Clifford-Fischer Theory Applied to Certain Groups Associated with Symplectic, Unitary and Thompson Groups

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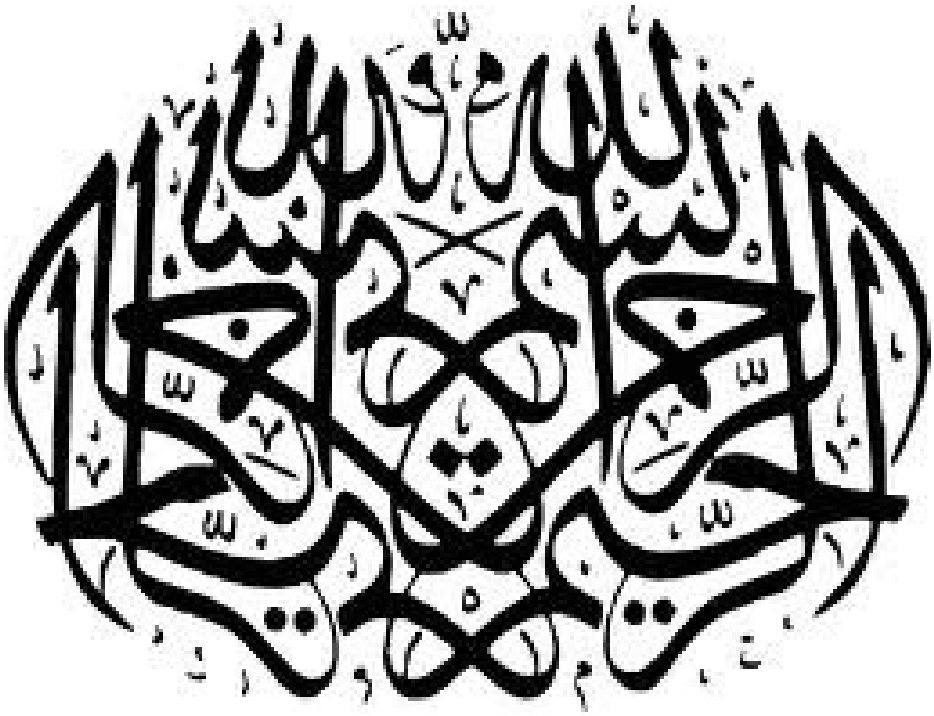


IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

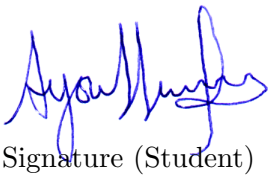
Abstract

The character table of a finite group is a very powerful tool to study the groups and to prove many results. Any finite group is either simple or has a normal subgroup and hence will be of extension type. The classification of finite simple groups, more recent work in group theory, has been completed in 1985. Researchers turned to look at the maximal subgroups and automorphism groups of simple groups. The character tables of all the maximal subgroups of the sporadic simple groups are known, except for some maximal subgroups of the Monster $\mathbb{M}$ and the Baby Monster $\mathbb{B}$. There are several well-developed methods for calculating the character tables of group extensions and in particular when the kernel of the extension is an elementary abelian group. Character tables of finite groups can be constructed using various theoretical and computational techniques. In this thesis we study the method developed by Bernd Fischer and known nowadays as the theory of *Clifford-Fischer matrices*. This method derives its fundamentals from the Clifford theory. Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a group extension. For each conjugacy class $[g_i]_G$, we construct a non-singular square matrix $\mathcal{F}_i$, called a Fischer matrix. Once we have all the Fischer matrices together with the character tables (ordinary or projective) and fusions of the inertia factor groups into G, the character table of $\overline{G}$ is then can be constructed easily. In this thesis we apply the coset analysis technique (this is a method to find the conjugacy classes of group extensions) together with theory of Clifford-Fischer matrices to calculate the ordinary character tables of seven groups of extensions type, in which four are non-split and three are split extensions. These groups are of the forms: $2_+^{1+8}{\cdot}A_9$, $3^7{:}Sp(6,2)$, $2^6{\cdot}Sp(6,2)$, $2^5{\cdot}GL(5,2)$, $2^{10}{:}(U_5(2){:}2)$, $2_-^{1+6}{:}((3^{1+2}{:}8){:}2)$ and $2^{2n}{\cdot}Sp(2n,2)$ and $2^8{\cdot}Sp(8,2)$. In addition we give some general results on the non-split group $2^{2n}{\cdot}Sp(2n,2)$.

Preface

The work described in this thesis was carried out under the supervision and direction of Professor Jamshid Moori, School of Mathematics, Statistics and Computer Sciences, University of KwaZulu-Natal, Pietermaritzburg, from January 2009 to April 2012.

The Thesis represent original work of the author and has not been otherwise been submitted in any form for any degree or diploma to any University. Where use has been made of the work of others it is duly acknowledged in the text.



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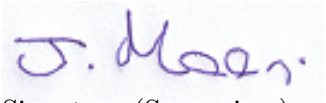

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Dedication

TO MY PARENTS, MY LOVELY WIFE MUNA, MY LOVELY DAUGHTER FATIMA, MY FAMILY AND TO THE BEST FRIEND I HAVE EVER GOT MUSA COMTOUR, I DEDICATE THIS WORK.

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List of Notations

$\mathbb{N}$	natural numbers
$\mathbb{Z}$	integer numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$\mathbb{F}$	field
$\mathbb{F}^*$	multiplicative group of $\mathbb{F}$
$\mathbb{F}_q$	Galois field of q elements
$\mathbb{V}$	vector space
$\mathbb{V}(n,q)$	vector space of dimension n over the field $\mathbb{F}_q$
dim	dimension of a vector space
det	determinant of a matrix
tr	trace of a matrix
G	finite group
$e,\ 1_G$	identity element of G
$\|G\|$	order of G
$o(g)$	order of $g \in G$
$\cong$	isomorphism of groups
$H \leq G$	H is a subgroup of G
$[G:H]$	index of H in G
$N \trianglelefteq G$	N is a normal subgroup of G
$K{\cdot}Q$	extension of a group K by a group Q

$K \times Q$	direct product of a group K by a group Q
$K{:}Q$	split extension of K by Q (semidirect product)
$K{\cdot}Q$	non-split extension of K by Q
G/N	quotient group
$\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_k$	distinct conjugacy classes of a finite group G
$[g],\ C_g$	conjugacy class of g in G
$C_G(g)$	centralizer of $g \in G$
G_x	stabilizer of $x \in X$ when G acts on X
x^G	orbit of $x \in X$
$\lvert Fix(g)\rvert$	number of elements in a set X fixed by $g \in G$ under the group action
$\mathrm{Aut}(G)$	automorphism group of G
G'	derived or commutator subgroup of G
$Z(G)$	center of G
h^g	conjugation of h by g
$M(G)$	Schur multiplier of G
$\mathfrak{C}(G)$	representation group or covering group of G
$[\alpha]$	equivalence class of factor sets containing α
$H^2(G, M)$	second cohomology group of a group G with coefficients in M
D_{2n}	dihedral group consisting of $2n$ elements
$p^{1+2m},\ p \neq 2$	extraspecial $p-$group of order p^{1+2m} with center $\mathbb{Z}_p$ and a quotient isomorphic to $\mathbb{V}(2m, p)$
2_+^{1+2m}	extraspecial $2-$group of order 2^{1+2m} of type "+"
2_-^{1+2m}	extraspecial $2-$group of order 2^{1+2m} of type "-"
$Syl_p(G)$	set of all Sylow $p-$subgroups of G
$\mathbb{Z}_n$	group $\{\overline{0}, \overline{1}, \cdots, \overline{n-1}\}$ under addition modulo n
S_n	symmetric group of n objects
A_n	alternating group of n objects
$GL(n, \mathbb{F})$	general linear group
$GL(n, q)$	finite general linear group
$SL(n, \mathbb{F})$	special linear group
$U_n(\mathbb{F})$	unitary group
$PGL(n, \mathbb{F})$	projective general linear group $GL(n, \mathbb{F})/Z(GL(n, \mathbb{F}))$

$PSL(n, \mathbb{F})$	projective special linear group $SL(n, \mathbb{F})/Z(SL(n, \mathbb{F}))$
$\mathrm{Aff}(n, \mathbb{F})$	affine group
$Sp(2n, \mathbb{F})$	symplectic group of dimension $2n$ over $\mathbb{F}$
Th	the sporadic Thompson simple group
Co_2	the sporadic Conway group of order 42 305 421 312 000
$\mathcal{C}(G)$	algebra of class functions on a finite group G
χ	character of a finite group
χ_ρ	character afforded by a representation ρ of G
$\mathbf{1}_G$	trivial character of G
deg	degree of a representation or a character
$\mathrm{Irr}(G)$	set of the ordinary irreducible characters of G
$\mathrm{IrrProj}(G, \alpha^{-1})$	set of irreducible projective characters of G with factor set α
(P, α)	projective representation of G with factor set α
$\langle\ ,\ \rangle$	inner product of class functions or a group generated by two elements (depends on the context of the discussion)
$\otimes$	tensor product of representations
$\oplus$, $\bigoplus$	direct sum
$\chi\uparrow_H^G$	character induced from a subgroup H to G
$\chi\downarrow_H^G$	character restricted from a group G to its subgroup H
$\mathbf{1}\uparrow_H^G$	permutation character of G on the cosets of G/H
$\inf(\chi)$	inflation (lift) of a character χ
$\ker\phi$	kernel of a homomorphism ϕ
$\mathrm{Im}(\phi)$	image of a function ϕ
χ_ρ	character afforded by a representation ρ of G
$\mathrm{Hom}(G, A)$	set of all homomorphisms from G to A

Chapter 1

Introduction

Since the classification of all finite simple groups, more recent work in group theory has involved methods of calculating character tables of maximal subgroups of finite simple groups. The character tables of all the maximal subgroups of the sporadic simple groups are known, except for some maximal subgroups of the Monster $\mathbb{M}$ and the Baby Monster $\mathbb{B}$. Most of these maximal subgroups are extensions of elementary abelian groups, so methods have been developed for calculating the character tables of extensions of elementary abelian groups. A knowledge of the character table of a group G provides considerable information about G and hence it is of importance in the Physical Sciences as well as in Pure Mathematics. Character tables of finite groups can be constructed using various techniques. For example, the Schreier-Sims algorithm, Todd-Coxeter coset enumeration method, the Burnside-Dixon algorithm and various other techniques. However Bernd Fischer presented a powerful and interesting technique for calculating the character tables of group extensions. This technique, which is known as *Clifford-Fischer matrices*, derives its fundamentals from the Clifford theory. Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a group extension. For each conjugacy class $[g_i]_G$, we construct a non-singular square matrix $\mathcal{F}_i$, called a Fischer matrix. Once we have all the Fischer matrices of G together with the character tables (ordinary or projective) and fusions of the inertia factor groups into G, the character table of $\overline{G}$ is then can be constructed easily. The first step in constructing the character table of any finite group is to find its conjugacy classes. If $\overline{G} = N{\cdot}G$ is an extension, the technique of the *coset analysis* can be used to calculate the conjugacy classes of $\overline{G}$. The idea of this technique is to consider for each conjugacy class $[g]_G$, one coset $N\overline{g}$, where $\overline{g}$ is a pre-image of g in $\overline{G}$. Then correspond to this class, we construct a number of conjugacy classes of $\overline{G}$. That is each conjugacy class of $\overline{G}$ corresponds uniquely to a

conjugacy class of G. In this thesis, we have applied the coset analysis technique together with the theory of Clifford-Fischer matrices to seven groups of extension type, in which four are *non-split* and the other three groups are *split* extensions. For both split and non-split cases, we considered the situation when the kernel N of $\overline{G}$ is abelian or non-abelian. Also in two examples $2_+^{1+8}{\cdot}A_9$ and $2^5{\cdot}GL(5,2)$, which are maximal in the Thompson group Th, the theory of *projective characters* is involved in the construction of the character tables of these two groups, while for the other five groups, we only use the ordinary character tables of the inertia factors to construct the character tables of these extensions. These seven groups are the context of the discussion of Chapters 6, 7, 8, 9 and 10. In fact the thesis is composed of 10 chapters and in addition we supply an Appendix, containing the character tables of many groups that have been discussed through the various chapters of this thesis. Next we give some details on the structures of these chapters.

Chapter 1 is an introductory chapter to this thesis.

In Chapter 2 we study the theory of group extensions. This chapter has been divided into three sections, where in Section 2.1 we give some definitions and basic results on the theory of group extensions. In Section 2.2 we give some results on a special type of extensions, namely the *semi-direct product* or *split extension*, where groups of this type satisfy further interesting properties. In Section 2.3 we discuss the conjugacy classes of elements of a group extension $\overline{G} = N{\cdot}G$ where N and G are as before. We describe the technique of the *coset analysis* for computing the conjugacy classes of $\overline{G}$. This technique was developed and used firstly by Moori in [47] and also in [48] and has since been widely used for computing the conjugacy classes of several group extensions.

In Chapter 3 we review the fundamental tools required for the theories of representations and characters, which will be used in the other chapters. This includes basic definitions and elementary results of representations and characters (Sections 3.1 and 3.2). Also we study some results on constructing new characters from characters we already know. In Section 3.3 we show that the product of two characters of a group G is again a character of G. In Section 3.4 we show that if G has a normal subgroup N, then the irreducible characters of the quotient G/N extend (lift) irreducibly to G. In Section 3.5 we study the dual operations *induction* and *restriction* of characters. We conclude Chapter 3 by studying an important type of characters of a group G known as the permutation character, which is associated with the group action. For instance permutation characters can be used to determine the fusions of conjugacy classes of subgroups into the main

group. If we have a subgroup $H \leq G$, then there exists a permutation character of G. Conversely if we have a permutation character of G, then under some certain conditions, we show the existence of a subgroup $H \leq G$.

In Chapter 4 we introduce projective representations and characters, and record a number of definitions and basic properties. These representations and characters are of great importance in the sequel of this thesis. We also introduce the Schur multiplier of a finite group G, which plays an important role in the study of projective representations and characters of G. In Section 4.1, we define and give some basic results on the Schur multiplier of a group G. The original motivation that inspired Schur to study the multiplier of a group G was to classify projective representations of G. Nowadays the formulation of his definition is the second cohomology group $H^2(G, \mathbb{C}^*)$. An ordinary representation of G is a homomorphism into the general linear group $GL(n, \mathbb{C})$, likewise a projective representation is also a homomorphism but into the projective general linear group $PGL(n, \mathbb{C})$. Schur (1904, 1907), see Rotman [64] for example, showed that to every finite group G, one can attach at least one finite group $\mathfrak{C}(G)$, called a *Schur cover*, with the property that every projective representation of G can be "lifted" to an ordinary representation of $\mathfrak{C}(G)$. The Schur cover is also known as a *representation group*, *covering group* or *Darstellungsgruppe.* The Schur covers of all the finite simple groups are known, and each is an example of a quasisimple group. The Schur cover of a perfect group is uniquely determined up to isomorphism, but the Schur cover of a general finite group is only determined up to "isoclinism" (see Defintion 4.2.6). Section 4.2 is devoted to the theory of projective representations of G. There is a relationship between the projective representations of a group G and the ordinary representations of the central extension $\alpha{\cdot}G$, where α is factor set of the Shcur multiplier of G. We also show that how projective representations of G can be obtained from the ordinary representations of the representation group or covering group of G. In Section 4.3 we discuss the projective characters and study the orthogonality relations analogous to the ones for ordinary characters. For further readings on projective representations and characters, readers are referred to Berkovich and Zhmud [15], Haggarty [32], Hoffman and Humphreys [34], Humphreys [35], Huppert [37], Isaacs [38], Karpilovsky [42], [43], Morris [51], [52], [53], [54], Nagao and Tsushima [56], Read [59], [60] and [61].

In Chapter 5 we study the theory of Clifford-Fischer matrices, a technique to calculate the character tables of group extensions. This method, which is based on Clifford Theory (see Clifford [18]), was developed by B. Fischer ([25], [26] and [27]). The chapter is divided into three sections, where in

Section 5.1 we discuss Clifford Theory and some of its consequences. Let $\overline{G} = N{\cdot}G$ be as before. The group $\overline{G}$ has dual action on the conjugacy classes of N and on the set $\mathrm{Irr}(N)$. Brauer Theorem (see Theorem 5.1.1) asserts that the number of orbits on the two actions is the same, but the lengths of the orbits may be different. Indeed if N is non-abelian, then

$$\sum_{k=1}^{t} |{\theta_k}^{\overline{G}}| = |\mathrm{Irr}(N)| \neq |N| = \sum_{k=1}^{t} |[n_k]_N^{\overline{G}}|,$$

where

- t is the number of orbits on the action of $\overline{G}$ on the conjugacy classes of N or on $\mathrm{Irr}(N)$,
- θ_k and n_k are respective class representatives of characters and conjugacy classes of N,
- ${\theta_k}^{\overline{G}}$ and $[n_k]_N^{\overline{G}}$ are orbits of N containing θ_k and n_k, respectively, on the action of $\overline{G}$ on $\mathrm{Irr}(N)$ and on the conjugacy classes of N respectively.

For every representative character θ_k of an orbit ${\theta_k}^{\overline{G}}$, two cases are distinguished. Either all θ_k (for all $1 \leq k \leq t$) are extendable to ordinary characters of their respective inertia groups $\overline{H}_k,\ 1 \leq k \leq t$, or some are non-extendable. In the case of extendability of every character θ_k, the set of irreducible characters of $\overline{G}$ is given by the formula (5.2). If θ_k, for some $k \in \{1, 2, \cdots, t\}$, is not extendable to an ordinary character of $\overline{H}_k$, then it is extendable to a projective character $\widetilde{\psi}_k$ of $\overline{H}_k$ with some factor set $\overline{\alpha}_k^{-1}$ of the Schur multiplier of $\overline{H}_k$. Thus a more general formula for Eq. (5.2) is given by Eq. (5.3). At the end of Section 5.1, we quote some results on extendability of characters of N to their respective inertia groups. In Section 5.2 we fix some notations and many settings to define the Fischer matrices of $\overline{G}$. For each conjugacy class $[g_i]_G$, we define a unique (up to the permutations of rows and columns) non-singular square matrix $\mathcal{F}_i$. This matrix is the Fischer matrix correspond to class $[g_i]_G$. We attach many information to this matrix in order to calculate its entries. For example the top and the bottom of $\mathcal{F}_i$ are labeled by the sizes of the centralizers of $g_{ij},\ 1 \leq j \leq c(g_i)$ in $\overline{G}$ and m_{ij} respectively, where

- $g_{ij},\ 1 \leq j \leq c(g_i)$ are the conjugacy classes of $\overline{G}$, corresponding to the class $[g_i]_G$, obtained by the coset analysis technique,
- m_{ij}'s are weights that can be computed through the formula (5.4).

Also the rows of the Fischer matrix $\mathcal{F}_i$ are labeled according to the fusions of the inertia factor groups $H_k \cong \overline{H}_k/N,\ 1 \leq k \leq t$, into the group G. Fischer matrices satisfy some interesting

properties, which help in the computations of their entries. These properties follow from the usual orthogonality relations for character tables. We gather these properties in Proposition 5.2.2. In fact we have never tried (in this thesis) to calculate the Fischer matrices directly from their definition. Rather, we only used their arithmetical properties, given by Proposition 5.2.2, to calculate their entries. In Section 5.3 we describe how to obtain the character table of $\overline{G}$. To do this we need to have

- the conjugacy classes of $\overline{G}$,
- the appropriate projective character tables of all the inertia factors H_k, with factor set α_k^{-1},
- the fusions of the $\alpha_k^{-1}-$regular classes of H_k into classes of $\overline{G}/N \cong G$,
- the Fischer matrices of $\overline{G}$.

The character table of $\overline{G}$ constructed using Clifford-Fischer theory, is partitioned into blocks corresponding to the inertia factor groups and to the cosets $N\overline{g}_i$, where $\overline{g}_i$ is a pre-image of g_i in $\overline{G}$.

In Chapter 6 we study a split extension group of the form $\overline{G} = 3^7{:}Sp(6,2)$, where in Section 6.1 we give a brief introduction and we show that $\overline{G}$ is a subgroup of $SL(8,3)$. In Section 6.2 we list the conjugacy classes of $\overline{G}$, obtained by applying the coset analysis technique, and see that correspond to the 30 conjugacy classes of $Sp(6,2)$, we get 118 classes for $\overline{G}$. In Section 6.3 through a long process we show that there are six inertia factors, namely $H_1 = Sp(6,2)$, $H_2 = U_4(2)$, $H_3 = 2^5{:}A_6$, $H_4 = A_7$, $H_5 = (3 \times A_6){:}2$ and $H_6 = (2^4{:}A_5){:}2$. In Section 6.4 we discuss the character tables of these inertia factor groups and we supply the tables in the Appendix. In Section 6.5 we calculate the Fischer matrices of $\overline{G}$ and we see that their sizes range between 2 and 10. In Section 6.6 we show how to obtain the character table of $\overline{G} = 3^7{:}Sp(6,2)$ via the Clifford-Fischer theory and in the Appendix (Table 11.8) we list the full character table of $\overline{G}$. We remark that a paper [8] entitled with "*On a group of the form* $3^7{:}Sp(6,2)$", based on the contents of this chapter, has been submitted for publication in 2012.

Chapter 7 is composed of two main sections, where in each section, we discuss a maximal subgroup of the Thompson sporadic simple group Th. In Section 7.1, we construct the character table of the second largest maximal subgroup of Th, namely the *Dempwolff group*, which is a non-split extension of the elementary abelian group 2^5 by the general linear group $GL(5,2)$. In Subsection

7.1.1 we give a brief introduction and a presentation of $\overline{G} = 2^{5\cdot}GL(5,2)$ in terms of generators of Th. In Subsection 7.1.2 we list the conjugacy classes of $\overline{G}$ obtained by applying the coset analysis technique and see that corresponding to the 27 conjugacy classes of $GL(5,2)$, we get 41 classes for $\overline{G}$. In Subsection 7.1.3 we show that there are two inertia factors, namely $H_1 = GL(5,2)$ and $H_2 = 2^4{:}GL(4,2)$. We also show that we will use the projective characters of H_2 with factor set α such that $\alpha \sim [2]$. In Subsection 7.1.4 we compute the fusion of classes of H_2 into classes of $\overline{G}$ and in Subsection 7.1.5 we calculate the Fischer matrices of $\overline{G}$. In Subsection 7.1.6 we show how to calculate the character table of $\overline{G}$ and we list the full character table of $\overline{G}$ in Table 11.9, which appears in the Appendix. We remark that a paper [9] entitled with "*Fischer matrices of Dempwolff group* $2^{5\cdot}GL(5,2)$", based on the contents of this chapter, has been submitted for publication in 2012.

In Section 7.2, we construct the character table of the third largest maximal subgroup of Th, which is a non-split extension of the extraspecial 2−group 2_+^{1+8} by the alternating group A_9. Subsection 7.2.1 is a brief introduction to the group $\overline{G} = 2_+^{1+8\cdot}A_9$. In Subsection 7.2.2 we compute the conjugacy classes of $\overline{G}$ using the coset analysis technique. In Subsection 7.2.3 we study the inertia factor groups of $\overline{G}$. One of the inertia factor groups is of the form $PSL(2,8)$:3 and we study its character table via Clifford-Fischer theory in Subsection 7.2.4. In Subsection 7.2.5 we supply the fusions of classes of the inertia factors into classes of A_9 and in Subsection 7.2.6 we calculate the Fischer matrices, which have sizes range between 1 and 5. The theory of projective characters is involved in the construction of the character table of $\overline{G}$. We remark that a paper [10] entitled with "*Fischer matrices of the group* $2_+^{1+8\cdot}A_9$", based on the contents of this chapter, has been submitted for publication in 2012.

In Chapter 8, we construct the character table of the non-split extension $\overline{G} = 2^{6\cdot}Sp(6,2)$ by means of Clifford-Fischer theory together with the coset analysis technique. In Section 8.1 we give a brief information on the existence of $\overline{G}$ and we also supply generators of $\overline{G}$ as well as generators for the kernel $N = 2^6$ in terms of permutations of the alternating group A_{128}. Section 8.2 is devoted to the construction of the conjugacy classes of $\overline{G}$ using the coset analysis technique. In Section 8.3 we determine the inertia factors and their fusions (into $Sp(6,2)$) and we supply the character tables of these inertia factors in the Appendix. In Section 8.4 we calculate the Fischer matrices of $\overline{G}$ and we see that these matrices are integral valued matrices with sizes range between 1 and 4. Finally in Section 8.5 we show how to construct the character table of $\overline{G}$ and in the Appendix we supply the

character table of $\overline{G}$, which will appear as Table 11.12. We remark that a paper [11] entitled with "*The character table of the non-split extension* $2^{6\cdot}Sp(6,2)$", based on the contents of this chapter, has been submitted for publication in 2012.

In Chapter 9 we first discuss some general results on the non-split extension $\overline{G}_n = 2^{2n\cdot}Sp(2n,2),\ n \geq 2$ and then we will focus mainly on the group $\overline{G} = 2^{8\cdot}Sp(8,2)$. Section 9.1 is an introductory section to the chapter. In Section 9.2 we supply a sequence of GAP commands that generate the group $2^{2n\cdot}Sp(2n,2)$ followed by some general results on the inertia factors of this group. In Section 9.3 we supply the generators of $\overline{G} = 2^{8\cdot}Sp(8,2)$ as well as generators of the normal subgroup $N = 2^8$ in terms of permutations of a set of cardinality 512 (more precisely, these generators are elements of the alternating group A_{512}). In Section 9.4 we construct the conjugacy classes of $\overline{G}_4$ using the coset analysis technique. In Section 9.5 we determine the inertia factor groups of $\overline{G}_4$ and we supply the number of irreducible characters for the cases $\overline{G}_n,\ n \in \{2,3,4,5,6\}$ (Table 9.1) as well as a general formula for this number for $\overline{G}_n$ (Eq. (9.1)). Also we supply the fusion of classes of the inertia factor group H_2 into classes of $Sp(8,2)$ (Table 9.3). In Section 9.6 we list the 81 Fischer matrices of $\overline{G}_4$. These matrices are integral valued with sizes range between 1 and 5. In Section 9.7 we describe how to obtain the full character table of $2^{8\cdot}Sp(8,2)$, which appears in the Appendix as Table 11.13. We remark that a paper [12] entitled with "*On the non-split extension* $2^{2n\cdot}Sp(2n,2)$ *and the character table of* $2^{8\cdot}Sp(8,2)$", based on the contents of this chapter, has been prepared and will be submitted for publication in 2012.

In Chapter 10 we are interested to calculate the ordinary character tables of two split extensions of the forms $2^{10}{:}(U_5(2){:}2)$ and $2^{1+6}_{-}{:}((3^{1+2}{:}8){:}2)$ by means of Clifford-Fischer theory. Section 10.1 is an introduction to the chapter where generators, in terms of 11×11 matrices over $\mathbb{F}_2$ for the groups $\overline{G} = 2^{10}{:}(U_5(2){:}2),\ N = 2^{10}$ and $G = U_5(2){:}2$ are supplied. In fact $\overline{G} \leq PSL(11,2)$. In Section 10.2 we supply the conjugacy classes of $\overline{G}$ using the coset analysis technique. In Section 10.3 we determine the inertia factor groups together with the fusions of classes of these inertia factors into classes of $G = U_5(2){:}2$. We also discuss the character tables of these groups in this section and we supply them in the Appendix. A considerable part of Section 10.3 is devoted to the construction of the character table of H_2 through Clifford-Fischer theory, where we will see that the Fischer matrices of this group satisfy some interesting properties (Lemmas 10.3.3 and 10.3.4). In Section 10.4 we list the Fischer matrices of $\overline{G}$. These matrices are integral valued with sizes range between 1 and 5. In Section 10.5 we show how to obtain the character table of $2^{10}{:}(U_5(2){:}2)$ and

its full character table is listed in Table 11.17. We remark that two papers [13] and [14] entitled respectively with "*On a group of the form* 2^{10}:$(U_5(2)$:$2)$" and "*Clifford-Fischer theory applied to a group of the form* 2^{1+6}_{-}:$((3^{1+2}$:$8)$:$2)$", based on the contents of this chapter, have been submitted for publication in 2012.

We would like to mention that the character tables of several additional groups are also supplied in the Appendix. These groups were encountered through the various chapters of the thesis.

Finally, we should add that 72 relevant references are listed under the Bibliography.

CHAPTER 2

Group Extensions

In the present chapter, we go over some results on group extensions that will be used in the sequel of this thesis. In Section 2.1 we give some definitions and basic results on theory of group extensions. Section 2.2 deals with results on a special type of extensions, namely the *semi-direct product* or *split extension*, where groups of this type satisfy further interesting properties. In Section 2.3 we discuss the conjugacy classes of elements of group extensions. We describe the technique of the *coset analysis* for computing the conjugacy classes of a group extension $\overline{G}$ of N by G, where N is a normal subgroup of $\overline{G}$. This technique was developed and used firstly by Moori in [47] and also in [48] and has since been widely used for computing the conjugacy classes of several group extensions. For example, it has been used in Ali [1], [2], [3], Barraclough [6], Moori and Basheer [8], [9], [10], [11], [12], [13], [14], Mpono [50], Rodrigues [62], Salleh [66] and Whitely [70].

In Sections 2.1 and 2.2 we mainly follow Mpono [55], Robinson [63] and Whitely [70].

2.1. Introduction

An ***extension*** $\overline{G}$ of a group N by a group G is a short exact sequence of groups and homomorphisms

$$1 \longrightarrow N \xrightarrow{\iota} \overline{G} \xrightarrow{\pi} G \longrightarrow 1\ , \tag{2.1}$$

where ι and π are the inclusion monomorphism and projection epimorphism respectively. Equivalently if N and G are arbitrary groups, then an extension of N by G is a group $\overline{G}$ possessing a normal subgroup N such that $\overline{G}/N \cong G$. An extension of N by G will be denoted by $N{\cdot}G$.

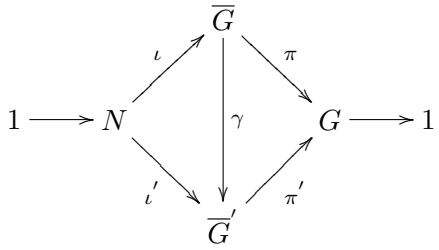
Example 2.1.1. An immediate trivial example of group extensions is the direct product of any

two arbitrary finite groups K and H.

Remark 2.1.1. Note from Eq. (2.1) that $\mathrm{Im}(\iota) = \ker(\pi) = N$. The group N is called the ***kernel*** of the extension and G is a ***complement*** of N with respect to the extension $\overline{G}$.

Remark 2.1.2. Complement of a normal subgroup N of $\overline{G}$ does not necessarily exist as a subgroup of $\overline{G}$ and even if exists, it need not to be unique. For example any group of order 2 serves as a complement of A_3 in S_3. However, if complements exist, they are unique up to isomorphism since they are all isomorphic to $\overline{G}/N$.

Definition 2.1.1. *Two extensions $\overline{G}$ and $\overline{G}'$ of N by G are said to be* **equivalent** if there is a homomorphism $\gamma : \overline{G} \longrightarrow \overline{G}'$ renders the commutativity of the diagram:



i.e., $\iota' = \gamma \circ \iota$ and $\pi = \pi' \circ \gamma$.

Remark 2.1.3. Such a homomorphism γ is necessarily an isomorphism by the ***five lemma***.

2.2. Semidirect Products and Split Extensions

Definition 2.2.1. *A group $\overline{G}$ is called a* **semi-direct product** of N by G, denoted by $N{:}G$, if N and G are subgroups of $\overline{G}$ such that

- $N \lhd \overline{G}$,
- $NG = \overline{G}$,
- $N \bigcap G = \{1_{\overline{G}}\}$.

The above definition asserts that every element $\overline{g} \in \overline{G}$ can be written uniquely as a product of an element of N together with an element of G. For the uniqueness, suppose that there exist $n_1, n_2 \in N$ and $g_1, g_2 \in G$ such that $n_1 g_1 = \overline{g}$ and also $n_2 g_2 = \overline{g}$. Then

$$n_1 g_1 = n_2 g_2 \Leftrightarrow n_2^{-1} n_1 = g_2 g_1^{-1} \Leftrightarrow n_2^{-1} n_1 = 1_{\overline{G}} \text{ and } g_2 g_1^{-1} = 1_{\overline{G}} \Leftrightarrow n_1 = n_2 \text{ and } g_1 = g_2.$$

Unlike the case of the direct product of groups, a semi-direct product of two groups N and G need not to be unique. That is if $\overline{G}_1$ and $\overline{G}_2$ are two groups both having isomorphic copies of N as a normal subgroup and G as a quotient group, and both $\overline{G}_1$ and $\overline{G}_2$ are semi-direct products of N by G, then it is not necessarily that $\overline{G}_1$ and $\overline{G}_2$ are isomorphic. For example there are two groups of order 16 that they are split extensions of $\mathbb{Z}_8$ and $\mathbb{Z}_2$. One of these two is D_{16} and the other one is a non abelian group of order 16, see GAP.

Now let $\mathrm{Aut}(N)$ denote the group of all automorphisms of N. The map $\psi : G \longrightarrow \mathrm{Aut}(N)$ defined by $\psi(g) = \psi_g$, where $\psi_g(n) = gng^{-1},\ \forall g \in G,\ n \in N$, is a group homomorphism. In fact for arbitrary finite groups N and G and for every homomorphism $\psi : G \longrightarrow \mathrm{Aut}(N)$ defined as above, a unique (up to isomorphism) semi-direct product, denoted by $N{:}_{\psi}G$, is determined. This semi-direct product is referred to the ***semi-direct product of*** N ***by*** G, ***realized by*** ψ.

Proposition 2.2.1. *A semi-direct product $N{:}G$ for arbitrary finite groups N and G exists if and only if there exists a homomorphism $\psi : G \longrightarrow \mathrm{Aut}(N)$.*

PROOF. Firstly assume that for arbitrary finite groups N and G, there exists a homomorphism $\psi : G \longrightarrow \mathrm{Aut}(N)$. We claim that there exists a semi-direct product $\overline{G}$ of N by G realized by ψ. Let K be the set consisting of all pairs $(n, g),\ n \in N,\ g \in G$. The operation $\star : K \longrightarrow K$ defined by $(n_1, g_1) \star (n_2, g_2) = (n_1\psi_{g_1}(n_2), g_1g_2),\ \forall n_1, n_2 \in N,\ g_1, g_2 \in G$, is clearly a binary operation on K. Moreover, $(K, \star)$ forms a group with $(1_N, 1_G)$ as the identity and $(\psi_{g^{-1}}(n^{-1}), g^{-1})$ as the inverse of (n, g) in K. It is not difficult to see that the set $\{(n, 1_G)|\ n \in N\}$ form a normal subgroup of K that is isomorphic to N, while the set $\{(1_N, g)|\ g \in G\}$ form a subgroup of K that is isomorphic to G. Clearly $\{(n, 1_G)|\ n \in N\} \bigcap \{(1_N, g)|\ g \in G\} = \{1_K\} = (1_N, 1_G)$. Hence K is a semi-direct product of N by G realized by ψ. We rename K to $\overline{G}$, that is $\overline{G} = N{:}_{\psi}G$.
Conversely, let $\overline{G} = N{:}G$ be a given semi-direct product of N by G. We know that for all $\overline{g} \in \overline{G}$, there exist two elements $n \in N$ and $g \in G$ such that $\overline{g} = ng$ uniquely. Let $\psi : G \longrightarrow \mathrm{Aut}(N)$ be the homomorphism given by $\psi(g) = \psi_g$, where $\psi_g(n) = gng^{-1},\ \forall g \in G,\ n \in N$. Thus $\overline{G} \cong N{:}_{\psi}G$ and this isomorphism is given by $ng \mapsto (n, g)$. ■

Remark 2.2.1. Proposition 2.2.1 shows that a semi-direct product is completely determined by a homomorphism $\psi : G \longrightarrow \mathrm{Aut}(N)$, that is to say, it is described by the way G acts on N.

In most of the cases, when we are dealing with a specific semi-direct product $N{:}_{\psi}G$ (we know how G acts on N), we always drop the homomorphism ψ from the context of the discussion and we only

use the notation $N{:}G$ instead of $N{:}_{\psi}G$ to denote the semi-direct product of N by G realized by ψ.

Now let $\overline{G}$ be any extension of N by G. Since $\overline{G}/N \cong G$, there exists an epimorphism $\pi : \overline{G} \longrightarrow G$ with $\ker(\pi) = N$. For each $g \in G$, we define a ***lifting*** of g to be an element $\overline{g} \in \overline{G}$ such that $\pi(\overline{g}) = g$. By choosing a lift to each $g \in G$, we get the set $\{\overline{g}|\ g \in G\}$, which is a transversal for N in $\overline{G}$ and consequently we also get a function $\lambda : G \longrightarrow \overline{G}$, which we refer to as a ***transversal function***. Thus if $\overline{g} \in \overline{G}$, the coset representative of $N\overline{g}$ is $\lambda(\pi(\overline{g})) = \lambda \circ \pi(\overline{g})$. In general, λ is not a homomorphism, but always satisfies the relation:

$$\lambda \circ \pi = 1. \tag{2.2}$$

Conversely any function λ satisfies Eq. (2.2) determines a transversal to N in $\overline{G}$, namely the set $\{\lambda(g)|\ g \in G\}$.

Definition 2.2.2. *An extension* $1 \longrightarrow N \xrightarrow{\iota} \overline{G} \xrightarrow{\pi} G \longrightarrow 1$ *is said to be* **split** *if there exists a transversal function* $\lambda : G \longrightarrow \overline{G}$ *which is a homomorphism. An extension* $\overline{G}$ *of* N *by* G *is said to be* **non-split** *if it is not a split extension.*

Theorem 2.2.2 (Robinson [63])**.** *The split extension* $1 \longrightarrow N \xrightarrow{\iota} \overline{G} \xrightarrow{\pi} G \longrightarrow 1$ *and the semi-direct product* $N{:}G$ *the are identical.*

PROOF. It is easy to see that every semi-direct product $N{:}G$ is a split extension as the function $\lambda : G \longrightarrow N{:}G$, given by $\lambda(g) = (1_N, g)$, is a transversal function, which is a homomorphism. This shows that any semi-direct product is a split extension. To establish the converse, suppose that the extension $1 \longrightarrow N \xrightarrow{\iota} \overline{G} \xrightarrow{\pi} G \longrightarrow 1$ splits via a homomorphism $\lambda : G \longrightarrow \overline{G}$. Then λ is a monomorphism. Now set

$$M := \lambda \circ \pi(\overline{G}) := \lambda(G) := \{\lambda(g)|\ g \in G\}.$$

Since $\lambda \circ \pi = 1$, we have $\pi(\lambda \circ \pi(\overline{g}^{-1})\overline{g}) = \pi(\overline{g}^{-1}\overline{g}) = \pi(\overline{g}^{-1})\pi(\overline{g}) = 1$, so that $\lambda \circ \pi(\overline{g}^{-1})\overline{g} \in \ker(\pi) = \mathrm{Im}(\iota) = N$ and $\overline{G} = NM$. Also since $\lambda \circ \pi(\overline{g}) \in M$ implies that $1 = \pi(\lambda \circ \pi(\overline{g})) = \pi(\overline{g})$, we obtain that $N \bigcap M = \{1\}$. Hence $\overline{G} = N{:}M \cong N{:}G$. This shows that every split extension is a semidirect product extension. ■

Note 2.2.1. For alternative proof of Theorem 2.2.2, see Theorem 2.2.10 of Mpono [55].

Theorem 2.2.3 (Schur-Zassenhaus)**.** *The extension* 2.1 *splits if the orders of* N *and* G *are finite and coprime.*

PROOF. See Gorenstein [31] or Rotman [64]. ■

We conclude this section by showing that even for a non-split extension of N by G, if N is abelian, then G acts on N.

Proposition 2.2.4. *Let $\overline{G}$ be an extension of N by G, with N abelian. Then there is a homomorphism $\psi : G \longrightarrow \mathrm{Aut}(N)$ such that $\psi_g(n) = \overline{g}n\overline{g}^{-1},\ n \in N$, and ψ is independent of the choice of the liftings $\{\overline{g}|\ g \in G\}$.*

PROOF. See Rotman [64] or Whitley [70]. ■

2.3. The Conjugacy Classes of Group Extensions

Since the number of the ordinary irreducible characters of any finite group G is same as the number of the conjugacy classes of G, having knowledge about the conjugacy classes of G is of no doubt of great importance to start constructing the character table of G. The conjugacy classes can be computed computationally or through various algorithms depending on the structure of the group. Moori in his PhD Thesis [47] and also in [48] developed a very interesting technique to calculate the conjugacy classes of any group extension $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$. This technique is known nowadays as the *coset analysis*. This method derives its strength as it can be used for both spilt and non-split group extensions and also whether the kernel of the extension is abelian or not. The coset analysis method has been used to determine the conjugacy classes of some group extensions by various authors such as Barraclough [6] and in particular by several MSc and PhD students, such as Ali [1], [2], [3], Mpono [50], Rodrigues [62], Whitely [70] and in many submitted and under preparation papers ([8], [9], [10], [11], [12], [13] and [14]) by the author and the supervisor of this thesis. In this section we give a shortened description on how the coset analysis can be used to determine the conjugacy classes of group extensions.

For each $g \in G$, let $\overline{g} \in \overline{G}$ map to g under the natural epimorphism $\pi : \overline{G} \longrightarrow G$ and let $g_1 = N\overline{g}_1, g_2 = N\overline{g}_2, \cdots, g_r = N\overline{g}_r$ be representatives for the conjugacy classes of $G \cong \overline{G}/N$. Therefore $\overline{g}_i \in \overline{G},\ \forall i$, and by convention we take $\overline{g}_1 = 1_{\overline{G}}$. For each conjugacy class $[g_i]_G,\ 1 \leq i \leq r$, we analyze one coset $N\overline{g}_i$ and the coset analysis constructs, corresponding to this class, a number of conjugacy classes of $\overline{G}$. That is each conjugacy class of $\overline{G}$ corresponds uniquely to a conjugacy class of G. This method can be described briefly in the following steps:

- For fixed $i \in \{1, 2, \cdots, r\}$, act N (by conjugation) on the coset $N\overline{g}_i$ and let the resulting orbits be $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$. If N is abelian (regardless to whether the extension is split or not), then $|Q_{i1}| = |Q_{i2}| = \cdots = |Q_{ik_i}| = \frac{|N|}{k_i}$.
- Act $\overline{G}$ on $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ and suppose f_{ij} orbits fuse together to form a new orbit Δ_{ij} and let the total number of the new resulting orbits in this action be $c(g_i)$ (that is $1 \leq j \leq c(g_i)$). Then $\overline{G}$ has a conjugacy class $[g_{ij}]_{\overline{G}}$ that contains Δ_{ij} and $|[g_{ij}]_{\overline{G}}| = |[g_i]_G| \times |\Delta_{ij}|$.
- Repeat the above two steps, for all $i \in \{1, 2, \cdots, r\}$.

Lemma 2.3.1. *For each $i \in \{1, 2 \cdots, r\}$, write $g_i = N\overline{g}_i = \bigcup_{j=1}^{c(g_i)} \left(N\overline{g}_i \bigcap [g_{ij}]_{\overline{G}}\right) = \bigcup_{j=1}^{c(g_i)} \Delta_{ij}$. Then $\{\overline{g}_{i1}, \overline{g}_{i2}, \cdots, \overline{g}_{ic(g_i)}\}$ is a complete set of representatives for the conjugacy classes of $\overline{G}$ that correspond (under the natural epimorphism) to $[g_i]_G$.*

PROOF. One can refer to Barraclough [6] with slight difference in notations. ■

Thus each $[g_i]_G$ affords $c(g_i)$ conjugacy classes in $\overline{G}$.

Remark 2.3.1. For fixed $i \in \{1, 2, \cdots, r\}$, the conjugacy class $[g_{ij}]_{\overline{G}}$ is partitioned into $|[g_i]_G|$ equal size subsets $\Delta_{ij1}, \Delta_{ij2}, \cdots, \Delta_{ij|[g_i]_G|}$, where $|\Delta_{iju}| = |\Delta_{ij}|$, for each $1 \leq u \leq |[g_i]_G|$ (we can take $\Delta_{ij1} = \Delta_{ij}$). Moreover, for fixed i and $s \in \{1, 2, \cdots, |[g_i]_G|\}$, the relation $\sum_{j=1}^{c(g_i)} |\Delta_{ijs}| = |N|$ holds. If the extension splits, then Δ_{i1s} is the intersection of $[g_{ij}]_{\overline{G}}$ with an element of $[g_i]_G$, for all $1 \leq s \leq |[g_i]_G|$.

Therefore information about every conjugacy class of $\overline{G}$ can be obtained by examining one coset $N\overline{g}_i = g_i \in G$ for each conjugacy class of G. The following two propositions relate the orders of the elements of $\overline{G}$ with those of G.

Proposition 2.3.2. *Let $\overline{G} = N{:}G$, where N is an abelian group. Also let $\overline{g} = ng \in \overline{G}$, for some $n \in N$ and $g \in G$. Then $o(g)|o(\overline{g})$.*

PROOF. Let $o(\overline{g})$ and $o(g)$ be k and m respectively. We have $1_{\overline{G}} = \overline{g}^k = (ng)^k = nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}} g^k$. Since G acts on N, we have $n, n^g, n^{g^2}, n^{g^3}, \cdots n^{g^{k-1}} \in N$ and therefore $nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}} \in N$. Now since $N \bigcap G = \{1_{\overline{G}}\}$ and $nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}} g^k = 1_{\overline{G}}$, we must have $nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}}$ and g^k equal to 1_N and 1_G respectively. Hence $m|k$. ■

Proposition 2.3.3. *With the settings of* Proposition 2.3.2 *and its proof, assume further that N is an elementary abelian $p-$group. Then $k \in \{m, pm\}$.*

PROOF. See Mpono [55] ■

Further results on the conjugacy classes of $\overline{G} = N{\cdot}G$, when N is abelian or the extension splits, can be found in many sources such as Ali [1], Barraclough [6], Moori [47], [48], Mpono [55], Rodrigues [62] or Whitely [70]. In particular Ali [1] and Mpono [55] have written some interesting computational programmes for the coset analysis method for CAYLEY, GAP and Magma and these programmes are available in their theses.

Chapter 3

Elementary Theories of Representations and Characters

From now on, G means a finite group unless otherwise stated.

The theories of representations and characters of finite groups were developed by the end of the 19th century. Frobenius, Burnside, Schur and Brauer have contributed largely to these theories.

"The year 1897 was marked by two important mathematical events: the publication of the first paper on representations of finite groups by Ferdinand Georg Frobenius (1849-1917) and the appearance of the first treatise in English on the theory of finite groups by William Burnside (1852-1927). Burnside soon developed his own approach to representations of finite groups. In the next few years, working independently, Frobenius and Burnside explored the new subject and its applications to finite group theory. They were soon joined in this enterprise by Issai Schur (1875-1941) and some years later, by Richard Brauer (1901-1977). These mathematicians' pioneering research is the subject of this book. $\cdots$" Curtis [20].

The material that will be covered in this chapter is to illustrate the basics and fundamentals of representations and characters of finite groups. As general references, this can be found in Curtis and Reiner [19], Isaacs [38], James [39], Moori [49] and Sagan [65].

In this chapter, we follow precisely the description given at the MSc dissertation [7] of the author of this thesis.

3.1. Preliminaries

There are two kinds of representations, namely ***permutation*** and ***matrix*** representations. An example of a permutation representation is given by the known Theorem of **Cayley**, which asserts that any group G (not necessarily finite) can be embedded into the Symmetric group S_G. The matrix representation of a finite group is of particular interest.

Definition 3.1.1. *Any homomorphism $\rho : G \longrightarrow GL(n, \mathbb{F})$, where $GL(n, \mathbb{F})$ is the group consisting of all $n \times n$ non-singular matrices is called a* **matrix representation** *or simply a* **representation** *of G. If $\mathbb{F} = \mathbb{C}$, then ρ is called an* **ordinary** *representation. The integer n is called the degree of ρ. Two representations ρ and σ are said to be* **equivalent** *if there exists $P \in GL(n, \mathbb{F})$ such that $\sigma(g) = P\rho(g)P^{-1},\ \forall g \in G$.*

From now on, we restrict ourselves to ordinary representations only, unless an explicit exception is made.

Definition 3.1.2. *If $\rho : G \longrightarrow GL(n, \mathbb{C})$ is a representation. Then ρ affords a complex valued function $\chi_\rho : G \longrightarrow \mathbb{C}$ defined by $\chi_\rho(g) = trace(\rho(g)), \forall g \in G$. The function χ_ρ is called a* **character** *afforded by the representation ρ of G or simply a character of G. The integer n is called the* **degree** *of χ_ρ. If $n = 1$, then χ_ρ is said to be* **linear**.

A function $\phi : G \longrightarrow \mathbb{C}$ which is invariant over every conjugacy class of G, that is $\phi(ghg^{-1}) = \phi(h),\ \forall g, h \in G$, is called a *class function* of G.

Proposition 3.1.1. *Any character of G is a class function.*

PROOF. Immediate since similar matrices have same trace. ■

Now over the set of class functions of a group G we define addition and multiplication of two class functions ψ_1 and ψ_2 by

$$\begin{aligned} (\psi_1 + \psi_2)(g) &= \psi_1(g) + \psi_2(g), \quad \forall g \in G, \\ \psi_1\psi_2(g) &= \psi_1(g)\psi_2(g), \quad \forall g \in G. \end{aligned}$$

It is clear that $\psi_1 + \psi_2$ and $\psi_1\psi_2$ are class functions of G. Also if $\lambda \in \mathbb{C}$, then $\lambda\psi$ is a class function of G whenever ψ is. Therefore the set of all class functions of a group G forms an ***algebra***, which

we denote by $\mathcal{C}(G)$. The set of all characters of G forms a ***subalgebra*** of $\mathcal{C}(G)$. However, it may not be clear that the product of two characters is again a character. This fact will be shown in Section 3.3. Now we prove that the sum of two characters is again a character.

Proposition 3.1.2. *If χ_ψ and χ_ϕ are two characters of G, then so is $\chi_\psi + \chi_\phi$.*

PROOF. Let ψ and ϕ be representations of G affording the characters χ_ψ and χ_ϕ respectively. Define the function ξ on G by $\xi(g) = \begin{pmatrix} \psi(g) & 0 \\ 0 & \phi(g) \end{pmatrix} = \psi(g) \oplus \phi(g)$. It is obvious that ξ is a homomorphism (representation) of G with $\chi_\xi = \chi_\psi + \chi_\phi$. ■

The above proposition motivates the following definition.

Definition 3.1.3. *A representation ρ of G is said to be* **irreducible** *if it is not a direct sum of other representations of G. Also a character χ of G is said to be* **irreducible** *if it is not a sum of other characters of G.*

Example 3.1.1. For any G, consider the function $\rho : G \longrightarrow GL(1, \mathbb{C})$ given by $\rho(g) = 1,\ \forall g \in G$. It is clear that ρ is a representation of G and $\chi_\rho(g) = 1,\ \forall g \in G$. Obviously ρ is irreducible. This character is called the *trivial character* and sometimes we may denote it by **1**.

The Theorem of Maschke and Schur's Lemma (see Theorem 5.1.6 and Corollary 5.1.9 of Moori [49]) are two pillars on which the edifice of representation theory rests. Maschke Theorem ensures that under certain conditions, any representation splits up into irreducible pieces. Schur's Lemma leads to the orthogonality of representations and hence characters. We mention the statement of Maschke Theorem only.

Theorem 3.1.3 (Maschke Theorem). *Let $\rho : G \longrightarrow GL(n, \mathbb{F})$ be a representation of G. If the characteristic of $\mathbb{F}$ is zero or does not divide $|G|$, then $\rho = \bigoplus_{i=1}^{r} \rho_i$, where ρ_i are irreducible representations of G.*

Over $\mathcal{C}(G)$ one can define an inner product $\langle , \rangle : \mathcal{C}(G) \times \mathcal{C}(G) \longrightarrow \mathbb{C}$ by

$$\langle \psi, \phi \rangle = \frac{1}{|G|} \sum_{g \in G} \psi(g)\overline{\phi(g)},$$

where $\overline{z}$ stands for the complex conjugate of z.

Among the important properties of characters of a group we can mention:

Proposition 3.1.4. 1. *Let χ_ρ be a character afforded by an irreducible representation ρ of G. Then $\langle \chi_\rho, \chi_\rho \rangle = 1$.*

2. *If χ_ρ and $\chi_{\rho'}$ are the irreducible characters of two non equivalent representations of G, then $\left\langle \chi_\rho, \chi_{\rho'} \right\rangle = 0$.*

3. *If $\rho \cong \bigoplus_{i=1}^{k} d_i \rho_i$, then $\chi_\rho = \sum_{i=1}^{k} d_i \chi_{\rho_i}$.*

4. *If $\rho \cong \bigoplus_{i=1}^{k} d_i \rho_i$, then $d_i = \langle \chi_\rho, \chi_{\rho_i} \rangle$.*

5. *ρ is irreducible if and only if $\langle \chi_\rho, \chi_\rho \rangle = 1$.*

PROOF. See Baker [5], James [40], Joshi [41] or Moori [49]. ■

We shall use the notation $\mathrm{Irr}(G)$ to denote the set of all ordinary irreducible characters of G.

Corollary 3.1.5. *The set $\mathrm{Irr}(G)$ forms an orthonormal basis for $\mathcal{C}(G)$ over $\mathbb{C}$.*

PROOF. Omitted. See James [40]. ■

Note 3.1.1. Observe that Corollary 3.1.5 asserts that if ψ is a class function of G, then $\psi = \sum_{i=1}^{k} \lambda_i \chi_i$, where $\lambda_i \in \mathbb{C}$ and $\mathrm{Irr}(G) = \{\chi_1, \chi_2, \cdots, \chi_k\}$. If $\lambda_i \in \mathbb{Z},\ \forall i$, then ψ is called a *generalized character*. Moreover, if $\lambda_i \in \mathbb{N} \cup \{0\}$, then ψ is a character of G.

The following theorem counts the number of irreducible characters of G.

Theorem 3.1.6. *The number of irreducible characters of G is equal to the number of conjugacy classes of G.*

PROOF. See Feit [24], James [40] or Moori [49]. ■

3.2. Character Tables and Orthogonality Relations

Definition 3.2.1 (**Character Table**). *The* **character table** *of a group G is a square matrix, its columns correspond to the conjugacy classes, while its rows correspond to the irreducible characters.*

The character table of G is very powerful tool to prove results about representations of G and G itself. For example, the character table of G enables us to

- decide the simplicity of G,
- determine all the normal subgroups and hence can help to decide solvability of the group (in particular we are able to find the center and commutator subgroup of G),
- determine the sizes of conjugacy classes of G,
- determine the degrees of all representations of G.

Corollary 3.2.1. *The character table of G is an invertible matrix.*

PROOF. Direct result from the fact that the irreducible characters, and hence the rows of the character table are linearly independent. ■

Proposition 3.2.2. *The following properties hold.*

1. $\chi(1_G)||G|,\ \forall \chi \in \mathrm{Irr}(G)$.
2. $\sum_{i=1}^{|\mathrm{Irr}(G)|} (\chi_i(1_G))^2 = |G|$.
3. *If* $\chi \in \mathrm{Irr}(G)$, *then* $\overline{\chi} \in \mathrm{Irr}(G)$, *where* $\overline{\chi}(g) = \overline{\chi(g)},\ \forall g \in G$.
4. $\chi(g^{-1}) = \overline{\chi(g)},\ \forall g \in G$. *In particular if* $g^{-1} \in [g]$, *then* $\chi(g) \in \mathbb{R},\ \forall \chi$.

PROOF. See James [40] or Moori [49]. ■

In addition to the properties mentioned in Proposition 3.2.2, the character table satisfies certain orthogonality relations mentioned in the next Theorem.

Theorem 3.2.3. *Let* $\mathrm{Irr}(G) = \{\chi_1, \chi_2, \cdots, \chi_k\}$ *and* $\{g_1, g_2, \cdots, g_k\}$ *be a collection of representatives for the conjugacy classes of G. For each* $1 \leq i \leq k$ *let* $C_G(g_i)$ *be the centralizer of* g_i. *Then we have the following relations:*

1. *The row orthogonality relation:*
   For each $1 \leq i, j \leq k$,
$$\sum_{s=1}^{k} \frac{\chi_i(g_s)\overline{\chi_j(g_s)}}{|C_G(g_s)|} = \langle \chi_i, \chi_j \rangle = \delta_{ij}.$$

2. *The column orthogonality relation:*

   For each $1 \leq i, j \leq k$,

$$\sum_{s=1}^{k} \frac{\chi_s(g_i)\overline{\chi_s(g_j)}}{|C_G(g_i)|} = \delta_{ij}.$$

PROOF.

1. Using Proposition 3.1.4(2) we have

$$\delta_{ij} = \langle \chi_i, \chi_j \rangle = \frac{1}{|G|} \sum_{g \in G} \chi_i(g)\overline{\chi_j(g)} = \frac{1}{|G|} \sum_{s=1}^{k} \frac{|G|}{|C_G(g_s)|} \chi_i(g_s)\overline{\chi_j(g_s)} = \sum_{s=1}^{k} \frac{\chi_i(g_s)\overline{\chi_j(g_s)}}{|C_G(g_s)|}.$$

2. For $1 \leq t \leq k$, define $\psi_t : G \to \mathbb{C}$ by $\psi_t(g) = \begin{cases} 1 & \text{if } g \in [g_t], \\ 0 & \text{otherwise.} \end{cases}$

   It is clear that ψ_t is a class function on G. Since $\mathrm{Irr}(G)$ form an orthonormal basis for $\mathcal{C}(G)$, then $\exists\ \lambda_s \in \mathbb{C}$ such that $\psi_t = \sum_{s=1}^{k} \lambda_s \chi_s$. Now for $1 \leq j \leq k$ we have

$$\lambda_j = \langle \psi_t, \chi_j \rangle = \frac{1}{|G|} \sum_{g \in G} \psi_t(g)\overline{\chi_j(g)} = \sum_{s=1}^{k} \frac{\psi_t(g_s)\overline{\chi_j(g_s)}}{|C_G(g_s)|} = \frac{\overline{\chi_j(g_t)}}{|C_G(g_t)|}.$$

   Hence $\psi_t = \sum_{j=1}^{k} \frac{\overline{\chi_j(g_t)}}{|C_G(g_t)|} \chi_j$. Thus we have the required formula :

$$\delta_{ts} = \psi_t(g_s) = \sum_{j=1}^{k} \frac{\chi_j(g_s)\overline{\chi_j(g_t)}}{|C_G(g_s)|}.$$

This completes the proof. ■

We conclude this section by giving the character table of the cyclic group $\mathbb{F}_q^*$.

Theorem 3.2.4. *The group* $\mathbb{F}_q^* = \langle \theta \rangle$ *has* $q-1$ *irreducible characters* $\chi_k,\ 0 \leq k \leq q-2$ *given at* θ^j, *by* $\chi_k(\theta^j) = e^{\frac{2\pi jk}{q-1} i}$.

PROOF. If $\rho(\theta) = (c)_{1\times 1} = c \in \mathbb{C}$ is a $1-$dimensional matrix representation, then the values of the representation ρ over all elements of $\mathbb{F}_q^*$ are determined by $\rho(\theta^j) = c^j$. By the definition of representation, we have

$$c^{q-1} = \rho(\theta^{q-1}) = \rho(1_{\mathbb{F}_q^*}) = 1.$$

It follows that c must be a $(q-1)th$ root of unity. Therefore each root of unity gives an irreducible representation and the result follows since $\chi_\rho = \rho$. ■

3.3. Tensor Product of Characters

In this section we follow precisely the description of Moori [49]. Given two matrices $P = (p_{ij})_{m\times m}$ and $Q = (q_{ij})_{n\times n}$, we define the *tensor product* of P and Q to be the $mn \times mn$ matrix $P \otimes Q$

$$P \otimes Q = (p_{ij})Q = \begin{pmatrix} p_{11}Q & p_{12}Q & \cdots & p_{1m}Q \\ p_{21}Q & p_{22}Q & \cdots & p_{2m}Q \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1}Q & p_{m2}Q & \cdots & p_{mm}Q \end{pmatrix}.$$

Then

$$trace(P \otimes Q) = p_{11}trace(Q) + p_{22}trace(Q) + \cdots + p_{mm}trace(Q) = trace(P)trace(Q).$$

Definition 3.3.1. *Let U and T be two representations of G. We define the tensor product of $T \otimes U$ by*

$$(T \otimes U)(g) = T(g) \otimes U(g),\ \forall g \in G.$$

Theorem 3.3.1. *Let T and U be representations of G. Then*

(i) *$T \otimes U$ is a representation of G,*

(ii) *$\chi_{T\otimes U} = \chi_T\chi_U$.*

PROOF.

(i) $\forall g, h \in G$, we have

$$\begin{aligned} (T \otimes U)(gh) &= T(gh) \otimes U(gh) \\ &= (T(g)T(h)) \otimes (U(g)U(h)) \\ &= (T(g) \otimes U(g))(T(h) \otimes U(h)) \\ &= (T \otimes U)(g)(T \otimes U)(h). \end{aligned}$$

(ii)

$$\begin{aligned} \chi_{T\otimes U}(g) &= trace((T \otimes U)(g)) \\ &= trace((T(g) \otimes U(g))) \\ &= trace(T(g))trace(U(g)) \\ &= \chi_T(g)\chi_U(g). \end{aligned}$$

Hence $\chi_{T\otimes U} = \chi_T\chi_U$.

This proves the Theorem. ■

Note 3.3.1. Observe that $T \otimes U \neq U \otimes T$ in general, but $\chi_{T\otimes U} = \chi_T\chi_U = \chi_U\chi_T = \chi_{U\otimes T}$. Thus the tensor product of characters is commutative.

Now we show that knowing the character tables of two groups K and H, then the tensor products can be used to obtain the character table of $K \times H$.

Theorem 3.3.2. *Let H_1 and H_2 be two groups with conjugacy classes $\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_r$ and $\mathcal{C}_1', \mathcal{C}_2', \cdots, \mathcal{C}_s'$ respectively. Suppose that $\mathrm{Irr}(H_1) = \{\chi_1, \chi_2, \cdots, \chi_r\}$ and $\mathrm{Irr}(H_2) = \{\chi_1', \chi_2', \cdots, \chi_s'\}$. The conjugacy classes of $H_1 \times H_2$ are $\mathcal{C}_i \times \mathcal{C}_j'$ and $\mathrm{Irr}(H_1 \times H_2) = \{\chi_i \times \chi_j' |\ \chi_i \in \mathrm{Irr}(H_1),\ \chi_j' \in \mathrm{Irr}(H_2)\}$ for $1 \leq i \leq r$ and $1 \leq j \leq s$.*

PROOF. For all $x, h_1 \in H_1$ and $y, h_2 \in H_2$, we have

$$(x, y)^{-1}(h_1, h_2)(x, y) = (x^{-1}h_1x, y^{-1}h_2y).$$

Therefore two elements (h_1, h_2) and (h_1', h_2') of $H_1 \times H_2$ are conjugate if and only if $h_1 \sim_{H_1} h_1'$ and $h_2 \sim_{H_2} h_2'$, where $\sim_H$ denotes the conjugation of two elements in a group H. Thus

$$\mathcal{C}_i \times \mathcal{C}_j', \quad 1 \leq i \leq r,\ 1 \leq j \leq s,$$

are the conjugacy classes of $H_1 \times H_2$. In particular, there are exactly rs conjugacy classes of $H_1 \times H_2$. On the other hand for all i, j, k, l,

$$\begin{aligned}
\left\langle \chi_i \times \chi_j', \chi_k \times \chi_l' \right\rangle &= \frac{1}{|H_1 \times H_2|} \sum_{h\in H_1,\ \widehat{h}\in H_2} \chi_i(h)\chi_j'(\widehat{h})\overline{\chi_k(h)\chi_l'(\widehat{h})} \\
&= \left(\frac{1}{|H_1|}\sum_{h\in H_1} \chi_i(h)\overline{\chi_k(h)}\right)\left(\frac{1}{|H_2|}\sum_{\widehat{h}\in H_2} \chi_j'(\widehat{h})\overline{\chi_l'(\widehat{h})}\right) \\
&= \langle \chi_i, \chi_k \rangle \left\langle \chi_j', \chi_l' \right\rangle = \delta_{ik}\delta_{jl}.
\end{aligned}$$

Thus the rs characters $\chi_i \times \chi_j'$ are distinct and irreducible. This completes the proof. ■

Note 3.3.2. Observe that if $\chi, \psi \in \mathrm{Irr}(G)$, then in general $\chi\psi \notin \mathrm{Irr}(G)$. In the special case when $\deg(\psi) = 1$, we have the following proposition.

Proposition 3.3.3. *Let ψ be a linear character of G and $\chi \in \mathrm{Irr}(G)$. Then $\chi\psi \in \mathrm{Irr}(G)$.*

PROOF. Suppose that ψ is a linear character of G. Then we know that $\psi(g)$ is a root of unity for any $g \in G$. In particular, we have $1 = |\psi(g)|^2 = \psi(g)\overline{\psi(g)}$ for every $g \in G$. Now assume that χ is an irreducible character of G. It follows that

$$\begin{aligned}\langle \psi\chi, \psi\chi \rangle &= \frac{1}{|G|}\sum_{g\in G} \psi\chi(g)\overline{\psi\chi(g)} \\ &= \frac{1}{|G|}\sum_{g\in G} \chi(g)\overline{\chi}(g)\psi(g)\overline{\psi}(g) \\ &= \frac{1}{|G|}\sum_{g\in G} \chi(g)\overline{\chi}(g) = \langle \chi, \chi \rangle = 1.\end{aligned}$$

Hence $\psi\chi$ is an irreducible character of G. ■

Proposition 3.3.4. *The number of linear characters of a group G is given by $|G|/|G'|$, where G' is the derived subgroup of G.*

PROOF. See Theorem 17.11 of James [40]. ■

3.4. Lifting of Characters

In this section, we present a method for constructing characters of G when it has a proper normal subgroup N. We may look at the quotient group G/N, which is of a smaller order than $|G|$. Therefore it becomes reasonable to assume that the irreducible characters of G/N are known. From this assumption we may construct characters of G in a process known as ***lifting of characters***. Thus the normal subgroups help to find characters of G and conversely the character table of G enables us to determine all the normal subgroups of G.

Proposition 3.4.1. *Let $N \lhd G$ and $\widetilde{\chi}$ be a character of G/N. The function $\chi : G \to \mathbb{C}$ defined by $\chi(g) = \widetilde{\chi}(gN), \forall g \in G$ is a character of G with $\deg(\chi) = \deg(\widetilde{\chi})$. Moreover; if $\widetilde{\chi} \in \mathrm{Irr}(G/N)$, then $\chi \in \mathrm{Irr}(G)$.*

PROOF. Assume that $\widetilde{\rho} : G/N \to GL(n, \mathbb{C})$ is a representation which affords the character $\widetilde{\chi}$. Define the function $\rho : G \to GL(n, \mathbb{C})$ by $\rho(g) = \widetilde{\rho}(gN),\ \forall g \in G$. Then ρ defines a representation on G since

$$\rho(gh) = \widetilde{\rho}(ghN) = \widetilde{\rho}(gNhN) = \widetilde{\rho}(gN)\widetilde{\rho}(hN) = \rho(g)\rho(h),\ \forall g, h \in G.$$

Hence the character χ, which is afforded by ρ, satisfies

$$\chi(g) = tr(\rho(g)) = tr(\widetilde{\rho}(gN)) = \widetilde{\chi}(gN)\ \forall g \in G.$$

and so χ is a character of G. For the degree of χ, we have

$$\deg(\chi) = \chi(1_G) = \widetilde{\chi}(1_G N) = \widetilde{\chi}(N) = \deg(\widetilde{\chi}).$$

Now let S be a transversal of N in G. Then

$$\begin{aligned}
1 = \langle \widetilde{\chi}, \widetilde{\chi} \rangle &= \frac{1}{|G/N|} \sum_{gN \in G/N} \widetilde{\chi}(gN)\widetilde{\chi}(gN)^{-1} \\
&= \frac{1}{|G|} \sum_{gN \in G/N} |N| \widetilde{\chi}(gN)\widetilde{\chi}(gN)^{-1} \\
&= \frac{1}{|G|} \sum_{g \in S} |N| \widetilde{\chi}(gN)\widetilde{\chi}(g^{-1}N) \\
&= \frac{1}{|G|} \sum_{g \in S} |N| \chi(g)\chi(g^{-1}) \\
&= \frac{1}{|G|} \sum_{g \in G} \chi(g)\chi(g^{-1}) \\
&= \langle \chi, \chi \rangle .
\end{aligned}$$

This completes the proof. ■

Definition 3.4.1. *The character χ defined in the above Proposition is called the* **lift** of $\widetilde{\chi}$ to G, or we say χ is the **inflation** of $\widetilde{\chi}$ and denoted by $\inf(\widetilde{\chi})$.

One of the advantages given by the character table of G is that it supplies us with all normal subgroups of G. This is the assertion of the next theorem.

Theorem 3.4.2. *Let $N \lhd G$. Then there exist irreducible characters $\chi_1, \chi_2, \cdots, \chi_s$ of G such that $N = \cap_{i=1}^{s} ker\chi_i$.*

PROOF. Firstly, we have the following observation. If $\chi_1, \chi_2, \cdots, \chi_k$ are the irreducible characters of G, then $\bigcap_{i=1}^{k} \ker \chi_i = \{1_G\}$. Now suppose that G/N has s distinct irreducible characters $\widetilde{\chi}_1, \widetilde{\chi}_2, \cdots, \widetilde{\chi}_s$. So $\bigcap_{i=1}^{s} \ker \widetilde{\chi}_i = \{N\}$. For $1 \leq i \leq s$, suppose that χ_i are the lifts to G of $\widetilde{\chi}_i$. Thus if $g \in \ker \chi_i$, then

$$\widetilde{\chi}_i(N) = \chi_i(1_G) = \chi_i(g) = \widetilde{\chi}_i(gN),$$

and hence $gN \in \ker \widetilde{\chi_i}$. Therefore if $g \in \bigcap_{i=1}^{|\mathrm{Irr}(G)|} \ker \chi_i$, then $gN \in \bigcap_{i=1}^{s} \ker \widetilde{\chi_i} = \{N\}$, and so $g \in N$. Hence $N = \bigcap_{i=1}^{s} \ker \chi_i$. ■

The converse of the above theorem is also true, *i.e.* every normal subgroup of G arises in this way.

Corollary 3.4.3. *G is simple if and only if for every $\chi_r \in \mathrm{Irr}(G)$, where $\chi_r \neq \chi_1$, and for all $1_G \neq g \in G$, we have $\chi_r(g) \neq \chi_r(1_G)$.*

PROOF. See Alperin [4] or Moori [49]. ■

Hence the character table can be used to decide whether G is simple group or not.

3.5. Restriction and Induction of Characters

Given a group G and a subgroup $H \leq G$, knowing characters of G, one can get some characters of H and vice versa. These two dual operations are known as ***restriction*** and ***induction*** of characters.

3.5.1 Restriction of Characters

Let $H \leq G$ and let $\rho : G \longrightarrow GL(n, \mathbb{C})$ be a representation of G. The ***restriction*** of ρ to H, denoted by $\rho{\downarrow}_H^G$ is defined by

$$\rho{\downarrow}_H^G(h) = \rho(h), \ \forall h \in H.$$

If χ_ρ is the character afforded by ρ, then it is not difficult to see that $\chi_\rho{\downarrow}_H^G$ is a character of H. Also if $\chi_\rho \in Irr(G)$, then it is not necessarily that $\chi_\rho{\downarrow}_H^G \in Irr(H)$.

Theorem 3.5.1. *Let $H \leq G$. Let $\chi \in \mathrm{Irr}(G)$ and let $\mathrm{Irr}(H) = \{\psi_1, \psi_2, \cdots, \psi_r\}$. Then $\chi{\downarrow}_H^G = \sum_{i=1}^{r} d_i\psi_i$, where $d_i \in \mathbb{N} \cup \{0\}$ and $\sum_{i=1}^{r} d_i^2 \leq [G : H]$. The equality holds in the previous $\sum$ if and only if $\chi(g) = 0, \ \forall g \in G \setminus H$.*

PROOF. We have

$$\sum_{i=1}^{r} d_i^2 = \langle \chi{\downarrow}_H^G, \chi{\downarrow}_H^G \rangle = \frac{1}{|H|} \sum_{h \in H} \chi(h)\overline{\chi(h)}.$$

Since $\chi \in \mathrm{Irr}(G)$, we have

$$\begin{aligned} 1 = \langle \chi, \chi \rangle_G &= \frac{1}{|G|} \sum_{g \in G} \chi(g)\overline{\chi(g)} \\ &= \frac{1}{|G|} \sum_{g \in H} \chi(g)\overline{\chi(g)} + \frac{1}{|G|} \sum_{g \in G \setminus H} \chi(g)\overline{\chi(g)} \\ &= \frac{|H|}{|G|} \sum_{i=1}^{r} d_i^2 + K, \end{aligned}$$

where $K = \frac{1}{|G|} \sum_{g \in G \setminus H} \chi(g)\overline{\chi(g)}$. Since $K = \frac{1}{|G|} \sum_{g \in G \setminus H} |\chi(g)|^2$, we have that $K \geq 0$. Thus

$$\frac{|H|}{|G|} \sum_{i=1}^{r} d_i^2 = 1 - K \leq 1,$$

so

$$\sum_{i=1}^{r} d_i^2 \leq \frac{|G|}{|H|} = [G : H].$$

Also

$$K = 0 \Longleftrightarrow |\chi(g)|^2 = 0 \Longleftrightarrow \chi(g) = 0, \ \forall g \in G \setminus H.$$

This completes the proof. ■

Theorem 3.5.1 asserts that the number of irreducible constituents of $\chi{\downarrow}_H^G$ is bounded above by $[G : H]$. Therefore if $[G : H]$ is fairly small, the character tables of H and G are closely related. For example if $[G : H] = 2$ and $\chi \in \mathrm{Irr}(G)$, then either $\chi{\downarrow}_H^G \in \mathrm{Irr}(H)$ or $\chi{\downarrow}_H^G = \psi_1 + \psi_2$ where $\psi_1, \psi_2 \in \mathrm{Irr}(H)$.

3.5.2 Induction of Characters

Let $H \leq G$ such that the set $\{x_1, x_2, \cdots, x_r\}$ is a transversal for H in G. Let ϕ be a representation of H of degree n. Then we define ϕ^* on G as follows:

$$\phi^*(g) = \begin{pmatrix} \phi(x_1 g x_1^{-1}) & \phi(x_1 g x_2^{-1}) & \cdots & \phi(x_1 g x_r^{-1}) \\ \phi(x_2 g x_1^{-1}) & \phi(x_2 g x_2^{-1}) & \cdots & \phi(x_2 g x_r^{-1}) \\ \vdots & \cdots & \ddots & \vdots \\ \phi(x_r g x_1^{-1}) & \phi(x_r g x_2^{-1}) & \cdots & \phi(x_r g x_r^{-1}) \end{pmatrix}$$

where $\phi(x_i g x_j^{-1})$ is $n \times n$ block satisfying the property that

$$\phi(x_i g x_j^{-1}) = 0_{n\times n} \quad \forall x_i g x_j^{-1} \notin H.$$

It is possible to show that ϕ^* is a representation of G of degree n.

Definition 3.5.1. *With the above, the representation ϕ^* is called the representation of G* **induced** *from the representation ϕ of H and is denoted by $\phi^* = \phi\uparrow_H^G$.*

Definition 3.5.2. *Let ϕ be a class function of H. Then $\phi\uparrow_H^G$, the* **induced** *class function on G, is defined by*

$$\phi\uparrow_H^G(g) = \frac{1}{|H|}\sum_{x\in G}\phi^0(xgx^{-1})$$

where ϕ^0 is defined on G by

$$\phi^0(h) = \begin{cases} \phi(h) & \text{if } h \in H, \\ 0 & \text{if } h \notin H. \end{cases}$$

Note that $\deg(\phi\uparrow_H^G) = [G : H]\deg(\phi)$.

Theorem 3.5.2. *If ϕ is a character of H where $H \leq G$, then $\phi\uparrow_H^G$ is a character of G.*

PROOF. See Moori [49] or Whitley [70]. ■

Theorem 3.5.3 (**Frobenius Reciprocity Theorem**)**.** *Let G be a group, $H \leq G$ and suppose that ϕ is a character of H and θ a character of G. Then*

$$\langle \phi, \theta\downarrow_H^G\rangle_H = \langle \phi\uparrow_H^G, \theta\rangle_G .$$

PROOF. We obtain that

$$\langle \phi\uparrow_H^G, \theta\rangle_G = \frac{1}{|G|}\sum_{g\in G}\phi\uparrow_H^G(g)\overline{\theta(g)} = \frac{1}{|G|}\frac{1}{|H|}\sum_{g\in G}\sum_{x\in G}\phi^0(xgx^{-1})\overline{\theta(g)}$$

Putting $y = xgx^{-1}$, then for fixed x, as g runs through G, so does y, and $\theta(y) = \theta(g)$, since θ is a class function on G. Hence

$$\begin{aligned} \langle \phi\uparrow_H^G, \theta\rangle_G &= \frac{1}{|G|}\frac{1}{|H|}\sum_{x\in G}\sum_{y\in G}\phi^0(y)\overline{\theta(y)} = \frac{1}{|G|}\frac{1}{|H|}\sum_{y\in G}\sum_{x\in G}\phi^0(y)\overline{\theta(y)} \\ &= \frac{1}{|H|}\sum_{y\in H}\phi(y)\overline{\theta(y)} = \langle \phi, \theta\downarrow_H^G\rangle_H . \end{aligned}$$

Hence the result. ■

Corollary 3.5.4. *Let* $\mathrm{Irr}(G) = \{\chi_1, \chi_2, \cdots, \chi_r\}$ *and* $\mathrm{Irr}(H) = \{\psi_1, \psi_2, \cdots, \psi_s\}$ *where* $H \leq G$. *Assume that* $\chi_i\downarrow_H^G = \sum_{j=1}^{s} a_{ij}\psi_j$ *and* $\psi_j\uparrow_H^G = \sum_{i=1}^{r} b_{ij}\chi_i$. *Then* $a_{ij} = b_{ij}$ *for all* i, j.

PROOF. Using the Frobenius Reciprocity Theorem we get $a_{ij} = \langle \chi_i\downarrow_H^G, \psi_j \rangle = \langle \chi_i, \psi_j\uparrow_H^G \rangle = b_{ij}$. ■

Next we compute the values of induced character $\phi\uparrow_H^G$ on classes of G.

Proposition 3.5.5. *Let* ϕ *be a character of* H *and let* $\phi\uparrow_H^G$ *be the induced character from* H *to* G. *Let* $g \in G$ *and suppose that* $[g]$ *breaks into* m *classes in* H *with representatives* $x_1, x_2, \cdots, x_m$. *If* $H \cap [g] = \varnothing$, *the empty set, then* $\phi\uparrow_H^G(g) = 0$, *while if* $H \cap [g] \neq \varnothing$, *then*

$$\phi\uparrow_H^G(g) = |C_G(g)| \sum_{i=1}^{m} \frac{\phi(x_i)}{|C_H(x_i)|}.$$

PROOF. We have

$$\phi\uparrow_H^G(g) = \frac{1}{|H|} \sum_{x \in G} \phi^0(xgx^{-1}).$$

If $H \cap [g] = \varnothing$, then $xgx^{-1} \notin H$ for all $x \in G$ and thus $\phi^0(xgx^{-1}) = 0$ for all $x \in G$ and $\phi\uparrow_H^G(g) = 0$. Now if $H \cap [g] \neq \varnothing$, then let $h \in H \cap [g]$. As x runs over G, we have $xgx^{-1} = h$ for exactly $|C_G(g)|$ times, so $\phi\uparrow_H^G(g) = \frac{|C_G(g)|}{|H|} \sum_{y \in [g]} \phi^0(y)$. Now $\phi^0(y) = 0$ if $y \notin H$, and $[g] \cap H$ contains $[H : C_H(x_i)]$ conjugates of each x_i. Therefore $\phi\uparrow_H^G(g) = |C_G(g)| \sum_{i=1}^{m} \frac{\phi(x_i)}{|C_H(x_i)|}$. ■

Proposition 3.5.6. *Let* $K_1 \leq K_2 \leq K_3$ *and let* ψ *be a class function on* K_1. *Then* $(\psi\uparrow_{K_1}^{K_2})\uparrow_{K_2}^{K_3} = \psi\uparrow_{K_1}^{K_3}$. *More generally if* $K_1 \leq K_2 \leq \cdots \leq K_n$ *is a nested sequence of subgroups of* K_n *and* ψ *is a class function on* K_1, *then* $(\psi\uparrow_{K_1}^{K_2})\uparrow_{K_2}^{K_3} \cdots \uparrow_{K_{n-1}}^{K_n} = \psi\uparrow_{K_1}^{K_n}$.

PROOF. We recall from the elementary theory of representations and characters of finite groups that if $H \leq G$ and ψ is a class function on H, then

$$\psi\uparrow_H^G(g) = \begin{cases} 0 & \text{if } H \bigcap [g]_G = \emptyset, \\ |C_G(g)| \sum_{i=1}^{t} \frac{\psi(x_i)}{|C_H(x_i)|} & \text{if } H \bigcap [g]_G \neq \emptyset, \end{cases} \tag{3.1}$$

where $x_1, x_2, \cdots, x_t$ are representatives of some conjugacy classes of H such that $H \bigcap [g]_G = \bigcup_{i=1}^{t} [x_i]_H$.

Now if $K_1 \leq K_2 \leq K_3$ and ψ is a character of K_1, then there are two cases:

Case I: Assume that $K_1 \bigcap [g]_{K_3} = \emptyset$. It follows by Eq. (3.1) that $\psi\uparrow_{K_1}^{K_3}(g) = 0$. Now consider the character $\psi\uparrow_{K_1}^{K_2}$ of K_2. Two subcases arise:

- $[g]_{K_3} \bigcap K_2 = \emptyset$: Here we get $\psi\uparrow_{K_1}^{K_2}\uparrow_{K_2}^{K_3}(g) = 0$ and therefore $\psi\uparrow_{K_1}^{K_2}\uparrow_{K_2}^{K_3}(g) = \psi\uparrow_{K_1}^{K_3}(g)$.
- $[g]_{K_3} \bigcap K_2 \neq \emptyset$: Let $y_1, y_2, \cdots, y_k$ be representatives of the conjugacy classes of K_2 such that $[g]_{K_3} \bigcap K_2 = \bigcup_{i=1}^{k} [y_i]_{K_2}$. Note that non of the classes represented by $y_1, y_2, \cdots, y_k$ intersect K_1, i.e. $[y_i]_{K_2} \bigcap K_1 = \emptyset, \ \forall i$ and consequently $\psi\uparrow_{K_1}^{K_2}(y_i) = 0, \forall i$. Now

$$\psi\uparrow_{K_1}^{K_2}\uparrow_{K_2}^{K_3}(g) = |C_{K_3}(g)| \sum_{i=1}^{k} \frac{\psi\uparrow_{K_1}^{K_2}(y_i)}{|C_{K_2}(y_i)|} = |C_{K_3}(g)| \sum_{i=1}^{k} \frac{0}{|C_{K_2}(y_i)|} = 0 = \psi\uparrow_{K_1}^{K_3}(g).$$

Therefore if $K_1 \bigcap [g]_{K_3} = \emptyset$, then $\psi\uparrow_{K_1}^{K_2}\uparrow_{K_2}^{K_3}(g) = \psi\uparrow_{K_1}^{K_3}(g)$.

Case II: Assume that $K_1 \bigcap [g]_{K_3} \neq \emptyset$. This implies that $K_1 \bigcap [g]_{K_3} = \bigcup_{i=1}^{m} [x_i]_{K_1}$ for some x_i and some positive integer m. Since $K_1 \leq K_2$ and $K_1 \bigcap [g]_{K_3} \neq \emptyset$, this implies that $K_2 \bigcap [g]_{K_3} \neq \emptyset$. Let $y_1, y_2, \cdots, y_k$ have the same meaning as in **Case I**. That is $K_2 \bigcap [g]_{K_3} = \bigcup_{i=1}^{k} [y_i]_{K_2}$. Now it is the case that some of $[y_i]_{K_2}$ intersect K_1 and other classes are not. After suitable re-arrangement and renaming of the classes represented by $\{y_1, y_2, \cdots, y_k\}$, let us assume that the set $\{y_1, y_2, \cdots, y_z\}$ consists of representatives of those classes of K_2 such that $[y_j] \bigcap K_1 \neq \emptyset$, for each $1 \leq j \leq z$ and $1 \leq z \leq k$, while let the set $\{y_{z+1}, y_{z+2}, \cdots, y_k\}$ consists of representatives of those classes of K_2 such that $[y_j] \bigcap K_1 = \emptyset$. Now for each $y_j \in \{y_1, y_2, \cdots, y_z\}$, let $y_{j1}, y_{j2}, \cdots, y_{js_j}$ be representatives of the conjugacy classes of K_1 such that $K_1 \bigcap [y_j]_{K_2} = \bigcup_{r=1}^{s_j} [y_{jr}]_{K_1}$. The key point here is that each $[y_{jr}]_{K_1}, \ 1 \leq r \leq s_j$, coincides with some $[x_i]_{K_1}$ for some $i \in \{1, 2, \cdots, m\}$ and hence $\sum_{j=1}^{z} s_j = m$. As a consequence of this coincidence, it follows that $\sum_{j=1}^{z} \sum_{r=1}^{s_j} \frac{\psi(y_{jr})}{|C_{K_1}(y_{jr})|} = \sum_{i=1}^{m} \frac{\psi(x_i)}{|C_{K_1}(x_i)|}$. Now

$$\begin{aligned}
\psi\uparrow_{K_1}^{K_2}\uparrow_{K_2}^{K_3}(g) &= |C_{K_3}(g)| \sum_{j=1}^{k} \frac{\psi\uparrow_{K_1}^{K_2}(y_j)}{|C_{K_2}(y_j)|} = |C_{K_3}(g)| \sum_{j=1}^{z} \frac{\psi\uparrow_{K_1}^{K_2}(y_j)}{|C_{K_2}(y_j)|} + |C_{K_3}(g)| \sum_{j=z+1}^{k} \frac{\psi\uparrow_{K_1}^{K_2}(y_j)}{|C_{K_2}(y_j)|} \\
&= |C_{K_3}(g)| \sum_{j=1}^{z} \frac{\psi\uparrow_{K_1}^{K_2}(y_j)}{|C_{K_2}(y_j)|} = |C_{K_3}(g)| \sum_{j=1}^{z} \left(\frac{1}{|C_{K_2}(y_j)|} \cdot |C_{K_2}(y_j)| \sum_{r=1}^{s_j} \frac{\psi(y_{jr})}{|C_{K_1}(y_{jr})|} \right) \\
&= |C_{K_3}(g)| \sum_{j=1}^{z} \sum_{r=1}^{s_j} \frac{\psi(y_{jr})}{|C_{K_1}(y_{jr})|} = |C_{K_3}(g)| \sum_{i=1}^{m} \frac{\psi(x_i)}{|C_{K_1}(x_i)|} = \psi\uparrow_{K_1}^{K_3}(g).
\end{aligned}$$

Hence $\psi\uparrow_{K_1}^{K_2}\uparrow_{K_2}^{K_3} = \psi\uparrow_{K_1}^{K_3}$. The proof of the general case can be proceeded by Mathematical Induction on n. ■

We conclude this section by remarking that the operations of restriction and induction of characters do not necessarily preserve irreducibility of characters.

3.6. Permutation Character

Let G acts on a finite set $\Omega = \{\omega_1, \omega_2, \cdots, \omega_k\}$ and for each $g \in G$ define the $k \times k$ matrix $\pi_g = (a_{ij})$ where

$$a_{ij} = \begin{cases} 1 & \text{if } \omega_i^g = \omega_j, \\ 0 & \text{otherwise.} \end{cases}$$

Then π_g is a permutation matrix of the action of g and $P : G \longrightarrow GL(k, \mathbb{C})$ given by $P(g) = \pi_g$ is a representation of G.

The character χ_P afforded by this representation is called a ***permutation character***, and $\chi_P(g) = |\{\omega \in \Omega |\ \omega^g = \omega\}|$, that is, $\chi_P(g)$ is the number of points of Ω left fixed by $g \in G$. Therefore $\chi_P(g) \in \mathbb{N} \cup \{0\}$, $\forall g \in G$.

Note 3.6.1. Observe that

$$\deg(\chi_P) = \chi_P(1_G) = |\{\omega \in \Omega |\ \omega^{1_G} = \omega\}| = |\Omega| = k,$$

since by definition of group action we have $\omega^{1_G} = \omega,\ \forall \omega \in \Omega$.

Recall that an action of G on a set X is called transitive if $\forall x, y \in X,\ \exists g \in G$ such that $x^g = y$. Now let $H \leq G$ and $S = \{a_1, a_2, \cdots, a_r\}$ be a left transversal for H in G. Then G acts on the set of left cosets of H in G by $(a_iH)^g = ga_iH$. It is clear that this action is transitive since for any $a_i, a_j \in S$, we have $(a_iH)^{a_j a_i^{-1}} = a_jH$. The resulting permutation character of this action is of degree $[G : H] = |S| = r$. In fact this permutation character is $\mathbf{1}\uparrow_H^G$. To see this we have

$$(a_iH)^g = a_iH \iff ga_iH = a_iH \iff a_i^{-1}ga_iH = H \iff a_i^{-1}ga_i \in H.$$

Thus

$$\chi_P(g) = \sum_{i=1}^{r} \phi^0(a_i^{-1}ga_i),$$

where

$$\phi^0(y) = \begin{cases} 1 & \text{if } y \in H, \\ 0 & \text{if } y \notin H. \end{cases}$$

Hence $\chi_P = \mathbf{1}\uparrow_H^G$. This shows that for any subgroup H, there exists a permutation character of G. Conversely, if G acts transitively on any set X, then the associated permutation character represents $\mathbf{1}\uparrow_H^G$ for some subgroup H of G. This is the assertion of the following theorem.

Theorem 3.6.1. *Let G act transitively on a set Ω and let $\omega \in \Omega$. Then $\mathbf{1}\uparrow_{G_\omega}^G$ is the permutation character of the action.*

PROOF. Since G act transitively on Ω, we have $\omega^G = \Omega$. It follows by the *Orbit-Stabilizer Theorem* (see Moori [49] for example) that there is a $1-1$ correspondence between Ω and the set of left cosets of G_ω in G, given by $\omega^t \longmapsto tG_\omega$ for $t \in G$. Now for $g \in G$ we have

$$(\omega^t)^g = \omega^t \Longleftrightarrow \omega^{t^{-1}gt} = \omega \Longleftrightarrow t^{-1}gt \in G_\omega \Longleftrightarrow tG_\omega = gtG_\omega \Longleftrightarrow tG_\omega = (tG_\omega)^g,$$

where G acts on the set of left cosets of G_ω in G as given above. Therefore the permutation character of the action of G on Ω is the same as the permutation character of the action of G on the left cosets of G_ω in G, which is $\mathbf{1}\uparrow_{G_\omega}^G$. ■

Corollary 3.6.2. *Let G act on Ω with a permutation character χ. Suppose Ω decomposes into exactly k orbits under the action of G. Then $\langle \chi, \mathbf{1} \rangle = k$, where $\mathbf{1}$ is the trivial character of G.*

PROOF. Write $\Omega = \bigcup_{i=1}^{k} \Delta_i$ where the Δ_i are orbits. Let χ_i be the permutation character of G on Δ_i so that $\chi = \sum_{i=1}^{k} \chi_i$. For $\omega \in \Delta_i$, we have $\chi_i = \mathbf{1}\uparrow_{G_\omega}^G$ by Theorem 3.6.1. Thus

$$\langle \chi_i, \mathbf{1} \rangle_G = \left\langle \mathbf{1}\uparrow_{G_\omega}^G, \mathbf{1} \right\rangle_G = \left\langle \mathbf{1}, \mathbf{1}\downarrow_{G_\omega}^G \right\rangle_{G_\omega} = 1$$

by Frobenius reciprocity. Therefore $\langle \chi, \mathbf{1} \rangle = \sum_{i=1}^{k} \langle \chi_i, \mathbf{1} \rangle = k$, completing the proof. ■

Lemma 3.6.3. *If G acts transitively on Ω, then all subgroups G_ω, $\omega \in \Omega$ of G are conjugate in G.*

PROOF. Since G acts transitively on Ω, there is some $h \in G$ such that $\omega^h = \kappa$ for any $\omega, \kappa \in \Omega$. Now

$$g \in G_\omega \Longleftrightarrow \omega^g = \omega \Longleftrightarrow \kappa^{gh^{-1}} = \kappa^{h^{-1}} \Longleftrightarrow \kappa^{hgh^{-1}} = \kappa \Longleftrightarrow hgh^{-1} \in G_\kappa \Longleftrightarrow g \in (G\kappa)^h.$$

Thus $G_\omega = (G_\kappa)^h$, which shows that $G_\omega = hG_\kappa h^{-1}$. That is G_ω and G_κ are conjugate in G. ■

Because $\mathbf{1}\uparrow_H^G$ is a transitive permutation character, it must satisfy certain necessary conditions mentioned in the following theorem.

Theorem 3.6.4. *Let $H \leq G$ and $\chi = \mathbf{1}\uparrow_H^G$. Then*

(i) $\deg(\chi)||G|$.

(ii) $\langle \chi, \psi \rangle \leq \deg(\psi),\ \forall \psi \in Irr(G)$.

(iii) $\langle \chi, \mathbf{1} \rangle = 1$.

(iv) $\chi(g) \in \mathbb{N} \cup \{0\},\ \forall g \in G$.

(v) $\chi(g) \leq \chi(g^m),\ \forall g \in G,\ \forall m \in \mathbb{N} \cup \{0\}$.

(vi) $o(g) \nmid \frac{|G|}{\chi(1_G)} \Longrightarrow \chi(g) = 0$.

(vii) $\chi(g)\frac{|[g]|}{\chi(1_G)} \in \mathbb{Z},\ \forall g \in G$.

PROOF. Let Ω be the set of the left cosets of H in G. Thus χ is the permutation character of G on Ω.

(i) Since $\deg(\chi) = [G : H]$, we have $\deg(\chi)||G|$.

(ii) Using Frobenius reciprocity we get $\langle \chi, \psi \rangle_G = \langle \mathbf{1}\uparrow_H^G, \psi \rangle_G = \langle \mathbf{1}\downarrow_H^G, \psi\downarrow_H^G \rangle_H \leq \deg(\psi)$.

(iii) Since χ is a transitive permutation character, it follows by Corollary 3.6.2 that $\langle \chi, \mathbf{1} \rangle = 1$.

(iv) This follows because $\chi(g)$ is the number of points left fixed by g and hence is non-negative.

(v) Let $g \in G_\omega$, that is $\omega^g = \omega$. It is clear that $\omega^{g^m} = \omega$. Thus any point of Ω left fixed by g is also fixed by g^m. Therefore the number of points fixed by g does not exceed the number of points fixed by g^m.

(vi) We know that $\frac{|G|}{\chi(1_G)} = |H|$ so if $o(g) \nmid |H|$, then $[g] \cap H = \varnothing$, the empty set. Hence $\mathbf{1}\uparrow_H^G(g) = 0$.

(vii) Let $\mathcal{B} = \{(\omega, x) |\ \omega \in \Omega,\ x \in [g],\ \omega^x = \omega\}$. Since χ is constant on $[g]$, we have

$$|[g]|\chi(g) = |\mathcal{B}| = \sum_{\omega \in \Omega} |[g] \cap G_\omega|.$$

By Lemma 3.6.3 all subgroups G_ω are conjugate in G. Thus $|[g] \cap G_\omega| = m$ is independent of ω, and $\chi(g)|[g]| = m|\Omega| = m\chi(1_G)$.

This completes the proof. ■

Corollary 3.6.5. *Let $H \leq G$ with $\chi = \mathbf{1}\uparrow_H^G$. Let $g \in G$ and assume that $[g]$ splits in H into m classes with representatives $h_1, h_2, \cdots, h_m$. Then*

$$\chi\uparrow_H^G(g) = \sum_{i=1}^{m} \frac{|C_G(g)|}{|C_H(h_i)|}.$$

PROOF. Immediate by Proposition 3.5.5. ■

Chapter 4

Schur Multiplier, Projective Representations and Characters

In this chapter we introduce the concept of the projective representations and characters of a finite group G, which are of great importance in the sequel of this thesis. We also introduce the Schur multiplier $M(G)$ of a group G, which plays an important role in the study of projective representations and characters of G. In Section 4.1, we give some definitions and basic results on the Schur multiplier of G. Section 4.2 is devoted to the theory of projective representations of G. There is a relationship between the projective representations of G and the ordinary representations of the central extension $M(G)\cdot G$, which is known as *covering group* or *representation group* of G. Every projective representation can be obtained from (or we say *lifted* to) an ordinary representation of $M(G)\cdot G$. In fact if α is factor set of the Shcur multiplier of G and ξ is a projective representation of G with associated factor set α, then ξ can be lifted to an ordinary representation of $\alpha\cdot G$ and we do not need to calculate the full covering group. In Section 4.3 we discuss the projective characters and study the orthogonality relations analogous to the ones for ordinary characters, given by Theorem 3.2.3. For further readings on projective representations and projective characters readers are referred to Berkovich and Zhmud [15], Haggarty [32], Hoffman and Humphreys [34], Humphreys [35], Huppert [37], Isaacs [38], Karpilovsky [42], [43], Morris [51], [52], [53], [54], Nagao and Tsushima [56], Read [59], [60] and [61].

4.1. Schur Multiplier

The Schur multiplier of a finite group G is of particular interest and importance to study all the projective representations of G. In this section we discuss some results that are useful in finding the Schur multiplier of G.

Definition 4.1.1. *A function $\alpha : G \times G \longrightarrow \mathbb{C}^*$ is called a* **factor set** of G if

$$\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z), \quad \forall\ x, y, z \in G. \tag{4.1}$$

Example 4.1.1. A trivial example of a factor set of any finite group G is the map 1, where $1(x, y) = 1,\ \forall\ x, y \in G$.

Next we define an equivalence relation amongst factor sets of a group G. Two factor sets α and β are said to be ***equivalent*** if there exists a function $\rho : G \longrightarrow \mathbb{C}^*$ such that

$$\beta(x, y) = \frac{\rho(x)\rho(y)}{\rho(xy)}\alpha(x, y), \quad \forall\ x, y \in G. \tag{4.2}$$

Clearly the above relation defines an equivalence relation amongst the factor sets. We denote the equivalence class containing the factor set α by $[\alpha]$. We define multiplication of two factor sets α and β by $(\alpha\beta)(x, y) = \alpha(x, y)\beta(x, y),\ \forall\ x, y \in G$. With no much difficulty we can see that $\alpha\beta$ satisfies Eq. (4.1) and thus $\alpha\beta$ is again a factor set. Moreover, we can see that for every factor set α, the function $\alpha^{-1} : G \times G \longrightarrow \mathbb{C}^*$, defined by $\alpha^{-1}(x, y) = (\alpha(x, y))^{-1} = \frac{1}{\alpha(x,y)}$ is also a factor set of G.

Definition 4.1.2. *The set of all equivalence classes of factor sets of a group G forms a group by defining $[\alpha][\beta] = [\alpha\beta]$. The identity of this group is $[1]$ and $[\alpha]^{-1} = [\alpha^{-1}]$. This group is called the* **Schur multiplier** *or* **multiplicator** *of G and we denote it by $M(G)$.*

Next we quote some results from Karpilovsky [42] on the Schur multipliers of finite groups.

Theorem 4.1.1. (i) *$M(G)$ is a finite abelian group.*

(ii) *If G is a cyclic group, then $M(G) = 1$.*

PROOF. See Nagao and Tsushima [56] or Rotman [64]. ■

Lemma 4.1.2. *Suppose that N is a normal subgroup of a finite group G. If $M(G) = 1$, then $M(G/N) \cong (N \bigcap G')/[N, G]$. In general, $|(N \bigcap G')/[N, G]|$ divides $|M(G/N)|$.*

PROOF. See Karpilovsky [43]. ■

Theorem 4.1.3. *Let G be a finite group and H be a subgroup of index m. Then $(M(G))^m \cong M(H)$, where $(M(G))^m$ is the group of all m^{th} powers of $M(G)$.*

PROOF. See Karpilovsky [43]. ■

Schur [68] reduced the problem of computing $M(G)$ to compute the Schur multiplier of the Sylow $p-$subgroups of G, where p is a prime dividing the order of G. The following theorem describes the Schur multiplier of a group G in terms of the Sylow $p-$subgroups of G.

Theorem 4.1.4 (**Schur** [68])**.** *Let S be a Sylow $p-$subgroup of G. Then the Sylow p-subgroup of $M(G)$ is isomorphic to a subgroup of $M(S)$.*

PROOF. See Karpilovsky [43]. ■

Theorem 4.1.5. *A group G has trivial Schur multiplier if and only if it has a set of subgroups with trivial Schur multipliers and relatively prime indices.*

PROOF. See Karpilovsky [43].

In GAP [30] and Magma [16] there are various commands to calculate the Schur multiplier of a finite group G. For example the commands "AbelianInvariantsMultiplier(G)" and "pMultiplicator(G,p)" of GAP and Magma respectively reveals the Schur multiplier of G and of a Sylow $p-$subgroup of G respectively. We deal with some examples in Chapters 7 and 8.

4.2. Projective Representations

In this section we follow mainly the description of Ali [1].

Definition 4.2.1. *Let G be a finite group. A* **projective representation** *of G is a map $P : G \longrightarrow GL(n, \mathbb{F})$ such that*

(i) $P(1_G) = I_n$, *where I_n is the identity $n \times n$ matrix,*

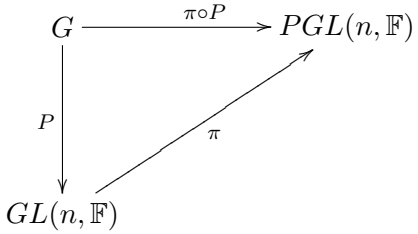
(ii) $P(x)P(y) = \alpha(x, y)P(xy)$ *for all $x, y \in G$, where $\alpha(x, y) \in \mathbb{F}^*$ depending on the pair (x, y).*

The function α is called the **factor set associated with** *P. Also the positive integer n is called the* **degree** of *P.*

Remark 4.2.1. From Definition 4.2.1, we observe that if $\alpha(x, y) = 1_{\mathbb{F}},\ \forall x, y \in G$, then we obtain that $P(xy) = P(x)P(y)$ and hence P becomes an ordinary representation of G. Sometimes a pair (P, α) is used to indicate a projective representation P and its associated factor set α.

Definition 4.2.2. *A projective representation $P : G \longrightarrow GL(n, \mathbb{F})$ is said to be* **irreducible** *if there are no non-trivial subspaces of the vector space $\mathbb{V}(n, \mathbb{F})$ which are sent into themselves by all the transformations $P(g),\ g \in G$.*

Now we look at projective representations from different point of view. It is an elementary result (see Basheer [7] for example) that the center $Z(GL(n, \mathbb{F}))$ consists of $\{\lambda I_n |\ \lambda \in \mathbb{F}^*\}$ and the quotient $GL(n, \mathbb{F})/Z(GL(n, \mathbb{F}))$ is called the **projective general linear group**, which is denoted by $PGL(n, \mathbb{F})$. Let $\pi : GL(n, \mathbb{F}) \longrightarrow PGL(n, \mathbb{F})$ be the natural epimorphism. If $P : G \longrightarrow GL(n, \mathbb{F})$ is a projective representation of a group G, then $\pi \circ P$ is a homomorphism of G into $PGL(n, \mathbb{F})$. Conversely if ρ is any homomorphism from G into $PGL(n, \mathbb{F})$ and if for each $g \in G$ we select a unique element $P(g)$ in the coset $\pi(g)$ of $Z(GL(n, \mathbb{F}))$ in $GL(n, \mathbb{F})$, choosing $P(1_G) = I_n$, then $g \mapsto P(g)$ is a projective representation of G over $\mathbb{V}(n, \mathbb{F})$.



Thus the projective $\mathbb{F}$-representations of G can be identified with the homomorphisms of G into the projective general linear group.

We would like to remark that the projective representations occur in an un-avoidable way in the study of ordinary representations.

Theorem 4.2.1. *Let $N \lhd G$ and suppose that χ is an irreducible $\mathbb{C}-$representation of N whose character is invariant in G. Then there exists a projective $\mathbb{C}-$representation $\mathfrak{X}$ of G such that for all $n \in N$ and $g \in G$ we have*

(i) $\mathfrak{X}(n) = \chi(n)$,

(ii) $\mathfrak{X}(ng) = \mathfrak{X}(n)\mathfrak{X}(g)$,

(iii) $\mathfrak{X}(gn) = \mathfrak{X}(g)\mathfrak{X}(n)$.

Furthermore, if $\mathfrak{X}_0$ *is another projective representation satisfying* (i), (ii) *and* (iii), *then* $\mathfrak{X}_0(g) = \mathfrak{X}(g)\mu(g)$ *for some function* $\mu : G \longrightarrow \mathbb{C}^*$, *which is constant on cosets of* N.

PROOF. See Isaacs [38]. ■

We now consider the associated factor sets with the projective representations.

Lemma 4.2.2. *Let* α *be the associated factor set with a projective representation* P *of* G. *Then* α *satisfies* $\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z)$ *for all* $x, y, z \in G$.

PROOF. By associativity we have

$$P(x)P(y)P(z) = \alpha(x, y)P(xy)P(z) = \alpha(x, y)\alpha(xy, z)P(xyz)$$

and

$$P(x)P(y)P(z) = \alpha(y, z)P(x)P(yz) = \alpha(y, z)\alpha(x, yz)P(xyz).$$

Now the result follows since $P(xyz)$ is invertible. ■

As with ordinary representations, we now define equivalence of projective representations. From now on, we will only be concerned with projective representations over the complex field $\mathbb{C}$.

Definition 4.2.3. *Two projective representations* P_1 *and* P_2 *of* G *are* **equivalent** *if there is a non-singular matrix* T *such that for all* $g \in G$, $P_1(g) = d(g)TP_2(g)T^{-1}$ *for some* $d(g) \in \mathbb{C}^*$. *If* $d(g) = 1$ *for all* $g \in G$ *then* P_1 *and* P_2 *are* **linearly equivalent**.

Having defined what is meant by linearly equivalence of projective representations, another formulation to Definition 4.2.2 can be given. A projective representation P is **irreducible** if it is not linearly equivalent to a projective representation of the form

$$\begin{pmatrix} \star & \star \\ 0 & \star \end{pmatrix}.$$

Lemma 4.2.3. *If two projective representations are equivalent, then they have equivalent factor sets; if they are linearly equivalent they have equal factor sets.*

PROOF. Let (P_1, α_1) and (P_2, α_2) be two equivalent projective representations of G. Suppose T is a non-singular matrix and $d : G \longrightarrow \mathbb{C}^*$ such that $P_1(g) = d(g)TP_2(g)T^{-1}$ for all $g \in G$. Now for $g, h \in G$,

$$\begin{aligned}
\alpha_1(g,h) &= P_1(g)P_1(h)(P_1(gh))^{-1} \\
&= d(g)TP_2(g)T^{-1}d(h)TP_2(h)T^{-1}(d(gh))^{-1}T(P_2(gh))^{-1}T^{-1} \\
&= d(g)d(h)(d(gh))^{-1}TP_2(g)P_2(h)(P_2(gh))^{-1}T^{-1} \\
&= d(g)d(h)(d(gh))^{-1}\alpha_2(g,h),
\end{aligned}$$

so α_1 and α_2 are equivalent. If P_1 and P_2 are linearly equivalent, then $d(g) = 1$ for all $g \in G$ in the above expressions, so $\alpha_1 = \alpha_2$. ■

Let $F[G, \mathbb{C}]$ be the set of all functions $\lambda : G \longrightarrow \mathbb{C}$. If P is a projective representation of G with factor set α and $\lambda \in F[G, \mathbb{C}]$, then $P' = \lambda P$, where $P'(g) = \lambda(g)P(g)$ for all $g \in G$, is a projective representation of G with factor set α', and α' satisfies the relation

$$\alpha'(x,y) = \lambda(x)\lambda(y)(\lambda(xy))^{-1}\alpha(x,y) \qquad \forall x, y \in G. \tag{4.3}$$

Remark 4.2.2. It follows from Eq. (4.3) that $\alpha \sim 1$ if and only if there exists $\lambda \in F[G, \mathbb{C}]$ such that

$$\alpha(x,y) = \lambda(x)\lambda(y)(\lambda(xy))^{-1} \qquad \forall\ x, y \in G.$$

The following result provides a close connection between the degrees of the irreducible projective characters with factor set α and the $o([\alpha])$.

Lemma 4.2.4. Berkovich and Zhmud [15] *Let (P, α) be a projective representation of G such that $\deg(P) = n$. If $o([\alpha]) = m$ then $m|n$.*

PROOF. We know that

$$P(x)P(y) = \alpha(x,y)P(xy).$$

Taking the determinant for both sides we obtain

$$\begin{aligned}
\det(P(x))\det(P(y)) &= \det(\alpha(x,y)P(xy)) \\
&= \alpha(x,y)^n \det(P(xy))
\end{aligned}$$

which implies

$$\alpha(x,y)^n = \det(P(x))\det(P(y))(\det(P(xy))^{-1}.$$

By Remark 4.2.2 we obtain $[\alpha]^n = 1$. Hence $m|n$. ■

As it has been mentioned in Ali [1] that projective representations of a group G can be obtained by three different ways. In this thesis we are only concerned with one method in which projective representations of a group G can be constructed from the ordinary representations of the so-called representation group or covering group. For the other methods one can refer to Ali [1] and more results on the construction of projective representations, by combining the three methods, can be found in a series of articles by Morris ([52], [53], [54]) and Read ([59], [60], [61]).

Definition 4.2.4. *A* **central extension** *of G is a group H together with a homomorphism ν of H onto G such that* $\ker \nu \leq Z(H)$.

Now let $\nu : \mathfrak{C}(G) \longrightarrow G$ be an epimorphism with kernel N and suppose that $\mathfrak{C}(G)$ is a central extension of N by G. Let $\tau : G \longrightarrow PGL(n, \mathbb{C})$ be a projective representation. If there is a homomorphism $\widetilde{\tau}$ renders the commutativity of the diagram

$$\begin{array}{ccc} \mathfrak{C}(G) & \xrightarrow{\ \nu\ } & G \\ \downarrow{\scriptstyle \widetilde{\tau}} & & \downarrow{\scriptstyle \tau} \\ GL(n,\mathbb{C}) & \xrightarrow{\ \pi\ } & PGL(n,\mathbb{C}) \end{array}$$

(i.e., $\tau \circ \nu = \pi \circ \widetilde{\tau}$), then τ is called has the property to be ***lifted*** to $\mathfrak{C}(G)$. We say $\mathfrak{C}(G)$ has the ***projective lifting property*** if every projective representation of G can be lifted to $\mathfrak{C}(G)$.

Remark 4.2.3. Note that the homomorphism $\widetilde{\tau}$ in above commutative diagram is an ordinary representation of $\mathfrak{C}(G)$ (i.e., a representation over the complex $\mathbb{C}$).

Definition 4.2.5. *If G is a group, then a* **cover** *or* **representation group** *of G is a central extension $\mathfrak{C}(G)$ of N (for some abelian group N) with the projective lifting property and with $N \leq \mathfrak{C}(G)'$.*

Schur proved (see Rotman [64]) that every finite group G has a cover $\mathfrak{C}(G)$ with $G \cong \mathfrak{C}(G)/M$ such that every projective representation of G determines an ordinary representation of $\mathfrak{C}(G)$; moreover, it turns out that $M \cong M(G)$ and $\mathfrak{C}(G)$ is the central extension of $M(G)$ by G. In fact the following theorem, due to Schur, shows the existence of covering groups of any finite groups.

Theorem 4.2.5 (Schur 1904). *Every finite group G has a cover $\mathfrak{C}(G)$, which is the central extension of $M(G)$ by G.*

PROOF. See Theorem 7.66 of Rotman [64]. ■

The above theorem affirms that every finite groups G, of order n, has at least one representation group $\mathfrak{C}(G)$ of order mn where $m = |M(G)|$ and the kernel of the extension is isomorphic to the Schur multiplier $M(G)$ of G.

Definition 4.2.6. *Let G_1 and G_2 be any two groups (not necessarily finite). We say G_1 is* **isoclinic** *to G_2 if the following conditions hold:*

(i) $G_1/Z(G_1) \cong G_2/Z(G_2)$,

(ii) $G_1' \cong G_2'$,

(iii) *If θ and ψ are the isomorphisms satisfying conditions* (i) *and* (ii) *respectively and if for all $a_1, b_1 \in G_1$ and $a_2, b_2 \in G_2$ we have*

$$\begin{aligned} \theta(a_1 Z(G_1)) &= a_2 Z(G_2), \\ \theta(b_1 Z(G_1)) &= b_2 Z(G_2), \end{aligned}$$

then $\psi(a_1^{-1} b_1^{-1} a_1 b_1) = a_2^{-1} b_2^{-1} a_2 b_2$.

Example 4.2.1.

- All the abelian groups are isoclinic, as if we considered any two abelian groups, they will satisfy the conditions in Definition 4.2.6
- Any two isomorphic groups are isoclinic, but the converse is not true, that is not any two isoclinic groups are isomorphic.

Remark 4.2.4. For a finite group G, the covering groups need not be unique, up to isomorphism, but are unique up to isoclinism. For example (see Rotman [64]) both D_8 and Q_8 are (non-isomorphic) covers of $\mathbb{Z}_2 \times \mathbb{Z}_2 \cong 2^2$. Note that one can verify the conditions of Definition 4.2.6 to check that the two groups D_8 and Q_8 are isoclinic.

Lemma 4.2.6. *Let (H, π) be a central extension of G with $A = \ker(\pi)$. Let X be a set of coset representatives for A in H, and write $X = \{x_g : g \in G\}$, where $\pi(x_g) = g$. Define $\alpha : G \times G \longrightarrow A$ by $x_g x_h = \alpha(g, h) x_{gh}$. Then α is an A-factor set of G and the equivalence class of α is independent of the choice of X.*

PROOF. See Issacs [38]. ■

Corollary 4.2.7. *Let H be a central extension of G with A, X and α be as in* Lemma 4.2.6. *Let T be an ordinary representation of H such that the restriction T_A is the scalar representation λI for some $\lambda \in \mathrm{Hom}(A, \mathbb{C}^*)$, that is*

$$T(a) = \begin{pmatrix} \lambda(a) & & & \\ & \lambda(a) & & \\ & & \ddots & \\ & & & \lambda(a) \end{pmatrix}_{n\times n} \quad \forall\ a \in A,$$

where $n = \deg(T)$. Define $P(g) = T(x_g)$ for $g \in G$. Then $(P, \lambda(\alpha))$ is a projective representation of G, where $\lambda(\alpha)(g,h) = \lambda(\alpha(g,h))$. Furthermore, P is irreducible if and only if T is and the equivalence class of P is independent of the choice of coset representatives X.

PROOF. See Isaacs [38].

Remark 4.2.5. Note that if T is an ordinary irreducible representation of H then the condition that T_A be scalar representation is satisfied by the Schur's lemma (see Moori [49]), since A lies in the center of H.

4.3. Projective Characters

Definition 4.3.1. *Let P be a projective representations of G with associated factor set α. Define $\xi(g) = Trace(P(g))$ for all $g \in G$. Then ξ is called a* **projective character** *of G. We say that ξ is* **irreducible** *if P is, and ξ has factor set α, where α is the factor set of P.*

Next correspond to a factor set α of G, elements of G are distinguished into two types, namely α-regular and α-irregular as in the following definition.

Definition 4.3.2. *Let α be a factor set of G. An element $g \in G$ is said to be* **α-regular** *if $\alpha(g,x) = \alpha(x,g),\ \forall x \in C_G(g)$. The element $g \in G$ is called* **α-irregular** *if it is not α-regular.*

Remark 4.3.1. Let α be a factor set of a group G. For each $g \in G$, we have

(i) if g is α-regular, then every conjugate of g is also α-regular.

(ii) An element g is α-regular if and only if g is α'-regular for every factor set α' equivalent to α.

Thus it makes sense to say a conjugacy class of G to be α-regular if every element in this class is α-regular.

We recall from Proposition 3.1.1 that all the ordinary characters of a finite group G are class functions. However, this no longer true for projective characters. For projective characters we have the following proposition.

Proposition 4.3.1. *Let* $\mathrm{IrrProj}(G, \alpha^{-1}) = \{\xi_1, \xi_2, \cdots, \xi_t\}$ *be set of all the projective characters of* G *with associated factor set* α *and suppose that* $\xi \in \mathrm{IrrProj}(G, \alpha^{-1})$. *If for any* α*-regular element* x *in* G *and for any* y *in* G *(not necessarily* α*-regular), the relation* $\alpha(x, y) = \alpha(y, y^{-1}xy)$ *satisfied, then* ξ *is a class function.*

PROOF. Refer to Proposition 2.2(iii) of Karpilovsky [43]. ■

The following theorem gives a criterion to see whether two projective representations P_1 and P_2 with factor set α are linearly equivalent or not.

Theorem 4.3.2. *Two projective representations* P_1 *and* P_2 *with factor set* α *are linearly equivalent if and only if they have the same projective character.*

PROOF. See Theorem 4.4 of Morris [51]. ■

If P is a projective representation of G with projective character ξ then there is an ordinary representation T of $\mathfrak{C}(G)$ such that $P(g) = T(x_g)$ for $g \in G$. Let χ be the character of $\mathfrak{C}(G)$ afforded by T, then $\xi(g) = \chi(x_g)$ for all $g \in G$.

We have seen that in Theorem 3.2.3 that the ordinary characters of a group G satisfy certain row and column orthogonality relations. The Projective characters of G also satisfy the usual orthogonality relations.

Theorem 4.3.3. *Let* $\mathrm{IrrProj}(G, \alpha^{-1})$ *be as in* Proposition 4.3.1 *and let* $\{C_1, C_2, \cdots, C_m\}$ *be the collection of the* α*-regular conjugacy classes of* G *with* g_i *being a representative of* C_i, $\forall i \in \{1, 2, \cdots, m\}$. *Then*

(i) $|\mathrm{IrrProj}(G, \alpha^{-1})| =$ *the number of* α*-regular conjugacy classes of* G *(i.e.,* $t = m$*).*

(ii)
$$\sum_{i=1}^{t} \xi_i(g_j)\overline{\xi_i(g_k)} = \delta_{jk}|C_G(g_j)| \quad \textit{for } j, k \in \{1, 2, \ldots, t\}.$$

(iii) *An element g of G is α-regular if and only if there is an irreducible projective character ξ of G with factor set α such that $\xi(g) \neq 0$.*

PROOF. See Haggarty and Humphreys [32]. ■

Now let G^0 be the set of all α-regular elements of the group G. We have the following theorem.

Theorem 4.3.4. *Let* $\mathrm{IrrProj}(G, \alpha^{-1}) = \{\xi_1, \xi_2, \cdots, \xi_t\}$ *be as before and let* $\{C_1, C_2, \cdots, C_t\}$ *be the collection of the α-regular conjugacy classes of G with g_i being a representative of C_i, $\forall i \in \{1, 2, \cdots, t\}$. Then*

$$\sum_{g \in G^0} \xi_i(g)\overline{\xi_j(g)} = |G|\delta_{ij}.$$

PROOF. See Karpilovsky [43]. ■

We conclude this chapter by remarking that to find the set $\mathrm{IrrProj}(G, \alpha^{-1})$, it is not necessarily to find the full covering group. Rather another group $\mathfrak{L}$ can be constructed, where every $\xi \in \mathrm{IrrProj}(G, \alpha^{-1})$ can be lifted to an ordinary character of $\mathfrak{L}$. This is has been shown in Haggarty and Humphreys [32]. To construct $\mathfrak{L}$, suppose that α is a factor set of G, such that $o([\alpha]) = r$ in the Schur multiplier $M(G)$. Let ω be an r^{th} root of unity and let α' be a representative of $[\alpha]$ whose values are powers of ω. For all $g, h \in G$, we define $\alpha'(g, h)$ by $\alpha'(g, h) = \omega^{\alpha(g,h)}$. Now for all $g \in G$, let

$$\mathfrak{L} = \left\langle x, x_g |\ o(x) = r,\ x^i x_g x^j x_h = x^{i+j} x^{\alpha(g,h)} x_{gh} \right\rangle.$$

Then $\mathfrak{L}$ is a quotient of the representation group H and any $\xi \in \mathrm{IrrProj}(G, \alpha^{-1})$ can be lifted to an ordinary representation of $\mathfrak{L}$. Thus the set $\mathrm{IrrProj}(G, \alpha^{-1})$ can be determined from the ordinary character table of $\mathfrak{L}$.

Chapter 5

The Theory of Clifford-Fischer Matrices

The theory of Clifford-Fischer matrices, which is based on Clifford Theory (see Clifford [18]), was developed by B. Fischer ([25], [26] and [27]). This technique, which is used to construct the character tables of group extensions, has also been discussed and applied to both split and non-split extensions in several publications, for example see Ali and Moori [2], [3], Barraclough [6], Basheer and Moori [8], [9], [10], [11], [12], [13], [14], Fischer [27], Moori [47], Moori and Mpono [50], Pahlings [57], Rodrigues [62], Seretlo [69], Whitely [70] and in a recent book by K. Lux and H. Pahlings [58].

5.1. The Clifford Theory

Definition 5.1.1. *Let N be any normal subgroup of $\overline{G}$ (not necessarily abelian) and let θ be any character of N. For $\overline{g} \in \overline{G}$, define another character $\theta^{\overline{g}}$ of N by $\theta^{\overline{g}}(n) = \theta(\overline{g}n\overline{g}^{-1})$, $\forall n \in N$. The character $\theta^{\overline{g}}$ is called a* **conjugate** *character of θ.*

Remark 5.1.1. From the above definition, it is not difficult to see that $\theta^{\overline{g}} \in \mathrm{Irr}(N)$ if and only if $\theta \in \mathrm{Irr}(N)$. Also it follows that $\overline{G}$ acts on $\mathrm{Irr}(N)$ by conjugation and since N acts trivially on $\mathrm{Irr}(N)$, $\mathrm{Irr}(N)$ is permuted by $\overline{G}/N$, by $\overline{g}N : \theta \mapsto \theta^{\overline{g}}$.

The group $\overline{G}$ has dual actions on the conjugacy classes of N and on $\mathrm{Irr}(N)$. The following important result by Brauer relates the number of orbits on the two actions.

Theorem 5.1.1 (**Brauer's Theorem**). *The number of $\overline{G}-$orbits on $\mathrm{Irr}(N)$ is same as the number of $\overline{G}-$orbits on the conjugacy classes of N.*

PROOF. See Lemma 4.5.2 of Gorenstein [31] or Theorem 5.1.5 of Mpono [55]. ■

Remark 5.1.2. Brauer Theorem affirms that when the group $\overline{G}$ act on $\mathrm{Irr}(N)$, it produces the same number of orbits as when $\overline{G}$ act on the conjugacy classes of N, but the orbit lengths may be different. Indeed if N is non-abelian, then

$$\sum_{k=1}^{t} |{\theta_k}^{\overline{G}}| = |\mathrm{Irr}(N)| \neq |N| = \sum_{k=1}^{t} |[n_k]_N^{\overline{G}}|,$$

where

- t is the number of orbits for the action of $\overline{G}$ on the conjugacy classes of N or on $\mathrm{Irr}(N)$,
- θ_k and n_k are respective representative character and element of a conjugacy class of N,
- ${\theta_k}^{\overline{G}}$ and $[n_k]_N^{\overline{G}}$ are orbits of N containing θ_k and n_k, respectively, for the action of $\overline{G}$ on $\mathrm{Irr}(N)$ and on the conjugacy classes of N respectively.

Definition 5.1.2. *Let G be a group and $K \leq G$ be any subgroup. Then for a character χ of K, we define*

$$I_G(\chi) = \{g \in N_G(K) |\ \chi^g = \chi\},$$

where χ^g is defined in a similar manner to Definition 5.1.1. *We call $I_G(\chi)$ the* **inertia group** *of χ in G. If K is normal in G, then*

$$I_G(\chi) = \{g \in G |\ \chi^g = \chi\}.$$

Now if $N \trianglelefteq \overline{G}$ and $\overline{H}_k$ is the inertia group of a character $\theta_k \in \mathrm{Irr}(N)$, it is easy to show that $N \trianglelefteq \overline{H}_k \leq \overline{G}$. The quotient $H_k \cong \overline{H}_k/N$ is referred to as the ***inertia factor group.*** The group H_k can be regarded as the inertia group of θ_k in the factor group $\overline{G}/N \cong G$. That is $H_{\theta_k} = H_k = \{g \in G |\ \theta_k^g = \theta_k\}$.

The following theorem, due to Clifford [18], is very important.

Theorem 5.1.2 (Clifford Theorem (1937)). *Let $\chi \in \mathrm{Irr}(\overline{G})$ and let $\theta_1, \theta_2, \cdots, \theta_t$ be representatives of orbits of $\overline{G}$ on $\mathrm{Irr}(N)$. For $k \in \{1, 2, \cdots, t\}$, let $\theta_k^{\overline{G}} = \{\theta_k = \theta_{k1}, \theta_{k2}, \cdots, \theta_{ks_k}\}$ and let $\overline{H}_k$ be the inertia group in $\overline{G}$ of θ_k. Then for some fixed $k \in \{1, 2, \cdots, t\}$ we have*

$$\chi{\downarrow}_N^{\overline{G}} = e_k \sum_{u=1}^{s_k} \theta_{ku}, \quad \textit{where} \quad e_k = \left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_k \right\rangle. \tag{5.1}$$

Moreover, for fixed k

$$\mathrm{Irr}(\overline{H}_k, \theta_k) := \left\{ \psi_k \in \mathrm{Irr}(\overline{H}_k) |\ \left\langle \psi_k{\downarrow}_N^{\overline{H}_k}, \theta_k \right\rangle \neq 0 \right\} \longleftrightarrow \left\{ \chi \in \mathrm{Irr}(\overline{G}) |\ \left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_k \right\rangle \neq 0 \right\} := \mathrm{Irr}(\overline{G}, \theta_k)$$

under the map $\psi_k \longmapsto \psi_k{\uparrow}_{\overline{H}_k}^{\overline{G}}$.

PROOF. We only show the first assertion of the theorem, namely Eq. (5.1). For the second part of the theorem, readers can refer to either Theorem 4.1.7 of Ali [1] or Theorem 3.3.2 of Whitely [70] and the readers have to be careful as there are differences in the notations. To verify Eq. (5.1), we compute (for fixed $k \in \{1, 2, \cdots, t\}$) the character $\theta_k{\uparrow}_N^{\overline{G}}{\downarrow}_N^{\overline{G}}$. For this, define θ_k^0 on $\overline{G}$ by

$$\theta_k^0(x) = \begin{cases} \theta_k(x), & \text{if } x \in N, \\ 0 & \text{if } x \notin N. \end{cases}$$

For $n \in N$, we have $\theta_k{\uparrow}_N^{\overline{G}}(n) = \frac{1}{|N|} \sum_{x \in \overline{G}} \theta_k^0(xnx^{-1})$. Since $xnx^{-1} \in N,\ \forall x \in \overline{G}$, we have $\theta_k{\uparrow}_N^{\overline{G}}(n) = \frac{1}{|N|} \sum_{x \in \overline{G}} \theta_k^x(n)$. Thus $|N|\theta_k{\uparrow}_N^{\overline{G}}{\downarrow}_N^{\overline{G}} = \sum_{x \in \overline{G}} \theta_k^x$, and if $\theta_l \in \mathrm{Irr}(N)$ such that $l \neq k$ (i.e., $\theta_l \notin \{\theta_{ku} |\ 1 \leq u \leq s_k\}$ then $0 = \left\langle \sum_{x \in \overline{G}} \theta_k^x, \theta_l \right\rangle$, so $\left\langle \theta_k{\uparrow}_N^{\overline{G}}{\downarrow}_N^{\overline{G}}, \theta_l \right\rangle = 0$. Now since χ is an irreducible constituent of $\theta_k{\uparrow}_N^{\overline{G}}$ by Frobenius reciprocity, it follows that $\left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_l \right\rangle = 0$. Hence all the irreducible constituents of $\chi{\downarrow}_N^{\overline{G}}$ are among the $\theta_{ku},\ 1 \leq u \leq s_k$. So $\chi{\downarrow}_N^{\overline{G}} = \sum_{u=1}^{s_k} \left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_{ku} \right\rangle \theta_{ku}$. But $\left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_{ku} \right\rangle = \left\langle \chi{\downarrow}_N^{\overline{G}}, \theta_k \right\rangle$ since θ_{ku} and θ_k are conjugate, so the proof is complete. ■

Definition 5.1.3. *Let $N \lhd \overline{G}$. A character $\theta \in \mathrm{Irr}(N)$ is said to be* **extendable** *to a character of its inertia group $\overline{H} = I_{\overline{G}}(\theta)$ if there exists $\psi \in \mathrm{Irr}(\overline{H})$ such that $\psi{\downarrow}_N^{\overline{H}} = \theta$.*

Theorem 5.1.3. *Further to the settings of* Theorem 5.1.2, *assume that for each $k \in \{1, 2, \cdots, t\}$, there exists $\psi_k \in \mathrm{Irr}(\overline{H}_k, \theta_k)$ such that $\psi_k{\downarrow}_N^{\overline{H}_k} = \theta_k$. Then*

$$\mathrm{Irr}(\overline{G}) = \bigcup_{k=1}^{t} \left\{ (\psi_k \inf(\zeta)){\uparrow}_{\overline{H}_k}^{\overline{G}} |\ \zeta \in \mathrm{Irr}(\overline{H}_k/N) \right\}. \tag{5.2}$$

PROOF. See Ali [1] or Whitley [70]. ■

Remark 5.1.3. It is by no means necessarily the case that there exists an extension ψ_k of θ_k to the inertia group $\overline{H}_k$ (that is the case $\mathrm{Irr}(\overline{H}_k, \theta_k) = \emptyset$, the empty set, or $\nexists\ \psi \in \mathrm{Irr}(\overline{H}_k)$ such that $\psi{\downarrow}_N^{\overline{H}_k} = \theta_k$, is feasible). However, there is always a *projective* extension $\widetilde{\psi}_k \in \mathrm{IrrProj}(\overline{H}_k, \overline{\alpha}_k^{-1})$ for

some factor set $\overline{\alpha}_k$ of the Schur multiplier of $\overline{H}_k$. Thus the more proper formula for Equation (5.2) is (see Remark 4.2.7 of Ali [1])

$$\mathrm{Irr}(\overline{G}) = \bigcup_{k=1}^{t} \left\{ (\widetilde{\psi}_k \inf(\zeta))\uparrow_{\overline{H}_k}^{\overline{G}} \mid \widetilde{\psi}_k \in \mathrm{IrrProj}(\overline{H}_k, \overline{\alpha}_k^{-1}),\ \zeta \in \mathrm{IrrProj}(\overline{H}_k/N, \alpha_k^{-1}) \right\}, \tag{5.3}$$

where the factor set α_k is obtained from $\overline{\alpha}_k$ as described in Corollary 7.3.3 of Whitely [70]. Hence the character table of $\overline{G}$ is partitioned into t blocks $\mathcal{K}_1, \mathcal{K}_2, \cdots, \mathcal{K}_t$, where each block $\mathcal{K}_k$ of characters (ordinary or projective) is produced from the inertia subgroup $\overline{H}_k$.

Note 5.1.1. Observe that if $\alpha_k \sim [1]$ in Equation (5.3), then we get Equation (5.2). That is $\mathrm{IrrProj}(\overline{H}_k, \overline{1}) = \mathrm{Irr}(\overline{H}_k)$ and $\mathrm{IrrProj}(H_k, 1) = \mathrm{Irr}(H_k)$.

By convention we take $\theta_1 = \mathbf{1}_N$, the trivial character of N. Thus $\overline{H}_{\theta_1} = \overline{H}_1 = \overline{G}$ and thus $\overline{H}_1/N \cong G$. Since $\{\mathbf{1}_{\overline{G}}\} \subseteq \mathrm{Irr}(\overline{G}, \mathbf{1}_N)$ and such that $\mathbf{1}_{\overline{G}}\downarrow_N^{\overline{G}} = \mathbf{1}_N$ (this means that the character θ_1 is extendable to a character of its inertia group $\overline{G}$), the block $\mathcal{K}_1$ will consists only of the ordinary irreducible characters of G.

In the rest of this section, we quote some results on extendability of characters of N to their respective inertia groups. We start with the following two well known results.

Theorem 5.1.4 (**Mackey's Theorem (1951)**). *Let $\overline{G} = N{:}G$ be a split extension and assume that N is abelian. If $\theta \in \mathrm{Irr}(N)$ is invariant in $\overline{G}$ (that is $\theta^{\overline{g}} = \theta,\ \forall\ \overline{g} \in \overline{G}$) then θ can be extended to a linear character of $\overline{G}$.*

PROOF. We follow the proof given by Whitely [70]. Let $\overline{G} = N{:}G$ be a split extension with N abelian. It follows that any $\overline{g} \in \overline{G}$ can be expressed uniquely as $\overline{g} = ng$, where $n \in N$ and $g \in G$. Define χ on $\overline{G}$ by $\chi(ng) = \theta(n)$. Since N is abelian, θ has degree 1 and thus is linear. The fact that θ is invariant in $\overline{G}$ implies that $\theta(n) = \theta(\overline{g}n\overline{g}^{-1}),\ \forall\ \overline{g} \in \overline{G}$. Now if $\overline{g}_1 = n_1 g_1$ and $\overline{g}_2 = n_2 g_2$ we obtain that

$$\begin{aligned} \chi(\overline{g}_1\overline{g}_2) &= \chi(n_1 g_1 n_2 g_2) = \chi(n_1 n_2^{g_1} g_1 g_2) = \theta(n_1 n_2^{g_1}) \\ &= \theta(n_1)\theta(n_2^{g_1}) = \theta(n_1)\theta(n_2) = \chi(\overline{g}_1)\chi(\overline{g}_2). \end{aligned}$$

Therefore χ is a linear character of $\overline{G}$ such that $\chi\downarrow_N^{\overline{G}} = \theta$. ■

Theorem 5.1.5 (**Gallagher Theorem (1962)**). *Let $\overline{G} = N{\cdot}G$ be any extension such that $\gcd(|N|, |G|) = 1$. Then every $\overline{G}-$invariant character of N is extendable to a character of $\overline{G}$.*

PROOF. See Theorem 6 of Gallagher [29]. ■

The above two results by Mackey and Gallagher are corollaries of a more general result by Karpilovsky [42] which we state without proof.

Theorem 5.1.6. *Let the group $\overline{G}$ contain a subgroup G such that $\overline{G} = NG$ for N normal in $\overline{G}$ and let $\theta \in \mathrm{Irr}(N)$ be invariant in $\overline{G}$. Then θ extends to an irreducible character of $\overline{G}$ if the following conditions hold:*

- $\gcd(\deg(\theta), |G|) = 1$,
- $N \bigcap G \leq N'$, *where N' is the derived subgroup of N.*

PROOF. See Karpilovsky [42]. ■

Another extension result is given by the following theorem.

Theorem 5.1.7. *Let $N \lhd \overline{G}$ and suppose that $\theta \in \mathrm{Irr}(N)$ such that θ is invariant in $\overline{G}$. Then θ is extendable to an ordinary character of $\overline{G}$ if* $\gcd\left([\overline{G} : N], \frac{|N|}{\deg(\theta)}\right) = 1$.

PROOF. See Gagola [28]. ■

The following theorem is very important and will be used later on in Subsection 10.3.3.

Theorem 5.1.8. *Suppose that $\overline{G}$ is a splitting extension of N by some group G. Then any linear character $\theta \in \mathrm{Irr}(N)$ is extendable to its inertia group $I_{\overline{G}}(\theta)$.*

PROOF. We follow the proof given by Mpono [55]. Let $\overline{G} = N{:}G$ and $\theta \in \mathrm{Irr}(N)$ be linear. Also let $\overline{H} = I_{\overline{G}}(\theta)$. Then we obtain that $\overline{H} = N{:}H$, where $H = I_G(\theta)$. Since $\overline{H}$ is a split extension, we obtain that $N \bigcap H = \{1\} \leq N'$. Also we have that $(\deg(\theta), |H|) = (1, |H|) = 1$ and clearly θ is $\overline{H}-$invariant. Now by applications of Theorem 5.1.14 of Mpono [55], it follows that the character θ is extendable to an ordinary character of $\overline{H}$. ■

Remark 5.1.4. Note that Mackey Theorem is reinforced by Theorem 5.1.8 since for N abelian, all the irreducible characters are linear and hence are extendable to their inertia groups.

There are many other results for extension of characters from a normal subgroup N of $\overline{G}$ to their inertia groups $\overline{H}$, for example see Ali [1], Karpilovsky [42], Mpono [55] or Whitely [70].

5.2. The Fischer Matrices

Let $\overline{G} = N \cdot G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a group extension. The idea in this section is to construct for every conjugacy class $[g_i]_G$, a unique (up to the permutation of rows and columns) matrix $\mathcal{F}_i$. We refer to this matrix as the ***Fischer matrix*** or ***Clifford matrix*** and it is very important for the computations of the character table of $\overline{G}$. Fischer matrices satisfy some interesting properties and certain orthogonality relations, which help in the computations of their entries. These properties will be mentioned at the end of this section and will be used in the rest of this thesis to calculate the Fischer matrices of all the groups under considerations.

We firstly fix some notations for the conjugacy classes.

- With π being the natural epimorphism from $\overline{G}$ onto G, we use the notation $U = \pi(\overline{U})$ for any subset $\overline{U} \subseteq \overline{G}$. We have seen from Section 2.3 that $\pi^{-1}([g_i]_G) = \bigcup_{j=1}^{c(g_i)} [g_{ij}]_{\overline{G}}$ for any $1 \leq i \leq r$. Let us assume that $\pi(g_{ij}) = g_i$ and by convention we may take $g_{11} = 1_{\overline{G}}$. Note that $c(g_1)$ is the number of $\overline{G}-$conjugacy classes obtained from N.
- $[g_{ij}]_{\overline{G}} \bigcap \overline{H}_k = \bigcup_{n=1}^{c(g_{ijk})} [g_{ijkn}]_{\overline{H}_k}$, where $g_{ijkn} \in \overline{H}_k$ and by $c(g_{ijk})$ we mean the number of $\overline{H}_k-$conjugacy classes that form a partition for $[g_{ij}]_{\overline{G}}$. Since $g_{11} = 1_{\overline{G}}$, we have $g_{11k1} = 1_{\overline{G}}$ and thus $c(g_{11k1}) = 1$ for all $1 \leq k \leq t$.
- $[g_i]_G \bigcap H_k = \bigcup_{m=1}^{c(g_{ik})} [g_{ikm}]_{H_k}$, where $g_{ikm} \in H_k$ and by $c(g_{ik})$ we mean the number of H_k-conjugacy classes that form a partition for $[g_i]_G$. Since $g_1 = 1_G$, we have $g_{1k1} = 1_G$ and thus $c(g_{1k1}) = 1$ for all $1 \leq k \leq t$. Also $\pi(g_{ijkn}) = g_{ikm}$ for some $m = f(j, n)$.

Proposition 5.2.1. *With the notations of* Theorem 5.1.3 *and the above settings, we have*

$$(\widetilde{\psi}_k \inf(\zeta))\uparrow^{\overline{G}}_{\overline{H}_k}(g_{ij}) = \sum_{m=1}^{c(g_{ik})} \zeta(g_{ikm}) \sum_{n=1}^{c(g_{ijk})} \frac{|C_{\overline{G}}(g_{ij})|}{|C_{\overline{H}_k}(g_{ijkn})|} \widetilde{\psi}_k(g_{ijkn}).$$

PROOF. See Ali [1] or Barraclough [6]. ■

We proceed to define the Fischer matrix $\mathcal{F}_i$ corresponding to the conjugacy class $[g_i]_G$. We label the top of the columns of $\mathcal{F}_i$ by the representatives of $[g_{ij}]_{\overline{G}}$, $1 \leq j \leq c(g_i)$ obtained by the coset

analysis and below each g_{ij} we put $|C_{\overline{G}}(g_{ij})|$. Thus there are $c(g_i)$ columns. The bottom of the columns of $\mathcal{F}_i$ are labeled by some ***weights*** m_{ij} that can be computed through the formula

$$m_{ij} = [N_{\overline{G}}(N\overline{g}_i) : C_{\overline{G}}(g_{ij})] = |N|\frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}. \tag{5.4}$$

To label the rows of $\mathcal{F}_i$ we define the set $\overline{J}_i$ to be (this is equivalent to the notation $R(g)$ used by Ali [1] (page 49), where g is a representative for a conjugacy class of G):

$$\overline{J}_i = \{(k, g_{ikm})|\ 1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ g_{ikm} \text{ is } \alpha_k^{-1} - \text{regular class}\},$$

or for more brevity we let

$$J_i = \{(k, m)|\ 1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ g_{ikm} \text{ is } \alpha_k^{-1} - \text{regular class}\}, \tag{5.5}$$

where t is the number of orbits on the action of $\overline{G}$ on $\mathrm{Irr}(N)$. Then each row of $\mathcal{F}_i$ is indexed by a pair $(k, g_{ikm}) \in \overline{J}_i$ or $(k, m) \in J_i$. For fixed $1 \leq k \leq t$, we let $\mathcal{F}_{ik}$ be a sub-matrix of $\mathcal{F}_i$ with rows correspond to the pairs $(k, g_{ik1}), (k, g_{ik2}), \cdots, (k, g_{ikr_{ik}})$ or for brevity $(k, 1), (k, 2), \cdots, (k, r_k)$. Now let

$$a_{ij}^{(k,m)} := \sum_{n=1}^{c(g_{ijk})} \frac{|C_{\overline{G}}(g_{ij})|}{|C_{\overline{H}_k}(g_{ijkn})|}\widetilde{\psi}_k(g_{ijkn}) \tag{5.6}$$

for which $\pi(g_{ijkn}) = g_{ikm}$. For each i, corresponding to the conjugacy class $[g_i]_G$, we define the Fischer matrix $\mathcal{F}_i = \left(a_{ij}^{(k,m)}\right)$, where $1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ 1 \leq j \leq c(g_i)$. The Fischer matrix $\mathcal{F}_i$,

$$\mathcal{F}_i = \left(a_{ij}^{(k,m)}\right) = \left(\begin{array}{c} \mathcal{F}_{i1} \\ \hline \mathcal{F}_{i2} \\ \hline \vdots \\ \hline \mathcal{F}_{it} \end{array}\right)$$

together with additional information required for their definition are presented as follows:

$\mathcal{F}_i$

g_i		g_{i1}	g_{i2}	$\cdots$	$g_{ic(g_i)}$
$\lvert C_{\overline{G}}(g_{ij})\rvert$		$\lvert C_{\overline{G}}(g_{i1})\rvert$	$\lvert C_{\overline{G}}(g_{i2})\rvert$	$\cdots$	$\lvert C_{\overline{G}}(g_{ic(g_i)})\rvert$
(k,m)	$\lvert C_{H_k}(g_{ikm})\rvert$				
$(1,1)$	$\lvert C_G(g_i)\rvert$	$a_{i1}^{(1,1)}$	$a_{i2}^{(1,1)}$	$\cdots$	$a_{ic(g_i)}^{(1,1)}$
$(2,1)$	$\lvert C_{H_2}(g_{i21})\rvert$	$a_{i1}^{(2,1)}$	$a_{i2}^{(2,1)}$	$\cdots$	$a_{ic(g_i)}^{(2,1)}$
$(2,2)$	$\lvert C_{H_2}(g_{i22})\rvert$	$a_{i1}^{(2,2)}$	$a_{i2}^{(2,2)}$	$\cdots$	$a_{ic(g_i)}^{(2,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(2,r_2)$	$\lvert C_{H_2}(g_{i2r_{i2}})\rvert$	$a_{i1}^{(2,r_2)}$	$a_{i2}^{(2,r_2)}$	$\cdots$	$a_{ic(g_i)}^{(2,r_2)}$
$(u,1)$	$\lvert C_{H_u}(g_{iu1})\rvert$	$a_{i1}^{(u,1)}$	$a_{i2}^{(u,1)}$	$\cdots$	$a_{ic(g_i)}^{(u,1)}$
$(u,2)$	$\lvert C_{H_u}(g_{iu2})\rvert$	$a_{i1}^{(u,2)}$	$a_{i2}^{(u,2)}$	$\cdots$	$a_{ic(g_i)}^{(u,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(u,r_u)	$\lvert C_{H_u}(g_{iur_{iu}})\rvert$	$a_{i1}^{(u,r_u)}$	$a_{i2}^{(u,r_u)}$	$\cdots$	$a_{ic(g_i)}^{(u,r_u)}$
$(t,1)$	$\lvert C_{H_t}(g_{it1})\rvert$	$a_{i1}^{(t,1)}$	$a_{i2}^{(t,1)}$	$\cdots$	$a_{ic(g_i)}^{(t,1)}$
$(t,2)$	$\lvert C_{H_t}(g_{it2})\rvert$	$a_{i1}^{(t,2)}$	$a_{i2}^{(t,2)}$	$\cdots$	$a_{ic(g_i)}^{(t,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(t,r_t)	$\lvert C_{H_t}(g_{itr_{it}})\rvert$	$a_{i1}^{(t,r_t)}$	$a_{i2}^{(t,r_t)}$	$\cdots$	$a_{ic(g_i)}^{(t,r_t)}$
m_{ij}		m_{i1}	m_{i2}	$\cdots$	$m_{ic(g_i)}$

From Eq. (5.6), we can see that entries of the Fischer matrices depend essentially on the irreducible characters of the inertia groups $\overline{H}_k$. In fact, in practice one will never compute the g_{ijkn} or try to compute the irreducible characters of the $\overline{H}_k$, simply because the character tables of these inertia groups are usually much larger and more complicated to compute than the character table of $\overline{G}$ itself. Rather we use the character tables of the inertia factor groups H_k together with the Fischer matrices to calculate the character table of $\overline{G}$. Fischer matrices satisfy some interesting properties, which help in the computations of their entries. These properties follow from the usual orthogonality relations for character tables. We gather these properties in the following Proposition.

Proposition 5.2.2. (i) $\sum_{k=1}^{t} c(g_{ik}) = c(g_i)$,

(ii) *$\mathcal{F}_i$ is non-singular for each i,*

(iii) $a_{ij}^{(1,1)} = 1,\ \forall\ 1 \leq j \leq c(g_i)$,

(iv) *If $N\overline{g}_i$ is a split coset, then $a_{i1}^{(k,m)} = \frac{|C_G(g_i)|}{|C_{H_k}(g_{ikm})|},\ \forall i \in \{1, 2, \cdots, r\}$. In particular for the identity coset we have $a_{11}^{(k,m)} = [G : H_k]\theta_k(1_N),\ \forall\ (k,m) \in J_1$,*

(v) *If $N\overline{g}_i$ is a split coset, then $|a_{ij}^{(k,m)}| \leq |a_{i1}^{(k,m)}|$ for all $1 \leq j \leq c(g_i)$. Moreover if $|N| = p^\alpha$, for some prime p, then $a_{ij}^{(k,m)} \equiv a_{i1}^{(k,m)} (\text{mod } p)$,*

(vi) *For each* $1 \leq i \leq r$, *the weights* m_{ij} *satisfy the relation* $\sum_{j=1}^{c(g_i)} m_{ij} = |N|$,

(vii) ***Column Orthogonality Relation:***

$$\sum_{(k,m)\in J_i} |C_{H_k}(g_{ikm})| a_{ij}^{(k,m)} \overline{a_{ij'}^{(k,m)}} = \delta_{jj'} |C_{\overline{G}}(g_{ij})|,$$

(viii) ***Row Orthogonality Relation:***

$$\sum_{j=1}^{c(g_i)} m_{ij} a_{ij}^{(k,m)} \overline{a_{ij}^{(k',m')}} = \delta_{(k,m)(k',m')} a_{i1}^{(k,m)} |N|.$$

PROOF. Proofs for many assertions of Proposition 5.2.2 can be founded in Moori's students theses, for example see Ali [1] or Mpono [55] and some other assertions are provided in Schiffer [67] as well as in Moori and Basheer [10] and Lux and Pahlings [58]. ■

We conclude this section on Fischer matrices by remarking that the Fischer matrices of any extension $\overline{G}$ are unique up to the ordering of the columns and rows, provided that the character tables of the $\overline{H}_k$ have been determined. Permuting the columns of the Fischer matrices will result in a corresponding permutation of the columns of the character table of $\overline{G}$.

5.3. The Character Tables of Group Extensions

Now we are in good position to describe the construction of the character table of $\overline{G} = N \cdot G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, through the theory of Clifford-Fischer matrices. To do this we need to have

- the conjugacy classes of $\overline{G}$,
- the appropriate projective character tables of the inertia factors H_k, with factor set α_k^{-1},
- the fusions of the $\alpha_k^{-1}-$regular classes of H_k into classes of $\overline{G}/N \cong G$,
- the Fischer matrices of $\overline{G}$.

Now with the above and for fixed $1 \leq i \leq r$ and $1 \leq k \leq t$, where r is the number of conjugacy classes of G, let $\mathcal{K}_{ik}$ be the fragment of the projective character table of H_k, with factor set α_k^{-1}, consisting of columns correspond to the conjugacy classes $g_{ik1}, g_{ik2}, \cdots, g_{ikr_{ik}}$ of H_k (which are the $\alpha_k^{-1}-$regular classes of H_k that fuse to $[g_i]_G$ and thus $r_{ik} = c(g_{ik})$). Then the characters of $\overline{G}$ on

the classes $[g_{ij}]_{\overline{G}}$, $1 \leq j \leq c(g_i)$, are given by the matrix $\mathcal{K}_{ik}\mathcal{F}_{ik}$, where $\mathcal{F}_{ik}$ is the sub-matrix of $\mathcal{F}_i$ defined previously with rows correspond to the pairs $(k, g_{ik1}), (k, g_{ik2}), \cdots, (k, g_{ikc(g_{ik})})$. Note that the size of $\mathcal{K}_{ik}$ is $|\mathrm{IrrProj}(H_k, \alpha_k^{-1})| \times r_{ik}$ and the size of $\mathcal{F}_{ik}$ is $r_{ik} \times c(g_i)$. Therefore the character table of $\overline{G}$ will have the form

	g_1				g_2				$\cdots$	g_r			
	g_{11}	g_{12}	$\cdots$	$g_{1c(g_1)}$	g_{21}	g_{22}	$\cdots$	$g_{2c(g_2)}$	$\cdots$	g_{r1}	g_{r2}	$\cdots$	$g_{rc(g_r)}$
$\mathcal{K}_1$	$\mathcal{K}_{11}\mathcal{F}_{11}$				$\mathcal{K}_{12}\mathcal{F}_{12}$				$\cdots$	$\mathcal{K}_{1r}\mathcal{F}_{1r}$			
$\mathcal{K}_2$	$\mathcal{K}_{21}\mathcal{F}_{21}$				$\mathcal{K}_{22}\mathcal{F}_{22}$				$\cdots$	$\mathcal{K}_{2r}\mathcal{F}_{2r}$			
$\vdots$	$\vdots$				$\vdots$				$\ddots$	$\vdots$			
$\mathcal{K}_t$	$\mathcal{K}_{t1}\mathcal{F}_{t1}$				$\mathcal{K}_{t2}\mathcal{F}_{t2}$				$\cdots$	$\mathcal{K}_{tr}\mathcal{F}_{tr}$			

Note 5.3.1. Observe that characters of $\overline{G}$ contained in the block $\mathcal{K}_1$ are just $\mathrm{Irr}(G)$ and therefore the size of $\mathcal{K}_{1i}\mathcal{F}_{1i}$, for each $1 \leq i \leq r$, is $|\mathrm{Irr}(G)| \times c(g_i)$. In particular, columns of $\mathcal{K}_{11}\mathcal{F}_{11}$ are the degrees of irreducible characters of G repeated themselves $c(g_1)$ times, where we know that $c(g_1)$ is the number of $\overline{G}-$conjugacy classes obtained from the normal subgroup N. Also note that $|\mathrm{Irr}(\overline{G})|$ satisfies the relation

$$|\mathrm{Irr}(\overline{G})| = \sum_{k=1}^{t} |\mathrm{IrrProj}(H_k, \alpha_k^{-1})| = |\mathrm{Irr}(G)| + \sum_{k=2}^{t} |\mathrm{IrrProj}(H_k, \alpha_k^{-1})|, \tag{5.7}$$

where α_k is a factor set of $M(H_k)$.

One of the biggest challenges in the theory of Clifford-Fischer matrices is the determination of the type of the character table of H_k (projective or ordinary), which is to be used in the construction of the character table of $\overline{G}$. We may firstly assume that all the irreducible characters of N are extendible to their respective inertia groups and consequently all the character tables of the H_k that we need to use are the ordinary ones. However, in general, there is no reason guaranteing that one can work with the ordinary characters of H_k, $2 \leq k \leq t$ (see Section 5.1 for some partial results on extendability of characters). Thus in practice making the right choice of the appropriate projective character table of H_k might be difficult unless the Schur multipliers of all the H_k are trivial. Otherwise one have to test all the possible choices, and each choice will lead to a contradiction, except for one case that will be successful. Sometimes Eq. (5.7) might also be useful to prove that we need to use projective characters of some of the inertia factors of a specific extension. For

example in Section 7.1 we have found that the group $\overline{G} = 2^{5\cdot}GL(5,2)$ has 41 conjugacy classes and we proved that the inertia factors are $H_1 = G = GL(5,2)$ and $H_2 = 2^4{:}GL(4,2)$, where we know that $|\mathrm{Irr}(H_1)| = 27$ and $|\mathrm{Irr}(H_2)| = 25$. Thus if we will use the ordinary character table of H_2 we get that $|\mathrm{Irr}(\overline{G})| = 25 + 27 = 52 \neq 41 =$ the number of conjugacy classes of $\overline{G}$. This simple argument tells that we have to make use of projective characters of H_2.

We conclude this section by remarking that from Pahlings and Lux [58], the character table of $\overline{G}$ in the format of Clifford-Fischer theory can be re-written as a product of two matrices, where the first one is a block diagonal matrix with the (projective) character tables of the inertia factors H_k on the diagonal and the second matrix being built up by the Fischer matrices and zeros, arranged in a suitable way.

Chapter 6

A Group of the Form $3^7{:}Sp(6,2)$

In this chapter we study a split extension group of the form $\overline{G} = 3^7{:}Sp(6,2)$, where in Section 6.1 we give a brief introduction and we generate $\overline{G}$ as a subgroup of $SL(8,3)$. In Section 6.2 we list the conjugacy classes of $\overline{G}$ obtained by applying the coset analysis technique and see that corresponding to the 30 conjugacy classes of $Sp(6,2)$, we get 118 classes for $\overline{G}$. In Section 6.3 through a long process we show that there are six inertia factors, namely $H_1 = Sp(6,2)$, $H_2 = U_4(2)$, $H_3 = 2^5{:}A_6$, $H_4 = A_7$, $H_5 = (3 \times A_6){:}2$ and $H_6 = (2^4{:}A_5){:}2$. In Section 6.4 we discuss the character tables of these groups and we supply these tables in the Appendix of this thesis. In Section 6.5 we calculate the 30 Fischer matrices of $\overline{G}$ and we see that the sizes of these matrices range between 2 and 10. In Section 6.6 we show how to obtain the character table of $\overline{G} = 3^7{:}Sp(6,2)$ via Clifford-Fischer theory. The full character table of $\overline{G}$ is given in the Appendix (see Table 11.8).

6.1. Introduction

Let $G = Sp(6,2)$ be the symplectic group of order 1451520. By the electronic Atlas of Wilson [71], the group G has a $7-$dimensional (absolutely) irreducible module over $GF(3) = \mathbb{F}_3 = \{0, 1, \xi\}$, where ξ is a primitive element of the field $\mathbb{F}_3$. Consequently a split extension of the form $3^7{:}Sp(6,2)$ does exist. Using the two 7×7 matrices over $\mathbb{F}_3$ that generate $Sp(6,2)$, supplied by the electronic Atlas of Wilson, we were able to construct G and then $\overline{G}$ inside GAP [30]. In fact we constructed the group $\overline{G}$ in GAP, in terms of 8×8 matrices over $\mathbb{F}_3$. The following three elements $\overline{g}_1, \overline{g}_2$ and $\overline{g}_3$ are $8-$dimensional matrices over $\mathbb{F}_3$ that generate $\overline{G}$.

$$\overline{g}_1 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & \xi & \xi & 0 & 0 & 0 & 0 \\ 0 & \xi & \xi & 0 & \xi & 0 & 0 & 0 \\ 0 & \xi & \xi & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{g}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & \xi & \xi & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \xi & 1 & \xi & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\overline{g}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

with $o(\overline{g}_1) = 2,\ o(\overline{g}_2) = 7$ and $o(\overline{g}_3) = 3$. However the group $\overline{G}$ can also be generated by the following two elements α_1 and α_2, where

$$\alpha_1 = \begin{pmatrix} \xi & \xi & 1 & \xi & \xi & 1 & 1 & 0 \\ 0 & 1 & 0 & \xi & \xi & \xi & 1 & 0 \\ 1 & \xi & 1 & 1 & 1 & \xi & 1 & 0 \\ \xi & \xi & 0 & 0 & 1 & 0 & \xi & 0 \\ \xi & 0 & 0 & 1 & \xi & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & \xi & 0 \\ 0 & \xi & \xi & \xi & 1 & 1 & 1 & 0 \\ 0 & 1 & \xi & 0 & 0 & 0 & 0 & \xi \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 1 & \xi & \xi & \xi & 1 & \xi & 0 & 0 \\ 1 & 0 & \xi & \xi & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & \xi & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & \xi & 0 & 1 & 0 \\ 1 & 1 & 0 & \xi & 1 & \xi & 0 & 0 \\ 1 & \xi & 1 & 1 & 1 & 0 & \xi & 0 \\ \xi & 0 & 0 & 0 & 0 & 0 & 1 & \xi \end{pmatrix},$$

with $o(\alpha_1) = 45$, $o(\alpha_2) = 45$ and $o(\alpha_1 \star \alpha_2) = 6$.

Corollary 6.1.1. $\overline{G} \leq SL(8,3)$ *with index 302 954 487 010 019 919 360.*

PROOF. This is readily verified since $\det(\alpha_1) = \det(\alpha_2) = 1$ and consequently $\det(\overline{g}) = 1$ for all $\overline{g} \in \overline{G}$. ■

Using GAP, one can easily check all the normal subgroups of $\overline{G}$. In fact the only proper normal subgroup of $\overline{G}$ is a group of order 2187 and thus must be isomorphic to the elementary abelian group $N = 3^7$. The following elements $n_1, n_2, \cdots, n_7$ are 8×8 matrices over $\mathbb{F}_3$ that are generators of N.

$$n_1 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 1 & \xi & \xi & 1 & 0 & \xi \end{pmatrix}, \quad n_2 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 0 & 1 & \xi & 1 & \xi & 0 & \xi & \xi \end{pmatrix},$$

$$n_3 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 0 & 1 & 1 & 0 & 1 & \xi & 0 & \xi \end{pmatrix}, \quad n_4 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & \xi \end{pmatrix},$$

$$n_5 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 1 & 0 & 1 & 0 & 0 & \xi & 1 & \xi \end{pmatrix}, \quad n_6 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 1 & \xi & 0 & 1 & \xi & \xi & \xi & \xi \end{pmatrix},$$

$$n_7 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 1 & 1 & 1 & 1 & \xi & 1 & 0 & \xi \end{pmatrix}.$$

In terms of 8×8 matrices over $\mathbb{F}_3$, the group $Sp(6,2)$ is generated by the following two elements g_1 and g_2 :

$$g_1 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 \\ 0 & \xi & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi \\ 0 & 0 & 1 & \xi & 1 & \xi & 0 & \xi \end{pmatrix},$$

with $o(g_1) = 2,\ o(g_2) = 7$ and $o(g_1 \star g_2) = 9$. Note that the group $Sp(6,2) = \langle g_1, g_2 \rangle$ together with the mentioned generators of N, gives the split extension $\overline{G} = 3^7{:}Sp(6,2)$.

6.2. The Conjugacy Classes of $\overline{G} = 3^7{:}Sp(6,2)$

In this section we use the coset analysis technique, discussed in Section 2.3, to construct the character table of $\overline{G} = 3^7{:}Sp(6,2)$. We have used GAP to build a small subroutine to find the values of $k_i = \chi(Sp(6,2)|3^7)$, (number of fixed points on the action of $N = \langle n_1, n_2, \cdots, n_7 \rangle$ on the coset $N\overline{g}_i = Ng_i$ and g_i is a representative of a conjugacy class of $Sp(6,2)$). We list these values in Table 6.1.

Table 6.1: Values of the fixed points of the action of $Sp(6,2)$ on 3^7

$[g_i]_G$	$[g_1]$	$[g_2]$	$[g_3]$	$[g_4]$	$[g_5]$	$[g_6]$	$[g_7]$	$[g_8]$	$[g_9]$	$[g_{10}]$	$[g_{11}]$	$[g_{12}]$	$[g_{13}]$
k_i	2187	3	27	243	27	243	27	27	27	27	3	3	27
$[g_i]_G$	$[g_{14}]$	$[g_{15}]$	$[g_{16}]$	$[g_{17}]$	$[g_{18}]$	$[g_{19}]$	$[g_{20}]$	$[g_{21}]$	$[g_{22}]$	$[g_{23}]$	$[g_{24}]$	$[g_{25}]$	$[g_{26}]$
k_i	27	3	27	3	27	3	3	3	3	3	3	3	3
$[g_i]_G$	$[g_{27}]$	$[g_{28}]$	$[g_{29}]$	$[g_{30}]$									
k_i	3	3	3	3									

In fact the subroutine we have used to find the values of k_i's can be developed further to find the values of f_{ij}'s (number of orbits (of the action of N on Ng_i) fused together under the action of $\overline{G} = \langle \alpha_1, \alpha_2 \rangle$), for each coset corresponding to $[g_i]_G$. A complete set of the f_{ij}'s and representatives for the conjugacy classes of $\overline{G}$ are given in Table 6.2.

Table 6.2: The conjugacy classes of $\overline{G} = 3^7$:$Sp(6,2)$

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{3^7:Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{3^7:Sp(6,2)}\rvert$	$\lvert C_{3^7:Sp(6,2)}(g_{ij})\rvert$
		$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	3174474240
		$f_{12} = 56$	$m_{12} = 56$	g_{12}	3	56	56687040
$g_1 = 1A$	$k_1 = 2187$	$f_{13} = 126$	$m_{13} = 126$	g_{13}	3	126	25194240
		$f_{14} = 576$	$m_{14} = 576$	g_{14}	3	576	5511240
		$f_{15} = 672$	$m_{15} = 672$	g_{15}	3	672	4723920
		$f_{16} = 756$	$m_{16} = 756$	g_{16}	3	756	4199040
$g_2 = 2A$	$k_2 = 3$	$f_{21} = 1$	$m_{21} = 729$	g_{21}	2	45927	69120
		$f_{22} = 2$	$m_{22} = 1458$	g_{22}	6	91854	34560
		$f_{31} = 1$	$m_{31} = 81$	g_{31}	2	25515	124416
$g_3 = 2B$	$k_3 = 27$	$f_{32} = 6$	$m_{32} = 486$	g_{32}	6	153090	20736
		$f_{33} = 8$	$m_{33} = 648$	g_{33}	6	204120	15552
		$f_{34} = 12$	$m_{34} = 972$	g_{34}	6	306180	10368
		$f_{41} = 1$	$m_{41} = 9$	g_{41}	2	8505	373248
		$f_{42} = 2$	$m_{42} = 18$	g_{42}	6	17010	186624
		$f_{43} = 8$	$m_{43} = 72$	g_{43}	6	68040	46656
		$f_{44} = 16$	$m_{44} = 144$	g_{44}	6	136080	23328
$g_4 = 2C$	$k_4 = 243$	$f_{45} = 16$	$m_{45} = 144$	g_{45}	6	136080	23328
		$f_{46} = 24$	$m_{46} = 216$	g_{46}	6	204120	15552
		$f_{47} = 32$	$m_{47} = 288$	g_{47}	6	272160	11664
		$f_{48} = 32$	$m_{48} = 288$	g_{48}	6	272160	11664
		$f_{49} = 48$	$m_{49} = 432$	g_{49}	6	408240	7776
		$f_{4,10} = 64$	$m_{4,10} = 576$	$g_{4,10}$	6	544320	5832
		$f_{51} = 1$	$m_{51} = 81$	g_{51}	2	306180	10368
$g_5 = 2D$	$k_5 = 27$	$f_{52} = 6$	$m_{52} = 486$	g_{52}	6	1837080	1728
		$f_{53} = 8$	$m_{53} = 648$	g_{53}	6	2449440	1296
		$f_{54} = 12$	$m_{54} = 972$	g_{54}	6	3674160	864
		$f_{61} = 1$	$m_{61} = 9$	g_{61}	3	6048	524880
		$f_{62} = 20$	$m_{62} = 180$	g_{62}	3	120960	26244
		$f_{63} = 30$	$m_{63} = 270$	g_{63}	3	181440	17496
$g_6 = 3A$	$k_6 = 243$	$f_{64} = 30$	$m_{64} = 270$	g_{64}	3	181440	17496
		$f_{65} = 12$	$m_{65} = 108$	g_{65}	9	72576	43740
		$f_{66} = 30$	$m_{66} = 270$	g_{66}	9	181440	17496
		$f_{67} = 120$	$m_{67} = 1080$	g_{67}	9	725760	4374

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Table 6.2 (continued)

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{3^7:Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{3^7:Sp(6,2)}\rvert$	$\lvert C_{3^7:Sp(6,2)}(g_{ij})\rvert$
		$f_{71}=1$	$m_{71}=81$	g_{71}	3	181440	17496
$g_7=3B$	$k_7=27$	$f_{72}=8$	$m_{72}=648$	g_{72}	3	1451520	2187
		$f_{73}=9$	$m_{73}=729$	g_{73}	9	1632960	1944
		$f_{74}=9$	$m_{74}=729$	g_{74}	9	1632960	1944
		$f_{81}=1$	$m_{81}=81$	g_{81}	3	1088640	2916
		$f_{82}=2$	$m_{82}=162$	g_{82}	3	2177280	1458
		$f_{83}=2$	$m_{83}=162$	g_{83}	9	2177280	1458
$g_8=3C$	$k_8=27$	$f_{84}=4$	$m_{84}=324$	g_{84}	9	4354560	729
		$f_{85}=6$	$m_{85}=486$	g_{85}	9	6531840	486
		$f_{86}=12$	$m_{86}=972$	g_{86}	9	13063680	243
		$f_{91}=1$	$m_{91}=81$	g_{91}	4	306180	10368
$g_9=4A$	$k_9=27$	$f_{92}=6$	$m_{92}=486$	g_{92}	12	1837080	1728
		$f_{93}=8$	$m_{93}=648$	g_{93}	12	2449440	1296
		$f_{94}=12$	$m_{94}=972$	g_{94}	12	3674160	864
		$f_{10,1}=1$	$m_{10,1}=81$	$g_{10,1}$	4	612360	5184
$g_{10}=4B$	$k_{10}=27$	$f_{10,2}=6$	$m_{10,2}=486$	$g_{10,2}$	12	3674160	864
		$f_{10,3}=8$	$m_{10,3}=648$	$g_{10,3}$	12	4898880	648
		$f_{10,4}=12$	$m_{10,4}=972$	$g_{10,4}$	12	734820	432
$g_{11}=4C$	$k_{11}=3$	$f_{11,1}=1$	$m_{11,1}=729$	$g_{11,1}$	4	5511240	576
		$f_{11,2}=2$	$m_{11,2}=1458$	$g_{11,2}$	12	11022480	288
$g_{12}=4D$	$k_{12}=3$	$f_{12,1}=1$	$m_{12,1}=729$	$g_{12,1}$	4	8266860	384
		$f_{12,2}=2$	$m_{12,2}=1458$	$g_{12,2}$	12	16533720	192
		$f_{13,1}=1$	$m_{13,1}=81$	$g_{13,1}$	4	3674160	864
		$f_{13,2}=2$	$m_{13,2}=162$	$g_{13,2}$	12	7348320	432
		$f_{13,3}=2$	$m_{13,3}=162$	$g_{13,3}$	12	7348320	432
$g_{13}=4E$	$k_{13}=27$	$f_{13,4}=2$	$m_{13,4}=162$	$g_{13,4}$	12	7348320	432
		$f_{13,5}=4$	$m_{13,5}=324$	$g_{13,5}$	12	14696640	216
		$f_{13,6}=4$	$m_{13,6}=324$	$g_{13,6}$	12	14696640	216
		$f_{13,7}=4$	$m_{13,7}=324$	$g_{13,7}$	12	14696640	216
		$f_{13,8}=8$	$m_{13,8}=648$	$g_{13,8}$	12	29393280	108

Continued on next page

Table 6.2 (continued)

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{3^7:Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{3^7:Sp(6,2)}\rvert$	$\lvert C_{3^7:Sp(6,2)}(g_{ij})\rvert$
		$f_{14,1}=1$	$m_{14,1}=81$	$g_{14,1}$	5	3919104	810
		$f_{14,2}=3$	$m_{14,2}=243$	$g_{14,2}$	15	11757312	270
		$f_{14,3}=3$	$m_{14,3}=243$	$g_{14,3}$	15	11757312	270
$g_{14}=5A$	$k_{14}=27$	$f_{14,4}=2$	$m_{14,4}=162$	$g_{14,4}$	15	7838208	405
		$f_{14,5}=6$	$m_{14,5}=486$	$g_{14,5}$	15	23514624	135
		$f_{14,6}=6$	$m_{14,6}=486$	$g_{14,6}$	15	23514624	135
		$f_{14,7}=6$	$m_{14,7}=486$	$g_{14,7}$	15	23514624	135
$g_{15}=6A$	$k_{15}=3$	$f_{15,1}=1$	$m_{15,1}=729$	$g_{15,1}$	6	7348320	432
		$f_{15,2}=2$	$m_{15,2}=1458$	$g_{15,2}$	6	14696640	216
		$f_{16,1}=1$	$m_{16,1}=81$	$g_{16,1}$	6	816480	3888
		$f_{16,2}=4$	$m_{16,2}=324$	$g_{16,2}$	6	3265920	972
$g_{16}=6B$	$k_{16}=27$	$f_{16,3}=4$	$m_{16,3}=324$	$g_{16,3}$	6	3265920	972
		$f_{16,4}=6$	$m_{16,4}=486$	$g_{16,4}$	6	4898880	648
		$f_{16,5}=12$	$m_{16,5}=972$	$g_{16,5}$	6	9797760	324
		$f_{17,1}=1$	$m_{17,1}=729$	$g_{17,1}$	6	14696640	216
$g_{17}=6C$	$k_{17}=3$	$f_{17,2}=1$	$m_{17,2}=729$	$g_{17,2}$	18	14696640	216
		$f_{17,3}=1$	$m_{17,3}=729$	$g_{17,3}$	18	14696640	216
		$f_{18,1}=1$	$m_{18,1}=81$	$g_{18,1}$	6	2449440	1296
		$f_{18,2}=2$	$m_{18,2}=162$	$g_{18,2}$	6	4898880	648
		$f_{18,3}=2$	$m_{18,3}=162$	$g_{18,3}$	6	4898880	648
$g_{18}=6D$	$k_{18}=27$	$f_{18,4}=4$	$m_{18,4}=324$	$g_{18,4}$	6	9797760	324
		$f_{18,5}=2$	$m_{18,5}=162$	$g_{18,5}$	18	4898880	648
		$f_{18,6}=4$	$m_{18,6}=324$	$g_{18,6}$	18	9797760	324
		$f_{18,7}=4$	$m_{18,7}=324$	$g_{18,7}$	18	9797760	324
		$f_{18,8}=8$	$m_{18,8}=648$	$g_{18,8}$	18	19595520	162
$g_{19}=6E$	$k_{19}=3$	$f_{19,1}=1$	$m_{19,1}=729$	$g_{19,1}$	6	39191040	108
		$f_{19,2}=2$	$m_{19,2}=1458$	$g_{19,2}$	6	78382080	54
$g_{20}=6F$	$k_{20}=3$	$f_{20,1}=1$	$m_{20,1}=729$	$g_{20,1}$	6	39191040	108
		$f_{20,2}=2$	$m_{20,2}=1458$	$g_{20,2}$	18	78382080	54
$g_{21}=6G$	$k_{21}=3$	$f_{21,1}=1$	$m_{21,1}=729$	$g_{21,1}$	6	88179840	36
		$f_{21,2}=2$	$m_{21,2}=1458$	$g_{21,2}$	18	176359680	18

Continued on next page

Table 6.2 (continued)

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{3^7:Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{3^7:Sp(6,2)}\rvert$	$\lvert C_{3^7:Sp(6,2)}(g_{ij})\rvert$
		$f_{22,1}=1$	$m_{22,1}=729$	$g_{22,1}$	7	151165440	21
$g_{22}=7A$	$k_{22}=3$	$f_{22,2}=1$	$m_{22,2}=729$	$g_{22,2}$	21	151165440	21
		$f_{22,3}=1$	$m_{22,3}=729$	$g_{22,3}$	21	151165440	21
$g_{23}=8A$	$k_{23}=3$	$f_{23,1}=1$	$m_{23,1}=729$	$g_{23,1}$	8	66134880	48
		$f_{23,2}=2$	$m_{23,2}=1458$	$g_{23,2}$	24	132269760	24
		$f_{24,1}=1$	$m_{24,1}=729$	$g_{24,1}$	8	66134880	48
$g_{24}=8B$	$k_{24}=3$	$f_{24,2}=1$	$m_{24,2}=729$	$g_{24,2}$	24	66134880	48
		$f_{24,3}=1$	$m_{24,3}=729$	$g_{24,3}$	24	66134880	48
		$f_{25,1}=1$	$m_{25,1}=729$	$g_{25,1}$	9	117573120	27
$g_{25}=9A$	$k_{25}=3$	$f_{25,2}=1$	$m_{25,2}=729$	$g_{25,2}$	9	117573120	27
		$f_{25,3}=1$	$m_{25,3}=729$	$g_{25,3}$	9	117573120	27
		$f_{26,1}=1$	$m_{26,1}=729$	$g_{26,1}$	10	105815808	30
$g_{26}=10A$	$k_{26}=3$	$f_{26,2}=1$	$m_{26,2}=729$	$g_{26,2}$	30	105815808	30
		$f_{26,3}=1$	$m_{26,3}=729$	$g_{26,3}$	30	105815808	30
$g_{27}=12A$	$k_{27}=3$	$f_{27,1}=1$	$m_{27,1}=729$	$g_{27,1}$	12	44089920	72
		$f_{27,2}=2$	$m_{27,2}=1458$	$g_{27,2}$	36	88179840	36
$g_{28}=12B$	$k_{28}=3$	$f_{28,1}=1$	$m_{28,1}=729$	$g_{28,1}$	12	44089920	72
		$f_{28,2}=2$	$m_{28,2}=1458$	$g_{28,2}$	12	88179840	36
		$f_{29,1}=1$	$m_{29,1}=729$	$g_{29,1}$	12	88179840	36
$g_{29}=12C$	$k_{29}=3$	$f_{29,2}=1$	$m_{29,2}=729$	$g_{25,2}$	36	88179840	36
		$f_{29,3}=1$	$m_{29,3}=729$	$g_{29,3}$	36	88179840	36
		$f_{30,1}=1$	$m_{30,1}=729$	$g_{30,1}$	15	70543872	45
$g_{30}=15A$	$k_{30}=3$	$f_{30,2}=1$	$m_{30,2}=729$	$g_{30,2}$	45	70543872	45
		$f_{30,3}=1$	$m_{30,3}=729$	$g_{30,3}$	45	70543872	45

Example 6.2.1. Consider the identity coset $N = 3^7$ as this coset is so important. Recall that N is abelian and thus each orbit of the action of N on itself consists of singleton. Therefore $k_1 = |N| = 2178$. Since we could present $\overline{G}$ and N in GAP in terms of 8×8 matrices over $\mathbb{F}_3$, it is easy to act $\overline{G}$ on N. In fact this action yielded six orbits of lengths 1, 56, 126, 576, 672 and 756 with representatives $g_{11}, g_{12}, g_{13}, g_{14}, g_{15}$ and g_{16} defined as follows:

$$g_{11} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \quad g_{12} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&1&0&0&\xi&\xi&1 \end{pmatrix},$$

$$g_{13} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&1&1&0&0&1 \end{pmatrix}, \quad g_{14} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&1&1&1&\xi&1 \end{pmatrix},$$

$$g_{15} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&\xi&1 \end{pmatrix}, \quad g_{16} = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&1 \end{pmatrix}.$$

Thus the identity coset affords six conjugacy classes in $\overline{G}$. These classes have sizes equal to the orbits lengths of $\overline{G}$ on N with respective representatives $g_{11}, g_{12}, g_{13}, g_{14}, g_{15}$ and g_{16}. Also the values of the f_{1j}'s are same as lengths of the corresponding conjugacy classes for all $1 \leq j \leq 6$. Clearly $g_{11} = 1_{\overline{G}}$ and thus $o(g_{11}) = 1$. Since $g_1 = 1_{Sp(6,2)}$ and $o(g_1) = 1$, it follows by applications of Proposition 2.3.3 that $o(g_{12}) = o(g_{13}) = o(g_{14}) = o(g_{15}) = o(g_{16}) = 3$. The orders of the preceding elements can also be seen directly since N is an elementary abelian 3−group.

6.3. Inertia Factor Groups of $\overline{G} = 3^7$:$Sp(6,2)$

In this section, through some favorable computations, we determine the inertia factor groups of the extension 3^7:$Sp(6,2)$. This determination is achieved by investigating the number of irreducible characters and fusions of conjugacy classes of some of the maximal subgroups of the maximal subgroups of $Sp(6,2)$ (sometimes we may go further and look at some of the maximal subgroups

of the maximal subgroups of the maximal subgroups of $Sp(6,2)$).

We have seen in Section 6.2 that the action of $\overline{G} = 3^7{:}Sp(6,2)$ (or just $G = Sp(6,2)$) on 3^7 produces six orbits of lengths 1, 56, 126, 576, 672 and 756. By Brauer Theorem (for example see Theorem 5.1.5 of Mpono [55]) the number of orbits of $\overline{G}$ (or just G) on $\mathrm{Irr}(3^7)$ will also be 6. Since $N = 3^7$ is a vector space, the action of G on $\mathrm{Irr}(3^7)$ can be viewed as the action of G on N^*, where N^* is the dual space of N. In fact we have found that the orbit lengths of G on $\mathrm{Irr}(3^7)$ are 1, 56, 126, 576, 672 and 756. Let $H_1, H_2, \cdots, H_6$ be the respective inertia factor groups of the representatives of characters from the previous orbits. We notice that these inertia factors have indices 1, 56, 126, 576, 672 and 756 respectively in $Sp(6,2)$. Clearly $H_1 = Sp(6,2)$. By looking at $\mathbb{ATLAS}$, the group $Sp(6,2)$ has 8 conjugacy classes of maximal subgroups. Let $M[1], M[2], \cdots, M[8]$ be representatives of these classes. That is $M[1] = U_4(2){:}2$, $M[2] = S_8$, $M[3] = 2^5{:}S_6$, $M[4] = U_3(3){:}2$, $M[5] = 2^6{:}L_3(2)$, $M[6] = (2^{1+4} \times 2^2){:}(S_3 \times S_3)$, $M[7] = S_3 \times S_6$ and $M[8] = PSL(2,8){:}3$. In Table 6.3 we give some information about the maximal subgroups of $M[1], M[2], \cdots, M[8]$. Now by considering the indices of the maximal subgroups of $Sp(6,2)$, we get the following possibilities for $H_1, H_2, \cdots, H_6$:

- $H_1 = Sp(6,2)$,
- $H_2 \leq M[1] = U_4(2){:}2$ with index 2,
- $H_3 \leq M[3] = 2^5{:}S_6$ with index 2,
- $H_4 \leq M[2] = S_8$ with index 16,
- $H_5 \leq M[1] = U_4(2){:}2$ with index 24 or a subgroup of $M[7] = S_3 \times S_6$ with index 2,
- $H_6 \leq M[1]$ with index 27, or a subgroup of $M[2]$ of index 21, or a subgroup of $M[3]$ with index 12.

Recall from Table 6.2 that the total number of conjugacy classes of $\overline{G}$ is 118. Using the results of Section 5.1, we deduce that the total contribution of irreducible characters from the six inertia factor groups must also be 118. That is

$$|\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| + |\mathrm{Irr}(H_4)| + |\mathrm{Irr}(H_5)| + |\mathrm{Irr}(H_6)| = 118. \tag{6.1}$$

6.3.1 First, Second, Third and Fourth Inertia Factor Groups

We recall that in Section 6.1 (Introduction), we represented the group $G = Sp(6,2)$ in terms of 8−dimensional matrices. For the sake of convenience in computations with GAP, we use 6−dimensional representations over $\mathbb{F}_2$ of $Sp(6,2)$.

We have used the following 6−dimensional matrices $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ over $\mathbb{F}_2$, that generate $Sp(6,2)$ (see [71]) to present $Sp(6,2)$ in GAP and then locate the maximal subgroups and the other required subgroups.

$$\widetilde{\alpha}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \widetilde{\alpha}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

We have mentioned that the first inertia factor group H_1 is $Sp(6,2)$, which has 30 irreducible characters. Since H_2 has an index 2 in $U_4(2)$:2, it is readily verified that $H_2 = M[11] = U_4(2)$, which has 20 irreducible characters. As a 6−dimensional subgroup of $Sp(6,2)$ over $\mathbb{F}_2$, the group H_2 is generated by σ_1 and σ_2, where

$$\sigma_1 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

The character table of $H_2 = U_4(2)$ is available in $\mathbb{ATLAS}$ and for convenience we have supplied it in Table 11.3. The fusion of classes of H_2 into classes of $Sp(6,2)$ are supplied in Table 6.6 or 6.11.

The third inertia factor group H_3 has an index 2 in 2^5:S_6. In Table 6.3 we supply some information on the maximal subgroups of the maximal subgroups of $Sp(6,2)$.

Table 6.3: Some information on the maximal subgroups of the maximal subgroups of $Sp(6,2)$

Maximal Subgroups of $Sp(6,2)$	$M[ij]$	$\lvert M[ij]\rvert$	$[M[i] : M[ij]]$	$\lvert\mathrm{Irr}(M[ij])\rvert$
$M[1] = U_4(2)$:2	$M[11] = U_4(2)$	25920	2	20
	$M[12] = (2^4{:}A_5){:}2$	1920	27	18
	$M[13] = 2 \times S_6$	1440	36	22
	$M[14] = ((((3 \times (3^2{:}2)){:}2){:}3){:}2){:}2$	1296	40	22
	$M[15] = ((3^{1+2}{:}Q_8){:}3){:}2$	1296	40	18
	$M[16] = (((((2 \times D_8){:}2){:}3){:}3){:}2){:}2$	1152	45	25
$M[2] = S_8$	$M[21] = A_8$	20160	2	14
	$M[22] = S_7$	5040	8	15
	$M[23] = 2 \times S_6$	1440	28	22
	$M[24] = (S_4 \times S_4){:}2$	1152	35	20
	$M[25] = S_5 \times S_3$	720	56	21
	$M[26] = ((((2 \times D_8){:}2){:}3){:}2){:}2$	384	105	20
	$M[27] = PSL(3,2){:}2$	336	120	9
$M[3] = 2^5{:}S_6$	$M[31] = 2^5{:}A_6$	11520	2	23
	$M[32] = (2^5{:}A_5){:}2$	3840	6	23
	$M[33] = 2 \times ((2^4{:}A_5){:}2)$	3840	6	36
	$M[34] = 2 \times ((S_4 \times S_4){:}2)$	2304	10	40
	$M[35] = (2^2 \times ((((2 \times D_8){:}2){:}3){:}2)){:}2$	1536	15	53
	$M[36] = ((((2 \times (2^4{:}2)){:}2){:}3){:}2){:}2$	1536	15	40
	$M[37] = 2 \times S_6$	1440	16	22
	$M[38] = 2 \times S_6$	1440	16	22
$M[4] = U_3(3)$:2	$M[41] = PSU(3,3)$	6048	2	14
	$M[42] = (3^{1+2}{:}8){:}2$	432	28	14
	$M[43] = PSL(3,2){:}2$	336	36	9
	$M[44] = ((4^2{:}3){:}2){:}2$	192	63	14
	$M[45] = (SL(2,3){:}4){:}2$	192	63	17

Continued on next page

Table 6.3 (continued)

Maximal Subgroups of $Sp(6,2)$	$M[ij]$	$\|M[ij]\|$	$[M[i] : M[ij]]$	$\|\mathrm{Irr}(M[ij])\|$
$M[5] = 2^6{:}L_3(2)$	$M[51] = (((2^2 \times ((2 \times D_8){:}2)){:}3){:}2){:}2$	1536	7	40
	$M[52] = ((((2 \times (2^4{:}2)){:}2){:}3){:}2){:}2$	1536	7	40
	$M[53] = 2^3{:}PSL(3,2)$	1344	8	11
	$M[34] = 2^3{\cdot}PSL(3,2)$	1344	8	11
	$M[55] = (2^6{:}7){:}3$	1344	8	16
$M[6] = (2^{1+4} \times 2^2){:}(S_3 \times S_3)$	$M[61] = ((2^2 \times (((2 \times D_8){:}2){:}3)){:}3){:}2$	2304	2	36
	$M[62] = ((2^2 \times (((2 \times D_8){:}2){:}3)){:}3){:}2$	2304	2	33
	$M[63] = ((2^2 \times (((2 \times D_8){:}2){:}3)){:}3){:}2$	2304	2	39
	$M[64] = (2^2 \times ((((2 \times D_8){:}2){:}3){:}2)){:}2$	1536	3	40
	$M[65] = (((2^2 \times ((2 \times D_8){:}2)){:}3){:}2){:}2$	1536	3	53
	$M[66] = (((((2 \times D_8){:}2){:}3){:}3){:}2){:}2$	1152	4	25
	$M[67] = 2 \times S_4 \times S_3$	288	16	30
$M[7] = S_3 \times S_6$	$M[71] = A_6 \times S_3$	2160	2	21
	$M[72] = (3 \times A_6){:}2$	2160	2	18
	$M[73] = 3 \times S_6$	2160	2	33
	$M[74] = 2 \times S_6$	1440	3	22
	$M[75] = S_3 \times S_5$	720	6	21
	$M[76] = S_5 \times S_3$	720	6	21
	$M[77] = ((S_3 \times S_3){:}2) \times S_3$	432	10	27
	$M[78] = 2 \times S_4 \times S_3$	288	15	30
	$M[79] = 2 \times S_3 \times S_4$	288	15	30
$M[8] = PSL(2,8){:}3$	$M[81] = PSL(2,8)$	504	3	9
	$M[82] = (2^3{:}7){:}3$	168	9	8
	$M[83] = (9{:}3){:}2$	54	28	10
	$M[84] = (7{:}3){:}2$	42	36	7

By checking the indices of the maximal subgroups of $M[3] = 2^5{:}S_6$, we can see that H_3 is in the conjugacy class of maximal subgroups of $M[3]$ containing $M[31] = 2^5{:}A_6$. Thus we can take $H_3 = M[31]$. Note that $M[31] = 2^5{:}A_6 = \langle \xi_1, \xi_2 \rangle$, where

$$\xi_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \xi_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Note 6.3.1. Observe that the group $H_3 = 2^5{:}A_6$ has 23 irreducible characters. The fusion of the conjugacy classes of $2^5{:}A_6$ into classes of $Sp(6,2)$ and the full character table of $2^5{:}A_6$ are listed in Tables 6.7 (or 6.11) and 11.4 respectively.

Next we determine the fourth inertia factor group H_4, which sits inside $M[2] = S_8$. Note that $[S_8 : H_4] = 16$ and thus (using Table 6.3) H_4 is either a subgroup of $M[21] = A_8$ with index 8, or it is a subgroup of $M[22] = S_7$ with index 2. In Table 6.4, we give some information about the maximal subgroups of $M[21]$ and $M[22]$.

Table 6.4: Some information on the maximal subgroups of $M[21]$ and $M[22]$

Maximal Subgroups of $M[12]$ & $M[22]$	$M[ijk]$	$\lvert M[ijk]\rvert$	$[M[ij] : M[ijk]]$	$\lvert\mathrm{Irr}(M[ijk])\rvert$
$M[21] = A_8$	$M[211] = A_7$	2520	8	9
	$M[212] = 2^3{:}PSL(3,2)$	1344	15	11
	$M[213] = 2^3{:}PSL(3,2)$	1344	15	11
	$M[214] = S_6$	720	28	11
	$M[215] = ((A_4 \times A_4){:}2){:}2$	576	35	16
	$M[216] = GL(2,4){:}2$	360	56	12
$M[22] = S_7$	$M[221] = A_7$	2520	2	9
	$M[222] = S_6$	720	7	11
	$M[223] = 2 \times S_5$	240	21	14
	$M[224] = S_4 \times S_3$	144	35	15
	$M[225] = (7{:}3){:}2$	42	120	7

From Table 6.4, we can see that a subgroup of $M[21] = A_8$ of index 8 must be isomorphic to A_7, while a subgroup of $M[22] = S_7$ of index 2 must also be isomorphic to A_7. Note that $M[211] = A_7 = \langle \beta_1, \beta_2 \rangle$ and $M[221] = A_7 = \langle \gamma_1, \gamma_2 \rangle$, where

$$\beta_1 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix},$$

$$\gamma_1 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Thus for the construction of the character table of $\overline{G}$, it will not make difference to which A_7 we choose. Hence we may take H_4 to be $M[211] \cong A_7$ and note that $|\mathrm{Irr}(H_4)| = |\mathrm{Irr}(A_7)| = 9$. The fusion of classes of A_7 into classes of $Sp(6,2)$ together with the character table of A_7 are given in Tables 6.8 (or 6.11) and 11.5 respectively.

6.3.2 Fifth and Sixth Inertia Factor Groups

From the last subsection we have seen that $|\mathrm{Irr}(H_1)| = 30$, $|\mathrm{Irr}(H_2)| = 20$, $|\mathrm{Irr}(H_3)| = 23$ and $|\mathrm{Irr}(H_4)| = 9$. Substituting these into Eq. (6.1), we get that $|\mathrm{Irr}(H_5)| + |\mathrm{Irr}(H_6)| = 36$.

Recall that H_5 is either an index 24 subgroup of $M[1] = U_4(2)$:2 or it is an index 2 subgroup of $M[7] = S_3 \times S_6$. If $H_5 \leq U_4(2)$:2 with index 24, then the only possibility (see Table 6.3) is that $H_5 \leq U_4(2)$ with $[U_4(2) : H_5] = 12$. However by looking at $\mathbb{ATLAS}$ we can see that the group $U_4(2)$ does not contain a subgroup of index 12. This leaves us with the other possibility, that is $H_5 \leq S_3 \times S_6$ and $[S_3 \times S_6 : H_5] = 2$. From Table 6.3, we can see that there are three classes of non-conjugate maximal subgroups of $S_3 \times S_6$, such that a subgroup of each class has an index 2. Therefore H_5 is either $M[71] = A_6 \times S_3$, $M[72] = (3 \times A_6)$:2 or $M[73] = 3 \times S_6$. Hence by the last column of Table 6.3, it follows that $|\mathrm{Irr}(H_5)| \in \{21, 18, 33\}$. We take this point into our considerations and we look at the group H_6.

The index of the sixth inertia factor group H_6 in $Sp(6,2)$ is 756. This forces H_6 to be either a subgroup of $U_4(2)$:2 with index 27, or a subgroup of S_8 of index 21, or a subgroup of 2^5:S_6 with index 12. However the second possibility ($H_6 \leq S_8$) is not feasible since S_8 does not contain a subgroup

of index that is a divisor of 27 (see Table 6.3). If H_6 is a subgroup of $U_4(2)$:2 of index 27, then it must be in the conjugacy class of maximal subgroups of $U_4(2)$:2 containing $M[12] = (2^4{:}A_5){:}2$. The group $M[12] = (2^4{:}A_5){:}2$ is generated by π_1 and π_2, where

$$\pi_1 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \quad \pi_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

and $|\mathrm{Irr}(H_6)| = 18$. On the other hand if $H_6 \leq 2^5{:}S_6$ such that $[2^5{:}S_6 : H_6] = 12$, then three possibilities arise (see Table 6.3):

- $H_6 \leq M[31] = 2^5{:}A_6$ with index 6,
- $H_6 \leq M[32] = (2^5{:}A_5){:}2$ with index 2 or
- $H_6 \leq M[33] = 2 \times ((2^4{:}A_5){:}2)$ with index 2.

In Table 6.5, we provide some information on the maximal subgroups of $M[31]$, $M[32]$ and $M[33]$.

From Table 6.5, we can see that there are 6 possibilities for H_6. This together with the option $H_6 = M[12] = (2^4{:}A_5){:}2$ gives that $H_6 \in \{M[12], M[311], M[312], M[321], M[331], M[332], M[333]\}$ and we notice (in a respective way) that $|\mathrm{Irr}(H_6)| \in \{18, 16, 24, 16, 18, 24, 18\}$. We recall that $|\mathrm{Irr}(H_5)| \in \{21, 18, 33\}$. Therefore possible pairs representing (H_5, H_6) are

$$\begin{aligned}(H_5, H_6) \in \{&(M[71], M[12]), (M[71], M[311]), (M[71], M[312]), (M[71], M[321]), (M[71], M[331]),\\ &(M[71], M[332]), (M[71], M[333]), (M[72], M[12]), (M[72], M[311]), (M[72], M[312]),\\ &(M[72], (M[321]), (M[72], M[331]), (M[72], M[332]), (M[72], M[333]), (M[73], M[12]),\\ &(M[73], (M[311]), (M[73], M[312]), (M[73], M[321]), (M[71], M[331]), (M[71], M[332]),\\ &(M[73], M[333])\}\end{aligned}$$

and it follows respectively that

$$\begin{aligned}(|\mathrm{Irr}(H_5)|, |\mathrm{Irr}(H_6)|) \in \{&(21, 18), (21, 16), (21, 24), (21, 16), (21, 18), (21, 24), (21, 18),\\ &(18, 18), (18, 16), (18, 24), (18, 16), (18, 18), (18, 24), (18, 18),\\ &(33, 18), (33, 16), (33, 24), (33, 16), (33, 18), (33, 24), (33, 18)\}.\end{aligned}$$

Table 6.5: Some information on the maximal subgroups of $M[31], M[32]$ and $M[33]$

Maximal Subgroups of $M[31]$, $M[32]$ & $M[33]$	$M[ijk]$	$\lvert M[ijk]\rvert$	$[M[ij] : M[ijk]]$	$\lvert\mathrm{Irr}(M[ijk])\rvert$
$M[31] = 2^5{:}A_6$	$M[311] = 2^5{:}A_5$	1920	6	16
	$M[312] = 2 \times (2^4{:}A_5)$	1920	6	24
	$M[313] = 2 \times ((A_4 \times A_4){:}4)$	1152	10	26
	$M[314] = (2 \times ((((2 \times D_8){:}2){:}3){:}2)){:}2$	768	15	31
	$M[315] = (((2 \times (2^4{:}2)){:}2){:}3){:}2$	768	15	23
	$M[316] = 2 \times A_6$	720	16	14
	$M[317] = 2 \times A_6$	720	16	14
$M[32] = (2^5{:}A_5){:}2$	$M[321] = 2^5{:}A_5$	1920	2	16
	$M[322] = (((2 \times (2^4{:}2)){:}2){:}3){:}2$	768	5	23
	$M[323] = 2 \times ((2^4{:}5){:}4)$	640	6	22
	$M[324] = 2 \times (((2^4{:}3){:}2){:}2)$	384	10	28
	$M[325] = 2 \times S_5$	240	16	14
	$M[326] = 2 \times S_5$	240	16	14
$M[33] = 2 \times ((2^4{:}A_5){:}2)$	$M[331] = (2^4{:}A_5){:}2$	1920	2	18
	$M[332] = 2 \times (2^4{:}A_5)$	1920	2	24
	$M[333] = (2^4{:}A_5){:}2$	1920	2	18
	$M[334] = 2 \times (((((2 \times D_8){:}2){:}3){:}2){:}2)$	768	5	40
	$M[335] = 2 \times ((2^4{:}5){:}4)$	640	6	22
	$M[336] = 2 \times S_4 \times D_8$	384	10	50
	$M[337] = 2 \times S_5$	240	16	14

Since $|\mathrm{Irr}(H_5)| + |\mathrm{Irr}(H_6)| = 36$, we deduce that

$$(|\mathrm{Irr}(H_5)|, |\mathrm{Irr}(H_6)|) \in \{(18,18), (18,18), (18,18)\},$$

that is

$$(H_5, H_6) \in \{(M[72], M[12]), (M[72], M[331]), (M[72], M[333])\}.$$

Hence $H_5 = M[72] = (3 \times A_6){:}2$ (in all the cases) and $H_6 = M[12] = (2^4{:}A_5){:}2,\ M[331] = (2^4{:}A_5){:}2$ or $M[333] = (2^4{:}A_5){:}2$. Note that the group $M[72] = (3 \times A_6){:}2$ is generated by μ_1 and μ_2, where

$$\mu_1 = \begin{pmatrix} 1&0&1&0&0&1\\ 0&1&1&0&0&1\\ 0&1&0&1&0&1\\ 1&1&0&0&1&1\\ 1&1&1&0&1&1\\ 0&1&0&1&1&1 \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} 0&1&1&0&0&1\\ 1&1&1&0&1&1\\ 1&1&0&1&1&1\\ 1&0&0&0&1&1\\ 1&0&1&0&0&1\\ 1&1&0&0&1&1 \end{pmatrix}.$$

The fusion of the conjugacy classes of $H_5 = (3 \times A_6)$:2 into classes of $Sp(6,2)$ and the full character table of H_5 are listed in Tables 6.9 (or 6.11) and 11.6 respectively. Next we determine the group H_6.

The groups $M[12]$, $M[331]$ and $M[333]$ are generated as follows: $M[12] = \langle \theta_1, \theta_2 \rangle$, $M[331] = \langle \epsilon_1, \epsilon_2 \rangle$ and $M[333] = \langle \delta_1, \delta_2 \rangle$, where

$$\theta_1 = \begin{pmatrix} 1&0&0&1&0&0\\ 0&1&0&1&1&0\\ 1&0&0&0&1&1\\ 0&0&0&1&0&0\\ 1&0&1&1&1&0\\ 1&0&0&1&1&0 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} 1&0&1&0&0&1\\ 0&1&0&0&1&1\\ 1&0&0&1&1&1\\ 0&0&0&1&0&0\\ 1&0&0&0&0&0\\ 0&0&1&0&0&0 \end{pmatrix},$$

$$\epsilon_1 = \begin{pmatrix} 1&0&1&0&0&1\\ 0&1&0&0&1&1\\ 1&0&0&1&1&1\\ 0&0&0&1&0&0\\ 1&0&0&0&0&0\\ 0&0&1&0&0&0 \end{pmatrix}, \quad \epsilon_2 = \begin{pmatrix} 1&0&0&1&0&0\\ 0&1&0&1&1&0\\ 1&0&0&0&1&1\\ 0&0&0&1&0&0\\ 1&0&1&1&1&0\\ 1&0&0&1&1&0 \end{pmatrix},$$

$$\delta_1 = \begin{pmatrix} 0&1&0&0&0&1\\ 0&0&1&0&1&0\\ 1&1&1&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&1&1\\ 0&0&0&1&0&0 \end{pmatrix}, \quad \delta_2 = \begin{pmatrix} 0&1&0&0&0&0\\ 1&1&1&1&0&1\\ 0&0&1&1&0&0\\ 1&1&0&0&1&0\\ 1&1&0&0&0&0\\ 1&1&0&1&1&1 \end{pmatrix}.$$

By looking at the fusion of the conjugacy classes of $M[12] = \langle \theta_1, \theta_2 \rangle$, we see that the unique conjugacy class of involutions with size 20, does fuse into the class $g_2 = 2A$ of $Sp(6,2)$. From Table 6.3 we see that $c(g_2) = 2$ and hence from the properties of the Fischer matrices (Section 5.1), we have $\sum_{k=1}^{t} c(g_{ik}) = c(g_i)$ and for $i = 2$ we get $\sum_{k=1}^{6} c(g_{2k}) = c(g_2) = 2$. From Table 6.11, we can see that the classes $2A$ of $H_1 = Sp(6,2)$ and $2a$ of H_3 are both fusing into the class $2A$

of $Sp(6,2)$. Therefore if $M[12]$ is the sixth inertia factor group, then we get a contradiction (the Fischer matrix $\mathcal{F}_2$ corresponds to $g_2 = 2A$ will be of size 3×2 contradicting Proposition 5.2.2(i)). By similar arguments we can show that the group $M[333]$ can not be H_6. Hence we deduce that $M[331] = (2^4{:}A_5){:}2$, with the generators ϵ_1 and ϵ_2, is the sixth inertia factor group H_6. The fusion of classes of $H_6 = (2^4{:}A_5){:}2$ into classes of $Sp(6,2)$ can be viewed in Table 6.10 or more clearly in Table 6.11. This completes our determination of the inertia factor groups of the extension $\overline{G} = 3^7{:}Sp(6,2)$.

Note 6.3.2. If θ_i is an orbit representative of the action of G on $\mathrm{Irr}(N)$, then H_i can be obtained by GAP as G_{θ_i}, the set stabilizer of θ_i in G.

6.3.3 Fusions of Inertia Factor Groups into $Sp(6,2)$

In this section we determine the fusions of classes of the inertia factor groups $H_2,\ H_3,\ H_4,\ H_5$ and H_6 into classes of $Sp(6,2)$. We have used the permutation characters of $Sp(6,2)$ on the inertia factor groups and the centralizer sizes to determine these fusions. We have used Proposition 3.5.6, which is very helpful in calculating the permutation characters $\chi(Sp(6,2)|H_i),\ 2 \leq i \leq 6$.

The decompositions of the permutation characters $\chi(Sp(6,2)|M[i]),\ 1 \leq i \leq 8$ are all given in $\mathbb{ATLAS}$. Also it is not difficult to calculate $\chi(M[i]|M[ij]),\ 1 \leq i \leq 8,\ j$ is the number of conjugacy classes of maximal subgroups of $M[i]$. Where it is needed, it is also not difficult to calculate the permutation character $\chi(M[ij]|M[ijk]),\ 1 \leq i \leq 8,\ j$ is the number of conjugacy classes of maximal subgroups of $M[i],\ k$ is the number of conjugacy classes of maximal subgroups of $M[ij]$. Thus with the applications of Proposition 3.5.6, the values of $\chi(Sp(6,2)|H_i),\ 2 \leq i \leq 6$ are easy to calculate. In fact we have listed these values at the bottom of Tables 6.6, 6.7, 6.8, 6.9 and 6.10. Now with the aid of the permutation characters, centralizer sizes and matrix conjugation in $Sp(6,2)$, we are able to determine all the fusions of classes of the inertia factor groups into classes of $Sp(6,2)$. We list these fusions in Tables 6.6, 6.7, 6.8, 6.9 and 6.10. We would like to remark that in each of the mentioned tables, we have listed only those classes of $Sp(6,2)$ that could be a possible candidate for fusions. For example since the group $H_2 = U_4(2)$ does not contain elements of orders 7, 8, 10 and 15, we omitted the classes $7A,\ 8A,\ 8B,\ 10A$ and $15A$ of $Sp(6,2)$ in Table 6.6. For convenience and for more clarity, we summarized all the fusions of classes of $H_2,\ H_3,\ H_4,\ H_5$ and H_6 into classes of $Sp(6,2)$, which are supplied in Tables 6.6, 6.7, 6.8, 6.9 and 6.10, into a single Table, namely Table 6.11.

Table 6.6: The fusion of classes of $H_2 = U_4(2)$ into classes of $G = Sp(6,2)$

	$[g]_G$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$4E$
	$\lvert C_G(g)\rvert$	1451520	23040	4608	1536	384	2160	648	108	384	192	192	128	32
$[h]_{H_2}$	$\lvert C_{H_2}(h)\rvert$													
$1a$	25920	[56]												
$2a$	96		240	48	[16]	4								
$2b$	576		40	[8]										
$3a$	108						[20]	6	1					
$3b$	648							[1]						
$3c$	648							[1]						
$3d$	54						40	12	[2]					
$4a$	8									48	24	24	16	[4]
$4b$	48									[8]	4	4		
	$\chi(G\vert H_2)$	56	0	8	16	0	20	2	2	8	0	0	0	4

Table 6.6 (continued)

	$[g]_G$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$9A$	$12A$	$12B$	$12C$
	$\lvert C_G(g)\rvert$	1451520	23040	4608	1536	384	2160	648	108	384	192	192	128
$[h]_{H_2}$	$\lvert C_{H_2}(h)\rvert$												
$5a$	5	[6]											
$6a$	36		4	[4]	2		1	1					
$6b$	36		4	[4]	2		1	1					
$6c$	72		2	2	[1]								
$6d$	72		2	2	[1]								
$6e$	18		8	8	4		2	[2]					
$6f$	12		12	12	6	[4]	3	3	1				
$9a$	9									[1]			
$9b$	9									[1]			
$12a$	12										2	2	[1]
$12b$	12										2	2	[1]
	$\chi(G\vert H_2)$	6	0	8	2	4	0	2	0	2	0	0	2

Table 6.7: The fusion of classes of $H_3 = 2^5{:}A_6$ into classes of $G = Sp(6,2)$

	$[g]_G$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$4E$
	$\lvert C_G(g)\rvert$	1451520	23040	4608	1536	384	2160	648	108	384	192	192	128	32
$[h]_{H_3}$	$\lvert C_{H_3}(h)\rvert$													
$1a$	11520	[126]												
$2a$	11520		[2]											
$2b$	768		30	[6]	2									
$2c$	768		30	6	[2]									
$2d$	64		360	72	[24]	6								
$2e$	64		360	72	24	[6]								
$3a$	72						[30]	9						
$3b$	18						120	36	[6]					
$4a$	64									6	3	3	[2]	
$4b$	64									[6]	3	3	2	
$4c$	16									24	12	12	8	[2]
$4d$	16									24	12	12	8	[2]
$4e$	16									24	[12]	12	8	2
	$\chi(G\vert H_3)$	126	2	6	26	6	30	0	6	6	12	0	2	4

Table 6.7 (continued)

	$[g]_G$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$8A$	$8B$	$10A$
	$\lvert C_G(g)\rvert$	30	144	144	72	48	36	36	12	16	16	10
$[h]_{H_3}$	$\lvert C_{H_3}(h)\rvert$											
$5a$	10	[3]										
$5b$	10	[3]										
$6a$	24		6	[6]	3	2						
$6b$	24		6	6	3	[2]						
$6c$	72		[2]	2	1							
$6d$	18		8	8	4		[2]	2				
$8a$	16									1	[1]	
$8b$	16									1	[1]	
$10a$	10											[1]
$10b$	10											[1]
	$\chi(G\vert H_3)$	6	2	6	0	2	2	0	0	0	2	2

Table 6.8: The fusion of classes of $H_4 = A_7$ into classes of $G = Sp(6,2)$

	$[g]_G$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$4E$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$7A$
	$\lvert C_G(g)\rvert$	1451520	23040	4608	1536	384	2160	648	108	384	192	192	128	32	30	144	144	72	48	36	36	12	7
$[h]_{H_4}$	$\lvert C_{H_4}(h)\rvert$																						
$1a$	2520	[576]																					
$2a$	24		960	192	[64]	16																	
$3a$	9						240	72	[12]														
$3b$	36						[60]	18	3														
$4a$	4									96	48	48	32	[8]									
$5a$	5														[6]								
$6a$	12															12	12	6	[4]	3	3	1	
$7a$	7																						[1]
$7b$	7																						[1]
	$\chi(G\vert H_4)$	576	0	0	64	0	60	0	12	0	0	0	0	8	6	0	0	0	4	0	0	0	2

Table 6.9: The fusion of classes of $H_5 = (3 \times A_6)$:2 into classes of $G = Sp(6,2)$

	$[g]_G$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$4E$
	$\lvert C_G(g)\rvert$	1451520	23040	4608	1536	384	2160	648	108	384	192	192	128	32
$[h]_{H_5}$	$\lvert C_{H_5}(h)\rvert$													
$1a$	2160	672												
$2a$	48		480	96	32	8								
$2b$	48		480	96	32	8								
$2c$	48		480	96	32	8								
$3a$	1080						2							
$3b$	54						40	12	2					
$3c$	27						80	24	4					
$3d$	27						80	24	4					
$3e$	54						40	12	2					
$4a$	24									16	8	8		
$4b$	8									48	24	24	16	4
	$\chi(G\lvert H_5)$	672	0	0	64	8	42	24	6	0	8	0	0	4

Table 6.9 (continued)

	$[g]_G$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$12A$	$12B$	$12C$	$15A$
	$\lvert C_G(g)\rvert$	30	144	144	72	48	36	36	12	24	24	12	15
$[h]_{H_5}$	$\lvert C_{H_5}(h)\rvert$												
$5a$	15	2											
$6a$	24		6	6	3	2							
$6b$	6		24	24	12	8	6	6	2				
$6c$	6		24	24	12	8	6	6	2				
$12a$	12									2	2	1	
$15a$	15												1
$15b$	15												1
	$\chi(G\lvert H_5)$	2	0	0	0	10	0	0	2	2	0	0	2

Table 6.10: The fusion of classes of $H_6 = (2^4{:}A_5){:}2$ into classes of $G = Sp(6,2)$

	$[g]_G$	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$4E$
	$\lvert C_G(g)\rvert$	1451520	23040	4608	1536	384	2160	648	108	384	192	192	128	32
$[h]_{H_6}$	$\lvert C_{H_6}(h)\rvert$													
$1a$	1920	756												
$2a$	192		120	24	8	2								
$2b$	384		60	12	4	1								
$2c$	32		720	144	48	12								
$2d$	32		720	144	48	12								
$2e$	96		240	48	16	4								
$3a$	24						90	27						
$4a$	32									12	6	6	4	1
$4b$	16									24	12	12	8	2
$4c$	32									12	6	6	4	1
$4d$	96									4	2	2		
$4e$	8									48	24	24	16	4
	$\chi(G\lvert H_6)$	756	0	12	72	12	90	0	0	12	6	2	0	6

Table 6.10 (continued)

	$[g]_G$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$8A$	$8B$	$12A$	$12B$	$12C$
	$\lvert C_G(g)\rvert$	30	144	144	72	48	36	36	12	16	16	24	24	12
$[h]_{H_6}$	$\lvert C_{H_6}(h)\rvert$													
$5a$	5	6												
$6a$	12		12	12	6	4	3	3	1					
$6b$	24		6	6	3	2								
$6c$	12		12	12	6	4	3	3	1					
$8a$	8									2	2			
$12a$	12											2	2	1
	$\chi(G\lvert H_6)$	6	0	12	0	6	0	0	0	2	0	0	2	0

Table 6.11: The fusion of conjugacy classes of the inertia factor groups into classes of $Sp(6,2)$

Inertia Factor Groups H_2, H_3, H_4, H_5 & H_6	Class of H_i	$\hookrightarrow$	Class of $Sp(6,2)$	Class of H_i	$\hookrightarrow$	Class of $Sp(6,2)$
$H_2 = M[11] = U_4(2)$	$1a = g_{121}$		$1A$	$6a = g_{16,21}$		$6B$
	$2a = g_{421}$		$2C$	$6b = g_{16,22}$		$6B$
	$2b = g_{321}$		$2B$	$6c = g_{17,21}$		$6C$
	$3a = g_{621}$		$3A$	$6d = g_{17,22}$		$6C$
	$3b = g_{721}$		$3B$	$6e = g_{20,21}$		$6F$
	$3c = g_{722}$		$3B$	$6f = g_{18,21}$		$6D$
	$3d = g_{821}$		$3C$	$9a = g_{25,21}$		$9A$
	$4a = g_{13,21}$		$4E$	$9b = g_{25,22}$		$9A$
	$4b = g_{921}$		$4A$	$12a = g_{29,21}$		$12C$
	$5a = g_{14,21}$		$5A$	$12b = g_{29,22}$		$12C$
$H_3 = M[31] = 2^5{:}A_6$	$1a = g_{131}$		$1A$	$4e = g_{10,31}$		$4B$
	$2a = g_{231}$		$2A$	$5a = g_{14,31}$		$5A$
	$2b = g_{331}$		$2B$	$5b = g_{14,32}$		$5A$
	$2c = g_{431}$		$2C$	$6a = g_{16,31}$		$6B$
	$2d = g_{432}$		$2C$	$6b = g_{18,31}$		$6D$
	$2e = g_{531}$		$2D$	$6c = g_{15,31}$		$6A$
	$3a = g_{631}$		$3A$	$6d = g_{19,31}$		$6E$
	$3b = g_{831}$		$3C$	$8a = g_{24,31}$		$8B$
	$4a = g_{12,31}$		$4D$	$8b = g_{24,32}$		$8B$
	$4b = g_{931}$		$4A$	$10a = g_{26,31}$		$10A$
	$4c = g_{13,31}$		$4E$	$10b = g_{26,32}$		$10A$
	$4d = g_{13,32}$		$4E$			
$H_4 = M[211] = A_7$	$1a = g_{141}$		$1A$	$5a = g_{14,41}$		$5A$
	$2a = g_{441}$		$2C$	$6a = g_{18,41}$		$6D$
	$3a = g_{841}$		$3C$	$7a = g_{22,41}$		$7A$
	$3b = g_{641}$		$3A$	$7b = g_{22,42}$		$7A$
	$4a = g_{13,41}$		$4E$			

Continued on next page

Inertia Factor Groups $H_2,\ H_3,$ $H_4,\ H_5$ & H_6	Class of H_i	$\hookrightarrow$ Class of $Sp(6,2)$	Class of H_i	$\hookrightarrow$ Class of $Sp(6,2)$
$H_5 = M[72] = (3 \times A_6)$:2	$1a = g_{151}$	$1A$	$4a = g_{10,51}$	$4B$
	$2a = g_{451}$	$2C$	$4b = g_{13,51}$	$4E$
	$2b = g_{452}$	$2C$	$5a = g_{14,51}$	$5A$
	$2c = g_{551}$	$2D$	$6a = g_{18,51}$	$6D$
	$3a = g_{651}$	$3A$	$6b = g_{18,52}$	$6D$
	$3b = g_{652}$	$3A$	$6c = g_{21,51}$	$6G$
	$3c = g_{851}$	$3C$	$12a = g_{27,51}$	$12A$
	$3d = g_{751}$	$3B$	$15a = g_{30,51}$	$15A$
	$3e = g_{852}$	$3C$	$15b = g_{30,52}$	$15A$
$H_6 = M[331] = (2^4$:$A_5)$:2	$1a = g_{161}$	$1A$	$4c = g_{10,61}$	$4B$
	$2a = g_{461}$	$2C$	$4d = g_{11,61}$	$4C$
	$2b = g_{361}$	$2B$	$4e = g_{13,62}$	$4E$
	$2c = g_{462}$	$2C$	$5a = g_{14,61}$	$5A$
	$2d = g_{561}$	$2D$	$6a = g_{16,61}$	$6B$
	$2e = g_{463}$	$2C$	$6b = g_{18,61}$	$6D$
	$3a = g_{661}$	$3A$	$6c = g_{18,62}$	$6D$
	$4a = g_{961}$	$4A$	$8a = g_{23,61}$	$8A$
	$4b = g_{13,61}$	$4E$	$12a = g_{28,61}$	$12B$

6.4. Character Tables of the Inertia Factor Groups

We recall that knowledge of the appropriate character tables of inertia factor groups is pivotal in calculating the full character table of any group extension. Since in our group $\overline{G} = 3^7$:$Sp(6,2)$, the normal subgroup 3^7 is abelian and the extension splits, it follows by Mackey's Theorem (see Theorem 5.1.4), that every character θ_k of 3^7 is extendible to a character of its inertia group $\overline{H}_k$. Thus all the factor sets $\overline{\alpha}_k$ are trivial and all the character tables of the inertia factor groups that we will use to construct the character table of $\overline{G}$ are the ordinary ones.

The character table of $H_1 = Sp(6,2)$ is available in $\mathbb{ATLAS}$. We have used GAP to construct the character tables of $H_2 = U_4(2),\ H_3 = 2^5$:$A_6,\ H_4 = A_7,\ H_5 = (3 \times A_6)$:2 and $H_6 = (2^4$:$A_5)$:2 since we know from Section 6.3 that $H_2 = \langle \sigma_1, \sigma_2 \rangle$, $H_3 = \langle \xi_1, \xi_2 \rangle$, $H_4 = \langle \beta_1, \beta_2 \rangle$, $H_5 = \langle \mu_1, \mu_2 \rangle$ and $H_6 = \langle \epsilon_1, \epsilon_2 \rangle$. The character tables of $H_2,\ H_3,\ H_4,\ H_5$ and H_6 are given as Tables 11.3, 11.4, 11.5, 11.6 and 11.7 (respectively), appear in the Appendix.

6.5. Fischer Matrices of $\overline{G} = 3^7$:$Sp(6,2)$

We recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of $g_{ij},\ 1 \leq j \leq c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 6.2 we supplied $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \leq i \leq 30,\ 1 \leq j \leq c(g_i)$. Also having obtained the fusions of the inertia factor groups $H_2,\ H_3,\ H_4,\ H_5$ and H_6 into $Sp(6,2)$, we are able to label the rows of the Fischer matrices as described in Section 5.1.

We have used the properties of Fischer matrices, given in Proposition 5.2.2, to calculate some of the entries of the Fischer matrices and also to build an algebraic system of equations. For example since the extension is split, then every coset $N\overline{g}_i$ (or just Ng_i) is a split coset (see Schiffer [67]) and it results that $a_{i1}^{(k,m)} = \frac{|C_{Sp(6,2)}(g_i)|}{|C_{H_k}(g_{ikm})|},\ \forall i \in \{1, 2, \cdots, 30\}$. With the help of the symbolic mathematical package Maxima [45], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G} = 3^7$:$Sp(6,2)$, which we list below:

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}
$o(g_{1j})$		1	3	3	3	3	3
$\lvert C_{\overline{G}}(g_{1j})\rvert$		3174474240	56687040	25194240	5511240	4723920	4199040
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$						
$(1,1)$	1451520	1	1	1	1	1	1
$(2,1)$	25920	56	−25	20	−7	2	2
$(3,2)$	11520	126	45	27	0	−9	0
$(4,1)$	2520	576	−72	0	27	−18	0
$(5,1)$	2160	672	24	−48	−21	−3	24
$(6,1)$	1920	756	27	0	0	27	−27
m_{1j}		1	56	126	576	672	756

$\mathcal{F}_2$

g_2		g_{21}	g_{22}
$o(g_{2j})$		2	6
$\lvert C_{\overline{G}}(g_{2j})\rvert$		69120	34560
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$		
$(1,1)$	23040	1	1
$(3,1)$	11520	2	−1
m_{2j}		729	1458

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}	g_{34}
$o(g_{3j})$		2	6	6	6
$\lvert C_{\overline{G}}(g_{2j})\rvert$		124416	20736	15552	10368
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$				
$(1,1)$	4608	1	1	1	1
$(2,1)$	576	8	−4	−1	2
$(3,1)$	768	6	3	−3	0
$(6,1)$	384	12	0	3	−3
m_{2j}		81	486	648	972

$\mathcal{F}_4$

g_4		$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{4,6}$	$g_{4,7}$	$g_{4,8}$	$g_{4,9}$	$g_{4,10}$
$o(g_{4j})$		2	6	6	6	6	6	6	6	6	6
$\lvert C_{\overline{G}}(g_{4j})\rvert$		373248	186624	46656	23328	23328	15552	11664	11664	7776	5832
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$										
$(1,1)$	1536	1	1	1	1	1	1	1	1	1	1
$(2,1)$	96	16	−8	10	−8	−5	4	4	−2	−2	1
$(3,1)$	768	2	−1	2	2	−1	2	−1	2	−1	−1
$(3,2)$	64	24	24	6	6	6	−3	6	−3	−3	−3
$(4,1)$	24	64	−32	−8	−8	4	−8	4	10	4	−5
$(5,1)$	48	32	32	−4	−4	−4	−4	−4	5	−4	5
$(5,2)$	48	32	−16	−16	2	8	8	−1	−4	−4	2
$(6,1)$	192	8	8	5	−4	5	2	−4	−1	2	−1
$(6,2)$	32	48	−24	12	−15	12	3	−15	−6	3	−6
$(6,3)$	96	16	16	−17	19	−17	−5	19	7	−5	7
m_{4j}		9	18	72	144	144	216	288	288	432	576

$\mathcal{F}_5$

g_5		g_{51}	g_{52}	g_{53}	g_{54}
$o(g_{5j})$		2	6	6	6
$\lvert C_{\overline{G}}(g_{5j})\rvert$		10368	1728	1296	864
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$				
$(1,1)$	384	1	1	1	1
$(3,1)$	64	6	3	−3	0
$(5,1)$	48	8	−4	−1	2
$(6,1)$	32	12	0	3	−3
m_{5j}		81	486	648	972

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}	g_{66}	g_{67}
$o(g_{6j})$		3	3	3	3	9	9	9
$\lvert C_{\overline{G}}(g_{6j})\rvert$		524880	26244	17496	17496	43740	17496	4374
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$							
$(1,1)$	2160	1	1	1	1	1	1	1
$(2,1)$	108	20	−7	2	2	−10	8	−1
$(3,1)$	72	30	3	−6	3	15	6	−3
$(4,1)$	36	60	6	−12	6	−15	−6	3
$(5,1)$	1080	2	2	2	2	−1	−1	−1
$(5,2)$	54	40	−14	4	4	10	−8	1
$(6,1)$	24	90	9	9	−18	0	0	0
m_{6j}		9	180	270	270	108	270	1080

$\mathcal{F}_7$

g_7		g_{71}	g_{72}	g_{73}	g_{74}
$o(g_{7j})$		3	3	9	9
$\lvert C_{\overline{G}}(g_{7j})\rvert$		17496	2187	1944	1944
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$				
$(1,1)$	648	1	1	1	1
$(2,1)$	648	1	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(2,2)$	648	1	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
$(5,1)$	27	24	-3	0	0
m_{7j}		81	648	729	729

$\mathcal{F}_8$

g_8		g_{81}	g_{82}	g_{83}	g_{84}	g_{85}	g_{86}
$o(g_{8j})$		3	3	9	9	9	9
$\lvert C_{\overline{G}}(g_{8j})\rvert$		2916	1458	1458	729	486	243
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$						
$(1,1)$	108	1	1	1	1	1	1
$(2,1)$	54	2	2	-1	-1	2	-1
$(3,1)$	18	6	-3	6	-3	0	0
$(4,1)$	9	12	-6	-6	3	0	0
$(5,1)$	27	4	4	-2	-2	-2	1
$(5,2)$	54	2	2	2	2	-1	-1
m_{8j}		81	162	162	324	486	972

$\mathcal{F}_9$

g_9		g_{91}	g_{92}	g_{93}	g_{94}
$o(g_{9j})$		4	12	12	12
$\lvert C_{\overline{G}}(g_{9j})\rvert$		10368	1728	1296	864
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$				
$(1,1)$	384	1	1	1	1
$(2,1)$	48	8	-4	-1	2
$(3,1)$	64	6	3	-3	0
$(6,1)$	32	12	0	3	-3
m_{9j}		81	486	648	972

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{10,4}$
$o(g_{10j})$		4	12	12	12
$\lvert C_{\overline{G}}(g_{10j})\rvert$		5184	864	648	432
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$				
$(1,1)$	192	1	1	1	1
$(3,1)$	16	12	0	3	-3
$(5,1)$	24	8	-4	-1	2
$(6,1)$	32	6	3	-3	0
m_{10j}		81	486	648	972

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$
$o(g_{11j})$		4	12
$\lvert C_{\overline{G}}(g_{11,j})\rvert$		576	288
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$		
$(1,1)$	192	1	1
$(6,1)$	96	2	-1
m_{11j}		729	1458

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$
$o(g_{12j})$		4	12
$\lvert C_{\overline{G}}(g_{12,j})\rvert$		384	192
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$		
$(1,1)$	128	1	1
$(3,1)$	64	2	-1
m_{12j}		729	1458

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$	$g_{13,5}$	$g_{13,6}$	$g_{13,7}$	$g_{13,8}$
$o(g_{13j})$		4	12	12	12	12	12	12	12
$\lvert C_{\overline{G}}(g_{13j})\rvert$		864	432	432	432	216	216	216	108
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$								
$(1,1)$	32	1	1	1	1	1	1	1	1
$(2,1)$	8	4	4	-2	-2	-2	1	-2	1
$(3,1)$	16	2	2	-1	2	-1	-1	2	-1
$(3,2)$	16	2	-1	2	2	-1	2	-1	-1
$(4,1)$	4	8	-4	-4	-4	2	2	2	-1
$(5,1)$	8	4	-2	4	-2	-2	-2	1	1
$(6,1)$	16	2	2	2	-1	2	-1	-1	-1
$(6,2)$	8	4	-2	-2	4	1	-2	-2	1
m_{13j}		81	162	162	162	324	324	324	648

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$	$g_{14,2}$	$g_{14,3}$	$g_{14,4}$	$g_{14,5}$	$g_{14,6}$	$g_{14,7}$
$o(g_{14j})$		5	15	15	15	15	15	15
$\lvert C_{\overline{G}}(g_{14j})\rvert$		810	270	270	405	135	135	135
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$							
$(1,1)$	30	1	1	1	1	1	1	1
$(2,1)$	5	6	0	0	-3	0	3	-3
$(3,1)$	10	3	$-\frac{3}{2}-\frac{3\sqrt{3}}{2}$	$-\frac{3}{2}+\frac{3\sqrt{3}}{2}$	3	0	0	0
$(3,2)$	10	3	$-\frac{3}{2}+\frac{3\sqrt{3}}{2}$	$-\frac{3}{2}-\frac{3\sqrt{3}}{2}$	3	0	0	0
$(4,1)$	5	6	0	0	-3	3	-3	0
$(5,1)$	15	2	2	2	2	-1	-1	-1
$(6,1)$	5	6	0	0	-3	-3	0	3
m_{14j}		81	243	243	162	486	486	486

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$
$o(g_{15j})$		6	6
$\lvert C_{\overline{G}}(g_{15,j})\rvert$		432	216
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$		
$(1,1)$	144	1	1
$(3,1)$	72	2	-1
m_{15j}		729	1458

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$	$g_{16,3}$	$g_{16,4}$	$g_{16,5}$
$o(g_{16j})$		6	6	6	6	6
$\lvert C_{\overline{G}}(g_{16j})\rvert$		3888	972	972	648	324
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$					
$(1,1)$	144	1	1	1	1	1
$(2,1)$	36	4	$-\frac{1}{2}-\frac{3\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{3\sqrt{3}}{2}$	-2	1
$(2,2)$	36	4	$-\frac{1}{2}+\frac{3\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{3\sqrt{3}}{2}$	-2	1
$(3,1)$	24	6	-3	-3	3	0
$(6,1)$	12	12	3	3	0	-3
m_{16j}		81	324	324	486	972

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$	$g_{17,3}$
$o(g_{17j})$		6	18	18
$\lvert C_{\overline{G}}(g_{17j})\rvert$		216	216	216
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$			
$(1,1)$	72	1	1	1
$(2,1)$	72	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(2,2)$	72	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{17j}		729	729	729

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$	$g_{18,3}$	$g_{18,4}$	$g_{18,5}$	$g_{18,6}$	$g_{18,7}$	$g_{18,8}$
$o(g_{18j})$		6	6	6	6	18	18	18	18
$\lvert C_{\overline{G}}(g_{18j})\rvert$		1296	648	648	324	648	324	324	162
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$								
$(1,1)$	48	1	1	1	1	1	1	1	1
$(2,1)$	12	4	-2	-2	1	4	-2	-2	1
$(3,1)$	24	2	2	-1	-1	2	-1	2	-1
$(4,1)$	12	4	4	-2	-2	-2	1	-2	1
$(5,1)$	24	2	2	2	2	-1	-1	-1	-1
$(5,2)$	6	8	-4	-4	2	-4	2	2	-1
$(6,1)$	24	2	-1	2	-1	2	2	-1	-1
$(6,2)$	12	4	-2	4	-2	-2	-2	1	1
m_{18j}		81	162	162	324	162	324	324	648

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		6	6
$\lvert C_{\overline{G}}(g_{19,j})\rvert$		108	54
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$		
$(1,1)$	36	1	1
$(3,1)$	18	2	-1
m_{19j}		729	1458

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$	$g_{20,2}$
$o(g_{20j})$		6	18
$\lvert C_{\overline{G}}(g_{20,j})\rvert$		108	54
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$		
$(1,1)$	36	1	1
$(2,1)$	18	2	-1
m_{20j}		729	1458

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$	$g_{21,2}$
$o(g_{21j})$		6	18
$\lvert C_{\overline{G}}(g_{21,j})\rvert$		36	18
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$		
$(1,1)$	12	1	1
$(5,1)$	6	2	-1
m_{21j}		729	1458

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$	$g_{22,2}$	$g_{22,3}$
$o(g_{22j})$		7	21	21
$\lvert C_{\overline{G}}(g_{22j})\rvert$		21	21	21
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$			
$(1,1)$	7	1	1	1
$(4,1)$	7	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(4,2)$	7	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{22j}		729	729	729

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$
$o(g_{23j})$		8	24
$\lvert C_{\overline{G}}(g_{23,j})\rvert$		48	24
(k,m)	$\lvert C_{H_k}(g_{231km})\rvert$		
$(1,1)$	16	1	1
$(6,1)$	8	2	-1
m_{23j}		729	1458

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$	$g_{24,2}$	$g_{24,3}$
$o(g_{24j})$		8	24	24
$\lvert C_{\overline{G}}(g_{24j})\rvert$		48	48	48
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$			
$(1,1)$	16	1	1	1
$(3,1)$	16	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(3,2)$	16	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{24j}		729	729	729

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$	$g_{25,2}$	$g_{25,3}$
$o(g_{25j})$		9	9	9
$\lvert C_{\overline{G}}(g_{25j})\rvert$		27	27	27
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$			
$(1,1)$	9	1	1	1
$(2,1)$	9	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(2,2)$	9	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{25j}		729	729	729

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$	$g_{26,2}$	$g_{26,3}$
$o(g_{26j})$		10	30	30
$\lvert C_{\overline{G}}(g_{26j})\rvert$		30	30	30
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$			
$(1,1)$	10	1	1	1
$(3,1)$	10	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(3,2)$	10	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{26j}		729	729	729

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$	$g_{27,2}$
$o(g_{27j})$		12	36
$\lvert C_{\overline{G}}(g_{27,j})\rvert$		72	36
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$		
$(1,1)$	24	1	1
$(5,1)$	12	2	-1
m_{27j}		729	1458

$\mathcal{F}_{28}$

g_{28}		$g_{28,1}$	$g_{28,2}$
$o(g_{28j})$		12	12
$\lvert C_{\overline{G}}(g_{28,j})\rvert$		72	36
(k,m)	$\lvert C_{H_k}(g_{28km})\rvert$		
$(1,1)$	24	1	1
$(6,1)$	12	2	-1
m_{28j}		729	1458

$\mathcal{F}_{29}$

g_{29}		$g_{29,1}$	$g_{29,2}$	$g_{29,3}$
$o(g_{29j})$		12	36	36
$\lvert C_{\overline{G}}(g_{29j})\rvert$		36	36	36
(k,m)	$\lvert C_{H_k}(g_{29km})\rvert$			
$(1,1)$	12	1	1	1
$(2,1)$	12	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(2,2)$	12	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{29j}		729	729	729

$\mathcal{F}_{30}$

g_{30}		$g_{30,1}$	$g_{30,2}$	$g_{30,3}$
$o(g_{30j})$		15	45	45
$\lvert C_{\overline{G}}(g_{30j})\rvert$		45	45	45
(k,m)	$\lvert C_{H_k}(g_{30km})\rvert$			
$(1,1)$	15	1	1	1
$(5,1)$	15	1	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$
$(5,2)$	15	1	$-\frac{1}{2}-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}$
m_{30j}		729	729	729

6.6. The Character Table of $\overline{G} = 3^7$:$Sp(6,2)$

Now we have

- the conjugacy classes of $\overline{G} = 3^7$:$Sp(6,2)$ (Table 6.2),
- the character tables of all the inertia factors (Tables 11.3, 11.4, 11.5, 11.6 and 11.7),
- the fusions of classes of the inertia factors into classes of $Sp(6,2)$ (Table 6.11),
- the Fischer matrices of $\overline{G}$ (Section 6.5).

By Section 5.3, it follows that the full character table of $\overline{G}$ can be constructed easily. We give some examples on how to construct the character table of $\overline{G}$, which is partitioned into blocks corresponding to the cosets and the inertia factor groups. For example we construct the parts $\mathcal{K}_{14,1}\mathcal{F}_{14,1},\ \mathcal{K}_{14,2}\mathcal{F}_{14,2},\ \mathcal{K}_{14,3}\mathcal{F}_{14,3},\ \mathcal{K}_{14,4}\mathcal{F}_{14,4},\ \mathcal{K}_{14,5}\mathcal{F}_{14,5}$ and $\mathcal{K}_{14,6}\mathcal{F}_{14,6}$ of the character table of $\overline{G}$ (this means that we are listing the values of all the irreducible characters of $\overline{G}$ on the $\overline{G}-$classes that correspond to the conjugacy class $5A$ of $Sp(6,2)$). The preceding parts can be derived as follows: From Table 6.11 we can see that there are fusions from all the inertia factor groups $H_1, H_2, \cdots, H_6$ into the class $g_{14} = [5A]_{Sp(6,2)}$. To construct the block $\mathcal{K}_{14,k}\mathcal{F}_{14,k},\ 1 \leq k \leq 6$, we select the column(s) of the character table of H_k corresponds to the class containing the elements of order 5 that fuse to $[5A]_{Sp(6,2)}$. Then we multiply this column(s) by the row(s) of the Fischer matrix $\mathcal{F}_{14}$ corresponding to the pairs $(k,m),\ 1 \leq m \leq c(g_{14,k})$. Thus we get a block in the character table of $\overline{G}$ of size $|\mathrm{Irr}(H_k)| \times c(g_{14,k})$. Next we apply this to all the blocks corresponding to the inertia factor groups $H_k,\ 1 \leq k \leq 6$. The values of the irreducible characters of $\overline{G}$ (corresponding to H_k) on the classes $g_{14,1},\ g_{14,2},\ g_{14,3},\ g_{14,4},\ g_{14,5},\ g_{14,6}$ and $g_{14,7}$ are given by

$$\mathcal{K}_{14,1}\mathcal{F}_{14,1} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 1 \\ 2 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ -2 \\ 0 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{14,1} & g_{14,2} & g_{14,3} & g_{14,4} & g_{14,5} & g_{14,6} & g_{14,7} \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{14,2}\mathcal{F}_{14,2} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 & -3 & 0 & 3 & -3 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{14,1} & g_{14,2} & g_{14,3} & g_{14,4} & g_{14,5} & g_{14,6} & g_{14,7} \end{matrix} \\ \begin{pmatrix} 6 & 0 & 0 & -3 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & -3 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6 & 0 & 0 & 3 & 0 & -3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6 & 0 & 0 & 3 & 0 & -3 & 3 \\ 6 & 0 & 0 & -3 & 0 & 3 & -3 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{14,3}\mathcal{F}_{14,3} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ A & A^\star \\ A^\star & A \\ -1 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ A^\star & A \\ A & A^\star \\ -1 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 3 & B & \widetilde{B} & 3 & 0 & 0 & 0 \\ 3 & \widetilde{B} & B & 3 & 0 & 0 & 0 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{14,1} & g_{14,2} & g_{14,3} & g_{14,4} & g_{14,5} & g_{14,6} & g_{14,7} \end{matrix} \\ \begin{pmatrix} 6 & -3 & -3 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3\overline{C} & 3C & 3 & 0 & 0 & 0 \\ 3 & 3C & 3\overline{C} & 3 & 0 & 0 & 0 \\ -6 & 3 & 3 & -6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3C & 3\overline{C} & 3 & 0 & 0 & 0 \\ 3 & 3\overline{C} & 3C & 3 & 0 & 0 & 0 \\ -6 & 3 & 3 & -6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array},$$

where $A = -\frac{1}{2} - \frac{\sqrt{5}}{2}i$, $A^\star = 1 - A$, $B = -\frac{3}{2} - \frac{3\sqrt{3}}{2}$ and $\widetilde{B} = -\frac{3}{2} + \frac{3\sqrt{3}}{2}$ and $C = b15 = -\frac{1}{2} + \frac{\sqrt{15}}{2}i$.

$$\mathcal{K}_{14,4}\mathcal{F}_{14,4} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ -1 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 & -3 & 3 & -3 & 0 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{14,1} & g_{14,2} & g_{14,3} & g_{14,4} & g_{14,5} & g_{14,6} & g_{14,7} \end{matrix} \\ \begin{pmatrix} 6 & 0 & 0 & -3 & 3 & -3 & 0 \\ 6 & 0 & 0 & -3 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6 & 0 & 0 & 3 & -3 & -3 & 0 \\ -6 & 0 & 0 & 3 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & -3 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{14,5}\mathcal{F}_{14,5} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ -2 \\ 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 & -1 & -1 & -1 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{14,1} & g_{14,2} & g_{14,3} & g_{14,4} & g_{14,5} & g_{14,6} & g_{14,7} \end{matrix} \\ \begin{pmatrix} 2 & 2 & 2 & 2 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 & -1 & -1 & -1 \\ 4 & 4 & 4 & 4 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 & -1 & -1 & -1 \\ -4 & -4 & -4 & -4 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{14,6}\mathcal{F}_{14,6} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 & -3 & -3 & 0 & 3 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{14,1} & g_{14,2} & g_{14,3} & g_{14,4} & g_{14,5} & g_{14,6} & g_{14,7} \end{matrix} \\ \begin{pmatrix} 6 & 0 & 0 & -3 & -3 & 0 & 3 \\ 6 & 0 & 0 & -3 & -3 & 0 & 3 \\ -6 & 0 & 0 & 3 & 3 & 0 & -3 \\ -6 & 0 & 0 & 3 & 3 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & -3 & -3 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array},$$

Similarly one can obtain all the other 174 blocks $\mathcal{K}_{ik}\mathcal{F}_{ik}$, $1 \leq i \leq 30,\ i \neq 14,\ 1 \leq k \leq 6$ and hence the full character table of $\overline{G}$, which is a 118×118 $\mathbb{C}-$valued matrix. We list the full character table of $\overline{G} = 3^7$:$Sp(6,2)$ in Table 11.8, appears in the Appendix.

Chapter 7

Two Maximal Subgroups of Thompson Group Th

The Thompson group Th has 16 conjugacy classes of maximal subgroups (see the electronic ATLAS of Wilson [71]). The second and third largest maximal subgroups of Th are both non-split extension groups of the forms $2^{5\cdot}GL(5,2)$ and $2_+^{1+8\cdot}A_9$ respectively. In this chapter we apply the Clifford-Fischer theory to obtain the character tables of the above mentioned groups. We list the Fischer matrices of the two groups and we will see that for the group $2_+^{1+8\cdot}A_9$, the Fischer matrices satisfy some special properties, which help in computing their entries.

7.1. Dempwolff Group $2^{5\cdot}GL(5,2)$

In this section we study the so-called *Dempwolff group*, which is a non-split extension of the elementary abelian group 2^5 by the general linear group $GL(5,2)$. In Subsection 7.1.1 we give a brief introduction and a presentation of $\overline{G} = 2^{5\cdot}GL(5,2)$ in terms of generators of the Thompson group Th. In Subsection 7.1.2 we list the 41 conjugacy classes of $\overline{G}$ obtained by applying the coset analysis technique using the 27 conjugacy classes of $GL(5,2)$. In Subsection 7.1.3 we show that there are two inertia factors, namely $H_1 = GL(5,2)$ and $H_2 = 2^4{:}GL(4,2)$. We also show that we need to use the projective characters of H_2 with factor set α such that $\alpha \sim [2]$. In Subsection 7.1.4 we compute the fusions of classes of H_2 into classes of $\overline{G}$ and in Subsection 7.1.5 we calculate the Fischer matrices of the extension. In Subsection 7.1.6 we show how to calculate the character table of $\overline{G}$. The full character table of $\overline{G}$ is listed in Table 11.9 which appears in the Appendix of this thesis.

7.1.1 Introduction

It is well known (cf. Huppert [36]) that $H^2(GL(n,q), V(n,q)) = 0,\ \forall\ q > 2$, where $H^2(K, M)$ is the second cohomology group of a group K with coefficients in M. Hence a non split extension of the form $V(n,q)\cdot GL(n,q),\ q > 2$ does not exist. In the case $q = 2$, Dempwolff proved in [21] that apart from the case $n = 5$, then

$$\dim_{\mathbb{F}_2} H^2(GL(n,2), \mathbb{F}_2^n) = \begin{cases} 1 & \text{if } n = 3 \text{ or } 4, \\ 0 & \text{otherwise.} \end{cases}$$

Hence non split extensions of the forms $2^{3}\cdot GL(3,2)$ and $2^{4}\cdot GL(4,2) \cong 2^{4}\cdot S_6$ exist and they are unique up to isomorphisms. In fact the group $2^{3}\cdot GL(3,2)$ is a maximal subgroup of the Chevalley group $G_2(3)$, while $2^{4}\cdot GL(4,2)$ sits maximally in the sporadic Conway group Co_3 (see $\mathbb{ATLAS}$ [17]). Later on in 1972, Dempwolff proved in [22] that a non split extension of the form $2^{5}\cdot GL(5,2)$ does exist and it is unique up to isomorphism. This group, which is known as *Dempwolff group* and we denote it in this section by $\overline{G}$, is the second largest maximal subgroup of the sporadic Thompson group Th (see $\mathbb{ATLAS}$). In the unique class of involutions $2A$ of Thompson group Th, there are 5 commuting involutions, say n_1, n_2, n_3, n_4 and n_5. The group $\langle n_1, n_2, n_3, n_4, n_5\rangle := K$ is an elementary abelian $2-$group of order 32 and $N_{\mathrm{Th}}(K) \cong 2^{5}\cdot GL(5,2) = \overline{G}$. Clearly the group $\overline{G}$ has order $32 \times 9\ 999\ 360 = 319\ 979\ 520$ and index $283\ 599\ 225$ in Th. In this section we are interested in the Fischer matrices of $\overline{G}$ and hence the re-computation of the character table of $\overline{G}$ using Clifford-Fischer theory.

Let x and y be in the $248-$dimensional matrix group over $\mathbb{F}_2$, that are generators of Th given by the electronic ATLAS of Wilson. Using MAGMA [16] it is possible to construct $\overline{G}$ inside Th since $\overline{G}$ is generated as follows

$$\overline{G} = \left\langle x, ((xyyxy)^2)^{((xy)^{15}(xyy)^{9}(xy)^{12}(xyy)^{16}(xy)^{17})} \right\rangle.$$

However $\overline{G} \leq GL(69,2)$ and generators a and b of $\overline{G}$ in terms of 69×69 matrices over $\mathbb{F}_2$ are also given by the electronic ATLAS of Wilson. Using this $69-$dimensional representation over $\mathbb{F}_2$, one can obtain the 5 generators n_1, n_2, n_3, n_4 and n_5 of the normal subgroup N of $\overline{G}$ using Magma or GAP [30]. The commands "Complements($\overline{G}$, N)" and "Complementclasses($\overline{G}$, N)" of Magma and GAP respectively reveal the complements of N in $\overline{G}$, where in our case, an empty set will be returned confirming that the extension is a non-split.

7.1.2 The Conjugacy Classes of $\overline{G} = 2^{5\cdot}GL(5,2)$

In this subsection we have used the method of the *coset analysis* discussed in Section 2.3, to find the conjugacy classes of $\overline{G} = 2^{5\cdot}GL(5,2)$. We list the classes of $\overline{G}$ in Table 7.1, where in this table,

- g_i is the i^{th} conjugacy class of $G = GL(5,2)$ as listed in the order given by the $\mathbb{ATLAS}$.
- k_i is the number of orbits $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ on the action of N on the coset $N\overline{g}_i = 2^5\overline{g}_i$, where $\overline{g}_i$ is a pre-image of g_i under the natural epimorphism $\pi : \overline{G} \longrightarrow G$. In particular, the action of N on the identity coset N produces 32 orbits each consists of singleton. Thus $k_1 = 32$.
- f_{ij} is the number of orbits fused together under the action of $\overline{G}$ on $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$. In particular, the action of $\overline{G}$ on the orbits $Q_{11}, Q_{12}, \cdots, Q_{1k_1}$ leaves invariant the orbit Q_{11} (this orbit assumed to consist of the identity element of N), while fuse the other 31 orbits (consisting of the involutions of N) into a single orbit. Thus $f_{11} = 1$ and $f_{12} = 31$.
- m_{ij} is the weight defined by Eq (5.4).
- g_{ij} is a representative of conjugacy class of $\overline{G}$ that correspond to the class g_i of G. In particular the action of $\overline{G}$ on N produces two conjugacy classes in $\overline{G}$ represented by $g_{11} = 1_{\overline{G}} = 1_N$ and g_{12} and these two classes have sizes 1 and 31 respectively.

Table 7.1: The conjugacy classes of $\overline{G} = 2^{5\cdot}GL(5,2)$

$[g]_{GL(5,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^{5\cdot}GL(5,2)}$	$o(g_{ij})$	$\lvert C_{2^{5\cdot}GL(5,2)}(g_{ij})\rvert$	$\lvert[g_{ij}]_{2^{5\cdot}GL(5,2)}\rvert$
$g_1 = 1A$	$k_1 = 32$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	319979520	1
		$f_{12} = 31$	$m_{12} = 31$	g_{12}	2	10321920	31
$g_2 = 2A$	$k_2 = 16$	$f_{21} = 8$	$m_{21} = 16$	g_{21}	4	43008	7440
		$f_{22} = 8$	$m_{22} = 16$	g_{22}	2	43008	7440
$g_3 = 2B$	$k_3 = 8$	$f_{31} = 8$	$m_{31} = 32$	g_{31}	4	1536	208320

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Table 7.1 (continued)

$[g]_{GL(5,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^5 \cdot GL(5,2)}$	$o(g_{ij})$	$\lvert C_{2^5 \cdot GL(5,2)}(g_{ij})\rvert$	$\lvert [g_{ij}]_{2^5 \cdot GL(5,2)}\rvert$
$g_4 = 3A$	$k_4 = 8$	$f_{41} = 1$	$m_{41} = 4$	g_{41}	3	4032	79360
		$f_{42} = 7$	$m_{42} = 28$	g_{42}	6	576	555520
$g_5 = 3B$	$k_5 = 2$	$f_{51} = 1$	$m_{51} = 16$	g_{51}	6	360	888832
		$f_{52} = 1$	$m_{52} = 16$	g_{52}	3	360	888832
$g_6 = 4A$	$k_6 = 8$	$f_{61} = 8$	$m_{61} = 32$	g_{61}	8	384	833280
$g_7 = 4B$	$k_7 = 4$	$f_{71} = 4$	$m_{71} = 32$	g_{71}	4	128	2499840
$g_8 = 4C$	$k_8 = 4$	$f_{81} = 4$	$m_{81} = 32$	g_{81}	8	32	9999360
$g_9 = 5A$	$k_9 = 2$	$f_{91} = 1$	$m_{91} = 16$	g_{91}	10	30	10665984
		$f_{92} = 1$	$m_{92} = 16$	g_{92}	5	30	10665984
		$f_{10,1} = 1$	$m_{10,1} = 8$	$g_{10,1}$	12	96	3333120
$g_{10} = 6A$	$k_{10} = 4$	$f_{10,2} = 1$	$m_{10,2} = 8$	$g_{10,2}$	12	96	3333120
		$f_{10,3} = 2$	$m_{10,3} = 16$	$g_{10,3}$	6	48	6666240
$g_{11} = 6B$	$k_{11} = 2$	$f_{11,1} = 2$	$m_{11,1} = 32$	$g_{11,1}$	12	12	26664960
$g_{12} = 7A$	$k_{12} = 4$	$f_{12,1} = 1$	$m_{12,1} = 8$	$g_{12,1}$	7	168	1904640
		$f_{12,2} = 3$	$m_{12,2} = 24$	$g_{12,2}$	14	56	5713920
$g_{13} = 7B$	$k_{13} = 4$	$f_{13,1} = 1$	$m_{13,1} = 8$	$g_{13,1}$	7	168	1904640
		$f_{13,2} = 3$	$m_{13,2} = 24$	$g_{13,2}$	14	56	5713920
$g_{14} = 8A$	$k_{14} = 2$	$f_{14,1} = 2$	$m_{14,1} = 32$	$g_{14,1}$	8	16	19998720
$g_{15} = 12A$	$k_{15} = 2$	$f_{15,1} = 1$	$m_{15,1} = 16$	$g_{15,1}$	24	24	13332480
		$f_{15,2} = 1$	$m_{15,2} = 16$	$g_{15,2}$	24	24	13332480
$g_{16} = 14A$	$k_{16} = 2$	$f_{16,1} = 1$	$m_{16,1} = 16$	$g_{16,1}$	28	28	11427840
		$f_{16,2} = 1$	$m_{16,2} = 16$	$g_{16,2}$	14	28	11427840
$g_{17} = 14B$	$k_{17} = 2$	$f_{17,1} = 1$	$m_{17,1} = 16$	$g_{17,1}$	28	28	11427840
		$f_{17,2} = 1$	$m_{17,2} = 16$	$g_{17,2}$	14	28	11427840

Continued on next page

Table 7.1 (continued)

$[g]_{GL(5,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^5 \cdot GL(5,2)}$	$o(g_{ij})$	$\lvert C_{2^5 \cdot GL(5,2)}(g_{ij})\rvert$	$\lvert [g_{ij}]_{2^5 \cdot GL(5,2)}\rvert$
$g_{18} = 15A$	$k_{18} = 2$	$f_{18,1} = 1$	$m_{18,1} = 16$	$g_{18,1}$	15	30	10665984
		$f_{18,2} = 1$	$m_{18,2} = 16$	$g_{18,2}$	30	30	10665984
$g_{19} = 15B$	$k_{19} = 2$	$f_{19,1} = 1$	$m_{19,1} = 16$	$g_{19,1}$	15	30	10665984
		$f_{19,2} = 1$	$m_{19,2} = 16$	$g_{19,2}$	30	30	10665984
$g_{20} = 21A$	$k_{20} = 1$	$f_{20,1} = 1$	$m_{20,1} = 32$	$g_{20,1}$	21	21	15237120
$g_{21} = 21B$	$k_{21} = 1$	$f_{21,1} = 1$	$m_{21,1} = 32$	$g_{21,1}$	21	21	15237120
$g_{22} = 31A$	$k_{22} = 1$	$f_{22,1} = 1$	$m_{22,1} = 32$	$g_{22,1}$	31	31	10321920
$g_{23} = 31B$	$k_{23} = 1$	$f_{23,1} = 1$	$m_{23,1} = 32$	$g_{23,1}$	31	31	10321920
$g_{24} = 31C$	$k_{24} = 1$	$f_{24,1} = 1$	$m_{24,1} = 32$	$g_{24,1}$	31	31	10321920
$g_{25} = 31D$	$k_{25} = 1$	$f_{25,1} = 1$	$m_{25,1} = 32$	$g_{25,1}$	31	31	10321920
$g_{26} = 31E$	$k_{26} = 1$	$f_{26,1} = 1$	$m_{26,1} = 32$	$g_{26,1}$	31	31	10321920
$g_{27} = 31F$	$k_{27} = 1$	$f_{27,1} = 1$	$m_{27,1} = 32$	$g_{27,1}$	31	31	10321920

7.1.3 The Inertia Groups of $\overline{G} = 2^{5 \cdot} GL(5,2)$

We have seen in Subsection 7.1.2 that the action of $\overline{G} = 2^{5 \cdot} GL(5,2)$ on $N = 2^5$ produced two orbits with lengths 1 and 31. By Theorem 5.1.1 of Brauer, it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce two orbits. These two orbits must necessarily have lengths 1 and 31 and the first orbit consists of the identity character $\mathbf{1}_N$ while the other orbit consists of the other non-trivial linear characters of N. Thus the corresponding inertia factor groups H_1 and H_2 have indices 1 and 31 respectively in $GL(5,2)$. By looking at the maximal subgroups of $GL(5,2)$ (see $\mathbb{ATLAS}$), it is readily verified that $H_1 = GL(5,2)$ and $H_2 = 2^4{:}GL(4,2)$ (note that there are two isomorphic non-conjugate subgroups of $GL(5,2)$ of the form $2^4{:}GL(4,2)$). In this subsection we show that we need to use projective characters, we also calculate the Schur multiplier of H_2 and supply the projective character table of H_2 with factor set $\alpha,\ \alpha \sim [2]$.

As a subgroup of $GL(5,2)$, the group $H_2 = 2^4{:}GL(4,2)$ is generated by the following two elements

$$h_1 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad h_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The following theorem characterizes the type of character table (ordinary or projective) of H_2 that we will use to construct the character table of Dempwolff group $\overline{G}$.

Theorem 7.1.1. *The character* $\sum_{i=2}^{32} \theta_i$, *where* $\theta_2, \theta_3, \cdots, \theta_{32}$ *are the non-trivial linear characters of* $N = 2^5$, *is not extendable to a character of its inertia group* $\overline{H}_2$.

PROOF. We recall from Table 7.1 that the number of conjugacy classes of $\overline{G}$ is 41. Thus we have to find 41 irreducible characters. From Section 5.3, these 41 characters are distributed into two blocks $\mathcal{K}_1$ and $\mathcal{K}_2$ corresponding to the inertia factor groups H_1 and H_2 respectively. From Note 5.3.1 we see that H_1 contributes with 27 characters to the character table of $\overline{G}$ (these 27 characters are the ordinary irreducible characters of $G = GL(5,2)$). If $\sum_{i=2}^{32} \theta_i$ is extendable to an ordinary character of $\overline{H}_2$, then we will use the ordinary character table of H_2 to construct the character table of $\overline{G}$. The two matrices h_1 and h_2 can be used to generate H_2 in either Magma or GAP and then one can ask for the character table of H_2, where we get $|\mathrm{Irr}(H_2)| = 25$. Using this, we obtain that $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| = 27 + 25 = 52$, contradicting the fact that $|\mathrm{Irr}(\overline{G})| = 41$. Therefore $\sum_{i=2}^{32} \theta_i$ is not extendable to an ordinary character of its inertia group $\overline{H}_2$. ■

Remark 7.1.1. Note that Theorem 7.1.1 asserts $\mathrm{Irr}(\overline{H}_2, \sum_{i=2}^{32} \theta_i) = \emptyset$ and hence we have to make the use of projective representations. Thus the projective character table of H_2 with factor set α^{-1}, that we will use to construct the ordinary character table of $\overline{G}$, must have 14 irreducible characters.

We note that the group $H_2 = 2^4{:}GL(4,2)$ has order $2^{10} \times 3^2 \times 5 \times 7$. We know by Schur (Theorem 4.1.4) that to find the Schur Multiplier $M(H_2)$ of H_2, we have to find the Multiplier of each Sylow $p-$subgroup of H_2, where p is a prime dividing $|H_2|$. To find these multipliers in Magma, we firstly convert the matrix group $H_2 = \langle h_1, h_2 \rangle$ into a permutation group. The group H_2 can be constructed in terms of permutations of a set of cardinality 32. The following two permutations $\widetilde{h}_1$

and $\widetilde{h}_2$ generate the group H_2 :

$$\begin{aligned}\widetilde{h}_1 &= (3\ \ 28\ \ 12\ \ 26\ \ 17\ \ 19\ \ 10)(4\ \ 27\ \ 11\ \ 25\ \ 18\ \ 20\ \ 9)(5\ \ 22\ \ 7\ \ 15\\ &\quad 14\ \ 24\ \ 30\ \ 6\ \ 21\ \ 8\ \ 16\ \ 13\ \ 23\ \ 29)(31\ \ 32),\\ \widetilde{h}_2 &= (3\ \ 19\ \ 30\ \ 22\ \ 4\ \ 20\ \ 29\ \ 21)(5\ \ 14\ \ 27\ \ 12\ \ 6\ \ 13\ \ 28\ \ 11)(7\ \ 32\\ &\quad 8\ \ 31)(9\ \ 23\ \ 17\ \ 16)(10\ \ 24\ \ 18\ \ 15)(25\ \ 26).\end{aligned}$$

The following sequence of Magma commands will produce the Schur multipliers of the Sylow $p-$subgroups for $p \in \{2, 3, 5, 7\}$. Also we use the command "pCove" to construct the central extension $p{\cdot}H_2$, but the above command works only for finitely presented groups and thus we used the command "FPGroup" to convert our permutation group H_2 into a finitely presented group.

```
> H:= PermutationGroup< 32 | (3,28,12,26,17,19,10)(4,27,11,25,18,20,9)
                             (5,22,7,15,14,24,30,6,21,8,16,13,23,29)(31,32),
                             (3,19,30,22,4,20,29,21) (5,14,27,12,6,13,28,11)
                             (7,32,8,31)(9,23,17,16)(10,24,18,15)(25,26)>;
> pMultiplicator(H,2);
[ 2 ]
> pMultiplicator(H,3);
[ 1 ]
> pMultiplicator(H,5);
[ 1 ]
> pMultiplicator(H,7);
[ 1 ]
> F := FPGroup(H);
> F2 := pCover(H, F, 2);
> Order(F2);
645120
> p, H:= CosetAction(F2, sub<F2|>);
> s:= SylowSubgroup(H, 3);
> p2, H1:= CosetAction(H, s);
> Order(H1);
645120
```

```
> ct:= CharacterTable(H1);
> ct;
```

From the above, we deduce that the Schur Multiplier of H_2 is $\mathbb{Z}_2 \times \mathbb{Z}_1 \times \mathbb{Z}_1 \times \mathbb{Z}_1 \cong \mathbb{Z}_2$, abbreviated to be just 2. Thus the only factor set α that $M(H_2)$ contains is $\alpha \sim [2]$. The covering group $M(H_2)\cdot H_2$ (central extension of $M(H_2)$ by H_2, see Definition 4.2.5) is isomorphic to $2\cdot(2^4{:}GL(4,2))$. To find the projective character table of H_2 with factor set α (since $\alpha \sim [2]$, α and α^{-1} are identical), it is sufficient to find the ordinary character table of the double cover group $2\cdot H_2$ of H_2 (in this situation, the 2-fold cover $2\cdot H_2$ is the full covering group $M(H_2)\cdot H_2 \cong 2\cdot(2^4{:}GL(4,2))$). From the above sequence of Magma commands, one can see that the group F2 is the double cover group $2\cdot H_2$ and it is clear that $|\text{F2}| = 645120 = 2 \times 322560 = 2 \times |H_2|$. Thus one can proceed computationally to obtain the character table of $2\cdot H_2$. Alternatively since $M(H_2)\cdot H_2$ is of extension type, the Clifford-Fischer theory can be applied recursively to obtain its character table. In fact the character table of this group is partitioned into two blocks $\widetilde{\mathcal{K}}_1$ and $\widetilde{\mathcal{K}}_2$, where $\widetilde{\mathcal{K}}_1$ consists of the 25 ordinary irreducible characters of H_2, while $\widetilde{\mathcal{K}}_2$ consists of a 14 irreducible characters (that is $|\text{Irr}(2\cdot(2^4{:}GL(4,2)))| = 39$). We are interested in these 14 characters contained in $\widetilde{\mathcal{K}}_2$, where these characters represent the projective character table of H_2 with factor set α, $\alpha \sim [2]$. Note that $|\text{Irr}(H_1)| + |\text{IrrProj}(H_2, 2)| = 27 + 14 = 41 = |\text{Irr}(\overline{G})|$. In Table 7.2 we list the projective character table of H_2 with factor set α, $\alpha \sim [2]$.

Table 7.2: Projective characters of $H_2 = 2^4{:}GL(4,2)$ with factor set α, $\alpha \sim [2]$

$[g_{ikm}]_{H_2}$	$1a$	$2a$	$2b$	$2c$	$4a$	$2d$	$4b$	$3a$	$3b$	$6a$	$4c$	$4d$	$4e$
$[g_{ikm}]_{H_2}$	g_{121}	g_{221}	g_{222}	g_{321}	g_{621}	g_{322}	g_{721}	g_{521}	g_{421}	$g_{10,21}$	g_{622}	g_{722}	g_{821}
$\lvert C_{H_2}(g_{ikm})\rvert$	322560	21504	1536	512	384	384	128	180	72	24	64	64	32
ψ_1	8	8	0	0	0	0	0	−4	2	2	0	0	0
ψ_2	24	24	0	0	0	0	0	−6	0	0	0	0	0
ψ_3	24	24	0	0	0	0	0	−6	0	0	0	0	0
ψ_4	48	48	0	0	0	0	0	6	0	0	0	0	0
ψ_5	56	56	0	0	0	0	0	−4	−1	−1	0	0	0
ψ_6	56	56	0	0	0	0	0	−4	−1	−1	0	0	0
ψ_7	56	56	0	0	0	0	0	2	2	2	0	0	0
ψ_8	56	56	0	0	0	0	0	2	2	2	0	0	0
ψ_9	64	64	0	0	0	0	0	4	−2	−2	0	0	0
ψ_{10}	120	−8	0	0	0	0	0	0	6	−2	0	0	0
ψ_{11}	120	−8	0	0	0	0	0	0	−3	1	0	0	0
ψ_{12}	120	−8	0	0	0	0	0	0	−3	1	0	0	0
ψ_{13}	360	−24	0	0	0	0	0	0	0	0	0	0	0
ψ_{14}	360	−24	0	0	0	0	0	0	0	0	0	0	0

Table 7.2 (continued)

$[g_{ikm}]_{H_2}$	$4f$	$8a$	$5a$	$6b$	$6c$	$12a$	$7a$	$14a$	$7b$	$14b$	$15a$	$15b$
$[g_{ikm}]_{H_2}$	g_{822}	$g_{14,21}$	g_{921}	$g_{11,21}$	$g_{10,22}$	$g_{15,21}$	$g_{12,21}$	$g_{16,21}$	$g_{13,21}$	$g_{17,21}$	$g_{18,21}$	$g_{19,21}$
$\lvert C_{H_2}(g_{ikm})\rvert$	16	16	15	12	12	12	14	14	14	14	15	15
ψ_1	0	0	2	0	0	0	1	1	1	1	1	1
ψ_2	0	0	1	0	0	0	A	A	$\overline{A}$	$\overline{A}$	-1	-1
ψ_3	0	0	1	0	0	0	$\overline{A}$	$\overline{A}$	A	A	-1	-1
ψ_4	0	0	2	0	0	0	-1	-1	-1	-1	1	1
ψ_5	0	0	-1	0	$-i\sqrt{3}$	$i\sqrt{3}$	0	0	0	0	1	1
ψ_6	0	0	-1	0	$i\sqrt{3}$	$-i\sqrt{3}$	0	0	0	0	1	1
ψ_7	0	0	-1	0	0	0	0	0	0	0	B	$\overline{B}$
ψ_8	0	0	-1	0	0	0	0	0	0	0	$\overline{B}$	B
ψ_9	0	0	1	0	0	0	1	1	1	1	-1	-1
ψ_{10}	0	0	0	0	0	0	1	-1	1	-1	0	0
ψ_{11}	0	0	0	0	$-i\sqrt{3}$	$-i\sqrt{3}$	1	-1	1	-1	0	0
ψ_{12}	0	0	0	0	$i\sqrt{3}$	$i\sqrt{3}$	1	-1	1	-1	0	0
ψ_{13}	0	0	0	0	0	0	A	$-A$	$\overline{A}$	$-\overline{A}$	0	0
ψ_{14}	0	0	0	0	0	0	$\overline{A}$	$-\overline{A}$	A	$-A$	0	0

where in Table 7.2, $A = -\frac{1}{2}\left(1 + i\sqrt{7}\right)$ and $B = -\frac{1}{2}\left(1 + i\sqrt{15}\right)$.

Table 7.2 indicates that classes $2b$, $2c$, $2d$, $4a$, $4b$, $4c$, $4d$, $4e$, $4f$, $6b$ and $8a$ are $\alpha-$irregular classes of H_2 by Theorem 4.3.3(iii). Thus H_2 has fourteen $\alpha-$regular classes and 14 characters with factor set α, as required by Theorem 4.3.3(i).

7.1.4 Fusion of Classes of H_2 into Classes of $GL(5,2)$

The permutation character $\chi(G|H_2)$ of $GL(5,2)$ on H_2 is of degree 31. From the $\mathbb{ATLAS}$, we see that $\chi(G|H_2)$ decomposes into the form $\mathbf{1a} + \mathbf{31a}$, where $\mathbf{1a}$ is the identity character of $GL(5,2)$ and $\mathbf{31a}$ is the irreducible character of $GL(5,2)$ of degree 31. With the aid of the permutation character and centralizer sizes, we were able to determine the fusion of classes of H_2 into classes of $GL(5,2)$. In Table 7.3 we show how to calculate the fusion of classes of H_2 into classes of $GL(5,2)$ and in Table 7.4 we listed the fusion. We remark that in Table 7.3 we have listed only those classes of $GL(5,2)$ that could be a possible candidate for fusions. For example since the group H_2 does not contain elements of orders 21 and 31, we omitted the classes $21A$, $21B$, $31A$, $31B$, $31C$, $31D$, $31E$ and $31F$, of $GL(5,2)$ in Table 7.4.

Table 7.3: The fusion of $H_2 = 2^4{:}GL(4,2)$ into $G = GL(5,2)$

	$[g]_{GL(5,2)}$	$1A$	$2A$	$2B$	$3A$	$3B$	$4A$	$4B$	$4C$	$5A$
	$\lvert C_{GL(5,2)}(g_i)\rvert$	9999360	21504	1536	504	180	384	128	32	15
$[g_{ikm}]_{H_2}$	$\lvert C_{H_2}(g_{ikm})\rvert$									
$1a = g_{121}$	322560	[31]								
$2a = g_{221}$	21504		[1]							
$2b = g_{222}$	1536		[14]	1						
$2c = g_{321}$	512		42	[3]						
$2d = g_{322}$	384		56	[4]						
$3a = g_{521}$	180					[1]				
$3b = g_{421}$	72				[7]					
$4a = g_{621}$	384						[1]			
$4b = g_{721}$	128						3	[1]		
$4c = g_{622}$	64						[6]	2		
$4d = g_{722}$	64						6	[2]		
$4e = g_{821}$	32						12	4	[1]	
$4f = g_{822}$	16						24	8	[2]	
$5a = g_{921}$	15									[1]
	$\chi(G\vert H_2)$	31	15	7	7	1	7	3	3	1

Table 7.3 (continued)

		$6A$	$6B$	$7A$	$7B$	$8A$	$12A$	$14A$	$14B$	$15A$	$15B$
	$\lvert C_{GL(5,2)}(g_i)\rvert$	24	12	42	42	16	12	14	14	15	15
$[g_{ikm}]_{H_2}$	$\lvert C_{H_2}(g_{ikm})\rvert$										
$6a = g_{10,21}$	24	[1]									
$6b = g_{11,21}$	12	2	[1]								
$6c = g_{10,22}$	12	[2]	1								
$7a = g_{12,21}$	14			[3]	3						
$7b = g_{13,21}$	14			3	[3]						
$8a = g_{14,21}$	16					[1]					
$12a = g_{15,21}$	12						[1]				
$14a = g_{16,21}$	14							[1]			
$14b = g_{17,21}$	14								[1]		
$15a = g_{18,21}$	15									[1]	1
$15b = g_{19,21}$	15									1	[1]
	$\chi(G\vert H_2)$	3	1	3	3	1	1	1	1	1	1

Table 7.4: The fusion of $H_2 = 2^4{:}GL(4,2)$ into $G = GL(5,2)$

Inertia Factor Group H_2	Class of H_2	$\hookrightarrow$	Class of $GL(5,2)$	Class of H_2	$\hookrightarrow$	Class of $GL(5,2)$
$H_2 = 2^4{:}GL(4,2)$	$1a = g_{121}$		$1A$	$5a = g_{921}$		$5A$
	$2a = g_{221}$		$2A$	$6a = g_{10,21}$		$6A$
	$2b = g_{222}$		$2A$	$6b = g_{11,21}$		$6B$
	$2c = g_{321}$		$2B$	$6c = g_{10,22}$		$6A$
	$2d = g_{322}$		$2B$	$7a = g_{12,21}$		$7A$
	$3a = g_{521}$		$3B$	$7b = g_{13,21}$		$7B$
	$3b = g_{421}$		$3A$	$8a = g_{14,21}$		$8A$
	$4a = g_{621}$		$4A$	$12a = g_{15,21}$		$12A$
	$4b = g_{721}$		$4B$	$14a = g_{16,21}$		$14A$
	$4c = g_{622}$		$4A$	$14b = g_{17,21}$		$14B$
	$4d = g_{722}$		$4B$	$15a = g_{18,21}$		$15A$
	$4e = g_{821}$		$4C$	$15b = g_{19,21}$		$15B$
	$4f = g_{822}$		$4C$			

7.1.5 Fischer Matrices of $2^{5\cdot}GL(5,2)$

We recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of $g_{ij},\ 1 \leq j \leq c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 7.1 we supplied $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \leq i \leq 27,\ 1 \leq j \leq c(g_i)$. Also having obtained the fusion of the inertia factor group H_2 into $GL(5,2)$, we are able to label the rows of the Fischer matrices as described in Section 5.3. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 7.1 that the sizes of the Fischer matrices of $\overline{G} = 2^{5\cdot}GL(5,2)$ range between 1 and 3 for every $i \in \{1, 2, \cdots, 27\}$.

We have used the arithmetical properties of Fischer matrices, given in Proposition 5.2.2, to calculate some of the entries of the Fischer matrices and also to build an algebraic system of equations. With the help of the symbolic mathematical package Maxima [45], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}
$o(g_{1j})$		1	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		319979520	10321920
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$		
$(1,1)$	9999360	1	1
$(2,1)$	322560	31	-1
m_{1j}		1	31

$\mathcal{F}_2$

g_2		g_{21}	g_{22}
$o(g_{2j})$		4	2
$\lvert C_{\overline{G}}(g_{2j})\rvert$		43008	43008
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$		
$(1,1)$	21504	1	1
$(2,1)$	21504	1	-1
m_{2j}		16	16

$\mathcal{F}_3$

g_3		g_{31}
$o(g_{3j})$		4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		1536
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$	
$(1,1)$	1536	1
m_{3j}		32

$\mathcal{F}_4$

g_4		g_{41}	g_{42}
$o(g_{4j})$		3	6
$\lvert C_{\overline{G}}(g_{4j})\rvert$		4032	576
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$		
$(1,1)$	504	1	1
$(2,1)$	72	7	-1
m_{4j}		4	28

$\mathcal{F}_5$

g_5		g_{51}	g_{52}
$o(g_{5j})$		6	3
$\lvert C_{\overline{G}}(g_{5j})\rvert$		360	360
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$		
$(1,1)$	180	1	1
$(2,1)$	180	1	-1
m_{5j}		16	16

$\mathcal{F}_6$

g_6		g_{61}
$o(g_{6j})$		8
$\lvert C_{\overline{G}}(g_{6j})\rvert$		384
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$	
$(1,1)$	384	1
m_{6j}		32

$\mathcal{F}_7$

g_7		g_{71}
$o(g_{7j})$		4
$\lvert C_{\overline{G}}(g_{7j})\rvert$		128
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$	
$(1,1)$	128	1
m_{7j}		32

$\mathcal{F}_8$

g_8		g_{81}
$o(g_{8j})$		8
$\lvert C_{\overline{G}}(g_{8j})\rvert$		32
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$	
$(1,1)$	32	1
m_{8j}		32

$\mathcal{F}_9$

g_9		g_{91}	g_{92}
$o(g_{9j})$		10	5
$\lvert C_{\overline{G}}(g_{9j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$		
$(1,1)$	15	1	1
$(2,1)$	15	1	-1
m_{9j}		16	16

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$
$o(g_{10j})$		12	12	6
$\lvert C_{\overline{G}}(g_{10j})\rvert$		96	96	48
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$			
$(1,1)$	24	1	1	1
$(2,1)$	24	1	1	-1
$(2,2)$	12	2	-2	0
m_{10j}		8	8	16

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$
$o(g_{11j})$		12
$\lvert C_{\overline{G}}(g_{11j})\rvert$		12
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$	
$(1,1)$	12	1
m_{11j}		32

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$
$o(g_{12j})$		7	14
$\lvert C_{\overline{G}}(g_{12j})\rvert$		168	56
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$		
$(1,1)$	42	1	1
$(2,1)$	14	3	-1
m_{12j}		8	24

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$
$o(g_{13j})$		7	14
$\lvert C_{\overline{G}}(g_{13j})\rvert$		168	56
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$		
$(1,1)$	42	1	1
$(2,1)$	14	3	-1
m_{13j}		8	24

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$
$o(g_{14j})$		8
$\lvert C_{\overline{G}}(g_{14j})\rvert$		16
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$	
$(1,1)$	16	1
m_{14j}		32

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$
$o(g_{15j})$		24	24
$\lvert C_{\overline{G}}(g_{15j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{15j}		16	16

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$
$o(g_{16j})$		28	14
$\lvert C_{\overline{G}}(g_{16j})\rvert$		28	28
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$		
$(1,1)$	28	1	1
$(2,1)$	28	1	-1
m_{16j}		16	16

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$
$o(g_{17j})$		28	14
$\lvert C_{\overline{G}}(g_{17j})\rvert$		28	28
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$		
$(1,1)$	28	1	1
$(2,1)$	28	1	-1
m_{17j}		16	16

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$
$o(g_{18j})$		15	30
$\lvert C_{\overline{G}}(g_{18j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	30	1	-1
m_{18j}		16	16

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		15	30
$\lvert C_{\overline{G}}(g_{19j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	30	1	-1
m_{19j}		16	16

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$
$o(g_{20j})$		21
$\lvert C_{\overline{G}}(g_{20j})\rvert$		21
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$	
$(1,1)$	21	1
m_{20j}		32

$\mathcal{F}_{21}$		
g_{21}		$g_{21,1}$
$o(g_{21j})$		21
$\lvert C_{\overline{G}}(g_{21j})\rvert$		21
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$	
$(1,1)$	21	1
m_{21j}		32

$\mathcal{F}_{22}$		
g_{22}		$g_{22,1}$
$o(g_{22j})$		31
$\lvert C_{\overline{G}}(g_{22j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$	
$(1,1)$	31	1
m_{22j}		32

$\mathcal{F}_{23}$		
g_{23}		$g_{23,1}$
$o(g_{23j})$		31
$\lvert C_{\overline{G}}(g_{23j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$	
$(1,1)$	31	1
m_{23j}		32

$\mathcal{F}_{24}$		
g_{24}		$g_{24,1}$
$o(g_{24j})$		31
$\lvert C_{\overline{G}}(g_{24j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$	
$(1,1)$	31	1
m_{24j}		32

$\mathcal{F}_{25}$		
g_{25}		$g_{25,1}$
$o(g_{25j})$		31
$\lvert C_{\overline{G}}(g_{25j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$	
$(1,1)$	31	1
m_{25j}		32

$\mathcal{F}_{26}$		
g_{26}		$g_{26,1}$
$o(g_{26j})$		31
$\lvert C_{\overline{G}}(g_{26j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$	
$(1,1)$	31	1
m_{26j}		32

$\mathcal{F}_{27}$		
g_{27}		$g_{27,1}$
$o(g_{27j})$		31
$\lvert C_{\overline{G}}(g_{27j})\rvert$		31
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$	
$(1,1)$	31	1
m_{27j}		32

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7.1.6 The Character Table of Dempwolff Group $\overline{G} = 2^{5\cdot}GL(5,2)$

Now we are in the position to construct the character table of Dempwolff group $\overline{G} = 2^{5\cdot}GL(5,2)$. In the previous subsections we have found

- the conjugacy classes of $\overline{G} = 2^{5\cdot}GL(5,2)$ (Table 7.1),
- the projective character table of the inertia factor $H_2 = 2^4{:}GL(4,2)$ with factor set $\alpha,\ \alpha \sim [2]$ (Table 7.2),

- the fusion of classes of the inertia factor H_2 into classes of $GL(5,2)$ (Table 7.4),
- the Fischer matrices of $\overline{G}$ (Subsection 7.1.5).

By Section 5.3, it follows that the full character table of $\overline{G}$ can be constructed easily. We give an example on how to construct the character table of $\overline{G}$, which is partitioned into 54 blocks corresponding to the 27 cosets and the two inertia factor groups. As an example we construct the parts $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$ and $\mathcal{K}_{10,2}\mathcal{F}_{10,2}$ of the character table of $\overline{G}$ (this means that we are listing the values of all the irreducible characters of $\overline{G}$ on the classes $g_{10,1}$, $g_{10,2}$ and $g_{10,3}$ of $\overline{G}$, which correspond to the conjugacy class $6A$ of $GL(5,2)$). The two parts $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$ and $\mathcal{K}_{10,2}\mathcal{F}_{10,2}$ can be derived as follows: From Table 7.4 we can see that there are two $\alpha-$regular classes, namely $6a = g_{10,21}$ and $6c = g_{10,22}$ of H_2 that fuse into the class $g_6 = [6A]_{GL(5,2)}$. To construct the part $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$, we multiply the column of the character table of $H_1 = GL(5,2)$ corresponds to the class $6A$ of $GL(5,2)$ (see the $\mathbb{ATLAS}$), by the first row of $\mathcal{F}_{10}$, namely $(1 \quad 1 \quad 1)$ and thus the part $\mathcal{K}_{10,1}\mathcal{F}_{10,1}$ of size 27×3, consists of the column of the character table of $GL(5,2)$ corresponds to the class $6A$ repeated 3 times. To construct the part $\mathcal{K}_{10,2}\mathcal{F}_{10,2}$, select the two columns of the projective character table of $H_2 = 2^4{:}GL(4,2)$, with factor set α, $\alpha \sim [2]$, correspond to the classes $6a$ and $6c$ of H_2 (see Table 7.2) and multiply these two columns by the two rows of $\mathcal{F}_{10}$ correspond to the pair $(2,1)$ and $(2,2)$. Thus we get a block in the character table of $\overline{G}$ of size 14×3. The above two parts have the following form:

$$\mathcal{K}_{10,1}\mathcal{F}_{10,1} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 2 \\ 1 \\ 1 \\ -2 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{10,1} & g_{10,2} & g_{10,3} \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{10,2}\mathcal{F}_{10,2} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & -i\sqrt{3} \\ -1 & i\sqrt{3} \\ 2 & 0 \\ 2 & 0 \\ -2 & 0 \\ -2 & 0 \\ 1 & -i\sqrt{3} \\ 1 & i\sqrt{3} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{10,1} & g_{10,2} & g_{10,3} \end{matrix} \\ \begin{pmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1-2i\sqrt{3} & -1+2i\sqrt{3} & 1 \\ -1+2i\sqrt{3} & -1-2i\sqrt{3} & 1 \\ 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 2 \\ -2 & -2 & 2 \\ 1-2i\sqrt{3} & 1+2i\sqrt{3} & -1 \\ 1+2i\sqrt{3} & 1-2i\sqrt{3} & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}.$$

Similarly one can construct all the other 52 parts $\mathcal{K}_{ik}\mathcal{F}_{ik},\ k \in \{1,2\},\ i \in \{1,2,\cdots,27\} \setminus \{10\}$. The full character table of $\overline{G} = 2^{5\cdot}GL(5,2)$ is available in many sources such as Magma, GAP or the book of G. Michler [46]. However, in Table 11.9, we supply the character table of $\overline{G}$ in the format of Clifford-Fischer theory (characters are organized in blocks corresponding to the inertia factors and the conjugacy classes of $\overline{G}$, where the conjugacy classes are arranged in blocks corresponding to the cosets $2^5\overline{g}_i$, where $\overline{g}_i$ is a pre-image of a representative g_i of a conjugacy class of $G = GL(5,2)$).

7.2. A Group of the Form $2_+^{1+8\cdot}A_9$

In this section we construct the character table of a maximal subgroup of the Thompson sporadic simple group Th. This group is a non-split extension of the extraspecial 2−group 2_+^{1+8} by the alternating group A_9. In the Subsection 7.2.2 we compute the conjugacy classes of the group $\overline{G} = 2_+^{1+8\cdot}A_9$ using the coset analysis technique. In Subsection 7.2.3 we study the inertia groups of $\overline{G}$. One of the inertia factor groups is a group of the form $PSL(2,8)$:3 and we study its character table via Clifford-Fischer theory in Subsection 7.2.4. In Subsection 7.2.5 we supply the fusion of classes of the inertia factors into classes of A_9 and in Subsection 7.2.6 we calculate the Fischer matrices of $\overline{G}$, which have sizes range between 1 and 5. The theory of projective characters is involved in the construction of the character table of $\overline{G}$.

7.2.1 Introduction

Let $\overline{G}$ be the centralizer of an involution in the sporadic Thompson group Th, which is the third largest maximal subgroup of Th (see $\mathbb{ATLAS}$). Then $\overline{G} = 2_+^{1+8\cdot}A_9$ is the non-split extension of the extraspecial 2−group of order 512, with outer automorphism group isomorphic to $O_8^+(2)$, by the Alternating group A_9. It has order 92 897 280 and index 976 841 775 in Th. Let x and y be in the 248−dimensional matrix group over $\mathbb{F}_2$, that are generators of Th given by the electronic ATLAS of Wilson. Using Magma and the programm supplied by Wilson, we constructed $\overline{G}$ inside Th. We were also able to locate a normal subgroup $N \lhd \overline{G}$ that is isomorphic to 2_+^{1+8} by checking the normal subgroups of $\overline{G}$ and the conditions of extraspecial p−groups.

Remark 7.2.1. A presentation for $\overline{G} = 2_+^{1+8\cdot}A_9$, due to Havas, Soicher and Wilson [33], has been restated in Lemmas 12.1.2 and 12.1.3 of Michler [46].

7.2.2 Conjugacy Classes of Group Extensions and of $\overline{G} = 2_+^{1+8}{\cdot}A_9$

In [46], G. Michler determined the conjugacy classes of $\overline{G} = 2_+^{1+8}{\cdot}A_9$ using the Algorithm of Kratzer (see page 294 of Michler [46]). One can also use Magma or GAP, with the presentation of $\overline{G}$ given by the ATLAS of Wilson, to compute its conjugacy classes. In this subsection, the conjugacy classes of $\overline{G} = 2_+^{1+8}{\cdot}A_9$ have been determined using the coset analysis method, discussed in Section 2.3, as we are interested to see the correspondence of the conjugacy classes of $\overline{G}$ with those classes of $G = A_9$.

Example 7.2.1. Consider the identity coset $N1_{A_9} = N = 2_+^{1+8}$ as this coset gives much information about the structure of the character table of $\overline{G}$. If $g = 1_{A_9}$, then the action of N on $N\overline{1_{A_9}} = N1_{A_9} = N$ produces the conjugacy classes of N, where we know that N has

- singleton conjugacy class consisting of 1_N,
- singleton conjugacy class consisting of the central involution σ of N,
- 135 conjugacy classes, each class consists of two non-central involutions,
- 120 conjugacy classes, each class consists of two elements each is of order 4.

Now using Magma, the action of $\overline{G}$ on the preceding orbits leaves invariant $\{1_N\} := \Delta_{11}$ and $\{\sigma\} := \Delta_{12}$, while fuse the 135 orbits of non-central involutions into a single orbit Δ_{13} and also fuse the 120 orbits of elements of order 4 altogether into a single orbit Δ_{14}. Thus the identity coset N produces four conjugacy classes $\Delta_{11},\ \Delta_{12},\ \Delta_{13}$ and Δ_{14} in $\overline{G}$, where $|\Delta_{11}| = |\Delta_{12}| = 1,\ |\Delta_{13}| = 270$ and $|\Delta_{14}| = 240$. We let $g_{11} = 1_{\overline{G}},\ g_{12} = \sigma,\ g_{13} \in \Delta_{13}$ and $g_{14} \in \Delta_{14}$ be representatives for the $\overline{G}-$conjugacy classes obtained from N.

Remark 7.2.2. One can go further in Example 7.2.1 and show that the corresponding point stabilizers of $g_{11},\ g_{12},\ g_{13}$ and g_{14} are $\overline{G},\ \overline{G},\ 2^8{\cdot}(2^3{:}GL(3,2))$ and $2^8{\cdot}(PSL(2,8){:}3)$ respectively, but this is irrelevant to our study.

Corollary 7.2.1. *The character table of $\overline{G}$ is composed of four blocks of characters.*

PROOF. By Brauer Theorem (see Theorem 5.1.1), the number of orbits of $\overline{G}$ on $\mathrm{Irr}(2_+^{1+8})$ is same as the number of orbits of $\overline{G}$ on 2_+^{1+8}. Hence the result. ■

In Table 7.5, we list the conjugacy classes of $\overline{G}$ together with the fusions of its classes into the classes of Thompson group Th.

Table 7.5: The conjugacy classes of $2_+^{1+8}.A_9$

$[g_i]_{A_9}$	m_{ij}	$[g_{ij}]_{2_+^{1+8}.A_9}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2_+^{1+8}.A_9}\rvert$	$\lvert C_{2_+^{1+8}.A_9}(g_{ij})\rvert$	$\hookrightarrow$ Th
	$m_{11} = 1$	g_{11}	1	1	92897280	$1A$
$g_1 = 1A$	$m_{12} = 1$	g_{12}	2	1	92897280	$2A$
	$m_{13} = 270$	g_{13}	2	270	344064	$2A$
	$m_{14} = 240$	g_{14}	4	240	387072	$4A$
$g_2 = 2A$	$m_{21} = 32$	g_{21}	4	12096	7680	$4B$
	$m_{22} = 480$	g_{22}	4	181440	512	$4B$
	$m_{31} = 32$	g_{31}	2	30240	3072	$2A$
$g_3 = 2B$	$m_{32} = 32$	g_{32}	4	30240	3072	$4A$
	$m_{33} = 192$	g_{33}	4	181440	512	$4B$
	$m_{34} = 256$	g_{34}	8	241920	384	$8A$
$g_4 = 3A$	$m_{41} = 256$	g_{41}	3	43008	2160	$3C$
	$m_{42} = 256$	g_{42}	6	43008	2160	$6A$
	$m_{51} = 64$	g_{51}	3	143360	648	$3B$
$g_5 = 3B$	$m_{52} = 64$	g_{52}	6	143360	648	$6C$
	$m_{53} = 384$	g_{53}	12	860160	108	$12C$
	$m_{61} = 16$	g_{61}	3	53760	1728	$3A$
	$m_{62} = 16$	g_{62}	6	53760	1728	$6B$
$g_6 = 3C$	$m_{63} = 288$	g_{63}	6	967680	96	$6B$
	$m_{64} = 96$	g_{64}	12	322560	288	$12A$
	$m_{65} = 96$	g_{65}	12	322560	288	$12B$
$g_7 = 4A$	$m_{71} = 128$	g_{71}	8	967680	96	$8B$
	$m_{72} = 384$	g_{72}	8	2903040	32	$8B$
	$m_{81} = 128$	g_{81}	4	1451520	64	$4B$
$g_8 = 4B$	$m_{82} = 128$	g_{82}	8	1451520	64	$8A$
	$m_{83} = 256$	g_{83}	8	2903040	32	$8B$

Continued on next page

Table 7.5 (continued)

$[g_i]_{A_9}$	m_{ij}	$[g_{ij}]_{2_+^{1+8}\cdot A_9}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2_+^{1+8}\cdot A_9}\rvert$	$\lvert C_{2_+^{1+8}\cdot A_9}(g_{ij})\rvert$	$\hookrightarrow$ Th
$g_9 = 5A$	$m_{91} = 256$	g_{91}	5	774144	120	$5A$
	$m_{92} = 256$	g_{92}	10	774144	120	$10A$
$g_{10} = 6A$	$m_{10,1} = 512$	$g_{10,1}$	12	3870720	24	$12D$
	$m_{11,1} = 128$	$g_{11,1}$	6	3870720	24	$6B$
	$m_{11,2} = 64$	$g_{11,2}$	12	1935360	48	$12A$
$g_{11} = 6B$	$m_{11,3} = 64$	$g_{11,3}$	12	1935360	48	$12B$
	$m_{11,4} = 128$	$g_{11,4}$	24	3870720	24	$24A$
	$m_{11,5} = 128$	$g_{11,5}$	24	3870720	24	$24B$
	$m_{12,1} = 64$	$g_{12,1}$	7	1658880	56	$7A$
	$m_{12,2} = 64$	$g_{12,2}$	14	1658880	56	$14A$
$g_{12} = 7A$	$m_{12,3} = 128$	$g_{12,3}$	14	3317760	28	$14A$
	$m_{12,4} = 128$	$g_{12,4}$	14	3317760	28	$14A$
	$m_{12,5} = 128$	$g_{12,5}$	28	3317760	28	$28A$
$g_{13} = 9A$	$m_{13,1} = 256$	$g_{13,1}$	9	5160960	18	$9C$
	$m_{13,2} = 256$	$g_{13,2}$	18	5160960	18	$18B$
	$m_{14,1} = 64$	$g_{14,1}$	9	1290240	72	$9A$
	$m_{14,2} = 64$	$g_{14,2}$	18	1290240	72	$18A$
$g_{14} = 9B$	$m_{14,3} = 128$	$g_{14,3}$	36	2580480	36	$36B$
	$m_{14,4} = 128$	$g_{14,4}$	36	2580480	36	$36A$
	$m_{14,5} = 128$	$g_{14,5}$	36	2580480	36	$36C$
$g_{15} = 10A$	$m_{15,1} = 512$	$g_{15,1}$	20	4644864	20	$20A$
$g_{16} = 12A$	$m_{16,1} = 256$	$g_{16,1}$	24	3870720	24	$24C$
	$m_{16,2} = 256$	$g_{16,2}$	24	3870720	24	$24D$
$g_{17} = 15A$	$m_{17,1} = 256$	$g_{17,1}$	15	3096576	30	$15A$
	$m_{17,2} = 256$	$g_{17,2}$	30	3096576	30	$30A$
$g_{18} = 15B$	$m_{18,1} = 256$	$g_{18,1}$	15	3096576	30	$15B$
	$m_{18,2} = 256$	$g_{18,2}$	30	3096576	30	$30B$

7.2.3 Inertia Factor Groups of $\overline{G} = 2_+^{1+8}{\cdot}A_9$

The action of $\overline{G} = 2_+^{1+8}{\cdot}A_9$ on $\mathrm{Irr}(2_+^{1+8})$ produces four orbits of lengths 1, 120, 135 and 1 with representatives $\theta_1,\ \theta_2,\ \theta_3$ and θ_4 respectively, where $\theta_1 = \mathbf{1}_N$ the trivial character of N and θ_4 is the unique faithful irreducible character of N of degree 16. Hence the inertia factor groups $H_1,\ H_2,\ H_3$ and H_4 of $\theta_1,\ \theta_2,\ \theta_3$ and θ_4 respectively have indices 1, 120, 135 and 1 respectively in $G \cong A_9$. A subgroup of A_9 of index 120 is a maximal subgroup isomorphic to $PSL(2,8)$:3 (by considering the maximal subgroups of A_9 given in $\mathbb{ATLAS}$). A subgroup of A_9 of index 135 can only be contained in a maximal subgroup that is isomorphic to A_8 and must have index 15 in A_8. Again by considering the maximal subgroups of A_8 given by $\mathbb{ATLAS}$, it turns out that this subgroup is isomorphic to $2^3{:}GL(3,2)$. Hence we have determined all the inertia factor groups $H_1,\ H_2,\ H_3$ and H_4. We list brief information on these inertia factors in the following:

$$\begin{aligned} H_1 &= A_9, & [A_9 : H_1] &= 1, & |\mathrm{Irr}(H_1)| &= 18, \\ H_2 &= PSL(2,8){:}3, & [A_9 : H_2] &= 120, & |\mathrm{Irr}(H_2)| &= 11, \end{aligned}$$

$$\begin{aligned} H_3 &= 2^3{:}GL(3,2), & [A_9 : H_3] &= 135, & |\mathrm{Irr}(H_3)| &= 11, \\ H_4 &= A_9, & [A_9 : H_4] &= 1, & |\mathrm{IrrProj}(H_4, 2)| &= 12. \end{aligned}$$

Note that in the above we have listed the number of ordinary irreducible characters of $H_1,\ H_2,\ H_3$ and the number of projective characters of H_4 with the factor set $\alpha^{-1},\ \alpha \sim [2]$ as we shall see later that this projective table is needed for the construction of the character table of $\overline{G} = 2_+^{1+8}{\cdot}A_9$. Therefore the character table of $\overline{G}$ is composed of four blocks of characters (as mentioned in Corollary 7.2.1) corresponds to the ordinary character tables of $A_9,\ PSL(2,8){:}3,\ 2^3{:}GL(3,2)$ and the projective table of A_9 with factor set $\alpha^{-1},\ \alpha \sim [2]$. Hence $\overline{G}$ has altogether 52 irreducible characters, which is exactly the same number of conjugacy classes we obtained in Subsection 7.2.2.

7.2.4 The Character Table of the Inertia Factor Group H_2

In this Subsection we construct the character table of $H_2 = PSL(2,8){:}3$, by means of Clifford-Fischer Theory.

The Conjugacy Classes of $H_2 = PSL(2,8)$:3

We apply the method of the coset analysis to compute the conjugacy classes of the split extension $H_2 = PSL(2,8)$:3. In Table 7.6 we give the character table of $PSL(2,8)$, although at this stage we are only interested in its conjugacy classes.

Table 7.6: The character table of $PSL(2,8)$

	$1a$	$2a$	$3a$	$7a$	$7b$	$7c$	$9a$	$9b$	$9c$
$\lvert C_{PSL(2,8)}(g)\rvert$	504	8	9	7	7	7	9	9	9
$\lvert [g]_{PSL(2,8)}\rvert$	1	63	56	72	72	72	56	56	56
ϑ_1	1	1	1	1	1	1	1	1	1
ϑ_2	7	-1	-2	0	0	0	1	1	1
ϑ_3	7	-1	1	0	0	0	D	E	F
ϑ_4	7	-1	1	0	0	0	E	F	D
ϑ_5	7	-1	1	0	0	0	F	D	E
ϑ_6	8	0	-1	1	1	1	-1	-1	-1
ϑ_7	9	1	0	A	C	B	0	0	0
ϑ_8	9	1	0	B	A	C	0	0	0
ϑ_9	9	1	0	C	B	A	0	0	0

where, $A = e^{\frac{6\pi}{7}i} + e^{\frac{8\pi}{7}i}$, $B = e^{\frac{4\pi}{7}i} + e^{\frac{10\pi}{7}i}$, $C = e^{\frac{2\pi}{7}i} + e^{\frac{12\pi}{7}i}$, $D = -e^{\frac{8\pi}{9}i} - e^{\frac{10\pi}{9}i}$, $E = -e^{\frac{4\pi}{9}i} - e^{\frac{14\pi}{9}i}$, $F = e^{\frac{4\pi}{9}i} + e^{\frac{8\pi}{9}i} + e^{\frac{10\pi}{9}i} + e^{\frac{14\pi}{9}i}$.

Since the extension splits, the action of H_2 on its normal subgroup $M = PSL(2,8)$ reduces to the action of M on itself followed by the action of $\mathbb{Z}_3$ on the resulting orbits of M on M. The resulting orbits of M on M are the conjugacy classes of $M = PSL(2,8)$. Thus $k_1 = 9$ (recall that k_i is the number of orbits obtained on the action of M on the coset $M\overline{g}_i = Mg_i$ as described by the coset analysis method). By regarding M as a subgroup of A_9, various orbits are:

$$
\begin{aligned}
Q_{11} &= 1^M,\ 1 \in 1a,\\
Q_{12} &= (2\ 3)(4\ 6)(5\ 9)(7\ 8)^M,\quad (2\ 3)(4\ 6)(5\ 9)(7\ 8) \in 2a,\\
Q_{13} &= (1\ 2\ 3)(4\ 7\ 5)(6\ 9\ 8)^M,\quad (1\ 2\ 3)(4\ 7\ 5)(6\ 9\ 8) \in 3a,\\
Q_{14} &= (3\ 4\ 5\ 6\ 7\ 8\ 9)^M,\quad (3\ 4\ 5\ 6\ 7\ 8\ 9) \in 7a,\\
Q_{15} &= (3\ 5\ 7\ 9\ 4\ 6\ 8)^M,\quad (3\ 5\ 7\ 9\ 4\ 6\ 8) \in 7b,\\
Q_{16} &= (3\ 6\ 9\ 5\ 8\ 4\ 7)^M,\quad (3\ 6\ 9\ 5\ 8\ 4\ 7) \in 7c,\\
Q_{17} &= (1\ 2\ 3\ 6\ 7\ 4\ 8\ 9\ 5)^M,\quad (1\ 2\ 3\ 6\ 7\ 4\ 8\ 9\ 5) \in 9a,\\
Q_{18} &= (1\ 2\ 3\ 8\ 5\ 7\ 9\ 6\ 4)^M,\quad (1\ 2\ 3\ 8\ 5\ 7\ 9\ 6\ 4) \in 9b,\\
Q_{19} &= (1\ 2\ 3\ 9\ 4\ 5\ 6\ 8\ 7)^M,\quad (1\ 2\ 3\ 9\ 4\ 5\ 6\ 8\ 7) \in 9c.
\end{aligned}
$$

Now let $\omega = \langle (1\ 2\ 3)(4\ 7\ 5)(6\ 9\ 8) \rangle$ and that $\mathbb{Z}_3 = \langle \omega \rangle$. The action by conjugation of $C_{\mathbb{Z}_3}(\omega) = \mathbb{Z}_3$ produces the following new orbits $\Delta_{11},\ \Delta_{12},\ \Delta_{13},\ \Delta_{14}$ and Δ_{15}, where

$$\Delta_{11} = Q_{11},\ \Delta_{12} = Q_{12},\ \Delta_{13} = Q_{13},\ \Delta_{14} = Q_{14} \bigcup Q_{15} \bigcup Q_{16},\ \Delta_{15} = Q_{17} \bigcup Q_{18} \bigcup Q_{19}.$$

Thus $|\Delta_{11}| = 1,\ |\Delta_{12}| = 63,\ |\Delta_{13}| = 56,\ |\Delta_{14}| = 216$ and $|\Delta_{15}| = 168$. We deduce that $f_{11} = f_{12} = f_{13} = 1$ and $f_{14} = f_{15} = 3$. Hence the coset $M = PSL(2,8)$ produces 5 conjugacy classes in $H_2 = PSL(2,8)$:3, namely $\Delta_{11},\ \Delta_{12},\ \Delta_{13},\ \Delta_{14}$ and Δ_{15} and we let the representatives of these classes be $1a,\ 2a,\ 3a,\ 7a$ and $9a$ respectively.

Now we form the coset $M\omega$ and we act M on $M\omega$ and then $\mathbb{Z}_3$ on the orbits of action of M on $M\omega$. The action of M on $M\omega$ produces 3 orbits with representative elements of order 3, 6 and 9 respectively. None of these orbits fuse together under the action of $\mathbb{Z}_3$. Thus we obtain from the coset $M\omega$ three conjugacy classes in H_2 represented by $3b,\ 6a$ and $9b$, and these classes are of size 84, 252 and 168 respectively. The same holds for the coset $M\omega^{-1}$ and we name the H_2-classes of $M\omega^{-1}$ by $3c,\ 6b$ and $9c$. Now complete information on the conjugacy classes of $H_2 = PSL(2,8)$:3 are listed in Table 7.7

Constructing the Character Table of $H_2 = PSL(2,8)$:3 Using Clifford Theory

Having obtained 11 conjugacy classes for H_2 we must obtain 11 irreducible characters of this group. We use Clifford Theory discussed in Section 5.1 to obtain these characters. We have to find the inertia groups of characters of $M = PSL(2,8)$. It is not difficult to see that the stabilizer of $\vartheta_1,\ \vartheta_2$

Table 7.7: The conjugacy classes of $H_2 = PSL(2,8)$:3

$[g_i]_{\mathbb{Z}_3}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{PSL(2,8):3}$	$\lvert[g_{ij}]_{PSL(2,8):3}\rvert$	$\lvert C_{PSL(2,8):3}(g_{ij})\rvert$
		$f_{11}=1$	$m_{11}=56$	$h_{11}=1a$	1	1512
		$f_{12}=1$	$m_{12}=56$	$h_{12}=2a$	63	24
$g_1=1_{\mathbb{Z}_3}$	9	$f_{13}=1$	$m_{13}=56$	$h_{13}=3a$	56	27
		$f_{14}=3$	$m_{14}=168$	$h_{14}=7a$	216	7
		$f_{15}=3$	$m_{15}=168$	$h_{15}=9a$	168	9
		$f_{21}=1$	$m_{21}=168$	$h_{21}=3b$	84	18
$g_2=\omega$	3	$f_{22}=1$	$m_{22}=168$	$h_{22}=6a$	252	6
		$f_{23}=1$	$m_{23}=168$	$h_{23}=9b$	168	9
		$f_{31}=1$	$m_{31}=168$	$h_{31}=3c$	84	18
$g_3=\omega^{-1}$	3	$f_{32}=1$	$m_{32}=168$	$h_{32}=6b$	252	6
		$f_{33}=1$	$m_{33}=168$	$h_{33}=9c$	168	9

and ϑ_6 is H_2 itself, while H_2 permutes $\{\vartheta_3, \vartheta_4, \vartheta_5\}$ together and also permutes $\{\vartheta_7, \vartheta_8, \vartheta_9\}$, where $\vartheta_1, \vartheta_2, \cdots, \vartheta_9$ are the characters of M listed in Table 7.6. Thus the inertia groups of the action of H_2 on $\mathrm{Irr}(N)$ are $\overline{T}_1 = \overline{T}_2 = \overline{T}_3 = H_2$ and $\overline{T}_4 = \overline{T}_5 = M$. Now let ξ_1, ξ_2 and ξ_3 be the irreducible characters of $\mathbb{Z}_3$, where ξ_1 is the trivial character and $\xi_2(\omega) = e^{\frac{2\pi}{3}i}$ with ω being the generator of $\mathbb{Z}_3$. Let $\widehat{\xi}_1, \widehat{\xi}_2$ and $\widehat{\xi}_3$ be the respective irreducible characters of H_2 that lift ξ_1, ξ_2 and ξ_3. This imply that $\widehat{\xi}_1, \widehat{\xi}_2$ and $\widehat{\xi}_3$ contain M in their kernels. Since the Schur multipliers of $H_2/H_2 \cong \{1_{\mathbb{Z}_3}\}$ and $H_2/N \cong \mathbb{Z}_3$ are trivial, we deduce that all characters of N are extendible to ordinary characters of their respective inertia groups. It follows that there exist $\widetilde{\vartheta}_1,\ \widetilde{\vartheta}_2,\ \widetilde{\vartheta}_6\ \in \mathrm{Irr}(H_2)$ and $\widetilde{\vartheta}_3,\ \widetilde{\vartheta}_7 \in \mathrm{Irr}(M)$ such that

$$\widetilde{\vartheta}_1{\downarrow}_M^{H_2} = \vartheta_1,\ \widetilde{\vartheta}_2{\downarrow}_M^{H_2} = \vartheta_2,\ \widetilde{\vartheta}_3{\downarrow}_M^{H_2} = \vartheta_3,\ \widetilde{\vartheta}_6{\downarrow}_M^{H_2} = \vartheta_6 \text{ and } \widetilde{\vartheta}_7{\downarrow}_M^{H_2} = \vartheta_7.$$

Now by Equation (5.2), the irreducible characters of H_2 are given by

$$\begin{aligned}
\mathrm{Irr}(H_2) &= \bigcup_{i=1}^{t}\Big\{(\widetilde{\vartheta}_i \inf(\xi))\uparrow_{\overline{T}_i}^{H_2} |\ \widetilde{\vartheta}_i \in \mathrm{Irr}(\overline{T}_i, \vartheta_i),\ \xi \in \mathrm{Irr}(\overline{T}_i/M)\Big\} \\
&= \bigcup_{i=1}^{5}\Big\{(\xi\psi_i)\uparrow_{\overline{T}_i}^{H_2} |\ \xi \in \mathrm{Irr}(\overline{T}_i)\Big\} \\
&= \Big\{\widetilde{\vartheta}_1\widehat{\xi}_1\uparrow_{H_2}^{H_2}, \widetilde{\vartheta}_1\widehat{\xi}_2\uparrow_{H_2}^{H_2}, \widetilde{\vartheta}_1\widehat{\xi}_3\uparrow_{H_2}^{H_2}\Big\}\bigcup\Big\{\widetilde{\vartheta}_2\widehat{\xi}_1\uparrow_{H_2}^{H_2}, \widetilde{\vartheta}_2\widehat{\xi}_2\uparrow_{H_2}^{H_2}, \widetilde{\vartheta}_2\widehat{\xi}_3\uparrow_{H_2}^{H_2}\Big\}\bigcup \\
&\quad \Big\{\widetilde{\vartheta}_6\widehat{\xi}_1\uparrow_{H_2}^{H_2}, \widetilde{\vartheta}_6\widehat{\xi}_2\uparrow_{H_2}^{H_2}, \widetilde{\vartheta}_6\widehat{\xi}_3\uparrow_{H_2}^{H_2}\Big\}\bigcup\Big\{\widetilde{\vartheta}_3\uparrow_{M}^{H_2}\Big\}\bigcup\Big\{\widetilde{\vartheta}_7\uparrow_{M}^{H_2}\Big\} \\
&= \Big\{\widehat{\xi}_1, \widehat{\xi}_2, \widehat{\xi}_3\Big\}\bigcup\Big\{\widetilde{\vartheta}_2\widehat{\xi}_1, \widetilde{\vartheta}_2\widehat{\xi}_2, \widetilde{\vartheta}_2\widehat{\xi}_3\Big\}\bigcup\Big\{\widetilde{\vartheta}_6\widehat{\xi}_1, \widetilde{\vartheta}_6\widehat{\xi}_2, \widetilde{\vartheta}_6\widehat{\xi}_3\Big\}\bigcup\Big\{\vartheta_3\uparrow_{M}^{H_2}\Big\}\bigcup\Big\{\vartheta_7\uparrow_{M}^{H_2}\Big\} \\
&= \Big\{\widehat{\xi}_1, \widehat{\xi}_2, \widehat{\xi}_3, \widetilde{\vartheta}_2\widehat{\xi}_1, \widetilde{\vartheta}_2\widehat{\xi}_2, \widetilde{\vartheta}_2\widehat{\xi}_3, \widetilde{\vartheta}_6\widehat{\xi}_1, \widetilde{\vartheta}_6\widehat{\xi}_2, \widetilde{\vartheta}_6\widehat{\xi}_3, \vartheta_3\uparrow_{M}^{H_2}, \vartheta_7\uparrow_{M}^{H_2}\Big\}.
\end{aligned}$$

Remark 7.2.3. Note that $\vartheta_3\uparrow_M^{H_2} = \vartheta_4\uparrow_M^{H_2} = \vartheta_5\uparrow_M^{H_2}$ and $\vartheta_7\uparrow_M^{H_2} = \vartheta_8\uparrow_M^{H_2} = \vartheta_9\uparrow_M^{H_2}$. Thus we get altogether a number of 11 irreducible characters of H_2, which is same as the number of conjugacy classes of H_2, given in Table 7.7.

Constructing the Character Table of $H_2 = PSL(2,8)$:3 Using Fischer Matrices

We have seen in the previous sub-subsection that the action of H_2 on $\mathrm{Irr}(M)$ produces 5 orbits $\{\vartheta_1\}$, $\{\vartheta_2\}$, $\{\vartheta_6\}$, $\{\vartheta_3, \vartheta_4, \vartheta_5\}$ and $\{\vartheta_7, \vartheta_8, \vartheta_9\}$. Therefore the identity Fischer matrix $\widehat{\mathcal{F}}_1$ is a 5×5 matrix. We know that the rows of $\widehat{\mathcal{F}}_1$ are the orbits sums of irreducible characters of M on the conjugacy classes of H_2 that are produced from M. Let $h_{11},\ h_{12},\ h_{13},\ h_{14}$ and h_{15} be representatives for the H_2-conjugacy classes obtained from M. From Sub-subsection 7.2.4 we get

$$\begin{aligned}
[h_{11}]_{H_2} &= [1a]_M,\ \ [h_{12}]_{H_2} = [2a]_M,\ \ [h_{13}]_{H_2} = [3a]_M, \\
[h_{14}]_{H_2} &= [7a]_M \bigcup [7b]_M \bigcup [7c]_M, \\
[h_{15}]_{H_2} &= [9a]_M \bigcup [9b]_M \bigcup [9c]_M.
\end{aligned}$$

The 5 rows of $\widehat{\mathcal{F}}_1$ will correspond to $\vartheta_1,\ \vartheta_2,\ \vartheta_6,\ \vartheta_3$ and ϑ_7 respectively. From Table 7.6, the values of the fourth row of $\widehat{\mathcal{F}}_1$ will be

$$\begin{aligned}
(\vartheta_3 + \vartheta_4 + \vartheta_5)(h_{11}) &= (\vartheta_3 + \vartheta_4 + \vartheta_5)(1a) = 7 + 7 + 7 = 21, \\
(\vartheta_3 + \vartheta_4 + \vartheta_5)(h_{12}) &= (\vartheta_3 + \vartheta_4 + \vartheta_5)(2a) = -1 + -1 + -1 = -3, \\
(\vartheta_3 + \vartheta_4 + \vartheta_5)(h_{13}) &= (\vartheta_3 + \vartheta_4 + \vartheta_5)(3a) = 1 + 1 + 1 = 3, \\
(\vartheta_3 + \vartheta_4 + \vartheta_5)(h_{14}) &= (\vartheta_3 + \vartheta_4 + \vartheta_5)(7a) = 0 + 0 + 0 = 0, \\
(\vartheta_3 + \vartheta_4 + \vartheta_5)(h_{15}) &= (\vartheta_3 + \vartheta_4 + \vartheta_5)(9a) = F + D + E = 0.
\end{aligned}$$

Similarly we can get all the rows of the identity Fischer matrix $\widehat{\mathcal{F}}_1$, which have the following form

$h_1 = 1_{\mathbb{Z}_3}$		h_{11}	h_{12}	h_{13}	h_{14}	h_{15}
$o(h_{ij})$		1	2	3	7	9
$\lvert C_{H_2}(h_{1j})\rvert$		1512	24	27	7	9
(k,m)	$\lvert C_{T_k}(h_{1km})\rvert$					
$(1,1)$	3	1	1	1	1	1
$(2,1)$	3	7	-1	-2	0	1
$(3,1)$	3	8	0	-1	1	-1
$(4,1)$	1	21	-3	3	0	0
$(5,1)$	1	27	3	0	-1	0
m_{1j}		56	56	56	168	168

where $T_k = H_2/\overline{T}_k$. Since we know that the Fischer matrices $\widehat{\mathcal{F}}_2$ and $\widehat{\mathcal{F}}_3$ (correspond to the elements ω and ω^{-1} of $\mathbb{Z}_3$ respectively) are 3×3 matrices, entries of these matrices can be easily calculated using properties of Fischer matrices listed in Proposition 5.2.2. Below are $\widehat{\mathcal{F}}_2$ and $\widehat{\mathcal{F}}_3$ respectively.

$h_2 = \omega$		h_{21}	h_{22}	h_{23}
$o(h_{ij})$		3	6	9
$\lvert C_{H_2}(h_{2j})\rvert$		18	6	9
(k,m)	$\lvert C_{T_k}(h_{2km})\rvert$			
$(1,1)$	3	1	1	1
$(2,1)$	3	1	-1	1
$(3,1)$	3	2	0	-1
m_{2j}		168	168	168

$h_3 = \omega^{-1}$		h_{31}	h_{32}	h_{33}
$o(h_{ij})$		3	6	9
$\lvert C_{H_2}(h_{3j})\rvert$		18	6	9
(k,m)	$\lvert C_{T_k}(h_{3km})\rvert$			
$(1,1)$	3	1	1	1
$(2,1)$	3	1	-1	1
$(3,1)$	3	2	0	-1
m_{3j}		168	168	168

Using the Fischer matrices and Section 5.3, the full character table of $H_2 = PSL(2,8)$:3 can be constructed easily. We supply this character table (which is also available in Magma and GAP) in Table 7.8.

Remark 7.2.4. We would like to indicate that the character table of $H_3 = 2^3{:}GL(3,2)$ was constructed, using Clifford-Fischer theory, in Chapter 5 of Whitely [70] which is available in GAP.

Table 7.8: The character table of $H_2 = PSL(2,8)$:3

	$1_{\mathbb{Z}_3}$					ω			ω^{-1}		
	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}	h_{21}	h_{22}	h_{23}	h_{31}	h_{32}	h_{33}
$o(h_{ij})$	1	2	3	7	9	3	6	9	3	6	9
$\lvert C_{H_2}(h_{ij})\rvert$	1512	24	27	7	9	18	6	9	18	6	9
$\lvert [h_{ij}]_{H_2}\rvert$	1	63	56	216	168	84	252	168	84	252	168
χ_1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_3	1	1	1	1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A
χ_4	7	-1	-2	0	1	1	-1	1	1	-1	1
χ_5	7	-1	-2	0	1	A	$-A$	A	$\overline{A}$	$-\overline{A}$	$\overline{A}$
χ_6	7	-1	-2	0	1	$\overline{A}$	$-\overline{A}$	$\overline{A}$	A	$-A$	A
χ_7	8	0	-1	1	-1	2	0	-1	2	0	-1
χ_8	8	0	-1	1	-1	$2A$	0	$-A$	$2\overline{A}$	0	$-\overline{A}$
χ_9	8	0	-1	1	-1	$2\overline{A}$	0	$-\overline{A}$	$2A$	0	$-A$
χ_{10}	21	-3	3	0	0	0	0	0	0	0	0
χ_{11}	27	3	0	-1	0	0	0	0	0	0	0

where in Table 7.8, $A = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

7.2.5 Fusion of the Inertia Factor Groups into A_9

In this subsection we determine the fusions of the inertia factor groups $H_2 = PSL(2,8){:}3$ and $H_3 = 2^3{:}GL(3,2)$ into A_9 using the permutation characters of A_9 on these groups. From $\mathbb{ATLAS}$, the permutation character of A_9 on H_2 is of the form

$$\chi(A_9|H_2) = \underline{1a} + \underline{35b} + \underline{84a}, \tag{7.1}$$

where $\underline{1a}$, $\underline{35b}$ and $\underline{84a}$ denote the characters χ_1, χ_8 and χ_{12} respectively of A_9 (in the order listed in $\mathbb{ATLAS}$). Also note that H_3 is a maximal subgroup of A_8 of index 15, which in turns is a maximal subgroup of A_9 of index 9.

Since we know the permutation characters of A_9 on A_8 and of A_8 on H_3, together with the fusions of classes of H_3 into A_8 and of A_8 into A_9, and with the help of Proposition 3.5.6 and $\mathbb{ATLAS}$, we are able to calculate the permutation character $\chi(A_9|H_3)$ as follows

$$\chi(A_9|H_3) = \mathbf{1}\uparrow_{H_3}^{A_9} = (\mathbf{1}\uparrow_{H_3}^{A_8})\uparrow_{A_8}^{A_9} = (\underline{1a} + \underline{14a})\uparrow_{A_8}^{A_9} = \underline{1a}\uparrow_{A_8}^{A_9} + \underline{14a}\uparrow_{A_8}^{A_9} = \underline{1a} + \underline{8a} + \underline{14a}\uparrow_{A_8}^{A_9}.$$

Now it is easy to evaluate the values of $\underline{14a}\uparrow_{A_8}^{A_9}$ using the induction formula for characters. In fact we have found that $\underline{14a}\uparrow_{A_8}^{A_9} = \underline{42a} + \underline{84a}$ and thus

$$\chi(A_9|H_3) = \underline{1a} + \underline{8a} + \underline{42a} + \underline{84a}. \tag{7.2}$$

Using Equations (7.1) and (7.2), we list in Table 7.9 the values of $\chi(A_9|H_2)$ and $\chi(A_9|H_3)$ on those classes of A_9, where there is possible fusions from classes of H_2 or H_3.

Table 7.9: The values of the permutation characters of A_9 on H_2 and H_3

$[g]_{A_9}$	$1A$	$2A$	$2B$	$3A$	$3B$	$3C$	$4A$	$4B$	$5A$	$6A$	$6B$	$7A$	$9A$	$9B$
$\chi(A_9\|H_2)$	120	0	8	0	3	6	0	0	0	0	2	1	0	3
$\chi(A_9\|H_3)$	135	15	7	0	0	9	3	3	0	0	1	2	0	0

Using the permutation characters of A_9 on H_2 and H_3 together with the size of centralizers, the fusions of H_2 and H_3 into A_9 are completely determined. Let $g_1, g_2, \cdots, g_{18}$ be as in Table 7.5. To be consistent with the notations of Chapter 5 we rename the classes of H_2 and H_3 according to the fusions of these classes into classes of A_9. We list these fusions in Table 7.10 and Table 7.11 respectively.

Table 7.10: The fusion of classes of $H_2 = PSL(2,8)$:3 into classes of A_9

	$[g]_{A_9}$	$1A$	$2A$	$2B$	$3A$	$3B$	$3C$	$4A$	$4B$	$5A$	$6A$	$6B$	$7A$	$9A$	$9B$
	$\lvert C_{A_9}(g)\rvert$	181440	480	192	1080	81	54	24	16	60	24	6	7	9	9
$[h_{ij}]_{H_2}$	$\lvert C_{H_2}(h_{ijm})\rvert$														
$h_{11} = g_{121}$	1512	[120]													
$h_{12} = g_{321}$	24		20	[8]											
$h_{13} = g_{521}$	27				40	[3]	2								
$h_{14} = g_{12,21}$	7												[1]		
$h_{15} = g_{14,22}$	9													1	[1]
$h_{21} = g_{621}$	18				60		[3]								
$h_{22} = g_{11,21}$	6										4	[1]			
$h_{23} = g_{14,24}$	9													1	[1]
$h_{31} = g_{622}$	18				60		[3]								
$h_{32} = g_{11,22}$	6										4	[1]			
$h_{33} = g_{14,23}$	9													1	[1]
	$\chi(A_9 \mid H_2)$	120	0	8	0	3	6	0	0	0	0	2	1	0	3

Table 7.11: The fusion of classes of $H_3 = 2^3$:$GL(3,2)$ into classes of A_9

	$[g]_{A_9}$	$1A$	$2A$	$2B$	$3A$	$3B$	$3C$	$4A$	$4B$	$5A$	$6A$	$6B$	$7A$
	$\lvert C_{A_9}(g)\rvert$	181440	480	192	1080	81	54	24	16	60	24	6	7
$[g]_{H_3}$	$\lvert C_{H_3}(g)\rvert$												
$1a = g_{131}$	1344	[135]											
$2a = g_{331}$	192			[1]									
$2b = g_{231}$	32		[15]	6									
$2c = g_{332}$	32		15	[6]									
$3a = g_{631}$	6				180		[9]						
$4a = g_{831}$	16								[1]				
$4b = g_{731}$	8							[3]	2				
$4c = g_{832}$	8							3	[2]				
$6a = g_{11,31}$	6										4	[1]	
$7a = g_{12,31}$	7												[1]
$7b = g_{12,32}$	7												[1]
	$\chi(A_9 \mid H_3)$	135	15	7	0	0	9	3	3	0	0	1	2

7.2.6 Fischer Matrices of $\overline{G} = 2_+^{1+8\cdot}A_9$

We have seen in Subsection 7.2.3 that the action of $\overline{G} = 2_+^{1+8\cdot}A_9$ on $\mathrm{Irr}(2_+^{1+8})$ produces four orbits with representatives $\theta_1,\ \theta_2,\ \theta_3$ and θ_4, where $|\theta_1^{\overline{G}}| = 1$, $|\theta_2^{\overline{G}}| = 120$, $|\theta_3^{\overline{G}}| = 135$ and $|\theta_4^{\overline{G}}| = 1$. It

follows by Proposition 5.2.2(i) that $c(g_1) = 4$. We determine the identity Fischer matrix $\mathcal{F}_1$ in some details as this matrix gives some information about the degrees of the irreducible characters of $\overline{G}$.

Lemma 7.2.2. *The matrix $\mathcal{F}_1$ is an integral matrix.*

PROOF. The result follows since the irreducible characters of N are an integer-values characters and the rows of $\mathcal{F}_1$ are the orbit sums of characters of N. ■

For $\mathcal{F}_1$, which is a 4×4 matrix, let its columns correspond to $g_{11} = 1_N,\ g_{12} = \sigma,\ g_{13}$ and g_{14} in this order, where σ is the central involution of N and g_{13} and g_{14} are elements of orders 2 and 4 respectively. Also let its rows correspond to $H_1 = A_9,\ H_2 = PSL(2,8){:}3,\ H_3 = 2^3{:}GL(3,2)$ and $H_4 = A_9$ in this order (Recall that $\mathcal{F}_1$ corresponds to $g_1 = 1_{A_9}$ and there is a fusion from each inertia factor and hence there is a row and only one row from each inertia factor group). Next we determine some of the entries of $\mathcal{F}_1$. The first row and column of $\mathcal{F}_1$ are determined by properties of Fischer matrices given by Proposition 5.2.2. Recall that the first column is

$$[|\theta_1^{\overline{G}}|\deg(\theta_1) \quad |\theta_2^{\overline{G}}|\deg(\theta_2) \quad |\theta_3^{\overline{G}}|\deg(\theta_3) \quad |\theta_4^{\overline{G}}|\deg(\theta_4)]^T = [1 \quad 120 \quad 135 \quad 16]^T.$$

The last row of $\mathcal{F}_1$ is

$$\begin{aligned}\left[\sum_{\theta_i\in\theta_4^{\overline{G}}}\theta_i(g_{11}) \quad \sum_{\theta_i\in\theta_4^{\overline{G}}}\theta_i(g_{12}) \quad \sum_{\theta_i\in\theta_4^{\overline{G}}}\theta_i(g_{13}) \quad \sum_{\theta_i\in\theta_4^{\overline{G}}}\theta_i(g_{14})\right] &= [\theta_4(g_{11}) \quad \theta_4(g_{12}) \quad \theta_4(g_{13}) \quad \theta_4(g_{14})] \\ &= [16 \quad -16 \quad 0 \quad 0].\end{aligned}$$

Recall that the orbits $\theta_1^{\overline{G}} = \{\theta_1\},\ \theta_2^{\overline{G}}$ and $\theta_3^{\overline{G}}$ consist of irreducible characters of N, which contain contain $Z(N) = \{1_N, \sigma\}$ in their kernel, while $\theta_4^{\overline{G}} = \{\theta_4\}$, where θ_4 is the unique faithful irreducible character of N of degree 16 that does not contain $Z(N)$ in its kernel. Since the second column of $\mathcal{F}_1$ corresponds to σ, entries of this column will coincide with entries of the first column point-wise except in the last row, where $\theta_4(\sigma) = -16 \neq 16 = \theta_4(1_N)$. Therefor the second column of $\mathcal{F}_1$ is $[1 \quad 120 \quad 135 \quad -16]^T$. Thus we have found the first two columns together with the first and last row of $\mathcal{F}_1$. The identity Fischer matrix $\mathcal{F}_1$ has the form

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}	g_{14}
$o(g_{1j})$		1	2	2	4
$\lvert C_{\overline{G}}(g_{1j})\rvert$		92897280	92897280	344064	387072
$\hookrightarrow$ Th		$1A$	$2A$	$2A$	$4A$
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$				
$(1,1)$	181440	1	1	1	1
$(2,1)$	1512	120	120	$a_{13}^{(2,1)}$	$a_{14}^{(2,1)}$
$(3,1)$	1344	135	135	$a_{13}^{(3,1)}$	$a_{14}^{(3,1)}$
$(4,1)$	181440	16	-16	0	0
m_{1j}		1	1	270	240

Using the orthogonality relations given by Proposition 5.2.2 (vi) & (vii), we get 10 equations in the unknowns $a_{13}^{(2,1)}$, $a_{14}^{(2,1)}$, $a_{13}^{(3,1)}$ and $a_{14}^{(3,1)}$. The unique integral solution of these 10 simultaneous equations reveals

$$a_{13}^{(2,1)} = -8, \quad a_{14}^{(2,1)} = 8, \quad a_{13}^{(3,1)} = 7, \quad a_{14}^{(3,1)} = -9.$$

Hence we have obtained the identity Fischer matrix.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}	g_{14}
$o(g_{1j})$		1	2	2	4
$\lvert C_{\overline{G}}(g_{1j})\rvert$		92897280	92897280	344064	387072
$\hookrightarrow$ Th		$1A$	$2A$	$2A$	$4A$
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$				
$(1,1)$	181440	1	1	1	1
$(2,1)$	1512	120	120	-8	8
$(3,1)$	1344	135	135	7	-9
$(4,1)$	181440	16	-16	0	0
m_{1j}		1	1	270	240

Having obtained the fusions of the inertia factors H_2 and H_3 into $G \cong A_9$ and with the properties of the Fischer matrices mentioned in Proposition 5.2.2, together with the help of Lemmas 7.2.4

and 7.2.5 we plan to compute all the Fischer matrices of the non-split extension $\overline{G} = 2_+^{1+8}{\cdot}A_9$. But firstly we need to decide whether we require to use the projective characters of any of the inertia factor groups.

Now for any $\chi_n \in \mathrm{Irr}(\mathrm{Th}),\ 1 \leq n \leq 48$, let $\chi_n^{(k)}$ denotes an irreducible character of $\overline{H}_k$ that $\chi_n^{(k)}{\uparrow}_{\overline{H}_k}^{\overline{G}}$ is a constituent or a zero of $\chi_n{\downarrow}_{\overline{G}}^{\mathrm{Th}}$. Since we have the fusions of $N = 2_+^{1+8}$ and $Z(N) = \{1, \sigma\}$ into Th, we have

$$\begin{aligned} \deg(\chi_n^{(1)}) &= \left\langle \chi_n{\downarrow}_N^{\mathrm{Th}}, \mathbf{1}_N \right\rangle_N, \\ \deg(\chi_n^{(4)}) &= \left\langle \chi_n{\downarrow}_{Z(N)}^{\mathrm{Th}}, -\mathbf{1}_{Z(N)} \right\rangle_{Z(N)} \\ &= \frac{1}{2}\left(\deg(\chi_n) - \chi_n(\sigma)\right), \end{aligned}$$

where $\mathbf{1}_N$ and $-\mathbf{1}_{Z(N)}$ denote the trivial character of N and the non-trivial character of $Z(N)$ respectively. In Table 7.12 we compute $\deg(\chi_n^{(1)})$ and $\deg(\chi_n^{(4)})$ for $2 \leq n \leq 48$. We use this table to determine whether we are required to use projective characters of inertia factor groups. From

Table 7.12: Degrees of some particular characters of some inertia groups

n	$\deg(\chi_n)$	$\deg(\chi_n^{(1)})$	$\deg(\chi_n^{(4)})$
2	248	0	8×16
3	4123	35	128×16
4	27000	120	840×16
5	27000	120	840×16
6	30628	28	960×16
$\vdots$	$\vdots$	$\vdots$	$r \times 16$
48	190373976	370008	5949288×16

Table 7.12 we can see that $\deg(\chi_n^{(4)}) = k \times 16$, where $k \in \mathbb{N},\ k \geq 2$. That is $\deg(\chi_n^{(4)}) \neq 16$ for all $n \in \{2, 3, \cdots, 48\}$. We deduce that $\overline{H}_4 = 2_+^{1+8}{\cdot}A_9 = \overline{G}$ has no character of degree 16 extends the faithful irreducible character θ_4 (of degree 16) of N. Thus we have to use a projective character table of $H_4 = A_9$ to construct the character table of $\overline{G}$. Since the Schur multiplier of A_9 is $\mathbb{Z}_2$, we have to consider the projective character table of A_9, which is available in $\mathbb{ATLAS}$.

Corollary 7.2.3. *The Schur multiplier of $\overline{G}$ is non-trivial and has a factor set $\alpha \sim [2]$.*

Since we will use the $\mathrm{IrrProj}(A_9, 2^{-1}) = \mathrm{IrrProj}(A_9, 2)$ to construct the character table of $\overline{G}$, there exists a character $\xi \in \mathrm{IrrProj}(\overline{G}, 2)$ such that $\xi{\downarrow}_N^{\overline{G}} = \theta_4$. Furthermore $\xi^2{\downarrow}_N^{\overline{G}} = \theta_4^2$ is the inflation of the regular character of $N/Z(N) \cong 2^8$ and hence the sum of the orbits of $\theta_1,\ \theta_2$ and θ_3 under the action $\overline{G}$. We deduce that there are uniquely determined linear characters $\psi_k \in \mathrm{Irr}(\overline{H}_k)$, $1 \leq k \leq 3$, such that

$$\xi^2 = \sum_{k=1}^{3} \psi_k{\uparrow}_{\overline{H}_k}^{\overline{G}} \quad \text{and} \quad \psi_k{\downarrow}_N^{\overline{H}_k} = \theta_k \ \text{ for } k \in \{1, 2, 3\}. \tag{7.3}$$

Hence $\theta_1,\ \theta_2$ and θ_3 are extendible to ordinary characters of their respective inertia groups. To summarize the construction of the character table of $\overline{G}$, we need the ordinary character tables of $H_1 = A_9,\ H_2 = PSL(2,8){:}3,\ H_3 = 2^3{:}GL(3,2)$ and the projective character table of $H_4 = A_9$ with factor set $\alpha^{-1},\ \alpha \sim [2]$. The character tables of A_9 and $2{\cdot}A_9$ are available in $\mathbb{ATLAS}$. The character table of H_3 has been computed, using Clifford-Fischer Theory, in details in Chapter 5 of Whitely [70] while the character table of H_2 has been supplied in Subsection 7.2.4 of this chapter using also the method of Clifford-Fischer. Note that from Subsection 7.2.3 we get 52 irreducible characters of $\overline{G}$, which is exactly the same number of the conjugacy classes of $\overline{G}$ given in Table 7.5.

Note 7.2.1. Observe that the character table of $2{\cdot}A_9$ indicates that classes of A_9 represented by $g_2 \sim 2A,\ g_3 \sim 2B,\ g_7 \sim 4A,\ g_8 \sim 4B,\ g_{10} \sim 6A$ and $g_{15} \sim 10A$ are $\alpha^{-1}-$irregular classes, where $\alpha \sim [2]$. This shows that the number of projective characters is same as the number of $\alpha^{-1}-$regular classes of A_9.

We proceed to compute the Fischer matrices of the extension $\overline{G} = 2_+^{1+8}{\cdot}A_9$. In what follows, by $\alpha^{-1}-$*regular Fischer matrix* we mean a Fischer matrix that corresponds to $\alpha^{-1}-$regular class of A_9. Since in our case $\alpha \sim [2]$, we only use the terms $\alpha-$*regular class* and $\alpha-$*regular Fischer matrix*.

Lemma 7.2.4. *For every $\alpha-$regular Fischer matrix $\mathcal{F}_i$, of size $c(g_i)$, the sum of the first $c(g_i) - 1$ rows equals the (componentwise) square of the last row.*

PROOF. The proof is similar to the proof of Lemma 6 of Pahlings [57]. ■

Lemma 7.2.5. *For every $\alpha-$regular Fischer matrix $\mathcal{F}_i$, we can order the g_{ij} for $1 \leq j \leq c(g_i)$ so that the last row of $\mathcal{F}_i$ is of the form $[q_i \ \ -q_i \ \ 0 \ \ \cdots \ \ 0]$ with q_i is a power of 2 and we may choose the $g_{i2} = \sigma g_{i1}$, where σ is the central involution in $\overline{G}$. Furthermore*

$$a_{i1}^{(k,m)} = a_{i2}^{(k,m)} = \frac{|C_{H_k}(g_{i11})|}{|C_{H_k}(g_{ikm})|} \quad \textit{for } 1 \leq k \leq 3,\ 1 \leq m \leq c(g_{ik}). \tag{7.4}$$

PROOF. The proof is similar to the proof of Lemma 7 of Pahlings [57], except that the ordinary character η of degree 2^{11} of $2_+^{1+22}{\cdot}Co_2$ (in Pahlings [57]) is replaced, in our Group $\overline{G} = 2_+^{1+8}{\cdot}A_9$, by a projective character ξ, with factor set $\alpha \sim [2]$, of degree 16. ■

Note 7.2.2. The proof of Lemma 7 of Pahlings [57] contained a very important piece of information that is the last row of every Fischer matrix of $2_+^{1+22}{\cdot}Co_2$ is $[\eta(g_{i1}) \quad \eta(g_{i2}) \quad \cdots \quad \eta(g_{is_i})]$, where s_i in his notation has the same meaning of $c(g_i)$ in our notation. In our group $\overline{G} = 2_+^{1+8}{\cdot}A_9$, the last row of every $\alpha-$regular Fischer matrix $\mathcal{F}_i$ is given by $[\xi(g_{i1}) \quad \xi(g_{i2}) \quad \cdots \quad \xi(g_{ic(g_i)})]$. Also the values of ξ on elements of $[g_{ij}]_{\overline{G}}$, where g_i is a representative of an $\alpha-$irregular conjugacy class, are zeros. That is $\xi(g_{ij}) = 0,\ \forall\ 1 \leq j \leq c(g_i),\ i \in \{2, 3, 7, 8, 10, 15\}$.

Note 7.2.3. Observe that with Lemma 7.2.4, Equation (7.4) and Note 7.2.2 we know the first two columns and the last row of every $\alpha-$regular Fischer matrix $\mathcal{F}_i$.

As an example, we compute the Fischer matrix $\mathcal{F}_6$, where $g_6 \in [3C]_{A_9}$. From Table 7.5 we see that g_6 has produced 5 conjugacy classes in $\overline{G}$, namely $g_{61},\ g_{62},\ g_{63},\ g_{64}$ and g_{65} with respective orders 3, 6, 6, 12 and 12, and respective centralizers 1728, 1728, 96, 288 and 288. Therefore $\mathcal{F}_6$ is a 5×5 matrix. Note that $|C_{A_9}(g_6)| = 54$. Now from Table 7.10 we can see that there are two classes namely $h_{14} = g_{621}$ and $h_{15} = g_{622}$ of $H_2 = PSL(2, 8){:}3$ that fuse to $[g_6]_{A_9}$. Note that $|C_{H_2}(g_{621})| = |C_{H_2}(g_{622})| = 18$. Also from Table 7.11 we can see that there is only one class namely $3a = g_{631}$ of $H_3 = 2^3{:}GL(3, 2)$ that fuse to $[g_6]_{A_9}$. Note that $|C_{H_3}(g_{631})| = 6$. Furthermore, g_6 is an $\alpha-$regular class (see $\mathbb{ATLAS}$) and therefore there is a row of $\mathcal{F}_6$ corresponds to $H_4 = A_9$. Now using Equation (5.4) we found that $m_{61} = m_{62} = 16,\ m_{63} = 288$ and $m_{64} = m_{65} = 96$. It follows that $\mathcal{F}_6$ has the form

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}
$o(g_{6j})$		3	6	6	12	12
$\lvert C_{\overline{G}}(g_{6j})\rvert$		1728	1728	96	288	288
(k, m)	$\lvert C_{H_k}(g_{6km})\rvert$					
$(1, 1)$	54	$a_{61}^{(1,1)}$	$a_{62}^{(1,1)}$	$a_{63}^{(1,1)}$	$a_{64}^{(1,1)}$	$a_{65}^{(1,1)}$
$(2, 1)$	18	$a_{61}^{(2,1)}$	$a_{62}^{(2,1)}$	$a_{63}^{(2,1)}$	$a_{64}^{(2,1)}$	$a_{65}^{(2,1)}$
$(2, 2)$	18	$a_{61}^{(2,2)}$	$a_{62}^{(2,2)}$	$a_{63}^{(2,2)}$	$a_{64}^{(2,2)}$	$a_{65}^{(2,2)}$
$(3, 1)$	6	$a_{61}^{(3,1)}$	$a_{62}^{(3,1)}$	$a_{63}^{(3,1)}$	$a_{64}^{(3,1)}$	$a_{65}^{(3,1)}$
$(4, 1)$	54	$a_{61}^{(4,1)}$	$a_{62}^{(4,1)}$	$a_{63}^{(4,1)}$	$a_{64}^{(4,1)}$	$a_{65}^{(4,1)}$
m_{6j}		16	16	288	96	96

Now using Proposition 5.2.2(iii), we get that $a_{61}^{(1,1)} = a_{62}^{(1,1)} = a_{63}^{(1,1)} = a_{64}^{(1,1)} = a_{65}^{(1,1)} = 1$. Using Equation (7.4) we get

$$\begin{aligned} a_{61}^{(2,1)} &= a_{62}^{(2,1)} = \frac{|C_{H_2}(g_6)|}{|C_{H_2}(g_{621})|} = \frac{54}{18} = 3, \\ a_{61}^{(2,2)} &= a_{62}^{(2,2)} = \frac{|C_{H_2}(g_6)|}{|C_{H_2}(g_{622})|} = \frac{54}{18} = 3, \\ a_{61}^{(3,1)} &= a_{62}^{(3,1)} = \frac{|C_{H_2}(g_6)|}{|C_{H_2}(g_{631})|} = \frac{54}{6} = 9. \end{aligned}$$

Using Lemmas 7.2.4 and 7.2.5 we get

$$(a_{61}^{(4,1)})^2 = q_6^2 = \sum_{k=1}^{3} \sum_{m=1}^{c(g_{6k})} a_{61}^{(k,m)} = 16.$$

We may assume, without loss of generality, that $a_{61}^{(4,1)} = 4$ and therefore $a_{62}^{(4,1)} = -q_6 = -4$. Also by Lemma 7.2.5 we have $a_{63}^{(4,1)} = a_{64}^{(4,1)} = a_{65}^{(4,1)} = 0$. So far the Fischer matrix $\mathcal{F}_6$ has the form

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}
$o(g_{6j})$		3	6	6	12	12
$\lvert C_{\overline{G}}(g_{6j})\rvert$		1728	1728	96	288	288
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$					
$(1,1)$	54	1	1	1	1	1
$(2,1)$	18	3	3	$a_{63}^{(2,1)}$	$a_{64}^{(2,1)}$	$a_{65}^{(2,1)}$
$(2,2)$	18	3	3	$a_{63}^{(2,2)}$	$a_{64}^{(2,2)}$	$a_{65}^{(2,2)}$
$(3,1)$	6	9	9	$a_{63}^{(3,1)}$	$a_{64}^{(3,1)}$	$a_{65}^{(3,1)}$
$(4,1)$	54	4	-4	0	0	0
m_{6j}		16	16	288	96	96

Using the columns and rows orthogonality relations (Proposition 5.2.2(vi)&(vii)) we obtain a set of 18 simultaneous equations. Solving these together with the consideration (Lemma 7.2.4) that

$$\begin{aligned} 1 + a_{63}^{(2,1)} + a_{63}^{(2,2)} + a_{63}^{(3,1)} &= 0, \\ 1 + a_{64}^{(2,1)} + a_{64}^{(2,2)} + a_{64}^{(3,1)} &= 0, \\ 1 + a_{65}^{(2,1)} + a_{65}^{(2,2)} + a_{65}^{(3,1)} &= 0, \end{aligned}$$

we get that

$$a_{63}^{(2,1)} = a_{63}^{(2,2)} = a_{64}^{(2,2)} = a_{65}^{(2,1)} = -1,\ a_{64}^{(2,1)} = a_{65}^{(2,2)} = 3,\ a_{64}^{(3,1)} = a_{65}^{(3,1)} = -3 \text{ and } a_{63}^{(3,1)} = 1.$$

Thus the Fischer matrix $\mathcal{F}_6$ is

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}
$o(g_{6j})$		3	6	6	12	12
$\lvert C_{\overline{G}}(g_{6j})\rvert$		1728	1728	96	288	288
$\hookrightarrow$ Th		$3A$	$6B$	$6B$	$12A$	$12B$
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$					
$(1,1)$	54	1	1	1	1	1
$(2,1)$	18	3	3	-1	3	-1
$(2,2)$	18	3	3	-1	-1	3
$(3,1)$	6	9	9	1	-3	-3
$(4,1)$	54	4	-4	0	0	0
m_{6j}		16	16	288	96	96

Using similar arguments we obtain all the other $\alpha-$regular Fischer matrices. Note that all the $\alpha-$irregular Fischer matrices are of small size (the size of every Fischer matrix (regular or irregular) is $c(g_i)$ and we have $c(g_i) \in \{1,2,3,4\},\ \forall\ i \in \{2,3,7,8,10,15\}$) and the columns and row orthogonality relations are sufficient to compute these matrices. Hence we have all the Fischer matrices of the group $\overline{G} = 2_+^{1+8}{\cdot}A_9$, which we list below

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}	g_{14}
$o(g_{1j})$		1	2	2	4
$\lvert C_{\overline{G}}(g_{1j})\rvert$		92897280	92897280	344064	387072
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$				
$(1,1)$	181440	1	1	1	1
$(2,1)$	1512	120	120	-8	8
$(3,1)$	1344	135	135	7	-9
$(4,1)$	181440	16	-16	0	0
m_{1j}		1	1	270	240

$\mathcal{F}_2$

g_2		g_{21}	g_{22}
$o(g_{2j})$		4	4
$\lvert C_{\overline{G}}(g_{2j})\rvert$		7680	5120
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$		
$(1,1)$	480	1	1
$(2,1)$	32	15	-1
m_{2j}		32	480

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}	g_{34}
$o(g_{3j})$		4	2	4	8
$\lvert C_{\overline{G}}(g_{3j})\rvert$		3072	3072	512	384
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$				
$(1,1)$	192	1	1	1	1
$(2,1)$	24	8	-8	0	0
$(3,1)$	192	1	1	1	-1
$(3,2)$	32	6	6	-2	0
m_{3j}		32	32	192	256

$\mathcal{F}_4$

g_4		g_{41}	g_{42}
$o(g_{4j})$		3	6
$\lvert C_{\overline{G}}(g_{4j})\rvert$		2160	2160
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$		
$(1,1)$	1080	1	1
$(4,1)$	1080	1	-1
m_{4j}		256	256

$\mathcal{F}_5$

g_5		g_{51}	g_{52}	g_{53}
$o(g_{5j})$		6	3	12
$\lvert C_{\overline{G}}(g_{5j})\rvert$		648	648	108
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$			
$(1,1)$	81	1	1	1
$(2,1)$	27	3	3	-1
$(4,1)$	81	2	-2	0
m_{5j}		64	64	384

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}
$o(g_{6j})$		3	6	6	12	12
$\lvert C_{\overline{G}}(g_{6j})\rvert$		1728	1728	96	288	288
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$					
$(1,1)$	54	1	1	1	1	1
$(2,1)$	18	3	3	-1	3	-1
$(2,2)$	18	3	3	-1	-1	3
$(3,1)$	6	9	9	1	-3	-3
$(4,1)$	54	4	-4	0	0	0
m_{6j}		16	16	288	96	96

$\mathcal{F}_7$

g_7		g_{71}	g_{72}
$o(g_{7j})$		8	8
$\lvert C_{\overline{G}}(g_{7j})\rvert$		96	32
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$		
$(1,1)$	24	1	1
$(3,1)$	8	3	-1
m_{7j}		128	384

$\mathcal{F}_8$

g_8		g_{81}	g_{82}	g_{83}
$o(g_{8j})$		8	4	8
$\lvert C_{\overline{G}}(g_{8j})\rvert$		64	64	32
(k,m)	$\lvert C_{H_8}(g_{8km})\rvert$			
$(1,1)$	16	1	1	1
$(3,1)$	16	1	1	-1
$(3,2)$	8	2	-2	0
m_{8j}		128	128	256

$\mathcal{F}_9$

g_9		g_{91}	g_{92}
$o(g_{9j})$		5	10
$\lvert C_{\overline{G}}(g_{9j})\rvert$		120	120
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$		
$(1,1)$	60	1	1
$(4,1)$	60	1	-1
m_{9j}		256	256

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$
$o(g_{10j})$		12
$\lvert C_{\overline{G}}(g_{10j})\rvert$		24
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$	
$(1,1)$	24	1
m_{10j}		512

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{11,4}$	$g_{11,5}$
$o(g_{11j})$		12	12	6	24	24
$\lvert C_{\overline{G}}(g_{11j})\rvert$		48	48	24	24	24
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$					
$(1,1)$	6	1	1	1	1	1
$(2,1)$	6	1	1	-1	1	-1
$(2,2)$	6	1	1	-1	-1	1
$(3,1)$	6	1	1	1	-1	-1
$(4,1)$	6	2	-2	0	0	0
m_{11j}		64	64	128	128	128

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{12,4}$	$g_{12,5}$
$o(g_{12j})$		7	14	14	14	28
$\lvert C_{\overline{G}}(g_{12j})\rvert$		56	56	28	28	28
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$					
$(1,1)$	7	1	1	1	1	1
$(2,1)$	7	1	1	-1	-1	1
$(3,1)$	7	1	1	-1	1	-1
$(3,2)$	7	1	1	1	-1	-1
$(4,1)$	7	2	-2	0	0	0
m_{12j}		64	64	128	128	128

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$
$o(g_{13j})$		9	18
$\lvert C_{\overline{G}}(g_{13j})\rvert$		18	18
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$		
$(1,1)$	9	1	1
$(4,1)$	9	1	-1
m_{13j}		256	256

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$	$g_{14,2}$	$g_{14,3}$	$g_{14,4}$	$g_{14,5}$
$o(g_{14j})$		18	9	36	36	36
$\lvert C_{\overline{G}}(g_{14j})\rvert$		72	72	36	36	36
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$					
$(1,1)$	9	1	1	1	1	1
$(2,1)$	9	1	1	-1	1	-1
$(2,2)$	9	1	1	1	-1	-1
$(2,3)$	9	1	1	-1	-1	1
$(4,1)$	9	2	-2	0	0	0
m_{14j}		64	64	128	128	128

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$
$o(g_{15j})$		20
$\lvert C_{\overline{G}}(g_{15j})\rvert$		20
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$	
$(1,1)$	20	1
m_{15j}		512

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$
$o(g_{16j})$		24	24
$\lvert C_{\overline{G}}(g_{16j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$		
$(1,1)$	12	1	1
$(4,1)$	12	1	-1
m_{16j}		256	256

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$
$o(g_{17j})$		15	30
$\lvert C_{\overline{G}}(g_{17j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$		
$(1,1)$	15	1	1
$(4,1)$	15	1	-1
m_{17j}		256	256

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$
$o(g_{18j})$		15	30
$\lvert C_{\overline{G}}(g_{18j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$		
$(1,1)$	15	1	1
$(4,1)$	15	1	-1
m_{18j}		256	256

7.2.7 The Character Table of $\overline{G} = 2_+^{1+8} \cdot A_9$

Through the various previous subsections, the ATLAS and Whitely Thesis [70], we have determined

- the conjugacy classes of $\overline{G} = 2_-^{1+8} \cdot A_9$ (Table 7.5),
- the character tables of the inertia factors $H_2,\ H_3$ and H_4 (Tables 7.8, the ATLAS and 5.4 of Whitely [70]),
- the fusions of classes of the inertia factors H_2 and H_3 into classes of A_9 (Tables 7.10, 7.11),
- the Fischer matrices of $\overline{G}$ (previous sub-subsection).

By Section 5.3, it follows that the full character table of $\overline{G}$ can be constructed easily. For example the part $\mathcal{K}_{62}\mathcal{F}_{62}$ of the character table of $\overline{G}$ can be derived as follows: From Table 7.8, two conjugacy classes namely $h_{21} = g_{621}$ and $h_{31} = g_{622}$ of $H_2 = PSL(2,8){:}\mathbb{Z}_3$ fuse to class $[g_6]_{A_9} = 3C$. Let $\mathcal{K}_{62}$ be the fragment of the character table of H_2 on these two classes, which can be extracted from Table 7.8. The Fischer matrix $\mathcal{F}_6$ has two rows correspond to the classes h_{21} and h_{31} of H_2. Let $\mathcal{F}_{62}$ be the partial Fischer matrix of $\mathcal{F}_6$ consisting of these two rows. Therefore the values of the irreducible characters of $\overline{G}$ (corresponding to H_2) on the classes $g_{61}, g_{62}, g_{63}, g_{64}$ and g_{65} are given by

$$\mathcal{K}_{62}\mathcal{F}_{62} = \begin{pmatrix} 1 & 1 \\ A & \overline{A} \\ \overline{A} & A \\ 1 & 1 \\ A & \overline{A} \\ \overline{A} & A \\ 2 & 2 \\ 2A & 2\overline{A} \\ 2\overline{A} & 2A \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 3 & 3 & -1 & 3 & -1 \\ 3 & 3 & -1 & -1 & 3 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{61} & g_{62} & g_{63} & g_{64} & g_{65} \end{matrix} \\ \begin{pmatrix} 6 & 6 & -2 & 2 & 2 \\ -3 & -3 & 1 & B & \overline{B} \\ -3 & -3 & 1 & \overline{B} & B \\ 6 & 6 & -2 & 2 & 2 \\ -3 & -3 & 1 & B & \overline{B} \\ -3 & -3 & 1 & \overline{B} & B \\ 12 & 12 & -4 & 4 & 4 \\ -6 & -6 & 2 & 2B & 2\overline{B} \\ -6 & -6 & 2 & 2\overline{B} & 2B \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array},$$

where $A = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $B = -1 + 2i\sqrt{3}$.

Similarly one can obtain all the 72 blocks $\mathcal{K}_{ik}\mathcal{F}_{ik}$, $1 \leq i \leq 18,\ 1 \leq k \leq 4$ and hence the full character table of $\overline{G}$, which is also available in Magma, GAP and Michler [46].

Corollary 7.2.6. *The group $\overline{G}$ has 12 faithful irreducible characters.*

PROOF. Note that $a_{11}^{(k,m)} = a_{12}^{(k,m)}$ for $1 \leq k \leq 3$, while $a_{11}^{(4,m)} = 16 \neq -16 = a_{12}^{(4,m)}$. Thus $\ker(\chi) \supset [g_{12}]_{\overline{G}}$, $\forall \chi \in \mathcal{K}_1,\ \mathcal{K}_2,\ \mathcal{K}_3$, where $\mathcal{K}_1,\ \mathcal{K}_2$ and $\mathcal{K}_3$ are the blocks of irreducible characters of $\overline{G}$ that

obtained through characters of the inertia factor groups $H_1,\ H_2$ and H_3 respectively. Hence if $\overline{G}$ has a faithful irreducible character χ, then $\chi \in \mathcal{K}_4 = \{\xi\beta_d |\ \beta_d \in \mathrm{IrrProj}(A_9, 2)\}$. Since the values of ξ are completely known (see Note 7.2.2) and since the character table of $2{\cdot}A_9$ is also known, we can see that

$$\xi\beta_d(g_{ij}) \neq \deg(\xi\beta_d),\ \forall\ 1 \leq i \leq 18,\ 1 \leq j \leq c(g_i),\ (i,j) \neq (1,1) \text{ and for all } d \in \{1, 2, \cdots, 12\}.$$

Thus $\overline{G}$ has 12 faithful irreducible characters as claimed. ■

Chapter 8

The Non-Split Extension $2^{6\cdot}Sp(6,2)$

In this chapter, we construct the character table of the non-split extension $\overline{G} = 2^{6\cdot}Sp(6,2)$ by means of Clifford-Fischer theory together with the coset analysis technique. In Section 8.1 we give a brief information on the existence of $\overline{G}$ and we also supply generators of $\overline{G}$ as well as for the kernel $N = 2^6$ of the extension in terms of permutations of the alternating group A_{128}. Section 8.2 is devoted to the construction of the conjugacy classes of $\overline{G}$ by means of the coset analysis technique. In Section 8.3 we determine the inertia factors and their fusions (into $Sp(6,2)$) and we supply the character tables of these inertia factors in the Appendix of this thesis. In Section 8.4 we calculate the Fischer matrices of $\overline{G}$ and we see that these matrices are integral valued matrices and their sizes range between 1 and 4. Finally in Section 8.5 we show how to construct the table of $\overline{G}$. The character table of $\overline{G}$ is given as Table 11.12 in the Appendix.

8.1. Introduction

Let $G = Sp(2n,q)$ be the symplectic group consisting of $2n \times 2n$ matrices over $\mathbb{F}_q$ that preserve a non-degenerate alternating bilinear form and let $V = q^{2n}$ be a $2n-$dimensional vector space over $\mathbb{F}_q$. In [23] Dempwolff proved that a non-split extension of the form $\overline{G}_n = 2^{2n\cdot}Sp(2n,2)$ does exist for all $n \geq 2$, where $\overline{G}_n/2^{2n} \cong Sp(2n,2)$ acts faithfully on 2^{2n}. Moreover such an extension is unique, up to isomorphism, since $\dim_{\mathbb{F}_2} H^2(Sp(2n,2), 2^{2n}) = 1$ for all $n \geq 2$, where $H^2(K,M)$ is the second cohomology group of a group K with coefficients in M. In the case $n = 2$, the non-split extension $\overline{G}_2 = 2^{4\cdot}Sp(4,2) \cong 2^{4\cdot}S_6$ is a maximal subgroup of the sporadic simple group Higman-Sims HS (see the $\mathbb{ATLAS}$ [17]). The character table of $\overline{G}_2$ is stored in a GAP library (see [30]) or one can

use the program supplied in the electronic ATLAS of Wilson [71] to construct its character table either in Magma [16] or GAP. In this chapter we are interested in calculating the character table of $\overline{G}_3 = 2^{6\cdot}Sp(6,2) := \overline{G}$ using Clifford-Fischer theory. It will turn out that the character table of $\overline{G}$ is a 67×67 integral matrix and coincides with the character table of the split extension $2^6{:}Sp(6,2)$, constructed in Mpono [55] and which is also available in GAP. We have built a small subroutine in GAP (see Section 9.2 and set $n = 3$ in the program therein) to construct the group $\overline{G}$ in terms of permutations of a set of cardinality 128. The following two elements $\overline{g}_1$ and $\overline{g}_2$ generate the group $\overline{G}$.

$$
\begin{aligned}
\overline{g}_1 = {} & (1\ 12)(2\ 111\ 86\ 50)(3\ 57\ 94\ 69)(4\ 95\ 23\ 121)(5\ 70\ 48\ 58)(6\ 71\ 21\ 59)(8\ 72\ 73\ 117)\\
& (9\ 108\ 33\ 74)(10\ 118\ 61\ 18)(11\ 119\ 31\ 19)(13\ 102\ 97\ 87)(14\ 100\ 27\ 101)(15\ 85\ 55\ 66)\\
& (16\ 115\ 88\ 67)(17\ 79\ 52\ 78)(20\ 93\ 120\ 104)(22\ 28\ 24\ 105)(25\ 106\ 47\ 29)(30\ 44\ 107\ 90)\\
& (32\ 45\ 34\ 91)(35\ 92\ 60\ 46)(36\ 65\ 64\ 37)(38\ 126\ 122\ 53)(39\ 42\ 116\ 89)(40\ 43\ 84\ 54)(41\\
& 63\ 56\ 125)(51\ 83\ 127\ 68)(62\ 96)(76\ 98)(77\ 99)(80\ 81\ 112\ 113)(82\ 114\ 128\ 103)(109\ 123)(110\ 124),\\
\overline{g}_2 = {} & (1\ 104\ 103\ 118\ 93\ 113\ 55\ 52\ 83\ 15\ 98\ 8\ 18\ 17\ 107)(2\ 73\ 12\ 69\ 68\ 91\ 57\ 81\ 89\ 86\\
& 114\ 42\ 123\ 30\ 45)(3\ 48\ 49\ 32\ 72\ 115\ 34\ 36\ 125\ 94\ 90\ 63\ 77\ 24\ 67)(4\ 53\ 35\ 127\ 102\ 99\\
& 106\ 47\ 60\ 59\ 112\ 14\ 82\ 6\ 27)(5\ 40\ 20\ 22\ 75\ 61\ 44\ 84\ 10\ 64\ 101\ 120\ 117\ 100\ 110)\\
& (9\ 50\ 88\ 43\ 79)(11\ 128\ 126\ 124\ 71\ 21\ 31\ 29\ 80\ 41\ 51\ 25\ 56\ 23\ 87)(13\ 105\ 96\ 28\ 97)\\
& (16\ 111\ 33\ 78\ 54)(19\ 116\ 74\ 66\ 92)(37\ 76\ 121)(38\ 70\ 62\ 58\ 122)(39\ 119\ 46\ 85\ 108)(65\ 109\ 95).
\end{aligned}
$$

with $o(\overline{g}_1) = 4,\ o(\overline{g}_2) = 15$ and $o(\overline{g}_1\overline{g}_2) = 9$.

Note 8.1.1. Note that the generators $\overline{g}_1$ and $\overline{g}_2$ fix the points 7 and 26. Thus $\overline{G}$ acts transitively on a set Ω of 126 points. Hence we have a permutation character $\chi(\overline{G}|\Omega) = \chi$ of degree 126. In Table 11.1 we list the values of χ on $\overline{G}-$classes and using the character table of $\overline{G}$ (Table 11.12) we can see that $\chi = \chi_1 + \chi_6 + \chi_7 + \chi_{31}$.

Now having the group $\overline{G}$ constructed in GAP, it is easy to obtain its normal subgroups. In fact the only non-trivial proper normal subgroup that $\overline{G}$ contains is a group of order 64 and thus must be isomorphic to the elementary abelian group $N = 2^6$. The following 6 permutations $n_1, n_2, \cdots, n_6$ generate the normal subgroup N.

$$
\begin{aligned}
n_1 = {} & (6\ 25)(11\ 35)(16\ 43)(19\ 46)(22\ 48)(29\ 59)(32\ 61)(37\ 65)(40\ 67)(49\ 75)(53\ 87)(55\ 89)(62\ 96)\\
& (68\ 103)(69\ 104)(71\ 106)(73\ 107)(74\ 108)(77\ 110)(79\ 111)(81\ 113)(83\ 114)(85\ 116)(90\ 117)(92\ 119)\\
& (94\ 120)(95\ 121)(97\ 122)(99\ 124)(101\ 125)(102\ 126)(127\ 128),
\end{aligned}
$$

n_2 = (5 24)(6 25)(10 34)(11 35)(15 42)(16 43)(18 45)(19 46)(21 47)(22 48)(28 58)(29 59)(31 60)(32 61)(36 64)(37 65)(39 66)(40 67)(54 88)(55 89)(70 105)(71 106)(76 109)(77 110)(80 112)(81 113)(84 115)(85 116)(91 118)(92 119)(98 123),

n_3 = (99 124) (4 23)(5 24)(6 25)(9 33)(10 34)(11 35)(14 41)(15 42)(16 43)(18 45)(19 46)(28 58)(29 59)(36 64)(37 65)(44 72)(50 78)(52 86)(54 88)(55 89)(57 93)(63 100)(69 104)(74 108)(80 112)(81 113)(83 114)(90 117)(95 121)(97 122)(102 126)(127 128),

n_4 = (3 20)(5 24)(6 25)(8 30)(10 34)(11 35)(13 38)(15 42)(16 43)(21 47)(22 48)(31 60)(32 61)(39 66)(40 67)(44 72)(50 78)(51 82)(54 88)(55 89)(57 93)(63 100)(69 104)(73 107)(79 111)(84 115)(85 116)(90 117)(94 120)(97 122)(101 125)(127 128),

n_5 = (2 17)(3 20)(4 23)(524)(8 30)(9 33)(10 34)(16 43)(19 46)(22 48)(27 56)(29 59)(32 61)(36 64)(39 66)(49 75)(50 78)(55 89)(62 96)(63 100)(69 104)(70 105)(77 110)(79 111)(80 112)(83 114)(84 115)(90 117)(91 118)(99 124)(101 125)(102 126),

n_6 = (1 12)(5 24)(6 25)(8 30)(9 33)(15 42)(16 43)(18 45)(19 46)(21 47)(22 48)(27 56)(36 64)(37 65)(39 66)(40 67)(50 78)(51 82)(52 86)(53 87)(57 93)(62 96)(70 105)(71 106)(79 111)(83 114)(90 117)(94 120)(95 121)(98 123)(99 124)(127 128).

In Magma or GAP one can check for the complements of $N = \langle n_1, n_2, \cdots, n_6 \rangle$ in $\overline{G} = \langle \overline{g}_1, \overline{g}_2 \rangle$, which in our case will return an empty list of complements. This shows that the group $\overline{G}$ constructed using the generators $\overline{g}_1$ and $\overline{g}_2$ is indeed a non-split extension of the elementary abelian group $N = 2^6$ by the symplectic group $Sp(6,2)$.

8.2. Conjugacy Classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$

We have used the method of the coset analysis, described in Section 2.3, to calculate the conjugacy classes of $\overline{G}$. We list these classes in Table 8.1 where in this table

- k_i's represent the number of fixed points of g_i on its action on $N = \langle n_1, n_2, \cdots, n_6 \rangle$,
- f_{ij}'s represent the number of orbits (of the action of N on $N\overline{g}_i$) fused together under the action of $\overline{G} = \langle \overline{g}_1, \overline{g}_2 \rangle$,
- m_{ij}'s are the weights defined by Eq. (5.4).

Table 8.1: The conjugacy classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^{6\cdot}Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{6\cdot}Sp(6,2)}\rvert$	$\lvert C_{2^{6\cdot}Sp(6,2}(g_{ij})\rvert$
$g_1 = 1A$	$k_1 = 64$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	92897280
		$f_{12} = 63$	$m_{12} = 63$	g_{12}	2	63	1474560
		$f_{21} = 13$	$m_{21} = 2$	g_{21}	4	126	737280
$g_2 = 2A$	$k_2 = 32$	$f_{22} = 15$	$m_{22} = 30$	g_{22}	4	1890	49152
		$f_{23} = 16$	$m_{23} = 32$	g_{23}	2	2016	46080
		$f_{31} = 1$	$m_{31} = 4$	g_{31}	2	1260	73728
$g_3 = 2B$	$k_3 = 16$	$f_{32} = 3$	$m_{32} = 12$	g_{32}	2	3780	24576
		$f_{33} = 12$	$m_{33} = 48$	g_{33}	4	15120	6144
		$f_{41} = 1$	$m_{41} = 4$	g_{41}	4	3780	24576
$g_4 = 2C$	$k_4 = 16$	$f_{42} = 3$	$m_{42} = 12$	g_{42}	4	11340	8192
		$f_{43} = 4$	$m_{43} = 16$	g_{43}	2	15120	6144
		$f_{44} = 8$	$m_{44} = 32$	g_{44}	4	30240	3072
		$f_{51} = 1$	$m_{51} = 8$	g_{51}	4	30240	3072
$g_5 = 2D$	$k_5 = 8$	$f_{52} = 1$	$m_{52} = 8$	g_{52}	2	30240	3072
		$f_{53} = 3$	$m_{53} = 24$	g_{53}	4	90720	1024
		$f_{54} = 3$	$m_{54} = 24$	g_{54}	4	90720	1024
$g_6 = 3A$	$k_6 = 16$	$f_{61} = 1$	$m_{61} = 4$	g_{61}	3	2688	34560
		$f_{62} = 15$	$m_{62} = 60$	g_{62}	6	40320	2304
$g_7 = 3B$	$k_7 = 1$	$f_{71} = 1$	$m_{71} = 64$	g_{71}	3	143360	648
$g_8 = 3C$	$k_8 = 4$	$f_{81} = 1$	$m_{81} = 16$	g_{81}	3	215040	432
		$f_{82} = 3$	$m_{82} = 48$	g_{82}	6	645120	144
$g_9 = 4A$	$k_9 = 4$	$f_{91} = 1$	$m_{91} = 16$	g_{91}	4	60480	1536
		$f_{92} = 3$	$m_{92} = 48$	g_{92}	4	181440	512
		$f_{10,1} = 1$	$m_{10,1} = 8$	$g_{10,1}$	8	60480	1536
$g_{10} = 4B$	$k_{10} = 8$	$f_{10,2} = 3$	$m_{10,2} = 24$	$g_{10,2}$	8	181440	512
		$f_{10,3} = 4$	$m_{10,3} = 32$	$g_{10,3}$	4	241920	384

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Table 8.1 (continued)

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^{6\cdot}Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{6\cdot}Sp(6,2)}\rvert$	$\lvert C_{2^{6\cdot}Sp(6,2)}(g_{ij})\rvert$
		$f_{11,1}=1$	$m_{11,1}=8$	$g_{11,1}$	8	60480	1536
$g_{11}=4C$	$k_{11}=8$	$f_{11,2}=3$	$m_{11,2}=24$	$g_{11,2}$	8	181440	512
		$f_{11,3}=4$	$m_{11,3}=32$	$g_{11,3}$	4	241920	384
		$f_{12,1}=1$	$m_{12,1}=16$	$g_{12,1}$	4	181440	512
$g_{12}=4D$	$k_{12}=4$	$f_{12,2}=1$	$m_{12,2}=16$	$g_{12,2}$	4	181440	512
		$f_{12,3}=2$	$m_{12,3}=32$	$g_{12,3}$	4	362880	256
		$f_{13,1}=1$	$m_{13,1}=16$	$g_{13,1}$	8	725760	128
$g_{13}=4E$	$k_{13}=4$	$f_{13,2}=1$	$m_{13,2}=16$	$g_{13,2}$	4	725760	128
		$f_{13,3}=1$	$m_{13,3}=16$	$g_{13,3}$	8	725760	128
		$f_{13,4}=1$	$m_{13,4}=16$	$g_{13,4}$	4	725760	128
$g_{14}=5A$	$k_{14}=4$	$f_{14,1}=3$	$m_{14,1}=48$	$g_{14,1}$	10	2322432	40
		$f_{14,2}=1$	$m_{14,2}=16$	$g_{14,2}$	5	774144	120
		$f_{15,1}=1$	$m_{15,1}=8$	$g_{15,1}$	12	80640	1152
$g_{15}=6A$	$k_{15}=8$	$f_{15,2}=3$	$m_{15,2}=24$	$g_{15,2}$	12	241920	384
		$f_{15,3}=4$	$m_{15,3}=32$	$g_{15,3}$	6	322560	288
$g_{16}=6B$	$k_{16}=4$	$f_{16,1}=1$	$m_{16,1}=16$	$g_{16,1}$	6	161280	576
		$f_{16,2}=3$	$m_{16,2}=48$	$g_{16,2}$	12	483840	192
$g_{17}=6C$	$k_{17}=1$	$f_{17,1}=1$	$m_{17,1}=64$	$g_{17,1}$	6	1290240	72
		$f_{18,1}=1$	$m_{18,1}=16$	$g_{18,1}$	12	483840	192
$g_{18}=6D$	$k_{18}=4$	$f_{18,2}=1$	$m_{18,2}=16$	$g_{18,2}$	6	483840	192
		$f_{18,3}=2$	$m_{18,3}=32$	$g_{18,3}$	12	967680	96
$g_{19}=6E$	$k_{19}=2$	$f_{19,1}=1$	$m_{19,1}=32$	$g_{19,1}$	12	1290240	72
		$f_{19,2}=1$	$m_{19,2}=32$	$g_{19,2}$	6	1290240	72
$g_{20}=6F$	$k_{20}=4$	$f_{20,1}=1$	$m_{20,1}=16$	$g_{20,1}$	6	645120	144
		$f_{20,2}=3$	$m_{20,2}=48$	$g_{20,2}$	6	1935360	48
$g_{21}=6G$	$k_{21}=2$	$f_{21,1}=1$	$m_{21,1}=32$	$g_{21,1}$	12	3870720	24
		$f_{21,2}=1$	$m_{21,2}=32$	$g_{21,2}$	6	3870720	24
$g_{22}=7A$	$k_{22}=1$	$f_{22,1}=1$	$m_{22,1}=64$	$g_{22,1}$	7	13271040	7

Continued on next page

Table 8.1 (continued)

$[g_i]_{Sp(6,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^{6\cdot}Sp(6,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{6\cdot}Sp(6,2)}\rvert$	$\lvert C_{2^{6\cdot}Sp(6,2)}(g_{ij})\rvert$
$g_{23} = 8A$	$k_{23} = 2$	$f_{23,1} = 1$	$m_{23,1} = 32$	$g_{23,1}$	8	2903040	32
		$f_{23,2} = 1$	$m_{23,2} = 32$	$g_{23,2}$	8	2903040	32
$g_{24} = 8B$	$k_{24} = 2$	$f_{24,1} = 1$	$m_{24,1} = 32$	$g_{24,1}$	8	2903040	32
		$f_{24,2} = 1$	$m_{24,2} = 32$	$g_{24,2}$	8	2903040	32
$g_{25} = 9A$	$k_{25} = 1$	$f_{25,1} = 1$	$m_{25,1} = 64$	$g_{25,1}$	9	10321920	9
$g_{26} = 10A$	$k_{26} = 2$	$f_{26,1} = 1$	$m_{26,1} = 32$	$g_{26,1}$	20	4644864	20
		$f_{26,2} = 1$	$m_{26,2} = 32$	$g_{26,2}$	10	4644864	20
$g_{27} = 12A$	$k_{27} = 2$	$f_{27,1} = 1$	$m_{27,1} = 32$	$g_{27,1}$	24	1935360	48
		$f_{27,2} = 1$	$m_{27,2} = 32$	$g_{27,2}$	12	1935360	48
$g_{28} = 12B$	$k_{28} = 2$	$f_{28,1} = 1$	$m_{28,1} = 32$	$g_{28,1}$	24	1935360	48
		$f_{28,2} = 1$	$m_{28,2} = 32$	$g_{28,2}$	12	1935360	48
$g_{29} = 12C$	$k_{29} = 1$	$f_{29,1} = 1$	$m_{29,1} = 64$	$g_{29,1}$	12	7741440	12
$g_{30} = 15A$	$k_{30} = 1$	$f_{30,1} = 1$	$m_{30,1} = 64$	$g_{30,1}$	15	6193152	15

Remark 8.2.1. Note that from Table 8.1, the group $\overline{G} = 2^{6\cdot}Sp(6,2)$ contains 6 conjugacy classes of involutions, while from Table 6.1 of Mpono [55], the split extension $T = 2^6{:}Sp(6,2)$ contains 8 conjugacy classes of involutions. This confirms that the group $\overline{G}$ constructed using the generators $\overline{g}_1$ and $\overline{g}_2$ is different from the group T but later it will be shown that the character tables of the two groups will be the same.

8.3. Inertia Factor Groups of $\overline{G} = 2^{6\cdot}Sp(6,2)$

We have seen in Section 8.2 that the action of $\overline{G}$ on N yielded two orbits of lengths 1 and 63, where the first orbit consists of the identity element 1_N while the other orbit consists of all the involutions of N. By Brauer Theorem (see Theorem 5.1.1), it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce two orbits. These two orbits must necessarily have lengths 1 and 63 and the first orbit consists of the identity character $\mathbf{1}_N$ while the other orbit consists of the other non-trivial linear

characters of N. Thus the corresponding inertia factor groups H_1 and H_2 have indices 1 and 63 respectively in $Sp(6,2)$. By looking at the maximal subgroups of $Sp(6,2)$ (see $\mathbb{ATLAS}$), it is readily verified that $H_1 = Sp(6,2)$ and $H_2 = 2^5{:}S_6$.

The character table of $H_1 = Sp(6,2)$ is available in the $\mathbb{ATLAS}$, the electronic ATLAS of Wilson [71] or can be obtained from Magma or GAP. The fusion of the conjugacy classes of $H_2 = 2^5{:}S_6$ into classes of $Sp(6,2)$ and the character table of H_2 can be found in many sources such as Ali [1], Magma, GAP or Mpono [55]. For the convenience of the reader, we supply the fusion as well as the character table of H_2 in Tables 8.2 and 11.11 respectively.

Table 8.2: The fusion of classes of $H_2 = 2^5{:}S_6$ into classes of $Sp(6,2)$

Inertia Factor Group H_2	Class of H_2	$\hookrightarrow$ Class of $Sp(6,2)$	Class of H_2	$\hookrightarrow$ Class of $Sp(6,2)$
$H_2 = 2^5{:}S_6$	$1a = g_{121}$	$1A$	$4g = g_{13,22}$	$4E$
	$2a = g_{221}$	$2A$	$4h = g_{11,22}$	$4C$
	$2b = g_{421}$	$2C$	$4i = g_{10,22}$	$4B$
	$2c = g_{321}$	$2B$	$4j = g_{13,23}$	$4E$
	$2d = g_{222}$	$2A$	$5a = g_{14,21}$	$5A$
	$2e = g_{422}$	$2C$	$6a = g_{16,21}$	$6B$
	$2f = g_{521}$	$2D$	$6b = g_{18,21}$	$6D$
	$2g = g_{522}$	$2D$	$6c = g_{15,21}$	$6A$
	$2h = g_{423}$	$2C$	$6d = g_{15,22}$	$6A$
	$2i = g_{523}$	$2D$	$6e = g_{18,22}$	$6D$
	$2j = g_{322}$	$2B$	$6f = g_{19,21}$	$6E$
	$3a = g_{621}$	$3A$	$6g = g_{21,21}$	$6G$
	$3b = g_{821}$	$3C$	$6h = g_{20,21}$	$6F$
	$4a = g_{11,21}$	$4C$	$8a = g_{23,21}$	$8A$
	$4b = g_{10,21}$	$4B$	$8b = g_{24,21}$	$8B$
	$4c = g_{12,21}$	$4D$	$10a = g_{26,21}$	$10A$
	$4d = g_{921}$	$4A$	$12a = g_{27,21}$	$12A$
	$4e = g_{13,21}$	$4E$	$12b = g_{28,21}$	$12B$
	$4f = g_{12,22}$	$4D$		

Note that from Table 8.1, the number of the conjugacy classes of $\overline{G}$ is 67. Thus $|\mathrm{Irr}(\overline{G})| = 67$. Since $|\mathrm{Irr}(H_1)| = |\mathrm{Irr}(Sp(6,2))| = 30$, the other inertia factor group $H_2 = 2^5{:}S_6$ must contribute with 37 characters in order to construct the ordinary character table of $\overline{G}$. Although we know that $|\mathrm{Irr}(H_2)| = |\mathrm{Irr}(2^5{:}S_6)| = 37$, which suggests to consider the set $\mathrm{Irr}(2^5{:}S_6)$, there is no reason, at this stage, allowing us to use the set $\mathrm{Irr}(2^5{:}S_6)$ as the group $\overline{G}$ does not split and consequently we do not know whether we will use the ordinary or a projective character table of H_2. To determine

the type of the character table (ordinary or a projective) we have to calculate the Schur multiplier of H_2, consider all the projective character tables and we may need to test all possible choices, and one is successful if all but one lead to a contradiction. The following sequence of Magma commands reveals the Schur multiplier of H_2 and also the ordinary character table of the full covering group of H_2.

```
> H2:= PermutationGroup< 62 | (1,2)(3,5)(4,7)(6,10)(12,16)(13,18)(14,20)(15,21)
                              (19,23)(24,28)(29,31)(30,32),(1,3,6,11)(2,4,8,13,
                               19,10,15,22,18,25,29,32)(5,9,14,21,16,23,27,20,
                               26,30,31,28)(7,12,17,24)>;
> Order(H2);
23040
> pMultiplicator(H2,2);
[ 2, 2 ]
> pMultiplicator(H2,3);
[ 1 ]
> pMultiplicator(H2,5);
[ 1 ]
> F := FPGroup(H2);
> F2 := pCover(H2, F, 2);
> Order(F2);
92160
> p, H:= CosetAction(F2, sub<F2|>);
> s:= SylowSubgroup(H, 3);
> p2, H1:= CosetAction(H, s);
> Order(H1);
92160
> ct:= CharacterTable(H1);
> ct;
```

From the above we can see that the Schur multiplier $M(H_2)$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_1 \times \mathbb{Z}_1 \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \cong 2^2$. Also the covering group $M(H_2)\cdot H_2$ (central extension of $M(H_2)$ by H_2, see Definition 4.2.5) is isomorphic to $2^2\cdot(2^5{:}S_6)$. The command "ct:= CharacterTable(H1);" will display the character table of $2^2\cdot(2^5{:}S_6)$, where we get 70 irreducible characters for this group, in which 37 of these are the ordinary characters of H_2 (see Table 11.11 for the ordinary characters of H_2). Thus we deduce that $|\mathrm{IrrProj}(H_2, \alpha^{-1})| \leq 33$, for any non-trivial factor set α of $M(H_2) = 2^2$. Therefore if we use a projective character table of H_2, with non-trivial factor set α, to construct the character table of $\overline{G}$, we obtain

$$|\mathrm{Irr}(H_1)| + |\mathrm{IrrProj}(H_2, \alpha^{-1})| \leq 30 + 33 = 63 < 67 = |\mathrm{Irr}(\overline{G})|.$$

This shows that we should use the set $\mathrm{Irr}(2^5{:}S_6)$ to construct the ordinary character table of $\overline{G}$.

This leads to the following corollary.

Corollary 8.3.1. *The second inertia group $\overline{H}_2$ has a character ψ of degree 63 such that $\psi{\downarrow}_N^{\overline{H}_2} = \sum_{i=2}^{64}\theta_i$, where $\theta_2, \theta_3, \cdots, \theta_{64}$ are the non-trivial linear characters of N.*

8.4. Fischer Matrices of $\overline{G} = 2^{6\cdot}Sp(6,2)$

We recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of $g_{ij},\ 1 \le j \le c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 8.1 we supplied $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \le i \le 30,\ 1 \le j \le c(g_i)$. Also having obtained the fusions of the inertia factor group H_2 into $Sp(6,2)$, we are able to label the rows of the Fischer matrices as described in Section 5.3. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 8.1 that the sizes of the Fischer matrices of $\overline{G} = 2^{6\cdot}Sp(6,2)$ range between 1 and 4 for every $i \in \{1, 2, \cdots, 30\}$. We have used the arithmetical properties of Fischer matrices, given in Proposition 5.2.2, to calculate some of the entries of the Fischer matrices and also to build an algebraic system of equations. With the help of the symbolic mathematical package Maxima [45], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}
$o(g_{1j})$		1	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		92897280	1474560
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$		
$(1,1)$	1451520	1	1
$(2,1)$	23040	63	-1
m_{1j}		1	63

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}
$o(g_{2j})$		4	4	2
$\lvert C_{\overline{G}}(g_{2j})\rvert$		737280	49152	46080
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$			
$(1,1)$	23040	1	1	1
$(2,1)$	23040	1	1	-1
$(2,2)$	768	30	-2	0
m_{2j}		2	30	32

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}
$o(g_{3j})$		2	2	4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		73728	24576	6144
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$			
$(1,1)$	4608	1	1	1
$(2,1)$	1536	3	3	-1
$(2,2)$	384	12	-4	0
m_{3j}		4	12	48

$\mathcal{F}_4$

g_4		g_{41}	g_{42}	g_{43}	g_{44}
$o(g_{4j})$		4	4	2	4
$\lvert C_{\overline{G}}(g_{4j})\rvert$		24576	8192	6144	3072
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$				
$(1,1)$	1536	1	1	1	1
$(2,1)$	1536	1	1	1	-1
$(2,2)$	768	2	2	-2	0
$(2,3)$	128	12	-4	0	0
m_{4j}		4	12	16	32

$\mathcal{F}_5$

g_5		g_{51}	g_{52}	g_{53}	g_{54}
$o(g_{5j})$		4	2	4	4
$\lvert C_{\overline{G}}(g_{5j})\rvert$		3072	3072	1024	1024
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$				
$(1,1)$	384	1	1	1	1
$(2,1)$	128	3	3	-1	-1
$(2,2)$	128	3	-3	-1	1
$(2,3)$	384	1	-1	1	-1
m_{5j}		8	8	24	24

$\mathcal{F}_6$

g_6		g_{61}	g_{62}
$o(g_{6j})$		3	6
$\lvert C_{\overline{G}}(g_{6j})\rvert$		34560	2304
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$		
$(1,1)$	2160	1	1
$(2,1)$	144	15	-1
m_{6j}		4	60

$\mathcal{F}_7$

g_7		g_{71}
$o(g_{7j})$		3
$\lvert C_{\overline{G}}(g_{7j})\rvert$		648
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$	
$(1,1)$	648	1
m_{7j}		64

$\mathcal{F}_8$

g_8		g_{81}	g_{82}
$o(g_{8j})$		3	6
$\lvert C_{\overline{G}}(g_{8j})\rvert$		432	144
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$		
$(1,1)$	108	1	1
$(2,1)$	36	3	-1
m_{8j}		16	48

$\mathcal{F}_9$

g_9		g_{91}	g_{92}
$o(g_{9j})$		4	4
$\lvert C_{\overline{G}}(g_{9j})\rvert$		1536	512
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$		
$(1,1)$	384	1	1
$(2,1)$	128	3	-1
m_{9j}		16	48

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$
$o(g_{10j})$		8	8	4
$\lvert C_{\overline{G}}(g_{10j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{10j}		8	24	32

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$	$g_{11,3}$
$o(g_{11j})$		8	8	4
$\lvert C_{\overline{G}}(g_{11j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{11j}		8	24	32

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$
$o(g_{12j})$		4	4	4
$\lvert C_{\overline{G}}(g_{12j})\rvert$		512	512	256
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$			
$(1,1)$	128	1	1	1
$(2,1)$	128	1	1	-1
$(2,2)$	64	2	-2	0
m_{12j}		16	16	32

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$
$o(g_{13j})$		8	4	8	4
$\lvert C_{\overline{G}}(g_{13j})\rvert$		128	128	128	128
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$				
$(1,1)$	32	1	1	1	1
$(2,1)$	32	1	1	-1	-1
$(2,2)$	32	1	-1	-1	1
$(2,3)$	32	1	-1	1	-1
m_{13j}		16	16	16	16

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$	$g_{14,2}$
$o(g_{14j})$		10	5
$\lvert C_{\overline{G}}(g_{14j})\rvert$		40	120
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	10	3	-1
m_{14j}		16	48

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$	$g_{15,3}$
$o(g_{15j})$		12	12	6
$\lvert C_{\overline{G}}(g_{15j})\rvert$		1152	384	288
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$			
$(1,1)$	144	1	1	1
$(2,1)$	144	1	1	-1
$(2,2)$	24	6	-2	0
m_{15j}		8	24	32

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$
$o(g_{16j})$		6	12
$\lvert C_{\overline{G}}(g_{16j})\rvert$		576	192
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$		
$(1,1)$	144	1	1
$(2,1)$	48	3	-1
m_{16j}		16	48

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$
$o(g_{17j})$		6
$\lvert C_{\overline{G}}(g_{17j})\rvert$		72
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$	
$(1,1)$	72	1
m_{17j}		64

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$	$g_{18,3}$
$o(g_{18j})$		12	6	12
$\lvert C_{\overline{G}}(g_{18j})\rvert$		192	192	96
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$			
$(1,1)$	48	1	1	1
$(2,1)$	48	1	1	-1
$(2,2)$	24	2	-2	0
m_{18j}		16	16	32

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		12	6
$\lvert C_{\overline{G}}(g_{19j})\rvert$		72	72
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$		
$(1,1)$	36	1	1
$(2,1)$	36	1	-1
m_{19j}		32	32

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$	$g_{20,2}$
$o(g_{20j})$		6	6
$\lvert C_{\overline{G}}(g_{20j})\rvert$		144	48
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$		
$(1,1)$	36	1	1
$(2,1)$	12	3	-1
m_{20j}		16	48

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$	$g_{21,2}$
$o(g_{21j})$		12	6
$\lvert C_{\overline{G}}(g_{21j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$		
$(1,1)$	12	1	1
$(2,1)$	12	1	-1
m_{21j}		32	32

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$
$o(g_{22j})$		7
$\lvert C_{\overline{G}}(g_{22j})\rvert$		7
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$	
$(1,1)$	7	1
m_{22j}		64

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$
$o(g_{23j})$		8	8
$\lvert C_{\overline{G}}(g_{23j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{23j}		32	32

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$	$g_{24,2}$
$o(g_{24j})$		8	8
$\lvert C_{\overline{G}}(g_{24j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{24j}		32	32

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$
$o(g_{25j})$		9
$\lvert C_{\overline{G}}(g_{25j})\rvert$		9
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$	
$(1,1)$	9	1
m_{25j}		64

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$	$g_{26,2}$
$o(g_{26j})$		20	10
$\lvert C_{\overline{G}}(g_{26j})\rvert$		20	20
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$		
$(1,1)$	10	1	1
$(2,1)$	10	1	-1
m_{26j}		32	32

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$	$g_{27,2}$
$o(g_{27j})$		24	12
$\lvert C_{\overline{G}}(g_{27j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{27j}		32	32

$\mathcal{F}_{28}$

g_{28}		$g_{28,1}$	$g_{28,2}$
$o(g_{28j})$		24	12
$\lvert C_{\overline{G}}(g_{28j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{28km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{28j}		32	32

$\mathcal{F}_{29}$

g_{29}		$g_{29,1}$
$o(g_{29j})$		12
$\lvert C_{\overline{G}}(g_{29j})\rvert$		12
(k,m)	$\lvert C_{H_k}(g_{29km})\rvert$	
$(1,1)$	12	1
m_{29j}		64

$\mathcal{F}_{30}$

g_{30}		$g_{30,1}$
$o(g_{30j})$		15
$\lvert C_{\overline{G}}(g_{30j})\rvert$		15
(k,m)	$\lvert C_{H_k}(g_{30km})\rvert$	
$(1,1)$	15	1
m_{30j}		64

Remark 8.4.1. We note that the Fischer matrix $\mathcal{F}_i,\ 1 \leq i \leq 30$, corresponds to class $[g_i]_{Sp(6,2)}$, of $\overline{G} = 2^{6\cdot}Sp(6,2)$ coincides with Fischer matrix corresponds to class $[g_i]_{Sp(6,2)}$ of $2^6{:}Sp(6,2)$ (see Mpono [55]), except for the classes $6A$ and $6B$ of $Sp(6,2)$, where the Fischer matrices correspond to these two classes are interchanged.

8.5. The Character Table of $\overline{G} = 2^{6\cdot}Sp(6,2)$

From Sections 8.2, 8.3 and 8.4 we have

- the conjugacy classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$ (Table 8.1),
- the fusion of classes of the inertia factor H_2 into classes of $Sp(6,2)$ (Table 8.2),
- the character table of the inertia factor H_2 (Table 11.11),
- the Fischer matrices (see Section 8.4).

By Section 5.3, it follows that the full character table of $\overline{G}$ can be constructed easily. We give an example on how to construct the character table of $\overline{G}$, which is partitioned into 60 parts corresponding to the 30 cosets and the two inertia factor groups. As an example we construct the parts $\mathcal{K}_{21}\mathcal{F}_{21}$ and $\mathcal{K}_{22}\mathcal{F}_{22}$ of the character table of $\overline{G}$ (this means that we are listing the values of all the irreducible characters of $\overline{G}$ on the classes $g_{21},\ g_{22}$, and g_{23} of $\overline{G}$, which correspond to the conjugacy class $2A$ of $Sp(6,2)$). The two parts $\mathcal{K}_{21}\mathcal{F}_{21}$ and $\mathcal{K}_{22}\mathcal{F}_{22}$ can be derived as follows: From Table 8.2 we can see that there are two classes, namely $2a = g_{221}$ and $2d = g_{222}$ of H_2 that fuse into the class $g_2 = [2A]_{Sp(6,2)}$. To construct the part $\mathcal{K}_{21}\mathcal{F}_{21}$, we multiply the column of the character table of $H_1 = Sp(6,2)$ corresponds to the class $2A$ of $Sp(6,2)$ (see the $\mathbb{ATLAS}$), by the first row of $\mathcal{F}_2$, namely $(1 \quad 1 \quad 1)$ and thus the part $\mathcal{K}_{21}\mathcal{F}_{21}$ of size 30×3, consists of the column of the character table of $Sp(6,2)$ corresponds to the class $2A$ repeated 3 times. To construct the part $\mathcal{K}_{22}\mathcal{F}_{22}$, select the two columns of the character table of $H_2 = 2^5{:}S_6$, correspond to the classes $2a$ and $2d$ of H_2 (see Table 11.11) and multiply these two columns by the two rows of $\mathcal{F}_2$ correspond to the pair $(2,1)$ and $(2,2)$. Thus we get a part in the character table of $\overline{G}$ of size 37×3. The above two parts have the following form:

$$\mathcal{K}_{21}\mathcal{F}_{21} = \begin{pmatrix} 1 \\ -5 \\ -5 \\ 9 \\ -11 \\ 15 \\ -5 \\ 15 \\ -24 \\ -10 \\ 4 \\ -35 \\ 25 \\ 5 \\ 40 \\ 40 \\ 21 \\ -51 \\ -39 \\ 50 \\ 10 \\ -24 \\ -40 \\ 40 \\ -45 \\ -16 \\ -30 \\ 45 \\ 20 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{21} & g_{22} & g_{23} \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 \\ -5 & -5 & -5 \\ -5 & -5 & -5 \\ 9 & 9 & 9 \\ -11 & -11 & -11 \\ 15 & 15 & 15 \\ -5 & -5 & -5 \\ 15 & 15 & 15 \\ -24 & -24 & -24 \\ -10 & -10 & -10 \\ 4 & 4 & 4 \\ -35 & -35 & -35 \\ 25 & 25 & 25 \\ 5 & 5 & 5 \\ 40 & 40 & 40 \\ 40 & 40 & 40 \\ 21 & 21 & 21 \\ -51 & -51 & -51 \\ -39 & -39 & -39 \\ 50 & 50 & 50 \\ 10 & 10 & 10 \\ -24 & -24 & -24 \\ -40 & -40 & -40 \\ 40 & 40 & 40 \\ -45 & -45 & -45 \\ -16 & -16 & -16 \\ -30 & -30 & -30 \\ 45 & 45 & 45 \\ 20 & 20 & 20 \\ 0 & 0 & 0 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{22}\mathcal{F}_{22} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 5 & -3 \\ 5 & 3 \\ 5 & -1 \\ 5 & 1 \\ -6 & -4 \\ -6 & 4 \\ 9 & -3 \\ 9 & 3 \\ -10 & -4 \\ -10 & -4 \\ 10 & -2 \\ -10 & 4 \\ -10 & 4 \\ 10 & 2 \\ 15 & 7 \\ 15 & -5 \\ 15 & -7 \\ 15 & 5 \\ 16 & 0 \\ -20 & 0 \\ -24 & 8 \\ -24 & -8 \\ -30 & -4 \\ 30 & -2 \\ 30 & 2 \\ -30 & 4 \\ -36 & 0 \\ -40 & 8 \\ -40 & -8 \\ -40 & 0 \\ -40 & 0 \\ 45 & -9 \\ 45 & 9 \\ 45 & 3 \\ 45 & -3 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 30 & -2 & 0 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{21} & g_{22} & g_{23} \end{matrix} \\ \begin{pmatrix} 31 & -1 & -1 \\ -29 & 3 & -1 \\ -85 & 11 & -5 \\ 95 & -1 & -5 \\ -25 & 7 & -5 \\ 35 & 3 & -5 \\ -126 & 2 & 6 \\ 114 & -14 & 6 \\ -81 & 15 & -9 \\ 99 & 3 & -9 \\ -130 & -2 & 10 \\ -130 & -2 & 10 \\ -50 & 14 & -10 \\ 110 & -18 & 10 \\ 110 & -18 & 10 \\ 70 & 6 & -10 \\ 225 & 1 & -15 \\ -135 & 25 & -15 \\ -195 & 29 & -15 \\ 165 & 5 & -15 \\ 16 & 16 & -16 \\ -20 & -20 & 20 \\ 216 & -40 & 24 \\ -264 & -8 & 24 \\ -150 & -22 & 30 \\ -30 & 34 & -30 \\ 90 & -38 & 30 \\ 90 & 26 & -30 \\ -36 & -36 & 36 \\ 200 & -56 & 40 \\ -280 & -24 & 40 \\ -40 & -40 & 40 \\ -40 & -40 & 40 \\ -225 & 63 & -45 \\ 315 & 27 & -45 \\ 135 & 39 & -45 \\ -45 & 51 & -45 \end{pmatrix} \end{array}.$$

Similarly one can obtain all the other 58 parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$, $1 \leq i \leq 30,\ i \neq 2,\ 1 \leq k \leq 2$ and hence the full character table of $\overline{G}$, which is a 67×67 $\mathbb{Z}-$valued matrix. We list the full character table of $\overline{G}$ in Table 11.12. By referring to Table 6.15 of Mpono [55], one can see that the entries of the character table of $2^6{:}Sp(6,2)$ coincide with the entries of the character table of $2^{6\cdot}Sp(6,2)$ constructed in this chapter.

CHAPTER 9

On the Extension $2^{2n\cdot}Sp(2n,2)$ and the Character Table of $2^{8\cdot}Sp(8,2)$

9.1. Introduction

Let $G_n = Sp(2n,q)$ be the symplectic group and let $V = q^{2n}$ be a $2n-$dimensional vector space over $\mathbb{F}_q$. In Chapter 8 we have mentioned the existence (by a paper of U. Dempwolff [23]) of a non-split extension of the form $\overline{G}_n = 2^{2n\cdot}Sp(2n,2)$ for all $n \geq 2$, where $\overline{G}_n/2^{2n} \cong Sp(2n,2)$ acts faithfully on 2^{2n}. Also such extension is unique, up to isomorphism. In Chapter 8 we have handled the case $\overline{G}_3 = 2^{6\cdot}Sp(6,2)$. In this chapter we give some general results on the group $\overline{G}_n$ and we focus mainly on $\overline{G}_4 = 2^{8\cdot}Sp(8,2) := \overline{G}$, where we will construct its character table using Clifford-Fischer theory. It will turn out that the character table of $\overline{G}$ is a 195×195 complex valued matrix and coincides with the character table of the split extension $2^8{:}Sp(8,2)$, which is available in a GAP library (see [30]). In Section 9.2 we supply a sequence of commands that generate $\overline{G}_n$ followed by some general results on the inertia factors of this extension. In particular we supply the number of irreducible characters for the cases $n \in \{2,3,4,5,6\}$ (Table 9.1) as well as a general formula for $|\mathrm{Irr}(\overline{G}_n)|$ (Eq. (9.1)). In Section 9.3 we supply the generators of the group $\overline{G} = 2^{8\cdot}Sp(8,2)$ as well as generators of the normal subgroup $N = 2^8$ in terms of permutations of a set of cardinality 512. In Section 9.4 we construct the conjugacy classes of $\overline{G}$ using the coset analysis technique. In Section 9.5 we determine the inertia factor groups of $\overline{G}$. Also in this section we determine the fusion of classes of the inertia factor group H_2 into classes of $Sp(8,2)$ (Table 9.3). In Section 9.6 we list the 81 Fischer matrices of $\overline{G}$. These matrices are integral valued and their sizes range between 1 and 5. In Section 9.7 we describe how to obtain the full character table of $2^{8\cdot}Sp(8,2)$, which appears in the Appendix

of this thesis as Table 11.13.

9.2. The Group $\overline{G}_n = 2^{2n}{\cdot}Sp(2n,2)$

The group $\overline{G}_n$ can be constructed in GAP in terms of permutations through the following sequence of commands.

```
gap> P:= ExtraspecialGroup(2^{2n+1}, "+");
gap> C:= CyclicGroup(4);
gap> D:= DirectProduct(P, C);
gap> NS := MinimalNormalSubgroups(D);
gap> N:= NS[k];
gap> R:= FactorGroup(D,N);
gap> A:= AutomorphismGroup(R);
gap> MS:= MaximalNormalSubgroups(A);
gap> S:= MS[r];
gap> iso:= IsomorphismPermGroup(S);
gap> image:= Image(iso);;
gap> NrMovedPoints(image);
gap> small:= SmallerDegreePermutationRepresentation(image);;
gap> Gn:= Image(small);
```

Remark 9.2.1. In the above programme, the command "MinimalNormalSubgroups(D)" always returns three copies of $\mathbb{Z}_2$ and the reader has to choose the appropriate $\mathbb{Z}_2$ in such away that the order of the automorphism group of the quotient of D by $\mathbb{Z}_2$ is $2^{2n+1} \times |Sp(2n,2)|$. Also the command "MaximalNormalSubgroups(A)" returns a list of those normal subgroups of A that are maximal, and the group $\overline{G}_n := S$ appears as an index 2 subgroup of A. Also the last five commands of the above programme are used to convert S from a finitely presented group into a group of permutations.

We also remark that the group $\overline{G}_n$ can be constructed in terms of permutations of a set of cardinality at least 2^{2n+1}, i.e., $\overline{G}_n \leq S_{2^{2n+1}}$, but $\overline{G}_n \not\leq S_{2^{2n+1}-1}$ (in fact $\overline{G}_n \leq A_{2^{2n+1}}$). In Chapter 8 we have seen that the generators of $\overline{G}_3 = 2^{6}{\cdot}Sp(6,2)$ are in terms of permutations of a set of cardinality 128.

The group $\overline{G}_n/2^{2n} \cong Sp(2n,2)$ acts faithfully on 2^{2n}, it yields two orbits of lengths 1 and $2^{2n}-1$. By Brauer Theorem (see Theorem 5.1.1), it follows that the action of $\overline{G}_n$ on $\mathrm{Irr}(N)$ will also produce two orbits. These two orbits must necessarily have lengths 1 and $2^{2n}-1$ and the first

orbit consists of the identity character $\mathbf{1}_N$ while the other orbit consists of the non-trivial linear characters of N. Thus the corresponding inertia factor groups H_1 and H_2 have indices 1 and $2^{2n}-1$ respectively in $Sp(2n,2)$. It is clear that $H_1 = G_n = Sp(2n,2)$, while a subgroup of $Sp(2n,2)$ of index $2^{2n}-1$ is the affine symplectic group $2^{2n-1}{:}Sp(2n-2,2)$, that is the stabilizer in $Sp(2n,2)$ of a non-zero vector of the vector space $\mathbb{V} = 2^{2n}$, which is maximal in $Sp(2n,2)$. From Section 5.3, it follows that the irreducible characters of $\overline{G}_n$ are distributed into two blocks of characters $\mathcal{K}_1$ and $\mathcal{K}_2$ corresponding to the ordinary characters of $H_1 = Sp(2n,2)$ and a projective character table of $H_2 = 2^{2n-1}{:}Sp(2n-2,2)$ respectively. Thus the number of irreducible characters of $\overline{G}_n$ is given by the following formula:

$$|\mathrm{Irr}(2^{2n}{\cdot}Sp(2n,2))| = |\mathrm{Irr}(Sp(2n,2))| + |\mathrm{IrrProj}(2^{2n-1}{:}Sp(2n-2,2),\alpha^{-1})|, \tag{9.1}$$

for some factor set α of the Schur multiplier of H_2. In his PhD Thesis [69], Seretlo constructed the ordinary character table of $\overline{G}_2 = 2^{4}{\cdot}Sp(4,2) \cong 2^{4}{\cdot}S_6$ by means of Clifford-Fischer theory, where he showed that there are two inertia factor groups $H_1 = S_6$ and $H_2 = 2 \times S_4$. He proved that for the construction of the character table of $\overline{G}_2$ we only need the ordinary character tables of H_1 and H_2. In Chapter 8 we showed that we need to use the ordinary character tables of the two inertia factors of $\overline{G}_3$. For the case $n = 4$, where the group $\overline{G}_4$ is the focus of the discussion of this chapter, the computations of the Schur multiplier of the second inertia factor group $2^7{:}Sp(6,2)$ reveals 2. By similar arguments used in Subsection 7.1.3 one can show that we need to use the ordinary character table of $2^7{:}Sp(6,2)$ to construct the character table of $\overline{G}_4$. By Proposition 5.2.2, it follows that the identity Fischer matrix of $\overline{G}_n$ for any $n \geq 2$ is of the form:

$\mathcal{F}_1$

$g_1 = 1_{Sp(2n,2)}$		g_{11}	g_{12}
$o(g_{1j})$		1	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		$\lvert\overline{G}_n\rvert$	$\lvert\overline{G}_n\rvert/2^{2n}-1$
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$		
$(1,1)$	$\lvert G_n\rvert$	1	1
$(2,1)$	$\lvert G_n\rvert/2^{2n}-1$	$2^{2n}-1$	-1
m_{1j}		1	$2^{2n}-1$

.

Thus the degree of any irreducible character of $\overline{G}_n$ contained in the block $\mathcal{K}_2$ is a multiple of $2^{2n}-1$. In the special cases $\overline{G}_5$ (resp. $\overline{G}_6$) the degrees of the irreducible characters contained in $\mathcal{K}_2$ correspond to $H_2 = 2^9{:}Sp(10,2)$ (resp. $H_2 = 2^{11}{:}Sp(12,2)$) are multiples of 1023 (resp. 4095). Using the programm supplied in Section 9.2 one can obtain the character tables of $\overline{G}_5$ (resp. $\overline{G}_6$),

where one can re-arrange (see Remark 9.2.2 below) the rows and columns in such away that the characters of $Sp(10,2)$ (resp. $Sp(12,2)$) form the block $\mathcal{K}_1$ and characters of $2^9{:}Sp(10,2)$ (resp. $2^{11}{:}Sp(12,2)$) form the other block $\mathcal{K}_2$. After this re-arrangement of characters, one can see that there is a character of $\overline{G}_5$ (resp. $\overline{G}_6$) contained in $\mathcal{K}_2$ of degree 1023 (resp. 4095). Recall from Lemma 4.2.4 that for any finite group K, if α is any non-trivial factor set of $M(K)$, then $\deg(\chi) > 1$ for any $\chi \in \mathrm{IrrProj}(K, \alpha^{-1})$. Now since $1023 | \deg(\chi)$ (resp. $4095 | \deg(\chi)$) for any $\chi \in \mathrm{Irr}(\overline{G}_5)$ (resp. $\chi \in \mathrm{Irr}(\overline{G}_6)$) such that χ is contained in $\mathcal{K}_2$ and since there is a character in $\mathcal{K}_2$ of degree 1023 (resp. 4095), it follows that the fragment (more precisely the column of degrees) of the projective character table of $2^9{:}Sp(10,2)$ (resp. $2^{11}{:}Sp(12,2)$) that we will use to construct (by means of Clifford-Fischer theory) the character table of $\overline{G}_5$ (resp. $\overline{G}_6$) contains a character of degree 1. This show that the associated factor set α of $M(2^9{:}Sp(10,2))$ (resp. $M(2^{11}{:}Sp(12,2))$) is trivial. Hence we will use the ordinary character table of $2^9{:}Sp(10,2)$ (resp. $2^{11}{:}Sp(12,2)$) to construct the character table of $\overline{G}_5$ (resp. $\overline{G}_6$).

From the above, Eq. (9.1) becomes

$$|\mathrm{Irr}(2^{2n}{\cdot}Sp(2n,2))| = |\mathrm{Irr}(Sp(2n,2))| + |\mathrm{Irr}(2^{2n-1}{:}Sp(2n-2,2))|, \quad 2 \leq n \leq 6. \tag{9.2}$$

In Table 9.1 we list the number of ordinary irreducible characters of $\overline{G}_n = 2^{2n}{\cdot}Sp(2n,2)$ for small values of n.

Table 9.1: The number of ordinary irreducible characters of $\overline{G}_n = 2^{2n}{\cdot}Sp(2n,2),\ n \in \{2,3,4,5,6\}$

n	$\lvert\mathrm{Irr}(Sp(2n,2))\rvert$	$\lvert\mathrm{Irr}(2^{2n-1}{:}Sp(2n-2,2))\rvert$	$\lvert\mathrm{Irr}(2^{2n}{\cdot}Sp(2n,2))\rvert$
2	11	10	21
3	30	37	67
4	81	114	195
5	198	322	520
6	477	839	1316

Remark 9.2.2. Note that for the group $\overline{G}_5$ (resp. $\overline{G}_6$), we have used the programm supplied in Section 9.2 to obtain its character table, which is not necessarily to be in the format of Clifford-Fischer theory. To re-arrange the characters of $\overline{G}_5$ (resp. $\overline{G}_6$) we only look at the first two columns corresponding to the identity of $\overline{G}_5$ (resp. $\overline{G}_6$) and to the class of $\overline{G}_5$ (resp. $\overline{G}_6$) consisting of the 1023 (resp. 4095) involutions contained in the normal subgroup $N = 2^{10}$ (resp. $N = 2^{12}$). If the degree of a character repeated in the second column, then this character must be in block $\mathcal{K}_1$,

otherwise it is in $\mathcal{K}_2$. Also note that we have used the information that we have on the character table of $\overline{G}_5$ (resp. $\overline{G}_6$) and the identity Fischer matrix of $\overline{G}_5$ (resp. $\overline{G}_6$) to determine the type of character table (projective or ordinary) of $2^9{:}Sp(10,2)$ (resp. $2^{11}{:}Sp(12,2)$) that is required.

We conclude this section by making the following conjecture.

Conjecture: We speculate that for any $n \in \mathbb{N}^{\geq 2}$ we only need to use the ordinary character table of $H_2 = 2^{2n-1}{:}Sp(2n-2,2)$ to construct the character table of $\overline{G}_n$ and thus the theory of projective representations is not involved in the construction of the character tables of $\overline{G}_n$. That is to say Eq. (9.2) is true for all $n \in \mathbb{N}^{\geq 2}$.

9.3. Generators of the Group $\overline{G} = 2^{8}{\cdot}Sp(8,2)$

From now on and up the end of this chapter, we let $\overline{G}$ be the group $\overline{G}_4 = 2^{8}{\cdot}Sp(8,2)$. The following two permutations of the alternating simple group A_{512} generate the group $\overline{G}$.

$\overline{g}_1$ = (1 46 211 15 32 257 208 91 169 133 271 394 436 127 463 438 286)(2 178 320 321 489 206 242 376 186 219 370 199 47 5 343 450 395)(3 90 126 293 388 254 55 331 241 119 147 82 400 330 160 490 297)(4 498 152 262 421 431 393 72 351 324 302 166 162 417 240 56 398)(6 193 474 491 347 40 244 336 377 415 103 488 319 424 426 98 461)(7 382 200 239 273 189 405 443 288 291 34 356 39 322 383 504 454)(8 221 470 411 99 384 153 85 73 479 227 111 427 458 422 408 440)(10 350 151 231 399 274 269 43 363 26 141 156 11 265 234 337 413)(12 483 33 509 70 407 284 494 249 154 457 195 183 287 281 275 130)(13 22 349 472 361 486 355 207 465 455 340 280 93 501 360 318 49)(14 445 30 97 81 364 129 505 180 123 277 448 508 477 334 95 374)(16 120 344 61 96 404 342 29 84 57 414 480 497 50 362 312 423),

$\overline{g}_2$ = (1 438 189 247 501 459 43 10 460 256 277 423 18 346 197)(2 410 75 4 12 122 11 255 137 322 329 326 171 384 398)(3 405 236 20 51 442 101 244 419 238 211 402 498 248 294)(5 387 17 92 452 68 89 381 235 90 404 482 428 135 106)(6 510 47 176 186 225 292 120 161 181 44 111 102 88 305)(7 14 480 467 313 407 321 86 59 163 70 140 267 295 325)(8 230 471 372 162 309 188 105 24 166 497 69 412 358 426)(13 282 152 254 411 429 445 40 60 394 368 190 262 192 25)(15 409 484 191 130 76 283 136 468 408 214 502 430 388 109)(16 312 103 391 511 354 112 48 355 400 418 286 71 213 327)(19 274 84 37 487 204 54 29 491 432 57 224 270 304 466)(21 500 82 375 142 343 246 22 311 202 241 350 62 144 356)(23 265 156 33 56 232 53 141 252 193 448 446 290 475 483)(26 330 494 447 380 357 195 67 222 349 125 276 119 253 110).

with $o(\overline{g}_1) = 17,\ o(\overline{g}_2) = 15$ and $o(\overline{g}_1\overline{g}_2) = 14$.

Note 9.3.1. Note that the generators $\overline{g}_1$ and $\overline{g}_2$ fix the points 9 and 42. Thus $\overline{G}$ acts transitively on a set Γ of 510 points. Hence we have a permutation character $\chi(\overline{G}|\Gamma) = \chi$ of degree 510. In Table 11.2 we list the values of χ on $\overline{G}-$classes and using the character table of $\overline{G}$ (Table 11.13) we can see that $\chi = \chi_1 + \chi_5 + \chi_6 + \chi_{82}$.

Now having the group $\overline{G}$ constructed in GAP, it is easy to obtain all its normal subgroups. In fact the only non-trivial proper normal subgroup that $\overline{G}$ contains is a group of order 256 and thus must be isomorphic to the elementary abelian group $N = 2^8$. The following 8 permutations $n_1, n_2, \cdots, n_8$ generate the normal subgroup N.

n_1 = (8 41)(15 61)(22 79)(27 88)(32 96)(35 99)(38 101)(47 121)(52 129)(55 132)(58 134)(65 144)(70 152)(73 155)(76 157)(82 168)(85 170)(90 174)(93 176)(102 184)(109 215)(111 217)(115 227)(118 229)(123 233)(126 235)(135 243)(138 253)(141 255)(146 259)(149 261)(158 269)(162 281)(164 283)(171 290)(177 297)(178 298)(180 300)(182 301)(183 302)(188 310)(193 322)(195 324)(200 336)(202 338)(205 340)(207 341)(209 343)(211 344)(213 346)(221 358)(223 360)(230 367)(236 374)(237 375)(239 377)(241 378)(242 379)(247 391)(249 393)(256 400)(262 407)(263 408)(265 410)(267 411)(268 412)(271 414)(273 415)(275 417)(277 418)(279 420)(284 421)(286 423)(288 424)(289 425)(291 426)(293 428)(295 429)(296 430)(304 433)(307 435)(312 438)(314 439)(316 441)(318 442)(320 444)(326 446)(328 447)(330 449)(332 450)(334 452)(348 455)(350 456)(352 458)(354 459)(356 461)(361 462)(363 464)(365 465)(366 466),

n_2 = (6 39)(8 41)(13 59)(15 61)(20 77)(22 79)(25 86)(27 88)(30 94)(32 96)(35 99)(37 100)(45 119)(47 121)(50 127)(52 129)(55 132)(57 133)(63 142)(65 144)(68 150)(70 152)(73 155)(75 156)(82 168)(84 169)(90 174)(92 175)(97 181)(102 184)(106 206)(108 214)(109 215)(111 217)(115 227)(117 228)(123 233)(125 234)(130 240)(135 243)(138 253)(140 254)(146 259)(148 260)(153 266)(158 269)(159 272)(161 280)(162 281)(164 283)(166 287)(171 290)(172 294)(177 297)(179 299)(182 301)(186 308)(188 310)(190 313)(192 321)(193 322)(195 324)(197 327)(199 335)(200 336)(202 338)(204 339)(207 341)(209 343)(212 345)(218 349)(220 357)(221 358)(223 360)(225 364)(230 367)(231 371)(236 374)(238 376)(241 378)(244 382)(246 390)(247 391)(249 393)(251 397)(256 400)(257 404)(262 407)(264 409)(267 411)(270 413)(273 415)(275 417)(278 419)(285 422)(288 424)(292 427)(295 429)(304 433)(306 434),

n_3 = (7 40)(14 60)(21 78)(26 87)(31 95)(34 98)(37 100)(46 120)(51 128)(54 131)(57 133)(64 143)(69 151)(72 154)(75 156)(81 167)(84 169)(89 173)(92 175)(102 184)(109 215)(110 216)(114 226)(117 228)(122 232)(125 234)(135 243)(137 252)(140 254)(145 258)(148 260)(158 269)(162 281)(163 282)(171 290)(177 297)(178 298)(179 299)(182 301)(183 302)(187 309)(193 322)(194 323)(200 336)(201 337)(204 339)(207 341)(208 342)(211 344)(212 345)(221 358)(222 359)(230 367)(236 374)(237 375)(238 376)(241 378)(242 379)(247 391)(248 392)(256 400)(262 407)(263 408)(264 409)(267 411)(268 412)(270 413)(273 415)(274 416)(277 418)(278 419)(284 421)(285 422)(288 424)(289 425)(291 426)(292 427)(295 429)(296 430)(303 432)(306 434)(311 437)(314 439)(315 440)(318 442)(319 443)(325 445)(328 447)(329 448)(332 450)(333 451)(347 454)(350 456)(351 457)(354 459)(355 460)(361 462)(362 463)(365 465)(366 466),

n_4 = (5 36)(8 41)(12 56)(15 61)(19 74)(22 79)(24 83)(27 88)(29 91)(32 96)(34 98)(38 101)(44 116)(47 121)(49 124)(52 129)(54 131)(58 134)(62 139)(65 144)(67 147)(70 152)(72 154)(76 157)(81 167)(85 170)(89 173)(93 176)(97 181)(102 184)(106 206)(107 210)(109 215)(111 217)(114 226)(118 229)(122 232)(126 235)(130 240)(135 243)(137 252)(141 255)(145 258)(149 261)(153 266)(158 269)(159 272)(160 276)(162 281)(164 283)(166 287)(171 290)(172 294)(177 297)(179 299)(183 302)(185 305)(188 310)(190 313)(191 317)(193 322)(195 324)(197 327)(198 331)(200 336)(202 338)(204 339)(208 342)(211 344)(213 346)(218 349)(219 353)(221 358)(223 360)(225 364)(230 367)(231 371)(236 374)(238 376)(242 379)(244 382)(245 386)(247 391)(249 393)(251 397)(256 400)(257 404)(262 407)(264 409)(268 412)(270 413)(274 416)(277 418)(279 420)(285 422)(289 425)(292 427)(296 430)(303 432)(307 435),

n_5 = (4 33)(11 53)(18 71)(29 91)(30 94)(31 95)(32 96)(49 124)(50 127)(51 128)(52 129)(67 147)(68 150)(69 151)(70 152)(80 165)(89 173)(90 174)(92 175)(93 176)(103 189)(105 203)(113 224)(122 232)(123 233)(125 234)(126 235)(136 250)(145 258)(146 259)(148 260)(149 261)(159 272)(160 276)(161 280)(162 281)(163 282)(164 283)(172 294)(177 297)(185 305)(186 308)(187 309)(188 310)(197 327)(198 331)(199 335)(200 336)(201 337)(202 338)(218 349)(219 353)(220 357)(221 358)(222 359)(223 360)(231 371)(236 374)(244 382)(245 386)(246 390)(247 391)(248 392)(249 393)(257 404)(262 407)(270 413)(271 414)(273 415)(274 416)(275 417)(277 418)(278 419)(279 420)(291 426)(292 427)(293 428)(295 429)(296 430)(303 432)(304 433)(306 434)(307 435)(325 445)(326 446)(328 447)(329 448)(330 449)(332 450)(333 451)(334 452)(347 454)(348 455)(350 456)(351 457)(352 458)(354 459)(355 460)(356 461)(368 467),

n_6 = (3 28)(4 33)(10 48)(11 53)(17 66)(18 71)(24 83)(25 86)(26 87)(27 88)(29 91)(30 94)(31 95)(32 96)(44 116)(45 119)(46 120)(47 121)(49 124)(50 127)(51 128)(52 129)(62 139)(63 142)(64 143)(65 144)(67 147)(68 150)(69 151)(70 152)(81 167)(82 168)(84 169)(85 170)(89 173)(90 174)(92 175)(93 176)(104 196)(105 203)(114 226)(115 227)(117 228)(118 229)(122 232)(123 233)(125 234)(126 235)(137 252)(138 253)(140 254)(141 255)(145 258)(146 259)(148 260)(149 261)(166 287)(171 290)(172 294)(177 297)(190 313)(191 317)(192 321)(193 322)(194 323)(195 324)(197 327)(198 331)(199 335)(200 336)(201 337)(202 338)(225 364)(230 367)(231 371)(236 374)(251 397)(256 400)(257 404)(262 407)(284 421)(285 422)(286 423)(288 424)(289 425)(291 426)(292 427)(293 428)(295 429)(296 430)(311 437)(312 438)(314 439)(315 440)(316 441)(318 442)(319 443)(320 444)(325 445)(326 446),

n_7 = (2 23)(4 33)(6 39)(7 40)(11 53)(13 59)(14 60)(17 66)(19 74)(22 79)(25 86)(26 87)(29 91)(32 96)(34 98)(38 101)(43 112)(45 119)(46 120)(49 124)(52 129)(54 131)(58 134)(62 139)(65 144)(68 150)(69 151)(73 155)(75 156)(80 165)(81 167)(85 170)(90 174)(92 175)(97 181)(102 184)(104 196)(107 210)(111 217)(113 224)(114 226)(118 229)(123 233)(125 234)(130 240)(135 243)(138 253)(140 254)(145 258)(149 261)(160 276)(164 283)(166 287)(171 290)(180 300)(182 301)(186 308)(187 309)(191 317)(195 324)(197 327)(199 335)(200 336)(201 337)(204 339)(209 343)(211 344)(212 345)(219 353)(223 360)(225 364)(230 367)(239 377)(241 378)(244 382)(246 390)(247 391)(248 392)(257 404)(262 407)(263 408)(264 409)(268 412)(270 413)(275 417)(277 418)(278 419)(286 423)(288 424)(291 426)(292 427)(296 430)(303 432)(307 435)(311 437)(316 441)(318 442)(319 443)(326 446)(328 447),

n_8 = (1 16)(5 36)(7 40)(10 48)(11 53)(13 59)(15 61)(19 74)(21 78)(24 83)(26 87)(29 91)(31 95)(35 99)(37 100)(43 112)(45 119)(47 121)(50 127)(52 129)(54 131)(58 134)(62 139)(64 143)(67 147)(69 151)(73 155)(75 156)(82 168)(84 169)(90 174)(92 175)(97 181)(102 184)(103 189)(104 196)(105 203)(108 214)(111 217)(113 224)(114 226)(118 229)(122 232)(126 235)(138 253)(140 254)(146 259)(148 260)(153 266)(158 269)(159 272)(160 276)(162 281)(163 282)(166 287)(171 290)(172 294)(177 297)(180 300)(183 302)(186 308)(188 310)(192 321)(195 324)(199 335)(202 338)(204 339)(207 341)(208 342)(213 346)(220 357)(223 360)(237 375)(238 376)(241 378)(244 382)(245 386)(247 391)(248 392)(251 397)(256 400)(257 404)(262 407)(265 410)(268 412)(271 414)(275 417)(277 418)(278 419)(286 423)(289 425)(293 428)(296 430)(303 432)(307 435)(311 437)(314 439)(315 440)(320 444)(325 445).

In Magma or GAP one can easily check for the complements of any normal subgroup N of $\overline{G}$. In our case the set of complements of $N = \langle n_1, n_2, \cdots, n_8 \rangle$ in $\overline{G} = \langle \overline{g}_1, \overline{g}_2 \rangle$, is empty. Also one can check that the quotient group $\overline{G}/N \cong Sp(8,2)$. This shows that the group $\overline{G}$ constructed using the generators $\overline{g}_1$ and $\overline{g}_2$ is indeed a non-split extension of the elementary abelian group $N = 2^8$ by the symplectic group $Sp(8,2)$.

9.4. The Conjugacy Classes of $\overline{G} = 2^{8}{\cdot}Sp(8,2)$

In this section we use the method of the coset analysis, discussed in Section 2.3, to construct the character table of $\overline{G}$. We supply the conjugacy classes of $\overline{G}$ in Table 9.2, where in this table:

- k_i's represent the number of fixed points on the action of $N = \langle n_1, n_2, \cdots, n_8 \rangle$ on the coset $N\overline{g}_i$,
- f_{ij}'s represent the number of orbits (of the action of N on $N\overline{g}_i$) fused together under the action of $\overline{G} = \langle \overline{g}_1, \overline{g}_2 \rangle$,
- m_{ij}'s are the weights defined by Eq. (5.4).

Table 9.2: The conjugacy classes of $\overline{G} = 2^{8}{\cdot}Sp(8,2)$

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
$g_1 = 1A$	$k_1 = 256$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	24257337753600
		$f_{12} = 255$	$m_{12} = 255$	g_{12}	2	255	47563407360
		$f_{21} = 1$	$m_{21} = 2$	g_{21}	4	510	23781703680
$g_2 = 2A$	$k_2 = 128$	$f_{22} = 63$	$m_{22} = 126$	g_{22}	4	32130	377487360
		$f_{23} = 64$	$m_{23} = 128$	g_{23}	2	32640	371589120
		$f_{31} = 1$	$m_{31} = 4$	g_{31}	2	21420	566231040
$g_3 = 2B$	$k_3 = 64$	$f_{32} = 15$	$m_{32} = 60$	g_{32}	2	321300	37748736
		$f_{33} = 48$	$m_{33} = 192$	g_{33}	4	1028160	11796480
		$f_{41} = 1$	$m_{41} = 4$	g_{41}	4	64260	188743680
$g_4 = 2C$	$k_4 = 64$	$f_{42} = 15$	$m_{42} = 60$	g_{42}	4	963900	12582912
		$f_{43} = 15$	$m_{43} = 64$	g_{43}	2	1028160	11796480
		$f_{44} = 32$	$m_{44} = 128$	g_{44}	4	2056320	5898240
$g_5 = 2D$	$k_5 = 16$	$f_{51} = 1$	$m_{51} = 16$	g_{51}	2	1028160	11796480
		$f_{52} = 15$	$m_{52} = 240$	g_{52}	4	15422400	786432
		$f_{61} = 1$	$m_{61} = 8$	g_{61}	4	2570400	4718592
		$f_{62} = 3$	$m_{62} = 24$	g_{62}	4	7711200	1572864
$g_6 = 2E$	$k_6 = 32$	$f_{63} = 4$	$m_{63} = 32$	g_{63}	2	10281600	1179648
		$f_{64} = 12$	$m_{64} = 96$	g_{64}	4	30844800	393216
		$f_{65} = 12$	$m_{65} = 96$	g_{65}	4	30844800	393216
		$f_{71} = 1$	$m_{71} = 16$	g_{71}	4	15422400	786432
$g_7 = 2F$	$k_7 = 16$	$f_{72} = 1$	$m_{72} = 16$	g_{72}	2	15422400	786432
		$f_{73} = 6$	$m_{73} = 96$	g_{73}	4	92534400	131072
		$f_{74} = 8$	$m_{74} = 128$	g_{74}	4	123379200	98304
$g_8 = 3A$	$k_8 = 64$	$f_{81} = 1$	$m_{81} = 4$	g_{81}	3	43520	278691840
		$f_{82} = 63$	$m_{82} = 252$	g_{82}	6	2741760	4423680
$g_9 = 3B$	$k_9 = 1$	$f_{91} = 1$	$m_{91} = 256$	g_{91}	3	155975680	77760
$g_{10} = 3C$	$k_{10} = 16$	$f_{10,1} = 1$	$m_{10,1} = 16$	$g_{10,1}$	3	58490880	207360
		$f_{10,2} = 15$	$m_{10,2} = 240$	$g_{10,2}$	6	877363200	13824

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Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
$g_{11}=3D$	$k_{11}=4$	$f_{11,1}=1$	$m_{11,1}=64$	$g_{11,1}$	3	779878400	15552
		$f_{11,2}=3$	$m_{11,2}=192$	$g_{11,2}$	6	2339635200	5184
$g_{12}=4A$	$k_{12}=32$	$f_{12,1}=1$	$m_{12,1}=8$	$g_{12,1}$	8	4112640	2949120
		$f_{12,2}=15$	$m_{12,2}=120$	$g_{12,2}$	8	61689600	196608
		$f_{12,3}=16$	$m_{12,3}=128$	$g_{12,3}$	4	65802240	184320
$g_{13}=4B$	$k_{13}=32$	$f_{13,1}=1$	$m_{13,1}=8$	$g_{13,1}$	4	4112640	2949120
		$f_{13,2}=15$	$m_{13,2}=120$	$g_{13,2}$	8	61689600	196608
		$f_{13,3}=16$	$m_{13,3}=128$	$g_{13,3}$	4	65802240	184320
$g_{14}=4C$	$k_{14}=16$	$f_{14,1}=1$	$m_{14,1}=16$	$g_{14,1}$	4	20563200	589824
		$f_{14,2}=3$	$m_{14,2}=48$	$g_{14,2}$	4	61689600	196608
		$f_{14,3}=12$	$m_{14,3}=192$	$g_{14,3}$	4	246758400	49152
$g_{15}=4D$	$k_{15}=16$	$f_{15,1}=1$	$m_{15,1}=16$	$g_{15,1}$	4	61689600	196608
		$f_{15,2}=3$	$m_{15,2}=48$	$g_{15,2}$	4	185068800	65536
		$f_{15,3}=4$	$m_{15,2}=64$	$g_{15,2}$	4	246758400	49152
		$f_{15,4}=8$	$m_{15,2}=128$	$g_{15,2}$	4	493516800	24576
$g_{16}=4E$	$k_{16}=8$	$f_{16,1}=1$	$m_{16,1}=32$	$g_{16,1}$	8	246758400	49152
		$f_{16,2}=3$	$m_{16,2}=96$	$g_{16,2}$	8	740275200	16384
		$f_{16,3}=4$	$m_{16,3}=128$	$g_{16,3}$	4	987033600	12288
$g_{17}=4F$	$k_{17}=8$	$f_{17,1}=1$	$m_{17,1}=32$	$g_{17,1}$	8	246758400	49152
		$f_{17,2}=3$	$m_{17,2}=96$	$g_{17,2}$	8	740275200	16384
		$f_{17,3}=4$	$m_{17,3}=128$	$g_{17,3}$	4	987033600	12288
$g_{18}=4G$	$k_{18}=8$	$f_{18,1}=1$	$m_{18,1}=32$	$g_{18,1}$	4	493516800	24576
		$f_{18,2}=1$	$m_{18,2}=32$	$g_{18,2}$	4	493516800	24576
		$f_{18,3}=3$	$m_{18,3}=96$	$g_{18,3}$	4	1480550400	8192
		$f_{18,4}=3$	$m_{18,4}=96$	$g_{18,4}$	4	1480550400	8192
$g_{19}=4H$	$k_{19}=16$	$f_{19,1}=1$	$m_{19,1}=16$	$g_{19,1}$	8	246758400	49152
		$f_{19,2}=3$	$m_{19,2}=48$	$g_{19,2}$	8	740275200	16384
		$f_{19,3}=4$	$m_{19,3}=64$	$g_{19,3}$	8	987033600	12288
		$f_{19,4}=4$	$m_{19,4}=64$	$g_{19,4}$	4	987033600	12288
		$f_{19,5}=4$	$m_{19,5}=64$	$g_{19,5}$	4	987033600	12288

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Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
		$f_{20,1}=1$	$m_{20,1}=32$	$g_{20,1}$	8	1480550400	8192
		$f_{20,2}=1$	$m_{20,2}=32$	$g_{20,2}$	8	1480550400	8192
$g_{20}=4I$	$k_{20}=8$	$f_{20,3}=2$	$m_{20,3}=64$	$g_{20,3}$	8	2961100800	4096
		$f_{20,4}=2$	$m_{20,4}=64$	$g_{20,4}$	4	2961100800	4096
		$f_{20,5}=2$	$m_{20,5}=64$	$g_{20,5}$	4	2961100800	4096
$g_{21}=4J$	$k_{21}=4$	$f_{21,1}=1$	$m_{21,1}=64$	$g_{21,1}$	4	3948134400	3072
		$f_{21,2}=3$	$m_{21,2}=192$	$g_{21,2}$	8	11844403200	1024
$g_{22}=4K$	$k_{22}=4$	$f_{22,1}=1$	$m_{22,1}=64$	$g_{22,1}$	8	5922201600	2084
		$f_{22,2}=1$	$m_{22,2}=64$	$g_{22,2}$	4	5922201600	2084
		$f_{22,3}=2$	$m_{22,3}=128$	$g_{22,3}$	8	11844403200	1024
		$f_{23,1}=1$	$m_{23,1}=64$	$g_{23,1}$	8	5922201600	2084
$g_{23}=4L$	$k_{23}=4$	$f_{23,2}=1$	$m_{23,2}=64$	$g_{23,2}$	4	5922201600	2084
		$f_{23,3}=2$	$m_{23,3}=128$	$g_{23,3}$	8	5922201600	2084
$g_{24}=5A$	$k_{24}=16$	$f_{24,1}=1$	$m_{24,1}=16$	$g_{24,1}$	5	210567168	57600
		$f_{24,2}=15$	$m_{24,2}=240$	$g_{24,2}$	10	3158507520	3840
$g_{25}=5B$	$k_{25}=1$	$f_{25,1}=1$	$m_{25,1}=256$	$g_{25,1}$	5	40428896256	300
		$f_{26,1}=1$	$m_{26,1}=8$	$g_{26,1}$	12	5483520	2211840
$g_{26}=6A$	$k_{26}=32$	$f_{26,2}=15$	$m_{26,2}=120$	$g_{26,2}$	12	82252800	147456
		$f_{26,3}=16$	$m_{26,3}=128$	$g_{26,3}$	6	87736320	138240
		$f_{27,1}=1$	$m_{27,1}=16$	$g_{27,1}$	6	54835200	221184
$g_{27}=6B$	$k_{27}=16$	$f_{27,2}=3$	$m_{27,2}=48$	$g_{27,2}$	6	164505600	73728
		$f_{27,3}=12$	$m_{27,3}=192$	$g_{27,3}$	12	658022400	18432
		$f_{28,1}=1$	$m_{28,1}=16$	$g_{28,1}$	12	164505600	73728
$g_{28}=6C$	$k_{28}=16$	$f_{28,2}=3$	$m_{28,2}=48$	$g_{28,2}$	12	493516800	24576
		$f_{27,3}=4$	$m_{28,3}=64$	$g_{28,3}$	6	658022400	18432
		$f_{28,4}=8$	$m_{28,4}=128$	$g_{28,4}$	12	1316044800	9216
$g_{29}=6D$	$k_{29}=16$	$f_{29,1}=1$	$m_{29,1}=16$	$g_{29,1}$	6	175472640	69120
		$f_{29,2}=15$	$m_{29,2}=240$	$g_{29,2}$	6	2632089600	4608
$g_{30}=6E$	$k_{30}=1$	$f_{30,1}=1$	$m_{30,1}=256$	$g_{30,1}$	6	7018905600	1728

Continued on next page

Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
$g_{31}=6F$	$k_{31}=2$	$f_{31,1}=1$	$m_{31,1}=128$	$g_{31,1}$	6	4679270400	2592
		$f_{31,2}=1$	$m_{31,2}=128$	$g_{31,2}$	12	4679270400	2592
		$f_{32,1}=1$	$m_{32,1}=32$	$g_{32,1}$	12	1316044800	9216
$g_{32}=6G$	$k_{32}=8$	$f_{32,2}=1$	$m_{32,2}=32$	$g_{32,2}$	6	1316044800	9216
		$f_{32,3}=3$	$m_{32,3}=96$	$g_{32,3}$	12	3948134400	3072
		$f_{32,4}=3$	$m_{32,4}=96$	$g_{32,4}$	12	3948134400	3072
		$f_{33,1}=1$	$m_{33,1}=32$	$g_{33,1}$	12	1754726400	6912
$g_{33}=6H$	$k_{33}=8$	$f_{33,2}=3$	$m_{33,2}=96$	$g_{33,2}$	12	5264179200	2304
		$f_{33,3}=4$	$m_{33,3}=128$	$g_{33,3}$	6	7018905600	1728
$g_{34}=6I$	$k_{34}=4$	$f_{34,1}=1$	$m_{34,1}=64$	$g_{34,1}$	6	3509452800	3456
		$f_{34,2}=3$	$m_{34,2}=192$	$g_{34,2}$	12	10528358400	1152
$g_{35}=6J$	$k_{35}=4$	$f_{35,1}=1$	$m_{35,1}=64$	$g_{35,1}$	6	7018905600	1728
		$f_{35,2}=3$	$m_{35,2}=192$	$g_{35,2}$	6	21056716800	576
$g_{36}=6K$	$k_{36}=1$	$f_{36,1}=1$	$m_{36,1}=256$	$g_{36,1}$	6	42113433600	288
		$f_{37,1}=1$	$m_{37,1}=64$	$g_{37,1}$	12	10528358400	1152
$g_{37}=6L$	$k_{37}=4$	$f_{37,2}=1$	$m_{37,2}=64$	$g_{37,2}$	6	10528358400	1152
		$f_{37,3}=2$	$m_{37,3}=128$	$g_{37,3}$	12	21056716800	576
$g_{38}=6M$	$k_{38}=4$	$f_{38,1}=1$	$m_{38,1}=64$	$g_{38,1}$	6	10528358400	1152
		$f_{38,2}=3$	$m_{38,2}=192$	$g_{38,2}$	12	31585075200	384
		$f_{39,1}=1$	$m_{39,1}=32$	$g_{39,1}$	12	5264179200	2304
$g_{39}=6N$	$k_{39}=8$	$f_{39,2}=3$	$m_{39,2}=96$	$g_{39,2}$	12	15792537600	768
		$f_{39,3}=4$	$m_{39,3}=128$	$g_{39,3}$	6	21056716800	576
$g_{40}=6O$	$k_{40}=2$	$f_{40,1}=1$	$m_{40,1}=128$	$g_{40,1}$	12	42113433600	288
		$f_{40,2}=1$	$m_{40,2}=128$	$g_{40,2}$	6	42113433600	288
		$f_{41,1}=1$	$m_{41,1}=64$	$g_{41,1}$	12	31585075200	384
$g_{41}=6P$	$k_{41}=4$	$f_{41,2}=1$	$m_{41,2}=64$	$g_{41,2}$	6	31585075200	384
		$f_{41,3}=2$	$m_{41,3}=128$	$g_{41,3}$	12	63170150400	192

Continued on next page

Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
$g_{42}=7A$	$k_{42}=4$	$f_{42,1}=1$	$m_{42,1}=64$	$g_{42,1}$	7	72194457600	168
		$f_{42,2}=3$	$m_{42,2}=192$	$g_{42,2}$	14	216583372800	56
		$f_{43,1}=1$	$m_{43,1}=32$	$g_{43,1}$	8	3948134400	3072
$g_{43}=8A$	$k_{43}=8$	$f_{43,2}=3$	$m_{43,2}=96$	$g_{43,2}$	8	11844403200	1024
		$f_{43,3}=4$	$m_{43,3}=128$	$g_{43,3}$	8	15792537600	768
		$f_{44,1}=1$	$m_{44,1}=32$	$g_{44,1}$	8	3948134400	3072
$g_{44}=8B$	$k_{44}=8$	$f_{44,2}=3$	$m_{44,2}=96$	$g_{44,2}$	8	11844403200	1024
		$f_{44,3}=4$	$m_{44,3}=128$	$g_{44,3}$	8	15792537600	768
		$f_{45,1}=1$	$m_{45,1}=64$	$g_{45,1}$	8	23688806400	512
$g_{45}=8C$	$k_{45}=4$	$f_{45,2}=1$	$m_{45,2}=64$	$g_{45,2}$	8	23688806400	512
		$f_{45,3}=2$	$m_{45,3}=128$	$g_{45,3}$	8	47377612800	256
		$f_{46,1}=1$	$m_{46,1}=64$	$g_{46,1}$	8	23688806400	512
$g_{46}=8D$	$k_{46}=4$	$f_{46,2}=1$	$m_{46,2}=64$	$g_{46,2}$	8	23688806400	512
		$f_{46,3}=2$	$m_{46,3}=128$	$g_{46,3}$	8	47377612800	256
$g_{47}=8E$	$k_{47}=2$	$f_{47,1}=1$	$m_{47,1}=128$	$g_{47,1}$	16	189510451200	64
		$f_{47,2}=1$	$m_{47,2}=128$	$g_{47,2}$	8	189510451200	64
$g_{48}=8F$	$k_{48}=2$	$f_{48,1}=1$	$m_{48,1}=128$	$g_{48,1}$	16	189510451200	64
		$f_{48,2}=1$	$m_{48,2}=128$	$g_{48,2}$	8	189510451200	64
$g_{49}=9A$	$k_{49}=4$	$f_{49,1}=1$	$m_{49,1}=64$	$g_{49,1}$	9	56151244800	216
		$f_{49,2}=3$	$m_{49,2}=192$	$g_{49,2}$	18	168453734400	72
$g_{50}=9B$	$k_{50}=1$	$f_{50,1}=1$	$m_{50,1}=256$	$g_{50,1}$	9	449209958400	27
		$f_{51,1}=1$	$m_{51,1}=32$	$g_{51,1}$	20	6317015040	1920
$g_{51}=10A$	$k_{51}=8$	$f_{51,2}=3$	$m_{51,2}=96$	$g_{51,2}$	20	18951045120	640
		$f_{51,3}=4$	$m_{51,3}=128$	$g_{51,3}$	10	25268060160	480
$g_{52}=10B$	$k_{52}=4$	$f_{52,1}=1$	$m_{52,1}=64$	$g_{52,1}$	10	12634030080	960
		$f_{52,2}=3$	$m_{52,2}=192$	$g_{52,2}$	20	37902090240	320

Continued on next page

Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
		$f_{53,1}=1$	$m_{53,1}=64$	$g_{53,1}$	20	37902090240	320
$g_{53}=10C$	$k_{53}=4$	$f_{53,2}=1$	$m_{53,2}=64$	$g_{53,2}$	10	37902090240	320
		$f_{53,3}=2$	$m_{53,3}=128$	$g_{53,3}$	20	75804180480	160
$g_{54}=10D$	$k_{54}=1$	$f_{54,1}=1$	$m_{54,1}=256$	$g_{54,1}$	10	606433443840	20
$g_{55}=12A$	$k_{55}=4$	$f_{55,1}=1$	$m_{55,1}=64$	$g_{55,1}$	12	2632089600	4608
		$f_{55,2}=3$	$m_{55,2}=192$	$g_{55,2}$	12	7896268800	1536
		$f_{56,1}=1$	$m_{56,1}=32$	$g_{56,1}$	24	2632089600	4608
$g_{56}=12B$	$k_{56}=8$	$f_{56,2}=3$	$m_{56,2}=96$	$g_{56,2}$	24	7896268800	1536
		$f_{56,3}=4$	$m_{56,3}=128$	$g_{56,3}$	12	10528358400	1152
		$f_{57,1}=1$	$m_{57,1}=32$	$g_{57,1}$	24	2632089600	4608
$g_{57}=12C$	$k_{57}=8$	$f_{57,2}=3$	$m_{57,2}=96$	$g_{57,2}$	24	7896268800	1536
		$f_{57,3}=4$	$m_{57,3}=128$	$g_{57,3}$	12	10528358400	1152
		$f_{58,1}=1$	$m_{58,1}=64$	$g_{58,1}$	12	7896268800	1536
$g_{58}=12D$	$k_{58}=4$	$f_{58,2}=1$	$m_{58,2}=64$	$g_{58,2}$	12	7896268800	1536
		$f_{58,3}=2$	$m_{58,3}=128$	$g_{58,3}$	12	15792537600	768
$g_{59}=12E$	$k_{59}=1$	$f_{59,1}=1$	$m_{59,1}=256$	$g_{59,1}$	12	84226867200	144
$g_{60}=12F$	$k_{60}=2$	$f_{60,1}=1$	$m_{60,1}=128$	$g_{60,1}$	24	42113433600	288
		$f_{60,2}=1$	$m_{60,2}=128$	$g_{60,2}$	12	42113433600	288
$g_{61}=12G$	$k_{61}=2$	$f_{61,1}=1$	$m_{61,1}=128$	$g_{61,1}$	24	42113433600	288
		$f_{61,2}=1$	$m_{61,2}=128$	$g_{61,2}$	12	42113433600	288
		$f_{62,1}=1$	$m_{62,1}=64$	$g_{62,1}$	24	31585075200	384
$g_{62}=12H$	$k_{62}=4$	$f_{62,2}=1$	$m_{62,2}=64$	$g_{62,2}$	24	31585075200	384
		$f_{62,3}=1$	$m_{62,3}=64$	$g_{62,3}$	12	31585075200	384
		$f_{62,4}=1$	$m_{62,4}=64$	$g_{62,4}$	12	31585075200	384
$g_{63}=12I$	$k_{63}=4$	$f_{63,1}=1$	$m_{63,1}=64$	$g_{63,1}$	12	42113433600	288
		$f_{63,2}=3$	$m_{63,2}=192$	$g_{63,2}$	12	126340300800	96
$g_{64}=12J$	$k_{64}=2$	$f_{64,1}=1$	$m_{64,1}=128$	$g_{64,1}$	24	126340300800	96
		$f_{64,2}=1$	$m_{64,2}=128$	$g_{64,2}$	12	126340300800	96

Continued on next page

Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
$g_{65}=12K$	$k_{65}=2$	$f_{65,1}=1$	$m_{65,1}=128$	$g_{65,1}$	24	126340300800	96
		$f_{65,2}=1$	$m_{65,2}=128$	$g_{65,2}$	12	126340300800	96
$g_{66}=12L$	$k_{66}=2$	$f_{66,1}=1$	$m_{66,1}=128$	$g_{66,1}$	12	252680601600	48
		$f_{66,2}=1$	$m_{66,2}=128$	$g_{66,2}$	12	252680601600	48
$g_{67}=12M$	$k_{67}=1$	$f_{67,1}=1$	$m_{67,1}=256$	$g_{67,1}$	12	505361203200	24
$g_{68}=14A$	$k_{68}=2$	$f_{68,1}=1$	$m_{68,1}=128$	$g_{68,1}$	28	433166745600	28
		$f_{68,2}=1$	$m_{68,2}=128$	$g_{68,2}$	14	433166745600	28
$g_{69}=15A$	$k_{69}=4$	$f_{69,1}=1$	$m_{69,1}=64$	$g_{69,1}$	15	33690746880	360
		$f_{69,2}=3$	$m_{69,2}=192$	$g_{69,2}$	30	101072240640	120
$g_{70}=15B$	$k_{70}=1$	$f_{70,1}=1$	$m_{70,1}=256$	$g_{70,1}$	15	134762987520	90
$g_{71}=15C$	$k_{71}=1$	$f_{71,1}=1$	$m_{71,1}=256$	$g_{71,1}$	15	808577925120	15
$g_{72}=17A$	$k_{72}=1$	$f_{72,1}=1$	$m_{72,1}=256$	$g_{72,1}$	17	713451110400	17
$g_{73}=17B$	$k_{73}=1$	$f_{73,1}=1$	$m_{73,1}=256$	$g_{73,1}$	17	713451110400	17
$g_{74}=18A$	$k_{74}=2$	$f_{74,1}=1$	$m_{74,1}=128$	$g_{74,1}$	36	336907468800	36
		$f_{74,2}=1$	$m_{74,2}=128$	$g_{74,2}$	18	336907468800	36
$g_{75}=20A$	$k_{75}=2$	$f_{75,1}=1$	$m_{75,1}=128$	$g_{75,1}$	40	151608360960	80
		$f_{75,2}=1$	$m_{75,2}=128$	$g_{75,2}$	20	151608360960	80
$g_{76}=20B$	$k_{76}=2$	$f_{76,1}=1$	$m_{76,1}=128$	$g_{76,1}$	40	151608360960	80
		$f_{76,2}=1$	$m_{76,2}=128$	$g_{76,2}$	20	151608360960	80
$g_{77}=21A$	$k_{77}=1$	$f_{77,1}=1$	$m_{77,1}=256$	$g_{77,1}$	21	577555660800	21
$g_{78}=24A$	$k_{78}=2$	$f_{78,1}=1$	$m_{78,1}=128$	$g_{78,1}$	24	126340300800	96
		$f_{78,2}=1$	$m_{78,2}=128$	$g_{78,2}$	24	126340300800	96
$g_{79}=24B$	$k_{79}=2$	$f_{79,1}=1$	$m_{79,1}=128$	$g_{79,1}$	24	126340300800	96
		$f_{79,2}=1$	$m_{79,2}=128$	$g_{79,2}$	24	126340300800	96

Continued on next page

Table 9.2 (continued)

$[g_i]_{Sp(8,2)}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{2^8\cdot Sp(8,2)}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^8\cdot Sp(8,2)}\rvert$	$\lvert C_{2^8\cdot Sp(8,2)}(g_{ij})\rvert$
$g_{80}=30A$	$k_{80}=2$	$f_{80,1}=1$	$m_{80,1}=128$	$g_{80,1}$	600	202144481280	60
		$f_{80,2}=1$	$m_{80,2}=128$	$g_{80,2}$	30	202144481280	60
$g_{81}=30B$	$k_{81}=1$	$f_{81,1}=1$	$m_{81,1}=256$	$g_{81,1}$	30	404288962560	30

Remark 9.4.1. Note that from Table 9.2, the group $\overline{G}=2^{8}{\cdot}Sp(8,2)$ contains 8 conjugacy classes of involutions, while from the character table of the split extension $2^8{:}Sp(8,2)$ (see GAP) there are 11 conjugacy classes of involutions. This confirms that the group $\overline{G}$ constructed using the generators $\overline{g}_1$ and $\overline{g}_2$ is different from the group $2^8{:}Sp(8,2)$ but later it will be shown that the character tables of the two groups are the same.

9.5. The Inertia Factor Groups of $\overline{G}=2^{8}{\cdot}Sp(8,2)$

From Table 9.1 we can see that $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(\overline{G}_4)| = |\mathrm{Irr}(2^{8}{\cdot}Sp(8,2))| = 195$, which coincides with the number of conjugacy classes of $\overline{G}$, listed in Table 9.2. Moreover these 195 characters of $\overline{G}$ are partitioned into two blocks of characters, where the first block consists of 81 characters correspond to $H_1 = G_4 = Sp(8,2)$, while the other block consists of 114 characters correspond to $H_2 = 2^7{:}Sp(6,2)$. The character table of $H_1 = Sp(8,2)$ is available in the $\mathbb{ATLAS}$, the electronic ATLAS of Wilson or can be obtained from Magma or GAP. We will use the character table of H_2 given in Table 6.7 of Ali [1] to construct the character table of $\overline{G}$. The fusion of the conjugacy classes of $H_2 = 2^7{:}Sp(6,2)$ into classes of $Sp(8,2)$ can be found in Table 6.9 of Ali [1] or can be obtain through Magma or GAP. For the convenience we supply the fusion of classes of H_2 in Table 9.3, which is taken from Ali [1] (Table 6.9), only that Ali was "capitalizing" the symbols for the conjugacy classes of H_2 while we use the small letter case for the classes in Table 9.3.

Table 9.3: The fusion of conjugacy classes of $H_2 = 2^7{:}Sp(6,2)$ into classes of $Sp(8,2)$

Class of $2^7{:}Sp(6,2)$	$\hookrightarrow$	Class of $Sp(8,2)$	Class of $2^7{:}Sp(6,2)$	$\hookrightarrow$	Class of $Sp(8,2)$	Class of $2^7{:}Sp(6,2)$	$\hookrightarrow$	Class of $Sp(8,2)$
$1a = g_{121}$		$1A$	$4t = g_{20,23}$		$4I$	$8c = g_{45,21}$		$8C$
$2a = g_{221}$		$2A$	$4u = g_{17,22}$		$4F$	$8d = g_{46,21}$		$8D$
$2b = g_{321}$		$2B$	$4v = g_{15,23}$		$4D$	$8e = g_{46,22}$		$8D$
$2c = g_{421}$		$2C$	$4w = g_{18,23}$		$4G$	$8f = g_{44,22}$		$8B$
$2d = g_{422}$		$2C$	$4x = g_{23,21}$		$4L$	$8g = g_{48,21}$		$8F$
$2e = g_{222}$		$2A$	$4y = g_{21,21}$		$4J$	$8h = g_{45,22}$		$8C$
$2f = g_{621}$		$2E$	$4z = g_{19,24}$		$4H$	$8i = g_{43,22}$		$8A$
$2g = g_{322}$		$2B$	$4aa = g_{20,24}$		$4I$	$8j = g_{47,21}$		$8E$
$2h = g_{622}$		$2E$	$4ab = g_{22,22}$		$4K$	$9a = g_{49,21}$		$9A$
$2i = g_{721}$		$2F$	$4ac = g_{23,22}$		$4L$	$10a = g_{51,21}$		$10A$
$2j = g_{521}$		$2D$	$5a = g_{24,21}$		$5A$	$10b = g_{53,21}$		$10C$
$2k = g_{423}$		$2C$	$6a = g_{26,21}$		$6A$	$10c = g_{52,21}$		$10B$
$2l = g_{623}$		$2E$	$6b = g_{27,21}$		$6B$	$10d = g_{53,22}$		$10C$
$2m = g_{722}$		$2F$	$6c = g_{28,21}$		$6C$	$10e = g_{51,22}$		$10A$
$2n = g_{723}$		$2F$	$6d = g_{31,21}$		$6F$	$12a = g_{56,21}$		$12B$
$2o = g_{624}$		$2E$	$6e = g_{33,21}$		$6H$	$12b = g_{57,21}$		$12C$

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Table 9.3 (continued)

Class of $2^7{:}Sp(6,2)$	$\hookrightarrow$	Class of $Sp(8,2)$	Class of $2^7{:}Sp(6,2)$	$\hookrightarrow$	Class of $Sp(8,2)$	Class of $2^7{:}Sp(6,2)$	$\hookrightarrow$	Class of $Sp(8,2)$
$3a = g_{821}$		$3A$	$6f = g_{33,21}$		$6I$	$12c = g_{58,21}$		$12D$
$3b = g_{11,21}$		$3D$	$6g = g_{37,21}$		$6L$	$12d = g_{55,21}$		$12A$
$3c = g_{10,21}$		$3C$	$6h = g_{28,22}$		$6C$	$12e = g_{58,22}$		$12D$
$4a = g_{13,21}$		$4B$	$6i = g_{26,22}$		$6A$	$12f = g_{62,21}$		$12H$
$4b = g_{12,21}$		$4A$	$6j = g_{32,21}$		$6G$	$12g = g_{61,21}$		$12G$
$4c = g_{15,21}$		$4D$	$6k = g_{27,22}$		$6B$	$12h = g_{60,21}$		$12F$
$4d = g_{14,21}$		$4C$	$6l = g_{32,22}$		$6G$	$12i = g_{64,21}$		$12J$
$4e = g_{15,22}$		$4D$	$6m = g_{34,21}$		$6J$	$12j = g_{65,21}$		$12K$
$4f = g_{19,21}$		$4H$	$6n = g_{40,21}$		$6O$	$12k = g_{62,22}$		$12H$
$4g = g_{16,21}$		$4E$	$6o = g_{28,23}$		$6C$	$12l = g_{57,22}$		$12C$
$4h = g_{17,21}$		$4F$	$6p = g_{32,23}$		$6G$	$12m = g_{62,23}$		$12H$
$4i = g_{18,21}$		$4G$	$6q = g_{29,21}$		$6D$	$12n = g_{56,22}$		$12B$
$4j = g_{20,21}$		$4I$	$6r = g_{39,21}$		$6N$	$12o = g_{63,21}$		$12I$
$4k = g_{14,22}$		$4C$	$6s = g_{38,21}$		$6M$	$12p = g_{66,21}$		$12L$
$4l = g_{18,22}$		$4G$	$6t = g_{41,21}$		$6P$	$14a = g_{68,21}$		$14A$
$4m = g_{22,21}$		$4K$	$6u = g_{37,22}$		$6L$	$15a = g_{69,21}$		$15A$
$4n = g_{19,22}$		$4H$	$6v = g_{33,22}$		$6H$	$18a = g_{74,21}$		$18A$
$4o = g_{13,22}$		$4B$	$6w = g_{41,22}$		$6P$	$20a = g_{75,21}$		$20A$
$4p = g_{16,22}$		$4E$	$6x = g_{39,22}$		$6N$	$20b = g_{76,21}$		$20B$
$4q = g_{20,22}$		$4I$	$7a = g_{42,21}$		$7A$	$24a = g_{78,21}$		$24A$
$4r = g_{19,23}$		$4H$	$8a = g_{43,21}$		$8A$	$24b = g_{79,21}$		$24B$
$4s = g_{12,22}$		$4A$	$8b = g_{44,21}$		$8B$	$30a = g_{80,21}$		$30A$

9.6. Fischer Matrices of $\overline{G} = 2^{8}\cdot Sp(8,2)$

In this section, we use the arithmetical properties of Fischer matrices, given in Proposition 5.2.2, to calculate some of the entries of the Fischer matrices and also to build an algebraic system of equations. To build these systems of equations, we firstly recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of $g_{ij},\ 1 \leq j \leq c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 9.2 we supplied $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \leq i \leq 81,\ 1 \leq j \leq c(g_i)$. Also having obtained the fusions of the inertia factor group H_2 into $Sp(8,2)$, we are able to label the rows of the Fischer matrices as described in Section 5.3. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 9.2 that the sizes of the Fischer matrices of $\overline{G}$ range between 1 and 5 for every $i \in \{1, 2, \cdots, 81\}$.

Now with the help of the symbolic mathematical package Maxima [45], we were able to solve the systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}
$o(g_{1j})$		1	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		12128668876800	47563407360
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$		
$(1,1)$	47377612800	1	1
$(2,1)$	185794560	255	-1
m_{1j}		1	255

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}
$o(g_{2j})$		4	4	2
$\lvert C_{\overline{G}}(g_{2j})\rvert$		23781703680	377487360	371589120
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$			
$(1,1)$	185794560	1	1	1
$(2,1)$	185794560	1	1	-1
$(2,2)$	1474560	126	-2	0
m_{2j}		2	126	128

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}
$o(g_{3j})$		2	2	4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		566231040	37748736	11796480
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$			
$(1,1)$	8847360	1	1	1
$(2,1)$	2949120	3	3	-1
$(2,2)$	147456	60	-4	0
m_{3j}		4	60	192

$\mathcal{F}_4$

g_4		g_{41}	g_{42}	g_{43}	g_{44}
$o(g_{4j})$		4	4	2	4
$\lvert C_{\overline{G}}(g_{4j})\rvert$		188743680	12582912	11796480	5898240
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$				
$(1,1)$	2949120	1	1	1	1
$(2,1)$	2949120	1	1	1	-1
$(2,2)$	1474560	2	2	-2	0
$(2,3)$	49152	60	-4	0	0
m_{4j}		4	60	64	128

$\mathcal{F}_5$

g_5		g_{51}	g_{52}
$o(g_{5j})$		2	4
$\lvert C_{\overline{G}}(g_{5j})\rvert$		11796480	786432
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$		
$(1,1)$	737280	1	1
$(2,1)$	49152	15	-1
m_{5j}		16	240

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}
$o(g_{6j})$		4	4	2	4	4
$\lvert C_{\overline{G}}(g_{6j})\rvert$		4718592	1572864	1179648	393216	393216
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$					
$(1,1)$	147456	1	1	1	1	1
$(2,1)$	49152	3	3	-3	1	-1
$(2,2)$	147456	1	1	-1	-1	1
$(2,3)$	49152	3	3	3	-1	-1
$(2,4)$	6144	24	-8	0	0	0
m_{6j}		8	24	32	96	96

$\mathcal{F}_7$

g_7		g_{71}	g_{72}	g_{73}	g_{74}
$o(g_{7j})$		4	2	4	4
$\lvert C_{\overline{G}}(g_{7j})\rvert$		786432	786432	131072	98304
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$				
$(1,1)$	49152	1	1	1	1
$(2,1)$	49152	1	1	1	-1
$(2,2)$	8192	6	6	-2	0
$(2,3)$	6144	8	-8	0	0
m_{7j}		16	16	96	128

$\mathcal{F}_8$

g_8		g_{81}	g_{82}
$o(g_{8j})$		3	6
$\lvert C_{\overline{G}}(g_{8j})\rvert$		278691840	4423680
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$		
$(1,1)$	4354560	1	1
$(2,1)$	69120	63	-1
m_{8j}		4	252

$\mathcal{F}_9$

g_9		g_{91}
$o(g_{9j})$		3
$\lvert C_{\overline{G}}(g_{9j})\rvert$		77760
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$	
$(1,1)$	77760	1
m_{9j}		256

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$
$o(g_{10j})$		3	6
$\lvert C_{\overline{G}}(g_{10j})\rvert$		207360	13824
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$		
$(1,1)$	12960	1	1
$(2,1)$	864	15	-1
m_{10j}		16	240

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$
$o(g_{11j})$		3	6
$\lvert C_{\overline{G}}(g_{11j})\rvert$		15552	5184
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$		
$(1,1)$	3888	1	1
$(2,1)$	1296	3	-1
m_{11j}		64	192

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$
$o(g_{12j})$		8	8	4
$\lvert C_{\overline{G}}(g_{12j})\rvert$		2949120	196608	184320
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$			
$(1,1)$	92160	1	1	1
$(2,1)$	92160	1	1	-1
$(2,2)$	3072	30	-2	0
m_{12j}		8	120	128

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$	$g_{13,3}$
$o(g_{13j})$		8	8	4
$\lvert C_{\overline{G}}(g_{13j})\rvert$		2949120	196608	184320
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$			
$(1,1)$	92160	1	1	1
$(2,1)$	92160	1	1	-1
$(2,2)$	3072	30	-2	0
m_{13j}		8	120	128

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$	$g_{14,2}$	$g_{14,3}$
$o(g_{14j})$		4	4	4
$\lvert C_{\overline{G}}(g_{14j})\rvert$		589824	196608	49152
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$			
$(1,1)$	36864	1	1	1
$(2,1)$	12288	3	3	-1
$(2,2)$	3072	12	-4	0
m_{14j}		16	48	192

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$	$g_{15,3}$	$g_{15,4}$
$o(g_{15j})$		4	4	4	4
$\lvert C_{\overline{G}}(g_{15j})\rvert$		196608	65536	49152	24576
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$				
$(1,1)$	12288	1	1	1	1
$(2,1)$	6144	2	2	-2	0
$(2,2)$	12288	1	1	1	-1
$(2,3)$	1024	12	-4	0	0
m_{15j}		16	48	64	128

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$	$g_{16,3}$
$o(g_{16j})$		8	8	4
$\lvert C_{\overline{G}}(g_{16j})\rvert$		49152	16384	12288
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$			
$(1,1)$	6144	1	1	1
$(2,1)$	6144	1	1	-1
$(2,2)$	1024	6	-2	0
m_{16j}		32	96	128

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$	$g_{17,3}$
$o(g_{17j})$		8	8	4
$\lvert C_{\overline{G}}(g_{17j})\rvert$		49152	16384	12288
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$			
$(1,1)$	6144	1	1	1
$(2,1)$	6144	1	1	-1
$(2,2)$	1024	6	-2	0
m_{17j}		32	96	128

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$	$g_{18,3}$	$g_{18,4}$
$o(g_{18j})$		4	4	4	4
$\lvert C_{\overline{G}}(g_{18j})\rvert$		24576	24576	8192	8192
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$				
$(1,1)$	3072	1	1	1	1
$(2,1)$	1024	3	3	-1	1
$(2,2)$	3072	1	-1	-1	-1
$(2,3)$	1024	3	-3	1	-1
m_{18j}		32	32	96	96

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$	$g_{19,3}$	$g_{19,4}$	$g_{19,5}$
$o(g_{19,j})$		8	8	8	4	4
$\lvert C_{\overline{G}}(g_{19j})\rvert$		49152	16384	12288	12288	12288
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$					
$(1,1)$	3072	1	1	1	1	1
$(2,1)$	3072	1	1	-1	1	-1
$(2,2)$	3072	1	1	-1	-1	1
$(2,3)$	3072	1	1	1	-1	-1
$(2,4)$	256	12	-4	0	0	0
m_{19j}		16	48	64	64	64

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$	$g_{20,2}$	$g_{20,3}$	$g_{20,4}$	$g_{20,5}$
$o(g_{20,j})$		8	8	8	4	4
$\lvert C_{\overline{G}}(g_{20j})\rvert$		8192	8192	4096	4096	4096
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$					
$(1,1)$	1024	1	1	1	1	1
$(2,1)$	1024	1	1	-1	1	-1
$(2,2)$	1024	1	1	-1	-1	1
$(2,3)$	1024	1	1	1	-1	-1
$(2,4)$	256	4	-4	0	0	0
m_{20j}		32	32	64	64	64

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$	$g_{21,2}$
$o(g_{21j})$		4	8
$\lvert C_{\overline{G}}(g_{21j})\rvert$		3072	1024
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$		
$(1,1)$	768	1	1
$(2,1)$	256	3	-1
m_{21j}		64	192

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$	$g_{22,2}$	$g_{22,3}$
$o(g_{22j})$		8	4	8
$\lvert C_{\overline{G}}(g_{22j})\rvert$		2048	2048	1024
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$			
$(1,1)$	512	1	1	1
$(2,1)$	512	1	1	-1
$(2,2)$	256	2	-2	0
m_{22j}		64	64	128

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$	$g_{23,3}$
$o(g_{23j})$		8	4	8
$\lvert C_{\overline{G}}(g_{23j})\rvert$		2048	2048	1024
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$			
$(1,1)$	512	1	1	1
$(2,1)$	512	1	1	-1
$(2,1)$	256	2	-2	0
m_{23j}		64	64	128

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$	$g_{24,2}$
$o(g_{24j})$		5	10
$\lvert C_{\overline{G}}(g_{24j})\rvert$		57600	3840
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$		
$(1,1)$	3600	1	1
$(2,1)$	240	15	-1
m_{24j}		16	240

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$
$o(g_{25j})$		5
$\lvert C_{\overline{G}}(g_{25j})\rvert$		300
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$	
$(1,1)$	300	1
m_{25j}		256

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$	$g_{26,2}$	$g_{26,3}$
$o(g_{26j})$		12	12	6
$\lvert C_{\overline{G}}(g_{26j})\rvert$		2211840	147456	138240
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$			
$(1,1)$	69120	1	1	1
$(2,1)$	69120	1	1	-1
$(2,2)$	2304	30	-2	0
m_{26j}		8	120	128

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$	$g_{27,2}$	$g_{27,3}$
$o(g_{27j})$		6	6	12
$\lvert C_{\overline{G}}(g_{27j})\rvert$		221184	73728	18432
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$			
$(1,1)$	13824	1	1	1
$(2,1)$	4608	3	3	-1
$(2,2)$	1152	12	-4	0
m_{27j}		16	48	192

$\mathcal{F}_{28}$

g_{28}		$g_{28,1}$	$g_{28,2}$	$g_{28,3}$	$g_{28,4}$
$o(g_{28j})$		12	12	6	12
$\lvert C_{\overline{G}}(g_{28j})\rvert$		73728	24576	18432	9216
(k,m)	$\lvert C_{H_k}(g_{28km})\rvert$				
$(1,1)$	4608	1	1	1	1
$(2,1)$	4608	1	1	1	-1
$(2,2)$	2304	2	2	-2	0
$(2,3)$	384	12	-4	0	0
m_{28j}		16	48	64	128

$\mathcal{F}_{29}$

g_{29}		$g_{29,1}$	$g_{29,2}$
$o(g_{29j})$		6	6
$\lvert C_{\overline{G}}(g_{29j})\rvert$		69120	4608
(k,m)	$\lvert C_{H_k}(g_{29km})\rvert$		
$(1,1)$	4320	1	1
$(2,1)$	288	15	-1
m_{29j}		16	240

$\mathcal{F}_{30}$

g_{30}		$g_{30,1}$
$o(g_{30j})$		6
$\lvert C_{\overline{G}}(g_{30j})\rvert$		1728
(k,m)	$\lvert C_{H_k}(g_{30km})\rvert$	
$(1,1)$	1728	1
m_{30j}		256

$\mathcal{F}_{31}$

g_{31}		$g_{31,1}$	$g_{31,2}$
$o(g_{31j})$		6	12
$\lvert C_{\overline{G}}(g_{31j})\rvert$		2592	2592
(k,m)	$\lvert C_{H_k}(g_{31km})\rvert$		
$(1,1)$	1296	1	1
$(2,1)$	1296	1	-1
m_{31j}		128	128

$\mathcal{F}_{32}$

g_{32}		$g_{32,1}$	$g_{32,2}$	$g_{32,3}$	$g_{32,4}$
$o(g_{32j})$		12	6	12	12
$\lvert C_{\overline{G}}(g_{32j})\rvert$		9216	9216	3072	3072
(k,m)	$\lvert C_{H_k}(g_{32km})\rvert$				
$(1,1)$	1152	1	1	1	1
$(2,1)$	384	3	3	-1	1
$(2,2)$	1152	1	-1	-1	-1
$(2,3)$	384	3	-3	1	-1
m_{32j}		32	32	96	96

$\mathcal{F}_{33}$

g_{33}		$g_{33,1}$	$g_{33,2}$	$g_{33,3}$
$o(g_{33j})$		12	12	6
$\lvert C_{\overline{G}}(g_{33j})\rvert$		6912	2304	1728
(k,m)	$\lvert C_{H_k}(g_{33km})\rvert$			
$(1,1)$	864	1	1	1
$(2,1)$	864	1	1	-1
$(2,2)$	144	6	-2	0
m_{33j}		32	96	128

$\mathcal{F}_{34}$

g_{34}		$g_{34,1}$	$g_{34,2}$
$o(g_{34j})$		6	12
$\lvert C_{\overline{G}}(g_{34j})\rvert$		3456	1152
(k,m)	$\lvert C_{H_k}(g_{34km})\rvert$		
$(1,1)$	864	1	1
$(2,1)$	288	3	-1
m_{34j}		64	192

$\mathcal{F}_{35}$

g_{35}		$g_{35,1}$	$g_{35,2}$
$o(g_{35j})$		6	6
$\lvert C_{\overline{G}}(g_{35j})\rvert$		1728	576
(k,m)	$\lvert C_{H_k}(g_{35km})\rvert$		
$(1,1)$	432	1	1
$(2,1)$	144	3	-1
m_{35j}		64	192

$\mathcal{F}_{36}$

g_{36}		$g_{36,1}$
$o(g_{36j})$		6
$\lvert C_{\overline{G}}(g_{36j})\rvert$		288
(k,m)	$\lvert C_{H_k}(g_{36km})\rvert$	
$(1,1)$	288	1
m_{36j}		256

$\mathcal{F}_{37}$

g_{37}		$g_{37,1}$	$g_{37,2}$	$g_{37,3}$
$o(g_{37j})$		12	6	12
$\lvert C_{\overline{G}}(g_{37j})\rvert$		1152	1152	576
(k,m)	$\lvert C_{H_k}(g_{37km})\rvert$			
$(1,1)$	288	1	1	1
$(2,1)$	288	1	1	-1
$(2,2)$	144	2	-2	0
m_{37j}		64	64	128

$\mathcal{F}_{38}$

g_{38}		$g_{38,1}$	$g_{38,2}$
$o(g_{38j})$		6	12
$\lvert C_{\overline{G}}(g_{38j})\rvert$		1152	384
(k,m)	$\lvert C_{H_k}(g_{38km})\rvert$		
$(1,1)$	288	1	1
$(2,1)$	96	3	-1
m_{38j}		64	192

$\mathcal{F}_{39}$

g_{39}		$g_{39,1}$	$g_{39,2}$	$g_{39,3}$
$o(g_{39j})$		12	12	6
$\lvert C_{\overline{G}}(g_{39j})\rvert$		2304	768	576
(k,m)	$\lvert C_{H_k}(g_{39km})\rvert$			
$(1,1)$	288	1	1	1
$(2,1)$	288	1	1	-1
$(2,2)$	48	6	-2	0
m_{39j}		32	96	128

$\mathcal{F}_{40}$

g_{40}		$g_{40,1}$	$g_{40,2}$
$o(g_{40j})$		12	6
$\lvert C_{\overline{G}}(g_{40j})\rvert$		288	288
(k,m)	$\lvert C_{H_k}(g_{40km})\rvert$		
$(1,1)$	144	1	1
$(2,1)$	144	1	-1
m_{40j}		128	128

$\mathcal{F}_{41}$

g_{41}		$g_{41,1}$	$g_{41,2}$	$g_{41,3}$
$o(g_{41j})$		12	6	12
$\lvert C_{\overline{G}}(g_{41j})\rvert$		384	384	192
(k,m)	$\lvert C_{H_k}(g_{41km})\rvert$			
$(1,1)$	96	1	1	1
$(2,1)$	96	1	1	-1
$(2,2)$	48	2	-2	0
m_{41j}		64	64	128

$\mathcal{F}_{42}$

g_{42}		$g_{42,1}$	$g_{42,2}$
$o(g_{42j})$		7	14
$\lvert C_{\overline{G}}(g_{42j})\rvert$		168	56
(k,m)	$\lvert C_{H_k}(g_{42km})\rvert$		
$(1,1)$	42	1	1
$(2,1)$	14	3	-1
m_{42j}		64	192

$\mathcal{F}_{43}$

g_{43}		$g_{43,1}$	$g_{43,2}$	$g_{43,3}$
$o(g_{43j})$		8	8	8
$\lvert C_{\overline{G}}(g_{43j})\rvert$		3072	1024	768
(k,m)	$\lvert C_{H_k}(g_{43km})\rvert$			
$(1,1)$	384	1	1	1
$(2,1)$	384	1	1	-1
$(2,2)$	64	6	-2	0
m_{43j}		32	96	128

$\mathcal{F}_{44}$

g_{44}		$g_{44,1}$	$g_{44,2}$	$g_{44,3}$
$o(g_{44j})$		8	8	8
$\lvert C_{\overline{G}}(g_{44j})\rvert$		3072	1024	768
(k,m)	$\lvert C_{H_k}(g_{44km})\rvert$			
$(1,1)$	384	1	1	1
$(2,1)$	384	1	1	-1
$(2,2)$	64	6	-2	0
m_{44j}		32	96	128

$\mathcal{F}_{45}$

g_{45}		$g_{45,1}$	$g_{45,2}$	$g_{45,3}$
$o(g_{45j})$		8	8	8
$\lvert C_{\overline{G}}(g_{45j})\rvert$		512	512	256
(k,m)	$\lvert C_{H_k}(g_{45km})\rvert$			
$(1,1)$	128	1	1	1
$(2,1)$	128	1	1	-1
$(2,2)$	64	2	-2	0
m_{45j}		64	64	128

$\mathcal{F}_{46}$

g_{46}		$g_{46,1}$	$g_{46,2}$	$g_{46,3}$
$o(g_{46j})$		8	8	8
$\lvert C_{\overline{G}}(g_{46j})\rvert$		512	512	256
(k,m)	$\lvert C_{H_k}(g_{46km})\rvert$			
$(1,1)$	128	1	1	1
$(2,1)$	128	1	1	-1
$(2,2)$	64	2	-2	0
m_{46j}		64	64	128

$\mathcal{F}_{47}$

g_{47}		$g_{47,1}$	$g_{47,2}$
$o(g_{47j})$		16	8
$\lvert C_{\overline{G}}(g_{47j})\rvert$		64	64
(k,m)	$\lvert C_{H_k}(g_{47km})\rvert$		
$(1,1)$	32	1	1
$(2,1)$	32	1	-1
m_{47j}		128	128

$\mathcal{F}_{48}$

g_{48}		$g_{48,1}$	$g_{48,2}$
$o(g_{48j})$		16	8
$\lvert C_{\overline{G}}(g_{48j})\rvert$		64	64
(k,m)	$\lvert C_{H_k}(g_{48km})\rvert$		
$(1,1)$	32	1	1
$(2,1)$	32	1	-1
m_{48j}		128	128

$\mathcal{F}_{49}$

g_{49}		$g_{49,1}$	$g_{49,2}$
$o(g_{49j})$		9	18
$\lvert C_{\overline{G}}(g_{49j})\rvert$		216	72
(k,m)	$\lvert C_{H_k}(g_{49km})\rvert$		
$(1,1)$	54	1	1
$(2,1)$	18	3	-1
m_{49j}		64	192

$\mathcal{F}_{50}$

g_{50}		$g_{50,1}$
$o(g_{50j})$		9
$\lvert C_{\overline{G}}(g_{50j})\rvert$		27
(k,m)	$\lvert C_{H_k}(g_{50km})\rvert$	
$(1,1)$	27	1
m_{50j}		256

$\mathcal{F}_{51}$

g_{51}		$g_{51,1}$	$g_{51,2}$	$g_{51,3}$
$o(g_{51j})$		20	20	10
$\lvert C_{\overline{G}}(g_{51j})\rvert$		1920	640	480
(k,m)	$\lvert C_{H_k}(g_{51km})\rvert$			
$(1,1)$	240	1	1	1
$(2,1)$	240	1	1	-1
$(2,2)$	40	6	-2	0
m_{51j}		32	96	128

$\mathcal{F}_{52}$

g_{52}		$g_{52,1}$	$g_{52,2}$
$o(g_{52j})$		10	20
$\lvert C_{\overline{G}}(g_{52j})\rvert$		960	320
(k,m)	$\lvert C_{H_k}(g_{52km})\rvert$		
$(1,1)$	240	1	1
$(2,1)$	80	3	-1
m_{52j}		64	192

$\mathcal{F}_{53}$

g_{53}		$g_{53,1}$	$g_{53,2}$	$g_{53,3}$
$o(g_{53j})$		20	10	20
$\lvert C_{\overline{G}}(g_{53j})\rvert$		320	320	160
(k,m)	$\lvert C_{H_k}(g_{53km})\rvert$			
$(1,1)$	80	1	1	1
$(2,1)$	80	1	1	-1
$(2,2)$	40	2	-2	0
m_{53j}		64	64	128

$\mathcal{F}_{54}$

g_{54}		$g_{54,1}$
$o(g_{54j})$		10
$\lvert C_{\overline{G}}(g_{54j})\rvert$		20
(k,m)	$\lvert C_{H_k}(g_{54km})\rvert$	
$(1,1)$	20	1
m_{54j}		256

$\mathcal{F}_{55}$

g_{55}		$g_{55,1}$	$g_{55,2}$
$o(g_{55j})$		12	12
$\lvert C_{\overline{G}}(g_{55j})\rvert$		4608	1536
(k,m)	$\lvert C_{H_k}(g_{55km})\rvert$		
$(1,1)$	1152	1	1
$(2,1)$	384	3	-1
m_{55j}		64	192

$\mathcal{F}_{56}$

g_{56}		$g_{56,1}$	$g_{56,2}$	$g_{56,3}$
$o(g_{56j})$		24	24	12
$\lvert C_{\overline{G}}(g_{56j})\rvert$		4608	1536	1152
(k,m)	$\lvert C_{H_k}(g_{56km})\rvert$			
$(1,1)$	576	1	1	1
$(2,1)$	576	1	1	-1
$(2,2)$	96	6	-2	0
m_{56j}		32	96	128

$\mathcal{F}_{57}$

g_{57}		$g_{57,1}$	$g_{57,2}$	$g_{57,3}$
$o(g_{57j})$		24	24	12
$\lvert C_{\overline{G}}(g_{57j})\rvert$		4608	1536	1152
(k,m)	$\lvert C_{H_k}(g_{57km})\rvert$			
$(1,1)$	576	1	1	1
$(2,1)$	576	1	1	-1
$(2,2)$	96	6	-2	0
m_{57j}		32	96	128

$\mathcal{F}_{58}$

g_{58}		$g_{58,1}$	$g_{58,2}$	$g_{58,3}$
$o(g_{58j})$		12	12	12
$\lvert C_{\overline{G}}(g_{58j})\rvert$		1536	1536	768
(k,m)	$\lvert C_{H_k}(g_{58km})\rvert$			
$(1,1)$	384	1	1	1
$(2,1)$	192	2	-2	0
$(2,2)$	384	1	1	-1
m_{58j}		64	64	128

$\mathcal{F}_{59}$

g_{59}		$g_{59,1}$
$o(g_{59j})$		12
$\lvert C_{\overline{G}}(g_{59j})\rvert$		144
(k,m)	$\lvert C_{H_k}(g_{59km})\rvert$	
$(1,1)$	144	1
m_{59j}		256

$\mathcal{F}_{60}$

g_{60}		$g_{60,1}$	$g_{60,2}$
$o(g_{60j})$		24	12
$\lvert C_{\overline{G}}(g_{60j})\rvert$		288	288
(k,m)	$\lvert C_{H_k}(g_{60km})\rvert$		
$(1,1)$	144	1	1
$(2,1)$	144	1	-1
m_{60j}		128	128

$\mathcal{F}_{61}$

g_{61}		$g_{61,1}$	$g_{61,2}$
$o(g_{61j})$		24	12
$\lvert C_{\overline{G}}(g_{61j})\rvert$		288	288
(k,m)	$\lvert C_{H_k}(g_{61km})\rvert$		
$(1,1)$	144	1	1
$(2,1)$	144	1	-1
m_{61j}		128	128

$\mathcal{F}_{62}$

g_{62}		$g_{62,1}$	$g_{62,2}$	$g_{62,3}$	$g_{62,4}$
$o(g_{62j})$		24	24	12	12
$\lvert C_{\overline{G}}(g_{62j})\rvert$		384	384	384	384
(k,m)	$\lvert C_{H_k}(g_{62km})\rvert$				
$(1,1)$	96	1	1	1	1
$(2,1)$	96	1	-1	1	-1
$(2,2)$	96	1	-1	-1	1
$(2,3)$	96	1	1	-1	-1
m_{62j}		64	64	64	64

$\mathcal{F}_{63}$

g_{63}		$g_{63,1}$	$g_{63,2}$
$o(g_{63j})$		12	12
$\lvert C_{\overline{G}}(g_{63j})\rvert$		288	96
(k,m)	$\lvert C_{H_k}(g_{63km})\rvert$		
$(1,1)$	72	1	1
$(2,1)$	24	3	-1
m_{63j}		64	192

$\mathcal{F}_{64}$

g_{64}		$g_{64,1}$	$g_{64,2}$
$o(g_{64j})$		24	12
$\lvert C_{\overline{G}}(g_{64j})\rvert$		96	96
(k,m)	$\lvert C_{H_k}(g_{64km})\rvert$		
$(1,1)$	48	1	1
$(2,1)$	48	1	-1
m_{64j}		128	128

$\mathcal{F}_{65}$

g_{65}		$g_{65,1}$	$g_{65,2}$
$o(g_{65j})$		24	12
$\lvert C_{\overline{G}}(g_{65j})\rvert$		96	96
(k,m)	$\lvert C_{H_k}(g_{65km})\rvert$		
$(1,1)$	48	1	1
$(2,1)$	48	1	-1
m_{65j}		128	128

$\mathcal{F}_{66}$

g_{66}		$g_{66,1}$	$g_{66,2}$
$o(g_{66j})$		12	12
$\lvert C_{\overline{G}}(g_{66j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{66km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{66j}		128	128

$\mathcal{F}_{67}$

g_{67}		$g_{67,1}$
$o(g_{67j})$		12
$\lvert C_{\overline{G}}(g_{67j})\rvert$		24
(k,m)	$\lvert C_{H_k}(g_{67km})\rvert$	
$(1,1)$	24	1
m_{67j}		256

$\mathcal{F}_{68}$

g_{68}		$g_{68,1}$	$g_{68,2}$
$o(g_{68j})$		28	14
$\lvert C_{\overline{G}}(g_{68j})\rvert$		28	28
(k,m)	$\lvert C_{H_k}(g_{68km})\rvert$		
$(1,1)$	14	1	1
$(2,1)$	14	1	-1
m_{68j}		128	128

$\mathcal{F}_{69}$

g_{69}		$g_{69,1}$	$g_{69,2}$
$o(g_{69j})$		15	30
$\lvert C_{\overline{G}}(g_{69j})\rvert$		360	120
(k,m)	$\lvert C_{H_k}(g_{69km})\rvert$		
$(1,1)$	90	1	1
$(2,1)$	30	3	-1
m_{69j}		64	192

$\mathcal{F}_{70}$

g_{70}		$g_{70,1}$
$o(g_{70j})$		15
$\lvert C_{\overline{G}}(g_{70j})\rvert$		90
(k,m)	$\lvert C_{H_k}(g_{70km})\rvert$	
$(1,1)$	90	1
m_{70j}		256

$\mathcal{F}_{71}$

g_{71}		$g_{71,1}$
$o(g_{71j})$		15
$\lvert C_{\overline{G}}(g_{71j})\rvert$		15
(k,m)	$\lvert C_{H_k}(g_{71km})\rvert$	
$(1,1)$	15	1
m_{71j}		256

$\mathcal{F}_{72}$

g_{72}		$g_{72,1}$
$o(g_{72j})$		17
$\lvert C_{\overline{G}}(g_{72j})\rvert$		17
(k,m)	$\lvert C_{H_k}(g_{72km})\rvert$	
$(1,1)$	17	1
m_{72j}		256

$\mathcal{F}_{73}$

g_{73}		$g_{73,1}$
$o(g_{73j})$		17
$\lvert C_{\overline{G}}(g_{73j})\rvert$		17
(k,m)	$\lvert C_{H_k}(g_{73km})\rvert$	
$(1,1)$	17	1
m_{73j}		256

$\mathcal{F}_{74}$

g_{74}		$g_{74,1}$	$g_{74,2}$
$o(g_{74j})$		36	18
$\lvert C_{\overline{G}}(g_{74j})\rvert$		36	36
(k,m)	$\lvert C_{H_k}(g_{74km})\rvert$		
$(1,1)$	18	1	1
$(2,1)$	18	1	-1
m_{74j}		128	128

$\mathcal{F}_{75}$

g_{75}		$g_{75,1}$	$g_{75,2}$
$o(g_{75j})$		40	20
$\lvert C_{\overline{G}}(g_{75j})\rvert$		80	80
(k,m)	$\lvert C_{H_k}(g_{75km})\rvert$		
$(1,1)$	40	1	1
$(2,1)$	40	1	-1
m_{75j}		128	128

$\mathcal{F}_{76}$

g_{76}		$g_{76,1}$	$g_{76,2}$
$o(g_{76j})$		40	20
$\lvert C_{\overline{G}}(g_{76j})\rvert$		80	80
(k,m)	$\lvert C_{H_k}(g_{76km})\rvert$		
$(1,1)$	40	1	1
$(2,1)$	40	1	-1
m_{76j}		128	128

$\mathcal{F}_{77}$

g_{77}		$g_{77,1}$
$o(g_{77j})$		21
$\lvert C_{\overline{G}}(g_{77j})\rvert$		21
(k,m)	$\lvert C_{H_k}(g_{77km})\rvert$	
$(1,1)$	21	1
m_{77j}		256

$\mathcal{F}_{78}$

g_{78}		$g_{78,1}$	$g_{78,2}$
$o(g_{78j})$		24	24
$\lvert C_{\overline{G}}(g_{78j})\rvert$		96	96
(k,m)	$\lvert C_{H_k}(g_{78km})\rvert$		
$(1,1)$	48	1	1
$(2,1)$	48	1	-1
m_{78j}		128	128

$\mathcal{F}_{79}$

g_{79}		$g_{79,1}$	$g_{79,2}$
$o(g_{79j})$		24	24
$\lvert C_{\overline{G}}(g_{79j})\rvert$		96	96
(k,m)	$\lvert C_{H_k}(g_{79km})\rvert$		
$(1,1)$	48	1	1
$(2,1)$	48	1	-1
m_{79j}		128	128

$\mathcal{F}_{80}$

g_{80}		$g_{80,1}$	$g_{80,2}$
$o(g_{80j})$		60	30
$\lvert C_{\overline{G}}(g_{80j})\rvert$		60	60
(k,m)	$\lvert C_{H_k}(g_{80km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	30	1	-1
m_{80j}		128	128

$\mathcal{F}_{81}$

g_{81}		$g_{81,1}$
$o(g_{81j})$		30
$\lvert C_{\overline{G}}(g_{81j})\rvert$		30
(k,m)	$\lvert C_{H_k}(g_{81km})\rvert$	
$(1,1)$	30	1
m_{81j}		256

9.7. The Character Table of $\overline{G} = 2^{8}{\cdot}Sp(8,2)$

Having obtained

- the conjugacy classes of $\overline{G} = 2^{8}{\cdot}Sp(8,2)$ (Table 9.2),
- the fusion of classes of the inertia factor H_2 into classes of $Sp(8,2)$ (Table 9.3),
- the Fischer matrices (see Section 9.6) and
- the character table of the inertia factor H_2 (Table 6.7 of Ali [1]),

the character table of $\overline{G} = 2^{8}{\cdot}Sp(8,2)$ can be constructed easily by following the description given in Section 5.3. The character table of $\overline{G}$ is a 195×195 complex valued matrix and it coincides with the character table of the Split extension $2^{8}{:}Sp(8,2)$. We supply the character table of $\overline{G}$ in Table 11.13, appears in the Appendix of this thesis.

Chapter 10

Two Groups of the Forms $2^{10}{:}(U_5(2){:}2)$ and $2^{1+6}_{-}{:}((3^{1+2}{:}8){:}2)$

In this chapter we are interested in calculating the ordinary character tables of two split extensions of the forms $\overline{G} = 2^{10}{:}(U_5(2){:}2)$ and $H_2 = 2^{1+6}_{-}{:}((3^{1+2}{:}8){:}2)$ by means of Clifford-Fischer theory. In Section 10.2 we supply the conjugacy classes of $\overline{G}$ using the coset analysis technique. In Section 10.3 we determine the inertia factor groups, the fusions of classes of these inertia factors into classes of $G = U_5(2){:}2$ and we supply the character tables of these groups. A considerable part of Section 10.3 is devoted to the construction of the character table of H_2 through Clifford-Fischer theory, where we will see that the Fischer matrices of this group satisfy some interesting properties (Lemmas 10.3.3 and 10.3.4). In Section 10.4 we list the 43 Fischer matrices of $\overline{G}$. These matrices are integral valued and sizes of these matrices range between 1 and 5. In Section 10.5 we show how to obtain the character table of $2^{10}{:}(U_5(2){:}2)$ and the full character table of $2^{10}{:}(U_5(2){:}2)$ is listed in Table 11.17.

10.1. Introduction

Let $U = U_5(2)$ be the special unitary group consisting of 5×5 matrices over $\mathbb{F}_4$ that preserves a non-singular Hermitian form. The outer automorphism of U is 2 (see the $\mathbb{ATLAS}$ [17]) and thus the full automorphism group of U is a group of the form $U_5(2){:}2$. We denote this group by G and we note that $|G| = 27\,371\,520$. By the electronic ATLAS of Wilson [71], the group G has a $10-$dimensional absolutely irreducible module over $\mathbb{F}_2$. Hence a split extension of the form $\overline{G} = 2^{10}{:}(U_5(2){:}2)$ does exist. In this chapter we are interested in calculating the ordinary character table of $\overline{G}$ using

Clifford-Fischer theory. It will turn out that the character table of $\overline{G}$ is a 109×109 complex valued matrix and it is partitioned into three blocks corresponding to three inertia factor groups $H_1 = U_5(2)$:2, $H_2 = 2^{1+6}_{-}$:$((3^{1+2}$:8):2) and $H_3 = O(5,2)$:2 (see Section 10.3). We have applied the same techniques of Clifford-Fischer theory and coset analysis to obtain the ordinary character table of the group H_2. Using the 10−dimensional matrices over $\mathbb{F}_2$ that generate $G = U_5(2)$:2, given at the electronic ATLAS of Wilson, we use GAP [30] to construct the split extension group $\overline{G}$. The following two 11×11 $\mathbb{F}_2$−matrices $\overline{g}_1$ and $\overline{g}_2$ generate $\overline{G}$.

$$\overline{g}_1 = \begin{pmatrix} 1&0&1&1&0&0&0&1&0&0&0\\ 0&1&1&1&0&0&1&1&1&0&0\\ 1&1&1&1&1&1&1&0&0&1&0\\ 1&0&0&1&0&0&1&0&0&0&0\\ 1&1&1&0&1&1&1&1&1&0&0\\ 1&1&1&1&1&0&0&1&1&0&0\\ 1&1&0&0&1&1&1&1&0&0&0\\ 0&0&1&1&0&0&0&1&1&1&0\\ 0&0&1&0&0&0&1&0&1&0&0\\ 1&1&1&1&0&0&0&1&0&0&0\\ 0&1&1&0&1&0&0&0&0&0&1 \end{pmatrix}, \quad \overline{g}_2 = \begin{pmatrix} 1&1&1&1&0&1&1&0&1&0&0\\ 0&1&1&0&0&1&1&1&1&0&0\\ 1&0&1&1&0&0&1&1&1&1&0\\ 0&0&1&0&0&0&1&1&0&0&0\\ 0&0&1&1&0&1&0&0&1&0&0\\ 1&0&1&1&1&1&1&1&0&1&0\\ 0&0&1&1&1&1&1&0&0&0&0\\ 0&1&0&0&0&1&1&0&1&1&0\\ 1&1&0&0&0&1&1&0&0&0&0\\ 1&0&0&0&0&0&1&1&1&1&0\\ 1&0&1&0&0&1&0&0&1&0&1 \end{pmatrix},$$

with $o(\overline{g}_1) = 24$, $o(\overline{g}_2) = 8$ and $o(\overline{g}_1\overline{g}_2) = 20$. It is easy to check that $\overline{G} \leq PSL(11,2)$.

Now having the group $\overline{G}$ constructed in GAP, it is easy to obtain all its normal subgroups. In fact $\overline{G}$ has only two non-trivial proper normal subgroups, one is of order 1024 and the other is of order 14 014 218 240. The normal subgroup of order 1024 is an elementary abelian 2−group and thus is isomorphic to N. The following 11−dimensional matrices over $\mathbb{F}_2$ generate the group $N = 2^{10}$.

$$n_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 0&1&0&1&1&1&1&0&0&1&1 \end{pmatrix}, \quad n_2 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 0&1&1&0&0&0&0&1&1&1&1 \end{pmatrix},$$

$$n_3 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 0&1&1&0&1&1&1&1&1&1&1 \end{pmatrix}, \quad n_4 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0&0&0&1 \end{pmatrix},$$

$$n_5 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&1&0&1&0&0&0&1 \end{pmatrix}, \quad n_6 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&1&1&1&1&1&0&1 \end{pmatrix},$$

$$n_7 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&0&0&1&1&1&0&0&0&0&1 \end{pmatrix}, \quad n_8 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&1&0&0&1&1&0&0&1&1&1 \end{pmatrix},$$

$$n_9 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&1&0&1&1&0&0&1&0&1&1 \end{pmatrix}, \quad n_{10} = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 1&1&1&0&0&1&0&0&1&0&1 \end{pmatrix}.$$

In Magma or GAP one can check for the complements of $N = \langle n_1, n_2, \cdots, n_{10}\rangle$ in $\overline{G} = \langle \overline{g}_1, \overline{g}_2\rangle$, where here we obtained only one complement G. The following two elements g_1 and g_2 generate the complement G of N in $\overline{G}$. Note that G is a subgroup of $\overline{G}$ isomorphic to the quotient $\overline{G}/N \cong U_5(2)$:2 and together with N creates the split extension $\overline{G}$ in consideration.

$$g_1 = \begin{pmatrix} 0&1&0&0&1&1&0&0&0&0&0\\ 0&1&1&1&1&0&1&1&1&0&0\\ 0&1&1&0&1&0&1&0&0&1&0\\ 1&1&1&0&0&1&1&1&0&0&0\\ 1&0&0&1&0&1&1&0&1&0&0\\ 1&1&1&0&0&1&0&1&0&1&0\\ 0&1&1&0&1&0&0&0&1&0&0\\ 1&1&0&0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&1&1&0&0&0\\ 1&1&1&0&1&0&1&1&0&1&0\\ 0&0&1&1&0&0&1&0&1&1&1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 1&0&0&0&0&1&0&1&0&0&0\\ 1&1&1&1&1&0&1&0&0&1&0\\ 0&0&1&1&0&0&0&1&1&1&0\\ 1&0&0&0&0&1&0&1&0&1&0\\ 0&0&0&1&1&1&0&1&1&1&0\\ 0&0&0&1&0&1&1&0&1&0&0\\ 1&0&0&0&0&0&0&0&0&1&0\\ 1&0&1&0&0&0&0&1&1&0&0\\ 0&0&0&1&1&0&0&1&1&0&0\\ 0&1&1&0&1&1&1&0&1&1&0\\ 1&1&0&0&0&1&0&0&1&0&1 \end{pmatrix},$$

with $o(g_1) = 16,\ o(g_2) = 16$ and $o(g_1g_2) = 12$.

10.2. Conjugacy Classes of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$

We have used the method of the *coset analysis*, discussed in Section 2.3, to calculate the conjugacy classes of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$. We supply these classes in Table 10.1, where in this table

- g_i is the i^{th} conjugacy class of $G = U_5(2)$:2 as listed in Table 11.14.
- k_i is the number of orbits $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ on the action of N on the coset $N\overline{g}_i = Ng_i = 2^{10}g_i$. In particular, the action of N on the identity coset N produces 1024 orbits each consists of singleton. Thus $k_1 = 1024$.

- f_{ij} is the number of orbits fused together under the action of $C_G(g_i)$ on $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$. In particular, the action of $C_G(g_1) = G$ on the orbits $Q_{11}, Q_{12}, \cdots, Q_{1k_1}$ affords three orbits of lengths 1, 495 and 528. Thus $f_{11} = 1,\ f_{12} = 495$ and $f_{13} = 528$.
- m_{ij} is the weight defined by Eq. (5.4).
- g_{ij} is a representative of conjugacy class of $\overline{G}$ that correspond to the class g_i of G. In particular the action of G on N produces three conjugacy classes in $\overline{G}$ represented by g_{11}, g_{12} and g_{13} and these three classes have sizes 1, 495 and 528 respectively.

Table 10.1: The conjugacy classes of $\overline{G} = 2^{10}$:$(U_5(2)$:2$)$

$[g_i]_{U_5(2):2}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{10}:(U_5(2):2)}\rvert$	$\lvert C_{2^{10}:(U_5(2):2)}(g_{ij})\rvert$
		$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	28028436480
$g_1 = 1A$	$k_1 = 1024$	$f_{12} = 495$	$m_{12} = 495$	g_{12}	2	495	56623104
		$f_{13} = 528$	$m_{13} = 528$	g_{13}	2	528	53084160
		$f_{21} = 1$	$m_{21} = 4$	g_{21}	2	660	42467328
$g_2 = 2A$	$k_2 = 256$	$f_{22} = 27$	$m_{22} = 108$	g_{22}	2	17820	1572864
		$f_{23} = 36$	$m_{23} = 144$	g_{23}	2	23760	1179648
		$f_{24} = 192$	$m_{24} = 768$	g_{24}	4	126720	221184
		$f_{31} = 1$	$m_{31} = 16$	g_{31}	2	47520	589824
$g_3 = 2B$	$k_3 = 64$	$f_{32} = 3$	$m_{32} = 48$	g_{32}	2	142560	196608
		$f_{33} = 24$	$m_{33} = 384$	g_{33}	4	1140480	24576
		$f_{34} = 36$	$m_{34} = 576$	g_{34}	4	1710720	16384
		$f_{41} = 1$	$m_{41} = 32$	g_{41}	2	608256	46080
$g_4 = 2C$	$k_4 = 32$	$f_{42} = 1$	$m_{42} = 32$	g_{42}	4	608256	46080
		$f_{43} = 15$	$m_{43} = 480$	g_{43}	4	9123840	3072
		$f_{44} = 15$	$m_{44} = 480$	g_{44}	4	9123840	3072
$g_5 = 3A$	$k_5 = 1$	$f_{51} = 1$	$m_{51} = 1024$	g_{51}	3	360448	77760

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$[g_i]_{U_5(2):2}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{10}:(U_5(2):2)}\rvert$	$\lvert C_{2^{10}:(U_5(2):2)}(g_{ij})\rvert$
		$f_{61}=1$	$m_{61}=16$	g_{61}	3	112640	248832
$g_6=3B$	$k_6=64$	$f_{62}=27$	$m_{62}=432$	g_{62}	6	3041280	9216
		$f_{63}=36$	$m_{63}=576$	g_{63}	6	4055040	6912
		$f_{71}=1$	$m_{71}=64$	g_{71}	3	4055040	6912
$g_7=3C$	$k_7=16$	$f_{72}=6$	$m_{72}=384$	g_{72}	6	24330240	1152
		$f_{73}=9$	$m_{73}=576$	g_{73}	6	36495360	768
$g_8=3D$	$k_8=4$	$f_{81}=1$	$m_{81}=256$	g_{81}	3	10813440	2592
		$f_{82}=3$	$m_{82}=768$	g_{82}	6	32440320	864
		$f_{91}=1$	$m_{91}=16$	g_{91}	4	190080	147456
		$f_{92}=3$	$m_{92}=3$	g_{92}	4	570240	49152
$g_9=4A$	$k_9=64$	$f_{93}=6$	$m_{93}=96$	g_{93}	4	1140480	24576
		$f_{94}=6$	$m_{94}=96$	g_{94}	4	1140480	24576
		$f_{95}=48$	$m_{95}=768$	g_{95}	4	9123840	3072
		$f_{10,1}=1$	$m_{10,1}=64$	$g_{10,1}$	4	2880960	12288
$g_{10}=4B$	$k_{10}=16$	$f_{10,2}=1$	$m_{10,2}=64$	$g_{10,2}$	4	2880960	12288
		$f_{10,3}=2$	$m_{10,3}=128$	$g_{10,3}$	4	4561920	6144
		$f_{10,4}=12$	$m_{10,4}=768$	$g_{10,4}$	4	27371520	1024
		$f_{11,1}=1$	$m_{11,1}=64$	$g_{11,1}$	4	9123840	3072
$g_{11}=4C$	$k_{11}=16$	$f_{11,2}=3$	$m_{11,2}=192$	$g_{11,2}$	4	27371520	1024
		$f_{11,3}=12$	$m_{11,3}=768$	$g_{11,3}$	8	109486080	256
		$f_{12,1}=1$	$m_{12,1}=128$	$g_{12,1}$	4	36495360	768
$g_{12}=4D$	$k_{12}=8$	$f_{12,2}=1$	$m_{12,2}=128$	$g_{12,2}$	4	36495360	768
		$f_{12,3}=3$	$m_{12,3}=384$	$g_{12,3}$	8	109486080	256
		$f_{12,4}=3$	$m_{12,4}=384$	$g_{12,4}$	8	109486080	256
$g_{13}=5A$	$k_{13}=4$	$f_{13,1}=1$	$m_{13,1}=256$	$g_{13,1}$	5	233570304	120
		$f_{13,2}=3$	$m_{13,2}=768$	$g_{13,2}$	10	700710912	40
$g_{14}=6A$	$k_{14}=1$	$f_{14,1}=1$	$m_{14,1}=1024$	$g_{14,1}$	6	16220160	1728
$g_{15}=6B$	$k_{15}=4$	$f_{15,1}=1$	$m_{15,1}=256$	$g_{15,1}$	6	5406720	5184
		$f_{15,2}=3$	$m_{15,2}=768$	$g_{15,2}$	12	16220160	1728

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$[g_i]_{U_5(2):2}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{10}:(U_5(2):2)}\rvert$	$\lvert C_{2^{10}:(U_5(2):2)}(g_{ij})\rvert$
		$f_{16,1}=1$	$m_{16,1}=64$	$g_{16,1}$	6	4055040	6912
$g_{16}=6C$	$k_{16}=16$	$f_{16,2}=3$	$m_{16,2}=192$	$g_{16,2}$	6	12165120	2304
		$f_{16,3}=12$	$m_{16,3}=768$	$g_{16,3}$	12	48660480	576
		$f_{17,1}=1$	$m_{17,1}=64$	$g_{17,1}$	6	4055040	6912
$g_{17}=6D$	$k_{17}=16$	$f_{17,2}=6$	$m_{17,2}=384$	$g_{17,2}$	6	24330240	1152
		$f_{17,3}=9$	$m_{17,3}=576$	$g_{17,3}$	6	36495360	768
$g_{18}=6E$	$k_{18}=1$	$f_{18,1}=1$	$m_{18,1}=1024$	$g_{18,1}$	6	97320960	288
$g_{19}=6F$	$k_{19}=4$	$f_{19,1}=1$	$m_{19,1}=256$	$g_{19,1}$	6	48660480	576
		$f_{19,2}=3$	$m_{19,2}=768$	$g_{19,2}$	12	145981440	192
$g_{20}=6G$	$k_{20}=4$	$f_{20,1}=1$	$m_{20,1}=256$	$g_{20,1}$	6	64880640	432
		$f_{20,2}=3$	$m_{20,2}=768$	$g_{20,2}$	6	194641920	144
$g_{21}=6H$	$k_{21}=4$	$f_{21,1}=1$	$m_{21,1}=256$	$g_{21,1}$	6	97320960	288
		$f_{21,2}=3$	$m_{21,2}=768$	$g_{21,2}$	6	291962880	96
$g_{22}=6I$	$k_{22}=2$	$f_{22,1}=1$	$m_{22,1}=512$	$g_{22,1}$	6	389283840	72
		$f_{22,2}=1$	$m_{22,2}=512$	$g_{22,2}$	12	389283840	72
		$f_{23,1}=1$	$m_{23,1}=128$	$g_{23,1}$	6	97320960	288
$g_{23}=6J$	$k_{23}=8$	$f_{23,2}=1$	$m_{23,2}=128$	$g_{23,2}$	12	97320960	288
		$f_{23,3}=3$	$m_{23,3}=384$	$g_{23,3}$	12	291962880	96
		$f_{23,4}=3$	$m_{23,4}=384$	$g_{23,4}$	12	291962880	96
		$f_{24,1}=1$	$m_{24,1}=128$	$g_{24,1}$	8	18247680	1536
$g_{24}=8A$	$k_{24}=8$	$f_{24,2}=3$	$m_{24,2}=384$	$g_{24,2}$	8	54743040	512
		$f_{24,3}=4$	$m_{24,3}=512$	$g_{24,3}$	8	72990720	384
		$f_{25,1}=1$	$m_{25,1}=128$	$g_{25,1}$	8	18247680	1536
$g_{25}=8B$	$k_{25}=8$	$f_{25,2}=3$	$m_{25,2}=384$	$g_{25,2}$	8	54743040	512
		$f_{25,1}=4$	$m_{25,3}=512$	$g_{25,3}$	8	72990720	384
		$f_{26,1}=1$	$m_{26,1}=128$	$g_{26,1}$	8	109486080	256
		$f_{26,2}=1$	$m_{26,2}=128$	$g_{26,2}$	8	109486080	256
$g_{26}=8C$	$k_{26}=8$	$f_{26,3}=2$	$m_{26,3}=256$	$g_{26,3}$	8	218972160	128
		$f_{26,4}=2$	$m_{26,4}=256$	$g_{26,4}$	8	218972160	128
		$f_{26,5}=2$	$m_{26,5}=256$	$g_{26,5}$	8	218972160	128

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$[g_i]_{U_5(2):2}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{10}:(U_5(2):2)}\rvert$	$\lvert C_{2^{10}:(U_5(2):2)}(g_{ij})\rvert$
		$f_{27,1}=1$	$m_{27,1}=256$	$g_{27,1}$	8	218972160	128
$g_{27}=8D$	$k_{27}=4$	$f_{27,2}=1$	$m_{27,2}=256$	$g_{27,2}$	8	218972160	128
		$f_{27,3}=2$	$m_{27,3}=512$	$g_{27,3}$	8	437944320	64
$g_{28}=9A$	$k_{28}=1$	$f_{28,1}=1$	$m_{28,1}=1024$	$g_{28,1}$	9	519045120	54
$g_{29}=9B$	$k_{29}=4$	$f_{29,1}=1$	$m_{29,1}=256$	$g_{29,1}$	9	259522560	108
		$f_{29,2}=3$	$m_{29,2}=768$	$g_{29,2}$	18	778567680	36
$g_{30}=10A$	$k_{30}=2$	$f_{30,1}=1$	$m_{30,1}=512$	$g_{30,1}$	10	1401421824	20
		$f_{30,2}=1$	$m_{30,2}=512$	$g_{30,2}$	20	1401421824	20
$g_{31}=11A$	$k_{31}=1$	$f_{31,1}=1$	$m_{31,1}=1024$	$g_{31,1}$	11	2548039680	11
$g_{32}=12A$	$k_{32}=1$	$f_{32,1}=1$	$m_{32,1}=1024$	$g_{32,1}$	12	194641920	144
$g_{33}=12B$	$k_{33}=4$	$f_{33,1}=1$	$m_{33,1}=256$	$g_{33,1}$	12	389283840	288
		$f_{33,2}=3$	$m_{33,2}=768$	$g_{33,2}$	12	291962880	96
		$f_{34,1}=1$	$m_{34,1}=64$	$g_{34,1}$	12	24330240	1152
		$f_{34,2}=3$	$m_{34,2}=192$	$g_{34,2}$	12	72990720	384
$g_{34}=12C$	$k_{34}=16$	$f_{34,3}=3$	$m_{34,3}=192$	$g_{34,3}$	12	72990720	384
		$f_{34,4}=3$	$m_{34,4}=192$	$g_{34,4}$	12	72990720	384
		$f_{34,5}=6$	$m_{34,5}=384$	$g_{34,5}$	12	145981440	192
$g_{35}=12D$	$k_{35}=1$	$f_{35,1}=1$	$m_{35,1}=1024$	$g_{35,1}$	12	1167851520	24
		$f_{36,1}=1$	$m_{36,1}=256$	$g_{36,1}$	12	291962880	96
$g_{36}=12E$	$k_{36}=4$	$f_{36,2}=1$	$m_{36,2}=256$	$g_{36,2}$	12	291962880	96
		$f_{36,3}=1$	$m_{36,3}=256$	$g_{36,3}$	12	291962880	96
		$f_{36,4}=1$	$m_{36,4}=256$	$g_{36,4}$	12	291962880	96
$g_{37}=12F$	$k_{37}=2$	$f_{37,1}=1$	$m_{37,1}=512$	$g_{37,1}$	12	1167851520	24
		$f_{37,2}=1$	$m_{37,2}=512$	$g_{37,2}$	12	1167851520	24
$g_{38}=15A$	$k_{38}=1$	$f_{38,1}=1$	$m_{38,1}=1024$	$g_{38,1}$	15	1868562432	15
$g_{39}=16A$	$k_{39}=2$	$f_{39,1}=1$	$m_{39,1}=512$	$g_{39,1}$	16	875888640	32
		$f_{39,2}=1$	$m_{39,2}=512$	$g_{39,2}$	16	875888640	32

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$[g_i]_{U_5(2):2}$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{2^{10}:(U_5(2):2)}\rvert$	$\lvert C_{2^{10}:(U_5(2):2)}(g_{ij})\rvert$
$g_{40} = 16B$	$k_{40} = 2$	$f_{40,1} = 1$	$m_{40,1} = 512$	$g_{40,1}$	16	875888640	32
		$f_{40,2} = 1$	$m_{40,2} = 512$	$g_{40,2}$	16	875888640	32
$g_{41} = 18A$	$k_{41} = 1$	$f_{41,1} = 1$	$m_{41,1} = 1024$	$g_{41,1}$	18	1557135360	18
$g_{42} = 24A$	$k_{42} = 2$	$f_{42,1} = 1$	$m_{42,1} = 512$	$g_{42,1}$	24	583925760	48
		$f_{42,2} = 1$	$m_{42,2} = 512$	$g_{42,2}$	24	583925760	48
$g_{43} = 24B$	$k_{43} = 2$	$f_{43,1} = 1$	$m_{43,1} = 512$	$g_{43,1}$	24	583925760	48
		$f_{43,2} = 1$	$m_{43,2} = 512$	$g_{43,2}$	24	583925760	48

10.3. Inertia Factor Groups of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$

We have seen from Section 10.2 that the action of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$ on $N = 2^{10}$ yielded three orbits of lengths 1, 495 and 528. By Theorem 5.1.1 of Brauer, it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce three orbits. Since $N = 2^{10}$ is a vector space, the action of $\overline{G}$ on $\mathrm{Irr}(2^{10})$ can be viewed as the action of $\overline{G}$ on N^*, where N^* is the dual space of N. In fact we act the group generated by the transposed matrices of the matrix generators of $\overline{G}$ on N. We have found that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ is self-dual to the action of $\overline{G}$ on N, that is the orbit lengths of $\overline{G}$ on $\mathrm{Irr}(N)$ are 1, 495 and 528. Let H_1, H_2 and H_3 be the respective inertia factor groups of the representatives of characters from the previous orbits. We notice that these inertia factors have indices 1, 495 and 528 respectively in $U_5(2)$:2. Clearly $H_1 = U_5(2)$:2 and the character table of this group is supplied in Table 11.14 of this thesis. By looking at $\mathbb{ATLAS}$, the group $U_5(2)$:2 has 7 conjugacy classes of maximal subgroups. Let $M[1], M[2], \cdots, M[7]$ be representatives of these classes. That is $M[1] = U_5(2)$, $M[2] = 2^{1+6}_-$:3^{1+2}_-:$2S_4$, $M[3] = (3 \times U_4(2))$:2, $M[4] = 2^{4+4}$:$(3 \times A_5)$:2, $M[5] = 3^4$:$(2 \times S_5)$, $M[6] = S_3 \times 3^{1+2}_+$:$2S_4$ and $M[7] = PSL(2, 11)$:2. Note that these maximal subgroups have indices 2, 165, 176, 297, 1408, 3520 and 20736 respectively in $G = U_5(2)$:2. By considering the indices of H_2 and H_3 in G, we can see that H_2 must be an index 3 subgroup of $M[2] = 2^{1+6}_-$:3^{1+2}_-:$2S_4$, while H_3 is either an index 264 subgroup of $M[1] = U_5(2)$ or of index 3 in $M[3] = (3 \times U_4(2))$:2. However the possibility $H_3 \leq M[1]$ is not feasible as we can see from the $\mathbb{ATLAS}$ that $U_5(2)$ does not contain a subgroup of index that is a divisor of 264. This leaves us with the other possibility that H_3 is an index 3 subgroup of $M[3]$. As subgroups of $G = \langle g_1, g_2 \rangle$,

the group $M[2]$ is generated by α_1 and α_2, while the group $M[3]$ is generated by β_1 and β_2, where

$$\alpha_1 = \begin{pmatrix} 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&1&0&1&1&1&1&1&0\\ 1&1&1&0&0&0&0&1&0&0&0\\ 1&1&0&1&1&1&0&0&0&0&0\\ 0&1&0&0&1&0&0&1&1&0&0\\ 1&1&1&1&0&1&1&0&1&0&0\\ 1&1&0&0&1&1&1&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&1&0\\ 0&1&1&0&1&0&0&1&1&0&0\\ 0&1&1&1&1&1&1&1&0&0&0\\ 1&0&0&1&1&0&1&1&1&0&1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0&1&1&0&0&1&0&0&1&0&0\\ 0&1&1&1&0&0&1&1&0&1&0\\ 1&1&1&0&0&1&0&1&1&1&0\\ 0&0&0&1&1&1&0&1&1&0&0\\ 1&1&1&1&1&0&0&1&1&0&0\\ 0&1&0&0&1&1&0&0&1&0&0\\ 0&1&0&0&1&1&0&1&1&1&0\\ 1&1&0&1&1&0&0&1&0&0&0\\ 1&0&0&0&1&1&1&0&1&1&0\\ 0&1&1&0&1&1&1&0&0&0&0\\ 1&0&1&0&0&1&1&0&0&1&1 \end{pmatrix},$$

$$\beta_1 = \begin{pmatrix} 1&0&0&0&0&1&0&1&1&1&0\\ 1&0&1&0&0&0&1&1&0&0&0\\ 0&1&0&1&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&1&0&1&0&0\\ 0&0&1&0&0&1&1&1&0&1&0\\ 0&0&1&0&0&1&0&0&0&0&0\\ 0&1&1&1&0&1&1&1&1&0&0\\ 0&0&0&0&1&1&1&1&0&0&0\\ 1&0&0&1&0&1&0&1&1&0&0\\ 1&1&1&0&0&0&1&0&0&1&0\\ 0&1&1&0&0&0&0&1&0&1&1 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 0&1&0&1&1&1&0&0&0&1&0\\ 0&1&1&0&1&1&0&1&1&0&0\\ 1&1&1&0&1&0&1&0&0&1&0\\ 1&1&1&0&1&0&0&1&1&0&0\\ 1&1&0&0&1&1&0&0&1&0&0\\ 1&1&1&0&0&0&1&0&0&1&0\\ 1&1&0&0&0&0&0&1&1&0&0\\ 0&0&1&0&0&0&1&0&0&1&0\\ 0&1&1&0&0&1&1&1&0&1&0\\ 1&0&0&0&0&0&0&0&0&1&0\\ 1&0&1&1&0&1&0&1&0&1&1 \end{pmatrix}.$$

Using GAP we were able to locate all the maximal subgroups of $M[2]$ and $M[3]$. We list brief information about these subgroups in Table 10.2. From Table 10.2 we can see that the two groups H_2 and H_3 are in the conjugacy classes of maximal subgroups containing $M[22]$ and $M[32]$ respectively. Note that from Table 10.1 the group $\overline{G}$ has 109 conjugacy classes. By the description of Section 5.3 the 109 irreducible characters of $\overline{G}$ are distributed into three blocks of characters correspond to the inertia factor groups. That is $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)|$. From Tables 11.14, 11.15 and 11.16, we can see that $|\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| = 43 + 41 + 25 = 109 = |\mathrm{Irr}(\overline{G})|$.

As subgroups of $G = \langle g_1, g_2 \rangle$, the group $M[22]$ is generated by μ_1 and μ_2, while the group $M[32]$ is generated by ζ_1 and ζ_2, where

Table 10.2: Some information on the maximal subgroups of $M[2]$ and $M[3]$

Maximal Subgroups of $M[2]$ & $M[3]$	$M[ij]$	$\lvert M[ij]\rvert$	$[M[i] : M[ij]]$	$\lvert\mathrm{Irr}(M[ij])\rvert$
$M[2] = 2^{1+6}_{-}$:3^{1+2}_{-}:$2S_4$	$M[21]$	82944	2	64
	$M[22]$	55296	3	41
	$M[23]$	41472	4	55
	$M[24]$	18432	9	50
	$M[25]$	2592	64	36
$M[3] = (3 \times U_4(2))$:2	$M[31]$	77760	2	60
	$M[32]$	51840	3	25
	$M[33]$	5760	27	30
	$M[34]$	4320	36	33
	$M[35]$	3888	40	39
	$M[36]$	3888	40	42
	$M[37]$	3456	45	48

$$\mu_1 = \begin{pmatrix} 0&1&0&1&0&1&0&1&1&1&0\\ 0&1&0&1&0&1&1&0&1&1&0\\ 1&1&0&0&0&1&0&0&0&0&0\\ 0&0&0&1&1&1&0&0&0&1&0\\ 1&1&0&1&1&1&0&0&0&0&0\\ 1&1&1&0&0&0&1&1&1&1&0\\ 0&1&0&0&0&1&0&0&1&0&0\\ 0&1&0&1&1&0&1&1&0&1&0\\ 0&1&0&1&0&0&0&0&1&1&0\\ 0&1&1&1&1&1&1&1&0&1&0\\ 0&0&0&1&0&0&0&0&1&1&1 \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} 0&1&1&0&1&0&0&1&1&0&0\\ 1&1&1&0&0&1&1&1&0&0&0\\ 1&1&0&0&0&0&0&1&1&0&0\\ 0&1&1&0&0&1&1&1&0&1&0\\ 0&0&1&0&0&0&0&0&1&0&0\\ 0&1&1&0&0&0&0&1&1&0&0\\ 0&0&0&1&0&0&1&0&1&0&0\\ 0&0&1&0&0&1&0&0&0&0&0\\ 1&0&1&1&0&1&1&1&0&0&0\\ 0&0&1&1&1&1&0&1&1&0&0\\ 1&0&1&1&0&0&0&0&0&0&1 \end{pmatrix},$$

with $o(\mu_1) = 24,\ o(\mu_2) = 16$ and $o(\mu_1\mu_2) = 12$,

$$\zeta_1 = \begin{pmatrix} 1&1&1&0&0&0&1&0&0&1&0\\ 0&1&0&1&1&1&0&0&1&1&0\\ 0&0&1&1&0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1&1&1&1&0\\ 1&1&1&1&0&1&0&0&1&0&0\\ 1&1&1&0&1&1&1&1&1&0&0\\ 0&0&1&1&1&0&0&0&0&0&0\\ 1&0&0&0&1&1&0&1&1&1&0\\ 1&0&0&0&0&1&1&1&1&0&0\\ 0&0&1&1&0&0&0&1&1&0&0\\ 0&0&0&0&1&0&1&1&1&1&1 \end{pmatrix}, \quad \zeta_2 = \begin{pmatrix} 1&1&0&0&0&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&1&1&1&0\\ 0&1&0&0&0&1&0&1&1&0&0\\ 0&0&1&0&1&1&1&0&0&1&0\\ 1&1&1&1&0&1&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&1&1&0\\ 1&1&1&1&1&0&0&1&1&1&0\\ 0&1&1&1&1&1&0&1&1&1&0\\ 0&1&1&0&0&0&1&0&1&1&0\\ 0&0&0&1&1&1&0&1&1&1&0\\ 1&1&1&0&0&1&1&0&0&1&1 \end{pmatrix},$$

with $o(\zeta_1) = 3,\ o(\zeta_2) = 4$ and $o(\zeta_1\zeta_2) = 10$.

Remark 10.3.1. With the aid of GAP we were able to determine the structures of all the normal subgroups of H_2 and H_3 and the corresponding quotient groups. In fact we have found that $H_2 \cong 2^{1+6}_{-}$:$((3^{1+2}$:$8)$:$2)$, while the group $H_3 = M[32] \cong O(5,3)$:$2 \cong U_4(2)$:$2 \cong Sp(4,3)$:2.

We recall that knowledge of the appropriate character tables of inertia factor groups is crucial in calculating the full character table of any group extension. Since in our extension $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$, the normal subgroup 2^{10} is abelian and the extension splits, it follows by applications of Mackey's Theorem (see Theorem 5.1.4), that every character ϑ_k of 2^{10} is extendible to an ordinary character of its respective inertia group $\overline{H}_k$. Thus all the factor sets $\overline{\alpha}_k$ are trivial and all the character tables of the inertia factor groups that we will use to construct the character table of $\overline{G}$ are the ordinary ones. The character table of H_3 is stored in GAP, or one can use any set of generators of $U_4(2)$:2 in the forms of 6, 8 or 14−dimensional $\mathbb{F}_2$−representations supplied at the electronic ATLAS of Wilson to generate this group in either Magma or GAP and then obtain its character table. Alternatively one can use the generators ζ_1 and ζ_2 to find the character table of H_3. The fusion of classes of H_3 into classes of $G = U(5,2)$:2 and the character table of H_3 are given in Tables 10.3 and 11.16 respectively.

The character table of $H_2 = 2^{1+6}_{-}$:$((3^{1+2}$:$8)$:$2)$ is not yet available in GAP. One can use the generators μ_1 and μ_2 of H_2 to generate H_2 inside Magma or GAP and then obtain its character table. However in this thesis we are interested to calculate the character table of H_2 using Clifford-Fischer technique together with the coset analysis method.

10.3.1 The Group $H_2 = 2^{1+6}_{-}$:$((3^{1+2}$:$8)$:$2)$

Let $K := 2^{1+6}_{-}$ and $Q := (3^{1+2}$:$8)$:$2 \cong H_2/2^{1+6}_{-}$. Then $H_2 = K$:Q. Since H_2 can be constructed in GAP, it is easy to obtain all its normal subgroups. In fact H_2 contains 11 normal subgroups of sizes 1, 2, 128, 384, 3456, 6912, 13824, 27648, 27648, 27648 and 55296. The normal subgroup of order 128 is an extra-special 2−group isomorphic to K. The following elements $\widetilde{k}_1, \widetilde{k}_2, \cdots, \widetilde{k}_7$ are 11−dimensional matrices over $\mathbb{F}_2$ that generate K.

Table 10.3: The fusion of classes of H_3 into classes of G

Class of H_3	$\hookrightarrow$	Class of $U_5(2)$:2	Class of H_3	$\hookrightarrow$	Class of $U_5(2)$:2
$1a = g_{131}$		$1A$	$6a = g_{14,31}$		$6D$
$2a = g_{431}$		$2C$	$6b = g_{20,31}$		$6G$
$2b = g_{231}$		$2A$	$6c = g_{23,31}$		$6J$
$2c = g_{331}$		$2B$	$6d = g_{16,31}$		$6C$
$2d = g_{432}$		$2C$	$6e = g_{22,31}$		$6I$
$3a = g_{731}$		$3C$	$6f = g_{21,31}$		$6H$
$3b = g_{831}$		$3D$	$6g = g_{23,32}$		$6J$
$3c = g_{631}$		$3B$	$8a = g_{26,31}$		$8C$
$4a = g_{12,31}$		$4D$	$9a = g_{29,31}$		$9B$
$4b = g_{933}$		$4A$	$10a = g_{30,31}$		$10A$
$4c = g_{12,32}$		$4D$	$12a = g_{37,31}$		$12F$
$4d = g_{11,31}$		$4C$	$12b = g_{34,31}$		$12C$
$5a = g_{13,31}$		$5A$			

$$\widetilde{k}_1 = \begin{pmatrix} 1&0&0&1&0&0&0&1&0&1&0 \\ 0&0&0&1&1&0&0&1&1&0&0 \\ 1&1&0&0&1&0&0&1&1&1&0 \\ 1&1&1&0&1&0&0&0&1&0&0 \\ 1&1&1&0&1&0&1&1&1&0&0 \\ 1&0&1&1&0&1&0&0&0&1&0 \\ 1&1&1&0&0&0&0&1&1&0&0 \\ 1&1&1&1&1&0&0&1&1&0&0 \\ 1&0&1&1&1&0&1&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&1&1 \end{pmatrix}, \quad \widetilde{k}_2 = \begin{pmatrix} 0&1&1&1&1&0&0&0&1&0&0 \\ 0&0&0&1&0&0&1&1&1&1&0 \\ 0&0&1&1&0&0&0&1&0&1&0 \\ 1&1&1&1&1&0&0&1&1&1&0 \\ 0&1&0&0&0&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&1&0&1&1&0 \\ 1&1&1&0&1&0&0&0&1&1&0 \\ 0&0&0&1&1&0&1&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 1&1&1&1&1&0&0&0&1&0&1 \end{pmatrix},$$

$$\widetilde{k}_3 = \begin{pmatrix} 0&0&1&0&0&1&0&1&0&0&0 \\ 1&1&1&1&0&0&0&1&1&1&0 \\ 1&0&0&1&0&0&0&1&1&1&0 \\ 1&1&1&1&0&1&1&0&0&0&0 \\ 1&1&1&0&1&1&1&1&1&0&0 \\ 1&1&1&1&0&1&1&0&1&1&0 \\ 0&1&0&1&0&1&0&1&1&1&0 \\ 1&1&1&0&0&1&1&1&0&0&0 \\ 1&1&1&0&0&1&1&0&1&0&0 \\ 1&1&1&1&0&0&1&0&1&0&0 \\ 1&1&1&0&0&1&1&1&1&0&1 \end{pmatrix}, \quad \widetilde{k}_4 = \begin{pmatrix} 1&1&0&1&0&1&1&0&0&1&0 \\ 0&0&0&1&0&1&1&1&1&1&0 \\ 0&1&1&1&0&1&1&1&1&1&0 \\ 1&0&1&0&0&0&0&1&1&1&0 \\ 1&0&1&0&1&1&0&0&1&0&0 \\ 1&0&1&1&0&1&0&0&0&1&0 \\ 1&1&1&0&0&1&0&0&0&0&0 \\ 1&0&1&1&0&0&0&0&1&1&0 \\ 1&0&1&1&0&0&0&1&0&1&0 \\ 1&0&1&1&0&0&0&0&0&0&0 \\ 1&0&1&0&0&1&0&0&1&0&1 \end{pmatrix},$$

$$\widetilde{k}_5 = \begin{pmatrix} 1&1&1&0&1&0&1&0&0&1&0\\ 1&1&0&0&0&1&1&0&0&1&0\\ 1&0&1&0&0&1&1&0&0&1&0\\ 1&0&1&1&1&1&1&0&1&0&0\\ 0&1&1&0&0&0&1&0&0&1&0\\ 1&0&1&1&0&1&0&0&0&1&0\\ 0&1&1&0&1&0&0&0&0&1&0\\ 0&0&0&1&1&1&1&1&1&1&0\\ 1&1&1&0&1&1&0&0&1&0&0\\ 0&1&0&0&0&0&1&0&1&1&0\\ 1&1&0&1&1&0&1&0&0&0&1 \end{pmatrix}, \quad \widetilde{k}_6 = \begin{pmatrix} 0&1&1&0&1&1&0&0&0&0&0\\ 0&0&1&0&1&0&1&0&0&1&0\\ 0&1&0&0&1&0&1&0&0&1&0\\ 0&1&1&0&0&1&0&0&1&0&0\\ 1&1&1&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&1&1&0&1&0&0\\ 1&1&1&0&1&1&1&0&0&0&0\\ 0&0&1&1&0&1&1&1&0&0&0\\ 1&0&0&0&0&1&1&0&1&1&0\\ 1&1&1&1&0&0&1&0&1&0&0\\ 1&0&1&0&1&1&1&0&1&0&1 \end{pmatrix},$$

$$\widetilde{k}_7 = \begin{pmatrix} 1&1&0&0&0&0&1&0&1&0&0\\ 1&0&1&1&0&0&1&0&1&1&0\\ 1&1&0&1&0&0&1&0&1&1&0\\ 1&0&1&0&0&0&0&0&0&1&0\\ 0&1&0&0&1&0&1&0&1&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&1&0&0\\ 1&0&1&1&0&0&0&1&0&1&0\\ 1&0&1&1&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&1&0&1&0&1 \end{pmatrix}.$$

Using GAP we have found that K has 4 complements in H_2, namely $(3^{1+2}$:$Q_8)$:2 (3 times) and $(3^{1+2}$:$8)$:2, where Q_8 is the quaternion group of order 8. Clearly we are interested in the fourth complement $(3^{1+2}$:$8)$:2, since it can be identified with Q. The following two elements $\overline{q}_1$ and $\overline{q}_2$ are $11-$dimensional matrices over $\mathbb{F}_2$ that generate the complement $Q = (3^{1+2}$:$8)$:2.

$$\overline{q}_1 = \begin{pmatrix} 1&0&0&1&1&1&1&1&0&0&0\\ 0&0&0&1&0&0&0&0&0&1&0\\ 1&1&0&1&0&1&0&1&0&1&0\\ 0&0&1&0&1&0&1&1&1&1&0\\ 1&0&1&1&1&1&0&0&0&1&0\\ 0&0&0&0&1&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&0&1&1&0\\ 0&0&0&1&1&1&0&1&1&1&0\\ 0&0&0&1&0&0&1&0&0&0&0\\ 1&0&0&1&0&0&1&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}, \quad \overline{q}_2 = \begin{pmatrix} 1&0&1&0&1&0&0&0&1&0&0\\ 1&0&1&0&0&1&1&0&0&1&0\\ 0&0&1&1&0&0&0&0&1&0&0\\ 1&1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&1&0&1&0&1&0\\ 1&1&1&0&0&0&1&0&0&1&0\\ 0&1&1&1&0&1&1&1&0&1&0\\ 1&1&0&0&0&0&0&0&0&1&0\\ 1&0&0&1&0&0&1&1&1&0&0\\ 1&1&1&0&1&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}.$$

The character table of the group $Q = (3^{1+2}$:$8)$:2 is given in Table 10.4.

Table 10.4: The character table of $Q = (3^{1+2}$:$8)$:2

	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}	q_{13}	q_{14}
	1_1	2_1	2_2	3_1	3_2	4_1	4_2	6_1	6_2	8_1	8_2	12_1	12_2	12_3
$\lvert C_Q(q_i)\rvert$	432	48	12	216	18	24	12	24	6	8	8	12	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	1	1	1	1	1	−1	−1	−1	1	1	1
χ_3	1	1	−1	1	1	1	−1	1	−1	1	1	−1	−1	1
χ_4	1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	1
χ_5	2	2	0	2	2	−2	0	2	0	0	0	0	0	−2
χ_6	2	−2	0	2	2	0	0	−2	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	0	0
χ_7	2	−2	0	2	2	0	0	−2	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	0	0
χ_8	6	−2	0	−3	0	2	−2	1	0	0	0	1	1	−1
χ_9	6	−2	0	−3	0	2	2	1	0	0	0	−1	−1	−1
χ_{10}	6	−2	0	−3	0	−2	0	1	0	0	0	$-i\sqrt{3}$	$i\sqrt{3}$	1
χ_{11}	6	−2	0	−3	0	−2	0	1	0	0	0	$i\sqrt{3}$	$-i\sqrt{3}$	1
χ_{12}	8	0	−2	8	−1	0	0	0	1	0	0	0	0	0
χ_{13}	8	0	2	8	−1	0	0	0	−1	0	0	0	0	0
χ_{14}	12	4	0	−6	0	0	0	−2	0	0	0	0	0	0

10.3.2 The Conjugacy Classes of $H_2 = 2^{1+6}_{-}$:$((3^{1+2}$:$8)$:$2)$

In this subsection we calculate the conjugacy classes of H_2 using the coset analysis technique as we are interested to organize the classes of H_2 corresponding to the classes of Q. Recall that K is a group of order $2^{1+6} = 128$ and has $2^6+1 = 65$ conjugacy classes. The action of $K = \left\langle \widetilde{k}_1, \widetilde{k}_2, \cdots, \widetilde{k}_7 \right\rangle$ on the identity coset $K1_Q = K$ produces the 65 conjugacy classes of K, where we know that K has

- singleton conjugacy class consisting of 1_K,
- singleton conjugacy class consisting of the central involution σ of K,
- 27 conjugacy classes, each class consists of two non-central involutions,
- 36 conjugacy classes, each class consists of two elements of order 4.

Using GAP, the action of $H_2 = \langle \mu_1, \mu_2 \rangle$ (or just $C_Q(1_Q) = Q = \langle \overline{q}_1, \overline{q}_2 \rangle$) on these 65 orbits leaves invariant $\{1_K\} := \Omega_1$ and $\{\sigma\} := \Omega_2$, while fuse the 27 orbits of non-central involutions into a single orbit Ω_3 and also fuse the 36 orbits of elements of order 4 altogether into a single orbit Ω_4. Thus in H_2, we get four conjugacy classes of sizes 1, 1, 54 and 72. Similarly one can apply this to the

other 13 cosets Kq_i, where q_i is a representative of a conjugacy class of Q (for the classes of Q, see Table 10.4). Corresponding to the 14 conjugacy classes of $Q = (3^{1+2}$:$8)$:2, we obtain 41 conjugacy classes for H_2. We list these classes in Table 10.5.

Table 10.5: The conjugacy classes of $H_2 = 2_-^{1+6}$:$((3^{1+2}$:$8)$:$2)$

$[q_i]_{(3^{1+2}:8):2}$	k_i	m_{ij}	$[q_{ij}]_{H_2}$	$o(q_{ij})$	$\lvert[q_{ij}]_{2_-^{1+6}:((3^{1+2}:8):2)}\rvert$	$\lvert C_{2_-^{1+6}:((3^{1+2}:8):2)}(q_{ij})\rvert$
		$m_{11} = 1$	q_{11}	1	1	55296
$q_1 = 1_1$	$k_1 = 65$	$m_{12} = 1$	q_{12}	2	1	55296
		$m_{13} = 54$	q_{13}	2	54	1024
		$m_{14} = 72$	q_{14}	4	72	768
		$m_{21} = 4$	q_{21}	2	36	1536
$q_2 = 2_1$	$k_2 = 17$	$m_{22} = 4$	q_{22}	2	36	1536
		$m_{23} = 24$	q_{23}	4	216	256
		$m_{24} = 96$	q_{24}	4	864	64
		$m_{31} = 8$	q_{31}	8	288	192
		$m_{32} = 8$	q_{32}	8	288	192
$q_3 = 2_2$	$k_3 = 9$	$m_{33} = 24$	q_{33}	2	576	96
		$m_{34} = 48$	q_{34}	8	1728	32
		$m_{35} = 48$	q_{35}	4	1728	32
$q_4 = 3_1$	$k_4 = 2$	$m_{41} = 64$	q_{41}	3	128	432
		$m_{42} = 64$	q_{42}	6	128	432
		$m_{51} = 16$	q_{51}	3	384	192
$q_5 = 3_2$	$k_5 = 5$	$m_{52} = 16$	q_{52}	6	384	192
		$m_{53} = 96$	q_{53}	12	2304	24
		$m_{61} = 16$	q_{61}	4	288	192
$q_6 = 4_1$	$k_6 = 5$	$m_{62} = 16$	q_{62}	4	288	192
		$m_{63} = 96$	q_{63}	8	1728	32

Continued on next page

Table 10.5 (continued)

$[q_i]_{(3^{1+2}:8):2}$	k_i	m_{ij}	$[q_{ij}]_{H_2}$	$o(q_{ij})$	$\lvert[q_{ij}]_{2_-^{1+6}:((3^{1+2}:8):2)}\rvert$	$\lvert C_{2_-^{1+6}:((3^{1+2}:8):2)}(q_{ij})\rvert$
		$m_{71}=16$	q_{71}	4	576	96
$q_7=4_2$	$k_7=5$	$m_{72}=16$	q_{72}	4	576	96
		$m_{73}=96$	q_{73}	8	3456	16
$q_8=6_1$	$k_8=2$	$m_{81}=64$	q_{81}	6	1152	48
		$m_{82}=64$	q_{82}	6	1152	48
		$m_{91}=32$	q_{91}	24	2304	24
$q_9=6_2$	$k_9=3$	$m_{92}=32$	q_{92}	24	2304	24
		$m_{93}=64$	q_{93}	6	4608	12
		$m_{10,1}=32$	$q_{10,1}$	8	1728	32
$q_{10}=8_1$	$k_{10}=3$	$m_{10,2}=32$	$q_{10,2}$	8	1728	32
		$m_{10,3}=64$	$q_{10,3}$	16	3456	16
		$m_{11,1}=32$	$q_{11,1}$	8	1728	32
$q_{11}=8_2$	$k_{11}=3$	$m_{11,2}=32$	$q_{11,2}$	8	1728	32
		$m_{11,3}=64$	$q_{11,3}$	16	3456	16
$q_{12}=12_1$	$k_{12}=2$	$m_{12,1}=64$	$q_{12,1}$	12	2304	24
		$m_{12,2}=64$	$q_{12,2}$	12	2304	24
$q_{13}=12_2$	$k_{13}=2$	$m_{13,1}=64$	$q_{13,1}$	12	2304	24
		$m_{13,2}=64$	$q_{13,2}$	12	2304	24
$q_{14}=12_3$	$k_{14}=2$	$m_{14,1}=64$	$q_{14,1}$	12	2304	24
		$m_{14,2}=64$	$q_{14,2}$	12	2304	24

10.3.3 The Inertia Factor Groups of $H_2 = 2_-^{1+6}$:$((3^{1+2}$:8):2)

We have seen in Subsection 10.3.2 that the action of H_2 on K produced four orbits of lengths 1, 1, 54 and 72. By Theorem 5.1.1 of Brauer, it follows that the action of H_2 on $\mathrm{Irr}(K)$ will also produce four orbits $\Delta_1,\ \Delta_2,\ \Delta_3$ and Δ_4 and let $\theta_1,\ \theta_2,\ \theta_3$ and θ_4 be respective representatives of these orbits. It is well known that any extra-special $p-$group of order p^{1+2m} has $p^{2m}+1$ irreducible characters (p^{2m} linear characters of the vector space p^{2m} are inflated to the full extension p^{1+2m} and $p-1$ faithful irreducible characters each of degree p^m). Thus the group $K = 2_-^{1+6}$ has 65 irreducible characters

in which 64 characters are linear and one unique faithful character θ_2 of degree 8 (the values of θ_2 are as follows: $\theta_2(1_K) = 8,\ \theta_2(\sigma) = -8$ and $\theta_2(t) = 0$ for any $t \in K \setminus \{1_K, \sigma\}$). It is necessary that the action of H_2 (or just Q) on $\mathrm{Irr}(K)$ leaves invariant the identity character $\theta_1 := \mathbf{1}_K$ and the character θ_2. Therefore the corresponding inertia factors of θ_1 and θ_2 are $T_1 = T_2 = Q = (3^{1+2}$:$8)$:2. Let $\Delta_1 = \{\mathbf{1}_K\}$ and $\Delta_2 = \{\theta_2\}$. Thus $|\Delta_3| + |\Delta_4| = 63$. Since $|Q| = 432 = 2^4 \times 3^3$, the set $\Omega = \{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 36, 48, 54, 72, 108, 144, 216, 432\}$ consists of all the divisors of $|Q|$. Since $|\Delta_3|, |\Delta_4| \in \Omega$ and $|\Delta_3| + |\Delta_4| = 63$, we can see that the only possibility for $(|\Delta_3|, |\Delta_4|)$ is that $(|\Delta_3|, |\Delta_4|) = (9, 54)$ or $(27, 36)$ (the cases $(54, 9)$ and $(36, 27)$ are excluded with out loss of generality). Let T_3 and T_4 be the respective inertia factor groups of characters from the orbits Δ_3 and Δ_4. The rest of this subsection is devoted to determine $(|\Delta_3|, |\Delta_4|)$ and (T_3, T_4). Clearly the identity character $\mathbf{1}_K$ is extendable to its inertia group. Using Magma or GAP one can easily obtain that the Schur multiplier $M(Q) = M((3^{1+2}$:$8)$:$2)$ is trivial. This shows that the character θ_2 is also extendable to an ordinary character of H_2. Now since the characters of Δ_3 and Δ_4 are linear characters of N, it follows by application of Theorem 5.1.8 that these characters are extendable to ordinary characters of their respective inertia groups. Thus for the construction of the character table of H_2, all the character tables of the inertia factors that we need to use are the ordinary ones. Using this and the fact that H_2 has 41 conjugacy classes (see Table 10.5) we deduce that $|\mathrm{Irr}(T_1)| + |\mathrm{Irr}(T_2)| + |\mathrm{Irr}(T_3)| + |\mathrm{Irr}(T_4)| = 41$. Since $|\mathrm{Irr}(T_1)| = |\mathrm{Irr}(T_2)| = 14$, we get that $|\mathrm{Irr}(T_3)| + |\mathrm{Irr}(T_4)| = 13$. This is a key result in determining the inertia factors T_3 and T_4.

Proposition 10.3.1. *The case* $(|\Delta_3|, |\Delta_4|) = (9, 54)$ *is not possible*

PROOF. Assume that $(|\Delta_3|, |\Delta_4|) = (9, 54)$. Then $|T_3| = 48$ and $|T_4| = 8$. Using GAP, it is easy to see that the group Q has four conjugacy classes of maximal subgroups, where each class is represented by $Q[1] := (3^{1+2}$:$4)$:2, $Q[2] := 3^{1+2}$:8, $Q[3] := 3^{1+2}$:Q_8 and $Q[4] := (3 \times Q_8)$:2. Note that the respective indices of $Q[1]$, $Q[2]$, $Q[3]$ and $Q[4]$ in Q are 2, 2, 2 and 9. This shows that $T_3 = Q[4] = (3 \times Q_8)$:$2$. Using GAP, we can see that $|\mathrm{Irr}(T_3)| = |\mathrm{Irr}((3 \times Q_8)$:$2)| = 12$. Using this and the fact that $|\mathrm{Irr}(T_3)| + |\mathrm{Irr}(T_4)| = 13$, we deduce that $|\mathrm{Irr}(T_4)| = 1$, but this clearly a contradiction as we know that $|T_4| = 8$. Thus $(|\Delta_3|, |\Delta_4|) \neq (9, 54)$. ■

From Proposition 10.3.1, it follows that $(|\Delta_3|, |\Delta_4|) = (27, 36)$, that is $|T_3| = 16$ and $|T_4| = 12$. From the elementary group theory, we know that there are, up to isomorphism, 14 groups of order 16 and 5 groups of order 12 (see GAP). Since we are interested in two groups T_3 and T_4 of orders 16 and 12 such that $|\mathrm{Irr}(T_3)| + |\mathrm{Irr}(T_4)| = 13$, it follows that there are 3 groups of order 16,

each having 7 characters and 2 groups of order 12 each having 6 characters. The three groups of order 16 each having 7 characters are $D_{16},\ QD_{16}$ and Q_{16}, while the two groups of order 12 each having 6 characters are 3:4 and D_{12}, where QD_{16} is the *quasihedral* group of order 16 and Q_{16} is the *quaternion* group of order 16. For conjugacy classes, character tables and structure of these groups, readers are advised to consult GAP. According to the above, possible pairs representing (T_3, T_4) are

$$(T_3, T_4) \in \{(D_{16}, 3{:}4), (D_{16}, D_{12}), (QD_{16}, 3{:}4), (QD_{16}, D_{12}), (Q_{16}, 3{:}4), (Q_{16}, D_{12})\}.$$

Proposition 10.3.2. *The pair $(T_3, T_4) = (QD_{16}, D_{12})$ is feasible.*

PROOF. Recall that the Fischer matrix $\mathcal{F}_{q_i}$ of H_2, corresponding to q_i, is of size $c(q_i)$ for each $i \in \{1, 2, \cdots, 14\}$. Also for each i, there are exactly two rows in $\mathcal{F}_{q_i}$ correspond to T_1 and T_2 and these two rows are labeled by $(1, 1)$ and $(2, 1)$. Therefore the two groups T_3 and T_4 contribute together $c(q_i) - 2$ rows to $\mathcal{F}_{q_i},\ \forall i \in \{1, 2, \cdots, 14\}$. From Table 10.5 we can deduce the values of all $c(q_i)$. In particular we have $c(q_2) = 4,\ c(q_3) = 5$ and $c(q_6) = c(q_7) = 3$. We will use this to show that (T_3, T_4) can be (QD_{16}, D_{12}). The group D_{16} has three classes of involutions while the two groups 3:4 and D_{12} have one and three classes of involutions respectively. Thus if $(T_3, T_4) = (D_{16}, 3{:}4)$ or (D_{16}, D_{12}), then the sizes (recall that we have two rows from T_1 and T_2) of the Fischer matrices $\mathcal{F}_{q_2}$ and $\mathcal{F}_{q_3}$ will be 6×4 and 8×5 respectively, but this clearly contradicts the fact that the Fischer matrices of any extension are squares (see Proposition 5.2.2(i)). Hence $(T_3, T_4) \notin \{(D_{16}, 3{:}4), (D_{16}, D_{12})\}$. The group Q_{16} has three classes of elements of order 4. For similar reasons if $(T_3, T_4) = (Q_{16}, 3{:}4)$ or (Q_{16}, D_{12}), then the Fischer matrices $\mathcal{F}_{q_6}$ and $\mathcal{F}_{q_7}$ will be of sizes 7×3 and 5×3, respectively, which is a contradiction again. Thus $(T_3, T_4) \notin \{(Q_{16}, 3{:}4), (Q_{16}, D_{12})\}$. Finally if $(T_3, T_4) = (QD_{16}, 3{:}4)$, then the Fischer matrices $\mathcal{F}_{q_6}$ and $\mathcal{F}_{q_7}$ will be of sizes 6×3 and 6×3, respectively, which is a contradiction again. Hence the case $(T_3, T_4) = (QD_{16}, D_{12})$ is feasible, that is $T_3 = QD_{16}$ and $T_4 = D_{12}$. ■

Proposition 10.3.2 asserts that (QD_{16}, D_{12}) could be a possible candidate for the inertia factor groups T_3 and T_4 respectively, but we are not yet sure whether the group $Q = (3^{1+2}{:}8){:}2 = \langle \overline{q}_1, \overline{q}_2 \rangle$ does contain subgroups of orders 16 and 12 that are isomorphic to QD_{16} and D_{12} respectively. By further computations (done through GAP) on the structures of the maximal subgroups $Q[1],\ Q[2],\ Q[3]$ and $Q[4]$ of Q, we have found that the group Q contains two subgroups of orders 16 and 12 such that these subgroups are isomorphic to QD_{16} and D_{12} respectively. Hence $T_3 = QD_{16}$

and $T_4 = D_{12}$. As subgroups of $Q = \langle \overline{q}_1, \overline{q}_2 \rangle$, the group $T_3 = QD_{16}$ is generated as follows:

$$c_1 = \begin{pmatrix} 0&0&1&0&1&0&1&0&1&1&0 \\ 0&0&1&1&0&1&1&0&1&0&0 \\ 0&1&0&0&1&0&0&1&1&1&0 \\ 1&0&1&1&1&0&0&1&0&0&0 \\ 0&1&1&1&1&1&0&1&0&0&0 \\ 0&0&1&0&1&1&0&0&1&1&0 \\ 1&0&1&0&1&0&0&0&1&1&0 \\ 1&1&0&0&0&0&1&1&1&0&0 \\ 1&1&0&1&1&1&0&0&1&1&0 \\ 0&0&1&0&1&0&0&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}, \quad c_2 = \begin{pmatrix} 1&0&1&0&1&0&0&0&1&0&0 \\ 0&1&1&0&1&0&0&0&1&0&0 \\ 1&1&0&1&0&1&1&0&1&1&0 \\ 0&0&1&0&0&1&1&0&0&1&0 \\ 1&1&1&1&1&1&1&0&1&1&0 \\ 0&0&1&0&1&1&0&0&1&0&0 \\ 0&0&1&0&1&0&1&0&1&0&0 \\ 0&1&0&1&1&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 1&1&1&0&1&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix},$$

while $T_4 = D_{12}$ is generated by:

$$d_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&1&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0 \\ 1&1&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}, \quad d_2 = \begin{pmatrix} 1&1&0&0&0&0&1&0&1&1&0 \\ 1&1&0&1&1&1&1&0&1&0&0 \\ 0&0&1&1&1&1&0&1&0&1&0 \\ 0&0&0&0&1&1&0&1&0&0&0 \\ 0&0&0&0&1&0&0&1&1&0&0 \\ 1&1&0&0&0&1&0&0&1&1&0 \\ 0&1&0&0&0&0&0&0&1&1&0 \\ 1&1&0&1&1&1&0&1&0&1&0 \\ 1&1&0&1&1&1&0&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix},$$

$$d_3 = \begin{pmatrix} 0&0&0&1&1&1&0&0&1&0&0 \\ 0&0&0&1&1&1&0&0&0&0&0 \\ 1&0&0&0&1&0&1&1&0&0&0 \\ 1&1&1&1&1&0&0&1&1&1&0 \\ 0&1&0&0&1&0&1&1&0&0&0 \\ 0&1&0&1&1&0&0&0&0&0&0 \\ 0&1&1&1&0&1&1&0&1&0&0 \\ 1&1&0&0&1&1&1&0&0&0&0 \\ 0&0&1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}.$$

The fusions of classes of $T_3 = QD_{16}$ and $T_4 = D_{12}$ into classes of $Q = (3^{1+2}$:8):2 are given in Table 10.6.

Table 10.6: The fusions of conjugacy classes of T_3 and T_4 into classes of Q

Inertia Factor Groups T_3 & T_4	Class of T_i	$\hookrightarrow$	Class of Q	Class of T_i	$\hookrightarrow$	Class of Q
$T_3 = QD_{16}$	q_{131}		1_1	q_{731}		4_2
	q_{231}		2_1	$q_{10,31}$		8_1
	q_{331}		2_2	$q_{11,31}$		8_2
	q_{631}		4_1			
$T_4 = D_{12}$	q_{141}		1_1	q_{241}		2_1
	q_{341}		2_2	q_{541}		3_2
	q_{342}		2_2	q_{941}		6_2

The character tables of T_3 and T_4 are well-known and they are available in GAP. For the convenience we supply the character tables of T_3 and T_4 in Tables 10.7 and 10.8 respectively.

Table 10.7: The character table of $T_3 = QD_{16}$

	q_{131}	q_{231}	q_{331}	q_{631}	q_{731}	$q_{10,31}$	$q_{11,31}$
$o(q_{i3m})$	1	2	2	4	4	8	8
$\lvert C_{T_3}(q_{ikm})\rvert$	16	16	4	8	4	8	8
χ_1	1	1	1	1	1	1	1
χ_2	1	1	−1	1	−1	1	1
χ_3	1	1	−1	1	1	−1	−1
χ_4	1	1	1	1	−1	−1	−1
χ_5	2	2	0	−2	0	0	0
χ_6	2	−2	0	0	0	$i\sqrt{2}$	$-i\sqrt{2}$
χ_7	2	−2	0	0	0	$-i\sqrt{2}$	$i\sqrt{2}$

10.3.4 Fischer Matrices of $H_2 = 2_-^{1+6}$:$((3^{1+2}$:8):2)

In this subsection we calculate the Fischer matrices of $H_2 = 2_-^{1+6}$:$((3^{1+2}$:8):2). From Section 5.2 we recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_{q_i}$, corresponding to q_i, by the sizes of the centralizers of $q_{ij},\ 1 \leq j \leq c(q_i)$, in H_2 and m_{ij} respectively. Also the rows of $\mathcal{F}_{q_i}$

Table 10.8: The character table of $T_4 = D_{12}$

	q_{141}	q_{341}	q_{342}	q_{241}	q_{541}	q_{941}
$o(q_{i4m})$	1	2	2	2	3	6
$\lvert C_{T_4}(q_{ikm})\rvert$	12	12	4	4	6	6
χ_1	1	1	1	1	1	1
χ_2	1	−1	1	−1	1	−1
χ_3	1	−1	−1	1	1	−1
χ_4	1	1	−1	−1	1	1
χ_5	2	−2	0	0	−1	1
χ_6	2	2	0	0	−1	−1

are partitioned into parts $\mathcal{F}_{ik},\ 1 \leq k \leq t$, corresponding to the inertia factors $H_1, H_2, \cdots, H_t$, where each $\mathcal{F}_{ik}$ consists of $c(q_{ik})$ rows correspond to the $\alpha_k^{-1}-$regular classes (those are the T_k-classes that fuse to class $[q_i]_Q$). Thus every row of $\mathcal{F}_{q_i}$ is labeled by the pair (k,m), where $1 \leq k \leq t$ and $1 \leq m \leq c(q_{ik})$. In Table 10.5 we supplied $|C_{H_2}(q_{ij})|$ and $m_{ij},\ 1 \leq i \leq 14,\ 1 \leq j \leq c(q_i)$. Also the fusions of classes of T_3 and T_4 are given in Table 10.6. Since the size of the Fischer matrix $\mathcal{F}_{q_i}$ is $c(q_i)$, it follows from Table 10.5 that the sizes of the Fischer matrices of $H_2 = 2^{1+6}_{-}$:((3^{1+2}:8):2) range between 2 and 5 for every $i \in \{1, 2, \cdots, 14\}$.

We have used the arithmetical properties of the Fischer matrices, given in Proposition 5.2.2, to calculate some of the entries of these matrices. In addition to these properties, we have the following important lemmas, which help us more in determining some entries of the Fischer matrices of H_2.

Lemma 10.3.3. *For every Fischer matrix $\mathcal{F}_{q_i}$, of size $c(q_i)$, the sum of the $1^{st}, 3^{rd}, 4^{th}, \cdots, c(q_i)^{th}$ rows equal (componentwise) the square of the modulus of the 2^{nd} row.*

PROOF. The proof is similar to the proof of Lemma 6 of Pahlings [57]. ■

Lemma 10.3.4. *For every Fischer matrix $\mathcal{F}_{q_i}$, we can order the q_{ij}, for $1 \leq j \leq c(q_i)$, so that the second row of $\mathcal{F}_{q_i}$ is of the form $[z_i \ \ -z_i \ \ 0 \ \ \cdots \ \ 0]$ and we may choose the $q_{i2} = \sigma q_{i1}$, where σ is the central involution in H_2. Furthermore*

$$a_{i1}^{(k,m)} = a_{i2}^{(k,m)} = \frac{|C_{H_k}(q_{i11})|}{|C_{H_k}(q_{ikm})|} \quad \text{for } k \in \{1,3,4\},\ 1 \leq m \leq c(g_{ik}). \tag{10.1}$$

PROOF. The proof is similar to the proof of Lemma 7 of Pahlings [57]. ■

Note 10.3.1. The proof of Lemma 7 of Pahlings [57] contained a very important piece of information, that is the last row of every Fischer matrix of $2^{1+22}_{+} \cdot Co_2$ is $[\eta(g_{i1}) \ \ \eta(g_{i2}) \ \ \cdots \ \ \eta(g_{is_i})]$, where

s_i in his notation has the same meaning of $c(q_i)$ in our notation. In our group H_2, the second row of every Fischer matrix $\mathcal{F}_{q_i}$ is given by $[\theta_2(q_{i1}) \quad \theta_2(q_{i2}) \quad 0 \quad \cdots \quad 0]$.

Note 10.3.2. Observe that with Lemma 10.3.3, Equation (10.1) and Note 10.3.1 we know the first two columns and the second row of every Fischer matrix $\mathcal{F}_{q_i}$. Also from Proposition 5.2.2(iii), we know the first row of every Fischer matrix $\mathcal{F}_{q_i}$. This reduces the number of unknowns in every Fischer matrix of size $c(q_i)$ to $c(q_i)^2 - 4c(q_i) + 4$.

Using the row and column orthogonality relations that the Fischer matrices satisfy (Proposition 5.2.2) we have built an algebraic system of equations. With the help of the symbolic mathematical package Maxima[45] we were able to solve these systems of equations and hence we have computed all the Fischer matrices of H_2, which we list below.

$\mathcal{F}_{q_1}$

q_1		q_{11}	q_{12}	q_{13}	q_{14}
$o(q_{1j})$		2	2	2	4
$\lvert C_{H_2}(q_{1j})\rvert$		55296	55296	1024	768
(k,m)	$\lvert C_{T_k}(q_{1km})\rvert$				
$(1,1)$	432	1	1	1	1
$(2,1)$	432	8	−8	0	0
$(3,1)$	16	27	27	−5	3
$(4,1)$	12	36	36	4	−4
m_{1j}		1	1	54	72

$\mathcal{F}_{q_2}$

q_2		q_{21}	q_{22}	q_{23}	q_{24}
$o(q_{2j})$		2	2	4	4
$\lvert C_{H_2}(q_{2j})\rvert$		1536	1536	256	64
(k,m)	$\lvert C_{T_k}(q_{2km})\rvert$				
$(1,1)$	48	1	1	1	1
$(2,1)$	48	4	−4	0	0
$(3,1)$	16	3	3	3	−1
$(4,1)$	4	12	12	−4	0
m_{2j}		4	4	24	96

$\mathcal{F}_{q_3}$

q_3		q_{31}	q_{32}	q_{33}	q_{34}	q_{35}
$o(q_{3j})$		8	8	2	4	8
$\lvert C_{H_2}(q_{3j})\rvert$		192	192	96	32	32
(k,m)	$\lvert C_{T_k}(q_{3km})\rvert$					
$(1,1)$	12	1	1	1	1	1
$(2,1)$	12	$2i\sqrt{2}$	$-2i\sqrt{2}$	0	0	0
$(3,1)$	4	3	3	3	−1	−1
$(4,1)$	12	1	1	−1	−1	1
$(4,2)$	4	3	3	−3	1	−1
m_{3j}		8	8	16	48	48

$\mathcal{F}_{q_4}$

q_4		q_{41}	q_{42}
$o(q_{4j})$		3	6
$\lvert C_{H_2}(q_{4j})\rvert$		432	432
(k,m)	$\lvert C_{T_k}(q_{4km})\rvert$		
$(1,1)$	216	1	1
$(2,1)$	216	−1	1
m_{4j}		64	64

$\mathcal{F}_{q_5}$

q_5		q_{51}	q_{52}	q_{53}
$o(q_{5j})$		3	6	12
$\lvert C_{H_2}(q_{5j})\rvert$		144	144	24
(k,m)	$\lvert C_{T_k}(q_{5km})\rvert$			
$(1,1)$	18	1	1	1
$(2,1)$	18	2	-2	0
$(4,1)$	6	3	3	-1
m_{5j}		16	16	96

$\mathcal{F}_{q_6}$

q_6		q_{61}	q_{62}	q_{63}
$o(q_{6j})$		4	4	8
$\lvert C_{H_2}(q_{6j})\rvert$		192	192	32
(k,m)	$\lvert C_{T_k}(q_{6km})\rvert$			
$(1,1)$	24	1	1	1
$(2,1)$	24	2	-2	0
$(3,1)$	8	3	3	-1
m_{6j}		16	16	96

$\mathcal{F}_{q_7}$

q_7		q_{71}	q_{72}	q_{73}
$o(q_{7j})$		4	4	8
$\lvert C_{H_2}(q_{7j})\rvert$		96	96	16
(k,m)	$\lvert C_{T_k}(q_{7km})\rvert$			
$(1,1)$	12	1	1	1
$(2,1)$	12	2	-2	0
$(3,1)$	4	3	3	-1
m_{7j}		16	16	96

$\mathcal{F}_{q_8}$

q_8		q_{81}	q_{82}
$o(q_{8j})$		6	6
$\lvert C_{H_2}(q_{8j})\rvert$		48	48
(k,m)	$\lvert C_{T_k}(q_{8km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{8j}		64	64

$\mathcal{F}_{q_9}$

q_9		q_{91}	q_{92}	q_{93}
$o(q_{9j})$		6	24	24
$\lvert C_{H_2}(q_{9j})\rvert$		24	24	12
(k,m)	$\lvert C_{T_k}(q_{9km})\rvert$			
$(1,1)$	6	1	1	1
$(2,1)$	6	$i\sqrt{2}$	$-i\sqrt{2}$	0
$(4,1)$	6	1	1	-1
m_{9j}		32	32	64

$\mathcal{F}_{q_{10}}$

q_{10}		$q_{10,1}$	$q_{10,2}$	$q_{10,3}$
$o(q_{10j})$		8	8	16
$\lvert C_{H_2}(q_{10j})\rvert$		32	32	16
(k,m)	$\lvert C_{T_k}(q_{10km})\rvert$			
$(1,1)$	8	1	1	1
$(2,1)$	8	$i\sqrt{2}$	$-i\sqrt{2}$	0
$(3,1)$	8	1	1	-1
m_{10j}		32	32	64

$\mathcal{F}_{q_{11}}$

q_{11}		$q_{11,1}$	$q_{11,2}$	$q_{11,3}$
$o(q_{11j})$		8	8	16
$\lvert C_{H_2}(q_{11j})\rvert$		32	32	16
(k,m)	$\lvert C_{T_k}(q_{11km})\rvert$			
$(1,1)$	8	1	1	1
$(2,1)$	8	$i\sqrt{2}$	$-i\sqrt{2}$	0
$(3,1)$	8	1	1	-1
m_{11j}		32	32	64

$\mathcal{F}_{q_{12}}$

q_{12}		$q_{12,1}$	$q_{12,2}$
$o(q_{12j})$		12	12
$\lvert C_{H_2}(q_{12j})\rvert$		24	24
(k,m)	$\lvert C_{T_k}(q_{12km})\rvert$		
$(1,1)$	12	1	1
$(2,1)$	12	1	-1
m_{12j}		64	64

$\mathcal{F}_{q_{13}}$			
q_{13}		$q_{13,1}$	$q_{13,2}$
$o(q_{13j})$		12	12
$\lvert C_{H_2}(q_{13j})\rvert$		24	24
(k,m)	$\lvert C_{T_k}(q_{13km})\rvert$		
$(1,1)$	12	1	1
$(2,1)$	12	1	-1
m_{13j}		64	64

$\mathcal{F}_{q_{14}}$			
q_{14}		$q_{14,1}$	$q_{14,2}$
$o(q_{14j})$		12	12
$\lvert C_{H_2}(q_{14j})\rvert$		24	24
(k,m)	$\lvert C_{T_k}(q_{14km})\rvert$		
$(1,1)$	12	1	1
$(2,1)$	12	1	-1
m_{14j}		64	64

Remark 10.3.2. In the above, if we omit the first column and the second row of every Fischer matrix $\mathcal{F}_{q_i}$, we obtain the corresponding Fischer matrix $\widetilde{\mathcal{F}}_{q_i}$ of the split extension $H = 2^6$:$((3^{1+2}$:$8)$:$2)$. This shows that the group $H = 2^6$:$((3^{1+2}$:$8)$:$2)$ has $\sum_{i=1}^{14} c(q_i) - 14 = 41 - 14 = 27$ conjugacy classes, which is also equal to $14 + 7 + 6 = |\mathrm{Irr}((3^{1+2}$:$8)$:$2)| + |\mathrm{Irr}(QD_{16})| + |\mathrm{Irr}(D_{12})|$ = the number of irreducible characters of H.

10.3.5 The Character Table of $H_2 = 2^{1+6}_{-}$:$((3^{1+2}$:$8)$:$2)$

Through Subsections 10.3.2, 10.3.3 and 10.3.4 we have found the following items:

- the conjugacy classes of $H_2 = 2^{1+6}_{-}$:$((3^{1+2}$:$8)$:$2)$ (Table 10.5),
- the fusions of classes of the inertia factors T_3 and T_4 into classes of Q (Table 10.6),
- the character tables of the inertia factors $T_1 = T_2$, T_3 and T_4 (Tables 10.4, 10.7 and 10.8 respectively),
- the Fischer matrices of H_2 (see Subsection 10.3.4).

By Section 5.3, it follows that the full character table of H_2 can be constructed easily. We give an example on how to construct the character table of H_2, which is partitioned into 56 parts corresponding to the 14 cosets and the four inertia factor groups. As an example we construct the parts $\mathcal{K}_{21}\mathcal{F}_{21}$, $\mathcal{K}_{22}\mathcal{F}_{22}$, $\mathcal{K}_{23}\mathcal{F}_{23}$ and $\mathcal{K}_{24}\mathcal{F}_{24}$ of the character table of H_2. This means that we are listing the values of all the irreducible characters of H_2 on the classes q_{21}, q_{22}, q_{23} and q_{24} of H_2, which correspond to the conjugacy class of Q represented by $q_2 = 2_1$. The four parts $\mathcal{K}_{21}\mathcal{F}_{21}$, $\mathcal{K}_{22}\mathcal{F}_{22}$ $\mathcal{K}_{23}\mathcal{F}_{23}$ and $\mathcal{K}_{24}\mathcal{F}_{24}$ can be derived as follows: From Table 10.6 we can see that there is a class of T_3, namely q_{231} and a class of T_4, namely q_{241} that fuse into the class $[q_2]_Q = [2_1]_Q$. To construct the

part $\mathcal{K}_{21}\mathcal{F}_{21}$, we multiply the column of the character table of $T_1 = Q = (3^{1+2}$:8):2 corresponding to the class 2_1 of Q (see Table 10.4), by the first row of $\mathcal{F}_{q_1}$, namely $(1 \quad 1 \quad 1 \quad 1)$ and thus the part $\mathcal{K}_{21}\mathcal{F}_{21}$ of size 14×4, consists of the column of the character table of Q corresponding to the class 2_1 repeated 4 times. To construct the part $\mathcal{K}_{22}\mathcal{F}_{22}$, we multiply again the column of the character table of $T_2 = Q$, corresponding to the class 2_1 of Q, by the row of $\mathcal{F}_{q_1}$ labeled by the pair $(2, 1)$. Thus we get a part in the character table of H_2 of size 14×4. Similar arguments can be used to construct the parts $\mathcal{K}_{23}\mathcal{F}_{23}$ and $\mathcal{K}_{24}\mathcal{F}_{24}$. The above four parts have the following forms:

$$\mathcal{K}_{21}\mathcal{F}_{21} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ 0 \\ 0 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{matrix} q_{21} & q_{22} & q_{23} & q_{24} \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 4 & 4 & 4 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{22}\mathcal{F}_{22} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ 0 \\ 0 \\ 4 \end{pmatrix} \begin{pmatrix} 4 & -4 & 0 & 0 \end{pmatrix} = \begin{array}{c} \begin{matrix} q_{21} & q_{22} & q_{23} & q_{24} \end{matrix} \\ \begin{pmatrix} 4 & -4 & 0 & 0 \\ 4 & -4 & 0 & 0 \\ 4 & -4 & 0 & 0 \\ 4 & -4 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ 8 & -8 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ -8 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 16 & -16 & 0 & 0 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{23}\mathcal{F}_{23} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ -2 \\ -2 \end{pmatrix} \begin{pmatrix} 3 & 3 & 3 & -1 \end{pmatrix} = \begin{array}{c} \begin{matrix} q_{21} & q_{22} & q_{23} & q_{24} \end{matrix} \\ \begin{pmatrix} 3 & 3 & 3 & -1 \\ 3 & 3 & 3 & -1 \\ 3 & 3 & 3 & -1 \\ 3 & 3 & 3 & -1 \\ 6 & 6 & 6 & -2 \\ -6 & -6 & -6 & 2 \\ -6 & -6 & -6 & 2 \end{pmatrix} \end{array},$$

$$\mathcal{K}_{24}\mathcal{F}_{24} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 4 & 0 \end{pmatrix} = \begin{array}{c} \begin{matrix} q_{21} & q_{22} & q_{23} & q_{24} \end{matrix} \\ \begin{pmatrix} 12 & 12 & 4 & 0 \\ 12 & 12 & 4 & 0 \\ -12 & -12 & -4 & 0 \\ -12 & -12 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

Similarly one can obtain all the other 52 parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$, $1 \leq i \leq 14,\ i \neq 2,\ 1 \leq k \leq 4$ and hence the full character table of H_2, which is a 41×41 $\mathbb{C}-$valued matrix. The full character table of H_2 is listed in Table 11.15. Alternatively one can use the generators μ_1 and μ_2 to generate H_2 inside GAP and then display its character table and then compare with Table 11.15, where one can see clearly the identifications after some suitable re-arrangement of columns and rows. We also determined the fusion of classes of H_2 into classes of $G = U_5(2)$:2 which are listed in Table 10.9

10.4. Fischer Matrices of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$

Similar to Subsection 10.3.4 we have calculated the Fischer matrices of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$. In Table 10.1 we supplied $|C_{\overline{G}}(g_{ij})|$ and $m_{ij},\ 1 \leq i \leq 43,\ 1 \leq j \leq c(g_i)$. Also the fusions of classes of H_2 and H_3 are given in Tables 10.9 and 10.3 respectively. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 10.1 that the sizes of the Fischer matrices of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$ range between 1 and 5 for every $i \in \{1, 2, \cdots, 43\}$.

We have used the arithmetical properties of the Fischer matrices, given in Proposition 5.2.2, to calculate some of the entries of these matrices and also to build an algebraic system of equations. For example since the extension splits and $N = 2^{10}$ is abelian, then every coset $N\overline{g}_i$ (or just Ng_i)

Table 10.9: The fusion of classes of H_2 into classes of G

Class of H_2	$\hookrightarrow$	Class of $U_5(2)$:2	Class of H_2	$\hookrightarrow$	Class of $U_5(2)$:2
$1a = g_{121}$		$1A$	$8a = g_{24,21}$		$8A$
$2a = g_{221}$		$2A$	$8b = g_{25,21}$		$8B$
$2b = g_{222}$		$2A$	$8c = g_{26,21}$		$8C$
$2c = g_{321}$		$2B$	$8d = g_{27,21}$		$8D$
$2d = g_{322}$		$2B$	$8e = g_{27,22}$		$8D$
$2e = g_{421}$		$2C$	$8f = g_{26,22}$		$8C$
$3a = g_{721}$		$3C$	$8g = g_{24,22}$		$8A$
$3b = g_{621}$		$3B$	$8h = g_{25,22}$		$8B$
$4a = g_{921}$		$4A$	$8i = g_{26,23}$		$8C$
$4b = g_{10,21}$		$4B$	$12a = g_{33,21}$		$12B$
$4c = g_{11,21}$		$4C$	$12b = g_{34,21}$		$12C$
$4d = g_{12,21}$		$4D$	$12c = g_{36,21}$		$12E$
$4e = g_{922}$		$4A$	$12d = g_{34,22}$		$12C$
$4f = g_{10,22}$		$4B$	$12e = g_{36,22}$		$12E$
$4g = g_{923}$		$4A$	$12f = g_{36,23}$		$12E$
$4h = g_{10,23}$		$4B$	$12g = g_{34,23}$		$12C$
$6a = g_{15,21}$		$6B$	$16a = g_{39,21}$		$16A$
$6b = g_{16,21}$		$6C$	$16b = g_{40,21}$		$16B$
$6c = g_{19,21}$		$6F$	$24a = g_{42,21}$		$24A$
$6d = g_{17,21}$		$6D$	$24b = g_{43,21}$		$24B$
$6e = g_{23,21}$		$6J$			

is a split coset (see Schiffer [67]) and it results that $a_{i1}^{(k,m)} = \frac{|C_{U_5(2):2}(g_i)|}{|C_{H_k}(g_{ikm})|},\ \forall i \in \{1, 2, \cdots, 43\}$. With the help of the symbolic mathematical package Maxima [45], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}
$o(g_{1j})$		1	2	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		28028436480	56623104	53084160
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$			
$(1,1)$	27371520	1	1	1
$(2,1)$	55296	495	15	-17
$(3,1)$	51840	528	-16	16
m_{1j}		1	495	528

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}	g_{24}
$o(g_{2j})$		2	2	2	4
$\lvert C_{\overline{G}}(g_{2j})\rvert$		42467328	1572864	1179648	221184
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$				
$(1,1)$	165888	1	1	1	1
$(2,1)$	55296	3	3	3	-1
$(2,2)$	1536	108	-20	12	0
$(3,1)$	1152	144	16	-16	0
m_{2j}		4	108	144	768

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}	g_{34}
$o(g_{3j})$		2	2	4	4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		589824	196608	24576	16384
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$				
$(1,1)$	9216	1	1	1	1
$(2,1)$	1536	6	6	2	-2
$(2,2)$	1024	9	9	-3	1
$(3,1)$	192	48	-16	0	0
m_{3j}		16	48	384	576

$\mathcal{F}_4$

g_4		g_{41}	g_{42}	g_{43}	g_{44}
$o(g_{4j})$		2	4	4	4
$\lvert C_{\overline{G}}(g_{4j})\rvert$		46080	46080	3072	3072
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$				
$(1,1)$	1440	1	1	1	1
$(2,1)$	96	15	15	-1	-1
$(3,1)$	1440	1	-1	1	-1
$(3,2)$	96	15	-15	-1	1
m_{4j}		32	32	480	480

$\mathcal{F}_5$

g_5		g_{51}
$o(g_{5j})$		3
$\lvert C_{\overline{G}}(g_{5j})\rvert$		77760
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$	
$(1,1)$	77760	1
m_{5j}		1024

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}
$o(g_{6j})$		3	6	6
$\lvert C_{\overline{G}}(g_{6j})\rvert$		248832	9216	6912
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$			
$(1,1)$	3888	1	1	1
$(2,1)$	144	27	3	-5
$(3,1)$	108	36	-4	4
m_{6j}		16	432	576

$\mathcal{F}_7$

g_7		g_{71}	g_{72}	g_{73}
$o(g_{7j})$		3	6	6
$\lvert C_{\overline{G}}(g_{7j})\rvert$		6912	1152	768
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$			
$(1,1)$	432	1	1	1
$(2,1)$	48	9	-3	1
$(3,1)$	72	6	2	-2
m_{7j}		64	384	576

$\mathcal{F}_8$

g_8		g_{81}	g_{82}
$o(g_{8j})$		3	6
$\lvert C_{\overline{G}}(g_{8j})\rvert$		2592	864
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$		
$(1,1)$	648	1	1
$(3,1)$	216	3	-1
m_{8j}		256	768

$\mathcal{F}_9$

g_9		g_{91}	g_{92}	g_{93}	g_{94}	g_{95}
$o(g_{9j})$		4	4	4	4	4
$\lvert C_{\overline{G}}(g_{9j})\rvert$		147456	49152	24576	24576	3072
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$					
$(1,1)$	2304	1	1	1	1	1
$(2,1)$	768	3	3	3	3	-1
$(2,2)$	192	12	12	-4	-4	0
$(2,3)$	96	24	-8	-8	8	0
$(3,1)$	96	24	-8	8	-8	0
m_{9j}		16	48	96	96	768

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{10,4}$
$o(g_{10j})$		4	4	4	4
$\lvert C_{\overline{G}}(g_{10j})\rvert$		12288	12288	6144	1024
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$				
$(1,1)$	768	1	1	1	1
$(2,1)$	256	3	3	3	-1
$(2,2)$	192	4	4	-4	0
$(2,3)$	96	8	-8	0	0
m_{10j}		64	64	128	768

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$	$g_{11,3}$
$o(g_{11j})$		4	4	8
$\lvert C_{\overline{G}}(g_{11j})\rvert$		3072	1024	256
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	64	3	3	-1
$(3,1)$	16	12	-4	0
m_{11j}		64	192	768

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{12,4}$
$o(g_{12j})$		4	4	8	8
$\lvert C_{\overline{G}}(g_{12j})\rvert$		768	768	256	256
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$				
$(1,1)$	96	1	1	1	1
$(2,1)$	32	3	3	-1	-1
$(3,1)$	96	1	-1	1	-1
$(3,2)$	32	3	-3	-1	1
m_{12j}		128	128	384	384

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$
$o(g_{13j})$		5	10
$\lvert C_{\overline{G}}(g_{13j})\rvert$		120	40
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$		
$(1,1)$	30	1	1
$(3,1)$	10	3	-1
m_{13j}		256	768

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$
$o(g_{14j})$		6
$\lvert C_{\overline{G}}(g_{14j})\rvert$		1728
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$	
$(1,1)$	1728	1
m_{14j}		1024

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$
$o(g_{15j})$		6	12
$\lvert C_{\overline{G}}(g_{15j})\rvert$		5184	1728
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$		
$(1,1)$	1296	1	1
$(2,1)$	432	3	-1
m_{15j}		256	768

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$	$g_{16,3}$
$o(g_{16j})$		6	6	12
$\lvert C_{\overline{G}}(g_{16j})\rvert$		6912	2304	576
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$			
$(1,1)$	432	1	1	1
$(2,1)$	144	3	3	-1
$(3,1)$	36	12	-4	0
m_{16j}		64	192	768

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$	$g_{17,3}$
$o(g_{17j})$		6	6	6
$\lvert C_{\overline{G}}(g_{17j})\rvert$		6912	1152	768
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$			
$(1,1)$	432	1	1	1
$(2,1)$	48	9	-3	1
$(3,1)$	72	6	2	-2
m_{17j}		64	384	576

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$
$o(g_{18j})$		6
$\lvert C_{\overline{G}}(g_{18j})\rvert$		288
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$	
$(1,1)$	288	1
m_{18j}		1024

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		6	12
$\lvert C_{\overline{G}}(g_{19j})\rvert$		576	192
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$		
$(1,1)$	144	1	1
$(2,1)$	48	3	-1
m_{19j}		256	768

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$	$g_{20,2}$
$o(g_{20j})$		6	6
$\lvert C_{\overline{G}}(g_{20j})\rvert$		432	144
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$		
$(1,1)$	108	1	1
$(3,1)$	36	3	-1
m_{20j}		256	768

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$	$g_{21,2}$
$o(g_{21j})$		6	6
$\lvert C_{\overline{G}}(g_{21j})\rvert$		288	96
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$		
$(1,1)$	72	1	1
$(3,1)$	24	3	-1
m_{21j}		256	768

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$	$g_{22,2}$
$o(g_{22j})$		6	12
$\lvert C_{\overline{G}}(g_{22j})\rvert$		72	72
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$		
$(1,1)$	36	1	1
$(3,1)$	36	1	-1
m_{22j}		512	512

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$	$g_{23,3}$	$g_{23,4}$
$o(g_{23j})$		6	12	12	12
$\lvert C_{\overline{G}}(g_{23j})\rvert$		288	288	96	96
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$				
$(1,1)$	36	1	1	1	1
$(2,1)$	12	3	3	-1	-1
$(3,1)$	36	1	-1	1	-1
$(3,2)$	12	3	-3	-1	1
m_{23j}		128	128	384	384

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$	$g_{24,2}$	$g_{24,3}$
$o(g_{24j})$		8	8	8
$\lvert C_{\overline{G}}(g_{24j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{24j}		128	384	512

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$	$g_{25,2}$	$g_{25,3}$
$o(g_{25j})$		8	8	8
$\lvert C_{\overline{G}}(g_{25j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{25j}		128	384	512

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$	$g_{26,2}$	$g_{26,3}$	$g_{26,4}$	$g_{26,5}$
$o(g_{26j})$		8	8	8	8	8
$\lvert C_{\overline{G}}(g_{26j})\rvert$		256	256	128	128	128
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$					
$(1,1)$	32	1	1	1	1	1
$(2,1)$	32	1	1	-1	1	-1
$(2,2)$	32	1	1	1	-1	-1
$(2,3)$	32	1	1	-1	-1	1
$(3,1)$	8	4	-4	0	0	0
m_{26j}		128	128	256	256	256

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$	$g_{27,2}$	$g_{27,3}$
$o(g_{27j})$		8	8	8
$\lvert C_{\overline{G}}(g_{27j})\rvert$		128	128	64
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$			
$(1,1)$	32	1	1	1
$(2,1)$	32	1	1	-1
$(2,2)$	16	2	-2	0
m_{27j}		256	256	512

$\mathcal{F}_{28}$

g_{28}		$g_{28,1}$
$o(g_{28j})$		9
$\lvert C_{\overline{G}}(g_{28j})\rvert$		54
(k,m)	$\lvert C_{H_k}(g_{28km})\rvert$	
$(1,1)$	54	1
m_{28j}		1024

$\mathcal{F}_{29}$

g_{29}		$g_{29,1}$	$g_{29,2}$
$o(g_{29j})$		9	18
$\lvert C_{\overline{G}}(g_{29j})\rvert$		108	36
(k,m)	$\lvert C_{H_k}(g_{29km})\rvert$		
$(1,1)$	27	1	1
$(3,1)$	9	3	-1
m_{29j}		256	768

$\mathcal{F}_{30}$

g_{30}		$g_{30,1}$	$g_{30,2}$
$o(g_{30j})$		10	20
$\lvert C_{\overline{G}}(g_{30j})\rvert$		20	20
(k,m)	$\lvert C_{H_k}(g_{30km})\rvert$		
$(1,1)$	10	1	1
$(3,1)$	10	1	-1
m_{30j}		512	512

$\mathcal{F}_{31}$

g_{31}		$g_{31,1}$
$o(g_{31j})$		11
$\lvert C_{\overline{G}}(g_{31j})\rvert$		11
(k,m)	$\lvert C_{H_k}(g_{31km})\rvert$	
$(1,1)$	11	1
m_{31j}		1024

$\mathcal{F}_{32}$

g_{32}		$g_{32,1}$
$o(g_{32j})$		12
$\lvert C_{\overline{G}}(g_{32j})\rvert$		144
(k,m)	$\lvert C_{H_k}(g_{32km})\rvert$	
$(1,1)$	144	1
m_{32j}		1024

$\mathcal{F}_{33}$

g_{33}		$g_{33,1}$	$g_{33,2}$
$o(g_{33j})$		12	12
$\lvert C_{\overline{G}}(g_{33j})\rvert$		288	96
(k,m)	$\lvert C_{H_k}(g_{33km})\rvert$		
$(1,1)$	72	1	1
$(2,1)$	24	3	-1
m_{33j}		256	768

$\mathcal{F}_{34}$

g_{34}		$g_{34,1}$	$g_{34,2}$	$g_{34,3}$	$g_{34,4}$	$g_{34,5}$
$o(g_{34j})$		12	12	12	12	12
$\lvert C_{\overline{G}}(g_{34j})\rvert$		1152	384	384	384	192
(k,m)	$\lvert C_{H_k}(g_{34km})\rvert$					
$(1,1)$	72	1	1	1	1	1
$(2,1)$	24	3	-1	-1	3	-1
$(2,2)$	24	3	-1	3	-1	-1
$(2,3)$	24	3	3	-1	-1	-1
$(3,1)$	12	6	-2	-2	-2	2
m_{34j}		64	192	192	192	384

$\mathcal{F}_{35}$

g_{35}		$g_{35,1}$
$o(g_{35j})$		12
$\lvert C_{\overline{G}}(g_{35j})\rvert$		24
(k,m)	$\lvert C_{H_k}(g_{35km})\rvert$	
$(1,1)$	24	1
m_{35j}		1024

$\mathcal{F}_{36}$

g_{36}		$g_{36,1}$	$g_{36,2}$	$g_{36,3}$	$g_{36,4}$
$o(g_{36j})$		12	12	12	12
$\lvert C_{\overline{G}}(g_{36j})\rvert$		96	96	96	96
(k,m)	$\lvert C_{H_k}(g_{36km})\rvert$				
$(1,1)$	24	1	1	1	1
$(2,1)$	24	1	-1	1	-1
$(2,2)$	24	1	1	-1	-1
$(2,3)$	24	1	-1	-1	1
m_{36j}		256	256	256	256

$\mathcal{F}_{37}$

g_{37}		$g_{37,1}$	$g_{37,2}$
$o(g_{37j})$		12	12
$\lvert C_{\overline{G}}(g_{37j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{37km})\rvert$		
$(1,1)$	12	1	1
$(3,1)$	12	1	-1
m_{37j}		512	512

$\mathcal{F}_{38}$

g_{38}		$g_{38,1}$
$o(g_{38j})$		15
$\lvert C_{\overline{G}}(g_{38j})\rvert$		15
(k,m)	$\lvert C_{H_k}(g_{38km})\rvert$	
$(1,1)$	15	1
m_{38j}		1024

$\mathcal{F}_{39}$

g_{39}		$g_{39,1}$	$g_{39,2}$
$o(g_{39j})$		16	16
$\lvert C_{\overline{G}}(g_{39j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{39km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{39j}		512	512

$\mathcal{F}_{40}$

g_{40}		$g_{40,1}$	$g_{40,2}$
$o(g_{40j})$		16	16
$\lvert C_{\overline{G}}(g_{40j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{40km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{40j}		512	512

$\mathcal{F}_{41}$

g_{41}		$g_{41,1}$
$o(g_{41j})$		18
$\lvert C_{\overline{G}}(g_{41j})\rvert$		18
(k,m)	$\lvert C_{H_k}(g_{41km})\rvert$	
$(1,1)$	18	1
m_{41j}		1024

$\mathcal{F}_{42}$

g_{42}		$g_{42,1}$	$g_{42,2}$
$o(g_{42j})$		24	24
$\lvert C_{\overline{G}}(g_{42j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{42km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{42j}		512	512

$\mathcal{F}_{43}$

g_{43}		$g_{43,1}$	$g_{43,2}$
$o(g_{43j})$		24	24
$\lvert C_{\overline{G}}(g_{43j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{43km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{43j}		512	512

10.5. The Character Table of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$

From Sections 10.2, 10.3 and 10.4 and the Appendix (Tables 11.15 and 11.16), we have

- the conjugacy classes of $\overline{G} = 2^{10}$:$(U_5(2)$:$2)$ (Table 10.1),

- the fusions of classes of the inertia factors H_2 and H_3 into classes of $U_5(2)$:2 (Tables 10.9 and 10.3 respectively),
- the character tables of the inertia factors H_2 and H_3 (Tables 11.15 and 11.16 respectively),
- the Fischer matrices of $\overline{G}$ (see Section 10.4).

By Section 5.3, it follows that the full character table of $\overline{G}$ can be constructed easily. One can apply similar arguments used in Subsection 10.3.5 to obtain the character table of $\overline{G}$, which is a 109×109 $\mathbb{C}-$valued matrix, partitioned into 129 parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$, where $1 \leq i \leq 43,\ 1 \leq k \leq 3$. The full character table of $\overline{G}$, in the format of Clifford-Fischer theory is given in Table 11.17. Alternatively for those who would like to see the character table of $\overline{G}$ (not necessarily in the format of Clifford-Fischer theory), one can use the generators $\overline{g}_1$ and $\overline{g}_2$, supplied at the introduction of this chapter, to generate $\overline{G}$ inside GAP and then ask for its character table.

Appendix

In Tables 11.1 and 11.2 respectively we supply the values of the permutation characters of $\overline{G} = 2^{6\cdot}Sp(6,2)$ (resp. $2^{8\cdot}Sp(8,2)$) on the set Ω (resp. Γ) discussed on Chapter 8 (resp. 9). Also we supply the character tables of the groups that have been discussed throughout the various chapters of this thesis, namely the character tables of the following groups:

Table 11.3	$H_2 = U_4(2)$	Chapter 6
Table 11.4	$H_3 = 2^5{:}A_6$	
Table 11.5	$H_4 = A_7$	
Table 11.6	$H_5 = (3 \times A_6){:}2$	
Table 11.7	$H_6 = (2^4{:}A_5){:}2$	
Table 11.8	$\overline{G} = 3^7{:}Sp(6,2)$	
Table 11.9	$2^{5\cdot}GL(5,2)$	Chapter 7
Table 11.10	$2_+^{1+8\cdot}A_9$	
Table 11.11	$H_2 = 2^5{:}S_6$	Chapter 8
Table 11.12	$\overline{G} = 2^{6\cdot}Sp(6,2)$	
Table 11.13	$2^{8\cdot}Sp(8,2)$	Chapter 9
Table 11.14	$H_1 = G = U_5(2){:}2$	Chapter 10
Table 11.15	$H_2 = 2_-^{1+6}{:}((3^{1+2}{:}8){:}2)$	
Table 11.16	$H_3 = Sp(4,3){:}2$	
Table 11.17	$\overline{G} = 2^{10}{:}(U_5(2){:}2)$	

Table 11.1: Values of the permutation character $\chi(\overline{G}|\Omega)$

$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$4g$	$2f$	$4h$	$4i$	$3a$
$\chi(\overline{G}\|\Omega)$	126	62	62	30	30	30	14	14	14	30	14	14	14	6	6	6	30
$[g]_{\overline{G}}$	$6a$	$3b$	$3c$	$6b$	$4j$	$4k$	$8a$	$8b$	$4l$	$8c$	$8d$	$4m$	$4n$	$4o$	$4p$	$8e$	$4q$
$\chi(\overline{G}\|\Omega)$	14	0	6	2	6	2	14	6	6	14	6	6	6	2	2	6	2
$[g]_{\overline{G}}$	$8f$	$4r$	$10a$	$5a$	$12a$	$12b$	$6c$	$6d$	$12c$	$6e$	$12d$	$6f$	$12e$	$12f$	$6g$	$6h$	$6i$
$\chi(\overline{G}\|\Omega)$	2	2	2	6	14	6	6	6	2	0	6	2	2	2	0	6	2
$[g]_{\overline{G}}$	$12g$	$6j$	$7a$	$8g$	$8h$	$8i$	$8j$	$9a$	$20a$	$10b$	$24a$	$12h$	$24b$	$12i$	$12j$	$15a$	
$\chi(\overline{G}\|\Omega)$	2	0	0	2	0	2	0	0	2	0	2	0	2	0	0	0	

Table 11.2: Values of the permutation character $\chi(\overline{G}|\Gamma)$

$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$
$\chi(\overline{G}\|\Gamma)$	510	254	254	126	126	126	62	62	126	62	62	62	30	14	62	30	30
$[g]_{\overline{G}}$	$4j$	$4k$	$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$
$\chi(\overline{G}\|\Gamma)$	30	30	30	14	14	14	126	62	0	30	14	6	2	62	30	30	62
$[g]_{\overline{G}}$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$	$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$
$\chi(\overline{G}\|\Gamma)$	30	30	30	14	14	30	14	14	14	14	6	6	14	6	6	14	6
$[g]_{\overline{G}}$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
$\chi(\overline{G}\|\Gamma)$	6	6	30	14	14	14	14	14	6	6	6	6	6	2	6	2	2

Table 11.2 (continued)

$[g]_{\overline{G}}$	$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$
$\chi(\overline{G}\|\Gamma)$	6	2	2	30	14	0	62	30	30	30	14	14	30	14	14
$[g]_{\overline{G}}$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$	$12k$	$12l$	$6m$	$6n$	$12m$
$\chi(\overline{G}\|\Gamma)$	14	30	14	0	2	2	14	6	6	6	14	6	6	6	2
$[g]_{\overline{G}}$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$
$\chi(\overline{G}\|\Gamma)$	6	2	0	6	2	2	6	2	14	6	6	2	0	6	2
$[g]_{\overline{G}}$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$	$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$
$\chi(\overline{G}\|\Gamma)$	2	6	2	14	6	6	14	6	6	6	2	2	6	2	2
$[g]_{\overline{G}}$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$
$\chi(\overline{G}\|\Gamma)$	2	0	2	0	6	2	0	14	6	6	6	2	6	2	2
$[g]_{\overline{G}}$	$10e$	$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$
$\chi(\overline{G}\|\Gamma)$	0	6	2	14	6	6	14	6	6	6	2	2	0	2	0
$[g]_{\overline{G}}$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$	$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$
$\chi(\overline{G}\|\Gamma)$	2	0	6	2	2	2	6	2	2	0	2	0	2	0	0
$[g]_{\overline{G}}$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
$\chi(\overline{G}\|\Gamma)$	2	0	6	2	0	0	0	0	2	0	2	0	2	0	0
$[g]_{\overline{G}}$	$24k$	$24l$	$24m$	$24n$	$60a$	$30b$	$30c$								
$\chi(\overline{G}\|\Gamma)$	2	0	2	0	2	0	0								

Table 11.3: Character table of $H_2 = U_4(2)$

	$1a$	$2a$	$2b$	$3a$	$3b$	$3c$	$3d$	$4a$	$4b$	$5a$	$6a$	$6b$	$6c$	$6d$	$6e$	$6f$	$9a$	$9b$	$12a$	$12b$
$\lvert C_{H_2}(g)\rvert$	25920	96	576	108	648	648	54	8	48	5	36	36	72	72	18	12	9	9	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	5	1	−3	−1	A	$\overline{A}$	2	−1	1	0	B	$-B$	C	$\overline{C}$	0	1	D	$\overline{D}$	$-\overline{D}$	$-D$
χ_3	5	1	−3	−1	$\overline{A}$	A	2	−1	1	0	$-B$	B	$\overline{C}$	C	0	1	$\overline{D}$	D	$-D$	$-\overline{D}$
χ_4	6	2	−2	3	−3	−3	0	0	2	1	1	1	1	1	−2	−1	0	0	−1	−1
χ_5	10	−2	2	1	E	$\overline{E}$	1	0	2	0	−1	−1	A	$\overline{A}$	−1	1	$-\overline{D}$	$-D$	$\overline{D}$	D
χ_6	10	−2	2	1	$\overline{E}$	E	1	0	2	0	−1	−1	$\overline{A}$	A	−1	1	$-D$	$-\overline{D}$	D	$\overline{D}$
χ_7	15	3	7	0	−3	−3	3	1	−1	0	−2	−2	1	1	1	0	0	0	−1	−1
χ_8	15	−1	−1	3	6	6	0	−1	3	0	−1	−1	2	2	2	−1	0	0	0	0
χ_9	20	4	4	5	2	2	−1	0	0	0	1	1	−2	−2	1	1	−1	−1	0	0
χ_{10}	24	0	8	0	6	6	3	0	0	−1	2	2	2	2	−1	0	0	0	0	0
χ_{11}	30	2	−10	3	3	3	3	0	−2	0	−1	−1	−1	−1	−1	−1	0	0	1	1
χ_{12}	30	2	6	−3	F	$\overline{F}$	0	0	2	0	$-B$	B	$\overline{C}$	C	0	−1	0	0	D	$\overline{D}$
χ_{13}	30	2	6	−3	$\overline{F}$	F	0	0	2	0	B	$-B$	C	$\overline{C}$	0	−1	0	0	$\overline{D}$	D
χ_{14}	40	0	−8	−2	G	$\overline{G}$	1	0	0	0	H	$\overline{H}$	H	$\overline{H}$	1	0	$-\overline{D}$	$-D$	0	0
χ_{15}	40	0	−8	−2	$\overline{G}$	G	1	0	0	0	$\overline{H}$	H	$\overline{H}$	H	1	0	$-D$	$-\overline{D}$	0	0
χ_{16}	45	−3	−3	0	I	$\overline{I}$	0	1	1	0	0	0	J	$\overline{J}$	0	0	0	0	$-D$	$-\overline{D}$
χ_{17}	45	−3	−3	0	$\overline{I}$	I	0	1	1	0	0	0	$\overline{J}$	J	0	0	0	0	$-\overline{D}$	$-D$
χ_{18}	60	4	−4	−3	6	6	−3	0	0	0	−1	−1	2	2	−1	1	0	0	0	0
χ_{19}	64	0	0	4	−8	−8	−2	0	0	−1	0	0	0	0	0	0	1	1	0	0
χ_{20}	81	−3	9	0	0	0	0	−1	−3	1	0	0	0	0	0	0	0	0	0	0

where in Table 11.3,

$A = \frac{1}{2} + \frac{3\sqrt{3}}{2}i$, $B = \sqrt{3}i$, $C = -\frac{3}{2} - \frac{\sqrt{3}}{2}i$, $D = \frac{1}{2} - \frac{\sqrt{3}}{2}i$, $E = -\frac{7}{2} + \frac{3\sqrt{3}}{2}i$,

$F = -\frac{3}{2} + \frac{9\sqrt{3}}{2}i$, $G = -5 - 3\sqrt{3}i$, $H = 1 - \sqrt{3}i$, $I = \frac{9}{2} + \frac{9\sqrt{3}}{2}i$, $J = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$.

Table 11.4: Character table of $H_3 = 2^5{:}A_6$

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$3a$	$3b$	$4a$	$4b$	$4c$	$4d$	$4e$	$5a$	$5b$	$6a$	$6b$	$6c$	$6d$	$8a$	$8b$	$10a$	$10b$
$\lvert C_{H_3}(g)\rvert$	11520	11520	768	768	64	64	72	18	64	64	16	16	16	10	10	24	24	72	18	16	16	10	10
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	5	5	5	5	1	1	2	−1	1	1	1	−1	−1	0	0	2	2	2	−1	−1	−1	0	0
χ_3	5	5	5	5	1	1	−1	2	1	1	1	−1	−1	0	0	−1	−1	−1	2	−1	−1	0	0
χ_4	6	−6	2	−2	2	−2	3	0	−2	2	0	0	0	1	1	1	−1	−3	0	0	0	−1	−1
χ_5	8	8	8	8	0	0	−1	−1	0	0	0	0	0	A	$A^\star$	−1	−1	−1	−1	0	0	A	$A^\star$
χ_6	8	8	8	8	0	0	−1	−1	0	0	0	0	0	$A^\star$	A	−1	−1	−1	−1	0	0	$A^\star$	A
χ_7	9	9	9	9	1	1	0	0	1	1	1	1	1	−1	−1	0	0	0	0	1	1	−1	−1
χ_8	10	10	10	10	−2	−2	1	1	−2	−2	−2	0	0	0	0	1	1	1	1	0	0	0	0
χ_9	10	−10	−2	2	2	−2	1	1	2	−2	0	−2	2	0	0	−1	1	−1	−1	0	0	0	0
χ_{10}	10	−10	−2	2	2	−2	1	1	2	−2	0	2	−2	0	0	−1	1	−1	−1	0	0	0	0
χ_{11}	10	−10	−2	2	−2	2	1	1	−2	2	0	0	0	0	0	−1	1	−1	−1	B	$-B$	0	0
χ_{12}	10	−10	−2	2	−2	2	1	1	−2	2	0	0	0	0	0	−1	1	−1	−1	$-B$	B	0	0
χ_{13}	15	15	−1	−1	−1	−1	3	0	3	3	−1	−1	−1	0	0	−1	−1	3	0	1	1	0	0
χ_{14}	15	15	−1	−1	3	3	3	0	−1	−1	−1	1	1	0	0	−1	−1	3	0	−1	−1	0	0
χ_{15}	18	−18	6	−6	−2	2	0	0	2	−2	0	0	0	$A^\star$	A	0	0	0	0	0	0	$-A^\star$	$-A$
χ_{16}	18	−18	6	−6	−2	2	0	0	2	−2	0	0	0	A	$A^\star$	0	0	0	0	0	0	$-A$	$-A^\star$
χ_{17}	24	−24	8	−8	0	0	3	0	0	0	0	0	0	−1	−1	1	−1	−3	0	0	0	1	1
χ_{18}	30	30	−2	−2	2	2	−3	0	2	2	−2	0	0	0	0	1	1	−3	0	0	0	0	0
χ_{19}	30	−30	10	−10	2	−2	−3	0	−2	2	0	0	0	0	0	−1	1	3	0	0	0	0	0
χ_{20}	40	−40	−8	8	0	0	−2	1	0	0	0	0	0	0	0	2	−2	2	−1	0	0	0	0
χ_{21}	40	−40	−8	8	0	0	1	−2	0	0	0	0	0	0	0	−1	1	−1	2	0	0	0	0
χ_{22}	45	45	−3	−3	−3	−3	0	0	1	1	1	1	1	0	0	0	0	0	0	−1	−1	0	0
χ_{23}	45	45	−3	−3	1	1	0	0	−3	−3	1	−1	−1	0	0	0	0	0	0	1	1	0	0

where in Table 11.4, $A = -2i$, $A^\star = 1 - A = 1 - 2i$ and $B = -\frac{1}{2} - \frac{\sqrt{5}}{2}i$.

Table 11.5: Character table of $H_4 = A_7$

	$1a$	$2a$	$3a$	$3b$	$4a$	$5a$	$6a$	$7a$	$7b$
$\lvert C_{H_4}(g)\rvert$	2520	24	9	36	4	5	12	7	7
χ_1	1	1	1	1	1	1	1	1	1
χ_2	6	2	3	0	0	1	-1	-1	-1
χ_3	10	-2	1	1	0	0	1	A	$\overline{A}$
χ_4	10	-2	1	1	0	0	1	$\overline{A}$	A
χ_5	14	2	2	-1	0	-1	2	0	0
χ_6	14	2	-1	2	0	-1	-1	0	0
χ_7	15	-1	3	0	-1	0	-1	1	1
χ_8	21	1	-3	0	-1	1	1	0	0
χ_9	35	-1	-1	-1	1	0	-1	0	0

where in Table 11.5, $A = -\frac{1}{2} - \frac{\sqrt{7}}{2}i$.

Table 11.6: Character table of $H_5 = (3 \times A_6){:}2$

	$1a$	$2a$	$2b$	$2c$	$3a$	$3b$	$3c$	$3d$	$3e$	$4a$	$4b$	$5a$	$6a$	$6b$	$6c$	$12a$	$15a$	$15b$
$\lvert C_{H_5}(g)\rvert$	2160	48	48	48	1080	54	27	27	54	24	8	15	24	6	6	12	15	15
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	−1	1	1	1	1	1	1	−1	1	1	−1	−1	1	1	1
χ_3	2	2	0	0	−1	2	−1	−1	2	2	0	2	−1	0	0	−1	−1	−1
χ_4	5	1	−1	3	5	−1	−1	2	2	−1	1	0	1	−1	0	−1	0	0
χ_5	5	1	1	−3	5	−1	−1	2	2	−1	−1	0	1	1	0	−1	0	0
χ_6	5	1	−3	1	5	2	2	−1	−1	−1	−1	0	1	0	1	−1	0	0
χ_7	5	1	3	−1	5	2	2	−1	−1	−1	1	0	1	0	−1	−1	0	0
χ_8	9	1	−3	−3	9	0	0	0	0	1	1	−1	1	0	0	1	−1	−1
χ_9	9	1	3	3	9	0	0	0	0	1	−1	−1	1	0	0	1	−1	−1
χ_{10}	10	2	0	0	−5	4	−2	1	−2	−2	0	0	−1	0	0	1	0	0
χ_{11}	10	2	0	0	−5	−2	1	−2	4	−2	0	0	−1	0	0	1	0	0
χ_{12}	10	−2	−2	2	10	1	1	1	1	0	0	0	−2	1	−1	0	0	0
χ_{13}	10	−2	2	−2	10	1	1	1	1	0	0	0	−2	−1	1	0	0	0
χ_{14}	16	0	0	0	16	−2	−2	−2	−2	0	0	1	0	0	0	0	1	1
χ_{15}	16	0	0	0	−8	−2	1	1	−2	0	0	1	0	0	0	0	A	$\overline{A}$
χ_{16}	16	0	0	0	−8	−2	1	1	−2	0	0	1	0	0	0	0	$\overline{A}$	A
χ_{17}	18	2	0	0	−9	0	0	0	0	2	0	−2	−1	0	0	−1	1	1
χ_{18}	20	−4	0	0	−10	2	−1	−1	2	0	0	0	2	0	0	0	0	0

where in Table 11.6, $A = -\frac{1}{2} - \frac{\sqrt{15}}{2}i$.

Table 11.7: Character table of $H_6 = (2^4{:}A_5){:}2$

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$3a$	$4a$	$4b$	$4c$	$4d$	$4e$	$5a$	$6a$	$6b$	$6c$	$8a$	$12a$
$\lvert C_{H_6}(g)\rvert$	1920	192	384	32	32	96	24	32	16	32	96	8	5	12	24	12	8	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1	1	−1	−1	−1
χ_3	4	4	4	0	−2	−2	1	0	0	−2	−2	0	−1	1	1	1	0	1
χ_4	4	4	4	0	2	2	1	0	0	2	2	0	−1	1	1	−1	0	−1
χ_5	5	5	5	1	−1	−1	−1	1	1	−1	−1	1	0	−1	−1	−1	1	−1
χ_6	5	5	5	1	1	1	−1	1	1	1	1	−1	0	−1	−1	1	−1	1
χ_7	5	1	−3	1	−1	3	2	1	−1	1	−3	1	0	0	−2	0	−1	0
χ_8	5	1	−3	1	1	−3	2	1	−1	−1	3	−1	0	0	−2	0	1	0
χ_9	6	6	6	−2	0	0	0	−2	−2	0	0	0	1	0	0	0	0	0
χ_{10}	10	−2	2	2	0	−4	1	−2	0	2	−2	0	0	−1	1	−1	0	1
χ_{11}	10	−2	2	−2	2	−2	1	2	0	0	−4	0	0	−1	1	1	0	−1
χ_{12}	10	−2	2	2	0	4	1	−2	0	−2	2	0	0	−1	1	1	0	−1
χ_{13}	10	−2	2	−2	−2	2	1	2	0	0	4	0	0	−1	1	−1	0	1
χ_{14}	10	2	−6	2	0	0	−2	2	−2	0	0	0	0	0	2	0	0	0
χ_{15}	15	3	−9	−1	1	−3	0	−1	1	−1	3	1	0	0	0	0	−1	0
χ_{16}	15	3	−9	−1	−1	3	0	−1	1	1	−3	−1	0	0	0	0	1	0
χ_{17}	20	−4	4	0	−2	−2	−1	0	0	2	2	0	0	1	−1	1	0	−1
χ_{18}	20	−4	4	0	2	2	−1	0	0	−2	−2	0	0	1	−1	−1	0	1

Table 11.8: The character table of $\overline{G} = 3^7{:}Sp(6,2)$

$[x]_G$		$1A$						$2A$		$2B$				$2C$									
$[g]_{\overline{G}}$		$1a$	$3a$	$3b$	$3c$	$3d$	$3e$	$2a$	$6a$	$2b$	$6b$	$6c$	$6d$	$2c$	$6e$	$6f$	$6g$	$6h$	$6i$	$6j$	$6k$	$6l$	$6m$
$\lvert C_{\overline{G}}(g)\rvert$	2	9	7	3	8	4	6	9	8	9	8	6	7	9	8	6	5	5	6	4	4	5	3
	3	11	8	9	9	10	11	3	3	5	4	5	4	6	6	6	6	6	5	6	6	5	6
	5	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		7	7	7	7	7	7	−5	−5	−1	−1	−1	−1	3	3	3	3	3	3	3	3	3	3
χ_3		15	15	15	15	15	15	−5	−5	7	7	7	7	3	3	3	3	3	3	3	3	3	3
χ_4		21	21	21	21	21	21	9	9	−3	−3	−3	−3	1	1	1	1	1	1	1	1	1	1
χ_5		21	21	21	21	21	21	−11	−11	5	5	5	5	5	5	5	5	5	5	5	5	5	5
χ_6		27	27	27	27	27	27	15	15	3	3	3	3	7	7	7	7	7	7	7	7	7	7
χ_7		35	35	35	35	35	35	15	15	11	11	11	11	7	7	7	7	7	7	7	7	7	7
χ_8		35	35	35	35	35	35	−5	−5	3	3	3	3	−5	−5	−5	−5	−5	−5	−5	−5	−5	−5
χ_9		56	56	56	56	56	56	−24	−24	−8	−8	−8	−8	8	8	8	8	8	8	8	8	8	8
χ_{10}		70	70	70	70	70	70	−10	−10	−10	−10	−10	−10	6	6	6	6	6	6	6	6	6	6
χ_{11}		84	84	84	84	84	84	4	4	20	20	20	20	4	4	4	4	4	4	4	4	4	4
χ_{12}		105	105	105	105	105	105	25	25	−7	−7	−7	−7	9	9	9	9	9	9	9	9	9	9
χ_{13}		105	105	105	105	105	105	−35	−35	1	1	1	1	5	5	5	5	5	5	5	5	5	5
χ_{14}		105	105	105	105	105	105	5	5	17	17	17	17	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{15}		120	120	120	120	120	120	40	40	−8	−8	−8	−8	8	8	8	8	8	8	8	8	8	8
χ_{16}		168	168	168	168	168	168	40	40	8	8	8	8	8	8	8	8	8	8	8	8	8	8
χ_{17}		189	189	189	189	189	189	−39	−39	21	21	21	21	1	1	1	1	1	1	1	1	1	1
χ_{18}		189	189	189	189	189	189	−51	−51	−3	−3	−3	−3	13	13	13	13	13	13	13	13	13	13
χ_{19}		189	189	189	189	189	189	21	21	−3	−3	−3	−3	−11	−11	−11	−11	−11	−11	−11	−11	−11	−11
χ_{20}		210	210	210	210	210	210	50	50	2	2	2	2	2	2	2	2	2	2	2	2	2	2
χ_{21}		210	210	210	210	210	210	10	10	−14	−14	−14	−14	10	10	10	10	10	10	10	10	10	10
χ_{22}		216	216	216	216	216	216	−24	−24	24	24	24	24	8	8	8	8	8	8	8	8	8	8
χ_{23}		280	280	280	280	280	280	40	40	24	24	24	24	8	8	8	8	8	8	8	8	8	8
χ_{24}		280	280	280	280	280	280	−40	−40	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8
χ_{25}		315	315	315	315	315	315	−45	−45	−21	−21	−21	−21	3	3	3	3	3	3	3	3	3	3
χ_{26}		336	336	336	336	336	336	−16	−16	16	16	16	16	−16	−16	−16	−16	−16	−16	−16	−16	−16	−16
χ_{27}		378	378	378	378	378	378	−30	−30	−6	−6	−6	−6	2	2	2	2	2	2	2	2	2	2
χ_{28}		405	405	405	405	405	405	45	45	−27	−27	−27	−27	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{29}		420	420	420	420	420	420	20	20	4	4	4	4	−12	−12	−12	−12	−12	−12	−12	−12	−12	−12
χ_{30}		512	512	512	512	512	512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$1A$						$2A$		$2B$				$2C$									
$[g]_{\overline{G}}$	$1a$	$3a$	$3b$	$3c$	$3d$	$3e$	$2a$	$6a$	$2b$	$6b$	$6c$	$6d$	$2c$	$6e$	$6f$	$6g$	$6h$	$6i$	$6j$	$6k$	$6l$	$6m$
χ_{31}	56	2	−7	20	2	−25	0	0	8	−4	−1	2	16	−8	10	−8	−5	4	4	−2	−2	1
χ_{32}	280	10	−35	100	10	−125	0	0	−24	12	3	−6	16	−8	10	−8	−5	4	4	−2	−2	1
χ_{33}	280	10	−35	100	10	−125	0	0	−24	12	3	−6	16	−8	10	−8	−5	4	4	−2	−2	1
χ_{34}	336	12	−42	120	12	−150	0	0	−16	8	2	−4	32	−16	20	−16	−10	8	8	−4	−4	2
χ_{35}	560	20	−70	200	20	−250	0	0	16	−8	−2	4	−32	16	−20	16	10	−8	−8	4	4	−2
χ_{36}	560	20	−70	200	20	−250	0	0	16	−8	−2	4	−32	16	−20	16	10	−8	−8	4	4	−2
χ_{37}	840	30	−105	300	30	−375	0	0	−8	4	1	−2	−16	8	−10	8	5	−4	−4	2	2	−1
χ_{38}	840	30	−105	300	30	−375	0	0	56	−28	−7	14	48	−24	30	−24	−15	12	12	−6	−6	3
χ_{39}	1120	40	−140	400	40	−500	0	0	32	−16	−4	8	64	−32	40	−32	−20	16	16	−8	−8	4
χ_{40}	1344	48	−168	480	48	−600	0	0	64	−32	−8	16	0	0	0	0	0	0	0	0	0	0
χ_{41}	1680	60	−210	600	60	−750	0	0	48	−24	−6	12	32	−16	20	−16	−10	8	8	−4	−4	2
χ_{42}	1680	60	−210	600	60	−750	0	0	48	−24	−6	12	32	−16	20	−16	−10	8	8	−4	−4	2
χ_{43}	1680	60	−210	600	60	−750	0	0	−80	40	10	−20	32	−16	20	−16	−10	8	8	−4	−4	2
χ_{44}	2240	80	−280	800	80	−1000	0	0	−64	32	8	−16	0	0	0	0	0	0	0	0	0	0
χ_{45}	2240	80	−280	800	80	−1000	0	0	−64	32	8	−16	0	0	0	0	0	0	0	0	0	0
χ_{46}	2520	90	−315	900	90	−1125	0	0	−24	12	3	−6	−48	24	−30	24	15	−12	−12	6	6	−3
χ_{47}	2520	90	−315	900	90	−1125	0	0	−24	12	3	−6	−48	24	−30	24	15	−12	−12	6	6	−3
χ_{48}	3360	120	−420	1200	120	−1500	0	0	−32	16	4	−8	64	−32	40	−32	−20	16	16	−8	−8	4
χ_{49}	3584	128	−448	1280	128	−1600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	4536	162	−567	1620	162	−2025	0	0	72	−36	−9	18	−48	24	−30	24	15	−12	−12	6	6	−3

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Table 11.8 (continued)

$[x]_G$	$1A$						$2A$		$2B$				$2C$									
$[g]_{\overline{G}}$	$1a$	$3a$	$3b$	$3c$	$3d$	$3e$	$2a$	$6a$	$2b$	$6b$	$6c$	$6d$	$2c$	$6e$	$6f$	$6g$	$6h$	$6i$	$6j$	$6k$	$6l$	$6m$
χ_{51}	126	0	0	27	−9	45	2	−1	6	3	−3	0	26	23	8	8	5	−1	5	−1	−4	−4
χ_{52}	630	0	0	135	−45	225	10	−5	30	15	−15	0	34	19	16	16	1	7	1	7	−8	−8
χ_{53}	630	0	0	135	−45	225	10	−5	30	15	−15	0	34	19	16	16	1	7	1	7	−8	−8
χ_{54}	756	0	0	162	−54	270	−12	6	−12	−6	6	0	52	46	16	16	10	−2	10	−2	−8	−8
χ_{55}	1008	0	0	216	−72	360	16	−8	48	24	−24	0	16	−8	16	16	−8	16	−8	16	−8	−8
χ_{56}	1008	0	0	216	−72	360	16	−8	48	24	−24	0	16	−8	16	16	−8	16	−8	16	−8	−8
χ_{57}	1134	0	0	243	−81	405	18	−9	54	27	−27	0	42	15	24	24	−3	15	−3	15	−12	−12
χ_{58}	1260	0	0	270	−90	450	20	−10	60	30	−30	0	−28	−58	8	8	−22	26	−22	26	−4	−4
χ_{59}	1260	0	0	270	−90	450	−20	10	12	6	−6	0	44	50	8	8	14	−10	14	−10	−4	−4
χ_{60}	1260	0	0	270	−90	450	−20	10	12	6	−6	0	44	50	8	8	14	−10	14	−10	−4	−4
χ_{61}	1260	0	0	270	−90	450	−20	10	12	6	−6	0	−52	−46	−16	−16	−10	2	−10	2	8	8
χ_{62}	1260	0	0	270	−90	450	−20	10	12	6	−6	0	−52	−46	−16	−16	−10	2	−10	2	8	8
χ_{63}	1890	0	0	405	−135	675	30	−15	−6	−3	3	0	−26	−23	−8	−8	−5	1	−5	1	4	4
χ_{64}	1890	0	0	405	−135	675	30	−15	−6	−3	3	0	70	73	16	16	19	−11	19	−11	−8	−8
χ_{65}	2268	0	0	486	−162	810	−36	18	−36	−18	18	0	−36	−54	0	0	−18	18	−18	18	0	0
χ_{66}	2268	0	0	486	−162	810	−36	18	−36	−18	18	0	−36	−54	0	0	−18	18	−18	18	0	0
χ_{67}	3024	0	0	648	−216	1080	−48	24	−48	−24	24	0	16	−8	16	16	−8	16	−8	16	−8	−8
χ_{68}	3780	0	0	810	−270	1350	60	−30	−12	−6	6	0	44	50	8	8	14	−10	14	−10	−4	−4
χ_{69}	3780	0	0	810	−270	1350	−60	30	−60	−30	30	0	68	38	32	32	2	14	2	14	−16	−16
χ_{70}	5040	0	0	1080	−360	1800	−80	40	48	24	−24	0	−16	8	−16	−16	8	−16	8	−16	8	8
χ_{71}	5040	0	0	1080	−360	1800	−80	40	48	24	−24	0	−16	8	−16	−16	8	−16	8	−16	8	8
χ_{72}	5670	0	0	1215	−405	2025	90	−45	−18	−9	9	0	18	27	0	0	9	−9	9	−9	0	0
χ_{73}	5670	0	0	1215	−405	2025	90	−45	−18	−9	9	0	−78	−69	−24	−24	−15	3	−15	3	12	12
χ_{74}	576	0	27	0	−18	−72	0	0	0	0	0	0	64	−32	−8	−8	4	−8	4	10	4	−5
χ_{75}	3456	0	162	0	−108	−432	0	0	0	0	0	0	128	−64	−16	−16	8	−16	8	20	8	−10
χ_{76}	5760	0	270	0	−180	−720	0	0	0	0	0	0	−128	64	16	16	−8	16	−8	−20	−8	10
χ_{77}	5760	0	270	0	−180	−720	0	0	0	0	0	0	−128	64	16	16	−8	16	−8	−20	−8	10
χ_{78}	8064	0	378	0	−252	−1008	0	0	0	0	0	0	128	−64	−16	−16	8	−16	8	20	8	−10
χ_{79}	8064	0	378	0	−252	−1008	0	0	0	0	0	0	128	−64	−16	−16	8	−16	8	20	8	−10
χ_{80}	8640	0	405	0	−270	−1080	0	0	0	0	0	0	−64	32	8	8	−4	8	−4	−10	−4	5
χ_{81}	12096	0	567	0	−378	−1512	0	0	0	0	0	0	64	−32	−8	−8	4	−8	4	10	4	−5
χ_{82}	20160	0	945	0	−630	−2520	0	0	0	0	0	0	−64	32	8	8	−4	8	−4	−10	−4	5

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Table 11.8 (continued)

$[x]_G$	$1A$						$2A$		$2B$				$2C$									
$[g]_{\overline{G}}$	$1a$	$3a$	$3b$	$3c$	$3d$	$3e$	$2a$	$6a$	$2b$	$6b$	$6c$	$6d$	$2c$	$6e$	$6f$	$6g$	$6h$	$6i$	$6j$	$6k$	$6l$	$6m$
χ_{83}	672	24	−21	−48	−3	24	0	0	0	0	0	0	64	16	−20	−2	4	4	−5	1	−8	7
χ_{84}	672	24	−21	−48	−3	24	0	0	0	0	0	0	0	48	12	−6	−12	−12	−3	9	0	3
χ_{85}	1344	48	−42	−96	−6	48	0	0	0	0	0	0	64	64	−8	−8	−8	−8	−8	10	−8	10
χ_{86}	3360	120	−105	−240	−15	120	0	0	0	0	0	0	128	−16	−52	2	20	20	−7	−7	−16	11
χ_{87}	3360	120	−105	−240	−15	120	0	0	0	0	0	0	−64	80	44	−10	−28	−28	−1	17	8	−1
χ_{88}	3360	120	−105	−240	−15	120	0	0	0	0	0	0	64	16	−20	−2	4	4	−5	1	−8	7
χ_{89}	3360	120	−105	−240	−15	120	0	0	0	0	0	0	0	48	12	−6	−12	−12	−3	9	0	3
χ_{90}	6048	216	−189	−432	−27	216	0	0	0	0	0	0	−64	80	44	−10	−28	−28	−1	17	8	−1
χ_{91}	6048	216	−189	−432	−27	216	0	0	0	0	0	0	128	−16	−52	2	20	20	−7	−7	−16	11
χ_{92}	6720	240	−210	−480	−30	240	0	0	0	0	0	0	64	64	−8	−8	−8	−8	−8	10	−8	10
χ_{93}	6720	240	−210	−480	−30	240	0	0	0	0	0	0	64	64	−8	−8	−8	−8	−8	10	−8	10
χ_{94}	6720	240	−210	−480	−30	240	0	0	0	0	0	0	0	−96	−24	12	24	24	6	−18	0	−6
χ_{95}	6720	240	−210	−480	−30	240	0	0	0	0	0	0	−128	−32	40	4	−8	−8	10	−2	16	−14
χ_{96}	10752	384	−336	−768	−48	384	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	10752	384	−336	−768	−48	384	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{98}	10752	384	−336	−768	−48	384	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	12096	432	−378	−864	−54	432	0	0	0	0	0	0	64	64	−8	−8	−8	−8	−8	10	−8	10
χ_{100}	13440	480	−420	−960	−60	480	0	0	0	0	0	0	−128	−128	16	16	16	16	16	−20	16	−20
χ_{101}	756	−27	0	0	27	27	0	0	12	0	3	−3	40	−32	25	7	7	−8	−11	−5	1	4
χ_{102}	756	−27	0	0	27	27	0	0	12	0	3	−3	72	0	9	9	−9	0	−9	−9	9	0
χ_{103}	3024	−108	0	0	108	108	0	0	48	0	12	−12	64	64	4	−14	4	16	−14	−8	16	−8
χ_{104}	3024	−108	0	0	108	108	0	0	48	0	12	−12	0	0	36	−18	36	0	−18	0	0	0
χ_{105}	3780	−135	0	0	135	135	0	0	−36	0	−9	9	8	−64	41	5	23	−16	−13	−1	−7	8
χ_{106}	3780	−135	0	0	135	135	0	0	−36	0	−9	9	104	32	−7	11	−25	8	−7	−13	17	−4
χ_{107}	3780	−135	0	0	135	135	0	0	60	0	15	−15	72	0	45	−9	27	0	−27	−9	9	0
χ_{108}	3780	−135	0	0	135	135	0	0	60	0	15	−15	104	32	29	−7	11	8	−25	−13	17	−4
χ_{109}	4536	−162	0	0	162	162	0	0	72	0	18	−18	−48	96	6	−48	42	24	−12	6	6	−12
χ_{110}	7560	−270	0	0	270	270	0	0	24	0	6	−6	16	−128	46	28	10	−32	−8	−2	−14	16
χ_{111}	7560	−270	0	0	270	270	0	0	24	0	6	−6	144	0	−18	36	−54	0	0	−18	18	0
χ_{112}	7560	−270	0	0	270	270	0	0	24	0	6	−6	−144	0	−18	−18	18	0	18	18	−18	0
χ_{113}	7560	−270	0	0	270	270	0	0	24	0	6	−6	−80	64	−50	−14	−14	16	22	10	−2	−8
χ_{114}	7560	−270	0	0	270	270	0	0	−72	0	−18	18	112	−32	34	16	−2	−8	−20	−14	10	4
χ_{115}	11340	−405	0	0	405	405	0	0	−108	0	−27	27	24	96	−21	−21	−3	24	−3	−3	15	−12
χ_{116}	11340	−405	0	0	405	405	0	0	−108	0	−27	27	−72	0	27	−27	45	0	−9	9	−9	0
χ_{117}	15120	−540	0	0	540	540	0	0	48	0	12	−12	0	0	−36	18	−36	0	18	0	0	0
χ_{118}	15120	−540	0	0	540	540	0	0	48	0	12	−12	−64	−64	−4	14	−4	−16	14	8	−16	8

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Table 11.8 (continued)

$[x]_G$		$2D$				$3A$							$3B$				$3C$						$4A$			
$[g]_{\overline{G}}$		$2d$	$6n$	$6o$	$6p$	$3f$	$3g$	$3h$	$3i$	$9a$	$9b$	$9c$	$3j$	$3k$	$9d$	$9e$	$3l$	$3m$	$9f$	$9g$	$9h$	$9i$	$4a$	$12a$	$12b$	$12c$
$\lvert C_{\overline{G}}(g)\rvert$	2	7	6	4	5	4	2	3	3	2	3	1	3	0	3	3	2	1	1	0	1	0	7	6	4	5
	3	4	3	4	3	8	8	7	7	7	7	7	7	7	5	5	6	6	6	6	5	5	4	3	4	3
	5	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		−1	−1	−1	−1	4	4	4	4	4	4	4	−2	−2	−2	−2	1	1	1	1	1	1	3	3	3	3
χ_3		−1	−1	−1	−1	0	0	0	0	0	0	0	−3	−3	−3	−3	3	3	3	3	3	3	−1	−1	−1	−1
χ_4		−3	−3	−3	−3	6	6	6	6	6	6	6	3	3	3	3	0	0	0	0	0	0	5	5	5	5
χ_5		−3	−3	−3	−3	6	6	6	6	6	6	6	3	3	3	3	0	0	0	0	0	0	1	1	1	1
χ_6		3	3	3	3	9	9	9	9	9	9	9	0	0	0	0	0	0	0	0	0	0	3	3	3	3
χ_7		3	3	3	3	5	5	5	5	5	5	5	−1	−1	−1	−1	2	2	2	2	2	2	−1	−1	−1	−1
χ_8		3	3	3	3	5	5	5	5	5	5	5	−1	−1	−1	−1	2	2	2	2	2	2	7	7	7	7
χ_9		0	0	0	0	11	11	11	11	11	11	11	2	2	2	2	2	2	2	2	2	2	0	0	0	0
χ_{10}		−2	−2	−2	−2	−5	−5	−5	−5	−5	−5	−5	7	7	7	7	1	1	1	1	1	1	2	2	2	2
χ_{11}		4	4	4	4	−6	−6	−6	−6	−6	−6	−6	3	3	3	3	3	3	3	3	3	3	4	4	4	4
χ_{12}		1	1	1	1	0	0	0	0	0	0	0	6	6	6	6	3	3	3	3	3	3	−3	−3	−3	−3
χ_{13}		1	1	1	1	15	15	15	15	15	15	15	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5	5
χ_{14}		−7	−7	−7	−7	0	0	0	0	0	0	0	6	6	6	6	3	3	3	3	3	3	−3	−3	−3	−3
χ_{15}		0	0	0	0	15	15	15	15	15	15	15	−6	−6	−6	−6	0	0	0	0	0	0	0	0	0	0
χ_{16}		8	8	8	8	6	6	6	6	6	6	6	6	6	6	6	−3	−3	−3	−3	−3	−3	0	0	0	0
χ_{17}		−3	−3	−3	−3	9	9	9	9	9	9	9	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	−3
χ_{18}		−3	−3	−3	−3	9	9	9	9	9	9	9	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	−3
χ_{19}		−3	−3	−3	−3	9	9	9	9	9	9	9	0	0	0	0	0	0	0	0	0	0	9	9	9	9
χ_{20}		−6	−6	−6	−6	15	15	15	15	15	15	15	3	3	3	3	0	0	0	0	0	0	−2	−2	−2	−2
χ_{21}		2	2	2	2	−15	−15	−15	−15	−15	−15	−15	−6	−6	−6	−6	3	3	3	3	3	3	6	6	6	6
χ_{22}		0	0	0	0	−9	−9	−9	−9	−9	−9	−9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}		0	0	0	0	−5	−5	−5	−5	−5	−5	−5	−8	−8	−8	−8	−2	−2	−2	−2	−2	−2	0	0	0	0
χ_{24}		8	8	8	8	10	10	10	10	10	10	10	10	10	10	10	1	1	1	1	1	1	0	0	0	0
χ_{25}		3	3	3	3	0	0	0	0	0	0	0	−9	−9	−9	−9	0	0	0	0	0	0	−5	−5	−5	−5
χ_{26}		0	0	0	0	6	6	6	6	6	6	6	−6	−6	−6	−6	0	0	0	0	0	0	0	0	0	0
χ_{27}		−6	−6	−6	−6	−9	−9	−9	−9	−9	−9	−9	0	0	0	0	0	0	0	0	0	0	6	6	6	6
χ_{28}		−3	−3	−3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	−3
χ_{29}		4	4	4	4	0	0	0	0	0	0	0	−3	−3	−3	−3	3	3	3	3	3	3	−4	−4	−4	−4
χ_{30}		0	0	0	0	−16	−16	−16	−16	−16	−16	−16	8	8	8	8	−4	−4	−4	−4	−4	−4	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$2D$				$3A$							$3B$				$3C$						$4A$			
$[g]_{\overline{G}}$	$2d$	$6n$	$6o$	$6p$	$3f$	$3g$	$3h$	$3i$	$9a$	$9b$	$9c$	$3j$	$3k$	$9d$	$9e$	$3l$	$3m$	$9f$	$9g$	$9h$	$9i$	$4a$	$12a$	$12b$	$12c$
χ_{31}	0	0	0	0	20	−7	2	2	−10	8	−1	2	2	−1	−1	2	2	−1	−1	2	−1	8	−4	−1	2
χ_{32}	0	0	0	0	−20	7	−2	−2	10	−8	1	1	1	−5	4	4	4	−2	−2	4	−2	8	−4	−1	2
χ_{33}	0	0	0	0	−20	7	−2	−2	10	−8	1	1	1	4	−5	4	4	−2	−2	4	−2	8	−4	−1	2
χ_{34}	0	0	0	0	60	−21	6	6	−30	24	−3	−6	−6	3	3	0	0	0	0	0	0	16	−8	−2	4
χ_{35}	0	0	0	0	20	−7	2	2	−10	8	−1	−7	−7	8	−1	2	2	−1	−1	2	−1	16	−8	−2	4
χ_{36}	0	0	0	0	20	−7	2	2	−10	8	−1	−7	−7	−1	8	2	2	−1	−1	2	−1	16	−8	−2	4
χ_{37}	0	0	0	0	60	−21	6	6	−30	24	−3	12	12	−6	−6	0	0	0	0	0	0	24	−12	−3	6
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	−6	−6	3	3	6	6	−3	−3	6	−3	−8	4	1	−2
χ_{39}	0	0	0	0	100	−35	10	10	−50	40	−5	4	4	−2	−2	−2	−2	1	1	−2	1	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	12	12	−6	−6	6	6	−3	−3	6	−3	0	0	0	0
χ_{41}	0	0	0	0	−60	21	−6	−6	30	−24	3	−3	−3	−12	15	0	0	0	0	0	0	16	−8	−2	4
χ_{42}	0	0	0	0	−60	21	−6	−6	30	−24	3	−3	−3	15	−12	0	0	0	0	0	0	16	−8	−2	4
χ_{43}	0	0	0	0	60	−21	6	6	−30	24	−3	6	6	−3	−3	6	6	−3	−3	6	−3	−16	8	2	−4
χ_{44}	0	0	0	0	−40	14	−4	−4	20	−16	2	−10	−10	14	−4	2	2	−1	−1	2	−1	0	0	0	0
χ_{45}	0	0	0	0	−40	14	−4	−4	20	−16	2	−10	−10	−4	14	2	2	−1	−1	2	−1	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	9	9	9	−18	0	0	0	0	0	0	8	−4	−1	2
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	9	9	−18	9	0	0	0	0	0	0	8	−4	−1	2
χ_{48}	0	0	0	0	−60	21	−6	−6	30	−24	3	12	12	−6	−6	−6	−6	3	3	−6	3	0	0	0	0
χ_{49}	0	0	0	0	80	−28	8	8	−40	32	−4	−16	−16	8	8	−4	−4	2	2	−4	2	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−24	12	3	−6

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Table 11.8 (continued)

$[x]_G$	$2D$				$3A$							$3B$				$3C$						$4A$			
$[g]_{\overline{G}}$	$2d$	$6n$	$6o$	$6p$	$3f$	$3g$	$3h$	$3i$	$9a$	$9b$	$9c$	$3j$	$3k$	$9d$	$9e$	$3l$	$3m$	$9f$	$9g$	$9h$	$9i$	$4a$	$12a$	$12b$	$12c$
χ_{51}	6	3	−3	0	30	3	−6	3	15	6	−3	0	0	0	0	6	−3	6	−3	0	0	6	3	−3	0
χ_{52}	6	3	−3	0	60	6	−12	6	30	12	−6	0	0	0	0	−6	3	−6	3	0	0	6	3	−3	0
χ_{53}	6	3	−3	0	−30	−3	6	−3	−15	−6	3	0	0	0	0	12	−6	12	−6	0	0	6	3	−3	0
χ_{54}	−12	−6	6	0	90	9	−18	9	45	18	−9	0	0	0	0	0	0	0	0	0	0	12	6	−6	0
χ_{55}	0	0	0	0	−30	−3	6	−3	−15	−6	3	0	0	0	0	−6	3	−6	3	0	0	0	0	0	0
χ_{56}	0	0	0	0	−30	−3	6	−3	−15	−6	3	0	0	0	0	−6	3	−6	3	0	0	0	0	0	0
χ_{57}	6	3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	3	−3	0
χ_{58}	−12	−6	6	0	30	3	−6	3	15	6	−3	0	0	0	0	6	−3	6	−3	0	0	−12	−6	6	0
χ_{59}	−12	−6	6	0	30	3	−6	3	15	6	−3	0	0	0	0	6	−3	6	−3	0	0	−12	−6	6	0
χ_{60}	−12	−6	6	0	30	3	−6	3	15	6	−3	0	0	0	0	6	−3	6	−3	0	0	−12	−6	6	0
χ_{61}	12	6	−6	0	30	3	−6	3	15	6	−3	0	0	0	0	6	−3	6	−3	0	0	12	6	−6	0
χ_{62}	12	6	−6	0	30	3	−6	3	15	6	−3	0	0	0	0	6	−3	6	−3	0	0	12	6	−6	0
χ_{63}	−6	−3	3	0	90	9	−18	9	45	18	−9	0	0	0	0	0	0	0	0	0	0	18	9	−9	0
χ_{64}	18	9	−9	0	90	9	−18	9	45	18	−9	0	0	0	0	0	0	0	0	0	0	−6	−3	3	0
χ_{65}	12	6	−6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−12	−6	6	0
χ_{66}	12	6	−6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−12	−6	6	0
χ_{67}	0	0	0	0	90	9	−18	9	45	18	−9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	12	6	−6	0	−90	−9	18	−9	−45	−18	9	0	0	0	0	0	0	0	0	0	0	12	6	−6	0
χ_{69}	−12	−6	6	0	−90	−9	18	−9	−45	−18	9	0	0	0	0	0	0	0	0	0	0	12	6	−6	0
χ_{70}	0	0	0	0	30	3	−6	3	15	6	−3	0	0	0	0	−12	6	−12	6	0	0	0	0	0	0
χ_{71}	0	0	0	0	−60	−6	12	−6	−30	−12	6	0	0	0	0	6	−3	6	−3	0	0	0	0	0	0
χ_{72}	6	3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−18	−9	9	0
χ_{73}	−18	−9	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	3	−3	0
χ_{74}	0	0	0	0	60	6	−12	6	−15	−6	3	0	0	0	0	12	−6	−6	3	0	0	0	0	0	0
χ_{75}	0	0	0	0	180	18	−36	18	−45	−18	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	60	6	−12	6	−15	−6	3	0	0	0	0	12	−6	−6	3	0	0	0	0	0	0
χ_{77}	0	0	0	0	60	6	−12	6	−15	−6	3	0	0	0	0	12	−6	−6	3	0	0	0	0	0	0
χ_{78}	0	0	0	0	120	12	−24	12	−30	−12	6	0	0	0	0	−12	6	6	−3	0	0	0	0	0	0
χ_{79}	0	0	0	0	−60	−6	12	−6	15	6	−3	0	0	0	0	24	−12	−12	6	0	0	0	0	0	0
χ_{80}	0	0	0	0	180	18	−36	18	−45	−18	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	−180	−18	36	−18	45	18	−9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	−60	−6	12	−6	15	6	−3	0	0	0	0	−12	6	6	−3	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$2D$				$3A$							$3B$				$3C$						$4A$			
$[g]_{\overline{G}}$	$2d$	$6n$	$6o$	$6p$	$3f$	$3g$	$3h$	$3i$	$9a$	$9b$	$9c$	$3j$	$3k$	$9d$	$9e$	$3l$	$3m$	$9f$	$9g$	$9h$	$9i$	$4a$	$12a$	$12b$	$12c$
χ_{83}	8	−4	−1	2	42	−12	6	6	9	−9	0	24	−3	0	0	6	6	0	0	−3	0	0	0	0	0
χ_{84}	−8	4	1	−2	42	−12	6	6	9	−9	0	24	−3	0	0	6	6	0	0	−3	0	0	0	0	0
χ_{85}	0	0	0	0	78	−30	6	6	21	−15	3	−24	3	0	0	0	0	6	6	0	−3	0	0	0	0
χ_{86}	−8	4	1	−2	90	−18	18	18	15	−21	−3	−24	3	0	0	6	6	−6	−6	−3	3	0	0	0	0
χ_{87}	8	−4	−1	2	90	−18	18	18	15	−21	−3	−24	3	0	0	6	6	−6	−6	−3	3	0	0	0	0
χ_{88}	−24	12	3	−6	−30	24	6	6	−15	3	−6	48	−6	0	0	0	0	6	6	0	−3	0	0	0	0
χ_{89}	24	−12	−3	6	−30	24	6	6	−15	3	−6	48	−6	0	0	0	0	6	6	0	−3	0	0	0	0
χ_{90}	−24	12	3	−6	18	18	18	18	−9	−9	−9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	24	−12	−3	6	18	18	18	18	−9	−9	−9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{92}	0	0	0	0	150	−66	6	6	45	−27	9	24	−3	0	0	−12	−12	0	0	6	0	0	0	0	0
χ_{93}	0	0	0	0	−90	18	−18	−18	−15	21	3	−48	6	0	0	12	12	6	6	−6	−3	0	0	0	0
χ_{94}	−16	8	2	−4	60	6	24	24	0	−18	−9	24	−3	0	0	6	6	0	0	−3	0	0	0	0	0
χ_{95}	16	−8	−2	4	60	6	24	24	0	−18	−9	24	−3	0	0	6	6	0	0	−3	0	0	0	0	0
χ_{96}	0	0	0	0	−48	60	24	24	−36	0	−18	−48	6	0	0	−12	−12	0	0	6	0	0	0	0	0
χ_{97}	0	0	0	0	−96	12	−24	−24	−12	24	6	24	−3	0	0	0	0	−6	−6	0	3	0	0	0	0
χ_{98}	0	0	0	0	−96	12	−24	−24	−12	24	6	24	−3	0	0	0	0	−6	−6	0	3	0	0	0	0
χ_{99}	0	0	0	0	−18	−18	−18	−18	9	9	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	0	0	60	−48	−12	−12	30	−6	12	−24	3	0	0	0	0	6	6	0	−3	0	0	0	0
χ_{101}	−12	0	−3	3	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	3	−3
χ_{102}	12	0	3	−3	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	3	−3
χ_{103}	24	0	6	−6	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{104}	−24	0	−6	6	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{105}	12	0	3	−3	180	18	18	−36	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	3	−3
χ_{106}	−12	0	−3	3	180	18	18	−36	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	3	−3
χ_{107}	−12	0	−3	3	−90	−9	−9	18	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	3	−3
χ_{108}	12	0	3	−3	−90	−9	−9	18	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	3	−3
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−24	0	−6	6
χ_{110}	0	0	0	0	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	−24	0	−6	6
χ_{111}	0	0	0	0	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	−24	0	−6	6
χ_{112}	24	0	6	−6	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	6	−6
χ_{113}	−24	0	−6	6	90	9	9	−18	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	6	−6
χ_{114}	0	0	0	0	−180	−18	−18	36	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	6	−6
χ_{115}	−12	0	−3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−12	0	−3	3
χ_{116}	12	0	3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−12	0	−3	3
χ_{117}	24	0	6	−6	−90	−9	−9	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	−24	0	−6	6	−90	−9	−9	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$		$4B$				$4C$		$4D$		$4E$								$5A$							$6A$	
$[g]_{\overline{G}}$		$4b$	$12d$	$12e$	$12f$	$4c$	$12g$	$4d$	$12h$	$4e$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$5a$	$15a$	$15b$	$15c$	$15d$	$15e$	$15f$	$6q$	$6r$
$\lvert C_{\overline{G}}(g)\rvert$	2	6	5	3	4	6	5	7	6	5	4	4	4	3	3	3	2	1	1	1	0	0	0	0	4	3
	3	4	3	4	3	2	2	1	1	3	3	3	3	3	3	3	3	4	3	3	4	3	3	3	3	3
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		1	1	1	1	−3	−3	−1	−1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	−2	−2
χ_3		−3	−3	−3	−3	1	1	3	3	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	−2	−2
χ_4		−1	−1	−1	−1	3	3	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	0	0
χ_5		−3	−3	−3	−3	−3	−3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−2	−2
χ_6		1	1	1	1	5	5	−1	−1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3
χ_7		5	5	5	5	1	1	3	3	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	3	3
χ_8		−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	1	1
χ_9		4	4	4	4	−4	−4	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	−3	−3
χ_{10}		2	2	2	2	2	2	2	2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	−1	−1
χ_{11}		0	0	0	0	0	0	4	4	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1	−2	−2
χ_{12}		−3	−3	−3	−3	−3	−3	−3	−3	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	4	4
χ_{13}		−1	−1	−1	−1	−5	−5	1	1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	1	1
χ_{14}		3	3	3	3	−1	−1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	2	2
χ_{15}		−4	−4	−4	−4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
χ_{16}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2	−2	−2	−2
χ_{17}		−5	−5	−5	−5	−1	−1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	3	3
χ_{18}		1	1	1	1	1	1	−3	−3	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−3	−3
χ_{19}		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−3	−3
χ_{20}		2	2	2	2	2	2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	−1	−1
χ_{21}		−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	1	1
χ_{22}		−4	−4	−4	−4	4	4	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	−3	−3
χ_{23}		4	4	4	4	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
χ_{24}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
χ_{25}		3	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0
χ_{26}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	2	2
χ_{27}		2	2	2	2	2	2	−2	−2	2	2	2	2	2	2	2	2	−2	−2	−2	−2	−2	−2	−2	3	3
χ_{28}		−3	−3	−3	−3	−3	−3	5	5	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
χ_{29}		0	0	0	0	0	0	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4
χ_{30}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	0	0

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Table 11.8 (continued)

$[x]_G$	$4B$				$4C$		$4D$		$4E$								$5A$							$6A$	
$[g]_{\overline{G}}$	$4b$	$12d$	$12e$	$12f$	$4c$	$12g$	$4d$	$12h$	$4e$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$5a$	$15a$	$15b$	$15c$	$15d$	$15e$	$15f$	$6q$	$6r$
χ_{31}	0	0	0	0	20	−7	2	2	−10	8	−1	2	2	−1	−1	2	2	−1	−1	2	−1	8	−4	−1	2
χ_{32}	0	0	0	0	−20	7	−2	−2	10	−8	1	1	1	−5	4	4	4	−2	−2	4	−2	8	−4	−1	2
χ_{33}	0	0	0	0	−20	7	−2	−2	10	−8	1	1	1	4	−5	4	4	−2	−2	4	−2	8	−4	−1	2
χ_{34}	0	0	0	0	60	−21	6	6	−30	24	−3	−6	−6	3	3	0	0	0	0	0	0	16	−8	−2	4
χ_{35}	0	0	0	0	20	−7	2	2	−10	8	−1	−7	−7	8	−1	2	2	−1	−1	2	−1	16	−8	−2	4
χ_{36}	0	0	0	0	20	−7	2	2	−10	8	−1	−7	−7	−1	8	2	2	−1	−1	2	−1	16	−8	−2	4
χ_{37}	0	0	0	0	60	−21	6	6	−30	24	−3	12	12	−6	−6	0	0	0	0	0	0	24	−12	−3	6
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	−6	−6	3	3	6	6	−3	−3	6	−3	−8	4	1	−2
χ_{39}	0	0	0	0	100	−35	10	10	−50	40	−5	4	4	−2	−2	−2	−2	1	1	−2	1	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	12	12	−6	−6	6	6	−3	−3	6	−3	0	0	0	0
χ_{41}	0	0	0	0	−60	21	−6	−6	30	−24	3	−3	−3	−12	15	0	0	0	0	0	0	16	−8	−2	4
χ_{42}	0	0	0	0	−60	21	−6	−6	30	−24	3	−3	−3	15	−12	0	0	0	0	0	0	16	−8	−2	4
χ_{43}	0	0	0	0	60	−21	6	6	−30	24	−3	6	6	−3	−3	6	6	−3	−3	6	−3	−16	8	2	−4
χ_{44}	0	0	0	0	−40	14	−4	−4	20	−16	2	−10	−10	14	−4	2	2	−1	−1	2	−1	0	0	0	0
χ_{45}	0	0	0	0	−40	14	−4	−4	20	−16	2	−10	−10	−4	14	2	2	−1	−1	2	−1	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	9	9	9	−18	0	0	0	0	0	0	8	−4	−1	2
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	9	9	−18	9	0	0	0	0	0	0	8	−4	−1	2
χ_{48}	0	0	0	0	−60	21	−6	−6	30	−24	3	12	12	−6	−6	−6	−6	3	3	−6	3	0	0	0	0
χ_{49}	0	0	0	0	80	−28	8	8	−40	32	−4	−16	−16	8	8	−4	−4	2	2	−4	2	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−24	12	3	−6

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Table 11.8 (continued)

$[x]_G$	$4B$				$4C$		$4D$		$4E$								$5A$							$6A$	
$[g]_{\overline{G}}$	$4b$	$12d$	$12e$	$12f$	$4c$	$12g$	$4d$	$12h$	$4e$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$5a$	$15a$	$15b$	$15c$	$15d$	$15e$	$15f$	$6q$	$6r$
χ_{51}	12	0	3	−3	0	0	2	−1	4	1	1	4	−2	1	1	−2	6	−3	−3	6	0	0	0	2	−1
χ_{52}	−12	0	−3	3	0	0	2	−1	0	3	−3	0	0	−3	3	0	0	0	0	0	0	0	0	4	−2
χ_{53}	−12	0	−3	3	0	0	2	−1	0	3	−3	0	0	−3	3	0	0	0	0	0	0	0	0	−2	1
χ_{54}	0	0	0	0	0	0	−4	2	0	0	0	0	0	0	0	0	6	−3	−3	6	0	0	0	−6	3
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	$\overline{A}$	A	3	0	0	0	−2	1
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	A	$\overline{A}$	3	0	0	0	−2	1
χ_{57}	12	0	3	−3	0	0	2	−1	4	1	1	4	−2	1	1	−2	−6	3	3	−6	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	−4	2	−4	−4	2	−4	2	2	−4	2	0	0	0	0	0	0	0	2	−1
χ_{59}	24	0	6	−6	0	0	4	−2	−4	2	−4	−4	2	−4	2	2	0	0	0	0	0	0	0	−2	1
χ_{60}	−24	0	−6	6	0	0	4	−2	4	−2	4	4	−2	4	−2	−2	0	0	0	0	0	0	0	−2	1
χ_{61}	0	0	0	0	0	0	−4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1
χ_{62}	0	0	0	0	0	0	−4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1
χ_{63}	−12	0	−3	3	0	0	6	−3	−4	−1	−1	−4	2	−1	−1	2	0	0	0	0	0	0	0	6	−3
χ_{64}	12	0	3	−3	0	0	−2	1	0	−3	3	0	0	3	−3	0	0	0	0	0	0	0	0	6	−3
χ_{65}	0	0	0	0	0	0	4	−2	0	0	0	0	0	0	0	0	3	A	$\overline{A}$	3	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	4	−2	0	0	0	0	0	0	0	0	3	$\overline{A}$	A	3	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−6	3	3	−6	0	0	0	−6	3
χ_{68}	0	0	0	0	0	0	4	−2	−4	−4	2	−4	2	2	−4	2	0	0	0	0	0	0	0	−6	3
χ_{69}	0	0	0	0	0	0	−4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	−3
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	−2
χ_{72}	−12	0	−3	3	0	0	−6	3	0	3	−3	0	0	−3	3	0	0	0	0	0	0	0	0	0	0
χ_{73}	12	0	3	−3	0	0	2	−1	4	1	1	4	−2	1	1	−2	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	8	−4	−4	−4	2	2	2	−1	6	0	0	−3	3	−3	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	−3	3	−3	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−6	0	0	3	−3	3	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−6	0	0	3	−3	3	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	−8	4	4	4	−2	−2	−2	1	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	−8	4	4	4	−2	−2	−2	1	6	0	0	−3	3	−3	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	8	−4	−4	−4	2	2	2	−1	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$4B$				$4C$		$4D$		$4E$								$5A$							$6A$	
$[g]_{\overline{G}}$	$4b$	$12d$	$12e$	$12f$	$4c$	$12g$	$4d$	$12h$	$4e$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$	$5a$	$15a$	$15b$	$15c$	$15d$	$15e$	$15f$	$6q$	$6r$
χ_{83}	8	−4	−1	2	0	0	0	0	4	−2	4	−2	−2	−2	1	1	2	2	2	2	−1	−1	−1	0	0
χ_{84}	8	−4	−1	2	0	0	0	0	−4	2	−4	2	2	2	−1	−1	2	2	2	2	−1	−1	−1	0	0
χ_{85}	16	−8	−2	4	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4	−2	−2	−2	0	0
χ_{86}	−8	4	1	−2	0	0	0	0	4	−2	4	−2	−2	−2	1	1	0	0	0	0	0	0	0	0	0
χ_{87}	−8	4	1	−2	0	0	0	0	−4	2	−4	2	2	2	−1	−1	0	0	0	0	0	0	0	0	0
χ_{88}	−8	4	1	−2	0	0	0	0	−4	2	−4	2	2	2	−1	−1	0	0	0	0	0	0	0	0	0
χ_{89}	−8	4	1	−2	0	0	0	0	4	−2	4	−2	−2	−2	1	1	0	0	0	0	0	0	0	0	0
χ_{90}	8	−4	−1	2	0	0	0	0	4	−2	4	−2	−2	−2	1	1	−2	−2	−2	−2	1	1	1	0	0
χ_{91}	8	−4	−1	2	0	0	0	0	−4	2	−4	2	2	2	−1	−1	−2	−2	−2	−2	1	1	1	0	0
χ_{92}	−16	8	2	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{93}	−16	8	2	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	0	0
χ_{99}	16	−8	−2	4	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	−4	−4	2	2	2	0	0
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	−6	−3	3	0	−2	1	0	0	−2	4	4	−5	1	1	1	−2	6	0	0	−3	−3	0	3	0	0
χ_{102}	6	3	−3	0	2	−1	0	0	6	0	0	3	3	−3	−3	0	6	0	0	−3	−3	0	3	0	0
χ_{103}	12	6	−6	0	4	−2	0	0	0	0	0	0	0	0	0	0	−6	0	0	3	3	0	−3	0	0
χ_{104}	−12	−6	6	0	−4	2	0	0	0	0	0	0	0	0	0	0	−6	0	0	3	3	0	−3	0	0
χ_{105}	−6	−3	3	0	6	−3	0	0	−6	0	0	−3	−3	3	3	0	0	0	0	0	0	0	0	0	0
χ_{106}	6	3	−3	0	−6	3	0	0	2	−4	−4	5	−1	−1	−1	2	0	0	0	0	0	0	0	0	0
χ_{107}	−6	−3	3	0	−2	1	0	0	6	0	0	3	3	−3	−3	0	0	0	0	0	0	0	0	0	0
χ_{108}	6	3	−3	0	2	−1	0	0	−2	4	4	−5	1	1	1	−2	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	−4	−4	−4	2	−4	2	2	2	6	0	0	−3	−3	0	3	0	0
χ_{110}	12	6	−6	0	−4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	−12	−6	6	0	4	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	−8	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	0	0	0	0	8	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	−4	−4	−4	2	−4	2	2	2	0	0	0	0	0	0	0	0	0
χ_{115}	6	3	−3	0	−6	3	0	0	−2	4	4	−5	1	1	1	−2	0	0	0	0	0	0	0	0	0
χ_{116}	−6	−3	3	0	6	−3	0	0	6	0	0	3	3	−3	−3	0	0	0	0	0	0	0	0	0	0
χ_{117}	−12	−6	6	0	−4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	12	6	−6	0	4	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$		$6B$					$6C$			$6D$								$6E$		$6F$		$6G$	
$[g]_{\overline{G}}$		$6s$	$6t$	$6u$	$6v$	$6w$	$6x$	$18a$	$18b$	$6y$	$6z$	$6aa$	$6ab$	$18c$	$18d$	$18e$	$18f$	$6ac$	$6ad$	$6ae$	$18g$	$6af$	$18h$
$\lvert C_{\overline{G}}(g)\rvert$	2	4	2	2	3	2	3	3	3	4	3	3	2	3	2	2	1	2	1	2	1	2	1
	3	5	5	5	4	4	3	3	3	4	4	4	4	4	4	4	4	3	3	3	3	2	2
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	1	1	−1	−1	−1	−1
χ_3		−2	−2	−2	−2	−2	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	−1	−1
χ_4		0	0	0	0	0	3	3	3	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0
χ_5		2	2	2	2	2	−1	−1	−1	2	2	2	2	2	2	2	2	−2	−2	2	2	0	0
χ_6		3	3	3	3	3	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
χ_7		−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	0	0	2	2	0	0
χ_8		−3	−3	−3	−3	−3	3	3	3	1	1	1	1	1	1	1	1	−2	−2	0	0	0	0
χ_9		1	1	1	1	1	−2	−2	−2	−1	−1	−1	−1	−1	−1	−1	−1	0	0	−2	−2	0	0
χ_{10}		−1	−1	−1	−1	−1	−1	−1	−1	3	3	3	3	3	3	3	3	−1	−1	−1	−1	1	1
χ_{11}		2	2	2	2	2	−1	−1	−1	−2	−2	−2	−2	−2	−2	−2	−2	1	1	−1	−1	1	1
χ_{12}		−4	−4	−4	−4	−4	2	2	2	0	0	0	0	0	0	0	0	1	1	−1	−1	1	1
χ_{13}		1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1
χ_{14}		2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1
χ_{15}		1	1	1	1	1	−2	−2	−2	−1	−1	−1	−1	−1	−1	−1	−1	−2	−2	−2	−2	0	0
χ_{16}		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	−1	−1	−1	−1
χ_{17}		3	3	3	3	3	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
χ_{18}		−3	−3	−3	−3	−3	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
χ_{19}		−3	−3	−3	−3	−3	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
χ_{20}		−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	2	2	2	2	0	0
χ_{21}		1	1	1	1	1	−2	−2	−2	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1
χ_{22}		−3	−3	−3	−3	−3	0	0	0	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0
χ_{23}		−3	−3	−3	−3	−3	0	0	0	−1	−1	−1	−1	−1	−1	−1	−1	−2	−2	0	0	0	0
χ_{24}		−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−1	−1	1	1	−1	−1
χ_{25}		0	0	0	0	0	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}		−2	−2	−2	−2	−2	−2	−2	−2	2	2	2	2	2	2	2	2	2	2	−2	−2	0	0
χ_{27}		3	3	3	3	3	0	0	0	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0
χ_{28}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}		4	4	4	4	4	1	1	1	0	0	0	0	0	0	0	0	−1	−1	1	1	1	1
χ_{30}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$6B$					$6C$			$6D$								$6E$		$6F$		$6G$	
$[g]_{\overline{G}}$	$6s$	$6t$	$6u$	$6v$	$6w$	$6x$	$18a$	$18b$	$6y$	$6z$	$6aa$	$6ab$	$18c$	$18d$	$18e$	$18f$	$6ac$	$6ad$	$6ae$	$18g$	$6af$	$18h$
χ_{31}	8	−1	−1	−4	2	2	−1	−1	4	−2	−2	1	4	−2	−2	1	0	0	2	−1	0	0
χ_{32}	0	−9	9	0	0	−3	0	3	4	−2	−2	1	4	−2	−2	1	0	0	0	0	0	0
χ_{33}	0	9	−9	0	0	−3	3	0	4	−2	−2	1	4	−2	−2	1	0	0	0	0	0	0
χ_{34}	8	−1	−1	−4	2	2	−1	−1	−4	2	2	−1	−4	2	2	−1	0	0	−4	2	0	0
χ_{35}	−8	1	1	4	−2	1	−5	4	4	−2	−2	1	4	−2	−2	1	0	0	−2	1	0	0
χ_{36}	−8	1	1	4	−2	1	4	−5	4	−2	−2	1	4	−2	−2	1	0	0	−2	1	0	0
χ_{37}	−8	1	1	4	−2	4	−2	−2	−4	2	2	−1	−4	2	2	−1	0	0	4	−2	0	0
χ_{38}	−16	2	2	8	−4	2	−1	−1	0	0	0	0	0	0	0	0	0	0	2	−1	0	0
χ_{39}	8	−1	−1	−4	2	−4	2	2	4	−2	−2	1	4	−2	−2	1	0	0	2	−1	0	0
χ_{40}	16	−2	−2	−8	4	4	−2	−2	0	0	0	0	0	0	0	0	0	0	−2	1	0	0
χ_{41}	0	9	−9	0	0	−3	3	0	−4	2	2	−1	−4	2	2	−1	0	0	0	0	0	0
χ_{42}	0	−9	9	0	0	−3	0	3	−4	2	2	−1	−4	2	2	−1	0	0	0	0	0	0
χ_{43}	−8	1	1	4	−2	−2	1	1	−4	2	2	−1	−4	2	2	−1	0	0	−2	1	0	0
χ_{44}	8	8	−10	−4	2	2	−4	2	0	0	0	0	0	0	0	0	0	0	2	−1	0	0
χ_{45}	8	−10	8	−4	2	2	2	−4	0	0	0	0	0	0	0	0	0	0	2	−1	0	0
χ_{46}	0	0	0	0	0	−3	6	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	−3	−3	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	−8	1	1	4	−2	4	−2	−2	4	−2	−2	1	4	−2	−2	1	0	0	−2	1	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$6B$					$6C$			$6D$								$6E$		$6F$		$6G$	
$[g]_{\overline{G}}$	$6s$	$6t$	$6u$	$6v$	$6w$	$6x$	$18a$	$18b$	$6y$	$6z$	$6aa$	$6ab$	$18c$	$18d$	$18e$	$18f$	$6ac$	$6ad$	$6ae$	$18g$	$6af$	$18h$
χ_{51}	6	−3	−3	3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	2	−1	0	0	0	0
χ_{52}	12	−6	−6	6	0	0	0	0	4	4	−2	−2	4	−2	4	−2	−2	1	0	0	0	0
χ_{53}	−6	3	3	−3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	4	−2	0	0	0	0
χ_{54}	6	−3	−3	3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	0	0	0	0	0	0
χ_{55}	−6	3	3	−3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	−2	1	0	0	0	0
χ_{56}	−6	3	3	−3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	−2	1	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	6	−3	−3	3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	2	−1	0	0	0	0
χ_{59}	−6	3	3	−3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	−2	1	0	0	0	0
χ_{60}	−6	3	3	−3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	−2	1	0	0	0	0
χ_{61}	−6	3	3	−3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	−2	1	0	0	0	0
χ_{62}	−6	3	3	−3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	−2	1	0	0	0	0
χ_{63}	−6	3	3	−3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	0	0	0	0	0	0
χ_{64}	−6	3	3	−3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	6	−3	−3	3	0	0	0	0	−2	−2	1	1	−2	1	−2	1	0	0	0	0	0	0
χ_{68}	6	−3	−3	3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	0	0	0	0	0	0
χ_{69}	−6	3	3	−3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	0	0	0	0	0	0
χ_{70}	−6	3	3	−3	0	0	0	0	2	2	−1	−1	2	−1	2	−1	4	−2	0	0	0	0
χ_{71}	12	−6	−6	6	0	0	0	0	−4	−4	2	2	−4	2	−4	2	−2	1	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	4	4	−2	−2	−2	1	−2	1	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	−4	−4	2	2	2	−1	2	−1	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	4	4	−2	−2	−2	1	−2	1	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	4	4	−2	−2	−2	1	−2	1	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	8	8	−4	−4	−4	2	−4	2	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	−4	−4	2	2	2	−1	2	−1	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	−4	−4	2	2	2	−1	2	−1	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	4	4	−2	−2	−2	1	−2	1	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	−4	−4	2	2	2	−1	2	−1	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$6B$					$6C$			$6D$								$6E$		$6F$		$6G$	
$[g]_{\overline{G}}$	$6s$	$6t$	$6u$	$6v$	$6w$	$6x$	$18a$	$18b$	$6y$	$6z$	$6aa$	$6ab$	$18c$	$18d$	$18e$	$18f$	$6ac$	$6ad$	$6ae$	$18g$	$6af$	$18h$
χ_{83}	0	0	0	0	0	0	0	0	10	−2	−2	4	−5	1	1	−2	0	0	0	0	−1	2
χ_{84}	0	0	0	0	0	0	0	0	−6	6	6	0	3	−3	−3	0	0	0	0	0	1	−2
χ_{85}	0	0	0	0	0	0	0	0	−2	−2	−2	−2	1	1	1	1	0	0	0	0	0	0
χ_{86}	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	−1	0	0	0	0	1	−2
χ_{87}	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	−1	0	0	0	0	−1	2
χ_{88}	0	0	0	0	0	0	0	0	10	−2	−2	4	−5	1	1	−2	0	0	0	0	0	0
χ_{89}	0	0	0	0	0	0	0	0	−6	6	6	0	3	−3	−3	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	−1	0	0	0	0	0	0
χ_{91}	0	0	0	0	0	0	0	0	2	2	2	2	−1	−1	−1	−1	0	0	0	0	0	0
χ_{92}	0	0	0	0	0	0	0	0	−2	−2	−2	−2	1	1	1	1	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	0	0	0	−2	−2	−2	−2	1	1	1	1	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	−12	0	0	−6	6	0	0	3	0	0	0	0	−1	2
χ_{95}	0	0	0	0	0	0	0	0	4	−8	−8	−2	−2	4	4	1	0	0	0	0	1	−2
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	0	0	0	−2	−2	−2	−2	1	1	1	1	0	0	0	0	0	0
χ_{100}	0	0	0	0	0	0	0	0	4	4	4	4	−2	−2	−2	−2	0	0	0	0	0	0
χ_{101}	12	3	3	0	−3	0	0	0	−2	1	−2	1	4	4	−2	−2	0	0	0	0	0	0
χ_{102}	12	3	3	0	−3	0	0	0	6	−3	6	−3	0	0	0	0	0	0	0	0	0	0
χ_{103}	12	3	3	0	−3	0	0	0	−2	1	−2	1	4	4	−2	−2	0	0	0	0	0	0
χ_{104}	12	3	3	0	−3	0	0	0	6	−3	6	−3	0	0	0	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	−4	2	−4	2	−4	−4	2	2	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	−4	2	−4	2	−4	−4	2	2	0	0	0	0	0	0
χ_{107}	−12	−3	−3	0	3	0	0	0	−6	3	−6	3	0	0	0	0	0	0	0	0	0	0
χ_{108}	−12	−3	−3	0	3	0	0	0	2	−1	2	−1	−4	−4	2	2	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	−12	−3	−3	0	3	0	0	0	−2	1	−2	1	4	4	−2	−2	0	0	0	0	0	0
χ_{111}	−12	−3	−3	0	3	0	0	0	6	−3	6	−3	0	0	0	0	0	0	0	0	0	0
χ_{112}	−12	−3	−3	0	3	0	0	0	6	−3	6	−3	0	0	0	0	0	0	0	0	0	0
χ_{113}	−12	−3	−3	0	3	0	0	0	−2	1	−2	1	4	4	−2	−2	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	4	−2	4	−2	4	4	−2	−2	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	12	3	3	0	−3	0	0	0	−6	3	−6	3	0	0	0	0	0	0	0	0	0	0
χ_{118}	12	3	3	0	−3	0	0	0	2	−1	2	−1	−4	−4	2	2	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$		$7A$			$8A$		$8B$			$9A$			$10A$			$12A$		$12B$		$12C$			$15A$		
$[g]_{\overline{G}}$		$7a$	$21a$	$21b$	$8a$	$24a$	$8b$	$24b$	$24c$	$9j$	$9k$	$9l$	$10a$	$30a$	$30b$	$12p$	$36a$	$12q$	$12r$	$12s$	$36b$	$36c$	$15g$	$45a$	$45b$
$\lvert C_{\overline{G}}(g)\rvert$	2	0	0	0	4	3	4	4	4	0	0	0	1	1	1	3	2	3	2	2	2	2	0	0	0
	3	1	1	1	1	1	1	1	1	3	3	3	1	1	1	2	2	2	2	2	2	2	2	2	2
	5	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
	7	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		0	0	0	−1	−1	1	1	1	1	1	1	0	0	0	−2	−2	0	0	0	0	0	−1	−1	−1
χ_3		1	1	1	1	1	−1	−1	−1	0	0	0	0	0	0	0	0	−2	−2	−1	−1	−1	0	0	0
χ_4		0	0	0	−1	−1	1	1	1	0	0	0	−1	−1	−1	2	2	0	0	−1	−1	−1	1	1	1
χ_5		0	0	0	−1	−1	−1	−1	−1	0	0	0	−1	−1	−1	0	0	0	0	1	1	1	1	1	1
χ_6		−1	−1	−1	1	1	−1	−1	−1	0	0	0	0	0	0	1	1	−1	−1	0	0	0	−1	−1	−1
χ_7		0	0	0	−1	−1	1	1	1	−1	−1	−1	0	0	0	−1	−1	1	1	−1	−1	−1	0	0	0
χ_8		0	0	0	1	1	1	1	1	−1	−1	−1	0	0	0	−1	−1	−1	−1	1	1	1	0	0	0
χ_9		0	0	0	0	0	0	0	0	−1	−1	−1	1	1	1	1	1	−1	−1	0	0	0	1	1	1
χ_{10}		0	0	0	0	0	0	0	0	1	1	1	0	0	0	−1	−1	−1	−1	−1	−1	−1	0	0	0
χ_{11}		0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	0	1	1	1	−1	−1	−1
χ_{12}		0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}		0	0	0	1	1	−1	−1	−1	0	0	0	0	0	0	−1	−1	1	1	−1	−1	−1	0	0	0
χ_{14}		0	0	0	1	1	−1	−1	−1	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0
χ_{15}		1	1	1	0	0	0	0	0	0	0	0	0	0	0	−1	−1	1	1	0	0	0	0	0	0
χ_{16}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
χ_{17}		0	0	0	−1	−1	1	1	1	0	0	0	1	1	1	1	1	−1	−1	0	0	0	−1	−1	−1
χ_{18}		0	0	0	1	1	1	1	1	0	0	0	−1	−1	−1	1	1	1	1	0	0	0	−1	−1	−1
χ_{19}		0	0	0	−1	−1	−1	−1	−1	0	0	0	1	1	1	1	1	1	1	0	0	0	−1	−1	−1
χ_{20}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	1	1	1	0	0	0
χ_{21}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
χ_{22}		−1	−1	−1	0	0	0	0	0	0	0	0	1	1	1	−1	−1	1	1	0	0	0	1	1	1
χ_{23}		0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	−1	−1	0	0	0	0	0	0
χ_{24}		0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}		0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
χ_{26}		0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	1	1	1
χ_{27}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	0	0	0	1	1	1
χ_{28}		−1	−1	−1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0
χ_{30}		1	1	1	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	−1	−1	−1

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Table 11.8 (continued)

$[x]_G$	$7A$			$8A$		$8B$			$9A$			$10A$			$12A$		$12B$		$12C$			$15A$		
$[g]_{\overline{G}}$	$7a$	$21a$	$21b$	$8a$	$24a$	$8b$	$24b$	$24c$	$9j$	$9k$	$9l$	$10a$	$30a$	$30b$	$12p$	$36a$	$12q$	$12r$	$12s$	$36b$	$36c$	$15g$	$45a$	$45b$
χ_{31}	0	0	0	0	0	0	0	0	−1	−1	2	0	0	0	0	0	0	0	2	−1	−1	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	0	−1	−1	2	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	1	−2	1	0	0	0	0	0	0	0	−1	2	−1	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	1	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	2	−1	−1	0	0	0	0	0	0	0	1	−2	1	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	−1	2	−1	0	0	0	0	0	0	0	1	1	−2	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	1	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	1	1	−2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−2	1	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	−2	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	−1	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	−1	2	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	2	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	2	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	2	−1	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	−1	−1	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$7A$			$8A$		$8B$			$9A$			$10A$			$12A$		$12B$		$12C$			$15A$		
$[g]_{\overline{G}}$	$7a$	$21a$	$21b$	$8a$	$24a$	$8b$	$24b$	$24c$	$9j$	$9k$	$9l$	$10a$	$30a$	$30b$	$12p$	$36a$	$12q$	$12r$	$12s$	$36b$	$36c$	$15g$	$45a$	$45b$
χ_{51}	0	0	0	0	0	2	−1	−1	0	0	0	2	−1	−1	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	1	B	$\overline{B}$	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	1	$\overline{B}$	B	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	2	−1	−1	0	0	0	−2	1	1	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	C	$-C$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	$-C$	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	2	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	−1	$-\overline{B}$	$-B$	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	−1	$-B$	$-\overline{B}$	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	2	−1	−1	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	2	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	2	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	−2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	−1	D	$D^\star$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	−1	$D^\star$	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	2	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.8 (continued)

$[x]_G$	$7A$			$8A$		$8B$			$9A$			$10A$			$12A$		$12B$		$12C$			$15A$		
$[g]_{\overline{G}}$	$7a$	$21a$	$21b$	$8a$	$24a$	$8b$	$24b$	$24c$	$9j$	$9k$	$9l$	$10a$	$30a$	$30b$	$12p$	$36a$	$12q$	$12r$	$12s$	$36b$	$36c$	$15g$	$45a$	$45b$
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	2	−1	−1
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	2	−1	−1
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	−2	1	1
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0	0	0
χ_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	−2	1	1
χ_{91}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	−2	1	1
χ_{92}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	−1
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	E	$E^\star$
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	$E^\star$	E
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	2	−1	−1
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	0	0	0	−2	1	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0
χ_{102}	0	0	0	2	−1	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0
χ_{103}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0
χ_{105}	0	0	0	2	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	−2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	0	0	0	2	−1	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0
χ_{108}	0	0	0	−2	1	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	2	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	−2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−1	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	0	0

where in Table 11.8

- $A = -3 * E(15)^7 - 3 * E(15)^{11} - 3 * E(15)^{13} - 3 * E(15)^{14} = -\frac{3}{2} + \frac{3\sqrt{15}}{2}i = 3b15$,
- $B = -E(15) - E(15)^2 - E(15)^4 - E(15)^8 = -\frac{1}{2} - \frac{\sqrt{15}}{2}i = -1 - b15$,
- $C = -2 * E(12)^7 + 2 * E(12)^{11} = 2 * ER(3) = 2\sqrt{3}i$,
- $C^\star = 1 - C = 1 - 2\sqrt{3}i$,
- $D = E(21)^2 + E(21)^8 + E(21)^{10} + E(21)^{11} + E(21)^{13} + E(21)^{19} = \frac{1}{2} - \frac{\sqrt{21}}{2}i = -b21$,
- $D^\star = 1 - D = 1 + b21$,
- $E = -2 * E(5) + E(5)^2 + E(5)^3 - 2 * E(5)^4 = \frac{1}{2} - \frac{3\sqrt{5}}{2}i = -1 - 3b5$.

Table 11.9: The character table of Dempwolff group $\overline{G} = 2^5 \cdot GL(5,2)$

$[x]_G$		$1A$		$2A$		$2B$	$3A$		$3B$		$4A$	$4B$	$4C$	$5A$		$6A$			$6B$	$7A$	
$[g]_{\overline{G}}$		$1a$	$2a$	$2b$	$4a$	$4b$	$3a$	$6a$	$6b$	$3b$	$8a$	$4c$	$8b$	$10a$	$5a$	$12a$	$12b$	$6c$	$12c$	$7a$	$14a$
$\lvert C_{\overline{G}}(g)\rvert$	2	15	15	11	11	9	6	6	3	3	7	7	5	1	1	5	5	4	2	3	3
	3	2	2	1	1	1	2	2	2	2	1	0	0	1	1	1	1	1	1	1	0
	5	1	1	0	0	0	0	0	1	1	0	0	0	1	1	0	0	0	0	0	0
	7	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	31	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		30	30	14	14	6	6	6	0	0	6	2	2	0	0	2	2	2	0	2	2
χ_3		124	124	28	28	12	1	1	4	4	4	4	0	−1	−1	1	1	1	0	−2	−2
χ_4		155	155	27	27	−5	8	8	5	5	3	−5	−1	0	0	0	0	0	1	1	1
χ_5		217	217	−7	−7	9	7	7	4	4	−7	1	1	2	2	−1	−1	−1	0	0	0
χ_6		280	280	56	56	8	7	7	−5	−5	8	0	0	0	0	−1	−1	−1	−1	0	0
χ_7		315	315	−21	−21	3	0	0	0	0	3	−1	−1	0	0	0	0	0	0	0	0
χ_8		315	315	−21	−21	3	0	0	0	0	3	−1	−1	0	0	0	0	0	0	0	0
χ_9		315	315	−21	−21	3	0	0	0	0	3	−1	−1	0	0	0	0	0	0	0	0
χ_{10}		315	315	−21	−21	3	0	0	0	0	3	−1	−1	0	0	0	0	0	0	0	0
χ_{11}		315	315	−21	−21	3	0	0	0	0	3	−1	−1	0	0	0	0	0	0	0	0
χ_{12}		315	315	−21	−21	3	0	0	0	0	3	−1	−1	0	0	0	0	0	0	0	0
χ_{13}		465	465	17	17	−15	3	3	0	0	1	1	1	0	0	−1	−1	−1	0	A	A
χ_{14}		465	465	17	17	−15	3	3	0	0	1	1	1	0	0	−1	−1	−1	0	$\overline{A}$	$\overline{A}$
χ_{15}		465	465	−31	−31	9	3	3	0	0	1	−3	1	0	0	−1	−1	−1	0	A	A
χ_{16}		465	465	−31	−31	9	3	3	0	0	1	−3	1	0	0	−1	−1	−1	0	$\overline{A}$	$\overline{A}$
χ_{17}		496	496	48	48	16	−8	−8	1	1	0	0	0	1	1	0	0	0	1	−1	−1
χ_{18}		651	651	−21	−21	−5	0	0	6	6	3	3	−1	1	1	0	0	0	−2	0	0
χ_{19}		651	651	−21	−21	−5	0	0	−3	−3	3	3	−1	1	1	0	0	0	1	0	0
χ_{20}		651	651	−21	−21	−5	0	0	−3	−3	3	3	−1	1	1	0	0	0	1	0	0
χ_{21}		868	868	−28	−28	4	7	7	1	1	−4	4	0	−2	−2	−1	−1	−1	1	0	0
χ_{22}		930	930	50	50	−6	6	6	0	0	−6	−2	−2	0	0	2	2	2	0	−1	−1
χ_{23}		930	930	−14	−14	−6	−3	−3	0	0	2	−2	2	0	0	1	1	1	0	B	B
χ_{24}		930	930	−14	−14	−6	−3	−3	0	0	2	−2	2	0	0	1	1	1	0	$\overline{B}$	$\overline{B}$
χ_{25}		960	960	64	64	0	−6	−6	0	0	0	0	0	0	0	−2	−2	−2	0	1	1
χ_{26}		1024	1024	0	0	0	−8	−8	4	4	0	0	0	−1	−1	0	0	0	0	2	2
χ_{27}		1240	1240	−8	−8	8	1	1	−5	−5	−8	0	0	0	0	1	1	1	−1	1	1

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Table 11.9 (continued)

$[x]_G$	$1A$		$2A$		$2B$	$3A$		$3B$		$4A$	$4B$	$4C$	$5A$		$6A$			$6B$	$7A$	
$[g]_{\overline{G}}$	$1a$	$2a$	$2b$	$4a$	$4b$	$3a$	$6a$	$6b$	$3b$	$8a$	$4c$	$8b$	$10a$	$5a$	$12a$	$12b$	$6c$	$12c$	$7a$	$14a$
χ_{28}	248	−8	−8	8	0	14	−2	4	−4	0	0	0	2	−2	2	2	−2	0	3	−1
χ_{29}	744	−24	−24	24	0	0	0	6	−6	0	0	0	1	−1	0	0	0	0	C	$-A$
χ_{30}	744	−24	−24	24	0	0	0	6	−6	0	0	0	1	−1	0	0	0	0	$\overline{C}$	$-\overline{A}$
χ_{31}	1488	−48	−48	48	0	0	0	−6	6	0	0	0	2	−2	0	0	0	0	−3	1
χ_{32}	1736	−56	−56	56	0	14	−2	−2	2	0	0	0	−1	1	2	2	−2	0	0	0
χ_{33}	1736	−56	−56	56	0	14	−2	−2	2	0	0	0	−1	1	2	2	−2	0	0	0
χ_{34}	1736	−56	−56	56	0	−7	1	4	−4	0	0	0	−1	1	$-\overline{D}$	$-D$	1	0	0	0
χ_{35}	1736	−56	−56	56	0	−7	1	4	−4	0	0	0	−1	1	$-D$	$-\overline{D}$	1	0	0	0
χ_{36}	1984	−64	−64	64	0	−14	2	−4	4	0	0	0	1	−1	−2	−2	2	0	3	−1
χ_{37}	3720	−120	8	−8	0	42	−6	0	0	0	0	0	0	0	−2	−2	2	0	3	−1
χ_{38}	3720	−120	8	−8	0	−21	3	0	0	0	0	0	0	0	D	$\overline{D}$	−1	0	3	−1
χ_{39}	3720	−120	8	−8	0	−21	3	0	0	0	0	0	0	0	$\overline{D}$	D	−1	0	3	−1
χ_{40}	11160	−360	24	−24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	$-A$
χ_{41}	11160	−360	24	−24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{C}$	$-\overline{A}$

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Table 11.9 (continued)

$[x]_G$		$7B$		$8A$	$12A$		$14A$		$14B$		$15A$		$15B$		$21A$	$21B$	$31A$	$31B$	$31C$	$31D$	$31E$	$31F$
$[g]_{\overline{G}}$		$7b$	$14b$	$8b$	$24a$	$24b$	$28a$	$14c$	$28b$	$14d$	$15a$	$30a$	$15b$	$30b$	$21a$	$21b$	$31a$	$31b$	$31c$	$31d$	$31e$	$31f$
$\lvert C_{\overline{G}}(g)\rvert$	2	3	3	4	3	3	2	2	2	2	1	1	1	1	0	0	0	0	0	0	0	0
	3	1	0	0	1	1	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
	7	1	1	0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0
	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		2	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1
χ_3		-2	-2	0	1	1	0	0	0	0	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_4		1	1	-1	0	0	-1	-1	-1	-1	0	0	0	0	1	1	0	0	0	0	0	0
χ_5		0	0	1	-1	-1	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0
χ_6		0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_7		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	E	F	G	$\overline{F}$	$\overline{G}$	$\overline{E}$
χ_8		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	F	G	$\overline{E}$	$\overline{G}$	E	$\overline{F}$
χ_9		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	G	$\overline{E}$	$\overline{F}$	E	F	$\overline{G}$
χ_{10}		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{E}$	$\overline{F}$	$\overline{G}$	F	G	E
χ_{11}		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{F}$	$\overline{G}$	E	G	$\overline{E}$	F
χ_{12}		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{G}$	E	F	$\overline{E}$	$\overline{F}$	G
χ_{13}		$\overline{A}$	$\overline{A}$	1	1	1	A	A	$\overline{A}$	$\overline{A}$	0	0	0	0	A	$\overline{A}$	0	0	0	0	0	0
χ_{14}		A	A	1	1	1	$\overline{A}$	$\overline{A}$	A	A	0	0	0	0	$\overline{A}$	A	0	0	0	0	0	0
χ_{15}		$\overline{A}$	$\overline{A}$	-1	1	1	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	0	0	0	0	A	$\overline{A}$	0	0	0	0	0	0
χ_{16}		A	A	-1	1	1	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	0	0	0	0	$\overline{A}$	A	0	0	0	0	0	0
χ_{17}		-1	-1	0	0	0	-1	-1	-1	-1	1	1	1	1	-1	-1	0	0	0	0	0	0
χ_{18}		0	0	-1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
χ_{19}		0	0	-1	0	0	0	0	0	0	H	H	$\overline{H}$	$\overline{H}$	0	0	0	0	0	0	0	0
χ_{20}		0	0	-1	0	0	0	0	0	0	$\overline{H}$	$\overline{H}$	H	H	0	0	0	0	0	0	0	0
χ_{21}		0	0	0	-1	-1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
χ_{22}		-1	-1	0	0	0	1	1	1	1	0	0	0	0	-1	-1	0	0	0	0	0	0
χ_{23}		$\overline{B}$	$\overline{B}$	0	-1	-1	0	0	0	0	0	0	0	0	$-A$	$-\overline{A}$	0	0	0	0	0	0
χ_{24}		B	B	0	-1	-1	0	0	0	0	0	0	0	0	$-\overline{A}$	$-A$	0	0	0	0	0	0
χ_{25}		1	1	0	0	0	1	1	1	1	0	0	0	0	1	1	-1	-1	-1	-1	-1	-1
χ_{26}		2	2	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_{27}		1	1	0	1	1	-1	-1	-1	-1	0	0	0	0	1	1	0	0	0	0	0	0

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Table 11.9 (continued)

$[x]_G$	$7B$		$8A$	$12A$		$14A$		$14B$		$15A$		$15B$		$21A$	$21B$	$31A$	$31B$	$31C$	$31D$	$31E$	$31F$
$[g]_{\overline{G}}$	$7b$	$14b$	$8b$	$24a$	$24b$	$28a$	$14c$	$28b$	$14d$	$15a$	$30a$	$15b$	$30b$	$21a$	$21b$	$31a$	$31b$	$31c$	$31d$	$31e$	$31f$
χ_{28}	3	-1	0	0	0	1	-1	1	-1	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{29}	$\overline{C}$	$-\overline{A}$	0	0	0	A	$-A$	$\overline{A}$	$-\overline{A}$	-1	1	-1	1	0	0	0	0	0	0	0	0
χ_{30}	C	$-A$	0	0	0	$\overline{A}$	$-\overline{A}$	A	$-A$	-1	1	-1	1	0	0	0	0	0	0	0	0
χ_{31}	-3	1	0	0	0	-1	1	-1	1	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	$\overline{H}$	$-\overline{H}$	H	$-H$	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	H	$-H$	$\overline{H}$	$-\overline{H}$	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	$-I$	I	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	I	$-I$	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{36}	3	-1	0	0	0	1	-1	1	-1	-1	1	-1	1	0	0	0	0	0	0	0	0
χ_{37}	3	-1	0	0	0	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	3	-1	0	I	$-I$	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	3	-1	0	$-I$	I	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	$\overline{C}$	$-\overline{A}$	0	0	0	$-A$	A	$-\overline{A}$	$\overline{A}$	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	C	$-A$	0	0	0	$-\overline{A}$	$\overline{A}$	$-A$	A	0	0	0	0	0	0	0	0	0	0	0	0

where in Table 11.9,

- $A = E(7) + E(7)^2 + E(7)^4 = (-1 + ER(-7))/2 = -\frac{1}{2} + \frac{\sqrt{7}}{2}i = b7$,
- $B = 2 * E(7) + 2 * E(7)^2 + 2 * E(7)^4 = -1 + ER(-7) = -1 + \sqrt{7}i = 2b7$,
- $C = 3 * E(7) + 3 * E(7)^2 + 3 * E(7)^4 = (-3 + 3 * ER(-7))/2 = -\frac{3}{2} + \frac{3\sqrt{7}}{2}i = 3b7$,
- $D = E(3) - 3 * E(3)^2 = 1 + 2 * ER(-3) = 1 + 2\sqrt{3}i$,
- $E = E(31)^3 + E(31)^6 + E(31)^{12} + E(31)^{17} + E(31)^{24}$,
- $F = E(31)^5 + E(31)^9 + E(31)^{10} + E(31)^{18} + E(31)^{20}$,
- $G = E(31)^{15} + E(31)^{23} + E(31)^{27} + E(31)^{29} + E(31)^{30}$,
- $H = -E(15)^7 - E(15)^{11} - E(15)^{13} - E(15)^{14} = -\frac{1}{2} + \frac{\sqrt{15}}{2}i = b15$,
- $I = E(3) - E(3)^2 = ER(-3) = \sqrt{3}i$.

Table 11.10: The character table of $\overline{G} = 2_+^{1+8}.A_9$

$[x]_G$		$1A$				$2A$		$2B$				$3A$		$3B$			$3C$					$4A$		$4B$		
$[g]_{\overline{G}}$		$1a$	$2a$	$2b$	$4a$	$4b$	$4c$	$4d$	$2c$	$4e$	$8a$	$3a$	$6a$	$6b$	$3b$	$12a$	$3c$	$6c$	$6d$	$12b$	$12c$	$8b$	$8c$	$8d$	$4f$	$8e$
$\lvert C_{\overline{G}}(g)\rvert$	2	15	15	14	11	9	9	10	10	9	7	4	4	3	3	2	6	6	5	5	5	5	5	6	6	5
	3	4	4	1	3	1	0	1	1	0	1	3	3	4	4	3	3	3	1	2	2	1	0	0	0	0
	5	1	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		8	8	8	8	4	4	0	0	0	0	5	5	−1	−1	−1	2	2	2	2	2	2	2	0	0	0
χ_3		21	21	21	21	1	1	−3	−3	−3	−3	−3	−3	3	3	3	0	0	0	0	0	−1	−1	1	1	1
χ_4		21	21	21	21	1	1	−3	−3	−3	−3	−3	−3	3	3	3	0	0	0	0	0	−1	−1	1	1	1
χ_5		27	27	27	27	7	7	3	3	3	3	9	9	0	0	0	0	0	0	0	0	1	1	−1	−1	−1
χ_6		28	28	28	28	4	4	−4	−4	−4	−4	10	10	1	1	1	1	1	1	1	1	0	0	0	0	0
χ_7		35	35	35	35	−5	−5	3	3	3	3	5	5	−1	−1	−1	2	2	2	2	2	−1	−1	−1	−1	−1
χ_8		35	35	35	35	−5	−5	3	3	3	3	5	5	−1	−1	−1	2	2	2	2	2	−1	−1	−1	−1	−1
χ_9		42	42	42	42	6	6	2	2	2	2	0	0	−3	−3	−3	3	3	3	3	3	0	0	2	2	2
χ_{10}		48	48	48	48	8	8	0	0	0	0	6	6	3	3	3	0	0	0	0	0	0	0	0	0	0
χ_{11}		56	56	56	56	−4	−4	0	0	0	0	11	11	2	2	2	2	2	2	2	2	−2	−2	0	0	0
χ_{12}		84	84	84	84	4	4	4	4	4	4	−6	−6	3	3	3	3	3	3	3	3	0	0	0	0	0
χ_{13}		105	105	105	105	5	5	1	1	1	1	15	15	−3	−3	−3	−3	−3	−3	−3	−3	−1	−1	1	1	1
χ_{14}		120	120	120	120	0	0	8	8	8	8	0	0	3	3	3	−3	−3	−3	−3	−3	0	0	0	0	0
χ_{15}		162	162	162	162	6	6	−6	−6	−6	−6	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2
χ_{16}		168	168	168	168	4	4	0	0	0	0	−15	−15	−3	−3	−3	0	0	0	0	0	−2	−2	0	0	0
χ_{17}		189	189	189	189	−11	−11	−3	−3	−3	−3	9	9	0	0	0	0	0	0	0	0	1	1	1	1	1
χ_{18}		216	216	216	216	−4	−4	0	0	0	0	−9	−9	0	0	0	0	0	0	0	0	2	2	0	0	0
χ_{19}		120	120	−8	8	0	0	8	−8	0	0	0	0	3	3	−1	6	6	−2	2	2	0	0	0	0	0
χ_{20}		120	120	−8	8	0	0	8	−8	0	0	0	0	3	3	−1	−3	−3	1	A	$\overline{A}$	0	0	0	0	0
χ_{21}		120	120	−8	8	0	0	8	−8	0	0	0	0	3	3	−1	−3	−3	1	$\overline{A}$	A	0	0	0	0	0
χ_{22}		840	840	−56	56	0	0	−8	8	0	0	0	0	−6	−6	2	6	6	−2	2	2	0	0	0	0	0
χ_{23}		840	840	−56	56	0	0	−8	8	0	0	0	0	−6	−6	2	−3	−3	1	A	$\overline{A}$	0	0	0	0	0
χ_{24}		840	840	−56	56	0	0	−8	8	0	0	0	0	−6	−6	2	−3	−3	1	$\overline{A}$	A	0	0	0	0	0
χ_{25}		960	960	−64	64	0	0	0	0	0	0	0	0	−3	−3	1	12	12	−4	4	4	0	0	0	0	0
χ_{26}		960	960	−64	64	0	0	0	0	0	0	0	0	−3	−3	1	−6	−6	2	B	$\overline{B}$	0	0	0	0	0
χ_{27}		960	960	−64	64	0	0	0	0	0	0	0	0	−3	−3	1	−6	−6	2	$\overline{B}$	B	0	0	0	0	0
χ_{28}		2520	2520	−168	168	0	0	−24	24	0	0	0	0	9	9	−3	0	0	0	0	0	0	0	0	0	0
χ_{29}		3240	3240	−216	216	0	0	24	−24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.10 (continued)

$[x]_G$	$1A$				$2A$		$2B$				$3A$		$3B$			$3C$					$4A$		$4B$		
$[g]_{\overline{G}}$	$1a$	$2a$	$2b$	$4a$	$4b$	$4c$	$4d$	$2c$	$4e$	$8a$	$3a$	$6a$	$6b$	$3b$	$12a$	$3c$	$6c$	$6d$	$12b$	$12c$	$8b$	$8c$	$8d$	$4f$	$8e$
χ_{30}	135	135	7	−9	15	−1	7	7	−1	−1	0	0	0	0	0	9	9	1	−3	−3	3	−1	3	−1	−1
χ_{31}	945	945	49	−63	−15	1	1	1	9	−7	0	0	0	0	0	9	9	1	−3	−3	−3	1	−3	1	1
χ_{32}	405	405	21	−27	−15	1	−3	−3	5	−3	0	0	0	0	0	0	0	0	0	0	3	−1	1	−3	1
χ_{33}	405	405	21	−27	−15	1	−3	−3	5	−3	0	0	0	0	0	0	0	0	0	0	3	−1	1	−3	1
χ_{34}	810	810	42	−54	30	−2	18	18	2	−6	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2
χ_{35}	945	945	49	−63	−15	1	17	17	−7	1	0	0	0	0	0	9	9	1	−3	−3	−3	1	1	−3	1
χ_{36}	945	945	49	−63	45	−3	−7	−7	1	1	0	0	0	0	0	9	9	1	−3	−3	3	−1	−3	1	1
χ_{37}	1080	1080	56	−72	0	0	8	8	8	−8	0	0	0	0	0	−9	−9	−1	3	3	0	0	0	0	0
χ_{38}	1890	1890	98	−126	30	−2	10	10	−6	2	0	0	0	0	0	−9	−9	−1	3	3	0	0	−2	−2	2
χ_{39}	2835	2835	147	−189	15	−1	−21	−21	3	3	0	0	0	0	0	0	0	0	0	0	−3	1	3	−1	−1
χ_{40}	2835	2835	147	−189	−45	3	3	3	−5	3	0	0	0	0	0	0	0	0	0	0	3	−1	−1	3	−1
χ_{41}	128	−128	0	0	0	0	0	0	0	0	−4	4	−2	2	0	8	−8	0	0	0	0	0	0	0	0
χ_{42}	128	−128	0	0	0	0	0	0	0	0	−4	4	−2	2	0	8	−8	0	0	0	0	0	0	0	0
χ_{43}	768	−768	0	0	0	0	0	0	0	0	6	−6	6	−6	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	768	−768	0	0	0	0	0	0	0	0	6	−6	6	−6	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	896	−896	0	0	0	0	0	0	0	0	−16	16	4	−4	0	8	−8	0	0	0	0	0	0	0	0
χ_{46}	1792	−1792	0	0	0	0	0	0	0	0	4	−4	8	−8	0	16	−16	0	0	0	0	0	0	0	0
χ_{47}	1920	−1920	0	0	0	0	0	0	0	0	0	0	6	−6	0	−12	12	0	0	0	0	0	0	0	0
χ_{48}	1920	−1920	0	0	0	0	0	0	0	0	0	0	6	−6	0	−12	12	0	0	0	0	0	0	0	0
χ_{49}	2560	−2560	0	0	0	0	0	0	0	0	−20	20	−4	4	0	−8	8	0	0	0	0	0	0	0	0
χ_{50}	2688	−2688	0	0	0	0	0	0	0	0	12	−12	−6	6	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	2688	−2688	0	0	0	0	0	0	0	0	12	−12	−6	6	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	3584	−3584	0	0	0	0	0	0	0	0	−4	4	−2	2	0	8	−8	0	0	0	0	0	0	0	0

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Table 11.10 (continued)

$[x]_G$		$5A$		$6A$	$6B$					$7A$					$9A$		$9B$					$10A$	$12A$	
$[g]_{\overline{G}}$		$5a$	$10a$	$12d$	$12e$	$12f$	$6e$	$24a$	$24b$	$7a$	$14a$	$14b$	$14c$	$28a$	$9a$	$18a$	$18b$	$9b$	$36a$	$36b$	$36c$	$20a$	$24c$	$24d$
$\lvert C_{\overline{G}}(g)\rvert$	2	3	3	3	4	4	3	3	3	3	3	2	2	2	1	1	3	3	2	2	2	2	3	3
	3	1	1	1	1	1	1	1	1	0	0	0	0	0	2	2	2	2	2	2	2	0	1	1
	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	7	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		3	3	1	0	0	0	0	0	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_3		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−1
χ_4		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−1
χ_5		2	2	1	0	0	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	2	1	1
χ_6		3	3	−2	−1	−1	−1	−1	−1	0	0	0	0	0	1	1	1	1	1	1	1	−1	0	0
χ_7		0	0	1	0	0	0	0	0	0	0	0	0	0	−1	−1	2	2	2	2	2	0	−1	−1
χ_8		0	0	1	0	0	0	0	0	0	0	0	0	0	2	2	−1	−1	−1	−1	−1	0	−1	−1
χ_9		−3	−3	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
χ_{10}		−2	−2	2	0	0	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	−2	0	0
χ_{11}		1	1	−1	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1	1	1	1
χ_{12}		−1	−1	−2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0
χ_{13}		0	0	−1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
χ_{14}		0	0	0	−1	−1	−1	−1	−1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{15}		−3	−3	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	1	0	0
χ_{16}		3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1
χ_{17}		−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1
χ_{18}		1	1	−1	0	0	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	1	−1	−1
χ_{19}		0	0	0	2	2	−2	0	0	1	1	−1	−1	1	0	0	3	3	−1	−1	−1	0	0	0
χ_{20}		0	0	0	−1	−1	1	$-C$	C	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0
χ_{21}		0	0	0	−1	−1	1	C	$-C$	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0
χ_{22}		0	0	0	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	2	$-D$	$-\overline{D}$	0	0	0
χ_{23}		0	0	0	1	1	−1	D	$-D$	0	0	0	0	0	0	0	0	0	−2	D	$\overline{D}$	0	0	0
χ_{24}		0	0	0	1	1	−1	$-D$	D	0	0	0	0	0	0	0	0	0	−2	$\overline{D}$	D	0	0	0
χ_{25}		0	0	0	0	0	0	0	0	1	1	−1	−1	1	0	0	0	0	2	$-D$	$-\overline{D}$	0	0	0
χ_{26}		0	0	0	0	0	0	0	0	1	1	−1	−1	1	0	0	0	0	2	$-\overline{D}$	$-D$	0	0	0
χ_{27}		0	0	0	0	0	0	0	0	1	1	−1	−1	1	0	0	−3	−3	1	1	1	0	0	0
χ_{28}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	$-\overline{D}$	$-D$	0	0	0
χ_{29}		0	0	0	0	0	0	0	0	−1	−1	1	1	−1	0	0	3	3	−1	−1	−1	0	0	0

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Table 11.10 (continued)

$[x]_G$	$5A$		$6A$	$6B$					$7A$					$9A$		$9B$					$10A$	$12A$	
$[g]_{\overline{G}}$	$5a$	$10a$	$12d$	$12e$	$12f$	$6e$	$24a$	$24b$	$7a$	$14a$	$14b$	$14c$	$28a$	$9a$	$18a$	$18b$	$9b$	$36a$	$36b$	$36c$	$20a$	$24c$	$24d$
χ_{30}	0	0	0	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	0	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	−1	−1	E	$-E$	1	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	−1	−1	$-E$	E	1	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	−2	−2	0	0	2	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	−1	−1	−1	1	1	2	2	0	0	−2	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	−2	2	0	0	0	0	0	0	2	−2	0	0	0	−1	1	4	−4	0	0	0	0	0	0
χ_{42}	−2	2	0	0	0	0	0	0	2	−2	0	0	0	2	−2	−2	2	0	0	0	0	0	0
χ_{43}	−2	2	0	0	0	0	0	0	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	−2	2	0	0	0	0	0	0	−2	2	0	0	0	−1	1	−2	2	0	0	0	0	0	0
χ_{45}	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	G	$-G$
χ_{46}	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	F	$-F$	0	0	0	2	−2	0	0	0	1	−1	2	−2	0	0	0	0	0	0
χ_{48}	0	0	0	$-F$	F	0	0	0	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	−2	2	0	0	0	1	−1	2	−2	0	0	0	0	0	0
χ_{50}	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-G$	G
χ_{51}	−2	2	0	0	0	0	0	0	0	0	0	0	0	−1	1	−2	2	0	0	0	0	0	0
χ_{52}	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.10 (continued)

$[x]_G$		$15A$		$15B$	
$[g]_{\overline{G}}$		$15a$	$30a$	$15b$	$30b$
$\lvert C_{\overline{G}}(g)\rvert$	2	1	1	1	1
	3	1	1	1	1
	5	1	1	1	1
	7	0	0	0	0
χ_1		1	1	1	1
χ_2		0	0	0	0
χ_3		H	H	$\overline{H}$	$\overline{H}$
χ_4		$\overline{H}$	$\overline{H}$	H	H
χ_5		-1	-1	-1	-1
χ_6		0	0	0	0
χ_7		0	0	0	0
χ_8		0	0	0	0
χ_9		0	0	0	0
χ_{10}		1	1	1	1
χ_{11}		1	1	1	1
χ_{12}		-1	-1	-1	-1
χ_{13}		0	0	0	0
χ_{14}		0	0	0	0
χ_{15}		0	0	0	0
χ_{16}		0	0	0	0
χ_{17}		-1	-1	-1	-1
χ_{18}		1	1	1	1
χ_{19}		0	0	0	0
χ_{20}		0	0	0	0
χ_{21}		0	0	0	0
χ_{22}		0	0	0	0
χ_{23}		0	0	0	0
χ_{24}		0	0	0	0
χ_{25}		0	0	0	0
χ_{26}		0	0	0	0
χ_{27}		0	0	0	0
χ_{28}		0	0	0	0
χ_{29}		0	0	0	0
χ_{30}		0	0	0	0
χ_{31}		0	0	0	0
χ_{32}		0	0	0	0
χ_{33}		0	0	0	0
χ_{34}		0	0	0	0
χ_{35}		0	0	0	0
χ_{36}		0	0	0	0
χ_{37}		0	0	0	0
χ_{38}		0	0	0	0
χ_{39}		0	0	0	0
χ_{40}		0	0	0	0
χ_{41}		1	-1	1	-1
χ_{42}		1	-1	1	-1
χ_{43}		$\overline{H}$	$-\overline{H}$	H	$-H$
χ_{44}		1	-1	1	-1
χ_{45}		1	-1	1	-1
χ_{46}		0	0	0	0
χ_{47}		0	0	0	0
χ_{48}		H	$-H$	$\overline{H}$	$-\overline{H}$
χ_{49}		-1	1	-1	1
χ_{50}		1	-1	1	-1
χ_{51}		-1	1	-1	1
χ_{52}		0	0	0	0

where in Table 11.10,

- $A = 3 * E(3) - E(3)^2 = -1 + 2 * ER(-3) = -1 + 2\sqrt{3}i = 1 + 4b3$,
- $B = 6 * E(3) - 2 * E(3)^2 = -2 + 4 * ER(-3) = -2 + 4\sqrt{3}i = 2 + 8b3$,
- $C = 2 * E(3) - 2 * E(3)^2 = 2 * ER(-3) = 2\sqrt{3}i$,
- $D = -2 * E(3) = 1 - ER(-3) = 1 - \sqrt{3}i$,
- $E = E(7) + E(7)^2 - E(7)^3 + E(7)^4 - E(7)^5 - E(7)^6 = \sqrt{7}i$,
- $F = 2 * E(3) - 2 * E(3)^2 = 2 * ER(-3) = 2\sqrt{3}i$,
- $G = E(24) + E(24)^{11} - E(24)^{17} - E(24)^{19} = ER(-6) = \sqrt{6}i$,
- $H = -E(15)^7 - E(15)^{11} - E(15)^{13} - E(15)^{14} = -\frac{1}{2} + \frac{\sqrt{15}}{2}i = b15$.

Table 11.11: The character table of $H_2 = 2^5{:}S_6$

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$2g$	$2h$	$2i$	$2j$	$3a$	$3b$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$
	g_{121}	g_{221}	g_{421}	g_{321}	g_{222}	g_{422}	g_{521}	g_{522}	g_{423}	g_{523}	g_{322}	g_{621}	g_{821}	$g_{11,21}$	$g_{10,21}$	$g_{12,21}$	g_{921}	$g_{13,21}$	$g_{12,22}$
$\lvert C_{2^5:S_6}(g_{ikm})\rvert$	23040	23040	1536	1536	768	768	128	128	128	384	384	144	36	192	192	128	128	32	64
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	1	−1
χ_3	5	5	5	5	3	3	3	1	1	−1	−1	2	−1	3	3	1	1	1	−1
χ_4	5	5	5	5	−3	−3	−3	1	1	1	1	2	−1	−3	−3	1	1	1	1
χ_5	5	5	5	5	−1	−1	−1	1	1	3	3	−1	2	−1	−1	1	1	1	3
χ_6	5	5	5	5	1	1	1	1	1	−3	−3	−1	2	1	1	1	1	1	−3
χ_7	6	−6	2	−2	4	−4	0	−2	2	0	0	3	0	2	−2	−2	2	0	0
χ_8	6	−6	2	−2	−4	4	0	−2	2	0	0	3	0	−2	2	−2	2	0	0
χ_9	9	9	9	9	−3	−3	−3	1	1	−3	−3	0	0	−3	−3	1	1	1	−3
χ_{10}	9	9	9	9	3	3	3	1	1	3	3	0	0	3	3	1	1	1	3
χ_{11}	10	−10	−2	2	−4	4	0	−2	2	−4	4	1	1	2	−2	2	−2	0	0
χ_{12}	10	−10	−2	2	−4	4	0	−2	2	4	−4	1	1	2	−2	2	−2	0	0
χ_{13}	10	−10	−2	2	4	−4	0	−2	2	−4	4	1	1	−2	2	2	−2	0	0
χ_{14}	10	−10	−2	2	4	−4	0	−2	2	4	−4	1	1	−2	2	2	−2	0	0
χ_{15}	10	10	10	10	−2	−2	−2	−2	−2	2	2	1	1	−2	−2	−2	−2	−2	2
χ_{16}	10	10	10	10	2	2	2	−2	−2	−2	−2	1	1	2	2	−2	−2	−2	−2
χ_{17}	15	15	−1	−1	−5	−5	3	−1	−1	3	3	3	0	−1	−1	3	3	−1	−1
χ_{18}	15	15	−1	−1	−7	−7	1	3	3	−3	−3	3	0	1	1	−1	−1	−1	1
χ_{19}	15	15	−1	−1	5	5	−3	−1	−1	−3	−3	3	0	1	1	3	3	−1	1
χ_{20}	15	15	−1	−1	7	7	−1	3	3	3	3	3	0	−1	−1	−1	−1	−1	−1
χ_{21}	16	16	16	16	0	0	0	0	0	0	0	−2	−2	0	0	0	0	0	0
χ_{22}	20	−20	−4	4	0	0	0	4	−4	0	0	2	2	0	0	−4	4	0	0
χ_{23}	24	−24	8	−8	−8	8	0	0	0	0	0	3	0	−4	4	0	0	0	0

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Table 11.11 (continued)

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$2g$	$2h$	$2i$	$2j$	$3a$	$3b$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$
	g_{121}	g_{221}	g_{421}	g_{321}	g_{222}	g_{422}	g_{521}	g_{522}	g_{423}	g_{523}	g_{322}	g_{621}	g_{821}	$g_{11,21}$	$g_{10,21}$	$g_{12,21}$	g_{921}	$g_{13,21}$	$g_{12,22}$
$\lvert C_{2^5:S_6}(g_{ikm})\rvert$	23040	23040	1536	1536	768	768	128	128	128	384	384	144	36	192	192	128	128	32	64
χ_{24}	24	−24	8	−8	8	−8	0	0	0	0	0	3	0	4	−4	0	0	0	0
χ_{25}	30	−30	10	−10	−4	4	0	−2	2	0	0	−3	0	−2	2	−2	2	0	0
χ_{26}	30	−30	10	−10	4	−4	0	−2	2	0	0	−3	0	2	−2	−2	2	0	0
χ_{27}	30	30	−2	−2	−2	−2	−2	2	2	−6	−6	−3	0	2	2	2	2	−2	2
χ_{28}	30	30	−2	−2	2	2	2	2	2	6	6	−3	0	−2	−2	2	2	−2	−2
χ_{29}	36	−36	12	−12	0	0	0	4	−4	0	0	0	0	0	0	4	−4	0	0
χ_{30}	40	−40	−8	8	8	−8	0	0	0	0	0	1	−2	−4	4	0	0	0	0
χ_{31}	40	−40	−8	8	−8	8	0	0	0	0	0	1	−2	4	−4	0	0	0	0
χ_{32}	40	−40	−8	8	0	0	0	0	0	8	−8	−2	1	0	0	0	0	0	0
χ_{33}	40	−40	−8	8	0	0	0	0	0	−8	8	−2	1	0	0	0	0	0	0
χ_{34}	45	45	−3	−3	9	9	1	1	1	−3	−3	0	0	−3	−3	−3	−3	1	1
χ_{35}	45	45	−3	−3	−9	−9	−1	1	1	3	3	0	0	3	3	−3	−3	1	−1
χ_{36}	45	45	−3	−3	−3	−3	5	−3	−3	−3	−3	0	0	−3	−3	1	1	1	1
χ_{37}	45	45	−3	−3	3	3	−5	−3	−3	3	3	0	0	3	3	1	1	1	−1

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Table 11.11 (continued)

	$4g$	$4h$	$4i$	$4j$	$5a$	$6a$	$6b$	$6c$	$6d$	$6e$	$6f$	$6g$	$6h$	$8a$	$8b$	$10a$	$12a$	$12b$
	$g_{13,22}$	$g_{11,22}$	$g_{10,22}$	$g_{13,23}$	$g_{14,21}$	$g_{16,21}$	$g_{18,21}$	$g_{15,21}$	$g_{15,22}$	$g_{18,22}$	$g_{19,21}$	$g_{21,21}$	$g_{20,21}$	$g_{23,21}$	$g_{24,21}$	$g_{26,21}$	$g_{27,21}$	$g_{28,21}$
$\lvert C_{H_2}(g_{ikm})\rvert$	32	32	32	32	10	48	48	144	24	24	36	12	12	16	16	10	24	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	−1	−1	1	1	1	1	1	1	−1	−1	1	−1	−1	−1	1	1	−1	−1
χ_3	1	1	−1	−1	0	2	2	2	0	0	−1	−1	−1	1	−1	0	0	0
χ_4	−1	−1	−1	−1	0	2	2	2	0	0	−1	1	1	−1	−1	0	0	0
χ_5	1	1	−1	−1	0	−1	−1	−1	−1	−1	2	0	0	1	−1	0	−1	−1
χ_6	−1	−1	−1	−1	0	−1	−1	−1	1	1	2	0	0	−1	−1	0	1	1
χ_7	−2	2	0	0	1	1	−1	-3	1	−1	0	0	0	0	0	−1	1	−1
χ_8	2	−2	0	0	1	1	−1	-3	−1	1	0	0	0	0	0	−1	−1	1
χ_9	1	1	1	1	−1	0	0	0	0	0	0	0	0	1	1	−1	0	0
χ_{10}	−1	−1	1	1	−1	0	0	0	0	0	0	0	0	−1	1	−1	0	0
χ_{11}	0	0	−2	2	0	−1	1	−1	−1	1	−1	−1	1	0	0	0	1	−1
χ_{12}	0	0	2	−2	0	−1	1	−1	−1	1	−1	1	−1	0	0	0	1	−1
χ_{13}	0	0	2	−2	0	−1	1	−1	1	−1	−1	−1	1	0	0	0	−1	1
χ_{14}	0	0	−2	2	0	−1	1	−1	1	−1	−1	1	−1	0	0	0	−1	1
χ_{15}	0	0	0	0	0	1	1	1	1	1	1	−1	−1	0	0	0	1	1
χ_{16}	0	0	0	0	0	1	1	1	−1	−1	1	1	1	0	0	0	−1	−1
χ_{17}	−1	−1	−1	−1	0	−1	−1	3	1	1	0	0	0	1	1	0	−1	−1
χ_{18}	−1	−1	1	1	0	−1	−1	3	−1	−1	0	0	0	1	−1	0	1	1
χ_{19}	1	1	−1	−1	0	−1	−1	3	−1	−1	0	0	0	−1	1	0	1	1
χ_{20}	1	1	1	1	0	−1	−1	3	1	1	0	0	0	−1	−1	0	−1	−1
χ_{21}	0	0	0	0	1	−2	−2	−2	0	0	−2	0	0	0	0	1	0	0
χ_{22}	0	0	0	0	0	−2	2	−2	0	0	−2	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	−1	1	−1	−3	1	−1	0	0	0	0	0	1	1	−1

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Table 11.11 (continued)

	$4g$	$4h$	$4i$	$4j$	$5a$	$6a$	$6b$	$6c$	$6d$	$6e$	$6f$	$6g$	$6h$	$8a$	$8b$	$10a$	$12a$	$12b$
	$g_{13,22}$	$g_{11,22}$	$g_{10,22}$	$g_{13,23}$	$g_{14,21}$	$g_{16,21}$	$g_{18,21}$	$g_{15,21}$	$g_{15,22}$	$g_{18,22}$	$g_{19,21}$	$g_{21,21}$	$g_{20,21}$	$g_{23,21}$	$g_{24,21}$	$g_{26,21}$	$g_{27,21}$	$g_{28,21}$
$\lvert C_{H_2}(g_{ikm})\rvert$	32	32	32	32	10	48	48	144	24	24	36	12	12	16	16	10	24	24
χ_{24}	0	0	0	0	−1	1	−1	−3	−1	1	0	0	0	0	0	1	−1	1
χ_{25}	−2	2	0	0	0	−1	1	3	−1	1	0	0	0	0	0	0	−1	1
χ_{26}	2	−2	0	0	0	−1	1	3	1	−1	0	0	0	0	0	0	1	−1
χ_{27}	0	0	0	0	0	1	1	−3	1	1	0	0	0	0	0	0	−1	−1
χ_{28}	0	0	0	0	0	1	1	−3	−1	−1	0	0	0	0	0	0	1	1
χ_{29}	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	−1	0	0
χ_{30}	0	0	0	0	0	−1	1	−1	−1	1	2	0	0	0	0	0	1	−1
χ_{31}	0	0	0	0	0	−1	1	−1	1	−1	2	0	0	0	0	0	−1	1
χ_{32}	0	0	0	0	0	2	−2	2	0	0	−1	−1	1	0	0	0	0	0
χ_{33}	0	0	0	0	0	2	−2	2	0	0	−1	1	−1	0	0	0	0	0
χ_{34}	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	1	1	0	0	0
χ_{35}	1	1	−1	−1	0	0	0	0	0	0	0	0	0	−1	1	0	0	0
χ_{36}	1	1	1	1	0	0	0	0	0	0	0	0	0	−1	−1	0	0	0
χ_{37}	−1	−1	1	1	0	0	0	0	0	0	0	0	0	1	−1	0	0	0

Table 11.12: The character table of $\overline{G} = 2^{6\cdot}Sp(6,2)$

	$1A$		$2A$			$2B$			$2C$				$2D$				$3A$	
	g_{11}	g_{12}	g_{21}	g_{22}	g_{23}	g_{31}	g_{32}	g_{33}	g_{41}	g_{42}	g_{43}	g_{44}	g_{51}	g_{52}	g_{53}	g_{54}	g_{61}	g_{62}
	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$4g$	$2f$	$4h$	$4i$	$3a$	$6a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	92897280	1474560	737280	46080	49152	73728	24576	6144	24576	8192	6144	3072	3072	3072	1024	1024	34560	2304
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	7	7	−5	−5	−5	−1	−1	−1	3	3	3	3	−1	−1	−1	−1	4	4
χ_3	15	15	−5	−5	−5	7	7	7	3	3	3	3	−1	−1	−1	−1	0	0
χ_4	21	21	−11	−11	−11	5	5	5	5	5	5	5	−3	−3	−3	−3	6	6
χ_5	21	21	9	9	9	−3	−3	−3	1	1	1	1	−3	−3	−3	−3	6	6
χ_6	27	27	15	15	15	3	3	3	7	7	7	7	3	3	3	3	9	9
χ_7	35	35	15	15	15	11	11	11	7	7	7	7	3	3	3	3	5	5
χ_8	35	35	−5	−5	−5	3	3	3	−5	−5	−5	−5	3	3	3	3	5	5
χ_9	56	56	−24	−24	−24	−8	−8	−8	8	8	8	8	0	0	0	0	11	11
χ_{10}	70	70	−10	−10	−10	−10	−10	−10	6	6	6	6	−2	−2	−2	−2	−5	−5
χ_{11}	84	84	4	4	4	20	20	20	4	4	4	4	4	4	4	4	−6	−6
χ_{12}	105	105	−35	−35	−35	1	1	1	5	5	5	5	1	1	1	1	15	15
χ_{13}	105	105	25	25	25	−7	−7	−7	9	9	9	9	1	1	1	1	0	0
χ_{14}	105	105	5	5	5	17	17	17	−3	−3	−3	−3	−7	−7	−7	−7	0	0
χ_{15}	120	120	40	40	40	−8	−8	−8	8	8	8	8	0	0	0	0	15	15
χ_{16}	168	168	40	40	40	8	8	8	8	8	8	8	8	8	8	8	6	6
χ_{17}	189	189	−39	−39	−39	21	21	21	1	1	1	1	−3	−3	−3	−3	9	9
χ_{18}	189	189	−51	−51	−51	−3	−3	−3	13	13	13	13	−3	−3	−3	−3	9	9
χ_{19}	189	189	21	21	21	−3	−3	−3	−11	−11	−11	−11	−3	−3	−3	−3	9	9
χ_{20}	210	210	50	50	50	2	2	2	2	2	2	2	−6	−6	−6	−6	15	15

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Table 11.12 (continued)

	$1A$		$2A$			$2B$			$2C$				$2D$				$3A$	
	g_{11}	g_{12}	g_{21}	g_{22}	g_{23}	g_{31}	g_{32}	g_{33}	g_{41}	g_{42}	g_{43}	g_{44}	g_{51}	g_{52}	g_{53}	g_{54}	g_{61}	g_{62}
	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$4g$	$2f$	$4h$	$4i$	$3a$	$6a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	92897280	1474560	737280	46080	49152	73728	24576	6144	24576	8192	6144	3072	3072	3072	1024	1024	34560	2304
χ_{21}	210	210	10	10	10	−14	−14	−14	10	10	10	10	2	2	2	2	−15	−15
χ_{22}	216	216	−24	−24	−24	24	24	24	8	8	8	8	0	0	0	0	−9	−9
χ_{23}	280	280	40	40	40	24	24	24	8	8	8	8	0	0	0	0	−5	−5
χ_{24}	280	280	−40	−40	−40	−8	−8	−8	−8	−8	−8	−8	8	8	8	8	10	10
χ_{25}	315	315	−45	−45	−45	−21	−21	−21	3	3	3	3	3	3	3	3	0	0
χ_{26}	336	336	−16	−16	−16	16	16	16	−16	−16	−16	−16	0	0	0	0	6	6
χ_{27}	378	378	−30	−30	−30	−6	−6	−6	2	2	2	2	−6	−6	−6	−6	−9	−9
χ_{28}	405	405	45	45	45	−27	−27	−27	−3	−3	−3	−3	−3	−3	−3	−3	0	0
χ_{29}	420	420	20	20	20	4	4	4	−12	−12	−12	−12	4	4	4	4	0	0
χ_{30}	512	512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−16	−16

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Table 11.12 (continued)

	$1A$		$2A$			$2B$			$2C$				$2D$				$3A$	
	g_{11}	g_{12}	g_{21}	g_{22}	g_{23}	g_{31}	g_{32}	g_{33}	g_{41}	g_{42}	g_{43}	g_{44}	g_{51}	g_{52}	g_{53}	g_{54}	g_{61}	g_{62}
	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$4g$	$2f$	$4h$	$4i$	$3a$	$6a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	92897280	1474560	737280	46080	49152	73728	24576	6144	24576	8192	6144	3072	3072	3072	1024	1024	34560	2304
χ_{31}	63	−1	31	−1	−1	15	−1	−1	15	−1	−1	−1	7	−1	−1	−1	15	−1
χ_{32}	63	−1	−29	3	−1	−9	7	−1	11	−5	3	−1	−1	−5	−1	3	15	−1
χ_{33}	315	−5	−85	11	−5	27	11	−5	11	−5	11	−5	−5	−13	3	3	30	−2
χ_{34}	315	−5	95	−1	−5	3	19	−5	23	7	−1	−5	11	7	−5	−1	30	−2
χ_{35}	315	−5	−25	7	−5	51	3	−5	15	−1	7	−5	3	−9	3	−1	−15	1
χ_{36}	315	−5	35	3	−5	−21	27	−5	19	3	3	−5	3	3	−5	3	−15	1
χ_{37}	378	−6	−126	2	6	−6	−6	2	34	2	−6	−2	−6	6	2	−2	45	−3
χ_{38}	378	−6	114	−14	6	−6	−6	2	18	−14	10	−2	−6	6	2	−2	45	−3
χ_{39}	567	−9	−81	15	−9	−9	39	−9	15	−1	15	−9	−9	−9	−1	7	0	0
χ_{40}	567	−9	99	3	−9	63	15	−9	27	11	3	−9	15	3	−1	−5	0	0
χ_{41}	630	−10	−130	−2	10	54	−10	−2	30	−2	−10	2	−10	10	−2	2	15	−1
χ_{42}	630	−10	−130	−2	10	−42	22	−2	30	−2	−10	2	−2	2	6	−6	15	−1
χ_{43}	630	−10	−50	14	−10	54	22	−10	−18	14	14	−10	−10	−2	6	−2	15	−1
χ_{44}	630	−10	110	−18	10	54	−10	−2	14	−18	6	2	−10	10	−2	2	15	−1
χ_{45}	630	−10	110	−18	10	−42	22	−2	14	−18	6	2	−2	2	6	−6	15	−1
χ_{46}	630	−10	70	6	−10	6	38	−10	−10	22	6	−10	−2	14	−2	−2	15	−1
χ_{47}	945	−15	225	1	−15	33	−15	1	49	1	−15	1	9	−15	1	1	45	−3
χ_{48}	945	−15	−135	25	−15	33	−15	1	−23	−7	9	1	9	9	1	−7	45	−3
χ_{49}	945	−15	−195	29	−15	−39	9	1	21	−27	13	1	9	−3	−7	5	45	−3
χ_{50}	945	−15	165	5	−15	−39	9	1	−3	13	−11	1	−15	−3	1	5	45	−3
χ_{51}	1008	−16	16	16	−16	48	48	−16	16	16	16	−16	0	0	0	0	−30	2

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Table 11.12 (continued)

	$1A$		$2A$			$2B$			$2C$				$2D$				$3A$	
	g_{11}	g_{12}	g_{21}	g_{22}	g_{23}	g_{31}	g_{32}	g_{33}	g_{41}	g_{42}	g_{43}	g_{44}	g_{51}	g_{52}	g_{53}	g_{54}	g_{61}	g_{62}
	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$4g$	$2f$	$4h$	$4i$	$3a$	$6a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	92897280	1474560	737280	46080	49152	73728	24576	6144	24576	8192	6144	3072	3072	3072	1024	1024	34560	2304
χ_{52}	1260	−20	−20	−20	20	12	12	−4	−52	12	−4	4	12	−12	−4	4	30	−2
χ_{53}	1512	−24	216	−40	24	−24	−24	8	−8	−8	24	−8	0	0	0	0	45	−3
χ_{54}	1512	−24	−264	−8	24	−24	−24	8	24	24	−8	−8	0	0	0	0	45	−3
χ_{55}	1890	−30	−150	−22	30	−30	−30	10	42	10	2	−10	−6	6	2	−2	−45	3
χ_{56}	1890	−30	−30	34	−30	−78	18	2	18	−14	2	2	−6	−6	−6	10	−45	3
χ_{57}	1890	−30	90	−38	30	−30	−30	10	26	−6	18	−10	−6	6	2	−2	−45	3
χ_{58}	1890	−30	90	26	−30	66	−30	2	26	−6	−6	2	18	−6	2	−6	−45	3
χ_{59}	2268	−36	−36	−36	36	−36	−36	12	−36	28	12	−12	12	−12	−4	4	0	0
χ_{60}	2520	−40	200	−56	40	24	24	−8	−24	−24	8	8	0	0	0	0	15	−1
χ_{61}	2520	−40	−280	−24	40	24	24	−8	8	8	−24	8	0	0	0	0	15	−1
χ_{62}	2520	−40	−40	−40	40	−72	56	−8	−8	−8	−8	8	8	−8	8	−8	−30	2
χ_{63}	2520	−40	−40	−40	40	120	−8	−8	−8	−8	−8	8	−8	8	−8	8	−30	2
χ_{64}	2835	−45	−225	63	−45	27	−21	3	−9	−25	15	3	3	−9	3	−1	0	0
χ_{65}	2835	−45	315	27	−45	−45	3	3	27	11	−21	3	3	3	−5	3	0	0
χ_{66}	2835	−45	135	39	−45	27	−21	3	−33	15	−9	3	−21	−9	11	−1	0	0
χ_{67}	2835	−45	−45	51	−45	−45	3	3	−45	3	3	3	3	27	−5	−5	0	0

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Table 11.12 (continued)

	$3B$	$3C$		$4A$		$4B$			$4C$			$4D$			$4E$				$5A$	
	g_{71}	g_{81}	g_{82}	g_{91}	g_{92}	$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$	$g_{14,1}$	$g_{14,2}$
	$3b$	$3c$	$6b$	$4j$	$4k$	$8a$	$8b$	$4l$	$8c$	$8d$	$4m$	$4n$	$4o$	$4p$	$8e$	$4q$	$8f$	$4r$	$10a$	$5a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	648	432	144	1536	512	1536	512	384	1536	512	384	512	512	256	128	128	128	128	40	120
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	−2	1	1	3	3	1	1	1	−3	−3	−3	−1	−1	−1	1	1	1	1	2	2
χ_3	−3	3	3	−1	−1	−3	−3	−3	1	1	1	3	3	3	1	1	1	1	0	0
χ_4	3	0	0	1	1	−3	−3	−3	−3	−3	−3	1	1	1	1	1	1	1	1	1
χ_5	3	0	0	5	5	−1	−1	−1	3	3	3	1	1	1	−1	−1	−1	−1	1	1
χ_6	0	0	0	3	3	1	1	1	5	5	5	−1	−1	−1	1	1	1	1	2	2
χ_7	−1	2	2	−1	−1	5	5	5	1	1	1	3	3	3	1	1	1	1	0	0
χ_8	−1	2	2	7	7	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0
χ_9	2	2	2	0	0	4	4	4	−4	−4	−4	0	0	0	0	0	0	0	1	1
χ_{10}	7	1	1	2	2	2	2	2	2	2	2	2	2	2	−2	−2	−2	−2	0	0
χ_{11}	3	3	3	4	4	0	0	0	0	0	0	4	4	4	0	0	0	0	−1	−1
χ_{12}	−3	−3	−3	5	5	−1	−1	−1	−5	−5	−5	1	1	1	−1	−1	−1	−1	0	0
χ_{13}	6	3	3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	1	1	1	1	0	0
χ_{14}	6	3	3	−3	−3	3	3	3	−1	−1	−1	1	1	1	−1	−1	−1	−1	0	0
χ_{15}	−6	0	0	0	0	−4	−4	−4	4	4	4	0	0	0	0	0	0	0	0	0
χ_{16}	6	−3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2
χ_{17}	0	0	0	−3	−3	−5	−5	−5	−1	−1	−1	1	1	1	−1	−1	−1	−1	−1	−1
χ_{18}	0	0	0	−3	−3	1	1	1	1	1	1	−3	−3	−3	1	1	1	1	−1	−1
χ_{19}	0	0	0	9	9	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1
χ_{20}	3	0	0	−2	−2	2	2	2	2	2	2	−2	−2	−2	−2	−2	−2	−2	0	0

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Table 11.12 (continued)

	$3B$	$3C$		$4A$		$4B$			$4C$			$4D$			$4E$				$5A$	
	g_{71}	g_{81}	g_{82}	g_{91}	g_{92}	$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$	$g_{14,1}$	$g_{14,2}$
	$3b$	$3c$	$6b$	$4j$	$4k$	$8a$	$8b$	$4l$	$8c$	$8d$	$4m$	$4n$	$4o$	$4p$	$8e$	$4q$	$8f$	$4r$	$10a$	$5a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	648	432	144	1536	512	1536	512	384	1536	512	384	512	512	256	128	128	128	128	40	120
χ_{21}	−6	3	3	6	6	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	0	0
χ_{22}	0	0	0	0	0	−4	−4	−4	4	4	4	0	0	0	0	0	0	0	1	1
χ_{23}	−8	−2	−2	0	0	4	4	4	−4	−4	−4	0	0	0	0	0	0	0	0	0
χ_{24}	10	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	−9	0	0	−5	−5	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	0	0
χ_{26}	−6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
χ_{27}	0	0	0	6	6	2	2	2	2	2	2	−2	−2	−2	2	2	2	2	−2	−2
χ_{28}	0	0	0	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5	1	1	1	1	0	0
χ_{29}	−3	3	3	−4	−4	0	0	0	0	0	0	−4	−4	−4	0	0	0	0	0	0
χ_{30}	8	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2

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Table 11.12 (continued)

	$3B$	$3C$		$4A$		$4B$			$4C$			$4D$			$4E$				$5A$	
	g_{71}	g_{81}	g_{82}	g_{91}	g_{92}	$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$	$g_{14,1}$	$g_{14,2}$
	$3b$	$3c$	$6b$	$4j$	$4k$	$8a$	$8b$	$4l$	$8c$	$8d$	$4m$	$4n$	$4o$	$4p$	$8e$	$4q$	$8f$	$4r$	$10a$	$5a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	648	432	144	1536	512	1536	512	384	1536	512	384	512	512	256	128	128	128	128	40	120
χ_{31}	0	3	−1	3	−1	7	−1	−1	7	−1	−1	3	−1	−1	3	−1	−1	−1	−1	3
χ_{32}	0	3	−1	3	−1	5	−3	1	−7	1	1	−1	3	−1	1	1	1	−3	−1	3
χ_{33}	0	−3	1	3	−1	−9	−1	3	−9	−1	3	3	−1	−1	−1	−1	3	−1	0	0
χ_{34}	0	−3	1	3	−1	−3	5	−3	9	1	−3	−1	3	−1	1	−3	1	1	0	0
χ_{35}	0	6	−2	3	−1	−7	1	1	5	−3	1	7	−5	−1	1	−3	1	1	0	0
χ_{36}	0	6	−2	3	−1	−5	3	−1	−5	3	−1	−5	7	−1	−1	−1	3	−1	0	0
χ_{37}	0	0	0	6	−2	2	2	−2	−14	2	2	−2	−2	2	2	−2	−2	2	−1	3
χ_{38}	0	0	0	6	−2	−2	−2	2	14	−2	−2	−2	−2	2	−2	2	2	−2	−1	3
χ_{39}	0	0	0	3	−1	3	−5	3	3	−5	3	−5	7	−1	3	−1	−1	−1	1	−3
χ_{40}	0	0	0	3	−1	9	1	−3	−3	5	−3	7	−5	−1	1	1	1	−3	1	−3
χ_{41}	0	3	−1	−6	2	−14	2	2	2	2	−2	2	2	−2	2	2	−2	−2	0	0
χ_{42}	0	3	−1	−6	2	10	−6	2	2	2	−2	2	2	−2	−2	−2	2	2	0	0
χ_{43}	0	3	−1	−6	2	−2	−2	2	−2	−2	2	2	−6	2	−2	2	−2	2	0	0
χ_{44}	0	3	−1	−6	2	14	−2	−2	−2	−2	2	2	2	−2	−2	−2	2	2	0	0
χ_{45}	0	3	−1	−6	2	−10	6	−2	−2	−2	2	2	2	−2	2	2	−2	−2	0	0
χ_{46}	0	3	−1	−6	2	2	2	−2	2	2	−2	−6	2	2	−2	2	−2	2	0	0
χ_{47}	0	0	0	−3	1	5	−3	1	5	−3	1	−3	1	1	1	1	−3	1	0	0
χ_{48}	0	0	0	9	−3	−7	1	1	−7	1	1	1	5	−3	−3	1	1	1	0	0
χ_{49}	0	0	0	−3	1	7	−1	−1	−5	3	−1	1	−3	1	−1	3	−1	−1	0	0
χ_{50}	0	0	0	9	−3	−5	3	−1	7	−1	−1	5	1	−3	−1	−1	−1	3	0	0
χ_{51}	0	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	3

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Table 11.12 (continued)

	$3B$	$3C$		$4A$		$4B$			$4C$			$4D$			$4E$				$5A$	
	g_{71}	g_{81}	g_{82}	g_{91}	g_{92}	$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$	$g_{14,1}$	$g_{14,2}$
	$3b$	$3c$	$6b$	$4j$	$4k$	$8a$	$8b$	$4l$	$8c$	$8d$	$4m$	$4n$	$4o$	$4p$	$8e$	$4q$	$8f$	$4r$	$10a$	$5a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	648	432	144	1536	512	1536	512	384	1536	512	384	512	512	256	128	128	128	128	40	120
χ_{52}	0	6	−2	12	−4	0	0	0	0	0	0	−4	−4	4	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	−4	−4	4	4	4	−4	0	0	0	0	0	0	0	1	−3
χ_{54}	0	0	0	0	0	4	4	−4	−4	−4	4	0	0	0	0	0	0	0	1	−3
χ_{55}	0	0	0	6	−2	2	2	−2	10	−6	2	−2	−2	2	−2	2	2	−2	0	0
χ_{56}	0	0	0	6	−2	2	2	−2	2	2	−2	6	−2	−2	−2	2	−2	2	0	0
χ_{57}	0	0	0	6	−2	−2	−2	2	−10	6	−2	−2	−2	2	2	−2	−2	2	0	0
χ_{58}	0	0	0	6	−2	−2	−2	2	−2	−2	2	−2	6	−2	−2	2	−2	2	0	0
χ_{59}	0	0	0	−12	4	0	0	0	0	0	0	4	4	−4	0	0	0	0	−1	3
χ_{60}	0	−6	2	0	0	4	4	−4	−4	−4	4	0	0	0	0	0	0	0	0	0
χ_{61}	0	−6	2	0	0	−4	−4	4	4	4	−4	0	0	0	0	0	0	0	0	0
χ_{62}	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	−9	3	−3	5	−3	9	1	−3	−5	−1	3	1	−3	1	1	0	0
χ_{65}	0	0	0	−9	3	−9	−1	3	−9	−1	3	−1	−5	3	−1	−1	3	−1	0	0
χ_{66}	0	0	0	3	−1	9	1	−3	−3	5	−3	−1	3	−1	1	1	1	−3	0	0
χ_{67}	0	0	0	3	−1	3	−5	3	3	−5	3	3	−1	−1	3	−1	−1	−1	0	0

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Table 11.12 (continued)

	$6A$			$6B$		$6C$	$6D$			$6E$		$6F$		$6G$		$7A$	$8A$		$8B$	
	$g_{15,1}$	$g_{15,2}$	$g_{15,3}$	$g_{16,1}$	$g_{16,2}$	$g_{17,1}$	$g_{18,1}$	$g_{18,2}$	$g_{18,3}$	$g_{19,1}$	$g_{19,2}$	$g_{20,1}$	$g_{20,2}$	$g_{21,1}$	$g_{21,2}$	$g_{22,1}$	$g_{23,1}$	$g_{23,2}$	$g_{24,1}$	$g_{24,2}$
	$12a$	$12b$	$6c$	$6d$	$12c$	$6e$	$12d$	$6f$	$12e$	$12f$	$6g$	$6h$	$6i$	$12g$	$6j$	$7a$	$8g$	$8h$	$8i$	$8j$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	1152	384	288	576	192	72	192	192	96	72	72	144	48	24	24	7	32	32	32	32
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	−2	−2	−2	2	2	2	0	0	0	1	1	−1	−1	−1	−1	0	−1	−1	1	1
χ_3	−2	−2	−2	−2	−2	1	0	0	0	1	1	1	1	−1	−1	1	1	1	−1	−1
χ_4	−2	−2	−2	2	2	−1	2	2	2	−2	−2	2	2	0	0	0	−1	−1	−1	−1
χ_5	0	0	0	0	0	3	−2	−2	−2	0	0	0	0	0	0	0	−1	−1	1	1
χ_6	3	3	3	3	3	0	1	1	1	0	0	0	0	0	0	−1	1	1	−1	−1
χ_7	3	3	3	−1	−1	−1	1	1	1	0	0	2	2	0	0	0	−1	−1	1	1
χ_8	1	1	1	−3	−3	3	1	1	1	−2	−2	0	0	0	0	0	1	1	1	1
χ_9	−3	−3	−3	1	1	−2	−1	−1	−1	0	0	−2	−2	0	0	0	0	0	0	0
χ_{10}	−1	−1	−1	−1	−1	−1	3	3	3	−1	−1	−1	−1	1	1	0	0	0	0	0
χ_{11}	−2	−2	−2	2	2	−1	−2	−2	−2	1	1	−1	−1	1	1	0	0	0	0	0
χ_{12}	1	1	1	1	1	1	−1	−1	−1	1	1	1	1	1	1	0	1	1	−1	−1
χ_{13}	4	4	4	−4	−4	2	0	0	0	1	1	−1	−1	1	1	0	−1	−1	−1	−1
χ_{14}	2	2	2	2	2	2	0	0	0	−1	−1	−1	−1	−1	−1	0	1	1	−1	−1
χ_{15}	1	1	1	1	1	−2	−1	−1	−1	−2	−2	−2	−2	0	0	1	0	0	0	0
χ_{16}	−2	−2	−2	2	2	2	2	2	2	1	1	−1	−1	−1	−1	0	0	0	0	0
χ_{17}	3	3	3	3	3	0	1	1	1	0	0	0	0	0	0	0	−1	−1	1	1
χ_{18}	−3	−3	−3	−3	−3	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1
χ_{19}	−3	−3	−3	−3	−3	0	1	1	1	0	0	0	0	0	0	0	−1	−1	−1	−1
χ_{20}	−1	−1	−1	−1	−1	−1	−1	−1	−1	2	2	2	2	0	0	0	0	0	0	0

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Table 11.12 (continued)

	$6A$			$6B$		$6C$	$6D$			$6E$		$6F$		$6G$		$7A$	$8A$		$8B$	
	$g_{15,1}$	$g_{15,2}$	$g_{15,3}$	$g_{16,1}$	$g_{16,2}$	$g_{17,1}$	$g_{18,1}$	$g_{18,2}$	$g_{18,3}$	$g_{19,1}$	$g_{19,2}$	$g_{20,1}$	$g_{20,2}$	$g_{21,1}$	$g_{21,2}$	$g_{22,1}$	$g_{23,1}$	$g_{23,2}$	$g_{24,1}$	$g_{24,2}$
	$12a$	$12b$	$6c$	$6d$	$12c$	$6e$	$12d$	$6f$	$12e$	$12f$	$6g$	$6h$	$6i$	$12g$	$6j$	$7a$	$8g$	$8h$	$8i$	$8j$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	1152	384	288	576	192	72	192	192	96	72	72	144	48	24	24	7	32	32	32	32
χ_{21}	1	1	1	1	1	−2	1	1	1	1	1	1	1	−1	−1	0	0	0	0	0
χ_{22}	−3	−3	−3	−3	−3	0	−1	−1	−1	0	0	0	0	0	0	−1	0	0	0	0
χ_{23}	1	1	1	−3	−3	0	−1	−1	−1	−2	−2	0	0	0	0	0	0	0	0	0
χ_{24}	2	2	2	−2	−2	−2	−2	−2	−2	−1	−1	1	1	−1	−1	0	0	0	0	0
χ_{25}	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1
χ_{26}	2	2	2	−2	−2	−2	2	2	2	2	2	−2	−2	0	0	0	0	0	0	0
χ_{27}	3	3	3	3	3	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	1	1
χ_{29}	−4	−4	−4	4	4	1	0	0	0	−1	−1	1	1	1	1	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

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Table 11.12 (continued)

	$6A$			$6B$		$6C$	$6D$			$6E$		$6F$		$6G$		$7A$	$8A$		$8B$	
	$g_{15,1}$	$g_{15,2}$	$g_{15,3}$	$g_{16,1}$	$g_{16,2}$	$g_{17,1}$	$g_{18,1}$	$g_{18,2}$	$g_{18,3}$	$g_{19,1}$	$g_{19,2}$	$g_{20,1}$	$g_{20,2}$	$g_{21,1}$	$g_{21,2}$	$g_{22,1}$	$g_{23,1}$	$g_{23,2}$	$g_{24,1}$	$g_{24,2}$
	$12a$	$12b$	$6c$	$6d$	$12c$	$6e$	$12d$	$6f$	$12e$	$12f$	$6g$	$6h$	$6i$	$12g$	$6j$	$7a$	$8g$	$8h$	$8i$	$8j$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	1152	384	288	576	192	72	192	192	96	72	72	144	48	24	24	7	32	32	32	32
χ_{31}	7	−1	−1	3	−1	0	3	−1	−1	1	−1	3	−1	1	−1	0	1	−1	1	−1
χ_{32}	−5	3	−1	3	−1	0	−1	3	−1	1	−1	−3	1	−1	1	0	−1	1	1	−1
χ_{33}	2	2	−2	6	−2	0	2	2	−2	−1	1	3	−1	1	−1	0	−1	1	−1	1
χ_{34}	2	2	−2	6	−2	0	2	2	−2	−1	1	−3	1	−1	1	0	1	−1	−1	1
χ_{35}	−7	1	1	−3	1	0	−3	1	1	2	−2	0	0	0	0	0	1	−1	−1	1
χ_{36}	5	−3	1	−3	1	0	1	−3	1	2	−2	0	0	0	0	0	−1	1	−1	1
χ_{37}	−9	−1	3	3	−1	0	1	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	3	−5	3	3	−1	0	−3	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	−1
χ_{41}	−7	1	1	−3	1	0	3	−1	−1	−1	1	3	−1	−1	1	0	0	0	0	0
χ_{42}	−7	1	1	−3	1	0	3	−1	−1	−1	1	−3	1	1	−1	0	0	0	0	0
χ_{43}	7	−1	−1	3	−1	0	3	−1	−1	1	−1	−3	1	−1	1	0	0	0	0	0
χ_{44}	5	−3	1	−3	1	0	−1	3	−1	−1	1	3	−1	−1	1	0	0	0	0	0
χ_{45}	5	−3	1	−3	1	0	−1	3	−1	−1	1	−3	1	1	−1	0	0	0	0	0
χ_{46}	−5	3	−1	3	−1	0	−1	3	−1	1	−1	3	−1	1	−1	0	0	0	0	0
χ_{47}	9	1	−3	−3	1	0	1	−3	1	0	0	0	0	0	0	0	−1	1	−1	1
χ_{48}	9	1	−3	−3	1	0	1	−3	1	0	0	0	0	0	0	0	1	−1	1	−1
χ_{49}	−3	5	−3	−3	1	0	−3	1	1	0	0	0	0	0	0	0	1	−1	−1	1
χ_{50}	−3	5	−3	−3	1	0	−3	1	1	0	0	0	0	0	0	0	−1	1	1	−1
χ_{51}	−2	−2	2	−6	2	0	−2	−2	2	−2	2	0	0	0	0	0	0	0	0	0

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Table 11.12 (continued)

	$6A$			$6B$		$6C$	$6D$			$6E$		$6F$		$6G$		$7A$	$8A$		$8B$	
	$g_{15,1}$	$g_{15,2}$	$g_{15,3}$	$g_{16,1}$	$g_{16,2}$	$g_{17,1}$	$g_{18,1}$	$g_{18,2}$	$g_{18,3}$	$g_{19,1}$	$g_{19,2}$	$g_{20,1}$	$g_{20,2}$	$g_{21,1}$	$g_{21,2}$	$g_{22,1}$	$g_{23,1}$	$g_{23,2}$	$g_{24,1}$	$g_{24,2}$
	$12a$	$12b$	$6c$	$6d$	$12c$	$6e$	$12d$	$6f$	$12e$	$12f$	$6g$	$6h$	$6i$	$12g$	$6j$	$7a$	$8g$	$8h$	$8i$	$8j$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	1152	384	288	576	192	72	192	192	96	72	72	144	48	24	24	7	32	32	32	32
χ_{52}	−2	−2	2	−6	2	0	2	2	−2	−2	2	0	0	0	0	0	0	0	0	0
χ_{53}	−9	−1	3	3	−1	0	1	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	3	−5	3	3	−1	0	−3	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	−3	5	−3	−3	1	0	3	−1	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	3	−5	3	3	−1	0	3	−1	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	9	1	−3	−3	1	0	−1	3	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	−9	−1	3	3	−1	0	−1	3	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	−7	1	1	−3	1	0	3	−1	−1	2	−2	0	0	0	0	0	0	0	0	0
χ_{61}	5	−3	1	−3	1	0	−1	3	−1	2	−2	0	0	0	0	0	0	0	0	0
χ_{62}	2	2	−2	6	−2	0	−2	−2	2	−1	1	3	−1	−1	1	0	0	0	0	0
χ_{63}	2	2	−2	6	−2	0	−2	−2	2	−1	1	−3	1	1	−1	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	−1
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−1	1
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1

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Table 11.12 (continued)

	$9A$	$10A$		$12A$		$12B$		$12C$	$15A$
	$g_{25,1}$	$g_{26,1}$	$g_{26,2}$	$g_{27,1}$	$g_{27,2}$	$g_{28,1}$	$g_{28,2}$	$g_{29,1}$	$g_{30,1}$
	$9a$	$20a$	$10b$	$24a$	$12h$	$24b$	$12i$	$12j$	$15a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	9	20	20	48	48	48	48	12	15
χ_1	1	1	1	1	1	1	1	1	1
χ_2	1	0	0	−2	−2	0	0	0	−1
χ_3	0	0	0	0	0	−2	−2	−1	0
χ_4	0	−1	−1	0	0	0	0	1	1
χ_5	0	−1	−1	2	2	0	0	−1	1
χ_6	0	0	0	1	1	−1	−1	0	−1
χ_7	−1	0	0	−1	−1	1	1	−1	0
χ_8	−1	0	0	−1	−1	−1	−1	1	0
χ_9	−1	1	1	1	1	−1	−1	0	1
χ_{10}	1	0	0	−1	−1	−1	−1	−1	0
χ_{11}	0	−1	−1	0	0	0	0	1	−1
χ_{12}	0	0	0	−1	−1	1	1	−1	0
χ_{13}	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	2	2	0	0
χ_{15}	0	0	0	−1	−1	1	1	0	0
χ_{16}	0	0	0	0	0	0	0	0	1
χ_{17}	0	1	1	1	1	−1	−1	0	−1
χ_{18}	0	−1	−1	1	1	1	1	0	−1
χ_{19}	0	1	1	1	1	1	1	0	−1
χ_{20}	0	0	0	−1	−1	−1	−1	1	0
χ_{21}	0	0	0	1	1	1	1	0	0
χ_{22}	0	1	1	−1	−1	1	1	0	1
χ_{23}	1	0	0	1	1	−1	−1	0	0
χ_{24}	1	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	1	0
χ_{26}	0	−1	−1	0	0	0	0	0	1
χ_{27}	0	0	0	−1	−1	−1	−1	0	1
χ_{28}	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	−1	0
χ_{30}	−1	0	0	0	0	0	0	0	−1

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Table 11.12 (continued)

	$9A$	$10A$		$12A$		$12B$		$12C$	$15A$
	$g_{25,1}$	$g_{26,1}$	$g_{26,2}$	$g_{27,1}$	$g_{27,2}$	$g_{28,1}$	$g_{28,2}$	$g_{29,1}$	$g_{30,1}$
	$9a$	$20a$	$10b$	$24a$	$12h$	$24b$	$12i$	$12j$	$15a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	9	20	20	48	48	48	48	12	15
χ_{31}	0	1	−1	1	−1	1	−1	0	0
χ_{32}	0	1	−1	−1	1	−1	1	0	0
χ_{33}	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	−1	1	−1	1	0	0
χ_{36}	0	0	0	1	−1	1	−1	0	0
χ_{37}	0	−1	1	−1	1	1	−1	0	0
χ_{38}	0	−1	1	1	−1	−1	1	0	0
χ_{39}	0	−1	1	0	0	0	0	0	0
χ_{40}	0	−1	1	0	0	0	0	0	0
χ_{41}	0	0	0	1	−1	−1	1	0	0
χ_{42}	0	0	0	1	−1	−1	1	0	0
χ_{43}	0	0	0	1	−1	1	−1	0	0
χ_{44}	0	0	0	−1	1	1	−1	0	0
χ_{45}	0	0	0	−1	1	1	−1	0	0
χ_{46}	0	0	0	−1	1	−1	1	0	0
χ_{47}	0	0	0	−1	1	−1	1	0	0
χ_{48}	0	0	0	−1	1	−1	1	0	0
χ_{49}	0	0	0	1	−1	1	−1	0	0
χ_{50}	0	0	0	1	−1	1	−1	0	0
χ_{51}	0	1	−1	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	0	0	0	0
χ_{53}	0	1	−1	−1	1	1	−1	0	0
χ_{54}	0	1	−1	1	−1	−1	1	0	0
χ_{55}	0	0	0	−1	1	1	−1	0	0
χ_{56}	0	0	0	−1	1	−1	1	0	0
χ_{57}	0	0	0	1	−1	−1	1	0	0
χ_{58}	0	0	0	1	−1	1	−1	0	0
χ_{59}	0	−1	1	0	0	0	0	0	0
χ_{60}	0	0	0	1	−1	−1	1	0	0
χ_{61}	0	0	0	−1	1	1	−1	0	0
χ_{62}	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0

Table 11.13: The character table of $\overline{G} = 2^{8\cdot}Sp(8,2)$

$[x]_G$		$1A$		$2A$			$2B$			$2C$				$2D$		$2E$				
$[g]_{\overline{G}}$		$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$	$4j$	$4k$
$\lvert C_{\overline{G}}(g)\rvert$	2	24	24	23	23	17	22	22	18	22	22	18	17	18	18	19	19	17	17	17
	3	5	4	4	2	4	3	2	2	2	1	2	2	2	1	2	1	2	1	1
	5	2	1	1	1	1	1	0	1	1	0	1	1	1	0	0	0	0	0	0
	7	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		35	35	−21	−21	−21	3	3	3	11	11	11	11	−5	−5	−5	−5	−5	−5	−5
χ_3		51	51	−21	−21	−21	19	19	19	11	11	11	11	11	11	−5	−5	−5	−5	−5
χ_4		85	85	−43	−43	−43	21	21	21	21	21	21	21	5	5	−11	−11	−11	−11	−11
χ_5		119	119	63	63	63	23	23	23	31	31	31	31	−1	−1	15	15	15	15	15
χ_6		135	135	63	63	63	39	39	39	31	31	31	31	15	15	15	15	15	15	15
χ_7		238	238	−98	−98	−98	−18	−18	−18	30	30	30	30	14	14	−2	−2	−2	−2	−2
χ_8		510	510	174	174	174	−2	−2	−2	46	46	46	46	30	30	14	14	14	14	14
χ_9		595	595	203	203	203	−13	−13	−13	43	43	43	43	−5	−5	−5	−5	−5	−5	−5
χ_{10}		595	595	35	35	35	−45	−45	−45	35	35	35	35	51	51	3	3	3	3	3
χ_{11}		918	918	−90	−90	−90	150	150	150	38	38	38	38	54	54	6	6	6	6	6
χ_{12}		1190	1190	182	182	182	166	166	166	54	54	54	54	70	70	22	22	22	22	22
χ_{13}		1275	1275	195	195	195	155	155	155	35	35	35	35	35	35	−13	−13	−13	−13	−13
χ_{14}		1512	1512	−504	−504	−504	−24	−24	−24	136	136	136	136	−24	−24	−24	−24	−24	−24	−24
χ_{15}		1785	1785	441	441	441	57	57	57	121	121	121	121	−55	−55	25	25	25	25	25
χ_{16}		1785	1785	−399	−399	−399	−103	−103	−103	81	81	81	81	65	65	1	1	1	1	1
χ_{17}		2295	2295	−657	−657	−657	87	87	87	143	143	143	143	15	15	−33	−33	−33	−33	−33
χ_{18}		2856	2856	840	840	840	40	40	40	200	200	200	200	−24	−24	40	40	40	40	40
χ_{19}		2856	2856	−504	−504	−504	296	296	296	136	136	136	136	40	40	−24	−24	−24	−24	−24
χ_{20}		2975	2975	−665	−665	−665	255	255	255	135	135	135	135	55	55	−41	−41	−41	−41	−41
χ_{21}		2975	2975	−385	−385	−385	95	95	95	−65	−65	−65	−65	15	15	31	31	31	31	31
χ_{22}		3213	3213	693	693	693	−147	−147	−147	85	85	85	85	69	69	−27	−27	−27	−27	−27
χ_{23}		3213	3213	−819	−819	−819	141	141	141	205	205	205	205	−51	−51	−51	−51	−51	−51	−51
χ_{24}		3400	3400	840	840	840	200	200	200	200	200	200	200	40	40	72	72	72	72	72
χ_{25}		3570	3570	210	210	210	50	50	50	−110	−110	−110	−110	66	66	−14	−14	−14	−14	−14
χ_{26}		3570	3570	882	882	882	178	178	178	178	178	178	178	−30	−30	18	18	18	18	18
χ_{27}		3570	3570	−798	−798	−798	−142	−142	−142	98	98	98	98	50	50	34	34	34	34	34
χ_{28}		3808	3808	−672	−672	−672	−32	−32	−32	96	96	96	96	−96	−96	−32	−32	−32	−32	−32
χ_{29}		4200	4200	840	840	840	360	360	360	200	200	200	200	40	40	40	40	40	40	40
χ_{30}		4760	4760	952	952	952	−104	−104	−104	56	56	56	56	−40	−40	−40	−40	−40	−40	−40

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Table 11.13 (continued)

$[x]_G$	$1A$		$2A$			$2B$			$2C$				$2D$		$2E$				
$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$	$4j$	$4k$
χ_{31}	5712	5712	336	336	336	336	336	336	80	80	80	80	16	16	80	80	80	80	80
χ_{32}	5950	5950	−770	−770	−770	−130	−130	−130	190	190	190	190	−50	−50	−34	−34	−34	−34	−34
χ_{33}	5950	5950	−210	−210	−210	190	190	190	−210	−210	−210	−210	30	30	14	14	14	14	14
χ_{34}	7140	7140	−252	−252	−252	484	484	484	4	4	4	4	100	100	−60	−60	−60	−60	−60
χ_{35}	8160	8160	−672	−672	−672	224	224	224	96	96	96	96	160	160	−32	−32	−32	−32	−32
χ_{36}	8925	8925	1365	1365	1365	−195	−195	−195	245	245	245	245	85	85	37	37	37	37	37
χ_{37}	8925	8925	−315	−315	−315	125	125	125	165	165	165	165	−155	−155	21	21	21	21	21
χ_{38}	8960	8960	−1792	−1792	−1792	−256	−256	−256	256	256	256	256	0	0	0	0	0	0	0
χ_{39}	9639	9639	567	567	567	−153	−153	−153	−9	−9	−9	−9	−105	−105	−9	−9	−9	−9	−9
χ_{40}	10200	10200	−1800	−1800	−1800	−40	−40	−40	120	120	120	120	−40	−40	24	24	24	24	24
χ_{41}	11900	11900	700	700	700	−260	−260	−260	60	60	60	60	220	220	−4	−4	−4	−4	−4
χ_{42}	11900	11900	−420	−420	−420	−260	−260	−260	220	220	220	220	−100	−100	−36	−36	−36	−36	−36
χ_{43}	13056	13056	2304	2304	2304	−256	−256	−256	256	256	256	256	0	0	0	0	0	0	0
χ_{44}	13600	13600	160	160	160	800	800	800	160	160	160	160	160	160	32	32	32	32	32
χ_{45}	14280	14280	840	840	840	−440	−440	−440	200	200	200	200	104	104	8	8	8	8	8
χ_{46}	14280	14280	−1848	−1848	−1848	328	328	328	72	72	72	72	40	40	−56	−56	−56	−56	−56
χ_{47}	14688	14688	2016	2016	2016	96	96	96	224	224	224	224	−96	−96	96	96	96	96	96
χ_{48}	16065	16065	−2583	−2583	−2583	−159	−159	−159	329	329	329	329	105	105	−39	−39	−39	−39	−39
χ_{49}	16065	16065	1449	1449	1449	33	33	33	329	329	329	329	−135	−135	57	57	57	57	57
χ_{50}	16065	16065	1953	1953	1953	129	129	129	−31	−31	−31	−31	−15	−15	−63	−63	−63	−63	−63
χ_{51}	16065	16065	441	441	441	417	417	417	89	89	89	89	−135	−135	−87	−87	−87	−87	−87
χ_{52}	16065	16065	−1071	−1071	−1071	−447	−447	−447	209	209	209	209	225	225	−15	−15	−15	−15	−15
χ_{53}	17850	17850	2730	2730	2730	−70	−70	−70	170	170	170	170	90	90	−22	−22	−22	−22	−22
χ_{54}	17850	17850	−630	−630	−630	570	570	570	10	10	10	10	−70	−70	74	74	74	74	74
χ_{55}	18360	18360	792	792	792	696	696	696	−104	−104	−104	−104	120	120	−72	−72	−72	−72	−72
χ_{56}	19040	19040	2016	2016	2016	352	352	352	224	224	224	224	160	160	96	96	96	96	96
χ_{57}	23800	23800	−1960	−1960	−1960	760	760	760	−40	−40	−40	−40	120	120	−8	−8	−8	−8	−8
χ_{58}	23800	23800	−3080	−3080	−3080	120	120	120	120	120	120	120	−40	−40	56	56	56	56	56
χ_{59}	26775	26775	1575	1575	1575	−105	−105	−105	−345	−345	−345	−345	−105	−105	−57	−57	−57	−57	−57
χ_{60}	26775	26775	−3465	−3465	−3465	−105	−105	−105	375	375	375	375	−105	−105	−9	−9	−9	−9	−9
χ_{61}	28560	28560	336	336	336	−368	−368	−368	336	336	336	336	80	80	16	16	16	16	16
χ_{62}	28917	28917	−2835	−2835	−2835	405	405	405	333	333	333	333	−99	−99	−99	−99	−99	−99	−99
χ_{63}	30464	30464	−1792	−1792	−1792	768	768	768	256	256	256	256	0	0	0	0	0	0	0
χ_{44}	32130	32130	2898	2898	2898	642	642	642	82	82	82	82	−30	−30	18	18	18	18	18
χ_{65}	32130	32130	882	882	882	258	258	258	−398	−398	−398	−398	−30	−30	18	18	18	18	18

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Table 11.13 (continued)

$[x]_G$	$1A$		$2A$			$2B$			$2C$				$2D$		$2E$				
$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$	$4j$	$4k$
χ_{66}	32130	32130	−2142	−2142	−2142	−318	−318	−318	−158	−158	−158	−158	210	210	66	66	66	66	66
χ_{67}	34425	34425	4185	4185	4185	−135	−135	−135	345	345	345	345	−135	−135	−39	−39	−39	−39	−39
χ_{68}	34425	34425	−2295	−2295	−2295	−135	−135	−135	−375	−375	−375	−375	−135	−135	105	105	105	105	105
χ_{69}	34560	34560	2304	2304	2304	768	768	768	256	256	256	256	0	0	0	0	0	0	0
χ_{70}	38080	38080	2240	2240	2240	−320	−320	−320	−320	−320	−320	−320	64	64	−64	−64	−64	−64	−64
χ_{71}	38556	38556	2268	2268	2268	−612	−612	−612	−36	−36	−36	−36	156	156	−36	−36	−36	−36	−36
χ_{72}	42525	42525	−2835	−2835	−2835	−675	−675	−675	45	45	45	45	45	45	45	45	45	45	45
χ_{73}	42525	42525	−2835	−2835	−2835	−675	−675	−675	45	45	45	45	45	45	45	45	45	45	45
χ_{74}	43520	43520	−512	−512	−512	512	512	512	−512	−512	−512	−512	0	0	0	0	0	0	0
χ_{75}	47600	47600	−1680	−1680	−1680	240	240	240	−400	−400	−400	−400	−80	−80	−16	−16	−16	−16	−16
χ_{76}	48195	48195	2835	2835	2835	−765	−765	−765	−45	−45	−45	−45	51	51	−45	−45	−45	−45	−45
χ_{77}	51408	51408	−1008	−1008	−1008	−48	−48	−48	16	16	16	16	144	144	−48	−48	−48	−48	−48
χ_{78}	53550	53550	−1890	−1890	−1890	−210	−210	−210	30	30	30	30	−210	−210	−66	−66	−66	−66	−66
χ_{79}	57120	57120	672	672	672	288	288	288	−352	−352	−352	−352	160	160	32	32	32	32	32
χ_{80}	65536	65536	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	68850	68850	1890	1890	1890	−270	−270	−270	−30	−30	−30	−30	−270	−270	66	66	66	66	66
χ_{82}	255	−1	127	−1	−1	63	−1	−1	63	−1	−1	−1	15	−1	31	−1	−1	−1	−1
χ_{83}	1785	−7	−623	17	−7	−39	25	−7	177	−15	17	−7	−15	1	−31	1	−23	9	1
χ_{84}	3825	−15	−615	25	−15	465	17	−15	185	−7	25	−15	105	−7	−23	9	−31	1	9
χ_{85}	5355	−21	−1365	43	−21	363	43	−21	299	−21	43	−21	75	−5	−85	11	−53	11	11
χ_{86}	5355	−21	1155	3	−21	−117	75	−21	99	35	3	−21	−45	3	−45	51	27	−5	−13
χ_{87}	6885	−27	1917	−3	−27	261	69	−27	477	29	−3	−27	45	−3	141	45	21	−11	−19
χ_{88}	7140	−28	−2044	4	28	228	−28	4	516	4	−28	−4	−60	4	−124	4	28	−4	4
χ_{89}	7140	−28	1988	−60	28	228	−28	4	452	−60	36	−4	−60	4	68	−60	28	−4	4
χ_{90}	8925	−35	1925	5	−35	765	61	−35	485	37	5	−35	165	−11	149	53	13	−19	−11
χ_{91}	8925	−35	−595	45	−35	285	93	−35	−275	45	45	−35	45	−3	45	−51	−3	−3	13
χ_{92}	9180	−36	−2052	−4	36	732	−36	−4	508	−4	−36	4	60	−4	−132	−4	36	4	−4
χ_{93}	9180	−36	1980	−68	36	732	−36	−4	444	−68	28	4	60	−4	60	−68	36	4	−4
χ_{94}	14280	−56	−2968	104	−56	−312	200	−56	488	−24	104	−56	−120	8	−56	−56	−88	40	8
χ_{95}	16065	−63	3969	1	−63	897	−63	1	961	1	−63	1	−15	1	225	1	−63	1	1
χ_{96}	16065	−63	−3591	121	−63	−543	33	1	601	−103	57	1	105	−7	9	41	−15	17	−23
χ_{97}	17850	−70	−1190	90	−70	−390	250	−70	410	26	90	−70	−150	10	−70	−6	−38	26	−6
χ_{98}	21420	−84	588	76	−84	1452	172	−84	332	76	76	−84	300	−20	140	12	−20	−20	12

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Table 11.13 (continued)

$[x]_G$	$1A$		$2A$			$2B$			$2C$				$2D$		$2E$				
$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$	$4j$	$4k$
χ_{99}	26775	−105	735	95	−105	1335	247	−105	−65	127	95	−105	255	−17	−145	79	7	−25	15
χ_{100}	26775	−105	3255	55	−105	−105	343	−105	695	119	55	−105	−105	7	119	87	55	−9	−41
χ_{101}	26775	−105	−4305	175	−105	375	311	−105	335	15	175	−105	15	−1	−65	−97	−121	39	31
χ_{102}	30600	−120	5160	40	−120	−120	392	−120	680	168	40	−120	−120	8	136	136	104	−24	−56
χ_{103}	42840	−168	7896	−296	168	−552	−40	24	856	−168	152	−24	120	−8	−40	−40	40	−24	24
χ_{104}	42840	−168	−8232	−40	168	−552	−40	24	1112	88	−104	−24	120	−8	−40	−40	40	−24	24
χ_{105}	42840	−168	5208	88	−168	984	472	−168	728	216	88	−168	120	−8	344	88	88	−40	−40
χ_{106}	48195	−189	−6237	291	−189	387	579	−189	867	35	291	−189	−45	3	−189	−93	−189	67	35
χ_{107}	48195	−189	2835	147	−189	387	579	−189	−429	275	147	−189	−45	3	−45	51	99	−29	−13
χ_{108}	48195	−189	−4725	267	−189	1827	483	−189	171	107	267	−189	315	−21	−165	−69	−141	19	59
χ_{109}	53550	−210	6510	110	−210	750	622	−210	430	302	110	−210	30	−2	14	206	142	−50	−50
χ_{110}	53550	−210	1470	190	−210	−210	686	−210	830	190	190	−210	−210	14	94	30	14	14	−34
χ_{111}	55080	−216	−2808	264	−216	2088	552	−216	648	136	264	−216	360	−24	−24	−24	−120	8	40
χ_{112}	64260	−252	9828	−412	252	−636	132	−28	1252	−284	132	28	−60	4	36	−92	60	−36	36
χ_{113}	64260	−252	−10332	−92	252	−636	132	−28	1572	36	−188	28	−60	4	−156	−28	60	−36	36
χ_{114}	71400	−280	5320	200	−280	2280	744	−280	840	328	200	−280	360	−24	168	168	72	−56	−24
χ_{115}	71400	−280	−4760	360	−280	360	872	−280	−280	232	360	−280	−120	8	40	−216	−88	40	40
χ_{116}	71400	−280	−280	−280	280	−1560	−24	40	1000	−24	40	−40	360	−24	−24	−24	24	−40	40
χ_{117}	80325	−315	−10395	485	−315	1605	−123	5	485	−219	165	5	165	−11	−27	133	−27	−27	5
χ_{118}	80325	−315	4725	245	−315	−1275	69	5	1205	−11	−75	5	405	−27	117	21	−27	37	−43
χ_{119}	80325	−315	12285	125	−315	165	−27	5	1565	93	−195	5	285	−19	333	−19	−75	21	−19
χ_{120}	80325	−315	−2835	365	−315	3045	−219	5	845	−115	45	5	45	−3	189	93	−75	−43	29
χ_{121}	80325	−315	−5355	405	−315	−315	1029	−315	405	213	405	−315	−315	21	−75	−171	−123	69	21
χ_{122}	85680	−336	−1680	368	−336	1968	944	−336	−656	368	368	−336	240	−16	−80	−80	−16	−16	48
χ_{123}	96390	−378	14742	150	−378	−378	6	6	1302	150	−234	6	−90	6	−138	54	−90	38	−10
χ_{124}	96390	−378	−15498	630	−378	−378	6	6	1782	−394	246	6	−90	6	−42	150	−90	38	−42
χ_{125}	96390	−378	−3402	438	−378	774	1158	−378	438	310	438	−378	−90	6	−234	−42	−90	38	22
χ_{126}	103275	−405	6075	315	−405	−405	1323	−405	315	507	315	−405	−405	27	27	123	171	−21	−69
χ_{127}	107100	−420	2940	380	−420	1500	1244	−420	−260	508	380	−420	60	−4	124	−4	92	−36	−4
χ_{128}	107100	−420	9660	−580	420	1500	−292	60	1340	−196	220	−60	−420	28	−4	−132	100	4	−4
χ_{129}	107100	−420	9660	−580	420	−420	−164	60	−580	−68	220	−60	60	−4	−260	124	−28	4	−4
χ_{130}	107100	−420	−10500	−260	420	1500	−292	60	1660	124	−100	−60	−420	28	−196	−68	100	4	−4
χ_{131}	107100	−420	−10500	−260	420	−420	−164	60	−260	252	−100	−60	60	−4	316	−68	−28	4	−4
χ_{132}	128520	−504	7560	−632	504	4488	−120	−56	776	−248	72	56	360	−24	−120	−120	120	56	−56
χ_{133}	128520	−504	−8568	−376	504	4488	−120	−56	1032	8	−184	56	360	−24	−120	−120	120	56	−56

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Table 11.13 (continued)

$[x]_G$	$1A$		$2A$			$2B$			$2C$				$2D$		$2E$				
$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$	$4j$	$4k$
χ_{134}	130560	−512	512	512	−512	1536	1536	−512	512	512	512	−512	0	0	0	0	0	0	0
χ_{135}	142800	−560	19600	−880	560	720	−304	80	1680	−368	400	−80	−240	16	80	−176	112	−16	16
χ_{136}	142800	−560	−20720	−240	560	720	−304	80	2320	272	−240	−80	−240	16	−304	−48	112	−16	16
χ_{137}	142800	−560	−560	−560	560	720	−304	80	−1840	208	80	−80	−240	16	80	80	−80	48	−48
χ_{138}	144585	−567	13041	369	−567	3753	−279	9	1809	81	−207	9	225	−15	513	33	−135	−39	33
χ_{139}	144585	−567	−9639	729	−567	−567	9	9	729	−231	153	9	585	−39	−135	153	9	9	−39
χ_{140}	160650	−630	9450	490	−630	330	−54	10	−470	170	−150	10	570	−38	−54	10	42	−22	10
χ_{141}	160650	−630	14490	410	−630	3210	−246	10	1050	154	−230	10	−150	10	−198	122	−150	−22	58
χ_{142}	160650	−630	14490	410	−630	−2550	138	10	1050	154	−230	10	330	−22	−102	−38	−54	74	−38
χ_{143}	160650	−630	−15750	890	−630	3210	−246	10	1530	−390	250	10	−150	10	−102	218	−150	−22	26
χ_{144}	160650	−630	−15750	890	−630	−2550	138	10	1530	−390	250	10	330	−22	−6	58	−54	74	−70
χ_{145}	160650	−630	−5670	730	−630	3210	−246	10	−1190	−38	90	10	330	−22	−198	122	42	−86	58
χ_{146}	171360	−672	7392	−800	672	1632	−416	96	−32	−32	224	−96	−480	32	352	−160	32	32	−32
χ_{147}	171360	−672	−8736	−544	672	1632	−416	96	224	224	−32	−96	−480	32	−416	96	32	32	−32
χ_{148}	183600	−720	19440	−1040	720	3120	48	−80	1520	−528	240	80	240	−16	48	−208	144	16	−16
χ_{149}	183600	−720	−20880	−400	720	3120	48	−80	2160	112	−400	80	240	−16	−336	−80	144	16	−16
χ_{150}	192780	−756	17388	−1044	756	−1908	396	−84	108	−404	204	84	−180	12	−276	108	−12	−44	44
χ_{151}	192780	−756	−18900	−468	756	−1908	396	−84	684	172	−372	84	−180	12	300	−84	−12	−44	44
χ_{152}	214200	−840	19320	−1160	840	−2760	−200	120	760	−264	440	−120	600	−40	−200	56	8	−56	56
χ_{153}	214200	−840	−21000	−520	840	−2760	−200	120	1400	376	−200	−120	600	−40	184	−72	8	−56	56
χ_{154}	240975	−945	29295	495	−945	1935	−177	15	3375	239	−465	15	−225	15	399	111	−177	15	−17
χ_{155}	240975	−945	−16065	1215	−945	1935	−177	15	−1665	−193	255	15	−225	15	255	−33	111	−81	31
χ_{156}	240975	−945	−23625	1335	−945	−2385	111	15	855	−489	375	15	135	−9	327	39	63	31	−89
χ_{157}	240975	−945	21735	615	−945	−2385	111	15	135	327	−345	15	135	−9	−393	87	63	31	−41
χ_{158}	257040	−1008	−21168	−688	1008	−2544	528	−112	2128	80	−432	112	−240	16	−240	16	48	−80	80
χ_{159}	257040	−1008	3024	976	−1008	2832	−240	16	976	−48	−48	16	720	−48	144	144	−48	−48	16
χ_{160}	257040	−1008	19152	−1328	1008	−2544	528	−112	1488	−560	208	112	−240	16	144	−112	48	−80	80
χ_{161}	321300	−1260	−1260	1300	−1260	660	−108	20	−3180	148	20	20	180	−12	84	−300	84	−44	84
χ_{162}	321300	−1260	8820	−1420	1260	2580	276	−140	−2700	−140	20	140	180	−12	−204	180	−84	76	−76
χ_{163}	321300	−1260	−11340	−1100	1260	2580	276	−140	−2380	180	−300	140	180	−12	372	−12	−84	76	−76
χ_{164}	385560	−1512	−31752	2040	−1512	−1512	24	24	888	−648	504	24	−360	24	24	24	−72	56	−40
χ_{165}	385560	−1512	28728	1080	−1512	−1512	24	24	−72	440	−456	24	−360	24	−168	−168	−72	56	24
χ_{166}	385560	−1512	−1512	−1512	1512	−3816	792	−168	792	−232	−168	168	−360	24	24	24	−24	−88	88
χ_{167}	428400	−1680	18480	−2000	1680	−1680	−656	240	1840	−208	560	−240	240	−16	112	−144	80	−48	48
χ_{168}	428400	−1680	−21840	−1360	1680	−1680	−656	240	2480	432	−80	−240	240	−16	−272	−16	80	−48	48

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Table 11.13 (continued)

$[x]_G$	$1A$		$2A$			$2B$			$2C$				$2D$		$2E$				
$[g]_{\overline{G}}$	$1a$	$2a$	$4a$	$4b$	$2b$	$2c$	$2d$	$4c$	$4d$	$4e$	$2e$	$4f$	$2f$	$4g$	$4h$	$4i$	$2g$	$4j$	$4k$
χ_{169}	428400	−1680	−1680	−1680	1680	2160	−912	240	2160	112	240	−240	−720	48	−144	−144	144	16	−16
χ_{170}	456960	−1792	30464	−2304	1792	−768	−768	256	−256	−256	768	−256	0	0	0	0	0	0	0
χ_{171}	456960	−1792	−34048	−1280	1792	−768	−768	256	768	768	−256	−256	0	0	0	0	0	0	0
χ_{172}	481950	−1890	−17010	2190	−1890	−1890	30	30	2190	−498	270	30	−450	30	−114	78	−162	94	−50
χ_{173}	481950	−1890	−1890	1950	−1890	−4770	222	30	990	−162	30	30	270	−18	−66	126	126	62	−130
χ_{174}	481950	−1890	13230	1710	−1890	−1890	30	30	1710	46	−210	30	−450	30	−210	−18	−162	94	−18
χ_{175}	481950	−1890	13230	1710	−1890	3870	−354	30	1710	46	−210	30	−450	30	654	78	−66	−66	14
χ_{176}	514080	−2016	6048	−2144	2016	6432	288	−224	−352	−352	−96	224	480	−32	288	−224	96	96	−96
χ_{177}	514080	−2016	−10080	−1888	2016	6432	288	−224	−96	−96	−352	224	480	−32	−480	32	96	96	−96
χ_{178}	571200	−2240	−2240	−2240	2240	−4800	−704	320	320	320	320	−320	960	−64	64	64	−64	−64	64
χ_{179}	578340	−2268	15876	−2556	2268	1188	−1116	324	−1404	132	612	−324	−540	36	−252	132	−36	60	−60
χ_{180}	578340	−2268	−20412	−1980	2268	1188	−1116	324	−828	708	36	−324	−540	36	324	−60	−36	60	−60
χ_{181}	578340	−2268	−2268	2340	−2268	−2268	36	36	−2268	36	36	36	−540	36	36	−348	36	36	36
χ_{182}	587520	−2304	29952	−2816	2304	768	768	−256	−768	−768	256	256	0	0	0	0	0	0	0
χ_{183}	587520	−2304	−34560	−1792	2304	768	768	−256	256	256	−768	256	0	0	0	0	0	0	0
χ_{184}	642600	−2520	17640	−2840	2520	−600	936	−280	360	−664	40	280	−120	8	−216	40	24	−40	40
χ_{185}	642600	−2520	27720	2120	−2520	1320	−216	40	−1080	456	−440	40	360	−24	−216	−216	−120	8	104
χ_{186}	642600	−2520	−32760	3080	−2520	1320	−216	40	−120	−632	520	40	360	−24	−24	−24	−120	8	40
χ_{187}	642600	−2520	−22680	−2200	2520	−600	936	−280	1000	−24	−600	280	−120	8	168	−88	24	−40	40
χ_{188}	642600	−2520	−2520	2600	−2520	7080	−600	40	−600	−88	40	40	−120	8	−216	40	−216	−88	168
χ_{189}	642600	−2520	−2520	2600	−2520	−4440	168	40	−600	−88	40	40	840	−56	−24	−280	−24	104	−24
χ_{190}	642600	−2520	−2520	−2520	2520	−2520	−984	360	−2520	552	360	−360	360	−24	168	168	−168	24	−24
χ_{191}	722925	−2835	19845	2565	−2835	1485	−243	45	−1755	357	−315	45	−315	21	−459	213	189	−99	21
χ_{192}	722925	−2835	−25515	3285	−2835	1485	−243	45	−1035	−459	405	45	−315	21	261	165	189	−99	−27
χ_{193}	722925	−2835	42525	2205	−2835	−2835	45	45	2205	477	−675	45	45	−3	189	93	45	45	−99
χ_{194}	722925	−2835	−2835	2925	−2835	−2835	45	45	−2835	45	45	45	45	−3	45	−51	333	−51	−51
χ_{195}	826200	−3240	−3240	−3240	3240	−3240	1368	−360	−3240	−168	−360	360	−360	24	216	216	−216	−24	24

Continued on next page

Table 11.13 (continued)

$[x]_G$		$2F$				$3A$		$3B$	$3C$		$3D$		$4A$			$4B$			$4C$			$4D$			
$[g]_{\overline{G}}$		$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$
$\lvert C_{\overline{G}}(g)\rvert$	2	18	18	17	15	15	15	6	9	9	6	6	16	16	12	16	16	12	16	16	14	16	16	14	13
	3	1	1	0	1	5	3	5	4	3	5	4	2	1	2	2	1	2	2	1	1	1	0	1	1
	5	0	0	0	0	1	1	1	1	0	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0
	7	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		3	3	3	3	14	14	5	2	2	−1	−1	−1	−1	−1	−9	−9	−9	7	7	7	−1	−1	−1	−1
χ_3		3	3	3	3	6	6	6	6	6	−3	−3	−9	−9	−9	−1	−1	−1	−1	−1	−1	7	7	7	7
χ_4		5	5	5	5	22	22	−5	4	4	4	4	−11	−11	−11	−11	−11	−11	5	5	5	5	5	5	5
χ_5		7	7	7	7	35	35	−1	5	5	2	2	11	11	11	19	19	19	11	11	11	3	3	3	3
χ_6		7	7	7	7	27	27	0	9	9	0	0	19	19	19	11	11	11	3	3	3	11	11	11	11
χ_7		−2	−2	−2	−2	49	49	−5	4	4	−5	−5	10	10	10	−22	−22	−22	14	14	14	−2	−2	−2	−2
χ_8		14	14	14	14	69	69	15	−3	−3	−3	−3	−6	−6	−6	26	26	26	14	14	14	−2	−2	−2	−2
χ_9		−13	−13	−13	−13	91	91	10	1	1	1	1	−5	−5	−5	35	35	35	23	23	23	−1	−1	−1	−1
χ_{10}		3	3	3	3	−35	−35	19	10	10	1	1	−5	−5	−5	−5	−5	−5	11	11	11	−5	−5	−5	−5
χ_{11}		6	6	6	6	−27	−27	0	18	18	0	0	−22	−22	−22	10	10	10	6	6	6	22	22	22	22
χ_{12}		22	22	22	22	−7	−7	20	11	11	2	2	26	26	26	−6	−6	−6	6	6	6	22	22	22	22
χ_{13}		−21	−21	−21	−21	15	15	15	15	15	6	6	35	35	35	−5	−5	−5	−9	−9	−9	15	15	15	15
χ_{14}		8	8	8	8	189	189	0	9	9	0	0	16	16	16	−64	−64	−64	24	24	24	−8	−8	−8	−8
χ_{15}		9	9	9	9	84	84	30	12	12	3	3	9	9	9	9	9	9	−7	−7	−7	−7	−7	−7	−7
χ_{16}		−7	−7	−7	−7	84	84	30	12	12	3	3	29	29	29	−11	−11	−11	−11	−11	−11	−3	−3	−3	−3
χ_{17}		7	7	7	7	216	216	0	−9	−9	0	0	−25	−25	−25	−65	−65	−65	27	27	27	3	3	3	3
χ_{18}		8	8	8	8	273	273	−24	3	3	−6	−6	0	0	0	80	80	80	24	24	24	−8	−8	−8	−8
χ_{19}		8	8	8	8	21	21	−6	21	21	−6	−6	−64	−64	−64	16	16	16	−8	−8	−8	24	24	24	24
χ_{20}		−1	−1	−1	−1	140	140	5	5	5	5	5	−65	−65	−65	−25	−25	−25	−5	−5	−5	19	19	19	19
χ_{21}		−17	−17	−17	−17	140	140	5	5	5	5	5	−25	−25	−25	−25	−25	−25	31	31	31	−1	−1	−1	−1
χ_{22}		−19	−19	−19	−19	189	189	0	9	9	0	0	−35	−35	−35	37	37	37	9	9	9	1	1	1	1
χ_{23}		13	13	13	13	189	189	0	9	9	0	0	−35	−35	−35	−35	−35	−35	−3	−3	−3	−3	−3	−3	−3
χ_{24}		40	40	40	40	190	190	10	−5	−5	7	7	40	40	40	40	40	40	8	8	8	8	8	8	8
χ_{25}		34	34	34	34	105	105	24	15	15	6	6	10	10	10	10	10	10	18	18	18	−14	−14	−14	−14
χ_{26}		−30	−30	−30	−30	231	231	15	6	6	6	6	50	50	50	50	50	50	2	2	2	2	2	2	2
χ_{27}		−30	−30	−30	−30	231	231	15	6	6	6	6	30	30	30	−50	−50	−50	18	18	18	2	2	2	2
χ_{28}		32	32	32	32	112	112	40	7	7	10	10	16	16	16	−16	−16	−16	0	0	0	0	0	0	0
χ_{29}		8	8	8	8	105	105	−30	15	15	−12	−12	80	80	80	0	0	0	−8	−8	−8	24	24	24	24
χ_{30}		−8	−8	−8	−8	287	287	−10	−1	−1	8	8	−16	−16	−16	64	64	64	40	40	40	8	8	8	8

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Table 11.13 (continued)

$[x]_G$	$2F$				$3A$		$3B$	$3C$		$3D$		$4A$			$4B$			$4C$			$4D$			
$[g]_{\overline{G}}$	$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$
χ_{31}	16	16	16	16	−84	−84	−30	6	6	15	15	0	0	0	0	0	0	16	16	16	16	16	16	16
χ_{32}	14	14	14	14	−35	−35	−35	10	10	10	10	30	30	30	30	30	30	−2	−2	−2	−2	−2	−2	−2
χ_{33}	14	14	14	14	175	175	−5	25	25	−8	−8	−10	−10	−10	−10	−10	−10	38	38	38	−10	−10	−10	−10
χ_{34}	4	4	4	4	−42	−42	−15	21	21	12	12	−4	−4	−4	−4	−4	−4	−12	−12	−12	20	20	20	20
χ_{35}	32	32	32	32	−48	−48	60	−21	−21	6	6	−16	−16	−16	16	16	16	0	0	0	0	0	0	0
χ_{36}	13	13	13	13	105	105	−30	15	15	15	15	−55	−55	−55	−15	−15	−15	−23	−23	−23	−15	−15	−15	−15
χ_{37}	13	13	13	13	−210	−210	60	15	15	−12	−12	−15	−15	−15	25	25	25	33	33	33	−7	−7	−7	−7
χ_{38}	0	0	0	0	392	392	−40	14	14	−4	−4	64	64	64	−64	−64	−64	0	0	0	0	0	0	0
χ_{39}	39	39	39	39	0	0	81	0	0	0	0	−9	−9	−9	−9	−9	−9	−33	−33	−33	15	15	15	15
χ_{40}	−8	−8	−8	−8	435	435	30	−15	−15	−6	−6	0	0	0	−80	−80	−80	40	40	40	8	8	8	8
χ_{41}	28	28	28	28	−70	−70	65	20	20	−7	−7	−20	−20	−20	−20	−20	−20	−20	−20	−20	−20	−20	−20	−20
χ_{42}	−4	−4	−4	−4	−280	−280	35	5	5	11	11	20	20	20	20	20	20	44	44	44	12	12	12	12
χ_{43}	0	0	0	0	456	456	24	6	6	−12	−12	−64	−64	−64	64	64	64	0	0	0	0	0	0	0
χ_{44}	32	32	32	32	−260	−260	10	10	10	−8	−8	0	0	0	0	0	0	32	32	32	32	32	32	32
χ_{45}	−24	−24	−24	−24	−210	−210	6	15	15	−3	−3	−40	−40	−40	−40	−40	−40	8	8	8	8	8	8	8
χ_{46}	40	40	40	40	294	294	−30	−21	−21	−3	−3	−40	−40	−40	−40	−40	−40	8	8	8	8	8	8	8
χ_{47}	32	32	32	32	216	216	0	−9	−9	0	0	−16	−16	−16	16	16	16	0	0	0	0	0	0	0
χ_{48}	17	17	17	17	378	378	0	−9	−9	0	0	41	41	41	−31	−31	−31	−3	−3	−3	−11	−11	−11	−11
χ_{49}	33	33	33	33	−189	−189	0	18	18	0	0	−19	−19	−19	−59	−59	−59	21	21	21	−19	−19	−19	−19
χ_{50}	−47	−47	−47	−47	378	378	0	−9	−9	0	0	41	41	41	41	41	41	33	33	33	1	1	1	1
χ_{51}	−15	−15	−15	−15	−189	−189	0	18	18	0	0	41	41	41	−31	−31	−31	21	21	21	−3	−3	−3	−3
χ_{52}	−15	−15	−15	−15	−189	−189	0	18	18	0	0	41	41	41	41	41	41	9	9	9	−7	−7	−7	−7
χ_{53}	10	10	10	10	525	525	−15	−15	−15	3	3	−10	−10	−10	70	70	70	10	10	10	−6	−6	−6	−6
χ_{54}	−54	−54	−54	−54	−105	−105	30	30	30	3	3	−50	−50	−50	30	30	30	−22	−22	−22	−6	−6	−6	−6
χ_{55}	−40	−40	−40	−40	27	27	0	9	9	0	0	64	64	64	−16	−16	−16	−24	−24	−24	8	8	8	8
χ_{56}	32	32	32	32	56	56	20	−37	−37	−4	−4	16	16	16	−16	−16	−16	0	0	0	0	0	0	0
χ_{57}	−40	−40	−40	−40	175	175	40	−5	−5	−14	−14	−80	−80	−80	0	0	0	−24	−24	−24	8	8	8	8
χ_{58}	−40	−40	−40	−40	490	490	−50	−5	−5	13	13	−40	−40	−40	−40	−40	−40	−8	−8	−8	−8	−8	−8	−8
χ_{59}	39	39	39	39	315	315	45	0	0	−9	−9	15	15	15	15	15	15	31	31	31	15	15	15	15
χ_{60}	−9	−9	−9	−9	315	315	45	0	0	−9	−9	15	15	15	15	15	15	−41	−41	−41	−9	−9	−9	−9
χ_{61}	16	16	16	16	−546	−546	−60	12	12	−6	−6	−16	−16	−16	−16	−16	−16	48	48	48	−16	−16	−16	−16
χ_{62}	21	21	21	21	0	0	0	0	0	0	0	−27	−27	−27	45	45	45	9	9	9	−15	−15	−15	−15
χ_{63}	0	0	0	0	−280	−280	−64	−10	−10	8	8	−64	−64	−64	64	64	64	0	0	0	0	0	0	0
χ_{44}	−14	−14	−14	−14	189	189	0	9	9	0	0	70	70	70	−10	−10	−10	−30	−30	−30	−14	−14	−14	−14
χ_{65}	−46	−46	−46	−46	189	189	0	9	9	0	0	10	10	10	10	10	10	42	42	42	−6	−6	−6	−6

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Table 11.13 (continued)

$[x]_G$	$2F$				$3A$		$3B$	$3C$		$3D$		$4A$			$4B$			$4C$			$4D$			
$[g]_{\overline{G}}$	$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$
χ_{66}	−14	−14	−14	−14	189	189	0	9	9	0	0	10	10	10	10	10	10	18	18	18	−14	−14	−14	−14
χ_{67}	−39	−39	−39	−39	405	405	0	0	0	0	0	−15	−15	−15	−15	−15	−15	−39	−39	−39	−7	−7	−7	−7
χ_{68}	9	9	9	9	405	405	0	0	0	0	0	−15	−15	−15	−15	−15	−15	33	33	33	17	17	17	17
χ_{69}	0	0	0	0	−216	−216	0	−18	−18	0	0	64	64	64	−64	−64	−64	0	0	0	0	0	0	0
χ_{70}	64	64	64	64	280	280	−44	10	10	10	10	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	−36	−36	−36	−36	0	0	81	0	0	0	0	−36	−36	−36	−36	−36	−36	12	12	12	12	12	12	12
χ_{72}	−3	−3	−3	−3	0	0	0	0	0	0	0	45	45	45	45	45	45	−27	−27	−27	21	21	21	21
χ_{73}	−3	−3	−3	−3	0	0	0	0	0	0	0	45	45	45	45	45	45	−27	−27	−27	21	21	21	21
χ_{74}	0	0	0	0	176	176	−40	−4	−4	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	48	48	48	48	140	140	50	20	20	17	17	0	0	0	0	0	0	−16	−16	−16	−16	−16	−16	−16
χ_{76}	3	3	3	3	0	0	−81	0	0	0	0	−45	−45	−45	−45	−45	−45	−21	−21	−21	27	27	27	27
χ_{77}	−48	−48	−48	−48	−378	−378	0	−18	−18	0	0	16	16	16	16	16	16	48	48	48	−16	−16	−16	−16
χ_{78}	30	30	30	30	−315	−315	−45	0	0	−18	−18	30	30	30	30	30	30	−10	−10	−10	6	6	6	6
χ_{79}	32	32	32	32	−84	−84	−30	6	6	−12	−12	0	0	0	0	0	0	−32	−32	−32	−32	−32	−32	−32
χ_{80}	0	0	0	0	−512	−512	64	−32	−32	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	−30	−30	−30	−30	−405	−405	0	0	0	0	0	−30	−30	−30	−30	−30	−30	−6	−6	−6	10	10	10	10
χ_{82}	15	−1	−1	−1	63	−1	0	15	−1	3	−1	31	−1	−1	31	−1	−1	15	−1	−1	15	−1	−1	−1
χ_{83}	9	25	−7	1	252	−4	0	15	−1	−6	2	25	−7	5	−95	1	5	45	−3	−3	−11	5	5	−3
χ_{84}	17	33	1	−7	0	0	0	45	−3	−9	3	−95	1	5	25	−7	5	−3	13	−3	53	5	−11	−3
χ_{85}	11	59	−5	−5	378	−6	0	0	0	9	−3	−101	−5	11	−101	−5	11	27	11	−5	27	11	−5	−5
χ_{86}	−21	27	−5	3	378	−6	0	0	0	9	−3	−21	11	−9	99	3	−9	63	−17	−1	7	−9	7	−1
χ_{87}	69	21	−11	−3	567	−9	0	0	0	0	0	45	13	−15	165	5	−15	57	9	−7	1	17	1	−7
χ_{88}	36	−28	4	−4	630	−10	0	15	−1	3	−1	−56	8	−4	−184	8	4	60	−4	−4	−4	−4	−4	4
χ_{89}	−28	36	4	−4	630	−10	0	15	−1	3	−1	56	−8	4	184	−8	−4	60	−4	−4	−4	−4	−4	4
χ_{90}	77	29	−3	−11	315	−5	0	30	−2	−3	1	165	5	−15	45	13	−15	9	25	−7	65	17	−15	−7
χ_{91}	−3	−51	13	−3	315	−5	0	30	−2	−3	1	−35	−3	5	−35	−3	5	69	−43	5	−11	5	−11	5
χ_{92}	28	−36	−4	4	378	−6	0	45	−3	0	0	−184	8	4	−56	8	−4	−12	−12	4	52	−12	4	−4
χ_{93}	−36	28	−4	4	378	−6	0	45	−3	0	0	184	−8	−4	56	−8	4	−12	−12	4	52	−12	4	−4
χ_{94}	40	40	−24	8	693	−11	0	30	−2	6	−2	96	−32	24	−144	−16	24	24	24	−8	−8	−8	24	−8
χ_{95}	49	−63	1	1	945	−15	0	45	−3	0	0	209	−15	1	209	−15	1	33	−15	1	33	−15	1	1
χ_{96}	−31	−15	17	−7	945	−15	0	45	−3	0	0	149	−11	1	−211	13	1	45	−3	−3	−11	5	5	−3
χ_{97}	10	42	−22	10	−315	5	0	15	−1	21	−7	50	−14	10	50	−14	10	42	10	−6	10	−22	26	−6
χ_{98}	76	12	12	−20	−378	6	0	45	−3	9	−3	4	4	−4	4	4	−4	60	−4	−4	92	28	−36	−4

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Table 11.13 (continued)

$[x]_G$	$2F$				$3A$		$3B$	$3C$		$3D$		$4A$			$4B$			$4C$			$4D$			
$[g]_{\overline{G}}$	$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$
χ_{99}	−57	55	23	−17	0	0	0	45	−3	18	−6	95	−1	−5	−25	7	−5	−45	3	3	43	27	−37	3
χ_{100}	55	39	−25	7	0	0	0	45	−3	18	−6	−65	31	−25	−65	31	−25	−9	39	−9	−41	7	23	−9
χ_{101}	39	23	−9	−1	945	−15	0	−45	3	−9	3	−65	−33	35	−185	−25	35	75	−5	−5	19	3	3	−5
χ_{102}	40	40	−24	8	945	−15	0	0	0	−18	6	−80	48	−40	160	32	−40	24	24	−8	−8	−8	24	−8
χ_{103}	−8	−8	−8	8	1890	−30	0	0	0	−9	3	−136	−8	16	376	−8	−16	120	−8	−8	−8	−8	−8	8
χ_{104}	−8	−8	−8	8	1890	−30	0	0	0	−9	3	136	8	−16	−376	8	16	120	−8	−8	−8	−8	−8	8
χ_{105}	120	−8	−8	−8	378	−6	0	−45	3	18	−6	40	40	−40	40	40	−40	24	24	−8	24	24	−8	−8
χ_{106}	51	99	−29	3	567	−9	0	0	0	0	0	−21	−53	51	−21	−53	51	3	51	−13	−29	19	19	−13
χ_{107}	−93	−45	19	3	567	−9	0	0	0	0	0	51	19	−21	51	19	−21	75	−69	11	−5	−21	−5	11
χ_{108}	3	51	19	−21	567	−9	0	0	0	0	0	−189	−29	39	−69	−37	39	−33	15	−1	55	39	−41	−1
χ_{109}	−34	62	−2	−2	945	−15	0	0	0	9	−3	110	46	−50	110	46	−50	−18	14	−2	−18	14	−2	−2
χ_{110}	62	30	−34	14	−945	15	0	45	−3	−18	6	−50	14	−10	−50	14	−10	102	6	−10	−42	−10	38	−10
χ_{111}	72	72	8	−24	−567	9	0	0	0	0	0	−144	−16	24	96	−32	24	24	24	−8	56	56	−40	−8
χ_{112}	−28	36	4	−4	1512	−24	0	45	−3	0	0	−160	32	−20	160	−32	20	−36	−36	12	−36	28	12	−12
χ_{113}	36	−28	4	−4	1512	−24	0	45	−3	0	0	160	−32	20	−160	32	−20	−36	−36	12	−36	28	12	−12
χ_{114}	72	72	8	−24	−315	5	0	−30	2	−24	8	160	32	−40	−80	48	−40	24	24	−8	56	56	−40	−8
χ_{115}	8	−120	8	8	630	−10	0	15	−1	30	−10	−40	−40	40	−40	−40	40	−24	−24	8	−24	−24	8	8
χ_{116}	−24	−24	−24	24	−1260	20	0	60	−4	3	−1	0	0	0	0	0	0	120	−8	−8	−8	−8	−8	8
χ_{117}	−59	21	21	−11	1890	−30	0	−45	3	0	0	−275	13	5	−275	13	5	69	21	−11	37	−11	21	−11
χ_{118}	69	21	21	−27	−945	15	0	90	−6	0	0	−155	5	5	−155	5	5	45	−3	−3	−67	13	13	−3
χ_{119}	149	−27	5	−19	1890	−30	0	−45	3	0	0	−95	1	5	265	−23	5	33	−15	1	−23	−7	9	1
χ_{120}	21	−27	5	−3	−945	15	0	90	−6	0	0	−215	9	5	145	−15	5	57	9	−7	81	−31	17	−7
χ_{121}	21	−27	−27	21	0	0	0	0	0	−27	9	45	−51	45	45	−51	45	−51	29	−3	−3	−51	45	−3
χ_{122}	−80	−80	48	−16	378	−6	0	0	0	−18	6	−16	−16	16	−16	−16	16	−48	−48	16	16	16	−48	16
χ_{123}	−138	−42	22	6	2835	−45	0	0	0	0	0	−54	10	−6	426	−22	−6	102	6	−10	−10	22	6	−10
χ_{124}	−42	54	−10	6	2835	−45	0	0	0	0	0	66	2	−6	−414	34	−6	54	−42	6	−26	6	−10	6
χ_{125}	−42	54	−10	6	−567	9	0	0	0	0	0	30	−34	30	30	−34	30	78	−18	−2	−34	−2	14	−2
χ_{126}	−69	−21	−21	27	0	0	0	0	0	0	0	−45	51	−45	−45	51	−45	−45	3	3	3	−77	51	3
χ_{127}	−36	−100	28	−4	0	0	0	45	−3	−9	3	20	20	−20	20	20	−20	−84	−20	12	−52	12	−20	12
χ_{128}	−4	60	28	−28	0	0	0	45	−3	−9	3	160	−32	20	−160	32	−20	−12	52	−12	−12	−12	−12	12
χ_{129}	92	−100	−4	4	1890	−30	0	0	0	18	−6	40	−24	20	200	8	−20	132	−60	4	4	4	4	−4
χ_{130}	60	−4	28	−28	0	0	0	45	−3	−9	3	−160	32	−20	160	−32	20	−12	52	−12	−12	−12	−12	12
χ_{131}	−100	92	−4	4	1890	−30	0	0	0	18	−6	−40	24	−20	−200	−8	20	132	−60	4	4	4	4	−4
χ_{132}	−24	−24	−24	24	−378	6	0	90	−6	0	0	376	−8	−16	−136	−8	16	−24	−24	8	104	−24	8	−8
χ_{133}	−24	−24	−24	24	−378	6	0	90	−6	0	0	−376	8	16	136	8	−16	−24	−24	8	104	−24	8	−8

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Table 11.13 (continued)

$[x]_G$	$2F$				$3A$		$3B$	$3C$		$3D$		$4A$			$4B$			$4C$			$4D$			
$[g]_{\overline{G}}$	$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$
χ_{134}	0	0	0	0	−1008	16	0	−60	4	24	−8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	−48	80	16	−16	3150	−50	0	−15	1	6	−2	80	−48	40	400	16	−40	48	48	−16	−16	−16	−16	16
χ_{136}	80	−48	16	−16	3150	−50	0	−15	1	6	−2	−80	48	−40	−400	−16	40	48	48	−16	−16	−16	−16	16
χ_{137}	16	16	16	−16	1260	−20	0	30	−2	−21	7	0	0	0	0	0	0	144	−112	16	16	16	16	−16
χ_{138}	201	−39	−7	−15	0	0	0	0	0	0	0	261	−27	9	−99	−3	9	45	−3	−3	69	−43	21	−3
χ_{139}	−39	105	41	−39	0	0	0	0	0	0	0	81	−15	9	81	−15	9	81	33	−15	−63	17	33	−15
χ_{140}	154	186	−6	−38	945	−15	0	45	−3	0	0	50	−14	10	50	−14	10	−54	42	−6	−86	10	26	−6
χ_{141}	−198	−38	26	10	945	−15	0	45	−3	0	0	430	−18	−10	−50	14	−10	−54	42	−6	26	−6	10	−6
χ_{142}	−102	−70	58	−22	945	−15	0	45	−3	0	0	−290	30	−10	−50	14	−10	−54	42	−6	26	−6	10	−6
χ_{143}	−102	58	−6	10	945	−15	0	45	−3	0	0	−410	38	−10	70	6	−10	−102	−6	10	10	−22	−6	10
χ_{144}	−6	26	26	−22	945	−15	0	45	−3	0	0	310	−10	−10	70	6	−10	−102	−6	10	10	−22	−6	10
χ_{145}	26	186	−6	−22	945	−15	0	45	−3	0	0	−70	−6	10	−70	−6	10	−30	66	−14	18	−14	34	−14
χ_{146}	−96	160	32	−32	0	0	0	45	−3	18	−6	−16	−16	16	16	16	−16	0	0	0	0	0	0	0
χ_{147}	160	−96	32	−32	0	0	0	45	−3	18	−6	16	16	−16	−16	−16	16	0	0	0	0	0	0	0
χ_{148}	−80	48	−16	16	1890	−30	0	−45	3	0	0	400	16	−40	80	−48	40	−48	−48	16	16	16	16	−16
χ_{149}	48	−80	−16	16	1890	−30	0	−45	3	0	0	−400	−16	40	−80	48	−40	−48	−48	16	16	16	16	−16
χ_{150}	108	−84	12	−12	2268	−36	0	0	0	0	0	−144	48	−36	144	−48	36	−12	−12	4	52	−12	4	−4
χ_{151}	−84	108	12	−12	2268	−36	0	0	0	0	0	144	−48	36	−144	48	−36	−12	−12	4	52	−12	4	−4
χ_{152}	24	−104	−40	40	1890	−30	0	45	−3	9	−3	−280	−24	40	40	40	−40	−72	56	−8	−8	−8	−8	8
χ_{153}	−104	24	−40	40	1890	−30	0	45	−3	9	−3	280	24	−40	−40	−40	40	−72	56	−8	−8	−8	−8	8
χ_{154}	63	−81	−17	15	2835	−45	0	0	0	0	0	135	−25	15	135	−25	15	−81	−33	15	−49	−1	−17	15
χ_{155}	15	−129	−1	15	2835	−45	0	0	0	0	0	−225	−1	15	−225	−1	15	135	−9	−9	23	7	7	−9
χ_{156}	−81	−225	63	−9	2835	−45	0	0	0	0	0	195	−29	15	−165	−5	15	3	51	−13	27	11	11	−13
χ_{157}	−33	207	−17	−9	2835	−45	0	0	0	0	0	−165	−5	15	195	−29	15	75	−69	11	51	−29	−13	11
χ_{158}	80	−48	16	−16	378	−6	0	45	−3	0	0	320	−64	40	160	32	−40	−48	−48	16	16	16	16	−16
χ_{159}	144	144	16	−48	−1890	30	0	−90	6	0	0	−16	−16	16	−16	−16	16	48	48	−16	−16	−16	48	−16
χ_{160}	−48	80	16	−16	378	−6	0	45	−3	0	0	−320	64	−40	−160	−32	40	−48	−48	16	16	16	16	−16
χ_{161}	180	−12	−12	−12	1890	−30	0	90	−6	0	0	20	20	−20	20	20	−20	132	−60	4	−60	4	4	4
χ_{162}	84	−108	−12	12	1890	−30	0	90	−6	0	0	200	8	−20	40	−24	20	60	60	−20	−68	−4	−20	20
χ_{163}	−108	84	−12	12	1890	−30	0	90	−6	0	0	−200	−8	20	−40	24	−20	60	60	−20	−68	−4	−20	20
χ_{164}	120	120	−72	24	2835	−45	0	0	0	0	0	144	16	−24	−96	32	−24	−24	−24	8	8	8	−24	8
χ_{165}	−72	−72	−8	24	2835	−45	0	0	0	0	0	−96	32	−24	144	16	−24	72	72	−24	40	40	8	−24
χ_{166}	24	24	24	−24	−2268	36	0	0	0	0	0	0	0	0	0	0	0	−24	−24	8	104	−24	8	−8
χ_{167}	−80	48	−16	16	−1890	30	0	−45	3	18	−6	−160	−32	40	−320	64	−40	48	48	−16	−16	−16	−16	16
χ_{168}	48	−80	−16	16	−1890	30	0	−45	3	18	−6	160	32	−40	320	−64	40	48	48	−16	−16	−16	−16	16

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Table 11.13 (continued)

$[x]_G$	$2F$				$3A$		$3B$	$3C$		$3D$		$4A$			$4B$			$4C$			$4D$			
$[g]_{\overline{G}}$	$4l$	$2h$	$4m$	$4n$	$3a$	$6a$	$3b$	$3c$	$6b$	$3d$	$6c$	$8a$	$8b$	$4o$	$8c$	$8d$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$
χ_{169}	48	48	48	−48	−3780	60	0	0	0	−9	3	0	0	0	0	0	0	240	−16	−16	−16	−16	−16	16
χ_{170}	0	0	0	0	2520	−40	0	−30	2	−24	8	−64	−64	64	64	64	−64	0	0	0	0	0	0	0
χ_{171}	0	0	0	0	2520	−40	0	−30	2	−24	8	64	64	−64	−64	−64	64	0	0	0	0	0	0	0
χ_{172}	−18	78	−50	30	−2835	45	0	0	0	0	0	90	26	−30	330	10	−30	78	−18	−2	−2	30	−18	−2
χ_{173}	−114	−18	46	−18	−2835	45	0	0	0	0	0	30	−34	30	30	−34	30	78	−18	−2	78	−18	−2	−2
χ_{174}	−114	−18	−18	30	−2835	45	0	0	0	0	0	−30	34	−30	−270	50	−30	126	30	−18	14	46	−2	−18
χ_{175}	78	−210	−18	30	−2835	45	0	0	0	0	0	−90	−26	30	−90	−26	30	54	−42	6	−26	6	−10	6
χ_{176}	−160	96	−32	32	−1512	24	0	−45	3	0	0	16	16	−16	−16	−16	16	0	0	0	0	0	0	0
χ_{177}	96	−160	−32	32	−1512	24	0	−45	3	0	0	−16	−16	16	16	16	−16	0	0	0	0	0	0	0
χ_{178}	−64	−64	−64	64	−2520	40	0	30	−2	−30	10	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{179}	132	−60	36	−36	0	0	0	0	0	0	0	144	−48	36	−144	48	−36	−180	12	12	12	12	12	−12
χ_{180}	−60	132	36	−36	0	0	0	0	0	0	0	−144	48	−36	144	−48	36	−180	12	12	12	12	12	−12
χ_{181}	228	36	−92	36	0	0	0	0	0	0	0	36	36	−36	36	36	−36	−108	84	−12	84	20	−12	−12
χ_{182}	0	0	0	0	1512	−24	0	−90	6	0	0	64	64	−64	−64	−64	64	0	0	0	0	0	0	0
χ_{183}	0	0	0	0	1512	−24	0	−90	6	0	0	−64	−64	64	64	64	−64	0	0	0	0	0	0	0
χ_{184}	72	−56	8	−8	−1890	30	0	45	−3	0	0	40	40	−40	−280	−24	40	−24	−24	8	−88	40	8	−8
χ_{185}	−120	−120	72	−24	945	−15	0	−90	6	0	0	160	32	−40	−80	48	−40	24	24	−8	−8	−8	24	−8
χ_{186}	72	72	8	−24	945	−15	0	−90	6	0	0	−80	48	−40	160	32	−40	−72	−72	24	−40	−40	−8	24
χ_{187}	−56	72	8	−8	−1890	30	0	45	−3	0	0	−40	−40	40	280	24	−40	−24	−24	8	−88	40	8	−8
χ_{188}	−120	8	8	8	−1890	30	0	45	−3	0	0	40	40	−40	40	40	−40	−24	−24	8	−24	−24	8	8
χ_{189}	72	−56	72	−56	−1890	30	0	45	−3	0	0	40	40	−40	40	40	−40	−24	−24	8	−24	−24	8	8
χ_{190}	−24	−24	−24	24	0	0	0	0	0	27	−9	0	0	0	0	0	0	24	−104	24	24	24	24	−24
χ_{191}	−99	237	−51	21	0	0	0	0	0	0	0	225	−63	45	−135	−39	45	9	−39	9	−15	1	−15	9
χ_{192}	−147	−195	29	21	0	0	0	0	0	0	0	−135	−39	45	225	−63	45	−63	81	−15	−39	41	9	−15
χ_{193}	93	45	−19	−3	0	0	0	0	0	0	0	−315	−27	45	−315	−27	45	−171	−27	21	−27	−11	−27	21
χ_{194}	45	−3	−3	−3	0	0	0	0	0	0	0	45	−51	45	45	−51	45	45	−3	−3	45	−3	−3	−3
χ_{195}	24	24	24	−24	0	0	0	0	0	0	0	0	0	0	0	0	0	72	72	−24	72	−56	−24	24

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Table 11.13 (continued)

$[x]_G$		$4E$			$4F$			$4G$				$4H$					$4I$					$4J$		$4K$		
$[g]_{\overline{G}}$		$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
$\lvert C_{\overline{G}}(g)\rvert$	2	14	14	12	14	14	12	13	13	13	13	14	14	12	12	12	13	13	12	12	12	10	10	11	11	10
	3	1	0	1	1	0	1	1	1	0	0	1	0	1	1	1	0	0	0	0	0	1	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		−5	−5	−5	3	3	3	−1	−1	−1	−1	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1	3	3	3
χ_3		3	3	3	−5	−5	−5	−1	−1	−1	−1	3	3	3	3	3	−1	−1	−1	−1	−1	3	3	−1	−1	−1
χ_4		−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5	5	5	−3	−3	−3	−3	−3	1	1	1	1	1
χ_5		7	7	7	−1	−1	−1	3	3	3	3	7	7	7	7	7	3	3	3	3	3	−1	−1	3	3	3
χ_6		−1	−1	−1	7	7	7	3	3	3	3	7	7	7	7	7	3	3	3	3	3	3	3	−1	−1	−1
χ_7		−6	−6	−6	−6	−6	−6	−2	−2	−2	−2	2	2	2	2	2	2	2	2	2	2	−2	−2	2	2	2
χ_8		10	10	10	10	10	10	−2	−2	−2	−2	2	2	2	2	2	2	2	2	2	2	−2	−2	2	2	2
χ_9		7	7	7	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−5	−5	−5	−5	−5	3	3	3	3	3
χ_{10}		3	3	3	3	3	3	−5	−5	−5	−5	−5	−5	−5	−5	−5	3	3	3	3	3	−1	−1	−1	−1	−1
χ_{11}		−6	−6	−6	−6	−6	−6	6	6	6	6	2	2	2	2	2	2	2	2	2	2	6	6	2	2	2
χ_{12}		10	10	10	10	10	10	6	6	6	6	2	2	2	2	2	2	2	2	2	2	6	6	2	2	2
χ_{13}		−1	−1	−1	7	7	7	−9	−9	−9	−9	−1	−1	−1	−1	−1	−5	−5	−5	−5	−5	−1	−1	−1	−1	−1
χ_{14}		−8	−8	−8	8	8	8	0	0	0	0	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0
χ_{15}		−15	−15	−15	−15	−15	−15	1	1	1	1	9	9	9	9	9	1	1	1	1	1	−3	−3	−3	−3	−3
χ_{16}		9	9	9	−15	−15	−15	5	5	5	5	1	1	1	1	1	−3	−3	−3	−3	−3	−3	−3	1	1	1
χ_{17}		−13	−13	−13	−5	−5	−5	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	3	3	3	3	3
χ_{18}		8	8	8	−8	−8	−8	0	0	0	0	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0
χ_{19}		8	8	8	−8	−8	−8	0	0	0	0	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0
χ_{20}		−5	−5	−5	−13	−13	−13	−5	−5	−5	−5	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_{21}		−1	−1	−1	−1	−1	−1	7	7	7	7	−9	−9	−9	−9	−9	−1	−1	−1	−1	−1	−5	−5	3	3	3
χ_{22}		−7	−7	−7	17	17	17	9	9	9	9	1	1	1	1	1	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{23}		13	13	13	13	13	13	−3	−3	−3	−3	13	13	13	13	13	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{24}		8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	0	0	0	0	0
χ_{25}		2	2	2	2	2	2	−6	−6	−6	−6	−6	−6	−6	−6	−6	2	2	2	2	2	6	6	−2	−2	−2
χ_{26}		−6	−6	−6	−6	−6	−6	−6	−6	−6	−6	2	2	2	2	2	−6	−6	−6	−6	−6	−2	−2	−2	−2	−2
χ_{27}		6	6	6	−10	−10	−10	−6	−6	−6	−6	−2	−2	−2	−2	−2	6	6	6	6	6	2	2	−2	−2	−2
χ_{28}		−16	−16	−16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}		−8	−8	−8	8	8	8	0	0	0	0	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0
χ_{30}		8	8	8	−8	−8	−8	0	0	0	0	−8	−8	−8	−8	−8	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$4E$			$4F$			$4G$				$4H$					$4I$					$4J$		$4K$		
$[g]_{\overline{G}}$	$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
χ_{31}	0	0	0	0	0	0	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	6	6	6	6	6	6	6	6	6	6	−2	−2	−2	−2	−2	−10	−10	−10	−10	−10	2	2	2	2	2
χ_{33}	−2	−2	−2	−2	−2	−2	6	6	6	6	−10	−10	−10	−10	−10	−2	−2	−2	−2	−2	2	2	2	2	2
χ_{34}	−4	−4	−4	−4	−4	−4	−12	−12	−12	−12	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	4	4	−4	−4	−4
χ_{35}	16	16	16	−16	−16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	−11	−11	−11	13	13	13	−7	−7	−7	−7	5	5	5	5	5	1	1	1	1	1	−3	−3	1	1	1
χ_{37}	−19	−19	−19	5	5	5	9	9	9	9	−3	−3	−3	−3	−3	−7	−7	−7	−7	−7	−3	−3	1	1	1
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	−9	−9	−9	−9	−9	−9	−9	−9	−9	−9	−9	−9	−9	−9	−9	7	7	7	7	7	−1	−1	7	7	7
χ_{40}	−8	−8	−8	8	8	8	0	0	0	0	−8	−8	−8	−8	−8	0	0	0	0	0	0	0	0	0	0
χ_{41}	12	12	12	12	12	12	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	4	4	4	4	4
χ_{42}	4	4	4	4	4	4	−4	−4	−4	−4	−12	−12	−12	−12	−12	4	4	4	4	4	4	4	−4	−4	−4
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	8	8	8	8	8	8	−8	−8	−8	−8	−8	−8	−8	−8	−8	8	8	8	8	8	0	0	0	0	0
χ_{46}	−8	−8	−8	−8	−8	−8	8	8	8	8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	0	0	0	0	0
χ_{47}	16	16	16	−16	−16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	5	5	5	−19	−19	−19	−3	−3	−3	−3	5	5	5	5	5	1	1	1	1	1	−3	−3	−3	−3	−3
χ_{49}	9	9	9	−15	−15	−15	−3	−3	−3	−3	1	1	1	1	1	−3	−3	−3	−3	−3	−3	−3	1	1	1
χ_{50}	1	1	1	1	1	1	9	9	9	9	−7	−7	−7	−7	−7	1	1	1	1	1	−3	−3	5	5	5
χ_{51}	21	21	21	−3	−3	−3	−3	−3	−3	−3	5	5	5	5	5	1	1	1	1	1	−3	−3	5	5	5
χ_{52}	−15	−15	−15	−15	−15	−15	9	9	9	9	−7	−7	−7	−7	−7	1	1	1	1	1	−3	−3	−3	−3	−3
χ_{53}	−2	−2	−2	14	14	14	−14	−14	−14	−14	−10	−10	−10	−10	−10	−2	−2	−2	−2	−2	2	2	−2	−2	−2
χ_{54}	−10	−10	−10	6	6	6	2	2	2	2	−2	−2	−2	−2	−2	6	6	6	6	6	−6	−6	−2	−2	−2
χ_{55}	−8	−8	−8	8	8	8	0	0	0	0	−8	−8	−8	−8	−8	0	0	0	0	0	0	0	0	0	0
χ_{56}	−16	−16	−16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	8	8	8	−8	−8	−8	0	0	0	0	−8	−8	−8	−8	−8	0	0	0	0	0	0	0	0	0	0
χ_{58}	8	8	8	8	8	8	8	8	8	8	−8	−8	−8	−8	−8	8	8	8	8	8	0	0	0	0	0
χ_{59}	−9	−9	−9	−9	−9	−9	−1	−1	−1	−1	−1	−1	−1	−1	−1	7	7	7	7	7	−5	−5	−5	−5	−5
χ_{60}	15	15	15	15	15	15	7	7	7	7	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	7	7	−1	−1	−1
χ_{61}	0	0	0	0	0	0	0	0	0	0	−16	−16	−16	−16	−16	0	0	0	0	0	0	0	0	0	0
χ_{62}	−15	−15	−15	9	9	9	−15	−15	−15	−15	9	9	9	9	9	5	5	5	5	5	−3	−3	5	5	5
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	14	14	14	−2	−2	−2	−6	−6	−6	−6	−10	−10	−10	−10	−10	−2	−2	−2	−2	−2	−6	−6	−2	−2	−2
χ_{65}	2	2	2	2	2	2	−6	−6	−6	−6	10	10	10	10	10	2	2	2	2	2	6	6	6	6	6

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Table 11.13 (continued)

$[x]_G$	$4E$			$4F$			$4G$				$4H$					$4I$					$4J$		$4K$		
$[g]_{\overline{G}}$	$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
χ_{66}	−14	−14	−14	−14	−14	−14	−6	−6	−6	−6	10	10	10	10	10	2	2	2	2	2	6	6	−2	−2	−2
χ_{67}	−15	−15	−15	−15	−15	−15	9	9	9	9	1	1	1	1	1	1	1	1	1	1	9	9	1	1	1
χ_{68}	9	9	9	9	9	9	−15	−15	−15	−15	1	1	1	1	1	−7	−7	−7	−7	−7	−3	−3	−3	−3	−3
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4
χ_{72}	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5
χ_{73}	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	−5	−5	3	3	3
χ_{77}	0	0	0	0	0	0	0	0	0	0	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0
χ_{78}	6	6	6	6	6	6	6	6	6	6	−2	−2	−2	−2	−2	6	6	6	6	6	2	2	−6	−6	−6
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	−6	−6	−6	−6	−6	−6	−6	−6	−6	−6	2	2	2	2	2	−6	−6	−6	−6	−6	6	6	−2	−2	−2
χ_{82}	7	−1	−1	7	−1	−1	7	−1	−1	−1	15	−1	−1	−1	−1	7	−1	−1	−1	−1	3	−1	3	−1	−1
χ_{83}	−19	5	1	5	−3	1	−3	−3	−3	5	13	−3	5	−7	1	1	−7	1	−3	5	−3	1	5	1	−3
χ_{84}	5	−3	1	−19	5	1	5	−11	5	−3	13	−3	5	1	−7	1	−7	1	5	−3	9	−3	1	−3	1
χ_{85}	−21	3	3	−21	3	3	−5	−13	3	3	11	−5	11	−5	−5	−5	−13	3	3	3	3	−1	3	−1	−1
χ_{86}	15	−9	3	−9	−1	3	−1	−17	−1	7	−9	7	−1	3	−5	−5	3	−5	7	−1	3	−1	3	7	−5
χ_{87}	33	−7	−3	9	1	−3	9	9	−7	1	25	9	1	−3	−11	13	5	−3	1	−7	−3	1	5	1	−3
χ_{88}	−16	0	4	16	0	−4	−4	4	4	−4	32	0	−8	4	−4	−8	8	0	4	−4	0	0	4	−4	0
χ_{89}	16	0	−4	−16	0	4	−4	4	4	−4	16	−16	8	−4	4	−8	8	0	−4	4	0	0	4	−4	0
χ_{90}	9	1	−3	33	−7	−3	17	1	1	−7	25	9	1	−11	−3	13	5	−3	−7	1	9	−3	1	−3	1
χ_{91}	−3	5	−3	−3	5	−3	13	5	−11	5	−19	−3	−3	5	5	−3	5	5	−3	−3	−3	1	5	9	−7
χ_{92}	16	0	−4	−16	0	4	−12	12	−4	4	32	0	−8	−4	4	−8	8	0	−4	4	0	0	−4	4	0
χ_{93}	−16	0	4	16	0	−4	−12	12	−4	4	16	−16	8	4	−4	−8	8	0	4	−4	0	0	−4	4	0
χ_{94}	−24	8	0	24	−8	0	0	0	0	0	8	8	8	−16	0	0	0	0	−8	8	0	0	0	0	0
χ_{95}	−7	1	1	−7	1	1	9	−15	1	1	49	1	−15	1	1	9	−15	1	1	1	−3	1	−3	1	1
χ_{96}	1	−7	5	−23	1	5	−3	−3	−3	5	9	−7	1	−11	13	5	−3	5	1	−7	−3	1	1	−3	1
χ_{97}	10	−6	2	10	−6	2	2	−14	2	2	−14	18	2	−6	−6	−6	10	−6	2	2	6	−2	−2	6	−2
χ_{98}	4	4	−4	4	4	−4	28	−4	−4	−4	4	4	4	−4	−4	4	4	4	−4	−4	12	−4	4	4	−4

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Table 11.13 (continued)

$[x]_G$	$4E$			$4F$			$4G$				$4H$					$4I$					$4J$		$4K$		
$[g]_{\overline{G}}$	$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
χ_{99}	−13	−5	7	11	−13	7	−21	−21	11	3	−13	3	−5	−1	7	−9	−1	−9	3	11	3	−1	−5	−1	3
χ_{100}	−17	7	−1	−17	7	−1	−9	15	−1	−1	15	−1	15	−9	−9	−1	−9	7	−1	−1	−9	3	−1	−5	3
χ_{101}	−29	11	−1	−5	3	−1	11	−5	−5	3	−13	3	11	−9	−1	−9	−1	7	−5	3	3	−1	3	7	−5
χ_{102}	24	−8	0	−24	8	0	0	0	0	0	8	8	8	0	−16	0	0	0	8	−8	0	0	0	0	0
χ_{103}	24	−8	0	24	−8	0	−8	8	8	−8	−8	−8	8	−16	16	−8	−8	8	0	0	0	0	0	0	0
χ_{104}	−24	8	0	−24	8	0	−8	8	8	−8	8	8	−8	16	−16	8	8	−8	0	0	0	0	0	0	0
χ_{105}	8	8	−8	8	8	−8	24	24	−8	−8	8	8	8	−8	−8	8	8	8	−8	−8	0	0	0	0	0
χ_{106}	3	−5	3	3	−5	3	−21	3	3	3	27	11	11	−13	−13	3	−5	−5	3	3	−9	3	−1	−5	3
χ_{107}	3	−5	3	3	−5	3	3	−21	−5	11	3	−13	−13	11	11	3	−5	−5	3	3	3	−1	11	7	−9
χ_{108}	−9	−1	3	−33	7	3	−9	−9	7	−1	−17	−1	7	3	−5	−13	−5	3	7	−1	3	−1	−5	−1	3
χ_{109}	6	−10	6	6	−10	6	−26	−10	6	6	−18	14	−2	−2	−2	−10	6	−10	6	6	−6	2	−6	2	2
χ_{110}	−10	6	−2	−10	6	−2	6	6	−10	6	−18	14	14	−10	−10	−10	6	6	−2	−2	−6	2	2	10	−6
χ_{111}	24	−8	0	−24	8	0	0	0	0	0	8	8	8	0	−16	0	0	0	8	−8	0	0	0	0	0
χ_{112}	−40	8	4	−8	8	−4	12	−12	4	−4	24	−8	0	−12	12	0	16	−8	4	−4	0	0	−4	4	0
χ_{113}	40	−8	−4	8	−8	4	12	−12	4	−4	24	−8	0	12	−12	−16	0	8	−4	4	0	0	−4	4	0
χ_{114}	−24	8	0	24	−8	0	0	0	0	0	8	8	8	−16	0	0	0	0	−8	8	0	0	0	0	0
χ_{115}	8	8	−8	8	8	−8	24	24	−8	−8	−8	−8	−8	8	8	8	8	8	−8	−8	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	−8	8	8	−8	−48	16	0	0	0	16	−16	0	0	0	0	0	−8	8	0
χ_{117}	−19	−11	13	−19	−11	13	21	−3	−3	−3	−35	−19	13	5	5	−3	5	5	−3	−3	−3	1	5	−7	1
χ_{118}	21	−3	−3	21	−3	−3	−27	−3	−3	13	−27	−11	5	5	5	5	13	−3	−3	−3	−3	1	5	−7	1
χ_{119}	13	5	−7	37	−3	−7	−15	−15	1	9	13	−3	−11	17	−7	9	1	−7	−3	5	−3	1	1	−3	1
χ_{120}	−27	−3	9	−3	−11	9	33	−15	1	−7	5	−11	−3	17	−7	1	−7	1	−3	5	−3	1	1	−3	1
χ_{121}	21	−3	−3	21	−3	−3	13	5	5	−11	−3	13	−3	−3	−3	5	13	−3	−3	−3	9	−3	−7	−3	5
χ_{122}	0	0	0	0	0	0	0	0	0	0	−16	−16	−16	16	16	0	0	0	0	0	0	0	0	0	0
χ_{123}	−6	10	−6	−6	10	−6	6	6	−10	6	−14	18	−14	18	−14	−14	2	2	2	2	6	−2	2	−6	2
χ_{124}	18	2	−6	18	2	−6	6	6	−10	6	10	−22	10	−14	18	10	−6	−6	2	2	6	−2	−6	2	2
χ_{125}	6	−10	6	6	−10	6	−18	−18	−2	14	30	−2	−2	−2	−2	6	−10	−10	6	6	−6	2	10	2	−6
χ_{126}	−21	3	3	−21	3	3	3	−21	11	−5	3	−13	3	3	3	−5	−13	3	3	3	15	−5	−1	−5	3
χ_{127}	4	4	−4	4	4	−4	−4	28	−4	−4	−12	−12	−12	12	12	4	4	4	−4	−4	−12	4	−4	−4	4
χ_{128}	−8	8	−4	−40	8	4	4	−4	−4	4	24	−8	0	12	−12	0	16	−8	−4	4	0	0	4	−4	0
χ_{129}	0	−16	12	0	16	−12	−12	12	12	−12	−32	0	8	−4	4	8	−8	0	−4	4	0	0	−4	4	0
χ_{130}	8	−8	4	40	−8	−4	4	−4	−4	4	24	−8	0	−12	12	−16	0	8	4	−4	0	0	4	−4	0
χ_{131}	0	16	−12	0	−16	12	−12	12	12	−12	−16	16	−8	4	−4	8	−8	0	4	−4	0	0	−4	4	0
χ_{132}	24	−8	0	24	−8	0	−24	24	−8	8	−8	−8	8	16	−16	−8	−8	8	0	0	0	0	0	0	0
χ_{133}	−24	8	0	−24	8	0	−24	24	−8	8	8	8	−8	−16	16	8	8	−8	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$4E$			$4F$			$4G$				$4H$					$4I$					$4J$		$4K$		
$[g]_{\overline{G}}$	$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	32	0	−8	−32	0	8	0	0	0	0	−16	−16	16	−8	8	0	0	0	−8	8	0	0	0	0	0
χ_{136}	−32	0	8	32	0	−8	0	0	0	0	16	16	−16	8	−8	0	0	0	8	−8	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	−16	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{138}	33	−7	−3	9	1	−3	21	−27	5	−3	9	−7	−15	−3	21	5	−3	−11	9	1	−3	1	1	−3	1
χ_{139}	−39	1	9	−39	1	9	−15	9	−7	9	33	−15	−15	9	9	9	−15	17	−7	−7	−3	1	−3	1	1
χ_{140}	26	10	−14	26	10	−14	−30	18	2	2	−30	2	−14	10	10	−6	10	−6	2	2	6	−2	−2	6	−2
χ_{141}	−2	14	−10	−2	14	−10	−6	−6	10	−6	−10	22	−10	−18	14	−10	6	6	−2	−2	−6	2	6	−2	−2
χ_{142}	−10	6	−2	38	−10	−2	18	18	2	−14	14	−18	14	6	−10	6	−10	−10	−2	14	−6	2	−2	6	−2
χ_{143}	22	6	−10	22	6	−10	−6	−6	10	−6	14	−18	14	14	−18	14	−2	−2	−2	−2	−6	2	−2	6	−2
χ_{144}	14	−2	−2	−34	14	−2	18	18	2	−14	−10	22	−10	−10	6	−18	−2	−2	14	−2	−6	2	6	−2	−2
χ_{145}	−14	2	2	−14	2	2	18	18	2	−14	−38	−6	−6	10	10	−14	2	2	2	2	6	−2	−2	6	−2
χ_{146}	16	16	−16	−16	−16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{147}	−16	−16	16	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{148}	−32	0	8	32	0	−8	0	0	0	0	−16	−16	16	8	−8	0	0	0	8	−8	0	0	0	0	0
χ_{149}	32	0	−8	−32	0	8	0	0	0	0	16	16	−16	−8	8	0	0	0	−8	8	0	0	0	0	0
χ_{150}	−24	24	−12	−24	−8	12	−12	12	−4	4	−24	8	0	−12	12	16	0	−8	4	−4	0	0	4	−4	0
χ_{151}	24	−24	12	24	8	−12	−12	12	−4	4	−24	8	0	12	−12	0	−16	8	−4	4	0	0	4	−4	0
χ_{152}	−8	−8	8	56	−8	−8	8	−8	−8	8	8	8	−8	−8	8	−8	−8	8	8	−8	0	0	0	0	0
χ_{153}	8	8	−8	−56	8	8	8	−8	−8	8	−8	−8	8	8	−8	8	8	−8	−8	8	0	0	0	0	0
χ_{154}	−33	−9	15	−33	−9	15	−9	15	−1	−1	23	7	−9	−1	−1	−1	−9	7	−1	−1	3	−1	−5	7	−1
χ_{155}	15	7	−9	15	7	−9	15	−9	−9	7	−49	−1	15	−1	−1	−17	7	−9	7	7	−9	3	−1	−5	3
χ_{156}	15	−9	3	−9	−1	3	−21	−21	11	3	−9	7	−1	−13	11	3	11	3	−1	−9	3	−1	−1	3	−1
χ_{157}	−9	−1	3	15	−9	3	27	−21	−5	3	−9	7	−1	11	−13	3	11	3	−9	−1	−9	3	−5	−1	3
χ_{158}	−16	16	−8	16	−16	8	0	0	0	0	−16	−16	16	8	−8	0	0	0	8	−8	0	0	0	0	0
χ_{159}	0	0	0	0	0	0	0	0	0	0	−16	−16	−16	16	16	0	0	0	0	0	0	0	0	0	0
χ_{160}	16	−16	8	−16	16	−8	0	0	0	0	16	16	−16	−8	8	0	0	0	−8	8	0	0	0	0	0
χ_{161}	−12	−12	12	−12	−12	12	−12	−12	−12	20	4	4	4	−4	−4	4	4	4	−4	−4	12	−4	−4	−4	4
χ_{162}	0	16	−12	0	−16	12	12	−12	4	−4	−32	0	8	4	−4	8	−8	0	4	−4	0	0	4	−4	0
χ_{163}	0	−16	12	0	16	−12	12	−12	4	−4	−16	16	−8	−4	4	8	−8	0	−4	4	0	0	4	−4	0
χ_{164}	−24	8	0	24	−8	0	0	0	0	0	−8	−8	−8	0	16	0	0	0	−8	8	0	0	0	0	0
χ_{165}	24	−8	0	−24	8	0	0	0	0	0	−8	−8	−8	16	0	0	0	0	8	−8	0	0	0	0	0
χ_{166}	0	0	0	0	0	0	−24	24	−8	8	−48	16	0	0	0	16	−16	0	0	0	0	0	8	−8	0
χ_{167}	−16	16	−8	16	−16	8	0	0	0	0	16	16	−16	8	−8	0	0	0	8	−8	0	0	0	0	0
χ_{168}	16	−16	8	−16	16	−8	0	0	0	0	−16	−16	16	−8	8	0	0	0	−8	8	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$4E$			$4F$			$4G$				$4H$					$4I$					$4J$		$4K$		
$[g]_{\overline{G}}$	$8e$	$8f$	$4x$	$8g$	$8h$	$4y$	$4z$	$4aa$	$4ab$	$4ac$	$8i$	$8j$	$8k$	$4ad$	$4ae$	$8l$	$8m$	$8n$	$4af$	$4ag$	$4ah$	$8o$	$8p$	$4ai$	$8q$
χ_{169}	0	0	0	0	0	0	−16	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{171}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{172}	−30	18	−6	18	2	−6	6	6	−10	6	−22	10	−22	18	2	−14	2	2	−6	10	6	−2	2	−6	2
χ_{173}	6	−10	6	6	−10	6	6	−42	6	6	−18	14	−2	−2	−2	6	22	6	−10	−10	−6	2	−6	2	2
χ_{174}	42	−6	−6	−6	10	−6	6	6	−10	6	2	−30	2	2	18	10	−6	−6	10	−6	6	−2	−6	2	2
χ_{175}	−18	−2	6	−18	−2	6	6	6	−10	6	−26	6	6	−2	−2	−18	−2	−2	6	6	−6	2	−6	2	2
χ_{176}	−16	−16	16	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{177}	16	16	−16	−16	−16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{178}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{179}	−24	−8	12	−24	24	−12	12	−12	−12	12	−24	8	0	12	−12	16	0	−8	−4	4	0	0	−4	4	0
χ_{180}	24	8	−12	24	−24	12	12	−12	−12	12	−24	8	0	−12	12	0	−16	8	4	−4	0	0	−4	4	0
χ_{181}	−12	−12	12	−12	−12	12	−12	−12	20	−12	−12	−12	−12	12	12	4	4	4	−4	−4	−12	4	4	4	−4
χ_{182}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{183}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{184}	56	−8	−8	−8	−8	8	24	−24	8	−8	8	8	−8	8	−8	−8	−8	8	−8	8	0	0	0	0	0
χ_{185}	−24	8	0	24	−8	0	0	0	0	0	8	8	8	−16	0	0	0	0	−8	8	0	0	0	0	0
χ_{186}	24	−8	0	−24	8	0	0	0	0	0	8	8	8	0	−16	0	0	0	8	−8	0	0	0	0	0
χ_{187}	−56	8	8	8	8	−8	24	−24	8	−8	−8	−8	8	−8	8	8	8	−8	8	−8	0	0	0	0	0
χ_{188}	8	8	−8	8	8	−8	−24	−24	8	8	8	8	8	−8	−8	8	8	8	−8	−8	0	0	0	0	0
χ_{189}	−8	−8	8	−8	−8	8	24	24	−8	−8	8	8	8	−8	−8	−8	−8	−8	8	8	0	0	0	0	0
χ_{190}	0	0	0	0	0	0	−8	8	8	−8	48	−16	0	0	0	−16	16	0	0	0	0	0	8	−8	0
χ_{191}	21	−19	9	−3	−11	9	33	33	−15	−7	21	5	−3	−15	9	9	1	−7	5	−3	−3	1	1	−3	1
χ_{192}	−3	−11	9	21	−19	9	−15	33	1	−7	21	5	−3	9	−15	9	1	−7	−3	5	9	−3	5	1	−3
χ_{193}	−3	5	−3	−3	5	−3	−27	−3	13	−3	−27	−11	21	−3	−3	−3	5	5	−3	−3	9	−3	9	−3	−3
χ_{194}	−3	37	−27	−3	37	−27	−3	−27	5	5	45	−3	−3	−3	−3	−3	−27	5	5	5	−3	1	−3	1	1
χ_{195}	0	0	0	0	0	0	−24	24	−8	8	48	−16	0	0	0	−16	16	0	0	0	0	0	−8	8	0

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Table 11.13 (continued)

$[x]_G$		$4L$			$5A$		$5B$	$6A$			$6B$			$6C$				$6D$		$6E$	$6F$		$6G$			
$[g]_{\overline{G}}$		$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$
$\lvert C_{\overline{G}}(g)\rvert$	2	11	11	10	8	8	2	14	14	10	13	13	11	13	13	11	10	9	9	6	5	5	10	10	10	10
	3	0	0	0	2	1	1	3	2	3	3	2	2	2	1	2	2	3	2	3	4	4	2	2	1	1
	5	0	0	0	2	1	2	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		−1	−1	−1	5	5	0	−6	−6	−6	6	6	6	2	2	2	2	0	0	−3	−3	−3	−2	−2	−2	−2
χ_3		3	3	3	1	1	1	−6	−6	−6	−2	−2	−2	2	2	2	2	4	4	−2	−3	−3	−2	−2	−2	−2
χ_4		1	1	1	5	5	0	−10	−10	−10	6	6	6	6	6	6	6	6	6	3	2	2	−2	−2	−2	−2
χ_5		−1	−1	−1	9	9	−1	15	15	15	11	11	11	7	7	7	7	5	5	−1	0	0	3	3	3	3
χ_6		3	3	3	5	5	0	15	15	15	3	3	3	7	7	7	7	9	9	0	0	0	3	3	3	3
χ_7		2	2	2	8	8	3	−11	−11	−11	9	9	9	−3	−3	−3	−3	−6	−6	3	1	1	1	1	1	1
χ_8		2	2	2	5	5	0	9	9	9	13	13	13	1	1	1	1	−5	−5	7	3	3	5	5	5	5
χ_9		−1	−1	−1	10	10	0	11	11	11	11	11	11	−5	−5	−5	−5	−1	−1	2	5	5	−5	−5	−5	−5
χ_{10}		7	7	7	0	0	5	5	5	5	−3	−3	−3	5	5	5	5	0	0	3	−1	−1	−3	−3	−3	−3
χ_{11}		2	2	2	−2	−2	3	−15	−15	−15	−3	−3	−3	−7	−7	−7	−7	0	0	0	0	0	−3	−3	−3	−3
χ_{12}		2	2	2	−5	−5	0	5	5	5	1	1	1	−3	−3	−3	−3	1	1	4	2	2	1	1	1	1
χ_{13}		3	3	3	0	0	0	15	15	15	−1	−1	−1	−1	−1	−1	−1	5	5	−1	6	6	−1	−1	−1	−1
χ_{14}		0	0	0	17	17	−3	−39	−39	−39	21	21	21	1	1	1	1	−9	−9	0	0	0	−3	−3	−3	−3
χ_{15}		−3	−3	−3	5	5	0	36	36	36	−12	−12	−12	4	4	4	4	0	0	6	9	9	4	4	4	4
χ_{16}		5	5	5	5	5	0	−24	−24	−24	−4	−4	−4	0	0	0	0	−10	−10	−10	−3	−3	4	4	4	4
χ_{17}		−1	−1	−1	10	10	0	−24	−24	−24	24	24	24	8	8	8	8	9	9	0	0	0	0	0	0	0
χ_{18}		0	0	0	16	16	1	45	45	45	25	25	25	5	5	5	5	−5	−5	−8	−6	−6	1	1	1	1
χ_{19}		0	0	0	1	1	−4	−39	−39	−39	−19	−19	−19	1	1	1	1	11	11	2	0	0	−3	−3	−3	−3
χ_{20}		3	3	3	0	0	0	−20	−20	−20	12	12	12	12	12	12	12	15	15	−3	1	1	4	4	4	4
χ_{21}		3	3	3	0	0	0	20	20	20	−4	−4	−4	4	4	4	4	5	5	5	−7	−7	4	4	4	4
χ_{22}		1	1	1	8	8	3	21	21	21	−3	−3	−3	−11	−11	−11	−11	−9	−9	0	0	0	−3	−3	−3	−3
χ_{23}		−3	−3	−3	8	8	3	−51	−51	−51	−3	−3	−3	13	13	13	13	9	9	0	0	0	−3	−3	−3	−3
χ_{24}		0	0	0	0	0	0	30	30	30	14	14	14	14	14	14	14	5	5	2	3	3	6	6	6	6
χ_{25}		−2	−2	−2	0	0	5	−15	−15	−15	−7	−7	−7	1	1	1	1	5	5	8	−6	−6	1	1	1	1
χ_{26}		−2	−2	−2	10	10	0	39	39	39	7	7	7	7	7	7	7	16	16	7	0	0	−9	−9	−9	−9
χ_{27}		−2	−2	−2	10	10	0	−21	−21	−21	−1	−1	−1	−13	−13	−13	−13	−4	−4	−1	−6	−6	7	7	7	7
χ_{28}		0	0	0	−7	−7	3	0	0	0	16	16	16	0	0	0	0	−5	−5	−8	−6	−6	−8	−8	−8	−8
χ_{29}		0	0	0	0	0	0	45	45	45	−15	−15	−15	5	5	5	5	15	15	−6	−6	−6	1	1	1	1
χ_{30}		0	0	0	5	5	0	−5	−5	−5	7	7	7	−13	−13	−13	−13	1	1	−2	−2	−2	−1	−1	−1	−1

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Table 11.13 (continued)

$[x]_G$	$4L$			$5A$		$5B$	$6A$			$6B$			$6C$				$6D$		$6E$	$6F$		$6G$			
$[g]_{\overline{G}}$	$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$
χ_{31}	0	0	0	−8	−8	−3	−36	−36	−36	12	12	12	−4	−4	−4	−4	−6	−6	−6	3	3	−4	−4	−4	−4
χ_{32}	2	2	2	0	0	0	−35	−35	−35	−19	−19	−19	13	13	13	13	−10	−10	5	4	4	5	5	5	5
χ_{33}	−6	−6	−6	0	0	0	15	15	15	−17	−17	−17	15	15	15	15	−5	−5	−5	6	6	−1	−1	−1	−1
χ_{34}	−4	−4	−4	−5	−5	0	30	30	30	22	22	22	−2	−2	−2	−2	−11	−11	1	0	0	6	6	6	6
χ_{35}	0	0	0	5	5	0	0	0	0	−16	−16	−16	0	0	0	0	−1	−1	−4	−6	−6	−8	−8	−8	−8
χ_{36}	5	5	5	0	0	0	45	45	45	−15	−15	−15	5	5	5	5	−15	−15	−6	−3	−3	1	1	1	1
χ_{37}	−3	−3	−3	0	0	0	−30	−30	−30	−10	−10	−10	−6	−6	−6	−6	5	5	−4	0	0	6	6	6	6
χ_{38}	0	0	0	5	5	0	−40	−40	−40	8	8	8	−8	−8	−8	−8	−16	−16	8	8	8	0	0	0	0
χ_{39}	−1	−1	−1	9	9	4	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	15	15	15	11	11	11	−9	−9	−9	−9	5	5	−10	0	0	3	3	3	3
χ_{41}	4	4	4	0	0	0	10	10	10	10	10	10	−6	−6	−6	−6	10	10	1	7	7	2	2	2	2
χ_{42}	−4	−4	−4	0	0	0	0	0	0	−8	−8	−8	16	16	16	16	−5	−5	−5	3	3	0	0	0	0
χ_{43}	0	0	0	1	1	−4	24	24	24	8	8	8	−8	−8	−8	−8	−16	−16	8	0	0	0	0	0	0
χ_{44}	0	0	0	0	0	0	−20	−20	−20	−4	−4	−4	−20	−20	−20	−20	−10	−10	2	−2	−2	−4	−4	−4	−4
χ_{45}	0	0	0	0	0	−5	30	30	30	−2	−2	−2	14	14	14	14	−5	−5	−2	3	3	−10	−10	−10	−10
χ_{46}	0	0	0	−10	−10	0	54	54	54	22	22	22	6	6	6	6	1	1	10	−3	−3	−2	−2	−2	−2
χ_{47}	0	0	0	−17	−17	3	−24	−24	−24	24	24	24	8	8	8	8	−9	−9	0	0	0	0	0	0	0
χ_{48}	1	1	1	−5	−5	0	−30	−30	−30	−6	−6	−6	2	2	2	2	−9	−9	0	0	0	−6	−6	−6	−6
χ_{49}	−3	−3	−3	−5	−5	0	39	39	39	−21	−21	−21	−1	−1	−1	−1	0	0	0	0	0	3	3	3	3
χ_{50}	5	5	5	−5	−5	0	−30	−30	−30	−6	−6	−6	2	2	2	2	9	9	0	0	0	−6	−6	−6	−6
χ_{51}	1	1	1	−5	−5	0	51	51	51	3	3	3	−13	−13	−13	−13	0	0	0	0	0	3	3	3	3
χ_{52}	5	5	5	−5	−5	0	−21	−21	−21	3	3	3	11	11	11	11	0	0	0	0	0	3	3	3	3
χ_{53}	−2	−2	−2	0	0	0	−15	−15	−15	5	5	5	−7	−7	−7	−7	5	5	−7	3	3	5	5	5	5
χ_{54}	−2	−2	−2	0	0	0	−45	−45	−45	15	15	15	−5	−5	−5	−5	0	0	6	−9	−9	−1	−1	−1	−1
χ_{55}	0	0	0	5	5	0	15	15	15	3	3	3	7	7	7	7	−9	−9	0	0	0	3	3	3	3
χ_{56}	0	0	0	−5	−5	0	−24	−24	−24	−8	−8	−8	8	8	8	8	−5	−5	4	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	35	35	35	7	7	7	11	11	11	11	−5	−5	−8	2	2	7	7	7	7
χ_{58}	0	0	0	0	0	0	10	10	10	−6	−6	−6	−6	−6	−6	−6	15	15	6	7	7	2	2	2	2
χ_{59}	3	3	3	0	0	0	−45	−45	−45	−21	−21	−21	3	3	3	3	0	0	−3	9	9	3	3	3	3
χ_{60}	−1	−1	−1	0	0	0	−45	−45	−45	−21	−21	−21	3	3	3	3	0	0	−3	−9	−9	3	3	3	3
χ_{61}	0	0	0	5	5	0	6	6	6	−2	−2	−2	6	6	6	6	10	10	4	−6	−6	−2	−2	−2	−2
χ_{62}	1	1	1	−18	−18	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	9	9	4	−40	−40	−40	−24	−24	−24	−8	−8	−8	−8	0	0	0	8	8	0	0	0	0
χ_{44}	−2	−2	−2	−10	−10	0	−39	−39	−39	21	21	21	1	1	1	1	9	9	0	0	0	−3	−3	−3	−3
χ_{65}	−2	−2	−2	−10	−10	0	−51	−51	−51	−3	−3	−3	13	13	13	13	−9	−9	0	0	0	−3	−3	−3	−3

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Table 11.13 (continued)

$[x]_G$	$4L$			$5A$		$5B$	$6A$			$6B$			$6C$				$6D$		$6E$	$6F$		$6G$			
$[g]_{\overline{G}}$	$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$
χ_{66}	−2	−2	−2	−10	−10	0	21	21	21	−3	−3	−3	−11	−11	−11	−11	9	9	0	0	0	−3	−3	−3	−3
χ_{67}	1	1	1	0	0	0	45	45	45	−27	−27	−27	−3	−3	−3	−3	0	0	0	0	0	−3	−3	−3	−3
χ_{68}	5	5	5	0	0	0	45	45	45	−27	−27	−27	−3	−3	−3	−3	0	0	0	0	0	−3	−3	−3	−3
χ_{69}	0	0	0	5	5	0	24	24	24	−24	−24	−24	−8	−8	−8	−8	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	−5	−40	−40	−40	−8	−8	−8	−8	−8	−8	−8	10	10	4	−10	−10	8	8	8	8
χ_{71}	−4	−4	−4	−9	−9	1	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0
χ_{72}	−3	−3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	−3	−3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	10	10	0	16	16	16	−16	−16	−16	16	16	16	16	−16	−16	8	−8	−8	0	0	0	0
χ_{75}	0	0	0	0	0	0	60	60	60	12	12	12	−4	−4	−4	−4	0	0	−6	3	3	−4	−4	−4	−4
χ_{76}	−5	−5	−5	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	−9	0	0	0	0	0	0
χ_{77}	0	0	0	−7	−7	3	30	30	30	6	6	6	−2	−2	−2	−2	0	0	0	0	0	6	6	6	6
χ_{78}	2	2	2	0	0	0	45	45	45	21	21	21	−3	−3	−3	−3	0	0	3	0	0	−3	−3	−3	−3
χ_{79}	0	0	0	10	10	0	−36	−36	−36	12	12	12	−4	−4	−4	−4	6	6	−6	6	6	−4	−4	−4	−4
χ_{80}	0	0	0	16	16	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	6	6	6	0	0	0	−45	−45	−45	27	27	27	3	3	3	3	0	0	0	0	0	3	3	3	3
χ_{82}	3	−1	−1	15	−1	0	31	−1	−1	15	−1	−1	15	−1	−1	−1	15	−1	0	1	−1	7	−1	−1	−1
χ_{83}	1	−3	1	30	−2	0	−56	8	−4	36	4	−4	0	0	8	−4	−15	1	0	−2	2	−4	−8	0	4
χ_{84}	5	1	−3	0	0	0	−60	4	0	−24	8	0	−4	−4	4	0	15	−1	0	−3	3	−8	−4	4	0
χ_{85}	3	−1	−1	15	−1	0	−54	10	−6	42	10	−6	26	−6	10	−6	30	−2	0	3	−3	2	−14	2	2
χ_{86}	−1	3	−1	15	−1	0	6	6	−6	18	18	−6	−18	14	6	−6	0	0	0	3	−3	−6	6	−2	2
χ_{87}	1	−3	1	30	−2	0	99	3	−9	63	15	−9	27	11	3	−9	0	0	0	0	0	15	3	−5	−1
χ_{88}	−4	4	0	45	−3	0	−130	−2	10	54	−10	−2	30	−2	−10	2	15	−1	0	−1	1	−10	10	2	−2
χ_{89}	−4	4	0	45	−3	0	110	−18	10	54	−10	−2	14	−18	6	2	15	−1	0	−1	1	−10	10	2	−2
χ_{90}	5	1	−3	0	0	0	95	−1	−5	3	19	−5	23	7	−1	−5	30	−2	0	−1	1	11	7	−1	−5
χ_{91}	−3	1	1	0	0	0	35	3	−5	−21	27	−5	19	3	3	−5	0	0	0	−1	1	3	3	3	−5
χ_{92}	4	−4	0	15	−1	0	−126	2	6	−6	−6	2	34	2	−6	−2	45	−3	0	0	0	−6	6	−2	2
χ_{93}	4	−4	0	15	−1	0	114	−14	6	−6	−6	2	18	−14	10	−2	45	−3	0	0	0	−6	6	−2	2
χ_{94}	0	0	0	15	−1	0	−79	17	−11	45	29	−11	−7	9	17	−11	−30	2	0	2	−2	−11	−7	1	5
χ_{95}	−3	1	1	45	−3	0	225	1	−15	33	−15	1	49	1	−15	1	45	−3	0	0	0	9	−15	1	1
χ_{96}	5	1	−3	45	−3	0	−135	25	−15	33	−15	1	−23	−7	9	1	−45	3	0	0	0	9	9	−7	1
χ_{97}	−2	6	−2	0	0	0	−35	−3	5	−27	−11	5	29	−19	−3	5	−15	1	0	7	−7	5	−11	5	−3
χ_{98}	4	4	−4	−15	1	0	−66	−2	6	6	−26	6	−34	−2	−2	6	−15	1	0	3	−3	−10	−2	−2	6

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Table 11.13 (continued)

$[x]_G$	$4L$			$5A$		$5B$	$6A$			$6B$			$6C$				$6D$		$6E$	$6F$		$6G$			
$[g]_{\overline{G}}$	$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$
χ_{99}	−1	3	−1	0	0	0	60	−4	0	24	−8	0	4	4	−4	0	−15	1	0	6	−6	8	4	−4	0
χ_{100}	−1	−5	3	0	0	0	120	−8	0	−48	16	0	8	8	−8	0	−15	1	0	6	−6	8	16	0	−8
χ_{101}	−1	3	−1	0	0	0	45	13	−15	57	41	−15	5	21	13	−15	15	−1	0	−3	3	1	5	−3	1
χ_{102}	0	0	0	0	0	0	45	13	−15	57	41	−15	5	21	13	−15	−30	2	0	−6	6	1	5	−3	1
χ_{103}	0	0	0	45	−3	0	90	−38	30	66	2	−6	−38	−6	2	6	−30	2	0	3	−3	2	−2	6	−6
χ_{104}	0	0	0	45	−3	0	−150	−22	30	66	2	−6	−22	10	−14	6	−30	2	0	3	−3	2	−2	6	−6
χ_{105}	0	0	0	−30	2	0	−54	10	−6	42	10	−6	26	−6	10	−6	−15	1	0	6	−6	2	−14	2	2
χ_{106}	−1	−5	3	−15	1	0	−81	15	−9	−9	39	−9	15	−1	15	−9	0	0	0	0	0	−9	−9	7	−1
χ_{107}	3	−1	−1	−15	1	0	−81	15	−9	−9	39	−9	15	−1	15	−9	0	0	0	0	0	−9	−9	7	−1
χ_{108}	−1	3	−1	−15	1	0	99	3	−9	63	15	−9	27	11	3	−9	0	0	0	0	0	15	3	−5	−1
χ_{109}	−6	2	2	0	0	0	−15	17	−15	33	49	−15	1	17	17	−15	30	−2	0	3	−3	−7	1	1	1
χ_{110}	−6	2	2	0	0	0	15	−17	15	−33	−49	15	−1	−17	−17	15	15	−1	0	−6	6	7	−1	−1	−1
χ_{111}	0	0	0	15	−1	0	−99	−3	9	−63	−15	9	−27	−11	−3	9	0	0	0	0	0	−15	−3	5	1
χ_{112}	4	−4	0	30	−2	0	216	−40	24	−24	−24	8	−8	−8	24	−8	−45	3	0	0	0	0	0	0	0
χ_{113}	4	−4	0	30	−2	0	−264	−8	24	−24	−24	8	24	24	−8	−8	−45	3	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	25	−7	5	−51	−3	5	−15	1	−7	5	0	0	0	−8	8	−3	9	1	−3
χ_{115}	0	0	0	0	0	0	70	6	−10	6	38	−10	−10	22	6	−10	15	−1	0	10	−10	−2	14	−2	−2
χ_{116}	8	−8	0	0	0	0	20	20	−20	−12	−12	4	52	−12	4	−4	0	0	0	−1	1	−12	12	−4	4
χ_{117}	5	−7	1	0	0	0	90	26	−30	66	−30	2	26	−6	−6	2	45	−3	0	0	0	18	−6	−6	2
χ_{118}	13	1	−7	0	0	0	135	−25	15	−33	15	−1	23	7	−9	−1	0	0	0	0	0	−9	−9	7	−1
χ_{119}	5	1	−3	0	0	0	90	26	−30	66	−30	2	26	−6	−6	2	−45	3	0	0	0	18	−6	−6	2
χ_{120}	−3	−7	5	0	0	0	−225	−1	15	−33	15	−1	−49	−1	15	−1	0	0	0	0	0	−9	15	−1	−1
χ_{121}	1	5	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	9	0	0	0	0
χ_{122}	0	0	0	15	−1	0	66	2	−6	−6	26	−6	34	2	2	−6	−30	2	0	−6	6	10	2	2	−6
χ_{123}	2	−6	2	45	−3	0	135	39	−45	27	−21	3	−33	15	−9	3	0	0	0	0	0	−21	−9	−1	11
χ_{124}	−6	2	2	45	−3	0	−225	63	−45	27	−21	3	−9	−25	15	3	0	0	0	0	0	3	−9	−1	3
χ_{125}	2	−6	2	−30	2	0	81	−15	9	9	−39	9	−15	1	−15	9	0	0	0	0	0	9	9	−7	1
χ_{126}	7	3	−5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	−4	−4	4	0	0	0	−120	8	0	48	−16	0	−8	−8	8	0	15	−1	0	−3	3	−8	−16	0	8
χ_{128}	−4	4	0	0	0	0	240	−16	0	−96	32	0	−16	−16	16	0	15	−1	0	3	−3	8	−8	−8	8
χ_{129}	4	−4	0	0	0	0	−150	−22	30	−30	34	−6	−22	10	−14	6	30	−2	0	−6	6	10	−10	−2	2
χ_{130}	−4	4	0	0	0	0	−240	16	0	−96	32	0	16	16	−16	0	15	−1	0	3	−3	8	−8	−8	8
χ_{131}	4	−4	0	0	0	0	90	−38	30	−30	34	−6	−38	−6	2	6	30	−2	0	−6	6	10	−10	−2	2
χ_{132}	0	0	0	−15	1	0	126	−2	−6	6	6	−2	−34	−2	6	2	0	0	0	0	0	6	−6	2	−2
χ_{133}	0	0	0	−15	1	0	−114	14	−6	6	6	−2	−18	14	−10	2	0	0	0	0	0	6	−6	2	−2

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Table 11.13 (continued)

$[x]_G$	$4L$			$5A$		$5B$	$6A$			$6B$			$6C$				$6D$		$6E$	$6F$		$6G$			
$[g]_{\overline{G}}$	$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$
χ_{134}	0	0	0	30	−2	0	−16	−16	16	−48	−48	16	−16	−16	−16	16	0	0	0	8	−8	0	0	0	0
χ_{135}	0	0	0	0	0	0	70	−58	50	78	14	−10	6	−26	−2	10	15	−1	0	−2	2	−10	10	2	−2
χ_{136}	0	0	0	0	0	0	−170	−42	50	78	14	−10	22	−10	−18	10	15	−1	0	−2	2	−10	10	2	−2
χ_{137}	0	0	0	0	0	0	−20	−20	20	−84	44	−4	44	−20	−4	4	−30	2	0	7	−7	−4	4	−12	12
χ_{138}	−3	−7	5	−45	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{139}	5	9	−7	−45	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{140}	−2	6	−2	0	0	0	−135	25	−15	33	−15	1	−23	−7	9	1	45	−3	0	0	0	9	9	−7	1
χ_{141}	6	−2	−2	0	0	0	165	5	−15	−39	9	1	−3	13	−11	1	45	−3	0	0	0	−15	−3	5	1
χ_{142}	−2	6	−2	0	0	0	165	5	−15	−39	9	1	−3	13	−11	1	−45	3	0	0	0	−15	−3	5	1
χ_{143}	−2	6	−2	0	0	0	−195	29	−15	−39	9	1	21	−27	13	1	45	−3	0	0	0	9	−3	5	−7
χ_{144}	6	−2	−2	0	0	0	−195	29	−15	−39	9	1	21	−27	13	1	−45	3	0	0	0	9	−3	5	−7
χ_{145}	−10	−2	6	0	0	0	225	1	−15	33	−15	1	49	1	−15	1	−45	3	0	0	0	9	−15	1	1
χ_{146}	0	0	0	−45	3	0	−240	16	0	96	−32	0	16	16	−16	0	−15	1	0	−6	6	−8	8	8	−8
χ_{147}	0	0	0	−45	3	0	240	−16	0	96	−32	0	−16	−16	16	0	−15	1	0	−6	6	−8	8	8	−8
χ_{148}	0	0	0	0	0	0	90	−38	30	−30	−30	10	26	−6	18	−10	45	−3	0	0	0	−6	6	−2	2
χ_{149}	0	0	0	0	0	0	−150	−22	30	−30	−30	10	42	10	2	−10	45	−3	0	0	0	−6	6	−2	2
χ_{150}	−4	4	0	15	−1	0	−36	−36	36	−36	−36	12	−36	28	12	−12	0	0	0	0	0	12	−12	4	−4
χ_{151}	−4	4	0	15	−1	0	−36	−36	36	−36	−36	12	−36	28	12	−12	0	0	0	0	0	12	−12	4	−4
χ_{152}	0	0	0	0	0	0	90	−38	30	−30	34	−6	−38	−6	2	6	−15	1	0	−3	3	10	−10	−2	2
χ_{153}	0	0	0	0	0	0	−150	−22	30	−30	34	−6	−22	10	−14	6	−15	1	0	−3	3	10	−10	−2	2
χ_{154}	−5	7	−1	0	0	0	315	27	−45	−45	3	3	27	11	−21	3	0	0	0	0	0	3	3	3	−5
χ_{155}	7	3	−5	0	0	0	315	27	−45	−45	3	3	27	11	−21	3	0	0	0	0	0	3	3	3	−5
χ_{156}	−5	−1	3	0	0	0	−45	51	−45	−45	3	3	−45	3	3	3	0	0	0	0	0	3	27	−5	−5
χ_{157}	−1	3	−1	0	0	0	−45	51	−45	−45	3	3	−45	3	3	3	0	0	0	0	0	3	27	−5	−5
χ_{158}	0	0	0	−30	2	0	−126	2	6	−6	−6	2	34	2	−6	−2	−45	3	0	0	0	−6	6	−2	2
χ_{159}	0	0	0	45	−3	0	−90	−26	30	−66	30	−2	−26	6	6	−2	0	0	0	0	0	−18	6	6	−2
χ_{160}	0	0	0	−30	2	0	114	−14	6	−6	−6	2	18	−14	10	−2	−45	3	0	0	0	−6	6	−2	2
χ_{161}	−4	−4	4	0	0	0	−30	34	−30	−78	18	2	18	−14	2	2	0	0	0	0	0	−6	−6	10	−6
χ_{162}	−4	4	0	0	0	0	−150	−22	30	−30	−30	10	42	10	2	−10	0	0	0	0	0	−6	6	−2	2
χ_{163}	−4	4	0	0	0	0	90	−38	30	−30	−30	10	26	−6	18	−10	0	0	0	0	0	−6	6	−2	2
χ_{164}	0	0	0	−45	3	0	135	39	−45	27	−21	3	−33	15	−9	3	0	0	0	0	0	−21	−9	−1	11
χ_{165}	0	0	0	−45	3	0	−225	63	−45	27	−21	3	−9	−25	15	3	0	0	0	0	0	3	−9	−1	3
χ_{166}	−8	8	0	30	−2	0	36	36	−36	36	36	−12	36	−28	−12	12	0	0	0	0	0	−12	12	−4	4
χ_{167}	0	0	0	0	0	0	150	22	−30	−66	−2	6	22	−10	14	−6	−15	1	0	−6	6	−2	2	−6	6
χ_{168}	0	0	0	0	0	0	−90	38	−30	−66	−2	6	38	6	−2	−6	−15	1	0	−6	6	−2	2	−6	6

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Table 11.13 (continued)

$[x]_G$	$4L$			$5A$		$5B$	$6A$			$6B$			$6C$				$6D$		$6E$	$6F$		$6G$			
$[g]_{\overline{G}}$	$8r$	$4aj$	$8s$	$5a$	$10a$	$5b$	$12a$	$12b$	$6d$	$6e$	$6f$	$12c$	$12d$	$12e$	$6g$	$12f$	$6h$	$6i$	$6j$	$12g$	$6k$	$12h$	$6l$	$12i$	$12j$
χ_{169}	0	0	0	0	0	0	60	60	−60	−36	−36	12	−36	28	12	−12	0	0	0	3	−3	12	−12	4	−4
χ_{170}	0	0	0	−45	3	0	−280	−24	40	24	24	−8	8	8	−24	8	0	0	0	8	−8	0	0	0	0
χ_{171}	0	0	0	−45	3	0	200	−56	40	24	24	−8	−24	−24	8	8	0	0	0	8	−8	0	0	0	0
χ_{172}	2	−6	2	0	0	0	−135	−39	45	−27	21	−3	33	−15	9	−3	0	0	0	0	0	21	9	1	−11
χ_{173}	−6	2	2	0	0	0	45	−51	45	45	−3	−3	45	−3	−3	−3	0	0	0	0	0	−3	−27	5	5
χ_{174}	−6	2	2	0	0	0	225	−63	45	−27	21	−3	9	25	−15	−3	0	0	0	0	0	−3	9	1	−3
χ_{175}	2	10	−6	0	0	0	−315	−27	45	45	−3	−3	−27	−11	21	−3	0	0	0	0	0	−3	−3	−3	5
χ_{176}	0	0	0	15	−1	0	−216	40	−24	24	24	−8	8	8	−24	8	−45	3	0	0	0	0	0	0	0
χ_{177}	0	0	0	15	−1	0	264	8	−24	24	24	−8	−24	−24	8	8	−45	3	0	0	0	0	0	0	0
χ_{178}	0	0	0	0	0	0	40	40	−40	72	−56	8	8	8	8	−8	30	−2	0	10	−10	−8	8	8	−8
χ_{179}	4	−4	0	45	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{180}	4	−4	0	45	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{181}	4	4	−4	45	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{182}	0	0	0	−15	1	0	−264	−8	24	−24	−24	8	24	24	−8	−8	0	0	0	0	0	0	0	0	0
χ_{183}	0	0	0	−15	1	0	216	−40	24	−24	−24	8	−8	−8	24	−8	0	0	0	0	0	0	0	0	0
χ_{184}	0	0	0	0	0	0	150	22	−30	30	30	−10	−42	−10	−2	10	45	−3	0	0	0	6	−6	2	−2
χ_{185}	0	0	0	0	0	0	−195	29	−15	−39	9	1	21	−27	13	1	0	0	0	0	0	9	−3	5	−7
χ_{186}	0	0	0	0	0	0	165	5	−15	−39	9	1	−3	13	−11	1	0	0	0	0	0	−15	−3	5	1
χ_{187}	0	0	0	0	0	0	−90	38	−30	30	30	−10	−26	6	−18	10	45	−3	0	0	0	6	−6	2	−2
χ_{188}	0	0	0	0	0	0	30	−34	30	78	−18	−2	−18	14	−2	−2	−45	3	0	0	0	6	6	−10	6
χ_{189}	0	0	0	0	0	0	30	−34	30	78	−18	−2	−18	14	−2	−2	45	−3	0	0	0	6	6	−10	6
χ_{190}	−8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	9	0	0	0	0
χ_{191}	5	1	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{192}	1	−3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{193}	1	−11	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{194}	−3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{195}	8	−8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$		$6H$			$6I$		$6J$		$6K$	$6L$			$6M$		$6N$			$6O$		$6P$			$7A$		$8A$		
$[g]_{\overline{G}}$		$12k$	$12l$	$6m$	$6n$	$12m$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$
$\lvert C_{\overline{G}}(g)\rvert$	2	8	8	6	7	7	6	6	5	7	7	6	7	7	8	8	6	5	5	7	7	6	3	3	10	10	8
	3	3	2	3	3	2	3	2	2	2	2	2	2	1	2	1	2	2	2	1	1	1	1	0	1	0	1
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		0	0	0	0	0	3	3	1	2	2	2	−2	−2	−2	−2	−2	1	1	0	0	0	0	0	−3	−3	−3
χ_3		0	0	0	4	4	1	1	2	2	2	2	2	2	−2	−2	−2	1	1	0	0	0	2	2	1	1	1
χ_4		−4	−4	−4	0	0	0	0	−1	0	0	0	2	2	−2	−2	−2	−2	−2	2	2	2	1	1	−3	−3	−3
χ_5		3	3	3	−1	−1	2	2	−1	1	1	1	−1	−1	3	3	3	0	0	1	1	1	0	0	5	5	5
χ_6		3	3	3	3	3	0	0	0	1	1	1	3	3	3	3	3	0	0	1	1	1	2	2	1	1	1
χ_7		4	4	4	0	0	3	3	−1	0	0	0	2	2	−2	−2	−2	1	1	−2	−2	−2	0	0	−2	−2	−2
χ_8		−3	−3	−3	1	1	1	1	3	1	1	1	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	2	2	2
χ_9		−1	−1	−1	−1	−1	5	5	−2	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	0	0	1	1	1
χ_{10}		2	2	2	−6	−6	−3	−3	3	2	2	2	0	0	0	0	0	3	3	0	0	0	0	0	−1	−1	−1
χ_{11}		6	6	6	6	6	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	1	1	2	2	2
χ_{12}		−1	−1	−1	7	7	−2	−2	4	3	3	3	1	1	1	1	1	−2	−2	1	1	1	0	0	−2	−2	−2
χ_{13}		−3	−3	−3	5	5	2	2	−1	−1	−1	−1	−1	−1	−1	−1	−1	2	2	−3	−3	−3	1	1	1	1	1
χ_{14}		3	3	3	3	3	0	0	0	1	1	1	−3	−3	−3	−3	−3	0	0	−1	−1	−1	0	0	−4	−4	−4
χ_{15}		0	0	0	0	0	3	3	2	4	4	4	−4	−4	4	4	4	1	1	0	0	0	0	0	−3	−3	−3
χ_{16}		0	0	0	−4	−4	−1	−1	2	0	0	0	2	2	−2	−2	−2	1	1	2	2	2	0	0	−1	−1	−1
χ_{17}		−3	−3	−3	−3	−3	0	0	0	−1	−1	−1	3	3	3	3	3	0	0	1	1	1	−1	−1	−3	−3	−3
χ_{18}		−3	−3	−3	1	1	−2	−2	0	−1	−1	−1	−3	−3	1	1	1	−2	−2	−1	−1	−1	0	0	4	4	4
χ_{19}		3	3	3	−1	−1	2	2	−2	1	1	1	1	1	−3	−3	−3	0	0	−1	−1	−1	0	0	4	4	4
χ_{20}		−5	−5	−5	3	3	−3	−3	1	−3	−3	−3	1	1	1	1	1	1	1	−1	−1	−1	0	0	−3	−3	−3
χ_{21}		−7	−7	−7	5	5	5	5	−3	1	1	1	−3	−3	1	1	1	1	1	1	1	1	0	0	3	3	3
χ_{22}		−3	−3	−3	−3	−3	0	0	0	1	1	1	3	3	3	3	3	0	0	−1	−1	−1	0	0	−1	−1	−1
χ_{23}		−3	−3	−3	−3	−3	0	0	0	1	1	1	−3	−3	−3	−3	−3	0	0	1	1	1	0	0	1	1	1
χ_{24}		3	3	3	−1	−1	−1	−1	−2	−1	−1	−1	1	1	−3	−3	−3	3	3	1	1	1	−2	−2	0	0	0
χ_{25}		3	3	3	−1	−1	2	2	0	−5	−5	−5	−3	−3	1	1	1	−2	−2	1	1	1	0	0	−2	−2	−2
χ_{26}		6	6	6	−2	−2	−2	−2	3	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2
χ_{27}		0	0	0	−4	−4	2	2	−1	2	2	2	2	2	−2	−2	−2	−2	−2	0	0	0	0	0	2	2	2
χ_{28}		−3	−3	−3	1	1	−2	−2	0	3	3	3	−3	−3	1	1	1	−2	−2	−1	−1	−1	0	0	0	0	0
χ_{29}		−3	−3	−3	−3	−3	0	0	−2	−1	−1	−1	1	1	1	1	1	−2	−2	−1	−1	−1	0	0	−4	−4	−4
χ_{30}		1	1	1	1	1	4	4	2	−1	−1	−1	−1	−1	−1	−1	−1	2	2	1	1	1	0	0	−4	−4	−4

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Table 11.13 (continued)

$[x]_G$	$6H$			$6I$		$6J$		$6K$	$6L$			$6M$		$6N$			$6O$		$6P$			$7A$		$8A$		
$[g]_{\overline{G}}$	$12k$	$12l$	$6m$	$6n$	$12m$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$
χ_{31}	6	6	6	−6	−6	3	3	−2	2	2	2	−2	−2	2	2	2	−1	−1	−2	−2	−2	0	0	0	0	0
χ_{32}	−2	−2	−2	2	2	2	2	1	−2	−2	−2	−2	−2	2	2	2	−4	−4	2	2	2	0	0	2	2	2
χ_{33}	−3	−3	−3	1	1	4	4	3	−3	−3	−3	3	3	−1	−1	−1	2	2	−1	−1	−1	0	0	2	2	2
χ_{34}	−3	−3	−3	1	1	4	4	1	1	1	1	1	1	−3	−3	−3	0	0	1	1	1	0	0	0	0	0
χ_{35}	−3	−3	−3	5	5	2	2	4	3	3	3	1	1	1	1	1	−2	−2	−1	−1	−1	−2	−2	0	0	0
χ_{36}	3	3	3	3	3	3	3	−2	−1	−1	−1	1	1	1	1	1	1	1	1	1	1	0	0	−1	−1	−1
χ_{37}	3	3	3	−1	−1	−4	−4	4	3	3	3	1	1	−3	−3	−3	0	0	1	1	1	0	0	3	3	3
χ_{38}	2	2	2	2	2	−4	−4	0	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3
χ_{40}	3	3	3	−1	−1	2	2	2	−3	−3	−3	−1	−1	3	3	3	0	0	1	1	1	1	1	4	4	4
χ_{41}	4	4	4	−8	−8	1	1	1	0	0	0	−2	−2	2	2	2	−1	−1	−2	−2	−2	0	0	0	0	0
χ_{42}	−3	−3	−3	1	1	−5	−5	−1	1	1	1	−1	−1	3	3	3	3	3	−1	−1	−1	0	0	0	0	0
χ_{43}	−6	−6	−6	2	2	−4	−4	0	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
χ_{44}	−2	−2	−2	2	2	−4	−4	−2	−2	−2	−2	−2	−2	2	2	2	2	2	2	2	2	−1	−1	0	0	0
χ_{45}	3	3	3	−5	−5	1	1	2	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	3	3	3	0	0	0	0	0
χ_{46}	3	3	3	−5	−5	1	1	−2	3	3	3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
χ_{47}	−3	−3	−3	−3	−3	0	0	0	−1	−1	−1	−3	−3	−3	−3	−3	0	0	−1	−1	−1	2	2	0	0	0
χ_{48}	3	3	3	3	3	0	0	0	−1	−1	−1	3	3	3	3	3	0	0	−1	−1	−1	0	0	3	3	3
χ_{49}	6	6	6	6	6	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	−5	−5	−5
χ_{50}	3	3	3	3	3	0	0	0	−1	−1	−1	−3	−3	−3	−3	−3	0	0	1	1	1	0	0	−3	−3	−3
χ_{51}	−6	−6	−6	−6	−6	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1
χ_{52}	−6	−6	−6	−6	−6	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
χ_{53}	3	3	3	−1	−1	−1	−1	−3	5	5	5	3	3	−1	−1	−1	−1	−1	1	1	1	0	0	−2	−2	−2
χ_{54}	0	0	0	0	0	3	3	2	−2	−2	−2	2	2	2	2	2	−1	−1	0	0	0	0	0	−2	−2	−2
χ_{55}	3	3	3	3	3	0	0	0	1	1	1	−3	−3	−3	−3	−3	0	0	−1	−1	−1	−1	−1	4	4	4
χ_{56}	−3	−3	−3	1	1	4	4	4	−1	−1	−1	1	1	−3	−3	−3	0	0	−1	−1	−1	0	0	0	0	0
χ_{57}	5	5	5	1	1	−2	−2	0	−1	−1	−1	−3	−3	1	1	1	−2	−2	−1	−1	−1	0	0	−4	−4	−4
χ_{58}	−5	−5	−5	3	3	−3	−3	2	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0
χ_{59}	0	0	0	0	0	3	3	−3	0	0	0	0	0	0	0	0	−3	−3	0	0	0	0	0	−1	−1	−1
χ_{60}	0	0	0	0	0	3	3	−3	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0	−1	−1	−1
χ_{61}	0	0	0	4	4	−2	−2	−4	0	0	0	2	2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3
χ_{63}	2	2	2	−6	−6	0	0	0	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	3	3	3	3	3	0	0	0	1	1	1	3	3	3	3	3	0	0	1	1	1	0	0	2	2	2
χ_{65}	−3	−3	−3	−3	−3	0	0	0	1	1	1	3	3	3	3	3	0	0	−1	−1	−1	0	0	−2	−2	−2

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Table 11.13 (continued)

$[x]_G$	$6H$			$6I$		$6J$		$6K$	$6L$			$6M$		$6N$			$6O$		$6P$			$7A$		$8A$		
$[g]_{\overline{G}}$	$12k$	$12l$	$6m$	$6n$	$12m$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$
χ_{66}	−3	−3	−3	−3	−3	0	0	0	1	1	1	−3	−3	−3	−3	−3	0	0	1	1	1	0	0	2	2	2
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	1	1	1
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	1	1	1
χ_{69}	−6	−6	−6	−6	−6	0	0	0	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
χ_{70}	2	2	2	−2	−2	−2	−2	4	−2	−2	−2	−2	−2	2	2	2	2	2	−2	−2	−2	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3
χ_{74}	4	4	4	−4	−4	−4	−4	0	4	4	4	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
χ_{75}	0	0	0	0	0	−3	−3	−2	−4	−4	−4	4	4	−4	−4	−4	−1	−1	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3
χ_{77}	6	6	6	6	6	0	0	0	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	6	6	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2
χ_{79}	−6	−6	−6	6	6	0	0	−2	2	2	2	−2	−2	2	2	2	2	2	2	2	2	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	2	2	2
χ_{82}	7	−1	−1	3	−1	3	−1	0	3	−1	−1	3	−1	7	−1	−1	1	−1	3	−1	−1	3	−1	7	−1	−1
χ_{83}	7	−1	−1	3	−1	6	−2	0	3	−1	−1	−3	1	−7	1	1	2	−2	−3	1	1	0	0	−9	−1	3
χ_{84}	9	1	−3	9	−3	3	−1	0	5	1	−3	3	−1	−5	3	−1	1	−1	−1	3	−1	3	−1	7	−1	−1
χ_{85}	−12	4	0	0	0	−3	1	0	−4	4	0	6	−2	2	2	−2	−1	1	2	2	−2	0	0	−9	−1	3
χ_{86}	0	0	0	0	0	9	−3	0	0	0	0	0	0	0	0	0	3	−3	0	0	0	0	0	−3	5	−3
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	11	3	−5
χ_{88}	−7	1	1	−3	1	3	−1	0	3	−1	−1	−3	1	−7	1	1	−1	1	3	−1	−1	0	0	−12	4	0
χ_{89}	5	−3	1	−3	1	3	−1	0	−1	3	−1	−3	1	5	−3	1	−1	1	−1	3	−1	0	0	12	−4	0
χ_{90}	2	2	−2	6	−2	−3	1	0	2	2	−2	6	−2	2	2	−2	−1	1	2	2	−2	0	0	−5	3	−1
χ_{91}	−10	6	−2	6	−2	9	−3	0	−2	6	−2	0	0	0	0	0	3	−3	0	0	0	0	0	5	−3	1
χ_{92}	−9	−1	3	3	−1	0	0	0	1	−3	1	3	−1	−9	−1	3	0	0	1	−3	1	3	−1	0	0	0
χ_{93}	3	−5	3	3	−1	0	0	0	−3	1	1	3	−1	3	−5	3	0	0	−3	1	1	3	−1	0	0	0
χ_{94}	2	2	−2	6	−2	−6	2	0	2	2	−2	−6	2	−2	−2	2	−2	2	−2	−2	2	0	0	−4	−4	4
χ_{95}	9	1	−3	−3	1	0	0	0	1	−3	1	−3	1	9	1	−3	0	0	1	−3	1	0	0	5	−3	1
χ_{96}	9	1	−3	−3	1	0	0	0	1	−3	1	3	−1	−9	−1	3	0	0	−1	3	−1	0	0	−5	3	−1
χ_{97}	−5	3	−1	3	−1	−3	1	0	−1	3	−1	−3	1	5	−3	1	−1	1	1	−3	1	0	0	2	2	−2
χ_{98}	9	1	−3	9	−3	−3	1	0	5	1	−3	−3	1	5	−3	1	−1	1	1	−3	1	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$6H$			$6I$		$6J$		$6K$	$6L$			$6M$		$6N$			$6O$		$6P$			$7A$		$8A$		
$[g]_{\overline{G}}$	$12k$	$12l$	$6m$	$6n$	$12m$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$
χ_{99}	−3	5	−3	9	−3	6	−2	0	1	5	−3	−3	1	−7	1	1	2	−2	−3	1	1	0	0	5	−3	1
χ_{100}	9	1	−3	9	−3	6	−2	0	5	1	−3	−3	1	5	−3	1	2	−2	1	−3	1	0	0	−9	−1	3
χ_{101}	3	−5	3	−9	3	3	−1	0	−1	−5	3	3	−1	7	−1	−1	1	−1	3	−1	−1	0	0	1	−7	5
χ_{102}	−12	4	0	0	0	−6	2	0	−4	4	0	−6	2	−2	−2	2	−2	2	−2	−2	2	3	−1	4	4	−4
χ_{103}	−12	4	0	0	0	3	−1	0	4	−4	0	6	−2	2	2	−2	−1	1	−2	−2	2	0	0	0	0	0
χ_{104}	12	−4	0	0	0	3	−1	0	−4	4	0	6	−2	2	2	−2	−1	1	−2	−2	2	0	0	0	0	0
χ_{105}	3	−5	3	−9	3	6	−2	0	−1	−5	3	−3	1	−7	1	1	2	−2	−3	1	1	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	−1	−1
χ_{107}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−5	3	−1
χ_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−7	1	1
χ_{109}	12	−4	0	0	0	−3	1	0	4	−4	0	6	−2	2	2	−2	−1	1	2	2	−2	0	0	2	2	−2
χ_{110}	9	1	−3	9	−3	−6	2	0	5	1	−3	3	−1	−5	3	−1	−2	2	−1	3	−1	0	0	−2	−2	2
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	4	4	−4
χ_{112}	−9	−1	3	3	−1	0	0	0	1	−3	1	−3	1	9	1	−3	0	0	−1	3	−1	0	0	0	0	0
χ_{113}	3	−5	3	3	−1	0	0	0	−3	1	1	−3	1	−3	5	−3	0	0	3	−1	−1	0	0	0	0	0
χ_{114}	−14	2	2	−6	2	0	0	0	−6	2	2	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	4
χ_{115}	−5	3	−1	3	−1	−6	2	0	−1	3	−1	3	−1	−5	3	−1	−2	2	−1	3	−1	0	0	0	0	0
χ_{116}	−4	−4	4	−12	4	−9	3	0	4	4	−4	0	0	0	0	0	3	−3	0	0	0	0	0	0	0	0
χ_{117}	−9	−1	3	3	−1	0	0	0	−1	3	−1	−3	1	9	1	−3	0	0	1	−3	1	0	0	−3	5	−3
χ_{118}	18	2	−6	−6	2	0	0	0	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	−7	1	1
χ_{119}	−9	−1	3	3	−1	0	0	0	−1	3	−1	3	−1	−9	−1	3	0	0	−1	3	−1	0	0	3	−5	3
χ_{120}	18	2	−6	−6	2	0	0	0	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	7	−1	−1
χ_{121}	0	0	0	0	0	9	−3	0	0	0	0	0	0	0	0	0	3	−3	0	0	0	0	0	−3	5	−3
χ_{122}	12	−4	0	0	0	−6	2	0	4	−4	0	−6	2	−2	−2	2	−2	2	−2	−2	2	0	0	0	0	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	2
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2
χ_{125}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	3	−5	3
χ_{127}	−3	5	−3	9	−3	3	−1	0	1	5	−3	3	−1	7	−1	−1	1	−1	3	−1	−1	0	0	0	0	0
χ_{128}	−9	−1	3	−9	3	3	−1	0	5	1	−3	−3	1	5	−3	1	−1	1	−1	3	−1	0	0	−12	4	0
χ_{129}	12	−4	0	0	0	6	−2	0	−4	4	0	−6	2	−2	−2	2	−2	2	2	2	−2	0	0	−12	4	0
χ_{130}	3	−5	3	−9	3	3	−1	0	1	5	−3	−3	1	−7	1	1	−1	1	3	−1	−1	0	0	12	−4	0
χ_{131}	−12	4	0	0	0	6	−2	0	4	−4	0	−6	2	−2	−2	2	−2	2	2	2	−2	0	0	12	−4	0
χ_{132}	−18	−2	6	6	−2	0	0	0	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{133}	6	−10	6	6	−2	0	0	0	−6	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$6H$			$6I$		$6J$		$6K$	$6L$			$6M$		$6N$			$6O$		$6P$			$7A$		$8A$		
$[g]_{\overline{G}}$	$12k$	$12l$	$6m$	$6n$	$12m$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$
χ_{134}	−4	−4	4	−12	4	0	0	0	−4	−4	4	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0
χ_{135}	7	−1	−1	3	−1	−6	2	0	−3	1	1	−3	1	−7	1	1	2	−2	3	−1	−1	0	0	0	0	0
χ_{136}	−5	3	−1	3	−1	−6	2	0	1	−3	1	−3	1	5	−3	1	2	−2	−1	3	−1	0	0	0	0	0
χ_{137}	−2	−2	2	−6	2	3	−1	0	2	2	−2	6	−2	2	2	−2	−1	1	−2	−2	2	0	0	0	0	0
χ_{138}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	−1	3
χ_{139}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	1	−3
χ_{140}	9	1	−3	−3	1	0	0	0	1	−3	1	−3	1	9	1	−3	0	0	1	−3	1	0	0	−2	−2	2
χ_{141}	−3	5	−3	−3	1	0	0	0	−3	1	1	−3	1	−3	5	−3	0	0	−3	1	1	0	0	2	2	−2
χ_{142}	−3	5	−3	−3	1	0	0	0	−3	1	1	3	−1	3	−5	3	0	0	3	−1	−1	0	0	2	2	−2
χ_{143}	−3	5	−3	−3	1	0	0	0	−3	1	1	−3	1	−3	5	−3	0	0	−3	1	1	0	0	−2	−2	2
χ_{144}	−3	5	−3	−3	1	0	0	0	−3	1	1	3	−1	3	−5	3	0	0	3	−1	−1	0	0	−2	−2	2
χ_{145}	9	1	−3	−3	1	0	0	0	1	−3	1	3	−1	−9	−1	3	0	0	−1	3	−1	0	0	2	2	−2
χ_{146}	3	−5	3	−9	3	6	−2	0	1	5	−3	3	−1	7	−1	−1	−2	2	−3	1	1	0	0	0	0	0
χ_{147}	−9	−1	3	−9	3	6	−2	0	5	1	−3	3	−1	−5	3	−1	−2	2	1	−3	1	0	0	0	0	0
χ_{148}	9	1	−3	−3	1	0	0	0	−1	3	−1	3	−1	−9	−1	3	0	0	1	−3	1	−3	1	0	0	0
χ_{149}	−3	5	−3	−3	1	0	0	0	3	−1	−1	3	−1	3	−5	3	0	0	−3	1	1	−3	1	0	0	0
χ_{150}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{151}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{152}	3	−5	3	−9	3	−3	1	0	1	5	−3	3	−1	7	−1	−1	1	−1	−3	1	1	0	0	0	0	0
χ_{153}	−9	−1	3	−9	3	−3	1	0	5	1	−3	3	−1	−5	3	−1	1	−1	1	−3	1	0	0	0	0	0
χ_{154}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−5	3	−1
χ_{155}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	−1	−1
χ_{156}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	−3	1
χ_{157}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−7	1	1
χ_{158}	−9	−1	3	3	−1	0	0	0	1	−3	1	−3	1	9	1	−3	0	0	−1	3	−1	0	0	0	0	0
χ_{159}	−18	−2	6	6	−2	0	0	0	−2	6	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{160}	3	−5	3	3	−1	0	0	0	−3	1	1	−3	1	−3	5	−3	0	0	3	−1	−1	0	0	0	0	0
χ_{161}	−6	10	−6	−6	2	0	0	0	−6	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{162}	6	−10	6	6	−2	0	0	0	−6	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{163}	−18	−2	6	6	−2	0	0	0	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{164}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	−4
χ_{165}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	4
χ_{166}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{167}	9	1	−3	9	−3	6	−2	0	−5	−1	3	3	−1	−5	3	−1	−2	2	1	−3	1	0	0	0	0	0
χ_{168}	−3	5	−3	9	−3	6	−2	0	−1	−5	3	3	−1	7	−1	−1	−2	2	−3	1	1	0	0	0	0	0

Continued on next page

Table 11.13 (continued)

$[x]_G$	$6H$			$6I$		$6J$		$6K$	$6L$			$6M$		$6N$			$6O$		$6P$			$7A$		$8A$		
$[g]_{\overline{G}}$	$12k$	$12l$	$6m$	$6n$	$12m$	$6o$	$6p$	$6q$	$12n$	$6r$	$12o$	$6s$	$12p$	$12q$	$12r$	$6t$	$12s$	$6u$	$12t$	$6v$	$12u$	$7a$	$14a$	$8t$	$8u$	$8v$
χ_{169}	0	0	0	0	0	−9	3	0	0	0	0	0	0	0	0	0	3	−3	0	0	0	0	0	0	0	0
χ_{170}	−10	6	−2	6	−2	0	0	0	2	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{171}	14	−2	−2	6	−2	0	0	0	−6	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{172}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2
χ_{173}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	2
χ_{174}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	2
χ_{175}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2
χ_{176}	9	1	−3	−3	1	0	0	0	−1	3	−1	−3	1	9	1	−3	0	0	−1	3	−1	0	0	0	0	0
χ_{177}	−3	5	−3	−3	1	0	0	0	3	−1	−1	−3	1	−3	5	−3	0	0	3	−1	−1	0	0	0	0	0
χ_{178}	−2	−2	2	−6	2	6	−2	0	2	2	−2	−6	2	−2	−2	2	−2	2	2	2	−2	0	0	0	0	0
χ_{179}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	−4	0
χ_{180}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−12	4	0
χ_{181}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{182}	−6	10	−6	−6	2	0	0	0	6	−2	−2	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0
χ_{183}	18	2	−6	−6	2	0	0	0	−2	6	−2	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0
χ_{184}	3	−5	3	3	−1	0	0	0	−3	1	1	3	−1	3	−5	3	0	0	−3	1	1	0	0	0	0	0
χ_{185}	6	−10	6	6	−2	0	0	0	6	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	4	4	−4
χ_{186}	6	−10	6	6	−2	0	0	0	6	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	4
χ_{187}	−9	−1	3	3	−1	0	0	0	1	−3	1	3	−1	−9	−1	3	0	0	1	−3	1	0	0	0	0	0
χ_{188}	−3	5	−3	−3	1	0	0	0	−3	1	1	3	−1	3	−5	3	0	0	3	−1	−1	0	0	0	0	0
χ_{189}	−3	5	−3	−3	1	0	0	0	−3	1	1	−3	1	−3	5	−3	0	0	−3	1	1	0	0	0	0	0
χ_{190}	0	0	0	0	0	−9	3	0	0	0	0	0	0	0	0	0	3	−3	0	0	0	0	0	0	0	0
χ_{191}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−5	3
χ_{192}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	−1	3
χ_{193}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	1	−3
χ_{194}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	5	−3
χ_{195}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0

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Table 11.13 (continued)

$[x]_G$		$8B$			$8C$			$8D$			$8E$		$8F$		$9A$		$9B$	$10A$			$10B$		$10C$			$10D$
$[g]_{\overline{G}}$		$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$	$10e$
$\lvert C_{\overline{G}}(g)\rvert$	2	10	10	8	9	9	8	9	9	8	6	6	6	6	3	3	0	7	7	5	6	6	6	6	5	2
	3	1	0	1	0	0	0	0	0	0	0	0	0	0	3	2	3	1	0	1	1	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		1	1	1	1	1	1	1	1	1	−1	−1	1	1	2	2	−1	−1	−1	−1	3	3	1	1	1	0
χ_3		−3	−3	−3	1	1	1	1	1	1	1	1	−1	−1	0	0	0	−1	−1	−1	−1	−1	1	1	1	1
χ_4		−3	−3	−3	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	−3	−3	−3	1	1	1	1	1	0
χ_5		1	1	1	1	1	1	1	1	1	1	1	−1	−1	2	2	−1	3	3	3	3	3	1	1	1	−1
χ_6		5	5	5	1	1	1	1	1	1	−1	−1	1	1	0	0	0	3	3	3	−1	−1	1	1	1	0
χ_7		2	2	2	−2	−2	−2	2	2	2	0	0	0	0	1	1	1	2	2	2	2	2	0	0	0	−1
χ_8		−2	−2	−2	2	2	2	−2	−2	−2	0	0	0	0	0	0	0	−1	−1	−1	3	3	1	1	1	0
χ_9		1	1	1	−3	−3	−3	1	1	1	−1	−1	1	1	1	1	1	−2	−2	−2	2	2	−2	−2	−2	0
χ_{10}		−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1
χ_{11}		−2	−2	−2	2	2	2	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−1
χ_{12}		2	2	2	−2	−2	−2	2	2	2	0	0	0	0	−1	−1	−1	−3	−3	−3	1	1	−1	−1	−1	0
χ_{13}		1	1	1	1	1	1	−3	−3	−3	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}		4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
χ_{15}		−3	−3	−3	1	1	1	1	1	1	−1	−1	−1	−1	0	0	0	1	1	1	−3	−3	1	1	1	0
χ_{16}		3	3	3	−1	−1	−1	−1	−1	−1	1	1	−1	−1	0	0	0	1	1	1	−3	−3	1	1	1	0
χ_{17}		−3	−3	−3	1	1	1	−3	−3	−3	1	1	−1	−1	0	0	0	−2	−2	−2	2	2	−2	−2	−2	0
χ_{18}		−4	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{19}		−4	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0
χ_{20}		−3	−3	−3	−3	−3	−3	1	1	1	−1	−1	1	1	−1	−1	−1	0	0	0	0	0	0	0	0	0
χ_{21}		3	3	3	−1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	0	0	0	0	0	0	0	0	0
χ_{22}		−1	−1	−1	−1	−1	−1	3	3	3	1	1	−1	−1	0	0	0	−2	−2	−2	−2	−2	0	0	0	−1
χ_{23}		1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	−4	−4	−4	−4	−4	0	0	0	−1
χ_{24}		0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	1	0	0	0	0	0	0	0	0	0
χ_{25}		−2	−2	−2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{26}		2	2	2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	2	2	2	−2	−2	−2	−2	−2	0
χ_{27}		−2	−2	−2	−2	−2	−2	2	2	2	0	0	0	0	0	0	0	2	2	2	−2	−2	−2	−2	−2	0
χ_{28}		0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	1	3	3	3	3	3	1	1	1	−1
χ_{29}		4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}		4	4	4	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−3	−3	−3	1	1	1	1	1	0

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Table 11.13 (continued)

$[x]_G$	$8B$			$8C$			$8D$			$8E$		$8F$		$9A$		$9B$	$10A$			$10B$		$10C$			$10D$
$[g]_{\overline{G}}$	$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$	$10e$
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	−4	−4	−4	0	0	0	1
χ_{32}	2	2	2	−2	−2	−2	−2	−2	−2	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
χ_{33}	2	2	2	2	2	2	2	2	2	0	0	0	0	−2	−2	1	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	−1	−1	−1	−1	−1	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	−1	−1	1	1	1	0
χ_{36}	−5	−5	−5	−1	−1	−1	−1	−1	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	3	3	3	−1	−1	1	1	1	0
χ_{39}	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	−3	−3	−3	−3	−3	1	1	1	0
χ_{40}	−4	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−1	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	1	1	1	0
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	−2	−2	2	2	2	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	−1	−1	−1	−1
χ_{48}	3	3	3	3	3	3	−1	−1	−1	−1	−1	1	1	0	0	0	−3	−3	−3	1	1	−1	−1	−1	0
χ_{49}	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	−1	0	0	0	−1	−1	−1	3	3	−1	−1	−1	0
χ_{50}	−3	−3	−3	1	1	1	1	1	1	−1	−1	−1	−1	0	0	0	3	3	3	−1	−1	−1	−1	−1	0
χ_{51}	−1	−1	−1	3	3	3	−1	−1	−1	−1	−1	1	1	0	0	0	1	1	1	−3	−3	−1	−1	−1	0
χ_{52}	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	0	0	0	−1	−1	−1	3	3	−1	−1	−1	0
χ_{53}	2	2	2	2	2	2	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	2	2	2	2	2	2	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	−4	−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	1	1	1	1	1	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−1	1	1	1	−3	−3	−1	−1	−1	0
χ_{57}	4	4	4	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
χ_{59}	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	−3	−3	1	1	1	0
χ_{62}	3	3	3	−1	−1	−1	3	3	3	1	1	−1	−1	0	0	0	0	0	0	0	0	−2	−2	−2	1
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	3	3	3	3	3	1	1	1	0
χ_{44}	−2	−2	−2	−2	−2	−2	2	2	2	0	0	0	0	0	0	0	−2	−2	−2	2	2	2	2	2	0
χ_{65}	−2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	2	2	2	−2	−2	2	2	2	0

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Table 11.13 (continued)

$[x]_G$	$8B$			$8C$			$8D$			$8E$		$8F$		$9A$		$9B$	$10A$			$10B$		$10C$			$10D$
$[g]_{\overline{G}}$	$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$	$10e$
χ_{66}	2	2	2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	−2	−2	−2	2	2	2	2	2	0
χ_{67}	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	3	3	1	1	1	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	−1
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	−1	−1	−1	1
χ_{72}	−3	−3	−3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	−3	−3	−3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−2	−2	−2	2	2	−2	−2	−2	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−1	0	0	0	0	0	0	0	0	0
χ_{76}	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	1
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	−3	−3	1	1	1	−1
χ_{78}	−2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	−2	−2	−2	−2	−2	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	1	0	0	0	0	0	0	0	0	0
χ_{81}	2	2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	7	−1	−1	3	−1	−1	3	−1	−1	1	−1	1	−1	3	−1	0	7	−1	−1	3	−1	3	−1	−1	0
χ_{83}	7	−1	−1	−1	3	−1	3	−1	−1	−1	1	1	−1	3	−1	0	2	2	−2	6	−2	2	2	−2	0
χ_{84}	−9	−1	3	3	−1	−1	−1	3	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	−9	−1	3	−1	3	−1	−1	3	−1	−1	1	−1	1	0	0	0	−5	3	−1	3	−1	−1	3	−1	0
χ_{86}	5	−3	1	−3	1	1	1	−3	1	−1	1	1	−1	0	0	0	−5	3	−1	3	−1	−1	3	−1	0
χ_{87}	−5	3	−1	3	−1	−1	−1	3	−1	1	−1	−1	1	0	0	0	2	2	−2	6	−2	2	2	−2	0
χ_{88}	0	0	0	4	−4	0	0	0	0	0	0	0	0	3	−1	0	−9	−1	3	3	−1	1	−3	1	0
χ_{89}	0	0	0	−4	4	0	0	0	0	0	0	0	0	3	−1	0	3	−5	3	3	−1	−3	1	1	0
χ_{90}	11	3	−5	−1	3	−1	3	−1	−1	−1	1	1	−1	−3	1	0	0	0	0	0	0	0	0	0	0
χ_{91}	5	−3	1	1	−3	1	1	−3	1	1	−1	1	−1	−3	1	0	0	0	0	0	0	0	0	0	0
χ_{92}	−12	4	0	0	0	0	4	−4	0	0	0	0	0	0	0	0	−7	1	1	−3	1	3	−1	−1	0
χ_{93}	12	−4	0	0	0	0	−4	4	0	0	0	0	0	0	0	0	5	−3	1	−3	1	−1	3	−1	0
χ_{94}	4	4	−4	0	0	0	0	0	0	0	0	0	0	−3	1	0	7	−1	−1	3	−1	3	−1	−1	0
χ_{95}	5	−3	1	1	−3	1	1	−3	1	−1	1	−1	1	0	0	0	9	1	−3	−3	1	1	−3	1	0
χ_{96}	7	−1	−1	−5	−1	3	3	−1	−1	1	−1	−1	1	0	0	0	9	1	−3	−3	1	1	−3	1	0
χ_{97}	2	2	−2	−2	−2	2	−2	−2	2	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−7	1	1	−3	1	−3	1	1	0

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Table 11.13 (continued)

$[x]_G$	$8B$			$8C$			$8D$			$8E$		$8F$		$9A$		$9B$	$10A$			$10B$		$10C$			$10D$
$[g]_{\overline{G}}$	$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$	$10e$
χ_{99}	−3	5	−3	1	−3	1	−3	1	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	−9	−1	3	−1	3	−1	−1	3	−1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	−7	1	1	1	−3	1	−3	1	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	−4	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	−1	3	3	−1	1	−3	1	0
χ_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−5	3	3	−1	−3	1	1	0
χ_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	2	−6	2	−2	−2	2	0
χ_{106}	7	−1	−1	3	−1	−1	3	−1	−1	1	−1	1	−1	0	0	0	−7	1	1	−3	1	−3	1	1	0
χ_{107}	−5	3	−1	−1	3	−1	−1	3	−1	−1	1	−1	1	0	0	0	5	−3	1	−3	1	1	−3	1	0
χ_{108}	1	−7	5	−3	1	1	1	−3	1	−1	1	1	−1	0	0	0	5	−3	1	−3	1	1	−3	1	0
χ_{109}	2	2	−2	−2	−2	2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	−2	−2	2	−2	−2	2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	−4	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	7	−1	−1	3	−1	3	−1	−1	0
χ_{112}	−12	4	0	0	0	0	4	−4	0	0	0	0	0	0	0	0	−2	−2	2	−6	2	2	2	−2	0
χ_{113}	12	−4	0	0	0	0	−4	4	0	0	0	0	0	0	0	0	−2	−2	2	−6	2	2	2	−2	0
χ_{114}	4	4	−4	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0
χ_{117}	−3	5	−3	−3	1	1	−3	1	1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	−7	1	1	−3	1	1	−3	1	1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	−9	−1	3	3	−1	−1	−5	−1	3	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	−5	3	−1	3	−1	−1	−5	−1	3	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	−3	5	−3	−3	1	1	−3	1	1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−5	3	−1	3	−1	−1	3	−1	0
χ_{123}	2	2	−2	−2	−2	2	2	2	−2	0	0	0	0	0	0	0	−3	5	−3	−3	1	−3	1	1	0
χ_{124}	−2	−2	2	2	2	−2	−2	−2	2	0	0	0	0	0	0	0	−3	5	−3	−3	1	−3	1	1	0
χ_{125}	2	2	−2	2	2	−2	2	2	−2	0	0	0	0	0	0	0	−2	−2	2	−6	2	−2	−2	2	0
χ_{126}	3	−5	3	3	−1	−1	3	−1	−1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{128}	0	0	0	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{129}	0	0	0	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	0	0	0	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	0	0	0	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−5	3	−1	3	−1	1	−3	1	0
χ_{133}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	−1	−1	3	−1	−3	1	1	0

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Table 11.13 (continued)

$[x]_G$	$8B$			$8C$			$8D$			$8E$		$8F$		$9A$		$9B$	$10A$			$10B$		$10C$			$10D$
$[g]_{\overline{G}}$	$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$	$10e$
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	2	2	−2	6	−2	2	2	−2	0
χ_{135}	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0
χ_{136}	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0
χ_{138}	3	−5	3	−5	−1	3	3	−1	−1	1	−1	−1	1	0	0	0	−9	−1	3	3	−1	−1	3	−1	0
χ_{139}	9	1	−3	1	−3	1	1	−3	1	−1	1	−1	1	0	0	0	−9	−1	3	3	−1	−1	3	−1	0
χ_{140}	−2	−2	2	2	2	−2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{141}	−2	−2	2	2	2	−2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{142}	−2	−2	2	−2	−2	2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{143}	2	2	−2	−2	−2	2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{144}	2	2	−2	2	2	−2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{145}	2	2	−2	2	2	−2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{146}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	5	−3	−3	1	3	−1	−1	0
χ_{147}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	1	−3	−3	1	−1	3	−1	0
χ_{148}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{149}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{150}	12	−4	0	0	0	0	−4	4	0	0	0	0	0	0	0	0	−7	1	1	−3	1	3	−1	−1	0
χ_{151}	−12	4	0	0	0	0	4	−4	0	0	0	0	0	0	0	0	5	−3	1	−3	1	−1	3	−1	0
χ_{152}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{153}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{154}	−5	3	−1	−1	3	−1	−1	3	−1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{155}	7	−1	−1	3	−1	−1	3	−1	−1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{156}	−7	1	1	1	−3	1	1	5	−3	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{157}	5	−3	1	1	5	−3	1	−3	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{158}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2	6	−2	−2	−2	2	0
χ_{159}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	1	−3	−3	1	1	−3	1	0
χ_{160}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	−2	6	−2	−2	−2	2	0
χ_{161}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{162}	−12	4	0	0	0	0	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{163}	12	−4	0	0	0	0	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{164}	−4	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−5	3	3	−1	3	−1	−1	0
χ_{165}	4	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−5	3	3	−1	3	−1	−1	0
χ_{166}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	2	−6	2	2	2	−2	0
χ_{167}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{168}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$8B$			$8C$			$8D$			$8E$		$8F$		$9A$		$9B$	$10A$			$10B$		$10C$			$10D$
$[g]_{\overline{G}}$	$8w$	$8x$	$8y$	$8z$	$8aa$	$8ab$	$8ac$	$8ad$	$8ae$	$16a$	$8af$	$16b$	$8ag$	$9a$	$18a$	$9b$	$20a$	$20b$	$10b$	$10c$	$20c$	$20d$	$10d$	$20e$	$10e$
χ_{169}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	0	9	1	−3	−3	1	−1	3	−1	0
χ_{171}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	0	−3	5	−3	−3	1	3	−1	−1	0
χ_{172}	−2	−2	2	−2	−2	2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{173}	−2	−2	2	2	2	−2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{174}	2	2	−2	2	2	−2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{175}	2	2	−2	2	2	−2	2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{176}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−7	1	1	−3	1	3	−1	−1	0
χ_{177}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	−3	1	−3	1	−1	3	−1	0
χ_{178}	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0
χ_{179}	0	0	0	−4	4	0	0	0	0	0	0	0	0	0	0	0	−9	−1	3	3	−1	1	−3	1	0
χ_{180}	0	0	0	4	−4	0	0	0	0	0	0	0	0	0	0	0	3	−5	3	3	−1	−3	1	1	0
χ_{181}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	5	−3	−3	1	−3	1	1	0
χ_{182}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	−1	−1	3	−1	−3	1	1	0
χ_{183}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−5	3	−1	3	−1	1	−3	1	0
χ_{184}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{185}	−4	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{186}	4	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{187}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{188}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{189}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{190}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{191}	−9	−1	3	−1	−5	3	−1	3	−1	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{192}	3	−5	3	−1	3	−1	−1	−5	3	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{193}	9	1	−3	1	−3	1	1	−3	1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{194}	−3	5	−3	−3	1	1	−3	1	1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{195}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$		$12A$		$12B$			$12C$			$12D$			$12E$	$12F$		$12G$		$12H$				$12I$	
$[g]_{\overline{G}}$		$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$
$\lvert C_{\overline{G}}(g)\rvert$	2	9	9	9	9	7	9	9	7	9	9	8	4	5	5	5	5	7	7	7	7	5	5
	3	2	1	2	1	2	2	1	2	1	1	1	2	2	2	2	2	1	1	1	1	2	1
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		−2	−2	−4	−4	−4	0	0	0	2	2	2	1	2	2	0	0	0	0	0	0	1	1
χ_3		2	2	0	0	0	−4	−4	−4	−2	−2	−2	2	0	0	2	2	0	0	0	0	−1	−1
χ_4		2	2	−2	−2	−2	−2	−2	−2	2	2	2	−1	−2	−2	−2	−2	2	2	2	2	2	2
χ_5		−1	−1	5	5	5	1	1	1	3	3	3	−1	−1	−1	1	1	1	1	1	1	2	2
χ_6		3	3	1	1	1	5	5	5	−1	−1	−1	0	1	1	−1	−1	1	1	1	1	0	0
χ_7		5	5	−5	−5	−5	−1	−1	−1	1	1	1	−1	−2	−2	2	2	−1	−1	−1	−1	−1	−1
χ_8		5	5	3	3	3	−1	−1	−1	1	1	1	−1	3	3	−1	−1	−1	−1	−1	−1	−1	−1
χ_9		−1	−1	7	7	7	−1	−1	−1	−1	−1	−1	2	1	1	−1	−1	−1	−1	−1	−1	−1	−1
χ_{10}		−7	−7	1	1	1	1	1	1	1	1	1	−1	−2	−2	−2	−2	1	1	1	1	−1	−1
χ_{11}		−3	−3	−1	−1	−1	−5	−5	−5	1	1	1	0	2	2	−2	−2	−1	−1	−1	−1	0	0
χ_{12}		−3	−3	−1	−1	−1	3	3	3	1	1	1	0	−1	−1	3	3	−1	−1	−1	−1	0	0
χ_{13}		3	3	−1	−1	−1	7	7	7	3	3	3	3	−1	−1	1	1	−1	−1	−1	−1	0	0
χ_{14}		−3	−3	−5	−5	−5	−1	−1	−1	1	1	1	0	1	1	−1	−1	−1	−1	−1	−1	0	0
χ_{15}		−4	−4	0	0	0	0	0	0	−4	−4	−4	2	0	0	0	0	0	0	0	0	−1	−1
χ_{16}		4	4	2	2	2	−2	−2	−2	0	0	0	−2	2	2	−2	−2	−2	−2	−2	−2	1	1
χ_{17}		0	0	−4	−4	−4	4	4	4	0	0	0	0	−1	−1	1	1	0	0	0	0	0	0
χ_{18}		−3	−3	3	3	3	−1	−1	−1	1	1	1	0	−3	−3	−1	−1	−1	−1	−1	−1	0	0
χ_{19}		1	1	−1	−1	−1	−5	−5	−5	−3	−3	−3	−2	−1	−1	1	1	−1	−1	−1	−1	−2	−2
χ_{20}		4	4	4	4	4	−4	−4	−4	4	4	4	1	1	1	−1	−1	0	0	0	0	1	1
χ_{21}		4	4	−4	−4	−4	−4	−4	−4	−4	−4	−4	1	−1	−1	−1	−1	0	0	0	0	1	1
χ_{22}		9	9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
χ_{23}		−3	−3	1	1	1	1	1	1	−3	−3	−3	0	1	1	1	1	1	1	1	1	0	0
χ_{24}		2	2	−2	−2	−2	−2	−2	−2	2	2	2	2	1	1	1	1	2	2	2	2	−1	−1
χ_{25}		9	9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	−3	−3	−3	−3	0	0
χ_{26}		−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	2	2	2	2	−1	−1	−1	−1	2	2
χ_{27}		3	3	−3	−3	−3	1	1	1	−1	−1	−1	3	0	0	−2	−2	1	1	1	1	0	0
χ_{28}		0	0	4	4	4	−4	−4	−4	0	0	0	0	1	1	−1	−1	0	0	0	0	0	0
χ_{29}		1	1	−1	−1	−1	3	3	3	−3	−3	−3	−2	−1	−1	−3	−3	−1	−1	−1	−1	−2	−2
χ_{30}		−5	−5	5	5	5	1	1	1	−1	−1	−1	−2	−1	−1	1	1	1	1	1	1	−2	−2

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Table 11.13 (continued)

$[x]_G$	$12A$		$12B$			$12C$			$12D$			$12E$	$12F$		$12G$		$12H$				$12I$	
$[g]_{\overline{G}}$	$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$
χ_{31}	4	4	0	0	0	0	0	0	4	4	4	−2	0	0	0	0	0	0	0	0	1	1
χ_{32}	1	1	−3	−3	−3	−3	−3	−3	1	1	1	1	0	0	0	0	1	1	1	1	−2	−2
χ_{33}	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	2	2
χ_{34}	−6	−6	2	2	2	2	2	2	2	2	2	−3	−1	−1	−1	−1	2	2	2	2	0	0
χ_{35}	0	0	−4	−4	−4	4	4	4	0	0	0	0	−1	−1	1	1	0	0	0	0	0	0
χ_{36}	1	1	−1	−1	−1	3	3	3	−3	−3	−3	−2	−1	−1	3	3	−1	−1	−1	−1	1	1
χ_{37}	6	6	0	0	0	4	4	4	2	2	2	0	−3	−3	1	1	0	0	0	0	0	0
χ_{38}	0	0	4	4	4	−4	−4	−4	0	0	0	0	−2	−2	2	2	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	−3	0	0	0	0	0	0	0	0	0	0
χ_{40}	−5	−5	−3	−3	−3	1	1	1	−1	−1	−1	−2	3	3	1	1	1	1	1	1	−2	−2
χ_{41}	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	1	−2	−2	−2	−2	2	2	2	2	1	1
χ_{42}	8	8	−4	−4	−4	−4	−4	−4	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	−1	−1
χ_{43}	0	0	−4	−4	−4	4	4	4	0	0	0	0	2	2	−2	−2	0	0	0	0	0	0
χ_{44}	−4	−4	0	0	0	0	0	0	−4	−4	−4	2	0	0	0	0	0	0	0	0	2	2
χ_{45}	2	2	2	2	2	2	2	2	2	2	2	2	−1	−1	−1	−1	−2	−2	−2	−2	−1	−1
χ_{46}	2	2	2	2	2	2	2	2	2	2	2	2	−1	−1	−1	−1	−2	−2	−2	−2	−1	−1
χ_{47}	0	0	−4	−4	−4	4	4	4	0	0	0	0	−1	−1	1	1	0	0	0	0	0	0
χ_{48}	6	6	2	2	2	2	2	2	−2	−2	−2	0	−1	−1	−1	−1	2	2	2	2	0	0
χ_{49}	3	3	5	5	5	1	1	1	−1	−1	−1	0	2	2	−2	−2	1	1	1	1	0	0
χ_{50}	6	6	2	2	2	2	2	2	−2	−2	−2	0	−1	−1	−1	−1	2	2	2	2	0	0
χ_{51}	3	3	−1	−1	−1	−1	−1	−1	3	3	3	0	2	2	2	2	−1	−1	−1	−1	0	0
χ_{52}	−9	−9	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	2	2	2	2	−1	−1	−1	−1	0	0
χ_{53}	1	1	−1	−1	−1	−5	−5	−5	−3	−3	−3	1	−1	−1	1	1	−1	−1	−1	−1	1	1
χ_{54}	−1	−1	1	1	1	−3	−3	−3	3	3	3	2	−2	−2	0	0	1	1	1	1	−1	−1
χ_{55}	3	3	1	1	1	5	5	5	−1	−1	−1	0	1	1	−1	−1	1	1	1	1	0	0
χ_{56}	0	0	4	4	4	−4	−4	−4	0	0	0	0	1	1	−1	−1	0	0	0	0	0	0
χ_{57}	3	3	1	1	1	−3	−3	−3	−1	−1	−1	0	1	1	3	3	1	1	1	1	0	0
χ_{58}	−2	−2	2	2	2	2	2	2	−2	−2	−2	−2	−1	−1	−1	−1	−2	−2	−2	−2	1	1
χ_{59}	−5	−5	3	3	3	3	3	3	3	3	3	1	0	0	0	0	−1	−1	−1	−1	1	1
χ_{60}	−5	−5	3	3	3	3	3	3	3	3	3	1	0	0	0	0	−1	−1	−1	−1	1	1
χ_{61}	−6	−6	2	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	−4	−4	−4	4	4	4	0	0	0	0	2	2	−2	−2	0	0	0	0	0	0
χ_{44}	−3	−3	−5	−5	−5	−1	−1	−1	1	1	1	0	1	1	−1	−1	−1	−1	−1	−1	0	0
χ_{65}	−3	−3	1	1	1	1	1	1	−3	−3	−3	0	1	1	1	1	1	1	1	1	0	0

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Table 11.13 (continued)

$[x]_G$	$12A$		$12B$			$12C$			$12D$			$12E$	$12F$		$12G$		$12H$				$12I$	
$[g]_{\overline{G}}$	$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$
χ_{66}	9	9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
χ_{67}	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5	0	0	0	0	0	1	1	1	1	0	0
χ_{68}	−3	−3	−3	−3	−3	−3	−3	−3	5	5	5	0	0	0	0	0	1	1	1	1	0	0
χ_{69}	0	0	4	4	4	−4	−4	−4	0	0	0	0	−2	−2	2	2	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	−3	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	−4	−4	0	0	0	0	0	0	−4	−4	−4	2	0	0	0	0	0	0	0	0	−1	−1
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0
χ_{77}	−6	−6	−2	−2	−2	−2	−2	−2	2	2	2	0	−2	−2	−2	−2	−2	−2	−2	−2	0	0
χ_{78}	5	5	−3	−3	−3	−3	−3	−3	−3	−3	−3	−1	0	0	0	0	1	1	1	1	2	2
χ_{79}	4	4	0	0	0	0	0	0	4	4	4	−2	0	0	0	0	0	0	0	0	−2	−2
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	3	3	3	3	3	3	3	3	−5	−5	−5	0	0	0	0	0	−1	−1	−1	−1	0	0
χ_{82}	3	−1	7	−1	−1	7	−1	−1	3	−1	−1	0	1	−1	1	−1	3	−1	−1	−1	3	−1
χ_{83}	0	0	−14	2	2	−2	−2	2	4	−4	0	0	1	−1	1	−1	−2	2	2	−2	0	0
χ_{84}	0	0	−2	−2	2	−14	2	2	−4	4	0	0	1	−1	1	−1	−2	2	−2	2	−3	1
χ_{85}	6	−2	−2	−2	2	−2	−2	2	6	−2	−2	0	−2	2	−2	2	2	2	−2	−2	3	−1
χ_{86}	−6	2	12	−4	0	0	0	0	−2	−2	2	0	0	0	0	0	0	−4	0	4	−3	1
χ_{87}	3	−1	9	1	−3	−3	5	−3	7	−5	−1	0	0	0	0	0	1	1	−3	1	0	0
χ_{88}	−6	2	−14	2	2	2	2	−2	2	2	−2	0	1	−1	−1	1	2	−2	−2	2	3	−1
χ_{89}	−6	2	14	−2	−2	−2	−2	2	2	2	−2	0	−1	1	1	−1	−2	2	2	−2	3	−1
χ_{90}	3	−1	−3	5	−3	9	1	−3	−1	3	−1	0	0	0	0	0	1	1	1	−3	−3	1
χ_{91}	3	−1	−5	3	−1	−5	3	−1	−5	7	−1	0	−2	2	−2	2	−1	3	−1	−1	3	−1
χ_{92}	6	−2	2	2	−2	−14	2	2	−2	−2	2	0	−1	1	1	−1	2	−2	2	−2	0	0
χ_{93}	6	−2	−2	−2	2	14	−2	−2	−2	−2	2	0	1	−1	−1	1	−2	2	−2	2	0	0
χ_{94}	−3	1	3	−5	3	−9	−1	3	1	−3	1	0	0	0	0	0	−1	−1	−1	3	0	0
χ_{95}	−3	1	5	−3	1	5	−3	1	−3	1	1	0	−1	1	−1	1	1	−3	1	1	0	0
χ_{96}	9	−3	−7	1	1	−7	1	1	1	5	−3	0	−1	1	−1	1	−3	1	1	1	0	0
χ_{97}	9	−3	−7	1	1	−7	1	1	1	5	−3	0	−1	1	−1	1	1	5	−3	−3	−3	1
χ_{98}	−6	2	−2	−2	2	−2	−2	2	2	−6	2	0	1	−1	1	−1	−2	−2	2	2	3	−1

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Table 11.13 (continued)

$[x]_G$	$12A$		$12B$			$12C$			$12D$			$12E$	$12F$		$12G$		$12H$				$12I$	
$[g]_{\overline{G}}$	$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$
χ_{99}	0	0	2	2	−2	14	−2	−2	4	−4	0	0	−1	1	−1	1	2	−2	2	−2	0	0
χ_{100}	0	0	4	4	−4	4	4	−4	−8	8	0	0	1	−1	1	−1	0	0	0	0	0	0
χ_{101}	−3	1	−5	3	−1	7	−1	−1	1	−3	1	0	1	−1	1	−1	−1	−1	3	−1	−3	1
χ_{102}	−3	1	−5	3	−1	7	−1	−1	1	−3	1	0	−2	2	−2	2	−1	−1	3	−1	0	0
χ_{103}	6	−2	14	−2	−2	−2	−2	2	−2	−2	2	0	2	−2	−2	2	−2	2	2	−2	−3	1
χ_{104}	6	−2	−14	2	2	2	2	−2	−2	−2	2	0	−2	2	2	−2	2	−2	−2	2	−3	1
χ_{105}	6	−2	−2	−2	2	−2	−2	2	6	−2	−2	0	1	−1	1	−1	2	2	−2	−2	0	0
χ_{106}	3	−1	3	−5	3	3	−5	3	−5	7	−1	0	0	0	0	0	3	−1	−1	−1	0	0
χ_{107}	3	−1	3	−5	3	3	−5	3	−5	7	−1	0	0	0	0	0	3	−1	−1	−1	0	0
χ_{108}	3	−1	9	1	−3	−3	5	−3	7	−5	−1	0	0	0	0	0	1	1	−3	1	0	0
χ_{109}	−3	1	−7	1	1	−7	1	1	−3	1	1	0	2	−2	2	−2	−3	1	1	1	3	−1
χ_{110}	3	−1	7	−1	−1	7	−1	−1	3	−1	−1	0	1	−1	1	−1	3	−1	−1	−1	0	0
χ_{111}	−3	1	−9	−1	3	3	−5	3	−7	5	1	0	0	0	0	0	−1	−1	3	−1	0	0
χ_{112}	0	0	−4	−4	4	4	4	−4	0	0	0	0	−1	1	1	−1	0	0	0	0	0	0
χ_{113}	0	0	4	4	−4	−4	−4	4	0	0	0	0	1	−1	−1	1	0	0	0	0	0	0
χ_{114}	−3	1	7	−1	−1	−5	3	−1	−7	5	1	0	−2	2	−2	2	−1	−1	−1	3	0	0
χ_{115}	−6	2	2	2	−2	2	2	−2	−6	2	2	0	−1	1	−1	1	−2	−2	2	2	0	0
χ_{116}	−12	4	0	0	0	0	0	0	4	4	−4	0	0	0	0	0	0	0	0	0	−3	1
χ_{117}	6	−2	−2	−2	2	−2	−2	2	−2	6	−2	0	1	−1	1	−1	−2	−2	2	2	0	0
χ_{118}	−9	3	7	−1	−1	7	−1	−1	−1	−5	3	0	−2	2	−2	2	3	−1	−1	−1	0	0
χ_{119}	6	−2	−2	−2	2	−2	−2	2	−2	6	−2	0	1	−1	1	−1	−2	−2	2	2	0	0
χ_{120}	3	−1	−5	3	−1	−5	3	−1	3	−1	−1	0	−2	2	−2	2	−1	3	−1	−1	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1
χ_{122}	6	−2	2	2	−2	2	2	−2	−2	6	−2	0	2	−2	2	−2	2	2	−2	−2	0	0
χ_{123}	3	−1	9	1	−3	−3	5	−3	−1	3	−1	0	0	0	0	0	1	1	−3	1	0	0
χ_{124}	−9	3	−3	5	−3	9	1	−3	−5	−1	3	0	0	0	0	0	1	1	1	−3	0	0
χ_{125}	−3	1	−3	5	−3	−3	5	−3	5	−7	1	0	0	0	0	0	−3	1	1	1	0	0
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	0	0	−4	−4	4	−4	−4	4	8	−8	0	0	−1	1	−1	1	0	0	0	0	−3	1
χ_{128}	0	0	4	4	−4	−4	−4	4	0	0	0	0	1	−1	−1	1	0	0	0	0	−3	1
χ_{129}	6	−2	10	−6	2	2	2	−2	−2	−2	2	0	−2	2	2	−2	−2	2	2	−2	0	0
χ_{130}	0	0	−4	−4	4	4	4	−4	0	0	0	0	−1	1	1	−1	0	0	0	0	−3	1
χ_{131}	6	−2	−10	6	−2	−2	−2	2	−2	−2	2	0	2	−2	−2	2	2	−2	−2	2	0	0
χ_{132}	−6	2	−2	−2	2	14	−2	−2	2	2	−2	0	−2	2	2	−2	−2	2	−2	2	0	0
χ_{133}	−6	2	2	2	−2	−14	2	2	2	2	−2	0	2	−2	−2	2	2	−2	2	−2	0	0

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Table 11.13 (continued)

$[x]_G$	$12A$		$12B$			$12C$			$12D$			$12E$	$12F$		$12G$		$12H$				$12I$	
$[g]_{\overline{G}}$	$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	−6	2	−10	6	−2	−2	−2	2	2	2	−2	0	−1	1	1	−1	2	−2	−2	2	0	0
χ_{136}	−6	2	10	−6	2	2	2	−2	2	2	−2	0	1	−1	−1	1	−2	2	2	−2	0	0
χ_{137}	−12	4	0	0	0	0	0	0	4	4	−4	0	0	0	0	0	0	0	0	0	3	−1
χ_{138}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{139}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{140}	9	−3	−7	1	1	−7	1	1	1	5	−3	0	−1	1	−1	1	−3	1	1	1	0	0
χ_{141}	9	−3	−5	3	−1	7	−1	−1	5	1	−3	0	1	−1	1	−1	−1	−1	3	−1	0	0
χ_{142}	9	−3	−5	3	−1	7	−1	−1	5	1	−3	0	1	−1	1	−1	−1	−1	3	−1	0	0
χ_{143}	−3	1	7	−1	−1	−5	3	−1	1	−3	1	0	1	−1	1	−1	−1	−1	−1	3	0	0
χ_{144}	−3	1	7	−1	−1	−5	3	−1	1	−3	1	0	1	−1	1	−1	−1	−1	−1	3	0	0
χ_{145}	−3	1	5	−3	1	5	−3	1	−3	1	1	0	−1	1	−1	1	1	−3	1	1	0	0
χ_{146}	0	0	−4	−4	4	4	4	−4	0	0	0	0	−1	1	1	−1	0	0	0	0	0	0
χ_{147}	0	0	4	4	−4	−4	−4	4	0	0	0	0	1	−1	−1	1	0	0	0	0	0	0
χ_{148}	6	−2	−2	−2	2	−10	6	−2	−2	−2	2	0	1	−1	−1	1	2	−2	2	−2	0	0
χ_{149}	6	−2	2	2	−2	10	−6	2	−2	−2	2	0	−1	1	1	−1	−2	2	−2	2	0	0
χ_{150}	−12	4	0	0	0	0	0	0	4	4	−4	0	0	0	0	0	0	0	0	0	0	0
χ_{151}	−12	4	0	0	0	0	0	0	4	4	−4	0	0	0	0	0	0	0	0	0	0	0
χ_{152}	6	−2	−10	6	−2	−2	−2	2	−2	−2	2	0	−1	1	1	−1	2	−2	−2	2	3	−1
χ_{153}	6	−2	10	−6	2	2	2	−2	−2	−2	2	0	1	−1	−1	1	−2	2	2	−2	3	−1
χ_{154}	−9	3	−9	−1	3	−9	−1	3	−1	−5	3	0	0	0	0	0	−1	3	−1	−1	0	0
χ_{155}	−9	3	−9	−1	3	−9	−1	3	−1	−5	3	0	0	0	0	0	−1	3	−1	−1	0	0
χ_{156}	3	−1	3	−5	3	3	−5	3	3	−1	−1	0	0	0	0	0	3	−1	−1	−1	0	0
χ_{157}	3	−1	3	−5	3	3	−5	3	3	−1	−1	0	0	0	0	0	3	−1	−1	−1	0	0
χ_{158}	6	−2	2	2	−2	−14	2	2	−2	−2	2	0	−1	1	1	−1	2	−2	2	−2	0	0
χ_{159}	−6	2	2	2	−2	2	2	−2	2	−6	2	0	2	−2	2	−2	2	2	−2	−2	0	0
χ_{160}	6	−2	−2	−2	2	14	−2	−2	−2	−2	2	0	1	−1	−1	1	−2	2	−2	2	0	0
χ_{161}	6	−2	2	2	−2	2	2	−2	6	−2	−2	0	2	−2	2	−2	−2	−2	2	2	0	0
χ_{162}	6	−2	2	2	−2	10	−6	2	−2	−2	2	0	2	−2	−2	2	−2	2	−2	2	0	0
χ_{163}	6	−2	−2	−2	2	−10	6	−2	−2	−2	2	0	−2	2	2	−2	2	−2	2	−2	0	0
χ_{164}	3	−1	9	1	−3	−3	5	−3	−1	3	−1	0	0	0	0	0	1	1	−3	1	0	0
χ_{165}	−9	3	−3	5	−3	9	1	−3	−5	−1	3	0	0	0	0	0	1	1	1	−3	0	0
χ_{166}	12	−4	0	0	0	0	0	0	−4	−4	4	0	0	0	0	0	0	0	0	0	0	0
χ_{167}	−6	2	14	−2	−2	−2	−2	2	2	2	−2	0	−1	1	1	−1	−2	2	2	−2	0	0
χ_{168}	−6	2	−14	2	2	2	2	−2	2	2	−2	0	1	−1	−1	1	2	−2	−2	2	0	0

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Table 11.13 (continued)

$[x]_G$	$12A$		$12B$			$12C$			$12D$			$12E$	$12F$		$12G$		$12H$				$12I$	
$[g]_{\overline{G}}$	$12v$	$12w$	$24a$	$24b$	$12x$	$24c$	$24d$	$12y$	$12z$	$12aa$	$12ab$	$12ac$	$24e$	$12ad$	$24f$	$12ae$	$24g$	$24h$	$12af$	$12ag$	$12ah$	$12ai$
χ_{169}	12	−4	0	0	0	0	0	0	−4	−4	4	0	0	0	0	0	0	0	0	0	3	−1
χ_{170}	0	0	−4	−4	4	4	4	−4	0	0	0	0	2	−2	−2	2	0	0	0	0	0	0
χ_{171}	0	0	4	4	−4	−4	−4	4	0	0	0	0	−2	2	2	−2	0	0	0	0	0	0
χ_{172}	−3	1	−9	−1	3	3	−5	3	1	−3	1	0	0	0	0	0	−1	−1	3	−1	0	0
χ_{173}	−3	1	−3	5	−3	−3	5	−3	−3	1	1	0	0	0	0	0	−3	1	1	1	0	0
χ_{174}	9	−3	3	−5	3	−9	−1	3	5	1	−3	0	0	0	0	0	−1	−1	−1	3	0	0
χ_{175}	9	−3	9	1	−3	9	1	−3	1	5	−3	0	0	0	0	0	1	−3	1	1	0	0
χ_{176}	0	0	4	4	−4	−4	−4	4	0	0	0	0	1	−1	−1	1	0	0	0	0	0	0
χ_{177}	0	0	−4	−4	4	4	4	−4	0	0	0	0	−1	1	1	−1	0	0	0	0	0	0
χ_{178}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{179}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{180}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{181}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{182}	0	0	4	4	−4	−4	−4	4	0	0	0	0	−2	2	2	−2	0	0	0	0	0	0
χ_{183}	0	0	−4	−4	4	4	4	−4	0	0	0	0	2	−2	−2	2	0	0	0	0	0	0
χ_{184}	−6	2	−2	−2	2	−10	6	−2	2	2	−2	0	1	−1	−1	1	2	−2	2	−2	0	0
χ_{185}	−3	1	7	−1	−1	−5	3	−1	1	−3	1	0	−2	2	−2	2	−1	−1	−1	3	0	0
χ_{186}	9	−3	−5	3	−1	7	−1	−1	5	1	−3	0	−2	2	−2	2	−1	−1	3	−1	0	0
χ_{187}	−6	2	2	2	−2	10	−6	2	2	2	−2	0	−1	1	1	−1	−2	2	−2	2	0	0
χ_{188}	−6	2	−2	−2	2	−2	−2	2	−6	2	2	0	1	−1	1	−1	2	2	−2	−2	0	0
χ_{189}	−6	2	−2	−2	2	−2	−2	2	−6	2	2	0	1	−1	1	−1	2	2	−2	−2	0	0
χ_{190}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1
χ_{191}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{192}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{193}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{194}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{195}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$		$12J$		$12K$		$12L$		$12M$	$14A$		$15A$		$15B$	$15C$	$17A$	$17B$	$18A$		$20A$		$20B$		$21A$
$[g]_{\overline{G}}$		$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
$\lvert C_{\overline{G}}(g)\rvert$	2	5	5	5	5	4	4	3	2	2	3	3	1	0	0	0	2	2	4	4	4	4	0
	3	1	1	1	1	1	1	1	0	0	2	1	2	1	0	0	2	2	0	0	0	0	1
	5	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0
	7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		−2	−2	0	0	−1	−1	−1	0	0	−1	−1	2	0	1	1	0	0	−1	−1	1	1	0
χ_3		0	0	−2	−2	−1	−1	0	0	0	1	1	1	1	0	0	0	0	1	1	−1	−1	−1
χ_4		0	0	0	0	0	0	1	−1	−1	2	2	−1	0	0	0	−1	−1	−1	−1	−1	−1	1
χ_5		1	1	−1	−1	0	0	−1	0	0	0	0	0	−1	0	0	0	0	1	1	−1	−1	0
χ_6		−1	−1	1	1	0	0	0	0	0	2	2	−1	0	−1	−1	0	0	−1	−1	1	1	−1
χ_7		0	0	0	0	1	1	1	0	0	−1	−1	−1	0	0	0	1	1	0	0	−2	−2	0
χ_8		1	1	1	1	1	1	1	−1	−1	−1	−1	2	0	0	0	0	0	−1	−1	1	1	−1
χ_9		1	1	−1	−1	−1	−1	0	0	0	1	1	1	0	0	0	−1	−1	0	0	0	0	0
χ_{10}		0	0	0	0	1	1	−1	0	0	0	0	0	−1	0	0	−1	−1	0	0	0	0	0
χ_{11}		0	0	0	0	0	0	0	1	1	−2	−2	−2	0	0	0	0	0	−2	−2	0	0	1
χ_{12}		1	1	1	1	0	0	0	0	0	−2	−2	1	0	0	0	−1	−1	1	1	−1	−1	0
χ_{13}		−1	−1	1	1	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{14}		1	1	−1	−1	0	0	0	0	0	−1	−1	−1	0	−1	−1	0	0	1	1	1	1	0
χ_{15}		0	0	0	0	1	1	0	0	0	−1	−1	2	0	0	0	0	0	−1	−1	−1	−1	0
χ_{16}		0	0	0	0	−1	−1	0	0	0	−1	−1	2	0	0	0	0	0	−1	−1	−1	−1	0
χ_{17}		−1	−1	1	1	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	−1
χ_{18}		−1	−1	1	1	0	0	0	0	0	−2	−2	−2	1	0	0	0	0	0	0	0	0	0
χ_{19}		−1	−1	1	1	0	0	0	0	0	1	1	1	−1	0	0	0	0	1	1	1	1	0
χ_{20}		1	1	−1	−1	1	1	−1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
χ_{21}		−1	−1	−1	−1	1	1	1	0	0	0	0	0	0	0	0	−1	−1	0	0	0	0	0
χ_{22}		−1	−1	−1	−1	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	2	2	0
χ_{23}		1	1	1	1	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0
χ_{24}		−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{25}		−1	−1	−1	−1	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0
χ_{26}		0	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{27}		0	0	2	2	0	0	−1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{28}		−1	−1	1	1	0	0	0	0	0	2	2	2	0	0	0	0	0	1	1	−1	−1	0
χ_{29}		1	1	−1	−1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
χ_{30}		−1	−1	1	1	0	0	0	0	0	2	2	−1	0	0	0	1	1	−1	−1	−1	−1	0

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Table 11.13 (continued)

$[x]_G$	$12J$		$12K$		$12L$		$12M$	$14A$		$15A$		$15B$	$15C$	$17A$	$17B$	$18A$		$20A$		$20B$		$21A$
$[g]_{\overline{G}}$	$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
χ_{31}	0	0	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
χ_{33}	1	1	1	1	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	−1	−1	−1	−1	0	0	1	0	0	−2	−2	1	0	0	0	0	0	1	1	1	1	0
χ_{35}	1	1	−1	−1	0	0	0	0	0	2	2	−1	0	0	0	0	0	−1	−1	1	1	1
χ_{36}	1	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	2	2	−1	0	1	1	−1	−1	−1	−1	1	1	0
χ_{39}	0	0	0	0	0	0	−1	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0
χ_{40}	1	1	−1	−1	0	0	0	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{41}	0	0	0	0	−1	−1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
χ_{42}	1	1	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	1	1	1	1	1	−1	0	0	0	0	1	1	−1	−1	1
χ_{44}	0	0	0	0	0	0	0	−1	−1	0	0	0	0	0	0	1	1	0	0	0	0	−1
χ_{45}	−1	−1	−1	−1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
χ_{46}	1	1	1	1	−1	−1	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0
χ_{47}	1	1	−1	−1	0	0	0	0	0	1	1	1	0	0	0	0	0	−1	−1	1	1	−1
χ_{48}	−1	−1	−1	−1	0	0	0	0	0	−2	−2	1	0	0	0	0	0	1	1	−1	−1	0
χ_{49}	0	0	0	0	0	0	0	0	0	1	1	−2	0	0	0	0	0	1	1	1	1	0
χ_{50}	1	1	1	1	0	0	0	0	0	−2	−2	1	0	0	0	0	0	1	1	1	1	0
χ_{51}	0	0	0	0	0	0	0	0	0	1	1	−2	0	0	0	0	0	1	1	−1	−1	0
χ_{52}	0	0	0	0	0	0	0	0	0	1	1	−2	0	0	0	0	0	1	1	1	1	0
χ_{53}	1	1	−1	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	2	2	0	0	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	1	1	−1	−1	0	0	0	1	1	2	2	−1	0	0	0	0	0	−1	−1	−1	−1	−1
χ_{56}	−1	−1	1	1	0	0	0	0	0	1	1	−2	0	0	0	0	0	1	1	−1	−1	0
χ_{57}	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	−1	−1	0	0	0	0	0
χ_{58}	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
χ_{59}	0	0	0	0	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	−1	−1	2	0	0	0	0	0	−1	−1	−1	−1	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1	−1	1	1	−1	−1	0
χ_{44}	−1	−1	1	1	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0
χ_{65}	−1	−1	−1	−1	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$12J$		$12K$		$12L$		$12M$	$14A$		$15A$		$15B$	$15C$	$17A$	$17B$	$18A$		$20A$		$20B$		$21A$
$[g]_{\overline{G}}$	$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
χ_{66}	1	1	1	1	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	−1
χ_{68}	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	−1
χ_{69}	0	0	0	0	0	0	0	1	1	−1	−1	2	0	−1	−1	0	0	−1	−1	1	1	1
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1	−1	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	−1	0	0	0	0	0	1	0	0	0	0	−1	−1	−1	−1	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	A	$A^\star$	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	$A^\star$	A	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	−1	−1	1	1	1	0	0	0	1	1	0	0	0	0	1
χ_{75}	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	1	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	1	1	1	1	0
χ_{78}	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	−2	−2	−2	−1	1	1	0	0	0	0	0	0	−1
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
χ_{82}	1	−1	1	−1	1	−1	0	1	−1	3	−1	0	0	0	0	1	−1	1	−1	1	−1	0
χ_{83}	−1	1	−1	1	0	0	0	0	0	−3	1	0	0	0	0	1	−1	0	0	0	0	0
χ_{84}	−1	1	−1	1	−1	1	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	1	−1	0	0	0	3	−1	0	0	0	0	0	0	−1	1	−1	1	0
χ_{86}	0	0	0	0	−1	1	0	0	0	3	−1	0	0	0	0	0	0	−1	1	−1	1	0
χ_{87}	0	0	0	0	0	0	0	−1	1	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	−1	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	−1	1	−1	1	1	−1	0
χ_{89}	1	−1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	−1	1	1	−1	−1	1	0
χ_{90}	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0	0
χ_{91}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0	0
χ_{92}	1	−1	−1	1	0	0	0	−1	1	3	−1	0	0	0	0	0	0	1	−1	−1	1	0
χ_{93}	−1	1	1	−1	0	0	0	−1	1	3	−1	0	0	0	0	0	0	−1	1	1	−1	0
χ_{94}	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	−1	1	1	−1	1	−1	0
χ_{95}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	0
χ_{96}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	0
χ_{97}	1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{98}	1	−1	1	−1	1	−1	0	0	0	−3	1	0	0	0	0	0	0	−1	1	−1	1	0

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Table 11.13 (continued)

$[x]_G$	$12J$		$12K$		$12L$		$12M$	$14A$		$15A$		$15B$	$15C$	$17A$	$17B$	$18A$		$20A$		$20B$		$21A$
$[g]_{\overline{G}}$	$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
χ_{99}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	−1	0
χ_{104}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	1	−1	−1	1	0
χ_{105}	−1	1	−1	1	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	−1	1	−1	1	0
χ_{107}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	1	−1	1	−1	0
χ_{108}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	1	−1	1	−1	0
χ_{109}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	−1	1	3	−1	0	0	0	0	0	0	1	−1	1	−1	0
χ_{112}	−1	1	1	−1	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	1	−1	−1	1	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{115}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{116}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0	0
χ_{117}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	−1	1	−1	1	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1	0
χ_{125}	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{126}	0	0	0	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{128}	1	−1	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{129}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	−1	1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	1	−1	−1	1	0
χ_{133}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	−1	1	1	−1	0

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Table 11.13 (continued)

$[x]_G$	$12J$		$12K$		$12L$		$12M$	$14A$		$15A$		$15B$	$15C$	$17A$	$17B$	$18A$		$20A$		$20B$		$21A$
$[g]_{\overline{G}}$	$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
χ_{134}	0	0	0	0	0	0	0	1	−1	−3	1	0	0	0	0	−1	1	0	0	0	0	0
χ_{135}	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{136}	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{137}	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{138}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1	0
χ_{139}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1	0
χ_{140}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{141}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{142}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{143}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{144}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{145}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{146}	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	−1	0
χ_{147}	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−1	1	0
χ_{148}	1	−1	−1	1	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{149}	−1	1	1	−1	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{150}	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	1	−1	−1	1	0
χ_{151}	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	−1	1	1	−1	0
χ_{152}	1	−1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{153}	−1	1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{154}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{155}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{156}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{157}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{158}	−1	1	1	−1	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{159}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	0
χ_{160}	1	−1	−1	1	0	0	0	0	0	3	−1	0	0	0	0	0	0	0	0	0	0	0
χ_{161}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{162}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{163}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{164}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	0
χ_{165}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	0
χ_{166}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{167}	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{168}	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$	$12J$		$12K$		$12L$		$12M$	$14A$		$15A$		$15B$	$15C$	$17A$	$17B$	$18A$		$20A$		$20B$		$21A$
$[g]_{\overline{G}}$	$24i$	$12aj$	$24j$	$12ak$	$12al$	$12am$	$12an$	$28a$	$14b$	$15a$	$30a$	$15b$	$15c$	$17a$	$17b$	$36a$	$18b$	$40a$	$20f$	$40b$	$20g$	$21a$
χ_{169}	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	−1	−1	1	0
χ_{171}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	1	−1	0
χ_{172}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{173}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{174}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{175}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{176}	−1	1	1	−1	0	0	0	0	0	3	−1	0	0	0	0	0	0	1	−1	−1	1	0
χ_{177}	1	−1	−1	1	0	0	0	0	0	3	−1	0	0	0	0	0	0	−1	1	1	−1	0
χ_{178}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0
χ_{179}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	1	−1	0
χ_{180}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	−1	1	0
χ_{181}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1	0
χ_{182}	0	0	0	0	0	0	0	−1	1	−3	1	0	0	0	0	0	0	−1	1	1	−1	0
χ_{183}	0	0	0	0	0	0	0	−1	1	−3	1	0	0	0	0	0	0	1	−1	−1	1	0
χ_{184}	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{185}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{186}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{187}	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{188}	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{189}	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{190}	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{191}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{192}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{193}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{194}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{195}	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.13 (continued)

$[x]_G$		$24A$		$24B$		$30A$		$30B$
$[g]_{\overline{G}}$		$24k$	$24l$	$24m$	$24n$	$60a$	$30b$	$30c$
$\lvert C_{\overline{G}}(g)\rvert$	2	5	5	5	5	2	2	1
	3	1	1	1	1	1	1	1
	5	0	0	0	0	1	1	1
	7	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1
χ_2		0	0	−2	−2	−1	−1	0
χ_3		−2	−2	0	0	−1	−1	−1
χ_4		0	0	0	0	0	0	1
χ_5		−1	−1	1	1	0	0	0
χ_6		1	1	−1	−1	0	0	−1
χ_7		1	1	−1	−1	−1	−1	−1
χ_8		−1	−1	1	1	−1	−1	0
χ_9		1	1	1	1	1	1	−1
χ_{10}		−1	−1	−1	−1	0	0	0
χ_{11}		−1	−1	1	1	0	0	0
χ_{12}		1	1	−1	−1	0	0	1
χ_{13}		1	1	1	1	0	0	0
χ_{14}		−1	−1	1	1	1	1	1
χ_{15}		0	0	0	0	1	1	0
χ_{16}		2	2	0	0	1	1	0
χ_{17}		0	0	0	0	1	1	−1
χ_{18}		1	1	−1	−1	0	0	0
χ_{19}		1	1	−1	−1	1	1	1
χ_{20}		0	0	0	0	0	0	0
χ_{21}		0	0	0	0	0	0	0
χ_{22}		−1	−1	−1	−1	1	1	1
χ_{23}		1	1	1	1	−1	−1	−1
χ_{24}		0	0	0	0	0	0	0
χ_{25}		1	1	1	1	0	0	0
χ_{26}		−1	−1	−1	−1	−1	−1	1
χ_{27}		−1	−1	1	1	−1	−1	1
χ_{28}		0	0	0	0	0	0	0
χ_{29}		−1	−1	1	1	0	0	0
χ_{30}		−1	−1	1	1	0	0	1
χ_{31}		0	0	0	0	−1	−1	−1
χ_{32}		−1	−1	−1	−1	0	0	0
χ_{33}		−1	−1	−1	−1	0	0	0
χ_{34}		0	0	0	0	0	0	−1
χ_{35}		0	0	0	0	0	0	−1
χ_{36}		−1	−1	1	1	0	0	0
χ_{37}		0	0	2	2	0	0	0
χ_{38}		0	0	0	0	0	0	−1
χ_{39}		0	0	0	0	0	0	0
χ_{40}		1	1	−1	−1	0	0	0
χ_{41}		0	0	0	0	0	0	0
χ_{42}		0	0	0	0	0	0	0
χ_{43}		0	0	0	0	−1	−1	−1
χ_{44}		0	0	0	0	0	0	0
χ_{45}		0	0	0	0	0	0	0
χ_{46}		0	0	0	0	−1	−1	1
χ_{47}		0	0	0	0	1	1	1
χ_{48}		0	0	0	0	0	0	1

Continued on next page

Table 11.13 (continued)

$[x]_G$	$24A$		$24B$		$30A$		$30B$
$[g]_{\overline{G}}$	$24k$	$24l$	$24m$	$24n$	$60a$	$30b$	$30c$
χ_{49}	1	1	−1	−1	−1	−1	0
χ_{50}	0	0	0	0	0	0	−1
χ_{51}	−1	−1	−1	−1	1	1	0
χ_{52}	1	1	1	1	−1	−1	0
χ_{53}	1	1	−1	−1	0	0	0
χ_{54}	1	1	−1	−1	0	0	0
χ_{55}	1	1	−1	−1	0	0	1
χ_{56}	0	0	0	0	1	1	0
χ_{57}	−1	−1	1	1	0	0	0
χ_{58}	0	0	0	0	0	0	0
χ_{59}	−1	−1	−1	−1	0	0	0
χ_{60}	−1	−1	−1	−1	0	0	0
χ_{61}	0	0	0	0	1	1	0
χ_{62}	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0
χ_{44}	−1	−1	1	1	1	1	−1
χ_{65}	1	1	1	1	−1	−1	1
χ_{66}	−1	−1	−1	−1	1	1	−1
χ_{67}	1	1	1	1	0	0	0
χ_{68}	1	1	1	1	0	0	0
χ_{69}	0	0	0	0	−1	−1	0
χ_{70}	0	0	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	1	1	−1
χ_{75}	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0
χ_{78}	1	1	1	1	0	0	0
χ_{79}	0	0	0	0	−1	−1	1
χ_{80}	0	0	0	0	0	0	0
χ_{81}	−1	−1	−1	−1	0	0	0
χ_{82}	1	−1	1	−1	1	−1	0
χ_{83}	0	0	−2	2	−1	1	0
χ_{84}	−2	2	0	0	0	0	0
χ_{85}	0	0	0	0	1	−1	0
χ_{86}	0	0	2	−2	1	−1	0
χ_{87}	−1	1	1	−1	−1	1	0
χ_{88}	0	0	0	0	0	0	0
χ_{89}	0	0	0	0	0	0	0
χ_{90}	1	−1	−1	1	0	0	0
χ_{91}	−1	1	−1	1	0	0	0
χ_{92}	0	0	0	0	−1	1	0
χ_{93}	0	0	0	0	−1	1	0
χ_{94}	−1	1	1	−1	1	−1	0
χ_{95}	−1	1	−1	1	0	0	0
χ_{96}	1	−1	1	−1	0	0	0
χ_{97}	−1	1	−1	1	0	0	0
χ_{98}	0	0	0	0	−1	1	0

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Table 11.13 (continued)

$[x]_G$	$24A$		$24B$		$30A$		$30B$
$[g]_{\overline{G}}$	$24k$	$24l$	$24m$	$24n$	$60a$	$30b$	$30c$
χ_{99}	2	−2	0	0	0	0	0
χ_{100}	0	0	0	0	0	0	0
χ_{101}	1	−1	−1	1	0	0	0
χ_{102}	1	−1	−1	1	0	0	0
χ_{103}	0	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	1	−1	0
χ_{106}	1	−1	1	−1	−1	1	0
χ_{107}	1	−1	1	−1	−1	1	0
χ_{108}	−1	1	1	−1	−1	1	0
χ_{109}	−1	1	−1	1	0	0	0
χ_{110}	1	−1	1	−1	0	0	0
χ_{111}	1	−1	−1	1	1	−1	0
χ_{112}	0	0	0	0	1	−1	0
χ_{113}	0	0	0	0	1	−1	0
χ_{114}	−1	1	1	−1	0	0	0
χ_{115}	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	0	0
χ_{118}	−1	1	−1	1	0	0	0
χ_{119}	0	0	0	0	0	0	0
χ_{120}	1	−1	1	−1	0	0	0
χ_{121}	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	1	−1	0
χ_{123}	1	−1	−1	1	0	0	0
χ_{124}	−1	1	1	−1	0	0	0
χ_{125}	−1	1	−1	1	1	−1	0
χ_{126}	0	0	0	0	0	0	0
χ_{127}	0	0	0	0	0	0	0
χ_{128}	0	0	0	0	0	0	0
χ_{129}	0	0	0	0	0	0	0
χ_{130}	0	0	0	0	0	0	0
χ_{131}	0	0	0	0	0	0	0
χ_{132}	0	0	0	0	1	−1	0
χ_{133}	0	0	0	0	1	−1	0
χ_{134}	0	0	0	0	−1	1	0
χ_{135}	0	0	0	0	0	0	0
χ_{136}	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0
χ_{138}	0	0	0	0	0	0	0
χ_{139}	0	0	0	0	0	0	0
χ_{140}	1	−1	1	−1	0	0	0
χ_{141}	−1	1	1	−1	0	0	0
χ_{142}	−1	1	1	−1	0	0	0
χ_{143}	1	−1	−1	1	0	0	0
χ_{144}	1	−1	−1	1	0	0	0
χ_{145}	−1	1	−1	1	0	0	0
χ_{146}	0	0	0	0	0	0	0
χ_{147}	0	0	0	0	0	0	0
χ_{148}	0	0	0	0	0	0	0
χ_{149}	0	0	0	0	0	0	0
χ_{150}	0	0	0	0	−1	1	0

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Table 11.13 (continued)

$[x]_G$	$24A$		$24B$		$30A$		$30B$
$[g]_{\overline{G}}$	$24k$	$24l$	$24m$	$24n$	$60a$	$30b$	$30c$
χ_{151}	0	0	0	0	−1	1	0
χ_{152}	0	0	0	0	0	0	0
χ_{153}	0	0	0	0	0	0	0
χ_{154}	1	−1	1	−1	0	0	0
χ_{155}	1	−1	1	−1	0	0	0
χ_{156}	−1	1	−1	1	0	0	0
χ_{157}	−1	1	−1	1	0	0	0
χ_{158}	0	0	0	0	−1	1	0
χ_{159}	0	0	0	0	0	0	0
χ_{160}	0	0	0	0	−1	1	0
χ_{161}	0	0	0	0	0	0	0
χ_{162}	0	0	0	0	0	0	0
χ_{163}	0	0	0	0	0	0	0
χ_{164}	1	−1	−1	1	0	0	0
χ_{165}	−1	1	1	−1	0	0	0
χ_{166}	0	0	0	0	1	−1	0
χ_{167}	0	0	0	0	0	0	0
χ_{168}	0	0	0	0	0	0	0
χ_{169}	0	0	0	0	0	0	0
χ_{170}	0	0	0	0	0	0	0
χ_{171}	0	0	0	0	0	0	0
χ_{172}	−1	1	1	−1	0	0	0
χ_{173}	1	−1	1	−1	0	0	0
χ_{174}	1	−1	−1	1	0	0	0
χ_{175}	−1	1	−1	1	0	0	0
χ_{176}	0	0	0	0	−1	1	0
χ_{177}	0	0	0	0	−1	1	0
χ_{178}	0	0	0	0	0	0	0
χ_{179}	0	0	0	0	0	0	0
χ_{180}	0	0	0	0	0	0	0
χ_{181}	0	0	0	0	0	0	0
χ_{182}	0	0	0	0	1	−1	0
χ_{183}	0	0	0	0	1	−1	0
χ_{184}	0	0	0	0	0	0	0
χ_{185}	1	−1	−1	1	0	0	0
χ_{186}	−1	1	1	−1	0	0	0
χ_{187}	0	0	0	0	0	0	0
χ_{188}	0	0	0	0	0	0	0
χ_{189}	0	0	0	0	0	0	0
χ_{190}	0	0	0	0	0	0	0
χ_{191}	0	0	0	0	0	0	0
χ_{192}	0	0	0	0	0	0	0
χ_{193}	0	0	0	0	0	0	0
χ_{194}	0	0	0	0	0	0	0
χ_{195}	0	0	0	0	0	0	0

where in Table 11.13,

- $A = E(17)^3+E(17)^5+E(17)^6+E(17)^7+E(17)^{10}+E(17)^{11}+E(17)^{12}+E(17)^{14} = -\frac{1}{2}-\frac{\sqrt{17}}{2} = -1 - b17$,
- $A^\star = 1 - A = 2 + b17$.

Table 11.14: The character table of $G = U_5(2){:}2$

	$1A$	$2A$	$2B$	$2C$	$3A$	$3B$	$3C$	$3D$	$4A$	$4B$	$4C$	$4D$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$
	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}	g_{17}	g_{18}	g_{19}
$\lvert C_G(g_i)\rvert$	27371520	165888	9216	1440	77760	3888	3888	648	2304	768	192	96	30	1728	1296	432	432	288	144
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1
χ_3	10	−6	2	0	−5	4	1	−2	2	−2	−2	0	0	3	3	0	−3	−1	−1
χ_4	10	−6	2	0	−5	4	1	−2	2	−2	−2	0	0	3	3	0	−3	−1	−1
χ_5	22	−10	6	0	4	4	−5	4	6	−2	−2	0	2	−4	−1	−4	−1	0	3
χ_6	44	12	−4	4	14	5	−1	2	4	4	0	0	−1	6	3	−3	3	2	−1
χ_7	44	12	−4	−4	14	5	−1	2	4	4	0	0	−1	6	3	−3	3	2	−1
χ_8	55	23	7	5	10	10	1	1	−1	−1	3	1	0	2	5	2	5	−2	1
χ_9	55	23	7	−5	10	10	1	1	−1	−1	3	−1	0	2	5	2	5	−2	1
χ_{10}	110	14	−2	0	−25	2	2	2	14	6	−2	0	0	−1	−4	2	2	−5	4
χ_{11}	110	−2	−10	0	−25	2	2	2	6	−6	2	0	0	7	−2	−2	−2	−1	2
χ_{12}	110	−2	−10	0	−25	2	2	2	6	−6	2	0	0	7	−2	−2	−2	−1	2
χ_{13}	120	24	8	−10	30	3	12	3	8	0	0	−2	0	6	6	3	0	2	2
χ_{14}	120	24	8	10	30	3	12	3	8	0	0	2	0	6	6	3	0	2	2
χ_{15}	132	36	20	0	−21	6	−3	6	4	−4	4	0	2	3	−9	6	−3	−1	−1
χ_{16}	165	−27	5	−5	30	3	3	−6	5	5	−3	3	0	6	−9	3	3	2	−1
χ_{17}	165	−27	5	5	30	3	3	−6	5	5	−3	−3	0	6	−9	3	3	2	−1
χ_{18}	176	−16	16	−4	−4	−4	14	5	0	0	0	−4	1	−4	2	−4	2	4	−2
χ_{19}	176	−16	16	4	−4	−4	14	5	0	0	0	4	1	−4	2	−4	2	4	−2
χ_{20}	220	−68	12	0	−5	22	4	4	−4	4	−4	0	0	−5	4	−2	−8	3	0
χ_{21}	220	60	12	0	−20	16	−5	−8	12	4	4	0	0	−12	−3	0	3	0	−3
χ_{22}	320	−64	0	0	−40	14	−4	−4	0	0	0	0	0	8	8	2	−4	0	0
χ_{23}	320	−64	0	0	−40	14	−4	−4	0	0	0	0	0	8	8	2	−4	0	0
χ_{24}	440	−72	−8	0	50	14	17	2	8	−8	0	0	0	−6	−9	6	−3	−2	−5
χ_{25}	440	88	8	−10	−10	17	8	−1	−8	0	0	−2	0	−2	−2	1	4	2	2
χ_{26}	440	−8	24	0	−10	−10	−19	8	8	8	0	0	0	−2	1	−2	1	−6	−3
χ_{27}	440	88	8	10	−10	17	8	−1	−8	0	0	2	0	−2	−2	1	4	2	2
χ_{28}	528	−48	−16	0	6	6	−12	6	16	0	0	0	−2	−18	6	6	0	2	2
χ_{29}	660	−12	4	0	75	−6	−15	−6	20	−4	4	0	0	3	−3	−6	−3	7	1
χ_{30}	660	84	20	−10	30	3	−15	3	−4	4	0	2	0	6	3	3	−3	2	−1

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Table 11.14 (continued)

	$1A$	$2A$	$2B$	$2C$	$3A$	$3B$	$3C$	$3D$	$4A$	$4B$	$4C$	$4D$	$5A$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$
	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}	g_{17}	g_{18}	g_{19}
$\lvert C_G(g_i)\rvert$	27371520	165888	9216	1440	77760	3888	3888	648	2304	768	192	96	30	1728	1296	432	432	288	144
χ_{31}	660	84	20	10	30	3	−15	3	−4	4	0	−2	0	6	3	3	−3	2	−1
χ_{32}	880	−80	16	0	−110	−2	16	−2	16	0	0	0	0	10	10	−2	4	−2	−2
χ_{33}	891	27	−21	−9	81	0	0	0	3	3	−1	3	1	9	0	0	0	−3	0
χ_{34}	891	27	−21	9	81	0	0	0	3	3	−1	−3	1	9	0	0	0	−3	0
χ_{35}	990	−162	30	0	45	18	−9	0	−2	−2	−2	0	0	−3	−9	−6	3	−3	3
χ_{36}	990	126	−18	0	45	18	−9	0	−2	−10	−2	0	0	−3	9	−6	3	−3	−3
χ_{37}	1024	0	0	16	64	−8	16	4	0	0	0	0	−1	0	0	0	0	0	0
χ_{38}	1024	0	0	−16	64	−8	16	4	0	0	0	0	−1	0	0	0	0	0	0
χ_{39}	1408	128	0	0	−104	4	4	4	0	0	0	0	−2	8	−16	−4	−4	0	0
χ_{40}	1760	96	32	0	20	−16	14	−10	0	0	0	0	0	−12	6	0	−6	−4	2
χ_{41}	1782	54	−42	0	−81	0	0	0	6	6	−2	0	2	−9	0	0	0	3	0
χ_{42}	1980	−36	12	0	−45	−18	−18	0	−4	−12	−4	0	0	3	0	6	6	3	0
χ_{43}	2430	−162	−18	0	0	0	0	0	−18	6	6	0	0	0	0	0	0	0	0

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Table 11.14 (continued)

	$6G$	$6H$	$6I$	$6J$	$8A$	$8B$	$8C$	$8D$	$9A$	$9B$	$10A$	$11A$	$12A$	$12B$	$12C$	$12D$	$12E$	$12F$	$15A$	$16A$	$16B$	$18A$	$24A$	$24B$
	g_{20}	g_{21}	g_{22}	g_{23}	g_{24}	g_{25}	g_{26}	g_{27}	g_{28}	g_{29}	g_{30}	g_{31}	g_{32}	g_{33}	g_{34}	g_{35}	g_{36}	g_{37}	g_{38}	g_{39}	g_{40}	g_{41}	g_{42}	g_{43}
$\lvert C_G(g_i)\rvert$	108	72	36	36	192	192	32	32	54	27	10	11	144	72	72	24	24	12	15	16	16	18	24	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	1	1	1	1	−1	1	−1	−1	1	−1	−1
χ_3	0	2	0	0	$-2i\sqrt{2}$	$2i\sqrt{2}$	0	0	−2	1	0	−1	−1	2	−1	1	1	0	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	$i\sqrt{2}$	$-i\sqrt{2}$
χ_4	0	2	0	0	$2i\sqrt{2}$	$-2i\sqrt{2}$	0	0	−2	1	0	−1	−1	2	−1	1	1	0	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	$-i\sqrt{2}$	$i\sqrt{2}$
χ_5	2	0	0	0	0	0	0	2	1	1	0	0	0	0	−3	−2	1	0	−1	0	0	−1	0	0
χ_6	0	2	−2	1	4	4	0	0	2	−1	−1	0	−2	1	1	0	1	0	−1	0	0	0	1	1
χ_7	0	2	2	−1	−4	−4	0	0	2	−1	1	0	−2	1	1	0	1	0	−1	0	0	0	−1	−1
χ_8	−1	1	−1	2	−3	−3	1	−1	1	1	0	0	2	2	−1	0	−1	1	0	−1	−1	−1	0	0
χ_9	−1	1	1	−2	3	3	−1	−1	1	1	0	0	2	2	−1	0	−1	−1	0	1	1	−1	0	0
χ_{10}	−4	−2	0	0	0	0	0	2	−1	−1	0	0	−1	2	2	1	0	0	0	0	0	−1	0	0
χ_{11}	−2	2	0	0	$-2i\sqrt{2}$	$2i\sqrt{2}$	0	0	−1	−1	0	0	3	0	0	−1	0	0	0	$i\sqrt{2}$	$-i\sqrt{2}$	1	$i\sqrt{2}$	$-i\sqrt{2}$
χ_{12}	−2	2	0	0	$2i\sqrt{2}$	$-2i\sqrt{2}$	0	0	−1	−1	0	0	3	0	0	−1	0	0	0	$-i\sqrt{2}$	$i\sqrt{2}$	1	$-i\sqrt{2}$	$i\sqrt{2}$
χ_{13}	3	−1	−1	−1	−2	−2	−2	0	0	0	0	−1	2	−1	2	0	0	1	0	0	0	0	1	1
χ_{14}	3	−1	1	1	2	2	2	0	0	0	0	−1	2	−1	2	0	0	−1	0	0	0	0	−1	−1
χ_{15}	0	2	0	0	0	0	0	0	0	0	0	0	−5	−2	1	1	−1	0	−1	0	0	0	0	0
χ_{16}	0	2	−2	1	−1	−1	−1	1	0	0	0	0	2	−1	−1	0	−1	0	0	1	1	0	−1	−1
χ_{17}	0	2	2	−1	1	1	1	1	0	0	0	0	2	−1	−1	0	−1	0	0	−1	−1	0	1	1
χ_{18}	−1	1	−1	2	0	0	0	0	−1	−1	1	0	0	0	0	0	0	−1	1	0	0	−1	0	0
χ_{19}	−1	1	1	−2	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	1	1	0	0	−1	0	0
χ_{20}	−2	0	0	0	0	0	0	0	1	1	0	0	−1	2	2	−1	−2	0	0	0	0	1	0	0
χ_{21}	0	0	0	0	0	0	0	0	−2	1	0	0	0	0	3	−2	1	0	0	0	0	0	0	0
χ_{22}	2	0	0	0	$-4i\sqrt{2}$	$4i\sqrt{2}$	0	0	−1	−1	0	1	0	0	0	0	0	0	0	0	0	−1	$-i\sqrt{2}$	$i\sqrt{2}$
χ_{23}	2	0	0	0	$4i\sqrt{2}$	$-4i\sqrt{2}$	0	0	−1	−1	0	1	0	0	0	0	0	0	0	0	0	−1	$i\sqrt{2}$	$-i\sqrt{2}$
χ_{24}	0	−2	0	0	0	0	0	0	2	−1	0	0	2	2	−1	0	1	0	0	0	0	0	0	0
χ_{25}	1	−1	−1	−1	2	2	2	0	−1	−1	0	0	−2	1	−2	0	0	1	0	0	0	1	−1	−1
χ_{26}	4	0	0	0	0	0	0	0	−1	−1	0	0	2	2	−1	0	−1	0	0	0	0	1	0	0
χ_{27}	1	−1	1	1	−2	−2	−2	0	−1	−1	0	0	−2	1	−2	0	0	−1	0	0	0	1	1	1
χ_{28}	0	2	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	0	0	0	1	0	0	0	0	0
χ_{29}	0	−2	0	0	0	0	0	0	0	0	0	0	−1	2	−1	1	−1	0	0	0	0	0	0	0
χ_{30}	−3	−1	−1	−1	−2	−2	2	0	0	0	0	0	2	−1	−1	0	1	−1	0	0	0	0	1	1

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Table 11.14 (continued)

	$6G$	$6H$	$6I$	$6J$	$8A$	$8B$	$8C$	$8D$	$9A$	$9B$	$10A$	$11A$	$12A$	$12B$	$12C$	$12D$	$12E$	$12F$	$15A$	$16A$	$16B$	$18A$	$24A$	$24B$
	g_{20}	g_{21}	g_{22}	g_{23}	g_{24}	g_{25}	g_{26}	g_{27}	g_{28}	g_{29}	g_{30}	g_{31}	g_{32}	g_{33}	g_{34}	g_{35}	g_{36}	g_{37}	g_{38}	g_{39}	g_{40}	g_{41}	g_{42}	g_{43}
$\lvert C_G(g_i)\rvert$	108	72	36	36	192	192	32	32	54	27	10	11	144	72	72	24	24	12	15	16	16	18	24	24
χ_{31}	−3	−1	1	1	2	2	−2	0	0	0	0	0	2	−1	−1	0	1	1	0	0	0	0	−1	−1
χ_{32}	−2	−2	0	0	0	0	0	0	1	1	0	0	−2	−2	−2	0	0	0	0	0	0	1	0	0
χ_{33}	0	0	0	0	3	3	−1	−1	0	0	1	0	−3	0	0	−1	0	0	1	−1	−1	0	0	0
χ_{34}	0	0	0	0	−3	−3	1	−1	0	0	−1	0	−3	0	0	−1	0	0	1	1	1	0	0	0
χ_{35}	0	0	0	0	0	0	0	−2	0	0	0	0	1	−2	1	1	1	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	2	0	0	0	0	1	−2	1	1	−1	0	0	0	0	0	0	0
χ_{37}	0	0	−2	−2	0	0	0	0	−2	1	1	1	0	0	0	0	0	0	−1	0	0	0	0	0
χ_{38}	0	0	2	2	0	0	0	0	−2	1	−1	1	0	0	0	0	0	0	−1	0	0	0	0	0
χ_{39}	2	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	−1	0	0
χ_{40}	0	2	0	0	0	0	0	0	2	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	−2	0	0	0	0	3	0	0	1	0	0	−1	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	−1	2	2	−1	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	2	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0

Table 11.15: The character table of $H_2 = 2_-^{1+6}{:}((3^{1+2}{:}8){:}2)$

	q_1				q_2				q_3					q_4		q_5			q_6			q_7		
	$1a$	$2a$	$2b$	$4a$	$2c$	$2d$	$4b$	$4c$	$8a$	$8b$	$2e$	$8c$	$4d$	$3a$	$6a$	$3b$	$6b$	$12a$	$4e$	$4f$	$8d$	$4g$	$4h$	$8e$
	q_{11}	q_{12}	q_{13}	q_{14}	q_{21}	q_{22}	q_{23}	q_{24}	q_{31}	q_{32}	q_{33}	q_{34}	q_{35}	q_{41}	q_{42}	q_{51}	q_{52}	q_{53}	q_{61}	q_{62}	q_{63}	q_{71}	q_{72}	q_{73}
$\lvert C_{H_2}(h)\rvert$	55296	55296	1024	768	1536	1536	256	64	192	192	96	32	32	432	432	144	144	24	192	192	32	96	96	16
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1
χ_4	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1
χ_5	2	2	2	2	2	2	2	2	0	0	0	0	0	2	2	2	2	2	−2	−2	−2	0	0	0
χ_6	2	2	2	2	−2	−2	−2	−2	0	0	0	0	0	2	2	2	2	2	0	0	0	0	0	0
χ_7	2	2	2	2	−2	−2	−2	−2	0	0	0	0	0	2	2	2	2	2	0	0	0	0	0	0
χ_8	6	6	6	6	−2	−2	−2	−2	0	0	0	0	0	−3	−3	0	0	0	2	2	2	−2	−2	−2
χ_9	6	6	6	6	−2	−2	−2	−2	0	0	0	0	0	−3	−3	0	0	0	2	2	2	2	2	2
χ_{10}	6	6	6	6	−2	−2	−2	−2	0	0	0	0	0	−3	−3	0	0	0	−2	−2	−2	0	0	0
χ_{11}	6	6	6	6	−2	−2	−2	−2	0	0	0	0	0	−3	−3	0	0	0	−2	−2	−2	0	0	0
χ_{12}	8	8	8	8	0	0	0	0	2	2	2	2	2	8	8	−1	−1	−1	0	0	0	0	0	0
χ_{13}	8	8	8	8	0	0	0	0	−2	−2	−2	−2	−2	8	8	−1	−1	−1	0	0	0	0	0	0
χ_{14}	12	12	12	12	4	4	4	4	0	0	0	0	0	−6	−6	0	0	0	0	0	0	0	0	0
χ_{15}	8	−8	0	0	4	−4	0	0	$2i\sqrt{2}$	$-2i\sqrt{2}$	0	0	0	−1	1	2	−2	0	2	−2	0	−2	2	0
χ_{16}	8	−8	0	0	4	−4	0	0	$-2i\sqrt{2}$	$2i\sqrt{2}$	0	0	0	−1	1	2	−2	0	2	−2	0	−2	2	0
χ_{17}	8	−8	0	0	4	−4	0	0	$-2i\sqrt{2}$	$2i\sqrt{2}$	0	0	0	−1	1	2	−2	0	2	−2	0	2	−2	0
χ_{18}	8	−8	0	0	4	−4	0	0	$2i\sqrt{2}$	$-2i\sqrt{2}$	0	0	0	−1	1	2	−2	0	2	−2	0	2	−2	0
χ_{19}	16	−16	0	0	−8	8	0	0	0	0	0	0	0	−2	2	4	−4	0	0	0	0	0	0	0
χ_{20}	16	−16	0	0	−8	8	0	0	0	0	0	0	0	−2	2	4	−4	0	0	0	0	0	0	0
χ_{21}	16	−16	0	0	8	−8	0	0	0	0	0	0	0	−2	2	4	−4	0	−4	4	0	0	0	0
χ_{22}	48	−48	0	0	−8	8	0	0	0	0	0	0	0	3	−3	0	0	0	4	−4	0	−4	4	0
χ_{23}	48	−48	0	0	−8	8	0	0	0	0	0	0	0	3	−3	0	0	0	4	−4	0	4	−4	0
χ_{24}	48	−48	0	0	−8	8	0	0	0	0	0	0	0	3	−3	0	0	0	−4	4	0	0	0	0
χ_{25}	48	−48	0	0	−8	8	0	0	0	0	0	0	0	3	−3	0	0	0	−4	4	0	0	0	0
χ_{26}	64	−64	0	0	0	0	0	0	$-4i\sqrt{2}$	$4i\sqrt{2}$	0	0	0	−8	8	−2	2	0	0	0	0	0	0	0
χ_{27}	64	−64	0	0	0	0	0	0	$4i\sqrt{2}$	$-4i\sqrt{2}$	0	0	0	−8	8	−2	2	0	0	0	0	0	0	0
χ_{28}	96	−96	0	0	16	−16	0	0	0	0	0	0	0	6	−6	0	0	0	0	0	0	0	0	0

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Table 11.15 (continued)

	q_1				q_2				q_3					q_4		q_5			q_6			q_7		
	$1a$	$2a$	$2b$	$4a$	$2c$	$2d$	$4b$	$4c$	$8a$	$8b$	$2e$	$8c$	$4d$	$3a$	$6a$	$3b$	$6b$	$12a$	$4e$	$4f$	$8d$	$4g$	$4h$	$8e$
	q_{11}	q_{12}	q_{13}	q_{14}	q_{21}	q_{22}	q_{23}	q_{24}	q_{31}	q_{32}	q_{33}	q_{34}	q_{35}	q_{41}	q_{42}	q_{51}	q_{52}	q_{53}	q_{61}	q_{62}	q_{63}	q_{71}	q_{72}	q_{73}
$\lvert C_{H_2}(h)\rvert$	55296	55296	1024	768	1536	1536	256	64	192	192	96	32	32	432	432	144	144	24	192	192	32	96	96	16
χ_{29}	27	27	−5	3	3	3	3	−1	−3	−3	−3	1	1	0	0	0	0	0	3	3	−1	3	3	−1
χ_{30}	27	27	−5	3	3	3	3	−1	3	3	3	−1	−1	0	0	0	0	0	3	3	−1	−3	−3	1
χ_{31}	27	27	−5	3	3	3	3	−1	−3	−3	−3	1	1	0	0	0	0	0	3	3	−1	−3	−3	1
χ_{32}	27	27	−5	3	3	3	3	−1	3	3	3	−1	−1	0	0	0	0	0	3	3	−1	3	3	−1
χ_{33}	54	54	−10	6	6	6	6	−2	0	0	0	0	0	0	0	0	0	0	−6	−6	2	0	0	0
χ_{34}	54	54	−10	6	−6	−6	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	54	54	−10	6	−6	−6	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	36	36	4	−4	−12	−12	4	0	−2	−2	2	2	−2	0	0	3	3	−1	0	0	0	0	0	0
χ_{37}	36	36	4	−4	−12	−12	4	0	2	2	−2	−2	2	0	0	3	3	−1	0	0	0	0	0	0
χ_{38}	36	36	4	−4	12	12	−4	0	4	4	−4	0	0	0	0	3	3	−1	0	0	0	0	0	0
χ_{39}	36	36	4	−4	12	12	−4	0	−4	−4	4	0	0	0	0	3	3	−1	0	0	0	0	0	0
χ_{40}	72	72	8	−8	0	0	0	0	−2	−2	2	−2	2	0	0	−3	−3	1	0	0	0	0	0	0
χ_{41}	72	72	8	−8	0	0	0	0	2	2	−2	2	−2	0	0	−3	−3	1	0	0	0	0	0	0

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Table 11.15 (continued)

	q_8		q_9			q_{10}			q_{11}			q_{12}		q_{13}		q_{14}	
	$6c$	$6d$	$24a$	$24b$	$6e$	$8f$	$8g$	$16a$	$8h$	$8i$	$16b$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$
	q_{81}	q_{82}	q_{91}	q_{92}	q_{93}	$q_{10,1}$	$q_{10,2}$	$q_{10,3}$	$q_{11,1}$	$q_{11,2}$	$q_{11,3}$	$q_{12,1}$	$q_{12,2}$	$q_{13,1}$	$q_{13,2}$	$q_{14,1}$	$q_{14,2}$
$\lvert C_{H_2}(h)\rvert$	48	48	24	24	12	32	32	16	32	32	16	24	24	24	24	24	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1
χ_3	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1
χ_4	1	1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	1	1
χ_5	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2
χ_6	−2	−2	0	0	0	$-i\sqrt{2}$	$-i\sqrt{2}$	$-i\sqrt{2}$	$i\sqrt{2}$	$i\sqrt{2}$	$i\sqrt{2}$	0	0	0	0	0	0
χ_7	−2	−2	0	0	0	$i\sqrt{2}$	$i\sqrt{2}$	$i\sqrt{2}$	$-i\sqrt{2}$	$-i\sqrt{2}$	$-i\sqrt{2}$	0	0	0	0	0	0
χ_8	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	−1	−1
χ_9	1	1	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1
χ_{10}	1	1	0	0	0	0	0	0	0	0	0	$-i\sqrt{3}$	$-i\sqrt{3}$	$i\sqrt{3}$	$i\sqrt{3}$	1	1
χ_{11}	1	1	0	0	0	0	0	0	0	0	0	$i\sqrt{3}$	$i\sqrt{3}$	$-i\sqrt{3}$	$-i\sqrt{3}$	1	1
χ_{12}	0	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	1	−1	$-i\sqrt{2}$	$i\sqrt{2}$	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	1	−1	1	−1	1	−1
χ_{16}	1	−1	$i\sqrt{2}$	$-i\sqrt{2}$	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	1	−1	1	−1	1	−1
χ_{17}	1	−1	$i\sqrt{2}$	$-i\sqrt{2}$	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	−1	1	−1	1	1	−1
χ_{18}	1	−1	$-i\sqrt{2}$	$i\sqrt{2}$	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	−1	1	−1	1	1	−1
χ_{19}	−2	2	0	0	0	2	−2	0	−2	2	0	0	0	0	0	0	0
χ_{20}	−2	2	0	0	0	−2	2	0	2	−2	0	0	0	0	0	0	0
χ_{21}	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	2
χ_{22}	1	−1	0	0	0	0	0	0	0	0	0	−1	1	−1	1	−1	1
χ_{23}	1	−1	0	0	0	0	0	0	0	0	0	1	−1	1	−1	−1	1
χ_{24}	1	−1	0	0	0	0	0	0	0	0	0	$-i\sqrt{3}$	$i\sqrt{3}$	$i\sqrt{3}$	$-i\sqrt{3}$	1	−1
χ_{25}	1	−1	0	0	0	0	0	0	0	0	0	$i\sqrt{3}$	$-i\sqrt{3}$	$-i\sqrt{3}$	$i\sqrt{3}$	1	−1
χ_{26}	0	0	$-i\sqrt{2}$	$i\sqrt{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	0	0	$i\sqrt{2}$	$-i\sqrt{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continued on next page

Table 11.15 (continued)

	q_8		q_9			q_{10}			q_{11}			q_{12}		q_{13}		q_{14}	
	$6c$	$6d$	$24a$	$24b$	$6e$	$8f$	$8g$	$16a$	$8h$	$8i$	$16b$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$
	q_{81}	q_{82}	q_{91}	q_{92}	q_{93}	$q_{10,1}$	$q_{10,2}$	$q_{10,3}$	$q_{11,1}$	$q_{11,2}$	$q_{11,3}$	$q_{12,1}$	$q_{12,2}$	$q_{13,1}$	$q_{13,2}$	$q_{14,1}$	$q_{14,2}$
$\lvert C_{H_2}(h)\rvert$	48	48	24	24	12	32	32	16	32	32	16	24	24	24	24	24	24
χ_{29}	0	0	0	0	0	-1	-1	1	-1	-1	1	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	-1	-1	1	-1	-1	1	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	1	1	-1	1	1	-1	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	1	1	-1	1	1	-1	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	$-i\sqrt{2}$	$-i\sqrt{2}$	$i\sqrt{2}$	$i\sqrt{2}$	$i\sqrt{2}$	$-i\sqrt{2}$	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	$i\sqrt{2}$	$i\sqrt{2}$	$-i\sqrt{2}$	$-i\sqrt{2}$	$-i\sqrt{2}$	$i\sqrt{2}$	0	0	0	0	0	0
χ_{36}	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0

Table 11.16: The character table of $H_3 = Sp(4,3)$:2

	$1a$	$2a$	$2b$	$2c$	$2d$	$3a$	$3b$	$3c$	$4a$	$4b$	$4c$	$4d$	$5a$	$6a$	$6b$	$6c$	$6d$	$6e$	$6f$	$6g$	$8a$	$9a$	$10a$	$12a$	$12b$
$\lvert C_{H_3}(g)\rvert$	51840	1440	1152	192	96	648	216	108	96	96	32	16	10	72	36	36	36	36	24	12	8	9	10	12	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	−1	1	1	−1	1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	−1	1
χ_3	6	−4	−2	2	0	−3	3	0	2	2	−2	0	1	1	1	2	−2	−1	−1	0	0	0	1	−1	−1
χ_4	6	4	−2	2	0	−3	3	0	−2	2	2	0	1	1	1	−2	−2	1	−1	0	0	0	−1	1	−1
χ_5	10	0	−6	2	0	1	−2	4	0	2	0	−2	0	−3	0	0	0	0	2	0	0	1	0	0	−1
χ_6	15	−5	−1	−1	3	6	3	0	−1	3	−1	−1	0	2	−1	−2	2	1	−1	0	1	0	0	−1	0
χ_7	15	−5	7	3	−1	−3	0	3	−3	−1	1	1	0	1	−2	1	1	−2	0	−1	1	0	0	0	−1
χ_8	15	5	−1	−1	−3	6	3	0	1	3	1	−1	0	2	−1	2	2	−1	−1	0	−1	0	0	1	0
χ_9	15	5	7	3	1	−3	0	3	3	−1	−1	1	0	1	−2	−1	1	2	0	1	−1	0	0	0	−1
χ_{10}	20	−10	4	4	−2	2	5	−1	−2	0	−2	0	0	−2	1	−1	1	−1	1	1	0	−1	0	1	0
χ_{11}	20	10	4	4	2	2	5	−1	2	0	2	0	0	−2	1	1	1	1	1	−1	0	−1	0	−1	0
χ_{12}	20	0	4	−4	0	−7	2	2	0	4	0	0	0	1	−2	0	−2	0	2	0	0	−1	0	0	1
χ_{13}	24	−4	8	0	−4	6	0	3	0	0	0	0	−1	2	2	−1	−1	2	0	−1	0	0	1	0	0
χ_{14}	24	4	8	0	4	6	0	3	0	0	0	0	−1	2	2	1	−1	−2	0	1	0	0	−1	0	0
χ_{15}	30	−10	−10	2	2	3	3	3	4	−2	0	0	0	−1	−1	−1	−1	−1	−1	−1	0	0	0	1	1
χ_{16}	30	10	−10	2	−2	3	3	3	−4	−2	0	0	0	−1	−1	1	−1	1	−1	1	0	0	0	−1	1
χ_{17}	60	−10	−4	4	−2	6	−3	−3	2	0	2	0	0	2	−1	−1	−1	−1	1	1	0	0	0	−1	0
χ_{18}	60	10	−4	4	2	6	−3	−3	−2	0	−2	0	0	2	−1	1	−1	1	1	−1	0	0	0	1	0
χ_{19}	60	0	12	4	0	−3	−6	0	0	4	0	0	0	−3	0	0	0	0	−2	0	0	0	0	0	1
χ_{20}	64	−16	0	0	0	−8	4	−2	0	0	0	0	−1	0	0	2	0	2	0	0	0	1	−1	0	0
χ_{21}	64	16	0	0	0	−8	4	−2	0	0	0	0	−1	0	0	−2	0	−2	0	0	0	1	1	0	0
χ_{22}	80	0	−16	0	0	−10	−4	2	0	0	0	0	0	2	2	0	2	0	0	0	0	−1	0	0	0
χ_{23}	81	9	9	−3	−3	0	0	0	3	−3	−1	−1	1	0	0	0	0	0	0	0	1	0	−1	0	0
χ_{24}	81	−9	9	−3	3	0	0	0	−3	−3	1	−1	1	0	0	0	0	0	0	0	−1	0	1	0	0
χ_{25}	90	0	−6	−6	0	9	0	0	0	2	0	2	0	−3	0	0	0	0	0	0	0	0	0	0	−1

Table 11.17: The character table of $\overline{G} = 2^{10}{:}(U_5(2){:}2)$

$[x]_G$		$1A$			$2A$				$2B$				$2C$				$3A$	$3B$			$3C$		
$[g]_{\overline{G}}$		$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$4a$	$2f$	$2g$	$4b$	$4c$	$2h$	$4d$	$4e$	$4f$	$3a$	$3b$	$6a$	$6b$	$3c$	$6c$	$6d$
$\lvert C_{\overline{G}}(g)\rvert$	2	21	17	21	19	19	17	13	16	16	13	14	10	10	10	10	6	10	8	10	8	7	8
	3	5	4	3	4	1	2	3	2	1	1	0	2	2	1	1	5	5	3	2	5	4	3
	5	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0
	11	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	1
χ_3		10	10	10	−6	−6	−6	−6	2	2	2	2	0	0	0	0	−5	4	4	4	1	1	1
χ_4		10	10	10	−6	−6	−6	−6	2	2	2	2	0	0	0	0	−5	4	4	4	1	1	1
χ_5		22	22	22	−10	−10	−10	−10	6	6	6	6	0	0	0	0	4	4	4	4	−5	−5	−5
χ_6		44	44	44	12	12	12	12	−4	−4	−4	−4	−4	−4	−4	−4	14	5	5	5	−1	−1	−1
χ_7		44	44	44	12	12	12	12	−4	−4	−4	−4	4	4	4	4	14	5	5	5	−1	−1	−1
χ_8		55	55	55	23	23	23	23	7	7	7	7	5	5	5	5	10	10	10	10	1	1	1
χ_9		55	55	55	23	23	23	23	7	7	7	7	−5	−5	−5	−5	10	10	10	10	1	1	1
χ_{10}		110	110	110	14	14	14	14	−2	−2	−2	−2	0	0	0	0	−25	2	2	2	2	2	2
χ_{11}		110	110	110	−2	−2	−2	−2	−10	−10	−10	−10	0	0	0	0	−25	2	2	2	2	2	2
χ_{12}		110	110	110	−2	−2	−2	−2	−10	−10	−10	−10	0	0	0	0	−25	2	2	2	2	2	2
χ_{13}		120	120	120	24	24	24	24	8	8	8	8	10	10	10	10	30	3	3	3	12	12	12
χ_{14}		120	120	120	24	24	24	24	8	8	8	8	−10	−10	−10	−10	30	3	3	3	12	12	12
χ_{15}		132	132	132	36	36	36	36	20	20	20	20	0	0	0	0	−21	6	6	6	−3	−3	−3
χ_{16}		165	165	165	−27	−27	−27	−27	5	5	5	5	−5	−5	−5	−5	30	3	3	3	3	3	3
χ_{17}		165	165	165	−27	−27	−27	−27	5	5	5	5	5	5	5	5	30	3	3	3	3	3	3
χ_{18}		176	176	176	−16	−16	−16	−16	16	16	16	16	4	4	4	4	−4	−4	−4	−4	14	14	14
χ_{19}		176	176	176	−16	−16	−16	−16	16	16	16	16	−4	−4	−4	−4	−4	−4	−4	−4	14	14	14
χ_{20}		220	220	220	−68	−68	−68	−68	12	12	12	12	0	0	0	0	−5	22	22	22	4	4	4
χ_{21}		220	220	220	60	60	60	60	12	12	12	12	0	0	0	0	−20	16	16	16	−5	−5	−5
χ_{22}		320	320	320	−64	−64	−64	−64	0	0	0	0	0	0	0	0	−40	14	14	14	−4	−4	−4
χ_{23}		320	320	320	−64	−64	−64	−64	0	0	0	0	0	0	0	0	−40	14	14	14	−4	−4	−4
χ_{24}		440	440	440	−72	−72	−72	−72	−8	−8	−8	−8	0	0	0	0	50	14	14	14	17	17	17
χ_{25}		440	440	440	−8	−8	−8	−8	24	24	24	24	0	0	0	0	−10	−10	−10	−10	−19	−19	−19
χ_{26}		440	440	440	88	88	88	88	8	8	8	8	−10	−10	−10	−10	−10	17	17	17	8	8	8
χ_{27}		440	440	440	88	88	88	88	8	8	8	8	10	10	10	10	−10	17	17	17	8	8	8
χ_{28}		528	528	528	−48	−48	−48	−48	−16	−16	−16	−16	0	0	0	0	6	6	6	6	−12	−12	−12
χ_{29}		660	660	660	84	84	84	84	20	20	20	20	10	10	10	10	30	3	3	3	−15	−15	−15
χ_{30}		660	660	660	−12	−12	−12	−12	4	4	4	4	0	0	0	0	75	−6	−6	−6	−15	−15	−15
χ_{31}		660	660	660	84	84	84	84	20	20	20	20	−10	−10	−10	−10	30	3	3	3	−15	−15	−15
χ_{32}		880	880	880	−80	−80	−80	−80	16	16	16	16	0	0	0	0	−110	−2	−2	−2	16	16	16
χ_{33}		891	891	891	27	27	27	27	−21	−21	−21	−21	9	9	9	9	81	0	0	0	0	0	0

Continued on next page

Table 11.17 (continued)

$[x]_G$	$1A$			$2A$				$2B$				$2C$				$3A$	$3B$			$3C$		
$[g]_{\overline{G}}$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$4a$	$2f$	$2g$	$4b$	$4c$	$2h$	$4d$	$4e$	$4f$	$3a$	$3b$	$6a$	$6b$	$3c$	$6c$	$6d$
χ_{34}	891	891	891	27	27	27	27	−21	−21	−21	−21	−9	−9	−9	−9	81	0	0	0	0	0	0
χ_{35}	990	990	990	126	126	126	126	−18	−18	−18	−18	0	0	0	0	45	18	18	18	−9	−9	−9
χ_{36}	990	990	990	−162	−162	−162	−162	30	30	30	30	0	0	0	0	45	18	18	18	−9	−9	−9
χ_{37}	1024	1024	1024	0	0	0	0	0	0	0	0	−16	−16	−16	−16	64	−8	−8	−8	16	16	16
χ_{38}	1024	1024	1024	0	0	0	0	0	0	0	0	16	16	16	16	64	−8	−8	−8	16	16	16
χ_{39}	1408	1408	1408	128	128	128	128	0	0	0	0	0	0	0	0	−104	4	4	4	4	4	4
χ_{40}	1760	1760	1760	96	96	96	96	32	32	32	32	0	0	0	0	20	−16	−16	−16	14	14	14
χ_{41}	1782	1782	1782	54	54	54	54	−42	−42	−42	−42	0	0	0	0	−81	0	0	0	0	0	0
χ_{42}	1980	1980	1980	−36	−36	−36	−36	12	12	12	12	0	0	0	0	−45	−18	−18	−18	−18	−18	−18
χ_{43}	2430	2430	2430	−162	−162	−162	−162	−18	−18	−18	−18	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	495	15	−17	111	−17	15	−1	15	15	−1	−1	−15	−15	1	1	0	27	3	−5	9	−3	1
χ_{45}	495	15	−17	111	−17	15	−1	15	15	−1	−1	15	15	−1	−1	0	27	3	−5	9	−3	1
χ_{46}	495	15	−17	111	−17	15	−1	15	15	−1	−1	−15	−15	1	1	0	27	3	−5	9	−3	1
χ_{47}	495	15	−17	111	−17	15	−1	15	15	−1	−1	15	15	−1	−1	0	27	3	−5	9	−3	1
χ_{48}	990	30	−34	222	−34	30	−2	30	30	−2	−2	0	0	0	0	0	54	6	−10	18	−6	2
χ_{49}	990	30	−34	−210	46	−18	−2	6	6	−10	6	0	0	0	0	0	54	6	−10	18	−6	2
χ_{50}	990	30	−34	−210	46	−18	−2	6	6	−10	6	0	0	0	0	0	54	6	−10	18	−6	2
χ_{51}	2970	90	−102	−198	58	−6	−6	42	42	−22	10	0	0	0	0	0	0	0	0	−27	9	−3
χ_{52}	2970	90	−102	−198	58	−6	−6	42	42	−22	10	0	0	0	0	0	0	0	0	−27	9	−3
χ_{53}	2970	90	−102	−198	58	−6	−6	42	42	−22	10	0	0	0	0	0	0	0	0	−27	9	−3
χ_{54}	2970	90	−102	−198	58	−6	−6	42	42	−22	10	0	0	0	0	0	0	0	0	−27	9	−3
χ_{55}	3960	120	−136	24	24	24	−8	72	72	−24	8	30	30	−2	−2	0	−27	−3	5	72	−24	8
χ_{56}	3960	120	−136	24	24	24	−8	24	24	8	−8	0	0	0	0	0	−27	−3	5	72	−24	8
χ_{57}	3960	120	−136	−456	56	−72	8	132	132	−28	4	0	0	0	0	0	54	6	−10	−9	3	−1
χ_{58}	3960	120	−136	−456	56	−72	8	−48	−48	−16	16	0	0	0	0	0	54	6	−10	−9	3	−1
χ_{59}	3960	120	−136	−456	56	−72	8	−48	−48	−16	16	0	0	0	0	0	54	6	−10	−9	3	−1
χ_{60}	3960	120	−136	−456	56	−72	8	48	48	16	−16	0	0	0	0	0	54	6	−10	−9	3	−1
χ_{61}	5940	180	−204	468	−44	84	−12	−27	−27	21	−11	−45	−45	3	3	0	0	0	0	−54	18	−6
χ_{62}	7920	240	−272	816	−208	48	16	−27	−27	21	−11	45	45	−3	−3	0	108	12	−20	−18	6	−2
χ_{63}	7920	240	−272	816	−208	48	16	−27	−27	21	−11	−45	−45	3	3	0	108	12	−20	−18	6	−2
χ_{64}	7920	240	−272	−912	112	−144	16	−27	−27	21	−11	45	45	−3	−3	0	108	12	−20	−18	6	−2
χ_{65}	13365	405	−459	405	21	117	−27	−36	−36	−36	28	−30	−30	2	2	0	0	0	0	0	0	0
χ_{66}	13365	405	−459	405	21	117	−27	108	108	12	−20	60	60	−4	−4	0	0	0	0	0	0	0
χ_{67}	13365	405	−459	405	21	117	−27	−36	−36	−36	28	30	30	−2	−2	0	0	0	0	0	0	0
χ_{68}	13365	405	−459	405	21	117	−27	108	108	12	−20	−60	−60	4	4	0	0	0	0	0	0	0
χ_{69}	17820	540	−612	−1188	348	−36	−36	−48	−48	−16	16	0	0	0	0	0	81	9	−15	0	0	0
χ_{70}	17820	540	−612	1404	−132	252	−36	−48	−48	−16	16	0	0	0	0	0	81	9	−15	0	0	0
χ_{71}	17820	540	−612	−1188	348	−36	−36	−48	−48	−16	16	0	0	0	0	0	81	9	−15	0	0	0

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Table 11.17 (continued)

$[x]_G$	$1A$			$2A$				$2B$				$2C$				$3A$	$3B$			$3C$		
$[g]_{\overline{G}}$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$4a$	$2f$	$2g$	$4b$	$4c$	$2h$	$4d$	$4e$	$4f$	$3a$	$3b$	$6a$	$6b$	$3c$	$6c$	$6d$
χ_{72}	17820	540	−612	1404	−132	252	−36	−48	−48	−16	16	0	0	0	0	0	81	9	−15	0	0	0
χ_{73}	23760	720	−816	720	−304	−48	48	−54	−54	42	−22	0	0	0	0	0	0	0	0	27	−9	3
χ_{74}	23760	720	−816	720	−304	−48	48	−126	−126	18	2	0	0	0	0	0	0	0	0	27	−9	3
χ_{75}	23760	720	−816	720	−304	−48	48	−126	−126	18	2	0	0	0	0	0	0	0	0	27	−9	3
χ_{76}	23760	720	−816	720	−304	−48	48	0	0	0	0	0	0	0	0	0	0	0	0	27	−9	3
χ_{77}	26730	810	−918	810	42	234	−54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	26730	810	−918	−486	282	90	−54	72	72	−24	8	−30	−30	2	2	0	0	0	0	0	0	0
χ_{79}	26730	810	−918	−486	282	90	−54	72	72	−24	8	30	30	−2	−2	0	0	0	0	0	0	0
χ_{80}	31680	960	−1088	−192	−192	−192	64	96	96	32	−32	0	0	0	0	0	−54	−6	10	−72	24	−8
χ_{81}	31680	960	−1088	−192	−192	−192	64	96	−32	0	0	−4	4	−4	4	0	−54	−6	10	−72	24	−8
χ_{82}	35640	1080	−1224	216	216	216	−72	96	−32	0	0	4	−4	4	−4	0	−81	−9	15	0	0	0
χ_{83}	35640	1080	−1224	216	216	216	−72	72	72	−24	8	−30	−30	2	2	0	−81	−9	15	0	0	0
χ_{84}	47520	1440	−1632	−2016	32	−480	96	24	24	8	−8	0	0	0	0	0	0	0	0	54	−18	6
χ_{85}	528	−16	16	144	16	−16	0	48	−16	0	0	16	−16	0	0	0	36	−4	4	6	2	−2
χ_{86}	528	−16	16	144	16	−16	0	48	−16	0	0	−16	16	0	0	0	36	−4	4	6	2	−2
χ_{87}	3168	−96	96	−288	−32	32	0	24	24	8	−8	0	0	0	0	0	0	0	0	−18	−6	6
χ_{88}	3168	−96	96	−288	−32	32	0	24	24	8	−8	0	0	0	0	0	0	0	0	−18	−6	6
χ_{89}	5280	−160	160	−864	−96	96	0	96	−32	0	0	0	0	0	0	0	144	−16	16	6	2	−2
χ_{90}	7920	−240	240	1008	112	−112	0	144	−48	0	0	20	−20	4	−4	0	108	−12	12	−18	−6	6
χ_{91}	7920	−240	240	1008	112	−112	0	144	−48	0	0	−20	20	−4	4	0	108	−12	12	−18	−6	6
χ_{92}	7920	−240	240	−144	−16	16	0	−48	16	0	0	−40	40	8	−8	0	0	0	0	36	12	−12
χ_{93}	7920	−240	240	−144	−16	16	0	−48	16	0	0	40	−40	−8	8	0	0	0	0	36	12	−12
χ_{94}	10560	−320	320	576	64	−64	0	−192	64	0	0	0	0	0	0	0	72	−8	8	−42	−14	14
χ_{95}	10560	−320	320	576	64	−64	0	192	−64	0	0	40	−40	8	−8	0	−36	4	−4	12	4	−4
χ_{96}	10560	−320	320	576	64	−64	0	192	−64	0	0	−40	40	−8	8	0	−36	4	−4	12	4	−4
χ_{97}	12672	−384	384	1152	128	−128	0	0	0	0	0	−64	64	0	0	0	108	−12	12	36	12	−12
χ_{98}	12672	−384	384	1152	128	−128	0	0	0	0	0	64	−64	0	0	0	108	−12	12	36	12	−12
χ_{99}	15840	−480	480	−1440	−160	160	0	96	−32	0	0	−20	20	12	−12	0	108	−12	12	18	6	−6
χ_{100}	15840	−480	480	−1440	−160	160	0	96	−32	0	0	20	−20	−12	12	0	108	−12	12	18	6	−6
χ_{101}	31680	−960	960	1728	192	−192	0	192	−64	0	0	0	0	0	0	0	0	0	0	−18	−6	6
χ_{102}	31680	−960	960	−576	−64	64	0	192	−64	0	0	40	−40	8	−8	0	−108	12	−12	36	12	−12
χ_{103}	31680	−960	960	−576	−64	64	0	192	−64	0	0	−40	40	−8	8	0	−108	12	−12	36	12	−12
χ_{104}	33792	−1024	1024	0	0	0	0	0	0	0	0	16	−16	16	−16	0	−72	8	−8	−48	−16	16
χ_{105}	33792	−1024	1024	0	0	0	0	0	0	0	0	−16	16	−16	16	0	−72	8	−8	−48	−16	16
χ_{106}	42240	−1280	1280	−2304	−256	256	0	0	0	0	0	0	0	0	0	0	72	−8	8	−60	−20	20
χ_{107}	42768	−1296	1296	1296	144	−144	0	−144	48	0	0	36	−36	−12	12	0	0	0	0	0	0	0
χ_{108}	42768	−1296	1296	1296	144	−144	0	−144	48	0	0	−36	36	12	−12	0	0	0	0	0	0	0
χ_{109}	47520	−1440	1440	−864	−96	96	0	−288	96	0	0	0	0	0	0	0	0	0	0	54	18	−18

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Table 11.17 (continued)

$[x]_G$		$3D$		$4A$					$4B$				$4C$			$4D$				$5A$		$6A$	$6B$	
$[g]_{\overline{G}}$		$3d$	$6e$	$4g$	$4h$	$4i$	$4j$	$4k$	$4l$	$4m$	$4n$	$4o$	$4p$	$4q$	$8a$	$4r$	$4s$	$8b$	$8c$	$5a$	$10a$	$6f$	$6g$	$12a$
$\lvert C_{\overline{G}}(g)\rvert$	2	5	5	14	14	13	13	10	12	12	11	10	10	10	8	8	8	8	8	3	3	6	6	6
	3	4	3	2	1	1	1	1	1	1	1	0	1	0	0	1	1	0	0	1	0	3	4	3
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1
χ_3		−2	−2	2	2	2	2	2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	3	3	3
χ_4		−2	−2	2	2	2	2	2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	3	3	3
χ_5		4	4	6	6	6	6	6	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	2	2	−4	−1	−1
χ_6		2	2	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	−1	−1	6	3	3
χ_7		2	2	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	−1	−1	6	3	3
χ_8		1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	3	3	3	1	1	1	1	0	0	2	5	5
χ_9		1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	3	3	3	−1	−1	−1	−1	0	0	2	5	5
χ_{10}		2	2	14	14	14	14	14	6	6	6	6	−2	−2	−2	0	0	0	0	0	0	−1	−4	−4
χ_{11}		2	2	6	6	6	6	6	−6	−6	−6	−6	2	2	2	0	0	0	0	0	0	7	−2	−2
χ_{12}		2	2	6	6	6	6	6	−6	−6	−6	−6	2	2	2	0	0	0	0	0	0	7	−2	−2
χ_{13}		3	3	8	8	8	8	8	0	0	0	0	0	0	0	2	2	2	2	0	0	6	6	6
χ_{14}		3	3	8	8	8	8	8	0	0	0	0	0	0	0	−2	−2	−2	−2	0	0	6	6	6
χ_{15}		6	6	4	4	4	4	4	−4	−4	−4	−4	4	4	4	0	0	0	0	2	2	3	−9	−9
χ_{16}		−6	−6	5	5	5	5	5	5	5	5	5	−3	−3	−3	3	3	3	3	0	0	6	−9	−9
χ_{17}		−6	−6	5	5	5	5	5	5	5	5	5	−3	−3	−3	−3	−3	−3	−3	0	0	6	−9	−9
χ_{18}		5	5	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4	1	1	−4	2	2
χ_{19}		5	5	0	0	0	0	0	0	0	0	0	0	0	0	−4	−4	−4	−4	1	1	−4	2	2
χ_{20}		4	4	−4	−4	−4	−4	−4	4	4	4	4	−4	−4	−4	0	0	0	0	0	0	−5	4	4
χ_{21}		−8	−8	12	12	12	12	12	4	4	4	4	4	4	4	0	0	0	0	0	0	−12	−3	−3
χ_{22}		−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	8
χ_{23}		−4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	8
χ_{24}		2	2	8	8	8	8	8	−8	−8	−8	−8	0	0	0	0	0	0	0	0	0	−6	−9	−9
χ_{25}		8	8	8	8	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	−2	1	1
χ_{26}		−1	−1	−8	−8	−8	−8	−8	0	0	0	0	0	0	0	−2	−2	−2	−2	0	0	−2	−2	−2
χ_{27}		−1	−1	−8	−8	−8	−8	−8	0	0	0	0	0	0	0	2	2	2	2	0	0	−2	−2	−2
χ_{28}		6	6	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−18	6	6
χ_{29}		3	3	−4	−4	−4	−4	−4	4	4	4	4	0	0	0	−2	−2	−2	−2	0	0	6	3	3
χ_{30}		−6	−6	20	20	20	20	20	−4	−4	−4	−4	4	4	4	0	0	0	0	0	0	3	−3	−3
χ_{31}		3	3	−4	−4	−4	−4	−4	4	4	4	4	0	0	0	2	2	2	2	0	0	6	3	3
χ_{32}		−2	−2	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	10	10	10
χ_{33}		0	0	3	3	3	3	3	3	3	3	3	−1	−1	−1	−3	−3	−3	−3	1	1	9	0	0

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Table 11.17 (continued)

$[x]_G$	$3D$		$4A$					$4B$				$4C$			$4D$				$5A$		$6A$	$6B$	
$[g]_{\overline{G}}$	$3d$	$6e$	$4g$	$4h$	$4i$	$4j$	$4k$	$4l$	$4m$	$4n$	$4o$	$4p$	$4q$	$8a$	$4r$	$4s$	$8b$	$8c$	$5a$	$10a$	$6f$	$6g$	$12a$
χ_{34}	0	0	3	3	3	3	3	3	3	3	3	−1	−1	−1	3	3	3	3	1	1	9	0	0
χ_{35}	0	0	−2	−2	−2	−2	−2	−10	−10	−10	−10	−2	−2	−2	0	0	0	0	0	0	−3	9	9
χ_{36}	0	0	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	−3	−9	−9
χ_{37}	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	0	0	0
χ_{38}	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	0	0	0
χ_{39}	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	8	−16	−16
χ_{40}	−10	−10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−12	6	6
χ_{41}	0	0	6	6	6	6	6	6	6	6	6	−2	−2	−2	0	0	0	0	2	2	−9	0	0
χ_{42}	0	0	−4	−4	−4	−4	−4	−12	−12	−12	−12	−4	−4	−4	0	0	0	0	0	0	3	0	0
χ_{43}	0	0	−18	−18	−18	−18	−18	6	6	6	6	6	6	6	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	39	7	−9	7	−1	15	−1	−1	−1	3	3	−1	−3	−3	1	1	0	0	0	3	−1
χ_{45}	0	0	39	7	−9	7	−1	15	−1	−1	−1	3	3	−1	3	3	−1	−1	0	0	0	3	−1
χ_{46}	0	0	−9	23	7	−9	−1	−1	15	−1	−1	3	3	−1	−3	−3	1	1	0	0	0	3	−1
χ_{47}	0	0	−9	23	7	−9	−1	−1	15	−1	−1	3	3	−1	3	3	−1	−1	0	0	0	3	−1
χ_{48}	0	0	−18	−18	14	14	−2	−2	−2	14	−2	6	6	−2	0	0	0	0	0	0	0	6	−2
χ_{49}	0	0	6	6	6	6	−2	−6	−6	−6	2	−6	−6	2	0	0	0	0	0	0	0	6	−2
χ_{50}	0	0	6	6	6	6	−2	−6	−6	−6	2	−6	−6	2	0	0	0	0	0	0	0	6	−2
χ_{51}	0	0	90	26	−6	26	−6	18	−14	−14	2	−6	−6	2	0	0	0	0	0	0	0	−9	3
χ_{52}	0	0	−6	58	26	−6	−6	−14	18	−14	2	−6	−6	2	0	0	0	0	0	0	0	−9	3
χ_{53}	0	0	−6	−6	26	26	−6	−14	−14	2	2	−6	−6	2	0	0	0	0	0	0	0	−9	3
χ_{54}	0	0	−6	−6	26	26	−6	−14	−14	2	2	−6	−6	2	0	0	0	0	0	0	0	−9	3
χ_{55}	0	0	24	24	24	24	−8	0	0	0	0	0	0	0	6	6	−2	−2	0	0	0	24	−8
χ_{56}	0	0	24	24	24	24	−8	8	−24	8	0	0	0	0	0	0	0	0	0	0	0	24	−8
χ_{57}	0	0	72	8	−24	8	0	12	12	12	−4	0	0	0	0	0	0	0	0	0	0	3	−1
χ_{58}	0	0	72	8	−24	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1
χ_{59}	0	0	−24	40	8	−24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1
χ_{60}	0	0	−24	40	8	−24	0	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	3	−1
χ_{61}	0	0	36	36	36	36	−12	45	−3	−3	−3	12	12	−4	3	3	−1	−1	0	0	0	−18	6
χ_{62}	0	0	0	0	0	0	0	45	−3	−3	−3	0	0	0	−3	−3	1	1	0	0	0	6	−2
χ_{63}	0	0	0	0	0	0	0	−3	45	−3	−3	0	0	0	3	3	−1	−1	0	0	0	6	−2
χ_{64}	0	0	−48	−48	16	16	0	−3	45	−3	−3	0	0	0	−3	−3	1	1	0	0	0	6	−2
χ_{65}	0	0	117	21	−27	21	−3	12	12	12	−4	−3	−3	1	6	6	−2	−2	0	0	0	0	0
χ_{66}	0	0	117	21	−27	21	−3	−12	−12	−12	4	−3	−3	1	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	−27	69	21	−27	−3	12	12	12	−4	−3	−3	1	−6	−6	2	2	0	0	0	0	0
χ_{68}	0	0	−27	69	21	−27	−3	−12	−12	−12	4	−3	−3	1	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	−12	−12	−12	−12	4	−48	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	−12	−12	−12	−12	4	16	−48	16	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	−12	−12	−12	−12	4	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.17 (continued)

$[x]_G$	$3D$		$4A$					$4B$				$4C$			$4D$				$5A$		$6A$	$6B$	
$[g]_{\overline{G}}$	$3d$	$6e$	$4g$	$4h$	$4i$	$4j$	$4k$	$4l$	$4m$	$4n$	$4o$	$4p$	$4q$	$8a$	$4r$	$4s$	$8b$	$8c$	$5a$	$10a$	$6f$	$6g$	$12a$
χ_{72}	0	0	−12	−12	−12	−12	4	16	16	−16	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	144	16	−48	16	0	−6	−6	42	−6	0	0	0	0	0	0	0	0	0	0	−9	3
χ_{74}	0	0	−48	80	16	−48	0	−18	−18	−18	6	0	0	0	0	0	0	0	0	0	0	−9	3
χ_{75}	0	0	−48	−48	16	16	0	−18	−18	−18	6	0	0	0	0	0	0	0	0	0	0	−9	3
χ_{76}	0	0	−48	−48	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	3
χ_{77}	0	0	−54	−54	42	42	−6	0	0	0	0	−6	−6	2	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	18	18	18	18	−6	0	0	0	0	6	6	−2	−6	−6	2	2	0	0	0	0	0
χ_{79}	0	0	18	18	18	18	−6	0	0	0	0	6	6	−2	6	6	−2	−2	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	−8
χ_{81}	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	4	−4	0	0	3	0	0	24	−8
χ_{82}	3	−1	−24	−24	−24	−24	8	0	0	0	0	0	0	0	−4	4	0	0	3	0	0	0	0
χ_{83}	9	−3	−24	−24	−24	−24	8	0	0	0	0	0	0	0	−4	4	−4	4	3	0	0	0	0
χ_{84}	9	−3	0	0	0	0	0	0	0	0	0	0	0	0	4	−4	4	−4	3	0	0	−18	6
χ_{85}	0	0	24	−8	8	−8	0	0	0	0	0	12	−4	0	−6	−6	2	2	0	−1	0	0	0
χ_{86}	0	0	24	−8	8	−8	0	−24	8	8	0	12	−4	0	0	0	0	0	0	−1	0	0	0
χ_{87}	0	0	48	−16	16	−16	0	−24	8	8	0	0	0	0	0	0	0	0	0	−1	0	0	0
χ_{88}	0	0	48	−16	16	−16	0	8	−24	8	0	0	0	0	0	0	0	0	0	−1	0	0	0
χ_{89}	−6	2	48	−16	16	−16	0	0	0	0	0	−24	8	0	0	0	0	0	0	0	0	0	0
χ_{90}	0	0	−24	8	−8	8	0	0	0	0	0	12	−4	0	0	0	−4	4	0	0	0	0	0
χ_{91}	0	0	−24	8	−8	8	0	0	0	0	0	12	−4	0	0	0	4	−4	0	0	0	0	0
χ_{92}	9	−3	72	−24	24	−24	0	0	0	0	0	−12	4	0	4	−4	0	0	0	0	0	0	0
χ_{93}	9	−3	72	−24	24	−24	0	0	0	0	0	−12	4	0	−4	4	0	0	0	0	0	0	0
χ_{94}	6	−2	96	−32	32	−32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	15	−5	0	0	0	0	0	0	0	0	0	0	0	0	8	−8	0	0	0	0	0	0	0
χ_{96}	15	−5	0	0	0	0	0	0	0	0	0	0	0	0	−8	8	0	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0
χ_{99}	9	−3	−48	16	−16	16	0	0	0	0	0	0	0	0	−4	4	4	−4	0	0	0	0	0
χ_{100}	9	−3	−48	16	−16	16	0	0	0	0	0	0	0	0	4	−4	−4	4	0	0	0	0	0
χ_{101}	−18	6	96	−32	32	−32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	−9	3	0	0	0	0	0	0	0	0	0	0	0	0	−8	8	0	0	0	0	0	0	0
χ_{103}	−9	3	0	0	0	0	0	0	0	0	0	0	0	0	8	−8	0	0	0	0	0	0	0
χ_{104}	12	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0
χ_{105}	12	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0
χ_{106}	−12	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	0	0	−72	24	−24	24	0	0	0	0	0	−12	4	0	0	0	4	−4	3	−1	0	0	0
χ_{108}	0	0	−72	24	−24	24	0	0	0	0	0	−12	4	0	0	0	−4	4	3	−1	0	0	0
χ_{109}	0	0	48	−16	16	−16	0	0	0	0	0	24	−8	0	0	0	0	0	0	0	0	0	0

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Table 11.17 (continued)

$[x]_G$		$6C$			$6D$			$6E$	$6F$		$6G$		$6H$		$6I$		$6J$				$8A$			$8B$		
$[g]_{\overline{G}}$		$6h$	$6i$	$12b$	$6j$	$6k$	$6l$	$6m$	$6n$	$12c$	$6o$	$6p$	$6q$	$6r$	$6s$	$12d$	$6t$	$12e$	$12f$	$12g$	$8d$	$8e$	$8f$	$8g$	$8h$	$8i$
$\lvert C_{\overline{G}}(g)\rvert$	2	8	8	6	8	7	8	5	6	6	4	4	5	5	3	3	5	5	5	5	9	9	7	9	9	7
	3	3	2	2	3	2	1	2	2	1	3	2	2	1	2	2	2	2	1	1	1	0	1	1	0	1
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_3		0	0	0	−3	−3	−3	−1	−1	−1	0	0	2	2	0	0	0	0	0	0	$-A$	$-A$	$-A$	A	A	A
χ_4		0	0	0	−3	−3	−3	−1	−1	−1	0	0	2	2	0	0	0	0	0	0	A	A	A	$-A$	$-A$	$-A$
χ_5		−4	−4	−4	−1	−1	−1	0	3	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_6		−3	−3	−3	3	3	3	2	−1	−1	0	0	2	2	2	2	−1	−1	−1	−1	−4	−4	−4	−4	−4	−4
χ_7		−3	−3	−3	3	3	3	2	−1	−1	0	0	2	2	−2	−2	1	1	1	1	4	4	4	4	4	4
χ_8		2	2	2	5	5	5	−2	1	1	−1	−1	1	1	−1	−1	2	2	2	2	−3	−3	−3	−3	−3	−3
χ_9		2	2	2	5	5	5	−2	1	1	−1	−1	1	1	1	1	−2	−2	−2	−2	3	3	3	3	3	3
χ_{10}		2	2	2	2	2	2	−5	4	4	−4	−4	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}		−2	−2	−2	−2	−2	−2	−1	2	2	−2	−2	2	2	0	0	0	0	0	0	$-A$	$-A$	$-A$	A	A	A
χ_{12}		−2	−2	−2	−2	−2	−2	−1	2	2	−2	−2	2	2	0	0	0	0	0	0	A	A	A	$-A$	$-A$	$-A$
χ_{13}		3	3	3	0	0	0	2	2	2	3	3	−1	−1	1	1	1	1	1	1	2	2	2	2	2	2
χ_{14}		3	3	3	0	0	0	2	2	2	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−2	−2	−2	−2	−2	−2
χ_{15}		6	6	6	−3	−3	−3	−1	−1	−1	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}		3	3	3	3	3	3	2	−1	−1	0	0	2	2	−2	−2	1	1	1	1	−1	−1	−1	−1	−1	−1
χ_{17}		3	3	3	3	3	3	2	−1	−1	0	0	2	2	2	2	−1	−1	−1	−1	1	1	1	1	1	1
χ_{18}		−4	−4	−4	2	2	2	4	−2	−2	−1	−1	1	1	1	1	−2	−2	−2	−2	0	0	0	0	0	0
χ_{19}		−4	−4	−4	2	2	2	4	−2	−2	−1	−1	1	1	−1	−1	2	2	2	2	0	0	0	0	0	0
χ_{20}		−2	−2	−2	−8	−8	−8	3	0	0	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{21}		0	0	0	3	3	3	0	−3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}		2	2	2	−4	−4	−4	0	0	0	2	2	0	0	0	0	0	0	0	0	$-B$	$-B$	$-B$	B	B	B
χ_{23}		2	2	2	−4	−4	−4	0	0	0	2	2	0	0	0	0	0	0	0	0	B	B	B	$-B$	$-B$	$-B$
χ_{24}		6	6	6	−3	−3	−3	−2	−5	−5	0	0	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}		−2	−2	−2	1	1	1	−6	−3	−3	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}		1	1	1	4	4	4	2	2	2	1	1	−1	−1	−1	−1	−1	−1	−1	−1	2	2	2	2	2	2
χ_{27}		1	1	1	4	4	4	2	2	2	1	1	−1	−1	1	1	1	1	1	1	−2	−2	−2	−2	−2	−2
χ_{28}		6	6	6	0	0	0	2	2	2	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}		3	3	3	−3	−3	−3	2	−1	−1	−3	−3	−1	−1	1	1	1	1	1	1	2	2	2	2	2	2
χ_{30}		−6	−6	−6	−3	−3	−3	7	1	1	0	0	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}		3	3	3	−3	−3	−3	2	−1	−1	−3	−3	−1	−1	−1	−1	−1	−1	−1	−1	−2	−2	−2	−2	−2	−2
χ_{32}		−2	−2	−2	4	4	4	−2	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}		0	0	0	0	0	0	−3	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	−3	−3	−3

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Table 11.17 (continued)

$[x]_G$	$6C$			$6D$			$6E$	$6F$		$6G$		$6H$		$6I$		$6J$				$8A$			$8B$		
$[g]_{\overline{G}}$	$6h$	$6i$	$12b$	$6j$	$6k$	$6l$	$6m$	$6n$	$12c$	$6o$	$6p$	$6q$	$6r$	$6s$	$12d$	$6t$	$12e$	$12f$	$12g$	$8d$	$8e$	$8f$	$8g$	$8h$	$8i$
χ_{34}	0	0	0	0	0	0	−3	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	3
χ_{35}	−6	−6	−6	3	3	3	−3	−3	−3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	−6	−6	−6	3	3	3	−3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0
χ_{39}	−4	−4	−4	−4	−4	−4	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	−6	−6	−6	−4	2	2	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	6	6	6	6	6	6	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	3	3	−1	9	−3	1	0	3	−1	0	0	0	0	0	0	−3	−3	1	1	−7	1	1	−7	1	1
χ_{45}	3	3	−1	9	−3	1	0	3	−1	0	0	0	0	0	0	3	3	−1	−1	7	−1	−1	7	−1	−1
χ_{46}	3	3	−1	9	−3	1	0	3	−1	0	0	0	0	0	0	−3	−3	1	1	5	−3	1	5	−3	1
χ_{47}	3	3	−1	9	−3	1	0	3	−1	0	0	0	0	0	0	3	3	−1	−1	−5	3	−1	−5	3	−1
χ_{48}	6	6	−2	18	−6	2	0	6	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	6	6	−2	−18	6	−2	0	−6	2	0	0	0	0	0	0	0	0	0	0	$-A$	A	0	C	$-C$	0
χ_{50}	6	6	−2	−18	6	−2	0	−6	2	0	0	0	0	0	0	0	0	0	0	C	$-C$	0	$-A$	A	0
χ_{51}	0	0	0	9	−3	1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	9	−3	1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	9	−3	1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	0	9	−3	1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	−3	−3	1	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	1	1	2	2	−2	2	2	−2
χ_{56}	−3	−3	1	0	3	0	0	0	0	0	0	0	0	0	0	3	3	−1	−1	−2	−2	2	−2	−2	2
χ_{57}	−6	−6	2	−9	6	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	$-D$	0	A	D	0	$-A$
χ_{58}	−6	−6	2	−9	−6	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	D	0	$-A$	$-D$	0	A
χ_{59}	−6	−6	2	−9	−6	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	$-B$	B	A	B	$-B$	$-A$
χ_{60}	−6	−6	2	−9	6	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	B	$-B$	$-A$	$-B$	B	A
χ_{61}	0	0	0	−18	0	−2	0	−6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	−12	−12	4	18	0	2	0	−6	2	0	0	0	0	0	0	0	0	0	0	12	−4	0	12	−4	0
χ_{63}	−12	−12	4	18	0	2	0	−6	2	0	0	0	0	0	0	0	0	0	0	−12	4	0	−12	4	0
χ_{64}	−12	−12	4	−18	0	−2	0	6	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−9	−1	3	−9	−1	3
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	1	−3	9	1	−3
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−5	3	3	−5	3
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	5	−3	−3	5	−3
χ_{69}	9	9	−3	0	3	0	0	0	0	0	0	0	0	0	0	3	3	−1	−1	2	2	−2	2	2	−2
χ_{70}	9	9	−3	0	3	0	0	0	0	0	0	0	0	0	0	3	3	−1	−1	−4	−4	4	−4	−4	4
χ_{71}	9	9	−3	0	3	0	0	0	0	0	0	0	0	0	0	−3	−3	1	1	−2	−2	2	−2	−2	2

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Table 11.17 (continued)

$[x]_G$	$6C$			$6D$			$6E$	$6F$		$6G$		$6H$		$6I$		$6J$				$8A$			$8B$		
$[g]_{\overline{G}}$	$6h$	$6i$	$12b$	$6j$	$6k$	$6l$	$6m$	$6n$	$12c$	$6o$	$6p$	$6q$	$6r$	$6s$	$12d$	$6t$	$12e$	$12f$	$12g$	$8d$	$8e$	$8f$	$8g$	$8h$	$8i$
χ_{72}	9	9	−3	0	3	0	0	0	0	0	0	0	0	0	0	−3	−3	1	1	4	4	−4	4	4	−4
χ_{73}	0	0	0	−9	0	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	−9	0	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	−9	0	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	−9	0	−1	0	3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-A$	A	0	C	$-C$	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	$-C$	0	$-A$	A	0
χ_{80}	6	6	−2	0	−6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-B$	$-B$	B	B	B	$-B$
χ_{81}	6	6	−2	0	2	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	B	B	$-B$	$-B$	$-B$	B
χ_{82}	−9	−9	3	0	2	0	0	0	0	0	0	3	−1	0	0	3	3	−1	−1	2	2	−2	2	2	−2
χ_{83}	−9	−9	3	0	2	0	0	0	0	0	0	−3	1	0	0	−3	−3	1	1	−2	−2	2	−2	−2	2
χ_{84}	0	0	0	18	2	2	0	−6	2	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	12	−4	0	6	0	−2	0	0	0	3	−1	0	0	1	−1	4	−4	0	0	0	0	0	0	0	0
χ_{86}	12	−4	0	6	3	−2	0	0	0	3	−1	0	0	−1	1	−4	4	0	0	0	0	0	0	0	0
χ_{87}	−24	8	0	6	3	−2	0	0	0	3	−1	0	0	−1	1	2	−2	2	−2	0	0	0	0	0	0
χ_{88}	−24	8	0	6	3	−2	0	0	0	3	−1	0	0	1	−1	−2	2	−2	2	0	0	0	0	0	0
χ_{89}	0	0	0	−18	−6	6	0	0	0	0	0	6	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{90}	12	−4	0	6	2	−2	0	0	0	−6	2	0	0	2	−2	2	−2	−2	2	0	0	0	0	0	0
χ_{91}	12	−4	0	6	2	−2	0	0	0	−6	2	0	0	−2	2	−2	2	2	−2	0	0	0	0	0	0
χ_{92}	24	−8	0	12	4	−4	0	0	0	−3	1	−3	1	−1	1	2	−2	2	−2	0	0	0	0	0	0
χ_{93}	24	−8	0	12	4	−4	0	0	0	−3	1	−3	1	1	−1	−2	2	−2	2	0	0	0	0	0	0
χ_{94}	−24	8	0	6	2	−2	0	0	0	−6	2	6	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	12	−4	0	−12	−4	4	0	0	0	3	−1	3	−1	1	−1	−2	2	2	−2	0	0	0	0	0	0
χ_{96}	12	−4	0	−12	−4	4	0	0	0	3	−1	3	−1	−1	1	2	−2	−2	2	0	0	0	0	0	0
χ_{97}	−12	4	0	12	4	−4	0	0	0	6	−2	0	0	2	−2	−4	4	0	0	0	0	0	0	0	0
χ_{98}	−12	4	0	12	4	−4	0	0	0	6	−2	0	0	−2	2	4	−4	0	0	0	0	0	0	0	0
χ_{99}	−12	4	0	−6	−2	2	0	0	0	−3	1	−3	1	1	−1	4	−4	0	0	0	0	0	0	0	0
χ_{100}	−12	4	0	−6	−2	2	0	0	0	−3	1	−3	1	−1	1	−4	4	0	0	0	0	0	0	0	0
χ_{101}	0	0	0	−18	−6	6	0	0	0	0	0	−6	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	−12	4	0	12	4	−4	0	0	0	−3	1	3	−1	1	−1	−2	2	2	−2	0	0	0	0	0	0
χ_{103}	−12	4	0	12	4	−4	0	0	0	−3	1	3	−1	−1	1	2	−2	−2	2	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	2	−2	2	−2	2	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−2	2	−2	2	−2	0	0	0	0	0	0
χ_{106}	24	−8	0	12	4	−4	0	0	0	6	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	−18	−6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table 11.17 (continued)

$[x]_G$		$8C$					$8D$			$9A$	$9B$		$10A$		$11A$	$12A$	$12B$		$12C$				
$[g]_{\overline{G}}$		$8j$	$8k$	$8l$	$8m$	$8n$	$8o$	$8p$	$8q$	$9a$	$9b$	$18a$	$10b$	$20a$	$11a$	$12h$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$
$\lvert C_{\overline{G}}(g)\rvert$	2	8	8	7	7	7	7	7	6	1	2	2	2	2	0	4	5	5	7	7	7	7	6
	3	0	0	0	0	0	0	0	0	3	3	2	0	0	0	2	2	1	2	1	1	1	1
	5	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	1	1
χ_3		0	0	0	0	0	0	0	0	−2	1	1	0	0	−1	−1	2	2	−1	−1	−1	−1	−1
χ_4		0	0	0	0	0	0	0	0	−2	1	1	0	0	−1	−1	2	2	−1	−1	−1	−1	−1
χ_5		0	0	0	0	0	2	2	2	1	1	1	0	0	0	0	0	0	−3	−3	−3	−3	−3
χ_6		0	0	0	0	0	0	0	0	2	−1	−1	1	1	0	−2	1	1	1	1	1	1	1
χ_7		0	0	0	0	0	0	0	0	2	−1	−1	−1	−1	0	−2	1	1	1	1	1	1	1
χ_8		1	1	1	1	1	−1	−1	−1	1	1	1	0	0	0	2	2	2	−1	−1	−1	−1	−1
χ_9		−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	0	0	0	2	2	2	−1	−1	−1	−1	−1
χ_{10}		0	0	0	0	0	2	2	2	−1	−1	−1	0	0	0	−1	2	2	2	2	2	2	2
χ_{11}		0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	3	0	0	0	0	0	0	0
χ_{12}		0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	3	0	0	0	0	0	0	0
χ_{13}		2	2	2	2	2	0	0	0	0	0	0	0	0	−1	2	−1	−1	2	2	2	2	2
χ_{14}		−2	−2	−2	−2	−2	0	0	0	0	0	0	0	0	−1	2	−1	−1	2	2	2	2	2
χ_{15}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	−5	−2	−2	1	1	1	1	1
χ_{16}		−1	−1	−1	−1	−1	1	1	1	0	0	0	0	0	0	2	−1	−1	−1	−1	−1	−1	−1
χ_{17}		1	1	1	1	1	1	1	1	0	0	0	0	0	0	2	−1	−1	−1	−1	−1	−1	−1
χ_{18}		0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0
χ_{19}		0	0	0	0	0	0	0	0	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0
χ_{20}		0	0	0	0	0	0	0	0	1	1	1	0	0	0	−1	2	2	2	2	2	2	2
χ_{21}		0	0	0	0	0	0	0	0	−2	1	1	0	0	0	0	0	0	3	3	3	3	3
χ_{22}		0	0	0	0	0	0	0	0	−1	−1	−1	0	0	1	0	0	0	0	0	0	0	0
χ_{23}		0	0	0	0	0	0	0	0	−1	−1	−1	0	0	1	0	0	0	0	0	0	0	0
χ_{24}		0	0	0	0	0	0	0	0	2	−1	−1	0	0	0	2	2	2	−1	−1	−1	−1	−1
χ_{25}		0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	2	2	2	−1	−1	−1	−1	−1
χ_{26}		2	2	2	2	2	0	0	0	−1	−1	−1	0	0	0	−2	1	1	−2	−2	−2	−2	−2
χ_{27}		−2	−2	−2	−2	−2	0	0	0	−1	−1	−1	0	0	0	−2	1	1	−2	−2	−2	−2	−2
χ_{28}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	−2	−2	−2	−2	−2	−2	−2
χ_{29}		−2	−2	−2	−2	−2	0	0	0	0	0	0	0	0	0	2	−1	−1	−1	−1	−1	−1	−1
χ_{30}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	2	2	−1	−1	−1	−1	−1
χ_{31}		2	2	2	2	2	0	0	0	0	0	0	0	0	0	2	−1	−1	−1	−1	−1	−1	−1
χ_{32}		0	0	0	0	0	0	0	0	1	1	1	0	0	0	−2	−2	−2	−2	−2	−2	−2	−2
χ_{33}		1	1	1	1	1	−1	−1	−1	0	0	0	−1	−1	0	−3	0	0	0	0	0	0	0

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Table 11.17 (continued)

$[x]_G$	$8C$					$8D$			$9A$	$9B$		$10A$		$11A$	$12A$	$12B$		$12C$				
$[g]_{\overline{G}}$	$8j$	$8k$	$8l$	$8m$	$8n$	$8o$	$8p$	$8q$	$9a$	$9b$	$18a$	$10b$	$20a$	$11a$	$12h$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$
χ_{34}	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	1	1	0	-3	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	2	2	2	0	0	0	0	0	0	1	-2	-2	1	1	1	1	1
χ_{36}	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	1	-2	-2	1	1	1	1	1
χ_{37}	0	0	0	0	0	0	0	0	-2	1	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	-2	1	1	1	1	1	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	3	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	2	2	2	2	2	2
χ_{43}	0	0	0	0	0	2	2	2	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
χ_{44}	-3	-3	1	1	1	3	-1	-1	0	0	0	0	0	0	0	3	-1	9	1	1	1	-3
χ_{45}	3	3	-1	-1	-1	3	-1	-1	0	0	0	0	0	0	0	3	-1	9	1	1	1	-3
χ_{46}	1	1	1	-3	1	-1	3	-1	0	0	0	0	0	0	0	3	-1	-3	-3	-3	5	1
χ_{47}	-1	-1	-1	3	-1	-1	3	-1	0	0	0	0	0	0	0	3	-1	-3	-3	-3	5	1
χ_{48}	0	0	0	0	0	-2	-2	2	0	0	0	0	0	0	0	6	-2	-6	2	2	-6	2
χ_{49}	0	0	A	0	$-A$	0	0	0	0	0	0	0	0	0	0	6	-2	0	0	0	0	0
χ_{50}	0	0	$-A$	0	A	0	0	0	0	0	0	0	0	0	0	6	-2	0	0	0	0	0
χ_{51}	0	0	0	0	0	6	-2	-2	0	0	0	0	0	0	0	0	0	-9	-1	-1	-1	3
χ_{52}	0	0	0	0	0	-2	6	-2	0	0	0	0	0	0	0	0	0	3	3	3	-5	-1
χ_{53}	0	0	0	0	0	-2	-2	2	0	0	0	0	0	0	0	0	0	3	$\overline{E}$	E	3	-1
χ_{54}	0	0	0	0	0	-2	-2	2	0	0	0	0	0	0	0	0	0	3	E	$\overline{E}$	3	-1
χ_{55}	2	2	-2	2	-2	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0
χ_{56}	0	0	$-A$	0	A	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-9	-1	-1	-1	3
χ_{58}	-4	-4	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-9	-1	-1	-1	3
χ_{59}	4	4	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	-5	-1
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	-5	-1
χ_{61}	-1	-1	-1	3	-1	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	1	1	1	-3	1	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	3	3	-1	-1	-1	1	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	-3	-3	1	1	1	1	-3	1	0	0	0	0	0	0	0	0	0	6	-2	-2	6	-2
χ_{65}	-2	-2	2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	2	2	-2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0

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Table 11.17 (continued)

$[x]_G$	$8C$					$8D$			$9A$	$9B$		$10A$		$11A$	$12A$	$12B$		$12C$				
$[g]_{\overline{G}}$	$8j$	$8k$	$8l$	$8m$	$8n$	$8o$	$8p$	$8q$	$9a$	$9b$	$18a$	$10b$	$20a$	$11a$	$12h$	$12i$	$12j$	$12k$	$12l$	$12m$	$12n$	$12o$
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0
χ_{73}	0	0	0	0	0	2	2	−2	0	0	0	0	0	0	0	0	0	9	1	1	1	−3
χ_{74}	0	0	A	0	$-A$	0	0	0	0	0	0	0	0	0	0	0	0	−3	−3	−3	5	1
χ_{75}	0	0	$-A$	0	A	0	0	0	0	0	0	0	0	0	0	0	0	−3	$-\overline{E}$	$-E$	−3	1
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−3	$-E$	$-\overline{E}$	−3	1
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	2	2	−2	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	−2	−2	2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	−2	−2	2	−2	2	0	0	0	0	3	−1	1	−1	0	0	0	0	6	−2	−2	−2	2
χ_{86}	0	0	$-A$	0	A	0	0	0	0	3	−1	−1	1	0	0	0	0	6	−2	−2	−2	2
χ_{87}	0	0	A	0	$-A$	0	0	0	0	0	0	1	−1	0	0	0	0	−6	2	2	2	−2
χ_{88}	0	0	A	0	$-A$	0	0	0	0	0	0	−1	1	0	0	0	0	−6	2	2	2	−2
χ_{89}	0	0	0	0	0	0	0	0	0	3	−1	0	0	0	0	0	0	−6	2	2	2	−2
χ_{90}	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−6	2	2	2	−2
χ_{91}	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−6	2	2	2	−2
χ_{92}	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{93}	4	−4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	6	−2	−2	−2	2
χ_{95}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	−2	−2	−2	2
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	−2	−2	−2	2
χ_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	−2	−2	−2	2
χ_{102}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0	0	0	3	−1	1	−1	0	0	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	0	3	−1	−1	1	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	−3	1	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	−4	4	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0
χ_{108}	4	−4	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−6	2	2	2	−2

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Table 11.17 (continued)

$[x]_G$		$12D$	$12E$				$12F$		$15A$	$16A$		$16B$		$18A$	$24a$		$24B$	
$[g]_{\overline{G}}$		$12p$	$12q$	$12r$	$12s$	$12t$	$12u$	$12v$	$15a$	$16a$	$16b$	$16c$	$16d$	$18b$	$24a$	$24b$	$24c$	$24d$
$\lvert C_{\overline{G}}(g)\rvert$	2	3	5	5	5	5	3	3	0	5	5	5	5	1	4	4	4	4
	3	1	1	1	1	1	1	1	1	0	0	0	0	2	1	1	1	1
	5	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2		1	1	1	1	1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1
χ_3		1	1	1	1	1	0	0	0	F	F	$-F$	$-F$	0	F	F	$-F$	$-F$
χ_4		1	1	1	1	1	0	0	0	$-F$	$-F$	F	F	0	$-F$	$-F$	F	F
χ_5		−2	1	1	1	1	0	0	−1	0	0	0	0	−1	0	0	0	0
χ_6		0	1	1	1	1	0	0	−1	0	0	0	0	0	−1	−1	−1	−1
χ_7		0	1	1	1	1	0	0	−1	0	0	0	0	0	1	1	1	1
χ_8		0	−1	−1	−1	−1	1	1	0	−1	−1	−1	−1	−1	0	0	0	0
χ_9		0	−1	−1	−1	−1	−1	−1	0	1	1	1	1	−1	0	0	0	0
χ_{10}		1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0
χ_{11}		−1	0	0	0	0	0	0	0	$-F$	$-F$	F	F	1	F	F	$-F$	$-F$
χ_{12}		−1	0	0	0	0	0	0	0	F	F	$-F$	$-F$	1	$-F$	$-F$	F	F
χ_{13}		0	0	0	0	0	−1	−1	0	0	0	0	0	0	−1	−1	−1	−1
χ_{14}		0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	1	1
χ_{15}		1	−1	−1	−1	−1	0	0	−1	0	0	0	0	0	0	0	0	0
χ_{16}		0	−1	−1	−1	−1	0	0	0	1	1	1	1	0	−1	−1	−1	−1
χ_{17}		0	−1	−1	−1	−1	0	0	0	−1	−1	−1	−1	0	1	1	1	1
χ_{18}		0	0	0	0	0	1	1	1	0	0	0	0	−1	0	0	0	0
χ_{19}		0	0	0	0	0	−1	−1	1	0	0	0	0	−1	0	0	0	0
χ_{20}		−1	−2	−2	−2	−2	0	0	0	0	0	0	0	1	0	0	0	0
χ_{21}		−2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}		0	0	0	0	0	0	0	0	0	0	0	0	−1	F	F	$-F$	$-F$
χ_{23}		0	0	0	0	0	0	0	0	0	0	0	0	−1	$-F$	$-F$	F	F
χ_{24}		0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}		0	−1	−1	−1	−1	0	0	0	0	0	0	0	1	0	0	0	0
χ_{26}		0	0	0	0	0	1	1	0	0	0	0	0	1	−1	−1	−1	−1
χ_{27}		0	0	0	0	0	−1	−1	0	0	0	0	0	1	1	1	1	1
χ_{28}		0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
χ_{29}		0	1	1	1	1	1	1	0	0	0	0	0	0	−1	−1	−1	−1
χ_{30}		1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}		0	1	1	1	1	−1	−1	0	0	0	0	0	0	1	1	1	1
χ_{32}		0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
χ_{33}		−1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
χ_{34}		−1	0	0	0	0	0	0	1	−1	−1	−1	−1	0	0	0	0	0
χ_{35}		1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}		1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}		0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0
χ_{38}		0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0
χ_{39}		0	0	0	0	0	0	0	1	0	0	0	0	−1	0	0	0	0
χ_{40}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}		1	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0
χ_{42}		−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}		0	3	−1	−1	−1	0	0	0	1	−1	1	−1	0	−1	1	−1	1
χ_{45}		0	3	−1	−1	−1	0	0	0	−1	1	−1	1	0	1	−1	1	−1
χ_{46}		0	−1	3	−1	−1	0	0	0	−1	1	−1	1	0	−1	1	−1	1
χ_{47}		0	−1	3	−1	−1	0	0	0	1	−1	1	−1	0	1	−1	1	−1
χ_{48}		0	−2	−2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}		0	0	0	0	0	0	0	0	$-F$	F	F	$-F$	0	0	0	0	0
χ_{50}		0	0	0	0	0	0	0	0	F	$-F$	$-F$	F	0	0	0	0	0
χ_{51}		0	−3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}		0	1	−3	1	1	0	0	0	0	0	0	0	0	0	0	0	0

Continued on next page

Table 11.17 (continued)

$[x]_G$	$12D$	$12E$				$12F$		$15A$	$16A$		$16B$		$18A$	$24a$		$24B$	
$[g]_{\overline{G}}$	$12p$	$12q$	$12r$	$12s$	$12t$	$12u$	$12v$	$15a$	$16a$	$16b$	$16c$	$16d$	$18b$	$24a$	$24b$	$24c$	$24d$
χ_{53}	0	1	1	G	$\overline{G}$	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	1	1	$\overline{G}$	G	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1
χ_{56}	0	-1	3	-1	-1	0	0	0	0	0	0	0	0	1	-1	1	-1
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	F	$-F$	$-F$	F
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	$-F$	F	F	$-F$
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	F	$-F$	$-F$	F
χ_{60}	0	-2	-2	2	2	0	0	0	0	0	0	0	0	$-F$	F	F	$-F$
χ_{61}	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	1	-1	1	-1	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	-3	1	1	1	0	0	0	0	0	0	0	0	-1	1	-1	1
χ_{70}	0	1	-3	1	1	0	0	0	0	0	0	0	0	-1	1	-1	1
χ_{71}	0	1	1	G	$\overline{G}$	0	0	0	0	0	0	0	0	1	-1	1	-1
χ_{72}	0	1	1	$\overline{G}$	G	0	0	0	0	0	0	0	0	1	-1	1	-1
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	F	$-F$	$-F$	F	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	$-F$	F	F	$-F$	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	F	$-F$	$-F$	F
χ_{81}	0	0	0	0	0	1	-1	0	0	0	0	0	0	$-F$	F	F	$-F$
χ_{82}	0	0	0	0	0	-1	1	0	0	0	0	0	0	-1	1	-1	1
χ_{83}	0	0	0	0	0	-1	1	0	0	0	0	0	0	1	-1	1	-1
χ_{84}	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{86}	0	3	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	0	3	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	-1	3	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{92}	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where in Table 11.17,

- $A = 2 * E(8) + 2 * E(8)^3 = 2 * ER(-2) = 2\sqrt{2}i$,
- $B = -4 * E(8) - 4 * E(8)^3 = -4 * ER(-2) = -4\sqrt{2}i = -2A$,
- $C = 6 * E(8) + 6 * E(8)^3 = 6 * ER(-2) = 6\sqrt{2}i$,
- $D = 8 * E(8) + 8 * E(8)^3 = 8 * ER(-2) = 8\sqrt{2}i$,
- $E = 5 * E(3) - 3 * E(3)^2 = -1 + 4 * ER(-3) = 3 + 8b3$,
- $F = -E(8) - E(8)^3 = -ER(-2) = -\sqrt{2}i$,
- $G = -E(3) + 3 * E(3)^2 = -1 - 2 * ER(-3) = -1 - 2\sqrt{3}i$.

Bibliography

[1] F. Ali, *Fischer-Clifford Theory for Split and Non-Split Group Extensions*, PhD Thesis, University of Natal, Pietermaritzburg, 2001.

[2] F. Ali and J. Moori, *The Fischer-Clifford matrices of a maximal subgroup of* Fi'_{24}, Journal of Representation Theory, **7** (2003), 300 - 321.

[3] F. Ali, *The Fischer-Clifford matrices of a maximal subgroup of the sporadic simple group of Held*, Algebra Colloquium, **14** (2007), 135 - 142.

[4] J. L. Alperin and R. B. Bell, *Groups and Representations*, Graduate Texts in Mathematics, **162**, Springer-Verlag, New York, 1995.

[5] A. Baker, *Representations of Finite Groups*, University of Glasgow, 2005. Available at: http://math.uchicago.edu/ margalit/repthy/

[6] R. W. Barraclough, *Some Calculations Related to the Monster Group*, PhD Thesis, University of Birmingham, Birmingham, 2005.

[7] A. B. M. Basheer, *Character Tables of the General Linear Group and Some of its Subroups*, MSc Dissertation, University of KwaZulu-Natal, Pietermaritzburg, 2009.

[8] A. B. M. Basheer and J. Moori, *On a group of the form* $3^7{:}Sp(6,2)$, submitted.

[9] A. B. M. Basheer and J. Moori, *Fischer matrices of Dempwolff group* $2^{5\cdot}GL(5,2)$, International Journal of Group Theory, **1** No 4 (2012) 43 - 63.

[10] A. B. M. Basheer and J. Moori, *Fischer matrices of the group* $2^{1+8\cdot}_{+}A_9$, submitted.

[11] A. B. M. Basheer and J. Moori, *The character table of the non-split extension* $2^{6\cdot}Sp(6,2)$, submitted.

[12] A. B. M. Basheer and J. Moori, *On the non-split extension* $2^{2n\cdot}Sp(2n,2)$ *and the character table of* $2^{8\cdot}Sp(8,2)$, to be submitted.

[13] A. B. M. Basheer and J. Moori, *On a group of the form* $2^{10}{:}(U_5(2){:}2)$, submitted.

[14] A. B. M. Basheer and J. Moori, *Clifford-Fischer theory applied to a group of the form* $2^{1+6}_{-}{:}((3^{1+2}{:}8){:}2)$, submitted.

[15] Y. Berkovich and E. Zhmud, *Characters of Finite Groups*. Part 1. Translations of Mathematical Monographs, **172**, Amer. Math. Soc., Providence, RI, 1998.

[16] W. Bosma and J. J. Cannon, *Handbook of Magma Functions*, Department of Mathematics, University of Sydeny, November 1994.

[17] J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker and R. A. Wilson, *Atlas of Finite Groups*, Clarendon Press, Oxford, 1985.

[18] A. H. Clifford, *Representations induced in an invariant subgroup*, Ann. of Math. **38** (1937), 533 - 550.

[19] C. W. Curtis and I. Reiner, *Methods of Representation Theory with Applications to Finite Groups and Orders*, **2**, John Wiley and Sons, New York, A Wiley-Interscience Series of Texts, Monographs and Tracts, 1987.

[20] C. W. Curtis, *Pioneers of Representation Theory: Frobenius, Burnside, Schur, and Brauer*, American Mathematical Society, London Mathematical Society, 1999.

[21] U. Dempwolff, *On extensions of elementary abelian groups of order* 2^n *by* $GL(n,2)$ *and the degree 2-cohomology of* $GL(n,2)$, Unpublished.

[22] U. Dempwolff, *On extensions of elementary abelian groups of order* 2^5 *by* $GL(5,2)$, Rend. Sem. Mat. Univ. Padova, **48** (1972), 359 - 364.

[23] U. Dempwolff, *Extensions of elementary abelian groups of order* 2^{2n} *by* $S_{2n}(2)$ *and the degree* $2-$*cohomology of* $S_{2n}(2)$, Illinois J. Math., **18** (1974), 451 - 468.

[24] W. Feit, *Characters of Finite Groups*, W. A. Benjamin, New York, 1967.

[25] B. Fischer, *Clifford matrizen*, manuscript (1982).

[26] B. Fischer, *Unpublished manuscript* (1985).

[27] B. Fischer, *Clifford matrices*, Representation theory of finite groups and finite-dimensional Lie algebras (eds G. O. Michler and C. M. Ringel; Birkhäuser, Basel, 1991), 1 - 16.

[28] S. M. Gagola, JR, *An extension theorem for characters*, Proc. Amer. Math. Soc., **83** (1981), 25 - 26.

[29] P. X. Gallagher, *Group characters and normal Hall subgroups*, Nagoya Math. Journal **21** (1962), 223 - 230.

[30] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.4.10; 2007.* (http://www.gap-system.org)

[31] D. Gorenstein, Finite Groups, Harper and Row Publishers, New York, 1968.

[32] R. J. Haggarty and J. F. Humphreys, *Projective characters of finite group*, Proc. London Math. Soc., **36** (1975), 176 - 192.

[33] G. Havas, L. H. Soicher and R. A. Wilson, *A presentation for the Thompson sporadic simple group*, in (William M. Kantor and Ákos Seress, eds.) Groups and Computation III. Ohio State University Math. Res. Inst., Berlin, New York, Walter de Gruyter, (1999), 192 - 200.

[34] P. N. Hoffman and J. F. Humphreys, *Projective Representations of the Symmetric Groups*, Clarendon Press, Oxford, 1992.

[35] J. F. Humphreys, *Projective character tables for the finite simple groups of order less than one million*, Comm. Algebra, **11** (1983), 725 - 751.

[36] B. Huppert, *Endliche Gruppen*, Springer-Verlag, Berlin-New York, 1967.

[37] B. Huppert, *Character Theory of Finite Groups*, Walter de Gruyter, Berlin, 1998.

[38] I. M. Isaacs, *Character Theory of Finite Groups*, Academic press, New York, San Fransisco, London, 1976.

[39] G. D. James, *Representations of the General Linear Groups*, London Mathematical Society, Cambridge University Press, **94**, Lecture Note Series, 1984.

[40] G. James and M. Liebeck, *Representations and Characters of Groups*, 2^{nd} edition, Cambridge Mathematical Textbooks, Cambridge University Press, 2001.

[41] A. W. Joshi, *Elements of Group Theory for Physicists*, 3^{rd} edition, Wiley Eastern Limited, New Delhi, India, 1982.

[42] G. Karpilovsky, *On extensions of characters from normal subgroups*, Proc. Edinburgh Math. Soc., **27** (1984), 7 - 9.

[43] G. Karpilovsky, *Projective Representations of Finite Groups*, Monographs and Textbooks in Pure and Applied Mathematics, **94**, (Marcel Dekker, New York), 1985.

[44] R. J. List, *On the characters of* $2^{n-\epsilon \cdot}S_n$, Arch. Math., **51** (1988), 118 - 124.

[45] Maxima, *A Computer Algebra System. Version 5.18.1; 2009.* (http://maxima.sourceforge.net)

[46] G. Michler, *Theory of Finite Simple Groups*, New Mathematical Monographs, **8**, Cambridge University Press, 2006.

[47] J. Moori, *On the Groups* G^+ *and* $\overline{G}$ *of the form* $2^{10}{:}M_{22}$ *and* $2^{10}{:}\overline{M}_{22}$, PhD Thesis, University of Birmingham, 1975.

[48] J. Moori, *On certain groups associated with the smallest Fischer group*, J. London Math. Soc., **2** (1981), 61 - 67.

[49] J. Moori, *Finite Groups and Representation Theory*, Lecture Notes, University of KawZulu-Natal, Pietermaritzburg, 2008.

[50] J. Moori and Z. Mpono, *The Fischer-Clifford matrices of the group* $2^6{:}SP_6(2)$. Quaest. Math., **22** (1999), 257 - 298.

[51] A. O. Morris, *Projective representations of finite groups*, Proc. on Conference on Clifford Algebra, Matscience, Madras, 1971.

[52] A. O. Morris, *Projective representations of Weyl groups*, J. London Math. Soc., **8** (1974), 125 - 133.

[53] A. O. Morris, *Projective representations of exceptional Weyl groups*, J. Algebra, **29** (1974), 567 - 586.

[54] A. O. Morris, *Projective representations of reflection groups*, Proc. London Math. Soc., **32** (1976), 403 - 420.

[55] Z. E. Mpono, *Fischer Clifford Theory and Character Tables of Group Extensions*, PhD Thesis, University of Natal, Pietermaritzburg, 1998.

[56] H. Nagao and Y. Tsushima, *Representations of Finite Groups*, Academic Press, San Diego, 1987.

[57] H. Pahlings, *The character table of* $2_+^{1+22}.Co_2$, Journal of Algebra, **315** (2007), 301 - 325.

[58] K. Lux and H. Pahlings, *Representations of Groups: A Computational Approach*, Cambridge University Press, Cambridge, 2010.

[59] E. W. Read, Projective characters of the Weyl group of type F_4, J. London Math. Soc., **8** (1974), 83 - 93.

[60] E. W. Read, The linear and projective characters of the finite reflection groups of type H_4, Quart. J. Math. Oxford, **25** (1974), 73 -79.

[61] E. W. Read, On projective representations of finite reflection groups of type B_l and D_l, J. London Math. Soc., **10** (1975) 129 -142.

[62] B. G. Rodrigues, *On The Theory and Examples of Group Extensions*, MSc Thesis, University of Natal, Pietermaritzburg, 1999.

[63] D. J. Robinson, *A Course in the Theory of Groups*, 2^{nd} edition, **80**, Springer-Verlag New York, Graduate Texts in Mathematics, 1996.

[64] J. J. Rotman, *An Introduction to the Theory of Groups*, 4^{th} edition, **148**, Springer-Verlag New York, Graduate Texts in Mathematics, 1995.

[65] B. E. Sagan, *The Symmetric Group Representataions, Combintorial Algorithms, and Symmetric Functions*, 2*nd* edition, **203**, Springer-Verlag New York, Graduate Texts in Mathematics, 2001.

[66] R. B. Salleh, *On the Construction of the Character Tables of Extension Groups*, PhD thesis, University of Birmingham, 1982.

[67] U. Schiffer, *Cliffordmatrizen*, Diplomarbeit, Lehrstul D Fur Matematik, RWTH, Aachen, 1995.

[68] I. Schur, *Über die darstellung der endlichen gruppen durch gebrochene lineare substitutionen*, J. für Math., **127** (1904), 20 - 50.

[69] T. T. Seretlo, *Fischer-Clifford Matrices and Character Tables of Certain Groups Associated with Simple Groups* $O_8^+(2)$, *HS and Ly*, PhD Thesis, University of KwaZulu-Natal, Pietermaitzburg, 2012.

[70] N. S. Whitely, *Fischer Matrices and Character Tables of Group Extensions*, MSc Thesis, University of Natal, Pietermaritzburg, 1993.

[71] R. A. Wilson et al., *Atlas of finite group representations*, http://brauer.maths.qmul.ac.uk/Atlas/v3/

[72] K. Zimba, *Fischer-Clifford Matrices of the Generalized Symmetric Group and Some Associated Groups*, PhD Thesis, University of KwaZulu-Natal, Pietermaritzburg, 2005.