An Investigation of the Informal Mathematical Knowledge and Competencies of Reception Class Entrants.

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Submitted in fulfilment of the academic requirements for the degree of Master of Education, in the Department of Educational Psychology, University of Natal

Pietermaritzburg

September 1996

Abstract

Recent research on the mathematical achievement of young children prompts one to question the widely accepted views of Piaget in this regard. Researchers have begun to concentrate on assessing the development of mathematical concepts in appropriate contexts. Aubrey (1993), Hughes (1986) and Gelman and Gallistel (1978) examined the mathematical competencies of pre-school children and suggest how this knowledge could inform instruction and curriculum development. This study investigates the mathematical knowledge and competencies of 40 reception class children from English speaking, working class homes in Pietermaritzburg, Kwazulu-Natal. The assessment tasks were adapted from those of Aubrey (1993), Young-Loveridge (1989) and Wright (1991). These are compatible with the key number activities in the "Learning Through Activity Programme" used in the reception class in this province. The tasks were presented during individual interviews, using everyday objects and familiar activities. Tasks included rote counting, understanding the cardinality rule, numeral recognition, written representation of numbers, ordering numbers, addition and subtraction with concrete objects, social sharing and multiplication, estimation, patterning and an understanding of shape, space, measurement, time, and ordinal numbers. The results confirm the findings of previous studies: most children enter the pre-school year with considerable knowledge about number. Low-attaining children had some basic number knowledge but could not cope with higher numbers or more abstract tasks. Higher scoring children were already competent in most areas of the reception class mathematics curriculum. As

the curriculum is suited to the low scorers, the majority of pupils are not provided with challenges to advance. Teachers may be unaware of the extent and range of children's mathematical knowledge, and the strategies used for manipulating numbers. Initial and ongoing assessment of each child's competence would enable teachers to develop and evaluate a meaningful curriculum. For every child to realise his/her potential implies instruction that is appropriate to the level and pace of learning. Further research should refine the assessment of children's mathematical knowledge and investigate the influences upon later mathematical achievements.

DECLARATION

I hereby state that the whole of this thesis unless specifically indicated to the contrary in the text, is my own original work and that it has not been submitted for any degree in any other university.

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September 1996

Acknowledgements

I wish to express my thanks and appreciation to all those who assisted me in the compilation of this thesis. In particular I would like to thank the following:-

- Mr Robin Farman, my supervisor, for his constant encouragement, supportive guidance and critical evaluation throughout the writing of this thesis.

- The principals, teachers and children of the schools I used in the study for their willing assistance and accommodating manner in which they entered into the assignment.

- My son, Andrew, for his invaluable assistance with the statistics required for this thesis and his constant and patient instruction on computer expertise.

- My daughter, Sarah, for her artistic representation of the pictures used in the assessment tasks.

- Finally, to my husband, Cees, for his constant support and encouragement which has had a significant positive influence on my work.

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Chapter 1 Purpose and Scope of the Study

1.1 Introduction

> *"We should take a new look at the abilities children possess before they start school, for it is pre-school children who have been most seriously underestimated. ...we should move away from the traditional Piagetian tasks of class-inclusion and conservation, and look instead at abilities which are more closely related to the kind of mathematics children learn in school. ..we should devise tasks which make sense to young children, so that we can look at their strengths rather than their weaknesses, at what they can do rather than at what they cannot. If we can get a clearer picture of what children actually know about number when they first come to school, we should be one step nearer to understanding what is subsequently going wrong."* (Hughes 1986 pp.23)

Over the last three decades there has been intense interest and concern by educationalists, parents, employers and pupils themselves about the teaching of mathematics in schools. First the British and then the American's influenced early childhood mathematics curricula with the introduction during the mid to late 1950s of the Cuisenaire approach to the teaching of number and then some ten years later to the 'new maths' which was characterised by the use of sets as a unifying theme in mathematics. The Cuisenaire approach used coloured rods to clarify the structure of the number system in an ordinal sense whereas the sets approach involved children in counting thus presenting number in a cardinal sense. The Cuisenaire approach emphasised mathematical principles and relationships focusing on each number in turn and the new maths used sets as a counting base to introduce operations thus emphasising the basic concepts and skills. These new approaches to mathematics were justified on the grounds that they focused on the development of meaning which merited change.

More recently Australian and New Zealand researchers have emphasised the need to review current research and practices based on recently developed models of young children's numerical development. Young-Loveridge (1989) and Wright (1992) conclude that the majority of children enter school with considerable understanding of number concepts and skills and are therefore under-challenged. They see the problem as one that starts in the first year of school. According to Wright (1991) "The observation that almost all of the children from the higher socio-economic kindergarten class were facile with the number word sequence in the range one to ten and were beyond the stage of perceptual counting suggests that the prenumber and number topics typically undertaken in the first six months of kindergarten, as indicated by state curricula and textbooks, are inappropriate for such children." (pp.9)

However, many children still find the subject daunting and feel intimidated by the prospect of learning new number concepts and developing logical thought processes. Employers find that their young employees lack basic number skills and need extra training in this field. Teachers of both junior and high school pupils find that new 'progressive' methods of teaching mathematics have not helped pupils to develop confidence in their ability to handle number concepts and many are not able to reach the required standards of achievement.

These problems together with "the burgeoning of science and technology have disclosed the limitations and the incompleteness of the curricula" (Leushina 1991 pp.22). Such questions have led many researchers to believe that the solution to these problems lies in an improved scheme of assessment of basic skills and knowledge. Only then will development and progress be based on levels of acquired knowledge and the use of varied strategies resulting in increased interest and enthusiasm, stimulated and maintained through the positive feedback from success. "Selfconfidence built on success is the most important objective of the mathematics curriculum" (National Research Council 1989, pp.45).

1.2 Value of Assessment

It is therefore considered to be important to fully understand what level of mathematical readiness has been reached before any effective mathematics program can be introduced. Researchers have repeatedly emphasised that learning situations must be structured in such a way that most pupils will experience success in a programme that challenge the pupil's abilities and ensures that pupils develop positive attitudes and interest. This will instil intrinsic motivation and enhance new learning (Williams 1965).

These questions then arise: Why do we need to assess the mathematical knowledge and competencies of very young children? How is this knowledge relevant to the introduction of formal mathematics at school and at what stage should any form of evaluation take place?

This study was designed to examine the informal knowledge and competencies of reception class entrants before they began on the programme of pre-mathematics skills. By studying the findings of this type of investigation over the past years, together with the results of this investigation, it may be possible to give specific proposals for rethinking and changing curriculum strategies in the teaching of number and for the instigating of ongoing research programmes in early childhood mathematics.

Before children begin any formal instruction in mathematics, Piaget views the development of children's intellectual growth as being governed by their actions - the active behaviour of the infant shapes his development. New experiences continually expand the scope of the child's actions, capabilities and skills and enlarge and direct his mastery of the world around him. Through *active participation* in play activities, children physically manipulate concrete objects thereby constructing their knowledge independently and spontaneously. At this stage the teacher's role is regarded as being "intellectually non-interventionist and relatively unimportant" (Hughes 1986 pp17). According to this view there is no need to assess the level of competence of each child for they will react and advance cognitively according to their stage of development. Instruction cannot influence the spontaneous course of development.

1.3 The Value of Instruction in the Early Years

Others question whether young children should be given the opportunity to develop spontaneously, or whether they should be guided in the process of understanding the world around them. Recent research has concluded that the cognitive potential of pre-school children is notably more extensive than had previously been assumed and this posed the question of how this potential could best be used. "The problem of instruction and development had to be considered in a new way" (Leushina 1991, pp.22). To link the informal with the formal is to explain the relationship between written methods and concrete representations of these methods through explicit instruction. "Too many children think of school mathematics as an artificial game with no relation to reality" (Ginsburg, 1977, pp.177). Children see mathematics as a purely academic subject that is useless, senseless and arbitrary where the only object is to get the right answer. For arithmetic to become meaningful, children need to be allowed to use their own ideas and gradually move to an understanding of how they relate to the mathematics of the school situation. Developmental instruction leads children from informal methods of representing number to abstract conceptual thought. It is only through instruction that children pass from lower to higher structures of intellectual activity (Leushina, 1991). New structures are only presented as a development based on the previously formed structures.

Soviet psychologists are convinced that instruction plays a specific role in children's development. They consider the mastery of knowledge as a process that advances the child to the next developmental level. They advocate that teaching should always be directed towards what children can do with the assistance and supervision of adults, rather than what they can manage by themselves. Leushina's principle of developmental instruction in fundamental mathematics for young children visualizes introducing the child to the understanding of quantitative, spatial, and temporal relationships. By looking at familiar objects in a new way the child learns a new attitude toward them, stimulating their cognitive interest and activating their logical thought. When new material is being studied, children should be given an opportunity to think and act independently as teachers provide instructions and demonstrations to direct exploration and thought., Teaching should always ensure that children reflect their knowledge through different solution methods and statements, thus proving that they have comprehended the problem and not memorized a stereotype response. Developmental instruction emphasises the use of knowledge previously gained in diverse situations that can be transferred and adapted to solve the current problem. Development according to this view concentrates on the processes that are forming and maturing and instruction forms the source of this development as it guides the progress. It is emphasised that methods of instruction, even for very young children, should not only communicate pre-existing knowledge but also develop children's ability to analyse, synthesise, generalise and classify that is to think logically and be able to apply knowledge. Instruction therefore must see that children's attention is not only focused on the content of the new material, but also on the methods of implementation. Leushina is convinced that only instruction directed by the teacher can influence the child's mental development and reports that: "All of the psychological studies done in the Soviet Union provide persuasive evidence that qualitative changes in the child's mental development occur during instruction. These studies show that pre-school children achieve higher levels in distinguishing attributes of objects (colour, shape, size) if they are instructed than they otherwise achieve" (Leushina 1991 pp25).

1.4 Knowledge of Individual Levels of Development

Once instruction of whatever sort begins and there is adult intervention, the need arises for knowledge of individual differences among children to be able to structure new knowledge on the skills and competencies already developed. At what stage this instruction should begin is still a matter of much debate. Mathematics programs based on Piagetian stage theory delay the introduction of number concepts until some presumed state of 'readiness' has been reached. Bruner (1960) suggests that our schools may be wasting time by postponing the introduction of a subject purely on the understanding that it is too difficult.

Young-Loveridge (1987) finds no evidence to show that numerical mathematics should be delayed until such competencies have been acquired. She proposes that initial mathematics instruction should build on children's existing knowledge about numbers and not allow it to develop randomly. For this reason she sees a need to ascertain the kinds of number skills which children have when they start instruction and for teachers to be aware of the differing mathematical skills which their pupils bring to the classroom. Her research showed that children's mathematical skills were consistently underestimated by teachers and this resulted in lower levels of achievement. Children with higher ability were hindered by the slow pace at which teachers moved and children who initially knew the least about number, made the largest advancement in learning during the first year.

All this points to the need for an early assessment of the abilities of young children. At the start of instruction it is essential that every child in the class is challenged with activities that build on their existing knowledge thus ensuring interest and motivation.

Research by Williams (1965) finds that: "Before an effective mathematics program can be introduced at the kindergarten level, it is imperative to know about the children's level of mathematical readiness at the time they enter school" (pp.261). Although this idea was recognised as long ago as the early 1960s, there was no thought given to procedure or rationale as the Russians did.

1.5 Test Design

From research findings over the past fifty years, this investigation aims to draw up a reliable test to determine the prenumber and early number knowledge possessed by children entering the reception class. Hughes (1986) suggests that we should move away from the type of tasks used in the Piagetian tests of class-inclusion and conservation (which examined logical thought rather than mathematical ideas) and concentrate on abilities which are more closely connected to the kind of mathematics learnt in school. If we are to understand and predict the kind of difficulties which some children experience in learning school mathematics, then the tasks presented during assessment should relate to the groundwork that needs to be covered in order that new concepts can be readily understood.

1.5.1 Context

Then too the tasks should be framed in the right context and appropriate language so that the question is clearly understood and makes sense to the young child. Piaget often used language and activities that were foreign to the young child thus making the task open to misinterpretation. Margaret Donaldson (1963) showed that how children solved problems in any testing situation was judged by the meanings they gave to the task. This decided the function of the context into which the children fitted that situation. If we are to understand how the child masters a subject matter, attention must be paid to the child's own way of defining, examining, recounting and discoursing. The child's interpretation of the facts might be quite different from what the adult's motives and intentions might be. There is a need to consider the situation from the child's point of view and not be overruled by the adult's idea of the situation. Children's knowledge of mathematics must be displayed in everyday activities using familiar objects for then it becomes more meaningful and is easier to comprehend.

1.5.2 Level of Development

It must be stressed that the aim of any such evaluation is to ascertain what the child *can* do and display their strengths rather than what they *cannot* do. Instruction in the pre-school situation will begin with the tasks that the child is able to master and move on to the more complex concepts, from the known to the unknown, from the simple to the complex. It is therefore essential that, to be of any value, a test must establish the level of mathematical development, the skills and strategies known to the child and the ability to apply this knowledge to the task at hand. If a test only shows a child's weaknesses and concepts not yet mastered, there will be nothing on which to base the new learning material and this 'missing link' will be the cause of future misunderstanding. Mathematics is a chain of knowledge where every link plays a vital role in determining the final strength of its ability.

1.5.3 Pupil's Sensory Perception

Test activities must take cognisance of the view that sensory processes are the basis of young children's comprehension of the qualitative and quantitative characteristics of objects and facts. Children come to understand the qualities and properties of an object by practical experiences of everyday activities: they recognise shapes and size by using their visual senses, they feel materials using tactile senses and kinaesthetic senses give awareness of their position and movement in space. As with all early learning experiences, "Sensory processes underlie the development of the first mathematical notions" (Leushina 1991 pp29).

1.5.4 Clinical Interview Method

A test of this nature would ideally be conducted on an individual basis. It is important to monitor the child's attention span and limit the distractions. When dealing with one child at a time the researcher is able to change activities when interest fades, move on to the next task when the child is unable to complete one thus maintaining motivation, encourage participation with praise and there is the opportunity to time the tasks to suit the level of ability. It may allow more time for each child to adjust to the conditions of the testing facility depending on their emotional state. The more nervous and ill at ease child can be drawn out of his shell and helped to relax by spending time playing with the equipment and familiarising himself with the situation or by talking to the researcher and developing confidence in his/her company. There may even be a need with the hyperactive or excitable child to stop the test at a particular point and continue at a later stage.

Individual testing also enables one to note strategies used and to record conversations that throw light onto the way children think and the reasons for their actions. Individual clinical interviews are one of the best ways to assess the processes that children use to solve word problems. When solving simple addition and subtraction problems, the interviewer can often infer the strategy a child is using by observing the child's actions with objects or fingers and watch how the child counts. In other situations the interviewer has to rely on the child's explanation of how the problem was solved or upon conversation as the child 'thinks aloud.' Romberg and Carpenter (1986) remind one that the objective of the research is not just to describe the strategy used by the child, but to observe the development of addition and subtraction concepts and skills and to build models that classify the knowledge necessary for accomplishment of each stage of development.

1.5.5 Relevance of Content to Curriculum

How then is this knowledge relevant to the formal instruction of school and what facts should the test present? "There can be little doubt that children enter school with considerable knowledge and understanding about numbers. The key question is how is this knowledge relevant to children's mathematics learning at school" (Young-Loveridge 1987, pp.163). Study by Wittrock (1986) suggests that cognitive learning takes place when a person builds perceptions and meanings for himself by making connections between new information and existing ideas. This theory is known as the 'principle of generative learning' and stresses the importance for learning of a person's present ideas or knowledge because this will influence which stimuli are selected and focused on, and the meaning given to the stimuli. If learning is to take place, then teachers need to determine just what ideas and knowledge a pupil has about the subject so that new material will relate to the pupil's experiences in suitable ways. Research by Hughes (1986), Gelman & Gallistel (1978), Wright (1991) et al. has emphasised the importance of teachers knowing the level of pre-schooler's mathematical competence and being aware of the rich informal knowledge they have acquired in the first few years of their lives. Only after careful assessment of each child's mathematical knowledge and competencies can the teacher organise the mathematics curriculum to capitalise on that knowledge. Clark (1962) emphasised that; "Evaluation is not an end in itself; rather it is a means to better learning experiences when closely linked with instructional procedures" (pp.101).

1.5.6 Recognition of Strategies used

Researchers such as Saxe (1985), Groen & Resnick (1977) and Hughes (1986) recognise that children invent their own strategies for solving mathematical problems which ties up with the concept of the generative learning theory. These strategies such as using fingers to count up or down the number sequence or the 'counting-on' strategy for doing addition are meaningful for the young child and they should be accounted for when instruction begins (Carpenter & Moser 1984 and Hughes 1986). Hughes suggests that teachers should help pupils to improve on the use of their strategies to make them more reliable and efficient and make the different methods of different children the focus of class discussion with praise for ingenious strategies. New methods cannot be forced on children, but instruction needs to be sequenced so that it builds on children's informal knowledge. Assessment of the child's strategies will go a long way to helping him/her to link the concrete experience to the formal manipulation of symbols.

Yet others believe that to allow young children to construct their own mathematical practices shows a naive view of children's cognitive development and a failure to recognise the differences between biologically primary and biologically secondary skills (Geary, 1994). Conceptual knowledge of number and counting is acquired through biologically primary skills, i.e. the natural inborn abilities, which are learned in the informal social interactions. However, procedural competencies and most mathematical problem-solving skills are biologically secondary skills which are shown in the child's ability to adapt and co-opt the biologically primary skills and attributes. This means that they require a different form of instruction, namely extensive practice on a wide variety of problems- a modified form of drill and practice. There is a need to teach children basic procedures and then to give plenty of practice. "The practice of basic procedures, especially when the practice is mixed with other types of procedures, should also provide the child with an opportunity to come to understand how the procedure works" (Geary, 1994, pp270). This method still allows children to use their own strategies for solving problems, but by giving attention to their errors, the teacher has an insight into the child's conceptual misunderstandings and is able to suggest alternative ways for solving the problem and so give instruction to solve conceptual errors. This psychological research implies that different teaching techniques are required for children to gain procedural and conceptual competencies. Even though this approach advocates instruction with a certain amount of drill, it still allows each child to develop his or her own techniques and understanding of mathematics so that the tasks remain interesting and engaging for the child.

1.5.7 Recognition of Errors

Similarly, an investigation into the errors children make will give a clue as to what process they might be using and how far they have developed in their arithmetic abilities. Ilg and Ames (1951) found that most young children's counting and writing errors were caused by normal immaturities and were not unique faults or flaws in the behaviour of any one child. The same types of errors occur repeatedly making it possible for the experienced teacher to be able to anticipate the errors the average child will make. Romberg and Carpenter (1986) argue that many errors are rule governed and therefore are the result of learning the wrong algorithm and not from failure to learn the correct algorithm. By diagnosing children's errors, it is possible to distinguish different 'bugs' or incorrect algorithms and identify the specific procedural rules that were not available and resulted in the 'bug'. Instruction can then be designed to prevent 'bugs' from occurring. If regular testing is part of the program, the teacher will know when these difficulties will occur and recognise the children who are most inclined to make them. Ilg and Ames (1951) find that: "One of our best clues to the child's stage of development appears to be the kinds of errors that he makes" (pp.24). The errors made when young children add and subtract will give clues as to what mental process is being used. If when adding the answer given is one more or one less than the correct answer, then he/she is probably counting to find the solution. This type of error at age 5 or 6 is acceptable, but if it continues to occur at a later stage, then it is proof of the use of a more immature method and would require special attention.

1.5.8 Language

".....many researchers have argued that young children in fact understand number conservation but fail the standard Piagetian conservation problem because of linguistic difficulties"(Sophian 1995, pp.559). Any research which investigates the meaning children give to certain mathematical problems, needs to determine their understanding of the language involved. Children's failure at a task may be because they do not understand the language used to present the problem rather than because they lack the required cognitive ability.

Donaldson (1978) sees most mathematics problems as being the result of the use of language that is 'disembodied' from the immediate context and this causes a difficulty whenever mathematics is taught. Piagetian tasks, which assessed children's ability to conserve number, used language in this way resulting in the child having to concentrate so much on the language of the adult that the problem to be solved was lost. Children are therefore required to think about the language used by the adult independently from the context in order to work out the meaning in their own terms and this makes the learning of mathematics difficult whenever it is taught. Researchers in this field of study (Hughes, 1986 and Groves & Stacey, 1990) have realised the importance of language in assessing children's mathematical competence and have accordingly modified their instructions to make the tasks clearer and to exclude the possibility of misunderstanding the requirements. Often an instruction is repeated in another form to ensure that the child has gleaned the correct interpretation of the task.

Hughes (1986) considers the idea that mathematics should be seen as a language. As a means of communication it is powerful, concise and unambiguous thus leading young learners of mathematics to feel that it is an unfamiliar foreign language and therefore difficult to comprehend. He quotes the four year old who when asked the question, "What does one and two make?", replied that he could not answer because he did not yet go to school. At this young age he was able to recognise the language of mathematics and know that this was not within his world of experience. Hughes worked with many young children of nursery school age both in the school situation and in their homes. He realised the importance of the context in which learning takes place and the need for children to understand what they are learning in their own terms. When assessing children's mathematical competence and skills it is therefore often necessary to think about their mathematical understanding in terms of their ability or lack thereof to perform the required translation first of the problem from its real life context into an appropriate mathematical calculation and then on completion of the process, back into the original context. If the language used is within the child's world of experience then, "The meaningful nature of the task almost certainly enabled the children to show their capabilities" (Hughes, 1986, pp.26).

Research carried out in natural settings, using familiar activities and objects and embedded in the real world context of the young child, will contribute greatly to providing an accurate reflection of the mathematical skills, knowledge and competencies of the child and encourage the use of individual strategies to solve problems. In such an everyday situation, the child will feel free to verbalise whilst working on a problem thus giving the teacher a world of information about the route used to reach the solution and just where any problems may arise. Ilg and Ames (1951) found that, "...what the child himself says, his own direct and unedited comments, tell us more about what is going on inside him, what he is thinking about and how he is responding, than any number of words that we may say about him" (pp.11). Even though the importance of this issue was emphasised as long ago as the middle of this century, there has been little reflection of these ideas in the practical learning situation.

1.6 Readiness for Learning

1.6.1 At Home

The part played by parents should be stressed. Afterall the education of the young child is performed in partnership with the school and the parents. Parents need to be made aware of the significant part they can play in these early years by increasing their child's opportunities to experience number through exposure to a wide range of problem-solving activities and encouragement to talk and think about numbers as they are presented in the everyday experiences of their children's lives. Parents should not underestimate their children's cognitive abilities. They should allow them to construct their own mathematical understanding and be encouraged to explain their interpretations. Children should be given activities involving assigning number to spatial, auditory and motor patterns, verbal number word sequence activities and counting of visible and invisibly objects (Wright, 1992). Researchers emphasise that, "To develop effective counting skills, young children require repetitive experience in counting" (Ginsburg, 1977, pp.20). Children enjoy counting and can be encouraged to play games which provide practice in basic skills and increase their general proficiency with number but in contexts which are both meaningful and enjoyable.

1.6.2 In School

If one follows Piaget's idea of the stages of development, then it is accepted that children who have not yet reached the concrete operational period cannot successfully complete the class-inclusion problem nor are they able to conserve number. Hence they are not capable of logical thought which is necessary if the child is to understand addition and subtraction. There has been a vast amount of criticism of these ideas over the past three decades and research has shown that it is not essential to delay mathematical instruction until children have reached a particular stage of readiness. Wright (1992) found that, "Any stage theory potentially applies a constraining effect on the teacher. One could reason that, because the child is at a given stage, only activities associated with that stage should be prescribed for the child" (pp.133). Such a stage theory can serve as a guide in the choice of suitable activities, but for advancement to occur the pupil must be presented with situations for which they do not have appropriate cognitive constructions. Teaching is not therefore the handing over of knowledge to the learner but rather the presenting of problem solving activities which are a necessary ingredient of learning. Teaching therefore has a crucial part to play in children's qualitative advancement of mathematical knowledge.

However there is still much controversy about when mathematics instruction should be introduced and at what stage in the child's development is he/she 'ready' to benefit from instruction in numerical mathematics (Ginsburg, 1977, Hughes, 1983, Wright, 1992). Young-Loveridge (1989) reports that although the majority of children in New Zealand enter school with considerable understanding of number concepts and skills, there was a tendency to underestimate their ability. Children were only taught number concepts such as enumeration and pattern recognition which led to lower standards of achievement because children were not challenged with activities which built on their existing knowledge.

There have been many attempts to design specific mathematical topics that can be assigned to the ages at which children should be able to complete the learning i.e. the child's readiness for learning the various mathematical functions. Hildreth (1935, pp.457) emphasises the importance of readiness when she says: "Associated with the problem of arithmetic deficiency is that of readiness for learning. The child may be bright, but lack the necessary background for profiting from initial arithmetic instruction. No amount of carefully integrated drill procedures compensates for this lack. The principle is violated more generally in the primary grades than in any others, though violation is flagrant in some schools setting arbitrary curriculum standards at every grade level. The preponderance of school failures in arithmetic as in reading demonstrates the minimal results obtained when the question 'Which children are now ready to begin arithmetic or proceed with the next step?' is overlooked."

She adds that testing for readiness for arithmetic cannot be in the form of a fragmentary readiness measurement which assumes that the 'whole' is the same as 'the sum of the parts'. It is not good enough for a test to determine the child's readiness for a given topic on records measured by an intelligence test that gives mere knowledge of prerequisite skills and a general level of mental growth. There should be an accurate evaluation of the child's concepts and experiences, his interests and requirements, and the measurement of the level of mental growth most closely associated with success in arithmetic.

Researchers find that there is a need to know more about the level of development of each child in order to know 'what' and 'how' the child is ready to learn. Arithmetic instruction will be improved when the level of instruction is suited to the actual abilities of the children to be taught. The actual abilities and disabilities of each child regarding the work that he/she should theoretically be able to do must be evaluated. There is a need for educators to determine more about the development within the child of the various processes of mathematics with attention focused on the meaning and understanding of the learner. This has placed value on the genetic point of view that stresses the importance of child growth and development and the various levels of maturation. As with other abilities, individual differences in mathematics are marked and it must be remembered that a child's chronological age will not necessary coincide with the stage at which he is able to function in mathematics. It is therefore important not only to know about each individual child's developmental rate in regard to mathematics, but also to know more about each individual child's particular processes, number systems, familiar numbers, and number combinations, short cuts, and methods which he uses to find solutions to problems.

Mathematics curricula should constantly consult the actual abilities of any one given child so that if this child cannot meet the generally accepted standards, it may necessitate the standards being altered to accommodate the child. Goals will then be related more closely to what the child will be able to achieve and not what the teacher would like the child to achieve. This stirs up many questions about the practicality of the situation when classrooms now have more children with a wide range of ability. Are teachers trained to consider the individual child and his/her level of development or is this simply not a feasible situation? As difficult as it may appear, there should be at least some attempt made to ensure that the content presented is closely assigned to the pre-existing knowledge of each child so that the basis is there on which to build the new ideas and concepts.

In this way there will be knowledge of the stages through which the child moves towards proficiency in any set mathematics process, the ability to spot immaturities which cause misunderstanding, identification of the stage which the child has reached, and a clear understanding of how far the child has to go before he/she will be competent at a particular calculation.

1.7 Mathematics Programme in Kwazulu-Natal

This study focuses on the pre-mathematics skills of children in the reception class of pre-primary school in the Province of Kwazulu-Natal in South Africa. This is a non-compulsory stage of the educational system that up to this time has accommodated only those children whose parents could afford to pay the fees levied by such schools. The State has given financial assistance in the form of the payment of two teacher's salaries, and guidance and advice from highly qualified and experienced advisors.

Instruction in mathematics begins in the first class of the junior school (Aged 5½-6½ years). In the year preceding this some children may have attended a pre-primary school where they would have experienced the enriched learning environment and participated in the 'Learning Through Activity Programme'. This year is generally known as the reception class, pre-school group or school readiness group and the children usually range between the ages of 5 to 6 years. During this year the children participate in a school readiness programme that presents suitable activities aimed at developing basic skills and concepts. Particular attention is given to the aims and objectives which underpin these basic skills, namely those necessary for formal learning such as pre-literacy skills, pre-writing skills and pre-mathematics skills.

This programme can be seen as the start of the Formative Phase and aims to prepare children for the demands of formal education. About 3/4 hour is allocated each day for the participation in this programme. Each concept is first introduced to the class as a whole and then the children are divided into sub-groups where they daily complete a variety of activities which focus on the concept to be understood. These activities include creative work, educational games and concrete experiences, all of which offer a wide variety of experiences to cater for the differing learning rates and ways of each child. The teacher is then able to work specifically with one small group every day while the other groups work independently. This enables the teacher to observe each child individually, assessing their level of achievement and assisting where there is uncertainty.

The pre-primary school environment incidentally nurtures a mathematical awareness through play, exploration and social interaction. The 'Learning Through Activity Programme' enhances this by providing a planned learning experience. The programme emphasises the following aspects of pre-mathematics skills:

a) language skills which will enable the child to express his/her thought processes.
b) numeracy with an understanding of number value. Counting experiences using concrete objects.
c) classification - the matching of like objects and the discriminating of unlike objects.
d) seriation and sequencing.
e) estimating and verifying.

Other aspects of the programme include relationships of size, length, height, mass, volume and capacity, spacial relationships, and exploration of mathematical concepts and relationships of shape.

The 'Learning through Activity Programme' offers guidelines for suitable activities and is in no ways prescriptive. The teacher is encouraged to explore concepts and use her own creative ability to make the 'lessons' stimulating and challenging whilst adhering to the philosophy of the 'Learning Through Activity Programme'. The idea is "to provide an enriched learning environment which enhances the child's individual potential and allows him to acquire knowledge, skills and attitudes for life" (Learning Through Activity 1993).

1.8 Factors affecting curriculum design

1.8.1 Assessment of levels of development

The teacher must be sure that the tasks she sets are relevant to the child's age and level of development. There is a need for some form of assessment to determine this. Many researchers such as Aubrey (1993), Hughes (1986), Gelman and Gallistel (1986) and Williams (1965) have found that it is essential to first establish what mathematical knowledge and competencies young children have before they start on the school readiness programme, for only then can teachers organise the mathematics curriculum so as to capitalise on that knowledge and ensure that new material introduced will relate to the children's experiences in appropriate ways. It is this issue that is central to this thesis.

What then is the level of children's mathematical readiness at the time when they participate in this 'Learning Through Activity Programme'? Are teachers who participate in this programme aware of the extent of number knowledge and skills possessed by these young children or is there only a vague notion of the number concepts that the child's previous four or five years have allowed him to acquire? Is there a wide range of ability or have most children developed what can be accepted as a normal number knowledge?

Although formal instruction in mathematics is not started until the first year of school i.e. when the child is 5½ to 6½ years of age, many researchers elsewhere such as Bjonerud (1960), Wright (1991) and Young-Loveridge (1989) question whether children in the younger age group would not benefit from a system which starts instruction during the pre-school year. Early instruction in numerical mathematics need not mean that this instruction must be very formal or inflexible but rather that it includes games which develop children's understanding about numbers and everyday classroom situations which entail numeracy and calculations. Young-Loveridge (1987) finds evidence that mathematics instruction should not be delayed until children have reached a particular stage of readiness. Every child is ready to learn something new about a subject as long as the new material is thoughtfully selected in accordance with the child's abilities and facilitated by the teacher.

1.8.2 Content of Pre-school Mathematics Programme

Wright (1992) questions whether there has been critical examination in recent years of the prenumber and early number topics that are presented in the kindergarten years. He accepts that these activities provide opportunities for experiential learning and language development and lead to the understanding of important early mathematics concepts, logical reasoning and discrimination skills. However, he questions whether these prenumber and early number activities should be the only or principal arithmetic activities presented in the pre-school programme. These types of activities exclude the children from participating in experiences which involve problem solving or abstract mathematics. Perhaps there is too much emphasis on other skills learnt in the mathematics lessons such as introducing the children into the situation of learning in teacher-directed and small group situations where social behaviour is stressed rather than struggling with mathematical problems. This raises the question whether the first instruction in mathematics is not sufficiently challenging and should include more problem solving and abstract mathematics activities.

Young-Loveridge (1989) examined the number concepts and skills of 81 five year old urban New Zealand children as they entered school and then again one year later. This research found that "large numbers of children were taught certain concepts (e.g. rote counting, enumeration, pattern recognition, ordinal numbers, numeral recognition) even though they already knew them, but were not taught addition and subtraction which they could also do, is a finding which is consistent with the idea that the curriculum is not well matched to the skills of the children" (pp.60).

Likewise Hunting and Sharpley (1988) assessed the fraction knowledge of 22 pre-school children (average age 4 years 5 months) and questioned the widely held belief that this concept should only be introduced when children were about 8 years old because the same basic process for learning whole numbers applies to learning fractions.

Wright (1992) therefore suggests that there is an urgent need to review the mathematics curricula for pre-school children because there is a general tendency to underestimate children's prior numerical knowledge and general cognitive abilities. The National Research Council (1989) in the United States agrees with this idea when it says, "Children *can* succeed in mathematics. If more is expected, more will be achieved" (pp.2).

Piaget (1952), on the other hand, points out that if we teach mathematical concepts too early when children are not ready and have not acquired the cognitive developmental ability, then it will be a waste of time, and could even be harmful creating negative attitudes and delay progress. Young-Loveridge (1989) claims that there is no evidence to support this theory. She relates the overwhelming evidence which shows that children enter school with considerable understanding of basic mathematical concepts and skills and there is no reason to believe that this ability is limited to one particular group within the pre-school population with all children performing successfully on at least some of the number tasks. If this is in fact the case, the key questions are; how is this knowledge utilised if at all and how is this knowledge and understanding about numbers relevant to the programme followed in the pre-school year.

If one accepts that mathematics concepts can and should be introduced at the pre-school level, it is surely vital that there be extensive evaluation of children's mathematics knowledge and skills so as to know how and when to adapt the programme to meet these needs. Williams (1965) finds that the new needs of society have caused changes in the field of mathematics and asks whether there should be an extension of mathematics instruction into the pre-school programme. However, he hastens to point out that such changes can only take place once there has been a thorough evaluation of the children's level of mathematical readiness before they start on any form of instruction. This information will enable the teacher to select appropriate content and determine the scope and sequence of the material to be presented. William's study aimed to ascertain the nature and extent of achievement of children in the pre-school group with respect to selected mathematical concepts, skills, and abilities. He concluded that the extent and nature of mathematical achievement of these children was far-ranging and affected by psychological and sociological factors. When this research was carried out, his views were disregarded in practice.

Bjonerud (1960) points out that where there is an "informal, incidental programme in elementary number concepts" (Bjonerud 1960, pp.347) educators need to know the extent of number knowledge and skills possessed by pre-school children before they start on such a programme. We know that some children of this age are different from others in their peer group because of family inheritance, the environment that the home has offered, the number of siblings and their age relative to the child, and the experiences of travel and stimulation that parents have given. Even though we accept these differences, there is often a tendency to overlook the specific differences in special areas of knowledge such as mathematics.

1.8.3 Evaluation of Competency Rather Than Inability

In evaluating the mathematical knowledge and competencies of children in the reception class many educationalists have dwelt on the cognitive shortcomings as they compare the pre-schooler's performance with that of the older child. Attention needs to be focused on the facilitating of investigations into the abilities of this age group without reference to children from other age groups. There is however a danger in only observing the ineptitude of the young child and minimising his/her ability to think logically and use abstract mathematics. By giving children of different ages the same task to test a given capacity, the child who completes it correctly is judged to have that particular capacity and the child who fails the test is described as not yet possessing the required ability. The emphasis being placed on the ability that the young child lacks.

Gelman and Gallistel (1978) consider the negative aspects of such an evaluation. They question the methodology where one performance of a single task determines if the child understands the number-invariance rule. The Piagetian number-conservation task is one method of testing a child for an understanding of the number-invariance rule but a failure on this task cannot be seen as a lack of understanding of this concept. Only when the same concept has been tested in a variety of different tasks, can any assessment of value be made. Secondly they question the theory that is developed from such a negative assessment. By noting the cognitive capacities of the 7 and 8 year old, and then stating that the pre-schooler lacks these capacities, there can be little understanding of the process of cognitive growth from the young child to the school-going child resulting in weak theory to describe the cognitive development during these years. It is important to understand how the child moves from what appears to be a lack of understanding to the next stage of development if one is to theorise on cognitive growth. If we are to understand how concepts such as conservation develop, we need to gain evidence of what knowledge and skills the pre-school child has before he participates in any training programme. Nunes and Bryant (1996) emphasise that children's understanding of mathematical concepts is generative and changes many times during childhood. Before they go to school most children have some understanding of how mathematical knowledge is structured and can generate knowledge that they have not learned. For Nunes and Bryant (1996) this further stressed the idea that, "The teacher should surely take this early knowledge into account and build on it, and that is one reason why it is important to know exactly what children already understand about mathematical concepts at the time that they start being taught about them" (pp.238).

1.8.4 Cognitive Development

Many theorists have the view that knowledge builds on knowledge (Schaeffer, Eggleston, and Scott 1974, Gelman and Gallistel 1978, Ginsburg 1983 and Hughes 1986) and an understanding of this process gives the theory about the way cognitive development proceeds. If children's cognitive development progresses through different stages, and each stage is dependent on the previous stage in various ways, only a careful analysis of each stage will allow the theorist to understand how the two are related. This relationship between the two stages can be one where the first stage serves as a catalyst, a component or a base on which to construct the later stage and therefore to know which relationship applies, one needs to have a clear idea of the accomplishments of both earlier and later stages. Obviously it will not be sufficient to describe any stage by what it does not contain but rather by what capabilities it has. As Gelman and Gallistel (1978) pointed out, "Stage theories contain an implicit or explicit assumption that pre-schoolers do things differently and not just that pre-schoolers can do fewer things" (pp.12).

Learning theorists also emphasise the importance of discovering the early cognitive capacities as they see development proceeding from experience. As children expand their experiences their responses are strengthened and impulses begin to control habits, resulting in the expansion of cognitive skills. Pre-schooler's inability to perform certain tasks is not a qualitative deficiency in cognitive ability but rather a lack of experience. Nunes and Bryant (1996) point out that one cannot analyse children's understanding of mathematical concepts as a purely cognitive matter, but rather a learning experience that is powerfully affected by social factors. If the level of training is advanced then pre-schoolers should be able to correctly complete more difficult cognitive tasks. Again this theory stresses the importance of knowing more about the earlier cognitive capacities.

Gelman and Gallistel (1978) sum up the need to investigate the informal knowledge and competencies of pre-school children when they say:

> *"As developmental psychologists - from the standpoint of both methodology and theory - we are committed to the empirical investigation of the pre-schooler's capabilities. We should avoid the tendency to compile a list of what pre-schoolers cannot do that older children can do. This tendency amounts to working backward from the full-fledged showing of a capacity. We are all aware of the danger of proceeding this way. There is no guarantee that the end state embodies the earlier stages of development. The emphasis must be on a consideration of the earlier stages in their own right. We must look for skills young children have - at least as much as we look for skills they lack"* (pp.12).

1.9 Aims and scope of the dissertation.

The purpose of this investigation is to examine some of the procedures suggested by researchers for the evaluation of mathematical knowledge of pre-schoolers and to discuss the implications of this analysis for theory and practice. This study, although based on certain educational policy in Kwazulu-Natal, raises questions that are relevant both in this context and which apply to more general universal issues.

This investigation aims to assess the level of mathematical knowledge and cognitive skills of young children aged five to six years in order to ascertain whether this knowledge of pupil's mathematical ability and understanding has been accounted for when planning a programme for reception class children.

This thesis comprises:-

-an examination of the range of competencies of young children dependent on the mediated learning experiences of the home environment;

-an examination of the mathematical knowledge and competencies of the children from working class homes whose home language is English;

-an examination of children's strategies used to solve numerical problems and the stages through which these pass;

-an examination of tasks which were included in the assessment to evaluate the child's knowledge and application of number, shape and space, algebra and handling data;

-an analysis of the tasks designed to evaluate whether they give a clear perspective on the structure of and relationship among kinds of knowledge essential for the development of children's mathematical thinking;

-an examination of the present Pre-Mathematics Skills learning experiences as presented in the 'Learning Through Activity Programme' in the pre-school groups in Kwazulu-Natal (See appendix E);

-an assessment as to whether or not there is a need to make changes in the content of the present mathematics curriculum. To review the ideas held on learning mathematics and the nature of instruction to keep pace with the dramatic advances in technology, and

-proposals for rethinking and changing curriculum in the teaching of number in early childhood and the implications this research offers for further investigation into the learning and teaching of mathematics.

1.10 A brief review of the scheme of the dissertation

In Chapter 2 there is a review of the relevant literature on recent research which aims to understand how mathematics is learnt and taught at an early age. Educationalists have analysed specific areas of mathematics content such as counting, addition and subtraction, shape and space, classification, estimation, measurement, etc. to establish young children's ability to perform these tasks and to record their invented strategies for solving such problems.

Chapter 3 explains the research design and method of investigation.

Chapter 4 gives a record of the results of each test and the observations made about individual strategies used to solve problems and complete the required tasks. Children's comments often gave a clear indication of their level of understanding and enabled the investigator to gain insight into their thought processes. The results included a comparison of children's ability in the different areas of mathematics content.

Chapter 5 reviews the research of this thesis and notes the way it relates to the findings of other investigations into the subject of the mathematical knowledge and competencies of the pre-school child.

Chapter 6 suggests ways to implement this theory with ideas on how the practical application of this type of assessment may demonstrate the wide range of competencies of young children and show how these should be accommodated in the conventional pre-school mathematics programme.
The strengths and limitations of this study are recorded with reference to the ways in which each test item could be altered to gain further insight into the child's understanding of the particular mathematical concept. The vast amount of research on each aspect of the test, has encouraged further adaptations to the tasks so that a more

accurate assessment can be made. There is a need to prove that the value of such an assessment will enable teachers to plan a curriculum that provides for children to move at their own pace through different stages of mathematical representation linking new information to what they already know.

Chapter 2 Historical Overview of the Development of Number Knowledge of the Young Child

2.1 Introduction

In the second half of the twentieth century there has been criticism of the ideas of Piaget whose work influenced the understanding of mathematics education with his theory of stages of development. Although Piaget's general principles about the need for children to understand what they are learning in their own terms, is acceptable; research has shown that he underestimates young children's ability by ignoring the context in which thinking takes place. Hughes (1986), Gelman & Gallistel (1978), Wright (1991), Young-Loveridge (1989) and Aubrey (1993) suggest that we should take a new look at the abilities children possess before they start school and see the relationship between this knowledge and the kind of mathematics children learn at school. They investigate what children can do rather than what they cannot do so that they have a clearer picture of what they know about number when they first come to school. This then should throw light onto why so many children have real difficulties in learning mathematics.

We know that in the first five years of a child's life he/she has absorbed a wide knowledge of facts and skills. This level of intelligence is dependent on his/her family inheritance, his/her environment, number of siblings and their age relative to the child, and the type of home life, travel, and personal experiences he/she is exposed to. These circumstances ensure that each young child is different from others in his peer group and accounts for the wide range of abilities of children in the first school year. Young-Loveridge (1989) and Aubrey (1993) question whether educators are guilty of taking children into schools with only a vague notion of the number concepts that his/her previous five years have allowed him/her to acquire. To apply an effective mathematics programme at any level of education it is imperative to know about the children's level of mathematical readiness at that particular time. This will ensure that the learning material is structured and presented at the appropriate level so that all the pupils can experience success; the programme will challenge the pupil's abilities; provide the right experience for the development of readiness, interest, and positive attitudes, and help to establish intrinsic motivation to further new learning.

The question educators ask is how do we ascertain the number knowledge and skills possessed by young children. For Gelman & Gallistel (1978) their work showed the importance of investigating both the child's ability to obtain representations of numerosity by counting and the child's ability to reason arithmetically. Like Hughes (1986) and Aubrey (1993), their research focused on detailed analyses of small specific areas of mathematics content, for instance, counting, addition and subtraction word problems, recognition of number words, estimation and the understanding of algebraic concepts such as classification, exploration of shape, measurement, space and time. Central to the young child's understanding of mathematics, is the representation of external objects and the manipulation of objects or their representations or symbols. In order to solve mathematical problems the child needs to be able to connect new information with existing knowledge. Children need to work from the known to the unknown. Therefore what is known already fundamentally shapes what will be learned. It therefore follows that the planned activities need to consider how or under what conditions construction of knowledge takes place.

For Aubrey this emphasises the need to consider understanding as 'situated cognition'(Aubrey 1993, pp.30). By observing exchanges within the physical and social world, we can think about and know the meaning of mathematics for young children. To assess young children's mathematical ability one needs to realise that it is established in the context of known situations and genuine activities and the planned activities for assessment must be appropriately structured. The use of familiar objects, activities, and everyday language will ensure that the child has understood the situation and feels confident to express his/her thoughts clearly and accurately. Many studies have attempted to determine the child's number knowledge on the basis of his/her verbal responses to questions involving number or with the use of unfamiliar objects and in strange situations. However if the child is to use the knowledge he/she already has, he/she must first recognise the situation as being within his/her world of experiences so that he/she can attempt to solve the problem. If not there will be some difference between what the child says he/she knows and how much he/she knows about what he/she says. It is often through his/her manipulation of objects rather than by his/her verbalisation of number names that we determine the child's true understanding of number concepts.

Like Hughes (1986), Gelman (1986) and Young-Loveridge (1989), Aubrey used the revised clinical interview method to assess the subjects participating in the research. This method was developed by Piaget (1952) to improve on the verbal method which needed the support of concrete objects to illustrate the problem to be solved and make it easier for the young child to conceptualise the situation. The use of familiar, acceptable and interesting objects and situations enabled the researcher to follow the intellectual activities used by the children in a variety of contexts, and to understand the cognitive processes which direct the child's thought and give reasons for performances on a wide range of tasks. If the examiner was able to be sure that the child had understood the problem in the way intended , then he would be sure that the evaluation of the child's cognitive competence represented the highest ability at his/her present stage of development. Even standardised instructions presented objectively, may not be understood in the way intended thus demanding clarification or modification of the instructions. Often to repeat an instruction or to rephrase the question may help to clarify the problem. In cross-cultural research the use of culture-specific materials may help to ensure that the problem is perceived as intended. Again this emphasises the importance of familiar objects and situations. The researcher also needs to be aware of the fact that young children often engage in 'romancing' and invent answers to please or amuse rather than giving serious well-thought-out replies. To avoid this situation it may be necessary to pose the same problem several times in different ways to determine the consistency of response. Similarly it may be necessary to determine the strength of the child's belief in a particular explanation by challenging the child's response with counter-suggestions to see if the child changes his/her response showing that he/she is not sure of his/her ideas. The test activities, apparatus and interview technique used by Aubrey in her assessment programme took cognisance of these aspects of the child's cognitive development so as to ensure that the results gave a true reflection of the child's ability.

Aubrey studied the work of fellow educationalists in America: (Gelman & Gallistel 1978, Ginsburg, 1977, Carpenter et al. 1982), and Britain: (Hughes 1986), to determine what activities would best demonstrate the numerical abilities and cognitive competence of the young child. This research led to an understanding of the way young children count, add and subtract, multiply and divide, estimate, represent written numbers, read numbers, classify, and recognise shape, patterns, measurement, time, and position in space and the connections between these types of knowledge required to develop this level of cognitive ability. Aubrey stressed that it was important that the study should give an understanding of these various kinds of knowledge and mathematical skills possessed by the young child so that it would throw light onto the link between formal, symbolic mathematics of school and the knowledge children develop out of school. By assessing the number knowledge of pre-school children it may be possible to determine which factors influence the early growth of number ideas. For Aubrey (1993), "The aim of such work has been to provide a framework for, and sequence to instruction" (pp.28). This perspective guided the design and structure of the tasks and aimed to study the relationship between kinds of knowledge and the way in which these elements influence the development of children's mathematical thinking. There was a need to ascertain whether there is a relationship between the pre-school child's ability to count and his concept of number conservation, and to find out how accurate the young child's concept of number is. The objective of the tasks was to find out whether children have the ability to understand mathematical concepts at a very young age and how important is it for the foundations to be laid at this age before an effective mathematics programme can be introduced at the school level.

Modern research into educational psychology shows that elementary mathematical concepts and rudimentary types of abstract thought are attainable by pre-schoolers. From an early age children start to develop ideas about the world around them while perceiving the attributes and properties of the objects around them and becoming actively involved with them. They observe colour, shape, quantity; the spatial arrangement and the number of objects; and relations among people and so build up the stock of sensory experience that forms the basis for elementary mathematical ideas and concepts. Should children be left to develop these skills and knowledge spontaneously or should the process be guided by instruction and development? Now that experts agree that the cognitive potential of children, even the very young, is considerably more extensive than had been previously supposed, it seemed certain that this potential must be efficiently used and its development supported in the best possible way.

2.2 Number

When children participate in the informal mathematics programme of the pre-school group, some of them may already have developed a sound mathematical knowledge from everyday situations in the home environment. Tizard and Hughes (1984) observed young children at home, recording their conversations with parents and siblings and found that in all social classes fundamental and extensive learning takes place. There were numerous examples of home activities that involved conversations about number, counting, money, shape, time, size, measurement, etc. showing that the young child had already acquired a sound knowledge of basic mathematical thought processes and could apply them to his everyday life experiences. But what is number? Is it a notion or a concept? (Leushina 1991). Because children use the number-words do they have an understanding of quantity? These issues lead to various points of view on the development and importance of counting for children from a very early age.

2.2.1 Counting

The development of counting skills and principles is a key aspect of pre-schoolers' mathematics (Baroody, 1992).

Does the ability to count and use number in their conversations indicate that the child has an understanding of the true value of number? Piaget is quite clear in his idea that "there is a very great psychological distance to be spanned between the child's learning to perform counts, however proficiently, and his attaining the first genuine, working idea of number in his mind." (Isaacs 1960, pp.11). To begin with the child is only interested in the activity of counting and pays no attention to the product. It is a process of inward growth or maturation that develops the idea of number as an explicit concept and enables the child to give an account of it in language. As the child pays more attention to the numbers he/she has counted, they begin to take on their own nature and properties. Although this is an internal growth, it is effected by the child's active relations with the world around him/her, his/her experiences and the way he/she interprets these into actions.

Learning to count involves learning the number words, applying them to things and understanding what counting is all about. Ginsburg (1983) describes this as a knowledge of the sequence number words which are produced in the conventional sequence order, the counting words where sequence number words are assigned to items and cardinal words where a number word describes the numerosity of a well-defined set of objects. But how does this develop and what develops first? The views of theorists differ widely on this issue.

The young child's environment is filled with opportunities for learning the counting words of his/her culture. However the sequence of number words is a little more difficult to master and often needs practice in the form of memorising songs and rhythms. For English speaking children, learning the words for numbers beyond 10 is particularly difficult for the number words are irregular and therefore need to be learnt whilst developing an understanding of the decade system. Mastering the number sequence is a combination of children's self-directed learning and the influence of parents and culture. Self-directed in that children choose to participate in number activities, are interested in expanding their knowledge and request advice from others. However children do not learn solely on their own. Culture also contributes by passing down basic concepts such as the nature of the number system, and it defines the context in which a child deals with a mathematical problem and so effects the way that the child tries to solve it (Bryant, 1994). Ginsburg (1977) notes how cultural experiences affect the ages at which children reach conservation and concludes that the Piagetian stages cannot be innate and only be influenced by biological maturation. "Arithmetic, like language, is very much a cultural product" (pp.50). Once the numbers one to twelve are learnt, children discover that the numbers from 13 onwards contain an underlying pattern which when applied will develop a few simple rules by which to name the numbers up to 100. Ginsburg (1977) points out that children's errors in counting are meaningful and informative, providing insight into what they are really trying to do e.g. "twenty-ten" shows how they try to apply the structure of the counting numbers. It must be remembered that children do not learn in only one way - some learning is done by rote and some by meaningfully applying the rules.

2.2.2 Numerosity

Once these number words have been learnt, children have to learn how these words relate to number concepts and how they are used to count. How children attach a number to a set of things has been investigated by many theorists who have tried to analyse the specific characteristics of the child's thought process. However just how and when young children are able to conserve number has been debated by researchers since early in this century. How does the child become aware of the fact that the word tags provide important information about the counted items? This involves an understanding of cardinality and ordinality (Brainerd 1979). The child must learn that the number word given to the last counted object of a group of items represents the total number of counted objects (cardinality) and that consecutive number words represent successively larger quantities (ordinality).

Very young children have shown that they are aware of and have some understanding of numerical invariance. Starkey & Cooper (1980) showed that infants have some awareness of the fact that the number of objects remains the same when the objects are rearranged, but changes as a result of the addition or subtraction of one or more objects. In their investigation, five month old infants were able to detect numerical differences in arrays which consist of small numbers of items (i.e. 2 or 3 items). The young child's verbal counting abilities would possibly grow from this numerical ability. Silverman & Rose (1980) questioned whether young children quantified small sets more accurately by subitizing or by counting and whether one process was preferred over the other. This research showed how children aged 3 years preferred to count given set sizes and the two quantifying activities produced very much the same responses. From this early conservation of small numbers of objects through a pattern-recognition procedure the child extends his number knowledge to counting larger numbers of objects before and after spatial transformations. In this way the child learns that spatial transformations do not change the cardinal value of an array.

Ginsburg (1977), however, suggested that children only learn to conserve number at about age six or seven. Younger children believe that rearrangement of sets changes the number value therefore indicating that counting and numbers do not have the same meaning for young children as they do for adults. For young children number value also changes when there is a change in the order in which numbers are counted.

These ideas suggest that number is a name rather than a concept describing a characteristic of a set.

2.2.3 Subitizing

How then do young children arrive at this 'name' or number given to a set? Are the objects first counted or is this process preceded by subitizing which is a perceptual mechanism used to judge numerosity?

Gelman & Gallistel (1978) point out that some theorists believe that young children's subitizing ability breaks down at about the point where adults appear to shift from subitizing to counting which leads to the conclusion that young children subitize rather than count. Further more young children have difficulty with larger sets because they cannot count. This idea suggests that young children subitize before they count a given number.

Gelman & Tucker (1975) suggest that number representations are first obtained by counting rather than by subitizing thus the practice of counting allows the child to skip the counting process and 'chunk' the array. This idea assigns subitizing an advanced organising role. It is further argued that children subitize by perceptual chunking or by subvocal counting. Gelman & Gallistel (1978) argue that pre-schoolers develop an ability to use perceptual strategies as they become sure of the results of the counting procedure and all short cut methods are diverse. Children seldom rely exclusively on a direct, perceptual pattern-recognition mechanism when abstracting number. They further reject the idea that subitizing operates independently of the counting procedure.

Ginsburg (1977) agrees that children first count objects laboriously with sets of any size and eventually over a period of time they learn to recognise or perceive small collections of objects. This is just one of the strategies which children spontaneously develop for efficient and economical counting.

Yet other researchers have presented evidence to show that young children are considerably competent with small numbers and that they therefore depend not on counting but on special perceptual methods of obtaining or representing specific numerosities in these situations (Fuson, 1988). These spatial perceptual methods include subitizing and the use of auditory, visual and kinaesthetic patterns. These perceptual processes used by young children may be similar to those used by animals in numerical tasks. This theory of perception of a group of objects, attached a standard shape to a group to assist with its identification, and thus it was the shape that was identified and not the quantity (Leushina, 1991). Children are not able to identify the group when there is a different arrangement of the same items

2.2.4 Conservation

What then is the relationship between counting procedures and understanding? Is counting only a mechanical rote exercise accomplished by the perfection of a skill or is it the demonstration of the development of an understanding of the principles of counting? Does an understanding of the value of a number develop from the ability to state the number sequence (the skill) or do young children have some innate knowledge for number which guides and expands all aspects of counting-skill development?

There has been much attention focused on the relationship between counting and the development of mathematical concepts, particularly cardinality. Piaget, (1952) being mainly interested in conceptual development, saw these two developments as quite independent and emphasised that counting does not play an important role in the development of conceptual knowledge about number. He found no connection between the ability to count and the development of an appreciation of equivalence and number conservation. In fact it is only when the child has an understanding of number conservation that counting can acquire meaning as a symbol to represent numerical relations. Piaget argued that in order to achieve a mature understanding of number conservation the child must understand that a change in the spatial position of a collection of items is compensated for by an equivalent change in a separation of the objects.

It was however questioned as to whether the child actually develops the ability to count before, at the same time as or after the child reaches an understanding of number conservation. Saxe (1979) determined the developmental relationship between children's use of counting as a notational symbol system and their understanding of number conservation. Young children's use of counting was prequantitative i.e. they used counting when they were required to compare or reproduce sets numerically, but they did not base their comparisons or reproductions on the products of their counting. By age 6 ½ years most children used counting as a symbolic tool to help understand numerical comparisons and reproductions.

Saxe (1979) demonstrated that quantitative counting strategies develop prior to the development of number-conservation concepts. However it is interesting to note that counting accuracy and counting strategy are partially independent from one another. Some children who use prequantitative counting strategies nevertheless count accurately on occasion and some children who use quantitative counting strategies still count inaccurately on occasion. Consistent accurate counting is not essential for number conservation but rather that the child extracts accurate numerical information from set of objects. These findings are contrary to Piaget's theory which considers the child's early counting experiences as merely rote knowledge.

Many theorists believe that the product of children's ability to count is number conservation. Gelman & Gallistel (1978) and Schaeffer, Eggleston,& Scott (1974) listed the basic principles of counting skills and stated that counting ability must reach a certain level of development before children depend on a numerical rather than a perceptual criterion for judging equivalence conservation. The ability to count objects will not necessary enable the child to conserve number, but the ability to count objects will permit the development of other skills and a variety of experiences which may lead to an understanding of number conservation.

2.2.5 Counting Skills

By focusing on the capabilities that pre-schoolers have, Briars & Siegler (1984) noticed that they are adept in executing the standard correct counting procedure. Focusing on one standard counting procedure, word/object correspondence, and four optional features: counting adjacent objects consecutively, pointing once to each object, starting at an end of a row, and proceeding in a left to right direction, children judged a puppet's counting as acceptable or unacceptable. Each child's ability to count rows of objects was also assessed. The majority of children were not limited to the standard counting procedure nor did they rely solely on the word/object correspondence rule, but had begun to learn which of the typical accompaniments of counting are essential and which are optional. Results demonstrated that children counted correctly before they consistently judged incorrect another individual's counting errors thus showing that counting skills precede knowledge of underlying principles. "Counting skills are learned by rote through imitation, practice, and reinforcement" (Baroody 1992,pp.100). This suggests that children learn to apply these skills in various counting contexts and this routine eventually enables them to generalise and abstract from it the common principles of counting. "Only after this has happened do children have principled knowledge" (Wynn, 1990, pp.158).

Sophian (1992) found that children showed an early concept of cardinality in their comprehension of number words but saw a need to dissociate cardinality from counting in early development with an integration a short time later. Counting is a socially transmitted activity and cardinality is a specific form of thought, both have separate origins but with development become integrated. Therefore although counting is not the basis for the initial construction of the concept of cardinality, Sophian suggests that counting contributes to later mathematical cognitive developments.

Perhaps this knowledge of counting skills is what Gelman & Meck (1983) refer to as implicit knowledge- a natural, inborn knowledge of the principles that a procedure must conform to in order to be a valid counting procedure. Children are able to verbalise the counting principles but do not have the explicit knowledge to be able to demonstrate or articulate the principles involved. "Counting starts out as a meaningless activity, something like a game of patty-cake, from which children abstract certain properties" (Wynn, 1990, pp.191). For her the development of children's understanding of counting is a complex and piecemeal process; an innate ability that must transfer the numerosities *one, two, three* to the correct number words. This idea was further implicated by Shipley and Shepperson (1990) who showed in a number of experiments that children have a very strong bias to both count, and respond to the 'oneness' of discrete, physically separate entities and this may help them in learning to count. They suggest that this disposition could be an underpinning for a broad range of human cognitive activities and account for the limited display of the counting principles.

Others have questioned whether counting experience was central to the development of an appreciation of equivalence and number conservation (Baroody and White, 1983). Do children reach a certain level of counting ability before they are able to conserve number? They found that not every child tested was proficient in all the counting skills before he or she conserved number indicating that the complex counting skills are probably not a necessary condition for number conservation.

Schaeffer, Eggleston, & Scott (1974) outlined the hierarchic integration of six number skills which develop the knowledge of number conservation and emphasised the mastery of counting skills. Counting involved the child's ability to understand the cardinality rule which states that the last number named during counting denotes the number of objects in an array, the counting procedure which is the consistent co-ordination of ordered number names and counted objects and the knowledge that x+1 is greater than x. In order for counting to reach this level of understanding, three number skills need to be acquired by young children: the acquisition of more x's which involves the ability to give, take, or ask for more x's, judgements of relative numerosity which is the ability to visualise that one array is greater in number than another and pattern recognition of small numbers. Observation has shown that children between the ages of 2 and 2½ learn to give, take or ask for more x's which is possibly a development of the concept of possession of 'more for me' (a typical sign of this egocentric age). They can also determine which of two arrays composed of 1-5 objects has more objects which is possibly a natural sign of their preoccupation with who has more of whatever is being displayed. Schaeffer, Eggleston & Scott (1974) give data to show that young children recognise small arrays of objects as number patterns possibly as a result of perceptual learning gained from observation and parental training.

Starkey & Cooper (1980) found that very young children have numerical abilities that enable them to use a rapid perceptual process called subitizing to distinguish among arrays containing fewer than four items. Therefore pattern recognition allows the child to use this visual number skill to take two objects from an array without counting the objects. It also allows them to apply the cardinality rule and to give number names to hidden arrays they have previously recognised. Then too children's pattern recognition skill allows them to visually discriminate between two number patterns they can recognise.

To master the counting procedure the child must be able to co-ordinate it's two components i.e. the ordered number series and the one-to- one correspondences between number names and objects. Children are quick to learn the ordered number series but have great difficulty with the one-to-one correspondences between number names and objects, perhaps because they find it difficult to remember which objects they have and have not counted. To overcome these difficulties the child resorts to pointing - a spontaneous action.

The counting procedure is affected by the number, nature, and arrangement of objects. More objects are harder to record in memory than are fewer objects. More complex and/ or less familiar objects are harder to number or group. Likewise, the spatial relations between objects determine whether the child can use a spatial plan to count an array systematically. This ability to use a spatial plan, increases with age. The counting procedure is therefore automarised with the increased use of pointing and spatial planning.

Schaeffer , Eggleston, & Scott's (1974) hierarchy of number skills proposed that, after children have learnt the counting procedure, they learn the cardinality and one-to-one correspondences- developing both these skills at the same time. Finally, these two skills integrate with the ability to judge the relative numerosity to learn that x+1 is greater than x. The child is now able to see two arrays, one of which has more objects than the other, and sets up one-to-one correspondences between the objects in both arrays. By applying the cardinality rule to both arrays the child notices that one array has more, and so he begins to learn that one specific count is greater by one than another specific count. Once the child has mastered these six number skills, he will have learnt to conserve number. This study by Schaeffer et al. (1974) assumes that number development is determined more by the application of number skills to object arrays than by spontaneous cognitive reorganisations. Children first count by rote and are gradually influenced by counting concepts (Briars and Siegler, 1984).

2.2.6 Principles of Counting

Contrary to this idea is the *principles-first* model which argues that the young child's ability to count is governed by several principles and that successful counting involves the co-ordinated application of all the principles (Gelman and Gallistel, 1878). They point out that the child's ability to count must not be based on adult criteria which requires the child to use conventional number words, instead value should be attached to the unique tags which mark or tick off the items in a collection. Baroody & Price (1983) showed that there was considerable evidence that young pre-schoolers used a stable nonconventional sequence. Rote counting can therefore develop without the understanding of the stable-order principle. These tags must be used in a fixed order and have an arbitrary status. Gelman and Gallistel (1978) believe that five principles govern and define counting, namely:

a) the one-one principle:

Every model of counting uses this principle which involves the ticking off of the items in an array so that one and only one tick is used for each item in the array. This principle involves the child in the processes of partitioning and tagging. Partitioning is the process of separating those items that have already been counted from those that are to be counted either mentally or physically. Tagging involves summing up, one at a time, distinct tags or counting words. These two processes are carried out in a rhythmic co-ordination - starting together, stopping together and staying in phase throughout their use.

b) the stable-order principle:

This principle involves the use of a stable or repeatable order of tags or lists which are used to correspond to items in an array. From an early age children develop numerical abilities which enable them to rote learn the first 12 or 13 number words.

c) the cardinal principle:

This principle shows an understanding of the value of a set by stating that the tag applied to the final item in the set, represents the number of items in the set.

d) the abstraction principle:

Once the how-to-count principles have been mastered, they are applied to the abstraction principle which concerns the range of entities to be counted. This principle makes no distinction between physical and non-physical entities and allows for the counting of any array. It has been argued that children only fully understand this principle at about the age of 7 years but Gelman & Gallistal (1978) believe this to be an underestimation.

e) the order-irrelevance principle:

This principle involves an understanding of the fact that the order in which the items are tagged is irrelevant. Children should know that a counting word can be assigned to any item and in any order so long as no count word is used more than once in a given count.

To test pre-schooler's ability to reason about number, Gelman conducted two types of studies: the magic experiment and videotaped counting experiments which brought out spontaneous counting and talk about number. These experiments showed that at a very early age children know the fundamentals of enumeration and adhere to all three counting principles when dealing with small set sizes (2 to 3). As set sizes increase, they begin to have trouble with the one-one principle, and they stop using the cardinal principle. When counting larger sets, they try to use the one-one principle but fail, however they continue to adhere with some success to the stable-order principle.

Gelman & Gallistel (1978) found that pre-schoolers do not normally place restrictions upon countable collections and are therefore able to carry out the abstraction principle. They readily group a variety of two-dimensional and three-dimensional materials together under the collection of "things to be counted". They found the order-irrelevance principle not that easy to apply and although most children had some idea of what was involved they clearly had further to go before they would reach a full understanding of this principle.

Likewise, Gelman and Meck (1986) agreed that before there could be skilled counting some form of understanding of principles had to take place. These five principles just described form the child's framework for or initial conceptual competence which is the basis for the task of acquiring counting skills.

2.2.7 Mutual-Development View

The 'mutual-development view' of Baroody (1992) sees the gradual evolving of number sequencing as the combination of an understanding of number-word counting with counting-skill development. Infants have some innate ability that informs and directs all aspects of counting-skill development particularly during the pre-school years when these skills are perfected by the emergence of new or stronger principles. Perhaps the 'mutual-development' view provides a middle ground between the skills-first view and 'some-principles-first' view and suggests that an understanding of number-word counting develops gradually and together with counting-skill development. Counting ability in the pre-school period may involve some innate ability for number competence but self-initiated learning and environmental factors will account for the perfection of counting procedures and the emergence of new and firmer principles.

At present research is still debating about the developmental relationship between counting principles and counting skills but there seems to be some evidence which suggests that pre-school children do understand the principles cited by Gelman and Gallistel (1978) but it is not clear how this understanding exists prior to the development of any counting skill. Do innate principles govern and inform children's earliest attempts to construct number-word sequences or are counting skills learned by rote through imitation, practice, and reinforcement?

The words of Droz (1992) may offer yet another perspective: "Children neither construct one notion of number nor one approach to number, but rather many notions and many approaches to multiple numbers that are known and unknown; that interact, overlap, and interpenetrate; and that can both complete each other and cancel each other out. Investigators in child development reduce this richness to one perspective, often highly congruent with what they find appropriate at a given point in time for reasons often known only to them." (pp.242).

2.2.8 Factors Influencing Cardinality

However most researchers would agree with the general Piagian position that counting alone is not sufficient for an adequate understanding of number and that in changed situations, operational thinking requires an ability to think in terms of quantity and therefore necessitates an understanding of numerosity.

For this reason researchers and theorists in the United States, (Gelman and Gallistel, 1978, Fuson, 1992, and Baroody, 1992), Australia, (Wright, 1992) and New Zealand, (Young-Loveridge, 1989) have focused on the role of counting in young children's number learning. Counting has assumed a more prominent role in the introduction of operations and number facts but this has not been accompanied by greater emphasis on the development of counting in the earlier activities of prenumber and early number (Wright, 1992). The importance of counting in children's numerical development is seen as essential because many of the commonly used thinking strategies involve counting. "Counting provides the representations of reality upon which the reasoning principles operate" (Gelman and Gallistel, 1978, pp.161). "The early skill at counting is guided by the availability of implicit counting principles" (Gelman, Meck and Merkin, 1986, pp.27).

At a very early age children seem to learn the difference between counting and non-counting words (Fuson (1988). Learning the number-word sequence continues long after the child is able to produce the number words correctly. Rote-counting, that is the production of the correct number word sequence follows an orderly succession of new abilities which Fuson called "the elaboration of the sequence." These five levels of elaboration are a lengthy process that ranges from age 4 to 7 or 8. Initially the number-word sequence is learnt as a connected and undifferentiated whole so that number words can only be produced by reciting the whole sequence. (Called the string level) Next comes the "unbreakable list level" where each word is separated, but because the sequence exists in a forward-directed form it can only be produced by starting at the beginning. Then comes "the breakable chain level" when children can start counting up from an arbitrary number in the sequence without saying the sequence from one. At the "numerical chain level" each word in the sequence can be as an equivalent single word or unit. At this level sets of sequence words can represent a numerical situation and can be counted or matched. Therefore to add five and three a child will say the first five sequence words and will then say three more sequence words, giving the final sequence word eight. Finally the "bi-directional chain level" allows the child to count up or down quickly from any word. The child's ability to say the correct sequence of number words is very strongly affected by the opportunity to learn and to practice this sequence. The characteristic form of incorrect sequences used by English speaking children suggest that to learn the number sequence involves a complex procedure and must be laboriously memorised.

Rote-counting is a complex process. By looking at children's rote counting errors, Young-Loveridge (1987) noted that they have an understanding of the decade structure of number. The most common stopping points in children's counting are at a number ending in 9 or 0 , entire decades are often omitted or repeated, new number words are constructed using rules (e.g. twenty-ten, twenty- eleven) and children are often able to count on from a particular number in the decade above their highest stopping point. Bryant (1994) feels that because of the difficulty experienced by pre-schoolers in understanding the decade system, their encounters with numbers may not at first be of much importance, as far as understanding of mathematics is concerned. In order for young children to thoroughly grasp the decade system, there must be instruction. There can only be a real breakthrough in understanding the number system when the structure of the decade system has been understood and not through learning to count.

Rational counting is a complex procedure and requires the child to enumerate or assign cardinal or ordinal meanings to items. For the child to demonstrate that he/she has mastered these skills the following four points should be adhered to:

a) one number directed toward each object,

b) each number must not be directed toward more than one object,

c) every object is numbered, and

d) no object is numbered more than once.

Fuson (1988) suggests that although young children are very good at this complex act, it is very difficult to consistently co-ordinate the pointing act with the number words and with the objects to be counted. Many variables influence the correspondence errors and counting research will have to pay careful attention to this fact.

At first rote-counting and rational-counting appear to be separate and different situations for children. When do children first indicate that they understand that counting has a result instead of just being an isolated activity? Fuson (1988) investigates the ideas of theorists and discovers that children seem to follow different routes to understanding the cardinality rule. Contrary to the theory proposed by Schaeffer et al. (1974) that children first discover last-word responses on subitizable sets and then later generalise such responses to larger sets, Fuson (1988) gave evidence that children rarely answered the how-many question by subitizing. Instead they gave the last word response with an incorrectly counted set even though they could have given the correct answer by subitizing. Accurate counting is therefore not required for last-word responding. Last-word responding was not influenced by set size as suggested by Gelman and Gallistel (1978). Children did not monitor their counting accuracy and stop giving last-word responses when they were not able to count accurately. Evidence supported the theory that children use the last-word rule or principle which is quickly learnt by observation or auditory "echoing" but this rule does not refer to the cardinality of the set. This transition of the child's use of the how-many-question rule to the understanding of the cardinal reference of a last-word response is an important developmental task for the pre-schooler but what moves children from one level of last-word responding to the more advanced level of cardinality is not clear.

Children not able to understand the cardinality rule, often re-count sets as many as seven times in response to each repeated question of "How many blocks are there?" rather than giving the final word from the count (Ginsburg 1983). This seems to indicate that they perceive the question as a request to count the objects rather than a request to give information gained from the counting act. Here the cardinality of the set has been given but has the concept been fully understood? To increase the understanding of the cardinality rule Markman (1979) reported how the use of a verbal manipulation assisted the process. Children hearing collective terms such as group, family, and team, focused their attention on the set as a whole rather than on the individual objects within it and this facilitated the appropriate use of the cardinal word to refer to the whole set. The way the question is posed will possibly lead to different inferences about the child's understanding of the cardinality rule.

Another factor that may lead to an apparent absence of the cardinality rule is forgetting (Ginsburg, 1983). If the child is asked the 'How many?' question after counting is completed, the failure to respond with the correct counting word may be due to a failure to remember what that word was rather than to a lack of understanding that the last counting word can also convey a cardinality meaning.

Results of all these studies are questioned by Gelman, Meck, & Merkin (1986) who interpret the evidence against the principle that assessments are erroneous if they do not account for communication factors which lead young children to fail. Studies emphasised the role of social factors that influence a child's assessment of a task. The constraints of a 'test' situation are more likely to yield misinterpretation thus affecting performance levels. Young children therefore require completely unambiguous instructions to avoid problems in assessing the task (utilisation competence).

Experiments that do not combine the relevant goal of obtaining the cardinal value of a set with their prior counting behaviour, show a problem in operational competence not in conceptual competence. One therefore cannot interpret the child's conceptual competence unless one is sure they have understood how to plan the solution. They go so far as to suggest that conceptual competence develops out of procedural competence. Children who count left to right are usually at an advantage over those who skip around and are less likely to miss or double count items. Because they have the utilisation competence they have developed conceptual competence.

"The sequence of counting words is one of the most important tools of early mathematics learning" (Brainerd 1982, p.89). Children show individual patterns of acquiring this structured process before the full conventional sequence is learned. Initially they acquire segments of the conventional number word sequence, then a relation between words in the sequence is established. Therefore the sequence is first used as a problem-solving tool in the process of counting objects and then later the counting words themselves become the objects that are counted. This number skill is then used as a tool in more sophisticated counting procedures and fundamental mathematical activities. Nunes and Bryant (1996) agree that children need to be encouraged to use counting in a variety of situations for solving problems. Counting as a problem solving strategy will make number more meaningful and enable young children to use counting as a thinking tool.

Researchers such as Carpenter & Moser (1984), Gelman & Gallistel (1978) and Williams (1965) understood clearly the need for more recognition to be given to the value of counting strategies and the way in which counting enables the child to connect a set of reasoning principles to reality. Young children are very interested in numbers and seem to be caught up in counting rituals. This natural fascination for numbers must surely be an incentive and assistance for learning to count! However because counting is , in its structure, a complex system of intercoordinated, individual operations, which are at first unknown to the child, it can only be accomplished as a result of adult organised instruction (Leushina, 1991). By imitating adults, the child only grasps some of the external operations of counting and needs to learn the more complex components such as the correlation of each item with a number-word, the number-word sequence and the numerosity of the set.

If counting is to be accompanied by an understanding of the concept of number, then instruction needs to lead the young child through the natural stages of development. Leushina (1991) sees the initial development at around the age of eighteen months as an observation of homogeneous objects either referred to as individual objects or collections of them which creates a basis for children to distinguish between singular and plural number. In this prenumber period of instruction, children are taught to not only distinguish between 'one' and 'many' but also to develop an idea of a set as a unit and the individual elements that make up that set. Such preliminary work with sets will introduce the child to the idea of number and enable him/her to learn counting more accurately in the future. Prenumber work with sets will develop counting skills but there is no need to rush into counting with number-words. Perceptual analyzers; visual, auditory, tactile, and kinaesthetic play various roles at different stages in the development of counting. At first the child accompanies homogeneous objects with identically repeated words and motions such as rhythmic movements with hands or head. Leushina observes how counting rhymes connect the first number-words and movement which shows the fundamental importance of motor analyzers in counting the elements of a set and in forming the first ideas of numerosity. However these number-words do not indicate counting and do not reflect comprehension of the meaning of number. This view stresses that early training in naming the number-words, even if the sequence is correct, does not assist in developing counting or a meaning of number. It is the interaction among the analyzers which promotes the perception of a set as a whole and the elements within it and leads to an understanding of a one-to-one correspondence and the development of counting with meaning.

Based on this theory, Leushina traces the development of counting in children. The first two stages involve the tagging each item of the set with a name and then comparing quantities of sets using words such as 'more', 'fewer', and 'equal'. In the third stage the sequential naming of number-words begins when the elements of sets are compared. Contrary to the ideas of others, Leushina sees the development of this stage as being mainly conditioned by teaching. Pre-school children have usually reached the fourth stage and are able to name the numerals in the correct sequence and to correlate a number-word with each element in a set. They have also learnt that the last number named gives the numerosity of the set and are not distracted by spacial or qualitative features. The last two stages see the development of counting groups or units and then counting by tens.

Children need help in developing segments in the natural number sequence. The naming of numbers is gradually learnt by first correctly naming the sequence up to five or ten and then going on to say the next numbers chaotically: 1,2,3,4,5,8,13,9,18. Development takes place as the segments of numbers that are remembered in sequence grow, and the children start to realize that each of the number-words always occupies the same place, although they do not understand why this is so. Counting is a formation of audio-vocal-motor connections between the numbers that are named with meaningless repetition. Because this word chain has been learnt, the connections cannot be disrupted and children are unable to start counting from any number other than 'one'. Gradually a set of numbers is ordered and named possibly with gaps but always in ascending order: 1,2,3,4,5,6,7,8,9,10,12,15,18,24,28,and 29. Once the numbers to 20 have been memorized, children learn that the first ten numerals are combined with the names for the tens to make the sequence 20,21,22,23,24,25,26,27,28,and 29, but often there is misunderstanding and the numbers are recited as 'twenty ten, twenty eleven'. Once this has been mastered, children need assistance to learn the words that start a new decade.

Leushina emphasises that without special instruction this process may be long and drawn out with some children 'pioneering' their way forward. This will account for the different levels of knowledge in children of the same age. Pre-schooler's knowledge of number does not always ensure that they are able to understand mathematics because they do not necessary have a thorough grasp of the decade system which at some stage must be taught (Bryant, 1994).

2.2.9 Ordinality

Number skills allow the child to demonstrate a knowledge of ordinality, or order relationships of equivalence, 'greater than' and 'less than'. To assess the child's knowledge of ordinal relationships Bullock and Gelman (1977) used the 'magic game' with 2½ to 5 year old children. In the first stage the children were shown 2 plates of toys. One plate displayed a single toy animal, and the other plate two animal toys. The child was asked to pick the winner either when the winner had more toys on the plate or less toys. In the second stage the researcher added one animal to the two-toy plate and 3 animals to the one-toy plate. Now children were asked to repeat the experiment and choose the winner on the basis of the relationship of more or less. "The results of this study suggest that children as young as 2½ years of age have an understanding of ordinal relationships" (Geary, 1994, pp.21). However, Geary questions whether a young child uses the same skill to realise that 10 is greater than 9. Does the development of ordinal knowledge for larger numbers involve simply joining number words to innate preverbal magnitudes (Gallistel and Gelman, 1992) or is this knowledge gleaned from learning and the use of conventional sequence of number words? (Fuson, 1988).

2.2.10 Evaluation of Counting Ability

In order to assess the number knowledge possessed by children beginning the kindergarten year of school, Wright (1991) developed a theoretical model of counting types based on the ideas of Steffe (1988) to determine the stage of each child so that learning programmes could be more closely attuned to the developmental levels of children. The qualitative differences in children's counting occur because there are differences in the nature of the unit items children are able to construct. He describes a progression of five distinct unit items: perceptual, figural, motor, verbal and abstract, and from each of these unit items develops a distinct counting type. The mental composition of 'unit' has a central role in the theory of counting types because the five counting types involve a progression in the most advanced 'unit items' that a child is able to build when counting. Wright developed a five-stage model of children's numerical development that could provide a basis for analysis and documentation of the differences in number knowledge among young children. Children in the first stage of this model can count only those items which they perceive, then at stage two they are no longer dependent on direct sensory input but still need to reconstruct or represent a sensory experience when counting, such as rhythmical motions of the hand or sequentially raised fingers. Then at stage three the child has developed an operational understanding of the meanings of number words and no longer relies on the links to represent experience. For example he/she has an understanding of the number seven and can count on from that number to find sums and missing addends. Stage four enables the child to focus on the collection of unit items as one thing as well as the individual abstract unit items. Therefore in a task such as 22-17, the number 17 is regarded as a composite unit and the child is able to count down from 22 to 17 to determine the difference. The fifth stage is characterised by the construction of the part-whole operation that is the simultaneous awareness of two number sequences and can dis-embed the smaller composite unit from the containing composite unit and compare them. e.g. 23+ =25

This five-stage model highlighted the need to work on the schemes counting-on, counting-up-to and counting-down-to instead of the standard paper-and-pencil work. His study therefore also saw the need to test the child's forward number word sequence (FNWS) and backward number word sequence (BNWS) and then to grade children on five levels according to their ability. Unlike other models, Wright graded the levels of FNWS and BNWS in corresponding similarities because he had observed children who, as a result of specific instruction, developed BNWS to almost the same extent as their FNWS and concluded that there appeared to be no theoretical reason why the construction of BNWS should lag behind FNWS. Each level or stage satisfies the following criteria: a) a distinctive ability remains constant throughout the stage, b) each stage incorporates the earlier stage, c) the stages form an uniform sequence, d) each new stage involves a theoretical reorganisation resulting from consideration and thought. Each level does not refer to a development of time but a certain elevation or improvement of performance.

By categorizing children according to their counting ability, Wright found that there was a wide range in the levels of number knowledge among children beginning the kindergarten year of school. This further emphasised the importance of teachers taking account of children's prior number knowledge and for "the urgent need for early childhood educators to rethink the content of the mathematics curriculum in the light of current research and for many children in the kindergarten year, to de-emphasise 'topics such as sorting, classifying, matching and patterning" (Wright, 1991, pp.14).

But is counting the only pathway to an understanding of number? Brissiaud (1992) focused on the type of behaviour in which children represent numerosity by a gesture after having formed a one-to-one correspondence with a corresponding set of fingers. He showed how the use of fingers is a meaningful way of showing numerosity and that it forms one of the basic developmental routes in the construction and acquisition of ways to represent numerosity. Often when young children are asked how old they are they will hold up the appropriate number of fingers but are not able to give the number word. Gestures precede the labelling of the quantity. In this way the child has invented a means to represent the numerosity required even though he did not know the 'number name'. Like counting, this method makes use of one-to-one correspondence but the quantity is represented by the set of fingers. This procedure for describing a given quantity of objects ensures that there is no period of time when counting is purely listing a sequence of number words. When asked to give the numerosity of a set of six objects, the child would use the one-to-one correspondence between objects and fingers thus giving an analogue representation of this quantity. "He thus knew that the word six represented a quantity, and he applied the cardinality rule on his first experience with counting. There was no time when counting was purely counting word tagging" (Brissiaud, 1992, pp.51). This pathway to understanding number never includes the rote learning of the sequence of number words. As each new number is learnt, the child uses an analogue representation of the quantity in the form of a finger symbol first so that the number has meaning and represents numerosity before the number name is learnt. This shows that before having learnt to count, the child has constructed a genuine conceptualization of numerosity based on the use of a gesticular system of analogue signs and not on a verbal system such as number words.

Brissiaud (1992) suggests that this pathway to number is partially the result of teaching methods and partially the result of a child's resistance to use counting words before he/she understood them. Children will always construct number no matter what path they choose, but perhaps the long-term consequences are affected by the initial pathway taken? Learning disabled children appear to have no difficulty in learning the correct sequence of number words but have difficulty in memorizing number facts. Brissiaud questions whether a different pathway to number might have produced different results in the long-term.

2.3 Addition and Subtraction

Addition and subtraction are fundamental activities in both school mathematics and everyday life. The importance of these mathematical operations was accepted by theorists such as Hughes (1986), Gelman & Gallistel (1978), Ilg and Ames (1951); Starkey and Gelman (1982), Groen and Resnick (1977) and Brush (1978) who believed that young children with no formal schooling in arithmetic do possess some understanding of addition and subtraction.

Wynn (1992) claims that babies as young as five months in age are able to add and subtract, and concludes that the basis of arithmetical understanding may be innate. She used a measure of surprise and found that babies looked longer at the inappropriate displays than at the appropriate ones, thus concluding that they could work out the results of simple additions and subtractions. Starkey (1982) likewise used nonverbal tasks to conclude that pre-school children can work out the results of simple additions and subtractions. He gave children aged 24 and 35 months two, three or four objects to put in a container. Then he either added or subtracted some objects himself or left the container untouched. The child was asked to remove all the objects from the container which was built in such a way that the child could only take out one object at a time. He found that on the whole they did reach into the box the right number of times.

However Bryant (1994) points out that when number words are introduced, young children begin to make serious mistakes. He concludes that pre-school children understand and use simple mathematical relations, and begin to learn about the number sequence, but that they have difficulty in combining these two very different types of mathematical achievement. He questions what causes children to quickly grasp and use quantitative relations and yet be so slow to come to terms with the basic meaning of number words. He suggests that the problem may lie in the informal instruction that they receive at home.

Piaget (1952) gave very little attention to the importance of addition and subtraction. He only demonstrated their relationship to his fundamental concepts of class-inclusion and conservation which he claimed were essential prerequisites for understanding addition and subtraction. Class-inclusion was a test of the child's ability to compare a set with a subset of itself or a whole with a part of that whole . The conservation of number was judged when the child gave the answer to the number of counters after they had been displaced so that they were no longer in one-to-one correspondence. From this information he argued that true understanding of addition and subtraction could not be attained before the onset of concrete operational thinking at around 7 years.

Gelman's magic studies (Gelman & Gallistel, 1978) claimed that Piaget's theory underestimated young children's abilities and ignored the context in which thinking takes place. By using small set sizes she showed that counting was the means to connecting a set of reasoning principles to reality. The magic experiments gave evidence that children as young as 3 years know that transformations involving displacements do not alter number and that transformations involving addition and subtraction do alter the numerical value of an array. Young children are already able to identify a number of operators that do not alter number such as lengthening, shortening, rotating a linear array or changing the colour. Likewise their numerical reasoning scheme includes operations that allow the child to deal with transformations that do alter numerosity such as addition and subtraction. When children notice an increase in numerosity they state that something has been added to the original array. Therefore to complete this operation the child realises that it involves the uniting of disjoint sets and he accordingly uses the same procedure that he uses to obtain a representation of any other numerosity - he counts beginning with the cardinal number of one of the sets and then adds by counting up from there. This process involves a step-by-step partitioning of the counted items from the to-be-counted items. This addition operation using the counting process would not work if the request was to add two non-disjoint sets.

Piaget found such a task requiring reasoning about numerosity to be beyond the scope of a young child. Likewise the process of subtraction is regarded by the child as the removal of items from a set and again the numerosity is obtained by counting the remaining objects. In the magic experiment the children noticed that objects had been removed from the set and realised that to return to the original numerosity of the set, the number of items removed would have to be added again thus involving a process of counting up from the remaining objects to the original number. It must be noted here that young children were only accurate about the number of items that needed to be added or subtracted when there was a deviation of one item but with a difference of two or more they were not so precise and used terms such as some more or some. It is however important to note that the magic experiment showed that these young children knew how to correct the difference by adding on to see what had been subtracted and subtracting to see what had been added on. Gelman refers to this as the solvability principle which is applied by using the counting procedure and involves the use of reasoning principles.

Several research studies (Starkey & Gelman 1982, Hughes 1986 and Brush 1978) have used the natural play situation of young children to ascertain their understanding of and ability to perform simple addition and subtraction. The tests used materials that were familiar to the children such as a box with blocks or coins held in the hand or marbles in a cylinder thus ensuring that the tasks were of a meaningful nature enabling the children to show their capabilities. Children aged 3 to 5 years were given a number of problems to solve each with the same basic structure. The number of objects in the container were first identified, then as others were added or taken away, so that the child could see and was told what had been done, he could work out the result and finally check his answer by looking in the container. As in other areas of number development, the children found small numbers easier to cope with than larger numbers. By the children's actions and comments as they worked on the problems, it was found that they used different strategies for small-number and large-number tasks. For small number tasks they would either simply name the final quantity of objects or count to that number as if they had constructed some sort of image of the objects in the container. Some children used their fingers to represent the screened objects while others seemed to rely on a direct visual image of the objects and tapped out the number on the container. Whilst using this strategy of counting up or down the number scale, starting from the initial contents of the container, children were just as successful on addition problems as on subtraction problems when dealing with small numbers. However, for large-number problems children were more successful with addition than with subtraction.

Brush (1978) questions whether subtraction is inherently more difficult than addition or whether children have encountered the phrase of 'more' more frequently than 'less' and that they have used numbers to count forward far more often than backward. These studies further supported the idea that the strategy used for large-number problems was one of counting-on from the initial quantity; quite a complex procedure which entails keeping track of how many steps up or down the scale they have moved. It should be noted here that the counting-on strategy is not usually one that children have been taught but rather one they have invented for themselves.

Groen and Resnick (1977) researched patterns of reaction times that emerge when children are taught a specific problem-solving procedure and then given extensive practice; showing how a drive for efficiency of performance resulted in children no longer using the algorithm they were originally taught but inventing a more efficient procedure.

In a three year longitudinal study of children's solutions to simple addition and subtraction word problems, Carpenter and Moser (1984) concluded that children are not entirely consistent in their choice of strategies and use them interchangeably rather than exclusively using the most efficient one. Once children have learned the more efficient strategy of counting-on from the larger number they often revert to the less efficient strategy of counting-all.

Hughes (1981) looked at a variety of task forms and how they were affected by age, social class, size of number and form of task presentation. The procedure began with the addition and subtraction of blocks in a box that were visible and progressed to them being invisible. Next the objects were removed and the child was asked a hypothetical question about the blocks and from there the question moved to an imaginary incident about people e.g. one child in a sweet shop and another comes in. Finally the problem was presented in a formal code i.e. without specific objects being mentioned. This study provides confirmation for Gelman & Gallistel's (1978) claim that pre-schoolers have a clear set of principles for reasoning about numerosity. Their competency includes an organised working knowledge of how small numbers are interrelated through the operations of addition and subtraction and can apply this knowledge to a variety of concrete and hypothetical situations. Task performance improves with age but shows a rapid increase between the ages of 3 and 5 years thus causing high variance between individual children. The size of the difference is often reported to be in the area of a years development and is often associated with social class which in turn is often attributed to differences in IQ. Hughes suggests that when children start at the nursery school there is already a marked inequality which pre-school education can do little to alter.

This research clearly demonstrated the significance of the form of task presentation. When addition and subtraction tasks included specific objects and events, either in sight or hypothetically, the task caused much less difficulty than when the problem was phased in the formal code of arithmetic. Hughes further considers the reasons for this difficulty with formal code presentations and suggests that there may be value in Piaget's thinking that young children lack understanding and are unable to move from concrete to abstract examples. However he argues that this theory fails to explain how children can solve hypothetical problems involving abstract concepts.

It is suggested that perhaps Donaldson's (1978) approach gives a clearer explanation which recognises that the young child may have adequate concepts for performing a variety of concrete and hypothetical additions and subtractions but lacks the ability to express these concepts in the formal code of arithmetic. The problem is therefore a linguistic one. The child's ability is therefore restricted to skills that are context-bound.

Later Hughes (1983) introduced pre-school children to a rudimentary form of arithmetic symbolism through the use of simple games with magnetic operator signs (+, -) and magnetic numerals (123...).
The idea was to find ways to help children free their thinking from the concrete so that they could express the concepts they already possessed in formal arithmetical symbolism. At the same time children may begin to understand the useful purpose served by formal symbolism. This study showed that pre-school children have considerable numerical competence and can grasp a rudimentary form of arithmetical symbolism in which numerals and operator signs are used to represent concrete quantities and events. It is suggested that perhaps young children may well possess many of the prerequisite skills required for learning arithmetic. If arithmetic symbols are introduced in a meaningful communicative situation such as games, it would make the transition from concrete objects and events to formal symbols much easier.

Research mentioned so far has all judged the child's understanding of addition and subtraction on the use of counting algorithms or some other type of algorithm but Starkey & Gelman (1982) look for evidence of the child's understanding of some of the basic definitions and properties of arithmetic. They considered the laws of inversion and compensation which had been emphasised by Piaget. Inversion reflects the inverse relation between addition and subtraction, i.e. to add a particular number of elements to an array can be negated by subtracting the same number of elements. Compensation shows how the initial numerical relation between two sets is altered by adding or subtracting elements to one of the sets and how the original number may be reinstated by adding or subtracting elements to the other set. They were careful to note whether a child solved an inversion problem using explicit knowledge of the operation or whether he/she resorted to using an accurate counting algorithm. Three year olds were capable of solving some of the simpler inversion problems without overt counting but possible using covert counting algorithms, or explicitly known inversion property, or memorised facts. Further studies complicated the task by using sets that were screened from view and where the experimenter gave the relative numerosity of the two sets or where the sets were placed in a one-to-one correspondence situation. These activities proved to be too difficult for most 3 year olds but 4 and 5 year olds correctly solved simple inversion and compensation problems. Comparing the results of simple inversion problems with those of simple compensation problems shows a close relationship suggesting that some common process is involved and that the two laws develop in tandem . To fully understand and develop competency in solving inversion and compensation problems, will be a slow drawn-out process but young children do have the ability to solve addition and subtraction problems using nonperceptual and noncounting procedures.

As with the acquisition of language, young children spontaneously develop an understanding of number and acquire counting algorithms and solutions to basic arithmetic problems. Research shows that some number abilities are natural human abilities that develop from the young child's knowledge of number words demonstrated by counting and an understanding of number conservation. Children therefore start school with considerable abilities in the area of simple addition and subtraction both in concrete and hypothetical situations but what the child cannot do is express his/her skills through the formal and context-free code of arithmetic. In the first years of school the primary objective for mathematics education is therefore to find ways in which the formal code of arithmetic can be introduced to the child in such a way that it is built onto the informal, context-bound skills and concepts which the child already possesses. For Hughes (1986), Brush (1978), Young-Loveridge (1989) and Carpenter & Moser (1984) the question remains as to why the primary school mathematics curriculum fails to capitalise on the rich informal mathematics that children bring to the classroom. Brush (1978) suggests that teachers should assess the level of a child's knowledge of arithmetic operations and indicate the areas of a child's difficulties by getting him to carry out a group of tasks. In this way the teacher would be guided toward an appropriate teaching strategy for each child.

2.4 Multiplication and Division

Very little research has been directed at the pre-schoolers use of the operations of multiplication and division but it is presumed that this understanding only develops after the understanding of addition and subtraction.

Gelman & Gallistel (1978) believe that the multiplication operation is slowly introduced into the numerical reasoning scheme through a long and variable developmental course that is structured by the influence of endogenous and exogenous developmental forces. The endogenous force is influenced by the demands the counting procedure makes on memory which results in the invention of a set of 'tag-generating' rules that represent large numerosities as products and sums of smaller numerosities. Fifty represents "five tens" which is the product of five and ten. In this way the limitations of memory and the conflict between the requirements of the counting system, result in the endogenous developmental forces inventing the multiplication operation. Situations in many different cultural environments also lead toward the use of multiplication operations. When there is a need to repeatedly count large sets such as the number of cattle in a field the likelihood of making a error or of losing one's place is greater. This leads to the operation of breaking up the set into smaller set sizes that could be counted accurately and then the number of sets containing x items each could be counted. Exogenous pressures would depend on the extent to which cultural environments influence young children. Factors such as currency transactions, groupings of people, food growing or purchasing animal herding etc. would all put pressures on children to discover the operation of multiplication. Throughout history the influence of trade in that culture has played an important part in the development of algorithms for determining multiplication, however today this is largely determined by the availability of schooling.

The development of the understanding of division is closely related and dependent upon an understanding of multiplication. The operation of division has always posed considerable difficulties even for the most able mathematicians. Being the inverse operation of multiplication, would make it more difficult to understand and would account for the fact that designers of curricula for the teaching of mathematics first ensure a clear understanding of the operation of multiplication before division is introduced. Gelman & Gallistel (1978) hesitate to comment further as they admit to knowing little about the psychological makeup of an understanding of the operations of multiplication and division.

However, Desforges & Desforges (1980) look at the relationship between early sharing behaviour and the more complex mathematical idea of division. They question the theory of Williams & Shuard (1970) which insists that social sharing is not mathematical sharing. Likewise they question the ideas of Copeland (1970) that multiplication and division should be taught simultaneously once the child has achieved 'reversibility of thought' and only after lots of experience with other number operations.

Because young children aged three years are able to conserve number providing the set size is small, recognise and use arithmetic operations and distinguish shape, size and colour, it may be argued that further progress in acquiring a more generalised notion of number conservation will develop from practice in contexts of limited set

size. It is also a known fact that very young children are able to participate in sharing acts and understand what sharing means. What is not known is how far set size influences their mathematical understanding of this procedure and how far a 'social sharing' procedure is understood in terms of it being a mathematical procedure.
In a study carried out by Desforges & Desforges (1980) young children aged 3½ to 6½ years were asked to share a number of objects between dolls and then given two conservation tests. From the sharing activity three main strategies were noted. The first involved distributing the set one by one between the dolls until all the objects were used up . The second strategy involved an attempt to divide the whole set into equal portions and give one portion to each doll and the third strategy was used by children who shared the set using small groups of two or three rather than one at a time . Each of these strategies could be divided into two types according to how the children assessed the numerical value of the subsets. Type one appeared to make no attempt to check or count as the sharing took place but simply dealt out the objects and assumed that dealing would lead to a fair process answer i.e. there was no reference to number or numerical checking by these children. Type two used the same strategies but the whole process was accompanied by careful checking and counting thus showing an overtly number based idea of sharing.

From this study the results showed quite clearly that conservation is not a necessary attainment for the development of a number-based idea of sharing as the younger group of children were non-conservers and yet predominantly used a number-checking strategy. However, the effect of set size was significant for the non-conserver in the young group. An increase in set size definitely increased the problems for these children. For the older group the set size made little difference.

The strategies used for dealing with remainders provide further insight into the understanding of children's comprehension of sharing. Some children asked for more to complete the share and make it even while others removed the excess. Another child suggested breaking the remainder in two or three in order to equalise the sharing while others simply ignored the remainder or added it to a group unaware of the unequal share. The strategy used by the youngest children was to ask for extra to make up fair shares and the oldest children always set the remainder aside.

Although there were differences in ability according to age, even the youngest group showed that they had some number-based notion of 'sharing' and knowing what was fair which could be related to the process of division. The older children demonstrated that without formal instruction they have a good grasp of sharing up to 30 amongst 2,3 and 5 with or without remainders and a clear understanding and approach to the process of sharing. .

A more recent investigation into the sharing skills of young children and the understanding of number equivalence was carried out by Frydman and Bryant (1988). They suggest that the proficiency shown by 3-year-olds in the studies of Desforges and Desforges (1980) is impressive, but that one should be careful about any claim that the children's successes demonstrate an understanding of the relation between one-to-one correspondence and quantity. This repetitive action which they have learnt from others may only be a drill which they apply without any understanding of its quantitative value. If children have an explicit understanding of the quantitative significance of sharing, they should be able to state the number of items in one shared set when they know the number in the other. Sharing would then be understood as a way of achieving numerical equality. Another way of testing this knowledge is to see whether they are able to adjust what they do when the quantities have to be shared in single units to one person but in pairs to the other.

Frydman and Bryant tested 4-year-olds and 5-year-olds to ascertain whether or not they connected sharing with number and their understanding of how to cope with units of different quantities in a sharing task. They were able to confirm that young pre-school children are able to share discontinuous material most efficiently using a form of temporal one-to-one correspondence. However, most of them are not able to deduce the equivalence of the respective cardinal values of the shared sets. These young children were neither able to adjust the way that they shared when they had to deal out units of varying quantities. The 5-year-olds on the other hand could cope with units of different quantities very well and were able to incorporate numerical information with temporal one-to-one correspondence. For Fryman and Bryant these results showed an early understanding of and a good grasp of the quantitative significance of temporal one-to-one correspondence, at an age when they are reported to have difficulty with the traditional tests of spatial one-to-one correspondence. It is interesting to note that 4-year-olds can be helped to incorporate number with sharing when colour cues are used to emphasise the use of one-to-one correspondence. In this way these children became aware of the quantitative significance of the difference between the units to be shared proving that they do have a basic understanding of the one-to-one correspondence but that they need guidance when a discrete quantity is changed from one object to two or more.

Now the question is asked: "Do young children first learn to share as a mere drill and with experience move to a genuine understanding of one-to-one correspondence?" or "Do children only adopt sharing as a result of some prior understanding of temporal one-to-one correspondence?" Whatever the answer is, it is quite clear that the common activities of sharing are important aspects of the study of the child's growing application of number and quantity.

2.5 Number Representation

2.5.1 Reading Numbers

Very young children observe and develop ideas about the many different aspects of texts they see around them and are able to distinguish number-shapes from letter-shapes (Lavine, 1977). Sinclair & Sinclair (1984) question whether children who are able to interpret a written representation of a number have any idea of what is being represented. For the child to identify a 3 as 'a 3 ' may only involve a process of naming an object and therefore tell us nothing about the child's basic knowledge of number concepts. Their research was to discover how young children (aged 4 to 6) interpret the written numerals they see around them and that this interpretation need not be linked to their skill at identifying and naming the various graphic shapes. Children were asked to read numbers on various objects in their environment that were familiar to them such as the numeral on a birthday cake, a bus stop sign, house number, runners T-shirt number, car licence plate and number in a lift. Their responses were classified according to how they interpreted the meaning of the numeral rather than their knowledge of the number shapes. Responses ranged from no response to a description of the numeral with no idea of it's function, to an understanding of the context in which it appears with a vague meaning but a rather unclear and general idea of it's function. Finally there is the response that shows that the numeral has a specific nature and serves to determine one possibility among others, i.e. the information obtained from the symbol directly deals with quantity, order, classification or grouping or one-to-one correspondence. From these responses they conclude that there is no sudden development from an understanding of the general ideas about the function of written material to a clear interpretation of the two different writing systems of numerals and letters and that numerals always give a certain kind of information namely quantity and value. Rather that this development is linked to the child's development of number concepts and alphabetic writing which goes together with the child's emergence of new ideas about spoken language. However children with no formal instruction in reading, writing and arithmetic are able to understand the specific nature of information provided by numerals and this ability is quite clearly established by the time they start school at age 6 years.

2.5.2 Writing Numbers

Although children are only introduced to written numerals when they begin school, they appear to have their own written representations of arithmetical concepts . Hughes (1986) looked at how young British children responded to representations of quantity. He asked children to "put something on paper" to show how many bricks were present. It was found that the variety of responses could be divided into four main categories: idiosyncratic, pictographic, iconic and symbolic.

Idiosyncratic responses were the children's representations that showed no signs of relating in any way to the number of objects present. These responses may have been meaningful to the child but were meaningless to the tester. In this type of response the most common representation was to cover the page with scribbles or to draw pictures of irrelevant objects.

A slightly more logical response was the pictographic where children tried to represent something of the appearance of what was in front of them as well as its numerosity. In this type of response the child indicated the shape, position, colour or orientation of the bricks. A typical pictographic response was to draw the bricks freehand or to place each brick in turn on the paper and draw around it. Perhaps this was a literal response or simply an attempt to be accurate.

Similarly the iconic response was based on one-to-one correspondence, but now the child uses a system whereby discrete marks of their own devise represents each brick. These responses took the form of simple tallies or other shapes like circles or houses which each represented a brick so while the individual elements may differ and are of no importance, the response to the task is correct in expressing the numerosity of the group.

The symbolic response was a representation of the number of bricks using numerals or number words.

Hughes (1986) reported that the method of response was fairly consistent so that if their first response was pictographic they would usually continue in this way for the other quantities. There was however a difference in the methods used by each age group. Three and four year olds favoured iconic and idiosyncratic methods while five and six year olds were more likely to produce pictographic and symbolic responses. Only once children have been taught arithmetic symbols at school, do these become the common type of response. Amongst pre-schoolers there was a high percentage of iconic representations, focusing entirely on number with no information about the type of object being represented but rather with the emphasis on whether the object was present or not. There seemed to be a link between the use of tallies and the widespread use of fingers to represent objects.

Accuracy was not always accomplished and children sometimes miscounted the bricks or lost the one-to-one correspondence but found it easier to work with small numbers (1,2 and 3). As to be expected, the older children were more accurate than the younger ones.

Hughes suggests that "any mode of representation, if used systematically, can be considered an acceptable written representation of number" (Hughes 1986, pp.61). Children may construct an idiosyncratic system which is meaningful to them but to the adult it appears not to show any resemblance to a number representation. However all these methods of representing number show a way of conveying information about number and often give additional information about facts such as shape, size, and colour. Symbolic and iconic systems usually tell one nothing about the objects being counted. An interesting exception to this rule was the child who used a symbolic system to represent the number of bricks presented but wrote the numerals in a vertical pattern to show that the bricks had been placed in a tower. In this way he had adapted the symbolic system to incorporate both iconic and pictographic elements.

A similar experiment was carried out by Sinclair, Siegrist & Sinclair (1983) with slightly different results. Children's notations were classified into six different categories. Forty-five children aged 4 to 6 who had had no formal instruction were asked to represent on paper the number of identical objects from one to eight that were displayed on a table. Unlike Hughes' research above, only one four-year-old produced uninterpretable notations. Notation-type 1 termed a global representation of quantity described the type of notation that neither represented the kind of object nor

the cardinality of the set but simply displayed a line of bars, hooks or squiggles of indeterminate number for all items with a cardinality of more than one.
Notation-type 2 showed an attempt to represent the object-kind without any indication of quantity. Children produced a drawing of the object displayed but gave no attention to the quantity they were expected to notice. Notation-type 3 was the same as the iconic response described by Hughes with a one-to-one correspondence where each object is represented by one abstract graphic symbol. A similar type of notation was the one-to-one correspondence with numerals replacing the abstract graphic symbol and either written down as 1234 for four balls or the cardinal value is written down the same number of times as its value; 4444 for four balls. The final two types of notation represent the cardinal value with one written numeral or the written numeral and a word or drawing to specify the object-kind. Sinclair et al. found that many children used several of these notation-types whereas Hughes reported that children were consistent in their method of response.

Young children find it difficult to respond when asked to represent the absence of quantity or nothing. When asked to put something on paper to show that there were no bricks on the table children found it hard to understand and could not see the purpose of the request but nevertheless they responded with a wide range of interesting representations. Those who used symbolic methods to represent quantity also used the conventional symbol '0' to represent the absence of bricks. Children who had used the iconic and pictographic methods invented their own symbols such as a dot or dash or an empty box or by leaving the page empty or by using the conventional symbol '0'. It was not clear, however, just what meaning, if any, these responses held for the children themselves.

To ascertain the meaning that children place on their representation of number and to be able to discover what they understand about what they have done, Hughes (1986) devised a game using tins containing different numbers of bricks. Initially the children had to guess the number of bricks in each tin and then it was explained that to be more accurate, the number of bricks in each tin could be written on the tin. The children did this themselves and discovered how their representation helped them to play the game and showed that the representations gave meaning to their choice and that they had a clear understanding of what they had done. When playing the game, children's representation of quantities differed from the previous study in two ways. Firstly, children found it easier to represent zero. Because the tin contained no bricks they would leave the paper blank or draw an empty tin or write a dash. Although the children seemed to regard the representation of no bricks as not any different from the other representations, it remained uncertain as to whether the representation of zero had meaning or whether it simply was an identification by means of default: that is, having identified definite quantities in the other tins they would know that the remaining tin contained nothing. Secondly, pictographic responses were less frequently used perhaps because the children knew that they had only to discriminate between different numbers of bricks and therefore the desire to represent other features of the bricks or tin was of less importance. Children realised that it was possible to represent the number of bricks in the tins by drawing the appropriate number of any object and this kind of response drew a wide range of solutions. If the children's representations were easily recognisable by an adult, they were generally successful themselves at identifying the tins. Most children who gave idiosyncratic responses were not able to recognise them but there were a few exceptions from those who seemed to give meaning to the mark they had put on the paper and were even able to recognise them a week later.

The question now arises that if children are universally able to represent number in the written form at a young age, and have a good understanding of the meaning of their representation, why do they have such a problem transferring their own written representations of simple arithmetical concepts to the new symbolism expressed in the abstract language of arithmetic? This process involves the child translating his concrete understanding of number that he has when he starts school to the new written form of representation using symbols of arithmetic - a difficult task for young children. Research has shown that pre-school children are able to represent small quantities and that their representations are based primarily on one-to-one correspondence involving counting procedures applied to real objects.

Hughes (1986) suggests that these findings have a number of important implications concerning the way we introduce written symbolism. There seems to be no connection between the child's representation of quantity and the system of symbols that he is required to learn. Ginsburg (1977) finds that young children often fail to understand the necessity or rationale for written methods which are imposed on them in school and they are required to use them. To understand arithmetic children need to be able to translate their knowledge of the concrete to the written representations of arithmetical problems. Games played with pre-school children can be an excellent way of introducing arithmetical symbols to children in contexts where the meaning and usefulness are immediately clear and comprehensible. Games will also encourage children to translate from the symbols back to the corresponding concrete situation whenever the need arises.

2.6 Estimation

How do young children estimate the numerical value of an array and does the ability to estimate accurately show mathematical knowledge and cognitive skills or is this a foreign concept only accomplished by guessing?

There are many situations where estimation rather than precise measurement or calculation is required and for this reason children need experiences to familiarise themselves with approximation. This will give them confidence to use their judgement to comprehend a problem. "Estimation is a process: it involves comprehending the problem, relating it to information that is already known, judging and verifying reasonableness and revising as necessary" (Harte & Glover 1993, pp.75). Estimation is therefore a mathematical way of thinking and communicating which needs to be encouraged as changes take place and symbol manipulation can be done so efficiently by machines. Estimation involves the use of higher-order thinking skills to solve problems by exploring number and spatial reasoning in real life situations.

Gelman (1972) distinguishes between the terms estimators and operators. Estimators are the processes that can be used to obtain a quantitative representation of a set. Operators are the processes that define the results of manipulating sets in different ways. Both processes involve an understanding of the problem and an informed judgement about the approximate numerical representation of the array. For Gelman and Gallistel (1978) these terms do not convey an accurate meaning of what it is we intend to incorporate in our working concepts. Estimate implies an approximate representation but we need to account for an exact representation as well which is why the term 'number abstractor' is preferred. Similarly the term 'reasoning principles' is substituted for operators to include not only deductions that involve operators but also those that concern the relations that hold between sets, i.e. the relations of equivalence, non-equivalence, and greater than or less than.

It has been shown that young children aged 3 and 4 years can accurately estimate the numerosity of set sizes of one to four (Smither, Smiley, & Rees 1974). For sets of five and beyond, the accuracy of numerical judgement falls off markedly. Gelman and Gallistel (1978) are challenged to investigate the factors which determine the child's ability to abstract number and the reasoning principles used to make numerical representations. To what extent is the child influenced by perceptual cues of length, density, arrangement of the array and heterogeneity of objects and duration of exposure?

Gelman and Tucker (1975) asked children aged 3 to 5 years to indicate how many 'things' they saw on a card displaying either homogeneous or heterogeneous set sizes of 2, 3, 4, 5, 7, 11, and 19. Each set size, either homogeneous or heterogeneous, was presented three times, once for only one second, once for five seconds and once for one minute. They found that young children performed best overall when the exposure time was longest and that the homogeneity-heterogeneity variable had no effect under any conditions. This may have been affected by the type of materials used and the procedure as these results have been disputed by others. In another study by Gelman and Tucker (1975) evidence was given that children's ability to cope with heterogeneous arrays can be affected by expectations. When one item from a homogeneous set was changed and replaced with an item of a different type, 3 and 4-year old children said that the numerosity had changed. However, they saw no change in numerosity when they were presented with a heterogeneous array and then with a homogeneous array of the same number.

The set sizes used by Gelman and Tucker gave interesting information about the accuracy of numerical estimation of pre-schoolers. As previously stated, their accuracy of numerical estimation falls off as numerosity becomes larger than 3 to 5 but given sufficient time when estimating these larger sets, pre-schoolers as young as 3 do better than chance when estimating numerosities as large as 11. When analysing the estimates given for larger sets, it could be seen that pre-schoolers use terms of number words that come later in the list of counting words showing that they have some idea of the fact that the serial list of number words represents larger and larger sets. These results also determined that they have the ability to differentiate set sizes larger than 5 and are able to trace the ordinal properties of set sizes to graded position in the order of number words even if they are not able to assign 'the' number word that accurately represents a given sets size.

This evidence of the young child's ability to represent larger set sizes by number words that come later in a serial list, prompts us to ask what processes bring about the child's ability to represent numerosity? Do pre-schoolers count to represent number or is there a perceptual mechanism often referred to as subitizing that enables them to accurately represent small numbers? Have young children not developed the ability to reason about number and therefore do not understand that transformations do not change the numerosity of a set? (Piaget, 1952).

Gelman & Gallistel (1978) argue that the young child has cognitive competence to carry out higher mental processes and cannot be compared to some birds, animals and primitive tribes who are able to recognise the differences among numerosities of small numbers by seeing the pattern as a whole. For Gelman, the magic experiments provide proof that the young child spontaneously counts to represent a given small number before taking advantage of a subitizing or perceptual grouping process. Gelman & Tucker (1975) report that young children are more accurate when estimating small sets when the conditions favour their chances to count, that is clearly displayed items and a longer exposure time. It is only after there has been practice at counting that children skip the counting process and use the advanced organising role of subitizing. Number is the salient cue. Young children are sensitive to number differences before they can make accurate number judgements and this process develops in a continuous orderly fashion. The use of cues of density and length depends not only on age and the importance of those dimensions but also on the magnitude of number and number differences (Smither, Smiley & Rees, 1974). Provided the array is sufficiently small, so that the child can accurately estimate its numerosity, then number will be the important cue and length will not necessarily influence the child's judgement (Lawson, Baron, & Siegel, 1974). Young children appear to lack all the necessary basics for distinguishing length and /or number from size and seem to apply the rule that when the numbers are beyond estimation range they use length for quantity but when numbers are within estimation range they use number for quantity (Siegel 1974).

Number judgement is a process that develops from the young child's focus on perceptual cues of length, area and density to the ability to count and abstract number as a dimension with the emphasis on cardinal value (Piaget 1952, Fuson & Hall 1983, Siegel 1982). The very young child often finds that the perceptual characteristics of very small arrays give accurate answers to the numerosity of a set (subitizing), but these early and successful subitizing experiences must later be 'unlearnt' when larger arrays of numerosity are judged and procedures of matching and counting are used.

Wright (1994) points out that one of the ways to understanding the complex conceptual structure of number is through the recognition of figural patterns and subitizing. Children learn to co-ordinate the names of number words with the sequential tagging of perceptual items and by imitation and reinforcement the response to the question 'how many' is learnt by referring to the 'one, two, three' names which emphasise the last number word in the sequence. Another way is to experientially acquire the number by recognising the character of the conceptual system without having any idea of the concept of number but simply through the manipulation of perceptual patterns. This notion implies that patterns and their number value develop independently of counting.
Wright goes on to explain that young children may also count spatial or temporal patterns to evaluate their numerosity

Harte and Glover (1993) see estimation as a way to encourage children to think mathematically and to explore number in everyday situations. "In the process of learning and practising estimation skills, our first grade students explored additional mathematics skills such as counting, place value, measuring and spatial reasoning; they actively and enthusiastically prepared for real-life problems." (pp75). Yet Gelman and Gallistel (1978) believe that the child's arithmetic reasoning is closely related to the representations of numerosity that are obtained by counting and see no value in the representations obtained by direct perception.

2.7 Classification

Does the child's ability to classify, categorise or sort items demonstrate his competence to think logically and display mathematical skills or is it dependent on available knowledge and the representation of that knowledge? In what ways does the method of presentation affect the child's ability to classify and is there a need to consider the nature of the stimuli?

From a very young age children learn to recognise and name the various objects in the world in which they live. From experience and observation these objects are recognised on the basis of certain physical properties, such as colour, size, shape or certain patterns of behaviour and through sensory perception they are classified into categories according to their unique characteristics or properties. This forming of concepts derived from their properties and relations is developed through perception which is the original source of cognition (Metlina, 1991). Young children soon learn the first stage of classification when sorting familiar objects such as toys, books, clothing, etc. and grouping them according which belong together. "The idea of sorting or classification is based on the idea of a relation" (Copeland 1979, pp.63). Whilst learning about the world in which they live, children investigate objects and perceive the criteria that enable them to solve simple classification problems. From this ability to single out qualitative attributes comes a shift to analysing the quantitative relations among them. Gibb (1975) finds that once the child is able to classify object and see the collections formed as entities, then he can classify sets of objects as equivalent or non-equivalent and order them on the basis of their numerosity.

Before the child can associate number with these processes, he must disregard the identities and attributes of the objects. Experience in classifying therefore provides the necessary groundwork for the later understanding of number in its abstract form.

According to Gibb (1975), the very young child first classifies when he accurately names an unfamiliar object belonging to a familiar identity class. For example, when given a new kind of toy he may never have seen before, he may readily identify it as a toy. In this way he almost automatically classifies every new object he perceives. Other classifying experiences are planned and presented to the child as a sorting task by an adult who sets the criterion, e.g. "Let's put all the socks in this drawer!" Therefore even if children do not have the precise language with which to label objects, they are nevertheless able to put together things that are alike or that belong together. The only knowledge that is needed is an understanding of the words 'put together', 'alike', and 'belong together'.

Piaget terms this the 'pre-classification stage' because children do not have the language skills to classify according to certain criteria but simply sort objects according to their visual form which gives them a 'graphical collection'. This cannot then in the true sense of the word be called a classification but rather a collection (Sime, 1973). These simple classification tasks are therefore solved only by perceptual structures which depend on sensory-motor schema rather than on forethought. So it is that through play a child lays a substructure for seriation and hence for logical thought. Stage 2 called 'quasi- classification' begins when children first enter the primary school and lasts for about two years. Now children are able to classify in the simplest sense of the word, that is they can sort elements out into their major classes such as colour, shape, and size. but they cannot see small classes within large classes. When children are shown a string of wooden beads they are not able to decide whether there are more wooden beads or more red wooden beads. For Piaget it is only in stage 3 that true classification takes place. Later during junior school life most of the complicated skills of classification are acquired. Now the child no longer relies on the immediate impact of visual form but is able to cross-classify and therefore uses logical reasoning to solve the problem.

For Piaget (1952) true classification is only acquired once the child is able to reason logically - a skill which is only developed in later junior school life. Gelman & Gallistel (1978) report that classification tasks require the child to learn to sort a set of stimuli according to attributes that the experimenter defines as correct and not be distracted by irrelevant attributes. There is much evidence to suggest that younger children seem less inclined than older children to focus on relevant information and this may explain why they have difficulty with discrimination tasks that involve several irrelevant dimensions. However, while watching pre-school children Gelman & Gallistel (1978) realised the importance of embedding the experimental task in a game that appeals to children of this age group and one that would maximise the likelihood of the child understanding what the experimenter wants him to do. They give the example of an occasion when a 2½ year-old child was shown toys varying in colour, shape, function, material etc. and he immediately picked out the red toys to play with but when asked subsequently to 'put together the ones that belonged together' the child did not respond. Did the child not understand the question or was it not a game the child wanted to play? Although the child had spontaneously shown that he was able to classify materials, he responded to the experimenters request with behaviour that has often been interpreted as an inability to classify. They give examples where the task is embodied in a detective game which makes it easier for the child to understand what the experimenter wants him to do and motivates the child to solve the problem. Young children are more likely to verbalise their thoughts and account for the criteria they are using to classify objects when tasks are designed to captivate the child's interest and encourage participation.

Besides designing the task to suit the child, Rosch (1976) points out that the child's ability to sort objects is also dependent on what types of objects they are asked to sort. Her work suggests that children are more likely to use consistent criteria if the sorting task involves 'natural' categories rather than arbitrary categories typically used in such tasks. Natural categories reflect real-world correlations that rely on basic levels of abstraction as these are most easily understood by children. The basic level will be the one that provides the most information with the least cognitive effort - a level at which objects share the most attributes that are relevant to humans.

To test whether children categorise basic objects of the natural category more readily than other objects, Rosch investigated the sorting of these objects into basic categories and into superordinate categories. Children from kindergarten to fifth grade were asked to sort pictures of objects that would fit into superordinate and basic categories. The superordinate categories were as follows: shoes, socks, shirts, pants (clothing), tables, chairs, beds (furniture), cars, trains, planes (vehicles). Children in the basic sorting condition received four clear pictures of one basic object from each of the three superordinate categories for example four tables, four cars and four pants. The results showed that the older children consistently used the superordinate criteria, while the younger children would do about as well on the basic-level sorting task. These results again emphasise that pre-schoolers are able to sort stimuli according to consistent criteria when consideration has been given to the nature of the stimuli used in the classification tasks.

For Chi (1983) the young child's inability to classify does not show a lack of classification skills nor a problem of access nor the lack of a hierarchical representation but rather it demonstrates the child's available knowledge and the ability to represent that knowledge.

The well-known findings on classification, categorisation and sorting tasks clearly indicate that : a) young children categorise on the basis of perceptual or concrete properties whereas older children categorise on the basis of abstract or functional features, b) younger children's categorisation shows no hierarchical representation but is shallow and linear, whereas older children categorisations are more hierarchical and c) young children use inconsistent criteria when sorting and older children are more consistent. To understand these findings and to explain their order of acquisition evidence shows that children's categorisation results are determined by the knowledge that they have and the representation that the knowledge takes, and not necessarily influenced by the lack of access, the lack of hierarchical representation, nor the lack of competence.

Chi (1983) demonstrated this by testing the classification skills of novice and expert children and comparing these results to see how they related to age differences. Secondly she investigated individual children's representations to compare the subjects' performances under two different stimulus conditions.

Young children's inability to classify in a class-inclusion manner has often been attributed to the lack of the notion of access. This assumes that the necessary knowledge is there but it cannot be accessed. Children are given secondary tasks such as a) asking young children to put all the members of a category together, or b) asking children to confirm that a statement such as 'A dog is in animal' is true or not, or c) finding out if they can ascribe attributes of a superordinate (dog) term to a nonsense word (such as 'fob') if children are told that fobs are dogs. Evidence shows that young children can do all these tasks successfully showing the presence of hierarchical class-inclusion representation and thus the idea that the inability to exhibit class-inclusion during classification is thought to be the result of limited access. Chi however claims that these secondary tasks assess only individual links or pieces of knowledge and not the entire interrelated knowledge structure. That the knowledge is there but not accessible for the task of classification is not valid when based on these secondary tasks. Another reason for young children's failure to classify in a class-inclusion way is when sets of stimuli are taken from the superordinate level and not the basic level. Rosch's results showed that basic level objects are those that are perceptually similar to each other and because young children can sort according to perceptual features they are successful at these tasks.

Chi's research found circumstances under which young children could demonstrate consistent, adult-like, hierarchical classification at the superordinate level. The classification skills of two groups of 7 year olds were compared and contrasted. One group of children had a large quantity of knowledge about dinosaurs and novice children had some idea of dinosaurs but could not identify any dinosaur correctly by name. The children were each presented with a set of 20 dinosaur pictures and asked to sort them into as many groups as they wished. The novice children sorted according to perceptual differences looking at the visual features of the dinosaurs thus forming basic-level categories and the expert children formed superordinate-level categories corresponding to categories such as the Duckbills, which are dinosaurs which have bills that look like ducks. The children's explanations for the groupings supported this interpretation. Again this supported the findings of Rosch that novice children would be able to categorise at this basic level. The expert children sorted them into functional or abstract features such as whether they were plant-eaters or meat-eaters and not according to perceptual features. Chi therefore provided evidence to prove that when the knowledge is available, as in the case of the expert children, they could classify hierarchically at the superordinate level and when the knowledge is not available, as in the case of the novice children, children of the same age tend to classify at the basic level, relying mainly on perceptual features. This emphasised the fact that classification skills rely on knowledge which is already organised in such a way as to allow a retrieval of this organisation and is not a particular intellectual skill developed with age as Piaget would have us believe.

The second test looked at an individual child's representation of a familiar domain. A 5 year old was asked to represent the class mates in different categories. Evidence was that children could be classified accurately into groups such as boys and girls, first-or second- grade or subgroups of names such as all the second-grade boys. However groupings did not conform to the orthodox representation but were based on the seating arrangement of the class. Nevertheless the classification was meaningful, hierarchical, consistent and available. These findings emphasise the fact that we must not only be concerned about whether the knowledge is available, but also whether it takes the orthodox form that the experimenter expects or an equally correct representation of the child.

Lastly Chi demonstrated that classification is not a skill that develops with age and intellect, but rather an activity that is a display of the knowledge and its representation that a child has. A 4-year old child, who was very knowledgeable about dinosaurs, was asked to categorise two subsets of dinosaurs, 20 in each subset. Although both subsets were well known to the child the one contained dinosaurs that were more familiar to the child. Categorisation of the more familiar subset was identical to those of the 7-year old experts previously mentioned who sorted according to criterion of meat- or plant-eating dinosaurs and the classification remained consistent and stable across the two trials. The other less familiar subset was sorted using inconsistent set for criteria, ranging from the diet to the habitat, to the locomotion. Over the three trials there was no sign of stability. The results of the latter could be construed as the classical developmental finding that young children use inconsistent criteria in sorting, but not in this case where the changes are shown by the same child in two subsets of a given domain that differed in the child's familiarity with each. The skill shown is not a fundamental ability that either exists or not in the child's repertoire, but rather, a characteristic of the particular representation that the existing knowledge takes.

These findings question the classical idea that young children have limited knowledge of classification and sorting because they lack access, classification skills, and hierarchical representation. If one accounts for the availability of content knowledge, together with an appropriate representation, it will be found that young children exhibit competencies that were previously only attributed to older children.

2.7.1 Seriation

We have seen how young children sort objects according to perceptual features recognising their similarities with ease. It is thought that throughout our lives we recognises differences before we recognise similarities and perhaps this accounts for the ease with which young children seriate with blocks.

Like classification tasks, the skill of seriation relies on the ability to sort objects according to their visual form. Because of this similarity Piaget (Sime 1973) found that young children develop the two skills at approximately the same time. Visual form has a strong impact on young children, motivating them as early as eighteen months to build a tower of blocks with descending sizes. This activity is dependent on sensory-motor schema rather than forethought but forms the start of seriation. So it is that through play the young child lays the foundation for seriation which leads to logical thought.

Copeland (1979) claims that young children are only able to seriate small numbers of objects and find difficulty in seriating as the number of objects becomes larger, or as the differences in size become slight. By age six or seven, the concrete operational level, children have developed a systematic way of solving the problem by using the logic of 'reversitivity' and 'transitivity'. Reversitivity is the ability to recognise that each stick is both longer than the preceding one and shorter than the one to follow. Transitivity means that they realise that if the third stick is longer than the second and the second is longer than the first, then the third must be longer than the first.
Seriation or ordering will therefore involve the ability to co-ordinate the relation of each object to the one before it and the one after it which according to Copeland children aged 4 to 5 are not able to do. At age 5 to 6 children have an intuitive idea of the series but construction is mainly by trial and error.

This early experience of classification and seriation forms a basis for intellectual growth and logical thought. The sensory-motor experiences involving relationships and correspondences lay the groundwork for the future operation of logical thought.

The place of language in this development must be stressed as it helps to accelerate both classification and seriation and improve accuracy. However the ability of very young children to handle these two tasks with competence emphasises the strength of perceptual knowledge available to them. They are able to put together things that are alike or that belong together before they have the precise language with which to describe or label the likeness of the objects. The only language that is necessary is the understanding of the concepts 'alike', 'put together' and 'belong together'. Classifying will develop language by encouraging children to describe the properties of objects and think about their utility and composition.

2.7.2 Patterning

The ability to recognise patterns of real objects, pictures and drawings is basic to mathematical insight. At first this will rely on perception to repeat or extend a pattern but children need to be encouraged to describe patterns and explain why they think a particular object comes next. Language ability plays an important part in this type of task and for this reason patterns need to be made up of concrete objects, pictures, and symbols that are familiar to the child. Patterns need not always be presented in linear form but must be strong and uncomplicated.

2.8 Space and Shape

From a very early age children explore space by visually and kinaesthetically experimenting with the everyday objects of their lives such as rattles, bottles, toys, etc. Although they may not consciously express what they see and feel, these experiences develop in them an awareness of the likenesses and differences in the shapes of a variety of objects, their size, and the object's position in space among other objects and in relation to themselves (spacial orientation). The child's understanding of space develops more rapidly as he adopts a vertical position and begins to move on his own. With this practical experience of space comes the gradual learning of words to express these concepts. A system of reference is formed which is based on how the child sees his body in relation to the object being observed. Leushina (1991) indicates that by the pre-school years the child has acquired a verbal reference system based on the fundamental spacial directions: forward- backward- up-down- right- left. Research has shown that children first relate directions to parts of their own body thus establishing a regular association such as 'up is where the head is, and down is where the feet are'. Children first master spacial relations by orientating them to their own bodies. The idea of space is developed through experience of direct movement within space. It is only through motor stimuli that visual stimuli acquire their vital meaning. Therefore as children acquire experience in spatial orientation, motor reactions expressed externally become intellectualised and form the beginnings of geometry.

As children's experiences of spacial orientation develop, they perceive space in a new and improved way. Understanding of space develops from the first orientation of themselves in an extremely limited area to a wider radius but still only locating objects in narrowly defined positions. By aged 5 years the child is able to define remote objects and the positions in a wider radius take on more meaning so that he is able to point out intermediate points in space such as front right, front left, and so on.

This orientation toward oneself is an indispensable condition in orientation in the arrangement of objects both away from oneself and away from other objects. To define the situation of objects, one always associates the surrounding objects with one's own position. Children will turn themselves around so that their bodies are in the same position before deciding that person's left and right sides. Orientation away from oneself assumes an ability to use a system where the origin of reference is oneself, but orientation away from other objects requires the calculation of the spatial situation of other objects as they refer to the particular object of reference. Here one must be able to work out various sides of the object such as the front, back, right and left, etc. These three areas of spacial development - spacial orientation toward oneself, away from oneself, and away from another object, occur during the pre-school years.

This development of the understanding of spacial relations of objects takes place through a sequence of events. At first the child perceives objects as 'separated entities' and is not aware of the connections that exist between them. Next the child begins to see the spatial relationship but the precision in evaluating these relationships is still comparative. Children still find it difficult to perceive the distance between objects or the space between the object and the reference point. The next stage sees the improvement in the perception of the spacial arrangement of objects. Now there is a more accurate evaluation of the relations among objects. This is mainly due to the fact that children have now mastered the significance of spatial prepositions and adverbs and are able to give a more accurate interpretation and evaluation of the arrangement of objects and their relation to one another.

Leushina (1991) suggests that working out the spatial relations among objects is a lengthy and complex process that is not completed at the end of the pre-school period but continues to develop during formal schooling. The child's understanding of the 'scheme of his own body' is the basis for expression of the basic spatial directions.

Again we see the importance of language development that gives children the tools of effective speech to describe the objects in the environment in their spatial relations.

Piaget (1952) suggests that the child's first geometric discoveries do not involve rigid shapes but are concerned with ideas such as separation, proximity, closure and order. The shapes the child sees are moving. His mother's face is not a rigid oval shape but a constantly changing shape as it's position moves from near to far or it turns to left or right. Copeland (1979) draws one's attention to the difference between Euclidean geometry, which includes the study of figures that could be called 'rigid' shapes and the mathematics of topology where figures are not fixed in shape but can be stretched or squeezed so that they assume a different shape. Shapes such as the square, circle, triangle and rectangle are equivalent topologically because they can be squeezed or changed to form each other and still remain as a simple closed figure. Gibb and Castaneda (1975) found that children first perceive figures topologically before they see sides and corners. A child asked to reproduce a square will draw a figure that has no corners but represents 'closedness' rather than shape. However it may be said that because of their age their response is governed by a lack of muscle control rather than by a perception error.

By examining the pattern of development, Robinson (1975) shows why these competencies can be considered to be evidence of geometric ideas. A child younger than four will reproduce a square by drawing it with 'ears' at one or more corners. The reason for this is not clear but we may guess that he notices the corners sticking out but is not sure how they should be represented. Therefore to show what he sees he draws them as separate entities. At age four he is able to draw the square more accurately consisting of four more or less straight lines and by age five most children begin at one corner (usually the upper left) and move the pencil in one continuous movement. The circle is usually drawn fairly accurately by about age three when the child can draw a circle stopping after a single revolution whereas the younger child will continue to go around and around. The equilateral triangle is more difficult to draw than either the square or the circle. Children seem to have a problem representing the diagonal stroke. From the age of about eighteen months children can represent the vertical and horizontal lines but appear to have difficulty with the oblique stroke even up to the age of six or seven. There seems to be no typical method of drawing the triangle with children using one, two or three separate strokes.

School readiness tests have shown that by age five most children can distinguish circles, squares and triangles from one another even though they may not know the names. This shows that they have the visual discrimination and are able to reproduce them with reasonable accuracy. The circle and square are usually drawn with one continues line which closes up the shape and does not retrace on the diagram. Children now also seem to have some sense of the direction of a line because the sides of the square are adequately horizontal and vertical.

Although children have some type of perceptual awareness and can reproduce shapes, they are not always able to name them or give an accurate verbal description of what is seen. Robinson (1975) concludes that by studying the changes in the drawings in which the type of error made by younger children disappears with age, one is led to the conclusion that the improvement is neither accidental nor solely attributable to better muscular co-ordination. The more accurate drawings must therefore represent a new perception that has developed with age.
"Can you teach 'shape' to young children, and if so, how?" asks Copeland (1979, pp. 99). Children learn about shapes through an interaction of the developmental processes and the experiences he has. Children should be given opportunities to handle and explore the shapes physically and not just told to look at them. Experiences of feeling shapes and moving in space will give meaning to the concept of shape as it is in the world around him.

Copeland agrees with Piaget that young children up to the age of seven are developmentally at the topological level and are not able to understand Euclidean shapes which stress the number of sides, length of sides and angles. Therefore in order to teach shapes, children should be given opportunities for physical exploration of the various shapes such as handling models of geometric objects, tracing outlines with their fingers and hands, and drawing them so that they construct an adequate mental representation of the objects. Simply to see and be told does not give understanding and true knowledge of the subject. Children need to construct their own mental ideas based on their own physical action and experiences of the objects. Geometric names can be given to the various shapes only after lengthy experimentation and exploration.

Mathematically the study of geometry provides an opportunity for the child to become acquainted with geometric properties and relations without being restricted by vocabulary. Properties such as straightness, closedness and connectedness have been experienced in the many activities but without the added complications of measurement and relations of size. Therefore by the time more formal instruction in geometry begins and a vocabulary needs to be developed, the children will already have a rich grounding of experience to associate with the words and concepts.

2.9 Measurement

Measuring as an activity or operation is one of the most frequently used number exercises in everyday life often involving physical objects in a concrete type of activity. We refer to the number of days of the week; the number of kilometres on a journey; the number of cents needed to buy something; the number of children present; the number that represents the temperature or rain level, etc. With its practical application and frequent use measurement should be an easy concept for young children to comprehend yet researchers have found that the teaching of measurement in the junior school presents many difficulties. How then do children develop an understanding of measurement and what are their ideas on the activity of measuring? What makes it such a difficult concept to perceive?

First we need to look at how a child approaches a problem situation that requires measurement. Piaget's studies gave many examples of a young child's inability to measure various everyday objects. Children given a piece of clay were first asked to roll it into a ball and then into a worm. When asked whether there was more clay in the ball or in the worm, they often gave one or other as the greater amount. Is there a misunderstanding about the meaning of the word 'more' or does this show an inability to conserve i.e. to recognise the constancy of matter over given perceptual transformations? Another experiment used two sticks of the same length that were placed along side each other. Children agreed that they were now the same length but when the same sticks were moved slightly so that the one was more to the right, the children now responded by saying that the one was longer than the other. Again we question whether the child understood what had to be compared or whether he was unable to conserve the quantity? Yet another experiment gave the child a container of water and he/she was asked to pour that same quantity of water into other containers of different shapes. The question was which container held the most water. The usual reply would be the container where the water is closest to the top. Similarly when asked to build two towers of blocks to the same height but with one on a table and the other on the floor, the report back will conclude that both are 'the same' and neither one is taller than the other. That the bases of the towers are not at the same level is not accounted for and the judgement is based on the visual comparison.

In each of these experiments the child is not comparing the 'right things' He/she has obviously not understood the basic idea of measurement. For Piaget these early stages in the development of conservation and measurement concepts are characterised by a complete inability to conserve or apply measurement processes and a total dependence on one-dimensional perceptual judgements.

If the child aged 5 to 7 is unable to conserve and therefore unable to measure should the study of linear measurement be postponed for children until they can conserve length? For Copeland (1979) there are stages through which a child develops an understanding of measurement and these are fully developed only by the age of 11 years. Before this time learning is of a perceptual sort, because measuring is a concrete type of activity but this does not qualify as a true understanding of the complex and elaborate concept of measurement. The four-year-old makes a visual estimate with no attempt to use a measuring instrument even if given one. The next stage is when the child uses a measuring instrument but incorrectly as he has no framework for comparing the two lengths or heights. When given a stick to measure the height of two towers of bricks, the child simply places the stick on top of the tower, thinking if it is level the towers are the same height. With no reference system to use he/she is unable to interpret the result and therefore prefers to use the visual perception to complete the task. He/she may later try to use his body in some way as a measuring tool, matching the tower to some point on his body and then comparing it to the other tower. Finally the child realises that his way is not convenient and he/she looks for another measuring tool that will be more accurate and easier to manipulate. Now for the first time the logic of mathematical relationships has been used.

The way children arrive at this final stage is determined by the experiences they have had. Both internal and external factors play a role in this development but the child is only able to assimilate whatever he/she is shown to his/her own schemata of representation and only remembers what he/she understands. He/she therefore only discovers the need for an independent common measure when he/she senses the difficulty of transferring sizes using his/her own body. The first measuring instrument is an object that is the same length as the tower to be measured and then he/she chooses one that is longer, marking on it the height of the tower. Finally he/she chooses a shorter rod and applies it the appropriate number of times along the tower to be measured. Only now is measurement intellectual with the operations involving logic: a subdivision into parts and a substitution of a part upon others. Measurement therefore involves a change of position either with a movement of the eye or a measuring instrument and the child must know that this movement does not change the length of the measuring instrument i.e. conservation or invariance of distance and length is fully understood.

Piaget therefore states quite clearly that young children's development of measurement notions is firmly related to the basic concept of conservation and therefore cannot be understood until the child reaches the formal operations stage at about 11 years when he/she now reasons with symbols or ideas rather than needing objects in the physical world as a basis for his thinking.

Other educationalists such as Bearison (1969), Smith, Trueblood, and Szabo (1981) and Kingsley and Hall (1967) support the role of measurement operations as a forerunner to conservation.

Zimiles (1963) has noted that the child relies less on perceptual cues once he gains proficiency with the rules of counting, cardination, and ordination because the ability to use a number system for estimates of numerosity provides for more precision, differentiation, and information which is universally used and easily communicated. Therefore it is thought that the understanding of quantity conservation is developed by the gradual emergence of a quantitative set to respond to conservation problems, and this replaces the tendency to concentrate solely on the perceptual cues of the problem. These ideas perhaps confirm the results of work done by Romberg & Gilbert (1972) to ascertain whether children's understanding of the concept of length would be increased if they were taught the concept of length as an attribute or property of objects. After three lessons the instructional experiment showed that although there were significant performance gains in some areas of the test, it was difficult to force or convince or teach young children to abandon perceptual clues.

Bearison's study (1969) aimed to identify and isolate specific factors involved in the development of conservation principles and his results showed that the numeration and comparison of single units of quantities resulted in the child's understanding and use of conservation. Measurement operations can therefore be used as a forerunner to conservation. By structuring the conservation problem within the context of their own actions with simple concrete measurement operations, the child's manipulation of the materials allowed them to recognise the principles of conservation. At first children were asked to make quantitative estimates using numerical basis for their judgements and then encouraged to count each unit so that their attention was directed to quantitative cues and the employment of quantifying operations. Through this training children began to realise that quantities were made up of their constituent units, but that the sum of these units was equal to the whole. The children therefore began to understand quantities in terms of their constituent units and were able to maintain their equivalence even when one of the two quantities was no longer separable into single elements. During the later stages children were given opportunities to compare judgements based upon their perceptual cues to those based upon quantifying operations and could then realise how deceptive the perceptive judgement could be. The experiences of these young children served as sufficient stimulus for the understanding of a generalised principle of conservation.

Smith, Trueblood, and Szabo (1981) set out to test Piaget's theory that children aged 5 to 7 would have difficulty learning to measure because they are unable to conserve length. If this was true then teachers would have to deal differently with children who are at two different stages of cognitive development i.e. the conservers and nonconservers of length. They therefore investigated the relationship between children's length conservation rank and their ability to achieve specific length measurement skills. Based on tests used by Piaget, children were classified as conservers and nonconservers and both groups were tested after each week's instruction to ascertain how well they had understood the concepts and skills that had been presented. Each week the children were made familiar with the relational terms- longer, shorter, and same length; measured block towers using non-standard units such as paper strips; given the use of a small non-standard unit to repeatedly measure the length of longer objects and the use of centimetre rulers to measure the length of objects. Children made important gains in linear measurement regardless of their developmental level or the manner in which instruction took place, thus refuting the suggestions made by some that linear measurement should be postponed until children are able to conserve length. Those children who were length conservers did not perform better than nonconservers on manipulative measurement criteria when mental ability was controlled thus suggesting that children aged 5 to 7 can be introduced to and will benefit from informal measuring activities of a practical nature.

Carpenter and Lewis (1976) questioned the value of concrete materials as a means for solving measurement problems. They suggested that concrete materials have a different meaning for children in the preoperational stage. They looked at children's identification of the importance of maintaining a standard unit of measure in a measurement operation and how they understand that the number of units measured is inversely related to the size of the unit. From this study they concluded that young children do have difficulties in dealing with measurement problems in which quantities are measured with different units of measure, but they are able to recognise the effect of changes in unit size and have some understanding of the connection between unit size and number of units. On the question of how this concept is understood, they found that children developed the idea of the inverse relationship before they realised that equal quantities were still equal even though they had measured a different number of units. This indicated that manipulations with different units of measure do not contribute to an understanding of the unit-size-number-of-units relationship and may tend to reinforce incorrect ideas of quantity. This conclusion can only be regarded as tentative. Numerical distracters may have influenced the perceptual information or the basic cognitive structures may not have developed sufficiently for the absorption of the new material.

Earlier work by Carpenter (1975) found that children are inclined to centre on a single dominant cue which can be a major factor in the development of measurement concepts. Numerical and perceptual distracters cause an equal number of errors and children are not dominated by the perceptual cues. When numerical cues are the correct ones to use, children find it easier to solve the problem than when measurement or conservation problems require perceptual cues to be used. His work therefore rejects the findings of Piaget (1952) on the fact that in all of the studies on which the researcher based his conclusions, the distracting cues were visual. This lack of experimental variability resulted in children focusing their attention on the immediate perceptual qualities of the event. Carpenter (1975) concludes that "although children have a number of misconceptions regarding the measurement process and often misapply measurement operations, measurement has some meaning for the majority of young children" (pp.11).

Robinson, Mahaffey and Nelson (1975) explore the nature of measurement, children's ideas and exploration of the concept of measurement and in so doing they hope to diagnose difficulties and plan suitable learning material. Initially children need to be introduced to the problem-solving approach where learning takes place as the child makes guesses and then tests them in the practical situation using his senses. This allows for growth in self-reliance and lets the child see that his mathematical experiences make sense when tested in the real world. In this way the child is gaining a basic understanding of the concept of measurement without being weighed down by numerical distracters. Only once this groundwork has been covered and there is a sound knowledge of the practical uses of measurement, will the need arise for more precision in comparison and therefore the introduction of numbers and the need to select the correct measure to suite the attribute of the object. "Thus understanding develops within the structures of the real world, and it is in the real world that we have need of measurement" (Robinson, Mahaffey and Nelson 1975, pp.250).

Young children are concerned with the issues of 'He's got more sweets than me!' and this drives them to reflect on strategies that assess relative quantity, volume, length, and so on. Gelman and Gallistel (1978) found that pre-school children are not too concerned with quantity when measuring, but do show an understanding of equality and the terms 'more than' and 'less than' and are aware of the quantitative nature of many measurement tasks. The understanding of terms such as largest, smallest, tallest, longest, most, closest and farthest are the introduction to premeasurement concepts (Bjonerud, 1960). He found that 80% of the pre-school children tested possessed a high degree of understanding of these terms. Young children were also able to recognise common instruments used in measurement such as clock, calendar, yardstick, scale and thermometer.

2.10 Time

How does the young child develop a "sense of time" and what is the basis for time perception? The perception and understanding of time show the way the idea of time exists in our emotion. The child needs to understand and recognise the various characteristics of time : 1) its fluidity - the fact that it is related to action; 2) its irreversibility, and 3) the absence of obvious form -it cannot be seen or heard. It is therefore a complex and difficult concept to comprehend and develops as experience is gained in differentiating time based on the activity and the way it is perceived.

The pre-school child's idea of time is based on sensory impressions or perceptions which Piaget refers to as intuitive time. A baby develops a wealth of sensory experiences without knowledge of the standards of time, for example he/she cries because it is feeding time and is content when satisfied. There is no idea or generalisation of the sense of time but only a connection with the specific activity to which it is related. The many practical activities of life help the child to develop this sense of time which begins to function by regulating activities. Later as the child is able to use logic rather than sensory data to determine the time, he/she develops an operational understanding of time.

To demonstrate this phenomenon Piaget placed two flasks of the same capacity, one on top of the other and filled the top flask with coloured water. Then at regular intervals fixed quantities are allowed to flow from top to bottom. The child is given a number of pieces of paper, each with a picture of the empty flasks on them and asked to record the level of water in each container after each flow. Next the drawings are shuffled and the child is asked to put them back in the order in which he drew them. According to Piaget (Copeland 1979), children at stage 1, from 5 to 7, are unable to correctly arrange these drawings in the sequence of events because they cannot fit them to separate points in marked time. At stage 2, 7 to 8 years, the child is now able to arrange the drawings in a single sequence. Because the essence of time is the co-ordination of at least two motions, the drawings are cut horizontally to separate the drawing of the upper flask from the lower flask and the child is expected to arrange both sections of the flask to match correctly. Only in the final stage at about 9 years of age, is the problem solved immediately showing an understanding of the operation of succession or order and the duration of time.
In all these stages children will use the method of trial and error to see if it looks right and often be successful but this perceptual technique does not show understanding. This then is the logic required to tell the time - the instinctive order of sequence of two actions - which children will learn by observing the motion of the clock hand as it measures some other action, such as going to bed.

To determine children's understanding of the duration of time that has past, Piaget (Copeland, 1979) again used the flasks and asked different questions. As the water dropped to the bottom flask, the child was asked if it took just as long for it to drop as it did to rise in the lower flask. It was found that children in the first stage based their answer on perceptual data and therefore said that there was a difference in time. The duration of time was judged on the size of the flask and the quantity of water in it. At the next stage the child uses his/her intuition and concludes that because time and pace are linked the liquid seems to run out of the top flask faster than it fills up the bottom flask. It must be realised that the child is still unable to co-ordinate duration of time with the order of events and lacks the operational thought necessary to identify the time taken for the flow of liquid. Only by age 8 or 9 is the child able to construct a time scale covering all moments and events. He/she understands that although the liquid appears to flow faster from the top flask, the time of transfer or duration of flow is the same. Again this shows the essence of time in which we co-ordinate the movement of the clock hands with the action.

Another aspect to understanding time is to be able to judge time when two actions of different speeds are fitted into a single time space. To test this children were shown two dolls at a starting line. At a signal they both hopped along the table with the one taking bigger strides, and then stopped on the second signal. Children were asked if the dolls had started at the same time and stopped at the same time. Children at stage one again based their answer on perceptual cues and therefore confuse time and space. For them the dolls do not stop at the same time and they may even think that they began at different times. At stage two they believe that the dolls started at the same time but still do not understand the duration of time and think that the one doll went slower than the other because it did not go as far. By age 7 to 9 years children have a clear idea of the physical aspect of time and immediately respond correctly.

Bjonerud (1960) found that pre-school children had difficulty recognising time when referring to a clock. About half of the children he tested were able to recognise time on the full hour. He however believed that this skill was not beyond the ability of most pre-schoolers if they were given the chance to use the clock in meaningful situations.

Copeland (1979) disagrees with this idea as he looks at the hands of the clock as they record the duration of an activity not as a single reading exercise which gives a numerical answer as the time, with little or no understanding of the concept. Copeland follows Piaget's thinking that children are unable to use watches or clocks accurately until about nine years of age. Younger children often think that the movement of the clock hands is related to the speed of the action being timed.

Therefore the development of a perception of time is reflected in two distinct but complementary ways-the sensation of duration of time and the mastering of the commonly accepted standards for evaluating time.

The development of time orientation is a difficult concept for young children to master, but instruction can facilitate this process and makes it possible to teach young children about time so that they develop a 'sense of time'.

Research has shown that children have a great deal of potential to master the different temporal notions and concepts and the ability to develop them will be enhanced when there is an understanding of their needs and an acceptance of and consideration for their 'sense of time'.

Leushina (1991) points out that children have difficulty in understanding the various concepts of time such as the duration, speed, movement, irreversibility and rhythm because it is an abstract idea which lacks any visual form. The words used to designate time are also relative rather than absolute and this further complicates the issue. Words such as today, tomorrow and yesterday refer to times that are continually moving and children find it difficult to grasp this idea. For this reason, very young children begin by relating the parts of the day to a characteristic activity for example we say 'Goodnight' when we go to bed or Saturday comes after the last school day of the week. Language also plays an important part in the development of a notion of time. It helps separate and generalise various time divisions according to their duration. The way children accurately use the terms second, minute, hour, day, week, month, year etc. depends on the basic attributes and characteristic of the content that is used to describe that particular time and these are often influenced by culture and geographic conditions such as weather and location; for example day will not always be the time when it is light or the time when a parent goes to work. Leushina believes that with proper adult guidance children can find visual means to point out the regular changes in days, seasons, etc. and that at age three or four they can be aware of the structure of a day and its duration. However in their speech children are better able to express ideas of speed and position of events in time and have more difficulty with the duration and sequence of time.

The understanding of these temporal relations increases slowly during the pre-school years and depends largely on the child's general mental and speech development. Educational psychologists stress that there is a need to assist young children in developing a 'sense of time'. Note should be taken of the three factors which affect children's assessment of time duration: the content of the activity, the degree of interest, and the age of the child. Time that is filled with a variety of events will hold the child's attention and pass quickly thus giving a shorter estimation of its length, whereas a monotonous activity will appear to pass more slowly. Likewise the degree of personal interest in the activity will also affect the time estimate. The activity that is of interest to the child will make the time pass faster and an estimation of it's duration will decrease. Estimations of time are therefore subjective and depend very much on the individual's personal interest in the task and on the richness and diversity of its content.

Leushina points out that children of the ages from 5 to 7 years show no differences in their estimation of the duration of time, thus indicating that this lack of progress is perhaps caused by deficiencies in their education. There is convincing evidence that when young children learn methods of determining duration of time, they develop a way of objectively estimating the duration of time segments of 1, 5, 10, or 15 minutes using an hourglass. This gives them the ability to develop a 'sense of time' which they are then able to use in their activity and behaviour. From this sensory experience of the duration of time comes the telling of time by means of first the hourglass and then the clock. Children's potential to master the various temporal ideas and concepts

is there but its development will depend on the experiences in life and guidance given by parents and teachers alike.
Leushina stresses that it is more important to develop the child's sense of time rather than merely teaching them to "tell" the time from the dial of a clock. This sensory perception will enable them to understand the connections between time and space; that is the longer the distance to be covered, the more time it will take. Measurement of time contributes to mathematical development through experiences of numerical representation of a 'sense of time', its duration, incessant movement, fluidity and irreversibility.

2.11 Conclusion

Earlier in this century many educational psychologists followed the ideas of Piaget, Dewey and Montessori who advocated an activity-type program for young children which encouraged physical exploration of objects to stimulate the intellect through the senses. These sensorimotor activities encourage spontaneous learning which prepares the pre-school child for the concrete operational stage from about 6 or 7 to 12 years of age and the beginnings of real logical thought. By rigidly defining the potential of children in each age group, the school curricula were set up to strictly conform with that potential thus restricting and limiting the possible development of both the slower learner and the more advanced learner. Enrichment of the programme will therefore stimulate cognitive growth but it does not attempt to accelerate the process. The curriculum should not take cognitive development for granted but should provide specific educational experiences based on the child's developmental level, to foster growth.

In many countries researchers have discovered that the cognitive potential of pre-school children is far more extensive than had been previously thought. How then could the learning programme of young children be adapted to cater for this potential?

How should instruction change to cater for this development? To make these adaptations it is necessary to understand and be aware of the rich informal knowledge of mathematics which young children possess and to know how this knowledge is developed.

Vygotsky (1985) stressed the importance of the environment in the acquisition of cognitive abilities and the need for teachers and parents to understand the basics of development which is a thorough knowledge of what the child is able to do and the level of development. Only then could the 'zone of proximal development' be implemented. This meant that the learning environment was structured so that the material presented was of a higher level than the child's development level i.e. the area between the child's development and the level of potential development. In this way instruction can be used to guide development and influence the spontaneous process. It is therefore the teacher's task to organise the children's activities in such a way that development occurs. The problem is presented and the child feels the need to solve it but does not have the new methods of action, behaviour, and thought to do so and therefore a conflict arises and this is the motivating force in development. In order for learning to occur, the teacher needs to determine just what knowledge children have so that they can introduce new material and relate it to children's experiences in appropriate ways. In this way instruction influences the child's mental development through activity directed by the teacher.

For Leushina (1991) instruction for children aged 5 to 6 will successfully develop concepts of number, quantity, relations between sets, and prove their judgements and conclusions. They develop the essential thought operations and pass rapidly from concrete to abstract conceptual thought. This however is only accomplished when instruction takes cognisance of the 'zone of proximal development' and ensures that learning is a conscious act and not arbitrary information, learnt through memorisation and without comprehension. Instruction therefore allows the child to move from 'the known' to 'the unknown' with qualitative changes in the child's mental development and conscious thought.

By discovering some of the specific number concepts possessed by the pre-school child, it is hoped that the teacher will be able to plan instruction based on the knowledge and needs of the young child and in this way encourage and foster enthusiasm for all to tread the path of number with confidence and success.

The research reported in this chapter indicates that pre-school children have considerably knowledge and understanding about number and that armed with the 'what' and 'how' of numerical development, teachers will be able to make a significant contribution to raising the levels of mathematics achievement and improving the methods of instruction. But first they will need to know the level of mathematical knowledge of their pupils and the strategies and representations used to exhibit these skills and competencies. New knowledge can then be grounded on what is already known and ensure a clear understanding which leads to cognitive growth.

Current research has stressed the importance of explaining the development of basic number concepts to determine how children solve problems, not simply to identify which problems are most difficult or how many children can solve a certain type of problem at a particular age. It is necessary to investigate the development of basic number concepts and to discover how children solve problems, the strategies they use and their interpretation of the problem, not just to record how many can correctly solve the problem. Carpenter (1980) suggests that there should be a study of the development of logical reasoning abilities to explain the development of early number concepts and then an analysis of the number skills such as counting, estimating, subitizing, comparing and matching. We need to know more about children's ability to learn and apply number skills. If research is to influence educational practice, it needs to concentrate on the selection and sequencing of content and on individualising instruction so that each child will develop concepts and processes at his own level of development. To determine the cognitive development of children, notice must be taken of individual differences and the effects of instruction. Research in mathematics education should therefore emphasise the processes and concepts that children acquire at certain points in the learning of important mathematical issues and describe how these concepts and processes evolve during the course of instruction. Individual strategies used and errors made on key tasks at each stage of instruction will throw light onto this development. Lastly it is necessary to record the changes in concepts, processes, and errors over the course of instruction to ascertain whether all children go through essentially the same basic sequence of development in learning certain concepts, that is whether there are key prerequisite facts that must be understood before they master a given concept. These studies suggest the need for longitudinal testing "to systematically monitor children's progress through a carefully designed sequence of instruction so that children's specific experiences can be identified" (Carpenter 1980, pp.195).

Gender issues have been researched with the emphasis on sex-related differences in mathematical learning. For many years it was believed that males learn mathematics better than females (Ilg and Ames, 1951) but Fennema and Behr (1980) suggest that most of this research has ignored nonsignificant findings for educational change such as nongenetic reasons for such differences and the fact that half the population copes less well in mathematics than the other half. Unfortunately these findings have negatively influenced females learning of mathematics and perpetuated the myth that they are less capable than males.

Wright (1991) found an interesting observation when assessing pre-school children's mathematical ability. Boys were overrepresented at either end of the scale of ability levels.

Young-Loveridge (1989) discovered that there were no statistically significant differences between the sexes either on total scores or for individual items. However an overall average of girls was slightly better than for the boys. (X=20.3 compared with 19.3) and they obtained a higher score on 24 of the 36 items (pp.53).

Geary (1994) confirms the ideas of many researchers across studies and across cultures that boy and girl infants do not differ in their ability to discriminate small numerosities (pp.191). Likewise pre-school children show no gender differences in biologically primary skills such as sensitivity to numerosity, understanding of basic number concepts, counting and arithmetic. "Boys are not biologically primed to outperform girls in basic mathematics" (pp.192). Gender differences only appear in adolescence and then in specific domains such as in mathematical problem solving in geometry and word problems but not in algebra. These differences appear to be caused by a combination of cognitive, psychosocial and biological factors which develop over the years at high school.

Research of these issues points to the fact that there is a need to review and reconstruct the teaching and learning of number in early childhood mathematics education in countries such as America, Britain, Australia and New Zealand.
What then is the situation in South Africa? Are the young children in the reception classes being challenged or is there an underestimation of their abilities? Is the curriculum well matched to the skills of these children or should there be a review of the informal knowledge and competencies of reception class entrants?
There is certainly a need to investigate the number knowledge, skills and strategies applied by pre-school children to ascertain whether or not they are being extended and to know if the present content of the curriculum and methods of instruction require adaptation to provide for the levels of mathematical ability.

The present study investigates the informal mathematical knowledge and competencies of reception class children before they embark on the informal programme of the pre-school. The research was carried out to determine the mathematical ability of 5 year old children from working class homes in the South African context to ascertain the standard of achievement of the pre-school child who up to this age has developed all he/she knows from the home environment.

Building on the work of Carpenter (1980), Aubrey (1993), Young-Loveridge (1989), Wright (1991) and Leushina (1991), this research examines the exploration of mathematical concepts and relationships as presented in the reception class in pre-primary schools in Kwazulu-Natal, South Africa and critically assesses whether the programme is allowing all children to refine and extend their understanding of mathematical concepts. Are the teacher's evaluations of children's ability too influenced by the work of Piaget or are educationalists aware of the more recent studies of the above which emphasise what children of this age can do rather than what they cannot do?

This research looks then at the situation in South Africa to see how changes can be made to expand the present programme to meet the needs and abilities of the children attending the pre-primary schools. Recent research of these issues questions whether teachers are aware of the abilities and competencies of these young children and plan their instruction on an appropriate level. Are there more feasible and practical ways of adapting the present programme to include an evaluation of children's mathematical competencies so that instruction will be based on individual ability which will ensure that children's mental development is maximised and expanded for them to reach their full potential?

3.1 Introduction

This investigation aims to assess the level of mathematical knowledge and cognitive skills of young children aged five to six years and to ascertain whether this knowledge of pupil's mathematical ability and understanding has been accounted for when planning a programme for reception class children. In this chapter research questions whether there is a wide range of competency among young children or whether those from one social group all start school with the same level of mathematical ability. The study involved children from working class homes because it was thought that their mediated learning experiences of the home environment would be similar. If this research was to influence the planning of a programme for reception class children, then the results would need to reflect the mathematical ability and competencies of the majority of the population of that age. To understand how children think and something about their level of development, these tests record the strategies used to solve numerical problems and the mistakes children make when performing these tasks. The tasks included in this assessment evaluated the child's knowledge and application of number, shape and space, classification, measurement, patterning and sequencing. By examining the results of each child and comparing their proficiency at each test, this investigation aimed to find whether or not there was any correlation between mathematical knowledge of one kind or another. Do we have a clear idea of what aspects of a young child's knowledge need to be extended and mediated in order for development to take place and for the child to gain the maximum benefit from the learning situation? If teachers' perceptions of children's ability are based on superficial knowledge, then there needs to be an investigation to establish the most reliable and effective way of assessing this ability so that this information can be used to produce a well planned and appropriate curriculum for the reception class.

This research expands on the ideas of Aubrey (1993), Hughes (1986), Gelman & Gallistel (1978) who all stressed the importance of evaluating the mathematical skills and competencies young children have so that curricula can expand these cognitive skills through experiences that are meaningful.

3.2 The Setting.

* The study was conducted in three pre-primary schools situated in working-class suburbs of the Pietermaritzburg area. Each school falls under the auspices of the Natal Kwazulu-Natal Education Department and therefore follows the curriculum as suggested in the 'Learning Through Activity Programme' (See appendix E). The schools cater for about 100 pupils and are fortunately able to employ staff who are qualified in pre-primary teaching. Children at each school range in age from three to six years and are divided into groups according to the number of years before formal schooling begins at age 5½ to 6½ years.

The schools follow a child-centred approach where activities are provided to encourage participation at the child's level of ability with teacher guidance to facilitate the learning process. Children are observed daily by their group teacher and detailed observations are recorded to ensure that teachers are aware of each child's social, emotional, physical and intellectual needs. There is a great emphasis on the whole child and his/her all round development. +These schools are all very well equipped with fairly large outside play areas containing apparatus to exercise large muscle control and development. Inside there are various rooms again well equipped to cater for creative art work, fantasy play, block constructions, a cognitive room with games, puzzles and construction sets and a library area. The morning is divided into periods of group activities led by the group teacher and free play times when children are able to move around the school and 'play' with any age group and in any area of the school. +The groups are divided according to their age and the number of years prior to starting formal schooling and each group contains approximately 25 children.

Group activities include an early morning greeting ring, a snack time, a music ring and a story ring at the end of the morning. All children go home at 12 noon. Some schools run an 'after school care group' for those children whose parents work a full day but this is not part of the school day and is run by an outsider who comes in to act only as a childminder.

3.3 The Subjects

Forty children participated in the study. Each child was selected by his/her group teacher on the basis of criteria given by the researcher. These criteria were that the child should come from a working class family where the parents had little or no academic tertiary education. Occupations of parents included bus drivers, hairdressers, car mechanics, and police, traffic and security officers. All the children were in the pre-school group which meant that they would be eligible for entering the junior school in the following year because they would then be of school going age which is 5½ to 6½ years. The last criterion was that their home language was English. This was to be a pre-requisite so that as far as was possible the children tested would understand the questions put to them. The school enrolment in each case included some children with home languages other than English. In most of these cases the home language was Afrikaans or Zulu. The issue of the investigation of the mathematical competence of children of these other languages is beyond the scope of this study. It would be desirable that subsequent studies address this issue using a similar procedure to that of the present study, in the language in which each child is fluent.

The majority of the children had been at some kind of pre-school for at least one year prior to this last year at pre-school, but there were a few children who had been at home and only started pre-school in the January before. Others had attended pre-school for two years, or had had playgroup experience.

None of the children who participated in the research had had any formal mathematics training in a school situation and would only be starting the pre-mathematics school readiness programme in the July of that year.

From the criteria listed above, teachers nominated 20 boys and 20 girls for inclusion in this study. The average age of the children was 5 years 6 months. The oldest child was 6 years 5 months and the youngest child was 5 years.

The parents of all the children to be tested were informed of the nature of the investigation and asked for their consent for their children to be included in the research.

3.4 Instruments

A range of ten tasks were selected to address the questions raised above.

Tasks required the children to use the apparatus they where given to answer the questions, which were developed to assess their number knowledge and gain some indication of their understanding of algebraic concepts such as shape and space, classification, measurement and seriation. The tasks were designed using the previous study of Aubrey (1993) as a basis and incorporating the work of Young-Loveridge (1989), Wright (1991) and Williams (1965). Tasks were assessed by records that noted the number of correct replies and the strategies used to complete the activity. Evaluation was designed to reflect the numerical ability of each child by measuring the number of correct responses and to study the methods used to calculate the answers. Note was taken of comments and actions which could throw light onto methods used to attain the answers and errors made. One of the best clues to the child's stage of development is often seen in the kind of errors that he/she makes and so it is not always enough to note whether or not the child gets the answers right or wrong but rather to reflect on the kind of errors that are made. It has been realised that types of errors are peculiar to the age of the child and to the processes being calculated and are therefore good clues of just what intellectual process the child is going through.

The researcher was guided in her choice of tasks by the main mathematical concepts explored in the present 'curriculum' of suggested activities for the pre-school group of children in the province of Kwazulu-Natal (See appendix E). The curriculum includes the exploration of mathematical concepts through the use of appropriate language and problem-solving activities designed to allow the child to build on existing knowledge by exploring numbers actively at his/her own pace in a manner determined by himself/herself. It was therefore important to know whether or not the child had already grasped these concepts and to understand the depth of his/her knowledge to be able to determine the level of development attained. The tasks involved the understanding of mathematical language and the practical application of concepts of number and algebraic terms. The record of the evaluation of each child's attempts was therefore not in the form of a simple 'yes / no', or 'right / wrong' answer but rather an observation of the way in which the problem was handled, the language used and the process followed to arrive at the end result. This would resolve the question whether there is a need to adapt the present mathematics curriculum to accommodate the level of mathematical competence as recorded in this evaluation.

3.5 Procedure

The assessment tasks were conducted with individual children in a secluded area where the children could actively participate by manipulating the test material and become completely involved in the situation. It was important that the children could not be seen by other children nor distracted by the activity and noise of the school situation. The tasks entailed the manipulation of everyday objects and activities familiar to children of this age so that as far as possible it could be assumed that the questions were meaningful and the results would therefore give a true reflection of the child's current level of knowledge and understanding. All the apparatus used in the test activities could be found either in a home or pre-primary school environment and would therefore not distract from the purpose of the investigation. Tests were administered to individual children starting soon after they had arrived at school in the morning and finishing at about 11 o'clock so as to be sure that the children were fresh and alert and before they could be tired by the normal school activities. The assessment lasted about 1½ hours depending on the child's ability to concentrate and the need for breaks. The researcher sat on the floor with each child to minimise their feelings of fear or uncertainty and to relax them as quickly as possible. This helped them to feel secure in familiar surroundings and at a level where they were accustomed to playing. Before the test was administered each participant was encouraged to examine the testing equipment and to chat freely to the interviewer about school and home activities, and likewise the interviewer explained who she was and the purpose of her visit. The interviewer then explained to the child what he/she would be expected to do, emphasising that there was no preconceived expectation of how well or how badly they would cope, but rather that they should 'play the games as well as they could'.

Individual assessment schedules were prepared so that scores could be entered for each response as well as notes on behaviour and notable verbal comments or actions (See appendix A).

A hand puppet was used to stimulate interest and set a relaxed and enjoyable atmosphere in which to work. Children relate well to a familiar bear puppet and are immediately removed from the 'test' situation into a type of fantasy world where the bear takes on the role of co-participant in the assessment and the child's confidence is enhanced. This corresponds with Donaldson's (1978) 'Naughty Teddy' who emerged from a box and messed up the game. When the array was transformed in this way the children were better able to conserve than with the standard presentation.

Assessment tasks were conducted by the researcher who holds a four year qualification in pre-primary school teaching and has taught in this phase of education for seventeen years. This experience means that she was familiar with this age group and felt confident and at ease when communicating with them as she was aware of their language ability, short concentration span and emotional immaturity. Having been involved in the early mathematics instruction of pre-school children, the importance of language was realised and special attention was given to ensuring that children understood the questions and that each child was addressed in the same manner. The assessment schedule facilitated reliability as each task was entered following the same procedure and results recorded as stated.

3.6 The Tasks

A sequence of tasks were presented to each child individually and completed at his/her own pace. In the case of tasks 1 to 9 the procedure followed that of the tests designed by Aubrey (1993). In the tenth task the procedure was changed and the apparatus altered but it still aimed to evaluate the same algebraic concepts. Children were not asked to build with 3-D shapes as it was thought that this could be a very time consuming activity and it would be difficult to measure ability in this way. Other activities such as those to assess an understanding of language of measurement, vocabulary of position on a line and in space, and recognising outcomes of common events would have used different materials in the assessment task as they were not described in detail by Aubrey (1993).

3.6.1 Test 1: Rote Counting.

The puppet asked the child to count as far as he/she was able in order to determine whether he/she knew the conventional order of the counting words and the highest number he/she was able to reach (Young-Loveridge 1989 & Aubrey 1993). Each child was given two trials, with the highest number being recorded on each count.

This was therefore a measure of the child's rote counting ability. All unconventional sequences with additions or omissions were recorded on the schedule as was the use of concrete materials such as fingers.

3.6.2 Test 2. Counting Objects in a set of 10.

The purpose of this task was to test the child's ability to count visible objects, pairing the number name with each countable object. Rational counting according to Gelman & Gallistel (1978) involves the application of the five principles which need to be co-ordinated for counting to be successful. It involves keeping track of items already counted and items yet to be counted which requires the co-ordination of partitioning and tagging whilst producing a series of names, one at a time, for each object. The procedure outlined by Aubrey (1993) was followed. This task required the children to count an array of three and seven small plastic blocks, first when placed in a line and then in a circle, and then to take out smaller subsets (four and ten) from the larger set of 12. The removing of subsets showed an understanding of the cardinality rule (Schaeffer et al., 1974) and a challenge to problem solving which provides a link between the child's environment and mathematical skills (Groves & Stacey 1990). The number of correct responses was recorded.

Making a set of a given number.

Counting a given array

3.6.3 Test 3: Order Invariance

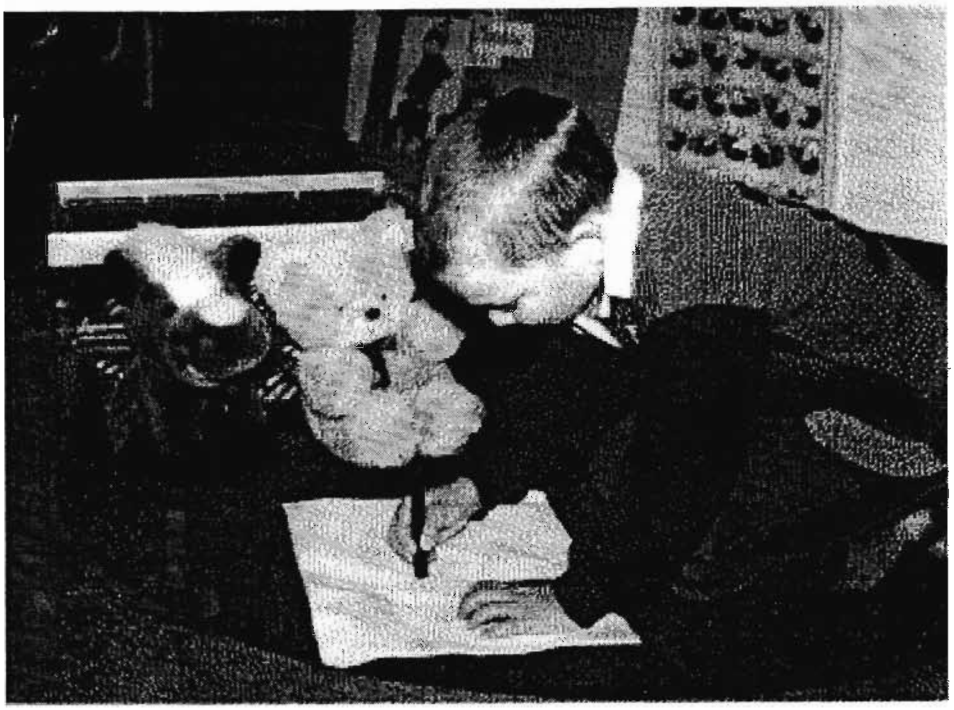
This task was designed to evaluate the child's understanding that arrays can be counted in various ways without altering the value of the set (Gelman & Gallistel 1978). Children were asked to count two sets, one of four and one of six blocks, first starting from the left, then from the right and finally from the middle. To accurately name the cardinality of the set, the child needed to understand that the last number named represented the number of items in the set regardless of the order in which the objects had been counted. Again the number of correct responses was recorded as well as any comments or actions of note.

3.6.4 Test 4: Reading Numbers.

This task required the child to recognise the written numerals from one to ten plus the numbers 12, 15, and 27 (Young-Loveridge 1989, Wright 1991). These numbers were displayed on pictures of everyday objects, people and animals (See appendix B). The number of correct readings out of 13 was recorded.

3.6.5 Test 5: Writing Numbers.

The purpose of this task was to assess how the child could represent the quantity of blocks in a written form (Hughes 1986). The child was given a large piece of paper and a thick crayon and asked to show the puppet how many blocks had been put down in front of him by writing something on the paper. Different quantities of blocks from one to ten were then displayed and the child was encouraged to draw something on the page that would depict the number of blocks. Notes were made of the method used to find the correct number and the ways in which the quantity was represented in the written form i.e. dots, lines, pictures, numbers, and blocks.



Representing the quantity of blocks in a written form.

3.6.6 Test 6: Ordering Numbers.

To test the child's number word sequence development, Wright (1991) emphasised that a distinction be made between the models of forward and backward number word sequence development even though there are similarities in the descriptions of the corresponding levels. The task therefore evaluated whether the child had developed strategies for counting on forwards or backwards thus showing the qualitative differences in children's counting skills as described by Wright in his 'Five stage Model' of children's construction and elaboration of the number sequence. The child was asked what number came after/before randomly presented numbers 1 to 20. The number of correct responses was recorded.

3.6.7 Test 7: Understanding Number Operations of Addition and Subtraction.

Simple problems involving 'adding to' and 'taking away' were presented in the form of sets being joined or items taken away from a set (Brush 1978, Young-Loveridge 1989, and Carpenter & Moser 1984). Two teddies were placed in front of the child and each bear was given an amount of sweets. The child was then given the problem to solve with the teddies putting their sweets together, thus x sweets from bear A would be added to the y sweets of bear B giving the total amount. This operation was repeated for 4+1, 3+1, 4+2, 5+2, 4+3, 2+3, 4+4, 3+4, 5+5, and 6+4.

A similar operation was used for subtraction, with the interviewer asking how many sweets would be left if bear A gave x of his y sweets to bear B. The problems included the following subtractions: 5-1, 6-2, 5-2, 5-3, 6-3, 6-4. 8-4, 6-5, and 9-5.

The number of correct responses was recorded as well as the strategies used to solve the problem.

3.6.8 Test 8: Division as Sharing and Multiplication as Continuous Addition.

The child participated in a social sharing activity and his/her understanding of the process at a physical level and as a mathematical concept was assessed (Desforges & Desforges 1980). The child was asked to share a number of sweets between the bears so that it would be 'fair'. Of particular importance was the strategies used to divide the sweets and the manner in which they dealt with the remainder. Set sizes of four, five, six and nine were used and were first divided among two bears and then among three bears.

The multiplication tasks were in the form of questions with no concrete materials presented. Each child was asked two questions: 'How many legs have two ducks got?' and 'How many wheels are there on three cars?' As they thought about the answer their actions and comments were noted to see whether there was any relationship between social sharing and multiplication as continuous addition.

Correct answers for the division scored a possible five and the multiplication a two. Again any noteworthy actions and conversations were noted.

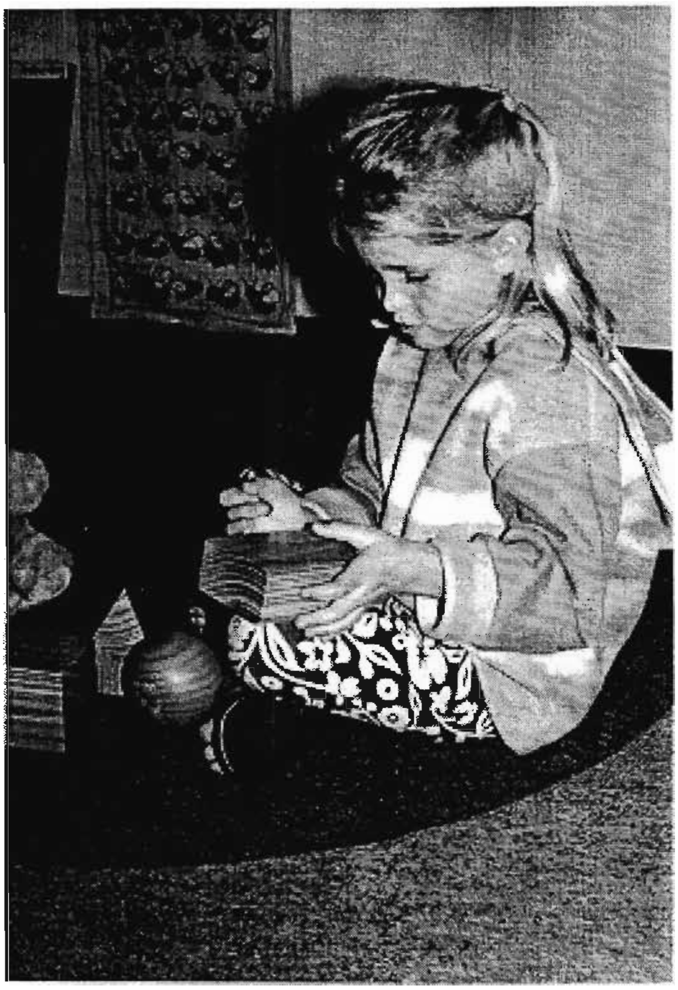
3.6.9 Test 9: Estimation

The process of quantitative judgements involves the use of perceptual cues of length, density, and arrangement of the array (Gelman & Tucker 1975). This task required the child to give the number of objects shown for a maximum of 3 seconds without counting. The first display was a bowl of six oranges and then a plate of 10 jelly baby sweets. The accuracy of each number judgement was noted as well as their sensitivity to number differences.

3.6.10 Test 10: Algebra

The tasks in this test investigated the child's ability to recognise patterns in different shapes and colours. The child was asked to copy and to continue first a pattern of alternative red and green plastic blocks and then a pattern of three different circular shapes. Lastly the child was asked to construct her/his own pattern using the three different circular shapes. Correct responses out of a possible four were recorded and a note made of the child's own pattern.

Then the child was presented with four wooden three-dimensional shapes and asked to name each one and then to match them to a two-dimensional shape. To evaluate their written representation of these shapes, the child was then asked to copy each shape as they saw it on paper using a fat crayon which encouraged large bold responses.



Examining and naming 3D shapes.

Their understanding of measurement was judged by an activity which involved arranging pieces of ribbon in order of length and then comparing them by using the language of measurement with words such as longest, shortest, longer, shorter and the same length. In order to make an accurate judgement about the difference in length and to demonstrate an understanding of the concept, the child would have to be sure to match the ends of the ribbon when lying them down on the floor. Their understanding of ordinal numbers was ascertained when they named the position of different Lego dolls standing in a line by using words such as first, last, second, third etc. Positioning of their bodies in space was recorded as they demonstrated their ability to move themselves to commands of next to, underneath, on top of, in front of and behind. Classification of objects according to certain criteria and representing them in different sets tested their ability to sort objects. They were then asked to give reasons for such a classification. Lastly, they were given cards depicting people engaged in everyday events and asked to arrange them in order of sequence and to relate the events in a logically occurring order.

3.7 Data Analysis.

To record the results of each child's test, pages were prepared detailing the information required. General particulars were first entered such as date, name, age, sex, number of years spent at school, occupation of father and mother and position in family. Then a short description of each test item was entered with space for results, comments and drawings or diagrams to describe the way in which tasks were completed (See appendix A). This structured schedule ensured that each child completed the tasks in the same order and gave some uniformity to the procedure. The four shapes: square, rectangle, triangle and circle were drawn on four separate sheets of paper with space left for the child's own interpretation.

The written recordings of each child's test results was reviewed as was the response to each task. Characteristic aspects of the child's response were recorded. Individual and group results were scored for each task. Tables were drawn up for each test to provide information on individual responses and to give an indication of the range and spread of scores.

4.1 Introduction

Results of the administration of tasks will be discussed by presenting the performances in terms of numbers of children who succeeded in accomplishing the task as presented to them. Instances of children who employed unusual or indirect approaches to performing the task are discussed below in the case of each task. The results from each participant for performances on each of the scores obtained during the procedure were further analysed by calculating the correlation of each test item to establish which of the scores would be the best predictor of the child's overall performance. The correlation coefficient of each test gave an idea of the relationship of that test to the average ability on all test scores.

Discussion of these findings and this relationship to the findings of other studies is provided in chapter 5.

4.2 Test Results

4.2.1 Test 1 Rote Counting

The Task

Each child was required to count as far as possible using the conventional counting words and the highest number reached after two attempts was recorded.

Rote counting ranged from five to forty-nine with only two children using fingers to assist them. Of those two who used their fingers to assist with counting, one only counted to 5 but the other to 29. There was a tendency to obtain a higher sequence length on the second opportunity to count but five counted further on the first attempt and eleven counted the same sequence on both occasions. As was noted by Fuson (1988) and Ginsburg (1977) a large number of children (21) finished counting with numbers ending in 9 or 0. Twenty children stopped at the end of the unit sequence of 9, and 1 child ended the sequence at the beginning of a decade which was 20. A few children reached their highest correct sequence and then continued to count in tens e.g. one child counted 1 to 12 and then 20,30,40,50,60,70,80,90, thereby showing some understanding of the decade system before learning the more difficult words which make up the numbers from 11 to 19. The mean sequence length was 24.2.

Number of Pupils (N=40)	Number Counted
2	1-10
18	11-20
13	21-30
4	31-40
3	41-50

Table 1. Test 1: Counting words.

4.2.2 Test 2: Counting Objects within 10.

The Task

Children were asked to count two small arrays that were first placed in a line and then in a circle, and then to extract two sub-sets from the larger set of 12.

Nearly all the children correctly counted the 3 and 7 blocks when placed in a line in front of them. Only one child did not manage to give the correct answer when counting the three blocks, both when placed in a line and again when arranged in a circle because he used the words one, three, two, but he was able to correctly count the 7 blocks in both instances.

Three children failed to count the 7 blocks when placed in a line, whereas 10 children gave the incorrect answer when the 7 blocks were arranged in a circle. As noted by

Gelman and Gallistel (1978), pre-school children find it more difficult to apply the one-to-one principle as set sizes increase. Slightly fewer children were able to manage the two tasks of taking a small sub-set out of the larger set of 12. Five failed to correctly extract the set of 4 and 12 failed on the set of 10. Most of the children pointed at the blocks as they counted them to make sure that they had the correct number. Only one child looked at the blocks and nodded his head as his eyes moved from one block to the next. One child counted from right to left and all the others worked from left to right.

The mean for the six possible correct answers was 5.2.

Number of Pupils (N=40)	Number correct (total 6)
0	1
1	2
2	3
8	4
7	5
22	6

Table II Test 2: Counting Objects within 10.

4.2.3 Test 3: Order Invariance.

The Task

The children were required to count an array of four and six blocks in three different orders to check whether each yielded the same value.

In this test the children found it relatively easy to count the two sets when starting at one or other end of the row but when asked to start counting from the middle of the row they could not accurately carry out the principle of partitioning and tagging and therefore gave the incorrect answer (Gelman & Gallistel 1978). One child was unable to count any of the arrays and another child could not count the set of 6 starting at either end or from the middle. It is interesting to note that both these children only managed to count to 5 in the first test. The mean score for the 6 possible correct answers was 5.1.

Number of Pupils (N=40)	Correct Responses (Total 6)
24	6
2	5
11	4
2	3
0	2
0	1
1	0

Table III Test 3: Order Invariance.

4.2.4 Test 4 Reading Numbers

The Task

The children were presented with pictures of everyday activities which displayed the numbers 1 to 10 randomly as well as the numbers 12, 15, and 27. Children were required to name the number as it was shown to them.

When numerals were randomly presented, three children could not recognise any numbers and three could only recognise one number. Only two children recognised numbers higher than 10 but no one was able to name the numeral 27 nor did they offer any suggestions as to how to interpret the number. One child read the numeral 12 as 3 showing that he had simply added the numbers one and two which he was able to recognise. Fifteen percent recognised 10 or more numbers, 53% recognised between 5 and 9 numbers, 18% recognised between 2 and 4 numbers, and 15% recognised one or none. Of the thirteen numbers presented the mean number of numerals recognised was 6.

There was also the idiosyncratic approach of a child who attempted to establish the number by counting on her fingers while looking at the number on display.

Numbers from 1 to 10 were randomly presented as were the numbers 12, 15, and 27.

Number of Pupils (N=40)	Number of Numerals Correctly Named
3	0
3	1
3	2
4	3
0	4
2	5
7	6
3	7
4	8
5	9
4	10
1	11
1	12

Table IV Test 4 Reading Randomly Presented Numerals.

Number of Pupils (N=40)	Numerals Recognised
35	1
27	2
27	3
31	4
30	5
27	6
18	7
19	8
8	9
10	10
2	12
1	15
0	27

Table V: Test 4 Reading Numerals

4.2.5 Test 5: Writing Numbers

The Task

Children were asked to represent the quantity of blocks presented by drawing on the piece of paper given (See appendix C).

All the children were willing to offer some sort of representation. Seventeen children represented the number of blocks given by drawing tags, pictures, circles, lines or shapes and 23 children wrote numerals as best they could. One child recorded the number by drawing horizontal lines on the page with the length of the line representing the size of the number. Another child put the blocks onto the page and drew round them, first as single entities and then later he placed them together and drew around the array. Another child carefully chose different pictures to represent each number i.e. 4 houses, 2 balloon, 3 faces, 5 trees, 6 crowns etc.

Those children who used numerals to represent number found 9,6, and 8 the most difficult to form and often wrote the numbers 2,3,7 and 5 in a reversed form or upside down. Three children were able to write the numbers 1 to 10. Thirteen of these 23 children who used numbers to represent the quantity, started with 1 and wrote all the numbers preceding the required number; e.g. 5 was represented by the numbers 1,2,3,4,5. A few children used a mixture of numbers and tags, writing the numbers they could form and then changed to invented representation when formal knowledge was exhausted. One child could only write the numbers 1, 2, and 5 but accurately recorded numbers up to six by completing the sequence with O's.

Seven children using pictures or tags to represent the number were a hundred percent correct on all values and six children who used numerals to represent the quantity were accurate on all accounts. Only three children were unable to record more than three numbers correctly.

One child appeared to have no idea of how to represent the numerosity of the number of blocks presented or she had not understood the question because she simply drew one block each time she was asked to represent a different number of blocks.

4.2.6 Test 6: Counting on Forwards and Backwards

The Task

This task assessed the child's ability to count on forwards and backwards. Each child was given a number from 1 to 20 in random order and asked to name the number that came after/before that number.

The children appeared to find this a fairly difficult task as it required them to think of the numbers in an abstract form without any concrete object on which to attach the numbers. Most children found it easier to give the number that came after randomly presented digits up to 20 with only 4 children unable to give any correct reply. The mean number of correct answers in the counting on task was 9.7. Only one child was able to give the correct reply for all 20 questions.

To count backwards appeared to be a more difficult task with 19 children unable to give even one correct reply. Children seemed to tire quickly and lost interest. The mean number of correct answers in the counting backwards task was 3 with 73% falling into the 0 to 5 number of correct answers.

A number of children used their fingers as they counted from one up to the number given and others just counted quietly. There seemed to be a need to repeat the sequence of numbers to work out the number that came before or after a given number.

Children were asked to say what number came after numbers 1 to 10 and then numbers 11 to 20.

Number of Children (N=40)	Number of Forward Counting Correct
8	0-5
11	6-10
16	11-15
5	16-20

Table VI Test 6 Counting on Forward

Children were asked to say what numbers came before numbers 1 to 10 and then numbers 11 to 20.

Number of Children (N=40)	Number of Backward Counting Correct
29	0-5
9	6-10
2	11-15
0	16-20

Table VII Test 6 Counting Backwards

4.2.7 Test 7: Addition and Subtraction

The Task

The children were given simple word problems to solve first 10 addition and then 9 subtraction sums using the concrete situation of teddies 'joining' and 'separating' their sweets.

Almost all the children pointed to the sweets as they added the two amounts together. Only a few children counted on from the one amount and only two gave the correct answer by just looking at the sweets. This suggests that either they had advanced to the counting stage (Gelman & Gallistel, 1978) or that they had not yet reached the stage of recognising patterns of the numbers displayed and were therefore not able to subitize (Ginsburg, 1977). The fact that the activity involved the teddy bears and brightly coloured sweets, interested most children and assisted in holding their attention but a few lost interest after completing the first three of four addition sums and carelessly skipped items when counting with no apparent concern about accuracy. Mistakes were more prevalent among the addition sums involving larger numbers with the most incorrect answers given for 4+3 and 6+4.

As was the case in previous research (Aubrey 1993), scores for subtraction were higher because the numbers involved were smaller. The task was quite a lengthy procedure and it was surprising how many children worked industriously at the counting of each set without decreasing the rate of accuracy and by careful partitioning off of the units counted and those still to be counted. The mean for correct addition of the ten sums was 8.5 with 22 children correctly finding the solution to all ten problems. The mean for the nine subtraction sums was 8.7 correct answers with 34 children being successful in all nine sums.

Number of Pupils (N=40)	Correct addition responses (total 10)
0	1
0	2
3	3
4	4
0	5
0	6
7	7
1	8
5	9
22	10

Table VIII Test 7 Addition of sets 1-6 with a maximum total of 10.

Number of Pupils (N=40)	Correct subtraction responses (total 9)
0	1
1	2
0	3
0	4
0	5
0	6
1	7
4	8
34	9

Table IX Test 7 Subtraction of sets 1-9

4.2.8 Test 8: Division and Multiplication

The Task

Using two or three teddies and a pile of sweets, the children were required to share a given number of sweets between the bears so that they would each have the same amount. Then the child was asked two questions which involved multiplication calculations but without the use of concrete materials.

Children seem to have a natural feeling for fairness and with small numbers and amounts, that provided for equal quantities when shared, they were nearly always successful. Twenty-one children gained a total of five out of five, fifteen only made one error in the problem that had a remainder and four children had a further difficulty with the larger amount of nine. Fourteen children shared the sweets to the bears in groups of twos or threes and the rest either dealt out the sweets one by one or used a combination of both these methods. The responses to the remainder ranged from a number of children who simply gave one bear 3 sweets and the other 2, being quite satisfied that they had shared out *all* the sweets they had been given and with no notice taken of the unequal number, to those who came up with ideas of what to do with the one left over. Eleven kept the remainder in their hand or said they would save it for the next day, two said they would give it back to mom, four placed the remainder in the middle of the two bears, and two decided to halve the remainder so that each bear would have two and a half sweets. Others asked for another sweet so that they could be fair to both bears.

Number of Pupils (N=40)	Correct responses (out of 5)
0	1
0	2
4	3
15	4
21	5

Table X Test 8: Division of 4,5,6,and 9 by 2, & 3.

The multiplication task was an abstract problem which most children found very difficult to solve. Two children were able to answer both questions correctly. Twenty-five children were only able to answer the first question correctly and the remaining 23 made no correct responses.

4.2.9 Test 9: Estimation

The Task

Each child was shown a bowl of six oranges and a plate of ten jelly babies for a maximum of three seconds and asked to estimate the number without counting the array.

Children's natural instinct appeared to be to count the array and they were confused when told to guess because this would not give them the correct answer. One child counted the first array of six oranges and then gave no answer for the second estimation because there was not enough time to count them. Four children were able to estimate both amounts correctly and a further ten were able to estimate six items. Only six children were not able to notice that the second array had a larger number of items than the first. A number of children gave estimates that were one and two points off the correct answer. Again, as with the other tasks, this task showed that children are more successful when dealing with small numbers and there was a strong urge to count so that the answer would give an accurate number.

Number of Pupils (N=40)	Estimate for six items
5	7-8
14	6
16	4-5
5	2-3

Table XI Test 9: Estimation of Six Items

Number of Pupils (N=40)	Estimate for 10 items
3	11-12
4	10
12	8-9
11	6-7
8	4-5
2	0-3

Table XII Test 9 Estimation of Ten Items

4.2.10 Test 10 Algebra Tasks

The Tasks

There were a variety of activities which ranged from copying and repeating a displayed pattern, making their own pattern, describing 3-D shapes and drawing 2-D shapes, and using appropriate language to describe concepts of measurement, classification, ordinal position in a line and position in space and the outcome of common events.

The task of patterning appeared to be very difficult either to complete or to understand. Because children coped better with the second patterning activity involving three circular shapes, it may be thought that they had a better grasp of what was required by the time they had seen the activity for the second time. A number of children tried to work from right to left or added to the pattern from both sides thus making it more difficult to complete correctly. Eighteen children managed to copy the first pattern and five were able to continue it whereas thirty-six children were able to copy the second pattern and twelve were able to continue it. Only seven children created a simple regular pattern on their own.

Three children completed all the patterning activities correctly and three were not able to do anything. One must question why the results of this task show such a poor performance when these children have coped so well at all the other areas of number, often applying their own strategies to solve problems. According to Sime (1973) this ability to recognise patterns is basic to mathematical insight yet these children, who have shown this ability in all other number activities, now appear to lack this knowledge. Language plays an important part in this type of task as does the type of objects used which leads one to suspect that the fault lay in this area and not altogether in the child's inability to cope with this activity.

Descriptions of regular shapes produced very few formal responses but fairly descriptive informal responses:- a sphere was identified as a ball, a circle, a round, with only three children naming it correctly, a cube was called a square or a block, a cuboid became a wall, a rectangle, a square, or a block, and a tetrahedron was called a triangle, a tent, an arrow, a star, or a zigzag. Matching 3-D to 2-D shapes showed a good understanding of the language of measurement and perceptual skills for identifying and decerning shapes. Only two children matched the cube with a rectangle and the cuboid with the square. The drawing of shapes was accomplished with relative ease with most children using one stroke to complete the shape. Eight children were unable to draw a triangle but made a rectangular shape with a point. One child drew a circle using two strokes so that it looked more like an oval. The square and rectangle were drawn either with one stroke or with four straight lines or with two sides in one stroke and the other two sides with another. One child was not able to make any of the shapes but tried to draw around the shape he was given to copy so that all lines were curved and no shape had corners.

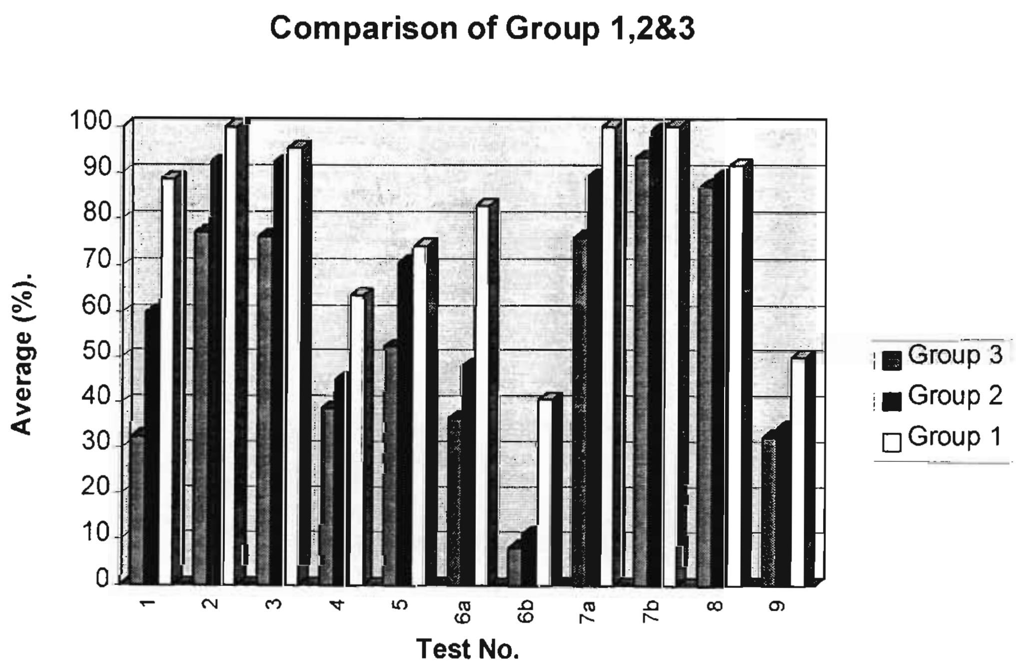
When judging the measurement of the lengths of tape all children knew which ones were the longest and the shortest but a few had difficulty with those of the same length. Not all the children knew to put the pieces of tape down on the floor with one end level with the others but were still able to give a correct answer. Position in a line produced a good response using ordinal numbers with the first and middle position gaining the most correct responses and the second position only found by half the children. Sorting objects was quickly and easily accomplished by all children. Three children classified the coloured beads into their colours and tried to sort the shells into ones with the same pattern showing a greater awareness of finer detail. The last activity drew mixed reactions but children were well able to place cards in order of the sequence of events. It could be clearly seen that some children were tired and had lost concentration and needed encouragement to complete the activity while others asked for more 'games' to play. Six children found it difficult to express themselves and explain what they saw on the cards, thus making it more difficult for them to arrange the cards in the correct sequence.

The seven children who reached the longest number word sequence in rote counting (three at 49, and four at 39) had all been at school for 1 to 2 years and gained consistently higher scores across all the number and algebra tests of recognising and writing numbers, counting on forwards and backwards and operations of addition, subtraction, division and multiplication. Of the twenty children who obtained rote counting scores below the mean, only one was able to score evenly well on all other tests. The two children, who scored the lowest sequence on rote counting, (5) also showed very poor ability on all other number tests; no recognition or writing of numbers, had difficulty counting sets of blocks, could not count on forwards or backwards and made many mistakes when adding, subtracting, dividing and multiplying. Neither of these children had been in a 'school situation' before this year and both were above the average age for the group. They also showed signs of immaturity of language (when describing shapes and completing other geometric exercises) and poor concentration (quickly tiring of an activity and easily distracted).

Of the thirteen children who scored above the mean for number word sequence in rote counting, (8 counted to 29, one to 28,and four to 26) eleven gained consistently high scores across almost all the number and algebra tests, only a few finding it difficult to count backwards. The other two children who scored above the mean could not recognise any numbers and had great difficulty counting on forwards and backwards, but coped fairly well with rational counting and simple number operations such as addition and subtraction, division and multiplication.

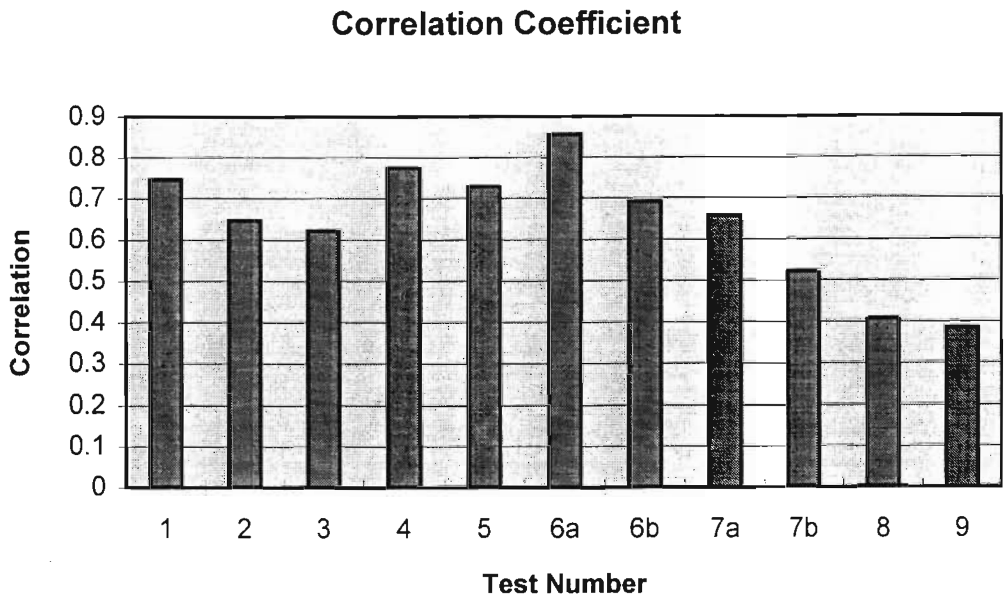
Many researchers have pointed out the importance of counting in children's numerical development (Wright 1992, Williams 1965, Ginsburg 1977, Fuson 1992, Steffe 1992). "Activities involving ascribing number to spacial, auditory, and motor patterns, number word sequence activities, and counting to establish the numerosity of visible and screened collections have a central role in the numerical development of children at the perceptual stage" (Wright, 1992. pp.138). These ideas provided systematic basis for the inclusion of graphs showing a division of participants into three groups according to their ability to count.

The following bar graph shows the average percentage score for each of the three groups for those tests where it is possible to determine such a score. These groups were determined by the longest number word sequences recorded in rote counting. Group 1 therefore have an average rote counting score of 39-49 (7children), group 2 have an average rote counting score of 26-29 (13 children) and group 3 an average rote counting score of 5-20 (20 children).



From these results it may be suggested that children who have a good knowledge of counting are also proficient at other tasks involving number and therefore are more advanced in mathematical competence and skills. If this is so, then there would be a case for the training and improving of children's counting ability as a basic and initial part of the pre-school programme. It would certainly be worth suggesting that perhaps children with poor counting ability should be exposed to more activities involving counting and encouraged to develop this skill and to use it in their everyday experiences. Many researchers have stressed the importance of counting as the basis of all arithmetic (Ginsburg 1977, Wright 1991, Young-Loveridge 1987 and Gelman and Gallistel 1978). It may be suggested that if those children who scored below the average on the counting test were able to improve this skill and become competent in their use of numbers, they would likewise increase their knowledge of mathematics and improve their scores on all the other tests.

The following table shows the correlation between each test and the average of all tests conducted, thus giving an idea of the value of each test in relation to the overall test result. This graph shows that test 6a, where children were required to give the forward number word sequence, correlated the best with each child's average score and therefore it was a good predictor of the child's ability to achieve in other test situations presented. Likewise, tests 4 (reading numbers) and 1 (counting) were well correlated to the child's overall performance.



4.3 Conclusion

The study shows that there is a wide range in levels of number knowledge among children beginning the pre-school year and that these children enter this year with considerable knowledge about number (See appendix D). These findings are consistent with the results of Aubrey (1993), Young-Loveridge (1989) and Wright (1991). Those with high levels of attainment were well beyond the prenumber and number topics typically undertaken in the pre-mathematics programme of the pre-school year. They are able to rote count well beyond twenty, understand conservation, recognise numerals, represent numerosity and are able to add and subtract with small numbers. All but two children were able to count beyond ten and most children had a reasonable knowledge of enumeration, number recognition, were able to represent numbers and by counting were able to complete addition and subtraction sums that were visually presented. The abstract tasks presented the most difficulty as did the increased size in numbers. This was particularly noticed in the counting forward and backward where the sequence of numbers had to be recalled without the use of concrete materials. Similarly the multiplication task was purely an abstract calculation which proved too difficult once the quantity rose above ten.

When comparing these results with those of other studies in Britain (Aubrey 1993, and Hughes 1986), America (Gelman & Gallistel 1978), New Zealand (Young-Loveridge 1989) and Australia (Wright 1991), there is a strong correlation between the number knowledge of these South African five year olds and those from other countries. These South African children from working class homes, demonstrated that they have number competencies that are underestimated by the reception class curriculum. They are already able to follow a programme of sorting, matching, classifying, joining and separating of sets, counting and ordering, recognising and writing numbers 0 to 10 and simple operations of addition and subtraction with the use of concrete materials, and topics such as measurement, shape and space recognition, sequencing and time recording. Whilst they may not be able to work as well in the abstract nor posses the formal conventions for representing number, they may have acquired much of this mathematical content and well be able to progress in these areas if such concepts were introduced at this stage (Hughes 1983 and Wright 1992).

As with the tasks presented by Aubrey(1993), children in this early stage of learning mathematics used their own strategies to solve problems thus showing their inventiveness and creativity which at this stage had not been stifled by formal instruction of rigid, singularly acceptable ways of resolving problems. This was best demonstrated in their response to the task of representing number in the written form where children's writing ranged from lines of varying length to pictures to formal numbers. Likewise, when adding two numbers their strategies varied from a rapid reply without any obvious counting, to counting on from the one number to include the second amount, to counting each individual item. There was obviously a range of alternative solutions which again demonstrated the various stages of development of mathematical knowledge and competencies of these young children.

In the following chapter there will be a review of the research of this thesis with reference to the ways in which it relates to the findings of other investigations into the subject of mathematical knowledge and competencies of the pre-school child.

Chapter 5 Discussion and Analysis of the Results of the Research

5.1 Introduction

The research of this thesis can now be reviewed in the light of the findings of other investigations into the subject of the mathematical knowledge and competencies of the pre-school child. Over the past two decades there has been a growing interest in children's numerical and mathematical development both in the scientific community and amongst the general public. Some basic arithmetic knowledge appears to be naturally acquired through inherent influences and cultural environments but the strongest influence on mathematical development is the instruction of formal education. Recently however educational, cognitive and developmental psychologists have worked on a concerted effort to pull together the findings on the diverse areas of children's numerical and mathematical development so that more effective teaching methods can be identified and curricula designed that best develop these skills and abilities (Geary 1994). Each of the tests in this research has attempted to evaluate the understanding and ability of young children's informal knowledge of number and arithmetic so that these results can be compared with the research of other educationalists and psychologists. It is hoped that in so doing there can be a better understanding of the level of young children's mathematical ability and the ways in which they execute their investigations into number activities so that the many dimensions of arithmetic development can be evaluated and recognised for optimal development to take place.

5.2 Analysis of Each Test Result

*5.2.1 Test 1 Rote Counting cf. Chapter 2 pp. 313*The first test in this research required the children to count as far as they were able and after two attempts the highest score was recorded. These results corroborated the findings of Buckingham and MacLatchy (1930), Bjonerud (1960), Aubrey (1993), and Young-Loveridge (1989) to reveal that young children's ability to count is well developed by the time informal instruction begins in the pre-school year. The ceiling of children's rote counting ability ranged from 5 to 49 confirming that there is a wide variation in the number knowledge of children beginning the pre-school year (Wright 1991 and Young-Loveridge 1989).

Did these children understand what they were doing when they counted? Did the high achievers in this test show mathematical ability or was this a 'parrot type' reply? Evidence given by researchers has produced two entirely different answers to the question by using different kinds of experiments with different criteria for understanding counting. Piaget's criteria were ordinality and cardinality and Gelman selected her five principles.

These results when compared with those of the tests that followed, showed that 78% of children who counted below the mean also had an average overall score below the mean and that 73% of children who counted above the mean also had an average overall score above the mean. This showed that most children with advanced counting skills demonstrated that they also had a firmer understanding of the principles of counting and were able to put these into practice. Their knowledge of number words was quite significant and could not be termed a mechanical, vacuous operation with no meaning (Piaget 1952). The correlation between the ability to rote count and successful mathematical achievement was researched by Williams (1965) when he ascertained the nature and extent of achievement of pre-school pupil's mathematical concepts, skills, and abilities. "Because rote counting ability is substantially related to successful mathematical achievement, and because pupils differ greatly in rote counting ability, activities designed to develop this ability should be made a part of the kindergarten program" (pp268). This idea is contrary to the findings of Brace & Nelson (1965) who sought to assess the number knowledge of pre-school children and to determine which factors influence the early growth of number ideas. Interrelations of counting scores with 'concepts of number' scores were calculated to determine the connection of counting to a knowledge of the concept of number and it was found that "The pre-school child's ability to count is not a reliable criterion of the extent to which he has developed the true concept of number" (pp.132). The results of these tests refute this idea as there was a strong correlation between those who counted to the highest number (rote counting),and those who were most successful in further tests which showed an understanding of the principles involved in rational counting.

The errors made in the counting task corresponded with the ideas of Fuson (1992) who noted that when young children first learn the sequence of counting words from one to twenty in English they find that because of the irregular relationship of the words, the sequence must be learnt as a rote list of meaningless words. As a result the typical errors made by the 3 and 4 year olds consist of a first section of number words in their correct order, followed by a stable section that is not correct, followed by an unstable list that varies each time number words are recalled. One child counted correctly to ten and then skipped out the number 16 on each of the two counts to 26. As was noted by Ginsburg (1977) and Fuson (1988) children tended to finish counting at the end of a decade on the number 9 or at the beginning of a decade on 0. Leushina (1991) suggests that children need help from an adult to be able to name the next decade. This was certainly the case for these children and the one who counted to 39 and then said 50.

Children's counting errors confirmed the developmental ideas of Leushina (1991) who traced children's notion of certain segments in the natural number sequence. All the children who were evaluated had learnt the number order for particular parts of the natural number sequence and then some produced sequences of numbers in ascending order but with gaps while a few gave numbers in a random and unstable order.

5.2.2 Test 2 Counting Objects in a set of 10. cf. Chapter 2 pp. 37

The second test evaluated the child's ability to count three and then seven blocks placed in a row and then to repeat the process when they were arranged in a circle. Schaeffer, Eggleston, and Scott (1974) traced the number development in young children and concluded that "to master the counting procedure the child must learn to co-ordinate its two components: the ordered number series and the one-to-one correspondences between number names and objects" (pp.365). Even though it was more difficult to remember which objects had been counted when the blocks were arranged in a circle, the children overcame the difficulty by pointing to each block as they correlated it with the number word sequence thus enabling 70% of the children tested to give the correct answer. Only one child was able to count the blocks without using a finger to point at each one as she counted. The errors made related to the inability to perform the one-to-one correspondence or not being able to remember which objects had been counted and which ones were still to be counted. This resulted in "tag duplication errors or a failure to count the last item in an array or using still another tag after all the items in the array had been tagged" (Gelman and Gallistel 1978, pp.89). This inability to carry out the one-one principle indicates, according to Gelman and Gallistel (1978) a lack of skill rather that the lack of understanding of the concept or rule. Errors occurred only with the larger number of blocks and more particularly when they were arranged in a circle. All the children except one used the conventional sequence of count words. Only one child repeatedly counted the three blocks using the words 'one, three, two'. According to Gelman and Gallistel (1978) this qualifies as the correct use of the stable-order principle because it shows that children are aware of the fact that counting uses a stable-ordering list of words i.e. "Numerons used in counting must be used in the same order in any one count as in any other count" (pp.94).

Wynn (1990) argues that this ability to count using the stable-order principle gives no evidence that the child understands the relationship between counting and the numerosity of the set. This skill is not guided by an understanding of the counting principle but rather a skill that develops from an innate ability to represent the principles and is therefore a routine that needs to be learnt. She points out that children are sensitive to many other stable orderings such as the alphabet and no evidence shows that children present the letters any differently than they do the numbers.

Bryant (1994) accepts that children know that they should count each object once and only once, but adds that this is not the only form of one-to-one correspondence that needs to be understood. He believes that children must also understand one-to-one correspondence between sets which will show that they have understood the quantitative significance of number words. This idea could not be tested as the children were not required to compare sets, however, the above ideas of Wynn and Bryant are refuted by the fact that these children proved in the second part of this test that they understood the concept of the number representing the numerosity of the set.

Likewise, Piaget (1952) and Bryant (1994) suggest that children know that numbers come in a certain order, but that the reason for this order is their increasing magnitude has not been proved. There is no evidence to prove that children understand the ordinality of the number sequence. Bryant (1994) concludes that children at first are merely practising a 'verbal routine' when they count and are not able to grasp the nature of a series that increases in quantity. The second part of this test again confirmed that by counting the number of objects in each set, the children showed that they understood the answer to the question 'how many' and realised that the number represented the numerosity of the set.

The second part of this test required the child to extract two sub-sets, of four and ten from the larger set of twelve. This activity involved not only the previous two counting principles of the unique tagging of items and the use of a repeatable list of tags but also the cardinal principle when counting enables one to determine how many items a set contains (Gelman and Gallistel 1978). Eleven children were unable to make a set of ten and five failed on the smaller set of four, with four children showing no idea of cardinality and making errors in both sets. As with Gelman's magic experiments, these results show that most pre-school children follow the three basic how-to-count principles of the counting model when dealing with sets of up to ten items and that increasing the set size increases their difficulty.

The nature and course of development of counting in young children can be seen by analysing the way in which these children applied the three basic principles. Their counting cannot be what Schaeffer, Eggleston, and Scott (1974) term as the pure acquisition of number words which are isolated from attempts to enumerate. Rather the evidence shows that young children have an understanding of and the ability to perform the staple-order, one-to-one correspondence, and cardinality principles and therefore know the fundamentals of enumeration, at least when the task deals with small numbers.

On the other hand, Piaget (1952) produced evidence to show that young children fail on all three tasks and do not understand cardinality or the meaning of number words. What then is the nature of this striking difference? Do children understand what they are saying when they count the items in an array? Each researcher gives evidence for his findings but the criteria that were used for understanding counting were quite different. Piaget's (1952) requirements were more demanding and concentrated on the relationship between sets of the same number and claimed that a child only understood the cardinality principle if he understood that a set of five objects is equal in number to any other set of five objects. Gelman's theory was based on less demanding requirements and only required the child to understand that the last number counted represented the number of the set. Bryant (1994) and Baroody (1992) believe that this is possible without the child really understanding the quantitative significance of the number. He sights evidence to show that not until children reach the age of 6 do they know how to use numbers to compare two different sets even though they are able to count proficiently (Michie 1984, and Sophian 1988). Sophian (1988) suggests that young children's numerical competencies are at first restricted to giving numerical values to individual sets and with further development they understand the numerical relations between sets either by counting or in a conservation context.

These counting tasks all used homogeneous and three-dimensional items that were visible to the children at all times. According to Steffe (1992) children's counting will show qualitative differences when there are differences in the nature of the unit items. Five distinct counting types therefore describe a progression which is based on the unit items that a child is able to construct when counting. Children in this test situation could not be assessed on this scale as there was no variation in the unit items and therefore all tests related to stage 1 where counting items were limited to items he/she could perceive.

Gelman and Gallistel (1978) referred to this as the 'abstraction principle' which deals with the definition of what is countable. Pre-school children were found to respect the abstraction principle and accept a wide variety of objects as countable. They noted that young children did not hesitate to use number words as tags when faced with collections of dissimilar objects and the heterogeneity of the array did not affect the accuracy of their performance either. They rejected the ideas of Piaget, (1952) who connected the development of complex classification skills to the development of a concept of number and in so doing gave the idea that children placed restrictions on what can be counted, i.e. objects that share notable perceptual properties. But are the requirements of Gelman for the understanding of cardinality too undemanding? These tests aimed to present the material in a context that would be familiar to the child and using language that the child would understand so that the evaluation would not divert the child's attention or produce misunderstanding of what was required. Under these circumstances the children showed an understanding of the cardinal principle.

Wynn (1990)studied the abstractness of children's mental representation of counting and found that at a very young age children begin to develop an abstract mental representation of what can generally be counted. Children counted objects accurately and were able to carry this principle over to counting actions and sounds which she suggests may point to unlearned abilities rather than knowledge of counting. Shipley and Shepperson (1990) would agree with this theory as their research finds that children have a strong bias to count discrete physical objects rather than parts of objects or individual objects that have been divided into separate parts. For them children are assisted in learning to count by this important innate desire to label the 'oneness' of discrete physical objects (pp.131), but found it difficult to count objects when these were broken up into different physical entities. When children were asked to count the forks and the forks had been broken up into physically separate entities, young children tended to count the physical pieces and not the complete forks. Surely this type of test involves other cognitive abilities and should not be confused with the ability to conserve? For Bryant (1994) this demonstrated that young children may realise when they do count that the last number is the important one, but they still do not seem to know what and when to count and have no idea why counting is important. Yet these children knew that in order to extract a set of a given number they would have to count the set which must surely show that they understood the value of the number to which they counted?

5.2.3 Test 3 Order Invariance. cf. Chapter 2 pp 43

The following test required the children to count two sets of blocks, one of four and one of six, starting first from the left, then from the right and finally from the middle. This activity evaluated the fifth principle which Gelman and Gallistel (1978) saw as part of the development of the understanding of counting. The order-irrelevance principle represents a combination of the first four principles which together show a full recognition of counting. To fully understand what counting is all about, the child must recognise that the order in which items are tagged is of no importance. It shows that the numerosity of a set, obtained from counting, is uniform with regard to the order in which the items in the set are counted. The consequences of counting are therefore shown to result from a) each item being tagged only once, b) tags always drawn from the same stable-ordered list, and c) the same final tag always representing the numerosity of the set. When different tags are reassigned to different objects the same cardinal number results. Gelman referred to this as the 'doesn't matter' principle for it showed that children were aware of the fact that the same item could be given two different number tags. To be sure that children were not assigning a particular number to an object, Gelman used an array of heterogeneous objects and on repeated occasions and with extensive questioning, children seemed indifferent to their order of tag task. Although children had an idea of what was involved, and understood more than was expected, they certainly had not reached a full understanding of the order-irrelevance principle.

Although this theory was only tested with homogeneous objects, sixty percent of the children tested were able to accurately count the two sets irrespective of the starting position. Most of the mistakes children made were when they were asked to count beginning from the middle block. Only one child was unable to count any of the sets correctly and two others made more mistakes when counting the larger of the two sets. These results would agree with Gelman and Gallistel (1978) that young children of this age have a reasonable understanding of the order-invariance principle and are not only able to apply the first four principles but also recognise the fact that much about counting is arbitrary.

When comparing each child's ability to count with his ability to do well on the order-invariance tasks it was found that most (but not all) children who were reasonably good counters showed explicit understanding of the order-invariance principle.

However, all the children who did well on the order-invariance tasks were not good counters. This finding was contrary to those of Gelman who claimed that all the children who did well on the order-invariance tasks were good counters. It cannot be said then that a good counting ability is necessary nor sufficient in order to understand the order-invariance principle.

The results of these counting-type tests have confirmed Gelman and Gallistel's theories about the counting abilities of pre-school children. The requirements of these tests are a reasonable match with those of the above theory and claim that the data collected measure the counting ability of pre-school children and their ability to reason about number. Both studies show that young children have an understanding of the basic principles of counting right from the start and that failure to put them into practice all the time is the result of a lack of skills which they acquire with experience and maturity. This can be concluded from the fact that all the children tested were able to carry out the required counting and understand the numerosity of sets when the numbers were small showing that they understand the basic principles of counting but make mistakes when they failed to put the principles into practice because the skills were not sufficiently well developed.

This evaluation would agree with Gelman and Gallistel (1978) that children who have mastered the counting process have not automatically developed an understanding of the reasoning principles. "Rather, counting provides the representation of reality upon which the reasoning principles operate" (pp.161). It is through the counting activity that children are able to apply a set of reasoning principles to reality and through practice in perfecting these skills they develop an understanding of the numerosity of number.

*5.2.4 Test 4 **Reading Numbers.** cf **Chapter 2 pp** 67*

What do children know about reading and writing numerals before they receive formal instruction at school and how is this knowledge acquired?

The aim of the following task was to assess children's ability to recognise written numerals (Saxe 1987). Most children tested recognised the numerals from 1 to 5 or 6 with others reading the numbers to 10 but very few understood the numbers larger than 10. All the children tested appeared to recognise the symbols as those which give information about quantity or tell of a numerical concept. This was gleaned from the conversation that accompanied the responses and the errors they made. Errors showed a confusion between 6 and 9 and a lack of understanding of the decade system so that 12 was referred to as 'a one' and 'a two'. Others said that they knew that 27 was a big number but could not name it.

Lavine (1977) showed how children's perception of different kinds of writing progresses from the recognition of an overgeneralization of conventional writing units to a more specific recognition of various aspects of writing. From these studies, with children aged 3 to 6 years, she showed that prior to instruction, children aged 5 are capable of distinguishing number-shapes from letter shapes. Children participating in this research appeared to understand that the conventional writing units presented a number and not a letter but this could not be proved as no letters appeared on the pictures.

Sinclair and Sinclair (1984) looked at children's varied ability to recognise written numerals and concluded that age and item differences play a part in young children's interpretation of written numerals. They saw little connection between children's ability to represent numbers in the written and verbal form thus concluding that the links between the two may be quite difficult to understand. These studies corroborated the findings of this research where children who had a good counting knowledge were not necessarily able to recognise and name more written numbers and conversely those who showed good recognition of written numbers were not always proficient at counting.

The age and item differences to responses of written numbers was not assessed in this research as the children's ages were similar and the items used were familiar to the children and well within their frame of reference.

Wright (1991) suggests that home influence would affect numeral recognition as parents are more likely to provide experiences for their children on this topic. Lavine (1977) finds that children learn more about number recognition on their own through exposure to graphic material and a natural fascination and interest for the written word.

These ideas would then account for the varied ability of children to recognise and name numbers.

5.2.5 Test 5 Written Numbers. cf Chapter2 pp. 68

The written representation of numbers elicited a wide variety of responses that were similar to those recorded by Hughes (1986) when he assessed this aspect of arithmetical concepts of pre-school children in Britain. Hughes saw the development of conventional number representation following a pattern that could be divided into four phases. The first phase was the idiosyncratic response where the representation of the number was not related in any way to the number of objects presented but simply a scribble covering the page. None of the children in this study fitted into this stage. The pictographic response was used by four children who drew the block shape to represent the numerosity. Eleven children used iconic responses by drawing shapes or tags to represent the number. Hughes sees this as a response to the one-to-one correspondence which satisfies the most important requirement: that of numerosity.

Most of the children used the symbolic response which was an attempt to write the conventional numerals. These results agree with Hughes' findings that a high percentage of pre-school children aged 5 and 6 years use pictographic and symbolic responses and that their method of response is consistent showing only one type of response repeated for each representation of a number. Hughes also recorded that accuracy was not always achieved as children miscounted the number of blocks to be represented especially when larger numbers were displayed. This was found to be a common error in this study.

A number of children responded in ways described by Sinclair, Siegrist & Sinclair (1983) in their assessment of 45 children aged 4 to 6. One child fitted the notation-type 2 described as an attempt to represent the object kind without any indication of quantity. This was shown by the fact that she simply drew the block shape without paying any attention to the representation of quantity. Sinclair et. al. placed children in the iconic or notation-type 3 stage if they used numerals instead of abstract graphic symbols to represent the one-to-one correspondence. To represent 4 blocks the child wrote 1234. A number of children tested in this research used this type of representation. This seemed to show that they were between the iconic and symbolic stages suggested by Hughes and perhaps there is a need for an intermediate stage when the child knows the value and graphics of the conventional number but still needs to represent it in the one-to-one correspondence form.

The most common error made in writing the conventional number symbols was the reversal of the figures, especially for the numbers 2,3,5,and 7, which confirms the findings of Hughes (1986) and Sinclair et al.(1983).

These findings are coherent with those of Hughes who developed a clear picture of the ability of pre-school children to represent small quantities based on one-to-one correspondence or symbolic representation.

5.2.6 Test 6 Counting on Forwards and Backwards. cf Chapter 2 pp. 52

Results of the child's acquisition and elaboration of number word sequences were closely related to those of Aubrey (1993) and Fuson, Richards & Briar (1982). They found that the largest percentage of children tested were able to give the number that came after the randomly presented digits up to 10. (82% of the children tested in this research) A lot fewer were able to say what number came before randomly presented digits up to 10. (40% of the children tested in this research). In each case the numbers from 10-20 proved to be more difficult than the numbers below 10. Both Aubrey (1993) and Carpenter et al. (1988) see this as a development towards the use of more abstract and adaptable strategies. Leushina (1991) recorded that many children are able to name the next number but still cannot name the preceding one because for them the natural number sequence seems to be moving forward. These children have formed a 'spatial image' of the natural number sequence but have not mastered a clear notion of the different relations between the *before* and *after* numbers. For Leushina this demonstrates that these children have not yet developed the ability to recognise the number sequence as a concept. This would certainly appear to be the case with the children tested in this research.

Wright (1991) used a similar method to investigate the pre-school child's knowledge of number word sequences but children were graded into five levels according to their capabilities and results were recorded separately for forward and backward number sequencing. Although Wright concluded that "there appears to be no theoretical reason why the construction of 'Backward Number Word Sequence' must lag behind 'Forward Number Word Sequence" (pp.4), he nevertheless considered it important to use separate tables to record children's ability. The descriptions of the five levels for each type of counting were similar. Fuson et al. (1982) assessed the two types of counting together as they thought that "they seem rarely to be separately acquired but rather result from a slow and laborious production from the forward sequence" (pp.68).

However one assessed the pre-school children on their ability to say the number after or before a given number, all the research (Wright 1991, Aubrey 1993, Young-Loveridge 1989 and Leushina 1991) agrees that the development of backward number word sequencing lags behind the development of forward number word sequencing. Wright reports that both will develop at the same time if specific instruction is given.

Most of the children tested in this research counted from one each time they were asked to give the number after or before a given number, either using their fingers or softly counting aloud. Wright grades these children on Level 2 and sees the progression to level 3 as the ability to produce the number word 1-10 without dropping back.

5.2.7 Test 7 Addition and Subtraction. cf Chapter 2 pp. 55

Many researchers in Britain, United States and New Zealand have shown that pre-school children have considerable abilities in the area of simple addition and subtraction, provided that the quantities are small (Hughes 1981, Brush 1978, Young-Loveridge 1989, Carpenter & Moser 1984, Aubrey 1993 and Gelman & Gallistel 1978). The present study proves this to be an accurate assessment of pre-school children's ability.

Gelman's magic studies (Gelman & Gallistel 1978) showed that young children realise that to join two sets the numerosity changes and therefore to find the new value the child uses the same procedure as for a single set and counts. This was true for almost all the children tested. Only a few knew some of the combinations and were able to give the answer without counting, e.g. 4=4 and 5=5 (Ilg & Ames 1951 and Carpenter & Moser 1983).

Carpenter & Moser (1983) investigated the strategies used by the children to solve the addition and subtraction problems and found that there were three basic levels. Almost all the children in this research sample used the most basic strategy - that of 'counting all'. They either physically joined the two sets by moving the sweets together or counted the total without physically joining the sets. To apply the one-to-one correspondence rule, children pointed to each sweet as they counted. Only one child counted as her eyes moved along the row of sweets. A few children used the second strategy - that of 'counting on'. This showed that they recognised that it was not necessary to reconstruct the entire counting sequence and so they began counting forward from the first addend in the problem. An advancement on this strategy is the ability to count forward from the larger of the two addends but none of the children tested in this research used this strategy. Four children used the third strategy but only for numbers where the two addends were equal e.g. 4=4 and 5=5 showing that they knew the number combination. Carpenter and Moser (1983) record that these are the first number combinations that are learned. "These solutions usually are based on doubles or numbers whose sum is 10" (pp.21).

The findings, showing that young children use strategies based on counting to solve addition and subtraction problems, confirm what others have found (Hughes 1986, Gelman and Gallistel 1976, Carpenter and Moser 1984 and Starkey & Gelman 1982). However, Carpenter and Moser (1984) point out that children are not always consistent in their choice of strategy and often use the most efficient one and then revert back to a less efficient strategy of counting-all. The development from one strategy to a more efficient one appears to be part of children's natural problem-solving strategy, (Groen & Resnick 1977) and unlike Piaget's theory, there can be no clearly defined stages which children enter and exit as they move to higher cognitive domains (Starkey and Gelman 1982).

Subtraction strategies also follow the same three levels (Carpenter & Moser 1983). Again these findings record that pre-school children operate on the first level and therefore remove the smaller quantity from the larger quantity and count the objects remaining. None of the children tested used the 'counting down from' strategy where the child counts backward beginning with the given larger number. The backward counting sequence contains as many counting number words as the smaller number in the subtraction problem. As children develop more advanced levels of strategies to solve addition and subtraction problems, there is an increase in abstract thinking and more flexibility in choice of strategy. The numbers used in the addition and subtraction problems were all below 10 and this may have influenced the fact that children scored higher on the subtraction (Aubrey 1993). Hughes (1986) and Young-Loveridge (1989) record that pre-school children were more successful with addition than with subtraction when larger number problems were presented.

The findings of this research were based on tasks which presented concrete physical objects visible at all times (Aubrey 1993). However, Hughes (1986) and Young-Loveridge (1989) tested pre-school children's addition and subtraction strategies and abilities using hidden objects thus requiring a certain amount of abstract thought. Both reported that pre-school children showed considerable ability when numbers involved were small.

As Aubrey (1993) noticed, children were not always accurate in their counting and errors were caused by not adhering to the one-to-one correspondence principle. Ilg and Ames (1951) reported that errors were mostly +1 or -1 and increased with the addition of larger numbers, and this was found to be so.

Starkey and Gelman (1982) assessed young children's ability to add and subtract using non- perceptual and noncounting procedures and concluded that by age 4 and 5 they were able to correctly solve simple inversion and compensation problems. (cf. Chapter 2 pp.61) This strategy was not applied by any of the children in this research.

5.2.8 Test 8 Division and Multiplication. cf Chapter 2 pp. 62

Most researchers who have evaluated the number knowledge possessed by pre-school children have not determined whether the operations of multiplication and division figure in the pre-school child's numerical reasoning (Wright 1991, Hughes 1986, Young-Loveridge 1989, Williams 1965, Scharz 1969, Bjonerud 1960 and Gelman & Gallistel 1978).

An understanding of multiplication and division is thought to develop after the understanding of addition and subtraction (Gelman & Gallistel 1978). The development that leads to an understanding of multiplication and division is further thought to be influenced by cultural and environmental factors which put pressure on the individual to discover numerical reasoning principles to overcome practical difficulties with large and accurate counts. In modern times, this development would depend on the instruction given in schools and this would account for the poor performance of children in this research evaluation as these children had had no formal schooling.

As with this research, Aubrey (1993) included two multiplication problems without concrete material and found that few children displayed the ability to think in the abstract, not even when small numbers were involved. Perhaps because children in this research were slightly older, there was a higher percentage of correct answers for the problem involving smaller numbers (63%) whereas only 5% answered both questions correctly.

Division is thought of as the inverse operation of multiplication, and is therefore dependent on an understanding of it and more difficult to solve than its source operation (Gelman & Gallistel 1978). "Multiplication and division should be taught simultaneously once the child has achieved reversibility of thought" (Copeland 1970, pp.113). These theories may apply to the abstract principle of division which has always posed considerable difficulties even for talented mathematicians. However, when used in the context of a practical social sharing activity it has vastly different consequences and questions the relationship between early sharing behaviour and the more complex mathematical idea of division (Desforges & Desforges 1980). Do young children have an idea of the numerical value of 'sharing' or is it simply a task that the child performs without realising how many objects there are, or how many they are to be shared between? (Williams & Shuard 1970).

The results of this research refute the ideas of Williams & Shuard (1970) who saw no connection between social sharing and mathematical sharing. Rather it is claimed here that the young children in this sample showed a strong number-based understanding and approach to the idea of sharing, especially when dealing with small set sizes (Desforges & Desforges 1980).

As described by Aubrey (1993) and Desforges & Desforges (1980), three types of strategies were used to solve the division or social sharing problems presented in this research. Sixteen children used the first strategy which involved distributing the set one by one between the bears until all the sweets were used up. The rest of the children used the other two strategies of either dividing the whole set into equal portions or into small groups of 2 or 3 and gave one portion to each bear. A few children used a combination of these strategies being influenced by the size of the set. For example when 6 was divided by 3, they gave each of the 3 bears 2 sweets but when 9 had to be divided by 3 they reverted to sharing one by one.

There were those children who dealt out the objects and assumed that dealing would lead to a fair answer and made no attempt to check or count the sharing process (Desforges & Desforges 1980). Then again others used the same strategies but accompanied the whole process with careful checking and counting thus showing an overtly number based idea of sharing. The sharing problem that required a strategy to deal with the remainder further proved that many children (nineteen) saw the need to involve numerosity in the final answer. These children either asked for another sweet to 'make it fair' or wanted to cut the remainder in half or simply removed the extra one by keeping it in their hand or placing it in-between the two bears or giving it back to the researcher. Children who simply added the extra sweet to the one bear's portion showed that they had not considered the number value of each set.

Frydman and Bryant (1988) showed that not before age 5 were pre-school children able to share discontinuous material using the one-to-one correspondence and incorporate numerical information on the size of sets. All these children tested were 5 years and older so this research appears to agree with this idea.

These results then corroborate the findings of Desforges & Desforges (1980), Aubrey (1993) and Frydman & Bryant (1988) which suggest, that prior to teaching, pre-school children have a number-based notion of sharing which is closely related to division. The everyday activity of sharing plays an important part in the study of the child's growing awareness of number and quantity.

5.2.9 Test 9 Estimation. cf Chapter 2 pp. 72

Estimation appears to be a foreign concept for young children. They found it difficult to carry out the task because their natural reaction and desire was to count (Aubrey 1993). This demonstrates their need for accuracy and the use of counting to represent number. Because the arrangement of the array of objects gave no clue as to the number, children could not use perceptual clues or subitizing to accurately record the numerosity of the sets.

As Gelman and Tucker (1975) found, children were more accurate when estimating small sets when their chances of counting were enhanced i.e. clearly displayed objects such as the oranges which facilitated counting. Fourteen children accurately estimated the number of oranges in the set of six whereas only four estimated the set of ten sweets that were displayed in a group on a plate. Aubrey (1993) also recorded a high percentage of accuracy on the first small set. The results showed that most children (37) were able to differentiate the set sizes and recognise that the second set contained more items than the first and therefore could assign a number word that comes later in the number word sequence to it even if the number word was not an accurate representation of the set. "The answers may be inaccurate, but they are orderly" (Gelman and Gallistel 1978, pp.62).

This again confirms that children are sensitive to number differences before they can make accurate number judgements (Smither, Smiley & Rees, 1974). In this way young children show some knowledge of the ordinal properties of number and their verbal representation. "This is the first demonstration that pre-schoolers are sensitive to the ordinal characteristics of larger numerosities and the ordinal characteristics of the number word sequence and the conventional relation between the two." (Gelman & Gallistel 1978, pp.62). The 'magic game' described by Bullock and Gelman (1977) offers further evidence that children aged 2½ to 5 years have an understanding of ordinal relationships (cf Chapter 2).

According to Gelman and Gallistel (1978) these children were able to estimate the numerosity of the set, e.g. give an approximate representation of the number. But if an exact representation is expected, then a number abstractor is required and this changes the requirements. Similarly the 'reasoning principles' used suggest that they were able to recognise the relation between the two sets and ascertain that one was greater than the other.

The research of this thesis confirms what many others have shown when analysing what children understand about small set sizes (Aubrey 1993, Gelman & Tucker 1975 and Wright 1994). Generally children aged 5 are able to estimate the number value of sets of 4 to 6 items but very few could enumerate sets larger than 6. It has also shown that pre-school children are able to represent larger set sizes by number words that come later in the serial list (Gelman & Gallistel 1978). The importance of counting as described by Gelman (Gelman 1972) appeared to be a salient behaviour that showed the role it plays in the way young children think about number. One child said that he could only give the answer if he could count the items and offered no response when the time of exposure did not allow for this. Fuson (1992) commented on the strong urge in many 5 and 6 year olds towards counting which makes them want to count even when objects are hidden.

It is widely accepted that activities with spatial patterns can make an important contribution to a child's numerical development (Harte & Glover 1993, Wright 1994 and Bjonerud 1960), and is a way of determining the child's development of number knowledge and the cardinality of a set. Yet Gelman and Gallistel (1978) only see the value of obtaining the numerosity of a set by counting which is certainly the preferred and seemingly natural way of the young child.

5.2.10 Test 10 Algebra Tasks. cf Chapter 2 pp. 76

5.2.10.1 Patterning. cf Chapter 2 pp. 84

According to Sime (1973) the ability to sequence shapes into a pattern is basic to mathematical insight and lays the groundwork for logical thought. Yet the children in this research appeared to lack the ability to sequence the shapes even though they had shown a range of abilities in other number activities. As many as 55% of those tested could neither copy nor continue a sequence using two different shapes. Aubrey (1993) also questions whether activities such as sequencing bear any relationship to the child's knowledge of number. She found that more than 50% of the pre-school children she tested could neither copy nor continue a pattern of two or three shapes.

A patterning activity as presented in this research is what most children found too difficult to complete. Most researchers emphasise the part played by language (Sime 1973 and Copeland 1979) which often accounts for errors in applying a concept rather than an inability to perform the task. This research showed a marked improvement on the second activity involving three different shapes where 88% copied the pattern and 33% copied and continued the pattern. What is not sure is whether the instructions were not understood the first time round and only became clearer on the second attempt or whether the children found it easier to recognise and repeat a pattern using three objects rather than two objects.

Four children gave no correct responses to any of the sequencing activities. The only other test in this research that elicited a nil response was the forward and backward number sequences. This observation may demonstrate the place of language where the concept has not been understood and the child is unaware of what is required. The words used by the researcher would have included 'pattern', 'copy' or 'repeat' which may not have been part of the child's vocabulary.

5.2.10.2 Shape. cf Chapter 2 pp. 87

Children's idea of shape showed that they were all familiar with the four regular shapes - sphere, cube, cuboid and tetrahedron and their description of these shapes was a combination of formal and informal responses. Aubrey (1993) found similar responses: for example the sphere being described as a round, a wheel, a ball, a circle and three children giving it the correct geometric term of a sphere. (All three children had been at the same pre-school for the past two years which suggests that it had been learnt at school.) Similarly the cube was referred to as a square and the cuboid as a rectangle or a block.

All the children were able to distinguish the various shapes from one another even if they were not able to name them (Robinson 1975). They had visual discrimination and could use perceptual awareness to differentiate one from the other. Further more only two children made errors when matching 3-D to 2-D shapes and incorrectly matched the cuboid and cube showing that children of this age have developed a sound knowledge of shapes and are able to discriminate between them.

When asked to draw the four regular shapes, most children had little difficulty and with one or two movements they reproduced a fairly accurate shape. Gibb and Castaneda (1975) found that children progressed from topological geometry to euclidean geometry: - a development from seeing shapes as closed with little attention to sides and corners to a recognition of shape and angles. For Copeland (1979) and

Piaget (1952) children up to the age of 7 are still at the topological level and are not able to understand euclidean shapes which stress the number of sides, length of sides and angles. However, the results of this research would rather agree with Robinson (1975) who considered children's increased ability to be evidence of development of geometric ideas rather than improved muscle control. He found that pre-school children had some idea of direction and line and were able to draw the square and rectangle with horizontal and vertical lines and accurate corners. The circle was the easiest to draw with only one child lifting the crayon to draw it in two strokes and thus ending up with an oblong shape. Robinson explains that this is the first shape which children aged three are able to draw in one stroke, stopping after a single revolution. He also noted that pre-school children find it difficult to draw the oblique strokes and even up to 6 and 7 years of age find the equilateral triangle difficult to draw. Eight of the children tested drew vertical or horizontal lines and could not get them to meet in the required shape of a triangle.

Only one child appeared to be at the topological geometric stage and drew simple closed figures where shapes were not rigid but rather stretched to take on the rough outline of the required shape with no corners or straight lines.

5.2.10.3 Measurement. cf Chapter 2 pp. 89

Piaget's (1952) experiments to test a young child's understanding of measurement showed that pre-schoolers were unable to compare the size or quantity of objects because they lacked the knowledge to conserve number i.e. to recognise the constancy of matter over given perceptual transformations. Young children were inclined to not compare the right things and were deemed to lack understanding of the basic idea of measurement. A total dependence on perceptual judgement seemed necessary as these young children had not developed the logical thought process that would enable them to conserve number and apply measurement procedures.

Only two participants in this research showed no understanding of the language or concept of measurement. Most of the children used perceptual clues and were ably to demonstrate a good understanding of the language of measurement (Aubrey 1993). They sorted the strips of tape and laid them on the floor in order of length, quickly naming the longest, shortest and those of the same length.

Copeland (1979) sees this as the first stage in the development towards an understanding of measurement. A concrete activity which applies a visual estimate with no accurate use of a measuring instrument. According to Copeland this cannot qualify as a true understanding of the complex and elaborate concept of measurement.

However, five children showed conservation knowledge and before comparing the length of the pieces of tape they made sure that all the ends were level thus obtaining an accurate measurement. The ability of these children indicates that an understanding of the concept of measurement can become a reality well before the formal operations stage at about 11 years as indicated by Piaget (1952). Bryant & Kopytyska (1976) also found that children aged 5 were able to measure the depth of a hole using a stick and on a further three different experiments the children's measuring ability confirmed this result.

Almost all the children tested showed a basic understanding of the concept of measurement when presented in a practical situation and when not weighed down by numerical distracters. After all, measurement need not only be a mathematical experience when numbers are included and accurate measurements are made (Robinson, Mahaffey &Nelson 1975).

This research shows that pre-school children have a sound knowledge of the practical uses of measurement and can move on to more precision in comparing quantities with the introduction of numbers and a measuring unit. According to Bjonerud (1960) these children possess a high degree of understanding of terms describing premeasurement concepts.

5.2.10.4 Ordinal Numbers. cf Chapter 2 pp. 52

The results of the test concerned with ordinal numbers were very similar to those recorded by Young-Loveridge (1989), Bjonerud (1960), Brace &Nelson (1965) and Williams (1965). They all found a very high percentage of children knew the first position which in this research was rated as 98% with the middle and last position gaining well over 50% and a considerable drop in the percentage of correct answers for the second position -between 35% and 53% . There was a marked drop in the number of those who knew the ordinal numbers of third, fourth or fifth. This was the one test that seemed to show little relationship between the ability to count and a knowledge of ordinal number which perhaps indicates that this is not a good judge of the child's understanding of number but rather an example of exposure to the language of number (Brace & Nelson 1965). Fuson (1992) has pointed out that many languages determine this special numerical context by using entirely different number words or by adding special letters to the usual counting word. Children who could count readily and conserve number had great difficulty with ordinal tasks because they did not know the ordinal words. In the United States, Beilin (1975) also found that children's knowledge of ordinal number lagged behind cardinal knowledge.

5.2.10.5 Spacial Awareness. cf Chapter 2 pp. 85

All the children participating in this research had acquired the three areas of spacial development as described by Leushina (1991). They were able to orientate towards themselves, away from themselves and away from objects. That is to say they were able to correlate surrounding objects with their own person; had the ability to use a system where the origin of reference was themselves and also to orientate away from objects thus making the object the origin of reference to which the spatial situation of other objects is determined.

The test showed that all children tested at this age could move their bodies to positions determined by the spacial situation of another object (a chair). Six children made one or two mistakes with positions such as next to, in front or underneath but were able to correctly position themselves for other commands such as behind and on top.

5.2.10.6 Classification. cf Chapter 2 pp. 76

All the children tested in this research were able to accurately classify and sort objects of four different materials and shapes. Does this imply that they are competent to think logically and display mathematical skills or is this an automatic and natural reaction with an understanding of the words 'put together', 'alike', or 'belong together' being the only knowledge required? (Gibb 1975).

Certainly the requirements of this test were straight forward and allowed all children to meet the criteria of Piaget's 'pre classification stage'. This meant that they could simply sort objects according to their visual form which gave them a collection of objects that looked the same. But Piaget (Sime 1973) claims that this is not true classification but rather a process of sorting things into a collection based only on perceptual structures which depend on sensory motor schema and not on logical thought. This then is not a display of mathematical skill. Perhaps this is an accurate assessment as the test only called for sorting according to visual likeness.

According to Piaget (Sime 1973) the development to the next stage only comes when a child enters formal school and is then able to sort elements into their major classes such as colour, shape and size. A number of children sorted the coloured beads into their colours while those less able simply put all the beads in one collection. Certainly the visual form played a major role in determining the classification for all children with some paying closer attention to detail such as finding shells with the identical shape or patterning. These children had progressed to the second stage- the 'quasiclassification' well before entry to formal school, thus proving to be more able than Piaget would have them be.

Rosch (1976) and Chi (1983) emphasised the importance of the nature of the stimuli used in the classification task. Younger children (those in the kindergarten) find it easier to sort objects on the basic level which only differentiates according to the visual stimuli (as was the case in this research), whereas older children use the superordinate criteria to divide objects into categories such as clothing, vehicles, and furniture. Classification is therefore an activity that displays the knowledge a young child has and his/her ability to represent that knowledge by the criteria he/she uses to classify objects. If one accounts for this availability of content knowledge, and the nature of the stimuli, then it will be found that young children have the ability and skills to classify and sort items. Rosch (1976) agrees then that pre-schoolers are able to classify objects if requested to sort according to basic criteria only. The items used in this research did not allow for the children to classify in ways other than by visual discrimination so the ideas of Rosch (1976) and Chi (1983) could not be assessed.

Gelman & Gallistel (1978) point out that classification tasks require the child to sort a set of stimuli according to attributes that the experimenter defines as correct and if the criteria for classification are not understood it may be interpreted as an inability to classify. In the case of this research the criteria for classification were basic and straight forward with little chance for misunderstanding. This would account for the high degree of accuracy shown by all children.

5.2.10.7 Sequence of Events. cf Chapter 2 pp. 96

The last activity focused on the child's ability to 'read'' the pictures accurately and then place them in the correct order according to the timing of the sequence of events and to discuss the time of day when it was most likely to have been performed. Most of the children showed adequate use of language to describe the scene and discuss the order in which they would carry out the routine. All were familiar with the terms ‘morning', 'night', 'before bed', 'after breakfast', and 'before supper'. Six children needed assistance and encouragement to assess the pictures and after questioning by the researcher they were guided towards completing the sequence in the correct order. These children had all scored below the average on most other test items perhaps indicating that this activity requires an ability to think logically and understand the concept of time.

Leushina (1991) points out that the development of temporal ideas increases during the pre-school years and depends on the child's general mental and speech development. This research found that the low achievers in this test lacked the language ability to express themselves adequately and their mental competence was below average on most other test items. As in all other areas of this research, children's potential to master the various temporal ideas and concepts is there but its development will depend on the experiences of the environment and guidance given by parents and teacher.

5.3 Conclusion

The results of these tasks show that reception class children, aged 5 years, from working class homes have considerable knowledge about numbers and that there is great variation in the amount of mathematical knowledge that children acquire before starting on the informal mathematics programme of the pre-school (Aubrey 1993, Wright 1991 and Young-Loveridge 1989). The demonstration of such early competencies and the wide range of number knowledge within one age group questions whether the reception class curriculum has accounted for this phenomenon in the construction of a programme that will best develop and extend the mathematical concepts of these children. Teachers need to be made aware of the informal mathematical knowledge brought into school, the wide range of ability, children's invented strategies for solving number problems and the stages through which they pass in their development of number concepts. For any learning experience to be worthwhile, activities need to be planned to offer children opportunities to extend their knowledge of number facts and stimulate logical thinking on a level that is sufficiently challenging. For this to be successful there needs to be a clear understanding of each child's level of development so that new learning is built on existing knowledge and children move at a pace which is appropriate for their individual rate of learning (Young-Loveridge 1989).

Wright (1991) points out that differences in children's mathematical competencies can be attributed to innate abilities and the opportunities for mathematical experiences provided by parents rather than pre-school experience or the lack thereof. Home influence has the greatest potential to bring about advancement in number knowledge that children develop prior to starting school. It therefore stands to reason that even if a group of children are from the same social class there will still be a wide range of abilities in all areas of the curriculum and these need to be accounted for. Young-Loveridge (1989) and Wright (1991) both emphasise the serious implications for teachers who start all children at the beginning of a programme and take them through every activity, regardless of whether it is appropriate for their level of achievement. When classroom mathematics activities are not well matched to children's current level of mathematical attainment, then achievement is lower.

Consistent with other research, (Geary 1994 and Brace and Nelson 1965) my exploratory analysis showed that there was no significant difference in the mathematical ability of boys and girls. Their total scores showed the same range of ability and there was little difference on individual items. However the results coincided with those of Young-Loveridge (1989) who found that the overall average of girls was slightly better than for boys. (X=65% compared with 62%) There was no evidence to show that boys were overrepresented at either end of the scale of ability levels as was suggested by Wright (1991).

The children who achieved high scores in this research would be able to master with relative ease the mathematics programme prescribed for children in the reception class in Kwazulu Natal, South Africa. These children had the ability to use mathematical language appropriately and with understanding, count showing knowledge of the sequence of number words and their quantitative value, recognise numerals and represent them, sequence events and objects in correct order of size, time or pattern and differentiate shapes and position in space. From this research it would appear that most of the children tested already have a clear understanding of the mathematical concepts that are presented in the school readiness programme. Even those children who did not score highly showed a wide range of informal competencies and an individualistic approach to solving number problems. Their strategies used for solving problems varied from concrete visual manipulation of number to more formal and even abstract calculations. For these children inaccuracy increased as the numbers became larger and the tasks more abstract.

Piaget's stage theory restricts teachers' efforts to only presenting activities associated with that stage which the child is at. By recording what mathematical competencies and abilities the young child has and the strategies used to solve these problems, one is lead to follow the approach of many educationalists such as Wright (1991), Aubrey (1993) and Young-Loveridge (1989) who accept the teachings of Vygotsky and stress the principles of his work. The 'zone of potential development' is the level of learning that the child has not yet attained but is likely to attain in an interactive teaching situation. With instruction the child will progress to this higher level and therefore teaching has a crucial function in children's qualitative advancements of mathematical knowledge (Leushina 1991). Under these circumstances there is a need for the level of instruction to fit the actual abilities of the children being taught. "A child's chronological age may be only a very slight clue as to the stage at which he is able to function in arithmetic" (Ilg & Ames 1951 pp.25). Besides the informal observations which teachers make, a more accurate and systematic assessment is required for planning mathematics instruction so that learning activities will move children from covering skills and concepts which they have already mastered and provide opportunities for incremental learning and cognitive growth. If the cognitive potential of pre-school children is considerably more extensive than had been previously supposed, then it is necessary to know how this potential can be most effectively used. Rather than relying on spontaneous development with maturity, instruction can influence this process and accelerate growth. Studies have shown that pre-school children achieve higher levels in distinguishing attributes of objects if they are instructed than they would otherwise achieve (Leushina 1991). It is the teacher's task to organise children's activities so that they present a new problem which requires the mastering of a new method of action, behaviour, or thought. The gap between what they know and the unknown causes a conflict which is the motivating force in development.

All this points to the importance of an assessment of children's abilities to structure learning on an appropriate level so that each child develops at his/her own rate and builds on previous knowledge thus forming a sound foundation on which to develop. "In my view these problems derive in large part from insufficient attention to explaining the connections between a new procedure and the knowledge the child already has" (Sophian 1992 pp.33). Mathematical knowledge must be taught in a strictly logical order, guiding children's actions and operations with mathematical material so as to develop a system of knowledge, abilities, and skills. Mathematics is a chain of knowledge that is broken when one link is missing. During play and work situations in everyday life, children interact with adults and acquire knowledge and abilities which develop their minds. However this form of development is fragmented and uncoordinated. Instructional lessons in mathematics provide a structured set of knowledge and abilities in a sequence and system of increasing complexity which develops children's thinking and promotes their understanding of the value of the knowledge they have acquired and reinforces their faith in their own ability (Leushina 1991).

The tests in this research confirmed the great range of individual differences in mathematical ability as reported by Aubrey (1993), Ilg & Ames (1951), Young-Loveridge(1989), Wright (1991) and Williams (1965). Considering what has been mentioned above, these individual differences call for a differentiated course of study for pre-school children to provide for their wide range of needs. Aubrey (1993) suggests that perhaps there is a need for individual tutoring in early mathematics just as this approach is accepted in the development of flexible reading strategies. These findings confirm the ideas of Young-Loveridge (1989) who looked at the serious implications for teachers who chose to take a 'lock-step' approach to teaching mathematics by starting all children at the beginning of a programme and taking them through every activity regardless of their level of mathematical ability. The practicality of this idea may be questioned but there is scope for some sort of differentiation in the mathematics programme to cater for this wide range in the levels of number knowledge. It is however important that teachers are aware of these differences and organise activities that offer opportunities to use the problem-solving skills children already possess so that their knowledge of number facts can be extended. Leushina (1991) emphasises the importance of the proper individual approach to ensure the presentation of new material at the correct tempo and level of work which will enable the child to achieve his/her maximum potential.

> *"In working with a group of children the teacher should study and know every child: the development of each child's memory and attention span, the rapidity of each child's perception of visual and verbal material, the nature of each child's interests and thinking, the degree of independence in practical activity and thought, the quality of each child's knowledge and level of general development, as well as mathematical concepts and speech, imagination, creativity, emotional-volitional manifestations, social orientation, and so forth"* (Leushina 1991, pp.180).

Williams (1965) suggests that this wide range of differences in mathematical achievement necessitates intraclass grouping of pupils according to their level of mathematical achievement which is judged from constant evaluation of the pupil's progress.

In order to design an appropriate reception class curriculum, the educationalist realises the need to understand the cognitive development of the pre-school child not from what he/she is not able to conceive as compared to the capacities of older children, but rather to experimentally uncover what the pre-schooler *can* do. Cognitive development proceeds in stages, each one integrated hierarchically into the subsequent stage. The manner in which each stage is connected may vary. The first stage may serve as a catalyst, a component, or a scaffold, but only a careful description of the accomplishment of both earlier and later stages will enable one to understand how the development takes place and so assist with this process. Gelman and Gallistel (1978) therefore stress the importance of empirical investigations that aim to start from the evaluation of mathematical knowledge of pre-schoolers to understand how they have progressed to this level. To see what they cannot do that older children can do will give us no idea of how these two stages are linked and how the development took place. For instruction to be meaningful, curriculum content and sequence should reflect the existing forms of children's mathematical competencies and knowledge and strategies used to solve problems.

Chapter 6 Conclusion

6.1 Introduction

"There can be little doubt that children enter school with considerable knowledge and understanding about numbers. The key question is how is this knowledge relevant to children's mathematics learning at school" (Young-Loveridge 1987, pp.163).

The scope of this chapter is to relate the findings of this investigation to the historical research of theorists and to point out how this knowledge could be of practical application in the educational system in South Africa today. The strengths and limitations of this study are emphasised and suggestions made as to how further study in this field would offer more conclusive evidence to encourage and convince teachers of the need to assess more accurately the levels of mathematical ability and competencies of pre-school children. This knowledge will guide teachers in their methodology and curriculum planning.

This research and that of other educationalists such as Hughes (1986), Wright (1992) and Aubrey (1993) has shown that young children display impressive mathematical ability before they start on the informal mathematics activities of the school readiness programme. Most of the children tested were able to display counting strategies, use conventional or invented systems of written number notation, read numbers, carry out simple addition and subtraction and social sharing using concrete apparatus, resolve multiplication and estimation problems with reasonable accuracy, and demonstrate a sound knowledge of geometric concepts of shape, space, measurement, patterning and sequence of events. They appear to be competent users of number when account is taken of their limitations: they are generally restricted to working with small numbers, are clearly influenced by the context in which problems are presented and their ability is affected by the environment and home situation of the years prior to entering school

(Hughes 1986). Researchers who do not take cognisance of the terms on which young children need to be assessed, are open to misinterpretation of the results and an unfair evaluation of the task presented.

6.2 Implications for Theory and Practice

Young children appear to be fascinated by number and show a natural enthusiasm to count and use number in the language of their everyday activities. Most young children demonstrate that they have the perseverance and interest to grapple with mathematical problems and use logic and knowledge of experience to calculate the answers. There is, however, such a contrast between this stage of development and the formal mathematics of the school situation where children battle to comprehend calculations with number. Perhaps there is misrepresentation of the stages of development with content not matching the acquired ability or not enough attention is being paid to individual differences in ability and strategies used to solve problems. Instruction needs to be informed by theories and methodology which have moved away from the idea that mathematics concepts occur naturally and spontaneously according to well defined stages of development. Somewhere along the line children have become lost either because they see no relevance to everyday life or meaning to the activities, or they are not allowed to solve problems in their own way. They are expected to carry out calculations which are not taxing enough or they have not understood the initial concept and have 'lost their way'. In other words as Hughes (1986) emphasises, the aims and objectives of early mathematics education need to be redefined to attend to the important link between the informal concrete mathematical knowledge which children bring to school and the formal symbolism of the school curriculum (Hughes 1986).

Bryant (1994) emphasises the importance of the context in which young children learn mathematics. He cites the achievements of the Brazilian street children who demonstrate considerable number ability when in informal settings but find it difficult to transfer this knowledge to the formal learning of mathematics. There seems to be a gap in the research to bridge these two types of mathematical achievement. Similarly Young-Loveridge (1987) finds that "there is now substantial research showing how children develop and use strategies over the early years of school, little is known about the effects of instruction on strategy use or about the transition from informal invented strategies to the formal algorithms and memorised number facts which are learnt as part of the mathematics curriculum" (pp.164). The children involved in this research certainly displayed a sound knowledge of numbers and were able to use this to solve simple problems requiring calculations and showed their own inventiveness to use strategies which made sense to them. But when formal schooling begins, many of these children will begin to flounder and not understand the mathematics prescribed for the first year of school.

Hughes (1986) offers guidelines in a number of areas to reduce the gap between the informal stage that emphasises the use of concrete experiences and formal manipulation of symbols. Both these elements are important but there needs to put more emphasis on the *links* between the two. These links could be established by translating their own concrete knowledge into the new language of formal mathematics. This can be accomplished by the recognition of the informal strategies children possess when they start school such as the use of fingers and counting up or down the number sequence. These strategies are meaningful to the child and should be the basis from which mathematics education starts and be used before new strategies are introduced. The need to recognise these diverse strategies was emphasised by Ginsburg (1977) when he stated that; "We need diversity in teaching. At the same time we should stress methods that allow children to make a connection between their informal knowledge and what is taught in school" (pp.177). The children participating in this research displayed their ability to use strategies that were meaningful to them by counting on their fingers or discovering ways to manipulate the objects to calculate the answer to the problem.

Then there is the idea that young children have difficulty using the conventional written symbolism of arithmetic and are therefore not ready for it in the first year of school. However this research and that of Hughes (1986) has shown that young children have an amazing capacity for written symbolism even if it is their own invented symbolism. Steffe and Cobb (1988) disagree with this idea and see this as a stumbling block for young children who are well acquainted with verbal number sequences but have difficulty translating their ideas into written form. They recommend that all work with standard paper-and-pencil algorithms should be abandoned and replaced with work on the schemes counting-on, counting-up-to and counting-down-to. There certainly appears to be scope for plenty of oral abstract counting activities (Wright 1991), but these children demonstrated that they have a sound knowledge of the written number system and are quite ready and capable of interpreting number in the written form. Ginsburg (1977) describes young children as 'functionally illiterate with respect to written symbolism' yet they are proficient in informal arithmetic. This may be the case for very young children, but those aged 5-6 years who participated in this research could not be described as above. If the two are not linked in a meaningful way connecting the concrete with the operations it can lead to a dread of mathematics. It is through language that such a link can be made. "The mathematical words can also be used in the context of concrete objects and manipulations on them" (Ginsburg 1977, pp.179). With the ability shown by the children in this research, there seems to be a need for both the introduction of the language of mathematics and the expanding of the written number which children at this age are developing as their small muscle co-ordination strengthens and allows for more accurate movements.

Considering the findings of this study it is important to shift the approach to early curriculum development and instruction as referred to in chapter 1 pp.4. Much of the research in this field has given light to what teachers should be doing but not much of this has been done. Teachers need not be working in the dark or be uncertain of their actions because these ideas have been well documented and researched. It is now necessary for teachers to take notice of this research and act on it so that mathematics education for the young child will lay the best possible foundation for each child's future success in this field.

There are ways in which the problem solving approach can be used to provide a link between the child's environment and mathematical concepts (Groves and Stacey 1990). They stress the importance of oral, written and symbolic language in mathematics, pointing out that children need plenty of opportunities which involve them in action and discussion so that concepts can be developed and refined. Current school mathematics places less importance on speed and accuracy with more attention to understanding and the utilising those facts that are known. "The pace of technological change has also emphasised the need for future citizens to be flexible, creative, independent thinkers and problem solvers" (Groves and Stacey 1990, pp.6). Problem solving activities play an important role in the development of mathematical thinking by providing a link between mathematics and the young child's reality. Challenges that are firmly embedded in the child's reality will capture their imagination and entice active participation in problem solving activities. The role of the teacher is then to assist children to structure their learning and interpret their experiences but at the same time to allow them to express their perception of the facts in a way that has meaning for them. This type of rich mathematical environment enables children to build a strong foundation for their understanding and gives them the confidence to take responsibility for their own learning while stimulating them to think and be creative. In this way mathematics in the classroom becomes closely linked to children's mathematical experiences outside the classroom and the gap between the two is diminished. The structure of this research demonstrates that children have an interest in and enthusiasm for solving problems relating to number in their world of experience and may then, if encouraged, use their own strategies to find solutions to more advanced mathematical problems. All the children involved in this research were willing to attempt almost every item of each test and showed an enthusiasm to work conscientiously at every task.

Ginsburg (1977) offers three ways to narrow the gap between informal and formal knowledge. First there is a need to resist judging children on the results of written work but rather to give them opportunities to solve verbal problems involving real objects. Secondly, through informal interviewing, teachers should identify children's unsuspected strengths in mathematical thinking because every child has some kind of basic strength on which development can proceed. Then lastly the gap will be narrowed if instruction is organised to build on this strength even if it is an informal skill it will lead to a deeper understanding and so help to bridge the gap. All these three ways of ensuring that there is a natural progression demonstrate the need for a form of assessment that will facilitate this process by determining the levels of ability. Instruction will then be based on development from the known to the unknown.

Leushina's (1991) principle of accessible instruction looks at the level and characteristics of children's mental development to ascertain what and how they can be taught. This principle suggests that instruction should be designed to proceed from the easy to the difficult, from the simple to the complex, from the known to the unknown. This will ensure that new knowledge and skills are attainable and therefore give the children feelings of success and an awareness of their own growth which then increases interest in the subject of mathematics. More recent research has shown that pre-school children's early mathematics experiences can involve problem solving and abstract thought as applied to elementary concepts. This study recognised that young children are fairly adept at solving abstract problems as they make use of their own strategies which give meaning to the problem.

This research questions whether young children in the reception class of a pre-school are sufficiently extended in their mathematical development and whether the curriculum suggestions as described in the "Activity Through Learning Programme" are designed with a clear knowledge of the levels of ability of these children. The researcher would agree with Wright (1992) when he questions whether the typical prenumber and early number activities of the reception class programme should be the only type of arithmetic activities offered to children of this age. There certainly is value in providing activities of matching, sorting, pairing and ordering for experiential learning, language development and the understanding of important mathematical concepts and even to develop logical reasoning and discrimination skills. These young children could, however, be extended further by activities which challenge them to think in abstract form and solve problems related to everyday experiences. Wright (1992) goes on to suggest that the general purpose of many pre-school programmes has been that of initiating the children into the processes of learning in the small-group situation directed by the teacher with emphasis on the development of social behaviour and self-discipline rather than tackling of mathematical problems.

We are reminded that "children do not need to be made ready for elementary arithmetic: they are already interested and engaged in it" (Ginsburg 1977, pp.74). This fact was certainly evident during the sessions spent with each child as he/she worked on the tests in this research programme. Young-Loveridge (1989) argues that mathematics in the pre-school year is not sufficiently challenging as the curriculum is not well matched to the skills and competencies of the children. Further evidence provided by Romberg and Carpenter (1986) points out that research on addition and subtraction shows how current pre-school programmes fail to capitalise on the rich informal mathematics that children bring to instruction. It is therefore not necessary to defer instruction on word problems until computational skills have been mastered but rather that word problems can be used as a basis for developing mathematical concepts. Instruction should explicitly assist children to move through successive stages in the development of mathematical skills and concepts.

According to Leushina (1991); "The principle of systematic and sequential teaching points out that knowledge must be taught in a strictly logical order and that children's actions and operations with mathematical material should be guided sequentially in order to develop a system of knowledge, abilities, and skills" (pp.169). This principle applies particularly to the teaching of mathematics as number knowledge is gained through a chain of understanding in which each link plays a vital part in the sequential development of cognitive powers and abilities and if broken or missing will result in a collapse of the developmental process. The teacher can only present new material once the child has mastered the previous stage and to do this there is a need to evaluate the child's level of development so that knowledge will be sequenced and ensure continuity. Of course a child acquires knowledge and abilities in everyday activities of play and work when interacting with the environment, but this knowledge is not co-ordinated or linked to other knowledge but remains local and particular to each child. However instruction provides lessons in a sequence and structure directed at a particular set of knowledge and abilities. It is therefore important to know the child's potential and be able to co-ordinate this with what is already known so that cognitive development will be encouraged. These ideas further confirm the need for a system of evaluation of mathematical ability to begin with the child who enters the first stage of informal classroom instruction. The present study has shown the wide range of mathematical ability of these children and the number knowledge and competence they have developed from the experiences of their home environment. All of these facts point to the value of such an assessment.

The findings of this study confirm the work of many researchers who during the last decade have recorded the mathematical competence of pre-schoolers and emphasised how important it is for reception class teachers to be aware of this so that they can organise the early mathematics curriculum to capitalise on this knowledge.
The theory of generative learning as describe by Wittrock (1974) (See chapter 1 pp.9) stresses the importance of understanding the number knowledge that young children bring to school so that instruction can be made relevant to children's mathematics learning at school. His theory of generative learning described how children construct and perceive meanings for themselves by linking new information with existing ideas.

Instruction therefore needs to be guided in its choice of content by existing knowledge so that new stimuli will be absorbed in appropriate ways.

Young-Loveridge (1989) noted that young children in their first year of school in Australia were taught concepts which they already knew but were not taught other concepts such as addition and subtraction which they could also understand. This finding indicated that the curriculum was not well matched to the skills and abilities of the children. As from the conclusions reached in this study, there is evidence to suggest that curriculum development at central, regional and local level should be more aware of the mathematical knowledge and competencies of reception class entrants. Again this points to the importance of continuous and accurate assessment of pupil's ability and level of development through the employment of reliable and efficient evaluations. Record books or computer updates would enable teachers to keep track of individual pupil's progress and account for lack of understanding or failure in a particular aspect of the work. Immediate intervention may address the problem by detecting the area where there has been a breakdown in the connection of 'known' facts to the 'unknown' and extra explanations and examples may help to overcome this failure before it affects all other areas of mathematical development.

The only justification for including prenumber and early number topics in the pre-school programme seems to be an inability to move away from the developmental theories of Piaget (Wright 1992). Researchers and theorists in the United States, Australia, Britain and Russia have emphasised the importance of a counting-based approach which shows how young children's number learning forms an essential basis on which to perform operations and build understanding of mathematical concepts and develop logical thought (Ginsburg 1977, Gelman and Gallistel 1978, Wright 1992, Steffe 1992 and Baroody 1992). Research has shown that the methods children use to do mathematics, the qualitative advancements children make over time, and the means by which children make those advancements all rest on the use of counting skills (Steffe et al. 1983). Children participating in this research also demonstrated their counting-based approach to solving problems. Their ability to count and conserve number was a determining factor in their competence to complete the tasks. This study would therefore accept the ideas of Wright and advocate the development of a curriculum which places more emphasis on the practice and expansion of counting skills.

Piaget's stage theory has a restraining effect on teachers because the child can only be taught activities prescribed for the stage at which he/she is at. This study suggests that the present reception class programme was based on the ideas of Piaget and has not accounted for the wide range of mathematical ability of the children in this age group thus restricting many who have developed more advanced mathematical knowledge from enriched home environments. There is evidence in this research which suggest that the counting-based approach agrees with Vygotsky's 'zone of potential development' which includes the learning that the child has not yet attained but is likely to attain in an interactive teaching situation. With a knowledge of counting the child is able to solve problems and calculate operations using the numbers he/she has learned and the strategies invented. Another principle which emerges from this theory is that teaching has a crucial function to play in children's qualitative advancements of mathematical knowledge. Instruction demands that the learner must be confronted with situations for which they do not have appropriate cognitive constructions thus initiating problem solving activities- a necessary ingredient of learning. The early mathematics curriculum should encourage teachers to organise instruction to capitalise on the wealth of knowledge the young child brings into the school situation.

If instruction is to match the content presented to the level and pace of learning of the child, then account should be taken of the wide range of mathematical ability of young children and the individual rates and styles of learning as demonstrated by this study. Young-Loveridge (1989) reports that the new 'Beginning School Mathematics Programme' which is used in New Zealand schools caters for this by "starting individual children at points in the programme where there is room for new (i.e.

incremental) learning and moving them through it at a pace which is appropriate for their individual rates of learning" (pp.61). However she points out that teachers are not necessary following this procedure and will need specific directions and assistance if this is to be followed.

Romberg and Carpenter (1986) suggest that research on individual differences and their lack of impact on instruction has resulted from little attention being given to how individual differences are related to how children learn, process information and the individualistic strategies they use to find answers to problems. There was ample evidence in this study of the various ways in which children attempted to calculate the answers to problems posed and this information indicated the level of proficiency reached in their understanding and working of number. An example of this was the addition problems which were solved in ways that ranged from a basic counting of all the objects presented to a more advanced understanding which enabled the child to start from the higher number and count on to include the smaller number. Attention to this aspect would show how instruction aims to compensate for differences and not to exacerbate the inequities of aptitude caused by social, cultural and innate differences. The importance of the individual in mathematics learning and teaching was underlined in a report in America by the National Research Council (1989) "Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding...All students engage in a great deal of invention as they learn mathematics; they impose their own interpretation on what is presented to create a theory that makes sense to them...Each student's knowledge of mathematics is uniquely personal.....Students retain best the mathematics that they learn by processes of internal construction and experience" (pp.58-59). Fennema and Behr (1980) stressed that mathematics educators were only interested in those individual differences related to the learning of mathematics and researched these aptitudes within both the cognitive and affective domain. However they emphasised that it was necessary to move from studying individual performance to studying the internal mental processes which in mathematics education turned to the study of problem solving. Researchers need to investigate further the implications this theory has for mathematics instruction. Fennema and Behr (1980) stress that; "It must be emphasised that the mere identification of traits on which individuals differ is not a particularly profitable area for research. It will become so only as the relation of these traits to the learning of mathematics is ascertained and the implications for instruction are delineated" (pp.350).

This study evaluated children's mathematical ability using the clinical interview method so that it would be possible to timeously note the various strategies used by the children to answer the problems presented. The correct answer was not the only important fact to be recorded but notes were taken on the child's conversation, gestures, movements and actions. This informed the researcher of the errors made and the reasons for such miscalculations so that there would be knowledge of the method of working and understanding of number which in turn could guide instruction and improve methodology.

Romberg and Carpenter (1986) assess the situation as follows; "Given that new information about learning and teaching is now available, that mathematics as a discipline is changing, and that future instruction will take into account the new technology, new assessment techniques must be developed if research is to improve" (pp.869). Assessment can no longer be guided by the ability to produce answers that are correct but should rather concentrate on the kind of knowledge the child has about a particular situation. It is important to understand how children try to organise and link new information to what they already know. Evaluation therefore needs to measure prior knowledge and the strategies children use as well as the errors they make and the number of correct answers obtained. Ginsburg (1977) believes that the informal interview is the best alternative to the standard test. By presenting the child with a specific concrete problem the researcher is able to observe the child's behaviour, record the 'out loud thinking' and work out the strategy used to solve the problem. Through questioning the researcher can check the interpretation by presenting a new problem or rephrasing the question.

The practicality of this means of testing has been questioned but as far as expertise and time are concerned surely teachers have the skills and understanding of the children they teach and the time spent will be valuable and economical if learning is accelerated and successful. This study together with the research of other educationalists could lead to the evolving of a standardised test for pre-school children which would give the teacher a guideline to the level of mathematical competence and ability of the children as they enter the reception class. For developmental activities to be well graded, such a form of systematic assessment as well as informal observations are necessary (Aubrey 1993, Ginsburg 1977, Young-Loveridge 1989, Wright 1991, Williams 1965, Hughes 1986 and Gelman and Gallistel 1978). With this knowledge teachers can group pupils according to their mathematical ability and vary the content material to suite the stages of development of each group. Constant evaluation of the pupil's progress would enable teachers to periodically dissolve and reconstitute groups to account for changes in children's ability over a wide range of concepts.

The results of the present study show that it is quite possible to assess fairly accurately the number knowledge of young children as they enter the pre-school group. It was also possible to ascertain the strategies they used to solve problems and the errors made when calculating these problems. By making this assessment early in the year, before the children had begun the mathematics programme of the pre-school group, it was possible to determine each child's level of number development which had been acquired from the home environment and experiences of accidental learning in the years prior to entering school. The results have shown a wide range of mathematics ability amongst the children tested and emphasised that the majority of children entered this reception class with considerable understanding of number concepts and skills. It was beyond the scope of this research to ascertain whether or not teachers have this knowledge of their pupils or whether all children in the group (placed there according to their chronological age) are taught every activity in the same sequence and at the same level of difficulty. If the curriculum is not well matched to the skills and ability of the children then activities will not offer challenges built on existing knowledge and will run the risk of children losing interest or if too complicated will result in others failing to understand the link to the new material.

6.3 Strengths and Limitations of this Study.

The results of this study give a picture of the number knowledge of these young children as they entered the reception class in a pre-primary school in Kwazulu-Natal. The clinical interview techniques proved most appropriate for this purpose and enabled the researcher to investigate their knowledge of number concepts, the strategies used and the errors made. Because children were assessed on an individual basis, it was possible to note and allow for each child's characteristic way of solving the problems presented and time each activity to suite the individual. This method also allowed for the adaptation of the questions to ensure that each child understood the requirements of each test. Language played an essential part in the evaluation of the child's ability and by interviewing them on an individual basis it was possible to rephrase questions so that one was quite sure that a lack of understanding was not construed as a lack of ability. By conducting the assessment in the child's school environment, it enabled the child to feel relaxed and familiar with the surroundings and equipment used in the test procedure. The individual interview technique also ensured that the child was not distracted by others as did the secluded area where the assessment took place.

findings

The results show that the test items used in this assessment demonstrate a fairly consistent level of ability for each child across the spectrum of concepts presented. Some test items were more closely correlated to the average than others, suggesting that some needed to be re-evaluated and adjusted to meet the interest or understanding of the child. The length of the evaluation was well suited to the attention span of the children of this age. Only one or two children out of the forty tested found the assessment too long and lost interest towards the end. All the other children maintained their concentration throughout the test and their attention was held because of the level of the tasks, the duration of each task and the variation of the activities which involved different apparatus and new challenges. The validity of the study was further enhanced by the fact that the same researcher interviewed all the children involved in the assessment thus ensuring that as far as possible there was consistency in the situation. Further more, only children whose home language was English participated in the research. This meant that as far as possible all children had an equal chance of understanding the problems presented and explaining their actions and answers. Participants were all from the same socio-economic group which was designed to exclude this aspect from the reasons for a wide range in the levels of number knowledge of children of the same age. Likewise the inclusion of an equal number of boys and girls gave the researcher ample opportunity to discover whether there seemed to be sex differences. The three schools that participated in the research were most willing and co-operative, offering their pupils and the use of their facilities to ensure that all the requirements had been met for a successful evaluation.

Aubrey's (1993) research facilitated a close replication for the children as each test item was clearly and accurately described by her with sufficient detail to enable the researcher to construct the test to comply as closely as possible with her work. The schedule drawn up enabled the researcher to carry out each evaluation using the same sequence of test items and to record the results in a systematic and detailed way.

As far as possible the test items were designed to cover the number concepts and problem solving techniques that are presented in the exploration of mathematical concepts and relationships as found in the 'Learning Through Activity Programme' that is used in the pre-primary schools. Without making the test too extensive but at the same time being sure to cover the relevant concepts, the test items gave a good indication of the stage of development and mathematical ability so that it could be ascertained how well the curriculum matched the level of competence and skills possessed by these young children. If educational practice is to be enriched then this type of evaluation will enable teachers to select and sequence content to match the stage of development of each child. This type of evaluation will help one to meet the requirements of a "good map of the cognitive development of key mathematical concepts and processes" (Carpenter 1980, pp.194).

Certain test items were limited in their ability to probe a clear understanding of the concept being tested. On the basis of the ideas of other researchers elsewhere it has become clear that some of the activities presented to the children could be extended or varied to include more concepts or the use of different materials. For example test 7 required the child to add the number of sweets given to two teddy bears on ten different occasions where the answer ranged from three to ten. This test would have given more insight into the child's understanding of the concept of addition and the use of counting in problem-solving contexts if some or all of the items to be added had been hidden (Wright 1991, Steffe and Cobb 1988 and Ginsburg 1977). Hughes (1986) found that children used different strategies when objects to be added were hidden, like counting fingers or a tapping movement which replaced the use of visual images. Likewise the subtraction task would have revealed interesting information about the strategies used if some of the tasks had involved hidden objects. Young-Loveridge (1989) tested young children's ability to think in the abstract by including addition and subtraction with imaginary objects. This would have given valuable information as to whether or not they were able to think in the abstract, a concept many researchers believe young children are not yet ready to handle.

Test 5 required the child to represent in written form the number of blocks displayed. Here it would have been interesting to see how children interpreted the absence of quantity, or 'none'. Would those who used the symbolic methods to represent quantity also use the conventional symbol 'O' and what would the idiosyncratic versions of this be? The multiplication tasks only involved hypothetical situations which the children found difficult to solve in the abstract form. Perhaps if there had been concrete material presented the children could have related better to the situation and more children would have been able to attempt to solve the problem and calculate the answer thus altering the results considerably.

Test 9 involved the estimation of two groups of objects presented on a plate and in a bowl. To determine the numerosity of the arrays children were asked not to count but simply to estimate the number of objects they saw in the space of the three seconds that was allowed. Because of the arrangement of the objects and the time allocated, it was not possible for children to use the process of subitizing nor counting but only allowed for estimation. It therefore only assessed the child's innate preverbal counting and timing systems which provides information on the relative quantities of sets of items (Gallistel and Gelman 1992). This ability to understand ordinal values develops from an interaction between innate sensitivities to numerosity and the child's experiences which accounts for the fact that 80% of the children tested showed this early sensitivity to ordinal relationships (Steffe et al 1983). To ascertain whether or not the child was able to use subitizing skills, it would be necessary to also present arrays of objects in a pattern formation. The perceptual process involved in subitizing makes young children sensitive to numerosity and allows for a more accurate assessment of quantity if there is a recognisable pattern. This would show whether or not they had developed beyond the counting stage and were now able to subitize (Gelman and Tucker 1975).

Task 10 required children to copy and continue a pattern using two colours of blocks which the children found difficult to do. Perhaps there was a need to make this activity more realistic by using coloured beads thread onto a cord to make a necklace or the use of a picture where the pattern had to be coloured in to copy the pattern made by the researcher. Because the children tested coped better with the second activity using three different coloured blocks; it may be that they did not understand the concept when first presented with the pattern showing that either the context was not suitable or the language used to describe the activity was not appropriate.

The task of classifying or sorting a number of objects into groups of similar objects was far too easy because of the choice of materials and it only relied on the recognition of the visual form of the different physical properties and not on forethought. All the children tested quickly sorted the objects into corks, polystyrene circles, shells, beads and plastic discs and were able to say why they had grouped them accordingly. This skill according to Piaget (Sime 1973) cannot be termed true classification but rather a way of sorting objects according to their visual form which gives them a 'graphical collection'. To test for a more advanced type of classification based on major classes such as colour, size and shape; children could have been presented with a variety of different coloured beads or balls of various sizes or different shapes and then asked to sort them according to whatever criteria they thought suitable. It would also have been valuable to test whether young children are more likely to sort objects into basic or superordinate categories as suggested by Rosch (1976). For such a test it would have been necessary to provide the children with pictures of clothing, vehicles, furniture and food so that those using the superordinate category could classify them into these groups while another set of pictures would display four drawings of one basic object such as four cars or four oranges. By changing the nature of the stimuli it would be possible to see which criteria are used by young children when they are required to classify objects and the attributes are not defined by the experimenter. The use of different materials and objects would therefore show the development of classification and the different ways of sorting objects into groups using either sensory-motor schema or logical thought.

Test 6 evaluated the child's ability to understand the Forward Number Word Sequence and Backward Number Word Sequence. This was an abstract problem solving activity and involved all the numbers in the range 1 to 20 for both sequences which meant that each child was asked forty abstract questions in this part of the test. Children soon tired of this activity because it was difficult to find the answer in the abstract and the task was fairly lengthy without any physical activity to break the monotony or to hold their attention. Either this activity could have been shortened by only asking for a sample of Forward Number Word Sequences and Backward Number Word Sequences in the range 1 to 20 or the activity could have been interspersed with tasks that required the manipulation of concrete materials and therefore involved action thus rekindling interest and attention.

These observations that came to light in the understanding of this study could be used as indicated above to refine the assessment procedure for further research and finally for application in the school situation.

6.4 Implications for Further Research

This research suggests that before entering the pre-school year, children have developed a much more quantitatively sophisticated knowledge of number than was previously thought because number , like language, is a natural field of human cognition and activity. Researchers over the past two decades have deliberated over the specifics of how number development occurs; the relative contributions of innate sensitivities and knowledge as opposed to the process of instruction and have debated about the empirical implications in this process. Likewise, this investigation and similar assessment procedures like those of Aubrey (1993), Young-Loveridge (1989) and Wright (1991) have shown that young children construct and invent their own mathematical knowledge and understanding from the experiences they encounter in their everyday lives. Evidence has been presented that stresses the importance of recognising the part played by counting in the child's numerical development. Surely then this constructivist research calls for the reconstruction of current early childhood mathematics curricula and further research into classroom practice and teacher training so that cognisance will be taken of young child's mathematics ability and there will be an understanding of the way in which number develops. This research argues that there is an urgent need to investigate and rebuild the teaching of number in early childhood mathematics education.

To review the present mathematics programme of the pre-school group, and to ascertain whether it is appropriately matched to the needs and ability of this aged child, interventionist research is suggested. Appropriate and valuable assessment schemes based on the research of Aubrey (1993), Young-Loveridge (1989), Wright (1994) and Ilg and Ames (1951) should be implemented to evaluate each child's level of mathematical development on entry into the reception class. Retesting these same children at the end of the pre-school year would enable the researcher to determine whether or not there had been worthwhile progress, which aspects of the programme had shown the most advancement and which children had achieved the highest rate of increase in mathematical ability. There is a need to establish applied research programmes in collaboration with teachers so that there will be a better understanding of the learning needs of the child as based on the level of ability and number knowledge present when they enter the school situation.

Research into the ideas teachers hold about how young children develop number knowledge as well as their knowledge of the level of mathematical ability and competency of children in the reception class would encourage teachers to concentrate on their methods of instruction and curriculum content. This type of research needs to be directed not to an investigation into teachers but rather an inquiry into learning and teaching with the assistance of teachers and aimed at helping them to be more successful in achieving their aim of improving the mathematical development of each child in their class. Teachers need to be made aware of the importance of evaluation of children's level of development as measurement for curriculum content and not as a means of grading pupils. "To revitalise the mathematics curriculum, it is necessary that assessment be aligned with the curriculum" (Schoen 1996, pp.12). Likewise, once teachers are aware of the wide range of mathematical ability in the class, there needs to be research into the best ways to accommodate this diversity in the methodology and content of the curriculum and teacher training informed of these theories.

This type of applied research programme, in collaboration with educational systems would aim to adapt curriculum of the reception class to meet the needs of these young children. With improved understanding of the development of number knowledge and the level of achievement in mathematics skills and competency of young pre-school children, there is potential for adaptations to the curriculum and teacher development programmes. This will encourage changes in standards of mathematical presentation, methods of presenting the material and adaptation to the content that will give children the basic knowledge needed to build on and lay a firm foundation for the formal development of mathematics.

It is important not to let mathematics education be driven by current ideology which is not based on practical situations in the classroom nor on proven facts that apply to the conditions in the country where it is to be adopted. There needs to be a combination of the strengths of current cognitive science research with concern for the realities of the classroom and focus on children's learning from instruction over a length of time. If research is in the form of classroom interventionist studies, then cognisance will be taken of the context in which children learn and this will add value and meaning to the results of such a study. Research of this type will provide a complete picture of how learning occurs in typical classrooms. There needs to be an understanding of how young children acquire mathematical skills and a clear set of assessment guidelines which will enable teachers to regulate the curriculum to meet the standards of the mathematical achievement of the children being taught. Research should therefore be aimed at finding the most suitable test items that will demonstrate the knowledge and mathematical competency of the child so that the learning and teaching situation will be most beneficial. Applied research programmes should work in collaboration with teachers to integrate research on learning and research on teaching. These investigations will test real classroom situations with the results moulding the curriculum and affecting the learning of content and the methods of instruction.

This type of applied research programme should include a systematic collaboration of the researchers both within individual schools and within the schools in the province or region so that there can be professional development of teachers and changes in school curricula and methods of instruction and learning. Studies of this nature need to be conducted by teams of researchers working in a wide range of socio-economical environments so that knowledge can be built up over time and ideas shared to find the most suitable set of test items that will best evaluate the mathematical ability and competencies of pre-school children and acknowledge the variety of strategies used in solving mathematical problems.

This research acknowledges the significance of the part played by parents in the young child's development of mathematical competence (See Chapter 1). A number enriched environment with exposure to a wide range of problem-solving activities and opportunities to talk and think about number will expand the young child's cognitive development and give him/her the confidence and encouragement to work with numbers. Parents should be reminded of the fact that young children have the ability and the interest to use number in their daily experiences and if encouraged and exposed to numerosity can become proficient in their use of numbers. Parents need to be made aware of research in this area and given guidance in the ways that they can help to make a difference in their child's mathematical development.

It was beyond the scope of this study to include an assessment of the mathematical ability of second language learners. Language plays a vital role in the evaluation of young children's mathematical ability both on the part of the questions asked by the researcher and the language used by the child to describe the strategies employed to solve the problem. This would surely be an important part of any future study on this subject, especially in a country such as this where most reception classes would be comprised of multi cultural and multi lingual groups. It is surprising to note that among all the researchers mentioned none of them have seen this as a centre of concern.

Perhaps future research needs to look more carefully at the constructivist's idea of how learning takes place and evaluate whether this idea provides a realistic basis on which to reconstruct current early childhood mathematics curriculum. Do children learn best when they construct their own mathematical understanding and invent strategies that make sense to them? Or are these ideas based on naive views of children's cognitive development which believe that "it is possible for students to construct for themselves the mathematical practices that, historically, took several thousand years to evolve" (Cobb et al. 1992, pp.28). Geary (1994) believes that procedural skills are a secondary biological skill and for this reason they are best learnt with drill and practice. It is only the conceptual knowledge which is a biologically primary skill that can be readily acquired under conditions that have the child think of the many different ways in which the problem can be solved.

Although there has been research into the development of mathematical understanding and the acquisition of number knowledge, little is known about the way one mathematical achievement relates to another. Is counting the basic structure on which all future number knowledge depends or will development still show meaningful progress if they are first introduced to experiences with relational comparisons? (Bryant 1994). Perhaps the number system in English makes it too difficult for children to understand the structure of the decade system and therefore the counting process needs to wait until children are able to comprehend this concept. There is a need for longitudinal research and intervention studies to bridge the gap between the various concepts of number and the way in which they affect future learning and teaching.

The development of number skills is certainly influenced by biological and environmental factors, but educational research can expand our knowledge about the ways young children learn to work with number concepts, the pathways they take to develop this skill and how teachers can expand this knowledge to its maximum potential. Research can contribute significantly to improved instructional techniques and enriched mathematical learning for young children if it directs its investigations to the goal of improving the mathematical development of children in their *first* year of instruction.

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Appendices

Appendix A: Assessment Form

Pupil		Number of years at school	
Age		Occupation of mother	
Gender		Occupation of father	
Date		Position in family	

TEST 1

Highest number counted	
Comments	

TEST 2

Count 3 blocks placed in a line	
Count 7 blocks placed in a line	
Count 3 blocks placed in a circle	
Count 7 blocks placed in a circle	
Extract a subset of 4 from the set of 12	
Extract a subset of 10 from the set of 12	

TEST 3

Count 4 blocks starting from the left	
Count 4 blocks starting from the right	
Count 4 blocks starting from the middle	
Count 6 blocks starting from the left	
Count 6 blocks starting from the right	
Count 6 blocks starting from the middle	

TEST 4

Read the numbers presented:

1 2 3 4 5 6 7 8 9 10 12 15 27

TEST 5

Record the number of blocks displayed: 1-10 (See attached sheet of paper)

Comments:

TEST 6

Give the number that comes *after* the number given:

1	6	3	9	7	4	2	5	8	10
11	15	13	18	12	16	14	17	20	19
Total number correct									
Comments									

Give the number that comes *before* the number given:

3	6	9	4	2	5	8	10	7	11
15	20	12	18	13	19	16	14	17	15
Total number correct									
Comments									

TEST 7 (a)

How many sweets would there be if the two bears joined theirs together?

4+1		2+3	
3+1		4+4	
4+2		3+4	
5+2		5+5	
4+3		6+4	
Total number correct			
Comments			

TEST 7 (b)

How many sweets would there be if one bear gave x number to his friend?

5-1		6-4	
6-2		8-4	
5-2		6-5	
5-3		9-5	
6-3			
Total number correct			
Comments			

TEST 8

Can you share these sweets between the *two* bears?

4÷2	
5÷2	
6÷2	
Total Number Correct	
Comments	

Can you share these sweets between the *three* bears?

6÷3	
9÷3	
Total Number Correct	
Comments	

How many legs have two ducks got?	
How many wheels are there on three cars?	

TEST 9

Estimate how many oranges there are in the bowl which contains 6 oranges.	
Estimate how many sweets there are on a plate which contains 10 sweets.	

TEST 10

a) Copy and continue a pattern of :

Alternate red and green blocks	
Three different circular shapes	

b) Make your own pattern using contrasting shape and colour:

c) Describe the 3-D shapes:

sphere	
cube	
cuboid	
tetrahedron	

d) Match the 2-D shape to the 3-D shape:

sphere-circle		cube-square	
cuboid-rectangle		tetrahedron-triangle	

e) Draw the 2-D shapes (see booklet attached):

	Comment
Square	
Circle	
Rectangle	
Triangle	

f) Observe the 4 different lengths of ribbon and describe them:

Comments

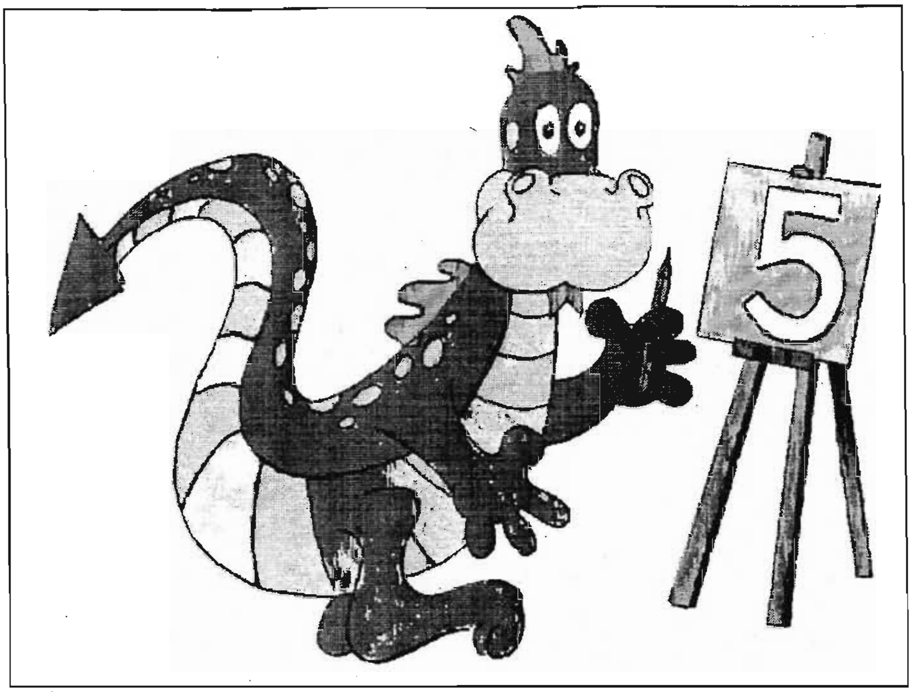
g)

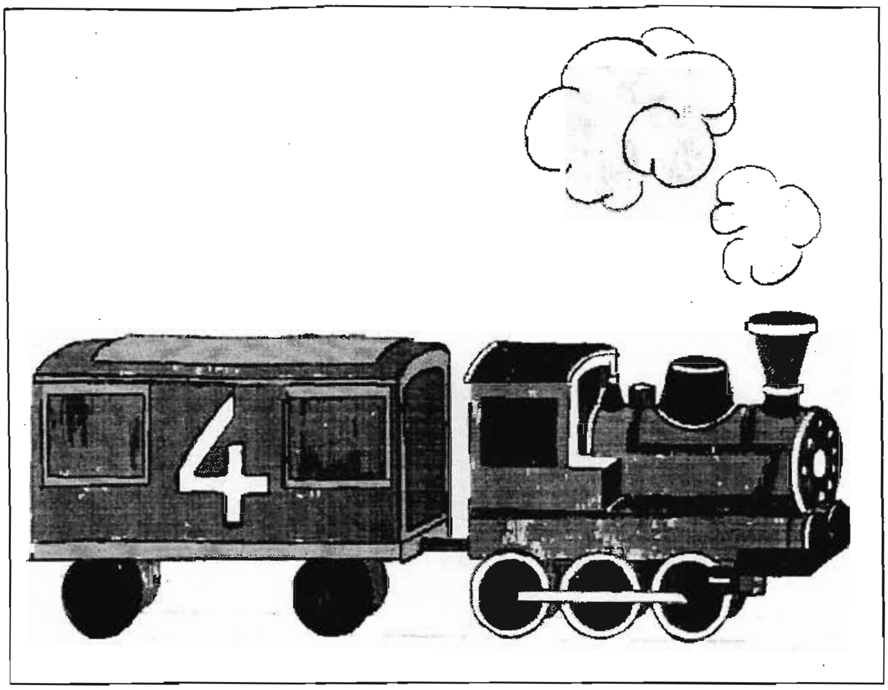
	Comment
Describe the positions of people in a line	
Understand the position of yourself in space.	

h)&i)

	Comment
Sort the given objects into categories and describe each set	
Sequence the events of these everyday activities.	

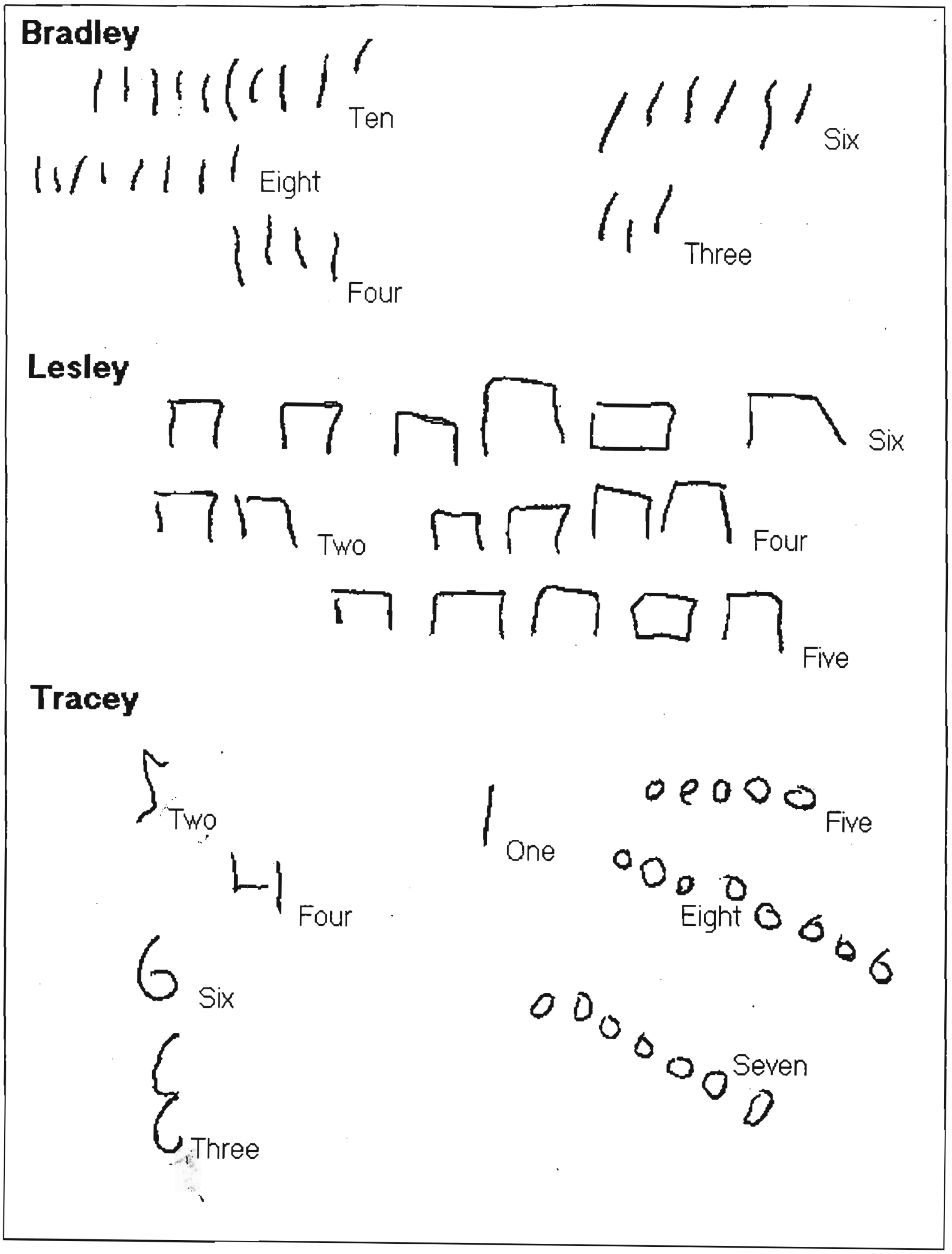
Appendix B: Number Cards:

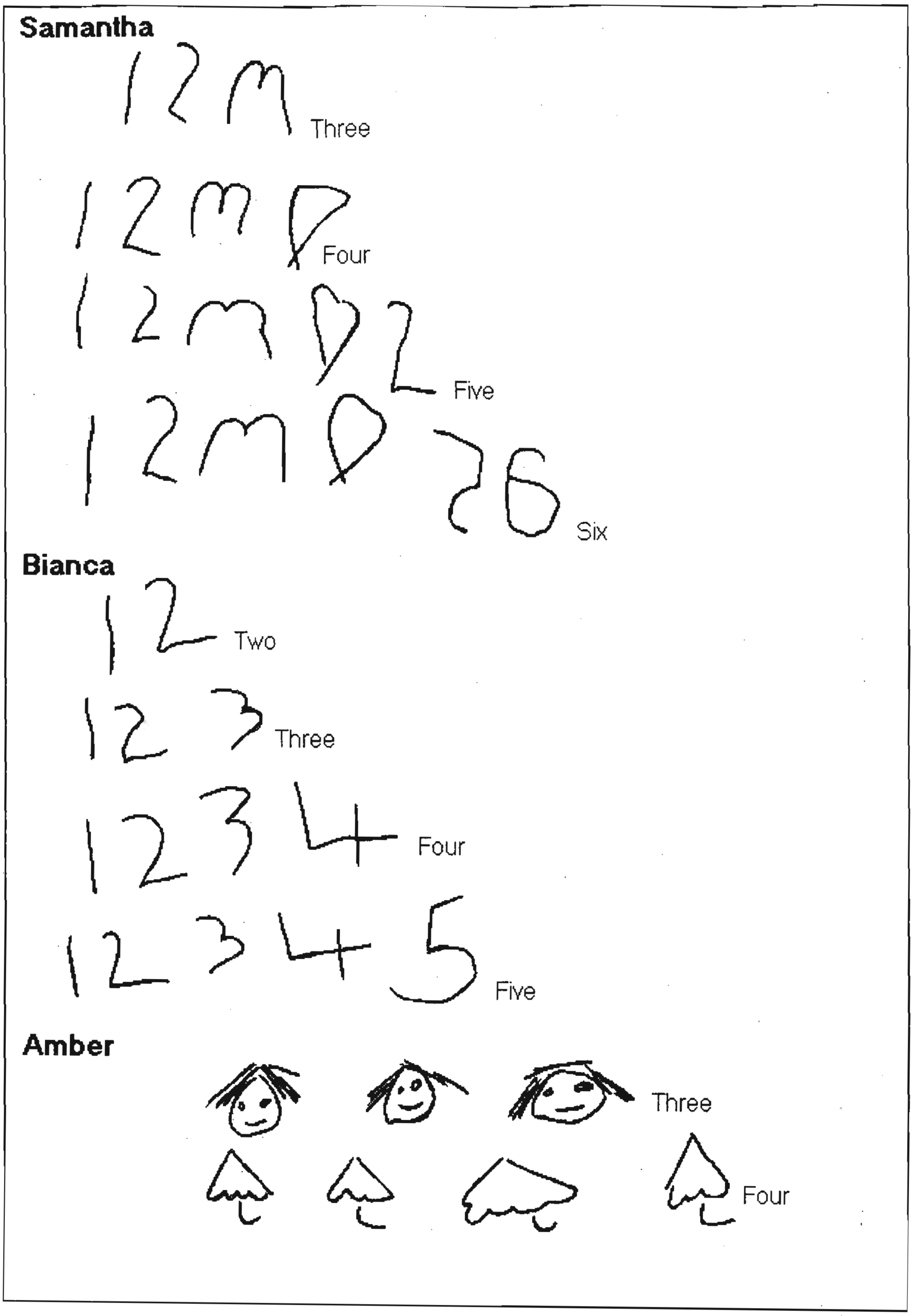



4


2

Appendix C: Children's Representation of Number




Samantha
Three
Four
Five
Six
Bianca
Two
Three
Four
Five
Amber
Three
Four

Appendix D: Test Results

No.	Name	Test1 Result	Test2 Result	Test3 Result	Test4 Result	Test5 Result	Test6a Result	Test6b Result	Test7a Result	Test7b Result	Test8 Result	Test9 Result	Average
1	Kayleigh	100	100	100	92	100	95	55	100	100	80	50	88.364
2	Stephanie	100	100	100	77	80	95	40	100	100	100	50	85.636
3	Fred	100	100	100	69	100	60	30	100	100	100	50	82.636
4	Calvin	80	100	100	85	80	100	70	100	100	100	50	87.727
5	Tarryn	80	100	67	54	70	90	45	100	100	80	50	76
6	Kyle 1	80	100	100	23	50	70	25	100	100	100	50	72.545
7	Kyle 2	80	100	100	46	40	70	20	100	100	80	50	71.455
8	Caron	60	100	100	69	90	80	35	100	100	100	50	80.364
9	Bradley	60	100	100	62	90	70	5	90	100	100	50	75.182
10	Samantha	60	83	100	77	100	55	0	100	100	100	50	75
11	Dane	60	100	66	69	100	65	35	90	100	60	50	72.273
12	Tracy	60	100	100	62	100	60	25	100	100	80	0	71.545
13	Lee	60	100	100	54	60	65	0	100	100	80	50	69.909
14	Christopher	60	100	100	23	100	40	25	60	100	100	50	68.909
15	James	60	100	100	23	70	35	20	100	100	100	50	68.909
16	Jacqueline	60	100	100	46	60	55	0	100	100	80	50	68.273
17	Rodney	60	83	100	46	70	55	0	70	89	100	0	61.182
18	Matthew	60	66	83	54	70	35	0	80	100	100	0	58.909
19	Craig	60	100	66	0	0	10	0	70	100	80	50	48.727
20	Mitchel	60	67	83	0	10	0	0	100	100	80	0	45.455
21	Amber	40	100	66	62	100	45	30	100	100	100	50	72.091
22	Kathryn	40	100	66	77	70	55	0	90	100	100	50	68
23	Darren	40	83	100	62	70	65	10	100	100	60	50	67.273
24	Jenna	40	83	100	46	50	50	0	100	100	100	50	65.364
25	Terence	40	66	100	46	40	60	5	100	100	100	50	64.273
26	Bradley	40	83	100	46	60	25	25	90	100	100	0	60.818
27	Jessica	40	100	100	23	40	30	0	100	100	80	0	55.727
28	Tessa	40	66	100	15	60	30	0	30	89	80	0	46.364
29	Rochelle	40	50	66	15	0	5	0	70	100	80	0	38.727
30	Tracey	30	66	66	77	100	60	30	100	100	100	100	75.364
31	Lesley	30	100	100	69	60	45	30	30	89	80	50	62.091
32	Bianca	30	100	100	38	50	35	0	100	100	100	0	59.364
33	Janita	30	100	100	15	50	60	0	100	100	80	0	57.727
34	Michael	30	83	66	8	20	0	0	90	100	100	100	54.273
35	Peta-Jane	30	33	66	54	40	65	30	70	100	100	0	53.455
36	Gaby	30	66	66	38	80	35	5	70	100	80	0	51.818
37	Shay-Lee	30	83	50	69	50	50	0	40	78	80	0	48.182
38	Ethan	30	66	66	8	40	15	0	40	89	100	50	45.818
39	Dwaine	10	50	50	8	30	0	0	70	100	60	50	38.909
40	Donavan	10	66	0	0	40	0	0	30	22	60	50	25.273
	Average	51.25	86.075	84.825	45.175	62.25	48.375	14.875	84.5	96.4	88.5	36.25	63.498
	Std. Deviation	22.667	18.213	22.081	26.402	28.328	27.254	18.345	22.753	12.943	13.502	27.706	
	Correlation Coefficien	0.7456	0.6456	0.6208	0.7735	0.728	0.8558	0.6903	0.6578	0.5187	0.4056	0.3848	1

Appendix E: Kwazulu-Natal pre-school Curriculum:

Learning Through Activity Programme.

6. EXPLORATION OF MATHEMATICAL CONCEPTS AND RELATIONSHIPS : NUMBER

Global Aim : To introduce the child to the problem-centred approach to mathematical concepts, allowing him to build on existing knowledge, thereby equipping him to explore numbers actively at his own pace in a manner determined by himself.

Many of the mathematical concepts covered in this section will have been taught incidentally throughout the year. It is necessary, however, to deal with them in greater depth in order to allow the child to refine and extend his understanding. Particular emphasis should be placed on the language of mathematics, as number exploration is the focus of this section.

Central to the new approach to mathematics is the understanding that verbalisation and problem-solving are an integral part of the process of understanding number.

The child should be given the opportunity to explore mathematical concepts freely, using concrete objects. Through experimentation, he will establish his own particular style of problem solving, whilst consolidating mathematical principles. In this way the child is encouraged to develop his own strategies to solve problems and to explain his deductive reasoning. Group interaction provides the opportunity to verify answers and shows that problems may be solved in a variety of ways.

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - use mathematical language appropriately and with understanding	6.1 The Language of Mathematics: Quantity words and comparative words, e.g. like/unlike, equal to, many/few, big/little, more than/less than, bigger than/smaller than, the same as, different, getting bigger, getting smaller, staying the same, etc.	The teacher introduces the language of mathematics through a comparative study of objects. The child notes similarities and differences. The teacher may use a negative questioning technique, e.g. "Find something that is not bigger/ smaller than .....", etc.	Concept Diagram What's in a Square? What Size? Lotto Begrippentaal

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Using dough, the child moulds people of various sizes. The teacher tells the children the story of "Goldilocks and the Three Bears." They then dramatizes the story and illustrate it, focusing on its comparative elements.	Colour/Shape: Two properties Maxi Bead Threading Kit Edim Classification Circles Fit-It Find-It Going for a Walk Wat Ontbreekt?

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - perceive similarities and differences when observing objects from a mathematical perspective - predict, with a degree of accuracy, how objects may be transformed - explain the reason for the transformation of an object	6.2 The Concept of Relationship The Relationship between objects, comparative differences and comparative likenesses Transformation of Size, i.e. getting bigger, getting smaller, staying the same, etc.	**Relationship/Transformation** Getting Bigger/Getting Smaller: The teacher presents an assortment of objects, including some of the following: e.g. two balloons, two candles, ball of wool and knitting needles, play-dough "cakes" and a knife, a glass and a jug of water, etc. Through observation and discussion, the child predicts which of these can get bigger or smaller and justifies his answers. Then, through experimentation, he is able to test and verify, e.g. if the balloon is blown it gets bigger, but if the air is released it will get smaller again. if the wool is used in knitting, the ball will get smaller and the knitted garment will become bigger.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - group objects according to one or more attributes	6.3. **Classification** Grouping of objects according to one common attribute Grouping of objects according to more than one common attribute	**Classification** Working in a sub-group, the child sorts a selection of cutlery according to their attributes, e.g. an assortment of spoons. He groups those with wooden handles, those made of metal or plastic, etc. He explains each time why he has grouped the items together or asks the rest of the sub-group to establish the reason for the grouping. This activity may be extended by selecting items which have more than one attribute in common. Negative questioning may also be employed to group items, e.g. all those that are not made of plastic, etc. The teacher presents pictures of wild animals. After discussion, the child draws all the animals	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - count by rote - understand that numbers occur in a specific order, increasing in value - commence the counting process from any given point - make some form of record which matches objects to marks on a page (tally) - compare groups of objects accurately with reference to number, i.e. more than, less than, the same as, etc.	6.4 Counting and Tallying Counting by rote as far as the children are able This can be extended by counting in groups, e.g. 2's, 3's, etc. Counting on The ability to pick up a number sequence from a given number, e.g. "count on from 4 to 15" The skill of counting on is a useful problem- solving tool in working out addition problems. It involves the child's being able to perceive the number of objects in one group and his counting on from there, to obtain a total. A child who has this skill solves addition problems more quickly	Counting The child counts objects in his school environment, e.g. chairs, children, etc. The Matal Classification Board may be used effectively to count children in various groups, e.g. boys; girls; children with long hair/short hair, etc. Each child may be given a box of assorted objects which he uses in discovery. The teacher may also pose problems, in order to direct his exploration, e.g. "Count out five blocks. How many blocks would be left if I took away two blocks?" etc.	Hi Ho Cherry O (Apfelchen) Collect the Chicks Geo Stacks Giro-number Colour Dominoes Number Bonds Jumbolino Picture Nines (Number and Picture Dominoes for Early Counting) Ludo Oranges and Lemons Snakes and Ladders Compendium Dice Games

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Counting on One child starts counting and stops at a given point, the next child continues and so on. A child counts on from a given point. Each child in the sub-group is given a card on which a certain number of small blocks are placed. The teacher instructs him to cover a given number of blocks, e.g. two. He then has to count on from there.	Blossoms Round the Castle Bird Game Sausage Snuffling Three to Match

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
	Tallying The ability to record graphically a given number of objects by means of dots, strokes or any pictorial representation	Tallying The teacher prepares a number of work cards. Each card graphically represents five objects which are found in the school environment. Each child in a sub-group is given his own card. He locates the objects in the school, counts them and completes his "tally sheet". The child tallies various objects from a large illustration and completes a "worksheet", pre-prepared by the teacher. This activity can be reversed so that the child draws a detailed, cumulative number picture, depicting the correct number of each object, as specified by a "tally card", e.g. one house, two trees, three chicks, four people.... etc.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Illustrated Baking Recipe: The child bakes biscuits following an illustrated recipe. Pot-of-Soup Pictures: Each child is given an illustrated recipe card. This shows various vegetable ingredients and indicates the required number of each vegetable. He draws these onto a large saucepan shape and applies a colour wash to complete the activity.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - compare two groups of objects by linking corresponding objects using woollen stands - compare visually one group of objects to another, by the process of matching each object to a corresponding object	6.5 One-to-One Correspondence Comparing groups of identical objects Comparing groups of disparate objects	One-to-One Correspondence Many opportunities for one-to-one correspondence are provided incidentally by the school routine and advantage should be taken of these, e.g. servers at snack time, distributing note books, setting out of School Readiness work by the children, etc. Draw One Object for Each Dot: The teacher pre-prepares a large page by folding it into quarters. The number of dots in each quarter indicates how many objects are to be drawn.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Hoops: Using two hoops, the teacher places a variety of objects in each. She may have the same/fewer/ more objects in each hoop. The child compares the number of objects in each hoop by linking corresponding objects, using wool, string or crocheted strands. The child then determines if the groups are equal/smaller than/ than/greater than each other. Groups of Objects: Each child is given a packet/bag/container of assorted objects, six to eight groups of objects of varying numbers. The child then sorts the various objects and compares the groups to ascertain the number of each of the objects. Suitable objects are dough cutters, corks, bottle tops, marbles, matches, beads, plastic ants, etc.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		What's In a Square?: The teacher prepares "What's In a Square"- type cards with strips of objects for the horizontal plane and strips of number dots for the vertical plane. It is recommended that the object strips have only two objects and the dot strips have four groups of dots. Using A4 paper, the teacher folds each page into eight rectangles: four down and two across. The strips are placed in position and the child draws the correct number of objects in the appropriate spaces.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to:	**6.6 Number Value and Ordering**	**Number Value**	Maxi Bead Threading Kit
- attach meaning to rote counting, i.e. to understand the value of each number, its constancy and progression	Number Value Understanding that each number has a value of its own, which is constant, separate and different from that of other numbers	Certain activities may be presented to convey individual number values, as well as the progressive value of numbers, e.g. number stories and songs:	Number Bands Duett Beetle Game
- recognise and identify number combinations and numerals	A number can be represented by dots which may or may not have a recognisable pattern.	Three: Three Bill Goats Gruff Four: Spot finds a Home Five: Five Little Elephants Balancing Six: One Little, two Little ...	Apple Orchard Match-a-Matics Heinevetter Einertrainer 1-5
- use said number combinations and numerals appropriately when tallying and recording	The value of each number is also represented and recognisable as a numeral	These may be used in a variety of ways, including puppet making, dramatization, story illustration, etc.	Whiskers and Waistcoats Begrippentaal
- recognise and name the ordinal position of objects	The reading of number words is incidental at pre-primary level		Build-a-Bear Domino Futura
- use correctly the language of ordering, e.g. relative size: tall, taller, tallest, etc.	Numbers are introduced individually and progressively from one to nine and the concept of zero should also be introduced		Number Charts

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - arrange groups of objects according to their quantitative value	The child's understanding of number value can be extended to include pairs, duet, twins, trio, triplets, quartet, etc.	Number Book: The child draws a picture to illustrate each number in group/set format. A variety of media may be used, e.g. painting, drawing, collage, etc. The child draws the appropriate numeral and number of dots on each picture. All the pictures are then assembled in a "Number Book", progressing from zero to nine. Grocery Shopping: The child chooses a selection of empty boxes from the anti-waste. Each box is priced, using number dots. The child plays shop, purchasing the items with coins, play money, numeral discs, etc.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Hide 'n Seek: The teacher sets out ten foil pie-plates on a table, arranging blocks underneath and on top of the pie-plates in various number combinations. A number card is placed next to each pie-plate. The child then deduces how many blocks are hidden under each pie-plate to equal the number specified on the card, e.g. "There are three blocks on top of a pie-plate, the numeral on the card is six. How many are hidden underneath?" Answer: Three.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
	Ordering The language of Ordering: long/longer/longest; tall/taller/tallest, etc. Ordinal Numbers: first, second, third, etc.	Ordering Each sub-group runs a race and the teacher takes note of the ordinal position of each child as he crosses the finish. She presents each one with a rosette on which the ordinal position is indicated with numerals. The child returns to his playroom and draws the results of his race. The teacher tells a story involving the members of a family arriving at the bus stop, e.g. "Granny arrives at the bus stop first. Granddad is on his way, but is overtaken by the little boy who stands second in line", etc. Each child uses cut-out pictures of a family to copy the order correctly. This story may also be illustrated by the children.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		The children in each sub-group measure each other and establish the progressive height order from shortest to tallest. The teacher cuts a large sheet of paper diagonally and divides it into sections. Each child selects the appropriate section according to his height and draws himself. The sub-group arrange their pictures in the correct order on the wall. The pictures are drawn to scale, but not actual life size. Each child is given a ball of dough which he moulds into three people - fat, fatter, fattest.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Cars and Garages: Ten empty milk cartons are painted or covered in bright colours. The shaped top is cut off each carton, to leave an opening for the door, when the "garages" are placed on their sides. Numeral cards are hinged onto the top (roof) of each garage, so that they can be lifted up to reveal the corresponding number of dots underneath, for verification. Small toy cars are each marked with number dots, using a permanent marking pen. The child first arranges the garages in the correct numerical order and then parks the cars appropriately. The zero-garage has no car.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Paper Clip Game: Number cards, each depicting numerals and corresponding number dots, are covered with clear contact. The child counts out and attaches paper clips appropriately, by sliding the correct number of clips over each card. This may be extended by grouping two cards together and recording or counting the total number of paper clips. Cards may also be compared to see which have more/ fewer paper clips.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		The Number Graph: Each child folds an A3 paper into as many halves as he can, both vertically and horizontally, to form a grid of small rectangles across the page. The vertical axis indicates the numbers in ascending order so that zero is at the top of the page. The horizontal plane will show a variety of objects. On the first day, no object is recorded in the space opposite zero. On the next day, one object is drawn in the space opposite the single dot. The graph is completed by drawing the correct number of objects opposite each number space, thus illustrating graphically the increasing number of objects.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Christmas Tree Decorations: Each child is given a photostatted outline of a Christmas tree drawn on A3 paper. At the bottom of the paper is a grid, on which several decorations are represented. Next to each decoration is an empty space for recording the number of decorations. The teacher asks the child to draw one star, two candles, three bells, etc. When the tree is fully decorated, the child counts and records the number of each kind of decoration. The child may tally using either dots or numerals. Number-to-Number/Dot-to-Dot pictures may be given to the child for completion to reinforce ordering of number.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - recognise that a given number is still the same number, irrespective of how it is constituted or rearranged	6.7 Conservation of Number	Conservation The teacher prepares a selection of circles with self-corrective cuts for each number value, e.g. four = two plus two; four = three plus one and four = four plus zero. The child matches the two halves and sorts the circles into their respective numbers. The teacher lays out three rows of beads with six beads in each row. Each row is arranged differently, i.e. in one row the beads are spread out, in another they are evenly spaced and in the third they touch each other. The child is then asked to identify which row has the most beads prior to counting the beads.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Aeroplanes and Hangars: The teacher makes nine hangars from wine sleeves cut in half, lengthwise. She numbers each hangar from one to nine, using numerals. Using twenty-nine party favour aeroplanes, she marks each one with a permanent marker on the wings according to the numerical conservation combinations, e.g. five = five plus zero; five = four plus one and five = three plus two. The hangars are marked with numerals and the aeroplane wings with number dots. The child adds the combinations and parks the aeroplanes in the appropriate hangars.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - arrange a series of events in time in a logical sequence - relate a sequence of events in a logical order - place sequence cards in a logical order with appropriate verbalisation	6.8 Sequencing Problem-solving and the deductive process are inherent in the new approach to Mathematics. At pre-primary level, it is necessary to work from the known within the child's world, exploring sequence with reference to the passage of time, prior to concentrating on mathematical elements Sequence of Time: Day and Night Days of the Week Seasons School Routines, etc Sequence of Events: Life cycles The progression of special events	Activities may be presented incidentally, throughout the year, to explore the sequence of the passing of time and events, e.g. days of the week songs, commemoration of special holidays, illustration of news on a daily basis, recording of weather on a daily basis, Life-Cycle Puzzles, etc. The child makes a "Time Book", drawing events in his day or events that are characteristic of each day of the week. The child makes a zig-zag or concertina page depicting an event of note in his life, i.e. what happened first, what happened next, what happened last. The child illustrates each phase.	Como Creche Sequences Life-Cycle Rolf Akti-Reakti Story Time Unamo Sequence Safety in the Home Four Seasons Puzzle Trio Sequence Puzzles

LTA Programme Exploration of Mathematical Concepts and Relationships : Number - Sequencing

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
	Language of Sequence: Next, last, every day, before, after, nearly, etc.	Afterwards it may be cut into three separate pictures, which other members of the sub-group arrange in a logical sequence to tell the story.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
The ability to: - arrange objects in order according to size, location and position - name with reasonable accuracy the ordinal position of an object in a series - recognise a repetitive pattern in a given series - predict what will come next given a partial series	6.9 Seriation The language of Seriation: Size: smallest to largest largest to smallest shortest to tallest thinnest to thickest Location: nearest to farthest farthest to nearest Position: first, second, third, etc. second to last penultimate last	Seriation The teacher arranges a group of children in a series, e.g. one child standing, one sitting, one standing, etc. The child is asked to predict what the next child in the series should do. The pattern is then completed as the rest of the children are included. Rhythmic Clapping Patterns: The teacher uses body percussion to form an auditory rhythm pattern, repeating three or four actions. The child is asked to copy the pattern and to then repeat it on his own.	Maxi Bead Threading Kit Geo-Stacks Tricky Fingers Patterning and Sequencing Cards Complete the Pattern

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Copy the Pattern: The child is given a Body Percussion Rhythmic Pattern card. Each child performs his pattern in front of the class. The class then copy the series, e.g. clap, clap, patchen, head-tap, etc. Shapes Patterns with Attribute Blocks: The teacher starts a shape series using Attribute Blocks and the child is asked to continue the pattern. When it is completed the child is asked to "read" it to the sub-group.	

Specific Objectives Evaluation Criteria	Content	Learning Experiences	Suggested Teaching Materials
		Bead Threading Pattern Cards: The child completes a bead pattern, following a partial example on a Pattern Card. The sub-group verify if his completed sequence pattern is correct. Similar patterns may be constructed using numerals.	