SMART BOARDS – SMART TEACHERS?

THE CASE OF TEACHING AND LEARNING OF ALGEBRAIC FUNCTIONS.

A Descriptive Study of the Use of Smart Boards in Teaching Algebraic Functions.

by

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ABSTRACT

This study set out to investigate how the use of a Smart Board impacts on the teaching and learning of algebraic functions. The research took place in a school equipped with Smart Boards in each Mathematics classroom. Data collection involved lesson observations in three classes over three lessons each. The teachers and learners were interviewed post observation and the data obtained were analysed according to Sfard's three-phase model framework to determine if the learners had a procedural or object view of a function after having been taught on a Smart Board. The findings show that by using a Smart Board learners had both procedural and object view of functions however, much of the teaching occurred in a way which would have been possible without the use of a Smart Board, indicating that teachers did not fully utilise the potential of such a technological tool. However, it emerged that visualisation played an important role in allowing learners to operate on functions as objects. So while the visualization that technology enables encouraged reification or allowed teachers and learners to operate on functions as a whole or even on families of functions, this appeared simply to 'speed up' the normal teaching-learning process rather than promote the explorative and investigative aspect of learning. Still, it must be acknowledged that this kind of practice is bound to strengthen these learners' function concepts as was evident in the ways they appeared to operate confidently on the objects as shown in the study. It must be acknowledged that teachers were extremely enthusiastic about the possibilities of the technology and were inspired to use technology more in their lessons to allow learners' visualisation of concepts. Positive comments made by learners showed that they too, were also motivated by the use of the Smart Board.

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DECLARATION

I, Charmaine Emmanuel, declare that:

i. The research reported in this dissertation, except where otherwise indicated, is my own work.

ii. This dissertation has not been submitted previously for any degree or examination at any other university or other higher education institution.

iii. This dissertation does not contain any other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.

Signed:

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Chapter 1 Introduction

The purpose of this study was to investigate the learning potentials of using Smart Board technology to teach algebraic functions and promote learning thereof.

Technology, as a medium, offers various opportunities to both teachers and learners to dispense and acquire mathematical knowledge and skills. Due to constant advancement in this field reasoning and critical thinking can be fostered, promoted and developed in learners in a number of ways. However, of relevance then, are the following issues:

To what extent should technology be used in the mathematics classroom?

What kinds of technology should teachers use?

When should teachers use it?

How can teachers use it as a teaching tool?

In general, technology includes the various levels of available technology: calculators, computers, laptops and interactive white boards (IWBS). It can be argued that perhaps the strongest reason to use technology, of any sort, in any classroom, especially mathematics classrooms, should be to introduce new topics and promote the discovery of properties thereof. Whilst it is acknowledged that educators will face many challenges as they try to keep up with or use technology- of fundamental concern to me in this study is how teachers can make maximum use of these innovative technological tools to assist in conceptual learning of algebraic functions.

1.1 Purpose of the study

I have been a Mathematics Educator for the past 10 years and over the years have had numerous experiences of learners protesting: “Where in real life is this needed?” or “What is the purpose of learning this?” This seems to suggest that perhaps we as teachers are not promoting and fostering investigative, explorative, applied or critical learning. Thomas and Holton state “For many years now the majority of teachers have been presenting

Mathematics as if it was just a set of rules that needed to be learnt. They have forgotten that mathematics is a live subject that exists to solve problems" (2003, p. 351). As a means to combat this, the research conducted by Lavigne & Lajoie (1996) suggests that a greater emphasis be placed on enabling learners to reason and think critically, to solve problems, and make more productive use of learning time to enrich their learning experiences.

To become an active and useful member of society, one must be empowered with the necessary skills and knowledge to be able to contribute positively and meaningfully. This theme of empowerment is advocated by the USA National Council of Teachers of Mathematics Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). The mathematics classroom is such a place where learners can be empowered to actively do meaningful mathematics and engage in critical thinking in the field of mathematics. This will then encourage learners to see the value of mathematics. It will allow them to gain and build confidence in their ability to do mathematics. Becoming mathematical problem solvers, being able to reason mathematically and to communicate effectively in mathematics will enable them to become contributing and valuable members of society. Accordingly, the role of the teacher must change from that of a dispenser of knowledge to that of a facilitator of learning. Activities and tasks must be designed by the teacher in such a way that the learners will be able to explore, conjecture, verify, generalize, and apply the results from their experiences to other settings and realistic problems. This is the essence of Mathematics and is the very activities in which mathematicians should be engaged. Mathematical power and ability can then be developed through understanding, using, and appreciating mathematics.

In the above-mentioned contexts, technology can then be seen as a valuable tool in the teaching and learning of mathematics, for it has that very ability to be used to enrich and empower mathematics learners as well as mathematics instructors. Research by Bransford, et al, 1996; Schoenfeld, 1982, 1989, 1992; Silver, 1987 concur that teachers too must be empowered. Wilson (2005) states that this sense of empowerment can be achieved "through the use of technology in mathematics exploration, open-ended problem solving, interpreting mathematics, developing understanding, and communicating about mathematics “.

Technology, especially computer and calculator technology, allows us a medium of providing the tools needed to enable such investigative or explorative activities, as pointed out by Owens & Waxman (1995). Michelich (2002) has found that learners “often have a difficult time in visualizing concepts and struggle to grasp information that is presented either verbally or in text”. Campbell, Lum, & Singh (2000), concur that by using auditory and visual methods of presenting information, learners will be able process the relevant information more quickly, which then fosters an enhanced learning process.

Whilst technology use offers many opportunities for teaching and learning in general, it can serve particular roles in the teaching and learning of mathematics. Kimmins (1995); Kimmins and Bouldin (1996), state these roles as aiding:

1. mathematical concept and skill development,
2. mathematical problem solving,
3. mathematical reasoning, and
4. mathematical communication.

This study explored the potential role of how Smart Board technology, whilst teaching functions, was used to facilitate in particular concept development and how it assisted learners to see that mathematics is dynamic and interactive. From the above-mentioned four categories, the technology used here aided in categories one and four, namely mathematical concept and skill development and mathematical communication. So far, little research has been done in this field. In addition, I have a personal motivation to explore this, since Smart Boards have been installed in all mathematics classrooms at the school where I teach. Please note that I have used the Smart Board and Interactive White Boards (IWBs) interchangeably throughout this thesis as this study involved the use of Smart Boards which is a type of IWB.

1.2 Research questions

Technology has become an integral part of many educational activities and can be used as a powerful tool to promote the understanding of fundamental concepts in mathematics. This research aims to explore how technology, specifically through the use of Smart Boards and graphical software, can assist learners in both conceptual understanding of a function and allow for possible experimentation with functions to gain a broader, in-depth understanding of the function concept.

The research questions that this study aimed to explore were:

- What kinds of learning environments are created when using Smart Board technology in teaching algebraic functions?
- How are teachers using Smart Board technology opportunities to promote active and meaningful learning of functions?
- Does visualisation aid in conceptual understanding?

1.3 Limitations of the study

As is often the case with descriptive data, the results and findings of this study apply only to the school which has participated in the study. Although the study wished to include other schools, time constraints led me to focus on one school. Secondly, the questions within the questionnaire were not exhaustive, and an open ended-response format was used wherever possible, making answers subjective. Interview sessions for interested participants had been planned. It was felt that these interviews would add to the depth of knowledge and provide insights in ways that a questionnaire could not. Again, due to time constraints, only one interview session with teachers and learners concerning the study was completed. Whilst it is acknowledged that my presence was likely to introduce a distortion of the natural situation (lesson observations) with learners and teachers, I was aware of this restriction and I tried to minimise it as much as possible. I set up the equipment, ensured it worked and left the rest of the lesson recording without my presence.

Chapter 2 Literature review

2.1 Introduction

The literature presented here will focus on international research undertaken as there is limited research regarding the use of IWBs, specifically Smart Boards in South African classrooms. The reason for this may certainly be the lack of infrastructure, lack of technology, and the lack of funding in South African schools. It is also important to note that although the literature reviewed is overwhelmingly positive about the impact and potential of IWBs, it is primarily based on the views of teachers and learners.

Firstly, I will review literature on the teaching and learning of functions. The literature is further subdivided into the following categories: the concept image; procedural and conceptual knowledge.

Secondly, I will focus on the general use of technology in teaching by discussing using technology as a visualisation tool; technology as an explorative tool and technology for reshaping learning opportunities.

Thirdly, I will discuss the use of IWB technology in particular.

2.2 Teaching and learning of the function concept

The concept of a function has been widely recognized as being foundational to school mathematics and mathematics in general, as argued by Romberg, Carpenter & Fennema, 1993. However, as pointed out by Eisenberg (1991, p. 140) the function concept is "... one of the most difficult concepts to master in the learning of school mathematics". A possible reason for this is that symbols are usually used to represent functions. The concept of a function represented by symbolic notation or any other form, for that matter, is an abstract concept. Therefore if a learner has any difficulty with the conceptualization or understanding of the symbolic representation used, or the context in which symbols are used, this will impact on the learner's understanding of the function concept. Another possible reason for learners' difficulties is the cohesion of fundamental ideas in the modern notion of functions. These involve input-calculation-output processes, co-variation and sets of ordered pairs, to mention a few. Further, change is often involved in a function concept and this is an advanced idea, since the independent variable could be continuously changing which then in turn affects the dependent variable.

2.3 Concept image

At this point it is necessary to explain and discuss the concept image. To do this I draw on Tall and Vinner's (1981) theory of concept image and concept definition. According to them, when we think of a concept we are reminded of something or recall a mental picture. Often these memories or recalled images do not necessarily correspond to a well-defined theoretical concept definition. This recollection of memories is termed the concept image. In Tall and Vinner's theory, concept image is the whole cognitive structure that is associated with the concept. Tall and Vinner 1981, Vinner 1991, argue that the concept images we have are formulated by: firstly, our previous experiences and conceptions and secondly by recalling tasks we did in which the concept definitions were tested. Thus mathematical experiences in everyday life and experiences of learning mathematics play an important role in our mathematical thinking.

Much of the literature regarding the teaching and learning of functions refers to the learners' concept image of function as found by (Vinner, 1983; Vinner and Dreyfus, 1989; Tall, 1989; Slavit 1997; Sfard, 1994; Thompson, 1994). Vinner and Dreyfus, in particular found that a learner may know the formal definition of functions yet not be able to fully apply it. This is the case, largely, because the correct application depends upon the learner's concept image of function, rather than the definition itself. When deciding whether an example is or is not a function, Vinner and Dreyfus, explain, a learner may apply a mental image believed to be a generic representative of a mathematical object. For instance, if a learner has only seen and worked with continuous functions, and a non-continuous function is given, it may be rejected as representative of the notion 'function'. Hence, from an instructional or pedagogical point of view, before a teacher can expect learners to use and apply functions accurately, the teacher must assist in accurately developing their concept images to fully encompass the definition. Simply providing the learners with definitions is insufficient; learners must be assisted in developing conceptual understanding of the mathematical object by using and operating on the mathematical objects. The theory of constructivism leads us to believe that the road to building or understanding more complete concept images must initially go through the incomplete or even incorrect attempts at using the mathematical object. Smart Board technology may therefore provide teachers and learners with a medium for such use and experimentation, which may help build up more complete concept images.

Dubinsky (1991) has suggested that an important way of understanding the concept of a function is to construct a process although it is acknowledged that the construction of a process is only taking one step of Sfard's duality of mathematical concepts (Sfard, 1991) into account.

Perhaps at this point the work of Sfard needs to be discussed to give greater clarity to the above-mentioned statement of understanding a function as a process. This entire framework will be discussed in greater detail in chapter 4, p. 33. Here, I will use as a starting point the distinction between procedural and conceptual knowledge.

2.4 Procedural and conceptual knowledge

The terms 'procedural knowledge' and 'conceptual knowledge' are commonly used to denote a difference between two forms of mathematical knowledge, state Hiebert and Lefevre (1986).

Procedural knowledge would involve numerical calculations or computational skills and knowledge of procedures for identifying mathematical components, algorithms and definitions. Procedural knowledge of mathematics has two parts:

a. Recognition, assimilation and use of the format and syntax of the symbolic representation, and

b. knowledge of rules and algorithms useful in breaking down and completing mathematical tasks.

Conceptual knowledge refers to knowledge of the underlying structure of Mathematics. It is characterised as knowledge rich in relationships and includes the understanding of mathematical concepts, definitions and factual knowledge. Hiebert and Lefevre (1986) state that both procedural and conceptual knowledge are considered necessary aspects of mathematical understanding. Sfard (1991) has extended this through her analysis of the development of various mathematical concepts, definitions and representations from a historical and a psychological perspective.

Sfard's analysis has shown that abstract representations such as functions can be developed in two fundamentally different ways: operationally, as processes, or structurally, as objects.

2.5 Technology as a visualization tool

Visualization is a powerful tool in Mathematics and Zimmerman & Cunningham (1991) have documented its importance as such a teaching tool. Eisenberg (1991) has shown that some success in furthering learners' concept images of the function concept can be achieved by introducing the function concept in a variety of representational contexts. Examples here include using visual representations which may be in the form of flow diagrams, tables and input–output machines. Other contexts would include graphs, or the use of algebraic representations in the form of ordered pairs or algebraic descriptions. The underlying psychological theme here, as indicated by Zimmerman & Cunningham, 1991; Presmeg, 1993; Parzysz, 1988, is the use of visual reasoning in mathematical knowledge. Thus the appropriate and skilled use of technology may support the understanding of the concept of a function by enabling learners to visualize a function through the picture presented by its graph. It may also allow for faster exploration and experimentation of a function's characteristics and properties.

The available technology tools such as calculators, computers and graphing software enable learners to construct mental, visual and symbolic representations of ideas and incorporate these into their approaches and thinking about problems. The technology-enabled visualizations should not be considered to be the end-product but rather should be used as a means of developing and strengthening the learners' mental images that help them to form, relate, and organize mathematical concepts. Dugdale (1993) finds that these tools have

> *raised the possibility of visual representations of functions playing a more important role in mathematical reasoning, investigation, and argument. Relationships among functions can be readily observed, conjectures can be made and tested, and reasoning can be refined through graphical investigation (p. 103).*

Further, Widmer and Sheffield (1994) have also shown that certain learning difficulties associated with the function concept can be addressed to a large extent by making use of function machines and function games together with calculators and computers. At this point however, it is important to acknowledge that while function games can be played without technology but rather the emphasis is that technology once again aids in and enhances visualization. Therefore it may be concluded that different representations of the concept of a function in a variety of contexts, and the processes they imply, could – but will not always - aid and so promote understanding.

2.6 Technology as an explorative tool

In their analysis of research on the teaching and learning of functions, Leinhardt, Zaslavsky, and Stein (1990, p. 7) argue that, "more than perhaps any other early mathematical topic, technology dramatically affects the teaching and learning of functions and graphs". Teachers can utilize technology such as computers and graphing calculators- but the same can be said of Smart Boards - to have learners make observations and conjectures within a variety of function representations such as equations/symbolic expressions, graphs, and tables. Learners can then begin to make connections among the different representations in order to further develop the concept image.

Although formal definitions could be part of building a concept image, it does not guarantee the understanding of the concept. When learners have formed their concept images, the definitions become unnecessary. Studies by Vinner and Dreyfus (1989), Sfard (1989) indicate that learners interpret mathematical concepts operationally as processes, even if the concepts were introduced structurally using definitions. The majority of learners do not use definitions when solving tasks because their normal everyday thought processes and instinctive habits take over and they are unaware of the need to consult the formal definitions. In most cases referring to the concept image is successful. However, Vinner (1991) suggests that only non-routine problems, like the identification of examples and non-examples of a given concept, problem solving and mathematical proofs, can encourage learners to use the formal concept definitions.

The use of technology as a tool can, in turn, allow learners to explore the connections among representations enabling the learning of functions to become investigative in nature. Of importance here is that the learning being investigative in nature, assists in assimilation and cementing of properties. However equally important is the ‘explore connections’ since it’s in what remains the same across representations that constitutes the concept.

As stated in one of the seven principles in the *Principles and Standards* developed for US teachers,

> *Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect those changes. Learners can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction (NCTM, 2000, p. 25).*

Technology may present the potential to fundamentally alter the order in which a learner develops a concept image for functions as the current graphing software available allows for the concurrent development of multiple representations in the mind of the learner. This in turn would then support the view presented by the President’s Committee of Advisors on Science and Technology, 1997 that the underlying pedagogical theme should be to “experience mathematics as problem solving, communication, reasoning, and building connections”.

2.7 Technology reshaping teaching opportunities and challenges

However, the question remains: how should teaching be reshaped to allow for these potentials to be realised in the classroom? Furthermore it must be noted that most of the research has taken place in developed contexts, and disadvantaged classrooms in South Africa may face challenges which have gone unnoticed or have been downplayed in these contexts.

Although one important finding from Moor & Zaskis (2000) is that technology improves more quickly than the possible ways to use it, it must be considered that the one important limitation to the full utilization of technology in education can be teacher inexperience in using computers and the Internet. Smith (2000) found that there is little support for teachers to learn how to use technologies to develop effective and enriching learning experiences for learners. Waits and Demana (2000, p. 53) support this and argue that adoption of technology by teachers requires professional development that focuses on both conceptual and pedagogical issues. They believe that there should be ongoing support in terms of "intensive start-up assistance and regular follow-up activities". In addition, studies of teachers' implementation of educational technology by Dwyer, Ringstaff & Sandholtz, 1991; Means & Olson, 1994, document that at least three to five years are needed for teachers to become competent and confident in teaching with technology .

Bell (1998), argues

> *while it is true that many new applications and tools exist and will continue to evolve which offer instructional applications using technology, it is also true that many teachers still need to be convinced of the value of these innovations and trained in their use. Technology can be incorporated into the teaching styles of many teachers who have previously been hesitant to test the waters using computers for instruction. For these reasons, the IWB is a device which is gaining popularity as a visual presenter and interactive teaching aid for use in multimedia instruction.*

2.8 Teaching with the Smart Board

The interactive electronic whiteboard system is composed of three parts: a computer, an LCD projector, and the interactive whiteboard system itself (see figure 1a).

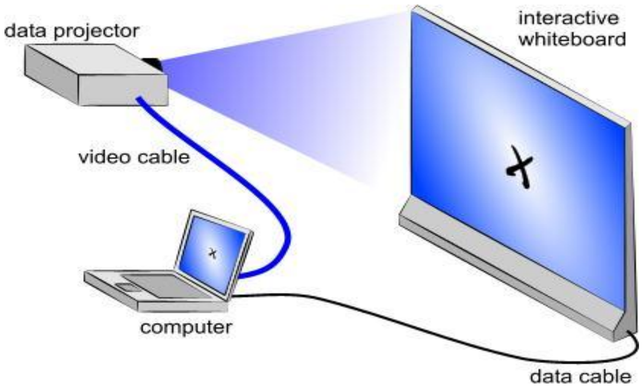

Figure 1: a (internet source)

Each aspect of this system is vital, and the full effects of the interactive whiteboard cannot be exploited without the computer or the LCD projector. The actual setup of the interactive electronic whiteboard in a classroom is shown in figures 1b, 1c, 1d and 1e.

Figure 1: b (data projector setup)

Figure 1: c (Smart Board setup)

Figure 1: d (laptop setup)

Figure 1: e (complete classroom setup)

Research by Weiser (2001) revealed that a variety of models of interactive electronic whiteboards are manufactured by numerous technology companies. This study focussed on the use of a Smart IWB. Weiser also found that many of the features of the Smart Board are offered by various competitors' interactive electronic whiteboards with minor differences in appearance and price.

Apart from the interactive electronic whiteboard surface being a huge screen display, it has interactive capabilities as it is touch sensitive. This was possible, by mapping the x, y coordinates and matching them against the computer screen. This would then allow a teacher or learner to "write" on the board with a pen or with their finger or activate characteristics of the board with the touch of a finger. We must at this point bear in mind, as with all technology, it is *how* it is used, not the technology itself, that will ultimately determine its value for teaching and learning.

The interactive quality of the board can lend itself to a degree of learner participation and involvement which might be limited by other presentation methods such as the chalkboard or overhead projector. Bjork (1978) argues that in interactive learning situations, the learner is a participant in the process rather than a spectator. Whilst it is acknowledged that learners can come up to a board and write on it making that type of lesson somewhat interactive, the Smart Board has the capabilities to make it interactive in a different way such that learners are able to manipulate objects for the whole class to see by combining the explorative elements of software with the display aspect of the board.

Since it is a relatively new product which has only recently been used in South African schools, there is a lack of research regarding its impact on instruction. Greiffenhagen (2002), points out that the IWBs were originally developed for office settings. The interactive electronic whiteboard shows promise in demonstrations but needs assessment in practice to determine its instructional value. Smith,Higgins, Wall & Miller (2005) have found that there is little academic literature available although it is slowly emerging. Smith et. al. also report there are available sources on the internet from reports and summaries of small-scale research projects. These were undertaken by various individual teachers, schools and higher education institutions across the UK, USA, Canada and Australia. A number of professional journals have published which contain descriptions of practice and teaching experience using IWBs. Research by (BECTA, 2003) suggests that teachers and learners value the surface features of the interactive electronic whiteboard such as motivation, engagement, involvement, participation and collaboration. So while international literature exists, insufficient research has been conducted to examine its effectiveness as a teaching tool in South African classrooms.

Knode (2006) states “Technologies have been coming in and out of our culture, and today one of the most ground-breaking advancements is the Smart Board. The interactive whiteboard has changed education for each and every learner that it has touched”. By exposing our learners to the technology of IWBs we will be assisting them to become versatile and creative thinkers. It will also prepare them for all other forms of technology they will be exposed to in their lives.

This study observed teachers and learners using the interactive whiteboard with certain computer software applications. Furthermore it tried to explore the impact it had on their conceptual understanding of functions. Jonassen (1996) defines mind tools as computer applications that can be used for developing reasoning and logical powers, which in turn, supports the theme of mathematics empowerment. A computer application, as a mind tool can be utilised to encourage critical thinking about the content being studied. Since the board is able to display various and multiple software applications whilst allowing shared use, it is able to provide the collaborative quality of a mind tool. This enables each learner to have a personal and individual responsibility for creating their own understanding, by allowing them time to construct their concept image or by allowing them to work at different levels, be it the process or object level of a function. Conceptual knowledge and procedural knowledge must therefore be emphasized together and technology tools could be used to reinforce their mutual development.

Knode (2006, no page number) points out that

> *Education opens doors for the future. By educating youth, by training teachers, by helping disadvantaged students, just by bringing in a new technology, our world can be shaped and molded into a better place for the future. Smart Boards have shown that by putting time and effort into changing some student's way of learning, they can revolutionize the way that they learn and grow. Interactive whiteboards are a key concept to the fresh new classroom of the future and they are only beginning.*

2.9 The role of technology in learning and teaching

Knode (2006) advocates that "Education in each individual's life is crucial to the future". The majority of jobs and careers generally require some level of technological experience or skill. Jokinen (2010) argues that "as technology in almost every industry is changing, so is it in education. Schools introduce pupils to the technology they'll meet in the work force".

By exposing them to the benefits of technology, learners can continue to learn and grow through each year in school. These technological experiences will stand them in good stead and allow them to be well-rounded technology users, as they enter the workforce later on. Knode (2006) supports this notion and states that “Smart Boards are becoming the classroom technology of tomorrow”.

The literature reviewed above covered: the teaching and learning of functions; the use of technology in teaching and looked at IWB technology in particular. The concept of a function was seen as foundational to school mathematics and mathematics in general but was seen to be one of the most difficult concepts to master. Among possible reasons for this were symbolic notation being used and the function concept is seen as an abstract one. The literature above showed that functions can be developed in two fundamentally different ways: operationally, as processes, or structurally, as objects. Technology as a teaching tool was therefore explored in the various roles: technology as a visualisation tool; technology as an explorative tool and technology as a tool for reshaping learning opportunities. Visualization was identified as a powerful tool in Mathematics and as such a teaching tool. It was shown that concept images of the function concept can be achieved by introducing the function concept in a variety of representational contexts and technology could facilitate this. The technology-enabled visualizations has the capacity to be used as a means of developing and strengthening the learners’ mental images that help them to form, relate, and organize mathematical concepts. The use of technology as a teaching tool can, in turn, allow learners to explore the connections among representations enabling the learning of functions to become investigative in nature thus promoting the theme of mathematical empowerment. Technologies have been advancing at an incredible pace and today one of the most ground-breaking advancements is the Smart Board which is a type of IWB. IWBs have the capacity to display various and multiple software applications which can assist in creating versatile and creative thinkers by promoting exploration and investigation of such topics such as the concept of a function. Although it is acknowledged it is only as useful in the hand of a skilled user, it has the potential to be a phenomenal teaching tool.

Chapter 3 Framework

3.1 Introduction

This study draws on a theoretical framework developed by Sfard (1991), which is called the three-phasemodel. This framework relates to principles of constructivist learning where knowledge is considered as a gradually built individual construction.

3.2 Sfards three-phasemodel

The three-phase model was developed by Sfard (1991) in an effort to understand the learners' mathematical concept formation. The three phases are: interiorization, condensation, and reification. She advocates that this three-phase model of concept formation is hierarchical as one stage cannot be reached before the former stages have been reached. Sfard, 1993 explains that, what appears to be a process at one level is later transformed into an abstract object at a higher level. Learners can then act on the object and so it becomes a building block of more advanced mathematical constructs.

According to Sfard, a mathematical concept formation has two sides, an operational one and a structural one. The learner has to first pass through operational phases until they develop a structural conception. She also points out that, *without the abstract objects all our mental activity would be more difficult*" (Sfard, 1991, p. 28).

It is perhaps necessary now to distinguish between an operational and a structural conception of the same mathematical notion. If a learner has acquired an operational conception, she or he will know how to operate with processes and actions. For a structural conception it is necessary to recognise the notion as a mathematical object. Sfard expects that the operational conception precedes the structural. In this process from operational to structural she argues that three steps must occur: *interiorization*, a process with familiar objects or actions performed on objects, *condensation*, where the former processes become separate entities and *reification* which means to see this new entity as an integrated, object-like whole (Sfard, 1991, p. 18).

While a learner can move gradually from interiorization to condensation, it is different when it comes to reification:

> *Reification (...) is defined as an ontological shift – a sudden ability to see something familiar in a totally new light. Thus, whereas interiorization and condensation are gradual, quantitative rather than qualitative changes, reification is an instantaneous quantum leap: a process solidifies into object, into a static structure (Sfard, 1991, p. 19 - 20).*

Sfard & Linchevski (1994) used the framework of the theory of reification to study the case of algebra. In particular, they focused on the transition from operational to structural regarding a variable as a fixed unknown on the one hand and in a functional context on the other hand. Sfard (1991) questions the movement to and recognition of a conceptual development and proposes:

> *It seems that we have no choice but to describe each phase in the formation of abstract objects in terms of such external characteristics as student's behaviour, attitudes and skills (Sfard, 1991, p. 18).*

According to Cottrill, Dubinsky, Nichols, Schwingendorf, Thomas & Vidakovic (1996) an object is derived from a process. It represents the process as a whole, and as something that can be acted upon. For example, a parabola can be drawn by substituting x values into a quadratic expression to get the corresponding y values. These plotted ordered pairs then enable the learner to the treat the graph of the parabola as an object.

Sfard clarifies that reification occurs when the learner is able to handle functions as objects. For example, when the ‘unknowns’ or ‘variables’ are functions and the learner has the ability to talk about general properties of different processes performed on functions one can say reification has occurred.

3.3 The three stages

According to Sfard, the process of concept learning includes three stages:

1. interiorization: a learner performs operations, processes or a series of actions on lower level mathematical objects,

2. condensation: a learner has an increased ability to recognise and work with different representations of a concept, and

3. reification: a learner can see and treat the mathematical concept as a complete object. At the stage of reification the function is detached from the process that produced it and the concept begins to receive its meaning as a member of certain category. However, if needed, the process level can always be invoked at any time.

3.3 A detailed discussion of the three phases

The first two phases are representative of the operational aspect of mathematical notation and the last phase shows the structural aspect of the mathematical notation. Sfard suggests that the structural conception of a mathematical notation is static whereas the operational conception is dynamic and detailed. Sfard also distinguishes between the words “concept” and “conception”.

> *… the word “concept” (sometimes replaced by “notion”) will be mentioned where a mathematical idea is concerned in its “official” form (Sfard 1991, p. 3).*

Sfard thus emphasises that mathematical concepts have an official, formal side. Ponte (1994) states that conceptions are regarded as a part of human knowledge. Sfard considers conceptions to be the private side of mathematical concepts, which every human being has in his or her mind (concept image):

- *the whole cluster of internal representations and associations evoked by the concept*

- *the concept’s counterpart in the internal, subjective “universe of human knowing”*

- *will be referred to as "conception" (Sfard 1991, p. 3).*

3.4 Functions as processes or objects

Sfard's analysis has shown that abstract notations such as functions can be conceived in two fundamentally different ways: operationally as processes or structurally as objects.

The process concept can also be likened to the action step. An action as defined by Weller, Dubinsky, McDonald, Brown, 2005, p. 5 is "any transformation of objects to obtain other objects". It is viewed as an explicit sequence of actions that are external to the observer. If an object is developed from an action, it is viewed as a set of operations that can be performed repeatedly for different input values. This is confirmed in the interiorization step of Sfard's framework as this occurs when the learner is capable of dealing with operational processes on numerical values, for example, the learners used the idea of variables in order to manipulate an expression and find values. Here they substitute values for x in the expression and find the corresponding y value, thereby enabling them to get the desired ordered pairs. In this stage, processes can be executed without necessarily running through all of the specific steps. For example, a learner is able to find that $f(4) = 10$ mentally when given the function $f(x) = 3x - 2$.

Again, a process is an action that has been interiorized and no longer requires an explicit sequence of steps. Weller et al., p. 5 expand on this by stating it is "characterized by an individual's ability to describe, to reflect upon, or to reverse the steps of a transformation without actually having to perform the steps explicitly".

3.5 Application of the framework

In my analysis and findings chapter, I have applied Sfard's Three-Phase Model of concept formation to determine the levels that learners worked on while learning functions and therefore establishing whether learners understand the function as an object or have a procedural view.

The learners’ development of the function concept was classified into the phases according to these criteria:

Interiorization: the learner could substitute values into a given expression and find the corresponding values, being able to plot the ordered pairs to identify the type of function.

Condensation: the learner deals with variables as with objects but does not see them as objects, the input and output values are of greater importance than the process itself

Reification: the function or family of functions are seen as independent objects and can be acted upon without going through the steps of substitution etc.

Important, too, was that Tall (1989) argues that the three steps using technology can be collapsed, whereas Sfard believes it is hierarchical. This will be expanded upon in the relevant discussion in chapter 7.

Chapter 4 Methodology

4.1 Introduction

According to Ott (1993, p.10), the “purpose of any descriptive study is to collect data about a population which can be organized and summarized in fashions which allow others to make sense of the results”. I chose a qualitative approach for my study as it offered me flexibility and robustness as “qualitative research is multi-method in focus, involving an interpretive, naturalistic approach to its subject matter”. (Denzin & Lincoln, 1994, p. 106). This qualitative approach occurred as observations and interviews which according to Denzin & Lincoln (1994, p.31), “allows one to investigate phenomena that span a period of time, yet it permits examination and interpretation of discrete events and activities within this temporal space”.

4.2 Setting

The study was conducted in a secondary school located in KwaZulu-Natal. This school has an enrolment of approximately 1 200 learners from grade 8 to 12. The school is open to all race groups and is a well-resourced school with learners from average to affluent backgrounds. The school is equipped with the latest technology in Mathematics classrooms and has qualified and experienced educators on staff. The mathematics department at this school received a substantial grant in 2008 from a company which set up a trust to promote Mathematics learning. A huge portion of the money was spent on the installation of interactive Smart Boards, data projectors and laptops. There has also been additional software purchased, namely a computer based tutorial program, Geometer’s Sketchpad and Autograph. Teachers have subsequently been trained in using these Smart Boards to teach.

4.3 Data collection tools

Data collection included:

- Nine lesson observations which were video recorded

- questionnaires filled out by a sample of six learners from the three classes (annexure a)
- questionnaires filled out by a sample of three teachers (annexure b)
- interviews with a sample of three learners (annexure c)
- interviews with a sample of three teachers (annexure d).

4.4 Observations

Lessons were observed and video recorded in the context of a natural situation. As Hoepfl points out,

> *Observation can lead to deeper understandings than interviews alone, because it provides a knowledge of the context in which events occur, and may enable the researcher to see things that participants themselves are not aware of, or that they are unwilling to discuss (1997, p.49).*

Of the three lessons observed, two were grade 11 lessons and one was a grade 10 lesson. The grade 11 lessons observed covered the parabolic function whilst the grade 10 lesson covered the straight line function. I chose lesson observations as a data collection tool because it enabled me to answer research questions one and three, namely:

What kinds of learning environments are created when using Smart Board technology in teaching algebraic functions?

Does visualisation aid in conceptual understanding?

4.5 Questionnaires

The questionnaires were filled out by learners directly after the first lesson on functions had been taught. They were then asked to redo the questionnaire once the function had been completely taught and explored. Learners were asked to rewrite their responses as to how the Smart Board technology aided their learning and understanding of functions.

4.6 Interviews

Whilst my questionnaires were designed to be self-administered, I backed it up with interviews which allowed me greater flexibility and more control of the questions. I also conducted interviews so as to ensure that any misinterpretation of questions or inappropriate responses was then clarified.

I felt that interviews were of vital importance as Bourque & Fielder (1995) point out that participants may not respond well to questionnaire questions which seem too long, too confusing, too sensitive, or are considered not interesting.

After the questionnaires had been completed and the lessons completely taught, three learners were selected and interviewed based on their responses in the questionnaires. They were interviewed in depth in terms of their conceptual understanding of functions and how technology aided in their understanding of the concept.

The interview questions for teachers were divided into four major categories, namely:

1. preparation and planning (what are the main things that teachers consider as they prepare to teach lessons on functions especially when they intend to use Smart Board technology?)

2. sources and uses of teaching tasks (where do teachers get their teaching activities/tasks from and how do they use these tasks, i.e. are the tasks investigative or explorative in nature?)

3. function representations (teachers’ presentation of the concept function and learners visualisation of the function concept using multiple representations), and

4. issues related to using the Smart Board as a teaching tool.

The first two categories, preparation and sources used, helped me to develop some insights into how teachers envision a lesson on functions in which technology is used and what outcomes they might expect. This was then analysed accordingly to determine the role of Smart Board technology aiding teaching and learning.

The latter two categories, function representations and using the Smart Board as a tool, helped to shed some light on the teachers' choices of representation in various contexts. This was then used to determine the kinds of 'understanding' learners are expected to have when dealing with the function concept.

Using Sfard's (1993) framework of process-object and conceptual development (1991), I analysed the teaching and learning of the function concept using the incorporated technology of the Smart Board by specifically discussing how the application of the process-object model to various ways of representing functions in a computer environment was utilised in teaching the function concept.

Both the questionnaires and interviews were selected as tools to enable me to answer research question 2 which was:

How are teachers using Smart Board technology opportunities to promote active and meaningful learning of functions?

4.7 Validity

Durrheim and Wassenaar (1999), state that in any research study it is important to consider the ethical principles of autonomy, non-maleficence and beneficence.

Autonomy refers to the consent of all participants, to voluntarily be a part of the study and participants were given the option to withdraw at any point in time, as can be seen in annexure a. Furthermore Guba & Lincoln (1985) state that there should be a personal and individualized approach to the interview process.. All my findings were discussed with the relevant participants for their input and clarity. It was also to ensure that the results match what they [the participants] intended to say. There was certainly no harm to any of the participants, verbally, socially or emotionally. Finally I hope my research will be of use for further studies in exploring Smart Board usage in Mathematics classrooms. I believe that it might be possible that by engaging the teachers in reflecting on their use of Smart Boards itself, this could inspire some developments in their teaching.

4.8 Paradigms

Of the paradigms considered, constructivism (Guba & Lincoln, 1989) which is often referred to as interpretivism is most closely aligned with the approach of this project. A constructivist epistemology can be defined as one in which “the investigator and the object of investigation are assumed to be interactively linked so that the ‘findings’ are literally created as the investigation proceeds” (Guba & Lincoln, 1989, p. 56).

Chapter 5 Analysis

5.1 Introduction

The analysis is divided into two parts. Firstly, based on the combination of the teachers' own perception of the utility of the Smart Board and the conceptual framework of Sfard, I analysed the classroom interactions as they were captured in the video recordings, supplemented where applicable with snapshots of the Smart Board during lessons. Secondly, I sum up by means of a table to show levels of working with a function as defined by Sfard and if the Smart Board technology facilitated this and/or if visualisation played a role. Lastly, I engage the views of the teachers and learners on the use of the Smart Board in teaching and learning functions to support the discussion that emerges from the analysis.

5.2 Analysis of the video recordings

After carefully viewing each of the lessons in detail, I have decided to code the relevant aspects in detail below.

In this section some examples from the video recordings show the ways in which the Smart Board was used and to what extent it enabled working through the three stages. I conclude by providing an overview of all the observations.

5.2.1 Extract I

The image (figure 2) seen below was used by a teacher to show input and output of a function via the 'function man'. That is, for a specific x value, there exists a specific y value. Learners were taken through examples like the one listed below:

$f(x) = -3x + 4$ therefore $f(2) = -3(2) + 4 = -2$. Here learners were directed through the procedural level of working with a function as certain input values of a function were substituted in the function to produce specific output values. Whilst substituting these values into the given function, learners were working on a procedural level. Once this is done it would enable the learners to use these values to produce ordered pairs which can be plotted, thereby representing the process as a set of objects – the ordered pairs and the corresponding points in the coordinate system.

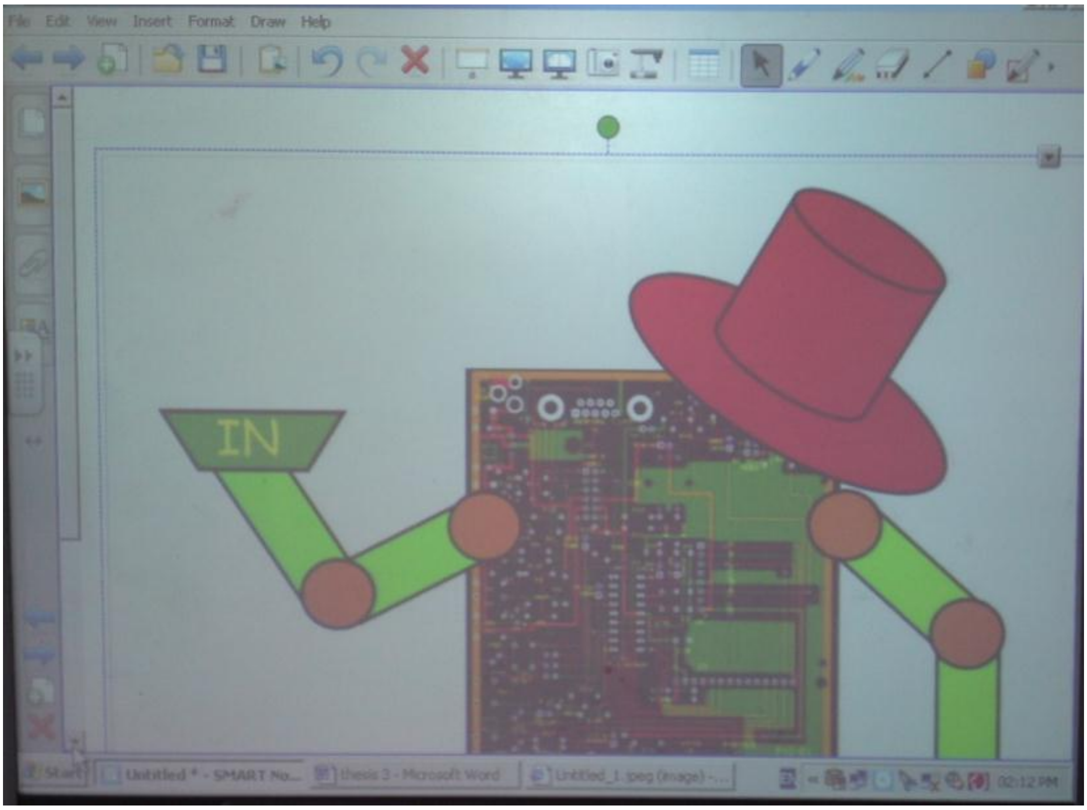

Figure 2

Using these ordered pairs, the teacher then introduced the straight line function as a number of these ordered pairs plotted, which allows learners to have an object view of the graph immediately instead of a process view. At this point it is important to remember that variables are intimately linked with ideas of functions. Here we see that the grade 10 learners had a process conception of a variable as they were able to replace the variable with a value from its domain (or multiple values from the domain one at a time), by doing this we see that they operated on the interiorisation step, when substituting values and moved to the condensation step of the three-phasemodel when the plotted ordered showed the straight line graph.

All in all, the lesson was not in substantive ways different from a class on this topic taught without the Smart Board. The board was used as a colourful and fast way of doing the work, but the content, order and conceptual engagement would not have been substantially different without the Smart Board. However the follow up interviews with teachers highlighted that the colourful and fast capabilities of the board stimulated interest and motivation in the lesson which made teaching the graphs easier.

5.2.2 Extract II

In an introductory lesson in one of the classrooms, the teacher showed the learners how to graph a parabola using the S mart Board parabolic function from the Smart gallery (see figure 3 below). The parabola as a function (or rather, as a family of functions) is defined in one form as $y = ax^2 + bx + c$, and here the teacher used an interactive application that drew the graph by altering values for a, b and c using sliders. It is apparent that the teacher treated the graph as a whole and as an object rather than sketch by plotting the x and y values using a table. According to Sfard this reflects reification as the graph was treated as an object rather than as the results of a process of steps to determine the y value for a particular x value and plotting these.

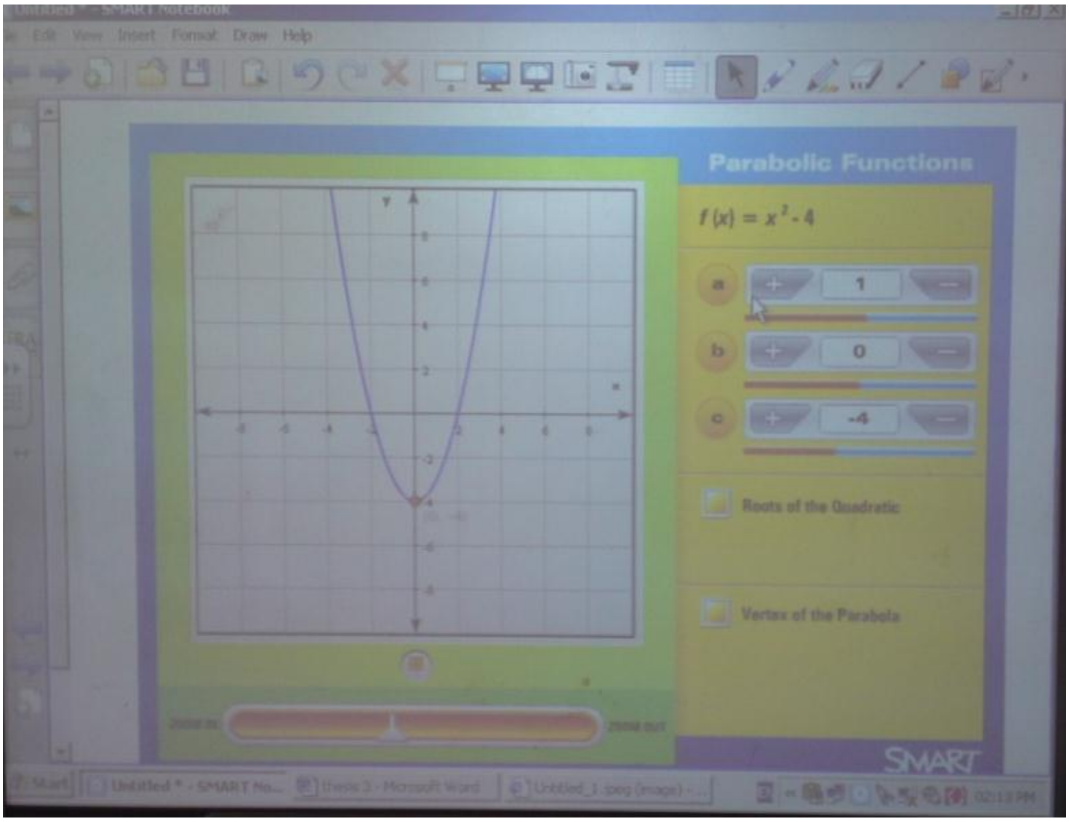

Figure 3

In the following lesson, she went on to show the learners and discuss how to find the axis of symmetry. Learners were also asked to note the turning point from the generated image of the graph. With regard to finding the equation of the axis of symmetry, the teacher asked the learners what symmetry meant. A learner responded with ‘when something is symmetrical you get mirror halves’. The teacher then probed further with ‘What do you think an axis of symmetry does?’ A learner responded with ‘cuts the graph in half’. Interestingly, we notice the procedural focus reflected even in the language; the teacher asks what the axis *does*, the learner responds with a reference to the axis of symmetry as an agent – it ‘cuts’.

The teacher then used the graphical images to show equal distances between symmetrical points. The teacher moved on to ask them to identify the x-intercepts and asked again what the axis of symmetry was. She also asked them to consider the relationship the x-intercepts had with the axis of symmetry. A learner responded with ‘the intercepts are the same distance away from the axis of symmetry.’ The teacher noted the response, and then asked what kind of a line was the axis of symmetry, to which the learner replied ‘a straight line’. The teacher acknowledged this but stated it was a vertical straight line which is defined by x = ’a particular value’. The teacher asked again, “So now how do you think we find the axis of symmetry”, to which a learner replied “find the distance between the x intercepts and halve it”. The teacher formalised this as $x = (x_1 + x_2)/2$. This seems to suggest that the teacher wants the learners to operate on a process level where the x intercept values are substituted into an expression to produce a particular value; learners would thus be working on an interiorisation phase.

A learner then promptly responded with, “So are we only able to find the axis of symmetry once we know what the x intercepts are?” “No, it can also be found using the formula $x = -b/2a$”. This too seems to suggest a procedural view where interiorisation occurs as the teacher once again prompts learners to substitute values into an expression.

The teacher continued, “Now what about the y value of the turning point? Look at the graph again. What do you notice, that this x from the axis of symmetry sits on the graph, so how can we get y?'' "Substitute that x value into the equation to find y", responded a learner.

From this entire extract above it can be argued that the learners moved from a pseudo-object view to a procedural view when substituting for finding the y-value and back to an object view when identifying the axis of symmetry from the diagram.

Next the learners took turns coming up to the Smart Board, "plugging in" their function and playing teacher. The learner at the board got her classmates involved in the different examples and asked for the axis of symmetry to be identified from the graph for each example and to confirm this turning point as an ordered pair on paper using the given expression. If the learners answered incorrectly it was the "teacher's" job to correct them and guide them toward the right answer. Once again, they worked on both an object (identifying the axis of symmetry) and a procedural level (confirming the coordinates of the turning point using an expression written on paper) whilst doing this task. The learners were actively involved and were later asked to create sketches of other graphs on graph paper by analysing shape and other relevant points and to write down the axis of symmetry. Thus we see the condensation phase illustrated, as analysis of graph properties were all that was needed to sketch. Using Sfards framework it can be seen that they were working on a procedural level when they plotted the graphs. This is identified as a procedural level as learners substituted x-values into the given expression and found the corresponding y-values to be able to plot the graph. Thereafter the ordered pairs plotted identified the shape of the graph which allowed the learners to treat the function as an object. The Smart Board thus allowed the teacher and learners a faster way of analysing a number of sketches. This may have been possible without a Smart Board but the interactive capacity of the board which allowed for sliders to change values, in turn allowed the learners to come up to the board and 'play' with graphs not given by the teacher.

From the above we see that this is one class which has utilised the Smart Board's features well. The learners could easily and visibly manipulate the variables in a continuous fashion; which is not possible without the technology. This class is different from what it would have been without the Smart Board, because it would not have been possible to operate on such a level of reification.

5.2.3 Extract III

In the other grade 11 lesson learners were offered the opportunity to move the sliders on the right to change the values of a, b and c in the function of $y = ax^2 + bx + c$ and to note and comment on the effect it had on the graph (see figure 4 below). They were asked to note the shape, vertical/horizontal shifts, change in the turning points and x/y intercepts.

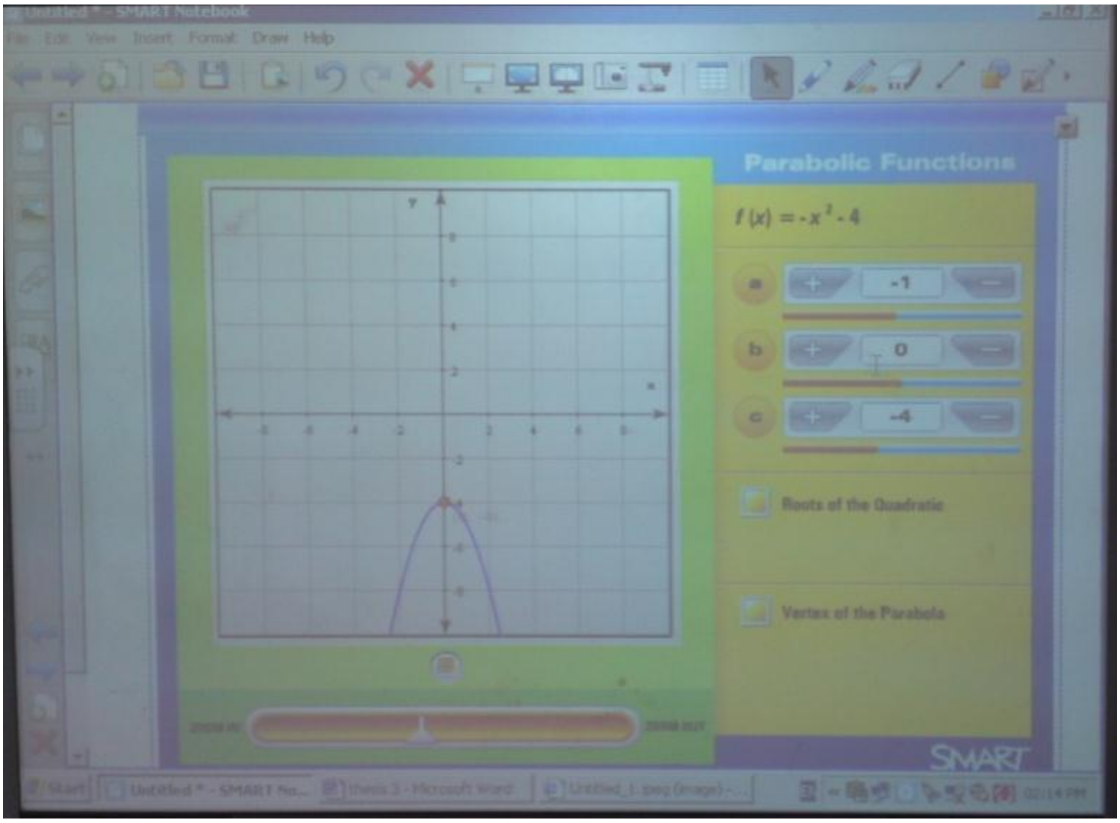

Figure 4

When a learner changes the value of a parameter in the function, the entire graph changes immediately. Thus, the graph is a changeable object. Instead of seeing a function as a process of calculating an output value (y) from an input value (x), or even seeing the function as the collection of all the resulting ordered pairs, the graph on the screen in a sense represents an entire family of functions, $y = ax^2 + bx + c$. This of course could be done using software on a computer linked to a projector, so the Smart Board only adds to this the easily observed actions of the person performing the parameter change.

It must also be noted that by being able to see the effects of the change in parameters of a, b and c, it allows for the linking of representations which facilitates easier reification of the function concept.

This extract too indicates greater use of the utilisation of the Smart Board's investigative, interactive and explorative features. The learners were able to perform parameter changes that manipulated the graphs. This would not have been possible without the technology. Thus the Smart Board capability facilitated easier ways to operate on levels of reification.

5.2.4 Extract IV

In this classroom the parabola in the form $y = a (x + p)^2 + q$ was given and without sketching learners were introduced to the parabola in the following manner:

Teacher: Girls many of us are familiar with the effect of q on a graph. You did this last year. Tell me what you remember?

Learner1: Miss, that's easy- the graph moves.

Teacher: How did the graph move?

Learner2: It moved up or down. If q was positive it moved up and if q was negative it moved down.

Teacher: Yes, that's correct, it moved up or down q units. Now girls what does p do to the graph? Lets take a look at two equations, $y = (x + 3)^2 + 4$ *and* $y = (x - 3)^2 + 4$*. Discuss at your desks what you think will happen and then confirm your answer with a sketch. This half of the class sketch the first graph, the others complete the second graph.*

The teacher's instruction to the learners was: *discuss at your desks what you think will happen and then confirm your answer with a sketch.* From this it is evident that the teacher encourages the learners to consider the graph as a whole; she also asks that they sketch their answer and not plot it carefully, so they operate on the function as an object.

From the response of learner 2 we notice that if the function is treated this way, that is moving up or down, it is considered an object. If the resultant graph is found by substituting values for q, we see that q has an effect of ‘operating’ on the values/points one ‘at a time’. This may be seen as procedural but just as easily the learners could take $y = f(x) + q$ which could be seen as operating on the function as an object. So this kind of understanding means that the learners are working on two levels of understanding the function concept - a pseudo-object level.

Once this activity was completed by the learners, the teacher graphed this on the Smart Board. The teacher asked them to compare their sketches to what she had just drawn using the Smart application. It turned out that it was contrary to what some learners had expected. The teacher then explored why. She looked at their conceptual understanding of the horizontal shifts as ‘imagined’ by the learners and then she used the ability of the interactive board to illustrate and demonstrate to them the direction of the shift. This allowed them to visualise accurately this type of shift without having to tediously plot these graphs themselves. At this point it is acknowledged that the hypothesising and checking could as easily have taken place in a classroom without the technology, but the process of plotting is speeded up, most certainly when engaging with an entire class at a time. This then reinforces treating the function as an object or a family of objects and eliminates getting ‘bogged down’ by plotting points.

It is important to note that the time ‘saved’ was not in any way used to deepen the mathematics to include proofs or reasoning around the result, though this might have linked the process and concept aspects of functions - unlike the work of Blomhøj (2003), where the learners are encouraged to first conjecture and then prove using Geometer’s Sketchpad. So while the learners may strengthen their conceptual knowledge of graphs, they are still learning this in a way that appears to be directed towards being able to master the skill of drawing graphs, not forming conjectures and proving them, nor understanding why they operate in such ways. Although this is not what my study is primarily analysing, it is interesting to note that by *not* engaging the learners in explanations of 'why', they are still kept from the regulating principles of the discipline (Dowling, 1993). They do, but they'll never quite know why they do it! Perhaps the ability of an IWB will allow such occurrences to be minimised as teachers could use its potential to be a more explorative and meaningful teaching tool.

In this extract, it appears that the teacher just used the Smart Board as a fast way to demonstrate/show what she would otherwise have had to do with several graphs on the board in a longer time frame.

5.2.5 Extract V

At this point I want to bring in what happened in the grade 10 classroom when dealing with shifts. Learners were asked to consider the following two graphs $y = x^2$ and $y = (x - 4)^2$ and asked to sketch what these functions would look like without actually plotting the ordered pairs. Not surprisingly many learners imagined that the graph of $y = (x - 4)^2$ would move 4 units to the left. The learners treated the graphs as objects as the whole graph was shifted without physically plotting points. Then the teacher drew both these graphs on the Smart Board, which greatly speeds up the process compared to classrooms without such facilities.

Learners were surprised to see the second graph shift 4 units to the right; this then led to an interaction which is examined in closer detail below:

Learner: *But the equation uses -4, so x is decreasing therefore it must move to the left and not the right, maybe the program is wrong.*

Teacher: *Okay before I explain this, I want you all to plot a few ordered pairs, using the table method. And then resketch.*

Learners went on to do this and came up with a shift 4 units to the right.

Learner: *Okay, so I was wrong, but why should it move right and not left when there is a minus sign?*

Teacher: *This is called the rule of horizontal translation, so here the graph shifts 4 units to the right and many of you can see that from plotting your ordered pairs.*

Learner: *So do we need to plot a table of values every time or is there a short way?*

Teacher: *Well according to the horizontal rule, the shift will happen in the opposite direction.*

Learner: *So what you are saying is that if it is minus, it moves right and if it is plus then it moves left.*

Teacher: *Yes, that is a way of remembering it. But remember it must be in brackets and not like this* $y = x^2 - 4$.

Learner: *Now, we know that, if you* ***only*** *add or subtract at the end the graphs just move up or down.*

The teacher went on to allow a few learners to come up to the board and draw some quadratic graphs with other horizontal shifts to 'cement' their understanding of horizontal shifts.

From the above interaction we see that the learner asks a question which treats the function as an object while the teacher keeps referring to the process of plotting a function and to a rule, which she does not justify, other than by examples. This seems to be a missed learning opportunity as the teacher could have used the potential of the IWB for further explorations and investigations, which would encourage learners to move from the basic level of procedural understanding to object and reification levels of working with the function concept. Instead, the teacher opted to keep the lesson simple and on a track that was easily handled - which would make complete sense in a class without the same opportunities to explore as that provided by IWB technology.

Thereafter learners used the white board in conjunction with Autograph to draw a few more graphs with horizontal shifts and many accepted the directional move because they were able to see the accurate graphs which were drawn. The teacher then used an interactive activity that allowed the learners to change sliders for p and q in the equation: $y = (x^2 + p) + q$. This clearly cemented the effects of p and q for the learners. It must be acknowledged here that the learners started from treating the function on a level as an object, used the process level to check their conjectures and then concluded on an object level again. In other words, it seems that the lesson is working within an object understanding of functions with the process aspect implied as already in place.

5.2.6 Extract VI

In one of the classrooms the teacher used an example to show how these functions can be applied to real life situations. To do this she used an example of car headlights.

> *A car headlight is an example of a Paraboloid of Revolution - taking a parabola and rotating it about its axis of symmetry. The smooth inner surface of the headlight is a glass reflector upon which bright aluminium has been deposited. This part is a powerful reflector.*
>
> *A parabolic reflector has the property that if a light source is placed at the focus of the reflector, the light rays will reflect from the mirror as rays parallel to the axis. This is used in auto headlights to give an intense concentrated beam of light.*

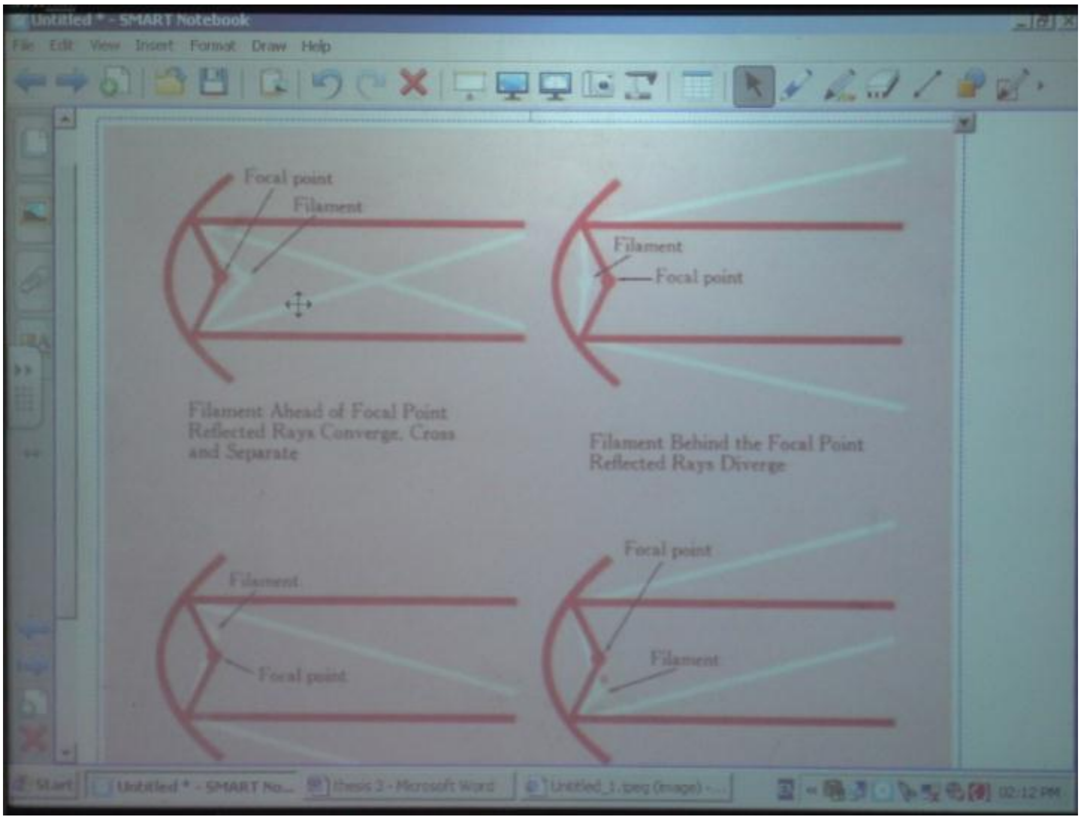

Figure 5a

For safer night driving, we do not want all the light rays to be parallel to the axis. Some light must be aimed far down the road, to the side, upward for signs or bridges. So we offset the filament from the focus and change the beam entirely.

In most of today's cars, a 4 Lamp System is used - 2 filament sealed beam units in 2 lamps. The position of the filaments accomplishes most of the desired illumination patterns. The rest is taken care of by special lenses which contain prisms to bend the light rays.

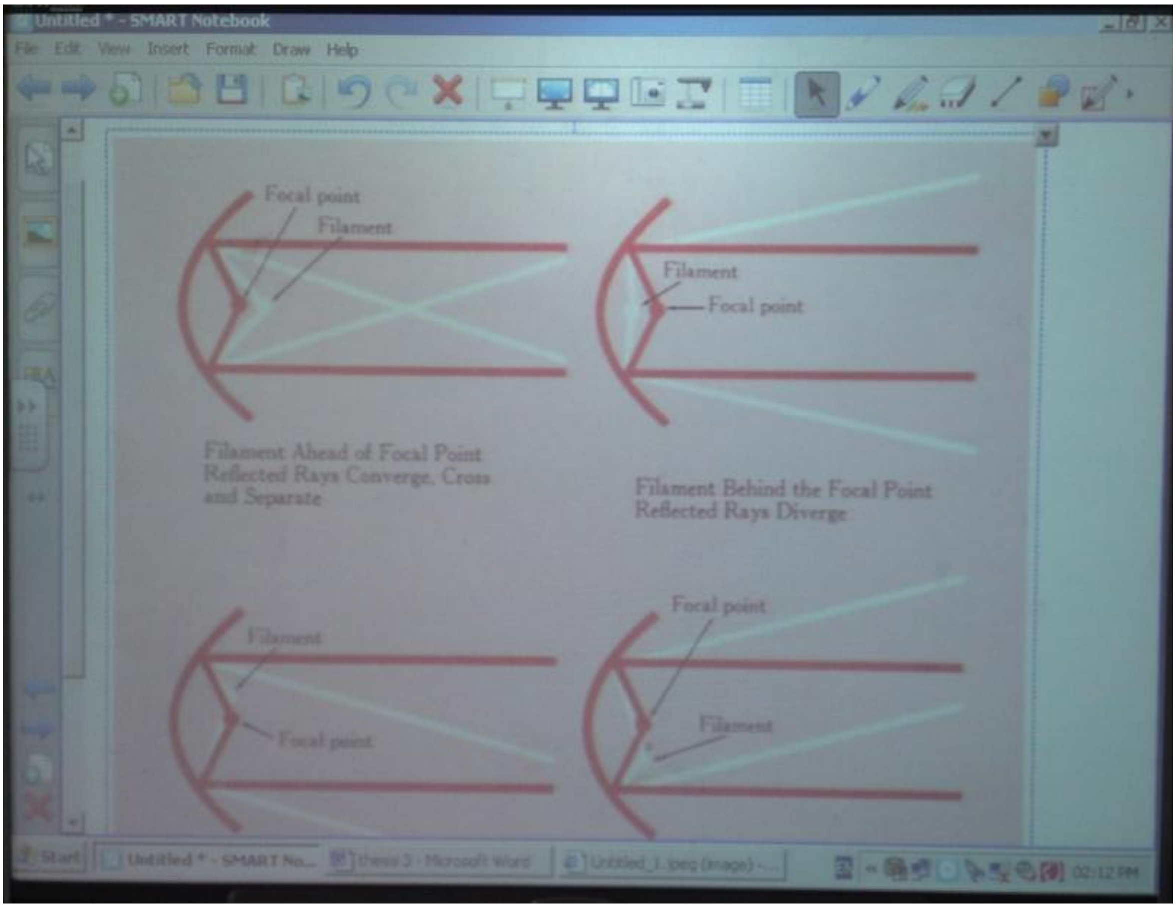

Figure 5b

Here it is relevant to note from the above figures that the teacher used a sketch of a real life application which could have been provided in other ways; this was not an interactive or manipulative application. According to the literature reviewed as discussed in chapter 2, the Smart Board allows you to set this up in a way which can be manipulated in three dimensions using the touch screen, showing the rays being reflected according to the laws of physics so it seems that in this lesson, the Smart Board is again just operating as a fancy screen. The teacher did not utilize its potential fully thereby once again missing an explorative learning opportunity. The Smart Board does, however, allow for interactive applications as well as being able to connect to the Internet for applications so whilst this was not done here, it does have that ability which again seems to be a missed learning opportunity here.

Again, from this extract it emerges that the Smart Board was only utilised as a fancy way of showing the same thing which otherwise would have had to be drawn. Its interactive capabilities would certainly have been relevant in allowing learners to investigate and explore the headlight example in more depth. However such capabilities of the board was not utilised.

5.2.7 Extract VII

After the initial introductory lesson, the learners in this classroom were given a number of different sketches which then had a number of equations listed. Some examples are shown below (see figures 6a and 6b):

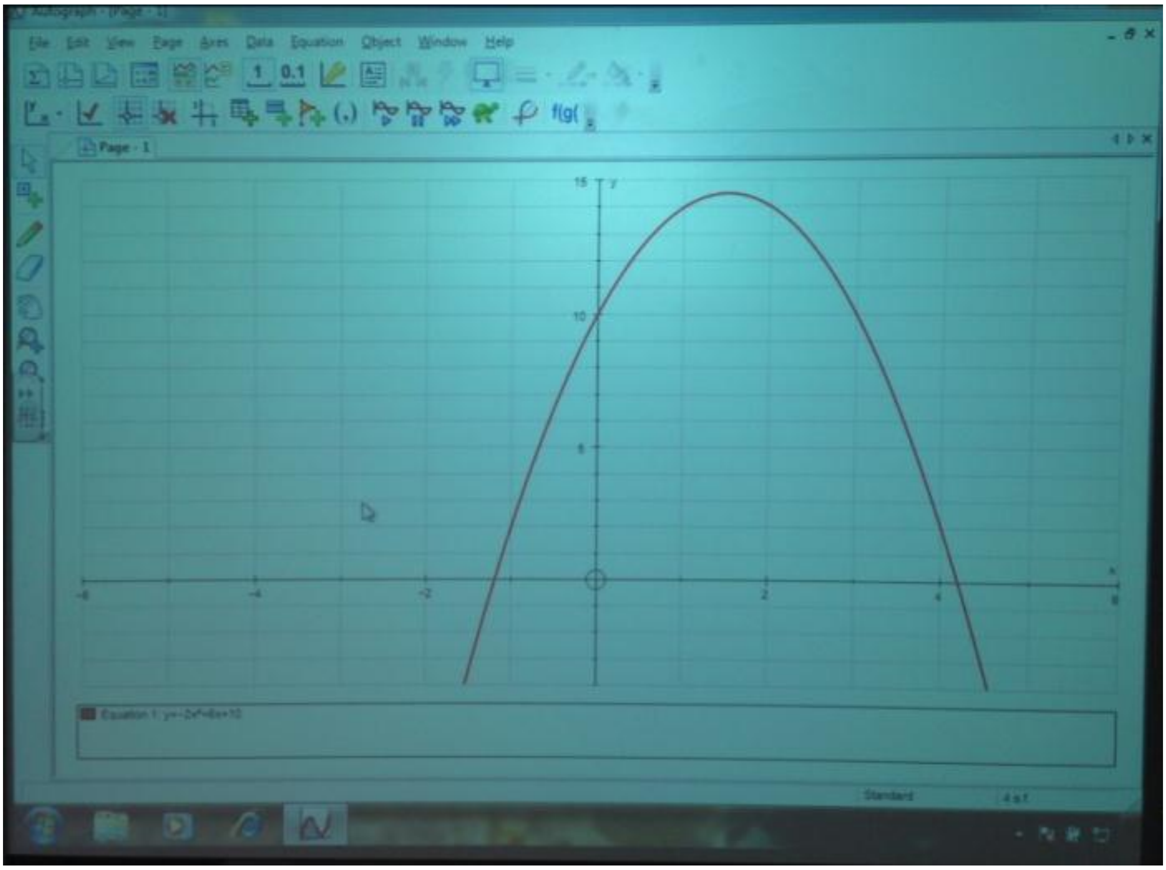

Figure 6a

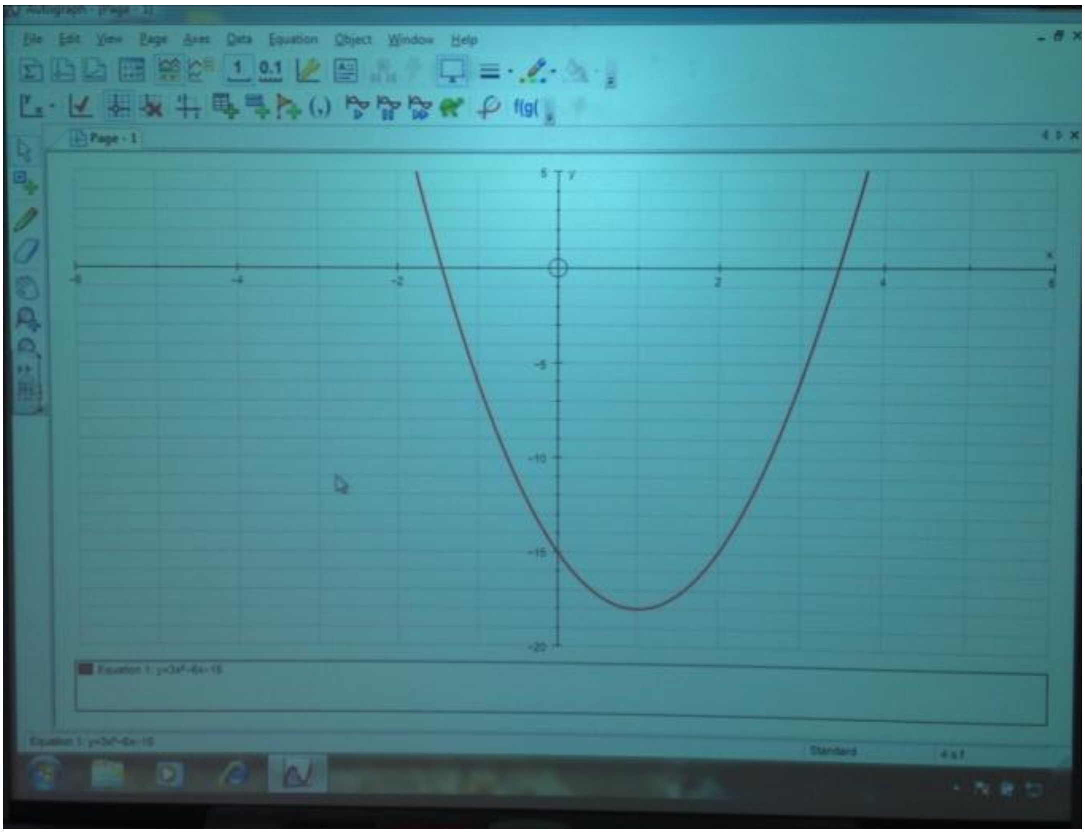

Figure 6b

The learners' task was to match the correct quadratic equation to the matching parabola. By engaging the learners in a task of this nature, the teacher enables the learners to develop a stronger object view of a function, as they were looking for properties to associate with each sketch and equation. The learners were encouraged to check assumptions on paper using the necessary calculations, again allowing for a process approach when working with a function – but then using this to consider the function as a whole. In that respect, this directed the learners to reify their conceptions. As the learners were trying to match the graph to the equation, the following questions from the teacher helped guide their thinking:

1. Can you identify the roots or solutions of the equation based on the graph?

2. Can you write the equation (expression) in factorised form?

Learners were able to 'look' and pick out values where the graph cut the x-axis. By doing this, the linking of representations helped learners to see roots as a property of function as a whole which could then be extended to a family of functions. This again can be done without a Smart Board and with a number of sketches instead, but the Smart Board facilitates time saving and various images that can be sketched in the classroom within a short time frame.

This shows the teacher enabling learners to move from an ‘interiorisation phase’ to a ‘condensation phase’. In these phases, processes were executed without running through all of the specific steps. For example, the Smart Board image enabled learners to pick out intercepts, calculate the value of the function at these points, and derive an expression in factored form. (Weller et al.,2005, p. 5) characterises a process “by an individual's ability to describe, to reflect upon, or to reverse the steps of a transformation without actually having to perform the steps explicitly”. So here learners analysed relevant information, linked representations, picked out necessary points and were able to give the equation in factored form. This then also allows for reification which strengthens the function concept as they work with it as an object.

From this extract it is evident that the teacher had a strong understanding of the function concept as she tried to foster the movement of conceptual understanding of functions by promoting movement to the condensation or reification levels. Although it could just as easily have been done in a classroom without the Smart Board technology, the teacher was able to use it to show more sketches, write down important points and link the learners ideas with the saving and recall feature of the boards.

5.2.8 Extract VIII

In a follow up lesson, learners had the chance to play on the Smart Board with different sketches of relations and functions. The learner then had a chance to determine which were functions and were able to physically see the number of y-values attached to each x-value. Sfard’s process of reification was encouraged as they visualised, speeding up the normal teaching-learning process.

The teacher went on to an interactive smart tool that drew graphs, and learners had to analyze the effect of changing the variables a, b and c in the parabolic function $y = ax^2 + bx + c$ (similar to what was seen in the grade 11 class). Learners had to develop a mental picture, some put it down on paper, and then the teacher confirmed/corrected their images

Figure 6a below shows the original function: $y = - x^2 - 4$

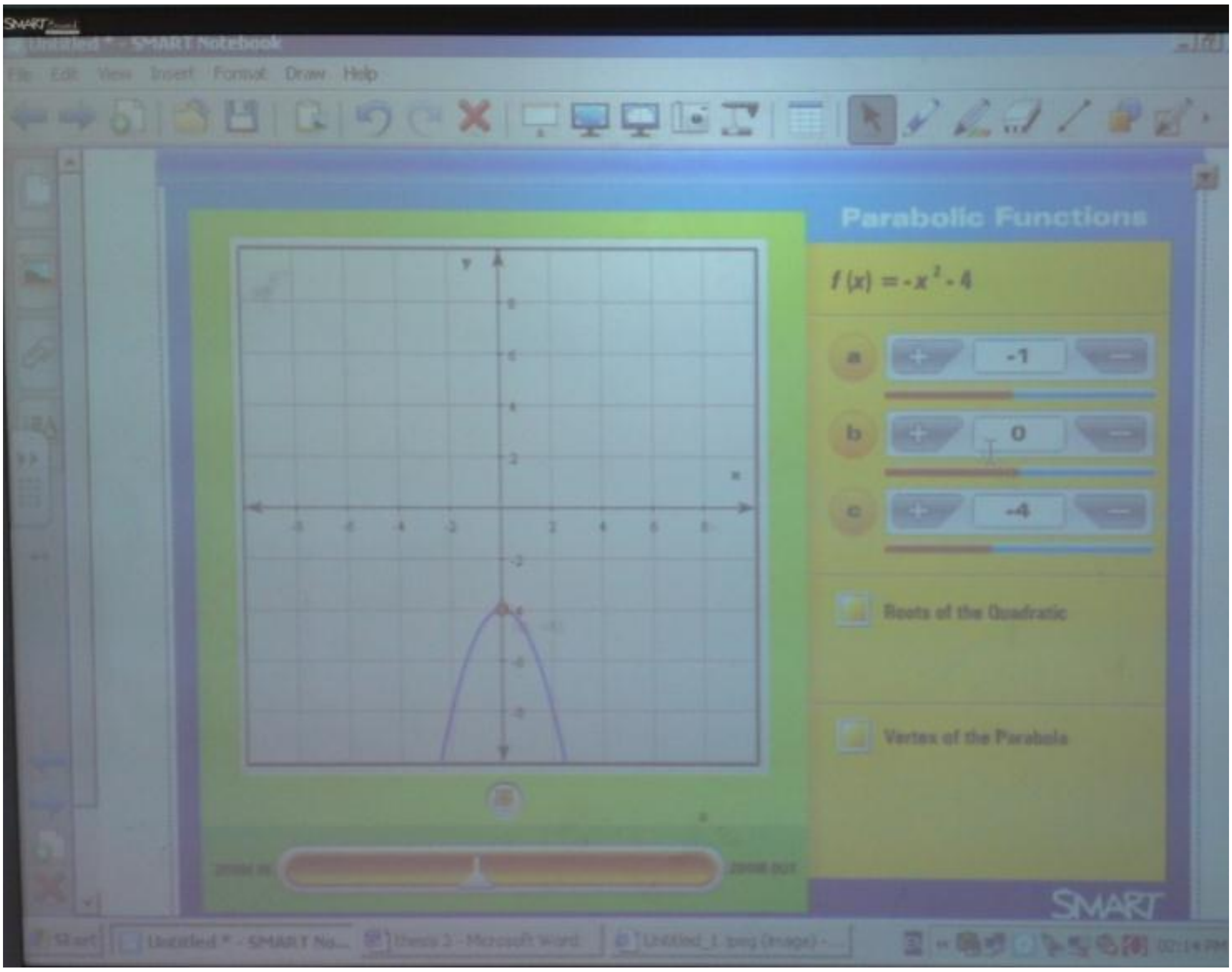

Figure 7a

By moving sliders which changed the values of a, b and c (here a = -1, b = 0 and c = -4) learners were able to see the physical changes to the graphs. See figure 6b below.

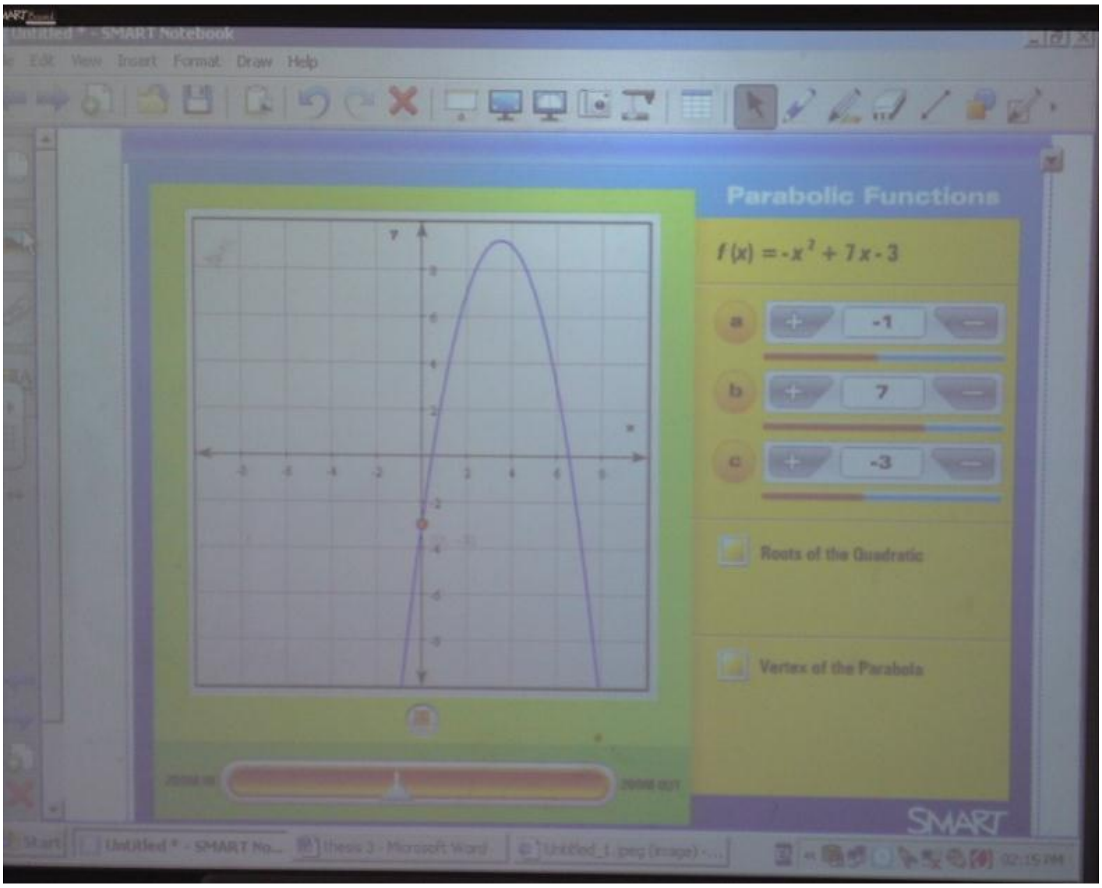

Figure 7b

Here the value of a remains as -1 so the learners observed the shape (direction) of the graph did not change. However b = 7 and c = -3 shows the original graph having shifted right and up.

Having completed the above activities, learners were taken on to analysing equations and identifying the family of functions associated with them. Learners were once again asked to work on paper and were able to give a rough sketch to show its representation and discuss relevant properties. For example, learners were able to say that the equation $y = 2x^2 + 5x - 9$ is a parabola that is “facing upwards” and will cut the y axis at -9, just by looking at its equation. We can see condensation and reification has taken place, as the learners did not go through the process of substituting values to plot ordered pairs and then say to which family of functions it belongs to but were immediately able to treat the function as an object with certain properties.

This extract too indicates greater use of the utilisation of the Smart Board’s investigative, interactive and explorative features. The learners were able to perform parameter changes that manipulated the graphs; this would not have been possible without the technology. It must also be noted that being able to see the effects of the change in parameters of a, b and c allows for the linking of representations which facilitates easier reification of the function concept.

Thus the Smart Board capability facilitated easier ways to operate on levels of reification. It must also be noted that much of the lesson in this class showed movement between the interiorization, condensation and reification levels of Sfard's framework thereby grounding the function concept to learners.

5.2.9 Extract VIX

In a final lesson in the grade 11 class we see the ability of the Smart Board as it is used to foster movement between all three levels of Sfard's framework thereby strengthening the learners' concept of a function. In the initial lessons, the learners displayed a process conception of a variable as they were able to view the variable as able to take on multiple values from its domain simultaneously when doing the quadratic function. Condensation occurred when the learners were not merely concerned with the act of substituting a value for the variable and as the lessons progressed they were able to recognize patterns that emerged in basic functional relationships. Condensation is explained as the learner having "developed the ability to use mapping as a whole, without looking into its specific values. Eventually, he or she can investigate functions, draw their graphs, and combine couples of functions" Sfard(1991). Here this was evident when the learners knew that the graph of $p(x) = 3x^2 + 8x - 4$ passes through (0; -4) and has two real and rational roots as seen in figure 7 below:

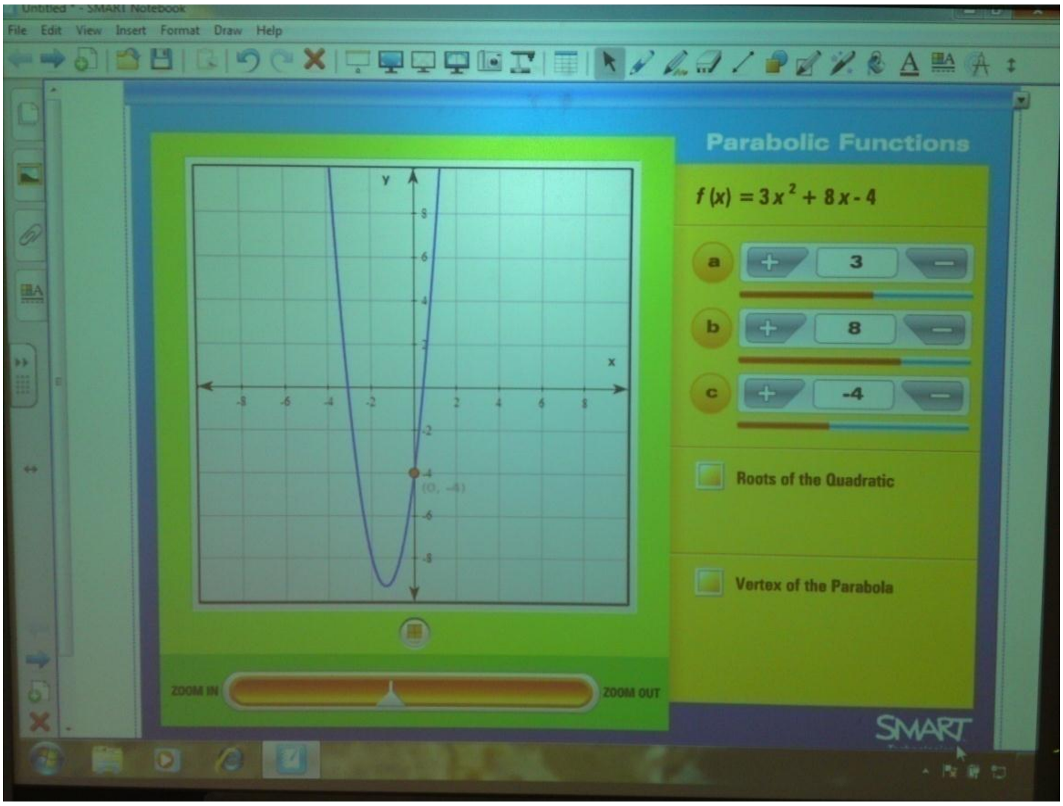

Figure 8a

The interactive capacity of the board then facilitated the changing of the function and enabling the learner to work out the vertex and the roots as shown in the following two figures for the above example (see figures 7b and 7c) below:

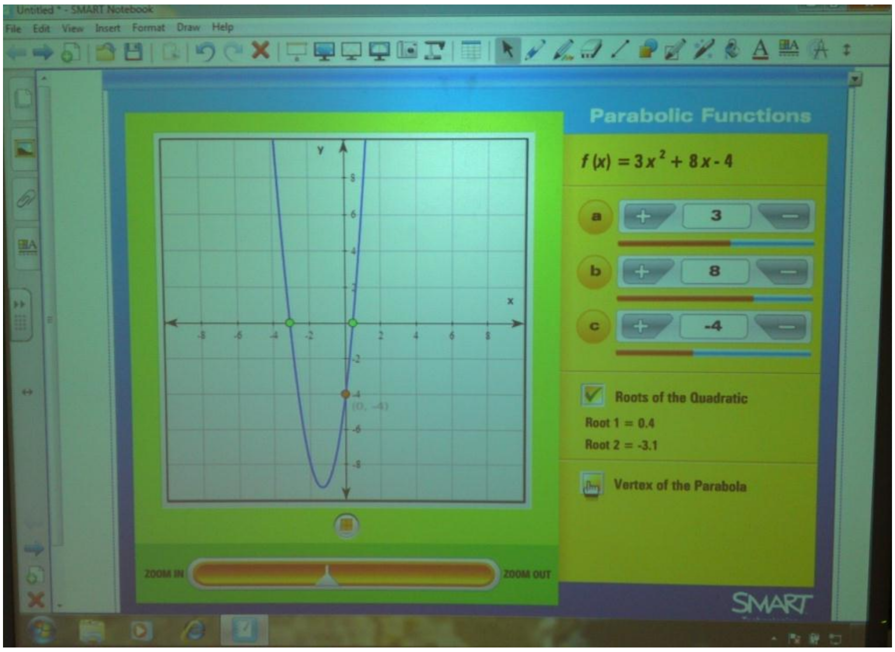

Figure 8b: (showing calculation of roots)

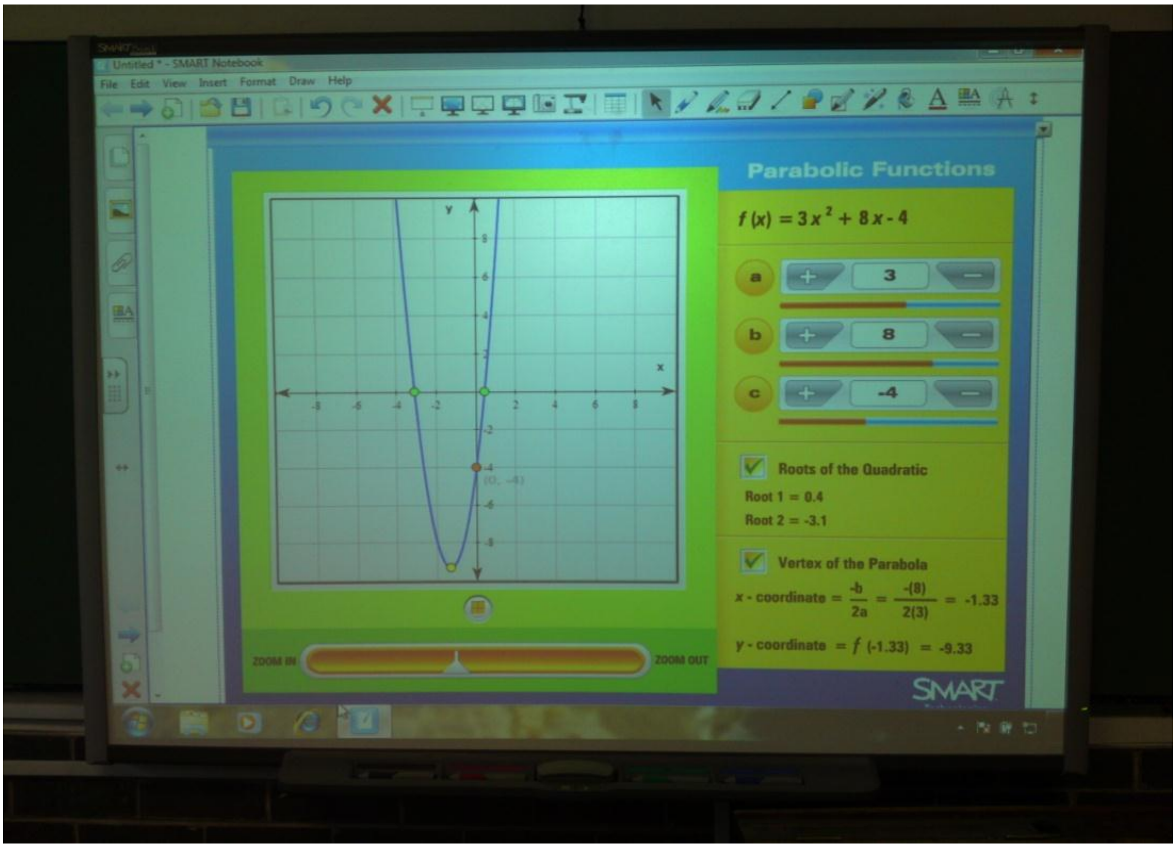

Figure 8c: (showing calculation of turning point)

5.3 Object/process duality of functions

As pointed out in Sfard's framework in chapter 4 that the function can be conceived in two fundamentally different ways: structurally as objects and operationally as processes. From the above analysis it seems that the duality of functions, i.e. as an object or as a process, can be facilitated by the use of technology such as Smart Boards. Different representations through visualisation have enabled learners to strengthen their function concept as they appear to work more on an object level of understanding than a procedural level. However, it is obviously an object level and not a pseudo-object level, as the learners shift into a process conception when testing a hypothesis, etc.

Thorough analysis of the stages in concept formation leads us to acknowledge that the transition from process to object is indeed difficult and of a higher level and, according to Sfard, can be accomplished in 3 steps, interiorisation, condensation and reification. However this analysis has shown that visualisation plays a role too and reinforces the idea of making a function a whole, meaning that the learners are encouraged to have an object view rather than a process view. This may well support the view of Tall (1989) as indicated in my literature review (see p. 29), that the stages in understanding a function concept may well be collapsed, as learners dealt with the function as an object initially before going through the hierarchical stages as described by Sfard. However, it is not possible to say with any certainty based on my observations, as the learners either worked in a way which demonstrated that they had already developed an object conception, or were engaged in tasks which facilitated condensation and reification by linking the object and process aspects.

Similarly, White (2009) argues "that appropriately structured learning experiences might support the development of structural conceptions independently of operational fluency". This is attributed to the fact that learners are exposed to the function concept in a technological environment which features multiple linked representations of the function concept. Indeed, different representations of functions afford different perspectives (as seen in the above extracts: an object view when looking at a graph and a process view when working with an equation or an expression). Studies by Schwartz & Yerushalmy (1992) found that representations of functions using symbolic notation tended to emphasize process aspects whilst other representations such as graphs highlighted the object-like properties of the function . This was indeed evident when analysing the above classroom extracts. Many of the approaches used showed how the Smart Board facilitated seeing and treating the function as a whole or an object with an underlying process which can be 'evoked' when needed, thus facilitating working on a structural rather than an operational level. Most teachers also used the Smart Board to operate on a family of functions which was generally used to reinforce an object-process link and a 'family' view.

To show the object-process duality more clearly and how the use of the Smart Board facilitated this, I have summarised the extracts by means of a table. I have indicated if understanding or working on the different levels as indicated by Sfard's three-phase model were present.

5.4 Summary

EXTRACT	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Process view	√				√			√	√
Object view		√	√	√	√		√	√	√
Interiorisation	√	√	√					√	√
Condensation	√	√	√				√	√	√
Reification		√	√				√	√	
Possible without Smart Board technology	Yes	No	No	Yes	Yes	Yes	No	No	No
Visualisation played a role	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

From the above table it is evident that the use of the Smart Board did facilitate the understanding of the function concept as learners were taken through the three stages of Sfard's model. Whilst four out of the nine lesson extracts would have been possible to do without a Smart Board, five lesson extracts show the potential that a Smart Board offers, especially in terms of being able to see 'difficult to grasp' concepts, emphasising the role of visualisation in aiding understanding of concepts.

Quick and definite conclusions about connections drawn between different and multiple forms of representations should not readily be made, suggests Thompson (1994). White (2009) concurs and explains that while a teacher might recognise a mathematical object such as a function being represented by a table or graph, a learner considering the same representation will not necessarily recognize the same object. Supporting this is Thompson & Sfard (1994), who maintain that learners may not be able to recognise the object and its properties across other representations. Thompson suggests that instead of enabling the learners to have an object conception of a function by highlighting the properties of such objects across different domains, attempts to engage learners with multiple representations of functions may simply leave them with disconnected conceptions of those various representations. In this way no continuity of the object is maintained or recognised. Thompson proposes that if teachers want to use activities that involve multiple representations, it should emphasise specific characteristics across the different representational forms rather than focusing on the abstract object in itself. In light of this the Smart Board, may in the future, be very useful as it has the potential to strengthen concept formation through linking representations as seen in extracts ii, iii, iv, vii, viii and ix as the learners displayed confidence in handling the functions as objects.

The interviews of learners and teachers are not analysed here but are referred to in my discussion of the results, to support the analysis of the videos.

Chapter 6 Findings

In the following I will draw from the analysis in order to answer the research questions.

6.1 What kinds of learning environments are created?

6.1.1 Learner interest and motivation

Research question one asked about the kinds of learning environments created when using Smart Boards in teaching algebraic functions. In this section, I will discuss these aspects of learning environments, namely: learner interest and motivation.

The “dynamic learning atmosphere” (Fernandez and Luftglass, 2003) created by the use of IWBs has led many (Bell, 2000, Ferl, 2003; Knight, 2003; Tate, 2002) to conclude that the greatest impact of the board’s use in a classroom is upon learner interest and motivation. Similar findings appear in this research study.

Learners displayed increased levels of motivation and enjoyed the interaction that the board offered as was seen in the above extracts. Learners paid greater attention, contributed more, offered to participate more, and to a great extent, enjoyed the Mathematics classroom that made use of the IWB.

From the analysis, and considering the responses of teachers and the learners, it can be concluded that the Smart Board created a positive and vibrant learning environment where learners were motivated and excited about their learning.

When interviewed, teachers commented that there was a noticeable improvement in learners’ attitudes. Two out of the three teachers noted that the most significant impact was the attention and motivation the learners had when working with the board. During the course of the lesson, there was an increase in levels of enthusiasm and the learners continued to want to complete most tasks using the board. The work ethic and interaction within the group improved, sustaining the learners’ motivation which in turn increased the level of participation of all learners. Although it is acknowledged that this could have been achieved in other ways, learners too said they were more motivated than before and said that their learning had benefited from the use of the IWB. They liked the idea of being able to *see* what happened when functions were manipulated instead of being *told* what to expect and what would happen, as shown in some of the above-mentioned extracts.

One learner, emphatically, stated “I was more interested and paid attention to the images”. It can therefore be argued that the teacher’s ability to use the board as a teaching tool led to the learner engaging with the usefulness of the board, causing greater positive attitudes towards the learning and teaching of Mathematics. The way in which information is gathered and presented, particularly through the use of colour, movement or involving animation of some sort is seen by the learners as exciting and therefore motivating. This then promotes increased levels of concentration and attention amongst learners.

6.1.2 Presentation and facilitation opportunities

From the videos and interviews discussed it is evident that the IWB enabled the teacher to provide input to the class as a whole. It also facilitated being able to go back and review previous lessons. The teachers were able to present from the front, whilst still observing the learners. When interviewed this was emphasised as they felt that they were in a better position to observe learners' responses. Accordingly the Smart Board was then used to reinforce certain ideas or to correct misconceptions by allowing learners to review concepts and examples of certain graph application exercises. Whilst it is acknowledged that teachers often deliver lessons from the front of a classroom, and are often in a position that allows them to observe learners’ responses, the use of the Smart Board allowed them access to a fast way of doing additional representations of a concept as shown in extracts iv and vi.

From this it can then be argued that IWBs have the potential to be effective tools for initiating, facilitating and providing a platform for support throughout the learning process. particularly where pupil participation and use of the board is encouraged. An important finding is that there is correlation between

IWBs and learners' views of learning, with visual learning being particularly significant as shown in extract vii.

Another learner noted, "It was fun and made the lesson more interesting as it came to life". Whilst another said, "We were able to see real life applications of functions, so it wasn't just a useless section that we had to learn". However, only one lesson actually contained such applications (see extract vi).

All three teachers who were video recorded and later interviewed said that they enjoyed having the attention of their learners and felt that learners benefited from visualisation. One went on to say "Learners certainly were more active participants and tended to ask more questions". Esarte-Sarries and Paterson (2003) refer to such broad learner participation as surface features of interactive teaching. However of importance here is that being able to view the function across representations prompted some learners to explore and ask further questions because they could engage with what was taught. This is shown in extract vii where a number of questions emerged.

It is important to remember the type of learning environment created using a Smart Board could be highly explorative and investigative. However teachers did not always utilise it in such ways as discussed in the analysis of extracts iv, v and vi. Upon interviewing one teacher, post lesson, it emerged that the teacher concerned did not feel confident enough at this stage to use the board in such ways. Teacher x commented: "I know that I did not do justice to the capabilities of the board. I think I would need more time and training to realise its full potential myself".

The investigative nature of the Smart Board too, was not fully utilised. As shown in extract iii, the teacher asked the learners to make a conjecture and to test them with hand drawn sketches. It must however be acknowledged that despite the potential of such technology in classrooms, it is pedagogically sound and essential that teachers do ask learners to work on paper at times since learners are tested on paper when they write exams such as the grade 12 national Senior certificate exam. So, although the use of a Smart Board aids faster teaching and learning since it has the ability to cover numerous examples in a shorter time span, credit must be given to teachers for insistence on written individual work to ensure that learners are able to answer questions as required in a written examination.

6.2 Teachers' use of the Smart Board

Research question 2 asked; "How are teachers using Smart Board technology opportunities to promote active and meaningful learning of functions?"

I will discuss this in two sections by analysing comments from the interviews made by teachers and learners.

Here I note the comment by a learner: "It was fun and made the lesson more interesting as it came to life". Learners were able to see that Mathematics is dynamic and can be embodied in real life contexts as in the illustration of the car headlights as seen in extract 6. (See figures 3a and 3b).

The interactivity of the board as a teaching tool must be acknowledged, as shown in extracts ii, iii, viii and ix which enabled learners to: Identify functions and contrast their properties from tables, graphs, or equations. It also allowed them to pick out important points and write the equation in different forms.

Of relevance here, is that the representations in different forms are linked. A change in one leads to a change in the other which then supports working on an object or family of functions level. Plotting, contrasting and comparing properties facilitated the movement between all three levels of Sfard's framework. This was illustrated in almost all the extracts analysed in the previous chapter.

Tall (1989) argues that the use of technology allows the process to lead to an object so quickly that the normal three stages are collapsed. As shown in extract viii, learners saw the effects of parameter changes and representations of a family of functions thereby forming an object view rather than a process view.

This again links in with accelerated learning techniques involving visual rather than verbal instruction which is made possible only by technology.

It was also interesting to note the following examples of 'missed opportunities':

a. Both teachers relied on explaining the types of shifts as a horizontal rule or a vertical rule. Further they used the board to visually demonstrate this with different examples. However, learners were not provided with a mathematical reason or description as to how these shifts occur and why they work in the 'opposite direction'.

b. Learners did not question the teacher any further about why the rule worked; they saw the graphs and mostly accepted a quick and short way of remembering the behaviour of the shift rather than trying to ask why or understand why it works that way.

c. One teacher had asked the learners to plot ordered pairs using a table of values to confirm their sketch, perhaps suggesting that teachers themselves need to examine their methods of explaining concepts to learners.

d. This in turn shows that perhaps some teachers themselves do not foster the movement of conceptual understanding of functions to move to the condensation or reification levels as they themselves, perhaps, do not always operate on these levels.

Thus it may seem that the teachers at times missed a learning opportunity for active and meaningful learning, where the potential of an interactive teaching tool could have been utilised to a greater extent. My personal impression is that the teachers in many ways have adjusted an approach they have used previously, without considering what other aspects of mathematics could have been brought in, now that there is more time for exploration, conjecturing, verifying/refuting, and explaining.

6.3 Visualisation

My third research question dealt with determining whether visualisation aids in concept formation. Before answering that question, specific mention of and discussion must be made about the essence of visualisation as a tool to aid conceptual understanding. There are many advantages to using Smart Board technology as discussed in the literature review in chapter 2 and shown in the analysis in chapter 6. In the interviews teachers claimed that the use of the Smart Board helped learners to visualize certain complex phenomena that helped in their conceptual understanding.

The theme of visualisation aiding conceptual understanding is of vital importance and must be expanded upon, and I refer to these comments of learners being able to visualise complex phenomena:

> *I liked being able to see what happens to the graphs rather than being told what would happen as we often are.*

> *I can actually see the way the values of a and c in the parabola equation* $y = ax^2 + bx + c$ *changes the graph. Now I know, a controls the shape or direction of the graph.*

Learners often have a difficult time in visualising concepts and struggle to grasp information that is presented verbally or in text form. Visualisation on the other hand is increasingly being accepted as an important aspect of mathematical reasoning.Zimmerman and Cunningham, 1991; Hershkowitz, Arcavi and Bruckheimer, 2001; Arcavi, 2003 state that visualisation is recognised as being a central component in mathematical activity. Studies by Malabar&Pountney (2002) have revealed that 'activities encouraging the construction of images can greatly enhance mathematics learning'. Indeed, this study revealed that Smart Board technology, assumed a very powerful and influential role in visual stimulation. Teachers stated that the IWB was great for demonstrations and as such proved to contribute to the depth of learners' understanding. This is indeed evident in the way the learners appear to confidently handle functions on an object rather than a procedural level as shown in extracts ii, iii, vii, viii and ix.

Zimmerman and Cunningham (1991) state that mathematical visualisation is not to equivalent to 'math appreciation through pictures’. They argue that images are simply a substitute for understanding on a superficial level. Rather, as discussed by Elliot (2000) of relevance is that visualisation supplies depth and meaning to understanding. It also serves as a reliable guide to problem solving, and can inspire creative discoveries. In order to achieve this level of understanding, however, it must be noted that visualisation alone cannot be isolated from the rest of Mathematics. The implication here then, is that symbolical, numerical and visual representations of ideas must be formulated and connected. Gaining increased focus in the fields of Mathematics and Mathematics Education is visualisation which is regarded as both the product and process of creating, interpreting and reflecting upon images. (Zimmerman and Cunningham, 1991; Arcavi, 2003). All of these aspects were present in the observed classes, as shown in the analysis. It has even been suggested that visual thinking may well become “the primary way of thinking in the future” (Hershkowitz and Markovits, 1992, p. 38).

This study thus seems to support the notion as pointed out by Tall (1989) that visual thinking and graphical representation should be linked to other modes of mathematical thinking and other forms of representation in an attempt to consolidate properties thereof.

Previous studies seem to indicate the visualisation goes beyond merely being able to see something. Mariottii and Pesci (1994) acknowledge *visualisation* occurrs when 'thinking is spontaneously accompanied and supported by images'. The connection between the seen and unseen is strengthened. Mason (1992) regards *visualising* as 'making the unseen visible' and *imagery* as 'the power to imagine the possible and the impossible'. 'In order to visualise there is a need to create many images to construct relationships that will facilitate visualisation and reasoning', state Solano and Presmeg (1995) thereby interpreting *visualisation* as 'the relationship between images' -. Espinosa (1997) warns that visualoisation is 'not a trivial cognitive activity: to visualise is not the same as to see'. She explains to *visualise* is the 'ability to create rich, mental images which the individual can manipulate in his mind, rehearse different representations of the concept and, if necessary, use paper or a computer screen to express the idea in question'. This study has not undertaken to measure whether seeing representations furthers the learners' ability to visualise and is acknowledged as another limitation to this study.

Unfortunately, despite the current views of researchers surrounding the importance of visualisation, there will be a tendency for visualisation to be undervalued in the South African mathematics classrooms as the majority of schools are classified as disadvantaged schools which cannot afford the luxury of such technological aids to show physical representations which can promote and cement visualisation.

Therefore, the increasing use of the Smart Board should be coupled with effective pedagogical practices that maximize its potential as a visualisation tool in mathematics education in general and the teaching of functions in particular.

Chapter 7 Discussion

The important question is: Does using Smart Board technology take the learning of the function concept to another level?

As previously stated in the literature review: as with all technology, it is *how* it is used, not the technology itself, which will ultimately determine its value for teaching and learning.

So it may seem, that technology can be powerful as a demonstration tool as it allows one the opportunity to make it easier and faster to do the work. If IWBS are used by skilled and knowledgeable teachers, the Smart Board technology then offers the potential to show properties of abstract and hard to grasp mathematical concepts such as the concept of a function.

On the reverse side, the limitation of the use of technology must be acknowledged. The visualisation method of doing mathematics can be promoted and pushed forward instead of fostering and encouraging the investigation of functional properties through analysis or exploration. From the extracts discussed, it certainly seems to support this notion as many of the extracts discussed showed that the teachers, to a large extent, just use it as a fancy way of illustrating what they would otherwise have done on a normal blackboard. Most of the lessons were not in substantive ways different from the way the content is ‘normally’ taught. Only, as pointed out in extract vii, where the learners got to manipulate the variables and see the effect on the graph showed how the technology used fostered such investigation and exploration. It was only here that the teacher seems to have made the shift to making the class more interactive by making use of the available technology.

The question that now emerges is; do Smart Boards allow for the ‘smarter’ teaching of functions?

From my study I would say that two answers emerged:

a. The Smart Board is used as a tool to aid teaching.

b. The Smart Board is used as a tool to aid learning through visualisation.

As a teaching tool, it can be used to enhance demonstrations which may stimulate greater learner interest, motivation and participation. As a learning tool, the highly interactive, explorative and investigative nature can be utilised.

Numerous benefits were previously identified in my literature review in chapter 2 about the potential use of Smart Boards as teaching tools.

The most important benefit is that, it has the ability to allow the visualisation of multiple representations at any given time. It enables fast exploration which proves its efficiency in saving time. As a technological teaching tool it promotes interactivity and participation in lessons.

A point to mention here is that when interviewed, teachers reported that Smart Boards allow for extended classroom possibilities. For example, if a teacher needs to cater for a range of needs within a lesson, this type of technology makes it possible for learners to work at different paces. Although this was not observed in many lessons, one teacher did put up an earlier application that a learner had not previously fully grasped. This may certainly prove to be useful when learners of different abilities are in one classroom as teachers will be able go back and review previously taught material which may be particularly beneficial for lower ability groups and learners with special needs.

At this point the most obvious distinction that emerges, between Smart Board technology and other technologies incorporating a data projector and a computer, is the ability to control the computer at the touch of the screen. When interviewed, post teaching, some teachers said that the touch-sensitive capacity of IWBs enabled them to present their lessons in a more efficient, interesting and professional manner. Being able to review previously prepared lessons use various programs and software and access the internet were regarded as being of great benefit to teachers. Movement within a lesson was facilitated as the teachers were able to move repeatedly from the visual to the verbal and vice versa. It is interesting to note here that many of these views expressed, seemed to focus on ways of improving presentation. It did not actually focus on utilising the available technology or its potential to alter their teaching styles or classroom interactions in a more profound way.

All teachers interviewed strongly agreed that the IWBs quickened the pace of lessons. This was seen in the various classroom extracts where time was not spent drawing a number of different graphs to illustrate a point. This again takes us back to the literature reviewed. Studies by Glover & Miller (2001); Greenwell (2002); Levy (2002); Ball (2003) acknowledged that although it can take time to prepare lessons with an IWB and to become technically accomplished, planning time would eventually be reduced. Research conducted by, Lee & Boyle (2003) supports this by stating that IWB technology facilitates the saving, sharing and re-use of lesson materials. Perhaps it will be useful if teachers could initially invest their time and effort in their lessons and then be able to reuse lessons. Thus the ability to save materials on an IWB will foster teacher development. Teachers will be in a position to self assess their teaching styles and delivery of lessons. Reflections of lessons can be on-going, not just from lesson to lesson, but also year to year. They can review and analyse what worked as opposed to what did not.

Also worth acknowledging, is that although some of the literature showed that the use of IWBs encourages learners' verbal and active participation in lessons, this study did not carry out an in-depth analysis of the level or quality of participation..However, it must be noted that some of the literature reviewed does consider the quality and depth of classroom interaction and participation in associating interactivity with more social constructivist views of education and learning. As seen in the above classroom extracts, only one of the teachers used the Smart Board to encourage learner participation around the conceptual understanding of functions as well as the mathematical practice of exploration and generalising.

Therefore the study shows that technology by itself cannot transform the quality of teaching. In fact, just as in the traditional approaches, technology risks being used like a flashcard with little student participation. There is therefore need for the adoption of IT to be done proportionately with the adoption of learner centred, constructivist (knowledge transformation and problem solving) approaches.

7.1 Implications for future data collection

The data collected in this study has enabled the evaluation of the use of the IWB in teaching algebraic functions and was aimed at understanding the development of learners’ conceptual knowledge of functions using such technology.

Although the literature reviewed is extremely positive about the potential and benefits offered by IWBs, the views and opinions expressed where those of the teachers and learners concerned in various studies. There is not enough evidence to identify and measure the actual impact of such technologies upon learning in terms of classroom interaction, or upon attainment and achievement.

I also believe that there was insufficient data to provide notable insight to investigate the ways in which the IWB technology acted as a tool in developing the learners' powers of visualisation. Furthermore this limited data cannot be used to comment on how such powers of visualisation might be initiated and developed by the use of mathematical software. So the next question that emerges is: to what extent can such technology encourage or stimulate visual thinking? Does such technology improve the visualisation skills of all learners? For future research, teachers using IWB technology can then be observed to determine the extent to which they have used visual methods in their teaching of functions and in lessons generally and what impact it has on teaching and learning.

7.2 Conclusion

The findings show that much of the teaching occurred in a way which would have been possible without the use of a Smart Board, showing that teachers did not fully utilise the potential of such a technological tool. However, visualisation played an important role in allowing learners to operate on functions as objects. So while the visualization that technology enables encouraged reification or allowed teachers and learners to operate on functions as a whole or even on families of functions, this appeared to simply ‘speed up’ the normal teaching-learning process rather than promote the explorative and investigative aspect. Still, this kind of practice is bound to strengthen these learners' function concepts as is evident in the ways they appear to operate confidently on the objects as shown in the study.

What must also be acknowledged is that the teachers were extremely enthusiastic about the possibilities of the technology and were inspired to use technology more in their lessons, in order to facilitate learners' visualisation of concepts. Learners too, were also motivated by the use of the Smart Board, thereby perhaps opening up the way for the full potential of technology enhanced lessons to be used in investigative, explorative and meaningful ways to enhance the teaching and learning of mathematics in the future.

It must be noted that the *Smart Board* itself does not enhance teaching and learning, the focus is on the way that it is used. Earle (2004) reinforces this by saying it can be another tool teachers can use to increase interactivity especially in science classes. Beauchamp & Parkinson (2005) argue that the real advantages of the IWB will only slowly emerge as teachers in conjunction with learners explore ways to use this new technology. This will allow for the mutual development of new teaching and learning strategies which will ultimately strengthen pedagogical practices.

Perhaps, in an advancing and modern world such as the one we live in, IWB technology may be influential in promoting active and dynamic mathematics teaching and learning. Osborne (1994) and Skamp (2004) have ascertained that IWBS do indeed have the potential, to integrate experiential activities with discussion and reflection to encourage the growth of coherent understanding.

Annexure A: Informed consent document

Topic: Smart Boards-Smart teachers? The use of Smart Boards to teach algebraic functions.

This research aims to:

Analyse the use of a Smart Board as it lends itself as a tool for teaching and learning of algebraic functions

Investigate the role of visualisation in developing the function concept, the roles of interactivity and exploration in teaching mathematics using technology aided tools like Smart Boards.

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Supervisor of the project: Professor Iben Christiansen

UKZN - School of Education

Participants will be selected from a school that has had Smart Boards installed in their teaching rooms. Teachers and learners will be purposively selected based on their questionnaire responses.

Participants are divided into teacher and learner categories

As a learner participant in this study, you will be asked to complete the following:

- Jot down your thoughts when taught using Smart Boards.

- Undergo one on one interview with researcher, answering specific questions. Estimated time 25 minutes for each interview.

As a teacher participant in this study, you will be asked to complete the following:

- Complete an initial brief survey describing your beliefs about mathematics and teaching mathematics, and the role of technology in teaching mathematics. Approximate time is 30 minutes
- undergo a one on one interview with researcher answering specific questions 45 minutes

Both as learners and teachers you will be observed in a natural setting, the classroom and the lesson will be video recorded. Three lessons per teacher will be recorded and analysed to determine how the Smart Board was used in the teaching of functions. The data collection will be carried out over a period of 2 weeks, interviews will be conducted thereafter. All interviews will be audio and video taped, though kept strictly confidential. Your name will be removed, the description of the school anonymised, also when results will be shared in written papers.

Benefits of participating in the study will allow you as teachers, the opportunity to see the potential that a Smart Board may present to teach algebraic functions and therefore may enable you to make maximum use of Smart Board technology to promote active learning in their/your maths classrooms. By participating in the research you will have the opportunity to design and reflect on a technology based lesson and the knowledge gained from the study will benefit the research base of mathematics education in providing insight into lesson planning with technology, specifically for teaching functions. There will be no risk associated with your participation in the project.

The data will be kept for the recommended period of five years and then all video tapes will be destroyed. All questionnaires and transcribed interviews will be shredded.

DECLARATION

I, ………………………………………………………… (full name of participant) or ………………………………………………………… (full name of parent/guardian), parent/guardian of ………………………… (full name of underage participant) hereby confirm that I understand the contents of this document and the nature of this research project, and I give consent to participate/my child/ward to participate in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire. I am also aware that my confidentiality and anonymity will be guaranteed. Should I wish to withdraw or not participate will not result in any form of disadvantage.

………………………………………………………………………………………

SIGNATURE OF PARTICIPANT DATE

Annexure B: Questionnaires for learners?

Questionnaire for learners:

1. Did the use of the Smart Board as a teaching tool help you understand the lesson that was taught?

2. How did it do this?

3. Did the use of the Smart Board in teaching maths change the way you see maths? How so?

4. Describe the impact the Smart Board had on you?

5. List a few words to describe the lesson with the use of a Smart Board.

6. How did the Smart Board help you understand the concept of a function? Explain

Annexure C: Questionnaires for teachers

1. Number of years teaching:

   Number of years teaching mathematics:

   Number of years teaching a course involving functions:

   Textbooks used/resources used:

   Previously

   Currently

2. What do you think about the lesson you just taught?

3. How do you feel it went?

4. What tasks did you plan for the learners for this lesson? Did you have to adapt them in any way? Why?

5. Describe your experiences teaching functions without the use of Smart Board technology.

Annexure D: Interview questions for learners

1. What do you think of the lesson that was just taught using the Smart Board?

2. Did the Smart Board help you learn?

3. (Follow up): Explain to me how it helped you understand the concept of a function better. Was there a particular moment where you realised something?

4. How do you feel about the lesson?

5. What was the most interesting aspect of the Smart Board?

6. How do you feel about the use of the Smart Board in teaching maths?

7. (Follow up): Did the use of the Smart Board change the way you think and feel about maths? How so?

8. What was the teacher trying to do when s/he …? How did that work for you?

9. How would you like to see the Smart Board used in future maths classes?

Annexure E: Interview questions for teachers:

1. (Follow up): How has it changed the learners' learning, if at all? How has it changed your teaching? How are you using it differently now than from when you first started using it?

2. What had you planned to do? How had you planned to use the Smart Board? Did you stick to your plan? Why not?

3. How do you plan to use the Smart Board in the future, what may you still explore that you haven't used so far? And is there any other technology besides Smart Boards you would choose to use in your teaching of functions if you had access to but currently do not?

4. What do you believe is the purpose or function of using technology in teaching and learning mathematics?

5. Where do you get your resources from? How do you adopt them to fit in with using the Smart Board?

6. Do you think it is important for learners to understand the function concept?

7. Why?

8. What do you think are the key aspects learners should understand about functions?

9. How do you feel the Smart Board assists in getting those key aspects across to the learners? Are there any ways in which it hinders it? What may this be?

Annexure F: Ethical clearance certificate

UNIVERSITY OF
KWAZULU-NATAL

09 October 2009

Faculty Research Committee
Faculty of Education
Edgewood Campus
University of KwaZulu-Natal

Dear Prof I Christiansen,

Consideration of Ethical Clearance for student:

Emmanuel, Charmaine - 953005163

Your student's ethical clearance application has met with approval in terms of the **internal review process** of the Faculty of Education.

Approval has been obtained from the Faculty Research Committee, and the application will be forwarded for ratification (MEd) or recommended in the case of PhD and Staff applications, to the Ethics Sub-Committee of the University of KwaZulu-Natal. All Masters applications approved by Faculty Research Committe may commence with research.

Both you and the student will be advised as to whether ethical clearance has been granted for the research thesis (PhD), once the Ethics Sub-Committee has reviewed the application. An ethical clearance certificate will be issued which you should retain with your records. The student should include the ethical clearance certificate in the final dissertation (appendixes).

Should you have any queries please contact Rishandhani Govender the Faculty Research Officer on (031) 260 3440 or on the email govender3@ukzn.ac.za

Yours faithfully

UNIVERSITY OF KWAZULU-NATAL
EDUCATION
2009 -10- 0 9
FACULTY RESEARCH OFFICE

Professor D. Bhana
Deputy Dean Postgraduate Studies and Research

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