prepare high-quality teachers. Using the WHOT model as a support between B and C in Figure 67 will allow them to become accomplished teachers as it is within this phase that they are taught mathematical problem solving strategies and engage with problem solving situations. The pre-service teachers were exposed to the various strategies in my lectures. They were set mini tasks where they had to research a few mathematical problem solving strategies. As the module progressed, I regularly provided mathematical problems and they had to use their researched problem solving strategies to solve the problems. I encouraged them to try more than one strategy when attempting to solve the problem.

With the WHOT model, pre-service teacher's development should commence at the higher education institutions. Understandably the modules are developed for the degrees to meet the requirements of the Department of Higher Education and lecturers are allocated to lecture the course material. In my initial statement above I used the word development but this is sadly ignored during lectures. Development and understanding should be an area of extreme importance in teacher training. The WHOT model can contribute immensely as the focus should be on least fifty percent of practical work. The practical work should be hands on activities involving strategic competence. According to National Research Council (2001) strategic competence refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what has been called problem solving and problem formulation in the literature of mathematics education. Through the WHOT model the pre-service teachers should be given opportunities to examine teaching problem solving methodologies and prepare activities to suit the mathematical content in schools. In becoming proficient at solving problem they learn how to form mental representations of problems, create mathematical relationships, and formulate solution methods (National Research Council, 2001). Once these methodologies have been assembled then the pre-service teachers should be exposed to microteaching.

Within this study I mentioned the importance of microteaching for pre-service teachers. It is through microteaching one can examine the strengths and weaknesses of a pre-service teachers teaching. The WHOT model can be positioned between the knowledge of content, knowledge of learning and knowledge of teaching of the teacher knowledge model (figure 67). This effective program of teacher preparation and professional development helps the pre-service teachers understand the mathematics they communicate to learners and how to facilitate that learning (National Research Council, 2001). In these programs, teachers are not given prescriptions for practice or readymade solutions to teaching problems. I used microteaching as a means to prepare the pre-service teachers for teaching practice. They had to do a presentation on how they will teach a chosen topic and it had to have a problem solving activity. The problem solving activity had to be completed by all present at the lecture. I found this very beneficial to all pre-service teachers. They were able to explore the use of many of the problem strategies that they had researched and also use those that were discussed in the lectures. Not only were the strengths and weaknesses of their teaching dissected during the microteaching, it armed the pre-service teachers with a large array of methodologies to teach problem solving. All of this placed the pre-service teachers at a distinct advantage for their teaching practice as the microteaching exercise now armed them to move away from the use impractical ways of teaching mathematics problem solving.

Another offshoot of the WHOT model is teaching practice. The presence of the pre-service teachers at schools must not be seen as an opportunity for schools to use them as warm bodies for serving relief for errant and absent teachers but rather to support their teaching development. This kind of support can linked to the Teacher Development Experiment (TDE) carried out by Simon, Tzur, Heinz, Kinzel, and Smith, (2000) which examined the interaction that occur in the teaching learning cycle. Teaching practice must be seen as a form of professional development with the aim of experienced teachers helping the pre-service teachers become efficient. This should be of a high quality, sustained and designed to improve mathematics teaching. The support should be positioned between knowledge of teaching and school content knowledge of the teacher knowledge model (figure 67). The support requires the allocation of time and resources and there is no better resource than an experienced teacher. The adage that experience is the best teacher remains valid. Having a qualified and experienced mathematics teacher adopt a pre-service teacher during teaching practice is priceless as the experience enables the teacher to assist the pre-service teacher to learn effectively. Using the WHOT model alongside the teacher knowledge model during the teaching and learning phases allow for collaboration and the sharing of real world learning experience. Using the pre-service teacher who summarised the entire mathematics lesson as an example, this could have been avoided if the mentor teacher had worked with the pre-service teacher in developing the lesson. It must be stressed that besides assisting the pre-service teachers to prepare for work, the mentor teacher has the ability to broaden the pre-service teacher's mathematical knowledge as they know which points to stress and which areas need emphasis in the mathematical curriculum. Mentor teachers have engaged with content and pedagogics over the years and they are in a position to foster the good methods onto the pre-service teachers. Acquiring the knowledge for teaching from their mentor teacher can lead to effective teaching of problem solving skills as the pre-service teachers will acquire the knowhow of the various strategies used when solving problems. The mentor teacher and pre-service teacher interaction will foster the development of mathematical problem solving proficiency over time.

Over and above pre-service training, there must be provision for in-service training, for the constant up-dating of professional knowledge (Gagne, 1974). A newly qualified pre-service teacher asked to teach mathematics for the first time can be daunting. The WHOT model supports the idea of in-service training in teacher's formative years of teaching as it can be used to rebuild teacher's confidence and expose them to novel techniques of teaching the mathematical content and assists in clearing up conceptual and procedural misconceptions. In-service training can help teachers raise the standards of their mathematical teaching problem solving as they will be supported by professionals who can guide their teaching methods.

Learners nowadays are growing up in a world already permeated by mathematics. The technologies used in and outside schools are all constructed on mathematical knowledge. Many educational and jobs require high levels of mathematical expertise (practical and theoretical), critical thinking and analysis. This study examined the correlation between teaching problem solving and how visualization benefits it. The third question examined how visual strategies affect the teaching and learning of problem solving. As visualization is sight and brain related it is something that pre-service teachers can use in their daily teaching and learning activities to employ solve both simple and complex problems. Visualization can be used within the context of the LCT model (figure 66) and teacher knowledge model (figure 67). It is such a powerful technique that it can now be applied to the field of teaching school mathematics (figure 66) to increase learner's ability to engage with mathematical problems from the school curriculum. This was evident when the pre-service teachers engaged their learners with visual stimuli (charts, video clips). Besides getting the learners to engage in the lesson, it was something that the learners referred to whilst working with their class activity. Using visual stimulation is crucially important in the teaching of mathematics as the development of understanding is hidden in it. The 'see and learn strategy' helps to reformulate the problem and makes it easier for the learners to translate the problem from written to visual. When people see they create a schema (visualize) in their mind. This schema enables learners to produce graphical representations to learn and understand the process involved in solving problems. This translation from a written form to a representation makes the solution clearer, enabling the learner to understand the process used in solving the problem. This method of visualization is used successfully in Singapore Maths where representations are used to illustrate mathematical problems. These representations are also used to test the various steps used to solve the problem. If used effectively in mathematics lesson visual strategies can make problem solving activities interesting for learners. This was evident in some of the observed lessons. During teaching practice the use of visual strategies by the pre-service teachers benefitted the learners immensely as they could see and understand what the pre-service teachers were referring too. According to Boonen et al (2016) pre-service teachers need to possess a repertoire of visual strategies and understand how to use it effectively to support problem solving. The active learner participation together with exemplary representations on the chalkboard created an active learner centred classroom. Good understanding enhanced their active participation in the learning process. The collected data indicated that the application of visual strategies to support the teaching of problem solving should be given more prominence in teacher education. Today it is vital that young people understand the mathematics they are learning. The pre-service teachers need to use their visual resources to engage the learner's imagination to think visually. Some of the pre-service teachers used visual resources, example, charts, video clips, worksheets, workbooks and the chalkboard to support their teaching of problem solving. Those who used teaching resources and concrete materials were able to focus their learner's attention on the problem. Charts showing schematic representations of concepts were made to explain the problem whilst some of them used rules alongside the problem to guide their learners through the problem. Video clips were used to show learners the procedures in solving the problems. There is an assumption that this enables the learners to follow and transfer the teaching method to new situations. According to the structural learning theory learners need to learn rules to guide them to calculate problems. The use of both charts and the video clips supported this theory. A few pre-service teachers, in teaching learners how to solve problems, drew their attention to the key words (concepts) in the problem. The focus on the key words can be used to develop conceptual understanding. The concepts need to be explained to the learners for a better mathematical understanding. This can be done using the Frayer Vocabulary Model (discussed in chapter 2). This Model requires the concept and a visual to be placed side by side to create a better understanding together with an explanation. An important pointer of this kind of conceptual understanding is being able to visually represent mathematical situations in varied ways and examining how the different representations can be useful for different purposes (National Research Council, 2001). Once the concept is understood then it gives the learners an opportunity to build on their understanding and work towards a solution for the problem. By working on the different conceptual representations, the learners are likely to explore the use of varying problem solving strategies to the solution to mathematical problems. This suggests that the pre-service teachers need to develop effective strategies for developing learner's understanding of what is required in the problem. Guler and Citlas (2011) stated that there is a positive relationship between visuals and problem solving and more time need to be spent using visual representations in problem solving. A few of pre-service teachers engaged their learners to work their solutions on the chalkboard. Those learners who used illustrations were asked to explain the use of it and how they used it to get the answer. These illustrations indicated the learner's thoughts and also provided a visual means to the teacher to rectify misconceptions or flaws in their solutions. It also gives the learners an opportunity to discuss the similarities and differences of their representations and how they can be connected to yield the same answer (National Research Council, 2001).

Technology is a powerful tool in the modern day mathematics classroom to aid visual stimulation. Technological skills allow for learners to engage effectively with the problem. They create their own representations or use technological created representations to construct knowledge. Thus the integration of technology in mathematics makes it a visualization tool to use for better creativity and active participation.

Research on visualization and the data obtained in this report, influences us that visual strategies is the cornerstone of teaching and learning of problem solving and that all learners can learn to solve problems through visualization.

6.3 RECOMMENDATIONS

Skills do not develop spontaneously but can be taught to enhance success in mathematics. The ability to solve problems can be efficiently taught with problem solving training. Using visual skills and problem solving strategies is also trainable. It is important that we emphasise this aspect in pre-service teachers training programme.

The central question is in what ways we can enhance mathematical problem solving through using visualization. The analysis of the data indicated that the pre-service teachers still pay little attention to the explicit teaching of problem solving skills and using visualization. Therefore it is necessary to improve the training of our future teachers (Ferreira and Arroio, 2007). In order to get a greater picture, an in-depth study is needed of the challenges faced by the pre-service teachers to teaching and using visualization in the CAPS curriculum so we can supply them with all the background knowledge to apply visualization in their teaching,

It is important to address the competencies and the needs of the pre-service teachers in future research. It is envisage that future research should investigate the duration of teaching training modules and the intensity of training needed to achieve a competent level in these modules for teaching purposes. One of the recommended modules that need to be included in their degree should be reading (Department of basic education, 2018).

A challenge to pre-service teachers is to learn how to use technology in a way that is pedagogically sound to support their teaching. Direct intervention is needed to educate pre-service teachers in the development of appropriate technological strategies in order to develop and enhance mathematical problem solving in schools.

According to Foucalt's Theory, visualization and its role in building knowledge is tied to the power and knowledge dynamics of our time (Ferreira and Arroio, 2007). An interesting field for future research will be to investigate the pre-service teacher's content and pedagogical knowledge in the use of visualization when teaching mathematical content. The qualitative analysis showed that despite having completed course material in problem solving the pre-service teacher's demonstrated deficit in mathematical content knowledge and how to teach problem solving.

Further research needs to be considered in regard to effective pre-service teacher preparation so that their training will facilitate adequate learner progress within the mathematics curriculum. It is important to note that the National Minister of Education called a Mathematics Indaba in 2016 for the sole purpose of "*overhauling of the South African pedagogical-content knowledge outlook in Mathematics*" so that quality learning and teaching takes by competent and qualified teachers to inspire learners with competencies for a changing world (Department of Basic Education, 2018:3).

The President's Ministerial Task Team (Department of Basic Education, 2013) had made recommendations in respect of the training of pre-service teachers in South Africa. To date nothing has been implemented. With the suggested recommendations by the task team on pre-service teacher development (Department of Basic Education, 2018) the National Education Minister needs to urgently look to overhauling pre-teacher training in South Africa. If nothing is not done in the foreseeable future, no matter how much of changes are made to the curriculum, South Africa will continue producing mediocre teachers who will continue with mediocre teaching in the classrooms.

6.4 LIMITATIONS

This study was limited by the sample of the participants. The participants within this study are students at a private higher education institution in the EThekwini metropolitan. This institution has a new teacher qualification programme endorsed by the Council for Higher Education (CHE) with their modules differing from other higher education institutions catering for teacher education. Future research should focus on a larger number of pre-service teachers possibly from other higher education institutions that offer teacher training programmes.

Due to the long time frame given for them to complete the questionnaire many of the participants only attempted to complete it when asked for it on the due date. The responses were rather brief or no responses were given.

The evaluation worksheets completed by learners at certain schools indicated that they had a language barrier. This impeded the completion of the worksheet.

Setting up the interviews with the pre-service teachers proved to be problematic as they had lectures or they left campus immediately after lectures due to the transport issues. Some were interviewed during the lecture break and for the others I had to negotiate with the lecturers to allow them to be interviewed during their schedule lecture.

6.5 CONCLUSION

The result of this study differs from previous problem solving literature and visualization. Data from this study indicated that the pre-service teachers have weak theoretical knowledge on visualization and its use in mathematics. According to Ferreira and Arroio (2007) pre-service teachers aren't yet cognizant to the impact of visualization and they don't know how to use them in a fruitful way. They need to understand and improve their knowledge on problem solving strategies and using visualization.

Mathematical problem solving is the most important aspect of teaching mathematics. The causative role of problem solving in the mathematics curriculum cannot be overtly stressed as it boosts the learning process. Its significance cannot be doubted as it increases the way learners think and build their cognitive ability in mathematics. The way you think is what the mind is seeing. Thinking and visualization shows a unique association. Guler and Citlas (2011) stated that visual representations assist learners to understand the problems. Therefore pre-service teachers need to develop their visual skills.

This study makes a unique contribution to the field of mathematics especially in the area of using problem solving strategies and visualization when solving problems. Even in the modern era teachers rely on the curriculum to teach and the only concern is to finish it timeously to meet the needs of the assessment programme. Whilst this may be true, there is growing evidence that students learn best when they are presented with academically challenging tasks that focuses on problem solving and skill building (National Research Council, 2001:335).

The literature within this study supported by the collected data has identified the benefits that visualization can bring to teaching and learning of mathematics. Problem solving strategies are one way in which teachers can support learner's mathematical potential. Teachers that use a variety of problem strategies and engage their learners to use visualization are bound to create powerful thinkers. According to Guler and Citlas (2011) teachers who use visualization more than other teachers have problem solving success. The visual imagery creates conceptual schemas and the strategies assist in making connections in the execution of the steps in solving the problem. The combination of both strategies and visualization provides learners with opportunities to become better problem solvers. Visualization helps the learners to understand and clarify before applying a procedure to the problem.

Innovative teaching is needed to improve learner's achievement in mathematics. This involves the use of mathematical problem solving strategies and visualization. Polya (1945) emphasised the need for teachers not to only discuss the problem with the learners but they also need to illustrate and practice the problem with them. Teaching learner's problem strategies gives them an opportunity to build their mathematical knowledge. They are able to relate with real world situations and this motivates the learners to connect their acquired knowledge with everyday life making understanding easier.

Visualization can be used innovatively in education to grasp the learner's attention. Visualization is indispensable as it allows the learners to explain themselves and demonstrate their understanding of concepts. It conveys the conceptual knowledge to the learners who may lack proficiency in solving problems as the learners are able to use visualization to connect the mathematical concepts to construct mathematical meaning. Guler and Citlas (2011) stated that learners who use visual representations mire in problem solving are more successful at problem solving.

Modern mathematics is difficult to learn as a language. Using different symbols, learning the various formulas and looking for relationships between them have made it difficult for learners to learn mathematics confidently. The application of the symbols and formulas in solving problems can assist learners become more prolific mathematicians. Using visualization will enable the learners to master them in problem solving. This will enable mathematics to become interesting as the learners will be able to apply the formulas and symbols to concepts making studying subject content relevant.

Teacher education needs to play an important part in preparing the pre-service teachers for the future. They need to become competent in utilising new technologies in their teaching. The emergence of educational technologies has the potential to change the way we teach mathematics and at the same time influence the way learners learn mathematics. It is important that these technologies are incorporated into teacher training programmes as this becomes an avenue for using visualization in the teaching and learning of mathematics.

In the Southern Africa and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) reports of 2000 (SACMEQ II) and 2007 (SACMEQ III) indicated that there were no improvement in South African learner's performance and this finding further is supported by the TIMMS study (Spaull, 2011). The 2007 SACMEQ III made a damning finding that the South African mathematics teachers have below basic levels of content knowledge. One of the problems that they had with learning mathematics was the lack of engagement with primary school mathematics. Considerable attention has to be given to the knowledge that the pre-service teachers need to acquire in order to teach mathematics. This will help alleviate the pre-service teachers contributing to the already poor reputation of the status of mathematics in South Africa. According to the findings of SACMEQ III (2007) report the given fact is that the teachers cannot teach what they do not know and the lack of both content and pedagogical content knowledge will hamper learner attainment in mathematics (Spaull, 2011). This will obviously snowball and have serious implications for the quality of mathematics education in South Africa. According to Hill and Ball (2009:68) "*good teachers know both content and how to "get it across" to their students*". Pre-service teachers need to have complete knowledge of the content in the mathematics curriculum as well as pedagogical content knowledge (PCK). PCK guides the teacher to teach the subject knowledge. The pre-service teachers need to have the necessary mathematical content knowledge (MCK) in order to understand or use the strategies themselves before engaging with their learners.

Higher education institutions need to reassess their education modules to ensure they are relevant and support teaching. CHE need to ensure that all higher education institutions offering teacher education have similar or related modules. This can assist in producing quality mathematic teachers.

The CAPS document dictates the policy on problem solving. Greater monitoring is required by the school's curriculum specialists to ensure that problem solving is taught regularly. Those mathematics teachers, especially those who do not have the qualification to teach the subject and are facing challenges, must be supported by the school based support team (SBST).

Literature supports the importance of using visualization in mathematics as a powerful tool to enhance learner's problem solving ability. The combination of both lays the foundation for better learning in mathematics therefore pre-service teachers will have to become better acquainted in using problem solving strategies and visualization to improve mathematics in the twenty first century.

In South Africa the CAPS mathematics curriculum has the five main content areas listed with many topics that need to be covered as the learner's progress through the grades. I had stated earlier in this study that with the changes in the curriculum, topics from the higher grades had filtered into the lower grades. A noticeable feature is the repetitiveness of the topics during the year. What is really needed in South Africa is a reduction in the topics in the content areas. In Singapore as a result of the TIMMS results, there was a thirty percent reduction in the content taught (Anderson, 2009). Likewise South Africa, as indicated by TIMMS, languished at the bottom and it will be prudent for the curriculum planners to consider reducing the repetitiveness of the topics in the content for the betterment of mathematics. According to The Department of Basic Education (2018:76) "*it is inevitable that curriculum adjustments should be undertaken*", wherever possible, "*to reorganise topics*" and "*the current mathematics curriculum can be reduced to make it more accessible to many learners who are not coping with the large amount of content*". Thus a review of the mathematics curriculum is urgently needed in South Africa.

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