A COMPARATIVE ANALYSIS OF PROBLEM-SOLVING PROCEDURES OF A SOUTH KOREAN AND A SOUTH AFRICAN GRADE SIX MATHEMATICS TEXTBOOK

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DECLARATION

I, Sathiaveni Duel Moodley, hereby declare that:

(i) The research reported in this thesis, except where otherwise indicated, is my work.

(ii) This thesis has not been submitted for any degree or examination at any other university.

(iii) This thesis, or any part of it, does not contain other persons' writing, unless specifically acknowledged as being sourced from other researchers. Where other written sources have been quoted, then:

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ABSTRACT

Mathematical tasks play a critical role in the teaching and learning of mathematics. Textbooks have been valued as an important tool in the teaching-learning process of Mathematics. This study aimed to analyse how problem-solving procedures are represented in selected mathematics textbooks in South Korea and South Africa using a composite framework. In the past few decades, international comparative studies have transformed the way mathematics education is perceived and has provided insight for improving student learning in many ways. In this study, 6th Grade South Korean and South African mathematics textbooks were compared with textbook analysis frameworks and Polya's 4-stage model being used to analyse data.

The comparison involved textbook design features, and the criteria for their quality (visual design, nature of approach, cognitive demand, content, learning, teaching, structure, organisation, linguistics characteristics and internal organisation). The focus included the basic structure, curriculum weighting, colour-coding or use of colour, topic representation, introduction and conclusion of topics and non- textual representations.

This study revealed similarities, such as the use of visuals, models, acting it out, guess and check and identify the pattern. The heuristics were present in both textbooks; however, the key difference being the South Korean textbook had been designed according to Polya's 4 stagemodel, with several heuristics being integrated in the design process, which created a strong foundation in developing critical thinking skills. The more salient features of the South African textbook are key words, mathematic ideas and 'did you know?' information boxes which aid second language students in understanding mathematical concepts. This may account for the differences in the Trends in International Mathematics and Science Study (TIMSS) results of the two countries, with South Korea scoring 1st while South African had been placed 47th.

The implementation of a model in the design process (e.g. Polya's 4 stagemodel and heuristics) by curriculum developers and textbook authors will result in the improvement of the quality of mathematical results as problem-based learning improves academic performance. The enhancement of students' attitude towards problem-solving and progress in mathematics results by including differentiated learning materials in the mathematics textbook. This will cater for their varying levels of ability and the development of critical thinking and cognitive domains of knowledge.

ABBREVIATIONS AND ACRONYMS

AMS	*American Management System*
ANA	*Annual National Assessments*
ANC	*African National Congress*
C2005	*Curriculum 2005*
CAPS	*Curriculum Assessments and Policy Statement*
CCSSO	*Common Core State Standards for Mathematics*
CU	*Conceptual Understanding*
CW	*Curriculum Weighting*
DBE	*Department of Basic Education*
DO	*Department of Education*
FP	*Foundation Phase*
GEI	*Georg Eckert Institute*
HOD	*Head of Department*
HOT	*Higher order thinking*
IAEP	*International Association of Equine Professionals*
ICME	*International Congress on Mathematical Education*
IEA	*International Association for Evaluation*
JET	*Joint Education Trust*
KLA	*Key Learning Areas*
MEHRD	*Ministry of Education and Human Resources Department*
MOE	*Ministry of Education*
MOFE	*Ministry of Finance and Economy*
MOL	*Ministry of Labour*
MOST	*Ministry of Science and Technology*
NAEA	*National Assessment of Educational Achievement*
NCCA	*National Council for Curriculum and Assessment*
NCS	*National Curriculum Statement*
NCTM	*National Council of Teachers of Mathematics*
NRC	*National Research Council*

NT	*Non-Textual*
OBE	*Outcomes Based Education*
OECD	*Organisation for Economic Co-operation and Development*
OTL	*Opportunity to Learn*
PISA	*Programme for International Assessments*
PP	*Procedurals Practice*
PS	*Problem-solving*
PSSM	*Principles and Standards for School Mathematics*
QCA	*Quality Content Analysis*
QUASAR	*Quantitative Understanding Amplifying Student Achievement and Reasoning*
RNCS	*Revised National Curriculum Statement*
RTL	*Recommended Textbook List*
SA	*South Africa*
SACMEQ	*South African Consortium for Monitoring Educational Quality*
SDGs	*Sustainable Development Goals*
SK	*South Korea*
SR	*Special requirements*
TIMSS	*Trends in International Mathematics and Science Study*
UKZN	*University of Kwazulu-Natal*
UNESCO	*United Nations Educational, Scientific and Cultural Organisation*
USA	*United States of America*

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CHAPTER 1: Introduction

1.1 Context of the study

This section provides some background information and a description of the study's context. Textbooks from two countries were selected, namely South Korea and South Africa, from which a comparative study was executed comparing backgrounds, curriculum and representations of problem-solving procedures in government approved Grade 6 mathematics textbooks.

1.2 Background to the study

Textbooks have a strong influence in mathematics teaching and learning, as it affects the content and the manner of instruction. The way in which key mathematical content is presented in accordance with curriculum orientations by educational instruments has become a point of concern for mathematics teachers (Simon, 2017). School textbooks are an integral resource in support of teaching and learning in many countries and have received increased attention from the international education community over the last decade (Kilpatrick, 2020).

In mathematics education, the growth of researchers' interest in this study is made apparent by the purposeful addition of the analysis of hundreds of textbooks and various curricular materials from 50 countries in the Trends in International Mathematics and Science Study (TIMSS). In 2004, the 10th International Congress on Mathematical Education (ICME-10) created a discussion group specifically dedicated to textbooks (DG14). This was the first time in the ICME's history in which textbooks were a discussion group's focus, and stirred many participants interest (Furinghetti, 2019).

Further studies have noted that mathematical tasks play a vital role in the teaching and learning of mathematics. Some studies (Gravemeijer, Stephan, Julie, Lin & Ohtani, 2017; Liu, Zhen, Ding, Liu, Wang, Jiang & Xu, 2018; Yuanita, Zulnaidi & Zakaria, 2018) analysed learning opportunities which textbooks provide according to task types, while others (Hadar, 2017; Koljonen, Ryve & Hemmi, 2018) explored the relationship between problem-solving procedures in textbooks and teaching and learning in the classroom. Sung (2017) researched the influence of visualization tendency on problem-solving ability and learning achievement of primary school students in South Korea, their research revealed that, visual tendency, and hands on activities play an important role in the acquisition of problem-solving abilities, critical thinking and the development of cognitive demand. Research conducted by Chirinda and Barmby, (2018) indicate that in South Africa, classrooms are overcrowded and students have a language barrier. English being the language of learning and teaching force teachers to strictly follow a set mathematics problem-solving process, which restricts students from developing critical thinking skills. Furthermore, the study revealed that teachers' adoption of a step-by-step strategy prohibits students' mastery of basic problem-solving procedures.

1.3 Rationale for the study

An interest to investigate this topic was triggered by the gap that exist between the South Korean and South African mathematics TIMSS results and textbook design. I was motivated to carry out a comparative analysis of problem-solving procedures of South Korean and South African Grade 6 mathematics textbooks when research by Liou (2017) showed the five Asian countries, namely Korea, Singapore, China, Hong Kong and Japan, which had the highest achievement scores at a Grade 8 level. The average achievement scores were above the highest international benchmark of 550, for mathematics and science.

Further research revealed that TIMSS uses the curriculum as the organising principle to provide educational opportunities to students. The curriculum model has three features namely the intended curriculum, the implemented curriculum and the attained curriculum. Mullis, Martin, Foy and Stanco (2012) describes the intended curriculum as defined by countries or the education system, the implemented curriculum taught by teachers, and the achieved curriculum being what students have learnt according to Curriculum Assessments and Policy Statements (CAPS). The implemented curriculum refers to how the educational system is rolled out for application (curriculum coverage) and the attained curriculum refers to what learners have learnt (learner achievement scores). I decided to explore the above mentioned curriculum and its' influence on the design of textbook.

Furthermore, the TIMSS study focused on the significance of the role of textbooks in mathematics, this being a key component of the programmatic curriculum that is the driving force behind the implementation of teaching and learning which takes place in the classroom. Mathematics and science are key areas of knowledge for developing individuals and the society (Gustafsson, Nilsen

& Hansen, 2018). Performance in these areas is one of the main indicators to assess a schooling system, with the TIMSS allowing participating nations to compare learner achievement across the world. This provides South Africa with an opportunity to benchmark itself against other countries, however, South Africa ranked penultimate in the results, which indicates poor problem-solving skills, and has become an issue of great concern over the past two decades (Mullis & Martin, 2017). Many tests have been implemented to detect the variation between the content in textbooks and the content being tested for example, through the Annual National Assessments (ANA) (Roberts & Barmby, 2016). These results are compared to surrounding countries such as Zambia, Namibia, and Mozambique and reveal that South African student perform very poorly compared to these countries. These factors further guided the direction of my study to develop a textbook framework which can aid policy makers to improve mathematics textbooks.

Charalambous *et al*. (2010); Fan, Xiong, Zhao and Niu (2018); Hong and Choi (2018) noted that the East-Asian students outperform Western students on numerical and arithmetical skills; specifically, on addition and subtraction tasks in the cross-national comparative studies. Fan, Xiong, Zhao and Niu (2018) state that cultural variables, language and the educational system could be attributes to the differences in performance. My decision to compare these two countries textbooks was borne out of their distinct language and educational diversity. The South Korean government introduced media literacy education at schools and social institutions to promote citizenship for responsible production and distribution of diverse content. South Korea also promotes a learning culture whereby their textbooks are written in Korean which is easily understood by the students (Heo, Leppisaari & Lee, 2018). The South African curriculum policy in Arts and Culture education acknowledges and promotes an understanding of cultural diversity. South Africa's culture is one of the most diverse in the world, with 11 official languages. The rich culture of each language brings its own dynamism to our assorted group of people while portraying our differences and development which is a priority in the education system (Rudwick, 2018).

In general terms, the TIMSS aims to assess students' knowledge, while the Programme for International Assessments (PISA) attempts to find what students can achieve with their knowledge. The TIMSS utilizes the curriculum as the major organizing concept. According to Spring (2017), textbooks govern what students learn and set the curriculum by indicating the specifics that are studied in most subjects. The public regards textbooks as authoritative, accurate and necessary and provide students with an early exposure to problem-solving procedures which to me was a major reason for specifically choosing mathematics textbooks for this study.

1.4 Focus of the study

The focus of this study is to gain an understanding of how the representation of problem-solving procedures in Grade 6 mathematics textbooks can develop students' cognitive demand and advance critical thinking. The focus is on how problem-solving procedures influences students' thinking abilities and how these strategies can ultimately improve mathematics results.

1.5 Significance of the study

This study will be beneficial to textbook designers, Government, teachers and students if the textbook design can aid in the development of problem-solving skills. According to O'Cadiz (2018), student success is the ultimate goal of educational reform efforts and should be an essential theme of policy. "While the structures of parent involvement programs, adequate resources and communication amongst stakeholders are all important, policymakers must ensure that the intended goal of education reform is successful for all students" (Moyer, Robison & Cai, 2018, p.115). Education policies also state that resources have to be made available to all students in order to provide quality education, with their implementation being done at a local level. National government and district policies are written for a wide variety of contexts, which states that policy language can remain flexible enough to accommodate diversity amongst students, families and communities (Son & Diletti, 2017). According to the TIMSS (2012), one of the developments of the reform is the increased number of textbook analysis studies that have led to more effective ways to improve students' performance in mathematics.

Tan and Liu (2018) revealed that education systems, which are influenced by culture, are strongly correlated to mathematics achievement, hence making this study significant due to the rich culture of South Korea and South Africa. Although there are various contributing factors that can account for inadequate results I chose to focus on textbooks because of its content and structure, which are important for accomplishing the desired outcome of the curriculum (O'Keeffe, 2013). This makes understanding how textbooks can contribute to effective learning important, as it could have serious implications for the teachers' ability to convey information, and the students' ability to engage with the material in a meaningful way.

In my personal teaching experience, I found that my students have visual, auditory and kinaesthetic learning styles that aid them in understanding and completing tasks in the textbook. The limitation of language, visual and hands-on activities in the textbook can lead to difficulty in problem-solving. This encouraged me to develop methods to accommodate their learning styles, and maximize their understanding using various resources and textbooks. I realize the presentation of the content in a textbook is vital in order to cater for varied learning styles. This study examined specifically the representation of textbook design and problem-solving procedures in South Korean and South African mathematics textbooks. It aimed to identify the quality of textbook design, how the representation of problem-solving procedures develops students' conceptual understanding, and ultimately, what can be done to improve mathematics results in South Africa in terms of textbook instruction. The study addressed factors that need to be considered to develop a problem-based textbook with 21st century skills that can improve mathematics results (Germaine, Richards, Koeller & Schubert-Irastorza, 2016).

1.6 Key research questions

This study was underpinned by the following research question:

How are problem-solving procedures presented in South Korean and South African Grade 6 mathematics textbooks?

Four sub questions emanated from the key research question viz.:

1. How are problem-solving procedures presented in the South Korean Grade 6 mathematics textbook?
2. How are problem-solving procedures presented in the South African Grade 6 mathematics textbook?
3. What are the similarities and differences in the representation of problem-solving procedures between the South Korean and the South African mathematics textbooks?
4. To what extent do the South Korean and the South African textbook address the principles required to develop Grade 6 cognitive demands and critical thinking?

5. Why are problem-solving procedures represented differently in the South Korean textbook as compared to the South African textbook?

1.7 Research design, methodology and paradigm

To acquire an improved understanding of how problem-solving procedures were represented in the selected textbooks, a comparative analysis was conducted using qualitative research, specifically content analysis, for data production.

This study included the interpretive paradigm, which aimed to understand the design process in each textbook and the challenges they presented in their respective contexts. It had been the most suitable choice as it supports the contention that the influence of metacognitive approaches and self-regulating procedures of solving math word problems can differ from one country to another. According to Jreisat (2017), comparative studies can either discover new knowledge or validate existing information, which enables textbook authors to improve and adopt better practices.

Esser and Vliegenthart (2017) explain that comparative analysis enriches the understanding of society by placing its familiar structures and routines against those of other systems, with comparison heightening the awareness of other systems, cultures, thinking and behaviour patterns. Geva-May, Hoffman and Muhleisen (2018) similarly states that "social scientists regard comparative studies as the methodological core of the humanistic and scientific methods" (p. 18). Andrews, Jablonka, Sojka and Xenofontos (2017) states that comparison aids in serving as an avenue to mediate and create relationships with other countries by building a solid foundation, as in sociology and history. This creates a challenge to focus on the interplay of factors, such as in history, philosophy and sociology, while foreign cultures approaches are analysed, with comparative thinking being the focus.

Kelly (2017) states the use of comparisons in education will assist teachers to make decisions on significant issues and factors to be considered to improve educational practice. The intention of this study is that when applied to education, the comparison will help textbook authors to use theories or models to explain educational phenomena and encourage interdisciplinary critique, which will be addressed in the recommendations.

1.8 Organisation of the thesis

Chapter 2. Background of the study: this chapter outlines the educational backgrounds of South Korea and South Africa, with special attention given to their historical context, curriculum and the development of mathematics textbooks. It portrays the challenges these countries have endured and their perseverance to create educational change.

Chapter 3. Literature Review: this chapter focuses on the development of textbook analysis, and in general, the various contributions that have been made to this field. It specifically focuses on mathematics textbook analysis and the number of aspects that are investigated in this field. The digital vs traditional textbooks are compared, and their advantages and disadvantages are highlighted in the respective textbooks. Improving the quality of textbooks is reviewed, in addition to, the influence of textbooks on the teaching and learning environment and the impact it has on students. The relevance of content in textbooks and their application to the real world is noted and followed by a conceptual understanding of problem-solving and the development of cognitive demand in textbooks. The importance of problem-solving is reviewed, and the representations of problem-solving tasks in textbooks are discussed. The importance of multiple representations in the textbook is explored, and the theoretical and conceptual framework presented.

Chapter 4. Methodology: this presents the research design and study methodology. The comparative study design entailed using document reviews and content analysis to compare the textbooks, with a focus on the theoretical framework developed in Chapter 3 being used to collect data. The research tools and criteria for the content analysis process of the way in which data had been collected is presented.

Chapters 5. Structural and didactical analysis of the South Korean mathematics textbook and in Chapter 6 the structural and didactical analysis of the South African mathematics textbook. These two chapters present the results of the content analysis individually. Five themes that were highlighted in these chapters, namely, the textbooks structure, organisation, layout, contents and language.

Chapter 7. Comparative analysis of South Korean and South African mathematics textbooks. This chapter compares the problem-solving procedures of the South Korean and South

African textbook. Twelve themes that were identified in this chapter, each one relating to the 4 stages of the Polya's problem-solving "understanding the problem, devising a plan; carrying out a plan, check and extend" and the 8 heuristics "act it out, change your point of view, draw a diagram, guess and check, look for a pattern, solve part of the problem, use a model and work backwards" (p.253). The theoretical and conceptual framework that influence the selection of the themes in this study is presented. Tables and visual representations are used to compare the results gathered from the South Korean and South African textbooks.

Chapter 8. Conclusion. The chapter reviews the extent to which the problem identified which required investigation has been addressed and if the aim has been achieved. It outlines the study limitation, the significance of the findings, and provides recommendations for further research and application.

1.9 Conclusion

In this chapter, the journey undertaken during the study has been outlined, and a brief background to the study has been provided. The researcher's personal motivation and rationale for embarking on this study have been described. The context of the study has been highlighted and explained and a brief description of the textbooks given. The focus and significance of the study, key research question, a brief overview of the research design, methods, paradigm and organisation of the thesis are included. The next chapter reviews the influence of the historical background on the South Korean and South African textbooks.

CHAPTER 2: Background of the study

2.1 Introduction

The previous chapter discussed the introduction and motivation for the study. This chapter reviews the background of South Korea and South Africa's educational system, mathematics curriculum and textbooks. The chronicled historical contexts and political remoulding that they have undergone over time, which have had a significant influence on the mathematics curriculum will also be discussed.

Park and Leung (2006) points out that mathematics textbooks strongly depend on the cultural traditions in the different countries. The study aimed to critically compare the Grade 6 mathematics textbooks of South Korea and South Africa, and to explore the implications of problem-solving procedures between these two countries, this chapter will focus on the historical influences of education in both countries.

2.2 Historical background of education in South Korea and South Africa

An historical perspective is provided to highlight relevant mathematical educational policies and practices in South Korea and South Africa, which have been influenced and dictated by the practices of their recent political dispensation. Steyn (2013) explains that within the new educational dispensations, the curriculum has been adapted to accommodate democracy along with the inclusion of 21st century knowledge and skills. The educational background of both countries is discussed as the literature indicates similar experiences in differing environments. It is apparent that South Korea and South Africa each went through educational transformations that birthed the development of a new education system, which also initiated varying views. The countries sought ways to deviate from the traditional[1] methods of teaching to a more democratic approach to education. Traditional methods rely mainly on textbooks, while modern ones use a more hands-on approach with other types of materials. "The traditional method of teaching is when a teacher directs students to learn through memorization and recitation techniques, thereby not developing their critical thinking, problem-solving and decision-making skills" (Santos-Trigo, 2020, p. 686). This contrasts with modern or constructivist approaches to teaching that involve a more interactive

[1] Traditional which refers to a curriculum that was obligatory, pre-given and unquestionable.

approach where the student is engaged with the material and takes more responsibility for their learning (Alamrani, Alammar, Alqahtani & Salem, 2018). According to Apple and Christian-Smith (2017), little attention has been given to textbooks which play an important role in whose culture is taught, this is also true about how mathematics problem-solving is presented.

Spring (2017) argues that political changes lead to educational policy revisions that are divergent, and that their variant environments gave rise to differences in resource materials and curriculum in general, and in the quality of textbooks in particular. Fan, *et al.* (2018) state that a cross-cultural study allows the researcher to explore certain values and norms amongst different countries, and to understand their associated cultural differences.

Many international studies have been conducted on mathematics education, the most prevalent being the Trends International Mathematics and Science Study (TIMSS) and the International Assessment of Educational Progress (IAEP), in which the mathematics curriculum and students' achievement in various countries have been compared (Martin; Mullis; & Hooper, 2017). Global comparisons of the TIMSS and IAEP focus on the curriculum which the majority of the school population are said to be following. Research has shown that students in South Africa performed poorly in the TIMSS (2012) compared to those in South Korea.

2.2.1 Historical background of education in South Korea

According to Choi, Lee, Lee and Kim (2017), during the period of the Three Kingdoms of Korea (57-688 AD), government education was heavily influenced by Confucianism and Buddhism. From 37 BC to 668 AD, Goguryeo, a form of writing, was used. Confucianism was the philosophy of education, with Buddhism a later addition. By contrast, the first educational establishment (372 AD) was Taehek followed by Baekje from 18 BC to 660 AD (Zhuang & Kidder, 2016), then Silla in the Three Kingdoms Period from 57 BC to 668 AD. Martial arts were taught, principally, to combat the invaders from other kingdoms. During the Goryeo dynasty (935 to 1392 AD) two major education types prevailed: Gwanhak, or government schools and Gukjagam (992 AD) the country's first university.

In the late 19th century, the government, patriots, and foreign missionaries began to establish schools adapted to contemporary culture, focusing on the right to education and social equality of education. Schools were mainly either Christian or civilian. In 1894, along with the Gabo Reform, education policy was changed. In July 1894, specialized schools were established. By 1894, Gojong of the Korean Empire officially promulgated that modern education should be accepted, with further clarifications and additions to the policy made in 1895 (Lee, 2017).

However, South Korean formal education was established on 15 August 1948 amid the aftershock of the Japanese occupation, which ended with Japan's defeat in World War II in 1945. From 1910 to 1945 the dictatorial Japanese government had exercised its power through an autocratic leadership style in order to ensure assimilation of the ethnic Korean society. The Japanese had created social segregation between local inhabitants (who were South Korean) and Japanese expatriates, and only allowed the teaching of subjects deemed important by the authorities, such as reading, writing, simple mathematics and poems (Kobayashi, 2018). Paper and textbooks were scarce, which resulted in students reciting their lessons until they memorized them. However, after World War II, the new independent government of South Korea established a modern educational framework based on that of the United States of America (USA). According to Rose-Ackerman and Palifka (2016) their reform made way for educational opportunities for all citizens by eliminating the segregated education system. South Korea promulgated the Basic Education Law to unify the different education system and created compulsory education for six years for all students from the age of seven. The curriculum is standardized, and the students now study subjects such as technology, mathematics and domestic science. The primary curriculum consists of nine principal subjects: moral education, Korean language, social studies, mathematics, science, physical education, music, fine arts, and practical arts.

The Ministry of Education (MOE) oversees general, specialised and official education of the country (South Korea), however, the Ministries of Finance and Economy (MOFE), Science and Technology (MOST), and Labour (MOL) all participate in framing and applying guidelines related to education and proficient training. For example, the MOFE allocates government funds for education, giving precedence to financial and community growth. Synott (2017) reveals the government's determination to make Korea a progressive nation in basic research and technology which has led the MOST’s to advance science and technology.

The Constitutional Law of the Republic of Korea (Article 31), which was promulgated in 1949, stipulates that all people have the same right to obtain an education equivalent to their abilities which is inculcated in the mathematics textbook design. While obligatory education is free, its liberation, competence, governmental neutrality and the self-sufficiency of organisations of higher education are guaranteed, as prescribed by law. Synott (2017) explains the state endorses lifetime schooling, and essential materials relating to the educational structure, including schools and lifelong education, management, funding, and the rank of teachers, are determined by law.

The country has made considerable progress in the development of their education system, with the average high school student generally attending class for eight hours. Two-thirds of South Korean students who attend preschools are enrolled in private kindergartens, with 50% receiving preschool education which results in early exposure to mathematics problem-solving procedures (Shim & Shin, 2019). Primary education is compulsory and is at no cost from the ages of six to twelve, which allows students to continue to develop their mathematical critical thinking skills. The language of instruction changed from Japanese to Korean in 1945, with textbooks and other resources being provided in the mother tongue, this allows students to understand the mathematics content easily. Learning environments in Korea at primary and post-primary level serve more than 6.5 million students. Class sizes vary at lower secondary level, with average class of 32 students per class (Heo, Leppisaari & Lee, 2018).

The South Korean government wanted to preserve their culture while promoting their development in line with the changing social demands (Shim & Shin, 2019). Furthermore, Joo, Lim and Kim (2016) noted that the Smart Education Initiative (SEI)[2] was announced for the KG to Grade 12 (K-12) education system in South Korea, which entails the use of technology. Li and Even (2011) stated that teachers' expertise can be improved in various ways, including providing continuous development of knowledge and skills in mathematics and pedagogy, undertaking and implementing changes in curriculum and instruction, and professional promotion. Gay (2018) explains that intercultural experiences positively influenced Korean teachers' beliefs about diversity, which has a substantial influence on their training of diverse teaching.

[2] SMART concept is the implementation of modern technology. Korean education is expected by many educational organizations to pioneer education into the 21st century by applying the smart technology.

According to Sandoval-Hernandez and Bialowolski (2016), based on global test scores, completion rates and the dominance of higher education seekers, South Korea is observed as having one of the greatest K-12 education system in the world. However, Lee, Puig, Lea and Lee (2013) claim that adolescents are constantly under pressure by schools, society and parents to perform well in external examinations. Advanced education is an overpoweringly serious issue in South Korean society, where it is regarded as one of the vital keystones of life (Heo, Leppisaari & Lee, 2018). Educational success holds a high cultural status as a necessity for socio-economic success, with academic achievements often being a source of pride for families. South Korea's quality education system is based on the foundation of four pillars, these being: 1. placing education at the center of a long-term development strategy, 2. requiring the correct people to become teachers, 3. developing these people into effective instructors, 4. prioritizing information and communications technology in education (Skovsmose, 2020).

The important endeavors of Korean mathematics education has received local and global acknowledgement, however, some ethnic and scholastic customs have raised disapproval and debates, e.g. extended learning hours, the substantial costs of extra lessons and exam-driven competitive teaching and learning (Carr & Wang, 2018). In addition, Lee, Cho and Lee (2019), highlighted certain problems in the assessments, apart from the under successes of South Korean students in emotional areas, which has required a plea for a support system for underachievers and for those from multi-cultural backgrounds.

2.2.2 Historical background of education in South Africa

The unequal education situation that prevails in South Africa at present, can be traced to the Apartheid era when the Nationalist government perpetuated race, class, gender and ethnic division, rather than common citizenship and nationhood (McKeever, 2017). Beginning with its colonial roots the London Missionary Society established mission schools in the Cape Colony after 1799. The Dutch Reformed Church built schools between 1806 and 1900, with Christian teaching at its core (Skinner, 2017). In rural areas, teachers taught basic literacy and math skills. Throughout the nineteenth century, Afrikaners resisted government policies aimed at spreading ~~of~~ the English language and British values. However, from 1900 to 1948, the British colonial government brought in thousands of teachers from the English-speaking world to educate children in the mother-tongue of English. By the early twentieth century, government funds were used primarily to educate Whites with minimal funding for black South Africans. When the Afrikaners gained power in 1948, they aggressively supported the use of Afrikaans as a medium of instruction, (Norman, 2017).

According to Tymbios (2017), throughout the nineteenth century, most religious schools in the Eastern Cape accepted Xhosa children who applied for admission. In the then named province of Natal, various Nguni-speaking groups sent their children to mission schools after the mid-nineteenth century. The government also financed teacher training classes for Africans as part of its pacification campaign. Until 1953, Black children were enrolled in mission schools offering identical instruction as in White-dominated schools. Under the Bantu Education Act (No. 47) of 1953, the government clamped down on such religious schools, obliging them either to close or sell their schools to the government, as funding was withheld. Christian National Education (CNE) became the government's stance on education. CNE subscribed to the notion of racial inferiority and superiority, promoted teaching of cultural diversity to justify the policy of separate development, and enforced mother-tongue instruction in primary schools thereby justifying the exclusion of or providing inferior curricula for mathematics and science subjects. In 1974, the "Afrikaans-medium decree" was issued by the Minister of Bantu Education and Development. Both English and Afrikaans were made compulsory in Black secondary schools (Tymbios, 2017). Violence erupted as a consequence of the resistance to this decree. On 16 June 1976, students protested against the language policy and led to an uprising that would gather momentum leading to the demise of apartheid. The National Policy for General Affairs Act (No. 76) of 1984 provided some improvements in Black education. However, overall separation was called for by the Bantu education system.

During this repressive era, the country was divided into nine administrative units based on race and culture (Mamdani, 2018). Each was managed by separate parliaments, including ministers of Education. Nine curricula deployed different textbooks, and resources, achieving varied outcomes. Some Bantustans re-established in South Africa in 1994 made use of own education departments. The education system, according to McKeever (2017) underwent a paradigm shift, from Bantu Education (1953), which applied rote learning, to the modern OBE system. The poor quality of education, it can be argued, emerged from the colonial and apartheid political and governance approaches that marginalised local languages and education provisioning.

Fiske and Ladd (2004) explain that the South African government abolished the nine racially defined Departments of Education and established a unified education system, which was consistent with the new constitution. The first major curriculum statement of a democratic South Africa was a National Curriculum Framework statement (1996), which stressed on lifelong learning. It stemmed from principles resulting from the White Paper on Education and Training (1995), the South African Qualifications Act (No 58 of 1995) and the National Education Policy Act (No 27 of 1996). The White Paper stressed the need for essential changes in education and training in South Africa in order to standardize and change teaching and learning. It emphasized the need for a change from the outdated goals-and-purposes to outcomes-based education (Jojo, 2019). It also encouraged a vision of a wealthy, integrated, democratic and globally competitive country with knowledgeable, inspired and creative people leading industrious, satisfying lives free of violence, judgement and bias.

(National Department of Education (NDE), 1997). According to Mouton, Rabie, de Coning and Cloete (2014), the period between 1994 and 2011 paved the way for a new historical era in the South African schooling system, as a new education system was introduced. The political standpoint of the newly formed democratic government in 1994 had been aimed to abolish previous educational policies that were systemically linked to a regime based on segregation. Lelliott, Mwakapenda, Doidge, Du Plessis, Mhlolo, Msimanga and Bowie (2009) stated that the political changes of 1994 were associated with policies that significantly changed the education sector across the country. According to Jojo (2019) South Africa’s poor results has resulted in mathematics being isolated as essential knowledge in South Africa.

The South African Department of Education embarked on curricular reform, which required teachers, including mathematics teachers, to be re-trained from the previous role of providing information that the students had been required to convert into knowledge to a facilitative and collaborative practitioner, where the student becomes responsible for their own learning. This reform led to the development of new textbooks being published (O'Cadiz, 2018).

The National Curriculum Statement Grades R-12 indicates the knowledge, skills and values that are vital to mathematics education in South African schools. This curriculum purposes to ensure that students obtain and implement fundamental knowledge and skills in an eloquent way. Furthermore, the intention is to exemplify rigour, high standards and create consistency in the mathematics curriculum so that all students are taught the crucial knowledge in the significant subjects (Jojo, 2019). This perspective has gained strength since the 1980's when the supposed neutrality of mathematics education started to be questioned.

South Africa aimed to preserve their culture as part of the mathematics curriculum, which influenced the decision to advance their mathematics system through indigenous knowledge. This refers to the understanding, skills and attitudes established by people with extensive history of interaction with nature (Mamdani, 2018). However, in order for an education system to be deemed successful it requires the necessary social-economic conditions, such as appropriate mathematics learning materials, libraries, learning aids, technical equipment and other necessary equipment, along with a foundation of early childhood development to ensure a higher chance of an educational success (O'Cadiz, 2018).

2.3 The mathematics curriculum

This section focuses on the influence of alterations in the mathematics curriculum in South Korea and South Africa. A school curriculum is a syllabus of carefully chosen content and learning experiences offered by a school that is proficient of either transforming or altering learner behaviour. Jitendra, Griffin and Xin (2010) explain curriculum design, development and assessment by utilising effective textbooks which are central to teaching and learning in the classroom. In a post-modern, technological environment, teachers find themselves academically and pedagogically challenged (Skovsmose, 2020). Teachers are faced with a teaching and learning environment that is constantly changing due to curriculum requirements, excessive paperwork, a growing demand for better performance from school administrator, accommodating for diverse learning needs and limited teacher knowledge. The curriculum reform in South Africa included the needs of students from diverse backgrounds and cultures, therefore, teachers had to adapt their instructional strategies to reach a wider range of students. According to Geary, van Marle, Chu, Rouder, Hoard and Nugent (2018), a higher quality mathematics curriculum has been created to guide instruction and meet the challenges of the 21st century learner.

Cottrell (2017) states that the mathematics curriculum along with the textbooks should be relevant and meaningful while also preparing students for real life experiences. They have to develop the students' creativity, critical thinking skills, communication skills and collaboration skills to assist them in a 21st century world. The mathematics curriculum of each country is discussed under the following sub-headings: Mathematics Curriculum (topics and sections), Curriculum Drivers, Ideological Orientation, Specific Aims of the Constitution, and Curriculum and Assessment Strategies.

2.3.1 The South Korean mathematics curriculum

The Ministry of Education and Human Resources Department (MEHRD) develops and monitors the South Korean national curriculum. The educational law recommends the curriculum and specifies criteria for the development of textbooks and instructional materials. The curriculum has undergone several revisions, one of which occurred during March 2000 (Li & Park, 2018). The trend being toward decentralisation in improving, diversifying, and applying the curriculum (The School Curriculum of the Republic of Korea, 2004).

2.3.1.1 Aims of the South Korean mathematics curriculum

According to Kim and Eom (2017), the curriculum is aimed at promoting self-sufficient, creative students who can lead the people of the world into the information age of the 21st century. The aim is to deliver all-inclusive basic education centered on students' competencies and talents, promoting all-encompassing values and inspiration while increasing ingenuity in order to adjust to globalisation and the information age.

According to Adams and Gottlieb (2017), the South Korean middle school mathematics curriculum, which was aimed at developing uniqueness and character, was revised in August 2011. The curriculum focused on prior learning from which students are able to understand mathematical concepts and make associations with their everyday lives. The manipulation activities were included to enable students to achieve a natural idea of what they are learning and develop their imagination and intellect to validate mathematical outcomes based on their knowledge and familiarity. Lew; Cho; Koh and Paek, (2012) states that since students were not able to take part in the intended mathematical process due to the large amounts of work forced upon them by the current curriculum, the curriculum had been revised and modified along with the removal of sections that were traditionally taught.

2.3.1.2 The South Korean curriculum choices and consequences

South Korea has a system of diagnostic assessments called the National Assessment of Educational Achievement (NAEA). Each year, achievement tests in two subjects are administered to all students in each of the Grades 6, 9 and 10 with the results being used to evaluate the performance of schools. The sole purpose of these tests is to provide students with information and the results are not given to them. The under-performing schools receive additional support and training from the government (OECD, 2011).

According to various international comparative studies of students' achievement e.g. TIMSS, (1995; 1999, 2003 & 2007); the Program for International Student Assessment (PISA, 2003) and the Organisation for Economic Co-operation and Development (OECD, 2006), South Korean school students perform at a very high mean score in mathematics. Particularly in 2003, Korean 8th Graders ranked 2nd in mathematics amongst 46 countries participating in the TIMSS and their achievement scores have been continuously improving (Lee, 2018). However, Lee, Puig, Lea and Lee (2013) contend that South Korean teenagers experience extensive anxiety due to an educational system that concentrates primarily on college entry testing and entails a considerable burden for academic accomplishment, which perpetually creates a competitive atmosphere in schools.

2.3.2 The South African mathematics curriculum

Morreira (2017) elucidates that the education system in South Africa changed with the publication of the Policy Framework for Education and Training in January 1994 (ANC, 1994), which formulated the politically driven goals for the education and training sectors which resulted in the Outcomes-Based Education (OBE) curriculum being selected. McKeever (2017) explains that the education system underwent a paradigm shift that began from the traditional system of rote learning to the OBE system, which was the Curriculum 2005 (C2005). This was followed by the

National Curriculum Statement (NCS), then the Revised National Curriculum Statement (RNCS), which led to the Foundations for Learning and the current Curriculum and Assessments Policy Statements (CAPS). The C2005 had been implemented in 1998, in an effort to prepare the school-going population to live meaningful adult lives. The OBE curriculum had been identified as the ideal vehicle to facilitate the achievement of that goal, then later changed. The education system in South Africa has transformed since the publication of the document, A Policy Framework for Education and Training in January 1994 (ANC 1994) in which the goals were communicated for the mathematics education and training sectors. The CAPS document explains that mathematics is a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making. Focus of content areas in mathematics curriculum in the Intermediate Phase (Grade 4, 5 and 6) covers five Content Areas:

- Numbers, Operations and Relationships;
- Patterns, Functions and Algebra;
- Space and Shape (Geometry);
- Measurement; and
- Data Handling.

Each content area contributes towards the acquisition of specific skills. The South African Ministerial Task Team suggested that the Department of Basic Education develop a clear and precise five-year plan to sustain the implementation of the National Curriculum Statement Grades R – 12 in schools and upgrade mathematics curriculum documents. Policy research indicates the ineffectiveness to hasten the implementation of curriculum reform. Kelly and Knowles (2016) noted the new mathematics national curriculum policy reform emphasizes that teachers must make connections between mathematics and the real world. This study also investigates how problem-solving procedures are represented in mathematics textbooks, whereby connections between mathematics and real world contexts are identified (Jojo, 2019).

2.3.2.1 Aims of the South African mathematics curriculum

According to You, Liu, Yu, Chen and Pan (2012) the objectives of the curriculum at high school level in South Africa are to develop each student's character and their expertise in order to strengthen the nation; to prepare them for jobs needed in society; to encourage each student's independence, emotional development, and critical thinking abilities to function in and out of school, building awareness of the important role that mathematics plays in real-life situations and lastly, to improve physical strength and foster a sound mind. CAPS is the national curriculum of South Africa, which was developed over a period of 17 years to change the curriculum that had been implemented during apartheid (pre-1994). The purposes of the Constitution of South Africa are to redress the inequalities of the past and create a society free of discrimination, providing social justice and fundamental human rights; to pave the way for societal reforms and individual growth; to create a more fertile atmosphere in which the government ensures the safety of all people; and ultimately to reposition themselves as a democratic South Africa in the global community (DoE, 2010). Mathematics is taught at foundation (grades 1–3), intermediate (grade 4–6), senior (grade 7–9) levels and further education and training (grades 10–12) phases. CAPS is prescriptive and demands uniformity in implementation across the country. The prescription is enforced, ostensibly, because the freedom associated with the implementation of changed curricular are counter-productive since learners' performance continues to be poor in mathematics.

Bantwini (2010) states that the CAPS Grades R – 12 aims to provide students with the necessary skills. Firstly, to identify and solve problems using critical and original thinking, students ability to work as individuals and in groups by organising their tasks dutifully and efficiently. Secondly, the skill to gather, examine, classify, judgmentally assess data, and converse effectively using visual, symbolic and/or language skills in various modes. Thirdly, the capability to use science and technology successfully by critically showing accountability towards the surroundings and the well-being of others. Lastly, the ability to establish an appreciation of the world by recognising the non-existence of problem-solving contexts in isolation. Inclusivity as a crucial part of the planning, preparing and teaching at every school, can be accomplished if teachers are able to recognise and address learning barriers while catering for a variety of learning abilities. The strategy which can be used by important school support structures and teachers is to manage comprehensive learning and to identify and address obstacles (DoE, 2010).

The CAPS Grades R - 12 is based on the following principles: firstly, social change, which ensures that the past educational inequalities are addressed, all people are afforded equal educational opportunities. Secondly, cognitive development approach to learning as opposed to rote learning, with minimum standards of knowledge and skills to be achieved at each Grade specifically. Thirdly, indicating the progression of content and context of all grades from simple to complex. Finally, human rights, inclusivity, environmental and social justice, infusing the principles and practices of social and environmental justice and human rights, in accordance with the Constitution of the Republic of South Africa (Rochester & Martin, 2005).

However, proper curriculum implementation is vital for the objectives to be achieved. Erden (2010) argues that teachers will fail in the implementation process if there is a break in understanding the curriculum framework. Makeleni and Sethusha (2014) explains that teachers' training, motivation, dedication and professional competence will ensure the effective implementation of the curriculum. Mathematics provides an effective way of building mental discipline, developing cognitive demand and mental awareness. In addition, mathematical knowledge plays a vital role in understanding the contents of other school subjects such as science, social studies, music and art.

2.3.2.2 The South African curriculum choices and consequences

Schoeman (2006) states that the African National Congress (ANC) government immediately began to realize that if the new-found democracy was to survive, education for democratic citizenship had to be taught to every future generation. The current curriculum in South Africa is CAP's, which is Curriculum and Assessment Policy statement. A national study was conducted in 2005, by the South African Consortium for Monitoring Educational Quality (SACMEQ), on teacher classroom assessment perceptions, beliefs and practices in collaboration with the Department of Education as part of a programme to implement an effective curriculum with affective assessments for improving learning in South African schools, including mathematics results. It is imperative to understand how assessments are applied in South African primary schools in order to serve students from different socio-economic backgrounds. The changing landscape of post-apartheid education in South Africa has seen significant reform in the mathematics curriculum and marked growth in the community of researchers investigating mathematics education.

The mathematics curriculum objective is created to provide for an information- and technology-based society which requires individuals who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics. The Annual National Assessments (ANA) is an assessment instrument introduced by the Department of Education (DOE) in 2011 to enable a systemic evaluation of education performance with the intention to enhance learner mathematics achievement. Roberts and Barmby (2016), states that the Department of Basic Education (DBE) is determined to improve the language and mathematics skills of the students. The ANA was intended to measure students' progress, and to establish their mathematics performance levels. According to Kanjee and Moloi (2014), the ANA is a tool to assess whether a child needs additional help or not and is used for learner progression in mathematics.

Furthermore, Sayed, Kanjee and Nkomo (2013) states that the ANA system creates a desire for students to be part of the high-performing schools, and this will influence teachers to improve learning and teaching in mathematics. However, teachers and schools do not want to be classified as under-performing due to the results attained through ANA, as it adds to the teaching and management trials at schools, especially in an environment of insufficient support and lack of assessment training (SADTU, 2011). According to Spaull and Kotze (2015), the problem is that these tests are being used as proof of improvements in mathematics when the ANA results do not cater for changes over time.

Spaull and Kotze (2015, p. 13) notes that "simply constructing an incentive structure of standards and testing around the expectation of steady improvements in mathematics performance is not a theory of improvement". A theory of enhancement has to reason for how students in schools learn what is required in order to meet the potentials of the responsibility system. The PISA and the TIMSS study are assessment instruments utilised in the South African education system. The TIMSS study, which tests mathematics and science, indicated the lack of improvement in Grade 8 mathematics and science achievement between 1995 and 2002. Spaull and Kotze (2015) explains that for disadvantaged students, there are insurmountable learning deficits that increase gradually between the mathematical knowledge required and the knowledge they possess. This shows that as time progresses, students fall further behind the curriculum leading to a situation where remediation is almost impossible in high school, as learning gaps were unaddressed for a long period of time. In this study, a comprehensive framework is created that will aid in the production of textbooks that will address the gaps that exist.

2.4 Mathematics textbooks

The programmatic curriculum becomes effective when the aims and objectives are linked to the content and is communicated in the schools. In theory, textbooks are developed on the basis of a programmatic curriculum, which guides the textbook evaluation. Matić and Gracin, (2016, p.349) used the term “the textbook as an artefact in the classroom” to describe the programmatic curriculum, which is the role of the textbook and other curriculum materials in a classroom. Increased attention is attributed to the quality of textbooks across the globe.

Textbooks are valuable in the teaching of mathematics due to their close relation to classroom instruction (van den Ham & Heinze, 2018). The textbooks identify the topics and are arranged in the manner in which students should explore them. The tasks and activities give direction to what is taught in the classroom. Hence, Hadar (2017) explains that textbooks are designed to assist teachers in organising their teaching.

2.4.1 South Korean mathematics textbooks

According to Ladnier-Hicks (2010), South Korean elementary mathematics textbooks have undergone various analysis and changes over 65 years. These textbooks are unique compared to those of western countries. Research conducted by Ju (2016) illustrates that Korean elementary mathematics textbooks have strengths in their coherence and systematic approaches. Textbooks and teacher’s guides are an important mediator between the national mathematics curriculum and what happens in the classroom. Instruction is textbook based instead of curriculum based, therefore, Ju (2016) explains that a higher quality of textbooks and curriculum materials are aligned with changes in the national curriculum in South Korea.

Furthermore, Ladnier-Hicks, McNeese and Johnson (2010) states that textbooks are aligned with the curriculum, as they are one of the primary materials used to implement its goals. South Korea modified its national mathematics curriculum eight times in the late 20th century, including the recent minor modification of the 7th curriculum. According to Ju (2016), the first three modifications focused mainly on the theoretical perspective of mathematics; the next three were considered to be the transition, and the last two focused on practical mathematics and differentiated mathematics, which include students' perspectives. The South Korean mathematics curriculum has been modified along with the expectations of their society. The main curriculum modifications were from teacher-centred instruction to learner-centred learning. Jung (2005) states that unlike the secondary mathematics teachers, Korean primary school teachers use only one mathematics textbook that is authorised by the MEHRD.

2.4.1.1 Textbook development in South Korea

Research conducted by Ladnier-Hicks *et al.* (2010) explains that the basic principle in developing elementary mathematics textbooks is to follow and specify what the curriculum intends. According to Germaine, Richards, Koeller and Schubert-Irastorza, (2016), the most recently developed seventh curriculum has a level-based differentiated structure and emphasizes students' learning activities in order to promote their mathematical power, which encompasses problem-solving ability, reasoning ability, communication skills, connections, and dispositions. This curriculum resulted from the repeated reflection that previous curricula were skill-oriented and fragmentary in conjunction with the expository method of instruction. Son and Diletti (2017) clarifies that the previous curriculum did not consider various differences amongst individual students with regard to mathematical abilities, needs, and interests.

In South Korea, like other countries, the curriculum is a vital document that serves as a guide for textbook development. The curriculum guides the criteria for teachers to choose the textbooks used. According to Jitendra, Griffen and Xin (2010), the main motivation behind the current curriculum include increasing concern for individual differences and the desire to provide maximum growth of individual students' abilities and needs. Ju (2016) illustrates that given the curriculum, mathematics textbooks intend to provide students with an increased amount of opportunities to nurture their own self-directed learning and improve their creativity.

To accomplish this, several directions were established in developing elementary mathematics textbooks. First, textbooks should consist of mathematical contents with which individual students can improve their own creative thinking and reasoning ability (Jitendra, Griffen & Xin 2010). At some point in a learning sequence, instructional resources are presented differently for individual variations in mathematical attainment (Romiszowski, 2016). Whereas high-achieving students have difficulties when confronted with advanced tasks including real-life complex situations, their low-achieving counterparts solve basic problems involving the fundamental understanding of mathematical concepts and principles (Germaine, Richards, Koeller & Schubert-Irastorza, 2016).

Second, textbooks consist of mathematical content that contributes to improving the process of teaching and learning. Textbooks have to underline a learning process by which students solve problems through individual exploration, small-group cooperation activities or discussion (Romiszowski, 2016). Third, textbooks are to be easy to use, interesting and convenient for students to follow. Instructions of games or activities in the textbooks should be specific in order for students to initiate it without a teacher's further explanation and demonstration (Kim, 2016).

Textbooks take into consideration students' various interests and stimulate their learning motivation (Bano, 2005). They allow for various editing, design and readability for students while deliberating the appropriate use of different multimedia learning resources (Ladnier-Hicks *et al*., 2010). Fourth, textbooks are to be flexible in ways that allow the teachers to adapt it to enhance the strengths of the students. A textbook should be recognized not as the sole material to be followed but as an illustration of embodying the idea of the curriculum (Romiszowski, 2016). The textbook development for Grade 1 through Grade 6 in South Korea had undergone revision. Students would have used the developed textbooks for Grades 1 and 2 in 2009, Grades 3 and 4 in 2010, and Grades 5 and 6 in 2011. As shown in Table 2.2, each curriculum has its own main focus. The contents of textbooks for each curriculum have reflected the spirit of each curriculum.

Table 2.1: Curriculum changes in South Korean mathematics (Revised from Jung, 2005, p. 3)

CURRICULUM	MAINSTREAM	REMARKS AND CHARACTERISTICS
1st Curriculum 1954-1962	Real life Centred which focuses on real life but the difficulty of Mathematics is relatively low.	Theoretical Mathematics: Real life centred curriculum. It focuses on linking real life to what is learnt in the textbook, but the activities cater for lower students. Textbooks also included real life scenarios.
2nd Curriculum 1963-1972	Mathematical Structure Centred	Complementary curriculum: focused on systematic learning and placed great value on the logical and theoretical aspects of mathematics. Authors of elementary mathematics textbooks emphasized the logic of the mathematics, which was unnecessarily difficult for young students.
3rd Curriculum 1973-1981	"New Math" Oriented	Was influenced by New Math, which introduced abstract but fundamental ideas, such as set theory in the elementary mathematics curriculum.
4th Curriculum 1982-1987	"Back to Basics" Oriented	Transition & Problem-solving Oriented Mathematics, being derived from the failure of New Math and the emergence of Back to the basics Movement. The textbooks also emphasized basic computation skills, which were weakened due to the structural approach in the 3rdcurriculum.
5th Curriculum 1988-1993	"Problem-solving" Oriented	The mathematics curriculum emphasized students' mathematical activities and the affective aspect of learning mathematics. The textbooks also consist of mathematical activities and operations of concrete materials.
6th Curriculum 1994-1999	Problem-solving & Information Society Oriented	As the trend of 1980s and 1990s, the curriculum stressed mathematical problem-solving, focusing on mathematical thinking. The textbooks included a problem-solving section in each Grade to foster students' problem-solving abilities.
7th Curriculum 2000-2008	Learner Centred & Differentiated	Practical and Activity Oriented Mathematics. The curriculum emphasizes learner's self-directed learning and differentiated learning for each student. It also stresses basic knowledge and skills of mathematics, mathematical thinking, and problem-solving. The textbooks include complementary mathematics sections for the lower achievers and in-depth mathematics sections for higher achievers.
Revised7th Curriculum 2009-	Learner Centred, Differentiated & Appropriated	Twenty-first century trend, knowledge is no longer fixed, and involved in various complex contexts. Competency such as the cognitive abilities, creativity, innovation, cooperative learning and skills which enable students to solve particular problems, as well as the motivational, volitional, and social readiness and capacity to use the solutions successfully and responsibly in variable situations' and to solve particular types of problems and deal with certain kinds of concrete situations.

The Koreans made changes between the seventh curriculum and the 2007 revision by adding differentiated workbooks according to student level. The textbook is aimed at average students, and the workbook is a supplement for upper- and lower-level students. In the 2009 revision, they decided not to use workbooks for the secondary level. Instead, they decided to raise the intellectual demand in the textbook with higher-levelled questions (Jung, 2005).

2.4.2 South African mathematics textbooks

The implementation of the new curricula in 1997 resulted in a change in the instructional approaches to develop problem-solving, which leads to high cognitive skills amongst students. Textbooks and workbooks have presented a serious issue within the South African education system. The Department of Basic Education (DBE), allotted considerable disbursement to supply Grades 1 to 6 students within public schools with literacy/language and numeracy/mathematics workbooks. The DBE workbooks which have been given to students comprise of 128 discrete worksheets, four per week with eight weeks per term (Department of Basic Education, 2011). Due to the considerable size and expenditure of an intercession, providing workbooks to students is a focal point of the South African government's strategy to improve learning outcomes.

According to the Joint Education Trust (JET), in a study carried out between January and June in 2011, educational researchers discovered a specific group of workbooks for Grade 6 mathematics proved to be efficient, the functionality had been equivalent to the presently permitted textbook used in South Africa. A crucial outcome of the report recommends that materials of acceptable quality, whether workbooks or textbooks, or a combination thereof, to be utilised continuously and methodically by all students, can enhance academic performance (Pretorius & Spaull, 2016). For Fleisch (2011), this result presents an enquiry as to if the currently permitted textbook is of acceptable quality, will the considerable funding in new materials prove to be impractical. The JET study had commenced in response to the Department of Basic Education's 2010-11 budget provision of R750 million for literacy and mathematics books for Grade 1 to 6 students (Fleisch, Taylor, Herholdt & Sapire, 2011).

2.4.2.1 Textbook development in South Africa

The Textbook Development Institute notes the concern in standard and efficacy of the textbooks utilized by educational institutes being disregarded and an undervalued factor impacting the quality of education in South Africa. Teachers are frequently answerable for the substandard calibre of education, while minimal recognition is given to the inferior quality of the resources which are mandatory for teachers to utilize, predominantly textbooks. (Taylor, 2007). Inferior standard of textbooks has been recognised as a key concern by the Presidency. The Department of Basic Education had been required by a Presidential injunction to develop and provide resources (and specifically workbooks) that would provide assistance to students and improve performance in the critical areas of literacy and numeracy. In 2011 the DBE, identifying the criticality of learning support materials in the learning process, had commenced a crucial initiative to supply Mathematics and Language workbooks to students to hasten the advancement in accessible and quality education for all students. (DBE, 2011).

Maskew, Miller, Longman and Heinemann (2012) are a component of Pearson, the world's foremost learning company, which offers education in the largest sense of the word. Through Maskew, Miller, Longman and Heinemann imprints, content is created in eleven South African languages for students from pre-school to Grade 12. They are partners in-service to the Department of Education, teachers, students and parents and are able to integrate substantial local knowledge with international proficiency to provide solutions which are at the forefront of education in the country. Their purpose is to create 21st century textbooks that assists students to develop critical thinking, creativity and collaboration skills, which are intended at refining learner outcomes.

The wide range of textbooks for South Africa that were developed and provided by the Pearson learning company covered the CAPS curriculum, which is specified by the DoE and informed by expert authors. Relevant illustrations and activities are included in order to develop results and motivate students. Extensive teacher support is provided to assist in saving time and to make teaching easier, including Extension and remediation worksheet books with Teacher's Guides (Cooper, Sidney & Alibali, 2018).

The content for each school term is covered within each of the five main content areas. A variation of tasks, feature boxes, challenges and games are included to develop students' enthusiasm about Mathematics in the classroom (Leonard, Mitchell, Barnes-Johnson, Unertl, Outka-Hill, Robinson & Hester-Croff, 2018). The colourful photographs and artwork are included to assist second language students to understand the concepts and topics. The Teacher's Guide includes a Mental Mathematics section with exercises for each day of the school year. The teacher is able to use the Extension and Remediation Worksheets to provide students with a challenge or to revise and consolidate the basic concepts.

Table 2.2: Curriculum changes in South Africa (1954-2009) based on table 1 by Jung, 2005

CURRICULUM	MAINSTREAM	REMARKS AND CHARACTERISTICS
1st Curriculum – Traditional Curriculum 1954-1995	Traditional method of mathematics teaching and learning.	Complementary curriculum: focused on systematic learning and placed great value on the logical and theoretical aspects of mathematics. Authors of elementary mathematics textbooks emphasized the logic of the mathematics, which was unnecessarily difficult for young students. Focused on rote learning.
2nd Curriculum – Curriculum 2005 (C2005) 1997 – 1998	Mathematical Structure as learner Centred	Theoretical Mathematics: Real life centred curriculum. It focused on real life, but the difficulty of mathematics is relatively low. Textbooks also included real life situations.
3rd Curriculum – National Curriculum Statement (NCS) 1999-2003	Mathematical Structure as learner Centred	Theoretical Mathematics: Real life centred curriculum. It focused on real life, but the difficulty of mathematics was relatively low. Textbooks also included real life situations.
4th Revised Curriculum – RNCS 2004-2008	Learner centred mathematical structure.	It was influenced by New Math, which introduced, which introduced abstract but fundamental ideas, such as set theory in the elementary mathematics curriculum.
5th Curriculum – CAPS 2009-current	Assessment based mathematical structure.	Transition & Problem-solving Oriented Mathematics: was derived from the failure of New Math and the emergence of Back to the basics Movement. The textbooks also emphasized basic computation skills, which were weakened due to the structural approach in the 3rdcurriculum.

2.5 Conclusion

As presented above, the variances amongst the several aspects of the textbooks in South Korea and South Africa reflect different societal and cultural values. As the textbook is an influential way through which students gain knowledge and values, these textbook distinctions are likely to allow for fundamental cultural differences. The previous discussion indicated that each of these countries' approaches has its own strengths and weaknesses. For instance, textbooks in South Korea may assist students in comprehending how useful mathematics can be in their lives, however, they may not be able to grasp concepts if the connection between a mathematical concept and the real life situation is unclear. In contrast, the South African textbooks may succeed in conveying ideas in an efficient way, developing understanding about the differences between their cultures which they can learn from each other, however students may have difficulty learning due to language barriers. The next chapter aims to critically review the literature on aligning textbooks to curriculum, including various aspects and presentation of problem-solving in mathematics textbooks.

CHAPTER 3: Literature review and theoretical framework

3.1 Introduction

Chapter 2 described and compared the backgrounds of South Korea and South Africa, necessary for understanding the study. This chapter presents an overview of various theoretical frameworks that have been developed to undertake textbook analysis. A review is presented of literature relevant to the topic. Here, the intention is to use selected material to develop a theoretical framework for evaluating Grade 6 mathematics textbooks. This chapter reviews textbook analysis, the development of textbook analysis, theoretical frameworks used to improve the quality of mathematics textbooks, conceptual understanding of problem-solving, multiple representations in mathematics textbooks, and relevant theoretical and conceptual frameworks.

3.2 The development of textbook analysis

China, Hong Kong, Japan, Korea and Singapore continue to outperform all participating countries in the TIMSS; and have featured in the top five performers over a 20-year period, according to Foy (2017). Furthermore, Yang and Wang (2016) discuss that the results have prompted researchers to compare their mathematics textbooks with these countries. Throughout the course of the past two decades, extensive attention has been paid to analysing, comparing and improving mathematics textbook content (Berisha, 2015). Eleven countries (Brazil, China, Cyprus, Finland, Greece, Ireland, Japan, Netherlands, Taiwan, United Kingdom and USA) are leading the studies in textbook analysis, and have stressed that the quality of textbooks can influence how students learn mathematics (see e.g., Delaney, 2010; Cai & Ni, 2011; Fan, 2013; Nie *et al*., 2013; Organisation for Economic Cooperation and Development [OECD], 2013; Charalambous, Hsu, Mesa & Delaney, 2010; Kim, 2014; Yang & Wang, 2016; Lessani, Suraya & Abu Baker, 2017).

International comparative studies in mathematics have provided a large body of knowledge on how the subject is studied in the context of the world's varied educational institutions. Textbooks, in particular, have been identified as an essential resource through which students have the chance to learn the subject. This has led to a range of research projects from different countries aimed at analysing the books, the studies having begun in the early 1990s (Ozer & Sezer, 2014). According to Son and Diletti (2017), this analysis is based on a variety of factors, which includes: the varying performance standards expected of students in different countries that participated in the TIMSS;

the procedural fluency of the mathematics textbook; how countries treat their mathematical content and problems, and how they prioritise conceptual understanding when releasing mathematics textbooks.

Data is obtained through textbook analysis that provides insight into the representation of mathematical content required to investigate challenges in the learning environment (Qadeer, 2013). Research conducted by Foy (2017) shows that there is a link between textbook analysis and performance evidence, including the TIMSS. Through insights acquired, there is an increasing number of textbook analysis studies seeking to enhance students' achievements in mathematics. However, Qadeer (2013) explains that there are insufficient meta-analysis studies that summarize textbook analysis studies in conjunction with suggested problem-solving techniques in the content. The methodical analysis of the structure, focus, and learning outcomes of textbooks, form the basis of textbook analysis (Qadeer, 2013).

A broad range of research projects has used textbook analysis in a bid to examine the opportunity to learn content (OTL). Researchers (Törnroos, 2005; Xin, 2007; Ding & Li, 2010; Zanten & Heuvel-Panhuizen, 2014; Wijaya, Heuvel-Panhuizen, & Doorman, 2015) and (Kolovou, Heuvel-Panhuizen, & Bakker, 2009; Wijaya *et al.*, 2015) have indicated that the lack of access to mathematics textbooks in Indonesian schools could possibly cause the students to experience challenges when handling context-based tasks. For USA students, a study indicated that their ability to address mathematics problems varied, depending on the problem types (Xin, 2007), which could be attributed to the textbooks' design, for example, imbalances in the distribution of word problems. The study helps to improve the quality of textbooks, which in turn supports improvement of mathematics results.

Son and Diletti (2017) discuss that, within the past three decades, there has been a growing global concern about the quality of mathematical textbooks content. Such results from the teaching strategies and practices continuously changing in the classroom, owing to textbook analysis. Yang and Wang (2016) explain that a comparative textbook analysis between China, Finland, Singapore, Taiwan, and the USA shows that there were significant differences in the representation forms, problem-type question format, and representation of visual form. The analysis has shown that mathematics textbooks are considered a useful resource in the teaching and learning environment

(Mohammadi & Abdi, 2014; Sunday, 2014; Bertram & Wassermann, 2015). Mathematics textbooks have attracted considerable attention from mathematics researchers around the world (Cai, Mok, Reedy & Stacey, 2016; Cai & Jiang 2017; Foy, 2017).

The South African National Council for Curriculum and Assessments (NCCA) scrutinised mathematics classes for primary schools in the country, the 2015 study indicating that teachers applied the didactic teaching style, with the textbook serving as a substantial aid in the classroom. Textbooks being essential in the classroom, such accessories have an undeniable impact on the learning process (Aineamani & Naicker, 2017). The NCCA analysed textbooks in order to establish whether they cater for differentiated learning styles[3] and processes − how such processes are represented in South African textbooks. Using textbooks with differentiated methods of teaching content is now encouraged, whereas previously, standard textbooks were used to teach content in a traditional manner. Differentiated instruction in textbooks has been used for the past 20 years. The term applied is the mixed-ability teaching method, which takes into account the significant differences amongst students according to their level of learning, learning styles, rate of learning, language proficiency, and literacy and numeracy skills. Mixed-ability methods imply an adaptation of the way the curriculum and learning tasks are presented in the content. Westwood (2018) posits that these significant differences in learning styles also determine the quantity of additional support resources specific students may require. Furthermore, Yang and Wang (2016) explain that textbook analysis proves that textbooks are one of the resources for promoting differentiated learning, such as catering for the various cognitive levels and the rate of learning.

The traditional didactical style of teaching (teacher-centred) is consistently being challenged in most countries, with the United Kingdom (UK) and United Arab Emirates (UAE) providing differentiating lessons that cater for different levels of understanding amongst students (high, medium, and low achievers) (Westwood, 2018). Some countries, for example, Switzerland, United Arab Emirates, and USA, are developing learner-centred activities, which are created according to the students' various abilities. Observing the learner-centredness or teacher-centredness of diverse textbooks, and the analysis of how different interactions (teacher-students, pairs,

[3] Learning styles - the visual learner, auditory learner, and kinesthetic learner.

individual work) are used in the textbook, can also prompt an argument on how the textbook content is being used by the teacher (Waltermann & Forel, 2015).

According to Tanujaya, Prahmana and Mumu (2017), teachers in Indonesia are still the sole source of information in the classroom, with no room being given to students for critical thinking or other methods of problem-solving procedures. Students are only allowed to use the method given by the teacher. However, Mohammadi and Abdi (2014) explain that textbook analysis shows that textbooks help beginner teachers design course activity, thereby ensuring that the class is well-structured with consistency and reasonable progress. In certain instances, teachers are involved in textbook design, exploring the means of enhancing explanatory understanding (Qadeer, 2013).

It is important to consider that there are aspects of the textbook that can have a negative effect on students' progress. When textbooks are designed as the sole source of information, students only see one perspective on a concept or issue. Students can overcome this issue if provided with multiple information sources, such as CD-ROMS, websites, and encyclopaedias (Lau, Lam, Kam, Nkhoma, Richardson & Thomas, 2018). Textbooks that are old or invalid provide students with irrelevant information. Such outdated material can be augmented by supplementary current materials. Textbooks which present low-level or fact-based questions may lead students to believe that learning is merely a collection of facts and numbers. Providing students with higher-level questions can stimulate innovative thinking and problem-solving skills.

The main focus of textbook analysis has always been advancing the quality of instruction in the classroom. The significance of comparative textbook research was emphasized by the League of Nations[4] as early as the 1920s, when they devoted serious attention to the question of international intellectual cooperation. This led to the creation of the International Committee on Intellectual Cooperation. From 1926, the International Committee was included in the International Institute of Intellectual Cooperation, which resulted in the creation of the International Educational Cinematographic Institute. The declared purpose of the United Nations Educational, Scientific and Cultural Organization (UNESCO) was to create peace and security in the world by promoting collaboration amongst nations through education, science, culture, and communication, in order to

[4] The League failed in its main purpose of keeping peace through the arbitration of international disputes. It was replaced by the United Nations.

create universal standards. Its aim was to combat discrimination, cultural or religious assimilation, or racial segregation in the field of education. The International Committee was the successor to the League of Nations' International Committee on Intellectual Cooperation[5], and the precursor to UNESCO.

UNESCO conducted further advancements in this area after World War II, and continued the emphasis on textbook analysis after the League was dissolved in 1946. The organisation began to collaborate with scholars from the Georg Eckert Institute (GEI), based in Braunschweig, Germany. The GEI conducts international, multidisciplinary and application-oriented research into school textbooks and educational media. Nicholls and Opal (2005) note that the GEI was founded in 1951 by Georg Eckert, an educationalist and historian of German descent, his institute receiving global recognition for its efforts in comparative textbook analysis.

Research conducted by Charalambous *et al*. (2010) led to the information being used to design textbooks that are effective and beneficial in the classroom (Charalambous *et al*., 2010; Yang & Lin, 2015). According to Pingel (2010), textbook analysis can be classified into didactic and content components of problem-solving procedures; the former being concerned with the teaching practice exhibited by the text, while the latter analysis addresses the content itself. Martin, Mullis, Foy and Stanco (2012) maintain that the earliest textbook analysis projects in the TIMSS sought to examine the content profiles, identifying how the textbooks were beneficial to the teaching and learning process. According to Fan (2013), textbook analysis attracted research interest, owing to its contribution to learning and teaching around the world, with studies having sought to identify their strengths and weaknesses in different countries.

UNESCO, together with the Council of Europe, has been instrumental in improving European history textbooks content by supporting numerous textbook analysis projects. Since 1949, history scholars and teachers have been invited to Pan-European conferences, and textbook authors have received publishing guidebooks to help eliminate prejudice and bias in their work. Wager (2014) states that inequalities continue to be presented to the readers, which happens when different groups, perspectives, and opinions influence the writing process.

[5] UNESCO took over from the League of Nations' International Committee on Intellectual Cooperation.

Lenon and Cleves (2015) state that textbooks that are most beneficial have the questioning set out according to Bloom's taxonomy, which aids in determining the level at which the high achievers are able to perform, bearing in mind the success and failure of educational systems in various countries. The authors (2015) suggest that South Africa could act in accordance with the education system from South Korea and Taiwan rather than the USA, as the former indicates a higher pass rate in international testing. The TIMSS has devised a common framework to compare the quality of textbooks. Therefore, when analysing mathematics textbooks, Bureau (2016, p.2) contended that the following factors need to be considered in order to improve the quality of the content:

- The structure (S), organisation (O) and layout (L)
- Content weighing (CW)
- Conceptual understanding (CU)
- Procedurals practice (PP)
- Problem-solving ability (PS)
- Special requirements (SR), which are mainly cognitive requirements.

The analysis of textbooks over time has aided in improving the quality of their content, with Yang and Wang (2016) focusing on content analysis, having found that there were significant differences in the representation of visual form.

Research conducted by Loughran and McDonald (2016) and MacKenzie *et al.* (2017) explains that textbook analysis has been executed in other subjects, such as history, accounting, geography, English and psychology. Grever and Van der Vlies (2017) further state that the composition, periodization, visual intertextuality, and chapters influence students' understanding of history, and that a detailed analysis is therefore carried out on the background to history textbooks. Textbooks have the potential to change the interpretation of historical events when the available information surrounding them changes. For example, Sleeter and Grant (2017) examined how history textbooks used in school presented issues by either ignoring or giving only passing attention to inequality based on race, gender, disability, and social class in the content. In addition, Vera (2018) suggests that textbooks are often used to reveal political and cultural changes that are evident in a country.

Chu, Reynolds, Tavares, Notari and Lee (2017) discuss crucial 21st century skills that would equip students to develop effective problem-solving skills. These researchers contend that mathematical proficiency requires a change wherein the content and pedagogy is combined to prepare the student with the necessary skills[6] in mathematical reasoning. Research conducted by Gravemeijer, Stephan, Julie, Lin and Ohtani (2017) reveals that these skills promote critical thinking, creativity, and collaboration, which develops conceptual understanding and differentiated lessons, by catering for the different learning styles present in the classroom. The continuous analysis of textbooks therefore aids in the transformation and development of textbook contents. Many mathematics textbook analysis projects have been conducted on algebra, geometry, and the representation of problem-solving (Gene, Lavidas, Zacharos, Koustourakis, 2018; Carvalho, 2018). One of the objectives of this study is to investigate how various problem-solving procedures can be used to develop students' knowledge, critical skills, and understanding of concepts taught.

3.2.1 Theoretical frameworks supporting mathematics textbook analysis

Various authors (Polya, 1945; Valverde *et al.*, 2002; Charalambous *et al.*, 2010; O'Keeffe, 2011b; O'Keefe, 2013; Morgan and Henning, 2013) have designed frameworks supporting mathematics textbook analysis (Figure 3.1) comprising four key elements (structural analysis, expectation analysis, content analysis, and language analysis). Content and structure are important for promoting a specific vision of a curriculum, which may have a significant impact on students, such features having positive or negative impacts on learning. However, conceptual understanding, procedural practice, problem-solving ability and special requirements are of particular significance with respect to the cognitive demands placed on students, these being aptly captured in Bloom's Taxonomy, discussed in detail under cognitive demand.

Furthermore, Polya's 4-stage model was selected for use as it impacts on problem-solving procedures. In 1945, Polya developed a set of 4 stages which allow students to use various strategies in solving problems (Polya, 1945, p. 253):

- Understand the problem
- Devise a plan

[6] 21st century skills are aimed at fostering specific competencies such as metacognition and reflective thinking, critical thinking, reasoning, and communication skills.

- Carry out the plan
- Check and extend

Polya's 4 stages (1945) are essential in elementary mathematics, and have a particular purpose in the curriculum. Valverde (2002, p. 13) designed the tripartite model of textbook analysis, highlighting a link between the intended and implemented curricula, indicating that the former is compiled by the educational system and national policy, comprising "content standards, curriculum guides, frameworks or other such documents" (Valverde, 2002, p. 9).

Table 3.1: Frameworks supporting mathematics textbook analysis

AUTHORS	FRAMEWORKS	DESCRIPTION OF FRAMEWORK	SIGNIFICANCE
Valverde *et al*. (2002, p.13)	Textbooks	Intended Implemented Attained	Tripartite Model *Intended analysis * Implemented analysis * Attained analysis
Charalambous *et al*. (2010)	Textbook analysis	Structure and content	*Horizontal analysis *Vertical analysis
TIMSS (2011)	Textbooks	Structure, content and performance	3 Dimensions *Structure *Performance expectations *Perspectives
O'Keeffe (2011b, p.5)	Initial development of the framework for mathematics textbook analysis	Structure: Structure and content expectations.	Significance of 2-stage model. *Structural analysis *Expectation analysis
O'Keeffe (2013)	Theoretical frameworks supporting mathematics textbook analysis	Provides overall structure for textbooks, reinforces the TIMSS framework.	Significance of 4-stage model. *Structural analysis *Expectation analysis *Content analysis *Language analysis
Morgan & Henning (2013, p.10)	The three-stage process: from data to literature to a tool and back to data.	Initial reading of data, writing annotations, colour-coding. Establishing semiotic mediation. Identify categories from the literature. Devise the plan. Finalising a design, applying the dimensions. Identifyng and consolidating themes.	Significance of 3-stage model. *Inductive analysis *Devising dimensions from the literature *Using the dimension model to analyse content, chapters

Charalambous *et al*. (2010) conducted a study of mathematics textbooks with the aim of analysing their content and topics at the level of cognitive demand, focusing on books from Cyprus, Ireland, and Taiwan. In order for a structure to impact positively, a number of key issues must be incorporated into the analysis process. Charalambous *et al*. (2010) formulated a framework (Table

3.2) for examining the learning opportunities presented in the content of these textbooks. It consisted of a number of components, these being horizontal and vertical, the former relating to background information and overall structure, such as the number of units, structure of units, topics, their sequencing and layout. The latter related to content, constructs of fractions, different methods of solving problems in the samples, cognitive demands of the assigned tests, and the kinds of answers students were able to present.

Table 3.2: Framework for textbook analysis (Charalambous *et al.*, 2010, p. 117)

<table>
<tr><td colspan="3">HORIZONTAL ANALYSIS OF THE TEXTBOOK</td></tr>
<tr><td colspan="2"><u>Background Information</u>
• Title
• Number of books
• Pages (number, density of pages)
• Profile of authors and advisory committee
• Publisher and year of publication
• Accompanying materials (e.g., teachers' guides and resource materials)</td><td><u>Overall Structure</u>
• Number of units/lessons and average number of pages per unit/lesson
• Structure of units/lessons
• Sequencing of topics
• Topics covered</td></tr>
<tr><td colspan="3">VERTICAL ANALYSIS OF THE TEXTBOOK (FOCUSING ON FRACTIONS)</td></tr>
<tr><td></td><td>What is presented?</td><td>What is expected?</td></tr>
<tr><td>Mathematical content</td><td><u>The content</u>
Different sub-constructs of fractions: part, whole, ratio, operator, quotient, measure.
<u>Ways of presenting the content</u>
Definitions,rules, conventions
Representations</td><td rowspan="3"><u>Potential cognitive demand</u>
• Memorization
• Procedures without connections
• Procedures with connections
• Conducting mathematics
Performance expectations
- answer only
Explanation
Justification
Evaluation</td></tr>
<tr><td>Mathematical practices</td><td>Worked out examples
Modelling thinking</td></tr>
<tr><td>Attitudes</td><td>View of mathematics</td></tr>
</table>

For Charalambous *et al.* (2010), the findings emphasized the need to examine textbooks in order to understand differences in instruction and achievement across countries.

According to O'Keefe (2013), textbooks are widely accepted as a common feature of classrooms worldwide; and are important vehicles for promoting the curricula. In her research on mathematics textbooks, O'Keeffe established a framework for textbook analysis based on the work of the TIMSS framework, which was adapted for mathematics curriculum analysis as a tool. The TIMSS framework for mathematics curriculum analysis is the main tool for textbook analysis, according to Martin, Mullis, Foy and Stanco (2017). It has three dimensions, namely, structure, performance expectations, and perspectives.

Morgan and Henning (2013) produced a five-dimensional tool for history textbook analysis that conceptualized and developed a three-stage process. The first stage consisted of initial reading content, writing annotations, colour-coding and themes taken from data that may be used to support the mathematics textbook analysis. The second stage entailed concepts identifying dimensions for the framework. Thereafter, the findings from this colour-coding process were combined with principles of text analysis, as derived from the literature, specifically focusing on the notion of semiotic mediation, as theorized by Vygotsky (1934).

McLeod (2018) purports that the work of Vygotsky (1934) has become the groundwork of much research and theory in cognitive development over the past several decades, these theoretical frameworks further supporting mathematics textbook analysis. Gauvain (2019) further offers that Vygotsky's theory stresses the fundamental role of social interaction in the development of cognition, essential in mathematics education (Vygotsky, 1978). Cognitive development through cooperative learning is one of the main 21st century skills required in mathematics. Lingard, Martino, Rezai-Rashti and Sellar (2015) contend that policymakers in many nations are involved in educational reforms in order to make effective educational decisions for the 21st century. Son and Diletti (2017) studied curriculum reforms by conducting an international comparative study that analysed learning opportunities presented in mathematics textbooks in the USA, and five high-achieving Asian education systems.

This study focused on specific aspects based on the work of Valverde *et al*. (2002), Charalambous *et al*. (2010), O'Keeffe (2011b), O'Keeffe (2013), and Morgan and Henning (2013), around textbook frameworks. However, for the purpose of this study, some aspects of Charalambous *et al.*'s (2010) framework were reinforced and new dimensions added to address problem-solving, specifically. New dimensions were in the form of Polya's 4-stage model and eight (8) heuristics, emphasizing particular importance to support educational reform.

3.3 Digital versus traditional textbooks

A study conducted by Joo, Park and Shin (2017) indicated that students preferred to use digital rather than traditional (printed) textbooks, although the comprehension results reflected an ideal outcome when reading traditional books. Evidence shows that students learn more effectively from traditional textbooks than from digital versions (Alexander & Trakhman, 2018). Therefore, textbook analysis, according to Powell, Driver, Roberts and Fall (2017), aims to improve the quality of textbooks. Nevertheless, printed textbooks are becoming increasingly obsolete in classrooms because digital materials are easily accessible. Digital material appears to focus on differentiated learning (different students in a class taught at different levels) and has discouraged the notion of 'one resource for all'. As with digital material, Lenon and Cleves (2015) suggest that textbooks, irrespective of their format, need to cater for diversity in the classroom, accommodating different learning styles to maximize learning and understanding in the classroom. Gracin (2018) announces that when analysing the level of demand exhibited by tasks in the textbooks, content, contextual, cognitive demands, and real-world applications must be taken into consideration.

Some countries are going through a digital shift, resulting in schools transitioning to digital textbooks (e-textbooks), incorporating 21st century skills, according to Palmer (2018). Grajek (2013) states that students have also become advanced in the use of technology and have easy access to information using their smartphones and tablets. Students' study practices apropos of textbooks usage are changing; and they are being guided and supported in the use of both digital learning resources and books. Fadel and Rajab (2017) suggest the didactic value behind these changes is to develop students with important 21st century skills, creativity, and critical thinking, with innovative abilities.

Morris, Lambe, Ciccone and Swinnerton (2016) provided evidence for anatomy and physiology educators showing that the textbooks can enrich students' awareness of their learning and satisfaction in the classroom when integrating mobile learning opportunities, with minimal support and training. Such opportunities further benefit students in terms of their engagement, convenience, attainment, and enjoyment. In 2012, the Communications Commission Chairman of the USA, Genachowski, stated that schools should change to digital textbooks within five years in order to develop interactive education. Such would be cost effective over a period of time compared with textbooks, and would guarantee the use of current and relevant content in the classrooms (Usdan & Gottheimer, 2012).

According to Botha (2018), the utilization of e-textbooks by students would prove advantageous, as it would assist in learning additional material timeously, and the effortless accessibility and referencing would aid in the enrichment of classroom activities, which could assist in improving the results of standardized testing. In contrast, this could result in health problems such as handheld contributed syndrome, which causes eye strain, headaches, carpal tunnel syndrome, neck and shoulder pains, and fibromyalgia (Morris, Lambe, Ciccone and Swinnerton, 2016). The use of e-textbooks would be more expensive than using printed textbooks in the short-term. People who read printed textbooks, according to Morris, Lambe, Ciccone and Swinnerton (2016), can comprehend more and learn more than those who read digital texts. In conclusion, (Van Laar, Van Deursen, Van Dijk and De Haan, 2017; Romel and Brucal, 2018) discuss the importance of 21st century skills which include the use of technology in the teaching and learning process in the classroom. It is likely that the central role of textbooks in the classroom will remain just that, despite the emphasis on e-learning and e-learning objects and curriculum initiates, such as those currently underway in South Korea (Turban, Outland, King, Lee, Liang & Turban, 2018). According to Botha (2018), digital resources can be used as complementary approaches to teaching and learning in South Africa. Robb (2019) opines that textbooks and technology are interactive and collaborative resources that can be integrated to present content in a less stagnant and innovative way, to meet the learning styles of students. Digital resources, such as graphics, can literally come alive, which can be extremely useful, although it is important to keep animations simple so that they do not become a distraction to students. This study intends to cater for the development of 21st century skills. Such skills would include students being given the opportunity of researching information in order to advance their problem-solving skills.

3.4 Improving the structure and content in textbooks

Textbooks that are used by teachers provide a link between practice and theory, stimulating a reflexive approach and the critical understanding of methodologies, while assisting in creating new contexts (Oliveira, 2017). These qualities portray the distinctiveness of textbooks as programmatic curriculum aids compared with other instructional media, which have their own notable characteristics. Printed textbooks are durable, easy to carry around, and are useful when electricity or electronic devices are absent (Sunday, 2014). Shannon (2010) states that textbooks are commodities, political objects, and cultural representations that help standardize instruction and provide structure, while attempting to maintain quality education and provide a learning resource or a syllabus for a programme. Alternatively, Garinger (2002) contends that textbooks provide a framework for students, and should therefore provide organized, objective-based instruction. Textbooks have been used as a vehicle to deliver curriculum, and to provide activities, homework tasks, and assessments in schools for many years.

Various studies have considered ways of improving the quality of textbooks, having focused on various aspects of content analysis. These include topic coverage, content distribution, the page space assigned to every topic, presentation, and visual representation (Törnroos, 2005; Delil, 2006; Grishchenko, 2009; Li & Even, 2011; Boonen, Reed, Schoonenboom & Jolles, 2016). However, there has been little research interest that aimed to analyse the content representation in textbooks (Fan, 2004). Stanic and Kilpatrick (1989) contend that mathematical problem-solving procedures are fundamental to learning and teaching at any given moment. Such procedures stir the students' interest in certain facets of the content, as well as the different techniques of presentation for processing information. Current research has changed little in terms of what students study compared with three decades ago. Various authors have analysed similar aspects in textbooks, irrespective of the timeline, in order to improve quality and to advance teaching and learning (Stanic & Kilpatrick, 1989; Törnroos, 2005; Delil, 2006 & Grishchenko, 2009; Li & Even, 2011).

3.4.1 The influence of textbooks on the teaching and learning environment

The accessibility of textbooks appears to be the single most encouraging factor in envisaging educational success (Chang & Silalahi, 2017), as textbooks enable teachers to follow curriculum content and provide structure and guidance. The instructional materials do not stand alone − they are cautiously incorporated into other policy and practice elements, either at the district or institution level. This offers the necessary guidance for teachers to fulfil the goals set for their teaching practice (Weinberg & Wiesner, 2011; Fan, 2013; Wijaya, van den Heuvel Panhuizen & Doorman, 2015; Gravemeijer, Stephan, Julie, Lin & Ohtani, 2017).

According to Reys, Reys and Chavez (2004), textbooks have a direct effect on tutors' choice of teaching methods and the content to be taught. The literature indicates that textbooks have a considerable influence on the level of learning opportunities, owing to their varying content and focus (Haggarty & Pepin, 2002; Törnroos, 2005). Hirsch et al. (2005) and Alajmi (2009) allow that textbooks influence the basis of not only what is being taught by teachers, but also homework activities that are assigned to students. Various studies, including those of Törnroos (2005) and Xin (2007), have strongly associated students' mathematical performance with the textbooks they have used.

Researchers have been in agreement for five decades that the instructional material provided in textbooks is essential for implementing the curriculum content (McKnight et al., 1987; Glidden, 1991; Robitaille & Travers, 1992; Tyson, 1997; McKnight, & Raizen, 1997). More recently, Gurney-Read (2016) stated that, according to the TIMSS, it is contended that the importance of instructional material endures. The researcher added that there were differences in the performance of USA Grade 8 students across varying topic areas that were associated with the kind of instructional materials used. The results showed that students performed better in sections that were covered by the content in the textbook, and that the materials that are developmental from grade to grade aid in the building of conceptual understanding.

One of the most major findings of the TIMSS (2012) is that students perform better when the content in the curriculum is taught using textbooks (Schmidt, Houang, & Cogan, 2012), whereas they performed poorly in the topics that were not taught with the assistance of such material. This suggests that hard-copy learning materials are essential in the mathematics classroom (National Research Council, 2002; Van Zoest & Bohl, 2002; Reys, Reys, Lapan, Holliday & Wasman, 2003). Instructional materials are concrete, and are used on a daily basis by both teachers and students, dissimilar to frameworks, objectives, and assessments (Ball & Cohen, 1996).

Based on the works by Törnroos (2005), Kajander and Lovric (2009), textbook content is regarded as extremely useful in the classroom setting, and provides the pillar of mathematics teaching. In many countries, textbooks are the basic teaching resource that promotes teacher instruction as well as conceptual understanding amongst students. The results of the TIMSS indicated that teachers choose their teaching methods depending on the mathematics textbook (Mullis, Martin, Foy &

Arora, 2012). In the same way, the textbooks used in mathematics education dictate which topics are addressed and in what manner. According to Stein, Remillard and Smith (2007), there are two important points regarding content; these being firstly, that the topic representation is a given text, and secondly, there is a particular manner in which problem-solving procedures are covered in textbooks. Such, together, initiate the teaching techniques and present a fitting platform for learning to occur.

Students can learn in a range of stimulating environments by using concrete objects, graphs, illustrations and tables. Learning may also occur outside textbook or classroom settings, and while such methods are not the sole learning resource available, they are of considerable importance in the learning environment, serving as teaching materials for the tutors. Students use textbooks for personal revision and to prepare for assessments. In contrast with many other innovations, textbook content is used in the classrooms. Such content is part of the routine of schools and has a significant impact on the education system (Behnke, 2018). Textbooks need to be interesting and easy for students to understand, otherwise they may become a hindrance to effective learning. Kim and Jang (2015) warned against separating the improvement of instructional materials from the overall improvement of the instruction process. School officials can examine the role of instructional materials and how they assist in improving mathematics instruction as a way of improving the understanding of mathematics content in the classroom (Weisel *et al.*, 2017).

Kochhar (1985) contends that students will benefit considerably from the selection of textbooks of a high standard. The researcher defines the criterion of a well-graded textbook as one that can be suitable for the students at their academic level and age. There can be simple, straightforward, and rationally outlined facts in each activity, which must cater for child-centred education. In the same vein, Aggarwal (2009) suggested the need for guidelines that could be used to test the relevance and adequacy of textbooks. Aggarwal (2009) discussed the selection, organisation, presentation of content, verbal language and visual communication, which entail the use of illustrations as vital themes for analysing a textbook. While sixteen years separated the two authors, they both focus on improving the quality of textbooks to maximize the chances of effective teaching and learning in the classroom. This study intends to improve the quality of South African textbooks by emulating and implementing the procedures that strengthen the South Korean textbooks.

3.4.2 Features related to the selection of content in textbooks

Almost two decades ago, Aggarwal (2009) identified specific features that related to the selection of textbook content in the classroom that is relevant, up-to-date, integrated, applicable to life, and adequately covers each topic linked to the curriculum. Son and Diletti (2017) focused on topic placement, which describes the section of the book in which a particular text appears. Comparative studies around the world have examined the learning opportunities available in mathematics textbooks used in USA schools, as well as in the high-performing education system of Asia. These comparative study findings suggest that students in the various countries are offered different mathematics and given diverse opportunities to learn mathematics, both of which are influenced by textbook and teacher (Boonlerts & Inprasitha, 2013; Choi & Park, 2013; Kang & Lee, 2014; Hong & Choi, 2018).

Maertens, Aggarwal, Moreels, Vermassen and Van Herzeele (2017) posit that relevance of content, integration of real-life experiences, and the link to the curriculum is essential. Content that is relevant to students has a compelling effect on their learning, and encourages enthusiasm on the student's part. When a country adopts textbooks from other countries, they need to consider the relevance of the content and expressions[7], such as the units of measurement and currency, which may differ. In a recent article which described how British schools used Chinese textbooks to improve their mathematics scores (Qin, 2017), the UK changed the content to suit their own country's currency and standards of measurements. Lenon and Cleves (2015) and McCabe and Whittaker (2017) contend that textbooks taken from countries where students perform well in mathematics, and adopted by countries where students are underperforming, produce significant improvement in students' results.

According to Wlodkowski and Ginsberg (2017), stories reinforce understanding of the problem-solving procedures taught in mathematics textbooks. Stories play a dynamic role in the growth and development of students. An author's purpose may be to inform the reader, to relate a story, or to recall events. Reading and sharing stories can aid students to learn sounds, words, and mathematical language, develop early literacy skills, and to learn the difference between 'real' and

[7]Expressions are a finite combination of symbols that are well-formed according to rules that depend on the context.

‘make-believe’. Research by Cavanaugh, Clemence, Teale, Rule and Montgomery (2017) reveal that students whose teachers employ stories and games in mathematics, vocabulary, comprehension, sequencing, memory development, and creative writing, all benefit from such storytelling. It is possible that such ploys improve listening skills which are essential to learning and understanding problem-solving procedures.

The interaction between students is a crucial part of the integration of cultural components into mathematical activities. The socio-cultural integration in textbooks includes a relationship between the teacher, student, and textbook, based on social interaction. Malegiannaki and Daradoumis (2017) observed that, when textbooks represented familiar socio-cultural stories, visuals and problem-solving procedures, the learning environment appeared more organised, and students became more involved in the learning material.

Presented appropriately, games can engage students’ attention and actively occupy them with mathematics concepts in a fun and non-threatening way. According to Tsekleves, Cosmas and Aggoun (2016), most games require problem-solving strategies. Planning and applying a range of strategies in a game, allows students to use their working memory to solve problems. Such aids in developing their mental reasoning.

The results of researchers van den Ham and Heinze (2018) on multilevel regression analyses revealed that mathematics textbook choice has a considerable effect on the students’ mathematics achievement; and that specific textbooks vary significantly in their effects. Furthermore, there are indications that the effect of textbook choice can be cumulative over the school years. Therefore, it would be wise to consider factors that could negatively impact a student’s ability to learn before realising that content can influence teaching and learning in either a positive or negative way. One of the objectives of this study is to identify the strengths and the weaknesses of the structure and problem-solving procedures of the chosen textbooks.

3.4.3 Application of content to the real world

Research on textbook analysis has also focused on the textbook, and how it relates to applications in the real-world[8] context. It was found that the Taiwanese teaching environment used mainly non-application problems in a textbook. These problems were not aligned with real-world contexts (Yang, Tseng & Wang, 2017). In a comparative study, which included Taiwan, with regard to the number of non-application problems, their textbooks scored the highest, with 96.7%, while books in the USA, Singapore and Finland gained 83.0%, 86.2%, and 79.4%, respectively (Yang, Tseng & Wang, 2017). The percentages were much lower for application problems, with Finland scoring 20.6%, Taiwan 3.3%, USA 17%, and Singapore 13.8%. Comparing the four series of textbooks, Taiwan had the lowest score in application problems, while Finland scored the highest (Yang, Tseng & Wang, 2017). World-related contexts contribute to mathematics, which is relevant to learning processes and to the lives of the students. This is owing to the research on textbooks which has advanced considerably over the past years (Kopka, 2010; Wijaya et al., 2015; Brehmer, Ryve & Van Steenbrugge, 2016; Yang, Tseng & Wang, 2017). Pictures which are presented as real-life images illustrate the relationship between the problems and how they relate to daily life (Root, Browder, Saunders & Lo, 2017). The content in South Korean and South African textbooks used in this study is linked to real-life contexts, which makes the learning process relevant.

3.5 Conceptual understanding of problem-solving

Conceptual understanding refers to a combined and purposeful appreciation of mathematical ideas. Students with conceptual understanding are acquainted with more than isolated facts and methods. They can identify the usefulness of mathematical ideas and their significance in various contexts. Students are able to systematically coordinate their knowledge, which enables them to scaffold new ideas by linking and building on existing information (Groth, 2017).

According to research conducted by Geary, van Marle, Chu, Rouder, Hoard and Nugent (2018), the development of conceptual understanding is vital for a student to solve everyday problems. This can be promoted and developed through the use of activities that advance students' thinking processes. A demonstration of an inquiry-based approach to learning would entail using manipulatives (concrete objects) to engage students with questions having unknown answers.

[8] Problems using real-life examples, e. g., teaching fractions using a pizza, something students can relate to.

Greene and Shorter (2017) explain that students are further challenged by opportunities to contribute to their own and others' understanding. This requires ending the urge to act hastily. Teachers have to slow down to cover content, becoming more creative on ways in which to focus on conceptual understandings. This also requires designing ways in which to demonstrate the evidence of applying these conceptual understandings. Extensive learning requires time and effort, which will aid in improving mathematics results (Bergsten, Engelbrecht & Kagesten, 2017).

Another complex task that explores the advantages of multiple representations, is estimation (Yuan, Prather, Mix & Smith, 2019). Estimation is often employed in textbooks, such as the AMS Foundation Activities; this begins by nurturing conceptual understanding, then progressing to procedural fluency. Encouraging students to employ multiple representations is highly likely to result in more open-ended problems, or, at the very least, students can embrace various problem-solving techniques. It is reasonable to point out that conceptual understanding and multiple representations cannot be isolated from one another.

Over the past decades, word problem-solving in mathematics has drawn the interest of researchers and educational practitioners (Campbell, 1992; Hegarty, Mayer & Monk, 1995; Hajer, 1996; Depaepe, Corte & Verschaffel, 2010; Moreno, Ozogul & Reisslein, 2011; Boonen, van der Schoot, van Wesel, de Vries & Jolles, 2013; Hickendorff, 2013; Swanson, Lussler & Orosco, 2013). Word problems comprise mathematical exercises in which the finer details are provided in the form of text. To successfully address these kinds of problems, the student must first understand the textual explanation; thereafter having the ability to perform the relevant mathematical operations (Lewis & Mayer, 1987; Hegarty, Mayer & Monk, 1995; Van der Schoot *et al*., 2009; Jitendra and Star, 2012; Highverdi & Wiest, 2016; Lewis, 2016). These two aspects are interconnected, meaning that students can only perform correct mathematical computations to solve the word problem if they first develop a deep understanding of the question. Unfortunately, comprehension may be a significant challenge when individuals are faced with language barriers (Lee, Ng & Ng, 2009; Thevenot, 2010; Boonen, van der Schoot, van Wesel, de Vries & Jolles, 2013; Vaughn, Martinez, Wanzek, Roberts, Swanson & Fall, 2017).

The use of Bloom's taxonomy questioning techniques can be beneficial in the development of students' conceptual understanding of problems. Students can grow from solving simple problems with a single step to solving complex problems; such involves multiple steps, presenting more of a challenge. However, it is just as important that textbook authors and reviewers actively apply conceptual and cognitive level guidelines in textbook development, a gap that this study intends to address.

3.5.1 The level of cognitive demands in textbooks

Cognitive demand is a major issue of concern when addressing problems or activities in the mathematics classroom. Cognitive demand indicates the importance of examining the level of mental capability required to solve mathematical problems in textbooks (Tanujaya, Prahmana & Mumu, 2017). In the learning environment, the level of cognitive demand needs to be considered for every task that is selected, to ensure that it offers the most suitable level of challenge for a diverse group of students. Almost three decades ago, Stein and Lane (1996) completed a project titled Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR). Their findings associated improved levels of performance in mathematics tests with two interconnected factors: the way the assessments are formulated, and their ability to involve the students in significant levels of cognitive thinking. Further research into the measure of cognitive demand arose from mathematics problems in the QUASAR project. The research indicates that students are constantly seeking opportunities to undertake activities that provide them with a greater understanding of concepts, relationships, and processes in mathematics (Stein, Smith, Henningsen & Silver, 2000).

Cognitive skills classifications emerged from the landmark 1956 publication by Bloom, being named the Taxonomy of Educational Objectives, the Classification of Educational Goals. A combination of ideas borrowed from this work assisted in developing Bloom’s taxonomy for categorizing cognitive skills. Bloom (1956) created the taxonomy in order to promote higher forms of thinking in learning and education. Using this taxonomy, cognitive skills can be classified into six groups. Such can be described as either lower-order or higher-order. Higher-order skills involve extensive learning with a higher degree of cognitive processing, while lower-order skills involve less cognitive processing. The categories range from the lowest knowledge, comprehension, application, analysis, and synthesis, to the highest, which is evaluation. It is, however, important to note that the term ‘higher-order thinking’ was not initially used by Bloom to describe the categories. This term was adopted over time (Adams, 2015). Hall (2018) presents a new version of Bloom's taxonomy comprising knowledge, comprehension, application, analysis, synthesis, evaluation, and (recently added) creativity.

Further research on cognitive science had been conducted after the original publication in 1956, leading to the rearrangement of the original classification of the cognitive processes. The categories, according to the later mode, were remember, understand, apply, analyse, evaluate, and create (Adam, 2015). The reorganized model placed synthesis at the top of the hierarchy in place of evaluation; and included an additional element for each of the six cognitive processes. It identified the four kinds of knowledge that are crucial during a given task. Rowley and Hartley (2017) explain firstly, factual knowledge, which would include terminology and discrete facts; and secondly, conceptual knowledge, consisting of principles, categories, models and theories. Thirdly is procedural knowledge, involving knowledge of methodologies or processes; and fourthly, metacognitive knowledge, which includes the knowledge of various learning methods, skills, and the ability to conduct self-assessments. The taxonomy of objectives has been incorporated into the production of many textbooks, which makes them beneficial in the classroom (Morton & Colbert-Getz, 2017).

Higher-order thinking (HOT) describes the kind of thinking that is beyond mere memorization of facts (Adams, 2015). HOT goes beyond what the students are familiar with, encouraging them to develop their higher-order thinking skills. This includes understanding the facts, making inferences from them, linking them to other relevant concepts, classifying them, manipulating them, and using innovative ways to combine and apply them in finding new methods of problem-solving (Thomas & Thorne, 2009). Apino and Retnawati (2017) place definitions of HOT into three categories, these being the transfer of knowledge, developing critical thinking, and problem-solving (Collins, 2014; Apino & Retnawati, 2017). Morton and Colbert-Getz (2017) aver that the use of multiple representations involves tasks that include decision-making. Such would include selecting the right form of representation, conducting the representations, and knowing the effect of one representation over the others.

Stein and Lane (1996) and Stein, Stein, Smith and Henningsen (2000) describe the four categories of cognitive demand. The first demand is memorization of content put together. The second demand refers to procedures without connections, which are classified as low cognitive-demand levels. The third category is procedures with connections; and the fourth group is studying of mathematics, classified as applying high cognitive-demand levels. Procedures without connections imply that the students do not connect the procedural or algorithmic knowledge they have to the relevant meanings and concepts. In contrast, procedures with connections indicate that students are expected to link their procedural and algorithmic know-how to the appropriate meanings and concepts (Tanujaya, Prahmana & Mumu, 2017). Lastly, studying mathematics indicates the students' ability to address mathematical problems using complex, non-algorithmic kinds of reasoning (Mhlolo, 2017). Textbooks may provide and promote the development of students' respective knowledge and skills, assisting them to reach the next level. According to Özer and Sezer (2014), textbooks are a valuable resource for those who seek to learn or to teach, in addition to being a good indicator for assessing learning opportunities. Students have to be provided with challenging activities in order to develop 21^{st} century skills. The intention of this study is to identify the extent to which the South Korean and the South African textbooks address the principles required to develop Grade 6 cognitive demands and critical thinking.

3.5.2 The importance of problem-solving in textbooks

It is acknowledged that problem-solving is the foundation for excellence when studying mathematics; and answering a range of chosen problems, which help to build, polish, and nurture creativity (Mhlolo, 2017). Globally, various studies (comparative or otherwise) have sought out ways of improving mathematical comprehension. This research includes the TIMSS and PISA. Arslan & Yazgan (2015) investigated the ability of students in Grades 6, 7 and 8 to demonstrate strategy flexibility when tackling non-routine problems. The researchers found that most students were able to identify the underlying pattern that yields the final solution in each problem. Furthermore, students tended to use various forms of visualization in their draft work; however, they occasionally made computational mistakes. The analysis section of this study focuses on the extent to which the textbooks allow students flexibility in the problem-solving procedures. Furthermore, mention is made of how various illustrations and concrete objects aid in problem-solving.

Conceptual understanding and communication, procedural fluency, strategic competence, and adaptive reasoning of mathematical content are important goals to be achieved in problem-solving in all grades. The researcher therefore decided to focus on these problem-solving procedures (Groth, 2017):

a. choosing and using the most suitable technique
b. adopting new techniques when the selected one fails to solve the problem
c. solving the problem using several methods
d. adopting new techniques to solve problems.

The acquisition of adaptive expertise when undertaking curricular subjects (explained by criteria iii and iv) is essential in mathematics-education-related psychology. Merriam-Webster (2017) defines communication as a method by which information is exchanged between persons which can support the growth of a wide range of cognitive skills. It is an exchange of thoughts which involves both listening and reading (comprehension) and both speaking and writing (expression). Somewhat exclusive to math, expression may also include the demonstration of mathematical ideas in non-linguistic ways. Strategies that aid students to communicate mathematical thinking efficiently would include both verbal, with the use of representations, and written work. Successful literacy strategies that can be used in the mathematics class would include word walls, modelling, shared writing, and revision.

Communication is a crucial part of conceptual understanding, as mathematics education involves sharing ideas and clarifying understanding. Cash (2017) discusses strategies for interactive learning. The researcher reminds that communication is an important concept, through which ideas become organized views. In deciding how to express those thoughts in words, students learn to think more deeply, to assess their own understanding, make associations, determine significance, and relate ideas. “The communication process also helps build meaning and permanence for ideas and makes them public” (National Council of Teachers of Mathematics [NCTM], 2000, p. 60).

Procedural fluency refers to knowledge of procedures, knowledge of when and how to use such appropriately, and skill in performing these procedures flexibly, correctly, and efficiently. Star and Rittle-Johnson (2009) provided a wider definition of flexibility, and described it as understanding a range of techniques in conjunction with their relative efficiency. This definition implies that the first step to achieving flexibility is being knowledgeable about multiple methods of problem-solving, with flexible students knowing more than one way of completing any given task. Secondly, the students should be aware of the efficiency level of the various techniques, and be able to identify the most efficient methods under certain circumstances. In the same way, Selter (2009) describes flexibility as the ability to move from one strategy to the other. Flexibility relates to communicating, reasoning, connecting, and presenting of mathematical content in various ways.

Aside from the aforementioned definitions, flexibility requires certain capabilities, such as the student's ability to apply several strategies in interpreting data, to be effective in problem-solving (Yin, Li, Gao, Lu & Zhang, 2017). Cottrell (2017) discusses students' ability to identify a variety of alternative interpretations of a given problem, adapting accordingly when the dynamics of the problem change; and modifying representations, switching from one method to another when required. Students' flexibility is portrayed when they are able to modify and adapt representations to suit a certain image, whether concrete/abstract, functional/structural, or principle-/surface-oriented.

Most flexibility research projects focus on specific issues of interest, for example, algebra or arithmetic. Consider the research conducted by Heinze, Marschick and Lipowsky (2009) on various groups of students in the third grade. The study addressed the adaptive application of problem-solving techniques. This was relative to the instructional methods shown in the textbooks, specifically, subtracting and adding three-digit numbers. Star and Newton (2013) examined flexibility as exhibited by experts when solving equations. The findings of the study indicate that prior knowledge is a reliable predictor of flexibility gains; and that gender is an important student background characteristic associated with the development of flexibility. In an earlier report, Star and Rittle-Johnson (2009) provided the findings of two intervention studies concerning the issues underlying flexibility levels when using different estimation methods. The report emphasized the need to encourage instructional practices that enhanced procedural fluency and increased students' understanding of concepts. The research report of Star and Rittle-Johnson (2009) clearly points out that adequate time, collaborative opportunities, and other resources made available to students will aid in this process.

Strategic competence refers to the ability to articulate mathematical problems, to represent, and to solve them (Goth, 2017; Sabilah & Siswono, 2018). This strand is similar to what has been called problem-solving and problem formulation, in the literature of mathematics education and cognitive science, and mathematical problem-solving, which has been studied at length. Research conducted by Liljedahl, Santos-Trigo, Malaspina and Bruder (2016) suggest connections which require students to study their solutions, reflecting on them as an interesting way, according to Polya, of concluding the stages of problem-solving. This means that the mathematically relevant observations that students make about their problem-solving solutions should be used to make connections to current and prior knowledge, as well as real-life contexts. This aids in the student discovering new learning; being able to relate the solution mathematically to one's own world. By means of this process, students are enabled to validate their deductions, communicating them to others, and reacting to the influences of others. Mathematically gifted students look closely to recognise a pattern or structure, and are able to create a model to present their results. Polya's 4-stage model is an enhancement of the principles of problem-solving by design. It not only makes clear the focus on past experiences and prior knowledge, but also presents these ideas in a very concise, logical, and teachable manner. This heuristic has become a popular, if not the most popular, instrument by which problem-solving is taught and learnt.

Schön (2017) declares that the development of problem-solving skills provides a person with the ability not only to address problems that they are familiar with, but to apply these skills in problems that they have not encountered. Research conducted by Tanujaya, Prahmana and Mumu (2017) reveals that using memorization methods to solve problems will enable a student to solve the initial steps of problems that they are presented with. However, such memorization will not enable students to adapt and solve problems that they have not previously encountered. Students require problem-solving skills and critical thinking to equip them to deal with life's challenges. Critical thinking is a learnt skill that requires instruction and practise. Asian mathematics teachers place great emphasis on thinking, and on understanding mathematical thinking (MT) which is an important feature of Chinese mathematics textbooks. Therefore, the Chinese students' exceptional performance in international assessment, such as the TIMSS, arises from the emphasis placed on MT (Li, Mok & Cao, 2019).

Polya (1945) devised a methodical procedure for solving problems that is now referred to by his name: the Polya 4-stage problem-solving process, which was used in the current study. This is relevant: this problem-solving process assists students improve an action plan for addressing problems that consist of details, main ideas, planning, and using of a method. As students work through each step, they may use ‘illustrations’ to consolidate their thoughts, provide evidence of their mathematical thinking, and show their plan for arriving at an answer. A problem-solving strategy is a plan of action used to find a solution, with diverse strategies having various action plans associated with them. For example, a familiar approach is trial and error, which allows students to persist in trying various solutions until they solve the problem.

Elia, Heuvel-Panhuizen and Kolovou’s (2009) study is one of the most outstanding works on problem-solving, having explored the methods applied by high-performing students in solving non-routine problems. The researchers classified strategy flexibility into intra-task and inter-task flexibility, the latter referring to the use of different techniques across problem-solving procedures. The former refers to strategy changes within problem-solving procedures, both types of flexibility being minimally represented by the selected sample. Additionally, the study results showed that intra-task strategy flexibility is of no use to students who seek the right answer to questions. On the other hand, students who score highly on inter-task strategy flexibility perform better than those who face challenges in this area.

A research project undertaken by Zhang and Mi (2010) using four non-routine problems sought to determine whether each student exhibits consistent performance across all subject areas. The study also aimed to identify the factors that affect students’ choice of techniques in diverse contexts, with intra-task flexibility being shown to have no significant impact on the attainment of correct answers to problems. This form of flexibility was further associated with an individual’s preference for certain methods and their confidence level. Moreover, the research exposed inconsistencies in the mathematics problem-solving behaviours of the same student in various subject areas.

Alajmi (2012) contends that textbooks can focus on mathematical contexts when presenting problems with an emphasis on real-world issues, such as those pertaining to fractions. When the students are unfamiliar with problems based on real-life scenarios, finding solutions becomes increasingly difficult (Wijaya, Van den Heuvel-Panhuizen & Doorman, 2015). Therefore, relying on real-world situations in the classroom provides significant benefits to the students and increases their understanding in the subject area (Gu, Huang, & Marton, 2004). Improved conceptual understanding amongst students can also be associated with the increased use of visual representations in mathematics problems (Fan & Zhu, 2007; Xin, 2007; Chang, Chang, Chiu, Liu, Chiang, Wen & Wu, 2017).

More recently, a study by Mahendra, Slamet and Budiyono (2017) showed that the problems given to a group of students could only be solved with proper conceptual understanding of the mathematical concepts and appropriate procedural knowledge of the proper algorithm. The research results indicated that each of the participating students applied at least three techniques in problem-solving. Of all ten methods that could be used in solving the eight non-routine problems, nine were employed at least once. The methods that were most frequently applied by the students were logical reasoning, systematic lists, diagram or a model, and looking for patterns, with optimal performers displaying expertise in the application of solution strategies.

Skills in reading, comprehension, and mental representation are essential for finding solutions to mathematical word problems. In the Realistic Math Education (RME) study, students first acquire representational skills that pertain to solving word problems. Those who undertake the curriculum facing challenges in addressing semantically complex word problems (Boonen et al., 2016; Lestari & Surya, 2017) showed that the use of textbooks impact on the students' ability or inability to solve certain types of problem. The representation of the contents in the textbook would either strengthen or weaken their ability in specific problem-solving skills.

It is important to explore ways of improving mathematics results through the representation of problem-solving in textbooks (OECD, 2010; Mullis *et al.*, 2012). Present theories of student's problem-solving behaviour consist of at least two elements (Boaler, Chen, Williams & Cordero, 2016; Cañas, Reiska & Möllits, 2017; Simpol, Shahrill, Li & Prahmana, 2017), the first being to identify and represent the problem. A problem may be represented as a concrete object, an abstract concept, or be formulated in only words, illustrations, or a combination of words and numbers. The second component involves choosing a suitable action plan to solve the problem elements (Boaler, Chen, Williams & Cordero, 2016; Cañas, Reiska & Möllits, 2017; Simpol, Shahrill, Li &

Prahmana, 2017). The studies addressed both components of problem-solving procedures in textbooks and were able to achieve both these components successfully. According to research conducted by Skovsmose (2020), problem-solving strategies that are represented as active, relevant, and at the level of the students, will create enthusiasm, encouraging students to collaborate and develop their own problem-solving strategies.

According to Groth (2017), adaptive thinking refers to the ability to think rationally about the associations amongst ideas and situations. Such reasoning originates from cautious reflection of alternatives, and embraces knowledge of how to substantiate the deductions. In mathematics, adaptive thinking is the link that connects everything together and promotes learning. Furthermore, it is used to connect facts, procedures, and concepts in finding a solution.

Research conducted by Cottrell (2017) suggests that reasoning is critical in learning mathematics. Students who sincerely make sense of mathematical concepts can apply them in problem-solving, and unfamiliar circumstances, using them as a foundation for prospective learning. Topic introductions can influence reasoning and make up the basis of mathematical proficiency and skill. The lack thereof leads to failure and disconnection in mathematics instruction. Thus, developing students' abilities with cognitive demand and intellect should be the ultimate goal of mathematics instruction. In order to achieve this goal, all mathematics classes should provide interesting, captivating introductions, and continuing opportunities for students to implement these procedures. Schneider, Beeres, Coban, Merz, Susan Schmidt, Stricker and De Smedt (2017) explain that cognitive demand is the process of manipulating and analysing objects, representations, diagrams, symbols, or statements to draw deductions based on evidence or assumptions. According to Pfadenhauer, Gerhardus, Mozygemba, Lysdahl, Booth, Hofmann and Rehfuess (2017), making sense is a method of understanding theories and concepts in order to accurately identify, describe, explain, and apply them. The analysis section of this study aims to identify the extent to which problem-solving procedures allow students to apply their understanding of theories and concepts.

3.5.3 The representation of problem-solving tasks in textbooks

The representation of problem-solving in mathematics education has been the main focus amongst researchers for many decades: it is important to know how students use the information in textbook to solve real-life problems. Brehmer, Ryve and Van Steenbrugge (2016) describe problem-solving as dealing with all mathematical tasks that pose an intellectual challenge to students, enhancing their overall understanding and growth in mathematics. In addition, the students are expected to explore the problems provided, and answer reasonably. Tanujaya, Prahmana and Mumu (2017) further indicated that problem-solving knowledge, and skills acquired in the mathematical field, enable students to deal with real-life situations. Students with suitable skills unearth underlying patterns to find solutions to different kinds of problems. Such students learn how to think critically, which enables them to interpret and analyse information that they are presented with, finding a solution (Nurdyansyah, Siti & Bachtiar; Cai & Jiang, 2017).

The Surya and Syahputra (2017) study aimed to improve the ability of high-level thinking by developing learning models based on problems faced by senior high school students. This study comprised teaching materials in the form of textbooks for students and a guide book for teachers that included the structured steps of solving mathematical problems based on problem solving which can develop high-level thinking. The outcome showed a significant improvement of students' problem solving abilities in three districts/cities in North Sumatra at four schools.

The Suastika (2017) study aimed to produce an open problem solving mathematics learning model to help students develop their creativity which meets the criteria of validity, practicality, and effectiveness. The study led to the development of an open problem solving math learning model that can develop students' creativity (PMT Model).

Research was conducted by Morin, Watson, Hester and Raver (2017) for students with mathematics difficulties (MD) where math word problem-solving was especially challenging. They examine the effects of a problem-solving strategy, bar model drawing, on the mathematical problem-solving skills of students with MD. The results showed that bar modeling drawing is an effective strategy for increasing elementary students' accuracy in solving math word problems and their ability to use cognitive strategies to solve the problems. One can assume that from the abovementioned studies, the use of a problem-solving model is beneficial to students' learning processes. The intention of this study is to implement a problem solving model in South African mathematics textbooks in order to improve results.

The following section discusses the role of mathematical problem-solving in textbooks, and the importance of multi-representation and conceptual understanding of problem-solving in order to improve mathematics results. Cooper, Sidney and Alibali (2018) suggest that students solve problems more accurately when visuals are included. Baek, Wickstrom, Tobias, Miller, Safak, Wessman-Enzinger and Kirwan (2017) further state that the non-textual elements are considered according to accuracy, connectivity, context, simplicity, and aesthetics. Such make up the central aspects that further aid in the problem-solving procedures throughout the textbook.

3.5.4 The representation of Polya's 4-stage model in textbooks

Polya's 4-stage model is used extensively in mathematics textbooks in Asian countries. This influenced my choice in this study. I examined the various components of Polya's 4-stage model and the eight (8) heuristics which impact greatly on problem-solving. Such led me to use the components, namely, understanding the problem, devising a plan, carrying out the plan, and checking and extending such, as my themes. I further added the eight (8) heuristics, namely, act it out, change your point of view, draw a diagram, guess and check, look for a pattern, solve part of the problem, use a model, and work backwards, as further themes to be investigated. Themes were derived from the theoretical and conceptual framework.

The students' ability to demonstrate problem-solving skills, according to Cai and Jiang (2017), refers to the students' ability to utilize learnt techniques with high levels of creativity and flexibility, as shown in textbooks. Problem-solving expertise is evident in an individual's display of flexible knowledge, with mathematics education stressing the need to recognize and encourage flexibility in the students' self-formulated techniques (Arslan & Yazgan, 2015).

In the study by Arslan and Yazgan (2015), students were able to apply two techniques with ease; firstly, being able to look for a pattern in the problem that is presented; and secondly, being able to make a drawing of the problem in order to solve it. The students were only required to draw a few diagrams, as the structural nature of the question made the patterns visible without any additional visual backing. However, the 'simplify the problem' technique proved to be challenging for the students, who demonstrated initiative to use the 'write an equation' technique (Arslan & Yazgan, 2015. p. 1519-1523). This study showed that students respond differently to problem-solving examples, and choose varying strategies. The implication is that textbooks should not be limited in the way problem-solving examples are presented, and the way in which instructions are given.

Various models have been established by researchers to describe the general problem-solving procedures used to address a problem (Dewey, 1933; Polya, 1971; Krutetskii, 1976; Lester, 1978; Bransford & Stein, 1984), with Polya's 4-stage model being the most frequently used method in mathematics instruction (Masingila & Moellwald, 1993; McCoy, 1994; Ng, 2002). The model entails problem understanding; creating a plan; implementing the plan, and counterchecking.

Polya's first principle, which is understand the problem, is sometimes avoided. Such results in students' efforts being frequently obstructed in finding solutions, mainly because they do not fully comprehend the problem. During this step, students can connect current knowledge with prior knowledge. This is followed by Polya's second principle, which is devise a plan, where there are many rational ways of finding solutions. The skill at choosing a suitable approach is best learnt by solving several problems. Choosing a strategy becomes increasingly easy when the unknown is identified, and a familiar problem can be created. Then, Polya's third principle, which is carrying out the plan, is usually easier than devising the plan. In general, care, patience, and the necessary skills are needed. Persisting with the chosen plan is important, and if it continues not to work, it should be discarded, and another chosen. The plan should be carried out by checking each step of the solution and ensuring that the step is correct. Finally, Polya's fourth principle which is looking back, is beneficial in evaluating the completed tasks by reflecting on what has been done, assessing what worked, and modifying what did not. This will allow students to decide on which strategy to use in solving future problems. A primary reason for students having trouble with problem-solving is that there is no single procedure that works all the time. For this reason, they should use Polya's model. Problems can change slightly and cause confusion. This means that problem-solving involves applied knowledge about the specific situation. If the problem or the underlying situation is misunderstood, misconceptions may occur.

3.5.5 The representation of heuristics in textbooks

The word heuristics originates from the Latin word heuristics, which means to *find out* or *discover* (Groner, Groner & Bischof, 2014). Heuristics are an experiential guide, and are mental shortcuts that help with the thinking processes in problem-solving procedures. According to Petherick

(2014), heuristics functions on the basis of trial and error, culminating in a realistic solution, although there is no assurance that the result will be definitive. The heuristics procedures are not the prescribed problem-solving model, but are used as a method of problem-solving where answers are not expected to yield an optimal explanation. Schoenfeld (1979) argued that specific problem-solving heuristics are commonly practised in actual problem-solving. The mathematics syllabuses in Singapore and the USA's National Council of Teachers of Mathematics (NCTM) standards list specific heuristics, and set different recommendations or requirements for classroom instructions at different levels of schooling (Jiang, Hwang & Cai, 2014).

3.6 Multiple representations in textbooks

Multiple representation aids students to communicate using various strategies, reasoning with different concrete objects, connecting problem-solving procedures with real-life visuals, and presenting mathematical content using visuals, sounds or objects. These multiple representations that are found in the textbooks guided the visual and structural analysis process in this study. Multiple representations of problem-solving tasks in textbooks stimulate differentiation in a classroom, thereby catering for students of different levels (Mhlolo, 2017). Students have different learning styles and abilities, with multiple representations being beneficial in maximizing the teaching and learning process in the classroom. The topics in a textbook are represented in various ways, using graphs, pictures, numbers, words, and tables to create lessons that clearly address mathematical ideas. Problem-solving methods need to be represented in a way that fosters understanding by the student (Evans & Swan, 2014; Sleeter & Carmona, 2017).

3.6.1 Content representation in textbooks

Siddiqui (2005) identified four important ways within which topics are represented in an attractive, motivating way, which is creative and interesting. Siddiqui identified four parameters within the presentation of language in the textbook, namely, correct spelling, appropriate vocabulary, correct punctuation, and short/simple sentences. Siddiqui (2005) goes on to say that textbooks must align illustrations, which are also non-textual elements, with the mental ability of the students and curriculum. Illustrations used must be easy to follow, current, relevant topics, purposeful representations, interesting introductions, simple, large enough for sight, and that have the ability to motivate students. Kochhar (1985) suggested over three decades ago that textbook topics should be graded according to appropriate levels, and be made to fit the capability of the targeted students. Topics should include simple, clear, and logically set out facts suitable for child-centred learning.

The textbook should capture the essential characteristics of the purpose of the topic in terms of skills to be learnt and their application (Cottrell, 2017). Educational intentions for a topic depend on students' knowledge and critical thinking. Such factors are needed in order to have a reasonable chance of success in achieving the intended learning outcomes (Gravemeijer, Stephan, Julie, Lin and Ohtani (2017). Learning outcomes are statements of the attributes and capabilities that a student should be able to display on successful completion of the topic. They provide the basis for determining student progress, and designing assessment strategies and methods linked to previous concepts. They also provide signposts pointing towards appropriate content and learning interactions, to help students achieve such, positioning them into context. As the nature of the topics may influence the way students think, it is important for the textbook to provide an equal number of problems for each topic (Gracin, 2018). The analysis of the structure of the respective textbooks will be carried out in this study. This process will include the identification of topics and how activities are represented in order to cater for the students' different ability levels.

3.6.2 Alignment of content weighting to curriculum

Curriculum and topic weighting influences the structure and content of the textbook which is taught to the student; it is intended to meet the requirements of the teaching process. The manner in which topics are introduced and concluded is planned to support their learning process by means of such appropriate topics and classroom activities. Two of the analysis chapters of this study focus on the structure of the textbook and its influence on the representation of problem-solving procedures.

3.6.3 Non-textual elements description

In the era of visual information, visual content is fundamental to people's everyday lives, with education resources having incorporated a variety of still and moving images. Visual analysis is the primary method of evaluating data in single-subject research (Wolfe & Slocum, 2015). Cooper, Sidney and Alibali (2018) state that students solve problems more accurately when visuals are included. Baek, Wickstrom, Tobias, Miller, Safak, Wessman-Enzinger and Kirwan (2017) further opine that the non-textual elements are considered according to accuracy, connectivity, context, simplicity, and aesthetics, which make up the central aspects. Such further aids in the problem-solving procedures throughout the textbook.

Combining concepts with meaningful pictures enables students to retain knowledge in their long-term memory (Gutiérrez, Puello & Galvis, 2015). Students are able to use various problem-solving strategies, with one of the most effective and flexible methods by making use of pictures and diagrams. Visuals and pictures aid students to understand a problem and identify a solution, while diagrams can also serve as a guide to assist them to keep track of what they need to find out in multi-step problems (Toneli & Petti, 2012). These strategies are beneficial to students: an equation can be found in every word problem which can be used to solve mathematical word problems. While word problems can be solved in many different ways, each method involves outlining the problem, a method which is being increasingly used (Toneli & Petti, 2012).

Research offers ways of showing that visual mathematics, represented in curriculum materials from preschool to Grade 12 can be greatly beneficial to students (Boaler, Chen, Williams & Cordero, 2016). Boaler, Chen, Williams and Cordero (2016) further state that it is a myth that visual mathematics is for low achievers or younger students. It is true that those who are given visual representation initially in their learning process then progress to advanced or abstract mathematics. Boalers' findings (2016) indicate that the brain is made up of distributed networks that is stimulated when knowledge is accessed and information exchanged. When working on mathematics specifically, the brain activity is spread out across a widely distributed network, which includes the ventral and dorsal visual pathways (Brown, Bosse, & Chandler, 2016). Neuroimaging has shown that, when people work on a numerical calculation such as 10×25, with symbolic digits 10 and 25, the mathematical thinking is grounded in visual processing, indicating that visual representation is required throughout learning, irrespective of the cognitive level or chronological age (Boaler, Chen, Williams & Cordero, 2016).

The mathematical representations that play an important part in education include symbolic or algebraic; written, verbal or oral; graphical and numeric; or tabular. According to Hollebrands (2003), tutors assess the students' understanding of the relevant mathematical concepts by examining their ways of generating, translating, interpreting, and using various mathematics expressions. Examining the students' level of understanding through their work representations is therefore essential for assessing their overall mathematical understanding (Brown, Bosse, & Chandler, 2016).

Yang, Tseng and Wang (2017) compared geometry problems in mathematics textbook for middle grade students from Finland, Taiwan, the USA, and Singapore. The four textbook series mainly focused on three forms of representation for mathematics problems: these being visual, verbal, and a visual-verbal combination. Minimal attention had been placed on purely mathematical problems which promote creativity, critical thinking, communication, and reasoning. The USA and Finnish textbooks had higher numbers of verbal (37.1% and 27.9%, respectively) and visual forms of problems (41.4% and 43.3%, respectively), while those from Singapore and Taiwan had higher numbers of the combined-form problems (33.7% and 69.4%, respectively). Of the four-textbook series, problems in the USA textbooks exhibited minimal use of mathematical expressions, estimated at 0.4%. Compared with the USA and Taiwanese books, those from Finland and Singapore had a greater balanced problem distribution in relation to the different representation forms. The findings indicate that the forms of representations in the textbook influence cognitive-demand levels and students' potential engagement when learning.

A highly effective Singapore mathematical programme utilizes bar models, which is one of the visualization strategies used in problem-solving. Students draw bars to represent simple and multi-step word problems, each bar indicating how the known elements of the problem relate to one another, using question marks to indicate what they need to find out. Bar models are used to represent ratios; and are a concrete method used in order to facilitate algebraic reasoning when solving abstract problems (Tonelli & Petti, 2012). This reveals that concrete methods are highly recommended for conceptual development and understanding.

A study carried out by Vula, Avdyli, Berisha, Saqipi and Elezi (2017) indicated that comprehension challenges for the conventional mathematical representations and symbols are likely to deter proper preparation amongst students, causing them to fail in algebra; for example, cases where students confuse the equal sign for an operation (Powell, 2012; Alibali *et al.*, 2013). Additionally, the research by Lannin *et al.* (2013) indicates that poor-performing students in the first and second-grade mathematics classes could not properly represent problem scenarios using formal representations. Such can be in the form of computer programmes, human thought processes, or informal and idiosyncratic written and physical representations. Multiple representation of problem-solving questions assists students to identify what is expected of them, by viewing various techniques of problem-solving strategies. This allows students to choose an appropriate method that they find suitable, allowing them to move forward with the problem-solving process. Multiple representation assists students to become creative in the strategies they choose, which allows them to formulate fresh and innovative ideas.

3.6.4 The benefits of colour-coding in textbooks

Students tend to become restless and lose interest when activities are above or below their level of competency, with the textbook level needing to accommodate interesting and challenging activities to keep the students engaged long enough to complete the tasks allocated to them. Nearly two decades ago, Aggarwal (2009) outlined the conditions of the general presentation format of the textbook as needing to be attractive and interesting to read. It should incorporate simple illustrations constructed in bright colours, having a colour-coding system in combination with thought-provoking photographs and graphs, containing current ideas, and being revised and reprinted frequently to incorporate changes. Lastly, charts, additional references, and a table of contents, make the book complete. More recently, Gao, Hui and Tian (2017) stated that people are visual beings; and that a large portion of the brain is tasked with visual processing.

People’s ability to process images depends on their reasoning and the capability of paying attention, as images have the power to attract attention with ease (Gao, Hui & Tian, 2017). The human brain processes images at a very rapid speed, and recognizes familiar items in as little as 100 milliseconds, with pictures being analysed very quickly. Therefore, knowing what an image means aids students in its immediate interpretation, thereby helping them to understand the question.

The brain is also attracted to bright colours (Gao, Hui & Tian, 2017), with vision being by far the most active of the senses. However, students are required to examine their textbooks and memorize words in sentences, which is not the most suitable strategy, considering that most are visual students who find it easier to memorize image-based content. Reading is sometimes a slow process and consumes a significant amount of time, with more time being spent reading a lengthy sentence than analysing a visual scene: people can quickly redevelop a particular image in their minds. The point of all the arguments above is that textbooks need to have multiple representations. The literature indicates that colour assists students to grasp concepts; it develops attention, retention, and memory (Dzulkifli & Mustafar, 2013). Olurinola and Tayo (2015) further state that colours used in the right proportion in mathematics textbooks can enhance learning, and benefit the retrieving process. The use of colour is considered in the analysis chapter of this study, considering that colour is an important stimulus which assists students to grasp concepts, and aids in retention and memory.

3.7 Theoretical and conceptual framework for this Study

Within the above context, the theoretical and conceptual framework used to guide this study consisted of three components: firstly, the practical aspects discussed above concerning multiple representations in the textbooks. The theoretical framework used in this study is the horizontal and vertical analysis which is divided into structural, content, and didactical analysis. The outline of the international guiding principles for designing textbooks, as produced by the Hong Kong Educational Bureau (2016), informs the horizontal and vertical contexts of the framework below.

The didactic analysis process includes Polyas's problem-solving model, which consists of 4 components, these being to understand, plan, act, and check a problem (Polya, 1945). This is supplemented by eight (8) heuristics, these being the methods used as problem-solving procedures which aided in the data-comparison process (Kahneman, Slovic, Slovic, & Tversky, 1982).

According to Peters (2018), theories are framed to explain, predict, and comprehend phenomena and, in many cases, to test and extend current knowledge within the confines of critical assumptions. In this study, the theoretical framework aims to demonstrate an understanding of models and concepts that is relevant to how problem-solving procedures are represented in specific textbooks, including the similarities and differences in the representations.

The conceptual framework is defined as a network of linked concepts. In this study the analytical tool consists of several variations and contexts used to make conceptual distinctions, and to organize structure and content in the chosen textbooks. Ovaskainen, Tikhonov, Norberg, Guillaume Blanchet, Duan, Dunson and Abrego (2017) present the advantages of conceptual framework analysis, its flexibility, its capability of amendment, and its emphasis on understanding instead of prediction.

Within the above context, the theoretical and conceptual framework used to guide this study consisted of three components, as indicated in Table 3.4. The study's theoretical framework entails the horizontal and vertical analysis, which is divided based on the work of various authors.

- The vertical analysis entailed didactic factors, or conceptual understanding in the textbooks.
- The didactic analysis process includes Polyas's problem-solving model, which consists of 4 components, these being to understand, plan, act, and check a problem (Polya, 1945).
- This is supplemented by eight (8) heuristics, these being the methods used as problem-solving procedures which aided in the data-comparison process (Kahneman, Slovic, Slovic, & Tversky, 1982).

The framework (Table 3.4) was used to identify how problem-solving procedures embrace these strategies, establishing whether students are given the opportunity of carrying out the required heuristics. Various types of problem-solving procedures were examined in the respective textbooks. This was in order to identify whether the questions are created to challenge students' critical thinking abilities, or whether they are simple straightforward problem-solving procedures. Further examination of the problem-solving procedures allows students to develop their communicating, reasoning, connecting, and presentation skills through the mathematical content.

Table 3.3: Textbook analysis framework

HORIZONTAL PRINCIPLES FOR TEXTBOOK DESIGN AND ANALYSIS	
1. Structure and organisation (Charalambous *et al*., 2010; Bureau, 2016) • Content sequencing - Keywords - Concepts - Textboxes • Structure - Table of Contents - Chapter Titles - Headings - Outline • Overview - Explained **2. Layout of the textbook** (Charalambous *et al*., 2010; Bureau, 2016) a. Logical & consistent b. Illustrations c. Lightweight paper d. Once-off use (TB) e. Font type **4. Language (**Bureau, 2016) a. Accuracy (spelling, proper terms) b. Coherent text c. Language is accurate d. Help provided (understand vocabulary) e. Text of high quality	**3. Content** (Bureau, 2016**)** a. Aims & objectives aligned (linked to curriculum) b. Supplementary materials provided c. Current content presented (linked to real life) d. Prior knowledge e. Content sequencing f. Concepts are correct: -- Simple illustrations/charts -- Bright colours -- Additional references g. Balance in depth & breadth: -- Content weighting -- Content sequencing h. Multiple representations i. 6 Content areas: -- Prisms and pyramids -- Division of fractions -- Division of decimals -- Percentages -- Width of circle -- Width & volume of a cube

VERTICAL CRITERIA FOR TEXTBOOK ANALYSIS (FOCUSING ON TEACHING & LEARNING PROBLEM-SOLVING)	
1. Didactic factors (Bureau, 2016) a. Cater for student diversity (various strategies) b. Conceptual understanding c. Student involvement (individual, peer & group) d. Interesting and motivating tasks e. Clear instructions f. Meaningful activities	**2. Conceptual understanding** (Charalambous, 2010; Bureau, 2016; Groth, 2017) a. Cognitive demand level (Bloom's taxonomy) b. Stimulating tasks c. Deeper thinking d. Flexibility e. Reading comprehension f. Problem-solving procedures − Conceptual understanding − Procedural fluency − Strategic competence − Adaptive reasoning − Level of competence

Polya's problem-solving model adapted from Fan & Zhu (2007, p. 66)

Polya's 4 stages	Description
1.Understand the problem	Identify the problem
2. Devise a plan	Plan a solution with a mathematical model
3. Carry out the plan	Resolve the problem as planned
4. Check and extend	Interpret solutions

Heuristics Adapted from Fan & Zhu (2007, p. 66)

Heuristics	Description
1. Act it out	Use people or objects to physically show what is described in the problem.
2. Change point of view	Approach a problem from another angle when a previous approach is not effective.
3. Draw a diagram	Draw a sketch based on the available information to visually represent the problem.

4. Guess and check	Make a reasonable guess at the answer; check to see whether it works; repeat to find the answer.
5. Look for a pattern	Identify patterns based on observation of characteristics, variations/differences in the problem.
6. Solve part of the problem	Divide a problem into several sub-problems; solve them 1 by 1; solve the original problem.
7. Use a model	Create visuals to model information on quantities/relationships, or changes in the problem
8. Work backwards	Approach a problem from its outcomes; work backwards to find what conditions need to be met.

The intention is to build on the framework of Charalambous *et al.* (2010) and add to her work by including the guidelines to international textbook design, Polya's 4-stage model, and the eight (8) additional heuristics. The development of the framework consists of three components, with the horizontal expanded to include the structure and organisation, content, layout and the use of mathematical language. The vertical component is expanded into two layers, namely, the concepts and cognitive demand, then the strategies used in problem-solving, including Polya's 4-stage model and the eight (8) heuristics.

3.8 Conclusion

The importance of textbook analysis and the various aspects on which this chapter focuses, for example, problem-solving, conceptual understanding, and representation of visuals in the textbooks, are a focus in many countries. Research has indicated that these are vital to developing a textbook that will aid in the development of students' learning. Textbook analysis is conducted on history, geography, accounting, psychology, mathematics, and science textbooks for various reasons. What students are exposed to, or what is missing in presentations, is of paramount importance. Textbooks have an undeniable influence on the learning and teaching process. This research therefore sought to identify how problem-solving procedures are represented in the South Korean and South African Grade 6 Mathematics textbooks, by analysing the said textbooks. Chapter 4 presents the methodology used to conduct the study on the South Korean and South African textbooks.

CHAPTER 4: Research design and methodology

4.1 Introduction

This study was aimed at identifying how problem-solving strategies are presented in Grade 6 South Korean and South African mathematics textbooks. The researcher employed structural and content analysis strategies to specific mathematics textbooks used in these countries to gain a clear understanding of the different strategies implemented in the teaching and learning process. This chapter outlines the methods used to address the study's objectives, which involved a systematic process of collecting and logically analysing data. Bryman (2016) maintains that the decision to choose a specific methodology should be based on its suitability in addressing the research problem, questions and aim. The interpretive research design and qualitative methodology used to obtain and analyse data were utilised for acquiring knowledge, trustworthiness and consistency (Flick, 2018).

4.2 Relevance of the interpretive paradigm

The interpretive paradigm was adopted to explore how problem-solving procedures are represented in mathematics textbooks in South Korea and South Africa, specifically to compare the textbooks' content relating to problem-solving. The following aspects are discussed in this chapter: research paradigm, research design, research methods, chosen textbooks, and the measures followed to ensure the trustworthiness of the instruments employed to collect data (Wilson, 2017).

According to Blaikie and Priest (2019) the use of this paradigm is suitable for studying context-specific, unique processes, and can help uncover interesting and relevant research questions and issues concerning teaching and learning in the classroom. Patten and Newhart (2017) explained that this paradigm facilitates text examination, helps establish meanings about how patterns of content and language shape the representation of a topic, and how relationships between those patterns express an underlying meaning, which is essential in this study. Research conducted by Veal (2017) reveals that the interpretive paradigm enables engagement, and interpretation of the materials being explored, which was a requirement in order to compare how problem-solving was represented in the chosen mathematics textbooks.

In this study, textbooks were used because they are representative of the purposes, goals and intentions, with their resolve being envisioned to accomplish a educational objectives. The aims being a destination for specific achievement focuses on creating understanding throughout the text. The intention is to identify how problem-solving develops critical thinking that involves everyday experience. Textbooks are therefore studied as active agents, this paradigm being used to capture the representation of problem-solving in both numbers and words (textual) and graphs, pictures, tables, diagrams and models (non-textual) representations.

Understanding the presentation of problem-solving procedures was important in this study, it was through the data gathering process that an understanding of the texts ensued. All interpretations were based on cultural relevance, authenticity and curriculum weightings, which are explained later in the chapter, with the interpretive paradigm aptly improving the understanding of a phenomenon (Clarke, Friese & Washburn, 2017). The interpretive paradigm was therefore used to explore how problem-solving procedures are represented in the textbooks of the respective countries, taking into consideration the contexts and age group of the students using these textbooks. Rahi (2017) explains the view attached to the interpretive paradigm is that only extensive interpretation of a subject can be used to obtain true knowledge, and in this case, the problem-solving procedures were analysed with the intention of gaining insight of their construction in the selected mathematics textbooks.

Alase (2017), states that the interpretive paradigm approach has an advantageous elements which allows the study to be expanded. During a textbook analysis process when comparing the representation of visuals and questions may require an extension due to the development of interpretive skills. As noted by Reiman, Thomson, Xiu, Mertens and Rosenbaum (2015), the skills consist of interpretation which was applied in this study by understanding selected textbooks, drawing inferences or by making comparisons between texts. The data was analysed and interpreted into meaningful themes to aid in answering the critical questions.

4.3 Research design

A research design is defined as a framework of methods and techniques chosen by the researcher to combine various components of research in a reasonably logical manner so that the research problem is efficiently handled. The research embraced textbook design which was based on the work of Valverde *et al*. (2002), Charalambous *et al*. (2010), O'Keeffe (2011b), O'Keeffe (2013), and Katalin and Henning (2013). In this study, the structure, content and research problem determine the research design and variables to gather information, and how the texts were used to produce and analyse data. A qualitative research design was used as it incorporated how theory relates, understands and interprets content using mathematical calculations that aided in answering my research questions (Alase, 2017).

The study entailed qualitative methods that used content analysis to understand the complexities of textbook studies and allowed a deeper understanding into the texts. According to Thiel, Thedinga, Barkhoff, Giel, Schweizer, Thiel and Zipfel (2018), while there is criticism about the objectivity and perceived bias associated with qualitative research, it allows researchers to stay close to the data. Kim, Sefcik and Bradway (2017) further states that different researchers will agree more readily on the same findings. Cohen, Manion and Morrison (2018) suggest that qualitative data includes information such as words, pictures, drawings, paintings and photographs. Here, a comparative study was used to gain an understanding of problem-solving in mathematics using visuals, pictures and photographs from the relevant textbooks.

The qualitative research focuses on the aspects that give meaning to the data that is being collected, on distinct features or arguments of what can be found in the information (Flick, 2018). Qualitative data analysis allows for the data to have a higher level of detail, which can afford more opportunities to gather insights from it during examination. Furthermore, Alase (2017) states that research must follow a specific pattern of data collection, analysis and information reporting. Qualitative research provides opportunities to gather important clues about the representation of visuals and questions in the text instead of being confined to a limited, and often self-fulfilling perspective. The study was underpinned by the aim of identifying how problem-solving procedures are represented in South Korean and South African Grade 6 mathematics textbooks. This study further incorporated a horizontal and vertical components, Polya's 4 stage model, 8 heuristics to create a textbook framework design. The framework was used to analyse the data. The implementation of an in-depth framework such as this, can provide insights into the structure, topics, problem-solving representations and levels of cognitive demand.

4.3.1 Purpose and benefits of comparative studies

The prominent educators of the 19th and 20th centuries have centred mostly on comparisons of national educational systems in an effort to encourage global understanding, enhance or reform education reception, and to determine what accounts for the differences amongst educational systems. More recently, comparative education aids in ensuring the standard of content and how it benefits the collaborating communities (Melnikova, Zaščerinska, Ahrens, Hariharan, Clipa, Sowinska-Milewska & Andreeva, 2017).

Comparative thinking is an important skill that helps develop the power to think deeply and comparatively about textbooks, the way they represent the curriculum and how education has impacted on that landscape. The event of comparative thinking and therefore the ability to use analytic frameworks for critique, was essential in this study. Comparative education may foster international understanding, individual growth, national development, global cooperation and higher order thinking, which is geared towards enrichment and improvement of students' performance (Holmes, 2018). Systematic critique and reflection must accompany curiosity, interest, and inquiry regarding things that are foreign. Comparative investigations, whether speculative and exploratory, or driven by more specific purposes, require framing and boundary (Cohen, Manion & Morrison, 2018).

It is becoming increasingly apparent that the complexity of issues facing schools around the world requires a comparative approach (Kelly, 2017). The Trends in International Maths and Science Study (TIMSS), which shows that East Asian countries are maintaining their 20-year exceptional results for students, has created much curiosity within the education field that has led to comparative studies (Hossain, 2017). There is a marked difference in the results between participating countries, with Hossain (2017) suggesting that educators need to build on people's inherent nature to compare their teaching materials with those of other countries. International comparisons challenged me to expand my understanding beyond the perspectives of our country. Consistent with Hong and Choi (2018), education is not only inclusive, supportable and excellent, but it always improves and meets the challenges of the evolving and unpredictable globalized world, which may be achieved through comparative studies which I aimed to achieve in this study. Furthermore, the intention was to produce a textbook analysis framework that educators could use to guide the development of mathematical textbooks.

Hong and Choi (2018) state that comparative methods are becoming increasingly popular and continues to evolve as their applications change. Nearly all disciplines have results from comparative research, which can be applied to almost all study topics, ranging from working conditions amongst nations to textbook studies in different countries (Uzunboylu & Karagözlü, 2017; Graneheim, Lindgren & Lundman, 2017; Riff, Lacy, Fico & Watson, 2019). I used South Korea and South Africa in the comparative study as their results differed drastically in the TIMSS study.

Comparisons have been used as a way to study human culture, history and society, with Karl Deutsch arguing that this form of investigation has been used for more than 2000 years, being important as a method of basic philosophic and scientific inquiry to draw comparisons between things (Haolader, 2017). The legacy of comparisons used in social theory can be traced back to the Greek Antiquity, with particularly cross-national comparison having increased, leading to increased research in the human and social sciences. According to Immelt (2017), comparative studies are a growing methodology due to developments in technological advancements and communications, together with internationalization tendencies. Internationalization is the designing of a product with the intention for it to meet the needs of users globally or be easily modified to meet the needs of a specific country.

Pollitt and Bouckaert (2017) describes comparative studies as an examination of similarities and differences of varied aspects, like national education system and structures. Consistent with Esser and Vliegenthart (2017), in recent years, comparative research in textbook studies have gained considerable ground, which may be interpreted as a symbol of these areas of research maturing as a discipline. A key issue in conducting comparative research was to make sure equivalence occurred. This was ensured by gathering visual and textual information that were comparable between the various contexts of both textbooks. This helped to avoid biases in measurement, instruments, and sampling by comparing information which I did during this study.

4.4 Selection of mathematics textbooks

Purposive sampling was used to select the books for inclusion (Flick, 2018), this being used to identify mathematics texts with particular characteristics. I used one South Korean and one South African Grade 6 textbook because problem-solving concepts are presented to a greater extend. Furthermore, these books were analysed as a whole, to address problem-solving procedures. The textbooks chosen for investigation were not randomly selected, their inclusion pertained to being approved by government, aligned to curriculum and ready to provide data to deal with the study aim (Laerd, 2012). This study required in-depth description and context, for which a large number of textbooks would have been irrelevant to the nature of this study. I chose these books (See, Table 4.1) because South Korean books are designed and prescribed by the Ministry of Education for state schools, with the revised 7th curriculum being used. The textbook from South Africa was on the Department of Education's recommended list and is CAPs compliant.

Table 4.1 Grade 6 mathematics textbooks used in the study

Title	Authors	Year of Publication	Country
Along with Shared Mathematics	B. Jong-Soo, P. Hyung-Mee, J. Jong-Ho, C. In-Sook, B. Sok-Yoon, D. Joo-Won, L. Kang-Sook, C. Song-I, P. Man-Koo, K. Nam-Joon, K. Hye-Jong, L. Kyung-Hee, K. Wan, K. Yoon-Son, S. Jong-Hwan, L. Jong-Min, A. Byung-Kon, O. Soo-Jin, I. See-Yon, J. In-Soo, J. Chae-Sook, K. Yong-Tae, N. In-Hye, S. Min, L. Jong-Hye, K.Nan-Ah, K. Jee-Yong, O. Jee-Yong, J. Hang-Mi, S. Jee-Won.	2015	South Korea
Platinum Mathematics CAPS	L. Bowie, C. Gleeson-Baird, R. Jones, H. Morgan, K. Morrison, M. Smallbones.	2012	South Africa

4.5 Data collection plan

A data collection schedule was developed by the researcher, which included guidelines to establish the quality of the textbook, Polya's 4 stagemodel and 8 heuristics with which to carry out the content analysis to enable a comparison. The SA textbook was presented in English while the South Korean textbook was presented in Korean. This entailed having the South Korean book translated into English by an accredited translator using age appropriate language to enable their content to be comparable. Thereafter, both books were read through repeatedly to enable a familiarity with the design and content, this being intended to integrate the idea of the 4 stages and the 8 heuristics into the content of the textbook. Each question from the respective selected textbooks was read to identify the aspect of the theory that could be used to solve the problem adequately. I manually went through the book with a pen and underlined sections concerning structure, content and problem-solving, then wrote them up, I also used a digital version of the books to copy visuals for the analysis chapter.

4.6 Benefits of content analysis in this study

Qualitative data analysis involves organizing, accounting for and explaining the data, making sense of it in terms of textual and non-textual representation in the textbooks, and noting patterns, themes and irregularities.

The school curriculum serves as a link between society and education, as they are based on certain values and knowledge that is designed to influence students in particular directions during their education (Tondeur, Van Braak, Ertmer & Ottenbreit-Leftwich, 2017). The content analysis method was chosen as it is suitable for various text materials and can reveal dominant content, making it an appropriate approach for analysing textbooks. Bryman (2016) argues content analysis is amongst the foremost important techniques to analyse texts and documents that aim to find revealing themes within the analysed materials. He defines content analysis as an approach that discovers meaning within a text, then classifies texts into various themes, which further aims to highlight the meaning of the context. Content analysis's object can be any type of recording, including protocols of observation, interviews/discourses transcripts, textbook, videotapes, newspaper articles, and written documents.

According to Graneheim, Lindgren and Lundman (2017), this analysis does not only involve extracting objective content from texts to look at meanings or counting words, but to manifest the content of the material, as well as its hidden content, which was utilized in the analysis process of this study. Its strength is its controlled method, using a step-by-step process for analyses and its ability to reveal international differences in the textbook content. The information was prepared by colour-coding and allocating themes which were identified within the texts. The subsequent steps were conducted: preparing the information, defining the unit of study, developing themes and a coding scheme, testing the coding scheme on a sample of text, coding all the text, assessing coding consistency, drawing conclusions from the coded data and reporting methods and findings.

There are two general types of content analysis: conceptual and relational, each one leading to different results, conclusions, interpretations and meanings. Conceptual analysis determines the existence and frequency of concepts in a text, the problem-solving stages and heuristics being chosen for examination and the analysis involving analysing problems, visuals and questions. To begin a conceptual content analysis, I first identified the research question and chose samples for analysis. Next, the text was coded into manageable content categories (topics, structural features), this being a process of selective reduction. By reducing the text to categories, the researcher focused on coding for specific words or patterns that informed the research question. This study focused on content analysis, where the process revolved around coding the gathered information from the mathematics textbooks, then categorising them into themes, which helped to answer the key questions.

Van Baren (2017) explains that relational content analysis is a technique used to explore the relationships of identified concepts in a text, being qualitative and involving non-statistical methods of analysing information that looks for correlations within the text. He further states that relational analysis focuses on looking at strengths, signs and directions found in words that have a common relationship, which are analysed. This study focused on the representations of words, visuals, games, stories and problem-solving procedures that helped strengthen the teaching and learning process for students.

According to Gaur and Kumar (2018), content analysis has become popular in most qualitative research, with three approaches being available, these being directed, conventional and summative. Gaur & Kumar (2018) developed two strategies for conducting directed qualitative content analysis (QCA), the first consisting of reading the textual data and highlighting those parts that, on first impression, appeared to be related to the predetermined codes dictated by a theory or prior research findings, which are then coded. The difference in the second strategy is in starting the coding process without initially highlighting the text, the first strategy being adopted in the analysis process that was effective. This process included reading both texts and highlighting features dealing with structure, content and didactic factors using selected colours which were then coded accordingly. In conventional content analysis, coding categories were derived directly from the text data, while summative analysis involved comparisons, usually of keywords or content, followed by interpreting of the underlying context (Assarroudi, Heshmati Nabavi, Armat, Ebadi & Vaismoradi, 2018).

In this study, inductive reasoning was used to identify themes from the content of the textbooks by careful examination and consistent comparison (Rahi, 2017). These themes were used to compare texts within and between the South Korean and South African textbooks. Furthermore, generating variables or concepts from previous studies, or theory, is very useful for qualitative research, and the data analysis inception in particular (Ary, Jacobs, Irvine & Walker, 2018). I looked at previous textbook studies and drew from the themes that were commonly analysed in order to improve the quality of textbooks, which guided my study.

4.6.1 Process of content analysis

The content analysis process was initiated during the early stages of the study, and entailed moving back and forth between the texts from the respective textbooks, which helped guide the selection of information to more useful themes to address the study objectives (Graneheim, Lindgren & Lundman, 2017). Content analysis involves a set of transparent and systematic steps for processing data to support valid and reliable inferences (Krippendorff, 2018, p.37). The content analysis process entailed the following steps described below:

1. Preparing the data
2. Defining the unit of analysis
3. Developing themes and a coding scheme
4. Testing the coding scheme on a sample of text
5. Coding all the text
6. Assessing coding consistency
7. Drawing conclusions from the coded data
8. Reporting methods and findings

Step 1: Preparing the data: This step entailed reading and re-reading the textbook in search of similarities and differences in content and highlighting important aspects that influenced the study. To carry out the process with the Korean textbook, it had to be translated into English to enable the data to be extracted and analysed. I then colour-coded the textbook, looking for the structure, and making notes of all the visuals and interesting ways problem-solving procedures were represented, which enabled me to create themes.

Step 2: Defining the unit of analysis: This step entailed defining the unit of analysis, which described what was being studied, the basic unit of analysis being the representations of problem-solving procedures, which were classified during content analysis, identified and colour-coded. Before coding, the data was grouped into subgroups/themes, and named to enable a comparison of the outcomes with other similar studies. The unit definition was therefore a crucial stage in the analysis. Distinct themes are generally used as the unit for analysis in content analysis, which are used in the analysis chapter of this study. In this study, the themes involved the structure of the textbook, weighting of the curriculum and the role of non-textual elements, after which I looked at specific topics followed by the 8 heuristics outlined in the conceptual framework (Neuendorf, 2016).

Step 3: Developing themes and a coding scheme: I looked at recent mathematics textbook studies to draw ideas about the themes and coding system that were used in problem-solving studies. I found that analyses of textbook tasks generally contained the usual themes, these being related to content, cognitive demands, question type and contextual features. The aim of this study was to include the fifth theme into the framework, that being the analysis of mathematical activities using the 4 stage model and the 8 heuristics (Veal, 2017; Baek, Wickstrom, Tobias, Miller, Safak, Wessman-Enzinger, & Kirwan, 2017; Glasnovic Gracin, 2018). The structure of the textbook, curriculum weighting, topic instruction, topic conclusion and visual representations were the initial themes. However, I aimed to create new themes from the data inductively. This is when I found that the textbooks were presented in a colour-coded style, filled with games and stories, which motivated me to add those themes into the study. It was ideal to use inductive content analysis for studies with theory development intentions.

Themes from the textbooks were created and comparisons were drawn between the respective textbooks (Graneheim, Lindgren & Lundman, 2017), as it was possible to distinguish themes. The constant comparative method essence consisted of: (1) compare every text assigned to a theme systematically with those already assigned to that theme to gain an understanding of the theoretical themes: and (2) integrate the themes and their properties by identifying visuals that are interpretive.

In some studies, researchers had a preliminary theory or models on which to base their inquiry, which they may change or update within the course of the analysis as new themes are developed inductively and then create themes from an initial list of coding (Graneheim, Lindgren & Lundman, 2017).

The adoption of coding schemes created in research has the advantage of supporting the comparison and accumulation of various findings. There is a need for themes to be mutually exclusive, as confounded variables would not follow assumptions of some statistical procedures in content analysis (Veal, 2017). However, a single theme assignment of a specific text may not be easy. In content analysis, one can assign a unit of text to various themes simultaneously (Titscher *et al*., 2000). To ensure that coding was consistent, particularly where several codes are included, I formulated a coding system that involved colour-codes and descriptions for code assignment and theme names (Veal, 2017). My coding system relied on the process of data analysis with the help of a constant comparative method.

Step 4: Testing the coding scheme on a sample of text: I used a standardised process for the analysis and used a coding scheme that was developed and validated earlier in the process. There was a need for me to recheck the coding consistency after the sample was coded, with the rules of the coding being revised if the level of consistency was low. Problems and doubts relating to the theme's definitions, categorization of specific cases, or coding rules were discussed and resolved between myself and my supervisors (Gaur & Kumar, 2018). Some of the important processes, included coding sample text, checking coding consistency and revising coding rules, continued until sufficient coding reliability had been achieved (Neuendorf, 2016).

Step 5: Coding all the text: Once adequate reliability had been achieved, I applied the rule of coding to the entire text. The coding was checked repeatedly to avoid straying from the aims that needed to be achieved (Blaikie & Priest, 2019). I continued to go through texts so that new themes and concepts developed that needed to be added to the coding system. I coded the themes as follows; Weighting of the curriculum (CW); Problem-solving (PS) and Non-textual (NT).

Step 6: Assessing coding consistency: I checked the coding consistency once more after coding all the data. It was unsafe to make assumptions that if a sample was reliably and consistently coded the coding of all texts was also consistent. I asked colleagues in a similar field to check the coding to ensure that there was consistency throughout the texts in order to address issues related to human error, and they took into consideration that coding rules could have changed over time (Neuendorf, 2016).

Step 7: Drawing conclusions from the coded data: In this stage, inferences were made about the identified themes and the data was presented in a table form as well as visually. I then compared and identified the relationships between them by uncovering patterns and themes and testing the themes against the full range of data (Neuendorf, 2016). My reasoning abilities led to the development of a suitable analysis process.

Step 8: Reporting methods and findings: Analytical processes and procedures were reported in a complete and truthful manner by myself so that this study can be repeated (Patton, 2002). The practices and decisions were reported using qualitative content analysis, which uncovered themes and patterns that were of significance to the study. However, the presentation of research findings from the qualitative content analysis was challenging. To justify the conclusions, the researcher used typical quotations (Gaur & Kumar, 2018) and adopted other alternatives of data display such as graphs, charts and conceptual networks (Neuendorf, 2016). Finally, the findings were reported according to the specific objectives of the research (Patton, 2002). The researcher strived for a balance between interpretation and description when presenting the results of the qualitative content analysis.

Denzin and Lincoln (2000) state that an informative and lengthy description must be provided so that readers have knowledge about the background and context, which is specified in this study. My personal and theoretical understanding of the phenomena of the study was represented by qualitative research, which is fundamentally interpretive. Inductive and deductive theme applications were used to develop a theme system, which means that I used themes from similar studies and created others as the information unfolded from the respective textbooks.

4.7 Ethical considerations

As the Aim of study is the structure and representation of problem-solving in textbooks, and no surveys, interviews or data obtained from people were used, the ethical considerations for this thesis were limited to plagiarism and self-plagiarism. This study is original, and the Grade 6 mathematics South Korean and South African textbooks were not compared previously. However, there were some research ethical steps that had to be followed. First, the application for an ethical clearance letter from UKZN for the study using the RIG system (see Appendix 1). The next research document was the translation letter, the Grade 6 South Korean textbook had to be translated to English (see Appendix 2). The translation letter was evidence that the South Korean textbook was translated professionally in order to ensure trustworthiness of the study.

4.7.1 Limitations of this study

A number of limitations may have affected this study:

- Content analysis depends on description, the limitations being that it described the findings rather than exposes reasons for the pattern observed.
- The availability of material limited the analysis, as I was only able to access one South Korean book, and the trends observed may not necessarily be a true representation of reality coding themes that are socially constructed and are therefore subjective. There is also the possibility that the meaning of the data may have been misinterpreted in the translation, as I needed to rely on a translator and an editor to check translations for accuracy.
- As only one textbook was used from each country, the small sample size and scope of this study may not have provided sufficient input as to how the framework can be further improved.
- I purchased the South Korean textbook via a South Korean contact based in Cape Town, South Africa because these specific textbooks were not available online. The textbook was translated in South Africa, which delayed my studies as well.

4.7.2 Trustworthiness of the study

The quality of the research is measured by trustworthiness, which is achieved through transferability, dependability, conformability and transferability. To enhance content analysis credibility, I ensured the trustworthiness while using open coding in order to analyse data from the selected textbooks (Graneheim, Lindgren & Lundman, 2017). Section by section analysis allowed verification of themes that maximised the identification of important themes. To improve the credibility of content analysis, strategies were designed to gather data that was able to apply the representation adequately. In addition, I designed processes that were transparent for drawing and coding conclusions from raw data.

Credibility refers to the adequate representation of what is being studied. According to Lock and Seele (2016), the process of open coding should adhere to the following principles that are broad. The first step involved reading through the textbook to get a general idea of the content. The next step entailed re-reading the chapters selected to identify the necessary themes and concepts from the text. Codes were selected according to the interpretation of the data and allocated code names were used to identify themes and concepts.

The related themes were analysed under similar codes to represent the findings of the study. Open coding helped to collect meaningful information and ideas while enhancing my understanding of the data used during analysis. I initially used a framework for gathering data on the general aspects, such as learning outcomes, weighing of curriculum strands, graphic representations, etc which influence the representation of problem-solving. This was followed by using a framework based on Polya's 4-stage model and the 8 heuristics to guide the open coding.

Activities that were engaged in to safeguard credibility included reading and re-reading of the text for purposes of attaining a full engagement with the text to avoid biased interpretations. This was supported by activities of continuously reading and re-reading of the texts by colleagues, which are identified as criteria for credibility (Korstjens & Moser, 2018). In addition, the quality of the research was improved by engaging other researchers in the field to check my results and research processes while at the same time checking and corroborating the results with the original content in the text (Merriam, 2009). According to Flick (2018), even if the study seems to be the description of a general approach to analysing documents using qualitative methods, content analysis is a sophisticated and concretely described method as well as an approach to analysing documents.

Ary, Jacobs, Irvine and Walker (2018) explain conformability as referring to the extent to which the characteristics of the data, as suggested by the researcher, can be confirmed by others who read or review the research results or the extent to which other researchers can confirm the findings of the research. To ensure internal coherence and consistency of the research process, continual reference was made to the notes of the research process, and a continuous checking with the texts as objects of study. These were notes that were made on the books on a continuous basis throughout the analysis process by myself. Checking the research product's internal coherence determines conformability. The research product's internal coherence includes the data, the interpretations, the findings, and the recommendations (Amado, Arce, Farina & Vilarino, 2016).

4.8 Conclusion

This chapter outlined research design and aimed to explain and justify the methodology used in this study. Using an interpretive paradigm to thematically analyse qualitative data in logical steps enabled me to develop themes. Although it was a time-consuming process, content analysis required re-reading, which was necessary to ensure that no meanings were missed, and that all the components of the Framework developed in Chapter 3 were identified. Colour coding the similarities and differences between the texts ensured the development of coding schemes, which was a helpful process.

The following chapter will explain the presentation of the data production process of a Grade 6 South Korean mathematics textbook. Initially, I analysed the two textbooks chapter by chapter, but then decided that addressing the structure of the respective textbooks would be beneficial before the 4 stage and 8 heuristics analysis. I then included horizontal and vertical aspects to the framework analysis that focuses on the structure, content and problem-solving aspects associated with the features of the textbooks. This comprises of Structure and organisation (SO), Layout (L), Content factors (CF) and Didactic factors (DF).

CHAPTER 5: Structural and didactical analysis of the South Korean mathematics textbook

5.1 Introduction

Problem-solving is regarded as occurring at 2 levels, namely the horizontal (structure and organisation, layout, content and language) and vertical (didactical factors, conceptual understanding. Polya's 4 stagemodel and problem-solving heuristics), with this chapter focusing on the horizontal level of analysis of the South Korean (SK) textbook. The overall book presentation impacts greatly on the 4 problem-solving steps and 8 heuristics, therefore, Chapter 5 (South Korea) and 6 (South Africa) will present a description of the books as a background to contextualize how problem-solving is enabled. Chapter 7 will compare their problem-solving (PS) and Conceptual understanding (CU) components. The vertical analysis addressed issues related to the conceptual understanding didactic factors and problem-solving procedures within the two textbooks.

According to Savides (2016), there are international guiding principles for designing textbooks, with the Hong Kong Educational bureau having produced and made available their design criteria, which will be reviewed to ascertain whether the SK mathematics textbooks meets them (Bureau, 2016). This chapter therefore reviews the horizontal analysis of the SK textbook, which consists of the structure and organisation, layout, representation of content and language.

5.2 Structure and organisation of the South Korean textbook

According to Lo, Hew and Chen (2017), the structure of a textbook allows for smooth and simple continuity of concepts and content. Each chapter needs to be inclusive of a specified learning outcome. The material should be organized, have clear visuals and be specific to the intended curriculum without deviating from the purpose. The South Korean (SK) text is compact, presents a clear structure, covers six topics and is in-depth. The structural presentation was easy for the researcher to follow as it was spaced well and easy to understand. The structure and organization of the South Korean textbook is represented in table 5.1 below, the structure highlights the organization of the content, structural features and overview of the textbook.

Table 5.1: South Korean textbook structure and organisation

Structure of the SK textbook	Physical features	Description
a. Content sequencing	Keywords	Beginning of each new unit
	Concepts	Beginning of each new unit
	Vocabulary	Beginning of the textbook
b. Structure	Table of contents	Beginning of the textbook
	Chapter titles	Beginning of each unit
	Headings and outlines	Beginning of each new unit
c. Overview	Students' guide	Introductory section
	Learning targets	Beginning of each unit
	Learning targets (recap)	Summary at the end of unit

5.2.1 Content sequencing

The content sequencing in the SK textbook is curriculum appropriate and logical. The key words and concepts are identified and highlighted in colour-coded form, which give students' direction and offers a sequence to the activities. The structure of the content is made apparent by means of functional information including table of contents, chapter titles, headings and outlines, which are either found at the introduction of the book or the beginning of each unit. The structure is further colour-coded with clear instructions and visuals. The textbook is presented in a colourful manner, which is beneficial to students in the learning process as it also makes for an attractive textbook.

Pictures are used to introduce a topic in a story format throughout the textbook, and the topic is concluded with constructive experiments, games and finishing activities. There is a systematic development of mathematical problem-solving procedures in all topics, which start with introducing visuals that give the students an idea of the problem-solving procedure that are going to be taught and end with activities that require understanding of the concept to be taught.

5.2.1.1 Representation of challenging activities

Research by Vong and Kaewurai (2017) explains that one of the skills which require improvement in the 21st century is critical thinking, being considered a skill which can be improved in a person's lifetime. Therefore, challenging activities develop students' critical thinking ability, which can be achieved by assigning more difficult activities or questions for a group of high-ability students in the classroom. The authors further explain that students exposed to challenging questions assisted in improving critical thinking skills. In the SK textbook, Figure 5.1 provides students with challenging activities that require additional problem-solving procedures which promote the improvement of critical thinking. The student is asked an open ended question in Figure 5.1, he is asked what is needed to solve the given problem. This requires the student to implement a problem-solving procedures. The following steps could be followed in solving the problem; Understanding the problem, what strategy can be used, carry out the plan and check the answer.

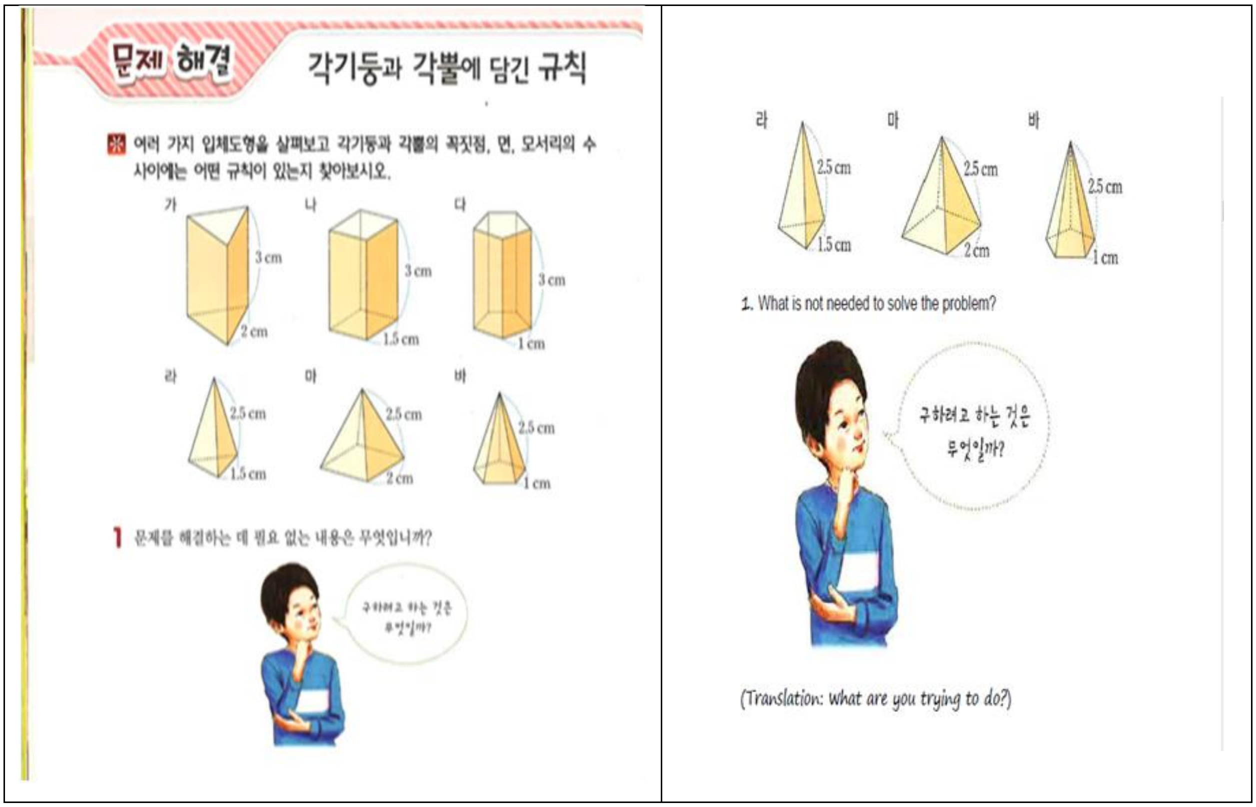


Figure 5.1: SK textbook: Challenges which accompany problems, p. 28

The challenge questions that are provided focus on allowing students to make connections instead of just accepting facts. Research conducted by Tarım (2017) states that word problems are presented to students during their schooling career with the intention of assisting their enhancement of skills, such as reasoning, representing, modelling and communication. The South Korean textbook provides advanced students with ample word problems and individual projects enabling their inquisition on a topic (See, Figure 5.1). There are two pages of problem-solving questions after every topic.

5.2.2 Structure of the SK textbook

The SK textbook has been designed to be a vessel of necessary information, prompts and strategies to learn and consider. There are pictures for students to relate to and problem-solving procedures to instigate the scaffolding of information (See, Figure 5.3). Most texts highlight the flow of the chapters which is done in a clear manner in the SK textbook (See, Figure 5.2). Each key concept starts with a clear description of the aim; which specifically labels the terminology to be incorporated, segments the concepts into manageable steps and provides visuals that prompt the student to follow the correct procedures through the use of headings, sub-headings and bold titles.

5.2.2.1 Representation of the table of contents

The SK textbook provides the same sequence of activities for each topic, the introduction has pictures that are presented with examples for student to follow in order to complete the given questions (See, Figures 5.5 to 5.9). Freedman, Leora and Plotnick (2019) explain that an introduction identifies the topic, it provides essential context, a particular focus and engages the students' interest. This affords them the chance to be aware of their learning style and discover relevant real-world applications, which is evident in the SK textbook (Treadwell, 2018).

The topics in the text are presented in a structured manner, which starts with a welcome message that informs teachers and students about the content of the textbook. It further explains that the content has been designed to develop students' knowledge, skills and understanding which is essential for Grade 6 level. The sections for each term in the SK textbook are presented with images (Figure 5.2) in the introduction with the relevant page numbers. Refer to Figure 5.2 below.

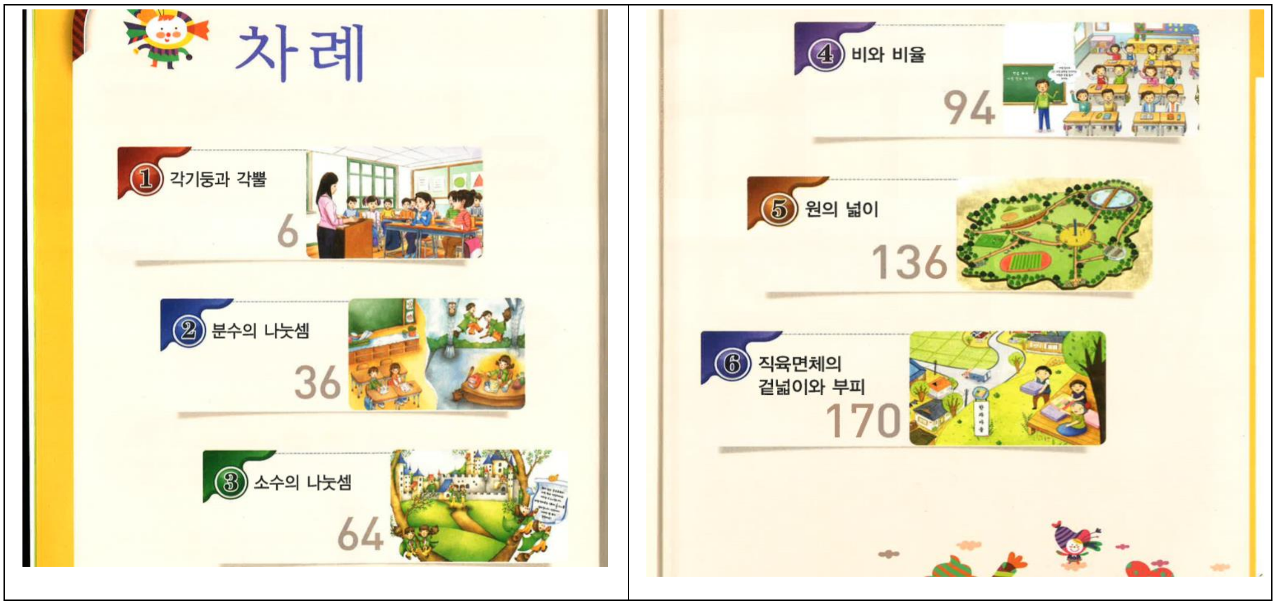


Figure 5.2: SK textbook: Content pages for each term, p. 1

The topics are linked to problem-solving procedures throughout the textbook, starting with real life stories and explanations (evident in Figure 5.11), and ending with investigations, problem-solving and games as activities (evident in Figure 5.12). There is a patterned development of mathematical problem-solving procedures in all topics. The topics started with question about prior knowledge aimed at linking it with current information and indicate the questions and the procedure required to solve problems, which provides guidance to students (Bureau, 2016).

5.2.2.2 Representation of questions

The textbook has approximately 3 to 5 questions per page, which shows that the focus is on quality of tasks, to aid in the growth of critical thinking and problem-solving abilities with the purpose of mastering the procedure. The textbook focuses on addressing developmental types of questions, with Figure 5.3 showing 3 questions on page 26 that build on each other.

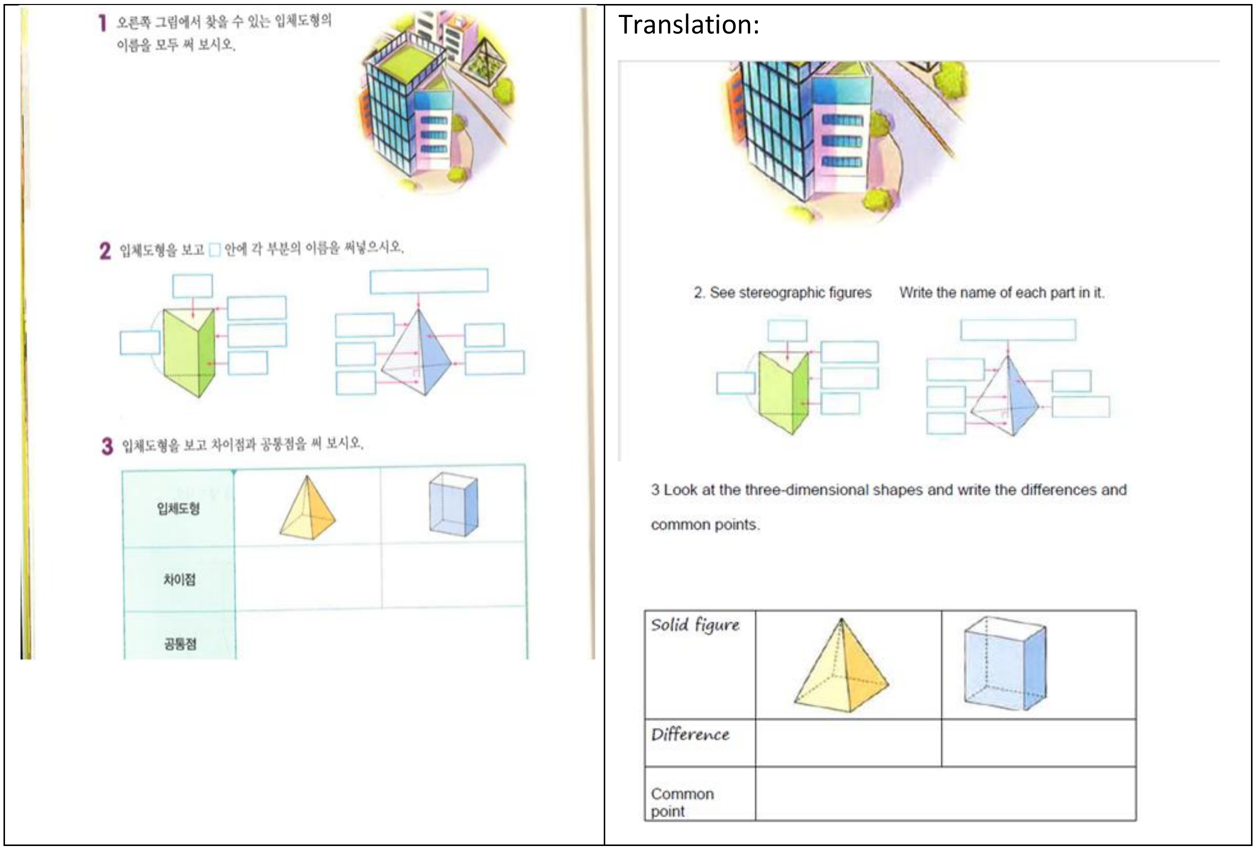


Figure 5.3: SK textbook: Number of problems on a single page, p. 26

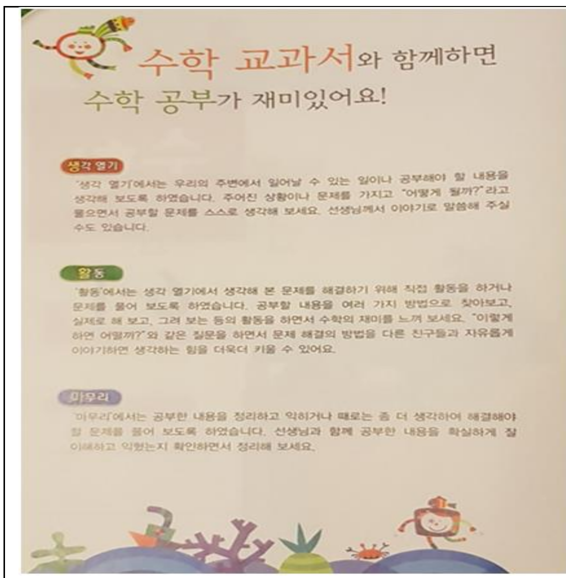
The text had few questions on a single page, probably as it is used as a textbook and a workbook, which allows the students to complete tasks. In the previous twenty years there had been a wide consensus about an importance of adaptive expertise in mathematics, which includes a flexible use of strategies. Consequently, researchers and educators have dealt with this phenomenon and examined instructional approaches that aim to improve the development and understanding of the students' strategy, being used in favor of a more adaptive manner, according to various authors (Heinze, Arend, Gruessing & Lipowsky, 2018; Torbeyns, Hickendorff, & Verschaffel, 2017; Torbeyns & Verschaffel, 2016). From an instructional perspective, teaching and learning material in the book are of notable attention in this situation as it supports mathematical classroom activities, and therefore textbooks provide a chance to understand adaptive strategy.

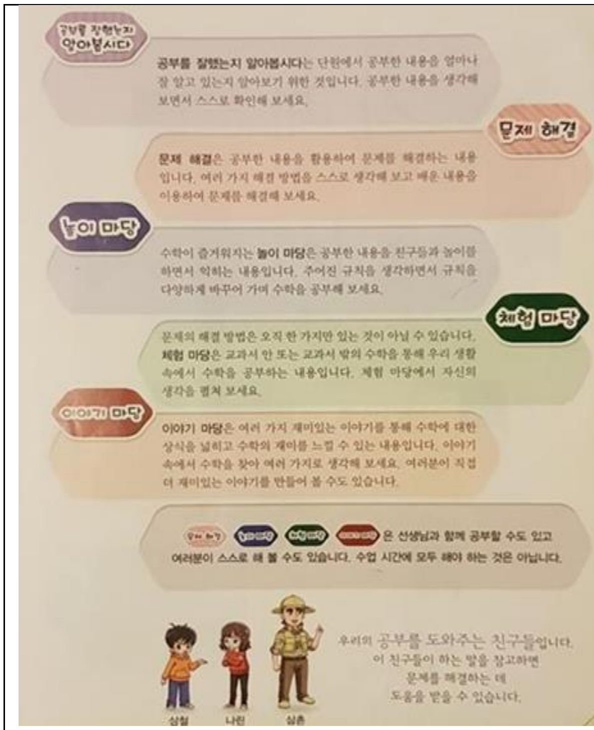
5.2.3 Overview of the learning targets in the SK textbook

An overview of the learning targets is put at the beginning and a summary provided at the end of each topic. A simple student's guide is available in the introductory section to demonstrate how they can use the textbook, evidence for these claims can be seen in Figure 5.4 below. In this study, overview relates to the presentation of colour-coding being a dynamic method of organisation of information which students require in order to learn, by using bright colours to capture the interest of visual students (Al-Ayash, Kane, Smith & Green-Armytage, 2016).

5.2.3.1 Representation of colour coding in the SK textbook

Colour coding is a system of using a certain colour in an area to represent a specific problem-solving procedure. Pan (2014) extended his previous study by verifying how colours influence the memory, with the study suggesting that the use of bright colours can result in an increased interest in learning and is effective in improving memory. Figure 5.4 presents a colour-coded sequence of mathematic topics, which are given at the beginning of the textbook and explained in detail and are indicated in the same style for each topic, making it easy for understanding and learning.

<table>
<tr><td rowspan="4">
</td><td>Mathematics Textbook in the company of mathematics study. Its fun!</td></tr>
<tr><td>Open your mind: In “open your mind”, we tried to think about what could happen around us or what we needed to study. If you have a given situation or a problem and you ask yourself “what will happen?” Think about the problem you're going to study. You could tell me a story.</td></tr>
<tr><td>Activity: In 'Activity', you can do your own activities to solve problems you have thought about. I tried to solve the problem. You can find out how to study in various ways, Feel the fun of mathematics while doing activities such as doing and drawing. Like this, how do I do it? Ask questions such as how to solve problems freely with other friends. If you talk, you can grow your strength more.</td></tr>
<tr><td>Finish: In 'finishing', you need to organize and study what you learnt. I tried to solve the problem. I am sure you have studied well with your teacher. Make sure you understand and get organized.</td></tr>
<tr><td rowspan="3"></td><td>Lets find out if you studied well. How much did you study in the unit? To know if you know well. Think about what you studied. Please check it yourself.</td></tr>
<tr><td>Problem solving: The contents of the study is used to solve the problem using problem solving steps. Think about: your own solutions and identify what you have learnt. Use it to solve the problem.</td></tr>
<tr><td>Math playground: The playground is where fun maths is played with friends. The content is learnt while playing. Think of a given rule to Study mathematics in a variety of ways.</td></tr>
</table>



Experiential: There may be only one way to solve the problem. Experiential yard is the life of our life through textbooks or mathematics out of textbooks. The contents of mathematics is studied in class. Experience it in the real world.

Open your mind. Story: This is about mathematics through many interesting stories. It is contents that can spread common sense and have fun with mathematics. Find Maths in a variety of ways. You can also create more interesting stories.

You can study with your teacher. You can do it yourself. I am happy in class . It does not have to be all. We have friends who help us study. If you refer to these friends, you solve the problem so that you can get help.

Figure 5.4: SK textbook: Colour coding identification, p. 2-3

The colour coding sequence is red, green, blue, light purple, light pink, navy blue, dark green and brown, this being the order of all tasks that are presented which is seen in Figure 5.4. This is followed by ‘an open your mind activity’ that links the concept and topic to real life contexts. These starter activities encourage students to create a story using different scenarios with the question ‘what will happen’. This activity possibly being designed to develop the creativity and imagination of the students.

Furthermore, the textbook pages are colour coded, firstly, light green pages have activities that represented problem-solving questions for that specific topic. Second, light purple represents the revision of the topic covered. Third, light pink which are the problem-solving procedures that enables consolidation opportunities. Fourth, dark green represents the investigation activities to inspire student to be creative, by encouraging the use of various techniques of the process. Fifth, navy blue symbolized the games that are to be played with peers to promote interaction with other students, which creates a context specific environment to use and indicates the subject specific terminology. Sixth, dark brown represents the cultural stories that encourage individual, peer and group activities to enhance co-operative learning and individual development. Colour assists students to grasp concepts that are taught, it develops attention, retention and memory, according to Dzulkifli and Mustafar (2013). Olurinola and Tayo (2015) further states that colours used in the right proportion in mathematics textbooks can enhance learning and benefit the retrieval process.

The topics in the SK textbook were presented in an interesting format, having started with general motivational instructions to encourage students to have fun, with games and playful activities being found in each section (See Figure 5.12). Gracin (2018) explained that the content of the topics may affect how students think, it is imperative that a textbook provides an equal amount of problem-solving procedures for each topic. The SK textbook provides the same sequence in each topic, which starts with introduction pictures and provides a similar number of questions for each topic.

5.3 Layout of the SK textbook

Textbooks across the globe vary significantly in size, length, and additional physical aspects as well as the curriculum they cover, with respect to the official mathematics curriculum of the country (Glatthorn, Boschee, Whitehead & Boschee, 2018). They also vary in the types of chapters and units they contain and in the way they are laid out. This section reviews the issues related to logical and consistent layout, representation of illustrations, lightweight paper, once-off use of the SK textbook and type of font found in the SK textbook.

5.3.1 Logical and consistent layout of the SK textbook

The sequencing of learning material is displayed in an accurate way to facilitate differentiation. The layout of the SK textbook displays size and spacing, number of pages, problem-solving procedures and colour-coding detail, which all contribute to logical and consistent appearance. The textbook has 215 pages, 15 of which consist of useful resources in the form of cut-outs, which are required for problem-solving activities.

5.3.2 Representation of illustrations in the SK textbook (Non-textual elements)

A non-textual aspect in this analysis represents visual representations which consist of elements which are not entirely verbal, numerical or mathematical symbols (Bureau, 2016). The types of non-textual components represented in the text are indicated in Table 5.2 showing 535 non-textual elements representing problem-solving procedures. Each topic begins with six pages of visuals, which are based on common folklore, and there are approximately seven visuals on each page throughout the text (Refer to Figures 5.5 to 5.9 in relation to visual found at the beginning of a topic, refer to Figure 7.7 for an example of graphs and refer to Figure 7.8 for an example of cut-out cards used in a game).

Table 5.2: The non-textual elements (illustrations) of the Grade 6 SK textbook

Types of Visuals	Number of Visuals
Graphs	71
Pictures	535
Additional resources (cut-out models)	15

Cooper, Sidney and Alibali (2018) explain that students solve problems more accurately when visuals are included. Non-textual elements include pictorial representations, such as pictures, diagrams, tables and mathematical representations such as graphs or mathematical figures. Baek, Wickstrom, Tobias, Miller, Safak, Wessman-Enzinger and Kirwan (2017) further state that the non-textual elements are considered significant for accuracy, connectivity, context, simplicity and aesthetics. The authors note that students have the ability to utilize pictures, models and diagrams to display large amounts of information, which makes it easier for them to reveal relationships and patterns and demonstrate understanding.

5.3.3 Physical elements of the textbook

The physical elements in the textbook have been incorporated into the layout, for the purpose of this analysis. The layout discusses the type of paper, textbook as a workbook, size, font and dimensions of the textbook.

5.3.3.1 Lightweight paper

The weight of textbooks can be reduced by using lightweight paper, and can be separated into a few thin volumes, printed in separate modules or bound in loose leaves. Light weight paper is thin and compact in size, and the relative ease of completion can help create a sense of satisfaction in students.

5.3.3.2 Textbook as a workbook

The design of a textbook that is also a workbook means that students can potentially use it on their own and interact with it on a regular basis. Students have the ability to customize their textbook by making memos, highlighting and merging linked sections which are based on their understanding and preliminary knowledge when the book is owned by each student, and does not have to be returned at year's end for use by another learner (Refer to Figures 7.4 and 7.7 for examples of the textbook being used as a workbook and textbook). The SK textbook provides additional resources that are used with specific activities, games and stories, which only allows for a once of use (refer to Figure 7.8 which shows play cards which are found as additional resources attached at the end of the book and Figure 7.5 which shows three dimensional shapes which are cut-outs as well).

5.3.3.3 Type of font and dimension of the textbook

According to the guiding principles of structure and organisation, the layout of the SK textbook is appropriate and logical, which is formally used by British imperial sizes (Winning, 2019). The text size is consistent with research on the effect of font size (12) on reading comprehension (Katzir, Hersnko & Halamish, 2013). Katzir, Hersnko and Halamish (2013) concluded that by Grade 5, students do not need larger 'childlike' fonts to facilitate comprehension. It is unclear whether decreased font size is a hindrance during specified time blocks for learning in school. Based on the 'Eyecare' Circular issued by the former SK Education Department (now the Education Bureau), the minimum font size is equivalent to size 12 in Microsoft Word for legibility. However, in order to avoid eye strain and for more comfort of reading, larger size fonts are recommended, especially for lower levels.

The dimensions of the book are 27cm by 21cm, which is slightly smaller than an A4 paper size (Wiltshire, 2019). The international standards for most countries are 27.7cm x 21cm for reference, the A4 size is standardized in English first language countries. The spine of textbook is glued and is strong and durable, the paper has not been laminated and feels like quality paper. The paper does not feel thin enough to rip or wear down with typical student use, and seems easy to write on and erase mistakes, without degrading. The SK textbook is divided into topics, which aids in the development ability in students to manage information as part of their learning (Table 5.3).

Table 5.3: South Korean textbook layout

Layout of the SK textbook	Physical features	Description
External features	Type of paper	Lightweight (0.69kg)
	Textbook and workbook	Once of use (TB)
	Size of Font	12
	Dimensions of the TB	27cm by 21cm

5.4 Representation of content in the SK textbook

Research conducted by Leonard (2018) states that mathematics content familiarizes students to concepts, skills and thinking strategies which are vital in daily life and encourage learning in the curriculum. It assists students in understanding the numbers, patterns and shapes which are seen in the surrounding world, it presents methods of handling data in an increasingly digital world and creates an essential contribution to their growth in becoming successful students. The representation of content in the SK textbook consists of aligning the aims and objectives, providing supplementary materials, current content presented (linked to real life), prior knowledge and multiple representation of problems.

5.4.1 Alignment of aims and objectives

A mathematics textbook presents the four components of the SK curriculum, which underpins didactic factors (aims, content, learning/teaching strategies, assessment) to enable student learning. Intended curriculum provides the aim that the pertinent national education authorities and society expect students to achieve at school in terms of knowledge, understanding, skills, values, and attitudes to be acquired and developed (Glatthorn, Boschee, Whitehead & Boschee, 2018). The objectives should encapsulate the essential characteristics of what the topic intends to do, as learning and teaching are purposeful activities. Educational intentions for a topic depend on pre-existing knowledge and skills that are required for a student to have a reasonable chance of success in achieving the intended learning objectives according to Cooper, Sidney and Alibali (2018). Every chapter in the SK textbook begins with activities which require students to recollect and to use existing knowledge, therefore reinforcing the relationships between current knowledge and the new ideas which are being learnt (Uzunboylu & Karagözlü, 2017). Six pages of visuals are presented which require recollecting prior knowledge or to create connections in students' minds between what was learnt and what needs to be learnt and preface every new topic (refer to Figures 5.5 to 5.9).

Objectives are statements of the attributes and capabilities that a student should be able to display on successful completion of the topic and need to be aligned to the aims. They provide the basis for determining student progress and designing assessment strategies and methods. They also provide signposts towards appropriate content and learning interactions to help students achieve those outcomes and fit them in context. In conclusion, it is apparent that the South Korean textbook is well aligned to its aims and objectives.

5.4.2 Provision of supplementary materials in the SK textbook

According to Bureau (2016), the content is self-contained and adequate to effectively focus on the learning objectives of the curriculum without the need for the use of supplementary materials which are associated with textbooks, to which the SK textbook adheres adequately. All the resources that are required for teaching and learning are provided as additional resources at the back of the textbook as cut outs, and each student is given a textbook for only their use, and are not required to return the textbook at year's end.

According to Dunning, Huchette and Lubin (2017), mathematical cut-outs for models are important as they help to explain known physical phenomena. Andersson and Palm (2017) discuss using cut-outs for model making as being vital in mathematics as it assists students in exploring the potentials of the project, as well as the possibilities of having different solutions. Furthermore, Starko (2017) states that it also helps the brain to think in 3 dimensions and advances the students' imagination. The models represent the real problem situations to aid in making decisions faster and more accurately and offer convenient and cost advantages over other means of obtaining the required information related to tactile experience. The South Korean textbook has 7% of its content as cut outs to be used in problem-solving procedures. Appropriate consideration is given to students’ prior knowledge and learning experience, and there is continuity in the development of concepts and skills to facilitate a smooth transition between different key stages of learning levels, which is in line with Retnowati (2017).

5.4.3 Presentation of current content (linked to real life)

A combination of relevant strategies can promote greater learning for more students, with a good introduction identifying the topic, providing essential visual context, indicating a particular focus and needing to engage the students' interest. This allows students to become aware of their ideal way of learning to search and find real world applications of what they are learning. Literature states that key topics in classroom instruction have to be presented in an accessible fashion (Good & Lavigne, 2017). Their purpose being to promote better understanding and transfer to practice, and to articulate the roles of teacher-centered pedagogy, student-centered instruction and project-based learning. The classroom is a familiar environment to the students and Figures 5.5 shows furniture that represented prisms and pyramids, with the teacher's table and the students' desks characterizing a rectangular prism. Figures 5.6 is a visual of a bakery, which would be familiar to students, the cakes being in the shape of hexagonal prisms and triangular prisms. The flowerpots in Figure 5.7 represent hexagonal shapes and the wedge at the door in Figure 5.7 is a triangular pyramid. The learning outcome specifies 3D shapes in the environment, with the content visuals depicting those that the students may be familiar with. While these visuals are consistent with the learning outcome, it is unsure whether students of the future will be familiar with a bakery, as online shopping and modern spending methods may vary depending on individual students' exposure to these 'real world' examples.

Figure 5.5: SK textbook: Shape identification, p. 6

Figure 5.6: SK textbook: Connecting shapes with real life objects, p. 8

Figure 5.7: SK textbook: Connecting shapes with real life objects, p. 9

Figures 5.8 and 5.9 have visuals of cakes in a triangular prism, triangular pyramid and hexagonal prism shapes, and shelves in the shape of cuboids and rectangular prisms. These visuals are relevant and connects to real life objects, which allows students to make a connection with what they are learning. Subsequently, a problem-solving question is presented, whereby the student has to create their own activity to solve problems previously thought about in the 'open your mind' task. According to Simões, Redondo and Vilas (2013), illustrations and collaboration with peers are promoted during the completion of this problem-solving procedure to enhance understanding, enjoyment and social skills. Once students complete their own understanding of the concept and topic, they are presented with activities to complete, for which drawings and pictures are provided that aids in the understanding process.

Figure 5.8: SK textbook: Shape identification, p. 10

Figure 5.9: SK textbook: Shape identification, p. 11

Each chapter ends with a ‘finish activity’, as the learning outcomes and concepts presented are reviewed to check for understanding and offer higher level critical thinking opportunities (See, Figure 5.10 below).

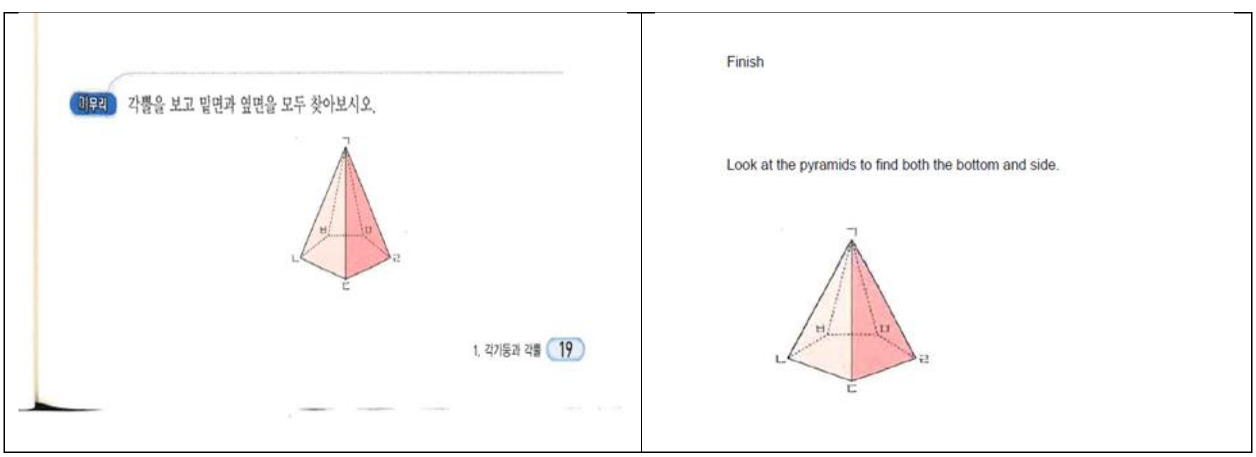


Figure 5.10: SK textbook: Finishing activity, p. 19

Open-ended questions are provided as prompts to help differentiate understanding. Each learner can reflect on what they have learnt and demonstrate their understanding with cooperative games, activities and a final story. The 'finish activity' provides diligent students with further opportunities for consolidation, reflection and to check for understanding. For others, the questions may prompt independent conversations or require further support from an educator, with student agency being required for the finish activities to be beneficial and reinforced. According to Bureau (2016), the content in the SK textbook is current, the information and concepts in the textbook are pertinent and precise, the ideas are coherent, there are ample examples and illustrations, which are interesting and is relevant to students' knowledge.

5.4.4 Linking learning to prior knowledge

There are visual representations before each topic begins, that are used to introduce each one in a relevant manner to enable students to relate to them easily (See Figures 5.5 to 5.9). Research conducted by Cooper, Sidney and Alibali (2018) suggests that prior knowledge, when linked with new problem-solving procedures, enables them to be easily understood with the aid of visual cues. Every topic in the SK textbook starts with questions that link prior knowledge that is attained through learning with current activities, and allows students to change from observations to reasoned interpretations based on the conceptual models available to them (Alcalde, Bond, Johnson, Butler, Cooper & Ellis, 2017).

5.4.5 Balancing the breadth and depth of content weighting

The breadth of the mathematics textbook would include the topics and time allocated to each topic. While the depth would make up the level of questioning in the textbook, e.g. making use of Blooms' taxonomy levels of questioning. The balance in breadth and depth of the South Korean curriculum includes content weighting and sequencing, the use of correct concepts and the six content areas (prisms and pyramids, fractions (division 1), fractions (division 2), percentages, width of a circle, width and volume of a cube) and the percentage that is allocated to each topic.

The content is usually designed to give appropriate balance to subject knowledge, application strategies, context and consolidation skills, and is organized into two main principles: this would entail the weight and sequence. Time is a major factor in determining the scope of content and the balance between breadth and depth (weight). The competencies used to influence the depth of the content would consist of mathematical thinking and reasoning, mathematical argumentation, mathematical communication, modelling, problem posing and solving, representation, symbols, and tools and technology. The South Korean Curriculum is used to enhance the learning experience of students in knowledge and application, which is purposed to improve the depth of the content in the mathematics textbooks.

The table of contents was examined in order to identify the topics of the curriculum, where larger units are segmented into manageable learning outcomes. The data was analysed and presented with description and percentages, and the items under each feature scored accordingly. For a significant comparison and better interpretation, it was essential to determine the percentage under every section. All the gathered data for the additional features were score aggregated and then reported in a percentage form. Table 5.4 portrays the weight of the curriculum, topic structure, page count and content areas in the SK mathematics textbook, as determined by the researcher.

Table 5.4: Content weighting in the textbook (215 pages)

Weighting of topics (215 pages)	Total page No. (%)	Topic description and content
Topic 1: Prisms and pyramids (20 pages) **Plus:** revision (2) investigation (2) story (2) problem-solving (2) games (2)	30 (14%)	Geometric shapes e.g. triangle, rectangle, pyramids
Topic 2: Fractions (Division 1) (18 pages) **Plus**: revision (2) investigation (2) story (2) problem-solving (2) games (2)	28 (13%)	Division of fractions e.g. (2 ÷ ¼)
Topic 3: Fractions (Division 2) (20 pages) **Plus:** revision (2) investigation (2) story (2) problem-solving (2) games (2)	30 (14%)	Division of decimal fractions e.g. 6.8 ÷ 0.9
Topic 4: Percentages (28 pages) **Plus**: revision (4) investigation (4) story (2) problem-solving (2) games (2)	42 (20%)	Percentages
Topic 5: Width of circle (22 pages) **Plus**: revision (2) investigation (2) story (2) problem-solving (2) games (2)	32 (15%)	Measurement (Diameter)
Topic 6: Width & volume of a cube (20 pages) **Plus:** revision (4) investigation (2) story (2) problem-solving (2) games (2)	32 (15%)	Measurement (Distance)
Overview / Contents	5 (2%)	Information pages
Glossary / Useful Resources	15 (7%)	Cut-outs for activities

The curriculum is presented in specific topics, their measurement being referred to as weighting, this being measured by the number of pages allocated to a specific topic, according to the table

5.4, there are six topics in the SK textbook. The topic on “percentages” is covered in 28 pages which translates into a weighting of 20%, while Prisms and Pyramids were allocated only 14% of the weighting. However, when Fractions (1) and (2) are combined, they have a higher weighting of 27% which is covered by 58 pages, more than a quarter of the content. Joutsenlahti and Perkkilä (2019) contend that Division of Fractions is a difficult concept to understand for both students and teachers. Adu-Gyamfi, Schwartz, Sinicrope and Bossé (2019) noted that the Division of Fractions is a crucial mathematical domain and highlighted the necessity for teachers to interpret the concept in different ways and to utilize these in multiple contexts. Xin, Tzur, Hord, Liu, Park and Si (2017) states that in the mathematics curriculum outcomes, fractions have raised expectations for schools and textbook designers.

The designers of the SK mathematics textbook have taken the importance of fractions into consideration (Doabler, Fien, Nelson-Walker & Baker, 2012). Weight is placed on Division of Fractions, which can typically be solved by using 'invert and multiplying' algorithms and are simple for students to calculate instinctively without an understanding, according to Sprenger (2018). It is difficult for students to understand why the usage of ‘the invert’ in fraction division is necessary, which involves an understanding of computational processes while using intricate fraction concepts. Students' understanding will differ, depending on the order in which they are presented and type of models which are used. The ‘fraction division conceptualization,’ ‘visual model’ and ‘algorithm’ are presented in the SK textbooks. While there is a significant focus on fraction division, fraction addition or subtraction were not addressed and it is unclear why these elements have been omitted (they may have been a focus on the previous Grades learning outcome), as they may be beneficial for review of strategies that can be utilized to problem solve.

While Division of Fractions holds the most weight, Percentages had 42 pages, which is 20% compared to 32 pages of the Width of a Circle and Width and Volume of a Cube, each holding a weight of 15%. Their similar weighting shows that the same amount of importance was placed on both these topics. Cut-outs were provided for students to create their own models in order to improve the problem-solving procedures under these topics.

The SK textbook has contextual learning from which students can grasp understanding of percentages while making connections to everyday life. Al Tamimi (2018) states that manipulative percentage activities, through which students may develop an instinctive idea of what they are learning, can possibly enhance their creativity. Reasoning to validate mathematical outcomes based on students' knowledge and experience was portrayed throughout the topic of percentages, with students being required to complete hands-on activities to show understanding through experience.

Prisms and Pyramids topic 1 are next in the weighting process, and consist of 30 pages (14%), the topic covering measurement of three-dimensional geometric shapes, and the textbook shows a visual representation for every problem within this topic. Baek, Wickstrom, Tobias, Miller, Safak, Wessman-Enzinger and Kirwan (2017) explain that pictorial strategies assist in the advancement of problem-solving skills. Sprenger (2018) also states that visual aids promote conceptual understanding and problem-solving skills, with an appropriate balance between breadth and depth in the presentation of the subject content textbook.

5.4.6 Multiple representation in the SK textbook

Multiple representations in mathematics are methods to represent, explain and refer to the same mathematical entity. Representations are used to understand, develop and connect dissimilar mathematical features of the same question or problem, along with identifying links between various strategies. According to Papageorgiou, Amariotakis and Spiliotopoulou (2017), numerous representations include graphs and diagrams, tables and grids, formulas, symbols, words, gestures, software code, videos, concrete models, physical and virtual manipulatives, pictures and sounds. Representations are thought-provoking tools for doing mathematics, and in the SK book, is presented in the form of stories, games, word problems, experiments, graphs, pictures, diagrams, tables, concrete objects and cut-out models.

5.4.6.1 Representation of stories

A conclusion to each learning outcome would possibly need to summarize the concept learnt, and aid in the identification of future challenges and directions concerning students' progress. A conclusion is the last part of the topic, which always ends by reviewing the problem-solving procedures taught. Figure 5.11 shows that the conclusion to each topic starts with pictures and ends with reinforcing the understanding of the problem-solving techniques taught with a story.

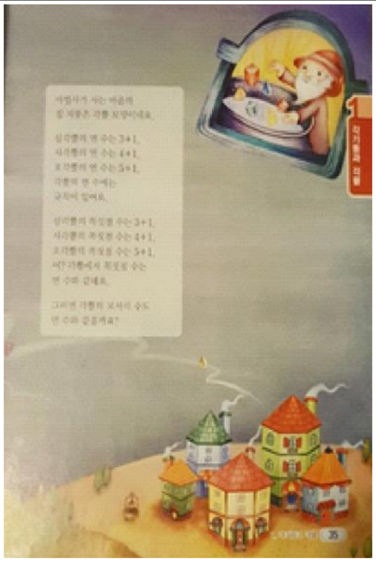
	Story: Magic village Of the town where the wizard lives The chimney of the house is prism. The number of triangles 3+2 The number of sides of a square 4+2 The number of faces 5+2 Name the prisms.

Figure 5.11: SK textbook: Story connecting to shapes, p. 34

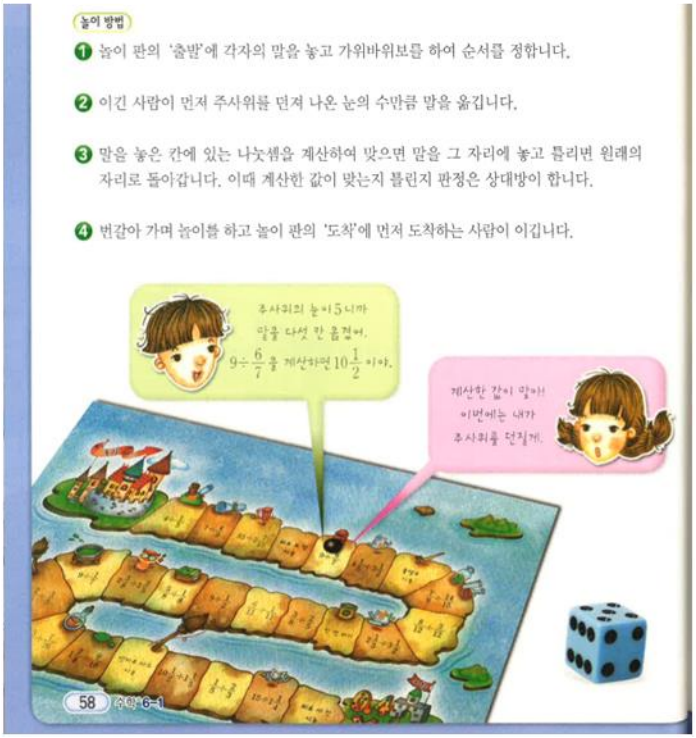
The SK textbook consistently concludes with a story, for example, the conclusion of Prisms and Pyramids ends with (represented on p. 34 & 35) a mystical story about the magic village, which indicates the shapes that have been studied in this chapter, highlighting all aspects of Prisms and Pyramids. The activity requires students to identify the various prisms in the illustration using the given information. The amount of questions provided in a textbook would determine the students' exposure to problem-solving techniques. According to Wlodkowski and Ginsberg (2017), connecting real life situations to the problem-solving procedures taught helps to make the learning process relevant. Students find stories easy to remember, as they help to organize data into meaningful patterns and encourage purposeful learning. Sprenger (2018) suggests that the brain is selective and tends to recall information which created a memorable pattern.

Stone (2017) explains that stories have an essential role in the growth and development of students, where an author's purpose may be to inform the reader, relate a story or recount events. Reading and sharing stories can assist students in learning sounds, words, mathematical language, develop early literacy skills and learn the difference between 'real' and 'make-belief. Research by Istenic Starčič, Cotic, Solomonides and Volk (2016) suggests that vocabulary, comprehension, sequencing, memory and creative writing all benefit from storytelling. Storytelling develops listening skills which are crucial in learning and to establish relationships between problem-solving procedures. It also encourages communication, creative thinking and problem-solving skills, with the level of difficulty of the content needing to be consistent with the curriculum requirements and the cognitive level of students.

Mathematical concepts which are based upon cultural perspectives permit students to not just reflect and appreciate their own culture, but also the culture and traditions of others (Leonard, 2018). Interactions amongst students are a crucial component of the incorporation of cultural components into mathematical activities. The socio-cultural integration in textbooks includes a connection between the teacher and student based on social interaction. Klem and Connell (2004) observed that when textbooks represented familiar socio-cultural stories using visuals and problem-solving procedures, the learning environment appeared more organized, and students tend to be increasingly involved in the learning material.

5.4.6.2 Representation of games

Instructional games are being accepted in classrooms, as the learning benefits of student engagement and immediate feedback are recognized. The findings of Huizenga, Ten Dam, Voogt and Admiraal (2017) showed that the majority of teachers who utilize games in class recognized student engagement and improved cognitive learning outcomes. Figure 5.12 provides students with simple games in which they are required to work with others. According to Lameras, Arnab, Dunwell, Stewart, Clarke and Petridis (2017) studies show how learning attributes and game mechanics can be prepared and used in the classroom.



Play method:

1.Set the order by placing your own words on the "departure" of the play board and lay a rock on the scissors.

2.The man who won is going to say the number of eyes when the player threw the dice.

3.If you calculate the end of the horse in the compartment and put the incorrect one in place of the original. Return to the seat. If the calculated value is correct, then the other party makes a decision.

4. The first person plays alternately and arrive at the ' Finish ' of the play board.

Boy: 'The Lord's son in law's eyes are 5, so he's moved 5 spaces. 9 ÷ 6/7 is 10 ½'.

Girl: 'I'll throw the Lord's son- in- law this time'.

Figure 5.12: SK textbook: Games, p. 58

Games presented in the SK textbook are used as a classroom strategy to develop interactive learning and are set for working in pairs and groups. Research by Awada and Gutiérrez-Colón (2019) claim that students may also develop cooperative learning and tolerance, and that they may also enjoy sharing and socializing with their peers by playing in groups. It is further mentioned by Ravyse, Blignaut, Leendertz and Woolner (2017) that when presented appropriately, games can grab students' attention and actively engage them with mathematics concepts in a fun and non-threatening way, which is evident in the SK textbook. Haydon (2017) explains that games offer a context for engaging practice, with numerous skills that students can improve through their playing, such as critical thinking skills, creativity, teamwork and good sportsmanship. The SK textbook is consistent by providing a game at the end of every topic.

5.5 Representation of language in the SK textbook

The language used in the SK textbook is suitable for the intended audience, including that it is written in the mother tongue language, which is Korean, makes it simpler for students to read and understand the instructions. The book makes use of simple language, which enables students to understand the instructions. The illustrations are age and culture appropriate, and the instructions are grammatically and structurally clear, which allows students to follow in general.

5.6 Conclusion

The SK textbook presented an appropriate structure of its content to facilitate learning, the sequence being appropriate and logical, with key words and concepts being identified and highlighted. The structure of the content is made apparent by means of functional devices, including table of contents, chapter titles, headings and outlines. An overview of the learning targets are presented at the beginning and a summary at the end of each topic. An easy student's guide is also offered in an introductory section to educate students on how to use the textbook. The use of each page is consistent with the criteria presented in Badea, Suditum & Presadă (2018).

An analysis had been carried out based on the guiding principles of the Japanese HK bureau, the process following the guidelines from previous studies (Molina-García & Martínez-Bello, 2014; Martínez-Bello & Martínez-Bello, 2016). Having defined the categories and themes, which were derived from the physical features of the SK mathematics textbook, the coding scheme and themes were identified, and the data was analysed and presented. The final coding scheme had 4 categories, namely the structure of the textbook; the representation of topics; the non-textual elements of the textbook and finally the weighting of the curriculum which were represented. Chapter 6 will follow the same process of analysis using a South African textbook.

CHAPTER 6: Structural and didactical analysis of the South African mathematics textbook

6.1 Introduction

The previous chapter focused on the structural features of the South Korean textbook, which influences the representation of problem-solving procedures. This chapter addresses the structural features of the South African textbook. The problem-solving procedures will be analysed before a comparison of the two texts in chapter seven. The same guiding principles outlined by Bureau (2016) that were used in chapter five, will also be utilized for chapter six. Quality textbook design criteria includes structure and organisation, layout, content, language, teaching and learning.

6.2 Structure and organisation of the South African textbook

According to Lo, Hew and Chen (2017), the structure of a textbook allows for smooth and simple continuity of concepts and content. Each chapter needs to be inclusive of a specified learning outcome. The material should be organized, have clear visuals and should be specific to the intended curriculum without deviating from the purpose.

The South African (SA) text is compact, presents a clear structure, covers 38 topics in the Grade 6 textbook which is shown in Table 6.1 below. This textbook was designed to be used as a resource, which means that it can be used over a number of years, by several students. Table 6.1 outlines content sequencing, structure and overview of the SA textbook.

Table 6.1: South African textbook structure and organisation

Structure of the SA textbook	Physical features	Description
Content sequencing	Keywords	Beginning of each new unit
	Concepts	Beginning of each new unit
	Vocabulary	Beginning of each new unit
Structure	Table of contents	Beginning of the textbook
	Chapter titles	Beginning of each unit
	Headings and outlines	Beginning of each new unit
Overview	Students' guide	Introductory section
	Learning targets	Beginning of each unit
	Learning targets (recap)	Summary at the end of unit

6.2.1 Content sequencing

The content sequencing included the representation of concepts and challenges, which provided students with additional information to aid in the comprehension process.

6.2.1.1 Representation of concepts

The South African textbook uses boxes with Math ideas, highlighting problem-solving techniques. Figure 6.1 provides an example of a Math idea, which provides students with guidance on how to approach a problem that they need to solve. This aids to enhance their comprehension of the topics and how to use the correct mathematical vocabulary when discussing problem-solving procedures.

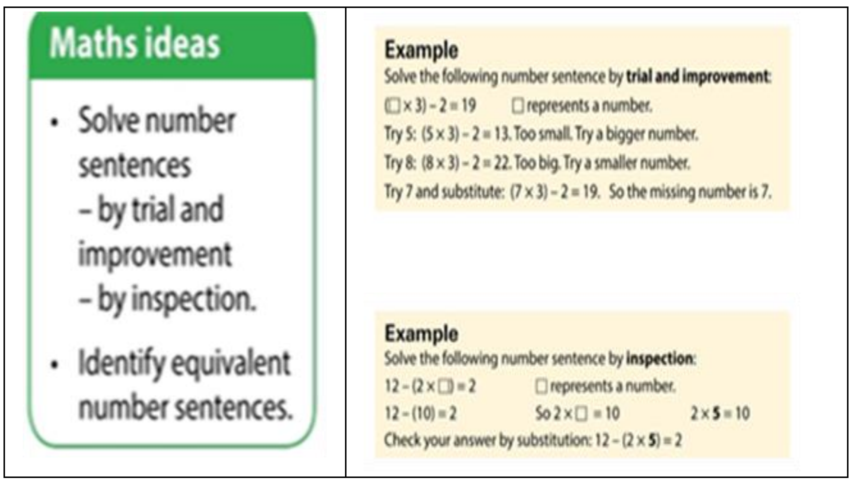


Figure 6.1: SA textbook: Math ideas, p. 8

The 'Maths idea' requires students' to solve problems by trial and improvement, then by inspection. In the example provided above, trial and improvement methods are highlighted to demonstrate that the learner can make use of various numbers in the solving process.

By inspection is a method used to check the answer by substitution. Holcomb and Cox (2017) elucidates that a stronger emphasis on 'Maths ideas' facilitates ease of use and understanding. Problem-based instruction, student-led solutions, risk taking, fun, and teamwork time, are provided by 'Maths ideas' in the SA textbooks.

6.2.1.2 Representation of challenge activities

Research by Tanujaya, Mumu and Margono (2017) explains that the difficulty of studying mathematics increases with the developmental progress in numerous fields, with the inclusion of advancements in mathematics. Therefore, challenging activities develop students' critical thinking ability, and this can be achieved by allocating more complex texts or questions for a group of high-ability students in the classroom. The above researchers further explain that textbooks should encourage students to change from factual to conceptual types of questions. In the South African textbook, Figure 6.2 provides students with challenge activities that require additional problem-solving procedures, which can promote the further development of critical thinking.

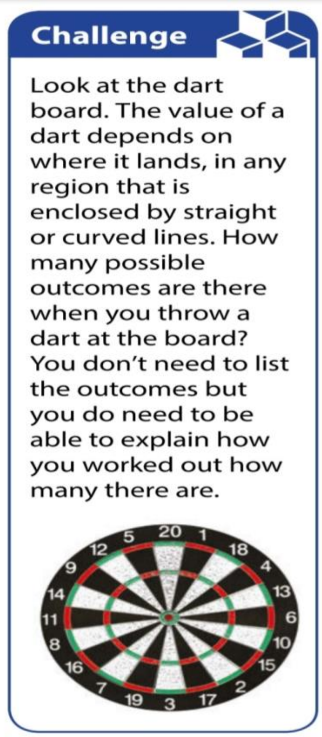


Figure 6.2: SA textbook: Challenges which accompany problems, p. 221

The challenge questions that are provided focus on allowing students to make connections instead of just accepting facts. Nilson (2016) explains that advanced students often learn new concepts promptly and they should be encouraged, to use a variety of texts, websites, blogs, etc. to investigate possible answers.

6.2.2 Structure of the SA textbook

The SA textbook has been designed to be a vessel of necessary information, prompts and strategies to learn and consider. There are pictures for students to relate to and problem-solving procedures to instigate the scaffolding of information and evidence for this claim is provided in Figure 6.6. Each key concept (See, Fgure 6.1) starts with a clear description of the aim; which specifically labels the terminology to be incorporated, segments the concepts into manageable steps and provides visuals that prompt the student to the correct procedures through the use of headings, sub-headings and bold titles (Bureau, 2016).

6.2.2.1 Representation of the table of contents

The SA textbook provides the same sequence for each topic, the introduction of each topic has pictures and are presented with examples for student to follow in order to complete the given exercises (See, Figure 6.3). Freedman, Leora and Plotnick (2019) explains that an introduction identifies the topic. It provides essential context, a particular focus and engages the students' interest. The introduction to a product-oriented approach to learning proved that students are able to connect to real world application easily according to Treadwell (2018).

The topics in the text are presented in a structured manner. It indicates that the content is divided into four school terms of ten weeks per term. Each term starts with a welcome message to the teacher, informing them about the content of the textbook. It further explains that the content has been designed to develop students' knowledge, skills and understanding that is essential for Grade 6 level. The evidence for the claim is provided in Figure 6.3 below. The images in Figure 6.3 are presented in the respective terms colour showing the structure of each term, and respective topics with the relevant page numbers.

Contents

Term 1

Term 2

Term 3

Term 4

Figure 6.3: SA textbook: Content pages for each term, p. 1

The topics are linked to problem-solving procedures throughout the textbook, starting with real life stories and explanations, and ending with revision, problem-solving, assignments, key words, 'did you know' questions and information, expanded opportunities and games as activities. There is a patterned development of mathematical problem-solving procedures in all topics, which start with two or three examples that indicate the questions and the procedure required to solve problems, thus providing guidance to students (Bureau, 2016).

6.2.2.2 Representation of questions

The textbook has approximately 14 questions per page, which shows that the focus is on quantity of tasks, possibly allowing students to practice various questions in order to master the procedure. Figure 6.4 shows that 17 questions were presented on page 39, with the textbook addressing similar types of questions but with a focus on quantity.

Draw and identify 2D shapes

EXERCISE 6.5

1. On grid paper, draw a polygon with five vertices.
   a) How many sides does your polygon have?
   b) What is the name of your polygon?
   c) Does your polygon have all sides equal?
   d) Did everyone in the class draw the same polygon? How are they the same or different?
2. Is it possible to have each of the following 2D shapes? If you think a shape is possible, draw a neat clear sketch of the shape on grid paper.
   a) a parallelogram with only one right angle
   b) a triangle with one right angle
   c) a triangle with two angles larger than a right angle
   d) a pentagon with two right angles
   e) a quadrilateral with all four angles smaller than a right angle
   f) a parallelogram with exactly two right angles
   g) a quadrilateral with two angles larger than right angles
   h) a parallelogram with opposite angles that are not equal
3. Look at all the real life examples of two-dimensional shapes that you may be familiar with. Identify each of the shapes in the pictures. Draw them on square dotted grid paper and then explain where you would see each of these objects or what the purpose of the object is.
   a) STOP
   b)
   c)
   d)

Figure 6.4: SA textbook: Number of problems on a single page, p. 39

The SA textbook had more questions on a single page, probably because it is used as a textbook only and allows for students to gain more practice. Research by Pepin, Xu, Trouche and Wang (2017) mentions that in terms of resources, the teacher makes use of various resources, including textbooks and teacher guides. They consider textbooks and teacher guides as instruments, which should be used for reference to provide the objectives for instruction that can be used with a variety of other resources.

Students in Manokwari, Indonesia, performed better in Mathematics after the use of Indonesian realistic mathematics education (RME), according to a study conducted by Tanujaya, Prahmana and Mumu (2017). Furthermore, this study revealed lesson material to be consistently presented with a standard pattern, such as formulas, examples of questions and practice questions, that were aligned and specific to the textbooks used. This implies that the implementation of effective learning material can improve mathematics results in South Africa.

6.2.3 Overview of the learning targets in the SA textbook

An overview of the learning target is put at the beginning and a revision exercise at the end of each topic. A simple student's guide is available in the introductory section, to demonstrate how students can use the textbook. Research conducted by Al-Ayash, Kane, Smith and Green-Armytage (2016) revealed that reading marks were substantially higher when using a hue of bright colours had a significant impact on students' emotions showing that blue amplified relaxation and feelings of calmness.

6.2.3.1 Representation of colour coding in the SA textbook

Colour coding is a system of using a certain colour in an area to represent a specific information box. Colour coding is a system of using certain colours in an area to represent a specific problem-solving procedure. Pan (2014) contended that colours can influence memory and produce a higher level of attention. The four examples in Figure 6.5 indicate that colours and images are used to differentiate the content for each of the four school terms. The pictures are represented by different colours according to each of the four school terms, making it easy to follow what will happen in each quarter of the year. Furthermore, there are images used to integrate various subjects with mathematics concepts.

First, term 1 (green) gives an image of a kite which is used to inform students about the history of kites. This link to history is further integrated into Mathematics through shapes, angles and measurement. Second, term 2 (red) has an image of the solar system showing that the planets revolve around the sun, it is used to discuss distance, circumference, 3D objects, 2D shapes and time. Third, term 3 (orange) shows an image of an iceberg whereby 90% of it is under water. The ice from South Pole and North Pole are discussed in the context of global warming, recycling, highlighting mass and position.

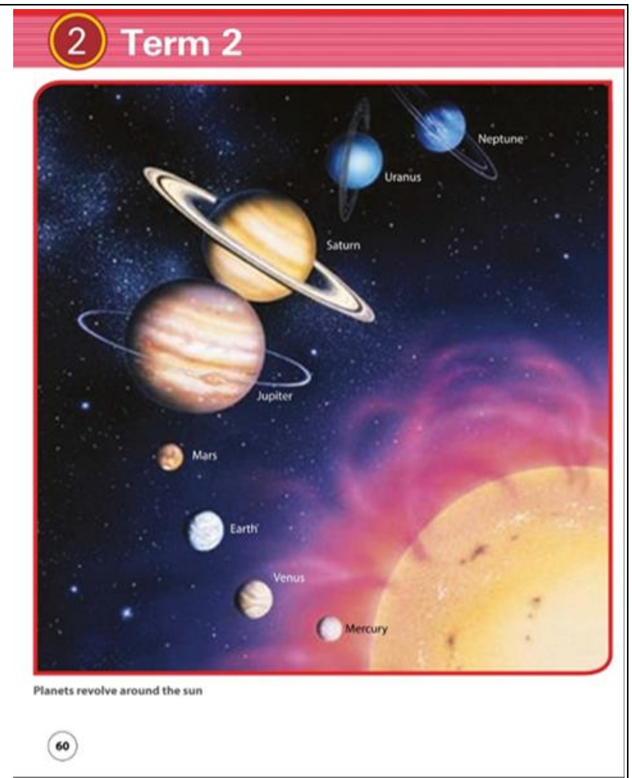
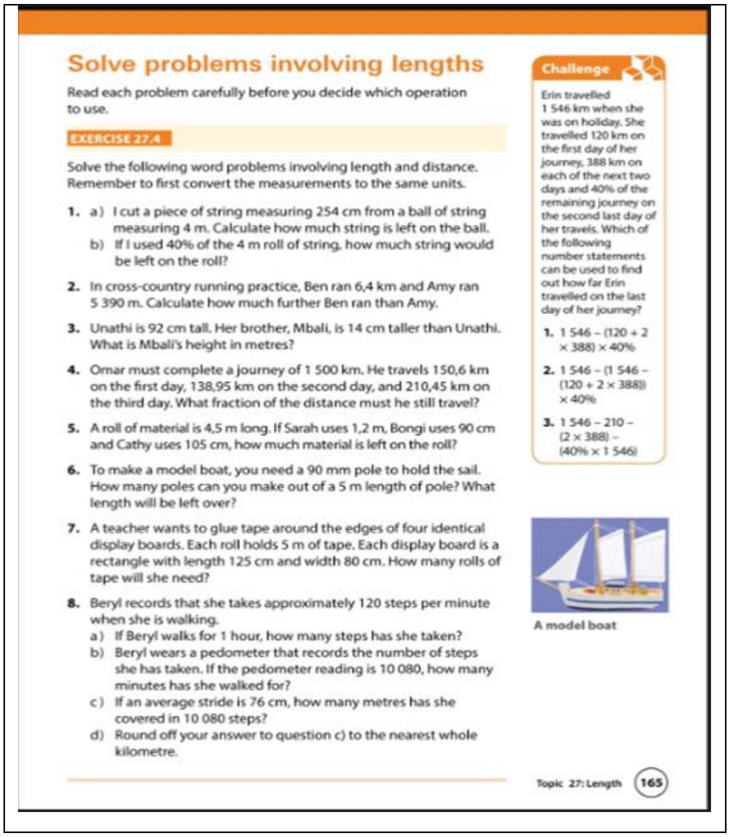


Figure 6.5: SA textbook: Colour representation pp. 2, 60, 110, 166

Lastly, term 4 has 4 images of cell phones being used in various situations, the way that they have changed our lives is discussed. Students are required to do a survey and gather information about how many students have access to cell phones and landline telephones, and entails data gathering and analysis using percentages and fractions. The above data show that subjects such as science, geography and technology are integrated into the mathematics textbook. This highlights that the mathematics subject is not taught in isolation, but it is integrated into other subjects (Thibaut, Ceuppens, De Loof, De Meester, Goovaerts, Struyf & Hellinckx, 2018).

6.3 Layout of the SA textbook

The South African textbook contains revision (evident in Figure 6.8), problem-solving (See, Figure 6.6), investigation questions refer to Figure 6.7, games and projects (See, Figures 6.10 and 6.8)



Solve problems involving lengths

Read each problem carefully before you decide which operation to use.

EXERCISE 27.4

Solve the following word problems involving length and distance. Remember to first convert the measurements to the same units.

1. a) I cut a piece of string measuring 254 cm from a ball of string measuring 4 m. Calculate how much string is left on the ball.
   b) If I used 40% of the 4 m roll of string, how much string would be left on the roll?
2. In cross-country running practice, Ben ran 6,4 km and Amy ran 5 390 m. Calculate how much further Ben ran than Amy.
3. Unathi is 92 cm tall. Her brother, Mbali, is 14 cm taller than Unathi. What is Mbali's height in metres?
4. Omar must complete a journey of 1 500 km. He travels 150,6 km on the first day, 138,95 km on the second day, and 210,45 km on the third day. What fraction of the distance must he still travel?
5. A roll of material is 4,5 m long. If Sarah uses 1,2 m, Bongi uses 90 cm and Cathy uses 105 cm, how much material is left on the roll?
6. To make a model boat, you need a 90 mm pole to hold the sail. How many poles can you make out of a 5 m length of pole? What length will be left over?
7. A teacher wants to glue tape around the edges of four identical display boards. Each roll holds 5 m of tape. Each display board is a rectangle with length 125 cm and width 80 cm. How many rolls of tape will she need?
8. Beryl records that she takes approximately 120 steps per minute when she is walking.
   a) If Beryl walks for 1 hour, how many steps has she taken?
   b) Beryl wears a pedometer that records the number of steps she has taken. If the pedometer reading is 10 080, how many minutes has she walked for?
   c) If an average stride is 76 cm, how many metres has she covered in 10 080 steps?
   d) Round off your answer to question c) to the nearest whole kilometre.

Challenge

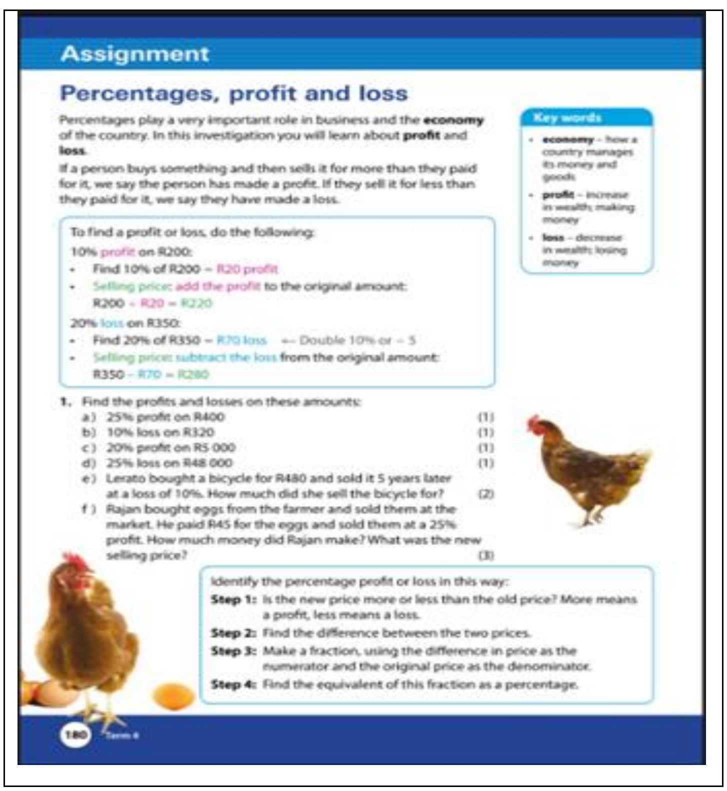
Erin travelled 1 546 km when she was on holiday. She travelled 120 km on the first day of her journey, 388 km on each of the next two days and 40% of the remaining journey on the second last day of her travels. Which of the following number statements can be used to find out how far Erin travelled on the last day of her journey?

1. 1 546 – (120 + 2 × 388) × 40%
2. 1 546 – (1 546 – (120 + 2 × 388)) × 40%
3. 1 546 – 210 – (2 × 388) – (40% × 1 546)

A model boat

Topic 27: Length 165

Figure 6.6: SA textbook: Problem-solving representation p.165



Assignment

Percentages, profit and loss

Percentages play a very important role in business and the **economy** of the country. In this investigation you will learn about **profit** and **loss**.

If a person buys something and then sells it for more than they paid for it, we say the person has made a profit. If they sell it for less than they paid for it, we say they have made a loss.

To find a profit or loss, do the following:

10% profit on R200:
- Find 10% of R200 = R20 profit
- Selling price: add the profit to the original amount: R200 + R20 = R220

20% loss on R350:
- Find 20% of R350 = R70 loss ← Double 10% or ÷ 5
- Selling price: subtract the loss from the original amount: R350 – R70 = R280

Key words
- **economy** – how a country manages its money and goods
- **profit** – increase in wealth; making money
- **loss** – decrease in wealth; losing money

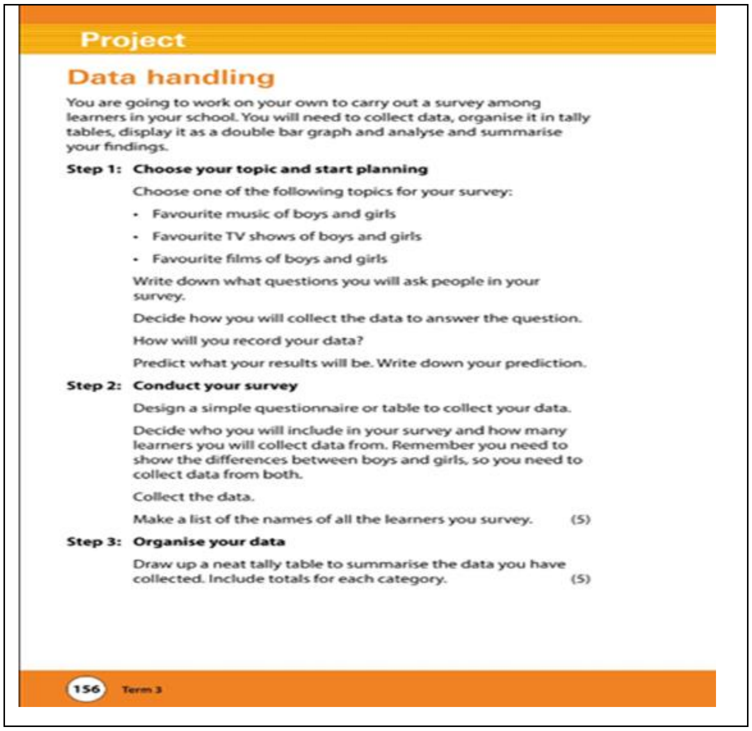
1. Find the profits and losses on these amounts:
   a) 25% profit on R400 (1)
   b) 10% loss on R320 (1)
   c) 20% profit on R5 000 (1)
   d) 25% loss on R48 000 (1)
   e) Lerato bought a bicycle for R480 and sold it 5 years later at a loss of 10%. How much did she sell the bicycle for? (2)
   f) Rajan bought eggs from the farmer and sold them at the market. He paid R45 for the eggs and sold them at a 25% profit. How much money did Rajan make? What was the new selling price? (3)

Identify the percentage profit or loss in this way:

Step 1: Is the new price more or less than the old price? More means a profit, less means a loss.
Step 2: Find the difference between the two prices.
Step 3: Make a fraction, using the difference in price as the numerator and the original price as the denominator.
Step 4: Find the equivalent of this fraction as a percentage.

180 Term 4

Figure 6.7: SA textbook: Investigation representation p.180



Project

Data handling

You are going to work on your own to carry out a survey among learners in your school. You will need to collect data, organise it in tally tables, display it as a double bar graph and analyse and summarise your findings.

Step 1: Choose your topic and start planning

Choose one of the following topics for your survey:

- Favourite music of boys and girls
- Favourite TV shows of boys and girls
- Favourite films of boys and girls

Write down what questions you will ask people in your survey.

Decide how you will collect the data to answer the question.

How will you record your data?

Predict what your results will be. Write down your prediction.

Step 2: Conduct your survey

Design a simple questionnaire or table to collect your data.

Decide who you will include in your survey and how many learners you will collect data from. Remember you need to show the differences between boys and girls, so you need to collect data from both.

Collect the data.

Make a list of the names of all the learners you survey. (5)

Step 3: Organise your data

Draw up a neat tally table to summarise the data you have collected. Include totals for each category. (5)

156 Term 3

Figure 6.8: SA textbook: Project representation p. 156

These elements are considered to be an important part of the curriculum design, these being an imperative component of the curriculum design. The South African textbook is not bulky and thick, and good quality printing, with bold and easy to read font. It is written in language that is easy to understand and user friendly. The paper used in the textbook was of good quality and the binding is strong and durable which allowed for students to use regularly.

6.3.1 Logical and consistent layout of the SA textbook

The sequencing of learning material has been presented in a clear and precise manner, to facilitate differentiation (See, Figure 6.3). The layout of the SA textbook shows size and spacing, number of pages in the textbook, number of questions and colour-coding detail, which all contribute to logical and consistent appearance (See, Figure 6.5). According to the guiding principles of structure and organisation, the layout of the SA textbook is appropriate and logical which is formally used by British imperial layout of textbook sizes (Wonning, 2019).

6.3.2 Representation of illustrations in the SA textbook (Non-textual elements)

Textbooks consist of text and non-textual components, the former being the written words that the students need to understand, and the latter being the visual stimuli that are not words. According to Root, Browder, Saunders and Lo (2017) using visuals in problem-solving procedures is an important strategy which improves memory. The non-textual elements in this study refer to visual representation which are not verbal, numerical, or mathematical symbolic representations. Table 6.2 indicates that there are 581 non-textual elements that represent problem-solving procedures in the SA textbook, these being pictorial representations such as pictures, diagrams, tables and mathematical representations, such as graphs or figures.

Table 6.2: The non-textual elements (illustrations) of the Grade 6 SA textbook

Types of Visuals	Number of Visuals
Graphs	71
Pictures	581
Additional resources (cut-out models	2

Table 6.2 indicates that there are visual representations, graphs and additional resources in the SA textbook, however, the distribution vary according to topics. These 581 illustrations are used to aid the students in a manner that they can relate to easily. Cooper, Sidney and Alibali (2018) elucidates that prior knowledge, when linked with new problem-solving procedures, are easily understood with the aid of visual learning.

Baek, Wickstrom, Tobias, Miller, Safak, Wessman-Enzinger and Kirwan (2017) explain that the textbook provides students with pictures, models and diagrams which indicate significant amounts of data in a manner that is simple to comprehend and assists in revealing relationships and patterns. The South African textbooks (Figure 6.9) represents rich visual images that are likely to be extremely useful and engaging to the students.

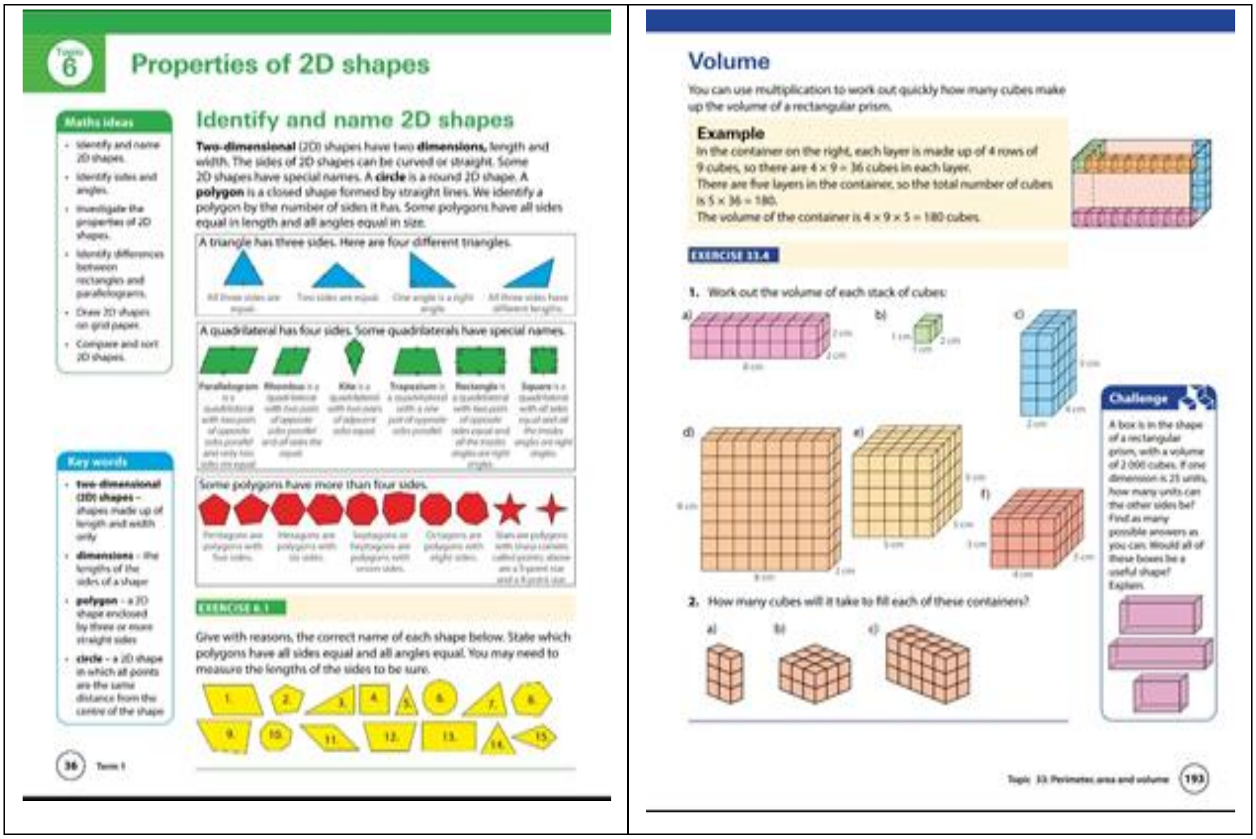


Figure 6.9: SA textbook: Visual representation, p. 36 & 193

There are visual representations at the start of the topics for each term, to introduce the content. Doyle and Zakrajsek (2018) states that combined with the brain's capacity for images, visual learning strategies assist students in understanding and retaining information which is presented to them. The South Africa textbook allows for visual learning by providing students with ample visuals in each topic.

6.3.3 Physical elements of the textbook

The physical elements of the textbook will be discussed in conjunction with the type of font, once-off use of the text, size of font and dimensions. The textbook includes two pages of useful resources that are used for the students' hands-on activities.

Table 6.3: South African textbook layout

Layout of the SA textbook	Physical features	Description
External features	Type of paper	Lightweight (0.39 kg)
	Textbook	Continuous use (TB)
	Size of Font	12
	Dimensions of the TB	27cm by 21cm

6.3.3.1 Lightweight paper

The South African Grade 6 textbook is a soft cover book consisting of 227 pages, with Table 6.3 indicating that the book weighs 0.39 kg and measures 27cm by 21cm. Paper with these specs can be very beneficial to students when carrying multiple books to school.

6.3.3.2 Once-off use of SA textbook

This textbook was designed to be used as a resource, which means that it can be used year after year by several students. It is not designed as a workbook, which means that students do not write it in, thus reducing the costs of buying new books each year. Therefore, designing a textbook which is creative, promotes effective mathematics learning by showing the problem-solving steps that are characteristic of a genuine word problem-solving approach can cost effective.

6.3.3.3 Type of font and dimension of the textbook

Katzir, Hersnko and Halamish (2013) states that by Grade 6, students do not need larger "childlike" fonts to facilitate comprehension, although the research test subjects were not timed for fluency and comprehension. It is unclear whether decreased font size is a hindrance during specified time blocks for learning in school. Based on the "Eyecare" Circular issued by the former Education Department (now the Education Bureau), the minimum font size is equivalent to size 12 in "Microsoft Word" for legibility. According to Wiltshire (2019), in order to avoid eye strain and for more comfort of reading, larger size fonts are suggested, especially for lower levels.

6.4 Representation of content in the SA textbook

The content in the SA textbook is dependent on the alignment of the Grade 6 mathematics curriculum. Though the content in the chosen South African textbook is aligned with the curriculum, the problem-solving procedures are limited. There is need for open-ended questions which would allow students to use their own problem solving strategy to find solutions. According to the Bureau (2016), a textbook should also have supplementary resources and multiple representations to enable students to complete all activities successfully. There is evidence of multiple representations with limited supplementary resources in the textbook due to it not being a workbook.

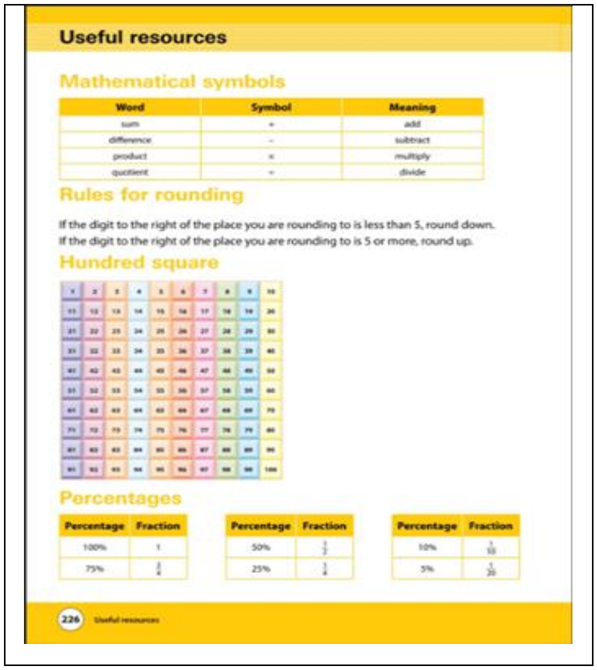
6.4.1 Alignment of aims and objectives

The Grade 6 mathematics textbook displays the four components of the SA curriculum that underpin didactic factors (aims, content, learning/teaching strategies, assessment), in student learning. For the purpose of this study, aims and assessment were discussed in Chapter 2, with the vertical analysis including content weighting, conceptual understanding, procedural practices, problem-solving and cognitive demand. The distinction among the several components of a differentiated curriculum have an importance that transcends the need for conceptual clarity (Glatthorn, Boschee, Whitehead & Boschee, 2018). This statement encapsulates the essential characteristics of what the topic intends to do, indicating that teaching and learning are purposeful activities. Cooper, Sidney and Alibali (2018) state that educational intentions for a topic depends on pre-existing knowledge and skills, which are required for a student to have a reasonable chance of success to achieve the intended learning objectives and outcomes.

Learning outcomes are statements of the attributes and capabilities that a student should be able to display on successful completion of the topic. They provide the basis for determining student progress and designing assessment strategies and methods. They also provide signposts towards appropriate content and learning interactions to help students achieve those outcomes and fit them in context.

6.4.2 Provision of supplementary materials in the SA textbook

Mathematical cut outs for models help to explain known physical phenomena and according to Simpol, Shahrill, Li and Prahmana (2017) photocopy resources help students to explore the potential of a problem-solving procedure, along with the possibilities of having different solutions. They also assist the brain to think in 3-dimensions and advance the students' imagination. Furthermore, Ibáñez and Delgado-Kloos (2018) explain that models represent the real problem situations that aid in making decisions faster and more accurately. The evidence for the provisional supplementary materials in the South African textbooks in in Figure 6.10 below.



Useful resources

Mathematical symbols

Word	Symbol	Meaning
sum	+	add
difference	−	subtract
product	×	multiply
quotient	÷	divide

Rules for rounding

If the digit to the right of the place you are rounding to is less than 5, round down.
If the digit to the right of the place you are rounding to is 5 or more, round up.

Hundred square

Percentages

Percentage	Fraction
100%	1
75%	$\frac{3}{4}$

Percentage	Fraction
50%	$\frac{1}{2}$
25%	$\frac{1}{4}$

Percentage	Fraction
10%	$\frac{1}{10}$
5%	$\frac{1}{20}$

226 Useful resources

Figure 6.10: SA textbook: Photocopy of resources representation, p. 226

The SA textbook has 3% of photocopy resources that are used in problem-solving procedures. It can be assumed according to the above research that these photocopy resources promote understanding in problems that are presented and enables students to achieve their learning outcomes. Research by Hughes and Acedo (2016) suggests the content in a textbook should provide additional resource to complete all given tasks. The textbook has minimal resources at the end of the book that are used to answer specific questions.

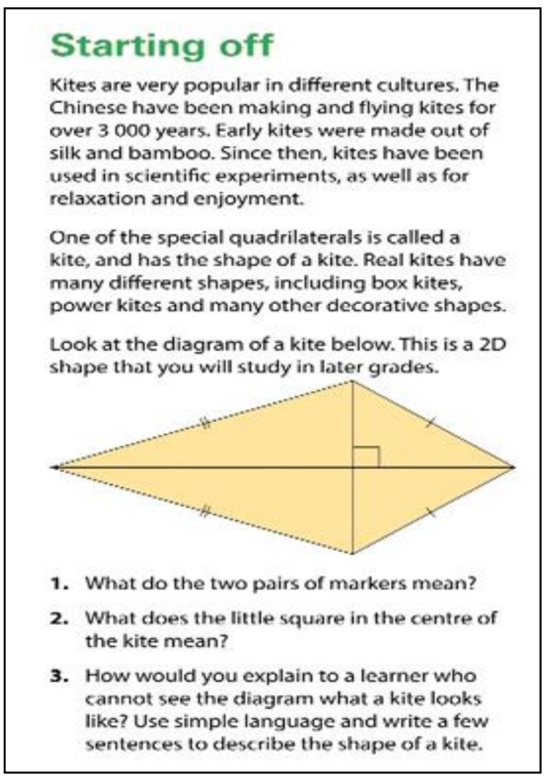
6.4.3 Presentation of current content (linked to real life)

The South African textbook is designed according to the number of school terms in South Africa, each term starting with two pages of pictures with real-life images[9] illustrating general knowledge topics that do not necessarily form a relationship between the many problem-solving questions (evident in Figure 6.4). Root, Browser, Saunders and Lo (2017) elucidate that the determination to educate students to identify problem-solving structures in word problems which are interrelated to real-world situations, daily life makes learning relevant.

Research conducted by Good and Lavigne (2017) states that an introduction to key topics in classroom instructions have to be presented in an accessible manner, its purpose being to promote

[9] The real-life image implies the representation of an actual object or people.

understanding. The textbook provides student-centered instructions and project-based learning and evidence is provided on Figure 6.11, which shows a combination of learning strategies. This task requires students to explore the cultural values and dimensions in kites, on which they work individually to calculate the dimensions and descriptions. The kite is used to connect with shapes and measurement to real life contexts, such as in China, indicating the application and relevance of mathematics across cultures. There is an emphasis on cultural dimensions, but it is used to reinforce mathematical understanding. The introduction of kite activity requires a description which necessitates writing skills used as a problem-solving procedure. This strategy is used to probably develop understanding, enjoyment and writing skills (Simões, Redondo & Vilas, 2013). One of the aims in the South African curriculum is to build an awareness of the important role that mathematics plays in real life situations including the personal development of the student which is accomplished in the textbook.



Starting off

Kites are very popular in different cultures. The Chinese have been making and flying kites for over 3 000 years. Early kites were made out of silk and bamboo. Since then, kites have been used in scientific experiments, as well as for relaxation and enjoyment.

One of the special quadrilaterals is called a kite, and has the shape of a kite. Real kites have many different shapes, including box kites, power kites and many other decorative shapes.

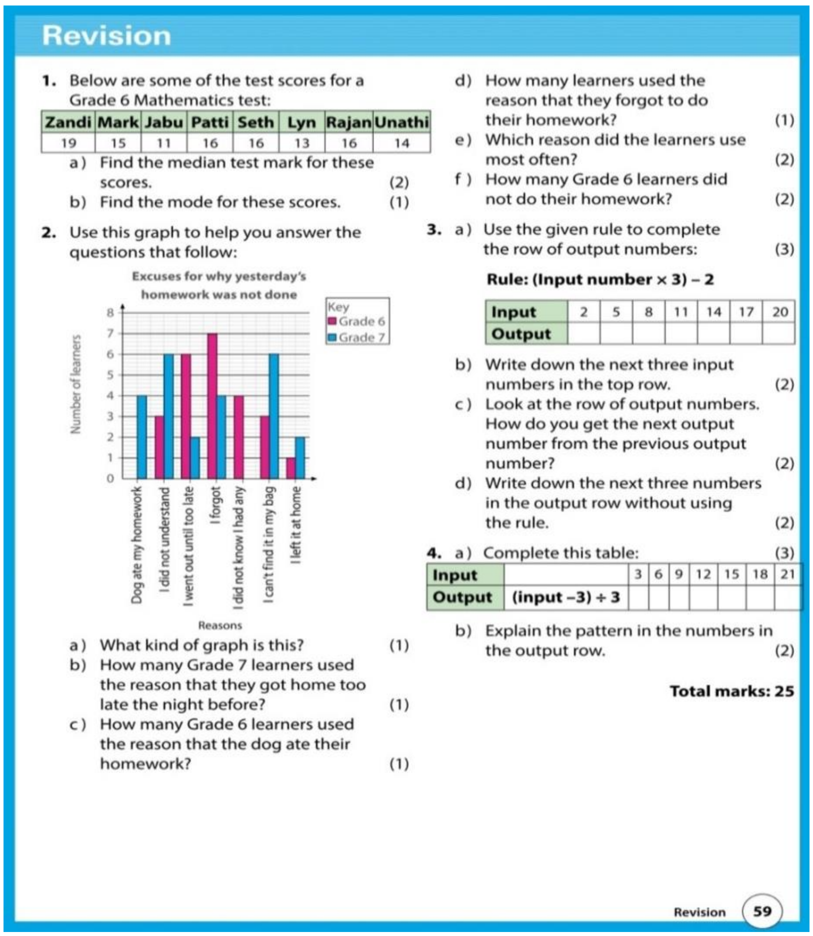
Look at the diagram of a kite below. This is a 2D shape that you will study in later grades.

1. What do the two pairs of markers mean?
2. What does the little square in the centre of the kite mean?
3. How would you explain to a learner who cannot see the diagram what a kite looks like? Use simple language and write a few sentences to describe the shape of a kite.

Figure 6.11: SA textbook: Introduction to term 1, p. 3

6.4.4 Linking learning to prior knowledge

A conclusion is the last part of the topic, which the book ends by summing up the problem-solving procedures that have been taught. Each topic ends with a revision exercise to reinforce an understanding of the problem-solving procedures. Students find visuals easy to remember because visual representations help them organize information into patterns and encourage meaningful learning. Sprenger (2018) states that the brain has the ability to memorise data that has a specific design. Figure 6.12 shows that the conclusion to each topic ends with a revision sheet to reinforce understanding and the use of visuals to aid in the process.



Revision

1. Below are some of the test scores for a Grade 6 Mathematics test:

Zandi	Mark	Jabu	Patti	Seth	Lyn	Rajan	Unathi
19	15	11	16	16	13	16	14

a) Find the median test mark for these scores. (2)
b) Find the mode for these scores. (1)

2. Use this graph to help you answer the questions that follow:

a) What kind of graph is this? (1)
b) How many Grade 7 learners used the reason that they got home too late the night before? (1)
c) How many Grade 6 learners used the reason that the dog ate their homework? (1)
d) How many learners used the reason that they forgot to do their homework? (1)
e) Which reason did the learners use most often? (2)
f) How many Grade 6 learners did not do their homework? (2)

3. a) Use the given rule to complete the row of output numbers: (3)

Rule: (Input number × 3) – 2

Input	2	5	8	11	14	17	20
Output							

b) Write down the next three input numbers in the top row. (2)
c) Look at the row of output numbers. How do you get the next output number from the previous output number? (2)
d) Write down the next three numbers in the output row without using the rule. (2)

4. a) Complete this table: (3)

Input		3	6	9	12	15	18	21
Output	(input −3) ÷ 3							

b) Explain the pattern in the numbers in the output row. (2)

Total marks: 25

Revision 59

Figure 6.12: SA textbook: Conclusion, p. 59

In the above figure 6.12, students are needed to analyse the data and interpret the graphs, which is an extension activity for higher achievers. Darling-Hammond, Hyler and Gardner (2017) contended that revision exercise provides teachers with opportunities to address common misconceptions student may have in understanding specific problems. In the South African textbook, an opportunity for presenting a conclusion to a topic taught would be to help consolidate a concept taught. Ideally the question required evidence of understanding of the concept that was just taught.

6.4.5 Balancing the breadth and depth of content weighting

The balancing of breadth and depth of the curriculum includes content weighting, content sequencing, the use of correct concepts and the six content areas. The CAPS document states that the weighting of the curriculum is divided into components to provide a platform for further mathematical development (Roberts, 2016). According to DoE (2011b), the weighting of mathematics content areas serves two primary purposes: First, provides specific time limits to complete tasks within each content area, guidance on the distribution of tasks for the final examination. Secondly, the weighting of the content areas are the same for each Grade in this phase. DoE further requires that all textbooks must comply to the requirements in order to be added to the Government list of textbooks. According to the CAPS document the curriculum is designed to allow students to obtain and improve knowledge, understanding, skills, values, and attitudes, which are to be aligned with the textbooks. Developing connections promotes improved mathematics understanding and assists with learning. Students are motivated to make links between various mathematics topics, across other content, skill areas and into the "real" world (English, 2017). The South African textbook weighting is determined by the number of pages allocated to a topic. Table 6.4 portray the weighting of the curriculum in the SA mathematics textbook according to the CAPS document, some topics are allocated more space than others. For example, number operations account for 50 % compared to data handling; patterns, functions and algebraic expressions, which is given the weighting of 10%. According to CAPS measurement, space and shape is allocated 15% of the space, however, the textbook allocation does vary from the CAPS documents.

Table 6.4: Content weighting in the textbook (227 pages)

Weighting of topics	Pages (%)	Topic description, content and outcomes
1. No operations and relationships (65 pages) **r**evision (4) + 2 Assignments + Investigation (2)	73 (32%)	Whole nos - ordering and comparing; addition and subtraction; multiplying and dividing. Common fractions; decimal fractions; percentages (6 topics)
2. Patterns, Functions and Algebraic expressions (23 pages.) Revision + starter activities + problem-solving Assignment (9)	32 (14%)	number sentences; patterns (2 Topics)
3. Measurement (30 pages) Revision, project + starter activity + problem-solving (9)	39 (17%)	Time; length; capacity & vol; mass; temp; area & vol, perimeter; history of measurement (7 Topics)
4. Space and shape (Geometry) (43 pages) **Plus**: revision (3) + Assignment (2) + starter activity (2) + problem-solving (1)	50 (22%)	Properties of 2D shapes; properties of 3D shapes; symmetry; viewing objects; transformations; position and movement (6 Topics)
5. Data handling (22 pages) **Plus**: revision (3) + starter activity (2)	27 (12%)	Collecting and organising data; representing data; interpreting data; analysing data; reporting data; probability (6 Topics)
Glossary / Useful Resources	6 (3%)	Useful resource activities
Total	227 (100%)	

In the Grade 6 SA textbook, more emphasis is placed on number operations (32%) at the start of the year. McGee, Wang and Polly (2013) explains that it is important for the students to be knowledgeable about basic concepts before proceeding on to other components of the curriculum. The number of topics in each section is represented in an uneven manner, lending more emphasis on some topics rather than others with Number operations having 6 main topics and equating to 32 % of the book. Space and shape; and Data handling had the same number of topics but with less than half the space allocated. The CAPS document (DoE, 2011b) states that the weighting of Number, Operations and Relationships has been increased to 50% for Grades 4, 5 and 6. This is an effort to guarantee that students are adequately proficient when they enter the Senior Phase (Roberts, 2016).

6.4.6 Multiple representation in the SA textbook

The SA textbook provides students with relevant games, information boxes, visuals, stories etc., which aid in developing comprehension of the concepts that are taught.

6.4.6.1 Representation of information boxes

According to Barton and Barton (2017), stories have an imperative role in the growth and development of students, their purpose possibly being to inform the reader, relate a story or recount events. They further emphasize the significance of storytelling as a teaching tool for indigenous students in narratives from early childhood development, such as in the SA textbooks 'did you know' facts in Figure 6.13. The 'did you know' boxes provide students with facts about indigenous knowledge system and mathematicians from various countries, aspects of culture, history, and relevant topics of mathematics. This allows students to become aware of mathematical contributions from different times in history, shaping an appreciation for the subject, developing personal connections and implying relevance to daily lives.

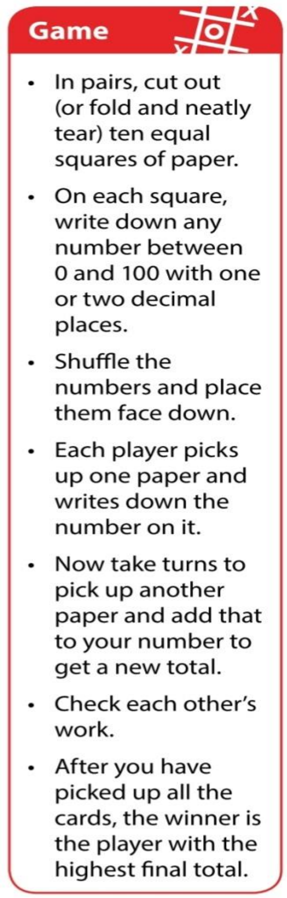
Figure 6.13: SA textbook: Additional information, p. 72 & 76

Research according to Gravemeijer, Stephan, Julie, Lin and Ohtani (2017) states that students need to be aware of real-world contexts. The ability to link learning content and pedagogy is required to prepare students for current-world readiness. This affords them the opportunity for reasoning and connections to problem-solving skills of the 21st century.

The CAPs policy states that valuing indigenous knowledge system is vital and recognizing the rich history and heritage of South Africa is essential, as it contributes to fostering the values that are in the Constitution; and Credibility, quality and efficiency (DoE, 2011b). The South African textbook provides cultural knowledge about South Africa as well as, various places like Egypt and China. Relaying cultural patterns provides interesting facts, rather than long stories. Research by Istenic Starčič, Cotic, Solomonides and Volk (2016) suggests that vocabulary, comprehension, sequencing, memory and creative writing all benefit from these short ‘did you know’ excerpts. Furthermore, word problems are written in story form which requires understanding, comprehension and critical problem-solving skills. Storytelling possibly enhances listening skills which are vital in learning and relationships between problem-solving procedures (Alkaaf & Al-Bulushi, 2017). Apino and Retnawati (2017) further explains that storytelling encourages communication, creative thinking and problem-solving skills.

6.4.6.2 Representation of games

Starko (2017) states that neurobiology creates motivation to learn. Particularly, the relationship between curiosity, imaginative play and opportunities for questioning. Figure 6.14 provides students with simple games in which they are required to work with others. According to Tsekleves, Cosmas and Aggoun (2016), students find immense enjoyment in playing games, therefore it aids in capturing the students’ attention allowing them to actively immerse in mathematics’ problems.



Game

- In pairs, cut out (or fold and neatly tear) ten equal squares of paper.
- On each square, write down any number between 0 and 100 with one or two decimal places.
- Shuffle the numbers and place them face down.
- Each player picks up one paper and writes down the number on it.
- Now take turns to pick up another paper and add that to your number to get a new total.
- Check each other's work.
- After you have picked up all the cards, the winner is the player with the highest final total.

Figure 6.14: SA textbook: Peer learning, p. 101

Kangas, Koskinen and Krokfors (2017) explains that games are also an important part of the structure of mathematics textbook. There are pedagogical foundations which reinforce the teacher's position and activities concerning the utilization of games in teaching. Game elements such as progress bar, rewards (stars), feedback, choice, challenge, and time were selected and carefully added to the game to increase its effectiveness and quality. Ravyse, Blignaut, Leendertz and Woolner (2017) further states that with a game, students can develop an understanding of a new concept or idea, have a changed perspective, or experiment with various alternatives or variables. Buehl (2017) suggests that games can be used as a classroom strategy to develop interactive learning. The game presented in the South African textbook includes examples that are set for working in pairs or a team.

6.5 Representation of language in the SA textbook

Language used in the SA is suitability for the intended audience, the fact that the majority of students do not speak English as their first language, has been accommodated in the book. The book makes use of vocabulary boxes, math ideas, visuals enables students to understand the instructions. The illustrations are age and culture appropriate, and the instructions are grammatically and structurally clear, which can be followed in general.

6.5.1 Representation of key words

The key words featured in the South African textbook provides the students with more information about what is required. Figure 6.15 presents the students with key words for topics, which may aid their comprehension of the topics and to use the correct mathematical vocabulary when learning and discussing problem-solving procedures.

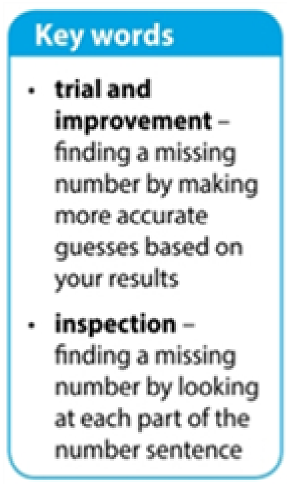


Figure 6.15: SA textbook: Keywords, p. 8

Key words that are highlighted and presented like in the above figure with a short definition are intended to guide the comprehension process in the textbook. Son and Dilette (2017) explains that textbooks can be tedious and are normally full of information and trying to work with a textbook without having a basic understanding of the central themes and topics the textbook reading can become increasingly difficult. Larsen-Freeman (2018) states that using key words help develop literacy in mathematics in second-language students. As South Africa has 11 official languages, multi-lingual language students may need assistance in developing correct terminology for proficiency in English. Working with key words will help students to immediately identify relevant information, and to aid in understanding the new concepts being taught.

6.6 Conclusion

The South African textbook provides ample opportunities for students to understand the topics that are presented. The textbook is designed to provide more practice and with information on the cultures of South Africa, and the history and culture of other countries as well. The text met the criteria laid out by Bureau (2016) regarding the structure, organisation, layout, content and language to a great degree. The SA book provides students with examples, guidelines, pictures, materials and illustrations that enable them to implement the 4 steps and 8 heuristics in problem-solving. It is designed as a problem-solving textbook whereby strategies have to be used to solve the given questions. The SA textbook provides students with investigative types of questions, examples and the necessary resources that require critical thinking skills. This level sets the context for problem-solving strategies which will be discussed in chapter 7.

CHAPTER 7: Comparative analysis of the South Korean and the South African mathematics textbook

7.1 Introduction

Chapter 5 presented the structure of the South Korean textbook and Chapter 6 that of the South Africa. This chapter identifies and compares the cross-national similarities and differences in the representation of the problem-solving procedures by addressing Objectives 1-3 and answering the four research questions:

1. How are problem-solving procedures presented in South Korean mathematics textbook?
2. How are problem-solving procedures presented in South African mathematics textbook?
3. What are the similarities and differences in the problem-solving procedures between the South Korean and South African mathematics textbooks?
4. To what extent do the South Korean and the South African textbooks address the principles required to develop Grade 6 cognitive demands and critical thinking?

7.2 Cognitive demand

The adapted analytical framework that was used to conduct a comparative analysis between the two textbooks is described in Chapter 3. Guidelines for textbook design (Bureau, 2016) discuss didactic factors, and state that classroom activities must cater for student diversity (various strategies), develop conceptual understanding, provide student involvement (individual, peer & group), provide activities that are interesting and motivating, have clear instructions and relevant meaning. The conceptual understanding aspects by Charalambous (2010), Bureau (2016) and Groth (2017) states that textbooks requires sufficient cognitive demand level according to Bloom et al. (1956). Furthermore, the textbook must provide stimulating tasks to encourage deeper thinking and flexibility when problem-solving. Reading comprehension during problem-solving must develop procedural fluency, strategic competence and adaptive reasoning, and improve the level of competency in mathematics.

7.3 Polya's problem-solving model in the textbooks

The analysis was informed by Polya's 4 stage model, with the 8 heuristics being identified as being relevant for problem-solving for mathematics (See Table 7.1). The heuristics that support each stage of the model are highlighted, along with the benefits of using specific one. An analytical framework and coding scheme were used to describe the presentation of problem-solving procedures represented in the textbooks with respect to these two components. This chapter is split into two parts; the first establishing the extent to which the Polya's 4 stages are addressed in both textbooks, and the second, how the 8 heuristics facilitate students to problem solve.

Table 7.1 Polya's 4 stages & 8 heuristics adapted from Fan and Zhu (2007, p. 66)

Polya's 4 Stages	The 8 Heuristics
1. Understand the problem	1. Act it out
2. Devise a Plan	2. Change your point of view
3. Carry out the Plan	3. Draw a diagram
4. Check and Extend	4. Guess and check
	5. Look for a pattern
	6. Solve part of the problem
	7. Use a model
	8. Work backwards

The South Korean (SK) and South African (SA) textbook were reviewed to determine how the 4 stages are represented and was achieved by taking all the tasks into consideration and dividing them into the various stages. The SK textbooks provided 576 questions whereas the SA textbooks provided 2317, with Table 7.2 indicating the breakdown of each problem-solving stage in the two textbooks.

Table 7.2: Number of questions in percentages in relation to Polya's 4 stages

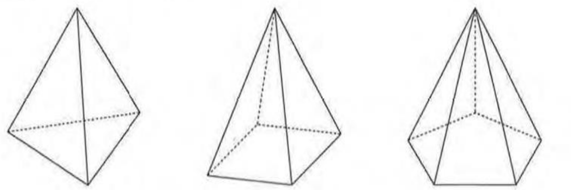
Polya's 4 stages	Description	SK	SA
1. Understand the problem	The ability to identify problems	43 (7%)	174 (8%)
2. Devise a Plan	Planning a solution with a mathematical model	102 (18%)	183 (8%)
3. Carry out the Plan	Resolving the problem as planned	330 (57%)	1786 (76%)
4. Check and Extend	Interpret solutions	101 (18%)	174 (8%)
TOTAL		576 (100%)	2317 (100%)

7.3.1 Stage 1: A comparison of the instructions for understanding the problem in the textbook

Step 1, defining and understanding the problem, is the initial stage of the problem-solving process. Before a problem can be solved, students need to know exactly what is expected, as any lack of clarity regarding expectations is likely to produce the incorrect results. The SK textbook had 43 (7%) activities that are designed to create understanding regarding any given problem-solving procedure, while the SA book had 174 (8%) examples to guide students understanding. There is a marked difference between both the percentage and numerical values, and although 174 is greater than 43, the percentages indicate similar values due to the number of tasks represented in each text. These numerical representations reveal that it is not ideal to compare numbers and percentages when there is a marked variance in the number of pages in the respective texts. Due to this discrepancy, it would be ideal to compare numbers instead of percentages, which I used. The SK and SA textbooks both connect the problems to real life objects, situations and scenarios to enhance students' knowledge of the problem. Figure 7.1 represents an example of step 1, 'understanding the problem' in both texts, which provides students with instructions to identify the shape of pyramids.

(Connecting to real life objects)

What is the shape of the underside of the pyramids?
Identify the names of the pyramids.



* Find the base of each pyramid and draw a line with a colour pen along the lines.
* What is the shape of the base?

SK textbook: examples of problems on 3D shapes, (PS1)

Identify 3D objects

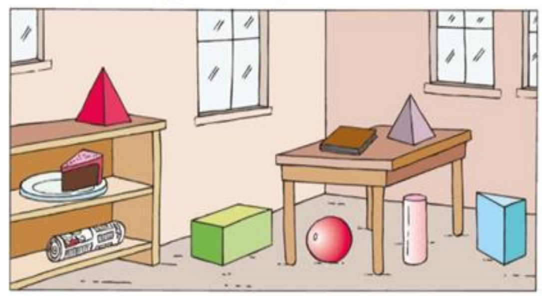
We live in a three-dimensional (3D) world, and the solid objects that you see around you are **3D objects**.

A **pyramid** is a 3D object with a base that is a polygon. All the other faces are triangles that meet at a common point.

Triangular-based pyramid: a pyramid that has four faces, all in the shape of a triangle (**tetrahedron** is another name for this pyramid)

Rectangular-based pyramid: a pyramid with a rectangular base

1. Identify and name the solids described below:
   a) It has a round base and a curved surface that comes to a point.
   b) It has a pentagonal base and five of its faces are triangles.
   c) It has seven faces.
2. Use the picture to answer these questions:
   a) Name two different cylinders in the picture.
   b) List three different objects with curved surfaces.
   c) Name all the prisms in the picture.
   d) Name all the pyramids in the picture.



SA textbook: examples of problems on 3D shapes, (PS1)

Figure 7.1: SK & SA textbook: Examples of problems on 3D shapes, (PS1)

Both texts represent pyramids as real-life objects to enable students to connect their learning to reality. However, the SK text leads the student to identify the pyramids, instead of giving them the information. The first instruction is for the students to identify the base of three given shapes (triangle, rectangle or pentagon). Second, they have to draw lines along the provided outlines to complete the pyramid. This instruction leads students to identifying the base shape; a triangular-, rectangular and the pentagonal-based pyramid. The mathematical instructions draw attention to the different perspectives, which leads to the identification of three specific types of pyramid. On the other hand, the SA textbook provides two shapes (triangle, rectangle) in contrast to SK's three shapes. The textbook design features presents sufficient representations, with the SA text providing instructions for students to utilize the given descriptors in order to identify the objects.

The initial instruction given to students in the SK textbook requires students to find the ‘underside’ of the pyramid before using the mathematical term ‘base’, making the assumption that they are familiar with the term ‘underside’, which is then connected to the term ‘base’. Connecting the specific use of language to appropriate mathematical terms aids in the understanding process. The SA textbook uses specific mathematical language, explaining that the base is called a polygon, which is beneficial to second language students. The SK textbook is designed to start every topic by creating an understanding, by asking questions about the concept being taught. However, the SA book starts every topic with an example of how students are required to solve the given exercise, provides key words, math ideas and information boxes to help them understand concepts.

The difference is that SA topics start with examples that allow student to solve problems, but not necessarily to aid their understanding and problem-solving adequately whereas, the SK topics start with real-life familiar objects so that students can conceptualize a 3D shape. This is followed by tasks related to pyramids in the SK textbook whilst the SA textbook foregoes the mathematical shapes and thereafter, the real-life examples are used in an application task. The SK tasks allow for students to link prior knowledge to current knowledge, while the SA text provides the current knowledge followed by prior knowledge. Both textbooks are addressing the same three dimensional shapes.

7.3.2 Stage 2: A comparison of the instructions for devising a plan in the textbook

Step 2, deciding on how a problem can be solved, is the subsequent step of the problem-solving procedure, and requires students to develop a strategy to work out the problem. Polya contends that there are many logical methods in which problems can be solved. Heuristic 1 (act it out), Heuristic 3 (draw a diagram), Heuristic 5 (look for a pattern) and Heuristic 7 (use a model) can be used to devise a strategy to solve the problem. This stage also allows for students to use any of the four operations (addition, subtraction, division and multiplication) to solve the problem.

The SK textbook provides 101 (18%) opportunities for students to utilize mathematical models to assist them in solving the given problems, while the SA book provides 183 (8%) opportunities, with a marked different between both the percentage and numerical values. Figure 7.2 represents an example of step 2, 'devising a plan' in both texts, the SK textbook providing students with an opportunity to measure ingredients to bake doughnuts, while the SA book uses fractions to divide sweets, which is practical in the SA context. Students in SA will have access to sweets as a resource and it related to their daily living,

Block 1 (girl): I need 2/25 cups of flour to make a doughnut.

Block 2 (boy): the flour we have is 4/5 short.

Block 3 (girl): Can you make the doughnuts out of this flour and share it equally?

Block 4 (boy): share between two.

SK textbook: measurements for baking, (PS2)

Look at the ingredients prepared above and answer the question T > L.

Material	Amount prepared(cup)
Flour	6 ¼
Brown sugar	4

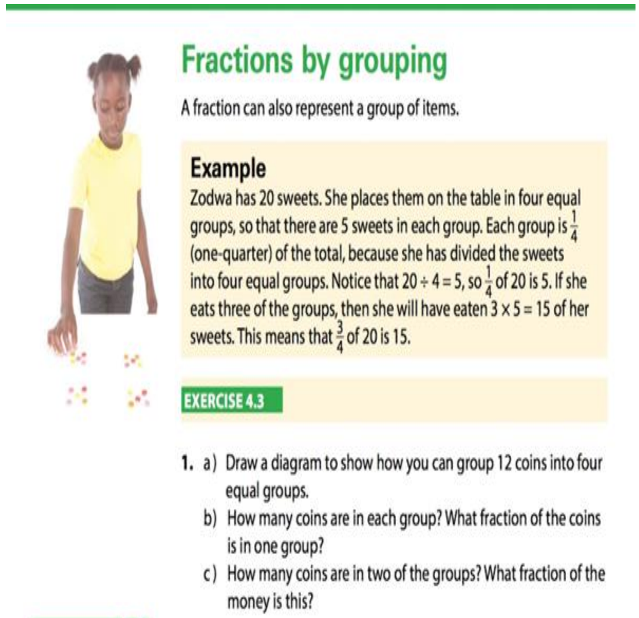
If you need a 2/7 cup of brown sugar, you can make it with brown sugar.

The number of individuals who are Min-Jun's pool underlined in the process

Why did you explain the part?

If the eclipse is 4 ÷ 2/7. If the natural number is a fraction, it becomes 28/7 ÷ 2/7, and the value of 28/7 ÷ 2/7 equals 28 ÷ 2. So 28 ÷ 2 = 14, so there are 14 ho-eaten that can be made.

4. If you need 5/8 scoops of flour to make a single doughnut, set the table for how many doughnuts you can make from flour and get a pack.



Fractions by grouping

A fraction can also represent a group of items.

Example

Zodwa has 20 sweets. She places them on the table in four equal groups, so that there are 5 sweets in each group. Each group is $\frac{1}{4}$ (one-quarter) of the total, because she has divided the sweets into four equal groups. Notice that $20 \div 4 = 5$, so $\frac{1}{4}$ of 20 is 5. If she eats three of the groups, then she will have eaten $3 \times 5 = 15$ of her sweets. This means that $\frac{3}{4}$ of 20 is 15.

EXERCISE 4.3

1. a) Draw a diagram to show how you can group 12 coins into four equal groups.
   b) How many coins are in each group? What fraction of the coins is in one group?
   c) How many coins are in two of the groups? What fraction of the money is this?

SA textbook: measurements for sweets, (PS2)

Figure 7.2: SK & SA textbook: Measurements of fractions, (PS2)

In the SK textbook, the students are short of flour and need to devise a plan to adapt the other measurements, accordingly, being required to complete the problem-solving procedure by using a mathematical model to find a solution. The SA textbook provides an example of how to use fractions to divide, followed by three additional exercise questions. The questions require a similar procedure to be followed, enabling consolidation opportunities to scaffold critical thinking, with the plan being provided for the students to follow. Whereas in the SK textbook the students are expected to create their own plan, a fraction wall in the SA book can be used to find equivalent fractions. The SK textbook is challenging in terms of the thought process expected to solve the problem, as 2/25 is extraordinarily complex for Grade 6 students. The SA texts give instructions that can be followed easily in order to derive the answer and provides the problem-solving process by enabling the students to focus on the content and not be concerned about the steps at this point. The problem has not been presented as difficult but manageable, which may lead to students being more successful (not caught up in the details).

7.3.3 Stage 3: A comparison of the instructions for carrying out the plan in the textbook

Step 3 carry out the plan (execute) is a fundamental part of the problem-solving process, as students are required to solve the problem using the procedure specified in step 2 'devise a plan'. Step 3 is usually easier than step 2, as students need to structure their thinking, and requires the student to follow the plan that has been devised. However, if it is unsuccessful, it will require the plan to be revised, discarded or redone. Heuristics 1 (act it out), 3 (draw a diagram), 5 (look for a pattern), 6 (solve part of the problem) and 7 (use a model) can be used to carry out the plan to solve the problem.

The SK textbook provides students with 330 (57%) problems to solve by carrying out a plan compared to 1786 (76%) in the SA text. In Figure 7.3, the SK students are required to create and carry out a plan to solve a problem of measurement, the instruction requiring the student to devise their own method using the image of the containers as a prompt. The SA textbook provides students with measurement and a method to measure the liquids in each jar and then provides an extension exercise.

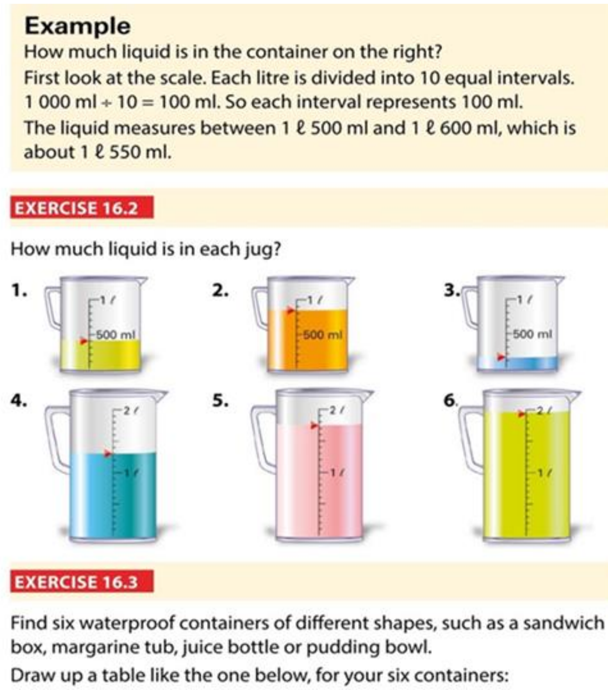
Translation (Man): Among the many ways, what is an easy and simple way to solve a problem?

(Girl): How many 1litre containers do I need to give you?

• Solve the problem in the way you were taught.

• Create new problems related to the fountain.

SK textbook: measurements for baking, (PS3)



SA textbook: measurements for volume, (PS3)

Figure 7.3: SK & SA textbook: Measurement of volume, (PS3)

The SK textbook visual shows a man deliberating about the way to solve the given problem. Students need to identify a simple method to fill a fountain using 1 litre containers, to devise and carry out the plan, and then create new scenarios related to filling the fountain. This process allows students to apply the problem-solving techniques to other questions and to further create new solutions related to the fountain. However, the SA textbook provides the procedure as an example and the students follow a similar procedure to create solutions, this process allows students to approach problems in multiple ways. Furthermore, students can use containers related to everyday life, demonstrating the link between measurement and different shapes, with correct mathematical terms being used concerning measurements in both textbooks.

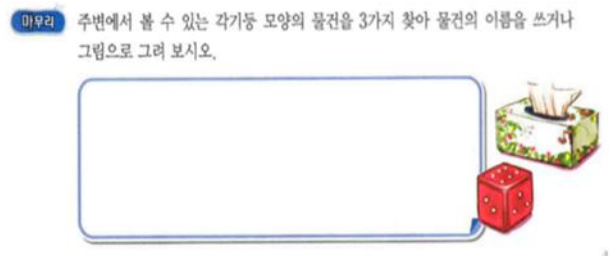
In conclusion, the SK and SA textbooks are similar because they both provide students with opportunities to carry out a plan and record the results. The differences is that the SK text requires a complex comprehension and application in various situations for practice. In comparison, the SA textbooks offers students the chance to learn to read and use measurement in a practical way. The follow-up activity of the SA textbook is similar to that of SK, as it requires using a vessel for filling and measuring (but on a different scale). The textbook used in SK represents problems that required cognitive demand and explanations; while in the SA book, the students had to document six different shaped vessels, draw a table and add data.

7.3.4 Stage 4: A comparison of the instructions for checking and extending in the textbook

Stage 4 emphasizes that when solving a problem, it is useful to look, check and see if the solution addressed the initial problem adequately (check and interpret). The textbook must provide students with the opportunity to determine if they used all the data provided and that the answer makes sense. If not, they need to reassess their strategy until they arrive at what seems to be a suitable answer. Polya notes that a lot can be achieved by taking time to reflect and look back at what was done, what worked and what did not, providing opportunity for students to decide what approach to take to find a solution to similar problems in future.

During the check and extend step, the students' understanding is identified when they make use of heuristics 2 (change your point of view), heuristic 4 (guess and check) and heuristic 8 (working backwards), to solve the problem. There were 101(18%) activities in the South Korean textbook and 174 (8%) in the South African, which allowed students the opportunity to check and extend problem-solving procedures, as indicated in Figure 7.4. The South Korean textbook provides more problem-solving activities compared to the South African textbook. Students are needed to use their prior knowledge to identify prismatic objects around them, the items (tissue box, dice) having to match 3D shapes and comply with the criteria. Two examples are provided, which allows the student to check and extend understanding and label additional objects.

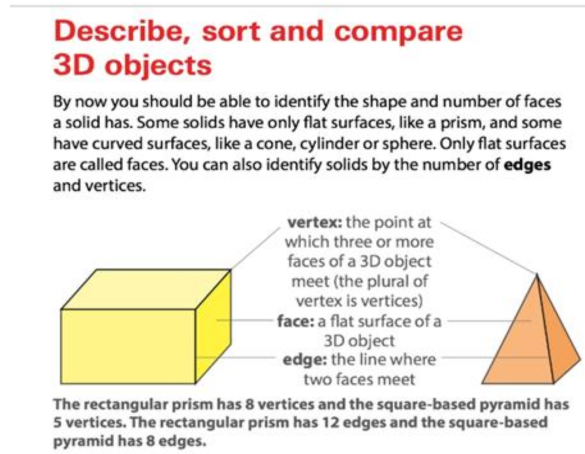
In the South Korean textbook, every section ends with a final activity, which provides students with an opportunity to reinforce their comprehension of the concept. This process extends understanding, allowing a further connection to real world objects, leading to intrinsic consolidation. There are a total of 11 pages of activities in the SK textbook which give students a chance to interpret solutions. The SA textbook presents an activity whereby students are expected to describe, sort and compare 3D objects, relying on the correct identification of the given shapes. They are required to sort 3D shapes according to prisms, cones and sphere, then compare the number of vertices and edges according to their attributes, with the correct mathematical language being used throughout the section.



Find 3 types of prismatic objects that you can see around you.

Draw 3 pictures of the objects.

SK textbook: prismatic identification, (PS4)



SA textbook: prismatic identification, (PS4)

Figure 7.4: SK & SA textbook: Prisms identification, (PS4)

The SA textbook provides student with the definition of the term's vertex, face and edges. It goes on to explain in detail exactly what these terms represent on three dimensional object, allowing students to obtain a better understanding of attribute labelling. In comparison, the starting activity in the SA textbook discusses the number of faces and surfaces of three dimensional shapes. In the SK textbook 'check and extend' section of identifying three dimensional prisms requires the students to identify three objects in the classroom that resemble the shapes studied, with the SA textbook having similar exercises.

7.4 Heuristics in the textbooks

Heuristic is a mental shortcut which permits people to work out problems and make informed decisions effectively and efficiently, thereby playing an essential part in mathematical problem-solving, which is vital to mathematics learning. Table 7.3 indicates the 8 heuristics used in this study, and the number of times they are used in the SK and SA textbooks. The numbers were obtained by the researcher counting each time one was addressed in the description box. For example, in heuristic 1, every question gives students the opportunity to use people or objects to demonstrate what is referred to in the problem. The 8 heuristics are addressed in terms of how they are used in the textbooks to assist with problem-solving.

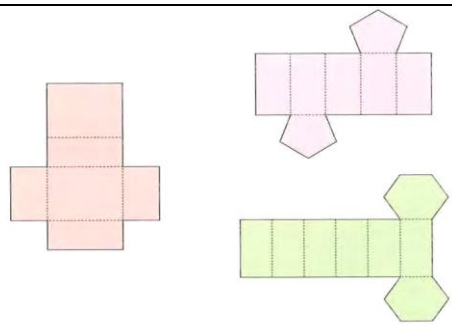
Table 7.3: The 8 heuristics adapted from Fan and Zhu (2007, p. 66)

Heuristics	Description	SK No. (%)	SA No. (%)
1. Act it out	Use people or objects to physically show what is described in the problem.	12 (25)	221 (10)
2. Change your point of view	Approach a problem from another angle when a previous approach is not effective.	14 (9)	166 (7)
3. Draw a diagram	Do a sketch based on the available information to visually represent the problem.	15 (6)	165 (7)
4. Guess and check	Make a reasonable guess of the answer and then check the result to see if it works. If necessary, repeat the procedure to find the answer, or at least a close approximation.	11 (10)	193 (8)
5. Look for a pattern	Identify patterns in the given pictures, based on careful observation of common characteristics, variations, or differences about numbers, shapes, etc. in the problem.	14 (25)	302 (13)
6. Solve part of the problem	Divide a problem into several sub-problems, then solve them one by one, and finally solve the original problem.	14 (7)	134 (6)
7. Use a model	Create visual representations (e.g. using points, lines or other easy-to-understand symbols) to model the information on quantities, relationships or changes that are involved in the problem	11 (11)	183 (8)
8.Work backwards	Approach a problem from its outcome or solutions and work backwards to find what conditions it eventually need to meet	14 (9)	136 (6)

7.4.1 A comparison of the instructions for 'acting it out' procedure

Acting out a problem is a strategy whereby students physically act out (use people or objects to demonstrate what is described) what is expected of them in the word-problem. Acting out a problem can make the problem realistic, aiding in students' conceptual comprehension of what the task entails and problem-solving. When given a word-problem in mathematics, the students discuss the crucial components of the text, as well as what the problem requires, while focusing on what is relevant and irrelevant. Problems can then be acted out with the aid of props or manipulatives to demonstrate what is being asked. Understanding word-problems can be challenging, due to various reasons, such as limited vocabulary, low reading levels, retaining and utilizing the multiple steps that may be required.

Figure 7.5 represents an example of heuristic 1, ‘Act it out’ in both the SK and SA textbook, with students being required to make a square pillar using prisms. This activity is designed to link the 3D shape to real life objects. There are 25% of *act it out* opportunities for students in the SK textbook and 10% in the SA texts, which allows students to relate to relevant situations, enabling their development of a greater insight of the concept that is being taught. Both the SK and SA students are required to make a paper-like prism, the former requiring them to draw the view of a square pillar. By building the square pillar from the SK textbook, students are able to link their learning to reality and encourages abstract thinking. The directives in the SK textbook are quite simple and direct, they require the use of cut outs (provided at the back of the textbook) to save the students time to complete the activity.

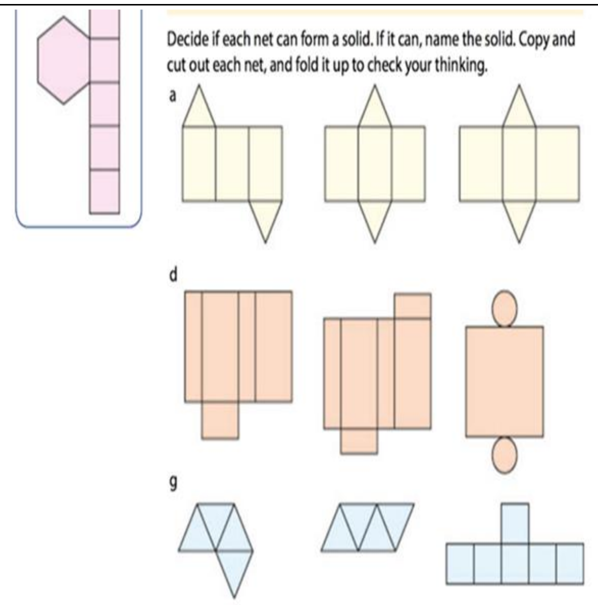


Open your mind:

Jungwoo: uses a paper-like prismatic shapes to make a model of a square pillar.

Let's draw a developed view of a square pillar.

SK textbook: model construction, (H1)



SA textbook: model construction, (H1)

Figure 7.5: SK & SA textbook: Model construction, (H1)

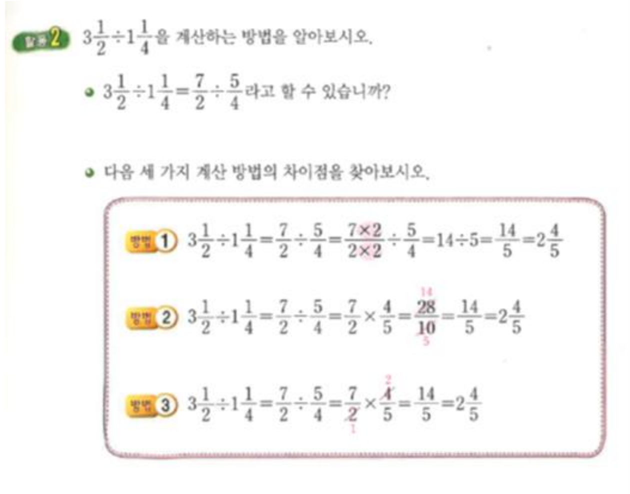
The SA textbook requires the students to make the model, with the cut-outs not being provided, and to think about if each net can form a solid. The SA book asks for copying (drawing), cutting, creating a model (concrete, hands on, authentic learning), with the guidelines allowing students to verify their thinking. Peranginangin and Surya (2017) explains that using ‘act it out’ activities allow students to relate what is being learnt to what they see in their environment. There are a few reasons why this type of learning can be considered important, for example, students will recall facts and skills effortlessly when they use them to solve actual problems. According to research, subtraction errors made by elementary students revealed that using examples is vital for gaining critical skills in problem-solving (Watson, Lopes, Oliveira & Judge, 2018).

While the SK and SA textbooks provide students with the opportunity to act out problem solving procedures by construction of models, the approach varies in these textbooks. The SK textbook have additional resources at the back of the book which allows student to remove and create a model time when required, whereas, the activity in the SA textbook expects students to create their own models from scratch. This process is time consuming and can hinder learning time.

7.4.2 A comparison of the instructions for 'changing your point of view' procedure

'Change your point of view' entails being able to approach a challenge from a different perspective when a prior method is not effective. There are at least three 'change your point of view' methods in the textbooks, giving the students several methods to solve a problem without restricting them to use a specific method, with Boaler and Anderson (2018) referring to this as the best practices in the classroom. Furthermore, there are 535 drawings of diagrams, pictures and sketches in the SK textbook, providing ample opportunities for the use of alternate methods to find solutions. There are 581 drawings, diagrams, pictures and sketches in the SA textbook, also providing students ample ideas of how-to problem solve. Then 166 activities are present for students to use various methods to find a solution to a problem. The SK and SA textbooks present similar number of activities that provide students with the opportunity to have alternate strategies to solve problems.

Figure 7.6 represents an example of heuristic 2, 'Change your point of view' in the SK and SA textbook, where students are required to calculate fractions and use the column method to add. Each text provides three examples, although the SA text clearly demonstrates three alternative methods for finding a solution, which aids in cognitive demand. The majority of fraction, multiplication and division problems in both curriculums require only procedural knowledge. However, multistep computational problems are more common in the SK than SA book, although, there are more questions in the latter than the former. The SA textbook provides students with opportunities to solve problems using more than one method in Figure 7.6, enabling them to have a place of reference for future problems.

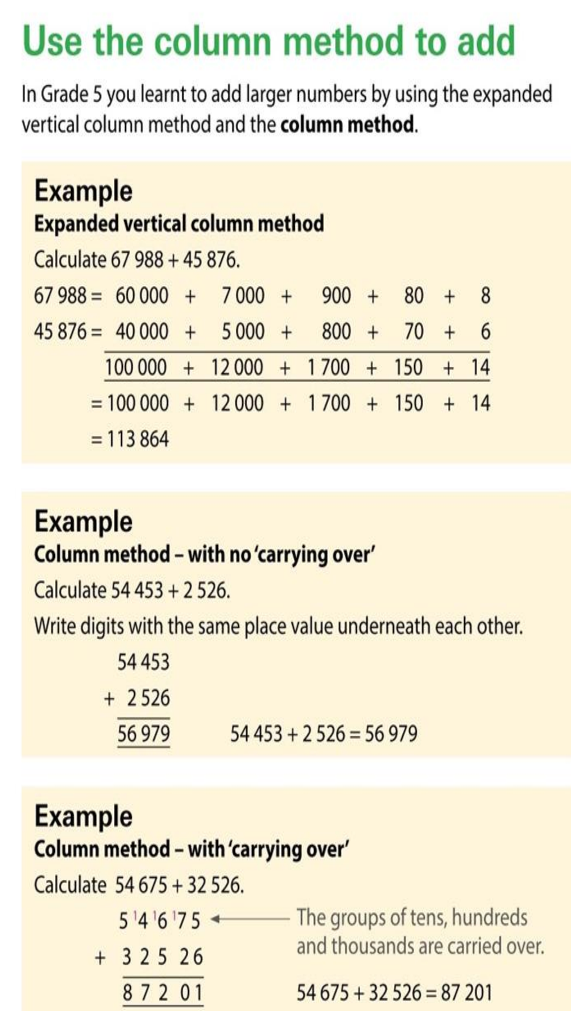


Learn how to calculate 3 ½ ÷1 ¼

Can I say that 3 ½ ÷1 ¼ = 7/2 ÷ 5/4?

Find out the differences between the three calculation methods with your friend:

SK textbook: multiple example representation, (H2)



Use the column method to add

In Grade 5 you learnt to add larger numbers by using the expanded vertical column method and the **column method**.

Example

Expanded vertical column method

Calculate 67 988 + 45 876.

67 988 =	60 000	+	7 000	+	900	+	80	+	8
45 876 =	40 000	+	5 000	+	800	+	70	+	6
	100 000	+	12 000	+	1 700	+	150	+	14
	= 100 000	+	12 000	+	1 700	+	150	+	14
	= 113 864								

Example

Column method – with no 'carrying over'

Calculate 54 453 + 2 526.

Write digits with the same place value underneath each other.

```
  54 453
+  2 526
  56 979
```

54 453 + 2 526 = 56 979

Example

Column method – with 'carrying over'

Calculate 54 675 + 32 526.

```
  5 ¹4 ¹6 ¹7 5  ←— The groups of tens, hundreds
+ 3  2  5  2 6      and thousands are carried over.
  8  7  2  0 1
```

54 675 + 32 526 = 87 201

SK textbook: multiple example representation, (H2)

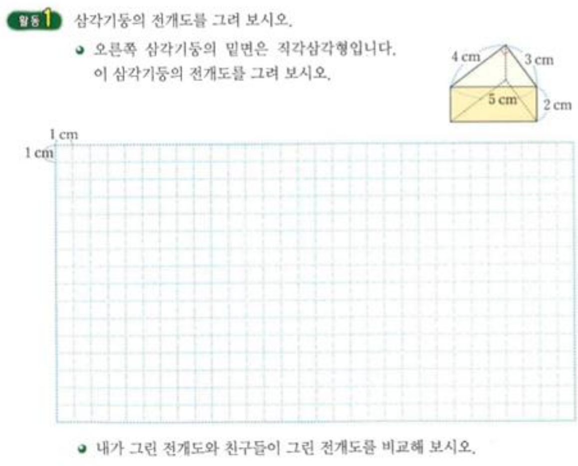
Figure 7.6: SK & SA textbook: Multiple example representation, (H2)

The instructions given in the SK textbook require the student to identify the differences between the three calculation methods, which the SA textbook also expects. The SK textbook portrays a more advanced level of expectation from the students learning process, while the use of mathematical language is more advanced in the SA textbook. The SK textbook is designed for student understanding of instruction and what is expected, with a number of solutions for a problem. The SK textbook provides guidance for students to select the most relevant solution, which helps in achieving cognitive, affective and psychomotor objectives. This process helps develop students into critical thinkers, giving them opportunities to think, design, plan, execute, change, re-evaluate and solve problems from an intrinsic perspective, which meets the needs of differentiated learning.

The South African textbook provides a variety of solved problem formulations, with steps and the final solution, with students being able to decide which example to use in their own problem-solving procedure. Anderson, Farrell, and Sauers (1984) and Pirolli and Anderson (1985) explain that low and medium level students, at the beginning, tend to ignore using advance heuristics, and rely on the examples provided for problem-solving. Students depend on context, prior knowledge, learnt skills, various ways of thinking and engagement with resources to create understanding about problem-solving procedures. They need to explore problem-solving procedures in context, which is possible if they can identify the differences in problem-solving procedures.

7.4.3 A comparison of the instructions for 'drawing a diagram' procedure

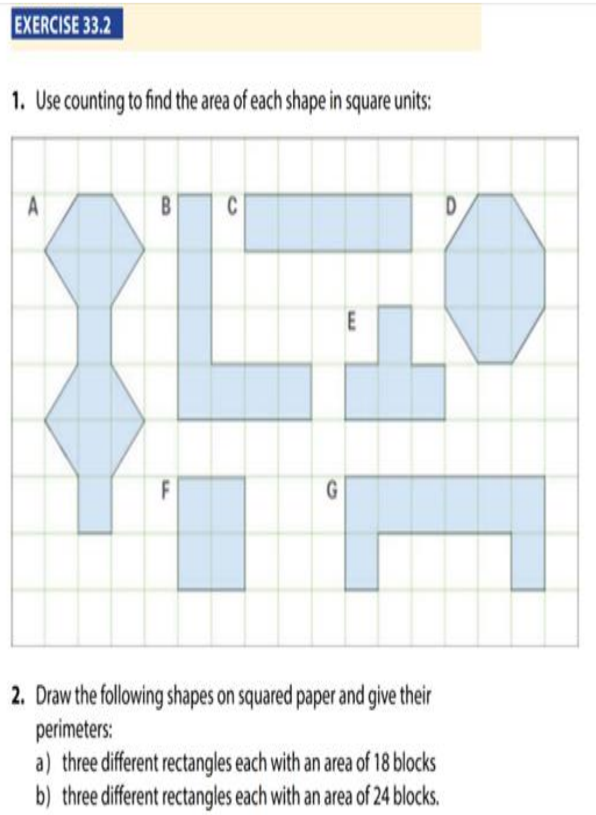
The 'draw a diagram' strategy is a method where students create a visual representation of a problem, which helps them to learn how to use a fundamental strategy which is effortlessly understood by students of all abilities. In Figure 7.7, students are presented with opportunities to draw diagrams as a procedure, the SK textbook giving them the chance to compare it with their friends' drawings, a process that allows them to develop communication, creative, cognitive and procedural skills. Illustrating a diagram requires students to draw a sketch based on the accessible data, to visually represent the problem, with SK providing 6% and SA 7% sketch activities for them to construct a diagram. According to Shin and Bryant (2017), various types of visual representations are often a good foundation for solving word problems, being a reward with purpose between language-as-text and the symbolic language of mathematics.



Draw the development of the triangular pillar - The bottom right triangular column is a right triangle.

Draw the development of this triangular pillar. Compare your developed drawing with your friends' developed drawing.

SK textbook: sketch development, (H3)



SA textbook: sketch development, (H3)

Figure 7.7: SK & SA textbook: Sketch development, (H3)

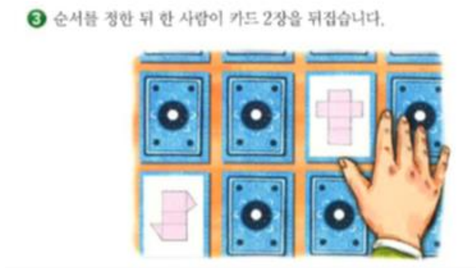
The visual and concrete representation of measurement and objects allows students to link real life objects to mathematics learning. This allows students realize that we use mathematics in our daily lives. Pictures and diagrams are a way of describing solutions and are an important part of mathematical communication, with both the SK and SA textbook requiring students to draw diagrams to represent different shapes. The SK textbook require them to draw the development of a triangular pillar, with further guidance on identifying the right angle at the bottom of a right triangular column, an example of what is expected is provided. The students are then expected to compare their developed drawing with their friends', the activity highlighting the identification of shapes and angles, ability to construct, methods of measurement and cooperative learning through comparing work.

In comparison, the SA textbook, instructs students to draw diagrams of three different rectangles on squared paper with an area of 18 and 24 blocks, while the textbook also provides examples to demonstrate exactly what is expected of them. This activity requires students to use what they have practiced to create the follow-up diagram. Spooner, Saunders, Root and Brosh (2017) explain that drawing allows students to create the required shapes, presenting them with hands-on activities and visual representations. The objective is to educate students to identify fundamental problem structures in word problems for improved applicability to real-world scenarios. Novriani and Surya (2017) contend that student should be given ample visual representations about the information so as to effectively accomplish the problem-solving procedures.

7.4.4 A comparison of the instructions for 'guessing and checking' procedure

'Guess and Check' is a problem-solving strategy which students utilize to solve mathematical problems by guessing the answer and then checking that it fits the conditions of the problem. Estimation is the most common topic, where students are required to guess a specific quantity before they weigh the object. Figure 7.8 is an example of a guess and check activity given in the SK and SA textbooks. The former provides the activity in the form of a game and the latter in the form of estimation questions with addition to various skills. SA provides opportunities to use the information to practice the skill, to extend the skill with step 2, decide which measurement to use in step 3, use real world objects in step 4 and evaluate which scale to use in step 5. The SK textbook has 10% and SA textbook 8% of guess and check problem-solving activities.

The SK textbook allow students to play a card game, the resources being provided at the back of the book as cut-out cards, with the game being played in pairs and the rules provided in the image. The SA textbook has an example of a guess and check problem that requires students to estimate and then measure the liquids using a measuring jug, this being a hands-on activity that allows them to get involved in the learning process. Firstly, they are expected to find objects in the classroom to weigh and to add the names to the table. Secondly, they need to decide which unit of measurement is appropriate for each object, grams or kilograms, requires some experience with weight, and to hold the object and to estimate its mass. Lastly, the object has to be weighed by choosing the best scale, with correct mathematical language being used throughout the activity.



Translation: Looking at the card removed from the pack of cards.

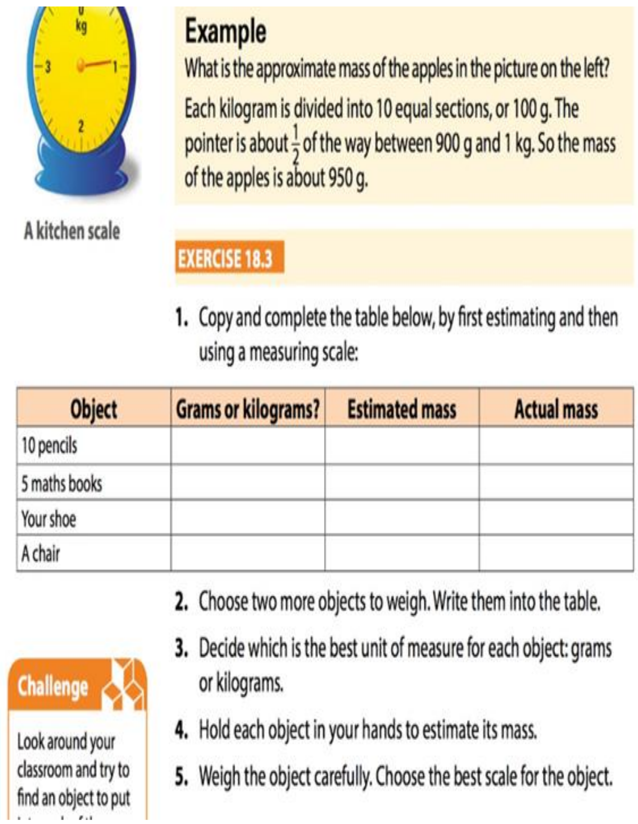
You: Describe the shape.

Your partner: Guess the shape.

You: Name of the prism or pyramid.

Place the card upside down on your desk.

SK textbook: 3D guess and check objects, (H4)



A kitchen scale

Example

What is the approximate mass of the apples in the picture on the left?

Each kilogram is divided into 10 equal sections, or 100 g. The pointer is about $\frac{1}{2}$ of the way between 900 g and 1 kg. So the mass of the apples is about 950 g.

EXERCISE 18.3

1. Copy and complete the table below, by first estimating and then using a measuring scale:

Object	Grams or kilograms?	Estimated mass	Actual mass
10 pencils			
5 maths books			
Your shoe			
A chair			

2. Choose two more objects to weigh. Write them into the table.
3. Decide which is the best unit of measure for each object: grams or kilograms.
4. Hold each object in your hands to estimate its mass.
5. Weigh the object carefully. Choose the best scale for the object.

Challenge

Look around your classroom and try to find an object to put

SA textbook: estimation of measurement, (H4)

Figure 7.8: SK & SA textbook: 3D guess and check objects, estimation of measurement, (H4)

7.4.5 A comparison of the instructions for 'looking for a pattern' procedure

'Look for a pattern' is a strategy in which students aim to identify patterns in the information to solve the problem, requiring them to find items, numbers or repetition of sequences. The SK textbook has 25% and SA 13% of problem-solving procedures that allows the students to look for a pattern using the concept represented. Figure 7.9 represents heuristic 5, the SK students being required to create a fountain using less than five shapes, which can be repeated. The SA textbook presents an activity requiring students to identify transformations in patterns, with learning being extended through specific questions.

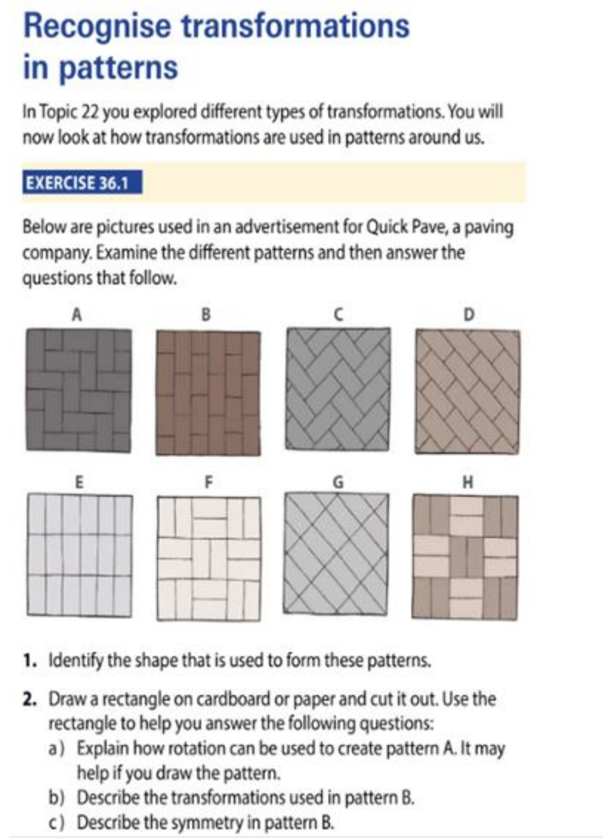
Experiment: Pieces made of shapes

Among the pieces of shapes
(Group 4) -Use a piece of resin shape

You want to create your own large fountain with less than 5 pieces which can be repeated.

•Build a fountain, know the relationship between the patterns (width of the shape piece).

SK textbook: pattern identification, (H5)



Recognise transformations in patterns

In Topic 22 you explored different types of transformations. You will now look at how transformations are used in patterns around us.

EXERCISE 36.1

Below are pictures used in an advertisement for Quick Pave, a paving company. Examine the different patterns and then answer the questions that follow.

1. Identify the shape that is used to form these patterns.
2. Draw a rectangle on cardboard or paper and cut it out. Use the rectangle to help you answer the following questions:
   a) Explain how rotation can be used to create pattern A. It may help if you draw the pattern.
   b) Describe the transformations used in pattern B.
   c) Describe the symmetry in pattern B.

SA textbook: pattern identification, (H5)

Figure 7.9: SK & SA textbook: Pattern identification, (H5)

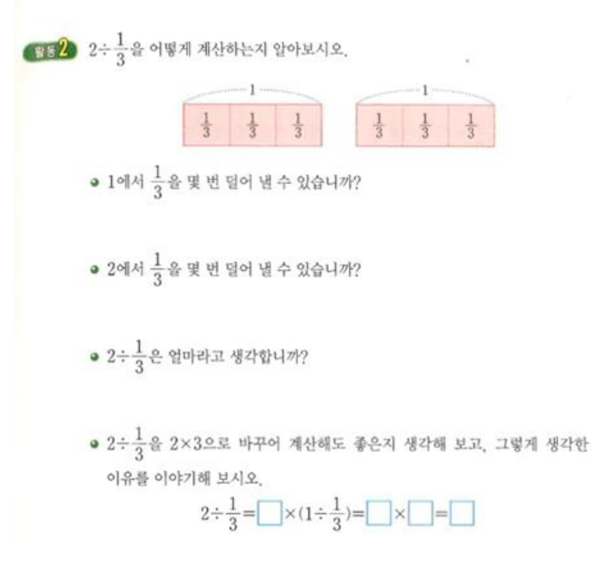
In the SK textbook (Figure 7.9) the 'look for a pattern' activities include using cooperative learning in groups to identify solutions, which helps students to express their thoughts, brainstorm ideas and discuss possibilities. The task requires the group to build a fountain, for which they must identify the relationship between the patterns, taking the width of the shape piece into consideration. The SA textbook also provides various problems for students to either create or complete a pattern. In Figure 7.9, the students are required to identify the shape that is used to create the various patterns, to match its shape as a cut-out, and explain how rotation can be used to create a pattern, being further encouraged to draw the pattern to extend learning. Students are asked to describe the transformation used and describe the symmetry in pattern B.

When students analyse math problems, they notice patterns, learn to generalize, form new ideas and activate their creative thinking. In order for a student to make the maximum use of their visual thinking capacity, they will have to initially master pattern recognition, which is presented in both SK and SA textbooks. Again, the SK textbook presents the activity in simple and easy to understand language, while the SA textbook uses proper mathematical language throughout the activity, which is important, as it provides second language students with additional information to enhance understanding.

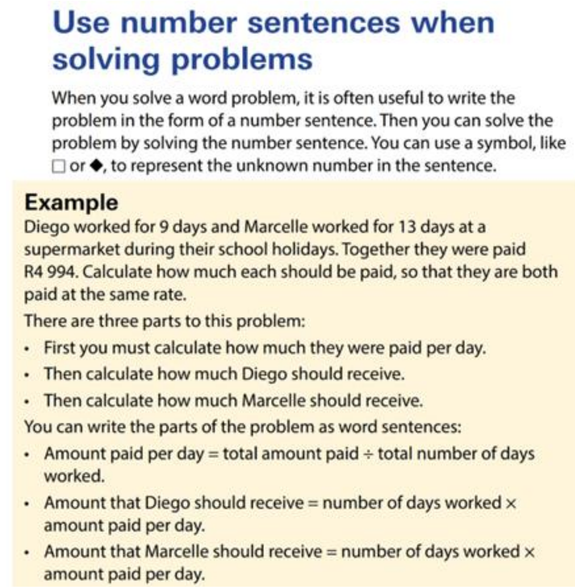
7.4.6 A comparison of the instructions for 'solving part of a problem' procedure

In Figure 7.10 of the SK text, students have the opportunity to solve part of a problem by breaking it down into manageable steps, enabling them to focus on achievable steps. The students are provided with 134 opportunities in the SK textbook, to segment procedures into sub-problems and then solve a portion at a time. In the SA textbook (Figure 7.10), there is a representation of a problem being broken up in various parts, with the students having to draw a table to show the possible outcomes that can be analysed once completed.

Both the SK and SA textbook allows for students to divide problems in order to make it easier to solve. However, the SA textbook provides more exercises which does not include open-ended problems. Students can gain from having effective problem-solving skills as a skill to prepare them for solving problems every day. Some of the problems in the SK and SA textbooks are more complex than others. Students must develop the skill to solve all problems efficiently, timeously and learn that there is no single method to solve all problems. However, there is always an element of the unknown in problem-solving, which can cause frustration when the problems are too complex, one way of dealing with them being to simplify the problem. The SK and SA textbook provide students with opportunities to try smaller numbers and solve parts of the problem, this allows them to segment issues in manageable steps, enabling the bigger problem to be more obtainable and not overwhelming.



SK textbook: addition of fractions, (H6)



Use number sentences when solving problems

When you solve a word problem, it is often useful to write the problem in the form of a number sentence. Then you can solve the problem by solving the number sentence. You can use a symbol, like □ or ◆, to represent the unknown number in the sentence.

Example

Diego worked for 9 days and Marcelle worked for 13 days at a supermarket during their school holidays. Together they were paid R4 994. Calculate how much each should be paid, so that they are both paid at the same rate.

There are three parts to this problem:

- First you must calculate how much they were paid per day.
- Then calculate how much Diego should receive.
- Then calculate how much Marcelle should receive.

You can write the parts of the problem as word sentences:

- Amount paid per day = total amount paid ÷ total number of days worked.
- Amount that Diego should receive = number of days worked × amount paid per day.
- Amount that Marcelle should receive = number of days worked × amount paid per day.

SA textbook: addition of fractions, (H6)

Figure 7.10: SK & SA textbook: Addition of fractions, (H6)

7.4.7 A comparison of the instructions for 'using a model' procedure

Mathematical modeling is a method of utilizing a variety of mathematical strategies, such as graphs, equations, diagrams, scatterplots and tree diagrams, to portray real-life scenarios. The model in Figure 7.11 from the SK and SA textbook offers a notion which reduces a problem to its vital characteristics. The SK textbook provides 11% and SA textbook 8% of problem-solving questions in which they are required to make models. The SK textbook presented students with cut-outs for all activities (as mentioned above), the representations being treated as an essential element that provides a connection with concepts and familiar objects in supporting students' understanding of mathematical concepts.

Students are required to make a prismatic building model in this activity, the relationships involved in communicating mathematical approaches, arguments and understandings in individual and group tasks being of importance. The problem-solving procedures continue to focus on identifying relationships amongst linked mathematical concepts and applying mathematics strategy to real-world scenarios. Figure 7.11 in the SA textbook gives an example of a model being used to develop understanding about the surface area of an object.

Using a model helps students to create a visual representation (e.g. using points, lines or other easy-to-understand symbols, cut-outs) of knowledge on quantities, connections or variations involved in the problem. Student are expected to measure straw and create a pyramid in the SA activity, in which they are given measurements and accurate steps to follow. When students obtain access to mathematical representations, they are equipped with a set of tools that substantially increase their ability to think mathematically. Mathematical modeling is part of the mathematics curriculum at all Grade levels, including preK-2, with the NCTM Principles and Standards for School Mathematics (PSSM) recommending that "*students should learn to make models to represent and solve problems*" (PSSM, p. 94).

Uncle is making a prismatic building model "I am trying to attach a window, let us look at the bottom and sides.

SK textbook: model construction, (H7)

3. Model a square-based pyramid

Step 1: Cut four straws to 6 cm lengths and four straws to 10 cm lengths.

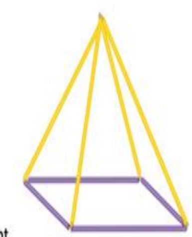
Step 2: Take the four shorter straws and thread them onto a piece of cotton.

Step 3: Tie the ends of the cotton tightly to form a square.

Step 4: Tie the cotton to each vertex of the square and thread a longer straw onto each piece.

Step 5: Tie the ends of the cotton together to form a square pyramid.

Step 6: Use glue or sticky tape to cover each side with different coloured tissue paper.



SA textbook: model of a square-based pyramid, (H7)

Figure 7.11: SK & SA textbook: Building a house representation, (H7)

The similarities and differences between the SK and SA text are the development of how skills are learnt. The SK and SA text requires the use of model making and the creation of visual representations, however in the SA text, it is required on a smaller scale. This process helps students to see and touch what they are learning conceptually. In SA, the presentation of exercises can improve the ability to retain and recall information, although some students may struggle to grasp the concept of an open-ended question. The question requires the student to make a model with paper and string. Novriani and Surya (2017) state that providing students with concrete objects that also cater for the kinesthetic and visual learning styles is beneficial in developing problem-solving skills.

7.4.8 A comparison of the instructions for 'working backwards' procedure

There are various methods to solve a math problem, equipping students to be successful problem solvers being as important as teaching computation and algorithms (Kong, (2019). Figure 7.12 illustrates a problem in the SK and SA textbook that requires students to work backwards, one step at a time to get to the starting point, or to simply check if the answers are correct. Working backwards is important to learn as it is a researched proven best practice strategy that aids in the development of meta-cognitive demand. The SK text provides students with 9% of problems that require 'working backwards' as a strategy to check work. The SA text offers 6% of the inverse operations to allow students to use one operation to check another.

Translation:

1. Plan for a building construction.
2. Draw a blueprint for the construction of the building and write down the preparations.

SK textbook: construction of a building (H8)

Inverse operations

Remember that multiplication and division are inverse operations. You can use one operation to check the answer of the other.

Example

Earlier, we used long division to find that 4 983 ÷ 215 = 23 remainder 38.

Use multiplication to check the answer: $(215 \times 23) + 38$

$= 215 \times (20 + 3) + 38$

$= (215 \times 2 \times 10) + (215 \times 3) + 38$

$= 4\,300 + 645 + 38$

$= 4\,938$

SA textbook: inverse operations (H8)

Figure 7.12: SK & SA textbook: Construction of a building, inverse operations, (H8)

The SK textbook provides a model of a building, the students being required to draw a blueprint for the construction and write the steps for the preparation process. The instructions are clear and need students to work backwards in order to create an exact building. The SA textbook provides activities that require inverse operations, allowing students to check their answers by working backwards e.g. 2x3=6; 6/3=2.5-3=2; 3+2=5. If working forwards requires addition, when working backwards subtraction will be required. If they multiply working forwards, they must divide when working backwards, this being important because it helps to integrate and build on other learnt skills. This requires students to be confident and knowledgeable with all operations, these techniques allowing students to check their answers for understanding and correctness.

Table 7.4: Summary of the similarities and differences of the SK and SA textbooks

No.		South Korea	South Africa
		Polya's 4 Stages	
1	Similarities	Real-life, familiar objects followed by tasks Mathematical objects followed by tasks	
	Differences	Prior knowledge followed by current knowledge	Current knowledge followed by prior knowledge
2	Similarities	Activities requires making a plan using materials	
	Differences	Activity is cognitively demanding.	Activity require following a given example to work out the answer.
3	Similarities	Activity requires the use of measuring using real objects	
	Differences	Requires, prior knowledge and critical thinking.	Scale given with measurements that can be used to solve the problem.
4	Similarities	Link prior knowledge to complete activity.	
	Differences	Real-life, familiar objects followed by task.	Mathematical objects followed by task.
		8 Heuristic	
1	Similarities	Math models used to complete a task.	
	Differences	Cut-out models provided to complete a task.	Models have to be made by students.
2	Similarities	A choice of three examples followed by tasks.	
	Differences	Tasks require analysis of the examples.	Tasks requires a choice of a method.
3	Similarities	Mathematical objects followed by task.	
	Differences	Math objects with measurements and grid paper provided for the task.	Measurements of the math objects are provided for the task.
4	Similarities	Guessing activity	
	Differences	Linking patterns and 3D shapes.	Linking prior knowledge with current knowledge.
5	Similarities	Math objects used to complete tasks	

	Differences	Current knowledge followed by prior knowledge.	Real-life, familiar objects followed by tasks.
6	Similarities	Clear step by step direction given to complete task.	
	Differences	Steps are the guide to complete the task.	An explanation followed by an example then task.
7	Similarities	Task leads to Current knowledge.	
	Differences	Real-life, familiar objects followed by tasks.	Mathematical objects followed by tasks.
8	Similarities	Current knowledge leads to prior knowledge.	
	Differences	Mathematical objects followed by tasks.	Math concepts followed by tasks.

7.5 Conclusion

The analysis process revealed strategies such a; drawing diagrams, making models, looking for patterns and working backwards, these techniques being utilized by both South Korean and South African texts to assist students to work towards mastery of skills. The differences found in performance requirements indicate that the South Korean textbook included more open-ended questions and a variety in solution requirements, while the South African textbook provides more guidance in how the questions can be answered. The results are relevant to documented cross - national differences in South Korean and South African students' mathematical performances. This study found theoretical and practical issues around non-textual elements, such as models, which are attached at the end of the textbook that may be useful for future research. Chapter 8 will conclude with the findings and recommendations of this study.

CHAPTER 8: Conclusion

8.1 Introduction

Chapter 5 addressed Objective 1, how problem-solving is represented in the South Korean textbook, specifically focusing on basic structural aspects, organisation, layout, content and language. While the structure of knowledge within a textbook is important, the physical structure will determine whether the intended presentation will be beneficial to the students. It included many aspects, such as those outlined by Valverde et al. (2002), area and framing, elements (pictorial, verbal, design), colour, non-colour and information levels. Chapter 6 addressed Objective 2, how problem-solving is represented in the South African textbook, which presented the structural analysis of the South African textbook, as done for the South Korean book.

Chapter 7 presented the findings for Objective 3 and 4 "the similarities and differences in the representation of problem-solving between South Korea and South Africa, which focused on the representation of problem-solving procedures in the South Korean and South African textbooks, highlighting the similarities and differences in relation to Polya's 4 stagemodel and 8 heuristics. The decision to analyse textbooks as a contributing factor of success was due to South Korean students featuring in the top five ranking of the 50 countries that participated in the TIMSS, they have excelled in Mathematics for the past 20 years. The South African students ranked penultimate on the TIMSS, indicating the need to explore the reasons for their poor performance.

This chapter will establish the extent to which the purpose was achieved by summarizing the study findings, outlining the significance of the study, identifying possible limitations, and present a number of recommendations for further investigation or application. Mathematics is a subject that teaches students standardized procedure to solve problems using numbers, with the manner in which instruction occurs, may vary around the word in terms of the curriculum and textbooks for each school grade. However, it is anticipated that by the time they leave school, all students globally will have achieved a certain level of mathematics literacy to enable them to not only function at a specific level, but to have sufficient background knowledge to enter tertiary academic institutions. The purpose of this study was to look at one point in that journey of instruction, Grade 6, to see if there were comparable areas of instruction in terms of content and the ability to solve problems using critical thinking associated with mathematics was being presented. The contention is that all the grades are important to equip students with knowledge and skills to reach their final year, with Grade 6 being selected due to their participation in National tests. The differences between the text books of SK and SA were explored to establish if this was one component of instruction that could account for the onward journey of students that affected not only the comparative marks of the students in that year, but at the end of their schooling. Significant differences in the problem-solving presentation of the books may suggest that this could be a contributing factor to the varying marks, and provide an opportunity to reflect on how the SA book could be modified to improve the critical thinking which is required to solve mathematics problems.

8.2 Findings surrounding textbook analysis

This study therefore aimed to explore the manner in which problem-solving is presented in the SK and SA Grade 6 mathematics textbook using a composite framework to establish the factors which contribute to students results. The results of the four objectives are presented to motivate the extent to which the aim was achieved.

8.2.1 Research objective 1: To identify how problem-solving procedures are presented in the South Korean Grade 6 mathematics textbooks

The first research objective sought to establish how problem-solving procedures were represented in a South Korean Grade 6 mathematics textbook. The results of this study have indicated; at Grade 6 level, the mathematics textbook has produced ample activities to develop expected level of conceptual understanding (according to Bloom's taxonomy) of the problem-solving procedures which are represented. According to TIMSS this is in line with the support content of the Grade 6 curriculum. I found that the visuals played a considerable role in the presentation of problem-solving procedures. The visual experiments and games were designed to allow students to become independent learners, which aids in cognitive, creative, co-operative, critical and social development. However, the SK textbook did not have key words, math ideas and information boxes which would aid second language students. The problem-solving procedures presented in the SK textbook were designed to develop the student using all of Polya's 4 stagemodel. In addition, relevant and interesting stories represent the concepts that are taught, which aids imagination and relevance, to help enhance student's creativity and critical thinking skills.

Knowledge always exists within context and worldview, which is what students would demonstrate in their daily problem-solving routines (Bardach & Patashnik, 2019). Students demonstrate familiarity with problem-solving procedures (as is presented throughout the textbook), which becomes part of their daily tasks. Furthermore, the results showed that Polya's 4 stage model was integrated into the textbook and that the heuristics were also used. The visuals were relevant to the Korean culture, which makes for easier association with the problem-solving procedures taught. Overall, the general picture that emerged was that the SK textbook was rich with problem-solving procedures, giving the students ample opportunities to practice the 4-stage model and the 8 heuristics. The content gave room for collaboration, with cognitive demand being at varying levels, giving opportunity for differentiation, understanding knowledge, and developing skills in line with their level of schooling.

8.2.2 Research objective 2: To identify how problem-solving procedures are presented in the South African Grade 6 mathematics textbooks

The South African Grade 6 mathematics textbook consisted of 2317 activities, with the main issue being overcrowding and having a set procedure to follow with examples. The problem-solving procedures adhere to the 8 heuristics with limitations, the students are given guidance as how to solve these problems using models and hands-on activities. Examples are presented at the beginning of each topic, which serve as a guide as to how to complete most of the exercises. Most of the problems are representations of the exercises, which mainly consist of routine and traditional questions. Less than 20% of the problems are related to real life situations and almost all the exercise problems have set problem-solving procedures and close-ended solutions.

Most exercise problems are presented either using number operations, algebraic expressions, count, order, compare, represent whole numbers, number sentences, common fractions and converting units of time. Only 1% of the problems entail the use of calculators; furthermore, the answers to the exercise problems are not provided at the back of the book. There are 69 'challenge yourself' activities which are intended to provide more challenges for students (especially for high-performing students) to further develop their critical thinking skills and creativity. The textbook design includes 33 'did you know' information boxes, which provides students with information about mathematicians, how gems and precious metals are weighed using carob seed, as well as measurement and information about the pyramids.

In addition to the challenge exercises and 'did you know' information boxes, three other types of boxes were included in the textbook design, namely: key words, math ideas and games boxes. There are 60 'key words' boxes, which are beneficial to the relevant topics presented. This feature is thought to be useful to second language students, as it provides them with more information about the purpose of the vocabulary. There are 38 topics in the South African textbook, each one starting with an additional feature of a 'math ideas' (38). These give students an idea of the various types of questions they will be asked e.g. to count in large numbers, recognize the place value of digits of up to six-digit numbers, order and compare up to six-digit numbers. These additional features contribute to the textbook's unique design, which helps student with additional information. Finally, there are seven games which allow students to collaborate and enjoy mathematics in the form of fun and competitiveness.

8.2.3 Research objective 3: To compare the similarities and differences in the problem-solving procedures between the South Korean and South African Grade 6 mathematics textbooks

This study examined a South Korean Grade 6 math textbook and a South African Grade 6 textbook, which both aim to support similar content. Both these textbooks were designed with the intention of maximizing teaching and learning in the classroom. However, the designs varied in many ways, which can impact on students learning. The purpose of the study is not to make an overall evaluation of the quality of the textbooks, but to investigate how the textbooks represent problem-solving. Moreover, as noted earlier, an analysis of textbooks is necessary to contribute to understanding of what really happens in classrooms during teaching.

Although the structure and organisation of the SK and SA textbooks are completely different, they provide for the needs of the students according to cultural context, content and language. The SK textbook is written in Korean to provide support to first language students, while the SA textbook is written in English but provides support texts to meet the needs of second language students. The SK textbook is developmental, using a set problem-based design structure for students from Grade 1 – 6, meaning that the same strategies are taught year after year using Polya 4 stage model and heuristic but with increasing cognitive demands. The SA textbook provides students with multiple examples which aid them in problem-solving strategies. While the SA textbook requires students to use heuristics to solve problems, the texts are not structured using a problem-solving model for Grade 6. The implementation of a problem-solving model in the structure of the textbooks will allow for the implementation of different levels of blooms' taxonomy questioning and development of critical thinking skills.

In the SK textbook, students have the opportunity to work with the 'big idea' before utilizing the other provided opportunities. In the SA textbook, the students are exposed to procedures of how problems are solved and less on how to understand, approach, extend and reflect on them, as the problem-solving process is given in the form of examples. Extension and reflection problems in the SA book are given in the form of revision, with more exercise problems which serve as practice. The procedures which are shown in the textbooks are usually routine, easy to work out and have limited need for exploration. This approach provides opportunity for teachers to compliment the lesson with open ended questions and discussion (which is not captured in the textbook). One can infer that these factors could account for the difference in the TIMMS results. A study conducted by Sibanda (2017) states that the bad mathematics performance was due to questions being difficult for students because it had complex grammatical patterns.

8.2.4 Research objective 4: To investigate the extent to which the South Korean and the South African textbooks address the principles required to develop Grade 6 cognitive demands and critical thinking

The SK textbook is designed using a problem-solving approach which is developmental, and which is not evident in the SA textbook. Student learning is greatest in classrooms where students engage in problem-solving tasks which are cognitively demanding (Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016). However, there are growing concerns that many SA students are given limited opportunities to engage in these types of tasks (Maddock & Maroun, 2018). According to Cottrell (2017) students exposed to differentiated tasks with higher-level cognitive demand lead to developing problem-solving proficiency. Furthermore, Cash (2017) states that differentiated tasks that are cognitively demanding will aid students to self-sufficiency in their learning experience. The SK text is designed to provide students with problem-posing tasks from the beginning to end, focusing on developing cognitive demand (Cai & Jiang, 2017). However, the dissemination of problem-posing tasks across content areas in the SA textbook suggests a need for a wider application in the design process. Pepin, Xu, Trouche and Wang (2017) state that tasks that are cognitively demanding lead to the development of deeper understanding which will aid in the advancement of mathematical expertise. Table 8.1 Indicates the application of the framework principles for the two books whether this has been good, satisfactory or poor, to provide an overview of their application and assist in understanding the possible role which the books could play in the growth of mathematics skills in Grade 6. Furthermore, it provides an overview of the researchers' conclusions about the extent to which the South Korean and the South African textbooks address the principles required to develop Grade 6 cognitive demands and critical thinking.

The South Korean textbook addressed problem-solving issues by providing practical higher order thinking geometry tasks, and in doing so provided students with opportunities to overcome or become familiar with known challenges. The results show that the South Korean textbook represented problem-solving procedures using international testing standards while the South African textbook demonstrated limitations in this area.,

Table 8.1. The application of the principles to develop cognitive demands and critical thinking

Principles	South Korea	South Africa
Approach		
HORIZONTAL PRINCIPLES FOR TEXTBOOK DESIGN AND ANALYSIS		
1. Structure and organisation: content, sequencing & overview.	Good	Good
2. Layout: logical, consistent, illustrations, font & type of paper.	Good	Satisfactory
3. Content: Aims, objectives, content sequencing, current content & prior knowledge.	Good	Satisfactory
4. Language: age appropriate and for 2nd English language student.	Satisfactory	Good
VERTICAL CRITERIA FOR TEXTBOOK ANALYSIS		
1. Didactic factors: diversity, meaningful and engaging.	Good	Good
2. Conceptual understanding: develop critical thinking.	Good	Good
3. Polya's 4 stages: develop cognitive demand and create independent thinking.	Good	Satisfactory
4. 8 Heuristics: develop conceptual understanding and create independent thinking	Good	Satisfactory

8.3 Discussions surrounding the horizontal principles

The textbook analysis consisted of two components which will be discussed in relation to teaching and learning in the classroom. Firstly, the horizontal principles for textbook analysis will be addressed. The horizontal components consisted of the following:

8.3.1 Structure and organisation of the SK and SA textbooks

The SK and SA textbook provide a suitable structure of the content, to facilitate learning. The SA textbook content is sequenced and logical, using key words, information boxes and challenging questions. The structure of content is made apparent by means of functional devices, including table of contents, chapter titles, headings and outlines in both SK and SA textbooks. An overview of the learning targets is presented at the beginning and a summary at the end, of each chapter. A simple student's guide is also presented in an introductory section to inform students how to use both textbooks. This guide enables students to use the book independently by following the steps, easy access to topics and it shows where the learning outcomes can be found which makes students learning easier.

8.3.2 Layout of the SK and SA textbooks

The SK textbook uses lightweight paper, which helps reduce the weight as their books are used only for the academic year and then discarded. Textbooks are separated into a few thin volumes and printed in separate modules to assist students to develop the ability to manage information as part of their learning. The books are thin and compact in size which provides students with a sense of satisfaction when completed. The design does not facilitate the reuse of the SK textbooks, for example, materials for once-off use (e.g. stickers, and cut outs).

The SK textbook uses the guiding principles which includes criteria for quality textbooks in areas such as structure, organisation, layout, content, language, teaching and learning, technical and functional requirements (Bureau, 2016). These principles are general and fundamental to textbooks for elementary students, all Key Learning Areas (KLA) and subjects at primary and secondary levels (Romel & Brucal, 2018). One can infer that these factors could be a major contribution to the excellent international results produced by South Korea.

The SA textbook is designed for repeated use and not suitable to be used as a workbook. It is high quality paper and colour filled and adheres to the guiding principles by providing students with various text boxes which includes challenges, word boxes, information boxes, games etc, which supplements the topics that are presented. In the SA textbook, photographs, drawings, mathematical objects are added to illustrate the integration of general knowledge subjects in the introduction section.

The layout is logical and consistent in both textbooks and the materials are well-organized with suitable use of space and borders for ease of reading. In the SK and SA textbook, illustrations such as photographs, pictures and graphs are accurate, appropriate, effective and suitably explained to motivate and facilitate learning. They serve to direct students to the instructional focus rather than distract them from it. However, in the SA textbook, illustrations are used to enhance the understanding of concepts due to language barriers.

8.3.3 Content of the SK and SA textbooks

The aims, targets and objectives are aligned with the relevant mathematics curriculum in both the SK and SA textbooks. The content in the SK textbook is sufficient to address the learning targets of the curriculum successfully, not requiring additional supplementary materials. The SA textbook provided additional resources to facilitate learning. The core elements of the curriculum are included in both the SK and SA textbooks. The content in the SK and SA textbooks are current, relevant and accurate; the mathematical concepts are correct and precise. There are sufficient examples and illustrations in both the texts. Examples and illustrations are interesting and related to students' daily experiences. The scaffolding process is used to develop concepts whereby the level of difficulty of the content is consistent with the curriculum requirements and the cognitive level expected of students. Appropriate consideration is given to students' prior knowledge and learning experience in both texts. There is continuity in the development of concepts and skills to facilitate a smooth transition between different key stages of learning, in both texts. However, the content in the SA textbook has to be structured using a problem-solving model, preferably Polya's 4 stage model.

8.3.4 Representation of language in the SK and SA textbooks

Textbooks are important sources of reading and are designed for students to facilitate independence and consolidate meaning. The SK textbook is written in the students' home language. The level of difficulty of the vocabulary is proportionate with the language ability of the target students, with new vocabulary progressively introduced in context at appropriate times. Reading passages are included to help students construct knowledge and process the content in a meaningful way. By using literacy as a tool, the students are able to process and extend meaning, to enrich understanding.

However, the SA textbook is written in English rather than the home language of the students, creating challenges for students with language barriers to understand the text. With assumed prior knowledge, the textbook may not relate to their culture or socio-economic context. For example, the analogies and examples may not be familiar to students' experience and or related to the rich diversity which is present in the classroom. However, the language is accurate and precise.

8.4 Discussions surrounding the vertical principles

The vertical analysis relates to the factors which influence teaching and learning, and relates to the didactic factors, conceptual understanding, Polya's 4 stages of problem-soling, and the 8 heuristics. As indicated at the beginning of this study, textbook design plays an important role in the analysis process which can be used to improve the quality of study materials. The vertical analysis component is integrated in the following discussion.

8.4.1 Didactic factors impact on textbook design and teaching

Teachers who follow textbooks in a passive way are likely to use tasks and information as solely presented in the book. If the textbook presents tasks in a passive dominant way, the teachers are likely to adopt the same approach. A good textbook design entails content which is aligned with the curriculum, provides opportunity for learning and teaching, structure and organisation, is easy to understand and has a good layout (Glatthorn, Boschee, Whitehead & Boschee, 2018). The following sections describe the main features that characterize quality textbooks and how the design of the South Korean and South African textbooks complied or lacked thereof.

The content of a textbook of a particular subject area manifests or translates the four components of the curriculum (aims, content, learning/teaching strategies, assessment) for the purpose of student learning. I found that the curriculum aims, targets and objectives were aligned with both SK and SA mathematics textbooks. The content in the SK textbook is self-contained and sufficient to effectively address the learning targets of the curriculum, without requiring the use of additional supplementary materials. In addition, the core elements of the curriculum are included, and the textbook does not cover unnecessary information. However, the SA textbook provides students with additional resources to solve problems. The materials which are included are non-core, non-foundation topics and serve for enrichment, being properly indicated in information boxes. The SA textbook provides additional information, which the SK textbooks do not provide. The reason for this is that the SA textbook is produced for second language learners unlike the SK textbook which is created for first language learners.

The content, information and data in the SK and SA textbook is current, applicable and precise. The concepts are correct and precise, the ideas are coherent, and there are satisfactory examples and illustrations which are interesting and relevant to students' experience. A continuous scaffolding process occur in order to develop conceptual understanding. There is an appropriate balance between breadth and depth in subject content concerning the level of critical thinking which is consistent with the cognitive level of students and curriculum demands.

The SK textbooks are designed for developmental progress, therefore prior knowledge and learning experience are highly considered. There is continuity in the development of concepts and skills to enable a smooth transition between different key stages of learning and Grades. In the SA textbook, the connections between related topics and problem-solving procedures are highlighted. The content and illustrations in both textbooks, do not carry any form of discrimination on the grounds of gender, age, race, religion, culture, disability etc. However, both textbooks do not provide websites to encourage and facilitate students to read larger amounts of materials on their own. The textbook should have additional pages with websites for students to access, this will serve as enrichment activities and additional research topics which would be beneficial for students' development of 21st century skills.

8.4.2 Polya's 4 stages develops conceptual understanding

It is important for South African textbooks to be aligned to the curriculum, with the aim of educating students of the 21st Century (Chu, Reynolds, Tavares, Notari & Lee, 2017). The SK Curriculum Development Institute conducted an all-inclusive review of the curriculum and created an adaptable curriculum framework which provides for differentiation. The current SA curriculum should aim to help students to learn how to develop problem-solving skills, promote positive values, attitudes and a commitment to life-long learning (Behnke, 2018). The SA textbook design could use the SK book as a model and focus more on the integrations of Polya's 4 stagemodel. This would assist in developing problem-solving skills, critical thinking, as well as procedural and conceptual understanding. According to Robertson and Graven (2019) the power of language can impact positively or negatively on opportunities for comprehending mathematical questions. There are many students in South Africa learning mathematics through English, which is not their primary language, and this could be one of the contributing factors to the poor performance in national and international assessments of mathematical proficiency. The SA textbook provides more exercise questions and can be improved in some aspects. The SA textbook needs to include more non-routine problems, non-traditional problems (e.g. projects), open-ended and application problems (especially using authentic real-life situations).

8.4.3 The 8 heuristics to develop conceptual understanding

According to Weaver, Chastain, DeCaro and DeCaro (2018) conceptual understanding is superior to memorized algorithms for solving tasks. They further state that a link between the memory and conceptual understanding is associated with connections between mathematical concepts and everyday life. All activities in the SK texts provides a continuous link between mathematical concepts taught and everyday objects, allowing problem-solving to be linked with reasoning. However, the SA textbook does not provide a continuous link between concepts and everyday objects which could result with most students having difficulty to understand and apply the concept of mathematics in a real-world context.

A study conducted by Jazuli, Sulthon and Kuswandi (2017) found that students who were taught to use a structured problem-solving strategy articulated a better understanding of concepts. This led the researchers to apply a similar strategy to contextual learning in order to improve problem-solving results. The results revealed that the contextual learning strategy significantly affected the conceptual understanding and the skill to solve problems in mathematics. Though heuristics are problem-solving strategies which are used in the SK and SA textbook, the SK textbook design incorporates these strategies in every activity resulting in higher performance levels.

8.5 Textbook design can develop critical thinking

The SK and SA textbooks are designed to develop knowledge, skills and understanding in mathematics. They are not a collection of useful material, but a guide for the student which will aid in mastering the subject area. Textbooks develop an understanding of the subject, usually integrating theory and practice for each topic as it covers the subject domain (Boonen et al., 2016; Lestari & Surya, 2017). Every topic in the SK textbook is designed to lead to mastery of the content. The SA textbook provides examples and exercises which enable students to learn each presented concept by following examples, then apply the concept in structured exercises or problems. Both the SK and SA textbooks provided examples for students to use in order to solve problems and exercises.

8.5.1 Relationship between textbook design and learning

Textbooks which use examples that are relevant and not too abstract enable students to relate to the task in a meaningful way. If the textbook presents tasks in an abstract and contextually irrelevant way, learning is made abstract and may be contextually irrelevant. In the SK and SA textbooks, the progress of common skills is adopted by engaging students in a diversity of learning tasks to assist students on how to learn.

The SK textbook provides a balanced coverage of cognitive skills of all levels, e.g. skills in information gathering, retaining focusing, organizing, integrating, analyzing and generating. Higher order thinking skills which require analysis and evaluation, not just remembering and comprehending facts, are gradually incorporated while giving attention to students' ability and developmental needs. The SK tasks require understanding, creative and critical thinking, which are motivated by involving students in less structured problems and more open-ended questions. Students are required to read further and experience the progression of learning by searching for information from several sources. Tasks require students to analyse, evaluate, develop their thinking processes and plan strategically, which aid in the development of Meta-cognitive skills

Learning strategies are suggested in learning activities in the SK textbook, which further promotes positive values and attitudes using learning and teaching through games, stories and investigation in the contexts of different topics. The strategies and activities on learning, teaching and assessment in the SK textbook, allow flexible use to cater for students' ability and learning styles. Appropriate support and challenging tasks are provided to cater for students' different abilities. Learning activities which are essential to achieving the learning targets are included. Students' performance in these activities provides feedback on how well they learn and give directions for further learning and teaching.

Learning activities are designed to assist students to actively integrate, practice and apply new knowledge. To achieve this, the Polya's 4 stage model and various heuristics are used in designing learning tasks in the South Korean textbook. The Polya 4 stage model consists of 4 stages: Link to students' prior knowledge, Organize new content, Reflect on what has been learnt, and Extend by applying knowledge to new contexts. Students foster life-long and life-wide learning, with real life exposure and use of authentic materials and community resources where appropriate.

In the SK textbook, the learning activities are designed to be interesting and motivate students to learn. They provide clear instructions accompanied by a variety of meaningful activities. Problem-solving procedures and activities are age appropriate, and have a workable number of tasks, consistent with the instructional strategies and learning targets. It provides opportunity for reflection, while individual, peer and group-assessment are integrated into learning objectives and tasks, to aid in improving learning. The activities in the SK text require understanding interesting and various answers with significant explanations. The activities help students recognize the objectives of the chapters/units as well as their own learning objectives, assess their own learning and reflect on their learning process. This can empower them to take further steps to develop their own learning.

The SA textbook provides mainly exercises using different levels of questioning, which caters for different levels of student ability. Differentiated leveled questions are provided and examples are given which the students can use in order to find a solution, these being the only strategies that are given to students in all topics. The content in the SA textbook is arranged in suitable learning chunks which can be used independently or in groups. The SA book is designed to provide exercises and practice throughout all topics, with revision exercises which serves to identify if students can complete tasks correctly.

8.5.2 Textbook design and usability

In chapter 5, it was discussed that the South Korean textbook is used as a workbook as well as a textbook. Whereas the SA textbook is not a workbook, rather it is used as a resource year after year. Appropriate textbook design and usability increases student satisfaction and productivity, leading to their improvement of critical thinking skills and improved results in tangible cost savings. Usability costs can be viewed according to cost of the textbook, 10% is allocated for the design process, 90% is assigned to techniques which help keep the product aligned with the curriculum aims and goals (Sulaiman & Mustafa, 2019). Good designs and usability have many benefits for textbooks developed for school use (Quiñones & Rusu, 2017). The well designed and usable textbooks will improve student results, decrease their errors, and make savings from the design changes in addition to decreasing the need for student support.

The usability of the textbook is an important part which will help to shape students' productivity. Usability is not solely about the easy use of a textbook, it involves a great deal more, such as, conceptual development, differentiated tasks, developing critical thinking, creativity, collaboration and innovation (Li, Mok & Cao, 2019). It is vital that the use of textbooks lead to students achieving their goals in an efficient manner in order to complete their goals successfully.

According to Yuriev, Naidu, Schembri and Short (2017), effectiveness is about whether students can achieve their goals with a high degree of accuracy. The effectiveness of a textbooks comes from the support provided to students when they work with the product, with efficiency. Efficiency is about speed, if students are able to complete tasks within specified time allocation which also refers to the level of engagement the problem-solving procedures offer to students in terms of proper layouts, readable texts, and the ability of students to use the textbook easily. The textbook should be designed with differentiated activities, taking students various learning styles and abilities into consideration. The best way to support ease of learning is to design problem-based learning to develop students existing mental abilities, which is a representation of something in the real world and how it is done from the students' perspective (Sabilah & Siswono, 2018).

8.6 Limitations of the study

A number of limitations need to be acknowledged. The first being that the study only investigated one textbook from both countries, and that the inclusion of more books may have given a broader perspective. However, as both books are on the recommended list of references, they are likely to be used by students, which means that the results of this study are still relevant. This study was limited by the knowledge of the extent in which they are used in both countries. Assumption cannot be made that textbooks are the sole reason for performance in SA Grade 6 students, or for the performance reported amongst SK students.

8.7 Recommendations

Based on the findings, the following recommendations are made:

8.7.1 The South African mathematics textbook:

- The textbook needs to place more focus on the general strategies of Polya's 4-stage model to develop students' cognitive demand.
- The heuristics could be used more frequently, and their distribution could be more balanced to ensure that the students are exposed to all 8 throughout the book.
- The framework developed in this study could serve as a useful guide to authors wanting to develop mathematics school textbooks at all levels, as the principles will apply across all grades, with the content and level of complexity needing to change to be contextually relevant.

8.7.2 General recommendations:

- Locally relevant textbooks' standards need to be developed to enable authors and publishers to follow them to ensure that problem-solving procedures are addressed in culturally and linguistically appropriate ways.

- The standards for textbooks should provide criteria for Governing bodies to select suitable textbooks for students and provide criteria for reviewers for the inclusion of textbooks in the Education Bureau's Recommended Textbook List (RTL).
- Textbooks and other instructional materials need to be analysed by an independent authority to establish the extent to which they promote mathematical proficiency, and to ensure that there is consistency of learning across the grades as student's progress to the higher levels.
- Government educational regulatory bodies should ensure that only approved textbooks are used in public primary schools.
- Further research could focus on textbooks used for mathematics to ensure that textbooks and instructional materials are in depth, cognitively developmental across all Grades, supporting all strands of mathematical proficiency.

8.8 Significance of the study

This study arose due to South Africa's Grade 6 students results in the TIMSS, with Arends, Winnaar and Mosimege (2017) noting that there are various explanations for their poor performance in the tests, i.e. home and school context, low socio income backgrounds, students enrolling in school at a late age, learning a subject in a second language and lack of resources. For the purpose of this study, I focused on Grade 6 SK and SA mathematics textbooks and aimed to explore the manner in which problem-solving is presented. This was accomplished by using a composite framework to identify the contributing factor to the difference in the students' marks with those in South Korea.

The analysis of textbooks intended to enable authors to develop quality learning material that would improve local, and therefore international, testing scores. It incorporated several textbook analysis frameworks, with the structural components also being derived from existing frameworks, with the problem-solving aspect consisting of Polya's 4 stage model and the 8 heuristics. The components derived from various frameworks were compiled to create a comprehensive model which can be used for textbook design and analysis. The relevance of this model is that it can be used to design the structure, layout, content, language, cognitive demand and problem-solving procedures in South African and other textbooks. Developing students' 21st century skills are vital, with a growing need for creativity, critical thinking in problem-solving, collaboration and innovative ideas, which need to be addressed in textbooks.

8.9 Conclusion

Curriculum interpretations influence textbook design according to instructional purposes, culture traditions, context and educational objectives. Furthermore, the method in which the textbook is used for instruction impacts on the students' mathematics learning. According to van den Ham and Heinz (2018) the TIMSS (2011) studies revealed that teachers repeatedly utilize mathematics textbooks as the central basis for their instruction, which may be due to expectations imposed on them to use and follow the text.

The South Korean textbook offers a solid groundwork for students to develop their capabilities in problem-solving. This includes applying theoretical understanding to work out problems, using multistep and challenging problems to improve logical and higher order thinking skills, engaging with an array of heuristics and learning new ideas and processes by problem-solving. The South African mathematics textbook provided vocabulary and additional information, such as 'did you know' boxes with additional knowledge about mathematicians, which can be useful to students who are English second language students. The SA textbook design tried to accommodate second language students as much as possible, which is difficult to do with the diversity that is present in many classes. In order to execute the curriculum efficiently, it is essential to have an educational environment that is conducive to successful learning. The environment necessitates, amongst other things, a curriculum which can be adjustable to the requirements of students and teachers sharing the same philosophy in the learning/teaching process, variability of quality learning experiences and the provision of quality textbooks to ensure that the students are able to reach their full potential and move seamlessly into the next year of schooling.

Continuous education has been a standpoint in the discourses of learning and suggested as a key education agenda in the Sustainable Development Goals (SDGs). SDG 4 aims to safeguard inclusivity and equal quality of education and encourage opportunities for lifelong learning for everyone. In daily life, the responsibility of the teacher is not to recognize problems and offer solutions or to provide preconceived notions and expertise, but to assist in the development of understanding the world. Facilitating students to think of their own ideas and solutions will empower students to improve their quality of life and experience gratification of success. At this point the teacher is similarly a learner during this process of discovery and understanding, as they think about learning beyond the formal education system. Students should be allowed to be active co-constructors in learning activities which focus on innovative, creative and cooperative practices which should be involved in education reform. To develop citizens who are prepared to deal with difficulties of the 21st Century, critical thinking and cognitive demand, are going to be key ingredients which will affect not only the way that they conduct their own lives, but the way in which they engage with the world and contribute to becoming future leaders.

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APPENDIX 1

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

Mrs Sathiaveni Duel Moodley (211559024)
School Of Education
Edgewood

Dear Mrs Sathiaveni Duel Moodley,

Protocol reference number: 00006943
Project title: A COMPARATIVE ANALYSIS OF PROBLEM-SOLVING PROCEDURES OF SOUTH KOREAN AND SOUTH AFRICAN GRADE SIX PRESCRIBED MATHEMATICS TEXTBOOKS

Exemption from Ethics Review

In response to your application received on , your school has indicated that the protocol has been granted **EXEMPTION FROM ETHICS REVIEW.**

Any alteration/s to the exempted research protocol, e.g., Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through an amendment/modification prior to its implementation. The original exemption number must be cited.

For any changes that could result in potential risk, an ethics application including the proposed amendments must be submitted to the relevant UKZN Research Ethics Committee. The original exemption number must be cited.

In case you have further queries, please quote the above reference number.

PLEASE NOTE:
Research data should be securely stored in the discipline/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours sincerely,

[redacted]

Prof Thamsanqa Thulani Bhengu
Academic Leader Research
School Of Education

UKZN Research Ethics Office
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
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APPENDIX 2

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I, Tamlyn Ferreira with the assistance of Khadija Mohamed have translated the Grade 6 Korean Mathematics textbook from Korean to English for the purpose of this study. Throughout the course of the translation process no additional words, ideas or thoughts were inserted, the translation from Korean to English is an exact rendition of the original content.

I, Roxanne Candice Barthus conducted the editing of the translation of the Korean mathematics textbook to ensure the reliability of the content.

Signed

Tamlyn Ferreira

[redacted]

Signed

Khadija Mohomed

[redacted]

Signed

R. Barthus

[redacted]

Directors: Roxanne Candice Barthus

APPENDIX 3

Durban
South Africa
1 June 2020

To whom it may concern

Title: A Comparative Analysis of Problem-Solving Procedures of South Korean and South African Grade 6 Prescribed Mathematics Textbooks.

Student: Sathiaveni Duel Moodley

Student No.: 211559024

I have edited the document and provided comment to the student for her to address. Once she has addressed my comments and made the changes as suggested I am confident that the document will be ready for submission.

Regards

[redacted]

Ms Carrin Martin
Academic Editor
MSocSci, PGDPH

APPENDIX 4

A COMPARATIVE ANALYSIS OF PROBLEM-SOLVING PROCEDURES OF SOUTH KOREAN AND SOUTH AFRICAN GRADE SIX PRESCRIBED MATHEMATICS TEXTBOOKS

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