Exploring pre-service teachers' views on the use of technology based teaching methods for teaching geometry

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By

Reginald Gerald Govender

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Supervisor: Dr J. Naidoo

Abstract

The purpose of this study was to explore pre-service teachers' views on the use of technology based teaching methods in geometry. In 2008, geometry and other topics formed part of the newly-introduced optional Mathematics paper 3. The main reason for an optional paper was that teachers lacked knowledge, resulting in poor learner performance. In 2011, geometry was re-instated in the South African mathematics curriculum as a compulsory topic. Information Communications Technologies (ICTs) offer essential features to the learning and teaching environment. Ongoing advancements in ICT, including software upgrades and fiber optic connections, have significantly enhanced institutions' capacity by expanding their range of services and capabilities. Both teachers and learners have an important role to play in using ICTs in education. However, in order to effectively integrate technology into mathematics education, teachers need to know when, where, and how to prepare and use such technology. In recent years, there has been an increase in teaching and communicating with learners through an online setting, with universities and teaching institutions investing resources in this effort. This study focuses on geometry applets that are loaded online, which pre-service teachers (students) are exposed too, in order to gauge their views on their usage and experience. Geometry topics from the General Education and Training (GET) and Further Education and Training (FET) band were selected for this investigation, namely, the theorem of Pythagoras and circle geometry. Data for this study were gathered through a literature search, administration of questionnaires in the form of worksheets, interviews, observations; and surveys. An interpretivist paradigm, which allows one to understand the phenomenon under study, was adapted within this case study, together with elements from an ethnographic study. The group of pre-service teachers averaged 79.6% for the theorem of Pythagoras and 80% for circle geometry using the selected technology. The themes emerging from the findings were: Motivation through the use of technology; visualisation is key when teaching geometry; using technology to teach is not for everyone; geometrical conceptual growth through technology; the use of technology promotes independent thinking and Geometric Habit of Mind (GHOM); and the quality of teacher training at university. A positive correlation was established between participants' understanding, views; and use of a technology based method when teaching geometry. Further analysis revealed three categories in which teachers can be classified based on their adaption of technology to geometry: Knowing the essentials; a tool in learning geometry; and a catalyst to learning geometry.

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Dedication

This thesis is dedicated to my:

Dear Mum, as I would not have made it thus far without her selfless sacrifices, support and inspiration.

AND

My Dad who passed away before seeing me achieve everything he knew I was capable of.

Acknowledgements

I would like to thank the greatest teacher Almighty God for helping me to complete this study. I have not seen Him and He is known by many names, but in my weakness He is my strength.

I am grateful to my supervisor, Dr J. Naidoo, for her patience, encouragement and meticulous academic guidance. You have imparted invaluable knowledge and research skills to me.

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All my lecturers at the School of Science, Mathematics and Technology Education who, during my prior studies, made me believe in myself and knew that work of this nature is indeed achievable.

My two sisters for their continuous support.

Last, but not least, the student teachers who participated in this study without whom this research would have not been possible. God bless you in your future teaching careers.

This study was a truly enriching and rewarding experience.

Declaration

I Reginald Gerald Govender declare that

(i) The research reported in this dissertation, except where otherwise indicated, is my original work.

(ii) This dissertation has not been submitted for any degree or examination at any other university.

(iii) This dissertation does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.

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Reginald Gerald Govender (207501841)

"As the candidate's Supervisor I agree/~~do not agree~~ to the submission of this dissertation."

Dr J. Naidoo

List of Abbreviations

BRICS: Brazil, Russia, India, China and South Africa

CAPS: Curriculum Assessment Policy Statement

DBE: Department of Basic Education

DOEs: Dynamic Online Environments

FET: Further Education and Training

FIFA: Fédération Internationale de Football Association

GET: General Education Training

GHOM: Geometric Habits of Mind

HCI: Human Communication Interaction

ICT/s: Information Communications Technology/ies

LTSM: Learner Teacher Support Materials

NSC: National Curriculum Statement

OBE: Outcomes Based Education

PC: Personal Computer

PRILS: Progress in International Reading Literacy Study

PST: Pre-Service Teacher

SAMEMEQ: Southern and East African Consortium for Monitoring Educational Quality

SRL: Self-Regulatory Learning

STEM: Science, Technology, Engineering, and Mathematics

TIMMS: Trends in International Mathematics and Science Study

TVET: Technical Vocational Educational and Training

ZPD: Zone of Proximal Development

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Chapter one

Introduction

1.1 Background

Mathematics can be described as a language which uses numbers and symbols to describe patterns and phenomena, in order to help one to understand the world around one. Mathematics comprises of various disciplines such as arithmetic, trigonometry, geometry, calculus; and measurement. In South Africa, geometry was reintroduced in the Mathematics paper 2 in 2011, with the introduction of the Curriculum and Assessment Policy Statement (CAPS). The first set of grade 12 learners who sat for Mathematics paper 2, which phased in the CAPS was the class of 2014. The Department of Basic Education's White Paper on e-Education aimed to transform the learning and teaching paradigm through the use of Information and Communication Technologies (ICTs). Technology has evolved over the years, and 21st century living includes computer literacy (Mayisela, 2013). Indeed, technology is a basic building block of today's highly globalised society. People are becoming increasingly dependent on technologies such as cellphones, email, the Internet, social networks, etc. Thus, it is imperative that such technologies are incorporated in the current generation's teaching and learning.

1.2 Purpose of study

Geometry was an optional subject in the South African curriculum from 2008 until the end of 2010. Learners' poor performance in mathematics, particularly geometry has been of great concern to all education stakeholders, including mathematicians, educators, parents and the government. The removal of this subject from the curriculum in 2008 was the result of teachers' lack of geometric knowledge which resulted in poor learner performance (Allais, Kitto, & Pillay, 2008; Govender, 2010). The reasons for teaching geometry in schools include that it connects mathematics with the physical world, enables the visualisation of ideas from other areas of mathematics and provides an example of mathematical systems (Usiskin, 2007).

Information and Communication Technologies is the collective term given to such technologies; it defines the wide range of technological tools and resources used to communicate, create, disseminate, store; and manage information (Mohanty, 2011). The effectiveness of these technologies is not based on the technology alone, but on the combination of the learning activity system that incorporates the technology. The Department of Basic Education (DBE) (2011) notes that content should be supported by available technologies to make and test conjectures in mathematics. However, they should not replace the teaching and learning process as this could result in loss of focus on the subject content. For example, in using calculators, if learners don't know the syntax of input, they will focus on the syntax rather than the mathematics being taught.

Computer technologies have been utilised in school mathematics since the invention of four-function calculators in the 1970s (Goose, 2010). Many advancements have since been made in teaching and learning environments such as smart boards, mathematical software, clickers, and mobile apps, etc. The influence of computer technologies on education appears to be more extensive in mathematics than other subjects. This could be the result of the close connection between the two disciplines, since computer science was previously a branch of mathematics, which was later, classified separately (Tatar, 2013).

Mathematics plays an important role in the development and status of a nation. The Fédération Internationale de Football Association (FIFA), the organisation that governs international football tournaments, drove the FIFA Legacy School Projects during the 2010 soccer World Cup which was hosted by South Africa. This initiative resulted in 1 650 schools being connected online (Ngobeni, 2015). Such initiatives enable the incorporation of technology in the classroom, raising the standard of education.

1.3 Focus of study

With the growing influence of technology in education, it is important to understand how South African pre-service teachers (PSTs) view its use when teaching geometry, a topic that was once omitted from the Mathematics curriculum. Our daily lives resolve around technology. Young and older teachers deliver the same content in different ways to suit the current generation of learners. The online platform has made information easily accessible anywhere in the world. Many institutions now offer all lessons and activities online (Moore & Kearsley, 2011), which promotes environmental sustainability and offers learners the flexibility of distance learning.

This study explored the views of PSTs on the use of technology in teaching geometry. For the purpose of the study, views are taken as subjective knowledge. Pre-service teachers' knowledge of and skills in technology and technology readiness are crucial as they prepare to integrate technologies in their future classrooms. The study focuses on Dynamic Online Environments (DOEs), the term assigned to the technology used in this research study. For the purpose of this study, DOEs are virtual spaces on the Internet which can be used for teaching and learning geometry. The key properties are interaction, visualisation and easy access. These pre-service teachers would soon obtain their qualification and apply teaching methodologies, which they were equipped with at university, to their technology savvy learners. For the purpose of this study, in-service and pre-service teachers are regarded as PSTs and both study at the School of Mathematics Education.

1.4 Technology in Education

The KwaZulu-Natal (KZN) educational summit 2015 theme was: “All doors of learning and culture shall be opened” which was adopted from the South African Freedom Charter drafted (Suttner & Cronin, 1986) more than 50 years ago. This theme underpinned the integration of ICTs into schools which was the focus of the summit. Globalising technology promotes international and local competition and development as well as the development of knowledge and skills. Teacher education institutions throughout the world have adopted educational reforms in response to this changing landscape (Cheng, 2005; Leendertz, Blignaut, Nieuwoudt, Els & Ellis, 2013). Raising educational standards makes a country’s labour force more efficient. This is significant in light of the high youth (15-24 years old) unemployment rate in South Africa, which is estimated at over 50% (Word Bank, 2015a). Thus, it is imperative to integrate technology in education, as it plays a major role in teaching and learning, thereby creating a ripple effect on the country’s economy.

In recent years, the South African DBE has rolled out the iBox supplied by Sangari[1] in schools. The aim is to improve learning in science, technology, engineering and mathematics, otherwise known as STEM subjects, in order to produce competent engineers, doctors and chartered accountants (Cook, 2014). This is in accordance with the 2014 Strategic Plan and the e-Learning Strategic Goal to move ICTs into the classroom. The iBox is the size of a suitcase and has a built in projector, laptop; and speakers. According to Legoete (the DBE co-ordinator for Sedibeng West in District 8), the district’s pass rate improved from 28% in 2009 to 68% in 2013; this is attributed to utilisation of the iBox (Fin24, 2014).

[1] Sangari is a supplier of educational resources to schools, colleges, universities and industrial training centres.

The recent textbook saga in Limpopo Province caused a major stir in the education sector. Delivery of Learner Teacher Support Materials (LTSMs) by the DBE is important for efficient teaching and learning. Many institutions are now opting for PC (Personal Computer[2]) tablets, as these devices offer electronic copies of textbooks which are loaded onto the device within a matter of seconds. They also offer many more functions that can be used in teaching and learning. Multimedia content such as video, audio, colour, hyperlinks and animation are part of some e-books. The White Paper on e-Education emphasises the integration of ICTs in schools. This would enable learners to become critical thinkers, efficient communicators, collaborative workers, problem-solvers, and managers of large amounts of information; and to be autonomous. In today's technology-driven age, technologies like electronic communication, wireless devices, the Internet, smartphones and PC tablets are used on a daily basis. Hence, the meaningful integration of technology in mathematics is imperative as it makes learning concepts in an understandable format available at universal level.

1.5 Research objectives

The aim of this study was to explore PSTs' views on using a technology based approach to the teaching and learning of geometry. To achieve this aim, the objectives were to explore:

- pre-service teachers' knowledge of technology based teaching methods for teaching geometry.
- pre-service teachers' awareness of technology based teaching methods for teaching geometry.
- pre-service teachers' views on technology based teaching methods to teach geometry.

In line with the purpose and objectives of this study, the following questions were investigated:

- What technology based teaching methods to teach geometry are pre-service teachers aware of?
- How do pre-service teachers view the use of technology based teaching methods to teach geometry?
- Why do pre-service teachers have these views on the use of technology based teaching methods to teach geometry?

[2] A computer that can be used by one person

1.6 Structure of the Dissertation

This study is organised into the following six chapters:

Chapter one introduced the study and discusses its scope and context. It presents the background to the study, and its purpose, focus and objectives. The motivation for the study, its significance, and the research assumptions are also discussed. The chapter concludes by outlining the structure of the dissertation.

Chapter two reviews the relevant literature which guided and supported this study. The nature of mathematics and how one interprets it is discussed, as well as the South African school curriculum and geometry. Theories and concepts in designing a geometry curriculum and the use of technology are also highlighted. Links are drawn between the relevant theories and concepts. The role of technology, including applets and the Internet (going online) is investigated. The relationship between metacognition and achievement based on self-regulation is examined. The chapter also reviews the literature on PSTs' understanding of Euclidean geometry.

Chapter three outlines the theoretical framework employed for this study, and the main concepts used in relation to the purpose of the study and the research questions. It reviews the pertinent literature that underpins the design of geometry content together with learner-centred instruction and technology, with a focus on the role of online technology in Mathematics teaching and in promoting conceptual understanding of geometry.

Chapter four sets out the rationale for the methodology selected for this study. It presents and describes the data collection methods and procedures, i.e., the design of the data collection instruments, the chronological order of the data collected, the location, ethical considerations; and informed consent. In addition, it outlines and justifies the research design and the trustworthiness of the study.

Chapter five presents the analysis of the data and the study's findings. Statistical procedures and the results of the data analysis are graphically represented. The findings are examined and interpreted, giving rise to relevant themes which are presented and discussed.

Finally, **chapter six** provides a conclusion based on the findings in Chapter five. This chapter discusses the results of the study in response to the three research questions. The study's limitations are also discussed, and recommendations are made for further research.

Chapter two

Literature review

2.1 Introduction

In today's world, it has become impossible to conclude a day without encountering some sort of mathematics or technology, such as computers, data projectors, PowerPoint slides, smart boards, smartphones, parking ticket dispensers, Internet access, social media; and so on. We are living in an ICT Age where a person can be regarded as either a digital native or a digital immigrant (Guo, 2008; Kanematsu & Barry, 2016). The current generation of learners are well versed in technology and its operations. Consequently, Nair (2012) and Tucker (2014) note that unless the gap between the current era of learners and technology in the teaching and learning environment is bridged, education will became irrelevant. It is imperative that teachers implement ICTs in this environment. This is the perfect opportunity to show learners how to use something that appeals to them, for their benefit.

Instructional Technology is a complex, integrated process which involves people carrying out procedures and ideas (Ely & Englewood, 1996). According to Kleitman, Stankov, Allwood, Young and Mak (2013), a person with high self-confidence has the potential to perform tasks better. Embedding mathematics learning into a digitalised environment could enhance learners' self-confidence, and motivate them to learn and improve their performance. Thus it is important to provide conditions that promote significant improvement in performance. Instructional Technology incorporates devices to analyse problems, as well as evaluate and manage solutions to problems, which are based on purposive and controlled learning scenarios (Ely & Englewood, 1996).

This study explored PSTs' views on the use of technology when teaching geometry. The teacher's understanding and delivery of content determines how well learners develop conceptual knowledge and execute procedures. Geometry was optional in the South African curriculum from 2008 until the end of 2010. Attention is drawn in this chapter to the CAPS and other relevant educational policy documents. Further consultation of research regarding pre-service teachers, geometry and ICTs used in education was examined.

2.2 Views of mathematics

According to Ernest (1991), theorems and theories of mathematics are important and useful, as they offer Science a foundation of truth and certainty. Many intellectual and physical changes have taken place throughout history, resulting in advances in the conceptual framework of mathematics. Mathematicians and philosophers such as Euclid, Proclus, Wallis, Pythagoras, Lobachevsky and al-Haytham, to name but a few, spent their lives trying to solve a single problem (Eder, 2000), using a particular view of mathematics. There are two views of mathematics: the absolutist view of mathematical knowledge which consists of certain and unchallengeable truths, and the fallibilist view which regards mathematical truth as imperfect and correctable (Davison & Mitchell, 2008; Ernest, 1991). Thus, teachers and learners could view mathematics as something that is rigid, or floppy and changeable. On one hand the external view of mathematics envisages mathematics as unchanging since conclusions are formed by known information (Ayalon & Even, 2010). Thus, knowledge is formed by deductive reasoning. On the other hand, the internal view of mathematics identifies constant changes and interconnection in mathematics (Ernest, 1991); this inductive reasoning creates knowledge.

Mathematics is becoming an essential tool to function effectively in everyday life. For example simple calculations are used in the kitchen for cooking, dietary evaluations, running household budgets, time management and so forth. Adopting a negative stance towards mathematics means that one dislikes it and thus avoids its use. Some people are afraid of the very essence of mathematical ideas and mathematicians, as they feel powerless and inferior and suffer from low self-esteem (Henderson, 1981). This has been the case throughout history and is prevalent in developing countries (Belbase, 2013). People tend to regard mathematics as an inherent natural ability (Fitzsimons, Jungwirth Maasz & Schloeglmann, 1996); therefore, lack of accomplishment in mathematics is thought to be permanent. This is evident in the low numbers of learners in South African schools that take pure Mathematics and Science (see Tables 1 and 2). Furthermore, few students further their studies in Mathematics and Science fields at higher education levels, resulting in many vacancies in these fields (Department of Higher Education and Training, 2014). Unpleasant experiences at school can be a contributing factor to students' lack of interest in STEM subjects/fields. As Perez-Felkner, McDonald and Schneider (2014) note, the level of school support and other factors have a strong influence on students' interest in STEM subjects at high school and courses after school. It is interesting to note that, following higher education, people who have jobs relating to mathematics, create perceptions of this subject as a well-stocked warehouse of ready-to-use formulae at their disposal, which advances their own theories (Sam & Ernest, 2000). In similar vein, Belbase (2013) observes that people who excel in school mathematics are highly respected and considered to be among the intelligent few.

Lack of motivation and interest which feeds low confidence is one of the critical reasons why learners feel that it is difficult to learn mathematics (Mji & Makgato, 2006). This may result in them giving up on learning and pursuing mathematics related fields of study. According to Ernest (1996), the negative attributes associated with mathematics include "difficult, cold, abstract, and in many cultures, largely masculine" (p. 802). Globally STEM subjects, particularly Mathematics are largely seen as male dominated (Perez-Felkner, McDonald, & Schneider, 2014; Pompa, 2015; Rothwell, 2013). This could be the result of a cultural barrier as most Mathematics, Science and Technology teachers in schools and the majority of mathematicians throughout history have been men. Negative experiences, and family, teachers and peers' attitudes and beliefs can also result in poor performance, influencing whether the person chooses Mathematics and other STEM subjects during school and in post-school education (Hill, Corbett & Rose, 2010; Talton & Simpson, 1986).

The DBE launched the Dinaledi project in 2001 that aimed to increase the number of learners studying and passing Mathematics and Sciences in Grades 10-12. The goals of the project were to improve the teaching of Mathematics and Science and the quality of passes and increase the enrolment of female learners in Mathematics, Science and Technology at school (Department of Education, 2005; Department of Education, 2014b). Despite this intervention, the number of female learners taking Mathematics in Grade 12 decreased from 142 990 in 2010 to 123 045 in 2014 (Department of Basic Education, 2014a). Likewise, 106 746 female learners were enrolled in Physical Science in 2010, compared to 88 729 in 2014 (Department of Basic Education, 2014a). The figures also show that 77.5% of male and 74.4% of female Grade 12 learners passed the National Senior Certificate in 2014 (NSC Examination Technical Report, 2014) and 82 7050 female learners qualified for Bachelor Studies at Higher Education Institutions (28.5%), compared to 68 047 male learners (28%) in 2014 (Department of Basic Education, 2014a). It is worthy of note that there was only a 0.5% difference between female and male achievements in Bachelor passes.

In 2014, 225 458 learners wrote the National Examination, with 53.5% achieving the pass mark of 30% and above (Department of Basic Education, 2014a). However, there has been a consistent decline in Mathematics and Physical Sciences enrolments (Tables 1 and 2).

Table 1: Mathematics and Physical Science Enrolments, 2010 to 2014 (Department of Basic Education, 2014a)

SUBJECT	2010	2011	2012	2013	2014
Mathematics	270 598	229 371	230 022	245 344	231 180
Physical Sciences	210 168	184 052	182 126	187 109	171 549
Number of enrolments	559166	511038	527572	575508	550127

Table 2: Mathematics and Physical Science Enrolments, 2010 to 2014 in % (Department of Basic Education, 2014a)

SUBJECT	2010	2011	2012	2013	2014
Mathematics	48%	45%	44%	43%	42%
Physical Sciences	38%	36%	35%	33%	31%
Number of enrolments	559166	511038	527572	575508	550127

In terms of the CAPS, it is compulsory for a learner to take either Mathematics or Mathematics Literacy (Department of Basic Education, 2011). The declining percentage of enrolments in Mathematics shown in Table 2 suggests that learners regard Mathematics as difficult and thus opt for Mathematics Literacy. Similarly, learners are opting to take subjects other than Physical Sciences, as mathematical calculations form part of this subject (Steklov, 1999). Atiyah, Dijkgraaf and Hitchin (2010) state that "the presence of physics in contemporary pure mathematics is undeniable" (p. 915). It is important to note that many regard Mathematics and Physical Science as gateway subjects. Furthermore, many schools require learners that choose Physical Science to also enrol for Mathematics.

2.3 The South African curriculum and geometry

Public[3] schools in South Africa are currently teaching in line with what is prescribed in CAPS, a government policy. The CAPS document was a result of the Curriculum 2005 review, which replaced Outcomes Based Education (OBE). The latter curriculum was determined by individual teachers' interpretations. It was designed by the National and Provincial Education Departments. This resulted in nine official curriculum documents,

[3] Public schools are managed by provincial education departments and are solely dependent on the state.

where every individual had their own notion of what the curriculum should be. American philosopher and educational reformist, John Dewey regarded education as transformative, thus enabling individuals to escape the limitations of their social group (Chambliss, 2003; Ross, 2000). This could explain the level of freedom and lack of clarity in Mathematics topics. It could also be a reaction to the repressive apartheid system, in that the new ideology seeks to give learners and teachers more control over the curriculum. However, past legacies continue (Wedekind, 2015) in that all schools are not treated the same by the Department of Education.

According to Schoenfeld (1987), when a teacher takes on a facilitator's role, this compels learners to focus on controlling their own decisions and the teacher's, thus promoting self-regulation. Learner-driven teaching systems result in greater learner involvement and control and more meaningful classroom interaction. As Estes (2004) notes, learners take control and responsibility for their own learning; thus, meaningful learning can be increased through learner-centered environments. In similar vein, Papert (1986) views learning as a construction rather than transmission of knowledge; thus, the notion that learning is effective when one is involved by actively investigating and constructing. However, in certain circumstances, this may not be the case. Woo and Reeves (2007) observe that not every interaction leads to increased learning.

The South African curriculum can be described as an open education system which is complex, difficult to understand and has unintentional consequences (Wedekind, 2015). There is no direct entry and exit, as there are many ways in which learners can come and go from the system. It therefore becomes chaotic as it is non-linear and complex. The different options include Technical Vocational Educational and Training (TVET) colleges offering National Certificates (NC) and NATED[4] engineering programs, Grade 9 School leaving certification; and private Further Education and Training (FET) colleges which allow the repetition of Grade 12. That this system is flawed is suggested by South Africa's poor performance at global level. The Third International Mathematics and Science Study (2011) ranked the country 44th out of the 45 countries that participated. The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACEMEQ III) (Spaull, 2011) and the Progress in International Reading Literacy Study (PRILS) (Howie, Van Staden, Tshele, Dowse & Zimmerman, 2012) produced similar results.

[4] A national qualification is often also referred to as a NATED, N1 to N6, or N-level qualification. It precedes a university degree, and can take the form of either a certificate or diploma.

The CAPS seeks to rectify the problems bedeviling the previous curriculum by bringing about standardisation. It is important to note that the CAPS contains much more detail than Curriculum 2005 on what needs to be taught and is prescriptive - for example, questions are set out within each topic. These questions are categorised into levels of difficulty: knowledge, routine procedure, complex procedure and problem solving which are aligned with Bloom's Taxonomy. This serves as a guide to the teacher about what should be taught and what questions learners should be exposed to. It can thus be concluded that the CAPS would bring about stratification of learners because of its prescriptive nature and emphasis on content. On the other hand, it could discourage creativity among learners and teachers as it is limited to the classroom and subject bound to what is prescribed. The CAPS contains elements of various curriculum models, namely, the product model, objectives driven curriculum, process model; and competent model. The influence of the product and objective model is evident in the Mathematics CAPS as there is a clarification column with clearly stated cognitive levels and examples that provides a clear structure from the beginning to the end of the topic. It therefore focuses very precisely on what must be taught and assessed. The process and competent curriculum models are also seen in the CAPS; group work, role play, use of resources, demonstrations, projects and so forth will still take place. As prescribed by the DBE (2011), besides tests and examinations, assessment can take the form of practical tasks, projects, oral presentations, demonstrations, performances and so on, aligning with the properties of the process and competent curriculum models.

Learners' assessments in the previous grade are taken into account in the CAPS, which demonstrates vertical demarcation in the learner's progression. As noted by the DBE (2011), a learner's academic possession provides proof of their readiness to be promoted within the grade or at the end of the year. Figure 1 presents a brief overview of the journey that geometry takes, as prescribed by the CAPS. Learners are exposed to the fundamentals of geometry at an early age with space and shape. They then focus on triangles, followed by the measuring of angles. Thus, the initial engagement with geometry is crucial, as it affects the learner's understanding later in schooling. It is important that teachers adopt a meticulous pedagogy at Grade R which has been identified as the critical teaching and learning space. It is proposed that all teachers in this phase (Early Childhood Development) attend further training offered by the Department of Education until 2018 (Department of Basic Education, 2015). This will result in learners being grounded in geometry at an early age at primary school, allowing for smooth transition to high school geometry. A learner's first encounter with geometry at high school is the Pythagorean Theorem in Grades 8 and 9. Later on in their career the focus moves to congruency, similarity and quadrilaterals. Circle geometry and its relevant theorems are mastered in Grade 11, followed by the Midpoint theorem. Figure 1 shows the progression of geometry from primary school to high school based on the CAPS.

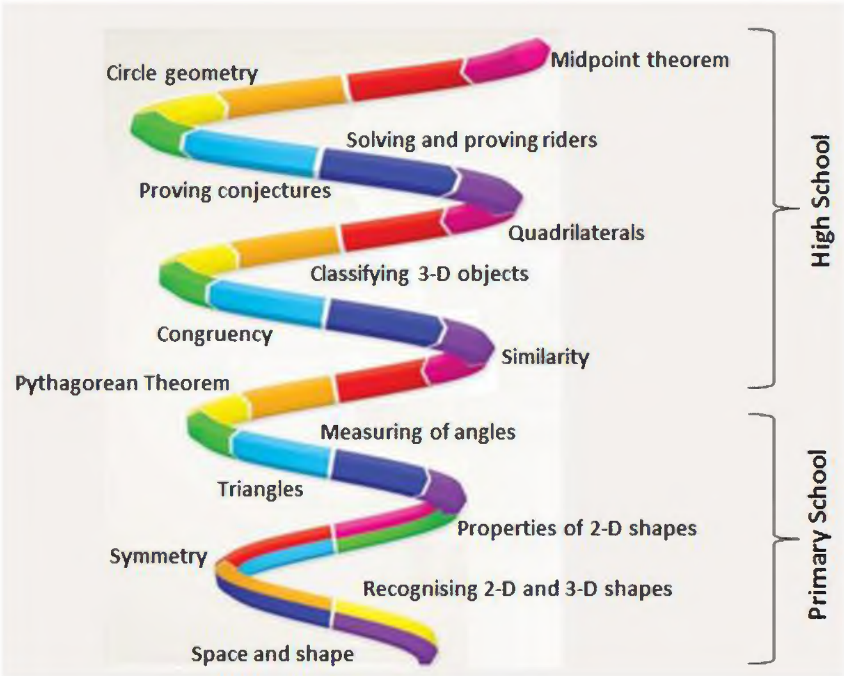


Figure 1: The journey of geometry at school according to the CAPS, Adapted from Department of Basic Education (2011)

Figure 1 clearly shows the conceptual growth of geometry through exposure to space and shape at primary school and being able to recognise shapes, identify their properties and classify them. In high school, there is a shift to a more structured geometry layout and thinking with the Pythagorean Theorem, congruency, similarity and proofs. As Mudaly (2010) notes, it is customarily for geometry instruction at primary school to start with the categorisation of shapes, which progresses to a formal deductive system at high school level. From 2008 to 2010, Euclidean geometry: proof constructions, solving of riders and theorem recognition was optional for learners in the FET phase (Dhlamini & De Villiers, 2012). With the introduction of the Mathematics CAPS, learners write two mathematical papers, with paper two covering Euclidean geometry. Prior to the implementation of the CAPS in 2011, Euclidean geometry was omitted from the mainstream Mathematics curriculum. About 300BC, Euclid of Alexandria (Eder, 2000) defined what constitutes flat-surface, otherwise known as 2-D /Euclidean geometry. Non-Euclidean geometries involve spherical, hyperbolic and elliptical geometry which are not part of the South African school curriculum.

Euclid stated five postulates/axioms of geometry which are used as the foundation for geometry proofs (Marshall & Scott, 2004). After many years, it is clear that this type of mathematical thinking in Euclidean geometry is of great importance; hence its return to the mainstream curriculum in South Africa. However, learners perform poorly in Euclidean geometry, as reflected in. the Department of Education's 2012 evaluation, the 2014 examiner's report and the Trends in International Mathematics and Science Study (TIMSS) in South Africa that found that learners under-performed in the geometry questions. The DBE conducts the Annual National Assessment (ANA) in order to measure learner progress. The ANA is part of the DBE's Action Plan to 2014: Towards the Realisation of Schooling 2025. These tests are administrated to learners in the senior phase (Grades 7 - 9), intermediate phase (Grades 4 – 6) and the foundation phase (Grades 1 – 3).

A 2014 analysis of the ANA found that Grade 9 learners faced challenges because of a lack of basic algebraic skills and their inability to solve geometry problems (Manuel, 2014). It should be noted that there is an unusual level of overconfidence among South African schools. According to the TIMMS (2011) study, 89% of South African Grade 9 teachers felt very confident teaching maths as compared to teachers in Japan – where the results were much better, but only 36% of the teachers felt the same. The use of technology would promote a sensible, easier geometric understanding among future PSTs that would enhance their own knowledge and enable them to pass it on to learners.

2.4 The understanding of geometry

According to the DBE (2011), one of the specific aims of the CAPS was to include the origins of mathematics, in order to show that it was a human creation. This would mean that the mathematics we understand might not necessarily be the same on another planet since it is a human creation/perspective. Geometry can be seen as an element of mathematics which deals with the study of shapes, either plane or solid. A plane shape is a geometrical form like a straight line joining two points that lies on a plane surface, whereas a solid shape is limited by surfaces which might be on a plane surface (Adolphus, 2011). Geometry is the study of spatial relationships, which can be easily likened to situations in real life. As continued progression in geometry develops, so does one's logical reasoning (Smith & Ulrich, 1957).

The construction of diagrams allows one to apply knowledge of geometry, geometric reasoning, and intuition to arithmetic and algebra problems. These include designs for an electronic circuit board, building plans, blueprints for a dress design, the layout of a webpage and so forth. Thus, understanding geometrical principles is essential. Geometry is a foundation for fields of study such as science, engineering, architecture, computer science, graphics, geology and astronomy (Banchoff, 1990). Learners require adequate knowledge of geometry as this provides them with the skills required to operate at the level of axiomatic thinking during and after school. Mathematical instruction in geometry is thus a necessity. For the purpose of this study, geometry refers to the mathematics of shape and space, which is traditionally incorporated in Euclidean geometry (Mudaly, 2002).

The conceptualisation of geometry starts at an early age when the child is able to orientate themselves in their daily surroundings (Freudenthal, 1991). This can be related to Piaget's second stage of development, preoperational (2-7 years). At this stage the child is able to engage in symbolic play, and mental representation of events and objects without any conversation. Familiarisation with their physical environment will eventually lead to more experiences that will pave the way to develop these definitions and theorems (Freudenthal, 1991). This is in relation to Piaget's third stage of development, concrete operational knowledge, as well as the fourth stage - formal operational knowledge. The concrete operational stage involves the use of rational and organised thinking to solve problems in a logical manner. When one attains Higher Order Thinking Skills (HOTS), one is able to solve problems abstractly or hypothetically, which is known as the formal operational stage. Piaget's stages and Freudenthal's ideology can be clearly identified in the CAPS in terms of the progressive way in which the geometry content is laid out – refer to Figure 1 (page 13).

Freudenthal and Piaget's concepts can be further extended to the Van Hiele levels where one's geometry development can be represented by mental development rather than age. These levels were identified by Dutch husband and wife team Pierre and Dina van Hiele. The Van Hiele levels are based on one's thought and concept development in geometry and comprise of five levels: Level 0 – Recognition, Level 1 – Analysis, Level 2 – Informal deduction/Order, Level 3 – Deduction; and Level 4 – Rigor. Table 3 below describes how one moves sequentially in one's geometric thinking process, as described by Van Hiele (2004) and Van Hiele (1986).

Table 3: The Van Hiele levels, Adapted from Van Hiele (2004) and Van Hiele (1986)

Level 0 - Recognition:	This level is the basic level, which involves a holistic and visual emphasis. Geometrical shapes are recognised based on their visual characteristics. Since an object is seen as a whole, there is no need for a deeper understanding of its characteristics.
Level 1 - Analysis:	At this level, analytical thinking is used in order to understand the concepts of the given object\s. This would include being able to study the object through observing, measuring, experimenting, drawing; and constructing. However, one is not capable of explaining the relationships among different geometrical objects.
Level 2 – Informal deduction:	This level involves rational thoughts, correlations and informal deduction, allowing one to correlate between different geometrical shapes. It also allows one to recognise the general characteristics of particular objects and to explain them in a hierarchical way.
Level 3 – Deduction:	The Deduction level is known as the formal deduction level. At this stage, one is able to make connections between geometrical objects. By now, one would already know how to sequence geometrical objects correctly.
Level 4 – Rigor:	On reaching this level, one is able to create meaningful debate by means of explanations and make comparisons of the axiomatic geometry system. Going forward, this person understands deductive reasoning, and will be able to provide evidence without proof or argument.

In terms of teaching geometry, the Van Hiele levels suggest that people at different levels of mathematical understanding speak, use and understand geometry differently. A common mistake that teachers are likely to make, is to treat every learner the same by using terms or concepts that can only be understood by a person/s who has progressed to the third or fourth level. For example, Figure 1 (page 13) clearly demonstrates that geometry is carefully structured from primary to high school in the CAPS. More importantly, the topic coverage and depth is aligned to the Van Hiele levels. This means that learners' first experience with geometry is of utmost important, as this lays a foundation for the progressive build up.

If one is placed below Level 3, one can provide proof only by memorisation (Van Hiele, 1986). This represents superficial understanding that lacks substance. However, studies carried out among Grade 12 South African learners have revealed that the majority of exiting high school learners have achieved concrete and visual levels rather than an abstract level in geometry, despite the fact that the national school exit examination clearly requires an understanding of the underlying abstract processes (Feza & Webb, 2005). The practical application of the Van Hiele levels starts with the exploration of geometric objects, rather than the traditional formulation of definitions and theorems. Therefore, these levels serve as a meaningful practical guideline when designing geometrical activities. Many learners only reach level 0: recognition and level 1: analysis, resulting in poor interpretation and understanding due to memorised learning (De Villiers, 2010; Vojkuvkova, 2012). This could be a result of the 'chalk and talk' style of teaching, where teachers present learners with static content, allowing no room for deductive reasoning or higher order explanations. Learners merely regurgitate content like theorems in tests and exams, without understanding it. It is imperative that teachers provide opportunities for learners to engage in activities that promote understanding, rather than simply concentrating on the problem at hand. The activities given to learners should form part of an investigation that allows them to discover the mathematical concept. In this way, learners will have a meaningful understanding when they apply the concept later on (Bansilal & Naidoo, 2012). The approach taken in this study is that exposing PSTs to technology when teaching geometry would draw out elements that stem from the Van Hiele levels.

2.5 Technology

2.5.1 The role of Information and Communications Technology (ICT)

Tatar (2013) notes that all technologies that are used to process, produce, use, share; and disseminate information are regarded as ICTs. Technology can take any form and has become an essential element of modern life. In terms of teaching, technological knowledge includes the ability to use computers, the Internet or digital material to teach learners (Leendertz, Blignaut, Nieuwoudt, Els & Ellis, 2013). Therefore, technology can be defined as a hardware device or software program that makes one's life easier, for example motor vehicles, coffee machines, e-mail services, smartphones, multimedia players and word processors. The integration of ICTs in mathematics is largely based on computer technology (Hennessy, Ruthven & Brindley, 2005; Tatar, 2013; Tay, Lim, Lim & Koh, 2012).

There have been impressive advances in teaching technologies over the years ranging from overhead projectors to PC tablets in the classroom. Educational technologies are constantly being updated or invented and have become so accessible that it is now non-negotiable to integrate them in a lesson. Such technologies enhance and create a captivating learning environment (Corrias, Buist, & Soong, 2014). The DBE (2004) observes that our world is changing, and ICTs are the pivot to promote change within the educational system. Using technologies in education allows for efficient development of learners' thinking, as compared to traditional teaching practices such as 'chalk and talk' (Chigona, Chigona, Kayongo & Kausa, 2010).

Multimedia content which mainly consists of audio and video has transformed how information is perceived by society (Qualman, 2012). It allows for in-depth understanding and interpretation of information, while paying great attention to detail (Mayer, 2003; Woo & Reeves, 2007). Similarly, multimedia will result in dramatic changes to the learning and teaching process, allowing for the expansion of new learning opportunities and access to educational resources beyond those of a traditional chalk, talk and textbook environment. The merging of modern-day and traditional teaching and learning strategies in mathematics will enhance the learning process (Naidoo & Govender, 2014; Tay, Lim, Lim & Koh, 2012). It is important to note, that the use of technology does not necessarily mean that it is being used critically and meaningfully in the classroom. One can easily fall short in demonstrating the use of technology to the extent that it is no different to traditional teaching methods. Strategic planning is a requirement for technology to be successfully adopted and for harmonious adoption in the classroom (Chigona, Chigona, Kayongo & Kausa, 2010).

2.5.2 Applets and Virtual manipulation and Kinetics

Living in a three dimensional world allows one to possess a remarkable amount of natural or untaught geometric knowledge (Lisi & Weatherall, 2014; Wohlhuter, 2013). From childhood, a person is constantly interacting with and making sense of space and shapes around them. As noted by Van Hiele (1982), the intuitive notions that children reveal when exposed to spatial situations should be capitalised. Thus, the teaching of three dimensional shapes using resources like text books and the chalkboard gives rise to a lack of in-depth learning of three dimensional shapes. This has a temporary impression on learners' minds as it does not provide a concrete foundation and negatively impacts on their mathematical careers. A degree of spatial awareness and related meanings are essential for mathematics (Yarmohammadian, 2014), especially geometry (Weckbacher & Okamoto, 2015; Yegambaran, 2009). A person learns best by doing. Hackathorna, Solomonb, Blankmeyerb, Tennialb and Garczynskib (2011) note that when learners are actively engaged, this promotes deeper levels of thinking which facilitate genuine encoding, longer storage, and quick retrieval, in contrast to a traditional chalk and talk lesson. Learners' interaction would result in authentic learning, allowing them to experience their learning rather than playing a passive role in the learning process.

More importantly, active learning enables learners to investigate issues that concern them, such as learning more about a current event, an experiment, scientific discoveries or exploring the Ice Age. One can use the Internet to investigate the topic at hand, participate in a virtual field trip to the event, and watch it as it unfolds without leaving one's comfort zone. Bansilal and Naidoo (2012) remark that visualisation promotes investigation and discovery through the use of concrete manipulations, models and diagrams. In addition, when learners are actively engaged, they construct knowledge from a combination of simulations like visuals and movement; hence knowledge is developed in a structured manner (Pettigrew & Shearman, 2014). This is important since the ability to imagine and manipulate forms in space plays a fundamental role in problem solving, especially in STEM subjects (Goldsmith, Simmons, Winner, Hetland, Hoyle & Brooks, 2014).

Ndlovu, Wessels and De Villiers (2011) note that technology is being integrated into mathematics education in many countries around the world. As a conceptual subject, face-to-face contact is regarded as necessary to convey mathematical concepts (Engelbrecht & Harding, 2005). Woo and Reeves (2007) maintain that learning that involves interaction is difficult as the use of technologies is more time-consuming than face-to-face contact. However, virtual manipulation will create a more concrete learning experience (Pettigrew & Shearman, 2014) because the visual and touch senses play a key role.

Applets are small programs that run on Java when accessed online via the Internet. They can be described as programs that run on the Internet allowing user interaction and thus virtual manipulation and visual simulation. Goose (2010) points out that "dynamic geometry packages and web-based applications that offer virtual learning environments, have changed the mathematics teaching and learning terrain" (p.67). Applets designed with dynamic properties allow for objects to be virtually touched and moved one by one, which can be used to introduce or reinforce a mathematical concept. This creates an environment where learners can pose and solve their own problems, by forming links between concepts and receiving immediate feedback, thus allowing for reflection (Arnold, 2013). The learner is actively involved in the learning process. Applets can be set up in stages using scaffolds. This is exemplified in the applet named Getting to know the circle (Appendix S). Designing applets around previous knowledge is important, as this offers support in grooming new knowledge. This led to the coining of the term Zone of Proximal Development (ZPD) (Bodrova, Germeroth, & Leong, 2013; Vygotsky, 1978). Thus, learning that starts from the known and moves to the unknown with the assistance of the applet would exhibit ZPD.

With regard to the applet design, Human-Computer Interaction (HCI) enables one to directly interact and manipulate objects, changing mathematical properties that are instantaneously represented on the screen. A level of confidence is achieved as one's visual senses are fed information (De Villiers, 2012), by noticing a change in measurement or orientation of the objects under study. Feeling in control and involved with the objects that are being manipulated immerses one in the learning environment.

In South Africa, adult illiteracy stands at just over 20% for young adults and school leavers (Wedekind, 2015). The language of instruction is undoubtedly an essential tool in the classroom as it promotes communication between the learner and teacher. According to De Villiers (1987), second language speakers find it difficult to shift from concrete to abstract levels of thinking. Geometry stresses the use of correct mathematical language with appropriate vocabulary to express the unique properties of the object under study (Atebe & Schäfer, 2010; Jamison, 2000). If a learner finds it difficult to define geometry's specialised terminology, poor performance results (De Villiers, 1987). Instruction via virtual manipulation offers more effective communication than traditional classroom instruction, where the teacher leads the learning process by reading out the instruction. Through visualisation, learners acquire the correct technical terms since they are able to see the definition and experience the meaning. Language will not pose a barrier to teaching and learning, as emphasis is placed on visualisation, allowing for the clear communication of geometry concepts. However, manipulatives can potentially be confusing if presented in a haphazard and disorganised way that lacks proper guidance (Arnold, 2013).

As noted by Bansilal and Naidoo (2012), research among high school students in the United States resulted in the identification of five types of imagery: concrete, kinaesthetic, dynamic, memory and pattern. It was found that dynamic imagery was the most effective but least used. The use of kinaesthetic and dynamic imagery has proven effective in the learning and reasoning process of geometry as the visual and touch/movement senses are exercised, bearing in mind that the human body is most responsive to these senses (Robles-De-La-Torre, 2006).

2.5.3 Going online

People on the African continent have difficulty in accessing basic services in their communities such as a post office, proper sanitation and paved roads (Hove, 2013; Freire, Lall & Leipziger, 2015; Schwab, 2014) to mention but a few. However, the Afrobarometer survey found that cellphones are more accessible than water in Africa (Business Report, 2014; Mitullah, Samson, Wambua, & Balongo, 2016). Thus, there is a fully functional communication network system in Africa, making Internet access more easily available.

South Africans who have access to the web have an ideal delivery system which can be used as an education medium (Naidoo & Govender, 2014). We are living in a technologically driven era, where many learners are surrounded by technology. Delivering lessons with the aid of technology presents the subject content in a recognisable format to learners. According to De Villiers (2012), the use of technology promotes understanding of the final expectation before one arrives at the solution. For example, in proving a geometry theorem, the learner already knows what to expect through discovery with the aid of technology. South Africa.info (2013) notes that South Africa is ranked 5th in Africa in terms of the number of people who are connected to the Internet and this is likely to increase. Woo and Reeves (2007) observe that web-based learning environments that incorporate interaction are more challenging than face-to-face learning because of time and distance factors. Nevertheless, due to the rapid development of ICTs, web-based learning has become a leading trend in education (Chen & Huang, 2013).

According to the Afrobarometer (2014) survey, cellphone coverage is almost universal in Africa, with 93% coverage among the 34 countries sampled. South Africa reached the 11 million Internet user mark at the end of 2013 (Enterprise New Media, 2012) and this was likely to grow. Recent research by the World Bank shows that 48.9 of every 100 South African are Internet users (The World Bank, 2015b), nearly half the population of 54,9 million (Statistics South Africa, 2015). These statistics suggest that the Internet is widely available on the African continent and in South Africa in particular. While access is paramount, data speed will be crucial in education delivery, as it provides access to the global knowledge that awaits learners and teachers. According to the Deputy Director General for Telecommunications and Postal Services, in 2013 25% of schools in South Africa were connected to the Internet (basic Internet of less than 5Mbps[5]). This was expected to increase to 50% by 2015 (at 5Mbps); and 100% by 2020 (running at 100 Mbps) (Ngobeni, 2015). The vision for 2030 is to reach staggering speeds of 1Gbps. This implies fixed networks and computer infrastructure.

It is important for education to be real and authentic for learners since this promotes meaningful understanding. Technology provides an opportunity to engage with learning activities and thus experience learning. For example, Science learners can collect accurate weather or chemical reaction data off the Internet and digitally trace trends and test hypotheses. It becomes easy to visualise mathematical concepts using online graphing calculators and software. Electronic communication tools allow learners to communicate with their peers and experts from other parts of the world through video conferencing, e-mails, and blogs, etc. Social networks create a learning environment that enables student-centred learning and end-user content creation and sharing (Cochrane, 2009). Group work becomes more manageable as learners can use mobile computing devices and wireless networks to create joint writing exercises and tasks, which automatically sync to the latest updated version. They can also read electronic books that allow them to explore related topics together as sharing becomes easy. Instead of relying on local libraries and hardcopy reference materials to research topics, learners have access to digital versions via the Internet and can consult libraries around the world. Art learners can view images of original artwork through the Internet. With appropriate software, they can create digital artwork and musical compositions. Learners in sports education can use online videos and simulations to learn about the relationship between the impact of physical movement and physiological changes.

The information on the web includes text, images, sound, interactions and video clips that, when integrated wisely in education, facilitate learners' understanding. Concepts are more likely to be lodged in long term memory; Clements, Julie, Yelland and Glass (2008) state that memories of practical activities create mental images and ideas that are connected because of the physical experience, promoting more meaningful understanding. Adapted and modified

[5] Mbps: megabit per second is a unit in which data speeds are measured. 1 Mega bit = 1 024 Kilobytes

from Lajoie (1993), Figure 2 highlights four major advantages of taking subject matter online. In no particular order, firstly it reduces memory load since there is a high level of visualisation, colour, and the option of navigating back and forth. Therefore one does not need to remember the findings from a previous step or to highlight an important point. Secondly, it reduces the time on laden computation. Tedious calculations can be skipped as one can focus on the learning concept at hand. Thirdly, it enhances logical reasoning, thus making it easy to test conjectures. There would be a high level of conviction before the actually discovery of the learning matter, for example a theorem. One would explore the objects in a virtual environment, thus allowing one to reach a definite conclusion. Finally, but most importantly, learners that are out of reach can still engage, and distance does not hinder the learning process.

Figure 2: Going online, Adapted and modified from Lajoie, (1993, p. 261).

Many teachers in South Africa do not have the time, knowledge and sometimes the resources to integrate such technology (Dixon, 2011; Mofokeng & Mji, 2009; Selepe, 2015). Working online gives teachers and learners unlimited access to these micro environments. As noted by the DBE (2004), there has been limited integration of ICTs in teaching and learning. Chigona, Chigona, Kayongo and Kausa (2010) suggest that teachers do not have the time to practice or the expertise to utilise ICTs; thus, they are not confident in integrating technologies. Many teachers would prefer ready-made technology based resources designed to accomplish the set goal or task (Leendertz, Blignaut, Nieuwoudt, Els, & Ellis, 2013; Lennex, 2010; Stoilescu, 2009). Hence, PSTs that are the focus of the current study were exposed to a dynamic online environment, making this technology available to them as a resource for future lessons, and encouraging them to investigate other technology based avenues when teaching mathematics, especially geometry. It is important to note that when learners work on geometry problems at home or during an examination, the teacher is not present to provide assistance. Thus, self-regulation is important when learners solve a problem.

2.6 Self-regulating and metacognition

Zimmerman (2002) maintains that self-regulated learning (SRL) is social in nature and origin; thus asking questions is a critical step in advancing in one's learning. This implies that although self-regulation focuses on the individual, interaction is valuable. As proposed by Polya (1957) the first principle is to understand the problem. This involves asking appropriate questions. Communication and asking questions are vital since teachers need to build on learners' ideas in ways that help them achieve more substantive understanding (Bransford, Brown, Cocking, Donovan, & Pellegrino, 2000). When the teacher is absent from the learning process, this results in learners self-engaging and making sense of the work or task at hand. Self-regulation can be defined as self-generated thoughts, feelings and behaviours that are directed at achieving goals (Zimmerman, 2002). Learners thus need to be empowered to become aware of their limitations and strengths. Teacher can promote this goal by making learners responsible for how they learn.

The way in which the teacher structures a lesson can motivate learners and enable them to achieve self-regulation. As noted by Zimmerman (2002), learners who find the subject matter interesting will desire mastery and be motivated to learn in a self-regulated fashion. Self-regulation can thus be termed a learning skill that one can attain by being taught how to control one's learning. The teacher creates awareness and promotes self-regulation by using various tools. Furthermore, each self-regulatory process or belief, such as goal setting, the use of strategies, and self-evaluation can be learnt from instruction and modelling through the use of technology (Zimmerman, 2002). While many studies have focused on developing effective systems that assist SRL (Chen & Huang, 2013), planning and proper implementation of instructional technologies is critical. When a person drives their decision making process by planning how to solve a problem, it builds confidence that they will succeed. Thus self-regulation is unintentional but becomes a natural process.

Technology can play a vital role in allowing one to control one's learning. Wilson, Fernandez and Hadaway (1993) refer to this process as metacognition, which involves thinking about one's own cognition and therefore promoting orderly, constructive reflection on a problem. The use of technology can bring about such order (Clapp & Swenson, 2013; Lim, Zhao, Tondeur, Chai & Tsai, 2013) Metacognition is the type of thinking that is required to connect pieces of knowledge in order to solve a problem effectively, especially in geometry, when prior theorems are used to solve current theorems. Learners that lack SRL are not able to deeply comprehend complex problems (Chen & Huang, 2013). Metacognition is knowledge of one's own thinking and the ability to monitor one's own understanding and problem-solving activity (Kilpatrick, Swafford & Findell, 2001). It has multiple and somewhat disjointed meanings that include knowledge of one's thought processes or self-regulation during problem solving (Schoenfeld, 1992). While there are numerous definitions of metacognition, knowledge and monitoring of one's own cognitive process are common to most. Technology gives one control of one's learning and exercises one's metacognition in becoming self-regulated. Thus, the manner in which technology is used in the classroom becomes important. In addition, the teacher needs to be knowledgeable of the subject content, ensuring the accuracy of the information provided. As Bransford, Brown, Cocking, Donovan, and Pellegrino (2000) remark, "teachers must come to teaching with the experience of in-depth study of the subject area themselves" (p.20). The teacher is the only person in the classroom who can model complex procedures (Schraw, Crippen & Hartley, 2006). However, when learners self-regulate, it helps them to think like mathematicians, where they pose their own mathematical questions and endeavor to solve them. In this way, they construct new knowledge.

In general it has been found that little time is spent on learning geometry as compared to algebra, although geometry is regarded as difficult (Department of Basic Education, 2014; Hansen, Gustafsson, Rosén, Sulkunen, Nissinen, Kupari, Ólafsson, Björnsson, Gronmo, Ronberg, & Mejding, 2014; Mji & Makgato, 2006). Learners become anxious and frustrated as they are expected to master selected theorems in the short space of time set by the curriculum. Being able to self-regulate is critically important; Zimmerman (2002) points out that a lack of self-regulation results in learners with vague self-evaluative standards that cannot gauge the level of academic preparation required for tasks like tests and examinations. When the learner is able to generate thoughts and reflect on their own thinking, they become critical thinkers. Schraw, Crippen and Hartley (2006) observe that critical reflection plays an important role in self-regulation. If learners can criticise their thinking, it is easier for them to identify ways to help them to achieve their goals. This results in a positive outlook on life since they know exactly what needs to be done to achieve their objectives. Critical reflection can assist in testing hypotheses and discovering theorems. Taken together with a dynamic geometry environment, it enables learners to take a second look at the role of proof (Mudaly, 2002).

Middleton and Spanias (2002) maintain that intrinsic motivation results in a person enjoying involvement in an activity, wanting to develop skills; and always applying themselves in order to achieve their goals. These are the qualities of a person who has a positive outlook – they always apply themselves and desire self-development. As Zimmerman (2002) points out, a self-regulated learner is likely to achieve better understanding of the subject matter and will display higher levels of self-efficacy. A learner that is intrinsically motivated will appreciate geometry. In terms of the five strands of mathematical proficiency, a learner with a productive disposition, is described as having the habitual inclination to regard mathematics as sensible, useful; and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick, Swafford & Findell, 2001) will regard geometry as valuable. A learner who is not intrinsically motivated would not appreciate geometry and give up more easily when posed with a problem that is underpinned by their geometry knowledge. Self-regulated learners are self-aware and self-motivated, and would apply such knowledge appropriately (Bodrova, Germeroth & Leong, 2013; Zimmerman, 2002).

Self-regulated learners sometimes adopt a defensive position during the self-reflection phase. This leads to withdrawal or avoiding opportunities to learn and perform, such as dropping a subject or being absent (Zimmerman, 2002). The learner behaves in this manner to protect their image (they might have excuses), thus giving them a positive outlook despite the fact that they have not admitted to failure. However, with the use of technology, this is less likely to occur since the learner will engage with the technology and make mistakes without being looked down upon by others. Self-regulated learners seek help as a corrective measure rather than the help which the teacher provides. These learners would approach the teacher when they have complex model queries. As proposed by Zimmerman (2002), "self-regulation is not a mental ability or an academic performance skill; rather it is the self-directive process by which learners transform their mental abilities into academic skills" (p. 65). In order for learners to self-regulate they need to adopt specific learning strategies or methods as they concurrently learn or complete a task. This justifies the use of technology, especially instructional technology, which is a dynamic online environment that offers the user control and can help in adopting such strategies or methods.

2.7 Pre-service teachers and teacher education

The DBE has adopted a set of Action Plans until 2019, Towards the Realisation of Schooling 2030 that aim to address weaknesses in the education system (KZN ICT Summit, 2015). The three main targets are increasing the number of bachelor passes in Grade 12, increasing the number of Mathematic passes in Grade 12; and increasing the number of learners who pass Physical Science (Department of Basic Education, 2015). In addition, the ANA projected a 50% pass rate for Mathematics and Science in 2016 and a 70% pass rate for Grade 12 (Sishi, 2015). Teachers will play a crucial role in the achievement of these results. According to Frank (1990), some teachers and communities subscribe to the myths that mathematics is about computation and is difficult; and that men are better at mathematics than women. These create a negative image of mathematics to the extent that one feels uncomfortable taking the subject and following mathematics and science career paths later in life. Thus, mathematics becomes unpopular because it is associated with difficulty and hopelessness.

In order to meet the DBE's targets, teachers will have to adopt a number of teaching techniques and methods that should be implemented in the lower grades in order to ensure that the goals are met by 2030. There are gaps in the content knowledge of teachers themselves, which are frequently accompanied by a lack of confidence and even a fear of mathematics education. Some teachers who teach STEM subjects do not have the relevant qualifications to teach these subjects at the required level (Dixon, 2011; Bansilal, Brijlall & Mkhwanazi, 2014; KZN Education Summit, 2015). Teaching should be regarded as a highly valued practice. Alternative careers such as in the financial sector, computer related fields and engineering that offer attractive salary packages are available at an increasing rate in South Africa and at the international level (Rothwell, 2013). This renders the teaching profession unattractive and is a reason why people do not study teaching (Armstrong, 2009; Hargreaves, Cunningham, Hansen, McIntyre, Oliver, & Pell, 2007; Mutshaeni, Denhere, & Ravhuhali, 2015; Symeonidis, 2015), resulting in teacher shortages.

Recent findings by Rothwell (2013) and Hanover Research (2014) show that STEM subjects are regarded as gateway subjects that offer a wide range of job opportunities and financial security. South Africa ranks third among the BRICS (Brazil, Russia, India, China and South Africa) economies in terms of education and infrastructure. However, the 2014 BRICS summit noted that South African higher education and training was rated 86th and thus as insufficient. Labour market efficiency was 113th which was affected by extremely rigid hiring practices, while firing practices were rated as 143rd in the world. On the other hand, wage inflexibility was rated 139th, with labour-employer relations rated at 144th. The latter is due to the strong influence of trade unions in the school sector. Chigona, Chigona, Kayongo and Kausa (2010) and Pompa (2014) note that education suffers from a lack of resources and escalating costs and that teachers find themselves forced to teach subjects that are not their specialisation.

Bansilal, Brijlall and Mkhwanazi's (2014) study assessed the current knowledge of Grade 12 mathematics teachers. The teachers obtained an average of 57% percent for this assessment. Furthermore, half of those sampled scored below 61%, while a quarter of the sample received below 39%. This raises serious concerns about the teaching of mathematics by FET (Grades 10 to 12) teachers (Bansilal, Brijlall & Mkhwanazi, 2014). The use of technology is the cornerstone of the current teaching and learning era. As noted by the DBE (2015), the current generation of learners has always lived in a digital world, and has been surrounded from birth by digital products driven by on-going technological developments.

Teachers at all levels can explore ways to use technology to engage their learners. While this may entail a simple adjustment in some instances, in many cases it will require further professional development in the pedagogical use of these new technologies (Galligan, Loch, McDonald & Taylor, 2010). According to Mofokeng and Mji (2009), many studies on ICT integration have found that lack of confidence is a barrier to ICT integration in mathematics. Many teachers find it challenging or impossible to integrate technology in their lessons as it is either not available or not easily accessible to them or their learners (Davies, 2013). Teachers' knowledge and willingness to adopt ICT is often associated with sociological factors such as age and teaching experience using ICT (Chigona, Chigona, Kayongo & Kausa, 2010; Cox & Marshall, 2007).

For novice teachers who are entering the profession, it is imperative to experience the excitement of being a part of a real classroom setting, getting to know learners, and planning and organising classroom tasks during teaching practice (Kiggundu & Nayimuli, 2009). Preparing teachers to use technology is emphasised in ICT policies and reports. Teachers need to know how to evaluate technology and to determine which is best suited to their learners, their classroom, the curriculum; and their teaching style.The KwaZulu-Natal (KZN)

Department of Education's e-Education strategy implementation 2014-2019 introduces teacher certification which is based on their teaching strategy and ICT implementation (e-Education strategy, 2015).

In order to understand how one thinks when posed with a problem, one needs to look back on previous, similar experiences (Harvard, 2013). John Dewey proposed that teachers should work with learners' current understanding by taking into account their prior knowledge (Chambliss, 2003). Miller (2011) explains that when one asks a teacher what they are doing tomorrow they are likely to answer: page 65, which often means they will write page 65 on the chalkboard and have learners copy it and recite it; this becomes the lesson. It is clear that many teachers are limiting their resources by utilising a single textbook to guide their lessons. This eliminates any meaningful link between prior and current and future knowledge. Learner engagement with the subject content is disregarded, and there is a tendency for teachers to treat learners like robots who are waiting to be fed with information on what needs to be done next in the curriculum.

In order for South Africa to provide quality education at global standards, graduate teachers must prepare themselves to engage with learners in the teaching and learning environment. A study conducted by Shapley, Sheehan, Maloney, and Walker (2010) found that the majority of teachers use available ICTs for only administrative purposes while learners only use the Internet for the completion of projects. The CAPS assigns four-and-a-half hours per week for the teaching of Mathematics in Senior FET (Grades 7-9), and Grades 10-12 receive ± 40 weeks of teaching (Department of Basic Education, 2011). It is impossible to integrate ICTs in every lesson as other school activities disrupt and reduce teaching time. A systemic evaluation conducted in 2012 by the DBE and the Trends in International Mathematics and Science Study in 2011 (Mullis, Martin, Foy & Arora, 2012) found that learners struggled to understand geometric concepts and performed extremely poorly in geometry. Furthermore, the DBE (2014b) NSC diagnostic report states,

> "more time needs to be spent on the teaching of geometry in all the grades. There needs to be enough time to explain the theorems properly and in a practical way – for learners to be able to see, to recognise the theorems featuring in a sketch; for learners to understand and practice the proofs of theorems; and for learners to practice enough how to apply the theorems in riders" (p. 117).

Pre-service teachers in the school of Mathematics Education are usually taught a set of core modules that are based on their phase specialisation. They are also required to complete a set of method modules that explore different methodologies for teaching Mathematics. Qualman

(2012) notes that half of the knowledge that students learn in the first half year of their four years of study at university will be outdated by the time they reach third year. In terms of geometry, some PSTs were not able to make the connection between content knowledge and demonstrating through conceptual application (Weber, 2001). The ability to recognise a theorem does not mean that one can apply it correctly. It is thus imperative that PSTs are exposed to a variety of technological advancements during their teacher training. This would enable them to respond and communicate with learners' needs in Mathematics in a meaningful way (Almenara & Diaz, 2012), and at the same time gain mastery of the subject content.

As noted previously, geometry was re-introduced into the curriculum in 2014 and all learners were assessed. In the first year, learners were assisted by providing a number of fill in questions, as well as other questions that were very straightforward. It cannot be assumed that this style of questioning will continue indefinitely. Over time, questions are likely to become more difficult as the geometry content becomes a norm among learners and teachers. Therefore it is imperative that teachers prepare their learners for basic questions (knowledge and routine) as well as higher level questions (complex and problem-solving) in Euclidean geometry (Department of Basic Education, 2014b).

Northcote and Lim (2009) suggest that, in order to improve teacher training, it should include world-wide networking through the Internet, web-site learning, interactive self-learning, multi-media facilities, learning materials; and video-conferencing for local and international sharing and exposure. Teacher training faculties and tertiary institutions as a whole are implementing e-learning to keep students abreast of modern developments and promote competitiveness. The teacher training program comprises compulsory method and teaching practice modules, which explore and incorporate the latest technology in classrooms. Technological trends produce new, challenging situations in the educational context in the training of both teachers and learners (Almenara & Diaz, 2012). This results in unlimited exposure to innovative teaching methods locally and globally, since the philosophy of the teacher determines the choice of the syllabus content and the teaching style (Lerman, 1983). For example the use of mobile phones in a class to find maps and locations, capture data and build computer games exercises learners' Higher Order Thinking Skills (HOTS). Furthermore, with the advancement of technology like mobile devices, they can be used to take measurements like lung capacity, oxygen in the air, heart rate, and distance, etc.

2.8 Conclusion

This chapter examined geometry in the South African curriculum and the understanding of geometry. Prospective mathematics teachers' readiness to teach geometry and teacher training were investigated. The individual's view of Mathematics is crucial, as it determines their future interaction with the subject. The current generation of learners in schools requires much more than traditional methods of teaching in order to engage meaningfully in the learning process. The integration of ICTs in education requires a timely, appropriate implementation process. Combining ICTs with effective pedagogy may be daunting for some teachers; therefore it is advisable that suitable available dynamic content should be used.

Organised integration of ICT in the classroom should benefit the teaching and learning of geometry. Proper planning and application of technology can help learners appreciate the importance and beauty of mathematics. Pre-service teachers should be ready to embrace the ever-changing technologically advancing world. The chapter also focused on technology, with specific reference to going online and using dynamic software. Dynamic creations that are placed online are termed applets, which are a combination of kinetics and visualisation, which is what a person sees and moves. These key senses enhance the learning experience and promote longer lasting recall and concrete understanding of the subject content at hand. While everyone imagines things differently, when given the same visual object, standardisation of the concept under study results, leading to similar conclusions.

The following chapter presents the theoretical framework that underpinned this study.

Chapter three

Theoretical framework

3.1 Introduction

This study explored PSTs' views on the use of technology based teaching methods to teach geometry. For the purpose of this study, technology is defined as something invented that is useful to solve a problem. With this in mind, two theoretical frameworks underpinned this study: Geometric Habits of Mind (GHOM) and Instructional Design (ID). According to Lim and Selden (2009), there are two categories of Habits of Mind: general Habits of Mind that can be applied to all subjects and content-specific Habits of Mind for a subject. Information Communications Technology includes any communication device or application, such as radio, television, cellular phones, computer hardware and software. Thus technology like Dynamic Online Environments (DOEs) can be classified as ICT since it is based on dynamic software that is placed online, creating an interactive applet. Information Communications Technology plays a crucial role in the education sector and the development of the country at large (John & Ali, 2015). Paquette (2014) notes that ICT-based tools and methods are crucial for instructional designers in planning the delivery of learning systems.

The Van Hiele's levels were consulted, as they are well-known pioneers of geometry thinking. These levels describe how an individual moves sequentially on their way to geometric thinking (Abul & Abidin, 2013). As noted and discussed in detail in Chapter 2, they consist of five distinct levels of geometric development in the following order: Level 0 – visualisation, Level 1 – analysis, Level 2 – informal deduction, Level 3 – deduction; and Level 4 – rigor (Dhlamini & De Villiers, 2012). It is of the utmost importance that teachers prepare young South Africans for life after school by equipping them with the tools required to apply their minds, reason, modify; and make sound decisions. This will help them to develop genuine mathematical ways of thinking (Cuoco, Goldenberg & Mark, 1996) to solve problems or challenges in and beyond the mathematics classroom.

3.2 General Habits of Mind

Habits of mind are productive approaches that promote reasoning and thus learning (Köse & Tanişli, 2012). The adaptation of one's mindset when confronted with a problem exercises one's rational thought. The cognitive process is ignited and learning thus takes place. These habits are normally applied to algebra and geometry and are termed Algebraic Habits of Mind (AHOM) or Geometry Habits of Mind (GHOM). While scholars interpret Habits of Mind in different ways, these interpretations are either similar or support one another. In general, Habits of Mind have the following properties: recognising figures, exploring, describing, discovering, visualising, conjecturing; and guessing (Cuoco, Goldenberg, & Mark, 1996; Driscoll, 2001; Driscoll, DiMatteo, Nikula, & Egan, 2007; Goldenberg, Shteingold & Feurzeig, 2003; Hull, Balka, & Miles, 2012; Mark, Cuoco, Goldenberg & Sword, 2010; Seaman & Szydlik, 2007). When an individual develops such Habits of Mind, they gain an in-depth rather than a superficial understanding of mathematics. In essence, they are able to develop an understanding of how mathematicians think.

Habits of Mind extend beyond the specific subject content and can thus be exercised outside the classroom. When a person develops a habit, they develop techniques that can be applied when faced with certain situations. A habit of mind can be likened to a cognitive disposition or a tendency to act mentally in a certain way, in response to certain situations (Lim, 2008). Developing such habits enables learners to smoothly transition to advanced mathematical study. At the same time, it benefits those who will not advance, but will nevertheless use these ways of thinking in other domains, such as investigative journalism, or diagnosis of a broken car or a sick person (Cuoco, Goldenberg & Mark, 1996). Therefore it is imperative to organise lessons and structure learning content around these ways of thinking in order for learners to be able to create meaningful thought processes which will lead to sound understanding.

3.3 Geometric Habits of Mind

Geometric Habits of Mind can be described as creative ways of thinking that promote the learning and application of geometry. This type of thinking involves exploring relationships, reasoning with these relationships, generalising, and investigating variants and invariants in these relationships (Driscoll, DiMatteo, Nikula, & Egan, 2007), thereby making sense of geometry. Geometry is a web of connected concepts and abstract thoughts which are represented through a system that relies on the visual senses (Battista, 2007). This study drew on the work of Driscoll, Zawojewski, Humez, Nikula, Goldsmith and Hammerman (2001) and Driscoll, DiMatteo, Nikula, Egan, Mark and Kelemanik (2008) that describes the following four GHOM that contribute to an understanding of geometry:

Reasoning with relationships:	Generalising geometric Ideas:	Investigating Invariants:	Balancing Exploration and Reflection:
actively looking for relationships within and between geometric figures and thinking about how the relationships can help one's understanding.	wanting to understand and describe the always and the every related to geometric phenomena. Does this happen every time? Why would this happen in every case?	finding what about a situation changes? and what stays the same?	trying various ways to approach a problem and regularly stepping back to reflect.

Figure 3: Fostering Geometric Habits of Mind adapted from Driscoll, DiMatteo, Nikula, Egan, Mark & Kelemanik (2008, pp. 7-12)

These four GHOM were consulted in the creation of the DOEs through dynamic software and the worksheets. The aim was to offer PSTs meaningful interaction with geometry with the aid of technology. It is hoped that the PSTs will reproduce their experiences during this study in their classrooms.

3.4 Instructional design

Instructional Design (ID) is a collection of theories and models that helps one to understand and apply instructional methods that foster learning (Paquette, 2014). It is a method or a process that facilitates strategic planning and organisation of learning and teaching activities, thus bringing order to a chaotic situation. This assists the teacher to assume the role of facilitator and the lesson becomes learner-centred. The adoption of technology supports the instructional design process. Paquette (2014) notes that this could include "web authoring tools and languages, knowledge modeling of instructional design methods, automated and guided instructional design, e-learning standards and social/semantic Web environments" (p.5).

Accompanying technology based learning are tips or hints which guide and probe one's understanding. These provide assistance in solving the problem and developing understanding which provides scaffolding. The Cognition and Technology Group at Vanderbilt (1997) termed these hints embedded data. This relates directly to the Zone of Proximal Development (ZPD) proposed by Vygotsky (1978). When one reaches a dead end or mental block, one can seek help from the embedded data. The embedded data/ hints draw on one's current understanding, taking one from the known to the unknown. These hints or data play a crucial role when instructional time is limited, allowing an individual to cover more work. Geometry is allocated three weeks in Grades 10 and 11 and two weeks in Grade 12 (Department of Education, 2011). Thus, the embedded data or hints are useful in the learning process, bearing in mind the number of teaching and learning disturbances such as sports, tours, public holidays and unforeseen events.

Instructional design can take the form of any type of multimedia such as web pages with photos, text, and video streaming, etc. This exposes the learner to multiple perspectives when solving a problem. There three main processes that take place during any learning are: Assimilation, when one encounters something similar to what they already know, including previous experience; Accommodation, when something new is learnt and this knowledge is accommodated; and Equilibrium, which is the balancing of Assimilation and Accommodation – 50/50 (Block, 1982; Piaget, 1952; Simatwa, 2010). Constructivism focuses on the learner and occurs while learning takes place. It is the result of an active process where the learner's prior knowledge is taken into consideration and is built on, thus causing cognitive conflict (Von Glasersfeld, 1989). Careful planning and use of ICTs together with ID could cause cognitive conflict, leading to realisation and the creation of strong, meaningful geometric thoughts. Such conflict would be enhanced by integrating GHOM in the design of the instructional material.

To promote proper implementation of technology in geometry, this study adapted the following instructional design principles by Mark, Cuoco, Goldenberg and Sword (2010) in alignment with GHOM to create instructional materials (DOEs):

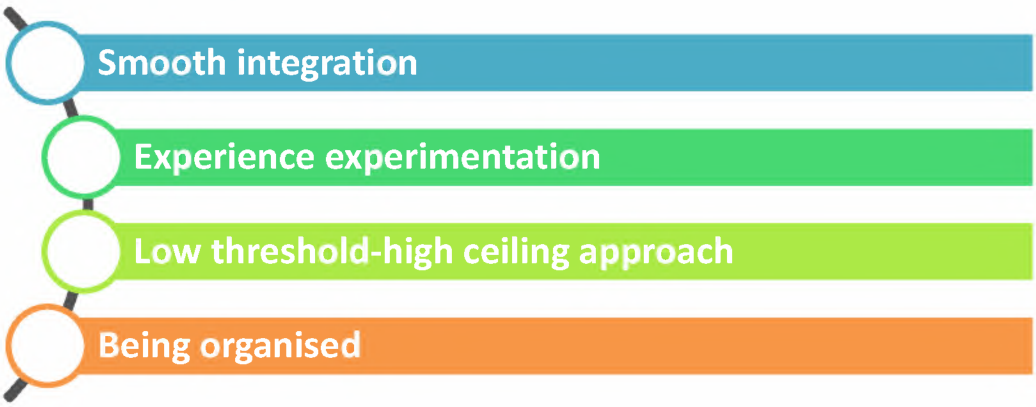


Figure 4: Instructional design principles adapted from Mark, Cuoco, Goldenberg & Sword, (2010, pp. 506-507)

The links between the GHOM levels and ID principles are as follows:

Smooth integration is key when using ICTs in Mathematics teaching and learning. Proper planning and development of ICTs is required before they can be utilised in the classroom.

Similarly, Mark, Cuoco, Goldenberg and Sword (2010) note that time is required to develop GHOM and they must be phased into learners' normal day-to-day activities.

Experience experimentation: one must be given the opportunity to experiment with the topic at hand such as a geometry theorem. The design of the learning material should allow for trial and error exploration of concrete problems before the formal mathematical development (Mark, Cuoco, Goldenberg & Sword, 2010), thus allowing for the integration of abstract and concrete concepts. The embedded data allows for exploration or discovery learning. Learners are given control of their learning, thus eliciting learner centeredness and responsibility. This is supported in the Van Hiele levels which indicate that one should engage before arriving at the formulae (Abu & Abidin, 2013). Experience experimentation is present in the GHOM levels, Reasoning with relationships, Investigating Invariants and Balancing Exploration and Reflection.

In the **low threshold-high ceiling approach**, the designed material is initially easy and becomes progressively more difficult, offering a challenge. For example, abstract concepts are introduced together with experimentation with specific numerical examples, enabling one to extend to deeper understanding (Mark, Cuoco, Goldenberg & Sword, 2010). This is seen when proving a theorem with unknown variables and actual measurements. This exercises one's thinking as one is constantly looking for patterns, spotting relationships, generalising and investigating. This is linked to the GHOM levels, Reasoning with relationships, Generalising geometric Ideas and Investigating Invariants. The experience enables one to feel competent and confident in their learning and at the same time be open to experiences that are more challenging.

Being organised means that development organised around GHOM helps teachers to manoeuvre through an accumulation of teaching methods and techniques gained over the years as well as what is prescribed in the curriculum. Some methods and techniques make for relevant learning only on a particular topic and have no meaning when moving onto another topic or subtopic. It is important that the bits and pieces of mathematical concepts and topics be meaningfully linked, as this can be used to show the interconnected network among the topics in Mathematics. This is more common in geometry, where previous knowledge is drawn on in higher grades. Examples include the use of Theorem of Pythagoras, congruency, similar triangles and the relationship of parallel lines which are all taught at junior grades and later required in senior grades. Meaningful exploration and reflection are promoted, which can lead to generalisation not only in geometry but other topics in Mathematics. This takes place in the following GHOM levels: Generalising geometric Ideas and Balancing Exploration and Reflection. Furthermore, as noted earlier, this type of thinking (GHOM) is not restricted to geometry, but can be applied to various experiences in one's daily life.

3.5 Integrating theory with the study

The DOEs and worksheets that PSTs engaged with were designed in line with the theories discussed in this chapter. These materials set out problems and activities that develop and emphasise the essential GHOM aligned with the use of the selected technology. It is hoped that the use of GHOM will enhance the ability to make connections and gain a deeper, less superficial understanding. Habits of Mind can be considered as practices; essentially these are things that mathematicians do (Bass, 2008). While learners are not expected to duplicate the modelling, thinking and behaviour of mathematicians and philosophers, it is imperative that they gain a deeper understanding that allows for grounded conceptual understanding. When faced with a non-routine problem, learners' procedural understanding is harnessed.

Geometric Habits of Mind practices include asking questions, identifying patterns or structure, consulting the literature and experts, making connections, using mathematical language with care and precision, seeking and analysing proofs, generalising, and exercising the visual senses (Bass, 2008; Driscoll, 2001; Goldenberg, Shteingold & Feurzeig, 2003). The integration of ICT promotes such practices, especially visual senses, which are important in STEM subjects (Goldsmith, Simmons, Winner, Hetland, Hoyle & Brooks, 2014). By arousing the learner's curiosity and encouraging the take-up of STEM subjects which are important in South Africa, learners' minds are harnessed through the use of well-planned ICTs. It is hoped that the DOE will be fruitful, as it is a web-based system that helps to develop and enhance performance via instructional design (Paquette, 2014).

Mark, Cuoco, Goldenberg and Sword (2010) argue that developing Habits of Mind is important in making the critical transition from arithmetic to algebra. This can be likened to the transition from one geometry theorem to the next. As the learner begins to ask questions, find patterns, explore relationships, etc., they begin to self-regulate. Learners should be encouraged to reflect on (a) what they have done after an action, and (b) what they are doing while enacting it, which Schön (1983) termed reflection-on-action and reflection-in-action, respectively. With respect to reflection-in-action, learners should routinely ask themselves: What do I know? What do I want? (Mason & Spence, 1999). This can be compared to the

Socratic Method, where one arrives at the truth by continually questioning, providing answers and criticising the answers. Mark, Cuoco, Goldenberg and Sword (2010) state that early algebraic thinking and moving toward to more formal investigation can typically be found in Grades 8 and 9 which is clear in the use of the Pythagorean Theorem and working with unknowns such as x and y in the CAPS (Department of Education, 2011).

3.6 Conclusion

This study adopted a case study approach involving an in-depth study of one particular case. The theories and concepts that underpinned the study were appropriate as the study sought to explore a group of PSTs' views on the use of technology based teaching methods for teaching geometry. Bertram (2010) states that the aim of a case study is to describe what it is like to be in a particular situation. The learning of geometry is as much about developing GHOM as it is about understanding established results in the discipline. Developing an individual's GHOM is imperative since the thought processes encompass ways of looking at things. The Habits of Mind used by mathematicians, computer scientists, and scientists are mirrored in systems that influence almost every aspect of our daily lives (Cuoco, Goldenberg & Mark, 1996). Learning should move progressively from easy to more difficult, thus advancing knowledge and skills. Active and explorative learning are fundamental to learning and are promoted in GHOM. As such, instructional design that is based on GHOM is a major paradigm for technology based learning (Paquette, 2014).

Chapter four

Research methodology

4.1 Introduction

This chapter describes the methodologies employed to conduct this study. This research commenced with an in-depth literature review on the teaching of Euclidean geometry and PSTs' views on technology. A case study design was selected to explore the problem. Data was collected from PSTs by administrating questionnaires in the form of worksheets, interviews, observations; and surveys that required responses in the form of a Likert scale. These instruments included open- and closed-ended questions. The data collection process took the form of a workshop, which unpacked the relevant circle geometry theorems prescribed by the CAPS through the use of a chosen technology. The study was both quantitative and qualitative in nature, with four methods of data collection employed with a view to enhancing the reliability of the results.

4.2 Research questions

In line with the study's purpose and objectives, the following questions were formulated:

- What technology based teaching methods to teach geometry are pre-service teachers aware of?
- How do pre-service teachers' view the use of technology based teaching methods to teach geometry?
- Why do pre-service teachers have these views on the use of technology based teaching methods to teach geometry?

4.3 Research approach

A case study methodology was employed to conduct this study. Mouton (2001) states that case studies aim to provide an in-depth description of a particular event or scenario. Similarly, Merriam (1988) notes that a "case study is an intensive, holistic, description and analysis of a single instance, phenomenon, or social unit" (p. 21). This study was structured in a natural setting for PSTs to use their understanding to make sense of or interpret the given phenomenon. As proposed by Stake (1995), a case study usually explores the uniqueness of a single case, gaining understanding of its activity within set parameters. The study explores PSTs' views on the use of technology in teaching Euclidean geometry. These views are based on human experiences which are realistic, in other words, volatile and not concrete. As noted by Yin (2009), when a context or situation is situated in reality and the phenomenon based on it is not clear, a case study is appropriate.

An interpretivist paradigm was adopted within this case study. Cohen (2001) maintains that a case study design should be approached from an interpretive paradigm as case studies are based on a particular situation or unique event. An interpretivist perspective allows for comprehensive holistic understanding. Cohen, Manion and Morrision (2000) note that: "...case studies investigate and report the complex dynamic and unfolding interactions of events, human relationships and other factors in a unique instance" (p. 181). Hence, it is important to interpret the participants' interactions in a specific situation, as well as their understanding of the phenomenon under study (Maree, 2007).

Generally, a case study examines a topic or phenomenon that exists before the research and will continue to exist after the study is complete (Denscombe, 2010). The PSTs were exposed to a computer technology which already exists and would remain accessible to them via the Internet after the completion of the study. The interpretative paradigm lends itself to an ethnographic study. Such a study sets out to understand and describe an unfamiliar setting in a natural, real world context. It is heavily based on description, thus eliciting qualitative data. Some properties of an ethnographic study are evident in this interpretive case study.

4.4 Qualitative and quantitative data

The use of both qualitative and quantitative data collection methods proved useful in this study. Stols, Mji, and Wessels, (2008) and Yin (2009) describe this as a mixed methods approach. Qualitative research is known as descriptive research, which deals with words and sentences, thus making it ideal in a case study. However quantitative data can also be used in a descriptive study (Boudah, 2011). An example of the quantitative data used in this study is the use of test scores and when participants rated their level of application during an activity (Boudah, 2011).

Check and Shutt (2012) identify three different types of mix methods studies: qualitative and quantitative data are equally important; quantitative data is given priority over qualitative data; and qualitative data is given priority over quantitative data. The last-mentioned approach was used in this study. In order to gain deeper understanding and explain the results of the quantitative data, it was necessary to use qualitative methodologies as they complemented each other. Furthermore, the mixed methods approach enabled the researcher to construct detailed descriptions and more complete explanations.

4.5 Data collection techniques

In order to answer the research questions and achieve the study's objectives, an appropriate topic/s in Euclidean geometry had to be selected. The topics chosen were the Theorem of Pythagoras which is taught in the GET phase, and circle geometry which is taught in the FET phase in line with the South African curriculum, CAPS.

Data was collected as follows:

- Completion of a questionnaire prior to exposure to technology based teaching methods. This served as a pre-survey where participants' responses were scrutinised, yielding qualitative data.
- Completion of a series of worksheets using applets online - DOE - on the Theorem of Pythagoras (Appendices H, I, J, K and L) and circle geometry (Appendix M) worksheets. These worksheets produced quantitative data by means of a solution which was either correct or incorrect, giving sub-totals for each section and grand totals.
- Observation of participants and video recording for later confirmation of what was observed. This enabled the collection of qualitative data.
- All participants completed a questionnaire (Appendix O, Appendix P - colour coded) which served as a post survey in the form of a Likert scale. By totaling each response, quantitative data was yielded.
- The workshop session concluded with a focus group interview.

4.5.1 The pre-survey questionnaire

It was imperative that the researcher understand the participants' prior experience and their interpretation of geometry and technology. A survey (Appendix G) was used to obtain background information on the participants. Denscombe (2010) notes that surveys are used to capture a snapshot at a particular point in time. The pre-survey data was gathered before any exposure to technology that engaged with the Theorem of Pythagoras or circle geometry. It consisted of 12 items with open- and closed-ended questions, focusing on geometry reasoning/thinking and the use of technology. The PSTs also provided information on the availability of computers/technologies, their usage and where they utilised them. This survey led up to the task based worksheets on the Theorem of Pythagoras and circle geometry.

4.5.2 Task based worksheets

The task based worksheets contained elements of a questionnaire in their design. There are two types of questionnaire designs, namely, descriptive and analytic (Oppenheim, 1966). A descriptive questionnaire is used to count a representative sample of the population. This allows for judgment on the entire population. It answers the question of how many participants have a certain trait. On the other hand, an analytic question seeks to answer the question, 'why'. Analytic questionnaires are known as relational surveys since they explore the relationships between variables (Oppenheim, 1966). A fixed choice answer is derived from a closed-ended question, while open-ended questions allow the participant freedom when responding (Denscombe, 2010). Both designs and question types were used in the design of the task based worksheets. The descriptive design was present when it came to solving the mathematical problems and there was a strong analytic design influence. Multiple question design techniques were integrated to meet the study's objectives and gather as much data as possible. The task based worksheets on circle geometry allowed participants to explore the DOE, resulting in the formation of proofs. The task based worksheets on the Theorem of Pythagoras together with the DOE assisted participants in understanding the theorem. The DOEs for both sets of worksheets were highly interactive and aimed to probe the participants' thinking.

The Theorem of Pythagoras worksheet

In general, learners at school describe the Pythagorean Theorem as $a^2+b^2=c^2$ where a, b, and c (c is the hypotenuse[6] side) are sides of a right angled triangle. However, they often don't know why this is true and most have never proved it, yet they use the theorem. Kondratieva (2011) notes that few secondary school students understand the fundaments of the statement: $a^2+b^2=c^2$ where a, b, and c (c is the hypotenuse side) are sides of a right angled triangle. The Pythagorean Theorem has been the most challenging in middle school (Habibi, 2010), which is Grades 8 and 9 in the South African context. This theorem has been controversial in many education systems because it lacks an appropriate introduction. It is also important to note that it is one of the initial theorems that learners are exposed to at high school; thus it is important how teachers introduce it. According to the ANA (2015) diagnostic report, about 84% of the 67 learners whose scripts were moderated could not calculate the length of a side of a right angled triangle using Pythagoras. The worksheets titled Identifying the sides (Appendix H), Pythagorean Puzzle (Appendix I), Half Circles (Appendix J), Magic Triangle (Appendix K) and Pythagorean tree (Appendix L) all reinforce Pythagorean relationships and

[6] The hypotenuse side is the longest side in a right angled triangle.

add meaning to the theorem. These activities placed emphasis on conceptual understanding, rather than the standard procedural understanding.

The circle geometry worksheet

The second topic was based on circle geometry which is a section that is introduced in Grade 11. This topic falls within Euclidean geometry, which was reintroduced into the South African syllabus in the FET phase with the implementation of the CAPS in 2011. This created serious concern; hence, the aim was to determine how PSTs would view teaching circle geometry using technology. Most of the PSTs in this study would have not studied circle geometry in the FET phase, as they completed Grade 12 prior to the implementation of the CAPS or they would have stopped teaching it from 2008 to 2012.

Four circle geometry theorems were selected for PSTs to engage with. The selection was based on the appearance and occurrence of the theorems directly and indirectly in the Mathematics NSC examinations from 2007 to 2013. The examiners' reports were also examined in order to detect problem areas and performance levels in circle geometry. The 2007 NSC written examination was the last year when the Mathematics paper 2 HG included geometry. In 2008, geometry and other topics formed part of the new Mathematics paper 3. The year 2013 was the last year that the Mathematics paper 3 was written.

The first theorem selected was: if a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord otherwise known as perpendicular from centre to chord (Applet: Appendix T). The proving and application of this theorem appeared in the 2013 NSC Mathematics paper 3. The examiner commented that learners and teachers were caught by surprise. Although some learners manage to score some marks with the application, this question was not answered well by the majority of learners (Department of Basic Education, 2014). It is clear that there is more emphasis on procedural understanding and application of the theorem rather than conceptualising its formation.

The next theorem selected for the worksheet was: if an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference, otherwise known as angle at centre is twice the angle at circumference (Appendix U). The proving of this question appeared in the 2009 NSC Mathematics paper 3. The third theorem selected was: if a quadrilateral is cyclic, then the opposite angles are supplementary, otherwise known as opposite angles of cyclic quad are supplementary (Appendix V). The proving of this theorem was in the last NSC Mathematics HG paper 2, in 2007. The final theorem selected was: the angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment, otherwise known as tangent- chord theorem (Appendix W). This theorem was in the NSC Mathematics paper 3 in 2010 and 2011. Application questions based on this theorem can be found in all Mathematics examinations papers from 2007 to 2013.

Further comments and suggestions included in the examiner's report were that learners did not learn these proofs as theorems; they did not know how to prove congruency; and that teachers' should include one question which requires bookwork testing theorem (Department of Basic Education, 2014). Thus, it is imperative to note that geometry is not a standalone topic; it is integrated with other mathematical knowledge when engaging with topics such as congruency, the Pythagorean Theorem, algebra, shapes, measurements and even trigonometry.

In most cases, one would need to know an earlier theorem in order to prove a conjecture. However due to strong application and procedural knowledge, one neglects conceptual understanding. This is a good example of what Freudenthal (1973) referred to as the anti-didactical inversion. De Villiers and Heideman (2014) describe this as "teaching only the final, polished mathematical product without showing its evolution over a period of time" (p.26). Where possible, in mathematics, and particularly in geometry, it is important that one does not teach theorems as individual items but allows for the linking of theorems/geometric concepts. As with geometry, prior knowledge is crucial in proving succeeding theorems. Geometry as a whole is seen as a vital component in many fields of study and is dominant in Engineering. Prior to the scrapping of the Mathematics Paper 3 in 2013, the Subject Report from Universities suggested that learners who wanted to pursue a career in Engineering should consider taking this paper (Department of Basic Education, 2011b). Pillay and De Villiers (2013) note that that "university lecturers in mathematics, science and engineering see geometry as an integral part of mathematics, and teaching it at school is important for university preparation" (p. 36).

4.5.3 Observations

Observation offers the researcher first hand evidence on the participants' experiences. Denscombe (2010) identifies three types of participant observation: total participation, participation in the normal setting; and participation as observer. Participation as observer was adopted in this study as it enabled the researcher to witness the details of the event first hand. This type of observation takes the form of shadowing a person or a group (Denscombe, 2010). It is the norm in observing multimedia interactive learning since it enables expert evaluation (Huysamen, 1994; Lee, 1999). The video recordings of the participants were analysed against the questionnaires, worksheets and interview data. These recordings confirmed or contradicted the participants' responses in other data collection instruments. In addition, playback enabled the researcher to note important points that could have been missed during data collection. Observation was not dependant on what was said, done or thought; rather, it provided primary evidence of the participants' experiences. The observation schedule (Appendix Q) included a checklist of common visual items (how many times did the PST ask a peer or tutor for help, emotions reflected in physical gestures, mastery and harmonious use of technology, and so on. A blank space was added to the schedule to note any other observations, affording the opportunity to describe any unusual event.

4.5.4 The post-survey questionnaire

The post survey took the form of a questionnaire with 30 items in the form of a Likert scale with options: strongly disagree - 1, disagree - 2, neutral - 3, agree - 4 or strongly agree - 5. Each participant was given a pre-test and a post-test in order to evaluate their holistic view of DOE technology and to obtain clear feedback on their perceptions of such technology. The 30 items are a subset to the following five themes: Use of the Internet and computer in Mathematics, Fluency of mathematics PSTs in computers and technology, Impact of DOE in teaching and learning of mathematics, DOE as a teaching tool, will DOE be utilised in future lessons; and DOE in everyday teaching. These themes were created to gain an understanding of the PSTs' experiences, to check against the pre-survey; and to note any change in views.

4.5.5 The focus group interview

The purpose of this type of interview is to gather information which is not directly observable (Patton, 1990). A focus group interview was held with the participants. Focus group interviews usually consist of a small number, not more than 50 participants who are brought together by the researcher to explore and express their views, attitudes and feelings (Check & Schutt, 2012). There are three types of interviews: structured, unstructured and semi-structured. Semi-structured interviews were used in this study together with a focus group interview schedule (Appendix N) which consisted of nine open-ended questions. This enabled the participants to share their experiences, perceptions and understanding of the topic under study. Probing the participants was fundamental as this led to in-depth qualitative data. This data reflected the PSTs' understanding of geometry through the use of DOEs and their views on this mode of teaching and learning.

The number of responses for each question was based on a cumulative approach. In this approach, participants' responses are welcomed until the researcher has accumulated sufficient information for the purpose of the study (Denscombe, 2010). Participants tend to become more responsive and engaged with one another in constructing individual and collective experiences when they are grouped (Check & Schutt, 2012; Cohen & Manion, 1987; Fontana & Frey, 1994). The focus group discussion thus provided rich, saturated qualitative data on the participants' beliefs about geometry as well as their feelings about what was done.

4.6 Designing the instructional materials

The core aim of this study was to explore PSTs' views and experiences when using technology based teaching methods to teach Euclidean geometry. The type of technology used is important when teaching or learning. The use of technology per se does not mean better teaching or understanding as it can be a distraction to both the educator and the learner (Mills, 2011). The layout and approach of the instructional materials were carefully designed.

4.6.1 The use of dynamic software

This study utilised dynamic software as a research tool as it is a powerful and versatile tool that offers learners an optimal learning environment. For the purpose of this study, dynamic software refers to dynamic geometry application software such as Geometer's Sketchpad, Cabri, Logo, Geometric Supposer, Geogebra and Wolfram's Mathematica, to name but a few. The key property of dynamic software is that it offers the learner freedom to move shapes and graphs, as compared to the static convention of sketches. It enables instantaneous movement of an object such as a parabola graph as the values change; a, b and c of $f(x) = ax^2 + bx + c$ for example. Take, for example, a part of the DOE for the theorem that states (Figure 5), if an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference otherwise known as angle at centre is twice the angle at circumference. This theorem is represented in three different ways in Figure 5 below, with "Figure 1" being the most basic representation. The click and drag action can be used such that Figure 5: "Figure 1" can be manipulated to identically represent Figure 5: “Figure 2” and Figure 5: “Figure 3”. In addition, one can notice the instantaneous change in the size of the angles.

The use of educational technologies such as dynamic software offers great opportunities for an exploratory style of teaching and learning of mathematics (De Villiers, 2011; Hölzl & Schäfer, 2013; Mavani & Mavani, 2014; Nyaumwe & Mtetwa, 2009; Patkin & Barkai, 2014; Stols, Mji & Wessels, 2008). However, the implementation of technology does not guarantee effective teaching and learning. As Davies (2013) remarks, the use of technology does not guarantee meaningful learning; neither does it mean that it is utilised in a meaningful way.

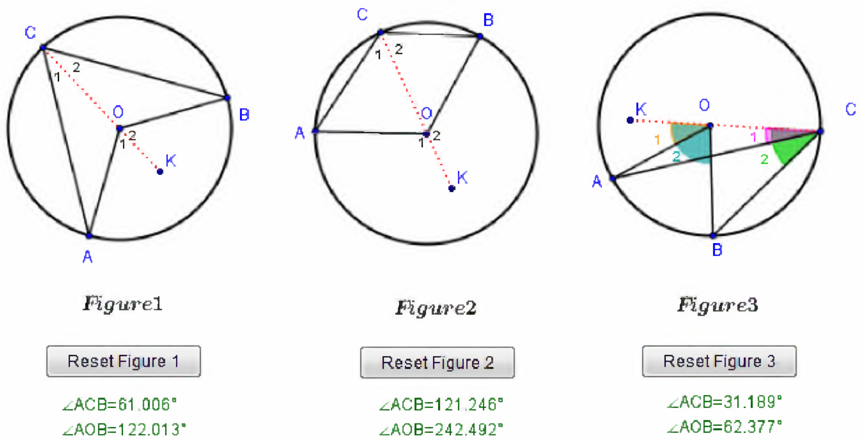


Figure 5: Part of the DOE for angle at centre is twice the angle at circumference created and uploaded later onto the website: http://govenderrg.wikispaces.com/Angle+at+centre+equals+twice+angle+at+circum

Exposure to educational software helps to make abstract concepts simple and tangible which results in knowledge fitting easily with a learner’s current knowledge structure. Mathematical exploration through the use of dynamic software should be a critical part of the high school curriculum (Mudaly, 2013). The worksheets designed covered the Theorem of Pythagoras (Appendices H, I, J, K and L) and circle geometry (Appendix M). For the purpose of this study, dynamic content was defined as the user’s freedom to make instantaneous changes to an object’s properties. Learners and teachers can manipulate the constructions made through the software dynamically as the software creates an environment which one can explore. The software Geogebra was used to create the mathematic dynamic content. Geogebra was created by Markus Hohenwarter in 2001 at the University of Salzburg (Geogebra, 2014) and is regarded as freeware software. This means that it can be downloaded free of charge from the Internet.

4.6.2 The Applets in this study

Applets are small programs that run on Java when accessed online through the Internet. They can be described as a program that runs on the Internet allowing user interaction. According to Goose (2010), "dynamic geometry packages; and web-based applications offering virtual learning environments have changed the mathematics teaching and learning terrain" (p.67). Taking dynamic software onto an online platform allows for online teaching, thus creating an applet. Hence the derived name: Dynamic Online Environment/s (DOE) in this study. The following website hosted the dynamic content online: www.govenderrg.wikispaces.com. The materials published on this site are under Creative Commons Copyright licensing, making them available to anyone with an Internet connection.These types of teaching and learning spaces offer informative, expertise-oriented learning, and personal instruction for the learner (Harvard, 2013). For the purpose of this study, DOE was chosen in order to accomplish the research objectives: pre-service teachers' knowledge, awareness and views of technology based teaching methods for teaching geometry.

Through the use of DOE, learners tend to take control of their own learning. This nurtures an analytical mind and meaningful engagement with mathematics content. In addition, the DOEs for both set of worksheets were highly interactive with colour, movement, pop-ups containing hints, and sliders. Patkin and Levenberg (2012) argue that the "development of visual links has importance for, as well as very great impact on, the level of comprehension of geometrical content" (p.14). Take, for example, the DOE for the Magic Triangle (Figure 6) with the use of sliders.

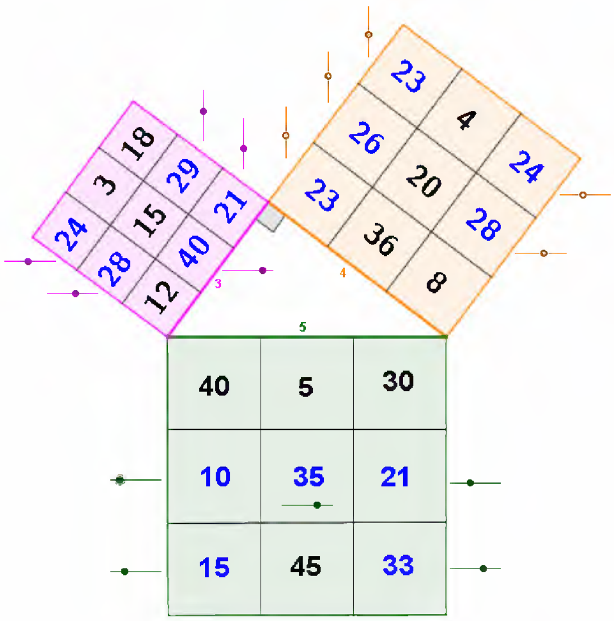


Figure 6: DOE for the Magic Triangle created and uploaded later onto the website: http://govenderrg.wikispaces.com/Magic+triangle

On completion of each square in the Magic Triangle, if the answer is correct, a smiley face will pop up (Figure 7).

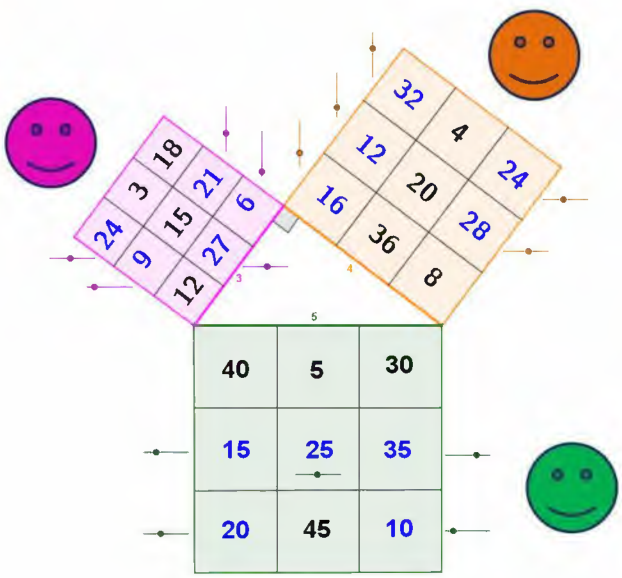


Figure 7: Magic Triangle answered online from http://govenderrg.wikispaces.com/Magic+triangle

4.7 Study setting

4.7.1 Location

The site chosen for this study was one university in KwaZulu-Natal, South Africa. At this university, PSTs have the option to register either for a Bachelor's Degree or Certificate in

Education. They can also further their studies in the General Education and Training (GET) or the Further Education and Training (FET) band. The data collection process took the form of a workshop where PSTs interacted with the task based worksheets and its relevant DOEs. This university's LAN was adequate to commence data collection as it offered well-equipped and updated computer technologies with Internet access. In addition it was convenient for the participants as the campus and surroundings were familiar to them.

4.7.2 Population and sample

The study adopted an exploratory sampling technique rather than representative sampling which is based on a large sample. According to Denscombe (2010), there is no absolute rule on the size of an exploratory sample since the researcher seeks depth and detail. Hence, the number of participants contributes to a case study design. The participants in this study were selected using a non-probability method known as purposive sampling. It is important to note that a non-probability sampling method seeks an exploratory understanding rather than a representative one. This method is the most convenient way of collecting data from students because the number of students available at the university on a particular day would be unknown. It is less complicated and time consuming and more economical (Huysamen, 1994). The location was convenient since it had a well-resourced computer room with easy access to PSTs. Check and Schutt (2012) explain that in purposive sampling, the elements in the sample possess a unique position or knowledge. The selection of the PSTs for the pilot and the main study was by invitation based on a set of requirements. Invitations were sent to 30 PSTs who met these requirements: mathematics PST who is studying within the School of Mathematics Education, in their 2nd, 3rd or 4th year of undergraduate study or continuing with postgraduate studies. The reason for excluding first year students was that all participants would have some teaching experience due to [7]TP. They would have already been exposed to the use of technology in the teaching and learning process due to the modules and course work designed by the university for students in their 2nd year and upward of study. This allowed for a good mix of views on teaching geometry using technology. It was possible that some participants might have completed Grade 12 between 2008 and 2010, when Euclidean geometry was excluded from Paper 2 and was an optional paper. This sample was able to provide valuable feedback regarding the use of computer technology in this techno era since they were learners in the recent past. The PSTs who were furthering their studies (postgraduate) would have practising/expertise experience. Thus, data was gathered from a broad range of PSTs from novice undergraduate teachers to practising postgraduate teachers.

[7] TP –Teaching Practice module offered at university for undergraduate students.

Twenty six PSTs accepted the invitation and ten were selected for the pilot study. Of the remaining 16, only ten participated in the main study. To summarise, each participant met the following criteria: 1. they were from the School of Mathematics Education, 2. phase specialisation in high school (upper senior FET or FET band), 3. in their 2^{nd}, 3^{rd} or 4^{th} year of study or postgraduate student.

4.7.3 The pilot study

As noted above, the pilot study was conducted with ten participants. The aim was to determine whether the data collection instruments elicited the desired data, thus testing the chosen procedures and materials. Following the outcome of the pilot study, the necessary changes were made in order for the data to be aligned to answer the research questions. Boudah (2011) notes that a pilot study gives a researcher a better understanding of the study and an opportunity to revise methods and procedures. Similarly, Huysamen (1994) states that a pilot study unearths any deficiencies in the measuring procedure before the actual study commences. Following the pilot study, certain questions were rephrased to eliminate ambiguity and ensure that the question was clear. The effectiveness and accuracy of the applets were ascertained as well as their limitations. As a result, two of the applets had to be modified/ re-designed and updated onto the host website.

4.8 Validity, Reliability and Rigour of this study

Lincoln and Guba (1985) note that no validity exists without reliability; thus ensuring validity also ensures reliability. Reliablity is an integral part of validity and trustworthness (Check & Schutt, 2012). It ensures that if the study were to be repeated, the same results would be obtained. The mixed methods approach and the use of multiple data sources (four data collection tools) in this study ensured that triangulation was achieved and hence validity, which prevents personal bias. Denscombe (2010) observes that the use of multiple methods in a case study facilitates the validation of data through triangulation.

The use of thick description promoted the study's trustworthiness and the transferability of information. Thick descriptions allowed for interpretation of what the participants said about their experiences in reality (Geertz, 1973). A characteristic of case studies is that their findings cannot be generalised. However, the use of thick description enables transferability to other similar cases. Although the sample was small, it was possible to collect data that made for interesting findings. Floyd and Fowler (2009) note that a smaller sample can describe a larger population with virtually the same degree of accuracy. The unanticipated responses from the PSTs were part of the richness of the case study, as a result from the micro surrounding forms thick descriptions. According to Denscombe (2010), one should "employ qualitative data through focus groups and interviews to improve the validity of a subsequent survey questionnaire that produces quantitative data" (p. 140). Thus, validity was achieved through the collection of qualitative data via the focus group interview and quantitative data via the post survey.

4.9 Permission and ethical considerations

According to the South African Bill of Rights (1996) no person may be discriminated against due to their race, religion, gender, language or birth. Throughout the research process, the researcher ensured that that the rights of the study participants were not compromised in any way. Permission was obtained from the relevant higher education bodies to conduct the research. A letter requesting permission to use the above-mentioned study population was sent to the Dean of the School of Education (Appendix A) at the selected university. In addition, the relevant school head (Appendix B) and students were approached to obtain their permission and cooperation (Appendix C). The participants were given a letter of consent to sign, containing details of the study with the option of participating and/or withdrawing at any stage of the research. Furthermore, the anonymity and confidentiality of participants were guaranteed by the use of pseudonyms. Permission was obtained to utilise the university's LAN. Prior to any letters requesting permission being sent out full ethical clearance had to be granted (Appendix D). In order to keep track of participants and safeguard their privacy and confidentiality, each participant was assigned a code starting with the letter "M" representing Mathematics student and a number. This code was written on their worksheets and surveys and had to be stated before a participant could respond during the focus group interview.

4.10 Conclusion

This chapter outlined the research methods and processes employed to conduct this study as well as the ethical considerations taken into account. Figure 8 shows a visual plan of the data collection process. A mixed methods approach was employed and the qualitative and quantitative data gathered allowed for in-depth comparison of the findings. The following website hosted the dynamic content online: www.govenderrg.wikispaces.com. All participants completed a pre-survey questionnaire at the beginning of the study without any prior intervention or exposure to technology, and a post-survey questionnaire at the end to evaluate if there was any change in mindset and understanding. The DOEs, together with the series of worksheets that explored selected topics in geometry provided a clear understanding of the perceptions and views that PSTs hold about using DOE to teach geometry.

All data collection instruments were piloted to check for clear directions to respondents, ambiguous statements, and sequencing of statements. The data collection instruments were revised based on feedback from the pilot study.

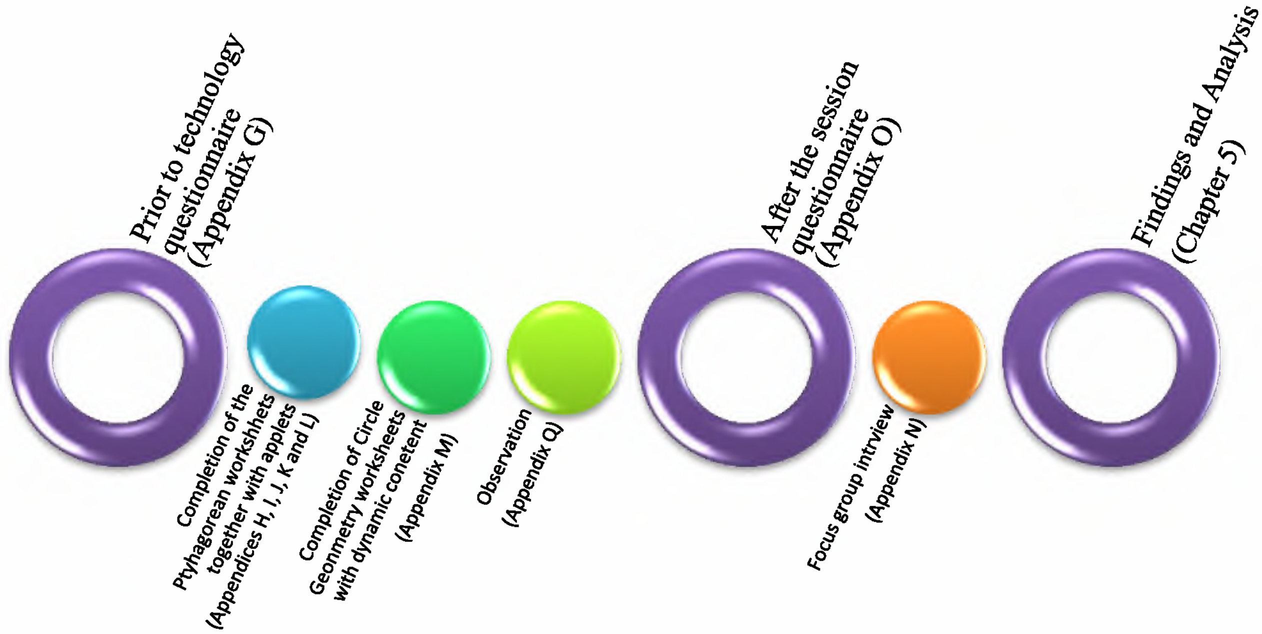


Figure 8: Overall view of the data collection process constructed by the researcher

The findings and analysis are discussed in detail in the following chapter. Although an arduous task due to the amount of data collected, conducting a mixed methods study was extremely rewarding. The quantitative methods corroborated and enhanced the qualitative methods, thus allowing for the precise findings and conclusions that are presented in the chapters that follow.

Chapter five

Findings and Analysis

5.1 Introduction

In line with the purpose of this study which was to explore PSTs' views on the use of technology to teach geometry, each participant completed a questionnaire which served as a pre-survey based on experience and current understanding of technology and geometry. Thereafter each participant completed a series of task based worksheets using the selected technology (DOE) for this study. The participants' reactions whilst completing the task based worksheets were closely observed. A second questionnaire was issued to the participants after exposure to the technology. This served as a post-survey which was followed by a focus group interview to further probe participants' perceptions of using technology when teaching geometry.

A combination of quantitative and qualitative methods was used to source data in order to fully address the research questions. The mixed methods approach allowed for the interaction of both types of data in order to reach common ground. The quantitative data was processed through the use of statistical software. Summary tables and graphical representations were created from the data captured through the software. This enabled visual representation of the findings, allowing for easier analysis and interpretation of the findings. The six themes that emerged during the analysis underpin the study's findings. Further analysis revealed three categories in which teachers can be classified based on their adaptation to technology when teaching geometry.

5.2 Data presentation and analysis

5.2.1 The pre-survey questionnaire

Prior to their exposure to technology, a questionnaire (Appendix G) was administered to the participants to gain a sense of their current understanding of geometry and perspectives of technology. Table 4 shows the subject combinations chosen by the PSTs. All the study participants were from the Mathematics discipline as per the criteria, with Mathematics being their first major. It was interesting to note that 60% of the PSTs' second majors fell under the Science discipline, particularly Physical Science at 40%. This emphasised the close relationship between Mathematics and Science in terms of calculations and logical reasoning. On the other hand, 20% of the PSTs selected a subject from the Commerce or Technology discipline.

Table 4: Subject specialisation/s groupings of respondents (Question A)

Discipline	Subject	% per subject	% per discipline
Science	Physical Science	40	60
	Life Science	10	
	Sport Science	10	
Commerce	Accounting	10	20
	Business Studies	10	
Technology	Technology	10	20
	Information Technology	10	

The majority of participants (60%) were in their fourth year of study, followed by postgraduate study (Table 5). As noted in Chapter 4, the criteria for participation in this study excluded first-year students as they would not possess the necessary background or experience.

Table 5: Respondents' year of study (Question B)

Year of study	%
1st	0
2nd	10
3rd	10
4th	60
Postgraduate	20

The data collection process took the form of a workshop at the participants' campus LAN (Local Area Network) in order to ensure a relaxed environment. As Denscombe (2010) and Bergold and Thomas (2012) note, a familiar environment enables participants to be relaxed and open to comment. This resulted in more productive collection of data.

Table 6: Workshop attendance of respondents (Question C)

Attended any workshops previously	%
Yes	20
No	80

Table 6 shows that 80% of the PSTs had not previously attended any workshops; this could be due to being students, with the majority in their final year of study. Hopefully, this workshop would encourage PSTs to attend future workshops and seminars, thus maintaining constant professional and academic growth as a teacher.

It was found that all the PSTs used textbooks as a resource when teaching (Table 7). This is most likely because the Department of Basic Education's Learning and Teaching Support Material (LTSM) provides a list of prescribed textbooks for each grade which are aligned with the CAPS (List of textbooks: Appendix X). According to Purcell, Heaps, Buchanan and Friedrich (2013), "the internet has a major impact on their (teachers) ability to access content, resources, and materials for their teaching" (p.2); as a result it was the second most common resource and was used by 80% of the participants. Only 10% of the PSTs used past examination papers when preparing for and teaching a lesson. Such papers are useful as they provide an exemplar format and layout of question types and levels in terms of what is expected in future assessments. However, it is pleasing to know that PSTs do not prepare learners for examinations but teach for the fulfillment of knowledge and understanding.

While collaboration is important, especially among budding teachers as it fosters new ideas and methods, only 30% of the participants reported that they consult other educators. Piercey (2010) notes that the reason teachers don't collaborate is simply because they won't or can't demonstrate and model the necessary attributes.

Table 7: Resources used by respondents (Question D)

Resources used when teaching	% per resource used
Textbooks	100
Internet	80
Past Year Papers	10
Other Educators	30

Table 8 shows that 50% of the PSTs did not do any geometry in school, which means that they were exposed to the subject for the first time at university.

Table 8: Did you do geometry at school? (Question E)

As a learner you did you do geometry at school?	%
Yes	50%
No	50%

Furthermore, 10% (Table 9) of the 50% that did not do geometry at school selected the Maths paper 3 at school. It is likely that half the PSTs completed high school prior to 2008 when geometry was part of the Mathematics paper 2. From the 90% that said No, 50% stated that they did geometry at school.

Table 9: Did you do Maths Paper 3 at school? (Question F)

Did you do Math's Paper 3 at school	%
Yes	10%
No	90%

Table 10 shows that 50% of PSTs were in the age bracket 22-30 years, followed by 30% who were <=21 years old (Table 10). This generation of teachers needs to prepare themselves to educate a tech generation of learners. The transition to and blending of technology with geometry would not be difficult for young teachers as they keep abreast with technological trends in their personal lives, with a ripple effect on their teaching (Purcell, Heaps, Buchanan and Friedrich, 2013). Lei (2009) describes such a generation of teachers as digital natives. They grew up with digital technology and computing devices like gaming consoles, smartphones, access to the Internet and other ICT. They are thus more comfortable with such technology than previous generations.

Table 10: Age of respondents (Question G)

Age	%
<=21	30%
22 to 30	50%
31 to 40	10%
>40	10%

As illustrated in Table 11, the most common available devices to aid teaching were USBs (90%), a portable laptop or notebook (80%) and a printer (70%). It was not surprising that the least available device was the desktop computer as we live a flexible, mobile and on-the-go life style. Portable laptops/notebooks make digital formatted information mobile and convenient.

Table 11: Devices available to respondents during teaching (Question 1)

Devices available when teaching	%
Desktop computers	30%
Portable laptops or notebooks	80%
Digital data projectors	40%
Printers	70%
Internet connections	50%
USB (memory) sticks	90%

Turning to devices available to the respondents at home, the results show that only 10% of the PSTs had digital data projectors and 20% had printers at home; thus the majority do not own these devices (Table 12). Table 11 shows that such devices are more likely to be available at school (digital data projectors at 40% and printers at 70%). The high number of participants who have USBs implies that digital format is favoured over files and huge paper trails.

Table 12: Devices available to respondents at home (Question 2)

Devices available at home	%
Desktop computers	30%
Portable laptop or notebooks	90%
Digital data projectors	10%
Printers	20%
Internet connections	40%
USB (memory) sticks	90%

Portable computers (desktop and portable) and USBs (Table 12) are likely to be the PSTs' personal belongings, as there seems to be a correlation between the percentages in Tables 11 and 12. This means that these devices are accessible during teaching or at home. Fifty per cent of the PSTs reported that they have Internet access during teaching, while 40% have access at home. Although many PSTs have Internet access, it is interesting to note that only 10% participate in online forums (Table 13) while the majority use the Internet to complete assignments, for e-mail usage, social networking and lesson planning. The 10% that participate in forums are aligned with the collaborative efforts of PSTs noted earlier.

Table 13: Respondents' activities on the Internet (Question 3)

Internet used for:	%
Play online games	20%
Collecting information for assignments	90%
Use e-mail	90%
Lesson plan preparation	80%
Browse the Internet for fun (such as watching videos, e.g., YouTube)	60%
Download music, movies, games or software from the Internet	70%
Social networking (e.g., Facebook and Twitter)	80%
Participate in online forums	10%

Table 14: Lesson preparation time allocated for the use of technology (Question 4)

A typical lesson preparation, how much time do you allocated for the use of technology?	%
I never use any technology in my lessons	10%
Only few minutes and not related to MATHEMATICS	10%
Only few minutes and related to MATHEMATICS	80%

Table 14 shows that 80% of the participants used technology which is related to mathematics in a lesson. This can be associated with the 70% that tried to integrate technology (Table 15) in their lesson preparation. The majority of the PSTs (80%) agreed that it is important to work with technology, while 50% (Table 15) felt that playing or working with technology is fun. Eady and Lockyer (2013) hold the view that providing access to technology is not enough, and the teacher still plays a crucial role in initiating the learning process. However, 20% (Table 15) of the PSTs felt that technology could be a distraction. On the other hand, 30% (Table 15) agreed that it is impossible to integrate technology meaningfully in the classroom.

Table 15: Respondents' agreements (Question 5)

Do you agree with these statements based on your experience with technology?	%
It is very important to me to work with a technology.	80%
I think playing or working with a technology is really fun.	50%
I try to incorporate the use of technology in my lessons preparations.	70%
I lose track of time when I am working with technology.	10%
Technology is a distraction in the classroom.	20%
It is impossible to integrate technology in a meaningful in the classroom.	30%

It was crucial to establish what the PSTs thought about geometry especially when it was removed from the curriculum and then reintroduced. The next question required them to describe geometry.

Table 16: Responses to geometry (Question 6)

Describe/discuss/draw what you think geometry is?	%
Left blank:	10%
Attempted:	90%

The 90% (Table 16) of the participants that attempted the question all provided correct definitions of geometry. The definitions included mainly text rather than drawings or mind mapping. It can be assumed that the way one defines geometry is how one understands it. The following are some of the descriptions and purposes of geometry expressed by PSTs in response to Question 6 (Table 16).

Question 6: **Describe/discuss/draw what you think geometry is?**

M04: "Geometry deals with the maintenance of building and can be easily linked to real world problems."

M26: "Geometry helps one to provide meaningful reasons."

M03: "Geometry is based on the relationship between points and lines."

In similar vein, **M14**, **M07** and **M13** all stated that geometry is a branch of mathematics that deals with the properties of shape and space; including size, positions and patterns that one encounters every day without even realising it. On the other hand, **M23** and **M19** expressed that geometry is imagery work, and one has to be an abstract thinker; this is a visual study which involves shapes, conjecture and patterns.

Table 17 shows that 10% of the participants drew the right angled triangle in an unusual way, in comparison to the normal drawing. This shows that most of the PSTs reflect on right angled triangles in a straightforward manner without any transformation or alternatives to the shape.

Table 17: Respondents' drawing of a right angled triangle (Question 7)

Draw a right angled triangle	%
Correct	100%
Normally drawn - either leftward or rightward	90%
Unusually drawn	10%

Figures 9 and 10 show right angled triangles drawn normally, with the 90 degree angle drawn at the base. Figure 10 demonstrates a leftward drawn right angled triangle while Figure 9 demonstrates a rightward right angled triangle. On the other hand, Figure 11 is an unusual drawing although it might be regarded as normal right angled triangle that has undergone some sort of transformation. However, whether drawn normally or unusually, they are both correct figures representing a right angled triangle.

Figure 9: M23's drawing

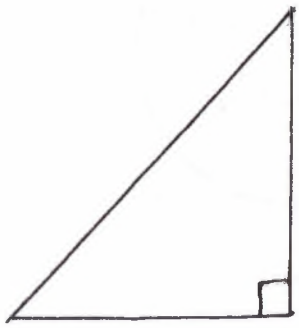


Figure 10: M14's drawing

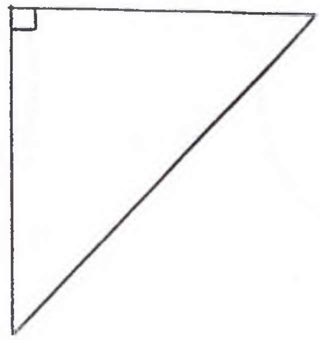


Figure 11: M19's drawing

It would be important to show learners the different ways of representing a figure as this allows for higher cognitive levels and thus being able to visualise with objects mentally going through change. Exposure to different representations of shapes can help later in an individual's geometry journey with complex geometric problems otherwise known as geometric riders. When learners are posed with an unseen problem, it is likely that they would rely on previous work done in class by sifting through different interpretations and representations of similar figures or scenarios.

Table 18: Respondents' agreements (Question 8)

In the triangle which side is the longest?	%
AB	100%
BC	0%
AC	0%

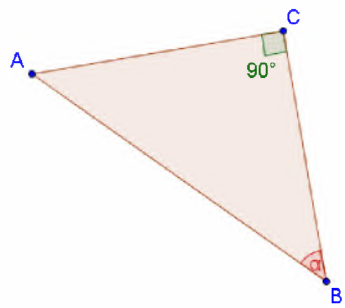


Figure 12: Right angled triangle

All the participants selected the correct answer which was side AB (Table 18) that was the longest side of the right angled triangle (Figure 12). In relation to this question, the next question (Table 19) was asked to gain more clarity on PSTs' understanding as to why this was the case. Here again all the PSTs responded correctly. Various methods were used to answer why AB was the longest side such as the side opposite the 90 degree angle is identified as the hypotenuse side by mere inspection; and by using a ruler to measure the longest side. Questions 7 and 8 are related to the content in the Pythagorean Theorem, transformation and measurements which are topics covered in the GET and Senior FET phases.

Table 19: Reasons for answer in question 8 (Question 9)

Explain how you arrived at your answer in question 8	%
Correct:	100%
Incorrect:	0%

The different representations (Figure 13) of the theorem that states that if an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference, otherwise known as angle at centre is twice the angle at circumference were given to the PSTs.

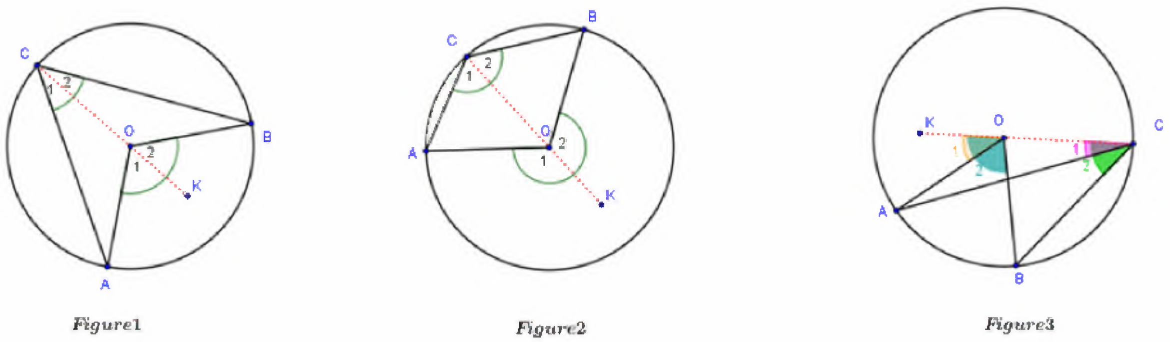


Figure 13: A screenshot of the different representations of the theorem: angle at centre is twice the angle at circumference created and uploaded later onto the web: http://govenderrg.wikispaces.com/Angle+at+centre+equals+twice+angle+at+circum.

Forty per cent of the PSTs knew that the theorem applies to Figures 1, 2 and 3 (Figure 13) that were the same figure while 60% provided an incorrect response, and 40% stated that only Figure 2 resembles Figure 1. This could be due to the fact that Figures 1 and 2 are not complex when compared to Figure 3. This can be related to question 7 (Table 17) since PSTs are familiar with drawing figures in one particular way. Thus, it becomes difficult to expect them to be able to understand and play with mental imagery to see that the same figure is shown differently. However, none of the participants agreed that no relationship exists between Figures 1, 2 and 3 (Table 20). The PSTs were able to supply correct answers to questions 7 and 8 as they involved basic shapes; however, when it came to complex geometric constructions like in Figure 13, it became difficult to interpret.

Table 20: Respondents' selection when comparing Figure 1 to Figure 2 and Figure 3 (Question 10)

Do Figures 2 and/or 3 resemble Figure 1?	**%**
Figure 2 and 3	40%
Figure 2	40%
Figure 3	20%
Neither	0%

Although 60% (Table 20, Figure 2: 40% + Figure 3: 60%) of the PSTs found no connection between Figures 2 and 3, 90% (Table 21) were able to support their choice with valid reasoning. In the description most were able to identify the theorem and state it in words, while some explained using transformation.

Table 21: Respondents' explanation for question 10 (Question 11)

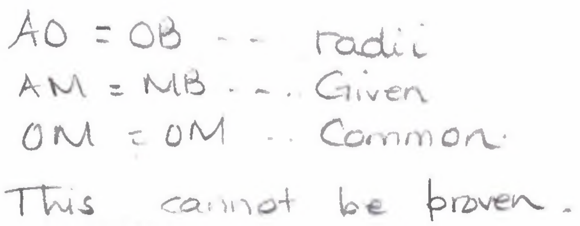
Explain how you arrived at your answer in question 10	**%**
Correct on the right track	90%
Incorrect explanation	10%

Table 22 shows that 60% of the PSTs were on the correct track with reference to their proof as to why OM ⊥ AB, and also provided valid reason/s.

Table 22: Respondents' explanation (Question 12)

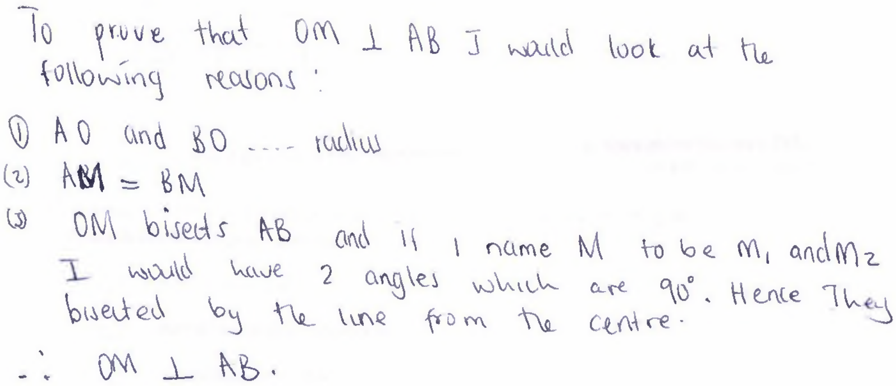
In the figure that follows, how would you prove that OM ⊥ AB? If you think it can't be proven state your reason/s why?	**%**
Correct - on the right track	60%
Incorrect explanation	40%

Most of the explanations included congruency which can be seen in Figures 14 and 15. However the reasoning in Figure 14 was partially correct in the beginning, but the participant could not complete the explanation correctly. It seemed as if they could not see the link or relation. A possible reason could have resulted from the way they laid out their explanation. On the other hand, few participants manage to correctly explain why OM ⊥ AB - this is seen in Figure 15.



AO = OB ... radii
AM = MB ... Given
OM = OM ... Common
This cannot be proven.

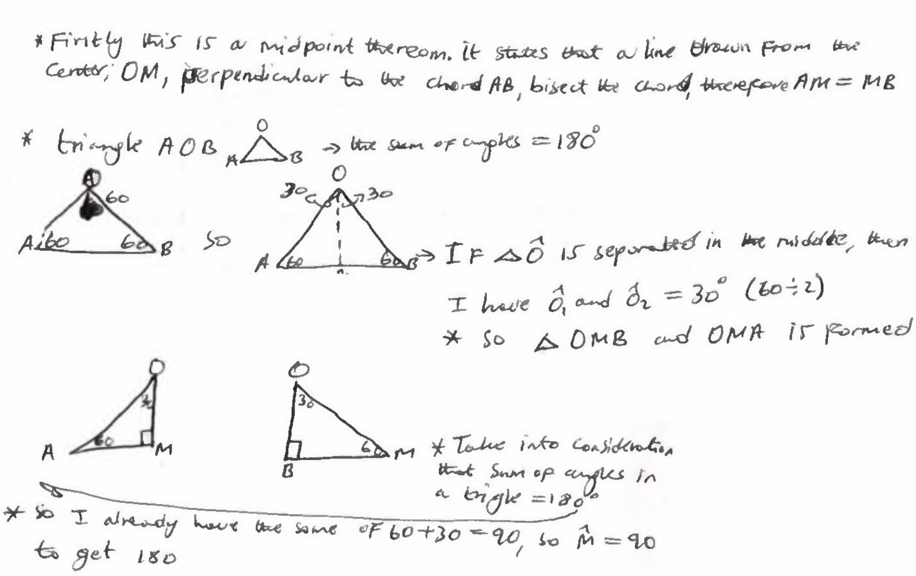
Figure 14: M19's explanation of why OM⊥AB



To prove that OM ⊥ AB I would look at the following reasons:
(1) AO and BO .... radius
(2) AM = BM
(3) OM biseds AB and if I name M to be M_1 and M_2 I would have 2 angles which are 90°. Hence They biseted by the line from the centre.
∴ OM ⊥ AB.

Figure 15: M14's explanation of why OM⊥AB

Some of the explanations from the participants showed a lack of geometric understanding and reasoning since 40% were incorrect (Table 22). This is exhibited in Figure 16, where incorrect theorems were referenced and assumptions were made that angles are equal to 60 degrees. The theorem under enquiry can be regarded as one of the early circle geometry theorems which lends itself to proving other theorems.



* Firstly this is a midpoint theroem. It states that a line drawn from the center, OM, perpendicular to the chord AB, bisect the chord, therefore AM = MB
* triangle AOB → the sum of angles = 180°
IF △Ô is separated in the middle, then I have $\hat{O}_1$ and $\hat{O}_2$ = 30° (60÷2)
* so △OMB and OMA is formed
* Take into consideration that sum of angles in a trigle = 180°
* so I already have the some of 60+30 = 90, so $\hat{M}$ = 90 to get 180

Figure 16: M03's explanation of why OM⊥AB

Following this questionnaire, a set of worksheets (Appendices H, I, J, K, L and M) which focused on some geometry thinking strategy in the form of a game or pattern were completed with the use of technology (DOEs). The worksheets were linked to the Theorem of Pythagoras or circle geometry, then assessed using a grading system: incorrect, partially correct and correct.

5.2.2 Task based worksheets

A grading system of 0 - incorrect, 1- partially correct and 2 - correct was used to mark the worksheets.

Table 23: Grading of respondents based on the Pythagorean Theorem worksheets

		Identifying the sides	Pythagoras Puzzle	Half circles			Magic Triangle					Pythagorean Tree
	Question →	**4**	**5**	**1**	**5**	**6**	**5**	**5.1**	**5.2**	**5.3**	**5.4**	**4**
	CODE ↓	Write your answers in the diagram below.	What can you conclude from the activities in steps 3 and 4 in terms of the small, medium and large square?	Remember the formula for a circle! What formula would you use for a half circle?	Find the area of all the half circles. Show all necessary working out.	When you add the two smaller half circles together what do you notice?	Write your answers in the figure below	The numbers written in the middle (central) boxes?	The numbers written in the correspondi ng boxes?	The sums of the numbers written in the 4 corners of each square?	Any other combination of numbers written within the squares?	Why do you think this is called a Pythagorean Tree?
1	M27	2	2	2	0	2	2	2	2	2	2	0
2	M03	2	2	2	1	2	2	2	0	0	0	2
3	M23	2	2	2	2	0	1	0	0	2	0	1
4	M19	2	2	2	2	2	2	2	2	2	2	2
5	M14	2	2	2	2	2	2	2	2	0	2	1
6	M01	2	2	0	1	2	2	2	2	2	2	1
7	M04	2	2	2	2	2	1	0	0	0	0	1
8	M07	2	2	1	2	2	2	2	2	2	2	2
9	M26	2	2	2	2	2	2	0	2	0	0	2
10	M13	2	2	2	2	2	2	2	2	2	2	2

Table 24: Average per question based on the Pythagorean Theorem worksheet

	Identifying the sides	Pythagoras Puzzle	Half circles			Magic Triangle					Pythagorean Tree	Average
Question→	4	5	1	5	6	5	5.1	5.2	5.3	5.4	4	
	Write your answers in the diagram below.	What can you conclude from the activities in steps 3 and 4 in terms of the small, medium and large square?	Remember the formula for a circle! What formula would you use for a half circle?	Find the area of all the half circles. Show all necessary working out.	When you add the two smaller half circles together what do you notice?	Write your answers in the figure below	The numbers written in the middle (central) boxes?	The numbers written in the corresponding boxes?	The sums of the numbers written in the 4 corners of each square?	Any other combination of numbers written within the squares?	Why do you think this is called a Pythagorean Tree?	
Incorrect	0%	0%	10%	10%	10%	0%	30%	30%	40%	40%	10%	
Partially	0%	0%	10%	20%	0%	20%	0%	0%	0%	0%	40%	
Correct	100%	100%	80%	70%	90%	80%	70%	70%	60%	60%	50%	
Correct per worksheet	100%	100%	80%			68%					50%	79.6%

The worksheet based tasks were answered in alignment with the relevant DOEs that contained high user interaction and visual aspects, with the aim of achieving a better understanding of the geometric concept. The PSTs achieved full marks for questions 4 and 5 (Table 24). They averaged 80% for the exploration of the Pythagorean Theorem via the Half circle worksheet, while the average for the Magic triangle worksheet was 68%. The Pythagorean Tree worksheet (50% average) relies heavily on visual representation as compared to the other worksheets, which require one to observe the size of the squares that are created as the tree grows. These worksheets stress the Pythagorean concept as it gives rise to other discoveries such as Pythagorean triplets and so forth. It was noted that the average percentages indicate an increase in difficulty from the first worksheet: Identifying the sides - 100% - to the last worksheet: Pythagorean Tree - 50% - as the arrangement of these worksheets required the participants to exercise their HOTs as they progressed. This indicates analytical progression in which one is required to develop, similar to the Low threshold–high ceiling approach expressed by Mark, Cuoco, Goldenberg and Sword’s (2010) instructional design principle. Similarly, the CAPS (2011) for Senior FET phase (Grades 7-9) states that, “geometry topics are much more inter-related than in the Intermediate Phase, especially those relating to constructions and geometry … hence care has to be taken regarding sequencing of topics” (p.27).

Table 25: Grading of respondents based on the Circle geometry worksheet

Theorem →		A	B	C	D	TOTAL	%
↓ CODE		If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. (Perpendicular from centre to chord)	If an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference.	If a quadrilateral is cyclic, then the opposite angles are supplementary, (Opp. ∠s of cyclic quad are supp.)	The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (tan.-chord theorem)	**8**	**100%**
1	M27	2	2	2	1	7	88
2	M03	2	1	2	1	6	75
3	M23	2	1	0	2	5	63
4	M19	2	2	2	1	7	88
5	M14	1	0	2	2	5	63
6	M01	2	2	2	2	8	100
7	M04	2	2	2	2	8	100
8	M07	2	2	2	2	8	100
9	M26	2	2	2	2	8	100
10	M13	2	2	2	2	8	100

Fifty per cent of the PSTs (derived from Table 25) scored full marks for the selected theorems. This finding correlates to the 50% of the PSTs who possess geometry knowledge from school to university. The geometry gap bridged from school to university can be depicted from the lowest score achieved for the circle geometry theorems which was 63%.

Table 26: Percentages incorrect, partially correct and correct per question based on the circle geometry theorems

Theorem →	A	B	C	D	Average
	If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. (Perpendicular from centre to chord)	If an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference.	If a quadrilateral is cyclic, then the opposite angles are supplementary, (Opp. ∠s of cyclic quad are supp.)	The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (tan.-chord theorem)	
Incorrect	0%	10%	10%	0%	5%
Partially	10%	20%	0%	30%	15%
Correct	90%	70%	90%	70%	80%

Table 26 shows that theorems labeled B and D were poorly answered, as the least number of PSTs got these theorems correct. However 90% were correct in proving the theorems labeled A and C. The average of incorrect and partially correct answers was 20%; thus, the average knowledge of the selected geometry theorems was 80%.

The set of worksheets together with the use of technology required the PSTs to prove the selected theorems or concept. It would be expected that 100% knowledge should be displayed. Average knowledge of the Pythagorean Theorem that is taught in the GET is 79,6%, while that of the circle theorems taught in the FET phase is 80%. If one does not understand the very concept of the theorem, it becomes difficult to apply it (procedural) and make sense of the application (conceptual). In some cases, the person would be able to apply the theorem to mediocre questions but find great difficultly when posed with a non-routine question, because of the lack of understanding of the theorem.

5.2.3 The post-survey questionnaire

The post survey questionnaire consisted of 30 statements that were categorised into six main questions (quality questions) which were pre-determined. The 30 statements were colour coded in order to group relevant quality questions before the survey was administered and after the completion of the worksheets and DOEs. This colour coded version was not available to participants (Appendix O, Appendix P: survey colour coded, Appendix R: participants' responses). The quality questions were as follows:

- GREEN CODE: Are the Internet and computers valuable in the teaching and learning of mathematics (geometry)?
- YELOW CODE: How fluent are mathematics pre-service teachers in using computers and other technology?
- PINK CODE: Does DOE/technology have an impact in the teaching and learning of mathematics (geometry)?
- BLUE CODE: Can DOE be used as a teaching tool?
- GREY CODE: Will DOE be utilised in future lessons?
- RED CODE: Can DOE be used in everyday teaching?

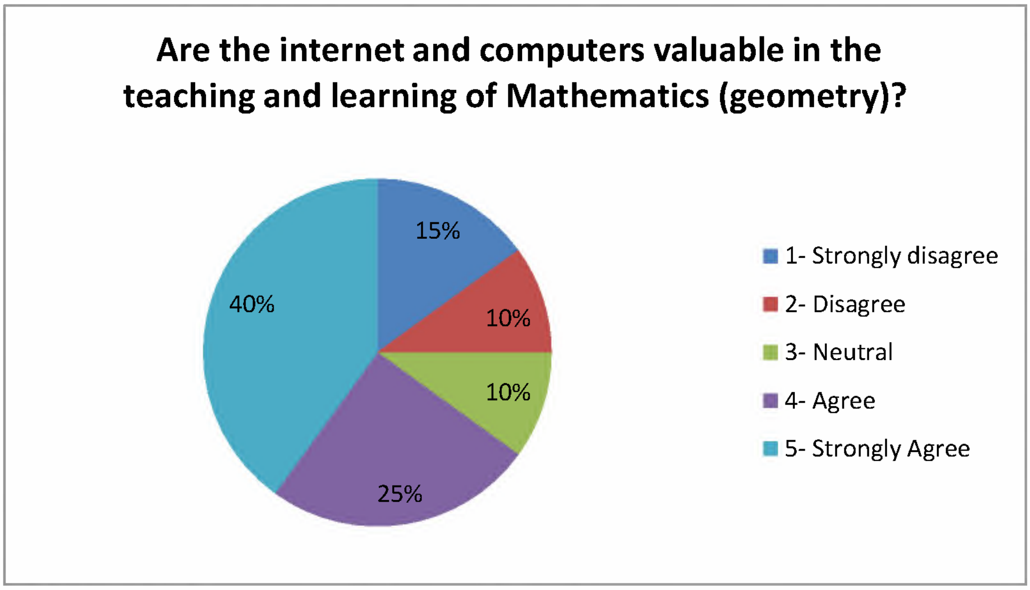
These quality questions were based on the study's research objectives: PSTs' knowledge, awareness and views of technology based teaching methods for teaching geometry. The statements were narrowed down to the focal technology used in this study. Participants were asked to select one of five options: 1- Strongly disagree, 2- Disagree, 3- Neutral, 4- Agree, 5- Strongly agree in the form of a Likert scale. The responses were then used to answer the six quality questions. For analysis purposes, the responses for strongly agree and agree were grouped. Similarly, strongly disagree and disagree were grouped; however, in some cases they were disaggregated for clarity.

Are the Internet and computers valuable in the teaching and learning of mathematics (geometry)?

Table 27: PSTs' Responses for GREEN CODE

GREEN CODE: Are the Internet and computers valuable in the teaching and learning of mathematics (geometry)?		
	RESPONSES	**% BASED ON 4 QUESTIONS**
1- Strongly Disagree	6	15%
2- Disagree	4	10%
3- Neutral	4	10%
4- Agree	10	25%
5- Strongly Agree	16	40%
TOTAL	**40**	**100%**

The Green code quality question was based on the responses from four statements. Sixty-five per cent of the PSTs favored the use of the Internet and computers in mathematics. It is important to note that 25% of the participants disagreed.



Graph 1: Pie chart showing responses for GREEN CODE

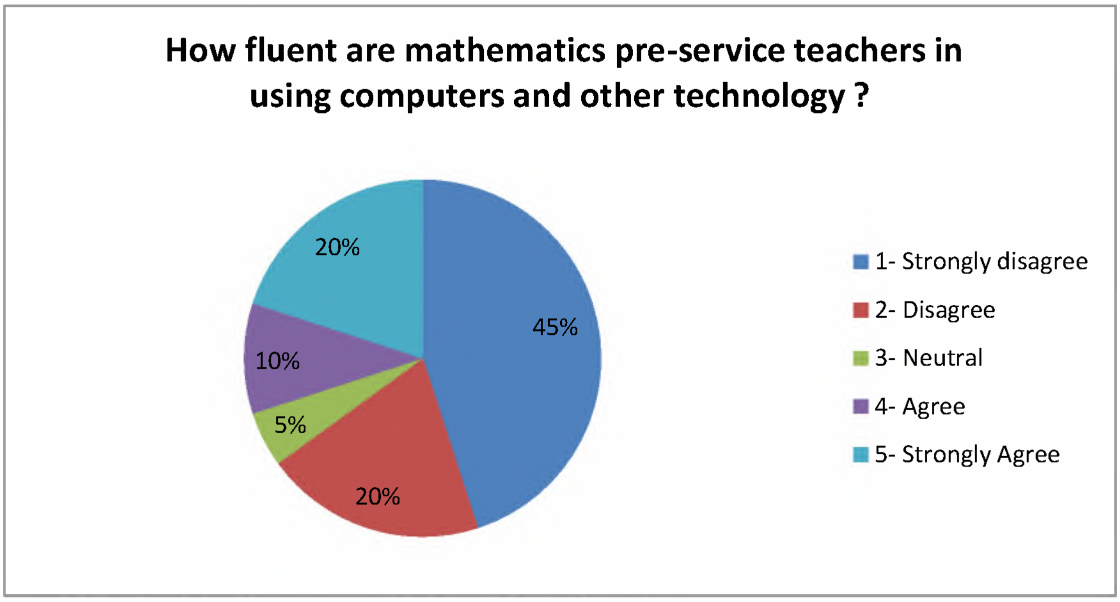
The manner in which one integrates technology into the classroom is important. Some of the PSTs held a neutral (10%) stance on the use of the Internet and computers in mathematics. It can thus be assumed that they were indecisive, since the aim of using technology in class is to aid learners' understanding while enhancing the lesson, while not replacing the teacher.

How fluent are Mathematics pre-service teachers in using computers and other educational technology?

Table 28: Responses for YELLOW CODE

YELOW CODE: How fluent are Mathematics pre-service teachers in using computers and other technology?		
	RESPONSES	**% BASED ON 4 QUESTIONS**
1- Strongly disagree	18	45%
2- Disagree	8	20%
3- Neutral	2	5%
4- Agree	4	10%
5- Strongly Agree	8	20%
TOTAL	**40**	**100%**

These quality questions consisted of four statements with a total of 40 responses. Derived from Table 28, 65% of the PSTs were viewed as being not fluent in the use of computers and other technology. This is cause for major concern as we live in fast-paced world, where technology is advancing, thus calling for the integration of computers and other technologies into the classroom. Five per cent of the PSTs were neutral, showing some level of knowledge of the utilisation of computers and technologies, but room for improvement. Only 30% were found to hold a view of being fluent in the use of technology. This is further confirmed by the observation that many participants struggled to perform basic computing tasks such as minimising/ maximising windows, refreshing a webpage, etc.



Graph 2: Pie chart showing responses for YELLOW CODE

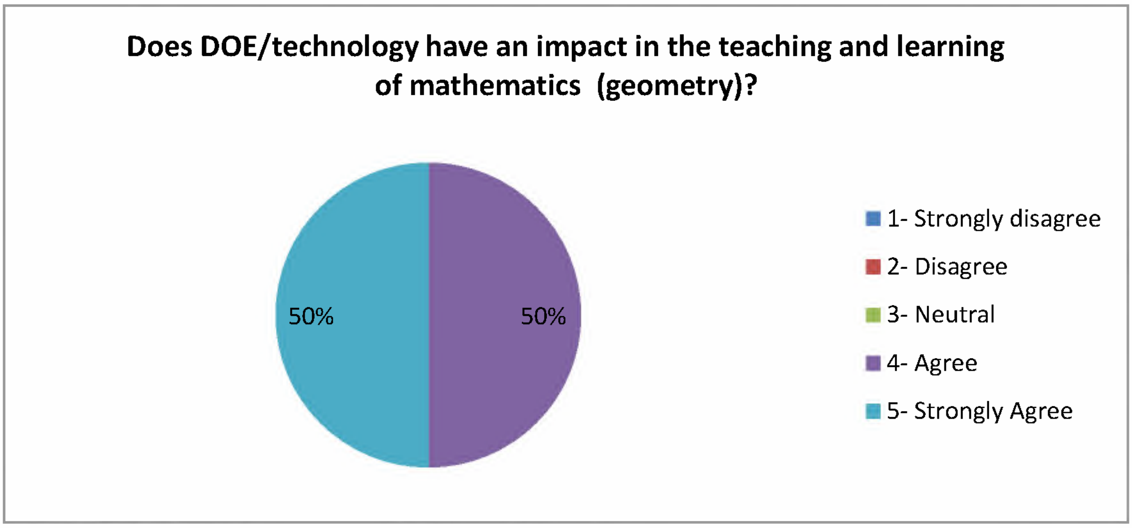
It was found that 65% (strongly disagree + disagree) of the PSTs were not fluent in the use of computers and other technologies and only 20% strongly agreed, meaning that they have mastery.

Does DOE/technology have an impact in the teaching and learning of mathematics (geometry)?

Table 29: Responses for PINK CODE

PINK CODE: Does DOE/technology have an impact in the teaching and learning of mathematics (geometry)?		
	RESPONSES	**% BASED ON 5 QUESTIONS**
1- Strongly disagree	0	0%
2- Disagree	0	0%
3- Neutral	0	0%
4- Agree	25	50%
5- Strongly Agree	25	50%
TOTAL	**50**	**100%**

All the PSTs favored the use of technology in the teaching and learning of geometry. This question was based on 50 responses from five statements.



Graph 3: Pie chart showing responses for PINK CODE

Graph 3 shows that none of the PSTs felt that DOE technology had no impact in the teaching and learning of geometry. While the majority (Table 28, Graph 2) did not possess knowledge and skills to use technology, they were aware that it is beneficial in the teaching and learning of geometry.

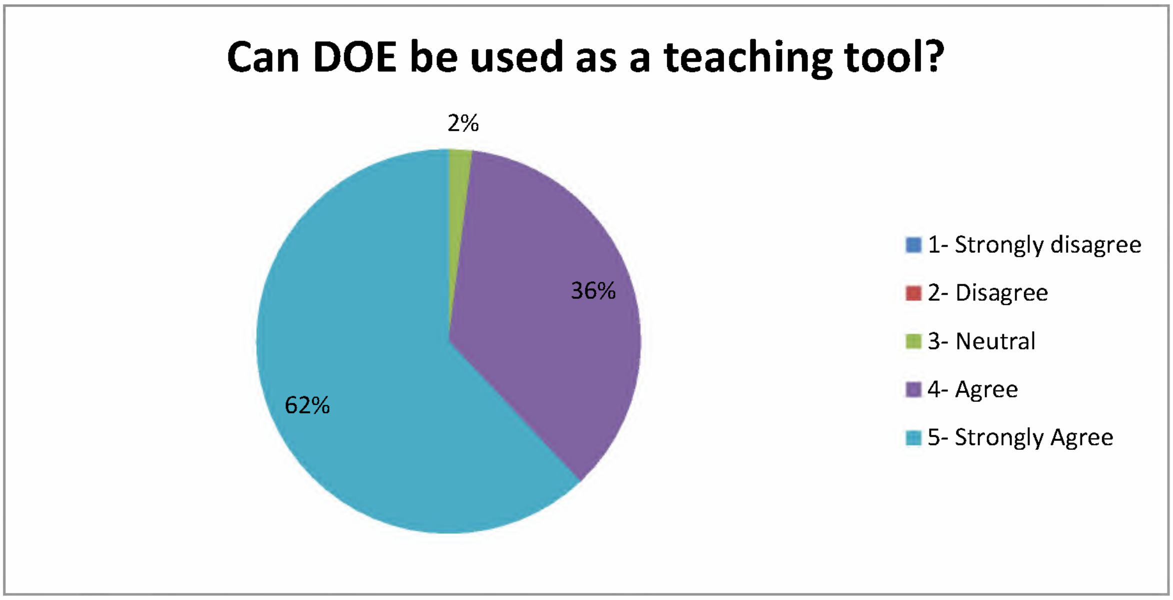
Can DOE be used as a teaching tool?

A total of 50 responses from five statements were used to address this question. Ninety-eight per cent (total of agree + strongly agree) of the PSTs agreed that the use of DOE as a teaching tool in class makes for meaningful teaching.

Table 30: Responses for BLUE CODE

BLUE CODE: Can DOE be used as a teaching tool?		
	RESPONSES	**% BASED ON 5 QUESTIONS**
1- Strongly disagree	0	0%
2- Disagree	0	0%
3- Neutral	1	2%
4- Agree	18	36%
5- Strongly Agree	31	62%
TOTAL	**50**	**100%**

However, 2% (neutral) of the participants were not entirely convinced of the benefits of using such technology when teaching.



Graph 4: Pie chart showing PST's responses for BLUE CODE

None of the participants felt that DOE does not have a positive effect as a teaching tool.

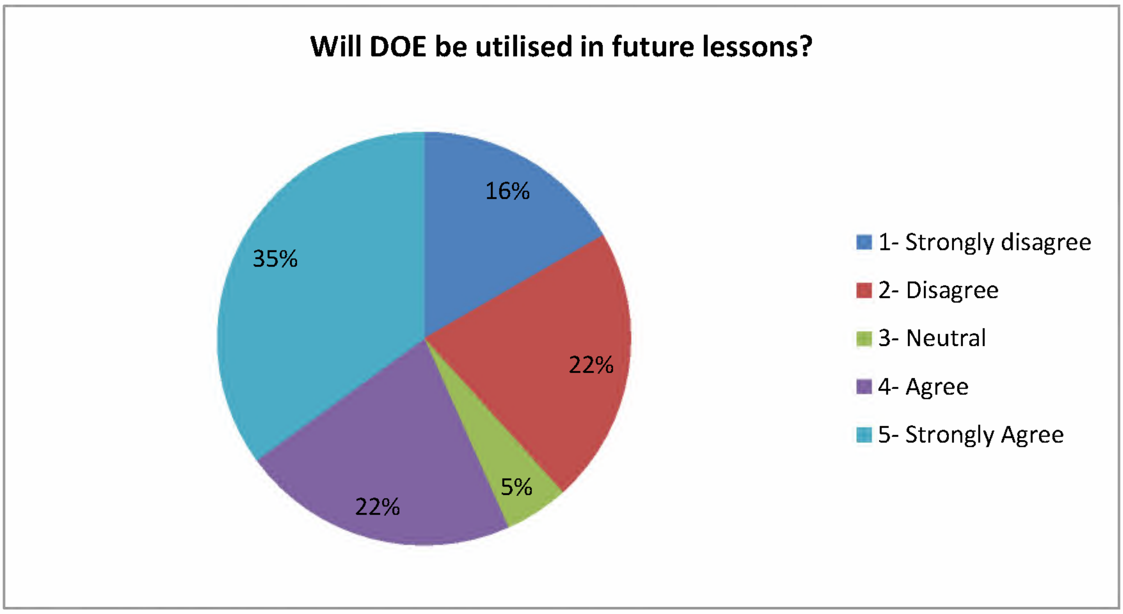
Will DOE be utilised in future lessons?

The response to this question is the result of six statements with a total of 60 responses. Despite the fact that none of the PSTs felt that DOE is not an effective teaching tool (Table 30, Graph 4), 39% (strongly disagree + disagree) indicated that they would not use DOE in their future lessons, while 5% (neutral) were indecisive.

Table 31: Responses for GREY CODE

GREY CODE: Will DOE be utilised in future lessons?		
	RESPONSES	**% BASED ON 6 QUESTIONS**
1- Strongly disagree	10	17%
2- Disagree	13	22%
3- Neutral	3	5%
4- Agree	13	22%
5- Strongly Agree	21	35%
TOTAL	**60**	**100%**

These findings could be the result of various contributory factors that hinder the use of DOE and technology in general in geometry. These factors were affirmed during the focus group interview and are discussed later in this chapter.



Graph 5: Pie chart showing responses for GREY CODE

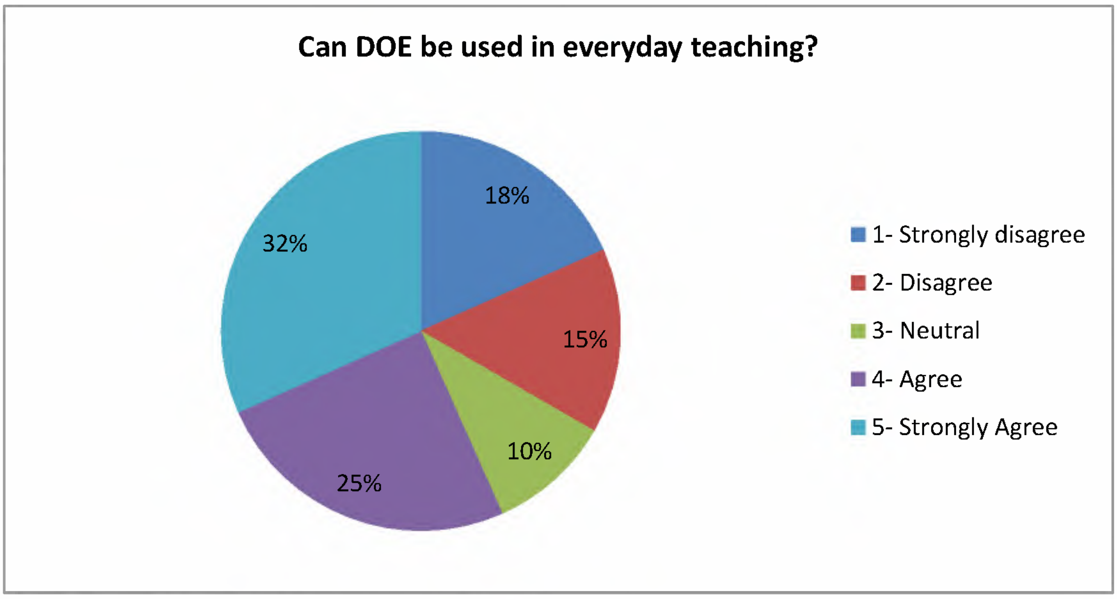
Fifty-seven per cent (total of agree + strongly agree) of the PSTs agreed that they were likely to use DOE in future lessons. It is assumed that that these PSTs have a different attitude and approach to teaching with technology and are likely to embrace different teaching methods. However, being in a well-resourced school can also affect the use of technology. If one focuses on their immediate surroundings without making a personal effort to bring about positive change, one would fail to make progress. Furthermore, 22% of the participants (Table 31) disagreed that that they would use DOE in future lessons. Some teachers tend to hold on to traditional methods of teaching and this leads to a level of resistance, since the old methods still work. The two major factors promoting technology integration are the technical skills of the teacher and most importantly, a positive attitude (Lei, 2009).

Can DOE be used in everyday teaching?

This question was based on six statements and a total of 60 responses. The time constraints and strict pacing set in the CAPS make it impossible to use technology in every lesson. Thirty-three per cent (total of strongly disagree+ disagree) of the participants disagreed that they would use DOE in everyday teaching, while 10% remained neutral. If technology such as DOEs cannot be used during class, learners can access it after school or while doing homework for consolidation of what was taught in class.

Table 32: Pie chart showing responses for RED CODE

RED CODE: Can DOE be used in everyday teaching?		
	RESPONSES	**% BASED ON 6 QUESTIONS**
1- Strongly disagree	11	18%
2- Disagree	9	15%
3- Neutral	6	10%
4- Agree	15	25%
5- Strongly Agree	19	32%
TOTAL	**60**	**100%**



Graph 6: Pie chart showing responses for RED CODE

Graph 6 shows that the majority (total of agree+ strongly agree) of PSTs agreed that DOE could be used in every day teaching.

5.2.4 Themes emerging from the data analysis

The data analysis revealed a range of factors that contributed to PSTs' views on the use of technology to teach geometry. These include their attitudes and beliefs about technology and their ability to use it effectively in teaching. The observations and focus group interview generated qualitative data, which was analysed holistically with the pre-questionnaire, worksheet based tasks and post-questionnaire to identify the emerging themes. This in-depth analysis coupled with thorough inspection identified consistencies and inconsistencies in the findings, which sought to provide responses to the critical questions in this study. Engaging with all collected data by means of coding and categorisation led to the identification of common themes. The following six themes emerged:

Motivation through the use of technology

The following response suggests that teachers are motivated to teach when exposed to technology like DOE. *"I* will use technology again in my future lessons because I believe that it does keep learners focused for a longer period of time and it actually makes learning exciting" (M14, Focus Group Interview). The PSTs found the use of DOEs highly stimulating and saw it as a valuable tool to use when teaching geometry. It was noted during their engagement with DOEs that they showed great interest and thoughtfulness. This was reinforced when examining their solutions to the worksheet based tasks, which included full, thoughtful explanations with the use of technology. It is important to note that while some people grew up constantly engaging with technology and are comfortable and confident with it, others were not exposed to technology at an early age and hence do not know much about it. The use of technology makes concepts clearer and allows for a new approach to thinking. A participant stated that technology *"makes me more confident, because I find that having* taught for 25 years there are new things that I learnt which I never known before and yet before I thought I knew everything and you see when you come with new things to the learners they have so much confidence in you and they become very much interested than they were before" (M26, Focus Group Interview). Even a seasoned teacher, who was pursuing studies in Mathematics Education, indicated that there is always room to adjust, change and improve the way one teaches; this teacher would have spent many years teaching geometry without tapping into the digital domain for assistance. During observation it was evident that all the PSTs showed great interest, excitement, enthusiasm and enlightenment when working with DOEs. One said, "*it is easy to understand mathematics… theorem of*

Pythagoras and there was something I experienced today that was exciting" (M04, Focus Group Interview).

The use of technology can be seen as source of teacher confidence in the classroom especially when it comes to the complex diagrams that PSTs encountered such as the angle at the centre is equal to twice the angle at the circumference theorem during the pre-questionnaire and post-questionnaire. A participant affirmed that it is "easier to explain from *what we saw in Figure 1 but once you move to other Figures, it's difficult to explain e*ven if *you do understand ... you are not like confident to process it to learners. But with DOE it shows us and it's easier to explain and understand"* (M03, Focus Group Interview). It was found that PSTs were confident when using DOEs; this was evident in the manner the questions were answered. Thus, it is clear that technology has an effect on both the learning and teaching of geometry; a PST stated that technology "makes you confident as a teacher because you are teaching the right things, and it makes the learners confident because they now have eradicated all these misconceptions and these errors that they make and they become more well-grounded individuals*"* (M13, Focus Group Interview).

The use of technology is likely to become a necessity when teaching certain mathematics topics. As a participant pointed out, "there are certain sections like geometry - it is time consuming to do without technology, as well functions - technology is required*"* (M23, Focus Group Interview). A major advantage of technology such as DOE in the Mathematics classroom is that it saves time and is undoubtedly precise, especially with routine tasks that learners have already mastered and reduces unnecessary repetition of drawings. It was noted that technology (DOE) "saves time and when drawing visuals or diagrams for an instance, it *is more accurate than the use of free hand"* (M07, Focus Group Interview). In addition, the fact that this technology is online means that it is readily available and does not require installation or advanced technical knowledge of using a computer, as the DOEs are loaded once the webpage is accessed.

Visualisation is key when teaching geometry

De Villiers (2012) points out that the integration of technology in education enables one to gain conviction through visualisation or empirical measurement. This was experienced by M01: *"...the visual interface of the Pythagoras theorem where* $r^2=x^2+y^2$, it gave me a better understanding of the relationship between r^2 and x^2 and y^2" (M01, Focus Group Interview).

Another PST said, "I really like the build-up on the theorem of Pythagoras because the way I taught - all along I wondered why learners do not get the gist ... *it allows learners to become* engaged in problem solving, because they can actually visualise. I think DOE has actually *brought light to the teachers"* (M26, Focus Group Interview).

The following comment was made in relation to the three figures from the theorem angle at the centre equals to twice the angle at the circumference when answered using technology: "I *did not realise that the will be the same diagram... I thought it was a different diagram applying the same theorem... but when you start using the DOE you start to realise that it is the same diagram but in a different way"* (M23, Focus Group Interview). This is affirmed by the fact that, before exposure to the technology, 60% (Table 20) of the PSTs were not able to recognise that Figures 1, 2 and 3 (Figure 13) are the same figure that underwent transformation. The use of technology such as DOEs enables visualisation that plays a critical role in the teaching of geometry as it allows for much deeper understanding than spoken explanations. Technology thus offers new or revised solutions and meaning to problems: "... it exposed me *to a different idea as to how that diagram came about"* (M23, Focus Group Interview).

It can be assumed that even a person who is proficient in geometry might integrate technology for their own knowledge and growth as technology like DOEs offers the opportunity to participate actively in one's own learning by means of visuals to see what they are doing rather than creating a mental picture. Assimilating current knowledge is thus easier. In geometry, no decoding of words/text is required in order to comprehend a problem as the essence of finding a solution to the problem depends largely on visualisation. Exposure to technology allows this "... as the abstract mathematics disappear and enables the learners to actually see what the teacher is talking about *more especially in Euclidean geometry"* (M26, Focus Group Interview). Furthermore, if technology "...had not been there, it would have *taken longer"* (M13, Focus Group Interview) to understand and complete the geometry worksheets.

Using technology to teach is not for everyone

While the literature review and this study's findings support the view that technology has a significant impact on teaching and learning in school in general, and especially in geometry, many direct and indirect factors prevent the use of technology in schools. A participant stated that although *"...technology* is more accurate than the use of free hand - disadvantages learners as they are not given the opportunity to draw a circle or construct a cyclic quad, they just sit in front of the co*mputer...*" (M07, Focus Group Interview). A similar argument was made by M04 who said that mathematical software "…sometimes beats us for example when you install Microsoft Mathematics you get everything there just insert equation and it shows you all the *steps, even plots the graph... you just enter what you want and it gives you all the solutions and what you need"* (M04, Focus Group Interview). Such software results in less mental engagement on the part of learners as solutions are provided at the click of a button. This implies that the use of technology to teach geometry does not result in increased satisfaction. On the other hand, teachers can become dependent on the use of technology when teaching. It is therefore imperative that they plan to use technology in a constructive manner, rather than using it for its own sake in the classroom. The principle goal of education is to create men and women who are capable of doing new things, not simply repeating what other generations have done (Piaget, 1964). Thus, while technology should be embraced, it is important how teachers use technology like DOEs. As noted earlier, 98% (Table 30) of the PSTs were of the opinion that DOE can be used as a teaching tool that can bring about change as compared to how geometry was taught without technology.

Despite the optimistic attitude of the majority of PSTs, M01 stated, "…when you learn too much about them (technology) they (teachers) *tend to misuse them"* (M01, Focus Group Interview). Any technological invention is human driven, because technology requires humans to feed it data, otherwise the results will be meaningless. Being able to use technology in geometry does not necessarily mean that one is using it critically, wisely, or meaningfully. It is of utmost importance that teachers allow learners to initially work with the problem by hand, and then gradually blend technology into the lesson (Naidoo and Govender, 2014), since assessments in schools including the final national examination do not permit the use of technology other than a scientific calculator.

Many South African schools do not have proper infrastructure, making it difficult to integrate technologies in the teaching and learning process. As a participant expressed, "*I don't think for me DOE would work because ... not all learners will have access to* Internet so I will be *the one with the computer"* (M01, Focus Group Interview). However, one can integrate DOEs differently despite difficult circumstances as "…you can make your student to see and observe if you can do the demonstration in front of them - they are able to come up with *conclusions"* (M03, Focus Group Interview). It is important to note that there is nothing wrong with the traditional approach of teaching without technology. Thirty-nine per cent of the participants did not feel that they would use technology like DOE in the future. However, M23 stated in terms of teaching geometry that "… it would have been much faster using the traditional approach but technol*ogy has showed me a better understanding"* (M23, Focus Group Interview).

Despite technology offering a better understanding of geometry "… if you look at the curriculum - it is more specific and therefore there is limited time to use it (technology-DOE)" (M03, Focus Group Interview). It is important to note that the Department of Education (2011) is very prescriptive and strict timelines are set. Therefore, it is unrealistic to use technology in every lesson. However, when presented with the opportunity to do so, one should use it. All the PSTs (Table 29) responded positively to technology like DOE having an impact on the learning of geometry.

According to M04, "…we cannot spend much of our money investing for other people so I think you can have the kno*wledge, but you can't help others because you cannot invest…*" (M04, Focus Group Interview). Previous studies have shown that the teaching profession offers unattractive salary packages; it seems that teachers are neglected and are ranked low among civil servants. However, teachers are required by the South African Council of Educators (SACE) to ensure that quality teaching takes place. As remarked by Parker (2002) in the SACE Handbook for the Code of Professional Ethics, a teacher must "acknowledge that the attitude, dedication, self-discipline, ideals, training and conduct of the teaching profession determine the quality of education in this country" (p.17). The way a teacher delivers the curriculum content is a catalyst to the learning process. The teacher's views and attitudes are crucial in adopting technology whether in a resourced or under-resourced school.

The integration of technology into geometry can result in learners drifting from the actual content of the lesson, losing teaching and learning time: "learners will just forget me and there will be less time" (M01, Focus Group Interview). Sixty per cent of the PSTs (Table 15) agreed that a teacher can lose track of time when working with technology, since it can be a distraction; this could lead some to believe that it is impossible to integrate technology meaningfully in the classroom. Once again, it is worth noting that careful planning is essential prior to the use of technology in any classroom. Its implementation must be controlled and monitored by the teacher while allowing learners the freedom to explore and discover on their own.

Geometrical conceptual growth through technology

Meaningful implementation and planning of technology such as DOEs and the worksheet based tasks in this study in geometry results in conceptual growth. A participant stated that, "... *what I understood today is that even if you learn something ... the next thing that* happens is just an extension of the first thing*"* (M01, Focus Group Interview). It was found that PSTs answered the worksheet based tasks with great insight whilst interacting with DOEs as their responses/solutions were more detailed than the responses/solutions in the pre-questionnaire. An example of conceptual growth is, "if you teach the sine rule and area rule cosine rule... if you start with them from the right angle triangle you will find the use of the DOE software will help you a lot to expand from right angle triangle to the rule" (M01, Focus Interview). Similarly, it is important to understand the circle geometry theorem: If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord otherwise known as perpendicular from centre to chord as it is used when proving other circle geometry theorems later on. Thus, a lack of conceptual understanding of an early theorem even from a previous year can result in difficultly when proving later geometry theorems that require previous knowledge.

Educational technologies can be used by teachers not only to enhance learners' understanding but to assist in refreshing geometry concepts themselves. They promote strong conceptual understanding amongst teachers and thus enhance confidence prior to teaching. A PST said that, "...at the moment as it stands I will use it for myself - to go to class, and as preparation *to go and teach. I will use it for my understanding for preparation to go to class and teach ... to have more understanding in order to teach the learners"* (M01, Focus Group Interview). This is reaffirmed by the fact that 80% of the participants (Table 14) used some kind of technology during lesson preparations.

Generally mathematic questions are placed in order of complexity from easy to difficult. As progression occurs in answering the questions greater mental involvement is required; i.e., exercising Higher Order Thinking Skills (HOTs). The use of GHOM and consultation of the Van Hiele levels when planning a geometry lesson can be fruitful as it can hasten the process of reaching a higher geometrical level of thinking, building and reinforcing the geometric concept and making it clearer and more understandable. A participant pointed out that, "some *situations are a challenge so when we use the DOE it's amazing that somehow … tend to understand, rather than using the chalkboard"* (M14, Focus Group Interview).

The development of misconceptions in mathematics can be catastrophic, especially in geometry when one theorem lends itself to proving other theorems. Misconceptions are likely to arise when there is a lack of grounding in the concept being taught. Some learners carry a misconception throughout schooling into tertiary education. A PST noted that, "… once they (learners) come to university level the lecturer tells them that what you know is completely wrong - *it disheartens them"* (M13, Focus Group Interview). It is difficult to eradicate misconceptions, especially when they have been accepted for a long time, thus allowing one to become complacent. Furthermore, an individual's views can impact on the way in which they counteract their misconception. As pointed out, "…by using DOE in the classroom it helps to eradicate misconceptions by the learner, because you can prove to them that by using the software that certain *things only happens in a certain way like for proofs and so on"* (M13, Focus Group Interview). Misconceptions in geometry can lead to incorrect problem solving paths, incomplete statements and insufficient reasoning which were noted earlier without the use of technology in Figures 14, 15 and 16 (Responses to Question 12, Table 22). A PST acknowledged that, "I had misconceptions which I need to eradicate in myself and how do I expect learners not to have these misconceptions if I have misconceptions myself?*"* (M26, Focus Group Interview). However, using technology, 90% of the participants (Table 26) were correct in proving the same theorem in Question 12 with no idea that their previous answers were partially correct or incorrect.

The use of technology promotes independent thinking and GHOM

The use of technology like DOE promotes engagement and a high level of meaningful interaction with geometry. This results in one thinking about the changes in an object as one interacts by selecting and dragging with a mouse, moving sliders and constant feedback. Self-reflection on one's thoughts is known as metacognition, and can be exercised when one interacts with technology in a meaningful way. A participant observed that, the DOE and the worksheets "gives you a little bit of thought when you are faced with an example that you are not used to. The worksheets made me self-reflect with more especially angle at the centre theorem because no one thought of trans*formation when looking"* (M23, Focus Group Interview). Exposure to technology in teaching allows for unseen geometry problems to be tackled with ease even when technology is not used later on in assessments as the four key elements of GHOM are groomed: Reasoning with relationships, Generalising geometric Ideas, Investigating Invariants and Balancing Exploration and Reflection. Mudaly (2013) describes the repetitive process of encountering a diagram as adapting it, then allowing oneself to think visually and analytically through reflection and interaction. Such thinking allows for new thoughts and the creation of new ideas when working through a problem.

It was noted that the PSTs were diverted from their common geometry thinking style. This was evident during observation and M23's comment above (previous paragraph) about realising that transformation can be used when proving circle geometry theorems. Stepping outside of one's comfort zone allows for new or modified thinking as explorative/discovery learning is promoted through the use of DOE when learning geometry. A PST stated, "…when using technology you are actually giving them (learners) a chance to research information on the Internet *by themselves"* (M07, Focus Group Interview). It is worth noting that technology like DOE is readily available on the Internet, allowing for easy access from anywhere around the world, and instant and autonomous learning.

When a learner is in control of their learning, metacognition takes place only if the engagement allows for it. Naidoo and Govender (2014) found that online technology such as DOE promotes self-regulation which in turn creates a learner-centered environment (Figure 17). Therefore, metacognition is directly related to control of one's learning behaviour (Finley, Tullis & Benjamin, 2010).

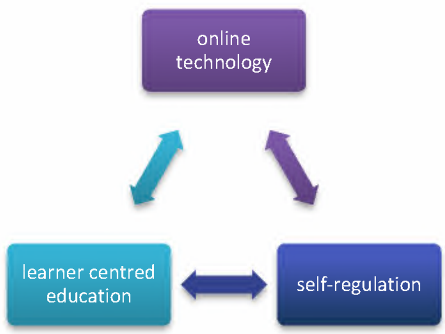


Figure 17: The effects of dynamic online technology on teaching and learning (Naidoo and Govender, 2014, p.46)

A participant pointed out that DOEs and the worksheets "…actually make me self-reflect *because as I was trying to make conjectures out of the questions. I got thinking … I never* *emphasised this… I had misconceptions which I need to eradicate in myself and how do I* expect learners not to have these misconceptions *if I have misconceptions myself …so I got to reflect"* (M26, Focus Group Interview). Engagement with technology offers in-depth geometrical understanding to the extent that it rectifies previous misunderstandings. One-on-one engagement with technology like DOE allows one to spend more time on concepts that are not understood and question oneself. As Finley, Tullis and Benjamin (2010) state, "learners control their studying based on the results of their monitoring, generally selecting to re-study and spend more time on items they have judged most difficult" (p. 111), thus promoting GHOM in geometry topics let alone metacognition and self-regulation in general.

Ören, Ormanci and Evreklia (2011) note that alternative teaching methods allow learners to reveal knowledge and skills through various means such as kinesthetics which involves human body movement. The understanding of a geometric shape's properties is enhanced when direct interaction takes place with the shape under study. With kinesthetics, the PSTs used their hands, fingers and the computer mouse to move objects on the screen in real time. This simple process involves movement and the visual sense to enact the learning experience. Such Human Computer Interaction (HCI) in the learning of geometry can undoubtedly induce self-reflection. As noted by M26, "*…when I looked at it... it didn't look like a diagram that* will be used to prove the angle at the centre theorem but technology did aid me to realise that by dragging the same diagram, it can be t*ransformed such that it appears in different forms"* (M26, Focus Group Interview).

The quality of teacher training at university

Amongst the 50% of the PSTs (Table 8) that did not do geometry at school, the findings show that university did bridge the gap as all the participants scored more than 50% in all geometry tasks. However despite the correct solutions/definitions in the response to the geometry questions (Table 16), there is limited understanding of geometry concepts among students (Makgato & Mji, 2006; Adolphus, 2011; Alex & Mammen, 2014) and PSTs (Ali, Bhagawati & Sarmah, 2014; De Villiers, 2004; Fujita & Jones, 2006). Pre-service teachers are kept up to date with current trends in technologies in education as it is explored in the course modules at university. In particular, the Teaching Practise (TP) module gives novice teachers an opportunity to gain first-hand experience in a classroom, with the freedom to devise and plan new ways of teaching subject content. Furthermore in the Mathematics Education course, PSTs are required to utilise dynamic software in some modules. The DOE used in this study is based on dynamic mathematical software. Such technology plays a pivotal role in helping learners to progress in geometry (Abu & Abidin, 2013; De Villiers, 2012; Idris, 2009; Mudaly, 2013).

While the use of technology is the norm at university where ICTs are widely used not only for teaching purposes but for communication, and accessing information, a participant noted that, "…learners who are born in rural areas have no clue about the structure of the *university and how lectures go about … now when you are using this technology ... DOE will* require them to use the Internet and they will have to use information on the Internet, then those leaners will be at an advantage being able to cope with the strategies used at university *because lecturers put information online and tell us to go and search this and that… "* (M07, Focus Group Interview). This suggests that teacher training enlightens and encourages teachers to use technology in their daily lives. It was noted that some PSTs were from non-urban communities and completed their schooling in these communities. However, despite their backgrounds, they persevered and tried to give of their best in their teaching. Thus, these soon to be graduate teachers will go into schools driving the use of technology like DOE to teach geometry in their classrooms. This would have a ripple effect where school leavers will be in an advantageous position as they will be able to cope with the strategies used outside of school, thus making a smooth transition to the real world. A PST observed: "I do believe that *it will benefit the kids …being able to access the information online b*y themselves unlike us who got to access information online only when we were here at u*niversity"* (M14, Focus Group Interview). The PSTs' readiness to teach geometry is revealed by the fact that, on average, 79, 6% scored correct answers in the Pythagorean Theorem and 80% in circle geometry. Since it could have been expected that they should all provide correct answers, this means that there is room for improvement in preparing them or that they are in transition.

It is important to note that everyone learns differently since an individual views and comprehends a concept in their own unique way. Therefore it can be argued that what does not make sense to one person will make perfect sense to another. As remarked by Ernest (1991), in "discussing problems and investigations … these concepts are ill defined and understood differently by different authors" (p. 284). Hence the training of geometry teachers is imperative so that they can correctly interpret a theorem or geometric relationship in order to describe it correctly to someone else. The use of DOE when teaching geometry can allow for common thought and understanding.

The ability to integrate different teaching and learning methods in geometry is the result of the Mathematic modules designed at university and the vision inculcated in PSTs. Careful planning and integration of technology in a lesson is crucial as it this would avoid going beyond the normal time limits of a school day. Nevertheless, M04 suggested: "… do not *focus on the time that you get by the school where they set a period … meet your learners on Saturdays, so you can do all sorts of things that you want like using the technology"* (M04, Focus Group Interview). These teachers are thus prepared to go the extra mile to hold special weekend classes that expose learners to different ways and learning strategies with the aid of technology.

Exposure to relevant technologies at university is significant as it incorporates the theoretical components. A participated stated that "… the knowledge acquired had made it possible to put that into practice (use technology) *at school"* (M26, Focus Group Interview). However, there was a sense of insufficient exposure to technology in mathematics as a whole, apart from geometry, during teacher training. It was found that there is a need to extend the footprint of technology like dynamic software into other avenues of mathematics besides geometry, like algebra and trigonometry. As a PST noted, "…the training I had, I can only use Sketchpad to teach Euclidean geometry but I would like to be able to teach other sections *with technology"* (M26, Focus Group Interview). This is supported by the fact that 65% of the participants (Table 28) were not fluent in technology for teaching. Naidoo (2012) proposes that "visual tools in mathematics has proven to be beneficial; it is recommended that pre-service institutions provide teachers on-going support in this area" (p.10).

5.2.5 Further analysis

The integration of technology must be thoroughly understood in order to harmoniously blend technological elements into the classroom. In general, the use of technology depends on the teacher's attitude, as teachers are the driving force in the learning environment. Further analysis in this study revealed three categories of views on the integration of technology in the classroom from a geometric perspective. Figure 18 below shows the views on technology in geometry.

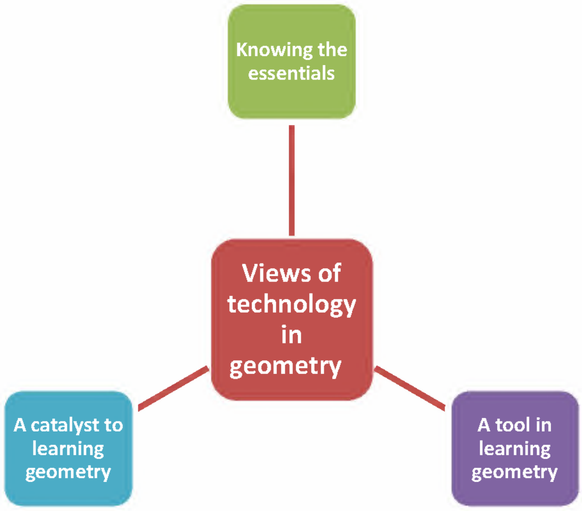


Figure 18: The role of technology in Geometry

Knowing the essentials

Using technology in a fairly harmonious manner but not necessarily on a daily basis would characterise one as being technology literate. The acquisition of basic computer principles and word processing skills forms the foundation for being computer literate (Department of Basic Education, 2004). Basic use of a computer which one might encounter during lesson preparation like switching it on, navigating through windows, opening files, editing files, saving, printing, downloading from the Internet; and so forth, would describe one as being computer literate. The responses to the statement: I know how to use a computer (Appendix R: Statement 2) averaged 4.6 on the Likert scale and a 1.6 average was found in response to I was frustrated sitting alone in front of a computer (Appendix R: Statement 7). Thus, it can be assumed that the majority of the participants have basic skills in using a computer and can be deemed computer literate. Furthermore it was noted that Computer Literacy is a compulsory module in the PSTs' first year of study and e-communication was used widely at the university. This could improve the efficiency of school administrative functions such as storing data electronically, in turn decreasing the time spent on administrative functions (Chigona, Chigona, Kayongo & Kausa, 2010) when PSTs are at school and later graduate. This enables teachers to spend more time on lesson planning and other activities.

However, teachers that are computer literate cannot design or construct their own geometry content but merely utilise available sources. An average score of 3, meaning a neutral response, was obtained for the statement DOE should only be used at university level

(Appendix R: Statement 15) for teacher training, while an average of 3.4 was obtained for DOE is too technical to use at school (Appendix R: Statement 5), suggesting that some sort of specialised knowledge is required.

A tool in learning geometry

The DBE (2004) notes that every teacher should not only have access to but the means to obtain a personal computer for personal use, for preparation of lessons and administration. In the following statements participants averaged 1.4 on the Likert scale: I dislike the use of technology when teaching mathematics (Appendix R: Statement 15) and 1.6: I have no intention to use DOE in my future mathematics lessons (Appendix R: Statement 15). This implies that technology is likely to be used to teach mathematics, let alone geometry. In addition, the majority of the participants agreed with the following statements: The DOE is simple and understandable to use (Appendix R: Statement 15, 4.3 average), DOE increases efficiency of Mathematics Education (Appendix R: Statement 9, 4.7 average) and Dynamic software can improve mathematics results (Appendix R: Statement 3, 4.4 average). Therefore, technology like DOE is seen as a valuable teaching tool in geometry in order to improve one's understanding. An analogy to technology as a geometry tool is the use of a hammer to knock a nail into wood. The tool which is the hammer refers to the chosen technology, for example, PowerPoint presentations, an overhead projector, mind map software, clickers, dynamic software, etc. The wood refers to the geometry subject content and the nail refers to the learner who pierces into the geometry subject content. This simply means that technology refers to the hammer being used to accomplish the task at hand, that is, to teach the learners who are the nails. It is in this category that one uses technology as a tool in the geometry teaching process, similar to using a ruler to draw a straight line and nothing more.

A catalyst to learning geometry

The integration of technology should not focus exclusively on computer skills or the utilisation of technology to complete a geometry task. Instead, it should be used more meaningfully within the context of the teacher's beliefs, which include how the teacher perceives effective teaching and how technology can alter the traditional roles of teachers and learners. As envisaged by the DBE (2004), "e-education is more than developing computer literacy, and the skills necessary to operate various types of information and communication technologies" (p.14). Hence, the teacher should demonstrate more than being technologically literate when using technology to enhance a geometry lesson. The majority of the participants agreed with the following statements: DOE increases the quality of Education (Appendix R: Statement 19, 4.4 average) and I would like to learn more about dynamic software in mathematics (Appendix R: Statement 20, 4.7 average). It can thus be concluded that PSTs regard technology as a valuable aid in learning geometry rather than as simply being used to transfer knowledge/a concept. The hints in the DOEs in this study act as a gateway when one hits a mental block. Polya (1957) recommends that each problem should be accompanied by a hint that will lead to the answer. The hints are a form of questioning of one's mental progression, which allows for self-correcting before intervention by the teacher.

The role of technology, commonly computers, in the teaching and learning of mathematics is becoming increasingly important, as it lays the foundation for the advancement of mathematics (Tatar, 2013). The adoption of technology should be driven by the desire to bring about change in the learning environment by integrating the chosen technology meaningfully in the classroom. It is important to note that the teacher should use technology to enhance the geometry learning process, rather than learning how to use the technology. As noted earlier, Visualisation is key when teaching geometry with an average score of 4.4 average obtained on the Likert scale for: Online mathematical content is convenient for my teaching preparation (Appendix R: Statement 16).

5.3 Conclusion

This chapter outlined the findings and the interpretation of the data collected by means of a pre-survey questionnaire, task based worksheets, observation schedule, post-survey questionnaire; and focus group interview. The findings were cross-checked with other data collected in order to promote the validity and reliability of this study. The aim of using a combination of data collection methods was to seek confirmation or contradiction within the data, thus providing in-depth information to support justifiable conclusions and applicable and meaningful recommendations.

Graphical representations of the findings were created where necessary, offering a visual interpretation. Data analysis identified a number of statistically significant relationships between aspects of teaching geometry and technology, as well as the attitudes and attributes of PSTs using such methods. Upon review of the findings, six major themes emerged: Motivation through the use of technology, Visualisation is key when teaching geometry, Using technology to teach is not for everyone, Geometrical conceptual growth through technology, The use of technology promotes independent thinking and GHOM; and The quality of teacher training at university. In addition, three categories were identified to classify teachers based on their adaption to technology when teaching geometry: Knowing the essentials, A tool in learning geometry; and A catalyst to learning geometry. The conclusions and recommendations are presented in the following chapter.

Chapter six

Conclusion

6.1 Introduction

This study explored PSTs' view on the use of technology to teach geometry. Two geometry topics were selected, the Pythagorean Theorem and circle geometry. The study participants were all studying in the field of Mathematics Education, and excluded first year students. The framework for the investigation was the Geometric Habits of Mind, together with the Van Hiele levels. Due to the integration of technology into geometry, an instructional design model by Mark, Cuoco, Goldenberg & Sword, 2010 was adapted and implemented. The objectives were to explore pre-service teachers' knowledge, awareness and views of technology based teaching methods for teaching geometry. In order to achieve these objectives, an interpretivist paradigm was adopted within this case study with some elements of an ethnographic study.

To drive this study, three critical questions were posed and data was collected by means of questionnaires, task based worksheets, a focus group interview and observations. The main themes emerging from this study were: Motivation through the use of technology, Visualisation is key when teaching geometry, Using technology to teach is not for everyone, Geometrical conceptual growth through technology, The use of technology promotes independent thinking and GHOM and The quality of teacher training at university. Further analysis revealed three categories in which teachers can be classified based on their adaption of technology to geometry: Knowing the essentials, A tool in learning geometry, and A catalyst to learning geometry.

6.2 Discussion and comments

The teacher plays a vital role in shaping how the learner interprets geometry, by means of planned pedagogy and technology which results in the promotion and motivation of the topic, making it fun, exciting, explorative, challenging, etc. High quality teacher training programs enable teachers and learners to compete nationally and globally. This includes pedagogical knowledge of integrating technology in geometry. Mathematical software and technology is utilised during teacher training modules, providing prospective teachers with skills that can be applied in the classroom. However, this exposure is insufficient for PSTs to continually investigate the latest trends and use of technology in teaching and learning in the future. The current Mathematics Education course seems to have not had the desired impact on PSTs for continual technology integration in geometry, as well as other mathematics topics, after the completion of their degree. Pre-service teachers require more specialised knowledge and skills to integrate technology effectively when teaching. It can be assumed that mathematics is foregrounded at university before introducing technology.

It is important to note that, based on their previous experience, the majority (80%) of the study participants felt that it is important to incorporate technology. Teacher training programs should not only incorporate geometry software packages in the teaching and learning process, but groom PSTs to acquire skills on how and when they should use it. They should be given more opportunities to create their own geometry lessons integrating technology before they enter teaching practice. The creation of scenario classrooms during teacher training would assist PSTs to prepare for technology integration in geometry in the real classroom and learner interaction via peer support. This would enable prospective teachers to become creators of content rather than simply accessing readymade materials. This type of interaction among PSTs will encourage collaboration, as few attend workshops and seminars and consult one another. This situation is likely even after they have qualified. Teaching training could include a module specifically designed for technology integration in geometry as well as other areas of mathematics. This would ensure that PSTs possess specialised knowledge and skills when using technology in class. The teaching of geometry should be taught as a full year course rather than being scattered and separated in the curriculum as this could result in scattered and disjointed geometry thinking. An individual's past experiences affect their approach when they use technology in geometry. This study has shown that an individual can be categorised in one of three categories based on their integration of technology in geometry, namely, Knowing the essentials, A tool in learning geometry, and A catalyst to learning geometry.

The Teaching Practice and Mathematics Method modules are important in preparing students to become teachers, as they enable them to explore cognitive strategies and ways of enhancing conceptual and procedural development, and to experience teaching and learning in a real, functional school environment. Pilot technology can assist with time management as it was noted that it is ineffective to use technology to teach in every lesson given the time frame of the South African curriculum. Planning is thus crucial. In addition, flaws can be identified before actually using technology in the lesson.

The use of technology in the study had a significant effect in developing PSTs' perceptions of technology in geometry and teaching in a positive manner. Current instruction on geometry in schools is inefficient and traditional teaching methods are inadequate in developing geometric understanding among learners. The PSTs held positive views with regard to incorporating technology like DOE when teaching geometry. It was found that they were better able to explain geometrical statements, reasons and the concepts of a theorem after engaging with technology.

Taking interactive geometry content and making it readily available to use online had a positive effect on the PSTs as it changed their views on using technology to teach geometry. Learners and teachers are given the freedom to discover concepts and cause-effect relationships through exploration and experimentation. Effective learning occurs when one interacts with the objects under study with the proper use of technology, by promoting geometric thinking through the development of GHOM. Developing learners' GHOM should be critical to the teacher as it prepares them to solve and make sense of complex geometric problems. GHOM can be achieved through the implementation of technology like DOEs which give instructions and feedback during one's interaction with the geometry concepts at hand. The prospective teachers' negative conceptions of geometry and their lack of confidence are altered when they realise the learning potential under the guidance of technology.

It is suggested that, wherever possible, teachers make use of visual and virtual reality technology, thus creating visual representation and promoting physical movement during the teaching of geometry. This improves understanding of geometrical relationships and properties more than presenting information in a verbal and static 2-D form. Blended learning is possible given the DBE's list of prescribed CAPS aligned textbooks that offer a mix of old and new teaching methods. An example is completion of the worksheet based tasks together with the DOE in this study.

The PSTs appreciated the interactive flow of the DOE, bearing in mind that it was underpinned by GHOM and instructional design, as adapted from Mark, Cuoco, Goldenberg and Sword (2010). In this study, the task based worksheets were completed hand-in-hand with the technology which offered hints, visual simulation; and drag and drop, allowing for identification and exploration. These modes of reasoning about specific geometric concepts were easily accomplished and could be done with confidence. It was possible to link GHOM and the instructional design to the Van Hiele model, whereby each learning period builds on and extends the thinking of the preceding level. The PSTs noticed that the sequencing of the activities was set out accordingly, offering a challenge from easy to difficult and thus requiring more thinking as they progressed.

The use of hints in the construction of these DOEs provides scaffolding, and gives one the opportunity to construct one's geometry knowledge in different ways. It is important to note that this depends on the individual's ability to recall, and understanding of previous content knowledge since it is not necessary to click on every hint. The individual is in control of their learning and can continue at their own pace. The hints can be skipped confidently knowing that the answer is correct via the visual feedback, resulting in understanding of the geometry concept/s leading to the theorem. The PSTs were of the opinion that DOEs allow the learner to self-assess their understanding and work at their own pace.

The use of technology allows one to self-regulate and establish one's geometry Habit of Mind without assistance from the teacher or a peer. Self-regulation is crucial to motivate learners and increase understanding, which is likely to impact one's academic performance, and geometric understanding of complex problems. There is a tendency to take the correction for granted without understanding and questioning: "why?" It is important for learners to self-assess their solutions and reasoning since learning from one's mistakes makes for better understanding.

The use of properly planned and thoughtfully integrated technologies has become essential in the teaching and learning of geometry. Technology like DOE offers support to teachers and promotes coherent mathematical reasoning. All the study participants felt that it was beneficial in the teaching and learning environment. The PSTs pointed out that effective feedback and instruction from the technology cultivated firm geometric reasoning. Teaching geometry with technology offers space for self-regulation and teacher intervention is only called for when required by the learner. Teachers are thus able to develop learners' cognitive strategies and offer support for purposeful inquiry. The PSTs felt that DOE allows for interaction and manipulation of geometric figures and exploration of relationships; and allows one to make generalisations leading to theorems/proofs. This might improve learners' understanding and serve as a refresher for teachers. The PSTs were unable to recognise the same geometric figure shown in three different forms during the pre-questionnaire without the use of technology. Furthermore, geometry is only taught once during the school academic year and it thus takes time to recognise the section again. The teacher would need to go over the topic content before going into class.

It was noted that DOE had a positive effect on the transfer time to discuss a geometric concept. Self-discovery plays a far more important role as a series of tasks is given, leading one to the emerging theorem/concept. The use of simulations in the DOE is not intended to replace classroom experience or the teacher. However, they provide learners with opportunities for repetition and exposure to multiple representations, which will help to deepen geometric understanding.

It was noted that before the exposure to technology, the participants' geometrical understanding required some attention, with specific reference to proving theorems, especially where there might be many line segments and constructions, making the diagram complex. Sixty per cent of the PSTs were unable to recognise geometric figures that had been transformed and link the figures to the same theorem (Chapter 5: Figure 13). Similarly, 90% drew the right angled triangle either facing leftward or rightward (Chapter 5: Table 17) and thus did not demonstrate mental capability to interpret and transform shapes. In addition, 40% of the PSTs provided incorrect or illogical mathematical explanations in proving the theorems (Chapter 5: Table 22, Figures 14, 15 and 16). As noted earlier, the main reason for the omission of geometry from the curriculum was a lack of teacher knowledge. If this trend continues, geometry will not be able to be part of the compulsory school curriculum. The National Examination Report (Department of Basic Education, 2014a) notes that learners leave out questions which ask them to prove complex diagrams in geometry. This suggests that teachers do not have sufficient knowledge to tackle these problems.

Having been exposed to technology in aiding geometry understanding, progressive readiness was observed among the PSTs to teach geometry. Through technological interventions (the use of DOE), 70% of the participants supplied correct statements and reasons and were able to notice the resemblance among the three figures (Chapter 5: Figure 13). The same question

(Chapter 5: Table 22) was investigated and 90% of the participants supplied correct answers. Overall, with the inclusion of technology an average mark of 80% (Chapter 5: Table 26) was obtained for the level of correctness in the selected circle geometry theorems and a 79, 6% (Chapter 5: Table 24) average for the level of correctness in Pythagorean Theorem questions. It can therefore be concluded that PSTs need to be well-versed in technological advancements and methods of teaching geometry as this will not only enable learners to understand and enjoy geometry but allow teachers to understand and teach it. The learning of geometry and the development of GHOM thus work concurrently since the development of productive ways of thinking is an integral part of such learning.

Many factors inhibit the use of technology in teaching geometry such as a lack of funding, vandalism, theft and the level of competency in technology. In the researcher's opinion, the most important factor is willingness to utilise such tools to teach geometry. It was found that 98% of the PSTs (Chapter 5: Table 31) affirmed technology as a good teaching and learning tool. However the presence of technology does not necessarily equate to improved learner performance or understanding. As Naidoo and Govender (2014) note, access to technology does not mean effective usage and achievement of the desired results in the teaching and learning environment.

It can be argued that technology replaces an individual's geometric thinking as it reduces the time and effort put into understanding a geometry concept. ICTs' potential can only be realised by teachers and learners when the chosen technology is properly adopted and integrated into the pedagogical process at school. This study found that PSTs are keen to integrate ICTs in their teaching and learning process, despite the fact that some noted that some schools don't have any ICTs, and that not all those which have them have effectively adopted educational technologies.

While it is possible to teach geometry without the use of technology since it worked for centuries, this will not match the extent to which technology is able to unmask and reveal geometric properties. Without technology the teaching of geometry becomes time consuming, as geometric diagrams have to be drawn and redrawn; this time could be used to complete other tasks. In order to benefit from the integration of technology in geometry, there needs to be a shift from traditional methods of teaching, bearing in mind that we live in a fast-paced technological world where people need to accept and adapt to change. New innovative forms of teaching have to be embraced in order to effectively maximise learners' understanding. As South Africa's National Development Plan (NDP) notes, the ICT sector will underpin the development of South Africa by 2030 (Education Summit, 2015). In years to come, teachers' level of proficiency in ICT will be included in the appraisal system and whole school evaluation (Department of Education e-Education strategy 2014-2019, 2015). Furthermore, newly-graduated teachers will require ICT certification before going into schools, over and above their teaching qualification (KZN ICT Summit, 2015). In conclusion, it is hoped that the findings from this study will have a positive impact on the teaching of geometry with the proficient integration and application of technology by not only PSTs, but qualified teachers and teacher training institutions.

6.3 Limitation and suggestions for further research

A shortcoming of this study is that it tends to be pertinent to PSTs that are based at or plan to teach in well-resourced schools. Table 11 in Chapter 5 showed that an Internet connection was only available to 50% of the participants during teaching. Thus, some schools are a long way off from introducing technology into geometry. There is a paucity of literature in this field that focuses on schools, teachers, and learners in disadvantaged areas (Chigona, Chigona, Kayongo & Kausa, 2010). However, the butterfly effect exists, where small actions in one part of the system ripple through the system. Therefore, every individual is accountable in working for change at the micro level, in order to see change at the macro level when teaching geometry for understanding.

Further research is required to gain a deeper understanding of individual challenges, and to solicit the views of PSTs from a wider range of teacher training institutions as those from another province or country might hold different views on the use of technology in geometry. While the sample size should be increased in order to generalise the findings, the current size was suitable for a case study. The driving force behind the integration of technology into geometry is the teacher; hence this study focused on PSTs. Including the views of learners that experience the use of technology when learning geometry, would provide insight from both perspectives. Further insight on the nature, content; and minimum duration of geometry together with the use of technology in each grade is also required to determine if the CAPS could be solely taught with technology based methods.

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Appendices

Appendix A - Letter to Dean

Professor G.H. Kamwendo
Dean of School of Education
University of KwaZulu-Natal
Edgewood Campus
Pinetown

RE: Permission to Conduct Research Study

Dear **Professor G.H. Kamwendo**

My name is Reginald Govender I am a Master's candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am writing to request permission to conduct my study at the School of Education.

We live in a technology driven society with people being able to recognise and use technology tools with ease. The key property of dynamic technology is that it offers the learner freedom to move shapes and graphs rather than the static convention of sketching. It offers instantaneous movement of an object such as a parabola graph - the values change a, b and c of $f(x) = ax^2 + bx + c$. This offers an era of teaching and learning whereby learners experience and discover mathematics online rather than just taking in given facts by the teacher. I am interested in exploring the use of dynamic online technology (applets) in the teaching of Mathematics. I am hereby seeking your consent to conduct research with a group of mathematics pre-service teachers from the School of Mathematic Education.
I have provided you with a copy of my proposal which includes copies of the measure and consent and assent forms to be used in the research process. Your approval to conduct this study will be greatly appreciated.

Yours sincerely

Reginald Govender
Student number: 207501841
Email: jaredgovender@ymail.com
Cell: 0613071742

My supervisor is **Dr. Jayaluxmi Naidoo** who is located at the School of Education, Edgewood campus of the University of KwaZulu-Natal.
Contact details: email: naidooj2@ukzn.ac.za Phone number: +27312601127

CONSENT FORM

I ____________________________________, Dean of School of Education, hereby consent to
Mr R.G Govender undertaking his research study at University of KwaZulu-Natal, Edgewood Campus.

Dean	**Date**	STAMP

Appendix B - Permission

Dr J Naidoo
Head of Discipline: Mathematic Education
University of KwaZulu-Natal
Edgewood Campus
Pinetown

RE: Permission to Conduct Research Study

Dear **Dr J Naidoo**

My name is Reginald Govender I am a Master's candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in exploring the use of dynamic online technology in the teaching of Mathematics. I am hereby seeking your consent to conduct research with a group of mathematics pre-service teachers from the School of Mathematic Education.

I have provided you with a copy of my proposal which includes copies of the measure and consent and assent forms to be used in the research process. Your approval to conduct this study will be greatly appreciated.

Yours sincerely

Reginald Govender
Student number: 207501841
Email: jaredgovender@ymail.com
Cell: 0613071742

CONSENT FORM

I Dr. J. Naidoo, Head of Discipline: Mathematic Education, hereby consent to Mr R.G Govender undertaking his research study at University of KwaZulu-Natal, Edgewood Campus, School of Mathematic Education.

Head of Discipline
Mathematic Education

20/09/14
Date

1

Appendix C - Informed consent letter

Mr R.G Govender
34 Vera Road
Malvern
Queensburgh
4030

Dear **Student**
School of Education, Mathematic Education, UKZN, Edgewood Campus

INFORMED CONSENT LETTER

My name is **Reginald Govender** I am a Master's candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in exploring the use of dynamic online technology in the teaching of Mathematics. To gather the information, I am interested in asking you some questions.
Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person, but reported only as a population member opinion.
- The interview may last for approximately 45 minutes.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in a filing cabinet at UKZN and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be prejudiced for taking such an action.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	Willing	Not willing
Audio equipment		
Photographic equipment		
Video equipment		

I can be contacted at:
Email: jaredgovender@ymail.com
Cell: 0613071742
My supervisor is **Dr. Jayaluxmi Naidoo** who is located at the School of Education, Edgewood campus of the University of KwaZulu-Natal.
Contact details: email: naidooj2@ukzn.ac.za Phone number: +27312601127.

You may also contact the Research Office through:
Ms P Ximba (HSSREC Research Office)
Tel: 031 260 3587
Email: ximbap@ukzn.ac.za
Thank you for your contribution to this research.

DECLARATION

I ....

(full names of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

....

SIGNATURE OF PARTICIPANT DATE

....

SIGNATURE OF PARENT DATE

(If participant is a minor)

Appendix D - Ethical clearance

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

21 October 2014

Mr Reginanld Gerald Govender (207501841)
School of Education
Edgewood Campus

Protocol reference number: HSS/1158/014M
Project title: Exploring pre-service teachers' views on the use of technology based teaching methods for teaching Geometry

Dear Mr Govender,

Full Approval – Expedited Application

In response to your application received on 12 September 2014, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Shenuka Singh (Chair)

/ms

Cc Supervisor: Dr J Naidoo
Cc Academic Leader Research: Professor P Morojele
Cc School Administrator: Mr Thoba Mthembu

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za
Website: www.ukzn.ac.za

1910 - 2010
100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses: Edgewood, Howard College, Medical School, Pietermaritzburg, Westville

Appendix E - Turn- it- in Report

Exploring pre-service teachers' views on the use of technology based teaching methods for teaching geometry

ORIGINALITY REPORT

11%	8%	4%	4%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Submitted to University of KwaZulu-Natal Student Paper	<1%
2	Submitted to Western Governors University Student Paper	<1%
3	ijedict.dec.uwi.edu Internet Source	<1%
4	my.unisa.ac.za Internet Source	<1%
5	www.pmena.org Internet Source	<1%
6	education.stateuniversity.com Internet Source	<1%
7	www.mathematik.uni-dortmund.de Internet Source	<1%
8	atcm.mathandtech.org Internet Source	<1%
9	Submitted to Manchester Metropolitan University Student Paper	<1%

Appendix F - Editor's Report

62 Ferguson Road
Glenwood
DURBAN 4001
Tel: 072 442 7896
Email: deanne.collins30@gmail.com
Income tax number: 0526066204

23 June 2016

This is to confirm that I have edited the dissertation, "Exploring pre-service teachers' views on the use of technology based teaching methods for teaching geometry", by Reginald Gerald Govender.

Yours sincerely,

(Ms) Deanne Collins (MA)
Professional Editor

Appendix G - Pre-service teacher questionnaire 1

Pre-Service teacher questionnaire 1

Personal information:

A. What are your subject specialisation/s?

____

B. What year of study are you in?

____

C. Have you attended any personal or professional development courses in the use of computers?
YES | NO *If YES list the course/s*

____

____

D. During teaching/teaching practise where do you source your questions, teaching materials, etc. from?

____

E. Did you do Geometry in grade 10, 11 and 12 (FET Phase) when you were at school?
YES | NO

F. Did you do Maths Paper 3 in grade 10, 11 and 12 (FET Phase) when you were at school?
YES | NO

G. Age:

Years	Tick
<=21	
22 to 30	
31 to 40	
>40	

For questions 1-5, 8 and 10 place a tick/s next to your response. You can have more than one tick.

1. Are any of these devices available for you to use during teaching/teaching practice?
(*Place tick/s next to your response*)

- ☐ Desktop computer
- ☐ Portable laptop or notebook
- ☐ Digital data projector
- ☐ Printer
- ☐ Internet connection
- ☐ USB (memory) stick

page 1 of 5

2. Are any of these devices available for you to use at home?
(*Place tick/s next to your response*)

- ☐ Desktop computer
- ☐ Portable laptop or notebook
- ☐ Digital data projector
- ☐ Printer
- ☐ Internet connection
- ☐ USB (memory) stick

3. I use the internet for following activities
(*Place tick/s next to your response*)

- ☐ Play online games
- ☐ Collecting information for assignments
- ☐ Use e-mail
- ☐ Lesson plan preparation
- ☐ Browse the Internet for fun (such as watching videos, e.g. YouTube)
- ☐ Download music, movies, games or software from the Internet
- ☐ Social networking (e.g. Facebook and Twitter)
- ☐ Participate in online forums

4. In a typical lesson preparation, how much time do you allocate or intend to use technology?
(*Place a tick next to your response*)

- ☐ I never use any technology in my lessons
- ☐ Only few minutes and not related to MATHEMATICS
- ☐ Only few minutes and related to MATHEMATICS

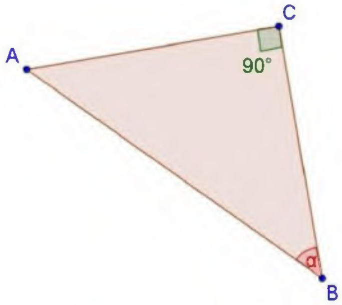
5. Thinking about your experience with technology: To what extent do you agree with the following statements?
(*Place tick/s next to your response*)

- ☐ It is very important to me to work with a technology.
- ☐ I think playing or working with a technology is really fun.
- ☐ I try to incorporate the use of technology in my lessons preparations.
- ☐ I lose track of time when I am working with technology.
- ☐ Technology is a distraction in the classroom.
- ☐ It is impossible to integrate technology in a meaningful in the classroom.

6. Describe/discuss/draw what you think Geometry is?

7. Draw a right angled triangle?

8. In the triangle below which side is the longest?
(*Place a tick next to your response*)



- ☐ AB
- ☐ BC
- ☐ AC

9. Explain how you arrived at your answer in question 8

10. Does Figures 2 and/or 3 resemble Figure 1?
(*Place a tick next to your response*)

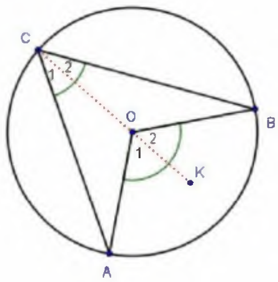


Figure1

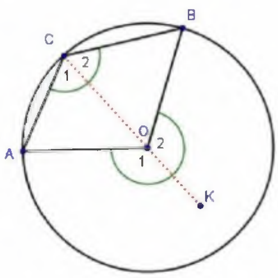


Figure2

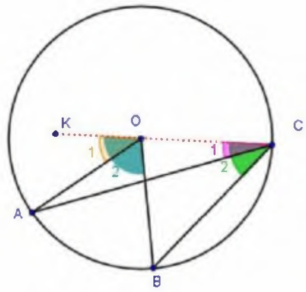
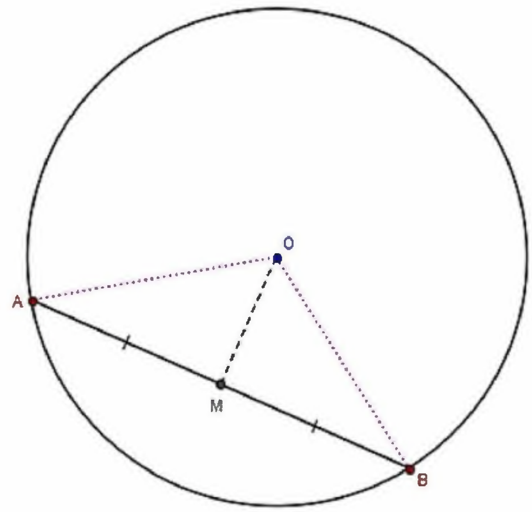


Figure3

- ☐ Figure 2 and 3
- ☐ Figure 2
- ☐ Figure 3
- ☐ Neither

11. Explain how you arrived at your answer in question 10.

12. In the figure that follows, how would you prove that OM ⊥ AB.
If you think it can't be proven state your reason/s why.



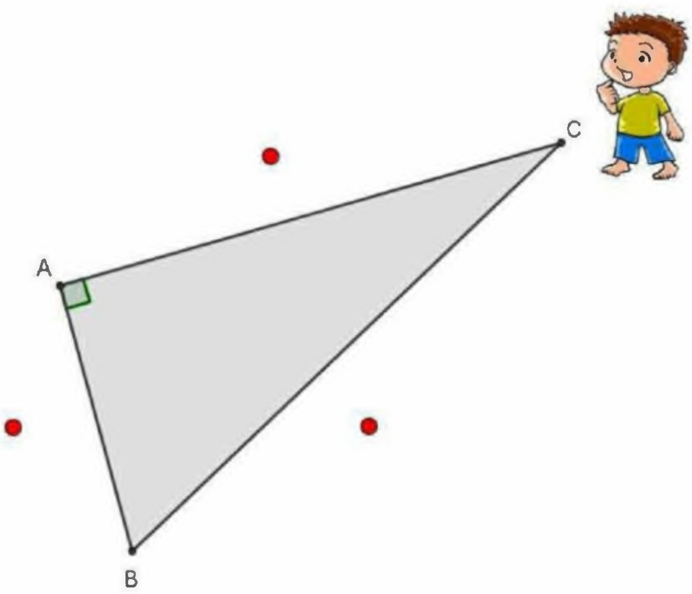
page 5 of 5

Appendix H - Identifying sides

Identifying the sides

STEPS:

1. Go to www.govenderrg.wikispaces.com
2. Click on the *Pythagoras* link along the left hand side.
3. Identify the sides of a right angled triangle by clicking the link *Identifying the sides of a RIGHT angle triangle*. Read the instructions carefully to complete the task. A smiley face will appear if the answer is correct.
4. Write your answers in the diagram below.



Appendix I - Pythagorean Puzzle

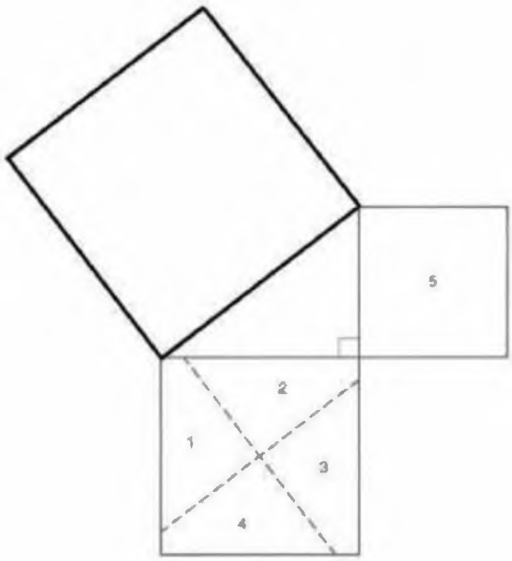
Pythagorean Puzzle

Adapted from Rose, Donaldson, Butain & McDonnell (2008, p.12)

Steps:

1. Go to www.govenderrg.wikispaces.com
2. Click on the *Pythagoras* link along the left hand side.
3. Click on Pythagorean Puzzle 1. Read the instructions carefully to complete the task.

The worksheet below is represented in Pythagorean Puzzle 1 applet if it was done by HAND.



- Cut off the small and medium squares. (Numbered 1-4 and 5)
- Cut the medium square along the dotted lines.
- Try to arrange the pieces (1-5) inside the larger, darker square.

4. Click on Pythagorean Puzzle 2. Read the instructions carefully.
5. What can you conclude from the activities in steps 3 and 4 in terms of the *small, medium and large square?*

____

____

____

Appendix J - Half Circles

Half Circles

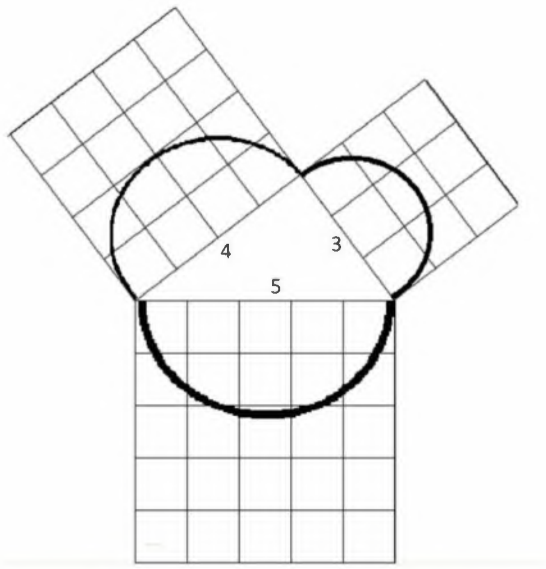
Adapted from Rose, Donaldson, Butain & McDonnell (2008, p.14)

Steps:

1. Remember the formula for a circle! What formula would you use for a half circle?

   ____

2. Go to www.govenderrg.wikispaces.com
3. Click on the *Pythagoras* link along the left hand side.
4. Click on the *Half circles* link. The figure below is represented in this applet.



5. Find the area of all the half circles. Show all necessary working out.

   ____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

6. When you add the two smaller half circles together what do you notice?

   ____

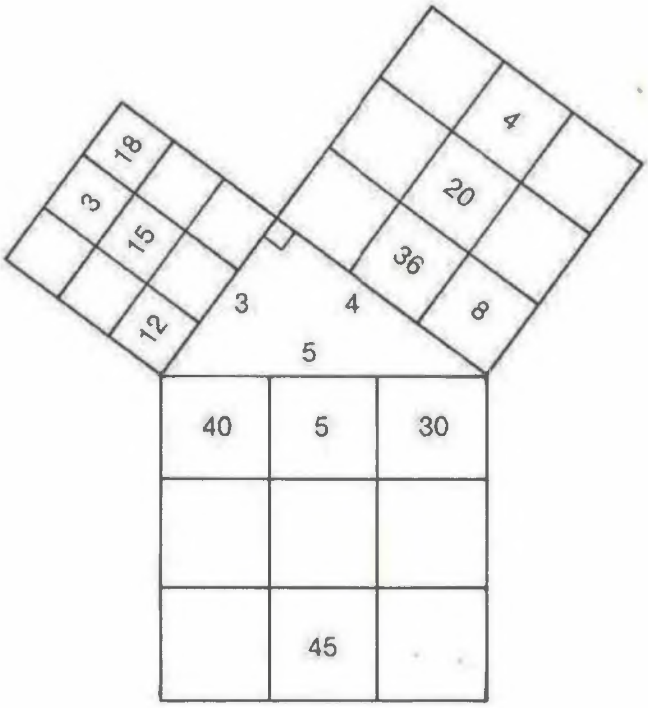
Appendix K - Magic Triangle

Magic Triangle

Adapted from Rose, Donaldson, Butain & McDonnell (2008, p.16)

Steps:

1. Go to www.govenderrg.wikispaces.com
2. Click on the *Pythagoras* link along the left hand side.
3. Click on the *Magic triangle*.
4. Complete the 3 magic squares. (The sum of each column, each row and each diagonal must be the same or equal.)
5. Write your answers in the figure below



5. Tick YES or NO. Does Theorem of Pythagoras apply to ...

5.1 The numbers written in the middle (central) boxes?	YES	NO
5.2 The numbers written in the corresponding boxes?	YES	NO
5.3 The sums of the numbers written in the 4 corners of each square?	YES	NO
5.4 Any other combination of numbers written within the squares?	YES	NO

Appendix L - Pythagorean Tree

Pythagorean Tree

Adapted from Rose, Donaldson, Butain & McDonnell (2008, p.19)

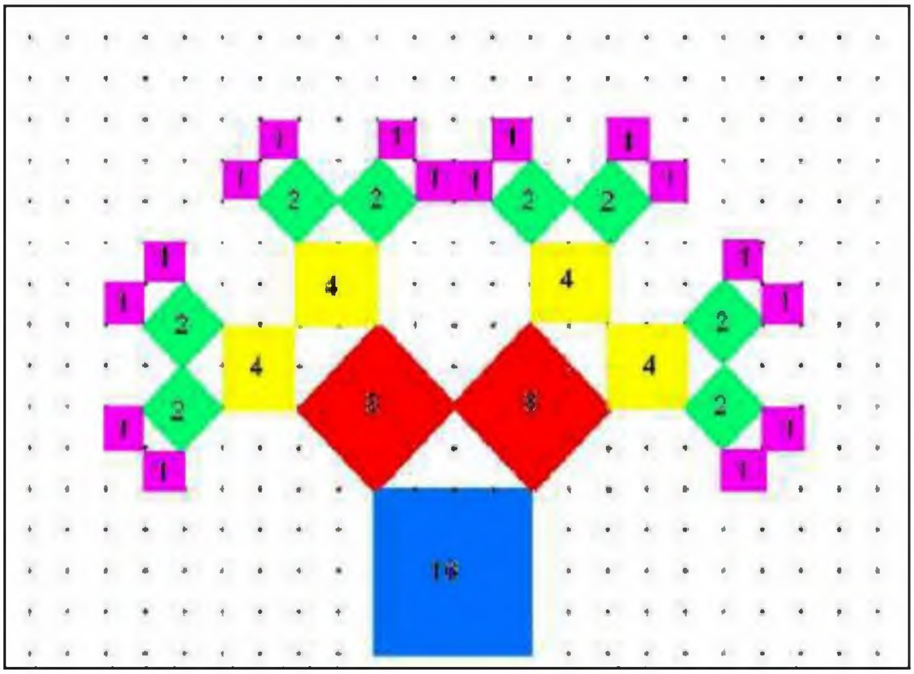
Steps:

1. Go to www.govenderrg.wikispaces.com
2. Click on the *Pythagoras* link along the left hand side.
3. Click on the *Pythagoras tree*. Use this applet to provide an answer in Step 4.
4. Why do you think this is called a Pythagorean Tree?

____

____

____



Appendix M - Circle geometry

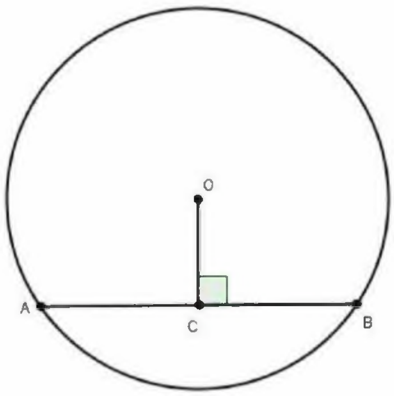
Circle Geometry

STEPS:

1. Go to www.govenderrg.wikispaces.com
2. Click on the *Circle Geometry* link along the left hand side.
3. Explore the circle by clicking on the link *Getting to know the circle*.
4. Go back to the *Circle Geometry* webpage.

5. Click on the relevant link on the *Circle Geometry* web page to complete the following proof: *If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord*.
6. Carefully read the instructions for the proof. Take note of what is given and required to prove.
7. Use this applet as a guide to prove the theorem. Show all necessary working, markings and constructions in SPACE provided below). DO NOT do the converse and corollary. However you may explore the converse and corollary.

A. If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord. (Perpendicular from centre to chord)



8. Go back to the *Circle Geometry* webpage.
9. Click on the relevant link on the *Circle Geometry* web page to complete the following proof: *If an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference*.
10. Carefully read the instructions for the proof. Take note of what is given and required to prove.
11. Use this applet as a guide to prove the theorem. Show all necessary working, markings and constructions in SPACE provided below). DO NOT do the converse and corollary. However you may explore the converse and corollary.

B. **If an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference.**

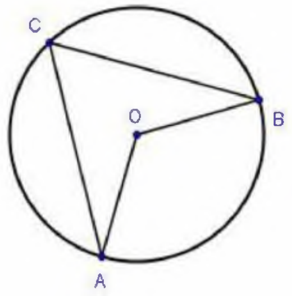


Figure1

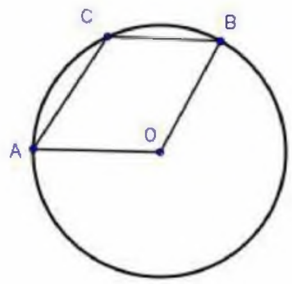


Figure2

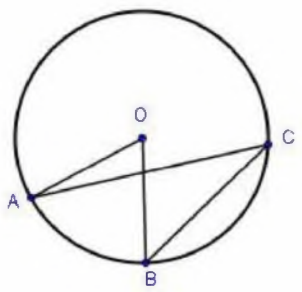


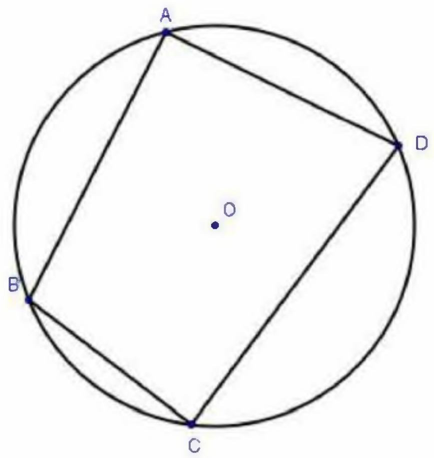
Figure3

Working for Figures 1 and 2

Working for Figure 3

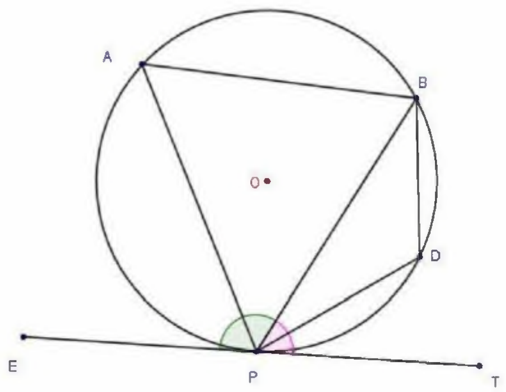
12. Go back to the *Circle Geometry* webpage.
13. Click on the relevant link on the *Circle Geometry* web page to complete the following proof: *If a quadrilateral is cyclic, then the opposite angles are supplementary*.
14. Carefully read the instructions for the proof. Take note of what is given and required to prove.
15. Use this applet as a guide to prove the theorem. Show all necessary working, markings and constructions in SPACE provided below). DO NOT do the converse and corollary. However you may explore the converse and corollary.

C. If a quadrilateral is cyclic, then the opposite angles are supplementary, (Opp. ∠s of cyclic quad are supp.)



16. Go back to the *Circle Geometry* webpage.
17. Click on the relevant link on the *Circle Geometry* web page to complete the following proof: *The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment*.
18. Carefully read the instructions for the proof. Take note of what is given and required to prove.
19. Use this applet as a guide to prove the theorem. Show all necessary working, markings and constructions in SPACE provided below). DO NOT do the converse and corollary. However you may explore the converse and corollary.

D. The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (tan.-chord theorem)



Appendix N - Focus group interview

Focus group Interview schedule

The use of Dynamic Online Environment in teacher education

Code: DOE- Dynamic Online Environment

1. What are your thoughts about the use of DOE at university?
2. Do you think being exposed to DOE in the mathematics curriculum at university (Higher Education) will help you become a better teacher at school?
3. Is it easier to understand mathematics (Theorem of Pythagoras and Circle Geometry) work through the use of DOE and Why? Does it change your attitude/perspective towards teaching mathematics?
4. What effect does the use of dynamic technology have on your mathematics understanding of concepts being taught? Do you think the DOE makes you confident in teaching mathematics?
5. What are your thoughts to the build-up of Theorem of Pythagoras? Do you think the worksheets enabled thinking? Did it get you thinking?
6. How do you feel about using technology (not only DOE) in class? Are you comfortable using technology in maths and why?
7. Did the use of DOE together with the worksheets make you self-reflect while answering? If YES or NO, Why?
8. Would the use of technology such as the one used in future lessons? Do you think it would benefit you as a teacher or your students later in life?
9. Would you have understood the lesson if technology was not used? Justify your answer?

Appendix O - Pre-service teacher questionnaire 2

Pre –Service teacher questionnaire 2

The use of Dynamic Online Environment in teacher education

Place a cross on the most appropriate:

CODES: *DOE*-Dynamic Online Environment

No.	Statement	Strongly disagree	Disagree	Neutral	Agree	Strongly Agree
1.	I frequently use the internet as a pre service teacher.					
2.	I know how to use a computer.					
3.	Dynamic software can improve mathematics results.					
4.	I have learnt (during the DOE session) as much as I would have learnt face to face.					
5.	Is the DOE is too technical to use at school.					
6.	Students at schools will benefit from DOE.					
7.	I was frustrated sitting alone in front of a computer.					
8.	It will be difficult to use the DOE in my teaching practice.					
9.	DOE increases efficiency of Mathematics Education.					
10.	Using DOE will make the Mathematic subject content interesting for students.					
11.	Class time is limited for the use of the DOE.					
12.	I dislike the use of technology when teaching mathematics.					
13.	My experience of schools is that they are adequately resourced to use the DOE when teaching.					
14.	The DOE is simple and understandable to use.					
15.	DOE should only be used at university level.					
16.	Online mathematical content is convenient for my teaching preparation.					
17.	Visualisation in DOE assists students in mathematics					
18.	I would recommend the DOE to my maths colleagues /friends.					
19.	DOE increases the quality of Education.					
20.	I would like to learn more about dynamic software in mathematics.					
21.	I have used this type of technology (DOE) or something similar previously.					
22.	I have no intention to use DOE in my future mathematics lessons.					
23.	I will share my experience and show students the DOE but will not use it.					
24.	Is it important for me as a university student to use DOE					
25.	I don't think I will ever need a computer/technology in my classroom.					
26.	If I had funds then I would invest in exposing my students to the DOE.					
27.	DOE can enhance students learning.					
28.	The layout of the content was a good build up to the Theorem of Pythagoras.					
29.	DOE will allow students to self-discover.					
30.	I use the computer less than three time a week					

Appendix P - Pre-service teacher Questionnaire 2 colour coded

Pre –Service teacher questionnaire 2

The use of Dynamic Online Environment in teacher education

Place a cross on the most appropriate:

CODES: *DOE*-Dynamic Online Environment

No.	Statement	Strongly disagree	Disagree	Neutral	Agree	Strongly Agree
1.	I frequently use the internet as a pre service teacher.					
2.	I know how to use a computer.					
3.	Dynamic software can improve mathematics results.					
4.	I have learnt (during the DOE session) as much as I would have learnt face to face.					
5.	Is the DOE is too technical to use at school.					
6.	Students at schools will benefit from DOE.					
7.	I was frustrated sitting alone in front of a computer.					
8.	It will be difficult to use the DOE in my teaching practice.					
9.	DOE increases efficiency of Mathematics Education.					
10.	Using DOE will make the Mathematic subject content interesting for students.					
11.	Class time is limited for the use of the DOE.					
12.	I dislike the use of technology when teaching mathematics.					
13.	My experience of schools is that they are adequately resourced to use the DOE when teaching.					
14.	The DOE is simple and understandable to use.					
15.	DOE should only be used at university level.					
16.	Online mathematical content is convenient for my teaching preparation.					
17.	Visualisation in DOE assists students in mathematics					
18.	I would recommend the DOE to my maths colleagues /friends.					
19.	DOE increases the quality of Education.					
20.	I would like to learn more about dynamic software in mathematics.					
21.	I have used this type of technology (DOE) or something similar previously.					
22.	I have no intention to use DOE in my future mathematics lessons.					
23.	I will share my experience and show students the DOE but will not use it.					
24.	Is it important for me as a university student to use DOE					
25.	I don't think I will ever need a computer/technology in my classroom.					
26.	If I had funds then I would invest in exposing my students to the DOE.					
27.	DOE can enhance students learning					
28.	The layout of the content was a good build up to the Theorem of Pythagoras.					
29.	DOE will allow students to self-discover.					
30.	I use the computer less than three time a week					

NOTE:

THIS IS FOR THE RESEARCHER ONLY- COPIES GIVEN TO PARTICIPANTS WILL NOT BE COLOUR CODED.

GREEN CODE: Does the internet and computers valuable in the teaching and learning of Mathematics (Geometry)?

YELOW CODE: How fluent are mathematics pre-service teachers in using computers and educational technology?

PINK CODE: Does DOE/technology have an impact in the teaching and learning of mathematics (Geometry)?

BLUE CODE: Can DOE be used as a teaching tool?

GREY CODE: Will DOE be utilised in future lessons?

RED CODE: Can DOE be used in everyday teaching?

Appendix Q - Participant observation schedule

Participant observation schedule

Adapted from Naidoo (2011)

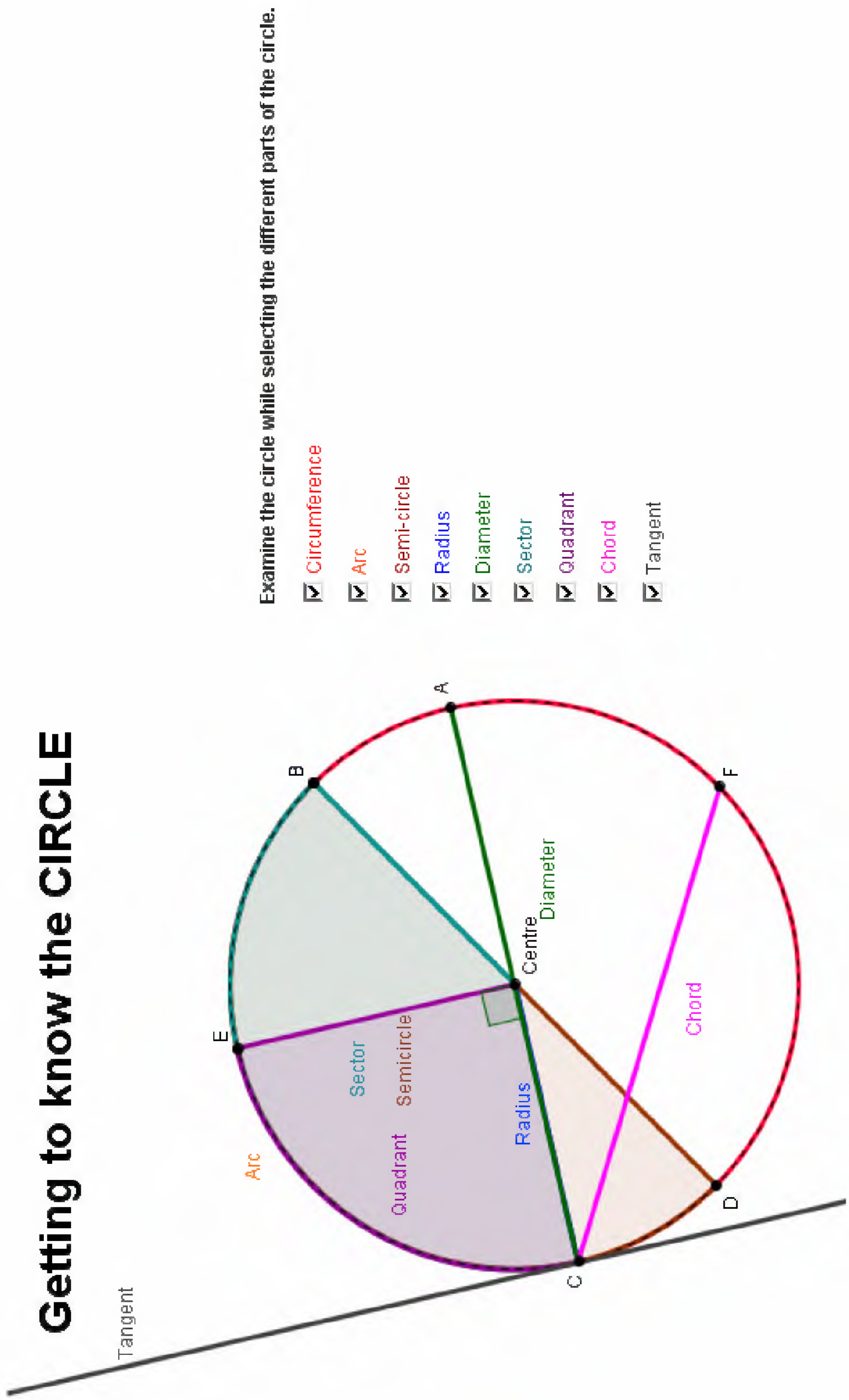
Indicator	Place tick
Enjoyment	
Excitement	
Boredom	
Understanding	
Confusion	
Indifference	
Inspiration	
Motivation	
Peer help	
Tutor help	
Lost	
Other (specify)	

Notes:

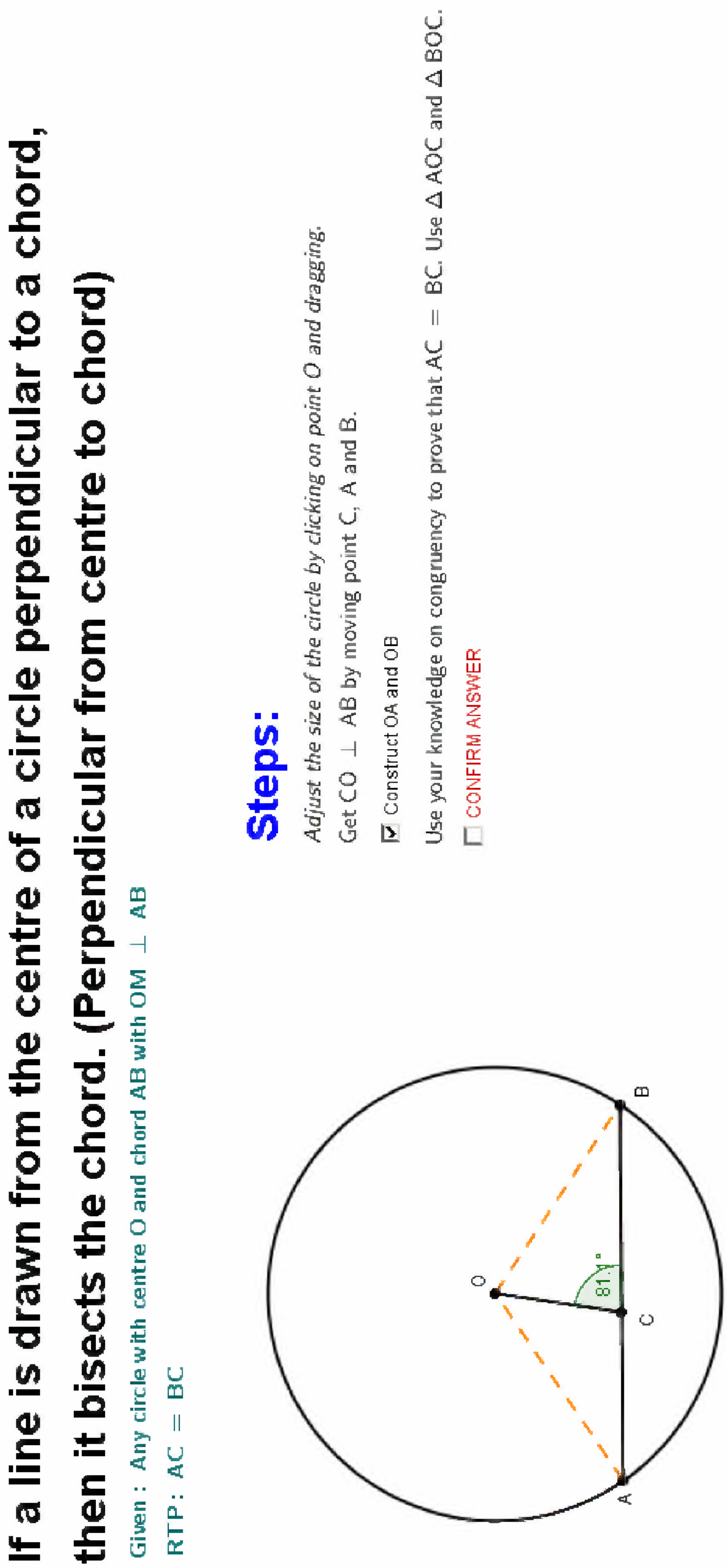
Appendix R - Participants' responses to questionnaire 2

		PARTICIPANTS										
		1	2	3	4	5	6	7	8	9	10	
	Statement	M27	M03	M23	M19	M14	M01	M04	M07	M26	M13	AVERAGE
1	I frequently use the internet as a pre service teacher.	2	4	3	4	5	4	3	5	1	5	3.6
2	I know how to use a computer.	2	4	5	5	5	5	5	5	5	5	4.6
3	Dynamic software can improve mathematics results.	4	4	4	5	4	5	4	5	5	4	4.4
4	I have learnt (during the DOE session) as much as I would have learnt face to face.	4	5	3	4	4	5	4	5	5	4	4.3
5	Is the DOE is too technical to use at school.	4	4	4	3	3	4	4	5	1	2	3.4
6	Students at schools will benefit from DOE.	5	5	4	4	5	4	5	5	5	4	4.6
7	I was frustrated sitting alone in front of a computer.	1	1	2	1	2	2	3	2	1	1	1.6
8	It will be difficult to use the DOE in my teaching practice.	1	2	4	4	3	1	4	2	1	1	2.3
9	DOE increases efficiency of Mathematics Education.	5	4	4	5	5	5	4	5	5	5	4.7
10	Using DOE will make the Mathematic subject content interesting for students.	5	4	4	5	5	5	5	5	5	5	4.8
11	Class time is limited for the use of the DOE.	3	4	5	4	3	1	4	2	1	2	2.9
12	I dislike the use of technology when teaching mathematics.	1	2	3	1	1	1	2	1	1	1	1.4
13	My experience of schools is that they are adequately resourced to use the DOE when teaching.	3	2	2	4	1	1	3	2	1	4	2.3
14	The DOE is simple and understandable to use.	4	4	4	4	4	5	4	5	5	4	4.3
15	DOE should only be used at university level.	2	2	4	3	4	5	2	5	1	2	3
16	Online mathematical content is convenient for my teaching preparation.	4	4	3	4	5	5	4	5	5	5	4.4
17	Visualisation in DOE assists students in mathematics	4	4	4	5	4	5	4	5	5	5	4.5
18	I would recommend the DOE to my maths colleagues /friends.	5	4	5	5	5	5	5	5	5	5	4.9
19	DOE increases the quality of Education.	4	4	4	5	4	5	4	5	5	4	4.4
20	I would like to learn more about dynamic software in mathematics.	5	4	5	5	4	5	5	5	5	4	4.7
21	I have used this type of technology (DOE) or something similar previously.	2	4	4	5	4	5	4	4	5	4	4.1
22	I have no intention to use DOE in my future mathematics lessons.	1	1	2	1	2	2	4	1	1	1	1.6
23	I will share my experience and show students the DOE but will not use it.	3	2	4	2	2	5	5	2	1	2	2.8
24	Is it important for me as a university student to use DOE	4	5	4	5	5	5	4	5	5	5	4.7
25	I don't think I will ever need a computer/technology in my classroom.	1	1	1	1	1	1	2	1	1	1	1.1
26	If I had funds then I would invest in exposing my students to the DOE.	5	4	4	2	5	5	5	5	5	5	4.5
27	DOE can enhance students learning.	5	4	4	5	5	5	4	5	5	5	4.7
28	The layout of the content was a good build up to the Theorem of Pythagoras.	5	4	4	5	5	5	5	5	5	5	4.8
29	DOE will allow students to self-discover.	5	4	4	5	5	5	5	5	5	5	4.8
30	I use the computer less than three time a week	2	1	2	2	1	5	3	1	1	1	1.9

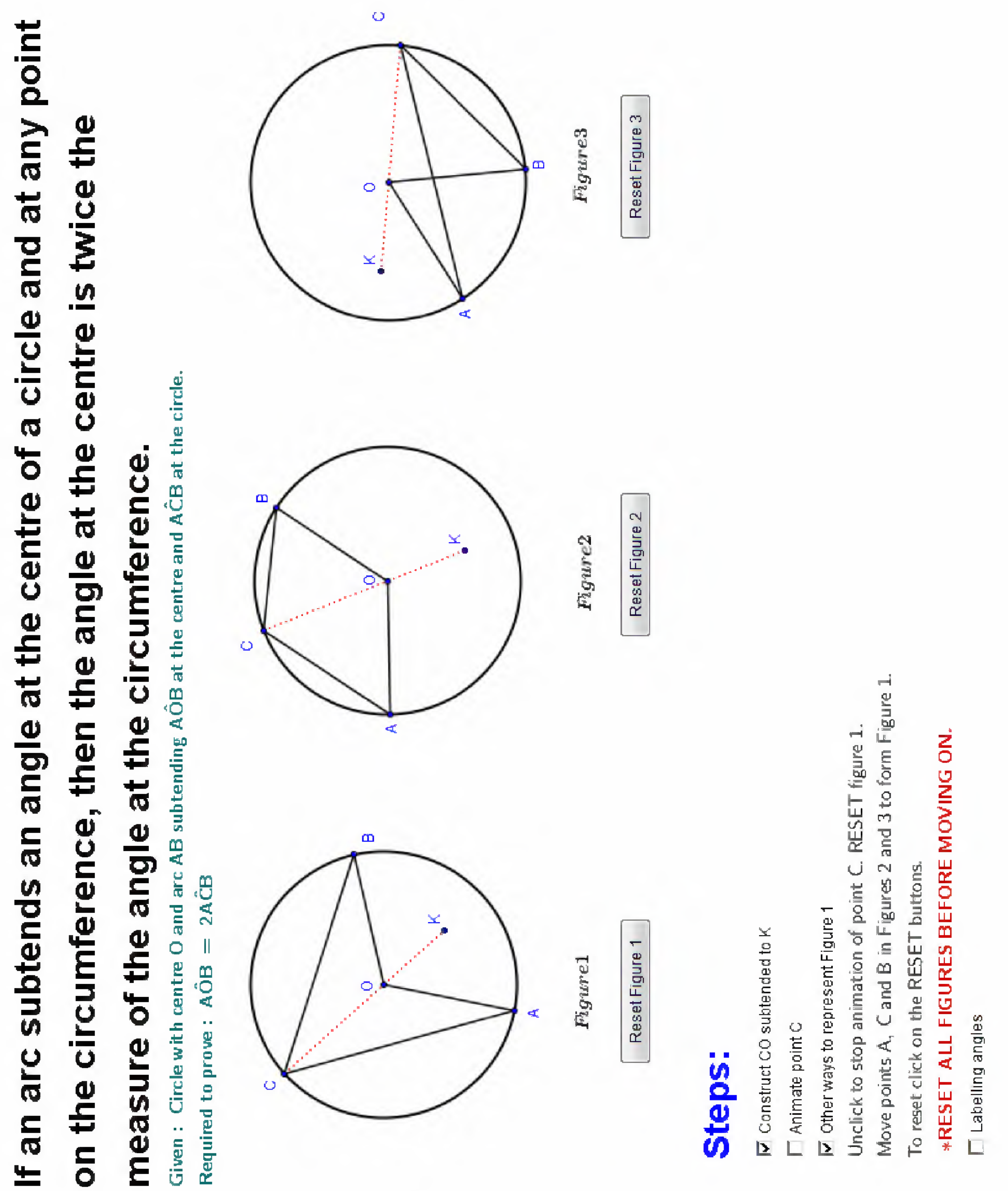
Appendix S - Getting to know the circle applet



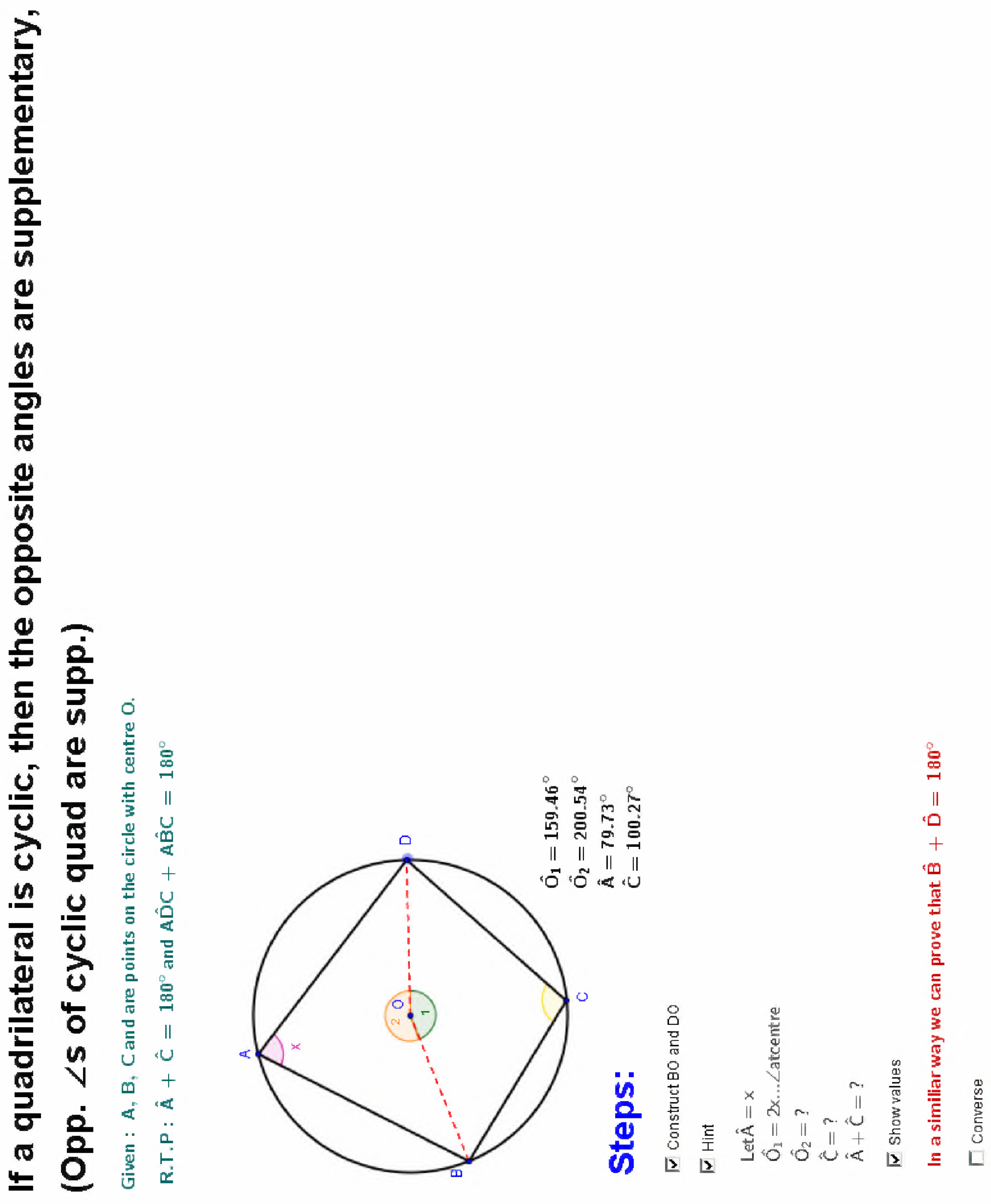
Appendix T - If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord otherwise known as perpendicular from centre to chord applet



Appendix U - If an arc subtends an angle at the centre of a circle and at any point on the circumference, then the angle at the centre is twice the measure of the angle at the circumference applet



Appendix V - If a quadrilateral is cyclic, then the opposite angles are supplementary, otherwise known as opposite angles of cyclic quad are supplementary applet



Appendix W - The angle between a tangent to a circle and a chord drawn from the point

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (tan. -chord theorem)

Given : Circle with centre O; PT a tangent at P; PB a chord and points A and D on the major and minor arcs respectively.

Required to prove : (i) $B\hat{P}T = \hat{A}$

(ii) $E\hat{P}B = \hat{D}$

Steps:

☑ For (i) & (ii) construct: OP and OB

☑ Hint (i)

Let $\hat{A} = x$

$\hat{O}_2 = 2x$ ...reason ?

$\hat{P}_3 = ?$ ...OP = OB radii

$B\hat{P}T = \hat{P}_{1+2} = 90^\circ - \hat{P}_3$

$B\hat{P}T = ?$

☐ Show values

☑ Hint (ii)

Let $\hat{D} = y$

$\hat{O}_1 = 2y$ ... ∠ at centre equals twice ∠ at circum.

$\hat{O}_2 = 360^\circ - \hat{O}_1$ ... revol.

$\hat{O}_2 = 360^\circ - 2y$

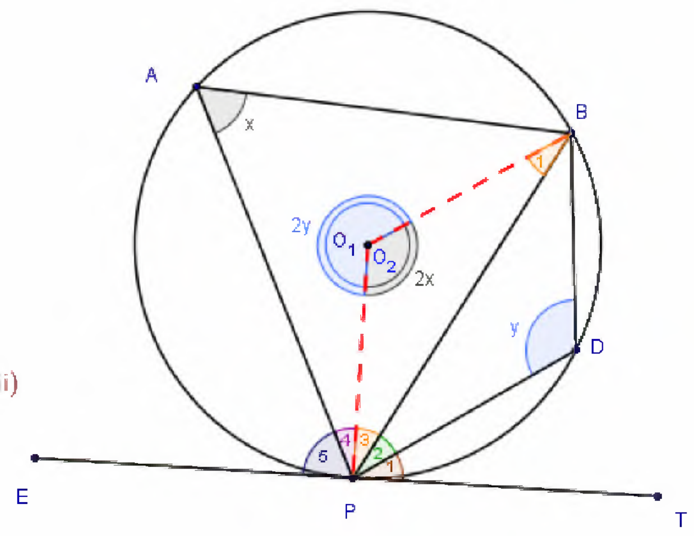
$\hat{P}_3 = (180^\circ - \hat{O}_2) \div 2$ ... issoc Δ (OB = OP radii)

$\hat{P}_3 = ?$

$\hat{P}_{4+5} = ?$

$\hat{P}_{3+4+5} = E\hat{P}B = ?$

☐ Converse:



Appendix X - List of prescribed CAPS Mathematics textbooks

Publisher	Grade	Title
Allcopy Publishers (Pty) Ltd	10	Mathematics Text Book Gr 10
Heinemann Publishers (Pty) Ltd	10	Classroom Mathematics Grade 10
Maskew Miller Longman (Pty) Ltd	10	Platinum Mathematics Grade 10
Lectio Publishers (Pty) Ltd	10	Mathematics Survival Series
Maskew Miller Longman (Pty) Ltd	11	Platinum Mathematics Grade 11
Via Afrika	11	Via Afrika Mathematics Grade 11
Macmillan South Africa (Pty) Ltd	11	Clever Keeping Mathematics Simple Grade 11
Allcopy Publishers (Pty) Ltd	11	Mind Action Series Mathematics
Berlut Books	12	Maths Handbook and Study Guide
Maskew Miller Longman (Pty) Ltd	12	Platinum Mathematics Grade 12
Macmillan South Africa (Pty) Ltd	12	Clever Keeping Mathematics Simple Grade 12
Via Afrika	12	Via Afrika Mathematics Grade 12
Allcopy Publishers (Pty) Ltd		Mind Action Series Mathematics
Heinemann Publishers (Pty) Ltd	12	Classroom Mathematics Grade 12

Appendix Y - Focus group interview transcript

Date: 18 October 2014
Time: 13:00

Researcher: **1. What are your thoughts about the use of DOE at university?**

M26: I think the use of DOE at university is going to produce teachers and educators who are well versed with modern technology and I think the type of teachers that will be produced is going to help South Africa produce learners who are very much well versed with the understanding of mathematics.

Researcher: Anyone else?

M26: Secondly I know I am very much articulate. I think it is going to help these teachers when they go to schools, because you see the learners are very much playful and they have a problem because mathematics is so abstract to them. They just not interested they feel it is very difficult. I think the use of DOE is going to help the teachers and help the learners understanding. It's going to catch their attention because the kids understand better when a playful method is used.

Researcher: Thank you

M13: My thoughts on the use of DOE at university is that as a student, when you are interacting with the software and when interacting with the Dynamic Online Environment- you actually bring that across into your teaching as well, because for example during my teaching practice I used sketch pad and taught the circle
theorems and I used it as well for teaching grade 10 trigonometry so it did help a lot and not only that - it also encourages learners to become more independent.
They don't rely so much on the teacher to spoon feed them and keep telling them what to do. Now they are forced to go and do it themselves and they are forced to come to their own conclusions and it's the same for students as well.
In our math 420 module we have to use Sketchpad. In all our lectures it happens here in this LAN. We use Sketchpad throughout for math's method 3 and we use Geogebra throughout.
It broadens your perspectives on teaching mathematics.

Researcher: Thank you; anyone else?

M26: Thirdly… probably the most active participant! I think the use of DOE must not be used alone. What I mean is that it is not meant to replace teachers. Teachers will always be there to support, guide and scaffold the learners, and for me I think it is very important that the use of DOE be used such that

mathematics is understood. I mean not that the use of technology is just used for the sake of using it I think teachers should be very much well equipped with how to merge DOE with mathematics because we all know guys that Mathematics is a discipline. Accordingly to Zeb and Belgin (2000) Mathematics is a discipline whose register, whose syntax is difficult to decipher. What I actually mean is that mathematics is so diverse there is a mathematics language in it.
There is English language in it so really the use of DOE - when it is used it must be used to enhance whatever abstract aspects are encountered in mathematics.

Researcher: Thank you; anyone else?

M01: What I noticed about this DOE for the student in the analytical situations when they learnt more about Sketchpad and Geogebra materials they tend to go to schools and use these materials as teaching aids not teaching materials. That's the problem! When you learn too much about them they tend to misuse them as teaching aids instead of teaching materials.

Researcher: Just to clarify teaching aid-your meaning of teaching aid would be?

M01: The meaning of teaching aid is everything that should be in the classroom like the basic material of the classroom like the chalkboard …. the curtain binders.
But teaching material should be the additional material to textbooks like Geogebra and Sketchpad - that's additional materials to textbooks … not everything that is in your cupboard.

Researcher: **2. Do you think being exposed to DOE in the mathematics curriculum at university (Higher Education) will help you become a better teacher at school?**

M04: Yes it will help but it will depend on the school situation. You can have knowledge of DOE but find out in your school there is no access of Internet for you to get online modem.
We cannot spend much of our money investing for other people so I think you can have the knowledge, but find out if you can help others because you cannot invest a lot.
That's my concern.

M03: I think it will help you as teacher because you will develop that demonstration.... You can make your student to see and observe and if you can do the demonstration in front of them they are able to come up with conclusions. However I still have a question based on the curriculum if you look at the curriculum - it is more specific and therefore is limited time to use it (DOE).

M04: Based on the point that M03 said about the time management. I think as a teacher you have to allocate yourself - do not focus on the time that you get by the school where they set period of time so I think you have to allocate yourself and meet your children on Saturdays, so you can do all sorts of things that you want to do using the technology and other stuff.

M19: I think it will be very helpful for us with DOE because as teachers - we are going to be taught how to use DOE and we also going to be taught different ways and different strategies that we can use DOE in a classroom.

Researcher: Thanks for that. Anybody else?

M23: I want to refer to M04's point using this is conducive to your school. Until you get to your school environment, then you understand the use of technology. In a sense you can have a Saturday class but the rule is you can't teach new content in a Saturday class. So you are not exposing the full component to the learners. Secondly, depends on the school as well. Like I said it must be conducive to the school.

So those are the factors that contribute- If you got a good school, its fine but else not.... so it depends on which school you are in.

Researcher: OK thanks anyone else wants to your view

M26: I think the use of DOE, you can become a better teacher in school because it allows learners to become engaged in problem solving, because they can actually visualise instead of the teacher drawing. At times you find that you draw a right angled triangle in one form and yet through DOE and through transformation learners will get an opportunity to drag and realise that the same sketch that can be drawn in other form ... can be the same one if its upside down or whatever. So for me visualisation use of sketches was done for many years, but it has remained as abstract to the learners. I think DOE has actually brought light to the teachers...

so that the abstract mathematics disappear and enables the learners to actually see what the teacher is talking about more especially in Euclidean geometry.

Researcher: Ok thanks for that. Anybody else would like add their view regarding point 2?

M03: I have not much to say but I can say that children learning will be better for teachers. We will be able to know different things to use in the classroom except the chalkboard and book. Also I think it will be helpful to our children by using different strategies and obtaining different strategies for children to solve problems in mathematics.

Researcher: Ok thanks. The next question is:

3. Is it easier to understand mathematics (Theorem of Pythagoras and Circle geometry) work through the use of DOE and Why? Does it change your attitude/perspective towards teaching mathematics when you use this type of technology?

M04: Yes it is easy to understand mathematics… theorem of Pythagoras and there was something I experienced today that was exciting. According to my experience at school I have known that the theorem of Pythagoras sits with $x^2+y^2=r^2$ but now I have that version of how it comes... I can now show it to my children ….so at the end of that vision … of how things come. What I have learnt today I can show my children evidence by first introducing DOE showing them the theorem of Pythagoras and then I can now tell them the experience that I have learnt that elsewhere so they know where things come from ($x^2+y^2=r^2$)

Researcher: So it's like you are adding meaning to that x squared plus y squared equals to something else isn't? [M04 nods yes]
Anybody else has any other points regarding 3 in regards to the theorem of Pythagoras and circle geometry?

M26: For me DOE is easier to understand because it is actually adding to knowledge that I have already acquired. DOE is skills facilitators… it enables one to work faster. You don't have to bring any mathematical tools because everything is within the software you are using plus for me it makes sense to use different teaching styles. I have 25 years' experience, and you find that for 25 years you are teaching one traditional pencil and paper method and producing learners who do not understand mathematics when it comes to tertiary level… but you just produced robots. For me it is important to produce learners who can understand mathematics and relate it to the real world, and can apply it to the real world where there are problems to be solved. Because that is the problem we are having with our learners. Even matrics may get As but when they come out to university they cannot apply mathematics that they learnt because they do have conceptual understanding. They only have procedural fluency where they just know the procedure only - that is all and as long as our learners are exposed to a procedural influence and not a conceptual understanding in this world, it means nothing. So DOE for me needs to be done and it is very important that we receive this with a positive attitude, so it will change us to have a positive attitude.

M23: In a nutshell it basically adds to our teaching knowledge. Also it actually becomes monotonous to teach one way teaching and with worksheets. Sometimes you need to engage in self-content as much we say as teachers one of the roles is to become a long-life learner. So therefore we are also learning in the process and we keep up with technology so there should be a possibility that where there are facilities - why not use it?

M14: I can say that the use of DOE makes learning for the learners easier. I have observed and found that some situations are a challenge so when we use the DOE it's amazing somehow learners tend to understand rather than the chalkboard. The use of DOE will make it easier.

Researcher: **4. What effect does the use of dynamic technology have on your mathematics understanding of concepts being taught? Do you think the DOE makes you confident in teaching mathematics?**
Anybody wants to share your view on that?

M13: I think that by using DOE in the classroom it helps to eradicate misconceptions by the learner, because you can prove to them that by using the software that certain things only happens in a certain way like for proofs and so on. Like for example the angle at the centre theorem - you can show them that no matter which way you orientate the diagram the angle at the centre will always be twice the angle at the circumference… What happens with learners is that they sometimes …they think that only the diagrams you give them ….that something will work. However if they do it on their own they get a different answers. So in this way even if you get them to construct it by themselves they may make mistakes like minor mistakes …like with the compass or using the protractor and things like that and then they think that you are wrong.

They have this misconception that if they carry on through other concepts in mathematics and once they come to university level the lecturer tells them that what you know is completely wrong it disheartens them. So in a way it makes you confident as a teacher because you are teaching the right things, and it makes the learners confident because they now have eradicated all these misconceptions and these errors that they make and they become more well-grounded individuals.

Researcher: Anyone else wants to share your viewpoint on question four? Do you think that DOE makes you confident in teaching mathematics?

M26: It makes me more confident, because I find that having taught for 25 years there are new things that I learnt which I never known before and yet before I thought I knew everything … and you see when you come with new things to the learners they have so much confidence in you and they become very much interested than they were before. I am saying that because I have discovered that through the use of DOE you find yourself conducting experiments where you are actually doing an empirical course instead of the traditional pencil and paper method, and for the mere fact that these experiments are in the form of investigations they are task based and they are explorative. They allow one to discover things on his/her own and I think once you discover something on your own it is so different - you have a fulfillment.

Just be repeating what you have read in the textbooks and what you have heard from other people. There is this Chinese proverb which says: "I hear I forget. I see I recall. I do I understand". That is how it feels about my confidence.

M01: I don't think for me DOE would work because to the school that we are going to, not all learners will have access to Internet so I will be the one with the computer. So the learners will just forget me and there will be less time … so learners will be like just fine with DOE without doing it.

Researcher: Would you then say that if you did not have DOE and if just used the traditional method maybe like the chalk and talk instead of DOE and then demonstrated it - do you think they will then understand?

M01: At the moment as it stands I will use it for myself to go to class and as preparation to go and teach. I will use it for my understanding for preparation to go to class and teach. Use it to have more understanding in order to teach the learners.

Researcher: So you would say that you will use the DOE as a recap-refresher before you go into the classroom?

M01: Yes at the moment I will use it for myself before teaching them.

Researcher: **5. What are your thoughts to the build-up of theorem of Pythagoras? Do you think the worksheets enabled thinking? Did it get you thinking? and the circle geometry as well. Do your think the worksheets enabled you to think.**

M23: It gives you a little bit of thought when you are faced with an example that you are not used to. The minute you come out of your comfort zone like m26 said with the use of traditional ways. You become set in your ways there is a possibility that this can get one thinking!

Researcher: OK; anybody else?

M01: Just what M04 said the visually interface of the Pythagoras theorem where $r^2=x^2+y^2$, it gave me a better understanding of the relationship between r^2 and x^2 and y^2

M03: For instance theorem two states angle at the centre equal twice the angle at the circumference. It's easier to explain from what we saw in Figure 1 but once you move to other Figures, it's difficult to explain even if you do understand … you are not like confident to process it to learners. But with DOE it shows us and it's easier to explain and understand.

M26: I really like the buildup on the theorem of Pythagoras. Because the way I taught it… all a long I wondered why learners do not get the jest of it. I can see why- it's the way I introduced it and besides I'd somewhere somewhat confused it when I say $r^2=x^2+y^2$. I was looking at this as the theorem of Pythagoras or is it part of analytical geometry where there is a triangle drawn on a Cartesian plane. But the way it was setup and built up … that magic triangles they actually got me thinking, insightful and exciting as a teacher. Which means I would be able to make learners think deeper than repeating the same thing… and boring the learners.

M23: What M26 said was actually… It actually integrates different sections. Magic squares integrates contingency tables and a lot of other mathematics are linked to it.

Researcher: Contingency would be part of probability.

M23: Yes and also the issue of addition and subtraction…. and broader thinking involved.

Researcher: **6. How do you feel about using technology (not only DOE) in class? Are *you comfortable using technology in math's and why?***

M04: Looking at the advantages and disadvantages. Advantages: it makes our work simpler and saves time…sometimes. As the school curriculum demands from us to work on time so the mathematics software sometimes…I am lost I'll get back to you!

M07: I think the use of mathematics programs in teaching I can say it's good as M04 said it saves time and when drawing visuals or diagrams for an instance, it is more accurate than the use of free hand. But there are some disadvantages now learners are not given the opportunity to draw a circle or construct a cyclic quad they just set in front of the computer with worksheet.

Researcher: But with the technology; are you comfortable using it?

M07: Yes I am as it saves time and drawings are more accurate.

Researcher: With the use of technology, do you think as a teacher you need to study first like how to use it before you go to classroom or do think you can just go into the classroom and use it?

M07: Say for an instance the worksheet you gave us regarding the Pythagoras theorem you can use the worksheets and ask learners to fill it in ask them to add the corresponding angles and that number square will give you that number squared. Then ask them to do it for all the numbers in the squares. Then you tell them that this is Pythagoras and then give them the formula. So you can use it initially to introduce the formula of Pythagoras.

M23: I think teaching is teaching. There are certain topics you need to use technology to emphasise and there others which you don't have to use. You have to integrate your teaching. It doesn't necessarily have to evolve around technology, but there are certain sections like geometry- it is time consuming to do without technology as well functions - technology is required, but certain things can do without technology. Comfortableness part of using technology as an educator - obviously any lesson you do, you do a lesson prep so you would have checked if your technology works.

Researcher: So in your lesson prep you should have checked how comfortable you are with the technology?

M23: You got to check if its working, what are the flaws, what are the mishaps within that technology so that you able to rectify.

M26: I am comfortable using any technology only because I have a positive attitude towards it. I will say I am both comfortable and uncomfortable. Why I am saying uncomfortable - because I feel that the knowledge I have is not adequate for me to use technology to teach algebra and trigonometry. For instance, I am using Sketchpad to teach Euclidean geometry but with the other sections, I don't want to lie, I do need intense training.

Researcher: Do you think the exposure at university enables you…you said that you need intense training. Do you think the exposure of technology at university is not sufficient for you?

M26: At the present moment it's not sufficient. With the training I had, I can only use Sketchpad to teach Euclidean geometry but I would like to be able to teach whatever other section with technology. I don't know how to use it to teach algebra and trigonometry and I would love to be exposed.

M04: This software sometimes beats us. For example when you install Microsoft you get everything there just insert equation and it shows you all the steps, even plots the graph… you just enter what you want and it gives you all the solutions and what you need.

Researcher: Is that the software Microsoft equations?

M04: No, Microsoft math.

M19: Sometimes it doesn't give any knowledge to our students. If they are lazy they just type the equation into the software gives them an answer without understanding.

Researcher: **7. Did the use of DOE together with the worksheets make you self-reflect while answering? If YES or NO Why?**

M26: With me the worksheets actually make me self-reflect because as I was trying to make conjectures out of the questions asked. I got thinking … I never emphasised this… I had misconceptions which I need to eradicate in myself and how do I expect learners not to have these misconceptions if I have misconceptions myself. So I got to reflect on myself very much trying to recall if I am on the right path... and so on... that's all.

Researcher: Anybody else got any other points?

M23: As M26 said that there was some misconceptions I did not realise that the angle at the center equals to twice the angle at the circumference will be the same diagram… I thought it was a different diagram applying the same theorem… But when you start using the DOE you start to realise that it is the same diagram but in a different way.

Researcher: Anyone else?

M26: The worksheets made me self-reflect with more especially angle at the centre theorem because no one thought of transformation when looking …When we were taught at school, we only had one form of the diagram and when the other forms appeared through transformation it appeared like something new. While to me it was not something new but something that I only learnt this year ... Why? Because I happened to come across it while I was collecting information for my dissertation ...only this year I learnt of it so it's actually makes one to self-reflect on how the different topics intertwine.... I think is very important instead of teaching them as different topics as different items. It's very important to realise all the integration in the same discipline among the different topics.

Researcher: **8. Would the use of technology such as the one used in future lessons? Do you think it would benefit you as a teacher or your students later in life?**

M01: I will use it. The reason is that because you see what I understood today is that even if you learn something … the next thing that happens is just an extension of the first thing. For example, if you teach the sine rule and area rule cosine rule... If you start with them form the right angle triangle you will find the use of the DOE software will help you a lot to expand from right angle triangle to the rule

Researcher: Would it benefit your students later in life when they are done with their schooling career?

M23: I would like to reiterate what I said - there are certain sections in mathematics that you require technology- it assists or benefits you. For example, the technology you use now in geometry (DOE), it has actually become a need. You'll find… geometry coming back into syllabus is that before learners where getting 80% 90%...they were good in mathematics applying to become engineers and so on.... but didn't have the geometry knowledge however they had to learn the geometry… secondly when you are an engineer I don't think you still use pen and paper ... or any job in this day and age you still use pen and paper. So the integration in teaching technology is a must.

Researcher: So you basically saying that when you as a teacher when you use technology in class you actually exposing learners to something that they might come a across later in their career after they finish grade 12?

M23: Yeah exactly… you are exposing them to the future and beyond grade 12. One of the educationally purposes is to create citizens so we do teach beyond schooling … and also there is a question often asked in school and sure as a math's teacher you often come across it: What do you use this for? There is a perfect answer you got to use in the future if you want to be an engineer.

Researcher: Thanks.

M14: I will use technology again in my future lessons because I believe that it does keep learners focused for a longer period of time and it actually makes learning exciting. Also I do believe that it will benefit the kids …being able to access the information online by themselves unlike us who got to access information online when we were here at University.

M04: To justify what M23 said... based on geometry; I think that this technology is very useful to us teachers and students because it helps you in the future. For instance those that are going to do engineering like building bridges. So if they build a bridge or a soccer stadium, they need to find the centre of gravity so it will balance… So it will benefit me a lot. I will even go out and tell others for instance geometry we use it in our daily lives ... we are here because of it even the house… the stairs that you walk on … yeah ok.

M26: The use of technology benefited me and I think it will also benefit the learners because it enables one to realise the relationships between and among angles and sides… It enables one to make conjectures and I really liked the different forms in which the diagram where drawn ... where you had to prove the angle at the centre theorem because it also looks at transformation as well. For me, it benefits me a lot because it makes me realise that it is very important to understand that understanding... and it reminds me of Skemp (1976) who stated that there are two types of understanding instrumental and relational understanding… for me I realised that for many years that I only considered instrumental understanding which I think was inadequate, because it is important to produce learners ...We produce learners that know exactly when they make conjectures they must be able to explain why they are true or not true, and they also need to understand that there would be cases where they have to use counter examples and see if the generalisations they have made still stand. Teaching is not only to make learners pass, we looking onto their future as well ...After they have used technologies we must realise that they will end up working …end up being employers or employees where they need to be self-managing people contributing workers and participating citizens in our democratic country of South Africa.

Researcher: Thanks for that; anybody else want to share your view points on point 8?

M07: I think … I can use technology when teaching future lessons and it can be used in life. Let's take for instance, learners who are born in rural areas have no clue about the structure of the university and how lectures go about … now when you are using this technology ...including math's DOE that will require them to use the Internet and they will have to use information on the Internet, then those leaners will be at an advantage being able to cope with the strategies used at University because lecturers put information online and tell us to go and search this and that. So when using that technology you are actually giving them a chance to research information on the Internet by themselves.

Researcher: **9. Would you have understood the lesson if technology was not used? Justify your answer?**

M23: I would have understood the lesson as a practicing teacher…but this came at a different angle so like I said earlier it enhancing your learning… at those diagrams being there without the technology I would have traditionally been able to complete.

Researcher: Would you have taken much more time, been quick …quicker without technology?

M23: Depends on the situation. Like the geometry, as I said earlier, I thought it was like a raider not the similar diagram … so now with the technology it actually exposed me to a different idea as to how that diagram came about.

Researcher: But in terms of you using … without technology, would you have done it much faster as compared to with technology.

M23: It would have been much faster the traditional approach but technology has showed me a better understanding.

M04: To understand it depends on the teacher that is teaching the subject, so if that teacher does not have that much knowledge of what he/she is teaching, then you won't understand it … So let's say you don't use any technology, you go and fetch other books that will assist you in order to understand what your teacher was teaching … because all understanding depends on what your teacher is telling you.

Researcher: Does anyone else want to raise any views?

M26: I wouldn't have understood the lesson I would have had an idea why am I saying that? Right angle triangle with squares and numbers around it …I thought I understood the language that was there but found that I misinterpreted the whole thing. Why? Because of the language barrier because English is my second language, and besides that there is a lot of ambiguity in mathematics because of the type of discipline that it is ...because you see, when you read whatever scenario that you given and you are reading it ...you might think at times you are reading it with comprehension. You'll find that you are misinterpreting the whole thing and

I found technology coming to my aid there... it really supported me it helped me fill in the gaps that I had. You'll find at times you just read the instruction… you think you understand it when you don't actually understand it, but follow the prompts and you'll end up with the right answer ...ahhh! I got it right at last because with the English language being a second language and with mathematics having its own language as well- it doesn't become that easy. At times, no matter how experienced you are, because when you read you need to read with comprehension. You must read with comprehension but because of the ambiguity there, you'll find that you misinterpreting what is required of you by doing something else which is not required.

Researcher So you saying whether it was with technology or not it all comes down to whether you understand what they trying to ask you in the question. The language use …

M26: What I am saying technology filled in the gaps where I did not understand what was being said … it helped me ...it supported me … filled in the gaps.

Researcher: So with the use of technology what is basically happening is that the language barrier is no more there.

M26: Yes that is what happened.

Researcher: Anyone else want to share your views with regards to question 9?

M13: I would had understood the lesson if technology was not used but that is only because I haven't finished school so long ago …so those ideas are still fresh in my mind and having come from teaching practice where I have taught these sections/that particular geometry theorem, it was fresh in my mind because I have taught these the children using technology as well ...I use Sketchpad to teach that theorem so I would have understood the lesson. Although if it had not been there, it would have taken longer.

M26: I am recalling something when we were required to prove the angle at the centre we are given three diagrams… it was one diagram in different forms. The use of technology helped me out more especially with the third diagram which appeared more complex … and when I looked at it... it didn't look like a diagram that will be used to prove the angle at the centre theorem but technology did aid me to realise that though; dragging the same diagram can be transformed such that it appears in different forms and it actually talks to the ....there is something I came across in the AMESA documents which was set by Gestrol 2007 where he stated - when you teach children there are four things which I cannot recall them now but I know one of them is that when the child deals with relationships between... among concepts like in the diagrams realising the relationship between the different angles and sides and making conjectures. I will say that technology did help me a lot.

Researcher: Anybody as any other thing you would like to comment about the technology used today or in general?

M23: In a nutshell it all depends on you what you prefer and what you feel is conducive to your teaching because at the end of the day you can't choose as to how one needs to teach and how one must teach ... teaching is teaching and you always use a method or aid or material which you feel comfortable with so it's made available for those who like it.

M26: When I looked at the teaching and learning style… for me I looked at it as constructivism where the teacher allows the… provides a task which is an experiment to be conducted... being task based for me it's a very powerful learning style because you don't spoon-feed, if you tell your learners what to do or how to do - it now you producing copy cats instead of producing student that would have end up being logical thinkers, and realise that we must not expect them to be problem solvers but also problem posers at the end. This reminded me we must not provide out learners with routine tasks and think we are providing them with problems that they need to solve. If you are giving them routine tasks that you find in textbooks ...they won't be able to be logical thinkers which is very important so for me technology... I have a positive attitude towards it and for me it adds to the mathematics that one already knows- and for me it's very important. We are in another era now where there is modern technology ...our children will end up adults they need to fit in the modern technology world and besides our children are so well versed with technology. Even at home, you buy a new gadget while you are still reading the instructions how to connect it…you don't even understand what they did they just come and connect everything they switch it on within no time... So for them it just happens automatically without their parent or teacher, teaching them … technology is the in thing.

Researcher: It's something natural to them?

M26: Yes … Its natural to them.

Researcher: Anybody else has any comments…closing comments?

M07: I just wanted to comment about the workshop today - it was my first time attending a math's workshop. I found it very interesting, very fruitful, it also encourages me as a future teacher to be willing to attend more maths's workshops because you pick up some teaching strategies that can be used ...to draw the attention of leaner. So it makes your mathematic lessons and concepts more interesting... so learners are more willing to come... get more active/practice when solving mathematical problems. So I gained a lot... so many strategies that I can use in teaching and the use of worksheets keeps the confidence… you can ask learners to answer the question and use the DOE by learners to answer, therefore its very useful.

M27: I think today was frustrating and interesting at the same time… especially the tan chord theorem - I only knew one way of proving the theorem, but today I learnt a new method of proving the tan chord theorem of which I think is easy and more easy to understand.

M26: For me …today's workshop was really powerful because it reminded me of Vygotsky (1978) - stated what you call the zone of proximal development where the teacher is supposed to fill in the gap, where the actual development takes place. For me technology fills in that gap, so I really appreciate it because as I am studying at UKZN. It is now making me realise what information I have collected, it has made me able to put all that into practice.

M01: I don't think for me it will work because the schools that we going to not all learners will have access to the computer. I will be the one with computer they will look at me sometimes they will do nothing.

Researcher: Thank you all!