Examining the gap between Grade 11 Mathematics teachers' content knowledge and its application in teaching Euclidean Geometry in selected secondary schools in the Ugu District.

BY

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DECLARATION

I, Bongiwe Princess Ntombela, declare that this is my original work, submitted in partial fulfilment of the degree of Master of Education in Curriculum Studies, entitled Understanding teachers' content knowledge of teaching Euclidean Geometry

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4 August 2025

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Date

As the candidate's supervisor, I agree to the submission of this thesis/dissertation for submission.

[redacted]

4 August 2025

Dr Lokesh R. Maharajh

Date

DEDICATION

This work is dedicated to

my son, Ndumiso Nhlakanipho Ntombela, and

all my family members, who are the source of my inspiration.

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ABSTRACT

This dissertation is an examination of the content knowledge of Grade 11 Mathematics teachers in the teaching of Euclidean Geometry in the Ugu District, KwaZulu-Natal. The study was motivated by the persistent poor performance of South African learners in Mathematics, particularly in Euclidean Geometry, which is a key component of the subject. The research aims to understand how Grade 11 Mathematics teachers use their content knowledge in the classroom. The theoretical framework for the study is the Van Hiele model, which describes a developmental progression of geometric understanding. The research adopted a qualitative approach with a multiple-case study design. Data was generated through tests and task-based interviews with ten Grade 11 Mathematics teachers from ten secondary schools. The findings indicate that poor performance in Euclidean Geometry is linked to teachers' lack of content knowledge and their inability to be innovative in lesson preparation and presentation. The study concludes that improving teachers' content knowledge and providing continuous professional development are critical to improving educational outcomes and fostering learner success in Euclidean Geometry. The recommendations include a need for further research on a larger scale and the implementation of accredited continuous professional development programs to address teachers' beliefs, attitudes, and practices.

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CHAPTER ONE

INTRODUCTION TO THE STUDY

1.1 INTRODUCTION

Euclidean Geometry is an essential component of mathematics education, particularly at the secondary level, where it fosters students' reasoning, spatial thinking, and problem-solving skills (Jablonski & Ludwig, 2023). Effective teaching of this topic requires that teachers possess deep and specialised content knowledge, which significantly impacts their ability to convey geometric concepts, engage students in meaningful mathematical discourse, and guide them in developing deductive reasoning (Ball et al., 2008).

Teachers' content knowledge, particularly in Geometry, can be categorised into two broad types: subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Shulman, 1986). Teachers' SMK encompasses understanding of geometric concepts, theorems, and proofs and their ability to solve geometric problems accurately and flexibly. Alternatively, PCK involves knowledge of how to make these concepts accessible to students by using effective representations, addressing common misconceptions, and employing strategies that foster a deep understanding of Geometry.

In the context of Euclidean Geometry, teachers need more than just a surface-level comprehension of definitions and postulates. They must know the underlying principles of geometric relationships, such as congruence, similarity, and the properties of triangles and circles, and be adept at explaining these concepts through multiple representations Furthermore, a teacher's ability to create meaningful connections between algebraic and geometric reasoning is crucial for helping students grasp the broader applications of Geometry (Hollebrands, 2003).

Research highlights those deficiencies in teachers' content knowledge often led to superficial instruction that focuses more on rote memorisation of procedures rather than conceptual understanding (Zohar, 2023; Chazan & Lueke, 2009). Therefore, improving teachers' content knowledge in Euclidean Geometry is essential to foster deeper student engagement with Geometry, enabling them to develop the logical thinking skills necessary for Advanced Mathematics.

This study aims to understand the depth of teachers' content knowledge in Euclidean Geometry and its use in their instructional practices. This will be through examining both SMK and PCK in this area; we can better understand how teachers' knowledge influences student outcomes and identify areas where professional development may be needed to enhance the teaching and learning of Euclidean Geometry.

This introductory chapter presents the background of this study, the problem statement, its context, and its objectives, followed by the research questions that guided this study. It further highlights the rationale, significance, scope, limitations, key concepts, and layout of this study. It also highlights an overview of Euclidean Geometry and the South African curriculum and ends with a conclusion.

1.2 BACKGROUND TO THE STUDY

Euclidean Geometry is a foundational component of secondary school Mathematics, crucial for developing students' reasoning, spatial thinking, and problem-solving skills. In South Africa, the Curriculum and Assessment Policy Statement (CAPS) identifies Euclidean Geometry as one of the ten core content topics in the Further Education and Training (FET) phase. It is taught in Grades 10, 11, and 12, where it constitutes a significant portion of Mathematics Paper Two, accounting for approximately 33% of the final examination in Grades 11 and 12.

However, persistent low performance by South African learners in Mathematics, particularly in the FET phase, has been a growing concern. Diagnostic reports from the National Senior Certificate (NSC) examinations consistently highlight poor learner scores in Euclidean Geometry questions. This underperformance points to a systemic challenge in the teaching and learning of this critical subject, which requires a deeper investigation into the factors at play. A key area of concern is the content knowledge of the teachers responsible for delivering this curriculum.

1.2.1. Initial teacher training programmes

Many teachers may not have received sufficient training in Euclidean Geometry during their tertiary education. Some teacher education programmes may focus more on general mathematics content and less on specialised areas such as Geometry (Gambini & Lénárt, 2021).

- Inadequate professional development: In-service professional development programmes may not adequately address the specific content areas where teachers are struggling (Govender et al., 2023). Programmes tend to focus on broad teaching strategies rather than on deepening teachers' knowledge of specific subjects (Taylor & Vinjevold, 1999).
- Historical challenges in mathematics education: The legacy of apartheid has left profound disparities in educational quality, particularly in under-resourced schools where teachers might not have access to the necessary training or materials for effective teaching (Spaull, 2013).

The deficiencies in teachers' content knowledge in Euclidean Geometry has significant implications for student learning outcomes. Poor teacher knowledge limits students' opportunities to engage with higher-order thinking and problem-solving tasks which are critical for success in Mathematics (Zohar & Ben-Ari, 2022). South Africa's low performance in international assessments, such as the Trends in International Mathematics and Science Study (TIMSS), underscores the urgent need to improve teachers' mathematical content knowledge to help elevate student achievement in Mathematics (Graham et al., 2024).

The exodus of learners to Mathematics Literacy in secondary schools (Kennedy et al., 2014) and the poor-quality passes in secondary school Mathematics over the years (Adler & Pillay, 2016) provided a justifiable foundation for conducting this study. The 2024 diagnostic report (p. 217, pp. 241-243) also agreed with the aforementioned statement of exodus of learners, as seen in Table 1.1.

Table 1.1: Mathematics performance trends (2020–2024)

Year	**No. wrote**	**No. achieved at 30% and above**	**%. achieved at 30% and above**	**No. achieved at 40% and above**	**%. achieved at 40% and above**
2020	233 315	125 526	53.8	82 964	35,6
2021	259 143	149 177	57.6	97,561	37.6
2022	269 734	148 346	55,0	07 041	36.0
2023	262 016	166 337	63.5	114 311	43.6
2024	251 488	173 774	69.1	120 430	47.9

As shown in Table 1.1, the number of candidates who sat for the mathematics examinations in 2024 decreased by 10 528, compared to that of 2023. The percentage of distinctions, over 80% improved from 3,4% in 2023 to 3,9% in 2024. The various support programmes employed by teachers, subject advisors and provincial education departments were continued in 2024, though there is a need for a structured support. This performance is mainly due to other topics of Mathematics because when looking closer at the performance of Euclidean Geometry in Paper Two, it is still performing far less than expectations since it is firstly taught in Grade 11 which means it has two consecutive years of teaching.

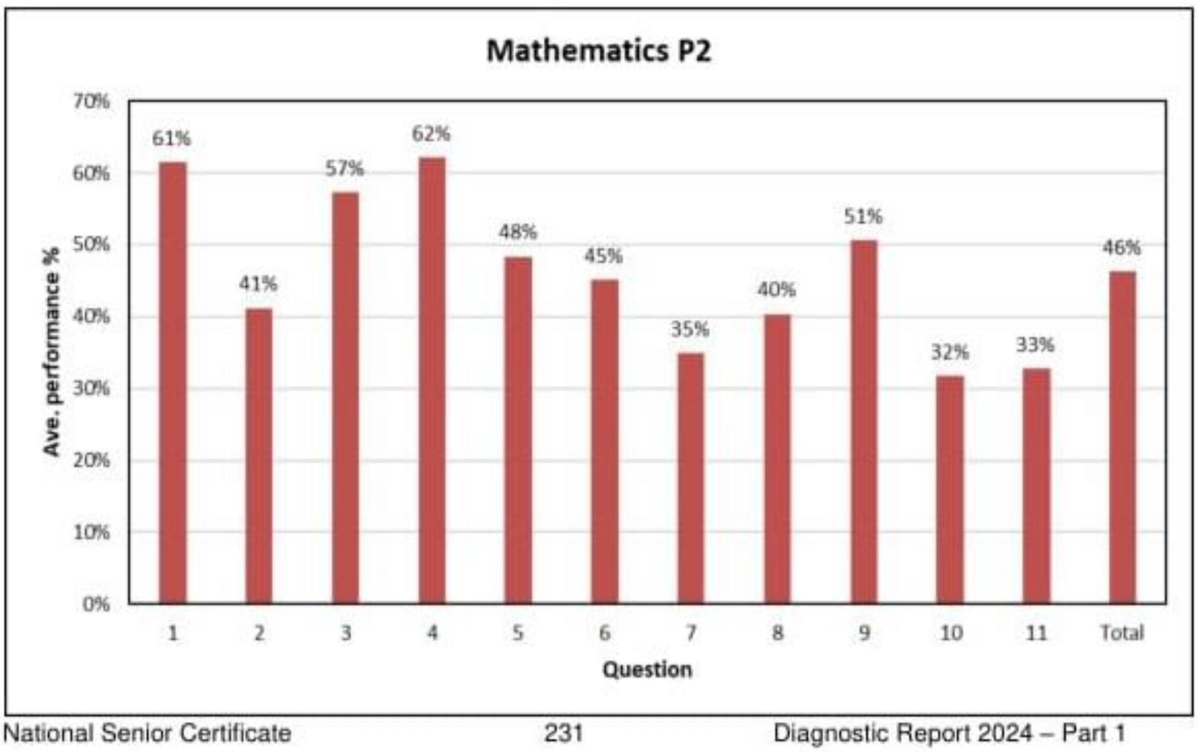

Figure 1.1: Performance of learners per question in Paper Two

Section 3 of the Diagnostic report (2024) is Diagnostic Question Analysis which is a report compiled from the internal moderators' reports received from all nine provinces for each question paper per subject which were consolidated, and the findings being summarised into this diagnostic report. Section 3 then includes A graphical representation of the average percentage marks obtained per question; A detailed analysis of learner performance on each specific question, indicating whether the question was answered well or poorly, along with an explanation for the response patterns; Common errors and misconceptions identified in learner responses; and Recommendations for improvement in teaching and learning, content and methodology, subject advisory support, and the utilisation of learning and teaching support materials.

Some of the common errors and misconceptions made by learners as indicated in section 3 of the diagnostic report question analysis by internal moderators still include assumptions and confusion in interpreting theorems. For example, in the Department of Basic Education's (DBE), diagnostic report book 1 (2024) many candidates for Q9.1 assumed that ECB was a tangent to the circle. Hence, candidates stated that $\hat{A}_1 = 86°$ instead of 40°. Some candidates confused the cyclic quadrilateral theorems and regarded the opposite angles of the cyclic quadrilateral to be equal instead of being supplementary. Other incorrect assumptions made were that $\hat{C}_1$ was the exterior angle of triangle ADC and that AB || DC and therefore the corresponding angles were equal.

Additionally, some candidates, in answering Q9.2, stated that $\hat{C}_2 = \hat{A}_1$ without proving it. Many candidates incorrectly used $\hat{B} = \frac{1}{2}\hat{A}_1$ instead of $\hat{A}_1 = \frac{1}{2}\hat{B}$. Some candidates incorrectly assumed that AC was the diameter of the circle and therefore incorrectly stated that $\hat{D} = 90°$. The following graphs illustrate the average performance of learners with a specific focus on questions 9, 10, and 11, which are for Euclidean Geometry.

Suggestions for improvements made by chief markers, internal moderators for mathematics in the 2024 diagnostic analysis report for Euclidean geometry included the following:

a) As mentioned in previous reports, more time needs to be spent on the teaching of Euclidean Geometry in all grades. Learners should read the given information carefully without making any assumptions. Exercises on Grade 11 and 12 Euclidean Geometry must include different activities and all levels of the taxonomy.

b) The need for learners to name the angles correctly has been mentioned in several reports previously. Teachers should not credit learners with marks in school-based assessment tasks if the angles are not named correctly.

c) Teach learners not to assume any facts in a geometry sketch but to use only what was given and that which had already been proven in earlier questions.

d) Learners need to be made aware that writing correct statements that are irrelevant to the answer in Euclidean Geometry will not earn them any marks in an examination”

According to Venkat and Graven (2017), most learners are timid to take Mathematics or do not take Mathematics in secondary schools because of their lack of confidence in their ability to do Mathematics and lack of motivation.

Lack of learner enthusiasm and interest and lack of basic content knowledge are some of the factors associated with learners’ poor performance in Mathematics (Tachie & Chireshe, 2013). According to Jones (2000), the successful teaching of Geometry depends on teachers’ content knowledge of Geometry as well as how to use content knowledge to teach effectively. A teacher’s knowledge of how to teach content is referred to as PCK. Kennedy et al. (2014) stated that teachers’ knowledge and skills affect their interaction in the classroom, this means that the knowledge that teachers possess and its use in their classrooms is the instrument of change in students’ learning. Hence, to effectively deal with learners’ challenges, teachers should have appropriate content knowledge as well as PCK. It was also confirmed that teachers require a deep and broad knowledge of Mathematics to be effective in their teaching (Hill, 2010).

Mathematics results in all grades within the education system indicate that the problems facing the teaching and learning of Mathematics have not yet been correctly addressed. According to Okitowamba et al. (2018), in South Africa, the national Mathematics average achievement at each grade within the system ranges between 40% and lower percentages, and this has been a serious problem in the South African basic education system since democracy in 1994 (Pournara & Barmby, 2019) and has been linked to many factors such as poor teaching facilities, learners’ negative attitudes towards Mathematics, a shortage of qualified Mathematics teachers, and poor teaching methods (Dube, 2016). In terms of Goal five in the action plan of the Department of Education towards school realisation 2030, the number of very good passes remains worryingly low. For example, in 2014, the number of Mathematics distinctions, or Mathematics passes achieving at least 80%, was 7 216. Despite the decrease in the size of the 2024 cohort, there is an increase in the total number of distinctions from 8 909 in 2023 to 9808 in 2024 and is still not sufficient to maintain the level of Mathematics skills the country needs, even though the percentage of distinctions over 80% improved from 3,4% in 2023 to 3,9% in 2024.

The DBE has recognised these issues and initiated various programmes aimed at strengthening teacher content knowledge, particularly in subjects such as Mathematics; however, addressing the content knowledge gap in Euclidean Geometry requires sustained efforts, including reviewing teacher education curricula, providing targeted in-service training, and developing better support systems for teachers. Despite extensive intervention programmes made by the Department of Education for both teachers and learners, Mathematics performance is still poor (Biyela et al., 2016).

According to Bansilal (2017), the 2017 NSC diagnostic report showed that only 35.1% which means that less than 35.1% of learners qualified for entry into science and engineering related careers, since the minimum required pass rate in Mathematics to these fields is 50% or more. When looking at the 2024 diagnostic report, which shows 47.9% of learners passed with 40% or more, there is a slight improvement, and this is a difference of 3,8% as compared to 2017 and to 2014 when the Curriculum and Assessment Policy Statement (CAPS) was implemented in Grade 12 for the first time. This means that it took the Department of Education's interventions eight years to improve by 3,8% in the quality of performance, and it also confirms that less than 47,9% learners qualify for science and engineering related careers. The percentage of Grade 12 mathematics learners in 2024 achieving at least 70%, a mark often considered a minimum for entry into university studies in a field such as engineering, is 9.5% (DBE, Diagnostic report, 2024, p. 218) These results show that there has been little improvement in Grade 12 mathematics learner performance for the past eight years compared to 2014 which is the first year of Euclidean Geometry assessment in Grade 12.

The DBE plans to increase the number of learners with bachelor's passes to 300 000 by 2024, with 350 000 learners passing Mathematics (Fourie, W. (2018) South Africa is falling short in fulfilling this goal. There were 251 488 Grade 12 learners who participated in 2024, and only 173 774 learners who achieved at 30% or more. This means that it is important to understand teachers' content knowledge in teaching Mathematics with specific reference to Euclidean Geometry, which seems to contribute more to poor performance.

Euclidean Geometry has been cited as one of the topics that is problematic for many learners, and teachers are said to be contributing factors in the poor performance in Mathematics (Novak & Tassell, 2017). Euclidean geometry was removed from the syllabus in 2006, but has been compulsory in the CAPS since 2012.

According to Steyn (2016), NSC mathematics diagnostic reports have indicated low learners' scores in the geometry questions, and Euclidean Geometry forms one-third of the Mathematics Paper Two examinations. CAPS' focus is more on content, and each topic contributes towards the acquisition of specific skills. Euclidean Geometry is one of the 10 main content topics taught in Mathematics in the Further Education and Training (FET) phase (DBE, 2011). It is taught in term three in grades 10 and 11 for about three weeks. As stated in the CAPS document, Euclidean Geometry forms part of Mathematics Paper Two and weighs about 20% in Grade 10 and 33% in Grades 11 and 12 (DBE, 2011).

Understanding and addressing the challenges related to teachers' content knowledge in Euclidean Geometry is critical to improving mathematics education in South Africa. Enhancing teachers' content knowledge will assist learners to develop strong reasoning and problem-solving skills, which are essential not only in Geometry but also in broader mathematical and scientific fields.

1.3 PROBLEM STATEMENT

Despite the foundational importance of Euclidean Geometry in the South African Mathematics curriculum, there is growing concern regarding learners' consistently poor performance in this specific content area. This challenge is evidenced by national diagnostic reports that consistently show low scores in geometry questions on the NSC examinations.

While various factors contribute to this issue, a significant gap in the literature and a central problem for this study is the suspected deficiency in teachers' content knowledge and its direct impact on learner performance. Preliminary findings suggest that many Mathematics teachers may lack a deep, conceptual understanding of Euclidean Geometry, including its core theorems, principles, and the interconnectedness of these ideas. This knowledge gap hinders their ability to facilitate meaningful learning and contributes to learners' difficulties in mastering the subject.

Furthermore, this deficiency in content knowledge can lead to pedagogical challenges. As noted in the study's abstract and findings, teachers' lack of content knowledge is linked to their inability to be innovative in lesson preparation and presentation. This suggests a disconnect between what teachers know and how they apply that knowledge in the classroom. This problem is exacerbated by factors such as a lack of adequate professional development opportunities and a teaching methodology that may be overly reliant on rote memorisation rather than fostering deductive reasoning. Therefore, there is a clear need to comprehensively assess the current state of Grade 11 Mathematics teachers' content knowledge of Euclidean Geometry and to understand the specific relationship between their knowledge and their teaching practices. This study aims to fill this gap by providing a detailed exploration of these issues within the Ugu District, KwaZulu-Natal.

1.4 RATIONALE

Teachers’ thorough understanding of subject matter, often referred to as content knowledge, is critical for effective teaching (Charalambous et al., 2020). In mathematics education, particularly in topics such as Euclidean Geometry, teachers’ content knowledge is essential for explaining concepts clearly, anticipating student misconceptions, and selecting appropriate instructional strategies. Research has shown that teachers with strong content knowledge are better able to help learners achieve higher levels of mathematical proficiency (Copur-Gencturk & Tolar, 2022; Jacob et al., 2020; Moh’d et al., 2021). In Euclidean Geometry, where abstract thinking and deductive reasoning are central, teachers’ strong grasp of content is even more critical.

Euclidean Geometry is a significant part of the South African mathematics curriculum, particularly in high school (Tachie, 2020; Machisi, 2021; Alex, 2019). According to the CAPS, Euclidean Geometry forms a substantial portion of the senior phase and FET phase mathematics curriculum. However, despite its importance, many South African learners struggle with Geometry, as reflected in matriculation results (Machisi, 2021). Teachers’ inadequate understanding of the subject may contribute to these low-performance levels, necessitating research on teachers' content knowledge (Sheppard & Wieman, 2020).

South Africa's education system, shaped by apartheid-era inequalities, continues to face challenges in providing quality mathematics education (Rais, 2024). Historically disadvantaged schools often have underqualified teachers, particularly in Mathematics, which has resulted in poor student outcomes (Grant & Brantlinger, 2022). Moreover, the abolition of Euclidean Geometry as a compulsory part of the curriculum in 2006, followed by its reinstatement in 2012, caused a gap in teacher training and expertise. These policy shifts more than likely exacerbated the lack of preparedness among teachers, which still impacts the quality of Geometry instruction today.

Studies have shown that many South African teachers lack both PCK and deep content knowledge in Mathematics (Dhlamini, 2022). In Euclidean Geometry, which involves a high level of cognitive demand due to its focus on logical reasoning, spatial understanding, and proof construction, teachers need strong foundational knowledge. However, research highlights that many South African teachers feel under-prepared to teach this content, often due to insufficient tertiary education or professional development opportunities (Waghid & van Wyk, 2024). Thus, understanding the current state of teachers' content knowledge in Geometry can help address gaps in teacher training programmes.

Improving teachers' content knowledge of teaching Euclidean Geometry would likely have a direct impact on learners' achievement levels. Research has shown that learners who are taught by teachers with strong content knowledge perform better in assessments, particularly in subjects like Mathematics that rely on conceptual understanding and problem-solving (Copur-Gencturk & Tolar, 2022). In the context of South Africa, where educational inequality persists, enhancing teachers' content knowledge for teaching Euclidean Geometry can help bridge the gap between under-performing and high-performing schools, ultimately promoting greater equity in mathematics educational outcomes.

Research into South African teachers' content knowledge of teaching Euclidean Geometry is critical given the subject's importance in the curriculum, the historical challenges faced in mathematics education, and the potential to improve learner outcomes. By understanding the current state of teacher content knowledge for teaching Euclidean geometry, this study can provide insights that inform teacher training programmes, professional development initiatives, and policies.

1.5 THE SIGNIFICANCE OF THIS RESEARCH

Studying teachers' content knowledge of teaching Euclidean Geometry in South Africa holds significant implications for education, pedagogy, and learner outcomes. The researcher presents several key points that follow, highlighting their importance.

Euclidean Geometry is a foundational component of the South African mathematics curriculum, particularly in the FET phase. Understanding teachers' content knowledge of teaching Euclidean Geometry in this area ensures that they can effectively deliver the curriculum and meet educational standards. Euclidean Geometry promotes spatial reasoning and critical thinking. If teachers possess strong content knowledge, they can foster deeper conceptual understanding among learners, which is crucial for success in Mathematics.

Research shows a direct correlation between teachers' content knowledge and learner achievement. Studies on teachers' content knowledge of Euclidean Geometry can reveal how this knowledge influences learners' performance in assessments, thereby guiding improvements in teaching practices. By understanding teachers' content knowledge of teaching Euclidean Geometry in South Africa, interventions can be designed to address specific weaknesses that may contribute to these gaps.

Insights from such studies can inform the development of targeted professional development programmes, ensuring that teacher training aligns with the needs identified in their content knowledge. Understanding the areas where teachers may lack content knowledge allows for the implementation of ongoing support systems, such as mentoring or collaborative teaching approaches to enhance their confidence and expertise in the teaching of Euclidean Geometry.

The study may reveal technological ways that leverage teachers' strong content knowledge, promoting innovative use of teachers' content knowledge in engaging students more effectively in Euclidean Geometry. Insights into teachers' content knowledge may encourage the integration of technology in the teaching of Euclidean Geometry, as teachers with a solid grasp of content are more likely to utilise digital tools and resources effectively.

South Africa's education system faces disparities across different regions and demographics. Understanding the content knowledge of teachers who teach Euclidean Geometry can help identify inequities and promote policies aimed at equitable access to quality mathematics education. Findings from such studies can inform policymakers about the current state of teachers' content knowledge, aiding in the development of curricula that align with teachers' expertise and the needs of learners. By understanding local challenges and strengths, educational authorities may ensure that South African curricula for Euclidean Geometry align with international best practices, potentially improving global competitiveness.

A study on teachers' content knowledge of teaching Euclidean Geometry in South Africa is crucial for enhancing educational quality and effectiveness. Ultimately, this research may help shape a more robust mathematical education framework in South Africa, empowering both teachers and learners to achieve their fullest potential in Euclidean Geometry and beyond.

1.6 TEACHING OF EUCLIDEAN GEOMETRY AND MATHEMATICS (CAPS)

CAPS was developed to be a complete and brief policy for grades R-12 (Motshekga, 2009). The policy gives clear guidelines on teaching and assessing learning for each grade. The Mathematics CAPS document has four sections. The first section highlights the background of the development of CAPS and describes the purpose and long-term goals of the South African curriculum in developing learners as critical thinkers. The second section defines Mathematics as the language of symbols, aiming at developing learners' skills of using number systems and mathematical language correctly, and it also clarifies the content to be taught. Section three shows specifications of the content to show progression in terms of concepts and skills per grade. The last section of the CAPS document for Mathematics has an assessment which then details forms of assessments and how they should be used, it also gives teachers a programme of assessment and cognitive levels. Reports and studies indicate poor achievement in Mathematics, particularly in Paper Two, which is the most poorly performed (Mullis et al., 2020; Department of Basic Education, 2021; van der Berg et al., 2016).

The teaching of Euclidean Geometry within the framework of the mathematics CAPS in South Africa is guided by specific principles and aims to develop students' spatial awareness, reasoning, and problem-solving skills.

As far as curriculum structure is concerned, Euclidean Geometry is typically included in the mathematics curriculum for grades 7 to 9. It emphasises the understanding of shapes, angles, properties of geometric figures, and theorems. The learning outcomes are that learners should understand geometric concepts. In other words, learners are expected to grasp fundamental concepts such as points, lines, angles, and shapes. The curriculum encourages learners to prove basic geometric theorems, enhancing their logical reasoning skills. There is an expectation that learners will apply geometric concepts in problem-solving contexts, including real-life scenarios. The assessment methods include formative assessments (ongoing assessments during the learning process) and summative assessments (final evaluations, such as exams). Questions in these assessments often involve practical applications of Geometry, requiring learners to demonstrate both procedural knowledge and conceptual understanding. The teaching is also guided by the fact that the CAPS promotes the integration of Geometry with other mathematical domains, such as measurement and Algebra, to provide a holistic understanding of Mathematics and includes activities involving the use of Geometry to solve problems related to areas, volumes, and coordinates.

There are suggestions for pedagogical approaches that involve active learning strategies, such as group work, discussions, and hands-on activities, which are encouraged to help students visualise and understand geometric concepts, but the selection of pedagogical approaches relies on teachers' strong content knowledge. Technology (such as geometry software) may also be incorporated to enhance understanding and engagement.

The teaching strategies suggested in the CAPS documents include the use of visual aids (incorporating diagrams, models, and software tools to help students visualise geometric concepts), real-life applications (relating geometric principles to everyday situations such as architecture and art, to show relevance and practical application, collaborative learning (encouraging group work and peer teaching to promote discussion and deeper understanding of geometric concepts), problem-based learning (presenting learners with complex problems that require learners to apply geometric reasoning to find solutions), and differentiated instruction (adapting teaching methods to accommodate different learning styles and abilities, ensuring all leaners can engage with the material effectively). The effective use of all these strategies is solely dependent on the teachers' content knowledge.

The teaching of Euclidean Geometry within the CAPS framework aims to build a strong foundation in geometric concepts while promoting critical thinking and problem-solving skills (Naidoo, 2020; Mudaly, 2018; Goba, 2019). Educators are encouraged to adopt varied pedagogical approaches to meet the diverse needs of learners and connect geometric principles to real-world contexts.

1.7 THE THEORETICAL FRAMEWORK

The van Hiele theory of levels provides a valuable framework for understanding how learners learn geometric concepts and how teachers can effectively impart that knowledge (Thompson & McNaughton, 2017). This theory, developed by Pierre van Hiele, outlines a developmental progression of geometric understanding of five levels, which can inform the study of teachers' content knowledge of teaching Euclidean Geometry (Clement, 2000). The van Hiele theory of levels serves as a robust theoretical framework for studying teachers' content knowledge of teaching Euclidean Geometry (Mason, 2008; Sowder, 2007). By understanding the developmental stages of geometric reasoning, teachers can improve their instructional practices, adapt to student needs, and ultimately foster deeper geometric understanding in their students. This theoretical perspective encourages a comprehensive examination of both teacher knowledge and learner learning trajectories, creating a more cohesive approach to geometry education.

1.8 LOCATION OF THE STUDY

The study was conducted within the Ugu District in the Mkhunya Circuit as well as the Braemar Circuit in KwaZulu-Natal. The Mkhunya Circuit is part of the Vulamehlo Local Municipality in the Ugu District of KwaZulu-Natal, South Africa. It is characterised by a predominantly rural landscape, with the community engaged in various forms of agriculture and local trade. The population of Mkhunya, as per the 2011 census, was around 2,529 residents, primarily made up of people of African descent, with a strong representation of the Zulu language and culture.

The Braemar Circuit is also located in the KwaZulu-Natal province of South Africa and is part of the Umzumbe local Municipality. The circuit is characterised by a predominantly rural landscape, with the community engaged in various forms of agriculture and local trade. The Braemar Circuit includes several schools and educational institutions. It is involved in various educational initiatives, including those focused on improving teaching practices and integrating real-life applications of subjects such as Mathematics.

1.9. RESEARCH OBJECTIVES

- To examine teachers' content knowledge of teaching Euclidean Geometry.
- To determine how Grade 11 Mathematics teachers use their content knowledge to teach Euclidean Geometry.

1.10 RESEARCH QUESTIONS

- What is Grade 11 Mathematics teachers' content knowledge of Euclidean Geometry?
- How do Grade 11 Mathematics teachers use their content knowledge to teach Euclidean Geometry?

1.11 RESEARCH METHODOLOGY

This study was conducted using a qualitative approach, as the researcher was interested in understanding Grade 11 teachers' content knowledge of teaching Euclidean Geometry and understanding how Grade 11 Mathematics teachers use their content knowledge to teach Euclidean Geometry. The qualitative approach allowed me to understand how individuals (Grade 11 Mathematics teachers) describe their worlds (Kamal, 2019).

The interpretive paradigm was considered the most relevant paradigm in this study. It emphasises the importance of social interaction as the basis for knowledge. The researcher used skills as a social being to understand how others understand their world (O'Donoghue, 2018). Therefore, knowledge was constructed by mutual negotiation and was specific to the situation being studied. Within the qualitative approach, the study used case studies to understand Grade 11 Mathematics teachers' content knowledge of teaching Euclidean Geometry. Case studies allow a phenomenon to be studied in a real-world context (Bass et al., 2018, p. 13). The data generation of the study included pre- and post-task-based interviews and a test. The test questions were aimed at revealing teachers' levels of understanding Euclidean Geometry in terms of van Hiele's theory**.**

As it was not possible to use all Grade 11 Mathematics teachers in the Mkhunya and Braemar Circuits in KwaZulu-Natal, a convenience sampling to select the teachers for the study was used. Participants were selected because they were easily and conveniently available and willing to provide data by virtue of their experience and knowledge (Robinson, 2014). The test results, where teachers as participants wrote a Euclidean Geometry test, were analysed as shown in Table 1.2. The teacher's responses were categorised as T1, T2, and so forth.

Table 1.2: Classification of teacher answers from their test scripts

CATEGORY	STATEMENT	No of times featured
T1	Correct response, correct reason.	
T2	Incorrect response, incorrect reason,	
T3	Correct response, incorrect reason	
T4	Incorrect response, correct reason	
T5	Correct response, no reason	
T6	Incomplete response, No reason	

The face-to-face individual interviews were analysed thematically. Audio recordings of the task interview were transcribed by the researcher and audited by the moderator. Task-based interview data were coded using computer-assisted qualitative data analysis software. Questionnaire responses were coded manually using a word processor. Coding in this study was a process of identifying information that addresses the variables of interest and assigning a label that best describes the information (Bazeley & Jackson, 2013).

Ten sampled participants in this study were coded to preserve their anonymity as well as their schools with the aim of protecting their privacy while collecting, analysing and interpreting data. The principals of 10 secondary schools in the Ugu District were approached for permission to conduct the study. They were given consent letters to sign. Subsequently, informed consent letters were given to the participants (the 10 Grade 11 Mathematics educators). The purpose of the study and participants' rights, such as withdrawal from participation, should they wish to do so, without being compelled, were explained.

1.12 CLARIFICATION OF TERMS

1.12.1 Mathematics Teachers

A Mathematics teacher is an educator who specialises in teaching Mathematics. Their role involves helping learners understand various mathematical concepts, theories, and problem-solving techniques, ranging from basic arithmetic to advanced subjects such as Algebra, Geometry, Calculus, Statistics, and more. Mathematics teachers in this study are full-time teachers with a Mathematics teaching qualification in the FET phase.

1.12.2 Teachers' Content Knowledge

In this study, teachers' content knowledge refers to the teachers' knowledge about specific content. This includes correct interpretation and applying mathematical concepts, facts, procedures, principles, ideas, and theories (Ramatlapana & Berger, 2018).

1.12.3 Teachers' Knowledge of Geometry

In this study, geometrical content knowledge refers to the teacher's ability to relate geometrical diagrams to their properties and prove and apply theorems when solving geometrical problems (Alex & Mammen, 2018).

1.12.4 Teachers' Euclidean Geometry Knowledge for Teaching

This refers to the teacher's ability to relate geometrical diagrams with their properties and to prove and apply theorems when solving geometrical problems (Alex & Mammen, 2018) and also to the knowledge of axiomatic systems where learners write proofs and solve problems (Yixuan et al., 2016).

1.13 CHAPTER OUTLINE

Chapter One

This chapter provides the introduction and background to the study. The researcher provided an overview of the background of the study, including poor learner performance in Mathematics, objectives, problem statements, research questions and clarification of terms.

Chapter Two

This chapter is based on a literature review representing different studies on teachers' content knowledge for teaching Euclidean Geometry. It highlights the importance of teachers' content knowledge and demonstrates the different views of other researchers. Teachers' content knowledge has been arranged according to van Hiele's levels of understanding Euclidean Geometry.

Chapter Three

This chapter describes the research design and methodological approach adopted in this study.

Chapter Four

This chapter presents the data and various methods used to analyse and interpret data. The test results are summarised in tabular form, while the participants' verbatim interviews are also presented and analysed.

Chapter Five

This chapter concludes the research study by analysing and discussing the findings as well as possible recommendations of the study.

1.14 CONCLUSION

This chapter provides an overview of the background of the study on teachers' content knowledge for teaching Euclidean Geometry. The next chapter discusses the literature on teachers' content knowledge to teach Euclidean Geometry.

Chapter Two

Literature Review on the Teaching and Learning of Euclidean Geometry

2.1 INTRODUCTION

My study is on teachers' content knowledge of Euclidean Geometry. This study aims to understand the depth of teachers' content knowledge in Euclidean Geometry and its impact on their instructional practices. This chapter presents a literature review of different studies on teachers' content knowledge of Euclidean Geometry. The objective is to understand teachers' content knowledge of Euclidean Geometry and determine how teachers use the content knowledge to teach Euclidean Geometry.

When teachers have limited content knowledge of Euclidean Geometry, they would likely not teach in order to help learners understand (Tachie, 2020). One of the main reasons Euclidean Geometry was made optional in South Africa in 2008 was that teachers were unfamiliar with the content (Bowie, 2009). It was, however, reintroduced in 2012 (DBE, 2011). Euclidean Geometry, a foundational component of mathematics education, plays a crucial role in developing learners' spatial reasoning and problem-solving skills. In South Africa, where the education system faces various challenges, the content knowledge of teachers for teaching Euclidean Geometry is paramount in ensuring effective instruction and learner understanding. The quality of teaching Euclidean Geometry is significantly influenced by teachers' mastery of the subject matter, which directly impacts learners' learning outcomes.

Recent studies highlight the importance of teacher content knowledge, particularly in Geometry, as it serves not only as a vehicle for learners 'engagement but also to connect mathematical concepts to real-world applications (Jablonski & Ludwig, 2023; Sunzuma & Maharaj, 2021). In South Africa, research indicates disparities in teacher preparedness and confidence in teaching Geometry, often linked to varying educational backgrounds and professional development opportunities (Tachie, 2020; Mthethwa et al., 2020; Taylor, 2019). Furthermore, the curriculum changes and the emphasis on critical thinking and problem-solving in mathematics education demand a deeper understanding of geometric principles among educators.

This literature review aims to explore existing research on South African teachers' content knowledge of Euclidean Geometry. It will examine the factors influencing teacher preparedness, the challenges they face in delivering Euclidean Geometry instruction, and the implications of their knowledge for learners studying. By synthesising current findings, this review sought to identify gaps in the literature and propose directions for future research, ultimately contributing to the enhancement of Euclidean Geometry education in South African schools. However, this will begin with a brief overview of the South African Mathematics Curriculum and Assessment Policy Statement.

2.2 THE SOUTH AFRICAN MATHEMATICS CAPS

The South African Curriculum and Assessment Policy Statement (CAPS) for Mathematics is a pivotal document that guides mathematics education from Grades R to 12. It outlines clear objectives, content specifications, and assessment methods aimed at improving the quality of mathematics teaching and learning across the country (Mpungose, 2020). Central to CAPS is an emphasis on inclusivity, ensuring that the curriculum accommodates diverse learning needs and socio-cultural contexts. It highlights the importance of culturally relevant mathematical content to enhance learner engagement and motivation, while also advocating for a deep conceptual understanding of mathematics rather than rote memorisation.

This pedagogical shift aligns with international best practices that balance procedural fluency with conceptual knowledge. CAPS encourages the integration of mathematical ideas into real-life contexts, helping learners see the relevance of mathematics in everyday life and fostering the development of problem-solving skills.

The structure of CAPS offers detailed guidance for teachers, with organised content across various grade levels and topics. In Grades 7 to 9, for example, learners engage with areas such as Number Patterns, Algebra, Geometry, and Data Handling. Each topic includes defined skills and knowledge outcomes, providing teachers with a clear roadmap for lesson planning and assessment. CAPS promotes a combination of formative assessments—used throughout the learning process—and summative assessments, which evaluate final learner performance. This dual approach supports a comprehensive understanding of learners' progress and reinforces the curriculum’s focus on critical thinking and problem-solving.

The implementation of CAPS carries significant implications for mathematics teaching. Teachers are encouraged to use a variety of instructional methods, such as cooperative learning, inquiry-based learning, and technology integration, which cater to different learning styles and promote active learner engagement. However, the success of these methods is contingent upon teachers' content knowledge for teaching. As such, effective implementation of the curriculum requires ongoing professional development (Johns & Sosibo, 2019). Teachers need support in unpacking curriculum expectations and strengthening their pedagogical and content knowledge to meet these demands.

Moreover, CAPS underscores the importance of collaboration among educators, parents, and the broader community. Involving parents in their children's learning process is seen as a way to reinforce the value of mathematics and its practical application beyond the classroom.

Despite its strengths, the CAPS framework has been met with critiques and faces several implementation challenges. A major concern lies in the disparity of resources and training across schools (Pillay, 2024; Meier & West, 2020). Many educators lack adequate preparation to deliver the curriculum effectively, leading to inconsistent learner experiences. Critics also note that the focus on standardised assessments may narrow the scope of instruction, prioritising test preparation over deeper mathematical understanding (Zhang et al., 2021; Schoenfeld, 2006). These concerns highlight the need for balanced assessment practices that value both formative and summative approaches. Additionally, significant inequities persist in learners' access to quality mathematics education, particularly across socio-economic lines. While the CAPS framework aims to address such disparities, challenges remain in ensuring that all learners benefit equitably from its intentions (Maringe et al., 2015; Morolong, 2019).

Overall, the Mathematics CAPS serves as a key instrument in shaping the direction of mathematics education in South Africa. By focusing on conceptual learning, real-world applications, and inclusive teaching practices, it seeks to prepare learners with essential mathematical skills for both personal development and professional pathways (Mudaly, 2018). Yet, its success relies heavily on addressing the persistent challenges of implementation, particularly those related to teacher capacity and equity in education provision. The following section will delve into the nature of mathematical knowledge and its significance in the professional development of teachers.

2.3 MATHEMATICS KNOWLEDGE

Mathematics knowledge in this study refers to the prescribed Mathematics CAPS curriculum and its implementation, with a particular focus on how it is delivered by teachers who are qualified to teach Mathematics in the Further Education and Training (FET) phase. These teachers are full-time professionals responsible for teaching Grade 11 Mathematics and are expected to possess a Mathematics or a mathematically related degree, followed by a formal teaching qualification. In South Africa, such qualifications may include the Postgraduate Certificate in Education, National Professional Diploma in Education, Higher Diploma in Education, Secondary Teachers Diploma, or a Bachelor of Education (Tarling & Ng'ambi, 2016; Tella, 2017). Tella (2017) further asserts that teachers with higher teaching qualifications tend to demonstrate deeper content knowledge and are more effective in lesson planning, learner guidance, and diagnostic assessment.

However, research has highlighted a persistent shortage of qualified and experienced Mathematics teachers in South Africa and globally (Straker, 1987; Akiba & Liang, 2013; Frank et al., 2021). In many schools, particularly those serving working-class communities and the poor, Mathematics is often taught by teachers who struggle with the content and its pedagogical application (Dube, 2016). Teacher qualification, which includes specialised knowledge and practical skills, is therefore central to effective teaching (Santagata & Sandholtz, 2018). Teachers are considered essential educational resources, especially in under-resourced environments, and their capacity to plan and deliver content meaningfully is a critical element of learner success (Steffe et al., 2016).

The role of Mathematics teachers extends beyond content delivery to include the integration of educational policies, theoretical frameworks, and reasoning into classroom practice. Teachers are key agents in interpreting and implementing the curriculum, effectively serving as the bridge between the prescribed curriculum and the one enacted in the classroom (Gvirtz & Beech, 2004). Their teaching should align with the aims and objectives of the curriculum, promoting critical and creative thinking, particularly in Geometry instruction (Chisholm, 2003).

Central to effective Mathematics instruction is teachers' content knowledge, which encompasses an understanding of the subject's concepts, facts, and principles. This includes both the specialised knowledge required for teaching and a broader, more substantive understanding of Mathematics as a discipline (Shulman, 1986; Ball et al., 2005; Brijlall & Isaac, 2011). According to Pournara and Barmby (2019), content knowledge involves knowing what to teach, such as Euclidean Geometry, and understanding the conceptual underpinnings of mathematical topics. Ramatlapana (2017) differentiates between substantive mathematical knowledge (SMK), which all mathematicians should possess, and the content knowledge specifically relevant to school teaching. Mewborn (2001) emphasised that the nature of knowledge required for teaching Mathematics has been a focus of research in the United States for decades, underscoring its significance.

To teach Mathematics effectively, teachers must grasp both the substantive and syntactic structures of the discipline (Gokalp, 2016). They need to be able to explain accepted facts, principles, and concepts within various branches of Mathematics and articulate their importance and interconnections (Tsang & Rowland, 2005). In high-performing education systems such as Finland, teachers are required to hold a master's degree in the subjects they teach, indicating the value placed on subject-specific expertise (Roberts-Hull et al., 2015).

Geometry, as an important area within Mathematics, demands particular attention to teachers' content knowledge. Geometry includes the study of points, lines, planes, space, and spatial figures and is foundational for developing critical thinking, deductive reasoning, and logical argumentation among learners (Kösa, 2016; Jupri, 2017). The origin of the term "geometry" from the Greek "geometrien"—with "geo" meaning place and "metric" meaning measure—emphasises its real-world applicability (Hendricks & Adu, 2016). Geometry helps learners interpret and navigate their environment and solve everyday problems (Oflaz et al., 2016), making the teacher's understanding of Geometry vital to learner success (Kovács et al., 2018). Yet, many South African teachers struggle with teaching Geometry due to a lack of subject content knowledge and underdeveloped cognitive skills (Adolphus, 2011). The challenges of visualising geometric objects, interpreting the language of the discipline, and developing deep conceptual understanding contribute to this difficulty.

Within the broader field of Geometry, Euclidean Geometry holds particular significance in the South African curriculum. Associated with the Greek mathematician Euclid and formalised in his seminal work *The Elements*, Euclidean Geometry represents a formal axiomatic system where learners are expected to write proofs and solve riders (Steyn, 2016; Yixuan et al., 2016). Despite its importance, the teaching and learning of Euclidean Geometry in South Africa has long been problematic (Sulistiowati et al., 2019). Many learners struggle with the topic, and it is often introduced thoroughly only in Grade 11, with Grade 12 serving as a year for revision. Based on personal teaching experience, Euclidean Geometry is one of the most difficult sections for learners to grasp.

A significant contributor to this challenge is the fact that many teachers currently teaching Mathematics in the FET phase were never adequately trained in Euclidean Geometry themselves (Machisi, 2021). This gap in their own learning hinders their ability to teach the topic effectively. Teachers' understanding of Euclidean Geometry is thus critical, as their content knowledge directly shapes learners' ability to grasp the concepts. As Wei et al. (2017) argue, learners are unlikely to understand Euclidean Geometry if it is taught by teachers who lack competence in the subject. Therefore, enhancing teachers' knowledge and confidence in this area is essential for improving learner outcomes and achieving the goals of the Mathematics curriculum.

2.4 THE ROLE OF MATHEMATICS TEACHERS’ CONTENT KNOWLEDGE IN SCHOOLING

In terms of the National Education Policy Act 27 of 1996, the Minister of Education outlined seven educator roles that serve as the norms for teacher development and form the core of all initial teacher qualifications and learning programmes. Several of these roles are deeply embedded in a teacher’s content knowledge, including the roles of learning interpreter and designer of learning programmes and materials, learning mediator, assessor, and subject specialist. It is evident that the effective fulfilment of these roles hinges on the teacher’s depth of subject knowledge; without adequate content knowledge, the competences attached to these roles are difficult to achieve.

When it comes to teaching Euclidean Geometry, teachers are expected to interpret and design lessons that align with the Curriculum and Assessment Policy Statement (CAPS), tailoring their instruction to accommodate diverse learner needs (DBE, 2011; Reeves, 2006). This involves understanding and unpacking the curriculum, selecting appropriate textual and visual resources, and sequencing the content in a way that supports differentiated learning. As Chisholm (2003) notes, teachers are required to design lessons that are meaningful and promote curriculum goals. In this context, content knowledge is essential in determining which physical resources—such as textbooks, workbooks, calculators, chalkboards—are appropriate for effective teaching. Without a strong grasp of content, teachers may struggle to make informed decisions about resource use, which compromises their ability to mediate learning effectively.

Assessment is another key area where a teacher's content knowledge is vital. According to Thompson (2006), teachers' views on mathematics education influence how they understand and implement assessment practices. A sound knowledge of Euclidean Geometry equips teachers to design assessments that go beyond rote learning and support the development of learners' reasoning skills. Weldeana (2008) advocates for problem-solving assessment approaches, which are more effective than teacher-centred methods because they foster analytical thinking. Darling-Hammond and Snyder (2000) similarly emphasise the importance of accurate and relevant assessment forms in mathematics education. However, research by Krishnannair and Christiansen (2013) found that many South African teachers cannot select assessment tasks that align with their teaching objectives. This points to a gap between content knowledge and assessment practice, which undermines the teacher's role as an assessor. Geometry, in particular, should be taught and assessed in ways that prioritise practical understanding over theoretical abstraction (Jones, 2000). The fact that some teachers struggle to achieve 60% on standardised tests in Grade 11 Euclidean Geometry underscores the need for in-service training to enhance their subject knowledge and assessment practices (Baumert et al., 2010; Berkvens et al., 2014).

In their role as subject specialists, mathematics teachers are expected to be well-versed in the knowledge, values, and methodologies relevant to their discipline. The concept of curriculum as a "plan for learning" (van den Akker et al., 2009) is useful in understanding the dual function of the intended and implemented curriculum. The intended curriculum refers to what is formally prescribed (Khoza, 2015), while the implemented curriculum reflects the content that is actually delivered in the classroom. Teachers' understanding of Euclidean Geometry thus forms part of the intended curriculum within CAPS (DBE, 2011), which has been designed to meet international standards (Hoadley & Jansen, 2012). This requires teachers not only to be familiar with the prescribed content but also to effectively implement it in ways that meet the curriculum's broader educational aims. According to Gvirtz and Beech (2004), the perceived curriculum is shaped by how teachers interpret and enact the intended curriculum. Therefore, a teacher's ability to use their content knowledge in the classroom is central to realising the goals of the curriculum and fostering learners' creative and critical thinking skills.

Equally important is the teacher's role as a learning mediator. This involves facilitating learning in a way that responds to the diverse needs of learners, including those with learning barriers. Shulman (1986) argues that teachers must not only be familiar with the factual and procedural aspects of geometry but must also grasp its conceptual foundations. This enables them to communicate geometric principles in accessible and engaging ways. Effective mediation entails contextualising abstract geometric concepts into practical, real-world applications. For instance, teachers can draw connections between geometry and fields such as architecture, engineering, or patterns in nature to illustrate the relevance of the subject (Bali et al., 2005). A deep understanding of Euclidean Geometry allows teachers to select pedagogical strategies that make the subject more engaging and meaningful for learners.

In conclusion, mathematics teachers' content knowledge of Euclidean Geometry is fundamental to their effectiveness in multiple professional roles as interpreters and designers of learning programmes, assessors, subject specialists, and learning mediators. Strengthening this knowledge base is essential for improving the quality of teaching and ultimately enhancing learner outcomes in Geometry (Baumert et al., 2010; Berkvens et al., 2014).

2.5 TEACHERS' KNOWLEDGE OF TEACHING

Teachers require a broad repertoire of knowledge beyond mere content expertise to effectively navigate the complexities of the teaching profession. In Mathematics, for instance, this includes understanding mathematical reasoning, fluency in the use of examples and terminology, and an awareness of what constitutes mathematical proficiency (Ball et al., 2008; Kilpatrick et al., 2001). Educational research has consistently demonstrated that the quality of learning opportunities provided by teachers significantly affects learners' understanding and motivation (Hattie, 2009; McCaffrey et al., 2004). Accordingly, increasing attention has been given to the interplay between teachers' content knowledge and their pedagogical content knowledge (PCK), both of which have been shown to influence instructional practice and learner outcomes (Baumert et al., 2010; Hill et al., 2005). PCK, as originally conceptualised by Shulman (1986), refers to the knowledge that enables teachers to make subject matter comprehensible to learners, while content knowledge refers to their understanding of the subject itself. Shulman (1986, p. 23) famously asserted that "mere content knowledge is more likely to be as useless pedagogically as content-free skill".

Although content knowledge and PCK are theoretically distinct, empirical findings suggest that in practice they often intersect. Hill et al. (2004) found that elementary teachers' content knowledge and PCK in Mathematics are often intertwined within a single construct termed Mathematical Knowledge for Teaching (MKT). MKT encompasses the specialised understanding required for effective Mathematics instruction (Ball et al., 2008). Within this framework, content knowledge is further broken down into three components: common content knowledge, specialised content knowledge, and horizon content knowledge. Common content knowledge refers to the mathematical knowledge used in non-teaching settings and allows teachers to verify the correctness of learners' responses and textbook content. Specialised content knowledge, by contrast, is unique to teaching and helps educators diagnose and address learners' errors. Horizon content knowledge involves an understanding of how mathematical topics interrelate across the curriculum (Ball et al., 2008). Luneta (2008) underscores the importance of teachers reflecting on and connecting their knowledge bases to enhance instruction, particularly in Geometry.

In the context of this study, content knowledge refers specifically to teachers' understanding of mathematical content, encompassing the correct interpretation and application of concepts, facts, procedures, principles, and theories (Ramatlapana & Berger, 2018). Research strongly indicates that a teacher's content knowledge directly affects their instructional quality and learner performance (Baumert & Kunter, 2013; Wang et al., 2018). Ball et al. (2008) emphasised that teachers who lack adequate subject knowledge are unlikely to facilitate meaningful student understanding. Professional development initiatives aimed at deepening teachers' grasp of geometric concepts have been found to enhance student engagement and understanding, particularly when such training occurs within professional learning communities (Reid & Reid, 2017). Shulman (1986) considered content knowledge fundamental to effective teaching, a sentiment echoed by Ponte and Chapman (2008), who described it as a cornerstone of instructional quality. Shulman (1986, p. 9) further posited that "those who understand knowledge growth in teaching" regard content knowledge as the root of all other knowledge domains necessary for educators.

Teachers with profound content knowledge are better equipped to guide learners toward deeper understanding by asking probing questions rather than simply providing answers. They are also capable of designing tasks at appropriate cognitive levels and adapting instruction to meet learners' needs (Anthony & Walshaw, 2009; Borko & Livingston, 2020; Kong & Li, 2021; Rosa & Orey, 2023; Zhang & Li, 2024). Kilic (2011) found that pre-service teachers with weak content knowledge struggled to identify and correct learners' misconceptions. However, when they possessed a deeper understanding of a topic, they were more successful in explaining mathematical reasoning and making interdisciplinary connections. Teachers' decisions regarding instructional resources and tasks also rely heavily on their depth of subject matter knowledge. The need for substantial content knowledge in Mathematics, particularly in Geometry, is widely acknowledged (Kleickmann et al., 2013; Ngwenya, 2014). Teachers with insufficient knowledge often lack the confidence to address unexpected learner questions, fail to detect errors in unfamiliar solution methods, and are less capable of anticipating learners' mistakes (Prendergast & O'Donoghue, 2014). Thus, teachers must have extensive subject knowledge to significantly impact classroom teaching and learning.

A wide range of factors influences teacher readiness to utilise their content knowledge effectively. These include their educational background, the nature of their training programmes, teaching experience, access to support systems, the school environment, and external socio-economic contexts. Research confirms that teachers with strong academic backgrounds in their subject areas tend to demonstrate greater confidence and effectiveness in the classroom (Koehler et al., 2014). Teachers holding advanced degrees often report higher self-efficacy and better student engagement outcomes (Webb et al., 2021). Darling-Hammond (2000) concluded that teachers trained specifically in their subject disciplines are better equipped to handle classroom demands, including adapting to new teaching modalities such as online instruction.

The structure and quality of training programmes are also pivotal. Programmes that integrate theoretical learning with practical teaching experiences significantly enhance teacher preparedness (Hammerness, 2005). Blume and Derry (2020) showed that training which emphasises not just content knowledge, but also how to apply it pedagogically, builds teacher confidence. Wilson et al. (2020) stressed that effective training should blend subject knowledge, pedagogy, and classroom practice. Such programmes should also support curriculum pacing and address the areas teachers find most challenging. The National Development Plan (2012, p. 315) advocates for the use of Information and Communication Technology (ICT) in teacher development, especially given the scale of the teaching workforce.

Experience is another essential factor. Ingersoll (2003) found that novice teachers often feel underprepared, a gap that mentorship can help bridge. More experienced teachers consistently report greater confidence and effectiveness. Lumpe (2012) confirmed that teaching experience correlates with improved classroom management and pedagogical decision-making. Experience also fosters self-efficacy, better adaptation to learner diversity, and increased participation in professional development, all of which contribute to improved teaching quality.

Support systems, including mentorship and ongoing professional development, are critical to strengthening teacher knowledge and confidence. Feiman-Nemser (2001) highlighted the benefits of mentoring, while Darling-Hammond et al. (2017) demonstrated how continuous learning opportunities enhance instructional quality. Ongoing training ensures teachers remain up to date with pedagogical innovations, thereby improving learning outcomes. More recent studies reaffirm these findings, indicating that mentoring and professional development significantly bolster teacher confidence and practice (Brown et al., 2024; Mgaiwa & Milinga, 2024; Woulfin & Jones, 2021).

The school environment also plays a central role. Thapa et al. (2013) found that teachers working in supportive environments felt more prepared to manage classroom challenges. Ostroff and Kozlowski (1992) highlighted that collaborative school cultures, rich in resources and characterised by strong leadership, positively influence teacher efficacy. Pearce (2006) linked effective school leadership to higher teacher confidence in applying content knowledge. Sullivan and McIntyre (2016) further observed that teachers who felt supported by their administrators and colleagues were more likely to express a sense of readiness.

External factors, such as socio-economic conditions and community engagement, equally affect teacher confidence. Coleman et al. (1966) first demonstrated that educational outcomes are shaped by broader social contexts. Teachers in better-resourced schools often report feeling more competent and supported. O'Day and Smith (2016) found that when schools actively foster relationships with families and communities, teachers report greater levels of preparedness. These insights suggest that creating strong school-community ties is crucial in helping teachers apply their content knowledge effectively.

In sum, teacher readiness is shaped by a complex interplay of academic background, training quality, professional experience, institutional support, and broader environmental factors. A cohesive approach that integrates these elements is essential for enhancing teachers' ability to utilise their content knowledge meaningfully and adaptively in today's dynamic classrooms.

2.6 THE CHALLENGES FACED IN TEACHING EUCLIDEAN GEOMETRY

Euclidean geometry plays a vital role in mathematics education, yet its teaching is often fraught with significant challenges. One of the primary difficulties stems from the cognitive demands the subject places on learners. Miller and Dyer (2020) highlight the importance of spatial reasoning in Geometry, noting that many learners struggle to develop this skill, which is fundamental not only for understanding geometric concepts but also for broader mathematical problem-solving and critical thinking. Similarly, Uttal et al. (2013) found that visualising and manipulating geometric figures can be a major hurdle for learners, particularly when grasping concepts such as congruence, similarity, and transformations. Gupta and Zheng (2020) underscore that cognitive load can be mitigated through instructional strategies like worked examples, which are especially effective for learners with low prior knowledge. They argue that carefully structured lessons aligned with students' existing schema can reduce overload and foster conceptual understanding. Compounding these cognitive challenges is the issue of teacher content knowledge. Fennell (2006) reported that many teachers feel inadequately prepared to teach Geometry, often lacking both confidence and depth in their subject knowledge. Ball et al. (2008) emphasise the need for teachers to possess a thorough understanding of Euclidean geometry in order to teach it effectively, calling for sustained professional development to enhance both content knowledge and pedagogical practices.

Curricular design and access to appropriate resources further influence the effectiveness of geometry teaching. Cai and Wang (2010) argue that many geometry curricula are overly focused on rote memorisation of formulas, neglecting the development of conceptual understanding. This can lead to disengagement and difficulty applying geometric principles to real-life situations. Battista (2008) advocates for the use of dynamic geometry software and physical manipulatives to promote deeper understanding. However, many teachers face barriers to implementing these tools due to limited access or insufficient training. Jablonski and Ludwig (2023) identify key themes in geometry education, including geometric thinking, argumentation and proof, teacher education, content, and digital tools. They note the persistent under-representation of Euclidean geometry in curriculum reforms and stress the need for integrating theoretical and practical approaches in teaching the subject.

Assessment practices also present significant obstacles. McClain and Cobb (2001) argue that traditional assessments often prioritise rote recall over conceptual understanding, thereby providing an inaccurate picture of students' knowledge. Stein and Smith (1998) add that misalignment between instructional goals and assessment methods leads to inconsistencies in evaluating student learning. More recent studies have explored innovative assessment techniques, such as using problem-posing tasks to develop higher-order thinking (Radmehr & Vos, 2020), employing augmented reality to enhance engagement and comprehension (Ibili et al., 2020), and utilising tools like PowerPoint to support geometry assessment (Mensah & Nabie, 2021). These studies underscore the importance of aligning assessments with learning goals and leveraging technology to improve outcomes.

Another pressing challenge in geometry education is learner motivation and engagement. Geometry is often perceived by learners as abstract and difficult, which can reduce motivation and hinder achievement (Hegarty et al., 2009). Lesh and Doerr (2003) assert that linking geometry instruction to real-world applications enhances learner understanding and interest, yet many teachers miss opportunities to contextualise their lessons effectively.

Equity and access remain critical concerns. Secada (1992) observed that systemic barriers disproportionately affect learners from marginalised backgrounds, limiting their access to high-quality geometry instruction and resources. Addressing these disparities requires targeted efforts to ensure that all students have equitable opportunities to engage with Euclidean geometry.

Teachers' knowledge has profound implications for learner understanding. Shulman (1986) first articulated the importance of content knowledge, pedagogical knowledge, and pedagogical content knowledge (PCK), framing these as essential components of effective teaching. Hill et al. (2005) demonstrated that teachers with a strong grasp of mathematical content are better equipped to improve learner performance. Ball et al. (2005) similarly found that knowledgeable teachers are more adept at anticipating student misconceptions and adjusting their instruction accordingly. Hill and Chin (2022) reaffirm this connection, showing that mathematical knowledge for teaching directly influences learner outcomes. Jacob et al. (2020) further support this, finding a significant correlation between teachers' professional competence and the quality of instruction, which in turn affects student achievement.

Pedagogical knowledge also plays a central role in shaping student outcomes. Koehler and Mishra (2009), through the Technological Pedagogical Content Knowledge (TPCK) framework, found that the integration of content, pedagogy, and technology can enrich instructional quality. Yuan and Lee (2020) highlight the effectiveness of pedagogical strategies like differentiated instruction and formative assessment in boosting learner engagement and success.

PCK, which bridges content and pedagogy, is particularly important for making subject matter comprehensible to learners. Shulman (1986) and Gess-Newsome (1999) underscore the role of PCK in translating complex concepts into accessible instruction. Fennell and Rowan (2001), along with Jacob et al. (2020) and Sintema and Marbán (2020), show that teachers with robust PCK can more effectively connect geometric concepts to students' lived experiences, thereby improving understanding.

The link between teacher knowledge and student achievement is well-documented. Darling-Hammond (2000) found that teacher qualifications, especially subject knowledge, are strong predictors of student learning. Hattie (2009) echoes this, identifying teacher efficacy rooted in knowledge and skill as a major influence on learner success. Hill and Chin (2020) also found that early-career teaching experience is associated with improved student outcomes in Mathematics and reading.

Professional development is a key mechanism for enhancing teacher knowledge. Desimone (2009) identified critical features of effective professional development, such as coherence, active learning, and adequate duration, that promote lasting changes in teaching practices. Garet et al. (2021) reinforce the value of collaborative, content-focused training. Darling-Hammond et al. (2020) reviewed 35 studies and concluded that sustained, high-quality professional development leads to improved teaching and better student outcomes.

Nonetheless, several challenges persist. Harris and Sass (2007) caution that teacher content knowledge can diminish without ongoing support, emphasising the need for continuous professional learning. Ball and Forzani (2011) argue that many teacher preparation programmes fail to sufficiently develop PCK, which hampers instructional effectiveness.

In conclusion, improving the teaching and learning of Euclidean geometry requires a comprehensive approach that addresses cognitive demands, strengthens teacher content and pedagogical knowledge, reforms curriculum and assessment practices, enhances learner engagement, and ensures equitable access to resources and instruction. Sustained professional development and further research are essential to support teachers and ultimately foster student success in Euclidean geometry.

2.7 CONCLUSION

The reviewed literature highlights the central role of teachers' content knowledge, pedagogical skills, and pedagogical content knowledge in the effective teaching of Euclidean geometry. It is evident that strong content knowledge not only enhances lesson planning and instructional delivery but also enables more accurate assessment of learner understanding. The quality of geometry instruction is closely tied to teachers' ability to address cognitive challenges, use appropriate resources, implement meaningful assessments, and foster learner engagement. Moreover, the literature confirms that learner achievement in Euclidean geometry is significantly influenced by the depth and application of the teacher's knowledge. Addressing persistent challenges such as inequities in access, insufficient teacher preparation, and limited use of real-world applications and digital tools requires a multifaceted and sustained effort. The following chapter introduces the van Hiele model, which provides the theoretical framework for this study and offers a structured perspective on how learners develop geometric understanding through hierarchical levels of reasoning.

CHAPTER THREE

THEORETICAL FRAMEWORK: THE VAN HIELE MODEL OF GEOMETRIC THOUGHT

3.1 INTRODUCTION

A theoretical framework is an essential part of any research project (Collins & Carrie, 2018). It links the research to existing theories and literature, placing it within a larger academic context. This helps demonstrate how the research advances, contradicts, or expands our current understanding. A theoretical framework also clarifies the study's ideas and variables, providing a lens through which to analyse the data. This helps the researcher identify patterns and connections within the data and interpret the results in light of the research objectives.

This study on teachers' content knowledge of teaching Euclidean Geometry uses the van Hiele model as its theoretical framework. Euclidean Geometry is a foundational part of mathematics education, crucial for developing students' spatial reasoning and geometric understanding. Effective teaching of this subject requires not only a deep understanding of the concepts but also the ability to convey them effectively.

The van Hiele model, proposed by Pierre van Hiele and Dina van Hiele-Geldof in the 1950s, offers a valuable framework for examining the development of geometric understanding (Machisi, 2020; Lwanga, 2022). The model posits that learners progress through five distinct levels of understanding: visualisation, analysis, informal deduction, formal deduction, and rigour**.** Each level builds on the previous one, highlighting the importance of teachers not only having strong content knowledge but also understanding their students' developmental stages of geometric thinking.

This study will explore Grade 11 teachers' content knowledge of Euclidean Geometry through the lens of the van Hiele model. By examining how teachers' understanding aligns with these levels of geometric reasoning, this research aims to shed light on the connections between teachers' knowledge and their pedagogical practices. Understanding these dynamics is essential for improving geometry instruction and fostering students' geometric reasoning skills.

The van Hiele model is a valuable framework for this study for several reasons:

Structured Progression: The model outlines a clear progression of geometric understanding, from simple visualisation to formal proof (Machisi, 2020). This helps educators assess their own and their students' understanding and identify areas for professional development.

Pedagogical Guidance: The model emphasises that teachers must not only understand geometric concepts but also be able to teach them effectively at different levels (Abdullah & Zakaria, 2013; Aldiabat & Yew, 2024). This understanding allows teachers to tailor their instruction to meet students' needs.

Promotes Reasoning Skills: The model helps develop reasoning skills by providing a structured framework for understanding geometric concepts (Pusey, 2003; Yalley et al., 2021). Examining how teachers reason about geometry can reveal gaps in their content knowledge.

Supports Constructivism: The model aligns with constructivist learning theories, which suggest that understanding is built through experiences (Piaget, 1973; Hiebert & Carpenter, 1992). Each stage of the model reflects the idea that knowledge is developed through active engagement.

Informs Assessments and Curriculum: The model provides a clear structure for creating assessments that evaluate geometric understanding and for aligning curriculum and instructional strategies with students' cognitive development (van Hiele, 1986; Battista, 2008).

Ultimately, using the van Hiele model as a framework will provide deeper insights into the complexities of teaching geometry and enhance the quality of geometry education.

3.2 Overview of the Van Hiele Model

The van Hiele model, developed by Dutch educators Pierre van Hiele and Dina van Hiele-Geldof in the 1950s, describes how students learn and understand geometry. It identifies five distinct levels of geometric reasoning that are also relevant for assessing and improving teachers' content knowledge. The levels are hierarchical, meaning a student must progress through each level to achieve a deeper understanding. Movement between levels requires appropriate instruction and experiences.

The five levels are:

Level 0: Visualisation (Recognition): At this level, individuals recognise geometric shapes based on their overall appearance. They can identify a square as a square but do not yet understand its specific properties.

Level 1: Analysis (Properties): Individuals begin to analyse shapes by understanding their properties (e.g., sides, angles, symmetry). They can classify shapes based on these characteristics but may not understand the relationships between different properties.

Level 2: Abstraction (Relationships): This level involves a deeper understanding where individuals can relate the properties of shapes to one another. They start to comprehend concepts like congruence, similarity, and the relationships between different classes of shapes (e.g., a square is also a rectangle).

Level 3: Deduction (Theorems): At this stage, individuals can reason logically about shapes, construct and understand geometric proofs, and work with formal definitions and theorems. They grasp the deductive nature of geometry.

Level 4: Rigour (Formal Systems): The highest level involves understanding geometry as a formal axiomatic system. Individuals can work with various axiomatic systems and appreciate the logical structure of geometry beyond a single system.

For teachers, the van Hiele model emphasizes that instruction should be aligned with the student’s current level of understanding. Teachers' content knowledge can be assessed through their own understanding of these levels, which enables them to recognize students' thinking and tailor instruction accordingly. The model highlights the importance of both content knowledge and pedagogical knowledge, a crucial duality for effective geometry instruction.

3.3 Empirical Studies that have used the Van Hiele Model

Several empirical studies have explored teachers' content knowledge of Euclidean Geometry through the lens of the van Hiele model.

Understanding of the Van Hiele Levels

Research has shown that teachers' awareness of the van Hiele levels significantly impacts their teaching. For example, a study by Dougherty and Bowers (2017) highlighted that teachers who were familiar with these levels were better equipped to facilitate student learning.

Content Knowledge and Pedagogical Content Knowledge (PCK)

Dindyal (2008) investigated the relationship between teachers' content knowledge and their PCK in geometry. The study found that teachers with a deeper understanding of Euclidean Geometry were more adept at addressing students' misconceptions and adapting instruction.

Impact on Student Learning

A study by Huang and Li (2015) found that teachers familiar with the van Hiele model were better at designing activities that promoted higher levels of student understanding, leading to better student outcomes.

Teacher Education Programs

Research by Steele et al. (2016) showed that integrating the van Hiele model into teacher training curricula improved pre-service teachers' content knowledge and confidence in teaching geometry.

Professional Development

Zazkis and Campbell (2014) found that professional development based on the van Hiele model significantly enhanced teachers' understanding of geometric concepts and their ability to teach them effectively.

Assessing Teacher Knowledge

Gürbüz et al. (2015) developed an assessment tool based on the van Hiele model to evaluate teachers' content knowledge, highlighting the need for valid assessment methods to inform professional development.

Case Studies

Case studies, such as those by Zazkis and Leikin (2008), have provided insights into how individual teachers' content knowledge, viewed through the van Hiele model, shapes their instructional practices and student engagement.

These studies collectively suggest that a strong grasp of the van Hiele model enhances teachers' content knowledge and pedagogical approaches, ultimately leading to better student understanding and engagement in geometry.

3.4 Critique of the Van Hiele Model

The van Hiele model is a widely recognised framework for understanding how students learn geometry. While it is particularly relevant for studying teachers' content knowledge in Euclidean Geometry, it has both strengths and limitations.

3.4.1 Strengths of the Van Hiele Model

Cognitive Development Focus: The model emphasises the progression of understanding, providing a structured way to assess teachers' content knowledge and their ability to teach concepts at various cognitive levels.

Clear Framework: The five distinct levels offer a clear framework for evaluating both student and teacher comprehension, which helps identify areas where teachers may struggle to convey concepts.

Pedagogical Implications: The model encourages teachers to design lessons that align with students' cognitive levels, thereby improving pedagogical practices and supporting differentiated instruction.

Assessment Tool: The model provides a basis for creating assessments to measure teachers' geometric understanding and its influence on their teaching.

3.4.2 Limitations of the Van Hiele Model

Rigid Stage Progression: The model's linear progression may oversimplify the complexities of how people learn geometry. Teachers and students might have strengths and weaknesses across different stages, which the model does not fully account for.

Contextual Factors: The model does not adequately consider contextual factors such as cultural influences, teaching experience, or school resources, which are especially crucial in diverse educational contexts like South Africa.

Emphasis on Formal Geometry: The model primarily focuses on Euclidean Geometry and may not fully encompass other important areas like transformational or coordinate geometry.

Neglect of Conceptual Understanding: While the model emphasises learning stages, it may under-emphasise the importance of a teacher's deep conceptual understanding of the underlying mathematical principles.

Lack of Attention to Instructional Strategies: The model does not directly address the specific instructional strategies teachers use to help students progress through the stages.

3.5 Conclusion

The van Hiele model is a robust theoretical framework for investigating teachers' content knowledge in Euclidean Geometry. Its focus on the developmental stages of geometric understanding provides a structured way to understand how teachers conceptualise and convey geometric concepts. This framework not only highlights the importance of teachers' content knowledge in improving student understanding but also underscores the necessity for targeted professional development. By adopting this model, we can gain a more nuanced understanding of the interplay between teachers' knowledge, instructional strategies, and student outcomes in geometry education.

CHAPTER FOUR

RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

This chapter outlines the research design and methods used in this qualitative study. The previous chapter presented the theoretical framework of the van Hiele model on teachers' content knowledge of Euclidean Geometry. This research focused on Grade 11 Mathematics teachers and sought to answer two key questions: What is Grade 11 Mathematics teachers content knowledge of Euclidean Geometry? And, how do Grade 11 Mathematics teachers use their content knowledge when teaching Euclidean Geometry? A framework of the descriptions and justifications of the research design and approaches is articulated in this chapter. The methodology followed concerning data collection, recording, summarising, analysing, and interpretation is presented. The participants, sampling procedures, and ethical considerations are also articulated. This discussion further clarifies the reasons for the chosen research methods and outlines the study's objectives.

4.2 RESEARCH APPROACH

This research study focused on exploring Grade 11 Mathematics teachers' content knowledge of teaching Euclidean Geometry. Hence, this study is qualitative (Taylor et al., 2015). In a qualitative study, a researcher intends to gain insights into participants' thoughts and feelings about a particular problem, which may provide the basis for future qualitative studies (Sutton & Austin, 2015). The research approach refers to the methodology or plan followed to discover or study a phenomenon to obtain data. The researcher used qualitative methods to collect in-depth details on a particular subject. A quantitative method is a scientific method that focuses on fresh data generation relating to the problem from a large population and analysis of the data. It ignores an individual's emotions, feelings, or environmental context (Kumar, 2019). Qualitative research tends to be field-focused (Eisner, 2017). According to Kamal (2019), qualitative researchers explore and are more concerned with how individuals or groups describe their worlds. Indeed, in this study, the researcher was also more concerned with how mathematical teachers teaching Euclidean Geometry in Grade 11 describe their understanding of teaching Euclidean Geometry.

According to Ary et al. (2018), qualitative research seeks a deeper understanding of the situation by concentrating on the total picture rather than numerically analysing it. This means that participants are studied in detail through in-depth questionnaires, interviews, and classroom observations. Jing and Huang (2015) maintained that qualitative research is a comprehensive exploration process that focuses more on understanding and interpreting social phenomena in natural settings to which people give meaning. Qualitative researchers use non-quantitative research methods to provide descriptions of current situations. Through the qualitative research approach, a researcher can analyse various conditions which may cause people to behave in a particular manner (Kapur, 2018). The researcher believed that understanding the teachers' content knowledge might help in finding solutions to address various conditions leading to poor performance in Mathematical Geometry in schools.

According to Kumar (2019), useful information cannot be reduced to numbers. People's ideas, feelings, emotions, beliefs, etc., can only be recorded as words. Taylor et al. (2015, p.7) referred to "qualitative research as the kind of research which produces language data, i.e., people's written responses or spoken words, and observing their actions". "Qualitative researchers ' research is not based on what they can measure or count; it is more concerned with the meaning of how people see things from their own real-life experiences" (Berger, 2015). This means that the researcher believed that teaching Euclidean Geometry was embedded in an individual's real-life experiences and could not be measured quantitatively. According to Holloway and Galvin (2016), researchers using a qualitative approach immerse themselves in a culture or group by observing its people and their interactions, often by participating in activities, interviewing key people, taking life histories, constructing case studies, and analysing existing documents or other cultural artefacts. The qualitative researcher aims to ascertain an insider's view of the group under study (Taylor et al., 2015).

4.3 RESEARCHER'S ROLE IN QUALITATIVE APPROACH

According to De Vos (2002), the researcher directly participated in the study using a qualitative research approach. The researcher intermingled with the participants, and they became the 'instrument'. The researcher directly participated in this study, interviewing the participants and collecting the data. The analysed data was based on the researcher's subjective interpretation and depended largely on the researcher's skills. From the aforementioned reasons, one concluded that a qualitative report could never exclude the researcher's perspectives. This was confirmed by Lewis (2015) when stating that the methods or procedures of qualitative research were usually seen as inductive and emerging and also shaped by the researcher's own subjective experience and meaning in collecting and analysing the data, and this concurred with the conduct of the researcher who was immensely involved in the data collection hence the analysis of the collected data had an element of subjective interpretation.

4.4 THE RATIONALE FOR THE QUALITATIVE APPROACH

The qualitative research approach was chosen based on the purpose of the study. Thus, collecting descriptive data from persons' written and spoken words will be preferred to explore teachers' understanding of Euclidean Geometry. Therefore, the qualitative approach is suitable for a research study of this nature, which seeks to understand teachers' content knowledge and how it can be used in mathematics education. Furthermore, the qualitative approach for this research was due to the flexibility and uniqueness of the research design. De Vos (2002) confirmed that no fixed patterns are followed, and the arrangement cannot be replicated exactly.

4.5 RELIABILITY OF THE QUALITATIVE APPROACH

Reliability refers to the degree to which a measurement instrument (a test and/or task-based interview) can yield the same results on repeated applications (Durrheim, 1999a). In this research, the instruments used in data generation were a test, a questionnaire, and a task-based interview. The test was first administered to five Grade 11 teachers in KwaZulu-Natal before the main participants wrote it. The task-based interview was conducted with the same population to increase reliability. The final questions were then produced after adjustments based on the pilot study. The trial's purpose was to determine whether the test instrument could help understand. The pilot study was based on the argument that protesting what they intended to investigate (Mouton, 2001) could help refine and improve the test instrument.

4.6 VALIDITY OF THE QUALITATIVE APPROACH

A valid research instrument measures what it is intended to measure (Ross, 2005). Ross further stated four important types of validity in education research: content validity, construct validity, external validity, and internal validity. This study used content validity. Teachers' subject matter content on circle geometry was investigated, as stated in Chapter 2. Validity was the most important characteristic when constructing or selecting research instruments.

4.7 RESEARCH PARADIGM

According to O'Donoghue (2018, pp. 23-24), "research paradigm is a framework or a pattern that guides a scientific study, and it functions as a map which clearly defines acceptable theories and methods to solve defined problems." This assisted the researcher in describing reality, knowledge, and truth (Rahi, 2017). Furthermore, the research paradigm affects the researcher's thinking about the topic (Kamal, 2019). A research paradigm helps the researcher define valid research (Ridder, 2017) since it represents researchers' thoughts, values, and beliefs about the world (Kamal, 2019).

The four major research paradigms used to guide research methods, i.e., positivism, interpretivism, critical, and postmodernism (Ryan, 2018), are based upon different assumptions about how knowledge is generated and accepted as valid (O'Donoghue, 2018). Ryan (2018) confirmed that being able to justify the decision to reject or adopt a paradigm forms the basis of any research; therefore, it is important to understand these paradigms and their origins and decide which is most appropriate for a particular study.

4.8 INTERPRETIVISM

The interpretive paradigm was considered the most relevant paradigm in this study. It emphasizes the importance of social interaction as the basis for knowledge. The researcher used skills as a social being to understand how others understand their world (O'Donoghue, 2018). Therefore, knowledge was constructed by mutual negotiation and was specific to the situation being investigated.

Interpretivism argues that truth and knowledge are culturally and historically situated. They are based on people's experiences and understanding (Ryan, 2018). Ryan (2018) further argued that one can never separate researchers from their values and beliefs, which will inform how data is collected, interpreted, and analysed. Interpretivists believe in deep understanding and interpretation of a concept, and they explore the understanding of the world in which they live by developing subjective meanings of their experiences (Rahi, 2017).

The purpose of using the interpretive paradigm was to provide information that allowed the researcher to "make sense" of the world from the perspective of participants (Bygstad et al., 2016, pp. 83-96). Munkvold and Bygstad (2016, pp. 83-96) further stated that interpretive research aims to "understand phenomena through accessing the meaning that participants assign to them." Researchers within this paradigm seek to understand rather than explain (Cohen et al., 2013). According to Rahi (2017), interpretive researchers emphasise a better understanding of the world through first-hand experience, truthful reporting, and quotations of actual conversations from insiders' perspectives. Researchers in this paradigm use data-gathering methods that are sensitive to context and allow rich and detailed descriptions of social phenomena by encouraging participants to speak freely (Le Grange, 2018). Researchers who follow the interpretive paradigm view the world through the experiences and perceptions of the participants, and they use those experiences to interpret and construct their understanding from collected data (Rahi, 2017). This study followed the interpretive paradigm since this study understands teachers' content knowledge of teaching Euclidean Geometry. Each participant was given questions, tested, and interviewed. The gathered data was used to construct and interpret participants' understanding.

4.9 RESEARCH STYLE

Taylor et al. (2015, p. 3) stated that "research design refers to the ways or methods of approaching problems and seeking solutions" or a framework created to find research questions. It is a guideline or an instrument to be followed when addressing the research problem (Leavy, 2017). Moreover, it expresses the techniques for conducting the study, including features such as from whom, when, and under what conditions the data will be collected. According to Durrheim (2002), it is the plan of how the researcher will systematically collect and analyse the data required to give reasonable solutions to research problems. The research design refers to the logical structure of the inquiry so that emphatic conclusions are afforded (De Vos, 2002; Pillay, 2004).

4.9.1 Case Studies

Case studies are defined as intensive studies about a person or a group of people to explore and understand a setting. This study explored Grade 11 Mathematics teachers' content knowledge of teaching Euclidean Geometry. A "case studies allow a phenomenon to be studied in its real-world context" (Bass et al., 2018, p. 13). According to Yin (1981, p. 98), "the need to use case studies arises whenever an empirical inquiry must examine a contemporary phenomenon in its real-life context, especially when the boundaries between phenomenon and context are not evident".

4.9.2 Multiple-Case Study

This study was built on a multiple-case study in which the researcher interacted with 10 Grade 11 Mathematics teachers in the Ugu district in KwaZulu-Natal. A multiple-case design is when conclusions are drawn from different schools (Nielsen et al., 2015). The rationale behind using a multiple-case study over a single-case study was to obtain a more comprehensive understanding of the subjects studied (Wahyuni, 2012, p. 72). Van der Berg and Gustafsson (2019) stated that a multiple-case study allows a wider exploration of research questions and creates stronger and more valuable findings.

4.10 DATA GENERATION

4.10.1 Data Generation Instruments

The data generation for the study included tests and a task-based interview as shown in Table 4.1.

Table 4.1: Data generation plan

Research question	Data collection method
1. What is Grade 11 Mathematics teachers' content knowledge of Euclidean Geometry?	Pre-task-based interview, Test, Task-based interview
2. How do Grade 11 Mathematics teachers use their content knowledge when teaching Euclidean Geometry?	Pre-task-based interview, Test, Task-based interview

4.10.2 Pre-Task-Based Interview

The researcher needed to find ways of understanding the participants, and it was necessary to find out some details about them. An interview task to identify teachers' conceptions of teaching was developed. This was created to ask participants about their background, understand their experiences, and gather information about their understanding of Euclidean Geometry. A list of questions was developed for the participants to interact with.

4.10.3 Test

The researcher designed and developed the paper-and-pen test with questions posed in English. The researcher adapted individual items in the test to comply with the requirements of this study. These items revealed teachers' van Hiele levels of understanding Euclidean Geometry with specific reference to Circle Geometry. This test consisted of 12 questions based on the first five categories of the first four levels of van Hiele's theory. The Harry Gwala District teachers were first administered the same paper-and-pen test. The aim was to test the validity of the van Hiele theory (Gutierrez et al., 1991; Senk, 1989; Mayberry, 1983). The test items were generated and carefully chosen. The test was designed with the following characteristics: The duration of the paper was two hours. The test consisted of several questions, but with no marks allocation since the study aimed to explore teachers' content knowledge of teaching circle Geometry and not grade them.

According to Gronlund (1998, p. 55), knowledge and comprehension items were used to "measure the degree to which previously learned material was recalled and determine whether teachers grasped the meaning of material without requiring them to apply it". Therefore, the research instrument used an extended-response question. The statement given had the information necessary for solving the extended-response question, as was customary in tests dealing with geometric riders (Van Putten et al., 2010). The items were categorised according to levels 1 through 4 of van Hiele's theory. The van Hiele level 1 is the most elementary of all levels and depends upon the recognition of shapes (De Villiers, 2010). Even though one might expect van Hiele level 1 to have been attained by teachers long before they enter the FET phase as learners themselves, van Hiele level 1 items were included to examine if they had mastered that level. The order of the items in the van Hiele levels was not random; teachers were required to think in a sequence of van Hiele levels, demonstrating their ability to access reasoning power at various levels rapidly. van Hiele levels were arranged in a cumulative hierarchy, as the groups of objectives were arranged in a complex ascending order. The complexity of tasks increased as each group of behaviours gathered all the behaviours of the low-complex groups (Krietzer & Maduas, 1994). Most of the questions were at levels 3 and 4.

4.10.4 Task-Based Interview

An interview was a discussion between the researcher and the respondent. Nevertheless, it was not the same as an everyday discussion because the researcher set the outline (task-based) and asked the questions (Cohen et al., 2007). It was a structured dialogue where the researcher had specific information from the respondent and designed questions (task-based interview) to be answered. An interview gives the interviewer an understanding of the meaning that can be further reviewed in initial responses during an interview. It pursued a collection of unbiased information that might be contingent on the study's attitudes, beliefs, and ethos. A total of 10 Mathematics teachers from 10 selected secondary schools in Ugu District, KwaZulu-Natal, were interviewed in this study. Participants were interviewed individually to understand individual levels of insight based on the van Hiele theory.

4.10.5 Data Generation Limitation

It was often challenging to overcome certain limitations of tests and interviews (Cohen et al., 2007), but it was significant to acknowledge and best minimise their influence on the research process. Bell (1987, p.43) noted that it is easier to "acknowledge that bias can creep in than to eliminate it together". The limitations must be viewed in the proposed general research design context. This study is not intended to duplicate the research design since each case study is unique, but extends the results and findings to teachers' understanding of Euclidean Geometry. For both data generation methods, the test and the task-based interview, questions were posed in English. The written test was answered strictly in English.

4.11 DATA ANALYSIS

Audio recordings of the task-based interviews were transcribed by the researcher and audited by the moderator. Task-based interview data were coded using Computer Assisted Qualitative Data Analysis Software. Questionnaire responses were coded manually using a word processor. In this study, coding was a process of identifying information that addresses the variables of interest and assigning a label (word or phrase) that best describes the information (Bazeley & Jackson, 2013).

The coded data can be a single word, a phrase, a full sentence, a picture, or a whole page of text (Saldana, 2013).

4.12 SAMPLING

4.12.1 Population

The research population is generally a substantial generation of individuals with similar characteristics, which serves as the central point of a scientific inquiry (Archibald, 2016). Due to large population sizes, researchers may select a subsection of the population known as the study population. This study population comprised 10 Grade 11 Mathematics teachers in the Ugu district.

4.12.2 Sample Method

The researcher selected a subsection of the population referred to as the study population due to its large population size. It is, therefore, from this population that the research sample was drawn. According to Walliman (2017), sampling is selecting a small group of cases from a large group.

4.12.3 Sample Size

The sample consisted of 10 high schools in this study, and only 1 Grade 11 Mathematics teacher per school would be selected. Sampling is central to the practice of qualitative research methods (Robinson, 2014). Convenience and purposive sampling were used in this research study. According to Valerio et al. (2016), the rationale for using convenience and purposive sampling was that it was impossible to use all Mathematics teachers, as their numbers are almost finite. Participants were selected because they were easily and conveniently available and willing to provide data by virtue of their experience and knowledge (Robinson, 2014). The convenience sampling was also based on affordability and accessibility (Etikan et al., 2016). Relevant participants were purposively identified due to their qualities (Orcher, 2016).

4.13 TRUSTWORTHINESS

To ensure credibility, I comprehensively examined these teachers' content knowledge of Euclidean Geometry, providing rich descriptions of their teaching practices (Johnson et al., 2020). This means I went beyond just observing and capturing the nuances of their lessons, their explanations, and how they interacted with students around geometric concepts. I used member checking, where I shared my initial interpretations with the teachers to see if they resonated (Korstjens & Moser, 2018). For transferability, while my sample is specific, I enhanced the potential for my findings to be relevant in other contexts by providing detailed information about the participating teachers and their school environments (Stalmeijer et al., 2024). This will allow readers to judge for themselves how similar these settings might be to their own (Varpio et al., 2021). Dependability is about showing that the research process is consistent and logical. I maintained a detailed audit trail in the form of a step-by-step account of the data generated and analysis (Carcary, 2020; McLeod, 2025). As explained in chapter 3, the selection of participants using convenience and purposive sampling, as well as the collection of the data (interviews, etc.), and its analysis. Finally, confirmability focuses on ensuring that the findings are rooted in the data and not your own biases. Triangulation (Farquhar, 2020), using multiple sources of data (such as test writing and teacher interviews) to explore the same themes, can strengthen this. I ensured confirmability by being transparent about my positionality (Olmos-Vega, 2023) as a researcher and how it might influence my interpretations. By thoughtfully addressing these components, I endeavoured to build a strong foundation of trustworthiness for the study.

4.14 ETHICAL CONSIDERATIONS

The principals of 10 secondary schools in the Ugu District sought permission to conduct the study. They were given consent letters to sign. Subsequently, informed consent letters will be given to the participants (10 Grade 11 Mathematics Educators). The purposes of the study and participants' rights, such as withdrawal from participation, should they wish to do so without being compelled to explain, were explained. According to Walliman (2017), working with human participants in research will always raise ethical issues about how well they are treated and honesty in collecting, analysing, and interpreting data. He added that participants should be treated with respect throughout the research process. In this study, all participants remained anonymous and were treated with respect. Each participant was required to sign a consent form explaining the study's purpose and rights, such as withdrawal from participating should they wish to do so without being compelled to explain. Codes were used in this study to preserve the anonymity of the 10 sampled schools and protect the 10 sampled participants (Rahim et al., 2017). According to Coffelt (2017), maintaining the anonymity of the study participants is an ethical practice designed to protect their privacy while collecting, analysing, and interpreting data.

4.15 CONCLUSION

In this chapter, the research approach is described, and a brief report of the research method is given. The methodology used in this research has been indicated as the interpretive research paradigm and employs a qualitative approach. The rationale for selecting these research methods has been discussed in the chapter. How validity and reliability were addressed in the qualitative approach. The role of the researcher and language in qualitative research has been dealt with. The research design, which involves a logical strategy for gathering evidence

CHAPTER FIVE

DATA ANALYSIS AND DISCUSSION OF FINDINGS

5.1 INTRODUCTION

The previous chapter detailed this study's qualitative research design and methods. This study aimed to understand Mathematics teachers' content knowledge of Euclidean Geometry. It sought to understand what the content knowledge of Mathematics teachers is and how Mathematics teachers use content knowledge in teaching Euclidean Geometry. There are limited empirical studies on how African Mathematics teachers use their content knowledge to teach Euclidean Geometry. Furthermore, there was a period when Geometry was made optional in the FET band in the South African curriculum. Many changes took place during the dawn of democracy, which led to the National Curriculum Statement. Another review conducted in 2009 led to CAPS, which was the third approach to reintroduce Geometry since 2012 in Grade 10, 2013 in Grade 11, and 2014 in Grade 12 (Goos et al., 2017). These curriculum "interferences" warranted a study on Mathematics teachers' content knowledge of Euclidean Geometry.

Teachers graduating from universities in 2013 had not studied Euclidean Geometry during their formal education, either in their secondary or tertiary training. These teachers' encounters with Euclidean Geometry may, therefore, be more problematic than those of teachers who have studied the subject and have been trained to teach Euclidean Geometry in the classroom. This under-teaching or non-teaching of Euclidean Geometry poses a severe threat to the performance of mathematics learning, hence the current study.

As described in Chapter Three, face-to-face semi-structured interviews and a test were used to generate data. These data generation methods gave much more detailed information and allowed for an in-depth understanding of the phenomena being researched, as participants could reflect and reason on various aspects of Euclidean geometry teaching differently. The critical questions that guided this study were: *What is teachers' content knowledge of teachers teaching Grade 11 Euclidean Geometry?* and *how do teachers use content knowledge in teaching Euclidean Geometry?*

In this chapter, the researcher presents the findings from the generated data through a given "test" to determine teachers' content knowledge of Euclidean Geometry. The researcher also conducted face-to-face individual interviews. This chapter adopted a thematic data analysis of the qualitative findings. However, in reporting on this research, the direct responses of the 10 participants who wrote the test and were interviewed were captured to illustrate the findings.

5.2 ANALYSIS OF TEACHERS' TEST RESULTS AND INTERPRETATION

In this part of the chapter, the teachers' test result is discussed. This section of the generated data responds to the first research question: What is teachers' content knowledge of teachers teaching Grade 11 Euclidean Geometry?

The participants were given an Euclidean Geometry test to ascertain their knowledge of Geometry. The test may be found in Appendix D3. The following are the test results of the participants.

5.2.1 Participants' Test Responses Per Question in Terms of the Outlined Codes and Van Hiele Levels

Table 5.1 shows the participants' test responses per question in terms of the outlined codes and van Hiele levels.

Table 5.1: Participants' test responses per question

Participant	*Q1*	*Q2*	*Q 3*	*Q 4*	*Q5*	*Q6.1*	*Q6.2*	*Q7.1*	*Q7.2.*	*Q8.1*	*Q8.2*	*Q9.1 & 9.2*	*Q10*	*Q11*	*Van Hiele*
TA	*T1*	*T1*	*T1*	*T1*	*T6,*	*T01*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*vhl3*
TB	*T1*	*T1*	*T1*	*T5*	*T6*	*T1*	*T5*	*T1*	*T2*	*T1*	*T1*	*T1*	*T1*	*T6*	*vhl2*
TC	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*Vhl3*
TD	*T1*	*T1*	*T1*	*T1*	*T6*	*T1*	*T1*	*T1*	*T5*	*T1*	*T3*	*T1*	*T1*	*T2*	*vhl1*
TE	*T1*	*T1*	*T1*	*T5*	*T6*	*T1*	*T3*	*T2*	*T2*	*T1*	*T3*	*T2*	*T1*	*T6*	*vhl1*
TF	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T1*	*T2*	*T2*	*T1*	*T2*	*T2*	*T1*	*T1*	*Vhl2*
TG	*T1*	*T1*	*T1*	*T5*	*T6*	*T1*	*T3*	*T1*	*T2*	*T1*	*T3*	*T1*	*T1*	*T6*	*vhl1*
TH	*T1*	*T1*	*T1*	*T1*	*T6*	*T1*	*T1*	*T1*	*T3*	*T1*	*T2*	*T1*	*T1*	*T2*	*vhl1*

TI	*T1*	*T1*	*T1*	*T5*	*T1*	*T1*	*T1*	*T1*	*T3*	*T1*	*T1*	*T1*	*T1*	*T1*	*vhl1*
TJ	*T1*	*T1*	*T1*	*T1*	*T6*	*T1*	*T1*	*T6*	*T2*	*T1*	*T3*	*T1*	*T1*	*T2*	*vhl1*
Total	*10*	*10*	*10*	*10*	*03*	*10*	*07*	*07*	*02*	*10*	*04*	*08*	*10*	*4*	
%	*100*	*100*	*100*	*60*	*30*	*100*	*70*	*70*	*20*	*90*	*40*	*80*	*100*	*40*	

The participants' test script responses were categorised as T1 to T5 in terms of the developed statement and reasons. The meanings of T1, T2, T3, T4, and T5 are presented in Table 5.2.

Table 5.2: Classification of teacher answers from their test scripts and colour coding

CATEGORY	STATEMENT	No of times featured	COLOUR CODING PER CATEGORY
T1	Correct response, correct reason.	90	
T2	Incorrect response, incorrect reason,	12	
T3	Correct response, incorrect reason	7	
T4	Incorrect response, correct reason	0	
T5	Correct response, no reason	6	
T6	Incomplete response, No reason	11	

Table 5.2 shows that for questions 1, 2 and 3, all 10 teachers provided the correct response and correct reasons. In Question 4, four teachers obtained a T5, i.e., they provided the correct response but gave no reason. In Question 5, seven teachers obtained a T6, meaning that they provided an incomplete response with no reason. In Question 6.2, there were two teachers with a T3 (correct response, incorrect reasons) and one teacher with a T5 (correct response, no reason). In Question 7.1, one teacher had a T2 (incorrect response and incorrect reason), two teachers had a T3 (correct response and incorrect reason), and one teacher had a T5 (correct response and no reason). In Question 7.2, there are five T2 (incorrect response and incorrect reason), two T3 (correct response and incorrect reason) and one T5 (correct response with no reason). Question 8 has two T2 (incorrect response and incorrect reason) and three T3 (correct response, incorrect reasons), while questions 9.1 and 9.2 have two T2 (incorrect response and incorrect reason). Question 11 has three T2 (incorrect response and incorrect reason) and three T6 (incomplete response with no reason).

There was a frequency of 90 for T1 (correct response and correct reasons), a frequency of 12 for T2 (incorrect response and incorrect reason), a frequency of 7 for T3 (correct response and incorrect reason), a frequency 6 for T5 (correct response with no reason); and a frequency of 11 for T6 (incomplete response with no reason). We notice from the aforementioned that there is a total frequency of 36 where an incorrect reason or no reason was given.

The participants' responses were also categorised in terms of the type of incorrect answers made, namely, conceptual and procedural incorrect answers.

Table 5.3: Classification of answers in terms of the type of incorrect answer made

Code	Category	Explanation
C	Conceptually incorrect answers	An insufficient mastery of basic facts, concepts, and skills causes a lack of knowledge of the concept.
P	Procedural incorrect answers	Participants know the concept and properties of figures but cannot apply it to the problem.

Lastly, participants' responses were then categorised (deductively) in terms of van Hiele's levels of Geometric thought (van Hiele 1986).

Table 5.4: Participants' responses were categorised in terms of van Hiele levels

Category number (van Hiele levels)	Description
VHL0	Naming and identification of common geometric shapes.
VHL1	Recognise geometric shapes based on their properties but cannot recognise relationships between classes of figures.
VHL2	Identification of class inclusion of shapes
VHL3	Can construct geometric proofs.
VHL4	Understand the relations between geometrical concepts.

Having presented the data from the Euclidean test, the researcher proceeded to present and discuss the data generated from the interviews with teachers.

5.2.2 Discussion of Test Scores of Participants

Table 5.1 indicates the results that the participants obtained in the 10 test questions. The participants' results, especially in questions 1, 2, and 3, revealed that all participants understood the calculation and interpretation of the theorem into a diagram. The first question in this section needed knowledge of the theorem that dealt with the relationship between a chord AB having centre O with C as a point on AB and a line drawn from the centre of a circle, such that ACO = 90^0. The second question wanted participants to draw a diagram representing a theorem that dealt with angles subtended by the chord at the circumference of the circle, on the same side of a chord are equal, and the third question also wanted participants to represent a statement in a rough sketch which states that if a line drawn from the centre of the circle is perpendicular to the chord, then it bisects the chord.

However, in Question 4, challenges started when all the participants did calculations correctly, but four participants could not state the reasons for their calculations. This means that though teachers teach Euclidean Geometry, sometimes they forget to emphasise the importance of writing reasons for their choices. Only 60% of the participants got correct answers with reasons. The other four participants did not provide reasons for their correct answers. This revealed a lack of logic in solving geometry questions. Participants were guessing the answer; hence, it became difficult to provide reasons. They were unable to make the relationship between ∠ and ∠, the given angle equal to 24°, deductively and present a logical, coherent argument. Ohtani (as cited in De Villiers, 1998) identified the traditional method of providing ready-made definitions in Geometry, which inhibits educators' competency in proof. Teachers should design assessment activities that promote higher-order thinking skills, which will result in deductive and logical reasoning, thereby enhancing learners' competency in proof questions.

In South Africa, the 2012 presentation by Linda Chisholm to the Minister of Basic Education highlights inadequate content and PCK (DBE, 2012). Teachers' lack of content knowledge results in teachers not explicitly explaining the concepts to the satisfaction of the learners. Conceptual, procedural, and orderly educators are directly responsible for the learning in class. They are the ones who create learning situations for learners in class, and I suspect that learners also lack the principles of Euclidean Geometry.

There were only three participants who answered Question 5 correctly and for the correct reasons. In this question, participants were given a sketch drawn not to scale where JLH was a tangent to a circle having centre O at L, K, and M were points on the circle and were required to prove the theorem, which states that ∠MLH= ∠K. This question revealed a gap in translating and using the content knowledge of the theorem because the question involved a number of theorems in the given diagram. The failure of seven participants revealed this, 70% of whom had failed to complete it.

Question 6.1. required participants to use the given diagram to prove the theorem, which states that an angle that an arc subtends at the centre is twice the size of an angle subtended by the same arc at the circumference of the circle, i.e. ∠VOW= ∠2P. All 10 participants could answer this question, and reasons were also provided, meaning they were familiar with this type of question.

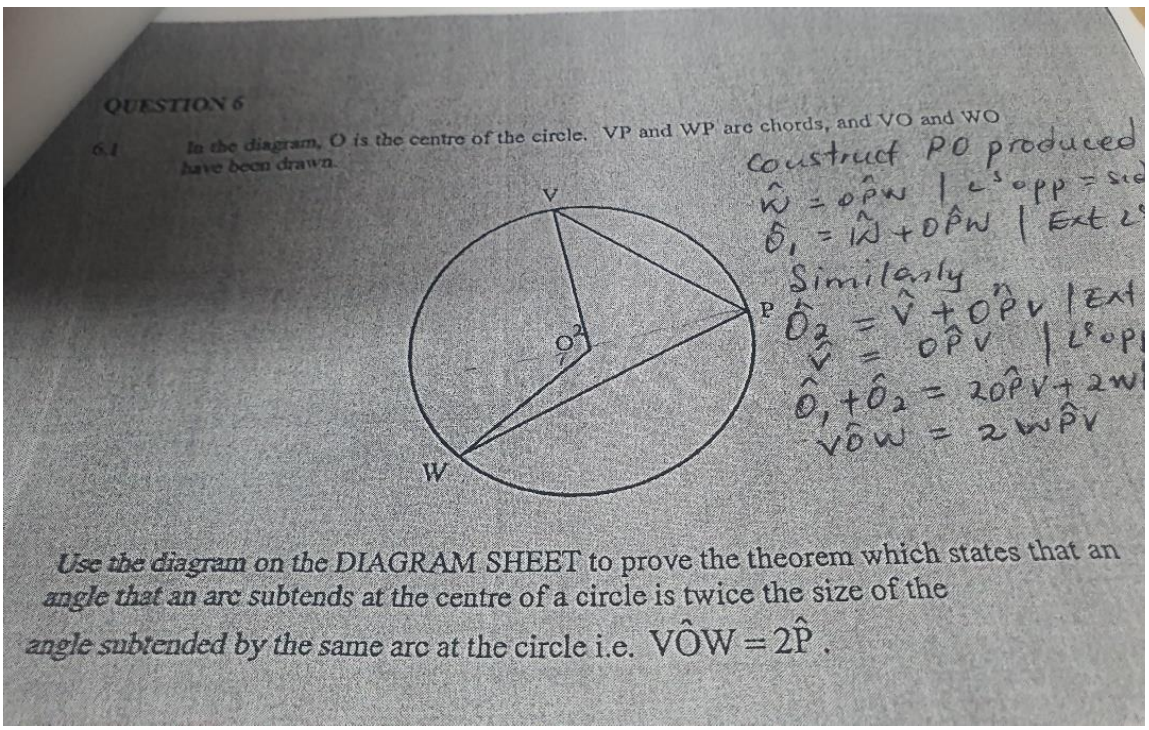

Figure 5.1: Question 6.1

In 6.2, the diagram states that O is the centre of a circle. A, B, C and F are points on the circumference. AC and BF intersect in E, and EF = FC, requiring the participants to prove that AB//FC and OBCE are cyclic quadrilaterals. This question required the participants to recall and use the knowledge of theorems to attempt it, and eight participants answered correctly. The other two did not complete it, and this may be caused by a failure to understand concepts and, hence, an inability to think deductively. According to Oberdorf and Taylor-Cox (1999), “lack of exposure to proper vocabulary” is one of the reasons for incorrect responses in Geometry. Therefore, the understanding of geometrical concepts is essential for logical and deductive interaction with geometrical figures.

This means that in terms of van Hiele levels, participants had attained level 2 but not the last levels of van Hiele theory, and teachers progressed and developed their knowledge of Geometry through these levels (van Hiele, 1986, 1999). The 1st level (level 0) is called visualisation: teachers are to identify a shape but are not able to provide its properties. The 2nd level (level 1) is called analysis: teachers can identify properties of shapes, but not in logical order. The 3rd level (level 2) is called informal deduction: teachers can combine the shapes and their properties to provide a precise definition as well as relate the shape to other shapes (van de Sandt, 2007). The 4th level (level 3) is called deduction: teachers can apply formal deductive arguments, such as in proofs. The 5th level (level 4) is called rigour: it is characterised by “formal reasoning about mathematical systems” (Yee, 2006). In terms of errors, they still have procedural errors, and this means that they will create challenges in the development of learners on procedural issues.

In Question 7, participants were required to calculate the unknown values on a circle having centre O. This diagram has A, B, C, D and E, which are the points on the circle having centre O, with DC produced to G. It also had a diameter AOD bisecting chord CE in F and intersects BE in S. It was noticeable that seven out of 10 answered 7.1 correctly whereas two participants did not attempt 7.1 and one participant calculated it wrongly for the wrong reason. The diagram could have been confusing due to many lines to participants that have content deficiency, participants who are still at 2nd level in the van Hiele levels (level 1), also called analysis, where participants can identify particular properties of shapes but not in logical order, cannot respond correctly on this diagram, this also gave at most eight participants in 7.2, which left two participants with correct responses for 7.2.

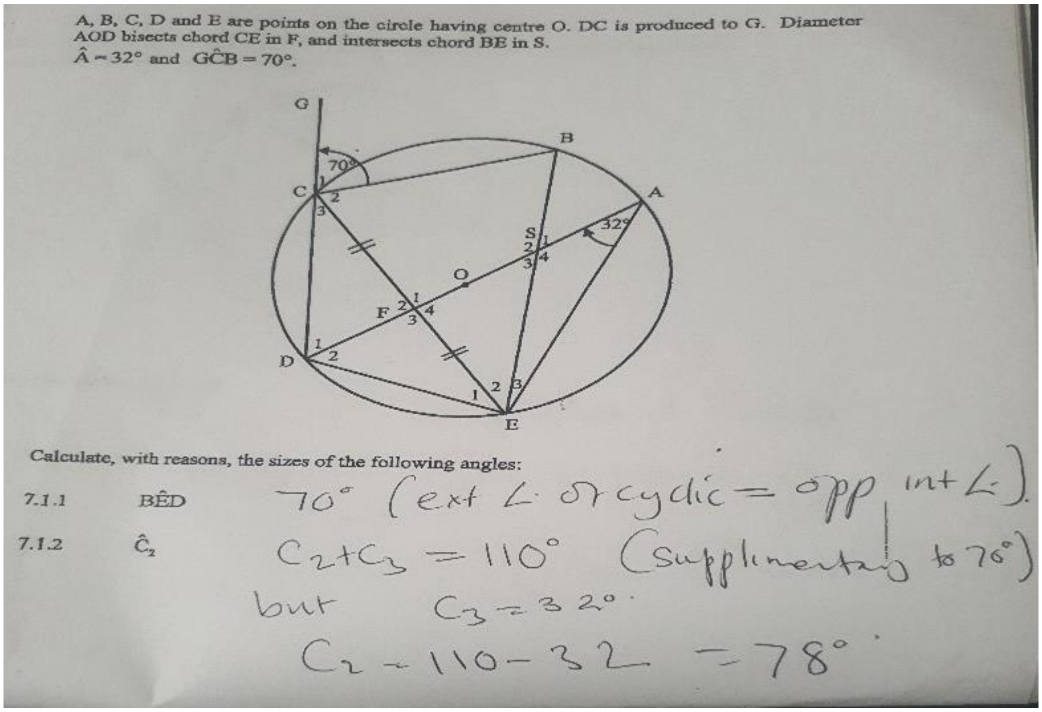

Figure 5.2: Question 7

The diagram in Question 7 had many lines, which means content knowledge of theorems formed the foundational basis for the participants to integrate relationships in these lines in relation to the correct calculations and reasons; hence complicated, participants were confused in choosing the correct theorems in their calculation, this clearly shows that achieving the van Hiele levels 3 and 4 remained problematic to participants as some did not complete questions that involved proofs or more than one procedural step.

Question 8 was based on the Tan-Chord theorem, and participants were required to calculate the size of angles K and H3 with reasons. Participants were familiar with this theorem, and thus, it became easier for them to calculate and provide the correct reason for Question 8.1 because H1 was given the value. This H1 was an angle in tangent, whereas K is an angle in a chord that is about H1. The challenge was in the calculation of Question 8.2. where participants were expected to find relationships to theorems that would lead to the correct answer. Four participants got correct answers and used correct reasons, while three of the participants got correct answers using wrong reasons, and the last two did not get it. Both calculations and reasons were wrong.

In Question 9, participants were required to calculate the values of angles O1 and O2 using theorems. Seven of them got it correct, and the reasons used were also correct, but three participants used the wrong theorem, which made the participants state the wrong value of O1; thus, O2 became incorrect. Participants attested that Question Nine can be viewed as complicated and tricky. In the post-interview conducted by the researcher, when asked which question was more challenging, most candidates indicated Question 9.

TA responded: *It's Question 9.1.2 since both O1 and O2. They are drawn from two equal chords, and those angles are at the centre. The one that I'm familiar with is that angles subtended by equal chords are always equal, but usually, they are on the circumference. But the theorem states that if two chords are equal, they subtend equal angles, whether it is circumference or whether at the circle.*

This shows that an in-depth content knowledge of a subject is needed if one must teach Euclidean Geometry. 70% of participants answered Question 9 correctly. The more lines added to a diagram; the more experience /content knowledge utilisation is needed. The participants demonstrated a deficiency in the content knowledge of Euclidean Geometry, and this poses a threat to the learning of Euclidean Geometry. It is also confirmed that learners' acquisition of geometric thought depends greatly upon the educator's mathematical content knowledge (Couto & Vale, 2014, p. 58). Content knowledge goes beyond knowledge of the facts or concepts of a domain to understand the structures of the subject matter. Nolan et al. (2015, p. 35) also propose that educators must know the content of the subject they teach thoroughly to be able to present it efficiently, to make the concepts accessible to a wide variety of learners and to engage learners in challenging work.

Question 10 wanted the participants to prove that LP+LS =180. This question needed the background of a cyclic quadrilateral, and to understand that the given information will generate solutions if used effectively, the participants, in this diagram, were given that O is the centre of the circle and PTSR is a cyclic quadrilateral. All 10 participants proved the theorem correctly and used the correct reasons. This demonstrated that participants have the knowledge to prove this theorem, and it is easily remembered, though two of them made silly mistakes of not putting signs of angles and degrees in calculations. This means that these participants do not emphasise signs in classroom situations. One of the participants attested that this question is like the question in the Mathematics Textbook that the participant used as an example of proving this theorem. This statement made the researcher link 100% correct responses to Question 10 with rote learning and showed that most of these participants believed in the textbook method.

In Question 11, the diagram was not drawn to scale, but the given information stated that O was the centre of the larger circle RTAP, OP was the diameter of the smaller circle PSO, NP was a tangent to both circles at P and that PR was perpendicular to NP. They were required to prove that PR bisects $\angle$ORN and to prove that $\angle$ROS=$\angle$PAT. Participants viewed Question 11 as more complicated. Only four participants completed the questions correctly for both questions 11.1 and 11.2. with correct reasons. The other six participants did not complete Question 11.1 and did not attempt Question 11.2.

TA indicated: *Question 11 is a little bit challenging, too, because. First of all, you need to prove to yourself that RP is parallel to OS. Without proving that on your own, you will not get it. That's number one. Number two. They said to show that RP bisects ORN. So, you need to understand what you must prove. You must prove that your angle ORP equals NRP.*
TB said: *A lot of information is needed to answer the question.*

The performance of participants in Question Twelve was good in 12.1 because the demonstration of their understanding of given statements that are normally used in the practice of Euclidean Geometry was clear, and all participants answered correctly. This means that participants have an in-depth understanding of the statements; however, when expected to calculate 12.2, one of the participants did not give reasons for the answer, and one participant got it wrong.

Table 5.5: Number of errors made by participants in terms of categories

	Q1	Q2	Q3	Q4	Q5	Q6.1	Q6.2	Q7.1	Q7.2	Q8.1	Q8.2	Q9.1	Q9.2	Q10	Q11
T2							02	02	05		02	02	02		02
T3							02		02	0	04				
T4															
T5			03				01		01						

T6					07			01							03

Most of the incorrect responses and statements were, therefore, due to conceptual errors. These were a result of participants' inability to derive meaning from the definitions or explanations provided and, therefore, did not adequately translate the concepts described and apply them to the geometry problems. Participants could not apply properties of Euclidean geometric concepts in correct contexts to simplify Euclidean geometry problems. Participants usually gave a reason unrelated to a problem. Luneta and Makonye (2010) affirmed when they argued that incorrect answers result if one fails to build procedures from conceptual knowledge. According to Marek et al. (2011), this results in the rote learning of Mathematics, where algorithms are learnt without connecting them to the underlying semantic information. According to Oberdorf and Taylor-Cox (1999), "lack of exposure to proper vocabulary" is one of the reasons for participants' incorrect responses in Euclidean Geometry.

The results were further coded in terms of themes and the van Hiele levels; incorrect answers were then analysed inductively and deductively. Table 5.1. clearly shows that achieving the van Hiele levels 2, 3 and 4 remains problematic for most of the participants. This is evident in the responses that participants got in questions that involved proof or more than one procedural step. Most of the participants find it easier to calculate the value of unknown angles than to prove equal angles, tangents, and cyclic quadrilaterals. Participants demonstrated the need for capacity in the study of deductive Euclidean geometry problems due to a lack of foundational concepts of Euclidean Geometry. Only two participants attained van Hiele level 3, two attained van Hiele level 2, and six participants remained at van Hiele level 1.

Based on the aforementioned, six out of 10 teachers teaching Euclidean Geometry in these schools have content knowledge deficiency in teaching Euclidean Geometry. According to Aslam et al. (2016), teacher qualification may be a special teaching skill, SMK, or teaching experience that makes a teacher suitable. These 10 participants all have a Bachelor of Education degree in the subject as the minimum requirement for teaching; however, as per the aforementioned table, some lack a background in Euclidean Geometry. Classroom teaching depends on the teacher's earlier experiences with learners, which are greatly influenced by the teacher's own time as a learner (Dean, 2019). This means that six participants did not have these experiences with the topic. The results of the test indicated a content deficiency in some of the educators teaching Euclidean Geometry for various reasons, but the main one is the failure to upgrade themselves in the content for teaching Euclidean Geometry. This also indicated that participants sometimes forget or have little knowledge of choosing the reasons for their calculations.

5.3 ANALYSIS OF TEACHERS' INTERVIEW DATA AND INTERPRETATION

The next section reports on the semi-structured face-to-face individual interviews with the 10 participants. The following common themes focusing on Euclidean Geometry were identified. The themes that emerged from the data are as follows:

(a) Teachers' content background in Euclidean Geometry

(b) Teachers importance of teachers understanding Euclidean geometrical theorems.

(c) Use of content knowledge in teaching Euclidean Geometry.

(d) The use of Euclidean content knowledge for meaningful reading and application.

(e) The use of Euclidean content knowledge in the development of lesson preparation.

(f) Factors leading to difficulty in the teaching of Euclidean Geometry.

(g) The language barrier in teaching Euclidean Geometry.

Four participants in this study agreed that Euclidean geometry activities /exercises were taken from textbooks. TC mentioned that teachers followed textbooks when teaching Euclidean geometry theorems, which were explained on the chalkboard. TD and TI also confirmed that teachers first explained Euclidean geometry theorems from textbooks, and after each theorem, application exercises were also given from textbooks. This means that they, through their learning experiences, adopt the same use of the content knowledge acquired in their teaching and learning of Euclidean Geometry. This is confirmed by their responses on using the content knowledge in teaching Euclidean Geometry.

TC: *I use examples from the textbook,*

TH: I develop activities with theorems I taught so learners will embark on given activities

Both TC and TH confirm that most teachers start by explaining the theorems from textbooks before allowing learners to interact with the diagrams. Looking at responses to Question 7.2, which is based on the reasons for poor performance.

TA: *Some teachers use textbook examples and expect learners to learn more*.

TC: *Teachers use textbooks to teach learners. Visual learning is not incorporated.*

Participants TA and TC believed that the textbook was insufficient for learners to pass. More is needed. This is also supported by Vidermanova and Vallo (2015), that learners who are taught according to what is written in the textbooks remain passive listeners, as textbooks may not meet the needs of all learners. Mathematics teachers should, at the most basic level, have background knowledge of mathematics content for the grade they are teaching, for the earlier grades, as well as for the several grades beyond that which they are expected to teach for them to deal adequately with learners' difficulties in Euclidean Geometry (Venkat & Spaull, 2015). This means that content knowledge is the basis for effective learning and development of learning material that will cater to learners' different abilities or cognitive levels.

TE*: Learners need teachers to clarify further*. *Some learners do not understand theorems.*

TG*: Learners lack the foundation of theorems*

TI*: Learners do not understand theorems*

TJ*: Learners do not link the theorems learned to the diagrams given.*

These responses from the aforementioned participants about the poor performance of learners in Euclidean Geometry also emphasise the importance of the role played by the teacher's content knowledge for learners to understand the lesson. Learners' content knowledge relies on the knowledge shared by their teachers. The teacher's understanding and expertise in the content area to be taught is the key to developing learners' knowledge of these concepts to be able to link theorems with diagrams because it encompasses a deep knowledge of the subject matter, including its key concepts, theories, principles, and skills. The teacher is consistently expected to provide clear explanations of the content. The teacher answers student questions accurately and provides feedback that furthers their learning. The responses provided by TC and TA demonstrate the failure to explain the content clearly. A deep understanding of the concepts assists in the use of content knowledge to develop lesson preparation of Euclidean Geometry.

5.3.1 Perceptions of Participants: Reasons Why Euclidean Geometry Performance Is So Poor in Schools

This study focuses on understanding Mathematics teachers' content knowledge of teaching Euclidean Geometry. The study focused mainly on what Mathematics teachers do in their classrooms when teaching Euclidean Geometry. However, the view of participant teachers on why Euclidean geometry performance is so poor in schools is as follows:

TA noted: *Some teachers use textbook examples and expect learners to learn more*

TC confirmed that *teachers use textbooks to teach learners. Visual learning is not incorporated.*

The aforementioned responses from the participants showed that Mathematics teachers taught Euclidean Geometry directly from the textbooks, while teachers' expectation of learners is to learn more. The aforementioned findings corroborate those of Lepik et al. (2015), who argued that, in some cases, mathematics classroom teaching is still generally organised around and delivered through Mathematics textbooks. The aforementioned assertion agrees with Weng et al. (2018), who, in their studies, argued that when learners are taught based on what is written in the textbook, they remain passive learners, as textbooks may not meet the needs of all learners. The role of Mathematics teachers is to facilitate learners' thinking and learning and should attempt to motivate learners to learn (Lessani et al., 2017). Accordingly, a plethora of studies argue that Euclidean Geometry continues to be regarded by learners as an area of the most significant challenge to learn and is difficult for teachers to teach (Abiam et al., 2016; Nojiyeza, 2019)

TC responded: *Most learners do not read the given statements. The given information. They jump into diagrams. And diagrams would be meaningless without them. Without the reading, the given information. And also, if they read the information, they read it without comprehension, without understanding. Sometimes, they will write the statement. They do not give the reason, but the memorandum said one mark for the statement and reason for both the statement and the reason one mark. So, in that case, it would be an incomplete solution, and they would lose marks.*

This means there is also a challenge in providing clear explanations regarding the importance of the principles of the content taught to learners. This is also confirmed by TJ and TI when their test responses neglected the inclusion of degree and angle signs.

5.3.2 Teachers' Content Background in Euclidean Geometry

Participants indicated during interviews that some did not study it at the high school or university level**.** The participating teachers also revealed they faced many challenges in teaching and learning Euclidean Geometry in schools. Regarding content knowledge of teaching, it was evident in the research findings that the teachers use their content knowledge based on experiences acquired at different levels of learning Euclidean Geometry. For instance, some of them confirmed the use of writing during teaching and learning without any explanation for the conceptual understanding of learners. The following quote provided evidence of the views generated by the participants:

TD said: *During my days in school, we mainly learned Euclidean Geometry with little or no explanation of the concept.*

Therefore, the findings from the aforementioned verbatim statements from the participants showed that they taught the Grade 11 learners' Geometry without soliciting the opinion/attention of the learners. Teachers completely dominate the teaching and learning processes. A teacher content knowledge is crucial to what is taught and learned. It is obvious that when Mathematics teachers are not well-grounded in effective content knowledge of the subject, the quality of mathematics teaching and learning will be compromised, and hence, mathematics learners' performance will be at risk. Mathematics teachers who lack content knowledge have a limited scope for preparing the content to be taught to learners.

Building student knowledge requires two-way communication between the teacher and the student, that is, engaging the learner's attention. Teacher-guided building of knowledge and understanding and consolidation of learning through application, practice, and reflection.

5.3.3 Lack of Content Knowledge of Teaching Euclidean Geometry

Content knowledge refers to information and skills relevant to a particular subject. Our content knowledge affects how we interpret the content goals we are expected to reach with our students. It affects how we hear and respond to our students' questions. It affects our ability to explain clearly and to ask good questions. It affects our ability to approach a mathematical idea flexibly with our students and to make connections. It affects our ability to push each student at that special moment when they are ready or curious. It also affects our ability to make those moments happen more often for our students.

It also emerged from this study that knowledge of instructional practices for participant teachers was not up to expected standards. The limited understanding and application of mathematical practices in teaching Euclidean Geometry were also revealed when teachers were interviewed. For instance, it became apparent that most participating teachers lacked practical skills, such as planning, demonstration, and organisational skills, as well as being able to develop their learners' understanding through involvement. The story of the participants is shown in the following quotes:

TB said: *I was not taught Euclidean Geometry with the basic tools, so sometimes I find it challenging to use some modern tools other than chalkboard, etc.*

Using content knowledge integrates subject expertise and skilled teaching, ensuring teachers deliver effective geometry teaching. The post-based test interview also revealed that some teachers apportion the blame on learners since learners found it difficult to remember some theorems and basic concepts, making it difficult for them to teach the idea effectively in schools.

TB stated: *Some of the teachers wait for a memorandum from the Department for marking common tests and cannot complete their memoranda.*

TF stated: *Teachers themselves have gaps. They are unable to give learners challenging diagrams.*

The research findings revealed that teachers' performance in Euclidean Geometry was due to a lack of mathematical knowledge and instructional practices. Thus, their lack of knowledge and application has led to the chalk-and-talk method being used predominantly guided by textbooks.

Teachers' attitudes towards Mathematics and the teaching of Euclidean Geometry, specifically, were affected. The study's findings have helped the researcher develop training for Mathematics teachers in collaboration with the DBE using mathematical modelling activities, as well as relevant skills and strategies in teaching Euclidean Geometry in schools for a better understanding of learners.

The following are the responses of the three participants from the individual structured face-to-face interviews regarding how they were taught Euclidean Geometry in high school. They further indicated they felt they were not fully prepared to teach the topic because they were not grounded in some aspects of the subject.

TF: *I am not confident about teaching some aspect of Euclidean Geometry because I was never taught Euclidean Geometry at the school level*

TB: *I find it a bit challenging to teach Geometry, especially in some areas, since I do not have a strong background in the topic.*

TH: *For several years, I have been teaching Geometry. I mostly skip some sections when teaching Euclidean Geometry.*

Teachers' knowledge and competence are based on the education and training taught during high school (Boiliu et al., 2021). The aforementioned responses show that some participants were uncomfortable with some geometry sections. Hence, some of them have no confidence in teaching some of the topics covered in Geometry. This finding indicates that inadequacy in content could hinder achieving set goals (Amolloh et al., 2018). This suggests that teachers who are not confident with the content covered in Euclidean Geometry prefer to skip or scratch the surface when teaching Euclidean Geometry.

5.3.4 Teachers' Importance of Understanding Euclidean Geometrical Theorems

Teachers who can construct thorough and instructive lesson plans can address misconceptions of mathematical concepts that learners experience in the classroom. They can explain difficulties that learners encounter during teaching and learning by pre-empting them. The analysis of the study further revealed that a lack of understanding of mathematical concepts and inadequate training in the teaching of Mathematics at school and higher education levels has led to teachers not being well-qualified or experienced in teaching Mathematics, particularly challenging topics such as Euclidean Geometry. This is in line with the assertion by Brown (1999) and Khoo and Clements (2001) that, in many instances, geometry teaching is based on immersive and adaptive instruction and learning where teachers shift from a conceptual approach (the sensible application of procedures) to a procedural approach (calculation accuracy).

The principle behind a mathematical theorem is essential for understanding and developing insight. This insight enables an individual to see the connections between different mathematical ideas and builds a more robust foundation for further learning and thus enabling a teacher to apply it to various problems and situations and be able to recognise when and where the theorem is applicable and how to modify it for more general cases. mathematical theorems are proven based on logical reasoning and a chain of steps derived from the underlying principles. Knowing the principles gives you the ability to follow and create rigorous mathematical proofs.

Solving complex problems and understanding the principles of the theorems can guide teachers in identifying the appropriate theorems to use and how to manipulate them to arrive at a solution. Knowing the principle behind a theorem helps prevent misapplication or misuse of the theorem. It ensures that you use the theorem correctly in relevant situations and avoid using it in inappropriate contexts. Accumulation of a deeper understanding of different principles creates a network of interconnected ideas that can enhance teachers' problem-solving abilities.

In summary, knowing the principle behind a mathematical theorem is the provision of a solid foundation for learning, problem-solving, and applying mathematical concepts effectively and efficiently.

5.3.5 The Use of Content Knowledge in Teaching Euclidean Geometry

From the participants’ comments, it is evident that they were taught Euclidean Geometry from textbooks. In particular, the participants mentioned that teachers followed textbooks when teaching Euclidean Geometry. The teacher participants made the following comments:

TE: *As for me, I was forced to teach Mathematics since I was redeployed to my current school. I only have a Grade 12 Mathematics background, and in most cases, I find it difficult to explain some Euclidean geometry concepts to the learner properly. I was taught to follow the textbook method. I wish the Department would send only teachers with Mathematics backgrounds from the University or diplomas to handle this critical subject.*

This was interviewed during a post-task-based interview conducted by the researcher and captured in the audio recording. The researcher realised that.

TD: *In my college, where I graduated as a teacher, my Mathematics Lecturer/Teacher always used telling and textbook methods to teach us, whereby individual attention was not given because he always complained about time and finishing the syllabus. I have stuck to that method as the only way I can teach my learners Geometry since I was not introduced to different teaching strategies during my training.*

TG: *Our teacher then usually teaches us how to prove some theorems in Geometry. He then asks us to refer to our textbooks to solve some examples.*

TA: *Whenever I am to do questions on Euclidean Geometry, in the diagram, learners need to know all the theorems in that diagram," all the families." We call them the families. If there is a centre, what needs to come to your mind? Okay. If it's a tangent, what needs to come to your mind? Cetera. What guides them is what they need to look for. Then, when it comes to a time when they are to answer the questions, they will have all the answers already. Then, they must try to ensure they write a formal response.*

Then, based on the questions asked, TA's response on using content knowledge confirmed that if the teacher has content knowledge, strategies to teach meaningfully become effective.

The aforementioned responses show that teachers read from the textbooks, hoping the whole class would understand what was being taught. This suggested there was no lesson preparation, content knowledge was never unpacked, and learners' content knowledge was never considered. Teachers seemed to depend on textbooks to write formulae and other theorems on the blackboard before explaining certain concepts to the learners. In many cases, teachers would request learners to open their textbooks to check answers to questions, not feeling confident enough to answer. This way of teaching modelled how teachers were taught at school, where the textbook is vital.

Restricting teaching to just the textbook without further application and discussion often results in learners' understanding not being fully developed (Tachie, 2020). Meanwhile, the findings from this study corroborate with the study (Tachie, 2020) that a lack of content knowledge of mathematical concepts and inadequate training in the teaching of Mathematics at school and higher education levels has led to teachers not being well-qualified or experienced in teaching Mathematics, particularly challenging topics such as Euclidean Geometry. In other words, relevant explanations were inadequate for teaching Geometry in schools.

On the other hand, current reform-driven mathematics documents stress the need for teachers to provide learning environments in which students will be challenged to engage with mathematics concepts and extend their understandings in meaningful ways. Although the study of teacher content knowledge has received considerable attention, there is less information about the teachers' content knowledge that impacts classroom practice. Renga et al. (2020) suggested that teachers must 'deconstruct' their content knowledge into more visible forms to help learners connect with their previous understandings and experiences. Documenting teachers' content knowledge for teaching Euclidean Geometry has received little attention in debates about teacher knowledge. There is limited information about how we might systematically characterise the key dimensions of the quality of teachers' mathematics content knowledge for teaching Euclidean Geometry and connections among these dimensions. Data collection using the mentioned methods in understanding teachers' content knowledge of Euclidean Geometry and using teachers' content knowledge in teaching Euclidean Geometry revealed that teachers content knowledge affects how we interpret the content goals we are expected to reach with our learners and how we hear and respond to our student's questions as well as affects teachers' ability to explain clearly.

5.3.6 Understanding Teachers' Content Knowledge of Euclidean Geometry

Educators in all 10 schools acknowledged that learners are not performing well in Euclidean Geometry. They further stated that it is because learners lack an information background from

the previous grades. According to the interviews, participants in some of these 10 schools did Euclidean Geometry in high school, and some did not do it, they only encountered it at the tertiary level. One of the participants indicated that he encountered Geometry as a teacher, not as a student at the school level as well as at the tertiary level; in other words, he was learning it with his learners. The following quote provided evidence of the views generated by the participants:
TF expressed: *As for me, I did not do Euclidean Geometry during my days at a higher institution. So, I can teach the topic based on the little knowledge I acquired in high school*.

Therefore, the aforementioned verbatim statements from participants showed that teachers who had not studied this topic in their secondary schooling or their teacher education programmes found themselves teaching the subject but with little knowledge and understanding. Their planning will be affected because preparing effective lessons for teaching Mathematics is influenced by how one understands mathematics concepts. Bobis et al. (2016) found that there is always a noteworthy weakening in the cognitive levels of mathematics content knowledge when a teacher is uncomfortable with the topic, so knowledge transfer is compromised. The same situation occurred with the teaching of Euclidean Geometry.

5.3.7 Factors Leading to Difficulty in the Teaching of Euclidean Geometry

Language of teaching and learning sometimes becomes a barrier, especially if it is not the home language; the presentation done in a language of teaching and learning which is not the home language demands the teacher to first think in their home language before interpreting it in English, teacher struggles in the effective preparation of the lesson. The teacher who is fluent in English will teach much better and will make /design lessons effectively. Euclidean Geometry needs an understanding of the question.

Inappropriate staffing of educators. Teachers find themselves teaching subjects they have limited knowledge of. Such a shortcoming can be mainly attributed to the high failure rate in Mathematics in Grade 12, which leads to fewer pupils pursuing Mathematics at the tertiary level. In most cases, the few who obtain a good pass in Mathematics in Grade 12 prefer to pursue careers in Engineering, Natural Sciences, Health Sciences, Computer Sciences, Technology and so forth.

TE: *I was forced to teach Mathematics since I was redeployed to my current school. I only have a Grade 12 Mathematics background, and in most cases, I find it difficult to explain some Euclidean geometry concepts to the learner properly. I was taught to follow the textbook method. I wish the Department would send only teachers with Mathematics backgrounds from the University or diplomas to handle this critical subject.*

This means that various factors influence the relationship between attitude and achievement enjoyment of Euclidean Geometry. Among these factors are classroom activities and the subject teacher. The nature of activities that happen during the lesson has a bearing on the overall enjoyment of the subject. If there are exciting activities during the lesson, pupils may tend to associate those with the subject. It should, therefore, be upon teachers to formulate activity-oriented exercises that are both suitable for and of interest to learners; this can only be achieved if the teacher has unlimited knowledge of the subject.

5.3.8 Failure of Teachers to Be Innovative in Lesson Preparation and Presentation

Almost all of the 10 teachers indicated that they did not develop lesson plans using current technology software, and this means that teachers have limited Euclidean content knowledge scope in terms of the impact of using technology on the use of content knowledge. Participants, as designers and interpreters of the learning, should indicate different ways of using resources in lesson preparation based on their knowledge of the subject.

5.4 CONCLUSION

This chapter presented, analysed, and interpreted data generated from 10 participants in this study. Research questions, the data generation plan, and themes that emerged from the collected data will be discussed in this chapter. The sources used to collect data were an open-ended questionnaire, semi-structured face-to-face interviews and lesson observations. Field notes were also used to collect data. Qualitative data were analysed through thematic analysis. The analysis of collected data demonstrated how the Grade 11 Mathematics teachers used their content knowledge when teaching Euclidean Geometry. Mathematics teachers need to be aware of their role in teaching Mathematics, especially Euclidean Geometry, so that learners may develop a solid understanding of Euclidean Geometry (Kösa, 2016). Furthermore, Mathematics teachers need to give learners enough confidence to discover Euclidean geometry problems and think critically to solve them (Bhagat & Chang, 2015). The discussion of the conclusion and recommendations will be presented in the next chapter.

CHAPTER SIX

CONCLUSION

6.1 INTRODUCTION

The previous chapter described and explained in detail the qualitative research design and methods employed in this study. In this chapter, a summary of all previous chapters is presented. The summary covers the aim, how each research question was addressed, the literature review and the methodology used. The chapter further accounts for the implications, limitations, and possible recommendations of this study.

6.2 SUMMARY OF THE STUDY

6.2.1 Aim, Objectives, and Research Question

This study seeks to understand Grade 11 teachers' content knowledge in the teaching of Euclidean Geometry.

The objectives of the study were

a) To examine teachers' content knowledge of teaching Euclidean Geometry.

b) To determine how teachers use content knowledge in teaching Euclidean Geometry.

The aforementioned objectives led to the formulation of the following research questions:

i. What is Grade 11 Mathematics teachers' content knowledge of Euclidean Geometry?

ii. How do Grade 11 Mathematics teachers use their content knowledge in the teaching of Euclidean Geometry among Grade 11 learners?

6.2.2 Summary of How Research Questions Were Addressed

This section summarises how the two research questions were answered based on the findings from the study.

6.2.2.1 Research Question Number One: *What is Grade 11 Mathematics Teachers' Content Knowledge of Euclidean Geometry?*

In developing learners' conceptual understanding of Euclidean Geometry, van Hiele's (1986) five levels of geometry thinking should be taken into consideration and followed. To understand

Grade 11 Mathematics teachers' content knowledge of Euclidean geometry tests, the scripts of 10 participants were analysed. It emerged from this study that the content knowledge of Euclidean Geometry of participant teachers for teaching and learning did not embrace the vital five levels and, therefore, both teaching and learning were poor. Only a few teachers exhibited acceptable standards of teaching.

This test showed that most of the participants had attained level 1 but not the last levels of van Hiele's theory, and teachers progressed and developed their knowledge of Geometry through these levels (Van Hiele, 1986, 1999). These confirmed arguments made, that topics that were removed previously and have now been reinstated will put teachers in the uncomfortable position of having to learn new topics (Phasha, 2016). The 1st level (level 0) is called visualisation: teachers were able to identify a shape but were not able to provide its properties. The 2nd level (level 1) is called analysis: teachers were able to identify properties of shapes, but not in logical order. The 3rd level (level 2) is called informal deduction: teachers were able to combine the shapes and their properties to provide a precise definition as well as relate the shape to other shapes (van de Sandt, 2007). This means that these participants become uncomfortable with applying deductive arguments. Most of the participants found it easier to calculate the value of unknown angles than to prove equal angles, tangents, and cyclic quadrilaterals. Participants demonstrated the need for capacity in the study of deductive Euclidean Geometry problems due to a lack of foundational concepts of Euclidean Geometry. Only two participants attained van Hiele level 3, two attained van Hiele level 2, and six participants remained at van Hiele level 1.

Based on the aforementioned, six out of 10 participants teaching Euclidean Geometry in these schools have content knowledge deficiency in teaching Euclidean Geometry. According to Aslam et al. (2016), teacher qualification may be a special teaching skill, SMK, or teaching experience that makes a teacher suitable. These 10 participants all have a Bachelor of Education degree in the subject as the minimum requirement for teaching; however, some lack a background in Euclidean Geometry. Classroom teaching depends on the teacher's earlier experiences with learners, which are greatly influenced by the teacher's own time as a learner (Dean, 2019). This means that six participants did not have these experiences with the topic.

The results of the test indicated a content deficiency in some of the participants teaching Euclidean Geometry for various reasons, but the main one is the failure to upgrade themselves in the content of teaching Euclidean Geometry. This also indicated that participants sometimes forget or have little knowledge of choosing the reasons for their calculations.

The analysis of the study further exposed that a lack of understanding of Euclidean geometry concepts and inadequate training in the teaching of Euclidean Geometry at school and higher education levels has led to teachers not being well-qualified or experienced in teaching Mathematics, particularly challenging topics such as Euclidean Geometry. Teachers who can make detailed and explanatory lesson plans can address misconceptions of mathematical concepts that learners experience in the classroom, as it is explained that content knowledge refers to the teachers' knowledge about specific content, correct interpretation and applying mathematical concepts, facts, procedures, principles, ideas, and theories (Ramatlapana & Berger, 2018).

This revealed that the main significance of the inclusion of Euclidean Geometry in the curriculum of the learners is to develop learners' critical thinking skills, deductive reasoning ability, and logical argument in preparation for the world of work, which is not attained due to a lack of understanding of content knowledge by teachers.

6.2.2.2 How do Grade 11 Mathematics Teachers use their content knowledge in the teaching of Euclidean Geometry among Grade 11 learners?

I needed to find ways of understanding the participants, and it was necessary to find out some details about them. A Task-based interview was developed to identify teachers' conceptions of teaching. This was created to ask participants about their background, understand their experiences, and gather information about their understanding of Euclidean Geometry. Pre- and post-task-based interviews with the 10 participants were analysed. These interviews with 10 participants assisted the researcher in this study in identifying common themes focusing on Euclidean Geometry.

6.2.2.2.1. Teachers as interpreters and designers of learning programmes and materials

Teachers are expected to design lessons that will be meaningful to the learners and promote the aims and objectives of the curriculum (Chisholm, 2003). Teachers' content knowledge is and will always influence the design of the learning programme as well as its interpretation.

However, four participants in this study agreed that when using content knowledge of Euclidean Geometry in preparing for lessons, activities/exercises were taken from textbooks. TC mentioned that teachers followed textbooks when teaching Euclidean geometry theorems, which were explained on the chalkboard. TD and TI also confirmed that teachers first explained Euclidean geometry theorems from textbooks, and after each theorem, application exercises were also given from textbooks. This means that these teachers, through their learning experiences, adopt the same use of the content knowledge which they acquired in their teaching and learning of Euclidean Geometry, with no further interpretation or design of new activities to cater for all learner abilities in the classroom and promotion of learner-centred education.

6.2.2.2.1 Teachers Use Euclidean Content Knowledge As An Assessor Of The Learning

Based on the test results in Chapter 4, six participants remained at van Hiele level 1, which means that most of the participants find it easier to calculate the value of unknown angles than to prove equal angles, tangents, and cyclic quadrilaterals. Participants demonstrated the need for capacity in the study of deductive Euclidean geometry problems due to a lack of foundational concepts of Euclidean Geometry. Only two participants attained van Hiele level 3, and two attained level 2. This further indicates that teachers, in their role as assessors, have a deficiency that may further need capacitation on content knowledge, as they are not going to be able to assess learners in all cognitive levels if they still have gaps, as per van Hiele's levels theory.

6.2.2.2.2 Teachers' Use of Euclidean Geometry Content Knowledge as Subject Specialists

Based on the results again in chapter four, it is evident that participants are not all well-grounded in the knowledge, skills, values, principles, methods, and procedures relevant to the discipline, subject, learning area, phase of the study, or professional or occupational practice as expected. This is revealed by the fact that some participants have correct responses but no reasons provided, and some participants provided incorrect reasons, as it should be when dealing with Euclidean Geometry. This also indicates gaps in the background of the Euclidean geometry content knowledge from the participants, and it also indicates that there will be poor mediation in the teaching and learning in the classroom situation.

6.3 RECOMMENDATIONS

a) There is a need to encourage Mathematics teachers to further their Euclidean geometry content knowledge by granting paid study leave to teachers. This will have an overall positive effect on learners' performance in Mathematics.

b) Professional development mathematics courses and mathematics workshops addressing Euclidean content knowledge, which is curriculum relevant and quality assured, should be ongoing, as this would help expand the horizon of Mathematics teachers since this topic was once omitted in the study of Mathematics in South Africa. Practical instruction in geometry teaching, learning and any topic in Mathematics for that matter, requires a teacher to develop sound mathematical knowledge as well as instructional skills and strategies, making use of helpful resources and activities that guide the teaching activities and further assist in the effective delivery of the lesson (Luneta, 2014). Teaching without good instructional skills, modelling activities, and sound knowledge of the concept usually puts the teacher in a challenging situation, and learners tend to lose interest in the topic. As a result, quality teaching is compromised. It is, therefore, crucial that Mathematics teachers are fully equipped with sound knowledge and are aware of the teaching environment and their learners. This will ensure that appropriate modelling activities are used in problem-solving.

c) Mathematics teachers should be given professional training on the active usage of technology in the teaching and learning of Euclidean Geometry. This has been very challenging to many teachers as it involves abstract and complex ideas. Building capacity in planning for using other resources, such as laptops and projectors, may help teachers develop their understanding of abstract geometry concepts through visualisation and graphic representation (Parrot & Eu, 2018). This will also help in the development of assessment activities since teachers are assessors of knowledge. Both pre- and in-service Mathematics teachers need to be trained to integrate technology in the use of Euclidean geometry content knowledge. Introducing technology in the lesson preparation may change traditional teaching to a more learner-centred one where learners are actively engaged in the construction of knowledge.

d) Mathematics learners should all be taught by qualified and experienced Mathematics teachers from primary school level through to secondary school level, as this will no doubt enhance their academic achievement.

e) Both pre-service and in-service Mathematics teachers need to be trained to integrate technology in the use of Euclidean geometry content knowledge. Introducing technology in the lesson preparation may change traditional teaching to a more learner-centred one, where learners are actively engaged in the construction of knowledge

At this point, knowledge of this study’s findings may not be enough; however, according to Geldenhuys and Oosthuizen (2015), changes need to be implemented in Mathematics teachers’ beliefs, attitudes and practices through accredited continuous professional development programmes.

Mathematics teachers need to be professionally developed for them to utilise content knowledge and skills to enhance their classroom mathematical abilities (Yow & Lotter, 2016 More research would be needed to investigate the effectiveness of these Mathematics teachers’ professional development in improving mathematics teachers’ classroom practices and hence learner performance in general (Gomez et al., 2015).

From the findings of this study, it is essential for further research of the same kind to be conducted on a large scale (that is, in a quantitative manner), where different opinions of teachers from other areas can be heard to establish whether the same problems occur when teaching Euclidean Geometry in schools. As South Africa needs suitably qualified Mathematics teachers who can teach the subject effectively to overcome the problem of poor performance and the high failure rate in Mathematics, further research is required. Findings from such a study could assist in ensuring quality mathematics education, which will develop the necessary skills needed by doctors, scientists, and many other scientifically oriented professionals.

6.4 SUGGESTIONS FOR FURTHER RESEARCH

Further studies could be extended to other aspects of Euclidean Geometry and could well include a bigger sample space that will accommodate more South African secondary schools and even more districts and provinces.

6.5 STUDY LIMITATIONS

As is typical of every research study, this research study also acknowledges some limitations that might have affected its results in one way or another. This research study was restricted to understanding Grade 11 teachers' content knowledge when teaching Euclidean Geometry. Only 10 secondary schools were sampled in the Ugu district, which comprises 160 secondary schools. This study was only conducted on the Euclidean Geometry topic, which is one of the 10 content areas in the Grade 11 mathematics curriculum. This current study had a very small sample of only 10 Grade 11 Mathematics teachers as participants, and these were only chosen from the Ugu district in KwaZulu-Natal. Therefore, the results and findings were only obtained from the 10 participants. Hence, the results of the current study may not be generalised. Different results might be obtained from various contexts with larger samples. Besides qualitative research, other research approaches and research instruments could have been employed.

6.6 CONCLUSION

The current study has contributed towards understanding content knowledge used by Grade 11 Mathematics teachers when teaching Euclidean Geometry. It has shown what teachers' content knowledge of Euclidean Geometry is and how Grade 11 Mathematics teachers use their content knowledge when teaching Euclidean Geometry. The current study may add value to the teaching and learning of Mathematics, especially geometry education. According to the analysis and findings, this study revealed that Mathematics teachers still use their content knowledge to plan for traditional methods when teaching Euclidean Geometry, and some have deficiencies in the content of Euclidean Geometry, which is an answer to poor performance of learners in this section. teachers follow textbooks when teaching Euclidean geometry theorems and explain basic skills and computational procedures on the chalkboard. It was also confirmed that teachers first explain Euclidean geometry theorems, and after each theorem, application exercises are given to learners. Content knowledge use, i.e., lesson preparation and assessment in teaching

Euclidean Geometry, where learners are active constructors of knowledge, may develop their thinking abilities and reasoning skills when solving Euclidean geometry problems, and this ought to be researched further.

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APPENDICES

Appendix A: Ethical clearance approval

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

15 April 2022

Bongiwe Princess Ntombela (208525007)
School Of Education
Edgewood Campus

Dear BP Ntombela,

Protocol reference number: HSSREC/00003962/2022
Project title: Teachers' content knowledge of teaching Euclidean geometry
Degree: Masters

Approval Notification – Expedited Application

This letter serves to notify you that your application received on 24 March 2022 in connection with the above, was reviewed by the Humanities and Social Sciences Research Ethics Committee (HSSREC) and the protocol has been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

This approval is valid until 15 April 2023.
To ensure uninterrupted approval of this study beyond the approval expiry date, a progress report must be submitted to the Research Office on the appropriate form 2 - 3 months before the expiry date. A close-out report to be submitted when study is finished.

All research conducted during the COVID-19 period must adhere to the national and UKZN guidelines.

HSSREC is registered with the South African National Research Ethics Council (REC-040414-040).

Yours sincerely,

[redacted]

Professor Dipane Hlalele (Chair)

/dd

Humanities and Social Sciences Research Ethics Committee
Postal Address: Private Bag X54001, Durban, 4000, South Africa
Telephone: +27 (0)31 260 8350/4557/3587 **Email:** hssrec@ukzn.ac.za **Website:** http://research.ukzn.ac.za/Research-Ethics

Appendix B: Letter of permission

Ms. Ntombela Bongiwe Princess

[redacted]m

[redacted]

P.O. Box 1723

Scottburgh

4180

The Principal

--------------------Secondary school

Sir

Re: Request for permission to do scholarly research in your district.

I hereby request permission to conduct scholarly research in the Ugu district, specifically to interview Grade 11 mathematics teachers. I am currently doing a Master's Degree in Education at the University of KwaZulu-Natal.

My topic seeks to explore teachers' content knowledge of grade 11 Euclidean geometry. I have asked the permission from the principals of the above schools and the participants. Participants in this study include teachers. Ethical issues will be observed throughout the study.

Should you wish to verify the above information concerning my research. You can contact my supervisor, Dr Lokesh Maharajh. His contact details are: 072 43 569 68, and his Email address is maharajhlr@ukzn.ac.za.

I look forward to your cooperation with my request.

Yours faithfully

Ms. Ntombela Bongiwe Princess

[redacted]

Student Number: 208525007

Appendix C: Letter of consent

To: Participant(s)

Research Project:

Teachers' content knowledge of teaching Euclidean Geometry

I, Bongiwe Princess Ntombela, am doing a study through the School of Education, Mathematics Education at the University of KwaZulu-Natal with Dr Lokesh Maharajh His contact number is 031 – 260 7252 (work). We would like to understand Grade 11 teachers' content knowledge of teaching Euclidean Geometry in KwaZulu-Natal: South Africa.

Teachers are requested to assist by participating in this research project as this will benefit our community and interested researchers in the field of medical science. However, participation is absolutely voluntary and has no impact on their employment. Participants will be asked to take part in the test and task-based interviews after the presentation has been completed. The whole session of the interview will be tape-recorded. All participants will be noted on transcripts and data collections by a pseudonym (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secured password and not been used for any other purpose except for the research.

Participants may leave the study at any time by telling the researcher. Participants may review any comment on any parts of the researchers' written reports.

(Researcher's Signature) (Date)

Participants

☐ AGREE

☐ DISAGREE

DECLARATION

I, *(Participant's NAME)* *(Signature)*

Appendix D1: Research instrument — pre-task-based interview

MATHEMATICS EDUCATION RESEARCH

RESEARCH INSTRUMENT – PRE TASK BASED INTERVIEW

TEACHERS' CONTENT KNOWLEDGE OF TEACHING EUCLIDEAN GEOMETRY

NAME OF THE PARTICIPANT: ________________________________
PARTICIPANT PSEUDONYM NAME: ________________________________
Focus and Purpose of the Study

- This study is based at Ugu District in KwaZulu-Natal
- This study investigates the sample of Grade 11 mathematics teachers' content knowledge of teaching Euclidean Geometry
- The study is underpinned by Van Hele levels of understanding Euclidean Geometry.
- The study's conclusion will make recommendations not generalisation, since the study is based in one District and sample size is small

INTERVIEW QUESTIONS

1. Did you study Euclidean Geometry in Grade 12? Yes/No____________ Explain how you were taught Euclidean Geometry in high school. If not, how did you capacitate yourself in Euclidean geometry?

____
____
____

2. Why do you think it is important to know theorems if one has to embark on activities of Euclidean Geometry?

____
____
____

3. How do you normally use theorems when teaching Euclidean geometry in class? Do you integrate visual learning?

____
____
____

4. Using your experience, why the performance of Euclidean geometry, is so poor in schools

____
____

5. Which Euclidean geometrical concepts were you familiar with at high school level?

____
____

Researcher's Reflections

____
____
____

Appendix D2: Research instrument — post task based interview

MATHEMATICS EDUCATION RESEARCH

TEACHERS' CONTENT KNOWLEDGE OF TEACHING EUCLIDEAN GEOMETRY

NAME OF THE PARTICIPANT: ________________________________

PARTICIPANT PSEUDONYM NAME: ________________________________

Focus and Purpose of the Study

- This study is based at Ugu District in KwaZulu-Natal
- This study investigates the sample of Grade 11 mathematics teachers' content knowledge of teaching Euclidean Geometry
- The study is underpinned by Van Hele levels of understanding Euclidean Geometry.
- The study's conclusion will make recommendations not generalisation, since the
- study is based in one District and sample size is small

INTERVIEW QUESTIONS

1. In Question 3 of your test, you were asked to draw a perpendicular line to a chord, how did you know the drawn line is perpendicular? And How did you know that the angle is bisected?

____

____

2. In Question 7, what conclusion can you draw with angle A and angle DCE, and why?

____

____

3. How did you arrive at the answer for Question 9.1,2?

____

____

4. How do you know if a line is a diameter of a circle?

____

____

5. When you are required to prove in a statement, what do you think is the best solution of doing it?

____

____

6. In terms of your responses, which question do you regard as little challenging? Why?

____

____

Researcher' s Reflections

Appendix D3: Research instrument — question paper

MATHEMATICS EDUCATION RESEARCH

RESEARCH INSTRUMENT

TEACHERS'CONTENT KNOWLEDGE OF TEACHING EUCLIDEAN GEOMETRY

NAME OF THE PARTICIPANT: ______________________

PARTICIPANT PSEUDONYM NAME: ______________________

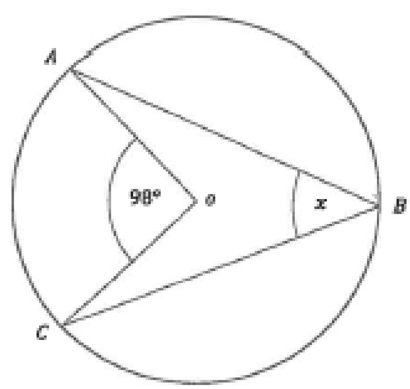
Focus and Purpose of the Study

• This study is based at Ugu District in KwaZulu-Natal
• This study investigates the sample of Grade 11 mathematics teachers' content knowledge of teaching Euclidean Geometry
• The study is underpinned by Van Hele Levels of understanding Euclidean Geometry.
• The study's conclusion will make recommendations not generalisation, since the study is based in one District and sample size is small

QUESTION 1

Points *A*, *B*, and *C* are all on the circumference of the circle.

O represents the centre.



Not drawn accurately

Calculate the angle *x*, giving a reason for your answer.

____

____

____

____

Answer ______________________

Question 2.

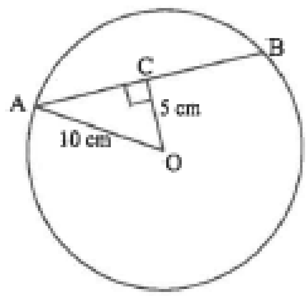
2.1. Angles subtended by the chord at the circumference of a circle, on the same side of a chord are equal. Denote the following statement with suitable rough sketches (drawings), as you will prepare for your lesson

QUESTION 3

3.1. If a line drawn from the centre of a circle is perpendicular to a chord, then it bisects the chord. Represent the following statement with suitable rough sketches (drawings), as you ll prepare for a lesson.

QUESTION 4

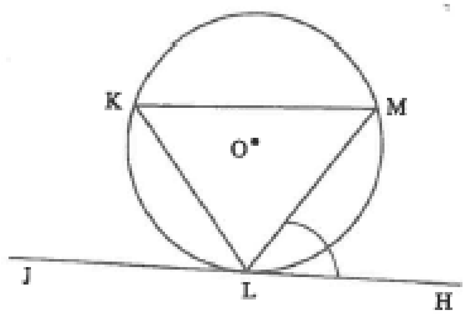
4.1 In the diagram, AB is a chord of the circle having centre O. C is a point on AB such that $A\hat{C}O = 90°$. OC = 5 cm and AO = 10 cm.



Calculate, with reasons, the length of AB.

QUESTION 5

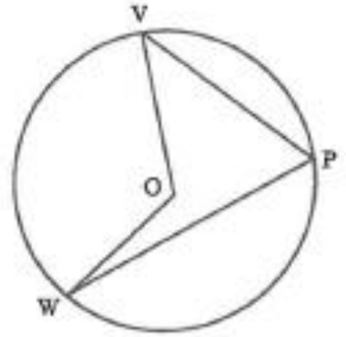
5.1 In the figure below, JLH is a tangent to the circle having centre O at L. K and M are points on the circle.



Prove the theorem which states that $M\hat{L}H = \hat{K}$.

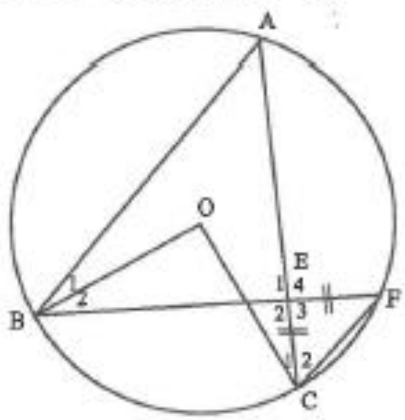
QUESTION 6

6.1 In the diagram, O is the centre of the circle. VP and WP are chords, and VO and WO have been drawn.



Use the diagram on the DIAGRAM SHEET to prove the theorem which states that an angle that an arc subtends at the centre of a circle is twice the size of the angle subtended by the same arc at the circle i.e. $V\hat{O}W = 2\hat{P}$.

6.2 In the diagram, O is the centre of the circle. A, B, C and F are points on the circumference. AC and BF intersect in E and EF = FC.



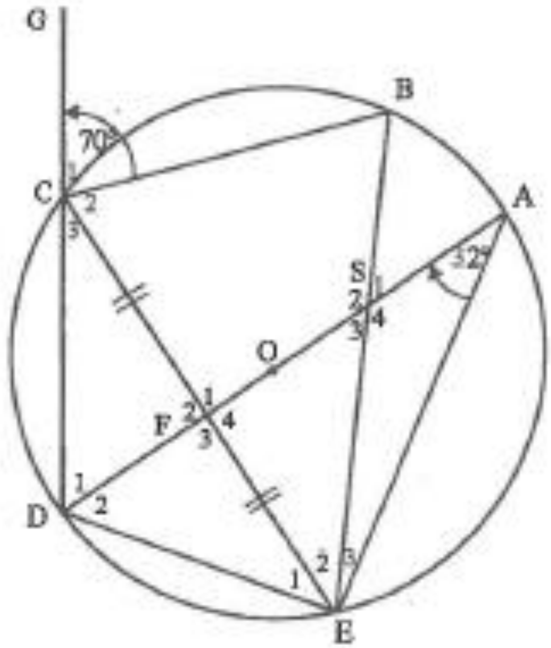
Prove that:

6.2.1 AB || FC.

6.2.2 OBCE is a cyclic quadrilateral.

QUESTION 7

A, B, C, D and E are points on the circle having centre O. DC is produced to G. Diameter AOD bisects chord CE in F, and intersects chord BE in S.
$\hat{A} = 32°$ and $G\hat{C}B = 70°$.



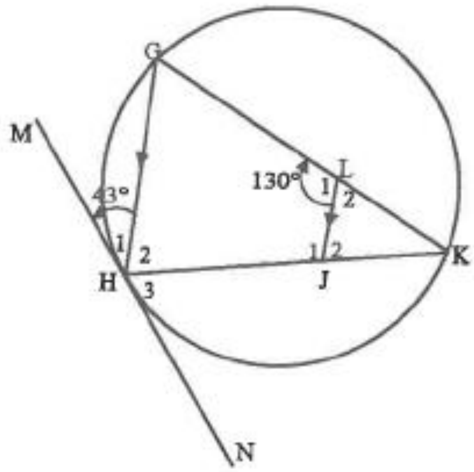
Calculate, with reasons, the sizes of the following angles:

7.1.1 $B\hat{E}D$

7.1.2 $\hat{C}_2$

QUESTION 8

MHN is a tangent to circle GHK at H. L is a point on GK and J a point on HK such that LJ is parallel to GH. $\hat{H}_1 = 43°$ and $\hat{L}_1 = 130°$.



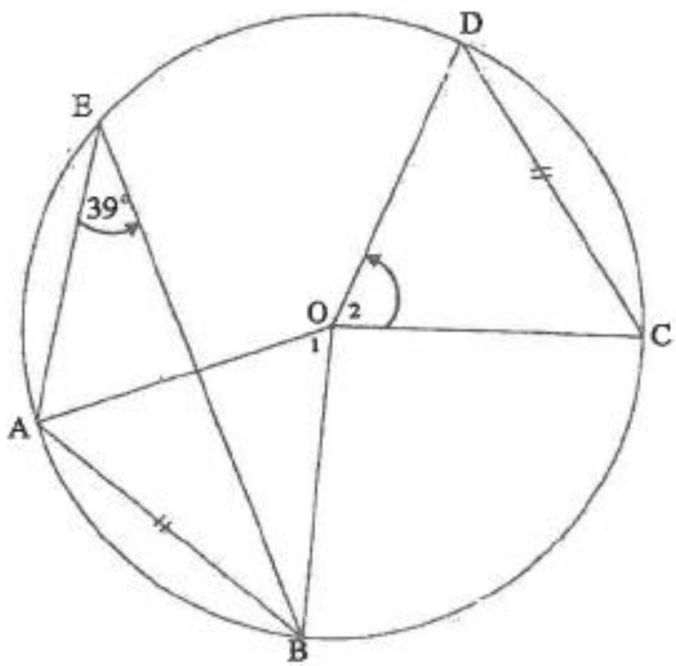
Calculate, with reasons, the size of:

8.1 $\hat{K}$

8.2 $\hat{H}_3$

QUESTION 9

9.1 In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $A\hat{E}B = 39°$.

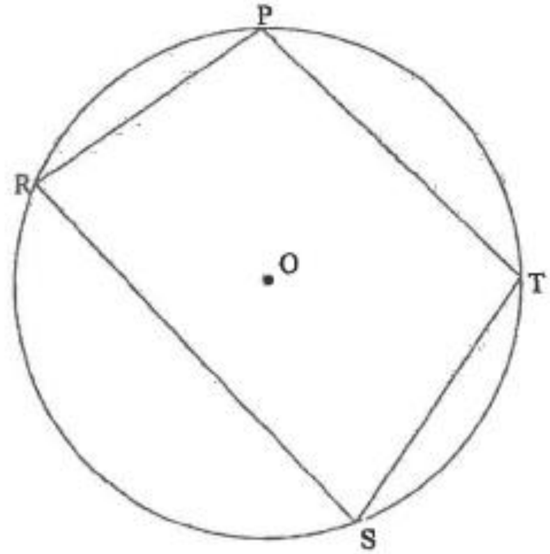


9.1.1 Determine the size of $\hat{O}_1$.

9.1.2 Determine the size of $\hat{O}_2$.

QUESTION 10

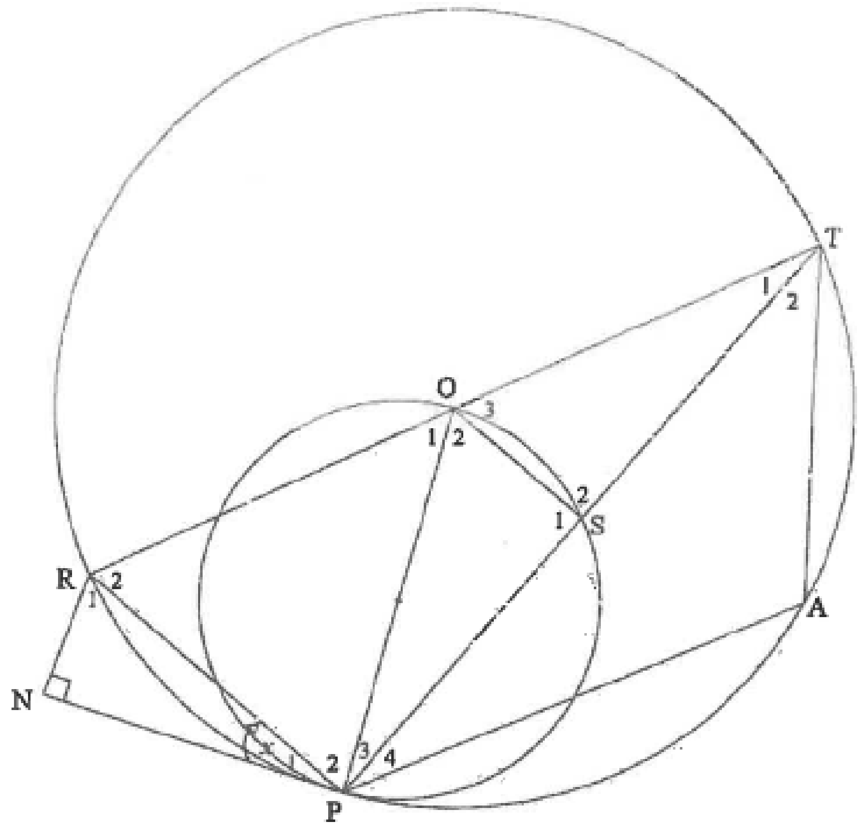
10.1 In the diagram below, O is the centre of the circle and PTSR is a cyclic quadrilateral.



Prove the theorem that states that $\hat{P} + \hat{S} = 180°$.

QUESTION 11

O is the centre of the larger circle RTAP. OP is the diameter of the smaller circle PSO. NP is a tangent to both circles at P. RN ⊥ NP.
Let $\hat{P}_1 = x$.



11.1 Prove that PR bisects $O\hat{R}N$.

11.2 Prove that $R\hat{O}S = P\hat{A}T$.

QUESTION 12

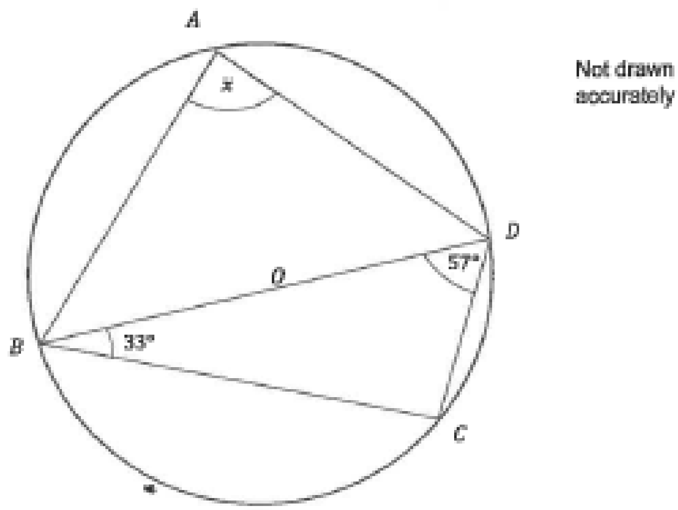
12. What is your understanding about these sentences/terms and its use in a given statement? Explain to your best knowledge

12.1. **Line drawn Perpendicular** to the chord
12.2. A line drawn from the centre bisecting the chord
12.3. An angle subtended by the diameter of the circle
12.4. Diameter of a circle
12.5. Cyclic quadrilateral

12.2. The diagram below shows a cyclic quadrilateral $ABCD$.

Points A, B, C and D touch the circumference of the circle.

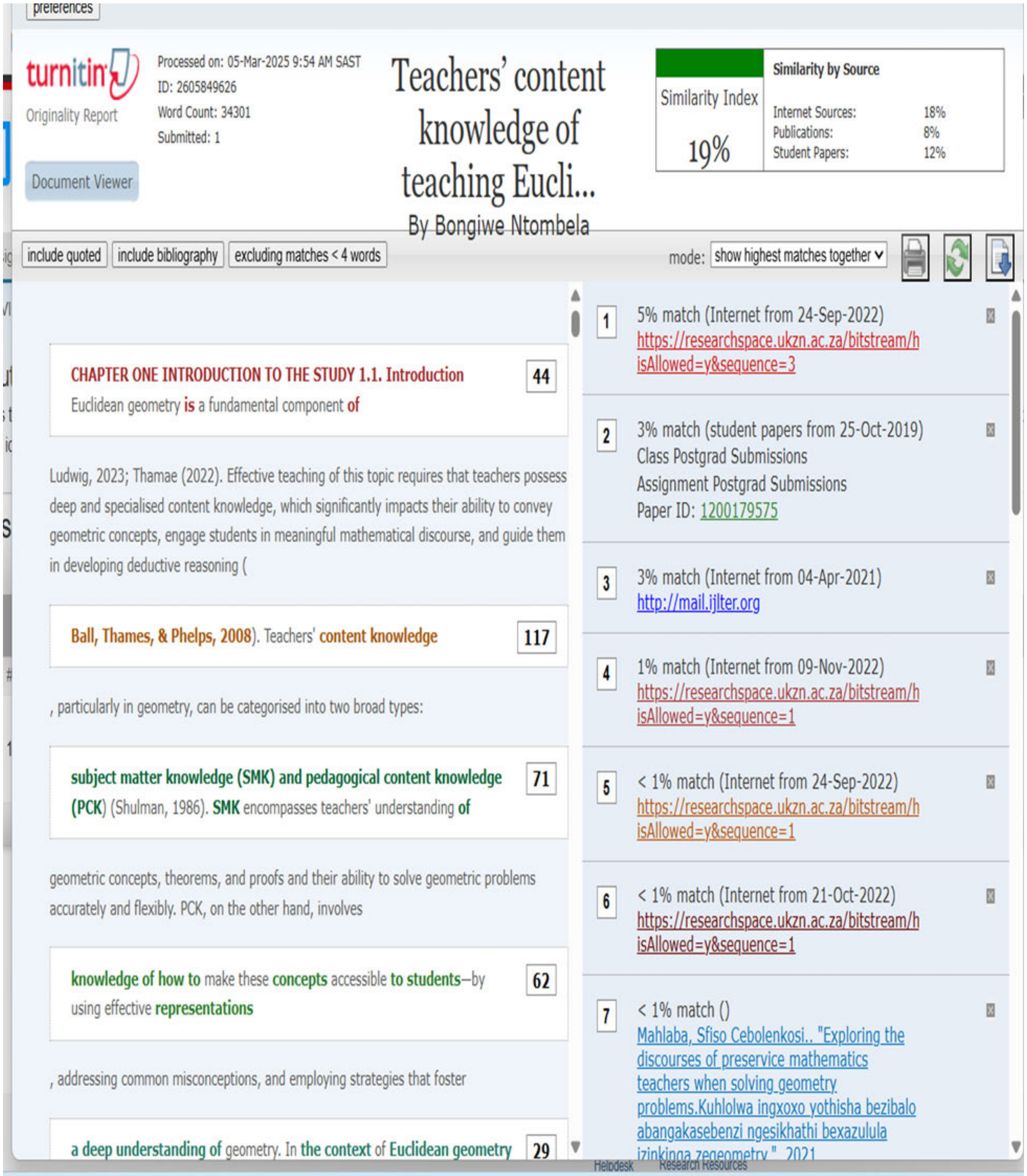
Line BD goes through centre O.



Work out the size of the angle marked x.

Explain your reasoning carefully.

Appendix E: Plagiarism report



Appendix F: Editor's certificate

Helen Bond

IMPELA EDITING SERVICES

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02 May 2025

CERTIFICATE

BONGIWE P. NTOMBELA

Dear Bongiwe

Thank you for using Impela Editing Services to edit your Master's thesis entitled "*TEACHERS' CONTENT KNOWLEDGE OF TEACHING EUCLIDEAN GEOMETRY*".

I have proofread for errors of grammar, punctuation, spelling, syntax and typing mistakes. I have formatted your work and checked the references (this means checking the formatting) according to APA 7th edition.

Please note that Impela Editing accepts no responsibility for work changed after issuing this certificate.

I wish you the very best in your submission and your career.

Kind regards

[redacted]