An exploration of the extent Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics

By

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Declaration:

I, Tawia Iddrisu Frimpong solemnly declare that:

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Supervisor's statement

This dissertation has been submitted with/without my approval

Supervisor's signature: [redacted] _____

Supervisor's name: Dr Zanele Ngcobo

Date: 04 August 2021

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Dedication

I dedicate this work to my parents Mr Iddrisu Frimpong (father) and Madam Asha Okyere - Frimpong (mother) for their support and encouragement. Their message to me has always been that:

> ***"Son do not come home with the same certificate you left home with".***

Abstract

This study explores the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. This qualitative study adopted an exploratory case study design and an interpretivist focal lens. The sampling was purposive, and the participants were three Grade 9 mathematics teachers, one from each of three high schools in the Harry Gwala District of KwaZulu-Natal province. The instruments used to generate data were semi-structured interviews and classroom observation.

The findings of the study revealed that the level or the extent to which teachers engage with learners' errors depends on 1) teachers' understanding of remedial teaching, which informs 2) teachers' ability to deal with learners' errors, and 3) teachers' mathematical knowledge, which is the content and pedagogical knowledge of teaching mathematics. Teachers also engage with learners' errors for the following reasons: 1) to provide remedial teaching, 2) to correct learners' mistakes or errors, 3) to provide feedback to learners, and 4) to promote peer learning. The findings of the study further revealed that teachers have limited time to engage with learners' errors, since they are time-bound to curriculum coverage or to finish the Annual Teaching Plan

However, it is important to note that the teachers who participated in the study do not have much knowledge about remedial teaching; therefore, they do not depend on or use remedial teaching to engage with learners' errors. The rationale for why teachers engage with learners' errors that emerged from the study includes to correct learners' errors, to provide feedback to learners, and to enhance remediation.

It is recommended that teachers must be educated on remedial teaching, how it is done, and its importance in helping learners to learn better. Also, the Department of Basic Education should design the Annual Teaching Plan to include remedial teaching for at least two to three hours, to be carried out before teaching moves on to the next topic. These three hours can be split further into lessons where a teacher can spend at least 10–15 minutes to explain errors before moving on to the next concept.

List of abbreviations

ANA	Annual National Assessment
ATP	Annual Teaching Plan
CAPS	Curriculum Assessment Policy Statement
CCK	Common Content Knowledge
DBE	Department of Basic Education
FET	Further Education and Training
GET	General Education and Training
HK	Horizon Knowledge
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KZN	KwaZulu-Natal
MKfT	Mathematical Knowledge for Teaching
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
SASA	South African Schools Act
SBA	School Based Assessment
SCK	Specialised Content Knowledge
UMALUSI	Council for Quality Assurance in General and Further Education and Training

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CHAPTER ONE

BACKGROUND AND INTRODUCTION TO THE STUDY

1.1 Introduction

According to Tsanwani and Juta (2015) the South African Schools Act (SASA) of post-apartheid South Africa, Act 84 of 1996, states that all schools in South Africa should provide a progressively high-quality education for all her citizenry up to the basic level. In order to achieve this constitutional mandate effectively, the educational system in South Africa at the basic school level is structured into the following phases by the Department of Basic Education (DBE): early childhood, foundation, intermediate, senior and further education and training (FET). While there is internal school-based assessment in the different phases, the performance of learners externally is mainly evaluated at the end of the FET phase (Grade 12), where learners from all corridors of the country write common National Senior Certificate (NSC) exams, which is administered by the DBE and certified by UMALUSI. In South Africa, UMALUSI is a Council set for Quality Assurance in General and Further Education and Training. Although UMALUSI is not a global examination body that regulates the standard of the NSC examination in South Africa, when learners finish the basic school year at matric level (Grade 12), the NSC issued to the learner by the accreditation board UMALUSI is globally competitive and accepted, which allows South African high school graduates to further their studies anywhere around the world.

Over the past years, either based on the analysis of internal school-based assessment (SBA) or externally based assessment, there has been an outcry regarding poor performance in mathematics. This poor performance is evident in DBE national performance reports for this school subject (2019, p. 8), which shows that the overall matric pass rate for mathematics from 2015 to 2019 was "49.1%, 51.1%, 51.9%, 58.0% and 54.6%" respectively. Even though the year 2018 recorded the highest pass rate in mathematics, there was a further decline of 3.4% in this rate in 2019. This needs to be improved.

In 2011 the DBE introduced an Annual National Assessment (ANA) in the foundation, intermediate and senior phase, with the aim of measuring the progress of learners towards a target of a 60% achievement threshold in literacy and numeracy by the end of Grades 3, 6 and 9 respectively. However, the last ANA was undertaken in 2014, whereafter it was abandoned by all education stakeholders for several reasons (Van der Berg, 2015). The DBE's outcomes of its qualitative analysis of the results in the ANA (DBE, 2013, 2014) made it evident that similar errors kept on repeating year after year, because those errors were not properly corrected. Figure 1.1 below shows Grade 9 ANA average percentage marks per content area in 2014 (DBE, 2014).

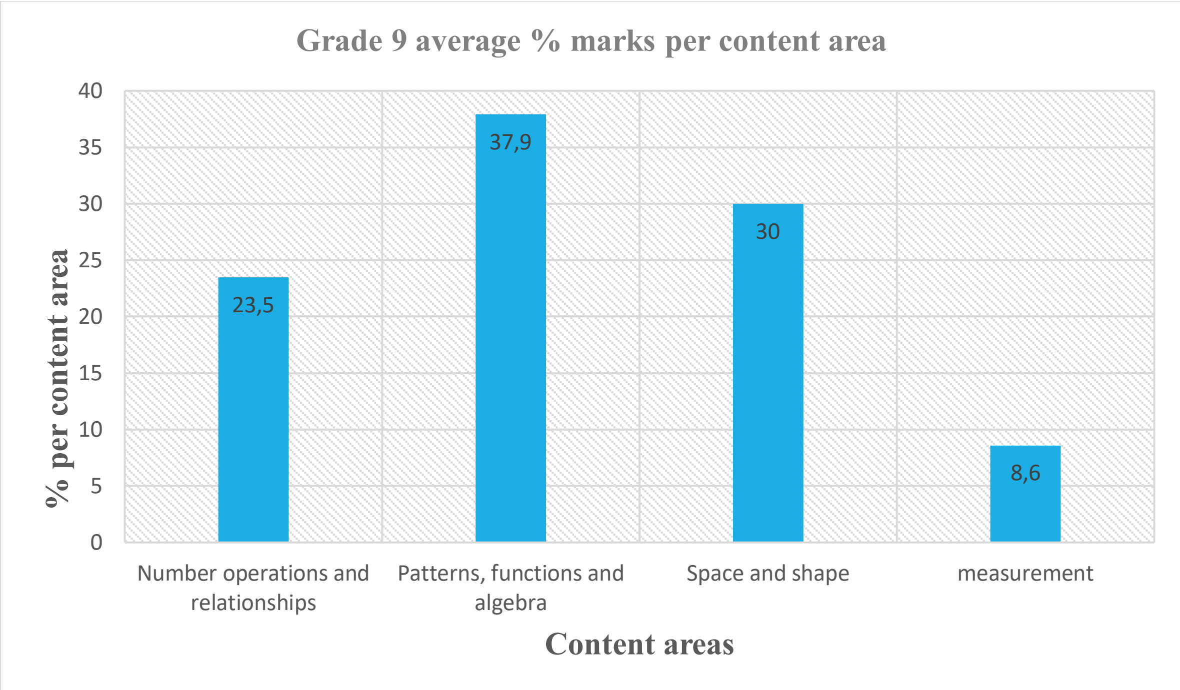

Figure 1.1 Average percentage mark in content areas as reported in the 2014 ANA report

Figure 1.1 indicates that in 2014, Grade 9 learners experienced the greatest difficulty in responding to questions on 'Measurement'. The second area of marked difficulty experienced by learners was 'Numbers, Operations and Relationships'. Learners found questions on 'Patterns, Functions and Algebra' relatively easier to respond (Figure 1.1). This was not different from findings in the 2013 report, which showed that the learners found topics on Geometry very difficult to answer. There were only a few areas where questions were answered well. According to the DBE (2014), this pattern of performance has not changed since the diagnostic analysis that was carried out in the 2013 ANA.

This means that there are several content areas in mathematics where learners need urgent attention and remediation in other to master the content. Pournara, Sanders, Adler, and Hodgen (2016) tracked Grade 9 to 11 learners' errors in algebra, and found identified errors similar to those mentioned in the DBE reports (2013, 2014). Pournara et al. (2016) findings suggest that the errors that learners commit seem to be persistent. While there can be many factors that result in these errors persisting, one wonders whether these errors were addressed during the teaching and learning process. What this study seeks to do is to explore how teachers engage with learners during the teaching and learning process.

Although there is no standardised common assessment in the grades other than Grade 12, the continually poor performance evident in the NSC report at the end of every year suggests that the lack of basic mathematical knowledge is one of the reasons why learners perform poorly at Grade 12 level. An example is, students' failure to simplify expressions, incorrect factorisation, and incorrect use of notations and parentheses (DBE, 2017). These concepts are taught mainly at Senior phase. The reports show that there is a need to pay attention in the Senior phase where learners make a transition from arithmetic to algebra, and also to the transition from concepts-oriented learning in the Senior phase to proof-oriented learning in the FET phase to improve mathematics results.

1.2 Problem statement

Over recent years the consistently poor performance in mathematics has revealed serious shortcomings in the process of the teaching and learning of this subject. According to Spaull and Kotze (2015) the uneven functioning of schools in South Africa due to its colonial past widens the learning gap – where those from underprivileged schools are continually at a disadvantage. With regard to teaching, the literature has highlighted issues associated with lack of teacher knowledge to teach mathematics (Arends, Winnaar, & Mosimege, 2017; Bansilal, Mkhwanazi, & Brijlall, 2014; Makhubele, Nkhoma, & Luneta, 2015; Pournara et al., 2016) as one of the main reasons leading to poor performance in the subject.

In terms of learning, the literature has identified issues associated with the amount of work learners are exposed to and the time they have available to spend on tasks (Metcalfe & Xu,

2018) . To address issues associated with teaching, there is evidence showing that attending to learners' errors during the process of teaching has the potential to improve learner performance (Herholdt & Sapire, 2014; Lam, Kaur, & Lee, 2013; Sapire, Shalem, Wilson-Thompson, & Paulsen, 2016). However, in reality it seems that what the literature advocates for is not being practised in the classroom, based on the evidence that common errors seem to be persisting in learners' responses over the years, as evident in the DBE diagnostic ANA (2013, 2014), and NSC (2015-2017) reports.

Common errors seem to be repeated by learners, and there also seems to be a continuation of gaps in the learners' knowledge construction from earlier grades (DBE, 2017). Based on the evidence in this report, it seems that in the process of teaching and learning, teachers do not pay much attention to addressing learners' errors for the purposes of improving learning. Ball, Thames, and Phelps (2008) argued that unlike ordinary mathematicians, mathematics teachers need to go beyond the solving of mathematics problems and engage with learners' errors. Furthermore, in her article uncovering the work of teaching, Ball (2017) emphasises teachers' engagement with learners' errors. Sapire et al. (2016) argue that engaging with learners' errors is no longer a professional requirement for teachers but is a responsibility; however, when reading moderators' diagnostic reports we see that common errors made by learners seem to be repeated year after year. One wonders whether, in the context of South Africa, teachers have incorporated this responsibility of engaging with learners' errors in the teaching and assessment of mathematics. The purpose of this study is therefore to explore the extent to which Grade 9 mathematics teachers engage with learners' errors during the process of teaching and assessment.

1.3 Background to the study

In South Africa research has paid much attention to exploring teachers' subject matter knowledge of mathematics, due to constant poor performance in mathematics (Arends et al., 2017; Bansilal et al., 2014; Mogari, 2014; Ndlovu, 2019; Ubah & Bansilal, 2018). Other studies have focused attention on implementing and evaluating interventions to curb the crisis of poor performance in mathematics (Mkhwanazi, Ndlovu, Ngema, & Bansilal, 2017; Pournara, Hodgen, Adler, & Pillay, 2015). However, limited research has been carried out to explore teaching in practice, mainly how teachers engage with learners' errors in the teaching and assessment of mathematics. This phenomenon has very little voice in the literature, with (Herholdt & Sapire, 2014; Sapire, Shalem, & Sorto, 2014; Sapire et al., 2016) having carried out small-scale studies on this.

When observing the trend in moderators' reports at the end of the year, one notices that year after year similar errors in particular topics keep on coming up. This suggests that there is inadequate or a lack of teachers' engagement with learners' errors in the teaching and assessment of mathematics. The literature has indicated that errors and misconceptions are part and parcel of learning (Brodie, 2014; Gardee, 2015; Hansen, Drews, Dudgeon, Lawton, & Surtees, 2020; Mahlabela, 2012) – and thus attending to learners' errors is a way to improve their learning in mathematics. While this argument may be correct, not many studies have focused on exploring teachers' use of learners' errors to inform teaching; this study hopes to bridge that gap.

This study explores the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics, and therefore seeks to bridge the gap of using learners' errors to inform their teaching in the long term.

1.3.1 What is error analysis?

Error analysis is the study of errors and their causes, and trying to find a corrective way to remedy such errors. One cannot talk about error analysis without acknowledging and referring to linguistics. In linguistics much has been written about error analysis (James, 2013; Jobeen et al, 2015; Khansir, 2012; Richard, 2015; & Rustipa, 2011) with current writers in linguistics, such as (Hinkel, 2018) defining error analysis as an area of linguistics that deals with the systematic and logical gathering and documentation of additional language errors in learners' language production. This mean error analysis is an aspect of linguistics that helps the learner to learn new language in the schooling system. However, there has been little to no study on error analysis in mathematics over recent years. Therefore, for the purposes of this study the researcher explains error analysis as a process whereby mathematics teachers identify the causes of learners' mathematical errors, and develop an approach or method of solving this problem to improve learners' mathematical knowledge.

In this study error analysis will be limited to studies around mathematics. According to Herholdt and Sapire (2014), error analysis is the study of errors in learners' work with a view to looking for possible explanations for these errors. The authors further argued that error analysis can be explained as error pattern analysis, which is the study of errors in learners' work with a view to finding explanations for these reasoning errors. McLaren et al. (2012) explained error analysis as a teaching strategy that has the potential of helping students to retain their learning.

The researcher of this study agrees with Herholdt and Sapire (2014) view of error analysis. At the same time, the researcher supports the argument made by McGuire (2013) that error analysis is not only about teachers analysing learners' correct, partially correct or incorrect procedures of finding answers to mathematical problems, but also implies the study of best practices for remediation. The author maintains that these best practices would require of the teacher a good knowledge of mathematical content, as well as a good grasp of learners' levels of mathematical understanding (McGuire, 2013).

In support of the above, it is worth mentioning that before teachers better engage with learners' errors, a few factors need to be addressed. Firstly, teachers need to have developed schema of the particular topic, because as (Ball et al., 2008) posit, teachers need to be able to solve the problems they expect learners to solve. Secondly, teachers must also be aware of errors learners make and also able to diagnose the errors with the aim of remedying them. Having such knowledge during teaching and assessment would make teachers attentive to learners and responsible for identifying any common errors and misconceptions they might have. Thirdly, teachers need to have the expertise to diagnose learners' errors and interpret them for the purpose of improving teaching, which (Ball et al., 2008) and McAuliffe (2013) refer to as specialised content knowledge (SCK). Fourthly, teachers must have a basic idea of mathematics and ability to evaluate whether or not those mathematical ideas are correct. This will be further explained in Chapter Two of this study.

1.3.2 Mathematics curriculum in South Africa

Mathematics is a subject that is necessary in life, and most countries in the world (including South Africa) through their ministries of education have made it compulsory. In South Africa there are three different forms of mathematics that learners can choose from after their Grade

9 school year: pure (core) mathematics, mathematical literacy, and technical mathematics. So after Grade 9 learners can proceed to Grade 10 and do one of these variations. The mathematics teacher must be knowledgeable enough in the subject; therefore, mathematical knowledge is a requirement for teachers to teach it. Also, teachers must have good pedagogical content knowledge in teaching mathematics.

Among other things, the Senior phase mathematics Curriculum and Assessment Policy Statement (CAPS) (DBE, 2012. p. 5) of South Africa emphasises the need to produce learners who can:

- Identify and solve problems and make decisions using critical and creative thinking;
- Work effectively as individuals and with others as members of a team;
- Organise and manage themselves and their activities responsibly and effectively;
- Collect, analyse, organise, and critically evaluate information;
- Communicate effectively using visual, symbolic and/or language skills in various modes;
- Use science and technology effectively and critically, showing responsibility towards the environment and the health of others; and
- Demonstrate an understanding of the world as a set of related systems, by recognising that problem-solving contexts do not exist in isolation.

These general aims read well on paper, but their implementation needs teachers whose mathematical knowledge for teaching is fully developed with all the strands of mathematical knowledge for teaching as advocated by Ball et al. (2008). Therefore, engagement with learners' errors can be one way in which teachers promote problem solving and making decisions using critical and creative thinking, because by engaging learners with errors they are forcing them to think deeply about their solutions and to engage critically with their thinking. Through engagement with learners' errors, teachers can use learners' errors to promote effective group work and efficient skills in the learning of mathematics, because when learners are not able to provide the solution but identify errors in their responses, then they start to reflect on their thinking. Such a strategy would automatically lead to talk about mathematics in the classroom, which Van de Walle (2011) emphasises as one way to improve teaching and learning, which is also continuously emphasised in the CAPS.

1.4 Rationale of the study

The rationale for doing the study is three-fold: firstly, it was driven by the researcher's experience as a former mathematics teacher in the South African context; secondly, to contribute to the pedagogy of teaching, as evidence shows that engaging with learners' errors might assist with improving learner performance (Sapire, 2014); and thirdly, because there is limited literature in this area, especially in-depth studies exploring teachers' engagement with learners' errors, regardless of the fact that this aspect of teaching is emphasised in the literature (Ball et al., 2008; Shalem et al., 2016; Aksu & Kul, 2016).

Engagement with learners' error analysis is a methodology that has previously been used mainly in the field of languages (Hinkel, 2018). Drawing from this, it has been noted that in mathematics error analysis has the potential as pedagogy to improve learner performance (Ball et al., 2008; Aksu & Kul 2016; Ball, 2017). Limited studies have been carried out to explore teachers' engagement with learners' errors in the South African context. Those that have been conducted paid attention to teachers' engagement with errors when teaching and not in assessment, while teaching and assessment are at the forefront in improving teaching and learning.

1.4.1 Researchers' experience as a mathematics teacher in the context of South Africa

Based on the anecdotal evidence of being a former mathematics teacher in the South African context, the researcher had noticed that the pressure to complete the curriculum sometimes compromises the emphasis on meaningful teaching in the classroom. The general trend that mathematics teachers chose to follow is to complete the syllabus. The researcher is not arguing that completing the syllabus is not important, because in the end summative assessment draws from completion of the syllabus; the argument being made is that completing the syllabus without learning taking place is a futile exercise. In the quest to complete the syllabus mathematics teachers find themselves following textbook exercises page by page. That form of teaching only pays attention to solving routine problems, followed by giving learners corrections when they fail to do so, without engaging with what led them to fail to solve the particular problem, or providing sums that are easy to solve.

Similar findings were evident in Makhubele and Luneta (2014) study, where the authors explored factors contributing to learners' poor performance in Grade 9. Without engaging with learners' errors, teachers attempt to offer remedial teaching that does not address learners' misconceptions but rather puts continuous emphasis on the rules that learners should have followed in solving problems. This is followed by teachers becoming frustrated as to what else they need to do in order to ensure that learning is taking place. However, drawing from the literature (Siyepu, 2013; Sapire et al., 2014; Shalem, et al., 2016; Aksu & Kul, 2016; Lam et al., 2017), there is evidence that engaging with learners' errors does assist in improving teaching and learning. Sapire et al. (2014) argue that learners' errors can be used to improve their learning; similarly, Ball et al. (2008) argue that teachers need to engage with learners' errors. It is therefore within these parameters that in this study the researcher decided to explore Grade 9 mathematics teachers' engagement with learners' errors in teaching and assessment in the mathematics classroom.

1.4.2 Error analysis as a pedagogy in mathematics

Drawing from Rushton's (2018) study on error analysis as a pedagogy for teaching mathematics, Rushton concluded that both teachers and students found error analysis as a useful method in the teaching and learning process of mathematics. For years, the method of teaching mathematics to learners has involved a teacher working out correct examples and using them as a starting point. Addressing learners' errors has been considered one of the pedagogies to improve teaching, especially in mathematics; however, this method of teaching has not been practised. Instead, mathematics teachers adopted the style of re-teaching a concept over and over again without paying attention to the source of the error. Linder, Poweer-Costello and Stegelin (2011) emphasises the development of mathematics talks to improve learning; addressing learners' errors is one method which teachers can adopt to develop mathematics talk, thus improving learning. Other authors (Tulis et al., 2016; Smith et al., 1993; Siyepu, 2014) have argued that learners' errors should form part of teaching and learning; however, if not attended to this causes a barrier to learning, while addressing it has the potential to enhance learning. In recent studies different methods, such as using errors learners make in solving mathematical problems, have been introduced to help learners learn mathematics better through error analysis (Rushton, 2018). While this study agrees with use of error analysis like Rushton (2018), he used quantitative data in his study while this study used qualitative data. In South Africa there are few studies exploring how teachers engage with learners' errors even, though the CAPS (DBE, 2012) document and other literature (Rushton, 2018) emphasised the importance of such in the teaching and learning of mathematics.

This study is of significance because the researcher strongly believes that when teachers engage with learners' errors, they can engage with their reasoning, which makes it possible to identify learners' misconceptions. Through identifying misconceptions, teachers would be able to devise appropriate remedial teaching strategies. In the long-term this can help to improve the national results for mathematics at the end of the NSC exams. This study will not only contribute to the literature but will help to show the significance of using error analysis as the methodology to teach mathematics, drawing on the fact that it has long been implemented in the languages.

1.4.3 Studies on the teaching and learning of mathematics in South Africa

In South Africa research has paid attention to factors leading to poor performance (Mji & Makgatho, 2006; Arend, Winnar, & Mosmege, 2017; Bansilal et al., 2014). Their studies mainly focus on exploring teachers' subject matter knowledge of mathematics. Others focus on implementing and evaluating interventions to curb the crisis in mathematics performance (Pournara et al., 2015; Mkhwanazi et al., 2018). However, limited research has been done to explore teaching in practice with an emphasis on how teachers engage with learners' errors in teaching and assessment in mathematics. When observing the trend in moderators' reports at the end of the year, one notices that similar errors keep coming up year after year, which might suggest lack of engagement with learners' errors in teaching practice.

The literature has echoed that errors and misconceptions are part and parcel of learning (Tulis et al.,2016; Bansilal & Mahlabela, 2015; Siyepu, 2013; Maharaj, 2014; Kadzunga & Bansilal, 2018). While their argument is true, not many studies focus on exploring teachers' use of learners' errors to inform teaching. Ndlovu et al. (2017) explored final-year pre-service teachers' competence to diagnose and interpret learners' errors. Their findings showed that although these pre-service teachers are at the exit level and soon to be in-service mathematics teachers, that aspect of knowledge to interrogate learners' errors in assessment is lacking. Shalem et al.'s (2016) study showed that generally teachers have difficulty in explaining learners reasoning, which suggests that the aspect of engaging with learners' errors is not common practice in South African mathematics classrooms.

1.5 Purpose of the study

According to Dean (2020), mathematics is a subject that is needed in every sphere of life and study. For example, mathematics is needed in areas of study such as the humanities, engineering, agriculture and science, health sciences, and management sciences, among others. The purpose of this study is exploring the extent to which Grade 9 mathematics teachers engage with learners' errors in teaching and assessment of mathematics. The researcher chose this purpose because of the global demand for mathematics in the 21st century, where technology is the key to sustainable economic growth. The way in which the mathematics teacher engages with learners' errors must be given critical attention by all sectors, particularly institutions of learning, to see whether learners' errors are engaged with and the impact that such engagement may have on learner performance in mathematics.

This is what this study seeks to do. This study primarily targets Grade 9 mathematics teachers, because in South Africa Grade 9 is the end of that phase of learning and it is in this grade that learners have to make a decision regarding subject choices. It is therefore in this grade that learner performance in mathematics is critical in terms of impacting their decision as to whether they pursue mathematics in Grade 10 or not. Secondly, although the ANA was last implemented in 2014, the ANA findings have implications for the teaching of mathematics in Grade 9 which remain relevant to date. In addition, according to Matcalfe (2015) findings from Jika IMfundo also revealed challenges with regard to curriculum coverage and Grade 9 mathematics teachers' willingness to reflect on those challenges at Grade 9 level to deal with those challenges. It is with these factors in mind that the researcher set out to explore the extent to which Grade 9 mathematics teachers engage with learners' errors during the teaching and assessment of mathematics.

1.6 Research objectives

The main purpose of this study was to explore how Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. Therefore, the following objectives were considered suitable for the study:

To explore how (the approach) Grade 9 mathematics teachers engage with learners' errors in the classroom: and

To examine why Grade 9 mathematics teachers engage or not engage with learners errors

1.7 Key research questions

In line with the above objectives, this study is underpinned by the following research questions, which enabled the researcher to understand the extent to which Grade 9 mathematics teachers engage with learners' errors in assessment and teaching:

To what extent do Grade 9 teachers engage with learners' errors in their teaching of mathematics?

How do Grade 9 teachers engage with learners' errors in teaching and assessment of mathematics?

Why do teachers in Grade 9 engage or not engage with learners' errors when teaching or assessing mathematics?

1.8 Location of the study

The study took place in the Harry Gwala District, specifically in the Ixopo circuit, of KwaZulu-Natal Province. Based on their context many schools that fall under Harry Gwala District are rural, and most are categorised as quintile[1] one and two based on the DBE definition and the resources available to them. Data for this study were generated from three schools, from which three teachers were selected, one from each school. For ethical purposes pseudonyms were used, and the names used for the schools in this study were Okyere High School, Mills High School, and Telecom High School.

The researcher decided to choose three schools, one from each of the quintiles one, two, and three, as per DBE categorisation. Each one of the three schools was selected from either quintile one, two or three. According to the Department of Education these are no-fee-paying schools and they have a shortage of resources in terms of infrastructure, learner-teacher support material and, in some cases, shortage of teachers. Okyere High School and Mills High School, which represent quintiles one and two respectively, years have been low-performing schools over the past years. In comparison, Telecom High School, which is a quintile three school,

[1] In south Africa, a quintile is a categorization of public schools by the Department of Basic Education into five groups from a poorly resource to less poorly resource schools. For example, quintile 1 schools are considered poorly resourced schools

seems to be performing better. Telecom High School was a semi-fee-paying school, and although it might have a shortage of resources, it has full human resources capacity. In terms of general performance, Telecom High School was an average-performing school. Since the schools were selected from different contexts, the population of learners in each differs.

Even though all three schools were dominated by African learners and African teachers, the schools in quintiles one and two were mostly populated by learners from a low socio-economic background compared to the one from quintile three. The school from quintile three has a balance between learners from a low socio-economic background and those of medium socio-economic status. The three schools admit learners from all walks of life. This study explores the extent Grade 9 to which teachers engage with learners' errors in the teaching and assessment of mathematics. One teacher from each of the three schools who teaches Grade 9 mathematics was asked to willingly participate in the study

1.9 Limitations of the study

Most researchers face problems while conducting their research. Some of the problems that the researcher encountered were related to time, proximity, and participants' commitment to participate fully in the research. Participants are the most important parties in all research; if they are not willing to participate in a study, the research cannot continue as planned by the researcher. For this reason, the researcher provided all of the participants with a letter of consent for them to sign indicating that they consented to participate in the study. However, participants in the study were entitled to withdraw their participation from the research at any time, if they so wished. The researcher extended an invitation to all of the Grade 9 mathematics teachers in each school to be participants. In the cases where there was only one teacher per school, a Grade 12 teacher who had taught Grade 9 in previous years was included, since the aim was to cater for potential withdrawals.

Another limitation, as with all other qualitative research, was that this study used a case study approach, and therefore its findings and results are applicable to a particular group and not the whole population. Therefore, the finding of this study cannot be generalised. However, the findings of this study can be used for transferability rather than generalisation. With regard to time, caution had to be taken with respect to conducting interviews to ensure that the study did not compromise teachers' teaching time. Participants were requested to stay after school for an interview; however, while all agreed to this, some did not honor the agreement. To eliminate such limitations and ensure that data collection took place the researcher took time to meet with some participants over the weekends.

1.10 Overview of the study

This study consists of seven chapters. Chapter One provides the introduction and background to the study, Chapter Two was covers the literature review, Chapter Three focuses on the theoretical framework used in the study, Chapter Four outlines the methodology used in the study, Chapter Five provides insight into the data collection and analysis, while Chapter Six describes the findings and discussion of the study, and finally Chapter Seven focuses on the limitations, implications and recommendations of the study as well as the conclusion. Each new chapter is a development of the previous chapter and aims to delve deeper into the phenomena being explored. The main aim of this study is to explore the extent to which Grade 9 mathematics teachers engage with learners' errors in assessment and teaching. An overview of the study is provided below.

1.10.1 Chapter One: Background and introduction to the study

Chapter One discussed the introduction and background to the study. In this chapter the focus was to provide a detailed explanation of the problem, purpose, and rationale for this study. In addition, the researcher discussed the objectives guiding the study as well as the research questions.

1.10.2 Chapter Two: Review of related literature

This chapter provides a review of the literature guiding this study. In line with the phenomena being studied, the researcher reviewed literature related to error analysis, errors and misconceptions, teachers' mathematical knowledge for teaching, teachers' engagement with errors, teachers' engagement with misconceptions, teachers' mathematical knowledge on error analysis, assessment of teaching, teachers' knowledge of assessment, and assessment of mathematics, among others.

1.10.3 Chapter three: Conceptual framework

The conceptual framework adopted for the study is a combination of error analysis by Sapire et al. (2014) and mathematical content knowledge as described by Ball et al. (2008), as shown in Figure 1.1 below. Arrows are used to show the connection between error analysis and mathematical content knowledge.

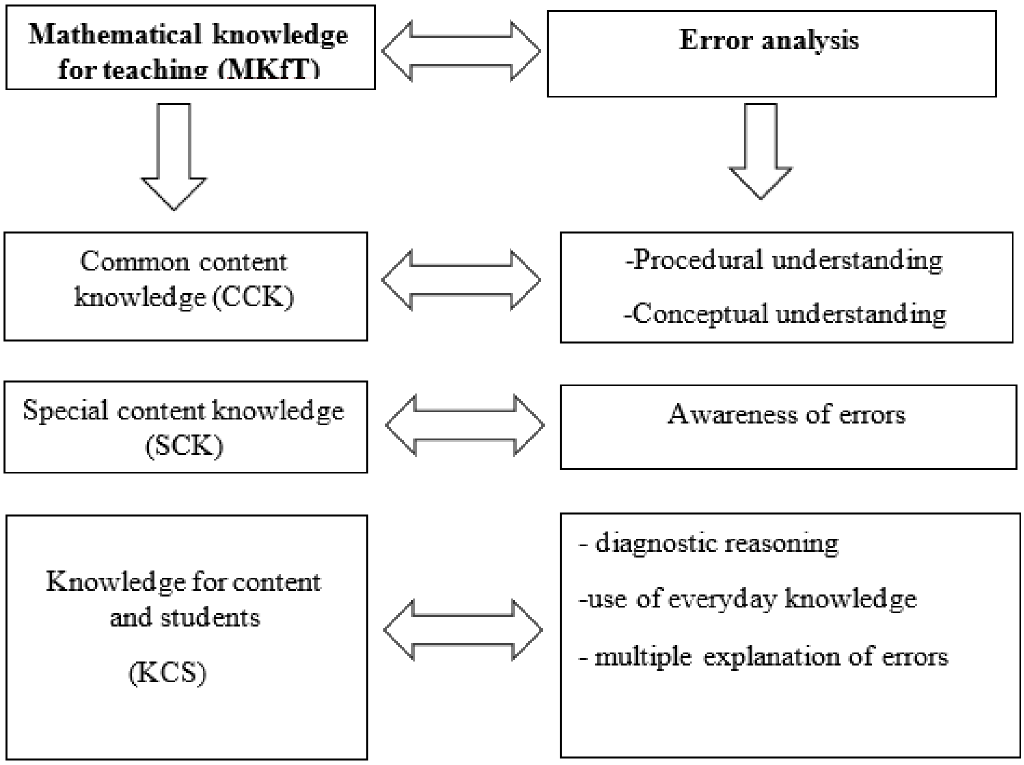

Figure 1.2 Conceptual framework enacted from error analysis by Sapire et al. (2014) and mathematical knowledge for teaching by Ball et al. (2008).

1.10.4 Chapter four: Research design and methodology

Chapter Four presents the research methodology used for this study in order to achieve the research objectives. The chapter presents the approach used for this study, which is the interpretivist paradigm, as well as the qualitative research design. It is also explained why case study was used in this study. This chapter also describes how and why the participants (three teachers) were chosen for the study. It outlines the sampling (purposive sampling), trustworthiness (credibility, transferability, dependability, and conformability), ethical issues and limitations of the study.

1.10.5 Chapter five: Data analysis and data presentation

Chapter Five describes the data analysis and presents the data which emerged. In this chapter data from the interviews, observations and document analysis are analysed and presented according to the responses given by the participants.

1.10.6 Chapter Six: Findings and discussion

Chapter Six is designated for the findings and discussion of this study. The findings are presented together with the discussion in order for readers to have a clear understanding of the study.

1.10.7 Chapter Seven: Overview, limitations, recommendations, and conclusion

This is the last chapter and provides an overview, limitations of the study, conclusions and recommendations that emerged from the study based on the findings in Chapter Six. This chapter concludes the study by describing the extent to which teachers engage with learners' errors in the teaching and assessment of mathematics in Grade 9.

1.11 Conclusion

This chapter provided the introduction and background to the study, as well as outlining the purpose and providing a general overview of the study. This chapter also gave a brief outline of the study. Chapter Two provides a review of the related relevant literature.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

Chapter One provided the introduction and background to the study, and comprehensively explained the purpose of the study and the related research questions. In this chapter the researcher focuses on a review of relevant literature relating to the study. Literature review is important chapter to research because it helps the researcher "to demonstrate skills in library searching, to show command of the subject area and understanding of the problem, to justify the research topic design and methodology" (Silverman, 2013, p. 342). Literature review also explains the key concepts of a study (Cohen et al., 2011). Silverman (2013) reveals that the literature review states points of agreement and disagreement between the current study and previous studies, and also provides the basis for the researcher to make claims later in the study.

In this chapter the researcher selected topics or themes related to the study and explored what scholarly researchers have to say about these matters. Understanding what has been discovered about the phenomena under study would assist the researcher to understand and interpret the current findings. The researcher based his argument on what previous researchers had said about the same theme or topic, and then agreed or disagreed with them, later relating the literature to the findings of the study (Silverman 2013).

In this study the researcher review literature to explored studies on error analysis and teachers' mathematical knowledge of teaching mathematics, since the purpose of the study was "to explore the extent Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics". The fundamentals or philosophy of Mathematics as a subject is a problem-solving discipline in nature. Therefore, the researcher holds the view that when teachers begin a process of solving mathematical problems errors will occur. Hence teachers require strong mathematical knowledge Ball et al.(2008) in other to effectively deal with learners' errors. Based on the aim of the study, the literature review looked at the following themes: teachers' mathematical knowledge for teaching, errors and misconceptions in mathematics, error analysis, teachers' knowledge of error analysis, error analysis in the teaching of mathematics, error analysis in assessment, assessment, , forms of assessment, feedback as a tool of assessment in mathematics, relationship between teaching and assessment in mathematics, teachers' knowledge of assessment in mathematics, teachers' mathematical knowledge for teaching, teachers' engagement with learners' errors in mathematics, teachers' engagement with learners' errors when assessing in mathematics, teachers' knowledge of error analysis and assessment in mathematics, among others.

2.2 Mathematical knowledge for teaching

Ball et al. (2008) have worked on two key research projects, namely the Mathematics Teaching and Learning to Teach, project and the Learning Mathematics for Teaching project. Through their work they have developed a model of teacher knowledge by examining the actual work of mathematics teaching in schools. This is a practice-based theory of what they call 'mathematical knowledge for teaching' – a kind of professional knowledge needed to teach mathematics to learners which is different from that mathematics demanded by other mathematically intensive occupations" (Ball et al., 2008) such as medicine, commerce, engineering and carpentry, and constitutes the mathematical knowledge needed to carry out the work of teaching mathematics (Ball et al., 2005; Ball et al., 2008).

Ball et al. (2008) have incorporated Shulman's subject matter content knowledge, PCK and curricular knowledge into the umbrella term "mathematical knowledge for teaching". Ball et al.'s model of mathematical knowledge for teaching uses Shulman's division between subject matter knowledge (SMK) and PCK and distinguishes three components of SMK, namely common content knowledge (CCK), specialised content knowledge (SCK) and horizon knowledge (HK). PCK comprises knowledge of content and teaching (KCT), knowledge of content and students (KCS) and knowledge of content and curriculum (KCC). A comparison between the two classifications of knowledge for teaching is presented in Figure 3.2 of Chapter Three.

The mathematical knowledge for teaching model proposed by Ball et al. (2008) (presented in Figure 3.2 in Chapter Three) is based on empirical evidence. According to McAuliffe (2013) the model was developed from analysing the work of teaching, and identifying the teaching tasks involved and the knowledge needed to teach mathematics effectively. It is based on the premise that teachers need to know mathematics and know how to use mathematics in the work of teaching learners (Ball et al., 2008). It is a theory of teacher knowledge "framed in relation to practice" and is composed of following domains of Mathematical knowledge.

McAuliffe (2013) reported that Ball and her colleagues were interested in identifying the recurrent tasks and problems of teaching mathematics and the mathematical knowledge, skills and sensibilities required to manage these tasks. It is an approach to developing theory about mathematical knowledge needed for teaching; it is not about what the teacher knows or might need to know, but rather a description of knowledge used in teaching. They define mathematical knowledge for teaching as mathematical knowledge entailed by teaching, in other words the mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to learners (Ball et al., 2008, p. 97).

2.3 Misconceptions and errors of mathematics

According to Hansen et al. (2020), a misconception is when a learner's conception is considered to be in conflict with the acceptable meaning and understanding of mathematical concepts. Ghani and Maat (2018) stated that misconceptions occur due to misunderstanding and misinterpretation of knowledge from inaccurate meanings. The authors further mentioned that misconception occurs because of a change of meaning by learners to suit the instructions given. Misconceptions are further exacerbated by continually being ignored in the process of teaching and learning, thus becoming a barrier to learning. While misconceptions can become a learning barrier for learners, they can also be used by teachers to improve teaching and learning. Therefore, misconceptions should not be ignored and relegated outside of the classroom, and instead should be attended to in the process of teaching and learning.

Various definitions of errors and misconceptions have been provided by many scholars (Ghani & Maat, 2018; Hansen et al., 2020; Luneta & Makonye, 2010; Makonye & Nhlanhla, 2014). While the authors express it in different terms, all agree that errors and misconceptions hinder learning because they are cognitively constructed, and further assert that teachers need to devise instruction strategies to eradicate them. In contrast, Hansen et al. (2020) hold the view that errors occur at the surface level of knowledge, are mostly procedural, and can easily be corrected by teachers. (Luneta & Makonye, 2010, p. 35): errors are “mistakes, slip, blunder, inaccuracy or deviation from an accurate procedure”. Similarly, Luneta (2015) posits that errors are symptoms of difficulties that students encounter during their learning experiences. Elbrink (2008) reiterated that errors occur as a result of carelessness, misreading of symbols or text; a lack of relevant experience or knowledge related to that mathematical topic, learning objective, or concept; and/or lack of awareness or inability to check the answer given.

While all of these views on definitions of misconceptions are correct, it is important to flag that misconceptions are not just mistakes that could be addressed by reteaching the concepts. Hansen et al (2020) hold a contradictory view on misconception; however, misconceptions go beyond the surface level and are cognitively constructed, which means that they are conceptual. As purported by Tulis et al., (2016) errors are the symptoms of underlying misconceptions – hence taking the view that errors are mostly procedural and can easily be corrected by teachers can be detrimental to the process of teaching and learning. The researcher concurs with Tulis et al., (2016) that it is safe to categorise mistakes, because not all mistakes are cognitively constructed; some are at the surface level, as held by Hansen et al. (2020) and Luneta and Makoye (2010), but others are symptoms of difficulties, as described by (Luneta ,2015).

Although some mistakes can be considered slips, as Luneta (2015) points out, they can cause difficulties and hinder the learning process. This means that even something that can be considered just a carelessness mistake might later develop into a cognitive error causing learning difficulties for learners if not attended to. To provide a distinction on errors, Sarwadi and Shahrill (2014) proposed that students’ errors are causally determined, mostly systematic, and occur as a result of students’ misconceptions. Mathematics teachers should consider using appropriate, varied teaching strategies so that these weaknesses will not be repeated, in order to improve teaching quality and facilitate learning activities (Alghazo & Alghazo, 2017).

Ada and Kurtuluş (2010) posit that instructional strategies such as repeating a lesson to emphasise a point does not help learners who have acquired an alternative conception or misconceived concepts in mathematics. This means that teachers need to explore alternative ways to eradicate learners' errors and misconceptions. However, while the literature raised the issues with regard to misconceptions, and a number of studies have explored learners' misconceptions (see Luneta & Makoye, 2010; Ogbonnayo, 2015; Ojose, 2015), there is limited research exploring teachers' engagement with learners' errors with the aim of addressing these misconceptions. It is therefore imperative to explore ways in which teachers engage with learners' errors and misconceptions during classroom instruction and assessment, to ascertain the effectiveness of their teaching strategies.

Sarwadi and Shahrill (2014) argued that acquiring mathematical knowledge is a continuous process that requires linking of new ideas to what has previously been taught. They further argue that if a learner is unable to assimilate and accommodate new ideas through previous ones, then there is a learning gap in the learning of new concepts, which contributes to mathematical errors and misconceptions (Sarwadi & Sharill, 2014). In addition, Baidoo (2019) reported that errors and misconceptions are barriers to teaching and learning and therefore cautions teachers not to downplay those made by learners during teaching and learning. Baidoo explained that errors and misconceptions affect learners negatively in learning new concepts and can lead to more confusion. This study shares the view of Baidoo and the other authors that mathematics teachers should attend to learners' errors and misconceptions timeously to avoid further confusion in the learners. It can be noted that errors and misconceptions are interconnected, in the sense that misconceptions of theories and ideas lead to errors.

Mbusi (2016) identified two forms of errors: 1) non-systematic errors; and 2) systematic errors. She explained that non-systematic errors as follows:

> slips and might exist due to students' carelessness, misreading information or forgetting some piece of information, unintentionally. Normally, students will easily correct such errors by themselves because there are no fundamental and faulty conceptual structures associated with them. (Mbusi, 2016, p. 388)

This conception of errors by Mbusi (2016) builds on what Tulis et al., (2016) emphasised in his study about handling pupils' misconceptions – which was that teachers need to be able to categorise types of mistakes made by learners. The ability to categorise them is useful in determining the appropriate strategy to use to address the mistake. For example, say a learner is given $23 - 7$ and they give 24 as the answer. While this can be seen as a slip, it is also possible that it is rooted in a misconception drawn from elementary level when dealing with whole numbers, where learners are taught that one cannot subtract a larger number from a smaller number. Thus, treating this as a slip without interrogating the learners' reasoning might not help address the difficulty – and the learner will keep repeating the error since the misconception is rooted cognitively.

Mbusi (2016) explained that systematic errors are those that are consistently applied, meaning the consistent application of faulty methods, algorithms, or rules. She also argued that the cause of systematic errors may relate to learners' procedural knowledge, conceptual knowledge, or links between these two types of knowledge. From the findings of her study about pre-service teachers' misconceptions and their related errors, Mbusi (2016,) warned that errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. Mbusi (2016) argued that errors can be as a result of consequence of definite processes whose nature must be discovered.

If one unpacks the findings of the study by Shalem et al. (2014) with 62 mathematics teachers in Gauteng in relation to Mbusi's categories, it could be argued that teachers do not engage with systematic errors in class. In this study the attention was on teachers' engagement with learners' errors, i.e. procedural and conceptual errors. They further explore teachers' awareness of errors, ability to diagnose errors and using everyday knowledge and multiple explanation of errors. It could be argued that their exploration of teachers' awareness of mainly procedural errors was more focused on how they engage with non-systematic errors as defined by Mbusi (2016); attention to conceptual errors was through exploring teachers' engagement with systematic errors and the incorporation of everyday knowledge while helping learners to make sense. Shalem et al.'s (2014) findings revealed that teachers mostly do not engage with learners' systematic errors, suggesting that the call by Baidoo (2019) for teachers not to downplay learners' errors does not seem to be implemented by mathematics teachers in their classrooms.

The lack of attention to learners' errors might be perpetuated by the fact that mathematics teaching is driven by rote learning focusing on imparting rules, procedures, and formulas. Sarwadi and Shahrill (2014) posit that the emphasis on rules, procedures and formulas promotes among learners the need to follow rigid procedures in order to arrive at the correct answer, instead of developing understanding of basic mathematical concepts. While procedural knowledge is the basic knowledge that learners need to have, for mathematical understanding they also need to conceptualise mathematical concepts, and one way in which teachers can help learners to achieve that is by interrogating their reasoning process, forcing them to engage with what they write by engaging with their errors.

Errors and misconceptions committed and held by learners are a result of their inability to connect their existing knowledge to new ideas which they are studying (Sarwadi & Shahrill, 2014). This means that errors may occur when learners fail to connect or link what they know to what they are learning. Here teachers need to be knowledgeable enough or have the competency to help correct learners' errors and misconception in mathematics. The literature has shown that although errors and misconceptions have been a major issue in educational research, errors are not entirely bad because they shape learners' learning. For example, Mbusi (2016) remarked that "making errors in computation is not completely bad because it becomes an important part of the learning process if these errors are dealt with diagnostically" (p. 388). Similarly, Brodie and Berger (2010) assert that "errors make sense when understood in relation to the current conceptual system of the student, which is usually a more limited version of a mature conceptual system" (p. 13).

The implication of Brodie and Berger's argument is that teachers should consider the existing knowledge of their learners in order to find out what the learners know and build on this to create the new knowledge. In agreement with the position of Brodie and Berger, the researcher believes that mathematics teachers can teach effectively and address learners' errors when concepts are taught by moving from the known to the unknown. This can be achieved through diagnostic assessment tasks. Analysis of diagnostic assessment tests may help teachers to detect learners' errors and find out the causes of those errors. Brodie and Berger (2010) further argue that attending to learners' errors regarding current conceptual structures should help them to become more powerful through increasing their understanding in a range of situations. Research around understanding learners' errors and misconceptions in mathematics is not a new field, and suggestions have been made as to how teachers can help to eradicate learners' errors in the mathematics classroom (Brodie & Berger, 2010; Maharaj, 2018; Ndlovu, 2019; Siyepu, 2013).

However, there is limited research focusing on understanding how teachers engage with learners' errors (Aksu & Kul, 2016; Gardee, 2015; Sapire et al., 2016; Sorto et al., 2014). The studies that started to look into teachers' engagement with errors pay more attention to classroom teaching. While teaching is the focal point, this study extends this field to include teachers' engagement with errors not only during classroom instruction but also when assessing. Teaching informs assessment and assessment informs teaching, and the two impact on learning. Therefore, if learners' errors are to be addressed in the process of learning, Gardee (2015) has emphasised that there is a need to research teachers' engagement with learners' errors when teaching and assessing.

According to Gardee (2015) errors are systematic and happen on a regular basis, and are unavoidable and persistent, often across contexts. While in agreement that errors are persistent since they are cognitively constructed, the researcher is of the view that they are avoidable because learners generally construct errors in the process of learning; this means if instructions are tailor-made to address errors, errors can therefore be avoided. Teachers need to have the expertise to engage with learners' errors. and Ball et al. (2008) called this kind of knowledge that is needed specialised content knowledge.

2.3.1 Learners' errors and misconceptions in mathematics

Errors and misconceptions have been a major topic in educational research, particularly in the area of mathematics. Past studies have shown that misconceptions lead to errors, and errors are symptoms of underlying misconceptions (Tulis et al., 2016). Makonye (2016) remarked that misconceptions are the underlying wrong beliefs and principles in one's mind that cause a series of errors. Makonye and Fakude (2016) explored Grade 8 learners' errors and misconceptions in the learning of addition and subtraction of directed numbers, and found that 83.3% of the learners who took part in the study had misconceptions of the topic, 16.7% had procedural errors, 67% strategic errors, and 28.6% logical errors. The authors remarked that these errors emanate from lack of reference to mediating artifacts. Makonye and Fakude (2016) also identify poor proficiency in English, which is the language for teaching and learning mathematics, as a factor contributing to learners' errors.

Baweja (2017) investigated secondary school learners' errors and misconceptions in relation to scientific attitude, and found that learners with an unfavourable scientific attitude commit more errors compared to their counterparts with a favourable scientific attitude. Baweja (2017) explained scientific attitude as an important correlate in the achievement of students. In relating this to mathematics, one can say that mathematics attitude is correlated with achievement. In addition, Sibanda (2006) studied misconceptions held and errors made by South African learners in answering science questions in the Trends in Mathematics and Science Study (TIMSS). This author avers that poor content coverage by teachers, learners' everyday experience as well as lack of learning resources and materials in some schools are some of the possible causes of learners' errors and misconceptions. It must be noted that manipulatives play a crucial role in concept development; I concur with Sibanda (2006) and Makonye and Fakude (2016) that lack of manipulatives in teaching and learning of mathematics leads to students' misconceptions and errors.

The aforementioned studies paid particular attention to describing learners' errors and the causes thereof, and all emphasise the importance of the teachers' role in addressing learners' errors. However, these studies are silent when it comes to understanding teachers' engagement with learners' errors. It is for these reasons that is deemed important this study be carried out.

2.4 Error analysis

According to James (2013), error analysis is a branch of applied linguistics. However, there are two schools of thought on error analysis in linguistics studies (James, 2013; Richards, 2015). The first holds that error analysis is a teaching methodology. It is argued that if the method of teaching is adequate, errors will not be made (Richard, 2015). In contrast, the second school of thought (James, 2013) holds the view that error errors can not be avoided, and teachers should accept that during teaching and do corrections for learners because no matter the kind of teaching strategy adopted because of the unequal society we find ourselves in errors and mistakes in learning will occur.

In mathematics, Herholdt and Sapire (2014) aver that error analysis is the study of errors in learners' work with the aim of looking for possible explanations as to why those errors were made. The authors explained further that teachers are required to analyse correct, partially correct and incorrect responses and to think of the learners so that conceivable remedial actions will be provided to deal with the errors. In their conclusion they said that analysing learners' errors requires teachers to have high levels of mathematical content knowledge and pedagogical content knowledge, which in the long-term will also help to increase the teachers' knowledge of mathematical reasoning and concept advancement. According to Larrain and Kaiser (2019) errors analysis is the ability of teachers to understand learnings thinking on mathematical concepts and be able to provide support for students to understand concepts better. The authors further argue that error analysis can be done through what they called "diagnostic competence". However, they said that diagnostic competence allows teachers to comprehend learners' ways of reasoning and adapt their teaching strategies accordingly to promote learning.

In her study uncovering the work of teaching, Ball (2017) avers that teachers must know before they teach. She further posits that the work of teaching goes beyond just knowing the mathematics, it includes the ability to diagnose, interpret and explain learners' errors. Therefore, drawing from the above it could be argued that engaging with learners' errors is an essential part of the work of teaching. Therefore, teachers must be more advanced in their mathematical knowledge for teaching because the aim of error analysis is to give a feedback that is constructive and that addresses learners' concerns (Hyland & Hyland, 2019). Adler

(2005) sees error analysis as a component of teaching and states that there is a need for teachers to develop the kind of mathematical knowledge for teaching that allows for engagement with learners' errors:

> What do teachers need to know and know how to do in order to deal with ranging learner responses, and in ways that produce what is usefully referred to as 'mathematical proficiency', a blend of conceptual understanding, procedural fluency and mathematical reasoning and problem solving skills. (Adler, 2005, p. 3)

The above statement emphasises that engaging with learners' errors does assist in improving learning. This is supported by McMillan (2017) when he mentioned that being wrong sometimes is experience that facilitates learning. McMillan (2017) further argues that being wrong or making mistakes (errors) is positive, not negative in teaching and learning. He further clarifies the positive benefits of being wrong, such as to establish the right classroom environment – a classroom in which there are norms and expectations about assessment that stress the value of learning from mistakes and errors. Such norms include a healthy climate of error management; limiting test anxiety; keeping assessment relevant, authentic, and fair; and involving learners in the assessment process. In contrast, Lai (2012) explained that error analysis is a method used by teachers to identify the cause of learners' errors when they repeatedly make mistakes. Lai (2012) further concludes that teachers' ability to identify errors or misconceptions is the first step towards them providing remedial teaching. While it is true that identifying errors and misconceptions is the first step, the researcher argue that teachers need to go beyond just identifying errors – they should interrogate the learners' reasoning in order to rectify the incorrect concept image that has been cognitively constructed.

Herholdt and Sapire (2014) supported this view, mentioning that learners' error analysis requires teachers to have strong mathematical content knowledge (MCK) and pedagogical content knowledge (PCK) to enable them to develop the correct methodology to help learners to correct their errors. They further claimed that in the long-term teachers' MCK and PCK would help to widen their knowledge of mathematical reasoning and mathematical growth. The researcher concurs with Herholdt and Sapire (2014) that in the attempt by mathematics teachers to engage with learners' errors, teachers need to have developed subject matter knowledge to help them develop an appropriate methodology to suit the needs of their learners, to help solve learners' mathematical problems in the classroom, topic by topic. This study holds the view that teachers' subject matter knowledge is the basic knowledge needed but, as mentioned by Ball et al. (2008) and Ball (2017), teachers need SCK in order to navigate the work of teaching.

Error analysis is not about pinpointing learners' errors, but rather to use those errors to improve learning. Therefore, if the aim is to improve learning it means teachers' knowledge about the subject plays a critical role in the ability to identify errors. López Valero et al. (2008) assert that mathematics teachers have to deal with two problems when correcting errors made by learners: firstly, they must find the cause of the error; and secondly, they must devise techniques that they can use effectively to engage with learners in correcting their errors. This is in agreement with Ball et al. (2008), but as mentioned by Ball (2017), the work of teaching goes beyond that because it includes teachers' ability to engage with learners' reasoning in order to identify the underlying misconception that gave rise to the errors evident in the learners' learning.

To detect discrepancies and links between and within the different aspects of teacher knowledge related to mathematical error analysis, Shalem et al. (2014) developed an instrument with six criteria based on aspects of teachers' knowledge related to explaining and diagnosing learners' errors. The six criteria are: procedural understanding of the correct answer, conceptual understanding of the correct answer, awareness of errors, diagnostic reasoning of learners' thinking in relation to error, use of everyday links in explanations of error, and multiple explanations of error. However, error analysis is not just about analysis of learners' correct, partially correct and incorrect steps towards finding a solution, but implies best practices for remediation (McGuire, 2013).

2.5 Teacher knowledge and learner achievement

Literature has shown the correlation between teachers' mathematical knowledge for teaching and learner achievement (e.g, Mji & Makgato. 2006; Pournara et al., 2015). The argument raised by these authors is that quality of mathematics teaching has a significant effect on students' learning gains. According to Barber et al., (2015) there are three components that are very important for learner achievement: teacher identity, subject matter knowledge and methodologies to teach. Pournara et al. (2015) iterates similar views and argues that teachers who have good subject matter knowledge tend to be more effective in their teaching and vice versa. Indeed, teacher knowledge plays a crucial part towards learner achievement. Moreover, it plays a crucial part in the teachers' ability to engage with learners' errors.

As it has been argued that attending to learners' errors should form part of teaching, it is teachers' competence with mathematical concepts that would make this possible in the classroom. For example, a teacher with sound mathematical knowledge to teach functions is able to understand types of errors made by learners in this area. However, if the same teacher is not competent in geometric concepts, he/she will struggle to engage with learners' errors in geometry. In this study the researcher is not exploring teachers' knowledge – however, the focus is on teachers' engagement with learners' errors, and thus teachers' knowledge plays a crucial part in allowing that to take place in the classroom.

2.6 Error analysis and mathematical knowledge for teaching

Researchers such as Ball et al. (2008), Adler (2010), McAuliffe (2013), Shalem et al. (2014), Bansilal et al. (2014), Scheiner (2015), Aksu and Kul (2016), Hine (2017) and Ball (2017) emphasise the importance of teacher mathematical knowledge for teaching in terms of learner achievement. In addition to the argument above, Ball et al. (2017) emphasised that the ability of teachers to engage with learners' errors includes teachers' ability to solve learners' mathematical problems. In their study with mathematics teachers, Shalem et al. (2014) extend this view and argue that teachers' use of everyday knowledge plays a crucial role in their ability to engage with learners' errors.

Scheiner (2015) distinguished between the substantive and syntactic dimensions of teacher SMK. Substantive knowledge encompasses the key facts, concepts, principles, structures and explanatory frameworks in a discipline, whereas syntactic knowledge concerns the rules of evidence and warrants of truth within that discipline. Drawing from the above scholars, it could be argued that error analysis is not a separate entity of teaching, but a daily activity that teachers need to perform while teaching and assessing with the aim of improving teaching. Therefore, ignoring learners' errors in the process of teaching and assessment means ignoring critical aspects of teachers' mathematical knowledge for teaching.

This same argument was echoed by Kazima and Adler (2006), who asserted that teachers' knowledge of error analysis should be seen or treated as a component of mathematics for teaching. Their argument is based on the premise that teachers need to recognise and know how to deal with learners' mathematical problems so that they can help develop learners' mathematical knowledge. However, the lack of content knowledge on the part of the teacher may prove the inclusion of error analysis in the teaching process to be counterproductive, because if teachers lack mathematical knowledge for teaching he/she might fail to interpret learners' errors and unearth the underlying misconceptions. Instead of enhancing learning, this might end up creating more learning barriers for learners. While most literature has shown that engaging with learners' errors is a critical part of teaching, Metcalfe (2017) warned that errors may be a counterproductive strategy for learning, because allowing people to make errors encourages them to practice incorrect and inefficient approaches that are difficult to overwrite later with the correct approaches. However, the researcher is of the view that engaging with learners is not encouraging learners to continually make errors, but rather is a way to ensure that errors that hinder learning are addressed. The errors manifest themselves in the process of teaching, learning and assessing, and it is at this point that teachers need to act decisively with the intention of enhancing teaching and learning.

2.7 Error analysis in the teaching of mathematics

As pointed out earlier, there is not much literature on error analysis in mathematics in South Africa. This study will draw literature from other parts of the world and link it to the South African context to indicate what other countries have to say about error analysis. According to Rohmah and Sutiarso (2018) "many learners see Mathematics as a field of study that is difficult to understand" (p. 671). They further argue that this happens because some teachers present mathematics to learners in a form that is less appealing and seems difficult for learners to learn; as a result, learners often feel bored and do not respond well to the lessons. In South Africa, mathematics is also considered as one of the subjects that is most often failed among learners across grades. This is supported by Makhubele and Luneta (2014, p. 61) when they said that "poor mathematics results at the primary and secondary level in South Africa severely limit the youth's capacity to exploit further training opportunities". The researcher agrees with the claims made by the Rohmah and Sutiarso (2018) and Makhubele and Luneta (2014) that mathematics is a difficult subject, and therefore supports what Rohmah and Sutiarso suggested as the solution to making mathematics easy.

They suggested that analysis of errors is needed to find out how students solve mathematical problems. They hold that if teachers can use error analysis effectively to find out how learners solve mathematical problems, it will help teachers to develop strategies to help learners to overcome their difficulties in mathematics. It is for this reason that in this study the researcher seeks to explore teachers' engagement with learners' errors when teaching and assessing, drawing from existing literature to unpack the use of error analysis in the teaching of mathematics to understand the practices teachers employ when engaging with learners' errors. Error analysis is crucial in the teaching of mathematics, and teachers must take a keen interest in using error analysis as one of the modern, useful ways to teach mathematic to learners across grades (Larrain & Kaiser, 2019). According to Ndlovu (2019) effective error analysis requires mathematics teachers to have strong mathematical content knowledge in order to carry out proper and useful error analysis of learner errors.

2.8 Error analysis in assessment

Assessment is an integral part of teaching and learning; therefore, it is imperative that research addresses these aspects together. The trend has been for researchers to focus on either teaching and learning or assessment; however, this study looks at teachers' engagement with learners' errors when teaching and assessing. Before one can look at literature about engagement with learners' errors when assessing, it is critical to understand how assessment is being understood in the context of teaching and learning, particularly of mathematics, as stated in the national CAPS (DBE, 2012) document.

2.8.1 Assessment

The national CAPS (DBE, 2012, p. 154) document explains assessment as follows:

> Assessment is a continuous planned process of identifying, gathering and interpreting information regarding the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this

> evidence; recording the findings and using this information to understand and thereby assist the learner's development in order to improve the process of learning and teaching. Assessment should be both informal and formal. In both cases regular feedback should be provided to learners to enhance their learning experience. This will assist the learner to achieve the minimum performance level of 40% to 49% required in Mathematics for promotion purposes.

This means that assessment can be simply defined as a day-to-day activity, process, that is an ongoing, integral part of teaching and learning. The CAPS (DBE, 2012, p 154) document further divides assessment into four types: baseline assessment, diagnostic assessment, formative assessment, and summative assessment.

Baseline assessment is defined as a type of assessment for mathematics teachers who might want to establish whether their learners meet the basic skills and knowledge levels required to learn a specific mathematics topic. Knowing learners' level of proficiency in a mathematics topic enables the teacher to plan her/his mathematics lesson appropriately, and to pitch it at the appropriate level. Baseline assessment, as the name suggests, should therefore be administered prior to teaching a mathematics topic.

Diagnostic assessment is not intended for promotion purposes but to inform the teacher about the learner's problem areas in mathematics that have the potential to hinder performance. Two broad areas form the basis of diagnostic assessment: content-related challenges, where learners find certain topics difficult to comprehend, and psychosocial factors such as negative attitudes, mathematics anxiety, poor study habits, poor problem-solving behaviour, etc. Appropriate interventions should be implemented to assist learners in overcoming these challenges early in their school careers.

Formative assessment is used to aid the teaching and learning processes, and hence is assessment for learning. It is the most commonly used type of assessment because it can be used in different forms at any time during a mathematics lesson (for example, short class work during or at the end of each lesson, and verbal questioning during the lesson). It is mainly informal and should not be used for promotion purposes. The fundamental distinguishing characteristic of formative assessment is constant feedback to learners, particularly with regard to their learning processes. The information provided by formative assessment can also be used by teachers to inform their methods of teaching.

Summative assessment is carried out after the completion of a mathematics topic or a cluster of related topics. It is therefore referred to as assessment **of** learning, according to CAPS (DBE,2012) since it focuses mainly on the products of learning. The results of summative assessment are recorded and used for promotion purposes. It is mainly formal by nature, comprising school-based assessment (SBA) and the end of year examination in South Africa. The forms of assessment presented in Table 2.1 are examples of summative assessment according to CAPS (DBE, 2012).

Table 2.1: minimum requirement for formative assessment at the senior phase mathematics

	Forms of Assessment	Minimum Requirements per term				Number of Tasks per Year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA	Test	1	1	1		3	**40%**
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	**2**	**3**	**3**	**2**	**10***	
End of the year Examination						1	**60%**

Drawing From the above definitions, the researcher holds the view that teachers can use diagnostic assessment and formative assessment when engaging with learners' errors in the teaching and assessment of mathematics. Classroom assessments are concepts and applications that explore how assessment is a key component of all aspects of the teaching and learning process, including organising and creating a classroom principle or rules, planning lessons, carrying out teaching, and assessing how students have understood concepts as a result of teaching.

The researcher in this study supports the explanation of assessment given by Airasian and Russel (2001), that assessment is an ongoing integral part of teaching which occurs on daily basis and adds that assessment is one of the key functional duties of teachers. Considering this, the researcher has defined assessment as a tool that is used to measure teaching and learning. This means that teachers always measure teaching and learning through assessment. Not far removed from the above views of assessment, Keeley (2015) believes that assessment can also be defined as a useful tool to inform teaching and learning as well as for measuring and documenting students' achievements.

Recently, other researchers who have written about assessment, such as Lund and Kirk (2019), defined assessment as part of the teaching process and stated that assessment results are critical for teachers to have a deeper reflection of what students have learned and what they still need to know. They believe that "good teaching is inseparable from good assessment" (Lund & Kirk, 2019, p. 19). This means that assessment and teaching go hand in hand, and whenever there is teaching there must be assessment. As teaching informs assessment and vice versa, this means it is not only teaching that impacts on learning, assessment also impacts on learning; therefore, errors manifest either during the teaching process or during the assessment process. Teachers therefore need to be able to diagnose errors during teaching and assessment and to formulate strategies to address them.

In contrast, Kalajahi and Abdullah (2016, p. 233) defined assessment as "the systematic process that provides an opportunity for teachers to meaningfully reflect on how learning is best delivered, collect respective evidences, and then use that information to improve their teaching". Recognising the above explanation of assessment, the researcher in this study agrees with the views given above by other researchers, but for the purposes of this study is more convinced by the definition given by Kalajahi and Abdullah that assessment gives teachers information about learners, and that the teachers use assessment to improve their teaching. When the assessment practices of teachers are improved, this might help them to engage with learners' errors in mathematics effectively and help learners to reduce or minimise the errors they make in mathematics.

Earl (2012) grouped assessment into three categories: assessment *for* learning (formative assessment), assessment *of* learning (summative assessment), and assessment *as* learning (peer assessment). For the purposes of this study, the researcher will not deal with these three categories of assessment much, but will rather pay attention to formative assessment, because in the teaching and learning process formative assessment provides immediate feedback on learners' work to teachers, in order for proper turn around strategies to be used to help improve learners' understanding. Considering the topic of this study, 'Exploring Grade 9 mathematics teachers' engagement with learners' errors in the teaching and assessment of mathematics', the researcher believes that formative assessment will be the most appropriate and effective assessment type to discuss.

2.8.2 Formative assessment as form of teaching mathematics

In this study, formative assessment is considered as a form of teaching mathematics. In the past, forms of teacher mathematics have not considered the role that formative assessment can play. This study seeks to bring something new to mathematics teachers. First the researcher will discuss some literature on formative assessment for better understanding.

According to Keeley (2015), formative assessment is a systematic process of collecting evidence about students' thinking and learning to inform instruction and provide feedback to the students while simultaneously prompting learning. She further argued that formative assessment is more assessment for learning rather than assessment of learning, and it can also be assessment as learning. Black and Wiliam (2010, p. 82) defined formative assessment (assessment for learning) as "activities undertaken by teachers and by their students in assessing themselves that provide information to be used as feedback to modify teaching and learning activities". This means that feedback plays a central role in order to modify teaching after assessment. In contrast, Ginsburg et al. (2017) defined formative assessment as putting together knowledge that can guide the teaching of individuals or groups of students. They further argue that this approach requires a sound understanding of children's thinking and learning, as well as effective methods for gaining understanding of mathematical concepts.

In a nutshell, one can say that formative assessment is important in the teaching and learning of mathematics because it helps the mathematics teacher to measure the level of learners' (students') understanding and provide immediate feedback for immediate action when errors are found in learners' work. While formative assessment is used to develop the learners' understanding of mathematics, Brown (2005) pointed out that diagnostic assessment is meant to improve the learners' experience and their level of achievement.

2.8.3 Feedback as a tool of assessment in mathematics

In the early years of the 21st century feedback has played an important role in the teaching and assessment of mathematics. Feedback can be provided at the task level (information on task performance), process level (information on processes required to master the task), self-regulatory level (information on the regulation of action), and self-level (information on the learner as a person, not related to task performance) (Hattie & Timperley, 2007). While feedback at the first three levels is associated with positive learning outcomes, feedback at the self-level usually contains too little task-related information to show positive effects on learning processes (Hattie & Timperley, 2007). The design of process-oriented feedback draws on Hattie and Timperley's (2007) ideas and tries to combine feedback at the task level, process level, and self-regulatory level by referring to specific tasks and focusing on cognitive and self-regulatory processes (Rakoczy et al., 2018).

In relation to formative assessment in mathematics, Hyland and Hyland (2019) defined feedback as a constructive judgement of assessment that reflects concern for students' future performance and progress of their learning development in mathematics. Feedback is an effective instrument for assessment to measure teaching and learning of mathematics. Good or effective feedback from teachers to learners helps them to work on areas of weakness in order to improve performance in mathematics.

However, during this time of using feedback to improve performance, the teacher must be able to not only provide feedback on what is wrong or right in learners' work, but also to engage and confront errors that learners make and help learners. This makes it clear that teachers must be knowledgeable in order to deal with learners' errors or mistakes effectively when providing feedback to them. This claim was made earlier by Brookhart (2017), when he reported that feedback speaks to learners and indicates that "somebody" cares enough about their work to check, read and mark it. Since teachers provide feedback and care about learners' work, teachers are expected to use feedback as part of assessment in teaching. For teachers to provide feedback to learners, they then become duty bound to analyse what they pick up from learners. This study supports Brookhart's claim on feedback and does not undermine what was said by Hyland and Hyland (2019).

Brookhart (2017) further reports that feedback is an important aspect of the formative assessment process. This claim by Brookhart is true most of the time because feedback comes after assessment; therefore, Brookhart further argues that formative assessment gives information to teachers and students about how students are doing relative to classroom learning goals. Formative assessment informs learning and therefore, according to Rakoczy et al. (2018, p. 155) is "information communicated to the learner that is intended to modify his or her thinking or behaviour for the purpose of improving learning." Similarly, Hattie and Timperley (2007) highlight that the main purpose of feedback is to highlight the discrepancy between current understanding and performance on one hand and the learning goal on the other, and to encourage and enable students to reduce this discrepancy. In support of Hattie and Timperley's claim, Rackoczy et al. (2018) added that for teachers to encourage and enable students to reduce the discrepancy between the learning goal and their current performance on a mathematics test, learners should be informed about whether or not they applied the mathematical operations needed to solve the tasks on the test.

2.8.4 Relationship between teaching and assessment in mathematics

According to Broadfoot (2012) assessment give direction to students and teachers in knowing whether learning has taken place. In other words, without assessment teachers and learners will not be able to know the outcomes of learning. For this reason, assessment is considered an integral part of teaching (Black & Wiliam, 2018). For example, when mathematics teachers prepare and teach a topic or concept, they use assessment to measure the understanding of learners to see if learning has taken place. If learners are able to provide correct answers to questions, then the objective of the lesson has been achieved. However, if learners fail to provide correct answers then teachers have to engage with the errors the learners make that cause them not to provide the correct answers.

According to Polly et al. (2013, p. 12) "Teachers' beliefs towards mathematics and their impressions of effective mathematics teaching have been associated with teachers' enacted instructional practices and their willingness to enact student-centered pedagogies". Drawing from the above, it is clear that teachers' beliefs play a critical role in the way they enact being teachers and engage with learners. Hence, if teachers believe that engaging with learners' errors would enhance teaching, they are more likely to use pedagogies aiming at addressing learners' errors in the process of both teaching and assessing. Researchers have found empirical links between specific instructional practices and student learning outcomes (Artzt et al., 2015; Kebritchi et al., 2010; Polly et al., 2013). These instructional practices reflect a student-centred view of teaching mathematics, where students engage in mathematically rich tasks and are supported by classroom teachers who pose questions and modify instruction based on students' mathematical thinking.

2.8 Teachers' engagement with errors and misconceptions when teaching and assessing in mathematics

Referring to the literature, Sapire et al. (2016) explained that error analysis is an integral part of teacher knowledge. Specifically in mathematics education, Hill and Ball (2009) argue that "engaging learners' errors is among the mathematical duties of teaching that recur across different curriculum materials or approaches to instruction" (p. 70). Key players in errors analysis in their fields of study, such as Ingram et al. (2013), argue that although teachers may not openly tell the learners that making errors is problematic, the way in which teachers deal with errors, by avoiding opportunities for learners to make and discuss mistakes in the classroom, indirectly suggests that errors are problematic (Heinze & Reiss, 2007).

According to Khansir (2012) error analysis can help provide a strong support for remedial teaching. He added that during the teaching programme it can reveal both the successes and the failures of the programme. Error analysis is useful in second language learning and mathematics, because it reveals the problem areas to teachers, syllabus designers and textbook writers. It can be used to design remedial exercises and focus more attention on the need for teachers to treat errors sensitively and productively, as errors can be used as tools – not only to motivate learners, but also to assist them in developing their conceptual knowledge by learning from their errors. Teachers may also regard errors as a failure on their part. This is reflected in Brodie and Berger's (2010) research, where teachers blamed the learners or themselves for the errors made in class.

2.9.1 Teachers' engaging with learners' errors when teaching mathematics

Teachers come across errors in their teaching in the mathematics classroom almost every day. When mathematics teachers try to respond to learners' errors in their classrooms, during or after teaching, they are engaging with learners' errors in mathematics using formative assessment (Black & Wiliam, 2006, as cited in Sapire et al., 2016). Sapire et al. (2016) further argued that teachers answering to learners' errors is a specific action of formative assessment, which depends on teachers' profound knowledge of mathematical content knowledge; it needs in-depth teachers' professional knowledge to make decisions on how to respond to learners' needs when teaching that content. Working with learners' errors diagnostically in context implies that the "cognitive architecture" (Hugo, 2015, p. 81) of teachers' mathematics knowledge is strong and that their knowledge is stored in their long-term memory in the form of "networked schemas", ready to be selected economically, for example, in the form of principles, representations and other symbolic forms. However, in South Africa there is empirical evidence showing that teachers' mathematical content knowledge and PCK is weak (Taylor et al., 2013).

Teachers must always assess learners' work, recognise the errors present in this work and try to help learners. Most teachers in South Africa and around the world do not get time to engage with learners' errors, even if they are able use their mathematical knowledge to identify learners' errors, because there is so much that they need to cover in the Annual Teaching Plan (ATP). Moreover, studies on teachers' engagement with learners' errors show that teachers' revealing position on an error makes it important to engage with learners' errors, meaning that teachers cannot simply reteach without engaging with the mathematical source of the error (Gagatsis & Kyriakides, 2000; Peng, 2010; Peng & Luo, 2009; Prediger, 2010). However, there is no literature that critically scrutinises the distinctive features that make up teacher pedagogical knowledge of errors, to engage learners' errors and their relation to subject matter knowledge SMK and knowledge about teaching. The understanding of this study is that knowledge of errors can be revealed to include both the substantive and syntactic dimensions of teacher SMK.

2.9.2 Teachers' engagement with learners' errors after mathematics assessment

Mathematics teachers engage with learners' errors effectively through assessment. Enu and Ngcobo (2020) stated that teachers can engage with learners' errors in mathematics through assessment. After learners have been assessed, then the mistakes they make inform the teacher to enable them to provide remedial action or teaching on a concept or topic, to engage with these errors. The authors mooted that formative assessment techniques such as questioning and feedback can be utilised in addressing learners' errors.

According to Bjork (2012) engaging with learners' errors is very strenuous, but the difficulty is desirable for learning. Teba (2017) explained that considering errors as part of learning, teachers are expected to pay attention to strategies for correcting such errors; assessment has been mentioned as one of such critical strategies in achieving that (Enu & Ngcobo, 2020). Lai (2012) posits that by highlighting learners' errors, teachers could provide teaching aiming at the learners' areas of learning needs.

2.10 Research into teachers' engagement with learners' errors

Research into learners' errors has gained momentum globally. A plethora of research dwells on explaining mathematical errors that learners make which hinder the learning process (Gur, 2009; Luneta & Makoye, 2010; Almog & Ilany, 2012; Durkin & Rittle-Johson, 2015; Mathaba & Bayaga, 2019). While research into learners' errors has gained momentum, this is not the case with research into teachers' engagement with learners' errors. Shalem (2014) conducted a study analysing teachers' knowledge of errors, with analysis of what criteria can be used to assess teachers' explanations of learners' errors in standardised mathematical assessments. The study was conducted with 62 mathematics teachers and findings showed that teachers draw on their mathematical knowledge to address learners' errors, with less attention to learners' everyday knowledge. Sapire et al. (2016) conducted a study on engaging with learners errors with 62 teachers.

The findings of this study revealed that teachers shied away from engaging with learners' errors during teaching. Where they made an attempt to engage, they were unable to come to grips with learners' errors let alone their thinking. The authors suggest that further research is needed in order to understand and explain the complexities of engaging with learners' errors Sapire et al. (2016). Similarly, Aksu and Kul (2016) conducted a study with 13 mathematics teachers investigating their knowledge of learners' difficulties and their ability to correct the errors. The findings revealed that while teachers have knowledge of the learners' errors, they are unable to correct errors and misconceptions evident in the learners' responses. Findings from the aforementioned studies reveal that this is an area that still needs to be researched, in order to understand how teachers can use learners' errors and misconceptions to improve learning. This study hopes to contribute to this limited field of research.

2.11 Conclusion

This chapter presented to readers some of the literature on the topic engaged by this study, which is "Exploring the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics". The literature that was reviewed under each theme of the study comprised previous work by authors who have carried out similar studies, for example on errors, misconceptions, assessment, and error analysis, among others. In the next chapter the conceptual framework used for the study is described.

CHAPTER THREE

CONCEPTUAL FRAMEWORK

3.1 Introduction

According to Adom, Hussein, and Agyem (2018) a conceptual framework offers researchers the freedom to adopt existing frameworks, and to modify them to suit the nature of the context of their research as well as the nature of their research questions. Ravitch and Carl (2019) aver that conceptual frameworks are reproductive frameworks that reflect the thinking of the entire research process. Akintoye (2015) posits that the conceptual framework is mostly used by researchers when existing theories are suitable for creating a firm structure for the study. Adom et al. (2018) add that a good conceptual framework must be expressed in writing in order to be better understood. The authors further suggest that diagrams can be used to define the constructs or variables of the research topic and their relationships. This means that the framework can be illustrated both textually and diagrammatically. The important thing is for the researcher to show constructs between the framework and the topic being studied.

This study aims to explore Grade 9 mathematics teachers' engagement with learners' errors in the teaching and assessment of mathematics. The ability of teachers to analyse errors requires them to have competence in mathematical knowledge for teaching. Therefore, two frameworks were deemed suitable for this study: Error analysis by Sapire et al. (2014), and Ball et al. (2008) framework of Mathematical Knowledge for Teaching (MKfT). In order to understand teachers' engagement with learners' errors, the two frameworks were combined to explore the way in which teachers use their knowledge for teaching to engage with learners' errors when teaching and assessing mathematics.

3.2 Error analysis

Error analysis is concerned with addressing the persistent errors which can be attributed to faults in reasoning, since they tend to reveal themselves in the sense-making process (Naidoo, 2012). It is important to note that not all errors are persistent; some are just careless mistakes, which are considered slips (as termed by Tulis et al., 2016). However, without engaging with the error it is not easy to tell whether it is a slip or a symptom of underlying misconceptions. Therefore, paying attention to errors and misconceptions is critical to improve learning (Smith et al., 1993; Adler 2005; Venkat & Adler, 2012).

Error analysis has its origins in the languages. For the purposes of this study in mathematics, literature can be drawn from the work of (Sapire et al., 2014) on error analysis. Although error analysis is not new in teaching and learning, it has currently regained attention with a focus on not just explaining errors that learners make which hinder their learning process; the new trends attempt to explain the extent to which teachers are able to engage with these errors during the process of teaching and learning (Bohlmann, Prince, & Deacon, 2017; Herholdt & Sapire, 2014; Larrain & Kaiser, 2019; Sapire et al., 2014; Sapire et al., 2016; Sisman & Aksu, 2016). Among the above authors, Sapire et al. (2016) developed a framework looking at what teachers focus on when analysing learners' errors. This study focused on the extent to which teachers engage with learners' errors, and the researcher therefore found the framework developed by Sapire et al. (2014) (see Figure 3.2) suitable for this study. The next section provides a brief explanation of this framework and its underpinning strands.

3.3 Error analysis framework

According to (Sapire et al., 2014, p. 2), "Error analysis is an integral part of teacher knowledge". This means that error analysis should be part of teachers' work and must be integrated into teaching. To analyse teacher knowledge of error analysis, (Sapire et al., 2014) developed a framework with six criteria, namely: procedural understanding, conceptual understanding, awareness of errors, diagnostic reasoning, use of everyday knowledge, and multiple explanations of errors. Sapire et al. (2014) explained procedural understanding as the quality of teachers' procedural explanation of mathematical concepts, for learners to grasp and become competent in working with procedures themselves. For example, in teaching exponents to Grade 9 learners, teachers are required to teach them the procedure of solving $4^3 \times 4^2 = 4^5$. Articulating the procedure, the teacher can explain to the learner that when you are multiplying an exponent with the same base, add the exponent. This becomes a procedure for learners to follow and use to competently solve problems of that nature anytime they come across them.

One can also term the procedural understanding of mathematical concepts as rules or steps to follow when solving mathematical concepts. Aydin (2018) refers to this kind of knowledge as the knowledge that encourages application of sequential action steps and techniques of solving a problem. While authors explain this kind of knowledge using different phrases, the common theme is that this knowledge requires competence to solve mathematical problems following routine/structured steps. Therefore, a teachers' competence in procedural knowledge is critical in assisting learners to know and understand the procedure to follow when solving a particular sum. For example, when asked to sketch a linear graph given $f(x) = 2x + 3$, the teacher needs to help the learners to know all the procedures to be followed to sketch the graph, such as using the table method, finding intercepts or using the gradient method.

In contrast, Sapire et al. (2014) posit that conceptual understanding is teachers' understanding of mathematical concepts, operations and relations, as well as the conceptual links that they make in their explanation of mathematical concepts to learners – which possibly can lead to generalisation of concepts by learners. For example, 4, 9, 16, 25, 36, 49, 64, 81 and 100 can be explained to learners as square numbers ($2^2 = 4$, $3^2 = 9$, $4^2 = 16$, etc.). This means, in brief, that a square of a number is the number multiplying itself two times. From here the learners can generalise this with conceptual understanding to say $10^2 = 100$. Conceptual understanding goes beyond carrying out procedures, but needs understanding of relationships between concepts: for example, when doing fractions, the understanding of why division is changed to multiplication and why one fraction is turned upside down. For teachers to assist learners with errors made, they need to be able to explain the 'Why?' question in mathematics. The third key element in Sapire et al.'s (2014) framework is the awareness of errors. Awareness of errors focuses on teachers' explanation of the actual mathematical errors and not the learners' reasoning (Sapire et al., 2014, p. 4). Here emphasis is on the quality of teachers' mathematical explanation of the actual mathematical errors that learners make.

Diagnostic reasoning was the fourth criterion for error analysis by (Sapire et al., 2014), and they explained it as the quality of teachers' attempts to provide a rationale for how learners reason mathematically when they choose a mathematical problem. This is what the author of this study calls "mathematical psychology". Mathematical psychology is the ability of the teacher to think or to find out the thinking capability or ability of the learner in providing wrong or correct answers.

The use of everyday knowledge in explaining learners' errors is what (Sapire et al., 2014) called the quality of teachers using everyday knowledge in explaining learners' errors, informed by links made to the mathematical understanding that the teacher want to impart to learners. Here it can be argued that mathematics should be taught by linking it to real-life situations. An example is using everyday language to teach fractions, by putting the problem into context rather than teaching abstractly or using word problems to teach equations, such as:

the sum of Ayanda and Anele's ages is 36. If Ayanda is 15 years old, how old is Anele?

Mathematically, this is the same as solving for x if $15 + X = 36$. The teacher can link this algebraic equation to real life, as in the case of Ayanda and Anele, and the learner can easily see that Anele is 21 years old before even doing the mathematical calculation. The use of everyday examples enables mathematical understanding (Sapire et al., 2014). The last of the six criteria is multiple explanation of errors, which (Sapire et al., 2014) explained as teachers' ability to provide alternative explanations for errors when they are engaging with learners' errors. This means that teachers must have different ways of dealing with learners' errors, so that learners can have alternatives to choose from which best work for them or which they understand best.

3.4 Mathematical knowledge for teaching (MKfT)

In the past decade, Ball et al. (2008) have worked on two key research projects, namely the Mathematics Teaching and Learning to Teach project and the Learning Mathematics for Teaching project. Through their work over the years, they have developed a model of teacher knowledge which is a practice-based theory of what they call "mathematical knowledge for teaching" (MKfT). As the name implies, this professional knowledge of mathematics is needed by teachers to carry out the work of teaching mathematics (Ball et al., 2005; Ball et al., 2008). Ball et al. (2008) extended the work of Shulman's subject matter knowledge (SMK), and pedagogical content knowledge (PCK) into MKfT. Ball's model of MKfT uses Shulman's division between SMK and PCK and distinguishes three components of SMK, namely common content knowledge (CCK), specialised content knowledge (SCK) and horizon knowledge (HK), as well as three components of PCK: knowledge of content and teaching (KCT), knowledge of content and students (KCS) and knowledge of content and curriculum (KCC), as shown in Figure 3.1.

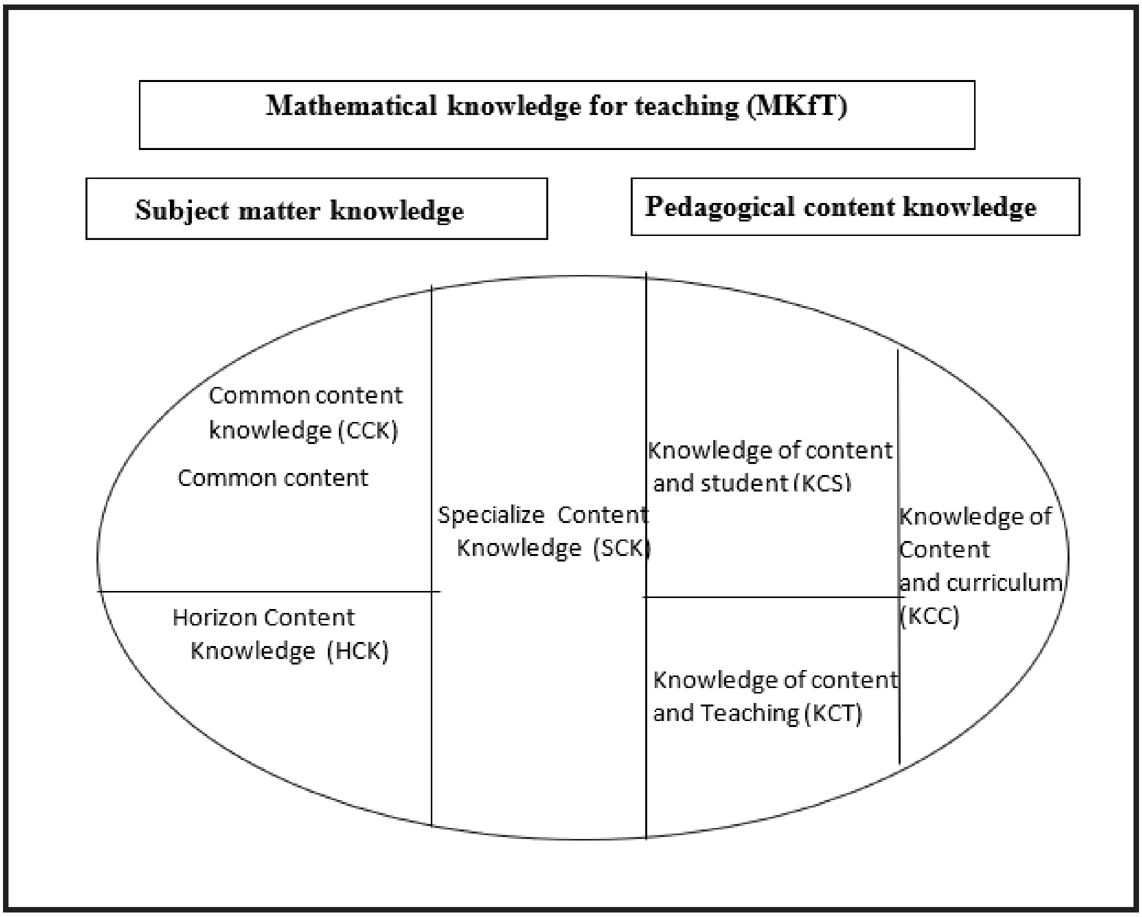

Figure 3.1 Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403)

This study adopted three of the six domains of (Ball et al., 2008) MKfT: CCK, SCK and KCS, and mapped it with the six strands of (Sapire et al., 2014) error analysis (Figure 3.2). While all of the six strands of mathematical knowledge are critical in the teaching of mathematics, the three domains used in this study were those which the researcher deemed useful in understanding teachers' engagement with learners' errors. The literature emphasised that learners' errors and misconceptions need to be identified, and further argued that learners' errors and misconceptions actually inform teaching and learning, meaning they are part of the teaching and learning process (Tulis et al., 2016; Siyepu, 2013; Ndlovu et al., 2017). However, the literature does not explicitly explain the type of knowledge that teachers need in order to be able to incorporate learners' errors and misconceptions in the process of teaching and learning. After interrogating Ball et al.'s (2008) strands of MKfT and Sapire et al.'s (2014) strands of analysing errors, the researcher found the three strands, namely CCK, SCK and KCS as critical in this process, thus leading to the new framework design shown in Figure 3.2. Drawing from Ball et al.'s (2008) framework to explain the types of mathematical knowledge needed for teaching mathematics, this involves identifying the teaching tasks involved and the knowledge needed to teach mathematics effectively. It is based on the premise that teachers need to know mathematics and know how to use mathematics in the work of teaching learners (Ball et al., 2008). Drawing from Ball et al. (2008) and Ball (2017) uncovering the work of teaching, the framework on MKfT encompasses understanding teachers' use of their teaching ability to improve teaching and learning, the ability to identify the recurrent tasks and problems in teaching mathematics, and the mathematical knowledge, skills, and sensibilities required to manage these tasks. Figure 3.2 provides a diagrammatic representation of the two frameworks combined to form the conceptual framework for this study.

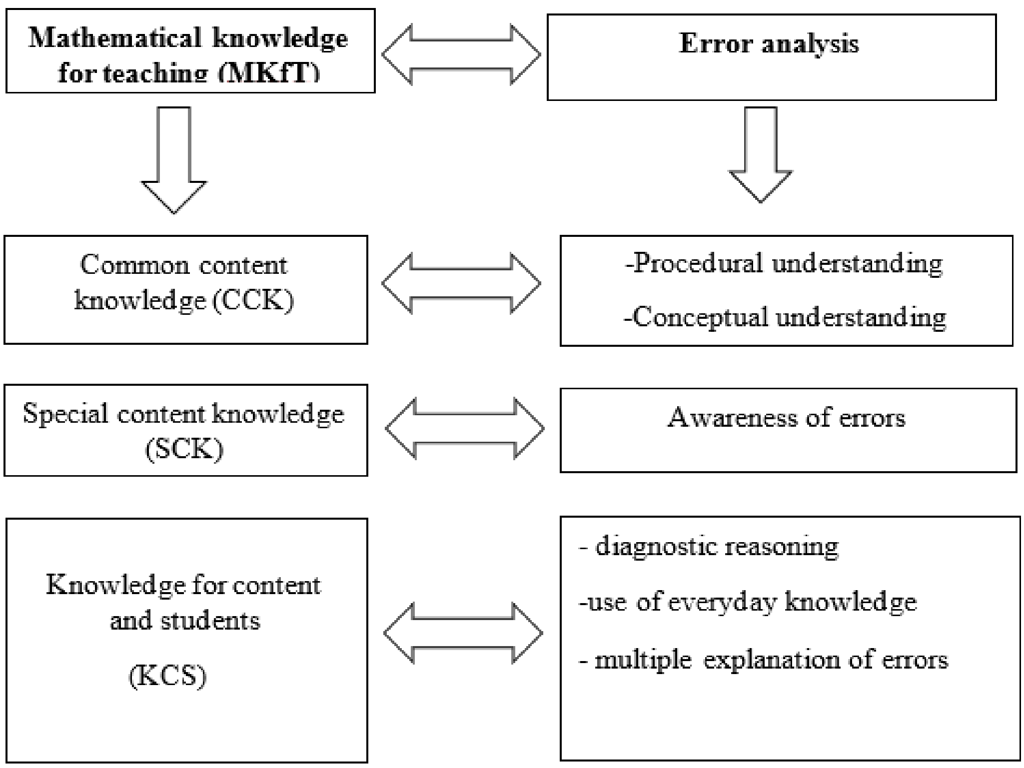

Figure 3.2 Conceptual framework: Enacted from error analysis by Sapire et al. (2014) and mathematical knowledge for teaching by Ball et al. (2008)

Looking at the combination of the two frameworks for this study in Figure 3.2, the researcher used mapping to link the six criteria of error analysis as purported by Sapire et al. (2014) (procedural understanding, conceptual understanding, awareness of error, diagnostic reasoning, everyday knowledge, and multiple explanations of errors) to three strands of MKfT as posited by Ball et al. (2008), i.e. CCK, SCK and KCS. Ball et al. (2008) focus on explaining teachers' mathematical knowledge needed for teaching, while Sapire et al. (2014) devise criteria to analyse learners' errors.

This study explores teachers' engagement with learners' errors. The ability to analyse errors requires an individual's knowledge to teach the subject, in this case mathematics. It is therefore within these parameters that the researcher finds the two conceptual frameworks useful to understand teachers' engagement with learners' errors. Teachers' ability to solve problems that they would expect his/her learners to solve requires their competence with CCK (Ball et al., 2008). However, the ability to analyse learners' errors also requires teachers' procedural and conceptual knowledge (Sapire et al., 2014). Therefore, the first key principle needed in the analysis of errors is teachers' CCK of the mathematics topic, which reveals teachers' procedural and conceptual knowledge of that topic.

The first block in Figure 3.2 above shows this alignment. In order to understand teachers' engagement, the researcher explored the extent of teachers' knowledge to solve the problems given to learners, translated into their ability to explain procedural and conceptual errors made by the learners when teaching and assessing. The second strand, which is the SCK, focuses on teachers' ability to interpret, diagnose and explain learners' errors. Possessing such knowledge would allow a teacher to become aware of learners' errors and to diagnose their reasoning that led to the error in the process of teaching and learning. Therefore, the second principle that grounds the understanding of teachers' engagement with learners' errors requires the fundamental knowledge of teachers' interpretation of errors. The third principle includes KCS, which is also needed in the diagnosis and explanation of errors. The three principles combining three strands of MKfT and strands for analysing errors are essential in exploring teachers' engagement with learners' errors.

In the next section the researcher provides an explanation of the three types of MKfT used in this study, and further discusses the error analysis framework. The researcher brought these two frameworks together to show their connectivity to each other as the lens underpinning this study.

3.5 Mapping teachers' mathematical knowledge for teaching and error analysis

According to (McGuire, 2013), error analysis is the study of best practices of remediation, which requires of teachers a good mathematical content knowledge (MCK), as well as a good grasp of learners' level of mathematical understanding (McGuire, 2013). In order for teachers to engage with learners' errors, this study designed a conceptual framework using two different frameworks which already existed: error analysis (Sapire et al., 2014) and MKfT (Ball et al., 2008). Although this study draws from existing frameworks, the uniqueness of the framework for this study is in its mapping of which aspect of teachers' MKfT is linked with which error analysis criteria are appropriate to deal with the kind of errors made by learners, as shown in Figure 3.2. To put it more simply, the researcher took the three domains from the MKfT framework and under each one of them further explained the error analysis criteria that fall under them (Figure 3.2). The amalgamation of error analysis and mathematical knowledge were deemed useful in unpacking the research questions framing this study, and thus in helping to interpret the findings. In the following section the researcher elaborates on the three strands of mathematical knowledge illustrated in the conceptual framework presented in Figure 3.2.

3.5.1 Common Content Knowledge (CCK)

Drawing from Ball et al. (2008), CCK is the basic knowledge that a mathematics teacher needs to have to teach mathematics. It is not unique to mathematics teachers, but it is the type of knowledge that any individual who has done mathematics should have. However, with respect to teachers, it is the kind of knowledge that involves the ability to correctly solve mathematical problems. This is supported by Lam (2017, p 26) Content knowledge describes a teacher's understanding of the structures of his or her subject, or "a deep understanding of the domain itself". Teachers need to know the work they must teach, recognise incorrect answers, and use mathematical terms and notations correctly. CCK is crucial in the teaching and learning of mathematics, and when properly managed in the classroom can go a long way to help in identifying and addressing learners' errors. This study connects teachers' CCK with Sapire et al.'s (2014) error analysis criteria (procedural understanding and conceptual understanding) as a strand for understanding the extent to which mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. The researcher of the current study explained the domains of error analysis that connect to CCK, as mentioned above.

3.5.1.1 Teachers' procedural understanding of errors

The relationship of procedural understanding of errors with CCK is the quality of teachers' procedural explanations of errors when analysing learners' errors during the process of teaching. According to (Sapire et al., 2014, p. 3) "teaching mathematics involves a great deal of procedural explanation, which should be done fully and accurately for the learners to grasp and become competent in working with the procedures themselves". This means that teachers need to be competent in the concept they teach, in order to be able to address the learners' errors effectively and be able to explain the critical stages required to obtain accurate answers.

3.5.1.2 Conceptual understanding

Sapire et al. (2014) reported that the importance of this strand is in the quality of the teachers' conceptual relations made in their explanations when they are analysing errors learners make in their attempt to do mathematics assessment in the classroom. Teaching mathematics involves conceptual explanations, which should be done with as many links as possible and in such a way that concepts can be generalised by learners and applied. The relationship between procedural and conceptual mathematics knowledge is complex, and recent research insists that the two need to be seen as integrated rather than polarised when thinking about mathematical ideas (Baroody et al., 2007; Long, 2005; Star, 2005).

Notwithstanding, some mathematical problems lend themselves more to procedural explanations, while in others the procedural and the conceptual are closely linked. There is a progression in the mathematical understanding of concepts: what may be conceptual for a Grade 3 learner (for example, basic addition of single-digit numbers) is procedural for a Grade

9 learner, who will have progressed to operations at a higher level. The two criteria are thus closely aligned and yet can be differentiated. This is not different from what most recent researchers such as Moru et al. (2014) reported: that in error analysis "Procedural errors are associated with procedural knowledge and conceptual errors are associated with conceptual knowledge" (p. 2).

In the process of assessment, learners solve problems but do not explain their thinking process; however, it is in the process of teaching that teachers could potentially get a deeper understanding of the learners' procedural and conceptual errors, by asking them to explain their reasoning. Several authors argue for the learners to explain their written work through verbal communication (Maharaj, 2014; Ndlovu, 2015, 2016); however, considering the constraints of the timeframe of teachers faced with trying to cover the curriculum, this might not always be possible during the process of teaching and learning within the classroom. It is therefore important that beyond engaging with learners' errors during teaching, teachers also pay particular attention to learners' errors when assessing. Therefore, for the purposes of this study, teachers' engagement with procedural and conceptual knowledge of learners would be explored in the way they engage with learners' written and verbal responses. Their CCK would also be explored through their engagement with learners' written work and their extent of engagement with learners' thinking after identifying their errors in the written work.

3.5.2 Specialised Content Knowledge (SCK)

Shulman (1987) proposed considering the relationship between two knowledge bases as the intersection of content and pedagogy, and introduced the notion of PCK. According to Shulman (1987), it is the special combination of content and pedagogy that teachers need to carry out their professional duties. Elaborating on what Shulman said, Ball et al. (2008) repositioned and defined SCK as the mathematical knowledge and skill that is exclusive to the teaching of mathematics. They further argued that teachers must be able to do a kind of mathematical work that learners cannot do, which involves sizing up the error, and interpreting and explaining the error. Drawing from Ball et al.'s (2008) arguments on SCK, the main characteristic for teachers' SCK is their ability to look for patterns in learners' errors, "sizing up whether a nonstandard approach would work in general" (p. 400).

Whereas teachers' knowledge of what counts as the explanation of the correct answer enables them to spot the error, looking for patterns in learners' errors enables them to interpret learners' solutions and evaluate their plausibility. This kind of knowledge not only enables the teacher to know the mathematics which they teach, it enables them to have the special kind of knowledge required to size up the source of a mathematical error and identify what mathematical steps would produce an error. It involves the ability to read beyond the procedure carried out, to the reasoning behind the procedure used. Understanding different interpretations of mathematical problems and solutions and helping learners to make sense of their work and that of others is crucial. It involves making features of content visible to and learnable by learners and explaining how mathematical language is used. Teachers must choose, make and use mathematical representations that are effective and help students to explain and justify their mathematical ideas. For the purposes of this study, the researcher maps the SCK with teachers' awareness of error as a strand in order to understand the extent to which mathematics teachers engage with learners' errors when they are aware of the errors made by learners.

3.5.2.1 Teachers' awareness of errors

As emphasised in the literature, errors are part of the learning process (Olivier, 1989; Siyepu, 2013; Sapire et al., 2016), so it is imperative that teachers not only understand that but actually practise it in their classrooms to incorporate learners in the teaching and learning process. Engaging with learners' errors would not only provide learners with the opportunity to engage with what they write, as purported by Maharaj (2014), it would help learners to develop what Van de Walle (2007) termed as "maths talk" – thus promoting communication in mathematics classrooms. Errors are part of learning and in every learning environment learners will make errors. Sapire et al. (2014) maintain that teachers' awareness of errors focuses on teachers' explanations of the actual mathematical error and not on learners' reasoning. Combining this with SCK, teachers would not only explain the actual errors but also the reasoning that potentially caused the error. In that way, teachers would not only focus on addressing the actual error, but also on the learners' reasoning that resulted in the error. This means that the teacher understands the error made by the learner and he/she explains the mathematical errors and provides strategies to prevent such errors from happening again in the future.

This was supported by Tulis et al., (2016 p.13) that “Mistakes are the steppingstones for learning” or “You can always learn from your mistakes”. Based on empirical findings, the consistent key argument is that errors initiate explanation and reflection processes in which deficient concepts are contrasted with correct concepts in order to establish accurate mental models (Tulis et al., 2016). This study draws its argument from the work of Sapire et al. (2014) and argues that teacher awareness of learner’s errors requires teachers use a balance of both CCK and SCK. Teachers who lack this knowledge find it difficult to be aware of the errors that learners make. For the purposes of this study teachers’ explanations and awareness of errors would be drawn from both the process of teaching and of assessing, exploring their engagement with learners’ errors in the assessment tasks and during the learning process, allowing learners to engage with what they wrote and their reasoning.

3.5.3 Knowledge of Content and Students (KCS)

Ball et al. (2008) defined KCS as the knowledge that combines knowing about learners and knowing about mathematics. They further maintained that it involves anticipating what learners are likely to think and what they will find confusing, as well as possible difficulties they may experience. This domain is crucial in the teaching of mathematics. However, over the years most writers who have written on teachers’ mathematical knowledge for teaching have ignored or spoken very little about this domain.

This study agrees with (Ball et al., 2008) and emphasises that the KCS requires teachers to anticipate the errors that learners are always likely to make in almost every topic. The teacher should be ready to engage with those errors as and when they become apparent in the teaching process. This means that during lesson preparation mathematics teachers must cater for such errors which learners are liable to make and address those errors timeously. Ball and colleagues (2008) further maintained that KCS involves recognising errors and identifying those which learners are likely to make, interpreting learner responses and developing learner justifications. Teachers are the managers of the classroom and therefore what they teach must be something they know, and they must predict errors and know how to solve those errors for learners who are learning concepts for the first time. Under this domain, the researcher in this study used the following strands – diagnostic reasoning, everyday knowledge and multiple explanations of errors – as error analysis tools, as derived from the work of Sapire et al. (2014).

3.5.3.1 Diagnostic reasoning

According to Sapire et al. (2014) the idea of teachers' explanation of errors goes beyond identifying the actual mathematical error ('awareness of error'). Therefore, these authors explained diagnostic reasoning as how teachers go beyond the mathematical error and explain the way learners were reasoning when they produced the error. The emphasis here is on the quality of the teachers' attempt to provide a rationale for how learners were reasoning mathematically when they made the errors. In South Africa, teachers are seen as carrying out diagnostic analysis where they go question by question after formal assessment and analyse the difficulties learners encountered in trying to answer them.

3.5.3.2 Everyday knowledge

When reporting on everyday knowledge, Sapire et al. (2014) reported that the emphasis in this strand is on the quality of the use of everyday knowledge in the explanation of the error, judged by the links made to the mathematical understanding that the teachers attempt to advance.

3.5.3.3 Multiple explanations of errors

With this criterion Sapire et al. (2014) mentioned that one of the challenges in the teaching of mathematics is that learners might need to hear more than one explanation of the error. This is because some explanations are more accurate or more accessible than others, and errors may need to be explained in different ways for different learners. This criterion examines the teachers' ability to offer alternative explanations of the error when they are engaging with learners' errors, which is aligned with Shulman's (1986) framework as an aspect of PCK related to "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9) in the context of error explanations.

3.6 Conclusion

This chapter presented the conceptual framework used for the study. It reported how two theories were joined together to create the conceptual framework used in this study. The two theories which were put together were error analysis (Sapire et al., 2014) and teachers' mathematical knowledge (Ball et al., 2008). In (Ball et al., 2008) three mathematical knowledge types – CCK, SCK and KCS – were linked with the six strands or criteria of error analysis of (Sapire et al., 2014). The next chapter presents the research methodology and design that were used in this study.

CHAPTER FOUR

RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

The previous chapter discussed the theoretical framework used in this study. This current chapter presents in detail the research methodology used in this study to achieve the research objectives. The researcher explains the paradigm underpinning this study, which is the interpretive paradigm, and its suitability for this study. The researcher further explains the research design, style, sample, and sampling techniques. In addition, the methods used to collect data and their suitability are discussed. When data have been collected they need to be analysed to understand the phenomena being studied. After the discussion of data collection methods, the researcher explains the data analysis procedure used in this study. Lastly, issues of trustworthiness considered for this study are discussed, followed by ethical issues and limitations.

4.2 Research approach

According to Creswell and Creswell (2017) a research approach is a plan and the procedures set out for research that span the steps from broad assumptions to detailed methods of data collection, analysis, and interpretation. Furthermore, these authors contend that this plan involves several decisions involving the choice of approach to be used to understand the phenomena of the study. They further argue that informing this decision should be the philosophical assumptions the researcher brings to the study, procedures of inquiry which they called research design, and specific research methods of data collection, analysis, and interpretation.

As mentioned by Creswell and Creswell (2017), the selection of research approach is also based on the nature of the research problem or issue being addressed, the researchers' personal experiences, and the audiences for the study. In this study the selection of the research approach was driven by the phenomena being studied, that aim to understand Grade 9 mathematics teachers' engagement with learners' errors. The aim is to gather evidence from teachers themselves and to observe their classroom practices to understand their engagement with learners' errors. Based on this, the researcher purposively chose mathematics teachers who teach Grade 9 mathematics as participants. There were other factors that informed the selection of the qualitative approach, which will be explained further below.

4.3 Methodological design

According to Kumar (2019), research methodology is a strategy, plan of action, process, or design that links the research paradigm to a research method. Creswell and Poth (2017) claim that research design and research methods are key aspects that represent a perspective of every research study. Creswell and Poth (2017) further elucidate that research design is a plan and procedure for research that takes the researcher from extensive assumptions to specified methods of data collection and analysis. In support of the above authors, the researcher holds the view that research design is a plan used to answer research questions and a research method is a strategy used by researchers to implement the plan in other to answer the research question.

This study explores the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. Therefore, the selection of research design and research methodology was based on the nature of the research problem or issue being addressed, the researcher's personal experiences, and the audiences for the study. This is a qualitative study framed in the interpretivist paradigm, as reported above. Therefore the researcher reached out to the settings of the participants to explore the extent to which these participants engage with learners' errors in the teaching and assessment of mathematics. The purpose of this study was to gain a deeper understanding of the extent to which mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. The researcher worked under the assumption that learners cannot avoid errors in mathematics, and teachers must be ready with the necessary knowledge and techniques to engage with such errors properly on a daily basis. In order to understand the phenomena being studied, the research set out to answer the following research questions:

- How do Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics?
- To what extent (if any) do Grade 9 teachers engage with learners' errors in the process of teaching and assessment in mathematics?
- Why do teachers in Grade 9 engage with learners' errors or not when teaching or assessing mathematics?

4.3.1 Research paradigm

The term paradigm as used in educational research is associated with the term 'worldview' (Scotland, 2012;Mackenzie & Knipe, 2006). Leacock (2020) explain that worldview is the perspective, or thinking, or school of thought, or set of shared beliefs, that informs the meaning or interpretation of research data. Leacock (2010) further explain that the paradigm is associated with the researcher's worldview; this means that a paradigm constitutes the abstract beliefs and principles that shape how a researcher sees the world, and how s/he interprets and acts within that world. The paradigm defines the researcher's philosophical orientation, which will influence the researcher's beliefs of constructing meaning from the data they have gathered, based on the experiences of the participants. Creswell (2014) defined the term worldview (paradigm) as the set of beliefs that guide action, which includes general orientation about the world and the nature of research held by the researcher. He classified worldviews (paradigms) into four taxonomies: interpretivist, positivist, critical and post-positivist. Drawing from the above, the researcher notes that paradigms are important in a research study because they provide the direction of beliefs and thoughts that guide as to how data are generated for a study and how they are interpreted to arrive at results. This study is framed in the interpretivist paradigm.

Leacock (2020) argue that the qualitative interpretivist paradigm assumes subjectivist epistemology, a relativist ontology, a naturalistic methodology, and balanced axiology. Subjectivist epistemology means that the researcher makes meaning of data through his/her own thinking and cognitive processing of the data, which is informed by his/her interactions with the participants. Relativist ontology means that the researcher believes that the situation studied has multiple realities, that can be explored through human interaction between the researcher and the participants. Leacock (2020) further explained that assuming a naturalist methodology means that the researcher makes use of data generated through an interview, observation and text, among others, where the researcher acts as a participant-observer. This means that the researcher does not influence the results of the study. Finally, they maintained that a balance of axiology assumes that the outcome of the research study will reflect the values of the researcher. These values ensure that the interpretivist researcher presents a balanced report of the findings. This is what this study seeks to do in order to ensure true validity (trustworthiness).

4.3.1.1 Paradigm underpinning this study

This study is underpinned by the interpretivist paradigm, since qualitative research is highly associated with interpretivism (Alharasheh,& Pius, 2020; Ritchie et al., 2013). Cohen et al. (2017) explain that the interpretive paradigm is a world view used to understand the subjectivity of human world experience. The authors noted that interpretive researchers begin with individuals and set out to understand their interpretation of the world around them. The authors further note that interpretive researchers aim to examine situations through the eyes of the participants rather than those of the researcher. Cohen et al. (2017) further argued that interpretive inquiry interprets and discovers the perspectives of the participants in the study, and answers to the inquiry are practically dependent on the context. This study aims to gain an in-depth understanding of the extent to which mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. This study supports what the above authors have said and believed regarding the goal of interpretivist researchers being to rely as much as possible on the participants' views of the situation being studied.

Some writers on interpretivism, such as Nind and Todd (2011), Creswell and Creswell (2017), and Creswell and Poth (2016) also argue that the interpretive paradigm allows researchers to view the world through the perceptions and experiences of the participants. Furthermore, O'Donoghue (2018) maintains that researchers in the interpretivist paradigm use their own knowledge as social beings to try to understand how others understand the world. He further argues that "knowledge is constructed by mutual negotiation and in specific to the situation being investigated" (p. 9). In contrast, Thanh (2015) argued that interpretive researchers do not seek the answers for their studies in rigid ways, but instead approach reality in a more subjective manner, typically from the point of view of people's own experiences and understanding of reality. Ormston et al. (2014) also pointed out that the interpretive paradigm is framed to reflect on the practices which emphasise the importance of understanding people's perspectives in the context of the conditions and circumstances of their lives. In this study, the researcher visited the site of each of the five participating schools. The aim was to capture the reality and engage with participants in their natural settings, to get rich feelings about the extent to which they engage with learners' errors in the teaching and assessment of mathematics.

The researcher used the interpretive paradigm in this study because he wanted to understand or deeply explore the extent to which mathematics teachers engage with learners' errors when teaching and in assessment. The aim is not to find the answers in a rigid way, but to explore through teachers' perspectives and practices the extent of their engagement with learners' errors in the teaching of mathematics. Rather than informing teachers how they should engage with learners' errors, this study set out to understand this phenomenon from the teachers' perspective, as it was argued by Cohen et al. (2017) that interpretivists aim to understand the phenomenon through the eyes of participants.

4.3.2 Qualitative research methodology

This study is built on the foundation of a qualitative research approach. According to Ritchie et al. (2013), qualitative research is largely associated with interpretivism. Alharasheh and Pius (2020) posit that qualitative research provides multiple ways of carrying out a study. In line with Alharasheh and Pius (2020) , this study uses multiple ways to collect data in order to get an in- depth understanding of the extent to which teachers engage with learners' errors when teaching and assessing. Since qualitative research is premised on the fact that there are multiple truths and people make meaning of the realities based on their settings, the researcher deemed it necessary to collect data in the participants' own setting where they engage with learners. Therefore, data for this study were collected from participants while they were in their own schools, in their own mathematics classrooms and observed while teaching, thus allowing the multiple realities to emerge.

Denzin and Lincoln (2011) propose that despite the inherent diversity within qualitative research, it can be described as "a set of interpretive, material practices that make the world visible. These practices transform the world. And turn the world into a series of representations, including field notes, interviews, conversations, observation, photographs, and recordings among others" (Denzin & Lincoln, 2011 p.3). The authors maintained that qualitative researchers study things in their natural settings, attempting to make sense of or interpret phenomena in terms of the meanings people bring to them. They further argued that qualitative research is often associated with specific kinds of data, usually involving words or images rather than numbers as in the quantitative study. In this study the researcher mainly used words to present data, to explain issues better for the benefit of readers.

Silverman (2011) asserted that the richness of qualitative data is often highlighted, as a result of distinctive approaches that qualitative researchers bring to the analysis and interpretation of data, and the kinds of output that derive from qualitative research. In this context, qualitative research is often distinguished by the fact that hypotheses are commonly generated from analysis of the data rather than stated at the beginning. Similarly, Check and Schutt (2011) added that qualitative researchers seek to capture the setting or people who produce text on their own terms, rather than those predefined by researchers' measures and hypotheses. What this means is that qualitative research tends to be more inductive, because the researcher identifies important categories in the data as well as patterns and relationships through the process of discovery (Check & Schutt, 2012). In this study some of the data were analysed using an inductive approach, since themes were generated from the study framework.

It is important to mention that qualitative research as purported by these authors (Check & Schutt, 2011; Creswell & Creswell, 2017; Silverman, 2011; Ritchie et al., 2013) focused mainly on key features of research design that may identify a study as 'qualitative', including a concern with 'what' 'why' and 'how' questions rather than 'how many', a focus on processes, and the flexible nature of qualitative research design. In this study the first research question looked at to 'what' extent Grade 9 mathematics teachers engage with learners' errors, while question two answers 'how' teachers engage with learners' errors, and finally question three was used to answer 'why' teachers engage with learners' errors in the teaching and assessment of mathematics. The justification for conducting this study using the qualitative approach is given in broader terms below.

Check and Schutt (2011) pointed out that qualitative study is associated with the interpretivist paradigm. Because this study falls under the interpretivist paradigm, a qualitative research method was appropriate. Ritchie et al. (2013) also agree with the fact that qualitative research is largely associated with interpretivism. According to Creswell and Creswell (2017), qualitative research is an approach for exploring and understanding the meaning individuals or groups assign to a social or human problem. On this basis, this study aimed to explore how mathematics teachers from five selected schools engage with learners' errors in the teaching and assessment of mathematics. Therefore, this study seeks to get an in-depth understanding of the following research problem, 'The extent to which mathematics teachers engage with learners' errors in the teaching and assessment of mathematics', from the perspective of the participants. Seven participants were selected from five high schools in the Ixopo education circuit in the Harry Gwala District.

Furthermore, Ndlovu (2016) asserts that qualitative studies use a variety of methods, such as interviews, observations, documents, etc., to gather data. In this study three different methods were used to gather or generate data: interview, observation, and document analysis. Moreover, in qualitative research data analysis can be both deductive and inductive, where facts are generated from themes gained from the participants and the researcher makes interpretations of the meaning of these data (Ormston et al., 2014). Since this study explores teachers' engagement with learners' errors, both inductive and deductive analysis were used.

4.4 Case study as a research style

A case study is a style or technique used in qualitative studies (Merriam, 2014). Yin (2017) defined a case study as an empirical method that explores a contemporary phenomenon more in-depth within its real-world context. Yin and Davis (2007) report that a case study assists the researcher to understand the real world, and assumes that such an understanding is likely to involve important contextual conditions relevant to the case. In this study, the researcher adopts a case study because he wants to explore how and why Grade 9 mathematics teachers engage with learners' errors.

According to Thomas (2021) case study helps the researcher to answer 'how' and 'why' type questions, while bearing in mind how a phenomenon is influenced by the context within which it is situated. The authors further argue that the qualitative case study provides tools for researchers to study complex phenomena within their contexts. Rule and John (2015, p. 4) also defined the case study as ''an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident". They further claim that a case study is ethnographic in nature because it focuses on understanding the theories and practices of the participants through their own experiences. It is within these parameters that the case study was deemed appropriate for this study, since it focuses on understanding teachers' knowledge of learners' errors and their practices when engaging with learners' errors.

Going back to Yin (2012), he categorises case studies as explanatory, exploratory, or descriptive. Yin explains the explanatory case study as answering a question that seeks to explain the presumed causal links in real-life interventions that are too complex for survey or experimental strategies. He also mentions that the exploratory case study is used to explore those situations in which the intervention being evaluated has no clear single set of outcomes. Finally, he explains the descriptive case study as used to describe an intervention or phenomenon and the real-life context in which it occurred. The researcher study used exploratory case study as explained by Yin (2012), since the study is about exploring the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics.

4.5 Site of the study

This study was conducted in the Harry Gwala District, one of the 11 districts located in the south-western part of KwaZulu-Natal Province in South Africa. Harry Gwala District Municipality (previously Sisonke District Municipality) comprises four local municipalities: Dr Nkosazana Dlamini Zuma, uMzimkhulu, Greater Kokstad, and Ubuhlebezwe (Ixopo). The educational head is situated in Kokstad. The study was carried out in three selected high schools around Ixopo. Ixopo is predominantly a farming area and is the political seat of the district, which is situated in the heart of Ubuhlebezwe Municipality. The map below demarcates where the study took place (Figure 4.1).

Figure 4.1 Map showing Harry Gwala District (source: http://www.harrygwaladm.gov.za)

The researcher used pseudonyms to represent the names of the three schools that were used for the study as follows: Okyere High School, Mills High School, and Royal High School,

4.5.1 Okyere High School

This is a fee-paying school with allowance to apply for government lenience for those parents who cannot afford school fees. Based on the classification of schools as stipulated by the DBE, this school is classified as quintile four. The school has a population of over 1000 high school learners, with more than 50 teaching and non-teaching staff. The school has achieved academic excellence over the years and the mathematics pass rate has been exceptional in recent years and currently.

4.5.2 Mills Combined School

Mills is classified as a quintile three school, which suggests that it has basic resources and relevant infrastructure to sustain itself, but also receives a subsidy from the DBE. The school has over 600 students together with about 37 educators. Since it has infrastructure, an average of 30 to 39 learners in a class is maintained to ensure effective teaching and learning. It is a combined school, starting from Grade R and going up to Grade 12. Grade R to Grade 7 have separate buildings to Grades 8 to 12, and the school has one principal with two deputy principals, one of whom is stationed in the Grade R to Grade 7 building.

4.5.3 Royal High School

Royal High School is considered to be a quintile 2 school. It has a population of about 140 learners with 11 teachers and one security guard. The school is in a rural area where the people who represent parents of the students are predominantly farmworkers. The school is faced with many challenges, such as absenteeism, coming to school late, poor turnout of parents at meetings, among others. Learners in this school are given free stationery, including calculators, and are given a meal each day.

4.6 Sampling

According to Check and Schutt (2012), there are two distinct forms of sampling: probability sampling and non-probability sampling. They further argued that probability sampling is mostly used in quantitative research, since it deals with large-scale numbers, while non-probability sampling is associated with both qualitative and quantitative (in cases where researchers cannot use probability sampling in their study) research. The sampling type that was used in this study was non-probability sampling. As Check and Schutt (2012, p. 102) reported, there are "four common non-probability sampling methods as available sampling, quota sampling, purposive sampling and snowball sampling". This study employed purposive sampling.

4.6.1 Purposive sampling

According to Check and Schutt (2012), purposive sampling targets participants who are largely knowledgeable about the issues under study or investigation. It is for this reason that the researcher chose Grade 9 teachers who have been teaching Grade 9 for more than one year, because it is believed that they would be able to provide rich data since they have experience of teaching mathematics. For example, teachers have been analysing the senior phase mathematics learners scripts to identify errors in their schools for the previous quarters /terms.

According to Cohen et al. (2011) purposive sampling is a key feature of qualitative research. The authors further argue that in many cases purposive sampling is used in order to access 'knowledgeable people' – people who have in-depth knowledge about particular issues, perhaps by virtue of their professional role, power, access to networks, expertise or experience (Cohen et al., 2011).

This study used the purposive nature of the sampling to select knowledgeable people who are teachers for the study; teachers are deemed to be knowledgeable and well informed about the phenomenon under study. Bertram and Christiansen (2014) also mentioned that purposive sampling is where the researcher makes specific choices about which people or groups of people are to be part of the sample. Three Grade 9 mathematics teachers – one from each of the three high schools selected – were involved in the study. Therefore, the participants for this study were three mathematics teachers from three schools.

In purposive sampling, the purpose of the study really influences how the researcher selects the participants. In this study Grade 9 mathematics teachers were chosen as participants, hence the purposive sampling because the purpose/objective of the study was:

1. To determine the extent of Grade 9 mathematics teachers' engagement with learners' errors during teaching and after assessment of mathematics.

2. To find out the approach/methodology used by Grade 9 mathematics teachers to engage with learners' errors: and
3. To explore why Grade 9 mathematics teachers, engage/ not engage with learners' errors.

With the above objectives, the researcher aimed to gain an in-depth understanding of how teachers engage with learners' errors; therefore, the researcher purposefully generated data from three Grade 9 mathematics teachers, as it was believed that they could provide rich information about the phenomenon being studied since on termly basis these teachers engage with learners script to identify errors and provide remedial actions to mitigate those errors.

4.6.2 Brief description of research participants

Three Grade 9 mathematics teachers were initially considered for this study. These teachers voluntarily consented and agreed to participate. All the three teachers had a degree to teach in high schools (Bachelor of education: FET phase). The criterion for selecting these participants was purposive. The study was about exploring the extent to which mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. For ethical reasons the identity of the participants was highly protected. As a result, the researcher used pseudonyms to represent the final participants in the study as follows: Kwaku, Anele and Zafira.

4.6.2.1 Kwaku

Kwaku is a male teacher at Okyere High school and one of the senior teachers at the school. He has Bachelor of education (B.Ed.) degree to teach FET phase with 15 years of teaching experience. He teaches Grade 8 and 9 mathematics and mathematical literacy in Grade 10. He is a passionate teacher who aspires to be one of the best teachers in the school, by helping learners – particularly the struggling learners – to love mathematics and change their thinking about mathematics as a difficult subject from the early grades like Grades 8 and 9.

4.6.2.2 Anele

Anele is another male teacher, hailing from Mills High School who has six years of teaching experience in the field of mathematics and science. He currently teaches mathematics from Grades 8 to 12 in the school. Anele holds Bachelor of education degree to teach FET

4.6.2.3 Zafira

Zafira is an experienced female teacher who has been teaching mathematics and Life Science for 13 years at Mills High School in Ixopo. She is currently the mathematics teacher for Grades 8 and 9 in the school. She teaches other science subjects from Grade 10 to 12. She plays a very important role in the school as the head of department (HOD) for mathematics and science. She has degree in education (B.Ed.) to teach FET phase.

4.7 Data collection procedure

Ritchie et al. (2013) posit that specific data generation methods such as observation, semi-structured interviews, document analysis, and focus group interviews are all methods which are suitable for qualitative research. However, in this study the researcher used observation and semi-structured interview to generate data. Table 4.1 indicates the data collection procedures used in this study to answer the research questions.

Table 4.1: Instruments and methods of data generation used to answer the research questions

Methods of generating data	Instruments used	Research question	Participants
Observation	Video recorder Field notes	How do Grade 9 teachers engage with learners' errors in teaching and assessment of mathematics?	Teachers
Semi-structured interviews and observation	Audio recorder Field notes	Why do teachers in Grade 9 engage or not engage with learners' errors when teaching or assessing mathematics?	Teachers
Semi-structured interviews	Learners' exercise books Audio recorder Field notes	To what extent do Grade 9 teachers engage with learners' errors in their teaching of mathematics?	Teachers

4.7.1 Interviews

The researcher holds the view that in a qualitative study an interview is a conversation between two persons, where one (the researcher or interviewer) develops an interview schedule to better understand the other (participant or interviewee) in a conversation using semi-structured questions. Bertram and Christiansen (2014, p. 80) hold a similar view that "An interview is a structured and focused conversation where the researcher has in mind particular information that he or she wants from the respondent and has designed particular questions to be answered". In this study the researcher used semi-structured interviews to explore teachers' understanding of the extent to which they engage with learners' errors in the teaching of mathematics. Using semi-structured interviews allowed the researcher to probe further for clarity and for the participant to express themselves without being restricted. This is supported by Creswell and Creswell (2014), when they assert that in a qualitative interview the more open-ended the questioning the better. The researcher was able to probe further for clarity as he listened to the participants in order to get in-depth information about the research questions. According to Brinkmann (2014), in a semi-structured interview the researcher gives some guidance based on his/her research interests and the interview guide but works flexibly with the guide and allows room for the respondents to express themselves to their best of ability. See Appendix A for the interview question guide used in this study.

4.7.2 Observation

According to Cohen et al. (2011), observation offers a researcher the opportunity to gather live data from naturally occurring social situations. In other words, through observation the researcher can view directly rather than relying on second-hand accounts; this is valuable because what people actually do may differ from what they say they do (Robson, 2002, as cited in Cohen et al., 2011). Therefore, observation was employed to enable the researcher to gain first-hand information on how Grade 9 teachers engage with learners' errors in the teaching and assessment of mathematics, and to provide a reality check in terms of what participants said during the interview. The researcher developed the observation instrument used it as one of the methods of generating data for this study and observed the participants one by one in their respective schools as they were teaching mathematics. The instrument which the researcher used to collect the data was a video recorder, because the researcher wanted to observe the participant in their contextual set up. This is supported by Griffin (2019) when he posits that video-recorded data provide the researcher with more contextual data.

The benefit of using a video recorder is that video recording allows the researcher to repeatedly view and play back the data and focus attention on things that have not been spoken to during data analysis. According to Griffin (2019), video recording allows the researcher to focus attention on things that were not seen at the time of taping or in previous viewings. The researcher went to the various schools to observe and record the teachers as they teach in their classrooms, and did not intervene in the teaching process during the observation. The participants gave their consent that a video recorder be used to record the lesson. The use of observation helped the researcher to answer the research question 'How do Grade 9 teachers engage with learners' errors in teaching and assessment of mathematics?'. See Appendix A for the observation data guide used in this study.

4.8 Data analysis procedures

According to Braun and Clarke. (2019), thematic analysis is a method for capturing patterns or 'themes' across qualitative datasets. Braun and Clarke. (2019, p. 57) further defined thematic analysis "as a method for systematically identifying, organizing and offing insight into patterns of meaning (themes) across data set". The authors mention that the purpose of data analysis is to identify themes across data sets in order to answer a research question. In a similar argument Javadi and Zarea (2016) also mentioned that thematic analysis is an approach for gaining meanings and concepts from data through identifying, scrutinising, and recording patterns or themes from a data set.

In this study, data were analysed as follows. Data generated from interviews and lesson observations were first transcribed. Data analysis was both inductive and deductive in nature. The researcher employed thematic codes through the inductive approach for the interview data. The transcript for each participant was first read, organised, and coded. The interview data were then arranged in a logical order and put into meaningful categories using Merriam's (2009) description of "category construction" (p. 178). Themes were identified by sorting and re-categorising the initial general categories into more in-depth ones. For coherence, responses from participants were cleaned. Data from classroom observations were deductively analysed using the themes generated from the framework, as discussed in Chapters Six and Seven.

4.8.1 Trustworthiness

According to Creswell and Poth (2017) trustworthiness refers to the accuracy and truthfulness of the findings in a qualitative study. In this study the researcher used multiple data sources to triangulate the data. The different data sources used were semi-structured interviews, observations, and document analysis. Cohen et al. (2011) mention that triangulation in qualitative research is a power tool which ensures that the findings of a study are truthful. The fact that different methods were used to generate data helped to ensure that the findings of this study are truthful and powerful.

4.8.2 Credibility

Credibility refers to the truth of the data or the participants' views and the interpretation and representation of them by the researcher (Polit & Beck, 2012). Bertram and Christiansen (2014) explain credibility as the extent to which the data generated in a study reflect participants' reality. Therefore, to ensure credibility in this study, the researcher applied the principle of triangulation which, according to Bertram and Christiansen (2014), is the use of different methods to collect or generate data from the participants in a study where the data reflect on participants' experiences or reality. In this study three methods of data collection were used, that is observation, interview, and document analysis. These methods helped to generate data to answer the research questions. Data were generated in the same settings but at different times. This study used interviews, observation, document analysis, and audio recording devices to generate data. The interviews were transcribed, and analysed word for word. The transcript of the interview was deemed accurate since it was recorded, and transcription was further compared to field notes.

According to Merriam (2014) a qualitative study is considered credible if individuals that share the same experience immediately recognize the explanations of human experience. This means that the researcher and the participant must both be part of the research. The credibility of this study is granted because the researcher used unstructured interviews and voice recordings to generate to capture the voices of the participants. This means that the exact things said by the participant were recorded and transcribed accordingly. The views of the researcher did not influence the results of the finding of this study. In addition, video recorder was used to capture the realities of the classroom during classroom observations. This was done to supplement the field notes.

4.8.3 Confirmability

This is another way of ensuring trustworthiness in qualitative research on the issue of confirmability. Confirmability means the researcher's ability to demonstrate that the data represent the participants' responses and not the researcher's biases or viewpoints (Polit & Beck, 2012; Tobin & Begley, 2004). Bertram and Christiansen (2014) also added that researchers can improve confirmability by making the research process transparent for readers to check-in if they would have reached the same conclusion. The researcher of this study holds the view that the position of the researcher was subjective and not objective. Therefore, the researcher was not biased in this study and remained neutral. Therefore, it is most appropriate that video recorder was used for recordings and transcribed them thereafter word for word from the participant. Providing verbatim responses of the transcripts ensure confirmability as these verbatim responses are exact words of the participants not researchers' interpretation of what participants said or did in the classroom.

4.8.4 Transferability

Researchers of a qualitative study (Tobin & Begley, 2004; Connelly, 2016;Houghton et al., 2013; Polit & Beck, 2012) have argued that transferability refers to findings that can be applied to other settings or groups. The researcher cannot know the sites that may wish to transfer the findings; however, the researcher is responsible for providing thick descriptions, so that those who seek to transfer the findings to their own site can judge transferability (Connelly, 2016). The researcher in this study endeavour to provide the realities of the participants with the hope that other researchers would find the findings useful in their own settings.

4.9 Ethical issues

Ethical issues in research are the pivots that revolve around qualitative research which involves human beings. According to Bertram and Christiansen (2014), Ethics are vital elements in research especially research that involves living things such as human beings and animals. This study agrees with Bertram and Christiansen and the following protocol was followed: First, the researcher sent gatekeeper's letter (see appendix B) to the principals of the schools to seek their blessings and assurance of safety for the study to be conducted in their school. As part of the ethical process, as protocol demands, the principals after accepting and signing the gatekeepers letter introduced the researcher to the teachers' concern.

The researcher then explained the topic and purpose of the study to the teachers and requested them voluntarily to be participants of the study. The teachers agreed to be participants and then they signed the consent form (see appendix C) to be part of the study voluntarily. The gatekeeper's letters from the principals and the participants' consent letters were attached to the proposal of this study and sent to the Humanities and Social Science Research Ethical Committee (HSSREC) for ethical approval. The HSSREC issued an ethical clearance certificated with protocol reference number: HSS/1603/018M which stated clearly the topic and where the study is to be conducted. See Appendix E.

Again, one of the ethical principles that were observed in this study was that of protecting the identity of the research site and participants through the use of pseudonyms. In addition, when describing the site and participants any inferences that could lead to the site being identified were carefully scrutinised. Since learners were not participants of the study the researcher that the video recorder is directed towards the teacher at all times thus to avoid capturing learners. In cases where this could not be avoided, learners' images and voices were immediately deleted.

4.10 Strengths and limitations of the study

One of the strengths in qualitative approaches, according to Creswell and Creswell (2017), is that they generate rich and detailed data from the participants' perspective. This study used different data collection methods (triangulation) to ascertain a better understanding of "how mathematics teachers engage with learners' errors in the teaching and assessment of mathematics". None of the participants were coerced to take part in the study, it was all on voluntary basis Although, all the eight participants agreed to take part in the study, only seven of the participants ended up being available for the interviews and only six were observed.

While some participants withdrew however, data generated from the six participants was considered in-depth enough and provided rich data to understand the phenomena being studied.

One of the challenges that the researcher faced was that at Okere High School, the participants were ready to be observed but the School head of department (HOD) for Mathematics and science denied the researcher to observe the participant. The failure to observe the teacher meant that the realities of the classroom practice of the teacher at Okere High School could not be reported on, only the perspective of the teacher help the researcher to construct meaning regarding the extent the teacher engages with learners' errors. The issue of being an outsider seem to be a cause for concern at Okere High School and a limitation of this study.

4.11 Conclusion

This chapter has presented the research methodology used in this study. The methods that were used to answer the research questions. The research design, population, sample and sampling procedure, data collection procedures, data analysis, trustworthiness issues, ethical issues and the limitations of my study. The next chapter will be data presentation and data analysis.

CHAPTER FIVE

DATA PRESENTATION AND DATA ANALYSIS

5.1 Introduction

This research study explores the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. In this chapter the researcher presents and analyses the data generated by the study. This chapter is presented in two sections. The first section presents data from the semi- structured interview and how they were inductively analysed, and is further split into two sub-sections, the first presenting data from the participants as transcribed in section 5.2, and the second describing how the interview was coded. The coding took place in three phases to arrive at the final themes which emerged from the analysis, as findings which seek to answer the research questions that aim to explore the extent to which Grade 9 mathematics teachers engage with learners' errors. In the second section of this chapter the researcher presents data from the lesson observation and how these were deductively analysed.

5.2 Data from the semi-structured interviews

The researcher presented data from the semi-structure interview responses from the participants as transcribed from the interview using semi-structured questions. The interview data were edited and cleaned with the aim of answering the research questions on the 'extent' of and 'how' teachers engage with learners' errors. The code RE was used to represent the researcher during the semi-structured interview conversation and the pseudonyms Kwaku, Anele and Zafira were used to represent the participants. This section is divided into two. The first section, 5.2.1, focuses on the extent to which Grade 9 mathematics teachers engage with learners' errors, and the second section, 5.2.2 presents conversations that aim to provide answers regarding how teachers engage with learners' errors.

5.2.1 The extent to which teachers engage with learners' errors

In line with the design of the study, it was deemed imperative to understand the extent to which teachers engage with learners' errors from the participants' perspectives. Therefore semi-structured interviews were conducted, and the extracts below present extracts from conversations that took place between the researcher and the participants.

RE: The Department of Basic Education has suggested to teachers to do remedial teaching. What is your understanding of remedial teaching?

> **Anele:** The concept of remedial teaching, from where I am sitting, I will look at the two words and start from the word remedial there. When you trying to sort out a problem or going about a cure or solution to something that is bothering you or something that is a problem, sometimes we speak of it when it comes to sicknesses and diseases. But when I am bringing it to our context, remedial teaching, this is teaching that is aimed at assessing out learners to understand a concept or a topic and most importantly to ensure that the anticipated skills of that objective of that topic are achieved. So basically, I will say it is that teaching that come after you have taught and you have assessed because after that assessment you can see that my students still lack A, B, C and D. These skills or objectives have not been met yet and now you start remedial teaching, you try to correct the errors and misconceptions or anything that the learners may have portrayed after you have assessed them.

> **Zafira:** My understanding about remedial teaching refers to using different types of media when teaching, because that enhance learners' understanding in different forms. For example, let's say you are teaching about maps or measurements, so you will have a map portrayed on the board, then you will have to show that or things like that but different media. Things that learners can see because learners have to understand better when they have seen something rather than if they are just being taught without them seeing anything or without them observing anything. So I believe that remedial teaching is using different media in teaching.

> **Kwaku:** Remedial teaching makes us so useful. When it is time for answering the questions then they continue doing a particular mistake. For example, yesterday during my lesson I was teaching algebraic equations involving exponents and I picked up a mistake from a learner that was pertaining to the integers, where the learner interchanged negative integer for positive integer. I have to reteach integers even though it was not the lesson I was teaching. Therefore, I can say that remedial teaching is reteaching something you have taught that learners failed to understand. You know what, with remedial teaching, you are emphasizing what learners need to put in mind.

RE: How do you do the remedial teaching when a learner fails to understand something? Do you have a strategy for doing remedial teaching?

Kwaku: … I used to do correction on the black board and then I give a similar activity for learners to do individually so that I can identify if all learners end up having a clear picture or a clear understanding of what I was saying to them. If I notice that most of them are still not getting it, then I put them in groups and identify one learner in a group who understands better to assist the others.

Anele: Being a mathematics teacher and us having been doing assessment every now and again, for example, when you give a homework or a classwork as part of assessment, and then from there after you have marked that task you can see that oh my learners still lack A, B, C and D. The skills that I want to impart to them, or the objective of this lesson has not been thoroughly met, then from there now you start your remedial teaching. You try to correct whatever errors learners have that did not make them understand during that short task, test or activity that you gave in the class. The normal ways I do it is I write correction on the board or sometimes I call some of the learners to the board to present their answers, then we correct it together. Is just marking and providing corrections to that activity on the board and going through each question step by step and trying to clarify. So, I mark and provide corrections.

Zafira: I first do the sums in class on the board like I am doing corrections; I always open opportunities for extra time if learners do not understand in class. I always tell them that they are more than welcome to come in break, they can come to me and we can discuss further or I can even conduct extra classes if it calls for that … yah so though with the Grade 9 we do not really get enough time so we don't really give them much of attention compared to the other grades like the FET Grades 10, 11, 12 because, perhaps you can agree with me on this one, 1 hour is not enough as a lesson. You cannot teach everything but then we try by all means and ask them if they do not understand anything or if they have missed some points then they come to us, they come to me during their spare time or break, so that I can clarify it for them. Sometimes I do re-teaching of concepts learners failed to understand, particularly in Grade 9 if there is availability of time since the ATP demands teaching throughout.

RE: Do you provide feedback to learners after assessment?

Kwaku: Yes.

Anele: Yes, I do sir.

Zafira: Yes, I do it after every assessment, for learners to do their correction.

RE: How do you give feedback to learners for them to understand the errors they are making?

Kwaku: Precisely yes, it's a way to give feedback to learners because I am of the view that my learners need to know as to how they did and where are their strong points and their weak points and what to improve and what to do to ensure that they pass eventually.

I revise the paper and try to use the board. I use the board so that I can explain so everyone can have the chance of looking at what is happing and, we do it step by step or we do it bit by bit up until we end up finishing. And then after seeing that am done with a particular portion, I also give them another activity to check if there are those who are still struggling, so that I can try to assist them. And just because of the time, it can happen that sometimes I ask other learners to do peer teaching.

Anele: I do it in order to provide feedback to my learners on the assessment they do, for them to know their mistakes so that they do not make the same mistake repeatedly. Usually, as a mathematics teacher who gives informal tasks every now and again, usually it is the oral feedback where I just talk to the learner or the class as to what was expected of them, where they went wrong and how they can improve. But for moderation purposes if it is a formal task we call analysis of learners' errors to check as to how the learners answered the questions and to pick up some errors they have done in the task.

Zafira: For example, if we had a test, they write a test and then after the test they revise the test and then I get to tell them that as I was marking, I noted some of you did this error and that error and make sure that when you write in exam you don't repeat the same errors. So that is how I encourage them not to repeat the same errors they've done in the assessment when I am giving them feedback. I do engage with learners' errors immediately I notice that an individual learner is making mistakes in answering questions in class. I ask other learners who understand me better to explain to those struggling and to me; it works perfectly. With formal quarterly tests I take a sample of the errors and speak to them when I am giving feedback on their performance in general. We do diagnostic analysis but is for filing and moderation purposes. Another thing, I use a method that my learners do not really like, but I do it because I believe in it. It encourages them so I just call out their names with their marks in front of other learners. By so doing I think that will encourage them to do better, because if you compare the marks in ascending order from a person who got the least to the person who got the highest marks, then those people who got the least marks will not want to get the least marks in the following test, they will want to do better. So although they don't like it, I do it anyway.

RE [probing]: What do you do if a particular learner or group of learners still repeat mistakes after the feedback? Do you interrogate their thinking process or what?

Kwaku: Yes, that particular issue depends on the ability of the learner. Sometimes you can ask the learner a question and the learner might give you a wrong answer because the learner is not focusing on what you are busy with. Busy processing his or her own thing on the side and then you end up asking the question and you try discipline him or her, and then if the learner gives you a wrong answer then that you say please stop what you are doing, do you see the consequence of not listening attentively in class, and participating actively in class? If he might change from there … This thing happens almost every day in my class. I also try every day to discipline the learner in different ways, but I didn't touch them. Fortunately, when I discipline learners, I see a huge difference in class, they participate actively with other learners and do the right thing, asking some questions in class. You are also supposed to give more activity pertaining to that particular thing which seems to be a problem to our learners.

Anele: Again, to be honest, with our classes of 48 learners and looking at the time frame, if a learner has given me a correct answer I do not interrogate their thinking. Really, I do not. I must be honest. If it is correct, I just tick it as correct. However, if I come across a learner who gives me incorrect answer, for starters, to ensure that I do not harm or affect their confidence, I do not generally say it is wrong. What I do, however, I ask how they may have arrived at that response. Yes, I am not saying your answer is wrong, but explain to us in class how you got that answer. And as the learner is explaining to me how they arrived, I try to pick up some misconceptions or some errors that may actually lead this learner to have incorrect answer. I am of the view that errors are results of underpinning misconceptions. If a learner continually does the same error over and over again, it means that there is underlying misconception that I must first address to ensure that I help that learner. So, to answer that question, I just address such misconceptions that are underpinning those errors in that class. And the next strategy which is working in my class is peer teaching, allowing another child who seems to be strong in that class or that topic to assist that learner who needs help.

Zafira: Well when learners get correct answers, I don't interrogate them, but when they get wrong answers I do interrogate them, because I get to ask them other questions to lead to the correct answer so that eventually the learner gets to the answer. But when they give me a correct answer I just pass on that because I believe that okay this learner knows this particular topic. You know what, I try to use different methods of teaching. Yeah, if I see that learners do errors on this particular topic perhaps I try to put them in groups so that other learners will help them to correct those errors, or even ask for assistance from other educators that can assist them in the school.

RE [probing]: What about the one who gives you the correct answer; do you try to interrogate the thinking process?

Kwaku: … mathematics is something that is sequential, that is why in most cases I say to my learners they should not use calculators when they are doing the sums for the first time, but they can use calculator to check the answer because calculator would not lie. You start by working things on your own, … and when you come to the last step and then you use calculator for checking. If the calculator does not give you the answer that you obtain when working on your own, then it means that you are wrong, and the learner will go back and check the mistakes so that he or she can rectify the mistake.

RE: Do you believe that teachers must have their content knowledge right before going to class, and why?

Kwaku: I am 100% in support that teachers must get their content knowledge right before they can teach. The truth is that mathematics is difficult [the participant in a small voice to the researcher] so when these learners notice that the teacher is not good enough to do some sums correctly, they lose confidence in the teacher and the subject. That is why I am saying that the teacher must get their content knowledge right.

Zafira: Yes, of course teachers must have content before they teach. I think that is something fundamentally important. Content is a very important part but then although a teacher must have content, but a way of delivering that content is more important than having content on its own. Yeah but content is very important, I do believe so, because you may know something but find it difficult to deliver it to somebody else. You may be an intelligent person but find it difficult to make another person be as intelligent as you are, yeah, because you process things faster than learners. So although you know something, you must put yourself on the level of the learners, try to think like them how they process the information. Yeah so that is why I am saying content information is important, but delivering that content information is the most important part.

Anele: The teacher content knowledge on the subject is not important but is very, very important sir, in the sense that, number one if you have thorough comprehension of your subject you can be Grade 9 teacher and you are able to know what has been done in a particular topic before [previous grade], what is expected of the learner now [current grade] and what is expected of the leaner in the next grade. Teachers' knowledge is crucial and is very, very important. I can safely say that the teacher with thorough knowledge of the subject will know the common errors learners come with in class, and as a result it will be easy for the teacher to address them even before learners display them.

RE: After an assessment do you by any chance do error analysis, and to what extent do you do that?

Kwaku: Yaa, after every assessment I do analyse the errors learners might have made and I give feedback to learners, so I pick up the errors, but I do not record it down. I simply write on a piece of paper, there is nothing which serve as document for showing the errors that learners are doing in my assessment. I simply pick up these things and write it down in that particular question paper. I simply indicate that, this question that seems to be difficult for learners so I should go back to reteach them up until they get it.

Anele: Yes, I do analysis of learners' errors because that will help me as a teacher to know where my learners are having problems. The errors will help me [determine] which method to use to help these learners, but to be honest I do those that are in formal task because it is part and parcel of our moderation tools.

Usually I look at three things: 1. Is just marking and providing corrections to that activity on the board and going through each question step by step and trying to clarify. 2. With me it is practically impossible to be in a class where my 40 learners are lost. It is usually the majority usually understands the concept, so I use the learners who are strong to come and present, to do what I call peer teaching in my class because of the understanding that sometimes learners learn better when they teach one another. So, I mark and provide corrections, I apply my peer teaching; so if after those two I don't get the desired outcome then I apply my reteaching, I start to teach as if I never taught the lesson, but I have never done that. To be honest I have never opted for reteaching since I started teaching. Usually, the first two strategies work for me.

Zafira: Precisely yes. because I am of the view that my learners need to know as to how they did and where are their strong points and their weak points and what to improve and what to do to ensure that they pass eventually. The analysis of errors helps me to give correct feedback to learners after assessment, to return the scripts back to them, and after they have seen what they did, correct or incorrect then after I will provide correction again, outlining step by step what was expected of them from that particular question asked.

RE: when you are making your lesson plan, do you by any chance take into account errors that learners will be making?

Kwaku: No, if am just teaching a topic I have to plan different strategies of working out the same thing. For example, I know that when teaching algebra one of the problematic areas is signs (integers), so at the back of my mind I know I will talk about integers but there is nowhere in my lesson plan that I will include integers, because it is not the topic I am going to teach.

However, within me I know I will talk about integers because that is where learners will make errors …

Anele: Yes, but I do not write it down in ink in the lesson plan how so it is that I am teaching mathematics Grade 8 and Grade 9 or let's say mathematics. And you will agree with me that mathematics is a continuation of what was done in the previous grade. So, in Grade 9 I am continuing from where the Grade 8 teacher left off. So, when I am preparing my lesson I do consider the common errors in this task. For my past years of teaching as a teacher and as a learner at some stage, I can say that these are some of the common errors that I expect or anticipate from my learners. So yes, when I am preparing my lesson, I do anticipate some common errors and some common misconceptions learners may carry from the previous grade.

Zafira: Yes, I do sir, you know in mathematics it is difficult to see the error that a learner will make without you as the educator doing the sum yourself, so you are trying to get the error that the learner can make in the particular topic. I do the sums that I will do in class and then when I do those sums, maybe when I do the sums as fast as possible I get to see that a learner can confuse the signs here or a learner can make an error there. And then when I get to class then I will have to clarify that you mustn't confuse the signs there, you mustn't do this error here, so I can just put it this way, when doing lesson plans I first do the sums aforetime before I get to the class, so that I can try to predict the errors that learners will make.

5.3 Analysis of data and generation of codes and themes

In this section the researcher describes how coding and themes were generated for the data from the semi-structured interview to answer the questions of the 'extent' to which and 'how' Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. The researcher employed thematic codes through the inductive approach to the semi-structured interview data. Colour codes were assigned to the themes throughout. The transcript of each participant was first read, organised, and coded. The interview data were then arranged in a logical order and placed into meaningful categories using Merriam's (2009) description of "category construction" (p. 178). For example, Figure 5.1 is an extract from the semi-structured interview between the researcher and Anele on the extent to which she engages with learners' errors, which illustrates the initial coding process for the study.

Anele's response:

... When I am bringing it to our context, remedial teaching, that is ***teaching that is aimed at assisting our learners to understand a concept*** *(**Learner support**) or a topic and most importantly to ensure that the anticipated skills of that objective of that topic are achieved. So basically I will say it is that teaching that comes after you have taught and you have assessed, because after that assessment you can see that my students still lack A, B, C and D. These skills or objectives have not been met yet and now you start remedial teaching (**redress of learning gap**), you try to correct the errors and misconceptions or anything that the learners may have portrayed after you have assessed them.*

Figure 5.1 Coding from Anele's response on the extent to which teachers engage with errors

This process was repeated in all sections for all responses generated from all of the participants in the study. Figure 5.2 shows an extract from Zafira's comments on how she engages with learners' errors and how her response was coded.

Zafira's response:

Precisely yes, because I am of the view that my learners need to know as to how they did (feedback) and where are their strong points and their weak points and what to improve and what to do to ensure that they pass eventually. The analysis of errors helps me to give correct feedback to learners after assessment, to give the feedback for me to return the scripts back to them, and after they have seen what they did, correct or incorrect then after I will provide correction (correction) again outlining step by step what was expected of them from that particular question asked (remedial teaching)

Figure 5.2 Coding from Zafira's responses on how teachers engage with learners' errors

Throughout the coding process the researcher assigned colour codes to specific categories, which assisted in generating themes to answer the research questions on the extent to which and how teachers engage with learners' errors. The process followed three phases: phase one shows how data was coded from the interview as presented in Table 5.1. Phase two presents how the coded data was categorise into meaningful theme in Table 5.2 and finally phase three show the final theme were generated. Data was then analysed in three sections using the three final themes generated in Table 5.3.

5.3.1 Inductive analysis of qualitative interview data

Table 5.1 Coding and explanation of codes from interview data

Participants	Codes			Codes/ explanation
Kwaku	MRT EoL	EAS C, R T, F	ICCK KFT, MFL	MRT: Meaning of remedial teaching. EAS: Error analysis strategies ICCK: Importance of common content knowledge EoL: Emphasis on learning C: Correction RT: Remedial teaching F: Feedback KFT: Knowledge for teaching MFL: Motivation for learning
Anele	DMT	F, RT,C, PL	KTIE, KFT, KFL	DMT: Different methods of teaching C: Correction RT: Remedial teaching F: Feedback PL: Peer learning KFT: Knowledge for teaching KFL: Knowledge for learning KTIE: Knowledge to identify errors
Zafira	LS, RLG	RT, IS, C, F	KFT, KFL	LS: Learner support RLG: Redress of learning gap C: Correction RT: Remedial teaching F: Feedback IS: Individual support KFT: Knowledge for teaching KFL: Knowledge for learning

In phase 2 of the analysis a constant comparative method was used and codes which were similar were collapsed to generate themes. In all, three themes were generated based on the categories, as presented in Table 5.2.

Table 5.2: Categories and themes generated from interview data coding

Codes	Categories	Themes
EOL LS / RLG	Emphasis of learning Learner support, Redress of learning gaps	***Teachers' conception of remedial teaching***
C, F, RT	Correction, Feedback, Remedial teaching	***Strategies for error analysis***
KFT, KFIE, MFL	Knowledge for teaching, Knowledge for identifying errors, Motivation for learning	***Usefulness of content knowledge***

Phase 3 speaks to Table 5.3, which is a summary of the final themes after the recategorisation of the themes from the semi-structured interviews.

Table 5.3: Final themes generated from interview data

FINAL THEMES
Theme 1: Usefulness of teachers' content knowledge
Theme 2: Teachers' conception of remedial teaching
Theme 3: Strategies for dealing with learners' errors

5.4 Classroom observation data

In this section the researcher presents data from the observation of lessons, which was aimed at determining the participants' classroom practices when it comes to engagement with learners' errors. Although all three participants were interviewed, at the time of classroom observation one participant could not be observed, since the Head of Department concerned felt that protocols were not properly followed since he was not informed, an only the participant and the principal knew about the intended classroom observation. The extracts below represent what was observed by the researcher in the two remaining participants' classrooms.

5.4.1 Observation of Zafira's lesson

Zafira, a Grade 9 mathematics teacher, started her lesson by checking learners' homework. She asked learners to present their homework on the chalk board, and then went to the board to check the work and quizzed them to see where they had gone wrong or got it right. The teacher in a way tried to correct the learners' mistakes and asked them to write corrections from the chalk board into their exercise books. The teacher then introduced a new topic, which was algebraic equations involving brackets (for example: $\frac{1}{2}(2x - 3) = 5$). After doing some activities with the learners, the teacher gave the learners a class activity to do on their own. She later asked three learners to present their work individually on the chalk board. The teacher asked all of the learners to mark their own work as she went through the answers presented by the three learners on the chalk board. In the process, the teacher highlighted some errors learners had made and corrected them while she was explaining to the whole class. Figure 5.3 is an example of a class activity presented by learner H.

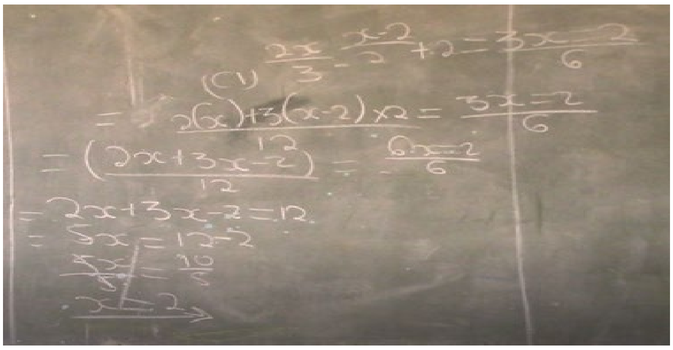

Figure 5.3 Sample of learner H's work in the class activity

5.4.2 Observation of Anele's lesson

Anele started the lesson by first writing the topic, which was algebraic equations, on the chalk board. The teacher then used questions and answers to discuss relevant previous knowledge that the learners had on the topic; for example, he used a question like ***"If you subtract your age from your mother's age, how old was your mother when she gave birth to you?"*** to check learners' previous knowledge which was relevant to the topic of algebraic equations. The teacher gave learners a few minutes to give their answers to the question. He then started to write some algebraic equations on the board. He first worked out some on the board and explained to learners how to solve algebraic equations, and later gave the learners an opportunity to ask questions. Figure 5.4 shows an example of an algebraic equation the teacher worked out on the chalk board.

Activity 1.1.4

Solve for x in the equation: $2x - 3(3 + x) = 5x + 9$

Solution:

$$2x - 3(3 + x) = 5x + 9$$

$$2x - 9 - 3x = 5x + 9$$

$$2x - 3x - 5x = 9 + 9$$

$$-6x = 18$$

$$\frac{-6x}{-6} = -\frac{18}{6}$$

$$x = -3$$

Figure 5.4 Example of Anele's lesson on algebraic equation in class

After explaining the activity in Figure 5.4, he then gave a learner exercise which was marked in class for learners to do their corrections.

Anele later introduced to learners the topic of exponential equations. He further reminded learners that they were still doing algebraic equations, but this time they were algebraic equations involving exponents. He gave the learners a worksheet which contained the rules or laws of exponents, which learners had already learned in the previous term. The teacher then told learners that the rules they had learnt in the previous lessons would be applied in the new topic which is algebraic equations involving exponents. Anele then wrote three questions on the board and asked learners to do them on their own. After few minutes he requested three learners to present their answers on the board. As the learners presented their work on the board and mistakes were identified, he then started engaging with the errors with the whole class. In some cases, Anele asked the learners to explain to the class how they arrived at the correct answers.

5.4.3 Analysis of data from lesson observation

Data from classroom observation were deductively analysed using the themes generated from the framework as shown and discussed in Chapter Three. The researcher consulted literature and the framework to develop the initial strategies which teachers employ in analysing learners' errors by identifying 1) the forms of error analysis, and 2) the strands of teachers' mathematical knowledge. To ensure alignment with the framework, participant practices were categorised using the strands of mathematical knowledge (Ball et al., 2008), and categorised for analysing learners' errors as mentioned by Sapire et al. (2014).

Table 5.4: Generating codes and themes from observation data.

Formulation of categories	Theme	Rationale
Recognition of mistakes or errors in the learners' steps. For example, Zafira recognised mistakes made by the learners. Anele worked on solutions with learners	Teacher knowledge and using procedural knowledge to address errors	To identify the types of error strategies teachers' use
Using explanation to address the error. For example, Zafira highlighted errors made and provided explanation	Knowledge teachers need to teach mathematics and understanding of conceptual errors	To establish the kind of mathematical knowledge teachers have

Drawing from one's knowledge to address the error. For example, Anele used his mathematical knowledge to identify learners' errors	Knowledge of students and the content in relation to teachers' awareness of commonly identified learners' errors	To establish the knowledge teachers have to identify learners' errors

Table 5.4 above shows how codes were generated from the lesson observations and how these were formulated into categories. Themes were then generated from the categories to help the researcher to answer the research questions using classroom observation. The rationale from the themes which were created support the already existing themes from the framework of the study.

Sapire et al. (2014) developed six criteria for error analysis, as follows: conceptual understanding, procedural understanding, awareness of errors, multiple explanation of errors, use of everyday knowledge, and diagnostic reasoning. However, the data from this study could be grouped into three, that is, teacher knowledge and procedural knowledge, teacher knowledge and understanding of conceptual errors, and awareness of errors. For effective engagement of learners' errors, it is important to note that teachers are supposed to have their mathematical knowledge in place in order to be able to deal with learners' errors. According to Ball et al. (2008) mathematical knowledge is the knowledge that teachers need in teaching mathematics. Shulman (1986), as cited in Ball et al. (2008), argued that knowing a subject for teaching requires more than facts and concepts. In other words, mathematics teachers must understand the principles and structures as well as the rules for establishing what is legitimate to do and say in the classroom.

5.5 Conclusion

This chapter presented the data and data analysis. Data from semi-structured interviews and lesson observations were presented and analysed to answer the main research questions of this study. The researcher used the interview data to find answers as to the 'extent' to which and 'how' teachers engage with learners' errors in the teaching of mathematics. Lesson observation was used to emphasis 'how' teachers engage with learners' errors in the teaching and assessment of mathematics.

In the next chapter, Chapter Six, the findings from the data are synthesised with discussion of the themes that emerged in order to answer the research questions.

CHAPTER SIX

FINDINGS AND DISCUSSION

6.1 Introduction

This study is a contribution to research studies in mathematics education in South Africa and beyond. The study focused on error analysis, and aimed at exploring the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics. In the previous chapter the researcher provided a presentation of the data and data analysis, and in this current chapter the findings and discussion of these are presented. In Chapter Five the themes that emerged during data presentation were discussed and supported by literature with a direct linkage to the framework of the study.

The analysis and ensuing results are based largely on lesson observations and transcribed interviews conducted with three participants from three schools who teach Grade 9 mathematics. Video recordings helped the researcher to make certain inferences about how teachers engage with learners' errors during the teaching of mathematics. The data from the lesson observations were verified or clarified through the interviews.

Detailed results for each of these analyses are organised according to the relevant mathematical concepts which are found in Chapter Five. This chapter presents a synthesis of the findings that transpired in the data presentation and analysis through the interviews and lesson observations outlined in Chapter Five. The method of analysis emerged through the analysis itself, and the research questions evolved as part of the process. The main questions that this study aimed to answer were:

- To what extent do Grade 9 teachers engage with learners' errors in their teaching of mathematics?
- How do Grade 9 teachers engage with learners' errors in the teaching and assessment of mathematics?

- Why do teachers in Grade 9 engage or not engage with learners' errors when teaching or assessing mathematics?

The researcher presents the main findings of the study, addressing each of the above research questions. The findings are divided into three different subsections, which answer the three main research questions. Data generated from participants through a semi-structured interview, lesson observations, and review of documents were triangulated to answer the three critical questions which guided the study. The subsequent sections present the findings

6.2 The extent to which Grade 9 teachers engage with learners' errors in the teaching and assessment of mathematics

When unpacking teachers' engagement with learners' errors, data that emerged assisted the researcher to generate themes. In the subsequent section the researcher elaborates on these themes in depth, thus unearthing the extent to which teachers engage with learners' errors.

6.2.1 Teachers' knowledge for remedial teaching

Teacher knowledge has been defined in different ways by many scholars, especially when it comes to teacher knowledge for teaching. Shulman (1986) is the forefather of studies on teacher knowledge, and Ball et al. (2008) extended his work, introducing a new dimension paying attention to mathematical knowledge for teaching. Building from these studies and many more, the findings of this study add a new dimension of teachers' knowledge for remedial teaching.

According to Wartono, Sumarjono, and John Rafafy (2018) remedial teaching is a relearning for students who face learning difficulty. Rini and Prabawanto (2020) also explained that remedial teaching plays an important role in the whole teaching programme. The authors further argue that through remedial teaching the teacher tries to provide learners with the optimum opportunity to create their mathematical capabilities. In agreement with the above authors, the researcher defines remedial teaching as a reteaching of concepts by the teacher to provide a relearning opportunity for the learner who struggled to understand a concept when it was first taught. To understand this definition from the participants' perspective, the interviews revealed some fundamental concepts, as illustrated in the teachers' verbatim responses below:

> **Kwaku:** … remedial teaching is reteaching something you have taught that learners failed to understand. You know what, with remedial teaching you are emphasizing what learners need to put in mind.
>
> **Zafira:** My understanding of remedial teaching refers to using different types of methods when teaching because that enhances learners' understanding in different forms.
>
> **Anele:** … is teaching that is aimed at assessing our learners to understand a concept or a topic and most importantly to ensure that the anticipated skills of that objective of that topic are achieved. So basically, I will say it is that teaching that comes after you have taught and you have assessed because after that assessment you can see that my students still lack A, B, C, and D. These skills or objectives have not been met yet and now you start remedial teaching, you try to correct the errors and misconceptions or anything that the learners may have to portray after you have assessed them.

As evident from the above extracts, the participants have a different conception of remedial teaching that is not aligned to the literature. For example, Kwaku posits that remedial teaching is emphasizing what learners need to put into their minds, which is not the same as relearning for students who face learning difficulties, as put by (Wartono et al., 2018). The emphasis in Kwaku's definition is on transferring knowledge to learners' minds, not on relearning the concepts. In contrast, Zafira's explanation put more emphasis on methods of teaching rather than remedial teaching, which suggests that she considers remedial teaching as another way to continuing teaching the concepts, with no emphasis on ensuring that relearning is taking place.

A different case ensued with Anele, as shown in the extract above which shows that his knowledge of remedial teaching to some extent speaks to the concept of relearning. This is based on his response that "it is the teaching that comes after you have taught and you have assessed and see that learners lack some skills". Sahito et al. (2017) mentioned that remedial teaching is providing a remedy or cure where it is needed most. According to Anele, remedial teaching is a means that is utilised by the teacher to provide a remedy to learners' knowledge gaps by unearthing the errors and misconceptions causing those knowledge gaps. This is contradictory to the two other participants, that is Kwaku and Zafira, who consider remedial teaching to be a continuation of the teaching. Anele's emphasis is grounded on correcting what has been identified as the missing link or correcting what has been identified as hindering knowledge construction.

6.2.2 Doing corrections to engage with learners' errors

Based on the findings, one of the common methods used by the participants in this study to attend to learners' errors was doing corrections in the class. This was evident in the responses during the interview sessions when they shared their views on how they engage with learners' errors, as illustrated in the extracts below:

> **Anele:** The normal ways I do it is I write correction on the board or sometimes I call some of the learners to the board to present their answers, then we correct it together. It is just marking and providing corrections to that activity on the board and going through each question step by step and trying to clarify. So, I mark and do corrections.
>
> **Kwaku:** … I used to do corrections on the black board and then I give a similar activity for learners to do individually so that I can identify if all learners end up having a clear picture or a clear understanding of what I was saying to them.
>
> **Zafira:** I first do the sums in class on the board like I am doing corrections; I am always open for opportunities for extra time if learners do not understand in class. I always tell them that they are more than welcome to come in break, they can come to me and we can discuss farther.

Participants mentioned that when they identify any errors in learners' work, they try to support the learners by correcting each stage of the process involved in the solving of the mathematics problem. This support assists the learners to improve on that concept. It was also evident in the above responses that the participants envisaged engaging with learners' errors as teaching the concepts rather than helping learners to construct the correct knowledge. For example, Zafira pointed out that when doing corrections the emphasis is on re-emphasising step-by-step procedures to ensure that learners follow. In the same vein, Kwaku emphasised working through a correction on the board to emphasise the steps. None of the participants seem to encourage learners to explain their thinking process or attempt to allow learners to engage with what they say or write. Their main strategy is doing corrections on the board and re-explaining the procedural steps.

6.2.3 Strategies to attend to learners' errors through peer support

Another result that emerged from the participants' responses during the data analysis process is peer support to deal with learners' errors. The researcher of this study has categorised peer support into peer teaching and peer assessment. Peer teaching is an effective method of learning (Gottlieb, Epstein, & Richards, 2017). In contrast, according to Panadero (2016) peer assessment is when people of equal status assess each other's work. From the findings, the researcher noticed that the participants used peer teaching to promote learning. Collectively the three participants hold the view that through peer teaching good learners can assist the weak ones. The teachers believe that at times learners learn best when they are instructed by their peers. Peer support as a method of dealing with learners' errors was shown in the comments by the participants outlined below.

Zafira noted that:

> .... other strategy which is working in my class is peer teaching, allowing another child who seems to be strong in that class or that topic to assist that learner who needs help.

This extract alludes to peer teaching as another strategy to engage with learners' errors.

Similarly, Kwaku remarked that:

> … Sometimes I ask learners to do peer teaching. Those who had the questions correct help their friends who got it wrong and ….

Furthermore, Anele commented as follows:

> I try to use different methods of teaching. Yeah, if I see that learners do errors on a particular topic perhaps because of the method I used to teach them and then they repeat the error then I try to put them in groups so that other learners will help them to correct those errors or even ask for assistance from other educators

As evident in the extracts above, the participants do use learners' errors to improve teaching; however, what is amiss is how they use it to improve learner performance – since none of them made mention of how they interrogate learners' responses to assessment tasks. Secondly, all the participants emphasise the incorporation of peer teaching, not peer assessment. Panadero (2016) argues that peer assessment is when people of the same status assess each other's work and that it provides an opportunity for learners to learn better; contradictory to that study, the participants used the able to assist the less able.

6.2.4 Teachers’ mathematical content knowledge

One of the themes that the researcher used to answer the first research question was teachers' content knowledge. According to Ball et al. (2008) mathematical content knowledge is the special content knowledge that teachers need to carry out their work as teachers of mathematics. The researcher in this study concurs with (Ball et al., 2008) and emphasises that the quality of mathematics teaching depends on teachers’ knowledge of the content, which can translate into teachers' ability to correct learners' errors. Participating teachers who were involved in the study claimed that for mathematics teachers to deal with learner errors effectively, teachers need to have in-depth mathematical content knowledge.

This claim by the participants has been supported by Ball et al. (2008), who maintained that for teachers to be able to interrogate and deal with learners’ errors they need first to develop their mathematical content knowledge. The notion of teachers’ mathematical content knowledge as a requirement in dealing with learners’ errors in mathematics was evident in the participants' responses from the data that were generated during data analysis, as shown in the extracts below:

> **Anele:** … teachers must have the content [knowledge] before they teach actually. I think that is something fundamentally important because the content is a very important part for [identifying] errors.
>
> **Kwaku:** … I am 100% in support that teachers must get their content knowledge right before they can teach. You, sir [referring to the researcher] mathematics is difficult [the participant in a small voice to the researcher during an interview] so when these learners notice that the teacher is not good enough to do some sums correctly, they lose confidence in the teacher and the subject. That is why I am saying that teachers must get their content knowledge right.

However, Zafira agrees with the fact the teachers must have content knowledge, but believes that having content knowledge only is not enough:

> … of course, teachers must have content. I think that is something fundamentally important. Content is a very important part but then although a teacher must have content, a way of delivering that content is more important than having content on its own. Yeah but the content is very important, I do believe so, because you may know something but find it difficult to deliver it to somebody else. You may be an intelligent person but find it difficult to make another person as intelligent as you are, yeah because you process things faster than learners. So although you know something you must put yourself on the level of the learners, try to think

like them, how they process the information. Yeah so that is why I am saying content information is important but delivering that content information is the most important part.

Zafira noted that content knowledge is not enough for the teacher to engage with learners' errors, and that teachers' pedagogical content knowledge of teaching is equally important when dealing with learners' errors. As clearly indicated in the framework of the study, Ball et al. (2008) argue that pedagogical content knowledge is the combination of content and pedagogy needed for teaching the subject (mathematics). The aspect of pedagogical content knowledge in the framework of (Ball et al., 2008), which the researcher used in this study, was termed knowledge of content and students (KCS). Ball et al. (2008) explained KCS as being the knowledge that combines knowing about learners and knowing about mathematics. The authors further maintained that KCS involves recognising errors and identifying the most likely errors learners make, interpreting learner responses, and developing learner justifications. In doing so, Sapire et al. (2014) posit that teachers would be using multiple ways of explaining the errors.

It can be noted that the findings are clear from the participants that for teachers to deal with learners' errors, they need to have the right mathematical content knowledge. Anele reveals that content knowledge is very important for teachers to identify learners' errors. Drawing from the framework of this study, Ball et al. (2008) maintain that mathematical content knowledge is important and crucial for the teaching and learning of mathematics. The finding of the study further reveals that one of the participants felt as follows:

> Kwaku: … mathematics is difficult so when learners notice that the mathematics teacher is not good enough, they lose confidence in both the subject and the teacher.

It therefore means that learners can learn better when the teacher before them has good mathematical content knowledge.

Drawing from the data from the interviews, it can be concluded that while the participants consider remedial teaching as a means to engage with learners' errors, their perception of what remedial teaching is are grounded on the basis of reteaching the concepts. This was further evident when they discussed the strategies used to engage with learners' errors. All of the participants emphasised doing corrections on the board and re-explaining the procedures to learners as the main strategy to engage with learners' errors. Secondly, the emphasis was on peer teaching by using the more knowledgeable to assist the less knowledgeable. Furthermore, they consider one's common content knowledge to be the key factor in the ability to attend to learners' errors, rather than SCK or KCS.

6.3 Why do teachers engage with learners' errors when teaching or assessing mathematics?

To ascertain the reasons why teachers engage with learners' errors from the participants' perspective, it was necessary for the researcher to interview the participants. In the section below verbatim responses illustrating the reasons why participant engage with learners' errors are presented. Of particular interest in the first phase of the interviews, the researcher ascertains their understanding of engaging with learners' errors and strategies they used in their classroom to engage with learners' errors. It was therefore important to understand the reasons behind the strategies and the participants' philosophical perspectives concerning learners' errors.

Table 6.1: Teachers' rationale for engaging with learners' errors

Reasons to engage with learners' errors		
Correcting mistakes	Feedback for learning	Remediation

From the data generated, it was clear that the participants engaged with learners' errors for three reasons: the intention to correct learners' mistakes, provide feedback to learners, and do remedial teaching. In a nutshell, these reasons are grounded in the quest to improve teaching and learning. Table 6.1 illustrates the reasons for engaging with learners' errors.

6.3.1 Correcting learners' mistakes or errors

Driven by the urge to correct learners' mistakes, the participants articulated in different ways how this is the driving force for them to engage with learners' errors:

> **Kwaku:** When I have given an assessment, I do the corrections on the chalk board then I do analyse those errors and tell them that you have made these errors and next time when you write the same paper or when you get to the exam you mustn't do this errors. I do this so that they know what they did wrong.
>
> **Zafira:** Yeah, I do engage with learners' errors in the form of corrections, you can put it like that, because doing corrections will help learners see where they went wrong.
>
> **Anele:** I engage with errors or mistakes because learners need to know what they did wrong. One of the ways I do engage with learners' errors is just marking and providing corrections to that activity on the board and going through each question step by step and trying to clarify.

Based on the above extracts, it was evident that Zafira, Kwaku and Anele are more concerned with letting learners know what they did wrong rather than why they are wrong and how they can improve in the future. Engaging with learners' errors is used as a corrective measure; however, the emphasis seems to be on developing procedural knowledge.

6.3.2 Providing feedback to promote learning

Another reason highlighted by the participants was based on the quest to give feedback on learning:

> **Zafira:** I get to tell them that as I was marking, I noted some of you did this error and that error and make sure that when you write in an examination you don't repeat the same errors. So that is how I encourage them not to repeat the same errors they've done in the assessment, to give feedback to learners.
>
> **Kwaku:** Precisely, yes, it's a way to give feedback to learners because I am of the view that my learners need to know as to how they did and where are their strong points and their weak points and what to improve and what to do to ensure that they pass eventually. I revise the paper and try to use the board. I use the board so that I can explain so everyone can have the chance

of looking at what is happening and, we do it step by step or we do it bit by bit up until we end up finishing. And then after seeing that am done with a particular portion, I also give them another activity to check if there are those who are still struggling so that I can try to assist them

Anele: I do it in order to provide feedback to my learners on the assessment they do, for them to know their mistakes so that they do not do the same mistake repeatedly. Usually, as a mathematics teacher who gives informal tasks now and again, it is oral feedback where I just talk to the learner or the class as to what was expected of them, where they went wrong, and how they can improve. But for moderation purposes, if it is a formal task, we call analysis of learner's errors to check as to how the learners answered the questions and to pick up some errors they have done in the task.

While all agreed that they are engaging with learners' errors to give feedback to learners, Anele added a new dimension of doing it for the purpose of informing the authorities about the performance of learners. As they mention, the basic drive is to give feedback to learners about their understanding of the concept taught or to report to authorities.

6.3.3 To reteach concepts

One of the reasons teachers engage with learners' errors is to provide remediation. In the interviews the participants mentioned remediation as reteaching of concepts:

Kwaku: I mark and provide corrections; I apply my peer teaching so if after those two and I don't get the desired outcome then I reteach. I start to teach as if I never taught the lesson…

Anele: I give feedback to learners after assessment by revising the paper in order to speak to learners' mistakes and I try to use the board. I use the board so that I can explain so everyone can have the chance of looking at what is happing and also, I do it step by step or we do it bit by bit up until we end up finishing. And then after seeing that am done with a particular portion, I also give them another activity to check if some are still struggling so that I can try to assist them. And just because of the time, it can happen that sometimes I ask other learners to do peer teaching.

Despite the above reasons why teachers engage with learners' errors, some of the participants also indicated challenges as to why they do not engage with learners' errors, for example, time constraints:

Kwaku: To be honest, time is a challenge for me to engage with my learners' errors properly, because the subject advisors and head of the department are all concerned with curriculum coverage. They don't want to hear anytime so I am so trying to finish the curriculum at the expense of learners.

Anele: … And just because of the time, it can happen that sometimes I don't do it, so I ask other learners who understand the concept to do peer teaching.

Based on the participants' responses it was evident that the quest to improve teaching and learning is the main reason they engage with learners' errors. In conclusion, the finding is clear that the reasons why Grade 9 mathematics teachers engage with learners' errors include but are not limited to providing feedback, correcting learners' mistakes, and remediation.

In the next section researcher presents findings from observing the participant practices. While it was necessary to hear the extent to which they engage with learners' errors the researcher deemed it equally important to explore if the said practices are indeed implemented in practice

6.4 Teacher mathematical knowledge and engagement with learners' errors in teaching and assessment of mathematics

A plethora of literature (e.g. Ball et al., 2008; Adler, 2010; Sapire et al., 2014; Ndlovu et al., 2017) has explored the importance of teacher knowledge in the teaching of mathematics. In addition, authors (e.g Sapire et al., 2014; 2016; Siyepu, 2014; Aksu & Kul., 2016; Ndlovu et al., 2017) have explored teachers' and pre-service teachers' engagement with learners' errors. Learners' errors are evident during the teaching and learning and assessment process, and therefore teacher knowledge plays a crucial part in the teachers' ability to address learners' errors. Moreover, attention to learners' errors is a way to improve learning, as emphasised in the CAPS document. The incorporation of diagnostic assessment as well as focus on addressing learners' errors thus emphasises that teacher knowledge plays a crucial role in addressing learners' errors. In the section below the researcher presents data from observing teachers engaging with learners' errors. The emphasis was on understanding the aspects of teachers' knowledge that the participants in this study drew on when engaging with learners' errors.

Enu and Ngcobo (2020) mentioned that through assessment teachers can engage with learners' errors in mathematics. This claim was earlier advanced by Gardner (2012) who sated that teachers' assessment could be used to correct identified errors and misconceptions. Therefore, in trying to answer research question two, participants' instructional and assessment practices were analysed to find out how they engage with learners' errors during teaching and assessment of mathematics. Combining the aspect of teacher knowledge (Ball et al., 2008) and categorisation of errors by Sapire et al. (2014), as explained in the framework, the researcher explores how teachers dealt with errors: procedural understanding of errors, conceptual understanding of errors, and teachers' awareness of errors. The sections that follow delineate how teachers engaged in learners' errors according to the three categories mentioned.

6.4.1 Teacher knowledge and using procedural knowledge to address errors

Using the framework that underpins this study, recognition of learners' errors lies under the domain of teachers' common content knowledge. During data analysis codes were assigned to teachers' statements and comments that addressed learners' errors during instructional activities. For example, during Zafira's lesson on algebra, after she has finished working on some examples with the learners she gave a class activity for them to try their hands at. After learners had worked for about 15 minutes, she called on one of the learners (learner H) to present her answer on the chalk board (see Figure 5.1, Chapter Five). It was evident in the learner's work that some procedural errors had been made.

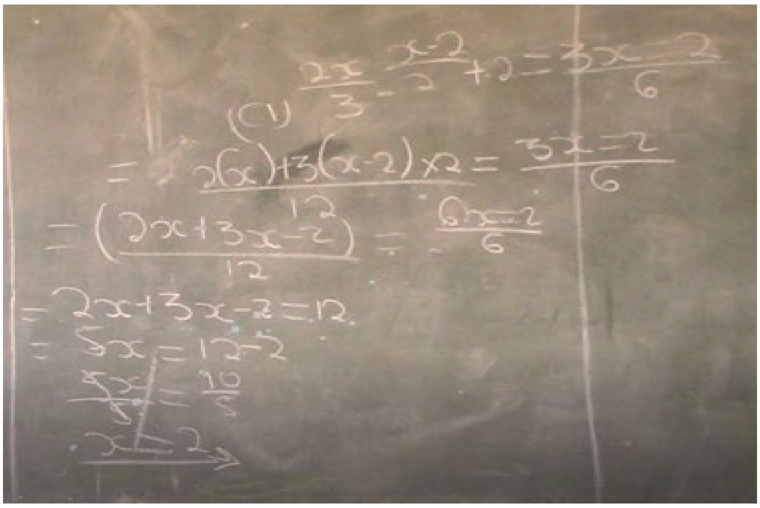

Figure 6.1 Sample of learner H's work in class activity

From the analysis of data, it was clear that Zafira became aware of and recognised some errors in the learner's work, and she was able to explain the steps needed to get to the correct answer. Hence she used the learner's work to engage the whole class on the errors identified, and she asked those who had their work wrong to make the necessary corrections. The excerpt that follows shows how the teacher engaged the class in dealing with the errors in the learner's work during the class observation. The teacher started by drawing learners' attention to the fact that, since the question involves fractions, the first thing to do is to find the least common dominator (LCD) of the equation, which she found to be 12. The conversation with her learners continued as follows:

Zafira:	How many times does three [the first divisor] go into 12 which is the LCD?
Learner A:	[Shouted from the back of the class] Four, ma'am.
Learner K:	[Raised hand) Three, ma'am.
Zafira:	Okay, how many of you agree on 4 and how many agree on 3?
Learners:	[12 learners raised their hand for 3, 10 raised their hand for 4, and 4 of them did not raise their hand at all]
Zafira:	4 is the correct number, class. As you can see from the board, learner H wrote 2 instead of 4 so we have to change the 2 to 4 then multiply 4 by (2x). In the next term let's see if learner H is correct.
Learners:	(All shouted) Okay, ma'am.
Zafira:	2 goes into 12 how many times?
Learner M:	6 times.
Zafira:	Any different answer, class?
Learner T:	[Raised hand]
Zafira:	Yes, learner T?
Learner T	Ma'am I agree with learner M, the answer is 6.
Zafira:	Okay, let's see if learner H is correct. What did learner H write here?
Learners:	[Whole class response] 3
Zafira:	Is learner H supposed to write 3?
Learners:	[Whole class response] No ma'am.
Zafira:	Okay, change 3 to 6 and multiply 6 by (x-2).
Learners:	Yes ma'am.
Zafira:	1 goes into 12 how many times?
Learners:	[Whole class response] 12 times ma'am.
Zafira:	Okay, then multiply 12 by 2.
Learners:	[Whole class response] Yes ma'am.

Zafira:	How many times will 6 go into 12?
Learners:	[Whole class response] 2 times ma'am.
Zafira:	Okay, multiply 2 by (3x-2).

From the excerpt, it can be seen that Zafira's engagement with the error was driven by the ability to solve the problem and the procedures that needed to be followed. While teacher Zafira recited these procedures, from the observation of the lesson it was evident that a fraction of the learners (for example, 12 out of 26, which represent 46% including learner H), could not make sense of the algebraic language used, e.g. how many times 3 can go into 12. While the teacher Anele continued with the explanation of the procedures with the rest of the class, it was evident that the language was not comprehended by all of the learners, for example, learner H who was puzzled by the answer.

The above scenario confirmed the data from the interview that the participant understanding of strategies to eradicate errors is anchored on doing corrections on the board rather than engaging with learner thinking. This was evident as teacher Anele, after noting the errors made by learner H, did not ask the learner to explain his/her thinking to unearth the underlying misconception, but opted to engage the whole class and started to reteach concepts to the whole class. While reteaching the concept to the whole class is important, learner H still could not work out the sum with the class, meaning that the method used has not assisted the learner to understand the concept better. Many scholars, such as Maharaj (2014), and Ndlovu and Brijlall (2015, 2016), have emphasised the importance of allowing learners to engage with what they write. While teacher Zafira used learner H's errors to re-emphasise the concepts, she did not engage with learners' thinking when addressing the errors made. The focus was more on re-emphasising the procedures to be followed.

Anele wrote an algebraic equation, 2x + 4 =10, on the chalk board and used a probing question to identify errors made by learners during his teaching, as follows:

Anele:	Learners, what do we do when a question like this given and you are to find unknown x?
Learner G:	[From the right corner] Sir you first group the like term.
Anele:	Is she correct?
Learners:	[Whole class response] Yes sir.

Anele: Okay learner M, come to the board and group the like term for us.

Learner M: [Goes to the board to present answer] 2x= -10+4

Anele: Is he correct?

Learners: [Some learners raised their hands]

Anele: Yes, learner D?

Learner D: No sir.

Anele: Okay come to the board and correct it for us to see.

Learner D: [Goes to the board] 2x = 10 – 4

Anele: Okay you are correct. Explain your answer to the class.

Learner D: [Laughed loudly] Sir, I transpose the 4 which is positive to the other side of the equation to where the 10 is and the 4 turned to negative. That is why I got 2x = 10 – 4.

Anele: Okay learners, do you understand?

Learner T: No sir.

Anele: Where are your difficulties here?

Learner T: Sir, everything on the board.

Learners: [Whole class laughs]

Anele: Okay learners, listen, remember we did integers in term one when transposing a number or when a number is crossing the equal sign and is positive, it changes to negative and positive turns to negative. From the equation here you can see that 4 is positive, therefore it will turn to negative. Therefore 10 -4 is correct on the other side of the equation. If the 4 was negative (-4) then it was going to 10 +4 on the other side of the equation.

Anele: Do you understand learner T?

Learners: Yes sir.

Anele: Yes, learner T, do you understand now?

Learner T: Yes sir.

Anele: Yes learner, what do we do next?

Learner P: Sir, you subtract 4 from 10 and get 6. So, the equation becomes 2x = 6. Then you divide 2x by 2 and 6 by 2.

Anele: Why do you divide by 2?

Learner P: Sir because we want x and 2 is Infront of x.

Anele: You are correct, sit down. Clap for him.

Learner: [All the learners clapped]

Anele: But learners, listen carefully, always we divide by the coefficient not necessarily 2 and yes in this equation the coefficient is 2. So always look for the coefficient and divide by both sides of the equation. Okay all of you divide 2 which is the coefficient and tell me the answer for x.

Learner J: Sir, the answer for x is 3.

Learners: [Whole class in loud voice] Yes sir x=3.

Anele: Yes, you correct x is 3. Clap for yourself.

From the above presentation, there was an engagement between Anele and his learners; it was observed that the teacher was probing the learners to get errors they made, in order to engage with them. The researcher observed that learner M could not transport positive numbers correctly. Therefore, the teacher used probing questions for learner D to pick up the error learner M had made and the teacher asked learner D to correct the error, which he did correctly. Again, the teacher realised that even though learner D had done it correctly, some learners, including learner T, still didn't understand, so he took time to explain.

It can be concluded that from the observation of Anele's class that some of the learners had a problem with transposing and integers; the teacher was able to use probing questions to deal with learners' errors, as used by Brodie (2014). Drawing from the above findings about the framework of the study, the procedural understanding (Sapire et al., 2014) that the participants had in dealing with learners' errors is because of their content knowledge (Ball et al., 2008) The researcher observed from the lessons that teachers had good content knowledge of the topic they were teaching.

6.4.2 Knowledge teachers need to teach mathematics and understanding of conceptual errors

Beyond recognition of the correctness of the answer, conceptual understanding refers to the ability to size up the error and explain the reasoning behind the error (Sapire et al. (2016). The ability to recognise the correctness of the answer and sizing up of the error span two of (Ball et al., 2008) the domains of teachers' mathematical knowledge: common content knowledge and specialised content knowledge. During lesson observation the emphasis was on understanding teachers' conceptual understanding of learners' errors, that is the ability to recognise the correctness of the answer and to size up learners' errors.

When Zafira was teaching algebraic equations involving fractions, she asked the class questions to ascertain the learners' understanding of how to deal with the lowest common multiple (LCM). At the end of this interaction, where learners expressed their thoughts about the meaning of the LCM, the teacher continued to do the first sums on the chalk board, to show the learners how to determine the LCM in an algebraic equation involving brackets. Figure 6.2 illustrates the example done in the class.

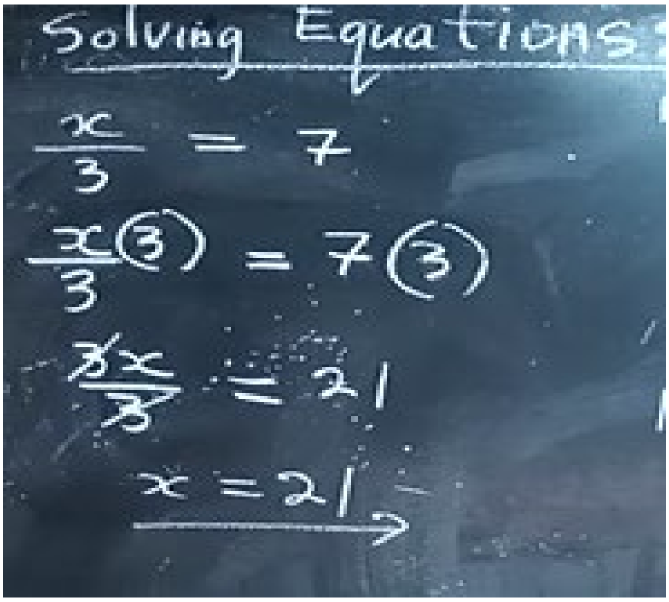

Figure 6.2 Solving of equations by Zafira

While doing the sum on the board, Zafira provided the following explanation when solving for *x* in the algebraic equation involving fraction $\frac{x}{3} = 7,$ as follows:

> *... okay we are solving for x here, but the x is the numerator, and the problem is we are diving this x by the denominator 3. so, to remove this denominator 3, we must multiply both sides of the equation by 3. That will be x divided by 3 multiplied by 3 and then 7 on the other side of the equals sign of the equation is also multiplied by 3.*

Zafira continued her explanation to learners as follows:

> *... put the three in a bracket, a bracket suggests multiplication. After the multiplication, you will have 3x divided by 3, which equals 7 multiplied by 3 which gives us 27. So you divide both sides of the question by 3 as* $\frac{3x}{3}$ *= 7.3 so 3x divides 3 to give us x and 7 multiplied by 3 is 21. Therefore, x is equal to 21.*

What was evident is that instead of allowing learners to solve the sum and then engage with the errors that were made, Zafira took over and retaught the concept. Also, the emphasis in the teaching was on explaining the procedures that learners needed to follow, not emphasising sense making – for example, asking learners the meaning of the equals sign, or using a trial and error method to determine the answer. The key thing was learners being able to identify the value of *x* that would make the statement true.

In his lesson presentation Anele also provided an explanation to learners, as follows:

Solve for *x* if $3^{x+1} = 9$

> *Okay learners, we want to solve for x and x is like the exponent, since on the left side we have 3 and 3 which is powers of the same base because we are multiplying powers of the same bases, we have to add the exponents to get something like* 3^{x+1} *on the left side. Again on the right side, you have to change the 9 to base 3 as* 3^2*=9 so in place of 9 you put* 3^2. *From there, your equation will look like* $3^{x+1} = 3^2$ *Now, learners, you can see that both sides of the equation have 3 as the same bases. Because the bases are the same you can equate the exponents, simplify, and then group like terms to solve for x. Your equation will then look like x+1=2. Group like terms by transposing I to the other side of the equation as x= 2-1. Therefore x = 1.*

Similarly to Zafira, Anele never gave learners a chance to solve the sum. While the researcher intended to observe the participants' engagement with learners' errors, what ended up happening in the classroom was teachers teaching the concept. Also, probing questions were not used during the teaching process to establish that learners were making sense of the concepts. While the participant explained in the interview how they engage with learners' errors, this was limited in the classroom setting. While Zafira and Anele were able to explain to the learners the procedural steps to be followed when solving algebraic equations, there was no engagement with learners' errors, and thus the researcher could not interpret their ability to size up learners' errors when teaching.

To further interrogate the engagement with learners' errors, an the extract from learners is assessed, where teachers mark learners' work is provided below (Figure 6.3A).

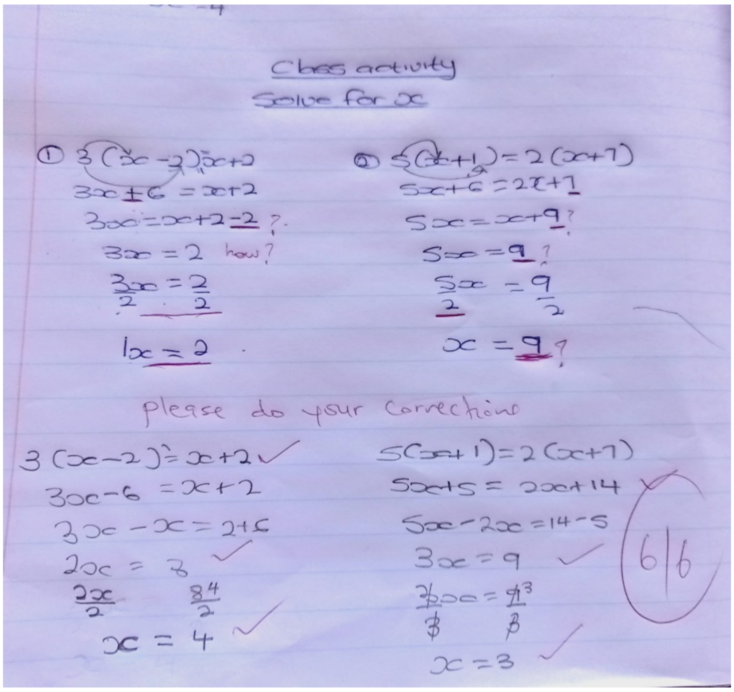

Figure 6.3A Extract of marked exercise from learner's exercise book

The extract shows an exercise marked by Anele, who then assisted learners to do corrections after marking. For example, in Figure 6.3A it could be seen that the learner failed to answer the question given as a class activity. It was observed that the learner had made a procedural error when expanding the bracket in both equations. Anele first highlighted the errors made by the learner in red pen in the learner's book. Anele further explained to the learners the procedural mistake that was made, but he did not go further to interrogate and engage with learners as to why they make such errors. For example, the researcher observed learner inability to group like terms and, most importantly, inability to divide 2 by 2 correctly as per the learner's own calculations; this illustrated a gap in content knowledge. In Grade 9 a learner is expected to divide a number by itself to get 1, but in this case the learner failed to do that. It was no different from question two, where the learner said 9 divided by 2 is 9. This means the errors that the learner was making go beyond errors made in the current topic; rather, the researcher observed that the learner lacks some knowledge of content from the lower grades, which contributed to errors made by the learner.

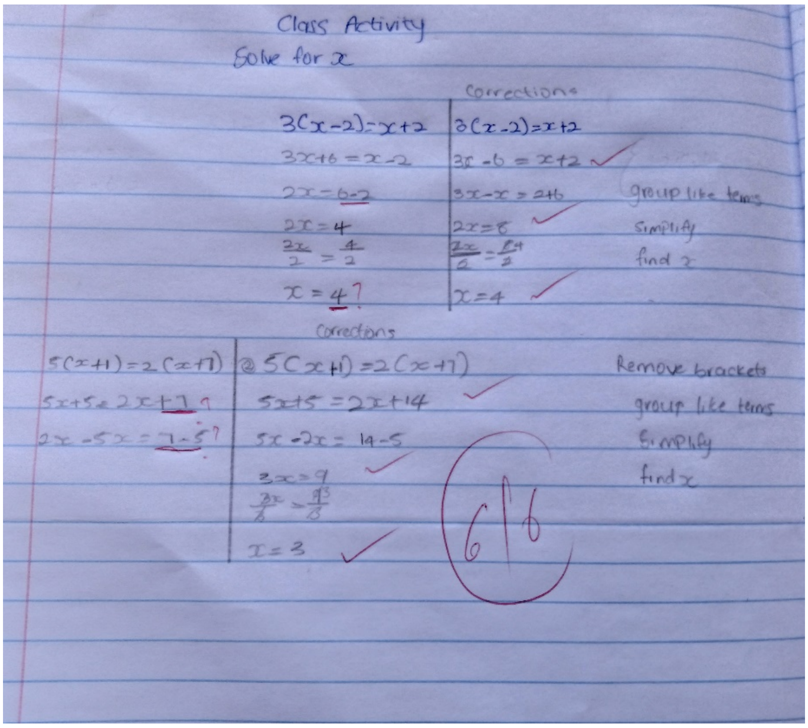

Figure 6.3B an extract of marked exercise from learner's exercise book

Again, from Figure 6.3B Anele marked the exercise and guided the leaner to do corrections. It was observed that Anele did not engage with the learner to find out how and why, for example, the learner can divide 4 by 2 and get 4. Anele simply wrote on the board and explained to learners how to do their corrections. Anele marked the corrections and took it to be the case that the learner now understands, because the learner can now have a tick in their exercise book. The researcher observed this trend in Anele's class, and found that Anele failed to interrogate and engage with errors that learners made before assisting the learners to do corrections.

6.4.3 Knowledge of students and content in relation to teachers' awareness of commonly identified errors

Sapire et al. (2016) argue that error analysis requires the teacher's awareness of learners' errors and the use of specific probing questions about learners' diagnostic reasoning during teaching and assessment. This involves teachers explaining specific mathematical content primarily from the perspective of how learners typically learn the topic or "the mistakes or misconceptions that commonly arise during the process of learning the topic" (Hill et al., 2008, p. 375). Sapire et al. (2016) maintain that because of social, environmental, and economic factors that affect the learning process of concepts, some learners learn through initial misconceptions; therefore, teachers need to develop a range of explanations with a view to address differences in the classroom. The excepts below illustrate what took place in the classrooms.

Zafira introduced her lesson on indices by first explaining some common errors that learners make in general, and presented them in a table, as shown in Table 6.2.

Table 6.2: Lesson presentation of errors learners make and correct method

Common errors learners make	**Correct method**
1.1 Solve for x: 2^X=8 2^x=8 $\frac{2^x}{2}=\frac{8}{2}$ X=4 Check: If you substitute x=4 in the equation you will get 2^4 which is 2.2.2.2=16, and not 8. Therefore x≠ 4	1.1a. Solve for x: $2^x = 8$ 2^x=8 2^x=2.2.2 $2^x=2^3$ X=3 Check: If you substitute x=3 in the equation you will get 2^3 which is 2.2.2=8. Therefore x=3

Throughout the activity the participating teacher pointed out certain common errors that learners make about the concept of exponential equations and procedures for correcting them. The teacher helped learners to learn mathematics through using other learners' errors. However, at no time were the learners in the class given time to solve the sums themselves in order to explore if they also make the same errors or whether new errors might emerge. While Zafira could be commended on drawing from the knowledge of content and students could assist the current learners to understand the concept, less could be said about her ability to engage with learners in instances that have the potential of arising in the classroom. What transpired in the classroom observation of Zafira is contrary to the argument made by Enu and Ngcobo (2020) that learners can be assisted to learn mathematics through the errors they make. The learners in Zafira's class were assisted to learn mathematics through the errors that have been commonly made in the past, without taking into consideration their own errors and addressing their own misconceptions.

In the interview, participants were asked if they cater for errors learners make in their lesson preparation. All three participants held the same view:

> *They do not include it in the lesson plan, but they explained that in the cause of teaching they have in mind the common errors learners make and remind learners not to repeat the same mistakes.*

Again, from the interview, when asked about their engagement with learners' errors, participants iterated the following:

> **Anele:** Usually, I look at three things: 1. Is just marking and providing corrections to that activity on the board and going through each question step by step and trying to clarify. 2. With me it is practically impossible to be in a class where my 40 learners are lost. I use the learners who are strong to come and present to do what I call peer teaching in my class, because of understanding that sometimes learners learn better when they teach one another.
>
> **Zafira:** For example, if we had a test I made them write the test, they write a test and then after the test they revise the test, and then I do engage with learners' errors immediately I notice that an individual learner is making a mistake in answering questions in class. I ask other learners who understand me better to explain to those struggling, and to me it works perfectly. With the formal quarterly test I take a sample of the errors and speak to them when I am giving feedback on their performance in general. We do diagnostic analysis but this is for filling and moderation purposes.
>
> **Kwaku:** Yeah, I do engage with learners' errors. I do error analysis when I am revising the task. When I have given an assessment when I am assessing on the board, then I do analyses of those errors in a form of correction, and tell them that you have made these errors and next time when you write the same paper or when you get to the exam you mustn't do these errors

Drawing from data from the classroom observations and interviews it could be argued that attending to learners' errors is part of teaching, and thus needs to happen in the classroom. However, when teachers have limited understanding of how to engage with learners' errors, the emphasis is shifted to reteaching the concepts, ignoring the aspect of learners relearning the concept. As evident from this study, teachers' limited knowledge of how to engage with learners' errors does impact on the classroom practices. When talking about engaging with learners' errors, the participants mostly emphasise reteaching of the concepts, and when explaining strategies used they mostly emphasise doing corrections on the board. While reteaching and doing corrections on the board is needed, allowing learners to explain their reasoning would go a long a way to ensuring that in-depth learning is taking place.

Based on the findings of this study, except for the ability to recognise the correctness of the answer, the participants did not engage with learners' errors cognitively. The findings of this study concur with those of Sapire et al. (2016), who posit that while teachers seem competent to identify learners' correct and incorrect responses, they lack the skill and knowledge to interrogate learners' reasoning. Similarly, Aksu and Kul (2016) also posit that teachers lack the required knowledge to address learners' errors. Ndlovu et al. (2017) argue that when people's subject matter knowledge has not fully developed, they generally struggle to engage with learners' errors. It is probably the case with these participants, that although they consider content knowledge to be the important driving factor in addressing learners' errors – as they mentioned in the interviews – their subject matter knowledge has not fully developed.

CHAPTER SEVEN

FINDINGS, RECOMMENDATIONS, AND CONCLUSION

7.1 Overview of the study

This study explores the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics through an exploratory case study. The sampling was purposive, with three Grade 9 mathematics teachers from three high schools in Harry Gwala District in KwaZulu-Natal province selected for study. The study is qualitative in nature and guided by the interpretivist paradigm. The interpretivist paradigm was selected because the researcher sought to gain in-depth knowledge on the extent to which Grade 9 mathematics teachers engage with learners in the teaching and assessment of mathematics. The data used in this study were generated using semi-structured interviews and classroom observation as the main data collection methods, as well as document analysis.

The aim of the use observation was to provide the researcher with first-hand information on how teachers engage with learners' errors during teaching and to corroborate the data generated from the interviews. Three participants were interviewed and two of them were observed, which assisted the researcher to generate data to answer the main research questions.

This thesis on the study consists of seven chapters. In this concluding chapter the researcher revisits the research questions and provide a summary of the main findings and the conclusion of the research. The researcher also highlights the implications of the study and suggests recommendations to improve the teaching and learning of mathematics in Grade 9 in the Harry Gwala District and in South Africa in general. Furthermore, suggestions are made for future research.

7.2 The main findings of the study

After analysing and discussing the data for this study, a conclusion on the main findings was drawn by linking the findings to the conceptual framework design in order to construct arguments to cement the findings coherently, to answer the following three main research questions of the study:

1. To what extent do Grade 9 teachers engage with learners' errors in their teaching of mathematics?
2. How do Grade 9 teachers engage with learners' errors in teaching and assessment of mathematics?
3. Why do teachers in Grade 9 engage or not engage with learners' errors when teaching or assessing mathematics?

The researcher answered the research questions, with the main focus on strategies, knowledge, and practices, as evident below.

7.2 Strategies teachers use to engage with learners' errors

In the quest to answer research question one, the findings of the study review the extent to which teachers engage with learners' errors. The findings reveal that participants consider remedial teaching as one of the ways to engage with learners' errors. Although participants advocated for remedial teaching, their practices reveal that whole-class corrections and peer teaching are those dominantly used. For example, the participating teachers (Zafira, Kwaku, and Anele) reported that they do corrections when they identify errors in learners' work, to assist learners to improve their knowledge of a concept.

The focus of the main strategy that the participants were comfortable with when doing corrections was to use peer teaching where the fast learners assist the slow learners using the classroom chalk board; then later the teacher re-explains the procedural steps of what learners got wrong. This finding resonates with one of the three categories of dealing with errors indicated in the literature by Brodie (2014): correction, probing, and embracing the errors. Data from the classroom observations revealed that the teachers engaged with learners' errors by providing corrections after assessment.

Furthermore, the findings revealed that the participants spend less time interrogating learners' incorrect responses than they do paying particular attention to learners who give correct answers. For example, in the process of doing corrections, if a learner gives an incorrect answer the teacher would then ask other learners or the class what the correct answer is. While they talk of remedial teaching, the findings show that they have limited understanding of remedial teaching and therefore could not implement it effectively in the classroom. What becomes more apparent is that while the participants effectively identify the incorrectness or correctness of learners' responses, demonstrating their development of the common content knowledge of the taught topic, they pay less attention to learners' reasonings. This contradicts the argument by Maharaj (2014) and Ndlovu and Brijlall (2015) of the importance of engaging learners with what they write. The findings not only revealed the lack of engaging learners with what they wrote, they also revealed the lack of engaging learners with what they say, as limited attention was paid to interrogating learner reasoning during class corrections.

For teachers to be able to address their learners' errors effectively, they must anticipate what their learners are likely to think and what they will find confusing. That is to say, knowledge of content and students (KCS) is useful in helping teachers hear and interpret learners' emerging and incomplete thinking. Grade 9 teachers in this study opined that for them to be able to deal with errors in teaching and assessment of mathematics, one fundamental thing required of every teacher is knowledge of the content. This finding can therefore be said to be partially in agreement with that of Gardee and Brodie (2015), that teachers should employ their professional knowledge to decide when and why it is appropriate to correct, probe, and embrace errors considering their knowledge of content and their learners.

7.3 Teachers' knowledge to engage with learners' errors

In answering research question two as to how Grade 9 teachers engage with learners' errors in teaching and assessment of mathematics, drawing from the framework, Sapire and Herholdt (2014) defined error analysis as the study of errors in learners' work with a view to finding explanations for these reasoning errors. This therefore means that error analysis is the study of identifying mistakes learners make during teaching and assessment. Even though the findings reveal certain discrepancies between the participants' practices and understanding of engaging with learners' errors when teaching and assessing mathematics, the researcher observed that participants are very aware of the learners' errors; however, they were not aware in terms of what they were doing to correct learners' errors.

Furthermore, the researcher noted during classroom observation that the participating teachers paid more attention to addressing and explaining the procedural steps that learners have performed incorrectly, suggesting development of their common content knowledge of the taught concepts. However, as mentioned above, the researcher found that the participating teachers failed to diagnose he reasons for learners' errors or to understand why learners make mistakes. Inability to diagnose learners' errors and reasoning leading to the incorrect answer means that the underlying misconceptions that govern the errors could not be rectified, thus becoming a perpetual barrier to learning.

7.4 Teachers' practices of engaging or not engaging with learners' errors

The researcher uses the classroom practices of the teachers to answer the question of whether they do or do not engage with learners' errors. The findings revealed that teachers do engage with learners' errors when teaching and assessing, when doing corrections and providing verbal feedback to learners. However, such engagement provides a short-term solution to the problem, since the underlying misconceptions have not yet been identified.

7.4.1 Teachers provide remediation to learners

According to James and Folorunso (2012) remediation refers to the process of leading learners to be aware of their errors and engaging in a possible correction. It is meant to correct deficiencies in learners, either individually or as a group. The role of remediation in the classroom is to serve as a levelling-up device (James & Folorunso, 2012), in the sense that students who failed to master certain materials are allowed or provided with the opportunity to level-up with those who had mastered them earlier. When teachers try to level-up to allow those who have not mastered a particular concept to do so, then they need to design a different strategy to reteach the concepts that have been taught already. The researcher noted from the findings that such practices were not being implemented by the participating teachers.

During the interviews one teacher mentioned that one of their duties as a teacher is to implement school policies, and remedial teaching is one of the school policies to improve performance; however, their understanding of the processes was minimal. Two participants (Kwaku and Zafira) held the view that remedial teaching is continuous teaching. Anele held a different view, that remedial teaching is the reteaching of concepts. None of the participants seem to consider that remedial teaching should go hand in hand with diagnosis and sizing of learners' errors. In the CAPS document (Department of Education 2012) emphasis is placed on the need for teachers to do remedial teaching after diagnostic analysis of learners' errors; however, as has transpired in this study, the implementation of remedial teaching should resonate with one's understanding of what remedial teaching is.

7.4.2 Importance of teachers' mathematical knowledge towards engagement with learners' errors

The researcher defines mathematical knowledge as the special classroom knowledge that teachers need to have to develop strategies to engage with learners' errors. The participating teachers (Zafira, Anele, and Kwaku) in this study agreed that the teacher's mathematical knowledge is important for teachers to do their work as mathematics teachers effectively. They held the view that teachers can only identify and correct learners' errors if the teachers themselves have good knowledge in their content area. However, one of the participants (Zafira) argued that teachers do not need only content knowledge; she claims that teachers' mathematical knowledge also encompasses the methodologies teachers use when teaching.

From the findings, the researcher noted that the participating teachers are aware of the errors that learners make before they could use strategies to correct them. It can therefore be concluded that to some extent the participating teachers have mathematical knowledge that enables them to identify when learners are giving incorrect or correct responses. However, due to limited attention given to engaging with learners' reasoning, the kind of feedback provided to learners does not serve to improve learning but rather to correct the misapplication of rules and procedures. Feedback from errors promotes learning and encourages learners to take responsibility for their performance. This means that feedback provides a reinforcement effect in learning (James & Folorunso, 2012). The findings of this study reveal that all of the participating teachers mostly used oral feedback to tell learners if they were wrong or correct during teaching, to give an immediate response to wrong answers given by learners. During the oral feedback questions and answers were used to engage with the learners. In addition, the form of feedback given on written work is marking the answer as correct or incorrect, without providing an explanation to learners as to why the answer is wrong and how they can improve.

7.5 Limitations of the study

The following limitations were associated with the study. First, this study was qualitative, and data were generated from three participants from three schools; therefore the results could not be generalised to the other schools. However, similar studies can be conducted in other schools in similar or different settings to ascertain to a greater extent teachers' strategies and practices in engaging with learners' errors. Although participants agreed to take part in the study, being an outsider meant that the researcher needed to spend a considerable amount of time at the research site to gain the trust of the participant and understand the dynamics of the school. While every attempt was made to spend considerable time at each research site, not being allowed to observe one participant in the classroom meant that the data collected at one of the research sites could not be triangulated. However, analysing learners' written work supplemented part of the data that could have been gained by observing the teacher in practice.

7.6 Recommendations and implications for teaching

The following are the recommendations that emerged from the study: error analysis as a teaching strategy for mathematics, teacher development on remedial teaching, and amendment of the annual teaching plan to include remedial teaching

7.6.1 Error analysis as a teaching strategy for mathematics

The researcher defines error analysis as the study of identifying an error in learners' answers and using mathematical knowledge to develop mathematical strategies to help deal with them. The researcher therefore recommends that error analysis be adopted as a teaching strategy in mathematics, as is done in linguistics. This is because the researcher holds the view that error analysis and mathematical knowledge for teaching should be seen as one component, and not isolated. This echoed by Herholdt and Sapire (2014), who posit that error analysis requires strong mathematical content knowledge and pedagogical knowledge to enable the teacher to adopt the right strategies to correct learners' errors.

This claim is supported by Kazima and Adler (2006), who asserted that teachers' understanding of error analysis should be seen as a component of teaching mathematics. The incorporation of error analysis has the potential to address the gaps identified in this study of the lack of teacher interrogation of learners' reasonings and proper implementation of remedial teaching. Based on the findings of the study, failure to engage with learner errors suggests that learners' misconceptions are not attended to, which later hinders learning; thus adopting error analysis as a teaching strategy would mean that teachers incorporate learners' errors in everyday teaching. In terms of further research, the vast literature has paid attention to exploring the causes of learners' poor performance in mathematics and identifying errors made; however, the findings showed that teachers' engagement with learners' errors is an area that needs to be investigated, because teachers' understanding of remediation is key in their planning of alternative strategies to address the errors.

7.6.2 Continual teacher developmental workshop on remedial teaching and teacher content knowledge

The findings of this study reveal that teachers have limited knowledge about remedial teaching and its implementation. The researcher recommends that since diagnosis of errors is included in the CAPS document, continual developmental workshops are necessary to ensure that teachers are equipped with knowledge and the know-how of how to implement it in the classroom. This is because teachers' involvement in, applying their mind in error analysis is an integral aspect of teacher knowledge (Sapire et al., 2016) of teaching. While continual teacher development for in-service teachers is critical, the researcher further recommends that institutions of higher learning that train teachers should do their best to train pre-service teachers on the meaning and importance of remedial teaching as an important instrument to correct learners' errors, to help learners to develop their conceptual knowledge of mathematical concepts.

7.6.3 Further studies

This study is a qualitative case study that involved three schools in the Harry Gwala District of Education in the KwaZulu-Natal province. However, engagement with learners' errors is a broad field and therefore further researcher is needed to solicit strategies that teachers use to engage with learners' errors, in order to improve learner performance in mathematics. Also, the study explored teachers when teaching certain mathematical concepts, not all mathematics concepts; hence it is recommended that further studies be conducted to understand the extent to which teachers engage with learners' errors when teaching concepts that have been identified as challenging for learners, such as geometry.

7.7 Conclusion of the study

The study adopted an exploratory case study, was qualitative and used an interpretivist lens. The sampling was purposive, with three Grade 9 mathematics teachers from three high schools in Harry Gwala District in KwaZulu-Natal province taking part. The data were generated through the use of semi-structured interviews and classroom observation as the main data collection methods, as well as document analysis. The study aimed to explore the extent to which Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics.

The findings of the study revealed that the level or the extent to which teachers engage with learners' errors is limited to doing whole-class corrections and peer teaching. It was also found that limited understanding of what remediation entails hindered the implementation of effective remedial strategies in the classroom. Of the six key elements articulated in the conceptual framework, that need to be incorporated in the classroom when teachers engage with learners' errors, the findings showed that the participating teachers only display awareness of the errors when dealing with procedural aspects of the learners' answers or solutions.

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APPENDICES

APPENDIX A: DATA COLLECTION INSTRUMENT

TOPIC OF THE STUDY:

Exploring grade 9 mathematics teachers' engagement with learners' errors in the teaching and assessment of mathematics at Harry Gwala District.

OBJECTIVES OF THE STUDY:

The objectives of the study are:

1. To determine the approach adopted by grade 9 mathematics teachers to engage with learners' errors in the classroom
2. To explore the extent grade 9 mathematics teachers, engage with learners' errors

RESEARCH QUESTIONS

1. What strategies used by grade 9 teachers to engage with learners' errors in the teaching and assessment of mathematics
2. How do grade 9 grade mathematics engage with learners' errors
3. Why do grade mathematics engage with learners' errors when teaching or assessing mathematics

DATA COLLECTION PROCEDURE:

- The researcher will first of foremost make an appointment with the participants to meet them in their respective schools
- The researcher will then request to conduct 15 to 20 minutes interview with each of the six participants individually
- After that the researcher will request for 10 to 15 minutes class observation as the participants (teachers) teach in their respective schools and classes each
- The researcher will then request for sample of learner's exercise books from the participants for document analysis purposes.
- The researcher will visit the participants and do a member check with them to see if the data generated is accurate and correct before publishing the study.

- After the study, the researcher will give one copy of the research to each of the participating schools for records

Data collection instruments:

Interview

Observation

Document analysis

INTERVIEW QUESTIONS:

1. Department of education suggest that teachers need to do remedial teaching. If I may ask what is your understanding of remedial teaching?
1. How do you do it in your class, let us say learners failed one of the assessment tasks.
2. What strategies do you use to understand learners' errors either when you are teaching or assessing?
3. After you have assessed your learners, how do you give feedback to make them understand the errors they have made
4. If learners continually do same mistakes repeatedly in different test, how do you help them address the errors they make
5. Do you perhaps do analysis of learners' errors after they have written a test? If you do may I request to see a sample perhaps for one or two tasks where analysis was done
6. Let us say Zafira (pseudoname of a learner) gave an incorrect response in a lesson or a correct response. How do you interrogate his thinking process?
7. In your lesson plan do you perhaps take account of learners' errors when preparing a lesson? If you do can explain how you do that? May I see some of your lesson plans if you do not mind.
8. Do you think it is necessary to identify learners' errors? Why?

OBSERVATION

During observation, the researcher will observe the following:

Strategies used to engage with errors

a) How does a teacher respond to learners correct and incorrect responses?
   If the teacher during teaching do engage with learners' response using the following codes adopted from Sapire et al and modified to suit this study:

a) Awareness of the error
b) Procedural knowledge of the error
c) Conceptual knowledge of the error
d) Unpacking the misconception governing the error

DOCUMENT ANALYSIS

The following document will be analysed

- Learners' exercise books and test scripts (10 % in each school, three learners book will be requested, top learner, average learner, weak learner. This will be at the discretion of the teachers
- Teacher lesson plan

In the learners book the researcher will analyse the marking.

a) Does the teacher explain to learners why the answer is incorrect?
b) Does the teacher focus on the method or the answer?
c) Are corrections checked? Do learners explained their thinking strategies. If not does the teacher emphasis this?

Teachers lesson plan

a) When planning lesson do teacher incorporate learners' errors
b) Are there any lesson plan targeting remedial teaching?

APPENDIX B: GATE KEEPER'S LETTER

APPENDIX B

KZN DEPARTMENT OF EDUCATION
LITTLE FLOWER COMBINED SCHOOL
PRIVATE BAG X553
IXOPO
3276
EMIS NO: 187 183
HARRY GWALA DISTRICT

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus

PERMISSION LETTER TO SCHOOL PRINCIPALS

TO: The principal

July 2018

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN YOUR SCHOOL

My name is TAWIA IDDRISU FRIMPONG who is a master's student at University of Kwazulu Natal with student number 216067505. I am a registered full time research masters student at the University. My interest of research is "exploring how grade 9 mathematics teachers engage with learners errors in teaching and assessment in Harry Gwala District particularly Ixopo circuit." Your school is among the favorite Schools selected for this study.

Mathematics teachers who teach grade 9 from your school are kindly requested to assist this study as participants. This study will use interview, observation and document analysis as a tool for data collection. Responses will be treated with confidentiality and pseudonyms will be used instead of the actual names of the school and participants for ethical reasons. Participants will be contacted in time to schedule for the interview. Participation will always remain voluntary which means that participants have a choice to withdraw from the study for any reason, anytime it they so wish without any penalties.

DECLARATION:

I ...Mr. J.Y.F. Vezi...(name), Principal of the school on this day of ...6...month...August...2018. Hereby grant permission to go ahead with the research in the school following the terms of reference noted in this request letter.

Signatu[redacted]

Date

6.08.18

3

APPENDIX B

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus

PERMISSION LETTER TO SCHOOL PRINCIPALS

TO: The principal

July 2018

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN YOUR SCHOOL

My name is TAWIA IDDRISU FRIMPONG who is a master's student at University of Kwazulu Natal with student number 216067505. I am a registered full time research master's student at the University. My interest of research is "exploring how grade 9 mathematics teachers engage with learners errors in teaching and assessment in Harry Gwala District particularly Ixopo circuit." Your school is among the favorite Schools selected for this study.

Mathematics teachers who teach grade 9 from your school are kindly requested to assist this study as participants. This study will use interview, observation and document analysis as a tool for data collection. Responses will be treated with confidentiality and pseudonyms will be used instead of the actual names of the school and participants for ethical reasons. Participants will be contacted in time to schedule for the interview. Participation will always remain voluntary which means that participants have a choice to withdraw from the study for any reason, anytime if they so wish without any penalties.

DECLARATION:

I FRANCIS NKOSINATHI HLONGWA (name), Principal of the school on this day of 27 month July 2018. Hereby grant permission to go ahead with the research in the school following the terms of reference noted in this request letter.

Signature of school principal

[redacted]

Date

27-07-2018

NKWENKWANE HIGH SCHOOL
P.O. ...
IXL ...
Cell: 0792081139

3

APPENDIX B

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus

PERMISSION LETTER TO SCHOOL PRINCIPALS

TO: The principal

July 2018

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN YOUR SCHOOL

My name is TAWIA IDDRISU FRIMPONG who is a master's student at University of Kwazulu Natal with student number 216067505. I am a registered full time research masters student at the University. My interest of research is "exploring how grade 9 mathematics teachers engage with learners errors in teaching and assessment in Harry Gwala District particularly Ixopo circuit." Your school is among the favorite Schools selected for this study.

Mathematics teachers who teach grade 9 from your school are kindly requested to assist this study as participants. This study will use interview, observation and document analysis as a tool for data collection. Responses will be treated with confidentiality and pseudonyms will be used instead of the actual names of the school and participants for ethical reasons. Participants will be contacted in time to schedule for the interview. Participation will always remain voluntary which means that participants have a choice to withdraw from the study for any reason, anytime if they so wish without any penalties.

DECLARATION:

I ....MANDY STOUTE..(name), Principal of the school on this day of ....2.... month August....2018. Hereby grant permission to go ahead with the research in the school following the terms of reference noted in this request letter.

Signature of school principal

[redacted]

Date

2-8-18

3

IXOPO HIGH SCHOOL
PRIVATE BAG 554
IXOPO 3276
TEL: 039 834 1063/4
FAX: 039 834 1005

APPENDIX C: CONSENT FORM FOR THE PARTICIPANTS

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,

Dear Participant, 1

Informed Consent Letter

My name is Tawia Iddrisu Frimpong. I am a Masters student studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in exploring how Grade 9 mathematics teachers engage in learner errors when teaching and assessment. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed, as your inputs will not be attributed to you in person but reported only as a population member opinion. The observation that will be made during the interview will only be used to deeply understand your engagement in learner errors when teaching and assessment mathematics and not for anything else, again they will not be attributed to you as a person.
- The interview may last for about 30 minutes
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalised for taking such an action.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment	X	
Photographic equipment	X	
Video equipment	X	

I can be contacted at:

Email: tawiafrimpong@gmail.com

Cell: 0676705019

My supervisor is Dr. Zanele Ndlovu who is a senior lecturer at the School of Education, Edgewood campus of the University of KwaZulu-Natal.

Contact details: email: Nlovuz3@ukzn.ac.za Phone number: +27312603784

You may also contact the Research Office through:

Ms P Ximba (HSSREC Research Office)

Tel: 031 260 3587

Email: ximbap@ukzn.ac.za)

Thank you for your contribution to this research.

Consent form for participants

Topic of the study:

Exploring the extent grade 9 mathematics teachers engage with learners' errors in teaching and assessment of mathematics.

DECLARATION BY PARTICIPANT

I hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project. I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT ...[redacted]... DATE 10/7/19

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,

Dear Participant, 2

Informed Consent Letter

My name is Tawia Iddrisu Frimpong. I am a Masters student studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in exploring how Grade 9 mathematics teachers engage in learner errors when teaching and assessment. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed, as your inputs will not be attributed to you in person but reported only as a population member opinion. The observation that will be made during the interview will only be used to deeply understand your engagement in learner errors when teaching and assessment mathematics and not for anything else, again they will not be attributed to you as a person.
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- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment	X	
Photographic equipment	X	
Video equipment	X	

I can be contacted at:

Email: tawiafrimpong@gmail.com

Cell: 0676705019

My supervisor is Dr. Zanele Ndlovu who is a senior lecturer at the School of Education, Edgewood campus of the University of KwaZulu-Natal.

Contact details: email: Nlovuz3@ukzn.ac.za Phone number: +27312603784

You may also contact the Research Office through:

Ms P Ximba (HSSREC Research Office)

Tel: 031 260 3587

Email: ximbap@ukzn.ac.za)

Thank you for your contribution to this research.

Consent form for participants

Topic of the study:

Exploring the extent grade 9 mathematics teachers engage with learners' errors in teaching and assessment of mathematics.

DECLARATION BY PARTICIPANT

I hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project. I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT ... [redacted] ....

DATE 09/7/2019

School of Education, College of Humanities,
University of KwaZulu-Natal,
Edgewood Campus,

Dear Participant, 3

Informed Consent Letter

My name is Tawia Iddrisu Frimpong. I am a Masters student studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in exploring how Grade 9 mathematics teachers engage in learner errors when teaching and assessment. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed, as your inputs will not be attributed to you in person but reported only as a population member opinion. The observation that will be made during the interview will only be used to deeply understand your engagement in learner errors when teaching and assessment mathematics and not for anything else, again they will not be attributed to you as a person.
- The interview may last for about 30 minutes
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalised for taking such an action.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment	✓	
Photographic equipment	✓	
Video equipment	✓	

I can be contacted at:

Email: tawiafrimpong@gmail.com

Cell: 0676705019

My supervisor is Dr. Zanele Ndlovu who is a senior lecturer at the School of Education, Edgewood campus of the University of KwaZulu-Natal.

Contact details: email: Nlovuz3@ukzn.ac.za Phone number: +27312603784

You may also contact the Research Office through:

Ms P Ximba (HSSREC Research Office)

Tel: 031 260 3587

Email: ximbap@ukzn.ac.za)

Thank you for your contribution to this research.

Consent form for participants

Topic of the study:

Exploring the extent grade 9 mathematics teachers engage with learners' errors in teaching and assessment of mathematics.

DECLARATION BY PARTICIPANT

I hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project. I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT [redacted] ………… DATE 11/07/2019

APPENDIX 1D1: SAMPLE CONSENT LETTER FOR LEARNERS

No 2 marry street

Ixopo, 3276

Kwa Zulu Natal

South Africa

05 Dec 2028

The learner

X School

Dear Learner,

<u>Re: Requesting permission to take a video of the lesson during classroom observation</u>

My name is **TAWIA IDDRISU FRIMPONG**. I am a master's student at University of Kwazulu Natal specializing in Mathematics Education with student number **216067506**. My interest of research is **"exploring how grade 9 mathematics teachers engage with learners errors in teaching and assessment in Harry Gwala District particularly Ixopo circuit."**

I kindly request your consent to take a video when observing the teacher in the classroom. The video will be focusing on the teacher since the teacher is the main participant but because it is a video you may be captured anytime. I therefore request that you sign this consent form for me.

The learner should take note of the following issues:

1. The learner is not a participant of the study
2. The participants are grade 9 mathematics teachers
3. The researcher will use video recorder to generate data from the teachers while observing them in the classroom

4. No information from the learner will be used as data for this study
5. Learners can be capture in the video camera randomly since the video will focus mainly on the teacher in the classroom
6. Learners whose image will be captured in the video will be deleted later and those images will not be published in any form in the study.

I Frimpong T.I signature [redacted] Date 03/07/2019

Kindly wish to request for your permission to take a video while I am observing the grade 9 mathematics teacher engaging with learners' errors in the classroom.

Yours faithfully
Tawia Iddrisu Frimpong

Masters in Mathematics Education
Student no.216067506
Cell: 0676705019
Email: tawiafrimpong@gmail.com

SUPERVISOR: **Dr. ZANELE NDLOVU** (Mathematics Education lecturer at the School of Education, Edgewood campus of the University of KwaZulu-Natal.)

Phone no: +27312603784
E-mail: ndlovuz3@ukzn.ac.za

DECLARATION

I ......Ayanda Zulu...... Do hereby confirm that I have read the above and **give my consent/not to give consent** that the researcher can use a video recorder in the classroom to generate data while observing my teacher. I do accept that with video recorder I can be captured anytime even though the camera is directed on the teacher and therefore I give my full consent.

Signature of student....[redacted]..... Date: ...10-7-2019...

APPENDIX 1D2: SAMPLE CONSENT LETTER FOR PARENTS

No 2 marry street

Ixopo, 3276

Kwa Zulu Natal

South Africa

The Parent / Guardians/ Caregiver

South Africa

<u>Re: Requesting Your Consent To Take A Video During Classroom Observation In The Class Where Your Child Is</u>

My name is **TAWIA IDDRISU FRIMPONG**. I am a master's student at University of Kwa Zulu Natal specializing in Mathematics Education with student number **216067506**. My interest of research is **"exploring how grade 9 mathematics teachers engage with learners errors in teaching and assessment in Harry Gwala District particularly Ixopo circuit."**

I kindly request your consent to take a video while your child is in the class during classroom observation. The teacher in the classroom is the main participant for this study. Whiles observing the teacher in the classroom, a video camera will be used to record the teacher and your child may be capture unaware.

Parents or guardians should take note of the following issues:

1. The learner is not a participant of the study
2. The participants are grade 9 mathematics teachers
3. The researcher will use video recorder to generate data from the teachers whiles observing them in the classroom

4. No information from the learner will be used as data for this study
5. Learners can be capture in the video camera randomly since the video will focus mainly on the teacher in the classroom
6. Learners whose image will be captured in the video will be deleted later and those images will not be published in any form in the study.

I Frimpong T.I signature [redacted] Date 03/07/2019

Kindly wish to request for your permission to take a video while I am observing the grade 9 mathematics teacher engaging with learners' errors in the classroom.

Yours faithfully

Tawia Iddrisu Frimpong

Masters in Mathematics Education
Student no.216067506
Cell: 0676705019
Email: tawiafrimpong@gmail.com

SUPERVISOR: **Dr. ZANELE NDLOVU** (Mathematics Education lecturer at the School of Education, Edgewood campus of the University of KwaZulu-Natal.)

Phone no: +27312603784
E-mail: ndlovuz3@ukzn.ac.za

DECLARATION

I......Mandisa Zulu.................................... (Full names of parent or guardian) I hereby confirm that I understand the contents of this document and the nature of the research project, and I **consent/not consent** to allowing the researcher to take a video of my child or dependent while is directly taking a video of the teacher in the classroom.

SIGNATURE OF PARENT OR GUARDIAN: ....[redacted].

DATE: 10-07-19

APPENDIX 2D1: SAMPLE CONSENT LETTER FOR LEARNERS IN ZULU

No.2 Marry Street

Ixopo, 3276

Kwa Zulu Natal

South Africa

05 Dec 2018

Mfundi

Isikole X

Mngeneli

Isicelo esiya sokuthatha ivideo ekilasini ngenkathi ngozobuka uthisha efundisa

Ngiwumfundi owenza ama Masters enyuvesi yakwaZulu Natal ngaphansi kweso likadokotela Zanele Ndlovu otholakala kanje, ucingo: 0724011275, email: ndlovuz3@ukzn.ac.za. Ngigxile ekucwaningeni imfundo ephakeme kulesisikhungo semfundo ephakeme ebhalwe ngenhla, ngifunda ngesihloko esithi: **Isihloko sesifundo sami sigxile ekubhekeni indlela othisha bezibalo abalungisa ngawo amaphutha ezingane ngesikhathi befundisa noma bemaka**

Umfundi ofunda ugrade 9 akasiyo indlenye yalesisifundo kodwa othisha abamfundisayo bayingxeye, ngakho kuzokwenzeka ukuthi ngenkathi ngithatha ivideo kathisha efundisa bese nomfundi avele.

Naku okubalulekile :

1. Othisha abafundisa izibalo kagrade 9 abazoba yingxenye yalesisifundo hhay abafundi.
2. Uma kwenzekile umfundi wavela kwivideo isithombe sakhe ngeke sisetshenziswa futhi sizocishwa.

Mina FRIMPONG T.I Sayi [redacted] Usuku 03/07/2019 Ngicela imvume yokuthi ngithathe ivideo ngesikhathi ngizobuka uthisha wakho efundisa

Tawia Iddrisu Frimpong
Masters in mathematics eduction
Student no.216067506
Cell: 0676705019
Email: tawiafrimpong@gmail.com

ABABHEKELELE: **Dokotela ZANELE NDLOVU (Ofundisa Izibalo Enyuvesi yakwaZulu Natal, email: ndlovuz3@ukzn.ac.za**

5

ISIBOPHEZELO

Mina........Ayanda Zulu........................ (**Amagama aphelele nesibongo**) ngiyaqinisekisa ukuthi ngiyakuqonda okudingwa yile ncwadi engiyibhalelwe. Ngiyavuma noma angivumi (**dwebela okukodwa**), ukuba nangephutha ngivele kulevideo

UKUSAYINDA: [redacted]

Usuku: 10-7-2019

APPENDIX 2D2: SAMPLE CONSENT LETTER FOR PARENTS IN ZULU

No.2 Marry Street

Ixopo, 3276

Kwa Zulu Natal

South Africa

05 Dec 2018

Mzali/ Mbheki

Isikole X

Isicelo esiya sokuthatha ivideo ekilasini lengane yakho ngenkathi ngozobuka uthisha efundisa

Ngiwumfundi owenza izifundo Masters enyuvesi yakwazulu Natali ngaphansi kweso likadokotela Zanele Ndlovu ongamthola nala: Ucingo 031 260 3784 noma emaile: ndlovuz3@ukzn.ac.za. Mina ngitholakala 0676705019. Ngigxile ekucwaningeni imfundo ephakeme kulesisikhungo semfundo ephakeme ebhalwe ngenhla, ngifunda ngesihloko esithi: **Isihloko sesifundo sami sigxile ekubhekeni indlela othisha bezibalo abalungisa ngawo amaphutha ezingane ngesikhathi befundisa noma bemaka.**

Umfundi ofunda ugrade 9 akasiyo indlenye yalesisifundo kodwa othisha abamfundisayo bayingxeye, ngakho kuzokwenzeka ukuthi ngenkathi ngithatha ivideo kathisha efundisa bese nomfundi avele.

Naku okubalulekile :

1. Othisha abafundisa izibalo kagrade 9 abazoba yingxenye yalesisifundo hhay abafundi.
2. Uma kwenzekile umfundi wavela kwivideo isithombe sakhe ngeke sisetshenziswa futhi sizocishwa.

Mina Frimpong T.I Sayi[redacted] Usuku 03/07/2019 Ngicela imvume yokuthi ngithathe ivideo ngesikhathi ngizobuka uthisha wakho efundisa

Tawia Iddrisu Frimpong
Masters in mathematics eduction
Student no.216067506
Cell: 0676705019
Email: tawiafrimpong@gmail.com

ABABHEKELELE: **Dokotela ZANELE NDLOVU (Ofundisa Izibalo Enyuvesi yakwaZulu Natal, email: ndlovuz3@ukzn.ac.za**

5

ISIBOPHEZELO

Mina Mzali/Mbheki ....Mandisa Zulu........(**Amagama aphelele**) ka....Ayanda Zulu...(**Amagama omfundi aphelele**) ngiyaqinisekisa ukuthi ngiyakuqonda okudingwa yile ncwadi engiyibhalelwe. Ngiyavuma noma angivumi (**dwebela okukodwa**), ukuba nangephutha umntwana wami avele kulevideo.

UKUSAYINDA: 10-07-19 **Usuku:** [redacted]

APPENDIX E: ETHICAL CLEARANCE CERTIFICATE

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

14 February 2019

Mr Tawia Iddrisu Frimpong (216067506)
School of Education
Edgewood Campus

Dear Mr Frimpong,

Protocol reference number: HSS/1603/018M
Project title: Exploring how Grade 9 Mathematics teachers' engage with learners' errors in the teaching and assessment of Mathematics at Harry Gwala District

Approval Notification – Expedited Application

In response to your application received on 10 September 2018, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

[redacted]

Dr Rosemary Sibanda (Chair)

/ms

Cc Supervisor: Dr Zanele Ndlovu
cc Academic Leader Research: Dr SB Khoza
cc School Administrator: Ms Sheryl Jeenarain

Humanities & Social Sciences Research Ethics Committee
Dr Rosemary Sibanda (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** [illegible] / [illegible] / [illegible]
Website: www.ukzn.ac.za

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APPENDIX F: TURNIT IN REPORT

Turnitin

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Mr By Tawia Frimpong

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APPENDIX G: EDITOR'S REPORT

L. Gething, M.Phil. (cum laude)
PO Box 1155, Milnerton 7441, South Africa; cell 072 212 5417

8 April 2021

Declaration of editing of a thesis towards an MEd.:

An exploration of the extent Grade 9 mathematics teachers engage with learners' errors in the teaching and assessment of mathematics

I hereby declare that I carried out language editing of the above thesis on behalf of Tawia Frimpong.

I am a professional writer and editor with many years of experience (e.g. 5 years on *SA Medical Journal*, 10 years heading the corporate communication division at the SA Medical Research Council), who specialises in Science and Technology editing - but am adept at editing in many different subject areas. I have edited a great deal of work for various academic journals and universities, including many theses.

I am a full member of the South African Freelancers' Association as well as of the Professional Editors' Association.

Yours sincerely

[redacted]

LEVERNE GETHING leverne@eject.co.za