

**Exploring pre-service mathematics teachers' knowledge and use of
mathematical modelling as a strategy for solving real-world
problems**

by

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PREFACE

The work described in this dissertation was carried out in the School of Science, Mathematics and Technology Education, University of KwaZulu-Natal, from **September 2006 to June 2008** under the supervision of **Dr Vimolan Mudaly**.

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

Eshara Dowlath
June 2008

Dr Vimolan Mudaly
(Supervisor)

ABSTRACT

Mathematical modelling is an area in mathematics education that has been much researched but conspicuously absent from the South African curriculum. The last few years have seen a move towards re-inclusion of mathematical modelling in the South African school curriculum. According to the National Curriculum Statement (2003a), “*mathematical modelling provides learners with the means to analyse and describe their world mathematically, and so allows learners to deepen their understanding of Mathematics while adding to their mathematical tools for solving real-world problems*”.

The purpose of this study was to explore pre-service mathematics teachers’ conception of mathematical modelling and the different strategies that pre-service mathematics teachers use when solving real-world mathematics problems. This study further investigated pre-service mathematics teachers’ ability to facilitate the understanding of specific mathematical modelling problems.

Twenty-one fourth year Further Education and Training students from the Faculty of Education, University of KwaZulu-Natal participated in this study. In order to obtain appropriate data to answer the research questions, the researcher designed three different research instruments. The open-ended questionnaire and the task-based questionnaire were administered to all the participants, whilst ten participants were chosen to be interviewed. The data that was collected was analysed qualitatively.

The research findings emanating from this study suggested that pre-service mathematics teachers did not have a suitable working knowledge of mathematical modelling, but were

nonetheless able to use their mathematical competencies to solve the three real-world problems that formed part of the task-based questionnaire. It was found that although the participants were aware of different strategies to solve these real-world mathematics problems, they choose to use the ones that they were most familiar with.

It is hoped that this study would prompt more universities to include mathematical modelling courses in the curriculum for prospective mathematics teachers.

DEDICATION

I dedicate this dissertation

To my father the late Ranjit Ramjettan (1932-1975)

A man who was an inspiration to everyone who knew him.

Daddy you are in my heart and thoughts.....always.

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1. My heartfelt thanks and gratitude go to my supervisor Dr Vimolan Mudaly. Your persistence and encouragement helped me see this project through.
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CHAPTER ONE

INTRODUCTION AND OVERVIEW

1.1 Introduction

Mathematical modelling has been and continues to be a central theme in mathematics education. The reason for this is because many questions and problems concerning human learning and the teaching of mathematics affect, and are affected by, relations between mathematics and the real world (Henn & Blum, 2004). According to Huntley and James (1990) mathematical modelling is not a new discovery, nor even a new activity. It has been with us throughout the ages and mathematicians such as Isaac Newton must be regarded as being outstanding mathematical modellers. One of the pioneers in the field of applications and modelling, Henry Pollak, called for an integration of modelling into mathematics teaching as early as in the sixties of the last century. It is, however, an area of mathematics education that has been much neglected, and the last few years have seen a move towards re-inclusion of mathematical modelling in the South African school curriculum.

This new emphasis has been noted by De Villier's (2007), and he pointed out that "*the new South African mathematics curriculum through all the grades strongly emphasizes a more relevant approach focusing a lot on applications of mathematics to the real world*", which indirectly points to the use of modelling skills. He further notes that this focus is in line with curriculum development in most other countries. Blum, Galbraith, Henn and Niss (2007) argue that whilst applications and modelling play more important roles in many countries' classrooms than in the past, there still exists a substantial gap between the ideals expressed in educational debate and innovative curricula on the one hand, and everyday teaching practice

on the other; and claim that genuine modelling activities are still rather rare in mathematics classrooms.

An important purpose of Mathematics in the Further Education and Training band is the establishment of proper connections between Mathematics as a discipline and the application of Mathematics in real-world contexts (Department of Education, 2003a). According to the National Curriculum Statement (2003a), “*mathematical modelling provides learners with the means to analyse and describe their world mathematically, and so allows learners to deepen their understanding of Mathematics while adding to their mathematical tools for solving real-world problems*”.

According to the National Curriculum Statement, learners should use mathematical models and analyse change in both real and abstract contexts; and the mathematical models of the situations may be represented in different ways such as a table of values, as a graph, or as a computational procedure, that is, a formula or expression (Department of Education, 2003a).

Specifically, if one examines the assessment standards of various grades, one is confronted with the following:

Learning Outcome 2: Functions & Algebra

** Use mathematical models to investigate problems that arise in real-life contexts:*

- *making conjectures, demonstrating and explaining their validity;*
- *expressing and justifying mathematical generalisations of situations;*
- *using the various representations to interpolate and extrapolate;*

- *describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and features.*

As a result of this strong emphasis in the new South African mathematics curriculum, the educator in the Further Education and Training band has to ensure that the learners who are interested in the subject or who intend to follow a career path in Mathematics, work towards being able to competently use mathematical process skills such as making conjectures, proving assertions and modelling situations (Department of Education, 2003a).

Whilst keeping in mind, the emphasis of the new South African mathematics curriculum as outlined in the National Curriculum Statement, one must remember that there are various other reasons for including mathematical modelling in the mathematics curriculum. This will be discussed in Chapter 3.

Educators are often faced with the situation where learners make no attempt to use their mathematics in the real-world situations outside school and hence they consider their school mathematics to be of little value. This according to Dossey, McCrone, Giordano and Weir (2002) is because many students believe mathematics is about memorising techniques and formulas, and beyond the grade they are in, they see little value in learning mathematics. Dossey et al. further point out that this view of mathematics arises because the teacher presents concepts and demonstrates techniques in the classroom and the student practices these techniques repeatedly but never understands why they are important or useful. Resnick (1987) comments that practical knowledge (real-life knowledge) and school knowledge are becoming mutually exclusive and that many students see little connection between what they learn in the

classroom with real life. Abrams (2001) suggests that the habits and technical skills involved in creating representations of everyday phenomena must be explicitly taught, and believes that students master them best in the context of an understanding of the modelling process.

Abrams (2001) maintains that in a vital democracy, a primary goal of schooling should be the development of thoughtful, informed and active citizens. He points out that mathematics is an indispensable tool for reaching this goal because with mathematics, we can ask and answer important social, scientific, and political questions and analyze the claims that policymakers advance. He calls this process of using mathematics to study questions from outside the discipline, mathematical modelling. Abrams explains that the methods and skills that we use to model a setting or situation provide us with a simplified representation, or model of it, which uses structures such as graphs, equations, or algorithms. Furthermore during these interdisciplinary inquiries, mathematics is neither the motivation for our work nor the result, instead, mathematics serves as an intellectual lens through which nonmathematical questions can be examined. He declares that when we teach our students to use mathematics in this way, we are providing them with an education that will serve them throughout their lives, to the benefit of society as a whole.

1.2 Rationale for the study

Although some teachers in schools use mathematical modelling, it is often limited to teach topics such as linear programming and certain trigonometric problems. Mathematical modelling could however, be used to teach other topics such as word problems involving algebraic theory, simultaneous equations, sequences and series and financial statements. This study attempted to determine whether pre-service mathematics teachers are able to fulfil the

needs of the new curriculum in South Africa with respect to mathematical modelling. At the same time, the research attempted to convince educators that modelling and its applications are essential components of mathematics learning; and that it is highly relevant to modern society.

As an educator herself, the researcher often used modelling in a limited way, and did not appreciate its relevance within mathematics learning and hence could not convey that to her learners. The researcher's interest in this research study was sparked in Semester 1 of 2006 when she taught a module in Primary Mathematics Education at the University of KwaZulu-Natal at the Edgewood Campus. As a component of the assessment for the module, pre-service Foundation-Intermediate and Intermediate-Senior phase students were required to submit an assignment. The assignment required the students to critique an article related to mathematical modelling. The students thereafter had to devise an activity from any learning outcome from the Revised National Curriculum Statement Mathematics Grade R-9, where learners would be required to solve a real-world problem using mathematical modelling. Only five percent of the students were able to use the given article and create a real-world problem that could be successfully modelled using mathematics.

Whilst talking with these pre-service students at the University of KwaZulu-Natal the researcher found that many students had the misconception that mathematical modelling meant solving any mathematics '*word problem*'. Upon further investigation the researcher found that South African based studies on pre-service mathematics teachers' conceptions and knowledge of mathematical modelling to be rare. Hence the interest in attempting to

investigate pre-service mathematics teachers' conception and knowledge of mathematical modelling.

Seeing that we are living in a data-drenched society, and Mathematics happens in many facets of our lives, for example, in commerce, in engineering in economics, and so on, teachers need to empower learners to be able to cope with this when they leave school and enter a tertiary institution or the workplace. In the words of Blum et al. (2007) "*all persons ought to learn mathematics as it provides a means for understanding the world around us, for coping with everyday problems, or for preparing for future professions. When addressing the question of how individuals acquire mathematical knowledge, we cannot avoid the role of its relationship to reality*". Data, graphs, tables, and statistics overwhelm us in almost every sphere of our lives; from medical reports to financial advice in newspapers. If teachers expose learners to mathematics within a modelling context at school, the learners should be able to analyse and interpret these models in a meaningful way.

In the following extract Huntley & James (1990) highlight some of the attitudes that learners and teachers have about mathematical modelling.

For students, the realisation that modelling does not comprise finding a unique answer to a well-defined mathematical problem, as previously encountered, takes a long time to dawn and requires a change of mental attitude towards problem solving. They generally lack confidence and feel insecure when faced with an imprecise problem which does not have a unique mathematical formulation or solution. A teacher new to modelling also faces problems, not least because it is a difficult subject to teach. In addition to developing, in the student, skills in the understanding of concepts and

techniques, the teacher needs to be concerned with developing innovative skills and an ability to interact between the abstract world of mathematics and the real world of the situation being modelled.

In the light of the above, the purpose of this study was to explore pre-service mathematics teachers' conception and knowledge of mathematical modelling and the different strategies that pre-service mathematics teachers use when solving real-world mathematics problems.

The researcher also attempted to determine whether pre-service mathematics teachers are suitably prepared to ensure that learners develop their ability to design, solve and use mathematical models; as well as to assess the opportunities and limitations of different models. The researcher largely locates this study within the framework of two theories, namely constructivism and Realistic Mathematics Education in order to understand the thinking and reasoning of the research participants.

1.3 Organisation of the study

This dissertation is organized into six chapters, a list of references and the appendices.

Chapter 1 is an introduction to the study and focuses on the rationale behind the study. The researcher also briefly explains the importance of mathematical modelling in today's curriculum.

Chapter 2 comprises a survey of the existing literature pertaining to mathematical modelling. The researcher presents some relevant definitions of mathematical modelling as well as other research that was conducted with reference to the need for prospective teachers to implement

mathematical modelling in secondary schools and some of the barriers that they are faced with.

Chapter 3 describes the three theories which underpin this research study.

Chapter 4 focuses on the research design and the research methodology that was used for data collection.

Chapter 5 discusses the research findings of this study.

Chapter 6 summarises the main findings of this research study and the implications of these findings, considers possible limitations of study and makes some recommendations for future research.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

According to Ostler (2003) we are now faced with the challenge of preparing secondary level mathematics teachers for a new century of instruction, namely, mathematical modelling. He adds that it is therefore important that these teachers of mathematics continue to grow with respect to the pedagogical techniques that have the greatest classroom potential. Ostler advises that researching these techniques, and making them usable for the classroom requires a great deal of effort, but argues that good teachers would certainly agree that the resources they bring to bear on behalf of their learners sets a foundation of success or failure for those learners.

The NSW Department of Education and Training (2001) advocates that students can learn mathematical modelling only by participating in the experience of constructing models and in the struggle to perfect their models. This they claim requires the teacher to allow the students to formulate, to test, to discuss and to adjust their thinking. Furthermore the role of the teacher is to provide the opportunity for this learning to take place and to guide the students through the modelling process, while allowing them the freedom within each stage. They advise that no amount of looking at or of applying models made by others can produce the confidence and skills that flow from making one's own models, no matter how crude. Allowing students to study a problem at the level of mathematics that they are comfortable using and guiding them through all the stages of the modelling process is needed if we are to equip our students with the confidence needed to apply mathematics in their day-to-day lives.

Dossey et al. (2002) view mathematical modelling as one of the richest forms of representation in mathematics. It requires students to work with and apply a variety of mathematical concepts, processes, and relationships. Fennel and Rowan (2001) advocate that using representations whether drawings, mental images, concrete materials or equations, helps students to organize their thinking and to try various approaches that may lead to a clearer understanding and solution.

Dossey et al. (2002) point out that since mathematical modelling is central to understanding the real world, while simultaneously developing worthwhile mathematics, students must be able to connect, through creative problem solving, their understanding of specific content to the modelling situation. They suggest that students at all levels should be given opportunities to model a wide variety of phenomena mathematically in ways that are appropriate to their level. Furthermore, they propose that if we are to realize this goal, then teachers' of mathematics must be able to select and direct activities that will forge their student's capabilities to form, interpret and apply mathematical models in a variety of settings.

The National Research Council (1989, as cited in Schielack, Chancellor and Childs, 2000) notes that questions on modelling also focuses children's attention on visual or symbolic representations that enable them to see important aspects of the problem. According to Fennel and Rowan (2001), representation is a process, an essential component of both, teaching and learning, a way to model mathematics, and a way for students to show their thinking about mathematics. They propose that teachers can use representation to clarify mathematical ideas to students, to access students' mathematical thinking and to help students translate a mathematical idea into a form that they can mentally or physically manipulate to gain

understanding. Consequently Fennel and Rowan point out that when students are able to represent a problem or mathematical situation in a way that is meaningful to them, the problem or situation becomes more accessible. Using representations whether drawings, mental images, concrete materials or equations, helps students organize their thinking and try various approaches that may lead to a clearer understanding and solution.

Lamon (1998, as cited in Nickson, 2004) suggests that modelling is also a powerful means of focussing pupil's attention on the variables and parameters that are inherent in problems and identifying the relationships between them.

An important consideration for the researcher in this study was whether pre-service mathematics teachers were able to facilitate the understanding of the solution of the mathematical modelling process to learners. This concern was necessitated by the need for mathematical modelling in commerce and industry, where the availability of fast and powerful computers has made it possible to 'mathematise' and 'computerise' a range of problems and activities previously unsolvable owing to their complexity. The research was also conducted mainly to determine whether mathematics teachers prepare learners for the increased opportunities for application of mathematics and statistics. The increase in the number of careers in industry and commerce which require a mathematical and statistical input has resulted in a very large and varied field of employment.

Edwards and Hamson (1989) stress that these opportunities can only be met if there are enough newly qualified professionals available with the right qualities to contribute; and teachers need to ensure that learners are adequately prepared to meet these challenges when

these learners are at tertiary institutions. They point out further that it is important for teachers to realise that learning to apply mathematics is a very different activity from learning mathematics, in view of the fact that the skills needed to be successful in applying mathematics are quite different from those needed to understand concepts, to prove theorems or to solve equations. Cross and Moscardini (1985) concur with this and add that modelling requires much more than just the ability to solve a set of equations, no matter how complex they may be.

Garfunkel (2004) agrees that mathematical modelling has become an indispensable tool for industry, government and academia. Furthermore he reports that to address the needs of society and to reform mathematics education to produce a more positive learning environment, the scientific, educational and business communities have consistently called for an increased emphasis on applications and mathematical modelling at secondary school level.

Similarly, Cross and Moscardini (1985) note that as mathematics, science and engineering graduates are more likely to become involved in the development and application of mathematical models, it is useful if they are exposed to the ideas involved in the modelling process.

2.2 Explanation of terms

It is perhaps important at this point to distinguish between physical models, mathematical modelling and mathematical models and applications in mathematics. Arora and Rogerson (1991) point out that irrespective of the different usages of the terms; they have one underlying feature in common, namely approximation to some '*real*' situation.

Arora and Rogerson (1991) explain that the word ‘*model*’ is often used in different contexts and it usually means ‘*a small object, built to scale, to represent some existing, or yet to be produced, object*’, such as scale models of an aeroplane or a building; or the model of a triangular pyramid. Intuitively most people understand what a model is in the physical sense, i.e. it is a scaled down replication of an object. The model shares properties of the original object, but because of its scaled down size, it can be manipulated and studied. In this dissertation, no reference will be made to this type of model.

Blum, Galbraith, Henn and Niss (2007a) remark that using mathematics to solve real world problems is often called *applying* mathematics and a real world problem which has been addressed by means of mathematics is called *application* of mathematics. In order to distinguish modelling from applications, Blum et al. point out that the term “modelling” tends to focus on the direction “reality \rightarrow mathematics” and emphasises the *processes* involved. In other words with modelling we are standing outside mathematics looking in and asking ourselves “Where can I find some mathematics to help me solve this problem?” In contrast to this, Blum et al. explain that the term “applications” tends to focus on the opposite direction “mathematics \rightarrow reality” and emphasises those parts of the real world which are made accessible to mathematical treatment and to which corresponding mathematical models exist. Put differently, with *applications* we are standing inside looking out and asking ourselves “Where can I use this particular piece of mathematical knowledge?”

Huntley and James (1990) maintain that it is important to distinguish between mathematical modelling and mathematical models. They argue that both of these have a distinctive role to play in the teaching of mathematics but there is a great difference between the passive

experience of seeing someone else's model and the active experience of formulating the model for oneself. In the passive role, the student is concerned with applying a mathematical technique to a set-piece model with emphasis on obtaining a solution to the model. Such exercises have an important role to play in the teaching of mathematics, as they help to illustrate the usefulness of, and increase student motivation towards, the technique being taught. However, they do not, on their own, provide insight into the process of model formulation and, therefore, should not be regarded as mathematical modelling.

The following are a few explanations and definitions of mathematical models. Swetz (1999) explains that the word *model* implies something that can be manipulated and that lends itself to experimentation. In a mathematical model, this manipulation and experimentation need not be physical – it can be intellectual. A sense of experimentation should be involved in the conception, realization, and eventual refinement of a mathematical model. According to Edwards and Hamson (1989) any *model* can be defined as a simplified representation of certain aspects of a real system. They thus provide the following as a definition of a mathematical model:

A mathematical model is a model created using mathematical concepts such as functions and equations. When we create mathematical models, we move from the real world into the abstract world of mathematical concepts which is where the model is built. We then manipulate the model using mathematical techniques or computer-aided numerical computation. Finally we re-enter the real-world, taking with us the solution to the mathematical problem, which is then translated into a useful solution to the real problem. The start and end of the problem is in the real world.

Arora and Rogerson (1991) defined a mathematical model as “*the representation or transformation of a real situation into mathematical terms, in order to understand more precisely, analyse and possibly predict there from*”.

Swetz and Hartzler (1991) defined a mathematical model as “*a mathematical structure that approximates the features of a phenomenon. The process of devising a mathematical model is called mathematical modelling*”. They point out some basic mathematical structures which lend themselves to modelling are graphs, equations, formulas, system of equations or inequalities, index numbers, numerical tables, and algorithms. According to Swetz and Hartzler a theoretical model of an object can similarly be constructed and explain that a theoretical model of an object or phenomena is a set of rules or laws that accurately represents that object or phenomena. When those rules or laws are mathematical in nature, a mathematical model has been developed.

It is appropriate at this point to define the term mathematical modelling. Arora and Rogerson (1991) define mathematical modelling as the “*art and exercise of building and working with mathematical models*”. A definition in much greater detail is given by Swetz (1999) as follows:

Mathematical modelling is a process and must be taught as a process. Certainly mathematical modelling involves problems, but it should not be considered as merely a collection of interesting problems and solution schemes. More important, modelling is a multistage process that evolves from the identification and mathematical articulation of a problem through its eventual solution and the testing of that solution in the

original problem situation. The challenge for teachers is to understand this process of mathematical modelling and apply it effectively.

He argues that mathematical modelling as a process should include conjecturing, modifying, and adapting mathematical theories to fit the real-world situation under consideration and then these modified theories can be tested to see whether they supply the required information or solution.

Berry and Houston (1995) describe the process of mathematical modelling as consisting of three main stages. One has to consider a problem set in the real world and *formulate* it as a mathematical problem; this together with any assumptions made is the mathematical model. The mathematical model is then *solved* and finally the solution is translated back into the original context so that the results produced by the model can be *interpreted* and used to help solve the real problem. Swetz (1999) outlines the process of mathematical modelling in more detail as follows:

The process of modelling consists of (1) identifying the problem, including the conditions and constraints under which it exists; (2) interpret the problem mathematically; (3) employing the theories and tools of mathematics to obtain a solution to the problem; (4) testing and interpreting the solution in the context of the problem; (5) refining the solution techniques to obtain a “better” answer to the problem under consideration, if necessary.

De Villiers (1993) describes mathematical modelling as very simply the process of using various mathematical structures such as graphs, equations, diagrams, scatter plots, tree diagrams and tables to represent real-world situations and points out that mathematical

modelling can be used to describe and study a real situation. According to De Villiers (1994), the process of mathematical modelling essentially consists of three steps, (1) construction of the mathematical model, (2) solution of the model and (3) interpretation and evaluation of the solution, as illustrated in Figure 1.

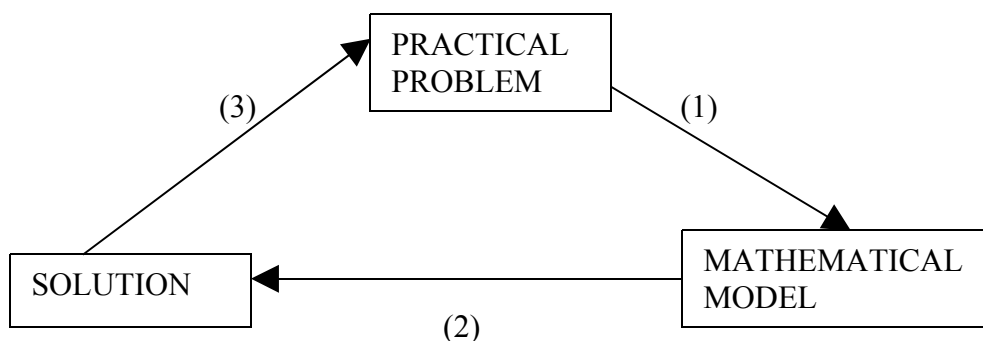


Figure 1

During the construction of the model several processes are necessary:

- the making of appropriate assumptions to simplify the situation
- data often has to be collected, tabulated, graphed, or transformed
- identification and symbolization of variables
- the construction of suitable formulae and/or representations like scale drawings.

During the solution process, we apply mathematical techniques such as factorization, differentiation and solution of equations. Lastly, in the interpretation and evaluation of the solution we need to check whether it is realistic by comparing it with the real world situation.

De Villiers (1994), explains that computer software and calculators can greatly assist us with the routine manipulations involved in the second step once an appropriate model has been

constructed. He however cautions us that the computer is usually of very little assistance in the first and last steps. De Villiers emphasizes that here human ingenuity and understanding is absolutely essential and suggests that should a model be inappropriate, the computer may produce an answer which is completely senseless, that is, computers and calculators can only do what they are told and are dependant on the accuracy of the data or model which is fed into them.

De Villiers (1993) distinguishes between three different categories of modelling:

1. Direct application

The immediate recognition of a certain (familiar) practical situation for which a model already exists or has already been developed, and the direct application of that model (as represented in Figure 2).

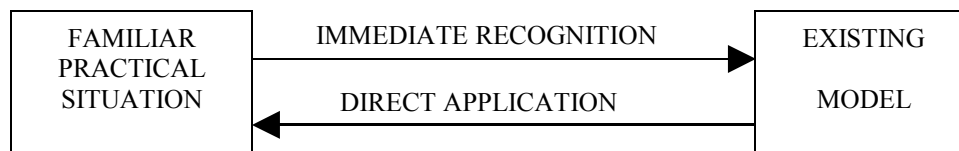


Figure 2

2. Analogical application

The construction of a model to represent an unfamiliar practical situation, after which it is recognized as analogous/similar to an existing model which was developed from another practical context. The existing techniques and methods of this model are then directly applied to the unfamiliar situation. This process is shown in Figure 3.

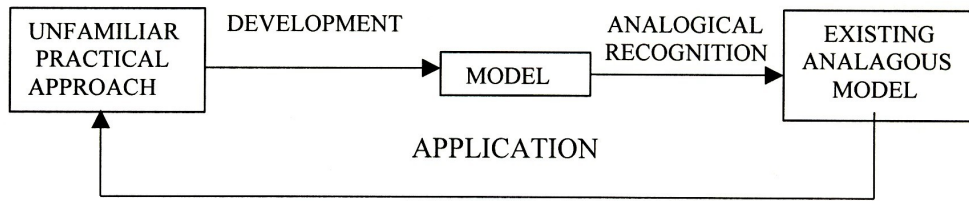


Figure 3

3. Creative application

This category, as illustrated in Figure 4, involves the creation of a previously unknown model consisting of totally new concepts and techniques, to represent an unfamiliar situation. This is one of the many ways in which new mathematics is created.

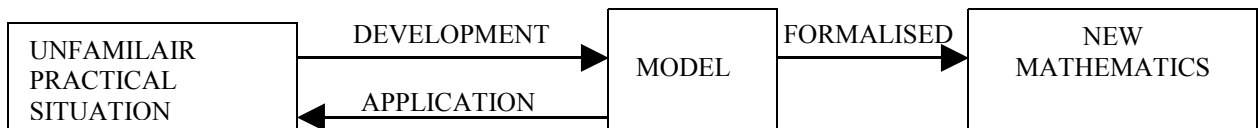


Figure 4

These three categories roughly increase in level of difficulty from category 1 to category 3. Category 3 suggests a powerful teaching strategy which is especially important if mathematics is at all to be represented to learners as a useful tool by which humans strive to understand their world. The real-world problems which were used in this research study were of category 3 level.

Similar models were suggested by Oberholzer (1992), Kaiser (2004) and Crisler and Simundza (2003). In their explanation of the model they isolated four distinct steps. In the first step one has to determine the important characteristics and make sure that the real-life

situation is understood. The data to use is decided upon and some assumptions have to be made. In the second step the mathematical model must be built. The third step consists of analysing the mathematical model using ones ability to do mathematics. Finally when the mathematical model has been analysed the results are expressed in mathematical terms, which then has to be translated back to the language of practice. The modeller then scrutinises the results in real terms and decides whether the results are plausible. If it is not possible, he knows that an error has occurred due to his initial assumptions or in the analysis of the mathematical model. Figure 5 below is a model as illustrated by Oberholzer (1992).

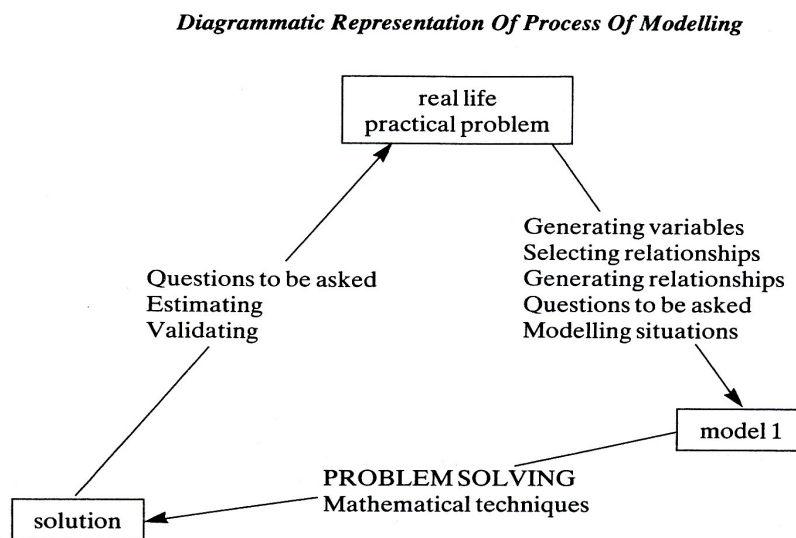


Figure 5

Figure 6 represents the mathematical modelling process as depicted by Dossey et al (2002). They portray the entire modelling process as a closed system. Dossey et al. describe the process as follows:

Given some real world system, sufficient data is gathered to formulate a model. Next the model is analyzed, and conclusions are reached. Finally conclusions about the real

world system are tested against new observations and data. It may be found there is a need to refine the model in order to improve its predictive or descriptive capabilities, or that the model doesn't 'fit' the real world at all, so a new model has to be formulated.

In addition to portraying the entire modelling process as a closed system, they describe model construction as an iterative process. This description is collaborated by Berry and Houston (1995) when they note that the process is started by examining some system and identifying the particular behaviour that is to be predicted or explained. Next, the variables and simplifying assumptions are identified, and a model is generated. Generally the process is started with a simple model, progressing through the modelling process, and then refining the model as the results dictate.

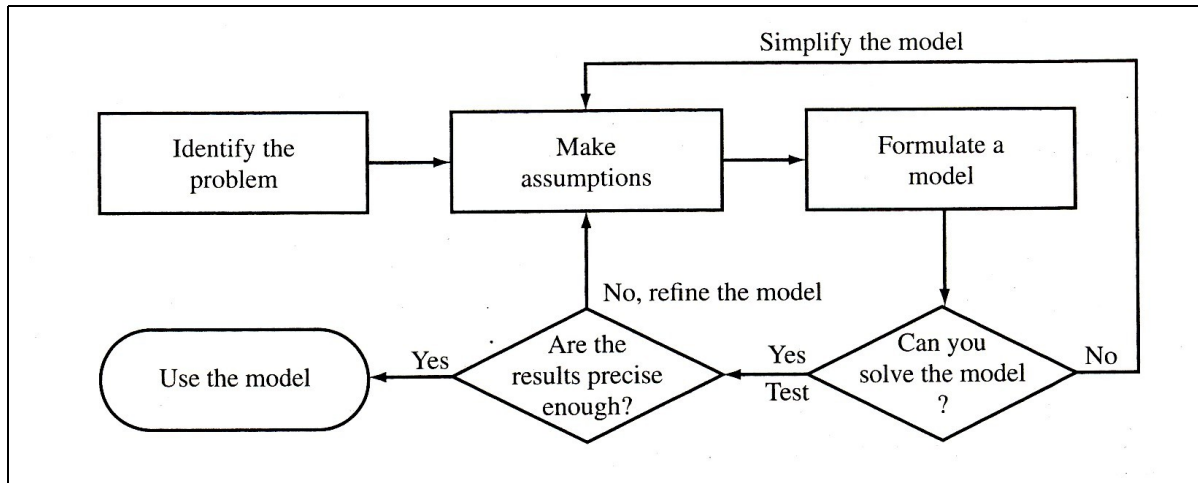


Figure 6

According to Galbraith (1989, as cited in NSW Department of Education and Training, 2001) although there are many variations of the modelling process, each one is essentially saying the same thing. He calls one of his approaches, the "*structured modelling approach*" which uses

real-life situations and the full modelling process from stages 1 to 7 as shown in Figure 7 below.

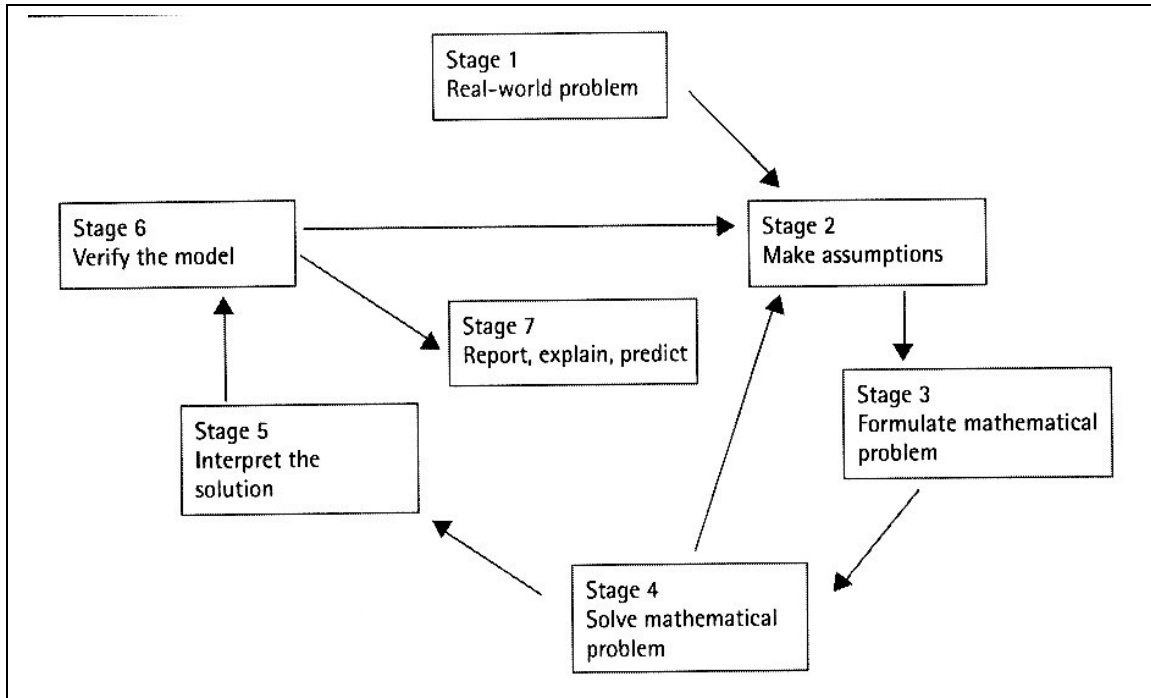


Figure 7

Technology opens many doors when it comes to creating representations. This research study did not require the participants to use a computer to obtain or to verify any of the solutions for the given tasks. However a research study by Mudaly (2004a) investigated the feasibility of introducing mathematical concepts within the context of dynamic geometry. Figure 8 used by Mudaly summarized the process of modelling when using a computer.

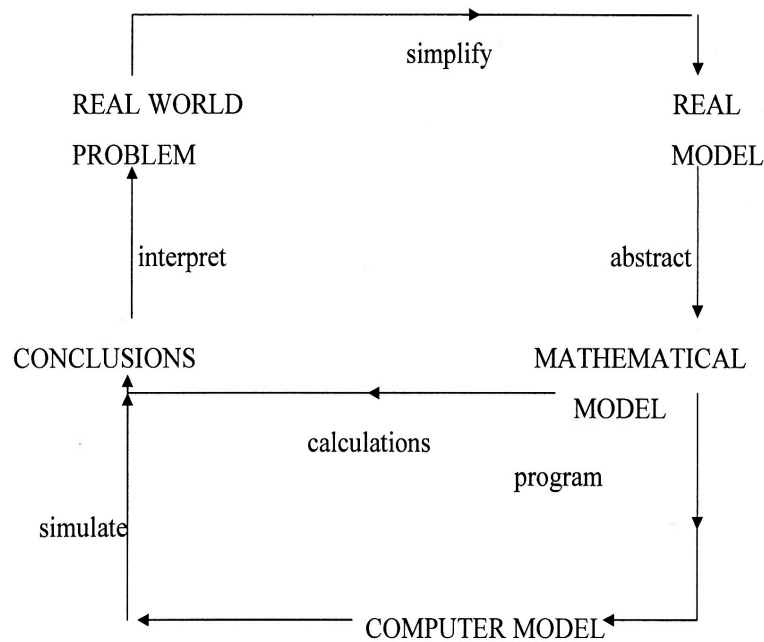


Figure 8

Mudaly (2004a) explains that Figure 8:

.....shows how a real-world problem is simplified, and using mathematical language and equations it is reduced to a mathematical model. Without the use of a computer, calculations may reveal the conclusions that would make the real-world problem much easier to understand. But, with the use of special computer programs, the model can be subjected to various tests within a few minutes. Although the program may not offer a detailed understanding of the problem, it will increase the conviction that the conclusion is valid or not.

Berry and Houston (1995) summarise the following points as key activities in the modelling process:

1. understand the problem
2. define variables to be used in your model

3. set up a mathematical model
4. formulate and solve the mathematical problem
5. interpret the solution, that is, describe the solution in words
6. compare with reality, that is, test the outcomes of your model with appropriate data.

2.3 The need for mathematical modelling

As noted by Verschaffel, De Corte and Greer (2001), a major argument for including verbal problems in the school curriculum has always been their potential role for the development, in students, of skills in knowing when and how to use their mathematical knowledge for approaching and solving problems in practical situations. They term this application of mathematics to solve problem situations in the real world, mathematical modelling. Verschaffel et al. (2001) and Henn (2007) view mathematical modelling as a vehicle for connecting problem solving to the real world. Furthermore, compared with the traditional approach to application word problems, they point out that the modelling perspective offers major advantages in that the process of modelling constitutes the bridge between mathematics as a set of tools for describing aspects of the real world, on the one hand, and mathematics as the analysis of abstract structures. Verschaffel et al. (2001) argue that given the increasing mathematisation of social as well as physical phenomena, they believe that an understanding of this aspect of mathematics is essential for informed citizenship. Moreover, taking aspects of reality into account in modelling the situations described in word problems is a potentially powerful way to connect pupils' mathematics problem solving to the real world, and to modify the belief and feeling of many children that mathematics is irrelevant in relation to their everyday lives.

According to Usiskin (1991) curriculum theorists generally feel that one should begin the building of a curriculum by agreeing on the aims, goals or objectives of that curriculum. Niss (1989, as cited in Usiskin, 1991), listed five aims of teaching applications and modelling.

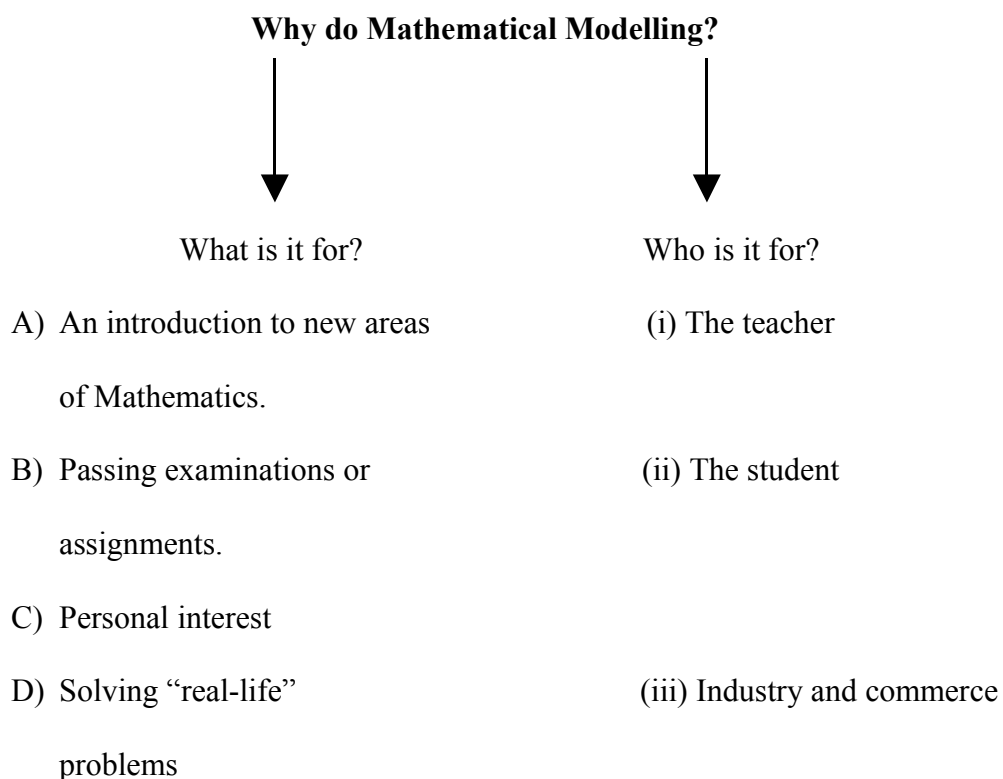
1. to foster creative and problem-solving attitudes, activities, and competencies;
2. to generate a critical potential towards the use and misuse of mathematics in applied contexts;
3. to provide the opportunity for students to practise applying mathematics that they would need as individuals, citizens, or professionals;
4. to contribute to a balanced picture of mathematics;
5. to assist in acquiring and understanding mathematical concepts.

Zbiek & Conner (2006) list the following points as reasons for engaging students in mathematical modelling activities:

1. to prepare students to work professionally with mathematical modelling;
2. to motivate students to study mathematics by showing them the real-world applicability of mathematical ideas; and
3. to provide students with opportunities to integrate mathematics with other areas of the curriculum.

Nickson (2004) asserts that by using a mathematical modelling approach, teachers may be able to identify pupils' thinking at different stages of reaching a solution to a problem depending upon the nature of the model. This he suggests can act as a diagnostic measure to gain feedback on learners' level of understanding.

In order to justify the use of mathematical modelling, Eyre (1991) uses the following diagram to explain who might do such an activity and why.



2.4 Real-world problems and the role of contexts in the mathematical modelling process

At this point it necessary to explain what is meant by *real-world*. Blum and Niss (1991) defined the concept of *real- world* as follows.

By *real world*, we mean the “rest of the world” outside mathematics, i.e. school, or university subjects or disciplines different from mathematics, or everyday life and the world around us. In contrast, with a *purely* mathematical problem, the defining situation is entirely embedded in some mathematical universe.

Albert and Antos (2000) point out that when children make connections between the real world and mathematical concepts, mathematics becomes relevant to them and as mathematics becomes relevant, students become more motivated to learn and more interested in the learning process. They offer the following suggestion for educators:

As educators, it is our responsibility to bring mathematics to life for children. For students to view mathematics with interest and enthusiasm, they must see the relevance between mathematics and everyday life. Teachers can nurture intellectual excitement by linking classroom activities with real-life experiences. As children begin to see and understand how mathematical concepts are used in their daily lives, they become more interested in learning mathematical processes. Their understanding of mathematical concepts and ideas is enhanced and strengthened. When teachers create and maintain instructional practices with which students can identify, ideas are reconstructed, confusion and conflicts are minimized, and understanding is achieved.

Galbraith (2007) distinguishes between word problems and real-world problems and argues that word problems are widely construed as close relatives of modelling problems, since both word problems and modelling problems are couched in verbal clothes. He however, maintains that the two terms differ markedly, specifically with respect to meaningfulness and purpose, seeing that modelling problems have real-world connections, which word problems often do not have. Galbraith explains that whilst a word problem may be couched in the language of the real world there is often no sense in how one is supposed to apply the decisions to one's real-world, since it does not show how mathematics can be applied to enhance understanding of real problems.

Boaler (1993) points out that many view the abstractness of mathematics as being a cold, detached, remote body of knowledge. She argues that this image may be broken down by the use of contexts which are more subjective and personal and also improves the ability of students to interpret events around them. Boaler explains that using real-world, local community and even individualised examples which students may analyse and interpret is thought to present mathematics as a means with which to understand reality and concludes that when learners make connections between the real world and mathematical concepts, mathematics becomes relevant to them. Bottle (2005) supports this view and states that by giving children a real context for their problem solving, gives them the best opportunity to become fluent in using mathematical skills and procedures. She concurs with Boaler and claims that if we make mathematics meaningful for children by relating it to their interests, then they will be more likely to see its relevance and use.

Mudaly (2004b) adds to this thought and states that “*the direct connection between classroom mathematics and real-world mathematics is a tenuous one, because it is often difficult to relate classroom mathematics to what happens in the real world*”. He goes on to say that “*if the word ‘real’ in this instance is not only interpreted as a connection to the real world, but as a reference to the problem situations which appear to be real in the learner’s mind, then the relationship between real-world and classroom mathematics becomes a bearable one*”.

According to De Villiers (2003), real-world situations are extremely complex and usually have to be simplified before mathematics can be meaningfully applied to them. Furthermore De Villiers (2007) explains that in the real world there are no perfectly straight lines, flat planes and spheres, nor can measurements be made with absolute precision. Learners, students and

teachers therefore must be able to make logical assumptions to simplify the original problem. Similarly Engel and Vogel (2004) view models as an over simplification of reality where part of the available information is discarded. This loss of information due to inherent assumptions, simplifications and abstractions does not invalidate the conclusions, rather it is intended to generalise the obtained results to hold true in other similar situations. In the problems (See Appendix B) given to the participants of this research study, various assumptions had to be made. A few of the assumptions are listed below:

PROBLEM 1

- A net profit of R25 and R30 per table and bookcase is not realistic.
- The carpenter assumes that he can sell all the tables and bookcases he produces each week. In reality this may not be always possible.

PROBLEM 2

- It is assumed that the mountain peak is perfectly vertical with respect to the line chosen to be the distance from the base of the mountain peak to the two observers.
- It is assumed that the height of the person is negligible compared to the height of the mountain peak.
- It is assumed that the two observers are standing 500m apart at the same level, that is, one may not be standing at a slightly higher level than the other.
- It is assumed that the angles of inclination were accurately measured.

PROBLEM 3

- It is assumed that the lawn is perfectly rectangular, i.e. there are no flower beds that encroach on to the lawn.

When working with real-world problems where assumptions have to be made, educators need to make learners aware of these assumptions.

2.5 Some obstacles to the implementation of mathematical modelling.

Research (Huntley and James, 1990; Doerr, 2007; Cross and Moscardini, 1985; and Burkhardt, 2004) shows that the attempts to implement mathematical modelling in schools around the world are met with various obstacles. These obstacles will be explained in the following paragraphs.

According to Cross and Moscardini (1985) teachers have to deal with their own problems associated with mathematical modelling. They explain that teachers have been used to a disciplined formal approach to mathematics and most teachers teach mathematics in the way that they have been taught. An important consideration for Doerr (2007) for research on teacher knowledge is the examination of how teachers' models for teaching mathematics develop. She remarks that it is clear that teachers come to their pre-service teacher programs with models of teaching already in place, based on years of apprenticeship as observers of practice. So according to Cross and Moscardini (1985), before teaching learners, teachers have to reorient themselves towards a new conceptual view of mathematics, one where mathematics is sublimated to the more general needs of solving a real world problem rather than just a set of mathematical equations.

Dossey et al. (2002) expand on this thought by stating that many teachers have only experience calculating model solutions- they have never built a model. As a result, they feel anxious about materials that call on them to lead learner investigations of real world problem situations or modelling projects, especially those for which no model is provided.

According to Cross and Moscardini (1985) the structure of many mathematics curricula is rather formal, that is, it is precise, well-structured and largely concerned with rigour. Students are accustomed to this, and Cross and Moscardini outline the following reasons why students find modelling disturbing:

1. the open-ended nature of problems;
2. the emphasis on providing some insight into a system rather than on elegant mathematics;
3. the lack of a 'unique' solution;
4. the necessity of combining a number of mathematical techniques rather than one obviously 'applicable' method;
5. coming to terms with the fact that real-life problems can rarely be solve in a short time.

Huntley and James (1990) also note that in order to effectively formulate a mathematical model, a student needs to have a good understanding of the mathematics being used. In addition to this they declare that there is general agreement amongst those that have been involved in the teaching of mathematical modelling, that it is not an appropriate vehicle for introducing new mathematical ideas and techniques.

2.6 The role of mathematical modelling in the secondary school mathematics curriculum

Swetz and Hartzler (1991) suggested the following reason for wanting to incorporate mathematical modelling into the secondary school curriculum. They argue that one of our ultimate aims as teachers is to prepare young people to function confidently and knowledgeably in real world situations. Seeing that mathematical modelling is a form of real world problem solving, a modelling approach to problem solving focuses a variety of mathematical skills on finding a solution and helps a learner see mathematics in a broad spectrum of applications. The strategies and skills learned in modelling exercises are easily transferable to new situations, and learners involved in modelling experiences obtain a greater appreciation of the power of mathematics. A similar thought is expressed by Zbiek (1998) when she proposes that a major reason to include modelling in the secondary school curriculum is to encourage deeper student understandings of mathematics through developing connections between mathematics and the real world.

Furthermore Klaoudatos (1994) points out that it is of their belief that modelling must constitute one of the essential components of teaching mathematics in secondary education. He adds that it is worth noting the fact, that while this process has attracted interest from at least the early 1980's, few researchers and educators believe in the teaching of mathematics through the modelling process. Klaoudatos explains that by being prompted by the problems which were spotted in the curriculum of Greek secondary education, educationists developed modelling-orientated teaching, a didactical model with which new mathematical concepts could be taught.

Voskoglou (1995) also emphasises the need for mathematical modelling in the curriculum and explains that until the end of the 1970's the process of modelling was a tool used only by those who worked with applied mathematics in industry and economics. But he points out that the failure of 'new mathematics' to give the student the opportunity to increase his capability to use mathematics in order to solve problems of the real world, turned the interests of specialists to using the process of mathematical modelling as a new method for teaching mathematics. Hence, the process of mathematical modelling is one of the introductory subjects in mathematics lectures for freshman at the School of Administration and Economics of the Technological Educational Institute of Mesolongi in Greece.

Furthermore, according to the NSW Department of Education and Training (2001) a concern voiced by the business sector of New South Wales; related to the need for mathematical modelling; drew attention to the perceived lack of transfer of school mathematical knowledge by students to the workplace. In their words "*business and economic interests are demanding that students be prepared as flexible problem solvers in order to be ready to meet the challenges and uncertainties of their fast-changing workplaces*". This appears to be similar to a proposal in the South African National Curriculum Statement for smarter, more adaptable and better educated workers. In order to address the concern by the business sector of New South Wales that although the school provided students with an extensive mathematical 'toolkit', they may have not prepared them adequately in its use; the importance of mathematical modelling is now reflected in the current Australian syllabuses, such as the inclusion of mathematical modelling within the Australian Years 9/10 Advanced Mathematics Syllabus.

Lingefjård (2004) points out that it was not until the middle of 1990 when the term mathematical modelling started to appear explicitly in the Swedish curriculum and quotes from the 2000 English version of the Swedish curriculum “*the school in its teaching of mathematics should aim to ensure that pupils develop their ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models*”. This statement can be easily likened to one of the purposes of teaching mathematics as outlined in the South African National Curriculum Statement.

2.7 What knowledge do teachers need to teach mathematical modelling?

Based on the call for contributions to the 14th ICMI *Study on Applications and Modelling in Mathematics Education*, Doerr (2007) observed that only rarely do mathematics teacher education programs include an orientation to mathematical modelling or the use of modelling in prospective teachers’ mathematics courses. She remarks that this suggests that one of the reasons for the limited use of applications and modelling at the primary and secondary levels of schooling is the lack of knowledge by those who are expected to teach mathematics through applications and modelling.

Similarly Makar and Confrey (2007) point out that a major challenge to widespread use of modelling and applications in secondary schools is teachers’ unfamiliarity with it. Consequently in order for teachers to facilitate the understanding of mathematical modelling, they need to know what mathematical modelling is.

The following list is a guide as outlined by Oberholzer (1992) to address any misconceptions that educators may have regarding the process of mathematical modelling:

Modelling is not:

- A chapter in a textbook that gets taught in weeks three and four in the fourth term;
- A topic that is so difficult that only the very best pupils get exposed to it and that only after the exam when nobody is really paying attention in any event;
- A series of work cards with textbook problems rephrased and the data altered from $r = 14\text{m}$ to $r = 13,85\text{m}$ so as to make it “*real-life*” values; throwing overboard all established methods and principles and replacing them by “*practical, real-life*” situations at all costs;
- A MUST for each and every period of the school year.

To examine the modelling knowledge of pre-service teachers, Doerr (2007) designed an undergraduate course in mathematical modelling where the primary goals of the course were to introduce pre-service teachers to some basic ideas and techniques in mathematical modelling by engaging them in the process of building mathematical models. With respect to subject matter knowledge in pre-service teacher education Doerr’s (2007) study yielded three significant findings.

1. All students were able to adjust their incorrect conceptions to mathematically correct ones through a process of explaining and justifying their models to each other. This suggests that mathematical modelling is a potentially powerful context for the mathematics learning of pre-service teachers.
2. Initially when pre-service teachers were asked to map the processes they had used to create a model, most of them created maps of a sequential, linear nature. However, later in the course, after the pre-service teachers engaged in extended discussions about

the mathematical modelling process, they made a striking shift from seeing modelling as a fairly linear, sequential activity to seeing modelling as a non-linear, cyclic activity.

3. The third finding argues that the nature of the modelling tasks, the range of tools available, the norms for augmentation, and the standards for quality of a solution were significant in influencing the types of modelling behaviour that occurred in each setting. This implies that pre-service teachers need to be exposed to a range of modelling activities that provide multiple opportunities for explanations and justifications of the modelling decisions that were made.

Doerr's (2007) research study described above, suggests two major implications for the pedagogical knowledge of the teacher when teaching with modelling tasks:

1. The teacher needs to have a broad and deep understanding of the diversity of approaches that learners might take. Modelling tasks provide the opportunity for learners to develop a diversity of approaches to expressing their interpretations of a given situation. While this created a rich source of mathematical discussion for the learners, it also placed tremendous pedagogical knowledge demands on the teacher. The teacher should be able to (1) listen for anticipated ambiguities, (2) offer useful representations of learner ideas, (3) hear unexpected approaches, and (4) support learners in making connections to other representations.
2. The second implication for the pedagogical knowledge of the teacher is illuminated in the shift that occurs in giving explanations and justifications. Rather than the teacher giving explanations and justifications to the learners, the discussion of the models created a learning context in which the learners were giving explanations and justifications to each other and to the teachers. The task of the teacher becomes one of

putting the learners in situations where they can interpret, explain, justify and evaluate the “goodness” of their models. The teacher gives the learners the task of refining and revising their models, rather than proceeding to evaluate them herself. This change in pedagogical strategy is a major shift from the more traditional instruction in mathematics where a primary role of the teacher is to evaluate learners’ work.

2.8 Barriers to the implementation of mathematical modelling

Antonius (2007) points out that modelling has had a central position in the Danish curricula in mathematics on upper secondary level for 15 years, but observes that the teaching does not yet reflect goals and intentions. One of the reasons that he provides for this is that the final examination still consists of a traditional written examination with a number of independent, standardised, pre-structured and rather closed tasks.

Burkhardt (2004) puts forward the following 4 barriers to large scale implementation of modelling in mainstream curricula:

1. In many countries the pattern of teaching and learning activities is still dominated by the EEE style of teaching (**E**xplanation, worked **E**xamples and imitative **E**xamples). The focus is on learnt facts, concepts and skills. A key lever to this barrier would be to have well-engineered materials to support assessment, teaching and professional development.
2. Many teachers view the real world as an unwelcome complication in many mathematics classrooms and say that teaching mathematics is demanding enough without the messiness of modelling reality, complicating the clean abstraction of mathematics. As a key lever to this barrier it would be essential to build public

understanding of modelling and its central role in providing students with important skills.

3. Burkhardt views the skills that teachers acquired during their pre-service education as inadequate to deliver a changing curriculum. A lever to this barrier would be the implementation of more powerful professional teacher development programs.
4. The fourth barrier is that research and development in education, as compared with other applied fields, is seldom turned into practice. In other words research in education is seldom generated into reliable solutions that can be implemented on a large scale. A key lever to this barrier would be to have products rather than papers as its key output.

Wheal (2004) describes his experience with South Australian students in their last years of secondary education. He remarks that typically secondary school mathematics has focused on the manipulation of other people's models and has given little attention to the situations which led to the construction of those models. Interpretation was limited to such things as identifying the nature of roots of equations and constructing of models was almost never undertaken as the aim was to implement specified algorithms. He adds that mathematical modelling were usually an additional component in a topic, undertaken when time permitted and after the skills considered essential had been mastered. Wheal views teachers as the key to any change in education and argues that any change in curriculum should be accompanied by a sustained teacher development and support program; the provision of suitable resources as well as complementary changes in the assessment of student achievement.

Lingefjård (2007) considers the rare use of the modelling process in mathematics courses as one of the obstacles in the successful implementation of mathematical modelling in the curriculum. He reports that the situation of mathematical modelling in the training of teachers is a complex situation and even if the national curriculum texts emphasise mathematical modelling, teacher education might be slow and resistant to follow. He argues that effective infusion of modelling in schools will take place if some form of modelling is provided for teachers in the course of their preparation. In the words of Lingefjård (2007):

The process of education of teachers with the aim of learning mathematical modelling should be viewed as a *gradual process achieved over along period* of time. The relevant knowledge required of mathematics teachers can not be met as the result of exposure to modelling in just one generic course. Instead modelling and technology should take place within both the teachers' content and methods courses. This, in turn, calls for collaborative efforts, joint planning, and a shared vision of mathematical modelling as something useful, beneficial, and valuable in mathematics and mathematics methods courses.

Although mathematical modelling now appears in the South African mathematics curriculum Brown & Schäfer (2006) suggest that mathematical modelling can very easily be incorporated into the new South Africa mathematical literacy curriculum by using the basic approach of mathematical modelling and applying it to more elementary mathematics. In a research study carried out by Brown & Schäfer (2006) with mathematical literacy teachers, it was found that the modelling approach worked well with the majority of reasonably and well skilled teachers. They however, found that many of the teachers with weaker mathematical skills took considerably longer to master the contexts and skills developed in the research activities. To

my knowledge there are currently no studies to explore the possibilities and obstacles experienced by South African secondary school mathematics teachers with reference to the implementation of mathematical modelling in the secondary school curriculum.

According to Ikeda (2007), there exists a variety of obstacles to teaching applications and modelling in different countries around the world in which applications and modelling are located within the national curriculum. The writer reports on the experiences of eight countries that exemplify general aspects of the issue of obstacles to teaching modelling. Some of the features of these obstacles are described below:

1. Germany: Being able to apply mathematics competently is an accepted overall aim of German mathematics teaching, but the question arises as to why mathematical modelling examples do not gain the same importance. The reasons for this can be viewed from various points of view: (a) The system: seeing that mathematics teaching is dominated by a subject-based understanding of theory, the lessons start from general concepts and then continue with general conclusions; (2) The teacher: mathematics teachers' perceptions of mathematics are dominated by an understanding of mathematics as a logical and consistent construction of thinking. The notions of usefulness and applications of mathematics play a minor role in the perceptions of mathematics and in the learning of mathematics; and (3) The student: the mathematical beliefs of most students are dominated by an understanding of mathematics as an accumulation of knowledge and the higher the age group, the lower the importance of application-based mathematical beliefs.
2. England: In England, everyone talks of the importance of being able to use mathematics, however, in most lower secondary school classrooms, few applications

are actually taught, and there is no modelling. Reasons for this include: (1) Specialist mathematics teachers focus on the concepts and procedural skills of pure mathematics. Applications, where they exist, are mainly seen as concept reinforcement; (2) the political emphasis is on 'basic skills' for weak students and schools; (3) the belief that you have to learn skills before you can apply them leads to an indefinite postponement of work on solving non-routine problems of any kind. Also many teachers have no experience of the teaching skills needed, and regard non-routine problems as 'unfair'; (4) The 1989 National Curriculum for mathematics and its tests introduced a view of mathematics as a checklist of narrowly-defined skills; (5) because of important testing at ages 7, 11, 14, 16, and 18 teaching focuses on fragments of mathematical performance that are tested; and finally (6) TIMSS has focuses on pure mathematics.

3. Romania: The most pre-eminent obstacle is the theoretical orientation of the teacher training pre-service and in-service programs. Their content is focussed on a highly theoretical level of mathematics, with a very low emphasis on aspects connected with teaching methodology, and even lower emphasis on modelling. With regard to learning, there is a discontinuity at the transition from primary to lower secondary education. The primary teachers teach a number of subjects, while the lower secondary teachers teach only mathematics. Learning is also compromised by the dual system of initial teacher training, where primary teachers are trained in colleges/high schools and are more pedagogically oriented than subject oriented; while the lower secondary teachers graduate at university level with a very low emphasis on the psychological and sociological aspects of teaching and learning. The discontinuity is also manifested at the level of curriculum interpretation. Teaching practice lags behind

the written curriculum recommendations due to insufficient training programs being provided to support the curriculum reform process.

4. Canada: In Canada, although each province is responsible for its own educational system, there are many similarities in the mathematics curricula. Most provinces have a curriculum and curriculum resources that support mathematical modelling. Mathematics educators in Canada realise that students need to see the relevance of mathematics, and mathematical modelling helps them to make the connections. However, in practice many teachers tend to use applications, modelling and problem solving as examples of uses of mathematics once the mathematical concepts are taught. Some of the obstacles to full implementation of mathematical modelling are teachers' understandings of modelling, their view of mathematics and their inexperience in doing mathematical modelling themselves. In Canada teachers are, however, moving along the continuum as professional development engages teachers in mathematical modelling activities.
5. Czech Republic: In the Czech Republic mathematical modelling is used to teach mathematics as well as to teach application and modelling in mathematics. The two problematic issues experienced are difficulties in choosing appropriate problems to model; and with difficulties with the language appropriate for applications and modelling.
6. Mozambique: In Mozambique (as in many lesser developed countries) applications and modelling are not perceived as an important part of the mathematics curriculum. The first obstacle is the existing curriculum which has a strong enforcing role on what is taught in Mozambican mathematics education. Secondly, the national examinations are an obstacle, as they do not contain any applications or modelling items. Another

obstacle is the predominately deductive approach used in mathematics education, whereby students are trained to memorize definitions and algorithms, resulting in little understanding, short-term retention, and low motivation. A further obstacle, is the large number of un(der)qualified teachers. There are thoughts on making the curriculum more practical, but few people in Mozambique have an idea as to what this means.

7. Netherlands: Experiences in the Netherlands suggest that a lack of adequate teacher enhancement, textbooks, and assessment can be a serious obstacle, even if modelling is mandated and exemplary tasks are developed. Although applications and modelling were a central trait of curriculum reform of the early 1990's, the Netherlands government did not facilitate much in-service teacher enhancement, and as a consequence, teachers seem to have developed a rather limited image of the innovation.
8. Japan: Here most teachers use applications and modelling as examples of uses of mathematics once mathematical concepts have been taught. There seems to be three dominant obstacles to the full implementation of modelling in the mathematic curriculum. The first is the influence of entrance examinations, in which modelling tasks hardly appear. The second is that there are too few modelling tasks for students to solve. The third obstacle is concerned with the belief and confidence of teachers. Teachers do not want to tackle modelling because modelling makes teaching more open and complex, and teachers have little experience in modelling.

From an analysis of the case studies described above, three situations can be identified. There are countries in which modelling is not perceived as an important part of the mathematics

curriculum; countries in which modelling is officially a central attribute of the mathematics curriculum; and yet other countries in which modelling is located in the national curriculum, but its role is not central.

In the following paragraphs the researcher describes four research studies involving pre-service mathematics teachers use of mathematical modelling that were conducted at four different universities around the world.

In Sweden, Lingefjärd (2004) had to convince mathematicians and administrators who favoured more traditional courses in mathematics as a suitable preparation for teachers of mathematics, to consider courses in mathematical modelling. In 2003 he attempted to find out if courses in mathematical modelling were actually being carried out at the 26 different departments in Sweden. Faculty members at departments of mathematics, departments of mathematics education, and departments of education as well as general administrators were given the opportunity of answering the question whether their department arranged and carried out courses in mathematical modelling. Within two weeks he received 200 responses. Only four out of the 26 universities could say that they gave a course in mathematical modelling, although two of the four offered the course in mathematical modelling as a voluntary course that only a few students chose to follow. Two other universities were planning to start mathematical modelling courses in the latter part of 2003, while 20 universities did not give any course in mathematical modelling for their prospective teachers.

According to Lingefjärd (2004) the main argument from the faculty members at these sites was that the curriculum was too crowded, and that the students first study algebra, abstract

algebra, calculus in one and several variables, discrete mathematics, geometry, linear algebra, logic reasoning and statistics. He added that the underlying argument often showed to be the lack of insight into mathematical modelling among faculty staff, the feeling that mathematical modelling by nature is an interdisciplinary subject, and therefore not “*real mathematics*”. Lingefjärd felt that the result of his survey was very disappointing, yet not surprising; and reasons that since mathematical modelling is a process rather than a precise body of knowledge, often needing to go outside the domain of mathematics makes this a major reason why many mathematics departments believe that mathematical modelling is less useful than other branches of mathematics in the preparation of teachers of mathematics. He makes the following comment regarding the implementation of mathematical modelling at university level:

To create, maintain, and sustain a course in mathematical modelling is indeed a difficult task. Even when the university teachers are interested in the subject, there are many different hurdles to pass. There is competition from other branches of mathematics, branches that are considered to be more “natural” in a teaching training program. There is a substantial demand of skills needed for both mastering a variety of problems and subjects as well as procedures for handling the teaching and assessment of the mathematical modelling process.

The new secondary school mathematics curriculum in Ontario, Canada made calls for using both modelling and applications in the classroom. The paper by Roulet and Suurtamm (2004) outlined observations based on their on-going research of teachers’ interpretations and implementation of the applications and mathematical modelling themes of this new curriculum. In particular they explored how images of mathematics supported or hindered

progress in the implementation of modelling. Roulet and Suurtamm (2004) concluded that when modelling was used, there was very little emphasis on analysis and interpretation of the model to determine its appropriateness for the context and that in most cases, investigation or inquiry was not used as a starting point for learning. During the research, Roulet and Suurtamm established that some teachers embraced the notion of mathematical modelling through inquiry to its fullest sense, whilst responses from others revealed that they were confused about the nature of modelling and how it fits within investigations.

Furthermore Roulet and Suurtamm (2004) observed that in incorporating mathematical modelling in the classroom, it was evident that teachers' conceptions of the discipline greatly influenced the materials they choose to use and how they employed them. They found that teachers with instrumentalist views of the subject and previous traditional experiences with the course content were inclined to incorporate applications as end-of-unit exercises, applying mathematics that had been previously studied, rather than as opportunities for modelling; whilst teachers with a constructivist epistemology and parallel views of teaching and learning were able to embrace the new curriculum and the notions of modelling. Roulet and Suurtamm, declared that "*for mathematical modelling to become a major focus of the curriculum, there is a need for a significant shift in teachers' views of mathematics*".

A study by Zbiek (1998) explored "*the strategies used by 13 prospective secondary school mathematics teachers to develop and validate functions as mathematical models of real world situations*". The students, enrolled in an elective mathematics course, had continuous access to curve fitters, graphing utilities, and other computing tools. Niss (1988, as cited in Zbiek, 1998), asserted that teachers were expected to play a central role in their learner's classroom

modelling experiences, although modelling as a complete and complex process had not been a central thread of teacher education and research. In Zbiek's (1998) study twelve of the thirteen participants had completed their mathematics and methods courses during the study and one student was beginning his advanced mathematics and methods courses. She discovered that the students' previous mathematics courses in both pure and applied mathematics devoted little time or no time to applying mathematics to real world situations whilst three students had encountered a modelling lesson in a mathematics education methods course completed prior to the study. Nine students who were concurrently enrolled in the methods course claimed that they had no idea what mathematical modelling was. All the prospective teachers in Zbiek's (1998) study entered the study with minimal knowledge of mathematical modelling. Although the participants in her study displayed an ability to use different strategies to develop and validate functions as mathematical models of real world situations, they struggled at times despite their mathematical backgrounds and familiarity with the phenomena to be modelled.

In another research study carried out in the mathematics departments in three colleges of education and three universities in Poland, Trelinski (1983) asked 223 graduate mathematics students preparing to be mathematics teachers to create a mathematical model of a situation from outside mathematics. Beside the general aim of determining whether the prospective teachers could create the mathematical model, the more practical purpose was according to Trelinski to "*access the degree to which a prospective teacher is prepared, through experience acquired in solving a modelling problem, to introduce his or her pupils to applying mathematics*". Once their solutions and strategies were analysed, Trelinski concluded that the students had found the task both difficult and unusual seeing that only 9 of them were able to

succeed fully. She concluded that each student tried only one idea of a suitable scheme. Many students seemed to follow no mathematically consistent line of reasoning, and no students presented complete solutions. These prospective teachers did not demonstrate a natural transfer of their, supposedly, strong mathematical understandings into acceptable models.

On a slightly different continuum Pournara and Lampsen (2007) reported on the implementation of a modelling course for pre-service teachers at the Wits School of Education, University of the Witwatersrand, South Africa. Pournara and Lampsen asserted that from their experience as mathematics teacher educators, there are two completing objects in mathematics teacher education: namely, mathematics and teaching. They argue that a modelling course for teachers should address both. Consequently they suggested that if modelling approaches were to be taken by teachers in secondary schools, then undergraduate teacher preparation must provide opportunities both to learn and to do modelling, and to engage with the realities of teaching mathematical modelling at school level.

Pournara and Lampsen (2007) first offered a course which focused on mathematical modelling in 2005 and then again in 2007. The course was intended for third/fourth year students because by the time students registered for the modelling course, they would have done courses in college algebra and functions, geometry and trigonometry, introductory calculus and linear algebra. Furthermore, if they were fourth year students they would also have done courses in statistics and financial mathematics. The principal goal of the course was to introduce student teachers to the practice of mathematical modelling. Bearing in mind that both lecturers were novices to modelling, they made use of tasks that drew on school-level mathematics. It was found that many students showed immense potential once they entered the

program. Pournara and Lampsen acknowledged that a single course on modelling was not likely to convince students of the value of modelling and admitted that there was insufficient attention to modelling across their courses at the Wits School of Education which they planned to address.

A final consideration by few intransigent mathematicians is whether mathematical modelling will undermine “real mathematics”? Steen, Turner and Burkhardt (2007) point out that many sceptics fear that modelling, if encouraged, will replace rigour and proof in mathematics classrooms. They however reassure the mathematics community that there are many valid reasons to ensure that reasoning and proof do not disappear from school mathematics. These reasons are that *“students need to learn that justification is a distinctive part of mathematics; that proof is more than plausibility or confirmation; that among the levels of convincing argument, mathematical proof alone yields certainty; and that the rigor of mathematical proof makes lengthy chains of logical argument reliable”*. Although mathematical modelling rarely emphasises formal proof, Steen, et al. (2007) maintain that it does emphasise the value of:

- accuracy at the end of a long chains of inferences and calculation;
- justifying findings, especially their applicability in relation to the problem content;
- explaining reasoning to team-mates and teachers;
- presenting conclusions coherently.

Through these means, Steen, et al. (2007) suggests that modelling both demonstrates and rehearses the importance of rigorous logical argument.

CHAPTER THREE

THEORETICAL FRAMEWORK

3.1 Introduction

This research study's theoretical orientation is informed by constructivism, Realistic Mathematics Education (RME) and Lave's Theory of situated learning.

3.2 Constructivism

Constructivism is a cognitive learning theory with a distinct focus on the mental processes that construct meaning. According to Van de Walle (2007) and Olivier (1992) the general principles of constructivism are based largely on Piaget's processes of assimilation and accommodation; where assimilation refers to the use of existing schemas to give meaning to experiences while accommodation is the process of altering existing ways of viewing things or ideas that contradict or do not fit into existing schemas.

Hanley (1994) describes a classroom based on the traditional model of teaching as a "*one-person show with a captive but often comatose audience*". She explains that classes are usually driven by "*teacher-talk*" and that there is a fixed world of knowledge that the learner must come to know. Hanley further remarks that "*teachers serve as pipelines and seek to transfer their thoughts and meanings to the passive student, leaving little room for student-initiated questions, independent thought or interaction between students*".

According to Orton (1994, 2004), the evolution of constructivism does not imply a rejection of earlier attempts to facilitate more effective learning within a cognitive learning environment. He argues that "*it is a misunderstanding of constructivism to suggest that there is little the*

teacher can do to facilitate learning simply because the construction must be carried out by the learner". Although literature contains many references to 'inquiry learning' as being best for the construction of understanding, the teacher still has to organize it. Orton points out that constructivism appears to suggest that the teacher needs to provide the "scaffolding" which allows the learner to progress, and it requires great skill to provide the best scaffolding for each learner. He remarks that "*a consistent policy of complete non-intervention by the teacher is therefore certainly not likely to be the best way to promote the construction of knowledge. However, a policy of non-intervention with a certain child at a particular point in time or with a particular group might be appropriate, especially when the responsibility of learning has been fully accepted by the child or the group*".

Echoing similar sentiments, Murphy (1997) observes that an important concept of constructivism is that of "*scaffolding which is a process of guiding the learner from what is presently known to what is to be known*". She points out that scaffolding allows learners to perform tasks that would ordinarily be slightly beyond their ability without the assistance and guidance from the teacher.

Within constructivism, the teacher acts as a facilitator of knowledge and must ensure that the learner has the ability to construct knowledge in their own minds through the process of discovery and problem-solving. According to Mouton (1996), a mathematical modelling strategy is in line with current constructivist perspectives on learning and is seen as a vehicle to a better understanding of mathematics. This is one of the primary reasons that the researcher has positioned this research study within this particular theoretical framework.

According to Van de Walle (2007) a commonly accepted goal among mathematics educators is that learners should understand mathematics, and the most widely accepted theory, which is constructivism, suggests that children must be active participants in the development of their own understanding. Lerman (1989); Njisane (1992) and Clements and Battista (1990) concur that in a constructivist teaching and learning environment, knowledge is actively constructed by the learner, and not passively received from the environment. Furthermore Van de Walle (2007) points out that constructivism provide us with insights concerning how children learn mathematics and guides us to use instructional strategies that begin with children rather than with ourselves. He finally notes that “*constructivism rejects the notion that children are blank slates..... they do not absorb ideas as teachers present them, rather, children are creators of their own knowledge*”.

Within the constructivist paradigm, teachers should create opportunities for children to create new mathematical knowledge by reflecting on the things that they do (their physical actions) and the ways that they think (their mental actions). According to Doerr (2007), a modelling approach to teaching mathematics calls for a major reversal in the usual roles of teachers and students, whereby students need to do more evaluating of their own ideas and teachers need to create opportunities where this evaluation can productively occur.

Constructivism makes provision for the fact that each individual person interprets and makes sense of the world in his or her own way. Within a constructivist teaching and learning environment, the learner should be able to make sense of a real-world problem in his or her own way. The interpretations that learners make will depend on their experience and upon social interaction with other people.

Developing learner's personal mathematical ideas is very important to the constructivist teacher when the teacher encourages learners to use various methods for solving problems. Through interaction with mathematical tasks and with others, the learner's own intuitive mathematical thinking gradually becomes more abstract and powerful. The researcher believes that mathematical modelling as a teaching strategy is the key to developing autonomous and self-motivated learners.

The following is a summary of some of the characteristics of a constructivist teacher as outlined by Brooks and Brooks (1993):

- Become one of many resources that the student may learn from, not the primary source of information.
- Engage students in experiences that challenge previous conceptions of their existing knowledge.
- Allow student responses to drive lessons and seek elaboration of students' initial responses. Allow student some thinking time after posing questions.
- Encourage the spirit of questioning by asking thoughtful, open-ended questions. Encourage thoughtful discussion among students.
- Use cognitive terminology such as "classify", "analyse" and "create" when framing tasks.
- Encourage and accept student autonomy and initiative. Be willing to let go of classroom control.
- Use raw data and primary sources, along with manipulative, interactive physical materials.

- Don't separate knowing from the process of finding out.
- Insist on clear expression from students. When students can communicate their understanding, then they have truly learned.

Jonassen (1991) suggests that educators create constructivist learning environments for their learners and has provided the following list of principles as a guideline:

- Create real-world environments that employ the context in which learning is relevant;
- Focus on realist approaches to solving real-world problems;
- The instructor is a coach and analyzer of the strategies used to solve these problems;
- Stress conceptual interrelatedness, providing multiple representations or perspectives on the content;
- Instructional goals and objectives should be negotiated and not imposed;
- Evaluation should serve as a self-analysis tool;
- Provide tools and environments that help learners interpret the multiple perspectives of the world;
- Learning should be internally controlled and mediated by the learner.

In providing support for Jonassen's (1991) principles listed above, Wilson and Cole (1991) outline the following concepts as central to constructivist design.

- The educator should embed learning in a rich authentic problem-solving environment;

- He should provide for authentic versus academic contexts for learning;
- There should be provision for learner control; and
- The educator should use errors as a mechanism to provide feedback on learners' understanding.

3.3 Realistic Mathematics Education

This research study has a strong leaning towards Realistic Mathematics Education because some of the views of the nature of mathematics as outlined in the South African National Curriculum Statement (2003a) are closely linked to the Realistic Mathematics Theory. These views include the notion that mathematics is seen as a human activity and that mathematical problems should include real-life situations. According to Vaid (2004), research and development teams at the Freudenthal Institute at the University of Utrecht in the Netherlands, are responsible for the development of the model of Realistic Mathematics Education, which has progressed significantly since 1971.

Van den Heuvel-Panhuizen (1998) explains that the present form of Realistic Mathematics Education has been mostly determined by Freudenthal's view on mathematics. He proposed that mathematics must be connected to reality, stay close to children's experience and be relevant to society, in order to be of human value. Instead of seeing mathematics as a subject to be transmitted, Freudenthal stressed the idea of mathematics as a human activity where mathematics lessons should give students the 'guided' opportunity to 're-invent' mathematics by doing it. Hence Van den Heuvel-Panhuizen summarises that this means that in mathematics education, the focal point should not be on mathematics as a closed system but on the activity,

on the process of mathematisation. Fauzan et al (2002) concur that Realistic Mathematics Education stresses the idea that mathematics is a human activity and mathematics must be connected to reality, and that it must be real to the learner using real-world context as a source of concept development. According to Benson (2004) Realistic Mathematics Education involves putting mathematics into recognizable, real life, contexts to allow the pupils to engage with the mathematics and generate solutions in a variety of forms, which encourage discussion in a more informal atmosphere whilst moving towards a more formal solution. This model is based on the principle, that learners see meaning in their schoolwork when they connect information with their own experience.

Treffers and Beishuizen (2000) state that in addition to illustrating the applicability and relevance of mathematics in real-world situations, realistic mathematics involves taking realistic context situations as the starting point or as the source for learning mathematics. They point out that this important realistic mathematics education viewpoint does not mean adding a few application problems to your mathematics lessons, but rather a complete reversal of the teaching/learning process, where the emphasis is no longer on the teacher transmitting knowledge and concepts, but on the children finding mathematical patterns and structures in realistic situations, and becoming active participants in the teaching/learning process. At this point one can draw parallels with the constructivist theory. Treffers and Beishuizen stress that 'realistic' also relates to mathematical activities which are experientially real to a child. According to Mudaly (2004) realistic mathematics educators place tremendous emphasis on the idea of making a mathematical idea real in the mind of the learner.

Similarly Zulkardi (2003) notes that mathematics must be close to children and be relevant to everyday life situations, and that ‘realistic’ refers not just to the connection with the real-world, but also refers to problem situations which are real in students’ minds.

Bottle (2005) also points out that ‘*realistic*’ does not just mean real-life situations. She explains that the term ‘realistic’ is taken from the Dutch word ‘*zich REALISEren*’ which can mean ‘to realise’ or ‘to imagine’. It therefore means that contexts that are realistic to children (although imaginary) are included. According to Bottle, this means that using stories can be useful in relating mathematical ideas to young children. Similarly games, television, videos and films can be considered as useful environments within which to develop mathematical learning, as all of these environments become real in a child’s imagination as they act them out.

Realistic Mathematics Education as a teaching and learning theory represents a significant departure from traditional ideas about teaching and learning mathematics. According to Meyer (2001) “perhaps the most obvious difference between an instructional sequence based on Realistic Mathematics Education and a more traditional sequence is their starting point. In explaining algebraic representations, most traditional algebra texts start with abstract expressions involving variables and rapidly move to formal equations and their manipulations. After attaining some degree of facility in manipulating equations, the student of algebra is invited to apply these skills to context-based problems. Materials based on Realistic Mathematics Education, however, reverse the progression. Instead of starting in the abstract

realm and moving toward the concrete application, the mathematic starts in contexts and gradually progresses to formal symbolism. This shift allows students, for example, to engage in meaningful, pre-formal algebraic activity in earlier grades than they traditionally have. Through a structured instructional sequence, students explore and rediscover significant mathematics that anticipates the more formal representations found in traditional algebra.

The usefulness of the Realistic Mathematics Education approach is evident in the success that the Netherlands achieved in the recent international comparisons. In 1995, 26 countries took part in the Third International Mathematics and Science Survey (TIMSS). Nine year old children in the Netherlands topped the scores from all the other European countries participating. It is interesting to note that in the Netherlands many children learn much of their mathematics through real-life problem-solving (Bottle, 2005).

The researcher views mathematical modelling as being very relevant within a Realistic Mathematics Education framework. Mathematical modelling engages learners in activity; they are able to apply mathematics to reality and they are able to see the usefulness of mathematics.

In summary, Meyer (2001) and Widjaja (n.d) lists the following as characteristics of realistic mathematics education:

1. use real life contexts as a starting point for learning- the starting point should be real to students, allowing them to immediately become engaged in the situation;

2. use models as a bridge between abstract and real that help students learn mathematics at different levels of abstraction- at first the model is a model of a situation that is familiar to students. By a process of generalizing and formalizing, the model eventually becomes an entity on its own and is possible to be used as a model for mathematical reasoning;
3. use student's own production or strategy as a result of doing mathematics. By making free production, students are guided to reflect on the path they themselves have taken in their learning process and, at the same time, to anticipate its continuation;
4. interaction is essential in learning mathematics, both between teacher and students, and between students and students. Explicit negotiation, intervention, discussion, cooperation, and evaluation are essential elements in a constructive learning process in which the student's informal methods are used as a lever to attain the formal ones;
5. connection among strands, to other disciplines, and to meaningful problems in the real world. Learning strands cannot be dealt with as separate entities; instead an intertwining of learning strands is exploited in problem solving.

Vaid (2004) points out that Realistic Mathematics Education demands that teachers establish a link between children's own world and the world of mathematical ideas. From a psychological perspective, this relates to Lave's theory of situated learning which argues that knowledge needs to be presented in an authentic context, unlike most classroom learning activities which involve knowledge that is abstract or out of context.

Eade (2004, as cited in Vaid, 2004) outlined the key aspects of Realistic Mathematics Education as follows:

- **It is realistic** – the term 'realistic' refers to situations which can be imagined by the learners. Realistic Mathematics Education initially presents knowledge within such a concrete context allowing pupils to develop informal strategies, but gradually through the process of guided 'mathematization', allows students to progress to more formal, abstract, standard strategies. These contexts are chosen to help students' mathematical development, not simply because they are interesting! The Realistic Mathematics Education problems, set in real world contexts, are presented, so that along with giving meaning and making mathematics more accessible to learners, they also illustrate the countless ways in which mathematics can be applied.
- **It involves 'mathematization'** - This can be split into two kinds.
 - HORIZONTAL MATHEMATIZATION: This is when the students' discover mathematical tools which can help to organize and solve a problem located in a real-life situation.
 - VERTICAL MATHEMATIZATION: This refers to the process of reorganisation within the mathematical system itself, for example refining and adjusting models or generalising to create more challenging mathematics and hence to a greater use of abstract strategies.
- **It is procedural vs. algorithmic** – Realistic Mathematics Education stresses understanding processes, rather than learning algorithms. Students 'discover' the mathematics for themselves, and so multiple solutions are encouraged and valued.

- **It incorporates the use of effective models** - The use of various models e.g. ratio tables and combination charts, provide a more visual process of doing mathematics.
- **It encourages 'guided reinvention'** - this implies beginning with the range of informal strategies provided by students, and building on these to promote the materialization of more sophisticated ways of symbolising and understanding.
- Due to students' directing the course of lessons, Realistic Mathematics Education requires a highly 'constructivist' approach to teaching, in which children are no longer seen as receivers of knowledge but the makers of it, and the role of the teacher is that of a facilitator.
- **It promotes 'historical simulation'** - Allowing the students to begin at the basics, using informal strategies and constructing the mathematics for themselves, simulates the discovery of the mathematics and allows them to appreciate the complexity of the mathematics.

3.4 Situated Learning Theory

The researcher found that the theory of situated learning was relevant to this study, in view of the fact that situated learning is emerging as a learning theory that is particularly relevant to teaching.

In the words of Clancey (1995) “*situated learning is the study of how human knowledge develops in the course of activity, and especially how people create and interpret description (representations) of what they are doing*”. Anderson, Reder & Simon (1996) add that situated

learning “*emphasises the idea that much of what is learned is specific to the situation in which it is learned*”. They find particularly important situated learning’s emphasis on the mismatch between typical school situations and the “real world” situations such as the workplace, where one needs to deploy mathematical knowledge. Anderson et al. agree that greater emphasis should be given to the relationship between what is learned in the classroom and what is needed outside of the classroom and view this as being a valuable contribution of the situated learning movement. In *Situated learning* (n.d), Lave argues that “*learning as it normally occurs is a function of the activity, context and culture in which it occurs (that is, it is situated)*”. This she points out is in contrast with most classroom learning activities which involve knowledge which is abstract and out of context.

Herrington and Oliver (1995) propose a model of instruction based on situated learning to be used in the design of learning environments. They suggest that the learning environment should:

- **Provide authentic context:** Context should reflect the way the knowledge will be used in real-life including the complexity of the real-world situation, providing purpose and the possibility for extended exploration.
- **Provide authentic activities:** Activities should demand that learners 'find' and 'solve' problems inherent in the situation and determine how they will accomplish the task.
- **Provide access to expert performances and the modelling of processes:** Observation of expert performances allow for the accumulation of narratives and strategies that use the social environment as a resource.

- **Provide multiple roles and perspectives:** Providing the learner with multiple opportunities to engage in an activity from differing perspectives will reveal different aspects of the situation.
- **Support collaborative construction of knowledge:** Activities should encourage collaborative searches for suggestions and solutions to promote critical thinking.
- **Provide coaching and scaffolding at critical times:** The learning environment should be available to intercept and offer hints and strategies when learners are unable to progress in the task.
- **Promote reflection to enable abstractions to be formed:** The environment presentation of problems should require that the learner take the entire environment or situation into consideration when problem solving.
- **Promote articulation to enable tacit knowledge to be made explicit:** Articulation of the vocabulary and the stories of a culture of practice that is an integral part of the situation presented within the learning environment deepens a learner's understanding of a topic.
- **Provide for integrated assessment of learning within the tasks:** Assessment and feedback on a learner's progress and during tasks should be offered without resorting to tests.

CHAPTER FOUR

RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

In this chapter, the methodology and the design of this research study is described by the researcher. This is done by discussing the method which was used to determine the sample, the appropriateness of the research instruments, the method of data analysis that was chosen and the ethical issues which had to be taken into consideration.

4.2 Research questions

This research is a descriptive/interpretative study of pre-service mathematics teachers' knowledge and use of mathematical modelling as a strategy for solving real-world problems as well as how it would influence their teaching in the Further Education and Training band. Merriam (2002) points out that in a basic interpretative and descriptive study the researcher is interested in understanding how participants make meaning of a situation; this meaning is mediated through the researcher; the strategy is inductive and the outcome is descriptive. With the aid of the research instruments that were designed for this study, the researcher attempted to answer the following critical questions:

1. What are pre-service mathematics teachers' conceptions of mathematical modelling?
2. Are pre-service mathematics teachers able to create and use mathematical models to solve real-world problems?
3. How do pre-service mathematics teachers translate real-world mathematics problems into mathematical models?

4. Are pre-service mathematics teachers able to facilitate the understanding of the solution of the mathematical modelling process?
5. Do pre-service mathematics teachers possess more than one strategy for solving real-world mathematics problems?

4.3 Selection of participants

Kumar (2005) emphasises that the accuracy of the findings in a research study largely depends upon the way you select your sample. The underlying premise in sampling is that, if a relatively small number of units are selected, it can provide – with a sufficiently high degree of probability- a fairly true reflection of the sampling population being studied.

The researcher therefore chose to employ non-random sampling for this research study since the study was not working towards representativeness or generalizability (Kumar, 2005). Cohen, Manion and Morrison (2005) point out that in a non-probability sample some members of the wider population definitely will be excluded and others definitely included; in other words every member of the wider population does not have an equal chance of being included in the sample. Specifically the researcher chose to use purposive or judgemental sampling where according to Kumar (2005) and Cohen, et al. (2005), the primary consideration is the judgement of the researcher as to who can provide the best information to achieve the objectives of the study. The researcher therefore only goes to those people who in his/her opinion are likely to have the required information and are willing to share it (Kumar, 2005).

The data for this research study was collected during the second semester of 2006 at the University of KwaZulu-Natal, Edgewood Campus. The participants chosen for this study were

all 21 third and fourth year pre-service mathematics teachers from the Faculty of Education of the University of KwaZulu-Natal. Keeping in mind the nature of this study, these third and fourth year pre-service mathematics teachers represented a primary source from which the data had to be collected. These students are currently following a Mathematics curriculum at the University which would eventually qualify them to teach Mathematics or/and Mathematical Literacy in the Further Education and Training band at a secondary school. The reasons for choosing these students as participants for this study were twofold. Firstly, I had constant access to them, as all of these students were part of a module for which I was responsible for during semester two of 2006. Secondly, the research tasks which were given by the researcher to the students were assumed to be within the mathematical competence of the chosen sample, following the warning from Cohen, et al. (2005) to “*avoid highbrow questions even with sophisticated respondents*”.

4.4 Data analysis methodology

Since this study explored pre-service mathematics teacher’s use of mathematical modelling as a strategy for solving real-world problems, a qualitative research methodology was utilized. Qualitative data is analysed thematically. O’Learly (2004) explains that thematic analysis can include analysis of words, concepts, literary devices and/or non-verbal cues. This is in contrast to quantitative data which is statistically analysed.

Rudestam and Newton (2001), point out that qualitative implies that the data are in the form of words as opposed to numbers and that qualitative data are usually reduced to themes or categories and evaluated subjectively. They further note that there is more emphasis on description and discovery and less emphasis on hypothesis testing and verification. Bell

(2005) concurs that researchers adopting a qualitative perspective are more concerned about the understanding of individuals' perceptions of the world and they seek insights rather than statistical perceptions of the world. Denzin and Lincoln's (2003), view is that the word qualitative implies an emphasis on the qualities of entities and on processes and meanings that are not experimentally examined in terms of quantity, amount, intensity or frequency.

Denzin and Lincoln (2003), purport that qualitative research is a situated activity that locates the researcher in the world of the participant and suggests that qualitative research involves an interpretative, naturalist approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or to interpret, phenomena in terms of meanings people bring to them.

According to Rudestam and Newton (2001), qualitative approaches are based on three fundamental assumptions:

1. *A Holistic View*: Qualitative methods seek to understand phenomena in their entirety in order to develop a complete understanding of a person, program, or situation. This is in contrast to the experimental paradigm, which aims to isolate and measure narrowly defined variables, and where understanding is tantamount to predication and control.
2. *An Inductive Approach*: Qualitative research begins with specific observations and moves toward the development of general patterns that emerge from the cases under study. The researcher does not impose much of an organizing structure or make assumptions about the interrelationships among the data prior to making the observations.

3. *Naturalistic inquiry*: Qualitative research is intended to understand phenomena in their naturally occurring states. It is discovery-orientated approach in the natural environment. Experimental research, by comparison, uses controlled conditions and a limited set of outcome variables.

4.5 Research Instruments

According to Kumar (2005) anything that becomes a means of collecting information for a study is called a '*research tool*' or a '*research instrument*'. In order to obtain appropriate data to answer the research questions, the researcher designed three different instruments for data collection, which is supported by Denzin and Lincoln (2003) when they stated that qualitative research is inherently multi-method in focus. This use of multi-methods, or triangulation, reflects an attempt to secure an in-depth understanding of the phenomenon in question.

4.5.1 The first instrument was a questionnaire (see APPENDIX A). This questionnaire was administered to all participants. The purpose of this questionnaire was to determine individual pre-service mathematics teachers' knowledge and conception of mathematical modelling. This open-ended questionnaire was administered to the respondents in a lecture theatre at the University of KwaZulu-Natal, Edgewood. This enabled the researcher to have personal contact with the study population and was able to explain the purpose, relevance and importance of the study. The participants were told to work independently and were also made aware that this questionnaire could be answered anonymously. The participants were given half an hour to answer the questions, and most of the participants were able to finish ahead of time. The responses obtained from the first question in the questionnaire assisted the researcher in providing support in answering the first research question.

Kumar (2005) describes a questionnaire as a written list of questions, the answers to which are recorded by the respondents. In the given questionnaire, the respondents were expected to read the questions, interpret what was expected and then write down the answers.

The questionnaire which was administered to the participants consisted of open-ended questions which allowed the participants to express themselves freely. When the questionnaire was administered to the participants, the researcher highlighted the following points as outlined by Rudestam and Newton (2001) to clarify the purpose of the research study and the intentions of the researcher:

1. the researcher was identified to the group of participants
2. the purpose of the research study was explained
3. how their confidentiality will be protected
4. the legitimacy of the study- that the researcher had obtained a letter of ethical clearance from the university.
5. request for cooperation - the researcher appealed to the participants for their help in making this research study a success.

According to Kumar (2005), a questionnaire has several advantages of which the researcher found the following to be most relevant to this study.

- *It is less expensive:* As you do not interview respondents, you save time, and human and financial resources. The use of a questionnaire, therefore, is comparatively convenient and inexpensive. Particularly, when it is administered collectively to a study population, it is an extremely inexpensive method of data collection.

The questionnaire administered in this research study allowed the researcher to obtain responses from the entire population in the group that was being investigated.

Owing to the fact that many of the participants in this research study were English second language students, the researcher took heed of the following guidelines as suggested by Kumar (2005) when formulating questions for the questionnaire:

- **Always use simple and everyday language.** Take care to use words that your respondents will understand as you have no opportunity to explain questions to them.
- **Do not use ambiguous questions.** An ambiguous question is one that contains more than one meaning and that can be interpreted differently by different respondents.
- **Do not ask double-barrelled questions.** A double-barrelled question is a question within a question, and the main problem is that one does not know which particular question a respondent has answered.
- **Do not ask leading questions.** A leading question leads a respondent to answer in a certain direction.

4.5.2 The second instrument (see APPENDIX B) was a set of appropriate tasks consisting of three real-world mathematics problems. The three real-world mathematics problems chosen to be given to the pre-service mathematics teachers was perceived to be suitable for data collection since all three problems are located within a real-world context and the solution of all three problems lends itself to the construction of mathematical models. Each of these mathematics problems could be solved using different strategies and approaches. Although the research was contextualised in a non-technological environment, the participants were allowed

to use calculators. The participants worked independently and were asked to record all ideas, attempts and partial solutions. The responses to these real-world problems helped the researcher to answer research questions two and three of this research study. As mentioned earlier, each of the real-world problems chosen to be given to the participants could be modelled using more than one strategy. The responses to the questions also indicated to the researcher whether the participants were capable of finding these different strategies to solve the problems, and hence this would assist the researcher to answer research question five. Finally, after all the responses to the real-world tasks were studied by the researcher, ten participants were chosen and interviewed.

4.5.3 The interview which was the third research instrument designed by the researcher was of a semi-structured nature, where the responses to the semi-structured questions allowed the researcher to answer research questions three, four and five. For the purpose of the semi-structured interview, the participants were chosen on the basis of their response to the real-world problems in the following manner:

1. Two participants who made no attempt to provide any solution to the questions.
2. Four participants who were able to provide solutions to all three tasks.
3. Four participants who only attempted to provide solutions to two or fewer of the real-world problems.

The interviews that the researcher conducted with the pre-service teachers formed the main data collection tool. The interviews were based on the responses made by the participants in the tasks. The questions in the interview schedule consisted of open-ended questions and this assisted the researcher in obtaining the respondents thoughts on the questions asked in the tasks, and their responses to these tasks.

Kumar (2005) points out that interviewing is a commonly used method of collecting information from people. He defines an interview as “*any person-to-person interaction between two or more individuals with a specific purpose in mind*”. The semi-structured interview schedule designed by the researcher bore characteristics of a structured interview except that the researcher had to be flexible with the questions when a participant had no response to the task, answered only one or two of the questions, and when all three problems were answered by the participants.

Kumar (2005) states that in a structured interview the researcher asks a predetermined set of questions, using the same wording and order of questions as specified in the interview schedule. An interview schedule is a written list of questions, open-ended or closed-ended, prepared for use by an interviewer in a person-to person interaction

According to Kumar (2005) one of the main advantages of the structured interview is that it provides uniform information, which assures the comparability of data, whilst Bell (2005) cites a major advantage of the interview as being its adaptability. She explains that a skilful interviewer can follow up ideas, probe responses and investigate motives and feelings, which the questionnaire can never do.

The researcher obtained permission (see Appendix E) from the participants to conduct and record the interviews. The researcher found tape-recording of the interviews to be particularly useful since the researcher had planned to engage in content analysis and needed to be able to listen to the tapes several times in order to be able to identify categories. It also allowed the researcher to code, summarise and to note particular comments which were of particular

interest (Bell, 2005). The recorded interviews were transcribed by the researcher in order to gain familiarity with the data.

4.6 Suitability of task-based questions

For the purpose of this research study the participants were presented with the three following task-based problems. These questions were based on work that the students would have covered in senior secondary school, or at university in Mathematics 110 and even Mathematics Method 3. The third question attempted to look at the students' creativity. The participants were asked to record all ideas, attempts and partial solutions; as well as to provide as many solutions as they see possible. They could use any suitable method of their choice.

The three pseudo real-world problems that the participants had to solve were:

1. A carpenter makes tables and bookcases for a net unit profit that he estimates as R25 and R30 respectively. He needs to determine how many units of furniture he should make each week. He has up to 690 sheets of timber to devote to the project weekly and up to 120 hours of labour. It requires 20 sheets of timber and 5 hours of labour to complete a table, and 30 sheets of timber and 4 hours of labour for a bookcase, and he can sell all the tables and bookcases he produces. The carpenter wants to determine a weekly production schedule for tables and bookcases that maximises his profits. (Dossey, McCrone, Giordano, & Weir, 2002).
2. Two observers are at sea level 500m apart and are in line with a distant mountain peak. They measure angles of 32° and 35° to the peak. How high is the peak above sea level? (Taylor & Sinclair, 2000).

3. Randy and his sister agreed that they would each mow one-half of the lawn. The lawn is a 25 m by 45 m rectangle. The lawn mower cuts a path half a metre wide. If Randy starts at one corner and mows a path completely around the outside, how many times should he go around to mow one half of the lawn? (Fesler, 1995).

These problems were chosen because they lend themselves to the process of mathematical modelling. There are also different strategies that may be used to solve these problems, and this assisted the researcher to answer critical question five. Drawing some kind of diagram either with or without graph paper is almost a necessity for these problems. Organising the information in some way that is meaningful to the student is helpful in discovering patterns. Although not absolutely necessary, a calculator may be useful for obtaining answers to these problems.

4.7 Ensuring reliability and validity

O'Leary (2004) explains that reliability is based on the notion that there is some sense of uniformity or standardization in what is being measured, and that methods need to consistently capture what is being explored. He thus defines reliability as “ *the extent to which a measure, procedure, or instrument provides the same result on repeated trials*”. As much as the researcher tried to make the research instruments reliable, there are several factors that may have affected the reliability of the research instruments.

Kumar (2005) maintains that these factors are impossible to control and notes the following as some of these factors.

- The wording of questions – a slight ambiguity in the wording of questions or statements can affect the reliability of a research instrument as respondents may interpret the questions differently at different times, resulting in different responses.
- The physical setting – in the case of an instrument being used in an interview, any change in the physical setting at the time of the repeat interview may affect the responses given by a respondent, which may affect reliability.
- The respondents' mood – a change in a respondent's mood when responding to questions or writing answers in a questionnaire can change and may affect the reliability of that instrument.
- The nature of interaction – in an interview situation, the interaction between the interviewer and the interviewee can affect responses significantly. During the repeat interview the responses given may be different due to a change in interaction, which could affect reliability.
- The regression effect of an instrument – when a research instrument is used to measure attitudes towards an issue, some respondents, after having expressed their opinion, may feel that they have been either too negative or too positive towards the issue. The second time that they may express their opinion differently, thereby affecting reliability.

Kumar (2005) defines validity as “*the ability of an instrument to measure what it is designed to measure*”. In qualitative data the subjectivity of respondents, their opinions, attitudes and perspectives together contribute to a degree of bias and hence validity should be seen as a matter of degree rather than as an absolute state (Cohen et al., 2005). In this

research study the researcher argues that the validity would be difficult to establish since should the task based questionnaire be re-administered to the participants, it would be human nature to try and answer the questions differently if the participants felt that their initial response was incorrect, and this would cause them to have different responses which would then influence the selection process for the interviews. Cohen et al. (2005) remark that in qualitative data validity might be addressed through the honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation and objectivity of the researcher.

Cohen et al. (2005) argue that “*whilst earlier versions of validity were based on the view that it was essentially a demonstration that a particular instrument in fact measures what it purports to measure, more recently validity has taken many forms*”. There are several different types of validity as outlined in Cohen et al. (2005). Catalytic validity, which prevailed through this research study, strives to ensure that research leads to action. Lather, Kincheloe and McLaren (1994, as cited in Cohen et al., 2005), “*suggest that the agenda for catalytic validity is to help participants to understand their worlds in order to transform them*”. The researcher is of the opinion that the participants in this researcher study were more aware of the process of mathematical modelling after the research process. The participants were also alerted to different strategies that can be used to solve real-world mathematics problems after going through the interview process.

4.8 Ethical considerations

According to O’Learly (2004) researchers are unconditionally responsible for the integrity of the research process. The respondents in this research study were assured of their

confidentiality throughout the research process. O'Leary (2004) states that confidentiality involves protecting the identity of those providing research data, whilst anonymity goes a step beyond confidentiality and refers to protection against identification from even the researcher and that data, information and responses collected anonymously cannot be identified with a particular respondent.

The participants in this research study could not be protected from anonymity since they were required to identify themselves on the task-based questionnaire. The reason for this was that the researcher had to choose 10 participants for the interview based on the responses from the participants as outlined earlier on in this chapter and the only way they could be identified by was by their names. Cohen et al. (2005) argue that a subject agreeing to a face-to-face interview can no way expect anonymity; at most the researcher can promise confidentiality. The participants in this research study were assured of confidentiality. Cohen et al. (2005) explain that although the researcher knew who has provided the information or are able to identify participants from the information given, she will in no way make the connection known publicly.

Rudestam and Newton (2001), argue that another key element of conducting ethical research involves obtaining informed consent from the participants. Before the research process could commence, the researcher ensured that the respondents had given informed consent to be involved in the research process. As O'Leary (2004) points out, the concept of informed consent emphasises the importance of researchers accurately informing respondents and participants of the nature of their research. O' Leary (2005); Rudestam and Newton (2001), further explain that participants can only give informed consent if they have a full

understanding of their requested involvement in a research project, including time commitment, type of activity, topics that will be covered, and all physical and emotional risks potentially involved. The participants should be informed about who is conducting the study and why the participants of the research study were singled out for participation.

The participants were also reminded that participation in the research study was voluntary and that they made aware of their right to discontinue at any time and that there would be no pressure for them to continue.

CHAPTER FIVE
ANALYSIS OF DATA

5.1 Introduction

In the first part of the data analysis the researcher did a quantitative analysis of the participants' responses to the questionnaire. Thereafter, a qualitative analyses of the participants' responses were conducted to determine what their concept of mathematical modelling was. The reason for choosing this method was that it indicated to the researcher, each participant's prior knowledge of mathematical modelling as well as their gained experiences, if any, in mathematical modelling.

5.2 Analysis of questionnaire

5.2.1 What knowledge do you have concerning mathematical modelling?

Fourteen percent of the participants said that they had no knowledge or no of idea what mathematical modelling is and responded as follows:

Andrew: *"No knowledge – not sure what mathematical modelling refers to."*

Udesh: *"None."*

Isabella did not respond to the question in any way. She handed in a blank sheet.

In hindsight, the researcher should have probed further to determine whether the above participants actually had an informal understanding of the concept mathematical modelling without knowing what the exact definition was.

The remaining eighty six percent of the participants attempted to give an explanation as to what they *thought* mathematical modelling was. Many explained that it had to do with real-world mathematics problems but were unable to capture the fact that it could involve

construction of graphs, equations, tables, and so on, to represent some real-world mathematics problem. None of the participants were able to explain the iterative nature of mathematical modelling. The following are some examples of the responses obtained:

Jacki: *“Mathematical modelling is where problems in maths are taught by the use of real-world problems.... Problems that could possibly occur in real-life.”*

Garth: *“It is the process of adapting theoretical mathematical principles to practical applications in the real-world.”*

Bruce: *“I would guess it would be the process of applying mathematics to real-life situations, for example creating a different equation that will predict what the population of rabbits in a certain country at a certain time.”*

Sibiya: *“Mathematical modelling is using tangible examples to learn mathematics concepts. It also takes into consideration teaching mathematics from real-life... real-world situation – that is putting context on mathematics for better understanding.”*

Ndonyela: *“To be able to apply the laws and principles of mathematics in real-life situations.”*

Michelle: *“My understanding of mathematical modelling is it is a branch of maths using real-life situations and applying maths to it.”*

Natasha: *“It is a mathematical way to express a real-world problem.”*

Although real-world contexts provide suitable modelling opportunities, it does not imply that any problem based on a real-world context is mathematical modelling. Consider the following example: *Virasha was given R300 by an uncle. She was asked to give one third of the money*

to her sister Ashmika. Although this problem uses basic mathematics it does not have to lend itself to the process of mathematical modelling.

Other participants viewed mathematical modelling as the construction of a physical or solid model. These are some of the responses.

Peter: *“I think this has something to do with devising a model for mathematics that will be suitable for different educational levels, for example in schools and universities in terms of curriculum development.”*

Kamal: *“It is a mathematically created model or behaviour of a system....that is using mathematics to describe a system and its behaviour patterns.”*

Ndlovu: *“It is where we use paper which we cut with scissors in order to make a shape to resemble the problem in the book.”*

Here it seems the students focussed on a physical model and not a process. This can be seen as an easy mistake.

In view of the fact that all the participants were exposed to mathematical modelling using the *Sketchpad* software in the module Mathematics for Educators 420, ten percent of the respondents felt that mathematical modelling meant using computer software such as *Sketchpad*.

Sam: *“Real-life situations that are represented in Maths using, for example, Sketchpad.”*

Michelle: *“We have used Geometer’s Sketchpad to solve problems related to real-life situations. It is using real-world situations in Maths.”*

Whilst *Sketchpad* or other mathematics software offers useful opportunities for modelling, it does not imply the mere use of these software is modelling. Computer software are effective tools in engaging in modelling.

There were other participants who viewed modelling as simply the construction of a diagram. Whilst drawing of a diagram may constitute a basic part of mathematical modelling, it is the mathematisation, interpretation and evaluation of the model that completes the process.

Ngidi: *“Mathematical modelling is where the learners have to solve problems by simplifying using a diagram.”*

Sanele: *“Mathematical modelling is when you analyse information into components by using diagrams or pictures.”*

Emmanuel: *“The way I found it has to do with making time-table, taking practical problem and put in more clear detail...graphs.”*

Finally some participants viewed modelling as another problem solving technique.

Sontsele: *“Mathematical modelling is about trying to solve the most difficult problem... you divide the problem to simplify it. It is about learning mathematical concepts. It is about working out ways to solve problems.”*

Bonguthando: *“It is trying to solve a problem in Maths that is difficult and it also written in words.”*

Moloi: *“Mathematical modelling is the reproduction of sums or arithmetic. Mathematical modelling is the representation of Maths.”*

None of the participants in this research study were able to give a precise definition of what mathematical modelling is. One participant, who came closest to having an idea of what mathematical modelling entailed, made the following response in the questionnaire:

Gareth: “...*intuitively linear programming seems to be a form of mathematical modelling.*”

Based on the responses obtained from the questionnaire the researcher drew the following hypothesis: Pre-service mathematics teachers at the University of KwaZulu-Natal have little or no idea of what mathematical modelling is. The researcher was able to make this claim based on the evidence provided in the questionnaire. Most of the participants were of the opinion that mathematical modelling merely consisted of the solution of word problems in mathematics.

Responses to the following questions from the questionnaire helped the researcher to understand why the participants did not have a clear conception of mathematical modelling.

5.2.2 Have you worked with mathematical modelling before? If yes, provide some details?

Thirty-three percent of the participants indicated that they had never worked with mathematical modelling before, or were not sure if they ever had been exposed to the topic. Although sixty seven percent of the participants pointed out that they had worked with mathematical modelling before they were not able to provide an adequate explanation for the term mathematical modelling. They provided the following evidence:

Jacki: “*Yes, I have done an assignment in Primary Mathematics 211 where we had to read articles based on mathematical modelling and then provide our own example of a*

mathematical model. I have also worked with mathematical modelling in Maths 420 in Geometer's Sketchpad. We were dealing with proofs".

Sam: *"Yes, activities in Sketchpad make use of mathematical modelling".*

Natasha: *"Yes, in Math 420 when we looked at distances in an equilateral triangle, where they relate it to a real-world problem of a ship wreck survivor and wants the shortest sum of distances to the shores for surfing.*

Gareth: *"Yes I taught it at teaching practice".*

Garth: *"In linear programming in Grade 12".*

Michelle: *"Yes, I have worked with mathematical modelling especially at university in Maths 420. In high school, we used mathematical modelling in linear programming – using real-life situations and variables such as money, decimals, etc. Also in calculus we used mathematical modelling to calculate maximum and minimum areas. Also in trigonometry we did examples with angles of elevation and depression (real-life examples using heights of towers etc). Most problem-solving questions in school involved real-life situations for example, calculating time and speed".*

Mbambo: *"I may have done it, but I don't know. The reason I don't know is that I have not been told that now you are doing mathematical modelling".*

The preceding seven participants seem to have been exposed to mathematical modelling although they may not have been aware it. Their responses indicated an awareness of the concept but not an actual understanding of what the concept meant. It seems that they had completed mathematical modelling in certain courses but had not actually investigated or understood its definition.

5.2.3 Can you recall whether your mathematics teacher used mathematical modelling in his/her class? If yes, can you remember how?

Keeping in mind that most of the participants in this research study think that mathematical modelling is solving real-world word problems, all the participants felt that their teachers had used mathematical modelling to teach various topics, but the participants all agreed that their teachers never used the words *mathematical modelling* in class when teaching these sections. Perhaps a possible reason for this is that the words *mathematical modelling* was never in the old curriculum, so teachers had no reason to place any emphasis on the actual words. This may change with the implementation of the new curriculum.

Andrew: *“They might have, but I am just not used to the wording”*.

Jacki: *“Yes, when doing Linear Programming and word problems in algebra”*.

Kamal: *“Yes, they used it to describe an optimal solution of the best case scenario concerning real-life problems”*.

Natasha: *“I think so... if maximising and minimising volumes etc has to do with mathematical modelling and linear programming”*.

Sontsele: *“Not sure. If modelling is what I think it is, my teacher did it to teach us calculus ... like we started in rate of change but we ended up doing optimisation”*.

Sansele: *“Yes in calculus in Grade 12 for word problem situations”*.

Bruce: *“Well, according to my definition, we did apply mathematics to real-life situations (problems) such as quadratic word problems, sequences and series, simultaneous equations, etc.”*

Gareth: *“If linear programming is, then yes. I might have done it, but I was never told by my teachers that this was mathematical modelling.”*

Michelle: *“Yes, but my teacher never referred to it as mathematical modelling. We just used real-life examples without knowing it was mathematical modelling. We used it a lot in calculus and a lot in trig”.*

It seems that the participants may have unwittingly done mathematical modelling in class without being told. This then raises the question about whether the teachers at schools, themselves are aware of the concept of mathematical modelling. This observation by the researcher is supported by Usiskin (2004) when he states that *“mathematical models are often used in primary and secondary mathematics classrooms, but the concepts and language of modelling are absent”.*

5.2.4 Do your university lecturers talk about or use mathematical modelling? If yes, during which modules did this occur?

Ten percent of the participants said that they *“can’t remember hearing the word mathematical modelling during lectures”*; four percent of the participants did not respond to this question and left the space blank; whilst eighty six percent of the participants indicated that their university lecturers spoke about or used mathematical modelling in some way during lectures. They mentioned various mathematics modules in which they encountered the term mathematical modelling. Some of the participants said that they were exposed to mathematical modelling during their mathematics method modules, many mentioned encountering mathematical modelling in the module Mathematics for Educators 420 and four percent of the students remembered the term from Primary Mathematics Education 210. The

following are some of the responses received that enabled the researcher to conclude that some of the lecturers of the participants used or mentioned mathematical modelling during the lectures. It would however appear that the students are of the opinion that the explanation of the term mathematical modelling during lectures is not lucid enough.

Andrew: *“I have heard it in Math 420 at Edgewood. I have come across the word but never had an explanation of it”*.

Jacki: *“Yes in Math 420 and Primary Mathematics 210”*.

Kamal: *“Yes. Math 420”*.

Sam: *“Yes in Math 420”*.

Ngidi: *“I think yes. In Mathematics Education”*.

Natasha: *“Yes, in Math 420”*.

Sontsele: *“Yes, in Math 220 in limits I think, and in Math 420”*.

Ndonyela: *“Yes, in Math Method”*.

Mbambo: *“Yes in Math 420”*.

Sibiya: *“...to an extent maths lecturers especially method lecturers are trying to give us a background on this concept except that it is not clear for me”*.

Sanele: *“Yes, in Math 310 the lecturer used it for optimisation problem”*.

Garth: *“Yes, in Math Method 2”*.

Emmanuel: *“In Math 420”*.

Bruce: *“Lecturer mentioned it Math 420”*.

Gareth: *“They mention the term, but expect us to know what it means, so it doesn't mean much”*.

This raises the question of why mathematical modelling is not given more prominence in the university education curriculum. Since mathematical modelling plays a significant part in the school curriculum one would expect that mathematical modelling would therefore be taught during teacher training courses. In support of this, Pournara and Lampen (2007) argue that “*if modelling approaches are to be taken up by teachers in secondary schools, undergraduate teacher preparation must provide opportunities both to learn and do modelling, and to engage with the realities of teaching mathematical modelling at school level.*”

Furthermore, it is also interesting to note that some students recall doing mathematical modelling in certain modules whilst others cannot even remember it. This may imply that these students are not concentrating sufficiently well in class.

5.2.5 Have you interacted with the mathematical modelling aspects in the National Curriculum Statement?

It was quite evident that all the participants have not studied the National Curriculum statement in great detail, since a hundred percent of the participants pointed out that they had not encountered any mathematical modelling aspects in the document. This implies that the National Curriculum Statement is not being discussed during lectures. This should, perhaps, be a compulsory part of one module. Some of the responses are as follows:

Udesh: “*None that I know of!*”

Peter: “*Not to my knowledge*”

Michelle: “*No. No lecturer has yet pointed out the mathematical modelling aspects of the NCS to us. We have not been shown how it appears in the form of outcomes and assessment standards.*”

Again this finding is significant because it implies that National Curriculum Statement document should be interrogated by students before they become qualified educators.

5.2.6 Do you think that mathematical modelling will play an important role in teaching mathematics in the FET phase?

Twenty four percent of the participants said that they were unsure whether mathematical modelling will play an important role in teaching mathematics in the FET phase, because they were themselves not sure what mathematical modelling entails. Seventy-six percent of the participants felt that mathematical modelling had an important role to play in the teaching of mathematics since the problems were real-world and that learners would find this more relevant and useful to them. The following are some of their views:

Jackie: *“Yes, it allows learners to be more familiar with how the real -world works and how maths plays a part in it”*.

Sam: *“Yes. As Maths needs to become more relevant teachers should make more use of mathematical modelling”*.

Natasha: *“Yes because the curriculum is now geared to being applicable to real-life. Since they want to make schooling more relevant to ‘real-life’, mathematics problems can be modelled as real-life problems to make it more applicable to real life”*.

Emmanuel: *“Yes, it will play an important role; because it is an integrated part of mathematics, and it can include other subjects like Physical Science, Chemistry etc. which will make student to be able to relate to the real-life situation”*.

Bruce: *“It would make learners more appreciative of mathematics. Generally many learners have a mental block with regard to Mathematics and are always asking ‘Where will I use this in life?’ I’m sure mathematical modelling will alleviate this type of question”*.

Michelle: *“Yes, I believe it is highly beneficial. Mathematical modelling allows you to relate Maths to the real-world. It is good for learners to be able to integrate what they learn in the classroom to their real-world”*.

Sontsele: *“Yes, of course. Learners in FET will have great chances of being able to employ what they learn in mathematics to real-life problems. Learners would have great understanding of what they do and they would be able to relate what they learn in Maths to real-life problems. Develop skills of modelling not only in Mathematics but also in different fields”*.

Others felt that by including mathematical modelling in the FET band, it would lead to a better understanding of mathematics since the learners will be able to visualise the problems (by the use of diagrams) that they are working with.

Kamal: *“Yes. By relating real-life scenarios and occurrence to mathematics it will help the learners to better visualise and understand mathematics”*.

Ngidi: *“Yes, by using diagrams it will help learners to find the answer for themselves”*.

Sanele: *“I think it would play a pivotal role since it encourages the use of pictures and diagrams. Because learners from OBE they construct knowledge for their own, the breaking down of information are very important”*.

This student indicated that the use of mathematical modelling would point to learners’ ability to understand the language of real-world word problems.

Garth: *“Yes, it will help in the assessment of learners’ comprehension skills”*.

None of the students really addressed this question sufficiently well. This could be due to the following reasons:

1. they did not have a good idea of mathematical modelling,
2. they could not think of the Further Education and Training curriculum as a whole because they did not interact with it,
3. and finally, this type of question was difficult for them to answer.

Somehow students related mathematical modelling to real-world problems only. Mathematical modelling could be used in purely mathematics problems too. Clearly the students' views were dictated by their idea of what mathematical modelling is. They expressed important views about the benefits of mathematical modelling, yet they claimed earlier that they had not come across mathematical modelling.

5.3 Analysis of task-based questionnaire

The questions were chosen in a way that two of them tested already acquired knowledge whilst the third attempted to look at the students' creativity. The analysis of each of the task-based questions, which the participants responded to, and the transcriptions of the semi-structured interviews were used to answer the following research questions.

Research question two: Are pre-service mathematics teachers able to create and use a mathematical model to solve real-world problems?

Question One: A carpenter makes tables and bookcases for a net unit profit that he estimates as R25 and R30 respectively. He needs to determine how many units of furniture he should make each week. He has up to 690 sheets of timber to devote to the project weekly and up to

120 hours of labour. It requires 20 sheets of timber and 5 hours of labour to complete a table, and 30 sheets of timber and 4 hours of labour for a bookcase, and he can sell all the tables and bookcases he produces. The carpenter wants to determine a weekly production schedule for tables and bookcases that maximises his profits. (Dossey et al, 2002).

As can be seen the question was based on work that the students would have done in matric, Mathematics 110 and even Mathematics Method 3 in the form of linear programming. Yet fourteen percent of the participants made no attempt to answer the above problem. Eighty-six percent of the participants made some attempt to provide an answer. The participants who attempted to provide a solution to the problem tried to construct some kind of mathematical model such as a table, an equation or graph, whilst others successfully used trial and error. Forty-three percent obtained a correct answer. This was in itself a significant result considering this is in fact a question from school mathematics. These prospective teachers could not solve it, and it raises the question as to how they were going to teach an entire section at schools the following year.

Some students were able to answer the question in a pure algebraic manner. No graphs were drawn, as is expected in the school curriculum when answering a question on linear programming. There is no problem with this method, but it is interesting to note the change in the procedure they have used previously. Figure 9 is an example of an algebraic solution:

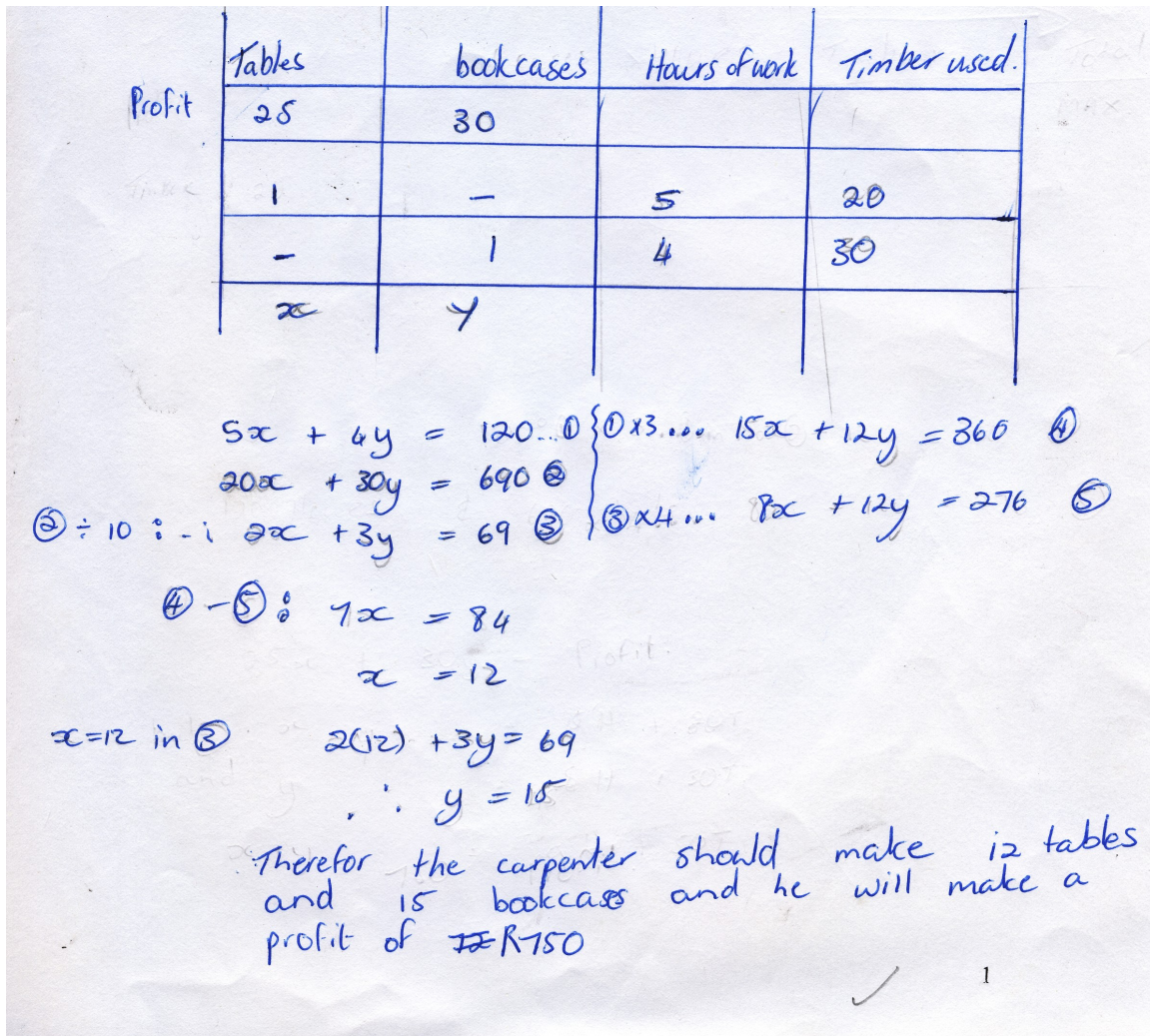


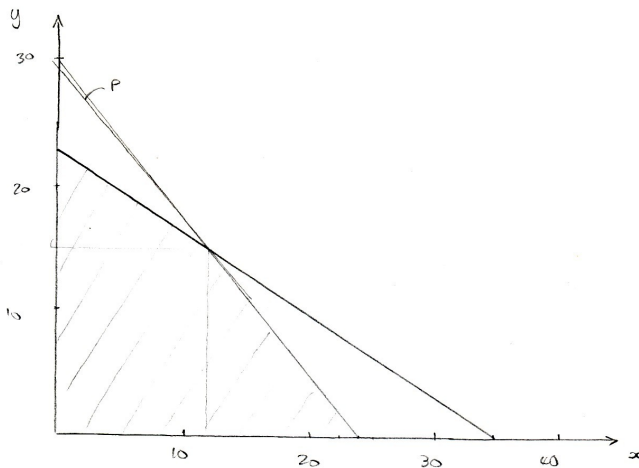
Figure 9

Other students did use graphical methods; as can be seen in Figure 10. It cannot be conclusively stated that these were more visual learners because they could have been engaging with the problem according to the procedural way in which they learned it.

Let the tables be represented by x
 Let the bookcases be represented by y

$$\therefore P = 25x + 30y$$

- ① $20x + 30y \leq 690$: $2x + 3y \leq 69$
 ② $5x + 4y \leq 120$: $5x + 4y \leq 120$
 ③ $x \geq 0, y \geq 0, x, y \in \mathbb{Z}$



$$30y = -25x + P$$

$$y = -\frac{5}{6}x + \frac{P}{30}$$

\therefore He should produce 12 tables and 15 bookcases weekly
 in order to maximize profit

Figure 10

And finally there were students who choose to answer the question using trial and error as depicted by Figure 11. These students obviously did not recognise the problem as being linear programming.

under and 4 hours of labour for a bookcase, and he can sell all the tables and bookcases he produces. The carpenter wants to determine a weekly production schedule for tables and bookcases that maximises his profits.

ATTEMPTS
↓

By Trial and Error:

① 690 sheets.

Bookcases

Use 350 sheets
Need 30 per bookcase.

$$30 \overline{)350} = 11,66 \\ = 11 \text{ complete.}$$

No. of hrs. \rightarrow 11 @ 4 hrs
= 44 hrs total.

Tables

Use 340 sheets
Need 20 per table.

$$20 \overline{)340} = 17 \text{ complete}$$

No. of hrs \rightarrow 17 @ 5
= 85 hrs.

$$\text{Total hrs} = 44 + 85 \\ = 129 \text{ hrs to many hrs.}$$

\therefore 1 less book case & 1 less table.

②

Use 300 sheets
Need 30 per bookcase

$$30 \overline{)300} = 10 \text{ complete.}$$

No. of hrs \rightarrow 10 @ 4 hrs
= 40 hrs.

Use 320 sheets
Need 20 per table.

$$20 \overline{)320} = 16 \text{ complete.}$$

No. of hrs \rightarrow 16 @ 5 hrs
= 80 hrs.

$$\text{Total hrs} = 40 + 80 \\ = 120 \text{ hrs.}$$

1

Now we have the correct number of hrs, i.e. 120 hrs used.

Possible Max Profit

Bookcases

10 bookcases @ R30
= R300

Tables

16 tables @ R25
= R400

Total Profit
= R300 + R400
= R700

Notes: Carpenter worked double hrs on tables than bookcases
∴ make hrs equal.

③

Bookcases

work 60 hrs
per bookcase = 4hrs

$4 \sqrt{60} = 15$ bookcases can make.

30 sheets needed per bookcase

∴ $15 \times 30 = 450$ sheets

Tables

work 60 hrs
per table = 5hrs

$5 \sqrt{60} = 12$ tables can make.

20 sheets needed per table

∴ $12 \times 20 = 240$ sheets.

Total hrs = 60 + 60
= 120 hrs max he has.

Total sheets = 450 + 240
= 690 sheets max he has.

Possible Max Profit

Bookcases

15 bookcases @ R30
= R450

Tables

12 tables @ R25
= R300

Total Profits
= R450 + R300
= R750

2

Figure 11

In all off these cases the students were able to arrive at a correct solution. In hindsight, the researcher realises that she should have asked these questions differently. The question should have been posed as follows: *In how many ways can we find the optimal solution?* This would have attempted to get the students to think of other creative solutions.

Question Two: Two observers are at sea level, 500m apart and are in line with a distant mountain peak. They measure angles of 32° and 35° to the peak. How high is the peak above sea level? (Taylor & Sinclair, 2000).

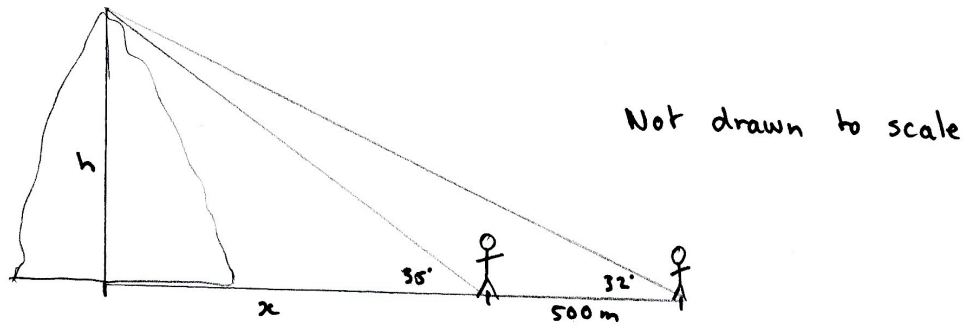
This type of question is answered between grades 10 and 12 at school. Students would have seen the question in various forms. Ninety-one percent of the participants attempted to provide a solution for this question. Nine percent of the respondents did not offer any solution. Only forty-seven percent of these participants were able to correctly answer the question. The researcher observed that many of the remaining fifty-three percent did not get a correct answer, purely because they did not apply their knowledge of trigonometry ratios correctly, could not recall the sine and cosine rules or encountered rounding off errors in their answers. The researcher therefore hypothesised that some of the participants did not obtain a correct answer, simply because they could not recall their previously learnt mathematics. Again, it is significant to note that a large percentage of the students could not solve the problem which they would themselves be teaching in their near future.

Most students, whether they solved the problem or not, drew a diagram to represent the problem as they understood it. This diagram probably allowed them to make meaning of the words that they read. Somehow, the visual aspect of the solution allowed them to understand

the problem. The following excerpt from an interview explains why the student found the diagram he drew useful.

Udesh: *“I didn’t really think much..... I just drew the diagram. When I draw it and once I can visualise it, then I know what’s happening..... Once I have drawn the diagram I can see all the information on the diagram.”*

Although fifty-three percent of the students did not arrive at the correct solution, as has been stated already, they made errors in calculation. It was apparent that they understood how to solve the problem but algebraically they erred. Figure 12 is an example of one correct solution.



$$h = x \tan 35^\circ \quad \text{and} \quad h = (500 + x) \tan 32^\circ$$

$$\text{but } x = \frac{h}{\tan 35^\circ}$$

$$\therefore h = \left(500 + \frac{h}{\tan 35^\circ} \right) \tan 32^\circ$$

$$= 500 \tan 32^\circ + \frac{h \tan 32^\circ}{\tan 35^\circ}$$

$$h - \frac{h \tan 32^\circ}{\tan 35^\circ} = 500 \tan 32^\circ$$

$$h \left(1 - \frac{\tan 32^\circ}{\tan 35^\circ} \right) = 500 \tan 32^\circ$$

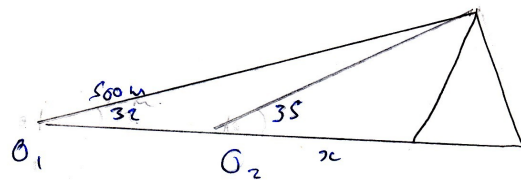
$$h = \frac{500 \tan 32^\circ}{\left(1 - \frac{\tan 32^\circ}{\tan 35^\circ} \right)}$$

$$h = 2903,8 \text{ m}$$

$$\underline{\underline{\approx 2904 \text{ m above sea level}}} \rightarrow \checkmark$$

Figure 12

Figure 13 represents a solution that contains a rounding off error in the calculation.



$$\tan 32^\circ = \frac{h}{500+x}$$

$$\tan 35^\circ = \frac{h}{x}$$

$$(\tan 32^\circ)(500+x) = (\tan 35^\circ)(x)$$

$$\frac{500+x}{x} = \frac{\tan 35^\circ}{\tan 32^\circ}$$

$$500+x = 1.12x$$

$$x = 4166,67 \text{ m}$$

$$h = x \tan 35^\circ$$

$$= 4166,67 \tan 35^\circ$$

$$= 2917,53 \text{ m}$$

Figure 13

Question Three: Randy and his sister agreed that they would each mow one-half of the lawn. The lawn is a 25 m by 45 m rectangle. The lawn mower cuts a path half a metre wide. If Randy starts at one corner and mows a path completely around the outside, how many times should he go around to mow one half of the lawn? (Fesler, 1995).

Eighty-six percent of the participants attempted Question three by at least attempting to draw a diagram or trying to generate a sequence. Only twenty-eight percent of all the participants who attempted the problem were able to obtain correct answers. The remaining seventy two percent obtained incorrect answers due to algebraic errors or the inability to recall the sum of an arithmetic series; or due to the fact that they could not proceed beyond the diagram stage.

Figure 14 is an example of a correct solution to problem 3.

Area of Rectangle = $l \times b$
 $= 45 \text{ m} \times 25 \text{ m}$
 $= 1125 \text{ m}^2$

Randy must mow $562,5 \text{ m}^2$

On first path - outer edge of lawn: Length: $0,5 \text{ m} \times 45 = 22,5$
 $22,5 \times 2 \text{ sides} = 45 \text{ m}^2$

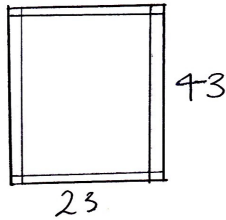
Breadth
 $25 - (\frac{1}{2} \text{ m} + \frac{1}{2} \text{ m})$ (already mowed)
 $= 24 \text{ m} \times 0,5 \text{ m} = 12$
 $12 \text{ m} \times 2 \text{ sides} = 24 \text{ m}^2$

On first path ; Randy mowed $45 \text{ m}^2 + 24 \text{ m}^2 = 69 \text{ m}^2$

Now rectangle is

Second path mowed
 $= 44 \text{ m}^2$
 $+ 23 \text{ m}^2$
 $\hline 67 \text{ m}^2$

3rd path mowed: Rectangle looks like this:



$$\begin{array}{r} 43\text{m}^2 \\ + 23\text{m}^2 \\ \hline 65\text{m}^2 \end{array}$$

Pattern Forming

Path	Total Length mowed in single path	Total Perimeter mowed in single path	Total mowed for that path	Total mowed overall
1	45	24	69	69
2	44	23	67	136
3	43	22	65	201
4	42	21	63	264
5	41	20	61	325
6	40	19	59	384
7	39	18	57	441
8	38	17	55	496
9	37	16	53	549

Up to the 9th time round he's mowed 549 m²
 he needs to mow 562,5 m² ∴ he needs
 13½ more metres² of area.

∴ Randy must go around ^{now} 9 times, and then
 on the 10th time around, he must stop after he
 has completed 27m of the length.
 (since 27m x 0,5m = 13,5m²)

Figure 14

Research question three: *How do pre-service mathematics teachers translate real-world mathematics problems into mathematical models?*

The researcher found that the participants used various methods to obtain solutions to the task-based questions. Many students chose to draw diagrams as they explain below:

Udesh: *“I didn’t really think much..... I just drew the diagram. When I draw it and once I can visualise it, then I know what’s happening..... I don’t really have to think that much once I have drawn the diagram I can see all the information on the diagram.”*

Sanele: *“What I did was I obviously drew a diagram. I then calculated the area of the lawn and found it to be 1125m². So half the area would be 562 ½ m².”*

Jackie: *“I drew a diagram.for me, I would never be able to answer that type of question just by using words and numbers. I would have to transfer that into some sort of picture or table or something in order to get the solution.”*

This in a sense may mean that the students converted the words that they read into a visual problem. From their explanations it appears that they understood this better.

Some participants chose to construct tables to display the information whilst others choose to use an algebraic method. Using a table could also be construed as creating a visual basis for their understanding. The following excerpts from the interviews give an idea why students chose to draw tables.

Udesh: *“I did it like this... I used a table because it was easier for me. They tell you what it costs to make a table and how much it costs to make a bookcase, how much of profit you get from each one and the time it takes to do it. I just associated variables with each one and drew them in a table with the required materials, time and profit and wrote down what is for each and then I set up my equations. I didn’t draw the graph because I didn’t think there was a need to draw the graph.... I solved it algebraically”.*

Andrew: *“A table makes more logical sense for somebody if you don’t understand where I got my equations from. A learner can see where they got the equations from”.*

Creating tables and diagrams were part of the modelling process. The students simply converted the problem into a version they understood and attempted a mathematical solution. Many students, who obtained correct answers, unknowingly completed the mathematical modelling process by checking their solutions. The following excerpts are some of the responses the researcher received when the participants were asked if they had checked their answers:

Researcher: *“Did you check your answer to see whether it is correct?”*

Jackie *“When working with this trial and error I used the calculator obviously and had to constantly check if I had the correct number of bookcases and number of tables. So yes, I can say I checked”.*

Garth: *“Yes, I substituted into the equations and got answers of 690 and 120”.*

Udesh: *“Yes, I substituted the values to see if I get a maximum profit”.*

Kamal: *“Yes....by drawing the graph, I found specific points and substituted into the inequation and checked”.*

Andrew: *“Yes, I did check I took my y and my x values and substituted it into my equations and looked if I could get the right answer again and that’s how I checked it..... through substitution”*.

Gareth: *“I did check two other points to determine whether they gave a greater profit margin, than the point that I had chosen and they didn’tand graphically that is the optimal point in terms of the two functions themselves”*.

Bruce: *“Yes well, I substituted different values for x and y into the profit and I saw that 12 and 15 was in fact the maximum”*.

Research question four: *Are pre-service mathematics teachers able to facilitate the understanding of the solution of the mathematical modelling process?*

Listed below are three examples of the kind of explanations obtained from the participants:

Garth: *“I said let x be the height that the height from the ground to the top of the mountain peak, because that ideally is what we are trying to find—the height of the peak above sea level. So let x be the height of the peak okay, which is length DC and now I’ve drawn my diagram obviously reflecting both of the people that are standing towards the peak okay. Obviously the person standing further away is going to have a smaller angle as opposed to the person standing a little closer and will be looking more upright. I have found two equations in terms of tan (i.e opposite over adjacent) using the variables as shown on the diagram, and I have solved simultaneously and found a value for y, which is the distance of the first person*

from the bottom of the mountain peak. Then using this small right –angled triangle over here, I solved for x and found the height of the mountain peak”.

Michelle: “Well, I firstly let the tables be x and the bookcases be y because those are the kind of variables that you going to have and then I used the time for the bookcases and the tables and then I took the hours of labour for the two variables and I used the sheets of timber for the two.....made those into inequalities because it has to be less than or equal to the total. I then used the straight line graph of y equals the two equations, plotted those on a graph and then for my maximumyou had to find the maximum profit and so I made an equation using the maximum profit and I found the gradient of that and that gradient I used to find my maximum point.....and then I read off my values from the graph”.

Gareth: “Basically what it is I did I took the initial area of the lawn and worked it out give the dimensions of the lawn and calculated what half the area would be.....then just basically because he’s mowing the strip that’s half metre wide....what happens is every lap that he goes round the lawn....the length and the breadth are each reduced by a metre which means then that the area is decreasing after every lap.....so what I did then was I calculated the area initially and after one lap.....two laps....and all the way down to 10 lapsand between 9 and 10 laps was essentially halfway”.

Research question five: *Do pre-service mathematics teachers possess more than one strategy for solving real-world mathematics problems?*

The participants were aware that there may be other strategies to work out each problem. There were a few that preferred to use the strategy that they were familiar with. They were not keen to change.

Jackie: *“Okay. Well the way I did it was through trial and error”*.

Jackie: *“Yes, I could use other strategies as working with variables for the tables and bookcases but I find trial and error easier. I don’t know that method very well and I haven’t been taught it very well”*.

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem”?*

Gareth: *“No. This is the only way I know. There are probably other ways to work it out, but I couldn’t”*.

Researcher: *“Why did you choose this particular solution”?*

Udesh: *“I used the sine rule here because it was the only way I could think of finding out DC and I used it again because I already was in the mood to use sine”*

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem”?*

Andrew: *“Probably is. I feel much more comfortable by using algebra like this. A table makes more logical sense for somebody if you don’t understand where I got my equations from. A learner can see where they got the equation from.”*

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem?”*

Michelle: *“I’m not too sure.”*

Based on the above responses the researcher claims that the students are not confident enough to experiment with new methods. Perhaps it might be useful to include in the teacher education curriculum aspects that would deal with a multi-pronged or divergent problem solving techniques using mathematical modelling.

And there were others that knew various strategies and simply choose one to work with. This indicates that although our trainee teachers may be aware of other strategies, they are comfortable using only methods which they have been taught. They are not prepared to attempt solutions in different ways. Again, this may point to a deficiency in the curriculum that they are exposed to.

Researcher: *“What thoughts came to mind when you read the first question?”*

Garth: *“Well, I thought two things. Firstly, I thought whether I would go about this drawing something graphically or working it out algebraically or whether I would use my maximum-minimum using derivatives. So the first thing was analysing my options and the second thing I thought was where did I remember doing this before”.*

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem?”*

Garth: *“Yes, I think there is”.*

Researcher: *“Tell me more”.*

Garth: *“You could work it out the opposite to what I’ve done. Instead of decreasing the lengths, you could increase the lengths, if you started at the centre”.*

Researcher: *“Okay, that’s sounds like a possibility”.*

Garth: *“I thought that there may be something else. I was looking for a formula or an equation and I couldn’t believe I couldn’t find anything. I looked at the difference between the small areas and I could see a common difference and kept thinking that there must be some formula”.*

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem?”*

Udesh: *“Yes. You could draw the two lines to intersect and you could do it using a graph. I didn’t draw the graph because I didn’t think there was a need to draw the graph.... I solved it algebraically.”*

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem?”*

Andrew: *“Yes. You could use the formula for an arithmetic series, unlike how I had to add to get the required area.”*

Researcher: *“Your solution is correct. Do you think that there are other strategies to solve the same problem?”*

Gareth: *“I suppose you could do it by trial and error and if I hadn’t known linear programming I probably would have ended up doing trial and error.”*

The researcher believes that the potential to mathematical model is present in the research participants. Her conjecture is that these students cannot adequately deal with mathematical modelling because they have not been formally exposed to the study of mathematical modelling in the teacher education curriculum.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

This research study explored pre-service mathematics teachers' knowledge and use of mathematical modelling as a strategy for solving real-world problems. The participants included senior students from the University of KwaZulu-Natal (Edgewood Campus) who were registered for the Further Education and Training phase. This chapter summarises the main findings and implications of these findings, limitations of study and some suggestions for future research.

6.2 Summary of main findings

This study has provided some significant results in terms of mathematical modelling in teacher education.

The empirical evidence showed that the participants did not have a suitable working knowledge of the term mathematical modelling. Sixty-seven percent of the students indicated that they have worked with mathematical modelling previously – either at school or in certain modules at university. Some of the participants felt that their teachers at school may have used mathematical modelling to teach certain topics but never used the words mathematical modelling in class. A few participants remembered hearing their university lecturers mention mathematical modelling in mathematics method modules and a course called Mathematics for Educators 420.

The following emerged after an analysis of the task-based questionnaire:

1. Eighty-six percent of the participants made an attempt to answer question one. Forty-three percent were able to correctly provide a solution.
2. With respect to question two, ninety-one percent made an effort to offer a solution, with forty-seven percent being able to correctly answer the question.
3. Question three proved to be the most challenging, considering that eighty-six percent of the participants made some attempt to understand the problem by drawing a diagram or trying to find a pattern using the terms of a sequence which they generated. Only twenty-eight percent of all the participants who attempted the problem were able to obtain correct answers. A possible reason for this is the fact that this type of question was never done at schools or at university. The first two types of questions were already covered previously. It can be hypothesised that students will attempt problems with which they are familiar. They are not trained nor are they confident to attempt problems that different and unfamiliar.

The researcher ascertained that the participants chose to use different methods to translate real-world mathematics problems into mathematical models. Some chose to use trial and error, drawing a diagram, constructing a table; whilst others chose to use a purely algebraic method.

The participants were aware that there may be other strategies to work out each problem. There were a few that preferred to use the strategy that they were familiar with. They were not keen to change.

The evidence from this study clearly indicates that pre-service mathematics teachers have the potential to construct mathematical models given real-world problems.

6.3 Limitations of the study

The task-based questionnaire administered to the respondents was presented in the form of a description. Students, to whom English is a second language, may have experienced difficulties with the description of the word problems. They may have not understood the problems sufficiently enough if they encountered problems with the text.

This research study focused on mathematical modelling in teacher education at the University of KwaZulu-Natal, South Africa. The researcher therefore did not deem it necessary to investigate all South African universities which offered mathematics teacher education programs. However, the work being done at the Wits School of Education, University of the Witwatersrand, South Africa was reviewed in chapter two.

It therefore must be stressed that the results of this research study apply only to the to the pre-service mathematics teachers at the University of KwaZulu-Natal, Edgewood Campus. No generalisation to other pre-service mathematics teachers at other South African universities is claimed.

6.4 Recommendations for future research

The consequent related research in the following areas is proposed by the researcher:

1. The researcher recommends that future studies be conducted which investigates the implementation of mathematical modelling in secondary schools in KwaZulu-Natal. The study could focus on how mathematical modelling is being implemented in South African schools and what are some of the difficulties, if any, being experienced. A longitudinal study should be carried out to determine if in-service teachers are in fact

implementing mathematical modelling as laid out in the National Curriculum Statement. A much broader and more adventurous study could probe the implementation of mathematical modelling at a national level.

2. Furthermore research could be conducted in the area of assessment in mathematical modelling. The researcher detected reservation in this area whilst reviewing literature for this study. This concern is highlighted by Antonius (2007) when he states that *“another question is how to design an examination which enables students to show their modelling skills and which also enable examiners to detect and to assess students’ modelling skills in the examination ‘products’ (written reports and/or oral presentation)”*.

With respect to the curriculum planners at the University of KwaZulu-Natal;

1. The curriculum planners should include a course in mathematical modelling for pre-service mathematics teachers.
2. Sections of these courses should focus mainly on tasks typical of the secondary school curriculum.

6.5 Conclusion

Whilst keeping in mind the rewards of mathematical modelling, De Villiers (1993) points out that not all mathematical topics are suited for development through modelling, and the source of new mathematics is not always the real world, but quite often results from further reflection on existing mathematics. He remarks that educators need to realise that modelling is an additional technique and that it cannot replace techniques such as drill and practice which are equally important. De Villiers (as cited in Amesa, n.d) however also argues that although a

mathematical modelling lesson can appear time-consuming, the rewards obtained with respect to the understanding of concepts and enjoyment more than justify the time spent. He states that “ *over the long term it actually saves time as a thorough understanding of the concept by the pupils in any mathematical situation make them naturally more versatile in their application and reduces the need for endless repetition before the pupils master a section*”.

He purports that mathematical modelling can be used very successfully whenever it is appropriate to connect the mathematics of the classroom with the mathematics world.

The researcher is of the opinion that pre-service mathematics teachers at the University of Kwa-zulu-Natal have the potential to mathematical model real-world problems. Although they do not have a suitable working knowledge of what mathematical modelling is, because they have not been formally exposed to the study of mathematical modelling when they were at school and in the teacher education curriculum, many of the participants would be able to facilitate the understanding of the mathematical modelling process.

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APPENDIX A

QUESTIONNAIRE: MATHEMATICAL MODELLING

Dear Student,

Thank you for consenting to answering this questionnaire. The purpose of this questionnaire is to determine pre-service teachers' knowledge of mathematical modelling. Although you are not compelled to answer these questions, your responses are vital for the process of change. Your answers will help to make a difference in Mathematics Education at schools as well as at the tertiary level.

Should you need more space, you may use the back of this questionnaire to write on.

ANSWER THE FOLLOWING QUESTIONS:

1. What knowledge do you have concerning mathematical modelling?

2. Have you worked with mathematical modelling before? If yes, provide some details.

3. Can you recall whether your mathematics teacher used mathematical modelling? If yes, can you remember how?

4. Do your university lecturers talk about or use mathematical modelling? If yes, during which modules did this occur?

5. Have you interacted with the mathematical modelling aspects in the National Curriculum Statement? If yes, can you explain how?

6. Do you think that mathematical modelling will play an important role in teaching mathematics in the FET phase?

7. Provide a motivation for your answer in question 6.

Thank you.

APPENDIX B

Dear Student,

Answer the following three problems in the spaces provided. You may provide as many solutions as you see possible and you may use any suitable method of your choice.

1. A carpenter makes tables and bookcases for a net unit profit that he estimates as R25 and R30 respectively. He needs to determine how many units of furniture he should make each week. He has up to 690 sheets of timber to devote to the project weekly and up to 120 hours of labour. It requires 20 sheets of timber and 5 hours of labour to complete a table, and 30 sheets of timber and 4 hours of labour for a bookcase, and he can sell all the tables and bookcases he produces. The carpenter wants to determine a weekly production schedule for tables and bookcases that maximises his profits.

2. Two observers are at sea level 500m apart and are in line with a distant mountain peak. They measure angles of 32° and 35° to the peak. How high is the peak above sea level?

3. Randy and his sister agreed that they would each mow one-half of the lawn. The lawn is a 25 m by 45 m rectangle. The lawn mower cuts a path half a metre wide. If Randy starts at one corner and mows a path completely around the outside, how many times should he go around to mow one half of the lawn?

APPENDIX C

INTERVIEW QUESTIONS

If solution was correct:

1. Can you explain what thoughts came to your mind when you read the first problem?
2. Did you find that the problem looked familiar to you in any way?
3. Did you attempt to solve such a problem previously? If so, when?
4. That answer is very interesting; could you please take me, step by step through your solution?
5. Your solution is correct. Do you think that there are other strategies to solve the same problem?
6. If you were given another similar problem, would you be able to answer it?
7. How would you explain such a solution to your learners?
8. Was it difficult to determine the answer?
9. Why did you choose this particular solution?
10. Did you check your answer to see if was correct?

If solution was incorrect:

1. Can you explain what thoughts came to your mind when you read the first problem?
2. Did you find that the problem looked familiar to you in any way?
3. Did you attempt to solve such a problem previously? If so, when?
4. I noticed that you did not get the correct answer I have one correct solution. Do you want to have a look at it?
5. Does this solution make sense to you?
6. Would you be able to solve such a problem in future?

APPENDIX D

SEEKING PERMISSION

5A Regent Place

Westville

3629

29 September 2006

Dear Prof. R. Vithal

Re: Permission to use Final year Pre- service students

My name is Eshara Dowlath. I am a Master of Education (Mathematics Education) student as well as a contract lecturer in the School of Science, Mathematics and Technology Education at Edgewood.

The focus of my study is Mathematical modelling in Teacher Education. I therefore wish to seek permission from you to interview final year pre-service mathematics teachers in order to obtain my data. If you require any further details you may contact Dr. V. Mudaly (SMTE).

Your assistance will be greatly appreciated.

Yours Sincerely

Mrs. E. Dowlath

Contact details: dowlathe@webmail.co.za

Cell phone: 083 7798229

APPENDIX E

INFORMED CONSENT

Date: _____

Dear _____

I am a Master of Education student in the School of Science, Mathematics and Technology Education at the University of KwaZulu-Natal. As part of the research module I am required to conduct a research project. This research project requires the participant to answer three questions and thereafter be interviewed by the researcher.

I would like your permission to involve you in this research process. The data collected from you will only be used for this research project only. No real names will be used in the write-up of the dissertation. You are guaranteed anonymity and confidentiality. You are not obliged to answer the questions and you are free to withdraw from the study at any given point.

If you have any questions, I can be reached on 083 779 8229.

Thank you for your assistance.

Yours sincerely

Eshara Dowlath

I have read the above and agree with the terms. I understand that my real name will not be used, and I may withdraw from the study at any time.

NAME: _____ SIGNATURE: _____

DATE: _____

APPENDIX F

INTERVIEW TRANSCRIPTIONS

INTERVIEW ONE: JACKY

R: RESEARCHER

J: JACKY

R: Can you explain what thoughts came to your mind when you first read the first question?

J: I first had to read the question a couple of times in order to actually find out what was needed required of me. When reading it..... what came to mind was that I had to work out what the maximum number of book cases and tables he needed to make in order to get his profit. In working that out I had to re-read the section on where 690 sheets were used, the hours, the total hours per table, per bookcase.

R: Did you find that the problem looked familiar to you in any way?

J: Familiar? no, not really. Well besides the fact that... I don't know. Yes...Yes... it was similar to a problem where we had almost exactly the same thing but working with sheep.he had to shave sheep and another animal of some sort. So yes.

R: Did you attempt to solve such a problem previously? If so, when.

J: No.

R: Could you please take me step by step through your solution?

J: Okay. Well the way I did it was through trial and error. I knew that I had a total number of 690 sheets so by working with just the bookcases on how many sheets one bookcase needs, I worked out that it needed 30 sheets, so I went 350 divided by the 30 and I got you can get a complete 11 bookcases there was a remainder but you can't have half a bookcase, so then working with hours I then said okay well if I got

11 complete bookcases working at for each one at 4 hours I would have a total of 44 hours then working with my table knowing that used then I used 350 sheets in my bookcase I the used 340 sheets left over from my table and each table needs 20 sheets so I divided 340 by 20 and I got a complete 17 tables and the number of hours working at it for 5 hours I get 85 hours. I then totalled my hours and found out that I had too many hours.. 9 hours too many so then I decided to subtract one bookcase and subtract one table because the addition of the two hours is 9 hours. My next attempt was to use 300 sheets for the bookcases where each bookcase uses 30 and 300 divided by 30 and I got 10 complete and

then that came to working for 40 hours and for my tables I used 320 sheets were it is 20 sheets per table that came to 16 complete tables and the number of hours were 80 hours, so I had 120 hours. I didn't use my maximum sheets but I used my maximum hours and that gave me a possible profit for my bookcases I had 10 full bookcases at R30 I'd get R300 and for the tables if had 16 full tables at R 25 .. I'd have R400. so my total profit would be R700. but I noticed that the carpenter worked double hours on the table than the bookcases therefore the hours should be equal. So instead of working on the sheet basis, I worked on the hours basis, so I gave the carpenter 60 hours to work on the bookcases and on a bookcase you need 4 hours so I went 60 hours divided by 4 hours and he has 15 bookcases to make in that time and if he needs 30 sheets per bookcase 30×15 gives me 450 sheets used in the bookcase and in the table again he had 60 hours and it is 5 hours per table so 60 divided by 5 .. he'll make 12 tables where he uses 20 sheets per table 12×20 is 240 so in total hours I used my 120 hours and the sheets I used my total sheets of 690. so that my possible maximum profit for the bookcase should be 15 b/cases at R30 is R450 and for the tables I used 12 tables at R25 each to give me R300. so my total is R750.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

J: Yes, I could use other strategies as working with variables for the tables and bookcases but I find trial and error easier. I don't know that method very well and I haven't been taught it very well.

R: If you were given another similar problem, would you be able to answer it?

J: If it was similar, yes.

R: Would you use the same method of trial and error?

J: Yes

R: How would you explain such a solution to your learners?

J: If I knew the variable method I would attempt the variable method before the trial and error purely because for time reasons. The trial and error method can be very long you know, but if a learner is more comfortable with the trial and error, I would obviously encourage them to find a quicker way of using the trial and error method.

R: Was it difficult to determine the answer?

J: Initially when first reading the question I thought lots of words and a couple of numbers so I did think it would be quite difficult but once actually thinking about what the different possibilities the carpenter could have done it was pretty easy thereafter.

R: Why did you choose this particular solution?

J: I find it easier to work with trial and error.

R: Did you check your answer to see whether it is correct?

J: When working with this trial and error I used the calculator obviously and had to constantly check if I had the correct number of bookcases and number of tables. So ja I can say I checked.

R: Can you explain what thoughts came to your mind when you first read the second problem?

J: What thoughts came to mind was that I felt that this wasn't a mathematical problem but more geared towards a Science problem, purely because it had dealt with distance and angle measurements.

R: Did the problem look familiar to you in any way?

J: It did. When doing Science in school I encountered a number of these problems.

R: Did you attempt to solve such a problem previously?

J: No, not in mathematics. Well, yes, in science in school.

R: Why did you not attempt this solution?

J: I attempted it in rough, not on this sheet. While attempting to find a solution or possible answer, every little scribble I made did not make sense. I decided not to write anything down because I could not make sense of my own working.

R: Seeing that you did not arrive at a correct solution, I have one possible correct solution. Would you like to have a look at it?

J: Yes

R: [Researcher explains solution to student]. Does this solution make sense to you?

J: Yes, it does. Your diagram is quite similar to what I had.

R: would you be able to solve such a problem in future?

J: Yes. Seeing that you have explained the solution to me and I have drawn a similar diagram.

R: How would you explain such a solution to your learners?

J: Obviously, I would tell them to revise their trigonometry, and refresh their memory about what they know and what they don't know. Then I would make sure that they actually understand what is asked of the question and how they interpret the question, and correct them and guide them to the correct answer, if it's wrong.

R: You mentioned earlier that you also drew a diagram to try to answer this question. Do you think that a diagram is necessary in such a problem?

J: Most definitelyfor me, I would never be able to answer that type of question just by using words and numbers. I would have to transfer that into some sort of picture or table or something in order to get the solution.

R: What thoughts came to mind when you first read the third question?

J: Very confused! Again lots of words.

R: Did the problem look familiar to in any way?

J: No. I did not have a problem like this before.

R: Did you attempt to solve a problem like this before?

J: Not previously. But I have tried something on this sheet.

R: Could you please explain what you have attempted on your sheet?

J: I drew a rectangular lawn, and then I have cut it into halves where Randy mowed half and his sister mowed the other half. I worked out the area of each half, which would obviously be the same, and the total area that I got was 1125m². I got that answer by being given the length and the width of this rectangular lawn. And then by working out each area Randy and his sister mowed, I divided it by 2 that is I divided 1125 by 2 and got 562 ½ m² and then I guessed from there on.

R: Let me show you a possible solution to this problem. [Researcher explains a possible solution to the student]. Does this solution make sense to you?

J: Yes. I am familiar with geometric series from school.

R: Would you be able to answer such a question in future?

J : Being told what method to use (in this case a geometric series) I would say yes. But not from straight reading that kind of question..... no, I'd need a hint or something.

R: Would you be able to explain this solution to your learners?

J: Yes, if I'm given the solution.

R: Again, seeing that you attempted the problem by drawing a diagram, do you think that drawing a diagram is important to such question?

J: It would be because it allows the learner to see the distance from where they start and where they finish that is half a metre from each edge. Also it would help them when calculating how much Randy has travelled around the lawn itself so by showing a picture of him doing first round, second round, third round and so on, it then develops a pattern which explains to you how to use the geometric series formula.

R: Do you think that there are any other strategies to work out this problem?

J: Yes, I suppose you could use the long method. By calculating the area that is being mowed each round and then adding it until you come to about $562 \frac{1}{2} \text{ m}^2$. You don't see a pattern.

R: Thank you.

INTERVIEW TWO: GARTH

R: RESEARCHER

G: GARTH

R: Can you explain what thoughts came to your mind when you first read the first question?

G: Well, I thought two things. Firstly, I thought whether I would go about this drawing something graphically or working it out algebraically or whether I would use my maximum-minimum using derivatives. So the first thing was analysing my options and the second thing I thought was where did I remember doing this before.

R: Did you find that the problem looked familiar to you in any way?

G: Yes, I did find it familiar. I had seen it before both in modules done here at college and at school.

R: Did you attempt to solve such a problem previously? If so, when.

G: Yes, at school.

R: Could you please take me step by step through your solution?

G: Initially, I just drew out a whole lot of equations using the information given, just to try and find out where they could actually relate and basically what I've done is I've said my tables are x and my bookcases are y , and then obviously then using x and y to formulate 2 equations. Because the profit is the main result, I've put the profit in terms of x and y and got $P = 25x + 30y$ because on each table he makes R25 profit so x (which will be the number of tables) times 25 is $25x$; and the profit on each bookcase is R30 so it would be $30y$ because y represents the number of bookcases, so finally the profit will be $25x + 30y$. To get the other two inequations, I said that the tables need 20 sheets of timber and the bookcases need 30 sheets of timber, I have a maximum amount of 690 sheets, so $20x + 30y$ has to be less than 690. Then it takes 5 hours to make a table and is the amount of time required to make the tables, and it takes 4 hours to make a bookcase so $4y$ is the time required to make the bookcases. Because the carpenter has a maximum amount of 120 hours I get $5x + 4y$ less than and equal to 120. Even though we dealing with linear inequations I use them as linear equations to sketch the graphs. I have drawn the graphs and once I have depicted those two linear equations I mark of this section (student shows feasible region) and I need to find the highest point of intersection, but that highest point needs to be determined by the gradient of my equation of this profit. This graphed

equation of the profit helped me to find the two points, but I also checked my answer algebraically and ended up with the same two points of 12 and 15.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

G: There should be. You could work it out algebraically only and not graphically. I tried using the derivative to get my maximum and minimum points, but with two variables I got no way.

R: If you were given another similar problem, would you be able to answer it?

G: Yes, but using the same method that I used in this problem.

R: How would you explain such a solution to your learners?

G: I would first give the learners the problem to attempt it, and see what they came up with- some may do it algebraically and some may attempt it graphically. Then I'd discuss the solution.

R: Was it difficult to determine the answer?

G: No, I don't think so.

R: Why did you choose this particular solution?

G: Because it was familiar to me and the derivative did not work.

R: Did you check your answer to see whether it is correct?

G: Yes, I substituted into the equations and got answers of 690 and 120.

R: Can you explain what thoughts came to your mind when you first read the second problem?

G: I enjoy this kind of question. It's one of those questions where you actually have to look beyond the question.

R: Did the problem look familiar to you in any way?

G: Yes, it did. I saw it recently in I think a Grade 10 class.(during Practice teaching).

R: Did you attempt to solve such a problem previously?

G: Yes, I did. When I was teaching Grade 10. Also I was at school about seven years ago and I remember these kinds of problems being vaguely familiar from school.

R: Could you please take me step by step through your solution?

G: I said let x be the height that the height from the ground to the top of the mountain peak, because that ideally is what we are trying to find—the height of the peak above sea

level. So let x be the height of the peak okay, which is length DC and now I've drawn my diagram obviously reflecting both of the people that are standing towards the peak okay. Obviously the person standing further away is going to have a smaller angle as opposed to the person standing a little closer and will be looking more upright. I have found two equations in terms of \tan (i.e opposite over adjacent) using the variables as shown on the diagram, and I have solved simultaneously and found a value for y , which is the distance of the first person from the bottom of the mountain peak. Then using this small right – angled triangle over here, I solved for x and found the height of the mountain peak.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

G: No. This is the only way I know. There are probably other ways to work it out, but I couldn't.

R: If you were given another similar problem, would you be able to answer it.

G: Yes, I would.

R: How would you explain such a solution to your learners?

G: I'd given them obviously the problem, but I think I'd probably word the question differently because of the possibility of the learners getting confused about when you standing in line with a distant mountain peak you could have them standing on either side and still have them in line, as opposed to one side.

R: Was it difficult to determine the answer?

G: No, I don't think so.

R: Why did you choose this particular solution?

G: Because it was familiar to me.

R: Did you check your answer to see whether it is correct?

G: No. I did not check my answer.

R: Can you explain what thoughts came to your mind when you first read the third problem?

G: First I thought it was an optimization problem, but when I really got into it, I realised it is not an optimization problem but rather natural thinking.

R: Did the problem look familiar to you in any way?

G: No. I have not really seen anything like this before.

R: Did you attempt to solve such a problem previously?

G: No not really. But I have worked with similar word problems before.

R: Could you please take me step by step through your solution?

G: What I did was I obviously drew a diagram. I then calculated the area of the lawn and found it to be 1125m². so half the area would be 562 ½ m². And initially was what I'd done was I'd gone through and mowed once around the lawn and I realised it decreased every time I went once round, and each time I had a different rectangle, a smaller rectangle, and then used the different lengths and found new areas and then just added until my area became nought. But the solution that I've got here I realised that we need to find half the area because that's how much he needs to complete. I then added each small area around the rectangle until I got about 562 ½ . after 9 rounds it was little less than 562 ½ and after 10 rounds it was too much so he has to mow 9 ¼ times.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

G: Yes, I think there is.

R: Tell me more.

G: You could work it out the opposite to what I've done. Instead of decreasing the lengths, you could increase the lengths, if you started at the centre.

R: Okay, that's sounds like a possibility.

G: I thought that there may be something else. I was looking for a formula or an equation and I couldn't believe I couldn't find anything. I looked at the difference between the small areas and I could see a common difference and kept thinking that there must be some formula.

R: If you were given another similar problem, would you be able to answer it.

G: Yes

R: How would you explain such a solution to your learners?

G: I'd give them the problem, but two things I would change if I may. One, I don't believe it is technically correct when mowing the lawn because the lawnmower cuts a path half a metre wide and you keep moving across, the practical application just doesn't make sense. It is going to be practically impossible to cut an absolute straight line, the measurement is going to be out. I think we must say that as much as it seems to be a real life problem, it

does not necessarily happen like in real life as much as we know that Randy is not a superhuman being to cut exactly half a metre at a time, lets allow it to happen for this example; and secondly I'd say well there are different methods to find the answer and I'd ask them to find a formula and try and get them frustrated, like I was frustrated.

R: Was it difficult to determine the answer?

G: No, it wasn't difficult, but more timeous.

R: Why did you choose this particular solution?

G: It was the only one that I felt secure with.

R: Did you check your answer to see whether it is correct?

G: Yes, I did.

R: Thank you.

INTERVIEW THREE: SANELE

R: RESEARCHER

S: SANELE

R: Can you explain what thoughts came to your mind when you first read the first problem?

S: I realise it took me back to 310 (Mathematics for educators 310 is a module at the University of KwaZulu-Natal, Edgewood campus) when I did calculus. But I'm trying to remember the section from 310, but I'm not sure.

R: Did you find that the problem looked familiar to you in any way?

S: Yes, it was familiar but because I thought this will take a lot of time to do I didn't do this one, I tried the others.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

S: Yes, I think I did, in school in calculus....I think maximum and minimum problems.

R: Why did you not attempt the question?

S: I think it would take a lot of time to work out because I have to draw a diagram first and then work it work.

R: I have one correct solution. Do you want me to explain it to you.

S: Yes, please.

R: [Researcher explains the solution to the student] Do you understand the solution?

S: It looks a little bit like linear programming... because we done linear programming last year in Maths 110... because I'm looking at constraints... these are constraints... this function must be less than and equal to 610.

R: If you were given another similar problem, would you be able to answer it?

S: Now that I have seen this solution, I think I will be able to work it out if I'm given enough time.

R: How would you explain such a solution to your learners?

S: I think it would be a problem for me....because at school I did Mathematics on SG and now when we come to university level they say that at school you done this and we not going back to school levels it's a problem for us who did Maths on SG... we didn't do all sections.

R: Can you explain what thoughts came to your mind when you first read the second problem?

S: I thought about lectures in Physics. That's what first came to my mind. Then I think about cosine rule and sine rule when I look at the problem because if you are talking about the sea or the rivers, we are talking about Physics vectors, so I thinking from a Science perspective.

R: Did you find that the problem looked familiar to you in any way?

S: Ya, it looked familiar to me because if you look at it, it is a triangle and it is in cosine rule we are using this construction... so it's familiar to me to make this side the height and put this side as a.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

S: Not really...like this, but something similar at school.

R: Could you please take me step by step through your solution ?

S: I first drew this diagram. This side was 500m...ok.. and I have to find the height so I used sine rule because $\sin A = \frac{a}{c}$. I take $a = c \sin A$. Then I cross multiply and then I work out the height.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

S: Yes, please.

R: [Researcher explains the solution to the student.] Do you understand the solution?

S: I thought the two observers were standing opposite each other, that's why I drew my diagram like this.

R: If you were given another similar problem, would you be able to answer it?

S: Yes, if I were given the diagram.. then I know this is a straight line and that these are the angles.

R: How would you explain such a solution to your learners?

S: Yes, I would be able to explain this to my learners, because I'm teaching Mathematics, and I'm using this diagram to teach cosine rule and sine rule because there is a similar diagram.

R: Can you explain what thoughts came to your mind when you first read the third problem?

S: I thought about optimization. But I have difficulties translating this form to data.... I have difficulty translating word problems to data, but I'm trying to improve.

R: Did you find that the problem looked familiar to you in any way?

S: Yes, because they talk about a rectangle and I'm thinking about in 310 how to use a diagram to relate to optimization .

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

S: Yes, I tried something similar, but this one was difficult, because we were given a diagram before....from previous experience in school we were given a diagram to work out, and I found it easier when we were given a diagram.

R: Why did you not attempt the question?

S: I had a problem to analyse the information that they were giving in the instruction.

R: I have one correct solution. Do you want me to explain it to you?

S: Yes, please.

R: [Researcher explains the solution to the student]. Do you understand the solution?

S: Yes, I see sequences and series. If I'm given the diagram and the sequence I can work it out. The problem is translating the information instruction into data.... In school we were given the sequence .. I can work out the problem.

R: If you were given another similar problem, would you be able to answer it?

S: As I said before I need a diagram.... if I have a diagram I can work it out.

R: How would you explain such a solution to your learners?

S: I will have a diagram.... if you maybe want to do it with children, then to see what's going on you must give the diagram or maybe to write the heading what is the section.

R: Thank you.

INTERVIEW FOUR

R: RESERACHER

U: UDESH

R: Can you explain what thoughts came to your mind when you first read the first problem?

U: The first thought was like to maximise profits...how much materials the carpenter should use, why they need to use so much material and how you can determine this.

R: Did you find that the problem looked familiar to you in any way?

U: Yes, it is linear programming.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

U: Yes, many times.... In school...in maths Method at university.

R: Could you please take me step by step through your solution ?

U: I did it like this... I used a table because it was easier for me. They tell you what it costs to make a table and how much it costs to make a bookcase, how much of profit you get from each one and the time it takes to do it. I just associated variables with each one and drew them in a table with the required materials, time and profit and wrote down what is for each and then I set up my equations. I didn't draw the graph because I didn't think there was a need to draw the graph.... I solved it algebraically.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

U: Yes. You could draw the two lines to intersect and you could do it using a graph but that takes time.

R: Was it difficult to determine the answer?

U: For me, no.

R: Why did you choose this particular solution?

U: It was simpler for me.... to do it algebraically. I don't like complicating things.

R: Did you check your answer to see whether it was correct?

U: Yes, I substituted the values to see if I get a maximum profit.

R: If you were given another similar problem, would you be able to answer it?

U: Yes, definitely.

R: How would you explain such a solution to your learners?

U: I'll explain what they are required to do first; and then... to every learner different methods will be simpler for them.... To some learners drawing a graph will be simpler than algebraically..... so I'll show them both methods and then I'll ask them to choose which method is easier for them. I'll also explain to them that drawing the table is important as it clearly show how much timber is needed time taken and so on.

R: Can you explain what thoughts came to your mind when you first read the second problem?

U: I didn't really think much..... I just drew the diagram. When I draw it and once I can visualise it, then I know what's happening..... I don't really have to think that much once I have drawn the diagram and can see all the information on the diagram.

R: Did you find that the problem looked familiar to you in any way?

U: Yes, I seen it many times at school in trig and in maths Method at university.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

U: Yes, at school and at university.

R: Could you please take me step by step through your solution ?

U: Okay, firstly I drew a diagram because for me drawing a diagram makes it easier. I used the given angles.... and placed it on the diagram as you can see. Then I used the sine rule to determine the length of this particular side and then I used the sine rule again to find out the length of that.... the height.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

U: yes, I think there maybe like using cos or tan.

R: If you were given another similar problem, would you be able to answer it?

U: Yes, I would be able to.

R: How would you explain such a solution to your learners?

U: Obviously your learners must be familiar with the sine rule and basic cos and tan functions and once you cover the basic right-angled triangle you move on to the one with two right-angled triangles. Then I will take it step by step with and I'll tell them why I found CD first because there is no way of determining h without knowing the length of AC

and the length of DB and you cannot work out the length of DB because they only give you that AD is 500 and they don't tell you the length of AB.

R: Was it difficult to determine the answer?

U: No, not really.

R: Why did you choose this particular solution?

U: I used the sine rule here because it was the only way I could think of finding out DC and I used it again because I already was in the mood to use sine.

R: Did you check your answer to see whether it was correct?

U: No, not really.

R: Can you explain what thoughts came to your mind when you first read the third problem?

U: I was thinking why must this boy go cut the grass like that.....if he wants to cut half of the grass, why doesn't he just cut half properly and leave the other half for his sister to cut. Does he now have to cut from the outer edge inwards?

R: Did you find that the problem looked familiar to you in any way?

U: No, not really, but I related it to sequences and series.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

U: No, I don't think so.

R: Could you please take me step by step through your solution ?

U: Okay, I drew a diagram because a diagram is important to me. First, I worked out the area of the whole lawn and then I calculated half of the area. Then I just worked out a new area each time he mowed half in around and then I found the differences between them and I set up a sequence. I wasn't sure I was doing the right thing. I used this thing here [student points to T1, T2, T3 etc.] to try and get an equation for the sequence, but I couldn't. Then at first I couldn't remember the sum formula for the sequence... and then I got it and then I just used the equation and solved it. But I'm not sure my answer is right.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you.

U: Yes.

R: [Researcher explains the solution to the student.]. Do you understand the solution?

U: Yes, I do. Now I can see where I went wrong.

R: If you were given another similar problem, would you be able to answer it?

U: Yes, definitely... after I have seen this solution.

R: How would you explain such a solution to your learners?

U: It would be difficult for me to explain it to my learners because I did not fully understand the question myself.

R: Thank you

INTERVIEW FIVE

R: RESEARCHER

K: KAMAL

R: Can you explain what thoughts came to your mind when you first read the first problem?

K: My first thought was that it is an optimization problem so I was thinking of using to solve it ... using just logic and sort of guessing it out and then figured I could use linear programming or differential equations to solve it.

R: Did you find that the problem looked familiar to you in any way?

K: Yeah, in past experience I've gone through such problems in mathematics in school when doing linear programming.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

K: Yes, in school and at university.

R: Could you please take me step by step through your solution?

K: First I allocated variables or letters to each respective item...that's the table and the bookcase and then using those I wrote out an equation to illustrate the profit. Then I used the alphabet or variable for the table or a bookcase to illustrate how many hours or how many sheets it would take to construct each table or each bookcase; so I wrote equations for that and I wrote out the total amount of sheets of timber and the total labour hours that there were and then I just solved them simultaneously and found the optimal solution.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

K: Yes. You could go and if you draw a graph or you could try differential equations and solve them that method.... which is longer.

R: Was it difficult to determine the answer?

K: No.

R: If you were given another similar problem, would you be able to answer it?

K: With ease.

R: Why did you choose this particular solution?

K: I thought it was the quickest way to reach the answer.

R: How would you explain such a solution to your learners?

K: I would try and ensure such that it is more real world problem rather than mathematical and try and develop the concept of this in their minds by using examples in graphs. I'll show them how to optimize a solution that by changing the number of tables and bookcases you can optimize the solution.

R: Did you check your answer to see whether it was correct?

K: Yes....by drawing the graph, I found specific points and substituted into the inequation and checked.

R: Can you explain what thoughts came to your mind when you first read the second problem?

K: When I first read it, I was a bit confused as to the positioning of the two observers, because at first I thought maybe that they could be standing not exactly in line with each other but on opposite sides of the bases....but then I figured that then it would be nearly impossible to solve the problem and so I opted to use the assumption that they were actually in line with each other and looking at the same mountain and then just used normal high school trigonometry to work it out.

R: Did you find that the problem looked familiar to you in any way?

K: Yeah....to a certain extent.... I think it was from a standard 9 or such high school problem in trigonometry.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

K: Yes, at high school and at university as well in a Maths Method module I think.

R: Could you please take me step by step through your solution ?

K: First I got a pictorial representation of what the real life problem would look like and then I sort of represented the distances in between the two observers and the mountain and the observers themselves and then solved them to find out what is the distance of the mountains above sea level.....so I just went and represented the distances between observer one and the mountain and observer two and the mountain using trigonometry and then solved for x which x would be the distance between observer two and the mountain base because you don't know that distance; so using that I then figured out what the distance between each observer is and the mountain and then used that solution to find out what the height of the mountain was.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

K: Yes. You could try calculus or something else but I'm not quite sure; maybe rates of change...

R: If you were given another similar problem, would you be able to answer it?

K: Yes, definitely.

R: How would you explain such a solution to your learners?

K: First I would tell them to draw a diagram of the situation...because I think it would be very difficult to work out these types of problems without a diagram. Then I would ask them to fill in all the given information on the diagram to see what is given and what has to be calculated. The learners will then have to recall their knowledge of trig.....like sin rule, sin, cos and tan functions... and then they will be guided to find the height of the mountain.

R: Was it difficult to determine the answer?

K: No...it was simple and straightforward.

R: Why did you choose this particular solution?

K: I thought it was the quickest way to reach the solution.

R: Did you check your answer to see whether it was correct.

K: Yes...I used the value of the height and tried to check.

R: Can you explain what thoughts came to your mind when you first read the third problem?

K: This is going to take a while.....that's what I thought.....because first when I looked at it, it looked relatively simple and then I read it again and realised that there are so many different ways of solving it but all the different ways will take a while.

R: Did you find that the problem looked familiar to you in any way?

K: Yes... I think we did some stuff like this in first year mathematics at university..... rates of change.....I think.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

K: Yes....this was in high school...in sequences and series and also at university.

R: Could you please take me step by step through your solution?

K: The first thing that I thought of was that every time this person cut around the lawn he's decreasing the total surface area of the lawn so I figured if I can find out the series at which the area of the lawn is being decreasing I could find out where half way was.

R: Why did you not attempt to write out a solution to the question?

K: I did not know how to write out a solution to what my thoughts and whether it was mathematically correct.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

K: Ok.

R: [Researcher explains solution to student]. Do you understand the solution?

K: Yes. I had an idea that it had to do with sequences and series but I did not know how to put it down.

R: If you were given another similar problem, would you be able to answer it?

K: Yes, I think so.

R: How would you explain such a solution to your learners?

K: I would tell them to relate this to sequences and series..... to show them that each time you cut the lawn the total area is decreasing and by how much it is decreasing.....and you can generate a series to show them that you can eventually cut to all the way to half of the lawn. I will also show them that drawing a diagram actually will help them to see how Randy is cutting the lawn in half a metre strips.

R: Thank you.

INTERVIEW SIX

R: RESEARCHER

A: ANDREW

R: Can you explain what thoughts came to your mind when you first read the first problem?

A: Okay, when I first read the problem I found that there was a lot of numbers that I had to memorise and get into a table because as soon I see a lot of numbers I was taught to make a table and then put them into a table and see what you can work out from there; and as I read through I see it had to do with profits, and then I had sheets of timber and I worked with hours.....so I separated them so that I can get a natural sum that I can work out....and do it two together to do it at the same time; because I got two different variables or two different things that I had to sort out.

R: Did you find that the problem looked familiar to you in any way?

A: Yeah, I had problems like this before....maybe not the same things used; but in similar ways....but I have done it.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

A: At school, yeah I have attempted to do problems like this previously. I actually found them very interesting. I've done them at school, at university and during extra classes when I help people.

R: Could you please take me step by step through your solution?

A: Okay, what I did was I first used the hours and I added them together to get a total hours of 120 hours of labour that he had to work in a week. My next one was the sheet of timber that he use and they specified that the amount of timber that he needed for a specific furniture that he had to make; so I made two equations and I set them; first multiplied because I didn't find anything equal because I deducted them from each other, so I can only get one variable so I can work it out; so I multiplied to get them the same or one variable the same so I can deduct them from each other and just have one left. So I ended up with x being 12 and I substituted into any one of my equations that I had and then you'll get the value of y .

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

A: I think there should be, if I give it more thought and take some time and actually work it out again.....you could go with a graph as well.....with the graph you can work out your values.... you can use your equations to draw graphs and use your graphs to get your values.

R: If you were given another similar problem, would you be able to answer it?

A: Well, I feel positive that I should be able to do it.

R: How would you explain such a solution to your learners?

A: Well, I'll explain to them that when you've got two problems at the same time; and you can form equations with them and they can be related to one another that you can work them out simultaneously and that's basically the way I obviously was taught like that I would follow that way; and if I was taught another way I could explain it to them differently. I feel much more comfortable by using algebra like this. A table makes more logical sense for somebody if you don't understand where I got my equations from. A learner can see where they got the equation from; so if they have a mistake then you go back to the table and explain to them how they actually got that problem.

R: Was it difficult to determine the answer?

A: In the start, when I did the table it was a bit confusing until I used colour coding here and then I picked up that you had your profit, your sheets and your hours, so I saw that I was going to get two equations and I directly saw that I can work the two equations and solve them simultaneously and then get to an answer.

R: Why did you choose this particular solution?

A: Just because I'm familiar with it and I've actually got to do a bit recently when I helped other people but if I had any other way I most probably would have tried that way first.

R: Did you check your answer to see whether it was correct?

A: Yeah, I did check I took my y and my x values and substituted it into my equations and looked if I could get the right answer again and that's how I checked it..... through substitution.

R: Can you explain what thoughts came to your mind when you first read the second problem?

A: Okay, the first thought that I had was that it's a horrible question because at school once we had these questions we had mix-ups we never really got to a final answer that actually worked out properly....so I think that's what gave me a negative attitude toward the question already... so when I read it I knew what I had to do.... I knew I had to use trig and using trig in the sense of working this out..... I hated that at school and I'm still hating it now until I find a method that I can actually work it out easier.

R: Did you find that the problem looked familiar to you in any way?

A: Yes, I had a question similar to this with a ship sailing on the sea with the toweryou had to measure or calculate the distance which I had in Grade 12.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

A: I have attempted to solve problems such as this. I have got the answer correctly but I don't feel confident in doing them because I don'tthe formulas that I had to use ...I was unsure of and that's why as well for this question using the formulas I had to use, I wasn't sure of.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

A: Yes, please.

R: [Researcher explains solution to student]. Do you understand the solution?

A: Yes, I do. The diagram clearly explains the situation. Like I said I am familiar with these problems so I understand the solution.... It's just that I have difficulty getting started with these problems.

R: If you were given another similar problem, would you be able to answer it?

A: Yeah, now that I have seen this and actually recap on everything I would most probably be able to do it and find it much easier.

R: How would you explain such a solution to your learners?

A: I would probably tell my learners that they need to be familiar with their trig ratios and sine and cosine rules. Also I think a diagram is important in these kinds of examples... to represent the x and the y and height. Visual aids help when you doing something like this. Even learners who don't understand can at least get familiar with what you trying to tell them so if they go and try it again they can actually see what you doing.

R: Can you explain what thoughts came to your mind when you first read the third problem?

A: I found this very interesting because all of us have to mow a lawn so you would like to know what you are doing.... and with this particular one I just found that the rectangle that he had to cut and every time you go around you are going to cut a rectangle but the rectangle you going to cut is only going to be half wide and then what you have to do is you going have to deduct the area of your rectangle with the rectangle that is left one on the inside that you haven't cut.... so I knew that you have had to deduct every time you'll have to work it out like that and then solve it.

R: Did you find that the problem looked familiar to you in any way?

A: Yeah, I think it is sequences and series.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

A: Yes, I helped someone who had a similar problem with numbers in sequences and series. I think it was in Maths 110.

R: Could you please take me step by step through your solution?

A: Okay what I did was I took the area of my whole rectangle which I got as 1125m² and then I found what the half was; because each person had to mow half of the lawn. So then I worked out every time they cut the lawn what the area would be so that I could get a sequence so that I could find the common difference of every time they mowed the lawn by how much it decreases....the amount that they have cut and till the amount that I gotthe sum of all the times that they go round calculates up to half of it and so that should be the amount that you should go round to get to half of cutting the rectangle. I knew that I had to use the formula to work this out after I got each individual thing so I added it in and this again were because I didn't have the formula with me... I did everything in there.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

A: Yes. You could use the formula for an arithmetic series if you could remember it, unlike how I had to add to get the required area. There may be other methods but I'm not sure about them.

R: If you were given another similar problem, would you be able to answer it?

A: Yeah, I really enjoyed this; even if it was a much more difficult one than this I would try and solve it.

R: How would you explain such a solution to your learners?

A: It seems that if you have a logical way of explaining it...they could understand it much better how much you cut as you go around. Maybe with the first time you could explain to them how much gets cut and the second time and they can actually understand that every time you go around you cut a bit less, a bit less and then you actually will get to a stage where you get to half of it.... It will maybe not be a round number as 6 or 7 or 9 but it could be $6\frac{1}{2}$ times or $6\frac{1}{4}$ times but you will get to half of it. I prefer drawing diagrams so that you have a visual thing so if you don't understand what I was doing here [student point to his solution] that you can at least understand what was going on there [student points to the diagram in his solution]. If a learner explains to you...if he can maybe not explain in words or mathematically but he can explain to you using a diagram what he was trying to do... so it can help him that he can get into a mathematical sense.

R: Was it difficult to determine the answer?

A: No, it was quite easy for me.

R: Why did you choose this particular solution?

A: I recognised that it was sequences and series, but I couldn't remember the formula....so this was the only way that I could find a solution.

R: Did you check your answer to see whether it was correct?

A: Yeah, I just counted it 9 times altogether to check if I would get about half of the area of the lawn.

R: Thank you.

INTERVIEW SEVEN

R: RESEARCHER

G: GARETH

R: Can you explain what thoughts came to your mind when you first read the first problem?

G: I don't know sometimes with Maths I don't actually have to think, it just somehow gets in my head, that why I often.....at school I used to battle with geometry because it just never used to pop into my head, but linear programming I always find quite easy at school because it just..... I don't know.....it just made sense to me. Essentially what I was looking at here is that I have two variables that I need to restrict and maximise obviously.....that being the timber and the labour so I drew up my table from that obviously tables and bookcases requiring different amounts for each of the two variables.

R: Did you find that the problem looked familiar to you in any way?

G: I did. We'd also been doing this..... we'd done this about two weeks before in Maths method 3 and we'd gone over linear programming.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

G: Yeah, I'd done so in matricit was part of our matric syllabus.

R: Could you please take me step by step through your solution?

G: Okay, what I did was because tables and bookcases required different amounts of timber and labour, what I did was I let variable x represent the number of tables and y represent the number of bookcases. Basically I then came up with the fractions thatin terms of total number of timber 690 sheets.....bookcases requiring 30 sheets of timber and tables requiring 20 sheets of timber...I drew up my inequality that 30 multiplied by the number of bookcases plus 20 multiplied by the number of tables has to be less or than equal to 690 obviously because it is restricted and I used the same sort of thinking to get my labour function if I may call it that....four multiplied by the number of bookcases plus five multiplied by the number of tables had to be less than or equal to 120. Then I just simplified those down and I plotted those on the first quadrant on a system of axes. And then to get my net profit function which is easy because he makes R25 on a table and R30 on a bookcase. The net profit equals $25x + 30y$obviously x and y being the number of tables and number of bookcases respectively...and then I took the intersection of the two

graphs, which in this case worked and I got it to be 12 tables and 15 bookcases which gave R750 profit. I also did check other points of intersection on my feasible region.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

G: I suppose you could do it by trial and error and if I hadn't known linear programming I probably would have ended up doing trial and error, because I don't like not knowing an answer to something.

R: If you were given another similar problem, would you be able to answer it?

G: Probably. I think the trickiest part is coming up with the table itself, sometimes I make a mistake or two in drawing up the table and that throws me; but once I've got that done, it's generally quite straight forward.

R: How would you explain such a solution to your learners?

G: I basically will go through my thought processes with them so that they can see..... maybe learn how to think in that kind of way.

R: Was it difficult to determine the answer?

G: No.

R: Why did you choose this particular solution?

G: It makes sense to me.... to put this stuff in the table, derive the equations from that, plot them and then get the answer.

R: Did you check your answer to see whether it was correct

G: I did check two other points to determine whether they gave a greater profit margin, than the point that I had chosen and they didn'tand oh graphically that is the optimal point in terms of the two functions themselves.

R: Can you explain what thoughts came to your mind when you first read the second problem?

G: This is very much like vectors in Physics which I did a lot of at school. I've taught a lot of it also.... So it kind of ... it makes sense to me that I'll have to use my trig ratios and that the height h is common to both triangles, obviously but it's just that the bases of the triangles that are different and the angles of inclination.

R: Did you find that the problem looked familiar to you in any way?

G: Yep, it did look familiar to me.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

G: Yes, at school.

R: Could you please take me step by step through your solution?

G: What I did was I drew out the problem, and represented the height by h . I drew a horizontal line where I marked off two separate points obviously cos the two people are in line and they are looking at the mountain the first distance was x and then that being from the centre of the base of the mountain to the first person and then between the first and second person I just isolated that distance as 500m. Then I just completed the two triangles...putting in the angles of inclination. Those two triangles are obviously right-angled triangles. Than I just represented the height h as a function of the distance from the observer to the base of the centre of the mountain multiplied by \tan of the angle of inclination..... and using simultaneous equations I calculated it out from there.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

G: Yes, I am sure there is.....like maybe using another trig ratio besides \tan .

R: If you were given another similar problem, would you be able to answer it?

G: Yes, definitely.

R: How would you explain such a solution to your learners?

G: Much the same way as my solution..... I'd have to get them to see that h because it is common to both triangles, you can use simultaneous equations to derive an answer from there.

R: Was it difficult to determine the answer?

G: It was not that tricky.

R: Why did you choose this particular solution?

G: Because it works.

R: Did you check your answer to see whether it was correct?

G: Actually, I didn'twhich I suppose is not really good.

R: Can you explain what thoughts came to your mind when you first read the third problem?

G: Well this one..... these kinds of problems I cannot actually do fully mathematically and I actually have to do every single step. I don't know if you can do it using calculus...I thought it might be possible to make it an optimization problem but it would have taken me too long to investigate the thought processes behind it.

R: Did you find that the problem looked familiar to you in any way?

G: Yes, it looks familiarlike maths problems we used to get in Maths Olympiads when I was in school.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

G: Yes, well having done Maths Olympiads, I would have attempted it.

R: Could you please take me step by step through your solution?

G: Basically what it is I did I took the initial area of the lawn and worked it out give the dimensions of the lawn and calculated what half the area would be.....then just basically because he's mowing the strip that's half metre wide....what happens is every lap that he goes round the lawn....the length and the breadth are each reduced by a metre which means then that the area is decreasing after every lap.....so what I did then was I calculated the area initially and after one lap.....two laps....and all the way down to 10 lapsand between 9 and 10 laps was essentially halfway.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

G: There probably are but I wasn't going to try.

R: If you were given another similar problem, would you be able to answer it?

G: I am sure I will be able to.

R: How would you explain such a solution to your learners?

G: Basically what I'd do is, I would.....I don't knowhow many children these days actually mow the lawn so I wouldn't actually be able to use that example. I would get them to understand that every time you go round to mow the lawn the distance that you have to travel around the perimeter itself is decreased.....therefore the area is obviously decreasing as you go around but every single time you go round the perimeter you doing 2m less.

R: Was it difficult to determine the answer?

G: Not really.

R: Why did you choose this particular solution?

G: It was what made sense to me.

R: Did you check your answer to see whether it was correct.

G: I didn't check my answer.

R: Thank you.

INTERVIEW EIGHT

R: RESEARCHER

M: MICHELLE

R: Can you explain what thoughts came to your mind when you first read the first problem?

M: I think I immediately knew that it was back to linear programming.

R: Did you find that the problem looked familiar to you in any way?

M: I think it just reminded me of things that we came across at high school.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

M: Yeah, in high school we did a lot.....I think in Grade 11.

R: Could you please take me step by step through your solution?

M: Well, I firstly let the tables be x and the bookcases be y cos those are the kind of variables that you going to have and then I used the time for the bookcases and the tables and then I took the hours of labour for the two variables and I used the sheets of timber for the two.....made those into inequalities cos it has to be less than or equal to the total. I then used the straight line graph of y equals the two equations, plotted those on a graph and then for my maximumyou had to find the maximum profit and so I made an equation using the maximum profit and I found the gradient of that and that gradient I used to find my maximum point.....and then I read off my values from the graph.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

M: Probably are....maybe a more algebraic method.

R: If you were given another similar problem, would you be able to answer it?

M: Yeah, I'm sure I would.

R: How would you explain such a solution to your learners?

M: I would try and show them that it's the variables of the tables and the bookcases that have to be used as the x and the ynot to add the labour and then the timber of the tables first and the labour and the timber of the bookcases. You need to understand from a logical perspective first, understand what the sum is actually telling you and what you have to work out and I think I will spend time trying to explain what the information that

you given is and then show them how you would apply the maths to the real life problem.

R: Was it difficult to determine the answer?

M: No, it wasn't.

R: Why did you choose this particular solution?

M: This was the way that we had been shown how to do linear programming at school.

R: Did you check your answer to see whether it was correct?

M: Not really.

R: Can you explain what thoughts came to your mind when you first read the second problem?

M: I think I just thought of that its.....well firstly I thought I needed to draw a diagram because I find it's much easier to firstly draw a diagram and then you get the points.... I can't visualise these things in my head.... I have to put them down on paper.

R: Did you find that the problem looked familiar to you in any way?

M: Yeah, I had an extra Math lesson and the girl has come with a problem similar to this. She's in Grade 11 and so I was very familiar with this.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

M: Yeah, we did.....in Grade 11 you do more of the basic ones like this and in Grade 12 it gets more 3-D.

R: Could you please take me step by step through your solution?

M: Well firstly, I tried to relate what the information was given in a diagram and so then formed two triangles, and then put what information was given so the angles from the horizontal upwards... I used my angles of elevation and then worked out the other angles using angle sum of a triangle so got as much information as I could onto my diagram before I started working out and then I looked atthere were actually three triangles that are formed but I firstly looked at the one where I had the most information. I needed to find PM eventually, but I knew that I would first need to find AP and then relate AP to PM....so using the sine rule in trig I found PM.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

M: I'm not too sure.

R: If you were given another similar problem, would you be able to answer it?

M: Yes, I am sure I will be able to.

R: How would you explain such a solution to your learners?

M: I would first start by drawing the diagram and guide them to see what they eventually prove but then guide them through the steps that they going to have to use i.e. they first have to work out AP, so that they will then be able to work out PM....so try and show them the logic in what they doing instead of just giving them numbers and saying that is the diagram.

R: Was it difficult to determine the answer?

M: No.

R: Why did you choose this particular solution?

M: I think my mind went back to the sine rule.....and I saw that it works.

R: Did you check your answer to see whether it was correct?

M: I didn't check if it was correct, but it did kind of looked like it seemed reasonable.

R: Can you explain what thoughts came to your mind when you first read the third problem?

M: My first thought was I don't know if I'm going to be able to do this. It took a while for me..... I started drawing the drawing..... I had to think through it.....it was just not straight forward.....you had to kind of think if I hadn't had the diagrams I think I would have been a bit confused with all the numbers

R: Did you find that the problem looked familiar to you in any way?

M: I think it reminded me of maximum and minimum, but I couldn't quite like.... I found it easier to do the diagrams which isn't usually ... usually I'll try to find an algebraic expression and just substitute ...for this one I just found it easier to draw the diagram...so in some ways it was familiar where I knew I'd done stuff like this but I wasn't able to apply a formula and things which I thought I could of.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

M: I think I've done similar ones.

R: Could you please take me step by step through your solution?

M: I started of with I worked out the entire area of the rectangle cos it said that Randy had to mow half...so I wanted to find out the whole area...to find out what half was and so I knew that's the number I'd be working up towards. I then worked out on his first path how many metres squared he would have mowed taking into account that it was only $\frac{1}{2}$ m that he was mowing each way and then if he's gone then we have to subtract the $\frac{1}{2}$ m from each other side.....and so I worked out on his first path, then on this second path, third pathand then worked it out and just drew up a table and then found out that on his 9th path he would have gone to 549 which is very close to his total but he needed an extra $13 \frac{1}{2}$ m..... I think so I knew it was 9 times around and he needed to mow 27m to get the $13 \frac{1}{2}$ m.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

M: I'm sure there are because like the way..... I thought this can't be the only way to work out this sum.

R: If you were given another similar problem, would you be able to answer it?

M: Yeah, I think I would.

R: How would you explain such a solution to your learners?

M: I would start of drawing the diagram. I'd probably get them to try it first.....get them to work out the first part and then the maybe total of the area for the first part and then second part, and third part and see if they can understand what the information that they given how to actually relate it to the diagram....but if I wanted them to get my solution, I'd probably would have drawn up a table like this for them and get them to see the pattern that does form.

R: Was it difficult to determine the answer?

M: It was... it was more difficult definitely than the other questions. I think I had to spend more time on this question. I think I did check this one more than I checked the other ones cos I wasn't too sure of my answer; so it was a little bit difficult. I wouldn't say it was easy.

R: Why did you choose this particular solution?

M: Cos I really couldn't really figure out another way to do it, it was the only way I was hoping that would be right.

R: Did you check your answer to see whether it was correct?

M: I did try to check it because I'd carried on with the table actually and I found out that eventually the total breadth became zero and found that my length when I added that up it actually equalled the other half of the area; so I did check it in the long way.

R: Thank you.

INTERVIEW NINE

R: RESEARCHER

B: BRUCE

R: Can you explain what thoughts came to your mind when you first read the first problem?

B: Well immediately I could see it had something to do with linear programming, cost and constraints.....

R: Did you find that the problem looked familiar to you in any way?

B: Yes, as soon as I saw it, I saw linear programming.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

B: Yes, it was in the syllabus at school.

R: Could you please take me step by step through your solution?

B: Well, we have two different variables here ...the table which I said we'll make that will be represented by x and the bookcases by y and obviously it makes sense that your profit will be R25 per table and R30 per bookcase so profit will be a combination of your products of 25 times table and 30 times bookcasesand constraints..... so you have 690 sheets of timber and it takes 20 sheets to make one table and 30 sheets to make I bookcase ...and the sum has to be less than and equal to 690....and the same with the second constraint...120 hours. It takes 5 hours to build one table and 4 hours to build one bookcase and total number of hours...maximum is 120 ...so the sum of hours has to be less than 120. And obviously you cannot have a negative amount of tables and negative amount of bookcases so those are your other two inequalities there and what I did I plotted all those onto a graph and found my feasible region and then I took this equation for the profit and made y the subject of the formulaand this would be your y -intercept and I said well if we take that gradient $-5/6$ and shift that so that our y -intercept is at a maximum we can take that y -intercept and find the maximum profit.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

B: Yes, you could not have a graph....just equations and work it out algebraically.

R: If you were given another similar problem, would you be able to answer it?

B: Yes, most definitely.

R: How would you explain such a solution to your learners?

B: I would first explain to them that they will have to draw up a table to be able to display all information that is given in the problem.... I will tell them that because we have two different things that we are working with i.e. tables and bookcases we have to use two variables such as x and y . They need to know about where we get our constraints from and how to obtain the maximum profit.

R: Was it difficult to determine the answer?

B: No, it was fairly straight forward.

R: Why did you choose this particular solution?

B: Because that's what I'm used to when I saw linear programming and that's what linear programming is.

R: Did you check your answer to see whether it was correct?

B: Yes well, I substituted different values for x and y into the profit and I saw that 12 and 15 was in fact the maximum.

R: Can you explain what thoughts came to your mind when you first read the second problem?

B: Obviously, trig to do with 3 dimensions.....you have three different planes in this case so I thought some how this length a here is common to both of these triangles and this length b is common to these two triangles and somehow there has to be some kind of relation between those two.

R: Did you find that the problem looked familiar to you in any way?

B: Yes, but slightly different because it wasn't that easy to work out for me....it wasn't. I know maybe I just was not seeing something, but we dealt with 3-D trig problems at school before but I don't think one quite like this.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

B: 3-D trig...yes, but one like this I don't think so.

R: Could you please take me step by step through your solution?

B: Here we have obviously the height and a person looking from a distance. That's going to be a right-angled triangle....so he's viewing at an angle of 35°another person 500m away is viewing at 32° and then I said well that height is common to both of them.....so I

said take both of them to right-angled triangle and we can work out this side using simple trig ratios and side a and then I took those values and used the sine rule to come up with something for sine A here and sine C and I just had two many equations and don't know what happen after that.....

R: Your solution is not entirely correct. Do you think that there are other strategies to solve the same problem?

B: Probably is.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

B: Yes, please.

R: [Researcher explains solution to student.] Do you understand the solution?

B: But in the question it doesn't say it's in a straight line though... and I didn't assume that... I also assume that they are standing at 2 different points.

R: If you were given another similar problem, would you be able to answer it?

B: Yes, now that I see this.

R: How would you explain such a solution to your learners?

B: From my experience they find 3- D trig very difficult...some see it straight away.... Some don't. I would find it difficult to explain it seeing that I still have difficulty in understanding it myself.

R: Can you explain what thoughts came to your mind when you first read the third problem?

B: When I read this question, first thing that came to mind is that ...they had to share the same area of lawn...so obviously the person who goes round the middle has got to do more laps than the person who goes round the outside.....but they have to have the same area if they mow exactly half...so calculating the area is easy...its length times breadth divided by 2 giving you half that area so they each get to mow that area and I worked out the area of each strip and I found a sequence. First outer strip was 69...then the next one 67 and so on...so I thought well first term would be 69.....difference would be -2 and plug that into your formula and we needed the sum of 562.5m²so that would be your sumnumber of times round would be nwe have a...we have d.... we can solve for n and we get 9,26 so about 9 ¼ times he has to mow.

R: Did you find that the problem looked familiar to you in any way?

B: Vaguely familiar

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

B: Similar kind I think...I can't remember.

R: Could you please take me step by step through your solution?

B: It's what I explained earlier on about creating a sequence and having your a value, d value and your sum, and plugging that into your formula and solving for n.

R: Your solution is correct. Do you think that there are other strategies to solve the same problem?

B: Probably are, I don't know for sure.

R: If you were given another similar problem, would you be able to answer it?

B: Yes I will be able to...if I apply my mind logically.

R: How would you explain such a solution to your learners?

B: I would ask them to draw a diagram, because it is necessary...anything imaginary.....if you see a whole lot of writing especially in Maths textbooks...you just don't want to learn but when you see diagrams and pictures every now and then it makes more interesting and to me it makes it more logical and easier to understand.

R: Was it difficult to determine the answer?

B: Not really....it made sense.

R: Why did you choose this particular solution?

B: That's how I saw it logically.....in terms of a pattern and generating a sequence and using the formula.

R: Did you check your answer to see whether it was correct?

B: Yes, I did...by checking how much was mowed each round and then added to see if I got about half the area.

INTERVIEW TEN

R: RESEARCHER

S: SIBIYA

R: Can you explain what thoughts came to your mind when you first read the first problem?

S: I think I have experience of this..... I just used the experience that I have on linear programming because I experience linear programming here at university...that's where I started knowing that such a thing exists before that I didn't know that such a thing exists.

R: Did you find that the problem looked familiar to you in any way?

S: Yes, it was familiar because I just thought linear programming and I said ya I need some constraints here I need to know what number of tables and what number of bookcases..... I think it was a lecturer here at university..... I did the foundation Maths 110 with him....that's where he introduced us to this things that we should have done in matric. He even told us that we should have done this in matric and then he gave us time.... I think he did a good job with me because I was like blank and he gave us class work, he gave us tuts, and then he gave us an assignment to go out and research linear programming.... I went out and did some research on it and yes I got understanding.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

S: Yes, yes I attempted some of these problems although sometimes I'm not sure if my especially when I have to draw now and find the feasible region.... I'm not sure which of the regions is my feasible region....yes I still have a problem there but ya.... I'm getting there.

R: Could you please take me step by step through your solution?

S: What I've done here I've just interpreted this statements mathematically...that's what I've tried to do....to say ...ok these words are telling me they have... this constraint is telling me 690 sheets..... that's the maximum he has so the 20 there....it requires 20 sheets of timber and 5 hours okay...so its 20 sheets of timber and I said my timbers are x ...there...so that's why I say $20x + 3y$ [student could not explain further]

R: Your solution is not entirely correct. Do you think that there are other strategies to solve the same problem?

S: Yes, I think so.....can't we use optimization here maybe...we could use simultaneous equations and solve for x and y.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

S: Yes, I'd like that.

R: [Researcher explains solution the student.] Do you understand the solution?

S: Yes, to some degree.....I see x and y greater than equal to 0 because we speak of units here...of things that have to be positive. The objective function is very important cos if we want to get my maximum minimum point.

R: If you were given another similar problem, would you be able to answer it?

S: Yes, I would.... the experience counts in mathematics. The more you engage; the more you get the confidence to do more.

R: How would you explain such a solution to your learners?

S: You need to start with practical examples before you come to numbers and stuff and let the learners understand what the statement implies in real life.

R: Can you explain what thoughts came to your mind when you first read the second problem?

S: When I was looking at this one I first thought of trigonometry.

R: Did you find that the problem looked familiar to you in any way?

S: Yes, I have seen it before.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

S: Yes, again here at university...because in matric I didn't even look at the question when I saw something like this.... I just turned the paper over and said that's not for me.....my teacher did not tell me to do this and our teachers actually avoided these things because they had problems with them...they couldn't solve them...that's the only thing that I think of...because why should you avoid helping your learners because you are there to teach them...ja so if you came with a problem they tell you go and try and solve it yourself and then you come again and again and they keep on postponing it until you get fed up and give up and think it is not important, and in the matric paper you get it and you fail.

R: Could you please take me step by step through your solution?

S: Here I looked at this bigger triangle. It had to be a right-angled triangle because I can easily apply my trig here.....it's convenient for me and then I looked at sum of angle of triangle = 180° ...I said ok because I want angle C and this is the height.

R: Your solution is not entirely correct. Do you think that there are other strategies to solve the same problem?

S: I think there may be.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

S: Yes, please.

R: [Researcher explains solution to student.] Do you understand the solution?

S: Yes, it makes sense to me.

R: If you were given another similar problem, would you be able to answer it?

S: Yes, I would now.....in school we didn't do too much of trig. The diagram is important the way you have in your solution.

R: How would you explain such a solution to your learners?

S: I think problems like this need experience. First I go through some measurements with them on things that they can see and visualize and then you can come to the problem.

R: Can you explain what thoughts came to your mind when you first read the third problem?

S: I'm visualising a rectangle.....25X45.

R: Did you find that the problem looked familiar to you in any way?

S: This problem.....I think I've seen some problems like this.....but not put in this context.... a river or a garden.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

S: I think ya...but the method we used was optimization in calculus.

R: Why did you not attempt the question?

S: I didn't know what to do.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

S: Yes, please.

R: [Researcher explains solution to the student.] Do you understand the solution?

S: Ya, now I do.....this is an arithmetic series.

R: If you were given another similar problem, would you be able to answer it?

S: I haven't done it before.

R: How would you explain such a solution to your learners?

S: I think I would use the pattern as you have explained to me. I would say 'What do you observe?' 'What is the difference?' I'd ask them questions like that.

INTERVIEW ELEVEN

R: RESEARCHER

E: EMMANUEL

R: Can you explain what thoughts came to your mind when you first read the first problem?

E: When I read the problem mam I found the two items which were like not necessarily the same and I then thought that I must just express the one in terms of y and the one in terms of x so that I can end up with the equation.

R: Did you find that the problem looked familiar to you in any way?

E: Not really, mam because I haven't done such problems at school; but when I read it I had the understanding of what was required.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

E: No, I never attempt because I never experience such problem before in school.....but I had an idea because I experience it in precalculus course.

R: Could you please take me step by step through your solution?

E: Firstly, I expressed the number of tables in terms of x and the bookcases in terms of y and then I said the profit I expressed it in terms of P . I know that the profit will definitely be equal to the number of tables plus bookcases.

R: Why did you not complete the question?

E: Cos I had 2 unknowns and it was impossible to solve for them. It would have been better if I had 2 equations ...it would have been better to solve simultaneously to find a solution I wasn't sure how to link the inequations.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you.

E: Yes, please mam.

R: [Researcher explains the solution to the student.] Do you understand the solution?

E: I understand it.....because I thought about it what if I draw the linear functions cos I have the values of x and I've got the values of y .

R: If you were given another similar problem, would you be able to answer it?

E: That's correct; I will be able to answer it cos I had the ideas that I did not express it down.

R: How would you explain such a solution to your learners?

E: I would you the same method that you have shown me.... I would tell my learners to draw a table and put all the numbers in the table....also it is easier to get the answer if you draw the linear functions.

R: Can you explain what thoughts came to your mind when you first read the second problem?

E: What came to my mind..... I thought about the triangle cos I was given that there was an angle at the peak and two angles like at the bottom and I said what if I express this in terms of a triangle....wouldn't I then be able to use trigonometry knowledge to attempt it.

R: Did you find that the problem looked familiar to you in any way?

E: I say yes mam. I did look familiar to me cos we did something of this nature.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

E: I did in a pre-calculus module.

R: Could you please take me step by step through your solution?

E: Firstly, I drew the triangle because I had the peak which was my A point here I was given the angle measurements of 32° and 35° to the peak. I assume that what they talking about the base angles of the triangle and I draw the triangle that you see here ABC with base angles B and C B was equal to 32° and C was equal to 35° and then what I said I express it in terms of tan. I said $\tan 32^\circ = \dots$ I know tan is equal to opposite over adjacent...apply that rule. I got the line perpendicular to BC and that line is opposite to the angle $B = 32^\circ$. That perpendicular height I express it as my height and then I said $\tan 32^\circ$ be equal to I then could not complete the problem.

R: Why did you not complete the question?

E: I didn't work out the value of x. I didn't know the value of x.

R: Your solution is not entirely correct. Do you think that there are other strategies to solve the same problem?

E: I don't know mam.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

E: Yes, mam

R: [Researcher explains solution the student.] Do you understand the solution?

E: I do understand it now that you explain it to me.

R: If you were given another similar problem, would you be able to answer it?

E: I'm not quite sure as I'm not that good in this kind of problem where you have to represent it in terms of a diagram.

R: How would you explain such a solution to your learners?

E: I would use the solution that you explained to me.

R: Can you explain what thoughts came to your mind when you first read the third problem?

E: The first thing that came in my mind I said oh I'm given that a mow here and they told me that the lawn is 25m X 45m rectangular and I draw my rectangle and then represent the sides in terms of the length which was given.

R: Did you find that the problem looked familiar to you in any way?

E: I'll say no..... I haven't seen such a problem before.

R: Did you attempt to solve such a problem previously to this occasion? If yes, when was that?

E: No...never....honest..... I never had done this at school.

R: Could you please take me step by step through your solution?

E: Firstly, mam I express it in terms of sequences and went the other way around. When I work it out the unit... each unit of the yard I determine that oh the first unit would be 69.... from 69 it moving by 2 units from 69 to 67; and from 67 to 65 and then when I look over it I see that it resemble what an arithmetic sequence and then I say I got a and d which is my common difference.

R: Why did you not complete the question?

E: Cos, mam I was confused and thought that this problem was pre-calculus; but I don't necessarily apply the arithmetic series..... I kept on questioning myself...should I use the length and the breadth to determine area.... I was just not sure that's why I stopped here.

R: I notice that you did not get the correct solution? I have one correct solution. Do you want me to explain it to you?

E: Yes, mam.

R: [Researcher explains the solution to the student.] Do you understand the solution?

E: I do understand it. My only problem was, I didn't link arithmetic series with the pre-calculus.

R: If you were given another similar problem, would you be able to answer it?

E: I think so.

R: How would you explain such a solution to your learners?

E: I'll ask them to draw the diagram, because I can see from your solution that the picture makes the problem clear and understandable. I'll ask them to think what sequences can they work out; and if they find it to be arithmetic sequence then they will apply their knowledge of arithmetic sequences to solve it.

R: Thank you.